Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.1-Sine/74-4.1.2.2-g-cos-^p-a+b-sin-^m-c+d-sin-^n

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 6:06am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1563]. This is test number [74].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.81 (1560)	0.19 (3)
Mathematica	96.93 (1515)	3.07 (48)
Maple	88.29 (1380)	11.71 (183)
Fricas	82.79 (1294)	17.21 (269)
Giac	77.74 (1215)	22.26 (348)
Mupad	72.36 (1131)	27.64 (432)
Maxima	62.89 (983)	37.11 (580)
Sympy	15.55 (243)	84.45 (1320)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

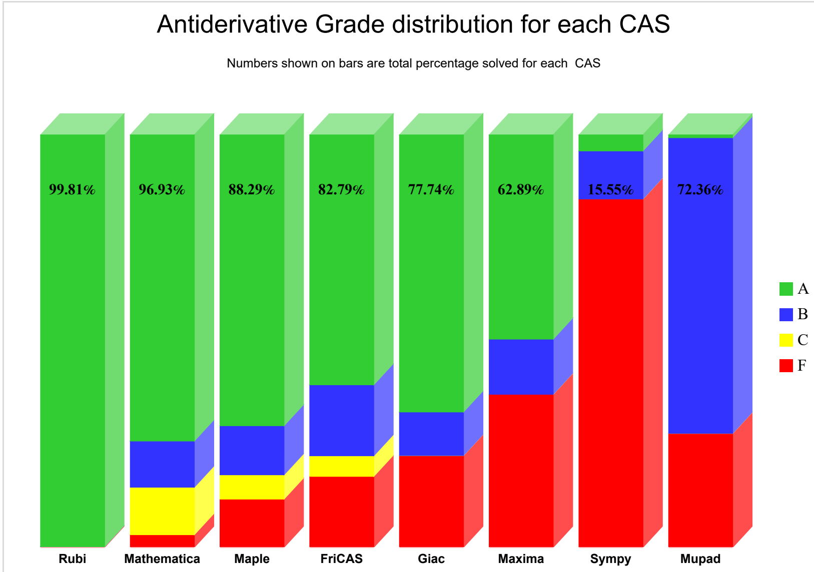
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

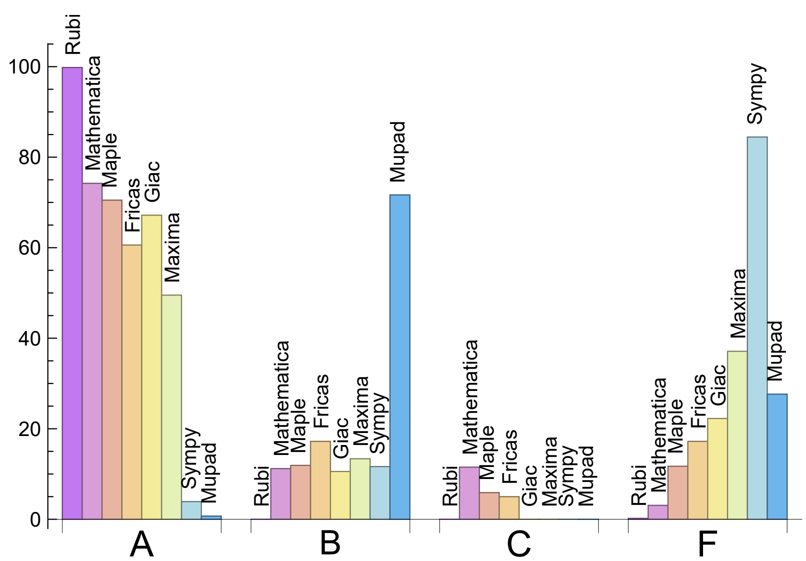
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.81	0.00	0.00	0.19
Mathematica	74.22	11.20	11.52	3.07
Maple	70.51	11.90	5.89	11.71
Giac	67.18	10.56	0.00	22.26
Fricas	60.59	17.21	4.99	17.21
Maxima	49.52	13.37	0.00	37.11
Sympy	3.90	11.64	0.00	84.45
Mupad	N/A	71.66	0.00	27.64

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00 %	0.00 %	0.00 %
Mathematica	48	68.75 %	31.25 %	0.00 %
Maple	183	100.00 %	0.00 %	0.00 %
Fricas	269	65.06 %	33.46 %	1.49 %
Giac	348	68.39 %	28.16 %	3.45 %
Maxima	580	74.14 %	4.66 %	21.21 %
Sympy	1320	25.76 %	45.53 %	28.71 %
Mupad	432	98.61 %	1.39 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

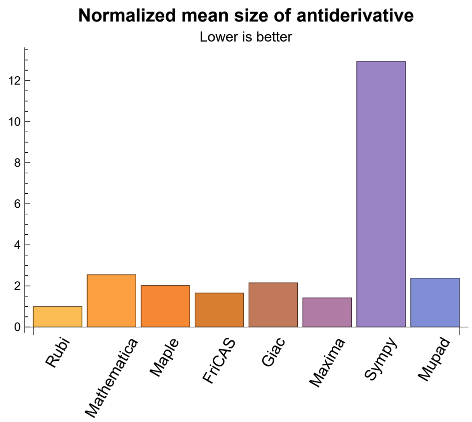
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

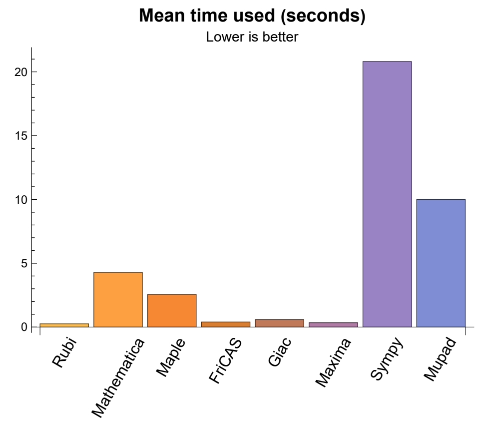
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.25	163.20	1.00	125.00	1.00
Mathematica	4.28	409.64	2.54	145.00	1.07
Maple	2.56	471.09	2.02	155.00	1.24
Maxima	0.34	164.24	1.42	119.00	1.05
Fricas	0.39	248.07	1.66	154.00	1.35
Sympy	20.80	1664.54	12.92	384.00	3.00
Giac	0.59	295.29	2.15	150.00	1.34
Mupad	10.00	359.72	2.38	208.00	1.97

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
12
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{1191, 1192, 1193, 1194, 1519, 1520, 1521, 1522, 1526, 1527, 1560\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {65, 70, 77, 78, 84, 85, 86, 87, 164, 165, 167, 168, 349, 488, 911, 937, 938, 945, 1030, 1042, 1049, 1091, 1188, 1189, 1190, 1276, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1437, 1438, 1439, 1440, 1441, 1442, 1479, 1480, 1516, 1561, 1562, 1563}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

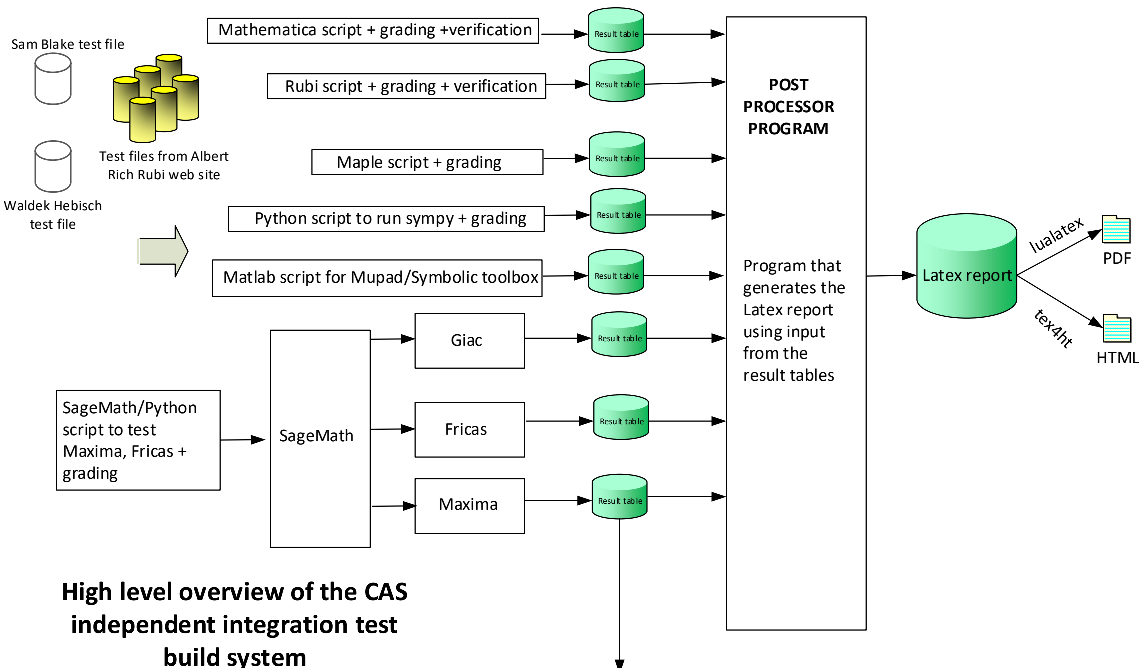
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563 }

B grade: { }

C grade: { }

F grade: { 1479, 1480, 1515 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 21, 22, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 69, 71, 72, 74, 75, 76, 79, 80, 81, 82, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 168, 173, 175, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 299, 301, 303, 304, 305, 306, 307, 308, 309, 316, 321, 322, 323, 324, 325, 329, 330, 331, 332, 333, 336, 337, 338, 339, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 409, 413, 414, 415, 417, 418, 419, 421, 425, 427, 428, 429, 430, 431, 432, 434, 442, 443, 444, 445, 446, 447, 448, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 473, 474, 475, 480, 490, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 583, 584, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 602, 603, 604, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 617, 619, 620, 621, 622, 623, 628, 629, 630, 631, 637, 638, 639, 640, 641, 642, 643, 647, 650, 651, 652, 653, 654, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 710, 711, 712, 713, 714, 715, 721, 727, 728, 729, 730, 731, 733, 735, 736, 737, 738, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 759, 762, 763, 764, 765, 766, 768, 769, 770, 772, 773, 776, 778, 779, 780, 781, 782, 783, 784, 787, 788, 789, 790, 791, 792, 795, 796, 797, 798, 799, 800, 801, 802, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 819, 820, 821, 822, 823, 824, 825, 826, 829, 830, 831, 832, 833, 834, 835, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 938, 942, 945, 953, 954, 955, 956, 957, 961, 962, 963, 965, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1037, 1038, 1039, 1040, 1041, 1050, 1051, 1052, 1053, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1101, 1102, 1105, 1106, 1107, 1108, 1109, 1110, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1151, 1152, 1160, 1168, 1169, 1170, 1176, 1177, 1178, 1184, 1185, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1450, 1451, 1452, 1453, 1454, 1456, 1457, 1458, 1459, 1460, 1461, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1492, 1493, 1494, 1495, 1496, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1516, 1517, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

B grade: { 13, 17, 23, 28, 35, 41, 42, 43, 44, 45, 77, 83, 167, 174, 257, 282, 292, 293, 297, 298, 300, 302, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 326, 327, 328, 334, 335, 340, 341, 342, 376, 377, 388, 403, 408, 410, 411, 412, 416, 420, 422, 423, 424, 426, 433, 435, 436, 437, 438, 439, 440, 441, 449, 468, 469, 470, 476, 477, 478, 479, 492, 567, 585, 586, 587, 600, 601, 611, 618, 624, 625, 626, 627, 632, 633, 634, 635, 636, 644, 645, 646, 648, 649, 699, 705, 706, 707, 708, 709, 716, 717, 718, 719, 720, 722, 723, 724, 725, 726, 732, 734, 739, 740, 741, 757, 758, 760, 761, 767, 771, 774, 775, 777, 785, 786, 793, 794, 803, 818, 827, 828, 836, 837, 911, 964, 966, 967, 968, 1042, 1049, 1065, 1066, 1074, 1075, 1091, 1103, 1104, 1111, 1122, 1135, 1136, 1183, 1250, 1268, 1292, 1293, 1294, 1307, 1308, 1309, 1327, 1328, 1357, 1358, 1449, 1455, 1462, 1479, 1480, 1497, 1505, 1518, 1561, 1562, 1563 }

C grade: { 6, 7, 53, 61, 65, 70, 73, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 108, 118, 129, 137, 146, 164, 165, 184, 270, 271, 343, 344, 345, 346, 347, 348, 349, 372, 373, 481, 482, 483, 484, 485, 486, 487, 488, 489, 581, 582, 858, 859, 860, 936, 937, 958, 959, 960, 998, 1030, 1036, 1054, 1055, 1099, 1100, 1145, 1146, 1147, 1148, 1149, 1150, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1171, 1172, 1173, 1174, 1175, 1179, 1180, 1181, 1182, 1186, 1187, 1188, 1189, 1190, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1437, 1438, 1439, 1440, 1441, 1442, 1489, 1490, 1491, 1515 }

F grade: { 66, 67, 68, 152, 153, 154, 159, 160, 161, 162, 163, 166, 169, 170, 171, 172, 182, 183, 491, 655, 939, 940, 941, 943, 944, 946, 947, 948, 949, 950, 951, 952, 1031, 1032, 1043, 1044, 1045, 1046, 1047, 1048, 1191, 1436, 1514, 1519, 1520, 1523, 1524, 1525 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 6, 9, 10, 11, 12, 14, 15, 18, 19, 20, 21, 22, 24, 25, 26, 30, 31, 32, 33, 34, 36, 37, 38, 39, 43, 47, 49, 50, 51, 52, 53, 57, 58, 59, 60, 62, 64, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 766, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 815, 816, 817, 818, 821, 822, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 863, 865, 866, 867, 868, 869, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 935, 936, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 972, 977, 978, 979, 995, 996, 997, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1146, 1154, 1162, 1172, 1180, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1374, 1376, 1380, 1382, 1386, 1388, 1393, 1395, 1400, 1406, 1410, 1418, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

B grade: { 5, 7, 8, 13, 16, 17, 23, 27, 28, 29, 35, 40, 41, 42, 44, 45, 46, 48, 54, 55, 56, 61, 63, 292, 506, 507, 508, 509, 510, 521, 522, 532, 627, 670, 671, 672, 673, 674, 675, 676, 709, 717, 765, 767, 775, 811, 812, 813, 814, 819, 820, 823, 824, 826, 862, 864, 870, 871, 872, 873, 874, 875, 894, 937, 958, 970, 971, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1091, 1143, 1144, 1145, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1210, 1211, 1212, 1213, 1214, 1258, 1268, 1276, 1277, 1278, 1303, 1322, 1408, 1409, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1505, 1515, 1542 }

C grade: { 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 174, 1370, 1371, 1372, 1373, 1375, 1377, 1378, 1379, 1381, 1383, 1384, 1385, 1387, 1389, 1390, 1391, 1392, 1394, 1396, 1397, 1398, 1399, 1401, 1402, 1403, 1404, 1405, 1407 }

F grade: { 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 258, 259, 260, 261, 262, 263, 264, 265, 490, 491, 492, 493, 565, 566, 567, 568, 569, 570, 571, 653, 654, 655, 697, 698, 699, 700, 701, 702, 703, 704, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1195, 1196, 1197, 1235, 1236, 1237, 1238, 1509, 1510, 1511, 1512, 1513, 1514, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.4 Maxima

A grade: { 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 311, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 435, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 774, 776, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1023, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1279, 1280, 1281, 1282, 1283, 1284, 1295, 1296, 1297, 1298, 1299, 1300, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1332, 1333, 1334, 1335, 1336, 1337, 1346, 1347, 1348, 1349, 1350, 1351, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1560 }

B grade: { 5, 14, 25, 38, 46, 52, 59, 73, 74, 75, 175, 176, 177, 185, 252, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 442, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 772, 773, 775, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 913, 914, 920, 921, 999, 1001, 1002, 1020, 1021, 1022, 1559 }

C grade: { }

F grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 34, 43, 45, 46, 47, 49, 55, 56, 62, 63, 64, 73, 74, 75, 81, 82, 83, 174, 181, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 253, 254, 255, 256, 257, 260, 261, 266, 267, 268, 269, 275, 276, 277, 278, 279, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 321, 322, 323, 324, 329, 330, 331, 336, 337, 338, 343, 344, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 379, 380, 381, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 443, 444, 453, 454, 455, 464, 465, 472, 473, 481, 482, 483, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 759, 760, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 807, 819, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 857, 858, 861, 862, 863, 865, 866, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 916, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 977, 978, 979, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 995, 996, 997, 998, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1017, 1018, 1021, 1022, 1023, 1033, 1034, 1035, 1040, 1041, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1085, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1135, 1136, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1256, 1257, 1258, 1259, 1260, 1266, 1267, 1268, 1269, 1270, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1320, 1321, 1322, 1323, 1324, 1326, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1346, 1347, 1348, 1349, 1350, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1471, 1472, 1473, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1560 }

B grade: { 17, 23, 28, 35, 41, 42, 44, 76, 79, 80, 175, 176, 177, 213, 220, 248, 252, 258, 259, 270, 271, 272, 273, 274, 280, 281, 282, 283, 284, 290, 291, 292, 293, 302, 311, 312, 313, 314, 317, 318, 319, 320, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 342, 345, 346, 347, 348, 349, 372, 373, 374, 375, 376, 385, 386, 387, 401, 416, 435, 436, 437, 438, 439, 440, 441, 442, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 470, 471, 474, 475, 476, 477, 478, 479, 480, 484, 485, 486, 487, 488, 489, 563, 564, 565, 566, 567, 581, 582, 583, 584, 585, 586, 587, 597, 598, 599, 601, 603, 616, 633, 697, 698, 699, 717, 753, 754, 755, 756, 757, 758, 761, 762, 763, 764, 768, 769, 770, 771, 802, 803, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 820, 856, 859, 860, 864, 867, 868, 869, 877, 913, 914, 915, 920, 921, 936, 937, 958, 964, 975, 976, 980, 993, 994, 999, 1015, 1016, 1020, 1054, 1055, 1056, 1057, 1058, 1064, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1090, 1099, 1100, 1101, 1102, 1103, 1132, 1133, 1134, 1137, 1138, 1139, 1140, 1141, 1142, 1231, 1232, 1233, 1234, 1235, 1236, 1250, 1251, 1252, 1253, 1254, 1255, 1261, 1262, 1263, 1264, 1265, 1271, 1272, 1273, 1274, 1275, 1293, 1306, 1308, 1309, 1318, 1319, 1325, 1327, 1328, 1329, 1331, 1344, 1345, 1351, 1369, 1468, 1469, 1470, 1474, 1475, 1476, 1477, 1556, 1557, 1558, 1559 }

C grade: { 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 1143, 1144, 1151, 1152, 1160, 1168, 1169, 1170, 1176, 1177, 1178, 1183, 1184, 1185, 1277 }

F grade: { 6, 7, 14, 15, 16, 24, 25, 26, 27, 36, 37, 38, 39, 40, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 84, 85, 86, 87, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 182, 183, 184, 189, 262, 263, 264, 265, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1145, 1146, 1147, 1148, 1149, 1150, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1171, 1172, 1173, 1174, 1175, 1179, 1180, 1181, 1182, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.6 Sympy

A grade: { 190, 191, 197, 198, 206, 207, 217, 218, 219, 224, 225, 226, 227, 350, 351, 352, 353, 381, 494, 495, 496, 497, 498, 511, 512, 513, 656, 657, 658, 659, 660, 661, 953, 954, 955, 956, 1059, 1061, 1069, 1105, 1106, 1107, 1116, 1191, 1198, 1199, 1200, 1201, 1202, 1215, 1216, 1243, 1279, 1280, 1528, 1529, 1530, 1531, 1536, 1537, 1538 }

B grade: { 208, 220, 232, 233, 234, 235, 240, 241, 242, 243, 244, 249, 250, 251, 252, 253, 258, 259, 260, 261, 266, 267, 268, 275, 276, 277, 285, 286, 294, 297, 298, 299, 300, 307, 308, 309, 310, 315, 316, 317, 321, 357, 358, 359, 365, 366, 367, 368, 378, 379, 380, 392, 393, 394, 395, 407, 409, 410, 411, 412, 420, 421, 422, 423, 424, 432, 433, 434, 440, 521, 522, 533, 534, 535, 544, 545, 546, 554, 555, 556, 565, 566, 567, 572, 573, 574, 575, 588, 589, 590, 591, 605, 606, 607, 608, 624, 625, 626, 627, 634, 635, 636, 644, 645, 646, 678, 679, 680, 681, 682, 683, 697, 698, 699, 706, 707, 708, 709, 725, 726, 741, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 935, 961, 962, 963, 969, 970, 971, 972, 977, 978, 979, 986, 987, 988, 989, 995, 996, 997, 1003, 1004, 1005, 1006, 1011, 1012, 1013, 1014, 1021, 1022, 1023, 1050, 1051, 1052, 1060, 1070, 1092, 1093, 1094, 1095, 1117, 1235, 1236, 1239, 1240, 1241, 1242, 1281, 1539, 1547, 1555 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 228, 229, 230, 231, 236, 237, 238, 239, 245, 246, 247, 248, 254, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 273, 274, 278, 279, 280, 281, 282, 283, 284, 287, 288, 289, 290, 291, 292, 293, 295, 296, 301, 302, 303, 304, 305, 306, 311, 312, 313, 314, 318, 319, 320, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 354, 355, 356, 360, 361, 362, 363, 364, 369, 370, 371, 372, 373, 374, 375, 376, 377, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 413, 414, 415, 416, 417, 418, 419, 425, 426, 427, 428, 429, 430, 431, 435, 436, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 514, 515, 516, 517, 518, 519, 520, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 536, 537, 538, 539, 540, 541, 542, 543, 547, 548, 549, 550, 551, 552, 553, 557, 558, 559, 560, 561, 562, 563, 564, 568, 569, 570, 571, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 628, 629, 630, 631, 632, 633, 637, 638, 639, 640, 641, 642, 643, 647, 648, 649, 650, 651, 652, 653, 654, 655, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 700, 701, 702, 703, 704, 705, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 957, 958, 959, 960, 964, 965, 966, 967, 968, 973, 974, 975, 976, 980, 981, 982, 983, 984, 985, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1007, 1008, 1009, 1010, 1015, 1016, 1017, 1018, 1019, 1020, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1053, 1054, 1055, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1192, 1193, 1194, 1195, 1196, 1197, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1237, 1238, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1532, 1533, 1534, 1535, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 278, 280, 281, 283, 285, 286, 287, 288, 290, 291, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 341, 342, 343, 344, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 413, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 487, 488, 489, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 559, 560, 561, 562, 563, 564, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 719, 720, 722, 723, 724, 725, 726, 727, 728, 729, 731, 732, 733, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 748, 749, 750, 751, 752, 754, 755, 756, 757, 758, 759, 760, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 935, 953, 954, 955, 956, 958, 959, 960, 961, 962, 963, 964, 969, 971, 972, 975, 976, 977, 978, 979, 981, 982, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1050, 1051, 1052, 1053, 1055, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1086, 1087, 1088, 1089, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1132, 1133, 1134, 1135, 1136, 1138, 1139, 1140, 1141, 1142, 1191, 1192, 1193, 1194, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1239, 1240, 1241, 1242, 1243, 1249, 1256, 1257, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1295, 1296, 1297, 1298, 1299, 1300, 1304, 1306, 1307, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1323, 1324, 1326, 1329, 1330, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1443, 1444, 1445, 1446, 1447, 1449, 1450, 1451, 1452, 1453, 1455, 1456, 1457, 1458, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1519, 1520, 1521, 1522, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1552, 1553, 1554, 1555, 1556, 1557, 1560 }

B grade: { 8, 17, 28, 41, 42, 55, 174, 175, 176, 177, 257, 270, 272, 279, 282, 284, 289, 292, 293, 302, 339, 340, 346, 347, 374, 375, 376, 385, 399, 408, 414, 416, 426, 486, 554, 555, 556, 557, 558, 565, 566, 567, 583, 584, 585, 587, 597, 601, 603, 627, 633, 641, 648, 697, 698, 699, 709, 717, 718, 721, 730, 734, 747, 753, 761, 824, 826, 870, 871, 872, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 936, 957, 965, 966, 967, 968, 970, 973, 974, 980, 983, 984, 985, 986, 987, 988, 989, 990, 991, 1001, 1002, 1020, 1021, 1022, 1023, 1040, 1041, 1054, 1056, 1071, 1072, 1073, 1085, 1090, 1101, 1102, 1103, 1108, 1121, 1130, 1137, 1235, 1236, 1244, 1245, 1246, 1247, 1248, 1250, 1251, 1252, 1253, 1254, 1255, 1258, 1268, 1285, 1286, 1294, 1301, 1302, 1303, 1305, 1308, 1309, 1320, 1321, 1322, 1325, 1327, 1328, 1331, 1344, 1369, 1448, 1454, 1459, 1469, 1477, 1550, 1551, 1558, 1559 }

C grade: { }

F grade: { 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 262, 263, 264, 265, 468, 469, 470, 471, 490, 491, 492, 493, 568, 569, 570, 571, 653, 654, 655, 700, 701, 702, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.1.8 Mupad

A grade: { 1191, 1192, 1193, 1194, 1519, 1520, 1521, 1522, 1526, 1527, 1560 }

B grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 35, 42, 43, 44, 45, 46, 47, 49, 62, 73, 74, 75, 76, 79, 80, 81, 82, 83, 175, 176, 177, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 913, 914, 915, 916, 920, 921, 922, 923, 928, 929, 930, 931, 935, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1020, 1021, 1022, 1023, 1033, 1034, 1035, 1040, 1041, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559 }

C grade: { }

F grade: { 6, 7, 8, 14, 15, 16, 17, 24, 25, 26, 27, 28, 36, 37, 38, 39, 40, 41, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 189, 262, 263, 264, 265, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 570, 571, 653, 654, 655, 703, 704, 911, 912, 917, 918, 919, 924, 925, 926, 927, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 1019, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1091, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1195, 1196, 1197, 1237, 1238, 1276, 1277, 1278, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1478, 1479, 1480, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1523, 1524, 1525, 1561, 1562, 1563 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	118	0	108	121
N.S.	1	1.00	1.13	1.45	0.00	1.28	0.00	1.17	1.32
time (sec)	N/A	0.251	0.413	17.878	0.000	0.359	0.000	0.498	11.449

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	94	106	0	103	0	108	110
N.S.	1	1.00	1.02	1.15	0.00	1.12	0.00	1.17	1.20
time (sec)	N/A	0.258	0.335	17.996	0.000	0.365	0.000	0.525	10.453

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	81	0	150	97
N.S.	1	1.00	0.90	0.98	0.00	0.88	0.00	1.63	1.05
time (sec)	N/A	0.272	0.282	0.157	0.000	0.369	0.000	0.559	1.714

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	59	55	0	59	0	102	64
N.S.	1	1.00	0.64	0.60	0.00	0.64	0.00	1.11	0.70
time (sec)	N/A	0.253	0.125	0.149	0.000	0.349	0.000	0.552	0.895

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	62	94	417	64	0	56	73
N.S.	1	1.00	1.38	2.09	9.27	1.42	0.00	1.24	1.62
time (sec)	N/A	0.193	0.226	0.160	0.549	0.383	0.000	0.530	8.959

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	121	141	0	0	0	101	-1
N.S.	1	1.00	1.22	1.42	0.00	0.00	0.00	1.02	-0.01
time (sec)	N/A	0.284	0.749	0.162	0.000	0.000	0.000	0.586	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	114	194	0	0	0	89	-1
N.S.	1	1.00	1.18	2.00	0.00	0.00	0.00	0.92	-0.01
time (sec)	N/A	0.293	0.559	0.164	0.000	0.000	0.000	0.574	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	90	96	0	86	0	96	-1
N.S.	1	1.00	1.88	2.00	0.00	1.79	0.00	2.00	-0.02
time (sec)	N/A	0.218	0.267	0.151	0.000	0.366	0.000	0.626	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	166	133	0	124	0	159	319
N.S.	1	1.00	1.19	0.95	0.00	0.89	0.00	1.14	2.28
time (sec)	N/A	0.346	0.809	0.193	0.000	0.376	0.000	0.493	12.429

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	156	116	0	109	0	208	122
N.S.	1	1.00	1.11	0.83	0.00	0.78	0.00	1.49	0.87
time (sec)	N/A	0.351	0.633	0.169	0.000	0.374	0.000	0.542	11.444

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	82	67	0	79	0	153	79
N.S.	1	1.00	0.59	0.48	0.00	0.56	0.00	1.09	0.56
time (sec)	N/A	0.345	0.395	0.165	0.000	0.373	0.000	0.580	1.477

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	83	90	0	81	0	104	97
N.S.	1	1.00	0.90	0.98	0.00	0.88	0.00	1.13	1.05
time (sec)	N/A	0.261	0.275	0.161	0.000	0.362	0.000	0.537	9.813

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	111	141	0	83	0	56	87
N.S.	1	1.00	2.47	3.13	0.00	1.84	0.00	1.24	1.93
time (sec)	N/A	0.214	0.382	0.157	0.000	0.370	0.000	0.475	9.398

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	130	171	914	0	0	139	-1
N.S.	1	1.00	0.88	1.16	6.22	0.00	0.00	0.95	-0.01
time (sec)	N/A	0.370	0.731	0.147	0.557	0.000	0.000	0.598	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	224	0	0	0	142	-1
N.S.	1	1.00	1.17	1.56	0.00	0.00	0.00	0.99	-0.01
time (sec)	N/A	0.376	0.694	0.167	0.000	0.000	0.000	0.534	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	191	276	0	0	0	126	-1
N.S.	1	1.00	1.30	1.88	0.00	0.00	0.00	0.86	-0.01
time (sec)	N/A	0.380	0.892	0.181	0.000	0.000	0.000	0.629	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	110	127	0	113	0	132	-1
N.S.	1	1.00	2.29	2.65	0.00	2.35	0.00	2.75	-0.02
time (sec)	N/A	0.229	0.941	0.153	0.000	0.349	0.000	0.538	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	118	152	0	137	0	124	236
N.S.	1	1.00	1.22	1.57	0.00	1.41	0.00	1.28	2.43
time (sec)	N/A	0.292	1.317	0.151	0.000	0.363	0.000	0.480	13.867

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	143	0	136	0	267	376
N.S.	1	1.00	0.94	0.76	0.00	0.72	0.00	1.42	2.00
time (sec)	N/A	0.422	1.417	0.198	0.000	0.364	0.000	0.562	12.623

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	87	77	0	108	0	216	179
N.S.	1	1.00	0.46	0.41	0.00	0.57	0.00	1.15	0.95
time (sec)	N/A	0.434	0.434	0.158	0.000	0.359	0.000	0.643	11.802

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	152	116	0	109	0	159	122
N.S.	1	1.00	1.09	0.83	0.00	0.78	0.00	1.14	0.87
time (sec)	N/A	0.358	0.682	0.172	0.000	0.386	0.000	0.543	11.517

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	106	0	103	0	108	108
N.S.	1	1.00	1.00	1.15	0.00	1.12	0.00	1.17	1.17
time (sec)	N/A	0.273	0.340	0.154	0.000	0.381	0.000	0.534	2.410

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	119	195	0	105	0	56	102
N.S.	1	1.00	2.64	4.33	0.00	2.33	0.00	1.24	2.27
time (sec)	N/A	0.208	0.622	0.150	0.000	0.355	0.000	0.499	1.875

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	140	218	0	0	0	148	-1
N.S.	1	1.00	0.73	1.13	0.00	0.00	0.00	0.77	-0.01
time (sec)	N/A	0.429	1.745	0.161	0.000	0.000	0.000	0.559	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	181	273	1214	0	0	179	-1
N.S.	1	1.00	0.94	1.42	6.32	0.00	0.00	0.93	-0.01
time (sec)	N/A	0.429	1.525	0.171	0.575	0.000	0.000	0.616	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	209	329	0	0	0	161	-1
N.S.	1	1.00	1.07	1.69	0.00	0.00	0.00	0.83	-0.01
time (sec)	N/A	0.435	1.768	0.171	0.000	0.000	0.000	0.501	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	234	401	0	0	0	166	-1
N.S.	1	1.00	1.21	2.08	0.00	0.00	0.00	0.86	-0.01
time (sec)	N/A	0.442	2.693	0.168	0.000	0.000	0.000	0.635	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	117	157	0	137	0	113	-1
N.S.	1	1.00	2.44	3.27	0.00	2.85	0.00	2.35	-0.02
time (sec)	N/A	0.234	2.758	0.138	0.000	0.373	0.000	0.544	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	130	191	0	175	0	164	317
N.S.	1	1.00	1.34	1.97	0.00	1.80	0.00	1.69	3.27
time (sec)	N/A	0.300	3.941	0.155	0.000	0.371	0.000	0.549	14.860

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	209	169	0	153	0	318	462
N.S.	1	1.00	0.89	0.72	0.00	0.65	0.00	1.35	1.96
time (sec)	N/A	0.492	3.636	0.254	0.000	0.411	0.000	0.567	13.533

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	97	87	0	125	0	267	247
N.S.	1	1.00	0.41	0.37	0.00	0.53	0.00	1.13	1.05
time (sec)	N/A	0.494	0.849	0.191	0.000	0.383	0.000	0.581	12.115

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	127	143	0	136	0	216	376
N.S.	1	1.00	0.68	0.76	0.00	0.72	0.00	1.15	2.00
time (sec)	N/A	0.415	1.585	0.193	0.000	0.377	0.000	0.635	12.548

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	133	0	123	0	159	319
N.S.	1	1.00	0.82	0.95	0.00	0.88	0.00	1.14	2.28
time (sec)	N/A	0.359	0.853	0.201	0.000	0.375	0.000	0.535	12.144

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	104	133	0	118	0	108	121
N.S.	1	1.00	1.13	1.45	0.00	1.28	0.00	1.17	1.32
time (sec)	N/A	0.276	0.372	0.168	0.000	0.351	0.000	0.521	11.485

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	142	245	0	119	0	56	113
N.S.	1	1.00	3.16	5.44	0.00	2.64	0.00	1.24	2.51
time (sec)	N/A	0.210	0.941	0.197	0.000	0.356	0.000	0.665	10.690

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	156	253	0	0	0	208	-1
N.S.	1	1.00	0.65	1.05	0.00	0.00	0.00	0.86	-0.00
time (sec)	N/A	0.522	4.285	0.201	0.000	0.000	0.000	0.590	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	196	305	0	0	0	187	-1
N.S.	1	1.00	0.82	1.28	0.00	0.00	0.00	0.79	-0.00
time (sec)	N/A	0.540	3.071	0.209	0.000	0.000	0.000	0.575	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	223	365	1514	0	0	198	-1
N.S.	1	1.00	0.93	1.53	6.33	0.00	0.00	0.83	-0.00
time (sec)	N/A	0.532	4.200	0.217	0.640	0.000	0.000	0.594	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	260	435	0	0	0	179	-1
N.S.	1	1.00	1.08	1.80	0.00	0.00	0.00	0.74	-0.00
time (sec)	N/A	0.535	5.725	0.212	0.000	0.000	0.000	0.582	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	437	493	0	0	0	169	-1
N.S.	1	1.00	1.80	2.03	0.00	0.00	0.00	0.70	-0.00
time (sec)	N/A	0.552	6.450	0.201	0.000	0.000	0.000	0.494	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	412	191	0	161	0	197	-1
N.S.	1	1.00	8.58	3.98	0.00	3.35	0.00	4.10	-0.02
time (sec)	N/A	0.229	6.495	0.155	0.000	0.365	0.000	0.558	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	419	233	0	205	0	213	373
N.S.	1	1.00	4.32	2.40	0.00	2.11	0.00	2.20	3.85
time (sec)	N/A	0.301	6.523	0.190	0.000	0.389	0.000	0.579	14.413

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	419	243	0	220	0	198	764
N.S.	1	1.00	2.89	1.68	0.00	1.52	0.00	1.37	5.27
time (sec)	N/A	0.377	6.562	0.206	0.000	0.381	0.000	0.635	14.929

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	134	195	0	105	0	56	96
N.S.	1	1.00	2.98	4.33	0.00	2.33	0.00	1.24	2.13
time (sec)	N/A	0.222	0.615	0.149	0.000	0.344	0.000	0.494	2.003

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	120	141	0	84	0	56	83
N.S.	1	1.00	2.67	3.13	0.00	1.87	0.00	1.24	1.84
time (sec)	N/A	0.213	0.346	0.154	0.000	0.362	0.000	0.474	9.964

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	62	90	418	64	0	56	67
N.S.	1	1.00	1.38	2.00	9.29	1.42	0.00	1.24	1.49
time (sec)	N/A	0.195	0.202	0.151	0.587	0.376	0.000	0.608	0.931

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	42	0	53	0	58	52
N.S.	1	1.00	1.02	0.98	0.00	1.23	0.00	1.35	1.21
time (sec)	N/A	0.190	0.201	0.145	0.000	0.356	0.000	0.547	8.869

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	104	136	0	0	0	58	-1
N.S.	1	1.00	1.93	2.52	0.00	0.00	0.00	1.07	-0.02
time (sec)	N/A	0.225	0.311	0.145	0.000	0.000	0.000	0.553	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	79	51	0	66	0	58	88
N.S.	1	1.00	1.88	1.21	0.00	1.57	0.00	1.38	2.10
time (sec)	N/A	0.217	0.362	0.158	0.000	0.352	0.000	0.491	9.771

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	163	252	0	0	0	226	-1
N.S.	1	1.00	0.68	1.05	0.00	0.00	0.00	0.95	-0.00
time (sec)	N/A	0.503	5.049	0.227	0.000	0.000	0.000	0.594	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	138	213	0	0	0	157	-1
N.S.	1	1.00	0.73	1.12	0.00	0.00	0.00	0.83	-0.01
time (sec)	N/A	0.463	1.569	0.188	0.000	0.000	0.000	0.564	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	134	172	914	0	0	132	-1
N.S.	1	1.00	0.92	1.19	6.30	0.00	0.00	0.91	-0.01
time (sec)	N/A	0.399	0.732	0.168	0.576	0.000	0.000	0.523	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	121	133	0	0	0	103	-1
N.S.	1	1.00	1.26	1.39	0.00	0.00	0.00	1.07	-0.01
time (sec)	N/A	0.308	0.734	0.176	0.000	0.000	0.000	0.520	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	102	134	0	0	0	58	-1
N.S.	1	1.00	2.00	2.63	0.00	0.00	0.00	1.14	-0.02
time (sec)	N/A	0.240	0.293	0.141	0.000	0.000	0.000	0.706	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	103	173	0	170	0	126	-1
N.S.	1	1.00	1.98	3.33	0.00	3.27	0.00	2.42	-0.02
time (sec)	N/A	0.240	0.379	0.392	0.000	0.382	0.000	0.500	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	163	245	0	345	0	181	-1
N.S.	1	1.00	1.57	2.36	0.00	3.32	0.00	1.74	-0.01
time (sec)	N/A	0.309	0.545	0.151	0.000	0.421	0.000	0.574	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	553	347	0	0	0	269	-1
N.S.	1	1.00	1.94	1.22	0.00	0.00	0.00	0.94	-0.00
time (sec)	N/A	0.601	6.465	0.175	0.000	0.000	0.000	0.593	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	179	307	0	0	0	199	-1
N.S.	1	1.00	0.76	1.30	0.00	0.00	0.00	0.84	-0.00
time (sec)	N/A	0.539	3.225	0.227	0.000	0.000	0.000	0.579	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	164	269	1214	0	0	175	-1
N.S.	1	1.00	0.86	1.41	6.36	0.00	0.00	0.92	-0.01
time (sec)	N/A	0.459	1.586	0.209	0.660	0.000	0.000	0.509	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	153	227	0	0	0	144	-1
N.S.	1	1.00	1.07	1.59	0.00	0.00	0.00	1.01	-0.01
time (sec)	N/A	0.393	0.735	0.174	0.000	0.000	0.000	0.559	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	113	188	0	0	0	94	-1
N.S.	1	1.00	1.16	1.94	0.00	0.00	0.00	0.97	-0.01
time (sec)	N/A	0.312	0.622	0.161	0.000	0.000	0.000	0.568	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	80	50	0	65	0	58	55
N.S.	1	1.00	1.86	1.16	0.00	1.51	0.00	1.35	1.28
time (sec)	N/A	0.225	0.353	0.130	0.000	0.345	0.000	0.562	9.261

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	163	247	0	339	0	180	-1
N.S.	1	1.00	1.57	2.38	0.00	3.26	0.00	1.73	-0.01
time (sec)	N/A	0.311	0.556	0.141	0.000	0.410	0.000	0.681	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	163	198	0	282	0	0	-1
N.S.	1	1.00	1.07	1.30	0.00	1.86	0.00	0.00	-0.01
time (sec)	N/A	0.371	0.587	0.161	0.000	0.408	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	2543	0	0	0	0	0	-1
N.S.	1	1.00	22.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	10.339	0.122	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	180.008	0.391	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	180.040	0.346	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	180.015	0.168	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.082	0.048	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	6442	0	0	0	0	0	-1
N.S.	1	1.00	83.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	19.213	0.552	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.142	0.943	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.102	1.165	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	695	0	586	409	0	0	1060
N.S.	1	1.00	2.85	0.00	2.40	1.68	0.00	0.00	4.34
time (sec)	N/A	0.409	6.439	0.122	0.623	0.415	0.000	0.000	14.970

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	149	0	447	255	0	0	528
N.S.	1	1.00	0.87	0.00	2.60	1.48	0.00	0.00	3.07
time (sec)	N/A	0.315	1.939	0.128	0.606	0.385	0.000	0.000	13.973

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	0	332	165	0	0	104
N.S.	1	1.00	1.04	0.00	3.10	1.54	0.00	0.00	0.97
time (sec)	N/A	0.229	0.394	0.119	0.566	0.398	0.000	0.000	1.720

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	116	0	0	68
N.S.	1	1.00	1.70	0.00	0.00	2.32	0.00	0.00	1.36
time (sec)	N/A	0.173	0.255	0.151	0.000	0.377	0.000	0.000	0.899

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	218	0	0	0	0	0	-1
N.S.	1	1.00	2.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.237	6.983	0.141	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	3174	0	0	0	0	0	-1
N.S.	1	1.00	40.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	6.453	0.160	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	116	0	0	68
N.S.	1	1.00	1.70	0.00	0.00	2.32	0.00	0.00	1.36
time (sec)	N/A	0.162	0.264	0.000	0.000	0.372	0.000	0.000	0.002

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	85	0	0	116	0	0	68
N.S.	1	1.00	1.70	0.00	0.00	2.32	0.00	0.00	1.36
time (sec)	N/A	0.162	0.251	0.178	0.000	0.369	0.000	0.000	9.110

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	176	0	0	111	0	0	334
N.S.	1	1.00	0.97	0.00	0.00	0.61	0.00	0.00	1.84
time (sec)	N/A	0.299	18.395	0.792	0.000	0.398	0.000	0.000	15.748

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	142	0	0	81	0	0	177
N.S.	1	1.00	1.25	0.00	0.00	0.71	0.00	0.00	1.55
time (sec)	N/A	0.234	14.226	1.018	0.000	0.385	0.000	0.000	10.599

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	109	0	0	51	0	0	101
N.S.	1	1.00	2.02	0.00	0.00	0.94	0.00	0.00	1.87
time (sec)	N/A	0.170	6.950	0.813	0.000	0.364	0.000	0.000	1.043

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	589	0	0	0	0	0	-1
N.S.	1	1.00	5.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.263	20.521	0.776	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	857	0	0	0	0	0	-1
N.S.	1	1.00	7.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.241	24.323	0.230	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	1255	0	0	0	0	0	-1
N.S.	1	1.00	10.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	16.987	0.178	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	2082	0	0	0	0	0	-1
N.S.	1	1.00	17.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	25.362	0.267	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	311	425	0	183	0	0	-1
N.S.	1	1.00	0.91	1.24	0.00	0.53	0.00	0.00	-0.00
time (sec)	N/A	1.107	6.705	33.365	0.000	0.136	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	281	392	0	169	0	0	-1
N.S.	1	1.00	0.97	1.35	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.931	1.388	0.250	0.000	0.157	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	255	372	0	143	0	0	-1
N.S.	1	1.00	1.09	1.58	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.712	1.243	0.268	0.000	0.115	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	249	344	0	124	0	0	-1
N.S.	1	1.00	1.40	1.93	0.00	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.503	1.677	0.298	0.000	0.116	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	197	362	0	120	0	0	-1
N.S.	1	1.00	1.61	2.97	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.342	1.444	0.241	0.000	0.106	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	211	2834	0	167	0	0	-1
N.S.	1	1.00	1.72	23.04	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.349	1.252	10.069	0.000	0.112	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	229	2040	0	225	0	0	-1
N.S.	1	1.00	1.26	11.21	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.542	1.483	1.602	0.000	0.142	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	256	966	0	287	0	0	-1
N.S.	1	1.00	1.08	4.08	0.00	1.21	0.00	0.00	-0.00
time (sec)	N/A	0.731	1.590	5.171	0.000	0.137	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	291	1126	0	350	0	0	-1
N.S.	1	1.00	1.00	3.86	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.912	1.940	0.260	0.000	0.140	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	193	382	0	174	0	0	-1
N.S.	1	1.00	0.55	1.09	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	1.066	0.896	0.263	0.000	0.141	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	113	356	0	147	0	0	-1
N.S.	1	1.00	0.38	1.21	0.00	0.50	0.00	0.00	-0.00
time (sec)	N/A	0.953	0.475	0.293	0.000	0.137	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	257	372	0	143	0	0	-1
N.S.	1	1.00	1.09	1.58	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.708	5.396	0.224	0.000	0.130	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	148	382	0	137	0	0	-1
N.S.	1	1.00	0.82	2.12	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.519	0.519	0.208	0.000	0.136	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	207	2892	0	185	0	0	-1
N.S.	1	1.00	1.14	15.89	0.00	1.02	0.00	0.00	-0.01
time (sec)	N/A	0.539	1.179	0.207	0.000	0.128	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	191	3498	0	233	0	0	-1
N.S.	1	1.00	1.03	18.81	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.544	0.961	0.217	0.000	0.118	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	218	2682	0	300	0	0	-1
N.S.	1	1.00	0.90	11.04	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.747	1.581	0.235	0.000	0.133	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	1136	0	364	0	0	-1
N.S.	1	1.00	1.55	3.79	0.00	1.21	0.00	0.00	-0.00
time (sec)	N/A	0.920	6.324	0.244	0.000	0.146	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1296	0	422	0	0	-1
N.S.	1	1.00	1.49	3.63	0.00	1.18	0.00	0.00	-0.00
time (sec)	N/A	1.133	6.361	2.490	0.000	0.161	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	120	366	0	181	0	0	-1
N.S.	1	1.00	0.30	0.90	0.00	0.45	0.00	0.00	-0.00
time (sec)	N/A	1.265	0.796	0.243	0.000	0.140	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	189	382	0	174	0	0	-1
N.S.	1	1.00	0.54	1.09	0.00	0.49	0.00	0.00	-0.00
time (sec)	N/A	1.084	0.887	0.267	0.000	0.144	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	281	392	0	169	0	0	-1
N.S.	1	1.00	0.97	1.35	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.903	1.478	0.241	0.000	0.128	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	158	415	0	160	0	0	-1
N.S.	1	1.00	0.68	1.77	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.703	1.223	0.208	0.000	0.129	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	240	2945	0	213	0	0	-1
N.S.	1	1.00	1.00	12.22	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.725	4.227	0.218	0.000	0.157	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	245	3549	0	264	0	0	-1
N.S.	1	1.00	1.01	14.60	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.734	1.670	0.231	0.000	0.138	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	246	4183	0	322	0	0	-1
N.S.	1	1.00	1.01	17.21	0.00	1.33	0.00	0.00	-0.00
time (sec)	N/A	0.737	1.810	0.232	0.000	0.139	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	3455	0	392	0	0	-1
N.S.	1	1.00	1.55	11.52	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.929	6.384	0.227	0.000	0.142	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	1313	0	456	0	0	-1
N.S.	1	1.00	1.49	3.68	0.00	1.28	0.00	0.00	-0.00
time (sec)	N/A	1.127	6.441	0.235	0.000	0.172	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1473	0	525	0	0	-1
N.S.	1	1.00	1.45	3.56	0.00	1.27	0.00	0.00	-0.00
time (sec)	N/A	1.324	6.471	0.297	0.000	0.187	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	226	392	0	202	0	0	-1
N.S.	1	1.00	0.49	0.85	0.00	0.44	0.00	0.00	-0.00
time (sec)	N/A	1.462	2.367	0.345	0.000	0.155	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	212	402	0	190	0	0	-1
N.S.	1	1.00	0.52	0.98	0.00	0.46	0.00	0.00	-0.00
time (sec)	N/A	1.299	1.984	0.279	0.000	0.155	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	360	425	0	183	0	0	-1
N.S.	1	1.00	1.05	1.24	0.00	0.53	0.00	0.00	-0.00
time (sec)	N/A	1.098	11.474	0.256	0.000	0.140	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	181	434	0	177	0	0	-1
N.S.	1	1.00	0.63	1.51	0.00	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.885	2.567	0.222	0.000	0.160	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	284	2996	0	229	0	0	-1
N.S.	1	1.00	0.97	10.19	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.894	6.392	0.244	0.000	0.145	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	267	3600	0	280	0	0	-1
N.S.	1	1.00	0.90	12.08	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.929	3.224	2.587	0.000	0.144	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	406	4236	0	339	0	0	-1
N.S.	1	1.00	1.35	14.12	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.931	6.432	0.250	0.000	0.150	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	464	4828	0	392	0	0	-1
N.S.	1	1.00	1.55	16.09	0.00	1.31	0.00	0.00	-0.00
time (sec)	N/A	0.941	6.478	0.235	0.000	0.165	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	532	3908	0	457	0	0	-1
N.S.	1	1.00	1.49	10.95	0.00	1.28	0.00	0.00	-0.00
time (sec)	N/A	1.154	6.532	0.237	0.000	0.172	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	600	1482	0	525	0	0	-1
N.S.	1	1.00	1.45	3.58	0.00	1.27	0.00	0.00	-0.00
time (sec)	N/A	1.328	6.551	0.259	0.000	0.177	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	668	1642	0	589	0	0	-1
N.S.	1	1.00	1.42	3.49	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	1.543	6.616	2.175	0.000	0.226	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	174	415	0	160	0	0	-1
N.S.	1	1.00	0.74	1.77	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.699	1.041	0.227	0.000	0.121	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	157	382	0	137	0	0	-1
N.S.	1	1.00	0.87	2.12	0.00	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.554	0.482	0.207	0.000	0.109	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	215	361	0	120	0	0	-1
N.S.	1	1.00	1.76	2.96	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.360	2.209	0.243	0.000	0.106	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	334	0	81	0	0	-1
N.S.	1	1.00	1.63	4.91	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.195	0.373	0.327	0.000	0.107	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	148	925	0	169	0	0	-1
N.S.	1	1.00	1.22	7.64	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.362	0.559	0.434	0.000	0.104	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	204	781	0	225	0	0	-1
N.S.	1	1.00	1.14	4.36	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.550	1.115	0.236	0.000	0.123	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	240	955	0	293	0	0	-1
N.S.	1	1.00	1.03	4.10	0.00	1.26	0.00	0.00	-0.00
time (sec)	N/A	0.742	1.760	0.214	0.000	0.139	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	282	2994	0	229	0	0	-1
N.S.	1	1.00	0.96	10.18	0.00	0.78	0.00	0.00	-0.00
time (sec)	N/A	0.906	6.410	11.566	0.000	0.132	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	238	2946	0	212	0	0	-1
N.S.	1	1.00	0.99	12.22	0.00	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.736	3.332	0.231	0.000	0.126	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	200	2890	0	184	0	0	-1
N.S.	1	1.00	1.10	15.88	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.528	1.114	0.204	0.000	0.120	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	213	2836	0	166	0	0	-1
N.S.	1	1.00	1.73	23.06	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.353	1.417	0.353	0.000	0.124	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	170	925	0	168	0	0	-1
N.S.	1	1.00	1.40	7.64	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.357	0.508	0.359	0.000	0.127	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	92	363	0	156	0	0	-1
N.S.	1	1.00	0.52	2.06	0.00	0.89	0.00	0.00	-0.01
time (sec)	N/A	0.568	0.529	0.309	0.000	0.171	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	134	877	0	252	0	0	-1
N.S.	1	1.00	0.57	3.70	0.00	1.06	0.00	0.00	-0.00
time (sec)	N/A	0.753	0.816	0.231	0.000	0.142	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	155	1177	0	315	0	0	-1
N.S.	1	1.00	0.53	4.00	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.944	1.082	0.229	0.000	0.203	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	356	3655	0	296	0	0	-1
N.S.	1	1.00	1.00	10.24	0.00	0.83	0.00	0.00	-0.00
time (sec)	N/A	1.127	6.533	2.151	0.000	0.187	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	250	3601	0	281	0	0	-1
N.S.	1	1.00	0.84	12.08	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.915	3.317	0.290	0.000	0.157	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	230	3550	0	264	0	0	-1
N.S.	1	1.00	0.95	14.61	0.00	1.09	0.00	0.00	-0.00
time (sec)	N/A	0.727	1.653	0.242	0.000	0.155	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	180	3497	0	233	0	0	-1
N.S.	1	1.00	0.97	18.80	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.545	0.842	0.220	0.000	0.139	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	230	2040	0	223	0	0	-1
N.S.	1	1.00	1.26	11.21	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.543	1.599	0.232	0.000	0.150	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	189	777	0	227	0	0	-1
N.S.	1	1.00	1.06	4.34	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.564	1.107	0.254	0.000	0.118	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	133	877	0	251	0	0	-1
N.S.	1	1.00	0.56	3.70	0.00	1.06	0.00	0.00	-0.00
time (sec)	N/A	0.738	0.804	0.235	0.000	0.210	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	104	395	0	170	0	0	-1
N.S.	1	1.00	0.36	1.36	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.958	0.822	0.240	0.000	0.137	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	171	947	0	277	0	0	-1
N.S.	1	1.00	0.49	2.71	0.00	0.79	0.00	0.00	-0.00
time (sec)	N/A	1.130	2.946	0.240	0.000	0.180	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	126	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	2.639	0.105	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	180.019	0.379	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	180.011	0.318	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	122.441	0.119	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.107	0.006	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.099	0.125	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.151	0.315	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	96	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.152	0.422	0.000	0.000	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	180.004	0.081	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	180.020	0.077	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	180.008	0.076	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	180.004	0.069	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.236	180.006	0.069	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	2320	0	0	0	0	0	-1
N.S.	1	1.00	20.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.243	162.397	0.069	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	754	0	0	0	0	0	-1
N.S.	1	1.00	7.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	20.705	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	180.014	0.089	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	382	0	0	0	0	0	-1
N.S.	1	1.00	3.11	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	22.311	0.300	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	202	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	8.784	0.260	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.253	112.275	0.134	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	23.336	0.077	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.238	177.727	0.138	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.246	130.028	0.186	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	30.493	0.115	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	155	8576	0	31	0	945	-1
N.S.	1	1.00	2.72	150.46	0.00	0.54	0.00	16.58	-0.02
time (sec)	N/A	0.145	70.337	4.631	0.000	0.377	0.000	1.768	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	0	1016	667	0	90024	1149
N.S.	1	1.00	1.03	0.00	5.00	3.29	0.00	443.47	5.66
time (sec)	N/A	0.452	4.770	0.169	0.833	0.402	0.000	51.935	17.207

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	0	511	310	0	29573	476
N.S.	1	1.00	1.13	0.00	4.02	2.44	0.00	232.86	3.75
time (sec)	N/A	0.274	1.009	0.457	0.601	0.382	0.000	18.335	18.128

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	96	0	221	137	0	4701	74
N.S.	1	1.00	1.66	0.00	3.81	2.36	0.00	81.05	1.28
time (sec)	N/A	0.107	0.510	0.136	0.541	0.395	0.000	5.690	9.417

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	50.725	0.438	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	135	0	0	0	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	36.627	0.142	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	136	0	0	0	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	31.677	0.143	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	94	0	0	50	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.98	0.00	0.00	-0.02
time (sec)	N/A	0.110	0.853	0.214	0.000	0.374	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	139.420	0.451	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.264	134.387	0.375	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	207	0	0	0	0	0	-1
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	25.593	0.544	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	155	88	0	0	53
N.S.	1	1.00	1.00	0.00	2.82	1.60	0.00	0.00	0.96
time (sec)	N/A	0.115	0.583	0.265	0.541	0.369	0.000	0.000	9.074

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	132	0	0	133	0	0	128
N.S.	1	1.00	1.06	0.00	0.00	1.06	0.00	0.00	1.02
time (sec)	N/A	0.270	24.742	0.335	0.000	0.377	0.000	0.000	10.157

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	183	0	0	191	0	0	887
N.S.	1	1.00	0.90	0.00	0.00	0.94	0.00	0.00	4.35
time (sec)	N/A	0.441	28.638	0.390	0.000	0.375	0.000	0.000	15.647

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	259	0	0	273	0	0	1623
N.S.	1	1.00	0.89	0.00	0.00	0.94	0.00	0.00	5.60
time (sec)	N/A	0.638	34.265	0.525	0.000	0.392	0.000	0.000	17.649

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	139	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.311	31.499	0.163	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	28	50	42	28	24
N.S.	1	1.00	1.00	0.85	0.85	1.52	1.27	0.85	0.73
time (sec)	N/A	0.030	0.009	0.125	0.278	0.349	0.186	0.445	0.051

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	30	28	28	39	41	28	24
N.S.	1	1.00	0.91	0.85	0.85	1.18	1.24	0.85	0.73
time (sec)	N/A	0.021	0.075	0.055	0.281	0.361	0.119	0.420	0.040

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	22	24	0	23	38
N.S.	1	1.00	1.08	0.83	0.92	1.00	0.00	0.96	1.58
time (sec)	N/A	0.016	0.031	0.087	0.283	0.353	0.000	0.455	8.814

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	24	25	33	0	26	55
N.S.	1	1.00	1.32	0.96	1.00	1.32	0.00	1.04	2.20
time (sec)	N/A	0.025	0.034	0.060	0.293	0.370	0.000	0.452	8.580

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	27	24	29	0	24	25
N.S.	1	1.00	0.97	0.90	0.80	0.97	0.00	0.80	0.83
time (sec)	N/A	0.030	0.017	0.092	0.276	0.346	0.000	0.445	8.517

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	39	0	26	39
N.S.	1	1.00	1.00	0.82	0.79	1.18	0.00	0.79	1.18
time (sec)	N/A	0.032	0.020	0.091	0.276	0.343	0.000	0.426	8.602

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	40	0	26	26
N.S.	1	1.00	1.00	0.82	0.79	1.21	0.00	0.79	0.79
time (sec)	N/A	0.030	0.017	0.110	0.289	0.367	0.000	0.449	8.565

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	45	45	72	63	45	36
N.S.	1	1.00	0.96	0.82	0.82	1.31	1.15	0.82	0.65
time (sec)	N/A	0.048	0.268	0.098	0.274	0.349	0.271	0.517	8.492

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	45	45	58	63	45	36
N.S.	1	1.00	0.69	0.82	0.82	1.05	1.15	0.82	0.65
time (sec)	N/A	0.034	0.048	0.079	0.277	0.343	0.195	0.438	8.505

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	34	41	43	0	42	119
N.S.	1	1.00	1.00	0.72	0.87	0.91	0.00	0.89	2.53
time (sec)	N/A	0.031	0.019	0.073	0.274	0.380	0.000	0.435	8.988

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	34	40	46	0	41	111
N.S.	1	1.00	0.88	0.79	0.93	1.07	0.00	0.95	2.58
time (sec)	N/A	0.039	0.020	0.082	0.287	0.416	0.000	0.461	8.892

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	36	43	62	0	44	111
N.S.	1	1.00	0.89	0.77	0.91	1.32	0.00	0.94	2.36
time (sec)	N/A	0.046	0.014	0.125	0.280	0.392	0.000	0.539	8.866

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	39	41	56	0	41	41
N.S.	1	1.00	0.67	1.30	1.37	1.87	0.00	1.37	1.37
time (sec)	N/A	0.040	0.019	0.108	0.286	0.369	0.000	0.457	8.903

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	57	0	43	43
N.S.	1	1.00	1.00	0.71	0.78	1.04	0.00	0.78	0.78
time (sec)	N/A	0.048	0.019	0.131	0.292	0.379	0.000	0.519	8.853

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	65	0	43	43
N.S.	1	1.00	1.00	0.71	0.78	1.18	0.00	0.78	0.78
time (sec)	N/A	0.047	0.020	0.145	0.274	0.341	0.000	0.467	8.918

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	39	43	70	0	43	43
N.S.	1	1.00	1.00	0.71	0.78	1.27	0.00	0.78	0.78
time (sec)	N/A	0.046	0.033	0.145	0.300	0.349	0.000	0.586	8.907

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	58	58	98	80	58	57
N.S.	1	1.00	1.10	0.79	0.79	1.34	1.10	0.79	0.78
time (sec)	N/A	0.051	0.247	0.168	0.260	0.345	0.714	0.484	0.065

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	70	58	58	85	82	58	57
N.S.	1	1.00	0.96	0.79	0.79	1.16	1.12	0.79	0.78
time (sec)	N/A	0.050	0.232	0.122	0.269	0.370	0.436	0.507	0.062

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	57	58	71	76	58	56
N.S.	1	1.00	0.67	1.27	1.29	1.58	1.69	1.29	1.24
time (sec)	N/A	0.032	0.097	0.099	0.265	0.378	0.301	0.475	0.060

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	44	55	59	0	56	102
N.S.	1	1.00	1.00	0.68	0.85	0.91	0.00	0.86	1.57
time (sec)	N/A	0.029	0.022	0.075	0.276	0.358	0.000	0.525	8.655

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	54	78	0	55	156
N.S.	1	1.00	1.00	0.74	0.87	1.26	0.00	0.89	2.52
time (sec)	N/A	0.041	0.024	0.105	0.288	0.390	0.000	0.512	8.593

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	44	54	77	0	55	163
N.S.	1	1.00	0.87	0.72	0.89	1.26	0.00	0.90	2.67
time (sec)	N/A	0.047	0.017	0.120	0.286	0.378	0.000	0.478	8.573

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	46	58	91	0	59	147
N.S.	1	1.00	0.88	0.71	0.89	1.40	0.00	0.91	2.26
time (sec)	N/A	0.049	0.017	0.128	0.278	0.369	0.000	0.500	8.616

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	20	49	54	72	0	54	54
N.S.	1	1.00	0.67	1.63	1.80	2.40	0.00	1.80	1.80
time (sec)	N/A	0.043	0.017	0.134	0.266	0.331	0.000	0.428	8.614

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	71	49	56	81	0	56	56
N.S.	1	1.00	1.16	0.80	0.92	1.33	0.00	0.92	0.92
time (sec)	N/A	0.043	0.026	0.148	0.271	0.338	0.000	0.491	8.604

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	56	86	0	56	56
N.S.	1	1.00	1.00	0.67	0.77	1.18	0.00	0.77	0.77
time (sec)	N/A	0.048	0.028	0.173	0.293	0.367	0.000	0.508	8.603

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	56	93	0	56	56
N.S.	1	1.00	1.00	0.67	0.77	1.27	0.00	0.77	0.77
time (sec)	N/A	0.049	0.025	0.153	0.278	0.354	0.000	0.483	8.665

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	71	71	124	97	71	70
N.S.	1	1.00	1.10	0.78	0.78	1.36	1.07	0.78	0.77
time (sec)	N/A	0.056	0.658	0.265	0.280	0.377	1.418	0.486	8.455

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	70	71	111	95	71	69
N.S.	1	1.00	1.02	0.80	0.81	1.26	1.08	0.81	0.78
time (sec)	N/A	0.056	0.631	0.218	0.274	0.355	0.947	0.498	8.446

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	70	71	97	95	71	69
N.S.	1	1.00	1.19	1.04	1.06	1.45	1.42	1.06	1.03
time (sec)	N/A	0.051	0.291	0.171	0.297	0.359	0.651	0.469	8.414

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	30	71	71	85	97	71	70
N.S.	1	1.00	0.67	1.58	1.58	1.89	2.16	1.58	1.56
time (sec)	N/A	0.032	0.076	0.139	0.269	0.344	0.551	0.476	0.048

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	54	68	72	0	69	118
N.S.	1	1.00	1.00	0.67	0.84	0.89	0.00	0.85	1.46
time (sec)	N/A	0.035	0.029	0.099	0.272	0.387	0.000	0.472	8.620

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	56	67	91	0	68	235
N.S.	1	1.00	1.00	0.72	0.86	1.17	0.00	0.87	3.01
time (sec)	N/A	0.046	0.026	0.115	0.271	0.360	0.000	0.533	8.647

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	56	66	98	0	67	207
N.S.	1	1.00	0.68	0.70	0.82	1.22	0.00	0.84	2.59
time (sec)	N/A	0.053	0.063	0.127	0.268	0.382	0.000	0.464	8.618

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	60	56	63	58	80	76	68
N.S.	1	1.00	0.71	0.66	0.74	0.68	0.94	0.89	0.80
time (sec)	N/A	0.064	0.120	0.135	0.284	0.348	0.652	0.429	8.456

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	46	53	48	66	64	56
N.S.	1	1.00	0.75	0.69	0.79	0.72	0.99	0.96	0.84
time (sec)	N/A	0.056	0.090	0.111	0.279	0.365	0.508	0.431	0.056

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	36	41	36	53	45	35
N.S.	1	1.00	0.78	0.73	0.84	0.73	1.08	0.92	0.71
time (sec)	N/A	0.050	0.047	0.090	0.277	0.354	0.327	0.456	0.047

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	26	30	26	37	31	26
N.S.	1	1.00	0.81	0.84	0.97	0.84	1.19	1.00	0.84
time (sec)	N/A	0.033	0.017	0.085	0.285	0.360	0.264	0.457	8.459

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	31	28	0	33	32
N.S.	1	1.00	1.00	0.84	0.97	0.88	0.00	1.03	1.00
time (sec)	N/A	0.027	0.013	0.083	0.285	0.367	0.000	0.449	8.609

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	43	51	0	45	55
N.S.	1	1.00	1.00	0.80	0.93	1.11	0.00	0.98	1.20
time (sec)	N/A	0.046	0.027	0.098	0.283	0.351	0.000	0.451	8.599

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	45	55	72	0	57	106
N.S.	1	1.00	1.00	0.71	0.87	1.14	0.00	0.90	1.68
time (sec)	N/A	0.054	0.034	0.110	0.273	0.357	0.000	0.454	8.823

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	57	65	102	0	67	139
N.S.	1	1.00	1.00	0.70	0.79	1.24	0.00	0.82	1.70
time (sec)	N/A	0.060	0.033	0.125	0.279	0.356	0.000	0.463	8.591

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	73	60	70	81	201	107	72
N.S.	1	1.00	0.84	0.69	0.80	0.93	2.31	1.23	0.83
time (sec)	N/A	0.065	0.452	0.207	0.281	0.373	1.009	0.490	0.055

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	48	59	72	170	90	59
N.S.	1	1.00	1.01	0.69	0.84	1.03	2.43	1.29	0.84
time (sec)	N/A	0.058	0.153	0.158	0.281	0.355	0.709	0.480	8.471

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	55	38	47	57	126	70	45
N.S.	1	1.00	1.06	0.73	0.90	1.10	2.42	1.35	0.87
time (sec)	N/A	0.053	0.165	0.150	0.295	0.355	0.539	0.467	0.083

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	27	28	34	40	95	56	34
N.S.	1	1.00	0.73	0.76	0.92	1.08	2.57	1.51	0.92
time (sec)	N/A	0.035	0.023	0.122	0.279	0.350	0.438	0.454	0.048

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	37	46	59	0	45	87
N.S.	1	1.00	0.69	0.71	0.88	1.13	0.00	0.87	1.67
time (sec)	N/A	0.033	0.046	0.146	0.289	0.348	0.000	0.434	8.576

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	45	51	68	104	0	69	136
N.S.	1	1.00	0.66	0.75	1.00	1.53	0.00	1.01	2.00
time (sec)	N/A	0.051	0.116	0.131	0.273	0.367	0.000	0.436	8.609

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	61	59	80	147	0	87	168
N.S.	1	1.00	0.72	0.69	0.94	1.73	0.00	1.02	1.98
time (sec)	N/A	0.063	0.152	0.194	0.274	0.362	0.000	0.442	8.613

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	98	69	90	195	0	103	202
N.S.	1	1.00	0.97	0.68	0.89	1.93	0.00	1.02	2.00
time (sec)	N/A	0.070	1.688	0.213	0.274	0.353	0.000	0.485	8.628

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	106	72	95	117	394	89	108
N.S.	1	1.00	0.95	0.65	0.86	1.05	3.55	0.80	0.97
time (sec)	N/A	0.074	0.556	0.273	0.265	0.356	2.410	0.454	0.129

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	78	62	81	107	347	73	91
N.S.	1	1.00	0.84	0.67	0.87	1.15	3.73	0.78	0.98
time (sec)	N/A	0.073	1.547	0.251	0.281	0.336	1.287	0.512	8.490

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	50	71	95	303	56	59
N.S.	1	1.00	0.95	0.68	0.96	1.28	4.09	0.76	0.80
time (sec)	N/A	0.059	0.313	0.203	0.274	0.355	0.914	0.456	0.074

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	42	60	75	257	45	44
N.S.	1	1.00	1.08	0.70	1.00	1.25	4.28	0.75	0.73
time (sec)	N/A	0.054	0.517	0.176	0.276	0.343	0.640	0.447	0.056

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	44	46	99	28	37
N.S.	1	1.00	1.00	1.10	1.47	1.53	3.30	0.93	1.23
time (sec)	N/A	0.029	0.025	0.144	0.279	0.331	0.609	0.492	0.053

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	49	72	104	0	59	148
N.S.	1	1.00	0.70	0.66	0.97	1.41	0.00	0.80	2.00
time (sec)	N/A	0.041	0.138	0.163	0.290	0.355	0.000	0.462	8.761

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	61	63	91	152	0	77	193
N.S.	1	1.00	0.68	0.70	1.01	1.69	0.00	0.86	2.14
time (sec)	N/A	0.057	0.320	0.172	0.278	0.363	0.000	0.471	8.652

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	71	73	103	196	0	86	227
N.S.	1	1.00	0.66	0.68	0.95	1.81	0.00	0.80	2.10
time (sec)	N/A	0.074	0.515	0.224	0.285	0.366	0.000	0.467	8.684

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	81	83	113	242	0	97	260
N.S.	1	1.00	0.64	0.66	0.90	1.92	0.00	0.77	2.06
time (sec)	N/A	0.076	3.790	0.228	0.273	0.363	0.000	0.495	8.676

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	119	74	105	144	588	84	114
N.S.	1	1.00	1.03	0.64	0.91	1.24	5.07	0.72	0.98
time (sec)	N/A	0.074	0.687	0.140	0.294	0.349	3.452	0.465	8.532

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	127	62	94	132	527	66	69
N.S.	1	1.00	1.34	0.65	0.99	1.39	5.55	0.69	0.73
time (sec)	N/A	0.066	6.460	0.283	0.301	0.353	1.724	0.497	0.123

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	61	54	83	112	466	55	54
N.S.	1	1.00	0.73	0.65	1.00	1.35	5.61	0.66	0.65
time (sec)	N/A	0.062	0.282	0.243	0.277	0.358	1.203	0.460	0.063

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	53	43	67	72	192	38	54
N.S.	1	1.00	1.77	1.43	2.23	2.40	6.40	1.27	1.80
time (sec)	N/A	0.043	0.154	0.205	0.294	0.339	1.051	0.487	8.483

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	30	33	57	62	129	28	37
N.S.	1	1.00	0.65	0.72	1.24	1.35	2.80	0.61	0.80
time (sec)	N/A	0.036	0.023	0.174	0.278	0.342	1.054	0.441	8.462

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	62	61	95	152	0	69	206
N.S.	1	1.00	0.64	0.63	0.98	1.57	0.00	0.71	2.12
time (sec)	N/A	0.047	0.257	0.204	0.279	0.345	0.000	0.471	9.526

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	75	114	201	0	87	251
N.S.	1	1.00	0.66	0.68	1.03	1.81	0.00	0.78	2.26
time (sec)	N/A	0.065	0.705	0.218	0.285	0.342	0.000	0.521	8.722

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	85	85	126	242	0	97	286
N.S.	1	1.00	0.65	0.65	0.96	1.85	0.00	0.74	2.18
time (sec)	N/A	0.082	2.547	0.224	0.303	0.372	0.000	0.484	8.690

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	118	42	61	86	0	89	43
N.S.	1	1.00	2.31	0.82	1.20	1.69	0.00	1.75	0.84
time (sec)	N/A	0.040	0.108	0.128	0.511	0.349	0.000	0.468	8.730

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	80	0	103	302	1833	127	370
N.S.	1	1.00	0.70	0.00	0.90	2.65	16.08	1.11	3.25
time (sec)	N/A	0.081	0.196	0.734	0.277	0.378	5.931	0.524	11.762

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	65	0	83	210	1061	101	242
N.S.	1	1.00	0.71	0.00	0.91	2.31	11.66	1.11	2.66
time (sec)	N/A	0.067	0.115	0.333	0.283	0.352	2.772	0.462	10.416

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	63	135	530	75	147
N.S.	1	1.00	0.74	0.00	0.93	1.99	7.79	1.10	2.16
time (sec)	N/A	0.059	0.145	0.273	0.275	0.369	1.535	0.434	9.641

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	0	39	62	190	45	67
N.S.	1	1.00	0.93	0.00	0.95	1.51	4.63	1.10	1.63
time (sec)	N/A	0.036	0.226	0.174	0.278	0.341	0.723	0.456	9.184

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.053	0.035	0.088	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.025	0.418	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.052	0.031	0.441	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	0.027	0.822	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	71	95	65	73	192	92	226
N.S.	1	1.00	0.68	0.90	0.62	0.70	1.83	0.88	2.15
time (sec)	N/A	0.110	0.162	0.141	0.282	0.357	0.453	0.482	12.131

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	54	77	52	62	144	62	198
N.S.	1	1.00	0.67	0.95	0.64	0.77	1.78	0.77	2.44
time (sec)	N/A	0.089	0.089	0.103	0.275	0.358	0.271	0.482	11.973

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	57	39	51	119	47	198
N.S.	1	1.00	0.65	0.88	0.60	0.78	1.83	0.72	3.05
time (sec)	N/A	0.063	0.076	0.086	0.276	0.351	0.201	0.450	12.332

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	74	55	57	60	0	87	160
N.S.	1	1.00	1.45	1.08	1.12	1.18	0.00	1.71	3.14
time (sec)	N/A	0.042	0.063	0.129	0.281	0.365	0.000	0.456	8.718

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	108
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	2.63
time (sec)	N/A	0.040	0.036	0.106	0.476	0.383	0.000	0.446	8.800

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	109	71	66	114	0	95	145
N.S.	1	1.00	2.10	1.37	1.27	2.19	0.00	1.83	2.79
time (sec)	N/A	0.054	0.041	0.125	0.479	0.364	0.000	0.455	8.676

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	95	72	61	119	0	115	111
N.S.	1	1.00	1.83	1.38	1.17	2.29	0.00	2.21	2.13
time (sec)	N/A	0.075	0.031	0.135	0.282	0.372	0.000	0.460	8.577

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	90	80	137	0	116	112
N.S.	1	1.00	1.82	1.22	1.08	1.85	0.00	1.57	1.51
time (sec)	N/A	0.095	0.042	0.171	0.284	0.375	0.000	0.488	8.593

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	177	110	92	169	0	144	143
N.S.	1	1.00	1.97	1.22	1.02	1.88	0.00	1.60	1.59
time (sec)	N/A	0.096	0.060	0.151	0.282	0.375	0.000	0.447	8.648

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	151	105	98	275	123	331
N.S.	1	1.00	0.64	1.12	0.78	0.73	2.04	0.91	2.45
time (sec)	N/A	0.174	0.392	0.207	0.288	0.351	0.671	0.518	12.308

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	76	142	93	85	309	106	257
N.S.	1	1.00	0.74	1.38	0.90	0.83	3.00	1.03	2.50
time (sec)	N/A	0.112	0.318	0.158	0.308	0.349	0.826	0.466	12.163

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	105	57	95	69	72	172	72	262
N.S.	1	1.15	0.63	1.04	0.76	0.79	1.89	0.79	2.88
time (sec)	N/A	0.091	0.159	0.127	0.276	0.347	0.314	0.488	12.132

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	73	75	86	0	101	188
N.S.	1	1.00	1.00	1.03	1.06	1.21	0.00	1.42	2.65
time (sec)	N/A	0.085	0.269	0.154	0.278	0.431	0.000	0.476	9.043

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	80	79	105	0	143	201
N.S.	1	1.00	1.27	1.08	1.07	1.42	0.00	1.93	2.72
time (sec)	N/A	0.075	0.421	0.151	0.484	0.405	0.000	0.458	8.811

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	102	103	104	143	0	128	213
N.S.	1	1.00	1.40	1.41	1.42	1.96	0.00	1.75	2.92
time (sec)	N/A	0.087	0.463	0.170	0.485	0.397	0.000	0.465	8.777

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	140	97	83	166	0	141	193
N.S.	1	1.00	1.92	1.33	1.14	2.27	0.00	1.93	2.64
time (sec)	N/A	0.151	0.427	0.189	0.478	0.362	0.000	0.442	8.937

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	209	143	130	155	0	164	161
N.S.	1	1.00	2.55	1.74	1.59	1.89	0.00	2.00	1.96
time (sec)	N/A	0.136	0.097	0.224	0.277	0.355	0.000	0.468	8.622

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	136	109	189	0	164	160
N.S.	1	1.00	1.89	1.36	1.09	1.89	0.00	1.64	1.60
time (sec)	N/A	0.141	0.698	0.209	0.280	0.352	0.000	0.495	8.629

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	200	187	227	0	228	339
N.S.	1	1.00	1.85	1.61	1.51	1.83	0.00	1.84	2.73
time (sec)	N/A	0.172	0.518	0.253	0.295	0.351	0.000	0.468	9.658

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	86	194	129	98	379	123	331
N.S.	1	1.00	0.65	1.47	0.98	0.74	2.87	0.93	2.51
time (sec)	N/A	0.202	0.485	0.218	0.281	0.365	0.730	0.465	12.159

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	133	76	156	106	85	328	106	349
N.S.	1	1.14	0.65	1.33	0.91	0.73	2.80	0.91	2.98
time (sec)	N/A	0.123	0.347	0.170	0.288	0.345	0.491	0.489	10.718

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	115	99	101	0	144	244
N.S.	1	1.00	0.83	1.16	1.00	1.02	0.00	1.45	2.46
time (sec)	N/A	0.112	0.504	0.175	0.279	0.374	0.000	0.481	10.357

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	106	94	93	121	0	162	264
N.S.	1	1.00	1.15	1.02	1.01	1.32	0.00	1.76	2.87
time (sec)	N/A	0.102	0.778	0.165	0.493	0.358	0.000	0.473	8.766

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	112	130	124	159	0	184	259
N.S.	1	1.00	1.14	1.33	1.27	1.62	0.00	1.88	2.64
time (sec)	N/A	0.100	0.799	0.184	0.480	0.359	0.000	0.480	8.681

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	148	125	117	180	0	161	249
N.S.	1	1.00	1.63	1.37	1.29	1.98	0.00	1.77	2.74
time (sec)	N/A	0.115	0.354	0.188	0.492	0.372	0.000	0.483	8.675

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	133	164	147	190	0	174	237
N.S.	1	1.00	1.33	1.64	1.47	1.90	0.00	1.74	2.37
time (sec)	N/A	0.149	0.435	0.221	0.485	0.381	0.000	0.504	8.893

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	267	185	155	190	0	196	291
N.S.	1	1.00	2.67	1.85	1.55	1.90	0.00	1.96	2.91
time (sec)	N/A	0.162	0.104	0.243	0.293	0.345	0.000	0.524	9.304

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	252	222	200	227	0	228	339
N.S.	1	1.00	2.03	1.79	1.61	1.83	0.00	1.84	2.73
time (sec)	N/A	0.193	2.872	0.233	0.293	0.364	0.000	0.497	9.675

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	151	182	128	85	381	106	349
N.S.	1	1.00	1.10	1.33	0.93	0.62	2.78	0.77	2.55
time (sec)	N/A	0.115	0.307	0.203	0.303	0.346	0.548	0.473	10.728

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	149	125	115	0	181	295
N.S.	1	1.00	0.81	1.27	1.07	0.98	0.00	1.55	2.52
time (sec)	N/A	0.143	0.787	0.204	0.281	0.363	0.000	0.494	10.419

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	136	136	117	135	0	194	295
N.S.	1	1.00	1.17	1.17	1.01	1.16	0.00	1.67	2.54
time (sec)	N/A	0.117	1.115	0.179	0.479	0.359	0.000	0.509	8.834

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	281	116	258	68	1360	114	107
N.S.	1	1.00	2.70	1.12	2.48	0.65	13.08	1.10	1.03
time (sec)	N/A	0.093	3.731	0.207	0.488	0.346	17.000	0.477	11.913

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	271	116	237	58	1049	114	79
N.S.	1	1.00	3.11	1.33	2.72	0.67	12.06	1.31	0.91
time (sec)	N/A	0.089	1.219	0.191	0.492	0.338	11.258	0.455	8.605

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	46	77	156	45	563	75	66
N.S.	1	1.00	0.74	1.24	2.52	0.73	9.08	1.21	1.06
time (sec)	N/A	0.084	0.065	0.158	0.488	0.365	4.832	0.452	10.464

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	161	77	133	34	366	72	33
N.S.	1	1.00	3.58	1.71	2.96	0.76	8.13	1.60	0.73
time (sec)	N/A	0.048	0.424	0.124	0.483	0.342	2.497	0.437	8.686

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	37	31	52	37	0	31	79
N.S.	1	1.00	1.68	1.41	2.36	1.68	0.00	1.41	3.59
time (sec)	N/A	0.046	0.070	0.142	0.495	0.352	0.000	0.468	8.770

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	69	44	70	62	0	65	25
N.S.	1	1.00	2.38	1.52	2.41	2.14	0.00	2.24	0.86
time (sec)	N/A	0.041	0.192	0.150	0.288	0.350	0.000	0.423	8.599

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	94	72	115	88	0	94	87
N.S.	1	1.00	1.77	1.36	2.17	1.66	0.00	1.77	1.64
time (sec)	N/A	0.074	0.318	0.196	0.285	0.347	0.000	0.457	8.653

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	126	96	153	119	0	128	119
N.S.	1	1.00	1.75	1.33	2.12	1.65	0.00	1.78	1.65
time (sec)	N/A	0.088	0.476	0.222	0.286	0.340	0.000	0.465	8.621

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	125	124	195	143	0	157	151
N.S.	1	1.00	1.32	1.31	2.05	1.51	0.00	1.65	1.59
time (sec)	N/A	0.092	0.804	0.257	0.287	0.366	0.000	0.447	8.675

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	189	150	234	173	0	187	291
N.S.	1	1.00	1.66	1.32	2.05	1.52	0.00	1.64	2.55
time (sec)	N/A	0.098	0.488	0.256	0.281	0.358	0.000	0.459	9.250

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	209	143	398	144	3580	145	147
N.S.	1	1.00	1.88	1.29	3.59	1.30	32.25	1.31	1.32
time (sec)	N/A	0.167	1.089	0.290	0.509	0.353	32.128	0.454	12.340

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	165	104	312	119	2263	106	120
N.S.	1	1.00	1.99	1.25	3.76	1.43	27.27	1.28	1.45
time (sec)	N/A	0.149	0.695	0.253	0.500	0.370	19.361	0.451	12.378

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	91	226	100	1248	91	95
N.S.	1	1.00	1.00	1.32	3.28	1.45	18.09	1.32	1.38
time (sec)	N/A	0.193	0.116	0.221	0.515	0.353	12.002	0.479	10.792

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	117	50	139	77	479	78	68
N.S.	1	1.00	2.49	1.06	2.96	1.64	10.19	1.66	1.45
time (sec)	N/A	0.048	0.250	0.208	0.487	0.352	6.906	0.429	8.865

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	115	34	55	103	0	38	39
N.S.	1	1.00	2.88	0.85	1.38	2.58	0.00	0.95	0.98
time (sec)	N/A	0.104	0.113	0.274	0.292	0.343	0.000	0.447	8.675

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	216	59	116	160	0	90	87
N.S.	1	1.00	4.00	1.09	2.15	2.96	0.00	1.67	1.61
time (sec)	N/A	0.071	0.569	0.259	0.290	0.346	0.000	0.467	8.686

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	364	87	161	246	0	116	120
N.S.	1	1.00	4.67	1.12	2.06	3.15	0.00	1.49	1.54
time (sec)	N/A	0.145	0.581	0.293	0.290	0.364	0.000	0.497	8.658

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	472	113	199	302	0	146	153
N.S.	1	1.00	5.19	1.24	2.19	3.32	0.00	1.60	1.68
time (sec)	N/A	0.168	0.923	0.359	0.278	0.366	0.000	0.503	8.645

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	197	123	314	163	2264	117	121
N.S.	1	1.00	2.03	1.27	3.24	1.68	23.34	1.21	1.25
time (sec)	N/A	0.181	0.812	0.340	0.487	0.350	34.636	0.445	12.071

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	96	84	228	144	1246	80	94
N.S.	1	1.00	1.26	1.11	3.00	1.89	16.39	1.05	1.24
time (sec)	N/A	0.120	0.485	0.280	0.494	0.339	21.045	0.453	11.322

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	145	67	142	124	529	60	54
N.S.	1	1.00	2.38	1.10	2.33	2.03	8.67	0.98	0.89
time (sec)	N/A	0.077	0.317	0.256	0.497	0.379	10.802	0.475	8.774

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	185	64	143	194	0	66	64
N.S.	1	1.00	2.72	0.94	2.10	2.85	0.00	0.97	0.94
time (sec)	N/A	0.128	0.280	0.281	0.285	0.360	0.000	0.479	8.900

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	255	89	202	279	0	109	145
N.S.	1	1.00	3.11	1.09	2.46	3.40	0.00	1.33	1.77
time (sec)	N/A	0.146	1.153	0.305	0.295	0.341	0.000	0.473	8.807

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	308	117	247	365	0	143	178
N.S.	1	1.00	2.91	1.10	2.33	3.44	0.00	1.35	1.68
time (sec)	N/A	0.178	4.404	0.371	0.298	0.354	0.000	0.484	8.660

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	171	130	355	263	1501	120	207
N.S.	1	1.00	1.19	0.90	2.47	1.83	10.42	0.83	1.44
time (sec)	N/A	0.110	0.695	0.423	0.328	0.367	70.819	0.537	9.056

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	109	85	0	151	0	157	-1
N.S.	1	1.00	0.56	0.44	0.00	0.78	0.00	0.81	-0.01
time (sec)	N/A	0.362	0.857	5.639	0.000	0.351	0.000	0.438	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	130	0	128	-1
N.S.	1	1.00	0.80	0.60	0.00	1.05	0.00	1.03	-0.01
time (sec)	N/A	0.223	0.408	4.365	0.000	0.349	0.000	0.436	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	89	65	0	111	0	99	-1
N.S.	1	1.00	0.97	0.71	0.00	1.21	0.00	1.08	-0.01
time (sec)	N/A	0.119	0.294	4.896	0.000	0.349	0.000	0.445	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	143	103	0	250	0	132	-1
N.S.	1	1.00	1.54	1.11	0.00	2.69	0.00	1.42	-0.01
time (sec)	N/A	0.222	0.152	4.948	0.000	0.368	0.000	0.453	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	279	0	149	-1
N.S.	1	1.00	2.31	1.40	0.00	3.13	0.00	1.67	-0.01
time (sec)	N/A	0.135	0.840	5.518	0.000	0.346	0.000	0.457	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	249	126	0	319	0	155	-1
N.S.	1	1.00	2.47	1.25	0.00	3.16	0.00	1.53	-0.01
time (sec)	N/A	0.262	0.598	5.671	0.000	0.350	0.000	0.495	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	285	144	0	361	0	184	-1
N.S.	1	1.00	2.08	1.05	0.00	2.64	0.00	1.34	-0.01
time (sec)	N/A	0.309	1.069	6.244	0.000	0.355	0.000	0.480	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	120	97	0	189	0	192	-1
N.S.	1	1.00	0.52	0.42	0.00	0.81	0.00	0.82	-0.00
time (sec)	N/A	0.467	2.824	4.878	0.000	0.342	0.000	0.432	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	166	0	162	-1
N.S.	1	1.00	0.71	0.56	0.00	1.06	0.00	1.04	-0.01
time (sec)	N/A	0.284	1.303	5.161	0.000	0.347	0.000	0.426	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	100	77	0	145	0	132	-1
N.S.	1	1.00	0.81	0.62	0.00	1.17	0.00	1.06	-0.01
time (sec)	N/A	0.170	1.056	5.377	0.000	0.360	0.000	0.434	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	145	123	0	282	0	165	-1
N.S.	1	1.00	1.18	1.00	0.00	2.29	0.00	1.34	-0.01
time (sec)	N/A	0.315	0.196	5.954	0.000	0.390	0.000	0.485	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	315	0	183	-1
N.S.	1	1.00	1.93	1.19	0.00	2.60	0.00	1.51	-0.01
time (sec)	N/A	0.207	0.555	6.247	0.000	0.365	0.000	1.292	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	271	151	0	359	0	185	-1
N.S.	1	1.00	2.07	1.15	0.00	2.74	0.00	1.41	-0.01
time (sec)	N/A	0.342	0.491	6.534	0.000	0.377	0.000	0.584	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	286	144	0	380	0	188	-1
N.S.	1	1.00	2.06	1.04	0.00	2.73	0.00	1.35	-0.01
time (sec)	N/A	0.386	0.659	6.954	0.000	0.361	0.000	0.611	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	97	74	0	136	0	103	-1
N.S.	1	1.00	0.61	0.47	0.00	0.86	0.00	0.65	-0.01
time (sec)	N/A	0.274	0.902	5.099	0.000	0.344	0.000	0.604	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	115	0	84	-1
N.S.	1	1.00	0.95	0.70	0.00	1.25	0.00	0.91	-0.01
time (sec)	N/A	0.228	0.265	3.588	0.000	0.348	0.000	0.508	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	54	0	96	0	65	-1
N.S.	1	1.00	1.28	0.90	0.00	1.60	0.00	1.08	-0.02
time (sec)	N/A	0.086	0.333	6.204	0.000	0.339	0.000	0.564	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	116	87	0	236	0	112	-1
N.S.	1	1.00	1.84	1.38	0.00	3.75	0.00	1.78	-0.02
time (sec)	N/A	0.147	0.104	5.416	0.000	0.356	0.000	0.579	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	263	0	132	-1
N.S.	1	1.00	2.23	1.66	0.00	4.24	0.00	2.13	-0.02
time (sec)	N/A	0.071	0.238	4.993	0.000	0.357	0.000	0.615	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	272	124	0	320	0	149	-1
N.S.	1	1.00	2.72	1.24	0.00	3.20	0.00	1.49	-0.01
time (sec)	N/A	0.207	1.305	7.290	0.000	0.361	0.000	0.527	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	367	0	168	-1
N.S.	1	1.00	2.16	1.07	0.00	2.72	0.00	1.24	-0.01
time (sec)	N/A	0.273	0.546	6.033	0.000	0.353	0.000	0.583	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	201	148	0	259	0	182	-1
N.S.	1	1.00	1.09	0.80	0.00	1.41	0.00	0.99	-0.01
time (sec)	N/A	0.395	1.343	5.357	0.000	0.351	0.000	0.607	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	114	0	236	0	144	-1
N.S.	1	1.00	1.07	0.81	0.00	1.69	0.00	1.03	-0.01
time (sec)	N/A	0.245	0.205	5.146	0.000	0.364	0.000	0.550	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	149	110	0	215	0	144	-1
N.S.	1	1.00	1.38	1.02	0.00	1.99	0.00	1.33	-0.01
time (sec)	N/A	0.113	0.424	5.448	0.000	0.380	0.000	0.549	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	130	97	0	291	0	152	-1
N.S.	1	1.00	1.53	1.14	0.00	3.42	0.00	1.79	-0.01
time (sec)	N/A	0.173	0.150	6.030	0.000	0.370	0.000	0.528	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	206	135	0	421	0	205	-1
N.S.	1	1.00	1.82	1.19	0.00	3.73	0.00	1.81	-0.01
time (sec)	N/A	0.154	1.572	7.029	0.000	0.376	0.000	0.648	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	508	0	224	-1
N.S.	1	1.00	2.02	1.07	0.00	3.32	0.00	1.46	-0.01
time (sec)	N/A	0.370	2.599	6.083	0.000	0.370	0.000	0.522	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	564	0	240	-1
N.S.	1	1.00	1.74	0.95	0.00	2.95	0.00	1.26	-0.01
time (sec)	N/A	0.478	1.613	6.872	0.000	0.380	0.000	0.546	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	92	50	72	90	50	49
N.S.	1	1.00	0.78	1.42	0.77	1.11	1.38	0.77	0.75
time (sec)	N/A	0.048	0.221	0.190	0.284	0.346	0.648	0.575	0.066

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	74	50	62	90	50	49
N.S.	1	1.00	0.78	1.14	0.77	0.95	1.38	0.77	0.75
time (sec)	N/A	0.048	0.160	0.135	0.291	0.379	0.451	0.507	0.062

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	58	54	50	51	66	50	49
N.S.	1	1.00	1.18	1.10	1.02	1.04	1.35	1.02	1.00
time (sec)	N/A	0.057	0.076	0.100	0.281	0.353	0.285	0.551	0.059

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	36	48	39	60	48	46
N.S.	1	1.00	0.98	0.80	1.07	0.87	1.33	1.07	1.02
time (sec)	N/A	0.023	0.013	0.108	0.277	0.370	0.205	0.550	0.055

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	45	47	51	0	48	92
N.S.	1	1.00	1.00	0.80	0.84	0.91	0.00	0.86	1.64
time (sec)	N/A	0.038	0.025	0.118	0.282	0.377	0.000	0.540	8.727

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	65	46	68	0	47	140
N.S.	1	1.00	1.00	1.23	0.87	1.28	0.00	0.89	2.64
time (sec)	N/A	0.040	0.030	0.106	0.283	0.361	0.000	0.552	8.696

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	60	67	45	69	0	46	146
N.S.	1	1.00	1.11	1.24	0.83	1.28	0.00	0.85	2.70
time (sec)	N/A	0.029	0.086	0.143	0.282	0.367	0.000	0.520	8.737

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	29	47	277	29	26
N.S.	1	1.00	0.76	0.81	0.78	1.27	7.49	0.78	0.70
time (sec)	N/A	0.066	0.107	0.119	0.304	0.337	8.682	0.611	0.045

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	30	29	37	224	29	26
N.S.	1	1.00	0.76	0.81	0.78	1.00	6.05	0.78	0.70
time (sec)	N/A	0.055	0.072	0.089	0.283	0.350	4.823	0.593	8.537

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	28	25	25	158	25	22
N.S.	1	1.00	0.75	0.88	0.78	0.78	4.94	0.78	0.69
time (sec)	N/A	0.028	0.031	0.079	0.287	0.362	2.491	0.504	0.041

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	23	28	27	25	0	28	71
N.S.	1	1.00	0.79	0.97	0.93	0.86	0.00	0.97	2.45
time (sec)	N/A	0.062	0.026	0.100	0.290	0.363	0.000	0.590	8.762

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	22	24	29	34	0	30	59
N.S.	1	1.00	0.73	0.80	0.97	1.13	0.00	1.00	1.97
time (sec)	N/A	0.056	0.029	0.096	0.297	0.363	0.000	0.526	8.697

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	24	25	26	30	0	26	23
N.S.	1	1.00	0.75	0.78	0.81	0.94	0.00	0.81	0.72
time (sec)	N/A	0.047	0.032	0.089	0.293	0.330	0.000	0.584	8.627

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	26	38	0	26	36
N.S.	1	1.00	0.76	0.78	0.70	1.03	0.00	0.70	0.97
time (sec)	N/A	0.061	0.042	0.109	0.283	0.337	0.000	0.558	8.757

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	28	29	26	41	0	26	25
N.S.	1	1.00	0.76	0.78	0.70	1.11	0.00	0.70	0.68
time (sec)	N/A	0.070	0.035	0.134	0.281	0.346	0.000	0.572	8.645

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	84	124	71	95	272	107	353
N.S.	1	1.00	0.59	0.87	0.50	0.66	1.90	0.75	2.47
time (sec)	N/A	0.131	0.205	0.278	0.289	0.369	1.503	0.664	12.343

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	71	106	61	84	248	92	320
N.S.	1	1.00	0.56	0.83	0.48	0.66	1.95	0.72	2.52
time (sec)	N/A	0.127	0.164	0.211	0.286	0.367	0.914	0.555	12.476

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	65	73	192	107	292
N.S.	1	1.00	0.79	0.85	0.63	0.71	1.86	1.04	2.83
time (sec)	N/A	0.094	0.158	0.147	0.287	0.373	0.620	0.628	12.274

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	68	52	62	167	92	292
N.S.	1	1.00	0.82	0.78	0.60	0.71	1.92	1.06	3.36
time (sec)	N/A	0.072	0.115	0.115	0.293	0.361	0.416	0.594	12.220

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	76	81	88	0	145	245
N.S.	1	1.00	0.91	0.85	0.91	0.99	0.00	1.63	2.75
time (sec)	N/A	0.063	0.115	0.121	0.298	0.386	0.000	0.652	10.189

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	77	94	90	107	0	142	244
N.S.	1	1.00	0.93	1.13	1.08	1.29	0.00	1.71	2.94
time (sec)	N/A	0.085	0.315	0.126	0.488	0.387	0.000	0.536	8.767

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	116	101	139	0	163	239
N.S.	1	1.00	1.00	1.23	1.07	1.48	0.00	1.73	2.54
time (sec)	N/A	0.080	0.558	0.139	0.494	0.391	0.000	0.592	8.708

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	228
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.78
time (sec)	N/A	0.059	0.041	0.134	0.496	0.361	0.000	0.673	8.699

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	104	107	180	0	153	217
N.S.	1	1.00	1.74	1.18	1.22	2.05	0.00	1.74	2.47
time (sec)	N/A	0.075	0.038	0.160	0.495	0.361	0.000	0.659	8.924

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	100	86	160	0	173	289
N.S.	1	1.00	1.82	1.35	1.16	2.16	0.00	2.34	3.91
time (sec)	N/A	0.091	0.029	0.161	0.284	0.362	0.000	0.639	10.434

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	175	118	106	187	0	201	337
N.S.	1	1.00	1.79	1.20	1.08	1.91	0.00	2.05	3.44
time (sec)	N/A	0.116	0.035	0.196	0.287	0.364	0.000	0.640	9.506

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	239	138	118	221	0	229	385
N.S.	1	1.00	2.10	1.21	1.04	1.94	0.00	2.01	3.38
time (sec)	N/A	0.118	0.063	0.214	0.290	0.398	0.000	0.583	9.946

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	279	156	138	239	0	201	337
N.S.	1	1.00	2.05	1.15	1.01	1.76	0.00	1.48	2.48
time (sec)	N/A	0.128	0.061	0.210	0.285	0.357	0.000	0.612	10.217

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	116	218	123	124	554	174	469
N.S.	1	1.00	0.63	1.18	0.66	0.67	2.99	0.94	2.54
time (sec)	N/A	0.255	0.532	0.300	0.298	0.360	1.938	0.701	12.211

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	86	162	101	111	335	123	437
N.S.	1	1.00	0.54	1.02	0.64	0.70	2.11	0.77	2.75
time (sec)	N/A	0.180	0.532	0.299	0.294	0.369	1.314	0.660	12.153

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	96	164	102	98	420	140	363
N.S.	1	1.00	0.68	1.16	0.72	0.70	2.98	0.99	2.57
time (sec)	N/A	0.198	0.380	0.237	0.289	0.365	0.942	0.599	12.351

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	106	82	85	223	123	388
N.S.	1	1.00	0.67	0.82	0.64	0.66	1.73	0.95	3.01
time (sec)	N/A	0.097	0.292	0.180	0.282	0.360	0.622	0.658	10.747

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	96	94	98	115	0	181	293
N.S.	1	1.00	0.81	0.79	0.82	0.97	0.00	1.52	2.46
time (sec)	N/A	0.103	0.736	0.204	0.295	0.362	0.000	0.511	10.215

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	83	136	128	135	0	210	310
N.S.	1	1.00	0.72	1.17	1.10	1.16	0.00	1.81	2.67
time (sec)	N/A	0.163	0.359	0.167	0.491	0.376	0.000	0.608	8.827

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	158	158	151	172	0	178	303
N.S.	1	1.00	1.61	1.61	1.54	1.76	0.00	1.82	3.09
time (sec)	N/A	0.108	1.436	0.203	0.490	0.365	0.000	0.634	8.911

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	191	146	139	192	0	209	293
N.S.	1	1.00	1.95	1.49	1.42	1.96	0.00	2.13	2.99
time (sec)	N/A	0.120	4.231	0.187	0.491	0.370	0.000	0.594	8.810

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	215	168	167	219	0	162	265
N.S.	1	1.00	1.85	1.45	1.44	1.89	0.00	1.40	2.28
time (sec)	N/A	0.139	0.924	0.225	0.491	0.365	0.000	0.691	8.770

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	130	124	239	0	207	275
N.S.	1	1.00	1.69	1.10	1.05	2.03	0.00	1.75	2.33
time (sec)	N/A	0.132	0.398	0.253	0.500	0.373	0.000	0.636	9.159

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	267	199	181	211	0	229	339
N.S.	1	1.00	2.02	1.51	1.37	1.60	0.00	1.73	2.57
time (sec)	N/A	0.180	0.077	0.260	0.289	0.366	0.000	0.563	9.754

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	291	256	233	271	0	293	319
N.S.	1	1.00	1.65	1.45	1.32	1.54	0.00	1.66	1.81
time (sec)	N/A	0.222	0.836	0.267	0.289	0.351	0.000	0.576	9.117

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	313	220	177	304	0	261	387
N.S.	1	1.00	1.86	1.31	1.05	1.81	0.00	1.55	2.30
time (sec)	N/A	0.195	0.943	0.282	0.288	0.364	0.000	0.590	12.071

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	353	310	283	340	0	357	395
N.S.	1	1.00	1.62	1.42	1.30	1.56	0.00	1.64	1.81
time (sec)	N/A	0.248	0.822	0.282	0.285	0.390	0.000	0.643	9.523

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	126	288	169	137	648	191	506
N.S.	1	1.00	0.62	1.42	0.83	0.67	3.19	0.94	2.49
time (sec)	N/A	0.276	0.690	0.428	0.292	0.379	4.697	0.716	11.720

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	116	252	149	124	595	174	572
N.S.	1	1.00	0.64	1.38	0.82	0.68	3.27	0.96	3.14
time (sec)	N/A	0.265	0.792	0.338	0.290	0.387	3.005	0.736	10.812

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	106	216	138	111	486	157	437
N.S.	1	1.00	0.67	1.36	0.87	0.70	3.06	0.99	2.75
time (sec)	N/A	0.219	0.651	0.323	0.286	0.388	2.086	0.593	12.280

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	96	178	115	98	440	140	461
N.S.	1	1.00	0.61	1.13	0.73	0.62	2.80	0.89	2.94
time (sec)	N/A	0.132	0.360	0.245	0.283	0.350	1.028	0.592	10.761

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	102	147	135	128	0	229	355
N.S.	1	1.00	0.71	1.03	0.94	0.90	0.00	1.60	2.48
time (sec)	N/A	0.145	0.707	0.217	0.287	0.377	0.000	0.604	10.510

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	148	150	141	147	0	226	356
N.S.	1	1.00	1.13	1.15	1.08	1.12	0.00	1.73	2.72
time (sec)	N/A	0.146	1.640	0.200	0.500	0.370	0.000	0.604	8.811

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	164	196	183	185	0	241	347
N.S.	1	1.00	1.20	1.43	1.34	1.35	0.00	1.76	2.53
time (sec)	N/A	0.131	1.961	0.200	0.490	0.396	0.000	0.668	8.808

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	201	184	185	206	0	250	339
N.S.	1	1.00	1.50	1.37	1.38	1.54	0.00	1.87	2.53
time (sec)	N/A	0.131	4.935	0.213	0.489	0.367	0.000	0.565	8.781

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	215	224	209	231	0	241	329
N.S.	1	1.00	1.56	1.62	1.51	1.67	0.00	1.75	2.38
time (sec)	N/A	0.140	0.927	0.235	0.490	0.374	0.000	0.627	8.782

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	216	190	180	252	0	226	554
N.S.	1	1.00	1.64	1.44	1.36	1.91	0.00	1.71	4.20
time (sec)	N/A	0.150	0.394	0.244	0.494	0.379	0.000	0.682	10.111

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	217	225	215	290	0	239	313
N.S.	1	1.00	1.29	1.34	1.28	1.73	0.00	1.42	1.86
time (sec)	N/A	0.193	0.751	0.224	0.489	0.377	0.000	0.563	9.380

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	363	241	206	247	0	261	387
N.S.	1	1.00	2.42	1.61	1.37	1.65	0.00	1.74	2.58
time (sec)	N/A	0.213	0.094	0.252	0.287	0.357	0.000	0.571	10.320

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	313	278	246	271	0	293	319
N.S.	1	1.00	1.78	1.58	1.40	1.54	0.00	1.66	1.81
time (sec)	N/A	0.239	4.364	0.257	0.288	0.358	0.000	0.582	9.221

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	313	316	268	304	0	325	357
N.S.	1	1.00	1.61	1.63	1.38	1.57	0.00	1.68	1.84
time (sec)	N/A	0.242	0.945	0.269	0.286	0.372	0.000	0.622	9.315

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	366	352	308	340	0	357	395
N.S.	1	1.00	1.69	1.63	1.43	1.57	0.00	1.65	1.83
time (sec)	N/A	0.278	1.782	0.293	0.288	0.393	0.000	0.704	9.552

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	116	306	186	124	746	174	572
N.S.	1	1.00	0.62	1.64	0.99	0.66	3.99	0.93	3.06
time (sec)	N/A	0.172	0.892	0.356	0.288	0.376	2.877	0.651	10.836

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	685	222	218	219	0	274	384
N.S.	1	1.00	4.89	1.59	1.56	1.56	0.00	1.96	2.74
time (sec)	N/A	0.169	6.310	0.236	0.486	0.394	0.000	0.590	8.805

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	168	380	80	2635	166	159
N.S.	1	1.00	0.64	1.24	2.81	0.59	19.52	1.23	1.18
time (sec)	N/A	0.145	0.196	0.228	0.494	0.364	46.489	0.563	11.375

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	377	155	339	70	2067	153	147
N.S.	1	1.00	3.22	1.32	2.90	0.60	17.67	1.31	1.26
time (sec)	N/A	0.141	3.531	0.193	0.503	0.342	28.694	0.495	11.309

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	258	129	278	60	1464	127	120
N.S.	1	1.00	2.84	1.42	3.05	0.66	16.09	1.40	1.32
time (sec)	N/A	0.116	1.738	0.166	0.492	0.346	17.522	0.587	12.009

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	219	129	257	50	1134	127	43
N.S.	1	1.00	3.00	1.77	3.52	0.68	15.53	1.74	0.59
time (sec)	N/A	0.083	1.130	0.144	0.490	0.345	9.104	0.539	8.798

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	60	87	156	57	0	88	136
N.S.	1	1.00	1.02	1.47	2.64	0.97	0.00	1.49	2.31
time (sec)	N/A	0.068	0.230	0.167	0.488	0.356	0.000	0.513	8.853

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	93	73	154	80	0	113	147
N.S.	1	1.00	1.90	1.49	3.14	1.63	0.00	2.31	3.00
time (sec)	N/A	0.085	0.323	0.181	0.485	0.376	0.000	0.571	8.723

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	102	84	138	104	0	103	159
N.S.	1	1.00	1.76	1.45	2.38	1.79	0.00	1.78	2.74
time (sec)	N/A	0.076	0.353	0.210	0.491	0.348	0.000	0.557	8.748

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	124	94	155	111	0	127	115
N.S.	1	1.00	2.14	1.62	2.67	1.91	0.00	2.19	1.98
time (sec)	N/A	0.066	0.375	0.184	0.283	0.352	0.000	0.544	8.669

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	98	154	132	0	129	119
N.S.	1	1.00	1.52	1.20	1.88	1.61	0.00	1.57	1.45
time (sec)	N/A	0.111	0.816	0.214	0.285	0.343	0.000	0.544	8.683

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	124	195	161	0	157	151
N.S.	1	1.00	1.89	1.24	1.95	1.61	0.00	1.57	1.51
time (sec)	N/A	0.120	0.441	0.225	0.288	0.372	0.000	0.789	8.754

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	176	274	188	0	216	339
N.S.	1	1.00	1.85	1.42	2.21	1.52	0.00	1.74	2.73
time (sec)	N/A	0.136	0.426	0.268	0.285	0.356	0.000	0.663	9.680

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	418	168	396	88	2895	166	160
N.S.	1	1.00	2.84	1.14	2.69	0.60	19.69	1.13	1.09
time (sec)	N/A	0.159	3.595	0.206	0.499	0.372	126.798	0.622	12.217

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	76	153	353	78	2271	153	146
N.S.	1	1.00	0.59	1.19	2.74	0.60	17.60	1.19	1.13
time (sec)	N/A	0.162	0.182	0.165	0.493	0.349	84.726	0.610	11.242

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	308	129	290	68	1608	127	89
N.S.	1	1.00	3.02	1.26	2.84	0.67	15.76	1.25	0.87
time (sec)	N/A	0.144	0.951	0.292	0.486	0.364	54.131	0.657	8.718

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	258	114	247	58	1153	114	79
N.S.	1	1.00	2.97	1.31	2.84	0.67	13.25	1.31	0.91
time (sec)	N/A	0.134	1.187	0.241	0.494	0.344	32.002	0.511	8.662

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	204	90	184	43	694	88	55
N.S.	1	1.00	2.91	1.29	2.63	0.61	9.91	1.26	0.79
time (sec)	N/A	0.080	0.625	0.198	0.490	0.355	18.352	0.751	8.637

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	46	48	82	45	0	52	97
N.S.	1	1.00	1.28	1.33	2.28	1.25	0.00	1.44	2.69
time (sec)	N/A	0.100	0.100	0.254	0.489	0.364	0.000	0.573	8.718

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	98	56	93	70	0	73	95
N.S.	1	1.00	2.80	1.60	2.66	2.00	0.00	2.09	2.71
time (sec)	N/A	0.103	0.282	0.244	0.490	0.360	0.000	0.769	8.832

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	72	115	93	0	98	84
N.S.	1	1.00	1.59	1.33	2.13	1.72	0.00	1.81	1.56
time (sec)	N/A	0.107	0.394	0.265	0.288	0.361	0.000	0.619	8.689

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	121	98	153	123	0	128	119
N.S.	1	1.00	1.83	1.48	2.32	1.86	0.00	1.94	1.80
time (sec)	N/A	0.092	0.669	0.273	0.281	0.382	0.000	0.796	8.665

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	116	124	195	149	0	157	151
N.S.	1	1.00	1.21	1.29	2.03	1.55	0.00	1.64	1.57
time (sec)	N/A	0.136	1.017	0.315	0.284	0.365	0.000	0.698	8.723

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	189	148	233	179	0	186	289
N.S.	1	1.00	1.69	1.32	2.08	1.60	0.00	1.66	2.58
time (sec)	N/A	0.150	0.557	0.333	0.286	0.361	0.000	0.674	9.304

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	229	176	275	204	0	215	339
N.S.	1	1.00	1.66	1.28	1.99	1.48	0.00	1.56	2.46
time (sec)	N/A	0.173	0.592	0.382	0.291	0.370	0.000	0.544	9.738

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	195	143	398	144	3578	145	146
N.S.	1	1.00	1.79	1.31	3.65	1.32	32.83	1.33	1.34
time (sec)	N/A	0.186	1.018	0.170	0.496	0.349	87.111	0.449	12.345

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	181	104	312	123	2264	106	121
N.S.	1	1.00	2.08	1.20	3.59	1.41	26.02	1.22	1.39
time (sec)	N/A	0.155	0.969	0.333	0.494	0.347	56.676	0.475	12.349

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	143	92	225	100	1244	91	94
N.S.	1	1.00	1.79	1.15	2.81	1.25	15.55	1.14	1.18
time (sec)	N/A	0.097	0.600	0.268	0.502	0.362	32.651	0.491	10.796

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	122	46	78	117	0	47	115
N.S.	1	1.00	2.71	1.02	1.73	2.60	0.00	1.04	2.56
time (sec)	N/A	0.126	0.206	0.303	0.501	0.365	0.000	0.458	8.729

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	156	59	116	165	0	90	87
N.S.	1	1.00	2.89	1.09	2.15	3.06	0.00	1.67	1.61
time (sec)	N/A	0.160	0.478	0.315	0.304	0.351	0.000	0.511	8.668

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	213	87	161	246	0	116	120
N.S.	1	1.00	2.73	1.12	2.06	3.15	0.00	1.49	1.54
time (sec)	N/A	0.168	4.409	0.335	0.289	0.355	0.000	0.476	8.693

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	251	113	199	302	0	146	153
N.S.	1	1.00	2.61	1.18	2.07	3.15	0.00	1.52	1.59
time (sec)	N/A	0.115	3.935	0.326	0.289	0.357	0.000	0.509	8.690

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	601	139	241	381	0	174	176
N.S.	1	1.00	5.14	1.19	2.06	3.26	0.00	1.49	1.50
time (sec)	N/A	0.206	6.136	0.377	0.285	0.363	0.000	0.552	9.131

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	143	100	269	211	901	92	157
N.S.	1	1.00	2.47	1.72	4.64	3.64	15.53	1.59	2.71
time (sec)	N/A	0.070	0.886	0.226	0.293	0.339	131.459	0.527	8.892

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	131	293	115	335	263	0	106	181
N.S.	1	1.47	3.29	1.29	3.76	2.96	0.00	1.19	2.03
time (sec)	N/A	0.318	1.849	0.228	0.305	0.358	0.000	0.595	9.051

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	195	130	401	315	0	120	205
N.S.	1	1.00	1.24	0.83	2.55	2.01	0.00	0.76	1.31
time (sec)	N/A	0.377	2.378	0.263	0.304	0.350	0.000	0.680	9.407

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	109	85	0	172	0	157	-1
N.S.	1	1.00	0.70	0.54	0.00	1.10	0.00	1.01	-0.01
time (sec)	N/A	0.276	2.548	5.003	0.000	0.350	0.000	0.450	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	99	75	0	151	0	128	-1
N.S.	1	1.00	0.80	0.60	0.00	1.22	0.00	1.03	-0.01
time (sec)	N/A	0.168	1.614	5.907	0.000	0.345	0.000	0.475	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	195	141	0	294	0	190	-1
N.S.	1	1.00	1.23	0.89	0.00	1.85	0.00	1.19	-0.01
time (sec)	N/A	0.311	0.248	5.506	0.000	0.360	0.000	0.556	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	258	162	0	320	0	208	-1
N.S.	1	1.00	1.74	1.09	0.00	2.16	0.00	1.41	-0.01
time (sec)	N/A	0.313	0.582	5.270	0.000	0.359	0.000	0.494	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	297	178	0	359	0	211	-1
N.S.	1	1.00	1.90	1.14	0.00	2.30	0.00	1.35	-0.01
time (sec)	N/A	0.258	0.739	5.885	0.000	0.381	0.000	0.468	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	380	0	211	-1
N.S.	1	1.00	1.90	1.04	0.00	2.33	0.00	1.29	-0.01
time (sec)	N/A	0.251	1.281	6.558	0.000	0.369	0.000	0.537	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	367	162	0	415	0	213	-1
N.S.	1	1.00	2.12	0.94	0.00	2.40	0.00	1.23	-0.01
time (sec)	N/A	0.343	1.935	6.866	0.000	0.386	0.000	0.547	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	403	180	0	461	0	242	-1
N.S.	1	1.00	1.93	0.86	0.00	2.21	0.00	1.16	-0.00
time (sec)	N/A	0.438	3.238	6.331	0.000	0.378	0.000	0.533	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	485	198	0	525	0	271	-1
N.S.	1	1.00	1.98	0.81	0.00	2.14	0.00	1.11	-0.00
time (sec)	N/A	0.523	5.802	7.577	0.000	0.377	0.000	0.503	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	191	216	0	567	0	300	-1
N.S.	1	1.00	0.68	0.77	0.00	2.02	0.00	1.07	-0.00
time (sec)	N/A	0.595	1.350	7.420	0.000	0.363	0.000	0.527	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	120	97	0	210	0	192	-1
N.S.	1	1.00	0.64	0.52	0.00	1.12	0.00	1.02	-0.01
time (sec)	N/A	0.330	6.573	4.776	0.000	0.351	0.000	0.515	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	110	87	0	189	0	162	-1
N.S.	1	1.00	0.71	0.56	0.00	1.21	0.00	1.04	-0.01
time (sec)	N/A	0.214	3.518	4.074	0.000	0.353	0.000	0.544	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	219	159	0	332	0	225	-1
N.S.	1	1.00	1.10	0.80	0.00	1.67	0.00	1.13	-0.01
time (sec)	N/A	0.459	0.511	7.030	0.000	0.376	0.000	0.474	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	283	180	0	360	0	243	-1
N.S.	1	1.00	1.59	1.01	0.00	2.02	0.00	1.37	-0.01
time (sec)	N/A	0.422	0.932	5.335	0.000	0.372	0.000	0.525	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	322	178	0	404	0	218	-1
N.S.	1	1.00	1.73	0.96	0.00	2.17	0.00	1.17	-0.01
time (sec)	N/A	0.363	0.767	5.408	0.000	0.357	0.000	0.468	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	424	0	246	-1
N.S.	1	1.00	1.70	0.99	0.00	2.15	0.00	1.25	-0.01
time (sec)	N/A	0.322	0.920	6.114	0.000	0.364	0.000	0.547	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	392	188	0	460	0	246	-1
N.S.	1	1.00	1.91	0.92	0.00	2.24	0.00	1.20	-0.00
time (sec)	N/A	0.465	0.972	6.883	0.000	0.377	0.000	0.477	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	404	180	0	488	0	248	-1
N.S.	1	1.00	1.88	0.84	0.00	2.27	0.00	1.15	-0.00
time (sec)	N/A	0.530	1.259	6.424	0.000	0.374	0.000	0.534	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	486	198	0	557	0	278	-1
N.S.	1	1.00	1.92	0.78	0.00	2.20	0.00	1.10	-0.00
time (sec)	N/A	0.606	1.907	7.286	0.000	0.375	0.000	0.483	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	522	216	0	600	0	308	-1
N.S.	1	1.00	1.79	0.74	0.00	2.06	0.00	1.06	-0.00
time (sec)	N/A	0.695	3.462	7.151	0.000	0.404	0.000	0.471	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	604	234	0	657	0	338	-1
N.S.	1	1.00	1.84	0.71	0.00	2.00	0.00	1.03	-0.00
time (sec)	N/A	0.792	4.520	7.006	0.000	0.395	0.000	0.546	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	74	0	155	0	103	-1
N.S.	1	1.00	1.15	0.60	0.00	1.25	0.00	0.83	-0.01
time (sec)	N/A	0.267	1.206	4.828	0.000	0.363	0.000	0.423	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	87	64	0	136	0	84	-1
N.S.	1	1.00	0.95	0.70	0.00	1.48	0.00	0.91	-0.01
time (sec)	N/A	0.132	1.103	5.211	0.000	0.338	0.000	0.438	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	169	123	0	279	0	157	-1
N.S.	1	1.00	1.30	0.95	0.00	2.15	0.00	1.21	-0.01
time (sec)	N/A	0.386	0.181	6.230	0.000	0.353	0.000	0.477	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	190	126	0	306	0	178	-1
N.S.	1	1.00	1.60	1.06	0.00	2.57	0.00	1.50	-0.01
time (sec)	N/A	0.339	0.339	5.251	0.000	0.363	0.000	0.464	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	296	150	0	346	0	0	-1
N.S.	1	1.00	2.37	1.20	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.367	2.479	5.926	0.000	0.373	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	369	0	0	-1
N.S.	1	1.00	2.16	1.07	0.00	2.73	0.00	0.00	-0.01
time (sec)	N/A	0.398	0.451	5.200	0.000	0.358	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	374	162	0	426	0	0	-1
N.S.	1	1.00	2.20	0.95	0.00	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.584	0.676	6.169	0.000	0.392	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	410	180	0	472	0	0	-1
N.S.	1	1.00	2.00	0.88	0.00	2.30	0.00	0.00	-0.00
time (sec)	N/A	0.756	0.762	6.520	0.000	0.377	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	102	77	0	161	0	103	-1
N.S.	1	1.00	0.50	0.38	0.00	0.79	0.00	0.50	-0.00
time (sec)	N/A	0.512	3.727	4.349	0.000	0.345	0.000	0.475	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	92	67	0	142	0	84	-1
N.S.	1	1.00	1.00	0.73	0.00	1.54	0.00	0.91	-0.01
time (sec)	N/A	0.242	2.763	4.853	0.000	0.359	0.000	0.566	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	82	57	0	121	0	65	-1
N.S.	1	1.00	1.37	0.95	0.00	2.02	0.00	1.08	-0.02
time (sec)	N/A	0.114	1.310	3.993	0.000	0.363	0.000	0.454	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	147	105	0	260	0	138	-1
N.S.	1	1.00	1.50	1.07	0.00	2.65	0.00	1.41	-0.01
time (sec)	N/A	0.241	0.198	4.641	0.000	0.368	0.000	0.490	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	220	123	0	291	0	165	-1
N.S.	1	1.00	2.34	1.31	0.00	3.10	0.00	1.76	-0.01
time (sec)	N/A	0.276	0.503	4.905	0.000	0.383	0.000	0.462	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	274	126	0	337	0	152	-1
N.S.	1	1.00	2.58	1.19	0.00	3.18	0.00	1.43	-0.01
time (sec)	N/A	0.329	1.567	5.734	0.000	0.357	0.000	0.487	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	383	0	168	-1
N.S.	1	1.00	2.04	1.00	0.00	2.66	0.00	1.17	-0.01
time (sec)	N/A	0.371	0.605	6.610	0.000	0.387	0.000	0.477	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	376	162	0	442	0	184	-1
N.S.	1	1.00	2.07	0.89	0.00	2.43	0.00	1.01	-0.01
time (sec)	N/A	0.497	0.758	6.810	0.000	0.362	0.000	0.513	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	412	180	0	492	0	200	-1
N.S.	1	1.00	1.87	0.82	0.00	2.24	0.00	0.91	-0.00
time (sec)	N/A	0.593	1.001	7.685	0.000	0.365	0.000	0.485	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	224	166	0	299	0	201	-1
N.S.	1	1.00	0.86	0.64	0.00	1.15	0.00	0.77	-0.00
time (sec)	N/A	0.903	0.973	6.378	0.000	0.370	0.000	0.493	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	225	166	0	280	0	201	-1
N.S.	1	1.00	1.01	0.75	0.00	1.26	0.00	0.91	-0.00
time (sec)	N/A	0.738	2.692	6.124	0.000	0.364	0.000	0.481	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	201	132	0	258	0	163	-1
N.S.	1	1.00	1.19	0.78	0.00	1.53	0.00	0.96	-0.01
time (sec)	N/A	0.294	2.108	4.984	0.000	0.371	0.000	0.539	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	177	130	0	239	0	163	-1
N.S.	1	1.00	1.29	0.95	0.00	1.74	0.00	1.19	-0.01
time (sec)	N/A	0.163	1.461	5.826	0.000	0.377	0.000	0.436	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	154	119	0	377	0	184	-1
N.S.	1	1.00	1.36	1.05	0.00	3.34	0.00	1.63	-0.01
time (sec)	N/A	0.264	0.282	5.299	0.000	0.403	0.000	0.477	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	170	133	0	421	0	205	-1
N.S.	1	1.00	1.50	1.18	0.00	3.73	0.00	1.81	-0.01
time (sec)	N/A	0.364	2.236	6.398	0.000	0.379	0.000	0.458	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	309	164	0	508	0	224	-1
N.S.	1	1.00	2.02	1.07	0.00	3.32	0.00	1.46	-0.01
time (sec)	N/A	0.497	2.877	7.477	0.000	0.377	0.000	0.470	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-2)	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	564	0	240	-1
N.S.	1	1.00	1.74	0.95	0.00	2.95	0.00	1.26	-0.01
time (sec)	N/A	0.630	1.694	6.547	0.000	0.393	0.000	0.474	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	414	200	0	643	0	256	-1
N.S.	1	1.00	1.81	0.87	0.00	2.81	0.00	1.12	-0.00
time (sec)	N/A	0.863	4.128	7.188	0.000	0.386	0.000	0.486	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.216	0.336	0.000	0.000	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.212	0.231	0.000	0.000	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	441	0	0	0	0	0	-1
N.S.	1	1.00	3.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	11.098	0.263	0.000	0.000	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	312	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	3.849	0.723	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	138	72	106	136	133	71
N.S.	1	1.00	1.00	1.42	0.74	1.09	1.40	1.37	0.73
time (sec)	N/A	0.062	0.324	0.332	0.290	0.368	2.907	0.535	8.718

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	87	120	72	95	136	118	71
N.S.	1	1.00	0.90	1.24	0.74	0.98	1.40	1.22	0.73
time (sec)	N/A	0.062	0.237	0.250	0.282	0.361	1.766	0.510	8.699

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	97	102	72	84	114	133	71
N.S.	1	1.00	1.20	1.26	0.89	1.04	1.41	1.64	0.88
time (sec)	N/A	0.085	0.259	0.258	0.276	0.354	1.636	0.494	0.054

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	84	72	73	114	118	71
N.S.	1	1.00	1.07	1.04	0.89	0.90	1.41	1.46	0.88
time (sec)	N/A	0.087	0.196	0.192	0.305	0.353	1.437	0.502	8.694

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	64	72	62	90	103	71
N.S.	1	1.00	1.20	0.98	1.11	0.95	1.38	1.58	1.09
time (sec)	N/A	0.059	0.121	0.129	0.293	0.363	0.594	0.534	8.699

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	65	69	74	0	70	126
N.S.	1	1.00	1.00	0.76	0.80	0.86	0.00	0.81	1.47
time (sec)	N/A	0.045	0.029	0.126	0.284	0.361	0.000	0.517	8.871

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	85	69	91	0	79	250
N.S.	1	1.00	1.00	1.02	0.83	1.10	0.00	0.95	3.01
time (sec)	N/A	0.054	0.031	0.117	0.282	0.365	0.000	0.536	8.878

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	105	68	102	0	82	229
N.S.	1	1.00	0.90	1.22	0.79	1.19	0.00	0.95	2.66
time (sec)	N/A	0.054	0.080	0.131	0.282	0.375	0.000	0.463	8.861

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	121	69	117	0	81	218
N.S.	1	1.00	0.89	1.42	0.81	1.38	0.00	0.95	2.56
time (sec)	N/A	0.048	0.108	0.147	0.284	0.363	0.000	0.454	8.794

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	101	69	110	0	82	207
N.S.	1	1.00	1.07	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.030	0.150	0.148	0.276	0.373	0.000	0.545	9.033

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	121	72	124	0	84	193
N.S.	1	1.00	1.07	1.41	0.84	1.44	0.00	0.98	2.24
time (sec)	N/A	0.050	0.120	0.161	0.288	0.362	0.000	0.493	8.898

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	70	100	0	70	69
N.S.	1	1.00	1.00	1.80	1.15	1.64	0.00	1.15	1.13
time (sec)	N/A	0.074	0.020	0.180	0.279	0.356	0.000	0.473	8.884

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	70	106	0	70	70
N.S.	1	1.00	1.00	1.97	1.08	1.63	0.00	1.08	1.08
time (sec)	N/A	0.084	0.020	0.185	0.280	0.353	0.000	0.498	8.834

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	70	109	0	70	70
N.S.	1	1.00	1.09	1.83	0.86	1.35	0.00	0.86	0.86
time (sec)	N/A	0.088	0.120	0.196	0.297	0.353	0.000	0.516	8.844

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	70	115	0	70	70
N.S.	1	1.00	1.09	2.05	0.86	1.42	0.00	0.86	0.86
time (sec)	N/A	0.088	0.090	0.199	0.285	0.405	0.000	0.528	8.884

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	70	122	0	70	70
N.S.	1	1.00	0.91	1.90	0.72	1.26	0.00	0.72	0.72
time (sec)	N/A	0.057	0.123	0.239	0.284	0.386	0.000	0.481	8.859

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	70	128	0	70	70
N.S.	1	1.00	0.91	2.08	0.72	1.32	0.00	0.72	0.72
time (sec)	N/A	0.057	0.135	0.233	0.287	0.363	0.000	0.479	8.976

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	158	97	111	189	168	96
N.S.	1	1.00	0.87	1.24	0.76	0.87	1.49	1.32	0.76
time (sec)	N/A	0.086	0.573	0.318	0.294	0.391	1.774	0.541	8.738

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	99	156	97	98	190	151	96
N.S.	1	1.00	0.91	1.43	0.89	0.90	1.74	1.39	0.88
time (sec)	N/A	0.083	0.532	0.275	0.318	0.379	1.298	0.506	8.762

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	102	97	85	139	134	96
N.S.	1	1.00	1.01	1.15	1.09	0.96	1.56	1.51	1.08
time (sec)	N/A	0.058	0.217	0.223	0.291	0.363	0.870	0.521	8.812

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	78	82	94	99	0	95	132
N.S.	1	1.00	0.66	0.69	0.79	0.83	0.00	0.80	1.11
time (sec)	N/A	0.069	0.059	0.179	0.291	0.372	0.000	0.511	9.066

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	114	121	94	118	0	107	333
N.S.	1	1.00	1.00	1.06	0.82	1.04	0.00	0.94	2.92
time (sec)	N/A	0.082	0.042	0.173	0.292	0.381	0.000	0.474	8.946

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	142	93	131	0	109	297
N.S.	1	1.00	0.66	1.22	0.80	1.13	0.00	0.94	2.56
time (sec)	N/A	0.085	0.126	0.187	0.282	0.366	0.000	0.499	8.944

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	74	177	93	115	0	107	288
N.S.	1	1.00	0.67	1.61	0.85	1.05	0.00	0.97	2.62
time (sec)	N/A	0.072	0.125	0.190	0.282	0.381	0.000	0.490	8.918

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	76	158	94	152	0	108	276
N.S.	1	1.00	0.66	1.36	0.81	1.31	0.00	0.93	2.38
time (sec)	N/A	0.046	0.303	0.208	0.299	0.368	0.000	0.488	8.796

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	76	193	94	153	0	109	267
N.S.	1	1.00	0.68	1.72	0.84	1.37	0.00	0.97	2.38
time (sec)	N/A	0.071	0.096	0.213	0.299	0.372	0.000	0.515	8.788

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	102	147	97	167	0	111	217
N.S.	1	1.00	0.86	1.24	0.82	1.40	0.00	0.93	1.82
time (sec)	N/A	0.083	0.030	0.251	0.286	0.386	0.000	0.625	8.975

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	110	208	110	111	255	168	109
N.S.	1	1.00	0.99	1.87	0.99	1.00	2.30	1.51	0.98
time (sec)	N/A	0.084	0.656	0.348	0.360	0.367	2.279	0.648	8.715

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	100	170	110	98	202	151	108
N.S.	1	1.00	1.12	1.91	1.24	1.10	2.27	1.70	1.21
time (sec)	N/A	0.061	0.468	0.306	0.305	0.371	1.289	0.524	0.073

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	88	131	107	112	0	108	178
N.S.	1	1.00	0.64	0.96	0.78	0.82	0.00	0.79	1.30
time (sec)	N/A	0.073	0.077	0.221	0.279	0.399	0.000	0.496	8.989

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	134	107	131	0	120	371
N.S.	1	1.00	0.65	1.01	0.80	0.98	0.00	0.90	2.79
time (sec)	N/A	0.086	0.122	0.195	0.306	0.401	0.000	0.616	9.185

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	174	106	145	0	120	342
N.S.	1	1.00	0.65	1.31	0.80	1.09	0.00	0.90	2.57
time (sec)	N/A	0.093	0.108	0.204	0.284	0.402	0.000	0.544	9.014

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	86	210	108	159	0	122	333
N.S.	1	1.00	0.66	1.60	0.82	1.21	0.00	0.93	2.54
time (sec)	N/A	0.078	0.182	0.211	0.284	0.391	0.000	0.499	8.982

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	86	210	108	159	0	121	322
N.S.	1	1.00	0.66	1.60	0.82	1.21	0.00	0.92	2.46
time (sec)	N/A	0.051	0.323	0.219	0.288	0.400	0.000	0.554	8.940

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	86	246	107	179	0	122	311
N.S.	1	1.00	0.65	1.85	0.80	1.35	0.00	0.92	2.34
time (sec)	N/A	0.077	0.124	0.255	0.280	0.398	0.000	0.544	8.946

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	215	108	180	0	122	296
N.S.	1	1.00	0.85	1.62	0.81	1.35	0.00	0.92	2.23
time (sec)	N/A	0.087	0.031	0.237	0.337	0.411	0.000	0.569	9.843

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	96	242	119	172	0	135	378
N.S.	1	1.00	0.66	1.67	0.82	1.19	0.00	0.93	2.61
time (sec)	N/A	0.082	0.117	0.224	0.294	0.398	0.000	0.589	8.970

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	96	243	120	144	0	134	368
N.S.	1	1.00	0.65	1.64	0.81	0.97	0.00	0.91	2.49
time (sec)	N/A	0.056	0.114	0.230	0.317	0.408	0.000	0.689	9.073

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	96	298	120	192	0	134	357
N.S.	1	1.00	0.66	2.04	0.82	1.32	0.00	0.92	2.45
time (sec)	N/A	0.080	0.131	0.235	0.376	0.397	0.000	0.629	9.086

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	49	67	981	49	57
N.S.	1	1.00	0.66	0.67	0.67	0.92	13.44	0.67	0.78
time (sec)	N/A	0.077	0.252	0.263	0.298	0.377	43.520	0.478	0.068

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	49	59	862	49	57
N.S.	1	1.00	0.66	0.67	0.67	0.81	11.81	0.67	0.78
time (sec)	N/A	0.114	0.165	0.203	0.291	0.400	25.958	0.464	0.063

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	49	49	741	49	57
N.S.	1	1.00	0.87	0.89	0.89	0.89	13.47	0.89	1.04
time (sec)	N/A	0.078	0.122	0.176	0.312	0.374	15.601	0.444	0.063

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	44	51	48	0	61	102
N.S.	1	1.00	0.75	0.68	0.78	0.74	0.00	0.94	1.57
time (sec)	N/A	0.067	0.038	0.184	0.295	0.407	0.000	0.451	9.051

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	45	46	52	65	0	65	146
N.S.	1	1.00	0.73	0.74	0.84	1.05	0.00	1.05	2.35
time (sec)	N/A	0.076	0.048	0.204	0.298	0.407	0.000	0.415	8.801

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	42	52	61	0	63	150
N.S.	1	1.00	0.75	0.70	0.87	1.02	0.00	1.05	2.50
time (sec)	N/A	0.076	0.068	0.202	0.307	0.406	0.000	0.481	8.927

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	44	50	75	0	62	138
N.S.	1	1.00	0.75	0.69	0.78	1.17	0.00	0.97	2.16
time (sec)	N/A	0.064	0.057	0.220	0.279	0.391	0.000	0.431	8.948

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	46	63	0	46	45
N.S.	1	1.00	0.59	0.96	0.90	1.24	0.00	0.90	0.88
time (sec)	N/A	0.064	0.037	0.204	0.285	0.359	0.000	0.511	8.932

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	49	46	71	0	46	46
N.S.	1	1.00	0.87	0.89	0.84	1.29	0.00	0.84	0.84
time (sec)	N/A	0.100	0.083	0.231	0.295	0.375	0.000	0.456	8.958

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	76	0	46	46
N.S.	1	1.00	0.66	0.67	0.63	1.04	0.00	0.63	0.63
time (sec)	N/A	0.079	0.076	0.256	0.279	0.365	0.000	0.491	8.942

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	46	84	0	46	46
N.S.	1	1.00	0.66	0.67	0.63	1.15	0.00	0.63	0.63
time (sec)	N/A	0.080	0.081	0.275	0.280	0.360	0.000	0.478	8.963

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	67	682	39	36
N.S.	1	1.00	0.69	0.71	0.71	1.22	12.40	0.71	0.65
time (sec)	N/A	0.073	0.391	0.141	0.285	0.387	68.193	0.459	0.061

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	59	588	39	36
N.S.	1	1.00	0.69	0.71	0.71	1.07	10.69	0.71	0.65
time (sec)	N/A	0.071	0.492	0.261	0.278	0.367	44.622	0.434	0.054

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	39	47	493	39	36
N.S.	1	1.00	0.69	0.71	0.71	0.85	8.96	0.71	0.65
time (sec)	N/A	0.048	0.208	0.236	0.293	0.366	27.746	0.468	8.832

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	34	39	36	0	47	120
N.S.	1	1.00	0.77	0.72	0.83	0.77	0.00	1.00	2.55
time (sec)	N/A	0.057	0.031	0.247	0.293	0.392	0.000	0.437	9.108

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	34	41	42	0	53	110
N.S.	1	1.00	0.74	0.79	0.95	0.98	0.00	1.23	2.56
time (sec)	N/A	0.072	0.035	0.263	0.287	0.394	0.000	0.502	8.919

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	38	36	40	55	0	52	104
N.S.	1	1.00	0.81	0.77	0.85	1.17	0.00	1.11	2.21
time (sec)	N/A	0.070	0.034	0.272	0.283	0.393	0.000	0.473	8.887

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	37	36	52	0	36	34
N.S.	1	1.00	0.65	1.19	1.16	1.68	0.00	1.16	1.10
time (sec)	N/A	0.056	0.033	0.257	0.288	0.366	0.000	0.481	8.931

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	57	0	36	36
N.S.	1	1.00	0.69	0.71	0.65	1.04	0.00	0.65	0.65
time (sec)	N/A	0.035	0.048	0.278	0.283	0.395	0.000	0.482	8.923

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	65	0	36	36
N.S.	1	1.00	0.69	0.71	0.65	1.18	0.00	0.65	0.65
time (sec)	N/A	0.062	0.056	0.277	0.277	0.372	0.000	0.499	8.941

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	36	72	0	36	36
N.S.	1	1.00	0.69	0.71	0.65	1.31	0.00	0.65	0.65
time (sec)	N/A	0.071	0.057	0.308	0.280	0.368	0.000	0.506	8.951

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	71	68	73	70	2558	193	83
N.S.	1	1.00	0.70	0.67	0.72	0.69	25.08	1.89	0.81
time (sec)	N/A	0.084	0.683	0.183	0.289	0.388	116.358	0.505	0.064

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	59	58	61	56	1698	167	69
N.S.	1	1.00	0.72	0.71	0.74	0.68	20.71	2.04	0.84
time (sec)	N/A	0.083	0.678	0.165	0.304	0.375	73.435	0.483	8.819

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	48	53	48	1102	141	57
N.S.	1	1.00	0.75	0.71	0.78	0.71	16.21	2.07	0.84
time (sec)	N/A	0.057	0.250	0.318	0.282	0.397	45.143	0.469	0.055

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	32	33	43	34	0	103	95
N.S.	1	1.00	0.71	0.73	0.96	0.76	0.00	2.29	2.11
time (sec)	N/A	0.062	0.029	0.323	0.281	0.423	0.000	0.500	8.919

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	39	44	52	0	101	71
N.S.	1	1.00	0.74	0.83	0.94	1.11	0.00	2.15	1.51
time (sec)	N/A	0.074	0.039	0.338	0.287	0.388	0.000	0.494	8.982

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	49	55	76	0	115	107
N.S.	1	1.00	0.75	0.75	0.85	1.17	0.00	1.77	1.65
time (sec)	N/A	0.081	0.057	0.348	0.279	0.382	0.000	0.524	8.879

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	59	65	106	0	145	139
N.S.	1	1.00	0.71	0.71	0.78	1.28	0.00	1.75	1.67
time (sec)	N/A	0.074	0.084	0.347	0.283	0.391	0.000	0.538	8.990

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	69	67	75	131	0	174	171
N.S.	1	1.00	0.72	0.70	0.78	1.36	0.00	1.81	1.78
time (sec)	N/A	0.048	0.210	0.370	0.286	0.397	0.000	0.508	8.877

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	79	85	161	0	204	203
N.S.	1	1.00	0.68	0.68	0.73	1.38	0.00	1.74	1.74
time (sec)	N/A	0.083	0.094	0.379	0.289	0.399	0.000	0.577	8.961

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	81	81	100	235	0	218	233
N.S.	1	1.00	0.68	0.68	0.83	1.96	0.00	1.82	1.94
time (sec)	N/A	0.058	0.483	0.421	0.307	0.386	0.000	0.508	8.886

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	89	110	283	0	248	266
N.S.	1	1.00	0.67	0.66	0.81	2.10	0.00	1.84	1.97
time (sec)	N/A	0.092	0.211	0.345	0.310	0.430	0.000	0.504	8.906

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	123	0	161	616	23312	769	923
N.S.	1	1.00	0.68	0.00	0.89	3.40	128.80	4.25	5.10
time (sec)	N/A	0.133	0.428	0.464	0.285	0.443	43.458	0.576	15.448

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	110	0	143	473	14997	576	819
N.S.	1	1.00	0.69	0.00	0.89	2.96	93.73	3.60	5.12
time (sec)	N/A	0.121	0.277	0.399	0.277	0.432	21.571	0.571	14.661

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	345	0	109	282	8675	379	550
N.S.	1	1.00	2.80	0.00	0.89	2.29	70.53	3.08	4.47
time (sec)	N/A	0.087	0.955	0.274	0.285	0.399	13.322	0.461	13.561

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	74	0	124	134	0	0	228
N.S.	1	1.00	0.81	0.00	1.36	1.47	0.00	0.00	2.51
time (sec)	N/A	0.095	0.516	0.323	0.322	0.424	0.000	0.000	10.205

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	0	81	105	0	0	146
N.S.	1	1.00	0.74	0.00	1.19	1.54	0.00	0.00	2.15
time (sec)	N/A	0.090	0.084	2.080	0.326	0.388	0.000	0.000	9.540

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	64	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.077	1.523	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	72	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.084	1.754	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	121	134	86	106	318	167	447
N.S.	1	1.00	0.73	0.81	0.52	0.64	1.93	1.01	2.71
time (sec)	N/A	0.146	0.387	0.337	0.284	0.427	2.561	0.519	11.872

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	101	116	76	95	294	137	414
N.S.	1	1.00	0.68	0.78	0.51	0.64	1.97	0.92	2.78
time (sec)	N/A	0.133	0.247	0.242	0.291	0.391	1.797	0.547	12.186

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	98	76	84	248	122	386
N.S.	1	1.00	0.73	0.78	0.61	0.67	1.98	0.98	3.09
time (sec)	N/A	0.111	0.198	0.226	0.281	0.390	1.302	0.535	12.357

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	78	63	73	223	122	96
N.S.	1	1.00	0.83	0.72	0.58	0.67	2.05	1.12	0.88
time (sec)	N/A	0.086	0.186	0.183	0.284	0.406	0.889	0.461	9.391

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	100	96	106	110	0	201	327
N.S.	1	1.00	0.79	0.76	0.83	0.87	0.00	1.58	2.57
time (sec)	N/A	0.078	0.088	0.144	0.281	0.404	0.000	0.493	10.707

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	98	114	121	129	0	198	313
N.S.	1	1.00	0.81	0.94	1.00	1.07	0.00	1.64	2.59
time (sec)	N/A	0.094	0.197	0.133	0.496	0.427	0.000	0.564	8.923

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	117	136	131	162	0	214	321
N.S.	1	1.00	0.87	1.01	0.98	1.21	0.00	1.60	2.40
time (sec)	N/A	0.106	1.865	0.161	0.501	0.384	0.000	0.451	8.858

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	174	154	122	182	0	220	310
N.S.	1	1.00	1.34	1.18	0.94	1.40	0.00	1.69	2.38
time (sec)	N/A	0.101	6.128	0.155	0.510	0.386	0.000	0.499	8.970

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	138	172	136	202	0	213	300
N.S.	1	1.00	1.03	1.28	1.01	1.51	0.00	1.59	2.24
time (sec)	N/A	0.094	1.367	0.162	0.496	0.411	0.000	0.479	8.817

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	164	129	125	222	0	199	291
N.S.	1	1.00	1.34	1.06	1.02	1.82	0.00	1.63	2.39
time (sec)	N/A	0.074	0.070	0.164	0.498	0.427	0.000	0.537	8.867

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	193	147	137	254	0	208	285
N.S.	1	1.00	1.51	1.15	1.07	1.98	0.00	1.62	2.23
time (sec)	N/A	0.106	0.044	0.191	0.531	0.414	0.000	0.506	9.460

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	175	128	106	210	0	228	385
N.S.	1	1.00	1.82	1.33	1.10	2.19	0.00	2.38	4.01
time (sec)	N/A	0.108	0.035	0.201	0.298	0.395	0.000	0.595	9.942

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	215	146	126	225	0	256	285
N.S.	1	1.00	1.76	1.20	1.03	1.84	0.00	2.10	2.34
time (sec)	N/A	0.140	0.046	0.229	0.281	0.389	0.000	0.516	9.154

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	301	166	138	259	0	256	285
N.S.	1	1.00	2.18	1.20	1.00	1.88	0.00	1.86	2.07
time (sec)	N/A	0.142	0.067	0.217	0.326	0.385	0.000	0.556	9.186

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	341	184	158	289	0	284	319
N.S.	1	1.00	2.13	1.15	0.99	1.81	0.00	1.78	1.99
time (sec)	N/A	0.161	0.062	0.250	0.330	0.411	0.000	0.541	9.400

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	363	202	168	320	0	340	387
N.S.	1	1.00	2.06	1.15	0.95	1.82	0.00	1.93	2.20
time (sec)	N/A	0.155	0.075	0.260	0.297	0.413	0.000	0.578	9.980

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	136	238	138	137	656	208	518
N.S.	1	1.00	0.65	1.14	0.66	0.66	3.14	1.00	2.48
time (sec)	N/A	0.274	0.964	0.484	0.311	0.425	3.534	0.595	11.118

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	172	116	124	384	191	543
N.S.	1	1.00	0.69	0.94	0.63	0.68	2.10	1.04	2.97
time (sec)	N/A	0.201	0.702	0.361	0.279	0.449	2.578	0.603	12.055

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	106	184	128	111	529	157	469
N.S.	1	1.00	0.64	1.12	0.78	0.67	3.21	0.95	2.84
time (sec)	N/A	0.222	0.458	0.297	0.299	0.384	1.863	0.527	12.125

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	106	116	93	98	282	157	501
N.S.	1	1.00	0.69	0.76	0.61	0.64	1.84	1.03	3.27
time (sec)	N/A	0.120	0.468	0.261	0.279	0.382	1.303	0.487	10.853

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	112	114	123	141	0	245	384
N.S.	1	1.00	0.70	0.71	0.76	0.88	0.00	1.52	2.39
time (sec)	N/A	0.126	0.284	0.231	0.276	0.421	0.000	0.495	10.840

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	110	166	173	161	0	274	401
N.S.	1	1.00	0.70	1.05	1.09	1.02	0.00	1.73	2.54
time (sec)	N/A	0.167	0.234	0.197	0.539	0.412	0.000	0.491	8.973

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	174	188	191	199	0	244	377
N.S.	1	1.00	1.24	1.34	1.36	1.42	0.00	1.74	2.69
time (sec)	N/A	0.152	3.602	0.222	0.505	0.401	0.000	0.529	8.939

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	209	224	190	219	0	274	384
N.S.	1	1.00	1.37	1.46	1.24	1.43	0.00	1.79	2.51
time (sec)	N/A	0.153	5.407	0.204	0.502	0.386	0.000	0.555	8.927

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	227	246	206	245	0	259	373
N.S.	1	1.00	1.48	1.61	1.35	1.60	0.00	1.69	2.44
time (sec)	N/A	0.155	0.901	0.216	0.501	0.399	0.000	0.550	8.940

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	264	217	184	265	0	272	363
N.S.	1	1.00	1.90	1.56	1.32	1.91	0.00	1.96	2.61
time (sec)	N/A	0.185	1.010	0.241	0.515	0.418	0.000	0.539	8.898

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	270	239	220	303	0	259	657
N.S.	1	1.00	1.72	1.52	1.40	1.93	0.00	1.65	4.18
time (sec)	N/A	0.169	1.435	0.253	0.514	0.403	0.000	0.535	11.010

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	262	173	154	323	0	270	351
N.S.	1	1.00	1.62	1.07	0.95	1.99	0.00	1.67	2.17
time (sec)	N/A	0.163	0.787	0.240	0.538	0.391	0.000	0.509	9.839

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	401	255	221	255	0	260	387
N.S.	1	1.00	2.20	1.40	1.21	1.40	0.00	1.43	2.13
time (sec)	N/A	0.226	0.084	0.268	0.277	0.394	0.000	0.534	11.164

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	313	192	155	291	0	324	357
N.S.	1	1.00	2.06	1.26	1.02	1.91	0.00	2.13	2.35
time (sec)	N/A	0.196	0.972	0.295	0.299	0.410	0.000	0.515	9.406

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	353	312	273	327	0	324	357
N.S.	1	1.00	1.55	1.37	1.20	1.43	0.00	1.42	1.57
time (sec)	N/A	0.275	0.846	0.322	0.332	0.427	0.000	0.558	9.435

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	187	248	197	360	0	388	433
N.S.	1	1.00	0.96	1.28	1.02	1.86	0.00	2.00	2.23
time (sec)	N/A	0.214	2.523	0.328	0.301	0.410	0.000	0.556	9.974

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	197	366	323	384	0	420	471
N.S.	1	1.00	0.73	1.36	1.20	1.42	0.00	1.56	1.74
time (sec)	N/A	0.307	3.124	0.336	0.337	0.426	0.000	0.504	10.358

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	146	308	184	150	748	225	612
N.S.	1	1.00	0.65	1.38	0.82	0.67	3.34	1.00	2.73
time (sec)	N/A	0.309	1.573	0.708	0.290	0.416	4.726	0.518	12.192

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	136	272	164	137	699	208	683
N.S.	1	1.00	0.65	1.30	0.78	0.66	3.34	1.00	3.27
time (sec)	N/A	0.303	1.085	0.557	0.284	0.426	3.531	0.488	11.023

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	126	236	164	124	597	191	543
N.S.	1	1.00	0.69	1.29	0.90	0.68	3.26	1.04	2.97
time (sec)	N/A	0.247	0.872	0.405	0.295	0.409	2.648	0.595	12.123

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	116	198	141	111	542	174	572
N.S.	1	1.00	0.64	1.09	0.78	0.61	2.99	0.96	3.16
time (sec)	N/A	0.148	0.674	0.314	0.289	0.402	1.856	0.570	10.963

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	122	177	171	154	0	277	429
N.S.	1	1.00	0.66	0.96	0.92	0.83	0.00	1.50	2.32
time (sec)	N/A	0.170	0.445	0.235	0.290	0.418	0.000	0.543	10.854

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	168	180	186	173	0	290	429
N.S.	1	1.00	0.97	1.04	1.08	1.00	0.00	1.68	2.48
time (sec)	N/A	0.186	1.583	0.211	0.495	0.424	0.000	0.527	9.378

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	664	236	239	212	0	306	438
N.S.	1	1.00	3.67	1.30	1.32	1.17	0.00	1.69	2.42
time (sec)	N/A	0.205	6.257	0.233	0.498	0.432	0.000	0.562	8.916

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	219	272	246	231	0	292	429
N.S.	1	1.00	1.24	1.55	1.40	1.31	0.00	1.66	2.44
time (sec)	N/A	0.155	1.011	0.250	0.500	0.406	0.000	0.567	8.994

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	235	312	268	258	0	313	419
N.S.	1	1.00	1.32	1.75	1.51	1.45	0.00	1.76	2.35
time (sec)	N/A	0.159	0.605	0.245	0.523	0.432	0.000	0.581	8.956

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	271	287	250	278	0	276	408
N.S.	1	1.00	1.55	1.64	1.43	1.59	0.00	1.58	2.33
time (sec)	N/A	0.204	1.264	0.277	0.582	0.417	0.000	0.596	8.924

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	289	323	275	316	0	307	396
N.S.	1	1.00	1.59	1.77	1.51	1.74	0.00	1.69	2.18
time (sec)	N/A	0.168	1.662	0.248	0.501	0.417	0.000	0.584	8.940

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	292	261	233	336	0	291	388
N.S.	1	1.00	1.70	1.52	1.35	1.95	0.00	1.69	2.26
time (sec)	N/A	0.194	1.002	0.251	0.512	0.402	0.000	0.594	9.044

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	279	296	265	362	0	302	389
N.S.	1	1.00	1.17	1.24	1.11	1.52	0.00	1.27	1.63
time (sec)	N/A	0.243	0.854	0.272	0.504	0.420	0.000	0.627	10.302

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	459	297	246	291	0	324	357
N.S.	1	1.00	2.30	1.48	1.23	1.46	0.00	1.62	1.78
time (sec)	N/A	0.250	0.097	0.289	0.284	0.401	0.000	0.645	9.404

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	365	334	286	327	0	356	395
N.S.	1	1.00	1.60	1.46	1.25	1.43	0.00	1.56	1.73
time (sec)	N/A	0.298	1.462	0.316	0.325	0.427	0.000	0.633	9.647

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	187	372	308	360	0	388	433
N.S.	1	1.00	0.76	1.51	1.25	1.46	0.00	1.58	1.76
time (sec)	N/A	0.309	2.821	0.358	0.303	0.417	0.000	0.630	9.976

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	197	408	348	384	0	420	471
N.S.	1	1.00	0.73	1.51	1.29	1.42	0.00	1.56	1.74
time (sec)	N/A	0.335	3.240	0.362	0.298	0.435	0.000	0.581	10.395

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	206	444	368	417	0	452	509
N.S.	1	1.00	0.72	1.55	1.29	1.46	0.00	1.58	1.78
time (sec)	N/A	0.338	4.359	0.405	0.296	0.420	0.000	0.576	10.900

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	229	320	294	245	0	324	474
N.S.	1	1.00	1.29	1.80	1.65	1.38	0.00	1.82	2.66
time (sec)	N/A	0.200	1.212	0.210	0.512	0.423	0.000	0.633	8.979

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	429	220	502	90	4318	218	211
N.S.	1	1.00	2.70	1.38	3.16	0.57	27.16	1.37	1.33
time (sec)	N/A	0.150	6.092	0.151	0.520	0.387	105.753	0.447	11.407

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	375	207	461	80	3580	205	199
N.S.	1	1.00	2.66	1.47	3.27	0.57	25.39	1.45	1.41
time (sec)	N/A	0.154	6.169	0.127	0.528	0.385	71.024	0.448	11.619

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	351	179	400	70	2773	179	172
N.S.	1	1.00	3.05	1.56	3.48	0.61	24.11	1.56	1.50
time (sec)	N/A	0.127	8.276	0.249	0.503	0.388	44.044	0.569	12.749

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	377	181	379	60	2307	179	173
N.S.	1	1.00	3.89	1.87	3.91	0.62	23.78	1.85	1.78
time (sec)	N/A	0.090	3.450	0.202	0.503	0.374	27.337	0.439	12.486

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	86	139	280	84	0	143	225
N.S.	1	1.00	0.85	1.38	2.77	0.83	0.00	1.42	2.23
time (sec)	N/A	0.086	0.272	0.233	0.513	0.408	0.000	0.446	10.434

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	122	121	277	104	0	147	229
N.S.	1	1.00	1.28	1.27	2.92	1.09	0.00	1.55	2.41
time (sec)	N/A	0.116	0.572	0.243	0.553	0.406	0.000	0.434	9.026

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	152	140	261	126	0	167	223
N.S.	1	1.00	1.43	1.32	2.46	1.19	0.00	1.58	2.10
time (sec)	N/A	0.114	0.361	0.236	0.521	0.406	0.000	0.471	8.976

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	138	125	240	148	0	157	212
N.S.	1	1.00	1.47	1.33	2.55	1.57	0.00	1.67	2.26
time (sec)	N/A	0.098	0.674	0.240	0.503	0.398	0.000	0.478	8.973

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	232	136	217	171	0	167	317
N.S.	1	1.00	2.27	1.33	2.13	1.68	0.00	1.64	3.11
time (sec)	N/A	0.098	0.480	0.242	0.500	0.383	0.000	0.493	9.419

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	189	148	234	155	0	187	183
N.S.	1	1.00	2.30	1.80	2.85	1.89	0.00	2.28	2.23
time (sec)	N/A	0.083	0.570	0.276	0.312	0.402	0.000	0.469	9.044

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	418	181	416	80	3046	179	173
N.S.	1	1.00	3.10	1.34	3.08	0.59	22.56	1.33	1.28
time (sec)	N/A	0.244	2.117	0.153	0.519	0.388	122.927	0.514	12.661

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	362	153	353	70	2271	153	146
N.S.	1	1.00	3.48	1.47	3.39	0.67	21.84	1.47	1.40
time (sec)	N/A	0.185	1.387	0.148	0.521	0.390	77.856	0.442	11.703

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	262	142	310	60	1720	140	81
N.S.	1	1.00	2.62	1.42	3.10	0.60	17.20	1.40	0.81
time (sec)	N/A	0.091	0.937	0.252	0.513	0.370	48.162	0.461	8.999

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	69	87	188	71	0	91	167
N.S.	1	1.00	0.95	1.19	2.58	0.97	0.00	1.25	2.29
time (sec)	N/A	0.140	0.236	0.284	0.499	0.389	0.000	0.445	9.469

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	116	112	202	88	0	131	175
N.S.	1	1.00	1.57	1.51	2.73	1.19	0.00	1.77	2.36
time (sec)	N/A	0.141	0.387	0.286	0.496	0.395	0.000	0.487	9.063

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	134	101	204	118	0	128	186
N.S.	1	1.00	1.84	1.38	2.79	1.62	0.00	1.75	2.55
time (sec)	N/A	0.152	0.696	0.291	0.517	0.398	0.000	0.483	9.104

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	124	110	176	139	0	137	261
N.S.	1	1.00	1.70	1.51	2.41	1.90	0.00	1.88	3.58
time (sec)	N/A	0.224	0.897	0.290	0.508	0.400	0.000	0.445	9.297

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	116	124	194	138	0	158	151
N.S.	1	1.00	1.41	1.51	2.37	1.68	0.00	1.93	1.84
time (sec)	N/A	0.201	0.938	0.299	0.295	0.394	0.000	0.538	9.027

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	122	195	167	0	157	149
N.S.	1	1.00	1.89	1.22	1.95	1.67	0.00	1.57	1.49
time (sec)	N/A	0.113	0.420	0.337	0.283	0.390	0.000	0.509	9.024

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	229	176	274	196	0	216	339
N.S.	1	1.00	1.85	1.42	2.21	1.58	0.00	1.74	2.73
time (sec)	N/A	0.223	0.492	0.355	0.293	0.393	0.000	0.516	10.110

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	366	168	373	78	2404	166	160
N.S.	1	1.00	2.84	1.30	2.89	0.60	18.64	1.29	1.24
time (sec)	N/A	0.170	1.501	0.181	0.516	0.401	180.597	0.513	11.556

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	310	129	290	68	1608	127	95
N.S.	1	1.00	2.95	1.23	2.76	0.65	15.31	1.21	0.90
time (sec)	N/A	0.145	1.656	0.168	0.561	0.381	121.483	0.486	9.034

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	105	255	129	267	58	1246	127	78
N.S.	1	1.25	3.04	1.54	3.18	0.69	14.83	1.51	0.93
time (sec)	N/A	0.111	1.028	0.174	0.541	0.364	78.333	0.476	9.004

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	63	87	161	59	0	89	150
N.S.	1	1.00	1.05	1.45	2.68	0.98	0.00	1.48	2.50
time (sec)	N/A	0.097	0.153	0.364	0.554	0.384	0.000	0.456	9.437

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	106	73	158	82	0	111	151
N.S.	1	1.00	2.16	1.49	3.22	1.67	0.00	2.27	3.08
time (sec)	N/A	0.112	0.362	0.370	0.507	0.417	0.000	0.470	9.479

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	126	84	138	109	0	108	161
N.S.	1	1.00	2.10	1.40	2.30	1.82	0.00	1.80	2.68
time (sec)	N/A	0.124	0.338	0.382	0.542	0.388	0.000	0.515	9.347

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	115	98	153	123	0	128	119
N.S.	1	1.00	1.60	1.36	2.12	1.71	0.00	1.78	1.65
time (sec)	N/A	0.133	0.922	0.366	0.297	0.381	0.000	0.498	9.246

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	125	124	195	149	0	156	151
N.S.	1	1.00	1.34	1.33	2.10	1.60	0.00	1.68	1.62
time (sec)	N/A	0.141	2.064	0.378	0.311	0.406	0.000	0.530	9.290

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	189	150	234	179	0	187	291
N.S.	1	1.00	1.66	1.32	2.05	1.57	0.00	1.64	2.55
time (sec)	N/A	0.135	1.298	0.389	0.339	0.384	0.000	0.499	9.847

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	188	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.463	0.440	0.000	0.000	0.000	0.000	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	164	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.168	0.209	0.471	0.000	0.000	0.000	0.000	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.229	0.289	0.000	0.000	0.000	0.000	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	117	166	94	128	184	163	93
N.S.	1	1.00	0.91	1.29	0.73	0.99	1.43	1.26	0.72
time (sec)	N/A	0.072	0.690	0.618	0.284	0.397	6.579	0.577	0.095

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	137	148	94	117	160	193	93
N.S.	1	1.00	1.21	1.31	0.83	1.04	1.42	1.71	0.82
time (sec)	N/A	0.102	0.504	0.466	0.273	0.389	4.850	0.543	8.818

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	127	130	94	106	160	178	93
N.S.	1	1.00	1.12	1.15	0.83	0.94	1.42	1.58	0.82
time (sec)	N/A	0.100	0.412	0.361	0.283	0.396	3.622	0.542	0.076

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	117	112	94	95	138	163	93
N.S.	1	1.00	1.21	1.15	0.97	0.98	1.42	1.68	0.96
time (sec)	N/A	0.098	0.429	0.276	0.284	0.392	2.629	0.558	0.076

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	94	94	84	138	133	93
N.S.	1	1.00	1.00	0.97	0.97	0.87	1.42	1.37	0.96
time (sec)	N/A	0.091	0.318	0.213	0.274	0.367	1.804	0.491	8.954

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	60	74	94	73	114	118	93
N.S.	1	1.00	0.74	0.91	1.16	0.90	1.41	1.46	1.15
time (sec)	N/A	0.065	0.268	0.195	0.303	0.376	1.359	0.472	9.024

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	85	91	96	0	92	160
N.S.	1	1.00	0.90	0.72	0.77	0.81	0.00	0.78	1.36
time (sec)	N/A	0.052	0.096	0.161	0.299	0.407	0.000	0.487	9.164

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	105	91	113	0	101	340
N.S.	1	1.00	0.89	0.92	0.80	0.99	0.00	0.89	2.98
time (sec)	N/A	0.063	0.099	0.142	0.313	0.390	0.000	0.485	9.325

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	125	90	124	0	104	311
N.S.	1	1.00	0.87	1.09	0.78	1.08	0.00	0.90	2.70
time (sec)	N/A	0.065	0.089	0.177	0.297	0.394	0.000	0.484	9.173

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	143	92	139	0	104	300
N.S.	1	1.00	0.87	1.21	0.78	1.18	0.00	0.88	2.54
time (sec)	N/A	0.064	0.151	0.161	0.278	0.408	0.000	0.483	9.071

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	105	161	92	142	0	103	290
N.S.	1	1.00	0.89	1.36	0.78	1.20	0.00	0.87	2.46
time (sec)	N/A	0.067	0.372	0.164	0.285	0.383	0.000	0.525	9.010

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	179	91	157	0	103	281
N.S.	1	1.00	0.89	1.56	0.79	1.37	0.00	0.90	2.44
time (sec)	N/A	0.059	0.145	0.175	0.288	0.391	0.000	0.479	9.096

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	111	143	91	158	0	104	267
N.S.	1	1.00	0.97	1.24	0.79	1.37	0.00	0.90	2.32
time (sec)	N/A	0.040	0.275	0.199	0.280	0.399	0.000	0.509	10.118

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	161	94	172	0	106	270
N.S.	1	1.00	0.97	1.35	0.79	1.45	0.00	0.89	2.27
time (sec)	N/A	0.060	0.284	0.218	0.277	0.392	0.000	0.478	9.316

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	138	92	131	0	92	91
N.S.	1	1.00	1.00	1.86	1.24	1.77	0.00	1.24	1.23
time (sec)	N/A	0.079	0.025	0.217	0.285	0.369	0.000	0.477	9.392

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	156	92	139	0	92	92
N.S.	1	1.00	1.00	1.93	1.14	1.72	0.00	1.14	1.14
time (sec)	N/A	0.088	0.049	0.210	0.283	0.373	0.000	0.540	9.229

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	176	92	144	0	92	92
N.S.	1	1.00	0.89	1.81	0.95	1.48	0.00	0.95	0.95
time (sec)	N/A	0.092	0.140	0.240	0.276	0.364	0.000	0.538	9.214

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	86	194	92	152	0	92	92
N.S.	1	1.00	0.89	2.00	0.95	1.57	0.00	0.95	0.95
time (sec)	N/A	0.089	0.114	0.245	0.283	0.379	0.000	0.549	9.277

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	212	92	153	0	92	92
N.S.	1	1.00	0.76	1.88	0.81	1.35	0.00	0.81	0.81
time (sec)	N/A	0.100	0.171	0.279	0.284	0.393	0.000	0.516	9.195

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	86	230	92	161	0	92	92
N.S.	1	1.00	0.76	2.04	0.81	1.42	0.00	0.81	0.81
time (sec)	N/A	0.099	0.159	0.281	0.283	0.397	0.000	0.502	9.355

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	86	248	92	166	0	92	92
N.S.	1	1.00	0.67	1.92	0.71	1.29	0.00	0.71	0.71
time (sec)	N/A	0.069	0.174	0.364	0.278	0.390	0.000	0.516	9.239

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	109	0	69	83
N.S.	1	1.00	0.62	0.63	0.63	1.00	0.00	0.63	0.76
time (sec)	N/A	0.093	0.437	0.184	0.285	0.400	0.000	0.468	0.080

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	99	2280	69	83
N.S.	1	1.00	0.62	0.63	0.63	0.91	20.92	0.63	0.76
time (sec)	N/A	0.089	0.646	0.161	0.322	0.388	222.254	0.475	8.986

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	69	89	2093	69	83
N.S.	1	1.00	0.62	0.63	0.63	0.82	19.20	0.63	0.76
time (sec)	N/A	0.088	0.299	0.166	0.308	0.374	157.084	0.442	8.940

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	69	79	1906	69	83
N.S.	1	1.00	0.75	0.76	0.76	0.87	20.95	0.76	0.91
time (sec)	N/A	0.114	0.398	0.131	0.287	0.372	100.928	0.458	0.059

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	69	69	1719	69	83
N.S.	1	1.00	0.75	0.76	0.76	0.76	18.89	0.76	0.91
time (sec)	N/A	0.114	0.211	0.135	0.286	0.374	74.921	0.482	8.962

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	69	59	1530	69	83
N.S.	1	1.00	0.93	0.95	0.95	0.81	20.96	0.95	1.14
time (sec)	N/A	0.080	0.190	0.227	0.288	0.393	45.574	0.428	9.127

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	65	67	49	1096	67	80
N.S.	1	1.00	0.97	0.96	0.99	0.72	16.12	0.99	1.18
time (sec)	N/A	0.044	0.135	0.228	0.289	0.379	24.652	0.452	9.137

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	68	64	71	70	0	88	140
N.S.	1	1.00	0.69	0.65	0.72	0.71	0.00	0.89	1.41
time (sec)	N/A	0.072	0.048	0.230	0.279	0.391	0.000	0.425	9.286

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	66	64	74	85	0	95	272
N.S.	1	1.00	0.69	0.67	0.78	0.89	0.00	1.00	2.86
time (sec)	N/A	0.082	0.091	0.249	0.284	0.373	0.000	0.468	9.364

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	64	74	91	0	94	231
N.S.	1	1.00	0.68	0.66	0.76	0.94	0.00	0.97	2.38
time (sec)	N/A	0.088	0.079	0.280	0.286	0.384	0.000	0.452	9.225

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	64	73	107	0	87	221
N.S.	1	1.00	0.68	0.66	0.75	1.10	0.00	0.90	2.28
time (sec)	N/A	0.086	0.120	0.280	0.293	0.381	0.000	0.443	9.168

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	62	72	104	0	83	214
N.S.	1	1.00	0.70	0.66	0.77	1.11	0.00	0.88	2.28
time (sec)	N/A	0.088	0.217	0.273	0.293	0.387	0.000	0.461	9.343

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	68	70	118	0	82	204
N.S.	1	1.00	0.68	0.68	0.70	1.18	0.00	0.82	2.04
time (sec)	N/A	0.071	0.080	0.267	0.305	0.379	0.000	0.515	9.240

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	66	96	0	66	63
N.S.	1	1.00	0.90	0.99	0.97	1.41	0.00	0.97	0.93
time (sec)	N/A	0.067	0.108	0.268	0.298	0.360	0.000	0.468	9.040

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	69	66	104	0	66	66
N.S.	1	1.00	0.93	0.95	0.90	1.42	0.00	0.90	0.90
time (sec)	N/A	0.104	0.115	0.287	0.339	0.362	0.000	0.496	8.974

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	66	107	0	66	66
N.S.	1	1.00	0.75	0.76	0.73	1.18	0.00	0.73	0.73
time (sec)	N/A	0.116	0.105	0.279	0.290	0.359	0.000	0.569	9.035

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	68	69	66	115	0	66	65
N.S.	1	1.00	0.75	0.76	0.73	1.26	0.00	0.73	0.71
time (sec)	N/A	0.117	0.119	0.342	0.290	0.364	0.000	0.479	9.041

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	120	0	66	66
N.S.	1	1.00	0.62	0.63	0.61	1.10	0.00	0.61	0.61
time (sec)	N/A	0.087	0.082	0.382	0.300	0.367	0.000	0.575	9.075

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	128	0	66	65
N.S.	1	1.00	0.62	0.63	0.61	1.17	0.00	0.61	0.60
time (sec)	N/A	0.088	0.086	0.400	0.279	0.373	0.000	0.486	9.041

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	66	131	0	66	65
N.S.	1	1.00	0.62	0.63	0.61	1.20	0.00	0.61	0.60
time (sec)	N/A	0.089	0.082	0.459	0.284	0.377	0.000	0.470	9.140

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	165	697	41196	1360	1130
N.S.	1	1.00	0.68	0.00	0.90	3.79	223.89	7.39	6.14
time (sec)	N/A	0.129	0.678	0.386	0.283	0.445	120.538	0.603	17.091

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	126	0	165	628	29818	1019	1142
N.S.	1	1.00	0.68	0.00	0.90	3.41	162.05	5.54	6.21
time (sec)	N/A	0.123	0.553	0.469	0.301	0.423	69.069	0.545	16.449

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	659	0	148	445	19968	674	901
N.S.	1	1.00	3.95	0.00	0.89	2.66	119.57	4.04	5.40
time (sec)	N/A	0.104	2.249	0.318	0.291	0.424	38.794	0.464	15.897

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	95	0	241	243	0	0	568
N.S.	1	1.00	0.69	0.00	1.76	1.77	0.00	0.00	4.15
time (sec)	N/A	0.117	0.222	0.404	0.321	0.389	0.000	0.000	14.291

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	117	0	126	169	0	0	280
N.S.	1	1.00	1.27	0.00	1.37	1.84	0.00	0.00	3.04
time (sec)	N/A	0.097	0.251	0.969	0.320	0.389	0.000	0.000	11.260

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	66	0	126	160	0	0	242
N.S.	1	1.00	0.72	0.00	1.37	1.74	0.00	0.00	2.63
time (sec)	N/A	0.096	0.150	0.701	0.304	0.395	0.000	0.000	10.615

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	104	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.186	1.662	0.000	0.000	0.000	0.000	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.159	0.853	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	518	311	705	110	0	309	303
N.S.	1	1.00	2.48	1.49	3.37	0.53	0.00	1.48	1.45
time (sec)	N/A	0.188	10.230	0.191	0.531	0.403	0.000	0.490	11.850

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	573	272	624	100	6409	270	263
N.S.	1	1.00	3.13	1.49	3.41	0.55	35.02	1.48	1.44
time (sec)	N/A	0.158	8.306	0.162	0.521	0.398	237.209	0.480	11.605

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	533	259	583	90	5501	257	251
N.S.	1	1.00	3.23	1.57	3.53	0.55	33.34	1.56	1.52
time (sec)	N/A	0.156	10.313	0.149	0.552	0.399	165.683	0.446	11.524

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	479	233	522	80	4490	231	224
N.S.	1	1.00	3.45	1.68	3.76	0.58	32.30	1.66	1.61
time (sec)	N/A	0.134	5.884	0.139	0.570	0.386	108.291	0.444	11.604

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	481	233	501	70	3888	231	225
N.S.	1	1.00	3.98	1.93	4.14	0.58	32.13	1.91	1.86
time (sec)	N/A	0.100	7.808	0.103	0.500	0.407	71.464	0.455	11.682

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	102	191	402	104	0	195	305
N.S.	1	1.00	0.71	1.34	2.81	0.73	0.00	1.36	2.13
time (sec)	N/A	0.097	0.196	0.287	0.507	0.388	0.000	0.433	11.830

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	177	379	124	0	199	296
N.S.	1	1.00	1.07	1.29	2.77	0.91	0.00	1.45	2.16
time (sec)	N/A	0.115	0.540	0.288	0.508	0.383	0.000	0.455	9.042

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	179	192	383	148	0	216	303
N.S.	1	1.00	1.19	1.28	2.55	0.99	0.00	1.44	2.02
time (sec)	N/A	0.127	0.379	0.299	0.507	0.385	0.000	0.510	9.019

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	197	177	362	168	0	228	290
N.S.	1	1.00	1.35	1.21	2.48	1.15	0.00	1.56	1.99
time (sec)	N/A	0.125	0.578	0.303	0.503	0.390	0.000	0.443	9.064

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	252	192	340	191	0	224	286
N.S.	1	1.00	1.68	1.28	2.27	1.27	0.00	1.49	1.91
time (sec)	N/A	0.125	0.536	0.296	0.534	0.409	0.000	0.470	9.034

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	264	179	319	211	0	217	279
N.S.	1	1.00	1.91	1.30	2.31	1.53	0.00	1.57	2.02
time (sec)	N/A	0.109	0.718	0.306	0.529	0.378	0.000	0.472	9.028

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	317	188	298	236	0	224	413
N.S.	1	1.00	2.23	1.32	2.10	1.66	0.00	1.58	2.91
time (sec)	N/A	0.122	0.719	0.340	0.499	0.399	0.000	0.488	10.387

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	284	200	315	198	0	244	387
N.S.	1	1.00	2.68	1.89	2.97	1.87	0.00	2.30	3.65
time (sec)	N/A	0.097	0.676	0.318	0.296	0.388	0.000	0.489	10.615

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	291	226	354	216	0	274	435
N.S.	1	1.00	2.17	1.69	2.64	1.61	0.00	2.04	3.25
time (sec)	N/A	0.154	1.072	0.303	0.279	0.397	0.000	0.491	11.388

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	313	228	355	249	0	273	435
N.S.	1	1.00	2.06	1.50	2.34	1.64	0.00	1.80	2.86
time (sec)	N/A	0.166	0.943	0.352	0.277	0.394	0.000	0.489	12.553

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	386	252	394	272	0	303	483
N.S.	1	1.00	2.19	1.43	2.24	1.55	0.00	1.72	2.74
time (sec)	N/A	0.179	1.073	0.399	0.286	0.400	0.000	0.523	13.995

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	187	302	475	302	0	360	579
N.S.	1	1.00	0.96	1.56	2.45	1.56	0.00	1.86	2.98
time (sec)	N/A	0.184	2.040	0.431	0.280	0.403	0.000	0.502	16.266

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	638	272	648	110	0	270	264
N.S.	1	1.00	3.14	1.34	3.19	0.54	0.00	1.33	1.30
time (sec)	N/A	0.284	7.663	0.191	0.513	0.403	0.000	0.579	11.509

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	585	257	605	100	0	257	250
N.S.	1	1.00	3.16	1.39	3.27	0.54	0.00	1.39	1.35
time (sec)	N/A	0.310	5.336	0.201	0.530	0.395	0.000	0.468	11.629

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	430	233	542	90	0	231	225
N.S.	1	1.00	2.70	1.47	3.41	0.57	0.00	1.45	1.42
time (sec)	N/A	0.262	3.739	0.165	0.516	0.382	0.000	0.541	11.681

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	481	203	479	80	3934	205	198
N.S.	1	1.00	3.41	1.44	3.40	0.57	27.90	1.45	1.40
time (sec)	N/A	0.255	2.606	0.167	0.547	0.390	182.102	0.463	11.703

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	418	194	436	70	3196	192	186
N.S.	1	1.00	3.37	1.56	3.52	0.56	25.77	1.55	1.50
time (sec)	N/A	0.095	3.791	0.164	0.504	0.383	126.643	0.461	12.652

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	93	152	333	94	0	156	262
N.S.	1	1.00	0.78	1.28	2.80	0.79	0.00	1.31	2.20
time (sec)	N/A	0.157	0.497	0.238	0.507	0.391	0.000	0.447	10.758

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	128	164	348	113	0	186	279
N.S.	1	1.00	1.10	1.41	3.00	0.97	0.00	1.60	2.41
time (sec)	N/A	0.220	1.072	0.234	0.502	0.386	0.000	0.479	9.212

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	158	140	330	140	0	168	270
N.S.	1	1.00	1.63	1.44	3.40	1.44	0.00	1.73	2.78
time (sec)	N/A	0.176	1.341	0.248	0.504	0.397	0.000	0.456	9.070

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	184	164	306	161	0	194	253
N.S.	1	1.00	1.90	1.69	3.15	1.66	0.00	2.00	2.61
time (sec)	N/A	0.182	1.700	0.270	0.503	0.397	0.000	0.491	9.156

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	219	127	263	187	0	159	232
N.S.	1	1.00	1.89	1.09	2.27	1.61	0.00	1.37	2.00
time (sec)	N/A	0.197	1.771	0.277	0.501	0.389	0.000	0.515	9.083

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	254	160	258	207	0	195	365
N.S.	1	1.00	2.15	1.36	2.19	1.75	0.00	1.65	3.09
time (sec)	N/A	0.204	0.813	0.280	0.485	0.398	0.000	0.493	9.851

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	145	176	275	183	0	215	339
N.S.	1	1.00	1.10	1.33	2.08	1.39	0.00	1.63	2.57
time (sec)	N/A	0.241	1.337	0.327	0.310	0.370	0.000	0.497	10.344

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	251	202	314	216	0	245	387
N.S.	1	1.00	2.02	1.63	2.53	1.74	0.00	1.98	3.12
time (sec)	N/A	0.179	0.780	0.319	0.281	0.403	0.000	0.528	10.841

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	291	228	355	239	0	273	435
N.S.	1	1.00	1.65	1.30	2.02	1.36	0.00	1.55	2.47
time (sec)	N/A	0.278	0.649	0.362	0.286	0.389	0.000	0.510	11.959

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	313	202	314	269	0	245	387
N.S.	1	1.00	1.86	1.20	1.87	1.60	0.00	1.46	2.30
time (sec)	N/A	0.259	1.248	0.432	0.305	0.404	0.000	0.527	12.558

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	353	278	435	294	0	331	531
N.S.	1	1.00	1.62	1.28	2.00	1.35	0.00	1.52	2.44
time (sec)	N/A	0.328	1.534	0.475	0.293	0.416	0.000	0.526	14.342

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	186	304	474	324	0	361	579
N.S.	1	1.00	0.89	1.45	2.26	1.54	0.00	1.72	2.76
time (sec)	N/A	0.284	3.796	0.501	0.283	0.406	0.000	0.489	16.548

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	482	220	499	90	0	218	212
N.S.	1	1.00	2.99	1.37	3.10	0.56	0.00	1.35	1.32
time (sec)	N/A	0.327	3.347	0.168	0.503	0.376	0.000	0.501	11.944

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	429	179	416	80	0	179	172
N.S.	1	1.00	3.23	1.35	3.13	0.60	0.00	1.35	1.29
time (sec)	N/A	0.273	8.093	0.167	0.575	0.368	0.000	0.488	12.639

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	366	181	393	70	2535	179	173
N.S.	1	1.00	2.79	1.38	3.00	0.53	19.35	1.37	1.32
time (sec)	N/A	0.120	1.127	0.151	0.516	0.392	216.159	0.482	12.493

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	125	269	84	0	129	222
N.S.	1	1.00	0.81	1.26	2.72	0.85	0.00	1.30	2.24
time (sec)	N/A	0.175	0.273	0.219	0.521	0.414	0.000	0.459	10.995

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	126	125	285	104	0	147	231
N.S.	1	1.00	1.37	1.36	3.10	1.13	0.00	1.60	2.51
time (sec)	N/A	0.156	0.688	0.238	0.513	0.403	0.000	0.476	9.124

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	144	140	267	130	0	172	228
N.S.	1	1.00	1.47	1.43	2.72	1.33	0.00	1.76	2.33
time (sec)	N/A	0.172	0.617	0.257	0.492	0.402	0.000	0.499	9.242

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	132	127	242	150	0	157	219
N.S.	1	1.00	1.43	1.38	2.63	1.63	0.00	1.71	2.38
time (sec)	N/A	0.185	1.768	0.271	0.491	0.379	0.000	0.494	9.257

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	165	136	218	164	0	166	315
N.S.	1	1.00	1.70	1.40	2.25	1.69	0.00	1.71	3.25
time (sec)	N/A	0.218	1.825	0.303	0.495	0.425	0.000	0.507	9.526

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	189	150	235	169	0	186	291
N.S.	1	1.00	1.89	1.50	2.35	1.69	0.00	1.86	2.91
time (sec)	N/A	0.233	1.231	0.307	0.284	0.422	0.000	0.539	9.602

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	242	176	274	196	0	216	339
N.S.	1	1.00	1.95	1.42	2.21	1.58	0.00	1.74	2.73
time (sec)	N/A	0.253	0.747	0.329	0.284	0.455	0.000	0.540	10.182

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	251	200	315	226	0	244	387
N.S.	1	1.00	1.79	1.43	2.25	1.61	0.00	1.74	2.76
time (sec)	N/A	0.170	0.697	0.337	0.287	0.385	0.000	0.522	10.858

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	317	228	354	249	0	274	435
N.S.	1	1.00	1.91	1.37	2.13	1.50	0.00	1.65	2.62
time (sec)	N/A	0.288	4.838	0.383	0.283	0.401	0.000	0.621	11.501

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	75	130	0	105	257
N.S.	1	1.00	1.00	1.27	0.91	1.59	0.00	1.28	3.13
time (sec)	N/A	0.095	0.238	0.167	0.577	0.374	0.000	0.430	14.820

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	62	104	0	90	160
N.S.	1	1.00	0.97	1.45	0.95	1.60	0.00	1.38	2.46
time (sec)	N/A	0.075	0.082	0.121	0.510	0.372	0.000	0.455	11.418

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	39	80	0	81	99
N.S.	1	1.00	1.21	1.51	1.00	2.05	0.00	2.08	2.54
time (sec)	N/A	0.078	0.031	0.089	0.506	0.396	0.000	0.484	9.213

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	32	60	0	29	24
N.S.	1	1.00	1.33	1.19	1.19	2.22	0.00	1.07	0.89
time (sec)	N/A	0.032	0.017	0.065	0.563	0.363	0.000	0.430	8.977

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	41	48	108	0	34	35
N.S.	1	1.00	1.56	1.14	1.33	3.00	0.00	0.94	0.97
time (sec)	N/A	0.057	0.029	0.143	0.302	0.387	0.000	0.441	8.951

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	68	61	59	165	0	87	77
N.S.	1	1.00	1.42	1.27	1.23	3.44	0.00	1.81	1.60
time (sec)	N/A	0.079	0.060	0.151	0.283	0.382	0.000	0.467	8.967

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	172	83	84	261	0	102	113
N.S.	1	1.00	2.29	1.11	1.12	3.48	0.00	1.36	1.51
time (sec)	N/A	0.096	1.085	0.161	0.273	0.380	0.000	0.440	8.898

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	205	102	98	308	0	130	144
N.S.	1	1.00	2.25	1.12	1.08	3.38	0.00	1.43	1.58
time (sec)	N/A	0.096	3.819	0.155	0.287	0.389	0.000	0.452	8.990

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	161	148	98	152	0	119	288
N.S.	1	1.00	1.81	1.66	1.10	1.71	0.00	1.34	3.24
time (sec)	N/A	0.134	0.369	0.141	0.506	0.351	0.000	0.527	14.710

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	84	125	0	102	183
N.S.	1	1.00	2.04	1.65	1.18	1.76	0.00	1.44	2.58
time (sec)	N/A	0.064	0.283	0.118	0.518	0.362	0.000	0.434	11.386

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	90	76	57	101	0	89	117
N.S.	1	1.00	2.09	1.77	1.33	2.35	0.00	2.07	2.72
time (sec)	N/A	0.037	0.261	0.126	0.497	0.383	0.000	0.436	9.180

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	69	58	65	126	0	38	39
N.S.	1	1.00	1.57	1.32	1.48	2.86	0.00	0.86	0.89
time (sec)	N/A	0.093	0.089	0.186	0.289	0.360	0.000	0.465	8.889

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	96	76	72	192	0	98	86
N.S.	1	1.00	1.66	1.31	1.24	3.31	0.00	1.69	1.48
time (sec)	N/A	0.154	0.299	0.192	0.289	0.355	0.000	0.462	8.927

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	124	117	124	300	0	116	124
N.S.	1	1.00	1.44	1.36	1.44	3.49	0.00	1.35	1.44
time (sec)	N/A	0.147	0.773	0.224	0.292	0.353	0.000	0.467	8.956

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	125	212	162	178	0	167	363
N.S.	1	1.00	1.13	1.91	1.46	1.60	0.00	1.50	3.27
time (sec)	N/A	0.121	0.573	0.174	0.509	0.367	0.000	0.490	14.701

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	117	154	0	119	288
N.S.	1	1.00	1.29	1.88	1.31	1.73	0.00	1.34	3.24
time (sec)	N/A	0.089	0.343	0.131	0.523	0.358	0.000	0.447	14.759

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	145	130	97	125	0	102	183
N.S.	1	1.00	2.16	1.94	1.45	1.87	0.00	1.52	2.73
time (sec)	N/A	0.045	0.350	0.116	0.500	0.349	0.000	0.476	11.393

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	74	77	84	151	0	49	112
N.S.	1	1.00	1.54	1.60	1.75	3.15	0.00	1.02	2.33
time (sec)	N/A	0.073	0.189	0.207	0.506	0.369	0.000	0.469	8.925

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	96	89	88	194	0	98	86
N.S.	1	1.00	1.71	1.59	1.57	3.46	0.00	1.75	1.54
time (sec)	N/A	0.095	0.453	0.205	0.287	0.381	0.000	0.468	8.968

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	124	128	135	300	0	116	125
N.S.	1	1.00	1.55	1.60	1.69	3.75	0.00	1.45	1.56
time (sec)	N/A	0.105	0.783	0.255	0.281	0.400	0.000	0.483	8.948

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	211	167	160	354	0	148	160
N.S.	1	1.00	2.15	1.70	1.63	3.61	0.00	1.51	1.63
time (sec)	N/A	0.134	6.168	0.246	0.282	0.369	0.000	0.501	8.972

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	148	99	236	80	0	125	129
N.S.	1	1.00	1.78	1.19	2.84	0.96	0.00	1.51	1.55
time (sec)	N/A	0.095	0.287	0.181	0.513	0.359	0.000	0.454	13.516

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	111	82	154	70	0	77	79
N.S.	1	1.00	1.59	1.17	2.20	1.00	0.00	1.10	1.13
time (sec)	N/A	0.079	0.266	0.171	0.570	0.353	0.000	0.457	11.178

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	90	47	0	68	47
N.S.	1	1.00	2.12	1.40	1.80	0.94	0.00	1.36	0.94
time (sec)	N/A	0.065	0.102	0.147	0.304	0.337	0.000	0.453	8.898

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	104	70	110	47	0	57	60
N.S.	1	1.00	2.81	1.89	2.97	1.27	0.00	1.54	1.62
time (sec)	N/A	0.065	0.104	0.136	0.279	0.332	0.000	0.445	8.906

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	149	79	136	115	0	83	92
N.S.	1	1.00	1.89	1.00	1.72	1.46	0.00	1.05	1.16
time (sec)	N/A	0.084	0.437	0.193	0.295	0.359	0.000	0.429	9.899

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	245	104	215	162	0	133	150
N.S.	1	1.00	2.63	1.12	2.31	1.74	0.00	1.43	1.61
time (sec)	N/A	0.113	0.444	0.231	0.294	0.350	0.000	0.468	9.114

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	191	168	421	132	0	160	172
N.S.	1	1.00	1.28	1.13	2.83	0.89	0.00	1.07	1.15
time (sec)	N/A	0.188	0.424	0.342	0.582	0.358	0.000	0.514	17.673

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	148	129	335	122	0	151	156
N.S.	1	1.00	1.23	1.08	2.79	1.02	0.00	1.26	1.30
time (sec)	N/A	0.184	0.400	0.269	0.515	0.363	0.000	0.470	15.384

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	143	112	249	112	0	103	105
N.S.	1	1.00	1.35	1.06	2.35	1.06	0.00	0.97	0.99
time (sec)	N/A	0.195	0.389	0.248	0.504	0.349	0.000	0.467	14.089

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	84	100	164	76	0	94	111
N.S.	1	1.00	1.27	1.52	2.48	1.15	0.00	1.42	1.68
time (sec)	N/A	0.178	0.193	0.221	0.288	0.338	0.000	0.480	9.275

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	86	100	184	77	0	94	132
N.S.	1	1.00	1.18	1.37	2.52	1.05	0.00	1.29	1.81
time (sec)	N/A	0.119	0.175	0.200	0.293	0.331	0.000	0.487	9.307

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	82	100	204	80	0	94	159
N.S.	1	1.00	1.15	1.41	2.87	1.13	0.00	1.32	2.24
time (sec)	N/A	0.072	0.171	0.204	0.296	0.342	0.000	0.451	9.386

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	196	109	250	168	0	109	117
N.S.	1	1.00	1.70	0.95	2.17	1.46	0.00	0.95	1.02
time (sec)	N/A	0.162	0.350	0.284	0.296	0.358	0.000	0.477	10.782

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	289	134	309	218	0	161	216
N.S.	1	1.00	2.22	1.03	2.38	1.68	0.00	1.24	1.66
time (sec)	N/A	0.209	0.552	0.318	0.279	0.373	0.000	0.483	10.779

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	328	162	354	260	0	187	191
N.S.	1	1.00	2.08	1.03	2.24	1.65	0.00	1.18	1.21
time (sec)	N/A	0.237	0.513	0.345	0.289	0.366	0.000	0.505	10.086

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	224	159	421	159	0	177	182
N.S.	1	1.00	1.48	1.05	2.79	1.05	0.00	1.17	1.21
time (sec)	N/A	0.249	0.470	0.370	0.510	0.357	0.000	0.531	15.500

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	214	142	335	150	0	129	131
N.S.	1	1.00	1.51	1.00	2.36	1.06	0.00	0.91	0.92
time (sec)	N/A	0.238	0.556	0.342	0.508	0.351	0.000	0.528	15.649

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	104	130	230	104	0	120	135
N.S.	1	1.00	1.02	1.27	2.25	1.02	0.00	1.18	1.32
time (sec)	N/A	0.227	0.345	0.293	0.288	0.346	0.000	0.558	9.692

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	104	130	250	102	0	120	158
N.S.	1	1.00	1.18	1.48	2.84	1.16	0.00	1.36	1.80
time (sec)	N/A	0.218	0.278	0.266	0.312	0.365	0.000	0.507	9.735

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	130	270	104	0	120	183
N.S.	1	1.00	1.01	1.26	2.62	1.01	0.00	1.17	1.78
time (sec)	N/A	0.171	0.272	0.246	0.305	0.352	0.000	0.492	10.152

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	104	130	290	106	0	120	206
N.S.	1	1.00	1.05	1.31	2.93	1.07	0.00	1.21	2.08
time (sec)	N/A	0.104	0.253	0.234	0.290	0.354	0.000	0.532	10.012

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	341	139	336	218	0	135	143
N.S.	1	1.00	2.26	0.92	2.23	1.44	0.00	0.89	0.95
time (sec)	N/A	0.202	0.301	0.339	0.297	0.366	0.000	0.468	11.331

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	351	164	395	265	0	187	274
N.S.	1	1.00	2.17	1.01	2.44	1.64	0.00	1.15	1.69
time (sec)	N/A	0.238	0.685	0.420	0.284	0.379	0.000	0.529	10.842

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	84	164	96	108	0	182	306
N.S.	1	1.00	0.72	1.40	0.82	0.92	0.00	1.56	2.62
time (sec)	N/A	0.099	0.294	0.191	0.518	0.366	0.000	0.505	14.962

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	154	87	98	0	134	243
N.S.	1	1.00	0.75	1.52	0.86	0.97	0.00	1.33	2.41
time (sec)	N/A	0.081	0.183	0.162	0.558	0.352	0.000	0.454	14.619

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	88	0	124	185
N.S.	1	1.00	1.12	1.36	0.90	1.22	0.00	1.72	2.57
time (sec)	N/A	0.054	0.037	0.147	0.578	0.364	0.000	0.475	12.718

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	69	88	55	75	0	74	117
N.S.	1	1.00	1.15	1.47	0.92	1.25	0.00	1.23	1.95
time (sec)	N/A	0.057	0.035	0.122	0.583	0.376	0.000	0.443	10.138

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	82	39	49	0	67	74
N.S.	1	1.00	1.00	1.82	0.87	1.09	0.00	1.49	1.64
time (sec)	N/A	0.070	0.030	0.126	0.282	0.345	0.000	0.459	9.030

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	36	26	50	0	53	50
N.S.	1	1.00	1.00	1.09	0.79	1.52	0.00	1.61	1.52
time (sec)	N/A	0.055	0.017	0.101	0.292	0.341	0.000	0.423	9.028

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	64	73	126	0	81	90
N.S.	1	1.00	1.25	0.94	1.07	1.85	0.00	1.19	1.32
time (sec)	N/A	0.060	0.095	0.204	0.287	0.368	0.000	0.446	9.941

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	109	90	83	170	0	129	145
N.S.	1	1.00	1.35	1.11	1.02	2.10	0.00	1.59	1.79
time (sec)	N/A	0.086	0.053	0.187	0.280	0.372	0.000	0.457	9.099

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	359	120	106	222	0	148	180
N.S.	1	1.00	3.26	1.09	0.96	2.02	0.00	1.35	1.64
time (sec)	N/A	0.100	6.085	0.235	0.281	0.375	0.000	0.477	8.984

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	120	159	186	120	196	0	135	287
N.S.	1	1.19	1.57	1.84	1.19	1.94	0.00	1.34	2.84
time (sec)	N/A	0.144	0.889	0.168	0.506	0.359	0.000	0.450	14.686

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	131	162	95	168	0	86	182
N.S.	1	1.00	1.52	1.88	1.10	1.95	0.00	1.00	2.12
time (sec)	N/A	0.174	0.750	0.173	0.493	0.338	0.000	0.476	12.162

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	79	114	71	141	0	67	102
N.S.	1	1.00	1.25	1.81	1.13	2.24	0.00	1.06	1.62
time (sec)	N/A	0.154	0.044	0.157	0.508	0.358	0.000	0.569	9.290

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	72	99	56	98	0	38	34
N.S.	1	1.00	1.20	1.65	0.93	1.63	0.00	0.63	0.57
time (sec)	N/A	0.062	0.213	0.142	0.277	0.352	0.000	0.425	9.143

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	142	81	90	231	0	73	98
N.S.	1	1.00	1.95	1.11	1.23	3.16	0.00	1.00	1.34
time (sec)	N/A	0.129	0.363	0.240	0.288	0.368	0.000	0.468	9.497

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	135	118	107	329	0	118	144
N.S.	1	1.00	1.55	1.36	1.23	3.78	0.00	1.36	1.66
time (sec)	N/A	0.178	0.653	0.252	0.306	0.366	0.000	0.463	9.454

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	190	164	160	428	0	150	182
N.S.	1	1.00	1.52	1.31	1.28	3.42	0.00	1.20	1.46
time (sec)	N/A	0.196	1.373	0.305	0.288	0.367	0.000	0.501	9.072

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	165	220	0	187	317
N.S.	1	1.00	1.49	2.24	1.39	1.85	0.00	1.57	2.66
time (sec)	N/A	0.141	1.475	0.179	0.506	0.359	0.000	0.502	14.924

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	159	246	145	196	0	135	287
N.S.	1	1.00	1.57	2.44	1.44	1.94	0.00	1.34	2.84
time (sec)	N/A	0.114	1.068	0.181	0.508	0.362	0.000	0.482	14.962

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	133	184	107	169	0	87	182
N.S.	1	1.00	1.73	2.39	1.39	2.19	0.00	1.13	2.36
time (sec)	N/A	0.143	0.897	0.171	0.552	0.356	0.000	0.500	12.299

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	107	126	84	143	0	67	102
N.S.	1	1.00	1.67	1.97	1.31	2.23	0.00	1.05	1.59
time (sec)	N/A	0.093	0.900	0.155	0.501	0.340	0.000	0.493	9.261

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	144	101	103	231	0	73	98
N.S.	1	1.00	2.00	1.40	1.43	3.21	0.00	1.01	1.36
time (sec)	N/A	0.093	0.466	0.267	0.288	0.362	0.000	0.535	9.226

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	135	131	123	334	0	118	144
N.S.	1	1.00	1.57	1.52	1.43	3.88	0.00	1.37	1.67
time (sec)	N/A	0.116	0.715	0.266	0.362	0.368	0.000	0.586	9.206

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	190	188	182	428	0	150	183
N.S.	1	1.00	1.73	1.71	1.65	3.89	0.00	1.36	1.66
time (sec)	N/A	0.135	1.386	0.270	0.351	0.360	0.000	0.636	9.143

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	287	232	205	528	0	194	239
N.S.	1	1.00	2.24	1.81	1.60	4.12	0.00	1.52	1.87
time (sec)	N/A	0.148	6.171	0.306	0.338	0.356	0.000	0.607	11.409

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	238	247	0	200	437
N.S.	1	1.00	1.76	2.52	1.66	1.73	0.00	1.40	3.06
time (sec)	N/A	0.152	1.094	0.250	0.603	0.356	0.000	0.700	16.825

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	158	268	158	197	0	135	287
N.S.	1	1.00	1.56	2.65	1.56	1.95	0.00	1.34	2.84
time (sec)	N/A	0.116	1.402	0.200	0.493	0.368	0.000	0.617	14.851

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	224	159	400	108	0	149	172
N.S.	1	1.00	1.91	1.36	3.42	0.92	0.00	1.27	1.47
time (sec)	N/A	0.112	0.514	0.289	0.503	0.370	0.000	0.736	18.669

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	191	127	318	98	0	130	131
N.S.	1	1.00	1.82	1.21	3.03	0.93	0.00	1.24	1.25
time (sec)	N/A	0.095	0.462	0.217	0.506	0.359	0.000	0.685	16.139

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	214	75	0	120	73
N.S.	1	1.00	1.54	1.88	3.10	1.09	0.00	1.74	1.06
time (sec)	N/A	0.068	0.219	0.204	0.289	0.346	0.000	0.780	9.550

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	106	115	234	75	0	120	86
N.S.	1	1.00	1.93	2.09	4.25	1.36	0.00	2.18	1.56
time (sec)	N/A	0.097	0.215	0.205	0.286	0.351	0.000	0.569	9.858

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	106	130	254	73	0	109	99
N.S.	1	1.00	1.45	1.78	3.48	1.00	0.00	1.49	1.36
time (sec)	N/A	0.108	0.255	0.197	0.280	0.346	0.000	0.588	10.159

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	106	130	274	75	0	120	112
N.S.	1	1.00	1.93	2.36	4.98	1.36	0.00	2.18	2.04
time (sec)	N/A	0.079	0.202	0.196	0.285	0.341	0.000	0.508	10.852

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	267	139	320	149	0	136	143
N.S.	1	1.00	2.32	1.21	2.78	1.30	0.00	1.18	1.24
time (sec)	N/A	0.094	0.479	0.260	0.298	0.380	0.000	0.499	11.529

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	341	164	379	194	0	178	257
N.S.	1	1.00	2.71	1.30	3.01	1.54	0.00	1.41	2.04
time (sec)	N/A	0.120	0.456	0.296	0.297	0.384	0.000	0.761	10.822

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	267	189	507	150	0	175	198
N.S.	1	1.00	1.72	1.22	3.27	0.97	0.00	1.13	1.28
time (sec)	N/A	0.203	0.585	0.444	0.539	0.363	0.000	0.615	17.730

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	257	172	421	140	0	155	156
N.S.	1	1.00	1.84	1.23	3.01	1.00	0.00	1.11	1.11
time (sec)	N/A	0.210	0.454	0.387	0.535	0.368	0.000	0.605	16.890

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	126	145	296	104	0	146	160
N.S.	1	1.00	1.48	1.71	3.48	1.22	0.00	1.72	1.88
time (sec)	N/A	0.199	0.202	0.351	0.308	0.355	0.000	0.770	12.368

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	316	103	0	146	184
N.S.	1	1.00	1.38	1.76	3.47	1.13	0.00	1.60	2.02
time (sec)	N/A	0.122	0.189	0.332	0.340	0.353	0.000	0.590	12.384

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	130	336	104	0	120	207
N.S.	1	1.00	1.38	1.43	3.69	1.14	0.00	1.32	2.27
time (sec)	N/A	0.202	0.244	0.316	0.368	0.370	0.000	0.694	14.065

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	126	160	356	103	0	146	231
N.S.	1	1.00	1.38	1.76	3.91	1.13	0.00	1.60	2.54
time (sec)	N/A	0.215	0.279	0.310	0.324	0.364	0.000	0.576	14.343

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	134	160	376	104	0	146	254
N.S.	1	1.00	1.44	1.72	4.04	1.12	0.00	1.57	2.73
time (sec)	N/A	0.086	0.221	0.300	0.305	0.355	0.000	0.582	14.350

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	352	169	422	200	0	161	169
N.S.	1	1.00	2.36	1.13	2.83	1.34	0.00	1.08	1.13
time (sec)	N/A	0.181	0.463	0.392	0.316	0.382	0.000	0.576	12.343

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	329	194	481	250	0	204	331
N.S.	1	1.00	2.01	1.18	2.93	1.52	0.00	1.24	2.02
time (sec)	N/A	0.228	5.604	0.435	0.364	0.370	0.000	0.617	11.238

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	277	222	526	292	0	238	243
N.S.	1	1.00	1.43	1.14	2.71	1.51	0.00	1.23	1.25
time (sec)	N/A	0.251	0.418	0.586	0.356	0.387	0.000	0.525	10.766

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	273	202	487	177	0	181	169
N.S.	1	1.00	1.53	1.13	2.74	0.99	0.00	1.02	0.95
time (sec)	N/A	0.254	0.409	0.293	0.559	0.385	0.000	0.669	17.808

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	185	190	362	130	0	172	184
N.S.	1	1.00	1.53	1.57	2.99	1.07	0.00	1.42	1.52
time (sec)	N/A	0.238	0.515	0.464	0.291	0.361	0.000	0.642	13.173

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	382	128	0	172	208
N.S.	1	1.00	1.76	1.81	3.64	1.22	0.00	1.64	1.98
time (sec)	N/A	0.237	0.260	0.387	0.316	0.362	0.000	0.630	13.192

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	185	175	402	130	0	159	232
N.S.	1	1.00	1.46	1.38	3.17	1.02	0.00	1.25	1.83
time (sec)	N/A	0.163	0.231	0.350	0.295	0.354	0.000	0.706	14.330

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	185	190	422	130	0	161	255
N.S.	1	1.00	1.76	1.81	4.02	1.24	0.00	1.53	2.43
time (sec)	N/A	0.224	0.250	0.366	0.294	0.355	0.000	0.635	16.124

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	185	190	442	130	0	172	279
N.S.	1	1.00	1.46	1.50	3.48	1.02	0.00	1.35	2.20
time (sec)	N/A	0.255	0.256	0.343	0.301	0.344	0.000	0.576	14.871

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	185	190	442	130	0	172	279
N.S.	1	1.00	1.50	1.54	3.59	1.06	0.00	1.40	2.27
time (sec)	N/A	0.117	0.233	0.339	0.310	0.346	0.000	0.602	15.001

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	204	199	508	250	0	187	195
N.S.	1	1.00	1.09	1.06	2.72	1.34	0.00	1.00	1.04
time (sec)	N/A	0.222	0.964	0.480	0.315	0.369	0.000	0.539	12.556

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	230	224	567	297	0	230	390
N.S.	1	1.00	1.15	1.12	2.84	1.48	0.00	1.15	1.95
time (sec)	N/A	0.260	0.468	0.577	0.308	0.377	0.000	0.621	11.316

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	166	190	488	153	0	172	279
N.S.	1	1.00	1.14	1.31	3.37	1.06	0.00	1.19	1.92
time (sec)	N/A	0.225	0.367	0.441	0.313	0.344	0.000	0.618	16.845

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	166	220	508	154	0	198	303
N.S.	1	1.00	1.14	1.52	3.50	1.06	0.00	1.37	2.09
time (sec)	N/A	0.276	0.359	0.400	0.329	0.354	0.000	0.627	15.893

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	184	166	218	528	153	0	198	327
N.S.	1	1.29	1.16	1.52	3.69	1.07	0.00	1.38	2.29
time (sec)	N/A	0.231	0.379	0.388	0.356	0.369	0.000	0.594	15.996

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	167	106	172	0	113	286
N.S.	1	1.00	1.00	1.26	0.80	1.29	0.00	0.85	2.15
time (sec)	N/A	0.080	0.359	0.207	0.281	0.400	0.000	0.568	9.817

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	121	95	159	0	101	235
N.S.	1	1.00	1.07	1.05	0.83	1.38	0.00	0.88	2.04
time (sec)	N/A	0.048	0.209	0.168	0.288	0.381	0.000	0.540	9.289

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	106	111	86	136	0	93	205
N.S.	1	1.00	1.01	1.06	0.82	1.30	0.00	0.89	1.95
time (sec)	N/A	0.066	0.185	0.160	0.281	0.382	0.000	0.554	9.259

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	98	86	136	0	90	167
N.S.	1	1.00	1.00	1.17	1.02	1.62	0.00	1.07	1.99
time (sec)	N/A	0.064	0.158	0.153	0.294	0.380	0.000	0.560	14.457

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	74	88	84	135	0	91	167
N.S.	1	1.00	0.88	1.05	1.00	1.61	0.00	1.08	1.99
time (sec)	N/A	0.065	0.024	0.145	0.271	0.367	0.000	0.533	14.456

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	74	80	84	135	0	91	167
N.S.	1	1.00	1.21	1.31	1.38	2.21	0.00	1.49	2.74
time (sec)	N/A	0.046	0.027	0.131	0.270	0.391	0.000	0.522	14.586

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	82	95	175	0	104	99
N.S.	1	1.00	0.85	0.70	0.81	1.50	0.00	0.89	0.85
time (sec)	N/A	0.076	0.171	0.214	0.288	0.390	0.000	0.535	0.095

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	76	101	114	229	0	121	118
N.S.	1	1.00	0.59	0.78	0.88	1.78	0.00	0.94	0.91
time (sec)	N/A	0.084	0.139	0.269	0.283	0.388	0.000	0.700	0.096

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	86	129	127	294	0	125	134
N.S.	1	1.00	0.60	0.90	0.89	2.06	0.00	0.87	0.94
time (sec)	N/A	0.092	0.523	0.275	0.289	0.399	0.000	0.582	9.312

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	90	147	138	343	0	149	145
N.S.	1	1.00	0.56	0.91	0.85	2.12	0.00	0.92	0.90
time (sec)	N/A	0.098	0.820	0.232	0.278	0.399	0.000	0.570	0.114

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	75	206	96	168	0	102	283
N.S.	1	1.00	0.63	1.73	0.81	1.41	0.00	0.86	2.38
time (sec)	N/A	0.059	0.187	0.240	0.279	0.396	0.000	0.513	9.227

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	67	201	83	154	0	88	225
N.S.	1	1.00	0.66	1.99	0.82	1.52	0.00	0.87	2.23
time (sec)	N/A	0.079	0.090	0.214	0.273	0.398	0.000	0.599	9.270

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	137	72	125	0	78	166
N.S.	1	1.00	1.05	1.57	0.83	1.44	0.00	0.90	1.91
time (sec)	N/A	0.081	0.319	0.189	0.276	0.387	0.000	0.597	9.296

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	39	167	72	125	0	77	123
N.S.	1	1.00	0.61	2.61	1.12	1.95	0.00	1.20	1.92
time (sec)	N/A	0.078	0.080	0.181	0.286	0.369	0.000	0.561	11.552

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	36	106	64	120	0	95	106
N.S.	1	1.00	0.56	1.66	1.00	1.88	0.00	1.48	1.66
time (sec)	N/A	0.059	0.061	0.174	0.274	0.378	0.000	0.475	11.126

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	100	84	166	0	91	91
N.S.	1	1.00	0.65	0.99	0.83	1.64	0.00	0.90	0.90
time (sec)	N/A	0.080	0.224	0.274	0.285	0.389	0.000	0.525	0.085

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	74	154	104	240	0	115	110
N.S.	1	1.00	0.64	1.33	0.90	2.07	0.00	0.99	0.95
time (sec)	N/A	0.094	0.198	0.289	0.287	0.395	0.000	0.695	9.050

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	84	166	119	302	0	125	126
N.S.	1	1.00	0.63	1.24	0.89	2.25	0.00	0.93	0.94
time (sec)	N/A	0.103	0.810	0.276	0.277	0.408	0.000	0.553	0.098

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	133	219	133	370	0	142	140
N.S.	1	1.00	0.89	1.46	0.89	2.47	0.00	0.95	0.93
time (sec)	N/A	0.109	6.058	0.294	0.278	0.386	0.000	0.517	9.055

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	73	302	96	141	0	242	321
N.S.	1	1.00	0.64	2.65	0.84	1.24	0.00	2.12	2.82
time (sec)	N/A	0.062	0.240	0.230	0.282	0.382	0.000	0.560	11.559

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	61	282	82	128	0	209	263
N.S.	1	1.00	0.64	2.94	0.85	1.33	0.00	2.18	2.74
time (sec)	N/A	0.079	0.289	0.214	0.274	0.377	0.000	0.681	10.929

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	223	70	110	0	178	205
N.S.	1	1.00	0.68	2.86	0.90	1.41	0.00	2.28	2.63
time (sec)	N/A	0.079	0.310	0.217	0.282	0.393	0.000	0.528	10.129

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	202	59	86	0	125	313
N.S.	1	1.00	0.70	3.16	0.92	1.34	0.00	1.95	4.89
time (sec)	N/A	0.074	0.186	0.188	0.286	0.382	0.000	0.628	9.707

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	181	42	44	0	32	30
N.S.	1	1.00	0.97	5.84	1.35	1.42	0.00	1.03	0.97
time (sec)	N/A	0.040	0.042	0.168	0.272	0.360	0.000	0.495	9.294

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	54	165	70	126	0	123	61
N.S.	1	1.00	0.70	2.14	0.91	1.64	0.00	1.60	0.79
time (sec)	N/A	0.069	0.322	0.268	0.290	0.370	0.000	0.538	9.117

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	63	168	90	185	0	166	80
N.S.	1	1.00	0.68	1.81	0.97	1.99	0.00	1.78	0.86
time (sec)	N/A	0.085	0.170	0.231	0.304	0.389	0.000	0.512	0.085

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	73	215	103	235	0	198	97
N.S.	1	1.00	0.66	1.94	0.93	2.12	0.00	1.78	0.87
time (sec)	N/A	0.091	0.540	0.268	0.281	0.378	0.000	0.572	0.093

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	153	143	209	207	0	179	567
N.S.	1	1.00	0.65	0.61	0.89	0.88	0.00	0.76	2.40
time (sec)	N/A	0.173	6.099	0.276	0.282	0.426	0.000	0.586	10.637

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	143	133	197	197	0	161	512
N.S.	1	1.00	0.65	0.60	0.90	0.90	0.00	0.73	2.33
time (sec)	N/A	0.162	6.117	0.280	0.275	0.429	0.000	0.578	10.337

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	133	121	186	187	0	147	485
N.S.	1	1.00	0.67	0.61	0.93	0.94	0.00	0.74	2.44
time (sec)	N/A	0.143	6.108	0.267	0.282	0.428	0.000	0.655	10.144

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	117	115	175	167	0	136	432
N.S.	1	1.00	0.62	0.61	0.93	0.89	0.00	0.72	2.30
time (sec)	N/A	0.128	2.604	0.421	0.308	0.412	0.000	0.641	9.360

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	101	115	175	167	0	136	388
N.S.	1	1.00	0.78	0.88	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.112	0.660	0.364	0.285	0.386	0.000	0.574	17.052

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	101	115	175	167	0	136	388
N.S.	1	1.00	0.75	0.86	1.31	1.25	0.00	1.01	2.90
time (sec)	N/A	0.150	0.663	0.341	0.295	0.390	0.000	0.520	17.117

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	92	103	173	167	0	136	388
N.S.	1	1.00	0.61	0.68	1.14	1.10	0.00	0.89	2.55
time (sec)	N/A	0.160	0.599	0.335	0.286	0.389	0.000	0.563	17.207

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	101	115	175	167	0	136	388
N.S.	1	1.00	0.67	0.77	1.17	1.11	0.00	0.91	2.59
time (sec)	N/A	0.155	0.470	0.331	0.290	0.391	0.000	0.544	17.148

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	92	103	175	167	0	136	388
N.S.	1	1.00	0.61	0.69	1.17	1.11	0.00	0.91	2.59
time (sec)	N/A	0.150	0.603	0.328	0.301	0.401	0.000	0.545	17.024

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	92	103	175	167	0	136	388
N.S.	1	1.00	0.62	0.70	1.18	1.13	0.00	0.92	2.62
time (sec)	N/A	0.141	0.369	0.345	0.277	0.372	0.000	0.600	17.116

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	92	103	175	167	0	136	388
N.S.	1	1.00	0.71	0.79	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.109	0.655	0.315	0.289	0.388	0.000	0.525	17.038

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	145	115	175	167	0	136	158
N.S.	1	1.00	0.88	0.70	1.06	1.01	0.00	0.82	0.96
time (sec)	N/A	0.092	0.363	0.346	0.293	0.391	0.000	0.480	0.235

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	122	187	202	0	149	191
N.S.	1	1.00	0.94	0.60	0.93	1.00	0.00	0.74	0.95
time (sec)	N/A	0.139	6.116	0.420	0.285	0.425	0.000	0.485	0.163

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	201	134	205	258	0	170	212
N.S.	1	1.00	0.93	0.62	0.94	1.19	0.00	0.78	0.98
time (sec)	N/A	0.159	6.102	0.312	0.293	0.423	0.000	0.507	9.329

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	213	142	217	311	0	182	223
N.S.	1	1.00	0.92	0.61	0.94	1.34	0.00	0.78	0.96
time (sec)	N/A	0.164	6.135	0.504	0.312	0.399	0.000	0.560	9.244

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	154	227	360	0	187	233
N.S.	1	1.00	0.91	0.61	0.90	1.42	0.00	0.74	0.92
time (sec)	N/A	0.186	6.107	0.758	0.296	0.427	0.000	0.605	9.264

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	139	248	91	115	0	138	215
N.S.	1	1.00	1.53	2.73	1.00	1.26	0.00	1.52	2.36
time (sec)	N/A	0.140	0.729	0.180	0.307	0.366	0.000	0.573	13.518

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	169	155	236	217	0	181	648
N.S.	1	1.00	0.64	0.59	0.89	0.82	0.00	0.69	2.45
time (sec)	N/A	0.194	6.122	0.353	0.305	0.434	0.000	0.611	11.204

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	159	147	225	207	0	167	595
N.S.	1	1.00	0.64	0.60	0.91	0.84	0.00	0.68	2.41
time (sec)	N/A	0.178	6.149	0.289	0.290	0.417	0.000	0.580	10.845

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	137	139	214	187	0	156	539
N.S.	1	1.00	0.59	0.60	0.92	0.80	0.00	0.67	2.31
time (sec)	N/A	0.157	4.656	0.266	0.281	0.415	0.000	0.609	9.400

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	122	139	214	187	0	156	497
N.S.	1	1.00	0.79	0.90	1.39	1.21	0.00	1.01	3.23
time (sec)	N/A	0.133	1.634	0.248	0.286	0.396	0.000	0.556	17.054

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	121	139	214	187	0	156	496
N.S.	1	1.00	0.76	0.87	1.34	1.17	0.00	0.98	3.10
time (sec)	N/A	0.176	2.123	0.266	0.292	0.423	0.000	0.539	16.770

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	124	127	214	187	0	156	496
N.S.	1	1.00	0.70	0.71	1.20	1.05	0.00	0.88	2.79
time (sec)	N/A	0.190	1.228	0.248	0.290	0.411	0.000	0.657	16.704

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	122	139	214	187	0	156	496
N.S.	1	1.00	0.69	0.79	1.22	1.06	0.00	0.89	2.82
time (sec)	N/A	0.178	1.915	0.259	0.301	0.403	0.000	0.566	16.645

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	116	127	214	187	0	156	496
N.S.	1	1.00	0.60	0.65	1.10	0.96	0.00	0.80	2.56
time (sec)	N/A	0.184	3.732	0.231	0.283	0.415	0.000	0.648	16.618

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	116	127	214	187	0	156	496
N.S.	1	1.00	0.60	0.66	1.11	0.97	0.00	0.81	2.58
time (sec)	N/A	0.173	4.238	0.233	0.280	0.410	0.000	0.518	16.705

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	104	115	214	187	0	156	496
N.S.	1	1.00	0.60	0.66	1.23	1.07	0.00	0.90	2.85
time (sec)	N/A	0.167	1.887	0.230	0.276	0.422	0.000	0.514	17.339

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	122	139	214	187	0	156	496
N.S.	1	1.00	0.71	0.81	1.24	1.09	0.00	0.91	2.88
time (sec)	N/A	0.152	1.801	0.228	0.317	0.401	0.000	0.604	16.881

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	127	214	187	0	156	496
N.S.	1	1.00	0.75	0.82	1.39	1.21	0.00	1.01	3.22
time (sec)	N/A	0.124	3.672	0.211	0.282	0.410	0.000	0.550	16.754

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	165	139	214	187	0	156	199
N.S.	1	1.00	0.79	0.66	1.02	0.89	0.00	0.74	0.95
time (sec)	N/A	0.117	0.867	0.224	0.294	0.397	0.000	0.471	9.409

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	228	146	226	222	0	169	231
N.S.	1	1.00	0.92	0.59	0.91	0.90	0.00	0.68	0.94
time (sec)	N/A	0.173	6.152	0.261	0.286	0.414	0.000	0.537	0.232

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	240	158	245	278	0	190	252
N.S.	1	1.00	0.92	0.60	0.94	1.06	0.00	0.73	0.96
time (sec)	N/A	0.192	6.174	0.401	0.297	0.419	0.000	0.639	9.456

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	254	166	257	331	0	202	263
N.S.	1	1.00	0.91	0.59	0.92	1.19	0.00	0.72	0.94
time (sec)	N/A	0.207	6.187	0.632	0.295	0.409	0.000	0.604	9.637

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	347	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	3.567	0.194	0.000	0.000	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.107	0.139	0.000	0.000	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	130	0	486	908	11900	1840	863
N.S.	1	1.00	0.74	0.00	2.78	5.19	68.00	10.51	4.93
time (sec)	N/A	0.140	0.447	0.453	0.310	0.465	39.001	0.534	18.138

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	105	0	313	550	5596	1001	617
N.S.	1	1.00	0.76	0.00	2.25	3.96	40.26	7.20	4.44
time (sec)	N/A	0.110	0.252	0.402	0.296	0.436	14.776	0.591	13.584

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	78	0	183	298	2159	463	302
N.S.	1	1.00	0.77	0.00	1.81	2.95	21.38	4.58	2.99
time (sec)	N/A	0.089	0.276	0.354	0.316	0.424	6.155	0.429	11.259

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	52	0	87	119	586	156	121
N.S.	1	1.00	0.85	0.00	1.43	1.95	9.61	2.56	1.98
time (sec)	N/A	0.055	0.370	0.201	0.285	0.370	1.642	0.439	10.082

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.057	0.105	0.000	0.000	0.000	0.000	0.000

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.051	0.506	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.096	0.052	0.530	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	143	0	476	750	9238	1845	1656
N.S.	1	1.00	0.84	0.00	2.80	4.41	54.34	10.85	9.74
time (sec)	N/A	0.124	0.528	0.566	0.298	0.426	23.837	0.497	16.867

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	307	408	4310	1003	703
N.S.	1	1.00	0.85	0.00	2.31	3.07	32.41	7.54	5.29
time (sec)	N/A	0.098	0.301	0.498	0.276	0.394	9.560	0.464	13.442

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	83	0	179	193	1622	462	305
N.S.	1	1.00	0.86	0.00	1.86	2.01	16.90	4.81	3.18
time (sec)	N/A	0.081	0.305	0.433	0.282	0.377	3.374	0.424	11.480

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	87	73	428	156	99
N.S.	1	1.00	0.86	0.00	1.47	1.24	7.25	2.64	1.68
time (sec)	N/A	0.047	0.096	0.211	0.286	0.363	1.279	0.440	9.764

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.076	0.118	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.077	0.596	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.061	0.610	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	61	0	0	0	0	0	-1
N.S.	1	1.13	1.13	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.056	0.097	0.000	0.000	0.000	0.000	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	150	0	159	197	2747	791	349
N.S.	1	1.00	1.12	0.00	1.19	1.47	20.50	5.90	2.60
time (sec)	N/A	0.083	1.036	0.346	0.319	0.389	17.526	0.426	12.194

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	94	0	119	140	1508	508	224
N.S.	1	1.00	0.87	0.00	1.10	1.30	13.96	4.70	2.07
time (sec)	N/A	0.070	0.423	0.612	0.275	0.422	7.408	0.504	10.741

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	0	84	93	697	287	138
N.S.	1	1.00	0.96	0.00	1.05	1.16	8.71	3.59	1.72
time (sec)	N/A	0.061	0.134	0.240	0.283	0.358	2.836	0.441	9.919

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	0	56	54	248	120	62
N.S.	1	1.00	0.80	0.00	1.04	1.00	4.59	2.22	1.15
time (sec)	N/A	0.036	0.021	0.030	0.267	0.356	1.156	0.447	9.419

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.042	0.073	0.000	0.000	0.000	0.000	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.040	0.060	0.000	0.000	0.000	0.000	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.046	0.069	0.000	0.000	0.000	0.000	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	84	64	96	80	76	199	87	276
N.S.	1	1.06	0.81	1.22	1.01	0.96	2.52	1.10	3.49
time (sec)	N/A	0.061	0.424	0.197	0.280	0.360	0.225	0.477	10.240

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	220	160	0	709	0	224	-1
N.S.	1	1.00	1.79	1.30	0.00	5.76	0.00	1.82	-0.01
time (sec)	N/A	0.337	2.042	8.421	0.000	0.447	0.000	0.511	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	208404	4461	0	2161	0	0	-1
N.S.	1	1.00	1478.04	31.64	0.00	15.33	0.00	0.00	-0.01
time (sec)	N/A	0.426	42.265	0.332	0.000	0.801	0.000	0.000	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.565	0.208	0.000	0.000	0.000	0.000	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.121	107.132	0.641	0.000	0.000	0.000	0.000	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	70.721	0.508	0.000	0.000	0.000	0.000	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	11.688	0.296	0.000	0.000	0.000	0.000	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	229	0	0	0	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.722	0.409	0.000	0.000	0.000	0.000	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	10.953	1.362	0.000	0.000	0.000	0.000	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	22.476	1.770	0.000	0.000	0.000	0.000	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	160	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.027	0.259	0.000	0.000	0.000	0.000	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.191	0.509	0.000	0.000	0.000	0.000	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.532	0.312	0.000	0.000	0.000	0.000	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	26.763	0.323	0.000	0.000	0.000	0.000	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	15.904	0.825	0.000	0.000	0.000	0.000	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	20.200	0.744	0.000	0.000	0.000	0.000	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	28.520	1.727	0.000	0.000	0.000	0.000	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	1.898	1.796	0.000	0.000	0.000	0.000	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	194	128	134	97	228	182	134
N.S.	1	1.00	1.45	0.96	1.00	0.72	1.70	1.36	1.00
time (sec)	N/A	0.101	0.629	0.476	0.344	0.375	1.407	0.511	0.122

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	130	108	104	81	178	145	102
N.S.	1	1.00	1.27	1.06	1.02	0.79	1.75	1.42	1.00
time (sec)	N/A	0.079	0.472	0.318	0.389	0.366	0.680	0.500	0.081

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	88	72	65	128	100	72
N.S.	1	1.00	1.00	1.13	0.92	0.83	1.64	1.28	0.92
time (sec)	N/A	0.060	0.590	0.221	0.372	0.355	0.295	0.448	9.007

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	44	42	48	75	52	40
N.S.	1	1.00	0.94	0.90	0.86	0.98	1.53	1.06	0.82
time (sec)	N/A	0.032	0.285	0.110	0.303	0.373	0.137	0.483	0.069

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	68	29	29	31	0	114	35
N.S.	1	1.00	2.00	0.85	0.85	0.91	0.00	3.35	1.03
time (sec)	N/A	0.038	0.035	0.158	0.306	0.364	0.000	0.444	0.069

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	260	110	55	90	0	84	43
N.S.	1	1.00	5.53	2.34	1.17	1.91	0.00	1.79	0.91
time (sec)	N/A	0.055	0.472	0.246	0.312	0.356	0.000	0.456	9.121

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	357	141	115	182	0	152	98
N.S.	1	1.00	3.57	1.41	1.15	1.82	0.00	1.52	0.98
time (sec)	N/A	0.079	1.126	0.280	0.390	0.363	0.000	0.495	0.137

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	451	169	171	222	0	201	155
N.S.	1	1.00	2.87	1.08	1.09	1.41	0.00	1.28	0.99
time (sec)	N/A	0.114	1.657	0.392	0.335	0.376	0.000	0.484	9.257

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	164	138	124	97	416	176	504
N.S.	1	1.00	1.19	1.00	0.90	0.70	3.01	1.28	3.65
time (sec)	N/A	0.093	0.787	0.368	0.301	0.370	1.101	0.523	10.725

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	120	118	98	81	306	133	391
N.S.	1	1.00	1.08	1.06	0.88	0.73	2.76	1.20	3.52
time (sec)	N/A	0.079	0.539	0.291	0.299	0.356	0.476	0.473	10.506

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	64	96	74	65	199	83	276
N.S.	1	1.00	0.76	1.14	0.88	0.77	2.37	0.99	3.29
time (sec)	N/A	0.065	0.647	0.189	0.292	0.360	0.219	0.418	10.553

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	85	54	56	73	0	36	33
N.S.	1	1.00	2.93	1.86	1.93	2.52	0.00	1.24	1.14
time (sec)	N/A	0.032	0.337	0.136	0.518	0.358	0.000	0.442	9.166

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	97	72	59	69	0	94	107
N.S.	1	1.00	1.94	1.44	1.18	1.38	0.00	1.88	2.14
time (sec)	N/A	0.045	0.506	0.220	0.289	0.342	0.000	0.481	9.247

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	223	102	86	112	0	225	224
N.S.	1	1.00	3.05	1.40	1.18	1.53	0.00	3.08	3.07
time (sec)	N/A	0.051	1.036	0.325	0.295	0.358	0.000	0.513	11.137

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	315	130	107	149	0	345	320
N.S.	1	1.00	3.28	1.35	1.11	1.55	0.00	3.59	3.33
time (sec)	N/A	0.054	1.645	0.332	0.295	0.350	0.000	0.522	12.639

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	407	158	126	185	0	465	416
N.S.	1	1.00	3.42	1.33	1.06	1.55	0.00	3.91	3.50
time (sec)	N/A	0.060	3.179	0.445	0.299	0.377	0.000	0.509	13.295

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	86	231	168	127	389	239	168
N.S.	1	1.00	0.64	1.72	1.25	0.95	2.90	1.78	1.25
time (sec)	N/A	0.122	0.807	0.681	0.382	0.386	1.917	0.596	9.199

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	201	142	109	309	202	140
N.S.	1	1.00	0.67	1.91	1.35	1.04	2.94	1.92	1.33
time (sec)	N/A	0.096	0.245	0.495	0.383	0.361	1.003	0.548	0.123

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	66	171	96	91	228	116	96
N.S.	1	1.00	0.85	2.19	1.23	1.17	2.92	1.49	1.23
time (sec)	N/A	0.082	0.294	0.316	0.291	0.363	0.468	0.504	9.075

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	75	68	72	117	88	66
N.S.	1	1.00	0.96	1.47	1.33	1.41	2.29	1.73	1.29
time (sec)	N/A	0.044	0.073	0.153	0.292	0.350	0.204	0.494	9.095

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	133	52	54	0	220	63
N.S.	1	1.00	0.85	2.22	0.87	0.90	0.00	3.67	1.05
time (sec)	N/A	0.058	0.065	0.192	0.304	0.357	0.000	0.423	0.075

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	189	37	55	0	112	44
N.S.	1	1.00	0.95	4.40	0.86	1.28	0.00	2.60	1.02
time (sec)	N/A	0.060	0.060	0.248	0.303	0.358	0.000	0.535	0.065

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	75	238	87	161	0	130	73
N.S.	1	1.00	0.97	3.09	1.13	2.09	0.00	1.69	0.95
time (sec)	N/A	0.086	0.104	0.303	0.306	0.381	0.000	0.481	0.109

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	90	304	148	271	0	209	136
N.S.	1	1.00	0.68	2.30	1.12	2.05	0.00	1.58	1.03
time (sec)	N/A	0.109	0.494	0.331	0.307	0.363	0.000	0.560	9.216

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	216	245	208	135	719	235	622
N.S.	1	1.00	1.10	1.25	1.06	0.69	3.67	1.20	3.17
time (sec)	N/A	0.140	3.688	0.582	0.389	0.380	1.424	0.550	10.806

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	171	215	171	115	539	192	494
N.S.	1	1.00	1.04	1.30	1.04	0.70	3.27	1.16	2.99
time (sec)	N/A	0.123	1.396	0.388	0.311	0.367	0.698	0.581	10.998

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	133	182	134	95	371	130	367
N.S.	1	1.00	0.99	1.36	1.00	0.71	2.77	0.97	2.74
time (sec)	N/A	0.114	0.860	0.256	0.311	0.373	0.332	0.525	10.465

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	91	123	104	128	0	125	110
N.S.	1	1.00	1.65	2.24	1.89	2.33	0.00	2.27	2.00
time (sec)	N/A	0.062	0.241	0.168	0.541	0.413	0.000	0.442	9.297

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	121	162	108	120	0	78	77
N.S.	1	1.00	1.66	2.22	1.48	1.64	0.00	1.07	1.05
time (sec)	N/A	0.080	0.019	0.238	0.303	0.371	0.000	0.471	9.148

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	178	231	147	113	0	192	175
N.S.	1	1.00	1.71	2.22	1.41	1.09	0.00	1.85	1.68
time (sec)	N/A	0.086	0.016	0.299	0.334	0.348	0.000	0.491	9.361

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	130	295	178	157	0	325	274
N.S.	1	1.00	1.01	2.29	1.38	1.22	0.00	2.52	2.12
time (sec)	N/A	0.093	0.306	0.307	0.307	0.359	0.000	0.480	12.231

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	359	207	197	0	461	370
N.S.	1	1.00	1.01	2.33	1.34	1.28	0.00	2.99	2.40
time (sec)	N/A	0.098	0.415	0.389	0.320	0.381	0.000	0.472	12.954

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	181	423	238	237	0	597	466
N.S.	1	1.00	1.01	2.36	1.33	1.32	0.00	3.34	2.60
time (sec)	N/A	0.108	0.804	0.566	0.321	0.398	0.000	0.477	13.960

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	86	345	182	155	530	283	177
N.S.	1	1.00	0.64	2.57	1.36	1.16	3.96	2.11	1.32
time (sec)	N/A	0.124	1.388	0.996	0.302	0.410	2.838	0.509	0.200

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	70	305	158	129	418	230	156
N.S.	1	1.00	0.67	2.90	1.50	1.23	3.98	2.19	1.49
time (sec)	N/A	0.120	0.384	0.677	0.303	0.427	1.446	0.554	0.144

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	53	265	126	115	313	172	126
N.S.	1	1.00	0.68	3.40	1.62	1.47	4.01	2.21	1.62
time (sec)	N/A	0.086	0.207	0.416	0.302	0.365	0.693	0.507	9.142

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	36	98	84	94	151	116	81
N.S.	1	1.00	0.71	1.92	1.65	1.84	2.96	2.27	1.59
time (sec)	N/A	0.041	0.076	0.202	0.320	0.371	0.284	0.489	9.073

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	198	73	77	0	289	100
N.S.	1	1.00	0.84	2.44	0.90	0.95	0.00	3.57	1.23
time (sec)	N/A	0.064	0.090	0.225	0.298	0.374	0.000	0.465	0.088

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	48	273	52	89	0	228	63
N.S.	1	1.00	0.77	4.40	0.84	1.44	0.00	3.68	1.02
time (sec)	N/A	0.070	0.116	0.283	0.287	0.373	0.000	0.479	0.084

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	337	47	49	0	82	36
N.S.	1	1.00	0.86	7.84	1.09	1.14	0.00	1.91	0.84
time (sec)	N/A	0.052	0.047	0.293	0.282	0.349	0.000	0.476	9.154

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	95	442	123	242	0	158	112
N.S.	1	1.00	0.90	4.21	1.17	2.30	0.00	1.50	1.07
time (sec)	N/A	0.092	0.240	0.305	0.295	0.376	0.000	0.501	9.142

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	151	542	185	353	0	237	172
N.S.	1	1.00	0.93	3.35	1.14	2.18	0.00	1.46	1.06
time (sec)	N/A	0.127	0.559	0.371	0.295	0.384	0.000	0.504	9.184

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	238	363	284	155	1042	273	711
N.S.	1	1.00	1.03	1.57	1.23	0.67	4.51	1.18	3.08
time (sec)	N/A	0.175	4.843	0.790	0.306	0.394	1.974	0.506	10.832

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	183	323	232	135	823	217	584
N.S.	1	1.00	0.92	1.62	1.16	0.68	4.12	1.08	2.92
time (sec)	N/A	0.159	1.600	0.528	0.308	0.384	1.078	0.616	10.690

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	146	279	199	111	588	165	451
N.S.	1	1.00	0.92	1.75	1.25	0.70	3.70	1.04	2.84
time (sec)	N/A	0.145	1.038	0.328	0.311	0.382	0.536	0.502	10.729

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	219	167	173	0	147	234
N.S.	1	1.00	0.90	2.41	1.84	1.90	0.00	1.62	2.57
time (sec)	N/A	0.070	0.189	0.197	0.539	0.369	0.000	0.476	11.512

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	121	248	164	167	0	93	140
N.S.	1	1.00	1.75	3.59	2.38	2.42	0.00	1.35	2.03
time (sec)	N/A	0.102	0.851	0.264	0.512	0.375	0.000	0.458	9.813

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	94	333	188	188	0	146	113
N.S.	1	1.00	0.88	3.11	1.76	1.76	0.00	1.36	1.06
time (sec)	N/A	0.105	0.136	0.233	0.297	0.371	0.000	0.475	11.305

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	135	435	228	146	0	260	213
N.S.	1	1.00	1.17	3.78	1.98	1.27	0.00	2.26	1.85
time (sec)	N/A	0.100	0.349	0.290	0.323	0.359	0.000	0.493	11.177

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	176	535	270	188	0	393	322
N.S.	1	1.00	1.26	3.82	1.93	1.34	0.00	2.81	2.30
time (sec)	N/A	0.104	0.477	0.364	0.315	0.377	0.000	0.515	12.213

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	69	107	104	84	3363	139	124
N.S.	1	1.00	0.66	1.02	0.99	0.80	32.03	1.32	1.18
time (sec)	N/A	0.106	0.160	0.339	0.290	0.376	42.499	0.459	0.108

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	75	72	66	1703	95	82
N.S.	1	1.00	0.91	0.95	0.91	0.84	21.56	1.20	1.04
time (sec)	N/A	0.081	0.110	0.254	0.287	0.379	14.990	0.452	9.201

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	44	43	44	49	588	51	47
N.S.	1	1.00	0.77	0.75	0.77	0.86	10.32	0.89	0.82
time (sec)	N/A	0.063	0.073	0.189	0.286	0.361	4.507	0.461	0.076

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	32	34	31	60	35	36
N.S.	1	1.00	0.86	0.89	0.94	0.86	1.67	0.97	1.00
time (sec)	N/A	0.045	0.026	0.106	0.298	0.405	0.269	0.420	9.233

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	62	58	73	0	79	43
N.S.	1	1.00	0.98	1.38	1.29	1.62	0.00	1.76	0.96
time (sec)	N/A	0.064	0.050	0.194	0.294	0.378	0.000	0.480	0.103

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	75	94	113	161	0	147	96
N.S.	1	1.00	0.82	1.03	1.24	1.77	0.00	1.62	1.05
time (sec)	N/A	0.097	0.212	0.260	0.296	0.382	0.000	0.492	0.128

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	105	132	165	194	0	192	151
N.S.	1	1.00	0.72	0.90	1.13	1.33	0.00	1.32	1.03
time (sec)	N/A	0.128	0.423	0.388	0.292	0.370	0.000	0.486	9.184

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	142	170	220	224	0	236	206
N.S.	1	1.00	0.69	0.83	1.07	1.09	0.00	1.15	1.00
time (sec)	N/A	0.173	0.690	0.556	0.328	0.387	0.000	0.511	9.347

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	82	83	82	2705	95	98
N.S.	1	1.00	0.66	1.04	1.05	1.04	34.24	1.20	1.24
time (sec)	N/A	0.082	0.122	0.273	0.315	0.366	71.655	0.465	0.075

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	58	61	64	1182	73	68
N.S.	1	1.00	0.67	1.14	1.20	1.25	23.18	1.43	1.33
time (sec)	N/A	0.070	0.046	0.333	0.312	0.364	27.185	0.519	9.122

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	55	54	48	1096	92	61
N.S.	1	1.00	0.77	0.83	0.82	0.73	16.61	1.39	0.92
time (sec)	N/A	0.070	0.070	0.292	0.296	0.365	8.948	0.524	0.073

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	37	43	45	121	76	41
N.S.	1	1.00	0.93	0.84	0.98	1.02	2.75	1.73	0.93
time (sec)	N/A	0.047	0.058	0.196	0.283	0.370	0.441	0.461	0.056

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	81	84	134	0	104	71
N.S.	1	1.00	0.97	1.14	1.18	1.89	0.00	1.46	1.00
time (sec)	N/A	0.075	0.092	0.296	0.288	0.376	0.000	0.437	9.248

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	87	113	139	230	0	169	121
N.S.	1	1.00	0.71	0.92	1.13	1.87	0.00	1.37	0.98
time (sec)	N/A	0.108	0.526	0.410	0.281	0.362	0.000	0.527	0.151

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	123	151	207	260	0	214	193
N.S.	1	1.00	0.69	0.84	1.16	1.45	0.00	1.20	1.08
time (sec)	N/A	0.143	0.496	0.534	0.294	0.390	0.000	0.527	9.499

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	160	189	252	290	0	258	240
N.S.	1	1.00	0.68	0.80	1.07	1.23	0.00	1.09	1.02
time (sec)	N/A	0.193	1.082	0.522	0.305	0.395	0.000	0.545	9.597

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.331	0.625	0.000	0.000	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	132	0	1250	342	0	1402	783
N.S.	1	1.00	0.83	0.00	7.86	2.15	0.00	8.82	4.92
time (sec)	N/A	0.113	0.554	0.415	0.339	0.442	0.000	0.489	17.899

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	103	0	669	228	22522	861	517
N.S.	1	1.00	0.84	0.00	5.44	1.85	183.11	7.00	4.20
time (sec)	N/A	0.094	0.294	0.362	0.320	0.407	150.019	0.462	15.820

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	0	298	140	5243	458	272
N.S.	1	1.00	1.00	0.00	3.20	1.51	56.38	4.92	2.92
time (sec)	N/A	0.083	0.223	0.329	0.307	0.401	17.086	0.456	11.741

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	51	0	87	73	428	156	99
N.S.	1	1.00	0.86	0.00	1.47	1.24	7.25	2.64	1.68
time (sec)	N/A	0.048	0.102	0.234	0.285	0.387	1.429	0.415	10.166

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.099	0.157	0.000	0.000	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	82	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.129	0.149	0.000	0.000	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	76	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.133	0.164	0.000	0.000	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.672	0.375	0.000	0.000	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.403	0.362	0.000	0.000	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	111	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.243	0.268	0.000	0.000	0.000	0.000	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	3925	0	0	0	0	0	-1
N.S.	1	1.00	31.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	6.417	0.148	0.000	0.000	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.464	0.154	0.000	0.000	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	3.136	0.173	0.000	0.000	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	160	0	0	204	0	0	441
N.S.	1	1.00	0.67	0.00	0.00	0.85	0.00	0.00	1.85
time (sec)	N/A	0.288	0.388	1.194	0.000	0.401	0.000	0.000	18.419

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	119	0	0	137	0	0	234
N.S.	1	1.00	0.71	0.00	0.00	0.82	0.00	0.00	1.39
time (sec)	N/A	0.205	0.208	1.101	0.000	0.424	0.000	0.000	11.940

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	89	0	0	129
N.S.	1	1.00	0.81	0.00	0.00	0.87	0.00	0.00	1.26
time (sec)	N/A	0.140	0.129	1.064	0.000	0.406	0.000	0.000	1.279

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	300	0	0	0	0	0	-1
N.S.	1	1.00	1.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	3.452	0.338	0.000	0.000	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	144	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.398	0.360	0.000	0.000	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	150	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.455	0.429	0.000	0.000	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	155	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.613	0.497	0.000	0.000	0.000	0.000	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	0	0	36	0	1878	33
N.S.	1	1.00	1.03	0.00	0.00	1.12	0.00	58.69	1.03
time (sec)	N/A	0.070	0.129	0.382	0.000	0.383	0.000	5.602	9.705

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	0	0	38	0	1863	35
N.S.	1	1.00	1.03	0.00	0.00	1.12	0.00	54.79	1.03
time (sec)	N/A	0.073	0.047	0.348	0.000	0.423	0.000	5.737	0.516

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	798	0	0	0	0	0	-1
N.S.	1	1.00	4.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	7.909	0.168	0.000	0.000	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	20.245	0.703	0.000	0.000	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	151	0	0	0	0	0	0	-1
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	5.024	0.346	0.000	0.000	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	155	0	0	0	0	0	0	-1
N.S.	1	1.04	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	8.054	0.255	0.000	0.000	0.000	0.000	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	11.695	0.722	0.000	0.000	0.000	0.000	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	16.232	0.707	0.000	0.000	0.000	0.000	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	153	0	0	0	0	0	0	-1
N.S.	1	1.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	21.266	1.199	0.000	0.000	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	852	0	0	0	0	0	-1
N.S.	1	1.00	4.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.273	8.821	0.230	0.000	0.000	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	77	95	65	73	192	92	153
N.S.	1	1.00	0.73	0.90	0.62	0.70	1.83	0.88	1.46
time (sec)	N/A	0.107	0.142	0.186	0.271	0.359	0.449	0.433	12.983

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	59	77	52	62	144	62	125
N.S.	1	1.00	0.73	0.95	0.64	0.77	1.78	0.77	1.54
time (sec)	N/A	0.094	0.079	0.135	0.284	0.380	0.308	0.466	12.892

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	57	39	51	119	47	125
N.S.	1	1.00	0.94	0.88	0.60	0.78	1.83	0.72	1.92
time (sec)	N/A	0.079	0.083	0.099	0.282	0.359	0.180	0.454	12.661

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	74	55	57	60	0	87	157
N.S.	1	1.00	1.45	1.08	1.12	1.18	0.00	1.71	3.08
time (sec)	N/A	0.048	0.050	0.119	0.279	0.394	0.000	0.442	9.874

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	158
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	3.85
time (sec)	N/A	0.040	0.033	0.107	0.493	0.368	0.000	0.451	9.545

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	109	71	66	114	0	95	151
N.S.	1	1.00	2.10	1.37	1.27	2.19	0.00	1.83	2.90
time (sec)	N/A	0.056	0.039	0.132	0.502	0.387	0.000	0.450	9.360

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	95	72	61	119	0	115	111
N.S.	1	1.00	1.83	1.38	1.17	2.29	0.00	2.21	2.13
time (sec)	N/A	0.076	0.033	0.138	0.286	0.395	0.000	0.538	9.309

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	90	80	137	0	116	112
N.S.	1	1.00	1.82	1.22	1.08	1.85	0.00	1.57	1.51
time (sec)	N/A	0.096	0.032	0.158	0.290	0.374	0.000	0.479	9.327

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	177	110	92	169	0	144	143
N.S.	1	1.00	1.97	1.22	1.02	1.88	0.00	1.60	1.59
time (sec)	N/A	0.095	0.058	0.171	0.286	0.376	0.000	0.466	9.333

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	132	150	104	104	275	141	233
N.S.	1	1.00	0.69	0.79	0.55	0.55	1.45	0.74	1.23
time (sec)	N/A	0.262	0.411	0.262	0.281	0.365	0.665	0.551	12.974

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	120	141	92	103	309	115	112
N.S.	1	1.00	0.74	0.87	0.56	0.63	1.90	0.71	0.69
time (sec)	N/A	0.276	0.169	0.221	0.282	0.400	0.492	0.529	9.615

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	94	68	73	172	82	180
N.S.	1	1.00	0.73	0.89	0.64	0.69	1.62	0.77	1.70
time (sec)	N/A	0.115	0.273	0.190	0.291	0.355	0.278	0.471	12.772

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	91	72	74	84	0	133	225
N.S.	1	1.00	1.01	0.80	0.82	0.93	0.00	1.48	2.50
time (sec)	N/A	0.171	0.161	0.207	0.294	0.400	0.000	0.459	9.868

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	116	79	79	118	0	148	277
N.S.	1	1.00	1.49	1.01	1.01	1.51	0.00	1.90	3.55
time (sec)	N/A	0.070	0.293	0.177	0.498	0.386	0.000	0.442	9.707

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	155	102	103	168	0	148	397
N.S.	1	1.00	1.74	1.15	1.16	1.89	0.00	1.66	4.46
time (sec)	N/A	0.195	0.623	0.217	0.485	0.378	0.000	0.470	10.198

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	538	96	82	167	0	167	231
N.S.	1	1.00	5.60	1.00	0.85	1.74	0.00	1.74	2.41
time (sec)	N/A	0.256	6.142	0.213	0.504	0.379	0.000	0.454	9.480

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	579	142	129	200	0	182	165
N.S.	1	1.00	4.71	1.15	1.05	1.63	0.00	1.48	1.34
time (sec)	N/A	0.231	6.136	0.259	0.280	0.379	0.000	0.485	9.328

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	236	135	108	195	0	222	187
N.S.	1	1.00	1.59	0.91	0.73	1.32	0.00	1.50	1.26
time (sec)	N/A	0.258	0.587	0.246	0.288	0.389	0.000	0.467	9.357

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	296	199	186	283	0	276	245
N.S.	1	1.00	1.74	1.17	1.09	1.66	0.00	1.62	1.44
time (sec)	N/A	0.259	0.586	0.300	0.286	0.368	0.000	0.477	9.444

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	157	196	131	141	394	166	455
N.S.	1	1.00	0.68	0.84	0.56	0.61	1.70	0.72	1.96
time (sec)	N/A	0.373	0.607	0.313	0.284	0.371	0.741	0.590	10.658

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	138	158	108	116	340	139	425
N.S.	1	1.00	0.85	0.97	0.66	0.71	2.09	0.85	2.61
time (sec)	N/A	0.201	0.579	0.241	0.274	0.368	0.511	0.556	10.746

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	117	101	116	0	293	567
N.S.	1	1.00	0.95	0.86	0.74	0.85	0.00	2.15	4.17
time (sec)	N/A	0.268	0.216	0.233	0.276	0.435	0.000	0.526	11.122

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	143	96	95	143	0	199	289
N.S.	1	1.00	1.40	0.94	0.93	1.40	0.00	1.95	2.83
time (sec)	N/A	0.100	0.932	0.207	0.500	0.396	0.000	0.497	9.474

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	192	132	128	216	0	272	585
N.S.	1	1.00	1.39	0.96	0.93	1.57	0.00	1.97	4.24
time (sec)	N/A	0.297	0.908	0.258	0.510	0.375	0.000	0.572	9.463

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	615	127	119	231	0	222	477
N.S.	1	1.00	4.46	0.92	0.86	1.67	0.00	1.61	3.46
time (sec)	N/A	0.309	6.164	0.250	0.507	0.382	0.000	0.553	10.709

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	690	166	149	265	0	234	348
N.S.	1	1.00	4.54	1.09	0.98	1.74	0.00	1.54	2.29
time (sec)	N/A	0.319	6.175	0.273	0.504	0.393	0.000	0.569	9.865

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	344	187	157	275	0	290	241
N.S.	1	1.00	1.88	1.02	0.86	1.50	0.00	1.58	1.32
time (sec)	N/A	0.375	0.941	0.301	0.299	0.372	0.000	0.524	9.439

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	369	224	202	310	0	354	292
N.S.	1	1.00	1.74	1.06	0.95	1.46	0.00	1.67	1.38
time (sec)	N/A	0.393	1.488	0.307	0.316	0.401	0.000	0.525	9.689

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	246	249	0	643	0	261	1688
N.S.	1	1.00	1.31	1.32	0.00	3.42	0.00	1.39	8.98
time (sec)	N/A	0.481	1.930	0.472	0.000	0.418	0.000	0.497	11.853

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	129	210	0	568	0	211	479
N.S.	1	1.00	0.84	1.37	0.00	3.71	0.00	1.38	3.13
time (sec)	N/A	0.324	0.316	0.403	0.000	0.414	0.000	0.444	10.400

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	130	151	0	479	0	191	269
N.S.	1	1.00	1.23	1.42	0.00	4.52	0.00	1.80	2.54
time (sec)	N/A	0.101	0.762	0.359	0.000	0.406	0.000	0.476	10.024

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	97	116	0	483	0	130	523
N.S.	1	1.00	1.05	1.26	0.00	5.25	0.00	1.41	5.68
time (sec)	N/A	0.158	0.163	0.468	0.000	0.428	0.000	0.412	10.162

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	139	156	0	768	0	218	1616
N.S.	1	1.00	1.21	1.36	0.00	6.68	0.00	1.90	14.05
time (sec)	N/A	0.285	0.537	0.487	0.000	0.455	0.000	0.491	11.414

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	196	206	0	1130	0	257	966
N.S.	1	1.00	1.25	1.31	0.00	7.20	0.00	1.64	6.15
time (sec)	N/A	0.490	2.082	0.545	0.000	0.544	0.000	0.473	9.895

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	385	264	0	1471	0	329	1089
N.S.	1	1.00	1.99	1.37	0.00	7.62	0.00	1.70	5.64
time (sec)	N/A	0.671	6.303	0.590	0.000	0.536	0.000	0.509	9.989

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	288	349	0	1058	0	535	2500
N.S.	1	1.00	1.08	1.31	0.00	3.98	0.00	2.01	9.40
time (sec)	N/A	0.563	4.188	0.718	0.000	0.449	0.000	0.526	17.722

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	159	289	0	919	0	302	3031
N.S.	1	1.00	0.88	1.61	0.00	5.11	0.00	1.68	16.84
time (sec)	N/A	0.367	0.864	0.640	0.000	0.420	0.000	0.472	15.788

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	289	252	0	793	0	256	2709
N.S.	1	1.00	1.73	1.51	0.00	4.75	0.00	1.53	16.22
time (sec)	N/A	0.183	1.436	0.533	0.000	0.419	0.000	0.487	13.930

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	253	0	996	0	277	1610
N.S.	1	1.00	1.00	1.64	0.00	6.47	0.00	1.80	10.45
time (sec)	N/A	0.311	0.822	0.652	0.000	0.579	0.000	0.483	12.439

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	195	299	0	1394	0	339	1762
N.S.	1	1.00	0.97	1.48	0.00	6.90	0.00	1.68	8.72
time (sec)	N/A	0.514	3.849	0.710	0.000	0.643	0.000	0.539	10.631

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	330	345	0	1922	0	526	1906
N.S.	1	1.00	1.23	1.28	0.00	7.14	0.00	1.96	7.09
time (sec)	N/A	0.740	6.230	0.765	0.000	0.858	0.000	0.512	11.006

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	3348	4675	0	0	0	0	-1
N.S.	1	1.00	9.65	13.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	29.589	15.618	0.000	0.000	0.000	0.000	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	124	71	95	272	107	223
N.S.	1	1.00	0.64	0.87	0.50	0.66	1.90	0.75	1.56
time (sec)	N/A	0.124	0.175	0.339	0.279	0.377	1.297	0.534	13.317

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	106	61	84	248	92	209
N.S.	1	1.00	0.61	0.83	0.48	0.66	1.95	0.72	1.65
time (sec)	N/A	0.118	0.141	0.261	0.282	0.372	0.914	0.509	13.166

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	88	88	65	73	192	107	181
N.S.	1	1.00	0.85	0.85	0.63	0.71	1.86	1.04	1.76
time (sec)	N/A	0.098	0.139	0.194	0.281	0.361	0.622	0.503	13.025

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	77	68	52	62	167	92	181
N.S.	1	1.00	0.89	0.78	0.60	0.71	1.92	1.06	2.08
time (sec)	N/A	0.076	0.121	0.149	0.293	0.357	0.410	0.535	12.860

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	109	76	81	88	0	145	242
N.S.	1	1.00	1.22	0.85	0.91	0.99	0.00	1.63	2.72
time (sec)	N/A	0.065	0.093	0.142	0.291	0.403	0.000	0.459	10.824

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	105	94	91	107	0	142	241
N.S.	1	1.00	1.27	1.13	1.10	1.29	0.00	1.71	2.90
time (sec)	N/A	0.085	0.249	0.123	0.488	0.358	0.000	0.442	9.388

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	132	116	101	139	0	163	236
N.S.	1	1.00	1.40	1.23	1.07	1.48	0.00	1.73	2.51
time (sec)	N/A	0.085	0.996	0.162	0.496	0.366	0.000	0.463	9.373

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	225
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.74
time (sec)	N/A	0.060	0.038	0.143	0.493	0.371	0.000	0.544	9.446

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	153	104	107	180	0	153	221
N.S.	1	1.00	1.74	1.18	1.22	2.05	0.00	1.74	2.51
time (sec)	N/A	0.075	0.038	0.161	0.502	0.407	0.000	0.482	9.674

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	135	100	86	160	0	173	174
N.S.	1	1.00	1.82	1.35	1.16	2.16	0.00	2.34	2.35
time (sec)	N/A	0.089	0.029	0.184	0.272	0.370	0.000	0.523	9.559

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	175	118	106	187	0	201	205
N.S.	1	1.00	1.79	1.20	1.08	1.91	0.00	2.05	2.09
time (sec)	N/A	0.115	0.036	0.209	0.282	0.386	0.000	0.486	9.522

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	239	138	118	221	0	229	399
N.S.	1	1.00	2.10	1.21	1.04	1.94	0.00	2.01	3.50
time (sec)	N/A	0.117	0.069	0.208	0.271	0.363	0.000	0.530	10.995

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	279	156	138	239	0	201	205
N.S.	1	1.00	2.05	1.15	1.01	1.76	0.00	1.48	1.51
time (sec)	N/A	0.135	0.064	0.233	0.278	0.362	0.000	0.512	9.877

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	144	161	100	116	335	142	309
N.S.	1	1.00	0.48	0.53	0.33	0.39	1.11	0.47	1.03
time (sec)	N/A	0.414	0.669	0.428	0.286	0.406	1.325	0.667	12.924

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	141	163	101	128	420	150	139
N.S.	1	1.00	0.51	0.59	0.36	0.46	1.51	0.54	0.50
time (sec)	N/A	0.411	0.440	0.345	0.272	0.388	0.953	0.658	9.943

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	132	105	81	85	223	141	256
N.S.	1	1.00	1.02	0.81	0.63	0.66	1.73	1.09	1.98
time (sec)	N/A	0.120	0.321	0.257	0.273	0.363	0.628	0.512	13.013

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	190	125	93	97	112	0	213	319
N.S.	1	1.64	1.08	0.80	0.84	0.97	0.00	1.84	2.75
time (sec)	N/A	0.286	0.347	0.253	0.281	0.392	0.000	0.489	11.123

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	167	135	127	155	0	274	578
N.S.	1	1.00	0.92	0.75	0.70	0.86	0.00	1.51	3.19
time (sec)	N/A	0.343	0.494	0.208	0.496	0.388	0.000	0.465	9.545

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	191	157	150	210	0	252	397
N.S.	1	1.00	1.01	0.83	0.79	1.11	0.00	1.33	2.10
time (sec)	N/A	0.312	2.257	0.255	0.502	0.371	0.000	0.493	9.444

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	293	145	138	218	0	241	584
N.S.	1	1.00	2.20	1.09	1.04	1.64	0.00	1.81	4.39
time (sec)	N/A	0.109	6.190	0.242	0.487	0.382	0.000	0.483	11.747

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	270	167	166	260	0	244	825
N.S.	1	1.00	1.52	0.94	0.93	1.46	0.00	1.37	4.63
time (sec)	N/A	0.299	2.527	0.260	0.521	0.381	0.000	0.511	10.894

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	285	129	123	241	0	263	346
N.S.	1	1.00	1.36	0.62	0.59	1.15	0.00	1.26	1.66
time (sec)	N/A	0.328	1.051	0.254	0.522	0.362	0.000	0.519	10.103

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	319	198	180	274	0	309	262
N.S.	1	1.00	1.35	0.84	0.76	1.16	0.00	1.31	1.11
time (sec)	N/A	0.392	0.590	0.304	0.310	0.365	0.000	0.516	9.669

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	322	163	134	248	0	347	302
N.S.	1	1.00	1.23	0.62	0.51	0.95	0.00	1.33	1.16
time (sec)	N/A	0.413	0.905	0.296	0.288	0.392	0.000	0.513	9.921

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	204	218	140	164	505	204	578
N.S.	1	1.00	0.58	0.62	0.40	0.46	1.43	0.58	1.63
time (sec)	N/A	0.585	0.906	0.490	0.286	0.393	1.352	0.676	10.773

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	189	180	117	136	456	184	552
N.S.	1	1.00	0.97	0.93	0.60	0.70	2.35	0.95	2.85
time (sec)	N/A	0.216	0.605	0.388	0.275	0.384	0.964	0.643	10.921

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	191	149	137	150	0	427	690
N.S.	1	1.00	0.76	0.60	0.55	0.60	0.00	1.71	2.76
time (sec)	N/A	0.439	0.376	0.268	0.282	0.416	0.000	0.504	11.574

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	194	152	143	179	0	345	674
N.S.	1	1.00	0.85	0.66	0.62	0.78	0.00	1.51	2.94
time (sec)	N/A	0.435	1.950	0.245	0.503	0.377	0.000	0.516	9.592

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	252	198	186	260	0	400	718
N.S.	1	1.00	1.09	0.86	0.81	1.13	0.00	1.73	3.11
time (sec)	N/A	0.440	6.127	0.270	0.503	0.397	0.000	0.532	9.595

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	355	186	187	293	0	421	405
N.S.	1	1.00	1.83	0.96	0.96	1.51	0.00	2.17	2.09
time (sec)	N/A	0.149	6.169	0.257	0.494	0.384	0.000	0.515	9.649

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	381	226	212	331	0	343	699
N.S.	1	1.00	2.04	1.21	1.13	1.77	0.00	1.83	3.74
time (sec)	N/A	0.427	6.248	0.283	0.488	0.381	0.000	0.570	9.500

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	405	192	182	334	0	356	1007
N.S.	1	1.00	1.78	0.85	0.80	1.47	0.00	1.57	4.44
time (sec)	N/A	0.464	0.879	0.333	0.495	0.399	0.000	0.546	12.284

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	408	227	217	373	0	399	446
N.S.	1	1.00	1.48	0.83	0.79	1.36	0.00	1.45	1.62
time (sec)	N/A	0.494	1.268	0.326	0.528	0.402	0.000	0.524	9.934

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	324	243	208	347	0	456	359
N.S.	1	1.00	1.07	0.80	0.69	1.15	0.00	1.50	1.18
time (sec)	N/A	0.564	0.655	0.328	0.286	0.377	0.000	0.536	9.906

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	268	280	248	384	0	457	381
N.S.	1	1.00	0.80	0.84	0.74	1.15	0.00	1.37	1.14
time (sec)	N/A	0.586	1.104	0.358	0.295	0.449	0.000	0.581	9.951

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	378	434	0	649	0	536	2390
N.S.	1	1.00	1.23	1.41	0.00	2.11	0.00	1.75	7.79
time (sec)	N/A	0.661	3.007	0.634	0.000	0.414	0.000	0.518	12.321

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	325	382	0	604	0	449	1003
N.S.	1	1.00	1.22	1.43	0.00	2.26	0.00	1.68	3.76
time (sec)	N/A	0.478	2.546	0.568	0.000	0.411	0.000	0.467	11.795

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	247	276	0	507	0	300	964
N.S.	1	1.00	1.52	1.69	0.00	3.11	0.00	1.84	5.91
time (sec)	N/A	0.204	1.646	0.454	0.000	0.448	0.000	0.468	11.526

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	193	0	516	0	286	2773
N.S.	1	1.00	1.18	1.41	0.00	3.77	0.00	2.09	20.24
time (sec)	N/A	0.183	0.506	0.547	0.000	0.495	0.000	0.476	11.153

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	182	201	0	675	0	260	2500
N.S.	1	1.00	1.18	1.31	0.00	4.38	0.00	1.69	16.23
time (sec)	N/A	0.180	1.272	0.530	0.000	0.464	0.000	0.475	12.091

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	180	191	209	0	804	0	275	675
N.S.	1	1.14	1.21	1.32	0.00	5.09	0.00	1.74	4.27
time (sec)	N/A	0.285	2.575	0.537	0.000	0.462	0.000	0.509	9.729

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	403	287	0	1149	0	356	973
N.S.	1	1.00	1.69	1.21	0.00	4.83	0.00	1.50	4.09
time (sec)	N/A	0.456	6.188	0.598	0.000	0.465	0.000	0.484	9.801

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	496	365	0	1578	0	461	1158
N.S.	1	1.00	1.70	1.25	0.00	5.40	0.00	1.58	3.97
time (sec)	N/A	0.675	6.284	0.688	0.000	0.626	0.000	0.526	9.829

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	1250	450	0	1110	0	540	2500
N.S.	1	1.00	3.78	1.36	0.00	3.35	0.00	1.63	7.55
time (sec)	N/A	0.636	7.037	0.476	0.000	0.476	0.000	0.557	16.596

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	1030	344	0	976	0	393	2034
N.S.	1	1.00	3.63	1.21	0.00	3.44	0.00	1.38	7.16
time (sec)	N/A	0.470	4.282	0.746	0.000	0.427	0.000	0.518	14.338

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	274	288	0	837	0	429	1743
N.S.	1	1.00	1.58	1.66	0.00	4.84	0.00	2.48	10.08
time (sec)	N/A	0.182	2.469	0.656	0.000	0.417	0.000	0.504	12.408

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	176	240	0	1042	0	275	2500
N.S.	1	1.00	1.01	1.37	0.00	5.95	0.00	1.57	14.29
time (sec)	N/A	0.188	1.255	0.749	0.000	0.594	0.000	0.487	13.505

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	184	214	0	1064	0	273	956
N.S.	1	1.00	1.01	1.18	0.00	5.85	0.00	1.50	5.25
time (sec)	N/A	0.302	1.903	0.698	0.000	0.467	0.000	0.503	9.878

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	319	283	0	1560	0	395	1100
N.S.	1	1.00	1.46	1.30	0.00	7.16	0.00	1.81	5.05
time (sec)	N/A	0.491	6.144	0.770	0.000	0.525	0.000	0.561	9.904

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	459	360	0	2027	0	451	1261
N.S.	1	1.00	1.59	1.25	0.00	7.01	0.00	1.56	4.36
time (sec)	N/A	0.700	6.161	0.863	0.000	0.651	0.000	0.559	10.044

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	347	431	0	2592	0	550	1487
N.S.	1	1.00	1.02	1.27	0.00	7.62	0.00	1.62	4.37
time (sec)	N/A	0.940	3.093	0.986	0.000	0.835	0.000	0.633	10.215

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	327	1619	0	633	0	0	-1
N.S.	1	1.00	0.71	3.50	0.00	1.37	0.00	0.00	-0.00
time (sec)	N/A	0.656	3.510	10.589	0.000	0.165	0.000	0.000	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	326	1356	0	584	0	0	-1
N.S.	1	1.00	0.98	4.08	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.409	2.797	10.247	0.000	0.170	0.000	0.000	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	435	1155	0	0	0	0	-1
N.S.	1	1.00	1.29	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	12.316	9.077	0.000	0.000	0.000	0.000	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	422	656	0	0	0	0	-1
N.S.	1	1.00	1.31	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	12.405	9.569	0.000	0.000	0.000	0.000	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	450	1364	0	0	0	0	-1
N.S.	1	1.00	1.30	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.562	12.300	10.208	0.000	0.000	0.000	0.000	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	473	1495	0	0	0	0	-1
N.S.	1	1.00	1.35	4.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	13.719	11.177	0.000	0.000	0.000	0.000	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	643	1761	0	0	0	0	-1
N.S.	1	1.00	1.56	4.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.794	16.415	11.240	0.000	0.000	0.000	0.000	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	545	2075	0	0	0	0	-1
N.S.	1	1.00	1.13	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.015	16.047	14.375	0.000	0.000	0.000	0.000	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	382	1801	0	689	0	0	-1
N.S.	1	1.00	0.72	3.41	0.00	1.30	0.00	0.00	-0.00
time (sec)	N/A	0.767	10.512	10.556	0.000	0.205	0.000	0.000	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	382	1619	0	632	0	0	-1
N.S.	1	1.00	0.97	4.11	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.532	8.050	10.819	0.000	0.175	0.000	0.000	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	477	1405	0	0	0	0	-1
N.S.	1	1.00	1.22	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.737	12.574	10.854	0.000	0.000	0.000	0.000	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	452	726	0	0	0	0	-1
N.S.	1	1.00	1.21	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.744	12.775	8.984	0.000	0.000	0.000	0.000	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	434	1379	0	0	0	0	-1
N.S.	1	1.00	1.13	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	12.200	12.526	0.000	0.000	0.000	0.000	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	486	1511	0	0	0	0	-1
N.S.	1	1.00	1.26	3.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.694	14.369	12.515	0.000	0.000	0.000	0.000	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	641	1760	0	0	0	0	-1
N.S.	1	1.00	1.57	4.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	16.467	13.040	0.000	0.000	0.000	0.000	0.000

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	544	2075	0	0	0	0	-1
N.S.	1	1.00	1.12	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.034	16.038	15.083	0.000	0.000	0.000	0.000	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0	-1
N.S.	1	1.00	1.40	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.242	16.524	20.346	0.000	0.000	0.000	0.000	0.000

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	450	1801	0	688	0	0	-1
N.S.	1	1.00	1.00	3.99	0.00	1.53	0.00	0.00	-0.00
time (sec)	N/A	0.686	14.988	10.828	0.000	0.207	0.000	0.000	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	521	1573	0	0	0	0	-1
N.S.	1	1.00	1.17	3.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.895	12.727	10.107	0.000	0.000	0.000	0.000	0.000

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	496	864	0	0	0	0	-1
N.S.	1	1.00	1.16	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.918	13.348	11.096	0.000	0.000	0.000	0.000	0.000

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	460	1520	0	0	0	0	-1
N.S.	1	1.00	1.07	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.903	13.749	12.152	0.000	0.000	0.000	0.000	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	466	1526	0	0	0	0	-1
N.S.	1	1.00	1.09	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.884	12.483	11.214	0.000	0.000	0.000	0.000	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	655	1777	0	0	0	0	-1
N.S.	1	1.00	1.46	3.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.963	16.482	13.727	0.000	0.000	0.000	0.000	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	545	2075	0	0	0	0	-1
N.S.	1	1.00	1.13	4.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.038	16.261	13.414	0.000	0.000	0.000	0.000	0.000

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	771	2458	0	0	0	0	-1
N.S.	1	1.00	1.40	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.256	16.600	18.957	0.000	0.000	0.000	0.000	0.000

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	382	1619	0	634	0	0	-1
N.S.	1	1.00	0.81	3.44	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.752	3.870	11.003	0.000	0.180	0.000	0.000	0.000

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	326	1356	0	585	0	0	-1
N.S.	1	1.00	0.80	3.35	0.00	1.44	0.00	0.00	-0.00
time (sec)	N/A	0.574	2.994	10.829	0.000	0.162	0.000	0.000	0.000

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	275	1190	0	537	0	0	-1
N.S.	1	1.00	0.97	4.20	0.00	1.90	0.00	0.00	-0.00
time (sec)	N/A	0.293	2.208	10.140	0.000	0.157	0.000	0.000	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	408	1018	0	0	0	0	-1
N.S.	1	1.00	1.42	3.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	12.491	10.219	0.000	0.000	0.000	0.000	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	416	704	0	0	0	0	-1
N.S.	1	1.00	1.46	2.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	12.410	18.773	0.000	0.000	0.000	0.000	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	443	913	0	0	0	0	-1
N.S.	1	1.00	1.44	2.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.437	12.273	22.273	0.000	0.000	0.000	0.000	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	475	1496	0	0	0	0	-1
N.S.	1	1.00	1.35	4.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.561	13.817	12.524	0.000	0.000	0.000	0.000	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	647	1761	0	0	0	0	-1
N.S.	1	1.00	1.57	4.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.785	16.448	12.700	0.000	0.000	0.000	0.000	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	326	1356	0	788	0	0	-1
N.S.	1	1.00	0.70	2.91	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.773	4.771	10.697	0.000	0.307	0.000	0.000	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	275	1190	0	723	0	0	-1
N.S.	1	1.00	0.69	2.97	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.582	3.637	9.470	0.000	0.203	0.000	0.000	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	222	943	0	659	0	0	-1
N.S.	1	1.00	0.85	3.61	0.00	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.272	2.811	9.132	0.000	0.180	0.000	0.000	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	419	1010	0	0	0	0	-1
N.S.	1	1.00	1.42	3.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	12.602	10.811	0.000	0.000	0.000	0.000	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	433	620	0	0	0	0	-1
N.S.	1	1.00	1.47	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	12.364	10.455	0.000	0.000	0.000	0.000	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	435	1349	0	0	0	0	-1
N.S.	1	1.00	1.19	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	13.317	11.457	0.000	0.000	0.000	0.000	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	468	1496	0	0	0	0	-1
N.S.	1	1.00	1.12	3.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.758	13.997	11.322	0.000	0.000	0.000	0.000	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	1044	2033	0	955	0	0	-1
N.S.	1	1.00	2.23	4.33	0.00	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.753	6.961	11.919	0.000	0.346	0.000	0.000	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	257	1642	0	873	0	0	-1
N.S.	1	1.00	0.63	4.00	0.00	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.569	5.648	11.483	0.000	0.272	0.000	0.000	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	211	1430	0	794	0	0	-1
N.S.	1	1.00	0.83	5.63	0.00	3.13	0.00	0.00	-0.00
time (sec)	N/A	0.268	4.422	10.705	0.000	0.221	0.000	0.000	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	443	1375	0	0	0	0	-1
N.S.	1	1.00	1.42	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	13.502	43.615	0.000	0.000	0.000	0.000	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	445	2112	0	0	0	0	-1
N.S.	1	1.00	1.29	6.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	13.734	11.677	0.000	0.000	0.000	0.000	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	622	2617	0	0	0	0	-1
N.S.	1	1.00	1.53	6.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.824	16.489	12.925	0.000	0.000	0.000	0.000	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	680	2870	0	0	0	0	-1
N.S.	1	1.00	1.48	6.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.977	16.558	15.016	0.000	0.000	0.000	0.000	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	1670	25053	0	0	0	0	-1
N.S.	1	1.00	3.27	49.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.201	6.413	9.118	0.000	0.000	0.000	0.000	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.086	180.006	0.325	0.000	0.000	0.000	0.000	0.000

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.058	4.879	0.247	0.000	0.000	0.000	0.000	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.065	3.976	0.421	0.000	0.000	0.000	0.000	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.066	4.511	0.359	0.000	0.000	0.000	0.000	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	623	195	0	0	0	0	0	-1
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.127	0.618	1.323	0.000	0.000	0.000	0.000	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	167	0	0	0	0	0	-1
N.S.	1	1.00	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	0.242	0.578	0.000	0.000	0.000	0.000	0.000

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.124	0.325	0.000	0.000	0.000	0.000	0.000

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	105	138	72	106	136	133	71
N.S.	1	1.00	1.08	1.42	0.74	1.09	1.40	1.37	0.73
time (sec)	N/A	0.082	0.304	0.527	0.274	0.369	2.578	0.547	0.083

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	94	120	72	95	136	118	71
N.S.	1	1.00	0.97	1.24	0.74	0.98	1.40	1.22	0.73
time (sec)	N/A	0.079	0.225	0.427	0.276	0.354	1.783	0.573	0.058

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	105	102	72	84	114	133	71
N.S.	1	1.00	1.30	1.26	0.89	1.04	1.41	1.64	0.88
time (sec)	N/A	0.090	0.218	0.360	0.268	0.372	1.273	0.503	0.057

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	94	84	72	73	114	118	71
N.S.	1	1.00	1.16	1.04	0.89	0.90	1.41	1.46	0.88
time (sec)	N/A	0.087	0.223	0.298	0.284	0.358	0.879	0.514	0.055

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	86	64	72	62	90	103	71
N.S.	1	1.00	1.32	0.98	1.11	0.95	1.38	1.58	1.09
time (sec)	N/A	0.063	0.167	0.214	0.280	0.363	0.592	0.466	11.594

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	86	65	69	74	0	70	126
N.S.	1	1.00	1.00	0.76	0.80	0.86	0.00	0.81	1.47
time (sec)	N/A	0.050	0.026	0.164	0.269	0.371	0.000	0.502	12.003

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	85	69	91	0	79	250
N.S.	1	1.00	1.00	1.02	0.83	1.10	0.00	0.95	3.01
time (sec)	N/A	0.061	0.025	0.146	0.280	0.361	0.000	0.474	11.927

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	77	105	68	102	0	82	229
N.S.	1	1.00	0.90	1.22	0.79	1.19	0.00	0.95	2.66
time (sec)	N/A	0.060	0.145	0.165	0.287	0.361	0.000	0.501	11.885

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	121	69	117	0	81	218
N.S.	1	1.00	0.89	1.42	0.81	1.38	0.00	0.95	2.56
time (sec)	N/A	0.054	0.111	0.168	0.284	0.361	0.000	0.451	11.911

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	87	101	69	110	0	82	207
N.S.	1	1.00	1.07	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.034	0.182	0.175	0.267	0.378	0.000	0.493	12.261

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	121	72	124	0	84	193
N.S.	1	1.00	1.07	1.41	0.84	1.44	0.00	0.98	2.24
time (sec)	N/A	0.054	0.134	0.191	0.275	0.361	0.000	0.518	11.803

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	110	70	100	0	70	69
N.S.	1	1.00	1.00	1.80	1.15	1.64	0.00	1.15	1.13
time (sec)	N/A	0.075	0.021	0.217	0.275	0.346	0.000	0.479	11.791

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	128	70	106	0	70	70
N.S.	1	1.00	1.00	1.97	1.08	1.63	0.00	1.08	1.08
time (sec)	N/A	0.082	0.022	0.236	0.272	0.362	0.000	0.525	11.606

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	148	70	109	0	70	70
N.S.	1	1.00	1.09	1.83	0.86	1.35	0.00	0.86	0.86
time (sec)	N/A	0.089	0.086	0.245	0.273	0.348	0.000	0.539	11.642

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	88	166	70	115	0	70	70
N.S.	1	1.00	1.09	2.05	0.86	1.42	0.00	0.86	0.86
time (sec)	N/A	0.087	0.090	0.274	0.282	0.357	0.000	0.494	11.669

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	184	70	122	0	70	70
N.S.	1	1.00	0.91	1.90	0.72	1.26	0.00	0.72	0.72
time (sec)	N/A	0.062	0.081	0.285	0.270	0.356	0.000	0.515	11.641

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	202	70	128	0	70	70
N.S.	1	1.00	0.91	2.08	0.72	1.32	0.00	0.72	0.72
time (sec)	N/A	0.063	0.079	0.314	0.278	0.359	0.000	0.505	11.686

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	169	155	108	121	190	173	108
N.S.	1	1.00	1.22	1.12	0.78	0.88	1.38	1.25	0.78
time (sec)	N/A	0.122	0.551	0.496	0.272	0.369	1.328	0.559	11.438

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	138	101	108	85	139	152	108
N.S.	1	1.00	1.00	0.73	0.78	0.62	1.01	1.10	0.78
time (sec)	N/A	0.084	0.501	0.400	0.286	0.352	0.953	0.507	11.468

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	105	81	105	96	0	118	153
N.S.	1	1.00	0.81	0.62	0.81	0.74	0.00	0.91	1.18
time (sec)	N/A	0.085	0.109	0.264	0.276	0.381	0.000	0.526	11.736

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	142	120	105	135	0	127	445
N.S.	1	1.00	1.14	0.96	0.84	1.08	0.00	1.02	3.56
time (sec)	N/A	0.100	0.041	0.243	0.269	0.386	0.000	0.559	12.016

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	103	141	104	160	0	140	331
N.S.	1	1.00	0.81	1.11	0.82	1.26	0.00	1.10	2.61
time (sec)	N/A	0.103	0.195	0.270	0.284	0.363	0.000	0.512	11.684

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	103	176	103	158	0	127	315
N.S.	1	1.00	0.86	1.47	0.86	1.32	0.00	1.06	2.62
time (sec)	N/A	0.092	0.190	0.273	0.270	0.356	0.000	0.531	11.712

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	107	157	105	177	0	138	310
N.S.	1	1.00	0.85	1.25	0.83	1.40	0.00	1.10	2.46
time (sec)	N/A	0.062	0.486	0.286	0.271	0.363	0.000	0.546	11.688

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	105	192	105	166	0	131	297
N.S.	1	1.00	0.85	1.55	0.85	1.34	0.00	1.06	2.40
time (sec)	N/A	0.095	0.126	0.300	0.278	0.372	0.000	0.519	11.713

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	107	146	108	183	0	134	274
N.S.	1	1.00	0.82	1.12	0.83	1.41	0.00	1.03	2.11
time (sec)	N/A	0.104	0.124	0.324	0.275	0.364	0.000	0.528	11.865

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	104	218	106	146	0	118	105
N.S.	1	1.00	0.81	1.69	0.82	1.13	0.00	0.91	0.81
time (sec)	N/A	0.103	0.145	0.322	0.285	0.346	0.000	0.527	11.794

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	108	173	106	148	0	118	107
N.S.	1	1.00	0.78	1.25	0.77	1.07	0.00	0.86	0.78
time (sec)	N/A	0.102	0.163	0.389	0.284	0.364	0.000	0.548	11.776

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	264	249	218	279	0	300	375
N.S.	1	1.00	1.12	1.06	0.93	1.19	0.00	1.28	1.60
time (sec)	N/A	0.188	1.514	0.376	0.276	0.406	0.000	0.468	0.135

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	225	198	184	246	0	249	254
N.S.	1	1.00	1.17	1.03	0.95	1.27	0.00	1.29	1.32
time (sec)	N/A	0.161	1.076	0.528	0.270	0.386	0.000	0.457	11.458

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	188	152	148	203	0	194	161
N.S.	1	1.00	1.20	0.97	0.94	1.29	0.00	1.24	1.03
time (sec)	N/A	0.107	0.725	0.481	0.278	0.388	0.000	0.504	0.088

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	119	118	189	0	154	338
N.S.	1	1.00	0.92	0.99	0.98	1.58	0.00	1.28	2.82
time (sec)	N/A	0.104	0.355	0.544	0.277	0.416	0.000	0.460	12.076

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	95	106	120	214	0	131	313
N.S.	1	1.00	0.87	0.97	1.10	1.96	0.00	1.20	2.87
time (sec)	N/A	0.115	0.433	0.608	0.286	0.408	0.000	0.470	12.101

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	116	129	147	335	0	190	280
N.S.	1	1.00	0.89	0.98	1.12	2.56	0.00	1.45	2.14
time (sec)	N/A	0.135	0.515	0.542	0.274	0.417	0.000	0.487	11.923

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	127	139	158	401	0	211	319
N.S.	1	1.00	0.86	0.95	1.07	2.73	0.00	1.44	2.17
time (sec)	N/A	0.135	1.248	0.549	0.308	0.384	0.000	0.475	11.717

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	187	172	189	542	0	278	439
N.S.	1	1.00	0.99	0.91	1.01	2.88	0.00	1.48	2.34
time (sec)	N/A	0.115	6.111	0.601	0.274	0.398	0.000	0.477	11.805

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	220	207	225	696	0	332	628
N.S.	1	1.00	0.97	0.92	1.00	3.08	0.00	1.47	2.78
time (sec)	N/A	0.180	2.806	0.701	0.273	0.425	0.000	0.477	11.902

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	139	0	178	572	18260	575	887
N.S.	1	1.00	0.82	0.00	1.05	3.36	107.41	3.38	5.22
time (sec)	N/A	0.144	0.523	0.634	0.282	0.417	21.774	0.561	18.921

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	97	0	109	282	8675	379	550
N.S.	1	1.00	0.79	0.00	0.89	2.29	70.53	3.08	4.47
time (sec)	N/A	0.090	0.153	0.335	0.283	0.405	11.759	0.487	16.337

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	133	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.413	0.415	0.000	0.000	0.000	0.000	0.000

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	143	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	0.312	1.444	0.000	0.000	0.000	0.000	0.000

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	210	225	135	161	488	214	441
N.S.	1	1.00	0.88	0.95	0.57	0.68	2.05	0.90	1.85
time (sec)	N/A	0.239	1.532	1.154	0.281	0.388	4.696	0.537	15.014

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	202	237	137	182	656	226	207
N.S.	1	1.00	0.81	0.95	0.55	0.73	2.62	0.90	0.83
time (sec)	N/A	0.228	1.050	1.004	0.286	0.394	3.540	0.687	13.555

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	197	171	115	128	384	217	386
N.S.	1	1.00	1.05	0.91	0.61	0.68	2.05	1.16	2.06
time (sec)	N/A	0.199	0.772	0.716	0.276	0.404	2.581	0.626	14.944

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	183	127	149	529	189	237
N.S.	1	1.00	0.96	0.91	0.63	0.74	2.63	0.94	1.18
time (sec)	N/A	0.167	0.624	0.555	0.269	0.383	1.863	0.565	11.928

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	161	115	92	97	282	176	332
N.S.	1	1.00	1.06	0.76	0.61	0.64	1.86	1.16	2.18
time (sec)	N/A	0.131	0.712	0.472	0.277	0.379	1.299	0.497	15.045

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	166	113	122	137	0	291	415
N.S.	1	1.00	1.06	0.72	0.78	0.87	0.00	1.85	2.64
time (sec)	N/A	0.133	0.200	0.341	0.276	0.403	0.000	0.544	13.710

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	220	165	172	188	0	368	683
N.S.	1	1.00	1.24	0.93	0.97	1.06	0.00	2.07	3.84
time (sec)	N/A	0.307	0.283	0.277	0.484	0.384	0.000	0.534	11.894

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	250	187	190	244	0	346	484
N.S.	1	1.00	1.39	1.04	1.06	1.36	0.00	1.92	2.69
time (sec)	N/A	0.196	6.128	0.287	0.515	0.389	0.000	0.541	11.757

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	336	223	189	252	0	366	665
N.S.	1	1.00	1.90	1.26	1.07	1.42	0.00	2.07	3.76
time (sec)	N/A	0.299	6.212	0.262	0.544	0.392	0.000	0.526	12.096

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	337	245	205	309	0	346	479
N.S.	1	1.00	1.94	1.41	1.18	1.78	0.00	1.99	2.75
time (sec)	N/A	0.183	6.178	0.293	0.489	0.388	0.000	0.519	11.743

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	351	216	183	306	0	337	888
N.S.	1	1.00	1.74	1.07	0.91	1.51	0.00	1.67	4.40
time (sec)	N/A	0.127	0.787	0.287	0.487	0.397	0.000	0.523	16.040

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	384	238	219	360	0	337	985
N.S.	1	1.00	2.19	1.36	1.25	2.06	0.00	1.93	5.63
time (sec)	N/A	0.176	0.726	0.335	0.499	0.382	0.000	0.568	15.062

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	280	172	153	320	0	356	379
N.S.	1	1.00	1.77	1.09	0.97	2.03	0.00	2.25	2.40
time (sec)	N/A	0.286	0.993	0.300	0.499	0.400	0.000	0.551	11.871

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	282	254	220	338	0	402	343
N.S.	1	1.00	1.77	1.60	1.38	2.13	0.00	2.53	2.16
time (sec)	N/A	0.205	0.570	0.384	0.291	0.395	0.000	0.558	12.689

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	204	191	154	291	0	408	373
N.S.	1	1.00	1.35	1.26	1.02	1.93	0.00	2.70	2.47
time (sec)	N/A	0.266	0.800	0.374	0.291	0.390	0.000	0.579	11.864

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	244	311	272	455	0	468	394
N.S.	1	1.00	1.16	1.48	1.30	2.17	0.00	2.23	1.88
time (sec)	N/A	0.225	1.009	0.423	0.288	0.393	0.000	0.576	11.934

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	250	247	196	363	0	502	448
N.S.	1	1.00	1.26	1.25	0.99	1.83	0.00	2.54	2.26
time (sec)	N/A	0.299	1.190	0.408	0.282	0.385	0.000	0.575	16.332

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	531	656	0	871	0	965	2500
N.S.	1	1.00	1.01	1.25	0.00	1.66	0.00	1.84	4.76
time (sec)	N/A	1.245	6.110	0.454	0.000	0.444	0.000	0.559	18.113

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	462	611	0	814	0	835	2500
N.S.	1	1.00	0.98	1.30	0.00	1.73	0.00	1.77	5.31
time (sec)	N/A	0.977	5.617	0.430	0.000	0.436	0.000	0.495	16.382

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	371	468	0	697	0	593	2500
N.S.	1	1.00	1.61	2.03	0.00	3.02	0.00	2.57	10.82
time (sec)	N/A	0.327	2.993	0.590	0.000	0.418	0.000	0.492	14.530

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	207	314	0	698	0	353	2500
N.S.	1	1.00	0.78	1.18	0.00	2.62	0.00	1.33	9.40
time (sec)	N/A	0.230	1.173	0.744	0.000	0.658	0.000	0.486	13.601

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	215	304	0	901	0	384	2500
N.S.	1	1.00	0.85	1.20	0.00	3.55	0.00	1.51	9.84
time (sec)	N/A	0.227	1.960	0.848	0.000	0.635	0.000	0.511	13.483

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	315	290	0	1210	0	463	2500
N.S.	1	1.00	1.25	1.16	0.00	4.82	0.00	1.84	9.96
time (sec)	N/A	0.222	6.149	0.777	0.000	0.694	0.000	0.509	12.902

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	428	327	0	1437	0	399	2500
N.S.	1	1.00	1.49	1.14	0.00	5.01	0.00	1.39	8.71
time (sec)	N/A	0.246	6.246	0.786	0.000	0.650	0.000	0.491	12.781

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	487	370	0	1576	0	475	1117
N.S.	1	1.00	1.61	1.22	0.00	5.20	0.00	1.57	3.69
time (sec)	N/A	0.769	6.259	0.750	0.000	0.596	0.000	0.521	12.032

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	361	465	0	2011	0	596	1424
N.S.	1	1.00	0.85	1.10	0.00	4.74	0.00	1.41	3.36
time (sec)	N/A	0.973	1.092	0.847	0.000	0.680	0.000	0.559	12.161

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	447	572	0	2588	0	736	1810
N.S.	1	1.00	0.93	1.19	0.00	5.39	0.00	1.53	3.77
time (sec)	N/A	1.241	1.123	1.071	0.000	0.914	0.000	0.569	12.579

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	536	536	631	696	0	1128	0	968	2500
N.S.	1	1.00	1.18	1.30	0.00	2.10	0.00	1.81	4.66
time (sec)	N/A	1.371	10.788	0.623	0.000	0.503	0.000	0.536	48.182

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	517	554	0	995	0	724	2500
N.S.	1	1.00	1.07	1.14	0.00	2.05	0.00	1.49	5.15
time (sec)	N/A	1.083	8.613	0.602	0.000	0.461	0.000	0.528	23.853

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	1250	472	0	837	0	581	2529
N.S.	1	1.00	5.27	1.99	0.00	3.53	0.00	2.45	10.67
time (sec)	N/A	0.305	5.699	0.531	0.000	0.442	0.000	0.519	16.795

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	243	359	0	1007	0	635	2500
N.S.	1	1.00	0.61	0.90	0.00	2.52	0.00	1.59	6.27
time (sec)	N/A	0.345	1.322	1.010	0.000	0.636	0.000	0.508	14.551

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	241	326	0	1171	0	461	2500
N.S.	1	1.00	0.77	1.04	0.00	3.73	0.00	1.47	7.96
time (sec)	N/A	0.322	4.977	1.035	0.000	0.621	0.000	0.539	13.695

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	384	357	0	1658	0	512	2500
N.S.	1	1.00	0.97	0.90	0.00	4.20	0.00	1.30	6.33
time (sec)	N/A	0.335	6.211	1.034	0.000	0.664	0.000	0.525	13.019

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	490	359	0	1553	0	478	1082
N.S.	1	1.00	1.49	1.09	0.00	4.72	0.00	1.45	3.29
time (sec)	N/A	0.834	6.194	1.007	0.000	0.506	0.000	0.510	12.413

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	363	455	0	2022	0	603	1275
N.S.	1	1.00	1.02	1.28	0.00	5.70	0.00	1.70	3.59
time (sec)	N/A	1.068	0.909	1.033	0.000	0.557	0.000	0.545	12.554

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	448	559	0	2571	0	731	1614
N.S.	1	1.00	0.91	1.14	0.00	5.23	0.00	1.49	3.28
time (sec)	N/A	1.350	1.186	1.111	0.000	0.739	0.000	0.564	12.626

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	728	775	0	3687	0	1018	2500
N.S.	1	1.00	1.21	1.29	0.00	6.14	0.00	1.70	4.17
time (sec)	N/A	1.995	1.905	0.932	0.000	0.897	0.000	0.566	13.286

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	1906	56846	0	0	0	0	-1
N.S.	1	1.00	2.68	79.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.668	6.681	2.799	0.000	0.000	0.000	0.000	0.000

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	161	127	372	0	155	0	0	-1
N.S.	1	1.01	0.80	2.34	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.314	32.510	0.000	0.104	0.000	0.000	0.000

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	105	616	0	0	0	0	-1
N.S.	1	1.00	0.54	3.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.290	0.456	0.467	0.000	0.000	0.000	0.000	0.000

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	65	67	71	105	68	64
N.S.	1	1.00	0.87	0.86	0.88	0.93	1.38	0.89	0.84
time (sec)	N/A	0.059	0.152	0.191	0.409	0.364	0.501	0.440	0.068

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	49	49	47	87	50	47
N.S.	1	1.00	0.89	0.89	0.89	0.85	1.58	0.91	0.85
time (sec)	N/A	0.052	0.079	0.114	0.375	0.382	0.399	0.442	0.065

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	33	33	31	65	34	31
N.S.	1	1.00	0.97	0.97	0.97	0.91	1.91	1.00	0.91
time (sec)	N/A	0.034	0.016	0.102	0.307	0.352	0.337	0.426	0.055

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	31	0	35	48
N.S.	1	1.00	1.00	0.97	0.97	0.91	0.00	1.03	1.41
time (sec)	N/A	0.026	0.013	0.117	0.295	0.358	0.000	0.467	11.826

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	34	47	56	0	49	89
N.S.	1	1.00	1.00	0.68	0.94	1.12	0.00	0.98	1.78
time (sec)	N/A	0.049	0.026	0.130	0.283	0.362	0.000	0.466	11.812

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	65	66	100	0	71	132
N.S.	1	1.00	1.00	0.90	0.92	1.39	0.00	0.99	1.83
time (sec)	N/A	0.059	0.035	0.184	0.297	0.379	0.000	0.450	11.785

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	177	355	0	427	0	467	376
N.S.	1	1.00	0.75	1.51	0.00	1.82	0.00	1.99	1.60
time (sec)	N/A	0.588	1.207	0.327	0.000	0.406	0.000	0.451	13.390

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	146	296	0	380	0	366	2616
N.S.	1	1.00	0.76	1.55	0.00	1.99	0.00	1.92	13.70
time (sec)	N/A	0.413	0.767	0.274	0.000	0.365	0.000	0.458	13.348

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	184	0	315	0	207	225
N.S.	1	1.00	0.88	1.24	0.00	2.13	0.00	1.40	1.52
time (sec)	N/A	0.293	0.191	0.256	0.000	0.367	0.000	0.480	12.478

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	104	146	0	275	0	159	190
N.S.	1	1.00	1.04	1.46	0.00	2.75	0.00	1.59	1.90
time (sec)	N/A	0.104	0.171	0.203	0.000	0.374	0.000	0.437	12.069

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	90	94	0	262	0	94	896
N.S.	1	1.00	1.20	1.25	0.00	3.49	0.00	1.25	11.95
time (sec)	N/A	0.119	0.071	0.279	0.000	0.404	0.000	0.463	11.975

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	108	109	0	314	0	129	204
N.S.	1	1.00	1.35	1.36	0.00	3.92	0.00	1.61	2.55
time (sec)	N/A	0.158	0.169	0.263	0.000	0.391	0.000	0.501	11.940

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	181	144	0	472	0	198	790
N.S.	1	1.00	1.59	1.26	0.00	4.14	0.00	1.74	6.93
time (sec)	N/A	0.289	0.604	0.349	0.000	0.449	0.000	0.465	12.453

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	351	205	0	591	0	270	749
N.S.	1	1.00	2.29	1.34	0.00	3.86	0.00	1.76	4.90
time (sec)	N/A	0.424	6.179	0.391	0.000	0.435	0.000	0.472	13.513

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	430	252	0	808	0	336	873
N.S.	1	1.00	2.22	1.30	0.00	4.16	0.00	1.73	4.50
time (sec)	N/A	0.595	6.214	0.427	0.000	0.514	0.000	0.510	12.069

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	506	336	0	959	0	444	1007
N.S.	1	1.00	2.13	1.41	0.00	4.03	0.00	1.87	4.23
time (sec)	N/A	0.781	1.294	0.495	0.000	0.512	0.000	0.496	12.081

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	147	131	127	0	149	133
N.S.	1	1.00	0.85	0.99	0.88	0.85	0.00	1.00	0.89
time (sec)	N/A	0.127	0.196	0.264	0.275	0.374	0.000	0.465	0.083

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	104	103	105	97	0	117	107
N.S.	1	1.00	0.87	0.87	0.88	0.82	0.00	0.98	0.90
time (sec)	N/A	0.111	0.258	0.259	0.280	0.372	0.000	0.481	11.623

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	83	79	78	0	85	78
N.S.	1	1.00	0.89	0.93	0.89	0.88	0.00	0.96	0.88
time (sec)	N/A	0.070	0.146	0.194	0.283	0.365	0.000	0.482	0.068

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	55	54	55	0	56	98
N.S.	1	1.00	0.90	0.93	0.92	0.93	0.00	0.95	1.66
time (sec)	N/A	0.074	0.051	0.217	0.294	0.422	0.000	0.471	11.861

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	59	57	69	0	59	118
N.S.	1	1.00	0.90	0.98	0.95	1.15	0.00	0.98	1.97
time (sec)	N/A	0.079	0.054	0.214	0.281	0.393	0.000	0.468	11.857

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	76	77	118	0	88	144
N.S.	1	1.00	0.77	0.90	0.92	1.40	0.00	1.05	1.71
time (sec)	N/A	0.061	0.108	0.263	0.284	0.377	0.000	0.486	11.758

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	274	512	0	526	0	726	600
N.S.	1	1.00	0.97	1.82	0.00	1.87	0.00	2.57	2.13
time (sec)	N/A	0.629	1.564	0.352	0.000	0.406	0.000	0.448	14.637

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	186	363	0	457	0	458	511
N.S.	1	1.00	0.79	1.54	0.00	1.94	0.00	1.95	2.17
time (sec)	N/A	0.463	1.401	0.322	0.000	0.390	0.000	0.470	13.369

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	155	309	0	414	0	371	453
N.S.	1	1.00	0.97	1.94	0.00	2.60	0.00	2.33	2.85
time (sec)	N/A	0.205	0.737	0.256	0.000	0.374	0.000	0.461	12.950

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	143	180	0	350	0	183	1320
N.S.	1	1.00	1.15	1.45	0.00	2.82	0.00	1.48	10.65
time (sec)	N/A	0.182	0.169	0.366	0.000	0.443	0.000	0.491	14.031

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	146	155	0	396	0	221	1167
N.S.	1	1.00	1.40	1.49	0.00	3.81	0.00	2.12	11.22
time (sec)	N/A	0.172	0.533	0.382	0.000	0.472	0.000	0.467	13.781

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	204	180	0	572	0	217	2718
N.S.	1	1.00	1.66	1.46	0.00	4.65	0.00	1.76	22.10
time (sec)	N/A	0.186	1.134	0.392	0.000	0.476	0.000	0.473	12.676

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	350	223	0	633	0	273	654
N.S.	1	1.00	2.27	1.45	0.00	4.11	0.00	1.77	4.25
time (sec)	N/A	0.282	6.128	0.412	0.000	0.474	0.000	0.452	12.424

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	433	291	0	904	0	375	953
N.S.	1	1.00	2.19	1.47	0.00	4.57	0.00	1.89	4.81
time (sec)	N/A	0.475	6.183	0.457	0.000	0.559	0.000	0.479	12.156

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	507	379	0	1051	0	484	1082
N.S.	1	1.00	2.08	1.55	0.00	4.31	0.00	1.98	4.43
time (sec)	N/A	0.656	1.256	0.528	0.000	0.591	0.000	0.517	12.181

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	180	256	205	199	0	261	236
N.S.	1	1.00	0.85	1.21	0.97	0.94	0.00	1.23	1.11
time (sec)	N/A	0.153	0.886	0.346	0.279	0.428	0.000	0.485	0.132

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	153	249	172	164	0	213	191
N.S.	1	1.00	0.85	1.38	0.96	0.91	0.00	1.18	1.06
time (sec)	N/A	0.130	0.515	0.328	0.280	0.380	0.000	0.477	11.782

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	128	160	139	142	0	165	150
N.S.	1	1.00	0.86	1.08	0.94	0.96	0.00	1.11	1.01
time (sec)	N/A	0.093	0.452	0.253	0.297	0.379	0.000	0.484	0.075

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	101	105	99	104	0	106	254
N.S.	1	1.00	0.94	0.98	0.93	0.97	0.00	0.99	2.37
time (sec)	N/A	0.093	0.095	0.281	0.282	0.401	0.000	0.475	12.098

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	86	90	91	133	0	105	233
N.S.	1	1.00	0.90	0.94	0.95	1.39	0.00	1.09	2.43
time (sec)	N/A	0.104	0.121	0.279	0.282	0.404	0.000	0.450	12.029

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	99	99	174	0	130	238
N.S.	1	1.00	0.92	0.94	0.94	1.66	0.00	1.24	2.27
time (sec)	N/A	0.114	0.265	0.298	0.309	0.398	0.000	0.446	11.980

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	110	111	113	198	0	151	227
N.S.	1	1.00	0.92	0.92	0.94	1.65	0.00	1.26	1.89
time (sec)	N/A	0.115	0.192	0.307	0.283	0.401	0.000	0.478	11.928

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	137	139	271	0	201	281
N.S.	1	1.00	0.78	0.93	0.94	1.83	0.00	1.36	1.90
time (sec)	N/A	0.089	0.683	0.348	0.288	0.393	0.000	0.513	11.828

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	179	167	170	346	0	251	381
N.S.	1	1.00	1.00	0.93	0.95	1.93	0.00	1.40	2.13
time (sec)	N/A	0.137	6.080	0.383	0.285	0.398	0.000	0.492	11.917

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	165	199	206	464	0	301	514
N.S.	1	1.00	0.78	0.94	0.97	2.19	0.00	1.42	2.42
time (sec)	N/A	0.159	2.581	0.503	0.302	0.403	0.000	0.493	12.314

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	403	797	0	706	0	1244	2500
N.S.	1	1.00	0.86	1.71	0.00	1.51	0.00	2.66	5.35
time (sec)	N/A	1.172	2.205	0.288	0.000	0.431	0.000	0.490	15.077

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	324	601	0	619	0	863	2500
N.S.	1	1.00	0.79	1.47	0.00	1.52	0.00	2.12	6.13
time (sec)	N/A	0.927	2.065	0.446	0.000	0.410	0.000	0.619	14.463

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	275	524	0	570	0	735	2500
N.S.	1	1.00	1.21	2.30	0.00	2.50	0.00	3.22	10.96
time (sec)	N/A	0.337	1.562	0.348	0.000	0.418	0.000	0.494	14.383

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	220	336	0	508	0	398	2500
N.S.	1	1.00	0.87	1.33	0.00	2.02	0.00	1.58	9.92
time (sec)	N/A	0.191	0.353	0.457	0.000	0.629	0.000	0.461	13.399

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	208	264	0	549	0	302	2500
N.S.	1	1.00	1.14	1.44	0.00	3.00	0.00	1.65	13.66
time (sec)	N/A	0.165	0.962	0.446	0.000	0.601	0.000	0.486	13.099

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	259	266	0	770	0	431	2500
N.S.	1	1.00	1.49	1.53	0.00	4.43	0.00	2.48	14.37
time (sec)	N/A	0.250	3.496	0.463	0.000	0.666	0.000	0.518	13.091

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	379	269	0	801	0	317	2500
N.S.	1	1.00	1.92	1.37	0.00	4.07	0.00	1.61	12.69
time (sec)	N/A	0.183	6.142	0.477	0.000	0.669	0.000	0.475	15.568

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	275	448	320	0	1034	0	396	2500
N.S.	1	1.41	2.30	1.64	0.00	5.30	0.00	2.03	12.82
time (sec)	N/A	0.195	6.158	0.507	0.000	0.680	0.000	0.495	13.398

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	307	504	390	0	1079	0	490	1099
N.S.	1	1.27	2.09	1.62	0.00	4.48	0.00	2.03	4.56
time (sec)	N/A	0.715	0.946	0.567	0.000	0.690	0.000	0.473	12.389

Problem 1329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	356	486	0	1462	0	627	1289
N.S.	1	1.00	0.98	1.34	0.00	4.03	0.00	1.73	3.55
time (sec)	N/A	0.943	1.022	0.619	0.000	0.825	0.000	0.501	12.337

Problem 1330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	442	604	0	1645	0	776	1513
N.S.	1	1.00	1.06	1.45	0.00	3.94	0.00	1.86	3.63
time (sec)	N/A	1.175	1.356	0.700	0.000	0.852	0.000	0.492	12.385

Problem 1331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	593	728	0	2082	0	948	1861
N.S.	1	1.00	1.25	1.53	0.00	4.37	0.00	1.99	3.91
time (sec)	N/A	1.389	2.324	0.660	0.000	1.315	0.000	0.508	12.665

Problem 1332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	87	82	100	0	85	134
N.S.	1	1.00	0.89	0.94	0.88	1.08	0.00	0.91	1.44
time (sec)	N/A	0.120	0.145	0.282	0.285	0.390	0.000	0.470	12.272

Problem 1333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	76	68	74	0	71	117
N.S.	1	1.00	0.90	0.95	0.85	0.92	0.00	0.89	1.46
time (sec)	N/A	0.103	0.047	0.217	0.333	0.383	0.000	0.444	11.940

Problem 1334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	87	71	65	63	0	71	91
N.S.	1	1.00	1.18	0.96	0.88	0.85	0.00	0.96	1.23
time (sec)	N/A	0.044	0.060	0.194	0.317	0.383	0.000	0.446	12.020

Problem 1335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	84	87	80	93	0	86	87
N.S.	1	1.00	0.90	0.94	0.86	1.00	0.00	0.92	0.94
time (sec)	N/A	0.098	0.073	0.259	0.274	0.411	0.000	0.445	0.171

Problem 1336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	102	95	143	0	113	98
N.S.	1	1.00	0.88	0.93	0.86	1.30	0.00	1.03	0.89
time (sec)	N/A	0.124	0.168	0.316	0.294	0.487	0.000	0.494	11.919

Problem 1337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	132	120	114	224	0	148	125
N.S.	1	1.00	1.00	0.91	0.86	1.70	0.00	1.12	0.95
time (sec)	N/A	0.138	0.343	0.368	0.289	0.561	0.000	0.485	11.855

Problem 1338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	221	208	0	521	0	208	2098
N.S.	1	1.00	0.82	0.78	0.00	1.94	0.00	0.78	7.83
time (sec)	N/A	0.272	1.040	0.391	0.000	0.400	0.000	0.473	16.815

Problem 1339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	186	150	0	431	0	173	1656
N.S.	1	1.00	1.02	0.82	0.00	2.36	0.00	0.95	9.05
time (sec)	N/A	0.178	0.784	0.335	0.000	0.418	0.000	0.478	14.278

Problem 1340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	152	130	0	369	0	131	1538
N.S.	1	1.00	1.14	0.98	0.00	2.77	0.00	0.98	11.56
time (sec)	N/A	0.120	0.591	0.287	0.000	0.386	0.000	0.493	14.046

Problem 1341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	152	112	0	305	0	107	148
N.S.	1	1.00	1.58	1.17	0.00	3.18	0.00	1.11	1.54
time (sec)	N/A	0.070	0.142	0.240	0.000	0.371	0.000	0.515	11.964

Problem 1342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	151	111	0	308	0	106	151
N.S.	1	1.00	1.84	1.35	0.00	3.76	0.00	1.29	1.84
time (sec)	N/A	0.069	0.133	0.223	0.000	0.390	0.000	0.458	12.019

Problem 1343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	191	122	0	457	0	135	659
N.S.	1	1.00	1.62	1.03	0.00	3.87	0.00	1.14	5.58
time (sec)	N/A	0.160	0.255	0.391	0.000	0.523	0.000	0.460	12.883

Problem 1344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	150	205	155	0	582	0	259	778
N.S.	1	1.17	1.60	1.21	0.00	4.55	0.00	2.02	6.08
time (sec)	N/A	0.193	0.719	0.458	0.000	0.524	0.000	0.496	13.682

Problem 1345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	212	261	199	0	878	0	245	1570
N.S.	1	1.17	1.44	1.10	0.00	4.85	0.00	1.35	8.67
time (sec)	N/A	0.232	2.094	0.593	0.000	0.729	0.000	0.498	15.348

Problem 1346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	121	142	157	0	177	217
N.S.	1	1.00	0.93	0.96	1.13	1.25	0.00	1.40	1.72
time (sec)	N/A	0.141	0.352	0.341	0.281	0.395	0.000	0.483	12.291

Problem 1347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	112	132	154	0	168	206
N.S.	1	1.00	0.93	0.97	1.14	1.33	0.00	1.45	1.78
time (sec)	N/A	0.155	0.320	0.302	0.287	0.397	0.000	0.487	12.216

Problem 1348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	162	112	132	155	0	170	208
N.S.	1	1.00	1.38	0.96	1.13	1.32	0.00	1.45	1.78
time (sec)	N/A	0.111	0.258	0.297	0.292	0.399	0.000	0.456	12.114

Problem 1349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	151	137	156	213	0	210	170
N.S.	1	1.00	0.97	0.88	1.00	1.37	0.00	1.35	1.09
time (sec)	N/A	0.162	0.465	0.422	0.288	0.689	0.000	0.467	12.417

Problem 1350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	174	152	200	287	0	279	195
N.S.	1	1.00	1.02	0.89	1.17	1.68	0.00	1.63	1.14
time (sec)	N/A	0.184	0.535	0.487	0.291	0.816	0.000	0.477	12.403

Problem 1351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	168	172	244	440	0	275	240
N.S.	1	1.00	0.85	0.87	1.24	2.23	0.00	1.40	1.22
time (sec)	N/A	0.210	0.951	0.601	0.296	1.117	0.000	0.514	12.479

Problem 1352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	195	214	0	476	0	241	372
N.S.	1	1.00	1.10	1.21	0.00	2.69	0.00	1.36	2.10
time (sec)	N/A	0.166	1.003	0.425	0.000	0.384	0.000	0.534	17.428

Problem 1353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	184	205	0	465	0	227	370
N.S.	1	1.00	1.30	1.44	0.00	3.27	0.00	1.60	2.61
time (sec)	N/A	0.152	1.014	0.473	0.000	0.383	0.000	0.487	17.158

Problem 1354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	200	207	0	470	0	229	375
N.S.	1	1.00	1.21	1.25	0.00	2.85	0.00	1.39	2.27
time (sec)	N/A	0.153	0.902	0.423	0.000	0.399	0.000	0.488	17.402

Problem 1355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	203	215	0	469	0	240	378
N.S.	1	1.00	1.47	1.56	0.00	3.40	0.00	1.74	2.74
time (sec)	N/A	0.145	0.899	0.396	0.000	0.387	0.000	0.477	17.061

Problem 1356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	334	221	0	680	0	308	2162
N.S.	1	1.00	1.72	1.14	0.00	3.51	0.00	1.59	11.14
time (sec)	N/A	0.274	3.216	0.634	0.000	0.764	0.000	0.511	16.203

Problem 1357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	247	450	253	0	831	0	357	2317
N.S.	1	1.12	2.05	1.15	0.00	3.78	0.00	1.62	10.53
time (sec)	N/A	0.313	6.319	0.681	0.000	0.770	0.000	0.507	17.384

Problem 1358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	947	298	0	1182	0	417	2500
N.S.	1	1.00	2.85	0.90	0.00	3.56	0.00	1.26	7.53
time (sec)	N/A	0.354	6.161	0.821	0.000	1.224	0.000	0.504	18.007

Problem 1359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	212	217	316	429	0	403	806
N.S.	1	1.00	0.88	0.90	1.32	1.79	0.00	1.68	3.36
time (sec)	N/A	0.448	1.976	0.759	0.382	0.639	0.000	0.527	14.185

Problem 1360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	198	203	303	351	0	384	627
N.S.	1	1.00	0.90	0.92	1.37	1.59	0.00	1.74	2.84
time (sec)	N/A	0.378	1.504	0.671	0.304	0.569	0.000	0.542	14.068

Problem 1361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	187	193	289	303	0	371	549
N.S.	1	1.00	0.90	0.93	1.39	1.46	0.00	1.78	2.64
time (sec)	N/A	0.369	1.128	0.598	0.321	0.596	0.000	0.536	13.381

Problem 1362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	189	288	261	0	343	498
N.S.	1	1.00	0.90	0.93	1.41	1.28	0.00	1.68	2.44
time (sec)	N/A	0.253	0.928	0.514	0.306	0.480	0.000	0.538	12.542

Problem 1363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	169	175	276	261	0	333	507
N.S.	1	1.00	0.89	0.92	1.45	1.37	0.00	1.75	2.67
time (sec)	N/A	0.304	1.063	0.509	0.299	0.470	0.000	0.527	12.455

Problem 1364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	166	176	267	260	0	326	471
N.S.	1	1.00	0.91	0.97	1.47	1.43	0.00	1.79	2.59
time (sec)	N/A	0.253	0.840	0.497	0.290	0.469	0.000	0.524	12.448

Problem 1365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	163	173	265	258	0	325	498
N.S.	1	1.00	0.92	0.97	1.49	1.45	0.00	1.83	2.80
time (sec)	N/A	0.251	0.816	0.469	0.296	0.458	0.000	0.511	12.515

Problem 1366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	244	170	267	255	0	323	483
N.S.	1	1.00	1.38	0.96	1.51	1.44	0.00	1.82	2.73
time (sec)	N/A	0.175	0.627	0.483	0.298	0.488	0.000	0.506	12.402

Problem 1367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	220	203	299	344	0	391	346
N.S.	1	1.00	0.94	0.87	1.28	1.48	0.00	1.68	1.48
time (sec)	N/A	0.261	1.816	0.687	0.303	1.300	0.000	0.483	12.428

Problem 1368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	234	219	361	425	0	418	373
N.S.	1	1.00	0.94	0.88	1.44	1.70	0.00	1.67	1.49
time (sec)	N/A	0.284	4.310	0.691	0.300	1.757	0.000	0.479	12.473

Problem 1369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	259	238	422	640	0	589	412
N.S.	1	1.00	0.95	0.87	1.54	2.34	0.00	2.15	1.50
time (sec)	N/A	0.318	6.092	0.940	0.316	2.453	0.000	0.511	12.699

Problem 1370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	816	1206	0	0	0	0	-1
N.S.	1	1.00	1.63	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.749	36.732	41.267	0.000	0.000	0.000	0.000	0.000

Problem 1371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	789	1345	0	0	0	0	-1
N.S.	1	1.00	1.76	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	36.494	44.556	0.000	0.000	0.000	0.000	0.000

Problem 1372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	372	882	0	0	0	0	-1
N.S.	1	1.00	1.01	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	16.367	38.823	0.000	0.000	0.000	0.000	0.000

Problem 1373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	351	693	0	0	0	0	-1
N.S.	1	1.00	1.03	2.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.457	16.282	42.012	0.000	0.000	0.000	0.000	0.000

Problem 1374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	534	186	0	0	0	0	-1
N.S.	1	1.00	1.50	0.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	23.481	16.746	0.000	0.000	0.000	0.000	0.000

Problem 1375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	1550	1266	0	0	0	0	-1
N.S.	1	1.00	3.58	2.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.593	57.635	78.509	0.000	0.000	0.000	0.000	0.000

Problem 1376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	1582	293	0	0	0	0	-1
N.S.	1	1.00	2.91	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.663	60.532	17.587	0.000	0.000	0.000	0.000	0.000

Problem 1377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	1991	1234	0	0	0	0	-1
N.S.	1	1.00	3.21	1.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.965	57.159	59.553	0.000	0.000	0.000	0.000	0.000

Problem 1378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	1953	1067	0	0	0	0	-1
N.S.	1	1.00	3.80	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	56.203	54.242	0.000	0.000	0.000	0.000	0.000

Problem 1379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	1909	888	0	0	0	0	-1
N.S.	1	1.00	4.48	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.648	44.776	44.178	0.000	0.000	0.000	0.000	0.000

Problem 1380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	484	216	0	0	0	0	-1
N.S.	1	1.00	1.10	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.766	16.454	18.804	0.000	0.000	0.000	0.000	0.000

Problem 1381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	2099	1451	0	0	0	0	-1
N.S.	1	1.00	4.48	3.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.824	57.100	75.704	0.000	0.000	0.000	0.000	0.000

Problem 1382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	2129	298	0	0	0	0	-1
N.S.	1	1.00	3.71	0.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.903	61.528	18.086	0.000	0.000	0.000	0.000	0.000

Problem 1383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	790	2207	0	0	0	0	-1
N.S.	1	1.00	1.30	3.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.886	34.151	54.829	0.000	0.000	0.000	0.000	0.000

Problem 1384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	824	1382	0	0	0	0	-1
N.S.	1	1.00	1.64	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	35.429	44.555	0.000	0.000	0.000	0.000	0.000

Problem 1385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	737	1597	0	0	0	0	-1
N.S.	1	1.00	1.78	3.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.621	35.874	51.911	0.000	0.000	0.000	0.000	0.000

Problem 1386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	484	257	0	0	0	0	-1
N.S.	1	1.00	1.14	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.759	43.370	20.598	0.000	0.000	0.000	0.000	0.000

Problem 1387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	1465	1465	0	0	0	0	-1
N.S.	1	1.00	3.17	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.819	58.016	79.136	0.000	0.000	0.000	0.000	0.000

Problem 1388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	1590	304	0	0	0	0	-1
N.S.	1	1.00	2.85	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.861	60.625	20.981	0.000	0.000	0.000	0.000	0.000

Problem 1389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1953	1086	0	0	0	0	-1
N.S.	1	1.00	3.84	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.977	45.835	42.808	0.000	0.000	0.000	0.000	0.000

Problem 1390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	1915	934	0	0	0	0	-1
N.S.	1	1.00	4.19	2.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.756	45.220	39.782	0.000	0.000	0.000	0.000	0.000

Problem 1391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	572	821	0	0	0	0	-1
N.S.	1	1.00	1.51	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	38.270	39.477	0.000	0.000	0.000	0.000	0.000

Problem 1392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	546	687	0	0	0	0	-1
N.S.	1	1.00	1.55	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	9.808	38.138	0.000	0.000	0.000	0.000	0.000

Problem 1393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	663	185	0	0	0	0	-1
N.S.	1	1.00	1.80	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.540	28.773	15.823	0.000	0.000	0.000	0.000	0.000

Problem 1394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	2093	1219	0	0	0	0	-1
N.S.	1	1.00	4.67	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	60.642	77.553	0.000	0.000	0.000	0.000	0.000

Problem 1395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	2129	301	0	0	0	0	-1
N.S.	1	1.00	3.82	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.687	59.402	19.346	0.000	0.000	0.000	0.000	0.000

Problem 1396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	584	584	820	1216	0	0	0	0	-1
N.S.	1	1.00	1.40	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	37.111	57.964	0.000	0.000	0.000	0.000	0.000

Problem 1397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	793	1023	0	0	0	0	-1
N.S.	1	1.00	1.56	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	37.161	67.405	0.000	0.000	0.000	0.000	0.000

Problem 1398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	785	1057	0	0	0	0	-1
N.S.	1	1.00	1.73	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.554	28.968	52.019	0.000	0.000	0.000	0.000	0.000

Problem 1399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	783	1104	0	0	0	0	-1
N.S.	1	1.00	1.90	2.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.586	28.718	57.178	0.000	0.000	0.000	0.000	0.000

Problem 1400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	507	507	1587	425	0	0	0	0	-1
N.S.	1	1.00	3.13	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.875	58.847	28.872	0.000	0.000	0.000	0.000	0.000

Problem 1401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	1635	1523	0	0	0	0	-1
N.S.	1	1.00	2.61	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.950	60.390	111.428	0.000	0.000	0.000	0.000	0.000

Problem 1402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	1958	1049	0	0	0	0	-1
N.S.	1	1.00	3.26	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.930	45.889	70.964	0.000	0.000	0.000	0.000	0.000

Problem 1403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	528	528	1193	1152	0	0	0	0	-1
N.S.	1	1.00	2.26	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	34.956	78.915	0.000	0.000	0.000	0.000	0.000

Problem 1404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	1184	940	0	0	0	0	-1
N.S.	1	1.00	2.53	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.580	34.323	70.083	0.000	0.000	0.000	0.000	0.000

Problem 1405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	1183	1117	0	0	0	0	-1
N.S.	1	1.00	2.74	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	33.973	81.331	0.000	0.000	0.000	0.000	0.000

Problem 1406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	2136	627	0	0	0	0	-1
N.S.	1	1.00	4.05	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.900	60.114	27.730	0.000	0.000	0.000	0.000	0.000

Problem 1407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	F(-2)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	2183	1571	0	0	0	0	-1
N.S.	1	1.00	3.35	2.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.008	61.246	127.346	0.000	0.000	0.000	0.000	0.000

Problem 1408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	1623	4649	0	0	0	0	-1
N.S.	1	1.00	1.75	5.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.155	56.824	13.204	0.000	0.000	0.000	0.000	0.000

Problem 1409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	176	3290	0	0	0	0	-1
N.S.	1	1.00	0.30	5.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.725	27.631	0.557	0.000	0.000	0.000	0.000	0.000

Problem 1410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	178	744	0	0	0	0	-1
N.S.	1	1.00	0.35	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	19.677	0.368	0.000	0.000	0.000	0.000	0.000

Problem 1411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	371	590	0	0	0	0	-1
N.S.	1	1.00	1.78	2.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	32.677	0.308	0.000	0.000	0.000	0.000	0.000

Problem 1412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	1619	2497	0	0	0	0	-1
N.S.	1	1.00	5.06	7.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.481	52.815	0.356	0.000	0.000	0.000	0.000	0.000

Problem 1413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	1645	2672	0	0	0	0	-1
N.S.	1	1.00	4.49	7.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.650	52.925	0.316	0.000	0.000	0.000	0.000	0.000

Problem 1414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	1726	6208	0	0	0	0	-1
N.S.	1	1.00	3.36	12.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.920	54.054	0.375	0.000	0.000	0.000	0.000	0.000

Problem 1415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	598	598	1768	6593	0	0	0	0	-1
N.S.	1	1.00	2.96	11.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.178	54.543	0.460	0.000	0.000	0.000	0.000	0.000

Problem 1416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	982	982	1898	2547	0	0	0	0	-1
N.S.	1	1.00	1.93	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.107	57.723	0.598	0.000	0.000	0.000	0.000	0.000

Problem 1417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	604	1926	0	0	0	0	-1
N.S.	1	1.00	0.99	3.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.659	29.180	0.380	0.000	0.000	0.000	0.000	0.000

Problem 1418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	178	944	0	0	0	0	-1
N.S.	1	1.00	0.31	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	21.379	0.310	0.000	0.000	0.000	0.000	0.000

Problem 1419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	1095	2587	0	0	0	0	-1
N.S.	1	1.00	3.41	8.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	37.703	0.322	0.000	0.000	0.000	0.000	0.000

Problem 1420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	1138	3014	0	0	0	0	-1
N.S.	1	1.00	2.62	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	40.037	0.303	0.000	0.000	0.000	0.000	0.000

Problem 1421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1165	5828	0	0	0	0	-1
N.S.	1	1.00	2.22	11.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.863	39.859	0.351	0.000	0.000	0.000	0.000	0.000

Problem 1422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	688	688	1210	6707	0	0	0	0	-1
N.S.	1	1.00	1.76	9.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.120	40.541	0.422	0.000	0.000	0.000	0.000	0.000

Problem 1423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	936	936	1615	6311	0	0	0	0	-1
N.S.	1	1.00	1.73	6.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.993	55.354	0.605	0.000	0.000	0.000	0.000	0.000

Problem 1424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	572	572	1399	5224	0	0	0	0	-1
N.S.	1	1.00	2.45	9.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.645	54.507	0.376	0.000	0.000	0.000	0.000	0.000

Problem 1425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	1611	5207	0	0	0	0	-1
N.S.	1	1.00	2.62	8.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	54.247	0.405	0.000	0.000	0.000	0.000	0.000

Problem 1426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	1656	4668	0	0	0	0	-1
N.S.	1	1.00	4.61	13.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	54.383	0.302	0.000	0.000	0.000	0.000	0.000

Problem 1427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	1734	10138	0	0	0	0	-1
N.S.	1	1.00	3.34	19.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	54.402	0.389	0.000	0.000	0.000	0.000	0.000

Problem 1428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	1776	10704	0	0	0	0	-1
N.S.	1	1.00	2.90	17.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.026	54.197	0.466	0.000	0.000	0.000	0.000	0.000

Problem 1429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	822	822	1850	17102	0	0	0	0	-1
N.S.	1	1.00	2.25	20.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.372	54.722	0.457	0.000	0.000	0.000	0.000	0.000

Problem 1430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	616	616	1318	2345	0	0	0	0	-1
N.S.	1	1.00	2.14	3.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	54.604	0.385	0.000	0.000	0.000	0.000	0.000

Problem 1431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	518	941	0	0	0	0	-1
N.S.	1	1.00	1.02	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	21.093	0.389	0.000	0.000	0.000	0.000	0.000

Problem 1432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	373	527	0	0	0	0	-1
N.S.	1	1.00	1.78	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	32.418	0.339	0.000	0.000	0.000	0.000	0.000

Problem 1433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	652	631	0	0	0	0	-1
N.S.	1	1.00	2.39	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	24.037	0.323	0.000	0.000	0.000	0.000	0.000

Problem 1434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	715	2286	0	0	0	0	-1
N.S.	1	1.00	2.23	7.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.542	25.314	0.345	0.000	0.000	0.000	0.000	0.000

Problem 1435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	1140	2987	0	0	0	0	-1
N.S.	1	1.00	2.69	7.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.766	40.667	0.394	0.000	0.000	0.000	0.000	0.000

Problem 1436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	0	4619	0	0	0	0	-1
N.S.	1	1.00	0.00	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.084	180.011	4.959	0.000	0.000	0.000	0.000	0.000

Problem 1437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	1648	2540	0	0	0	0	-1
N.S.	1	1.00	4.35	6.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	52.284	0.457	0.000	0.000	0.000	0.000	0.000

Problem 1438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	1274	2539	0	0	0	0	-1
N.S.	1	1.00	3.41	6.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	41.090	0.430	0.000	0.000	0.000	0.000	0.000

Problem 1439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	1279	2559	0	0	0	0	-1
N.S.	1	1.00	3.37	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	41.517	0.329	0.000	0.000	0.000	0.000	0.000

Problem 1440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	1707	3104	0	0	0	0	-1
N.S.	1	1.00	3.01	5.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.925	54.465	0.354	0.000	0.000	0.000	0.000	0.000

Problem 1441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	1727	3315	0	0	0	0	-1
N.S.	1	1.00	2.57	4.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.149	54.778	0.450	0.000	0.000	0.000	0.000	0.000

Problem 1442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	717	3490	0	0	0	0	-1
N.S.	1	1.00	2.17	10.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	25.317	0.630	0.000	0.000	0.000	0.000	0.000

Problem 1443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	104	75	72	0	119	112
N.S.	1	1.00	1.00	1.27	0.91	0.88	0.00	1.45	1.37
time (sec)	N/A	0.090	0.333	0.164	0.499	0.587	0.000	0.464	18.223

Problem 1444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	63	94	62	61	0	104	98
N.S.	1	1.00	0.97	1.45	0.95	0.94	0.00	1.60	1.51
time (sec)	N/A	0.069	0.137	0.131	0.571	0.390	0.000	0.493	15.966

Problem 1445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	39	47	0	58	55
N.S.	1	1.00	1.24	1.55	1.03	1.24	0.00	1.53	1.45
time (sec)	N/A	0.041	0.026	0.115	0.514	0.392	0.000	0.490	12.480

Problem 1446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	32	32	36	0	43	41
N.S.	1	1.00	1.33	1.19	1.19	1.33	0.00	1.59	1.52
time (sec)	N/A	0.030	0.013	0.102	0.518	0.400	0.000	0.438	11.935

Problem 1447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	56	41	48	65	0	48	52
N.S.	1	1.00	1.56	1.14	1.33	1.81	0.00	1.33	1.44
time (sec)	N/A	0.051	0.029	0.156	0.281	0.409	0.000	0.457	11.936

Problem 1448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	68	61	59	96	0	103	92
N.S.	1	1.00	1.42	1.27	1.23	2.00	0.00	2.15	1.92
time (sec)	N/A	0.074	0.065	0.145	0.270	0.406	0.000	0.476	11.907

Problem 1449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	172	83	84	128	0	116	127
N.S.	1	1.00	2.29	1.11	1.12	1.71	0.00	1.55	1.69
time (sec)	N/A	0.090	0.258	0.184	0.271	0.440	0.000	0.472	11.929

Problem 1450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	104	147	97	91	0	172	149
N.S.	1	1.00	1.11	1.56	1.03	0.97	0.00	1.83	1.59
time (sec)	N/A	0.120	0.321	0.184	0.490	0.365	0.000	0.499	18.494

Problem 1451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	83	81	0	137	145
N.S.	1	1.00	0.82	1.23	0.88	0.86	0.00	1.46	1.54
time (sec)	N/A	0.088	0.322	0.151	0.491	0.374	0.000	0.540	17.039

Problem 1452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	66	75	56	59	0	82	81
N.S.	1	1.00	1.57	1.79	1.33	1.40	0.00	1.95	1.93
time (sec)	N/A	0.039	0.228	0.170	0.518	0.368	0.000	0.443	12.253

Problem 1453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	70	58	57	64	77	0	57	62
N.S.	1	1.52	1.26	1.24	1.39	1.67	0.00	1.24	1.35
time (sec)	N/A	0.106	0.154	0.243	0.281	0.519	0.000	0.507	11.849

Problem 1454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	102	75	71	113	0	128	108
N.S.	1	1.00	1.73	1.27	1.20	1.92	0.00	2.17	1.83
time (sec)	N/A	0.187	0.253	0.239	0.299	0.371	0.000	0.501	11.872

Problem 1455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	124	238	116	123	186	0	157	148
N.S.	1	1.24	2.38	1.16	1.23	1.86	0.00	1.57	1.48
time (sec)	N/A	0.168	0.341	0.309	0.292	0.385	0.000	0.475	11.854

Problem 1456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	196	136	123	192	0	204	194
N.S.	1	1.00	1.88	1.31	1.18	1.85	0.00	1.96	1.87
time (sec)	N/A	0.146	0.720	0.304	0.295	0.389	0.000	0.471	11.862

Problem 1457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	147	214	164	135	0	336	323
N.S.	1	1.00	0.75	1.09	0.83	0.69	0.00	1.71	1.64
time (sec)	N/A	0.171	0.542	0.190	0.550	0.405	0.000	0.497	16.023

Problem 1458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	119	116	0	207	249
N.S.	1	1.00	0.77	1.16	0.82	0.79	0.00	1.42	1.71
time (sec)	N/A	0.130	0.547	0.181	0.480	0.358	0.000	0.504	16.657

Problem 1459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	132	99	90	0	148	219
N.S.	1	1.00	1.21	1.76	1.32	1.20	0.00	1.97	2.92
time (sec)	N/A	0.044	0.434	0.181	0.496	0.375	0.000	0.481	16.288

Problem 1460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	79	86	99	0	86	154
N.S.	1	1.00	1.06	1.01	1.10	1.27	0.00	1.10	1.97
time (sec)	N/A	0.094	0.231	0.291	0.481	0.382	0.000	0.512	11.930

Problem 1461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	114	91	90	131	0	148	120
N.S.	1	1.00	1.31	1.05	1.03	1.51	0.00	1.70	1.38
time (sec)	N/A	0.127	0.287	0.273	0.299	0.565	0.000	0.519	11.906

Problem 1462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	267	130	137	196	0	179	166
N.S.	1	1.00	2.02	0.98	1.04	1.48	0.00	1.36	1.26
time (sec)	N/A	0.145	0.410	0.309	0.272	0.428	0.000	0.565	11.913

Problem 1463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	287	169	162	252	0	245	218
N.S.	1	1.00	1.75	1.03	0.99	1.54	0.00	1.49	1.33
time (sec)	N/A	0.176	0.989	0.319	0.281	0.379	0.000	0.696	11.888

Problem 1464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	236	184	0	724	0	264	2500
N.S.	1	1.00	1.06	0.83	0.00	3.26	0.00	1.19	11.26
time (sec)	N/A	0.240	1.460	0.551	0.000	0.425	0.000	0.646	19.769

Problem 1465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	162	155	0	534	0	222	276
N.S.	1	1.00	0.76	0.73	0.00	2.52	0.00	1.05	1.30
time (sec)	N/A	0.188	0.719	0.467	0.000	0.388	0.000	0.584	15.276

Problem 1466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	169	162	0	569	0	251	313
N.S.	1	1.00	0.84	0.81	0.00	2.84	0.00	1.26	1.56
time (sec)	N/A	0.215	0.631	0.434	0.000	0.426	0.000	0.564	15.961

Problem 1467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	169	164	0	557	0	243	310
N.S.	1	1.00	1.27	1.23	0.00	4.19	0.00	1.83	2.33
time (sec)	N/A	0.138	0.648	0.424	0.000	0.409	0.000	0.570	16.322

Problem 1468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	203	185	0	957	0	314	2076
N.S.	1	1.00	0.89	0.81	0.00	4.18	0.00	1.37	9.07
time (sec)	N/A	0.201	1.620	0.701	0.000	0.904	0.000	0.606	14.862

Problem 1469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	254	215	0	1355	0	523	2151
N.S.	1	1.00	1.02	0.87	0.00	5.46	0.00	2.11	8.67
time (sec)	N/A	0.243	2.330	0.759	0.000	0.903	0.000	0.591	13.332

Problem 1470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	356	261	0	1844	0	423	2302
N.S.	1	1.00	1.21	0.88	0.00	6.25	0.00	1.43	7.80
time (sec)	N/A	0.264	6.350	0.799	0.000	1.450	0.000	0.560	14.093

Problem 1471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	195	222	0	906	0	351	585
N.S.	1	1.00	0.50	0.57	0.00	2.34	0.00	0.90	1.51
time (sec)	N/A	0.393	2.357	0.787	0.000	0.423	0.000	0.687	18.233

Problem 1472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	204	243	0	917	0	377	583
N.S.	1	1.00	0.56	0.66	0.00	2.51	0.00	1.03	1.59
time (sec)	N/A	0.329	2.509	0.698	0.000	0.417	0.000	0.555	17.851

Problem 1473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	258	0	934	0	384	627
N.S.	1	1.00	0.61	0.74	0.00	2.67	0.00	1.10	1.79
time (sec)	N/A	0.384	2.377	0.618	0.000	0.432	0.000	0.689	18.207

Problem 1474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	206	242	0	895	0	365	624
N.S.	1	1.00	1.01	1.19	0.00	4.39	0.00	1.79	3.06
time (sec)	N/A	0.240	2.244	0.619	0.000	0.424	0.000	0.644	18.107

Problem 1475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	278	278	0	1623	0	411	2500
N.S.	1	1.00	0.69	0.69	0.00	4.04	0.00	1.02	6.22
time (sec)	N/A	0.351	5.390	0.971	0.000	1.872	0.000	0.565	17.794

Problem 1476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	379	311	0	2140	0	633	2500
N.S.	1	1.00	0.89	0.73	0.00	5.05	0.00	1.49	5.90
time (sec)	N/A	0.397	6.257	1.100	0.000	1.620	0.000	0.714	15.777

Problem 1477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	432	355	0	2672	0	900	2500
N.S.	1	1.00	0.92	0.76	0.00	5.69	0.00	1.91	5.32
time (sec)	N/A	0.426	6.917	1.286	0.000	2.769	0.000	0.664	16.824

Problem 1478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	198	650	0	0	0	0	-1
N.S.	1	1.00	1.25	4.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	23.749	3.545	0.000	0.000	0.000	0.000	0.000

Problem 1479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	0	4593	2313	0	0	0	0	-1
N.S.	1	0.00	14.72	7.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.114	32.899	0.315	0.000	0.000	0.000	0.000	0.000

Problem 1480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	366	0	4665	2426	0	0	0	0	-1
N.S.	1	0.00	12.75	6.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	32.704	0.405	0.000	0.000	0.000	0.000	0.000

Problem 1481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	177	132	149	0	135	346
N.S.	1	1.00	1.01	1.14	0.85	0.96	0.00	0.87	2.23
time (sec)	N/A	0.110	0.446	0.260	0.306	0.373	0.000	0.585	12.271

Problem 1482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	167	121	138	0	124	304
N.S.	1	1.00	0.99	1.24	0.90	1.02	0.00	0.92	2.25
time (sec)	N/A	0.092	0.282	0.224	0.293	0.376	0.000	0.581	12.157

Problem 1483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	121	108	114	0	108	261
N.S.	1	1.00	1.06	1.04	0.93	0.98	0.00	0.93	2.25
time (sec)	N/A	0.074	0.224	0.215	0.292	0.445	0.000	0.529	12.099

Problem 1484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	106	111	100	104	0	100	221
N.S.	1	1.00	1.03	1.08	0.97	1.01	0.00	0.97	2.15
time (sec)	N/A	0.085	0.239	0.200	0.296	0.363	0.000	0.590	12.030

Problem 1485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	84	98	89	93	0	81	144
N.S.	1	1.00	1.14	1.32	1.20	1.26	0.00	1.09	1.95
time (sec)	N/A	0.091	0.196	0.208	0.270	0.357	0.000	0.550	18.080

Problem 1486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	88	86	91	0	78	144
N.S.	1	1.00	1.00	1.19	1.16	1.23	0.00	1.05	1.95
time (sec)	N/A	0.093	0.019	0.174	0.355	0.370	0.000	0.535	18.069

Problem 1487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	80	75	80	0	67	158
N.S.	1	1.00	1.00	1.08	1.01	1.08	0.00	0.91	2.14
time (sec)	N/A	0.071	0.019	0.182	0.290	0.359	0.000	0.519	18.539

Problem 1488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	82	109	125	0	113	116
N.S.	1	1.00	1.00	0.83	1.10	1.26	0.00	1.14	1.17
time (sec)	N/A	0.074	0.855	0.252	0.276	0.369	0.000	0.518	11.874

Problem 1489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	76	101	126	159	0	134	130
N.S.	1	1.00	0.66	0.88	1.10	1.38	0.00	1.17	1.13
time (sec)	N/A	0.103	0.273	0.264	0.297	0.366	0.000	0.575	11.887

Problem 1490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	129	140	211	0	133	146
N.S.	1	1.00	0.64	0.96	1.04	1.56	0.00	0.99	1.08
time (sec)	N/A	0.110	0.404	0.279	0.281	0.374	0.000	0.521	0.111

Problem 1491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	147	151	248	0	160	157
N.S.	1	1.00	0.58	0.95	0.97	1.60	0.00	1.03	1.01
time (sec)	N/A	0.115	0.489	0.260	0.267	0.375	0.000	0.638	11.913

Problem 1492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	186	266	180	198	0	198	433
N.S.	1	1.00	0.98	1.41	0.95	1.05	0.00	1.05	2.29
time (sec)	N/A	0.238	1.123	0.335	0.345	0.391	0.000	0.533	12.333

Problem 1493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	164	205	157	178	0	175	377
N.S.	1	1.00	1.01	1.27	0.97	1.10	0.00	1.08	2.33
time (sec)	N/A	0.185	1.440	0.296	0.358	0.396	0.000	0.607	12.224

Problem 1494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	151	200	148	151	0	157	332
N.S.	1	1.00	1.01	1.33	0.99	1.01	0.00	1.05	2.21
time (sec)	N/A	0.189	0.688	0.273	0.327	0.395	0.000	0.550	12.211

Problem 1495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	129	136	123	127	0	130	247
N.S.	1	1.00	1.11	1.17	1.06	1.09	0.00	1.12	2.13
time (sec)	N/A	0.147	0.270	0.280	0.270	0.368	0.000	0.509	12.087

Problem 1496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	85	166	120	124	0	124	191
N.S.	1	1.00	0.91	1.78	1.29	1.33	0.00	1.33	2.05
time (sec)	N/A	0.123	0.542	0.257	0.278	0.359	0.000	0.487	17.052

Problem 1497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	215	105	97	104	0	89	183
N.S.	1	1.00	2.99	1.46	1.35	1.44	0.00	1.24	2.54
time (sec)	N/A	0.068	2.413	0.254	0.296	0.381	0.000	0.464	18.871

Problem 1498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	137	99	130	144	0	134	131
N.S.	1	1.00	1.09	0.79	1.03	1.14	0.00	1.06	1.04
time (sec)	N/A	0.143	0.639	0.400	0.273	0.371	0.000	0.501	0.134

Problem 1499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	162	153	163	202	0	186	169
N.S.	1	1.00	0.96	0.91	0.97	1.20	0.00	1.11	1.01
time (sec)	N/A	0.229	1.930	0.401	0.281	0.490	0.000	0.561	11.869

Problem 1500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	182	165	183	285	0	190	194
N.S.	1	1.00	0.98	0.89	0.99	1.54	0.00	1.03	1.05
time (sec)	N/A	0.261	2.416	0.396	0.309	0.370	0.000	0.556	0.135

Problem 1501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	199	304	217	238	0	251	512
N.S.	1	1.00	0.99	1.50	1.07	1.18	0.00	1.24	2.53
time (sec)	N/A	0.225	0.696	0.355	0.274	0.401	0.000	0.618	12.119

Problem 1502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	174	284	190	208	0	221	449
N.S.	1	1.00	0.98	1.60	1.07	1.18	0.00	1.25	2.54
time (sec)	N/A	0.240	0.392	0.328	0.342	0.404	0.000	0.559	12.109

Problem 1503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	147	225	173	176	0	188	356
N.S.	1	1.00	1.04	1.58	1.22	1.24	0.00	1.32	2.51
time (sec)	N/A	0.195	0.320	0.345	0.290	0.375	0.000	0.519	12.094

Problem 1504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	140	204	151	156	0	168	299
N.S.	1	1.00	0.97	1.42	1.05	1.08	0.00	1.17	2.08
time (sec)	N/A	0.165	0.273	0.312	0.336	0.376	0.000	0.522	12.136

Problem 1505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	370	183	140	143	0	142	228
N.S.	1	1.00	4.11	2.03	1.56	1.59	0.00	1.58	2.53
time (sec)	N/A	0.073	0.994	0.314	0.289	0.370	0.000	0.532	16.993

Problem 1506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	157	167	160	173	0	175	169
N.S.	1	1.00	0.95	1.01	0.97	1.05	0.00	1.06	1.02
time (sec)	N/A	0.166	0.391	0.423	0.272	0.369	0.000	0.541	11.906

Problem 1507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	161	170	188	226	0	210	182
N.S.	1	1.00	0.94	0.99	1.10	1.32	0.00	1.23	1.06
time (sec)	N/A	0.253	0.836	0.368	0.368	0.393	0.000	0.568	0.134

Problem 1508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	190	217	217	337	0	240	221
N.S.	1	1.00	0.86	0.98	0.98	1.52	0.00	1.09	1.00
time (sec)	N/A	0.296	2.070	0.410	0.276	0.422	0.000	0.566	11.949

Problem 1509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	164	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.143	1.209	0.000	0.000	0.000	0.000	0.000

Problem 1510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	158	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.113	0.681	0.000	0.000	0.000	0.000	0.000

Problem 1511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	158	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.147	0.630	0.000	0.000	0.000	0.000	0.000

Problem 1512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.083	0.182	0.000	0.000	0.000	0.000	0.000

Problem 1513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	241	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.311	0.209	0.000	0.000	0.000	0.000	0.000

Problem 1514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.385	18.419	0.126	0.000	0.000	0.000	0.000	0.000

Problem 1515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	B	F	F	F(-1)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	502	0	1600	5434	0	0	0	0	-1
N.S.	1	0.00	3.19	10.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	33.448	0.918	0.000	0.000	0.000	0.000	0.000

Problem 1516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	573	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.811	6.328	0.619	0.000	0.000	0.000	0.000	0.000

Problem 1517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	398	0	0	0	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	3.639	0.339	0.000	0.000	0.000	0.000	0.000

Problem 1518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	301	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.551	0.086	0.000	0.000	0.000	0.000	0.000

Problem 1519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F(-2)	F(-1)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.127	180.010	0.417	0.000	0.000	0.000	0.000	0.000

Problem 1520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	F(-1)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.123	180.006	2.108	0.000	0.000	0.000	0.000	0.000

Problem 1521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.074	5.670	0.224	0.000	0.000	0.000	0.000	0.000

Problem 1522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.114	78.131	0.174	0.000	0.000	0.000	0.000	0.000

Problem 1523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.017	9.157	0.664	0.000	0.000	0.000	0.000	0.000

Problem 1524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	375	373	0	0	0	0	0	0	-1
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	3.295	0.330	0.000	0.000	0.000	0.000	0.000

Problem 1525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.235	0.079	0.000	0.000	0.000	0.000	0.000

Problem 1526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.082	3.079	0.389	0.000	0.000	0.000	0.000	0.000

Problem 1527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	41.782	1.696	0.000	0.000	0.000	0.000	0.000

Problem 1528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	151	128	151	106	228	182	156
N.S.	1	1.00	0.80	0.68	0.80	0.56	1.21	0.97	0.83
time (sec)	N/A	0.169	0.868	0.639	0.279	0.384	1.303	0.533	0.127

Problem 1529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	116	108	116	88	178	145	118
N.S.	1	1.00	0.81	0.76	0.81	0.62	1.24	1.01	0.83
time (sec)	N/A	0.117	0.320	0.405	0.278	0.439	0.625	0.577	12.057

Problem 1530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	80	70	128	100	83
N.S.	1	1.00	0.82	0.91	0.82	0.72	1.32	1.03	0.86
time (sec)	N/A	0.075	0.264	0.303	0.278	0.357	0.267	0.521	11.991

Problem 1531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	44	45	51	75	52	44
N.S.	1	1.00	0.87	0.85	0.87	0.98	1.44	1.00	0.85
time (sec)	N/A	0.035	0.059	0.168	0.277	0.350	0.116	0.520	12.037

Problem 1532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	68	71	64	66	0	67	53
N.S.	1	1.00	1.06	1.11	1.00	1.03	0.00	1.05	0.83
time (sec)	N/A	0.069	0.026	0.231	0.293	0.380	0.000	0.516	0.121

Problem 1533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	110	78	92	0	84	63
N.S.	1	1.00	0.92	1.86	1.32	1.56	0.00	1.42	1.07
time (sec)	N/A	0.052	0.166	0.296	0.288	0.386	0.000	0.442	0.110

Problem 1534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	82	141	112	114	0	114	91
N.S.	1	1.00	0.93	1.60	1.27	1.30	0.00	1.30	1.03
time (sec)	N/A	0.061	0.425	0.361	0.297	0.367	0.000	0.531	0.141

Problem 1535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	169	143	135	0	139	120
N.S.	1	1.00	0.88	1.43	1.21	1.14	0.00	1.18	1.02
time (sec)	N/A	0.071	0.616	0.407	0.286	0.390	0.000	0.450	12.442

Problem 1536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	295	229	238	174	389	279	236
N.S.	1	1.00	0.85	0.66	0.68	0.50	1.11	0.80	0.68
time (sec)	N/A	0.276	0.998	0.875	0.287	0.415	1.834	0.607	0.180

Problem 1537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	227	199	184	147	309	231	180
N.S.	1	1.00	0.98	0.86	0.80	0.64	1.34	1.00	0.78
time (sec)	N/A	0.187	0.343	0.667	0.280	0.400	0.938	0.616	0.107

Problem 1538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	111	169	128	120	228	168	127
N.S.	1	1.00	0.84	1.28	0.97	0.91	1.73	1.27	0.96
time (sec)	N/A	0.120	0.172	0.435	0.272	0.357	0.435	0.489	12.298

Problem 1539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	73	74	92	117	86	71
N.S.	1	1.00	0.76	1.35	1.37	1.70	2.17	1.59	1.31
time (sec)	N/A	0.056	0.049	0.208	0.266	0.356	0.189	0.463	0.066

Problem 1540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	81	131	109	111	0	129	80
N.S.	1	1.00	0.86	1.39	1.16	1.18	0.00	1.37	0.85
time (sec)	N/A	0.119	0.146	0.281	0.285	0.378	0.000	0.459	12.370

Problem 1541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	174	187	122	136	0	146	118
N.S.	1	1.00	1.55	1.67	1.09	1.21	0.00	1.30	1.05
time (sec)	N/A	0.125	1.053	0.368	0.277	0.427	0.000	0.558	12.363

Problem 1542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	186	236	171	173	0	187	181
N.S.	1	1.00	1.52	1.93	1.40	1.42	0.00	1.53	1.48
time (sec)	N/A	0.107	1.281	0.426	0.276	0.378	0.000	0.526	12.378

Problem 1543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	242	302	211	203	0	229	220
N.S.	1	1.00	1.51	1.89	1.32	1.27	0.00	1.43	1.38
time (sec)	N/A	0.142	1.033	0.394	0.279	0.398	0.000	0.529	12.445

Problem 1544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	218	506	366	366	0	511	435
N.S.	1	1.00	0.69	1.61	1.16	1.16	0.00	1.62	1.38
time (sec)	N/A	0.255	0.596	0.424	0.270	0.424	0.000	0.520	12.366

Problem 1545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	148	283	220	222	0	286	253
N.S.	1	1.00	0.73	1.40	1.09	1.10	0.00	1.42	1.25
time (sec)	N/A	0.178	0.295	0.312	0.296	0.412	0.000	0.541	0.098

Problem 1546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	125	112	112	0	129	122
N.S.	1	1.00	0.80	1.13	1.01	1.01	0.00	1.16	1.10
time (sec)	N/A	0.114	0.273	0.279	0.275	0.381	0.000	0.535	0.078

Problem 1547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	40	40	38	104	41	41
N.S.	1	1.00	0.95	0.98	0.98	0.93	2.54	1.00	1.00
time (sec)	N/A	0.048	0.036	0.154	0.269	0.374	0.354	0.470	0.063

Problem 1548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	99	89	79	88	0	87	89
N.S.	1	1.00	1.10	0.99	0.88	0.98	0.00	0.97	0.99
time (sec)	N/A	0.106	0.141	0.288	0.273	0.615	0.000	0.455	0.313

Problem 1549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	197	149	175	234	0	260	197
N.S.	1	1.00	1.24	0.94	1.10	1.47	0.00	1.64	1.24
time (sec)	N/A	0.203	0.538	0.428	0.277	0.601	0.000	0.486	0.523

Problem 1550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	321	256	367	413	0	539	427
N.S.	1	1.00	1.22	0.97	1.40	1.57	0.00	2.05	1.62
time (sec)	N/A	0.321	0.891	0.664	0.311	1.115	0.000	0.529	12.955

Problem 1551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	565	393	632	643	0	907	729
N.S.	1	1.00	1.48	1.03	1.65	1.68	0.00	2.37	1.90
time (sec)	N/A	0.478	1.703	1.179	0.306	2.448	0.000	0.537	13.435

Problem 1552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	396	467	377	508	0	570	682
N.S.	1	1.00	1.22	1.44	1.16	1.57	0.00	1.76	2.10
time (sec)	N/A	0.276	1.076	0.510	0.285	0.435	0.000	0.483	12.149

Problem 1553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	234	258	229	322	0	328	290
N.S.	1	1.00	1.14	1.25	1.11	1.56	0.00	1.59	1.41
time (sec)	N/A	0.184	1.432	0.640	0.292	0.414	0.000	0.491	0.120

Problem 1554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	117	118	178	0	188	128
N.S.	1	1.00	0.98	1.04	1.04	1.58	0.00	1.66	1.13
time (sec)	N/A	0.114	0.392	0.502	0.279	0.413	0.000	0.462	12.099

Problem 1555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	47	48	54	178	80	48
N.S.	1	1.00	0.88	0.98	1.00	1.12	3.71	1.67	1.00
time (sec)	N/A	0.053	0.063	0.277	0.277	0.370	0.555	0.541	0.063

Problem 1556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	178	128	147	283	0	205	131
N.S.	1	1.00	1.32	0.95	1.09	2.10	0.00	1.52	0.97
time (sec)	N/A	0.139	0.970	0.545	0.284	0.510	0.000	0.507	0.417

Problem 1557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	246	191	346	598	0	335	327
N.S.	1	1.00	1.08	0.84	1.52	2.62	0.00	1.47	1.43
time (sec)	N/A	0.235	1.211	0.824	0.297	0.865	0.000	0.577	12.662

Problem 1558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	297	659	881	0	761	615
N.S.	1	1.00	0.99	0.80	1.77	2.37	0.00	2.05	1.65
time (sec)	N/A	0.401	3.538	1.334	0.311	1.909	0.000	0.557	13.462

Problem 1559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	766	435	1083	1244	0	1185	1024
N.S.	1	1.00	1.39	0.79	1.97	2.26	0.00	2.15	1.86
time (sec)	N/A	0.652	6.150	1.890	0.399	4.696	0.000	0.668	14.385

Problem 1560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	7.363	0.382	0.000	0.000	0.000	0.000	0.000

Problem 1561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	5085	0	0	0	0	0	-1
N.S.	1	1.00	15.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	33.969	0.740	0.000	0.000	0.000	0.000	0.000

Problem 1562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	12568	0	0	0	0	0	-1
N.S.	1	1.00	24.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	54.301	1.713	0.000	0.000	0.000	0.000	0.000

Problem 1563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	5113	0	0	0	0	0	-1
N.S.	1	1.00	16.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	26.836	0.841	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [45]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	38	0.079
2	A	3	3	1.00	38	0.079
3	A	3	3	1.00	38	0.079
4	A	3	3	1.00	38	0.079
5	A	2	2	1.00	38	0.053
6	A	5	5	1.00	38	0.132
7	A	5	5	1.00	38	0.132
8	A	2	2	1.00	38	0.053
9	A	4	3	1.00	38	0.079
10	A	4	3	1.00	38	0.079
11	A	4	3	1.00	38	0.079
12	A	3	3	1.00	38	0.079
13	A	2	2	1.00	38	0.053
14	A	6	5	1.00	38	0.132
15	A	6	6	1.00	38	0.158
16	A	6	5	1.00	38	0.132
17	A	2	2	1.00	38	0.053
18	A	3	3	1.00	38	0.079
19	A	5	3	1.00	38	0.079
20	A	5	3	1.00	38	0.079
21	A	4	3	1.00	38	0.079
22	A	3	3	1.00	38	0.079
23	A	2	2	1.00	38	0.053
24	A	7	5	1.00	38	0.132
25	A	7	6	1.00	38	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	6	1.00	38	0.158
27	A	7	5	1.00	38	0.132
28	A	2	2	1.00	38	0.053
29	A	3	3	1.00	38	0.079
30	A	6	3	1.00	38	0.079
31	A	6	3	1.00	38	0.079
32	A	5	3	1.00	38	0.079
33	A	4	3	1.00	38	0.079
34	A	3	3	1.00	38	0.079
35	A	2	2	1.00	38	0.053
36	A	8	5	1.00	38	0.132
37	A	8	6	1.00	38	0.158
38	A	8	6	1.00	38	0.158
39	A	8	6	1.00	38	0.158
40	A	8	5	1.00	38	0.132
41	A	2	2	1.00	38	0.053
42	A	3	3	1.00	38	0.079
43	A	4	3	1.00	38	0.079
44	A	2	2	1.00	38	0.053
45	A	2	2	1.00	38	0.053
46	A	2	2	1.00	38	0.053
47	A	2	2	1.00	38	0.053
48	A	4	4	1.00	38	0.105
49	A	2	2	1.00	38	0.053
50	A	8	5	1.00	38	0.132
51	A	7	5	1.00	38	0.132
52	A	6	5	1.00	38	0.132
53	A	5	5	1.00	38	0.132
54	A	4	4	1.00	38	0.105
55	A	3	3	1.00	38	0.079
56	A	4	4	1.00	38	0.105
57	A	9	6	1.00	38	0.158
58	A	8	6	1.00	38	0.158
59	A	7	6	1.00	38	0.158
60	A	6	6	1.00	38	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	38	0.132
62	A	2	2	1.00	38	0.053
63	A	4	4	1.00	38	0.105
64	A	5	4	1.00	38	0.105
65	A	5	5	1.00	34	0.147
66	A	4	4	1.00	34	0.118
67	A	4	4	1.00	34	0.118
68	A	4	4	1.00	32	0.125
69	A	3	3	1.00	21	0.143
70	A	3	3	1.00	34	0.088
71	A	4	4	1.00	34	0.118
72	A	4	4	1.00	34	0.118
73	A	5	3	1.00	36	0.083
74	A	4	3	1.00	36	0.083
75	A	3	3	1.00	36	0.083
76	A	2	2	1.00	36	0.056
77	A	4	4	1.00	36	0.111
78	A	4	4	1.00	36	0.111
79	A	2	2	1.00	36	0.056
80	A	2	2	1.00	36	0.056
81	A	4	3	1.00	38	0.079
82	A	3	3	1.00	38	0.079
83	A	2	2	1.00	38	0.053
84	A	5	5	1.00	38	0.132
85	A	5	5	1.00	38	0.132
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	38	0.132
88	A	8	4	1.00	42	0.095
89	A	7	4	1.00	42	0.095
90	A	6	4	1.00	42	0.095
91	A	5	4	1.00	42	0.095
92	A	4	4	1.00	42	0.095
93	A	4	4	1.00	42	0.095
94	A	5	5	1.00	42	0.119
95	A	6	5	1.00	42	0.119

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	7	5	1.00	42	0.119
97	A	8	4	1.00	42	0.095
98	A	7	4	1.00	42	0.095
99	A	6	4	1.00	42	0.095
100	A	5	4	1.00	42	0.095
101	A	5	5	1.00	42	0.119
102	A	5	4	1.00	42	0.095
103	A	6	5	1.00	42	0.119
104	A	7	5	1.00	42	0.119
105	A	8	5	1.00	42	0.119
106	A	9	4	1.00	42	0.095
107	A	8	4	1.00	42	0.095
108	A	7	4	1.00	42	0.095
109	A	6	4	1.00	42	0.095
110	A	6	5	1.00	42	0.119
111	A	6	5	1.00	42	0.119
112	A	6	4	1.00	42	0.095
113	A	7	5	1.00	42	0.119
114	A	8	5	1.00	42	0.119
115	A	9	5	1.00	42	0.119
116	A	10	4	1.00	42	0.095
117	A	9	4	1.00	42	0.095
118	A	8	4	1.00	42	0.095
119	A	7	4	1.00	42	0.095
120	A	7	5	1.00	42	0.119
121	A	7	5	1.00	42	0.119
122	A	7	5	1.00	42	0.119
123	A	7	4	1.00	42	0.095
124	A	8	5	1.00	42	0.119
125	A	9	5	1.00	42	0.119
126	A	10	5	1.00	42	0.119
127	A	6	4	1.00	42	0.095
128	A	5	4	1.00	42	0.095
129	A	4	4	1.00	42	0.095
130	A	3	3	1.00	42	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	42	0.095
132	A	5	4	1.00	42	0.095
133	A	6	4	1.00	42	0.095
134	A	7	5	1.00	42	0.119
135	A	6	5	1.00	42	0.119
136	A	5	5	1.00	42	0.119
137	A	4	4	1.00	42	0.095
138	A	4	4	1.00	42	0.095
139	A	5	4	1.00	42	0.095
140	A	6	4	1.00	42	0.095
141	A	7	4	1.00	42	0.095
142	A	8	5	1.00	42	0.119
143	A	7	5	1.00	42	0.119
144	A	6	5	1.00	42	0.119
145	A	5	4	1.00	42	0.095
146	A	5	5	1.00	42	0.119
147	A	5	4	1.00	42	0.095
148	A	6	4	1.00	42	0.095
149	A	7	4	1.00	42	0.095
150	A	8	4	1.00	42	0.095
151	A	4	4	1.00	38	0.105
152	A	4	4	1.00	38	0.105
153	A	4	4	1.00	38	0.105
154	A	4	4	1.00	36	0.111
155	A	3	3	1.00	25	0.120
156	A	4	4	1.00	38	0.105
157	A	4	4	1.00	38	0.105
158	A	4	4	1.00	38	0.105
159	A	4	4	1.00	40	0.100
160	A	4	4	1.00	40	0.100
161	A	4	4	1.00	40	0.100
162	A	4	4	1.00	40	0.100
163	A	4	4	1.00	40	0.100
164	A	4	4	1.00	40	0.100
165	A	4	4	1.00	40	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	4	1.00	40	0.100
167	A	4	4	1.00	42	0.095
168	A	4	4	1.00	42	0.095
169	A	4	4	1.00	42	0.095
170	A	4	4	1.00	40	0.100
171	A	4	4	1.00	42	0.095
172	A	4	4	1.00	42	0.095
173	A	4	4	1.00	36	0.111
174	A	4	4	1.00	42	0.095
175	A	3	2	1.00	40	0.050
176	A	2	2	1.00	40	0.050
177	A	1	1	1.00	40	0.025
178	A	4	4	1.00	40	0.100
179	A	4	4	1.00	40	0.100
180	A	4	4	1.00	40	0.100
181	A	3	3	1.00	40	0.075
182	A	4	4	1.00	45	0.089
183	A	4	4	1.00	45	0.089
184	A	4	4	1.00	45	0.089
185	A	1	1	1.00	43	0.023
186	A	2	2	1.00	45	0.044
187	A	3	2	1.00	45	0.044
188	A	4	2	1.00	45	0.044
189	A	5	5	1.00	36	0.139
190	A	4	3	1.00	25	0.120
191	A	4	3	1.00	23	0.130
192	A	3	2	1.00	17	0.118
193	A	4	3	1.00	23	0.130
194	A	3	3	1.00	25	0.120
195	A	4	3	1.00	25	0.120
196	A	4	3	1.00	25	0.120
197	A	4	3	1.00	27	0.111
198	A	4	3	1.00	25	0.120
199	A	3	2	1.00	19	0.105
200	A	4	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	27	0.111
202	A	3	3	1.00	27	0.111
203	A	4	3	1.00	27	0.111
204	A	4	3	1.00	27	0.111
205	A	4	3	1.00	27	0.111
206	A	4	3	1.00	27	0.111
207	A	4	3	1.00	27	0.111
208	A	4	3	1.00	25	0.120
209	A	3	2	1.00	19	0.105
210	A	4	3	1.00	25	0.120
211	A	4	3	1.00	27	0.111
212	A	4	3	1.00	27	0.111
213	A	3	3	1.00	27	0.111
214	A	4	4	1.00	27	0.148
215	A	4	3	1.00	27	0.111
216	A	4	3	1.00	27	0.111
217	A	4	3	1.00	27	0.111
218	A	4	3	1.00	27	0.111
219	A	4	3	1.00	27	0.111
220	A	4	3	1.00	25	0.120
221	A	3	2	1.00	19	0.105
222	A	4	3	1.00	25	0.120
223	A	4	3	1.00	27	0.111
224	A	4	3	1.00	27	0.111
225	A	4	3	1.00	27	0.111
226	A	4	3	1.00	27	0.111
227	A	4	3	1.00	25	0.120
228	A	4	4	1.00	19	0.210
229	A	4	3	1.00	25	0.120
230	A	4	3	1.00	27	0.111
231	A	4	3	1.00	27	0.111
232	A	4	3	1.00	27	0.111
233	A	4	3	1.00	27	0.111
234	A	4	3	1.00	27	0.111
235	A	4	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	2	1.00	19	0.105
237	A	4	3	1.00	25	0.120
238	A	4	3	1.00	27	0.111
239	A	4	3	1.00	27	0.111
240	A	4	3	1.00	27	0.111
241	A	4	3	1.00	27	0.111
242	A	4	3	1.00	27	0.111
243	A	4	3	1.00	27	0.111
244	A	3	3	1.00	25	0.120
245	A	3	2	1.00	19	0.105
246	A	4	3	1.00	25	0.120
247	A	4	3	1.00	27	0.111
248	A	4	3	1.00	27	0.111
249	A	4	3	1.00	27	0.111
250	A	4	3	1.00	27	0.111
251	A	4	3	1.00	27	0.111
252	A	3	3	1.00	27	0.111
253	A	4	3	1.00	25	0.120
254	A	3	2	1.00	19	0.105
255	A	4	3	1.00	25	0.120
256	A	4	3	1.00	27	0.111
257	A	4	4	1.00	21	0.190
258	A	3	2	1.00	27	0.074
259	A	3	2	1.00	27	0.074
260	A	3	2	1.00	27	0.074
261	A	3	2	1.00	25	0.080
262	A	2	2	1.00	27	0.074
263	A	2	2	1.00	27	0.074
264	A	2	2	1.00	27	0.074
265	A	2	2	1.00	27	0.074
266	A	8	6	1.00	27	0.222
267	A	7	6	1.00	27	0.222
268	A	6	6	1.00	25	0.240
269	A	6	6	1.00	23	0.261
270	A	7	6	1.00	19	0.316

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	5	5	1.00	25	0.200
272	A	5	5	1.00	27	0.185
273	A	6	6	1.00	27	0.222
274	A	7	6	1.00	27	0.222
275	A	12	7	1.00	29	0.241
276	A	5	4	1.00	29	0.138
277	A	5	5	1.15	27	0.185
278	A	9	8	1.00	25	0.320
279	A	8	6	1.00	21	0.286
280	A	7	6	1.00	27	0.222
281	A	8	7	1.00	29	0.241
282	A	9	6	1.00	29	0.207
283	A	10	7	1.00	29	0.241
284	A	12	6	1.00	29	0.207
285	A	15	7	1.00	29	0.241
286	A	6	5	1.14	27	0.185
287	A	12	9	1.00	25	0.360
288	A	10	7	1.00	21	0.333
289	A	10	7	1.00	27	0.259
290	A	10	6	1.00	29	0.207
291	A	11	8	1.00	29	0.276
292	A	12	7	1.00	29	0.241
293	A	14	7	1.00	29	0.241
294	A	6	4	1.00	21	0.190
295	A	15	10	1.00	25	0.400
296	A	12	6	1.00	21	0.286
297	A	6	4	1.00	29	0.138
298	A	6	4	1.00	29	0.138
299	A	5	4	1.00	29	0.138
300	A	4	4	1.00	27	0.148
301	A	3	3	1.00	25	0.120
302	A	4	4	1.00	21	0.190
303	A	5	5	1.00	27	0.185
304	A	5	4	1.00	29	0.138
305	A	6	4	1.00	29	0.138

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	4	1.00	29	0.138
307	A	12	7	1.00	29	0.241
308	A	9	7	1.00	29	0.241
309	A	8	7	1.00	29	0.241
310	A	3	2	1.00	27	0.074
311	A	5	4	1.00	25	0.160
312	A	7	5	1.00	21	0.238
313	A	9	7	1.00	27	0.259
314	A	11	7	1.00	29	0.241
315	A	9	7	1.00	29	0.241
316	A	4	4	1.00	29	0.138
317	A	3	3	1.00	27	0.111
318	A	7	5	1.00	25	0.200
319	A	10	7	1.00	21	0.333
320	A	11	8	1.00	27	0.296
321	A	5	4	1.00	27	0.148
322	A	7	7	1.00	31	0.226
323	A	4	4	1.00	31	0.129
324	A	3	3	1.00	29	0.103
325	A	5	5	1.00	27	0.185
326	A	4	4	1.00	23	0.174
327	A	5	5	1.00	29	0.172
328	A	6	6	1.00	31	0.194
329	A	8	7	1.00	31	0.226
330	A	5	4	1.00	31	0.129
331	A	4	3	1.00	29	0.103
332	A	6	5	1.00	27	0.185
333	A	5	5	1.00	23	0.217
334	A	6	5	1.00	29	0.172
335	A	6	5	1.00	31	0.161
336	A	6	6	1.00	31	0.194
337	A	4	4	1.00	31	0.129
338	A	2	2	1.00	29	0.069
339	A	4	4	1.00	27	0.148
340	A	4	4	1.00	23	0.174

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	5	5	1.00	29	0.172
342	A	6	5	1.00	31	0.161
343	A	8	7	1.00	31	0.226
344	A	5	5	1.00	31	0.161
345	A	4	4	1.00	29	0.138
346	A	6	5	1.00	27	0.185
347	A	6	5	1.00	23	0.217
348	A	8	6	1.00	29	0.207
349	A	9	6	1.00	31	0.194
350	A	4	3	1.00	27	0.111
351	A	4	3	1.00	27	0.111
352	A	6	5	1.00	25	0.200
353	A	3	2	1.00	19	0.105
354	A	4	3	1.00	25	0.120
355	A	4	3	1.00	25	0.120
356	A	3	2	1.00	19	0.105
357	A	4	3	1.00	29	0.103
358	A	5	3	1.00	27	0.111
359	A	2	1	1.00	21	0.048
360	A	4	3	1.00	27	0.111
361	A	4	3	1.00	27	0.111
362	A	5	4	1.00	21	0.190
363	A	4	3	1.00	27	0.111
364	A	4	3	1.00	29	0.103
365	A	9	6	1.00	27	0.222
366	A	9	6	1.00	27	0.222
367	A	8	6	1.00	27	0.222
368	A	7	6	1.00	25	0.240
369	A	8	6	1.00	25	0.240
370	A	9	8	1.00	27	0.296
371	A	9	7	1.00	25	0.280
372	A	9	7	1.00	19	0.368
373	A	7	5	1.00	25	0.200
374	A	6	5	1.00	27	0.185
375	A	7	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	8	6	1.00	27	0.222
377	A	9	6	1.00	27	0.222
378	A	16	6	1.00	29	0.207
379	A	13	7	1.00	29	0.241
380	A	14	6	1.00	29	0.207
381	A	6	5	1.00	27	0.185
382	A	11	8	1.00	27	0.296
383	A	13	7	1.00	29	0.241
384	A	12	8	1.00	27	0.296
385	A	12	7	1.00	21	0.333
386	A	13	6	1.00	27	0.222
387	A	10	7	1.00	29	0.241
388	A	11	6	1.00	29	0.207
389	A	14	6	1.00	29	0.207
390	A	13	7	1.00	29	0.241
391	A	16	6	1.00	29	0.207
392	A	19	6	1.00	29	0.207
393	A	19	7	1.00	29	0.241
394	A	17	7	1.00	29	0.241
395	A	7	5	1.00	27	0.185
396	A	15	9	1.00	27	0.333
397	A	15	7	1.00	29	0.241
398	A	15	8	1.00	27	0.296
399	A	14	8	1.00	21	0.381
400	A	15	7	1.00	27	0.259
401	A	15	6	1.00	29	0.207
402	A	14	8	1.00	29	0.276
403	A	14	7	1.00	29	0.241
404	A	16	7	1.00	29	0.241
405	A	17	7	1.00	29	0.241
406	A	19	7	1.00	29	0.241
407	A	8	5	1.00	29	0.172
408	A	17	8	1.00	21	0.381
409	A	8	6	1.00	29	0.207
410	A	8	6	1.00	29	0.207

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	7	6	1.00	29	0.207
412	A	6	6	1.00	27	0.222
413	A	6	6	1.00	27	0.222
414	A	6	6	1.00	29	0.207
415	A	5	5	1.00	27	0.185
416	A	5	5	1.00	21	0.238
417	A	6	6	1.00	27	0.222
418	A	7	6	1.00	29	0.207
419	A	8	6	1.00	29	0.207
420	A	11	5	1.00	29	0.172
421	A	12	5	1.00	29	0.172
422	A	10	5	1.00	29	0.172
423	A	10	5	1.00	29	0.172
424	A	4	4	1.00	27	0.148
425	A	4	4	1.00	27	0.148
426	A	6	5	1.00	29	0.172
427	A	8	6	1.00	27	0.222
428	A	9	6	1.00	21	0.286
429	A	10	5	1.00	27	0.185
430	A	10	5	1.00	29	0.172
431	A	12	5	1.00	29	0.172
432	A	12	7	1.00	29	0.241
433	A	9	7	1.00	29	0.241
434	A	4	4	1.00	27	0.148
435	A	5	4	1.00	27	0.148
436	A	7	6	1.00	29	0.207
437	A	9	7	1.00	27	0.259
438	A	11	6	1.00	21	0.286
439	A	14	7	1.00	27	0.259
440	A	2	2	1.00	27	0.074
441	A	18	4	1.47	29	0.138
442	A	24	4	1.00	29	0.138
443	A	5	4	1.00	31	0.129
444	A	4	3	1.00	29	0.103
445	A	9	9	1.00	29	0.310

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	8	8	1.00	31	0.258
447	A	7	7	1.00	29	0.241
448	A	7	7	1.00	23	0.304
449	A	9	6	1.00	29	0.207
450	A	11	6	1.00	31	0.194
451	A	13	6	1.00	31	0.194
452	A	15	6	1.00	31	0.194
453	A	6	4	1.00	31	0.129
454	A	5	3	1.00	29	0.103
455	A	12	12	1.00	29	0.414
456	A	10	10	1.00	31	0.323
457	A	9	9	1.00	29	0.310
458	A	8	8	1.00	23	0.348
459	A	11	9	1.00	29	0.310
460	A	12	9	1.00	31	0.290
461	A	14	9	1.00	31	0.290
462	A	16	9	1.00	31	0.290
463	A	18	9	1.00	31	0.290
464	A	5	4	1.00	31	0.129
465	A	3	3	1.00	29	0.103
466	A	13	10	1.00	29	0.345
467	A	11	8	1.00	31	0.258
468	A	11	8	1.00	29	0.276
469	A	11	7	1.00	23	0.304
470	A	15	8	1.00	29	0.276
471	A	17	8	1.00	31	0.258
472	A	12	7	1.00	31	0.226
473	A	4	4	1.00	31	0.129
474	A	2	2	1.00	29	0.069
475	A	6	6	1.00	29	0.207
476	A	9	6	1.00	31	0.194
477	A	8	6	1.00	29	0.207
478	A	10	6	1.00	23	0.261
479	A	12	6	1.00	29	0.207
480	A	14	6	1.00	31	0.194

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	18	9	1.00	31	0.290
482	A	16	9	1.00	31	0.290
483	A	6	5	1.00	31	0.161
484	A	5	4	1.00	29	0.138
485	A	9	6	1.00	29	0.207
486	A	12	7	1.00	31	0.226
487	A	14	8	1.00	29	0.276
488	A	16	8	1.00	23	0.348
489	A	18	8	1.00	29	0.276
490	A	5	2	1.00	29	0.069
491	A	3	2	1.00	27	0.074
492	A	3	2	1.00	29	0.069
493	A	5	4	1.00	29	0.138
494	A	4	3	1.00	27	0.111
495	A	4	3	1.00	27	0.111
496	A	7	5	1.00	27	0.185
497	A	7	5	1.00	27	0.185
498	A	6	5	1.00	25	0.200
499	A	4	3	1.00	25	0.120
500	A	4	3	1.00	27	0.111
501	A	4	3	1.00	27	0.111
502	A	4	3	1.00	25	0.120
503	A	3	2	1.00	19	0.105
504	A	4	3	1.00	25	0.120
505	A	6	5	1.00	27	0.185
506	A	6	5	1.00	27	0.185
507	A	7	5	1.00	27	0.185
508	A	7	5	1.00	27	0.185
509	A	4	3	1.00	27	0.111
510	A	4	3	1.00	27	0.111
511	A	4	3	1.00	29	0.103
512	A	4	3	1.00	29	0.103
513	A	4	3	1.00	27	0.111
514	A	4	3	1.00	27	0.111
515	A	4	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	4	3	1.00	29	0.103
517	A	4	3	1.00	27	0.111
518	A	3	2	1.00	21	0.095
519	A	4	3	1.00	27	0.111
520	A	4	3	1.00	29	0.103
521	A	4	3	1.00	29	0.103
522	A	4	3	1.00	27	0.111
523	A	4	3	1.00	27	0.111
524	A	4	3	1.00	29	0.103
525	A	4	3	1.00	29	0.103
526	A	4	3	1.00	27	0.111
527	A	3	2	1.00	21	0.095
528	A	4	3	1.00	27	0.111
529	A	4	3	1.00	29	0.103
530	A	4	3	1.00	27	0.111
531	A	3	2	1.00	21	0.095
532	A	4	3	1.00	27	0.111
533	A	4	3	1.00	29	0.103
534	A	7	3	1.00	29	0.103
535	A	6	5	1.00	27	0.185
536	A	4	3	1.00	27	0.111
537	A	4	3	1.00	29	0.103
538	A	4	3	1.00	29	0.103
539	A	4	3	1.00	27	0.111
540	A	5	4	1.00	21	0.190
541	A	6	5	1.00	27	0.185
542	A	4	3	1.00	29	0.103
543	A	4	3	1.00	29	0.103
544	A	4	3	1.00	29	0.103
545	A	4	3	1.00	29	0.103
546	A	4	3	1.00	27	0.111
547	A	4	3	1.00	27	0.111
548	A	4	3	1.00	29	0.103
549	A	4	3	1.00	29	0.103
550	A	3	3	1.00	27	0.111

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	3	2	1.00	21	0.095
552	A	4	3	1.00	27	0.111
553	A	4	3	1.00	29	0.103
554	A	4	3	1.00	29	0.103
555	A	4	3	1.00	29	0.103
556	A	4	3	1.00	27	0.111
557	A	4	3	1.00	27	0.111
558	A	4	3	1.00	29	0.103
559	A	4	3	1.00	29	0.103
560	A	4	3	1.00	27	0.111
561	A	3	2	1.00	21	0.095
562	A	4	3	1.00	27	0.111
563	A	3	2	1.00	21	0.095
564	A	4	3	1.00	27	0.111
565	A	3	2	1.00	29	0.069
566	A	3	2	1.00	29	0.069
567	A	3	2	1.00	27	0.074
568	A	3	2	1.00	29	0.069
569	A	3	2	1.00	29	0.069
570	A	4	3	1.00	29	0.103
571	A	4	4	1.00	29	0.138
572	A	10	6	1.00	27	0.222
573	A	10	6	1.00	27	0.222
574	A	9	6	1.00	27	0.222
575	A	8	6	1.00	25	0.240
576	A	9	6	1.00	25	0.240
577	A	10	8	1.00	27	0.296
578	A	11	8	1.00	27	0.296
579	A	11	7	1.00	27	0.259
580	A	11	8	1.00	25	0.320
581	A	11	7	1.00	19	0.368
582	A	9	5	1.00	25	0.200
583	A	7	5	1.00	27	0.185
584	A	8	6	1.00	27	0.222
585	A	9	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	10	6	1.00	27	0.222
587	A	10	6	1.00	27	0.222
588	A	18	6	1.00	29	0.207
589	A	14	7	1.00	29	0.241
590	A	16	6	1.00	29	0.207
591	A	7	5	1.00	27	0.185
592	A	12	8	1.00	27	0.296
593	A	17	7	1.00	29	0.241
594	A	16	7	1.00	29	0.241
595	A	17	8	1.00	29	0.276
596	A	16	8	1.00	27	0.296
597	A	15	7	1.00	21	0.333
598	A	17	6	1.00	27	0.222
599	A	12	7	1.00	29	0.241
600	A	13	6	1.00	29	0.207
601	A	12	7	1.00	29	0.241
602	A	16	6	1.00	29	0.207
603	A	14	7	1.00	29	0.241
604	A	18	6	1.00	29	0.207
605	A	21	6	1.00	29	0.207
606	A	21	7	1.00	29	0.241
607	A	19	7	1.00	29	0.241
608	A	8	5	1.00	27	0.185
609	A	17	9	1.00	27	0.333
610	A	19	7	1.00	29	0.241
611	A	17	8	1.00	29	0.276
612	A	15	8	1.00	29	0.276
613	A	16	8	1.00	27	0.296
614	A	16	7	1.00	21	0.333
615	A	18	7	1.00	27	0.259
616	A	18	6	1.00	29	0.207
617	A	17	8	1.00	29	0.276
618	A	16	7	1.00	29	0.241
619	A	18	7	1.00	29	0.241
620	A	19	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
621	A	21	7	1.00	29	0.241
622	A	21	6	1.00	29	0.207
623	A	22	7	1.00	29	0.241
624	A	9	6	1.00	29	0.207
625	A	9	6	1.00	29	0.207
626	A	8	6	1.00	29	0.207
627	A	7	6	1.00	27	0.222
628	A	8	6	1.00	27	0.222
629	A	9	8	1.00	29	0.276
630	A	9	7	1.00	29	0.241
631	A	8	7	1.00	29	0.241
632	A	7	5	1.00	27	0.185
633	A	6	5	1.00	21	0.238
634	A	13	8	1.00	29	0.276
635	A	6	5	1.00	29	0.172
636	A	5	4	1.00	27	0.148
637	A	10	9	1.00	27	0.333
638	A	9	7	1.00	29	0.241
639	A	8	7	1.00	29	0.241
640	A	9	8	1.00	29	0.276
641	A	10	7	1.00	27	0.259
642	A	11	5	1.00	21	0.238
643	A	13	7	1.00	27	0.259
644	A	14	5	1.00	29	0.172
645	A	12	5	1.00	29	0.172
646	A	5	5	1.25	27	0.185
647	A	7	6	1.00	27	0.222
648	A	7	6	1.00	29	0.207
649	A	8	6	1.00	29	0.207
650	A	10	6	1.00	29	0.207
651	A	12	6	1.00	27	0.222
652	A	12	5	1.00	21	0.238
653	A	6	2	1.00	29	0.069
654	A	5	2	1.00	29	0.069
655	A	3	2	1.00	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	4	3	1.00	27	0.111
657	A	8	6	1.00	27	0.222
658	A	8	6	1.00	27	0.222
659	A	7	5	1.00	27	0.185
660	A	7	5	1.00	27	0.185
661	A	6	5	1.00	25	0.200
662	A	4	3	1.00	25	0.120
663	A	4	3	1.00	27	0.111
664	A	4	3	1.00	27	0.111
665	A	4	3	1.00	27	0.111
666	A	4	3	1.00	27	0.111
667	A	4	3	1.00	25	0.120
668	A	3	2	1.00	19	0.105
669	A	4	3	1.00	25	0.120
670	A	6	5	1.00	27	0.185
671	A	6	5	1.00	27	0.185
672	A	7	5	1.00	27	0.185
673	A	7	5	1.00	27	0.185
674	A	8	6	1.00	27	0.222
675	A	8	6	1.00	27	0.222
676	A	4	3	1.00	27	0.111
677	A	4	3	1.00	29	0.103
678	A	4	3	1.00	29	0.103
679	A	4	3	1.00	29	0.103
680	A	7	5	1.00	29	0.172
681	A	7	5	1.00	29	0.172
682	A	6	5	1.00	27	0.185
683	A	3	2	1.00	21	0.095
684	A	4	3	1.00	27	0.111
685	A	4	3	1.00	29	0.103
686	A	4	3	1.00	29	0.103
687	A	4	3	1.00	29	0.103
688	A	4	3	1.00	29	0.103
689	A	4	3	1.00	27	0.111
690	A	6	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
691	A	6	5	1.00	27	0.185
692	A	7	5	1.00	29	0.172
693	A	7	5	1.00	29	0.172
694	A	4	3	1.00	29	0.103
695	A	4	3	1.00	29	0.103
696	A	4	3	1.00	29	0.103
697	A	3	2	1.00	29	0.069
698	A	3	2	1.00	29	0.069
699	A	3	2	1.00	27	0.074
700	A	3	2	1.00	29	0.069
701	A	3	2	1.00	29	0.069
702	A	3	2	1.00	29	0.069
703	A	8	4	1.00	29	0.138
704	A	4	4	1.00	29	0.138
705	A	11	6	1.00	29	0.207
706	A	10	6	1.00	29	0.207
707	A	10	6	1.00	29	0.207
708	A	9	6	1.00	29	0.207
709	A	8	6	1.00	27	0.222
710	A	9	6	1.00	27	0.222
711	A	10	8	1.00	29	0.276
712	A	11	8	1.00	29	0.276
713	A	11	7	1.00	29	0.241
714	A	11	8	1.00	29	0.276
715	A	10	7	1.00	29	0.241
716	A	9	5	1.00	27	0.185
717	A	7	5	1.00	21	0.238
718	A	8	6	1.00	27	0.222
719	A	9	6	1.00	29	0.207
720	A	10	6	1.00	29	0.207
721	A	10	6	1.00	29	0.207
722	A	15	7	1.00	29	0.241
723	A	17	7	1.00	29	0.241
724	A	14	8	1.00	29	0.276
725	A	15	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	6	4	1.00	27	0.148
727	A	12	9	1.00	27	0.333
728	A	14	8	1.00	29	0.276
729	A	13	9	1.00	29	0.310
730	A	13	8	1.00	29	0.276
731	A	14	7	1.00	29	0.241
732	A	11	8	1.00	29	0.276
733	A	12	7	1.00	27	0.259
734	A	19	5	1.00	21	0.238
735	A	15	7	1.00	27	0.259
736	A	14	8	1.00	29	0.276
737	A	17	7	1.00	29	0.241
738	A	15	7	1.00	29	0.241
739	A	18	8	1.00	29	0.276
740	A	16	8	1.00	29	0.276
741	A	6	5	1.00	27	0.185
742	A	13	10	1.00	27	0.370
743	A	11	8	1.00	29	0.276
744	A	11	8	1.00	29	0.276
745	A	11	7	1.00	29	0.241
746	A	12	9	1.00	29	0.310
747	A	13	8	1.00	29	0.276
748	A	15	8	1.00	27	0.296
749	A	17	4	1.00	21	0.190
750	A	18	8	1.00	27	0.296
751	A	8	7	1.00	27	0.259
752	A	8	7	1.00	25	0.280
753	A	5	5	1.00	19	0.263
754	A	5	4	1.00	23	0.174
755	A	6	6	1.00	25	0.240
756	A	7	6	1.00	27	0.222
757	A	8	7	1.00	27	0.259
758	A	8	7	1.00	27	0.259
759	A	12	8	1.00	27	0.296
760	A	6	5	1.00	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	2	1.00	25	0.080
762	A	9	7	1.00	27	0.259
763	A	10	8	1.00	29	0.276
764	A	12	7	1.00	29	0.241
765	A	11	6	1.00	27	0.222
766	A	8	6	1.00	21	0.286
767	A	2	2	1.00	25	0.080
768	A	4	3	1.00	27	0.111
769	A	6	5	1.00	29	0.172
770	A	8	6	1.00	29	0.207
771	A	10	6	1.00	29	0.207
772	A	7	5	1.00	29	0.172
773	A	6	4	1.00	27	0.148
774	A	5	4	1.00	21	0.190
775	A	5	4	1.00	25	0.160
776	A	7	5	1.00	27	0.185
777	A	8	6	1.00	29	0.207
778	A	15	10	1.00	29	0.345
779	A	13	8	1.00	29	0.276
780	A	12	8	1.00	29	0.276
781	A	11	7	1.00	27	0.259
782	A	10	5	1.00	21	0.238
783	A	4	4	1.00	25	0.160
784	A	11	8	1.00	27	0.296
785	A	12	8	1.00	29	0.276
786	A	15	8	1.00	29	0.276
787	A	16	10	1.00	29	0.345
788	A	16	9	1.00	29	0.310
789	A	14	8	1.00	29	0.276
790	A	14	7	1.00	27	0.259
791	A	14	5	1.00	21	0.238
792	A	5	4	1.00	25	0.160
793	A	14	10	1.00	27	0.370
794	A	14	10	1.00	29	0.345
795	A	9	7	1.00	27	0.259

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	9	7	1.00	25	0.280
797	A	8	5	1.00	19	0.263
798	A	6	4	1.00	25	0.160
799	A	5	4	1.00	27	0.148
800	A	5	4	1.00	25	0.160
801	A	7	5	1.00	25	0.200
802	A	8	6	1.00	27	0.222
803	A	9	7	1.00	27	0.259
804	A	4	4	1.19	21	0.190
805	A	7	7	1.00	27	0.259
806	A	4	4	1.00	29	0.138
807	A	4	4	1.00	27	0.148
808	A	5	5	1.00	27	0.185
809	A	7	7	1.00	29	0.241
810	A	8	8	1.00	29	0.276
811	A	10	7	1.00	21	0.333
812	A	8	6	1.00	27	0.222
813	A	5	5	1.00	29	0.172
814	A	4	4	1.00	27	0.148
815	A	6	4	1.00	27	0.148
816	A	8	6	1.00	29	0.207
817	A	10	7	1.00	29	0.241
818	A	12	7	1.00	29	0.241
819	A	13	7	1.00	21	0.333
820	A	8	6	1.00	29	0.207
821	A	8	5	1.00	29	0.172
822	A	8	5	1.00	27	0.185
823	A	6	5	1.00	21	0.238
824	A	6	5	1.00	27	0.185
825	A	7	4	1.00	29	0.138
826	A	6	5	1.00	27	0.185
827	A	7	5	1.00	27	0.185
828	A	8	6	1.00	29	0.207
829	A	14	8	1.00	29	0.276
830	A	13	8	1.00	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
831	A	11	7	1.00	27	0.259
832	A	10	6	1.00	21	0.286
833	A	12	6	1.00	27	0.222
834	A	12	6	1.00	29	0.207
835	A	4	3	1.00	27	0.111
836	A	11	8	1.00	27	0.296
837	A	12	8	1.00	29	0.276
838	A	15	8	1.00	29	0.276
839	A	17	9	1.00	29	0.310
840	A	14	8	1.00	29	0.276
841	A	14	7	1.00	27	0.259
842	A	14	5	1.00	21	0.238
843	A	15	6	1.00	27	0.222
844	A	15	6	1.00	29	0.207
845	A	5	3	1.00	27	0.111
846	A	14	10	1.00	27	0.370
847	A	14	10	1.00	29	0.345
848	A	17	5	1.00	21	0.238
849	A	18	6	1.00	27	0.222
850	A	8	4	1.29	29	0.138
851	A	4	3	1.00	25	0.120
852	A	3	2	1.00	19	0.105
853	A	4	3	1.00	25	0.120
854	A	5	4	1.00	27	0.148
855	A	5	4	1.00	27	0.148
856	A	5	4	1.00	25	0.160
857	A	4	3	1.00	25	0.120
858	A	4	3	1.00	27	0.111
859	A	4	3	1.00	27	0.111
860	A	4	3	1.00	27	0.111
861	A	3	2	1.00	21	0.095
862	A	4	3	1.00	27	0.111
863	A	4	3	1.00	29	0.103
864	A	5	4	1.00	29	0.138
865	A	5	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
866	A	4	3	1.00	27	0.111
867	A	4	3	1.00	29	0.103
868	A	4	3	1.00	29	0.103
869	A	4	3	1.00	29	0.103
870	A	3	2	1.00	21	0.095
871	A	4	3	1.00	27	0.111
872	A	4	3	1.00	29	0.103
873	A	4	3	1.00	29	0.103
874	A	3	3	1.00	27	0.111
875	A	4	3	1.00	27	0.111
876	A	4	3	1.00	29	0.103
877	A	4	3	1.00	29	0.103
878	A	4	3	1.00	29	0.103
879	A	4	3	1.00	29	0.103
880	A	4	3	1.00	29	0.103
881	A	4	3	1.00	27	0.111
882	A	8	5	1.00	21	0.238
883	A	8	6	1.00	27	0.222
884	A	9	6	1.00	29	0.207
885	A	9	6	1.00	29	0.207
886	A	9	6	1.00	29	0.207
887	A	9	6	1.00	29	0.207
888	A	8	6	1.00	27	0.222
889	A	4	3	1.00	21	0.143
890	A	4	3	1.00	27	0.111
891	A	4	3	1.00	29	0.103
892	A	4	3	1.00	29	0.103
893	A	4	3	1.00	29	0.103
894	A	11	5	1.00	29	0.172
895	A	4	3	1.00	29	0.103
896	A	4	3	1.00	29	0.103
897	A	4	3	1.00	27	0.111
898	A	9	5	1.00	21	0.238
899	A	9	6	1.00	27	0.222
900	A	10	6	1.00	29	0.207

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
901	A	10	6	1.00	29	0.207
902	A	11	7	1.00	29	0.241
903	A	11	7	1.00	29	0.241
904	A	10	6	1.00	29	0.207
905	A	10	6	1.00	29	0.207
906	A	9	6	1.00	27	0.222
907	A	4	3	1.00	21	0.143
908	A	4	3	1.00	27	0.111
909	A	4	3	1.00	29	0.103
910	A	4	3	1.00	29	0.103
911	A	5	4	1.00	33	0.121
912	A	3	3	1.00	31	0.097
913	A	3	2	1.00	31	0.065
914	A	3	2	1.00	31	0.065
915	A	3	2	1.00	31	0.065
916	A	3	2	1.00	29	0.069
917	A	2	2	1.00	31	0.065
918	A	2	2	1.00	31	0.065
919	A	2	2	1.00	31	0.065
920	A	3	2	1.00	31	0.065
921	A	3	2	1.00	31	0.065
922	A	3	2	1.00	31	0.065
923	A	3	2	1.00	29	0.069
924	A	2	2	1.00	31	0.065
925	A	2	2	1.00	31	0.065
926	A	2	2	1.00	31	0.065
927	A	3	3	1.13	27	0.111
928	A	4	3	1.00	27	0.111
929	A	4	3	1.00	27	0.111
930	A	4	3	1.00	27	0.111
931	A	4	3	1.00	25	0.120
932	A	2	2	1.00	19	0.105
933	A	3	3	1.00	25	0.120
934	A	3	3	1.00	27	0.111
935	A	4	4	1.06	29	0.138

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	6	6	1.00	35	0.171
937	A	6	6	1.00	37	0.162
938	A	4	4	1.00	33	0.121
939	A	3	3	1.00	33	0.091
940	A	3	3	1.00	33	0.091
941	A	3	3	1.00	31	0.097
942	A	4	4	1.00	33	0.121
943	A	3	3	1.00	33	0.091
944	A	3	3	1.00	33	0.091
945	A	4	4	1.00	33	0.121
946	A	3	3	1.00	33	0.091
947	A	3	3	1.00	31	0.097
948	A	3	3	1.00	33	0.091
949	A	4	4	1.00	33	0.121
950	A	3	3	1.00	33	0.091
951	A	3	3	1.00	33	0.091
952	A	3	3	1.00	33	0.091
953	A	3	2	1.00	29	0.069
954	A	3	2	1.00	29	0.069
955	A	3	2	1.00	29	0.069
956	A	3	2	1.00	27	0.074
957	A	3	2	1.00	27	0.074
958	A	4	3	1.00	29	0.103
959	A	4	3	1.00	29	0.103
960	A	4	3	1.00	29	0.103
961	A	6	4	1.00	29	0.138
962	A	5	4	1.00	29	0.138
963	A	4	4	1.00	29	0.138
964	A	2	2	1.00	29	0.069
965	A	3	3	1.00	29	0.103
966	A	3	2	1.00	29	0.069
967	A	3	2	1.00	29	0.069
968	A	3	2	1.00	29	0.069
969	A	3	2	1.00	31	0.065
970	A	3	2	1.00	31	0.065

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	3	2	1.00	31	0.065
972	A	3	2	1.00	29	0.069
973	A	3	2	1.00	29	0.069
974	A	3	2	1.00	31	0.065
975	A	4	3	1.00	31	0.097
976	A	4	3	1.00	31	0.097
977	A	7	5	1.00	31	0.161
978	A	6	5	1.00	31	0.161
979	A	5	5	1.00	31	0.161
980	A	3	2	1.00	31	0.065
981	A	4	4	1.00	31	0.129
982	A	4	3	1.00	31	0.097
983	A	4	3	1.00	31	0.097
984	A	4	3	1.00	31	0.097
985	A	4	3	1.00	31	0.097
986	A	3	2	1.00	31	0.065
987	A	3	2	1.00	31	0.065
988	A	3	2	1.00	31	0.065
989	A	3	2	1.00	29	0.069
990	A	3	2	1.00	29	0.069
991	A	3	2	1.00	31	0.065
992	A	2	2	1.00	31	0.065
993	A	4	3	1.00	31	0.097
994	A	4	3	1.00	31	0.097
995	A	8	5	1.00	31	0.161
996	A	7	5	1.00	31	0.161
997	A	6	5	1.00	31	0.161
998	A	2	2	1.00	31	0.065
999	A	4	4	1.00	31	0.129
1000	A	4	4	1.00	31	0.129
1001	A	4	3	1.00	31	0.097
1002	A	4	3	1.00	31	0.097
1003	A	3	2	1.00	31	0.065
1004	A	3	2	1.00	31	0.065
1005	A	3	2	1.00	31	0.065

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1006	A	3	2	1.00	29	0.069
1007	A	4	3	1.00	29	0.103
1008	A	4	3	1.00	31	0.097
1009	A	4	3	1.00	31	0.097
1010	A	4	3	1.00	31	0.097
1011	A	3	2	1.00	31	0.065
1012	A	3	2	1.00	31	0.065
1013	A	3	2	1.00	31	0.065
1014	A	3	2	1.00	29	0.069
1015	A	4	3	1.00	29	0.103
1016	A	4	3	1.00	31	0.097
1017	A	4	3	1.00	31	0.097
1018	A	4	3	1.00	31	0.097
1019	A	4	4	1.00	33	0.121
1020	A	3	2	1.00	31	0.065
1021	A	3	2	1.00	31	0.065
1022	A	3	2	1.00	31	0.065
1023	A	3	2	1.00	29	0.069
1024	A	3	3	1.00	29	0.103
1025	A	3	3	1.00	31	0.097
1026	A	3	3	1.00	31	0.097
1027	A	4	4	1.00	31	0.129
1028	A	4	4	1.00	31	0.129
1029	A	4	4	1.00	31	0.129
1030	A	4	4	1.00	31	0.129
1031	A	4	4	1.00	31	0.129
1032	A	4	4	1.00	31	0.129
1033	A	4	3	1.00	38	0.079
1034	A	3	3	1.00	38	0.079
1035	A	2	2	1.00	38	0.053
1036	A	4	4	1.00	38	0.105
1037	A	4	4	1.00	36	0.111
1038	A	4	4	1.00	38	0.105
1039	A	4	4	1.00	38	0.105
1040	A	1	1	1.00	40	0.025

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1041	A	1	1	1.00	40	0.025
1042	A	4	4	1.00	35	0.114
1043	A	3	3	1.03	35	0.086
1044	A	3	3	1.04	33	0.091
1045	A	3	3	1.04	35	0.086
1046	A	3	3	1.03	35	0.086
1047	A	3	3	1.03	35	0.086
1048	A	3	3	1.03	35	0.086
1049	A	5	5	1.00	35	0.143
1050	A	8	6	1.00	27	0.222
1051	A	7	6	1.00	27	0.222
1052	A	6	6	1.00	25	0.240
1053	A	6	6	1.00	23	0.261
1054	A	7	6	1.00	19	0.316
1055	A	5	5	1.00	25	0.200
1056	A	5	5	1.00	27	0.185
1057	A	6	6	1.00	27	0.222
1058	A	7	6	1.00	27	0.222
1059	A	10	8	1.00	29	0.276
1060	A	9	8	1.00	29	0.276
1061	A	5	4	1.00	27	0.148
1062	A	6	6	1.00	25	0.240
1063	A	9	7	1.00	21	0.333
1064	A	6	6	1.00	27	0.222
1065	A	6	6	1.00	29	0.207
1066	A	8	8	1.00	29	0.276
1067	A	9	9	1.00	29	0.310
1068	A	9	8	1.00	29	0.276
1069	A	10	9	1.00	29	0.310
1070	A	6	4	1.00	27	0.148
1071	A	7	7	1.00	25	0.280
1072	A	11	9	1.00	21	0.429
1073	A	7	7	1.00	27	0.259
1074	A	7	7	1.00	29	0.241
1075	A	7	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1076	A	9	9	1.00	29	0.310
1077	A	10	10	1.00	29	0.345
1078	A	9	9	1.00	29	0.310
1079	A	8	8	1.00	29	0.276
1080	A	5	5	1.00	27	0.185
1081	A	8	8	1.00	25	0.320
1082	A	8	7	1.00	21	0.333
1083	A	9	8	1.00	27	0.296
1084	A	10	8	1.00	29	0.276
1085	A	9	8	1.00	29	0.276
1086	A	8	8	1.00	29	0.276
1087	A	6	6	1.00	27	0.222
1088	A	8	7	1.00	25	0.280
1089	A	9	8	1.00	21	0.381
1090	A	10	8	1.00	27	0.296
1091	A	5	5	1.00	35	0.143
1092	A	9	6	1.00	27	0.222
1093	A	9	6	1.00	27	0.222
1094	A	8	6	1.00	27	0.222
1095	A	7	6	1.00	25	0.240
1096	A	8	6	1.00	25	0.240
1097	A	9	8	1.00	27	0.296
1098	A	9	7	1.00	25	0.280
1099	A	9	7	1.00	19	0.368
1100	A	7	5	1.00	25	0.200
1101	A	6	5	1.00	27	0.185
1102	A	7	6	1.00	27	0.222
1103	A	8	6	1.00	27	0.222
1104	A	9	6	1.00	27	0.222
1105	A	10	8	1.00	29	0.276
1106	A	9	8	1.00	29	0.276
1107	A	6	4	1.00	27	0.148
1108	A	6	6	1.64	27	0.222
1109	A	6	6	1.00	29	0.207
1110	A	6	6	1.00	27	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1111	A	13	9	1.00	21	0.429
1112	A	6	6	1.00	27	0.222
1113	A	6	6	1.00	29	0.207
1114	A	8	8	1.00	29	0.276
1115	A	9	9	1.00	29	0.310
1116	A	10	8	1.00	29	0.276
1117	A	7	4	1.00	27	0.148
1118	A	7	6	1.00	27	0.222
1119	A	7	6	1.00	29	0.207
1120	A	7	6	1.00	27	0.222
1121	A	17	10	1.00	21	0.476
1122	A	7	6	1.00	27	0.222
1123	A	7	6	1.00	29	0.207
1124	A	7	6	1.00	29	0.207
1125	A	9	8	1.00	29	0.276
1126	A	10	9	1.00	29	0.310
1127	A	9	7	1.00	29	0.241
1128	A	8	7	1.00	29	0.241
1129	A	6	6	1.00	27	0.222
1130	A	6	6	1.00	27	0.222
1131	A	6	6	1.00	29	0.207
1132	A	7	7	1.14	27	0.259
1133	A	8	7	1.00	21	0.333
1134	A	9	7	1.00	27	0.259
1135	A	9	7	1.00	29	0.241
1136	A	8	7	1.00	29	0.241
1137	A	6	5	1.00	27	0.185
1138	A	6	6	1.00	27	0.222
1139	A	7	7	1.00	29	0.241
1140	A	8	7	1.00	27	0.259
1141	A	9	7	1.00	21	0.333
1142	A	10	7	1.00	27	0.259
1143	A	10	9	1.00	31	0.290
1144	A	8	7	1.00	29	0.241
1145	A	10	10	1.00	29	0.345

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1146	A	10	10	1.00	31	0.323
1147	A	10	10	1.00	29	0.345
1148	A	10	10	1.00	23	0.435
1149	A	11	11	1.00	29	0.379
1150	A	12	11	1.00	31	0.355
1151	A	11	9	1.00	31	0.290
1152	A	9	7	1.00	29	0.241
1153	A	11	10	1.00	29	0.345
1154	A	11	10	1.00	31	0.323
1155	A	11	10	1.00	29	0.345
1156	A	11	11	1.00	23	0.478
1157	A	11	10	1.00	29	0.345
1158	A	12	11	1.00	31	0.355
1159	A	13	11	1.00	31	0.355
1160	A	10	7	1.00	29	0.241
1161	A	12	10	1.00	29	0.345
1162	A	12	10	1.00	31	0.323
1163	A	12	10	1.00	29	0.345
1164	A	12	11	1.00	23	0.478
1165	A	12	11	1.00	29	0.379
1166	A	12	10	1.00	31	0.323
1167	A	13	11	1.00	31	0.355
1168	A	10	8	1.00	31	0.258
1169	A	9	8	1.00	31	0.258
1170	A	7	6	1.00	29	0.207
1171	A	9	9	1.00	29	0.310
1172	A	9	9	1.00	31	0.290
1173	A	9	9	1.00	29	0.310
1174	A	10	10	1.00	23	0.435
1175	A	11	10	1.00	29	0.345
1176	A	10	8	1.00	31	0.258
1177	A	9	8	1.00	31	0.258
1178	A	7	7	1.00	29	0.241
1179	A	9	9	1.00	29	0.310
1180	A	9	9	1.00	31	0.290

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1181	A	10	10	1.00	29	0.345
1182	A	11	10	1.00	23	0.435
1183	A	10	8	1.00	31	0.258
1184	A	9	8	1.00	31	0.258
1185	A	7	6	1.00	29	0.207
1186	A	9	9	1.00	29	0.310
1187	A	10	10	1.00	31	0.323
1188	A	11	10	1.00	29	0.345
1189	A	12	10	1.00	23	0.435
1190	A	8	7	1.00	35	0.200
1191	A	0	0	0.00	0	0.000
1192	A	0	0	0.00	0	0.000
1193	A	0	0	0.00	0	0.000
1194	A	0	0	0.00	0	0.000
1195	A	8	6	1.00	29	0.207
1196	A	7	6	1.00	29	0.207
1197	A	3	2	1.00	27	0.074
1198	A	4	3	1.00	27	0.111
1199	A	4	3	1.00	27	0.111
1200	A	7	5	1.00	27	0.185
1201	A	7	5	1.00	27	0.185
1202	A	6	5	1.00	25	0.200
1203	A	4	3	1.00	25	0.120
1204	A	4	3	1.00	27	0.111
1205	A	4	3	1.00	27	0.111
1206	A	4	3	1.00	25	0.120
1207	A	3	2	1.00	19	0.105
1208	A	4	3	1.00	25	0.120
1209	A	6	5	1.00	27	0.185
1210	A	6	5	1.00	27	0.185
1211	A	7	5	1.00	27	0.185
1212	A	7	5	1.00	27	0.185
1213	A	4	3	1.00	27	0.111
1214	A	4	3	1.00	27	0.111
1215	A	4	3	1.00	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1216	A	4	3	1.00	27	0.111
1217	A	4	3	1.00	27	0.111
1218	A	4	3	1.00	29	0.103
1219	A	4	3	1.00	29	0.103
1220	A	4	3	1.00	27	0.111
1221	A	3	2	1.00	21	0.095
1222	A	4	3	1.00	27	0.111
1223	A	4	3	1.00	29	0.103
1224	A	4	3	1.00	29	0.103
1225	A	4	3	1.00	29	0.103
1226	A	4	3	1.00	29	0.103
1227	A	4	3	1.00	29	0.103
1228	A	4	3	1.00	27	0.111
1229	A	4	3	1.00	27	0.111
1230	A	4	3	1.00	29	0.103
1231	A	4	3	1.00	29	0.103
1232	A	4	3	1.00	27	0.111
1233	A	3	2	1.00	21	0.095
1234	A	4	3	1.00	27	0.111
1235	A	3	2	1.00	29	0.069
1236	A	3	2	1.00	27	0.074
1237	A	5	4	1.00	29	0.138
1238	A	5	4	1.00	29	0.138
1239	A	12	7	1.00	29	0.241
1240	A	12	9	1.00	29	0.310
1241	A	11	7	1.00	29	0.241
1242	A	11	8	1.00	29	0.276
1243	A	7	4	1.00	27	0.148
1244	A	9	5	1.00	27	0.185
1245	A	12	8	1.00	29	0.276
1246	A	11	8	1.00	29	0.276
1247	A	12	9	1.00	29	0.310
1248	A	12	9	1.00	27	0.333
1249	A	16	10	1.00	21	0.476
1250	A	11	9	1.00	27	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1251	A	9	5	1.00	29	0.172
1252	A	9	9	1.00	29	0.310
1253	A	9	5	1.00	29	0.172
1254	A	11	10	1.00	29	0.345
1255	A	10	5	1.00	29	0.172
1256	A	11	8	1.00	29	0.276
1257	A	10	8	1.00	29	0.276
1258	A	7	6	1.00	27	0.222
1259	A	16	11	1.00	27	0.407
1260	A	16	11	1.00	29	0.379
1261	A	16	11	1.00	29	0.379
1262	A	17	10	1.00	29	0.345
1263	A	9	7	1.00	27	0.259
1264	A	10	7	1.00	21	0.333
1265	A	11	7	1.00	27	0.259
1266	A	11	8	1.00	29	0.276
1267	A	10	8	1.00	29	0.276
1268	A	7	6	1.00	27	0.222
1269	A	20	11	1.00	27	0.407
1270	A	20	11	1.00	29	0.379
1271	A	21	11	1.00	29	0.379
1272	A	9	7	1.00	29	0.241
1273	A	10	7	1.00	27	0.259
1274	A	11	7	1.00	21	0.333
1275	A	13	7	1.00	29	0.241
1276	A	8	7	1.00	35	0.200
1277	A	9	9	1.01	37	0.243
1278	A	6	6	1.00	37	0.162
1279	A	4	3	1.00	27	0.111
1280	A	4	3	1.00	27	0.111
1281	A	4	3	1.00	25	0.120
1282	A	4	4	1.00	19	0.210
1283	A	4	3	1.00	25	0.120
1284	A	4	3	1.00	27	0.111
1285	A	10	8	1.00	29	0.276

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1286	A	9	8	1.00	29	0.276
1287	A	8	8	1.00	29	0.276
1288	A	5	5	1.00	27	0.185
1289	A	6	6	1.00	25	0.240
1290	A	7	7	1.00	21	0.333
1291	A	8	8	1.00	27	0.296
1292	A	9	8	1.00	29	0.276
1293	A	10	8	1.00	29	0.276
1294	A	11	8	1.00	29	0.276
1295	A	4	3	1.00	29	0.103
1296	A	4	3	1.00	29	0.103
1297	A	4	3	1.00	27	0.111
1298	A	4	3	1.00	27	0.111
1299	A	4	3	1.00	27	0.111
1300	A	3	2	1.00	21	0.095
1301	A	9	7	1.00	29	0.241
1302	A	8	7	1.00	29	0.241
1303	A	6	5	1.00	27	0.185
1304	A	6	6	1.00	27	0.222
1305	A	6	6	1.00	29	0.207
1306	A	6	6	1.00	27	0.222
1307	A	7	7	1.00	21	0.333
1308	A	8	7	1.00	27	0.259
1309	A	9	7	1.00	29	0.241
1310	A	4	3	1.00	29	0.103
1311	A	4	3	1.00	29	0.103
1312	A	4	3	1.00	27	0.111
1313	A	4	3	1.00	27	0.111
1314	A	4	3	1.00	29	0.103
1315	A	4	3	1.00	29	0.103
1316	A	4	3	1.00	27	0.111
1317	A	3	2	1.00	21	0.095
1318	A	4	3	1.00	27	0.111
1319	A	4	3	1.00	29	0.103
1320	A	11	7	1.00	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1321	A	10	7	1.00	29	0.241
1322	A	7	5	1.00	27	0.185
1323	A	14	9	1.00	27	0.333
1324	A	13	10	1.00	29	0.345
1325	A	6	6	1.00	29	0.207
1326	A	13	9	1.00	29	0.310
1327	A	15	8	1.41	27	0.296
1328	A	9	7	1.27	21	0.333
1329	A	10	7	1.00	27	0.259
1330	A	11	7	1.00	29	0.241
1331	A	12	7	1.00	29	0.241
1332	A	4	3	1.00	27	0.111
1333	A	4	3	1.00	25	0.120
1334	A	3	2	1.00	19	0.105
1335	A	4	3	1.00	25	0.120
1336	A	4	3	1.00	27	0.111
1337	A	4	3	1.00	27	0.111
1338	A	14	13	1.00	29	0.448
1339	A	12	11	1.00	29	0.379
1340	A	9	8	1.00	27	0.296
1341	A	8	7	1.00	21	0.333
1342	A	5	5	1.00	25	0.200
1343	A	10	9	1.00	27	0.333
1344	A	13	11	1.17	29	0.379
1345	A	17	12	1.17	29	0.414
1346	A	4	3	1.00	21	0.143
1347	A	5	4	1.00	27	0.148
1348	A	5	4	1.00	27	0.148
1349	A	4	3	1.00	27	0.111
1350	A	4	3	1.00	29	0.103
1351	A	4	3	1.00	29	0.103
1352	A	13	9	1.00	21	0.429
1353	A	10	9	1.00	27	0.333
1354	A	10	9	1.00	29	0.310
1355	A	6	5	1.00	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1356	A	12	10	1.00	27	0.370
1357	A	15	12	1.12	29	0.414
1358	A	20	13	1.00	29	0.448
1359	A	6	4	1.00	29	0.138
1360	A	6	4	1.00	29	0.138
1361	A	6	4	1.00	27	0.148
1362	A	5	3	1.00	21	0.143
1363	A	6	4	1.00	27	0.148
1364	A	6	5	1.00	29	0.172
1365	A	6	5	1.00	29	0.172
1366	A	6	4	1.00	27	0.148
1367	A	4	3	1.00	27	0.111
1368	A	4	3	1.00	29	0.103
1369	A	4	3	1.00	29	0.103
1370	A	21	14	1.00	33	0.424
1371	A	18	13	1.00	33	0.394
1372	A	15	12	1.00	33	0.364
1373	A	12	10	1.00	31	0.323
1374	A	16	11	1.00	31	0.355
1375	A	19	14	1.00	33	0.424
1376	A	25	15	1.00	33	0.454
1377	A	24	16	1.00	33	0.485
1378	A	20	15	1.00	33	0.454
1379	A	13	11	1.00	31	0.355
1380	A	21	16	1.00	31	0.516
1381	A	24	17	1.00	33	0.515
1382	A	30	18	1.00	33	0.546
1383	A	24	16	1.00	33	0.485
1384	A	20	15	1.00	33	0.454
1385	A	13	11	1.00	31	0.355
1386	A	21	16	1.00	31	0.516
1387	A	24	17	1.00	33	0.515
1388	A	30	18	1.00	33	0.546
1389	A	23	15	1.00	33	0.454
1390	A	19	14	1.00	33	0.424

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1391	A	15	13	1.00	33	0.394
1392	A	12	10	1.00	31	0.323
1393	A	16	11	1.00	31	0.355
1394	A	19	14	1.00	33	0.424
1395	A	25	15	1.00	33	0.454
1396	A	22	15	1.00	33	0.454
1397	A	18	14	1.00	33	0.424
1398	A	15	13	1.00	33	0.394
1399	A	13	11	1.00	31	0.355
1400	A	21	16	1.00	31	0.516
1401	A	25	18	1.00	33	0.546
1402	A	22	16	1.00	33	0.485
1403	A	18	14	1.00	33	0.424
1404	A	15	13	1.00	33	0.394
1405	A	13	11	1.00	31	0.355
1406	A	21	16	1.00	31	0.516
1407	A	25	18	1.00	33	0.546
1408	A	31	15	1.00	37	0.405
1409	A	19	14	1.00	37	0.378
1410	A	16	12	1.00	37	0.324
1411	A	5	4	1.00	37	0.108
1412	A	9	8	1.00	37	0.216
1413	A	11	9	1.00	37	0.243
1414	A	16	9	1.00	37	0.243
1415	A	19	9	1.00	37	0.243
1416	A	31	16	1.00	37	0.432
1417	A	19	15	1.00	37	0.405
1418	A	18	14	1.00	37	0.378
1419	A	8	7	1.00	37	0.189
1420	A	12	9	1.00	37	0.243
1421	A	15	9	1.00	37	0.243
1422	A	20	9	1.00	37	0.243
1423	A	31	17	1.00	37	0.460
1424	A	19	15	1.00	37	0.405
1425	A	20	16	1.00	37	0.432

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1426	A	10	9	1.00	37	0.243
1427	A	15	10	1.00	37	0.270
1428	A	18	10	1.00	37	0.270
1429	A	24	10	1.00	37	0.270
1430	A	19	14	1.00	37	0.378
1431	A	15	11	1.00	37	0.297
1432	A	4	3	1.00	37	0.081
1433	A	7	6	1.00	37	0.162
1434	A	9	7	1.00	37	0.189
1435	A	13	8	1.00	37	0.216
1436	A	31	17	1.00	37	0.460
1437	A	10	9	1.00	37	0.243
1438	A	11	10	1.00	37	0.270
1439	A	11	10	1.00	37	0.270
1440	A	16	12	1.00	37	0.324
1441	A	19	12	1.00	37	0.324
1442	A	8	7	1.00	37	0.189
1443	A	8	7	1.00	27	0.259
1444	A	8	7	1.00	25	0.280
1445	A	7	5	1.00	19	0.263
1446	A	5	4	1.00	23	0.174
1447	A	6	6	1.00	25	0.240
1448	A	7	6	1.00	27	0.222
1449	A	8	7	1.00	27	0.259
1450	A	8	7	1.00	27	0.259
1451	A	11	9	1.00	21	0.429
1452	A	4	3	1.00	25	0.120
1453	A	8	8	1.52	27	0.296
1454	A	7	6	1.00	29	0.207
1455	A	10	9	1.24	29	0.310
1456	A	8	7	1.00	29	0.241
1457	A	17	8	1.00	27	0.296
1458	A	14	10	1.00	21	0.476
1459	A	3	3	1.00	25	0.120
1460	A	11	8	1.00	27	0.296

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1461	A	12	9	1.00	29	0.310
1462	A	14	9	1.00	29	0.310
1463	A	15	8	1.00	29	0.276
1464	A	12	7	1.00	29	0.241
1465	A	12	7	1.00	27	0.259
1466	A	12	7	1.00	21	0.333
1467	A	6	6	1.00	25	0.240
1468	A	13	8	1.00	27	0.296
1469	A	15	10	1.00	29	0.345
1470	A	17	11	1.00	29	0.379
1471	A	18	8	1.00	29	0.276
1472	A	18	8	1.00	27	0.296
1473	A	18	8	1.00	21	0.381
1474	A	7	6	1.00	25	0.240
1475	A	19	9	1.00	27	0.333
1476	A	21	11	1.00	29	0.379
1477	A	23	12	1.00	29	0.414
1478	A	2	2	1.00	35	0.057
1479	F	0	0	N/A	0.	N/A
1480	F	0	0	N/A	0.	N/A
1481	A	11	8	1.00	27	0.296
1482	A	10	8	1.00	25	0.320
1483	A	7	5	1.00	19	0.263
1484	A	7	5	1.00	25	0.200
1485	A	6	5	1.00	27	0.185
1486	A	6	6	1.00	27	0.222
1487	A	6	6	1.00	25	0.240
1488	A	8	6	1.00	25	0.240
1489	A	10	8	1.00	27	0.296
1490	A	10	8	1.00	27	0.296
1491	A	11	8	1.00	27	0.296
1492	A	9	6	1.00	27	0.222
1493	A	8	5	1.00	21	0.238
1494	A	9	6	1.00	27	0.222
1495	A	7	5	1.00	29	0.172

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1496	A	5	5	1.00	29	0.172
1497	A	6	5	1.00	27	0.185
1498	A	6	5	1.00	27	0.185
1499	A	6	4	1.00	29	0.138
1500	A	6	4	1.00	29	0.138
1501	A	8	5	1.00	21	0.238
1502	A	9	6	1.00	27	0.222
1503	A	8	6	1.00	29	0.207
1504	A	7	6	1.00	29	0.207
1505	A	5	5	1.00	27	0.185
1506	A	6	5	1.00	27	0.185
1507	A	6	4	1.00	29	0.138
1508	A	6	4	1.00	29	0.138
1509	A	6	4	1.00	29	0.138
1510	A	5	4	1.00	29	0.138
1511	A	5	4	1.00	29	0.138
1512	A	4	3	1.00	27	0.111
1513	A	10	3	1.00	29	0.103
1514	A	17	5	1.00	29	0.172
1515	F	0	0	N/A	0.	N/A
1516	A	11	8	1.00	35	0.229
1517	A	10	7	1.00	33	0.212
1518	A	2	2	1.00	23	0.087
1519	A	0	0	0.00	0	0.000
1520	A	0	0	0.00	0	0.000
1521	A	0	0	0.00	0	0.000
1522	A	0	0	0.00	0	0.000
1523	A	11	8	1.00	33	0.242
1524	A	10	7	0.99	31	0.226
1525	A	2	2	1.00	21	0.095
1526	A	0	0	0.00	0	0.000
1527	A	0	0	0.00	0	0.000
1528	A	3	2	1.00	29	0.069
1529	A	3	2	1.00	29	0.069
1530	A	3	2	1.00	29	0.069

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1531	A	3	2	1.00	27	0.074
1532	A	5	4	1.00	27	0.148
1533	A	3	3	1.00	29	0.103
1534	A	4	4	1.00	29	0.138
1535	A	5	4	1.00	29	0.138
1536	A	3	2	1.00	31	0.065
1537	A	3	2	1.00	31	0.065
1538	A	3	2	1.00	31	0.065
1539	A	3	2	1.00	29	0.069
1540	A	6	4	1.00	29	0.138
1541	A	5	4	1.00	31	0.129
1542	A	4	4	1.00	31	0.129
1543	A	5	5	1.00	31	0.161
1544	A	3	2	1.00	31	0.065
1545	A	3	2	1.00	31	0.065
1546	A	3	2	1.00	31	0.065
1547	A	3	2	1.00	29	0.069
1548	A	3	2	1.00	29	0.069
1549	A	4	3	1.00	31	0.097
1550	A	5	3	1.00	31	0.097
1551	A	6	3	1.00	31	0.097
1552	A	3	2	1.00	31	0.065
1553	A	3	2	1.00	31	0.065
1554	A	3	2	1.00	31	0.065
1555	A	3	2	1.00	29	0.069
1556	A	3	2	1.00	29	0.069
1557	A	4	3	1.00	31	0.097
1558	A	5	3	1.00	31	0.097
1559	A	6	3	1.00	31	0.097
1560	A	0	0	0.00	0	0.000
1561	A	4	2	1.00	35	0.057
1562	A	5	2	1.00	35	0.057
1563	A	5	3	1.00	35	0.086

Chapter 3

Listing of integrals

Local contents

3.1 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=92

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a+a\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{6cf}$$

```
[Out] -1/15*a*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/6*co
s(f*x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.25, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{6cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/15*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(c*f*Sqrt[a + a*Sin[e + f
*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2))/
(6*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{ac}$$
$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{6cf}$$
$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e}{}$$

Mathematica [A]
time = 0.41, size = 104, normalized size = 1.13

$$\frac{c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(405\cos(2(e+fx))+90\cos(4(e+fx))-5\cos(6(e+fx))+1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2
),x]
```

```
[Out] (c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(405*
Cos[2*(e + f*x)] + 90*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] + 1080*Sin[e +
f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A]
time = 17.88, size = 133, normalized size = 1.45

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)\sqrt{a\,(1+\sin(fx+e))}\,(5(\cos^8(fx+e))+3(\cos^6(fx+e))\sin(fx+e)+4(\cos^6(fx+e))+7(\cos^4(}{30f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(5*cos
(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)+4*cos(f*x+e)^6+7*cos(f*x+e)^4*sin(f*x+e
)+7*cos(f*x+e)^2*sin(f*x+e)-7*cos(f*x+e)^2+28*sin(f*x+e)+28)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)
^2, x)
```

Fricas [A]
time = 0.36, size = 118, normalized size = 1.28

$$-\frac{\left(5\,c^{3}\cos\left(fx+e\right)^{6}-30\,c^{3}\cos\left(fx+e\right)^{4}+25\,c^{3}+2\,\left(9\,c^{3}\cos\left(fx+e\right)^{4}-8\,c^{3}\cos\left(fx+e\right)^{2}-16\,c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*c^3*cos(f*x + e)^6 - 30*c^3*cos(f*x + e)^4 + 25*c^3 + 2*(9*c^3*cos
(f*x + e)^4 - 8*c^3*cos(f*x + e)^2 - 16*c^3)*sin(f*x + e))*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 108, normalized size = 1.17

$$-\frac{32\,\left(5\,c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}-6\,c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -32/15*(5*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12 - 6*c^3*sgn(cos(-1/4*pi + 1/2*
```

```
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1
/2*e)^10)*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 11.45, size = 121, normalized size = 1.32

$$\frac{c^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(405\,\cos\left(e+f\,x\right)+495\,\cos\left(3\,e+3\,f\,x\right)+85\,\cos\left(5\,e+5\,f\,x\right)-5\,\cos\left(7\,e+7\,f\,x\right)+1100\,\sin\left(2\,e+2\,f\,x\right)-16\,\sin\left(4\,e+4\,f\,x\right)-36\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(405*cos(e
+ f*x) + 495*cos(3*e + 3*f*x) + 85*cos(5*e + 5*f*x) - 5*cos(7*e + 7*f*x) +
1100*sin(2*e + 2*f*x) - 16*sin(4*e + 4*f*x) - 36*sin(6*e + 6*f*x)))/(960*f*
(cos(2*e + 2*f*x) + 1))
```

3.2 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=92

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a+a\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{5cf}$$

```
[Out] -1/10*a*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/5*co
s(f*x+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.26, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{5cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/10*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(c*f*Sqrt[a + a*Sin[e + f
*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))/
(5*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(}{5cf}$$

$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{10cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e}{}$$

Mathematica [A]
time = 0.33, size = 94, normalized size = 1.02

$$\frac{c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(20\cos(2(e+fx))+5\cos(4(e+fx))+70\sin(e+fx)+5\sin(3(e+fx))-\sin(5(e+fx)))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(20*C
os[2*(e + f*x)] + 5*Cos[4*(e + f*x)] + 70*Sin[e + f*x] + 5*Sin[3*(e + f*x)]
 - Sin[5*(e + f*x)]))/(80*f)
```

Maple [A]
time = 18.00, size = 106, normalized size = 1.15

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}}\sin(fx+e)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(2(\cos^6(fx+e))+(\cos^4(fx+e))\sin(fx+e)+2(\cos^4(fx+e))+3(\cos^2(}{10f\cos(fx+e)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/10/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(2*cos
(f*x+e)^6+cos(f*x+e)^4*sin(f*x+e)+2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
6*sin(f*x+e)+6)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)
^2, x)
```

Fricas [A]
time = 0.36, size = 103, normalized size = 1.12

$$\frac{\left(5\,c^2\cos\left(fx+e\right)^4-5\,c^2-2\,\left(c^2\cos\left(fx+e\right)^4-2\,c^2\cos\left(fx+e\right)^2-4\,c^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/10*(5*c^2*cos(f*x + e)^4 - 5*c^2 - 2*(c^2*cos(f*x + e)^4 - 2*c^2*cos(f*x
+ e)^2 - 4*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)/(f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.52, size = 108, normalized size = 1.17

$$-\frac{8\left(4\,c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}-5\,c^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\right)\sqrt{a}\,\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -8/5*(4*c^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10 - 5*c^2*sgn(cos(-1/4*pi + 1/2*f*
```

```
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^8)*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 10.45, size = 110, normalized size = 1.20

$$\frac{c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)+75\,\sin\left(2\,e+2\,f\,x\right)+4\,\sin\left(4\,e+4\,f\,x\right)-\sin\left(6\,e+6\,f\,x\right)\right)}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e +
 f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) + 75*sin(2*e + 2*f*x) + 4*
sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

3.3 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=92

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a+a\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{4cf}$$

```
[Out] -1/6*a*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/4*cos
(f*x+e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.27, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{4cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(c*f*Sqrt[a + a*Sin[e + f*
x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))/(
4*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{4cf}$$

$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{6cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e}{}$$

Mathematica [A]
time = 0.28, size = 83, normalized size = 0.90

$$\frac{c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(12\cos(2(e+fx))+3\cos(4(e+fx))+8(9\sin(e+fx)+\sin(3(e+fx))))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] (c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(12*Cos
[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)]))
)/(96*f)
```

Maple [A]
time = 0.16, size = 90, normalized size = 0.98

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(3(\cos^4(fx+e))+(\cos^2(fx+e))\sin(fx+e)+4(\cos^2(fx+e))+5\sin(fx}{12f\cos(fx+e)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/12/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(3*cos
(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+e)^2+5*sin(f*x+e)+5)/cos(f*x+e)
^3
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)
^2, x)
```

<u>Fricas</u> [A]
time = 0.37, size = 81, normalized size = 0.88

$$\frac{\left(3\,c\cos\left(fx+e\right)^{4}+4\,\left(c\cos\left(fx+e\right)^{2}+2\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/12*(3*c*cos(f*x + e)^4 + 4*(c*cos(f*x + e)^2 + 2*c)*sin(f*x + e) - 3*c)*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

<u>Sympy</u> [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*cos(e +
f*x)**2, x)
```

<u>Giac</u> [A]
time = 0.56, size = 150, normalized size = 1.63

$$-\frac{4\left(3\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-8\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+6\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -4/3*(3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*c*cos(-1/4*pi + 1/2*f*x + 1/2*e
)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
 + 6*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 1.71, size = 97, normalized size = 1.05

$$\frac{c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)+80\,\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e + f
*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) + 80*sin(2*e + 2*f*x) + 8*si
n(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

3.4 $\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=92

$$-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a+a\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}{3cf}$$

```
[Out] -1/3*a*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/3*cos
(f*x+e)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.25, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}{3cf}-\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] -1/3*(a*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(c*f*Sqrt[a + a*Sin[e + f*
x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(
3*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,a}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e}{3cf}$$

$$= -\frac{a\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{3cf\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e}{}$$

Mathematica [A]
time = 0.13, size = 59, normalized size = 0.64

$$\frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(9\sin(e+fx)+\sin(3(e+fx)))}{12f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(9*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f)
```

Maple [A]
time = 0.15, size = 55, normalized size = 0.60

method	result	size
default	$\frac{(\cos^2(fx+e)+2)\sqrt{-c(\sin(fx+e)-1)}\,\sin(fx+e)\sqrt{a(1+\sin(fx+e))}}{3f\cos(fx+e)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^2+2)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e
)))^(1/2)/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
, x)
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.64

$$\frac{\left(\cos\left(fx+e\right)^{2}+2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(cos(f*x + e)^2 + 2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x
)**2, x)
```

Giac [A]
time = 0.55, size = 102, normalized size = 1.11

$$\frac{4\left(2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 4/3*(2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqr
t(a)*sqrt(c)/f
```

Mupad [B]
time = 0.90, size = 64, normalized size = 0.70

$$\frac{(10\sin(2e+2fx)+\sin(4e+4fx))\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}}{12f(\cos(2e+2fx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] ((10*sin(2*e + 2*f*x) + sin(4*e + 4*f*x))*(a*(sin(e + f*x) + 1))^(1/2)*(-c*
(sin(e + f*x) - 1))^(1/2))/(12*f*(cos(2*e + 2*f*x) + 1))
```

3.5 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 62, normalized size = 1.38

$$-\frac{\sec(e+fx)(\cos(2(e+fx))-4\sin(e+fx))\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{4cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/4*(Sec[e + f*x]*(Cos[2*(e + f*x)] - 4*Sin[e + f*x])*Sqrt[a*(1 + Sin[e +
f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(39) = 78$.
time = 0.16, size = 94, normalized size = 2.09

method	result	size
default	$\frac{\sin(fx+e)\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))+\sin(fx+e)+2\cos(fx+e)-1)}{2f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,(1-\cos(fx+e)+\sin(fx+e))}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)
^2+sin(f*x+e)+2*cos(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(1-cos(f*x+e)+sin(f
*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(42) = 84$.
time = 0.55, size = 417, normalized size = 9.27

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{a}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{c+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}{c+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}\right)}{c+\frac{2c\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{c\sin(fx+e)^4}{(\cos(fx+e)+1)^4}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(
a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)*sqrt(c)*sin(f*x + e)^2
```

```
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(c + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + c*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [**A**]
time = 0.38, size = 64, normalized size = 1.42

$$-\frac{\left(\cos\left(fx+e\right)^{2}-2\,\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,cf\,\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/2*(cos(f*x + e)^2 - 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cos^{2}\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
 1)), x)
```

Giac [**A**]
time = 0.53, size = 56, normalized size = 1.24

$$-\frac{2\,\sqrt{a}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sqrt{c}\,f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -2*sqrt(a)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))/(sqrt(c)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 8.96, size = 73, normalized size = 1.62

$$-\frac{\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-4\,\sin\left(2\,e+2\,f\,x\right)\right)}{4\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x)
+ cos(3*e + 3*f*x) - 4*sin(2*e + 2*f*x)))/(4*c*f*(cos(2*e + 2*f*x) + 1))
```

3.6 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*a*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$-\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[
c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} + \frac{(2a\cos(e+fx))\int \frac{\cos(e}{c-c\sin}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c}}$$

$$= -\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{(2a\cos(e+fx))\text{Subst}\left(\int \frac{1}{c+}\right.}{cf\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2a\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{a+}}{cf\sqrt{c-c\sin}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.75, size = 121, normalized size = 1.22

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(-2\log\left(e^{i(e+fx)}\right)+4\log\left(-i+e^{i(e+fx)}\right)+\sin(e+fx)\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(3
/2),x]

[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-2*L
og[E^(I*(e + f*x))] + 4*Log[-I + E^(I*(e + f*x))] + Sin[e + f*x]))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(3/2)))

Maple [A]
time = 0.16, size = 141, normalized size = 1.42

method	result
default	$-\frac{\left(2\ln\left(\frac{2}{1+\cos(fx+e)}\right)-4\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\sin(fx+e)\right)\left(\cos(fx+e)\sin(fx+e)-\left(\cos^{2}(fx+e)\right)-2\sin(fx+e)-\cos(fx\right.}{f(1-\cos(fx+e)+\sin(fx+e))(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)

[Out] -1/f*(2*ln(2/(1+cos(f*x+e)))-4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-s
in(f*x+e))*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(
a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/
2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")

[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2
/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cos^{2}\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))
**(3/2), x)
```

Giac [A]
time = 0.59, size = 101, normalized size = 1.02

$$\frac{2\,\sqrt{a}\,\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 2*sqrt(a)*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^2*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) + log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.7 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)+a*cos(f*x+e)*l
n(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2818, 2816, 2746, 31}

$$\frac{a\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(3/2)) +
(a*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{(a\cos(e+fx))\int \frac{\cos(e+\ldots}{c-c\sin(e\ldots}}{c\sqrt{a+a\sin(e+fx)}\sqrt{c-c\ldots}}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{1}{c+x}\,dx\ldots\right.}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c\ldots}}$$

$$= \frac{\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{a\cos(e+fx)\log(1-\sin\ldots}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 114, normalized size = 1.18

$$\frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\left(2-\log\left(e^{i(e+fx)}\right)+2\log\left(-i+e^{i(e+fx)}\right)+\left(\log\left(e^{i(e+fx)}\right)-2\log\left(-i+e^{i(e+fx)}\right)\right)\sin(e+fx)\right)}{c^2f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(5
/2),x]
```

[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(2 - Log[E^(I*(e + f*x))] + 2*Log[-I + E^(I*(e + f*x))] + (Log[E^(I*(e + f*x))] - 2*Log[-I + E^(I*(e + f*x))])*Sin[e + f*x]))/(c^2*f*Sqrt[c - c*Sin[e + f*x]])

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(89) = 178$.
time = 0.16, size = 194, normalized size = 2.00

method	result
default	$-\frac{\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(\frac{2}{1+\cos(fx+e)}\right)+2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+}{f(1-\cos(fx+e)+\sin(fx+e))(-c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/f*(ln(2/(1+cos(f*x+e)))*sin(f*x+e)-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln(2/(1+cos(f*x+e)))+2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e))*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(5/2)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\,\cos^{2}\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))
**(5/2), x)
```

Giac [A]
time = 0.57, size = 89, normalized size = 0.92

$$-\frac{\left(2\,\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}\right)\sqrt{a}}{c^{\frac{5}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -(2*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e)) + sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/sin(-1/4*pi + 1/2*f*x + 1/2*e)^
2)*sqrt(a)/(c^(5/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.8 $\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/c/f/(c-c*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.22, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*a*c*f*(c - c*Sin[e + f*x])^(5/
2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{4acf(c-c\sin(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 90, normalized size = 1.88

$$\frac{\sin(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}{c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^(7
/2),x]
[Out] (Sin[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])/(c^4*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(42) = 84$.
time = 0.15, size = 96, normalized size = 2.00

method	result	size
default	$-\frac{\sqrt{a(1+\sin(fx+e))}\,\sin(fx+e)(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2\sin(fx+e)-\cos(fx+e)+2)}{f(-c(\sin(fx+e)-1))^{\frac{7}{2}}(1-\cos(fx+e)+\sin(fx+e))}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
[Out] -1/f*(a*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^
2-2*sin(f*x+e)-cos(f*x+e)+2)/(-c*(sin(f*x+e)-1))^(7/2)/(1-cos(f*x+e)+sin(f*
x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(7/
2), x)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 1.79

$$-\frac{\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{c^4f\cos(fx+e)^3+2\,c^4f\cos(fx+e)\sin(fx+e)-2\,c^4f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(c^4*f*cos
(f*x + e)^3 + 2*c^4*f*cos(f*x + e)*sin(f*x + e) - 2*c^4*f*cos(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(45) = 90.
time = 0.63, size = 96, normalized size = 2.00

$$\frac{\left(2\,\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{4\,c^4f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/4*(2*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
1/2*e)^2 - sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^4*f*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.9 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=140

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{21cf}-\frac{\cos(e+\ldots}{}$$

```
[Out] -1/7*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-4/105*a^2
*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-2/21*a*cos(f*
x+e)*(c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.35, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{7cf}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{21cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(105*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^
(9/2))/(21*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f
*x])^(9/2))/(7*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{\cdots}}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\,\mathrm{s}\cdots}{7cf}$$
$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-\cdots}{21cf}$$
$$= -\frac{4a^2\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{105cf\sqrt{a+a\sin(e+fx)}} - \cdots$$

Mathematica [A]
time = 0.81, size = 166, normalized size = 1.19

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(1050\cos(2(e+fx))+420\cos(4(e+fx))+70\cos(6(e+fx))+4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5(e+fx))-15\sin(7(e+fx)))}{6720f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*
Sin[e + f*x]]*(1050*Cos[2*(e + f*x)] + 420*Cos[4*(e + f*x)] + 70*Cos[6*(e +
 f*x)] + 4725*Sin[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 1
5*Sin[7*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A]
time = 0.19, size = 133, normalized size = 0.95

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(15\left(\cos^8(fx+e)\right)+5\left(\cos^6(fx+e)\right)\sin(fx+e)+16\left(\cos^6(fx+e)\right)+13\left(\cos^4(fx+e\cdots\right.\right.}{105f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/105/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(15*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+16*cos(f*x+e)^6+13*cos(f*x+e)^4*sin(f
*x+e)+16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)+58)/cos(f*x+
e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.38, size = 124, normalized size = 0.89

$$\frac{\left(35\,ac^{3}\cos\left(fx+e\right)^{6}-35\,ac^{3}-\left(15\,ac^{3}\cos\left(fx+e\right)^{6}-24\,ac^{3}\cos\left(fx+e\right)^{4}-32\,ac^{3}\cos\left(fx+e\right)^{2}-64\,ac^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/105*(35*a*c^3*cos(f*x + e)^6 - 35*a*c^3 - (15*a*c^3*cos(f*x + e)^6 - 24*a
*c^3*cos(f*x + e)^4 - 32*a*c^3*cos(f*x + e)^2 - 64*a*c^3)*sin(f*x + e))*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 159, normalized size = 1.14

$$\frac{128\left(15\,ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}-35\,ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}+21\,ac^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\right)\sqrt{a}\,\sqrt{c}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 128/105*(15*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14 - 35*a*c^3*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^12 + 21*a*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)*sqrt(a)*sqrt(c)/
f
```

Mupad [B]
time = 12.43, size = 319, normalized size = 2.28

$$\frac{e^{-e\,7i-f\,x\,7i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{5\,a\,c^3\,e^{e\,7i+f\,x\,7i}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}+\frac{a\,c^3\,e^{e\,7i+f\,x\,7i}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{8\,f}+\frac{a\,c^3\,e^{e\,7i+f\,x\,7i}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{48\,f}+\frac{19\,a\,c^3\,e^{e\,7i+f\,x\,7i}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}+\frac{a\,c^3\,e^{e\,7i+f\,x\,7i}\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}-\frac{a\,c^3\,e^{e\,7i+f\,x\,7i}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}+\frac{45\,a\,c^3\,e^{e\,7i+f\,x\,7i}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((5*a*c^3*exp(e*7i + f*x*7
i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(16*f) + (a*c^3*exp(e*7i +
f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(8*f) + (a*c^3*exp(e*7
i + f*x*7i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(48*f) + (19*a*c^3
*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (
a*c^3*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*
f) - (a*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))
/(224*f) + (45*a*c^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(e + f*x))^(
1/2))/(32*f)))/(2*cos(e + f*x))
```

3.10 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=140

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a+a\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{15cf}-\frac{\cos(e+f}{}$$

```
[Out] -1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/c/f-1/15*a^2*
cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-2/15*a*cos(f*x
+e)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.35, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{6cf}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{15cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/15*(a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2))/(15*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*
x])^(7/2))/(6*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\mathrm{s}}{6cf}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-}{15cf}$$

$$= -\frac{a^2\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{15cf\sqrt{a+a\sin(e+fx)}} - \frac{2}{}$$

Mathematica [A]
time = 0.63, size = 156, normalized size = 1.11

$$\frac{c^2(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(75\cos(2(e+fx))+30\cos(4(e+fx))+5\cos(6(e+fx))+600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx)))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (c^2*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
f*x]]*(75*Cos[2*(e + f*x)] + 30*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 600
*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(960*f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

Maple [A]
time = 0.17, size = 116, normalized size = 0.83

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left(5\left(\cos^6(fx+e)\right)+\left(\cos^4(fx+e)\right)\sin(fx+e)+6\left(\cos^4(fx+e)\right)+3\left(\cos^2(fx+e)\right)\sin\right.}{30f\cos(fx+e)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/30/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(5*cos
(f*x+e)^6+cos(f*x+e)^4*sin(f*x+e)+6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e)+
8*cos(f*x+e)^2+11*sin(f*x+e)+11)/cos(f*x+e)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [**A**]
time = 0.37, size = 109, normalized size = 0.78

$$\frac{\left(5\,ac^{2}\cos\left(fx+e\right)^{6}-5\,ac^{2}+2\,\left(3\,ac^{2}\cos\left(fx+e\right)^{4}+4\,ac^{2}\cos\left(fx+e\right)^{2}+8\,ac^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a*c^2*cos(f*x + e)^6 - 5*a*c^2 + 2*(3*a*c^2*cos(f*x + e)^4 + 4*a*c^
2*cos(f*x + e)^2 + 8*a*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 208, normalized size = 1.49

$$\frac{16\left(10\,ac^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-36\,ac^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+45\,ac^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-20\,ac^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 16/15*(10*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 36*a*c^2*cos(-1/4*pi + 1/2
*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e)) + 45*a*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 20*a*c^2*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 11.44, size = 122, normalized size = 0.87

$$\frac{a\,c^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\,f\,x\right)+5\,\cos\left(7\,e+7\,f\,x\right)+700\,\sin\left(2\,e+2\,f\,x\right)+112\,\sin\left(4\,e+4\,f\,x\right)+12\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (a*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos(e
 + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x) +
 700*sin(2*e + 2*f*x) + 112*sin(4*e + 4*f*x) + 12*sin(6*e + 6*f*x)))/(960*f
*(cos(2*e + 2*f*x) + 1))
```

3.11 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a+a\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{\cos(e+f}{}$$

```
[Out] -1/5*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/c/f-2/15*a^2*
cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/5*a*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.34, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}{5cf}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}{5cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(15*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2))/(5*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2))/(5*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\,\mathrm{s}}{5cf}$$
$$= -\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c}{5cf}$$
$$= -\frac{2a^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{15cf\sqrt{a+a\sin(e+fx)}} -$$

Mathematica [A]
time = 0.40, size = 82, normalized size = 0.59

$$-\frac{c\sec^3(e+fx)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\,(150\sin(e+fx)+25\sin(3(e+fx))+3\sin(5(e+fx)))}{240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/240*(c*Sec[e + f*x]^3*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*S
qrt[c - c*Sin[e + f*x]]*(150*Sin[e + f*x] + 25*Sin[3*(e + f*x)] + 3*Sin[5*(
e + f*x)]))/f
```

Maple [A]
time = 0.16, size = 67, normalized size = 0.48

method	result	size
default	$\frac{(3(\cos^4(fx+e))+4(\cos^2(fx+e))+8)(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}}{15f\cos(fx+e)^3}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/15/f*(3*cos(f*x+e)^4+4*cos(f*x+e)^2+8)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+
e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [**A**]
time = 0.37, size = 79, normalized size = 0.56

$$\frac{\left(3\,ac\cos\left(fx+e\right)^{4}+4\,ac\cos\left(fx+e\right)^{2}+8\,ac\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/15*(3*a*c*cos(f*x + e)^4 + 4*a*c*cos(f*x + e)^2 + 8*a*c)*sqrt(a*sin(f*x +
 e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [**A**]
time = 0.58, size = 153, normalized size = 1.09

$$-\frac{16\left(6\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-15\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+10\,ac\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -16/15*(6*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 15*a*c*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) + 10*a*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 1.48, size = 79, normalized size = 0.56

$$\frac{a\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(175\,\sin\left(2\,e+2\,f\,x\right)+28\,\sin\left(4\,e+4\,f\,x\right)+3\,\sin\left(6\,e+6\,f\,x\right)\right)}{240\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(175*sin(2*
e + 2*f*x) + 28*sin(4*e + 4*f*x) + 3*sin(6*e + 6*f*x)))/(240*f*(cos(2*e + 2
*f*x) + 1))
```

3.12 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=92

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}$$

```
[Out] 1/6*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/4*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.26, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}{4af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*f*Sqrt[c - c*Sin[e + f*x]]
) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(4*a
*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin}}{4af}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e}{}$$

Mathematica [A]
time = 0.27, size = 83, normalized size = 0.90

$$\frac{a\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-12\cos(2(e+fx))-3\cos(4(e+fx))+8(9\sin(e+fx)+\sin(3(e+fx))))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-12*Co
s[2*(e + f*x)] - 3*Cos[4*(e + f*x)] + 8*(9*Sin[e + f*x] + Sin[3*(e + f*x)])
))/(96*f)
```

Maple [A]
time = 0.16, size = 90, normalized size = 0.98

method	result
default	$-\frac{\sqrt{-c(\sin(fx+e)-1)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}(-3(\cos^4(fx+e))+(\cos^2(fx+e))\sin(fx+e)-4(\cos^2(fx+e))+5}{12f\cos(fx+e)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(-3*c
os(f*x+e)^4+cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2+5*sin(f*x+e)-5)/cos(f*x+
e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A]
time = 0.36, size = 81, normalized size = 0.88

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{4}-4\,\left(a\cos\left(fx+e\right)^{2}+2\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{12\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/12*(3*a*cos(f*x + e)^4 - 4*(a*cos(f*x + e)^2 + 2*a)*sin(f*x + e) - 3*a)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*cos(e +
f*x)**2, x)
```

Giac [A]
time = 0.54, size = 104, normalized size = 1.13

$$\frac{4\left(3\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,a\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 4/3*(3*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 4*a*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 9.81, size = 97, normalized size = 1.05

$$-\frac{a\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(12\,\cos\left(e+f\,x\right)+15\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)-80\,\sin\left(2\,e+2\,f\,x\right)-8\,\sin\left(4\,e+4\,f\,x\right)\right)}{96\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*cos(e +
f*x) + 15*cos(3*e + 3*f*x) + 3*cos(5*e + 5*f*x) - 80*sin(2*e + 2*f*x) - 8*s
in(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))
```

3.13 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 111 vs. 2(45) = 90.
time = 0.38, size = 111, normalized size = 2.47

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}(-6\cos(2(e+fx))+15\sin(e+fx)-\sin(3(e+fx)))}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{3}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-6*Cos
[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(39) = 78.
time = 0.16, size = 141, normalized size = 3.13

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{3}{2}}\left((\cos^2(fx+e))\sin(fx+e)+\cos^3(fx+e)-3\cos(fx+e)\sin(fx+e)+2(\cos^2(fx+e))-\sin(fx+e)-4\cos(f\right.}{3f\sqrt{-c(\sin(fx+e)-1)}\,(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e)+\cos(fx+e)-2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+
e)^3-3*cos(f*x+e)*sin(f*x+e)+2*cos(f*x+e)^2-sin(f*x+e)-4*cos(f*x+e)+1)/(-c*
(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(
f*x+e)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [A]
time = 0.37, size = 83, normalized size = 1.84

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{2}+\left(a\cos\left(fx+e\right)^{2}-4\,a\right)\sin\left(fx+e\right)-3\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*(3*a*cos(f*x + e)^2 + (a*cos(f*x + e)^2 - 4*a)*sin(f*x + e) - 3*a)*sqr
t(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a(\sin\left(e+fx\right)+1)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)}{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x
) - 1)), x)
```

Giac [A]
time = 0.48, size = 56, normalized size = 1.24

$$-\frac{8\,a^{\frac{3}{2}}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,\sqrt{c}\,f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -8/3*a^(3/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))/(sqrt(c)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 9.40, size = 87, normalized size = 1.93

$$-\frac{a\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)-14\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)\right)}{12\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e + f
*x) + 6*cos(3*e + 3*f*x) - 14*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(12*c*f
*(cos(2*e + 2*f*x) + 1))
```

3.14 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=147

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a+a\sin(\ldots}{2cf\sqrt{c-c\sin(e\ldots}}$$

```
[Out] -1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-4*a^2*cos
(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-
2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$-\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f
*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c
*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,d}{c}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e}{2cf\sqrt{c-c\sin(e+}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e}{2cf\sqrt{c-c\sin(e+}}$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e}{2cf\sqrt{c-c\sin(e+}}$$

$$= -\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{}}{cf\sqrt{c-c}}$$

Mathematica [A]

time = 0.73, size = 130, normalized size = 0.88

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(-\cos(2(e+fx))+32\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+12\sin(e+fx)\right)}{4cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
Cos[2*(e + f*x)] + 32*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 12*Sin[e +
 f*x]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [A]
time = 0.15, size = 171, normalized size = 1.16

method	result
default	$\frac{\left(\cos^2(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)-16\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-6\sin(fx+e)-1\right)\left(\cos(fx+e)\sin(fx+e)-\left(\cos^2(fx+e)\right)-2\sin\right.}{2f(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e)+\cos(fx+e)-2)(-c(\sin(fx+e)-1))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*(cos(f*x+e)^2+8*ln(2/(1+cos(f*x+e)))-16*ln(-(-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))-6*sin(f*x+e)-1)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x
+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e
)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. 2(143) = 286.
time = 0.56, size = 914, normalized size = 6.22

$$\frac{\frac{16a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{3}{2}}}-\frac{8a^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{c^{\frac{3}{2}}}+\frac{10a^{\frac{3}{2}}-\frac{11a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{15a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{20a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{7a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{c^{\frac{3}{2}}-\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{2c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{10a^{\frac{3}{2}}-\frac{13a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{25a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{20a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{9a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{c^{\frac{3}{2}}-\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{2c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{2\left(\frac{5a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{5a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{8a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{5a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{c^{\frac{3}{2}}-\frac{2c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{4c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{2c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/2*(16*a^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(3/2) - 8*a^(3/2
)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(3/2) + (10*a^(3/2) - 11*a
^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*a^(3/2)*sin(f*x + e)^2/(cos(f*x
 + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^(3/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 7*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
```

```
 + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^(3/2)*s
in(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*c^(3/2)*sin(f*x
 + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)
 - (10*a^(3/2) - 13*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*a^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 20*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 15*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 9*a^(3/2)*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)
^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6) + 2*(5*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 5*a
^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^(3/2)*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 - 5*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*a^(3/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(c^(3/2) - 2*c^(3/2)*sin(f*x + e)/(co
s(f*x + e) + 1) + 3*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^(3/2)
*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 - 2*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + c^(3/2)*sin(f*x
 + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x +
e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin{(e + fx)} + 1))^{\frac{3}{2}} \cos^{2}{(e + fx)}}{(-c(\sin{(e + fx)} - 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*cos(e + f*x)**2/(-c*(sin(e + f*x) -
1))**(3/2), x)
```

Giac [A]
time = 0.60, size = 139, normalized size = 0.95

$$\frac{2\,a^{\frac{3}{2}}\sqrt{c}\left(\frac{2\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+2\,c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{4}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 2*a^(3/2)*sqrt(c)*(2*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))) + (c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^4)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.15 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a+a\sin(}}{c^2f\sqrt{c-c\sin(e+f}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*cos(f*x+
e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a
*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$\frac{4a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (4*a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) + (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e}}{c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e}}{c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{2a\cos(e+fx)\sqrt{a+a\sin(e}}{c^2f\sqrt{c-c\sin(e+fx}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)\log(1-}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c}}$$

Mathematica [A]
time = 0.69, size = 169, normalized size = 1.17

$$\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(7+\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(2-16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(7 +
Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (2 - 16*Lo
g[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.17, size = 224, normalized size = 1.56

method	result
default	$\frac{\left(4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-8\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)+\cos^2(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)+8\ln\left(-\frac{-1+\cos(fx+e)+}{\sin(fx+}\right.\right.}{f(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e)+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(4*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-8*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))*sin(f*x+e)+cos(f*x+e)^2-4*ln(2/(1+cos(f*x+e)))+8*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))+5*sin(f*x+e)-1)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)
^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)*sin(f*x+
e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*c
os(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*cos(e + f*x)**2/(-c*(sin(e + f*x) -
1))**(5/2), x)
```

Giac [A]
time = 0.53, size = 142, normalized size = 0.99

$$-\frac{2\,a^{\frac{3}{2}}\sqrt{c}\left(\frac{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{1}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -2*a^(3/2)*sqrt(c)*(cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^3*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))) + 2*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 1/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
1)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+fx)^2 (a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.16 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=147

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^2f(c-c\sin(e+fx))^{3/2}}-\frac{a^2\cos(e+fx)\log(1-\sin(e+}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)-a*cos(f*x+
e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-a^2*cos(f*x+e)*ln(1-
sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2818, 2816, 2746, 31}

$$-\frac{a^2\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^2*f*(c - c*Sin[e + f*x])^(
3/2)) - (a^2*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e-}}{c^2f(c-c\sin(e+fx))^3}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e-}}{c^2f(c-c\sin(e+fx))^3}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e-}}{c^2f(c-c\sin(e+fx))^3}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{a\cos(e+fx)\sqrt{a+a\sin(e-}}{c^2f(c-c\sin(e+fx))^3}$$

Mathematica [A]
time = 0.89, size = 191, normalized size = 1.30

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(-2-3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -((a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-2
 - 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*Log[Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]] + 4*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]])*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Si
n[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(133) = 266$.
time = 0.18, size = 276, normalized size = 1.88

method	result
default	$-\frac{\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^2(fx+e)\right)-2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)+2\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-4\ln\left(-\frac{-1+\cos(}{s}\right.\right.}{f(\cos(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*cos(f*x+e)^2+2*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-4*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+2*cos(f*x+e)^2-2*ln(2/(1+cos(f*x+e)))
+4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2)*(cos(f*x+e)*sin(f*x+e)-cos
(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)*s
in(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e
)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.63, size = 126, normalized size = 0.86

$$\frac{\left(4\,a\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{4\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}\right)\sqrt{a}}{2\,c^{\frac{7}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 1/2*(4*a*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e)) + (4*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*pi + 1/2*f*x
+ 1/2*e)^4)*sqrt(a)/(c^(7/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.17 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A]
time = 0.23, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*a*c*f*(c - c*Sin[e + f*x])^(7/
2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{6acf(c-c\sin(e+fx))^{7/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.
time = 0.94, size = 110, normalized size = 2.29

$$-\frac{a(-5+3\cos(2(e+fx)))\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] -1/6*(a*(-5 + 3*Cos[2*(e + f*x)])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*S
qrt[a*(1 + Sin[e + f*x])])/(c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1
 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(42) = 84$.
time = 0.15, size = 127, normalized size = 2.65

method	result	size
default	$-\frac{(\cos^2(fx+e)-4)(a(1+\sin(fx+e)))^{\frac{3}{2}}\sin(fx+e)(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2\sin(fx+e)-\cos(fx+e)+2)}{3f(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e)+\cos(fx+e)-2)(-c(\sin(fx+e)-1))^{\frac{9}{2}}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3/f*(cos(f*x+e)^2-4)*(a*(1+sin(f*x+e)))^(3/2)*sin(f*x+e)*(cos(f*x+e)*sin
(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e)*sin(f*x+e)+cos(
f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(45) = 90$.
time = 0.35, size = 113, normalized size = 2.35

$$\frac{\left(3\,a\cos\left(fx+e\right)^{2}-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\left(3\,c^{5}f\cos\left(fx+e\right)^{3}-4\,c^{5}f\cos\left(fx+e\right)-\left(c^{5}f\cos\left(fx+e\right)^{3}-4\,c^{5}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*a*cos(f*x + e)^2 - 4*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(3*c^5*f*cos(f*x + e)^3 - 4*c^5*f*cos(f*x + e) - (c^5*f*cos(f*x + e)
^3 - 4*c^5*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(45) = 90$.
time = 0.54, size = 132, normalized size = 2.75

$$-\frac{\left(3\,a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-3\,a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{6\,c^{5}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] -1/6*(3*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^4 - 3*a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e)^2 + a*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a
)/(c^5*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)
^6)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(9/2),
x)
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

3.18 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c/f/(c-c*sin(f*x+e))^(9/2)+1/48*cos
(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/c^2/f/(c-c*sin(f*x+e))^(7/2)
```

Rubi [A]
time = 0.29, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{48ac^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*a*c^2*f*(c - c*Sin[e +
f*x])^(7/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}+\frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{8ac^2}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{8acf(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a+a\sin(e+f}{48ac^2f(c-c\sin(e+fx)}\end{aligned}$$

Mathematica [A]
time = 1.32, size = 118, normalized size = 1.22

$$-\frac{a\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\,(5-3\cos(2(e+fx))+4\sin(e+fx))}{12c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] -1/12*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]
*(5 - 3*Cos[2*(e + f*x)] + 4*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[
(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.15, size = 152, normalized size = 1.57

method	result
default	$\frac{((\cos^2(fx+e))\sin(fx+e)-4(\cos^2(fx+e))-4\sin(fx+e)+10)(a(1+\sin(fx+e)))^{\frac{3}{2}}\sin(fx+e)(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2}{6f(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e)+\cos(fx+e)-2)(-c(\sin(fx+e)-1))^{\frac{11}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/6/f*(cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2-4*sin(f*x+e)+10)*(a*(1+sin(f*
x+e)))^(3/2)*sin(f*x+e)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-co
s(f*x+e)+2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/
(-c*(sin(f*x+e)-1))^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [A]
time = 0.36, size = 137, normalized size = 1.41

$$-\frac{\left(3\,a\cos\left(fx+e\right)^{2}-2\,a\sin\left(fx+e\right)-4\,a\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{6\left(c^{6}f\cos\left(fx+e\right)^{5}-8\,c^{6}f\cos\left(fx+e\right)^{3}+8\,c^{6}f\cos\left(fx+e\right)+4\left(c^{6}f\cos\left(fx+e\right)^{3}-2\,c^{6}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -1/6*(3*a*cos(f*x + e)^2 - 2*a*sin(f*x + e) - 4*a)*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 +
 8*c^6*f*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin
(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 124, normalized size = 1.28

$$-\frac{\left(6\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-8\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+3\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{48\,c^{\frac{11}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] -1/48*(6*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^4 - 8*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^2 + 3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(11/2)*f*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8)
```

Mupad [B]
time = 13.87, size = 236, normalized size = 2.43

$$\frac{\sqrt{c-c\sin(e+fx)}\left(\frac{40\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\sqrt{a+a\sin(e+fx)}}{3\,c^6\,f}-\frac{8\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\cos(2\,e+2\,f\,x)\sqrt{a+a\sin(e+fx)}}{c^6\,f}+\frac{32\,a\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\sin(e+f\,x)\sqrt{a+a\sin(e+fx)}}{3\,c^6\,f}\right)}{84\cos(e+fx)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}-54\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\cos(3\,e+3\,f\,x)+2\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\cos(5\,e+5\,f\,x)-96\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\sin(2\,e+2\,f\,x)+16\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\sin(4\,e+4\,f\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((40*a*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^
(1/2))/(3*c^6*f) - (8*a*exp(e*5i + f*x*5i)*cos(2*e + 2*f*x)*(a + a*sin(e +
f*x))^(1/2))/(c^6*f) + (32*a*exp(e*5i + f*x*5i)*sin(e + f*x)*(a + a*sin(e +
 f*x))^(1/2))/(3*c^6*f)))/(84*cos(e + f*x)*exp(e*5i + f*x*5i) - 54*exp(e*5i
 + f*x*5i)*cos(3*e + 3*f*x) + 2*exp(e*5i + f*x*5i)*cos(5*e + 5*f*x) - 96*ex
p(e*5i + f*x*5i)*sin(2*e + 2*f*x) + 16*exp(e*5i + f*x*5i)*sin(4*e + 4*f*x))
```

3.19 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=188

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a+a\sin(e+fx)}}-\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{3a\cos(e$$

```
[Out] -3/28*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/8*co
s(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/35*a^3*cos(f*x
+e)*(c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-1/14*a^2*cos(f*x+e)*(
c-c*sin(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.42, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{14cf}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9/2}}{8cf}-\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{28cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] -1/35*(a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
9/2))/(14*c*f) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(9/2))/(28*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*S
in[e + f*x])^(9/2))/(8*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/}}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin}{8cf}$$
$$= -\frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{28cf}$$
$$= -\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\,s}{14cf}$$
$$= -\frac{a^3\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{a^2}{}$$

Mathematica [A]
time = 1.42, size = 176, normalized size = 0.94

$$-\frac{c^3(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(1960\cos(2(e+fx))+980\cos(4(e+fx))+280\cos(6(e+fx))+35\cos(8(e+fx))+19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+fx))+80\sin(7(e+fx)))}{35840f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] -1/35840*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c
*Sin[e + f*x]]*(1960*Cos[2*(e + f*x)] + 980*Cos[4*(e + f*x)] + 280*Cos[6*(e
 + f*x)] + 35*Cos[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)]
 + 784*Sin[5*(e + f*x)] + 80*Sin[7*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.20, size = 143, normalized size = 0.76

method	result

default	$\frac{(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}(35(\cos^8(fx+e))+5(\cos^6(fx+e))\sin(fx+e)+40(\cos^6(fx+e))+13(\cos^4(fx+e}{280f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/280/f*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(35*c
os(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)+40*cos(f*x+e)^6+13*cos(f*x+e)^4*sin(f
*x+e)+48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+64*cos(f*x+e)^2+93*sin(f*x
+e)+93)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.36, size = 136, normalized size = 0.72

$$\frac{\left(35\,a^{2}c^{3}\cos\left(fx+e\right)^{8}-35\,a^{2}c^{3}+8\left(5\,a^{2}c^{3}\cos\left(fx+e\right)^{6}+6\,a^{2}c^{3}\cos\left(fx+e\right)^{4}+8\,a^{2}c^{3}\cos\left(fx+e\right)^{2}+16\,a^{2}c^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] 1/280*(35*a^2*c^3*cos(f*x + e)^8 - 35*a^2*c^3 + 8*(5*a^2*c^3*cos(f*x + e)^6
 + 6*a^2*c^3*cos(f*x + e)^4 + 8*a^2*c^3*cos(f*x + e)^2 + 16*a^2*c^3)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 267, normalized size = 1.42

$$-\frac{32 \left(35 a^2 c^3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{16} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 160 a^2 c^3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{14} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 280 a^2 c^3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{12} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) - 224 a^2 c^3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{10} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) + 70 a^2 c^3 \cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)^{8} \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2} f x + \frac{1}{2} e\right)\right)\right) \sqrt{a} \sqrt{c}}{35 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] -32/35*(35*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 160*a^2*c^3*cos(-1/4*pi
 + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + 280*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 224*a^2*
c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 70*a^2*c^3*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 12.62, size = 376, normalized size = 2.00

$$\frac{e^{-e\,8i-f\,x\,8i} \sqrt{c-c\,\sin\left(e+f\,x\right)} \left(\frac{35\,a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{7\,a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{128\,f}+\frac{a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\cos(8\,e+8\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{512\,f}+\frac{7\,a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^2\,c^3\,e^{e\,8i+f\,x\,8i}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^2*c^3*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*a^2*c^3*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^2*c^3
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) +
(a^2*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) + (a^2*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^2*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^2*c^3*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
 a*sin(e + f*x))^(1/2))/(160*f) + (a^2*c^3*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.20 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a+a\sin(e+fx)}}-\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}{35cf}-a\cos($$

```
[Out] -1/7*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/c/f-1/7*cos
(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/c/f-2/35*a^3*cos(f*x+
e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/35*a^2*cos(f*x+e)*(c
-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.43, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}{35cf}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}{7cf}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}{7cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*a^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(35*c*f*Sqrt[a + a*Sin[e +
 f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])
^(7/2))/(35*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e
+ f*x])^(7/2))/(7*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(c - c*Si
n[e + f*x])^(7/2))/(7*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/}}{ac}$$
$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin}{7cf}$$
$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c\,\mathrm{s}}{7cf}$$
$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c}{35cf}$$
$$= -\frac{2a^3\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{35cf\sqrt{a+a\sin(e+fx)}} - \frac{4a}{}$$

Mathematica [A]
time = 0.43, size = 87, normalized size = 0.46

$$\frac{a^2c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(1225\sin(e+fx)+245\sin(3(e+fx))+49\sin(5(e+fx))+5\sin(7(e+fx)))}{2240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^2*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
1225*Sin[e + f*x] + 245*Sin[3*(e + f*x)] + 49*Sin[5*(e + f*x)] + 5*Sin[7*(e
 + f*x)]))/(2240*f)
```

Maple [A]
time = 0.16, size = 77, normalized size = 0.41

method	result	size
default	$\frac{\left(5\left(\cos^6(fx+e)\right)+6\left(\cos^4(fx+e)\right)+8\left(\cos^2(fx+e)\right)+16\right)\left(-c(\sin(fx+e)-1)\right)^{\frac{5}{2}}\sin(fx+e)\left(a(1+\sin(fx+e))\right)^{\frac{5}{2}}}{35f\cos(fx+e)^5}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/35/f*(5*cos(f*x+e)^6+6*cos(f*x+e)^4+8*cos(f*x+e)^2+16)*(-c*(sin(f*x+e)-1)
)^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [**A**]
time = 0.36, size = 108, normalized size = 0.57

$$\frac{\left(5\,a^2c^2\cos\left(fx+e\right)^6+6\,a^2c^2\cos\left(fx+e\right)^4+8\,a^2c^2\cos\left(fx+e\right)^2+16\,a^2c^2\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{35\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] 1/35*(5*a^2*c^2*cos(f*x + e)^6 + 6*a^2*c^2*cos(f*x + e)^4 + 8*a^2*c^2*cos(f
*x + e)^2 + 16*a^2*c^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
sin(f*x + e)/(f*cos(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 216, normalized size = 1.15

$$\frac{32\left(20\,a^2c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-70\,a^2c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+84\,a^2c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-35\,a^2c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 32/35*(20*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 70*a^2*c^2*cos(-1/4*pi +
 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e)) + 84*a^2*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 35*a^2*c^2*
cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 11.80, size = 179, normalized size = 0.95

$$\frac{\frac{1225\,a^2\,c^2\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{245\,a^2\,c^2\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{49\,a^2\,c^2\,\sin\left(5\,e+5\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}+\frac{5\,a^2\,c^2\,\sin\left(7\,e+7\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{32}}{70\,f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] ((1225*a^2*c^2*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2))/32 + (245*a^2*c^2*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2)*(c -
c*sin(e + f*x))^(1/2))/32 + (49*a^2*c^2*sin(5*e + 5*f*x)*(a + a*sin(e + f*x
))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32 + (5*a^2*c^2*sin(7*e + 7*f*x)*(a +
a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/32)/(70*f*cos(e + f*x))
```

3.21 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{15af}+\frac{\cos(e+f}{}$$

```
[Out] 1/6*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/a/f+1/15*c^2*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)+2/15*c*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.36, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}}{6af}+\frac{2c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{15af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(15*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
])/(15*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])
^(3/2))/(6*a*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx &= \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/}}{ac} \\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(}{6af} \\ &= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c}}{15af} \\ &= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{2c\,\mathrm{co}}{}\end{aligned}$$

Mathematica [A]
time = 0.68, size = 152, normalized size = 1.09

$$-\frac{c(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\,(-75\cos(2(e+fx))-30\cos(4(e+fx))-5\cos(6(e+fx))+600\sin(e+fx)+100\sin(3(e+fx))+12\sin(5(e+fx)))}{960f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] -1/960*(c*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e
 + f*x]]*(-75*Cos[2*(e + f*x)] - 30*Cos[4*(e + f*x)] - 5*Cos[6*(e + f*x)] +
 600*Sin[e + f*x] + 100*Sin[3*(e + f*x)] + 12*Sin[5*(e + f*x)]))/(f*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [A]
time = 0.17, size = 116, normalized size = 0.83

method	result
default	$-\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(-5\left(\cos^6(fx+e)\right)+\left(\cos^4(fx+e)\right)\sin(fx+e)-6\left(\cos^4(fx+e)\right)+3\left(\cos^2(fx+e)\right)\right.}{30f\cos(fx+e)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/30/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-5*c
os(f*x+e)^6+cos(f*x+e)^4*sin(f*x+e)-6*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)-8*cos(f*x+e)^2+11*sin(f*x+e)-11)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.39, size = 109, normalized size = 0.78

$$-\frac{\left(5\,a^{2}c\cos\left(fx+e\right)^{6}-5\,a^{2}c-2\,\left(3\,a^{2}c\cos\left(fx+e\right)^{4}+4\,a^{2}c\cos\left(fx+e\right)^{2}+8\,a^{2}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/30*(5*a^2*c*cos(f*x + e)^6 - 5*a^2*c - 2*(3*a^2*c*cos(f*x + e)^4 + 4*a^2
*c*cos(f*x + e)^2 + 8*a^2*c)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c
*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 159, normalized size = 1.14

$$-\frac{16\left(10\,a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-24\,a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+15\,a^{2}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -16/15*(10*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 24*a^2*c*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e)) + 15*a^2*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 11.52, size = 122, normalized size = 0.87

$$-\frac{a^2\,c\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(75\,\cos\left(e+f\,x\right)+105\,\cos\left(3\,e+3\,f\,x\right)+35\,\cos\left(5\,e+5\,f\,x\right)+5\,\cos\left(7\,e+7\,f\,x\right)-700\,\sin\left(2\,e+2\,f\,x\right)-112\,\sin\left(4\,e+4\,f\,x\right)-12\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(a^2*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*cos(
e + f*x) + 105*cos(3*e + 3*f*x) + 35*cos(5*e + 5*f*x) + 5*cos(7*e + 7*f*x)
- 700*sin(2*e + 2*f*x) - 112*sin(4*e + 4*f*x) - 12*sin(6*e + 6*f*x)))/(960*
f*(cos(2*e + 2*f*x) + 1))
```

3.22 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=92

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}$$

```
[Out] 1/10*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/5*cos
(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.27, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}}{5af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(5*
a*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(}{5af}$$
$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e-}{}$$

Mathematica [A]
time = 0.34, size = 92, normalized size = 1.00

$$-\frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(20\cos(2(e+fx))+5\cos(4(e+fx))-70\sin(e+fx)-5\sin(3(e+fx))+\sin(5(e+fx)))}{80f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/80*(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]
*(20*Cos[2*(e + f*x)] + 5*Cos[4*(e + f*x)] - 70*Sin[e + f*x] - 5*Sin[3*(e +
 f*x)] + Sin[5*(e + f*x)]))/f
```

Maple [A]
time = 0.15, size = 106, normalized size = 1.15

method	result
default	$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(-2(\cos^6(fx+e))+(\cos^4(fx+e))\sin(fx+e)-2(\cos^4(fx+e))+3(c\right.}{10f\cos(fx+e)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/10/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-2*c
os(f*x+e)^6+cos(f*x+e)^4*sin(f*x+e)-2*cos(f*x+e)^4+3*cos(f*x+e)^2*sin(f*x+e
)+6*sin(f*x+e)-6)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A]
time = 0.38, size = 103, normalized size = 1.12

$$-\frac{\left(5\,a^{2}\cos\left(fx+e\right)^{4}-5\,a^{2}+2\left(a^{2}\cos\left(fx+e\right)^{4}-2\,a^{2}\cos\left(fx+e\right)^{2}-4\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{10\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(f*x + e)^4 - 5*a^2 + 2*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x
 + e)^2 - 4*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e
) + c)/(f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.53, size = 108, normalized size = 1.17

$$\frac{8\left(4\,a^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-5\,a^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 8/5*(4*a^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 5*a^2*cos(-1/4*pi + 1/2*f*x + 1
```

```
/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 2.41, size = 108, normalized size = 1.17

$$-\frac{a^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(20\,\cos\left(e+f\,x\right)+25\,\cos\left(3\,e+3\,f\,x\right)+5\,\cos\left(5\,e+5\,f\,x\right)-75\,\sin\left(2\,e+2\,f\,x\right)-4\,\sin\left(4\,e+4\,f\,x\right)+\sin\left(6\,e+6\,f\,x\right)\right)}{80\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(20*cos(e
+ f*x) + 25*cos(3*e + 3*f*x) + 5*cos(5*e + 5*f*x) - 75*sin(2*e + 2*f*x) - 4
*sin(4*e + 4*f*x) + sin(6*e + 6*f*x)))/(80*f*(cos(2*e + 2*f*x) + 1))
```

3.23 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 119 vs. 2(45) = 90.
time = 0.62, size = 119, normalized size = 2.64

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}(-28\cos(2(e+fx))+\cos(4(e+fx))+56\sin(e+fx)-8\sin(3(e+fx)))}{32f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-28*Co
s[2*(e + f*x)] + Cos[4*(e + f*x)] + 56*Sin[e + f*x] - 8*Sin[3*(e + f*x)]))/
(32*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(39) = 78.
time = 0.15, size = 195, normalized size = 4.33

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}\left(\cos^4(fx+e)-(\cos^3(fx+e))\sin(fx+e)-4(\cos^3(fx+e))-3(\cos^2(fx+e))\sin(fx+e)-4(\cos^2(fx+e))+7\right.}{4f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(\cos^3(fx+e)-(\cos^2(fx+e))\sin(fx+e)-3(\cos^2(fx+e))-2\cos(fx+e)\sin(fx+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*
x+e)-4*cos(f*x+e)^3-3*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^2+7*cos(f*x+e)*s
in(f*x+e)+8*cos(f*x+e)+sin(f*x+e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)^
3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(f*x+e)*sin(f*x+e)-2*cos(f*x+
e)+4*sin(f*x+e)+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.
time = 0.35, size = 105, normalized size = 2.33

$$\frac{\left(a^{2} \cos \left(f x+e\right)^{4}-8 \, a^{2} \cos \left(f x+e\right)^{2}+7 \, a^{2}-4 \, {\left(a^{2} \cos \left(f x+e\right)^{2}-2 \, a^{2}\right)} \sin \left(f x+e\right)\right)} \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c}}{4 \, c f \cos \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/4*(a^2*cos(f*x + e)^4 - 8*a^2*cos(f*x + e)^2 + 7*a^2 - 4*(a^2*cos(f*x + e
)^2 - 2*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
c)/(c*f*cos(f*x + e))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.50, size = 56, normalized size = 1.24

$$-\frac{4 \, a^{\frac{5}{2}} \cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)^{8} \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right)}{\sqrt{c} f \mathrm{sgn}\left(\sin \left(-\frac{1}{4} \, \pi + \frac{1}{2} \, f x + \frac{1}{2} \, e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -4*a^(5/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))/(sqrt(c)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 1.88, size = 102, normalized size = 2.27

$$-\frac{a^2\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)-48\,\sin\left(2\,e+2\,f\,x\right)+8\,\sin\left(4\,e+4\,f\,x\right)\right)}{32\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e
+ f*x) + 27*cos(3*e + 3*f*x) - cos(5*e + 5*f*x) - 48*sin(2*e + 2*f*x) + 8*s
in(4*e + 4*f*x)))/(32*c*f*(cos(2*e + 2*f*x) + 1))
```

3.24 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a+a\sin}{cf\sqrt{c-c\sin(e}}$$

```
[Out] -a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-1/3*cos(f*x
+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)-8*a^3*cos(f*x+e)*ln(1
-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*a^2*cos(f*
x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$-\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{4a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-8*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) - (4*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c
*f*Sqrt[c - c*Sin[e + f*x]]) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/
(c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/
(3*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,d}{c}$$

$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e+}{3cf\sqrt{c-c\sin(e+}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin}{cf\sqrt{c-c\sin(e+}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin}{cf\sqrt{c-c\sin(e+}$$

$$= -\frac{4a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{a\cos(e+fx)(a+a\sin}{cf\sqrt{c-c\sin(e+}$$

$$= -\frac{8a^3\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4a^2\cos(e+fx)}{cf\sqrt{c-}$$

Mathematica [A]
time = 1.75, size = 140, normalized size = 0.73

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-12\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+87\sin(e+fx)-\sin(3(e+fx))\right)}{12cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(
-12*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 87*Si
n[e + f*x] - Sin[3*(e + f*x)]))/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.16, size = 218, normalized size = 1.13

method	result

default	$\frac{\left(\left(\cos^2(fx+e)\right)\sin(fx+e)+6\left(\cos^2(fx+e)\right)+24\ln\left(\frac{2}{1+\cos(fx+e)}\right)-48\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-22\sin(fx+e)-6\right)\left(\cos(fx+e)\right.}{3f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)-\left(\cos^3(fx+e)\right)+2\cos(fx+e)\sin(fx+e)+3\left(\cos^2(fx+e)\right)-4\sin(fx+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)+6*cos(f*x+e)^2+24*ln(2/(1+cos(f*x+e)))-48*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-22*sin(f*x+e)-6)*(cos(f*x+e)*sin(f
*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(5/2)/(cos
(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f
*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.56, size = 148, normalized size = 0.77

$$\frac{4\,a^{\frac{5}{2}}\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{2\,c^{4}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+3\,c^{4}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+6\,c^{4}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{6}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 4/3*a^(5/2)*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (2*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6
+ 3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 6*c^4*cos(-1/4*pi + 1/2*f*x + 1/
2*e)^2)/(c^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.25 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=192

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2\cos(e+fx)\sqrt{a+a\,\mathrm{s}}}{c^2f\sqrt{c-c\sin(e+}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(3/2)+3/2*a*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+12*a^3*cos(f*x+e)*ln
(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6*a^2*co
s(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$\frac{12a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (12*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (6*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]
)/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])
^(3/2))/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f\sqrt{c-c\sin(e+}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin}}{c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin}}{c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{6a^2\cos(e+fx)\sqrt{a+a\sin}}{c^2f\sqrt{c-c\sin(e+f}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{12a^3\cos(e+fx)\log(1}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

Mathematica [A]
time = 1.53, size = 181, normalized size = 0.94

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(44+18\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(39-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{8c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(44
 + 18*Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (39
 - 192*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e +
f*x)]))/(8*c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^
2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.17, size = 273, normalized size = 1.42

method	result

default	$-\frac{\left(-(\cos^2(fx+e))\sin(fx+e)+48\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-9(\cos^2(fx+e))-48\ln(\right.}{2f((\cos^2(fx+e))\sin(fx+e)-(\cos^3(fx+e))+2\cos(fx+e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/f*(-cos(f*x+e)^2*sin(f*x+e)+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))*sin(f*x+e)-24*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-9*cos(f*x+e)^2-48*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-25*sin(f*x+e)+24*ln(2/(1+cos(f*x+e)))+9
)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(
f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e
)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(187) = 374$.
time = 0.57, size = 1214, normalized size = 6.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -1/6*(144*a^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(5/2) - 72*a^(
5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(5/2) + (46*a^(5/2) - 1
21*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*a^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 - 179*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*a
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 43*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 15*a^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - (46*a^(5/2) - 19
9*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*a^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 509*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*a^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 373*a^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 63*a^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 - 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2
```

```
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + c^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + 6*(13*a^(5/2)*s
in(f*x + e)/(cos(f*x + e) + 1) - 39*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 55*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 74*a^(5/2)*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 55*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 - 39*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*a^(5/2)*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7)/(c^(5/2) - 4*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
 1) + 8*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 12*c^(5/2)*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 14*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 12*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*c^(5/2)*sin(f*x + e)^6
/(cos(f*x + e) + 1)^6 - 4*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + c^(
5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos
(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.62, size = 179, normalized size = 0.93

$$-\frac{2\,a^{\frac{5}{2}}\sqrt{c}\left(\frac{6\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{c^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+4\,c^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{6}}-\frac{2}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)c^{3}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -2*a^(5/2)*sqrt(c)*(6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))) + (c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^6 - 2/((cos(-1/4*pi + 1/2*f*x + 1/2*e
)^2 - 1)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.26 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=195

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{3a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}}-\frac{6a^3\cos(e+fx)\log(1-\sin(e}{c^3f\sqrt{a+a\sin(e+fx)}\sqrt{c-cs}$$

[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)-3/2*a*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-6*a^3*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-3*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A]
time = 0.43, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{6a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{3a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) - (3*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x])^(3/2)) - (6*a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{2c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f(c-c\sin(e+fx}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f(c-c\sin(e+fx}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f(c-c\sin(e+fx}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f(c-c\sin(e+fx}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{3a\cos(e+fx)(a+a\sin(e}{2c^2f(c-c\sin(e+fx}$$

Mathematica [A]
time = 1.77, size = 209, normalized size = 1.07

$$-\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(-28-72\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\cos(2(e+fx))\left(-1+6\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+\left(41+96\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{4c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] -1/4*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])
]*(-28 - 72*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 4*Cos[2*(e + f*x)]*(
-1 + 6*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) + (41 + 96*Log[Cos[(e + f*
x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f*x)]))/(c^3*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [A]
time = 0.17, size = 329, normalized size = 1.69

method	result

default	$\frac{\left(12\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))+(\cos^2(fx+e))\sin(fx+e)-6\ln\left(\frac{2}{1+\cos(fx+e)}\right)(\cos^2(fx+e))+24\ln\left(-\frac{-1+\cos(fx+e)}{\sin(fx}\right.\right.}{f((\cos^2(fx+e))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(12*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2+cos(f*x+e)^
2*sin(f*x+e)-6*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+24*ln(-(-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))*sin(f*x+e)-12*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-10*cos(f*
x+e)^2-24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-6*sin(f*x+e)+12*ln(2/(
1+cos(f*x+e)))+10)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*cos
(f*x+e)*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x
+e)-1))^(7/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
7/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(
f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*
sin(f*x + e)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.50, size = 161, normalized size = 0.83

$$\frac{a^{\frac{5}{2}}\sqrt{c}\left(\frac{2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}{c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\log\left(-\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+1\right)}{c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{6\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-5}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}-1\right)^{2}c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] a^(5/2)*sqrt(c)*(2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^4*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))) + 6*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^4*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (6*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 5)
/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^4*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.27 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=193

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}+\frac{a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{c^3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)+a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)+a^3*cos(f*x+e)*ln(1-sin(
f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2818, 2816, 2746, 31}

$$\frac{a^3\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f(c-c\sin(e+fx))^{3/2}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^2f(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x
])^(5/2)) + (a^2*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*(c - c*Sin[e
 + f*x])^(3/2)) + (a^3*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*Sqrt[a +
a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e-}{2c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e-}{2c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e-}{2c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e-}{2c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{a\cos(e+fx)(a+a\sin(e-}{2c^2f(c-c\sin(e+fx)}$$

Mathematica [A]
time = 2.69, size = 234, normalized size = 1.21

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(34+30\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-18\cos(2(e+fx))\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-9\left(4+5\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+3\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{6c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(34
 + 30*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 18*Cos[2*(e + f*x)]*(1 + L
og[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) - 9*(4 + 5*Log[Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]])*Sin[e + f*x] + 3*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*Sin[3*(e + f*x)]))/(6*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(173) = 346$.
time = 0.17, size = 401, normalized size = 2.08

method	result
default	$\left(6\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\left(\cos^2(fx+e)\right)-3\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\left(\cos^2(fx+e)\right)-18\ln\left(-\frac{-1+\cos(fx+e)+\sin(f}{\sin(fx+e)}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(6*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2
-3*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2-18*ln(-(-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*cos(f*x+e)^2*sin(f*x+e)+9*ln(2/(1+cos(f*x
+e)))*cos(f*x+e)^2-24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)
+12*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+6*cos(f*x+e)^2+24*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+14*sin(f*x+e)-12*ln(2/(1+cos(f*x+e)))-6)*(cos(f*x+e)*
sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(5/2)
/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)+3*cos(f*x+e)
^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*co
s(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^
5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 166, normalized size = 0.86

$$-\frac{\left(12\,a^2\log\left(\left|\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{18\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4-9\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+2\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6}\right)\sqrt{a}}{6\,c^{\frac{9}{2}}\,f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] -1/6*(12*a^2*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e)) + (18*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
1/2*f*x + 1/2*e)^4 - 9*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
+ 1/2*f*x + 1/2*e)^2 + 2*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/sin(-1/4*
pi + 1/2*f*x + 1/2*e)^6)*sqrt(a)/(c^(9/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (e+f\,x)^2\,(a+a\,\sin (e+f\,x))^{5/2}}{(c-c\,\sin (e+f\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

3.28 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/8*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A]
time = 0.23, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*a*c*f*(c - c*Sin[e + f*x])^(9/
2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{8acf(c-c\sin(e+fx))^{9/2}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.
time = 2.76, size = 117, normalized size = 2.44

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\,(-7\sin(e+fx)+\sin(3(e+fx)))}{4c^5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-7
*Sin[e + f*x] + Sin[3*(e + f*x)]))/(4*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*
x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(42) = 84.
time = 0.14, size = 157, normalized size = 3.27

method	result
default	$-\frac{(\cos^2(fx+e)-2)\left(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2\sin(fx+e)-\cos(fx+e)+2\right)\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{5}{2}}}{f((\cos^2(fx+e))\sin(fx+e)-(\cos^3(fx+e))+2\cos(fx+e)\sin(fx+e)+3(\cos^2(fx+e))-4\sin(fx+e)+2\cos(fx+e)-4)(-c(\sin(fx+e)-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/f*(cos(f*x+e)^2-2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(
f*x+e)+2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(
f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4
)/(-c*(sin(f*x+e)-1))^(11/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(45) = 90$.
time = 0.37, size = 137, normalized size = 2.85

$$-\frac{(a^2\cos(fx+e)^2-2a^2)\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{c^6f\cos(fx+e)^5-8c^6f\cos(fx+e)^3+8c^6f\cos(fx+e)+4(c^6f\cos(fx+e)^3-2c^6f\cos(fx+e))\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] -(a^2*cos(f*x + e)^2 - 2*a^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e)/(c^6*f*cos(f*x + e)^5 - 8*c^6*f*cos(f*x + e)^3 + 8*c^6*f
*cos(f*x + e) + 4*(c^6*f*cos(f*x + e)^3 - 2*c^6*f*cos(f*x + e))*sin(f*x + e
))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(45) = 90$.
time = 0.54, size = 113, normalized size = 2.35

$$-\frac{\left(4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^6-6\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^4+4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{8\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4c^{\frac{11}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] -1/8*(4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^6 - 6*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^4 + 4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^(5/2)*sgn(cos(-1/4*pi + 1/2*
```

```
f*x + 1/2*e))/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^4*c^(11/2)*f*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(e+fx)^2 (a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(11/2)
, x)
```

3.29 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}$$

```
[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c/f/(c-c*sin(f*x+e))^(11/2)+1/80*c
os(f*x+e)*(a+a*sin(f*x+e))^(7/2)/a/c^2/f/(c-c*sin(f*x+e))^(9/2)
```

Rubi [A]
time = 0.30, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{80ac^2f(c-c\sin(e+fx))^{9/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^(13/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(1
1/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*a*c^2*f*(c - c*Sin[e
+ f*x])^(9/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{10ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{10acf(c-c\sin(e+fx))^{11/2}} + \frac{\cos(e+fx)(a+a\sin(e+f}{80ac^2f(c-c\sin(e+fx)}$$

Mathematica [A]
time = 3.94, size = 130, normalized size = 1.34

$$\frac{a^2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\,(14-10\cos(2(e+fx))+35\sin(e+fx)-5\sin(3(e+fx)))}{40c^6f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^6\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(14
 - 10*Cos[2*(e + f*x)] + 35*Sin[e + f*x] - 5*Sin[3*(e + f*x)]))/(40*c^6*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e
 + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(85) = 170.
time = 0.16, size = 191, normalized size = 1.97

method	result
default	$\frac{(\cos^4(fx+e)+5(\cos^2(fx+e))\sin(fx+e)-17(\cos^2(fx+e))-10\sin(fx+e)+26)(a(1+\sin(fx+e)))^{\frac{5}{2}}\sin(fx+e)(\cos(fx+e)\sin(fx+e)-}{10f((\cos^2(fx+e))\sin(fx+e)-(\cos^3(fx+e))+2\cos(fx+e)\sin(fx+e)+3(\cos^2(fx+e))-4\sin(fx+e)+2\cos(fx+e)-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/10/f*(cos(f*x+e)^4+5*cos(f*x+e)^2*sin(f*x+e)-17*cos(f*x+e)^2-10*sin(f*x+e
)+26)*(a*(1+sin(f*x+e)))^(5/2)*sin(f*x+e)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)
^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*cos(f
*x+e)*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e
)-1))^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
13/2), x)
```

Fricas [A]
time = 0.37, size = 175, normalized size = 1.80

$$-\frac{\left(5 a^{2} \cos \left(f x+e\right)^{2}-6 a^{2}+5 \left(a^{2} \cos \left(f x+e\right)^{2}-2 a^{2}\right) \sin \left(f x+e\right)\right) \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c}}{10 \left(5 c^{7} f \cos \left(f x+e\right)^{5}-20 c^{7} f \cos \left(f x+e\right)^{3}+16 c^{7} f \cos \left(f x+e\right)-\left(c^{7} f \cos \left(f x+e\right)^{5}-12 c^{7} f \cos \left(f x+e\right)^{3}+16 c^{7} f \cos \left(f x+e\right)\right) \sin \left(f x+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")
```

```
[Out] -1/10*(5*a^2*cos(f*x + e)^2 - 6*a^2 + 5*(a^2*cos(f*x + e)^2 - 2*a^2)*sin(f*
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x
 + e)^5 - 20*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x
+ e)^5 - 12*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 164, normalized size = 1.69

$$\frac{\left(10 a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{6}-20 a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+15 a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{2}-4 a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)\right) \sqrt{a}}{80 c^{\frac{13}{2}} f \mathrm{sgn}\left(\sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")
```

```
[Out] 1/80*(10*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^6 - 20*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^4 + 15*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^2 - 4*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(13/2)
*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)
```

Mupad [B]
time = 14.86, size = 317, normalized size = 3.27

$$\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,112{}\mathrm{i}}{5\,c^7\,f}+\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,56{}\mathrm{i}}{c^7\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,16{}\mathrm{i}}{c^7\,f}-\frac{a^2\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,8{}\mathrm{i}}{c^7\,f}\right)}{\cos\left(e+f\,x\right)\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,264{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)\,220{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)\,20{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)\,330{}\mathrm{i}+{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)\,88{}\mathrm{i}-{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)\,2{}\mathrm{i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(13/2)
,x)
```

```
[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^(
1/2)*112i)/(5*c^7*f) + (a^2*exp(e*6i + f*x*6i)*sin(e + f*x)*(a + a*sin(e +
f*x))^(1/2)*56i)/(c^7*f) - (a^2*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(a + a*
sin(e + f*x))^(1/2)*16i)/(c^7*f) - (a^2*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)
*(a + a*sin(e + f*x))^(1/2)*8i)/(c^7*f)))/(cos(e + f*x)*exp(e*6i + f*x*6i)*
264i - exp(e*6i + f*x*6i)*cos(3*e + 3*f*x)*220i + exp(e*6i + f*x*6i)*cos(5*
e + 5*f*x)*20i - exp(e*6i + f*x*6i)*sin(2*e + 2*f*x)*330i + exp(e*6i + f*x*
6i)*sin(4*e + 4*f*x)*88i - exp(e*6i + f*x*6i)*sin(6*e + 6*f*x)*2i)
```

3.30 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=236

$$-\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a+a\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{11/2}}{105cf}-\frac{a^2\,\text{co}}{}$$

```
[Out] -1/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(11/2)/c/f-4/4
5*a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(11/2)/c/f-1/10*cos(
f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(11/2)/c/f-4/315*a^4*cos(f*x
+e)*(c-c*sin(f*x+e))^(11/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/105*a^3*cos(f*x+e)
*(c-c*sin(f*x+e))^(11/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{11/2}}{105cf}-\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{11/2}}{15cf}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{11/2}}{10cf}-\frac{4a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{11/2}}{45cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-4*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(11/2))/(315*c*f*Sqrt[a + a*Sin[e
 + f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(11/2))/(105*c*f) - (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*
Sin[e + f*x])^(11/2))/(15*c*f) - (4*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/
2)*(c - c*Sin[e + f*x])^(11/2))/(45*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x
])^(7/2)*(c - c*Sin[e + f*x])^(11/2))/(10*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
```

```
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{9/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{11}}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin}{10cf}$$

$$= -\frac{4a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c}{45cf}$$

$$= -\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-c}{15cf}$$

$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c}{105cf}$$

$$= -\frac{4a^4\cos(e+fx)(c-c\sin(e+fx))^{11/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4}{}$$

Mathematica [A]
time = 3.64, size = 209, normalized size = 0.89

$$\frac{a^3c^4(-1+\sin(e+fx))^4(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,(13230\cos(2(e+fx))+7560\cos(4(e+fx))+2835\cos(6(e+fx))+630\cos(8(e+fx))+63\cos(10(e+fx))+158760\sin(e+fx)+35280\sin(3(e+fx))+9072\sin(5(e+fx))+1620\sin(7(e+fx))+140\sin(9(e+fx)))}{322560f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(9
/2),x]
```

```
[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x
])]*Sqrt[c - c*Sin[e + f*x]]*(13230*Cos[2*(e + f*x)] + 7560*Cos[4*(e + f*x)
] + 2835*Cos[6*(e + f*x)] + 630*Cos[8*(e + f*x)] + 63*Cos[10*(e + f*x)] + 1
58760*Sin[e + f*x] + 35280*Sin[3*(e + f*x)] + 9072*Sin[5*(e + f*x)] + 1620*
Sin[7*(e + f*x)] + 140*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A]
time = 0.25, size = 169, normalized size = 0.72

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{9}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}(63(\cos^{10}(fx+e))+7\sin(fx+e)(\cos^{8}(fx+e))+70(\cos^{8}(fx+e))+17(\cos^{6}(fx+e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/630/f*(-c*(sin(f*x+e)-1))^(9/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(63*c
os(f*x+e)^10+7*sin(f*x+e)*cos(f*x+e)^8+70*cos(f*x+e)^8+17*cos(f*x+e)^6*sin(
f*x+e)+80*cos(f*x+e)^6+33*cos(f*x+e)^4*sin(f*x+e)+96*cos(f*x+e)^4+65*cos(f*
x+e)^2*sin(f*x+e)+128*cos(f*x+e)^2+193*sin(f*x+e)+193)/cos(f*x+e)^9
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.41, size = 153, normalized size = 0.65

$$\frac{\left(63\,a^{3}c^{4}\cos\left(fx+e\right)^{10}-63\,a^{3}c^{4}+2\,\left(35\,a^{3}c^{4}\cos\left(fx+e\right)^{8}+40\,a^{3}c^{4}\cos\left(fx+e\right)^{6}+48\,a^{3}c^{4}\cos\left(fx+e\right)^{4}+64\,a^{3}c^{4}\cos\left(fx+e\right)^{2}+128\,a^{3}c^{4}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{630\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] 1/630*(63*a^3*c^4*cos(f*x + e)^10 - 63*a^3*c^4 + 2*(35*a^3*c^4*cos(f*x + e)
^8 + 40*a^3*c^4*cos(f*x + e)^6 + 48*a^3*c^4*cos(f*x + e)^4 + 64*a^3*c^4*cos
(f*x + e)^2 + 128*a^3*c^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 318, normalized size = 1.35

$$\frac{256\left(126a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{20}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-700a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{18}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+1575a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{16}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-1800a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+1050a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-252a^3c^4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] 256/315*(126*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^20*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 700*a^3*c^4*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^18*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 1575*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 1800*
a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 1050*a^3*c^4*cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - 252*a^3*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 13.53, size = 462, normalized size = 1.96

$$\frac{\mathrm{e}^{-e\,10\mathrm{i}-f\,x\,10\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{63\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{21\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{256\,f}+\frac{3\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{9\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\cos(6\,e+6\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{512\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\cos(8\,e+8\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{256\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\cos(10\,e+10\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{2560\,f}+\frac{7\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{9\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\sin(5\,e+5\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{9\,a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\sin(7\,e+7\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{896\,f}+\frac{a^3\,c^4\,\mathrm{e}^{e\,10\mathrm{i}+f\,x\,10\mathrm{i}}\,\sin(9\,e+9\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{1152\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] (exp(- e*10i - f*x*10i)*(c - c*sin(e + f*x))^(1/2)*((63*a^3*c^4*exp(e*10i +
 f*x*10i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (21*a^3*c^4*exp
(e*10i + f*x*10i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(256*f) + (3
*a^3*c^4*exp(e*10i + f*x*10i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/
(64*f) + (9*a^3*c^4*exp(e*10i + f*x*10i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*
x))^(1/2))/(512*f) + (a^3*c^4*exp(e*10i + f*x*10i)*cos(8*e + 8*f*x)*(a + a*
sin(e + f*x))^(1/2))/(256*f) + (a^3*c^4*exp(e*10i + f*x*10i)*cos(10*e + 10*
f*x)*(a + a*sin(e + f*x))^(1/2))/(2560*f) + (7*a^3*c^4*exp(e*10i + f*x*10i)
*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (9*a^3*c^4*exp(e*10i
 + f*x*10i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (9*a^3*c
^4*exp(e*10i + f*x*10i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(896*f
) + (a^3*c^4*exp(e*10i + f*x*10i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))^(1/
2))/(1152*f)))/(2*cos(e + f*x))
```

3.31 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=236

$$-\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a+a\sin(e+fx)}}-\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}{63cf}-\frac{2a^2\cos}{}$$

```
[Out] -2/21*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/9*
a*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(9/2)/c/f-1/9*cos(f*x+
e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(9/2)/c/f-8/315*a^4*cos(f*x+e)*(
c-c*sin(f*x+e))^(9/2)/c/f/(a+a*sin(f*x+e))^(1/2)-4/63*a^3*cos(f*x+e)*(c-c*s
in(f*x+e))^(9/2)*(a+a*sin(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.49, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$-\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a\sin(e+fx)+a}}-\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}{63cf}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{9/2}}{21cf}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{9/2}}{9cf}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{9/2}}{9cf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-8*a^4*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(315*c*f*Sqrt[a + a*Sin[e
+ f*x]]) - (4*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(9/2))/(63*c*f) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c - c*S
in[e + f*x])^(9/2))/(21*c*f) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2)*(
c - c*Sin[e + f*x])^(9/2))/(9*c*f) - (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/
2)*(c - c*Sin[e + f*x])^(9/2))/(9*c*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
```

```
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{7/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{9/}}{ac}$$

$$= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin}{9cf}$$

$$= -\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}(c-c\,s}{9cf}$$

$$= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}(c-}{21cf}$$

$$= -\frac{4a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}\,(c-c}{63cf}$$

$$= -\frac{8a^4\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{315cf\sqrt{a+a\sin(e+fx)}} - \frac{4a}{}$$

Mathematica [A]
time = 0.85, size = 97, normalized size = 0.41

$$\frac{a^3c^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(39690\sin(e+fx)+8820\sin(3(e+fx))+2268\sin(5(e+fx))+405\sin(7(e+fx))+35\sin(9(e+fx)))}{80640f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
39690*Sin[e + f*x] + 8820*Sin[3*(e + f*x)] + 2268*Sin[5*(e + f*x)] + 405*Si
n[7*(e + f*x)] + 35*Sin[9*(e + f*x)]))/(80640*f)
```

Maple [A]
time = 0.19, size = 87, normalized size = 0.37

method	result
default	$\frac{(35(\cos^8(fx+e))+40(\cos^6(fx+e))+48(\cos^4(fx+e))+64(\cos^2(fx+e))+128)(-c(\sin(fx+e)-1))^{\frac{7}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}}{315f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)

[Out] 1/315/f*(35*cos(f*x+e)^8+40*cos(f*x+e)^6+48*cos(f*x+e)^4+64*cos(f*x+e)^2+128)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/cos(f*x+e)^7

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2, x)

Fricas [**A**]
time = 0.38, size = 125, normalized size = 0.53

$$\frac{(35\,a^3c^3\cos(fx+e)^8+40\,a^3c^3\cos(fx+e)^6+48\,a^3c^3\cos(fx+e)^4+64\,a^3c^3\cos(fx+e)^2+128\,a^3c^3)\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{315\,f\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out] 1/315*(35*a^3*c^3*cos(f*x + e)^8 + 40*a^3*c^3*cos(f*x + e)^6 + 48*a^3*c^3*cos(f*x + e)^4 + 64*a^3*c^3*cos(f*x + e)^2 + 128*a^3*c^3)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(f*cos(f*x + e))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(7/2),x)

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 267, normalized size = 1.13

$$-\frac{256\left(70\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{18}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-315\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{16}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+540\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-420\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+126\,a^3c^3\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\right)\sqrt{a}\sqrt{c}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] -256/315*(70*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^18*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 315*a^3*c^3*cos(-1/4*
pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + 540*a^3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 420*a^
3*c^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 126*a^3*c^3*cos(-1/4*pi + 1/2*f*x +
1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 12.12, size = 247, normalized size = 1.05

$$\frac{e^{-e\,9i-f\,x\,9i}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{63\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}+\frac{7\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{9\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(5\,e+5\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{9\,a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(7\,e+7\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{896\,f}+\frac{a^3\,c^3\,e^{e\,9i+f\,x\,9i}\,\sin\left(9\,e+9\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{1152\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*9i - f*x*9i)*(c - c*sin(e + f*x))^(1/2)*((63*a^3*c^3*exp(e*9i + f*
x*9i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) + (7*a^3*c^3*exp(e*9i
 + f*x*9i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (9*a^3*c^3
*exp(e*9i + f*x*9i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) +
(9*a^3*c^3*exp(e*9i + f*x*9i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/
(896*f) + (a^3*c^3*exp(e*9i + f*x*9i)*sin(9*e + 9*f*x)*(a + a*sin(e + f*x))
^(1/2))/(1152*f)))/(2*cos(e + f*x))
```

3.32 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=188

$$\frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{3c\cos(e+$$

```
[Out] 3/28*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(3/2)/a/f+1/8*cos
(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(5/2)/a/f+1/35*c^3*cos(f*x+
e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/14*c^2*cos(f*x+e)*(a
+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.42, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{c^3\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{35af\sqrt{c-c\sin(e+fx)}}+\frac{c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{14af}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{5/2}}{8af}+\frac{3c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{28af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(35*a*f*Sqrt[c - c*Sin[e + f*
x]]) + (c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]
])/(14*a*f) + (3*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f
*x])^(3/2))/(28*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[
e + f*x])^(5/2))/(8*a*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{7/}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(}{8af}$$
$$= \frac{3c\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\,\mathrm{si}}{28af}$$
$$= \frac{c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c}}{14af}$$
$$= \frac{c^3\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{35af\sqrt{c-c\sin(e+fx)}} + \frac{c^2\,\mathrm{co}}{}$$

Mathematica [A]
time = 1.59, size = 127, normalized size = 0.68

$$\frac{a^3c^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-1960\cos(2(e+fx))-980\cos(4(e+fx))-280\cos(6(e+fx))-35\cos(8(e+fx))+19600\sin(e+fx)+3920\sin(3(e+fx))+784\sin(5(e+fx))+80\sin(7(e+fx)))}{35840f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (a^3*c^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(
-1960*Cos[2*(e + f*x)] - 980*Cos[4*(e + f*x)] - 280*Cos[6*(e + f*x)] - 35*C
os[8*(e + f*x)] + 19600*Sin[e + f*x] + 3920*Sin[3*(e + f*x)] + 784*Sin[5*(e
 + f*x)] + 80*Sin[7*(e + f*x)]))/(35840*f)
```

Maple [A]
time = 0.19, size = 143, normalized size = 0.76

method	result
default	$-\frac{(-c(\sin(fx+e)-1))^{\frac{5}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(-35\left(\cos^8(fx+e)\right)+5\left(\cos^6(fx+e)\right)\sin(fx+e)-40\left(\cos^6(fx+e)\right)+13\left(\cos^4(fx-\right.\right.}{280f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/280/f*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-35
*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-40*cos(f*x+e)^6+13*cos(f*x+e)^4*sin
(f*x+e)-48*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)-64*cos(f*x+e)^2+93*sin(f
*x+e)-93)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.38, size = 136, normalized size = 0.72

$$-\frac{\left(35\,a^{3}c^{2}\cos\left(fx+e\right)^{8}-35\,a^{3}c^{2}-8\left(5\,a^{3}c^{2}\cos\left(fx+e\right)^{6}+6\,a^{3}c^{2}\cos\left(fx+e\right)^{4}+8\,a^{3}c^{2}\cos\left(fx+e\right)^{2}+16\,a^{3}c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{280\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] -1/280*(35*a^3*c^2*cos(f*x + e)^8 - 35*a^3*c^2 - 8*(5*a^3*c^2*cos(f*x + e)^
6 + 6*a^3*c^2*cos(f*x + e)^4 + 8*a^3*c^2*cos(f*x + e)^2 + 16*a^3*c^2)*sin(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 216, normalized size = 1.15

$$\frac{32 \left(35 a^{3} c^{2} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{16} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)-120 a^{3} c^{2} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{14} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)+140 a^{3} c^{2} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{12} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)-56 a^{3} c^{2} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{10} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)\right) \sqrt{a} \sqrt{c}}{35 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 32/35*(35*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^16*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 120*a^3*c^2*cos(-1/4*pi
+ 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) + 140*a^3*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 56*a^3*c^
2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 12.55, size = 376, normalized size = 2.00

$$\frac{{\mathrm{e}}^{-e\,8{}\mathrm{i}-f\,x\,8{}\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{35\,a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}-\frac{7\,a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}-\frac{7\,a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{128\,f}-\frac{a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\cos\left(6\,e+6\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{64\,f}-\frac{a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\cos\left(8\,e+8\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{512\,f}+\frac{7\,a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}+\frac{7\,a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^3\,c^2\,{\mathrm{e}}^{e\,8{}\mathrm{i}+f\,x\,8{}\mathrm{i}}\,\sin\left(7\,e+7\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*a^3*c^2*exp(e*8i + f*
x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*a^3*c^2*exp(e*8i
 + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*a^3*c^2
*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) -
(a^3*c^2*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(6
4*f) - (a^3*c^2*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1
/2))/(512*f) + (7*a^3*c^2*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e
+ f*x))^(1/2))/(32*f) + (7*a^3*c^2*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a +
 a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2*exp(e*8i + f*x*8i)*sin(7*e + 7*f
*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

3.33 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{4c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{21af}+\frac{\cos(e+f$$

```
[Out] 1/7*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(3/2)/a/f+4/105*c^2*
cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+2/21*c*cos(f*x
+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.36, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{4c^2\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{105af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}(c-c\sin(e+fx))^{3/2}}{7af}+\frac{2c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{21af}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (4*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(105*a*f*Sqrt[c - c*Sin[e +
 f*x]]) + (2*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f
*x]])/(21*a*f) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*(c - c*Sin[e + f*
x])^(3/2))/(7*a*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{5/}}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}(c-c\sin(}{7af}$$
$$= \frac{2c\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c}}{21af}$$
$$= \frac{4c^2\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{105af\sqrt{c-c\sin(e+fx)}} + \frac{2c\,c}{}$$

Mathematica [A]
time = 0.85, size = 115, normalized size = 0.82

$$\frac{a^3c\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-1050\cos(2(e+fx))-420\cos(4(e+fx))-70\cos(6(e+fx))+4725\sin(e+fx)+665\sin(3(e+fx))+21\sin(5(e+fx))-15\sin(7(e+fx)))}{6720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (a^3*c*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-1
050*Cos[2*(e + f*x)] - 420*Cos[4*(e + f*x)] - 70*Cos[6*(e + f*x)] + 4725*Si
n[e + f*x] + 665*Sin[3*(e + f*x)] + 21*Sin[5*(e + f*x)] - 15*Sin[7*(e + f*x
)]))/(6720*f)
```

Maple [A]
time = 0.20, size = 133, normalized size = 0.95

method	result
default	$-\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(-15\left(\cos^8(fx+e)\right)+5\left(\cos^6(fx+e)\right)\sin(fx+e)-16\left(\cos^6(fx+e)\right)+13\left(\cos^4(fx-\right.\right.}{105f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/105/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-15
*cos(f*x+e)^8+5*cos(f*x+e)^6*sin(f*x+e)-16*cos(f*x+e)^6+13*cos(f*x+e)^4*sin
(f*x+e)-16*cos(f*x+e)^4+29*cos(f*x+e)^2*sin(f*x+e)+58*sin(f*x+e)-58)/cos(f*
x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(3/2)*cos(f*x +
e)^2, x)
```

Fricas [A]
time = 0.37, size = 123, normalized size = 0.88

$$-\frac{\left(35\,a^{3}c\cos\left(fx+e\right)^{6}-35\,a^{3}c+\left(15\,a^{3}c\cos\left(fx+e\right)^{6}-24\,a^{3}c\cos\left(fx+e\right)^{4}-32\,a^{3}c\cos\left(fx+e\right)^{2}-64\,a^{3}c\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{105\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/105*(35*a^3*c*cos(f*x + e)^6 - 35*a^3*c + (15*a^3*c*cos(f*x + e)^6 - 24*
a^3*c*cos(f*x + e)^4 - 32*a^3*c*cos(f*x + e)^2 - 64*a^3*c)*sin(f*x + e))*sq
rt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 159, normalized size = 1.14

$$-\frac{128\left(15\,a^{3}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-35\,a^{3}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+21\,a^{3}c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -128/105*(15*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^14*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 35*a^3*c*cos(-1/4*pi +
1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e)) + 21*a^3*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)
/f
```

Mupad [B]
time = 12.14, size = 319, normalized size = 2.28

$$-\frac{\mathrm{e}^{-e\,7\mathrm{i}-f\,x\,7\mathrm{i}}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{5\,a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{16\,f}+\frac{a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{8\,f}+\frac{a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(6\,e+6\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{48\,f}-\frac{19\,a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{96\,f}-\frac{a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{160\,f}+\frac{a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(7\,e+7\,f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{224\,f}-\frac{45\,a^3\,c\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(e+f\,x\right)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{32\,f}\right)}{2\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -(exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((5*a^3*c*exp(e*7i + f*x*
7i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(16*f) + (a^3*c*exp(e*7i +
 f*x*7i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(8*f) + (a^3*c*exp(e*
7i + f*x*7i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(48*f) - (19*a^3*
c*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) -
(a^3*c*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160
*f) + (a^3*c*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2)
)/(224*f) - (45*a^3*c*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(e + f*x))^
(1/2))/(32*f)))/(2*cos(e + f*x))
```

3.34 $\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}$

Optimal. Leaf size=92

$$\frac{c\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}$$

```
[Out] 1/15*c*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)+1/6*cos
(f*x+e)*(a+a*sin(f*x+e))^(9/2)*(c-c*sin(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.28, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2819, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}\sqrt{c-c\sin(e+fx)}}{6af}+\frac{c\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{15af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(15*a*f*Sqrt[c - c*Sin[e + f*x]
]) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2)*Sqrt[c - c*Sin[e + f*x]])/(6*
a*f)
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{9/2}(c-c\sin(e+fx))^{3/2}}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(}}{6af}$$

$$= \frac{c\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{15af\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e-}{}$$

Mathematica [A]
time = 0.37, size = 104, normalized size = 1.13

$$\frac{a^3\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\,(-405\cos(2(e+fx))-90\cos(4(e+fx))+5\cos(6(e+fx))+1080\sin(e+fx)+20\sin(3(e+fx))-36\sin(5(e+fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (a^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-405
*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)] + 1080*Sin[e +
 f*x] + 20*Sin[3*(e + f*x)] - 36*Sin[5*(e + f*x)]))/(960*f)
```

Maple [A]
time = 0.17, size = 133, normalized size = 1.45

method	result
default	$-\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(-5\left(\cos^8(fx+e)\right)+3\left(\cos^6(fx+e)\right)\sin(fx+e)-4\left(\cos^6(fx+e)\right)+7(\right.}{30f\cos(fx+e)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/30/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(-5*c
os(f*x+e)^8+3*cos(f*x+e)^6*sin(f*x+e)-4*cos(f*x+e)^6+7*cos(f*x+e)^4*sin(f*x
+e)+7*cos(f*x+e)^2*sin(f*x+e)+7*cos(f*x+e)^2+28*sin(f*x+e)-28)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)
^2, x)
```

Fricas [A]
time = 0.35, size = 118, normalized size = 1.28

$$\frac{\left(5\,a^{3}\cos\left(fx+e\right)^{6}-30\,a^{3}\cos\left(fx+e\right)^{4}+25\,a^{3}-2\,\left(9\,a^{3}\cos\left(fx+e\right)^{4}-8\,a^{3}\cos\left(fx+e\right)^{2}-16\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/30*(5*a^3*cos(f*x + e)^6 - 30*a^3*cos(f*x + e)^4 + 25*a^3 - 2*(9*a^3*cos(
f*x + e)^4 - 8*a^3*cos(f*x + e)^2 - 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 108, normalized size = 1.17

$$\frac{32\,\left(5\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-6\,a^{3}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}\,\sqrt{c}}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 32/15*(5*a^3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^12*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*a^3*cos(-1/4*pi + 1/2*f*x +
```

```
 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)))*sqrt(a)*sqrt(c)/f
```

Mupad [B]
time = 11.49, size = 121, normalized size = 1.32

$$-\frac{a^3\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(405\,\cos\left(e+f\,x\right)+495\,\cos\left(3\,e+3\,f\,x\right)+85\,\cos\left(5\,e+5\,f\,x\right)-5\,\cos\left(7\,e+7\,f\,x\right)-1100\,\sin\left(2\,e+2\,f\,x\right)+16\,\sin\left(4\,e+4\,f\,x\right)+36\,\sin\left(6\,e+6\,f\,x\right)\right)}{960\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(405*cos(e
 + f*x) + 495*cos(3*e + 3*f*x) + 85*cos(5*e + 5*f*x) - 5*cos(7*e + 7*f*x) -
 1100*sin(2*e + 2*f*x) + 16*sin(4*e + 4*f*x) + 36*sin(6*e + 6*f*x)))/(960*f
*(cos(2*e + 2*f*x) + 1))
```

3.35 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/5*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(5*a*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{9/2}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{5af\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 142 vs. $2(45) = 90$.
time = 0.94, size = 142, normalized size = 3.16

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\,(-120\cos(2(e+fx))+10\cos(4(e+fx))+210\sin(e+fx)-45\sin(3(e+fx))+\sin(5(e+fx)))}{80f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 +
 Sin[e + f*x])]*(-120*Cos[2*(e + f*x)] + 10*Cos[4*(e + f*x)] + 210*Sin[e +
f*x] - 45*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/(80*f*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(39) = 78$.
time = 0.20, size = 245, normalized size = 5.44

method	result
default	$\frac{\sin(fx+e)(a(1+\sin(fx+e)))^{\frac{7}{2}}\left(\cos^5(fx+e)+(\cos^4(fx+e))\sin(fx+e)+4(\cos^4(fx+e))-5(\cos^3(fx+e))\sin(fx+e)-12(\cos^3(fx+e))-\right.}{5f\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4(\cos^2(fx+e))\sin(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/5/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(cos(f*x+e)^5+cos(f*x+e)^4*sin(f*
x+e)+4*cos(f*x+e)^4-5*cos(f*x+e)^3*sin(f*x+e)-12*cos(f*x+e)^3-7*cos(f*x+e)^
2*sin(f*x+e)-8*cos(f*x+e)^2+15*cos(f*x+e)*sin(f*x+e)+16*cos(f*x+e)+sin(f*x+
e)-1)/(-c*(sin(f*x+e)-1))^(1/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos
(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-
4*cos(f*x+e)+8*sin(f*x+e)+8)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/sqrt(-c*sin(f*x + e) +
c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(42) = 84$.

time = 0.36, size = 119, normalized size = 2.64

$$\frac{\left(5 a^{3} \cos \left(f x+e\right)^{4}-20 a^{3} \cos \left(f x+e\right)^{2}+15 a^{3}+\left(a^{3} \cos \left(f x+e\right)^{4}-12 a^{3} \cos \left(f x+e\right)^{2}+16 a^{3}\right) \sin \left(f x+e\right)\right) \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c}}{5 c f \cos \left(f x+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 15*a^3 + (a^3*cos(f*x +
 e)^4 - 12*a^3*cos(f*x + e)^2 + 16*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c)/(c*f*cos(f*x + e))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.67, size = 56, normalized size = 1.24

$$-\frac{32 a^{\frac{7}{2}} \cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{10} \operatorname{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{5 \sqrt{c} f \operatorname{sgn}\left(\sin \left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -32/5*a^(7/2)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^10*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))/(sqrt(c)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]

time = 10.69, size = 113, normalized size = 2.51

$$-\frac{a^{3} \sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(120\,\cos\left(e+f\,x\right)+110\,\cos\left(3\,e+3\,f\,x\right)-10\,\cos\left(5\,e+5\,f\,x\right)-165\,\sin\left(2\,e+2\,f\,x\right)+44\,\sin\left(4\,e+4\,f\,x\right)-\sin\left(6\,e+6\,f\,x\right)\right)}{80\,c\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(120*cos(e
 + f*x) + 110*cos(3*e + 3*f*x) - 10*cos(5*e + 5*f*x) - 165*sin(2*e + 2*f*x)
 + 44*sin(4*e + 4*f*x) - sin(6*e + 6*f*x)))/(80*c*f*(cos(2*e + 2*f*x) + 1))
```

3.36 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a+a}{cf\sqrt{c-c\sin(e}$$

```
[Out] -2*a^2*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)-2/3*a*c
os(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)-1/4*cos(f*x+e)*
(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(1/2)-16*a^4*cos(f*x+e)*ln(1-si
n(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-8*a^3*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$-\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{8a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{cf\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4cf\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (-16*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]]) - (8*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(
c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3
/2))/(c*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x]
)^(5/2))/(3*c*f*Sqrt[c - c*Sin[e + f*x]]) - (Cos[e + f*x]*(a + a*Sin[e + f*
x])^(7/2))/(4*c*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac} \\
&= -\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf\sqrt{c-c\sin(e+fx)}} + \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,d}{c} \\
&= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3cf\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)(a+a\sin(e}{4cf\sqrt{c-c\sin(e+}} \\
&= -\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a\cos(e+fx)(a+a\,\mathrm{s}}{3cf\sqrt{c-c\sin(e}} \\
&= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\,\mathrm{s}}{cf\sqrt{c-c\sin(e}} \\
&= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\,\mathrm{s}}{cf\sqrt{c-c\sin(e}} \\
&= -\frac{8a^3\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{cf\sqrt{c-c\sin(e+fx)}} - \frac{2a^2\cos(e+fx)(a+a\,\mathrm{s}}{cf\sqrt{c-c\sin(e}} \\
&= -\frac{16a^4\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{8a^3\cos(e+fx)}{cf\sqrt{c-}}
\end{aligned}
$$

Mathematica [A]
time = 4.29, size = 156, normalized size = 0.65

$$
-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a(1+\sin(e+fx))}\left(-276\cos(2(e+fx))+3\cos(4(e+fx))+8\left(384\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+195\sin(e+fx)-5\sin(3(e+fx))\right)\right)}{96cf\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] -1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]
*(-276*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 8*(384*Log[Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]] + 195*Sin[e + f*x] - 5*Sin[3*(e + f*x)])))/(c*f*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.20, size = 253, normalized size = 1.05

method	result
default	$\frac{\left(3\left(\cos^4(fx+e)\right)-20\left(\cos^2(fx+e)\right)\sin(fx+e)-72\left(\cos^2(fx+e)\right)+384\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+200\sin(fx+e)-192\ln\left(\frac{2}{1+\cos(}\right.\right.}{12f(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/12/f*(3*cos(f*x+e)^4-20*cos(f*x+e)^2*sin(f*x+e)-72*cos(f*x+e)^2+384*ln(-(
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+200*sin(f*x+e)-192*ln(2/(1+cos(f*x+e)
))+69)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1
+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*
cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)
+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 208, normalized size = 0.86

$$\frac{4\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{12\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{3\,c^{6}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+4\,c^{6}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+6\,c^{6}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+12\,c^{6}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{8}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 4/3*a^(7/2)*sqrt(c)*(12*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^2*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (3*c^6*cos(-1/4*pi + 1/2*f*x + 1/2*e)^8
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c^6*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 6*c^6*cos(-1/4*pi + 1/2*f*x + 1/2*
e)^4*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*c^6*cos(-1/4*pi + 1/2*f*x + 1
/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/c^8)*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(3/2),
 x)
```

3.37 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=238

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}}+\frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{16a^3\cos(e+fx)\sqrt{a+a}}{c^2f\sqrt{c-c\sin(e+}}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(3/2)+4*a^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+4/3*a*cos(f*x+e)*(a+
a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)+32*a^4*cos(f*x+e)*ln(1-sin
(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+16*a^3*cos(f*x
+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$\frac{32a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{16a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^2f\sqrt{c-c\sin(e+fx)}}+\frac{4a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3c^2f\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{cf(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(c*f*(c - c*Sin[e + f*x])^(3/2))
+ (32*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]]) + (16*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a^2*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(3/2))/(c^2*f*Sqrt[c - c*Sin[e + f*x]]) + (4*a*Cos[e + f*x]*(a + a*Sin[
e + f*x])^(5/2))/(3*c^2*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,d\ldots}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a\cos(e+fx)(a+a\sin(e\ldots}{3c^2f\sqrt{c-c\sin(e+\ldots}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{4a^2\cos(e+fx)(a+a\sin(\ldots}{c^2f\sqrt{c-c\sin(e+\ldots}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\,\mathrm{si}\ldots}}{c^2f\sqrt{c-c\sin(e+\ldots}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\,\mathrm{si}\ldots}}{c^2f\sqrt{c-c\sin(e+\ldots}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{16a^3\cos(e+fx)\sqrt{a+a\,\mathrm{si}\ldots}}{c^2f\sqrt{c-c\sin(e+\ldots}}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{cf(c-c\sin(e+fx))^{3/2}} + \frac{32a^4\cos(e+fx)\log(1\ldots}{c^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{\ldots}}$$

Mathematica [A]
time = 3.07, size = 196, normalized size = 0.82

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(-177-172\cos(2(e+fx))+\cos(4(e+fx))-1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-396\sin(e+fx)+1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-16\sin(3(e+fx))\right)}{24c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/24*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x]
)]*(-177 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]] - 396*Sin[e + f*x] + 1536*Log[Cos[(e + f*x)/2] - Si
n[(e + f*x)/2]]*Sin[e + f*x] - 16*Sin[3*(e + f*x)]))/(c^2*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.21, size = 305, normalized size = 1.28

method	result
default	$\frac{\left(\cos^4(fx+e)-8(\cos^2(fx+e))\sin(fx+e)+192\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-96\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-44(\cos^2(fx}{3f(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4(\cos^2(fx-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^4-8*cos(f*x+e)^2*sin(f*x+e)+192*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))*sin(f*x+e)-96*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-44*cos(f*x+
e)^2-192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-91*sin(f*x+e)+96*ln(2/(
1+cos(f*x+e)))+43)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos
(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-
4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)
*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 187, normalized size = 0.79

$$-\frac{8\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{12\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+1\right)}{c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{3}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-1\right)c^3\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{c^6\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^6+3\,c^6\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+9\,c^6\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{c^9\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -8/3*a^(7/2)*sqrt(c)*(12*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^3*sg
n(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 3/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 -
 1)*c^3*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + (c^6*cos(-1/4*pi + 1/2*f*x +
 1/2*e)^6 + 3*c^6*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 9*c^6*cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2)/(c^9*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(5/2),
 x)
```

3.38 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=239

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}}-\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}}-\frac{24a^4\cos(e+fx)\log(1-\sin(e}{c^3f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin}$$

```
[Out] 1/2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(5/2)-2*a*cos(f*
x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)-3*a^2*cos(f*x+e)*(
a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)-24*a^4*cos(f*x+e)*ln(1-s
in(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-12*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{24a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{12a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{3a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f\sqrt{c-c\sin(e+fx)}}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{c^2f(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(7/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(c^2*f*(c - c*Sin[e + f*x
])^(3/2)) - (24*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*S
in[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (12*a^3*Cos[e + f*x]*Sqrt[a + a*Si
n[e + f*x]])/(c^3*f*Sqrt[c - c*Sin[e + f*x]]) - (3*a^2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(3/2))/(c^3*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx}{c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{2cf(c-c\sin(e+fx))^{5/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+ \cdots}{c^2f(c-c\sin(e+fx)) \cdots}$$

Mathematica [A]
time = 4.20, size = 223, normalized size = 0.93

$$\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(273+\cos(4(e+fx))+\cos(2(e+fx))\left(106-384\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+1152\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-320\sin(e+fx)-1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)-24\sin(3(e+fx))\right)}{16c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])]*(27
3 + Cos[4*(e + f*x)] + Cos[2*(e + f*x)]*(106 - 384*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2]]) + 1152*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 320*Sin
[e + f*x] - 1536*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 24
*Sin[3*(e + f*x)]))/(16*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + S
in[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.22, size = 365, normalized size = 1.53

method	result
default	$-\frac{\left(-\left(\cos^4(fx+e)\right)+96\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\left(\cos^2(fx+e)\right)+12\left(\cos^2(fx+e)\right)\sin(fx+e)-48\ln\left(\frac{2}{1+\cos(fx+e)}\right)\left(\cos^2(fx+e\right.}{2f(\cos^4(fx+e)+(\cos}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/f*(-cos(f*x+e)^4+96*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x
+e)^2+12*cos(f*x+e)^2*sin(f*x+e)-48*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2+192*l
n(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-96*ln(2/(1+cos(f*x+e))
)*sin(f*x+e)-73*cos(f*x+e)^2-192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
-58*sin(f*x+e)+96*ln(2/(1+cos(f*x+e)))+74)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e
)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f
*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-
4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1514 vs. $2(233) = 466$.
time = 0.64, size = 1514, normalized size = 6.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] 1/30*(1440*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 720*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + (334*a^(7/2)
- 1449*a^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 2693*a^(7/2)*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 3278*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
 3199*a^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2014*a^(7/2)*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 315*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
 + 10*a^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 525*a^(7/2)*sin(f*x + e
)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/
(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x +
 e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/
(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7
/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10) - (334*a^(7/2) - 2079*a^(7/2)*si
```

```
n(f*x + e)/(cos(f*x + e) + 1) + 6203*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2 - 10698*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15049*a^(7/2)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 15354*a^(7/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5 + 12165*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 7410*a^(7
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 2985*a^(7/2)*sin(f*x + e)^8/(cos(
f*x + e) + 1)^8 - 555*a^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2)
 - 6*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 32*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^
(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 46*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/
2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 - 6*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(
f*x + e)^10/(cos(f*x + e) + 1)^10) + 10*(75*a^(7/2)*sin(f*x + e)/(cos(f*x +
 e) + 1) - 375*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 854*a^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1257*a^(7/2)*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + 1534*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1257*a^(7/2)
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 854*a^(7/2)*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7 - 375*a^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 75*a^(7/2)*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos
(f*x + e) + 1) + 17*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 32*c^(7/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 46*c^(7/2)*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4 - 52*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 46*c^(7/2)*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e
) + 1)^7 + 17*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 6*c^(7/2)*sin(f
*x + e)^9/(cos(f*x + e) + 1)^9 + c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)
^10))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4
*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 198, normalized size = 0.83

$$\frac{2\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{12\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{8\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-7}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{2}c^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{c^{4}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+6\,c^{4}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{c^{8}}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x, alg
orithm="giac")
```

```
[Out] 2*a^(7/2)*sqrt(c)*(12*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^4*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))) - (8*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 7)/
((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*c^4*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))) + (c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) + 6*c^4*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)))/c^8)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(7/2),
 x)
```

3.39 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=241

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}}-\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}}+\frac{2a^2\cos(e+fx)(a+a\sin(e+fx))}{c^3f(c-c\sin(e+fx))^{3/2}}$$

```
[Out] 1/3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(7/2)-2/3*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)+2*a^2*cos(f*x+e)
*(a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)+8*a^4*cos(f*x+e)*ln(1-
sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*a^3*cos(f
*x+e)*(a+a*sin(f*x+e))^(1/2)/c^4/f/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$\frac{8a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{4a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f\sqrt{c-c\sin(e+fx)}}+\frac{2a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{c^3f(c-c\sin(e+fx))^{3/2}}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3c^2f(c-c\sin(e+fx))^{5/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(9/2),
x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)
) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f
*x])^(5/2)) + (2*a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(c^3*f*(c - c
*Sin[e + f*x])^(3/2)) + (8*a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*a^3*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(c^4*f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{4\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx}{3c^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{3cf(c-c\sin(e+fx))^{7/2}} - \frac{2a\cos(e+fx)(a+a\sin(e+}{3c^2f(c-c\sin(e+fx)}$$

Mathematica [A]
time = 5.73, size = 260, normalized size = 1.08

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}\left(-563+3\cos(4(e+fx))-960\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+48\cos(2(e+fx))\left(5+12\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+690\sin(e+fx)+1440\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+18\sin(3(e+fx))-96\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(3(e+fx))\right)}{24c^4f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^4\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] -1/24*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x]
)]*(-563 + 3*Cos[4*(e + f*x)] - 960*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 48*Cos[2*(e + f*x)]*(5 + 12*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]) +
 690*Sin[e + f*x] + 1440*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f
*x] + 18*Sin[3*(e + f*x)] - 96*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin
[3*(e + f*x)]))/(c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e +
f*x])^4*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.21, size = 435, normalized size = 1.80

method	result
default	$\frac{\left(24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)\left(\cos^2(fx+e)\right)-48\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)\left(\cos^2(fx+e)\right)+3\left(\cos^4(fx+e)\right)-72\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(24*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2-48*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+3*cos(f*x+e)^4-72*ln(2/(1
+cos(f*x+e)))*cos(f*x+e)^2+144*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*c
os(f*x+e)^2+49*cos(f*x+e)^2*sin(f*x+e)-96*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+1
92*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-63*cos(f*x+e)^2+96
*ln(2/(1+cos(f*x+e)))-192*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-76*sin
(f*x+e)+60)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*
(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)
^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f
*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c
^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 179, normalized size = 0.74

$$-\frac{2\,a^{\frac{7}{2}}\sqrt{c}\left(\frac{3\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{c^{5}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{12\,\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{c^{5}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{18\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-30\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+13}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)^{3}c^{5}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] -2/3*a^(7/2)*sqrt(c)*(3*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(c^5*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))) + 12*log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(c^
5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - (18*cos(-1/4*pi + 1/2*f*x + 1/2*e)
^4 - 30*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 13)/((cos(-1/4*pi + 1/2*f*x + 1/
2*e)^2 - 1)^3*c^5*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2),
 x)
```

3.40 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=243

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}}-\frac{a\cos(e+fx)(a+a\sin(e+fx))^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}+\frac{a^2\cos(e+fx)(a+a\sin(e+fx))^{3}}{2c^3f(c-c\sin(e+fx))^{5/2}}$$

```
[Out] 1/4*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*cos(f*x+
e)*(a+a*sin(f*x+e))^(3/2)/c^3/f/(c-c*sin(f*x+e))^(5/2)-a^3*cos(f*x+e)*(a+a*
sin(f*x+e))^(1/2)/c^4/f/(c-c*sin(f*x+e))^(3/2)-a^4*cos(f*x+e)*ln(1-sin(f*x+
e))/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2818, 2816, 2746, 31}

$$-\frac{a^4\cos(e+fx)\log(1-\sin(e+fx))}{c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{c^4f(c-c\sin(e+fx))^{3/2}}+\frac{a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{2c^3f(c-c\sin(e+fx))^{5/2}}-\frac{a\cos(e+fx)(a\sin(e+fx)+a)^{5/2}}{3c^2f(c-c\sin(e+fx))^{7/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(11/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(4*c*f*(c - c*Sin[e + f*x])^(9/2)
) - (a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c^2*f*(c - c*Sin[e + f*x
])^(7/2)) + (a^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^3*f*(c - c*S
in[e + f*x])^(5/2)) - (a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^4*f*(c
 - c*Sin[e + f*x])^(3/2)) - (a^4*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^5*f
*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{9/2}}\,dx}{ac} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{\int \frac{(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx}{c^2} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{7/2}}{4cf(c-c\sin(e+fx))^{9/2}} - \frac{a\cos(e+fx)(a+a\sin(e}{3c^2f(c-c\sin(e+fx}
\end{aligned}
$$

Mathematica [A]
time = 6.45, size = 437, normalized size = 1.80

$$
\frac{4\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}} - \frac{32\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}} + \frac{12\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}} - \frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}} - \frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{11/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(11/2),x]
```

```
[Out] (4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (32
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (12
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (8*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) - (2*Log[C
os[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^1
```

```
1*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*
(c - c*Sin[e + f*x])^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(217) = 434$.

time = 0.20, size = 493, normalized size = 2.03

method	result
default	$\frac{\left(6\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^4(fx+e))-3\ln\left(\frac{2}{1+\cos(fx+e)}\right)(\cos^4(fx+e))+24\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)(\cos^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/3/f*(6*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^4-3*ln(2/(1+
cos(f*x+e)))*cos(f*x+e)^4+24*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin
(f*x+e)*cos(f*x+e)^2-12*ln(2/(1+cos(f*x+e)))*sin(f*x+e)*cos(f*x+e)^2-8*cos(
f*x+e)^4-48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-8*cos(f
*x+e)^2*sin(f*x+e)+24*ln(2/(1+cos(f*x+e)))*cos(f*x+e)^2-48*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+24*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+28
*cos(f*x+e)^2+48*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*sin(f*x+e)-24
*ln(2/(1+cos(f*x+e)))-20)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-
cos(f*x+e)+2)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e
)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(
f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(11/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
11/2), x)
```

Fricas [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="fricas")
```

```
[Out] integral((3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^4
 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(c^6*cos(f*x + e)^6 - 18*c^6*cos(f*x + e)^4 + 48*c^6*cos(f*x
 + e)^2 - 32*c^6 + 2*(3*c^6*cos(f*x + e)^4 - 16*c^6*cos(f*x + e)^2 + 16*c^6
)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 169, normalized size = 0.70

$$\frac{\sqrt{2}\, a^{\frac{7}{2}} \sqrt{c} \left(\frac{12 \sqrt{2} \log\left(-2 \cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)^2+2\right)}{c^6 \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)} - \frac{\sqrt{2} \left(48 \cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)^6-108 \cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)^4+88 \cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)^2-25\right)}{\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)^2-1\right)^4 c^6 \mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)}\right) \mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)}{24 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x, al
gorithm="giac")
```

```
[Out] 1/24*sqrt(2)*a^(7/2)*sqrt(c)*(12*sqrt(2)*log(-2*cos(-1/4*pi + 1/2*f*x + 1/2
*e)^2 + 2)/(c^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - sqrt(2)*(48*cos(-1/4
*pi + 1/2*f*x + 1/2*e)^6 - 108*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 88*cos(-1
/4*pi + 1/2*f*x + 1/2*e)^2 - 25)/((cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^4*
c^6*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))/f
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2)
, x)
```

3.41 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=48

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

[Out] 1/10*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(11/2)

Rubi [A]
time = 0.23, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(13/2),x]

[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(10*a*c*f*(c - c*Sin[e + f*x])^(11/2))

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{ac} = \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{10acf(c-c\sin(e+fx))^{11/2}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 412 vs. $2(48) = 96$.
time = 6.50, size = 412, normalized size = 8.58

$$\frac{16\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} - \frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} + \frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} - \frac{4\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}} + \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(13/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) -
(8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + (8*
(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) - (4*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(42) = 84$.
time = 0.16, size = 191, normalized size = 3.98

method	result
default	$-\frac{(\cos^4(fx+e)-12(\cos^2(fx+e))+16)(a(1+\sin(fx+e)))^{\frac{7}{2}}\sin(fx+e)(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-}{5f(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-4\cos(fx+e)\sin(fx+e)-4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] -1/5/f*(cos(f*x+e)^4-12*cos(f*x+e)^2+16)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e
)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e
)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(
```

```
f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+
e)-1))^(13/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
13/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(45) = 90$.
time = 0.36, size = 161, normalized size = 3.35

$$\frac{\left(5\,a^{3}\cos\left(fx+e\right)^{4}-20\,a^{3}\cos\left(fx+e\right)^{2}+16\,a^{3}\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{5\,\left(5\,c^{7}f\cos\left(fx+e\right)^{5}-20\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\cos\left(fx+e\right)-\left(c^{7}f\cos\left(fx+e\right)^{5}-12\,c^{7}f\cos\left(fx+e\right)^{3}+16\,c^{7}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="fricas")
```

```
[Out] 1/5*(5*a^3*cos(f*x + e)^4 - 20*a^3*cos(f*x + e)^2 + 16*a^3)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^7*f*cos(f*x + e)^5 - 20*c^7*f*cos(
f*x + e)^3 + 16*c^7*f*cos(f*x + e) - (c^7*f*cos(f*x + e)^5 - 12*c^7*f*cos(f
*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(45) = 90$.
time = 0.56, size = 197, normalized size = 4.10

$$-\frac{\left(5\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-10\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+10\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-5\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{10\,c^{\frac{13}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x, al
gorithm="giac")
```

```
[Out] -1/10*(5*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^8 - 10*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x +
 1/2*e)^6 + 10*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*
x + 1/2*e)^4 - 5*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^2 + a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(13/2)
*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^10)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{13/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(13/2)
,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(13/2)
, x)
```

3.42 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=97

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/12*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(13/2)+1/120*
cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^2/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A]
time = 0.30, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{120ac^2f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(15/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(12*a*c*f*(c - c*Sin[e + f*x])^(1
3/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(120*a*c^2*f*(c - c*Sin[e
 + f*x])^(11/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{ac} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{11/2}}\,dx}{12ac^2} \\
&= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{12acf(c-c\sin(e+fx))^{13/2}} + \frac{\cos(e+fx)(a+a\sin(e+
}{120ac^2f(c-c\sin(e+fx
}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 419 vs. 2(97) = 194.
time = 6.52, size = 419, normalized size = 4.32

$$\frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}-\frac{32\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}+\frac{6\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}-\frac{8\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}+\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{7/2}}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(15/2),x]
```

```
[Out] (8*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(3
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (
32*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5
*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (
6*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(f*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (8*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(85) = 170.
time = 0.19, size = 233, normalized size = 2.40

method	result
default	$\frac{\left(3\left(\cos^4(fx+e)\right)\sin(fx+e)-18\left(\cos^4(fx+e)\right)-36\left(\cos^2(fx+e)\right)\sin(fx+e)+116\left(\cos^2(fx+e)\right)+48\sin(fx+e)-128\right)\left(a(1+\sin(fx+e))\right)^{\frac{7}{2}}}{30f\left(\cos^4(fx+e)+\left(\cos^3(fx+e)\right)\sin(fx+e)+3\left(\cos^3(fx+e)\right)-4\left(\cos^2(fx+e)\right)\sin(fx+e)-8\left(\cos^2(fx+e)\right)-4\cos(fx+e)\sin\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/30/f*(3*cos(f*x+e)^4*sin(f*x+e)-18*cos(f*x+e)^4-36*cos(f*x+e)^2*sin(f*x+e
)+116*cos(f*x+e)^2+48*sin(f*x+e)-128)*(a*(1+sin(f*x+e)))^(7/2)*sin(f*x+e)*(
cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(cos(f*x+e)^4
+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x
+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-
1))^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(7/2)*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(
15/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(91) = 182.
time = 0.39, size = 205, normalized size = 2.11

$$-\frac{\left(15\,a^{3}\cos\left(fx+e\right)^{4}-60\,a^{3}\cos\left(fx+e\right)^{2}+48\,a^{3}-4\left(5\,a^{3}\cos\left(fx+e\right)^{2}-8\,a^{3}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{30\left(c^{8}f\cos\left(fx+e\right)^{7}-18\,c^{8}f\cos\left(fx+e\right)^{5}+48\,c^{8}f\cos\left(fx+e\right)^{3}-32\,c^{8}f\cos\left(fx+e\right)+2\left(3\,c^{8}f\cos\left(fx+e\right)^{5}-16\,c^{8}f\cos\left(fx+e\right)^{3}+16\,c^{8}f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="fricas")
```

```
[Out] -1/30*(15*a^3*cos(f*x + e)^4 - 60*a^3*cos(f*x + e)^2 + 48*a^3 - 4*(5*a^3*co
s(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^8*f*cos(f*x + e)^7 - 18*c^8*f*cos(f*x + e)^5 + 48*c^8*f*cos(
f*x + e)^3 - 32*c^8*f*cos(f*x + e) + 2*(3*c^8*f*cos(f*x + e)^5 - 16*c^8*f*c
os(f*x + e)^3 + 16*c^8*f*cos(f*x + e))*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(15/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(91) = 182.
time = 0.58, size = 213, normalized size = 2.20

$$-\frac{\left(15\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-40\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+45\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-24\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+5\,a^{3}\sqrt{c}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{120\,c^{8}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x, al
gorithm="giac")
```

```
[Out] -1/120*(15*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^8 - 40*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-
1/4*pi + 1/2*f*x + 1/2*e)^6 + 45*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 24*a^3*sqrt(c)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 5*a^3*sqrt(c)*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^8*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^12)
```

Mupad [**B**]
time = 14.41, size = 373, normalized size = 3.85

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{504\,a^{3}\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^{8}\,f}+\frac{576\,a^{3}\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(e+f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{5\,c^{8}\,f}-\frac{96\,a^{3}\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(2\,e+2\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^{8}\,f}+\frac{8\,a^{3}\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos(4\,e+4\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{c^{8}\,f}-\frac{64\,a^{3}\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin(3\,e+3\,f\,x)\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{3\,c^{8}\,f}\right)}{-858\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}+858\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)-130\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)+2\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\cos\left(7\,e+7\,f\,x\right)+1144\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(2\,e+2\,f\,x\right)-416\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(4\,e+4\,f\,x\right)+24\,\mathrm{e}^{e\,7\mathrm{i}+f\,x\,7\mathrm{i}}\,\sin\left(6\,e+6\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(15/2)
,x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((504*a^3*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^(1/2))/(5*c^8*f) + (576*a^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(a + a*sin(
e + f*x))^(1/2))/(5*c^8*f) - (96*a^3*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(a
 + a*sin(e + f*x))^(1/2))/(c^8*f) + (8*a^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f
*x)*(a + a*sin(e + f*x))^(1/2))/(c^8*f) - (64*a^3*exp(e*7i + f*x*7i)*sin(3*
e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^8*f)))/(858*exp(e*7i + f*x*7i)*
cos(3*e + 3*f*x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x
*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*
7i + f*x*7i)*sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 2
4*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x))
```

3.43 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx$

Optimal. Leaf size=145

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}$$

```
[Out] 1/14*cos(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c/f/(c-c*sin(f*x+e))^(15/2)+1/84*c
os(f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^2/f/(c-c*sin(f*x+e))^(13/2)+1/840*cos(
f*x+e)*(a+a*sin(f*x+e))^(9/2)/a/c^3/f/(c-c*sin(f*x+e))^(11/2)
```

Rubi [A]
time = 0.38, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{840ac^3f(c-c\sin(e+fx))^{11/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{84ac^2f(c-c\sin(e+fx))^{13/2}}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^(17/2)
,x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(14*a*c*f*(c - c*Sin[e + f*x])^(1
5/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(84*a*c^2*f*(c - c*Sin[e
+ f*x])^(13/2)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(9/2))/(840*a*c^3*f*(c
 - c*Sin[e + f*x])^(11/2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{17/2}}\,dx = \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{15/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\int \frac{(a+a\sin(e+fx))^{9/2}}{(c-c\sin(e+fx))^{13/2}}\,dx}{7ac^2}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+ }{84ac^2f(c-c\sin(e+fx}$$

$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{9/2}}{14acf(c-c\sin(e+fx))^{15/2}} + \frac{\cos(e+fx)(a+a\sin(e+ }{84ac^2f(c-c\sin(e+fx}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 419 vs. 2(145) = 290.
time = 6.56, size = 419, normalized size = 2.89

$$\frac{16\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{7f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} - \frac{16\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} + \frac{24\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} - \frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}} + \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{7/2}}{3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x])^
(17/2),x]
```

```
[Out] (16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(
7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
(16*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(
3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) +
(24*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(
5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) -
(2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))
```

Maple [**A**]
time = 0.21, size = 243, normalized size = 1.68

method	result
default	$\frac{(9(\cos^6(fx+e))+63(\cos^4(fx+e))\sin(fx+e)-216(\cos^4(fx+e))-406(\cos^2(fx+e))\sin(fx+e)+790(\cos^2(fx+e))+448\sin(fx+e)-68}{105f(\cos^4(fx+e)+(\cos^3(fx+e))\sin(fx+e)+3(\cos^3(fx+e))-4(\cos^2(fx+e))\sin(fx+e)-8(\cos^2(fx+e))-4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x,method=_R
ETURNVERBOSE)
```

```
[Out] 1/105/f*(9*cos(f*x+e)^6+63*cos(f*x+e)^4*sin(f*x+e)-216*cos(f*x+e)^4-406*cos
(f*x+e)^2*sin(f*x+e)+790*cos(f*x+e)^2+448*sin(f*x+e)-688)*(a*(1+sin(f*x+e))
)^(7/2)*sin(f*x+e)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x
+e)+2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e)+3*cos(f*x+e)^3-4*cos(f*x+e)^2*
sin(f*x+e)-8*cos(f*x+e)^2-4*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)
+8)/(-c*(sin(f*x+e)-1))^(17/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 0.38, size = 220, normalized size = 1.52

$$-\frac{\left(35\,a^3\cos\left(fx+e\right)^4-154\,a^3\cos\left(fx+e\right)^2+128\,a^3-14\,\left(5\,a^3\cos\left(fx+e\right)^2-8\,a^3\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{105\,\left(7\,c^9f\cos\left(fx+e\right)^7-56\,c^9f\cos\left(fx+e\right)^5+112\,c^9f\cos\left(fx+e\right)^3-64\,c^9f\cos\left(fx+e\right)-\left(c^9f\cos\left(fx+e\right)^7-24\,c^9f\cos\left(fx+e\right)^5+80\,c^9f\cos\left(fx+e\right)^3-64\,c^9f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, al
gorithm="fricas")
```

```
[Out] -1/105*(35*a^3*cos(f*x + e)^4 - 154*a^3*cos(f*x + e)^2 + 128*a^3 - 14*(5*a^
3*cos(f*x + e)^2 - 8*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c)/(7*c^9*f*cos(f*x + e)^7 - 56*c^9*f*cos(f*x + e)^5 + 112*c^9
*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e) - (c^9*f*cos(f*x + e)^7 - 24*c^9*
f*cos(f*x + e)^5 + 80*c^9*f*cos(f*x + e)^3 - 64*c^9*f*cos(f*x + e))*sin(f*x
 + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(17/2),x)

[Out] Timed out

Giac [A]
time = 0.64, size = 198, normalized size = 1.37

$$-\frac{\left(35\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}-105\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+126\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}-70\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+15\,a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{a}}{840\,c^{\frac{17}{2}}f\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(17/2),x, algorithm="giac")

[Out] -1/840*(35*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 - 105*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 126*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 - 70*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 15*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/(c^(17/2)*f*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^14)

Mupad [B]
time = 14.93, size = 764, normalized size = 5.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(17/2),x)

[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^9*f) + (64*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(3*c^9*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*1088i)/(15*c^9*f) - (576*a^3*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c^9*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*5472i)/(35*c^9*f) + (576*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(5*c^9*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*1088i)/(15*c^9*f) - (64*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(3*c^9*f) + (a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/

```
2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^9*f)))/(exp(e*1i + f*x*1i)*
14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350i + 910*exp(e*4i + f*x*
4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x*6i) - exp(e*7i + f*x*7
i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i + f*x*10i) + exp(e*11i
 + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13i)*350i +
90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i - exp(e*16i + f*x*16i) +
 1)
```

3.44 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/4*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/4*(Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(c*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(45) = 90.
time = 0.62, size = 134, normalized size = 2.98

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\,(28\cos(2(e+fx))-\cos(4(e+fx))+56\sin(e+fx)-8\sin(3(e+fx)))}{32f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c
*Sin[e + f*x]]*(28*Cos[2*(e + f*x)] - Cos[4*(e + f*x)] + 56*Sin[e + f*x] -
8*Sin[3*(e + f*x)]))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqrt[a*(
1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(39) = 78.
time = 0.15, size = 195, normalized size = 4.33

method	result
default	$\sin(fx+e)(-c(\sin(fx+e)-1))^{\frac{5}{2}}\left(\left(\cos^3(fx+e)\right)\sin(fx+e)+\cos^4(fx+e)+3\left(\cos^2(fx+e)\right)\sin(fx+e)-4\left(\cos^3(fx+e)\right)-7\cos(fx+e)\sin(\right.$ / $4f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\left(\left(\cos^2(fx+e)\right)\sin(fx+e)+\cos^3(fx+e)+2\cos(fx+e)\sin(fx+e)-3(\cos^2(fx+e\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(5/2)*(cos(f*x+e)^3*sin(f*x+e)+cos(f*x
+e)^4+3*cos(f*x+e)^2*sin(f*x+e)-4*cos(f*x+e)^3-7*cos(f*x+e)*sin(f*x+e)-4*co
s(f*x+e)^2-sin(f*x+e)+8*cos(f*x+e)-1)/(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^
2*sin(f*x+e)+cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+
e)-2*cos(f*x+e)+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

time = 0.34, size = 105, normalized size = 2.33

$$-\frac{\left(c^{2}\cos\left(fx+e\right)^{4}-8\,c^{2}\cos\left(fx+e\right)^{2}+7\,c^{2}+4\left(c^{2}\cos\left(fx+e\right)^{2}-2\,c^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{4\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -1/4*(c^2*cos(f*x + e)^4 - 8*c^2*cos(f*x + e)^2 + 7*c^2 + 4*(c^2*cos(f*x +
e)^2 - 2*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)/(a*f*cos(f*x + e))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]

time = 0.49, size = 56, normalized size = 1.24

$$\frac{4\,c^{\frac{5}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}}{\sqrt{a}\,f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 4*c^(5/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^8/(sqrt(a)*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]

time = 2.00, size = 96, normalized size = 2.13

$$-\frac{c^{2}\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(28\,\cos\left(e+f\,x\right)+27\,\cos\left(3\,e+3\,f\,x\right)-\cos\left(5\,e+5\,f\,x\right)+48\,\sin\left(2\,e+2\,f\,x\right)-8\,\sin\left(4\,e+4\,f\,x\right)\right)}{64\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(c^2*(-c*(sin(e + f*x) - 1))^(1/2)*(28*cos(e + f*x) + 27*cos(3*e + 3*f*x)
- cos(5*e + 5*f*x) + 48*sin(2*e + 2*f*x) - 8*sin(4*e + 4*f*x)))/(64*f*(a*(s
in(e + f*x) + 1))^(1/2)*(sin(e + f*x) - 1))
```

3.45 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/3*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/3*(Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(c*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(45) = 90.
time = 0.35, size = 120, normalized size = 2.67

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\,(6\cos(2(e+fx))+15\sin(e+fx)-\sin(3(e+fx)))}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] -1/12*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c -
 c*Sin[e + f*x]]*(6*Cos[2*(e + f*x)] + 15*Sin[e + f*x] - Sin[3*(e + f*x)]))
/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(39) = 78.
time = 0.15, size = 141, normalized size = 3.13

method	result
default	$\frac{(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)\left(\cos^3(fx+e)-\left(\cos^2(fx+e)\right)\sin(fx+e)+2\left(\cos^2(fx+e)\right)+3\cos(fx+e)\sin(fx+e)-4\cos(fx+e)+\sin}{3f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(\cos^2(fx+e)-\cos(fx+e)\sin(fx+e)+\cos(fx+e)+2\sin(fx+e)-2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(cos(f*x+e)^3-cos(f*x+e)^2*sin(f
*x+e)+2*cos(f*x+e)^2+3*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+sin(f*x+e)+1)/(a*
(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(
f*x+e)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/sqrt(a*sin(f*x + e) +
a), x)
```

Fricas [A]
time = 0.36, size = 84, normalized size = 1.87

$$\frac{\left(3\,c\cos\left(fx+e\right)^{2}-\left(c\cos\left(fx+e\right)^{2}-4\,c\right)\sin\left(fx+e\right)-3\,c\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{3\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/3*(3*c*cos(f*x + e)^2 - (c*cos(f*x + e)^2 - 4*c)*sin(f*x + e) - 3*c)*sqrt
(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c(\sin\left(e+fx\right)-1)\right)^{\frac{3}{2}}\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*cos(e + f*x)**2/sqrt(a*(sin(e + f*x
) + 1)), x)
```

Giac [A]
time = 0.47, size = 56, normalized size = 1.24

$$\frac{8\,c^{\frac{3}{2}}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}}{3\,\sqrt{a}\,f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 8/3*c^(3/2)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2
*e)^6/(sqrt(a)*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 9.96, size = 83, normalized size = 1.84

$$-\frac{c\,\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(6\,\cos\left(e+f\,x\right)+6\,\cos\left(3\,e+3\,f\,x\right)+14\,\sin\left(2\,e+2\,f\,x\right)-\sin\left(4\,e+4\,f\,x\right)\right)}{24\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -(c*(-c*(sin(e + f*x) - 1))^(1/2)*(6*cos(e + f*x) + 6*cos(3*e + 3*f*x) + 14
*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(24*f*(a*(sin(e + f*x) + 1))^(1/2)*(
sin(e + f*x) - 1))
```

3.46 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -1/2*(Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(c*f*Sqrt[a + a*Sin[e + f*x]
])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx}{ac}\\ &= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2cf\sqrt{a+a\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 62, normalized size = 1.38

$$\frac{\sec(e+fx)\sqrt{a(1+\sin(e+fx))}\,(\cos(2(e+fx))+4\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]
],x]
```

```
[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(Cos[2*(e + f*x)] + 4*Sin[e + f*x]
)*Sqrt[c - c*Sin[e + f*x]])/(4*a*f)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(39) = 78$.
time = 0.15, size = 90, normalized size = 2.00

method	result	size
default	$\frac{\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\sin(fx+e)\left(\cos^{2}(fx+e)+\cos(fx+e)\sin(fx+e)-2\cos(fx+e)+\sin(fx+e)+1\right)}{2f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(-1+\cos(fx+e)+\sin(fx+e)\right)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(cos(f*x+e)^2+cos(f*x+e)*sin(f*x
+e)-2*cos(f*x+e)+sin(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)+sin(
f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(42) = 84$.
time = 0.59, size = 418, normalized size = 9.29

$$-\frac{\frac{2\sqrt{a}\sqrt{c}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}-\frac{2\sqrt{a}\sqrt{c}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3\sqrt{a}\sqrt{c}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}}{a+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}+\frac{2\left(\frac{\sqrt{a}\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1}-\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{\sqrt{a}\sqrt{c}\sin(fx+e)^{3}}{(\cos(fx+e)+1)^{3}}\right)}{a+\frac{2a\sin(fx+e)^{2}}{(\cos(fx+e)+1)^{2}}+\frac{a\sin(fx+e)^{4}}{(\cos(fx+e)+1)^{4}}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*((2*sqrt(a)*sqrt(c) + sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1)
+ sqrt(a)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin
(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - (2*sqrt(a)*sqrt(c) - sqrt(a)
*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(a)*sqrt(c)*sin(f*x + e)^2
```

```
/(cos(f*x + e) + 1)^2 - sqrt(a)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3
)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^4/(cos(f*x
+ e) + 1)^4) + 2*(sqrt(a)*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) - sqrt(a)
*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(a)*sqrt(c)*sin(f*x + e)
^3/(cos(f*x + e) + 1)^3)/(a + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

Fricas [A]
time = 0.38, size = 64, normalized size = 1.42

$$\frac{\left(\cos\left(fx+e\right)^{2}+2\,\sin\left(fx+e\right)-1\right)\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{2\,af\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 1/2*(cos(f*x + e)^2 + 2*sin(f*x + e) - 1)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c)/(a*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\cos^{2}\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/sqrt(a*(sin(e + f*x) +
 1)), x)
```

Giac [A]
time = 0.61, size = 56, normalized size = 1.24

$$\frac{2\,\sqrt{c}\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}}{\sqrt{a}\,f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 2*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e
)^4/(sqrt(a)*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 0.93, size = 67, normalized size = 1.49

$$-\frac{\sqrt{-c\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)+4\,\sin\left(2\,e+2\,f\,x\right)\right)}{8\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((-c*(sin(e + f*x) - 1))^(1/2)*(cos(e + f*x) + cos(3*e + 3*f*x) + 4*sin(2*
e + 2*f*x)))/(8*f*(a*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*x) - 1))
```

3.47 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] -((Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\
&= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{cf\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 44, normalized size = 1.02

$$\frac{\sin(2(e+fx))}{2f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
),x]
```

```
[Out] Sin[2*(e + f*x)]/(2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.14, size = 42, normalized size = 0.98

method	result	size
default	$\frac{\cos(fx+e)\sin(fx+e)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)), x)
```

Fricas [A]
time = 0.36, size = 53, normalized size = 1.23

$$\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{acf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)/(a*c*f*cos(
f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x)
- 1))), x)
```

Giac [A]
time = 0.55, size = 58, normalized size = 1.35

$$-\frac{2\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}{\sqrt{a}\sqrt{c}\,f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2/(sqrt(a)*sqrt(c)*f*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 8.87, size = 52, normalized size = 1.21

$$-\frac{\sin(2e+2fx)\sqrt{-c(\sin(e+fx)-1)}}{2cf\sqrt{a(\sin(e+fx)+1)}(\sin(e+fx)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] -(sin(2*e + 2*f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(2*c*f*(a*(sin(e + f*x) +
 1))^(1/2)*(sin(e + f*x) - 1))
```

3.48 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)
```

Rubi [A]
time = 0.22, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2920, 2816, 2746, 31}

$$-\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]
```

```
[Out] -((Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac}\\
&= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{c-c\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{\cos(e+fx)\text{Subst}\left(\int \frac{1}{c+x}\,dx, x, -c\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}\\
&= -\frac{\cos(e+fx)\log(1-\sin(e+fx))}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.31, size = 104, normalized size = 1.93

$$
-\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\,(c-c\sin(e+fx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/
2)),x]
```

```
[Out] (-2*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Sqrt[a*(1 + Sin[e + f*x
])]*(c - c*Sin[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(50) = 100$.
time = 0.14, size = 136, normalized size = 2.52

method	result
default	$\frac{(-1+\cos(fx+e)-\sin(fx+e))\left(-2\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{2}{1+\cos(fx+e)}\right)\right)(\cos^2(fx+e)-\cos(fx+e)\sin(fx+e)+\cos(fx+}{2f(-1+\cos(fx+e))\sqrt{a\,(1+\sin\,(fx+e))}\,(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*(-1+cos(f*x+e)-sin(f*x+e))*(-2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+ln(2/(1+cos(f*x+e))))*(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2
*sin(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))
^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3
/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^2*sin(f*x
 + e) - a*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}\,(-c(\sin(e+fx)-1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1)
)**(3/2)), x)
```

Giac [A]
time = 0.55, size = 58, normalized size = 1.07

$$\frac{2 \log\left(\left|\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right|\right)}{\sqrt{a}c^{\frac{3}{2}}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 2*log(abs(sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sqrt(a)*c^(3/2)*f*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

3.49 $\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}$$

```
[Out] cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$\frac{\cos(e+fx)}{cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] Cos[e + f*x]/(c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)}{cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 79, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\sqrt{a(1+\sin(e+fx))}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/
2)),x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]))/(f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/2))
```

Maple [A]
time = 0.16, size = 51, normalized size = 1.21

method	result	size
default	$-\frac{(\sin(fx+e)-1)\cos(fx+e)\sin(fx+e)}{f\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,(-c(\sin(fx+e)-1))^{\frac{5}{2}}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(sin(f*x+e)-1)*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin
(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5
/2)), x)
```

Fricas [A]
time = 0.35, size = 66, normalized size = 1.57

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac^3f\cos\left(fx+e\right)\sin\left(fx+e\right)-ac^3f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^3*f*cos(f*x + e)*s
in(f*x + e) - a*c^3*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1)
)**(5/2)), x)
```

Giac [A]
time = 0.49, size = 58, normalized size = 1.38

$$-\frac{1}{2\sqrt{a}\,c^{\frac{5}{2}}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -1/2/(sqrt(a)*c^(5/2)*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)
```

Mupad [B]
time = 9.77, size = 88, normalized size = 2.10

$$-\frac{2\sqrt{-c\left(\sin\left(e+f\,x\right)-1\right)}\left(4\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+\sin\left(2\,e+2\,f\,x\right)-2\right)}{c^3\,f\,\sqrt{a\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(12\sin\left(e+f\,x\right)^2-15\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(sin(2*e + 2*f*x) + 4*sin(e/2 + (f*x)/2)^
2 - 2))/(c^3*f*(a*(sin(e + f*x) + 1))^(1/2)*(sin(3*e + 3*f*x) - 15*sin(e +
f*x) + 12*sin(e + f*x)^2 + 4))
```

3.50 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{16c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+2/3*c*co
s(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)+1/4*cos(f*x+e)*(
c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(1/2)+16*c^4*cos(f*x+e)*ln(1+sin
(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+8*c^3*cos(f*x+e)
*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$\frac{16c^4\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (16*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*S
qrt[c - c*Sin[e + f*x]]) + (8*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/
2))/(a*f*Sqrt[a + a*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*(c - c*Sin[e + f*x])
^(5/2))/(3*a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x
])^(7/2))/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+}{4af\sqrt{a+a\sin(e+}$$

$$= \frac{2c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c\cos(e+fx)(c-c\sin}{3af\sqrt{a+a\sin(e}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin}{af\sqrt{a+a\sin(e+}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin}{af\sqrt{a+a\sin(e+}$$

$$= \frac{8c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2c^2\cos(e+fx)(c-c\sin}{af\sqrt{a+a\sin(e+}$$

$$= \frac{16c^4\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{8c^3\cos(e+fx)\sqrt{}}{af\sqrt{a+a}}$$

Mathematica [A]
time = 5.05, size = 163, normalized size = 0.68

$$\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}\,\left(276\cos(2(e+fx))-3\cos(4(e+fx))-8\left(384\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-195\sin(e+fx)+5\sin(3(e+fx))\right)\right)}{96f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])^3*Sqrt[c -
 c*Sin[e + f*x]]*(276*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)] - 8*(384*Log[Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]] - 195*Sin[e + f*x] + 5*Sin[3*(e + f*x)])
))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/
2))
```

Maple [A]
time = 0.23, size = 252, normalized size = 1.05

method	result
default	$-\frac{\left(3\left(\cos^4(fx+e)\right)+20\left(\cos^2(fx+e)\right)\sin(fx+e)-72\left(\cos^2(fx+e)\right)+384\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-200\sin(fx+e)-192\ln\left(\frac{2}{1+\cos(fx\right.\right.}{12f\left((\cos^3(fx+e))\sin(fx+e)-(\cos^4(fx+e))-4(\cos^2(fx+e))\sin(fx+e)-3(\cos^3(fx+e))-4\cos(fx+e)\sin(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/12/f*(3*cos(f*x+e)^4+20*cos(f*x+e)^2*sin(f*x+e)-72*cos(f*x+e)^2+384*ln((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-200*sin(f*x+e)-192*ln(2/(1+cos(f*x+e))
)+69)*(-c*(sin(f*x+e)-1))^(7/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f
*x+e)+cos(f*x+e)-2)/(cos(f*x+e)^3*sin(f*x+e)-cos(f*x+e)^4-4*cos(f*x+e)^2*si
n(f*x+e)-3*cos(f*x+e)^3-4*cos(f*x+e)*sin(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e)
+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 226, normalized size = 0.95

$$-\frac{2 \sqrt{2} \sqrt{a} c^{\frac{7}{2}} \left(\frac{12 \sqrt{2} \log\left(-\sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{2}+1\right)}{a^{2} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}+\frac{3 \sqrt{2} a^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{8}+4 \sqrt{2} a^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{6}+6 \sqrt{2} a^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{4}+12 \sqrt{2} a^{6} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right) \sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)^{2}}{a^{8}}\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4} \pi+\frac{1}{2} f x+\frac{1}{2} e\right)\right)}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -2/3*sqrt(2)*sqrt(a)*c^(7/2)*(12*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e
)^2 + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + (3*sqrt(2)*a^6*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^8 + 4*sqrt(2)*a
^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 6
*sqrt(2)*a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^4 + 12*sqrt(2)*a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^2)/a^8)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.51 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{8c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{c\cos(e+fx)(c-c\sin(e$$

```
[Out] c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+1/3*cos(f*x+
e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(1/2)+8*c^3*cos(f*x+e)*ln(1+
sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*c^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$\frac{8c^3\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (8*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (4*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*
f*Sqrt[a + a*Sin[e + f*x]]) + (c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(
a*f*Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(
3*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+f}{3af\sqrt{a+a\sin(e+fx}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+}{af\sqrt{a+a\sin(e+f}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+}{af\sqrt{a+a\sin(e+f}$$

$$= \frac{4c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{c\cos(e+fx)(c-c\sin(e+}{af\sqrt{a+a\sin(e+f}$$

$$= \frac{8c^3\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{4c^2\cos(e+fx)\sqrt{}}{af\sqrt{a+a\mathrm{s}}}$$

Mathematica [A]
time = 1.57, size = 138, normalized size = 0.73

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\left(-12\cos(2(e+fx))+192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)-87\sin(e+fx)+\sin(3(e+fx))\right)}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-12*
Cos[2*(e + f*x)] + 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 87*Sin[e
+ f*x] + Sin[3*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*
(1 + Sin[e + f*x]))^(3/2))
```

Maple [A]
time = 0.19, size = 213, normalized size = 1.12

method	result

default	$\frac{\left((\cos^2(fx+e))\sin(fx+e)-6(\cos^2(fx+e))+48\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-22\sin(fx+e)-24\ln\left(\frac{2}{1+\cos(fx+e)}\right)+6\right)(-c(\sin(fx+e)}{3f((\cos^2(fx+e))\sin(fx+e)+\cos^3(fx+e)+2\cos(fx+e)\sin(fx+e)-3(\cos^2(fx+e))-4\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(cos(f*x+e)^2*sin(f*x+e)-6*cos(f*x+e)^2+48*ln((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))-22*sin(f*x+e)-24*ln(2/(1+cos(f*x+e)))+6)*(-c*(sin(f*x+e)-1))
^(5/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(
f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)-3*cos(f*x+e)^2-4*s
in(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f
*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.56, size = 157, normalized size = 0.83

$$-\frac{2\,\sqrt{2}\,\sqrt{a}\,c^{\frac{5}{2}}\left(\frac{6\,\sqrt{2}\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{\sqrt{2}\,\left(2\,a^{4}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+3\,a^{4}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+6\,a^{4}\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\right)}{a^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -2/3*sqrt(2)*sqrt(a)*c^(5/2)*(6*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(2)*(2*a^4*sin(-1/4
*pi + 1/2*f*x + 1/2*e)^6 + 3*a^4*sin(-1/4*pi + 1/2*f*x + 1/2*e)^4 + 6*a^4*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^6*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.52 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}+\frac{\cos(e+fx)(c-c\sin(e-}{2af\sqrt{a+a\sin(e+}$$

```
[Out] 1/2*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(1/2)+4*c^2*cos(
f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2
*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(3/2),
x]
```

```
[Out] (4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]]) + (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*
Sqrt[a + a*Sin[e + f*x]]) + (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*a*
f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+f}{2af\sqrt{a+a\sin(e+fx}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+f}{2af\sqrt{a+a\sin(e+fx}$$

$$= \frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{\cos(e+fx)(c-c\sin(e+f}{2af\sqrt{a+a\sin(e+fx}$$

$$= \frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c\cos(e+fx)\sqrt{c}}{af\sqrt{a+a\,\mathrm{si}}}$$

Mathematica [A]

time = 0.73, size = 134, normalized size = 0.92

$$\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-1+\sin(e+fx))\left(\cos(2(e+fx))-32\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+12\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*(Cos[2*(e +
f*x)] - 32*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 12*Sin[e + f*x])*Sqrt
[c - c*Sin[e + f*x]])/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
Sin[e + f*x]))^(3/2))
```

Maple [A]
time = 0.17, size = 172, normalized size = 1.19

method	result
default	$-\frac{\left(\cos^2(fx+e)+8\ln\left(\frac{2}{1+\cos(fx+e)}\right)-16\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+6\sin(fx+e)-1\right)\left(-c(\sin(fx+e)-1)\right)^{\frac{3}{2}}(\cos(fx+e)\sin(fx+e)-}{2f(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2\sin(fx+e)-\cos(fx+e)+2)(a(1+\sin(fx+e)))}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/f*(cos(f*x+e)^2+8*ln(2/(1+cos(f*x+e)))-16*ln((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))+6*sin(f*x+e)-1)*(-c*(sin(f*x+e)-1))^(3/2)*(cos(f*x+e)*sin(f*x+
e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(f*x+e)*sin(f*x+e)-cos(f*x+e
)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(141) = 282$.
time = 0.58, size = 914, normalized size = 6.30

$$-\frac{\frac{16c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{3}{2}}}-\frac{8c^{\frac{3}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{3}{2}}}-\frac{10c^{\frac{3}{2}}+\frac{13c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{25c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{15c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{9c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{a^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}+\frac{10c^{\frac{3}{2}}+\frac{11c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{15c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{20c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{7c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{a^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}-\frac{2\left(\frac{5c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{8c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{5c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)}{a^{\frac{3}{2}}+\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{a^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] -1/2*(16*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 8*c^(3/
2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - (10*c^(3/2) + 13*
c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 25*c^(3/2)*sin(f*x + e)^2/(cos(f*
x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*c^(3/2)
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 9*c^(3/2)*sin(f*x + e)^5/(cos(f*x +
```

```
e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 2*a^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^
6) + (10*c^(3/2) + 11*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^(3/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 20*c^(3/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7*c^(3/2)*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(cos(f*x
+ e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2)*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1
)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*x + e)^
6/(cos(f*x + e) + 1)^6) - 2*(5*c^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5*
c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*c^(3/2)*sin(f*x + e)^3/(cos
(f*x + e) + 1)^3 + 5*c^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*c^(3/2
)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(3/2) + 2*a^(3/2)*sin(f*x + e)/(c
os(f*x + e) + 1) + 3*a^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^(3/2
)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 + 2*a^(3/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(3/2)*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e
) - 2*a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e+fx)-1))^{\frac{3}{2}}\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*cos(e + f*x)**2/(a*(sin(e + f*x) +
1))**(3/2), x)
```

Giac [A]
time = 0.52, size = 132, normalized size = 0.91

$$-\frac{2\left(\sqrt{a}\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-4\,\sqrt{a}\,c\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+4\,\sqrt{a}\,c\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{c}}{a^{2}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] -2*(sqrt(a)*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f*x +
1/2*e)) - 4*sqrt(a)*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 4*sqrt(a)*c*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.53 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{2c\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}}$$

```
[Out] 2*c*cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2819, 2816, 2746, 31}

$$\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a\sin(e+fx)+a}}+\frac{2c\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (2*c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*Sqrt[a
 + a*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{2\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{(2c\cos(e+fx))\int \frac{\cos(e+}{a+a\sin(}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-cs}}$$

$$= \frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af\sqrt{a+a\sin(e+fx)}} + \frac{(2c\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\right.}{af\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

$$= \frac{2c\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\cos(e+fx)\sqrt{c-}}{af\sqrt{a+a\sin}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.73, size = 121, normalized size = 1.26

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(2\log\left(e^{i(e+fx)}\right)-4\log\left(i+e^{i(e+fx)}\right)+\sin(e+fx)\right)\sqrt{c-c\sin(e+fx)}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3/2),x]

[Out] -(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(2*Log[E^(I*(e + f*x))] - 4*Log[I + E^(I*(e + f*x))] + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)))

Maple [A]
time = 0.18, size = 133, normalized size = 1.39

method	result
default	$\frac{\left(2\ln\left(\frac{2}{1+\cos(fx+e)}\right)-4\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\sin(fx+e)\right)\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(\cos(fx+e)\sin(fx+e)+\cos^{2}(fx+e\right.}{f(-1+\cos(fx+e)+\sin(fx+e))(a(1+\sin(fx+e)))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/f*(2*ln(2/(1+cos(f*x+e)))-4*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\,\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/(a*(sin(e + f*x) + 1))
**(3/2), x)
```

Giac [A]
time = 0.52, size = 103, normalized size = 1.07

$$\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{a}\,\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right))-2\,\sqrt{2}\,\sqrt{a}\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\mathrm{sgn}(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right))\right)\sqrt{c}}{a^{2}f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2)*sqrt(a)*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e)) - 2*sqrt(2)*sqrt(a)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*
e)))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(c)/(a^2*f*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2),
 x)
```

3.54 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*ln(1+sin(f*x+e))/a/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.24, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2920, 2816, 2746, 31}

$$\frac{\cos(e+fx)\log(\sin(e+fx)+1)}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]
```

```
[Out] (Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{ac}\\
&= \frac{\cos(e+fx)\int \frac{\cos(e+fx)}{a+a\sin(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\mathrm{Subst}\left(\int \frac{1}{a+x}\,dx,x,a\sin(e+fx)\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\cos(e+fx)\log(1+\sin(e+fx))}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 102, normalized size = 2.00

$$\frac{2\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] (2*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(f*(a*(1 + Sin[e + f*x]))^(
3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(47) = 94$.
time = 0.14, size = 134, normalized size = 2.63

method	result
default	$-\frac{(-1+\cos(fx+e)+\sin(fx+e))\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\right)(\cos(fx+e)\sin(fx+e)+\cos^2(fx+e)-2\sin(fx+e}{2f(-1+\cos(fx+e))(a(1+\sin(fx+e)))^{\frac{3}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/f*(-1+cos(f*x+e)+sin(f*x+e))*(ln(2/(1+cos(f*x+e)))-2*ln((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e)))*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+c
os(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*c*sin(f*x
+ e) + a^2*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*
x) - 1))), x)
```

Giac [A]
time = 0.71, size = 58, normalized size = 1.14

$$-\frac{2\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{\frac{3}{2}}\sqrt{c}\,f\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] -2*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(3/2)*sqrt(c)*f*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),
 x)
```

3.55 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{acf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{acf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}\\
&= \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\
&= \frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{acf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 103, normalized size = 1.98

$$
\frac{\cos^3(e+fx)\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]))/(c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(48) = 96.
time = 0.39, size = 173, normalized size = 3.33

method	result
default	$\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\right)\left(\cos(fx+e)\sin(fx+e)-\left(\cos^2(fx+e)\right)-2\sin(fx+e)-\cos(fx+e)+2\right)}{2f(-1+\cos(fx+e))(a(1+\sin(fx+e)))^{\frac{3}{2}}(-c(\sin(fx+e)-1))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/2/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-ln((1-cos(f*x+e)+sin(f*x+
e))/sin(f*x+e)))*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e
)+2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos
(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Fricas [A]
time = 0.38, size = 170, normalized size = 3.27

$$\left[\frac{\sqrt{ac}\log\left(-\frac{ac\cos\left(fx+e\right)^{3}-2\,ac\cos\left(fx+e\right)-2\,\sqrt{ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{3}}\right)}{2\,a^{2}c^{2}f},-\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos\left(fx+e\right)\sin\left(fx+e\right)}\right)}{a^{2}c^{2}f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/2*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x +
e)^3)/(a^2*c^2*f), -sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*s
qrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))/(a^2*c^2*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) -
 1))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(52) = 104.
time = 0.50, size = 126, normalized size = 2.42

$$\frac{\sqrt{a}\,\sqrt{c}\left(\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{2}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{2\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{2}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/2*sqrt(a)*sqrt(c)*(log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^2*c^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*
log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*c^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(e+fx)^2}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

3.56 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=104

$$\frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2ac^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arct
anh(sin(f*x+e))*cos(f*x+e)/a/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2920, 2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2acf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] Cos[e + f*x]/(2*a*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
+ (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a*c^2*f*Sqrt[a + a*Sin[e + f*x]]*
Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\int -\ldots}{\sqrt{\ldots}}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\ldots}{2ac^2\ldots}$$

$$= \frac{\cos(e+fx)}{2acf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} + \frac{\ldots}{2ac^2\ldots}$$

Mathematica [A]
time = 0.54, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(1-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2c^2f(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 - Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c^2*f*(-1 + Si
n[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(92) = 184.
time = 0.15, size = 245, normalized size = 2.36

method	result
default	$-\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+\ln\left(\frac{1-\cos(fx-}{\sin}\right.\right.}{4f(-1+\cos(fx}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/4/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+sin(f*x+e))*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(5/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)

Fricas [A]
time = 0.42, size = 345, normalized size = 3.32

$$\left[\frac{\sqrt{ac}\,(\cos(fx+e)\sin(fx+e)-\cos(fx+e))\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\,\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)-2\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{4\,(a^2c^3f\cos(fx+e)\sin(fx+e)-a^2c^3f\cos(fx+e))},\,-\frac{\sqrt{-ac}\,(\cos(fx+e)\sin(fx+e)-\cos(fx+e))\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{2\,(a^2c^3f\cos(fx+e)\sin(fx+e)-a^2c^3f\cos(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) - cos(f*x + e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)*sin(f*x + e) - a^2*c^3*f*cos(f*x + e))]

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.57, size = 181, normalized size = 1.74

$$\frac{\sqrt{c}\left(\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{\frac{3}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{2\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{\frac{3}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{1}{\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{\frac{3}{2}}c^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 1/4*sqrt(c)*(log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^(3/2)*c^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*log(
abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^(3/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 1/((cos(-1/4*pi + 1/2*f*x
 + 1/2*e)^2 - 1)*a^(3/2)*c^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),
 x)
```

3.57 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=285

$$-\frac{80c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}-\frac{10c^3\cos(e+fx)(c-}{a^2f\sqrt{a+a\sin}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/a/f/(a+a*sin(f*x+e))^(3/2)-10*c^3*cos(f*
x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-10/3*c^2*cos(f*x+e
)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-5/4*c*cos(f*x+e)*(c-c
*sin(f*x+e))^(7/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-80*c^5*cos(f*x+e)*ln(1+sin(
f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-40*c^4*cos(f*x+
e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{80c^5\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}-\frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-80*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (40*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^3*Cos[e + f*x]*(c - c*Sin[e + f*
x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (10*c^2*Cos[e + f*x]*(c - c*S
in[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (5*c*Cos[e + f*x]*
(c - c*Sin[e + f*x])^(7/2))/(4*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f
*x]*(c - c*Sin[e + f*x])^(9/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{11/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \frac{(c-c\sin(e+fx))^{9/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))^{9/2}}{af(a+a\sin(e+fx))^{3/2}}$$

$$= -\frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{5c\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{4a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^2\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{10c^3\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{80c^5\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{40c^4\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [A]
time = 6.46, size = 553, normalized size = 1.94

$$-\frac{32\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{9/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}+\frac{47\cos(2(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}-\frac{\cos(4(e+fx))\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}{32f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}-\frac{160\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}+\frac{203\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sin(e+fx)(c-c\sin(e+fx))^{9/2}}{4f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}-\frac{7\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}\sin(3(e+fx))}{12f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2))/(f
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (4
7*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f
*x])^(9/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*
x]))^(5/2)) - (Cos[4*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c
```

- c*Sin[e + f*x])^(9/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - (160*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (203*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - (7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)])/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [A]
time = 0.18, size = 347, normalized size = 1.22

method	result
default	$-\frac{\left(-3\left(\cos^4(fx+e)\right)\sin(fx+e)+25\left(\cos^4(fx+e)\right)+116\left(\cos^2(fx+e)\right)\sin(fx+e)+960\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-1920\ln\left(\frac{1-\cos(f}{s}\right.\right.}{12f\left(\left(\cos^4(fx+e)\right)\sin(fx+e)+\cos^5(fx+e)+4\left(\cos^3(fx+e)\right)\sin(fx+e)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/12/f*(-3*cos(f*x+e)^4*sin(f*x+e)+25*cos(f*x+e)^4+116*cos(f*x+e)^2*sin(f*x+e)+960*ln(2/(1+cos(f*x+e)))*sin(f*x+e)-1920*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-500*cos(f*x+e)^2+960*ln(2/(1+cos(f*x+e)))-1920*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+859*sin(f*x+e)+475)*(-c*(sin(f*x+e)-1))^(9/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(f*x+e)^4*sin(f*x+e)+cos(f*x+e)^5+4*cos(f*x+e)^3*sin(f*x+e)-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*cos(f*x+e)*sin(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x+e)))^(5/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((-c*sin(f*x + e) + c)^(9/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral(-(c^4*cos(f*x + e)^6 - 8*c^4*cos(f*x + e)^4 + 8*c^4*cos(f*x + e)^2
 + 4*(c^4*cos(f*x + e)^4 - 2*c^4*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f
*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3
*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 269, normalized size = 0.94

$$\frac{2\,\sqrt{2}\,\sqrt{a}\,c^{\frac{9}{2}}\left(\frac{60\,\sqrt{2}\,\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{12\,\sqrt{2}}{\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}-1\right)a^{3}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{3\,\sqrt{2}\,a^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{8}+8\,\sqrt{2}\,a^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{6}+18\,\sqrt{2}\,a^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{4}+48\,\sqrt{2}\,a^{9}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}}{a^{12}}\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 2/3*sqrt(2)*sqrt(a)*c^(9/2)*(60*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 12*sqrt(2)/((sin(-1/4*p
i + 1/2*f*x + 1/2*e)^2 - 1)*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + (3*s
qrt(2)*a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^8 + 8*sqrt(2)*a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e)^6 + 18*sqrt(2)*a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^4 + 48*sqrt(2)*a^9*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/a^12)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2),
x)

[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.58 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{32c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}-\frac{4c^2\cos(e+fx)(c-c}{a^2f\sqrt{a+a\sin}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2)-4*c^2*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-4/3*c*cos(f*x+e)*(c
-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-32*c^4*cos(f*x+e)*ln(1+si
n(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-16*c^3*cos(f*
x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{32c^4\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{4c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-32*c^4*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (16*c^3*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]
)/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x
])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (4*c*Cos[e + f*x]*(c - c*Sin[e
 + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*S
in[e + f*x])^(7/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac} \\
&= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{7/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{4\int \frac{(c-c\sin(e+fx))^{7/2}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} \\
&= -\frac{4c\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{3a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e+fx))}{af(a+a\sin(e+fx)} \\
&= -\frac{4c^2\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c\cos(e+fx)(c-c\sin}{3a^2f\sqrt{a+a\sin(e}} \\
&= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin}{a^2f\sqrt{a+a\sin(e}} \\
&= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin}{a^2f\sqrt{a+a\sin(e}} \\
&= -\frac{16c^3\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{4c^2\cos(e+fx)(c-c\sin}{a^2f\sqrt{a+a\sin(e}} \\
&= -\frac{32c^4\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{16c^3\cos(e+fx}{a^2f\sqrt{a+}}
\end{aligned}
$$

Mathematica [A]
time = 3.22, size = 179, normalized size = 0.76

$$
\frac{c^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\left(-177-172\cos(2(e+fx))+\cos(4(e+fx))-1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+396\sin(e+fx)-1536\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin(e+fx)+16\sin(3(e+fx))\right)}{24f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-177
 - 172*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 1536*Log[Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + 396*Sin[e + f*x] - 1536*Log[Cos[(e + f*x)/2] + Sin[(e + f
*x)/2]]*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(24*f*(Cos[(e + f*x)/2] - Sin[
(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A]
time = 0.23, size = 307, normalized size = 1.30

method	result
default	$\frac{\left(-(\cos^4(fx+e))-8(\cos^2(fx+e))\sin(fx+e)+192\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-96\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+44(\cos^2(}{3f((\cos^3(fx+e))\sin(fx+e)-(\cos^4(fx+e))-4(\cos^2(fx+e))\sin(fx+e)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(-cos(f*x+e)^4-8*cos(f*x+e)^2*sin(f*x+e)+192*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))*sin(f*x+e)-96*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+44*cos(f*x+e
)^2+192*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-91*sin(f*x+e)-96*ln(2/(1+c
os(f*x+e)))-43)*(-c*(sin(f*x+e)-1))^(7/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)
^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(f*x+e)^3*sin(f*x+e)-cos(f*x+e)^4-4*cos(f
*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*cos(f*x+e)*sin(f*x+e)+8*cos(f*x+e)^2+8*
sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(7/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^4
 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin
(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)
*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 199, normalized size = 0.84

$$\frac{4 \sqrt{2} \sqrt{a} c^{\frac{7}{2}} \left(\frac{12 \sqrt{2} \log\left(-\sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{2}+1\right)}{a^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)} - \frac{3 \sqrt{2}}{\left(\sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{2}-1\right) a^{3} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)} + \frac{\sqrt{2} \left(a^{6} \sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{6}+3 a^{6} \sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{4}+9 a^{6} \sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)^{2}\right)}{a^{9} \mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)}\right) \mathrm{sgn}\left(\sin\left(-\frac{1}{4} \pi+\frac{1}{2} fx+\frac{1}{2} e\right)\right)}{3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 4/3*sqrt(2)*sqrt(a)*c^(7/2)*(12*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/2*e)
^2 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 3*sqrt(2)/((sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^2 - 1)*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + sqrt(
2)*(a^6*sin(-1/4*pi + 1/2*f*x + 1/2*e)^6 + 3*a^6*sin(-1/4*pi + 1/2*f*x + 1/
2*e)^4 + 9*a^6*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2)/(a^9*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.59 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{12c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}-\frac{3c\cos(e+fx)(c-c}{2a^2f\sqrt{a+a\sin}}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/f/(a+a*sin(f*x+e))^(3/2)-3/2*c*cos(f*x
+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)-12*c^3*cos(f*x+e)*l
n(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6*c^2*c
os(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{12c^3\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-12*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]
]*Sqrt[c - c*Sin[e + f*x]]) - (6*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])
/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (3*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^
(3/2))/(2*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*
x])^(5/2))/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
```

```
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{5/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{3\int \frac{(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}}{a^2}$$

$$= -\frac{3c\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{2a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e}{af(a+a\sin(e+f}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin}{2a^2f\sqrt{a+a\sin(e}}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin}{2a^2f\sqrt{a+a\sin(e}}$$

$$= -\frac{6c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{3c\cos(e+fx)(c-c\sin}{2a^2f\sqrt{a+a\sin(e}}$$

$$= -\frac{12c^3\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{6c^2\cos(e+f}{a^2f\sqrt{a}}$$

Mathematica [A]
time = 1.59, size = 164, normalized size = 0.86

$$\frac{c^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\left(-44-18\cos(2(e+fx))-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(39-192\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)+\sin(3(e+fx))\right)}{8f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(-44
- 18*Cos[2*(e + f*x)] - 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (39
- 192*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x] + Sin[3*(e + f
*x)]))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5
/2))
```

Maple [A]
time = 0.21, size = 269, normalized size = 1.41

method	result

default	$-\frac{\left(-\left(\cos^2(fx+e)\right)\sin(fx+e)+48\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-24\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+9\left(\cos^2(fx+e)\right)+48\ln\left(\frac{1-}{}\right.\right.}{2f\left(\left(\cos^2(fx+e)\right)\sin(fx+e)+\cos^3(fx+e)+2\cos(fx+e)\sin\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/2/f*(-cos(f*x+e)^2*sin(f*x+e)+48*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)*sin(f*x+e)-24*ln(2/(1+cos(f*x+e)))*sin(f*x+e)+9*cos(f*x+e)^2+48*ln((1-cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))-25*sin(f*x+e)-24*ln(2/(1+cos(f*x+e)))-9)*(-
c*(sin(f*x+e)-1))^(5/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+co
s(f*x+e)-2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)-3
*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. $2(186) = 372$.
time = 0.66, size = 1214, normalized size = 6.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] 1/6*(144*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 72*c^(5
/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - (46*c^(5/2) + 19
9*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 335*c^(5/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 509*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 496*c^
(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 373*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 219*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*c^(
5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)
/(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^
(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(
f*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2
)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
 e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) + (46*c^(5/2) + 1
21*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 149*c^(5/2)*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 179*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 148*c
^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 43*c^(5/2)*sin(f*x + e)^5/(cos
(f*x + e) + 1)^5 + 33*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 15*c^(5
/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/
(cos(f*x + e) + 1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(
5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f
*x + e) + 1)^4 + 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)
```

```
*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x +
e) + 1)^7 + a^(5/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8) - 6*(13*c^(5/2)*si
n(f*x + e)/(cos(f*x + e) + 1) + 39*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 55*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 74*c^(5/2)*sin(f*x +
 e)^4/(cos(f*x + e) + 1)^4 + 55*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
 + 39*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 13*c^(5/2)*sin(f*x + e)
^7/(cos(f*x + e) + 1)^7)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 8*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 12*a^(5/2)*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 14*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 +
 12*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 8*a^(5/2)*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 + 4*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + a^(5
/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8))/f
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*co
s(f*x + e)^2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos
(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.51, size = 175, normalized size = 0.92

$$\frac{2\left(\sqrt{a}\,c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-6\,\sqrt{a}\,c^2\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+12\,\sqrt{a}\,c^2\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+\frac{2\,\sqrt{a}\,c^2\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}\right)\sqrt{c}}{a^3f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] 2*(sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^4*sgn(sin(-1/4*pi + 1/2*f*x +
 1/2*e)) - 6*sqrt(a)*c^2*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*pi +
```

```
 1/2*f*x + 1/2*e)) + 12*sqrt(a)*c^2*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e))
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*sqrt(a)*c^2*sgn(sin(-1/4*pi + 1/2
*f*x + 1/2*e))/cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^3*f*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.60 $\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}}-\frac{\cos(e+fx)(c-c\sin(}{af(a+a\sin(e+}$$

```
[Out] -cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a/f/(a+a*sin(f*x+e))^(3/2)-4*c^2*cos(f*x
+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2*
c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2920, 2818, 2819, 2816, 2746, 31}

$$-\frac{4c^2\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a\sin(e+fx)+a}}-\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(5/2),
x]
```

```
[Out] (-4*c^2*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]
*Sqrt[c - c*Sin[e + f*x]]) - (2*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a
^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/
(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)(c-c\sin(e+fx))^{3/2}}{af(a+a\sin(e+fx))^{3/2}} - \frac{2\int \frac{(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e}{af(a+a\sin(e+f}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e}{af(a+a\sin(e+f}$$

$$= -\frac{2c\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cos(e+fx)(c-c\sin(e}{af(a+a\sin(e+f}$$

$$= -\frac{4c^2\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2c\cos(e+fx}{a^2f\sqrt{a}}$$

Mathematica [A]
time = 0.74, size = 153, normalized size = 1.07

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}\left(7+\cos(2(e+fx))+16\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+2\left(-1+8\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]]*(7
 + Cos[2*(e + f*x)] + 16*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(-1 +
 8*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(f*(Cos[(e + f*
x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A]
time = 0.17, size = 227, normalized size = 1.59

method	result
default	$-\frac{\left(8\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-4\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)+\cos^2(fx+e)+8\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-5\sin(fx+}{f(\cos(fx+e)\sin(fx+e)-(\cos^2(fx+e))-2\sin(fx+e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(8*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-4*ln(2/(1+cos(f
*x+e)))*sin(f*x+e)+cos(f*x+e)^2+8*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
5*sin(f*x+e)-4*ln(2/(1+cos(f*x+e)))-1)*(-c*(sin(f*x+e)-1))^(3/2)*(cos(f*x+e
)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(cos(f*x+e)*sin(f*x+e)
-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] integral((c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*sqrt(a*sin(f*x
+ e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*co
s(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c(\sin{\left(e+fx \right)}-1)\right)^{\frac{3}{2}}\cos^{2}{\left(e+fx \right)}}{\left(a\left(\sin{\left(e+fx \right)}+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*cos(e + f*x)**2/(a*(sin(e + f*x) +
1))**(5/2), x)
```

Giac [A]
time = 0.56, size = 144, normalized size = 1.01

$$-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{a}\,c\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-4\sqrt{2}\sqrt{a}\,c\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)-\frac{\sqrt{2}\sqrt{a}\,c\,\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^{2}}\right)\sqrt{c}}{a^{3}f\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] -sqrt(2)*(sqrt(2)*sqrt(a)*c*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 4*sqrt(2)*sqrt(a)*c*log(abs(cos(-1/4*pi + 1/2*f*x +
 1/2*e)))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - sqrt(2)*sqrt(a)*c*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))/cos(-1/4*pi + 1/2*f*x + 1/2*e)^2)*sqrt(c)/(a^3*f
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+fx)^2\,(c-c\,\sin(e+fx))^{3/2}}{(a+a\,\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.61 $\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}}$$

```
[Out] -c*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.31, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2818, 2816, 2746, 31}

$$-\frac{c\cos(e+fx)\log(\sin(e+fx)+1)}{a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -((c*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])) - (Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*f*(a
+ a*Sin[e + f*x])^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2816

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.)
+ (f_.)*(x_)]], x_Symbol] :> Dist[a*c*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x
]]*Sqrt[c + d*Sin[e + f*x]])), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2818

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(2*n + 1))), x] - Dist[b*((2*m - 1)/(d*(
2*n + 1))), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] &&  !(ILtQ[m + n, 0] && GtQ[2*m + n
 + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\int \frac{\cos(}{a+a\sin}}{a\sqrt{a+a\sin(e+fx)}\sqrt{c-}}$$

$$= -\frac{\cos(e+fx)\sqrt{c-c\sin(e+fx)}}{af(a+a\sin(e+fx))^{3/2}} - \frac{(c\cos(e+fx))\text{Subst}\left(\int \frac{1}{a+x}\right.}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{}}$$

$$= -\frac{c\cos(e+fx)\log(1+\sin(e+fx))}{a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{\cos(e+fx)\sqrt{c}}{af(a+a\sin}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 113, normalized size = 1.16

$$\frac{\sec(e+fx)\sqrt{c-c\sin(e+fx)}\left(\log\left(e^{i(e+fx)}\right)-2\left(1+\log\left(i+e^{i(e+fx)}\right)\right)+\left(\log\left(e^{i(e+fx)}\right)-2\log\left(i+e^{i(e+fx)}\right)\right)\sin(e+fx)\right)}{a^2f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(5
/2),x]
```

[Out] (Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]]*(Log[E^(I*(e + f*x))] - 2*(1 + Log[I + E^(I*(e + f*x))]) + (Log[E^(I*(e + f*x))] - 2*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(a^2*f*Sqrt[a*(1 + Sin[e + f*x])])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(89) = 178$.
time = 0.16, size = 188, normalized size = 1.94

method	result
default	$-\frac{\left(\ln\left(\frac{2}{1+\cos(fx+e)}\right)\sin(fx+e)-2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)+\ln\left(\frac{2}{1+\cos(fx+e)}\right)-2\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)+2\sin(fx}{f(-1+\cos(fx+e)+\sin(fx+e))(a(1+\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/f*(ln(2/(1+cos(f*x+e)))*sin(f*x+e)-2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+ln(2/(1+cos(f*x+e)))-2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate(sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] integral(-sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*cos(f*x + e)^2/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e+fx)-1)}\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)**2/(a*(sin(e + f*x) + 1))
**(5/2), x)
```

Giac [**A**]
time = 0.57, size = 94, normalized size = 0.97

$$\frac{\left(2\sqrt{a}\log\left(\left|\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+\frac{\sqrt{a}\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2}\right)\sqrt{c}}{a^3f\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] (2*sqrt(a)*log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(sin(-1/4*pi + 1/2*f
*x + 1/2*e)) + sqrt(a)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))/cos(-1/4*pi + 1/
2*f*x + 1/2*e)^2)*sqrt(c)/(a^3*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^2\,\sqrt{c-c\,\sin(e+f\,x)}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(5/2),
x)
```

```
[Out] int((cos(e + f*x)^2*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(5/2),
 x)
```

3.62 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -cos(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2817}

$$-\frac{\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]
[Out] -(Cos[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int \frac{\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx}{ac}\\ &= -\frac{\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 80, normalized size = 1.86

$$-\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{f(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3)/(f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]]))
```

Maple [A]
time = 0.13, size = 50, normalized size = 1.16

method	result	size
default	$\frac{(1+\sin(fx+e))\cos(fx+e)\sin(fx+e)}{f(a(1+\sin(fx+e)))^{\frac{5}{2}}\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(1+sin(f*x+e))*cos(f*x+e)*sin(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(
f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) +
 c)), x)
```

Fricas [A]
time = 0.34, size = 65, normalized size = 1.51

$$-\frac{\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{a^3cf\cos\left(fx+e\right)\sin\left(fx+e\right)+a^3cf\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] -sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*c*f*cos(f*x + e)*s
in(f*x + e) + a^3*c*f*cos(f*x + e))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e + f*
x) - 1))), x)
```

Giac [A]
time = 0.56, size = 58, normalized size = 1.35

$$\frac{1}{2\,a^{\frac{5}{2}}\sqrt{c}\,f\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] 1/2/(a^(5/2)*sqrt(c)*f*cos(-1/4*pi + 1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))
```

Mupad [B]
time = 9.26, size = 55, normalized size = 1.28

$$-\frac{2\,\cos(e+f\,x)\,\sqrt{-c\,(\sin(e+f\,x)-1)}}{a^2\,c\,f\,(\cos(2\,e+2\,f\,x)+1)\,\sqrt{a\,(\sin(e+f\,x)+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)),
x)
```

```
[Out] -(2*cos(e + f*x)*(-c*(sin(e + f*x) - 1))^(1/2))/(a^2*c*f*(cos(2*e + 2*f*x)
+ 1)*(a*(sin(e + f*x) + 1))^(1/2))
```

3.63 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2a^2cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/2*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)+1/2*arc
tanh(sin(f*x+e))*cos(f*x+e)/a^2/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2920, 2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2cf\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)),
x]
```

```
[Out] -1/2*Cos[e + f*x]/(a*c*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]
]) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x
]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx}{ac}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int -}{\sqrt{}}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{2a^2c}$$

$$= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{2a^2c}$$

Mathematica [**A**]
time = 0.56, size = 163, normalized size = 1.57

$$\frac{\cos^3(e+fx)\left(1+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\left(\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin(e+fx)\right)}{2cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (Cos[e + f*x]^3*(1 + Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2]] + (Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] -
 Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]))/(2*c*f*(-1 + Sin[
e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(92) = 184.
time = 0.14, size = 247, normalized size = 2.38

method	result
default	$\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)-\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)\sin(fx+e)+\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)-\ln\left(\frac{1-\cos(fx+}{\sin(}\right.\right.}{4f(-1+\cos(fx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)-ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)+ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))-ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-sin(f*x+e))*(cos(f*x+e)*sin(
f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(cos(f*x+e)*sin(f*x+e)+cos(f
*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)
/(-c*(sin(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(3/2)), x)
```

Fricas [A]
time = 0.41, size = 339, normalized size = 3.26

$$\left[\frac{\sqrt{ac}\,(\cos(fx+e)\sin(fx+e)+\cos(fx+e))\log\left(-\frac{ac\cos(fx+e)^3-2\,ac\cos(fx+e)-2\sqrt{ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{\cos(fx+e)^3}\right)-2\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{4\,(a^3c^2f\cos(fx+e)\sin(fx+e)+a^3c^2f\cos(fx+e))},\ -\frac{\sqrt{-ac}\,(\cos(fx+e)\sin(fx+e)+\cos(fx+e))\arctan\left(\frac{\sqrt{-ac}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)+\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{2\,(a^3c^2f\cos(fx+e)\sin(fx+e)+a^3c^2f\cos(fx+e))}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*log(-(a*c*cos(f*
x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) +
 a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c
^2*f*cos(f*x + e)), -1/2*(sqrt(-a*c)*(cos(f*x + e)*sin(f*x + e) + cos(f*x +
 e))*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(
a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c))/(a^3*c^2*f*cos(f*x + e)*sin(f*x + e) + a^3*c^2*f*cos(f*x + e))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.68, size = 180, normalized size = 1.73

$$\frac{\sqrt{a}\,\sqrt{c}\left(\frac{\log\left(-\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}+1\right)}{a^{3}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{2\,\log\left(\left|\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right|\right)}{a^{3}c^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}+\frac{1}{a^{3}c^{2}\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, alg
orithm="giac")
```

```
[Out] 1/4*sqrt(a)*sqrt(c)*(log(-cos(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)/(a^3*c^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*
log(abs(cos(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*c^2*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))) + 1/(a^3*c^2*cos(-1/4*pi +
1/2*f*x + 1/2*e)^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),
 x)
```

3.64 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{}{2a^2c^2}$$

```
[Out] -1/2*cos(f*x+e)/a/c/f/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+1/2*cos
(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*arctanh(s
in(f*x+e))*cos(f*x+e)/a^2/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)
```

Rubi [A]
time = 0.37, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2920, 2822, 2820, 3855}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{\cos(e+fx)}{2a^2cf\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{\cos(e+fx)}{2acf(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] -1/2*Cos[e + f*x]/(a*c*f*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3
/2)) + Cos[e + f*x]/(2*a^2*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x]
)^(3/2)) + (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*a^2*c^2*f*Sqrt[a + a*Sin
[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2820

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_
.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]
]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c,
d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{1}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx}{ac} \\
&= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \cdots \\
&= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \cdots \\
&= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \cdots \\
&= -\frac{\cos(e+fx)}{2acf(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \cdots
\end{aligned}
$$

Mathematica [A]
time = 0.59, size = 163, normalized size = 1.07

$$
\frac{\sec(e+fx)\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\cos(2(e+fx))\left(-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)+2\sin(e+fx)\right)}{4a^2c^2f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (Sec[e + f*x]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]] + Cos[2*(e + f*x)]*(-Log[Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 2*Sin[e + f*x]))/
(4*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [A]
time = 0.16, size = 198, normalized size = 1.30

method	result
default	$\frac{\left(\ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-\ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right)(\cos^2(fx+e))-\sin(fx+e)\right)(\cos(fx+e)\sin(fx+e)-(\cos}{4f(-1+\cos(fx+e))(a(1+\sin(fx+e)))^{\frac{5}{2}}(-c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/f*(ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2-sin(f*x+e))*(cos(f*x+e)*sin(f*x+e
)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)
^2-2*sin(f*x+e)+cos(f*x+e)-2)/(-1+cos(f*x+e))/(a*(1+sin(f*x+e)))^(5/2)/(-c*
(sin(f*x+e)-1))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^
(5/2)), x)
```

Fricas [A]
time = 0.41, size = 282, normalized size = 1.86

$$\left[\frac{\sqrt{ac}\cos\left(fx+e\right)^{3}\log\left(-\frac{ac\cos(fx+e)^{3}-2ac\cos(fx+e)-2\sqrt{ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin(fx+e)}{\cos(fx+e)^{3}}\right)+2\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{4a^{3}c^{3}f\cos\left(fx+e\right)^{3}}, -\frac{\sqrt{-ac}\arctan\left(\frac{\sqrt{-ac}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}}{ac\cos(fx+e)\sin(fx+e)}\right)\cos\left(fx+e\right)^{3}-\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\sin\left(fx+e\right)}{2a^{3}c^{3}f\cos\left(fx+e\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*cos(f*x + e)^3*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e
) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x
+ e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*arctan(sqrt(-a
*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*si
n(f*x + e)))*cos(f*x + e)^3 - sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
 + c)*sin(f*x + e))/(a^3*c^3*f*cos(f*x + e)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),
 x)
```

3.65 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=114

$$\frac{2^{\frac{3}{2}+n}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(-1-2n);\frac{1}{2}(5+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}-n}(a+}{f(3+2m)}$$

```
[Out] 2^(3/2+n)*c^2*cos(f*x+e)^3*hypergeom([3/2+m, -1/2-n],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/f/(
3+2*m)
```

Rubi [A]
time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2920, 2824, 2768, 72, 71}

$$\frac{c^2 2^{n+\frac{3}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(-2n-1);\frac{1}{2}(2m+5);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(3/2 + n)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (-1 - 2*n)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 - n)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(f*(3 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{1+n}}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\sin\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+fx))\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}+n}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+\right.}{}$$

$$= \frac{2^{\frac{3}{2}+n}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(-1-2\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 10.34, size = 2543, normalized size = 22.31

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (-16*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*A
ppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2
 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + P
i/2 - f*x)/2]*Sin[(-e + Pi/2 - f*x)/2]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
+ f*x])^n*Tan[(-e + Pi/2 - f*x)/4])/(f*(n*(AppellF1[1/2 + n, -2*m, 2*(1 + m
 + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] +
8*AppellF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n),
3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*Appel
lF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2])*Csc[(-e + Pi/2 - f*x)/4]*Sec[(-e + Pi/2 - f*x)/4]
^3 + (AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m +
n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*A
ppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2
 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Csc[(-e + P
i/2 - f*x)/2]*Sec[(-e + Pi/2 - f*x)/4]^2*Sec[(-e + Pi/2 - f*x)/2] + 2*(m +
n)*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m + n)
, 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*App
ellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -T
an[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2 +
 n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/
2 - f*x)/4]^2*Sec[(-e + Pi/2 - f*x)/2]*Tan[(-e + Pi/2 - f*x)/4] - 4*m*(Appe
llF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Ta
n[(-e + Pi/2 - f*x)/4]^2] + 8*AppellF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 +
n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/
2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2] - 4*AppellF1[1/2 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Sec[(-e + Pi/2 - f*x)
/2]^2*Tan[(-e + Pi/2 - f*x)/4] + 32*m*(AppellF1[1/2 + n, -2*m, 2*(1 + m + n
), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*Ap
pellF1[1/2 + n, -2*m, 2*(2 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -
Tan[(-e + Pi/2 - f*x)/4]^2] - 5*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2
+ n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*AppellF1[
1/2 + n, -2*m, 5 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2])*Sec[e + f*x]^2*Sin[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + P
```

```
i/2 - f*x)/4] - (2*(m*(1 + 2*n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1 + m + n), 5
/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 5*(1/2 +
 n)*(2*m*AppellF1[3/2 + n, 1 - 2*m, 3 + 2*(m + n), 5/2 + n, Tan[(-e + Pi/2
- f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + (3 + 2*(m + n))*AppellF1[3/2 +
n, -2*m, 2*(2 + m + n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]) - 4*(1/2 + n)*(2*m*AppellF1[3/2 + n, 1 - 2*m, 5 + 2*(m + n)
, 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + (5 +
2*(m + n))*AppellF1[3/2 + n, -2*m, 2*(3 + m + n), 5/2 + n, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]) + (1 + m + n)*(1 + 2*n)*AppellF1[
3/2 + n, -2*m, 3 + 2*m + 2*n, 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 16*(1/2 + n)*(m*AppellF1[3/2 + n, 1 - 2*m, 2*(2 + m
+ n), 5/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + (
2 + m + n)*AppellF1[3/2 + n, -2*m, 5 + 2*m + 2*n, 5/2 + n, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Csc[(-e + Pi/2 - f*x)/2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Sec[(-e + Pi/2 - f*x)/2]*Tan[(-e + Pi/2 - f*x)/4]^2)/(3
/2 + n)))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
fricas")
```

[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (-c(\sin(e+fx)-1))^n \cos^2(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*cos(e + f*x)*
*2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorithm="
giac")

[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)

[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)

3.66 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{2^{\frac{3}{2}+m}a^4c^3\cos^9(e+fx)\,{}_2F_1\left(\frac{9}{2},-\frac{1}{2}-m;\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{-4+m}}{9f}$$

```
[Out] -1/9*2^(3/2+m)*a^4*c^3*cos(f*x+e)^9*hypergeom([9/2, -1/2-m],[11/2],1/2-1/2*
sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f
```

Rubi [A]
time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^4c^32^{m+\frac{3}{2}}\cos^9(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{9}{2},-m-\frac{1}{2};\frac{11}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{9f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] -1/9*(2^(3/2 + m)*a^4*c^3*Cos[e + f*x]^9*Hypergeometric2F1[9/2, -1/2 - m, 1
1/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x
])^(-4 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \left(a^3c^3\right)\int \cos^8(e+fx)(a+a\sin(e+fx))^{-3+
$$

$$= \frac{\left(a^5c^3\cos^9(e+fx)\right)\text{Subst}\left(\int (a-ax)^{7/2}(a+
}{f(a-a\sin(e+fx))^{9/2}(a+a\sin
}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^5c^3\cos^9(e+fx)(a+a\sin(e+fx))^{-
}{}$$

$$= -\frac{2^{\frac{3}{2}+m}a^4c^3\cos^9(e+fx)\,{}_2F_1\left(\frac{9}{2},-\frac{1}{2}-m;\frac{11}{2};
}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]
[Out] $Aborted
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m(c-c\sin(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(3*c^3*cos(f*x + e)^4 - 4*c^3*cos(f*x + e)^2 - (c^3*cos(f*x + e)^
4 - 4*c^3*cos(f*x + e)^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx))^m (c-c\sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)
```

3.67 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{2^{\frac{3}{2}+m}a^3c^2\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{-3-}}{7f}$$

```
[Out] -1/7*2^(3/2+m)*a^3*c^2*cos(f*x+e)^7*hypergeom([7/2, -1/2-m],[9/2],1/2-1/2*s
in(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f
```

Rubi [A]
time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^3c^22^{m+\frac{3}{2}}\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{1}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/7*(2^(3/2 + m)*a^3*c^2*Cos[e + f*x]^7*Hypergeometric2F1[7/2, -1/2 - m, 9
/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x]
)^(-3 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = (a^2c^2)\int \cos^6(e+fx)(a+a\sin(e+fx))^{-2+m}$$

$$= \frac{(a^4c^2\cos^7(e+fx))\,\text{Subst}\left(\int(a-ax)^{5/2}(a+a$$

$$f(a-a\sin(e+fx))^{7/2}(a+a\sin$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^4c^2\cos^7(e+fx)(a+a\sin(e+fx))^{-3}\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+m}a^3c^2\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{1}{2}-m;\frac{9}{2};\frac{1}{2}\right.}{}$$

Mathematica [F]
time = 180.04, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
[Out] $Aborted
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m\,(c-c\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(c^2*cos(f*x + e)^4 + 2*c^2*cos(f*x + e)^2*sin(f*x + e) - 2*c^2*c
os(f*x + e)^2)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int (a\sin(e+fx)+a)^m\cos^2(e+fx)\,dx+\int \left(-2(a\sin(e+fx)+a)^m\sin(e+fx)\cos^2(e+fx)\right)\,dx+\int (a\sin(e+fx)+a)^m\sin^2(e+fx)\cos^2(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] c**2*(Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2, x) + Integral(-2*(a
*sin(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2, x) + Integral((a*sin(e
+ f*x) + a)**m*sin(e + f*x)**2*cos(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx))^m (c-c\sin(e+fx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)
```

3.68 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=84

$$-\frac{2^{\frac{3}{2}+m}a^2c\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{-2+}}{5f}$$

```
[Out] -1/5*2^(3/2+m)*a^2*c*cos(f*x+e)^5*hypergeom([5/2, -1/2-m],[7/2],1/2-1/2*sin
(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f
```

Rubi [A]
time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^2c2^{m+\frac{3}{2}}\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{1}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/5*(2^(3/2 + m)*a^2*c*Cos[e + f*x]^5*Hypergeometric2F1[5/2, -1/2 - m, 7/2
, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^
(-2 + m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = (ac)\int \cos^4(e+fx)(a+a\sin(e+fx))^{-1+m}\,dx$$

$$= \frac{(a^3c\cos^5(e+fx))\,\text{Subst}\left(\int (a-ax)^{3/2}(a+ax\right.}{f(a-a\sin(e+fx))^{5/2}(a+a\sin(}$$

$$= \frac{\left(2^{\frac{1}{2}+m}a^3c\cos^5(e+fx)(a+a\sin(e+fx))^{-2+}\right.}{}$$

$$= -\frac{2^{\frac{3}{2}+m}a^2c\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{1}{2}-m;\frac{7}{2};\frac{1}{2}(1\right.}{}$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
[Out] $Aborted
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(c*cos(f*x + e)^2*sin(f*x + e) - c*cos(f*x + e)^2)*(a*sin(f*x + e
) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int\left(-(a\sin{(e+fx)}+a)^{m}\cos^{2}{(e+fx)}\right)\,dx+\int(a\sin{(e+fx)}+a)^{m}\sin{(e+fx)}\cos^{2}{(e+fx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] -c*(Integral(-(a*sin(e + f*x) + a)**m*cos(e + f*x)**2, x) + Integral((a*sin
(e + f*x) + a)**m*sin(e + f*x)*cos(e + f*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx))^m (c-c\sin(e+fx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

3.69 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=81

$$-\frac{2^{\frac{3}{2}+m}a\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{-1+m}}{3f}$$

```
[Out] -1/3*2^(3/2+m)*a*cos(f*x+e)^3*hypergeom([3/2, -1/2-m],[5/2],1/2-1/2*sin(f*x
+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-1+m)/f
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{3}{2}}\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/3*(2^(3/2 + m)*a*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1
 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(-1
+ m))/f
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{(a^2\cos^3(e+fx))\,\text{Subst}\Big(\int\sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}\,dx,x,\sin(e+f}{f(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}$$
$$= \frac{\Big(2^{\frac{1}{2}+m}a^2\cos^3(e+fx)(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(e+fx)}{a}\right)^{-\frac{1}{2}-}}{f(a-a\sin(e+}$$
$$= -\frac{2^{\frac{3}{2}+m}a\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+}{3f}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 0.96

$$-\frac{2^{\frac{3}{2}+m}\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{3}{2}-m}(a(1+\sin(e+fx)))^m}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
[Out] -1/3*(2^(3/2 + m)*Cos[e + f*x]^3*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
 Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int\left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (e+fx)+1))^m \cos^2 (e+fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos \left (e+f\,x\right)}^2\,{\left (a+a\,\sin \left (e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

3.70 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=77

$$-\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^m}{cf}$$

```
[Out] -2^(3/2+m)*cos(f*x+e)*hypergeom([1/2, -1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+
sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/c/f
```

Rubi [A]
time = 0.12, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2919, 2731, 2730}

$$-\frac{2^{m+\frac{3}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{2},-m-\frac{1}{2};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, -1/2 - m, 3/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(c*f))
```

Rule 2730

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}\,dx}{ac}$$
$$= \frac{((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m)\int (1+\sin(e+fx))^{1+m}}{c}$$
$$= -\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},-\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin}{cf}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 19.21, size = 6442, normalized size = 83.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(\cos^2(fx+e))\,(a+a\sin(fx+e))^m}{c-c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{(a\sin(e+fx)+a)^m\cos^2(e+fx)}{\sin(e+fx)-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] -Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(sin(e + f*x) - 1), x)/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(e+f\,x\right)}^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{c-c\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)
```

3.71 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=81

$$\frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{1+m}}{ac^2f}$$

```
[Out] 2^(3/2+m)*hypergeom([-1/2, -1/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+
sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f
```

Rubi [A]
time = 0.15, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2919, 2768, 72, 71}

$$\frac{2^{m+\frac{3}{2}}\sec(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{1}{2},-m-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{ac^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(a*
c^2*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^{2+m}\,dx}{a^2c^2}$$
$$= \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\Big(}{c^2f}$$
$$= \frac{\left(2^{\frac{1}{2}+m}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\,(a+a\sin(e+fx))^{1+m}\,\Big(\right.}{c^2f}$$
$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\text{s}}{ac^2f}$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 1.09

$$\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a(1+\sin(e+fx)))^m}{c^2f(1-\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/2 - m, 1/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(c^2*f*
(1 - Sin[e + f*x]))
```

Maple [F]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c^2*cos(f*x + e)^2 + 2*c^2
*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(a\sin{(e+fx)}+a)^m\cos^2{(e+fx)}}{\sin^2{(e+fx)}-2\sin{(e+fx)}+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m*cos(e + f*x)**2/(sin(e + f*x)**2 - 2*sin(e
 + f*x) + 1), x)/c**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m}{(c-c\,\sin(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x)
```

3.72 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+fx))^{2+m}}{3a^2c^3f}$$

```
[Out] 1/3*2^(3/2+m)*hypergeom([-3/2, -1/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e
)^3*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f
```

Rubi [A]
time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2919, 2768, 72, 71}

$$\frac{2^{m+\frac{3}{2}}\sec^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{3}{2},-m-\frac{1}{2};-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3a^2c^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*
Sec[e + f*x]^3*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/
(3*a^2*c^3*f)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{\int \sec^4(e+fx)(a+a\sin(e+fx))^{3+m}\,dx}{a^3c^3}$$

$$= \frac{\left(\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\right) \text{Subst}}{ac^3f}$$

$$= \frac{\left(2^{\frac{1}{2}+m}\sec^3(e+fx)(a-a\sin(e+fx))^{3/2}(a+a\sin(e+fx))^{2+m}\right.}{ac^3}$$

$$= \frac{2^{\frac{3}{2}+m}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec^3(e+fx)(1-}{3a^2c^3f}$$

Mathematica [A]
time = 0.10, size = 91, normalized size = 1.06

$$\frac{2^{\frac{3}{2}+m}\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a(1+\sin(e+fx)))^m}{3c^3f(1-\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (2^(3/2 + m)*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/2 - m, -1/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a*(1 + Sin[e + f*x]))^m)/(3*c^3
*f*(1 - Sin[e + f*x])^2)
```

Maple [F]
time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] -integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(3*c^3*cos(f*x + e)^2 - 4*c
^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate(-(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(c*sin(f*x + e) - c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+fx)^2 (a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x)
```

3.73 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=244

$$\frac{768c^3\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(7+2m)(9+2m)\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}}+\frac{192c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-}}{af(9+2m)\left(35+24m+4m^2\right)}$$

```
[Out] 24*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(3/2)/a/f/(4*m^2+32
*m+63)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(5/2)/a/f/(9+2*
m)+768*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(4*m^2+16*m+15)/(4*m^2+32*
m+63)/(c-c*sin(f*x+e))^(1/2)+192*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c
*sin(f*x+e))^(1/2)/a/f/(8*m^3+84*m^2+286*m+315)
```

Rubi [A]
time = 0.41, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2920, 2819, 2817}

$$\frac{768c^3\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)(2m+9)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{192c^2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af(2m+9)\left(4m^2+24m+35\right)}+\frac{24c\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+32m+63\right)}+\frac{2\cos(e+fx)(c-c\sin(e+fx))^{5/2}(a\sin(e+fx)+a)^{m+1}}{af(2m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (768*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(9 + 2*m
)*(15 + 16*m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (192*c^2*Cos[e + f*x]*(a
+ a*Sin[e + f*x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(9 + 2*m)*(35 + 24
*m + 4*m^2)) + (24*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
 + f*x])^(3/2))/(a*f*(63 + 32*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*Sin[e +
f*x])^(1 + m)*(c - c*Sin[e + f*x])^(5/2))/(a*f*(9 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
```

```
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))}{ac}$$

$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{s}}{af(9+2m)}$$

$$= \frac{24c\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-}{af\left(63+32m+4m^2\right)}$$

$$= \frac{192c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{}}{af(5+2m)\left(63+32m+4m\right.}$$

$$= \frac{768c^3\cos(e+fx)(a+a\sin(e+f}{af(3+2m)(5+2m)\left(63+32m+4m^2\right)\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.44, size = 695, normalized size = 2.85

$$\frac{(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{5/2}\left(\frac{(2205+590m+108m^2+8m^3)\left(\left(\frac{3}{8}+\frac{3i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)+\left(\frac{3}{8}-\frac{3i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)(9+2m)}+\frac{(2205+590m+108m^2+8m^3)\left(\left(\frac{3}{8}-\frac{3i}{8}\right)\cos\left(\frac{1}{2}(e+fx)\right)+\left(\frac{3}{8}+\frac{3i}{8}\right)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)(9+2m)}+\frac{(191m+48m^2+4m^3)\left((1-i)\cos\left(\frac{3}{2}(e+fx)\right)-(1+i)\sin\left(\frac{3}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)(9+2m)}+\frac{(191m+48m^2+4m^3)\left((1+i)\cos\left(\frac{3}{2}(e+fx)\right)-(1-i)\sin\left(\frac{3}{2}(e+fx)\right)\right)}{(3+2m)(5+2m)(7+2m)(9+2m)}+\frac{(21+2m)\left(\left(\frac{3}{2}+\frac{3i}{2}\right)\cos\left(\frac{5}{2}(e+fx)\right)+\left(\frac{3}{2}-\frac{3i}{2}\right)\sin\left(\frac{5}{2}(e+fx)\right)\right)}{(5+2m)(7+2m)(9+2m)}+\frac{(21+2m)\left(\left(\frac{3}{2}-\frac{3i}{2}\right)\cos\left(\frac{5}{2}(e+fx)\right)+\left(\frac{3}{2}+\frac{3i}{2}\right)\sin\left(\frac{5}{2}(e+fx)\right)\right)}{(5+2m)(7+2m)(9+2m)}+\frac{(15+2m)\left(\left(\frac{3}{16}-\frac{3i}{16}\right)\cos\left(\frac{7}{2}(e+fx)\right)-\left(\frac{3}{16}+\frac{3i}{16}\right)\sin\left(\frac{7}{2}(e+fx)\right)\right)}{(7+2m)(9+2m)}+\frac{(15+2m)\left(\left(\frac{3}{16}+\frac{3i}{16}\right)\cos\left(\frac{7}{2}(e+fx)\right)-\left(\frac{3}{16}-\frac{3i}{16}\right)\sin\left(\frac{7}{2}(e+fx)\right)\right)}{(7+2m)(9+2m)}+\frac{\left(-\frac{1}{16}+\frac{i}{16}\right)\cos\left(\frac{9}{2}(e+fx)\right)-\left(\frac{1}{16}+\frac{i}{16}\right)\sin\left(\frac{9}{2}(e+fx)\right)}{9+2m}+\frac{\left(-\frac{1}{16}-\frac{i}{16}\right)\cos\left(\frac{9}{2}(e+fx)\right)-\left(\frac{1}{16}-\frac{i}{16}\right)\sin\left(\frac{9}{2}(e+fx)\right)}{9+2m}\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2),
x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2205 + 590*m + 108*
m^2 + 8*m^3)*((3/8 + (3*I)/8)*Cos[(e + f*x)/2] + (3/8 - (3*I)/8)*Sin[(e + f
*x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((2205 + 590*m + 108*m
^2 + 8*m^3)*((3/8 - (3*I)/8)*Cos[(e + f*x)/2] + (3/8 + (3*I)/8)*Sin[(e + f*
x)/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^
3)*((1 - I)*Cos[(3*(e + f*x))/2] - (1 + I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m
)*(5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((191*m + 48*m^2 + 4*m^3)*((1 + I)*Cos[(
```

```
3*(e + f*x))/2] - (1 - I)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 +
2*m)*(9 + 2*m)) + ((21 + 2*m)*((3/2 + (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2
- (3*I)/2)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((21 +
2*m)*((3/2 - (3*I)/2)*Cos[(5*(e + f*x))/2] + (3/2 + (3*I)/2)*Sin[(5*(e + f*
x))/2]))/((5 + 2*m)*(7 + 2*m)*(9 + 2*m)) + ((15 + 2*m)*((3/16 - (3*I)/16)*C
os[(7*(e + f*x))/2] - (3/16 + (3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(
9 + 2*m)) + ((15 + 2*m)*((3/16 + (3*I)/16)*Cos[(7*(e + f*x))/2] - (3/16 - (
3*I)/16)*Sin[(7*(e + f*x))/2]))/((7 + 2*m)*(9 + 2*m)) + ((-1/16 + I/16)*Cos
[(9*(e + f*x))/2] - (1/16 + I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*m) + ((-1/16
 - I/16)*Cos[(9*(e + f*x))/2] - (1/16 - I/16)*Sin[(9*(e + f*x))/2])/(9 + 2*
m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(244) = 488$.
time = 0.62, size = 586, normalized size = 2.40

$$-\frac{2\left(\left(8m^3+108m^2+526m+957\right)a^m c^{\frac{5}{2}} - \frac{3\left(8m^3+76m^2+142m-315\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} - \frac{24\left(4m^2+16m-81\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + \frac{16\left(4m^3+36m^2+95m+315\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^3}{\left(\cos\left(fx+e\right)+1\right)^3} - \frac{6\left(8m^3+60m^2+206m-567\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4} - \frac{6\left(8m^3+60m^2+206m-567\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^5}{\left(\cos\left(fx+e\right)+1\right)^5} + \frac{16\left(4m^3+36m^2+95m+315\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^6}{\left(\cos\left(fx+e\right)+1\right)^6} - \frac{24\left(4m^2+16m-81\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^7}{\left(\cos\left(fx+e\right)+1\right)^7} - \frac{3\left(8m^3+76m^2+142m-315\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^8}{\left(\cos\left(fx+e\right)+1\right)^8} + \frac{\left(8m^3+108m^2+526m+957\right)a^m c^{\frac{5}{2}}\sin\left(fx+e\right)^9}{\left(\cos\left(fx+e\right)+1\right)^9}\right)e^{\left(2m\log\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+1\right) - m\log\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)\right)}}{\left(16m^4+192m^3+824m^2+1488m + \frac{2\left(16m^4+192m^3+824m^2+1488m+945\right)\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2} + \frac{\left(16m^4+192m^3+824m^2+1488m+945\right)\sin\left(fx+e\right)^4}{\left(\cos\left(fx+e\right)+1\right)^4} + 945\right)f\left(\frac{\sin\left(fx+e\right)^2}{\left(\cos\left(fx+e\right)+1\right)^2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
```

```
[Out] -2*((8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2) - 3*(8*m^3 + 76*m^2 + 142*m
 - 315)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) - 24*(4*m^2 + 16*m - 81
)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 16*(4*m^3 + 36*m^2 + 95
*m + 315)*a^m*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*(8*m^3 + 60*m
^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*(8*m^
3 + 60*m^2 + 206*m - 567)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 +
 16*(4*m^3 + 36*m^2 + 95*m + 315)*a^m*c^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*(4*m^2 + 16*m - 81)*a^m*c^(5/2)*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 - 3*(8*m^3 + 76*m^2 + 142*m - 315)*a^m*c^(5/2)*sin(f*x + e)^8/(cos(f*x
 + e) + 1)^8 + (8*m^3 + 108*m^2 + 526*m + 957)*a^m*c^(5/2)*sin(f*x + e)^9/(
cos(f*x + e) + 1)^9)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*lo
g(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 192*m^3 + 824*m^2 +
1488*m + 2*(16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^2/(cos(
```

```
f*x + e) + 1)^2 + (16*m^4 + 192*m^3 + 824*m^2 + 1488*m + 945)*sin(f*x + e)^
4/(cos(f*x + e) + 1)^4 + 945)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2))
```

Fricas [A]
time = 0.41, size = 409, normalized size = 1.68

$$-\frac{2\left(\left(8c^2m^3+60c^2m^2+142c^2m+105c^2\right)\cos\left(fx+e\right)^5-\left(8c^2m^3+108c^2m^2+334c^2m+285c^2\right)\cos\left(fx+e\right)^4-2\left(8c^2m^3+84c^2m^2+334c^2m+339c^2\right)\cos\left(fx+e\right)^3-384c^2\cos\left(fx+e\right)-96\left(2c^2m-c^2\right)\cos\left(fx+e\right)^2-768c^2+\left(\left(8c^2m^3+60c^2m^2+142c^2m+105c^2\right)\cos\left(fx+e\right)^4+2\left(8c^2m^3+84c^2m^2+238c^2m+195c^2\right)\cos\left(fx+e\right)^3-384c^2\cos\left(fx+e\right)-96\left(2c^2m+3c^2\right)\cos\left(fx+e\right)^2-768c^2\right)\sin\left(fx+e\right)\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{16fm^4+192fm^3+824fm^2+1488fm+\left(16fm^4+192fm^3+824fm^2+1488fm+945f\right)\cos\left(fx+e\right)-\left(16fm^4+192fm^3+824fm^2+1488fm+945f\right)\sin\left(fx+e\right)+945f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] -2*((8*c^2*m^3 + 60*c^2*m^2 + 142*c^2*m + 105*c^2)*cos(f*x + e)^5 - (8*c^2*
m^3 + 108*c^2*m^2 + 334*c^2*m + 285*c^2)*cos(f*x + e)^4 - 2*(8*c^2*m^3 + 84
*c^2*m^2 + 334*c^2*m + 339*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*
(2*c^2*m - c^2)*cos(f*x + e)^2 - 768*c^2 + ((8*c^2*m^3 + 60*c^2*m^2 + 142*c
^2*m + 105*c^2)*cos(f*x + e)^4 + 2*(8*c^2*m^3 + 84*c^2*m^2 + 238*c^2*m + 19
5*c^2)*cos(f*x + e)^3 - 384*c^2*cos(f*x + e) - 96*(2*c^2*m + 3*c^2)*cos(f*x
 + e)^2 - 768*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m/(16*f*m^4 + 192*f*m^3 + 824*f*m^2 + 1488*f*m + (16*f*m^4 + 192*f*m^3
 + 824*f*m^2 + 1488*f*m + 945*f)*cos(f*x + e) - (16*f*m^4 + 192*f*m^3 + 824
*f*m^2 + 1488*f*m + 945*f)*sin(f*x + e) + 945*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2
, x)
```

Mupad [B]
time = 14.97, size = 1060, normalized size = 4.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
3*c^2*exp(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^m*(48*m + 4*m^2 + 63))/(f*(m*1488i + m^2*824i + m^3*192i +
m^4*16i + 945i)) - (c^2*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2))^m*(m*142i + m^2*60i + m^3*8i + 105i))/(8*f*(m*1488i + m^2*82
4i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*2i + f*x*2i)*(a + a*((exp(-
 e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*48i + m^2*4i + 63i
))/(f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 945i)) - (c^2*exp(e*9i + f
*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*
(142*m + 60*m^2 + 8*m^3 + 105))/(8*f*(m*1488i + m^2*824i + m^3*192i + m^4*1
6i + 945i)) + (3*c^2*exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2))^m*(270*m + 92*m^2 + 8*m^3 + 225))/(8*f*(m*14
88i + m^2*824i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*8i + f*x*8i)*(a
 + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*270i +
 m^2*92i + m^3*8i + 225i))/(8*f*(m*1488i + m^2*824i + m^3*192i + m^4*16i +
945i)) + (3*c^2*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^m*(590*m + 108*m^2 + 8*m^3 + 2205))/(4*f*(m*1488i
 + m^2*824i + m^3*192i + m^4*16i + 945i)) + (3*c^2*exp(e*4i + f*x*4i)*(a +
a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*590i + m^
2*108i + m^3*8i + 2205i))/(4*f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 9
45i)) + (2*c^2*m*exp(e*3i + f*x*3i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2))^m*(48*m + 4*m^2 + 191))/(f*(m*1488i + m^2*824i +
 m^3*192i + m^4*16i + 945i)) + (2*c^2*m*exp(e*6i + f*x*6i)*(a + a*((exp(- e
*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*48i + m^2*4i + 191i)
)/(f*(m*1488i + m^2*824i + m^3*192i + m^4*16i + 945i))))/(exp(e*5i + f*x*5i
) + (exp(e*4i + f*x*4i)*(1488*m + 824*m^2 + 192*m^3 + 16*m^4 + 945))/(m*148
8i + m^2*824i + m^3*192i + m^4*16i + 945i))
```

3.74 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{64c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(7+2m)\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+}}{af\left(35+24m+4m^2\right)}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(3/2)/a/f/(7+2*m)+64*c
^2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(8*m^3+60*m^2+142*m+105)/(c-c*sin(
f*x+e))^(1/2)+16*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(1/2)
/a/f/(4*m^2+24*m+35)
```

Rubi [A]

time = 0.31, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2920, 2819, 2817}

$$\frac{64c^2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+7)\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{16c\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+24m+35\right)}+\frac{2\cos(e+fx)(c-c\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{m+1}}{af(2m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (64*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(7 + 2*m)*(15 + 16*
m + 4*m^2)*Sqrt[c - c*Sin[e + f*x]]) + (16*c*Cos[e + f*x]*(a + a*Sin[e + f*
x])^(1 + m)*Sqrt[c - c*Sin[e + f*x]])/(a*f*(35 + 24*m + 4*m^2)) + (2*Cos[e
+ f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(3/2))/(a*f*(7 + 2
*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{5/}}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin}{af(7+2m)}$$
$$= \frac{16c\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-}}{af\left(35+24m+4m^2\right)}$$
$$= \frac{64c^2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\left(35+24m+4m^2\right)\sqrt{c-c\sin(e}}$$

Mathematica [**A**]
time = 1.94, size = 149, normalized size = 0.87

$$-\frac{c\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\left(-157-80m-12m^2+\left(15+16m+4m^2\right)\cos(2(e+fx))+4\left(27+24m+4m^2\right)\sin(e+fx)\right)}{f(3+2m)(5+2m)(7+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2),
x]
```

```
[Out] -((c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[
c - c*Sin[e + f*x]]*(-157 - 80*m - 12*m^2 + (15 + 16*m + 4*m^2)*Cos[2*(e +
f*x)] + 4*(27 + 24*m + 4*m^2)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)*(7 + 2*
m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m\,(c-c\sin(fx+e))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(173) = 346$.
time = 0.61, size = 447, normalized size = 2.60

$$-\frac{2\left((4\,m^2+32\,m+71)a^m c^{\frac{3}{2}}-\frac{(4\,m^2-105)a^m c^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}-\frac{(12\,m^2+64\,m-91)a^m c^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(12\,m^2+32\,m+245)a^m c^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{(12\,m^2+32\,m+245)a^m c^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{(12\,m^2+64\,m-91)a^m c^{\frac{3}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{(4\,m^2-105)a^m c^{\frac{3}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{(4\,m^2+32\,m+71)a^m c^{\frac{3}{2}}\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)e^{\left(2\,m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(8\,m^3+60\,m^2+142\,m+\frac{2\,(8\,m^3+60\,m^2+142\,m+105)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(8\,m^3+60\,m^2+142\,m+105)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+105\right)f\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] -2*((4*m^2 + 32*m + 71)*a^m*c^(3/2) - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e
)/(cos(f*x + e) + 1) - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 + (12*m^2 + 32*m + 245)*a^m*c^(3/2)*sin(f*x + e)^4/(cos(f*x +
 e) + 1)^4 - (12*m^2 + 64*m - 91)*a^m*c^(3/2)*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - (4*m^2 - 105)*a^m*c^(3/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4
*m^2 + 32*m + 71)*a^m*c^(3/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*e^(2*m*l
og(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))/((8*m^3 + 60*m^2 + 142*m + 2*(8*m^3 + 60*m^2 + 142*m + 105)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (8*m^3 + 60*m^2 + 142*m + 105)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 105)*f*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
 + 1)^(3/2))
```

Fricas [A]
time = 0.38, size = 255, normalized size = 1.48

$$\frac{2\left(\left(4\,cm^2+16\,cm+15\,c\right)\cos\left(fx+e\right)^4+\left(4\,cm^2+32\,cm+39\,c\right)\cos\left(fx+e\right)^3+8\left(2\,cm-c\right)\cos\left(fx+e\right)^2+32\,c\cos\left(fx+e\right)-\left(\left(4\,cm^2+16\,cm+15\,c\right)\cos\left(fx+e\right)^3-8\left(2\,cm+3\,c\right)\cos\left(fx+e\right)^2-32\,c\cos\left(fx+e\right)-64\,c\right)\sin\left(fx+e\right)+64\,c\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^m}{8\,fm^3+60\,fm^2+142\,fm+\left(8\,fm^3+60\,fm^2+142\,fm+105\,f\right)\cos\left(fx+e\right)-\left(8\,fm^3+60\,fm^2+142\,fm+105\,f\right)\sin\left(fx+e\right)+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] 2*((4*c*m^2 + 16*c*m + 15*c)*cos(f*x + e)^4 + (4*c*m^2 + 32*c*m + 39*c)*cos
(f*x + e)^3 + 8*(2*c*m - c)*cos(f*x + e)^2 + 32*c*cos(f*x + e) - ((4*c*m^2
+ 16*c*m + 15*c)*cos(f*x + e)^3 - 8*(2*c*m + 3*c)*cos(f*x + e)^2 - 32*c*cos
(f*x + e) - 64*c)*sin(f*x + e) + 64*c)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
 + e) + a)^m/(8*f*m^3 + 60*f*m^2 + 142*f*m + (8*f*m^3 + 60*f*m^2 + 142*f*m
+ 105*f)*cos(f*x + e) - (8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)*sin(f*x + e)
 + 105*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] integrate((-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2
, x)
```

Mupad [B]
time = 13.97, size = 528, normalized size = 3.07

$$-\frac{\sqrt{c-c\,\sin\left(e+f\,x\right)}\,\left(\frac{c\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,4{}\mathrm{i}+m\,16{}\mathrm{i}+15{}\mathrm{i}\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,7{}\mathrm{i}+f\,x\,7{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+16\,m+15\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(4\,m^2+48\,m+63\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(12\,m^2+80\,m-35\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{c\,{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,4{}\mathrm{i}+m\,48{}\mathrm{i}+63{}\mathrm{i}\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{c\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,12{}\mathrm{i}+m\,80{}\mathrm{i}-35{}\mathrm{i}\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{c\,{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(12\,m^2+112\,m+525\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{c\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(m^2\,12{}\mathrm{i}+m\,112{}\mathrm{i}+525{}\mathrm{i}\right)}{4\,f\,\left(8\,m^3+60\,m^2+142\,m+105\right)}\right)}{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\left(m^3\,8{}\mathrm{i}+m^2\,60{}\mathrm{i}+m\,142{}\mathrm{i}+105{}\mathrm{i}\right)}{8\,m^3+60\,m^2+142\,m+105}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((c*(a + a*sin(e + f*x))^m*(m*16i + m^2*4i + 1
5i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*7i + f*x*7i)*(a + a*si
n(e + f*x))^m*(16*m + 4*m^2 + 15))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (
c*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(48*m + 4*m^2 + 63))/(4*f*(142*
m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*5i + f*x*5i)*(a + a*sin(e + f*x))^m*(
80*m + 12*m^2 - 35))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) + (c*exp(e*6i + f
*x*6i)*(a + a*sin(e + f*x))^m*(m*48i + m^2*4i + 63i))/(4*f*(142*m + 60*m^2
+ 8*m^3 + 105)) + (c*exp(e*2i + f*x*2i)*(a + a*sin(e + f*x))^m*(m*80i + m^2
*12i - 35i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105)) - (c*exp(e*3i + f*x*3i)*(
a + a*sin(e + f*x))^m*(112*m + 12*m^2 + 525))/(4*f*(142*m + 60*m^2 + 8*m^3
+ 105)) + (c*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(m*112i + m^2*12i +
525i))/(4*f*(142*m + 60*m^2 + 8*m^3 + 105))))/(exp(e*4i + f*x*4i) - (exp(e*
3i + f*x*3i)*(m*142i + m^2*60i + m^3*8i + 105i))/(142*m + 60*m^2 + 8*m^3 +
105))
```

3.75 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}$ d

Optimal. Leaf size=107

$$\frac{8c\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}}{af(5+2m)}$$

```
[Out] 8*c*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(4*m^2+16*m+15)/(c-c*sin(f*x+e))^
(1/2)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(1/2)/a/f/(5+2*m
)
```

Rubi [A]

time = 0.23, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2920, 2819, 2817}

$$\frac{8c\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af\left(4m^2+16m+15\right)\sqrt{c-c\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{c-c\sin(e+fx)}\,(a\sin(e+fx)+a)^{m+1}}{af(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (8*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(15 + 16*m + 4*m^2)*Sq
rt[c - c*Sin[e + f*x]]) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt
[c - c*Sin[e + f*x]])/(a*f*(5 + 2*m))
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2819

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^
(m - 1)*((c + d*Sin[e + f*x])^n/(f*(m + n))), x] + Dist[a*((2*m - 1)/(m + n
)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IG
tQ[m - 1/2, 0] &&  !LtQ[n, -1] &&  !(IGtQ[n - 1/2, 0] && LtQ[n, m]) &&  !(I
LtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
```

```
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{3/2}}{ac}$$
$$= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin}}{af(5+2m)}$$
$$= \frac{8c\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af\left(15+16m+4m^2\right)\sqrt{c-c\sin(e+fx)}} +$$

Mathematica [A]
time = 0.39, size = 111, normalized size = 1.04

$$-\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^m\sqrt{c-c\sin(e+fx)}\,(-7-2m+(3+2m)\sin(e+fx))}{f(3+2m)(5+2m)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (-2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m*Sqrt[c
 - c*Sin[e + f*x]]*(-7 - 2*m + (3 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2
*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m\sqrt{c-c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(109) = 218$.
time = 0.57, size = 332, normalized size = 3.10

$$-\frac{2\left(a^m\sqrt{c}\,(2m+7)+\frac{a^m\sqrt{c}\,(2m+15)\sin(fx+e)}{\cos(fx+e)+1}-\frac{2a^m\sqrt{c}\,(2m-5)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2a^m\sqrt{c}\,(2m-5)\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^m\sqrt{c}\,(2m+15)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{a^m\sqrt{c}\,(2m+7)\sin(fx+e)^5}{(\cos(fx+e)+1)^5}\right)e^{\left(2m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left(4m^2+16m+\frac{2(4m^2+16m+15)\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{(4m^2+16m+15)\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+15\right)f\sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] -2*(a^m*sqrt(c)*(2*m + 7) + a^m*sqrt(c)*(2*m + 15)*sin(f*x + e)/(cos(f*x +
e) + 1) - 2*a^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a
^m*sqrt(c)*(2*m - 5)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^m*sqrt(c)*(2*m
 + 15)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^m*sqrt(c)*(2*m + 7)*sin(f*x
+ e)^5/(cos(f*x + e) + 1)^5)*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1
) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 16*m + 2*(4*m
^2 + 16*m + 15)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + (4*m^2 + 16*m + 15)*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 + 15)*f*sqrt(sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 1))
```

Fricas [A]
time = 0.40, size = 165, normalized size = 1.54

$$\frac{2\left((2m+3)\cos\left(fx+e\right)^{3}+(2m-1)\cos\left(fx+e\right)^{2}+\left((2m+3)\cos\left(fx+e\right)^{2}+4\cos\left(fx+e\right)+8\right)\sin\left(fx+e\right)+4\cos\left(fx+e\right)+8\right)\sqrt{-c\sin\left(fx+e\right)+c}\left(a\sin\left(fx+e\right)+a\right)^{m}}{4fm^{2}+16fm+\left(4fm^{2}+16fm+15f\right)\cos\left(fx+e\right)-\left(4fm^{2}+16fm+15f\right)\sin\left(fx+e\right)+15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] 2*((2*m + 3)*cos(f*x + e)^3 + (2*m - 1)*cos(f*x + e)^2 + ((2*m + 3)*cos(f*x
 + e)^2 + 4*cos(f*x + e) + 8)*sin(f*x + e) + 4*cos(f*x + e) + 8)*sqrt(-c*si
n(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 16*f*m + (4*f*m^2 + 16*f*
m + 15*f)*cos(f*x + e) - (4*f*m^2 + 16*f*m + 15*f)*sin(f*x + e) + 15*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\sqrt{-c\left(\sin\left(e+fx\right)-1\right)}\cos^{2}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*sqrt(-c*(sin(e + f*x) - 1))*cos(e + f*x)
**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2,
x)
```

Mupad [B]
time = 1.72, size = 104, normalized size = 0.97

$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,\left(25\,\cos(e+f\,x)+3\,\cos(3\,e+3\,f\,x)+8\,\sin(2\,e+2\,f\,x)+6\,m\,\cos(e+f\,x)+2\,m\,\cos(3\,e+3\,f\,x)\right)}{2\,f\,(\sin(e+f\,x)-1)\,(4\,m^2+16\,m+15)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(25*cos(e + f*x) +
 3*cos(3*e + 3*f*x) + 8*sin(2*e + 2*f*x) + 6*m*cos(e + f*x) + 2*m*cos(3*e +
 3*f*x)))/(2*f*(sin(e + f*x) - 1)*(16*m + 4*m^2 + 15))
```

3.76 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2920, 2817}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^m}{f(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]], x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(51) = 102.
time = 0.38, size = 116, normalized size = 2.32

$$-\frac{2\left(\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2\right)\sqrt{-c\sin(fx+e)+c}\,(a\sin(fx+e)+a)^m}{2cfm+3cf+(2cfm+3cf)\cos(fx+e)-(2cfm+3cf)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m \cos^2(e+fx)}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 0.90, size = 68, normalized size = 1.36

$$-\frac{(a(\sin(e+fx)+1))^m \sqrt{-c(\sin(e+fx)-1)}\,(2\cos(e+fx)+\sin(2e+2fx))}{c f (2m+3)(\sin(e+fx)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(c*f*(2*m + 3)*(sin(e + f*x) - 1))
```

3.77 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{acf(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e)
)^(1+m)/a/c/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2920, 2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{acf(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(a*c*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
```

```
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{c-c\sin(e+fx)}}\,dx}{ac} \\
&= \frac{\cos(e+fx)\int \sec(e+fx)(a+a\sin(e+fx))^{\frac{3}{2}+m}\,dx}{ac\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{\cos(e+fx)\mathrm{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{cf\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} \\
&= \frac{\cos(e+fx)\,{}_2F_1\left(1,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(}{acf(3+2m)\sqrt{c-c\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(76) = 152.
time = 6.98, size = 218, normalized size = 2.87

$$-\frac{2^{-\frac{5}{2}-2m}\cos^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(-4^{1+m}\,{}_2F_1\left(1,2+2m;3+2m;\cos\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+{}_2F_1\left(2+2m,2+2m;3+2m;\frac{1}{2}\left(1-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\right)\sec^4\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\sec^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)^{2m}\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a+a\sin(e+fx))^m}{f(1+m)(c-c\sin(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -((2^(-5/2 - 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*(-(4^(1 + m)*Hypergeometric2F1
[1, 2 + 2*m, 3 + 2*m, Cos[(-e + Pi/2 - f*x)/2]]) + Hypergeometric2F1[2 + 2*
m, 2 + 2*m, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2]*Sec[(-e + Pi/2 - f
*x)/4]^4*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(c - c*Sin[e + f*x])^(3/2)))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(3/2)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\cos^2\left(e+fx\right)}{\left(-c\left(\sin\left(e+fx\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/(-c*(sin(e + f*x) - 1))*
*(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2), x)
```

3.78 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{2ac^2f(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/2*cos(f*x+e)*hypergeom([2, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*
x+e))^(1+m)/a/c^2/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2920, 2824, 2746, 70}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+\frac{3}{2};m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{2ac^2f(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[2, 3/2 + m, 5/2 + m, (1 + Sin[e + f*x])/2]*
(a + a*Sin[e + f*x])^(1 + m))/(2*a*c^2*f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]]
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
```

```
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx &= \frac{\int \frac{(a+a\sin(e+fx))^{1+m}}{(c-c\sin(e+fx))^{3/2}}\,dx}{ac}\\ &= \frac{\cos(e+fx)\int \sec^3(e+fx)(a+a\sin(e+fx))^{\frac{5}{2}+m}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{(a\cos(e+fx))\text{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{c^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\\ &= \frac{\cos(e+fx)\,{}_2F_1\left(2,\frac{3}{2}+m;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(}{2ac^2f(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.45, size = 3174, normalized size = 40.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] (2^(-3/2 - 2*m)*(-(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Cos[(-e + Pi/2 -
f*x)/2]]) + Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)
/4]^2)/2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(a + a*Sin[e + f*x])^m)/(f*m*(c - c*Sin[e + f*x])^(5/2)) - ((Cos
[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a +
 a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -
```

Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi
/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Co
t[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)
/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 -
f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan
[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e +
 Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/(4*S
qrt[2]*f*(c - c*Sin[e + f*x])^(5/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[P
i/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*Cos[(-e + Pi/2 - f*x)/4]*(Cos[(-e + P
i/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -2*m, 2*m
, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi
/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m,
2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2
- f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4
]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[
1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e +
 Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 -
f*x)/4]^4)^(2*m))/(1 + 2*m)))/Sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)
*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4])
/2 + m*AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/
2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^
3 + (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/2*(m*
AppellF1[2, 1 - 2*m, 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*Appell
F1[2, -2*m, 1 + 2*m, 3, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/
4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + (m*AppellF1
[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*
Cot[(-e + Pi/2 - f*x)/4]^3*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + m*Appel
lF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^
2]*Cot[(-e + Pi/2 - f*x)/4]^3*(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m)*(
Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m
) + (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2
- f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m
)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(2*(1 - Cot[(-e + Pi/2 - f*x)/4]^
2)^(2*m)) - (Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*
((m*AppellF1[2, 1 - 2*m, 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/
2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2 + (m*
AppellF1[2, -2*m, 1 + 2*m, 3, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 -
 f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]*Csc[(-e + Pi/2 - f*x)/4]^2)/2)*(1 - Ta
n[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (
m*AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f
*x)/4]^2]*Csc[(-e + Pi/2 - f*x)/4]*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sec[(
-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m))/(1 - Cot[(

```
-e + Pi/2 - f*x)/4]^2)^(2*m) + (AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan
[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2
- f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m)
)/(2^(2*m)*(1 + 2*m)) + (2^(1 - 2*m)*(-1/2*((1 + 2*m)*AppellF1[2 + 2*m, 2*m
, 2, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)
/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2 + 2*m) - (m*
(1 + 2*m)*AppellF1[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)
/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-
e + Pi/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - T
an[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m) - (2^(2 - 2*m)*m*AppellF1[1 + 2
*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(-1 + T
an[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(-1 + 2*m))/(1
+ 2*m)))/(8*Sqrt[2])))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{\left(c-c\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="maxima")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^(5/2)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="fricas")
```

```
[Out] integral(-sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(
3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x, algorit
hm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(si
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2), x)
```

3.79 $\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(3+2*m)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2920, 2817}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}}{af(2m+3)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(3 + 2*m)*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}\sqrt{c-c\sin(e+fx)}\,dx}{ac}\\ &= \frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}}{af(3+2m)\sqrt{c-c\sin(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^m}{f(3+2m)\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(51) = 102.
time = 0.37, size = 116, normalized size = 2.32

$$-\frac{2\left(\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2\right)\sqrt{-c\sin(fx+e)+c}\,(a\sin(fx+e)+a)^m}{2\,cfm+3\,cf+(2\,cfm+3\,cf)\cos(fx+e)-(2\,cfm+3\,cf)\sin(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(2*c*f*m + 3*c*f + (2*c*f*m
+ 3*c*f)*cos(f*x + e) - (2*c*f*m + 3*c*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e+fx)+1))^m \cos^2(e+fx)}{\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-c*(sin(e + f*x) -
1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 0.00, size = 68, normalized size = 1.36

$$-\frac{(a\,(\sin(e+f\,x)+1))^m\,\sqrt{-c\,(\sin(e+f\,x)-1)}\,(2\,\cos(e+f\,x)+\sin(2\,e+2\,f\,x))}{c\,f\,(2\,m+3)\,(\sin(e+f\,x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(c*f*(2*m + 3)*(sin(e + f*x) - 1))
```

3.80 $\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=50

$$\frac{2\cos(e+fx)(c+c\sin(e+fx))^{1+m}}{cf(3+2m)\sqrt{a-a\sin(e+fx)}}$$

```
[Out] 2*cos(f*x+e)*(c+c*sin(f*x+e))^(1+m)/c/f/(3+2*m)/(a-a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2920, 2817}

$$\frac{2\cos(e+fx)(c\sin(e+fx)+c)^{m+1}}{cf(2m+3)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]
```

```
[Out] (2*Cos[e + f*x]*(c + c*Sin[e + f*x])^(1 + m))/(c*f*(3 + 2*m)*Sqrt[a - a*Sin
[e + f*x]])
```

Rule 2817

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_.) + (f
_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])^
n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(e+fx)(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx &= \frac{\int \sqrt{a-a\sin(e+fx)}\,(c+c\sin(e+fx))^{1+m}\,dx}{ac}\\
&= \frac{2\cos(e+fx)(c+c\sin(e+fx))^{1+m}}{cf(3+2m)\sqrt{a-a\sin(e+fx)}}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 85, normalized size = 1.70

$$\frac{2\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{3}(c(1+\sin(e+fx)))^{m}}{f(3+2m)\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],
x]
```

```
[Out] (2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])^3*(c*(1 + Sin[e + f*x]))^m)/(f*(3 + 2*m)*Sqrt[a - a*Sin[e + f*x]])
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int\frac{\left(\cos^{2}\left(fx+e\right)\right)\left(c+c\sin\left(fx+e\right)\right)^{m}}{\sqrt{a-a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(51) = 102.
time = 0.37, size = 116, normalized size = 2.32

$$-\frac{2\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2\right)\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{m}}{2\,afm+3\,af+\left(2\,afm+3\,af\right)\cos\left(fx+e\right)-\left(2\,afm+3\,af\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

```
[Out] -2*(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)*sq
rt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(2*a*f*m + 3*a*f + (2*a*f*m
+ 3*a*f)*cos(f*x + e) - (2*a*f*m + 3*a*f)*sin(f*x + e))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e+fx)+1))^m \cos^2(e+fx)}{\sqrt{-a(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral((c*(sin(e + f*x) + 1))**m*cos(e + f*x)**2/sqrt(-a*(sin(e + f*x) -
1)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.11, size = 68, normalized size = 1.36

$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(c\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(2\,\cos\left(e+f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\right)}{a\,f\,\left(2\,m+3\right)\,\left(\sin\left(e+f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2),x)
```

```
[Out] -((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^m*(2*cos(e + f*x) +
sin(2*e + 2*f*x)))/(a*f*(2*m + 3)*(sin(e + f*x) - 1))
```

3.81 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx$

Optimal. Leaf size=182

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-4-m}}{acf(7+2m)}+\frac{2\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(}{ac^2f\,(35+24m+4m^2)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-4-m)/a/c/f/(7+2*m)+2*c
os(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-3-m)/a/c^2/f/(4*m^2+24*
m+35)+2*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c^3/f/(
8*m^3+60*m^2+142*m+105)
```

Rubi [A]

time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^3f(2m+7)\left(4m^2+16m+15\right)}+\frac{2\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{ac^2f\left(4m^2+24m+35\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-4}}{acf(2m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-4 - m))/(
a*c*f*(7 + 2*m)) + (2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[
e + f*x])^(-3 - m))/(a*c^2*f*(35 + 24*m + 4*m^2)) + (2*Cos[e + f*x]*(a + a*
Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(a*c^3*f*(7 + 2*m)*(15
 + 16*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
```

```
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx)}{ac}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{si}}{acf(7+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{si}}{acf(7+2m)}$$
$$= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{si}}{acf(7+2m)}$$

Mathematica [A]
time = 18.39, size = 176, normalized size = 0.97

$$\frac{2^{-2-m}\cos^3\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-7-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-5-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-5-m}\left(4(6+5m+m^2)+\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)-2(5+2m)\sin(e+fx)\right)}{f(3+2m)(5+2m)(7+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 -
m),x]
```

```
[Out] (2^(-2 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-7 - 2*m)*
(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-5 - m)*(4*(6 + 5*m + m^2) + C
os[2*(-e + Pi/2 - f*x)] - 2*(5 + 2*m)*Sin[e + f*x]))/(f*(3 + 2*m)*(5 + 2*m)
*(7 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-5 - m)))
```

Maple [F]
time = 0.79, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m(c-c\sin(fx+e))^{-5-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 5)*cos(f*x + e
)^2, x)
```

Fricas [A]
time = 0.40, size = 111, normalized size = 0.61

$$-\frac{\left(2\,\cos\left(fx+e\right)^{5}+2\,\left(2\,m+5\right)\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-\left(4\,m^{2}+20\,m+25\right)\cos\left(fx+e\right)^{3}\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-5}}{8\,fm^{3}+60\,fm^{2}+142\,fm+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="fricas")
```

```
[Out] -(2*cos(f*x + e)^5 + 2*(2*m + 5)*cos(f*x + e)^3*sin(f*x + e) - (4*m^2 + 20*
m + 25)*cos(f*x + e)^3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m -
5)/(8*f*m^3 + 60*f*m^2 + 142*f*m + 105*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-5-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-5-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 5)*cos(f*x + e
)^2, x)
```

Mupad [B]
time = 15.75, size = 334, normalized size = 1.84

$$\frac{\cos\left(e+f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(24\,m^2+120\,m+140\right)}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}-\frac{\cos\left(5\,e+5\,f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{\cos\left(3\,e+3\,f\,x\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(8\,m^2+40\,m+45\right)}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{\sin\left(4\,e+4\,f\,x\right)\,\left(m\,4\mathrm{i}+10\mathrm{i}\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,1\mathrm{i}}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}+\frac{\sin\left(2\,e+2\,f\,x\right)\,\left(m\,8\mathrm{i}+20\mathrm{i}\right)\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,1\mathrm{i}}{8\,f\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+5}\,\left(8\,m^3+60\,m^2+142\,m+105\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 5),x)
```

```
[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(120*m + 24*m^2 + 140))/(8*f*(c - c*si
n(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) - (cos(5*e + 5*f*x)*(a
+ a*sin(e + f*x))^m)/(8*f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*
m^3 + 105)) + (sin(4*e + 4*f*x)*(m*4i + 10i)*(a + a*sin(e + f*x))^m*1i)/(8*
f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) + (sin(2*e +
 2*f*x)*(m*8i + 20i)*(a + a*sin(e + f*x))^m*1i)/(8*f*(c - c*sin(e + f*x))^(
m + 5)*(142*m + 60*m^2 + 8*m^3 + 105)) + (cos(3*e + 3*f*x)*(a + a*sin(e + f
*x))^m*(40*m + 8*m^2 + 45))/(8*f*(c - c*sin(e + f*x))^(m + 5)*(142*m + 60*m
^2 + 8*m^3 + 105))
```

3.82 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx$

Optimal. Leaf size=114

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-3-m}}{acf(5+2m)}+\frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e}{ac^2f\left(15+16m+4m^2\right)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-3-m)/a/c/f/(5+2*m)+cos
(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c^2/f/(4*m^2+16*m+
15)
```

Rubi [A]
time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2920, 2822, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{ac^2f\left(4m^2+16m+15\right)}+\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-3}}{acf(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-3 - m))/(
a*c*f*(5 + 2*m)) + (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e
+ f*x])^(-2 - m))/(a*c^2*f*(15 + 16*m + 4*m^2))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2822

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] + Dist[(m + n + 1)/(a*(2*m + 1)
), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &&
ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] ||  !
SumSimplerQ[n, 1])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx)}{ac} \\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{si}}{acf(5+2m)} \\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\,\mathrm{si}}{acf(5+2m)}\end{aligned}$$

Mathematica [A]
time = 14.23, size = 142, normalized size = 1.25

$$-\frac{2^{-1-m}\cos^3\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\sin^{-5-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-4-m)}(-2(2+m)+\sin(e+fx))(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-4-m}}{f(3+2m)(5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 -
m),x]
```

```
[Out] -((2^(-1 - m)*Cos[(-e + Pi/2 - f*x)/2]^3*Sin[(-e + Pi/2 - f*x)/2]^(-5 - 2*m
)*(-2*(2 + m) + Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(
-4 - m))/(f*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-
4 - m))))
```

Maple [F]
time = 1.02, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-4-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)*cos(f*x + e
)^2, x)
```

Fricas [A]
time = 0.39, size = 81, normalized size = 0.71

$$\frac{\left(2\,(m+2)\cos\left(fx+e\right)^{3}-\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)+a\right)^{m}\left(-c\sin\left(fx+e\right)+c\right)^{-m-4}}{4\,fm^{2}+16\,fm+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="fricas")
```

```
[Out] (2*(m + 2)*cos(f*x + e)^3 - cos(f*x + e)^3*sin(f*x + e))*(a*sin(f*x + e) +
a)^m*(-c*sin(f*x + e) + c)^(-m - 4)/(4*f*m^2 + 16*f*m + 15*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-4-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-4-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)*cos(f*x + e
)^2, x)
```

Mupad [B]
time = 10.60, size = 177, normalized size = 1.55

$$-\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(2\,\sin\left(2\,e+2\,f\,x\right)+\sin\left(4\,e+4\,f\,x\right)+48\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+16\,\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2+12\,m\,\left(2\,\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2-1\right)+4\,m\,\left(2\,\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-1\right)-32\right)}{c^4\,f\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(4\,m^2+16\,m+15\right)\,\left(56\,\sin\left(e+f\,x\right)^2-56\,\sin\left(e+f\,x\right)-2\,\sin\left(2\,e+2\,f\,x\right)^2+8\,\sin\left(3\,e+3\,f\,x\right)+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 4),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(2*sin(2*e + 2*f*x) + sin(4*e + 4*f*x) + 48*sin(
e/2 + (f*x)/2)^2 + 16*sin((3*e)/2 + (3*f*x)/2)^2 + 12*m*(2*sin(e/2 + (f*x)/
2)^2 - 1) + 4*m*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) - 32))/(c^4*f*(-c*(sin(e
 + f*x) - 1))^m*(16*m + 4*m^2 + 15)*(8*sin(3*e + 3*f*x) - 56*sin(e + f*x) -
 2*sin(2*e + 2*f*x)^2 + 56*sin(e + f*x)^2 + 8))
```

3.83 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=54

$$\frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))^{-2-m}}{acf(3+2m)}$$

```
[Out] cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c-c*sin(f*x+e))^(-2-m)/a/c/f/(3+2*m)
```

Rubi [A]
time = 0.17, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2920, 2821}

$$\frac{\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c-c\sin(e+fx))^{-m-2}}{acf(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m),x]
```

```
[Out] (Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c - c*Sin[e + f*x])^(-2 - m))/(
a*c*f*(3 + 2*m))
```

Rule 2821

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(
(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1))), x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && Ne
Q[m, -2^(-1)]
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\begin{aligned}\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx &= \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx\\ &\,\,\overline{ac}\\ &= \frac{\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c-c}{acf(3+2m)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(54) = 108.
time = 6.95, size = 109, normalized size = 2.02

$$\frac{2^{-m}\cos^{-3-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-m}\sin^3\left(\frac{1}{4}(2e+\pi+2fx)\right)}{c^3f(3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 -
m),x]
```

```
[Out] (Cos[(2*e + Pi + 2*f*x)/4]^(-3 - 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])
^(2*m)*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4]^3)/(2^m*c^3*f*(3
+ 2*m)*(c - c*Sin[e + f*x])^m)
```

Maple [F]
time = 0.81, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^m(c-c\sin(fx+e))^{-3-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e
)^2, x)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.94

$$\frac{(a\sin(fx+e)+a)^m(-c\sin(fx+e)+c)^{-m-3}\cos(fx+e)^3}{2\,fm+3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="fricas")
```

```
[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e)^3/(2*f*m
 + 3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 3)*cos(f*x + e
)^2, x)
```

Mupad [**B**]
time = 1.04, size = 101, normalized size = 1.87

$$-\frac{(a\,(\sin\left(e+f\,x\right)+1))^m\,\left(6\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2+2\sin\left(\frac{3\,e}{2}+\frac{3\,f\,x}{2}\right)^2-4\right)}{c^3\,f\,\left(2\,m+3\right)\,\left(-c\,\left(\sin\left(e+f\,x\right)-1\right)\right)^m\,\left(12\sin\left(e+f\,x\right)^2-15\,\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 3),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(6*sin(e/2 + (f*x)/2)^2 + 2*sin((3*e)/2 + (3*f*x
)/2)^2 - 4))/(c^3*f*(2*m + 3)*(-c*(sin(e + f*x) - 1))^m*(sin(3*e + 3*f*x) -
 15*sin(e + f*x) + 12*sin(e + f*x)^2 + 4))
```

3.84 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=113

$$\frac{2^{-\frac{1}{2}-m}\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+a\,\mathrm{s}}{f(3+2m)}$$

```
[Out] 2^(-1/2-m)*cos(f*x+e)^3*hypergeom([3/2+m, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e)
)*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2*
m)
```

Rubi [A]
time = 0.26, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2824, 2768, 72, 71}

$$\frac{2^{-m-\frac{1}{2}}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m),x]
```

```
[Out] (2^(-1/2 - m)*Cos[e + f*x]^3*Hypergeometric2F1[(3 + 2*m)/2, (3 + 2*m)/2, (5
 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
+ f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{3}{2}-m}c\cos^{1-2m+2(1+m)}(e+fx)(a+a\,s\right.}{}$$

$$= \frac{2^{-\frac{1}{2}-m}\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(3+2m),\frac{1}{2}(3-\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.52, size = 589, normalized size = 5.21

$$-\frac{2^{1-m}(-3+2m)\cos^2\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\cot\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\csc^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(-\left((1+2m)F_1\left(\frac{1}{2}-m;-2(1+m),1;\frac{3}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\right)+(-1+2m)\cot^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\,{}_2F_1\left(-\frac{1}{2}-m,-2(1+m);\frac{1}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\right)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-2-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}}{f\left(-1+4m^2\right)\left(8(1+m)F_1\left(\frac{3}{2}-m;-1-2m,1;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+4F_1\left(\frac{3}{2}-m;-2(1+m),2;\frac{5}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+(-3+2m)\left(2F_1\left(\frac{1}{2}-m;-2(1+m),1;\frac{3}{2}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right),-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\cot^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)-\csc^4\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)(1+\sin(e+fx))\left(1-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^{2m}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 -
m),x]
```

```
[Out] -((2^(1 - m)*(-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/2]^2*Cot[(-e + Pi/2 - f*x)/4]
*Csc[(-e + Pi/2 - f*x)/4]^2*(-((1 + 2*m)*AppellF1[1/2 - m, -2*(1 + m), 1, 3
/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]) + (-1 + 2
*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeometric2F1[-1/2 - m, -2*(1 + m), 1/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2])*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 - m))/(f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^(2*(-2 - m))*(8*(1 + m)*AppellF1[3/2 - m, -1 - 2*m,
1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*Ap
pellF1[3/2 - m, -2*(1 + m), 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2] + (-3 + 2*m)*(2*AppellF1[1/2 - m, -2*(1 + m), 1, 3/2
- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/
2 - f*x)/4]^2 - Csc[(-e + Pi/2 - f*x)/4]^4*(1 + Sin[e + f*x])*(1 - Tan[(-e
+ Pi/2 - f*x)/4]^2)^(2*m)))))
```

Maple [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(f x +e \right)\right) \left(a +a \sin \left(f x +e \right)\right)^{m} \left(c -c \sin \left(f x +e \right)\right)^{-2-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e)
^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)*cos(f*x + e
)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{m+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 2),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 2), x
)
```

3.85 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=114

$$\frac{2^{\frac{1}{2}-m}c\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+a\,\mathrm{s}}{f(3+2m)}$$

```
[Out] 2^(1/2-m)*c*cos(f*x+e)^3*hypergeom([1/2+m, 3/2+m],[5/2+m],1/2+1/2*sin(f*x+e
))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(3+2
*m)
```

Rubi [A]
time = 0.24, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2824, 2768, 72, 71}

$$\frac{c2^{\frac{1}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m+1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m),x]
```

```
[Out] (2^(1/2 - m)*c*Cos[e + f*x]^3*Hypergeometric2F1[(1 + 2*m)/2, (3 + 2*m)/2, (
5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+f\ldots}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-\ldots\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+\ldots\right.}{\ldots}$$

$$= \frac{\left(2^{-\frac{1}{2}-m}c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\ldots\right.}{\ldots}$$

$$= \frac{2^{\frac{1}{2}-m}c\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m),\frac{1}{2}(3+\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 24.32, size = 857, normalized size = 7.52

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 -
m),x]
```

```
[Out] -((2^(2 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 4*AppellF1[1/2 - m, -2*m, 2
, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 4*
AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e
 - Pi + 2*f*x)/8]^2])*Cos[(2*e + Pi + 2*f*x)/4]^(1 - 2*m)*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^(2*(1 + m))*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2
*f*x)/4]^2)/(c*f*(-1 + 2*m)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^m*((-3
 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -
Tan[(2*e - Pi + 2*f*x)/8]^2] + 2*((6 - 4*m)*AppellF1[1/2 - m, -2*m, 2, 3/2
- m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + (-6 + 4*
m)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(
2*e - Pi + 2*f*x)/8]^2] + (2*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(
-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 8*m*AppellF1[3/2 -
 m, 1 - 2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f
*x)/8]^2] + 8*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f
*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + AppellF1[3/2 - m, -2*m, 2, 5/2 -
m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 8*AppellF1
[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi +
2*f*x)/8]^2] + 12*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-2*e + Pi - 2*f*
x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Tan[(2*e - Pi + 2*f*x)/8]^2))))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{-1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e)
^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x, algori
thm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 1)*cos(f*x + e
)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+fx)^2 (a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 1),x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 1), x
)
```

3.86 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=116

$$\frac{2^{\frac{3}{2}-m}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+\ldots}{f(3+2m)}$$

```
[Out] 2^(3/2-m)*c^2*cos(f*x+e)^3*hypergeom([3/2+m, -1/2+m],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(
3+2*m)
```

Rubi [A]
time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2920, 2824, 2768, 72, 71}

$$\frac{c^2 2^{\frac{3}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-1),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(3/2 - m)*c^2*Cos[e + f*x]^3*Hypergeometric2F1[(-1 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx))}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-c\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+f\right.}{}$$

$$= \frac{\left(2^{\frac{1}{2}-m}c^3\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(\right.}{}$$

$$= \frac{2^{\frac{3}{2}-m}c^2\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-1+2m),\frac{1}{2}(3\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 16.99, size = 1255, normalized size = 10.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,x]
```

```
[Out] (2^(3 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e + Pi -
 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 5*AppellF1[1/2 - m, -2*m, 3,
3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 8*Ap
pellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e -
 Pi + 2*f*x)/8]^2] - 4*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-2*e + Pi -
 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2])*Cos[(2*e + Pi + 2*f*x)/4]^(3 -
 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a*(1 + Sin[e + f*x]))^m*
Sin[(2*e + Pi + 2*f*x)/4]^2)/(f*(-1 + 2*m)*(c - c*Sin[e + f*x])^m*((-3 + 2*
m)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(
2*e - Pi + 2*f*x)/8]^2] - 5*(-3 + 2*m)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m,
Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] + 8*(-3 + 2*m)*
AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e
 - Pi + 2*f*x)/8]^2] + 12*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-2*e + P
i - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] - 8*m*AppellF1[1/2 - m, -2*m
, 5, 3/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2] +
 4*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -
Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f*x)/8]^2 - 20*m*AppellF1[3/
2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi +
2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f*x)/8]^2 + 32*m*AppellF1[3/2 - m, 1 - 2*m,
4, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan
[(2*e - Pi + 2*f*x)/8]^2 - 16*m*AppellF1[3/2 - m, 1 - 2*m, 5, 5/2 - m, Tan[
(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f
*x)/8]^2 + 4*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]
^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f*x)/8]^2 - 30*AppellF1
[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi +
2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f*x)/8]^2 + 64*AppellF1[3/2 - m, -2*m, 5, 5/
2 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan[(2*e
 - Pi + 2*f*x)/8]^2 - 40*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-2*e + Pi
 - 2*f*x)/8]^2, -Tan[(2*e - Pi + 2*f*x)/8]^2]*Tan[(2*e - Pi + 2*f*x)/8]^2))
```

Maple [F]

time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right) \left(a+a\sin\left(fx+e\right)\right)^m \left(c-c\sin\left(fx+e\right)\right)^{-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, algorithm
="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)^2/(-c*sin(f*x + e) + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+fx)^2 \left(a+a\sin(e+fx)\right)^m}{\left(c-c\sin(e+fx)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)
```

```
[Out] int((cos(e + f*x)^2*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x)
```

3.87 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=116

$$\frac{2^{\frac{5}{2}-m}c^3\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(3+2m);\frac{1}{2}(5+2m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1}{2}+m}(a+}{f(3+2m)}$$

```
[Out] 2^(5/2-m)*c^3*cos(f*x+e)^3*hypergeom([-3/2+m, 3/2+m],[5/2+m],1/2+1/2*sin(f*
x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m)/f/(
3+2*m)
```

Rubi [A]
time = 0.25, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2920, 2824, 2768, 72, 71}

$$\frac{c^3 2^{\frac{5}{2}-m}\cos^3(e+fx)(1-\sin(e+fx))^{m+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-2}\,{}_2F_1\left(\frac{1}{2}(2m-3),\frac{1}{2}(2m+3);\frac{1}{2}(2m+5);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(5/2 - m)*c^3*Cos[e + f*x]^3*Hypergeometric2F1[(-3 + 2*m)/2, (3 + 2*m)/2
, (5 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2824

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e
+ f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPart[m]/Cos[e + f*x]^(2*FracP
art[m])), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] ||  !FractionQ[n])
```

Rule 2920

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(a^(p/
2)*c^(p/2)), Int[(a + b*Sin[e + f*x])^(m + p/2)*(c + d*Sin[e + f*x])^(n + p
/2), x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[b*c + a*d, 0] && E
qQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \frac{\int (a+a\sin(e+fx))^{1+m}(c-c\sin(e+fx)}{ac}$$

$$= \left(\cos^{-2m}(e+fx)(a+a\sin(e+fx))^m(c-\right.$$

$$= \frac{\left(c^2\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin(e+\right.}{}$$

$$= \frac{\left(2^{\frac{3}{2}-m}c^4\cos^{1-2m+2(1+m)}(e+fx)(a+a\sin\right.}{}$$

$$= \frac{2^{\frac{5}{2}-m}c^3\cos^3(e+fx)\,{}_2F_1\left(\frac{1}{2}(-3+2m),\frac{1}{2}(3\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 25.36, size = 2082, normalized size = 17.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m
),x]
```

```
[Out] (2^(5 - m)*(-3 + 2*m)*(AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 -
 f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 6*AppellF1[1/2 - m, -2*m, 4, 3/2
 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 13*AppellF
1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] - 12*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 4*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f
*x)/4]^3*Cos[(-e + Pi/2 - f*x)/2]^2*Sin[(-e + Pi/2 - f*x)/4]*Sin[(-e + Pi/2
 - f*x)/2]^(4 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(
f*(-1 + 2*m)*((-3 + 2*m)*AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 18*
AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 - 12*m*AppellF1[1/2 - m, -2*m,
 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(
-e + Pi/2 - f*x)/4]^2 - 39*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 2
6*m*AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-
e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 36*AppellF1[1/2 - m, -2*
m, 6, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos
[(-e + Pi/2 - f*x)/4]^2 - 24*m*AppellF1[1/2 - m, -2*m, 6, 3/2 - m, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2
 - 12*AppellF1[1/2 - m, -2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2]*Cos[(-e + Pi/2 - f*x)/4]^2 + 8*m*AppellF1[1/2 - m,
-2*m, 7, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
Cos[(-e + Pi/2 - f*x)/4]^2 + 72*AppellF1[3/2 - m, -2*m, 7, 5/2 - m, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Cos[(-e + Pi/2 - f*
x)/2]) + 4*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4
]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 24*m*AppellF
1[3/2 - m, 1 - 2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 52*m*AppellF1[3/2 - m, 1 - 2*m,
5, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-
e + Pi/2 - f*x)/4]^2 - 48*m*AppellF1[3/2 - m, 1 - 2*m, 6, 5/2 - m, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2
 + 16*m*AppellF1[3/2 - m, 1 - 2*m, 7, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 6*AppellF1[3/2 -
m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2]*Sin[(-e + Pi/2 - f*x)/4]^2 - 48*AppellF1[3/2 - m, -2*m, 5, 5/2 - m, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/
4]^2 + 130*AppellF1[3/2 - m, -2*m, 6, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2,
-Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^2 + 56*AppellF1[3/2 -
```

```
 m, -2*m, 8, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*Sin[(-e + Pi/2 - f*x)/4]^2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(1
 - m)))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e)
^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1)*cos(f*x + e
)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m),x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m), x)
```

3.88 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx$

Optimal. Leaf size=343

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^4g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac}{}$$

```
[Out] 10/77*a*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e)
)^(1/2)+2/33*a*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f
*x+e))^(1/2)-2/11*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f/g/(a+a*si
n(f*x+e))^(1/2)+2/3*a*c^4*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(
c-c*sin(f*x+e))^(1/2)+2*a*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^
(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/7*a*c^3*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.11, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2ac^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}+\frac{10ac^2(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{77fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{33fg\sqrt{a\sin(e+fx)+a}}-\frac{2a(c-c\sin(e+fx))^{7/2}(g\cos(e+fx))^{5/2}}{11fg\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7
/2),x]
```

```
[Out] (2*a*c^4*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (2*a*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
+ (2*a*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*f*g*Sqrt[a +
 a*Sin[e + f*x]]) + (10*a*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(77*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c
- c*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e
+ f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx)}{11fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))}{33fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{10ac^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx}{77fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2ac^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx}}{7fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}$$

$$= \frac{2ac^4(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.70, size = 311, normalized size = 0.91

$$\frac{c^4e^{-5i(e+fx)}\left(-i+e^{i(e+fx)}\right)g\sqrt{g\cos(e+fx)}\left(\sqrt{1+e^{2i(e+fx)}}\left(-21i+154e^{i(e+fx)}+423ie^{2i(e+fx)}-308e^{3i(e+fx)}+1374ie^{4i(e+fx)}-7392e^{5i(e+fx)}+1374ie^{6i(e+fx)}+308e^{7i(e+fx)}+423ie^{8i(e+fx)}-154e^{9i(e+fx)}-21ie^{10i(e+fx)}\right)+4928e^{7i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{3696\left(i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(7/2),x]
```

```
[Out] (c^4*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(-21*I + 154*E^(I*(e + f*x)) + (423*I)*E^((2*I)*(e + f*x)) - 308*E^(
(3*I)*(e + f*x)) + (1374*I)*E^((4*I)*(e + f*x)) - 7392*E^((5*I)*(e + f*x))
+ (1374*I)*E^((6*I)*(e + f*x)) + 308*E^((7*I)*(e + f*x)) + (423*I)*E^((8*I)
*(e + f*x)) - 154*E^((9*I)*(e + f*x)) - (21*I)*E^((10*I)*(e + f*x))) + 4928
*E^((7*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))]
)*Sqrt[a*(1 + Sin[e + f*x])])/(3696*E^((5*I)*(e + f*x))*(I + E^(I*(e + f*x)
))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 33.36, size = 425, normalized size = 1.24

method	result
default	$-\frac{2(-c(\sin(fx+e)-1))^{\frac{7}{2}}\left(21\left(\cos^6(fx+e)\right)\sin(fx+e)+231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/231/f*(-c*(sin(f*x+e)-1))^(7/2)*(21*cos(f*x+e)^6*sin(f*x+e)+231*I*sin(f*
x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e))
)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)-77*cos(f*x+e)^6+231*I*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)-231*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f
*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-132*cos(f*x+e)^4*sin(f*x+e)
+154*cos(f*x+e)^4+154*cos(f*x+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*
(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+
4)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 183, normalized size = 0.53

$$\frac{-231i\sqrt{2}\sqrt{acg}\,c^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,c^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\left(21c^3g\cos(fx+e)^4-132c^3g\cos(fx+e)^2+77\left(c^3g\cos(fx+e)^2-c^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{231f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/231*(-231*I*sqrt(2)*sqrt(a*c*g)*c^3*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
c^3*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) - 2*(21*c^3*g*cos(f*x + e)^4 - 132*c^3*g*cos(f*x + e)^2 + 77*(
c^3*g*cos(f*x + e)^2 - c^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(7/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(7/2), x)
```

3.89 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=290

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^3g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a}{}$$

```
[Out] 2/21*a*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(
1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))
^(1/2)+22/45*a*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin
(f*x+e))^(1/2)+22/15*a*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/105*a*c^2*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22ac^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{105fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{21fg\sqrt{a\sin(e+fx)+a}}-\frac{2a(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (22*a*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) + (22*a*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]]) + (22*a*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(105*f*g
*Sqrt[a + a*Sin[e + f*x]]) + (2*a*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2))/(21*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)
*(c - c*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\begin{aligned}
\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx}{9fg\sqrt{a+a\sin(e+fx)}} \\
&= \frac{2ac(g\cos(e+fx))^{5/2}(c-c\sin(e+fx)}{21fg\sqrt{a+a\sin(e+fx)}} \\
&= \frac{22ac^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+}}{105fg\sqrt{a+a\sin(e+fx)}} \\
&= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e}} \\
&= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e}} \\
&= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e}} \\
&= \frac{22ac^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 1.39, size = 281, normalized size = 0.97

$$-\frac{c^3 e^{-4i(e+fx)} \left(-i+e^{i(e+fx)}\right) g \sqrt{g\cos(e+fx)} \left(\sqrt{1+e^{2i(e+fx)}} \left(-35-180ie^{i(e+fx)}+238e^{2i(e+fx)}-540ie^{3i(e+fx)}+3696e^{4i(e+fx)}-540ie^{5i(e+fx)}-238e^{6i(e+fx)}-180ie^{7i(e+fx)}+35e^{8i(e+fx)}\right)-2464e^{6i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{2520\left(i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\, f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] -1/2520*(c^3*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*
I)*(e + f*x))]*(-35 - (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) - (
540*I)*E^((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) - (540*I)*E^((5*I)*(e
 + f*x)) - 238*E^((6*I)*(e + f*x)) - (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8
*I)*(e + f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(E^((4*I)*(e + f*x))*(I
 + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[e + f*x]
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 392, normalized size = 1.35

method	result
default	$-\frac{2(-c(\sin(fx+e)-1))^{\frac{5}{2}}\left(231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,-231i\sin(f\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/315/f*(-c*(sin(f*x+e)-1))^(5/2)*(231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+35*cos
(f*x+e)^6+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+90*cos(f*x+e)^4*sin(f*x+e)-112*cos(f*x+e)^4-154*cos(f*
x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f
*x+e)^2+2*sin(f*x+e)-2)/sin(f*x+e)/cos(f*x+e)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 169, normalized size = 0.58

$$\frac{-231i\,\sqrt{2}\,\sqrt{acg}\,c^{2}g\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+231i\,\sqrt{2}\,\sqrt{acg}\,c^{2}g\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))+2\left(90\,c^{2}g\cos(fx+e)^{2}-7\left(5\,c^{2}g\cos(fx+e)^{2}-11\,c^{2}g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/315*(-231*I*sqrt(2)*sqrt(a*c*g)*c^2*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
c^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) + 2*(90*c^2*g*cos(f*x + e)^2 - 7*(5*c^2*g*cos(f*x + e)^2 - 11*
c^2*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

3.90 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=235

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{6ac}{}$$

```
[Out] -2/7*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(1/
2)+2/5*a*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+6/5*a*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+
a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+6/35*a*c*(g*cos(f*x+e))^(5/2)*(c
-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6ac^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}+\frac{6ac\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{35fg\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2*a*c^2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (6*a*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) + (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\begin{aligned}\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}(c-c\sin(e+fx)}{7fg\sqrt{a+a\sin(e+fx)}} \\ &= \frac{6ac(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx}}{35fg\sqrt{a+a\sin(e+fx)}} \\ &= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+}} \\ &= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+}} \\ &= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+}} \\ &= \frac{2ac^2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.24, size = 255, normalized size = 1.09

$$\frac{c^2e^{-3i(e+fx)}\left(-i+e^{i(e+fx)}\right)g\sqrt{g\cos(e+fx)}\left(\sqrt{1+e^{2i(e+fx)}}\left(5i-14e^{i(e+fx)}+15ie^{2i(e+fx)}-168e^{3i(e+fx)}+15ie^{4i(e+fx)}+14e^{5i(e+fx)}+5ie^{6i(e+fx)}\right)+112e^{5i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{140\left(i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*
x])^(3/2),x]
```

```
[Out] (c^2*(-I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e +
f*x))]*(5*I - 14*E^(I*(e + f*x)) + (15*I)*E^((2*I)*(e + f*x)) - 168*E^((3*I
)*(e + f*x)) + (15*I)*E^((4*I)*(e + f*x)) + 14*E^((5*I)*(e + f*x)) + (5*I)*
E^((6*I)*(e + f*x))) + 112*E^((5*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4,
7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(140*E^((3*I)*(e +
f*x))*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[c - c*Sin[
e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 372, normalized size = 1.58

method	result
default	$-\frac{2(-c(\sin(fx+e)-1))^{\frac{3}{2}}\left(-21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}+21i\sin(\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/35/f*(-c*(sin(f*x+e)-1))^(3/2)*(-21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)+21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5*cos(f*
x+e)^4*sin(f*x+e)-21*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*sin(f*x+e)
*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-7*cos(f*x+e)^4-14*cos(f*x+e)^2+21*cos(f*x+e))*(g
*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)^3/sin
(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 143, normalized size = 0.61

$$\frac{-21i\sqrt{2}\,\sqrt{acg}\,cg\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+21i\sqrt{2}\,\sqrt{acg}\,cg\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))+2\left(5\,cg\cos(fx+e)^2+7\,cg\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/35*(-21*I*sqrt(2)*sqrt(a*c*g)*c*g*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*sqrt(a*c*g)*c*g*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
 e))) + 2*(5*c*g*cos(f*x + e)^2 + 7*c*g*sin(f*x + e))*sqrt(g*cos(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

3.91 $\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e-$

Optimal. Leaf size=178

$$\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g}{}$$

```
[Out] 2/5*a*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1
/2)+6/5*a*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f
*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/5*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x
+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{2a\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}+\frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6acg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] (2*a*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (6*a*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) -
(2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*Sin
[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
```

```
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f}}$$

$$= \frac{2ac(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+f}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.68, size = 249, normalized size = 1.40

$$\frac{(g\cos(e+fx))^{3/2}\csc\left(\frac{e}{2}\right)\sec\left(\frac{e}{2}\right)\sec^3(e+fx)\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}\,\left(-11\cos(fx)-13\cos(2e+fx)+\cos(2e+3fx)-\cos(4e+3fx)+12\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2ifx}(\cos(e)+i\sin(e))^2\right)(\cos(fx)-i\sin(fx))\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}+4\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2ifx}(\cos(e)+i\sin(e))^2\right)(\cos(fx)+i\sin(fx))\sqrt{1+\cos(2(e+fx))+i\sin(2(e+fx))}\right)}{40f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e/2]*Sec[e/2]*Sec[e + f*x]^3*Sqrt[a*(1 + Sin[e
+ f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-11*Cos[f*x] - 13*Cos[2*e + f*x] + Cos[2
```

```
*e + 3*f*x] - Cos[4*e + 3*f*x] + 12*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^(
(2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] - I*Sin[f*x])*Sqrt[1 + Cos[2*(
e + f*x)] + I*Sin[2*(e + f*x)]] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((
2*I)*f*x)*(Cos[e] + I*Sin[e])^2)]*(Cos[f*x] + I*Sin[f*x])*Sqrt[1 + Cos[2*(e
 + f*x)] + I*Sin[2*(e + f*x)]]))/(40*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.30, size = 344, normalized size = 1.93

method	result
default	$-\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(3i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(-c*(sin(f*x+e)-1))^(1/2)*(3*I*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-3*I*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(
-1+cos(f*x+e))/sin(f*x+e),I)-3*I*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f
*x+e)^4+2*cos(f*x+e)^2-3*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e))
)^(1/2)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 124, normalized size = 0.70

$$\frac{2\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,g\sin\left(fx+e\right)-3i\sqrt{2}\,\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3i\sqrt{2}\,\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{5\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)*g*sin(f*x + e) - 3*I*sqrt(2)*sqrt(a*c*g)*g*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(a*
c*g)*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))))/f
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x +
 e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

3.92 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2ag\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/3*a*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/
2)+2*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e)
)^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]]) + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
```

```
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + a\int \frac{}{\sqrt{a+}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(ag\cos(e}{\sqrt{a+a\sin}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\left(ag\sqrt{\cos(}\right.}{\sqrt{}}$$

$$= -\frac{2a(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2ag\sqrt{\cos(}}{f\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.44, size = 197, normalized size = 1.61

$$-\frac{i\sqrt{ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}\,g\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g}\left(-i\sqrt{1+e^{2i(e+fx)}}\left(1-6ie^{i(e+fx)}+e^{2i(e+fx)}\right)+4e^{3i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{3c\left(1+e^{2i(e+fx)}\right)^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((-1/3*I)*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*g*Sqrt[((1 +
 E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((-I)*Sqrt[1 + E^((2*I)*(e + f*x)
)]*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + 4*E^((3*I)*(e + f*x)
)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e
 + f*x])])/(c*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 362, normalized size = 2.97

method	result
default	$\frac{2\sqrt{a\left(1+\sin\left(fx+e\right)\right)}\,\left(g\cos(fx+e)\right)^{\frac{3}{2}}\left(3i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/3/f*(a*(1+sin(f*x+e)))^(1/2)*(g*cos(f*x+e))^(3/2)*(3*I*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))
^(1/2)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f
*x+e)))^(1/2)-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+3*cos(f*x+e))/(1+sin(f
*x+e))/cos(f*x+e)/sin(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x +
 e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 0.98

$$\frac{-3i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+3i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)-2\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g}{3cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*sqrt(2)*sqrt(a*c*g)*g*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(a*c*g)*g*weierstr
```

```
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) -
 2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
g)/(c*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

3.93 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-
6*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))
^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$\frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)) - (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(3a)\int \cdots}{\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(3ag\cos(e\cdots}{c\sqrt{a+a\sin\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\left(3ag\sqrt{\cos(e\cdots}\right.}{c\sqrt{\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{6ag\sqrt{\cos(e\cdots}}{cf\sqrt{a+\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.25, size = 211, normalized size = 1.72

$$-\frac{2g\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g}\left(\left(i-5e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}+2e^{2i(e+fx)}\left(-i+e^{i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{c\sqrt{ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}\left(i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (-2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((I - 5*E^(I*(e +
 f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] + 2*E^((2*I)*(e + f*x))*(-I + E^(I*(e
 + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1
```

```
 + Sin[e + f*x])])/(c*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*
(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

Maple [C] Result contains complex when optimal does not.
time = 10.07, size = 2834, normalized size = 23.04

method	result	size
default	Expression too large to display	2834

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1+cos(f*x+e))*(ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/si
n(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(3/2)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*
x+e)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x
+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+6*I*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
2*sin(f*x+e)-2*cos(f*x+e)^2*sin(f*x+e)+4*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(3/2)-4*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*co
s(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+6*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)-6*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+4*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(3/2)-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-4*ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(3/2)+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)
```

```
/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+12*I*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^2-12*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2
+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos
(f*x+e)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-6*I*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*cos(f*x+e)^3+10*cos(f*x+e)^2-2*cos(f*x+e)^3+ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/
2)-ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(3/2)-3*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)+3*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(3/2)-3*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*cos(f*x+e)^
2*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2
)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+
e)))^(1/2)/(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)^2-2*sin(f*x+e)+cos(f*x+e)-2)/(
-c*(sin(f*x+e)-1))^(3/2)/cos(f*x+e)/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 167, normalized size = 1.36

$$-\frac{4\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g+3\sqrt{acg}\left(-i\sqrt{2}\,g\sin\left(fx+e\right)+i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))+3\sqrt{acg}\left(i\sqrt{2}\,g\sin\left(fx+e\right)-i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))}{c^2f\sin\left(fx+e\right)-c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -(4*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*g + 3*sqrt(a*c*g)*(-I*sqrt(2)*g*sin(f*x + e) + I*sqrt(2)*g)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqr
t(a*c*g)*(I*sqrt(2)*g*sin(f*x + e) - I*sqrt(2)*g)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(c^2*f*sin(f*x +
e) - c^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

3.94 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{6ag\sqrt{\cos(e}}{5c^2f\sqrt{a}}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
)-6/5*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^
(1/2)+6/5*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{6ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6a(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2)) - (6*a*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*
x]]*(c - c*Sin[e + f*x])^(3/2)) + (6*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{}{\sqrt{a}}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{}{5cfg\sqrt{a-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{}{5cfg\sqrt{a-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{}{5cfg\sqrt{a-}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} - \frac{}{5cfg\sqrt{a-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.48, size = 229, normalized size = 1.26

$$\frac{4ig\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g}\left(\left(5+4ie^{i(e+fx)}-3e^{2i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}+e^{i(e+fx)}\left(-i+e^{i(e+fx)}\right)^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{5c\left(ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2\right)^{3/2}\left(i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 + (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(-I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(c*((I*c*(-I + E^(I*(e +
f*x)))^2)/E^(I*(e + f*x)))^(3/2)*(I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

Maple [C] Result contains complex when optimal does not.
time = 1.60, size = 2040, normalized size = 11.21

method	result	size
default	Expression too large to display	2040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/10/f*(a*(1+sin(f*x+e)))^(1/2)*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^3*(si
n(f*x+e)-1)*(12*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)+5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*sin(f*x+e)-5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)-12*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^2-4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*si
n(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-12*
I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)
-24*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*
x+e)+24*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*co
s(f*x+e)+12*I*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*EllipticF(I
```

```
*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(
f*x+e)))^(1/2)-12*I*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+24*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x
+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(1+cos(f*x+e)))^(1/2)-24*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+24*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-24*I*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+12*I*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)-12*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*si
n(f*x+e)*cos(f*x+e)^2+8*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-12*I*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+12*I*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+5*cos(f*x+e)^3*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^3*ln(-2*(2*
cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+5*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)+8*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+20*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)*cos(f*x+e)-20*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3)/(1+sin(
f*x+e))/sin(f*x+e)^7/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)/(-c*(sin(f*x+e)-1
))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 225, normalized size = 1.24

$$-\frac{2\,\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(3\,g\sin\left(fx+e\right)-g\right)+3\,\left(i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}+2i\,\sqrt{2}\,g\sin\left(fx+e\right)-2i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3\,\left(-i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}-2i\,\sqrt{2}\,g\sin\left(fx+e\right)+2i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{5\,\left(c^{3}f\cos\left(fx+e\right)^{2}+2\,c^{3}f\sin\left(fx+e\right)-2\,c^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)*(3*g*sin(f*x + e) - g) + 3*(I*sqrt(2)*g*cos(f*x + e)^2 + 2*I*sqrt(2)*g
*sin(f*x + e) - 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(-I*sqrt(2)*g*cos(f*x
 + e)^2 - 2*I*sqrt(2)*g*sin(f*x + e) + 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(
c^3*f*cos(f*x + e)^2 + 2*c^3*f*sin(f*x + e) - 2*c^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

3.95 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=237

$$\frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}-\frac{\quad}{15c^2fg\sqrt{\ }}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
)-2/15*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))
^(1/2)-2/15*a*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(
f*x+e))^(1/2)+2/15*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elli
pticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^3
/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g
*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (2*a*g*Sqrt[Cos[e +
 f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a
*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{a\int \cdots}{\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{\cdots}{15cfg\sqrt{a\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{\cdots}{15cfg\sqrt{a\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{\cdots}{15cfg\sqrt{a\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{\cdots}{15cfg\sqrt{a\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} - \frac{\cdots}{15cfg\sqrt{a\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.59, size = 256, normalized size = 1.08

$$\frac{4e^{3i(e+fx)}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g\right)^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}\left(-29i+e^{i(e+fx)}+15ie^{2i(e+fx)}-3e^{3i(e+fx)}\right)+\left(-i+e^{i(e+fx)}\right)^5{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{45c^3\left(-i+e^{i(e+fx)}\right)^4\sqrt{ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}\left(i+e^{i(e+fx)}\right)\left(1+e^{2i(e+fx)}\right)^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(7/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-29*I + E^(I*(e + f*x)) + (15*I)*E^((2*I)
*(e + f*x)) - 3*E^((3*I)*(e + f*x))) + (-I + E^(I*(e + f*x)))^5*Hypergeomet
ric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(4
5*c^3*(-I + E^(I*(e + f*x)))^4*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^2)/E^(I*(e
+ f*x))]*(I + E^(I*(e + f*x)))*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] Result contains complex when optimal does not.
time = 5.17, size = 966, normalized size = 4.08

method	result	size
default	Expression too large to display	966

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x,method=_RETURNVERBOSE)

[Out] 2/45/f*(a*(1+sin(f*x+e)))^(1/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)-sin(f*x+e)+1)*(-12*I*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9*I*cos(f*x+e)^2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+12*I*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*cos(f*x+e)^4*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*cos(f*x+e)^4*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+6*I*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9*I*cos(f*x+e)^2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*cos(f*x+e)^4+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+10*cos(f*x+e)^3+6*cos(f*x+e)^2*sin(f*x+e)-19*cos(f*x+e)^2-16*cos(f*x+e)*sin(f*x+e)-4*cos(f*x+e)+10*sin(f*x+e)+10)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(7/2)/cos(f*x+e)/sin(f*x+e)^5

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 287, normalized size = 1.21

$$-\frac{2 \, {\left(3 \, g \cos\left(f x + e\right)^{2} + 9 \, g \sin\left(f x + e\right) + g\right)} \sqrt{g \cos\left(f x + e\right)} \sqrt{a \sin\left(f x + e\right) + a} \sqrt{-c \sin\left(f x + e\right) + c} - 3 \, {\left(-3 i \, \sqrt{2} g \cos\left(f x + e\right)^{2} + {\left(i \, \sqrt{2} g \cos\left(f x + e\right)^{2} - 4 i \, \sqrt{2} g\right)} \sin\left(f x + e\right) + 4 i \, \sqrt{2} g\right)} \sqrt{a c g} {\rm weierstrassZeta}\left(-4, 0, {\rm weierstrassPInverse}\left(-4, 0, \cos\left(f x + e\right) + i \, \sin\left(f x + e\right)\right)\right) - 3 \, {\left(3 i \, \sqrt{2} g \cos\left(f x + e\right)^{2} + {\left(-i \, \sqrt{2} g \cos\left(f x + e\right)^{2} + 4 i \, \sqrt{2} g\right)} \sin\left(f x + e\right) - 4 i \, \sqrt{2} g\right)} \sqrt{a c g} {\rm weierstrassZeta}\left(-4, 0, {\rm weierstrassPInverse}\left(-4, 0, \cos\left(f x + e\right) - i \, \sin\left(f x + e\right)\right)\right)}{45 \, {\left(3 \, c^{4} f \cos\left(f x + e\right)^{2} - 4 \, c^{4} f - {\left(c^{4} f \cos\left(f x + e\right)^{2} - 4 \, c^{4} f\right)} \sin\left(f x + e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] -1/45*(2*(3*g*cos(f*x + e)^2 + 9*g*sin(f*x + e) + g)*sqrt(g*cos(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) - 3*(-3*I*sqrt(2)*g*cos(f
*x + e)^2 + (I*sqrt(2)*g*cos(f*x + e)^2 - 4*I*sqrt(2)*g)*sin(f*x + e) + 4*I
*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(f*x + e) + I*sin(f*x + e))) - 3*(3*I*sqrt(2)*g*cos(f*x + e)^2 + (-I*sqrt
(2)*g*cos(f*x + e)^2 + 4*I*sqrt(2)*g)*sin(f*x + e) - 4*I*sqrt(2)*g)*sqrt(a*
c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e))))/(3*c^4*f*cos(f*x + e)^2 - 4*c^4*f - (c^4*f*cos(f*x + e)^2 - 4*
c^4*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{a+a\,\sin\left(e+f\,x\right)}}{(c-c\,\sin\left(e+f\,x\right))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

3.96 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=292

$$\frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{39cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}-\frac{}{65c^2fg\sqrt{}}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(1/
2)-2/39*a*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)
)^(1/2)-2/65*a*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin
(f*x+e))^(1/2)-2/65*a*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+2/65*a*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(
1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{2ag\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{65c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{65c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{2a(g\cos(e+fx))^{5/2}}{39cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e + f*x])^
(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2))/(13*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(9/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(39*c*f*g*Sqrt[a + a*Sin[e +
f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (2*a*(g*Cos[e + f*x])^(5/2))/(65*c^2*f*
g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (2*a*(g*Cos[e + f*
x])^(5/2))/(65*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))
 + (2*a*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(65*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{(3a)\int \cdots}{\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}}{13fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}} - \frac{\cdots}{39cfg\sqrt{a \cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.94, size = 291, normalized size = 1.00

$$\frac{4e^{3i(e+fx)}\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g\right)^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}\left(-1+149ie^{i(e+fx)}+44e^{2i(e+fx)}-64ie^{3i(e+fx)}+21e^{4i(e+fx)}+3ie^{5i(e+fx)}\right)-i\left(-i+e^{i(e+fx)}\right)^7{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{a(1+\sin(e+fx))}}{195c^4\left(1-ie^{i(e+fx)}\right)\left(-i+e^{i(e+fx)}\right)^6\sqrt{ice^{-i(e+fx)}\left(-i+e^{i(e+fx)}\right)^2}\left(1+e^{2i(e+fx)}\right)^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[a + a*Sin[e + f*x]])/(c - c*Sin[e +
f*x])^(9/2),x]
```

```
[Out] (4*E^((3*I)*(e + f*x))*(((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x)))^(3/2
)*(Sqrt[1 + E^((2*I)*(e + f*x))]*(-1 + (149*I)*E^(I*(e + f*x)) + 44*E^((2*I
)*(e + f*x)) - (64*I)*E^((3*I)*(e + f*x)) + 21*E^((4*I)*(e + f*x)) + (3*I)*
E^((5*I)*(e + f*x))) - I*(-I + E^(I*(e + f*x)))^7*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[a*(1 + Sin[e + f*x])])/(195*c^4*(1 - I*
E^(I*(e + f*x)))*(-I + E^(I*(e + f*x)))^6*Sqrt[(I*c*(-I + E^(I*(e + f*x)))^
2)/E^(I*(e + f*x))]*(1 + E^((2*I)*(e + f*x)))^(3/2)*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 1126, normalized size = 3.86

method	result	size
default	Expression too large to display	1126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/195/f*(a*(1+sin(f*x+e)))^(1/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+
e)-cos(f*x+e)-sin(f*x+e)+1)*(9*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e))
)^(1/2)+12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-12*I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+21*I*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-12*I*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+co
s(f*x+e)))^(1/2)+12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*I*cos(f*x+e)^6
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(1+cos(f*x+e)))^(1/2)-9*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^
(1/2)+27*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^6*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)
^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-18
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*cos(f*x+e)^4*sin(f*x+e)-27*I*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+18*I*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^
(1/2)-9*cos(f*x+e)^4-5*cos(f*x+e)^3*sin(f*x+e)-24*cos(f*x+e)^3-10*cos(f*x+e
)^2*sin(f*x+e)+45*cos(f*x+e)^2+42*cos(f*x+e)*sin(f*x+e)+18*cos(f*x+e)-30*si
n(f*x+e)-30)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(1+sin(f*x+e))/(-c*(sin(f*x+e)-1
))^(9/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e)
+ c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 350, normalized size = 1.20

$$\frac{2\left(12\,g\cos\left(fx+e\right)^{2}-\left(3\,g\cos\left(fx+e\right)^{2}-23\,g\right)\sin\left(fx+e\right)+7\,g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}-3\left(i\sqrt{2}\,g\cos\left(fx+e\right)^{4}-8i\sqrt{2}\,g\cos\left(fx+e\right)^{2}+4\left(i\sqrt{2}\,g\cos\left(fx+e\right)^{2}-2i\sqrt{2}\,g\right)\sin\left(fx+e\right)+8i\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)-3\left(-i\sqrt{2}\,g\cos\left(fx+e\right)^{4}+8i\sqrt{2}\,g\cos\left(fx+e\right)^{2}+4\left(-i\sqrt{2}\,g\cos\left(fx+e\right)^{2}+2i\sqrt{2}\,g\right)\sin\left(fx+e\right)-8i\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{195\left(c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f+4\left(c^{5}f\cos\left(fx+e\right)^{2}-2\,c^{5}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] 1/195*(2*(12*g*cos(f*x + e)^2 - (3*g*cos(f*x + e)^2 - 23*g)*sin(f*x + e) +
7*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c
) - 3*(I*sqrt(2)*g*cos(f*x + e)^4 - 8*I*sqrt(2)*g*cos(f*x + e)^2 + 4*(I*sqr
t(2)*g*cos(f*x + e)^2 - 2*I*sqrt(2)*g)*sin(f*x + e) + 8*I*sqrt(2)*g)*sqrt(a
*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e))) - 3*(-I*sqrt(2)*g*cos(f*x + e)^4 + 8*I*sqrt(2)*g*cos(f*x + e)^
2 + 4*(-I*sqrt(2)*g*cos(f*x + e)^2 + 2*I*sqrt(2)*g)*sin(f*x + e) - 8*I*sqrt
(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*
x + e) - I*sin(f*x + e))))/(c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^2 +
 8*c^5*f + 4*(c^5*f*cos(f*x + e)^2 - 2*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{a+a\sin(e+fx)}}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

3.97 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=352

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{2c}{}$$

```
[Out] 2/33*a^2*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))
^(1/2)-14/99*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f
*x+e))^(1/2)-2/11*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*
x+e))^(1/2)/f/g+14/45*a^2*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)+14/15*a^2*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*co
s(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/15*a^2*c^
2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2a^2c^2\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}}+\frac{2a^2c(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{33fg\sqrt{a\sin(e+fx)+a}}-\frac{14a^2(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{99fg\sqrt{a\sin(e+fx)+a}}-\frac{2a\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{11fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (14*a^2*c^3*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin
[e + f*x])^(3/2))/(33*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^2*(g*Cos[e + f*
x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[a + a*Sin[e + f*x]]) - (
2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5
/2))/(11*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+}}{11fg}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+}{99fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx}{33fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+}}{15fg\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= \frac{14a^2c^3(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

Mathematica [A]
time = 0.90, size = 193, normalized size = 0.55

$$\frac{c^2(g\cos(e+fx))^{3/2}(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\left(3696E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(450\cos(e+fx)+225\cos(3(e+fx))+45\cos(5(e+fx))+836\sin(2(e+fx))+110\sin(4(e+fx)))\right)}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^(3
/2)*Sqrt[c - c*Sin[e + f*x]]*(3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e +
 f*x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] + 836
*Sin[2*(e + f*x)] + 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 382, normalized size = 1.09

method	result

default	$-\frac{2\left(45\left(\cos^{6}(fx+e)\right)\sin(fx+e)-231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}+2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/495/f*(45*cos(f*x+e)^6*sin(f*x+e)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-55*co
s(f*x+e)^6-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e
)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x+e)^2+231*cos(f*x+e))*(-c*
(sin(f*x+e)-1))^(5/2)*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(sin(f*
x+e)-1)/sin(f*x+e)/cos(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 174, normalized size = 0.49

$$\frac{-231i\sqrt{2}\sqrt{acg}\,ac^{2}g\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,ac^{2}g\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(45\,ac^{2}g\cos(fx+e)^{4}+11\left(5\,ac^{2}g\cos(fx+e)^{2}+7\,ac^{2}g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{495\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/495*(-231*I*sqrt(2)*sqrt(a*c*g)*a*c^2*g*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g
)*a*c^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e))) + 2*(45*a*c^2*g*cos(f*x + e)^4 + 11*(5*a*c^2*g*cos(f*x + e
```

```
)^2 + 7*a*c^2*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

3.98 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=295

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+$$

```
[Out] -2/9*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(
1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/
2)/f/g+14/45*a^2*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+14/15*a^2*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))
^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/15*a^2*c*(g*cos(f*
x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.95, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}}+\frac{2a^2c\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}}-\frac{2a\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{9fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (14*a^2*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (14*a^2*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (2*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(15*f
*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2))/(9*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\begin{aligned}
\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+}}{9fg} \\
&= -\frac{2a^2(g\cos(e+fx))^{5/2}(c-c\sin(e+f}{9fg\sqrt{a+a\sin(e+fx)}} \\
&= \frac{2a^2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+}}{15fg\sqrt{a+a\sin(e+fx)}} \\
&= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}} \\
&= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}} \\
&= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}} \\
&= \frac{14a^2c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}
\end{aligned}$$

Mathematica [A]

time = 0.47, size = 113, normalized size = 0.38

$$-\frac{c(g\cos(e+fx))^{3/2}(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}\left(168E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(38\sin(2(e+fx))+5\sin(4(e+fx)))\right)}{180f\cos^{\frac{9}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] -1/180*(c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))
^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(168*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e
 + f*x]]*(38*Sin[2*(e + f*x)] + 5*Sin[4*(e + f*x)])))/(f*Cos[e + f*x]^(9/2)
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 356, normalized size = 1.21

method	result
default	$\frac{2(-c(\sin(fx+e)-1))^{\frac{3}{2}}\left(21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,-21i\sin(fx+e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(-c*(sin(f*x+e)-1))^(3/2)*(21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)-21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5*cos(f*x+
e)^6+21*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*sin(f*x+e)*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-2*cos(f*x+e)^4-14*cos(f*x+e)^2+21*cos(f*x+e))*(g*cos(f*x+e))^
(3/2)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^5/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 147, normalized size = 0.50

$$\frac{-21i\sqrt{2}\sqrt{acg}\,acg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+21i\sqrt{2}\sqrt{acg}\,acg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(5\,acg\cos(fx+e)^2+7\,acg\right)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}\,\sin(fx+e)}{45\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*sqrt(a*c*g)*a*c*g*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*sqrt(a*c*g)*a*c
*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f
*x + e))) + 2*(5*a*c*g*cos(f*x + e)^2 + 7*a*c*g)*sqrt(g*cos(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{3/2}\,(c-c\,\sin(e+f\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

3.99 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+$

Optimal. Leaf size=235

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-6a$$

```
[Out] 2/7*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(1/2
)-2/5*a^2*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)+6/5*a^2*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a
*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-6/35*a*c*(g*cos(f*x+e))^(5/2)*(a+
a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6a^2cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{6ac\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{35fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^2*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (6*a^2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (6*a*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3}\cdots}{7fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{6ac(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+\cdots}}{35fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e\cdots}}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e\cdots}}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e\cdots}}$$

$$= -\frac{2a^2c(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.40, size = 257, normalized size = 1.09

$$\frac{ia^2e^{-3i(e+fx)}\left(i+e^{i(e+fx)}\right)g\sqrt{g\cos(e+fx)}\left(\sqrt{1+e^{2i(e+fx)}}\left(5-14ie^{i(e+fx)}+15e^{2i(e+fx)}-168ie^{3i(e+fx)}+15e^{4i(e+fx)}+14ie^{5i(e+fx)}+5e^{6i(e+fx)}\right)+112ie^{5i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{140\left(-i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((I/140)*a^2*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I
)*(e + f*x))]*(5 - (14*I)*E^(I*(e + f*x)) + 15*E^((2*I)*(e + f*x)) - (168*I
)*E^((3*I)*(e + f*x)) + 15*E^((4*I)*(e + f*x)) + (14*I)*E^((5*I)*(e + f*x))
 + 5*E^((6*I)*(e + f*x))) + (112*I)*E^((5*I)*(e + f*x))*Hypergeometric2F1[1
/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(E^((3*I)*(e
 + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 372, normalized size = 1.58

method	result
default	$-\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/35/f*(-c*(sin(f*x+e)-1))^(1/2)*(21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(
f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+21*I*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+
5*cos(f*x+e)^4*sin(f*x+e)+7*cos(f*x+e)^4+14*cos(f*x+e)^2-21*cos(f*x+e))*(g*
cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(3/2)/(1+sin(f*x+e))/cos(f*x+e)^3/sin(
f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 143, normalized size = 0.61

$$\frac{-21i\sqrt{2}\sqrt{acg}\,ag\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))+21i\sqrt{2}\sqrt{acg}\,ag\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))-2\left(5\,ag\cos\left(fx+e\right)^2-7\,ag\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}}{35\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/35*(-21*I*sqrt(2)*sqrt(a*c*g)*a*g*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*sqrt(a*c*g)*a*g*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
 e))) - 2*(5*a*g*cos(f*x + e)^2 - 7*a*g*sin(f*x + e))*sqrt(g*cos(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

3.100 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] -14/15*a^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)+14/5*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin
(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-14*a^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (2*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg\sqrt{c-c\sin(e+fx)}} + \frac{1}{5}(7a)\int \frac{(g\,\mathrm{co}}{}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos}{}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos}{}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{2a(g\cos}{}$$

$$= -\frac{14a^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14a^2g\sqrt{}}{5f}$$

Mathematica [A]
time = 0.52, size = 148, normalized size = 0.82

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{3/2}\left(-42E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(20\cos(e+fx)+3\sin(2(e+fx)))\right)}{15f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/15*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 +
 Sin[e + f*x]))^(3/2)*(-42*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(
```

```
20*Cos[e + f*x] + 3*Sin[2*(e + f*x)]))))/(f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 382, normalized size = 2.12

method	result
default	$\frac{2(a(1+\sin(fx+e)))^{\frac{3}{2}}(g\cos(fx+e))^{\frac{3}{2}}\left(21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/15/f*(a*(1+sin(f*x+e)))^(3/2)*(g*cos(f*x+e))^(3/2)*(21*I*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)-21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-21*I*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)-3*cos(f*x+e)^4+10*cos(f*x+e)^2*sin(f*x+e)+24*cos(f*x+e)^
2-21*cos(f*x+e))/(cos(f*x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)/sin(f*x+e)/(-c*(s
in(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 137, normalized size = 0.76

$$\frac{-21i\sqrt{2}\sqrt{acg}\,ag\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+21i\sqrt{2}\sqrt{acg}\,ag\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2(3ag\sin(fx+e)+10ag)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{15cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/15*(-21*I*sqrt(2)*sqrt(a*c*g)*a*g*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*sqrt(a*c*g)*a*g*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
 e))) - 2*(3*a*g*sin(f*x + e) + 10*a*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(c*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^{3/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

3.101 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{14a^2g\sqrt{\cos(e+}}{cf\sqrt{a+a}}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
14/3*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))
^(1/2)-14*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(s
in(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{14a^2(g\cos(e+fx))^{5/2}}{3cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{4a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(f*g*(c - c*Sin[e + f
*x])^(3/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(3*c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(7a)\int \frac{(g\cos(e+f}{\sqrt{}}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a}{3cfg\sqrt{a+a\sin}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a}{3cfg\sqrt{a+a\sin}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a}{3cfg\sqrt{a+a\sin}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{14a}{3cfg\sqrt{a+a\sin}}$$

Mathematica [A]
time = 1.18, size = 207, normalized size = 1.14

$$-\frac{2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(-21E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)(12+\cos(e+fx))-(-12+\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)(a(1+\sin(e+fx)))^{3/2}}{3cf\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(-21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) - (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(a*(1 + Sin[e + f*x]))^(3/2))/(3*c*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 2892, normalized size = 15.89

method	result	size
default	Expression too large to display	2892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f
*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)-cos(f*x+e)^3*sin(f*x+e)-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(
f*x+e)))^(1/2)-21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9*cos(f
*x+e)^2*sin(f*x+e)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e
)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+1
2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x
+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-12*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(3/2)+18*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
```

```
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-18*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(3/2)+12*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)
/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(3/2)*sin(f*x+e)-12*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+42*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2
)-42*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1
/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*
x+e)))^(1/2)-21*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*
x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(
1/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+33*cos(
f*x+e)^2-8*cos(f*x+e)^3+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(3/2)-9*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+9*cos(f*x+e)*sin
(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(3/2)+3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-9*cos(f*x+
e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+9*cos(f*x+e)^2*sin(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+
```

```
cos(f*x+e))^2)^(3/2)+cos(f*x+e)^4)*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^
(3/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*cos(f*x+e)*sin
(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(sin(f...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 185, normalized size = 1.02

$$\frac{2\,(ag\sin(fx+e)-13\,ag)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}-21\left(-i\sqrt{2}\,ag\sin(fx+e)+i\sqrt{2}\,ag\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))-21\left(i\sqrt{2}\,ag\sin(fx+e)-i\sqrt{2}\,ag\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))}{3\,(c^2f\sin(fx+e)-c^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a*g*sin(f*x + e) - 13*a*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e)
 + a)*sqrt(-c*sin(f*x + e) + c) - 21*(-I*sqrt(2)*a*g*sin(f*x + e) + I*sqrt(
2)*a*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f
*x + e) + I*sin(f*x + e))) - 21*(I*sqrt(2)*a*g*sin(f*x + e) - I*sqrt(2)*a*g
)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e
) - I*sin(f*x + e))))/(c^2*f*sin(f*x + e) - c^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

3.102 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}+\frac{42a^2g\sqrt{\cos(e\ \cdots}}{5c^2f\sqrt{a+\cdots}}$$

```
[Out] -28/5*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)
)^(1/2)+4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+
e))^(5/2)+42/5*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$\frac{42a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{4a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*f*g*(c - c*Sin[e +
 f*x])^(5/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e +
 f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (42*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Co
s[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sq
rt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(7a)\int \frac{(g\cos(e+f\ldots}{\ldots}}{\ldots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a\ldots}{5cfg\sqrt{a+a\sin\ldots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a\ldots}{5cfg\sqrt{a+a\sin\ldots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a\ldots}{5cfg\sqrt{a+a\sin\ldots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{28a\ldots}{5cfg\sqrt{a+a\sin\ldots}}$$

Mathematica [A]
time = 0.96, size = 191, normalized size = 1.03

$$-\frac{a\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\sqrt{a(1+\sin(e+fx))}\left(-42E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+8\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)-2\sin\left(\frac{3}{2}(e+fx)\right)\right)\right)}{5c^2f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/5*(a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])
]*(-42*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 +
8*Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] + Sin[(e +
f*x)/2] - 2*Sin[(3*(e + f*x))/2])))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^3*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 3498, normalized size = 18.81

method	result	size
default	Expression too large to display	3498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(21*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos
(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)+42*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-63*I*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f
*x+e)))^(1/2)+63*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)-10*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(3/2)+10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+5*cos(
f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^4*sin(f*x+e)*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^3*sin(f*x+e)+21*I*cos(f*x+e)^3*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)-46*cos(f*x+e)^2*sin(f*x+e)+10*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(3/2)-10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+40*ln(
-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(
```

```
f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-40*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
3/2)+35*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*c
os(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-10*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/
2)*sin(f*x+e)-35*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+
e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+10*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(3/2)*sin(f*x+e)+38*cos(f*x+e)^2-9*cos(f*x+e)^3+5*cos(f*x+e)^5*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1
)/sin(f*x+e)^2)-5*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+10*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(3/2)-10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-35*cos(f*x+e)*sin(f*x+e)*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)+35*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-25
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+
e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+25*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(3/2)-45*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 233, normalized size = 1.25

$$-\frac{8\left(4\,ag\sin\left(fx+e\right)-3\,ag\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+21\left(i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}+2i\sqrt{2}\,ag\sin\left(fx+e\right)-2i\sqrt{2}\,ag\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+21\left(-i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}-2i\sqrt{2}\,ag\sin\left(fx+e\right)+2i\sqrt{2}\,ag\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{5\left(c^{3}f\cos\left(fx+e\right)^{2}+2\,c^{3}f\sin\left(fx+e\right)-2\,c^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(8*(4*a*g*sin(f*x + e) - 3*a*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c) + 21*(I*sqrt(2)*a*g*cos(f*x + e)^2 + 2*I*
sqrt(2)*a*g*sin(f*x + e) - 2*I*sqrt(2)*a*g)*sqrt(a*c*g)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*(-I*sqr
t(2)*a*g*cos(f*x + e)^2 - 2*I*sqrt(2)*a*g*sin(f*x + e) + 2*I*sqrt(2)*a*g)*s
qrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
 I*sin(f*x + e))))/(c^3*f*cos(f*x + e)^2 + 2*c^3*f*sin(f*x + e) - 2*c^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

3.103 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$\frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{45cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{\quad}{15c^2fg\sqrt{a+}}$$

```
[Out] -28/45*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e
))^(1/2)+14/15*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a
*sin(f*x+e))^(1/2)+4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c
-c*sin(f*x+e))^(7/2)-14/15*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))
^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{45cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{4a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(9*f*g*(c - c*Sin[e +
 f*x])^(7/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(45*c*f*g*Sqrt[a + a*Sin[e
+ f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(15*c
^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx &= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(7a)\int \frac{(g\cos(e+}{}\\
&= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{2}{45cfg\sqrt{a+a}}\\
&= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{2}{45cfg\sqrt{a+a}}\\
&= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{2}{45cfg\sqrt{a+a}}\\
&= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{2}{45cfg\sqrt{a+a}}\\
&= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{2}{45cfg\sqrt{a+a}}
\end{aligned}
$$

Mathematica [A]
time = 1.58, size = 218, normalized size = 0.90

$$\frac{a\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\sqrt{a(1+\sin(e+fx))}\left(84E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+\sqrt{\cos(e+fx)}\left(-74\cos\left(\frac{1}{2}(e+fx)\right)-15\cos\left(\frac{3}{2}(e+fx)\right)+21\cos\left(\frac{5}{2}(e+fx)\right)-74\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+21\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{90c^3f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (a*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*(84
*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + Sqrt[C
os[e + f*x]]*(-74*Cos[(e + f*x)/2] - 15*Cos[(3*(e + f*x))/2] + 21*Cos[(5*(e
 + f*x))/2] - 74*Sin[(e + f*x)/2] + 15*Sin[(3*(e + f*x))/2] + 21*Sin[(5*(e
+ f*x))/2])))/(90*c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-1 + Sin[e
 + f*x])^3*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 2682, normalized size = 11.04

method	result	size
default	Expression too large to display	2682

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x,me
thod=_RETURNVERBOSE)

[Out] 1/90/f*(1+cos(f*x+e))*(-1+cos(f*x+e))^4*(45*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e
)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-168*I*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+90*cos(f*
x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*
x+e)-90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*c
os(f*x+e)*sin(f*x+e)+12*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)*cos
(f*x+e)^2-248*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)*cos(f*x+e)+16
4*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-80*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)+84*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-84*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+168*I*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*sin(f*x+e)-168*I*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3
*sin(f*x+e)-84*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(
1/2)*cos(f*x+e)^2*sin(f*x+e)+84*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)-336*I*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+336*I*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f
*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+
e)-168*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)+168*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)+168*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1
+cos(f*x+e)))^(1/2)-336*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x
+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(1+cos(f*x+e)))^(1/2)+336*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+336*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
```

```
I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-336*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+90*cos(f*x+e)^3*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^3*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+90*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)-80*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+88*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)*cos(f*x+e)-88*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3-84*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4+168*I*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-168*I*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-45*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+180*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)*cos(f*x+e)^3*sin(f*x+e))*(sin(f*x+e)-1)*(g*cos(f*x+e))^(3/
2)*(a*(1+sin(f*x+e)))^(3/2)/sin(f*x+e)^9/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/
2)/(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 300, normalized size = 1.23

$$\frac{2\left(21\,ag\cos\left(fx+e\right)^{2}+18\,ag\sin\left(fx+e\right)-38\,ag\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+21\left(3i\,\sqrt{2}\,ag\cos\left(fx+e\right)^{2}-4i\,\sqrt{2}\,ag+\left(-i\,\sqrt{2}\,ag\cos\left(fx+e\right)^{2}+4i\,\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+21\left(-3i\,\sqrt{2}\,ag\cos\left(fx+e\right)^{2}+4i\,\sqrt{2}\,ag+\left(i\,\sqrt{2}\,ag\cos\left(fx+e\right)^{2}-4i\,\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{45\left(3\,c^{4}f\cos\left(fx+e\right)^{2}-4\,c^{4}f-\left(c^{4}f\cos\left(fx+e\right)^{2}-4\,c^{4}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(21*a*g*cos(f*x + e)^2 + 18*a*g*sin(f*x + e) - 38*a*g)*sqrt(g*cos(f
*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 21*(3*I*sqrt(
2)*a*g*cos(f*x + e)^2 - 4*I*sqrt(2)*a*g + (-I*sqrt(2)*a*g*cos(f*x + e)^2 +
4*I*sqrt(2)*a*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*(-3*I*sqrt(2)*a*g*co
s(f*x + e)^2 + 4*I*sqrt(2)*a*g + (I*sqrt(2)*a*g*cos(f*x + e)^2 - 4*I*sqrt(2
)*a*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(3*c^4*f*cos(f*x + e)^2 - 4*c^4*f
- (c^4*f*cos(f*x + e)^2 - 4*c^4*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

3.104 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{117cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}+\frac{}{195c^2fg\sqrt{a}}$$

```
[Out] -28/117*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+
e))^(1/2)+14/195*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a
+a*sin(f*x+e))^(1/2)+14/195*a^2*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e
))^(3/2)/(a+a*sin(f*x+e))^(1/2)+4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e)
)^(1/2)/f/g/(c-c*sin(f*x+e))^(9/2)-14/195*a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(g*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.92, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{195c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{117cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{4a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(13*f*g*(c - c*Sin[e
+ f*x])^(9/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(117*c*f*g*Sqrt[a + a*Sin[
e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(19
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (14*a^2*(g
*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[
(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(7a)\int \frac{(g\cos(e+\cdots}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{2\cdots}{117cfg\sqrt{a+\cdots}}$$

Mathematica [A]
time = 6.32, size = 464, normalized size = 1.55

$$-\frac{14(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{3/2}}{195f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{9/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(\frac{14}{195}+\frac{8}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{64}{117\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}+\frac{14}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{16\sin\left(\frac{1}{2}(e+fx)\right)}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}-\frac{128\sin\left(\frac{1}{2}(e+fx)\right)}{117\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{3/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(3/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(14
/195 + 8/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 64/(117*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^4) + 14/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2) + (16*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
- (128*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (2
8*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin
[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
```

```
f*x]))^(3/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*
x])^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1136, normalized size = 3.79

method	result	size
default	Expression too large to display	1136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/585/f*(a*(1+sin(f*x+e)))^(3/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+
e)-cos(f*x+e)-sin(f*x+e)+1)*(-84*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-147*I*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+63*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(1+cos(f*x+e)))^(1/2)-63*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*
x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2
)-126*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)^6*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)+84*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+21*I*cos(f*x+e)^6*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos
(f*x+e)))^(1/2)+126*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^
4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-84*I*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(1/(1+cos(f*x+e)))^(1/2)+147*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)+84*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+21*cos(f*x+e)^4*sin(f*x+
e)-189*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+189*I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+c
os(f*x+e)))^(1/2)-63*cos(f*x+e)^4+160*cos(f*x+e)^3*sin(f*x+e)+222*cos(f*x+e
)^3-265*cos(f*x+e)^2*sin(f*x+e)-75*cos(f*x+e)^2-96*cos(f*x+e)*sin(f*x+e)-26
4*cos(f*x+e)+180*sin(f*x+e)+180)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^
2-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(9/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 364, normalized size = 1.21

$$-\frac{2\left(84\,ag\cos\left(fx+e\right)^{2}-146\,ag-\left(21\,ag\cos\left(fx+e\right)^{2}+34\,ag\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+21\left(-i\sqrt{2}\,ag\cos\left(fx+e\right)^{4}+8i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}-8i\sqrt{2}\,ag+4\left(-i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}+2i\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+21\left(i\sqrt{2}\,ag\cos\left(fx+e\right)^{4}-8i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}+8i\sqrt{2}\,ag+4\left(i\sqrt{2}\,ag\cos\left(fx+e\right)^{2}-2i\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{585\left(c^{5}f\cos\left(fx+e\right)^{4}-8\,c^{5}f\cos\left(fx+e\right)^{2}+8\,c^{5}f+4\left(c^{5}f\cos\left(fx+e\right)^{2}-2\,c^{5}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] -1/585*(2*(84*a*g*cos(f*x + e)^2 - 146*a*g - (21*a*g*cos(f*x + e)^2 + 34*a*
g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(
f*x + e) + c) + 21*(-I*sqrt(2)*a*g*cos(f*x + e)^4 + 8*I*sqrt(2)*a*g*cos(f*x
 + e)^2 - 8*I*sqrt(2)*a*g + 4*(-I*sqrt(2)*a*g*cos(f*x + e)^2 + 2*I*sqrt(2)*
a*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*(I*sqrt(2)*a*g*cos(f*x + e)^4 -
8*I*sqrt(2)*a*g*cos(f*x + e)^2 + 8*I*sqrt(2)*a*g + 4*(I*sqrt(2)*a*g*cos(f*x
 + e)^2 - 2*I*sqrt(2)*a*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(c^5*f*cos(f*x
 + e)^4 - 8*c^5*f*cos(f*x + e)^2 + 8*c^5*f + 4*(c^5*f*cos(f*x + e)^2 - 2*c^
5*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

3.105 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{221cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{9/2}}+\frac{}{663c^2fg\sqrt{a}}$$

```
[Out] -28/221*a^2*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+
e))^(1/2)+14/663*a^2*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a
+a*sin(f*x+e))^(1/2)+14/1105*a^2*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+
e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+14/1105*a^2*(g*cos(f*x+e))^(5/2)/c^4/f/g/(
c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+4/17*a*(g*cos(f*x+e))^(5/2)*(a
+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(11/2)-14/1105*a^2*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*c
os(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{14a^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{1105c^5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{1105c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{14a^2(g\cos(e+fx))^{5/2}}{663c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{28a^2(g\cos(e+fx))^{5/2}}{221cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}+\frac{4a\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(17*f*g*(c - c*Sin[e
 + f*x])^(11/2)) - (28*a^2*(g*Cos[e + f*x])^(5/2))/(221*c*f*g*Sqrt[a + a*Sin
[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(6
63*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*a^2*(
g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
 + f*x])^(5/2)) + (14*a^2*(g*Cos[e + f*x])^(5/2))/(1105*c^4*f*g*Sqrt[a + a*S
in[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (14*a^2*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(1105*c^5*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(7a)\int \frac{(g\cos(e+}{}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{2}{221cfg\sqrt{a+}}$$

Mathematica [A]
time = 6.36, size = 532, normalized size = 1.49

$$-\frac{14(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{3/2}}{1105f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{11/2}} + \frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{14}{1105}+\frac{8}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8}-\frac{80}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}+\frac{14}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}+\frac{14}{1105\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{16\sin\left(\frac{1}{2}(e+fx)\right)}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}-\frac{160\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{1105\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{28\sin\left(\frac{1}{2}(e+fx)\right)}{1105\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{3/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(3/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-14*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(3/2))/(1105*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(14/1105 + 8/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 80/(221*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^6) + 14/(663*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^4) + 14/(1105*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (16*Sin[(e
```

```
 + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (160*Sin[(e + f*
x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (28*Sin[(e + f*x)/2]
)/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (28*Sin[(e + f*x)/2])/(11
05*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (28*Sin[(e + f*x)/2])/(1105*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(3/2))/(f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(11/2))
```

Maple [C] Result contains complex when optimal does not.
time = 2.49, size = 1296, normalized size = 3.63

method	result	size
default	Expression too large to display	1296

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/3315/f*(a*(1+sin(f*x+e)))^(3/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x
+e)-cos(f*x+e)-sin(f*x+e)+1)*(780+780*sin(f*x+e)-948*cos(f*x+e)+168*I*(1/(1
+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-168*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+523*cos(f*x+e)^3*sin(f
*x+e)+21*I*cos(f*x+e)^6*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+84*cos(f*x+e)^
4*sin(f*x+e)-775*cos(f*x+e)^2*sin(f*x+e)+21*cos(f*x+e)^6-605*cos(f*x+e)^2+9
41*cos(f*x+e)^3-612*cos(f*x+e)*sin(f*x+e)-35*cos(f*x+e)^5-21*I*cos(f*x+e)^6
*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-189*I*cos(f*x+e)^4*sin(f*x+e)*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+189*I*cos(f*x+e)^4*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+
336*I*cos(f*x+e)^2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-336*I*cos(f*x+e)^2*
sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-84*I*cos(f*x+e)^6*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+84*I*cos(f*x+e)^6*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+336*I*cos(f*x+e)^4*(1/(1+
cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)-336*I*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-420*I*cos(
f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellipti
cF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+420*I*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+168*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
```

```
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-168*I*sin(f*x+e)*(1/(1+cos(
f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)-154*cos(f*x+e)^4)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)
^2-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(11/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 422, normalized size = 1.18

$$\frac{2\left(21\,ag\cos\left(fx+e\right)^4-266\,ag\cos\left(fx+e\right)^2+502\,ag+\left(105\,ag\cos\left(fx+e\right)^2+278\,ag\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+21\left(5i\sqrt{2}\,ag\cos\left(fx+e\right)^4-20i\sqrt{2}\,ag\cos\left(fx+e\right)^2+16i\sqrt{2}\,ag+\left(-i\sqrt{2}\,ag\cos\left(fx+e\right)^4+12i\sqrt{2}\,ag\cos\left(fx+e\right)^2-16i\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+21\left(-5i\sqrt{2}\,ag\cos\left(fx+e\right)^4+20i\sqrt{2}\,ag\cos\left(fx+e\right)^2-16i\sqrt{2}\,ag+\left(i\sqrt{2}\,ag\cos\left(fx+e\right)^4-12i\sqrt{2}\,ag\cos\left(fx+e\right)^2+16i\sqrt{2}\,ag\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{3315\left(5\,c^6f\cos\left(fx+e\right)^4-20\,c^6f\cos\left(fx+e\right)^2+16\,c^6f-\left(c^6f\cos\left(fx+e\right)^4-12\,c^6f\cos\left(fx+e\right)^2+16\,c^6f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] 1/3315*(2*(21*a*g*cos(f*x + e)^4 - 266*a*g*cos(f*x + e)^2 + 502*a*g + (105*
a*g*cos(f*x + e)^2 + 278*a*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 21*(5*I*sqrt(2)*a*g*cos(f*x + e)
^4 - 20*I*sqrt(2)*a*g*cos(f*x + e)^2 + 16*I*sqrt(2)*a*g + (-I*sqrt(2)*a*g*c
os(f*x + e)^4 + 12*I*sqrt(2)*a*g*cos(f*x + e)^2 - 16*I*sqrt(2)*a*g)*sin(f*x
 + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*
x + e) + I*sin(f*x + e))) + 21*(-5*I*sqrt(2)*a*g*cos(f*x + e)^4 + 20*I*sqrt
(2)*a*g*cos(f*x + e)^2 - 16*I*sqrt(2)*a*g + (I*sqrt(2)*a*g*cos(f*x + e)^4 -
 12*I*sqrt(2)*a*g*cos(f*x + e)^2 + 16*I*sqrt(2)*a*g)*sin(f*x + e))*sqrt(a*c
*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(
f*x + e))))/(5*c^6*f*cos(f*x + e)^4 - 20*c^6*f*cos(f*x + e)^2 + 16*c^6*f -
(c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^2 + 16*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^{3/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

3.106 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=406

$$\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{195f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/
f/g+2/39*a^3*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x
+e))^(1/2)-14/117*a^3*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*
sin(f*x+e))^(1/2)-2/13*a^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)*(a+a
*sin(f*x+e))^(1/2)/f/g+154/585*a^3*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*
x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+154/195*a^3*c^3*g*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^
(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+
22/195*a^3*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x
+e))^(1/2)
```

Rubi [A]

time = 1.26, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{195fg\sqrt{a\sin(e+fx)+a}}-\frac{14a^3(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{117fg\sqrt{a\sin(e+fx)+a}}+\frac{2a^3c(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{13fg}-\frac{2a(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{13fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (154*a^3*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt
[c - c*Sin[e + f*x]]) + (154*a^3*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]]) + (22*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]
)/(195*f*g*Sqrt[a + a*Sin[e + f*x]]) + (2*a^3*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]) - (14*a^3*(g*Cos[
e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(117*f*g*Sqrt[a + a*Sin[e + f*x
]]) - (2*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e +
 f*x])^(5/2))/(13*f*g) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(
3/2)*(c - c*Sin[e + f*x])^(5/2))/(13*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+f}{13fg}$$
$$= -\frac{2a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e}}{13fg}$$
$$= -\frac{14a^3(g\cos(e+fx))^{5/2}(c-c\sin(e+}{117fg\sqrt{a+a\sin(e+fx)}}$$
$$= \frac{2a^3c(g\cos(e+fx))^{5/2}(c-c\sin(e+f}{39fg\sqrt{a+a\sin(e+fx)}}$$
$$= \frac{22a^3c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e}}{195fg\sqrt{a+a\sin(e+fx)}}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$
$$= \frac{154a^3c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

Mathematica [A]
time = 0.80, size = 120, normalized size = 0.30

$$\frac{a^2c^2(g\cos(e+fx))^{3/2}\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\left(7392E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(1897\sin(2(e+fx))+400\sin(4(e+fx))+45\sin(6(e+fx)))\right)}{9360f\cos^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (a^2*c^2*(g*Cos[e + f*x])^(3/2)*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e
 + f*x]]*(7392*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1897*Sin[2*(
e + f*x)] + 400*Sin[4*(e + f*x)] + 45*Sin[6*(e + f*x)])))/(9360*f*Cos[e + f
*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 366, normalized size = 0.90

method	result
default	$\frac{2(-c(\sin(fx+e)-1))^{\frac{5}{2}}\left(-45(\cos^8(fx+e))+231i\sin(fx+e)\cos(fx+e)\operatorname{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+}{1+\cos(fx}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/585/f*(-c*(sin(f*x+e)-1))^(5/2)*(-45*cos(f*x+e)^8+231*I*(1/(1+cos(f*x+e))
)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)-10*cos(f*x+e)^6+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*
(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x+e)^2+231*c
os(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/sin(f*x+e)/cos(f*x
+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 181, normalized size = 0.45

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^2c^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^2c^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(45a^2c^2g\cos(fx+e)^4+55a^2c^2g\cos(fx+e)^2+77a^2c^2g\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\sin(fx+e)}{585f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/585*(-231*I*sqrt(2)*sqrt(a*c*g)*a^2*c^2*g*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c
*g)*a^2*c^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e
```

```
) - I*sin(f*x + e))) + 2*(45*a^2*c^2*g*cos(f*x + e)^4 + 55*a^2*c^2*g*cos(f*
x + e)^2 + 77*a^2*c^2*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt
(-c*sin(f*x + e) + c)*sin(f*x + e))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

3.107 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=352

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] -2/33*a*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e)
)^(1/2)+14/99*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(
f*x+e))^(1/2)-14/45*a^3*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)
/(c-c*sin(f*x+e))^(1/2)+14/15*a^3*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(
f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/15*a^2*c^2*
(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)+2/11
*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/f/g
```

Rubi [A]
time = 1.08, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{14a^3c^2(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^3c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{15fg\sqrt{c-c\sin(e+fx)}}-\frac{2ac^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{33fg\sqrt{c-c\sin(e+fx)}}+\frac{14c^2(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{99fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{11fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-14*a^3*c^2*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^3*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^2*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
5*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Si
n[e + f*x])^(3/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (14*c^2*(g*Cos[e + f
*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(99*f*g*Sqrt[c - c*Sin[e + f*x]]) +
(2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]])/(11*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))\cdots}{11fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx\cdots}{99fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e+\cdots}{33fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e\cdots}}{15fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}\cdots}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(\cdots}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}\cdots}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(\cdots}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}\cdots}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(\cdots}}$$

$$= -\frac{14a^3c^2(g\cos(e+fx))^{5/2}\cdots}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(\cdots}}$$

Mathematica [A]
time = 0.89, size = 189, normalized size = 0.54

$$\frac{c(g\cos(e+fx))^{3/2}(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}\left(-3696E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(450\cos(e+fx)+225\cos(3(e+fx))+45\cos(5(e+fx))-836\sin(2(e+fx))-110\sin(4(e+fx)))\right)}{3960f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(5/2)*
Sqrt[c - c*Sin[e + f*x]]*(-3696*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*
x]]*(450*Cos[e + f*x] + 225*Cos[3*(e + f*x)] + 45*Cos[5*(e + f*x)] - 836*Si
n[2*(e + f*x)] - 110*Sin[4*(e + f*x)])))/(3960*f*Cos[e + f*x]^(3/2)*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 382, normalized size = 1.09

method	result

default	$\frac{2\left(-45\left(\cos^{6}(fx+e)\right)\sin(fx+e)+231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}-23}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/495/f*(-45*cos(f*x+e)^6*sin(f*x+e)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-55*co
s(f*x+e)^6+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e
)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x+e)^2+231*cos(f*x+e))*(-c*
(sin(f*x+e)-1))^(3/2)*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(1+sin(
f*x+e))/cos(f*x+e)^5/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 174, normalized size = 0.49

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^{2}cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^{2}cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\left(45a^{2}cg\cos(fx+e)^{4}-11\left(5a^{2}cg\cos(fx+e)^{2}+7a^{2}cg\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{495f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] 1/495*(-231*I*sqrt(2)*sqrt(a*c*g)*a^2*c*g*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g
)*a^2*c*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e))) - 2*(45*a^2*c*g*cos(f*x + e)^4 - 11*(5*a^2*c*g*cos(f*x + e
```

```
)^2 + 7*a^2*c*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^{5/2}\,(c-c\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

3.108 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+}$

Optimal. Leaf size=290

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] -2/21*a*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^
(1/2)+2/9*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e)
)^(1/2)-22/45*a^3*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*si
n(f*x+e))^(1/2)+22/15*a^3*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*
e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1
/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/105*a^2*c*(g*cos(f*x
+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{105fg\sqrt{c-c\sin(e+fx)}}-\frac{2ac(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-22*a^3*c*(g*Cos[e + f*x])^(5/2))/(45*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]]) + (22*a^3*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(15*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
f*x]]) - (22*a^2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(105*f*
g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e +
f*x])^(3/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2
)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5}}{9fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{21fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{22a^2c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+}}{105fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{22a^3c(g\cos(e+fx))^{5/2}}{45fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.48, size = 281, normalized size = 0.97

$$\frac{a^3e^{-4i(e+fx)}\left(i+e^{i(e+fx)}\right)g\sqrt{g\cos(e+fx)}\left(\sqrt{1+e^{2i(e+fx)}}\left(-35+180ie^{i(e+fx)}+238e^{2i(e+fx)}+540ie^{3i(e+fx)}+3696e^{4i(e+fx)}+540ie^{5i(e+fx)}-238e^{6i(e+fx)}+180ie^{7i(e+fx)}+35e^{8i(e+fx)}\right)-2464e^{6i(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{2520\left(-i+e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\,f\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] (a^3*(I + E^(I*(e + f*x)))*g*Sqrt[g*Cos[e + f*x]]*(Sqrt[1 + E^((2*I)*(e + f
*x))]*(-35 + (180*I)*E^(I*(e + f*x)) + 238*E^((2*I)*(e + f*x)) + (540*I)*E^
((3*I)*(e + f*x)) + 3696*E^((4*I)*(e + f*x)) + (540*I)*E^((5*I)*(e + f*x))
- 238*E^((6*I)*(e + f*x)) + (180*I)*E^((7*I)*(e + f*x)) + 35*E^((8*I)*(e +
f*x))) - 2464*E^((6*I)*(e + f*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I
)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(2520*E^((4*I)*(e + f*x))*(-I + E^
(I*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 392, normalized size = 1.35

method	result
default	$\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\left(231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/315/f*(-c*(sin(f*x+e)-1))^(1/2)*(231*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-35*cos(
f*x+e)^6+231*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)+90*cos(f*x+e)^4*sin(f*x+e)+112*cos(f*x+e)^4+154*cos(f*x
+e)^2-231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*
x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 169, normalized size = 0.58

$$\frac{-231i\,\sqrt{2}\,\sqrt{acg}\,a^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\,\sin(fx+e)))+231i\,\sqrt{2}\,\sqrt{acg}\,a^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\,\sin(fx+e)))-2\left(90\,a^2g\cos(fx+e)^2+7\left(5\,a^2g\cos(fx+e)^2-11\,a^2g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\,\sqrt{a\sin(fx+e)+a}\,\sqrt{-c\sin(fx+e)+c}}{315\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/315*(-231*I*sqrt(2)*sqrt(a*c*g)*a^2*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
a^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) - 2*(90*a^2*g*cos(f*x + e)^2 + 7*(5*a^2*g*cos(f*x + e)^2 - 11*
a^2*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*
sin(f*x + e) + c))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{5/2}\,\sqrt{c-c\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

3.109 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] -2/7*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(1/
2)-22/15*a^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)+22/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a
*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/35*a^2*(g*cos(f*x+e))^(5/2)*(a
+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{35fg\sqrt{c-c\sin(e+fx)}}-\frac{2a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (22*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipt
icE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]
) - (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(35*f*g*Sqrt[c
 - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(7*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{7fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{7}(11a)\int \frac{(g}{}$$

$$= -\frac{22a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{35fg\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e}{7f}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2(g\,c}{}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2(g\,c}{}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{22a^2(g\,c}{}$$

$$= -\frac{22a^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22a^3g\sqrt{}}{5f}$$

Mathematica [A]
time = 1.22, size = 158, normalized size = 0.68

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}\left(-924E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(515\cos(e+fx)-15\cos(3(e+fx))+126\sin(2(e+fx)))\right)}{210f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/210*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1
+ Sin[e + f*x]))^(5/2)*(-924*EllipticE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]
*(515*Cos[e + f*x] - 15*Cos[3*(e + f*x)] + 126*Sin[2*(e + f*x)])))/(f*Cos[e
 + f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*
x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 415, normalized size = 1.77

method	result
default	$\frac{2(a(1+\sin(fx+e)))^{\frac{5}{2}}(g\cos(fx+e))^{\frac{3}{2}}\left(231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/105/f*(a*(1+sin(f*x+e)))^(5/2)*(g*cos(f*x+e))^(3/2)*(231*I*sin(f*x+e)*cos
(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)+231*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)-15*cos(f*x+e)^4*sin(f*x+e)-63*cos(f*x+e)^4+140*cos(
f*x+e)^2*sin(f*x+e)+294*cos(f*x+e)^2-231*cos(f*x+e))/(cos(f*x+e)^2*sin(f*x+
e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/cos(f*x+e)/sin(f*x+e)/(-c*(sin(f*x+e)-1))
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 160, normalized size = 0.68

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\,(15\,a^2g\cos(fx+e)^2-63\,a^2g\sin(fx+e)-140\,a^2g)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{105\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/105*(-231*I*sqrt(2)*sqrt(a*c*g)*a^2*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
a^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) + 2*(15*a^2*g*cos(f*x + e)^2 - 63*a^2*g*sin(f*x + e) - 140*a^2
*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
)/(c*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

3.110 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}}+\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(}}{5cf\sqrt{a}}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
154/15*a^3*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e
))^(1/2)-154/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(
a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/5*a^2*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{154a^3(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)|2\right)\sqrt{g\cos(e+fx)}}{5cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5cfg\sqrt{c-c\sin(e+fx)}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (154*a^3*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*C
os[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (22*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e +
f*x]])/(5*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(11a)\int \frac{(g\cos(}{\sqrt{}}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22a^2(g\cos(e}{5cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{1}{15cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{1}{15cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{1}{15cfg\sqrt{a+}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{1}{15cfg\sqrt{a+}}$$

Mathematica [A]
time = 4.23, size = 240, normalized size = 1.00

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2(a(1+\sin(e+fx)))^{5/2}\left(-924E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{\cos(e+fx)}\left(520\cos\left(\frac{1}{2}(e+fx)\right)+37\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{5}{2}(e+fx)\right)+520\sin\left(\frac{1}{2}(e+fx)\right)-37\sin\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{30cf\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] -1/30*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(a*(1
 + Sin[e + f*x]))^(5/2)*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]*(520*Cos[(e + f*x)/2] + 37*Cos[(3*(
e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] + 520*Sin[(e + f*x)/2] - 37*Sin[(3*(e
 + f*x))/2] + 3*Sin[(5*(e + f*x))/2])))/(c*f*Cos[e + f*x]^(3/2)*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^5*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 2945, normalized size = 12.22

method	result	size
default	Expression too large to display	2945

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/15/f*(-1+cos(f*x+e))*(231*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-17*cos(f*x+e)^3*sin(f*x+e)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*cos(f*x+e)^4*sin(f*x+e)-111*cos(f*x+e)^2*sin(f*x+e)+120*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-120*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+180*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-180*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+120*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-120*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+351*cos(f*x+e)^2-94*cos(f*x+e)^3+3*cos(f*x+e)^5-231*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+

```
e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-462*I*cos(f*x+e)^2*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)+231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-231*I*cos(f*x+e)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-90*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+90*cos(f*x+e)*sin(f
*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(3/2)+30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*cos(f*x
+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+90*cos(f*x+e)^2*sin(f*x+e)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(3/2)+20*cos(f*x+e)^4)*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+
e)))^(5/2)/(cos(f*x+e)^4+cos(f*x+e)^3*sin(f*x+e...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 213, normalized size = 0.88

$$-\frac{2\left(3\,a^2g\cos\left(fx+e\right)^2-17\,a^2g\sin\left(fx+e\right)+137\,a^2g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(-i\sqrt{2}\,a^2g\sin\left(fx+e\right)+i\sqrt{2}\,a^2g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+231\left(i\sqrt{2}\,a^2g\sin\left(fx+e\right)-i\sqrt{2}\,a^2g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{15\left(c^2f\sin\left(fx+e\right)-c^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(3*a^2*g*cos(f*x + e)^2 - 17*a^2*g*sin(f*x + e) + 137*a^2*g)*sqrt(
g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(-
I*sqrt(2)*a^2*g*sin(f*x + e) + I*sqrt(2)*a^2*g)*sqrt(a*c*g)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(I
*sqrt(2)*a^2*g*sin(f*x + e) - I*sqrt(2)*a^2*g)*sqrt(a*c*g)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(c^2*f*s
in(f*x + e) - c^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

3.111 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}}-\frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{5cfg(c-c\sin(e+fx))^{3/2}}-\frac{154a^3}{15c^2fg\sqrt{a+a\sin}}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(5/2
)-44/5*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e
))^(3/2)-154/15*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-
c*sin(f*x+e))^(1/2)+154/5*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^
(1/2)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$-\frac{154a^3(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{44a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5cfg(c-c\sin(e+fx))^{3/2}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(5*c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(15
*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (154*a^3*g*Sq
rt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*S
qrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(11a)\int \frac{(g\cos(}{}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e}{5cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e}{5cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e}{5cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e}{5cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{44a^2(g\cos(e}{5cfg}$$

Mathematica [A]
time = 1.67, size = 245, normalized size = 1.01

$$-\frac{a^2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{a(1+\sin(e+fx))}\left(-924E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(226\cos\left(\frac{1}{2}(e+fx)\right)+327\cos\left(\frac{3}{2}(e+fx)\right)-5\cos\left(\frac{5}{2}(e+fx)\right)+226\sin\left(\frac{1}{2}(e+fx)\right)-327\sin\left(\frac{3}{2}(e+fx)\right)-5\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{30c^2f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/30*(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos
[(3*(e + f*x))/2] - 5*Cos[(5*(e + f*x))/2] + 226*Sin[(e + f*x)/2] - 327*Sin
[(3*(e + f*x))/2] - 5*Sin[(5*(e + f*x))/2])))/(c^2*f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e +
 f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 3549, normalized size = 14.60

method	result	size
default	Expression too large to display	3549

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-45*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)+45*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-65*cos
(f*x+e)^3*sin(f*x+e)+231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*(
1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)+5*cos(f*x+e)^4*sin(f*x+e)+486*cos(f*x+e)^2*sin(f*x+e)-90*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-360*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+360*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
3/2)-315*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*
cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+90*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3
/2)*sin(f*x+e)+315*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(3/2)*sin(f*x+e)-438*cos(f*x+e)^2+89*cos(f*x+e)^3+231*I*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)-693*I*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
```

```
*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)+693*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)
))^(1/2)-462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e
)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)-45*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+45*cos(f*x+e)
^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^5-90*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2
)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(3/2)-231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)+462*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-462*I*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+693*I*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)-693*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)+315*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-315*cos(f*x+e)
*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 264, normalized size = 1.09

$$-\frac{2 \left(5 a^2 g \cos (fx+e)^2+166 a^2 g \sin (fx+e)-142 a^2 g\right) \sqrt{g \cos (fx+e)} \sqrt{a \sin (fx+e)+a} \sqrt{-c \sin (fx+e)+c}+231 \left(i \sqrt{2} a^2 g \cos (fx+e)^2+2 i \sqrt{2} a^2 g \sin (fx+e)-2 i \sqrt{2} a^2 g\right) \sqrt{acg}\, \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos (fx+e)+i \sin (fx+e)))+231 \left(-i \sqrt{2} a^2 g \cos (fx+e)^2-2 i \sqrt{2} a^2 g \sin (fx+e)+2 i \sqrt{2} a^2 g\right) \sqrt{acg}\, \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos (fx+e)-i \sin (fx+e)))}{15 \left(c^3 f \cos (fx+e)^2+2 c^3 f \sin (fx+e)-2 c^3 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(5*a^2*g*cos(f*x + e)^2 + 166*a^2*g*sin(f*x + e) - 142*a^2*g)*sqrt
(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(
I*sqrt(2)*a^2*g*cos(f*x + e)^2 + 2*I*sqrt(2)*a^2*g*sin(f*x + e) - 2*I*sqrt(
2)*a^2*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) + 231*(-I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 2*I*s
qrt(2)*a^2*g*sin(f*x + e) + 2*I*sqrt(2)*a^2*g)*sqrt(a*c*g)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(c^3*f*c
os(f*x + e)^2 + 2*c^3*f*sin(f*x + e) - 2*c^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

3.112 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=243

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}}-\frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{45cfg(c-c\sin(e+fx))^{5/2}}+\frac{308a}{45c^2fg\sqrt{a+a\sin}}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(7/2
)+308/45*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f
*x+e))^(1/2)-44/45*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c
-c*sin(f*x+e))^(5/2)-154/15*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e)
)^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$-\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{45c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{44a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{45cfg(c-c\sin(e+fx))^{5/2}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(45*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/(4
5*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^3*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{(11a)\int \frac{(g\cos(e\text{-}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+}{45cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+}{45cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+}{45cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+}{45cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{44a^2(g\cos(e+}{45cfg}$$

Mathematica [A]
time = 1.81, size = 246, normalized size = 1.01

$$-\frac{a^2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{a(1+\sin(e+fx))}\left(-924E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+2\sqrt{\cos(e+fx)}\left(182\cos\left(\frac{1}{2}(e+fx)\right)+195\cos\left(\frac{3}{2}(e+fx)\right)-93\cos\left(\frac{5}{2}(e+fx)\right)+182\sin\left(\frac{1}{2}(e+fx)\right)-195\sin\left(\frac{3}{2}(e+fx)\right)-93\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{90c^3f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^3\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] -1/90*(a^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^5 + 2*Sqrt[Cos[e + f*x]]*(182*Cos[(e + f*x)/2] + 195*C
os[(3*(e + f*x))/2] - 93*Cos[(5*(e + f*x))/2] + 182*Sin[(e + f*x)/2] - 195*
Sin[(3*(e + f*x))/2] - 93*Sin[(5*(e + f*x))/2])))/(c^3*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin
[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 4183, normalized size = 17.21

method	result	size
default	Expression too large to display	4183

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/45/f*(-1+cos(f*x+e))*(-135*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+
e)^2)+135*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+1350*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(3/2)-1350*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+
135*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-135*cos(f*x+e)^4*sin(
f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-348*cos(f*x+e)^3*sin(f*x+e)-462*I*cos(f*
x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-90*cos(f*x+e)^4*sin(f*x+e)+1768
*cos(f*x+e)^2*sin(f*x+e)-2025*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+2025
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-1890*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
```

```
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(3/2)+540*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+1890*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(3/2)-540*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-1928*cos(f*x+e)
^2+268*cos(f*x+e)^3-810*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+810*cos(f*x+
e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-90*cos(f*x+e)^5-540*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)+540*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+1890*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)-1890*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+945*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^
3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-945*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(3/2)+2295*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+
e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)-2295*cos(f*x+e)^2*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+1182*cos(
f*x+e)^4+1848*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-1848*I*cos(f*x+e)^4*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-2772*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 322, normalized size = 1.33

$$\frac{4\left(93\,a^2g\cos\left(fx+e\right)^2+144\,a^2g\sin\left(fx+e\right)-164\,a^2g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(3i\,\sqrt{2}\,a^2g\cos\left(fx+e\right)^2-4i\,\sqrt{2}\,a^2g+\left(-i\,\sqrt{2}\,a^2g\cos\left(fx+e\right)^2+4i\,\sqrt{2}\,a^2g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+231\left(-3i\,\sqrt{2}\,a^2g\cos\left(fx+e\right)^2+4i\,\sqrt{2}\,a^2g+\left(i\,\sqrt{2}\,a^2g\cos\left(fx+e\right)^2-4i\,\sqrt{2}\,a^2g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{45\left(3\,c^4f\cos\left(fx+e\right)^2-4\,c^4f-\left(c^4f\cos\left(fx+e\right)^2-4\,c^4f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] 1/45*(4*(93*a^2*g*cos(f*x + e)^2 + 144*a^2*g*sin(f*x + e) - 164*a^2*g)*sqrt
(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(
3*I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 4*I*sqrt(2)*a^2*g + (-I*sqrt(2)*a^2*g*co
s(f*x + e)^2 + 4*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(-
3*I*sqrt(2)*a^2*g*cos(f*x + e)^2 + 4*I*sqrt(2)*a^2*g + (I*sqrt(2)*a^2*g*cos
(f*x + e)^2 - 4*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(3*c^4*f
*cos(f*x + e)^2 - 4*c^4*f - (c^4*f*cos(f*x + e)^2 - 4*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

3.113 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}}-\frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{117cfg(c-c\sin(e+fx))^{7/2}}+\frac{308a}{585c^2fg\sqrt{a+a\,s}}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(9/
2)+308/585*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin
(f*x+e))^(1/2)-154/195*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3
/2)/(a+a*sin(f*x+e))^(1/2)-44/117*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))
^(1/2)/c/f/g/(c-c*sin(f*x+e))^(7/2)+154/195*a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/
2)*(g*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195c^4f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{195c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{585c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}-\frac{44a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{117cfg(c-c\sin(e+fx))^{7/2}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])
/(117*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))/
(585*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (154*a^
3*(g*Cos[e + f*x])^(5/2))/(195*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[
e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(195*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - (11a)\int \frac{(g\cos(}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{44a^2(g\cos(e}{117cf}$$

Mathematica [A]
time = 6.38, size = 464, normalized size = 1.55

$$\frac{154(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}{195f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(-\frac{154}{195}+\frac{16}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{232}{117\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}+\frac{236}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{32\sin\left(\frac{1}{2}(e+fx)\right)}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}-\frac{464\sin\left(\frac{1}{2}(e+fx)\right)}{117\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+\frac{472\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{195\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))/(195*f*Cos[e + f*x]^(3/2)*
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)) + ((g*C
os[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-1
54/195 + 16/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 232/(117*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^4) + 236/(195*(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^2) + (32*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
)^7) - (464*Sin[(e + f*x)/2])/(117*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)
 + (472*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (
308*Sin[(e + f*x)/2])/(195*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 +
Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin
[e + f*x])^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 3455, normalized size = 11.52

method	result	size
default	Expression too large to display	3455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/1170/f*(a*(1+sin(f*x+e)))^(5/2)*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)-1)*(-1+
cos(f*x+e))^4*(1+cos(f*x+e))*(1755*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-3696*I*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+3696*I*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+924*I*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-924*I*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)
+3696*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-3696
*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+2340*cos(
f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(
f*x+e)-2340*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^
2)*cos(f*x+e)*sin(f*x+e)-2752*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+
e)*cos(f*x+e)^2-2256*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)*cos(f*
x+e)-1008*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+2772*I*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*
x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x
+e)-2772*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*c
os(f*x+e)^4*sin(f*x+e)+5544*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)^3*sin(f*x+e)-5544*I*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*sin(f*x+e)-924*I*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1
```

```
/2)+924*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(1+cos(f*x+e)))^(1/2)-7392*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+7392*I*EllipticF(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+1440*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)+1848*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)^5-1848*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3696*I*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4+3696*I*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4-9240*I*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3+9240*I*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3-924*
I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)
^2+924*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos
(f*x+e)^2+7392*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(
1/2)*cos(f*x+e)-7392*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+
e)))^(1/2)*cos(f*x+e)+924*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^4
*sin(f*x+e)+2925*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)-2925*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 392, normalized size = 1.31

$$\frac{2\left(339a^{2}g\cos\left(fx+e\right)^{2}-436a^{2}g-\left(231a^{2}g\cos\left(fx+e\right)^{2}-796a^{2}g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}-231\left(i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{4}-8i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{2}+8i\sqrt{2}a^{2}g+4\left(i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{2}-2i\sqrt{2}a^{2}g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)-231\left(-i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{4}+8i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{2}-8i\sqrt{2}a^{2}g+4\left(-i\sqrt{2}a^{2}g\cos\left(fx+e\right)^{2}+2i\sqrt{2}a^{2}g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{585\left(c^{5}f\cos\left(fx+e\right)^{4}-8c^{5}f\cos\left(fx+e\right)^{2}+8c^{5}f+4\left(c^{5}f\cos\left(fx+e\right)^{2}-2c^{5}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] 1/585*(2*(339*a^2*g*cos(f*x + e)^2 - 436*a^2*g - (231*a^2*g*cos(f*x + e)^2
- 796*a^2*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(-c*sin(f*x + e) + c) - 231*(I*sqrt(2)*a^2*g*cos(f*x + e)^4 - 8*I*sqrt(2)
*a^2*g*cos(f*x + e)^2 + 8*I*sqrt(2)*a^2*g + 4*(I*sqrt(2)*a^2*g*cos(f*x + e)
^2 - 2*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 231*(-I*sqrt(2)*
a^2*g*cos(f*x + e)^4 + 8*I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 8*I*sqrt(2)*a^2*g
 + 4*(-I*sqrt(2)*a^2*g*cos(f*x + e)^2 + 2*I*sqrt(2)*a^2*g)*sin(f*x + e))*sq
rt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
I*sin(f*x + e))))/(c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^2 + 8*c^5*f
+ 4*(c^5*f*cos(f*x + e)^2 - 2*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

3.114 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}}-\frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{221cfg(c-c\sin(e+fx))^{9/2}}+\frac{308a}{1989c^2fg\sqrt{a+a\,\mathrm{s}}}$$

```
[Out] 4/17*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(11
/2)+308/1989*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*s
in(f*x+e))^(1/2)-154/3315*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))
^(5/2)/(a+a*sin(f*x+e))^(1/2)-154/3315*a^3*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-
c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-44/221*a^2*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(9/2)+154/3315*a^3*g*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2
))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*
sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{154a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3315c^5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{3/2}}-\frac{154a^3(g\cos(e+fx))^{5/2}}{3315c^3fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{5/2}}+\frac{308a^3(g\cos(e+fx))^{5/2}}{1989c^2fg\sqrt{a\sin(e+fx)+a}(c-c\sin(e+fx))^{7/2}}-\frac{44a^2\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{221cfg(c-c\sin(e+fx))^{9/2}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (308*a^3*(g*Cos[e + f*x])^(5/2))
/(1989*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) - (154*
a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) - (154*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (154*a^3*g*Sqrt[Cos[e + f*
x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3315*c^5*f*Sqrt[a + a*
Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(11a)\int \frac{(g\cos(e\cdots}{\cdots}}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{44a^2(g\cos(e+\cdots}{221cfg\cdots}$$

Mathematica [A]
time = 6.44, size = 532, normalized size = 1.49

$$\frac{154(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{5/2}}{3315f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{11/2}} + \frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(-\frac{154}{3315}+\frac{16}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^8}-\frac{296}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}+\frac{1172}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}-\frac{154}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{32\sin\left(\frac{1}{2}(e+fx)\right)}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}-\frac{592\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}+\frac{2344\sin\left(\frac{1}{2}(e+fx)\right)}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}-\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(-154/3315 + 16/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 296/(221*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 1172/(1989*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^4) - 154/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (3
2*Sin[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (592*Sin
```

```
[(e + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (2344*Sin[(e
 + f*x)/2])/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (308*Sin[(e +
f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (308*Sin[(e + f*x
)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^
(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(11/
2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 1313, normalized size = 3.68

method	result	size
default	Expression too large to display	1313

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -2/9945/f*(a*(1+sin(f*x+e)))^(5/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x
+e)-cos(f*x+e)-sin(f*x+e)+1)*(4680-1848*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1
/2)+1848*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-924*I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+c
os(f*x+e)))^(1/2)+924*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e
)^6*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3696*I*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3696*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)-4620*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+4620*I*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(1+cos(f*x+e)))^(1/2)+4680*sin(f*x+e)-2832*cos(f*x+e)+1848*I*EllipticE(I*(-
1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x
+e)))^(1/2)-1848*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+4192*cos(f*x+e)^3*sin(f*x+e)+2
31*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*sin(f*x+e)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-924*cos(f*x+e)^4*sin
(f*x+e)-1420*cos(f*x+e)^2*sin(f*x+e)-231*cos(f*x+e)^6-9920*cos(f*x+e)^2+622
4*cos(f*x+e)^3-6528*cos(f*x+e)*sin(f*x+e)-2930*cos(f*x+e)^5-231*I*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*sin(f*x+e)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-2079*I*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1
/(1+cos(f*x+e)))^(1/2)+2079*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos
(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)+3696*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+
```

```
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3696*I*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+5009*cos(f*x+e)^4)*(cos(f*x+e)^
2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/(
-c*(sin(f*x+e)-1))^(11/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(11/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 456, normalized size = 1.28

$$-\frac{2\left(231\,a^2g\cos\left(fx+e\right)^4+389\,a^2g\cos\left(fx+e\right)^2-1108\,a^2g+\left(1155\,a^2g\cos\left(fx+e\right)^2-3572\,a^2g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}-231\left(-5i\sqrt{2}\,a^2g\cos\left(fx+e\right)^4+20i\sqrt{2}\,a^2g\cos\left(fx+e\right)^2-16i\sqrt{2}\,a^2g+\left(i\sqrt{2}\,a^2g\cos\left(fx+e\right)^4-12i\sqrt{2}\,a^2g\cos\left(fx+e\right)^2+16i\sqrt{2}\,a^2g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)-231\left(5i\sqrt{2}\,a^2g\cos\left(fx+e\right)^4-20i\sqrt{2}\,a^2g\cos\left(fx+e\right)^2+16i\sqrt{2}\,a^2g+\left(-i\sqrt{2}\,a^2g\cos\left(fx+e\right)^4+12i\sqrt{2}\,a^2g\cos\left(fx+e\right)^2-16i\sqrt{2}\,a^2g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{9945\left(5\,c^6f\cos\left(fx+e\right)^4-20\,c^6f\cos\left(fx+e\right)^2+16\,c^6f-\left(c^6f\cos\left(fx+e\right)^4-12\,c^6f\cos\left(fx+e\right)^2+16\,c^6f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")
```

```
[Out] -1/9945*(2*(231*a^2*g*cos(f*x + e)^4 + 389*a^2*g*cos(f*x + e)^2 - 1108*a^2*
g + (1155*a^2*g*cos(f*x + e)^2 - 3572*a^2*g)*sin(f*x + e))*sqrt(g*cos(f*x +
 e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) - 231*(-5*I*sqrt(2)
*a^2*g*cos(f*x + e)^4 + 20*I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 16*I*sqrt(2)*a^
2*g + (I*sqrt(2)*a^2*g*cos(f*x + e)^4 - 12*I*sqrt(2)*a^2*g*cos(f*x + e)^2 +
 16*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 231*(5*I*sqrt(2)*a^
2*g*cos(f*x + e)^4 - 20*I*sqrt(2)*a^2*g*cos(f*x + e)^2 + 16*I*sqrt(2)*a^2*g
 + (-I*sqrt(2)*a^2*g*cos(f*x + e)^4 + 12*I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 1
6*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(5*c^6*f*cos(f*x + e)^
4 - 20*c^6*f*cos(f*x + e)^2 + 16*c^6*f - (c^6*f*cos(f*x + e)^4 - 12*c^6*f*c
os(f*x + e)^2 + 16*c^6*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
11/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

3.115 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}}-\frac{44a^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{357cfg(c-c\sin(e+fx))^{11/2}}+\frac{44a^3(}{663c^2fg\sqrt{a+a\sin}}$$

```
[Out] 4/21*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))^(13
/2)+44/663*a^3*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(9/2)/(a+a*sin
(f*x+e))^(1/2)-22/1989*a^3*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(7
/2)/(a+a*sin(f*x+e))^(1/2)-22/3315*a^3*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*si
n(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)-22/3315*a^3*(g*cos(f*x+e))^(5/2)/c^5
/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-44/357*a^2*(g*cos(f*x+e)
)^(5/2)*(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(11/2)+22/3315*a^3*g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^6/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 1.32, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{22a^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3315c^6f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{22a^3(g\cos(e+fx))^{5/2}}{1989c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{44a^3(g\cos(e+fx))^{5/2}}{663c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}-\frac{44a^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{357cfg(c-c\sin(e+fx))^{11/2}}+\frac{4a(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (44*a^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]]
)/(357*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2))
/(663*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(9/2)) - (22*a^
3*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin
[e + f*x])^(7/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) - (22*a^3*(g*Cos[e + f*x])^(5/2
))/(3315*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (22
*a^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
3315*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(11a)\int \frac{(g\cos(e\cdots}{\cdots}}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{44a^2(g\cos(e+\cdots}{357cfg\cdots}$$

Mathematica [A]
time = 6.47, size = 600, normalized size = 1.45

$$\frac{22(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}(a(1+\sin(e+fx)))^{5/2}}{3315f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{13/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}\left(-\frac{22}{3315}+\frac{16}{21\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}}-\frac{120}{119\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{8}}+\frac{84}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{6}}-\frac{22}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{4}}-\frac{22}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2}}+\frac{32\sin\left(\frac{1}{2}(e+fx)\right)}{21\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}-\frac{240\sin\left(\frac{1}{2}(e+fx)\right)}{119\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{9}}+\frac{168\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}}-\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{5}}-\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{3}}-\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{3315\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{5/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(5/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(5/2))/(3315*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(-22/3315 + 16/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 120/(119*(Co
```

```
s[(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 84/(221*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^6) - 22/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) - 22/(331
5*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (32*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (240*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (168*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (44*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^5) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] -
 Sin[(e + f*x)/2])^3) - (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^5*(c - c*Sin[e + f*x])^(13/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.30, size = 1473, normalized size = 3.56

method	result	size
default	Expression too large to display	1473

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/69615/f*(a*(1+sin(f*x+e)))^(5/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x
+e)-cos(f*x+e)-sin(f*x+e)+1)*(-26520+385*sin(f*x+e)*cos(f*x+e)^5+3696*I*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^8*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*
x+e)^8*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3234*I*(1/(1+cos(f*x+e)))^
(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+3234*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9471*I*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9471*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-10164*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+10164*I*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)-26520*sin(f*x+e)+22824*cos(f*x+e)-3696*I*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+3696*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-24488*cos(f*x+e)^3*sin(
f*x+e)+1155*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*cos(f*x
+e)^6*sin(f*x+e)+2618*cos(f*x+e)^4*sin(f*x+e)+18020*cos(f*x+e)^2*sin(f*x+e)
+1155*cos(f*x+e)^6+43600*cos(f*x+e)^2-35284*cos(f*x+e)^3+30216*cos(f*x+e)*s
```

```
in(f*x+e)+11998*cos(f*x+e)^5-1155*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-5775*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*sin(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5775*I*
(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f
*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+8316*I*(1/(1+cos(f*x+e)))
^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-8316*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)-17773*cos(f*x+e)^4-3696*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*
sin(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(13/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 525, normalized size = 1.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] 1/69615*(2*(1386*a^2*g*cos(f*x + e)^4 + 5607*a^2*g*cos(f*x + e)^2 - 10796*a
^2*g - (231*a^2*g*cos(f*x + e)^4 - 4081*a^2*g*cos(f*x + e)^2 + 15724*a^2*g)
*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c) - 231*(I*sqrt(2)*a^2*g*cos(f*x + e)^6 - 18*I*sqrt(2)*a^2*g*cos(
f*x + e)^4 + 48*I*sqrt(2)*a^2*g*cos(f*x + e)^2 - 32*I*sqrt(2)*a^2*g + 2*(3*
I*sqrt(2)*a^2*g*cos(f*x + e)^4 - 16*I*sqrt(2)*a^2*g*cos(f*x + e)^2 + 16*I*s
qrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 231*(-I*sqrt(2)*a^2*g*cos(
f*x + e)^6 + 18*I*sqrt(2)*a^2*g*cos(f*x + e)^4 - 48*I*sqrt(2)*a^2*g*cos(f*x
 + e)^2 + 32*I*sqrt(2)*a^2*g + 2*(-3*I*sqrt(2)*a^2*g*cos(f*x + e)^4 + 16*I*
sqrt(2)*a^2*g*cos(f*x + e)^2 - 16*I*sqrt(2)*a^2*g)*sin(f*x + e))*sqrt(a*c*g
```

```
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*
x + e))))/(c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*
x + e)^2 - 32*c^7*f + 2*(3*c^7*f*cos(f*x + e)^4 - 16*c^7*f*cos(f*x + e)^2 +
 16*c^7*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{5/2}}{(c-c\,\sin(e+f\,x))^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(13/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x)
)^(13/2), x)
```

3.116 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=463

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^4c^3g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{195f\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/15*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f
/g-2/39*a^2*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*
x+e))^(1/2)-14/585*a*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c
-c*sin(f*x+e))^(1/2)+14/195*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)
/f/g/(c-c*sin(f*x+e))^(1/2)-154/585*a^4*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*s
in(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+154/195*a^4*c^3*g*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(
1/2)-22/195*a^3*c^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*si
n(f*x+e))^(1/2)+22/195*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c
*sin(f*x+e))^(1/2)/f/g
```

Rubi [A]

time = 1.46, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^4c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{195f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3c^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{195fg\sqrt{c-c\sin(e+fx)}}-\frac{2a^2c^3(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{39fg\sqrt{c-c\sin(e+fx)}}-\frac{14ac^3(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{585fg\sqrt{c-c\sin(e+fx)}}+\frac{14c^3(a\sin(e+fx)+a)^{7/2}(g\cos(e+fx))^{5/2}}{195fg\sqrt{c-c\sin(e+fx)}}+\frac{22c^2(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{195fg}+\frac{2c(a\sin(e+fx)+a)^{7/2}(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{15fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-154*a^4*c^3*(g*Cos[e + f*x])^(5/2))/(585*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqr
t[c - c*Sin[e + f*x]]) + (154*a^4*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(195*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*
Sin[e + f*x]]) - (22*a^3*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]
])/(195*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*c^3*(g*Cos[e + f*x])^(5/2)*(
a + a*Sin[e + f*x])^(3/2))/(39*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^3*(g
*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(585*f*g*Sqrt[c - c*Sin[e
+ f*x]]) + (14*c^3*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(195*
f*g*Sqrt[c - c*Sin[e + f*x]]) + (22*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e
 + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(195*f*g) + (2*c*(g*Cos[e + f*x])^
(5/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(15*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{5/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))}{15fg}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{195fg}$$

$$= \frac{14c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{195fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14ac^3(g\cos(e+fx))^{5/2}(a+a\sin(e+}{585fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^2c^3(g\cos(e+fx))^{5/2}(a+a\sin(e+}{39fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{22a^3c^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e}}{195fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{154a^4c^3(g\cos(e+fx))^{5/2}}{585fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

Mathematica [A]
time = 2.37, size = 226, normalized size = 0.49

$$-\frac{a^3c^2(g\cos(e+fx))^{3/2}(-1+\sin(e+fx))^2(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\left(-14784E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(1365\cos(e+fx)+819\cos(3(e+fx))+273\cos(5(e+fx))+39\cos(7(e+fx))-3794\sin(2(e+fx))-800\sin(4(e+fx))-90\sin(6(e+fx)))\right)}{18720f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(5/2),x]
```

```
[Out] -1/18720*(a^3*c^2*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])^2*(1 + Sin[e +
 f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-14784*Ellipt
icE[(e + f*x)/2, 2] + Sqrt[Cos[e + f*x]]*(1365*Cos[e + f*x] + 819*Cos[3*(e
+ f*x)] + 273*Cos[5*(e + f*x)] + 39*Cos[7*(e + f*x)] - 3794*Sin[2*(e + f*x)
] - 800*Sin[4*(e + f*x)] - 90*Sin[6*(e + f*x)])))/(f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
7)
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 392, normalized size = 0.85

method	result
default	$\frac{2\left(-39\sin(fx+e)\left(\cos^8(fx+e)\right)-45\left(\cos^8(fx+e)\right)+231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{}{1}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/585/f*(-39*sin(f*x+e)*cos(f*x+e)^8-45*cos(f*x+e)^8+231*I*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-10*cos(f*x+e)^6+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I
*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+e)^4-154*cos(f*x+e)^2+231*
cos(f*x+e))*(-c*(sin(f*x+e)-1))^(5/2)*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)
))^(7/2)/(1+sin(f*x+e))/sin(f*x+e)/cos(f*x+e)^7
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 202, normalized size = 0.44

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^3c^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^3c^2g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\left(39a^3c^2g\cos(fx+e)^6-\left(45a^3c^2g\cos(fx+e)^4+55a^3c^2g\cos(fx+e)^2+77a^3c^2g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{585f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/585*(-231*I*sqrt(2)*sqrt(a*c*g)*a^3*c^2*g*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c
```

```
*g)*a^3*c^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e
) - I*sin(f*x + e))) - 2*(39*a^3*c^2*g*cos(f*x + e)^6 - (45*a^3*c^2*g*cos(f
*x + e)^4 + 55*a^3*c^2*g*cos(f*x + e)^2 + 77*a^3*c^2*g)*sin(f*x + e))*sqrt(
g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(5/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(5/2), x)
```

3.117 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=409

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14a^4c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{13f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\cdot$$

```
[Out] -10/143*a^2*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*
x+e))^(1/2)-14/429*a*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c
-c*sin(f*x+e))^(1/2)+14/143*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)
/f/g/(c-c*sin(f*x+e))^(1/2)-14/39*a^4*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin
(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+14/13*a^4*c^2*g*(cos(1/2*f*x+1/2*e)^2
)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)
^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
-2/13*a^3*c^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+
e))^(1/2)+2/13*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e
))^(1/2)/f/g
```

Rubi [A]
time = 1.30, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14a^4c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{13f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c^2\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{13fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{143fg\sqrt{c-c\sin(e+fx)}}-\frac{14ac^2(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{429fg\sqrt{c-c\sin(e+fx)}}+\frac{14c^2(a\sin(e+fx)+a)^{7/2}(g\cos(e+fx))^{5/2}}{143fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)^{7/2}\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{13fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-14*a^4*c^2*(g*Cos[e + f*x])^(5/2))/(39*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (14*a^4*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]
]*EllipticE[(e + f*x)/2, 2])/(13*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) - (2*a^3*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(1
3*f*g*Sqrt[c - c*Sin[e + f*x]]) - (10*a^2*c^2*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(3/2))/(143*f*g*Sqrt[c - c*Sin[e + f*x]]) - (14*a*c^2*(g*Cos
[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(429*f*g*Sqrt[c - c*Sin[e + f*
x]]) + (14*c^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(143*f*g*
Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x]
)^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(13*f*g)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}(c-c\sin(e+fx))^{3/2}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx}{13fg}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+f}{143fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14ac^2(g\cos(e+fx))^{5/2}(a+a\sin(e}{429fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{10a^2c^2(g\cos(e+fx))^{5/2}(a+a\sin(e}{143fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^3c^2(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e}}{13fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

$$= -\frac{14a^4c^2(g\cos(e+fx))^{5/2}}{39fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin}}$$

Mathematica [A]
time = 1.98, size = 212, normalized size = 0.52

$$\frac{a^3c(g\cos(e+fx))^{3/2}(-1+\sin(e+fx))(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}\left(-7392E\left(\frac{1}{2}(e+fx)|2\right)+\sqrt{\cos(e+fx)}(1560\cos(e+fx)+780\cos(3(e+fx))+156\cos(5(e+fx))-1507\sin(2(e+fx))-88\sin(4(e+fx))+33\sin(6(e+fx)))\right)}{6864f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (a^3*c*(g*Cos[e + f*x])^(3/2)*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3*Sqrt
[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-7392*EllipticE[(e + f*x)/
2, 2] + Sqrt[Cos[e + f*x]]*(1560*Cos[e + f*x] + 780*Cos[3*(e + f*x)] + 156*
Cos[5*(e + f*x)] - 1507*Sin[2*(e + f*x)] - 88*Sin[4*(e + f*x)] + 33*Sin[6*(
e + f*x)])))/(6864*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [C] Result contains complex when optimal does not.
time = 0.28, size = 402, normalized size = 0.98

method	result
default	$-\frac{2(-c(\sin(fx+e)-1))^{\frac{3}{2}}\left(33\left(\cos^8(fx+e)\right)-78\left(\cos^6(fx+e)\right)\sin(fx+e)-88\left(\cos^6(fx+e)\right)+231i\sin(fx+e)\cos(fx+e)\operatorname{EllipticF}\left(\frac{i(-1}{s}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/429/f*(-c*(sin(f*x+e)-1))^(3/2)*(33*cos(f*x+e)^8-78*cos(f*x+e)^6*sin(f*x
+e)-88*cos(f*x+e)^6+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2
31*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*
cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22*cos(f*x+
e)^4-154*cos(f*x+e)^2+231*cos(f*x+e))*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)
))^(7/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)^5/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 190, normalized size = 0.46

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^3cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^3cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\left(78a^3cg\cos(fx+e)^4+11\left(3a^3cg\cos(fx+e)^4-5a^3cg\cos(fx+e)^2-7a^3cg\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{429f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] 1/429*(-231*I*sqrt(2)*sqrt(a*c*g)*a^3*c*g*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g
)*a^3*c*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
```

```
I*sin(f*x + e))) - 2*(78*a^3*c*g*cos(f*x + e)^4 + 11*(3*a^3*c*g*cos(f*x + e
)^4 - 5*a^3*c*g*cos(f*x + e)^2 - 7*a^3*c*g)*sin(f*x + e))*sqrt(g*cos(f*x +
e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e
) + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(3/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(3/2), x)
```

3.118 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+$

Optimal. Leaf size=343

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2a^4cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3}{}$$

```
[Out] -10/77*a^2*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e
))^(1/2)-2/33*a*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(
f*x+e))^(1/2)+2/11*c*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(7/2)/f/g/(c-c*s
in(f*x+e))^(1/2)-2/3*a^4*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/
(c-c*sin(f*x+e))^(1/2)+2*a^4*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))
^(1/2)/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-2/7*a^3*c*(g*cos(f*x
+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.10, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2a^4cg\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2a^3c\sqrt{a\sin(e+fx)+a}(g\cos(e+fx))^{5/2}}{7fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2c(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{77fg\sqrt{c-c\sin(e+fx)}}-\frac{2ac(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{33fg\sqrt{c-c\sin(e+fx)}}+\frac{2c(a\sin(e+fx)+a)^{7/2}(g\cos(e+fx))^{5/2}}{11fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*
x]],x]
```

```
[Out] (-2*a^4*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
c*Sin[e + f*x]]) + (2*a^4*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (2*a^3*c*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(7*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*c*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^
(3/2))/(77*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*c*(g*Cos[e + f*x])^(5/2)*(a
 + a*Sin[e + f*x])^(5/2))/(33*f*g*Sqrt[c - c*Sin[e + f*x]]) + (2*c*(g*Cos[e
 + f*x])^(5/2)*(a + a*Sin[e + f*x])^(7/2))/(11*f*g*Sqrt[c - c*Sin[e + f*x]]
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}\sqrt{c-c\sin(e+fx)}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))}{11fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2ac(g\cos(e+fx))^{5/2}(a+a\sin(e+f}{33fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{10a^2c(g\cos(e+fx))^{5/2}(a+a\sin(e+}{77fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^3c(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e}}{7fg\sqrt{c-c\sin(e+fx)}}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

$$= -\frac{2a^4c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 11.47, size = 360, normalized size = 1.05

$$\frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\left(\frac{(2+2i)a^4e^{\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\left(e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)\right)^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}+\left(-1+e^{2ie}\right){}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(e+fx)}\right)\right)}{\left(-1+e^{2ie}\right)\sqrt{-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(1+e^{2i(e+fx)}\right)^{3/2}f}-\frac{a^3\sqrt{\cos(e+fx)}\sqrt{a(1+\sin(e+fx))}\,(1374\cos(e+fx)+423\cos(3(e+fx))-7(3\cos(5(e+fx))-528\cot(e)+44\sin(2(e+fx))-22\sin(4(e+fx))))}{1848f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)}{\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[
e + f*x]],x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(((2 + 2*I)*a^4*E^((I/2)*(
e + f*x))*(I + E^(I*(e + f*x)))*((1 + E^((2*I)*(e + f*x)))/E^(I*(e + f*x)))
^(3/2)*(Sqrt[1 + E^((2*I)*(e + f*x))] + (-1 + E^((2*I)*e))*Hypergeometric2F
1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))]))/((-1 + E^((2*I)*e))*Sqrt[((-I)*a*
(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(1 + E^((2*I)*(e + f*x)))^(3/2)*f
) - (a^3*Sqrt[Cos[e + f*x]]*Sqrt[a*(1 + Sin[e + f*x])]*(1374*Cos[e + f*x] +
 423*Cos[3*(e + f*x)] - 7*(3*Cos[5*(e + f*x)] - 528*Cot[e] + 44*Sin[2*(e +
f*x)] - 22*Sin[4*(e + f*x)])))/(1848*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
))))/(Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 425, normalized size = 1.24

method	result
default	$-\frac{2\left(21\left(\cos^6(fx+e)\right)\sin(fx+e)+77\left(\cos^6(fx+e)\right)-231i\sin(fx+e)\cos(fx+e)\,\text{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{c}{1+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/231/f*(21*cos(f*x+e)^6*sin(f*x+e)+77*cos(f*x+e)^6-231*I*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)-132*cos(f*x+e)^4*sin(f*x+e)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin
(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-154*cos(f*x+e)^4-154*cos(
f*x+e)^2+231*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(g*cos(f*x+e))^(3/2)*(a*
(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-
4)/cos(f*x+e)^3/sin(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 183, normalized size = 0.53

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(21a^3g\cos(fx+e)^4-132a^3g\cos(fx+e)^2-77\left(a^3g\cos(fx+e)^2-a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{231f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/231*(-231*I*sqrt(2)*sqrt(a*c*g)*a^3*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
a^3*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) + 2*(21*a^3*g*cos(f*x + e)^4 - 132*a^3*g*cos(f*x + e)^2 - 77*(
a^3*g*cos(f*x + e)^2 - a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin
(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/f
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)*(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x
 + e) + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{7/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(1/2),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^
(1/2), x)
```

3.119 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=288

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22a^4g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-22$$

```
[Out] -10/21*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/f/g/(c-c*sin(f*x+e))
^(1/2)-2/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e
))^(1/2)-22/9*a^4*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+22/3*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/21*a^3*(g*cos(f*x+e))^(5/
2)*(a+a*sin(f*x+e))^(1/2)/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$-\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{2a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Sin[e +
f*x]],x]
```

```
[Out] (-22*a^4*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (22*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(21*f*g*Sqrt[c
- c*Sin[e + f*x]]) - (10*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3
/2))/(21*f*g*Sqrt[c - c*Sin[e + f*x]]) - (2*a*(g*Cos[e + f*x])^(5/2)*(a + a
*Sin[e + f*x])^(5/2))/(9*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{\sqrt{c-c\sin(e+fx)}}\,dx &= -\frac{2a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg\sqrt{c-c\sin(e+fx)}}+\frac{1}{3}(5a)\int \ldots \\
&= -\frac{10a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{2a(g\cos(e \ldots}{9 \ldots} \\
&= -\frac{22a^3(g\cos(e+fx))^{5/2}\sqrt{a+a\sin(e+fx)}}{21fg\sqrt{c-c\sin(e+fx)}}-\frac{10a^2(g\cos(\ldots}{21 \ldots} \\
&= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3(g\cos \ldots}{2 \ldots} \\
&= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3(g\cos \ldots}{2 \ldots} \\
&= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{22a^3(g\cos \ldots}{2 \ldots} \\
&= -\frac{22a^4(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^4g\sqrt{c \ldots}}{3f\sqrt{\ldots}}
\end{aligned}
$$

Mathematica [A]
time = 2.57, size = 181, normalized size = 0.63

$$
-\frac{a^3(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\left(-1848E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(1128\cos(e+fx)-72\cos(3(e+fx))+350\sin(2(e+fx))-7\sin(4(e+fx)))\right)}{252f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\sqrt{c-c\sin(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/Sqrt[c - c*Si
n[e + f*x]],x]
```

```
[Out] -1/252*(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1
 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2,
 2] + Sqrt[Cos[e + f*x]]*(1128*Cos[e + f*x] - 72*Cos[3*(e + f*x)] + 350*Sin
[2*(e + f*x)] - 7*Sin[4*(e + f*x)])))/(f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 434, normalized size = 1.51

method	result
default	$-\frac{2(a(1+\sin(fx+e)))^{\frac{7}{2}}(g\cos(fx+e))^{\frac{3}{2}}\left(231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e}{1+\cos(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/63/f*(a*(1+sin(f*x+e)))^(7/2)*(g*cos(f*x+e))^(3/2)*(231*I*sin(f*x+e)*cos
(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)+7*cos(f*x+e)^6+231*I*sin(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*
I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-36*cos(f*x+e)^4*sin(f*x+e)-98*cos(f*
x+e)^4+168*cos(f*x+e)^2*sin(f*x+e)+322*cos(f*x+e)^2-231*cos(f*x+e))/(cos(f*
x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)+8)/cos(f*x+e)/
sin(f*x+e)/(-c*(sin(f*x+e)-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x
 + e) + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 177, normalized size = 0.61

$$\frac{-231i\sqrt{2}\sqrt{acg}\,a^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,a^3g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\left(36\,a^3g\cos(fx+e)^2-168\,a^3g+7\left(a^3g\cos(fx+e)^2-13\,a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{63\,cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/63*(-231*I*sqrt(2)*sqrt(a*c*g)*a^3*g*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*a
^3*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin
(f*x + e))) + 2*(36*a^3*g*cos(f*x + e)^2 - 168*a^3*g + 7*(a^3*g*cos(f*x + e
)^2 - 13*a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(-c*sin(f*x + e) + c))/(c*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{\sqrt{c-c\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(1/2), x)
```

3.120 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^4g\sqrt{\cos(e+}{cf\sqrt{a+a}}$$

```
[Out] 4*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)+
30/7*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e))
^(1/2)+22*a^4*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*
x+e))^(1/2)-66*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(
a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+66/7*a^3*(g*cos(f*x+e))^(5/2)*
(a+a*sin(f*x+e))^(1/2)/c/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{22a^4(g\cos(e+fx))^{5/2}}{cfg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{66a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{7cfg\sqrt{c-c\sin(e+fx)}}+\frac{30a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{7cfg\sqrt{c-c\sin(e+fx)}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(f*g*(c - c*Sin[e +
 f*x])^(3/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(c*f*g*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e
+ f*x]]*EllipticE[(e + f*x)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (66*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt[a + a*Sin[e + f*x]])/
(7*c*f*g*Sqrt[c - c*Sin[e + f*x]]) + (30*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*
Sin[e + f*x])^(3/2))/(7*c*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} - \frac{(15a)\int \frac{(g\cos(\cdots}{\cdots}}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{30a^2(g\cos(e\cdots}{7cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{66a^3(g\cos(e\cdots}{7cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22\cdots}{cfg\sqrt{a+a\,s\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22\cdots}{cfg\sqrt{a+a\,s\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22\cdots}{cfg\sqrt{a+a\,s\cdots}}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{fg(c-c\sin(e+fx))^{3/2}} + \frac{22\cdots}{cfg\sqrt{a+a\,s\cdots}}$$

Mathematica [A]
time = 6.39, size = 284, normalized size = 0.97

$$-\frac{66(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}}{f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{3/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{7/2}\left(32+\frac{109}{14}\cos(e+fx)-\frac{1}{14}\cos(3(e+fx))+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)}+\sin(2(e+fx))\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 +
 Sin[e + f*x]))^(7/2)*(32 + (109*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14 + (
64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + Sin[2*(e + f*x
)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 2996, normalized size = 10.19

method	result	size
default	Expression too large to display	2996

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/7/f*(-1+cos(f*x+e))*(462*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-462*I*
cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))
^(1/2)-231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^3*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1
+cos(f*x+e)))^(1/2)-231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-sin(f*x+e)
*cos(f*x+e)^5-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)+21*cos(f*x+e)^3*sin(f*x+e)-231*I*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*sin(f*x+e)-7*cos(f*x+e)^4*sin(f*x+e)+119*cos(f*x+e)^2
*sin(f*x+e)+231*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+cos(f*x+e
)^6-112*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*c
os(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+112*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(3/2)-168*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/si
n(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+168*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(3/2)-112*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+112*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
```

```
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)-28*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-343*cos(f*x+e)^2+98*cos(f*x+e
)^3-6*cos(f*x+e)^5-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/s
in(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3
/2)+231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-231*I*sin(f*x+e
)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+84*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(3/2)-84*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*
sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-28*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(3/2)+84*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)-84*cos(f*x+e)^2*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-28*cos(f*x+e)^4)
*(g*cos(f*x+e))^(3/2)*(a*(1+sin(f*x+e)))^(7/2)/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.15, size = 229, normalized size = 0.78

$$-\frac{2\left(6\,a^3g\cos\left(fx+e\right)^2+133\,a^3g+\left(a^3g\cos\left(fx+e\right)^2-21\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(-i\sqrt{2}\,a^3g\sin\left(fx+e\right)+i\sqrt{2}\,a^3g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+231\left(i\sqrt{2}\,a^3g\sin\left(fx+e\right)-i\sqrt{2}\,a^3g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{7\left(c^2f\sin\left(fx+e\right)-c^2f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/7*(2*(6*a^3*g*cos(f*x + e)^2 + 133*a^3*g + (a^3*g*cos(f*x + e)^2 - 21*a^
3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c) + 231*(-I*sqrt(2)*a^3*g*sin(f*x + e) + I*sqrt(2)*a^3*g)*sqr
t(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I
*sin(f*x + e))) + 231*(I*sqrt(2)*a^3*g*sin(f*x + e) - I*sqrt(2)*a^3*g)*sqrt
(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(c^2*f*sin(f*x + e) - c^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(3/2), x)
```

3.121 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}}-\frac{12a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{cfg(c-c\sin(e+fx))^{3/2}}-\frac{154a}{5c^2fg\sqrt{a+a\sin}}$$

```
[Out] 4/5*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2
)-12*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e))
^(3/2)-154/5*a^4*(g*cos(f*x+e))^(5/2)/c^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+462/5*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-66/5*a^3*(g*cos(f*x+
e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$-\frac{154a^4(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{462a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{5c^2fg\sqrt{c-c\sin(e+fx)}}-\frac{12a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{cfg(c-c\sin(e+fx))^{3/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(5*f*g*(c - c*Sin[e
 + f*x])^(5/2)) - (12*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2)
)/(c*f*g*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*(g*Cos[e + f*x])^(5/2))/(5*
c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (462*a^4*g*Sqr
t[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sq
rt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (66*a^3*(g*Cos[e + f*x])
^(5/2)*Sqrt[a + a*Sin[e + f*x]])/(5*c^2*f*g*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{(3a)\int \frac{(g\cos(e}{}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{5fg(c-c\sin(e+fx))^{5/2}} - \frac{12a^2(g\cos(e}{cfg}$$

Mathematica [A]
time = 3.22, size = 267, normalized size = 0.90

$$-\frac{a^3(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{a(1+\sin(e+fx))}\left(-1848E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(487\cos\left(\frac{1}{2}(e+fx)\right)+633\cos\left(\frac{3}{2}(e+fx)\right)-17\cos\left(\frac{5}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)+487\sin\left(\frac{1}{2}(e+fx)\right)-633\sin\left(\frac{3}{2}(e+fx)\right)-17\sin\left(\frac{5}{2}(e+fx)\right)-\sin\left(\frac{7}{2}(e+fx)\right)\right)\right)}{20c^2f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] -1/20*(a^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*S
qrt[a*(1 + Sin[e + f*x])]*(-1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Co
s[(3*(e + f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] + 487*S
in[(e + f*x)/2] - 633*Sin[(3*(e + f*x))/2] - 17*Sin[(5*(e + f*x))/2] - Sin[
(7*(e + f*x))/2])))/(c^2*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 2.59, size = 3600, normalized size = 12.08

method	result	size
default	Expression too large to display	3600

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(-1+cos(f*x+e))*(-sin(f*x+e)*cos(f*x+e)^5+231*I*cos(f*x+e)^4*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-693*I*cos(f
*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+693*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1
/2)-462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+462*I*cos(f*x+e)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(
f*x+e)))^(1/2)-231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellipti
cE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+693*
I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-693*I*cos(f*x+e)^2*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)-462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-80*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2
)+40*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-40*cos(f*x+e)^4*sin(
f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+69*cos(f*x+e)^3*sin(f*x+e)+231*I*cos(f*x
+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-9*cos(f*x+e)^4*sin(f*x+e)-478*co
s(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^6+80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3
/2)-80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^
```

```
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(
f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+320*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)-320*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f
*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+280*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(3/2)-80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-280*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+
446*cos(f*x+e)^2-85*cos(f*x+e)^3+40*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-
40*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-8*cos(f*x+e)^5+80*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(3/2)-80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-280*cos(f*x+e)*sin(f*x+e)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(3/2)+280*cos(f*x+e)*sin(f*x+e...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.14, size = 280, normalized size = 0.94

$$-\frac{2\left(8a^3g\cos(fx+e)^2-146a^3g+\left(a^3g\cos(fx+e)^2+162a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}+231\left(i\sqrt{2}a^3g\cos(fx+e)^2+2i\sqrt{2}a^3g\sin(fx+e)-2i\sqrt{2}a^3g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231\left(-i\sqrt{2}a^3g\cos(fx+e)^2-2i\sqrt{2}a^3g\sin(fx+e)+2i\sqrt{2}a^3g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{5\left(c^3f\cos(fx+e)^2+2c^3f\sin(fx+e)-2c^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(2*(8*a^3*g*cos(f*x + e)^2 - 146*a^3*g + (a^3*g*cos(f*x + e)^2 + 162*a
^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*s
in(f*x + e) + c) + 231*(I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 2*I*sqrt(2)*a^3*g*
sin(f*x + e) - 2*I*sqrt(2)*a^3*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(-I*sqrt(2)*a^3*
g*cos(f*x + e)^2 - 2*I*sqrt(2)*a^3*g*sin(f*x + e) + 2*I*sqrt(2)*a^3*g)*sqrt
(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(c^3*f*cos(f*x + e)^2 + 2*c^3*f*sin(f*x + e) - 2*c^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(5/2), x)
```

3.122 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=300

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}}-\frac{4a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{3cfg(c-c\sin(e+fx))^{5/2}}+\frac{44a^3(g\cos(e+fx}{3c^2fg(c-}$$

```
[Out] 4/9*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2
)-4/3*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x+e)
)^(5/2)+44/3*a^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*s
in(f*x+e))^(3/2)+154/9*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(a+a*sin(f*x+e))^(1
/2)/(c-c*sin(f*x+e))^(1/2)-154/3*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f
*x+e))^(1/2)/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{154a^4(g\cos(e+fx))^{5/2}}{9c^3fg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{3c^2fg(c-c\sin(e+fx))^{3/2}}-\frac{4a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{3cfg(c-c\sin(e+fx))^{5/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(7/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(9*f*g*(c - c*Sin[e
 + f*x])^(7/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2))
/(3*c*f*g*(c - c*Sin[e + f*x])^(5/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*Sqrt
[a + a*Sin[e + f*x]])/(3*c^2*f*g*(c - c*Sin[e + f*x])^(3/2)) + (154*a^4*(g*
Cos[e + f*x])^(5/2))/(9*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(3*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{7/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - (5a)\int \frac{(g\cos(e \cdots}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{9fg(c-c\sin(e+fx))^{7/2}} - \frac{4a^2(g\cos(e+\cdots}{3cfg\cdots}$$

Mathematica [**A**]
time = 6.43, size = 406, normalized size = 1.35

$$-\frac{154(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{7/2}}{3f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{7/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7\left(\frac{112}{3}+\frac{2}{3}\cos(e+fx)+\frac{32}{9\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}-\frac{32}{3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{9\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}-\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}+\frac{224\sin\left(\frac{1}{2}(e+fx)\right)}{3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(7/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(112
/3 + (2*Cos[e + f*x])/3 + 32/(9*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) -
32/(3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(9*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (64*Sin[(e + f*x)/2])/(3*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^3) + (224*Sin[(e + f*x)/2])/(3*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2]))))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.25, size = 4236, normalized size = 14.12

method	result	size
default	Expression too large to display	4236

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2),x,me
thod=_RETURNVERBOSE)

[Out] -2/9/f*(-1+cos(f*x+e))*(-3*sin(f*x+e)*cos(f*x+e)^5-54*cos(f*x+e)^5*sin(f*x+
e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)+54*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f
*x+e)^2)+540*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-540*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)+54*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x
+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-
54*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-162*cos(f*x+e)^3*sin
(f*x+e)+231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*(1/(1+cos(f*x+
e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-57*cos(f*x+
e)^4*sin(f*x+e)+908*cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^6-810*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(3/2)+810*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-756
ln(-2(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+
e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+216*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin
(f*x+e)+756*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)
*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-216*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3
/2)*sin(f*x+e)-940*cos(f*x+e)^2+146*cos(f*x+e)^3-324*cos(f*x+e)^5*(-cos(f*x

```
+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
1)/sin(f*x+e)^2)+324*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+231*I*cos(f*x
+e)^5*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I*cos(f*x+e)^5*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+924*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924*I*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(1/(1+co
s(f*x+e)))^(1/2)-1386*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(1/(
1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1386*I*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)-924*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2
)+924*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)-54*cos(f*x+e)^5-216*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(3/2)-231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*(1/(
1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2
31*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+1386*I*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-1386*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)+924*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-92
4*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.15, size = 339, normalized size = 1.13

$$\frac{2\left(177\,a^3g\cos\left(fx+e\right)^2-316\,a^3g-3\left(a^3g\cos\left(fx+e\right)^2-100\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(3i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2-4i\,\sqrt{2}\,a^3g+\left(-i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2+4i\,\sqrt{2}\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+231\left(-3i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2+4i\,\sqrt{2}\,a^3g+\left(i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2-4i\,\sqrt{2}\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{9\left(3\,c^4f\cos\left(fx+e\right)^2-4\,c^4f-\left(c^4f\cos\left(fx+e\right)^2-4\,c^4f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(177*a^3*g*cos(f*x + e)^2 - 316*a^3*g - 3*(a^3*g*cos(f*x + e)^2 - 10
0*a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-
c*sin(f*x + e) + c) + 231*(3*I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 4*I*sqrt(2)*a
^3*g + (-I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 4*I*sqrt(2)*a^3*g)*sin(f*x + e))*
sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
+ I*sin(f*x + e))) + 231*(-3*I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 4*I*sqrt(2)*a
^3*g + (I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 4*I*sqrt(2)*a^3*g)*sin(f*x + e))*s
qrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
 I*sin(f*x + e))))/(3*c^4*f*cos(f*x + e)^2 - 4*c^4*f - (c^4*f*cos(f*x + e)^
2 - 4*c^4*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(7/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(7/2), x)
```

3.123 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=300

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}}-\frac{20a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{39cfg(c-c\sin(e+fx))^{7/2}}+\frac{44a^3(g\cos(e+fx)}{39c^2fg(c-}$$

```
[Out] 4/13*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(9/
2)-20/39*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x
+e))^(7/2)-308/39*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(3/2)/(
a+a*sin(f*x+e))^(1/2)+44/39*a^3*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)
/c^2/f/g/(c-c*sin(f*x+e))^(5/2)+154/13*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g
*cos(f*x+e))^(1/2)/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{13c^4f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{308a^4(g\cos(e+fx))^{5/2}}{39c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{39c^2fg(c-c\sin(e+fx))^{5/2}}-\frac{20a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{39cfg(c-c\sin(e+fx))^{7/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(9/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(13*f*g*(c - c*Sin[
e + f*x])^(9/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(39*c*f*g*(c - c*Sin[e + f*x])^(7/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*S
qrt[a + a*Sin[e + f*x]])/(39*c^2*f*g*(c - c*Sin[e + f*x])^(5/2)) - (308*a^4
*(g*Cos[e + f*x])^(5/2))/(39*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e
+ f*x])^(3/2)) + (154*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(13*c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f
*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{(15a)\int \frac{(g\cos(}{}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{13fg(c-c\sin(e+fx))^{9/2}} - \frac{20a^2(g\cos(e}{39cf}$$

Mathematica [A]

time = 6.48, size = 464, normalized size = 1.55

$$\frac{154(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{7/2}}{13f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{9/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9\left(-\frac{128}{13}+\frac{32}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6}-\frac{224}{39\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4}+\frac{80}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}-\frac{448\sin\left(\frac{1}{2}(e+fx)\right)}{39\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5}+\frac{160\sin\left(\frac{1}{2}(e+fx)\right)}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{256\sin\left(\frac{1}{2}(e+fx)\right)}{13\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(c-c\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
 + f*x])^(9/2),x]
```

```
[Out] (154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(13*f*Cos[e + f*x]^(3/2)*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(-12
8/13 + 32/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) - 224/(39*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^4) + 80/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^2) + (64*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
 - (448*Sin[(e + f*x)/2])/(39*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (16
0*Sin[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) - (256*Sin
[(e + f*x)/2])/(13*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e +
f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x
])^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 4828, normalized size = 16.09

method	result	size
default	Expression too large to display	4828

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/39/f*(-1+cos(f*x+e))*(231*I*cos(f*x+e)^6*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-231*I
*cos(f*x+e)^6*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-462*I*cos(f*x+e)^5*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e
)))^(1/2)+462*I*cos(f*x+e)^5*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*
x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-2541*I*cos(f*x+e)^4*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(1/(1+cos(f*x+e)))^(1/2)+2541*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-924
*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+924*I*cos(f*x+e)^3*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+2772*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE
```

(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-2772*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-39*sin(f*x+e)*cos(f*x+e)^5+546*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-546*cos(f*x+e)^5*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-2184*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+2184*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-858*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+858*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+284*cos(f*x+e)^3*sin(f*x+e)+231*I*cos(f*x+e)^5*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+660*cos(f*x+e)^4*sin(f*x+e)-1896*cos(f*x+e)^2*sin(f*x+e)+39*cos(f*x+e)^6-546*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+546*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+2184*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-2184*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+2184*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-624*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-2184*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+624*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+1800*cos(f*x+e)^2-332*cos(f*x+e)^3+78*cos(f*x+e)^6*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)^2+2*cos(f*

```
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-78*cos(f*x+e)^
6*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+1092*cos(f*x+e)^5*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)-1092*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 392, normalized size = 1.31

$$\frac{16\left(57a^3g\cos(fx+e)^2-74a^3g-8\left(3a^3g\cos(fx+e)^2-10a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}-231\left(i\sqrt{2}a^3g\cos(fx+e)^4-8i\sqrt{2}a^3g\cos(fx+e)^2+8i\sqrt{2}a^3g+4\left(i\sqrt{2}a^3g\cos(fx+e)^2-2i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-231\left(-i\sqrt{2}a^3g\cos(fx+e)^4+8i\sqrt{2}a^3g\cos(fx+e)^2-8i\sqrt{2}a^3g+4\left(-i\sqrt{2}a^3g\cos(fx+e)^2+2i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{39\left(c^5f\cos(fx+e)^4-8c^5f\cos(fx+e)^2+8c^5f+4\left(c^5f\cos(fx+e)^2-2c^5f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="fricas")
```

```
[Out] 1/39*(16*(57*a^3*g*cos(f*x + e)^2 - 74*a^3*g - 8*(3*a^3*g*cos(f*x + e)^2 -
10*a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c) - 231*(I*sqrt(2)*a^3*g*cos(f*x + e)^4 - 8*I*sqrt(2)*a^
3*g*cos(f*x + e)^2 + 8*I*sqrt(2)*a^3*g + 4*(I*sqrt(2)*a^3*g*cos(f*x + e)^2
- 2*I*sqrt(2)*a^3*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 231*(-I*sqrt(2)*a^3
*g*cos(f*x + e)^4 + 8*I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 8*I*sqrt(2)*a^3*g +
4*(-I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 2*I*sqrt(2)*a^3*g)*sin(f*x + e))*sqrt(
a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))))/(c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^2 + 8*c^5*f + 4
*(c^5*f*cos(f*x + e)^2 - 2*c^5*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(9/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(9/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(9/2), x)
```

3.124 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=357

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}}-\frac{60a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{221cfg(c-c\sin(e+fx))^{9/2}}+\frac{220a^3(g\cos(e+f}{663c^2fg(c}$$

```
[Out] 4/17*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(11
/2)-60/221*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f
*x+e))^(9/2)-308/663*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(5/2
)/(a+a*sin(f*x+e))^(1/2)+154/221*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin(
f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+220/663*a^3*(g*cos(f*x+e))^(5/2)*(a+a*
sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(7/2)-154/221*a^4*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*c
os(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^5/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{154a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{221c^5f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{154a^4(g\cos(e+fx))^{5/2}}{221c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{308a^4(g\cos(e+fx))^{5/2}}{663c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{220a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{663c^2fg(c-c\sin(e+fx))^{7/2}}-\frac{60a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{221cfg(c-c\sin(e+fx))^{9/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(11/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(17*f*g*(c - c*Sin[
e + f*x])^(11/2)) - (60*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(221*c*f*g*(c - c*Sin[e + f*x])^(9/2)) + (220*a^3*(g*Cos[e + f*x])^(5/2
)*Sqrt[a + a*Sin[e + f*x]])/(663*c^2*f*g*(c - c*Sin[e + f*x])^(7/2)) - (308
*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*S
in[e + f*x])^(5/2)) + (154*a^4*(g*Cos[e + f*x])^(5/2))/(221*c^4*f*g*Sqrt[a
+ a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (154*a^4*g*Sqrt[Cos[e + f*x
]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(221*c^5*f*Sqrt[a + a*Si
n[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{11/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{(15a)\int \frac{(g\cos(e\cdots}{\cdots}}{\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{17fg(c-c\sin(e+fx))^{11/2}} - \frac{60a^2(g\cos(e+\cdots}{221cf\cdots}$$

Mathematica [A]
time = 6.53, size = 532, normalized size = 1.49

$$-\frac{154(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}(a(1+\sin(e+fx)))^{7/2}}{221f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{11/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}\left(\frac{154}{221}+\frac{32}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{8}}-\frac{864}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{6}}+\frac{2096}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{4}}-\frac{288}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2}}+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{17\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{9}}-\frac{1728\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}}+\frac{4192\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{5}}-\frac{576\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{3}}+\frac{308\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(11/2),x]
```

```
[Out] (-154*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^11*(a*(1 + Sin[e + f*x]))^(7/2))/(221*f*Cos[e + f*x]^(3/2
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + ((
g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11
*(154/221 + 32/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 864/(221*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 2096/(663*(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2])^4) - 288/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Si
n[(e + f*x)/2])/(17*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (1728*Sin[(e
 + f*x)/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (4192*Sin[(e +
```

```
f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) - (576*Sin[(e + f*x)
/2])/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (308*Sin[(e + f*x)/2])
/(221*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))
/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 3908, normalized size = 10.95

method	result	size
default	Expression too large to display	3908

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] -1/1326/f*(a*(1+sin(f*x+e)))^(7/2)*(g*cos(f*x+e))^(3/2)*(sin(f*x+e)-1)*(-1+
cos(f*x+e))^4*(1+cos(f*x+e))*(5304*cos(f*x+e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-3696*I*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3696*I*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^6*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-7392*I*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+7
392*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))
^(1/2)+7392*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2
)*cos(f*x+e)^4-7392*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*cos(f*x+e)^4+22176*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-22176*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3696*I*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2-3696*I*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+7392*I*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-7392*I*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-14784*
I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*El
```

```
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)
+14784*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*cos
(f*x+e)-7392*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)+7392*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+53
04*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^
2)*sin(f*x+e)-5304*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(
f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+752*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*si
n(f*x+e)*cos(f*x+e)^2-9888*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)*
cos(f*x+e)+6928*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-2496*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+14784*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*sin(f*x+e)-14784*I*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)+924*I*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^6
*sin(f*x+e)-924*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)*cos(f*x+e)^6*sin(f*x+e)+1848*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^5*sin(f*x+e)-1848*I*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2
)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5*sin(f*x+e)-6468*I*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4
*sin(f*x+e)+6468*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*s
in(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)-14784*I*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*sin(f*x+e)+14784*I*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="maxima")
```

[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(11/2), x)

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 457, normalized size = 1.28

$$\frac{2\left(231\,a^3g\cos\left(fx+e\right)^4-1600\,a^3g\cos\left(fx+e\right)^2+1544\,a^3g+4\left(123\,a^3g\cos\left(fx+e\right)^2-230\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(5i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^4-20i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2+16i\,\sqrt{2}\,a^3g+\left(-i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^4+12i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2-16i\,\sqrt{2}\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+231\left(-5i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^4+20i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2-16i\,\sqrt{2}\,a^3g+\left(i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^4-12i\,\sqrt{2}\,a^3g\cos\left(fx+e\right)^2+16i\,\sqrt{2}\,a^3g\right)\sin\left(fx+e\right)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{663\left(5\,c^6f\cos\left(fx+e\right)^4-20\,c^6f\cos\left(fx+e\right)^2+16\,c^6f-\left(c^6f\cos\left(fx+e\right)^4-12\,c^6f\cos\left(fx+e\right)^2+16\,c^6f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="fricas")

[Out] 1/663*(2*(231*a^3*g*cos(f*x + e)^4 - 1600*a^3*g*cos(f*x + e)^2 + 1544*a^3*g
 + 4*(123*a^3*g*cos(f*x + e)^2 - 230*a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x +
e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(5*I*sqrt(2)*a
^3*g*cos(f*x + e)^4 - 20*I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 16*I*sqrt(2)*a^3*
g + (-I*sqrt(2)*a^3*g*cos(f*x + e)^4 + 12*I*sqrt(2)*a^3*g*cos(f*x + e)^2 -
16*I*sqrt(2)*a^3*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(-5*I*sqrt(2)*a^
3*g*cos(f*x + e)^4 + 20*I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 16*I*sqrt(2)*a^3*g
 + (I*sqrt(2)*a^3*g*cos(f*x + e)^4 - 12*I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 16
*I*sqrt(2)*a^3*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(5*c^6*f*cos(f*x + e)^4
 - 20*c^6*f*cos(f*x + e)^2 + 16*c^6*f - (c^6*f*cos(f*x + e)^4 - 12*c^6*f*co
s(f*x + e)^2 + 16*c^6*f)*sin(f*x + e))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
11/2),x)

[Out] Timed out

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(11/
2),x, algorithm="giac")

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(11/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(11/2), x)
```

3.125 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=414

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}}-\frac{20a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{119cfg(c-c\sin(e+fx))^{11/2}}+\frac{220a^3(g\cos(e+}{1547c^2fg}$$

```
[Out] 4/21*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(13
/2)-20/119*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f
*x+e))^(11/2)-220/1989*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(7
/2)/(a+a*sin(f*x+e))^(1/2)+22/663*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin
(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+22/663*a^4*(g*cos(f*x+e))^(5/2)/c^5/f
/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+220/1547*a^3*(g*cos(f*x+e)
)^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(9/2)-22/663*a^4*g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^6/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.33, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{663c^6f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{663c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{220a^4(g\cos(e+fx))^{5/2}}{1989c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}+\frac{220a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{1547c^2fg(c-c\sin(e+fx))^{9/2}}-\frac{20a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{119cfg(c-c\sin(e+fx))^{11/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(13/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(21*f*g*(c - c*Sin[
e + f*x])^(13/2)) - (20*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/
2))/(119*c*f*g*(c - c*Sin[e + f*x])^(11/2)) + (220*a^3*(g*Cos[e + f*x])^(5/
2)*Sqrt[a + a*Sin[e + f*x]])/(1547*c^2*f*g*(c - c*Sin[e + f*x])^(9/2)) - (2
20*a^4*(g*Cos[e + f*x])^(5/2))/(1989*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(663*c^4*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (22*a^4*(g*Cos[e + f*x])^
(5/2))/(663*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) -
(22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
)/(663*c^6*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{13/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{(5a)\int \frac{(g\cos(e}{}}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{21fg(c-c\sin(e+fx))^{13/2}} - \frac{20a^2(g\cos(e}{119c}$$

Mathematica [A]
time = 6.55, size = 600, normalized size = 1.45

$$-\frac{22(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}(a(1+\sin(e+fx)))^{7/2}}{663f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{13/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{13}\left(\frac{22}{663}+\frac{32}{21\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{10}}-\frac{352}{119\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{8}}+\frac{464}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{6}}-\frac{1216}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{4}}+\frac{22}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2}}+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{21\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}-\frac{704\sin\left(\frac{1}{2}(e+fx)\right)}{119\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{9}}+\frac{928\sin\left(\frac{1}{2}(e+fx)\right)}{221\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}}-\frac{2432\sin\left(\frac{1}{2}(e+fx)\right)}{1989\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{5}}+\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{3}}+\frac{44\sin\left(\frac{1}{2}(e+fx)\right)}{663\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)}\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(13/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^13*(a*(1 + Sin[e + f*x]))^(7/2))/(663*f*Cos[e + f*x]^(3/2)
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2)) + ((g
*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13*
(22/663 + 32/(21*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) - 352/(119*(Cos[
```

```
(e + f*x)/2] - Sin[(e + f*x)/2])^8) + 464/(221*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^6) - 1216/(1989*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(66
3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(21*(Cos
[(e + f*x)/2] - Sin[(e + f*x)/2])^11) - (704*Sin[(e + f*x)/2])/(119*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^9) + (928*Sin[(e + f*x)/2])/(221*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^7) - (2432*Sin[(e + f*x)/2])/(1989*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3) + (44*Sin[(e + f*x)/2])/(663*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^7*(c - c*Sin[e + f*x])^(13/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 1482, normalized size = 3.58

method	result	size
default	Expression too large to display	1482

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/2),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] 2/13923/f*(a*(1+sin(f*x+e)))^(7/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x
+e)-cos(f*x+e)-sin(f*x+e)+1)*(10608-4256*sin(f*x+e)*cos(f*x+e)^5+3696*I*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^8*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*
x+e)^8*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3234*I*(1/(1+cos(f*x+e)))^
(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)+3234*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9471*I*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-9471*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-10164*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+10164*I*(1/(1+cos(f*
x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)+10608*sin(f*x+e)-14304*cos(f*x+e)-3696*I*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+3696*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+12640*cos(f*x+e)^3*sin(
f*x+e)+1155*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*si
n(f*x+e)*cos(f*x+e)^6*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*cos(f*x
+e)^6*sin(f*x+e)+7259*cos(f*x+e)^4*sin(f*x+e)-19108*cos(f*x+e)^2*sin(f*x+e)
+1155*cos(f*x+e)^6-12092*cos(f*x+e)^2+20408*cos(f*x+e)^3-6912*cos(f*x+e)*si
```

```
n(f*x+e)-6566*cos(f*x+e)^5-1155*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^6*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)-5775*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)*cos(f*x+e)^4*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5775*I*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x
+e)^4*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+8316*I*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)-8316*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)^2*EllipticE(I*(-1+cos(f*x+e))/si
n(f*x+e),I)+791*cos(f*x+e)^4-3696*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I))*(
cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(
f*x+e)^2+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(13/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(13/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 525, normalized size = 1.27

$$-\frac{2\left(1386a^3g\cos(fx+e)^4-8316a^3g\cos(fx+e)^2+7768a^3g-\left(231a^3g\cos(fx+e)^4+560a^3g\cos(fx+e)^2-2840a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}+231\left(-i\sqrt{2}a^3g\cos(fx+e)^6+18i\sqrt{2}a^3g\cos(fx+e)^4-48i\sqrt{2}a^3g\cos(fx+e)^2+32i\sqrt{2}a^3g+2\left(-3i\sqrt{2}a^3g\cos(fx+e)^4+16i\sqrt{2}a^3g\cos(fx+e)^2-16i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231\left(i\sqrt{2}a^3g\cos(fx+e)^6-18i\sqrt{2}a^3g\cos(fx+e)^4+48i\sqrt{2}a^3g\cos(fx+e)^2-32i\sqrt{2}a^3g+2\left(3i\sqrt{2}a^3g\cos(fx+e)^4-16i\sqrt{2}a^3g\cos(fx+e)^2+16i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{13923\left(c^7f\cos(fx+e)^6-18c^7f\cos(fx+e)^4+48c^7f\cos(fx+e)^2-32c^7f+2\left(3c^7f\cos(fx+e)^4-16c^7f\cos(fx+e)^2+16c^7f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="fricas")
```

```
[Out] -1/13923*(2*(1386*a^3*g*cos(f*x + e)^4 - 8316*a^3*g*cos(f*x + e)^2 + 7768*a
^3*g - (231*a^3*g*cos(f*x + e)^4 + 560*a^3*g*cos(f*x + e)^2 - 2840*a^3*g)*s
in(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c) + 231*(-I*sqrt(2)*a^3*g*cos(f*x + e)^6 + 18*I*sqrt(2)*a^3*g*cos(f
*x + e)^4 - 48*I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 32*I*sqrt(2)*a^3*g + 2*(-3*
I*sqrt(2)*a^3*g*cos(f*x + e)^4 + 16*I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 16*I*s
qrt(2)*a^3*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(I*sqrt(2)*a^3*g*cos(f
*x + e)^6 - 18*I*sqrt(2)*a^3*g*cos(f*x + e)^4 + 48*I*sqrt(2)*a^3*g*cos(f*x
+ e)^2 - 32*I*sqrt(2)*a^3*g + 2*(3*I*sqrt(2)*a^3*g*cos(f*x + e)^4 - 16*I*sq
rt(2)*a^3*g*cos(f*x + e)^2 + 16*I*sqrt(2)*a^3*g)*sin(f*x + e))*sqrt(a*c*g)*
```

```
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x
+ e))))/(c^7*f*cos(f*x + e)^6 - 18*c^7*f*cos(f*x + e)^4 + 48*c^7*f*cos(f*x
+ e)^2 - 32*c^7*f + 2*(3*c^7*f*cos(f*x + e)^4 - 16*c^7*f*cos(f*x + e)^2 + 1
6*c^7*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
13/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(13/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^{7/2}}{(c-c\,\sin(e+f\,x))^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(13/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(13/2), x)
```

3.126 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx$

Optimal. Leaf size=471

$$\frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}}-\frac{4a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{3/2}}{35cfg(c-c\sin(e+fx))^{13/2}}+\frac{44a^3(g\cos(e+fx}{595c^2fg(c}$$

```
[Out] 4/25*a*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(15
/2)-4/35*a^2*(g*cos(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(3/2)/c/f/g/(c-c*sin(f*x
+e))^(13/2)-44/1105*a^4*(g*cos(f*x+e))^(5/2)/c^3/f/g/(c-c*sin(f*x+e))^(9/2)
/(a+a*sin(f*x+e))^(1/2)+22/3315*a^4*(g*cos(f*x+e))^(5/2)/c^4/f/g/(c-c*sin(f
*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+22/5525*a^4*(g*cos(f*x+e))^(5/2)/c^5/f/
g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+22/5525*a^4*(g*cos(f*x+e))^
(5/2)/c^6/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+44/595*a^3*(g*c
os(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2)/c^2/f/g/(c-c*sin(f*x+e))^(11/2)-22/
5525*a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^7/f/(a+a*sin(
f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.54, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$-\frac{22a^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5525c^7f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^6fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{5525c^5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{22a^4(g\cos(e+fx))^{5/2}}{3315c^4fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{44a^4(g\cos(e+fx))^{5/2}}{1105c^3fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{9/2}}+\frac{44a^3\sqrt{a\sin(e+fx)+a}\,(g\cos(e+fx))^{5/2}}{595c^2fg(c-c\sin(e+fx))^{11/2}}-\frac{4a^2(a\sin(e+fx)+a)^{3/2}(g\cos(e+fx))^{5/2}}{35cfg(c-c\sin(e+fx))^{13/2}}+\frac{4a(a\sin(e+fx)+a)^{5/2}(g\cos(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e + f*x]
)^(15/2),x]
```

```
[Out] (4*a*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(5/2))/(25*f*g*(c - c*Sin[
e + f*x])^(15/2)) - (4*a^2*(g*Cos[e + f*x])^(5/2)*(a + a*Sin[e + f*x])^(3/2
))/(35*c*f*g*(c - c*Sin[e + f*x])^(13/2)) + (44*a^3*(g*Cos[e + f*x])^(5/2)*
Sqrt[a + a*Sin[e + f*x]])/(595*c^2*f*g*(c - c*Sin[e + f*x])^(11/2)) - (44*a
^4*(g*Cos[e + f*x])^(5/2))/(1105*c^3*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Si
n[e + f*x])^(9/2)) + (22*a^4*(g*Cos[e + f*x])^(5/2))/(3315*c^4*f*g*Sqrt[a +
 a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (22*a^4*(g*Cos[e + f*x])^(5/
2))/(5525*c^5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2
2*a^4*(g*Cos[e + f*x])^(5/2))/(5525*c^6*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c
*Sin[e + f*x])^(3/2)) - (22*a^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5525*c^7*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{7/2}}{(c-c\sin(e+fx))^{15/2}}\,dx = \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{(3a)\int \frac{(g\cos(e}{}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

$$= \frac{4a(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{5/2}}{25fg(c-c\sin(e+fx))^{15/2}} - \frac{4a^2(g\cos(e+}{35cf g}$$

Mathematica [A]

time = 6.62, size = 668, normalized size = 1.42

$$-\frac{22(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{15}(a(1+\sin(e+fx)))^{7/2}}{5525f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{15/2}} + \frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{15}\left(\frac{22}{5525}+\frac{32}{25(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{12}}-\frac{416}{175(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{10}}+\frac{4656}{2975(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{8}}-\frac{2144}{5525(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{6}}+\frac{22}{3315(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{4}}+\frac{22}{5525(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{2}}+\frac{64\sin(\frac{1}{2}(e+fx))}{25(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{13}}-\frac{832\sin(\frac{1}{2}(e+fx))}{175(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{11}}+\frac{9312\sin(\frac{1}{2}(e+fx))}{2975(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{9}}-\frac{4288\sin(\frac{1}{2}(e+fx))}{5525(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{7}}+\frac{44\sin(\frac{1}{2}(e+fx))}{3315(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{5}}+\frac{44\sin(\frac{1}{2}(e+fx))}{5525(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))^{3}}+\frac{44\sin(\frac{1}{2}(e+fx))}{5525(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))}\right)(a(1+\sin(e+fx)))^{7/2}}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{7}(c-c\sin(e+fx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/2))/(c - c*Sin[e
+ f*x])^(15/2),x]
```

```
[Out] (-22*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^15*(a*(1 + Sin[e + f*x]))^(7/2))/(5525*f*Cos[e + f*x]^(3/2
```

)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^15*(22/5525 + 32/(25*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^12) - 416/(175*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10) + 4656/(2975*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8) - 2144/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6) + 22/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4) + 22/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2) + (64*Sin[(e + f*x)/2])/(25*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^13) - (832*Sin[(e + f*x)/2])/(175*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11) + (9312*Sin[(e + f*x)/2])/(2975*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9) - (4288*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7) + (44*Sin[(e + f*x)/2])/(3315*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3) + (44*Sin[(e + f*x)/2])/(5525*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))

Maple [C] Result contains complex when optimal does not.
time = 2.18, size = 1642, normalized size = 3.49

method	result	size
default	Expression too large to display	1642

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/2),x,method=_RETURNVERBOSE)

[Out] -2/116025/f*(a*(1+sin(f*x+e)))^(7/2)*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)-sin(f*x+e)+1)*(-74256+20895*sin(f*x+e)*cos(f*x+e)^5-7392*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+7392*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-74256*sin(f*x+e)+81648*cos(f*x+e)-85052*cos(f*x+e)^3*sin(f*x+e)+231*I*cos(f*x+e)^8*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1386*cos(f*x+e)^6*sin(f*x+e)-29673*cos(f*x+e)^4*sin(f*x+e)+99836*cos(f*x+e)^2*sin(f*x+e)+231*cos(f*x+e)^8-4004*cos(f*x+e)^6+22176*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-22176*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+7392*I*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-7392*I*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+112324*cos(f*x+e)^2-130804*cos(f*x+e)^3+66864*cos(f*x+e)*sin(f*x+e)+50003*cos(f*x+e)^5-385*cos(f*x+e)^7-1386*I*cos(f*x+e)^8*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+1386*I*cos(f*x+e)^8*(cos(f*x+e)/(1+cos(

```
f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)+10164*I*cos(f*x+e)^6*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x
+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-10164*I*cos(f*x+e)^6*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)-23562*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+23
562*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-34757*cos(f*x+e)^4-231*I*cos(f
*x+e)^8*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/
2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-4389*I*cos(f*x+e)^6*sin(f*x+e)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)+4389*I*cos(f*x+e)^6*sin(f*x+e)*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)+15246*I*cos(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-15246*I*
cos(f*x+e)^4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-18480*I*cos(f*x+e)^2*sin(
f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+18480*I*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I))*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(cos(f*x+e)^4-4*cos(f*x+e)^2
*sin(f*x+e)-8*cos(f*x+e)^2+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(15/2)/cos(f
*x+e)/sin(f*x+e)^5
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e
) + c)^(15/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 589, normalized size = 1.25

$$\frac{2\left(231a^3g\cos(fx+e)^6-5698a^3g\cos(fx+e)^4+42044a^3g\cos(fx+e)^2-42056a^3g+7\left(231a^3g\cos(fx+e)^4+1544a^3g\cos(fx+e)^2-4600a^3g\right)\sin(fx+e)\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}+231\left(7i\sqrt{2}a^3g\cos(fx+e)^6-56i\sqrt{2}a^3g\cos(fx+e)^4+112i\sqrt{2}a^3g\cos(fx+e)^2-64i\sqrt{2}a^3g+\left(-i\sqrt{2}a^3g\cos(fx+e)^6+24i\sqrt{2}a^3g\cos(fx+e)^4-80i\sqrt{2}a^3g\cos(fx+e)^2+64i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231\left(-7i\sqrt{2}a^3g\cos(fx+e)^6+56i\sqrt{2}a^3g\cos(fx+e)^4-112i\sqrt{2}a^3g\cos(fx+e)^2+64i\sqrt{2}a^3g+\left(i\sqrt{2}a^3g\cos(fx+e)^6-24i\sqrt{2}a^3g\cos(fx+e)^4+80i\sqrt{2}a^3g\cos(fx+e)^2-64i\sqrt{2}a^3g\right)\sin(fx+e)\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{116025\left(7c^8f\cos(fx+e)^6-56c^8f\cos(fx+e)^4+112c^8f\cos(fx+e)^2-64c^8f-\left(c^8f\cos(fx+e)^6-24c^8f\cos(fx+e)^4+80c^8f\cos(fx+e)^2-64c^8f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="fricas")
```

```
[Out] 1/116025*(2*(231*a^3*g*cos(f*x + e)^6 - 5698*a^3*g*cos(f*x + e)^4 + 42044*a
^3*g*cos(f*x + e)^2 - 42056*a^3*g + 7*(231*a^3*g*cos(f*x + e)^4 + 1544*a^3*
g*cos(f*x + e)^2 - 4600*a^3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*si
n(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(7*I*sqrt(2)*a^3*g*cos(f*x
+ e)^6 - 56*I*sqrt(2)*a^3*g*cos(f*x + e)^4 + 112*I*sqrt(2)*a^3*g*cos(f*x +
e)^2 - 64*I*sqrt(2)*a^3*g + (-I*sqrt(2)*a^3*g*cos(f*x + e)^6 + 24*I*sqrt(2)
*a^3*g*cos(f*x + e)^4 - 80*I*sqrt(2)*a^3*g*cos(f*x + e)^2 + 64*I*sqrt(2)*a^
3*g)*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(-7*I*sqrt(2)*a^3*g*cos(f*x + e
)^6 + 56*I*sqrt(2)*a^3*g*cos(f*x + e)^4 - 112*I*sqrt(2)*a^3*g*cos(f*x + e)^
2 + 64*I*sqrt(2)*a^3*g + (I*sqrt(2)*a^3*g*cos(f*x + e)^6 - 24*I*sqrt(2)*a^3
*g*cos(f*x + e)^4 + 80*I*sqrt(2)*a^3*g*cos(f*x + e)^2 - 64*I*sqrt(2)*a^3*g)
*sin(f*x + e))*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(f*x + e) - I*sin(f*x + e))))/(7*c^8*f*cos(f*x + e)^6 - 56*c^8*f*cos(f
*x + e)^4 + 112*c^8*f*cos(f*x + e)^2 - 64*c^8*f - (c^8*f*cos(f*x + e)^6 - 2
4*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^2 - 64*c^8*f)*sin(f*x + e))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(7/2)/(c-c*sin(f*x+e))**(
15/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(7/2)/(c-c*sin(f*x+e))^(15/
2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{15/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(15/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x)
)^(15/2), x)
```

3.127 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=234

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{22c^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+22c^2$$

```
[Out] 2/7*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(1/2
)+22/15*c^3*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)+22/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elliptic
E(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+22/35*c^2*(g*cos(f*x+e))^(5/2)*(c-
c*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35fg\sqrt{a\sin(e+fx)+a}}+\frac{2c(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{7fg\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (22*c^3*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (22*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35*f*g*Sqrt[a
+ a*Sin[e + f*x]]) + (2*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)
)/(7*f*g*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{7}(11c)\int \frac{(g\,c \cdots}{\cdots}$$

$$= \frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35fg\sqrt{a+a\sin(e+fx)}} + \frac{2c(g\cos(e+\cdots}{7fg \cdots}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\,co\cdots}{3\cdots}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\,co\cdots}{3\cdots}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^2(g\,co\cdots}{3\cdots}$$

$$= \frac{22c^3(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{22c^3g\sqrt{c\cdots}}{5f\sqrt{\cdots}}$$

Mathematica [A]
time = 1.04, size = 174, normalized size = 0.74

$$\frac{c^2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\left(924E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}(515\cos(e+fx)-3(5\cos(3(e+fx))+42\sin(2(e+fx))))\right)}{210f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin
[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2] + Sqrt
[Cos[e + f*x]]*(515*Cos[e + f*x] - 3*(5*Cos[3*(e + f*x)] + 42*Sin[2*(e + f*
x)]))))/(210*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*S
qrt[a*(1 + Sin[e + f*x])])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 415, normalized size = 1.77

method	result
default	$-\frac{2(g\cos(fx+e))^{\frac{3}{2}}\left(231i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}-231i\sin(fx+e)\,c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/105/f*(g*cos(f*x+e))^(3/2)*(231*I*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)-231*I*sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+231*I*sin(f
*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)-231*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+
15*cos(f*x+e)^4*sin(f*x+e)-63*cos(f*x+e)^4-140*cos(f*x+e)^2*sin(f*x+e)+294*
cos(f*x+e)^2-231*cos(f*x+e))*(-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^2*sin(f*
x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e))
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 160, normalized size = 0.68

$$\frac{-231i\sqrt{2}\sqrt{acg}\,c^2g\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231i\sqrt{2}\sqrt{acg}\,c^2g\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\left(15c^2g\cos(fx+e)^2+63c^2g\sin(fx+e)-140c^2g\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{105af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/105*(-231*I*sqrt(2)*sqrt(a*c*g)*c^2*g*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*I*sqrt(2)*sqrt(a*c*g)*
c^2*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*si
n(f*x + e))) - 2*(15*c^2*g*cos(f*x + e)^2 + 63*c^2*g*sin(f*x + e) - 140*c^2
*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
)/(a*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

3.128 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=180

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2c(g$$

```
[Out] 14/15*c^2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^
(1/2)+14/5*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*si
n(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(
f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2c\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e +
f*x]],x]
```

```
[Out] (14*c^2*(g*Cos[e + f*x])^(5/2))/(15*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) + (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(5*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 + (2*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*Sqrt[a + a*
Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg\sqrt{a+a\sin(e+fx)}} + \frac{1}{5}(7c)\int \frac{(g\cos}{}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(}{5}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(}{5}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2c(g\cos(}{5}$$

$$= \frac{14c^2(g\cos(e+fx))^{5/2}}{15fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{14c^2g\sqrt{c}}{5f\sqrt{}}$$

Mathematica [A]
time = 0.48, size = 157, normalized size = 0.87

$$-\frac{c(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}\left(42E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sqrt{\cos(e+fx)}\,(20\cos(e+fx)-3\sin(2(e+fx)))\right)}{15f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\sqrt{a(1+\sin(e+fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Si
n[e + f*x]],x]
```

```
[Out] -1/15*(c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 +
 Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]*(42*EllipticE[(e + f*x)/2, 2] + Sqr
```

```
t[Cos[e + f*x]]*(20*Cos[e + f*x] - 3*Sin[2*(e + f*x)])))/(f*Cos[e + f*x]^(3
/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 382, normalized size = 2.12

method	result
default	$-\frac{2(g\cos(fx+e))^{\frac{3}{2}}\left(21i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\,-21i\sin(fx+e)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(g*cos(f*x+e))^(3/2)*(21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))
^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)-21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*sin(f*x+e
)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-
1+cos(f*x+e))/sin(f*x+e),I)-21*I*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f
*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*cos
(f*x+e)^4+10*cos(f*x+e)^2*sin(f*x+e)-24*cos(f*x+e)^2+21*cos(f*x+e))*(-c*(si
n(f*x+e)-1))^(3/2)/(cos(f*x+e)^2+2*sin(f*x+e)-2)/cos(f*x+e)/sin(f*x+e)/(a*(
1+sin(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x
 + e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 137, normalized size = 0.76

$$\frac{-21i\sqrt{2}\sqrt{acg}\,cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+21i\sqrt{2}\sqrt{acg}\,cg\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))-2\,(3\,cg\sin(fx+e)-10\,cg)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}}{15\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/15*(-21*I*sqrt(2)*sqrt(a*c*g)*c*g*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*I*sqrt(2)*sqrt(a*c*g)*c*g*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x +
 e))) - 2*(3*c*g*sin(f*x + e) - 10*c*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x
 + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,(c-c\,\sin\left(e+f\,x\right))^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

3.129 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=122

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2/3*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
)+2*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))
^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2930, 2921, 2721, 2719}

$$\frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{2cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*
x]],x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(5/2))/(3*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]]) + (2*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e +
 f*x)/2, 2])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
```

```
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + c\int \frac{}{\sqrt{a+a}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{(cg\cos(e}{\sqrt{a+a\sin}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{\left(cg\sqrt{\cos(e}\right.}{\sqrt{a}}$$

$$= \frac{2c(g\cos(e+fx))^{5/2}}{3fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{2cg\sqrt{\cos(e}}{f\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.21, size = 215, normalized size = 1.76

$$\frac{\sqrt{-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)}(g\cos(e+fx))^{3/2}\left(\sqrt{1+e^{2i(e+fx)}}\left(1-6ie^{i(e+fx)}+e^{2i(e+fx)}\right)+12ie^{i(e+fx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{3a\left(1+e^{2i(e+fx)}\right)^{3/2}f\cos^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[
e + f*x]],x]
```

```
[Out] (Sqrt[((-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*Sqrt[(1 + E^((2*I)*
(e + f*x)))/E^(I*(e + f*x))]*(g*Cos[e + f*x])^(3/2)*(Sqrt[1 + E^((2*I)*(e +
 f*x))]*(1 - (6*I)*E^(I*(e + f*x)) + E^((2*I)*(e + f*x))) + (12*I)*E^(I*(e
+ f*x))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(e + f*x))])*Sqrt[c - c
*Sin[e + f*x]])/(3*a*(1 + E^((2*I)*(e + f*x)))^(3/2)*f*Cos[e + f*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 361, normalized size = 2.96

method	result
default	$-\frac{2\sqrt{-c\left(\sin\left(fx+e\right)-1\right)}\,\left(g\cos(fx+e)\right)^{\frac{3}{2}}\left(3i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\,\sqrt{1\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(-c*(sin(f*x+e)-1))^(1/2)*(g*cos(f*x+e))^(3/2)*(3*I*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)+3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos
(f*x+e)))^(1/2)+cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2+3*cos(f*x+e))/(sin(f
*x+e)-1)/cos(f*x+e)/sin(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x +
 e) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 120, normalized size = 0.98

$$\frac{-3i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))+3i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))+2\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,g}{3af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/3*(-3*I*sqrt(2)*sqrt(a*c*g)*g*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*I*sqrt(2)*sqrt(a*c*g)*g*weierstr
```

```
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))) +
 2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*
g)/(a*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(1/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(1/2), x)
```

3.130 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=68

$$\frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1
/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2921, 2721, 2719}

$$\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x
]]),x]
```

```
[Out] (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(f*
Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{\left(g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\right)\int\sqrt{\cos(e+fx)}\,dx}{\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$
$$= \frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

Mathematica [A]
time = 0.37, size = 111, normalized size = 1.63

$$\frac{2(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{f\cos^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(f*Cos[e + f*x]^(3/2)
*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.33, size = 334, normalized size = 4.91

method	result
default	$\frac{2(g\cos(fx+e))^{\frac{3}{2}}\left(i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)},i\right)\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}-i\sin(fx+e)\cos(fx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/f*(g*cos(f*x+e))^(3/2)*(I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f
*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+co
s(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(
```

```
f*x+e)))^(1/2)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1
+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-cos(f*x+e)^2+cos(f*x+
e))/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(f*x+e
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 81, normalized size = 1.19

$$\frac{-i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+i\sqrt{2}\sqrt{acg}\,g\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(a*c*g)*g*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) + I*sqrt(2)*sqrt(a*c*g)*g*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a*c*f)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(g\cos(e+fx))^{\frac{3}{2}}}{\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e
 + f*x) - 1))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

3.131 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=121

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{cf\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-2*
g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c/f/(a+a*sin(f*x+e))^(1/
2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$\frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x
])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x
)/2, 2])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\int \frac{\cdots}{\sqrt{a+\cdots}}}{\cdots}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{(g\cos\cdots}{c\sqrt{a+\cdots}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{\left(g\sqrt{\cos\cdots}\right.}{\cdots}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}} - \frac{2g\sqrt{\cos\cdots}}{cf\sqrt{\cdots}}$$

Mathematica [A]
time = 0.56, size = 148, normalized size = 1.22

$$\frac{2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(-\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{cf\cos^{\frac{3}{2}}(e+fx)\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(EllipticE[
(e + f*x)/2, 2]*(-Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]
*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))/(c*f*Cos[e + f*x]^(3/2)*Sqrt[a*(1
+ Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 925, normalized size = 7.64

method	result	size
default	Expression too large to display	925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(sin(f*x+e)-1)*(4*I*(1/(1+cos(
f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^
2-4*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)*cos(f*x+e)^2+8*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*sin(f*x+e)*cos(f*x+e)-8*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+4*I*(1/(1+cos(f*x+e)))^(1/2)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(1+cos(f*x+e)))^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)*sin(f*x+e)*cos(f*x+e)+ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)-cos(f*x+e)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)-4*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)*sin(f*x+e)+4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))/(1+cos(f*x+e))/sin(f*
x+e)^5/(a*(1+sin(f*x+e)))^(1/2)/(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)/(-c*(s
in(f*x+e)-1))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 169, normalized size = 1.40

$$-\frac{2\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,g-\sqrt{acg}\left(i\sqrt{2}\,g\sin(fx+e)-i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-\sqrt{acg}\left(-i\sqrt{2}\,g\sin(fx+e)+i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{ac^2f\sin(fx+e)-ac^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*g - sqrt(a*c*g)*(I*sqrt(2)*g*sin(f*x + e) - I*sqrt(2)*g)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - sqrt(a*c
*g)*(-I*sqrt(2)*g*sin(f*x + e) + I*sqrt(2)*g)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a*c^2*f*sin(f*x + e)
 - a*c^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}}{\sqrt{a+a\,\sin(e+f\,x)}\,(c-c\,\sin(e+f\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(3/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(3/2)), x)
```

3.132 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=179

$$\frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e}}{5c^2f\sqrt{a}}$$

```
[Out] 2/5*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+
2/5*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2
)-2/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f
*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^2/f/(a+a*sin(f*x
+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
5/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(5*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*Sqrt[a + a*Sin[e + f*x]]*
(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*
EllipticE[(e + f*x)/2, 2])/(5*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin
[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{\int \frac{}{\sqrt{a}}}{}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{5cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{5cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{5cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{5fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}} + \frac{}{5cfg\sqrt{}}$$

Mathematica [A]
time = 1.11, size = 204, normalized size = 1.14

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-2E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(3\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)+4\sin^3\left(\frac{1}{2}(e+fx)\right)\right)\right)}{5c^2f\cos^{\frac{3}{2}}(e+fx)(-1+\sin(e+fx))^2\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(5/2)),x]
```

[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(-2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3*(e
+ f*x))/2] + 4*Sin[(e + f*x)/2]^3)))/(5*c^2*f*Cos[e + f*x]^(3/2)*(-1 + Sin[
e + f*x])^2*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 781, normalized size = 4.36

method	result
default	$-\frac{2(g\cos(fx+e))^{\frac{3}{2}}(\cos(fx+e)\sin(fx+e)-\cos(fx+e)-\sin(fx+e)+1)\left(i(\cos^{4}(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \mathrm{Elliptic}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)

[Out] -2/5/f*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)-sin(f*x+e)+1)
*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e
)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-2*I*EllipticE(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^
(1/2)+2*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(
1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^2+2*cos
(f*x+e)-sin(f*x+e)-1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2
)/(-c*(sin(f*x+e)-1))^(5/2)/sin(f*x+e)^5/cos(f*x+e)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 225, normalized size = 1.26

$$\frac{2\,\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\left(g\sin\left(fx+e\right)-2\,g\right)+\left(i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}+2i\,\sqrt{2}\,g\sin\left(fx+e\right)-2i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+\left(-i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}-2i\,\sqrt{2}\,g\sin\left(fx+e\right)+2i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{5\,\left(ac^{3}f\cos\left(fx+e\right)^{2}+2\,ac^{3}f\sin\left(fx+e\right)-2\,ac^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)*(g*sin(f*x + e) - 2*g) + (I*sqrt(2)*g*cos(f*x + e)^2 + 2*I*sqrt(2)*g*si
n(f*x + e) - 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + (-I*sqrt(2)*g*cos(f*x + e)
^2 - 2*I*sqrt(2)*g*sin(f*x + e) + 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a*c^3
*f*cos(f*x + e)^2 + 2*a*c^3*f*sin(f*x + e) - 2*a*c^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{a+a\,\sin\left(e+f\,x\right)}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

3.133 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=233

$$\frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}+\frac{2(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{15c^2fg\sqrt{}}$$

```
[Out] 2/9*(g*cos(f*x+e))^(5/2)/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+
2/15*(g*cos(f*x+e))^(5/2)/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/
2)+2/15*(g*cos(f*x+e))^(5/2)/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)
)^(1/2)-2/15*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/c^3/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{15c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(e+fx))^{5/2}}{15cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(
7/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(9*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2))/(15*c^2*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*S
qrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(15*c^3*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{\int \frac{}{\sqrt{a+}}}{}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{}{15cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{}{15cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{}{15cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{}{15cfg\sqrt{}}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{9fg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}} + \frac{}{15cfg\sqrt{}}$$

Mathematica [A]
time = 1.76, size = 240, normalized size = 1.03

$$\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(12E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5+\sqrt{\cos(e+fx)}\left(-32\cos\left(\frac{1}{2}(e+fx)\right)-15\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{5}{2}(e+fx)\right)-32\sin\left(\frac{1}{2}(e+fx)\right)+15\sin\left(\frac{3}{2}(e+fx)\right)+3\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{90c^3f\cos^{\frac{3}{2}}(e+fx)(-1+\sin(e+fx))^3\sqrt{a(1+\sin(e+fx))}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f
*x])^(7/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])*(12*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^5 + Sqrt[Cos[e + f*x]]*(-32*Cos[(e + f*x)/2] - 15*Cos[(
3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 32*Sin[(e + f*x)/2] + 15*Sin[(3*
(e + f*x))/2] + 3*Sin[(5*(e + f*x))/2])))/(90*c^3*f*Cos[e + f*x]^(3/2)*(-1
+ Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 955, normalized size = 4.10

method	result	size
default	Expression too large to display	955

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+e)-cos(f*x+e)-sin(f*x+e)+1)
*(6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)
))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*cos(f*x+e)^4*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)-6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x
+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-12*I*(1/(1+c
os(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^2-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+12*I*(1/(1+co
s(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)^2+6*I*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-6
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-9*I*EllipticE(I*(-1+cos(f*x+e))/s
in(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
1+cos(f*x+e)))^(1/2)+6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+co
s(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+9*I*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)^4*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)-3*cos(f*x+e)^4+5*cos(f*x+e)^3-6*cos(f*x+e)^2*sin(f*x+e)+4*cos(f*x+
e)^2+cos(f*x+e)*sin(f*x+e)-11*cos(f*x+e)+5*sin(f*x+e)+5)*(cos(f*x+e)^2+2*co
s(f*x+e)+1)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(7/2)/cos(f*x+e)/s
in(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e)
 + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 293, normalized size = 1.26

$$\frac{2\left(3\,g\cos\left(fx+e\right)^{2}+9\,g\sin\left(fx+e\right)-14\,g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+3\left(3i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}+\left(-i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}+4i\,\sqrt{2}\,g\right)\sin\left(fx+e\right)-4i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)\right)+3\left(-3i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}+\left(i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}-4i\,\sqrt{2}\,g\right)\sin\left(fx+e\right)+4i\,\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)\right)\right)}{45\left(3\,ac^{4}f\cos\left(fx+e\right)^{2}-4\,ac^{4}f-\left(ac^{4}f\cos\left(fx+e\right)^{2}-4\,ac^{4}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(3*g*cos(f*x + e)^2 + 9*g*sin(f*x + e) - 14*g)*sqrt(g*cos(f*x + e))
*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 3*(3*I*sqrt(2)*g*cos(
f*x + e)^2 + (-I*sqrt(2)*g*cos(f*x + e)^2 + 4*I*sqrt(2)*g)*sin(f*x + e) - 4
*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(f*x + e) + I*sin(f*x + e))) + 3*(-3*I*sqrt(2)*g*cos(f*x + e)^2 + (I*sq
rt(2)*g*cos(f*x + e)^2 - 4*I*sqrt(2)*g)*sin(f*x + e) + 4*I*sqrt(2)*g)*sqrt(
a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))))/(3*a*c^4*f*cos(f*x + e)^2 - 4*a*c^4*f - (a*c^4*f*cos(f*x + e
)^2 - 4*a*c^4*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

3.134 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^4g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-66c$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)
-30/7*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a/f/g/(a+a*sin(f*x+e)
)^(1/2)-22*c^4*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f
*x+e))^(1/2)-66*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-66/7*c^3*(g*cos(f*x+e))^(5/2)
*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.91, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$-\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{66c^3\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{7afg\sqrt{a\sin(e+fx)+a}}-\frac{30c^2(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{7afg\sqrt{a\sin(e+fx)+a}}-\frac{4c(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-22*c^4*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c
*Sin[e + f*x]]) - (66*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellipti
cE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
 - (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(7*a*f*g*Sqrt[a
 + a*Sin[e + f*x]]) - (30*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(
3/2))/(7*a*f*g*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c -
 c*Sin[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(15c)\int \frac{(g\cos(e}{\sqrt{}}}{}$$

$$= -\frac{30c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e-}{fg}$$

$$= -\frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{7afg\sqrt{a+a\sin(e+fx)}} - \frac{30c^2(g\cos(e}{7af}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos}{7}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos}{7}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^3(g\cos}{7}$$

$$= -\frac{22c^4(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{66c^4g\sqrt{\mathrm{co}}}{af\sqrt{}}$$

Mathematica [A]
time = 6.41, size = 282, normalized size = 0.96

$$-\frac{66(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{7/2}}{f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{3/2}}+\frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(c-c\sin(e+fx))^{7/2}\left(-32-\frac{109}{14}\cos(e+fx)+\frac{1}{14}\cos(3(e+fx))+\frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)}+\sin(2(e+fx))\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^7(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (-66*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + S
in[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(7/2))/(f*Cos[e + f*x]^(3/2)*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)) + ((g*Cos[e
 + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*
Sin[e + f*x])^(7/2)*(-32 - (109*Cos[e + f*x])/14 + Cos[3*(e + f*x)]/14 + (6
4*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sin[2*(e + f*x)
]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

Maple [C] Result contains complex when optimal does not.
time = 11.57, size = 2994, normalized size = 10.18

method	result	size
default	Expression too large to display	2994

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/7/f*(-1+cos(f*x+e))*(sin(f*x+e)*cos(f*x+e)^5-28*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(3/2)+28*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f
*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-21*cos(f*x+e)^3*
sin(f*x+e)-231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f
*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+7*cos(f*
x+e)^4*sin(f*x+e)-119*cos(f*x+e)^2*sin(f*x+e)+231*I*sin(f*x+e)*cos(f*x+e)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)+cos(f*x+e)^6-112*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(3/2)+112*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)
*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-168*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(3/2)+168*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-112*ln(-2*(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-28*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+1
12*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+
e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*
x+e)-343*cos(f*x+e)^2+98*cos(f*x+e)^3-6*cos(f*x+e)^5-28*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(3/2)+28*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)
```

```
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e))
)^(1/2)-231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+
e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+462*I*cos(f*x+e)^2*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1
/(1+cos(f*x+e)))^(1/2)-462*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*c
os(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)+231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-231*I*sin(f*x+e
)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(
1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-84*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(3/2)+84*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-28*ln(-
2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*
sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+28*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(3/2)-84*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)+84*cos(f*x+e)^2*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-28*cos(f*x+e)^4)
*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e)-1))^(7/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.13, size = 229, normalized size = 0.78

$$-\frac{2 \left(6 c^3 g \cos \left(f x+e\right)^2+133 c^3 g-\left(c^3 g \cos \left(f x+e\right)^2-21 c^3 g\right) \sin \left(f x+e\right)\right) \sqrt{g \cos \left(f x+e\right)}\, \sqrt{a \sin \left(f x+e\right)+a}\, \sqrt{-c \sin \left(f x+e\right)+c}+231 \left(-i \sqrt{2}\, c^3 g \sin \left(f x+e\right)-i \sqrt{2}\, c^3 g\right) \sqrt{a c g}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(f x+e\right)+i \sin \left(f x+e\right)\right)\right)+231 \left(i \sqrt{2}\, c^3 g \sin \left(f x+e\right)+i \sqrt{2}\, c^3 g\right) \sqrt{a c g}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(f x+e\right)-i \sin \left(f x+e\right)\right)\right)}{7 \left(a^2 f \sin \left(f x+e\right)+a^2 f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/7*(2*(6*c^3*g*cos(f*x + e)^2 + 133*c^3*g - (c^3*g*cos(f*x + e)^2 - 21*c^
3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c) + 231*(-I*sqrt(2)*c^3*g*sin(f*x + e) - I*sqrt(2)*c^3*g)*sqr
t(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I
*sin(f*x + e))) + 231*(I*sqrt(2)*c^3*g*sin(f*x + e) + I*sqrt(2)*c^3*g)*sqrt
(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(a^2*f*sin(f*x + e) + a^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos \left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin \left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin \left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

3.135 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{154c^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(3/2)
-154/15*c^3*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+
e))^(1/2)-154/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellip
ticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-22/5*c^2*(g*cos(f*x+e))^(5/2)
*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$-\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{154c^3g\sqrt{\cos(e+fx)}E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{22c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5afg\sqrt{a\sin(e+fx)+a}}-\frac{4c(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) - (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (22*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*f*g
*Sqrt[a + a*Sin[e + f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x
])^(3/2))/(f*g*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(11c)\int \frac{(g\cos(e}{\sqrt{}}}{}$$

$$= -\frac{22c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg\sqrt{a+a\sin(e+fx)}} - \frac{4c(g\cos(e+}{fg(}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\,c}{}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\,c}{}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{22c^2(g\,c}{}$$

$$= -\frac{154c^3(g\cos(e+fx))^{5/2}}{15afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{154c^3g}{5a}$$

Mathematica [A]
time = 3.33, size = 238, normalized size = 0.99

$$-\frac{c^2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2(-1+\sin(e+fx))^2\sqrt{c-c\sin(e+fx)}\left(924E\left(\frac{1}{2}(e+fx)|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{\cos(e+fx)}\left(520\cos\left(\frac{1}{2}(e+fx)\right)+37\cos\left(\frac{3}{2}(e+fx)\right)+3\cos\left(\frac{5}{2}(e+fx)\right)-520\sin\left(\frac{1}{2}(e+fx)\right)+37\sin\left(\frac{3}{2}(e+fx)\right)-3\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{30f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] -1/30*(c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(
-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e + f*x]]*(520*Cos[(e +
f*x)/2] + 37*Cos[(3*(e + f*x))/2] + 3*Cos[(5*(e + f*x))/2] - 520*Sin[(e + f
*x)/2] + 37*Sin[(3*(e + f*x))/2] - 3*Sin[(5*(e + f*x))/2])))/(f*Cos[e + f*x
]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(3/2
))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 2946, normalized size = 12.22

method	result	size
default	Expression too large to display	2946

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(462*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-462*I
*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e))
)^(1/2)-231*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos(f*x+e)^3*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(
1+cos(f*x+e)))^(1/2)-231*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(
1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+231*I*cos
(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+30*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+17*cos(f*x+e)^3*s
in(f*x+e)+231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
sin(f*x+e)*cos(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*cos(f*x+e
)^4*sin(f*x+e)+111*cos(f*x+e)^2*sin(f*x+e)+120*ln(-2*(2*cos(f*x+e)^2*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(3/2)-120*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+
e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+180*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(3/2)-180*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+120*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x
+e)-120*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos
(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+c
```

```
os(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*s
in(f*x+e)+351*cos(f*x+e)^2-94*cos(f*x+e)^3+3*cos(f*x+e)^5-231*I*cos(f*x+e)^
2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-30*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(3/2)-231*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)+90*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*cos(f*x+e)*sin(f
*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+30*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(3/2)-30*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+90*cos(f*x
+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*cos(f*x+e)^2*sin(f*x+e)*ln(
-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(3/2)+20*cos(f*x+e)^4)*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e
)-1))^(5/2)/(cos(f*x+e)^4-cos(f*x+e)^3*sin(f*x+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 212, normalized size = 0.88

$$-\frac{2 \left(3 c^{2} g \cos \left(f x+e\right)^{2}+17 c^{2} g \sin \left(f x+e\right)+137 c^{2} g\right) \sqrt{g \cos \left(f x+e\right)} \sqrt{a \sin \left(f x+e\right)+a} \sqrt{-c \sin \left(f x+e\right)+c}+231 \left(-i \sqrt{2} c^{2} g \sin \left(f x+e\right)-i \sqrt{2} c^{2} g\right) \sqrt{a c g}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(f x+e\right)+i \sin \left(f x+e\right)\right)\right)+231 \left(i \sqrt{2} c^{2} g \sin \left(f x+e\right)+i \sqrt{2} c^{2} g\right) \sqrt{a c g}\, \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos \left(f x+e\right)-i \sin \left(f x+e\right)\right)\right)}{15 \left(a^{2} f \sin \left(f x+e\right)+a^{2} f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(3*c^2*g*cos(f*x + e)^2 + 17*c^2*g*sin(f*x + e) + 137*c^2*g)*sqrt(
g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(-
I*sqrt(2)*c^2*g*sin(f*x + e) - I*sqrt(2)*c^2*g)*sqrt(a*c*g)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(I
*sqrt(2)*c^2*g*sin(f*x + e) + I*sqrt(2)*c^2*g)*sqrt(a*c*g)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*f*s
in(f*x + e) + a^2*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{5/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

3.136 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}\,\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}-4c$$

```
[Out] -14/3*c^2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e)
)^(1/2)-14*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(
f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.53, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$-\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{14c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(3/2),x]
```

```
[Out] (-14*c^2*(g*Cos[e + f*x])^(5/2))/(3*a*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c -
 c*Sin[e + f*x]]) - (14*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ellip
ticE[(e + f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]
]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(f*g*(a + a*Sin[
e + f*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{fg(a+a\sin(e+fx))^{3/2}} - \frac{(7c)\int \frac{(g\cos(e\cdots}{\cdots}}{\cdots}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos\cdots}{\cdots}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos\cdots}{\cdots}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos\cdots}{\cdots}$$

$$= -\frac{14c^2(g\cos(e+fx))^{5/2}}{3afg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} - \frac{14c^2g\sqrt{\cdots}}{af\cdots}$$

Mathematica [A]
time = 1.11, size = 200, normalized size = 1.10

$$\frac{2c(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(21E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)+\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)(12+\cos(e+fx))+(-12+\cos(e+fx))\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)(-1+\sin(e+fx))\sqrt{c-c\sin(e+fx)}}{3f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(3/2),x]
```

```
[Out] (2*c*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(21*Ell
ipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + Sqrt[Cos[e +
 f*x]]*(Cos[(e + f*x)/2]*(12 + Cos[e + f*x]) + (-12 + Cos[e + f*x])*Sin[(e
+ f*x)/2]))*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*f*Cos[e + f*x]
^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(3/2)
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 2890, normalized size = 15.88

method	result	size
default	Expression too large to display	2890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f
*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(3/2)+cos(f*x+e)^3*sin(f*x+e)-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(
f*x+e)))^(1/2)-21*I*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+9*cos(f
*x+e)^2*sin(f*x+e)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e
)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+1
2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x
+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-12*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(3/2)+18*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
```

```
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-18*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(3/2)+12*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)
/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*ln(-2*(2*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(3/2)*sin(f*x+e)-12*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+33*cos(f*x+e)^2-8*cos(f*x+e)^3+42*I*cos(f*
x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-42*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I
)+21*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)+21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)^3*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(3/2)+9*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-9*cos(f*x+e)*sin
(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos
(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)
^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+9*cos(f*x+
e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-9*cos(f*x+e)^2*sin(f*x+e)*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+
```

```
cos(f*x+e))^2)^(3/2)+cos(f*x+e)^4)*(g*cos(f*x+e))^(3/2)*(-c*(sin(f*x+e)-1))
^(3/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*cos(f*x+e)*sin(f*x+e)-3*cos(
f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/sin(f*x+e...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 184, normalized size = 1.01

$$-\frac{2\left(cg\sin\left(fx+e\right)+13\,cg\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+21\left(-i\,\sqrt{2}\,cg\sin\left(fx+e\right)-i\,\sqrt{2}\,cg\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+21\left(i\,\sqrt{2}\,cg\sin\left(fx+e\right)+i\,\sqrt{2}\,cg\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{3\left(a^{2}f\sin\left(fx+e\right)+a^{2}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(c*g*sin(f*x + e) + 13*c*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e
) + a)*sqrt(-c*sin(f*x + e) + c) + 21*(-I*sqrt(2)*c*g*sin(f*x + e) - I*sqrt
(2)*c*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) + I*sin(f*x + e))) + 21*(I*sqrt(2)*c*g*sin(f*x + e) + I*sqrt(2)*c*
g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x +
e) - I*sin(f*x + e))))/(a^2*f*sin(f*x + e) + a^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

3.137 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -4*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)
-6*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e)
)^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$-\frac{4c(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(3/2),x]
```

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Si
n[e + f*x]]) - (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{}{\sqrt{c}}}{}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3cg\cos}{a\sqrt{a+a}}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(3cg\sqrt{\cos}\right.}{a}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{6cg\sqrt{\cos(}}{af\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.42, size = 213, normalized size = 1.73

$$\frac{2g\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g}\left(-\left(\left(i+5e^{i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}\right)+2e^{2i(e+fx)}\left(i+e^{i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{a\left(-i+e^{i(e+fx)}\right)\sqrt{-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\sqrt{1+e^{2i(e+fx)}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(3/2),x]
```

```
[Out] (2*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*(-((I + 5*E^(I*(e
+ f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))]) + 2*E^((2*I)*(e + f*x))*(I + E^(I*(
e + f*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(e + f*x))])*Sqrt[c -
```

```
 c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*a*(I + E^(I*(e + f*x
)))^2)/E^(I*(e + f*x))]*Sqrt[1 + E^((2*I)*(e + f*x))]*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.35, size = 2836, normalized size = 23.06

method	result	size
default	Expression too large to display	2836

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1+cos(f*x+e))*(-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+ln(-(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(3/2)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos
(f*x+e)))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)-2*cos(f*x+e)^2*sin(f*x+e)-4*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+4
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)
^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-6*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(3/2)+6*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*c
os(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-4*ln(-2*(2*cos(f*x+e)^2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(3/2)-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+
e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+4*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(3/2)+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x
+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+12*I*(1/(1+cos(f*x+e
)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*cos(f*x+e)^2-12*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2+6*I*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos
```

```
(f*x+e))/sin(f*x+e),I)*cos(f*x+e)-6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)+
6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3-6*I*(1/(1+cos(f*x+e)))^(1/2)*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)^3-10*cos(f*x+e)^2+2*cos(f*x+e)^3-ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e
)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(3/2)-3*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*
cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-ln(-2*(2*cos(f*x+e)^2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-
3*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-
1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+3*cos(f*x+e)^2*sin(f*
x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(3/2)-6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)+6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+
e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-6*I*(1/(1+cos
(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e))*(-c*(sin(f*x+e)-1))^(1/2)*(g*cos(
f*x+e))^(3/2)/(cos(f*x+e)^2-cos(f*x+e)*sin(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2
)/sin(f*x+e)/cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 166, normalized size = 1.35

$$-\frac{4\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g+3\sqrt{acg}\left(-i\sqrt{2}\,g\sin\left(fx+e\right)-i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3\sqrt{acg}\left(i\sqrt{2}\,g\sin\left(fx+e\right)+i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{a^2f\sin\left(fx+e\right)+a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -(4*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*g + 3*sqrt(a*c*g)*(-I*sqrt(2)*g*sin(f*x + e) - I*sqrt(2)*g)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*sqr
t(a*c*g)*(I*sqrt(2)*g*sin(f*x + e) + I*sqrt(2)*g)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*f*sin(f*x +
e) + a^2*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(3/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(3/2), x)
```

3.138 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=121

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2)-2
*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/f/(a+a*sin(f*x+e))^(1
/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[
e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
```

```
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\int \ldots}{\sqrt{a-\ldots}}$$
$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{(g\,\mathrm{co}\ldots}{a\sqrt{a+\ldots}}$$
$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{\left(g\sqrt{\mathrm{co}\ldots}\right.}{\ldots}$$
$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{2g\sqrt{\mathrm{co}\ldots}}{af\ldots}$$

Mathematica [A]
time = 0.51, size = 170, normalized size = 1.40

$$-\frac{2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)+E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\cos^{\frac{3}{2}}(e+fx)(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^2*(Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) + EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/
2])))/(f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e +
 f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.36, size = 925, normalized size = 7.64

method	result	size
default	Expression too large to display	925

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 1/2/f*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))^2*(4*I*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(1/(1+cos(f*x+e)))^
(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-4*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(1/
(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)+8*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*
sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-8*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)+4*I*(1/(1+cos(f*x+e)))^(1/2)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-4*I*(1/(1+cos(f*x+e)))^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+4*cos(f*x+e)^2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)+ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)*sin(f*x+e)*cos(f*x+e)-cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)-1)/sin(f*x+e)^2)*sin(f*x+e)+4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)*sin(f*x+e)-4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2))/(1+cos(f*x+e))/sin(f*
x+e)^5/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)/(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 168, normalized size = 1.39

$$-\frac{2\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\,g-\sqrt{acg}\left(i\sqrt{2}\,g\sin\left(fx+e\right)+i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))-\sqrt{acg}\left(-i\sqrt{2}\,g\sin\left(fx+e\right)-i\sqrt{2}\,g\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))}{a^{2}cf\sin\left(fx+e\right)+a^{2}cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)
*g - sqrt(a*c*g)*(I*sqrt(2)*g*sin(f*x + e) + I*sqrt(2)*g)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - sqrt(a*c
*g)*(-I*sqrt(2)*g*sin(f*x + e) - I*sqrt(2)*g)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^2*c*f*sin(f*x + e)
 + a^2*c*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

3.139 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(}}{acf\sqrt{}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2)+2
*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)-2
*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c/f/(a+a*sin(f*x+e))^
(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$\frac{2(g\cos(e+fx))^{5/2}}{afg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{3/2}}-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{acf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(3/2)) + (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*Sqrt[a + a*Sin[e + f*x]]*(
c - c*Sin[e + f*x])^(3/2)) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(a*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
+ f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{\int \frac{}{\sqrt{}}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{}}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}} + \frac{}{afg\sqrt{}}$$

Mathematica [**A**]
time = 0.53, size = 92, normalized size = 0.52

$$-\frac{2(g\cos(e+fx))^{5/2}\left(-\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)+\sin(e+fx)\right)}{cfg(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(3/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2)*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
 + Sin[e + f*x]))/(c*f*g*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*S
qrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.31, size = 363, normalized size = 2.06

method	result
default	$-\frac{2(1+\cos(fx+e))^2(g\cos(fx+e))^{\frac{3}{2}}(1+\sin(fx+e))(-1+\cos(fx+e))^2(\sin(fx+e)-1)\left(i\sin(fx+e)\cos(fx+e)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(fx}{\sin(fx+e}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))^2
*(sin(f*x+e)-1)*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin
(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-I*Ellipt
icF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e))
)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-cos(f*x+e)+1)/(a*(1+sin(f*x+e)
))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 156, normalized size = 0.89

$$\frac{i\sqrt{2}\sqrt{acg}\,g\cos(fx+e)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-i\sqrt{2}\sqrt{acg}\,g\cos(fx+e)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))+2\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,g\sin(fx+e)}{a^2c^2f\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(a*c*g)*g*cos(f*x + e)^2*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - I*sqrt(2)*sqrt(a*c*g)*g*co
s(f*x + e)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e)
 - I*sin(f*x + e))) + 2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(
-c*sin(f*x + e) + c)*g*sin(f*x + e))/(a^2*c^2*f*cos(f*x + e)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(3/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(3/2)), x)
```

3.140 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=237

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{6(g\cos(e+fx))^{5/2}}{5afg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{5/2}}+\frac{}{5acfg\sqrt{}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)+6
/5*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)
+6/5*(g*cos(f*x+e))^(5/2)/a/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(
1/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c^2/f/(a+a*si
n(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5ac^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5acfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{6(g\cos(e+fx))^{5/2}}{5afg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*Sqrt[a + a*Sin[e + f*x]]
*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*Sqrt[a
 + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g*Sqrt[Cos[e + f*x]]*Sq
rt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a*c^2*f*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{3\int -}{\quad}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{5afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{5afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{5afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{5afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}} + \frac{}{5afg}$$

Mathematica [A]
time = 0.82, size = 134, normalized size = 0.57

$$-\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(1-3\cos(2(e+fx))-6\sin(e+fx))+E\left(\frac{1}{2}(e+fx)\middle|2\right)(6\cos(e+fx)-3\sin(2(e+fx)))\right)}{5c^2f(-1+\sin(e+fx))^2(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] -1/5*(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(1 - 3*
Cos[2*(e + f*x)] - 6*Sin[e + f*x]) + EllipticE[(e + f*x)/2, 2]*(6*Cos[e + f
*x] - 3*Sin[2*(e + f*x)])))/(c^2*f*(-1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*
x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 877, normalized size = 3.70

method	result	size
default	Expression too large to display	877

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(-1+cos(f*x+e))^2*(1+sin(f*x+e)
)*(sin(f*x+e)-1)*(3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*
sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*I*c
os(f*x+e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+3*I*cos(f*x+e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^3*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*I*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*
x+e)))^(1/2)-3*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1
+cos(f*x+e)))^(1/2)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*I*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos
(f*x+e)))^(1/2)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*si
n(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*cos(f
*x+e)*sin(f*x+e)+2*sin(f*x+e)+3*cos(f*x+e)-3)/(a*(1+sin(f*x+e)))^(3/2)/(-c*
(sin(f*x+e)-1))^(5/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 252, normalized size = 1.06

$$-\frac{2\left(3\,g\cos\left(fx+e\right)^{2}+3\,g\sin\left(fx+e\right)-2\,g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+3\left(-i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+3\left(i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{5\left(a^{2}c^{3}f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-a^{2}c^{3}f\cos\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(2*(3*g*cos(f*x + e)^2 + 3*g*sin(f*x + e) - 2*g)*sqrt(g*cos(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 3*(-I*sqrt(2)*g*cos(f*
x + e)^2*sin(f*x + e) + I*sqrt(2)*g*cos(f*x + e)^2)*sqrt(a*c*g)*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*
(I*sqrt(2)*g*cos(f*x + e)^2*sin(f*x + e) - I*sqrt(2)*g*cos(f*x + e)^2)*sqrt
(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(a^2*c^3*f*cos(f*x + e)^2*sin(f*x + e) - a^2*c^3*f*cos(f*x
+ e)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}\,(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

3.141 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=294

$$-\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}+\frac{10(g\cos(e+fx))^{5/2}}{9afg\sqrt{a+a\sin(e+fx)}\,(c-c\sin(e+fx))^{7/2}}+\frac{}{3acfg\sqrt{}}$$

```
[Out] -2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2)+1
0/9*(g*cos(f*x+e))^(5/2)/a/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2
)+2/3*(g*cos(f*x+e))^(5/2)/a/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^
(1/2)+2/3*(g*cos(f*x+e))^(5/2)/a/c^2/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*
x+e))^(1/2)-2/3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a/c^3/f/
(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{3ac^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{3ac^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{2(g\cos(e+fx))^{5/2}}{3acfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{10(g\cos(e+fx))^{5/2}}{9afg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
f*x])^(7/2)) + (10*(g*Cos[e + f*x])^(5/2))/(9*a*f*g*Sqrt[a + a*Sin[e + f*x]
]*(c - c*Sin[e + f*x])^(7/2)) + (2*(g*Cos[e + f*x])^(5/2))/(3*a*c*f*g*Sqrt[
a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (2*(g*Cos[e + f*x])^(5/2)
)/(3*a*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (2*g*
Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(3*a*c^3
*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{5\int}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}} + \frac{}{9af}$$

Mathematica [A]
time = 1.08, size = 155, normalized size = 0.53

$$\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(3E\left(\frac{1}{2}(e+fx)\middle|2\right)(5\cos(e+fx)-\cos(3(e+fx))-4\sin(2(e+fx)))+\sqrt{\cos(e+fx)}\,(4-12\cos(2(e+fx))-17\sin(e+fx)+3\sin(3(e+fx)))\right)}{18c^3f(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{3/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(3*EllipticE[(e + f*x)/2, 2]*(5*
Cos[e + f*x] - Cos[3*(e + f*x)] - 4*Sin[2*(e + f*x)]) + Sqrt[Cos[e + f*x]]*
(4 - 12*Cos[2*(e + f*x)] - 17*Sin[e + f*x] + 3*Sin[3*(e + f*x)])))/(18*c^3*
f*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x
]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 1177, normalized size = 4.00

method	result	size
default	Expression too large to display	1177

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/9/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))
^2*(sin(f*x+e)-1)*(3*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+6*
I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)-6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1
/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-6*I*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*cos(f*x+e)^3-6*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-6*I*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+
e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-6*I*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+
e)))^(1/2)+6*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x
+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+6*I*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3
```

```
+6*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)+3*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)+6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+co
s(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^3*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*sin(f*x+e)-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*cos(f*x+e)^3-
2*cos(f*x+e)^2+6*cos(f*x+e)*sin(f*x+e)-6*cos(f*x+e)-4*sin(f*x+e)+5)/(a*(1+s
in(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(7/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 315, normalized size = 1.07

$$-\frac{2\left(6\,g\cos\left(fx+e\right)^{2}-\left(3\,g\cos\left(fx+e\right)^{2}-5\,g\right)\sin\left(fx+e\right)-4\,g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+3\left(-i\,\sqrt{2}\,g\cos\left(fx+e\right)^{4}-2i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+2i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))+3\left(i\,\sqrt{2}\,g\cos\left(fx+e\right)^{4}+2i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2i\,\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{9\left(a^{2}c^{4}f\cos\left(fx+e\right)^{4}+2\,a^{2}c^{4}f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-2\,a^{2}c^{4}f\cos\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(6*g*cos(f*x + e)^2 - (3*g*cos(f*x + e)^2 - 5*g)*sin(f*x + e) - 4*g
)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) +
 3*(-I*sqrt(2)*g*cos(f*x + e)^4 - 2*I*sqrt(2)*g*cos(f*x + e)^2*sin(f*x + e)
 + 2*I*sqrt(2)*g*cos(f*x + e)^2)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*(I*sqrt(2)*g*cos(f*
x + e)^4 + 2*I*sqrt(2)*g*cos(f*x + e)^2*sin(f*x + e) - 2*I*sqrt(2)*g*cos(f*
x + e)^2)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) - I*sin(f*x + e))))/(a^2*c^4*f*cos(f*x + e)^4 + 2*a^2*c^4*f*cos(f
*x + e)^2*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos{(e+f\,x)})^{3/2}}{(a+a\,\sin{(e+f\,x)})^{3/2}\,(c-c\,\sin{(e+f\,x)})^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

3.142 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=357

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^5g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+$$

```
[Out] 76/5*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))
^(3/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(7/2)/f/g/(a+a*sin(f*x+e
))^(5/2)+114/7*c^3*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a^2/f/g/(a+a
*sin(f*x+e))^(1/2)+418/5*c^5*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^
(1/2)/(c-c*sin(f*x+e))^(1/2)+1254/5*c^5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*co
s(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+1254/35
*c^4*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a^2/f/g/(a+a*sin(f*x+e))^(
1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^5g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{1254c^4\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{35a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{114c^3(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{7a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{76c^2(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}}-\frac{4c(c-c\sin(e+fx))^{7/2}(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (418*c^5*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (1254*c^5*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (1254*c^4*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(35
*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (114*c^3*(g*Cos[e + f*x])^(5/2)*(c - c
*Sin[e + f*x])^(3/2))/(7*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]) + (76*c^2*(g*Cos
[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(7/2))/(5*f*g*(a
+ a*Sin[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{9/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(19c)\int \frac{(g\cos}{}}{}$$

$$= \frac{76c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+}{5fg}$$

$$= \frac{114c^3(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{7a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{76c^2(g\cos}{5a}$$

$$= \frac{1254c^4(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{35a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{114c^3(g\cos}{7}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g}{}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g}{}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^4(g}{}$$

$$= \frac{418c^5(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{1254c^5g}{5a^2}$$

Mathematica [A]
time = 6.53, size = 356, normalized size = 1.00

$$\frac{1254(g\cos(e+fx))^{3/2}E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}}{5f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}} + \frac{(g\cos(e+fx))^{3/2}\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5(c-c\sin(e+fx))^{9/2}\left(\frac{736}{5}+\frac{221}{14}\cos(e+fx)-\frac{1}{14}\cos(3(e+fx))+\frac{128\sin\left(\frac{1}{2}(e+fx)\right)}{5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}-\frac{64}{5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2}-\frac{1472\sin\left(\frac{1}{2}(e+fx)\right)}{5\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}-\frac{7}{5}\sin(2(e+fx))\right)}{f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^9(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (1254*(g*Cos[e + f*x])^(3/2)*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(5*f*Cos[e + f*x]^(3/2)*(Co
s[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + ((g*Co
s[e + f*x])^(3/2)*Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c -
 c*Sin[e + f*x])^(9/2)*(736/5 + (221*Cos[e + f*x])/14 - Cos[3*(e + f*x)]/14
 + (128*Sin[(e + f*x)/2])/(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3) - 64/
(5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2) - (1472*Sin[(e + f*x)/2])/(5*(C
```

```
os[(e + f*x)/2] + Sin[(e + f*x)/2])) - (7*Sin[2*(e + f*x)])/5))/(f*(Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 2.15, size = 3655, normalized size = 10.24

method	result	size
default	Expression too large to display	3655

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/35/f*(-1+cos(f*x+e))*(39*sin(f*x+e)*cos(f*x+e)^5-1400*ln(-2*(2*cos(f*x+e)
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)+1400*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1
)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-700*cos(f
*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+700*cos(f*x+e)^4*sin(f*x+e)*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-1)/sin(f*x+e)^2)-1351*cos(f*x+e)^3*sin(f*x+e)+8778*I*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-5*cos(f*x+e)^6*sin(f*x+e)+231*cos(f*x+e)
^4*sin(f*x+e)+9002*cos(f*x+e)^2*sin(f*x+e)+44*cos(f*x+e)^6+1400*ln(-2*(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(3/2)-1400*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+5
600*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-5600*ln(-(2*cos(f*x+e)^2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(3/2)+4900*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(
f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+1400*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)*sin(f*x+e)-4900*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
```

```
/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-1400*l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+8554*cos(f*x+e)^2-1575*cos(f*x+e)^3+700
*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-700*cos(f*x+e)^5*(-cos(f*x+e)/(1+co
s(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(
f*x+e)^2)-192*cos(f*x+e)^5+5*cos(f*x+e)^7-8778*I*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)+4389*I*sin(f*x+e)*cos(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/
2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)-4389*I*sin(f*x+e)*cos(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+
cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+13167*I*sin(f*
x+e)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-13167*I*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(1+cos(f*x+e)))^(1/2)+1400*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-1400*ln(-(2*cos
(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(3/2)-4389*I*cos(f*x+e)^4*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+4389*I*Ellipt
icE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+13167*I*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1
/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)-13167*I*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+8778*I*(1/(1+cos(f*x+e)
))^(1/2)*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)-8778*I*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+4900*cos(f
*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+co...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(9/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 296, normalized size = 0.83

$$-\frac{2\left(5\,c^4g\cos\left(fx+e\right)^4-192\,c^4g\cos\left(fx+e\right)^2+2814\,c^4g+\left(39\,c^4g\cos\left(fx+e\right)^2+3038\,c^4g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}-4389\left(-i\sqrt{2}\,c^4g\cos\left(fx+e\right)^2+2i\sqrt{2}\,c^4g\sin\left(fx+e\right)+2i\sqrt{2}\,c^4g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)-4389\left(i\sqrt{2}\,c^4g\cos\left(fx+e\right)^2-2i\sqrt{2}\,c^4g\sin\left(fx+e\right)-2i\sqrt{2}\,c^4g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{35\left(a^3f\cos\left(fx+e\right)^2-2\,a^3f\sin\left(fx+e\right)-2\,a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/35*(2*(5*c^4*g*cos(f*x + e)^4 - 192*c^4*g*cos(f*x + e)^2 + 2814*c^4*g +
(39*c^4*g*cos(f*x + e)^2 + 3038*c^4*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*s
qrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) - 4389*(-I*sqrt(2)*c^4*g*
cos(f*x + e)^2 + 2*I*sqrt(2)*c^4*g*sin(f*x + e) + 2*I*sqrt(2)*c^4*g)*sqrt(a
*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*si
n(f*x + e))) - 4389*(I*sqrt(2)*c^4*g*cos(f*x + e)^2 - 2*I*sqrt(2)*c^4*g*sin
(f*x + e) - 2*I*sqrt(2)*c^4*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 -
2*a^3*f*sin(f*x + e) - 2*a^3*f)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

3.143 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{462c^4g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{66}{}$$

```
[Out] 12*c^2*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/a/f/g/(a+a*sin(f*x+e))^(
3/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))
^(5/2)+154/5*c^4*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)+462/5*c^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/
2)/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+66/5*c^3*(g*cos(f*x+
e))^(5/2)*(c-c*sin(f*x+e))^(1/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{462c^4g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{66c^3\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}}+\frac{12c^2(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{afg(a\sin(e+fx)+a)^{3/2}}-\frac{4c(c-c\sin(e+fx))^{5/2}(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (154*c^4*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
 - c*Sin[e + f*x]]) + (462*c^4*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*El
lipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e
 + f*x]]) + (66*c^3*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a^2
*f*g*Sqrt[a + a*Sin[e + f*x]]) + (12*c^2*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[
e + f*x])^(3/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])
^(5/2)*(c - c*Sin[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(3c)\int \frac{(g\cos(e+}{(}}{}$$

$$= \frac{12c^2(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+}{5fg(}$$

$$= \frac{66c^3(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5a^2fg\sqrt{a+a\sin(e+fx)}} + \frac{12c^2(g\cos(e+}{afg(}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos}{5a}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos}{5a}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{66c^3(g\cos}{5a}$$

$$= \frac{154c^4(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{462c^4g\sqrt{}}{5a^2f}$$

Mathematica [A]
time = 3.32, size = 250, normalized size = 0.84

$$\frac{c^3(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\sin(e+fx)}\left(1848E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(487\cos\left(\frac{1}{2}(e+fx)\right)+633\cos\left(\frac{3}{2}(e+fx)\right)-17\cos\left(\frac{5}{2}(e+fx)\right)+\cos\left(\frac{7}{2}(e+fx)\right)-487\sin\left(\frac{1}{2}(e+fx)\right)+633\sin\left(\frac{3}{2}(e+fx)\right)+17\sin\left(\frac{5}{2}(e+fx)\right)+\sin\left(\frac{7}{2}(e+fx)\right)\right)\right)}{20f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^3*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(1848*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(487*Cos[(e + f*x)/2] + 633*Cos[(3*(e +
 f*x))/2] - 17*Cos[(5*(e + f*x))/2] + Cos[(7*(e + f*x))/2] - 487*Sin[(e + f
*x)/2] + 633*Sin[(3*(e + f*x))/2] + 17*Sin[(5*(e + f*x))/2] + Sin[(7*(e + f
*x))/2])))/(20*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(
a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.29, size = 3601, normalized size = 12.08

method	result	size
default	Expression too large to display	3601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/5/f*(-1+cos(f*x+e))*(-sin(f*x+e)*cos(f*x+e)^5+231*I*cos(f*x+e)^4*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+
e))/sin(f*x+e),I)-231*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-693*I*cos(f*
x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+693*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/
2)-462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+462*I*cos(f*x+e)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f
*x+e)))^(1/2)+80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-80*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(3/2)+40*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(
f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^
2)-40*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+69*cos(f*x+e)^3*s
in(f*x+e)+231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*(1/(1+cos(f*
x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-9*cos(f*x
+e)^4*sin(f*x+e)-478*cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^6-80*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(3/2)+80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-320*ln
(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+320*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*
x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(3/2)-280*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2
```

```
)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-80*ln(-2*(2*cos(f*x+e)^2*
(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(3/2)*sin(f*x+e)+280*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(
f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+80*ln(-(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(3/2)*sin(f*x+e)-446*cos(f*x+e)^2+85*cos(f*x+e)^3-40*cos(f*x+e)^5*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^
(1/2)-1)/sin(f*x+e)^2)+40*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+8*cos(f*
x+e)^5-80*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+80*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-231*I*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*c
os(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+462*I*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos
(f*x+e)))^(1/2)*sin(f*x+e)-462*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*sin(f
*x+e)+693*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f*x+e)
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-693*I*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*
x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-280*cos(f*x+e)*sin(f*x+e)*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)+280*cos(f*x+e)*sin(f*x+e)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.16, size = 281, normalized size = 0.94

$$\frac{2\left(8\,c^{3}g\cos\left(fx+e\right)^{2}-146\,c^{3}g-\left(c^{3}g\cos\left(fx+e\right)^{2}+162\,c^{3}g\right)\sin\left(fx+e\right)\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+231\left(-i\sqrt{2}\,c^{3}g\cos\left(fx+e\right)^{2}+2i\sqrt{2}\,c^{3}g\sin\left(fx+e\right)+2i\sqrt{2}\,c^{3}g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))+231\left(i\sqrt{2}\,c^{3}g\cos\left(fx+e\right)^{2}-2i\sqrt{2}\,c^{3}g\sin\left(fx+e\right)-2i\sqrt{2}\,c^{3}g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))}{5\left(a^{3}f\cos\left(fx+e\right)^{2}-2\,a^{3}f\sin\left(fx+e\right)-2\,a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/5*(2*(8*c^3*g*cos(f*x + e)^2 - 146*c^3*g - (c^3*g*cos(f*x + e)^2 + 162*c^
3*g)*sin(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*si
n(f*x + e) + c) + 231*(-I*sqrt(2)*c^3*g*cos(f*x + e)^2 + 2*I*sqrt(2)*c^3*g*
sin(f*x + e) + 2*I*sqrt(2)*c^3*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 231*(I*sqrt(2)*c^3*g
*cos(f*x + e)^2 - 2*I*sqrt(2)*c^3*g*sin(f*x + e) - 2*I*sqrt(2)*c^3*g)*sqrt(
a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*s
in(f*x + e))))/(a^3*f*cos(f*x + e)^2 - 2*a^3*f*sin(f*x + e) - 2*a^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

3.144 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=243

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}+\ldots$$

```
[Out] -4/5*c*(g*cos(f*x+e))^(5/2)*(c-c*sin(f*x+e))^(3/2)/f/g/(a+a*sin(f*x+e))^(5/
2)+154/15*c^3*(g*cos(f*x+e))^(5/2)/a^2/f/g/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(
f*x+e))^(1/2)+154/5*c^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*E
llipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/
a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+44/5*c^2*(g*cos(f*x+e))
^(5/2)*(c-c*sin(f*x+e))^(1/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)
```

Rubi [A]
time = 0.73, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2930, 2921, 2721, 2719}

$$\frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{154c^3g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{44c^2\sqrt{c-c\sin(e+fx)}(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}}-\frac{4c(c-c\sin(e+fx))^{3/2}(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (154*c^3*(g*Cos[e + f*x])^(5/2))/(15*a^2*f*g*Sqrt[a + a*Sin[e + f*x]]*Sqrt[
c - c*Sin[e + f*x]]) + (154*c^3*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*E
llipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[
e + f*x]]) + (44*c^2*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*a*
f*g*(a + a*Sin[e + f*x])^(3/2)) - (4*c*(g*Cos[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(3/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2930

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && GtQ[m, 0] && NeQ[m + n + p, 0] &&  !LtQ[0, n, m] && IntegersQ[2*
m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(11c)\int \frac{(g\cos(e}{}}{}$$

$$= \frac{44c^2(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5afg(a+a\sin(e+fx))^{3/2}} - \frac{4c(g\cos(e+f}{5fg(a}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\,\mathrm{co}}{5}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\,\mathrm{co}}{5}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{44c^2(g\,\mathrm{co}}{5}$$

$$= \frac{154c^3(g\cos(e+fx))^{5/2}}{15a^2fg\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}} + \frac{154c^3g\sqrt{}}{5a^2}$$

Mathematica [A]
time = 1.65, size = 230, normalized size = 0.95

$$\frac{c^2(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{c-c\sin(e+fx)}\left(924E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(226\cos\left(\frac{1}{2}(e+fx)\right)+327\cos\left(\frac{3}{2}(e+fx)\right)-5\cos\left(\frac{5}{2}(e+fx)\right)-226\sin\left(\frac{1}{2}(e+fx)\right)+327\sin\left(\frac{3}{2}(e+fx)\right)+5\sin\left(\frac{5}{2}(e+fx)\right)\right)\right)}{30f\cos^{\frac{3}{2}}(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

```
[Out] (c^2*(g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c
- c*Sin[e + f*x]]*(924*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(226*Cos[(e + f*x)/2] + 327*Cos[(3*(e +
f*x))/2] - 5*Cos[(5*(e + f*x))/2] - 226*Sin[(e + f*x)/2] + 327*Sin[(3*(e +
f*x))/2] + 5*Sin[(5*(e + f*x))/2])))/(30*f*Cos[e + f*x]^(3/2)*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 3550, normalized size = 14.61

method	result	size
default	Expression too large to display	3550

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/15/f*(-1+cos(f*x+e))*(231*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-231*I
*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-693*I*cos(f*x+e)^2*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e
)))^(1/2)+693*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)-462*I*cos(f*x+e)*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1
/(1+cos(f*x+e)))^(1/2)+462*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*E
llipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+90*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2
)+45*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*co
s(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2
*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-45*cos(f*x+e)^4*sin(
f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+65*cos(f*x+e)^3*sin(f*x+e)+231*I*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-5*cos(f*x+e)^4*sin(f*x+e)-486*co
s(f*x+e)^2*sin(f*x+e)-90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1
)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+90*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-360*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/
2)+360*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-315*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(3/2)-90*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)
-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*
x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+315*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(3/2)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
```

```
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/
sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-438*cos(f*x+e
)^2+89*cos(f*x+e)^3-45*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln
(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f
*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+45*cos(f*x+e)
^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/
(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x
+e))^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^5-90*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2
)+90*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(3/2)-231*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^3*(1/(1+cos(f*x+e)))^(1/2)
*sin(f*x+e)+462*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-462*I*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+693*I*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)-693*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(1/(1+cos(f*x+e)))^(1/2)*sin(
f*x+e)-315*cos(f*x+e)*sin(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+315*cos(f*x+e)
*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 264, normalized size = 1.09

$$\frac{2\left(5c^2g\cos(fx+e)^2-166c^2g\sin(fx+e)-142c^2g\right)\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}+231\left(-i\sqrt{2}c^2g\cos(fx+e)^2+2i\sqrt{2}c^2g\sin(fx+e)+2i\sqrt{2}c^2g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))+231\left(i\sqrt{2}c^2g\cos(fx+e)^2-2i\sqrt{2}c^2g\sin(fx+e)-2i\sqrt{2}c^2g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{15\left(a^3f\cos(fx+e)^2-2a^3f\sin(fx+e)-2a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(5*c^2*g*cos(f*x + e)^2 - 166*c^2*g*sin(f*x + e) - 142*c^2*g)*sqrt(
g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 231*(-
I*sqrt(2)*c^2*g*cos(f*x + e)^2 + 2*I*sqrt(2)*c^2*g*sin(f*x + e) + 2*I*sqrt(
2)*c^2*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(f*x + e) + I*sin(f*x + e))) + 231*(I*sqrt(2)*c^2*g*cos(f*x + e)^2 - 2*I*sq
rt(2)*c^2*g*sin(f*x + e) - 2*I*sqrt(2)*c^2*g)*sqrt(a*c*g)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^3*f*co
s(f*x + e)^2 - 2*a^3*f*sin(f*x + e) - 2*a^3*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

3.145 $\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=186

$$\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{42c^2g\sqrt{\cos(e+fx)}\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5a^2f\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)}}-\frac{4c(}{}$$

```
[Out] 28/5*c^2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))
^(1/2)+42/5*c^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE
(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a
+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-4/5*c*(g*cos(f*x+e))^(5/2)*(c-c
*sin(f*x+e))^(1/2)/f/g/(a+a*sin(f*x+e))^(5/2)
```

Rubi [A]
time = 0.55, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2929, 2921, 2721, 2719}

$$\frac{42c^2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] (28*c^2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c
- c*Sin[e + f*x]]) + (42*c^2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e +
 f*x]]) - (4*c*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])/(5*f*g*(a +
 a*Sin[e + f*x])^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c-c\sin(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}{5fg(a+a\sin(e+fx))^{5/2}} - \frac{(7c)\int \frac{(g\cos(e-}{}}{}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos}{5}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos}{5}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} - \frac{4c(g\cos}{5}$$

$$= \frac{28c^2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}} + \frac{42c^2g\sqrt{}}{5a^2}$$

Mathematica [A]

time = 0.84, size = 180, normalized size = 0.97

$$\frac{c\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}\left(42E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+8\sqrt{\cos(e+fx)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{3}{2}(e+fx)\right)\right)\right)}{5f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3(a(1+\sin(e+fx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e
+ f*x])^(5/2),x]
```

[Out] (c*Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]*(42*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 8*Sqrt[Cos[e + f*x]]*(Cos[(e + f*x)/2] + 2*Cos[(3*(e + f*x))/2] - Sin[(e + f*x)/2] + 2*Sin[(3*(e + f*x))/2])))/(5*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(5/2))

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 3497, normalized size = 18.80

method	result	size
default	Expression too large to display	3497

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/5/f*(-1+cos(f*x+e))*(-10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-5*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^4*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^3*sin(f*x+e)+21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+46*cos(f*x+e)^2*sin(f*x+e)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+40*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-40*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+35*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)-35*ln(-(2*cos(f*x+e)

```
^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(3/2)-10*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(
1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/si
n(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*sin(f*x+e)+38*cos(f*x+e)^2
-9*cos(f*x+e)^3+5*cos(f*x+e)^5*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^5*(-c
os(f*x+e)/(1+cos(f*x+e))^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos
(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2
)^(1/2)-1)/sin(f*x+e)^2)+10*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e
))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2
)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-10*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(3/2)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)^4*(co
s(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)-42*I*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*El
lipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+42*I*cos(f
*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(
I*(-1+cos(f*x+e))/sin(f*x+e),I)-63*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)
+63*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+35*cos(f*x+e)*sin(f*x+e)*ln(-2
*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*
x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(
1+cos(f*x+e))^2)^(3/2)-35*cos(f*x+e)*sin(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+c
os(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+2
5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2
*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x
+e)^3*sin(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(3/2)-25*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)^3*sin(f*x+e)*(-cos
(f*x+e)/(1+cos(f*x+e))^2)^(3/2)+45*cos(f*x+e)^2*sin(f*x+e)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e
) + a)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 233, normalized size = 1.25

$$-\frac{8\left(4\,cg\sin\left(fx+e\right)+3\,cg\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}-21\left(-i\sqrt{2}\,cg\cos\left(fx+e\right)^{2}+2i\sqrt{2}\,cg\sin\left(fx+e\right)+2i\sqrt{2}\,cg\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))-21\left(i\sqrt{2}\,cg\cos\left(fx+e\right)^{2}-2i\sqrt{2}\,cg\sin\left(fx+e\right)-2i\sqrt{2}\,cg\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))}{5\left(a^{3}f\cos\left(fx+e\right)^{2}-2\,a^{3}f\sin\left(fx+e\right)-2\,a^{3}f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(8*(4*c*g*sin(f*x + e) + 3*c*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x +
e) + a)*sqrt(-c*sin(f*x + e) + c) - 21*(-I*sqrt(2)*c*g*cos(f*x + e)^2 + 2*I
*sqrt(2)*c*g*sin(f*x + e) + 2*I*sqrt(2)*c*g)*sqrt(a*c*g)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 21*(I*sqr
t(2)*c*g*cos(f*x + e)^2 - 2*I*sqrt(2)*c*g*sin(f*x + e) - 2*I*sqrt(2)*c*g)*s
qrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) -
 I*sin(f*x + e))))/(a^3*f*cos(f*x + e)^2 - 2*a^3*f*sin(f*x + e) - 2*a^3*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

3.146 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}+\frac{6cg\sqrt{\cos}}{5a^2f\sqrt{}}$$

```
[Out] -4/5*c*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/
2)+6/5*c*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))
^(1/2)+6/5*c*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a+a*
sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2929, 2931, 2921, 2721, 2719}

$$\frac{6cg\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}+\frac{6c(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^
(5/2),x]
```

```
[Out] (-4*c*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*
Sin[e + f*x]]) + (6*c*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])
^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + (6*c*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e +
 f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c
- c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2929

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2
*b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(2*n + p + 1))), x] - Dist[b*((2*m + p - 1)/(d*(2*n + p + 1))),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] &&
 EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LtQ[n, -1] && NeQ[2*n + p + 1, 0] && Int
egersQ[2*m, 2*n, 2*p]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{(3c)\int \frac{}{(a}}{}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{5afg(a-}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{5afg(a-}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{5afg(a-}$$

$$= -\frac{4c(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{}{5afg(a-}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.60, size = 230, normalized size = 1.26

$$\frac{4ig\sqrt{e^{-i(e+fx)}\left(1+e^{2i(e+fx)}\right)g}\left(\left(5-4ie^{i(e+fx)}-3e^{2i(e+fx)}\right)\sqrt{1+e^{2i(e+fx)}}+e^{i(e+fx)}\left(i+e^{i(e+fx)}\right)^3{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(e+fx)}\right)\right)\sqrt{c-c\sin(e+fx)}}{5a\left(-i+e^{i(e+fx)}\right)\left(-iae^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2\right)^{3/2}\sqrt{1+e^{2i(e+fx)}}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e +
f*x])^(5/2),x]
```

```
[Out] (((4*I)/5)*g*Sqrt[((1 + E^((2*I)*(e + f*x)))*g)/E^(I*(e + f*x))]*((5 - (4*I
)*E^(I*(e + f*x)) - 3*E^((2*I)*(e + f*x)))*Sqrt[1 + E^((2*I)*(e + f*x))] +
E^(I*(e + f*x))*(I + E^(I*(e + f*x)))^3*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(e + f*x))])*Sqrt[c - c*Sin[e + f*x]])/(a*(-I + E^(I*(e + f*x)))*((
(-I)*a*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x)))^(3/2)*Sqrt[1 + E^((2*I)*(e
 + f*x))]*f)
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 2040, normalized size = 11.21

method	result	size
default	Expression too large to display	2040

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -1/10/f*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))^3*(-c*(sin(f*x+
e)-1))^(1/2)*(12*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)-5*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+
e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/
2)-1)/sin(f*x+e)^2)*sin(f*x+e)+5*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(
f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)
^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)*sin(f*x+e)+12*(-cos(f*x+e)/(1+cos(f*x+e)
)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^2+4*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*s
in(f*x+e)*cos(f*x+e)-8*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-12
*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e
)+24*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)*(-co
s(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+co
s(f*x+e)))^(1/2)-24*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
cos(f*x+e)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+12*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
```

```
,I)*sin(f*x+e)*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e
)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-12*I*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*
EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)^2+8*(-cos(f
*x+e)/(1+cos(f*x+e))^2)^(1/2)+12*I*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f*x+e))
^2)^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+
e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-12*I*cos(f*x+e)^4*(-cos(f*x+e)/(1+cos(f
*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+24*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-24*I*EllipticF(I*(-1+cos(f*
x+e))/sin(f*x+e),I)*cos(f*x+e)^3*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-24*I*(-cos(f*x+e)/(1+
cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)+24*I*(-cos(f*x+e)
/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*cos(f*x+e)-12*I*(-cos(f*
x+e)/(1+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+12*I*(-cos(f*x+e)/(1
+cos(f*x+e))^2)^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+5*cos(f*x+e)^3*ln(-(2*cos(f
*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^3*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)+5*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)-1)/sin(f*x+e)^2)*cos(f*x+e)-8*(-cos(f*x
+e)/(1+cos(f*x+e))^2)^(1/2)*sin(f*x+e)+20*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1
/2)*cos(f*x+e)-20*(-cos(f*x+e)/(1+cos(f*x+e))^2)^(1/2)*cos(f*x+e)^3)/(sin(f
*x+e)-1)/sin(f*x+e)^7/(a*(1+sin(f*x+e)))^(5/2)/(-cos(f*x+e)/(1+cos(f*x+e))^
2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 223, normalized size = 1.23

$$-\frac{2\sqrt{g\cos(fx+e)}\sqrt{a\sin(fx+e)+a}\sqrt{-c\sin(fx+e)+c}\,(3g\sin(fx+e)+g)-3\left(-i\sqrt{2}\,g\cos(fx+e)^2+2i\sqrt{2}\,g\sin(fx+e)+2i\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)+i\sin(fx+e)))-3\left(i\sqrt{2}\,g\cos(fx+e)^2-2i\sqrt{2}\,g\sin(fx+e)-2i\sqrt{2}\,g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(fx+e)-i\sin(fx+e)))}{5\left(a^3f\cos(fx+e)^2-2a^3f\sin(fx+e)-2a^3f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e)
+ c)*(3*g*sin(f*x + e) + g) - 3*(-I*sqrt(2)*g*cos(f*x + e)^2 + 2*I*sqrt(2)*
g*sin(f*x + e) + 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*(I*sqrt(2)*g*cos(f*x
 + e)^2 - 2*I*sqrt(2)*g*sin(f*x + e) - 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(
a^3*f*cos(f*x + e)^2 - 2*a^3*f*sin(f*x + e) - 2*a^3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(
5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,\sqrt{c-c\,\sin(e+f\,x)}}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(5/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)
)^(5/2), x)
```

3.147 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=179

$$-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2g\sqrt{\cos}}{5a^2f}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)
-2/5*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/
2)-2/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*
x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{2g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2f\sqrt{a\sin(e+fx)+a}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f
*x]]),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Si
n[e + f*x]]) - (2*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^(3/
2)*Sqrt[c - c*Sin[e + f*x]]) - (2*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]
*EllipticE[(e + f*x)/2, 2])/(5*a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Si
n[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
```

```
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} + \frac{\int \frac{}{(a+a}}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{5afg(}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{5afg(}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{5afg(}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}} - \frac{}{5afg(}$$

Mathematica [A]
time = 1.11, size = 189, normalized size = 1.06

$$-\frac{(g\cos(e+fx))^{3/2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\left(2E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3+\sqrt{\cos(e+fx)}\left(3\cos\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{3}{2}(e+fx)\right)-4\sin^3\left(\frac{1}{2}(e+fx)\right)\right)\right)}{5f\cos^{\frac{3}{2}}(e+fx)(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin
[e + f*x]]),x]
```

[Out] -1/5*((g*Cos[e + f*x])^(3/2)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(2*EllipticE[(e + f*x)/2, 2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + Sqrt[Cos[e + f*x]]*(3*Cos[(e + f*x)/2] + Cos[(3*(e + f*x))/2] - 4*Sin[(e + f*x)/2]^3)))/(f*Cos[e + f*x]^(3/2)*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 777, normalized size = 4.34

method	result
default	$-\frac{2(g\cos(fx+e))^{\frac{3}{2}}(\cos(fx+e)\sin(fx+e)+\cos(fx+e)-\sin(fx+e)-1)\left(i(\cos^4(fx+e))\sqrt{\frac{1}{1+\cos(fx+e)}}\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \text{Ellipti}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)

[Out] -2/5/f*(g*cos(f*x+e))^(3/2)*(cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-sin(f*x+e)-1)*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-2*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+2*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^2-2*cos(f*x+e)-sin(f*x+e)+1)*(cos(f*x+e)^2+2*cos(f*x+e)+1)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)^5/cos(f*x+e)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*
x + e) + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 227, normalized size = 1.27

$$\frac{2\sqrt{g\cos\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{-c\sin\left(fx+e\right)+c}\left(g\sin\left(fx+e\right)+2g\right)-\left(-i\sqrt{2}g\cos\left(fx+e\right)^{2}+2i\sqrt{2}g\sin\left(fx+e\right)+2i\sqrt{2}g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)))-\left(i\sqrt{2}g\cos\left(fx+e\right)^{2}-2i\sqrt{2}g\sin\left(fx+e\right)-2i\sqrt{2}g\right)\sqrt{acg}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)))}{5\left(a^{3}cf\cos\left(fx+e\right)^{2}-2a^{3}cf\sin\left(fx+e\right)-2a^{3}cf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) +
 c)*(g*sin(f*x + e) + 2*g) - (-I*sqrt(2)*g*cos(f*x + e)^2 + 2*I*sqrt(2)*g*s
in(f*x + e) + 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - (I*sqrt(2)*g*cos(f*x + e)
^2 - 2*I*sqrt(2)*g*sin(f*x + e) - 2*I*sqrt(2)*g)*sqrt(a*c*g)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^3*c
*f*cos(f*x + e)^2 - 2*a^3*c*f*sin(f*x + e) - 2*a^3*c*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\sqrt{c-c\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(1/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(1/2)), x)
```

3.148 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}-\frac{6(g\cos(e+fx))^{5/2}}{5cfg(a+a\sin(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}-\frac{}{5acfg(a+}$$

```
[Out] 2*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2)-6/
5*(g*cos(f*x+e))^(5/2)/c/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2)-
6/5*(g*cos(f*x+e))^(5/2)/a/c/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1
/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/c/f/(a+a*sin
(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5a^2cf\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5acfg(a\sin(e+fx)+a)^{3/2}\sqrt{c-c\sin(e+fx)}}-\frac{6(g\cos(e+fx))^{5/2}}{5cfg(a\sin(e+fx)+a)^{5/2}\sqrt{c-c\sin(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{fg(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(3/2)),x]
```

```
[Out] (2*(g*Cos[e + f*x])^(5/2))/(f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f
*x])^(3/2)) - (6*(g*Cos[e + f*x])^(5/2))/(5*c*f*g*(a + a*Sin[e + f*x])^(5/2
)*Sqrt[c - c*Sin[e + f*x]]) - (6*(g*Cos[e + f*x])^(5/2))/(5*a*c*f*g*(a + a*
Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) - (6*g*Sqrt[Cos[e + f*x]]*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*c*f*Sqrt[a + a*Sin[e +
f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx = \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} + \frac{3\int \frac{}{(a}}{}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{5cfg(}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{5cfg(}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{5cfg(}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{5cfg(}$$

$$= \frac{2(g\cos(e+fx))^{5/2}}{fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}} - \frac{}{5cfg(}$$

Mathematica [A]
time = 0.80, size = 133, normalized size = 0.56

$$\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(\sqrt{\cos(e+fx)}\,(-1+3\cos(2(e+fx))-6\sin(e+fx))+3E\left(\frac{1}{2}(e+fx)\middle|2\right)(2\cos(e+fx)+\sin(2(e+fx)))\right)}{5cf(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(3/2)),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(Sqrt[Cos[e + f*x]]*(-1 + 3*Cos[
2*(e + f*x)] - 6*Sin[e + f*x]) + 3*EllipticE[(e + f*x)/2, 2]*(2*Cos[e + f*x
] + Sin[2*(e + f*x)])))/(5*c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(
5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 877, normalized size = 3.70

method	result	size
default	Expression too large to display	877

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))
^2*(sin(f*x+e)-1)*(3*I*cos(f*x+e)^3*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),
I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+3*I*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+co
s(f*x+e)))^(1/2)+3*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)-3*I*sin(f*x+e)*(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^2*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-3*I*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*(1/(1+cos(f*x+e)))^(1/2)+3*I*cos(f*x+
e)^2*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3
*I*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*sin(f*x+e)*(cos(f*x+
e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)+3*I*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*cos(
f*x+e)*sin(f*x+e)-3*cos(f*x+e)+2*sin(f*x+e)+3)/(a*(1+sin(f*x+e)))^(5/2)/(-c
*(sin(f*x+e)-1))^(3/2)/cos(f*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(3/2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 251, normalized size = 1.06

$$-\frac{2\left(3\,g\cos\left(fx+e\right)^{2}-3\,g\sin\left(fx+e\right)-2\,g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}+3\left(-i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)-i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)+i\sin\left(fx+e\right)\right)\right)+3\left(i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+i\sqrt{2}\,g\cos\left(fx+e\right)^{2}\right)\sqrt{acg}\,\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(fx+e\right)-i\sin\left(fx+e\right)\right)\right)}{5\left(a^{3}c^{2}f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)+a^{3}c^{2}f\cos\left(fx+e\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="fricas")
```

```
[Out] -1/5*(2*(3*g*cos(f*x + e)^2 - 3*g*sin(f*x + e) - 2*g)*sqrt(g*cos(f*x + e))*
sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c) + 3*(-I*sqrt(2)*g*cos(f*
x + e)^2*sin(f*x + e) - I*sqrt(2)*g*cos(f*x + e)^2)*sqrt(a*c*g)*weierstrass
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 3*
(I*sqrt(2)*g*cos(f*x + e)^2*sin(f*x + e) + I*sqrt(2)*g*cos(f*x + e)^2)*sqrt
(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*
sin(f*x + e))))/(a^3*c^2*f*cos(f*x + e)^2*sin(f*x + e) + a^3*c^2*f*cos(f*x
+ e)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(3/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(3/2)), x)
```

3.149 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}-\frac{2(g\cos(e+fx))^{5/2}}{afg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{5/2}}+\frac{\cdots}{5a^2fg\cdots}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2)
-2*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2)
+6/5*(g*cos(f*x+e))^(5/2)/a^2/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(
1/2)+6/5*(g*cos(f*x+e))^(5/2)/a^2/c/f/g/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x
+e))^(1/2)-6/5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e))^(1/2)/a^2/c^2/f
/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.96, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {2931, 2921, 2721, 2719}

$$-\frac{6g\sqrt{\cos(e+fx)}\,E\left(\left.\frac{1}{2}(e+fx)\right|2\right)\sqrt{g\cos(e+fx)}}{5a^2c^2f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{6(g\cos(e+fx))^{5/2}}{5a^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}-\frac{2(g\cos(e+fx))^{5/2}}{afg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{5/2}}-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(5/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(5/2)) - (2*(g*Cos[e + f*x])^(5/2))/(a*f*g*(a + a*Sin[e + f*x])^(3/
2)*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2))/(5*a^2*f*g*Sqrt
[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + (6*(g*Cos[e + f*x])^(5/2
))/(5*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) - (6*g
*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*a^2*
c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} + \frac{\int}{(a}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{5/2}} - \frac{}{afg}$$

Mathematica [A]

time = 0.82, size = 104, normalized size = 0.36

$$\frac{(g\cos(e+fx))^{3/2}\sec^3(e+fx)\left(-12\cos^{\frac{5}{2}}(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)+7\sin(e+fx)+3\sin(3(e+fx))\right)}{10a^2c^2f\sqrt{a(1+\sin(e+fx))}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(5/2)),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]^3*(-12*Cos[e + f*x]^(5/2)*EllipticE[(e
 + f*x)/2, 2] + 7*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(10*a^2*c^2*f*Sqrt[a*
(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 395, normalized size = 1.36

method	result
default	$-\frac{2(1+\cos(fx+e))^2(g\cos(fx+e))^{\frac{3}{2}}(1+\sin(fx+e))(-1+\cos(fx+e))^2(\sin(fx+e)-1)\left(3i(\cos^3(fx+e))\sqrt{\frac{\cos(fx+e)}{1+\cos(fx+e)}}\ \text{EllipticE}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] -2/5/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))
^2*(sin(f*x+e)-1)*(3*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*Ellip
ticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-3*
I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+3*I*EllipticE(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(1+cos(f*x+e)))^(1/2)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e
)))^(1/2)-3*cos(f*x+e)^3+2*cos(f*x+e)^2+1)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(si
n(f*x+e)-1))^(5/2)/sin(f*x+e)^5/cos(f*x+e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(5/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 170, normalized size = 0.58

$$\frac{3i\sqrt{2}\,\sqrt{acg}\,g\cos\left(fx+e\right)^{4}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)))-3i\sqrt{2}\,\sqrt{acg}\,g\cos\left(fx+e\right)^{4}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(fx+e\right)-i\,\sin\left(fx+e\right)))+2\left(3\,g\cos\left(fx+e\right)^{2}+g\right)\sqrt{g\cos\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)+a}\,\sqrt{-c\sin\left(fx+e\right)+c}\,\sin\left(fx+e\right)}{5\,a^{3}c^{3}f\cos\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="fricas")
```

```
[Out] 1/5*(3*I*sqrt(2)*sqrt(a*c*g)*g*cos(f*x + e)^4*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) - 3*I*sqrt(2)*sqrt(a*c
*g)*g*cos(f*x + e)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
f*x + e) - I*sin(f*x + e))) + 2*(3*g*cos(f*x + e)^2 + g)*sqrt(g*cos(f*x + e
))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/(a^3*c^
3*f*cos(f*x + e)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(g\,\cos\left(e+f\,x\right)\right)^{3/2}}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(5/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(5/2)), x)
```

3.150 $\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}-\frac{14(g\cos(e+fx))^{5/2}}{5afg(a+a\sin(e+fx))^{3/2}(c-c\sin(e+fx))^{7/2}}+\frac{\cdots}{9a^2fg\cdots}$$

```
[Out] -2/5*(g*cos(f*x+e))^(5/2)/f/g/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2)
-14/5*(g*cos(f*x+e))^(5/2)/a/f/g/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(7
/2)+14/9*(g*cos(f*x+e))^(5/2)/a^2/f/g/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e
))^(1/2)+14/15*(g*cos(f*x+e))^(5/2)/a^2/c/f/g/(c-c*sin(f*x+e))^(5/2)/(a+a*s
in(f*x+e))^(1/2)+14/15*(g*cos(f*x+e))^(5/2)/a^2/c^2/f/g/(c-c*sin(f*x+e))^(3
/2)/(a+a*sin(f*x+e))^(1/2)-14/15*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x
+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(g*cos(f*x+e
))^(1/2)/a^2/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2931, 2921, 2721, 2719}

$$-\frac{14g\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15a^2c^3f\sqrt{a\sin(e+fx)+a}\,\sqrt{c-c\sin(e+fx)}}+\frac{14(g\cos(e+fx))^{5/2}}{15a^2c^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{3/2}}+\frac{14(g\cos(e+fx))^{5/2}}{15a^2cfg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{5/2}}+\frac{14(g\cos(e+fx))^{5/2}}{9a^2fg\sqrt{a\sin(e+fx)+a}\,(c-c\sin(e+fx))^{7/2}}-\frac{14(g\cos(e+fx))^{5/2}}{5afg(a\sin(e+fx)+a)^{3/2}(c-c\sin(e+fx))^{7/2}}-\frac{2(g\cos(e+fx))^{5/2}}{5fg(a\sin(e+fx)+a)^{5/2}(c-c\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])
^(7/2)),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2))/(5*f*g*(a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e
+ f*x])^(7/2)) - (14*(g*Cos[e + f*x])^(5/2))/(5*a*f*g*(a + a*Sin[e + f*x])^
(3/2)*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])^(5/2))/(9*a^2*f*g*
Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2)) + (14*(g*Cos[e + f*x])
^(5/2))/(15*a^2*c*f*g*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2))
+ (14*(g*Cos[e + f*x])^(5/2))/(15*a^2*c^2*f*g*Sqrt[a + a*Sin[e + f*x]]*(c -
 c*Sin[e + f*x])^(3/2)) - (14*g*Sqrt[Cos[e + f*x]]*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(15*a^2*c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*S
in[e + f*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2921

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[g*
(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])), Int[(g*
Cos[e + f*x])^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && EqQ[
b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 2931

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b
^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&  !LtQ[m, n, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}}\,dx = -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} + \frac{7\int}{}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

$$= -\frac{2(g\cos(e+fx))^{5/2}}{5fg(a+a\sin(e+fx))^{5/2}(c-c\sin(e+fx))^{7/2}} - \frac{}{5af}$$

Mathematica [A]
time = 2.95, size = 171, normalized size = 0.49

$$-\frac{\sqrt{\cos(e+fx)}\,(g\cos(e+fx))^{3/2}\left(42E\left(\frac{1}{2}(e+fx)\middle|2\right)\left(-3\cos(e+fx)-\cos(3(e+fx))+4\cos^3(e+fx)\sin(e+fx)\right)+\sqrt{\cos(e+fx)}\,(-9+28\cos(2(e+fx))+21\cos(4(e+fx))+98\sin(e+fx)+42\sin(3(e+fx)))\right)}{180c^3f(-1+\sin(e+fx))^3(a(1+\sin(e+fx)))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e +
 f*x])^(7/2)),x]
```

```
[Out] -1/180*(Sqrt[Cos[e + f*x]]*(g*Cos[e + f*x])^(3/2)*(42*EllipticE[(e + f*x)/2
, 2]*(-3*Cos[e + f*x] - Cos[3*(e + f*x)] + 4*Cos[e + f*x]^3*Sin[e + f*x]) +
 Sqrt[Cos[e + f*x]]*(-9 + 28*Cos[2*(e + f*x)] + 21*Cos[4*(e + f*x)] + 98*Si
n[e + f*x] + 42*Sin[3*(e + f*x)])))/(c^3*f*(-1 + Sin[e + f*x])^3*(a*(1 + Si
n[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 947, normalized size = 2.71

method	result	size
default	Expression too large to display	947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] 2/45/f*(1+cos(f*x+e))^2*(g*cos(f*x+e))^(3/2)*(1+sin(f*x+e))*(-1+cos(f*x+e))
^2*(sin(f*x+e)-1)*(-21*I*cos(f*x+e)^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*
x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticE
(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2
)+21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1
/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-21*I*cos(f*x+e)^3*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*(1/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+21*I*cos(f*x+e)^5*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)-21*I*cos(f*x+e)^5*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*(1/(1+cos(f*x+e)))^(1/2)+21*I*cos(f*x+e)^2*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(1+
cos(f*x+e)))^(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e
)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)+2
1*I*cos(f*x+e)^4*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(1+cos(f*x+e)))^(1/2)
*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*EllipticE(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*cos(f*x+e)^2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1
/(1+cos(f*x+e)))^(1/2)-21*I*cos(f*x+e)^3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(1+cos(f*x+e)))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*cos(f
*x+e)^3*sin(f*x+e)+21*cos(f*x+e)^3+14*cos(f*x+e)^2*sin(f*x+e)-14*cos(f*x+e)
^2+2*sin(f*x+e)-7)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(7/2)/cos(f
*x+e)/sin(f*x+e)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x +
e) + c)^(7/2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 277, normalized size = 0.79

$$-\frac{2 \left(21 g \cos \left(fx+e\right)^{4}-14 g \cos \left(fx+e\right)^{2}+7 \left(3 g \cos \left(fx+e\right)^{2}+g\right) \sin \left(fx+e\right)-2 g\right) \sqrt{g \cos \left(fx+e\right)}\, \sqrt{a \sin \left(fx+e\right)+a}\, \sqrt{-c \sin \left(fx+e\right)+c}+21 \left(-i \sqrt{2}\, g \cos \left(fx+e\right)^{4} \sin \left(fx+e\right)+i \sqrt{2}\, g \cos \left(fx+e\right)^{4}\right) \sqrt{acg}\, \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(fx+e\right)+i \sin \left(fx+e\right)\right)\right)+21 \left(i \sqrt{2}\, g \cos \left(fx+e\right)^{4} \sin \left(fx+e\right)-i \sqrt{2}\, g \cos \left(fx+e\right)^{4}\right) \sqrt{acg}\, \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(fx+e\right)-i \sin \left(fx+e\right)\right)\right)}{45 \left(a^{3} c^{4} f \cos \left(fx+e\right)^{4} \sin \left(fx+e\right)-a^{3} c^{4} f \cos \left(fx+e\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="fricas")
```

```
[Out] -1/45*(2*(21*g*cos(f*x + e)^4 - 14*g*cos(f*x + e)^2 + 7*(3*g*cos(f*x + e)^2
 + g)*sin(f*x + e) - 2*g)*sqrt(g*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqr
t(-c*sin(f*x + e) + c) + 21*(-I*sqrt(2)*g*cos(f*x + e)^4*sin(f*x + e) + I*s
qrt(2)*g*cos(f*x + e)^4)*sqrt(a*c*g)*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(f*x + e) + I*sin(f*x + e))) + 21*(I*sqrt(2)*g*cos(f*x + e)^
4*sin(f*x + e) - I*sqrt(2)*g*cos(f*x + e)^4)*sqrt(a*c*g)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(f*x + e) - I*sin(f*x + e))))/(a^3*c^4*f
*cos(f*x + e)^4*sin(f*x + e) - a^3*c^4*f*cos(f*x + e)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(
7/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(7/2
),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(7/2)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))
^(7/2)), x)
```

3.151 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$\frac{2^{\frac{9}{4}+n}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(-1-4n);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}-n}(\ldots}{fg(5+4m)}$$

```
[Out] 2^(9/4+n)*c*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -1/4-n],[9/4+m],1/2+1/2*
sin(f*x+e))*(1-sin(f*x+e))^(-1/4-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1
+n)/f/g/(5+4*m)
```

Rubi [A]
time = 0.20, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2932, 2768, 72, 71}

$$\frac{c2^{n+\frac{9}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{-n-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(-4n-1);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(9/4 + n)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (-1 -
4*n)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 - n)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))\right.}{}$$

$$= \frac{\left(2^{\frac{1}{4}+n}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+\right.}{}$$

$$= \frac{2^{\frac{9}{4}+n}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\right.}{}$$

Mathematica [A]
time = 2.64, size = 126, normalized size = 1.06

$$-\frac{8g\sqrt{g\cos(e+fx)}\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\csc^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{\frac{3}{2}+m+n}{}_2F_1\left(\frac{5}{4}+n,\frac{5}{2}+m+n;\frac{9}{4}+n;-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{f(5+4n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^n,x]
```

```
[Out] (-8*g*Sqrt[g*Cos[e + f*x]]*Cos[(2*e + Pi + 2*f*x)/4]^2*(Csc[(2*e + Pi + 2*f
*x)/4]^2)^(3/2 + m + n)*Hypergeometric2F1[5/4 + n, 5/2 + m + n, 9/4 + n, -T
an[(2*e - Pi + 2*f*x)/4]^2]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n
)/(f*(5 + 4*n))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a+a\sin(fx+e))^m (c-c\sin(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
n*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x
)
```

3.152 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{9}{4}+m}a^4c^3(g\cos(e+fx))^{17/2}\,{}_2F_1\left(\frac{17}{4},-\frac{1}{4}-m;\frac{21}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+}{17fg^7}$$

[Out] -1/17*2^(9/4+m)*a^4*c^3*(g*cos(f*x+e))^(17/2)*hypergeom([17/4, -1/4-m],[21/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-4+m)/f/g^7

Rubi [A]
time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^4c^3 2^{m+\frac{9}{4}}(g\cos(e+fx))^{17/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-4}\,{}_2F_1\left(\frac{17}{4},-m-\frac{1}{4};\frac{21}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{17fg^7}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3,x]

[Out] -1/17*(2^(9/4 + m)*a^4*c^3*(g*Cos[e + f*x])^(17/2)*Hypergeometric2F1[17/4, -1/4 - m, 21/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(-4 + m))/(f*g^7)

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^3\,dx = \frac{(a^3c^3)\int (g\cos(e+fx))^{15/2}(a+a\sin(e+}{g^6}$$

$$= \frac{(a^5c^3(g\cos(e+fx))^{17/2})\,\text{Subst}\Big(\int (a-a}{fg^7(a-a\sin(e+fx))^{17/}}$$

$$= \frac{\Big(2^{\frac{1}{4}+m}a^5c^3(g\cos(e+fx))^{17/2}(a+a\sin(}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^4c^3(g\cos(e+fx))^{17/2}\,{}_2F_1\big(\frac{17}{4},-}{}$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^3,x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) +
a)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-(3*c^3*g*cos(f*x + e)^3 - 4*c^3*g*cos(f*x + e) - (c^3*g*cos(f*x +
 e)^3 - 4*c^3*g*cos(f*x + e))*sin(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x
 + e) + a)^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) +
a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g \cos(e+fx))^{3/2} (a+a \sin(e+fx))^m (c-c \sin(e+fx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x
)
```

3.153 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx$

Optimal. Leaf size=93

$$-\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{13/2}\,{}_2F_1\left(\frac{13}{4},-\frac{1}{4}-m;\frac{17}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+}{13fg^5}$$

```
[Out] -1/13*2^(9/4+m)*a^3*c^2*(g*cos(f*x+e))^(13/2)*hypergeom([13/4, -1/4-m],[17/
4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-3+m)/f/g^
5
```

Rubi [A]
time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^3c^2 2^{m+\frac{9}{4}}(g\cos(e+fx))^{13/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{13}{4},-m-\frac{1}{4};\frac{17}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{13fg^5}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] -1/13*(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(13/2)*Hypergeometric2F1[13/4,
-1/4 - m, 17/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*
Sin[e + f*x])^(-3 + m))/(f*g^5)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^2\,dx = \frac{(a^2c^2)\int (g\cos(e+fx))^{11/2}(a+a\sin(e+}{g^4}$$

$$= \frac{(a^4c^2(g\cos(e+fx))^{13/2})\,\text{Subst}\Big(\int (a-a}{fg^5(a-a\sin(e+fx))^{13/4}}$$

$$= \frac{\Big(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{13/2}(a+a\sin(}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{13/2}\,{}_2F_1\big(\frac{13}{4},-}{}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^2,x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a
)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a
)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g \cos(e+fx))^{3/2} (a+a \sin(e+fx))^m (c-c \sin(e+fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x
)
```

3.154 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$

Optimal. Leaf size=91

$$-\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{9/2}\,{}_2F_1\left(\frac{9}{4},-\frac{1}{4}-m;\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx$$
$$9fg^3$$

```
[Out] -1/9*2^(9/4+m)*a^2*c*(g*cos(f*x+e))^(9/2)*hypergeom([9/4, -1/4-m],[13/4],1/
2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-2+m)/f/g^3
```

Rubi [A]
time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2919, 2768, 72, 71}

$$-\frac{a^2c2^{m+\frac{9}{4}}(g\cos(e+fx))^{9/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{9}{4},-m-\frac{1}{4};\frac{13}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{9fg^3}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]),x]
```

```
[Out] -1/9*(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(9/2)*Hypergeometric2F1[9/4, -1/4
- m, 13/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e
 + f*x])^(-2 + m))/(f*g^3)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx = \frac{(ac)\int (g\cos(e+fx))^{7/2}(a+a\sin(e+fx}{g^2}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{9/2}\right)\text{Subst}\left(\int (a-ax)\right.}{fg^3(a-a\sin(e+fx))^{9/4}(}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{9/2}(a+a\sin(e+\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{9/2}\,{}_2F_1\left(\frac{9}{4},-\frac{1}{4}-\right.}{}$$

Mathematica [F]
time = 122.44, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
), x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}\,(a+a\sin(fx+e))^m\,(c-c\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)
^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)
^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m\,(c-c\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)
```

3.155 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^{-}}{5fg}$$

```
[Out] -1/5*2^(9/4+m)*a*(g*cos(f*x+e))^(5/2)*hypergeom([5/4, -1/4-m],[9/4],1/2-1/2
*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/g
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2768, 72, 71}

$$-\frac{a2^{m+\frac{9}{4}}(g\cos(e+fx))^{5/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{5}{4},-m-\frac{1}{4};\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -1/5*(2^(9/4 + m)*a*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m,
 9/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*
x])^(-1 + m))/(f*g)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
```

```
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\,dx = \frac{\left(a^2(g\cos(e+fx))^{5/2}\right)\text{Subst}\left(\int \sqrt[4]{a-ax}\,(a+ax)^{\frac{1}{4}+m}\,dx, x,\right.}{fg(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{5/4}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx))^{-1+m}\left(\frac{a+a\sin(\right.\right.}{a}}{fg(a-a\,s}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(e+\right.}{5fg}$$

Mathematica [A]
time = 0.11, size = 85, normalized size = 0.97

$$-\frac{2^{\frac{9}{4}+m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{5}{4},-\frac{1}{4}-m;\frac{9}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{5}{4}-m}(a(1+\sin(e+fx)))^m}{5fg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m,x]
[Out] -1/5*(2^(9/4 + m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[5/4, -1/4 - m, 9
/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-5/4 - m)*(a*(1 + Sin[e + f*x
]))^m)/(f*g)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m,x)
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m, x)
```

3.156 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(\frac{1}{4},-\frac{1}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^m}{cf}$$

```
[Out] -2^(9/4+m)*g*hypergeom([1/4, -1/4-m],[5/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e
))^(-1/4-m)*(a+a*sin(f*x+e))^m*(g*cos(f*x+e))^(1/2)/c/f
```

Rubi [A]
time = 0.18, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2919, 2768, 72, 71}

$$-\frac{g2^{m+\frac{9}{4}}\sqrt{g\cos(e+fx)}\,(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(\frac{1}{4},-m-\frac{1}{4};\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x]),x]
```

```
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m
)/(c*f))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
```

```
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\begin{aligned}\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{c-c\sin(e+fx)}\,dx &= \frac{g^2\int \frac{(a+a\sin(e+fx))^{1+m}}{\sqrt{g\cos(e+fx)}}\,dx}{ac}\\ &= \frac{\left(ag\sqrt{g\cos(e+fx)}\right)\text{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{3/4}}\,dx,x,\sin(e+fx)\right)}{cf\sqrt[4]{a-a\sin(e+fx)}\sqrt[4]{a+a\sin(e+fx)}}\\ &= \frac{\left(2^{\frac{1}{4}+m}ag\sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^m\left(\frac{a+a\sin(e+fx)}{a}\right.\right.}{cf\sqrt[4]{a-a\sin(e+}}\\ &= -\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(\frac{1}{4},-\frac{1}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(e+fx}{cf}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 84, normalized size = 1.00

$$-\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(\frac{1}{4},-\frac{1}{4}-m;\frac{5}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a(1+\sin(e+fx)))^m}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x]),x]
[Out] -((2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[1/4, -1/4 - m, 5/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(c*f))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m}{c-c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c*sin
(f*x + e) - c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{c-c\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x
)
```

3.157 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{2^{\frac{9}{4}+m}g^3\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^{1+m}}{3ac^2f(g\cos(e+fx))^{3/2}}$$

```
[Out] 1/3*2^(9/4+m)*g^3*hypergeom([-3/4, -1/4-m],[1/4],1/2-1/2*sin(f*x+e))*(1+sin
(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/(g*cos(f*x+e))^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2919, 2768, 72, 71}

$$\frac{g^3 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{4},-m-\frac{1}{4};\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^2,
x]
```

```
[Out] (2^(9/4 + m)*g^3*Hypergeometric2F1[-3/4, -1/4 - m, 1/4, (1 - Sin[e + f*x])/
2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3*a*c^2*f*(
g*Cos[e + f*x])^(3/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^2}\,dx = \frac{g^4\int \frac{(a+a\sin(e+fx))^{2+m}}{(g\cos(e+fx))^{5/2}}\,dx}{a^2c^2}$$

$$= \frac{\left(g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{3/4}\right)\operatorname{Subst}\left(\int \frac{(a+}{(a}\right.}{c^2f(g\cos(e+fx))^{3/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^3(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{1+m}\left(\frac{a+a\sin}{}\right.\right.}{c^2f(g\cos(e+}$$

$$= \frac{2^{\frac{9}{4}+m}g^3\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+}{3ac^2f(g\cos(e+fx))^{3/2}}$$

Mathematica [A]
time = 0.15, size = 96, normalized size = 1.03

$$-\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{4}-m;\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a(1+\sin(e+fx)))^m}{3c^2f(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^2,x]
```

```
[Out] -1/3*(2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-3/4, -1/4 - m,
1/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*
x]))^m)/(c^2*f*(-1 + Sin[e + f*x]))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(c^2*c
os(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c
)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,
x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,
 x)
```

3.158 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{2^{\frac{9}{4}+m}g^5\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4}-m;-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^{2+m}}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

```
[Out] 1/7*2^(9/4+m)*g^5*hypergeom([-7/4, -1/4-m],[-3/4],1/2-1/2*sin(f*x+e))*(1+si
n(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f/(g*cos(f*x+e))^(7/2)
```

Rubi [A]

time = 0.19, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2919, 2768, 72, 71}

$$\frac{g^5 2^{m+\frac{9}{4}}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{7}{4},-m-\frac{1}{4};-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^3,
x]
```

```
[Out] (2^(9/4 + m)*g^5*Hypergeometric2F1[-7/4, -1/4 - m, -3/4, (1 - Sin[e + f*x])
/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(2 + m))/(7*a^2*c^3*
f*(g*Cos[e + f*x])^(7/2))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2919

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[
a^m*(c^m/g^(2*m)), Int[(g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n -
 m), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] &
& EqQ[a^2 - b^2, 0] && IntegerQ[m] &&  !(IntegerQ[n] && LtQ[n^2, m^2])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx = \frac{g^6\int \frac{(a+a\sin(e+fx))^{3+m}}{(g\cos(e+fx))^{9/2}}\,dx}{a^3c^3}$$

$$= \frac{\left(g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{7/4}\right)\text{Subst}\left(\int \frac{(a+}{(a-}\right.}{ac^3f(g\cos(e+fx))^{7/2}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^5(a-a\sin(e+fx))^{7/4}(a+a\sin(e+fx))^{2+m}\left(\frac{a+a\,\text{si}}{}\right.\right.}{ac^3f(g\cos(e+}$$

$$= \frac{2^{\frac{9}{4}+m}g^5\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4}-m;-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e}{7a^2c^3f(g\cos(e+fx))^{7/2}}$$

Mathematica [**A**]
time = 0.15, size = 96, normalized size = 1.03

$$\frac{2^{\frac{9}{4}+m}g\sqrt{g\cos(e+fx)}\,{}_2F_1\left(-\frac{7}{4},-\frac{1}{4}-m;-\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a(1+\sin(e+fx)))^m}{7c^3f(-1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^3,x]
```

```
[Out] (2^(9/4 + m)*g*Sqrt[g*Cos[e + f*x]]*Hypergeometric2F1[-7/4, -1/4 - m, -3/4,
 (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a*(1 + Sin[e + f*x]))
^m)/(7*c^3*f*(-1 + Sin[e + f*x])^2)
```

Maple [**F**]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}\,(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] -integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(3*c^3
*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] integrate(-(g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) -
c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,
 x)
```

3.159 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=114

$$-\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{15/2}\,{}_2F_1\left(\frac{15}{4},-\frac{1}{4}-m;\frac{19}{4};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a$$
$$15fg^6$$

```
[Out] -1/15*2^(9/4+m)*a^3*c^2*(g*cos(f*x+e))^(15/2)*hypergeom([15/4, -1/4-m],[19/
4],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^
(-3+m)*(c-c*sin(f*x+e))^(1/2)/f/g^6
```

Rubi [A]
time = 0.23, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{a^3c^22^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{15/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{15}{4},-m-\frac{1}{4};\frac{19}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{15fg^6}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(5/2
),x]
```

```
[Out] -1/15*(2^(9/4 + m)*a^3*c^2*(g*Cos[e + f*x])^(15/2)*Hypergeometric2F1[15/4,
-1/4 - m, 19/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4
 - m)*(a + a*Sin[e + f*x])^(-3 + m)*Sqrt[c - c*Sin[e + f*x]])/(f*g^6)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] || !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{5/2}\,dx = \frac{\left(a^2c^2\sec(e+fx)\sqrt{a+a\sin(e+fx)}\ \sqrt{}\right.}{}$$

$$= \frac{\left(a^4c^2(g\cos(e+fx))^{15/2}\sec(e+fx)\sqrt{c}\right.}{fg^6(a-}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^4c^2(g\cos(e+fx))^{15/2}\sec(e+f\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a^3c^2(g\cos(e+fx))^{15/2}\,{}_2F_1\left(\frac{15}{4},-\right.}{}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(5/2),x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e
) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
[Out] integral(-(c^2*g*cos(f*x + e)^3 + 2*c^2*g*cos(f*x + e)*sin(f*x + e) - 2*c^2
*g*cos(f*x + e))*sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x
+ e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(5/2)
,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e
) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m\,(c-c\sin(e+fx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2
), x)
```

3.160 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{11/2}\,{}_2F_1\left(\frac{11}{4},-\frac{1}{4}-m;\frac{15}{4};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a}{11fg^4}$$

```
[Out] -1/11*2^(9/4+m)*a^2*c*(g*cos(f*x+e))^(11/2)*hypergeom([11/4, -1/4-m],[15/4]
,1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-
2+m)*(c-c*sin(f*x+e))^(1/2)/f/g^4
```

Rubi [A]
time = 0.24, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{a^2c2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{11/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{11}{4},-m-\frac{1}{4};\frac{15}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{11fg^4}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3/2
),x]
```

```
[Out] -1/11*(2^(9/4 + m)*a^2*c*(g*Cos[e + f*x])^(11/2)*Hypergeometric2F1[11/4, -1
/4 - m, 15/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 -
 m)*(a + a*Sin[e + f*x])^(-2 + m)*Sqrt[c - c*Sin[e + f*x]])/(f*g^4)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3/2}\,dx = \frac{\left(ac\sec(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c\cdots}\right.}{\cdots}$$

$$= \frac{\left(a^3c(g\cos(e+fx))^{11/2}\sec(e+fx)\sqrt{c-\cdots}\right.}{fg^4(a-\cdots}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^3c(g\cos(e+fx))^{11/2}\sec(e+fx\cdots\right.}{\cdots}$$

$$= -\frac{2^{\frac{9}{4}+m}a^2c(g\cos(e+fx))^{11/2}\,{}_2F_1\left(\frac{11}{4},-\frac{1}{4}\cdots\right.}{\cdots}$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(3/2),x]
[Out] $Aborted
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (g\cos (fx+e))^{\frac{3}{2}} (a+a\sin (fx+e))^{m} (c-c\sin (fx+e))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
```

```
[Out] integral(-(c*g*cos(f*x + e)*sin(f*x + e) - c*g*cos(f*x + e))*sqrt(g*cos(f*x
 + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e
) + a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2} (a+a\sin(e+fx))^m (c-c\sin(e+fx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2
), x)
```

3.161 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+$

Optimal. Leaf size=109

$$-\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{1}{4}-m;\frac{11}{4};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a}{7fg^2}$$

```
[Out] -1/7*2^(9/4+m)*a*(g*cos(f*x+e))^(7/2)*hypergeom([7/4, -1/4-m],[11/4],1/2-1/
2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)*(c
-c*sin(f*x+e))^(1/2)/f/g^2
```

Rubi [A]
time = 0.21, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{a2^{m+\frac{9}{4}}\sec(e+fx)\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{7/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{7}{4},-m-\frac{1}{4};\frac{11}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{7fg^2}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e + f*x]],
x]
```

```
[Out] -1/7*(2^(9/4 + m)*a*(g*Cos[e + f*x])^(7/2)*Hypergeometric2F1[7/4, -1/4 - m,
 11/4, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Sin[e + f*x])^(-1/4 - m)*(a
+ a*Sin[e + f*x])^(-1 + m)*Sqrt[c - c*Sin[e + f*x]])/(f*g^2)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m\sqrt{c-c\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\right.}{}$$

$$= \frac{\left(a^2(g\cos(e+fx))^{7/2}\sec(e+fx)\sqrt{c-c\right.}{fg^2(a-a\mathrm{s}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2(g\cos(e+fx))^{7/2}\sec(e+fx)(a\right.}{}$$

$$= -\frac{2^{\frac{9}{4}+m}a(g\cos(e+fx))^{7/2}\,{}_2F_1\left(\frac{7}{4},-\frac{1}{4}-m\right.}{}$$

Mathematica [**F**]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*Sqrt[c - c*Sin[e +
f*x]],x]
[Out] $Aborted
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m\sqrt{c-c\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a
)^m*g*cos(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e)
+ a)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m\,\sqrt{c-c\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2
),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2
), x)
```

3.162 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2^{\frac{9}{4}+m} a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*a*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/3*(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3
/4, -1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a +
 a*Sin[e + f*x])^(-1 + m))/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int \sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x, \right.}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\sin}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-1}\right.}{f(a-a\sin(e+fx)}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1\right.}{3f\sqrt{c-}}$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2), x)
```

3.163 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{2^{\frac{9}{4}+m}g^2\cos(e+fx)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^m}{cf\sqrt{g\cos(e+fx)}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 2^(9/4+m)*g^2*cos(f*x+e)*hypergeom([-1/4, -1/4-m],[3/4],1/2-1/2*sin(f*x+e))
*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^m/c/f/(g*cos(f*x+e))^(1/2)/(c-c*s
in(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$\frac{g^2 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{4},-m-\frac{1}{4};\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{cf\sqrt{c-c\sin(e+fx)}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(3
/2),x]
```

```
[Out] (2^(9/4 + m)*g^2*Cos[e + f*x]*Hypergeometric2F1[-1/4, -1/4 - m, 3/4, (1 - S
in[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^m)/(c*f*
Sqrt[g*Cos[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] || !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{3/2}}\,dx = \frac{\left(g^3\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{3}{2}+m}}{(g\cos(e+fx))^{3/2}}\,dx}{ac\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}\right)\operatorname{Subst}\left(\int \frac{(a+ax)^{\frac{1}{4}+m}}{(a-ax)^{5/4}}\,d\ldots\right.}{cf\sqrt{g\cos(e+fx)}\,\sqrt[4]{a+a\sin(e+fx)}\,\sqrt{c-c\sin(\ldots}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}ag^2\cos(e+fx)\sqrt[4]{a-a\sin(e+fx)}\,(a+a\sin(e+fx\ldots\right.}{cf\sqrt{g\cos(e+fx\ldots}}$$

$$= \frac{2^{\frac{9}{4}+m}g^2\cos(e+fx)\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4}-m;\frac{3}{4};\frac{1}{2}(1-\sin(e+fx))\right)\ldots}{cf\sqrt{g\cos(e+fx)}\,\sqrt{c-c\,\mathrm{s}\ldots}}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(3/2),x]

[Out] $Aborted
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m}{(c-c\sin(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="maxima")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="fricas")
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(3/2)
,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(3/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3
/2), x)
```

3.164 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2^{\frac{9}{4}+m}g^4\cos(e+fx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4}-m;-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^{1+m}}{5ac^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin(e+fx)}}$$

```
[Out] 1/5*2^(9/4+m)*g^4*cos(f*x+e)*hypergeom([-5/4, -1/4-m],[-1/4],1/2-1/2*sin(f*
x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/(g*cos(f*x+e))
^(5/2)/(c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$\frac{g^4 2^{m+\frac{9}{4}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{5}{4},-m-\frac{1}{4};-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{5ac^2f\sqrt{c-c\sin(e+fx)}\,(g\cos(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^(5
/2),x]
```

```
[Out] (2^(9/4 + m)*g^4*Cos[e + f*x]*Hypergeometric2F1[-5/4, -1/4 - m, -1/4, (1 -
Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a + a*Sin[e + f*x])^(1 + m)
)/(5*a*c^2*f*(g*Cos[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx = \frac{\left(g^5\cos(e+fx)\right)\int \frac{(a+a\sin(e+fx))^{\frac{5}{2}+m}}{(g\cos(e+fx))^{7/2}}\,dx}{a^2c^2\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx))^{3/4}\right)}{c^2f(g\cos(e+fx))^{5/2}\sqrt{c-c\sin}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}g^4\cos(e+fx)(a-a\sin(e+fx))^{5/4}(a+a\sin(e+fx}{c^2f(g\cos(e+fx)}$$

$$= \frac{2^{\frac{9}{4}+m}g^4\cos(e+fx)\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4}-m;-\frac{1}{4};\frac{1}{2}(1-\sin(e+fx)\right.}{5ac^2f(g\cos(e+fx))^{5/2}\sqrt{c-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 162.40, size = 2320, normalized size = 20.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^(5/2),x]
```

```
[Out] ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cos[(-e + Pi/2 - f*x)/2]*(g*Cos[e + f*x
])^(3/2)*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2,
 -Tan[(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)
/4]^4)*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(a + a*Sin[e + f*x])^m*Tan[(-e + Pi/2 - f*x)/4])/(60*Sqrt[2]*f*S
qrt[Cos[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2
] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^4*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2
]*(-1/240*((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF
1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/
4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + P
i/2 - f*x)/4]^2)^(2 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]^2)/(2 - 2*Tan[(-e + Pi/
2 - f*x)/4]^2)^(3/2) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x
]]*(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[
(-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e +
 Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)
*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2 + 2*m))/(480*Sqrt[2 - 2*Tan[(-e + Pi/2 - f
*x)/4]^2]) + (m*(Cos[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sqrt[Cos[e + f*x]]*
(5*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
 + Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(S
ec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]^2)/(120*Sqrt[
2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(5
*AppellF1[3/4, -1/2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2] + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec
[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Sin[e + f*x]*Tan[(-e + Pi/2 - f*x)/4])/(
240*Sqrt[Cos[e + f*x]]*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2]) - ((1 + 2*m)
*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(5*AppellF1[3/4, -1/
2 - 2*m, 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]
 + 3*AppellF1[-5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^4)*(Sec[(-e + Pi/2 - f*x)
/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]^2)/(240*Sqrt[2 - 2*Tan[(-e + Pi/2
 - f*x)/4]^2]) - ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Sqrt[Cos[e + f*x]]*(Se
c[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2 - f*x)/4]*(-3*AppellF1[-
5/4, -1/2 - 2*m, 2*m, -1/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^3*Csc[(-e + Pi/2 - f*x)/4]^2 + 3*Cot[(-e
 + Pi/2 - f*x)/4]^4*(-5*m*AppellF1[-1/4, -1/2 - 2*m, 1 + 2*m, 3/4, Tan[(-e
+ Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2
*Tan[(-e + Pi/2 - f*x)/4] + (5*(-1/2 - 2*m)*AppellF1[-1/4, 1/2 - 2*m, 2*m,
3/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/
2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/2) + 5*((-3*m*AppellF1[7/4, -1/2 -
2*m, 1 + 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/7 + (3*(-1/2 - 2*m)*
```

```
AppellF1[7/4, 1/2 - 2*m, 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/14))
)/(120*Sqrt[2 - 2*Tan[(-e + Pi/2 - f*x)/4]^2])))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^{m}}{(c-c\sin(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4
*c^3)*sin(f*x + e)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(5/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(5/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5
/2), x)
```

3.165 $\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(a+a\sin(e+fx))^{-1+m}}{3f\sqrt{c-c\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*a*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(a+a*sin(f*x+e))^(-1+m)/f/(
c-c*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{a2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e + f*x]
],x]
```

```
[Out] -1/3*(2^(9/4 + m)*a*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3
/4, -1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(a +
 a*Sin[e + f*x])^(-1 + m))/(f*Sqrt[c - c*Sin[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m}{\sqrt{c-c\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,(a+a\sin(e+fx))^{\frac{1}{2}+m}\,d}{\sqrt{a+a\sin(e+fx)}\,\sqrt{c-c\sin(e+fx)}}$$

$$= \frac{\left(a^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\text{Subst}\left(\int\frac{(a+ax)^{\frac{1}{4}+m}}{\sqrt[4]{a-ax}}\,dx, x\right.}{f(a-a\sin(e+fx))^{3/4}(a+a\sin(e+fx))^{5/4}\sqrt{c-c\,\text{si}}}$$

$$= \frac{\left(2^{\frac{1}{4}+m}a^2\cos(e+fx)(g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^{-}\right.}{f(a-a\sin(e+fx}$$

$$= -\frac{2^{\frac{9}{4}+m}a\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}\right.}{3f\sqrt{c-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 20.71, size = 754, normalized size = 7.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/Sqrt[c - c*Sin[e
+ f*x]],x]
```

```
[Out] (352*Sqrt[2]*Cos[(-e + Pi/2 - f*x)/4]^6*(g*Cos[e + f*x])^(3/2)*Cot[(-e + Pi
/2 - f*x)/4]^2*Csc[(-e + Pi/2 - f*x)/2]*Sec[e + f*x]*(Cos[(e + f*x)/2] - Si
```

```
n[(e + f*x)/2])*(a + a*Sin[e + f*x])^m*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)*(-7
*AppellF1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
-e + Pi/2 - f*x)/4]^2] + 3*AppellF1[7/4, -1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e
 + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^
2))/(3*f*(56*(3 + 2*m)*AppellF1[11/4, -1/2 - 2*m, 4 + 2*m, 15/4, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 28*(1 + 4*m)*AppellF1[11/4
, 1/2 - 2*m, 3 + 2*m, 15/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f
*x)/4]^2] - 22*Cot[(-e + Pi/2 - f*x)/4]^2*(4*(3 + 2*m)*AppellF1[7/4, -1/2 -
 2*m, 4 + 2*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2] + 2*(1 + 4*m)*AppellF1[7/4, 1/2 - 2*m, 3 + 2*m, 11/4, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 7*(AppellF1[7/4, -1/2 - 2*m, 3 + 2
*m, 11/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - Appell
F1[3/4, -1/2 - 2*m, 3 + 2*m, 7/4, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2)))*Sqrt[c - c*Sin[e + f*x]])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(a+a\sin(fx+e))^m}{\sqrt{c-c\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e
) + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m*g*cos(f*x + e)/(c*sin(f*x + e) - c), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/(c-c*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/(c-c*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(a+a\,\sin(e+f\,x))^m}{\sqrt{c-c\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1
/2), x)
```

3.166 $\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2^{\frac{9}{4}+m}c\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{4}-m}(c+c\sin}{3f\sqrt{a-a\sin(e+fx)}}$$

```
[Out] -1/3*2^(9/4+m)*c*cos(f*x+e)*(g*cos(f*x+e))^(3/2)*hypergeom([3/4, -1/4-m],[7
/4],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/4-m)*(c+c*sin(f*x+e))^(-1+m)/f/(
a-a*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$-\frac{c2^{m+\frac{9}{4}}\cos(e+fx)(g\cos(e+fx))^{3/2}(\sin(e+fx)+1)^{-m-\frac{1}{4}}(c\sin(e+fx)+c)^{m-1}\,{}_2F_1\left(\frac{3}{4},-m-\frac{1}{4};\frac{7}{4};\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt{a-a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]
],x]
```

```
[Out] -1/3*(2^(9/4 + m)*c*Cos[e + f*x]*(g*Cos[e + f*x])^(3/2)*Hypergeometric2F1[3
/4, -1/4 - m, 7/4, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/4 - m)*(c +
 c*Sin[e + f*x])^(-1 + m))/(f*Sqrt[a - a*Sin[e + f*x]])
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^m}{\sqrt{a-a\sin(e+fx)}}\,dx = \frac{(g\cos(e+fx))\int\sqrt{g\cos(e+fx)}\,(c+c\sin(e+fx))^{\frac{1}{2}+m}\,dx}{\sqrt{a-a\sin(e+fx)}\,\sqrt{c+c\sin(e+fx)}}$$

$$= \frac{\left(c^2\cos(e+fx)(g\cos(e+fx))^{3/2}\right)\operatorname{Subst}\left(\int\frac{(c+cx)^{\frac{1}{4}+m}}{\sqrt[4]{c-cx}}\,dx, x,\right.}{f\sqrt{a-a\sin(e+fx)}\,(c-c\sin(e+fx))^{3/4}(c+c\sin(e}$$

$$= \frac{\left(2^{\frac{1}{4}+m}c^2\cos(e+fx)(g\cos(e+fx))^{3/2}(c+c\sin(e+fx))^{-1}\right.}{f\sqrt{a-a\sin(e+f}}$$

$$= -\frac{2^{\frac{9}{4}+m}c\cos(e+fx)(g\cos(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},-\frac{1}{4}-m;\frac{7}{4};\frac{1}{2}(\right.}{3f\sqrt{a-}}$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e
+ f*x]],x]
```

```
[Out] $Aborted
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^{\frac{3}{2}}(c+c\sin(fx+e))^{m}}{\sqrt{a-a\sin(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e
) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) +
c)^m*g*cos(f*x + e)/(a*sin(f*x + e) - a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2)
,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x,
 algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,(c+c\,\sin(e+f\,x))^m}{\sqrt{a-a\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1
/2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1
/2), x)
```

3.167 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{-\frac{3}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(11+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+m}}{c^2fg(5+4m)}$$

```
[Out] 2^(-3/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 11/4+m],[9/4+m],1/2+1/2*s
in(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-
m)/c^2/f/g/(5+4*m)
```

Rubi [A]

time = 0.26, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2932, 2768, 72, 71}

$$\frac{2^{-m-\frac{3}{4}}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+11);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{c^2fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
 - m),x]
```

```
[Out] (2^(-3/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (11 + 4
*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c^2*f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+f$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{11}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin\right.}{}$$

$$= \frac{2^{-\frac{3}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 382 vs. 2(123) = 246.
time = 22.31, size = 382, normalized size = 3.11

$$\frac{2^{-4-m}(g\cos(e+fx))^{3/2}\left((-3+8m+16m^2)\cot^4\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right){}_2F_1\left(-\frac{3}{2}-2m,-\frac{7}{4}-m;-\frac{3}{4}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+(7+4m)\left(2(-1+4m)\cot^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right){}_2F_1\left(-\frac{3}{2}-2m,-\frac{3}{4}-m;\frac{1}{4}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)+(3+4m)\,{}_2F_1\left(-\frac{3}{2}-2m,\frac{1}{4}-m;\frac{5}{4}-m;\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)\right)\right)\sec^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\sec(e+fx)\sin^{-2m}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-3-m)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3-m}\left(1-\tan^2\left(\frac{1}{4}\left(-e+\frac{\pi}{2}-fx\right)\right)\right)^{-\frac{1}{2}-2m}}{f(-1+4m)(3+4m)(7+4m)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-3 - m),x]
```

```
[Out] (2^(-4 - m)*(g*Cos[e + f*x])^(3/2)*((-3 + 8*m + 16*m^2)*Cot[(-e + Pi/2 - f*
x)/4]^4*Hypergeometric2F1[-3/2 - 2*m, -7/4 - m, -3/4 - m, Tan[(-e + Pi/2 -
f*x)/4]^2] + (7 + 4*m)*(2*(-1 + 4*m)*Cot[(-e + Pi/2 - f*x)/4]^2*Hypergeomet
```

```
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (3 + 4*
m)*Hypergeometric2F1[-3/2 - 2*m, 1/4 - m, 5/4 - m, Tan[(-e + Pi/2 - f*x)/4]
^2]))*Sec[(-e + Pi/2 - f*x)/4]^2*Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c
*Sin[e + f*x])^(-3 - m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1/2 - 2*m))/(f*(
-1 + 4*m)*(3 + 4*m)*(7 + 4*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)
/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{-3-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 3)*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-3-m
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,(a+a\,\sin\left(e+f\,x\right))^m}{(c-c\,\sin\left(e+f\,x\right))^{m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 3),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 3), x)
```

3.168 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4m),\frac{1}{4}(7+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+m}(a}{cfg(5+4m)}$$

```
[Out] 2^(1/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 7/4+m],[9/4+m],1/2+1/2*sin
(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)
/c/f/g/(5+4*m)
```

Rubi [A]
time = 0.26, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2932, 2768, 72, 71}

$$\frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+5),\frac{1}{4}(4m+7);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{cfg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
 - m),x]
```

```
[Out] (2^(1/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(5 + 4*m)/4, (7 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2-m}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+f$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{7}{4}-m}(g\cos(e+fx))^{5/2}(a+a\sin\right.}{}$$

$$= \frac{2^{\frac{1}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(5+4\right.}{}$$

Mathematica [A]
time = 8.78, size = 202, normalized size = 1.64

$$\frac{2^{-1-m}g\sqrt{g\cos(e+fx)}\cos^{-2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\csc^2\left(\frac{1}{8}(-2e+\pi-2fx)\right)\,{}_2F_1\left(-\frac{3}{2}-2m,-\frac{3}{4}-m;\frac{1}{4}-m;\tan^2\left(\frac{1}{8}(-2e+\pi-2fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2(2+m)}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^{-m}\left(1-\tan^2\left(\frac{1}{8}(2e-\pi+2fx)\right)\right)^{-\frac{1}{2}-2m}}{c^2f(3+4m)(-1+\sin(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-2 - m),x]

[Out] (2^(-1 - m)*g*Sqrt[g*Cos[e + f*x]]*Csc[(-2*e + Pi - 2*f*x)/8]^2*Hypergeomet
ric2F1[-3/2 - 2*m, -3/4 - m, 1/4 - m, Tan[(-2*e + Pi - 2*f*x)/8]^2]*(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2])^(2*(2 + m))*(a*(1 + Sin[e + f*x]))^m*(1 - Ta
```

```
n[(2*e - Pi + 2*f*x)/8]^2)^(-1/2 - 2*m))/(c^2*f*(3 + 4*m)*Cos[(2*e + Pi + 2
*f*x)/4]^(2*m)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^m)
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{-2-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 2)*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-2-m
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 2),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 2), x)
```

3.169 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$

Optimal. Leaf size=120

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(3+4m),\frac{1}{4}(5+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+m}(a}{fg(5+4m)}$$

```
[Out] 2^(5/4-m)*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, 3/4+m],[9/4+m],1/2+1/2*sin
(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)
/f/g/(5+4*m)
```

Rubi [A]
time = 0.25, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2932, 2768, 72, 71}

$$\frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m+3),\frac{1}{4}(4m+5);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
- m),x]
```

```
[Out] (2^(5/4 - m)*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(3 + 4*m)/4, (5 + 4*m
)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a +
a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+f$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+\right.}{}$$

$$= \frac{\left(2^{-\frac{3}{4}-m}c(g\cos(e+fx))^{5/2}(a+a\sin\right.}{}$$

$$= \frac{2^{\frac{5}{4}-m}(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(3+4\right.}{}$$

Mathematica [F]
time = 112.28, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(-1 - m), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (g\cos (fx+e))^{\frac{3}{2}} (a+a\sin (fx+e))^m (c-c\sin (fx+e))^{-1-m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m - 1)*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1-m
),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m),x
, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^{3/2}\,(a+a\sin(e+fx))^m}{(c-c\sin(e+fx))^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 1),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m
 + 1), x)
```

3.170 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$

Optimal. Leaf size=121

$$\frac{2^{\frac{9}{4}-m}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-1+4m),\frac{1}{4}(5+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+m}}{fg(5+4m)}$$

```
[Out] 2^(9/4-m)*c*(g*cos(f*x+e))^(5/2)*hypergeom([-1/4+m, 5/4+m],[9/4+m],1/2+1/2*
sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1
-m)/f/g/(5+4*m)
```

Rubi [A]
time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 2768, 72, 71}

$$\frac{c2^{\frac{9}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-1),\frac{1}{4}(4m+5);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*x])^m,
x]
```

```
[Out] (2^(9/4 - m)*c*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-1 + 4*m)/4, (5 +
4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a
 + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+fx\right.}{}$$

$$= \frac{\left(2^{\frac{1}{4}-m}c^2(g\cos(e+fx))^{5/2}(a+a\sin(e\right.}{}$$

$$= \frac{2^{\frac{9}{4}-m}c(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\tfrac{1}{4}(-1+4\right.}{}$$

Mathematica [F]
time = 23.34, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m)/(c - c*Sin[e + f*
x])^m, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a+a\sin(fx+e))^{m} (c-c\sin(fx+e))^{-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*g*cos(f*x + e)/(-c*sin
(f*x + e) + c)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m/((c-c*sin(f*x+e))**m),x
)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m/((c-c*sin(f*x+e))^m),x, a
lgorithm="giac")
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) +
c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{\left(c-c\,\sin\left(e+f\,x\right)\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
x)
[Out] int(((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,
 x)
```

3.171 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{13}{4}-m}c^2(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-5+4m),\frac{1}{4}(5+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+}}{fg(5+4m)}$$

[Out] 2^(13/4-m)*c^2*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -5/4+m],[9/4+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1-m)/f/g/(5+4*m)

Rubi [A]
time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2932, 2768, 72, 71}

$$\frac{c^2 2^{\frac{13}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-5),\frac{1}{4}(4m+5);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m),x]
```

```
[Out] (2^(13/4 - m)*c^2*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-5 + 4*m)/4, (5 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+f\right.}{}$$

$$= \frac{\left(2^{\frac{5}{4}-m}c^3(g\cos(e+fx))^{5/2}(a+a\sin(\right.}{}$$

$$= \frac{2^{\frac{13}{4}-m}c^2(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-5-\right.}{}$$

Mathematica [F]
time = 177.73, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m),x]

[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(1 - m), x]
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{\frac{3}{2}} (a+a\sin(fx+e))^{m} (c-c\sin(fx+e))^{1-m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 1)*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1-m)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{1-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
 m),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 -
 m), x)
```

3.172 $\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$

Optimal. Leaf size=123

$$\frac{2^{\frac{17}{4}-m}c^3(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-9+4m),\frac{1}{4}(5+4m);\frac{1}{4}(9+4m);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{-\frac{1}{4}+\cdots}}{fg(5+4m)}$$

```
[Out] 2^(17/4-m)*c^3*(g*cos(f*x+e))^(5/2)*hypergeom([5/4+m, -9/4+m],[9/4+m],1/2+1
/2*sin(f*x+e))*(1-sin(f*x+e))^(-1/4+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^
(-1-m)/f/g/(5+4*m)
```

Rubi [A]
time = 0.25, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2932, 2768, 72, 71}

$$\frac{c^3 2^{\frac{17}{4}-m}(g\cos(e+fx))^{5/2}(1-\sin(e+fx))^{m-\frac{1}{4}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{-m-1}\,{}_2F_1\left(\frac{1}{4}(4m-9),\frac{1}{4}(4m+5);\frac{1}{4}(4m+9);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(4m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 -
 m),x]
```

```
[Out] (2^(17/4 - m)*c^3*(g*Cos[e + f*x])^(5/2)*Hypergeometric2F1[(-9 + 4*m)/4, (5
 + 4*m)/4, (9 + 4*m)/4, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(-1/4 + m)
*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*g*(5 + 4*m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{5/2}(a+a\sin(e+f\right.}{}$$

$$= \frac{\left(2^{\frac{9}{4}-m}c^4(g\cos(e+fx))^{5/2}(a+a\sin(\right.}{}$$

$$= \frac{2^{\frac{17}{4}-m}c^3(g\cos(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(-9-\right.}{}$$

Mathematica [F]
time = 130.03, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x]
)^(2 - m), x]
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{\frac{3}{2}} \left(a+a\sin\left(fx+e\right)\right)^{m} \left(c-c\sin\left(fx+e\right)\right)^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

```
[Out] int((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^
(-m + 2)*g*cos(f*x + e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2-m)
,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m),x,
 algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) +
c)^(-m + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^{3/2}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2-m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
 m),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 -
 m), x)
```

3.173 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{2^{\frac{1}{2}+n+\frac{p}{2}}c(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2n-p),\frac{1}{2}(1+2m+p);\frac{1}{2}(3+2m+p);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e}{fg(1+2m+p)}$$

```
[Out] 2^(1/2+n+1/2*p)*c*(g*cos(f*x+e))^(1+p)*hypergeom([1/2-n-1/2*p, 1/2+m+1/2*p]
,[3/2+m+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2-n-1/2*p)*(a+a*sin(f*
x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/g/(1+2*m+p)
```

Rubi [A]

time = 0.20, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2932, 2768, 72, 71}

$$\frac{c2^{n+\frac{p}{2}+\frac{1}{2}}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{p+1}(1-\sin(e+fx))^{\frac{1}{2}(-2n-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2n-p+1),\frac{1}{2}(2m+p+1);\frac{1}{2}(2m+p+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(2m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n + p/2)*c*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 - 2*n -
p)/2, (1 + 2*m + p)/2, (3 + 2*m + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e +
f*x])^((1 - 2*n - p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n
))/(f*g*(1 + 2*m + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n+\frac{p}{2}}c^2(g\cos(e+fx))^{1+p}(a+a\sin(\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n+\frac{p}{2}}c(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2n\right.}{}$$

Mathematica [A]
time = 30.49, size = 133, normalized size = 0.99

$$\frac{2(g\cos(e+fx))^p\,{}_2F_1\left(1+m+n+p,\frac{1}{2}(1+2n+p);\frac{1}{2}(3+2n+p);-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{m+n+p}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f(1+2n+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]

[Out] (2*(g*Cos[e + f*x])^p*Hypergeometric2F1[1 + m + n + p, (1 + 2*n + p)/2, (3
+ 2*n + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(
m + n + p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi +
2*f*x)/4])/(f*(1 + 2*n + p))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p (a+a\sin(fx+e))^m (c-c\sin(fx+e))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+f\,x))^p\,(a+a\sin(e+f\,x))^m\,(c-c\sin(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

3.174 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx$

Optimal. Leaf size=57

$$-\frac{g(g\cos(e+fx))^{-2m}\log(1-\sin(e+fx))(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m}{cf}$$

```
[Out] -g*ln(1-sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m/c/f/((g*cos(f*x+e
))^(2*m))
```

Rubi [A]
time = 0.14, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2922, 12, 2746, 31}

$$-\frac{g\log(1-\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{cf}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + m),x]
```

```
[Out] -((g*Log[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(
c*f*(g*Cos[e + f*x])^(2*m)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2922

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d
, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*
m + p - 1, 0] && EqQ[m - n - 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+m}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+$$

$$= (g(g\cos(e+fx))^{-2m}(a+a\sin(e+$$

$$= -\frac{(g(g\cos(e+fx))^{-2m}(a+a\sin(e}{}$$

$$= -\frac{g(g\cos(e+fx))^{-2m}\log(1-\sin(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 155 vs. 2(57) = 114.
time = 70.34, size = 155, normalized size = 2.72

$$\frac{2^m g(g\cos(e+fx))^{-2m}\cos^{2m}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(4\log\left(\csc^2\left(\frac{1}{16}(2e+7\pi+2fx)\right)\right)-\log\left(\tan^2\left(\frac{1}{16}(-2e+\pi-2fx)\right)\right)-2\log\left(-1+\tan^2\left(\frac{1}{16}(-2e+\pi-2fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2m}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^m}{cf}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^(-1 + m),x]
```

```
[Out] (2^m*g*Cos[(2*e + Pi + 2*f*x)/4]^(2*m)*(4*Log[Csc[(2*e + 7*Pi + 2*f*x)/16]^
2] - Log[Tan[(-2*e + Pi - 2*f*x)/16]^2] - 2*Log[-1 + Tan[(-2*e + Pi - 2*f*x
)/16]^2])*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^m)/(c*f*(g*Cos[e +
f*x])^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 4.63, size = 8576, normalized size = 150.46

method	result	size
risch	Expression too large to display	8576

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m),x,met
hod=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^(m - 1), x)
```

Fricas [A]
time = 0.38, size = 31, normalized size = 0.54

$$-\frac{a\left(\frac{ac}{g^2}\right)^{m-1}\log\left(-\sin\left(fx+e\right)+1\right)}{fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="fricas")
```

```
[Out] -a*(a*c/g^2)^(m - 1)*log(-sin(f*x + e) + 1)/(f*g)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-1
+m),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(63) = 126.
time = 1.77, size = 945, normalized size = 16.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+m)
,x, algorithm="giac")
```

```
[Out] 1/2*(4*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c))
+ log(abs(g)))*floor(1/4*(pi + 2*f*x - 4*pi*floor(1/2*(pi + f*x + e)/pi) +
2*e)/pi)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c
) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*floor(-1/
4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^2 + 4*pi*e^(m*log(abs(a)) +
m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*floor(1/2*(pi
+ f*x + e)/pi)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4
*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*flo
or(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^2 + 2*pi*e^(m*log(abs(
a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*sgn(tan(
1/2*f*x + 1/2*e)^2 - 1)*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*f
loor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g)
 - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^2 + 3*pi*e^(m
*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))
*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) +
 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*floor(-1/4*sgn(c)
 + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g))^2 - 4*pi*e^(m*log(abs(a)) + m*log(ab
s(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*floor(1/4*(pi + 2*f*x
- 4*pi*floor(1/2*(pi + f*x + e)/pi) + 2*e)/pi) - 4*pi*e^(m*log(abs(a)) + m*
log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*floor(1/2*(pi +
f*x + e)/pi) - 2*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs(g)) - lo
g(abs(c)) + log(abs(g)))*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 4*e^(m*log(abs(a
)) + m*log(abs(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)))*log(2*(ta
n(1/2*f*x + 1/2*e)^2 - 2*tan(1/2*f*x + 1/2*e) + 1)/(tan(1/2*f*x + 1/2*e)^2
+ 1))*tan(1/4*pi + pi*m*floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) +
 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*floor(-1/4*s
gn(c) + 1) - 1/4*pi*sgn(c) + 1/4*pi*sgn(g)) - 2*e^(m*log(abs(a)) + m*log(ab
s(c)) - 2*m*log(abs(g)) - log(abs(c)) + log(abs(g)) + 1)*tan(1/4*pi + pi*m*
floor(-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) +
1/4*pi*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(
c) + 1/4*pi*sgn(g))^2 - 3*pi*e^(m*log(abs(a)) + m*log(abs(c)) - 2*m*log(abs
(g)) - log(abs(c)) + log(abs(g))) + 2*e^(m*log(abs(a)) + m*log(abs(c)) - 2*
m*log(abs(g)) - log(abs(c)) + log(abs(g)) + 1))/(f*tan(1/4*pi + pi*m*floor(
-1/4*sgn(a) + 1/2) + pi*m*floor(-1/4*sgn(c) + 1) + 1/4*pi*m*sgn(a) + 1/4*pi
*m*sgn(c) - 1/2*pi*m*sgn(g) - pi*floor(-1/4*sgn(c) + 1) - 1/4*pi*sgn(c) + 1
/4*pi*sgn(g))^2 + f)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int (g\,\cos\left(e+f\,x\right))^{1-2\,m}\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^{m-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
(m - 1),x)
```

```
[Out] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
(m - 1), x)
```

3.175 $\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=203

$$-\frac{8a^3(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx))^{-3+m}(c-c\sin(e+fx))^n}{fg(3-m+n)(4-m+n)(5-m+n)}-\frac{4a^2(g\cos(e+fx))^{6-2m}(a+a\sin(e+}{fg(4-m+n)(}$$

```
[Out] -8*a^3*(g*cos(f*x+e))^(6-2*m)*(a+a*sin(f*x+e))^(-3+m)*(c-c*sin(f*x+e))^n/f/
g/(3-m+n)/(4-m+n)/(5-m+n)-4*a^2*(g*cos(f*x+e))^(6-2*m)*(a+a*sin(f*x+e))^(-2
+m)*(c-c*sin(f*x+e))^n/f/g/(4-m+n)/(5-m+n)-a*(g*cos(f*x+e))^(6-2*m)*(a+a*si
n(f*x+e))^(-1+m)*(c-c*sin(f*x+e))^n/f/g/(5-m+n)
```

Rubi [A]
time = 0.45, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2925, 2923}

$$-\frac{8a^3(a\sin(e+fx)+a)^{m-3}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+3)(-m+n+4)(-m+n+5)}-\frac{4a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+4)(-m+n+5)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{6-2m}}{fg(-m+n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-8*a^3*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-3 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(3 - m + n)*(4 - m + n)*(5 - m + n)) - (4*a^2*(g*Cos[e +
 f*x])^(6 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin[e + f*x])^n)/(f*g
*(4 - m + n)*(5 - m + n)) - (a*(g*Cos[e + f*x])^(6 - 2*m)*(a + a*Sin[e + f*
x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(5 - m + n))
```

Rule 2923

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])
^n/(f*g*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rule 2925

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
```

```
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simp
lify[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0
] && GtQ[Simplify[2*m + n + 3*(p/2) + 1], 0])
```

Rubi steps

$$\int (g\cos(e+fx))^{5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{6-2m}(a+a\sin(e+fx}{fg(5-m+}$$
$$= -\frac{4a^2(g\cos(e+fx))^{6-2m}(a+a\sin(e+}{fg(4-m+n)(5}$$
$$= -\frac{8a^3(g\cos(e+fx))^{6-2m}(a+a\sin(e+}{fg(3-m+n)(4-m+}$$

Mathematica [A]
time = 4.77, size = 210, normalized size = 1.03

$$\frac{e^{n(-2\log(\cos(e+fx))+\log(a(1+\sin(e+fx)))+\log(c-c\sin(e+fx)))}g^5\cos^{2n}(e+fx)(g\cos(e+fx))^{-2m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^6(a(1+\sin(e+fx)))^{m-n}\left(-76+29m-3m^2-29n+6mn-3n^2+(12+m^2+7n+n^2-m(7+2n))\cos(2(e+fx))-4(18-9m+m^2+9n-2mn+n^2)\sin(e+fx)\right)}{2f(3-m+n)(4-m+n)(5-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*g^5*Cos[e + f*x]^(2*n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(a*
(1 + Sin[e + f*x]))^(m - n)*(-76 + 29*m - 3*m^2 - 29*n + 6*m*n - 3*n^2 + (1
2 + m^2 + 7*n + n^2 - m*(7 + 2*n))*Cos[2*(e + f*x)] - 4*(18 - 9*m + m^2 + 9
*n - 2*m*n + n^2)*Sin[e + f*x]))/(2*f*(3 - m + n)*(4 - m + n)*(5 - m + n)*(
g*Cos[e + f*x])^(2*m))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{5-2m}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(211) = 422$.
time = 0.83, size = 1016, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] ((m^2 - m*(2*n + 11) + n^2 + 11*n + 32)*a^m*c^n*g^5 - 2*(m^2 - m*(2*n + 15)
 + n^2 + 15*n + 60)*a^m*c^n*g^5*sin(f*x + e)/(cos(f*x + e) + 1) - (3*m^2 -
m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 + 8*(m^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 + 2*(m^2 - m*(2*n - 5) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4*(3*m^2 - m*(6*n + 13) + 3*n^2 + 13*n +
 116)*a^m*c^n*g^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 2*(m^2 - m*(2*n - 5
) + n^2 - 5*n + 160)*a^m*c^n*g^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 8*(m
^2 - m*(2*n + 7) + n^2 + 7*n - 20)*a^m*c^n*g^5*sin(f*x + e)^7/(cos(f*x + e)
 + 1)^7 - (3*m^2 - m*(6*n + 1) + 3*n^2 + n - 160)*a^m*c^n*g^5*sin(f*x + e)^
8/(cos(f*x + e) + 1)^8 - 2*(m^2 - m*(2*n + 15) + n^2 + 15*n + 60)*a^m*c^n*g
^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + (m^2 - m*(2*n + 11) + n^2 + 11*n +
 32)*a^m*c^n*g^5*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*e^(2*n*log(sin(f*x
+ e)/(cos(f*x + e) + 1) - 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1
) + m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 1))/(((m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m) + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 +
 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 10*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*
n - 60)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 10*(m^3 - 3*m^2*(n +
4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)
^6/(cos(f*x + e) + 1)^6 + 5*(m^3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47
)*m - 12*n^2 - 47*n - 60)*g^(2*m)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + (m^
3 - 3*m^2*(n + 4) - n^3 + (3*n^2 + 24*n + 47)*m - 12*n^2 - 47*n - 60)*g^(2*
m)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(211) = 422$.
time = 0.40, size = 667, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] -((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^3 - (m^2 - (2*m - 11)*n
 + n^2 - 11*m + 24)*cos(f*x + e)^2 - 2*(m^2 - (2*m - 9)*n + n^2 - 9*m + 22)
*cos(f*x + e) - ((m^2 - (2*m - 7)*n + n^2 - 7*m + 12)*cos(f*x + e)^2 + 2*(m
^2 - (2*m - 9)*n + n^2 - 9*m + 18)*cos(f*x + e) - 8)*sin(f*x + e) - 8)*(g*c
os(f*x + e))^(-2*m + 5)*(a*sin(f*x + e) + a)^m*e^(2*n*log(g*cos(f*x + e)) -
 n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(4*f*m^3 - 4*f*n^3 - (f*m^3 -
f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n
 - 60*f)*cos(f*x + e)^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 - 3*(f*m^3 - f*n^3
- 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n - 60*
f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n + 2*(f*m^3 - f*
n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m + 47*f)*n -
 60*f)*cos(f*x + e) + (4*f*m^3 - 4*f*n^3 - 48*f*m^2 + 12*(f*m - 4*f)*n^2 -
(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*f*m
+ 47*f)*n - 60*f)*cos(f*x + e)^2 + 188*f*m - 4*(3*f*m^2 - 24*f*m + 47*f)*n
+ 2*(f*m^3 - f*n^3 - 12*f*m^2 + 3*(f*m - 4*f)*n^2 + 47*f*m - (3*f*m^2 - 24*
f*m + 47*f)*n - 60*f)*cos(f*x + e) - 240*f)*sin(f*x + e) - 240*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90024 vs. 2(211) = 422.
time = 51.94, size = 90024, normalized size = 443.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] 4*(m^2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)
)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f
*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log
(abs(c)) - 2*m*log(abs(g)) - 5*log(2) + 5*log(4*abs(tan(1/8*pi - 1/4*f*x -
1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 5*log(abs(g)))*tan(-5/8*pi
 + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2
) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi
```

```
+ 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c)
+ 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x
+ 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4
*pi*m - 1/4*pi*n - 5/4*f*x - 5/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2
*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 5/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2
) - 5/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 5/4*pi*sgn(g) - 5/4*e)^2*tan(1
/2*f*x + 1/2*e)^10 - 2*m*n*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*p
i - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*ab
s(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m
*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - 5*log(2) + 5*log(4*abs(tan
(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 5*log(
abs(g)))*tan(-5/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi
+ 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x
/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi
*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*
n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*p
i*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1
/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 5/4*f*x - 5/2*pi*floor(1/2*f*x/pi +
1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 5/2*pi*floor(1/2*f*x
/pi + 1/2*e/pi + 1/2) - 5/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 5/4*pi*sgn
(g) - 5/4*e)^2*tan(1/2*f*x + 1/2*e)^10 + n^2*e^(m*log(2) - n*log(2) - 2*m*l
og(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 +
1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x -
1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - 5*log(2)
 + 5*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e
)^2 + 1)) + 5*log(abs(g)))*tan(-5/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi -
 floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/
pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/
2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*s
gn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2
*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1
/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 5/4*f*x - 5/2*pi*f
loor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 5/
2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 5/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 5/4*pi*sgn(g) - 5/4*e)^2*tan(1/2*f*x + 1/2*e)^10 - 2*m^2*e^(m*log(2
) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1
/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan
(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*lo
g(abs(g)) - 5*log(2) + 5*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*
pi - 1/4*f*x - 1/4*e)^2 + 1)) + 5*log(abs(g)))*tan(-5/8*pi + pi*m*floor(1/2
*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor
(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*f
loor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2)
 - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*s
gn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) -
```

```
 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n
- 5/4*f*x - 5/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/p
i + 1/2) + 1/4) - 5/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 5/4*pi*sgn(ta
n(1/2*f*x + 1/2*e)^2 - 1) - 5/4*pi*sgn(g) - 5/4*e)^2*tan(1/2*f*x + 1/2*e)^9
 + 4*m*n*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*
e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4
*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*l
og(abs(c)) - 2*m*log(abs(g)) - 5*log(2) + 5*log(4*abs(tan(1/8*pi - 1/4*f*x
- 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 5*log(abs(g)))*tan(-5/8*
pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2)
+ 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1
/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/p
i + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c
) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - ...
```

Mupad [B]
time = 17.21, size = 1149, normalized size = 5.66

$$\frac{(c-c\sin(e+f\,x))^n\left(-\frac{\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(m^2-2\,m\,n-7\,m+n^2+7\,n+12)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}-\frac{e^{e\,5i+f\,x\,5i}\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(m^2\,1i-m\,n\,2i-m\,7i+n^2\,1i+n\,7i+12i)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}+\frac{e^{e\,2i+f\,x\,2i}\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(2\,m^2-4\,m\,n-22\,m+2\,n^2+22\,n+80)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}+\frac{e^{e\,4i+f\,x\,4i}\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(3\,m^2-6\,m\,n-29\,m+3\,n^2+29\,n+60)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}+\frac{e^{e\,1i+f\,x\,1i}\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(m^2\,3i-m\,n\,6i-m\,29i+n^2\,3i+n\,29i+60i)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}+\frac{e^{e\,3i+f\,x\,3i}\left(g\left(\frac{e^{-e\,1i-f\,x\,1i}}{2}+\frac{e^{e\,1i+f\,x\,1i}}{2}\right)\right)^{5-2\,m}(a+a\sin(e+f\,x))^m\,(m^2\,2i-m\,n\,4i-m\,22i+n^2\,2i+n\,22i+80i)}{f\,(-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60)}\right)}{5\,e^{e\,1i+f\,x\,1i}-10\,e^{e\,3i+f\,x\,3i}+e^{e\,5i+f\,x\,5i}+\frac{-m^3\,1i+m^2\,n\,3i+m^2\,12i-m\,n^2\,3i-m\,n\,24i-m\,47i+n^3\,1i+n^2\,12i+n\,47i+60i}{-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60}-\frac{10\,e^{e\,2i+f\,x\,2i}\,(-m^3\,1i+m^2\,n\,3i+m^2\,12i-m\,n^2\,3i-m\,n\,24i-m\,47i+n^3\,1i+n^2\,12i+n\,47i+60i)}{-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60}+\frac{5\,e^{e\,4i+f\,x\,4i}\,(-m^3\,1i+m^2\,n\,3i+m^2\,12i-m\,n^2\,3i-m\,n\,24i-m\,47i+n^3\,1i+n^2\,12i+n\,47i+60i)}{-m^3+3\,m^2\,n+12\,m^2-3\,m\,n^2-24\,m\,n-47\,m+n^3+12\,n^2+47\,n+60}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] ((c - c*sin(e + f*x))^n*((exp(e*2i + f*x*2i)*(g*(exp(- e*1i - f*x*1i)/2 + e
xp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(22*n - 22*m - 4*m*n
 + 2*m^2 + 2*n^2 + 80))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m
^2 - m^3 + 12*n^2 + n^3 + 60)) - (exp(e*5i + f*x*5i)*(g*(exp(- e*1i - f*x*1
i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(n*7i - m*7i
 - m*n*2i + m^2*1i + n^2*1i + 12i))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*
m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60)) - ((g*(exp(- e*1i - f*x*1i)/2 +
exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(7*n - 7*m - 2*m*n
+ m^2 + n^2 + 12))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 -
m^3 + 12*n^2 + n^3 + 60)) + (exp(e*4i + f*x*4i)*(g*(exp(- e*1i - f*x*1i)/2
+ exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(29*n - 29*m - 6*
m*n + 3*m^2 + 3*n^2 + 60))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 1
2*m^2 - m^3 + 12*n^2 + n^3 + 60)) + (exp(e*1i + f*x*1i)*(g*(exp(- e*1i - f*
x*1i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x))^m*(n*29i -
m*29i - m*n*6i + m^2*3i + n^2*3i + 60i))/(f*(47*n - 47*m - 24*m*n - 3*m*n^2
 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60)) + (exp(e*3i + f*x*3i)*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(5 - 2*m)*(a + a*sin(e + f*x
))^m*(n*22i - m*22i - m*n*4i + m^2*2i + n^2*2i + 80i))/(f*(47*n - 47*m - 24
*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60))))/(5*exp(e*1i
 + f*x*1i) - 10*exp(e*3i + f*x*3i) + exp(e*5i + f*x*5i) + (n*47i - m*47i -
m*n*24i - m*n^2*3i + m^2*n*3i + m^2*12i - m^3*1i + n^2*12i + n^3*1i + 60i)/
```

```
(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 6
0) - (10*exp(e*2i + f*x*2i)*(n*47i - m*47i - m*n*24i - m*n^2*3i + m^2*n*3i
+ m^2*12i - m^3*1i + n^2*12i + n^3*1i + 60i))/(47*n - 47*m - 24*m*n - 3*m*n
^2 + 3*m^2*n + 12*m^2 - m^3 + 12*n^2 + n^3 + 60) + (5*exp(e*4i + f*x*4i)*(n
*47i - m*47i - m*n*24i - m*n^2*3i + m^2*n*3i + m^2*12i - m^3*1i + n^2*12i +
 n^3*1i + 60i))/(47*n - 47*m - 24*m*n - 3*m*n^2 + 3*m^2*n + 12*m^2 - m^3 +
12*n^2 + n^3 + 60))
```

3.176 $\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$-\frac{2a^2(g\cos(e+fx))^{4-2m}(a+a\sin(e+fx))^{-2+m}(c-c\sin(e+fx))^n}{fg(2-m+n)(3-m+n)}-\frac{a(g\cos(e+fx))^{4-2m}(a+a\sin(e+f$$

```
[Out] -2*a^2*(g*cos(f*x+e))^(4-2*m)*(a+a*sin(f*x+e))^(-2+m)*(c-c*sin(f*x+e))^n/f/
g/(2-m+n)/(3-m+n)-a*(g*cos(f*x+e))^(4-2*m)*(a+a*sin(f*x+e))^(-1+m)*(c-c*sin
(f*x+e))^n/f/g/(3-m+n)
```

Rubi [A]
time = 0.27, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2925, 2923}

$$-\frac{2a^2(a\sin(e+fx)+a)^{m-2}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+2)(-m+n+3)}-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{4-2m}}{fg(-m+n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] (-2*a^2*(g*Cos[e + f*x])^(4 - 2*m)*(a + a*Sin[e + f*x])^(-2 + m)*(c - c*Sin
[e + f*x])^n)/(f*g*(2 - m + n)*(3 - m + n)) - (a*(g*Cos[e + f*x])^(4 - 2*m)
*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e + f*x])^n)/(f*g*(3 - m + n))
```

Rule 2923

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])
^n/(f*g*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rule 2925

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-
b)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*
x])^n/(f*g*(m + n + p))), x] + Dist[a*((2*m + p - 1)/(m + n + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 -
b^2, 0] && IGtQ[Simplify[m + p/2 - 1/2], 0] &&  !LtQ[n, -1] &&  !(IGtQ[Simp
```

```
lify[n + p/2 - 1/2], 0] && GtQ[m - n, 0]) &&  !(ILtQ[Simplify[m + n + p], 0
] && GtQ[Simplify[2*m + n + 3*(p/2) + 1], 0])
```

Rubi steps

$$\int (g\cos(e+fx))^{3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{4-2m}(a+a\sin(e+ \ldots}{fg(3-m+ \ldots}$$

$$= -\frac{2a^2(g\cos(e+fx))^{4-2m}(a+a\sin(e+ \ldots}{fg(2-m+n)(\ldots}$$

Mathematica [A]
time = 1.01, size = 143, normalized size = 1.13

$$-\frac{e^{n(-2\log(\cos(e+fx))+\log(a(1+\sin(e+fx)))+\log(c-c\sin(e+fx)))}g^3\cos^{2n}(e+fx)(g\cos(e+fx))^{-2m}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^4(a(1+\sin(e+fx)))^{m-n}(4-m+n+(2-m+n)\sin(e+fx))}{f(2-m+n)(3-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] -((E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e
 + f*x]]))*g^3*Cos[e + f*x]^(2*n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(
a*(1 + Sin[e + f*x]))^(m - n)*(4 - m + n + (2 - m + n)*Sin[e + f*x]))/(f*(2
 - m + n)*(3 - m + n)*(g*Cos[e + f*x])^(2*m)))
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{3-2m}\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c-c\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(132) = 264$.
time = 0.60, size = 511, normalized size = 4.02

$$\frac{\left(a^m c^n g^3(m-n-4) - \frac{2a^m c^n g^3(m-n-6)\sin(fx+e)}{\cos(fx+e)+1} - \frac{a^m c^n g^3(m-n+12)\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{4a^m c^n g^3(m-n+2)\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{a^m c^n g^3(m-n+12)\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{2a^m c^n g^3(m-n-6)\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{a^m c^n g^3(m-n-4)\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)e^{\left(2n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-2m\log\left(-\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)+m\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)-n\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)\right)}}{\left((m^2-m(2n+5)+n^2+5n+6)g^{2m} + \frac{3(m^2-m(2n+5)+n^2+5n+6)g^{2m}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3(m^2-m(2n+5)+n^2+5n+6)g^{2m}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{(m^2-m(2n+5)+n^2+5n+6)g^{2m}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g^3*(m - n - 4) - 2*a^m*c^n*g^3*(m - n - 6)*sin(f*x + e)/(cos(f*x
+ e) + 1) - a^m*c^n*g^3*(m - n + 12)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
4*a^m*c^n*g^3*(m - n + 2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^m*c^n*g^3
*(m - n + 12)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2*a^m*c^n*g^3*(m - n -
6)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^m*c^n*g^3*(m - n - 4)*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)
- 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/(((
m^2 - m*(2*n + 5) + n^2 + 5*n + 6)*g^(2*m) + 3*(m^2 - m*(2*n + 5) + n^2 + 5
*n + 6)*g^(2*m)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*(m^2 - m*(2*n + 5)
+ n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + (m^2 - m*(2*
n + 5) + n^2 + 5*n + 6)*g^(2*m)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(132) = 264$.
time = 0.38, size = 310, normalized size = 2.44

$$\frac{\left((m-n-2)\cos\left(fx+e\right)^{2}+(m-n-4)\cos\left(fx+e\right)+\left((m-n-2)\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-2\right)\left(g\cos\left(fx+e\right)\right)^{-2\,m+3}\left(a\sin\left(fx+e\right)+a\right)^{m}e^{\left(2\,n\log\left(g\cos(fx+e)\right)-n\log\left(a\sin(fx+e)+a\right)+n\log\left(\frac{ac}{g^{2}}\right)\right)}}{2\,fm^{2}+2\,fn^{2}-\left(fm^{2}+fn^{2}-5\,fm-(2\,fm-5\,f)n+6\,f\right)\cos\left(fx+e\right)^{2}-10\,fm-2\,(2\,fm-5\,f)n+\left(fm^{2}+fn^{2}-5\,fm-(2\,fm-5\,f)n+6\,f\right)\cos\left(fx+e\right)+\left(2\,fm^{2}+2\,fn^{2}-10\,fm-2\,(2\,fm-5\,f)n+\left(fm^{2}+fn^{2}-5\,fm-(2\,fm-5\,f)n+6\,f\right)\cos\left(fx+e\right)+12\,f\right)\sin\left(fx+e\right)+12\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] ((m - n - 2)*cos(f*x + e)^2 + (m - n - 4)*cos(f*x + e) + ((m - n - 2)*cos(f
*x + e) + 2)*sin(f*x + e) - 2)*(g*cos(f*x + e))^(-2*m + 3)*(a*sin(f*x + e)
+ a)^m*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g
^2))/(2*f*m^2 + 2*f*n^2 - (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*c
os(f*x + e)^2 - 10*f*m - 2*(2*f*m - 5*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*
m - 5*f)*n + 6*f)*cos(f*x + e) + (2*f*m^2 + 2*f*n^2 - 10*f*m - 2*(2*f*m - 5
*f)*n + (f*m^2 + f*n^2 - 5*f*m - (2*f*m - 5*f)*n + 6*f)*cos(f*x + e) + 12*f
)*sin(f*x + e) + 12*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 29573 vs. $2(132) = 264$.
time = 18.33, size = 29573, normalized size = 232.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] 2*(m*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/
(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x
 - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(a
bs(c)) - 2*m*log(abs(g)) - 3*log(2) + 3*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/
4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 3*log(abs(g)))*tan(-3/8*pi +
 pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/
4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2)
+ 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi +
1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) +
1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x +
1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*p
i*m - 1/4*pi*n - 3/4*f*x - 3/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f
*x/pi + 1/2*e/pi + 1/2) + 1/4) - 3/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2)
- 3/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3/4*pi*sgn(g) - 3/4*e)^2*tan(1/2
*f*x + 1/2*e)^6 - n*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4
*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1
/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(ab
s(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - 3*log(2) + 3*log(4*abs(tan(1/8*pi
 - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 3*log(abs(g))
)*tan(-3/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e
/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floo
r(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor
(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m
*sgn(g) + 1/4*pi*m - 1/4*pi*n - 3/4*f*x - 3/2*pi*floor(1/2*f*x/pi + 1/2*e/p
i - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 3/2*pi*floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) - 3/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 3/4*pi*sgn(g) - 3
/4*e)^2*tan(1/2*f*x + 1/2*e)^6 + 4*m*e^(m*log(2) - n*log(2) - 2*m*log(4*abs
(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*
n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2
 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - 3*log(2) + 3*log
(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)
) + 3*log(abs(g)))*tan(-3/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1
/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - flo
```

```
or(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi +
 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) +
1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*
sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 3/4*f*x - 3/2*pi*floor(1/2
*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 3/2*pi*flo
or(1/2*f*x/pi + 1/2*e/pi + 1/2) - 3/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) -
3/4*pi*sgn(g) - 3/4*e)^2*tan(1/2*f*x + 1/2*e)^5 - 4*n*e^(m*log(2) - n*log(2
) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/
4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1
/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) -
 3*log(2) + 3*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*
x - 1/4*e)^2 + 1)) + 3*log(abs(g)))*tan(-3/8*pi + pi*m*floor(1/2*f*x/pi + 1
/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi
 + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*
x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*flo
or(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*
f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*s
gn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 3/4*f*x -
 3/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
1/4) - 3/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 3/4*pi*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) - 3/4*pi*sgn(g) - 3/4*e)^2*tan(1/2*f*x + 1/2*e)^5 + 2*m*e^(m
*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*
pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)
)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) -
2*m*log(abs(g)) - 3*log(2) + 3*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(ta
n(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 3*log(abs(g)))*tan(-3/8*pi + pi*m*flo
or(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n
*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) +
pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi
+ 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*
pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n...
```

Mupad [B]
time = 18.13, size = 476, normalized size = 3.75

$$-\frac{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^n\,\left(\frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3-2\,m}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(n-m+2\right)}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}-\frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3-2\,m}\,\left(\cos\left(3\,e+3\,f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\,1{}\mathrm{i}\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(-m\,1{}\mathrm{i}+n\,1{}\mathrm{i}+2{}\mathrm{i}\right)}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}-\frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3-2\,m}\,\left(\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,1{}\mathrm{i}\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(-m\,1{}\mathrm{i}+n\,1{}\mathrm{i}+6{}\mathrm{i}\right)}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}+\frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3-2\,m}\,\left(\cos\left(2\,e+2\,f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(n-m+6\right)}{f\,\left(m^2-2\,m\,n-5\,m+n^2+5\,n+6\right)}\right)}{3\,\cos\left(e+f\,x\right)+\sin\left(e+f\,x\right)\,3{}\mathrm{i}-\cos\left(3\,e+3\,f\,x\right)-\sin\left(3\,e+3\,f\,x\right)\,1{}\mathrm{i}+\frac{m^2\,1{}\mathrm{i}-m\,n\,2{}\mathrm{i}-m\,5{}\mathrm{i}+n^2\,1{}\mathrm{i}+n\,5{}\mathrm{i}+6{}\mathrm{i}}{m^2-2\,m\,n-5\,m+n^2+5\,n+6}-\frac{3\,\left(\cos\left(2\,e+2\,f\,x\right)+\sin\left(2\,e+2\,f\,x\right)\,1{}\mathrm{i}\right)\,\left(m^2\,1{}\mathrm{i}-m\,n\,2{}\mathrm{i}-m\,5{}\mathrm{i}+n^2\,1{}\mathrm{i}+n\,5{}\mathrm{i}+6{}\mathrm{i}\right)}{m^2-2\,m\,n-5\,m+n^2+5\,n+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] -((c - c*sin(e + f*x))^n*(((g*cos(e + f*x))^(3 - 2*m)*(a + a*sin(e + f*x))^
m*(n - m + 2))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) - ((g*cos(e + f*x))^
(3 - 2*m)*(cos(3*e + 3*f*x) + sin(3*e + 3*f*x)*1i)*(a + a*sin(e + f*x))^m*(
```

```
n*1i - m*1i + 2i))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) - ((g*cos(e + f*
x))^(3 - 2*m)*(cos(e + f*x) + sin(e + f*x)*1i)*(a + a*sin(e + f*x))^m*(n*1i
 - m*1i + 6i))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6)) + ((g*cos(e + f*x))^
(3 - 2*m)*(cos(2*e + 2*f*x) + sin(2*e + 2*f*x)*1i)*(a + a*sin(e + f*x))^m*(
n - m + 6))/(f*(5*n - 5*m - 2*m*n + m^2 + n^2 + 6))))/(3*cos(e + f*x) + sin
(e + f*x)*3i - cos(3*e + 3*f*x) - sin(3*e + 3*f*x)*1i + (n*5i - m*5i - m*n*
2i + m^2*1i + n^2*1i + 6i)/(5*n - 5*m - 2*m*n + m^2 + n^2 + 6) - (3*(cos(2*
e + 2*f*x) + sin(2*e + 2*f*x)*1i)*(n*5i - m*5i - m*n*2i + m^2*1i + n^2*1i +
 6i))/(5*n - 5*m - 2*m*n + m^2 + n^2 + 6))
```

3.177 $\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=58

$$-\frac{a(g\cos(e+fx))^{2-2m}(a+a\sin(e+fx))^{-1+m}(c-c\sin(e+fx))^n}{fg(1-m+n)}$$

```
[Out] -a*(g*cos(f*x+e))^(2-2*m)*(a+a*sin(f*x+e))^(-1+m)*(c-c*sin(f*x+e))^n/f/g/(1
-m+n)
```

Rubi [A]
time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2923}

$$-\frac{a(a\sin(e+fx)+a)^{m-1}(c-c\sin(e+fx))^n(g\cos(e+fx))^{2-2m}}{fg(-m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
n,x]
```

```
[Out] -((a*(g*Cos[e + f*x])^(2 - 2*m)*(a + a*Sin[e + f*x])^(-1 + m)*(c - c*Sin[e
+ f*x])^n)/(f*g*(1 - m + n)))
```

Rule 2923

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])
^n/(f*g*(m - n - 1))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m - n -
1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^{1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = -\frac{a(g\cos(e+fx))^{2-2m}(a+a\sin(e+fx$$

$$fg(1-m+$$

Mathematica [A]
time = 0.51, size = 96, normalized size = 1.66

$$\frac{e^{n(-2\log(\cos(e+fx))+\log(a(1+\sin(e+fx)))+\log(c-c\sin(e+fx)))}g\cos^{2n}(e+fx)(g\cos(e+fx))^{-2m}(-1+\sin(e+fx))(a(1+\sin(e+fx)))^{m-n}}{f(1-m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
f*x])^n,x]
```

```
[Out] (E^(n*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*g*Cos[e + f*x]^(2*n)*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(m
 - n))/(f*(1 - m + n)*(g*Cos[e + f*x])^(2*m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(g \cos \left(f x +e \right)\right)^{1-2 m} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(c -c \sin \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(60) = 120.
time = 0.54, size = 221, normalized size = 3.81

$$\frac{\left(a^{m} c^{n} g - \frac{2 \, a^{m} c^{n} g \sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} + \frac{a^{m} c^{n} g \sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}}\right)} e^{\left(2 \, n \log\left(\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} - 1\right) - 2 \, m \log\left(-\frac{\sin\left(f x + e\right)}{\cos\left(f x + e\right) + 1} + 1\right) + m \log\left(\frac{\sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}} + 1\right) - n \log\left(\frac{\sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}} + 1\right)\right)}}{{\left(g^{2 \, m} {\left(m - n - 1\right)} + \frac{g^{2 \, m} {\left(m - n - 1\right)} \sin\left(f x + e\right)^{2}}{{\left(\cos\left(f x + e\right) + 1\right)}^{2}}\right)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="maxima")
```

```
[Out] (a^m*c^n*g - 2*a^m*c^n*g*sin(f*x + e)/(cos(f*x + e) + 1) + a^m*c^n*g*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2)*e^(2*n*log(sin(f*x + e)/(cos(f*x + e) + 1) -
 1) - 2*m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) + m*log(sin(f*x + e)^2/
(cos(f*x + e) + 1)^2 + 1) - n*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))
/((g^(2*m)*(m - n - 1) + g^(2*m)*(m - n - 1)*sin(f*x + e)^2/(cos(f*x + e) +
 1)^2)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(60) = 120.
time = 0.40, size = 137, normalized size = 2.36

$$\frac{\left(g \cos\left(f x + e\right)\right)^{-2 \, m + 1} {\left(a \sin\left(f x + e\right) + a\right)}^{m} {\left(\cos\left(f x + e\right) - \sin\left(f x + e\right) + 1\right)} e^{\left(2 \, n \log\left(g \cos\left(f x + e\right)\right) - n \log\left(a \sin\left(f x + e\right) + a\right) + n \log\left(\frac{a c}{g^{2}}\right)\right)}}{f m - f n + {\left(f m - f n - f\right)} \cos\left(f x + e\right) + {\left(f m - f n - f\right)} \sin\left(f x + e\right) - f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^(-2*m + 1)*(a*sin(f*x + e) + a)^m*(cos(f*x + e) - sin(f*x
+ e) + 1)*e^(2*n*log(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*
c/g^2))/(f*m - f*n + (f*m - f*n - f)*cos(f*x + e) + (f*m - f*n - f)*sin(f*x
 + e) - f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4701 vs. $2(60) = 120$.
time = 5.69, size = 4701, normalized size = 81.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, a
lgorithm="giac")
```

```
[Out] -(e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(ta
n(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x -
1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(
c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(
tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(-1/8*pi + pi*m*flo
or(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n
*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) +
pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi
+ 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*
pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2
- 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4
*pi*n - 1/4*f*x - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + 1/4) - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 1/4*pi*
sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*sgn(g) - 1/4*e)^2*tan(1/2*f*x + 1/
2*e)^2 - 2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/
4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1
/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n
```

*log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(-1/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 1/4*f*x - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*sgn(g) - 1/4*e)^2*tan(1/2*f*x + 1/2*e) - 2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(-1/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 1/4*f*x - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*sgn(g) - 1/4*e)*tan(1/2*f*x + 1/2*e)^2 + e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(-1/8*pi + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*n*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - pi*m*floor(-1/4*sgn(a) + 1/2) - pi*n*floor(-1/4*sgn(c) + 1) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/2*pi*n*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*m*sgn(a) - 1/4*pi*n*sgn(c) + 1/2*pi*m*sgn(g) + 1/4*pi*m - 1/4*pi*n - 1/4*f*x - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) - 1/2*pi*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) - 1/4*pi*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*sgn(g) - 1/4*e)^2 - e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(1/2*f*x + 1/2*e)^2 + 2*e^(m*log(2) - n*log(2) - 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4

```
*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*n*log(4*abs(tan(1/8*pi - 1/
4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)) + n*
log(abs(c)) - 2*m*log(abs(g)) - log(2) + log(4*abs(tan(1/8*pi - 1/4*f*x - 1
/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + log(abs(g)))*tan(-1/8*pi +
pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*...
```

Mupad [B]
time = 9.42, size = 74, normalized size = 1.28

$$-\frac{g\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,{\left(a\left(\sin\left(e+f\,x\right)+1\right)\right)}^m\,{\left(-c\left(\sin\left(e+f\,x\right)-1\right)\right)}^n}{2\,f\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{2\,m}\,\left(\sin\left(e+f\,x\right)+1\right)\,\left(n-m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1 - 2*m)*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^
n,x)
```

```
[Out] -(g*(cos(2*e + 2*f*x) + 1)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))
^n)/(2*f*(g*cos(e + f*x))^(2*m)*(sin(e + f*x) + 1)*(n - m + 1))
```

3.178 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=81

$$\frac{(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,-m+n;1-m+n;\frac{1}{2}(1-\sin(e+fx))\right)(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{2fg(m-n)}$$

```
[Out] 1/2*hypergeom([1, -m+n],[1-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c
*sin(f*x+e))^n/f/g/(m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A]

time = 0.15, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 12, 2746, 70}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-2m}\,{}_2F_1\left(1,n-m;-m+n+1;\frac{1}{2}(1-\sin(e+fx))\right)}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (Hypergeometric2F1[1, -m + n, 1 - m + n, (1 - Sin[e + f*x])/2]*(a + a*Sin[e
 + f*x])^m*(c - c*Sin[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)$$

$$= \frac{((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)}{}$$

$$= -\frac{(c(g\cos(e+fx))^{-2m}(a+a\sin(e+f}{}$$

$$= \frac{(g\cos(e+fx))^{-2m}\,{}_2F_1(1,-m+n;1-}{}$$

Mathematica [A]
time = 50.72, size = 115, normalized size = 1.42

$$\frac{(g\cos(e+fx))^{-2m}\,{}_2F_1\left(-m+n,-m+n;1-m+n;-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-m+n}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{2fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[-m + n, -m + n, 1 - m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]
*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin
[e + f*x])^n)/(2*f*g*(m - n)*(g*Cos[e + f*x])^(2*m))
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-2m}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\sin(e + fx))^m (c - c\sin(e + fx))^n}{(g\cos(e + fx))^{2m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 1), x)
```

3.179 $\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=85

$$\frac{c(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-1-m+n;-m+n;\frac{1}{2}(1-\sin(e+fx))\right)(a+a\sin(e+fx))^m(c-c\sin(e+f}{4fg^3(1+m-n)}$$

```
[Out] 1/4*c*hypergeom([2, -1-m+n],[-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m*(
c-c*sin(f*x+e))^(-1+n)/f/g^3/(1+m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A]

time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 12, 2746, 70}

$$\frac{c(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(2,-m+n-1;n-m;\frac{1}{2}(1-\sin(e+fx))\right)}{4fg^3(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c*Hypergeometric2F1[2, -1 - m + n, -m + n, (1 - Sin[e + f*x])/2]*(a + a*Si
n[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))/(4*f*g^3*(1 + m - n)*(g*Cos[e
+ f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-3-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)$$

$$= \frac{((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)}{}$$

$$= -\frac{(c^3(g\cos(e+fx))^{-2m}(a+a\sin(e+}{}$$

$$= \frac{c(g\cos(e+fx))^{-2m}\,{}_2F_1\big(2,-1-m+}{}$$

Mathematica [A]
time = 36.63, size = 135, normalized size = 1.59

$$\frac{(g\cos(e+fx))^{-2m}\cot^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\,{}_2F_1\left(-2-m+n,-1-m+n;-m+n;-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-m+n}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{8fg^3(1+m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-3 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^2*Hypergeometric2F1[-2 - m + n, -1 - m + n, -m +
 n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n)*(a
*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(8*f*g^3*(1 + m - n)*(g*Cos[
e + f*x])^(2*m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-3-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-3-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-3-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 3)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^n}{\left(g\,\cos\left(e+f\,x\right)\right)^{2\,m+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 3),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m +
 3), x)
```

3.180 $\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{c^2(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-2-m+n;-1-m+n;\frac{1}{2}(1-\sin(e+fx))\right)(a+a\sin(e+fx))^m(c-c\sin(e$$
$$8fg^5(2+m-n)$$

```
[Out] 1/8*c^2*hypergeom([3, -2-m+n],[-1-m+n],1/2-1/2*sin(f*x+e))*(a+a*sin(f*x+e))
^m*(c-c*sin(f*x+e))^(-2+n)/f/g^5/(2+m-n)/((g*cos(f*x+e))^(2*m))
```

Rubi [A]
time = 0.17, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2932, 12, 2746, 70}

$$\frac{c^2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-2m}\,{}_2F_1\left(3,-m+n-2;-m+n-1;\frac{1}{2}(1-\sin(e+fx))\right)}{8fg^5(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^n,x]
```

```
[Out] (c^2*Hypergeometric2F1[3, -2 - m + n, -1 - m + n, (1 - Sin[e + f*x])/2]*(a
+ a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n))/(8*f*g^5*(2 + m - n)*(g*
Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-5-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)$$

$$= \frac{((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)}{}$$

$$= -\frac{(c^5(g\cos(e+fx))^{-2m}(a+a\sin(e+}{}$$

$$= \frac{c^2(g\cos(e+fx))^{-2m}\,{}_2F_1\big(3,-2-m+}{}$$

Mathematica [A]
time = 31.68, size = 136, normalized size = 1.55

$$\frac{(g\cos(e+fx))^{-2m}\cot^4\left(\frac{1}{4}(2e-\pi+2fx)\right)\,{}_2F_1\left(-4-m+n,-2-m+n;-1-m+n;-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{-m+n}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{32fg^5(2+m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-5 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^n,x]
```

```
[Out] (Cot[(2*e - Pi + 2*f*x)/4]^4*Hypergeometric2F1[-4 - m + n, -2 - m + n, -1 -
 m + n, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(-m + n
)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(32*f*g^5*(2 + m - n)*(g
*Cos[e + f*x])^(2*m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{-5-2m}\left(a+a\sin\left(fx+e\right)\right)^m\left(c-c\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e
) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-5-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-5-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

[Out] integrate((g*cos(f*x + e))^(-2*m - 5)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^n}{(g\,\cos(e+f\,x))^{2\,m+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m + 5),x)

[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(2*m + 5), x)

3.181 $\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}(\sin(e+fx))(g\cos(e+fx))^{-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m}{fg}$$

```
[Out] arctanh(sin(f*x+e))*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m/f/g/((g*cos(f*x+e
))^(2*m))
```

Rubi [A]
time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2926, 12, 3855}

$$\frac{\tanh^{-1}(\sin(e+fx))(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^m(g\cos(e+fx))^{-2m}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])
^m,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^m)/(f*g*
(g*Cos[e + f*x])^(2*m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2926

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
&& EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p + 1, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-2m}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^m\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)$$

$$= \frac{((g\cos(e+fx))^{-2m}(a+a\sin(e+fx)}{}$$

$$= \frac{\tanh^{-1}(\sin(e+fx))(g\cos(e+fx))^{-2}}{}$$

Mathematica [A]
time = 0.85, size = 94, normalized size = 1.84

$$\frac{e^{m(-2\log(\cos(e+fx))+\log(a(1+\sin(e+fx)))+\log(c-c\sin(e+fx)))}\sin^{-1}(\sec(e+fx))\cos^{2(1+m)}(e+fx)(g\cos(e+fx))^{-1-2m}\csc(e+fx)\sqrt{-\tan^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - 2*m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e +
 f*x])^m,x]
```

```
[Out] (E^(m*(-2*Log[Cos[e + f*x]] + Log[a*(1 + Sin[e + f*x])] + Log[c - c*Sin[e +
 f*x]]))*ArcSin[Sec[e + f*x]]*Cos[e + f*x]^(2*(1 + m))*(g*Cos[e + f*x])^(-1
 - 2*m)*Csc[e + f*x]*Sqrt[-Tan[e + f*x]^2])/f
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-2m}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

Fricas [A]
time = 0.37, size = 50, normalized size = 0.98

$$\frac{\left(\frac{ac}{g^2}\right)^m \log\left(\sin\left(fx+e\right)+1\right) - \left(\frac{ac}{g^2}\right)^m \log\left(-\sin\left(fx+e\right)+1\right)}{2\,fg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="fricas")
```

```
[Out] 1/2*((a*c/g^2)^m*log(sin(f*x + e) + 1) - (a*c/g^2)^m*log(-sin(f*x + e) + 1)
)/(f*g)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-2*m)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**m,
x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-2*m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^m,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-2*m - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
e) + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,\left(c-c\,\sin\left(e+f\,x\right)\right)^m}{\left(g\,\cos\left(e+f\,x\right)\right)^{2\,m+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^m)/(g*cos(e + f*x))^(2*m +
 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^m)/(g*cos(e + f*x))^(2*m +
 1), x)
```

3.182 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$

Optimal. Leaf size=134

$$\frac{2^{3-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(-4+m-n),\frac{m-n}{2};\frac{1}{2}(2+m-n);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx}{fg(m-n)}$$

```
[Out] 2^(3-1/2*m+1/2*n)*c^3*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, -2+1/2*
m-1/2*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(
a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A]

time = 0.27, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2932, 2768, 72, 71}

$$\frac{c^3 2^{-\frac{m}{2}+\frac{n}{2}+3}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-4),\frac{m-n}{2};\frac{1}{2}(m-n+2);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(3 + n),x]
```

```
[Out] (2^(3 - m/2 + n/2)*c^3*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-4 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+$$

$$= \frac{\left(c^2(g\cos(e+fx))^{-m-n}(a+a\sin\right.}{}$$

$$= \frac{\left(2^{2-\frac{m}{2}+\frac{n}{2}}c^4(g\cos(e+fx))^{-m-n}(\right.}{}$$

$$= \frac{2^{3-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}\,{}_2F}{}$$

Mathematica [F]
time = 139.42, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{3+n}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n),x]

[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(3 + n), x]
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-m-n} (a+a\sin(fx+e))^m (c-c\sin(fx+e))^{3+n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(3
+n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin\left(e+f\,x\right))^m\,(c-c\,\sin\left(e+f\,x\right))^{n+3}}{(g\,\cos\left(e+f\,x\right))^{m+n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 3))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 3))/(g*cos(e + f*x))^
(m + n + 1), x)
```

3.183 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$

Optimal. Leaf size=134

$$\frac{2^{2-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(-2+m-n),\frac{m-n}{2};\frac{1}{2}(2+m-n);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx)}{fg(m-n)}$$

```
[Out] 2^(2-1/2*m+1/2*n)*c^2*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, -1+1/2*
m-1/2*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(
a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A]
time = 0.26, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2932, 2768, 72, 71}

$$\frac{c^2 2^{-\frac{m}{2}+\frac{n}{2}+2}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{1}{2}(m-n-2),\frac{m-n}{2};\frac{1}{2}(m-n+2);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(2 + n),x]
```

```
[Out] (2^(2 - m/2 + n/2)*c^2*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(-2 + m
- n)/2, (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^
((m - n)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx = ((g\cos(e+fx))^{-2m}(a+a\sin(e+$$

$$= \frac{\left(c^2(g\cos(e+fx))^{-m-n}(a+a\sin\right.}{}$$

$$= \frac{\left(2^{1-\frac{m}{2}+\frac{n}{2}}c^3(g\cos(e+fx))^{-m-n}(\right.}{}$$

$$= \frac{2^{2-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}\,{}_2F}{}$$

Mathematica [F]
time = 134.39, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{2+n}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(2 + n), x]
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int (g\cos (fx+e))^{-1-m-n} (a+a\sin (fx+e))^{m} (c-c\sin (fx+e))^{2+n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(2
+n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\,\sin(e+f\,x))^m\,(c-c\,\sin(e+f\,x))^{n+2}}{(g\,\cos(e+f\,x))^{m+n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 2))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 2))/(g*cos(e + f*x))^
(m + n + 1), x)
```

3.184 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx$

Optimal. Leaf size=131

$$\frac{2^{1-\frac{m}{2}+\frac{n}{2}}c(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(2+m-n);\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{m-n}{2}}(a+a\sin(e+fx))^m\ldots}{fg(m-n)}$$

```
[Out] 2^(1-1/2*m+1/2*n)*c*(g*cos(f*x+e))^(-n-m)*hypergeom([1/2*m-1/2*n, 1/2*m-1/2
*n],[1+1/2*m-1/2*n],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2*m-1/2*n)*(a+a*s
in(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A]

time = 0.25, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2932, 2768, 72, 71}

$$\frac{c2^{-\frac{m}{2}+\frac{n}{2}+1}(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(1-\sin(e+fx))^{\frac{m-n}{2}}(g\cos(e+fx))^{-m-n}\,{}_2F_1\left(\frac{m-n}{2},\frac{m-n}{2};\frac{1}{2}(m-n+2);\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(1 + n),x]
```

```
[Out] (2^(1 - m/2 + n/2)*c*(g*Cos[e + f*x])^(-m - n)*Hypergeometric2F1[(m - n)/2,
 (m - n)/2, (2 + m - n)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((m - n
)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(f*g*(m - n))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{1+n}\,dx = \left((g\cos(e+fx))^{-2m}(a+a\sin(e+\right.$$

$$= \frac{\left(c^2(g\cos(e+fx))^{-m-n}(a+a\sin\right.}{}$$

$$= \frac{\left(2^{-\frac{m}{2}+\frac{n}{2}}c^2(g\cos(e+fx))^{-m-n}(a\right.}{}$$

$$= \frac{2^{1-\frac{m}{2}+\frac{n}{2}}c(g\cos(e+fx))^{-m-n}\,{}_2F_1}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 25.59, size = 207, normalized size = 1.58

$$\frac{ic(g\cos(e+fx))^{-m-n}\left({}_2F_1\left(1,1-m+n;2-m+n;-\frac{i(-1+\tan(\frac{1}{2}(e+fx)))}{1+\tan(\frac{1}{2}(e+fx))}\right)-{}_2F_1\left(1,1-m+n;2-m+n;\frac{i(-1+\tan(\frac{1}{2}(e+fx)))}{1+\tan(\frac{1}{2}(e+fx))}\right)\right)(-1+\sin(e+fx))(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{fg(-1+m-n)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(1 + n),x]
```

```
[Out] (I*c*(g*Cos[e + f*x])^(-m - n)*(Hypergeometric2F1[1, 1 - m + n, 2 - m + n,
((-I)*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])] - Hypergeometric2F1[
1, 1 - m + n, 2 - m + n, (I*(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])
```

```
])*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/(f*
g*(-1 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]))
```

Maple [**F**]
time = 0.54, size = 0, normalized size = 0.00

$$\int \left(g \cos \left(f x +e \right)\right)^{-1-m -n} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(c -c \sin \left(f x +e \right)\right)^{1+n}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x +
 e) + c)^(n + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(1
+n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1+n)
,x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + f x))^m (c - c \sin(e + f x))^{n+1}}{(g \cos(e + f x))^{m+n+1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 1))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n + 1))/(g*cos(e + f*x))^
(m + n + 1), x)
```

3.185 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=55

$$\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n}{fg(m-n)}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f/g/(m-n)
```

Rubi [A]
time = 0.12, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = $\{2927\}$

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(
f*g*(m - n))
```

Rule 2927

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(m - n))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+f$$

Mathematica [A]
time = 0.58, size = 55, normalized size = 1.00

$$\frac{(g\cos(e+fx))^{-m-n}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n}{fg(m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^n,x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)
/(f*g*(m - n))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int (g\cos (fx+e))^{-1-m-n} (a+a\sin (fx+e))^{m} (c-c\sin (fx+e))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(58) = 116.
time = 0.54, size = 155, normalized size = 2.82

$$\frac{a^{m}c^{n}g^{-m-n-1}e^{\left(m\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)+2\,n\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)-m\log\left(-\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)-n\log\left(-\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)\right)}}{f(m-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="maxima")
```

```
[Out] a^m*c^n*g^(-m - n - 1)*e^(m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*lo
g(sin(f*x + e)/(cos(f*x + e) + 1) + 1) + 2*n*log(sin(f*x + e)/(cos(f*x + e)
 + 1) - 1) - m*log(-sin(f*x + e)/(cos(f*x + e) + 1) + 1) - n*log(-sin(f*x +
 e)/(cos(f*x + e) + 1) + 1))/(f*(m - n))
```

Fricas [A]
time = 0.37, size = 88, normalized size = 1.60

$$\frac{(g\cos (fx+e))^{-m-n-1} (a\sin (fx+e)+a)^{m} \cos (fx+e)\, e^{\left(2\,n\log(g\cos(fx+e))-n\log(a\sin(fx+e)+a)+n\log\left(\frac{ac}{g^2}\right)\right)}}{fm-fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*cos(f*x + e)*e^(2*n*lo
g(g*cos(f*x + e)) - n*log(a*sin(f*x + e) + a) + n*log(a*c/g^2))/(f*m - f*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,
x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x,
algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^n, x)
```

Mupad [**B**]
time = 9.07, size = 53, normalized size = 0.96

$$\frac{(a\,(\sin\left(e+f\,x\right)+1))^m\,(-c\,(\sin\left(e+f\,x\right)-1))^n}{f\,g\,(g\,\cos\left(e+f\,x\right))^{m+n}\,\,(m-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n)/(g*cos(e + f*x))^(m + n
 + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/(f*g*(g*cos(e + f*x))^
(m + n)*(m - n))
```

3.186 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx$

Optimal. Leaf size=125

$$\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}}{fg(2+m-n)}+\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m}{cfg(m-n)(2+m-}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/g/(2+m-n
)+(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/c/f/g/(m-n)/(
2+m-n)
```

Rubi [A]

time = 0.27, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2928, 2927}

$$\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{fg(m-n+2)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{cfg(m-n)(m-n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-1 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1
+ n))/(f*g*(2 + m - n)) + ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m
*(c - c*Sin[e + f*x])^n)/(c*f*g*(m - n)*(2 + m - n))
```

Rule 2927

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(m - n))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2928

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-1+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e-}{fg(2+r}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e-}{fg(2+r}$$

Mathematica [A]
time = 24.74, size = 132, normalized size = 1.06

$$-\frac{2^{-1+n}(g\cos(e+fx))^{-m-n}\cos^{2(-1+n)}\left(\frac{1}{4}(2e+\pi+2fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2-2n}(a(1+\sin(e+fx)))^m(-1-m+n+\sin(e+fx))(c-c\sin(e+fx))^{-1+n}}{fg(m-n)(2+m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-1 + n),x]
```

```
[Out] -((2^(-1 + n)*(g*Cos[e + f*x])^(-m - n)*Cos[(2*e + Pi + 2*f*x)/4]^(2*(-1 +
n))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2 - 2*n)*(a*(1 + Sin[e + f*x]))^
m*(-1 - m + n + Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 + n))/(f*g*(m - n)*(
2 + m - n)))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-m-n}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^{-1+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.38, size = 133, normalized size = 1.06

$$\frac{((m-n+1)\cos(fx+e)-\cos(fx+e)\sin(fx+e))(g\cos(fx+e))^{-m-n-1}(a\sin(fx+e)+a)^{m}e^{\left(2(n-1)\log(g\cos(fx+e))-(n-1)\log(a\sin(fx+e)+a)+(n-1)\log\left(\frac{ac}{g^{2}}\right)\right)}}{fm^{2}+fn^{2}+2fm-2(fm+f)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="fricas")
```

```
[Out] ((m - n + 1)*cos(f*x + e) - cos(f*x + e)*sin(f*x + e))*(g*cos(f*x + e))^(-m
 - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 1)*log(g*cos(f*x + e)) - (n - 1)
*log(a*sin(f*x + e) + a) + (n - 1)*log(a*c/g^2))/(f*m^2 + f*n^2 + 2*f*m - 2
*(f*m + f)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
1+n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 1), x)
```

Mupad [B]
time = 10.16, size = 128, normalized size = 1.02

$$\frac{(a(\sin(e+fx)+1))^{m}(-c(\sin(e+fx)-1))^{n}(2\cos(e+fx)-\sin(2e+2fx)+2m\cos(e+fx)-2n\cos(e+fx))}{cfg(g\cos(e+fx))^{m+n}(2\cos(e+fx)-\sin(2e+2fx))(m^{2}-2mn+2m+n^{2}-2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 1))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n*(2*cos(e + f*x) - sin(2
*e + 2*f*x) + 2*m*cos(e + f*x) - 2*n*cos(e + f*x)))/(c*f*g*(g*cos(e + f*x))
^(m + n)*(2*cos(e + f*x) - sin(2*e + 2*f*x))*(2*m - 2*n - 2*m*n + m^2 + n^2
))
```

3.187 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx$

Optimal. Leaf size=204

$$\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}}{fg(4+m-n)}+\frac{2(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^n}{cfg(2+m-n)(4+m}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/f/g/(4+m-n
)+2*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/c/f/g/
(2+m-n)/(4+m-n)+2*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^n/c^2/f/g/(m-n)/(2+m-n)/(4+m-n)
```

Rubi [A]
time = 0.44, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2928, 2927}

$$\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^2fg(m-n)(m-n+2)(m-n+4)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-m-n}}{fg(m-n+4)}+\frac{2(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{cfg(m-n+2)(m-n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-2 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2
+ n))/(f*g*(4 + m - n)) + (2*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-1 + n))/(c*f*g*(2 + m - n)*(4 + m - n)) + (2*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^2*f
*g*(m - n)*(2 + m - n)*(4 + m - n))
```

Rule 2927

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(m - n))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2928

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
```

```
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(4+}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(4+}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(4+}$$

Mathematica [A]
time = 28.64, size = 183, normalized size = 0.90

$$\frac{2^{-2+n}\cos(e+fx)(g\cos(e+fx))^{-1-m-n}\sin^{-4+2n}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-2+n)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-2+n}\left(3+4m+m^2-4n-2mn+n^2+\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)-2(2+m-n)\sin(e+fx)\right)}{f(m-n)(2+m-n)(4+m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-2 + n),x]
```

```
[Out] (2^(-2 + n)*Cos[e + f*x]*(g*Cos[e + f*x])^(-1 - m - n)*Sin[(-e + Pi/2 - f*x
)/2]^(-4 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 + n)*(3 + 4
*m + m^2 - 4*n - 2*m*n + n^2 + Cos[2*(-e + Pi/2 - f*x)] - 2*(2 + m - n)*Sin
[e + f*x]))/(f*(m - n)*(2 + m - n)*(4 + m - n)*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])^(2*(-2 + n)))
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-m-n}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^{-2+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 0.38, size = 191, normalized size = 0.94

$$-\frac{\left(2\,\cos\left(fx+e\right)^{3}+2\,(m-n+2)\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(m^{2}-2\,(m+2)n+n^{2}+4\,m+4\right)\cos\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^{m}e^{\left(2\,(n-2)\log(g\cos(fx+e))-(n-2)\log(a\sin(fx+e)+a)+(n-2)\log\left(\frac{ac}{g^{2}}\right)\right)}}{fm^{3}-fn^{3}+6\,fm^{2}+3\,(fm+2\,f)n^{2}+8\,fm-\left(3\,fm^{2}+12\,fm+8\,f\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="fricas")
```

```
[Out] -(2*cos(f*x + e)^3 + 2*(m - n + 2)*cos(f*x + e)*sin(f*x + e) - (m^2 - 2*(m
+ 2)*n + n^2 + 4*m + 4)*cos(f*x + e))*(g*cos(f*x + e))^(-m - n - 1)*(a*sin(
f*x + e) + a)^m*e^(2*(n - 2)*log(g*cos(f*x + e)) - (n - 2)*log(a*sin(f*x +
e) + a) + (n - 2)*log(a*c/g^2))/(f*m^3 - f*n^3 + 6*f*m^2 + 3*(f*m + 2*f)*n^
2 + 8*f*m - (3*f*m^2 + 12*f*m + 8*f)*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
2+n),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 2), x)
```

Mupad [B]
time = 15.65, size = 887, normalized size = 4.35

$$-{\mathrm{e}}^{-e\,3{}\mathrm{i}-f\,x\,3{}\mathrm{i}}\,{\left(c-c\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^{n-2}\,\left(\frac{{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}+\frac{{\mathrm{e}}^{e\,6{}\mathrm{i}+f\,x\,6{}\mathrm{i}}\,{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}-\frac{{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m\,\left(2\,m^2-4\,m\,n+8\,m+2\,n^2-8\,n+5\right)}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}-\frac{{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}\,{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m\,\left(2\,m^2-4\,m\,n+8\,m+2\,n^2-8\,n+5\right)}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m\,\left(m\,2{}\mathrm{i}-n\,2{}\mathrm{i}+4{}\mathrm{i}\right)}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}-\frac{{\mathrm{e}}^{e\,5{}\mathrm{i}+f\,x\,5{}\mathrm{i}}\,{\left(a+a\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)\right)}^m\,\left(m\,2{}\mathrm{i}-n\,2{}\mathrm{i}+4{}\mathrm{i}\right)}{4\,f\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^{m+n+1}\,\left(m^3-3\,m^2\,n+6\,m^2+3\,m\,n^2-12\,m\,n+8\,m-n^3+6\,n^2-8\,n\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 2))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] -exp(- e*3i - f*x*3i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(n - 2)*((a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^m/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m
+ n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^
3)) + (exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
 f*x*1i)*1i)/2))^m)/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2)
)^(m + n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2
 - n^3)) - (exp(e*2i + f*x*2i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^m*(8*m - 8*n - 4*m*n + 2*m^2 + 2*n^2 + 5))/(4*f*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*
m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)) - (exp(e*4i + f*x*4i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(8*m -
 8*n - 4*m*n + 2*m^2 + 2*n^2 + 5))/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*
1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^
2 + m^3 + 6*n^2 - n^3)) + (exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i - f*x*1i)
*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*2i - n*2i + 4i))/(4*f*(g*(exp(- e
*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(8*m - 8*n - 12*m*n +
3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)) - (exp(e*5i + f*x*5i)*(a +
a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*2i - n*2i
 + 4i))/(4*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1
)*(8*m - 8*n - 12*m*n + 3*m*n^2 - 3*m^2*n + 6*m^2 + m^3 + 6*n^2 - n^3)))
```

3.188 $\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx$

Optimal. Leaf size=290

$$\frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}}{fg(6+m-n)}+\frac{3(g\cos(e+fx))^{-m-n}(a+a\sin(e+fx))^n}{cfg(4+m-n)(6+m}$$

```
[Out] (g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n)/f/g/(6+m-n
)+3*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-2+n)/c/f/g/
(4+m-n)/(6+m-n)+6*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))
^(-1+n)/c^2/f/g/(2+m-n)/(4+m-n)/(6+m-n)+6*(g*cos(f*x+e))^(-n-m)*(a+a*sin(f*
x+e))^m*(c-c*sin(f*x+e))^n/c^3/f/g/(m-n)/(2+m-n)/(4+m-n)/(6+m-n)
```

Rubi [A]
time = 0.64, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2928, 2927}

$$\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^n(g\cos(e+fx))^{-m-n}}{c^3fg(m-n)(m-n+2)(m-n+4)(m-n+6)}+\frac{6(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\cos(e+fx))^{-m-n}}{c^2fg(m-n+2)(m-n+4)(m-n+6)}+\frac{(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-3}(g\cos(e+fx))^{-m-n}}{fg(m-n+6)}+\frac{3(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-2}(g\cos(e+fx))^{-m-n}}{cfg(m-n+4)(m-n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x
])^(-3 + n),x]
```

```
[Out] ((g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3
+ n))/(f*g*(6 + m - n)) + (3*(g*Cos[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])
^m*(c - c*Sin[e + f*x])^(-2 + n))/(c*f*g*(4 + m - n)*(6 + m - n)) + (6*(g*C
os[e + f*x])^(-m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 + n))
/(c^2*f*g*(2 + m - n)*(4 + m - n)*(6 + m - n)) + (6*(g*Cos[e + f*x])^(-m -
n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/(c^3*f*g*(m - n)*(2 + m -
 n)*(4 + m - n)*(6 + m - n))
```

Rule 2927

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
f*g*(m - n))), x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + p + 1, 0] && NeQ[m, n]
```

Rule 2928

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b*
(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(a*
```

```
f*g*(2*m + p + 1))), x] + Dist[(m + n + p + 1)/(a*(2*m + p + 1)), Int[(g*Co
s[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /
; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2
- b^2, 0] && ILtQ[Simplify[m + n + p + 1], 0] && NeQ[2*m + p + 1, 0] && (Su
mSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m-n}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\,dx = \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(6+}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(6+}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(6+}$$
$$= \frac{(g\cos(e+fx))^{-m-n}(a+a\sin(e}{fg(6+}$$

Mathematica [A]
time = 34.26, size = 259, normalized size = 0.89

$$-\frac{2^{-4+n}\cos(e+fx)(g\cos(e+fx))^{-1-m-n}\sin^{-6+2n}\left(\frac{1}{2}\left(-e+\frac{\pi}{2}-fx\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-3+n)}(a+a\sin(e+fx))^m(c-c\sin(e+fx))^{-3+n}\left(-30-46m-18m^2-2m^3+46n+36mn+6m^2n-18n^2-6mn^2+2n^3-6(3+m-n)\cos\left(2\left(-e+\frac{\pi}{2}-fx\right)\right)+3\cos\left(3\left(-e+\frac{\pi}{2}-fx\right)\right)+3(15+2m^2-4m(-3+n)-12n+2n^2)\sin(e+fx)\right)}{f(m-n)(2+m-n)(4+m-n)(6+m-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m - n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e
 + f*x])^(-3 + n),x]
```

```
[Out] -((2^(-4 + n)*Cos[e + f*x]*(g*Cos[e + f*x])^(-1 - m - n)*Sin[(-e + Pi/2 - f
*x)/2]^(-6 + 2*n)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 + n)*(-30
 - 46*m - 18*m^2 - 2*m^3 + 46*n + 36*m*n + 6*m^2*n - 18*n^2 - 6*m*n^2 + 2*n
^3 - 6*(3 + m - n)*Cos[2*(-e + Pi/2 - f*x)] + 3*Cos[3*(-e + Pi/2 - f*x)] +
3*(15 + 2*m^2 - 4*m*(-3 + n) - 12*n + 2*n^2)*Sin[e + f*x]))/(f*(m - n)*(2 +
 m - n)*(4 + m - n)*(6 + m - n)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-
3 + n))))
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-m-n}(a+a\sin(fx+e))^m(c-c\sin(fx+e))^{-3+n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

```
[Out] int((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**A**]
time = 0.39, size = 273, normalized size = 0.94

$$-\frac{\left(6\,(m-n+3)\cos\left(fx+e\right)^3-\left(m^3+3\,(m+3)n^2-n^3+9\,m^2-\left(3\,m^2+18\,m+26\right)n+26\,m+24\right)\cos\left(fx+e\right)-3\left(2\,\cos\left(fx+e\right)^3-\left(m^2-2\,(m+3)n+n^2+6\,m+8\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{-m-n-1}\left(a\sin\left(fx+e\right)+a\right)^m e^{\left(2\,(n-3)\log\left(g\cos\left(fx+e\right)\right)-(n-3)\log\left(a\sin\left(fx+e\right)+a\right)+(n-3)\log\left(\frac{ac}{g^2}\right)\right)}}{fm^4+fn^4+12\,fm^3-4\,(fm+3\,f)n^3+44\,fm^2+2\left(3\,fm^2+18\,fm+22\,f\right)n^2+48\,fm-4\left(fm^3+9\,fm^2+22\,fm+12\,f\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="fricas")
```

```
[Out] -(6*(m - n + 3)*cos(f*x + e)^3 - (m^3 + 3*(m + 3)*n^2 - n^3 + 9*m^2 - (3*m^
2 + 18*m + 26)*n + 26*m + 24)*cos(f*x + e) - 3*(2*cos(f*x + e)^3 - (m^2 - 2
*(m + 3)*n + n^2 + 6*m + 8)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^(-
m - n - 1)*(a*sin(f*x + e) + a)^m*e^(2*(n - 3)*log(g*cos(f*x + e)) - (n - 3
)*log(a*sin(f*x + e) + a) + (n - 3)*log(a*c/g^2))/(f*m^4 + f*n^4 + 12*f*m^3
 - 4*(f*m + 3*f)*n^3 + 44*f*m^2 + 2*(3*f*m^2 + 18*f*m + 22*f)*n^2 + 48*f*m
- 4*(f*m^3 + 9*f*m^2 + 22*f*m + 12*f)*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m-n)*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**(-
3+n),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m-n)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-3+n
),x, algorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(-m - n - 1)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x
+ e) + c)^(n - 3), x)
```

Mupad [B]
time = 17.65, size = 1623, normalized size = 5.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(n - 3))/(g*cos(e + f*x))^
(m + n + 1),x)
```

```
[Out] -exp(- e*4i - f*x*4i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2))^(n - 3)*((3*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^m)/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))
^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i -
m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^
2*6i)) - (3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^m)/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1
i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3
*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i +
m^2*n^2*6i)) - (exp(e*2i + f*x*2i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2))^m*(36*m - 36*n - 12*m*n + 6*m^2 + 6*n^2 + 42))/(8
*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i -
 n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i +
m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*6i +
f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m
*(36*m - 36*n - 12*m*n + 6*m^2 + 6*n^2 + 42))/(8*f*(g*(exp(- e*1i - f*x*1i)
/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36
i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i
- n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*1i + f*x*1i)*(a + a*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*6i - n*6i + 18i))/(8*f*
(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*
48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3
*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)) + (exp(e*7i + f*x
*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m
*6i - n*6i + 18i))/(8*f*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))
^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i -
m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^
2*6i)) - (exp(e*3i + f*x*3i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2))^m*(m*86i - n*86i - m*n*72i + m*n^2*12i - m^2*n*12i + m^
```

```
2*36i + m^3*4i + n^2*36i - n^3*4i + 42i))/(8*f*(g*(exp(- e*1i - f*x*1i)/2 +
 exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i - m*n*88i + m*n^2*36i -
m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i + m^4*1i + n^2*44i - n^
3*12i + n^4*1i + m^2*n^2*6i)) - (exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^m*(m*86i - n*86i - m*n*72i + m*n
^2*12i - m^2*n*12i + m^2*36i + m^3*4i + n^2*36i - n^3*4i + 42i))/(8*f*(g*(e
xp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(m + n + 1)*(m*48i - n*48i -
 m*n*88i + m*n^2*36i - m^2*n*36i - m*n^3*4i - m^3*n*4i + m^2*44i + m^3*12i
+ m^4*1i + n^2*44i - n^3*12i + n^4*1i + m^2*n^2*6i)))
```

3.189 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx$

Optimal. Leaf size=138

$$\frac{2^{\frac{1}{2}+n-\frac{p}{2}}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m-p),\frac{1}{2}(1-2n+p);\frac{1}{2}(3+2m-p);\frac{1}{2}(1+\sin(e+fx))\right)(g\sec(e+fx))}{f(1+2m-p)}$$

```
[Out] 2^(1/2+n-1/2*p)*c*cos(f*x+e)*hypergeom([1/2+m-1/2*p, 1/2-n+1/2*p],[3/2+m-1/
2*p],1/2+1/2*sin(f*x+e))*(g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/2-n+1/2*p)*(a+a
*sin(f*x+e))^m*(c-c*sin(f*x+e))^(-1+n)/f/(1+2*m-p)
```

Rubi [A]
time = 0.31, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {3005, 2932, 2768, 72, 71}

$$\frac{c2^{n-\frac{p}{2}+\frac{1}{2}}\cos(e+fx)(a\sin(e+fx)+a)^m(c-c\sin(e+fx))^{n-1}(g\sec(e+fx))^p(1-\sin(e+fx))^{\frac{1}{2}(-2n+p+1)}\,{}_2F_1\left(\frac{1}{2}(2m-p+1),\frac{1}{2}(-2n+p+1);\frac{1}{2}(2m-p+3);\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m-p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 + n - p/2)*c*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m - p)/2, (1 - 2
*n + p)/2, (3 + 2*m - p)/2, (1 + Sin[e + f*x])/2]*(g*Sec[e + f*x])^p*(1 - S
in[e + f*x])^((1 - 2*n + p)/2)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^
(-1 + n))/(f*(1 + 2*m - p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2932

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*
x])^FracPart[m]/(g^(2*IntPart[m])*(g*Cos[e + f*x])^(2*FracPart[m]))), Int[(
g*Cos[e + f*x])^(2*m + p)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] &
& (FractionQ[m] ||  !FractionQ[n])
```

Rule 3005

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(g*Cos[e + f*x])^p),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c-c\sin(e+fx))^n\,dx = ((g\cos(e+fx))^p(g\sec(e+fx))^p)\int (g\cos($$

$$= \left((g\cos(e+fx))^{-2m+p}(g\sec(e+fx))^p(a+\right.$$

$$= \frac{\left(c^2\cos(e+fx)(g\sec(e+fx))^p(a+a\sin(e\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+n-\frac{p}{2}}c^2\cos(e+fx)(g\sec(e+fx))^p(a\right.}{}$$

$$= \frac{2^{\frac{1}{2}+n-\frac{p}{2}}c\cos(e+fx)\,{}_2F_1\left(\frac{1}{2}(1+2m-p),\frac{1}{2}(\right.}{}$$

Mathematica [A]
time = 31.50, size = 139, normalized size = 1.01

$$\frac{2\,{}_2F_1\left(1+m+n-p,\frac{1}{2}+n-\frac{p}{2};\frac{3}{2}+n-\frac{p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(g\sec(e+fx))^p\sec^2\left(\frac{1}{4}(2e-\pi+2fx)\right)^{m+n-p}(a(1+\sin(e+fx)))^m(c-c\sin(e+fx))^n\tan\left(\frac{1}{4}(2e-\pi+2fx)\right)}{f(1+2n-p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n,
x]
```

```
[Out] (2*Hypergeometric2F1[1 + m + n - p, 1/2 + n - p/2, 3/2 + n - p/2, -Tan[(2*e
 - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^p*(Sec[(2*e - Pi + 2*f*x)/4]^2)^(m +
n - p)*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n*Tan[(2*e - Pi + 2*f*
x)/4])/(f*(1 + 2*n - p))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(g \sec \left(f x +e \right)\right)^{p} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(c -c \sin \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n,
 x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c-c*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)
```

```
[Out] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)
```

3.190 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}$$

[Out] 1/3*a*sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d

Rubi [A]
time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]

[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (ax^2+x^3)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a\sin^3(c+dx)}{3d} + \frac{a\sin^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{a\sin^3(c+dx)}{3d} + \frac{a\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.12, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4}+\frac{a\left(\sin^3(dx+c)\right)}{3}}{d}$	28
default	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4}+\frac{a\left(\sin^3(dx+c)\right)}{3}}{d}$	28
risch	$\frac{a\sin(dx+c)}{4d}+\frac{a\cos(4dx+4c)}{32d}-\frac{a\sin(3dx+3c)}{12d}-\frac{a\cos(2dx+2c)}{8d}$	59
norman	$\frac{\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*a*sin(d*x+c)^4+1/3*a*sin(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.85

$$\frac{3\,a\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 50, normalized size = 1.52

$$\frac{3\,a\cos\left(dx+c\right)^{4}-6\,a\cos\left(dx+c\right)^{2}-4\,\left(a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(3*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 - 4*(a*cos(d*x + c)^2 - a)*si
n(d*x + c))/d
```

Sympy [A]
time = 0.19, size = 42, normalized size = 1.27

$$\begin{cases}\frac{a\sin^{4}\left(c+dx\right)}{4d}+\frac{a\sin^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\ x(a\sin\left(c\right)+a)\sin^{2}\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
[Out] Piecewise((a*sin(c + d*x)**4/(4*d) + a*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)**2*cos(c), True))
```

Giac [A]
time = 0.45, size = 28, normalized size = 0.85

$$\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3)/d
```

Mupad [B]
time = 0.05, size = 24, normalized size = 0.73

$$\frac{a\sin\left(c+d\,x\right)^{3}\,\left(3\,\sin\left(c+d\,x\right)+4\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
[Out] (a*sin(c + d*x)^3*(3*sin(c + d*x) + 4))/(12*d)
```

3.191 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 1/2*a*sin(d*x+c)^2/d+1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {2912, 12, 45}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}(\int x(a+x)\,dx, x, a\sin(c+dx))}{a^2 d}$$
$$= \frac{\text{Subst}(\int (ax+x^2)\,dx, x, a\sin(c+dx))}{a^2 d}$$
$$= \frac{a\sin^2(c+dx)}{2d} + \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.07, size = 30, normalized size = 0.91

$$\frac{-3a\cos(2(c+dx)) + 4a\sin^3(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (-3*a*Cos[2*(c + d*x)] + 4*a*Sin[c + d*x]^3)/(12*d)
```

Maple [A]
time = 0.06, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}+\frac{a\left(\sin^2(dx+c)\right)}{2}}{d}$	28
default	$\frac{\frac{a\left(\sin^3(dx+c)\right)}{3}+\frac{a\left(\sin^2(dx+c)\right)}{2}}{d}$	28
risch	$\frac{a\sin(dx+c)}{4d} - \frac{a\sin(3dx+3c)}{12d} - \frac{a\cos(2dx+2c)}{4d}$	44
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a*sin(d*x+c)^3+1/2*a*sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.85

$$\frac{2\,a\sin\left(dx+c\right)^3 + 3\,a\sin\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 39, normalized size = 1.18

$$-\frac{3\,a\cos\left(dx+c\right)^2+2\,\left(a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*a*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.12, size = 41, normalized size = 1.24

$$\begin{cases} \frac{a\sin^3\left(c+dx\right)}{3d}+\frac{a\sin^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x(a\sin\left(c\right)+a)\sin\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] Piecewise((a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x
*(a*sin(c) + a)*sin(c)*cos(c), True))
```

Giac [A]
time = 0.42, size = 28, normalized size = 0.85

$$\frac{2\,a\sin\left(dx+c\right)^3+3\,a\sin\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2)/d
```

Mupad [B]
time = 0.04, size = 24, normalized size = 0.73

$$\frac{a\sin\left(c+d\,x\right)^2\,\left(2\,\sin\left(c+d\,x\right)+3\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x)),x)
[Out] (a*sin(c + d*x)^2*(2*sin(c + d*x) + 3))/(6*d)
```

3.192 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2786, 45}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]

[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cot(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.08

$$\frac{a(\log(\cos(c+dx))+\log(\tan(c+dx))+\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

Maple [A]
time = 0.09, size = 20, normalized size = 0.83

method	result	size
derivativedivides	$\frac{a(\sin(dx+c)+\ln(\sin(dx+c)))}{d}$	20
default	$\frac{a(\sin(dx+c)+\ln(\sin(dx+c)))}{d}$	20
risch	$-iax-\frac{2iac}{d}+\frac{a\ln(e^{2i(dx+c)}-1)}{d}+\frac{a\sin(dx+c)}{d}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*(sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.92

$$\frac{a\log\left(\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 24, normalized size = 1.00

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos\left(c+dx\right)\csc\left(c+dx\right)dx+\int \sin\left(c+dx\right)\cos\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c + d
*x)*csc(c + d*x), x))
```

Giac [A]
time = 0.46, size = 23, normalized size = 0.96

$$\frac{a\log\left(\left|\sin\left(dx+c\right)\right|\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

Mupad [B]
time = 8.81, size = 38, normalized size = 1.58

$$\frac{a\left(\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x),x)
[Out] (a*(log(tan(c/2 + (d*x)/2)) + sin(c + d*x) - log(tan(c/2 + (d*x)/2)^2 + 1))
)/d
```

3.193 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=25

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2912, 12, 45}

$$\frac{a\log(\sin(c+dx))}{d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{a+x}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a}{x^2}+\frac{1}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d} + \frac{a\log(\sin(c+dx))}{d}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.32

$$-\frac{a\csc(c+dx)}{d} + \frac{a(\log(\cos(c+dx)) + \log(\tan(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) + (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d
```

Maple [A]
time = 0.06, size = 24, normalized size = 0.96

method	result	size
derivativedivides	$\frac{a\left(\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$	24
default	$\frac{a\left(\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$	24
risch	$-iax-\frac{2iac}{d}-\frac{2ia\,e^{i(dx+c)}}{d(e^{2i(dx+c)}-1)}+\frac{a\ln(e^{2i(dx+c)}-1)}{d}$	61
norman	$\frac{-\frac{a}{2d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*(ln(sin(d*x+c))-1/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 25, normalized size = 1.00

$$\frac{a\log\left(\sin\left(dx+c\right)\right) - \frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) - a/sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 33, normalized size = 1.32

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-a}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c))*sin(d*x + c) - a)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^{2}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**2, x))
```

Giac [A]
time = 0.45, size = 26, normalized size = 1.04

$$\frac{a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{a}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) - a/sin(d*x + c))/d
```

Mupad [B]
time = 8.58, size = 55, normalized size = 2.20

$$-\frac{a\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] -(a*(tan(c/2 + (d*x)/2) - 2*log(tan(c/2 + (d*x)/2)) + 2*log(tan(c/2 + (d*x)
/2)^2 + 1) + 1/tan(c/2 + (d*x)/2)))/(2*d)
```

3.194 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a+a\sin(c+dx))^2}{2ad}$$

[Out] -1/2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2/a/d

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 37}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^2}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]

[Out] -1/2*(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2)/(a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{a+x}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^2}{2ad}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 0.97

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.09, size = 27, normalized size = 0.90

method	result	size
derivativedivides	$\frac{a\left(-\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$	27
default	$\frac{a\left(-\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$	27
risch	$-\frac{2ia\left(i e^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}$	54
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] a/d*(-1/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 24, normalized size = 0.80

$$-\frac{2\,a\sin\left(dx+c\right)+a}{2\,d\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

Fricas [A]
time = 0.35, size = 29, normalized size = 0.97

$$\frac{2 a \sin (dx+c)+a}{2 \left(d \cos (dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*sin(d*x + c) + a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \cos (c+dx) \csc^3 (c+dx)\, dx + \int \sin (c+dx) \cos (c+dx) \csc^3 (c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**3, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**3, x))
```

Giac [A]
time = 0.45, size = 24, normalized size = 0.80

$$-\frac{2 a \sin (dx+c)+a}{2 d \sin (dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*sin(d*x + c) + a)/(d*sin(d*x + c)^2)
```

Mupad [B]
time = 8.52, size = 25, normalized size = 0.83

$$-\frac{\frac{a}{2}+a\,\sin \left(c+d\,x\right)}{d\sin \left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] -(a/2 + a*sin(c + d*x))/(d*sin(c + d*x)^2)
```

3.195 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] -1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d
```

Rubi [A]

time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2912, 12, 45}

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{a+x}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{a}{x^4}+\frac{1}{x^3}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.09, size = 27, normalized size = 0.82

method	result	si
derivativedivides	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$	27
default	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$	27
risch	$\frac{2a\left(4i\,e^{3i(dx+c)}+3\,e^{4i(dx+c)}-3\,e^{2i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$	55
norman	$\frac{-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(-1/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.79

$$-\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

Fricas [A]
time = 0.34, size = 39, normalized size = 1.18

$$\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*sin(d*x + c) + 2*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^{4}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**4, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**4, x))
```

Giac [A]
time = 0.43, size = 26, normalized size = 0.79

$$-\frac{3\,a\sin\left(dx+c\right)+2\,a}{6\,d\sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*sin(d*x + c) + 2*a)/(d*sin(d*x + c)^3)
```

Mupad [B]
time = 8.60, size = 39, normalized size = 1.18

$$-\frac{\frac{5\,a\,\sin(c+d\,x)}{16}+\frac{a\,\left(\frac{3\,\sin(3\,c+3\,d\,x)}{16}+1\right)}{3}}{d\sin\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] -((5*a*sin(c + d*x))/16 + (a*((3*sin(3*c + 3*d*x))/16 + 1))/3)/(d*sin(c + d
*x)^3)
```

3.196 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}$$

```
[Out] -1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
```

Rubi [A]

time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2912, 12, 45}

$$-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{a+x}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(\frac{a}{x^5}+\frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{a\csc^3(c+dx)}{3d} - \frac{a\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.11, size = 27, normalized size = 0.82

method	result
derivativedivides	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
default	$\frac{a\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
risch	$\frac{4ia\left(3i\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{5i(dx+c)}-2\,\mathrm{e}^{3i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}-\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{5a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a(\mathrm{ta}}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] a/d*(-1/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.79

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

Fricas [A]
time = 0.37, size = 40, normalized size = 1.21

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos\left(c+dx\right)\csc^{5}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)*csc(c + d*x)**5, x) + Integral(sin(c + d*x)*cos(c
+ d*x)*csc(c + d*x)**5, x))
```

Giac [A]
time = 0.45, size = 26, normalized size = 0.79

$$-\frac{4\,a\sin\left(dx+c\right)+3\,a}{12\,d\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*a*sin(d*x + c) + 3*a)/(d*sin(d*x + c)^4)
```

Mupad [B]
time = 8.56, size = 26, normalized size = 0.79

$$-\frac{\frac{a}{4}+\frac{a\,\sin\left(c+d\,x\right)}{3}}{d\,\sin\left(c+d\,x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] -(a/4 + (a*sin(c + d*x))/3)/(d*sin(c + d*x)^4)
```

3.197 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^5(c+dx)}{5d}$$

```
[Out] 1/3*a^2*sin(d*x+c)^3/d+1/2*a^2*sin(d*x+c)^4/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(2*d) + (a^2*Sin[c + d*x]
^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a^2x^2+2ax^3+x^4)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin^4(c+dx)}{2d} + \frac{a^2\sin^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.27, size = 53, normalized size = 0.96

$$\frac{a^2(15\cos(4(c+dx)) + 104\sin^3(c+dx) - 12\cos(2(c+dx))\,(5+2\sin^3(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(15*Cos[4*(c + d*x)] + 104*Sin[c + d*x]^3 - 12*Cos[2*(c + d*x)]*(5 + 2
*Sin[c + d*x]^3)))/(240*d)
```

Maple [A]
time = 0.10, size = 45, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^2\left(\sin^5(dx+c)\right)}{5}+\frac{a^2\left(\sin^4(dx+c)\right)}{2}+\frac{a^2\left(\sin^3(dx+c)\right)}{3}}{d}$	45
default	$\frac{\frac{a^2\left(\sin^5(dx+c)\right)}{5}+\frac{a^2\left(\sin^4(dx+c)\right)}{2}+\frac{a^2\left(\sin^3(dx+c)\right)}{3}}{d}$	45
risch	$\frac{3a^2\sin(dx+c)}{8d}+\frac{a^2\sin(5dx+5c)}{80d}+\frac{a^2\cos(4dx+4c)}{16d}-\frac{7a^2\sin(3dx+3c)}{48d}-\frac{a^2\cos(2dx+2c)}{4d}$	84
norman	$\frac{\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{176a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/5*a^2*sin(d*x+c)^5+1/2*a^2*sin(d*x+c)^4+1/3*a^2*sin(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.82

$$\frac{6\,a^2\sin\left(dx+c\right)^5 + 15\,a^2\sin\left(dx+c\right)^4 + 10\,a^2\sin\left(dx+c\right)^3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

Fricas [A]
time = 0.35, size = 72, normalized size = 1.31

$$\frac{15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2+2\,\left(3\,a^2\cos\left(dx+c\right)^4-11\,a^2\cos\left(dx+c\right)^2+8\,a^2\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^2 + 2*(3*a^2*cos(d*x + c)
^4 - 11*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.27, size = 63, normalized size = 1.15

$$\begin{cases} \frac{a^2\sin^5\left(c+dx\right)}{5d}+\frac{a^2\sin^4\left(c+dx\right)}{2d}+\frac{a^2\sin^3\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x(a\sin\left(c\right)+a)^2\sin^2\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*sin(c + d*x)**5/(5*d) + a**2*sin(c + d*x)**4/(2*d) + a**2*s
in(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c), Tru
e))
```

Giac [A]
time = 0.52, size = 45, normalized size = 0.82

$$\frac{6\,a^2\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4+10\,a^2\sin\left(dx+c\right)^3}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3)
/d
```

Mupad [B]
time = 8.49, size = 36, normalized size = 0.65

$$\frac{a^2\sin\left(c+d\,x\right)^3\,\left(6\sin\left(c+d\,x\right)^2+15\,\sin\left(c+d\,x\right)+10\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
[Out] (a^2*sin(c + d*x)^3*(15*sin(c + d*x) + 6*sin(c + d*x)^2 + 10))/(30*d)
```

3.198 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{4d}$$

```
[Out] 1/2*a^2*sin(d*x+c)^2/d+2/3*a^2*sin(d*x+c)^3/d+1/4*a^2*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2912, 12, 45}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int (a^2x+2ax^2+x^3)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{a^2\sin^2(c+dx)}{2d} + \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.69

$$\frac{a^2\sin^2(c+dx)\left(6+8\sin(c+dx)+3\sin^2(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*Sin[c + d*x]^2*(6 + 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*d)
```

Maple [A]
time = 0.08, size = 45, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4}+\frac{2a^2\left(\sin^3(dx+c)\right)}{3}+\frac{a^2\left(\sin^2(dx+c)\right)}{2}}{d}$	45
default	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4}+\frac{2a^2\left(\sin^3(dx+c)\right)}{3}+\frac{a^2\left(\sin^2(dx+c)\right)}{2}}{d}$	45
risch	$\frac{a^2\sin(dx+c)}{2d}+\frac{a^2\cos(4dx+4c)}{32d}-\frac{a^2\sin(3dx+3c)}{6d}-\frac{3a^2\cos(2dx+2c)}{8d}$	67
norman	$\frac{\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{16a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/4*a^2*sin(d*x+c)^4+2/3*a^2*sin(d*x+c)^3+1/2*a^2*sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.82

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.34, size = 58, normalized size = 1.05

$$\frac{3\,a^2\cos\left(dx+c\right)^4-12\,a^2\cos\left(dx+c\right)^2-8\,\left(a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] 1/12*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 - 8*(a^2*cos(d*x + c)^2
- a^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.20, size = 63, normalized size = 1.15

$$\begin{cases}\frac{a^2\sin^4\left(c+dx\right)}{4d}+\frac{2a^2\sin^3\left(c+dx\right)}{3d}+\frac{a^2\sin^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x(a\sin\left(c\right)+a)^2\sin\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
[Out] Piecewise((a**2*sin(c + d*x)**4/(4*d) + 2*a**2*sin(c + d*x)**3/(3*d) + a**2
*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c), True
))
```

Giac [A]
time = 0.44, size = 45, normalized size = 0.82

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2)/d
```

Mupad [B]
time = 8.51, size = 36, normalized size = 0.65

$$\frac{a^2\sin\left(c+d\,x\right)^2\,\left(3\sin\left(c+d\,x\right)^2+8\,\sin\left(c+d\,x\right)+6\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
[Out] (a^2*sin(c + d*x)^2*(8*sin(c + d*x) + 3*sin(c + d*x)^2 + 6))/(12*d)
```

3.199 $\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2786, 45}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^2\,dx&=\frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.00

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(
2*d)
```

Maple [A]
time = 0.07, size = 34, normalized size = 0.72

method	result	size
derivativedivides	$\frac{a^2\left(\frac{\left(\sin^2(dx+c)\right)}{2}+2\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$	34
default	$\frac{a^2\left(\frac{\left(\sin^2(dx+c)\right)}{2}+2\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$	34
risch	$-ia^2x-\frac{a^2 e^{2i(dx+c)}}{8d}-\frac{a^2 e^{-2i(dx+c)}}{8d}-\frac{2ia^2c}{d}+\frac{a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{2a^2\sin(dx+c)}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] a^2/d*(1/2*sin(d*x+c)^2+2*sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.87

$$\frac{a^2\sin\left(dx+c\right)^2+2\,a^2\log\left(\sin\left(dx+c\right)\right)+4\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 43, normalized size = 0.91

$$-\frac{a^2\cos\left(dx+c\right)^2-2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a^2*sin(d*x + c)
)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos(c+dx)\csc(c+dx)\,dx+\int 2\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx+\int\sin^2(c+dx)\cos(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(2*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc(c + d
*x), x))
```

Giac [A]
time = 0.44, size = 42, normalized size = 0.89

$$\frac{a^2\sin(dx+c)^2+2\,a^2\log(|\sin(dx+c)|)+4\,a^2\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a^2*sin(d*x + c)
)/d
```

Mupad [B]
time = 8.99, size = 119, normalized size = 2.53

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (2*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c
/2 + (d*x)/2)^3 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 1)) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.200 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)}{d}$$

```
[Out] -a^2*csc(d*x+c)/d+2*a^2*ln(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d + (a^2*Sin[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}+\frac{2a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc(c+dx)}{d} + \frac{2a^2\log(\sin(c+dx))}{d} + \frac{a^2\sin(c+dx}{d}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.88

$$a^2\left(-\frac{\csc(c+dx)}{d} + \frac{2\log(\sin(c+dx))}{d} + \frac{\sin(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] a^2*(-(Csc[c + d*x]/d) + (2*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

Maple [A]
time = 0.08, size = 34, normalized size = 0.79

method	result
derivativedivides	$\frac{a^2\left(\sin(dx+c)+2\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
default	$\frac{a^2\left(\sin(dx+c)+2\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
risch	$-2ia^2x - \frac{ia^2e^{i(dx+c)}}{2d} + \frac{ia^2e^{-i(dx+c)}}{2d} - \frac{4ia^2c}{d} - \frac{2ia^2e^{i(dx+c)}}{d(e^{2i(dx+c)}-1)} + \frac{2a^2\ln(e^{2i(dx+c)}-1)}{d}$
norman	$\frac{-\frac{a^2}{2d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{2a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{2a^2\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] a^2/d*(sin(d*x+c)+2*ln(sin(d*x+c))-1/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.93

$$\frac{2\,a^2\log\left(\sin\left(dx+c\right)\right)+a^2\sin\left(dx+c\right)-\frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(sin(d*x + c)) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

Fricas [A]
time = 0.42, size = 46, normalized size = 1.07

$$-\frac{a^2 \cos \left(dx+c\right)^2 - 2\,a^2 \log \left(\frac{1}{2}\, \sin \left(dx+c\right)\right) \sin \left(dx+c\right)}{d \sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c))*sin(d*x + c))/(d*sin(d*x
 + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \cos \left(c+dx\right) \csc^2 \left(c+dx\right) dx + \int 2 \sin \left(c+dx\right) \cos \left(c+dx\right) \csc^2 \left(c+dx\right) dx + \int \sin^2 \left(c+dx\right) \cos \left(c+dx\right) \csc^2 \left(c+dx\right) dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc
(c + d*x)**2, x))
```

Giac [A]
time = 0.46, size = 41, normalized size = 0.95

$$\frac{2\,a^2 \log \left(\left|\sin \left(dx+c\right)\right|\right) + a^2 \sin \left(dx+c\right) - \frac{a^2}{\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*log(abs(sin(d*x + c))) + a^2*sin(d*x + c) - a^2/sin(d*x + c))/d
```

Mupad [B]
time = 8.89, size = 111, normalized size = 2.58

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (3*a^2*tan(c/2 + (d*x)/2)^2 - a^2)/(d*(
2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^3)) - (a^2*tan(c/2 + (d*x)/2))/
(2*d) - (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.201 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=47

$$-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d+a^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) + (a^2*Log[Sin[c + d*x
]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} + \frac{a^2\log(\sin(c+d}{d}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.89

$$a^2\left(-\frac{2\csc(c+dx)}{d} - \frac{\csc^2(c+dx)}{2d} + \frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*((-2*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + Log[Sin[c + d*x]]/d)
```

Maple [A]
time = 0.12, size = 36, normalized size = 0.77

method	result
derivativedivides	$\frac{a^2\left(\ln(\sin(dx+c))-\frac{2}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
default	$\frac{a^2\left(\ln(\sin(dx+c))-\frac{2}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2ia^2\left(ie^{2i(dx+c)}+2\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}$
norman	$\frac{-\frac{a^2}{8d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(ln(sin(d*x+c))-2/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.91

$$\frac{2a^2\log\left(\sin\left(dx+c\right)\right)-\frac{4a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(sin(d*x + c)) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.39, size = 62, normalized size = 1.32

$$\frac{4a^2\sin\left(dx+c\right)+a^2+2\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*sin(d*x + c) + a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*
x + c)))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int2\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x)**3, x) + Integral(2*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc
(c + d*x)**3, x))
```

Giac [A]
time = 0.54, size = 44, normalized size = 0.94

$$\frac{2a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{4a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(sin(d*x + c))) - (4*a^2*sin(d*x + c) + a^2)/sin(d*x + c)
^2)/d
```

Mupad [B]
time = 8.87, size = 111, normalized size = 2.36

$$\frac{a^2 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{8}+a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (cot(c
/2 + (d*x)/2)^2*(a^2/8 + a^2*tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/
2))/d - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.202 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^3(c+dx)(a+a\sin(c+dx))^3}{3ad}$$

[Out] -1/3*csc(d*x+c)^3*(a+a*sin(d*x+c))^3/a/d

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 37}

$$-\frac{\csc^3(c+dx)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]

[Out] -1/3*(Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(a*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)(a+a\sin(c+dx))^3}{3ad}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.67

$$-\frac{a^2(1+\csc(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*(1 + Csc[c + d*x])^3)/d
```

Maple [A]
time = 0.11, size = 39, normalized size = 1.30

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{\sin(dx+c)}-\frac{1}{\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
default	$\frac{a^2\left(-\frac{1}{\sin(dx+c)}-\frac{1}{\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
risch	$-\frac{2ia^2\left(3\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}+6ie^{4i(dx+c)}+3\,e^{i(dx+c)}-6ie^{2i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{-\frac{a^2}{24d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{17a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{23a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{23a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{17a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a^2\left(\tan^9\left(\right.\right.}{4}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1/sin(d*x+c)-1/sin(d*x+c)^2-1/3/sin(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 1.37

$$-\frac{3\,a^2\sin\left(dx+c\right)^2+3\,a^2\sin\left(dx+c\right)+a^2}{3\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

Fricas [A]
time = 0.37, size = 56, normalized size = 1.87

$$-\frac{3\,a^2\cos\left(dx+c\right)^2-3\,a^2\sin\left(dx+c\right)-4\,a^2}{3\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^2 - 3*a^2*sin(d*x + c) - 4*a^2)/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos\left(c+dx\right)\csc^4\left(c+dx\right)dx+\int 2\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^4\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cos\left(c+dx\right)\csc^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)*csc(c + d*x)**4, x) + Integral(2*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**4, x) + Integral(sin(c + d*x)**2*cos(c + d*x)*csc
(c + d*x)**4, x))
```

Giac [A]
time = 0.46, size = 41, normalized size = 1.37

$$-\frac{3\,a^2\sin\left(dx+c\right)^2+3\,a^2\sin\left(dx+c\right)+a^2}{3\,d\sin\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/(d*sin(d*x + c)^3)
```

Mupad [B]
time = 8.90, size = 41, normalized size = 1.37

$$-\frac{a^2\sin\left(c+d\,x\right)^2+a^2\,\sin\left(c+d\,x\right)+\frac{a^2}{3}}{d\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] -(a^2*sin(c + d*x) + a^2/3 + a^2*sin(c + d*x)^2)/(d*sin(c + d*x)^3)
```

3.203 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}$$

```
[Out] -1/2*a^2*csc(d*x+c)^2/d-2/3*a^2*csc(d*x+c)^3/d-1/4*a^2*csc(d*x+c)^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2912, 12, 45}

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Csc[c + d*x]^2)/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(\frac{a^2}{x^5}+\frac{2a}{x^4}+\frac{1}{x^3}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*Csc[c + d*x]^2)/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d
*x]^4)/(4*d)
```

Maple [A]
time = 0.13, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
default	$\frac{a^2\left(-\frac{1}{4\sin(dx+c)^4}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
risch	$\frac{2a^2\left(3\,\mathrm{e}^{6i(dx+c)}-12\,\mathrm{e}^{4i(dx+c)}+8i\mathrm{e}^{5i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}-8i\mathrm{e}^{3i(dx+c)}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}-\frac{7a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{5a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{5a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{5a^2\left(\tan^9\left(\frac{dx}{2}+\right.\right.}{12d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1/4/sin(d*x+c)^4-2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.78

$$-\frac{6\,a^2\sin\left(dx+c\right)^2+8\,a^2\sin\left(dx+c\right)+3\,a^2}{12\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

Fricas [A]
time = 0.38, size = 57, normalized size = 1.04

$$\frac{6\,a^2\cos\left(dx+c\right)^2-8\,a^2\sin\left(dx+c\right)-9\,a^2}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*cos(d*x + c)^2 - 8*a^2*sin(d*x + c) - 9*a^2)/(d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 43, normalized size = 0.78

$$-\frac{6\,a^2\sin\left(dx+c\right)^2+8\,a^2\sin\left(dx+c\right)+3\,a^2}{12\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*a^2*sin(d*x + c)^2 + 8*a^2*sin(d*x + c) + 3*a^2)/(d*sin(d*x + c)^4
)
```

Mupad [B]
time = 8.85, size = 43, normalized size = 0.78

$$-\frac{\frac{a^2\sin(c+d\,x)^2}{2}+\frac{2\,a^2\,\sin(c+d\,x)}{3}+\frac{a^2}{4}}{d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] -((2*a^2*sin(c + d*x))/3 + a^2/4 + (a^2*sin(c + d*x)^2)/2)/(d*sin(c + d*x)^
4)
```

3.204 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^5(c+dx)}{5d}$$

```
[Out] -1/3*a^2*csc(d*x+c)^3/d-1/2*a^2*csc(d*x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Csc[c + d*x]^3)/d - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^5\text{Subst}\left(\int \frac{(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{a^2}{x^6}+\frac{2a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{2d} - \frac{a^2\csc^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{2d} - \frac{a^2\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*Csc[c + d*x]^3)/d - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x
]^5)/(5*d)
```

Maple [A]
time = 0.14, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{2\sin(dx+c)^4}\right)}{d}$
default	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3}-\frac{1}{5\sin(dx+c)^5}-\frac{1}{2\sin(dx+c)^4}\right)}{d}$
risch	$\frac{8ia^2\left(5\,e^{7i(dx+c)}-22\,e^{5i(dx+c)}+15ie^{6i(dx+c)}+5\,e^{3i(dx+c)}-15ie^{4i(dx+c)}\right)}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a^2}{160d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}-\frac{41a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}-\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{163a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}-\frac{61a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{61a^2\left(\tan^8 \right.}{9}}{\tan\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1/3/sin(d*x+c)^3-1/5/sin(d*x+c)^5-1/2/sin(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.78

$$-\frac{10\,a^2\sin\left(dx+c\right)^2+15\,a^2\sin\left(dx+c\right)+6\,a^2}{30\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)
^5)
```

Fricas [A]
time = 0.34, size = 65, normalized size = 1.18

$$\frac{10\,a^2\cos\left(dx+c\right)^2-15\,a^2\sin\left(dx+c\right)-16\,a^2}{30\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*a^2*cos(d*x + c)^2 - 15*a^2*sin(d*x + c) - 16*a^2)/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.47, size = 43, normalized size = 0.78

$$-\frac{10\,a^2\sin\left(dx+c\right)^2+15\,a^2\sin\left(dx+c\right)+6\,a^2}{30\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(10*a^2*sin(d*x + c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/(d*sin(d*x + c)
^5)
```

Mupad [B]
time = 8.92, size = 43, normalized size = 0.78

$$-\frac{\frac{a^2\sin(c+d\,x)^2}{3}+\frac{a^2\,\sin(c+d\,x)}{2}+\frac{a^2}{5}}{d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
[Out] -((a^2*sin(c + d*x))/2 + a^2/5 + (a^2*sin(c + d*x)^2)/3)/(d*sin(c + d*x)^5)
```

3.205 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^6(c+dx)}{6d}$$

```
[Out] -1/4*a^2*csc(d*x+c)^4/d-2/5*a^2*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Csc[c + d*x]^4)/d - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{(a+x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{a^2}{x^7}+\frac{2a}{x^6}+\frac{1}{x^5}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^2\csc^4(c+dx)}{4d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\csc^6(c+dx)}{6d}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 1.00

$$-\frac{a^2\csc^4(c+dx)}{4d} - \frac{2a^2\csc^5(c+dx)}{5d} - \frac{a^2\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(a^2*Csc[c + d*x]^4)/d - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d
*x]^6)/(6*d)
```

Maple [A]
time = 0.14, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{2}{5\sin(dx+c)^5}\right)}{d}$
default	$\frac{a^2\left(-\frac{1}{6\sin(dx+c)^6}-\frac{1}{4\sin(dx+c)^4}-\frac{2}{5\sin(dx+c)^5}\right)}{d}$
risch	$-\frac{4a^2\left(15\,e^{8i(dx+c)}-70\,e^{6i(dx+c)}+48i e^{7i(dx+c)}+15\,e^{4i(dx+c)}-48i e^{5i(dx+c)}\right)}{15d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$-\frac{a^2}{384d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}-\frac{7a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}-\frac{7a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{21a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{7a^2\left(\tan^7\left(\frac{dx}{2}\right.\right.}{16d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/d*(-1/6/sin(d*x+c)^6-1/4/sin(d*x+c)^4-2/5/sin(d*x+c)^5)
```

Maxima [A]
time = 0.30, size = 43, normalized size = 0.78

$$-\frac{15\,a^2\sin\left(dx+c\right)^2+24\,a^2\sin\left(dx+c\right)+10\,a^2}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

Fricas [A]
time = 0.35, size = 70, normalized size = 1.27

$$-\frac{15\,a^2\cos\left(dx+c\right)^2-24\,a^2\sin\left(dx+c\right)-25\,a^2}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*a^2*cos(d*x + c)^2 - 24*a^2*sin(d*x + c) - 25*a^2)/(d*cos(d*x + c
)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.59, size = 43, normalized size = 0.78

$$-\frac{15\,a^2\sin\left(dx+c\right)^2+24\,a^2\sin\left(dx+c\right)+10\,a^2}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/(d*sin(d*x + c
)^6)
```

Mupad [B]
time = 8.91, size = 43, normalized size = 0.78

$$-\frac{\frac{a^2\sin(c+d\,x)^2}{4}+\frac{2\,a^2\,\sin(c+d\,x)}{5}+\frac{a^2}{6}}{d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] -((2*a^2*sin(c + d*x))/5 + a^2/6 + (a^2*sin(c + d*x)^2)/4)/(d*sin(c + d*x)^
6)
```

3.206 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^7(c+dx)}{7d}$$

```
[Out] 1/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+1/2*a^3*sin(d*x+c)^6/d+1/7*a^
3*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*
x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int x^3(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^4d}$$

$$= \frac{\text{Subst}\left(\int \left(a^3x^3+3a^2x^4+3ax^5+x^6\right)\,dx, x, a\sin(c+dx\right.}{a^4d}$$

$$= \frac{a^3\sin^4(c+dx)}{4d} + \frac{3a^3\sin^5(c+dx)}{5d} + \frac{a^3\sin^6(c+dx)}{2d} + $$

Mathematica [A]
time = 0.25, size = 80, normalized size = 1.10

$$-\frac{a^3(-350+805\cos(2(c+dx))-280\cos(4(c+dx))+35\cos(6(c+dx))-1015\sin(c+dx)+525\sin(3(c+dx))-119\sin(5(c+dx))+5\sin(7(c+dx)))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2240*(a^3*(-350 + 805*Cos[2*(c + d*x)] - 280*Cos[4*(c + d*x)] + 35*Cos[6
*(c + d*x)] - 1015*Sin[c + d*x] + 525*Sin[3*(c + d*x)] - 119*Sin[5*(c + d*x
)] + 5*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.17, size = 58, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{a^3\left(\sin^7(dx+c)\right)}{7}+\frac{a^3\left(\sin^6(dx+c)\right)}{2}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{a^3\left(\sin^4(dx+c)\right)}{4}}{d}$
default	$\frac{\frac{a^3\left(\sin^7(dx+c)\right)}{7}+\frac{a^3\left(\sin^6(dx+c)\right)}{2}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{a^3\left(\sin^4(dx+c)\right)}{4}}{d}$
risch	$\frac{29a^3\sin(dx+c)}{64d}-\frac{a^3\sin(7dx+7c)}{448d}-\frac{a^3\cos(6dx+6c)}{64d}+\frac{17a^3\sin(5dx+5c)}{320d}+\frac{a^3\cos(4dx+4c)}{8d}-\frac{15a^3\sin(3dx+3c)}{64d}$
norman	$\frac{\frac{4a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{44a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{44a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{96a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{1984a^3\left(\tan^7\left(\right.\right.}{35d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/7*a^3*sin(d*x+c)^7+1/2*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+1/4*a^3
*sin(d*x+c)^4)
```

Maxima [A]
time = 0.26, size = 58, normalized size = 0.79

$$\frac{20\,a^3\sin(dx+c)^7+70\,a^3\sin(dx+c)^6+84\,a^3\sin(dx+c)^5+35\,a^3\sin(dx+c)^4}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.34, size = 98, normalized size = 1.34

$$-\frac{70\,a^3\cos(dx+c)^6-245\,a^3\cos(dx+c)^4+280\,a^3\cos(dx+c)^2+4\left(5\,a^3\cos(dx+c)^6-36\,a^3\cos(dx+c)^4+57\,a^3\cos(dx+c)^2-26\,a^3\right)\sin(dx+c)}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(70*a^3*cos(d*x + c)^6 - 245*a^3*cos(d*x + c)^4 + 280*a^3*cos(d*x +
c)^2 + 4*(5*a^3*cos(d*x + c)^6 - 36*a^3*cos(d*x + c)^4 + 57*a^3*cos(d*x + c
)^2 - 26*a^3)*sin(d*x + c))/d
```

Sympy [A]
time = 0.71, size = 80, normalized size = 1.10

$$\begin{cases}\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^3\sin^3(c)\cos(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**7/(7*d) + a**3*sin(c + d*x)**6/(2*d) + 3*a**3
*sin(c + d*x)**5/(5*d) + a**3*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c
) + a)**3*sin(c)**3*cos(c), True))
```

Giac [A]
time = 0.48, size = 58, normalized size = 0.79

$$\frac{20\,a^3\sin(dx+c)^7+70\,a^3\sin(dx+c)^6+84\,a^3\sin(dx+c)^5+35\,a^3\sin(dx+c)^4}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(20*a^3*sin(d*x + c)^7 + 70*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)^
5 + 35*a^3*sin(d*x + c)^4)/d
```

Mupad [B]
time = 0.07, size = 57, normalized size = 0.78

$$\frac{\frac{a^3\sin(c+d\,x)^7}{7}+\frac{a^3\sin(c+d\,x)^6}{2}+\frac{3\,a^3\sin(c+d\,x)^5}{5}+\frac{a^3\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^4)/4 + (3*a^3*sin(c + d*x)^5)/5 + (a^3*sin(c + d*x)^6)/2
 + (a^3*sin(c + d*x)^7)/7)/d
```

3.207 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^6(c+dx)}{6d}$$

```
[Out] 1/3*a^3*sin(d*x+c)^3/d+3/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+1/6*a^
3*sin(d*x+c)^6/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (3*a^3*Sin[c +
d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^3}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^2+3a^2x^3+3ax^4+x^5\right)\,dx, x, a\sin(c+dx\right.}{a^3d}$$
$$= \frac{a^3\sin^3(c+dx)}{3d} + \frac{3a^3\sin^4(c+dx)}{4d} + \frac{3a^3\sin^5(c+dx)}{5d} +$$

Mathematica [A]
time = 0.23, size = 70, normalized size = 0.96

$$-\frac{a^3(-45+870\cos(2(c+dx))-240\cos(4(c+dx))+10\cos(6(c+dx))-1200\sin(c+dx)+520\sin(3(c+dx))-72\sin(5(c+dx)))}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(a^3*(-45 + 870*Cos[2*(c + d*x)] - 240*Cos[4*(c + d*x)] + 10*Cos[6*
(c + d*x)] - 1200*Sin[c + d*x] + 520*Sin[3*(c + d*x)] - 72*Sin[5*(c + d*x)]
))/d
```

Maple [A]
time = 0.12, size = 58, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{a^3\left(\sin^6(dx+c)\right)}{6}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+\frac{a^3\left(\sin^3(dx+c)\right)}{3}}{d}$
default	$\frac{\frac{a^3\left(\sin^6(dx+c)\right)}{6}+\frac{3a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+\frac{a^3\left(\sin^3(dx+c)\right)}{3}}{d}$
risch	$\frac{5a^3\sin(dx+c)}{8d}-\frac{a^3\cos(6dx+6c)}{192d}+\frac{3a^3\sin(5dx+5c)}{80d}+\frac{a^3\cos(4dx+4c)}{8d}-\frac{13a^3\sin(3dx+3c)}{48d}-\frac{29a^3\cos(2dx+2c)}{64d}$
norman	$\frac{\frac{8a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{136a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{136a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{12a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{12a^3\left(\tan^8\left(\frac{d}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*a^3*sin(d*x+c)^6+3/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+1/3*a^3
*sin(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 0.79

$$\frac{10\,a^3\sin\left(dx+c\right)^6+36\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 85, normalized size = 1.16

$$-\frac{10\,a^3\cos\left(dx+c\right)^6-75\,a^3\cos\left(dx+c\right)^4+120\,a^3\cos\left(dx+c\right)^2-4\left(9\,a^3\cos\left(dx+c\right)^4-23\,a^3\cos\left(dx+c\right)^2+14\,a^3\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*a^3*cos(d*x + c)^6 - 75*a^3*cos(d*x + c)^4 + 120*a^3*cos(d*x + c)
^2 - 4*(9*a^3*cos(d*x + c)^4 - 23*a^3*cos(d*x + c)^2 + 14*a^3)*sin(d*x + c)
)/d
```

Sympy [A]
time = 0.44, size = 82, normalized size = 1.12

$$\begin{cases}\frac{a^3\sin^6\left(c+dx\right)}{6d}+\frac{3a^3\sin^5\left(c+dx\right)}{5d}+\frac{3a^3\sin^4\left(c+dx\right)}{4d}+\frac{a^3\sin^3\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x(a\sin\left(c\right)+a)^3\sin^2\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**6/(6*d) + 3*a**3*sin(c + d*x)**5/(5*d) + 3*a*
*3*sin(c + d*x)**4/(4*d) + a**3*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin
(c) + a)**3*sin(c)**2*cos(c), True))
```

Giac [A]
time = 0.51, size = 58, normalized size = 0.79

$$\frac{10\,a^3\sin\left(dx+c\right)^6+36\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4
 + 20*a^3*sin(d*x + c)^3)/d
```

Mupad [B]
time = 0.06, size = 57, normalized size = 0.78

$$\frac{\frac{a^3\sin(c+dx)^6}{6}+\frac{3a^3\sin(c+dx)^5}{5}+\frac{3a^3\sin(c+dx)^4}{4}+\frac{a^3\sin(c+dx)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^3)/3 + (3*a^3*sin(c + d*x)^4)/4 + (3*a^3*sin(c + d*x)^5)
/5 + (a^3*sin(c + d*x)^6)/6)/d
```

3.208 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=45

$$-\frac{(a+a\sin(c+dx))^4}{4ad}+\frac{(a+a\sin(c+dx))^5}{5a^2d}$$

```
[Out] -1/4*(a+a*sin(d*x+c))^4/a/d+1/5*(a+a*sin(d*x+c))^5/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^5}{5a^2d}-\frac{(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(a + a*Sin[c + d*x])^4/(a*d) + (a + a*Sin[c + d*x])^5/(5*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^3+(a+x)^4\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^4}{4ad}+\frac{(a+a\sin(c+dx))^5}{5a^2d}$$

Mathematica [A]
time = 0.10, size = 30, normalized size = 0.67

$$\frac{a^3(1+\sin(c+dx))^4(-1+4\sin(c+dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(1 + Sin[c + d*x])^4*(-1 + 4*Sin[c + d*x]))/(20*d)
```

Maple [A]
time = 0.10, size = 57, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+a^3\left(\sin^3(dx+c)\right)+\frac{a^3\left(\sin^2(dx+c)\right)}{2}}{d}$
default	$\frac{\frac{a^3\left(\sin^5(dx+c)\right)}{5}+\frac{3a^3\left(\sin^4(dx+c)\right)}{4}+a^3\left(\sin^3(dx+c)\right)+\frac{a^3\left(\sin^2(dx+c)\right)}{2}}{d}$
risch	$\frac{7a^3\sin(dx+c)}{8d}+\frac{a^3\sin(5dx+5c)}{80d}+\frac{3a^3\cos(4dx+4c)}{32d}-\frac{5a^3\sin(3dx+3c)}{16d}-\frac{5a^3\cos(2dx+2c)}{8d}$
norman	$\frac{\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{112a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{18a^3\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/5*a^3*sin(d*x+c)^5+3/4*a^3*sin(d*x+c)^4+a^3*sin(d*x+c)^3+1/2*a^3*sin
(d*x+c)^2)
```

Maxima [A]
time = 0.26, size = 58, normalized size = 1.29

$$\frac{4\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3+10\,a^3\sin\left(dx+c\right)^2}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 71, normalized size = 1.58

$$\frac{15\,a^3\cos\left(dx+c\right)^4-40\,a^3\cos\left(dx+c\right)^2+4\left(a^3\cos\left(dx+c\right)^4-7\,a^3\cos\left(dx+c\right)^2+6\,a^3\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(15*a^3*cos(d*x + c)^4 - 40*a^3*cos(d*x + c)^2 + 4*(a^3*cos(d*x + c)^4
 - 7*a^3*cos(d*x + c)^2 + 6*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.
time = 0.30, size = 76, normalized size = 1.69

$$\begin{cases} \frac{a^3\sin^5\left(c+dx\right)}{5d}+\frac{3a^3\sin^4\left(c+dx\right)}{4d}+\frac{a^3\sin^3\left(c+dx\right)}{d}+\frac{a^3\sin^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x(a\sin\left(c\right)+a)^3\sin\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*sin(c + d*x)**5/(5*d) + 3*a**3*sin(c + d*x)**4/(4*d) + a**3
*sin(c + d*x)**3/d + a**3*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) +
a)**3*sin(c)*cos(c), True))
```

Giac [A]
time = 0.47, size = 58, normalized size = 1.29

$$\frac{4\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3+10\,a^3\sin\left(dx+c\right)^2}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/20*(4*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
+ 10*a^3*sin(d*x + c)^2)/d
```

Mupad [B]
time = 0.06, size = 56, normalized size = 1.24

$$\frac{\frac{a^3\sin(c+d\,x)^5}{5}+\frac{3\,a^3\sin(c+d\,x)^4}{4}+a^3\sin\left(c+d\,x\right)^3+\frac{a^3\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (3*a^3*sin(c + d*x)^4)/4 + (
a^3*sin(c + d*x)^5)/5)/d
```

3.209 $\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx)}{3d}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(
d*x+c)^3/d
```

Rubi [A]

time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2786, 45}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(}{3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.00

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)
/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.08, size = 44, normalized size = 0.68

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a^3\left(\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$
risch	$-ia^3x-\frac{3a^3e^{2i(dx+c)}}{8d}-\frac{3a^3e^{-2i(dx+c)}}{8d}-\frac{2ia^3c}{d}+\frac{a^3\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{13a^3\sin(dx+c)}{4d}-\frac{a^3\sin(3dx+3c)}{12d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3/d*(1/3*sin(d*x+c)^3+3/2*sin(d*x+c)^2+3*sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+6\,a^3\log\left(\sin\left(dx+c\right)\right)+18\,a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c))
+ 18*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.91

$$-\frac{9\,a^3\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\,\left(a^3\cos\left(dx+c\right)^2-10\,a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(a^3*cos(d*x +
 c)^2 - 10*a^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\cos\left(c+dx\right)\csc\left(c+dx\right)dx+\int3\sin\left(c+dx\right)\cos\left(c+dx\right)\csc\left(c+dx\right)dx+\int3\sin^2\left(c+dx\right)\cos\left(c+dx\right)\csc\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\cos\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(3*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)*csc(c +
 d*x), x) + Integral(sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x), x))
```

Giac [A]
time = 0.53, size = 56, normalized size = 0.86

$$\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x +
c))) + 18*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 8.65, size = 102, normalized size = 1.57

$$\frac{10\,a^3\,\sin\left(c+d\,x\right)}{3\,d}-\frac{a^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a^3\,{\cos\left(c+d\,x\right)}^2}{2\,d}-\frac{a^3\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (10*a^3*sin(c + d*x))/(3*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a^3*cos(c + d*x)^2)/(2*d)
- (a^3*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.210 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=62

$$-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}$$

```
[Out] -a^3*csc(d*x+c)/d+3*a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x
+c)^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(3a+\frac{a^3}{x^2}+\frac{3a^2}{x}+x\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d} + \frac{3a^3\log(\sin(c+dx))}{d} + \frac{3a^3\sin(c+d}{d}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 1.00

$$-\frac{a^3\csc(c+dx)}{d} + \frac{3a^3\log(\sin(c+dx))}{d} + \frac{3a^3\sin(c+dx)}{d} + \frac{a^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x]
)/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.10, size = 46, normalized size = 0.74

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\sin^2(dx+c)\right)}{2}+3\sin(dx+c)+3\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
default	$\frac{a^3\left(\frac{\left(\sin^2(dx+c)\right)}{2}+3\sin(dx+c)+3\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
risch	$-3ia^3x-\frac{a^3e^{2i(dx+c)}}{8d}-\frac{3ia^3e^{i(dx+c)}}{2d}+\frac{3ia^3e^{-i(dx+c)}}{2d}-\frac{a^3e^{-2i(dx+c)}}{8d}-\frac{6ia^3c}{d}-\frac{2ia^3e^{i(dx+c)}}{d(e^{2i(dx+c)}-1)}+\frac{3a^3}{}$
norman	$\frac{\frac{2a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{2d}+\frac{4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{9a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3\left(\tan^8\left(\frac{dx}{2}\right.\right.}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(1/2*sin(d*x+c)^2+3*sin(d*x+c)+3*ln(sin(d*x+c))-1/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.87

$$\frac{a^3 \sin(dx+c)^2 + 6\,a^3 \log(\sin(dx+c)) + 6\,a^3 \sin(dx+c) - \frac{2\,a^3}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)) + 6*a^3*sin(d*x + c) - 2*
a^3/sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 78, normalized size = 1.26

$$-\frac{12\,a^3 \cos(dx+c)^2 - 12\,a^3 \log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c) - 8\,a^3 + \left(2\,a^3 \cos(dx+c)^2 - a^3\right)\sin(dx+c)}{4\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a^3*cos(d*x + c)^2 - 12*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) - 8
*a^3 + (2*a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \cos(c+dx)\csc^2(c+dx)\,dx + \int 3\sin(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx + \int 3\sin^2(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx + \int \sin^3(c+dx)\cos(c+dx)\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)*c
sc(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x)**2,
 x))
```

Giac [A]
time = 0.51, size = 55, normalized size = 0.89

$$\frac{a^3 \sin(dx+c)^2 + 6\,a^3 \log(|\sin(dx+c)|) + 6\,a^3 \sin(dx+c) - \frac{2\,a^3}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x + c))) + 6*a^3*sin(d*x + c)
 - 2*a^3/sin(d*x + c))/d
```

Mupad [B]
time = 8.59, size = 156, normalized size = 2.52

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+10\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (10*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*ta
n(c/2 + (d*x)/2)^3 + 11*a^3*tan(c/2 + (d*x)/2)^4 - a^3)/(d*(2*tan(c/2 + (d*
x)/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) - (a^3*tan(c/2 +
(d*x)/2))/(2*d) - (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.211 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$-\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+3*a^3*ln(sin(d*x+c))/d+a^3*sin(d
*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (3*a^3*Log[Sin[c + d
*x]])/d + (a^3*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}+\frac{3a^2}{x^2}+\frac{3a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} + \frac{3a^3\log(\sin(c+$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.87

$$a^3\left(-\frac{3\csc(c+dx)}{d} - \frac{\csc^2(c+dx)}{2d} + \frac{3\log(\sin(c+dx))}{d} + \frac{\sin(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
[Out] a^3*((-3*Csc[c + d*x])/d - Csc[c + d*x]^2/(2*d) + (3*Log[Sin[c + d*x]])/d +
 Sin[c + d*x]/d)
```

Maple [A]
time = 0.12, size = 44, normalized size = 0.72

method	result
derivativedivides	$\frac{a^3\left(\sin(dx+c)+3\ln(\sin(dx+c))-\frac{3}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
default	$\frac{a^3\left(\sin(dx+c)+3\ln(\sin(dx+c))-\frac{3}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
risch	$-3ia^3x - \frac{ia^3e^{i(dx+c)}}{2d} + \frac{ia^3e^{-i(dx+c)}}{2d} - \frac{6ia^3c}{d} - \frac{2ia^3\left(ie^{2i(dx+c)}+3e^{3i(dx+c)}-3e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2} + \frac{3a^3\ln(e^{2i(dx+c}}{d}$
norman	$\frac{-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{4a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^3\left(\tan^{10}\left(\frac{dx}{2}\right.\right.}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3/d*(sin(d*x+c)+3*ln(sin(d*x+c))-3/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.89

$$\frac{6\,a^3\log\left(\sin\left(dx+c\right)\right)+2\,a^3\sin\left(dx+c\right)-\frac{6\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a^3*log(sin(d*x + c)) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c) + a
^3)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 77, normalized size = 1.26

$$\frac{a^3+6\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(a^3\cos\left(dx+c\right)^2+2\,a^3\right)\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3 + 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 2*(a^3*cos(
d*x + c)^2 + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int 3\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\cos\left(c+dx\right)\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)*csc(c + d*x)**3, x) + Integral(3*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**3, x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)*c
sc(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x)**3,
 x))
```

Giac [A]
time = 0.48, size = 55, normalized size = 0.90

$$\frac{6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a^3\sin\left(dx+c\right)-\frac{6\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(sin(d*x + c))) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) + a^3)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 8.57, size = 163, normalized size = 2.67

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - ((a^3*tan(c/2 + (d*x)/2)^2)/2 - 2*a^3*t
an(c/2 + (d*x)/2)^3 + a^3/2 + 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*
x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*
a^3*tan(c/2 + (d*x)/2))/(2*d) - (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.212 $\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=65

$$-\frac{3a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-3/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d+a^3*ln(si
n(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) + (a^3*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{a^3}{x^4}+\frac{3a^2}{x^3}+\frac{3a}{x^2}+\frac{1}{x}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.88

$$a^3\left(-\frac{3\csc(c+dx)}{d}-\frac{3\csc^2(c+dx)}{2d}-\frac{\csc^3(c+dx)}{3d}+\frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
[Out] a^3*((-3*Csc[c + d*x])/d - (3*Csc[c + d*x]^2)/(2*d) - Csc[c + d*x]^3/(3*d)
+ Log[Sin[c + d*x]]/d)
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.71

method	result
derivativedivides	$\frac{a^3\left(\ln(\sin(dx+c))-\frac{3}{\sin(dx+c)}-\frac{3}{2\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
default	$\frac{a^3\left(\ln(\sin(dx+c))-\frac{3}{\sin(dx+c)}-\frac{3}{2\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}\right)}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}-\frac{2ia^3\left(9\,e^{5i(dx+c)}-22\,e^{3i(dx+c)}+9i e^{4i(dx+c)}+9\,e^{i(dx+c)}-9i e^{2i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{a^3\ln\left(e^{2i(dx+c)}-1\right)}{d}$
norman	$\frac{-\frac{a^3}{24d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{7a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{53a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{59a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{53a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{7a^3\left(\tan^1\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3/d*(ln(sin(d*x+c))-3/sin(d*x+c)-3/2/sin(d*x+c)^2-1/3/sin(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 0.89

$$\frac{6\,a^3\log\left(\sin\left(dx+c\right)\right)-\frac{18\,a^3\sin\left(dx+c\right)^2+9\,a^3\sin\left(dx+c\right)+2\,a^3}{\sin\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*log(sin(d*x + c)) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x + c)
+ 2*a^3)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 1.40

$$-\frac{18\,a^3\cos\left(dx+c\right)^2-9\,a^3\sin\left(dx+c\right)-20\,a^3-6\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(18*a^3*cos(d*x + c)^2 - 9*a^3*sin(d*x + c) - 20*a^3 - 6*(a^3*cos(d*x
+ c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*s
in(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 59, normalized size = 0.91

$$\frac{6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{18\,a^3\sin\left(dx+c\right)^2+9\,a^3\sin\left(dx+c\right)+2\,a^3}{\sin\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(abs(sin(d*x + c))) - (18*a^3*sin(d*x + c)^2 + 9*a^3*sin(d*x
+ c) + 2*a^3)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 8.62, size = 147, normalized size = 2.26

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{3}\right)}{8\,d}-\frac{13\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^3
*tan(c/2 + (d*x)/2)^3)/(24*d) - (cot(c/2 + (d*x)/2)^3*(13*a^3*tan(c/2 + (d*
x)/2)^2 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2)))/(8*d) - (13*a^3*tan(c/2 + (d*x
)/2))/(8*d) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.213 $\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{4ad}$$

```
[Out] -1/4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 37}

$$-\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{4ad}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 0.67

$$-\frac{a^3(1+\csc(c+dx))^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
[Out] -1/4*(a^3*(1 + Csc[c + d*x])^4)/d
```

Maple [A]
time = 0.13, size = 49, normalized size = 1.63

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}-\frac{3}{2\sin(dx+c)^2}-\frac{1}{\sin(dx+c)^3}\right)}{d}$
default	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}-\frac{3}{2\sin(dx+c)^2}-\frac{1}{\sin(dx+c)^3}\right)}{d}$
risch	$-\frac{2ia^3\left(3i e^{6i(dx+c)}+e^{7i(dx+c)}-8i e^{4i(dx+c)}-7 e^{5i(dx+c)}+3i e^{2i(dx+c)}+7 e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a^3}{64d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{31a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{5a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{31a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{11a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{31a^3\left(\tan^9 \cdots\right.}{8 \cdots}}{\tan\left(\frac{dx}{2} \cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3/d*(-1/4/sin(d*x+c)^4-1/sin(d*x+c)-3/2/sin(d*x+c)^2-1/sin(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin(dx+c)^3+6\,a^3\sin(dx+c)^2+4\,a^3\sin(dx+c)+a^3}{4\,d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.
time = 0.33, size = 72, normalized size = 2.40

$$\frac{6\,a^3\cos\left(dx+c\right)^2-7\,a^3+4\,\left(a^3\cos\left(dx+c\right)^2-2\,a^3\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*cos(d*x + c)^2 - 7*a^3 + 4*(a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x
+ c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.43, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin\left(dx+c\right)^3+6\,a^3\sin\left(dx+c\right)^2+4\,a^3\sin\left(dx+c\right)+a^3}{4\,d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^
3)/(d*sin(d*x + c)^4)
```

Mupad [B]
time = 8.61, size = 54, normalized size = 1.80

$$-\frac{4\,a^3\sin\left(c+d\,x\right)^3+6\,a^3\sin\left(c+d\,x\right)^2+4\,a^3\,\sin\left(c+d\,x\right)+a^3}{4\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] -(4*a^3*sin(c + d*x) + a^3 + 6*a^3*sin(c + d*x)^2 + 4*a^3*sin(c + d*x)^3)/(
4*d*sin(c + d*x)^4)
```

3.214 $\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=61

$$\frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{20ad}-\frac{\csc^5(c+dx)(a+a\sin(c+dx))^4}{5ad}$$

```
[Out] 1/20*csc(d*x+c)^4*(a+a*sin(d*x+c))^4/a/d-1/5*csc(d*x+c)^5*(a+a*sin(d*x+c))^
4/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2912, 12, 47, 37}

$$\frac{\csc^4(c+dx)(a\sin(c+dx)+a)^4}{20ad}-\frac{\csc^5(c+dx)(a\sin(c+dx)+a)^4}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(a + a*Sin[c + d*x])^4)/(20*a*d) - (Csc[c + d*x]^5*(a + a*S
in[c + d*x])^4)/(5*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^5\text{Subst}\left(\int \frac{(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^5(c+dx)(a+a\sin(c+dx))^4}{5ad} - \frac{a^4\text{Subst}\left(\int \frac{(a+x)^3}{x^5}\right.}{}$$
$$= \frac{\csc^4(c+dx)(a+a\sin(c+dx))^4}{20ad} - \frac{\csc^5(c+dx)(a+a}{5ad}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.16

$$-\frac{a^3\csc^2(c+dx)}{2d} - \frac{a^3\csc^3(c+dx)}{d} - \frac{3a^3\csc^4(c+dx)}{4d} - \frac{a^3\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
[Out] -1/2*(a^3*Csc[c + d*x]^2)/d - (a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^
4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.15, size = 49, normalized size = 0.80

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{5\sin(dx+c)^5}\right)}{d}$
default	$\frac{a^3\left(-\frac{1}{\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{5\sin(dx+c)^5}\right)}{d}$
risch	$\frac{2a^3\left(20i e^{7i(dx+c)}+5\,e^{8i(dx+c)}-56i e^{5i(dx+c)}-45\,e^{6i(dx+c)}+20i e^{3i(dx+c)}+45\,e^{4i(dx+c)}-5\,e^{2i(dx+c)}\right)}{5d\left(e^{2i(dx+c)}-1\right)^5}$

norman	$-\frac{a^3}{160d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{7a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{29a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{37a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{89a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{47a^3\left(\tan^8\right.}{1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(-1/sin(d*x+c)^3-1/2/sin(d*x+c)^2-3/4/sin(d*x+c)^4-1/5/sin(d*x+c)^5)
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.92

$$-\frac{10\,a^3\sin\left(dx+c\right)^3+20\,a^3\sin\left(dx+c\right)^2+15\,a^3\sin\left(dx+c\right)+4\,a^3}{20\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

Fricas [A]
time = 0.34, size = 81, normalized size = 1.33

$$\frac{20\,a^3\cos\left(dx+c\right)^2-24\,a^3+5\,\left(2\,a^3\cos\left(dx+c\right)^2-5\,a^3\right)\sin\left(dx+c\right)}{20\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/20*(20*a^3*cos(d*x + c)^2 - 24*a^3 + 5*(2*a^3*cos(d*x + c)^2 - 5*a^3)*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.49, size = 56, normalized size = 0.92

$$-\frac{10\,a^3\sin\left(dx+c\right)^3+20\,a^3\sin\left(dx+c\right)^2+15\,a^3\sin\left(dx+c\right)+4\,a^3}{20\,d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20*(10*a^3*sin(d*x + c)^3 + 20*a^3*sin(d*x + c)^2 + 15*a^3*sin(d*x + c)
+ 4*a^3)/(d*sin(d*x + c)^5)
```

Mupad [B]
time = 8.60, size = 56, normalized size = 0.92

$$-\frac{10\,a^3\sin\left(c+d\,x\right)^3+20\,a^3\sin\left(c+d\,x\right)^2+15\,a^3\,\sin\left(c+d\,x\right)+4\,a^3}{20\,d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(15*a^3*sin(c + d*x) + 4*a^3 + 20*a^3*sin(c + d*x)^2 + 10*a^3*sin(c + d*x)
^3)/(20*d*sin(c + d*x)^5)
```

3.215 $\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6(c+dx)}{6d}$$

```
[Out] -1/3*a^3*csc(d*x+c)^3/d-3/4*a^3*csc(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/6*a
^3*csc(d*x+c)^6/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*(a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a+x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{(a+x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{a^3}{x^7}+\frac{3a^2}{x^6}+\frac{3a}{x^5}+\frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc^3(c+dx)}{3d} - \frac{3a^3\csc^4(c+dx)}{4d} - \frac{3a^3\csc^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 1.00

$$-\frac{a^3\csc^3(c+dx)}{3d} - \frac{3a^3\csc^4(c+dx)}{4d} - \frac{3a^3\csc^5(c+dx)}{5d} - \frac{a^3\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*(a^3*Csc[c + d*x]^3)/d - (3*a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c +
 d*x]^5)/(5*d) - (a^3*Csc[c + d*x]^6)/(6*d)
```

Maple [A]
time = 0.17, size = 49, normalized size = 0.67

method	result	size
derivativedivides	$\frac{a^3\left(-\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}-\frac{3}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}\right)}{d}$	49
default	$\frac{a^3\left(-\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}-\frac{3}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}\right)}{d}$	49
risch	$\frac{4ia^3\left(45i e^{8i(dx+c)}+10\,e^{9i(dx+c)}-130i e^{6i(dx+c)}-102\,e^{7i(dx+c)}+45i e^{4i(dx+c)}+102\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}\right)}{15d\left(e^{2i(dx+c)}-1\right)^6}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/d*(-3/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3-3/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6
)
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.77

$$-\frac{20\,a^3\sin(dx+c)^3+45\,a^3\sin(dx+c)^2+36\,a^3\sin(dx+c)+10\,a^3}{60\,d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.37, size = 86, normalized size = 1.18

$$-\frac{45\,a^3\cos\left(dx+c\right)^2-55\,a^3+4\,\left(5\,a^3\cos\left(dx+c\right)^2-14\,a^3\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*a^3*cos(d*x + c)^2 - 55*a^3 + 4*(5*a^3*cos(d*x + c)^2 - 14*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.51, size = 56, normalized size = 0.77

$$-\frac{20\,a^3\sin\left(dx+c\right)^3+45\,a^3\sin\left(dx+c\right)^2+36\,a^3\sin\left(dx+c\right)+10\,a^3}{60\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(20*a^3*sin(d*x + c)^3 + 45*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c)
+ 10*a^3)/(d*sin(d*x + c)^6)
```

Mupad [B]
time = 8.60, size = 56, normalized size = 0.77

$$-\frac{a^3\,\left(-20\sin\left(c+d\,x\right)^6+20\sin\left(c+d\,x\right)^3+45\sin\left(c+d\,x\right)^2+36\,\sin\left(c+d\,x\right)+10\right)}{60\,d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(a^3*(36*sin(c + d*x) + 45*sin(c + d*x)^2 + 20*sin(c + d*x)^3 - 20*sin(c +
 d*x)^6 + 10))/(60*d*sin(c + d*x)^6)
```

3.216 $\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=73

$$-\frac{a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{a^3\csc^7(c+dx)}{7d}$$

```
[Out] -1/4*a^3*csc(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/2*a^3*csc(d*x+c)^6/d-1/7*a
^3*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^3\csc^7(c+dx)}{7d}-\frac{a^3\csc^6(c+dx)}{2d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(a^3*Csc[c + d*x]^4)/d - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a+x)^3}{x^8}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^7\text{Subst}\left(\int \frac{(a+x)^3}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{a^3}{x^8}+\frac{3a^2}{x^7}+\frac{3a}{x^6}+\frac{1}{x^5}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^3\csc^4(c+dx)}{4d} - \frac{3a^3\csc^5(c+dx)}{5d} - \frac{a^3\csc^6(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 1.00

$$-\frac{a^3\csc^4(c+dx)}{4d} - \frac{3a^3\csc^5(c+dx)}{5d} - \frac{a^3\csc^6(c+dx)}{2d} - \frac{a^3\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
[Out] -1/4*(a^3*Csc[c + d*x]^4)/d - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a^3*Csc[c + d
*x]^6)/(2*d) - (a^3*Csc[c + d*x]^7)/(7*d)
```

Maple [A]
time = 0.15, size = 49, normalized size = 0.67

method	result	siz
derivativedivides	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^6}-\frac{1}{7\sin(dx+c)^7}-\frac{3}{5\sin(dx+c)^5}\right)}{d}$	49
default	$\frac{a^3\left(-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^6}-\frac{1}{7\sin(dx+c)^7}-\frac{3}{5\sin(dx+c)^5}\right)}{d}$	49
risch	$-\frac{4a^3\left(168ie^{9i(dx+c)}+35\,e^{10i(dx+c)}-496ie^{7i(dx+c)}-385\,e^{8i(dx+c)}+168ie^{5i(dx+c)}+385\,e^{6i(dx+c)}-35\,e^{4i(dx+c)}\right)}{35d\left(e^{2i(dx+c)}-1\right)^7}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] a^3/d*(-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^6-1/7/sin(d*x+c)^7-3/5/sin(d*x+c)^5
)
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin(dx+c)^3 + 84\,a^3\sin(dx+c)^2 + 70\,a^3\sin(dx+c) + 20\,a^3}{140\,d\sin(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

Fricas [A]
time = 0.35, size = 93, normalized size = 1.27

$$-\frac{84\,a^3\cos\left(dx+c\right)^2-104\,a^3+35\,\left(a^3\cos\left(dx+c\right)^2-3\,a^3\right)\sin\left(dx+c\right)}{140\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/140*(84*a^3*cos(d*x + c)^2 - 104*a^3 + 35*(a^3*cos(d*x + c)^2 - 3*a^3)*s
in(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.48, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin\left(dx+c\right)^3+84\,a^3\sin\left(dx+c\right)^2+70\,a^3\sin\left(dx+c\right)+20\,a^3}{140\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/140*(35*a^3*sin(d*x + c)^3 + 84*a^3*sin(d*x + c)^2 + 70*a^3*sin(d*x + c)
 + 20*a^3)/(d*sin(d*x + c)^7)
```

Mupad [B]
time = 8.66, size = 56, normalized size = 0.77

$$-\frac{35\,a^3\sin\left(c+d\,x\right)^3+84\,a^3\sin\left(c+d\,x\right)^2+70\,a^3\,\sin\left(c+d\,x\right)+20\,a^3}{140\,d\sin\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] -(70*a^3*sin(c + d*x) + 20*a^3 + 84*a^3*sin(c + d*x)^2 + 35*a^3*sin(c + d*x
)^3)/(140*d*sin(c + d*x)^7)
```

3.217 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=91

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{a^4\sin^9(c+dx)}{9d}$$

```
[Out] 1/5*a^4*sin(d*x+c)^5/d+2/3*a^4*sin(d*x+c)^6/d+6/7*a^4*sin(d*x+c)^7/d+1/2*a^
4*sin(d*x+c)^8/d+1/9*a^4*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^4\sin^9(c+dx)}{9d}+\frac{a^4\sin^8(c+dx)}{2d}+\frac{6a^4\sin^7(c+dx)}{7d}+\frac{2a^4\sin^6(c+dx)}{3d}+\frac{a^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^5)/(5*d) + (2*a^4*Sin[c + d*x]^6)/(3*d) + (6*a^4*Sin[c +
d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(2*d) + (a^4*Sin[c + d*x]^9)/(9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)^4}{a^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^4(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^4+4a^3x^5+6a^2x^6+4ax^7+x^8\right)\,dx, x, a\,\text{s}}{a^5 d}$$
$$= \frac{a^4\sin^5(c+dx)}{5d} + \frac{2a^4\sin^6(c+dx)}{3d} + \frac{6a^4\sin^7(c+dx)}{7d}$$

Mathematica [A]
time = 0.66, size = 100, normalized size = 1.10

$$\frac{a^4(4095-42840\cos(2(c+dx))+18900\cos(4(c+dx))-4200\cos(6(c+dx))+315\cos(8(c+dx))+52290\sin(c+dx)-30660\sin(3(c+dx))+9828\sin(5(c+dx))-1395\sin(7(c+dx))+35\sin(9(c+dx)))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(4095 - 42840*Cos[2*(c + d*x)] + 18900*Cos[4*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 315*Cos[8*(c + d*x)] + 52290*Sin[c + d*x] - 30660*Sin[3*(c + d*
x)] + 9828*Sin[5*(c + d*x)] - 1395*Sin[7*(c + d*x)] + 35*Sin[9*(c + d*x)]))
/(80640*d)
```

Maple [A]
time = 0.26, size = 71, normalized size = 0.78

method	result
derivativedivides	$\frac{\frac{a^4\left(\sin^9(dx+c)\right)}{9}+\frac{a^4\left(\sin^8(dx+c)\right)}{2}+\frac{6a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{a^4\left(\sin^5(dx+c)\right)}{5}}{d}$
default	$\frac{\frac{a^4\left(\sin^9(dx+c)\right)}{9}+\frac{a^4\left(\sin^8(dx+c)\right)}{2}+\frac{6a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{a^4\left(\sin^5(dx+c)\right)}{5}}{d}$
risch	$\frac{83a^4\sin(dx+c)}{128d}+\frac{a^4\sin(9dx+9c)}{2304d}+\frac{a^4\cos(8dx+8c)}{256d}-\frac{31a^4\sin(7dx+7c)}{1792d}-\frac{5a^4\cos(6dx+6c)}{96d}+\frac{39a^4\sin(5dx+5}{320d}$
norman	$\frac{\frac{32a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{4736a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{99136a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d}+\frac{4736a^4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{32a^4\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{1}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/9*a^4*sin(d*x+c)^9+1/2*a^4*sin(d*x+c)^8+6/7*a^4*sin(d*x+c)^7+2/3*a^4
*sin(d*x+c)^6+1/5*a^4*sin(d*x+c)^5)
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.78

$$\frac{70\,a^{4}\sin\left(dx+c\right)^{9}+315\,a^{4}\sin\left(dx+c\right)^{8}+540\,a^{4}\sin\left(dx+c\right)^{7}+420\,a^{4}\sin\left(dx+c\right)^{6}+126\,a^{4}\sin\left(dx+c\right)^{5}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.38, size = 124, normalized size = 1.36

$$\frac{315\,a^{4}\cos\left(dx+c\right)^{8}-1680\,a^{4}\cos\left(dx+c\right)^{6}+3150\,a^{4}\cos\left(dx+c\right)^{4}-2520\,a^{4}\cos\left(dx+c\right)^{2}+2\,\left(35\,a^{4}\cos\left(dx+c\right)^{8}-410\,a^{4}\cos\left(dx+c\right)^{6}+1083\,a^{4}\cos\left(dx+c\right)^{4}-1076\,a^{4}\cos\left(dx+c\right)^{2}+368\,a^{4}\right)\sin\left(dx+c\right)}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/630*(315*a^4*cos(d*x + c)^8 - 1680*a^4*cos(d*x + c)^6 + 3150*a^4*cos(d*x
+ c)^4 - 2520*a^4*cos(d*x + c)^2 + 2*(35*a^4*cos(d*x + c)^8 - 410*a^4*cos(d
*x + c)^6 + 1083*a^4*cos(d*x + c)^4 - 1076*a^4*cos(d*x + c)^2 + 368*a^4)*si
n(d*x + c))/d
```

Sympy [A]
time = 1.42, size = 97, normalized size = 1.07

$$\begin{cases} \frac{a^{4}\sin^{9}{\left(c+dx \right)}}{9d}+\frac{a^{4}\sin^{8}{\left(c+dx \right)}}{2d}+\frac{6a^{4}\sin^{7}{\left(c+dx \right)}}{7d}+\frac{2a^{4}\sin^{6}{\left(c+dx \right)}}{3d}+\frac{a^{4}\sin^{5}{\left(c+dx \right)}}{5d} & \text{for}\: d \neq 0 \\ x\left(a\sin{\left(c \right)}+a\right)^{4}\sin^{4}{\left(c \right)}\cos{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**9/(9*d) + a**4*sin(c + d*x)**8/(2*d) + 6*a**4
*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + a**4*sin(c + d*x)**
5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**4*cos(c), True))
```

Giac [A]
time = 0.49, size = 71, normalized size = 0.78

$$\frac{70\,a^{4}\sin\left(dx+c\right)^{9}+315\,a^{4}\sin\left(dx+c\right)^{8}+540\,a^{4}\sin\left(dx+c\right)^{7}+420\,a^{4}\sin\left(dx+c\right)^{6}+126\,a^{4}\sin\left(dx+c\right)^{5}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*a^4*sin(d*x + c)^9 + 315*a^4*sin(d*x + c)^8 + 540*a^4*sin(d*x + c
)^7 + 420*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5)/d
```

Mupad [B]
time = 8.46, size = 70, normalized size = 0.77

$$\frac{\frac{a^4\sin(c+dx)^9}{9}+\frac{a^4\sin(c+dx)^8}{2}+\frac{6a^4\sin(c+dx)^7}{7}+\frac{2a^4\sin(c+dx)^6}{3}+\frac{a^4\sin(c+dx)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^5)/5 + (2*a^4*sin(c + d*x)^6)/3 + (6*a^4*sin(c + d*x)^7)
/7 + (a^4*sin(c + d*x)^8)/2 + (a^4*sin(c + d*x)^9)/9)/d
```

3.218 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=88

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^8(c+dx)}{8d}$$

```
[Out] 1/4*a^4*sin(d*x+c)^4/d+4/5*a^4*sin(d*x+c)^5/d+a^4*sin(d*x+c)^6/d+4/7*a^4*si
n(d*x+c)^7/d+1/8*a^4*sin(d*x+c)^8/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^4)/(4*d) + (4*a^4*Sin[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*
x]^6)/d + (4*a^4*Sin[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x]^8)/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^4}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^3+4a^3x^4+6a^2x^5+4ax^6+x^7\right)dx, x, a\,\text{s}\right.}{a^4d}$$
$$= \frac{a^4\sin^4(c+dx)}{4d} + \frac{4a^4\sin^5(c+dx)}{5d} + \frac{a^4\sin^6(c+dx)}{d} +$$

Mathematica [A]
time = 0.63, size = 90, normalized size = 1.02

$$\frac{a^4(36400-69720\cos(2(c+dx))+26460\cos(4(c+dx))-4200\cos(6(c+dx))+105\cos(8(c+dx))+87360\sin(c+dx)-47040\sin(3(c+dx))+12096\sin(5(c+dx))-960\sin(7(c+dx)))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(36400 - 69720*Cos[2*(c + d*x)] + 26460*Cos[4*(c + d*x)] - 4200*Cos[6*
(c + d*x)] + 105*Cos[8*(c + d*x)] + 87360*Sin[c + d*x] - 47040*Sin[3*(c + d
*x)] + 12096*Sin[5*(c + d*x)] - 960*Sin[7*(c + d*x)]))/(107520*d)
```

Maple [A]
time = 0.22, size = 70, normalized size = 0.80

method	result
derivativedivides	$\frac{\frac{a^4\left(\sin^8(dx+c)\right)}{8}+\frac{4a^4\left(\sin^7(dx+c)\right)}{7}+a^4(\sin^6(dx+c))+\frac{4a^4\left(\sin^5(dx+c)\right)}{5}+\frac{a^4\left(\sin^4(dx+c)\right)}{4}}{d}$
default	$\frac{\frac{a^4\left(\sin^8(dx+c)\right)}{8}+\frac{4a^4\left(\sin^7(dx+c)\right)}{7}+a^4(\sin^6(dx+c))+\frac{4a^4\left(\sin^5(dx+c)\right)}{5}+\frac{a^4\left(\sin^4(dx+c)\right)}{4}}{d}$
risch	$\frac{13a^4\sin(dx+c)}{16d}+\frac{a^4\cos(8dx+8c)}{1024d}-\frac{a^4\sin(7dx+7c)}{112d}-\frac{5a^4\cos(6dx+6c)}{128d}+\frac{9a^4\sin(5dx+5c)}{80d}+\frac{63a^4\cos(4dx+4c}{256d}$
norman	$\frac{\frac{128a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{5248a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{5248a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{128a^4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{4a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^4(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8*a^4*sin(d*x+c)^8+4/7*a^4*sin(d*x+c)^7+a^4*sin(d*x+c)^6+4/5*a^4*sin
(d*x+c)^5+1/4*a^4*sin(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.81

$$\frac{35\,a^4\sin\left(dx+c\right)^8+160\,a^4\sin\left(dx+c\right)^7+280\,a^4\sin\left(dx+c\right)^6+224\,a^4\sin\left(dx+c\right)^5+70\,a^4\sin\left(dx+c\right)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.36, size = 111, normalized size = 1.26

$$\frac{35\,a^4\cos\left(dx+c\right)^8-420\,a^4\cos\left(dx+c\right)^6+1120\,a^4\cos\left(dx+c\right)^4-1120\,a^4\cos\left(dx+c\right)^2-32\left(5\,a^4\cos\left(dx+c\right)^6-22\,a^4\cos\left(dx+c\right)^4+29\,a^4\cos\left(dx+c\right)^2-12\,a^4\right)\sin\left(dx+c\right)}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/280*(35*a^4*cos(d*x + c)^8 - 420*a^4*cos(d*x + c)^6 + 1120*a^4*cos(d*x +
c)^4 - 1120*a^4*cos(d*x + c)^2 - 32*(5*a^4*cos(d*x + c)^6 - 22*a^4*cos(d*x
+ c)^4 + 29*a^4*cos(d*x + c)^2 - 12*a^4)*sin(d*x + c))/d
```

Sympy [A]
time = 0.95, size = 95, normalized size = 1.08

$$\begin{cases} \frac{a^4\sin^8(c+dx)}{8d}+\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^6(c+dx)}{d}+\frac{4a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{4d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^4\sin^3(c)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**8/(8*d) + 4*a**4*sin(c + d*x)**7/(7*d) + a**4
*sin(c + d*x)**6/d + 4*a**4*sin(c + d*x)**5/(5*d) + a**4*sin(c + d*x)**4/(4
*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**3*cos(c), True))
```

Giac [A]
time = 0.50, size = 71, normalized size = 0.81

$$\frac{35\,a^4\sin\left(dx+c\right)^8+160\,a^4\sin\left(dx+c\right)^7+280\,a^4\sin\left(dx+c\right)^6+224\,a^4\sin\left(dx+c\right)^5+70\,a^4\sin\left(dx+c\right)^4}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/280*(35*a^4*sin(d*x + c)^8 + 160*a^4*sin(d*x + c)^7 + 280*a^4*sin(d*x + c
)^6 + 224*a^4*sin(d*x + c)^5 + 70*a^4*sin(d*x + c)^4)/d
```

Mupad [B]
time = 8.45, size = 69, normalized size = 0.78

$$\frac{\frac{a^4\sin(c+d\,x)^8}{8}+\frac{4\,a^4\sin(c+d\,x)^7}{7}+a^4\sin\left(c+d\,x\right)^6+\frac{4\,a^4\sin(c+d\,x)^5}{5}+\frac{a^4\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^4)/4 + (4*a^4*sin(c + d*x)^5)/5 + a^4*sin(c + d*x)^6 + (
4*a^4*sin(c + d*x)^7)/7 + (a^4*sin(c + d*x)^8)/8)/d
```

3.219 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=67

$$\frac{(a+a\sin(c+dx))^5}{5ad}-\frac{(a+a\sin(c+dx))^6}{3a^2d}+\frac{(a+a\sin(c+dx))^7}{7a^3d}$$

```
[Out] 1/5*(a+a*sin(d*x+c))^5/a/d-1/3*(a+a*sin(d*x+c))^6/a^2/d+1/7*(a+a*sin(d*x+c)
)^7/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^7}{7a^3d}-\frac{(a\sin(c+dx)+a)^6}{3a^2d}+\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a + a*Sin[c + d*x])^5/(5*a*d) - (a + a*Sin[c + d*x])^6/(3*a^2*d) + (a + a*
Sin[c + d*x])^7/(7*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^4}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2(a+x)^4 - 2a(a+x)^5 + (a+x)^6\right)\,dx, x, a$$
$$= \frac{(a+a\sin(c+dx))^5}{5ad} - \frac{(a+a\sin(c+dx))^6}{3a^2d} + \frac{(a+a\,\text{si}$$

Mathematica [A]
time = 0.29, size = 80, normalized size = 1.19

$$-\frac{a^4(-630+5460\cos(2(c+dx))-1680\cos(4(c+dx))+140\cos(6(c+dx))-7245\sin(c+dx)+3395\sin(3(c+dx))-609\sin(5(c+dx))+15\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
[Out] -1/6720*(a^4*(-630 + 5460*Cos[2*(c + d*x)] - 1680*Cos[4*(c + d*x)] + 140*Co
s[6*(c + d*x)] - 7245*Sin[c + d*x] + 3395*Sin[3*(c + d*x)] - 609*Sin[5*(c +
 d*x)] + 15*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.17, size = 70, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{6a^4\left(\sin^5(dx+c)\right)}{5}+a^4\left(\sin^4(dx+c)\right)+\frac{a^4\left(\sin^3(dx+c)\right)}{3}}{d}$
default	$\frac{\frac{a^4\left(\sin^7(dx+c)\right)}{7}+\frac{2a^4\left(\sin^6(dx+c)\right)}{3}+\frac{6a^4\left(\sin^5(dx+c)\right)}{5}+a^4\left(\sin^4(dx+c)\right)+\frac{a^4\left(\sin^3(dx+c)\right)}{3}}{d}$
risch	$\frac{69a^4\sin(dx+c)}{64d}-\frac{a^4\sin(7dx+7c)}{448d}-\frac{a^4\cos(6dx+6c)}{48d}+\frac{29a^4\sin(5dx+5c)}{320d}+\frac{a^4\cos(4dx+4c)}{4d}-\frac{97a^4\sin(3dx+3c}{192d}$
norman	$\frac{\frac{16a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{736a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{3888a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{736a^4(\text{t}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(1/7*a^4*sin(d*x+c)^7+2/3*a^4*sin(d*x+c)^6+6/5*a^4*sin(d*x+c)^5+a^4*sin
(d*x+c)^4+1/3*a^4*sin(d*x+c)^3)
```

Maxima [A]
time = 0.30, size = 71, normalized size = 1.06

$$\frac{15\,a^4\sin\left(dx+c\right)^7+70\,a^4\sin\left(dx+c\right)^6+126\,a^4\sin\left(dx+c\right)^5+105\,a^4\sin\left(dx+c\right)^4+35\,a^4\sin\left(dx+c\right)^3}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)
^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.36, size = 97, normalized size = 1.45

$$-\frac{70\,a^4\cos\left(dx+c\right)^6-315\,a^4\cos\left(dx+c\right)^4+420\,a^4\cos\left(dx+c\right)^2+\left(15\,a^4\cos\left(dx+c\right)^6-171\,a^4\cos\left(dx+c\right)^4+332\,a^4\cos\left(dx+c\right)^2-176\,a^4\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/105*(70*a^4*cos(d*x + c)^6 - 315*a^4*cos(d*x + c)^4 + 420*a^4*cos(d*x +
c)^2 + (15*a^4*cos(d*x + c)^6 - 171*a^4*cos(d*x + c)^4 + 332*a^4*cos(d*x +
c)^2 - 176*a^4)*sin(d*x + c))/d
```

Sympy [A]
time = 0.65, size = 95, normalized size = 1.42

$$\begin{cases}\frac{a^4\sin^7\left(c+dx\right)}{7d}+\frac{2a^4\sin^6\left(c+dx\right)}{3d}+\frac{6a^4\sin^5\left(c+dx\right)}{5d}+\frac{a^4\sin^4\left(c+dx\right)}{d}+\frac{a^4\sin^3\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x\left(a\sin\left(c\right)+a\right)^4\sin^2\left(c\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**7/(7*d) + 2*a**4*sin(c + d*x)**6/(3*d) + 6*a*
*4*sin(c + d*x)**5/(5*d) + a**4*sin(c + d*x)**4/d + a**4*sin(c + d*x)**3/(3
*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**2*cos(c), True))
```

Giac [A]
time = 0.47, size = 71, normalized size = 1.06

$$\frac{15\,a^4\sin\left(dx+c\right)^7+70\,a^4\sin\left(dx+c\right)^6+126\,a^4\sin\left(dx+c\right)^5+105\,a^4\sin\left(dx+c\right)^4+35\,a^4\sin\left(dx+c\right)^3}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

[Out] 1/105*(15*a^4*sin(d*x + c)^7 + 70*a^4*sin(d*x + c)^6 + 126*a^4*sin(d*x + c)^5 + 105*a^4*sin(d*x + c)^4 + 35*a^4*sin(d*x + c)^3)/d

Mupad [B]
time = 8.41, size = 69, normalized size = 1.03

$$\frac{\frac{a^4\sin(c+d\,x)^7}{7}+\frac{2\,a^4\sin(c+d\,x)^6}{3}+\frac{6\,a^4\sin(c+d\,x)^5}{5}+a^4\sin\left(c+d\,x\right)^4+\frac{a^4\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^4,x)

[Out] ((a^4*sin(c + d*x)^3)/3 + a^4*sin(c + d*x)^4 + (6*a^4*sin(c + d*x)^5)/5 + (2*a^4*sin(c + d*x)^6)/3 + (a^4*sin(c + d*x)^7)/7)/d

3.220 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=45

$$-\frac{(a+a\sin(c+dx))^5}{5ad}+\frac{(a+a\sin(c+dx))^6}{6a^2d}$$

```
[Out] -1/5*(a+a*sin(d*x+c))^5/a/d+1/6*(a+a*sin(d*x+c))^6/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^6}{6a^2d}-\frac{(a\sin(c+dx)+a)^5}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/5*(a + a*Sin[c + d*x])^5/(a*d) + (a + a*Sin[c + d*x])^6/(6*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^4+(a+x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^5}{5ad}+\frac{(a+a\sin(c+dx))^6}{6a^2d}$$

Mathematica [A]
time = 0.08, size = 30, normalized size = 0.67

$$\frac{a^4(1+\sin(c+dx))^5(-1+5\sin(c+dx))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^5*(-1 + 5*Sin[c + d*x]))/(30*d)
```

Maple [A]
time = 0.14, size = 71, normalized size = 1.58

method	result
derivativedivides	$\frac{\frac{a^4\left(\sin^6(dx+c)\right)}{6}+\frac{4a^4\left(\sin^5(dx+c)\right)}{5}+\frac{3a^4\left(\sin^4(dx+c)\right)}{2}+\frac{4a^4\left(\sin^3(dx+c)\right)}{3}+\frac{a^4\left(\sin^2(dx+c)\right)}{2}}{d}$
default	$\frac{\frac{a^4\left(\sin^6(dx+c)\right)}{6}+\frac{4a^4\left(\sin^5(dx+c)\right)}{5}+\frac{3a^4\left(\sin^4(dx+c)\right)}{2}+\frac{4a^4\left(\sin^3(dx+c)\right)}{3}+\frac{a^4\left(\sin^2(dx+c)\right)}{2}}{d}$
risch	$\frac{3a^4\sin(dx+c)}{2d}-\frac{a^4\cos(6dx+6c)}{192d}+\frac{a^4\sin(5dx+5c)}{20d}+\frac{7a^4\cos(4dx+4c)}{32d}-\frac{7a^4\sin(3dx+3c)}{12d}-\frac{69a^4\cos(2dx+2c}{64d}$
norman	$\frac{\frac{32a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{32a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{32a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{288a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{288a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{32a^4\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*a^4*sin(d*x+c)^6+4/5*a^4*sin(d*x+c)^5+3/2*a^4*sin(d*x+c)^4+4/3*a^4
*sin(d*x+c)^3+1/2*a^4*sin(d*x+c)^2)
```

Maxima [A]
time = 0.27, size = 71, normalized size = 1.58

$$\frac{5\,a^4\sin\left(dx+c\right)^6+24\,a^4\sin\left(dx+c\right)^5+45\,a^4\sin\left(dx+c\right)^4+40\,a^4\sin\left(dx+c\right)^3+15\,a^4\sin\left(dx+c\right)^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.
time = 0.34, size = 85, normalized size = 1.89

$$-\frac{5\,a^4\cos\left(dx+c\right)^6-60\,a^4\cos\left(dx+c\right)^4+120\,a^4\cos\left(dx+c\right)^2-8\,\left(3\,a^4\cos\left(dx+c\right)^4-11\,a^4\cos\left(dx+c\right)^2+8\,a^4\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/30*(5*a^4*cos(d*x + c)^6 - 60*a^4*cos(d*x + c)^4 + 120*a^4*cos(d*x + c)^
2 - 8*(3*a^4*cos(d*x + c)^4 - 11*a^4*cos(d*x + c)^2 + 8*a^4)*sin(d*x + c))/
d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(34) = 68$.
time = 0.55, size = 97, normalized size = 2.16

$$\begin{cases} \frac{a^4\sin^6\left(c+dx\right)}{6d}+\frac{4a^4\sin^5\left(c+dx\right)}{5d}+\frac{3a^4\sin^4\left(c+dx\right)}{2d}+\frac{4a^4\sin^3\left(c+dx\right)}{3d}+\frac{a^4\sin^2\left(c+dx\right)}{2d} & \text{for}\ d\neq 0 \\ x(a\sin\left(c\right)+a)^4\sin\left(c\right)\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*sin(c + d*x)**6/(6*d) + 4*a**4*sin(c + d*x)**5/(5*d) + 3*a*
*4*sin(c + d*x)**4/(2*d) + 4*a**4*sin(c + d*x)**3/(3*d) + a**4*sin(c + d*x)
**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)*cos(c), True))
```

Giac [A]
time = 0.48, size = 71, normalized size = 1.58

$$\frac{5\,a^4\sin\left(dx+c\right)^6+24\,a^4\sin\left(dx+c\right)^5+45\,a^4\sin\left(dx+c\right)^4+40\,a^4\sin\left(dx+c\right)^3+15\,a^4\sin\left(dx+c\right)^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(5*a^4*sin(d*x + c)^6 + 24*a^4*sin(d*x + c)^5 + 45*a^4*sin(d*x + c)^4
+ 40*a^4*sin(d*x + c)^3 + 15*a^4*sin(d*x + c)^2)/d
```

Mupad [B]

time = 0.05, size = 70, normalized size = 1.56

$$\frac{\frac{a^4 \sin(c+d x)^6}{6} + \frac{4 a^4 \sin(c+d x)^5}{5} + \frac{3 a^4 \sin(c+d x)^4}{2} + \frac{4 a^4 \sin(c+d x)^3}{3} + \frac{a^4 \sin(c+d x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^4,x)
```

```
[Out] ((a^4*sin(c + d*x)^2)/2 + (4*a^4*sin(c + d*x)^3)/3 + (3*a^4*sin(c + d*x)^4)
/2 + (4*a^4*sin(c + d*x)^5)/5 + (a^4*sin(c + d*x)^6)/6)/d
```

3.221 $\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=81

$$\frac{a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^4(c+dx)}{4d}$$

```
[Out] a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+3*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*
x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 45}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(4a^3+\frac{a^4}{x}+6a^2x+4ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(}{3c}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 81, normalized size = 1.00

$$\frac{a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{3a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Log[Sin[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (3*a^4*Sin[c + d*x]^2)
/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.10, size = 54, normalized size = 0.67

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{4\left(\sin^3(dx+c)\right)}{3}+3\left(\sin^2(dx+c)\right)+4\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a^4\left(\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{4\left(\sin^3(dx+c)\right)}{3}+3\left(\sin^2(dx+c)\right)+4\sin(dx+c)+\ln(\sin(dx+c))\right)}{d}$
risch	$-ia^4x-\frac{13a^4e^{2i(dx+c)}}{16d}-\frac{13a^4e^{-2i(dx+c)}}{16d}-\frac{2ia^4c}{d}+\frac{a^4\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{5a^4\sin(dx+c)}{d}+\frac{a^4\cos(4dx+4c}{32d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/d*(1/4*sin(d*x+c)^4+4/3*sin(d*x+c)^3+3*sin(d*x+c)^2+4*sin(d*x+c)+ln(sin
(d*x+c)))
```

Maxima [A]
time = 0.27, size = 68, normalized size = 0.84

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+36\,a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(\sin\left(dx+c\right)\right)+48\,a^4\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(sin(d*x + c)) + 48*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 72, normalized size = 0.89

$$\frac{3\,a^4\cos\left(dx+c\right)^4-42\,a^4\cos\left(dx+c\right)^2+12\,a^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-16\,\left(a^4\cos\left(dx+c\right)^2-4\,a^4\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^4*cos(d*x + c)^4 - 42*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x
 + c)) - 16*(a^4*cos(d*x + c)^2 - 4*a^4)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int \cos(c+dx)\csc(c+dx)\,dx + \int 4\sin(c+dx)\cos(c+dx)\csc(c+dx)\,dx + \int 6\sin^2(c+dx)\cos(c+dx)\csc(c+dx)\,dx + \int 4\sin^3(c+dx)\cos(c+dx)\csc(c+dx)\,dx + \int \sin^4(c+dx)\cos(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(cos(c + d*x)*csc(c + d*x), x) + Integral(4*sin(c + d*x)*cos(
c + d*x)*csc(c + d*x), x) + Integral(6*sin(c + d*x)**2*cos(c + d*x)*csc(c +
 d*x), x) + Integral(4*sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x), x) + Inte
gral(sin(c + d*x)**4*cos(c + d*x)*csc(c + d*x), x))
```

Giac [A]
time = 0.47, size = 69, normalized size = 0.85

$$\frac{3\,a^4\sin(dx+c)^4 + 16\,a^4\sin(dx+c)^3 + 36\,a^4\sin(dx+c)^2 + 12\,a^4\log(|\sin(dx+c)|) + 48\,a^4\sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 36*a^4*sin(d*x + c)^2
+ 12*a^4*log(abs(sin(d*x + c))) + 48*a^4*sin(d*x + c))/d
```

Mupad [B]
time = 8.62, size = 118, normalized size = 1.46

$$\frac{16\,a^4\,\sin(c+d\,x)}{3\,d} - \frac{a^4\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d} + \frac{a^4\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{7\,a^4\cos(c+d\,x)^2}{2\,d} + \frac{a^4\cos(c+d\,x)^4}{4\,d} - \frac{4\,a^4\cos(c+d\,x)^2\,\sin(c+d\,x)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x),x)
```

```
[Out] (16*a^4*sin(c + d*x))/(3*d) - (a^4*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^4*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a^4*cos(c + d*x)^2)/(2*d)
+ (a^4*cos(c + d*x)^4)/(4*d) - (4*a^4*cos(c + d*x)^2*sin(c + d*x))/(3*d)
```

3.222 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=78

$$-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{a^4\sin^3(c+dx)}{3d}$$

```
[Out] -a^4*csc(d*x+c)/d+4*a^4*ln(sin(d*x+c))/d+6*a^4*sin(d*x+c)/d+2*a^4*sin(d*x+c
)^2/d+1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{6a^4\sin(c+dx)}{d}-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(6a^2+\frac{a^4}{x^2}+\frac{4a^3}{x}+4ax+x^2\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 1.00

$$-\frac{a^4\csc(c+dx)}{d}+\frac{4a^4\log(\sin(c+dx))}{d}+\frac{6a^4\sin(c+dx)}{d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{a^4\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -((a^4*Csc[c + d*x])/d) + (4*a^4*Log[Sin[c + d*x]])/d + (6*a^4*Sin[c + d*x]
)/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.12, size = 56, normalized size = 0.72

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\sin^3(dx+c)\right)}{3}+2\left(\sin^2(dx+c)\right)+6\sin(dx+c)+4\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
default	$\frac{a^4\left(\frac{\left(\sin^3(dx+c)\right)}{3}+2\left(\sin^2(dx+c)\right)+6\sin(dx+c)+4\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}\right)}{d}$
risch	$-4ia^4x-\frac{a^4e^{2i(dx+c)}}{2d}-\frac{25ia^4e^{i(dx+c)}}{8d}+\frac{25ia^4e^{-i(dx+c)}}{8d}-\frac{a^4e^{-2i(dx+c)}}{2d}-\frac{8ia^4c}{d}-\frac{2ia^4e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)}+\frac{4a^4}{}$
norman	$\frac{-\frac{a^4}{2d}+\frac{19a^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{101a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{101a^4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{19a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+8a^4(}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/d*(1/3*sin(d*x+c)^3+2*sin(d*x+c)^2+6*sin(d*x+c)+4*ln(sin(d*x+c))-1/sin(
d*x+c))
```

Maxima [A]
time = 0.27, size = 67, normalized size = 0.86

$$\frac{a^4\sin\left(dx+c\right)^3+6\,a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(\sin\left(dx+c\right)\right)+18\,a^4\sin\left(dx+c\right)-\frac{3\,a^4}{\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) +
 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.17

$$\frac{a^4\cos\left(dx+c\right)^4-20\,a^4\cos\left(dx+c\right)^2+12\,a^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+16\,a^4-3\,\left(2\,a^4\cos\left(dx+c\right)^2-a^4\right)\sin\left(dx+c\right)}{3\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(a^4*cos(d*x + c)^4 - 20*a^4*cos(d*x + c)^2 + 12*a^4*log(1/2*sin(d*x +
c))*sin(d*x + c) + 16*a^4 - 3*(2*a^4*cos(d*x + c)^2 - a^4)*sin(d*x + c))/(d
*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\cos\left(c+dx\right)\csc^2\left(c+dx\right)\,dx+\int 4\sin\left(c+dx\right)\cos\left(c+dx\right)\csc^2\left(c+dx\right)\,dx+\int 6\sin^2\left(c+dx\right)\cos\left(c+dx\right)\csc^2\left(c+dx\right)\,dx+\int 4\sin^3\left(c+dx\right)\cos\left(c+dx\right)\csc^2\left(c+dx\right)\,dx+\int\sin^4\left(c+dx\right)\cos\left(c+dx\right)\csc^2\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(cos(c + d*x)*csc(c + d*x)**2, x) + Integral(4*sin(c + d*x)*c
os(c + d*x)*csc(c + d*x)**2, x) + Integral(6*sin(c + d*x)**2*cos(c + d*x)*c
sc(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*cos(c + d*x)*csc(c + d*x)**
2, x) + Integral(sin(c + d*x)**4*cos(c + d*x)*csc(c + d*x)**2, x))
```

Giac [A]
time = 0.53, size = 68, normalized size = 0.87

$$\frac{a^4\sin\left(dx+c\right)^3+6\,a^4\sin\left(dx+c\right)^2+12\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^4\sin\left(dx+c\right)-\frac{3\,a^4}{\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

[Out] 1/3*(a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c))) + 18*a^4*sin(d*x + c) - 3*a^4/sin(d*x + c))/d

Mupad [B]
time = 8.65, size = 235, normalized size = 3.01

$$\frac{8\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}-\frac{8\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{4\,a^4\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{4\,a^4\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{28\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{16\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{8\,a^4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{23\,a^4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x)^2,x)

[Out] (8*a^4*cos(c/2 + (d*x)/2)^2)/d - (8*a^4*cos(c/2 + (d*x)/2)^4)/d - (4*a^4*log(1/cos(c/2 + (d*x)/2)^2))/d + (4*a^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (28*a^4*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a^4*cos(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a^4*cos(c/2 + (d*x)/2)^7)/(3*d*sin(c/2 + (d*x)/2)) + (23*a^4*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^4*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 + (d*x)/2))

3.223 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=80

$$-\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{6a^4\log(\sin(c+dx))}{d}+\frac{4a^4\sin(c+dx)}{d}+\frac{a^4\sin^2(c+dx)}{2d}$$

```
[Out] -4*a^4*csc(d*x+c)/d-1/2*a^4*csc(d*x+c)^2/d+6*a^4*ln(sin(d*x+c))/d+4*a^4*sin
(d*x+c)/d+1/2*a^4*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^4\sin^2(c+dx)}{2d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{6a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (6*a^4*Log[Sin[c + d
*x]])/d + (4*a^4*Sin[c + d*x])/d + (a^4*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{a^2\text{Subst}\left(\int \frac{(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^2\text{Subst}\left(\int \left(4a+\frac{a^4}{x^3}+\frac{4a^3}{x^2}+\frac{6a^2}{x}+x\right)\,dx, x, a\sin(c+d\right.}{d}$$

$$= -\frac{4a^4\csc(c+dx)}{d} - \frac{a^4\csc^2(c+dx)}{2d} + \frac{6a^4\log(\sin(c+d}{d}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.68

$$-\frac{a^4(8\csc(c+dx)+\csc^2(c+dx)-12\log(\sin(c+dx))-8\sin(c+dx)-\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/2*(a^4*(8*Csc[c + d*x] + Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] - 8*Sin[c
 + d*x] - Sin[c + d*x]^2))/d
```

Maple [A]
time = 0.13, size = 56, normalized size = 0.70

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)+6\ln(\sin(dx+c))-\frac{4}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
default	$\frac{a^4\left(\frac{\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)+6\ln(\sin(dx+c))-\frac{4}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}\right)}{d}$
risch	$-6ia^4x-\frac{a^4e^{2i(dx+c)}}{8d}-\frac{2ia^4e^{i(dx+c)}}{d}+\frac{2ia^4e^{-i(dx+c)}}{d}-\frac{a^4e^{-2i(dx+c)}}{8d}-\frac{12ia^4c}{d}-\frac{2ia^4\left(ie^{2i(dx+c)}+4e^{3i(dx}\right.}{d\left(e^{2i(dx+c)}-\right.}$
norman	$\frac{-\frac{a^4}{8d}-\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^4\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^4/d*(1/2*sin(d*x+c)^2+4*sin(d*x+c)+6*ln(sin(d*x+c))-4/sin(d*x+c)-1/2/sin(
d*x+c)^2)
```

Maxima [A]
time = 0.27, size = 66, normalized size = 0.82

$$\frac{a^4\sin(dx+c)^2+12\,a^4\log(\sin(dx+c))+8\,a^4\sin(dx+c)-\frac{8\,a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(sin(d*x + c)) + 8*a^4*sin(d*x + c) - (
8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 98, normalized size = 1.22

$$-\frac{2\,a^4\cos(dx+c)^4-16\,a^4\cos(dx+c)^2\sin(dx+c)-3\,a^4\cos(dx+c)^2-a^4-24\left(a^4\cos(dx+c)^2-a^4\right)\log\left(\frac{1}{2}\sin(dx+c)\right)}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^4*cos(d*x + c)^4 - 16*a^4*cos(d*x + c)^2*sin(d*x + c) - 3*a^4*cos
(d*x + c)^2 - a^4 - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)))/(d
*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 67, normalized size = 0.84

$$\frac{a^4\sin(dx+c)^2+12\,a^4\log(|\sin(dx+c)|)+8\,a^4\sin(dx+c)-\frac{8\,a^4\sin(dx+c)+a^4}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(a^4*sin(d*x + c)^2 + 12*a^4*log(abs(sin(d*x + c))) + 8*a^4*sin(d*x + c
) - (8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 8.62, size = 207, normalized size = 2.59

$$\frac{6\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-24\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{15\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-16\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^4}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}-\frac{6\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^4)/sin(c + d*x)^3,x)
```

```
[Out] (6*a^4*log(tan(c/2 + (d*x)/2)))/d - (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^4
*tan(c/2 + (d*x)/2)^2 - 16*a^4*tan(c/2 + (d*x)/2)^3 - (15*a^4*tan(c/2 + (d*
x)/2)^4)/2 - 24*a^4*tan(c/2 + (d*x)/2)^5 + a^4/2 + 8*a^4*tan(c/2 + (d*x)/2)
)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2
)^6)) - (2*a^4*tan(c/2 + (d*x)/2))/d - (6*a^4*log(tan(c/2 + (d*x)/2)^2 + 1)
)/d
```

3.224 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=85

$$\frac{\log(1+\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a
/d+1/4*sin(d*x+c)^4/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) -
 Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3+a^2x-ax^2+x^3+\frac{a^4}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\log(1+\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad} - \frac{\sin^3(c+dx)}{3ad} + \frac{\sin^4(c}{4a}$$

Mathematica [A]
time = 0.12, size = 60, normalized size = 0.71

$$\frac{12\log(1+\sin(c+dx)) - 12\sin(c+dx) + 6\sin^2(c+dx) - 4\sin^3(c+dx) + 3\sin^4(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
[Out] (12*Log[1 + Sin[c + d*x]] - 12*Sin[c + d*x] + 6*Sin[c + d*x]^2 - 4*Sin[c +
d*x]^3 + 3*Sin[c + d*x]^4)/(12*a*d)
```

Maple [A]
time = 0.14, size = 56, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(1+\sin(dx+c))}{da}$
default	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(1+\sin(dx+c))}{da}$
risch	$-\frac{ix}{a}+\frac{5ie^{i(dx+c)}}{8ad}-\frac{5ie^{-i(dx+c)}}{8ad}-\frac{2ic}{ad}+\frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{\sin(3dx+3c)}{12ad}-\frac{3\cos(2dx+2c)}{8ad}$
norman	$\frac{\frac{2}{ad}+\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{10\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{10\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{38\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{38}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2-sin(d*x+c)+ln(1+s
in(d*x+c)))
```

Maxima [A]
time = 0.28, size = 63, normalized size = 0.74

$$\frac{\frac{3 \sin(dx+c)^4-4 \sin(dx+c)^3+6 \sin(dx+c)^2-12 \sin(dx+c)}{a}+\frac{12 \log(\sin(dx+c)+1)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 12*sin(d*x
+ c))/a + 12*log(sin(d*x + c) + 1)/a)/d
```

Fricas [A]
time = 0.35, size = 58, normalized size = 0.68

$$\frac{3 \cos(dx+c)^4-12 \cos(dx+c)^2+4 \left(\cos(dx+c)^2-4\right) \sin(dx+c)+12 \log(\sin(dx+c)+1)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 4)*sin(d*x
 + c) + 12*log(sin(d*x + c) + 1))/(a*d)
```

Sympy [A]
time = 0.65, size = 80, normalized size = 0.94

$$\begin{cases} \frac{\log(\sin(c+dx)+1)}{ad}+\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad} & \text{for } d\neq 0 \\ \frac{x\sin^4(c)\cos(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**4/(4*a*d) - sin(c +
d*x)**3/(3*a*d) + sin(c + d*x)**2/(2*a*d) - sin(c + d*x)/(a*d), Ne(d, 0)),
(x*sin(c)**4*cos(c)/(a*sin(c) + a), True))
```

Giac [A]
time = 0.43, size = 76, normalized size = 0.89

$$\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a}+\frac{3 a^3 \sin(dx+c)^4-4 a^3 \sin(dx+c)^3+6 a^3 \sin(dx+c)^2-12 a^3 \sin(dx+c)}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c) + 1))/a + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d
*x + c)^3 + 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c))/a^4)/d
```

Mupad [B]
time = 8.46, size = 68, normalized size = 0.80

$$\frac{\frac{\ln(\sin(c+d\,x)+1)}{a}-\frac{\sin(c+d\,x)}{a}+\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}+\frac{\sin(c+d\,x)^4}{4\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(sin(c + d*x) + 1)/a - sin(c + d*x)/a + sin(c + d*x)^2/(2*a) - sin(c +
d*x)^3/(3*a) + sin(c + d*x)^4/(4*a))/d
```

3.225 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=67

$$-\frac{\log(1+\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/
a/d
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d
) + Sin[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2 - ax + x^2 - \frac{a^3}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= -\frac{\log(1+\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.09, size = 50, normalized size = 0.75

$$\frac{-6\log(1+\sin(c+dx)) + 6\sin(c+dx) - 3\sin^2(c+dx) + 2\sin^3(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
[Out] (-6*Log[1 + Sin[c + d*x]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

Maple [A]
time = 0.11, size = 46, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \frac{\left(\sin^2(dx+c)\right)}{2} + \sin(dx+c) - \ln(1+\sin(dx+c))}{da}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \frac{\left(\sin^2(dx+c)\right)}{2} + \sin(dx+c) - \ln(1+\sin(dx+c))}{da}$
risch	$\frac{ix}{a} - \frac{5i e^{i(dx+c)}}{8ad} + \frac{5i e^{-i(dx+c)}}{8ad} + \frac{2ic}{ad} - \frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad} - \frac{\sin(3dx+3c)}{12ad} + \frac{\cos(2dx+2c)}{4ad}$
norman	$\frac{-\frac{5}{3ad} - \frac{20\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} - \frac{20\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} - \frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad} + \frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad} - \frac{16\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} - \frac{16\left(\tan^5\right.}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2+sin(d*x+c)-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.79

$$\frac{\frac{2\,\sin(dx+c)^3-3\,\sin(dx+c)^2+6\,\sin(dx+c)}{a}-\frac{6\,\log(\sin(dx+c)+1)}{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 6*sin(d*x + c))/a - 6*log(sin(d
*x + c) + 1)/a)/d
```

Fricas [A]
time = 0.37, size = 48, normalized size = 0.72

$$\frac{3\,\cos\left(dx+c\right)^2-2\,\left(\cos\left(dx+c\right)^2-4\right)\sin\left(dx+c\right)-6\,\log\left(\sin\left(dx+c\right)+1\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 4)*sin(d*x + c) - 6*log(sin(d*x
 + c) + 1))/(a*d)
```

Sympy [A]
time = 0.51, size = 66, normalized size = 0.99

$$\begin{cases} -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin^3\left(c+dx\right)}{3ad}-\frac{\sin^2\left(c+dx\right)}{2ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for}\ d\neq 0\\ \frac{x\sin^3\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) - sin(c +
 d*x)**2/(2*a*d) + sin(c + d*x)/(a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*si
n(c) + a), True))
```

Giac [A]
time = 0.43, size = 64, normalized size = 0.96

$$-\frac{\frac{6\,\log(|\sin(dx+c)+1|)}{a}-\frac{2\,a^2\,\sin(dx+c)^3-3\,a^2\,\sin(dx+c)^2+6\,a^2\,\sin(dx+c)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a - (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*
x + c)^2 + 6*a^2*sin(d*x + c))/a^3)/d
```

Mupad [B]
time = 0.06, size = 56, normalized size = 0.84

$$-\frac{\frac{\ln(\sin(c+d\,x)+1)}{a}-\frac{\sin(c+d\,x)}{a}+\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] -(log(sin(c + d*x) + 1)/a - sin(c + d*x)/a + sin(c + d*x)^2/(2*a) - sin(c +
 d*x)^3/(3*a))/d
```

3.226 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{\log(1+\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2912, 12, 45}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\log(1+\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.78

$$\frac{2\log(1+\sin(c+dx)) - 2\sin(c+dx) + \sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (2*Log[1 + Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a*d)
```

Maple [A]
time = 0.09, size = 36, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(1+\sin(dx+c))}{da}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(1+\sin(dx+c))}{da}$
risch	$-\frac{ix}{a}+\frac{i\,e^{i(dx+c)}}{2ad}-\frac{i\,e^{-i(dx+c)}}{2ad}-\frac{2ic}{ad}+\frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad}-\frac{\cos(2dx+2c)}{4ad}$
norman	$\frac{\frac{2}{ad}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/2*sin(d*x+c)^2-sin(d*x+c)+ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.84

$$\frac{\frac{\sin(dx+c)^2-2\,\sin(dx+c)}{a}+\frac{2\,\log(\sin(dx+c)+1)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a + 2*log(sin(d*x + c) + 1)/a)/d
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.73

$$-\frac{\cos\left(dx+c\right)^{2}-2\,\log\left(\sin\left(dx+c\right)+1\right)+2\,\sin\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(cos(d*x + c)^2 - 2*log(sin(d*x + c) + 1) + 2*sin(d*x + c))/(a*d)
```

Sympy [A]
time = 0.33, size = 53, normalized size = 1.08

$$\begin{cases} \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin^{2}\left(c+dx\right)}{2ad}-\frac{\sin\left(c+dx\right)}{ad} & \text{for}\ d\neq 0 \\ \frac{x\sin^{2}\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Piecewise((log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**2/(2*a*d) - sin(c + 
d*x)/(a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a), True))
```

Giac [A]
time = 0.46, size = 45, normalized size = 0.92

$$\frac{\frac{2\,\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}+\frac{a\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a + (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/a^2)/d
```

Mupad [B]
time = 0.05, size = 35, normalized size = 0.71

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)-\sin\left(c+d\,x\right)+\frac{\sin\left(c+d\,x\right)^{2}}{2}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
[Out] (log(sin(c + d*x) + 1) - sin(c + d*x) + sin(c + d*x)^2/2)/(a*d)
```

3.227 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$-\frac{\log(1+\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}$$

```
[Out] -ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2912, 12, 45}

$$\frac{\sin(c+dx)}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Log[1 + Sin[c + d*x]]/(a*d)) + Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{\log(1+\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.81

$$\frac{-\log(1+\sin(c+dx))+\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (-Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.08, size = 26, normalized size = 0.84

method	result	si
derivativedivides	$\frac{\sin(dx+c)-\ln(1+\sin(dx+c))}{da}$	26
default	$\frac{\sin(dx+c)-\ln(1+\sin(dx+c))}{da}$	26
risch	$\frac{ix}{a}-\frac{i\mathrm{e}^{i(dx+c)}}{2ad}+\frac{i\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{2ic}{ad}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}$	76
norman	$\frac{-\frac{2}{ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(sin(d*x+c)-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -(log(sin(d*x + c) + 1)/a - sin(d*x + c)/a)/d
```

Fricas [A]
time = 0.36, size = 26, normalized size = 0.84

$$-\frac{\log\left(\sin\left(dx+c\right)+1\right)-\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(sin(d*x + c) + 1) - sin(d*x + c))/(a*d)
```

Sympy [A]
time = 0.26, size = 37, normalized size = 1.19

$$\begin{cases} -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for } d\neq 0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Piecewise((-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Ne(d, 0)), (x
*sin(c)*cos(c)/(a*sin(c) + a), True))
```

Giac [A]
time = 0.46, size = 31, normalized size = 1.00

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a}-\frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c) + 1))/a - sin(d*x + c)/a)/d
```

Mupad [B]
time = 8.46, size = 26, normalized size = 0.84

$$-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)-\sin\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x)),x)
[Out] -(log(sin(c + d*x) + 1) - sin(c + d*x))/(a*d)
```

3.228 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2786, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.08, size = 27, normalized size = 0.84

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))-\ln(1+\sin(dx+c))}{ad}$	27
default	$\frac{\ln(\sin(dx+c))-\ln(1+\sin(dx+c))}{ad}$	27
risch	$-\frac{2\ln(e^{i(dx+c)}+i)}{ad}+\frac{\ln(e^{2i(dx+c)}-1)}{ad}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(ln(sin(d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 31, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.37, size = 28, normalized size = 0.88

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos\left(c+dx\right)\csc\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 33, normalized size = 1.03

$$-\frac{\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

Mupad [B]
time = 8.61, size = 32, normalized size = 1.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
[Out] (log(tan(c/2 + (d*x)/2)) - 2*log(tan(c/2 + (d*x)/2) + 1))/(a*d)
```

3.229 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d+ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2912, 12, 46}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{1}{a^2x} + \frac{1}{a^2(a+x)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.00

$$-\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*
d)
```

Maple [A]
time = 0.10, size = 37, normalized size = 0.80

method	result	size
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))+\ln(1+\sin(dx+c))}{ad}$	37
default	$\frac{-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))+\ln(1+\sin(dx+c))}{ad}$	37
risch	$-\frac{2i\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{ad}$	74
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-1/sin(d*x+c)-ln(sin(d*x+c))+ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.93

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a} - \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] (log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a - 1/(a*sin(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 51, normalized size = 1.11

$$-\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right) - \log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+1}{ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) - log(sin(d*x + c) + 1)*sin(d*x + c) +
 1)/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc^2\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 45, normalized size = 0.98

$$\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a} - \frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a - 1/(a*sin(d*x + c
)))/d
```

Mupad [B]
time = 8.60, size = 55, normalized size = 1.20

$$-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*log(tan(c/2 + (d*x)/2)) - 4*log(tan(c/2 + (d*x)/2) + 1) + tan(c/2 + (d*
x)/2) + 1/tan(c/2 + (d*x)/2))/(2*a*d)
```

3.230 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d+ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{a^2x^2}+\frac{1}{a^3x}-\frac{1}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.00

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) + Log[Sin[c + d*x]]/(a*d) - Log
[1 + Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.11, size = 45, normalized size = 0.71

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)}-\ln(1+\sin(dx+c))}{ad}$	45
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)}-\ln(1+\sin(dx+c))}{ad}$	45
risch	$\frac{2i\left(-i\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{ad}$	97
norman	$\frac{-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-1/2/sin(d*x+c)^2+ln(sin(d*x+c))+1/sin(d*x+c)-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 55, normalized size = 0.87

$$-\frac{\frac{2\log(\sin(dx+c)+1)}{a}-\frac{2\log(\sin(dx+c))}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c) + 1)/a - 2*log(sin(d*x + c))/a - (2*sin(d*x + c) -
 1)/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.14

$$\frac{2\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-2\sin\left(dx+c\right)+1}{2\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos\left(c+dx\right)\csc^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 57, normalized size = 0.90

$$-\frac{\frac{2\log(|\sin(dx+c)+1|)}{a}-\frac{2\log(|\sin(dx+c)|)}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a - 2*log(abs(sin(d*x + c)))/a - (2*sin(
d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

Mupad [B]
time = 8.82, size = 106, normalized size = 1.68

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*log(tan(c
/2 + (d*x)/2) + 1))/(a*d) + tan(c/2 + (d*x)/2)/(2*a*d) + (cot(c/2 + (d*x)/2
)^2*(2*tan(c/2 + (d*x)/2) - 1/2))/(4*a*d)
```

3.231 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d-ln(sin(d*x+c))/a/
d+ln(1+sin(d*x+c))/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{a^3\text{Subst}\left(\int \frac{1}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{a^2x^3}+\frac{1}{a^3x^2}-\frac{1}{a^4x}+\frac{1}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1 \cdots}{\cdots}$$

Mathematica [**A**]
time = 0.03, size = 82, normalized size = 1.00

$$-\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\log(1+\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) - L
og[Sin[c + d*x]]/(a*d) + Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [**A**]
time = 0.12, size = 57, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^2}-\ln(\sin(dx+c))+\ln(1+\sin(dx+c))}{ad}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^2}-\ln(\sin(dx+c))+\ln(1+\sin(dx+c))}{ad}$
risch	$-\frac{2i\left(3\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}-3ie^{4i(dx+c)}+3\,e^{i(dx+c)}+3ie^{2i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}-1\right)^3}+\frac{2\ln\left(e^{i(dx+c)}+i\right)}{ad}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\ln\left(\tan\left(\cdots\right.\right.}{a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/d*(-1/3/sin(d*x+c)^3-1/sin(d*x+c)+1/2/sin(d*x+c)^2-ln(sin(d*x+c))+ln(1+
sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.79

$$\frac{\frac{6 \log(\sin(dx+c)+1)}{a} - \frac{6 \log(\sin(dx+c))}{a} - \frac{6 \sin(dx+c)^2 - 3 \sin(dx+c) + 2}{a \sin(dx+c)^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c) + 1)/a - 6*log(sin(d*x + c))/a - (6*sin(d*x + c)^2
- 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.36, size = 102, normalized size = 1.24

$$-\frac{6 \left(\cos \left(dx+c\right)^2-1\right) \log \left(\frac{1}{2} \sin \left(dx+c\right)\right) \sin \left(dx+c\right)-6 \left(\cos \left(dx+c\right)^2-1\right) \log \left(\sin \left(dx+c\right)+1\right) \sin \left(dx+c\right)+6 \cos \left(dx+c\right)^2+3 \sin \left(dx+c\right)-8}{6 \left(ad \cos \left(dx+c\right)^2-ad\right) \sin \left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 6*(cos(d*
x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 6*cos(d*x + c)^2 + 3*sin
(d*x + c) - 8)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos \left(c+dx\right) \csc^4 \left(c+dx\right)}{\sin \left(c+dx\right)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 67, normalized size = 0.82

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a} - \frac{6 \log(|\sin(dx+c)|)}{a} - \frac{6 \sin(dx+c)^2 - 3 \sin(dx+c) + 2}{a \sin(dx+c)^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c)))/a - (6*sin(d
*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

Mupad [B]
time = 8.59, size = 139, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - log(tan(c/2
+ (d*x)/2))/(a*d) + (2*log(tan(c/2 + (d*x)/2) + 1))/(a*d) - (5*tan(c/2 + (d
*x)/2))/(8*a*d) - (cot(c/2 + (d*x)/2)^3*(5*tan(c/2 + (d*x)/2)^2 - tan(c/2 +
 (d*x)/2) + 1/3))/(8*a*d)
```

3.232 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$-\frac{4\log(1+\sin(c+dx))}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{\sin^3(c+dx)}{3a^2d}-\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^2/d+3*sin(d*x+c)/a^2/d-sin(d*x+c)^2/a^2/d+1/3*sin(d*x
+c)^3/a^2/d-1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^2(c+dx)}{a^2d}+\frac{3\sin(c+dx)}{a^2d}-\frac{1}{d\,(a^2\sin(c+dx)+a^2)}-\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^2*d) + (3*Sin[c + d*x])/(a^2*d) - Sin[c + d*x
]^2/(a^2*d) + Sin[c + d*x]^3/(3*a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2 - 2ax + x^2 + \frac{a^4}{(a+x)^2} - \frac{4a^3}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^2 d} + \frac{3\sin(c+dx)}{a^2 d} - \frac{\sin^2(c+dx)}{a^2 d} + \frac{\sin^3(c+dx)}{3a^2 d} -$$

Mathematica [A]
time = 0.45, size = 73, normalized size = 0.84

$$\frac{-12\log(1+\sin(c+dx)) - \frac{3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + 9\sin(c+dx) - 3\sin^2(c+dx) + \sin^3(c+dx)}{3a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
[Out] (-12*Log[1 + Sin[c + d*x]] - 3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 9*
Sin[c + d*x] - 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(3*a^2*d)
```

Maple [A]
time = 0.21, size = 60, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \left(\sin^2(dx+c)\right) + 3\sin(dx+c) - \frac{1}{1+\sin(dx+c)} - 4\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \left(\sin^2(dx+c)\right) + 3\sin(dx+c) - \frac{1}{1+\sin(dx+c)} - 4\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$\frac{4ix}{a^2} - \frac{13i e^{i(dx+c)}}{8d\,a^2} + \frac{13i e^{-i(dx+c)}}{8d\,a^2} + \frac{8ic}{d\,a^2} - \frac{2i e^{i(dx+c)}}{d\,a^2\left(e^{i(dx+c)}+i\right)^2} - \frac{8\ln\left(e^{i(dx+c)}+i\right)}{d\,a^2} - \frac{\sin(3dx+3c)}{12d\,a^2} + \frac{\cos(2dx}{2d}$
norman	$\frac{\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad} + \frac{8\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{72\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{72\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{16\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{128(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 a(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/3*sin(d*x+c)^3-sin(d*x+c)^2+3*sin(d*x+c)-1/(1+sin(d*x+c))-4*ln(1
+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.80

$$-\frac{\frac{3}{a^2\sin(dx+c)+a^2}-\frac{\sin(dx+c)^3-3\sin(dx+c)^2+9\sin(dx+c)}{a^2}+\frac{12\log(\sin(dx+c)+1)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3/(a^2*sin(d*x + c) + a^2) - (sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 9*s
in(d*x + c))/a^2 + 12*log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.93

$$\frac{2\cos(dx+c)^4-16\cos(dx+c)^2-24(\sin(dx+c)+1)\log(\sin(dx+c)+1)+\left(4\cos(dx+c)^2+17\right)\sin(dx+c)+11}{6\left(a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 16*cos(d*x + c)^2 - 24*(sin(d*x + c) + 1)*log(sin(d
*x + c) + 1) + (4*cos(d*x + c)^2 + 17)*sin(d*x + c) + 11)/(a^2*d*sin(d*x +
c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(75) = 150$.
time = 1.01, size = 201, normalized size = 2.31

$$\begin{cases}-\frac{12\log(\sin(c+dx)+1)\sin(c+dx)}{3a^2d\sin(c+dx)+3a^2d}-\frac{12\log(\sin(c+dx)+1)}{3a^2d\sin(c+dx)+3a^2d}+\frac{\sin^4(c+dx)}{3a^2d\sin(c+dx)+3a^2d}-\frac{2\sin^3(c+dx)}{3a^2d\sin(c+dx)+3a^2d}+\frac{6\sin^2(c+dx)}{3a^2d\sin(c+dx)+3a^2d}-\frac{12}{3a^2d\sin(c+dx)+3a^2d} & \text{for } d\neq 0\\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**2*d*sin(c + d*x) +
3*a**2*d) - 12*log(sin(c + d*x) + 1)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + s
in(c + d*x)**4/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 2*sin(c + d*x)**3/(3*a*
*2*d*sin(c + d*x) + 3*a**2*d) + 6*sin(c + d*x)**2/(3*a**2*d*sin(c + d*x) +
3*a**2*d) - 12/(3*a**2*d*sin(c + d*x) + 3*a**2*d), Ne(d, 0)), (x*sin(c)**4*
cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.49, size = 107, normalized size = 1.23

$$-\frac{\frac{(a\sin(dx+c)+a)^3\left(\frac{6a}{a\sin(dx+c)+a}-\frac{18a^2}{(a\sin(dx+c)+a)^2}-1\right)}{a^5}-\frac{12\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{3}{(a\sin(dx+c)+a)a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*((a*sin(d*x + c) + a)^3*(6*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x
+ c) + a)^2 - 1)/a^5 - 12*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)
^2*abs(a)))/a^2 + 3/((a*sin(d*x + c) + a)*a))/d
```

Mupad [B]
time = 0.05, size = 72, normalized size = 0.83

$$-\frac{\frac{1}{a^2\,\sin(c+d\,x)+a^2}+\frac{4\,\ln(\sin(c+d\,x)+1)}{a^2}-\frac{3\,\sin(c+d\,x)}{a^2}+\frac{\sin(c+d\,x)^2}{a^2}-\frac{\sin(c+d\,x)^3}{3\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(1/(a^2*sin(c + d*x) + a^2) + (4*log(sin(c + d*x) + 1))/a^2 - (3*sin(c + d
*x))/a^2 + sin(c + d*x)^2/a^2 - sin(c + d*x)^3/(3*a^2))/d
```

3.233 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{3\log(1+\sin(c+dx))}{a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin^2(c+dx)}{2a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 3*ln(1+sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d+1/d/(a^2
+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2912, 12, 45}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{1}{d\,(a^2\sin(c+dx)+a^2)}+\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*Log[1 + Sin[c + d*x]])/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]
^2/(2*a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+x-\frac{a^3}{(a+x)^2}+\frac{3a^2}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{3\log(1+\sin(c+dx))}{a^2d} - \frac{2\sin(c+dx)}{a^2d} + \frac{\sin^2(c+dx)}{2a^2d} + \frac{1}{d\,(a^2+a^2\sin(c+$$

Mathematica [A]
time = 0.15, size = 71, normalized size = 1.01

$$\frac{2+6\log(1+\sin(c+dx))+(-4+6\log(1+\sin(c+dx)))\sin(c+dx)-3\sin^2(c+dx)+\sin^3(c+dx)}{2a^2d(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2 + 6*Log[1 + Sin[c + d*x]] + (-4 + 6*Log[1 + Sin[c + d*x]])*Sin[c + d*x]
- 3*Sin[c + d*x]^2 + Sin[c + d*x]^3)/(2*a^2*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.16, size = 48, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-2\sin(dx+c)+\frac{1}{1+\sin(dx+c)}+3\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-2\sin(dx+c)+\frac{1}{1+\sin(dx+c)}+3\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$-\frac{3ix}{a^2}+\frac{i e^{i(dx+c)}}{d\,a^2}-\frac{i e^{-i(dx+c)}}{d\,a^2}-\frac{6ic}{d\,a^2}+\frac{2i e^{i(dx+c)}}{d\,a^2\left(e^{i(dx+c)}+i\right)^2}+\frac{6\ln\left(e^{i(dx+c)}+i\right)}{d\,a^2}-\frac{\cos(2dx+2c)}{4d\,a^2}$
norman	$\frac{-\frac{12\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{12\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{26\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{26\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{38\left(\mathrm{ta}\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/2*sin(d*x+c)^2-2*sin(d*x+c)+1/(1+sin(d*x+c))+3*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.84

$$\frac{\frac{2}{a^2\sin(dx+c)+a^2}+\frac{\sin(dx+c)^2-4\sin(dx+c)}{a^2}+\frac{6\log(\sin(dx+c)+1)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2/(a^2*sin(d*x + c) + a^2) + (sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 6
*log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.03

$$\frac{6\cos(dx+c)^2+12(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(2\cos(dx+c)^2+7)\sin(dx+c)-3}{4(a^2d\sin(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 + 12*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (2*co
s(d*x + c)^2 + 7)*sin(d*x + c) - 3)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(61) = 122.
time = 0.71, size = 170, normalized size = 2.43

$$\begin{cases}\frac{6\log(\sin(c+dx)+1)\sin(c+dx)}{2a^2d\sin(c+dx)+2a^2d}+\frac{6\log(\sin(c+dx)+1)}{2a^2d\sin(c+dx)+2a^2d}+\frac{\sin^3(c+dx)}{2a^2d\sin(c+dx)+2a^2d}-\frac{3\sin^2(c+dx)}{2a^2d\sin(c+dx)+2a^2d}+\frac{6}{2a^2d\sin(c+dx)+2a^2d} & \text{for } d\neq 0\\ \frac{x\sin^3(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x) + 2*
a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) + sin(
c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) - 3*sin(c + d*x)**2/(2*a**2*
d*sin(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Ne(d, 0)
), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.48, size = 90, normalized size = 1.29

$$-\frac{\frac{(a\sin(dx+c)+a)^2\left(\frac{6a}{a\sin(dx+c)+a}-1\right)}{a^4}+\frac{6\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}-\frac{2}{(a\sin(dx+c)+a)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((a*sin(d*x + c) + a)^2*(6*a/(a*sin(d*x + c) + a) - 1)/a^4 + 6*log(abs
(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 - 2/((a*sin(d*x +
 c) + a)*a))/d
```

Mupad [B]
time = 8.47, size = 59, normalized size = 0.84

$$\frac{\frac{1}{a^2\,\sin(c+d\,x)+a^2}+\frac{3\,\ln(\sin(c+d\,x)+1)}{a^2}-\frac{2\,\sin(c+d\,x)}{a^2}+\frac{\sin(c+d\,x)^2}{2\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (1/(a^2*sin(c + d*x) + a^2) + (3*log(sin(c + d*x) + 1))/a^2 - (2*sin(c + d*
x))/a^2 + sin(c + d*x)^2/(2*a^2))/d
```

3.234 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{2\log(1+\sin(c+dx))}{a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] -2*ln(1+sin(d*x+c))/a^2/d+sin(d*x+c)/a^2/d-1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin(c+dx)}{a^2d}-\frac{1}{d\,(a^2\sin(c+dx)+a^2)}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Log[1 + Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2*d) - 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a+x)^2}-\frac{2a}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= -\frac{2\log(1+\sin(c+dx))}{a^2d} + \frac{\sin(c+dx)}{a^2d} - \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A]
time = 0.16, size = 55, normalized size = 1.06

$$\frac{-8\log(1+\sin(c+dx)) - \frac{4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + 4\sin(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-8*Log[1 + Sin[c + d*x]] - 4/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 4*S
in[c + d*x])/(4*a^2*d)
```

Maple [A]
time = 0.15, size = 38, normalized size = 0.73

method	result
derivativedivides	$\frac{\sin(dx+c)-\frac{1}{1+\sin(dx+c)}-2\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{\sin(dx+c)-\frac{1}{1+\sin(dx+c)}-2\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$\frac{2ix}{a^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^2}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^2}+\frac{4ic}{d\,a^2}-\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2}-\frac{4\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^2}$
norman	$\frac{\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{20\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{20\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{16\left(\tan^3\left(\frac{d}{}\right.\right.}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(sin(d*x+c)-1/(1+sin(d*x+c))-2*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 47, normalized size = 0.90

$$-\frac{\frac{1}{a^2\sin(dx+c)+a^2}+\frac{2\log(\sin(dx+c)+1)}{a^2}-\frac{\sin(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(1/(a^2*sin(d*x + c) + a^2) + 2*log(sin(d*x + c) + 1)/a^2 - sin(d*x + c)/a
^2)/d
```

Fricas [A]
time = 0.35, size = 57, normalized size = 1.10

$$-\frac{\cos(dx+c)^2+2(\sin(dx+c)+1)\log(\sin(dx+c)+1)-\sin(dx+c)}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 + 2*(sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - sin(d*x + c
))/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(44) = 88$.
time = 0.54, size = 126, normalized size = 2.42

$$\begin{cases} -\frac{2\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d}+\frac{\sin^2(c+dx)}{a^2d\sin(c+dx)+a^2d}-\frac{2}{a^2d\sin(c+dx)+a^2d} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-2*log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**
2*d) - 2*log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + sin(c + d*x
)**2/(a**2*d*sin(c + d*x) + a**2*d) - 2/(a**2*d*sin(c + d*x) + a**2*d), Ne(
d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.47, size = 70, normalized size = 1.35

$$\frac{\frac{2\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{a\sin(dx+c)+a}{a^3}-\frac{1}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a^2 + (a*si
n(d*x + c) + a)/a^3 - 1/((a*sin(d*x + c) + a)*a))/d
```

Mupad [B]
time = 0.08, size = 45, normalized size = 0.87

$$\frac{\sin\left(c+d\,x\right)^2-2}{a^2\,d\,\left(\sin\left(c+d\,x\right)+1\right)}-\frac{2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2 - 2)/(a^2*d*(sin(c + d*x) + 1)) - (2*log(sin(c + d*x) + 1))
/(a^2*d)
```

3.235 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=37

$$\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{1}{d\,(a^2\sin(c+dx)+a^2)}+\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^2}+\frac{1}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.73

$$\frac{\log(1+\sin(c+dx))+\frac{1}{1+\sin(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*d)
```

Maple [A]
time = 0.12, size = 28, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{1}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{\frac{1}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{d\,a^2}+\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^2}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{2\ln(\tan$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/(1+sin(d*x+c))+ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.92

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}+\frac{\log(\sin(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) + log(sin(d*x + c) + 1)/a^2)/d
```

Fricas [A]
time = 0.35, size = 40, normalized size = 1.08

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(31) = 62$.
time = 0.44, size = 95, normalized size = 2.57

$$\begin{cases} \frac{\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d} + \frac{\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d} + \frac{1}{a^2d\sin(c+dx)+a^2d} & \text{for } d \neq 0 \\ \frac{x\sin(c)\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((log(sin(c + d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d
) + log(sin(c + d*x) + 1)/(a**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c
+ d*x) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.45, size = 56, normalized size = 1.51

$$-\frac{\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a} - \frac{1}{a\sin(dx+c)+a}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*sin
(d*x + c) + a))/(a*d)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.92

$$\frac{1}{a^2\,d\,(\sin(c+d\,x)+1)} + \frac{\ln(\sin(c+d\,x)+1)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
[Out] 1/(a^2*d*(sin(c + d*x) + 1)) + log(sin(c + d*x) + 1)/(a^2*d)
```

3.236 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^2/d-ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2786, 46}

$$\frac{1}{d\,(a^2\sin(c+dx)+a^2)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx&=\frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\&=\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\,(a^2+a^2\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 36, normalized size = 0.69

$$\frac{\log(\sin(c+dx))-\log(1+\sin(c+dx))+\frac{1}{1+\sin(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

Maple [A]
time = 0.15, size = 37, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^2d}$	37
default	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^2d}$	37
risch	$\frac{2i\,e^{i(dx+c)}}{d\,a^2\left(e^{i(dx+c)}+i\right)^2}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{d\,a^2}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(ln(sin(d*x+c))+1/(1+sin(d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

Fricas [A]
time = 0.35, size = 59, normalized size = 1.13

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right)-(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1),
x)/a**2
```

Giac [A]
time = 0.43, size = 45, normalized size = 0.87

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3}+\frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

Mupad [B]
time = 8.58, size = 87, normalized size = 1.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) -
 (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2 + 2*a^2*tan(c/2
+ (d*x)/2)))
```

3.237 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(1+\sin(c+dx))}{a^2d}-\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] -csc(d*x+c)/a^2/d-2*ln(sin(d*x+c))/a^2/d+2*ln(1+sin(d*x+c))/a^2/d-1/d/(a^2+
a^2*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.120$, Rules used = {2912, 12, 46}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + (2*Log[1 + Sin[c
+ d*x]])/(a^2*d) - 1/(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2}{a^3x} + \frac{1}{a^2(a+x)^2} + \frac{2}{a^3(a+x)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{a^2d} - \frac{2\log(\sin(c+dx))}{a^2d} + \frac{2\log(1+\sin(c+dx))}{a^2d} - \frac{1}{d\,(a^2+a^2\sin}$$

Mathematica [A]
time = 0.12, size = 45, normalized size = 0.66

$$-\frac{\csc(c+dx)+2\log(\sin(c+dx))-2\log(1+\sin(c+dx))+\frac{1}{1+\sin(c+dx)}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (1 + Sin[
c + d*x])^(-1))/(a^2*d))
```

Maple [A]
time = 0.13, size = 51, normalized size = 0.75

method	result	size
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-2\ln(\sin(dx+c))-\frac{1}{1+\sin(dx+c)}+2\ln(1+\sin(dx+c))}{a^2d}$	51
default	$\frac{-\frac{1}{\sin(dx+c)}-2\ln(\sin(dx+c))-\frac{1}{1+\sin(dx+c)}+2\ln(1+\sin(dx+c))}{a^2d}$	51
risch	$-\frac{4i\left(i\,e^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^2a^2d}+\frac{4\ln\left(e^{i(dx+c)}+i\right)}{d\,a^2}-\frac{2\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	112
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{9\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{9\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^2}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(-1/sin(d*x+c)-2*ln(sin(d*x+c))-1/(1+sin(d*x+c))+2*ln(1+sin(d*x+c))
)
```

Maxima [A]
time = 0.27, size = 68, normalized size = 1.00

$$-\frac{\frac{2\sin(dx+c)+1}{a^2\sin(dx+c)^2+a^2\sin(dx+c)}-\frac{2\log(\sin(dx+c)+1)}{a^2}+\frac{2\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((2*sin(d*x + c) + 1)/(a^2*sin(d*x + c)^2 + a^2*sin(d*x + c)) - 2*log(sin(
d*x + c) + 1)/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.53

$$-\frac{2\left(\cos\left(dx+c\right)^2-\sin\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^2-\sin\left(dx+c\right)-1\right)\log\left(\sin\left(dx+c\right)+1\right)-2\sin\left(dx+c\right)-1}{a^2d\cos\left(dx+c\right)^2-a^2d\sin\left(dx+c\right)-a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*(cos(d*x + c)^2 - sin(d*x + c) - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x
+ c)^2 - sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) - 1)/(a^2
*d*cos(d*x + c)^2 - a^2*d*sin(d*x + c) - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos\left(c+dx\right)\csc^2\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.44, size = 69, normalized size = 1.01

$$-\frac{\frac{2\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{1}{(a\sin(dx+c)+a)a}-\frac{1}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 1/((a*sin(d*x + c) + a)*a)
- 1/(a^2*(a/(a*sin(d*x + c) + a) - 1)))/d
```

Mupad [B]
time = 8.61, size = 136, normalized size = 2.00

$$\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2\,d}-\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}-\frac{-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1}{d\left(2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (4*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*
d) - (2*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)^2 + 1)/(d*(4*a^2*tan(c/2
+ (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan(c/2 + (d*x)/2))) - ta
n(c/2 + (d*x)/2)/(2*a^2*d)
```

3.238 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(1+\sin(c+dx))}{a^2d}+\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+3*ln(sin(d*x+c))/a^2/d-3*ln(1+sin
(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{3\log(\sin(c+dx))}{a^2d}-\frac{3\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (3*Log[Sin[c + d*x]])
/(a^2*d) - (3*Log[1 + Sin[c + d*x]])/(a^2*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{2}{a^3x^2} + \frac{3}{a^4x} - \frac{1}{a^3(a+x)^2} - \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{3\log(\sin(c+dx))}{a^2d} - \frac{3\log(1+\sin(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.15, size = 61, normalized size = 0.72

$$\frac{4\csc(c+dx) - \csc^2(c+dx) + 6\log(\sin(c+dx)) - 6\log(1+\sin(c+dx)) + \frac{2}{1+\sin(c+dx)}}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c +
d*x]] + 2/(1 + Sin[c + d*x]))/(2*a^2*d)
```

Maple [A]
time = 0.19, size = 59, normalized size = 0.69

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{2}{\sin(dx+c)}+3\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-3\ln(1+\sin(dx+c))}{a^2d}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{2}{\sin(dx+c)}+3\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-3\ln(1+\sin(dx+c))}{a^2d}$
risch	$\frac{2i\left(3i e^{4i(dx+c)}+3 e^{5i(dx+c)}-3i e^{2i(dx+c)}-4 e^{3i(dx+c)}+3 e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^2 a^2 d}-\frac{6\ln\left(e^{i(dx+c)}+i\right)}{d a^2}+\frac{3\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$
norman	$\frac{-\frac{1}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{6\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^2}-$ 6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(-1/2/sin(d*x+c)^2+2/sin(d*x+c)+3*ln(sin(d*x+c))+1/(1+sin(d*x+c))-3
*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 0.94

$$\frac{\frac{6\,\sin(dx+c)^2+3\,\sin(dx+c)-1}{a^2\sin(dx+c)^3+a^2\sin(dx+c)^2}-\frac{6\,\log(\sin(dx+c)+1)}{a^2}+\frac{6\,\log(\sin(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((6*sin(d*x + c)^2 + 3*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^3 + a^2*sin(
d*x + c)^2) - 6*log(sin(d*x + c) + 1)/a^2 + 6*log(sin(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.36, size = 147, normalized size = 1.73

$$\frac{6\,\cos\left(dx+c\right)^2+6\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)-1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-6\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)-1\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,\sin\left(dx+c\right)-5}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d+\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cos(d*x + c)^2 + 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - 1)*log(1/2*sin(d*x + c)) - 6*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*si
n(d*x + c) - 1)*log(sin(d*x + c) + 1) - 3*sin(d*x + c) - 5)/(a^2*d*cos(d*x
+ c)^2 - a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc^3\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.44, size = 87, normalized size = 1.02

$$\frac{\frac{6\,\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{2}{(a\sin(dx+c)+a)a}-\frac{\frac{6\,a}{a\sin(dx+c)+a}-5}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 2/((a*sin(d*x + c) + a)*
a) - (6*a/(a*sin(d*x + c) + a) - 5)/(a^2*(a/(a*sin(d*x + c) + a) - 1)^2))/d
```

Mupad [B]
time = 8.61, size = 168, normalized size = 1.98

$$\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^2\,d}-\frac{6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}+\frac{-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+8\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (6*l
og(tan(c/2 + (d*x)/2) + 1))/(a^2*d) + (3*tan(c/2 + (d*x)/2) + (15*tan(c/2 +
 (d*x)/2)^2)/2 - 4*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)
^2 + 8*a^2*tan(c/2 + (d*x)/2)^3 + 4*a^2*tan(c/2 + (d*x)/2)^4)) + tan(c/2 +
(d*x)/2)/(a^2*d)
```

3.239 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{3\csc(c+dx)}{a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{\csc^3(c+dx)}{3a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(1+\sin(c+dx))}{a^2d}-\frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

```
[Out] -3*csc(d*x+c)/a^2/d+csc(d*x+c)^2/a^2/d-1/3*csc(d*x+c)^3/a^2/d-4*ln(sin(d*x+
c))/a^2/d+4*ln(1+sin(d*x+c))/a^2/d-1/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{a^2d}-\frac{3\csc(c+dx)}{a^2d}-\frac{4\log(\sin(c+dx))}{a^2d}+\frac{4\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{1}{x^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{a^2x^4} - \frac{2}{a^3x^3} + \frac{3}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right)\,dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{3\csc(c+dx)}{a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{\csc^3(c+dx)}{3a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log}{}$$

Mathematica [A]
time = 1.69, size = 98, normalized size = 0.97

$$-\frac{3\csc(c+dx)}{a^2d} + \frac{\csc^2(c+dx)}{a^2d} - \frac{\csc^3(c+dx)}{3a^2d} - \frac{4\log(\sin(c+dx))}{a^2d} + \frac{4\log(1+\sin(c+dx))}{a^2d} - \frac{1}{a^2d(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*Csc[c + d*x])/(a^2*d) + Csc[c + d*x]^2/(a^2*d) - Csc[c + d*x]^3/(3*a^2*
d) - (4*Log[Sin[c + d*x]])/(a^2*d) + (4*Log[1 + Sin[c + d*x]])/(a^2*d) - 1/
(a^2*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.21, size = 69, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{1}{\sin(dx+c)^2}-\frac{3}{\sin(dx+c)}-4\ln(\sin(dx+c))-\frac{1}{1+\sin(dx+c)}+4\ln(1+\sin(dx+c))}{a^2d}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{1}{\sin(dx+c)^2}-\frac{3}{\sin(dx+c)}-4\ln(\sin(dx+c))-\frac{1}{1+\sin(dx+c)}+4\ln(1+\sin(dx+c))}{a^2d}$
risch	$-\frac{8i\left(3i e^{6i(dx+c)}+3 e^{7i(dx+c)}-8i e^{4i(dx+c)}-7 e^{5i(dx+c)}+3i e^{2i(dx+c)}+7 e^{3i(dx+c)}-3 e^{i(dx+c)}\right)}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^2a^2d}+\frac{8\ln\left(e^{i(dx+c)}+i\right)}{d\,a^2}-4$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{33\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{33\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/d*(-1/3/sin(d*x+c)^3+1/sin(d*x+c)^2-3/sin(d*x+c)-4*ln(sin(d*x+c))-1/(
1+sin(d*x+c))+4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 90, normalized size = 0.89

$$-\frac{\frac{12\sin(dx+c)^3+6\sin(dx+c)^2-2\sin(dx+c)+1}{a^2\sin(dx+c)^4+a^2\sin(dx+c)^3}-\frac{12\log(\sin(dx+c)+1)}{a^2}+\frac{12\log(\sin(dx+c))}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 6*sin(d*x + c)^2 - 2*sin(d*x + c) + 1)/(a^2*sin(
d*x + c)^4 + a^2*sin(d*x + c)^3) - 12*log(sin(d*x + c) + 1)/a^2 + 12*log(si
n(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.35, size = 195, normalized size = 1.93

$$\frac{6\cos(dx+c)^2-12\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+12\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^2-1\right)\sin(dx+c)+1\right)\log(\sin(dx+c)+1)+2\left(6\cos(dx+c)^2-5\right)\sin(dx+c)-7}{3\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d-\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x +
c)^2 - 1)*sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 - (cos(d*x + c)^2 - 1)*sin(d*x + c) + 1)*log(sin(d*x + c) +
1) + 2*(6*cos(d*x + c)^2 - 5)*sin(d*x + c) - 7)/(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos(c+dx)\csc^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.48, size = 103, normalized size = 1.02

$$-\frac{\frac{12\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^2}+\frac{3}{(a\sin(dx+c)+a)a}+\frac{\frac{30\,a}{a\sin(dx+c)+a}-\frac{18\,a^2}{(a\sin(dx+c)+a)^2}-13}{a^2\left(\frac{a}{a\sin(dx+c)+a}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^2 + 3/((a*sin(d*x + c) + a
)*a) + (30*a/(a*sin(d*x + c) + a) - 18*a^2/(a*sin(d*x + c) + a)^2 - 13)/(a^
2*(a/(a*sin(d*x + c) + a) - 1)^3))/d
```

Mupad [B]
time = 8.63, size = 202, normalized size = 2.00

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\,\left(8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+16\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^2\,d}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a^2*d) - tan(c/2 + (d*x)/2)^3/(24*a^2*d) - ((28*tan
(c/2 + (d*x)/2)^2)/3 - (4*tan(c/2 + (d*x)/2))/3 + 24*tan(c/2 + (d*x)/2)^3 -
 3*tan(c/2 + (d*x)/2)^4 + 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 16*a^2*tan(
c/2 + (d*x)/2)^4 + 8*a^2*tan(c/2 + (d*x)/2)^5)) - (4*log(tan(c/2 + (d*x)/2)
))/(a^2*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) - (13*tan(c/2 + (d*x)/
2))/(8*a^2*d)
```

3.240 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=111

$$-\frac{10\log(1+\sin(c+dx))}{a^3d}+\frac{6\sin(c+dx)}{a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{\sin^3(c+dx)}{3a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{}{d\,(a^3+a}$$

```
[Out] -10*ln(1+sin(d*x+c))/a^3/d+6*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*si
n(d*x+c)^3/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2-5/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{6\sin(c+dx)}{a^3d}-\frac{5}{d\,(a^3\sin(c+dx)+a^3)}-\frac{10\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-10*Log[1 + Sin[c + d*x]])/(a^3*d) + (6*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
 d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x
])^2) - 5/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$

$$= \frac{\text{Subst}\left(\int \left(6a^2-3ax+x^2-\frac{a^5}{(a+x)^3}+\frac{5a^4}{(a+x)^2}-\frac{10a^3}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$

$$= -\frac{10\log(1+\sin(c+dx))}{a^3 d}+\frac{6\sin(c+dx)}{a^3 d}-\frac{3\sin^2(c+dx)}{2a^3 d}+\frac{\sin^3(c+dx)}{3a^3 d}$$

Mathematica [A]

time = 0.56, size = 106, normalized size = 0.95

$$\frac{-417-960\log(1+\sin(c+dx))-6(-21+320\log(1+\sin(c+dx)))\sin(c+dx)+(1023-960\log(1+\sin(c+dx)))\sin^2(c+dx)+320\sin^3(c+dx)-80\sin^4(c+dx)+32\sin^5(c+dx)}{96a^3d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-417 - 960*Log[1 + Sin[c + d*x]] - 6*(-21 + 320*Log[1 + Sin[c + d*x]])*Sin
[c + d*x] + (1023 - 960*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + 320*Sin[c +
 d*x]^3 - 80*Sin[c + d*x]^4 + 32*Sin[c + d*x]^5)/(96*a^3*d*(1 + Sin[c + d*x
])^2)
```

Maple [A]

time = 0.27, size = 72, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{3\left(\sin^2(dx+c)\right)}{2}+6\sin(dx+c)-\frac{5}{1+\sin(dx+c)}-10\ln(1+\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{3\left(\sin^2(dx+c)\right)}{2}+6\sin(dx+c)-\frac{5}{1+\sin(dx+c)}-10\ln(1+\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
risch	$\frac{10ix}{a^3}+\frac{i e^{3i(dx+c)}}{24d\,a^3}+\frac{3\,e^{2i(dx+c)}}{8d\,a^3}-\frac{25i e^{i(dx+c)}}{8d\,a^3}+\frac{25i e^{-i(dx+c)}}{8d\,a^3}+\frac{3\,e^{-2i(dx+c)}}{8d\,a^3}-\frac{i e^{-3i(dx+c)}}{24d\,a^3}+\frac{20ic}{d\,a^3}-\frac{2i(-5}{}$
norman	$\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{20\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{80\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{80\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{680\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{680\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{154}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/d/a^3*(1/3*sin(d*x+c)^3-3/2*sin(d*x+c)^2+6*sin(d*x+c)-5/(1+sin(d*x+c))-10
*ln(1+sin(d*x+c))+1/2/(1+sin(d*x+c))^2)

Maxima [A]
time = 0.27, size = 95, normalized size = 0.86

$$-\frac{\frac{3\,(10\,\sin(dx+c)+9)}{a^3\sin(dx+c)^2+2\,a^3\sin(dx+c)+a^3}-\frac{2\,\sin(dx+c)^3-9\,\sin(dx+c)^2+36\,\sin(dx+c)}{a^3}+\frac{60\,\log(\sin(dx+c)+1)}{a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/6*(3*(10*sin(d*x + c) + 9)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^
3) - (2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 36*sin(d*x + c))/a^3 + 60*log(s
in(d*x + c) + 1)/a^3)/d

Fricas [A]
time = 0.36, size = 117, normalized size = 1.05

$$\frac{10\,\cos\left(dx+c\right)^4+115\,\cos\left(dx+c\right)^2-120\,\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,\left(2\,\cos\left(dx+c\right)^4-24\,\cos\left(dx+c\right)^2+37\right)\sin\left(dx+c\right)-80}{12\,\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\sin\left(dx+c\right)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/12*(10*cos(d*x + c)^4 + 115*cos(d*x + c)^2 - 120*(cos(d*x + c)^2 - 2*sin(
d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*(2*cos(d*x + c)^4 - 24*cos(d*x + c)
^2 + 37)*sin(d*x + c) - 80)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) -
2*a^3*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(97) = 194$.
time = 2.41, size = 394, normalized size = 3.55

$$\begin{cases} - \frac{60 \log{\left(\sin{\left(c + dx \right)} + 1 \right)} \sin^{2}{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} - \frac{120 \log{\left(\sin{\left(c + dx \right)} + 1 \right)} \sin{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} - \frac{60 \log{\left(\sin{\left(c + dx \right)} + 1 \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} + \frac{2 \sin^{5}{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} - \frac{5 \sin^{4}{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} + \frac{20 \sin^{3}{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} - \frac{120 \sin{\left(c + dx \right)}}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} - \frac{90}{6 a^{3} d \sin^{2}{\left(c + dx \right)} + 12 a^{3} d \sin{\left(c + dx \right)} + 6 a^{3} d} & \text{for}\: d \neq 0 \\\frac{x \sin^{5}{\left(c \right)} \cos{\left(c \right)}}{\left(a \sin{\left(c \right)} + a\right)^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((-60*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**3*d*sin(c + d*x)
2 + 12*a3*d*sin(c + d*x) + 6*a**3*d) - 120*log(sin(c + d*x) + 1)*sin(c
+ d*x)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 60*
log(sin(c + d*x) + 1)/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c + d*x) +
6*a**3*d) + 2*sin(c + d*x)**5/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d*sin(c +
d*x) + 6*a**3*d) - 5*sin(c + d*x)**4/(6*a**3*d*sin(c + d*x)**2 + 12*a**3*d
*sin(c + d*x) + 6*a**3*d) + 20*sin(c + d*x)**3/(6*a**3*d*sin(c + d*x)**2 +

```
12*a**3*d*sin(c + d*x) + 6*a**3*d) - 120*sin(c + d*x)/(6*a**3*d*sin(c + d*x
)**2 + 12*a**3*d*sin(c + d*x) + 6*a**3*d) - 90/(6*a**3*d*sin(c + d*x)**2 +
12*a**3*d*sin(c + d*x) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c
) + a)**3, True))
```

Giac [A]
time = 0.45, size = 89, normalized size = 0.80

$$-\frac{\frac{60\,\log\left(|\sin(dx+c)+1|\right)}{a^3}+\frac{3\,(10\,\sin(dx+c)+9)}{a^3(\sin(dx+c)+1)^2}-\frac{2\,a^6\,\sin(dx+c)^3-9\,a^6\,\sin(dx+c)^2+36\,a^6\,\sin(dx+c)}{a^9}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 + 3*(10*sin(d*x + c) + 9)/(a^3*(sin
(d*x + c) + 1)^2) - (2*a^6*sin(d*x + c)^3 - 9*a^6*sin(d*x + c)^2 + 36*a^6*s
in(d*x + c))/a^9)/d
```

Mupad [B]
time = 0.13, size = 108, normalized size = 0.97

$$\frac{6\,\sin\left(c+d\,x\right)}{a^3\,d}-\frac{5\,\sin\left(c+d\,x\right)+\frac{9}{2}}{d\,\left(a^3\,\sin\left(c+d\,x\right)^2+2\,a^3\,\sin\left(c+d\,x\right)+a^3\right)}-\frac{10\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3\,d}-\frac{3\,\sin\left(c+d\,x\right)^2}{2\,a^3\,d}+\frac{\sin\left(c+d\,x\right)^3}{3\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^5)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (6*sin(c + d*x))/(a^3*d) - (5*sin(c + d*x) + 9/2)/(d*(2*a^3*sin(c + d*x) +
a^3 + a^3*sin(c + d*x)^2)) - (10*log(sin(c + d*x) + 1))/(a^3*d) - (3*sin(c
+ d*x)^2)/(2*a^3*d) + sin(c + d*x)^3/(3*a^3*d)
```

3.241 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$\frac{6\log(1+\sin(c+dx))}{a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{\sin^2(c+dx)}{2a^3d}-\frac{1}{2ad(a+a\sin(c+dx))^2}+\frac{4}{d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] 6*ln(1+sin(d*x+c))/a^3/d-3*sin(d*x+c)/a^3/d+1/2*sin(d*x+c)^2/a^3/d-1/2/a/d/
(a+a*sin(d*x+c))^2+4/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^2(c+dx)}{2a^3d}-\frac{3\sin(c+dx)}{a^3d}+\frac{4}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{6\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Log[1 + Sin[c + d*x]])/(a^3*d) - (3*Sin[c + d*x])/(a^3*d) + Sin[c + d*x]
^2/(2*a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 4/(d*(a^3 + a^3*Sin[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a+x+\frac{a^4}{(a+x)^3}-\frac{4a^3}{(a+x)^2}+\frac{6a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{6\log(1+\sin(c+dx))}{a^3 d} - \frac{3\sin(c+dx)}{a^3 d} + \frac{\sin^2(c+dx)}{2a^3 d} - \frac{1}{2ad(a+a\sin(c+$$

Mathematica [A]
time = 1.55, size = 78, normalized size = 0.84

$$\frac{96\log(1+\sin(c+dx)) + \frac{56}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + 8\sin^2(c+dx) + \sin(c+dx)\left(-48+\frac{64}{(1+\sin(c+dx))^2}\right)}{16a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (96*Log[1 + Sin[c + d*x]] + 56/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 8*
Sin[c + d*x]^2 + Sin[c + d*x]*(-48 + 64/(1 + Sin[c + d*x])^2))/(16*a^3*d)
```

Maple [A]
time = 0.25, size = 62, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-3\sin(dx+c)+\frac{4}{1+\sin(dx+c)}+6\ln(1+\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-3\sin(dx+c)+\frac{4}{1+\sin(dx+c)}+6\ln(1+\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
risch	$-\frac{6ix}{a^3}-\frac{e^{2i(dx+c)}}{8d\,a^3}+\frac{3ie^{i(dx+c)}}{2d\,a^3}-\frac{3ie^{-i(dx+c)}}{2d\,a^3}-\frac{e^{-2i(dx+c)}}{8d\,a^3}-\frac{12ic}{d\,a^3}+\frac{2i\left(7ie^{2i(dx+c)}+4\,e^{3i(dx+c)}-4\,e^{i(dx+c)}\right)}{d\,a^3\left(e^{i(dx+c)}+i\right)^4}$
norman	$-\frac{12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{12\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{124\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{124\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{416\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{416\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/2*sin(d*x+c)^2-3*sin(d*x+c)+4/(1+sin(d*x+c))+6*ln(1+sin(d*x+c))-
1/2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 81, normalized size = 0.87

$$\frac{\frac{8 \sin(dx+c)+7}{a^3 \sin(dx+c)^2+2\,a^3 \sin(dx+c)+a^3}+\frac{\sin(dx+c)^2-6 \sin(dx+c)}{a^3}+\frac{12 \log(\sin(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((8*sin(d*x + c) + 7)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 (sin(d*x + c)^2 - 6*sin(d*x + c))/a^3 + 12*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.34, size = 107, normalized size = 1.15

$$-\frac{2 \cos\left(dx+c\right)^4+19 \cos\left(dx+c\right)^2-24 \left(\cos\left(dx+c\right)^2-2 \sin\left(dx+c\right)-2\right) \log\left(\sin\left(dx+c\right)+1\right)+2 \left(4 \cos\left(dx+c\right)^2-3\right) \sin\left(dx+c\right)-8}{4 \left(a^3 d \cos\left(dx+c\right)^2-2\,a^3 d \sin\left(dx+c\right)-2\,a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(d*x + c)^4 + 19*cos(d*x + c)^2 - 24*(cos(d*x + c)^2 - 2*sin(d*x
 + c) - 2)*log(sin(d*x + c) + 1) + 2*(4*cos(d*x + c)^2 - 3)*sin(d*x + c) -
8)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(80) = 160$.
time = 1.29, size = 347, normalized size = 3.73

$$\begin{cases} \frac{12 \log\left(\sin\left(c+dx\right)+1\right) \sin^2\left(c+dx\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}+\frac{24 \log\left(\sin\left(c+dx\right)+1\right) \sin\left(c+dx\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}+\frac{12 \log\left(\sin\left(c+dx\right)+1\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}+\frac{\sin^4\left(c+dx\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}-\frac{4 \sin^3\left(c+dx\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}+\frac{24 \sin\left(c+dx\right)}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d}+\frac{18}{2a^3 d \sin^2\left(c+dx\right)+4a^3 d \sin\left(c+dx\right)+2a^3 d} & \text{for}\: d \neq 0 \\ \frac{x \sin^4\left(c\right) \cos\left(c\right)}{\left(a \sin\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((12*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 12*log(
sin(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d) + sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
2*a**3*d) - 4*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c +
d*x) + 2*a**3*d) + 24*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin
(c + d*x) + 2*a**3*d) + 18/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x
) + 2*a**3*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.51, size = 73, normalized size = 0.78

$$\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^3}+\frac{8 \sin(dx+c)+7}{a^3(\sin(dx+c)+1)^2}+\frac{a^3 \sin(dx+c)^2-6\,a^3 \sin(dx+c)}{a^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 + (8*sin(d*x + c) + 7)/(a^3*(sin(d*x
 + c) + 1)^2) + (a^3*sin(d*x + c)^2 - 6*a^3*sin(d*x + c))/a^6)/d
```

Mupad [B]
time = 8.49, size = 91, normalized size = 0.98

$$\frac{6\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3\,d}+\frac{4\,\sin\left(c+d\,x\right)+\frac{7}{2}}{d\,\left(a^3\,{\sin\left(c+d\,x\right)}^2+2\,a^3\,\sin\left(c+d\,x\right)+a^3\right)}-\frac{3\,\sin\left(c+d\,x\right)}{a^3\,d}+\frac{{\sin\left(c+d\,x\right)}^2}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (6*log(sin(c + d*x) + 1))/(a^3*d) + (4*sin(c + d*x) + 7/2)/(d*(2*a^3*sin(c
+ d*x) + a^3 + a^3*sin(c + d*x)^2)) - (3*sin(c + d*x))/(a^3*d) + sin(c + d*
x)^2/(2*a^3*d)
```

3.242 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$-\frac{3\log(1+\sin(c+dx))}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{3}{d\,(a^3+a^3\sin(c+dx))}$$

```
[Out] -3*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2-3/d/(
a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2912, 12, 45}

$$\frac{\sin(c+dx)}{a^3d}-\frac{3}{d\,(a^3\sin(c+dx)+a^3)}-\frac{3\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) - 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a+x)^3}+\frac{3a^2}{(a+x)^2}-\frac{3a}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= -\frac{3\log(1+\sin(c+dx))}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{}{d\,(a^3+}$$

Mathematica [A]
time = 0.31, size = 70, normalized size = 0.95

$$\frac{-12\log(1+\sin(c+dx))+4\sin(c+dx)+\frac{-9-10\sin(c+dx)}{(1+\sin(c+dx))^2}+\frac{\sin^2(c+dx)}{(1+\sin(c+dx))^2}}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Log[1 + Sin[c + d*x]] + 4*Sin[c + d*x] + (-9 - 10*Sin[c + d*x])/(1 + S
in[c + d*x])^2 + Sin[c + d*x]^2/(1 + Sin[c + d*x])^2)/(4*a^3*d)
```

Maple [A]
time = 0.20, size = 50, normalized size = 0.68

method	result
derivativedivides	$\frac{\sin(dx+c)-\frac{3}{1+\sin(dx+c)}-3\ln(1+\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
default	$\frac{\sin(dx+c)-\frac{3}{1+\sin(dx+c)}-3\ln(1+\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
risch	$\frac{3ix}{a^3}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^3}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^3}+\frac{6ic}{d\,a^3}-\frac{2i\left(-3\,\mathrm{e}^{i(dx+c)}+5i\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{3i(dx+c)}\right)}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^4}-\frac{6\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}$
norman	$\frac{\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{6\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{110\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{110\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{156\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{156\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{24(\mathrm{ta}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4a^2(\mathrm{t}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(sin(d*x+c)-3/(1+sin(d*x+c))-3*ln(1+sin(d*x+c))+1/2/(1+sin(d*x+c))^
2)
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.96

$$-\frac{\frac{6\,\sin(dx+c)+5}{a^3\,\sin(dx+c)^2+2\,a^3\,\sin(dx+c)+a^3}+\frac{6\,\log(\sin(dx+c)+1)}{a^3}-\frac{2\,\sin(dx+c)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*((6*sin(d*x + c) + 5)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)
+ 6*log(sin(d*x + c) + 1)/a^3 - 2*sin(d*x + c)/a^3)/d
```

Fricas [A]
time = 0.35, size = 95, normalized size = 1.28

$$\frac{4\,\cos\left(dx+c\right)^2-6\,\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)+2\,\left(\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)+1}{2\,\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\sin\left(dx+c\right)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/2*(4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x
 + c) + 1) + 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2
 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(63) = 126$.
time = 0.91, size = 303, normalized size = 4.09

$$\begin{cases} -\frac{6\log\left(\sin\left(c+dx\right)+1\right)\sin^2\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{12\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{6\log\left(\sin\left(c+dx\right)+1\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{2\sin^3\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{12\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{9}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d} & \text{for}\: d\neq 0 \\ \frac{x\sin^3\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
[Out] Piecewise((-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d
*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(s
in(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3
*d) + 2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
 2*a**3*d) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d
*x) + 2*a**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a
**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.46, size = 56, normalized size = 0.76

$$-\frac{\frac{6\,\log(|\sin(dx+c)+1|)}{a^3}-\frac{2\,\sin(dx+c)}{a^3}+\frac{6\,\sin(dx+c)+5}{a^3(\sin(dx+c)+1)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 2*sin(d*x + c)/a^3 + (6*sin(d*x +
c) + 5)/(a^3*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 0.07, size = 59, normalized size = 0.80

$$\frac{\sin(c+d\,x)}{a^3\,d}-\frac{3\,\ln\left(\sin(c+d\,x)+1\right)}{a^3\,d}-\frac{3\,\sin(c+d\,x)+\frac{5}{2}}{a^3\,d\,\left(\sin(c+d\,x)+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] sin(c + d*x)/(a^3*d) - (3*log(sin(c + d*x) + 1))/(a^3*d) - (3*sin(c + d*x)
+ 5/2)/(a^3*d*(sin(c + d*x) + 1)^2)
```

3.243 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$\frac{\log(1+\sin(c+dx))}{a^3d}-\frac{1}{2ad(a+a\sin(c+dx))^2}+\frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

[Out] ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(a+a*sin(d*x+c))^2+2/d/(a^3+a^3*sin(d*x+c))

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2912, 12, 45}

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]

[Out] Log[1 + Sin[c + d*x]]/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) + 2/(d*(a^3 + a^3*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+x)^3} - \frac{2a}{(a+x)^2} + \frac{1}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\log(1+\sin(c+dx))}{a^3 d} - \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

Mathematica [A]
time = 0.52, size = 65, normalized size = 1.08

$$\frac{8\log(1+\sin(c+dx)) + \frac{12}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{16\sin(c+dx)}{(1+\sin(c+dx))^2}}{8a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (8*Log[1 + Sin[c + d*x]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + (16
*Sin[c + d*x])/(1 + Sin[c + d*x])^2)/(8*a^3*d)
```

Maple [A]
time = 0.18, size = 42, normalized size = 0.70

method	result
derivativedivides	$\frac{\frac{2}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
default	$\frac{\frac{2}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{d\,a^3}+\frac{2i\left(3i\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}-2\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^4}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{14\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{26\left(\tan^{4}\left(\right.\right.}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(2/(1+sin(d*x+c))+ln(1+sin(d*x+c))-1/2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 60, normalized size = 1.00

$$\frac{\frac{4\,\sin(dx+c)+3}{a^3\,\sin(dx+c)^2+2\,a^3\,\sin(dx+c)+a^3}+\frac{2\,\log(\sin(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) +
 2*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.34, size = 75, normalized size = 1.25

$$\frac{2\,\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-4\,\sin\left(dx+c\right)-3}{2\,\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\sin\left(dx+c\right)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 4*sin(
d*x + c) - 3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(49) = 98$.
time = 0.64, size = 257, normalized size = 4.28

$$\begin{cases} \frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin^2\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{4\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{2\log\left(\sin\left(c+dx\right)+1\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{4\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{3}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d} & \text{for}\: d \neq 0 \\ \frac{x\sin^2\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**
2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x
)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin
(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d
) + 4*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a*
*3*d) + 3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Ne
(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.45, size = 45, normalized size = 0.75

$$\frac{\frac{2\,\log(|\sin(dx+c)+1|)}{a^3}+\frac{4\,\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 + (4*sin(d*x + c) + 3)/(a^3*(sin(d*x
+ c) + 1)^2))/d
```

Mupad [B]
time = 0.06, size = 44, normalized size = 0.73

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3\,d}+\frac{2\,\sin\left(c+d\,x\right)+\frac{3}{2}}{a^3\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] log(sin(c + d*x) + 1)/(a^3*d) + (2*sin(c + d*x) + 3/2)/(a^3*d*(sin(c + d*x)
 + 1)^2)
```

3.244 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^2(c+dx)}{2ad(a+a\sin(c+dx))^2}$$

```
[Out] 1/2*sin(d*x+c)^2/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 37}

$$\frac{\sin^2(c+dx)}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\sin^2(c+dx)}{2ad(a+a\sin(c+dx))^2}$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 1.00

$$\frac{\sin^2(c+dx)}{2ad(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d*(a + a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.14, size = 33, normalized size = 1.10

method	result	size
derivativedivides	$\frac{-\frac{1}{1+\sin(dx+c)}+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$	33
default	$\frac{-\frac{1}{1+\sin(dx+c)}+\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^3}$	33
risch	$-\frac{2i\left(i\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^4}$	57
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/(1+sin(d*x+c))+1/2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 44, normalized size = 1.47

$$-\frac{2\sin(dx+c)+1}{2\left(a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/((a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)*
d)
```

Fricas [A]
time = 0.33, size = 46, normalized size = 1.53

$$\frac{2\sin\left(dx+c\right)+1}{2\left(a^3d\cos\left(dx+c\right)^2-2\,a^3d\sin\left(dx+c\right)-2\,a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a
^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(24) = 48$.
time = 0.61, size = 99, normalized size = 3.30

$$\begin{cases} -\frac{2\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}-\frac{1}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d} & \text{for}\ d\neq 0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-2*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x
) + 2*a**3*d) - 1/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d), Ne(d, 0)), (x*sin(c)*cos(c)/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.49, size = 28, normalized size = 0.93

$$-\frac{2\sin\left(dx+c\right)+1}{2\,a^3d(\sin\left(dx+c\right)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*sin(d*x + c) + 1)/(a^3*d*(sin(d*x + c) + 1)^2)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 1.23

$$\frac{1}{2\,a^3\,d\left(\sin\left(c+d\,x\right)+1\right)^2}-\frac{1}{a^3\,d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
[Out] 1/(2*a^3*d*(sin(c + d*x) + 1)^2) - 1/(a^3*d*(sin(c + d*x) + 1))
```

3.245 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=74

$$\frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{1}{d(a^3+a^3\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^3/d-ln(1+sin(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+1/d/
(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 46}

$$\frac{1}{d(a^3\sin(c+dx)+a^3)} + \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(\sin(c+dx)+1)}{a^3d} + \frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{}{d\,(a^3+a^3}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.70

$$\frac{2\log(\sin(c+dx)) - 2\log(1+\sin(c+dx)) + \frac{3+2\sin(c+dx)}{(1+\sin(c+dx))^2}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A]
time = 0.16, size = 49, normalized size = 0.66

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^3d}$	49
default	$\frac{\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^3d}$	49
risch	$\frac{2i\left(-e^{i(dx+c)}+3ie^{2i(dx+c)}+e^{3i(dx+c)}\right)}{d\,a^3\left(e^{i(dx+c)}+i\right)^4} - \frac{2\ln\left(e^{i(dx+c)}+i\right)}{d\,a^3} + \frac{\ln\left(e^{2i(dx+c)}-1\right)}{d\,a^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(ln(sin(d*x+c))+1/2/(1+sin(d*x+c))^2+1/(1+sin(d*x+c))-ln(1+sin(d*x+
c)))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.97

$$\frac{\frac{2\,\sin(dx+c)+3}{a^3\,\sin(dx+c)^2+2\,a^3\,\sin(dx+c)+a^3} - \frac{2\,\log(\sin(dx+c)+1)}{a^3} + \frac{2\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.41

$$\frac{2\left(\cos\left(dx+c\right)^{2}-2\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^{2}-2\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-2\sin\left(dx+c\right)-3}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2a^{3}d\sin\left(dx+c\right)-2a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3
*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.46, size = 59, normalized size = 0.80

$$-\frac{\frac{2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}}-\frac{2\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{3}}-\frac{2\sin\left(dx+c\right)+3}{a^{3}\left(\sin\left(dx+c\right)+1\right)^{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 8.76, size = 148, normalized size = 2.00

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^{3}d}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+4a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+6a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+4a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^{3}\right)}-\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (4*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*tan(
c/2 + (d*x)/2)^3 + a^3*tan(c/2 + (d*x)/2)^4 + a^3 + 4*a^3*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
```

3.246 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(1+\sin(c+dx))}{a^3d}-\frac{1}{2ad(a+a\sin(c+dx))^2}-\frac{2}{d\left(a^3+a^3\sin(c+dx)\right)}$$

```
[Out] -csc(d*x+c)/a^3/d-3*ln(sin(d*x+c))/a^3/d+3*ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(
a+a*sin(d*x+c))^2-2/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 46}

$$-\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{3\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (3*Log[1 + Sin[c
+ d*x]])/(a^3*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 2/(d*(a^3 + a^3*Sin[c
 + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{a^3x^2} - \frac{3}{a^4x} + \frac{1}{a^2(a+x)^3} + \frac{2}{a^3(a+x)^2} + \frac{3}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{\csc(c+dx)}{a^3d} - \frac{3\log(\sin(c+dx))}{a^3d} + \frac{3\log(1+\sin(c+dx))}{a^3d} - \frac{1}{2ad(a+a\sin}$$

Mathematica [A]
time = 0.32, size = 61, normalized size = 0.68

$$-\frac{2\csc(c+dx)+6\log(\sin(c+dx))-6\log(1+\sin(c+dx))+\frac{1}{(1+\sin(c+dx))^2}+\frac{4}{1+\sin(c+dx)}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(2*Csc[c + d*x] + 6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (1 +
 Sin[c + d*x])^(-2) + 4/(1 + Sin[c + d*x]))/(a^3*d)
```

Maple [A]
time = 0.17, size = 63, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-3\ln(\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}-\frac{2}{1+\sin(dx+c)}+3\ln(1+\sin(dx+c))}{a^3d}$
default	$\frac{-\frac{1}{\sin(dx+c)}-3\ln(\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}-\frac{2}{1+\sin(dx+c)}+3\ln(1+\sin(dx+c))}{a^3d}$
risch	$-\frac{2i\left(9i e^{4i(dx+c)}+3e^{5i(dx+c)}-9i e^{2i(dx+c)}-10e^{3i(dx+c)}+3e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^4 d a^3}+\frac{6\ln\left(e^{i(dx+c)}+i\right)}{d a^3}-\frac{3\ln\left(e^{2i(dx+c)}-1\right)}{d a^3}$
norman	$\frac{\frac{13\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{13\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{67\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{67\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(-1/sin(d*x+c)-3*ln(sin(d*x+c))-1/2/(1+sin(d*x+c))^2-2/(1+sin(d*x+c
))+3*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.01

$$-\frac{\frac{6 \sin(dx+c)^2+9 \sin(dx+c)+2}{a^3 \sin(dx+c)^3+2 a^3 \sin(dx+c)^2+a^3 \sin(dx+c)} - \frac{6 \log(\sin(dx+c)+1)}{a^3} + \frac{6 \log(\sin(dx+c))}{a^3}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((6*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 2*a^3*s
in(d*x + c)^2 + a^3*sin(d*x + c)) - 6*log(sin(d*x + c) + 1)/a^3 + 6*log(sin
(d*x + c))/a^3)/d
```

Fricas [A]
time = 0.36, size = 152, normalized size = 1.69

$$-\frac{6 \cos(dx+c)^2 + 6 \left(2 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 2\right) \sin(dx+c) - 2\right) \log\left(\frac{1}{2} \sin(dx+c)\right) - 6 \left(2 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 2\right) \sin(dx+c) - 2\right) \log(\sin(dx+c)+1) - 9 \sin(dx+c) - 8}{2 \left(2 a^3 d \cos(dx+c)^2 - 2 a^3 d + \left(a^3 d \cos(dx+c)^2 - 2 a^3 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*cos(d*x + c)^2 + 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 2)*sin(d*x
 + c) - 2)*log(1/2*sin(d*x + c)) - 6*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 -
2)*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 9*sin(d*x + c) - 8)/(2*a^3*d*c
os(d*x + c)^2 - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx) \csc^2(c+dx)}{\sin^3(c+dx)+3 \sin^2(c+dx)+3 \sin(c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.47, size = 77, normalized size = 0.86

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a^3} - \frac{6 \log(|\sin(dx+c)|)}{a^3} - \frac{6 \sin(dx+c)^2+9 \sin(dx+c)+2}{a^3 (\sin(dx+c)+1)^2 \sin(dx+c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c)))/a^3 - (6*s
in(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c))
)/d
```

Mupad [B]
time = 8.65, size = 193, normalized size = 2.14

$$\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1}{d\left(2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}+\frac{6\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) + 16*tan(c/2 + (d*x)/2)^3 +
11*tan(c/2 + (d*x)/2)^4 - 1)/(d*(8*a^3*tan(c/2 + (d*x)/2)^2 + 12*a^3*tan(c/
2 + (d*x)/2)^3 + 8*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*x)/2)^5 +
2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (6*log(t
an(c/2 + (d*x)/2) + 1))/(a^3*d) - tan(c/2 + (d*x)/2)/(2*a^3*d)
```

3.247 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{3\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(1+\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+a\sin(c+dx))^2}+\frac{}{d\,(a^3+}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+6*ln(sin(d*x+c))/a^3/d-6*ln(1+sin
(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+3/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$\frac{3}{d\,(a^3\sin(c+dx)+a^3)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{6\log(\sin(c+dx))}{a^3d}-\frac{6\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (6*Log[Sin[c + d*x]])
/(a^3*d) - (6*Log[1 + Sin[c + d*x]])/(a^3*d) + 1/(2*a*d*(a + a*Sin[c + d*x]
)^2) + 3/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3}{a^4x^2} + \frac{6}{a^5x} - \frac{1}{a^3(a+x)^3} - \frac{3}{a^4(a+x)^2} - \frac{6}{a^5(a+x)}\right)\,dx, x, a\sin(\cdots}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{6\log(\sin(c+dx))}{a^3d} - \frac{6\log(1+\sin(c+dx))}{a^3d} \cdots$$

Mathematica [A]
time = 0.51, size = 71, normalized size = 0.66

$$\frac{6\csc(c+dx) - \csc^2(c+dx) + 12\log(\sin(c+dx)) - 12\log(1+\sin(c+dx)) + \frac{1}{(1+\sin(c+dx))^2} + \frac{6}{1+\sin(c+dx)}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c
+ d*x]] + (1 + Sin[c + d*x])^(-2) + 6/(1 + Sin[c + d*x]))/(2*a^3*d)
```

Maple [A]
time = 0.22, size = 73, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+6\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{3}{1+\sin(dx+c)}-6\ln(1+\sin(dx+c))}{a^3d}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+6\ln(\sin(dx+c))+\frac{1}{2(1+\sin(dx+c))^2}+\frac{3}{1+\sin(dx+c)}-6\ln(1+\sin(dx+c))}{a^3d}$
risch	$\frac{4i\left(9i e^{6i(dx+c)}+3e^{7i(dx+c)}-16i e^{4i(dx+c)}-13e^{5i(dx+c)}+9i e^{2i(dx+c)}+13e^{3i(dx+c)}-3e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^4 d a^3}-\frac{12\ln\left(e^{i(dx+c)}+i\right)}{d a^3}+\cdots$
norman	$\frac{-\frac{24\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{24\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{7\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{251\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\cdots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(-1/2/sin(d*x+c)^2+3/sin(d*x+c)+6*ln(sin(d*x+c))+1/2/(1+sin(d*x+c))
^2+3/(1+sin(d*x+c))-6*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 103, normalized size = 0.95

$$\frac{\frac{12 \sin(dx+c)^3+18 \sin(dx+c)^2+4 \sin(dx+c)-1}{a^3 \sin(dx+c)^4+2 a^3 \sin(dx+c)^3+a^3 \sin(dx+c)^2} - \frac{12 \log(\sin(dx+c)+1)}{a^3} + \frac{12 \log(\sin(dx+c))}{a^3}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(
d*x + c)^4 + 2*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2) - 12*log(sin(d*x +
c) + 1)/a^3 + 12*log(sin(d*x + c))/a^3)/d
```

Fricas [A]
time = 0.37, size = 196, normalized size = 1.81

$$-\frac{18 \cos(dx+c)^2 - 12 \left(\cos(dx+c)^4 - 3 \cos(dx+c)^2 - 2 \left(\cos(dx+c)^2 - 1\right) \sin(dx+c) + 2\right) \log\left(\frac{1}{2} \sin(dx+c)\right) + 12 \left(\cos(dx+c)^4 - 3 \cos(dx+c)^2 - 2 \left(\cos(dx+c)^2 - 1\right) \sin(dx+c) + 2\right) \log(\sin(dx+c) + 1) + 4 \left(3 \cos(dx+c)^2 - 4\right) \sin(dx+c) - 17}{2 \left(a^3 d \cos(dx+c)^4 - 3 a^3 d \cos(dx+c)^2 + 2 a^3 d - 2 \left(a^3 d \cos(dx+c)^2 - a^3 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(18*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 4*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 17)/(a^3*d*cos(d*x + c)^
4 - 3*a^3*d*cos(d*x + c)^2 + 2*a^3*d - 2*(a^3*d*cos(d*x + c)^2 - a^3*d)*sin
(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.47, size = 86, normalized size = 0.80

$$-\frac{\frac{12 \log(|\sin(dx+c)+1|)}{a^3} - \frac{12 \log(|\sin(dx+c)|)}{a^3} - \frac{12 \sin(dx+c)^3+18 \sin(dx+c)^2+4 \sin(dx+c)-1}{\left(\sin(dx+c)^2+\sin(dx+c)\right)^2 a^3}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c)))/a^3 - (
12*sin(d*x + c)^3 + 18*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/((sin(d*x + c)^
2 + sin(d*x + c))^2*a^3))/d
```

Mupad [B]
time = 8.68, size = 227, normalized size = 2.10

$$\frac{6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^3\,d}+\frac{-26\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-\frac{65\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{2}+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+24\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+16\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (4*t
an(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^3 - (65*
tan(c/2 + (d*x)/2)^4)/2 - 26*tan(c/2 + (d*x)/2)^5 - 1/2)/(d*(4*a^3*tan(c/2
+ (d*x)/2)^2 + 16*a^3*tan(c/2 + (d*x)/2)^3 + 24*a^3*tan(c/2 + (d*x)/2)^4 +
16*a^3*tan(c/2 + (d*x)/2)^5 + 4*a^3*tan(c/2 + (d*x)/2)^6)) - (12*log(tan(c/
2 + (d*x)/2) + 1))/(a^3*d) + (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

3.248 $\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{6\csc(c+dx)}{a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{\csc^3(c+dx)}{3a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(1+\sin(c+dx))}{a^3d}-\frac{1}{2ad(a+a\sin}$$

```
[Out] -6*csc(d*x+c)/a^3/d+3/2*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^3/d-10*ln(sin
(d*x+c))/a^3/d+10*ln(1+sin(d*x+c))/a^3/d-1/2/a/d/(a+a*sin(d*x+c))^2-4/d/(a^
3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$-\frac{4}{d(a^3\sin(c+dx)+a^3)}-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{6\csc(c+dx)}{a^3d}-\frac{10\log(\sin(c+dx))}{a^3d}+\frac{10\log(\sin(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-6*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (10*Log[Sin[c + d*x]])/(a^3*d) + (10*Log[1 + Sin[c + d*x]])/(a^3
*d) - 1/(2*a*d*(a + a*Sin[c + d*x])^2) - 4/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^4}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{1}{x^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{a^3x^4} - \frac{3}{a^4x^3} + \frac{6}{a^5x^2} - \frac{10}{a^6x} + \frac{1}{a^4(a+x)^3} + \frac{4}{a^5(a+x)^2} + \frac{10}{a^6(a+x)}\right)dx, x, \right.}{d}$$
$$= -\frac{6\csc(c+dx)}{a^3d} + \frac{3\csc^2(c+dx)}{2a^3d} - \frac{\csc^3(c+dx)}{3a^3d} - \frac{10\log(\sin(c+dx))}{a^3d} + \frac{10}{}$$

Mathematica [A]
time = 3.79, size = 81, normalized size = 0.64

$$-\frac{36\csc(c+dx) - 9\csc^2(c+dx) + 2\csc^3(c+dx) + 60\log(\sin(c+dx)) - 60\log(1+\sin(c+dx)) + \frac{3(9+8\sin(c+dx))}{(1+\sin(c+dx))^2}}{6a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(36*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 60*Log[Sin[c
+ d*x]] - 60*Log[1 + Sin[c + d*x]] + (3*(9 + 8*Sin[c + d*x]))/(1 + Sin[c +
d*x])^2)/(a^3*d)
```

Maple [A]
time = 0.23, size = 83, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{3}{2\sin(dx+c)^2}-\frac{6}{\sin(dx+c)}-10\ln(\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}-\frac{4}{1+\sin(dx+c)}+10\ln(1+\sin(dx+c))}{a^3d}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{3}{2\sin(dx+c)^2}-\frac{6}{\sin(dx+c)}-10\ln(\sin(dx+c))-\frac{1}{2(1+\sin(dx+c))^2}-\frac{4}{1+\sin(dx+c)}+10\ln(1+\sin(dx+c))}{a^3d}$
risch	$-\frac{4i(45ie^{8i(dx+c)}+15e^{9i(dx+c)}-125ie^{6i(dx+c)}-80e^{7i(dx+c)}+125ie^{4i(dx+c)}+138e^{5i(dx+c)}-45ie^{2i(dx+c)}-80e^{3i(dx+c)}+}{3(e^{2i(dx+c)}-1)^3(e^{i(dx+c)}+i)^4d\,a^3}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan(\frac{dx}{2}+\frac{c}{2})}{6ad}-\frac{5(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{3ad}-\frac{5(\tan^9(\frac{dx}{2}+\frac{c}{2}))}{3ad}+\frac{\tan^{10}(\frac{dx}{2}+\frac{c}{2})}{6ad}-\frac{\tan^{11}(\frac{dx}{2}+\frac{c}{2})}{24ad}+\frac{325(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{8ad}+\frac{325(\tan^7(}{8a}}{\tan(\frac{dx}{2}+\frac{c}{2})^3a^2(\tan(\frac{dx}{2}+\frac{c}{2})+1)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3/d*(-1/3/sin(d*x+c)^3+3/2/sin(d*x+c)^2-6/sin(d*x+c)-10*ln(sin(d*x+c))-
1/2/(1+sin(d*x+c))^2-4/(1+sin(d*x+c))+10*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 113, normalized size = 0.90

$$-\frac{\frac{60 \sin(dx+c)^4+90 \sin(dx+c)^3+20 \sin(dx+c)^2-5 \sin(dx+c)+2}{a^3 \sin(dx+c)^5+2 a^3 \sin(dx+c)^4+a^3 \sin(dx+c)^3}-\frac{60 \log(\sin(dx+c)+1)}{a^3}+\frac{60 \log(\sin(dx+c))}{a^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*((60*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*
x + c) + 2)/(a^3*sin(d*x + c)^5 + 2*a^3*sin(d*x + c)^4 + a^3*sin(d*x + c)^3
) - 60*log(sin(d*x + c) + 1)/a^3 + 60*log(sin(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(120) = 240.
time = 0.36, size = 242, normalized size = 1.92

$$-\frac{60 \cos(dx+c)^4-140 \cos(dx+c)^2+60 \left(2 \cos(dx+c)^4-4 \cos(dx+c)^2+\left(\cos(dx+c)^4-3 \cos(dx+c)^2+2\right) \sin(dx+c)+2\right) \log\left(\frac{1}{2} \sin(dx+c)\right)-60 \left(2 \cos(dx+c)^4-4 \cos(dx+c)^2+\left(\cos(dx+c)^4-3 \cos(dx+c)^2+2\right) \sin(dx+c)+2\right) \log(\sin(dx+c)+1)-5 \left(18 \cos(dx+c)^2-17\right) \sin(dx+c)+82}{6 \left(2 a^3 d \cos(dx+c)^4-4 a^3 d \cos(dx+c)^2+2 a^3 d+\left(a^3 d \cos(dx+c)^4-3 a^3 d \cos(dx+c)^2+2 a^3 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*cos(d*x + c)^4 - 140*cos(d*x + c)^2 + 60*(2*cos(d*x + c)^4 - 4*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log
(1/2*sin(d*x + c)) - 60*(2*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c
)^4 - 3*cos(d*x + c)^2 + 2)*sin(d*x + c) + 2)*log(sin(d*x + c) + 1) - 5*(18
*cos(d*x + c)^2 - 17)*sin(d*x + c) + 82)/(2*a^3*d*cos(d*x + c)^4 - 4*a^3*d*
cos(d*x + c)^2 + 2*a^3*d + (a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^2 +
 2*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx) \csc^4(c+dx)}{\sin^3(c+dx)+3 \sin^2(c+dx)+3 \sin(c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.50, size = 97, normalized size = 0.77

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^3}-\frac{60 \log(|\sin(dx+c)|)}{a^3}-\frac{60 \sin(dx+c)^4+90 \sin(dx+c)^3+20 \sin(dx+c)^2-5 \sin(dx+c)+2}{a^3(\sin(dx+c)+1)^2 \sin(dx+c)^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c)))/a^3 - (6
0*sin(d*x + c)^4 + 90*sin(d*x + c)^3 + 20*sin(d*x + c)^2 - 5*sin(d*x + c) +
 2)/(a^3*(sin(d*x + c) + 1)^2*sin(d*x + c)^3))/d
```

Mupad [B]
time = 8.68, size = 260, normalized size = 2.06

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{10\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{20\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}-\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{-55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{250\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\,\left(8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+48\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+32\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (10*
log(tan(c/2 + (d*x)/2)))/(a^3*d) + (20*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
 - (25*tan(c/2 + (d*x)/2))/(8*a^3*d) - (15*tan(c/2 + (d*x)/2)^2 - (5*tan(c/
2 + (d*x)/2))/3 + (250*tan(c/2 + (d*x)/2)^3)/3 + (175*tan(c/2 + (d*x)/2)^4)
/3 - 47*tan(c/2 + (d*x)/2)^5 - 55*tan(c/2 + (d*x)/2)^6 + 1/3)/(d*(8*a^3*tan
(c/2 + (d*x)/2)^3 + 32*a^3*tan(c/2 + (d*x)/2)^4 + 48*a^3*tan(c/2 + (d*x)/2)
^5 + 32*a^3*tan(c/2 + (d*x)/2)^6 + 8*a^3*tan(c/2 + (d*x)/2)^7))
```

3.249 $\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=116

$$\frac{10\log(1+\sin(c+dx))}{a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{\sin^2(c+dx)}{2a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{5}{2d\left(a^2+a^2\sin(c+dx)\right)^2}+$$

[Out] 10*ln(1+sin(d*x+c))/a^4/d-4*sin(d*x+c)/a^4/d+1/2*sin(d*x+c)^2/a^4/d+1/3/a/d /(a+a*sin(d*x+c))^3-5/2/d/(a^2+a^2*sin(d*x+c))^2+10/d/(a^4+a^4*sin(d*x+c))

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin^2(c+dx)}{2a^4d}-\frac{4\sin(c+dx)}{a^4d}+\frac{10}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{10\log(\sin(c+dx)+1)}{a^4d}-\frac{5}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]

[Out] (10*Log[1 + Sin[c + d*x]])/(a^4*d) - (4*Sin[c + d*x])/(a^4*d) + Sin[c + d*x]^2/(2*a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 5/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 10/(d*(a^4 + a^4*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{a^5(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a+x-\frac{a^5}{(a+x)^4}+\frac{5a^4}{(a+x)^3}-\frac{10a^3}{(a+x)^2}+\frac{10a^2}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{10\log(1+\sin(c+dx))}{a^4d} - \frac{4\sin(c+dx)}{a^4d} + \frac{\sin^2(c+dx)}{2a^4d} + \frac{1}{3ad(a+a\sin(c+$$

Mathematica [A]
time = 0.69, size = 119, normalized size = 1.03

$$\frac{47+60\log(1+\sin(c+dx))+9(9+20\log(1+\sin(c+dx)))\sin(c+dx)+9(-1+20\log(1+\sin(c+dx)))\sin^2(c+dx)+(-63+60\log(1+\sin(c+dx)))\sin^3(c+dx)-15\sin^4(c+dx)+3\sin^5(c+dx)}{6a^4d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (47 + 60*Log[1 + Sin[c + d*x]] + 9*(9 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d
*x] + 9*(-1 + 20*Log[1 + Sin[c + d*x]])*Sin[c + d*x]^2 + (-63 + 60*Log[1 +
Sin[c + d*x]])*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 + 3*Sin[c + d*x]^5)/(6*a^
4*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.14, size = 74, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-4\sin(dx+c)+\frac{1}{3(1+\sin(dx+c))^3}+\frac{10}{1+\sin(dx+c)}+10\ln(1+\sin(dx+c))-\frac{5}{2(1+\sin(dx+c))^2}}{d\,a^4}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-4\sin(dx+c)+\frac{1}{3(1+\sin(dx+c))^3}+\frac{10}{1+\sin(dx+c)}+10\ln(1+\sin(dx+c))-\frac{5}{2(1+\sin(dx+c))^2}}{d\,a^4}$
risch	$-\frac{10ix}{a^4}-\frac{e^{2i(dx+c)}}{8a^4d}+\frac{2ie^{i(dx+c)}}{a^4d}-\frac{2ie^{-i(dx+c)}}{a^4d}-\frac{e^{-2i(dx+c)}}{8a^4d}-\frac{20ic}{a^4d}+\frac{2i\left(-154\,e^{3i(dx+c)}-105ie^{2i(dx+c)}+105i\right.}{3d\,a^4\left(e^{i(dx\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/2*sin(d*x+c)^2-4*sin(d*x+c)+1/3/(1+sin(d*x+c))^3+10/(1+sin(d*x+c
))+10*ln(1+sin(d*x+c))-5/2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.29, size = 105, normalized size = 0.91

$$\frac{\frac{60\sin(dx+c)^2+105\sin(dx+c)+47}{a^4\sin(dx+c)^3+3a^4\sin(dx+c)^2+3a^4\sin(dx+c)+a^4}+\frac{3\left(\sin(dx+c)^2-8\sin(dx+c)\right)}{a^4}+\frac{60\log(\sin(dx+c)+1)}{a^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^2 + 105*sin(d*x + c) + 47)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 3*(sin(d*x + c)^2 - 8*sin(d*
x + c))/a^4 + 60*log(sin(d*x + c) + 1)/a^4)/d
```

Fricas [A]
time = 0.35, size = 144, normalized size = 1.24

$$\frac{30\cos(dx+c)^4-87\cos(dx+c)^2+120\left(3\cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)+1)-3\left(2\cos(dx+c)^4+39\cos(dx+c)^2+10\right)\sin(dx+c)-34}{12\left(3a^4d\cos(dx+c)^2-4a^4d+\left(a^4d\cos(dx+c)^2-4a^4d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 87*cos(d*x + c)^2 + 120*(3*cos(d*x + c)^2 + (cos(
d*x + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 3*(2*cos(d*x + c)
^4 + 39*cos(d*x + c)^2 + 10)*sin(d*x + c) - 34)/(3*a^4*d*cos(d*x + c)^2 - 4
*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(100) = 200.
time = 3.45, size = 588, normalized size = 5.07

$$\begin{cases} \frac{60\log(\sin(c+dx)+1)\sin^3(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{180\log(\sin(c+dx)+1)\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{180\log(\sin(c+dx)+1)\sin(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{60\log(\sin(c+dx)+1)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{3\sin^5(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}-\frac{15\sin^4(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{180\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{270\sin(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{110}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} & \text{for}\ d\neq 0 \\ \frac{x\sin^5(c)\cos(c)}{(a\sin(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((60*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)*
*3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*l
og(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*
sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 180*log(sin(c + d*x)
 + 1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
18*a**4*d*sin(c + d*x) + 6*a**4*d) + 60*log(sin(c + d*x) + 1)/(6*a**4*d*sin
(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*
d) + 3*sin(c + d*x)**5/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**
2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 15*sin(c + d*x)**4/(6*a**4*d*sin(c
 + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d)
```

```
 + 180*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**
2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 270*sin(c + d*x)/(6*a**4*d*sin(c +
 d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) +
 110/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(
c + d*x) + 6*a**4*d), Ne(d, 0)), (x*sin(c)**5*cos(c)/(a*sin(c) + a)**4, Tru
e))
```

Giac [A]
time = 0.47, size = 84, normalized size = 0.72

$$\frac{\frac{60\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^4}+\frac{60\sin\left(dx+c\right)^2+105\sin\left(dx+c\right)+47}{a^4\left(\sin\left(dx+c\right)+1\right)^3}+\frac{3\left(a^4\sin\left(dx+c\right)^2-8a^4\sin\left(dx+c\right)\right)}{a^8}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 + (60*sin(d*x + c)^2 + 105*sin(d*x +
 c) + 47)/(a^4*(sin(d*x + c) + 1)^3) + 3*(a^4*sin(d*x + c)^2 - 8*a^4*sin(d*
x + c))/a^8)/d
```

Mupad [B]
time = 8.53, size = 114, normalized size = 0.98

$$\frac{10\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^4\,d}-\frac{4\sin\left(c+d\,x\right)}{a^4\,d}+\frac{10\sin\left(c+d\,x\right)^2+\frac{35\sin\left(c+d\,x\right)}{2}+\frac{47}{6}}{d\,\left(a^4\sin\left(c+d\,x\right)^3+3\,a^4\sin\left(c+d\,x\right)^2+3\,a^4\sin\left(c+d\,x\right)+a^4\right)}+\frac{\sin\left(c+d\,x\right)^2}{2\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^5)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] (10*log(sin(c + d*x) + 1))/(a^4*d) - (4*sin(c + d*x))/(a^4*d) + ((35*sin(c
+ d*x))/2 + 10*sin(c + d*x)^2 + 47/6)/(d*(3*a^4*sin(c + d*x) + a^4 + 3*a^4*
sin(c + d*x)^2 + a^4*sin(c + d*x)^3)) + sin(c + d*x)^2/(2*a^4*d)
```

3.250 $\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=95

$$-\frac{4\log(1+\sin(c+dx))}{a^4d}+\frac{\sin(c+dx)}{a^4d}-\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{2}{d\left(a^2+a^2\sin(c+dx)\right)^2}-\frac{6}{d\left(a^4+a^4\sin(c+dx)\right)}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^4/d+sin(d*x+c)/a^4/d-1/3/a/d/(a+a*sin(d*x+c))^3+2/d/(
a^2+a^2*sin(d*x+c))^2-6/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{\sin(c+dx)}{a^4d}-\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{4\log(\sin(c+dx)+1)}{a^4d}+\frac{2}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^4*d) + Sin[c + d*x]/(a^4*d) - 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 2/(d*(a^2 + a^2*Sin[c + d*x])^2) - 6/(d*(a^4 + a^4*Sin[
c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{a^4(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{(a+x)^4}-\frac{4a^3}{(a+x)^3}+\frac{6a^2}{(a+x)^2}-\frac{4a}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^4d}+\frac{\sin(c+dx)}{a^4d}-\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{}{d\,(a^2+}$$

Mathematica [A]
time = 6.46, size = 127, normalized size = 1.34

$$-\frac{1}{24a^4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}-\frac{3(1+2\sin(c+dx))^2}{16a^4d(1+\sin(c+dx))^3}-\frac{192\log(1+\sin(c+dx))-48\sin(c+dx)+\frac{197+444\sin(c+dx)+252\sin^2(c+dx)}{(1+\sin(c+dx))^3}}{48a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/24*1/(a^4*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*(1 + 2*Sin[c +
 d*x])^2)/(16*a^4*d*(1 + Sin[c + d*x])^3) - (192*Log[1 + Sin[c + d*x]] - 48
*Sin[c + d*x] + (197 + 444*Sin[c + d*x] + 252*Sin[c + d*x]^2)/(1 + Sin[c +
d*x])^3)/(48*a^4*d)
```

Maple [A]
time = 0.28, size = 62, normalized size = 0.65

method	result
derivativedivides	$\frac{\sin(dx+c)-\frac{1}{3(1+\sin(dx+c))^3}-\frac{6}{1+\sin(dx+c)}-4\ln(1+\sin(dx+c))+\frac{2}{(1+\sin(dx+c))^2}}{d\,a^4}$
default	$\frac{\sin(dx+c)-\frac{1}{3(1+\sin(dx+c))^3}-\frac{6}{1+\sin(dx+c)}-4\ln(1+\sin(dx+c))+\frac{2}{(1+\sin(dx+c))^2}}{d\,a^4}$
risch	$\frac{4ix}{a^4}-\frac{i\mathrm{e}^{i(dx+c)}}{2a^4d}+\frac{i\mathrm{e}^{-i(dx+c)}}{2a^4d}+\frac{8ic}{a^4d}-\frac{4i\left(30i\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}-30i\mathrm{e}^{2i(dx+c)}-44\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^4\left(\mathrm{e}^{i(dx+c)}+i\right)^6}-8$
norman	$\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{48\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{48\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{464\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{464\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{1112}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(sin(d*x+c)-1/3/(1+sin(d*x+c))^3-6/(1+sin(d*x+c))-4*ln(1+sin(d*x+c)
)+2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.99

$$-\frac{\frac{18 \sin(dx+c)^2+30 \sin(dx+c)+13}{a^4 \sin(dx+c)^3+3 a^4 \sin(dx+c)^2+3 a^4 \sin(dx+c)+a^4}+\frac{12 \log(\sin(dx+c)+1)}{a^4}-\frac{3 \sin(dx+c)}{a^4}}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((18*sin(d*x + c)^2 + 30*sin(d*x + c) + 13)/(a^4*sin(d*x + c)^3 + 3*a^
4*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 12*log(sin(d*x + c) + 1)/a^4
 - 3*sin(d*x + c)/a^4)/d
```

Fricas [A]
time = 0.35, size = 132, normalized size = 1.39

$$-\frac{3 \cos(dx+c)^4+3 \cos(dx+c)^2+12 \left(3 \cos(dx+c)^2+\left(\cos(dx+c)^2-4\right) \sin(dx+c)-4\right) \log(\sin(dx+c)+1)-9 \left(\cos(dx+c)^2+2\right) \sin(dx+c)-19}{3 \left(3 a^4 d \cos(dx+c)^2-4 a^4 d+\left(a^4 d \cos(dx+c)^2-4 a^4 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 + 12*(3*cos(d*x + c)^2 + (cos(d*x
 + c)^2 - 4)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 9*(cos(d*x + c)^2 +
2)*sin(d*x + c) - 19)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x +
c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(82) = 164$.
time = 1.72, size = 527, normalized size = 5.55

$$\begin{cases} -\frac{12 \log(\sin(c+dx)+1)\sin^3(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{36 \log(\sin(c+dx)+1)\sin^2(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{36 \log(\sin(c+dx)+1)\sin(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{12 \log(\sin(c+dx)+1)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}+\frac{3 \sin^4(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{36 \sin^2(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{54 \sin(c+dx)}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d}-\frac{22}{3a^4 d\sin^3(c+dx)+9a^4 d\sin^2(c+dx)+9a^4 d\sin(c+dx)+3a^4 d} & \text{for}\ d \neq 0 \\ \frac{x\sin^4(c)\cos(c)}{(a\sin(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)
**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin
(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)
*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
*d*sin(c + d*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*
x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 3*si
n(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4
```

```
*d*sin(c + d*x) + 3*a**4*d) - 36*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3
+ 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 54*sin(c +
 d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c
 + d*x) + 3*a**4*d) - 22/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)*
*2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**4*cos(c)/(a*s
in(c) + a)**4, True))
```

Giac [A]
time = 0.50, size = 66, normalized size = 0.69

$$-\frac{\frac{12\log\left(|\sin\left(dx+c\right)+1|\right)}{a^4}-\frac{3\sin\left(dx+c\right)}{a^4}+\frac{18\sin\left(dx+c\right)^2+30\sin\left(dx+c\right)+13}{a^4\left(\sin\left(dx+c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 3*sin(d*x + c)/a^4 + (18*sin(d*x
+ c)^2 + 30*sin(d*x + c) + 13)/(a^4*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 0.12, size = 69, normalized size = 0.73

$$\frac{\sin\left(c+d\,x\right)}{a^4\,d}-\frac{4\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^4\,d}-\frac{6\sin\left(c+d\,x\right)^2+10\,\sin\left(c+d\,x\right)+\frac{13}{3}}{a^4\,d\left(\sin\left(c+d\,x\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^4)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] sin(c + d*x)/(a^4*d) - (4*log(sin(c + d*x) + 1))/(a^4*d) - (10*sin(c + d*x)
 + 6*sin(c + d*x)^2 + 13/3)/(a^4*d*(sin(c + d*x) + 1)^3)
```

3.251 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=83

$$\frac{\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{3}{2d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{3}{d\left(a^4+a^4\sin(c+dx)\right)}$$

```
[Out] ln(1+sin(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3-3/2/d/(a^2+a^2*sin(d*x+c)
)^2+3/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\log(\sin(c+dx)+1)}{a^4d}-\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 3/(2*d*(
a^2 + a^2*Sin[c + d*x])^2) + 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^4d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+x)^4}+\frac{3a^2}{(a+x)^3}-\frac{3a}{(a+x)^2}+\frac{1}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^4d} \\
&= \frac{\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{3}{2d\left(a^2+a^2\sin(c+dx)\right)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.28, size = 61, normalized size = 0.73

$$
\frac{11+27\sin(c+dx)+18\sin^2(c+dx)+6\log(1+\sin(c+dx))(1+\sin(c+dx))^3}{6a^4d(1+\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (11 + 27*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 6*Log[1 + Sin[c + d*x]]*(1 + Si
n[c + d*x])^3)/(6*a^4*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.24, size = 54, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{1}{3(1+\sin(dx+c))^3}+\frac{3}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))-\frac{3}{2(1+\sin(dx+c))^2}}{d\,a^4}$
default	$\frac{\frac{1}{3(1+\sin(dx+c))^3}+\frac{3}{1+\sin(dx+c)}+\ln(1+\sin(dx+c))-\frac{3}{2(1+\sin(dx+c))^2}}{d\,a^4}$
risch	$-\frac{ix}{a^4}-\frac{2ic}{a^4d}+\frac{2i\left(-40\,e^{3i(dx+c)}-27ie^{2i(dx+c)}+27ie^{4i(dx+c)}+9\,e^{5i(dx+c)}+9\,e^{i(dx+c)}\right)}{3d\,a^4\left(e^{i(dx+c)}+i\right)^6}+\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^4d}$
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{12\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{12\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{228\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{228\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{230(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(1/3/(1+sin(d*x+c))^3+3/(1+sin(d*x+c))+ln(1+sin(d*x+c))-3/2/(1+sin(
d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 1.00

$$\frac{\frac{18 \sin(dx+c)^2+27 \sin(dx+c)+11}{a^4 \sin(dx+c)^3+3 a^4 \sin(dx+c)^2+3 a^4 \sin(dx+c)+a^4}+\frac{6 \log(\sin(dx+c)+1)}{a^4}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((18*sin(d*x + c)^2 + 27*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4
*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) + 6*log(sin(d*x + c) + 1)/a^4)/
d
```

Fricas [A]
time = 0.36, size = 112, normalized size = 1.35

$$\frac{18 \cos(dx+c)^2+6\left(3 \cos(dx+c)^2+\left(\cos(dx+c)^2-4\right) \sin(dx+c)-4\right) \log(\sin(dx+c)+1)-27 \sin(dx+c)-29}{6\left(3 a^4 d \cos(dx+c)^2-4 a^4 d+\left(a^4 d \cos(dx+c)^2-4 a^4 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
 + c) - 4)*log(sin(d*x + c) + 1) - 27*sin(d*x + c) - 29)/(3*a^4*d*cos(d*x +
 c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(70) = 140$.
time = 1.20, size = 466, normalized size = 5.61

$$\begin{cases} \frac{6 \log(\sin(c+dx)+1) \sin^3(c+dx)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{18 \log(\sin(c+dx)+1) \sin^2(c+dx)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{18 \log(\sin(c+dx)+1) \sin(c+dx)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{6 \log(\sin(c+dx)+1)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{18 \sin^2(c+dx)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{27 \sin(c+dx)}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} + \frac{11}{6a^4 d \sin^3(c+dx)+18a^4 d \sin^2(c+dx)+18a^4 d \sin(c+dx)+6a^4 d} & \text{for } d \neq 0 \\ \frac{x \sin^3(c) \cos(c)}{(a \sin(c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((6*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**
3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log
(sin(c + d*x) + 1)*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*si
n(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) +
1)*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*
a**4*d*sin(c + d*x) + 6*a**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c +
 d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) +
 18*sin(c + d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 +
 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 27*sin(c + d*x)/(6*a**4*d*sin(c + d*x
)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 11/
```

```
(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d
*x) + 6*a**4*d), Ne(d, 0)), (x*sin(c)**3*cos(c)/(a*sin(c) + a)**4, True))
```

Giac [A]
time = 0.46, size = 55, normalized size = 0.66

$$\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a^4} + \frac{18 \sin(dx+c)^2+27 \sin(dx+c)+11}{a^4(\sin(dx+c)+1)^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 + (18*sin(d*x + c)^2 + 27*sin(d*x + c
) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 0.06, size = 54, normalized size = 0.65

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^4\,d}+\frac{3\sin\left(c+d\,x\right)^2+\frac{9\sin(c+d\,x)}{2}+\frac{11}{6}}{a^4\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] log(sin(c + d*x) + 1)/(a^4*d) + ((9*sin(c + d*x))/2 + 3*sin(c + d*x)^2 + 11
/6)/(a^4*d*(sin(c + d*x) + 1)^3)
```

3.252 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(c+dx)}{3ad(a+a\sin(c+dx))^3}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d/(a+a*sin(d*x+c))^3
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 37}

$$\frac{\sin^3(c+dx)}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d*(a + a*Sin[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\sin^3(c+dx)}{3ad(a+a\sin(c+dx))^3}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 1.77

$$\frac{-5+3\cos(2(c+dx))-6\sin(c+dx)}{6a^4 d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-5 + 3*Cos[2*(c + d*x)] - 6*Sin[c + d*x])/(6*a^4*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^6)
```

Maple [A]
time = 0.20, size = 43, normalized size = 1.43

method	result
derivativedivides	$\frac{-\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{1+\sin(dx+c)}+\frac{1}{(1+\sin(dx+c))^2}}{d\,a^4}$
default	$\frac{-\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{1+\sin(dx+c)}+\frac{1}{(1+\sin(dx+c))^2}}{d\,a^4}$
risch	$-\frac{2i\left(3\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}+6ie^{4i(dx+c)}+3\,e^{i(dx+c)}-6ie^{2i(dx+c)}\right)}{3d\,a^4\left(e^{i(dx+c)}+i\right)^6}$
norman	$\frac{\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^4\left(\frac{dx}{2}\cdots\right.\right.}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/3/(1+sin(d*x+c))^3-1/(1+sin(d*x+c))+1/(1+sin(d*x+c))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(28) = 56.
time = 0.29, size = 67, normalized size = 2.23

$$-\frac{3\,\sin\left(dx+c\right)^2+3\,\sin\left(dx+c\right)+1}{3\left(a^4\sin\left(dx+c\right)^3+3\,a^4\sin\left(dx+c\right)^2+3\,a^4\sin\left(dx+c\right)+a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*s
in(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.
time = 0.34, size = 72, normalized size = 2.40

$$-\frac{3\cos\left(dx+c\right)^{2}-3\sin\left(dx+c\right)-4}{3\left(3a^{4}d\cos\left(dx+c\right)^{2}-4a^{4}d+\left(a^{4}d\cos\left(dx+c\right)^{2}-4a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 - 3*sin(d*x + c) - 4)/(3*a^4*d*cos(d*x + c)^2 - 4*a^
4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(24) = 48$.
time = 1.05, size = 192, normalized size = 6.40

$$\begin{cases} -\frac{3\sin^{2}\left(c+dx\right)}{3a^{4}d\sin^{3}\left(c+dx\right)+9a^{4}d\sin^{2}\left(c+dx\right)+9a^{4}d\sin\left(c+dx\right)+3a^{4}d}-\frac{3\sin\left(c+dx\right)}{3a^{4}d\sin^{3}\left(c+dx\right)+9a^{4}d\sin^{2}\left(c+dx\right)+9a^{4}d\sin\left(c+dx\right)+3a^{4}d}-\frac{1}{3a^{4}d\sin^{3}\left(c+dx\right)+9a^{4}d\sin^{2}\left(c+dx\right)+9a^{4}d\sin\left(c+dx\right)+3a^{4}d} & \text{for}\: d\neq 0 \\ \frac{x\sin^{2}\left(c\right)\cos\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-3*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c +
d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 3*sin(c + d*x)/(3*a**4*d*sin(
c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d)
- 1/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c +
 d*x) + 3*a**4*d), Ne(d, 0)), (x*sin(c)**2*cos(c)/(a*sin(c) + a)**4, True))
```

Giac [A]
time = 0.49, size = 38, normalized size = 1.27

$$-\frac{3\sin\left(dx+c\right)^{2}+3\sin\left(dx+c\right)+1}{3a^{4}d(\sin\left(dx+c\right)+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 + 3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

Mupad [B]
time = 8.48, size = 54, normalized size = 1.80

$$\frac{1}{a^4\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^2}-\frac{1}{a^4\,d\,\left(\sin\left(c+d\,x\right)+1\right)}-\frac{1}{3\,a^4\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] 1/(a^4*d*(sin(c + d*x) + 1)^2) - 1/(a^4*d*(sin(c + d*x) + 1)) - 1/(3*a^4*d*
(sin(c + d*x) + 1)^3)
```

3.253 $\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=46

$$\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{1}{2d\left(a^2+a^2\sin(c+dx)\right)^2}$$

```
[Out] 1/3/a/d/(a+a*sin(d*x+c))^3-1/2/d/(a^2+a^2*sin(d*x+c))^2
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{1}{3ad(a\sin(c+dx)+a)^3}-\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{a(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+x)^4}+\frac{1}{(a+x)^3}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{1}{3ad(a+a\sin(c+dx))^3} - \frac{1}{2d\left(a^2+a^2\sin(c+dx)\right)^2}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.65

$$-\frac{1+3\sin(c+dx)}{6a^4d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + a*Sin[c + d*x])^4,x]
[Out] -1/6*(1 + 3*Sin[c + d*x])/(a^4*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.17, size = 33, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^4}$
default	$\frac{\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{2(1+\sin(dx+c))^2}}{d\,a^4}$
risch	$\frac{\frac{4i\,e^{3i(dx+c)}}{3}+2\,e^{4i(dx+c)}-2\,e^{2i(dx+c)}}{d\,a^4\left(e^{i(dx+c)}+i\right)^6}$
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{22\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{22\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{10\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{26\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d/a^4*(1/3/(1+sin(d*x+c))^3-1/2/(1+sin(d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 57, normalized size = 1.24

$$-\frac{3 \sin (dx+c)+1}{6 \left(a^4 \sin (dx+c)^3+3 a^4 \sin (dx+c)^2+3 a^4 \sin (dx+c)+a^4\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + 3*a
^4*sin(d*x + c) + a^4)*d)
```

Fricas [A]
time = 0.34, size = 62, normalized size = 1.35

$$\frac{3 \sin (dx+c)+1}{6 \left(3 a^4 d \cos (dx+c)^2-4 a^4 d+\left(a^4 d \cos (dx+c)^2-4 a^4 d\right) \sin (dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*sin(d*x + c) + 1)/(3*a^4*d*cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x
 + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(37) = 74.
time = 1.05, size = 129, normalized size = 2.80

$$\begin{cases} -\frac{3 \sin (c+dx)}{6a^4 d \sin^3 (c+dx)+18a^4 d \sin^2 (c+dx)+18a^4 d \sin (c+dx)+6a^4 d} - \frac{1}{6a^4 d \sin^3 (c+dx)+18a^4 d \sin^2 (c+dx)+18a^4 d \sin (c+dx)+6a^4 d} & \text{for } d \neq 0 \\ \frac{x \sin (c) \cos (c)}{(a \sin (c)+a)^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((-3*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*
x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) - 1/(6*a**4*d*sin(c + d*x)**3 +
18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Ne(d, 0)),
(x*sin(c)*cos(c)/(a*sin(c) + a)**4, True))
```

Giac [A]
time = 0.44, size = 28, normalized size = 0.61

$$-\frac{3 \sin (dx+c)+1}{6 a^4 d(\sin (dx+c)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*sin(d*x + c) + 1)/(a^4*d*(sin(d*x + c) + 1)^3)
```

Mupad [B]
time = 8.46, size = 37, normalized size = 0.80

$$\frac{1}{3\,a^4\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^3}-\frac{1}{2\,a^4\,d\,{\left(\sin\left(c+d\,x\right)+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + a*sin(c + d*x))^4,x)
```

```
[Out] 1/(3*a^4*d*(sin(c + d*x) + 1)^3) - 1/(2*a^4*d*(sin(c + d*x) + 1)^2)
```

3.254 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=97

$$\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{1}{2d\left(a^2+a^2\sin(c+dx)\right)^2}+\frac{1}{d\left(a^4+a^4\sin(\right.}$$

```
[Out] ln(sin(d*x+c))/a^4/d-ln(1+sin(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3+1/2/
d/(a^2+a^2*sin(d*x+c))^2+1/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 46}

$$\frac{1}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{\log(\sin(c+dx))}{a^4d}-\frac{\log(\sin(c+dx)+1)}{a^4d}+\frac{1}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^4*d) - Log[1 + Sin[c + d*x]]/(a^4*d) + 1/(3*a*d*(a + a
*Sin[c + d*x])^3) + 1/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(d*(a^4 + a^4*Si
n[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^4x} - \frac{1}{a(a+x)^4} - \frac{1}{a^2(a+x)^3} - \frac{1}{a^3(a+x)^2} - \frac{1}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^4d} - \frac{\log(1+\sin(c+dx))}{a^4d} + \frac{1}{3ad(a+a\sin(c+dx))^3} + \frac{}{2d\,(a^2+a}$$

Mathematica [A]
time = 0.26, size = 62, normalized size = 0.64

$$\frac{6\log(\sin(c+dx)) - 6\log(1+\sin(c+dx)) + \frac{11+15\sin(c+dx)+6\sin^2(c+dx)}{(1+\sin(c+dx))^3}}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (6*Log[Sin[c + d*x]] - 6*Log[1 + Sin[c + d*x]] + (11 + 15*Sin[c + d*x] + 6*
Sin[c + d*x]^2)/(1 + Sin[c + d*x])^3)/(6*a^4*d)
```

Maple [A]
time = 0.20, size = 61, normalized size = 0.63

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{3(1+\sin(dx+c))^3}+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^4d}$	61
default	$\frac{\ln(\sin(dx+c))+\frac{1}{3(1+\sin(dx+c))^3}+\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{a^4d}$	61
risch	$\frac{2i\left(-28\,e^{3i(dx+c)}-15ie^{2i(dx+c)}+15ie^{4i(dx+c)}+3\,e^{5i(dx+c)}+3\,e^{i(dx+c)}\right)}{3d\,a^4\left(e^{i(dx+c)}+i\right)^6}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^4d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^4d}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4/d*(ln(sin(d*x+c))+1/3/(1+sin(d*x+c))^3+1/2/(1+sin(d*x+c))^2+1/(1+sin(
d*x+c))-ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 95, normalized size = 0.98

$$\frac{\frac{6\,\sin(dx+c)^2+15\,\sin(dx+c)+11}{a^4\sin(dx+c)^3+3\,a^4\sin(dx+c)^2+3\,a^4\sin(dx+c)+a^4} - \frac{6\,\log(\sin(dx+c)+1)}{a^4} + \frac{6\,\log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((6*sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*sin(d*x + c)^3 + 3*a^4*
sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + a^4) - 6*log(sin(d*x + c) + 1)/a^4 +
6*log(sin(d*x + c))/a^4)/d
```

Fricas [A]
time = 0.35, size = 152, normalized size = 1.57

$$\frac{6 \cos(dx+c)^2 + 6\left(3 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 6\left(3 \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log(\sin(dx+c)+1) - 15 \sin(dx+c) - 17}{6\left(3 a^4 d \cos(dx+c)^2 - 4 a^4 d + \left(a^4 d \cos(dx+c)^2 - 4 a^4 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cos(d*x + c)^2 + 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*x
+ c) - 4)*log(1/2*sin(d*x + c)) - 6*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4
)*sin(d*x + c) - 4)*log(sin(d*x + c) + 1) - 15*sin(d*x + c) - 17)/(3*a^4*d*
cos(d*x + c)^2 - 4*a^4*d + (a^4*d*cos(d*x + c)^2 - 4*a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6
*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.47, size = 69, normalized size = 0.71

$$-\frac{\frac{6 \log(|\sin(dx+c)+1|)}{a^4} - \frac{6 \log(|\sin(dx+c)|)}{a^4} - \frac{6 \sin(dx+c)^2+15 \sin(dx+c)+11}{a^4(\sin(dx+c)+1)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(sin(d*x + c) + 1))/a^4 - 6*log(abs(sin(d*x + c)))/a^4 - (6*
sin(d*x + c)^2 + 15*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 9.53, size = 206, normalized size = 2.12

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4 d} - \frac{2 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4 d} - \frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + \frac{80\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + 18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 15a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 20a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 15a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 6a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + a*sin(c + d*x))^4),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^4*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d) -
 (6*tan(c/2 + (d*x)/2) + 18*tan(c/2 + (d*x)/2)^2 + (80*tan(c/2 + (d*x)/2)^3
)/3 + 18*tan(c/2 + (d*x)/2)^4 + 6*tan(c/2 + (d*x)/2)^5)/(d*(15*a^4*tan(c/2
+ (d*x)/2)^2 + 20*a^4*tan(c/2 + (d*x)/2)^3 + 15*a^4*tan(c/2 + (d*x)/2)^4 +
6*a^4*tan(c/2 + (d*x)/2)^5 + a^4*tan(c/2 + (d*x)/2)^6 + a^4 + 6*a^4*tan(c/2
 + (d*x)/2)))
```

3.255 $\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=111

$$-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(1+\sin(c+dx))}{a^4d}-\frac{1}{3ad(a+a\sin(c+dx))^3}-\frac{1}{d\left(a^2+a^2\sin(c+dx)\right.}$$

```
[Out] -csc(d*x+c)/a^4/d-4*ln(sin(d*x+c))/a^4/d+4*ln(1+sin(d*x+c))/a^4/d-1/3/a/d/(
a+a*sin(d*x+c))^3-1/d/(a^2+a^2*sin(d*x+c))^2-3/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 46}

$$-\frac{3}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc(c+dx)}{a^4d}-\frac{4\log(\sin(c+dx))}{a^4d}+\frac{4\log(\sin(c+dx)+1)}{a^4d}-\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(Csc[c + d*x]/(a^4*d)) - (4*Log[Sin[c + d*x]])/(a^4*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^4*d) - 1/(3*a*d*(a + a*Sin[c + d*x])^3) - 1/(d*(a^2 + a^2*Sin[c
 + d*x])^2) - 3/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^2}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{1}{x^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{a^4x^2} - \frac{4}{a^5x} + \frac{1}{a^2(a+x)^4} + \frac{2}{a^3(a+x)^3} + \frac{3}{a^4(a+x)^2} + \frac{4}{a^5(a+x)}\right)\,dx, x, a\sin\right.}{d}$$
$$= -\frac{\csc(c+dx)}{a^4d} - \frac{4\log(\sin(c+dx))}{a^4d} + \frac{4\log(1+\sin(c+dx))}{a^4d} - \frac{1}{3ad(a+a\sin}$$

Mathematica [A]
time = 0.70, size = 73, normalized size = 0.66

$$-\frac{3\csc(c+dx)+12\log(\sin(c+dx))-12\log(1+\sin(c+dx))+\frac{1}{(1+\sin(c+dx))^3}+\frac{3}{(1+\sin(c+dx))^2}+\frac{9}{1+\sin(c+dx)}}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/3*(3*Csc[c + d*x] + 12*Log[Sin[c + d*x]] - 12*Log[1 + Sin[c + d*x]] + (1
 + Sin[c + d*x])^(-3) + 3/(1 + Sin[c + d*x])^2 + 9/(1 + Sin[c + d*x]))/(a^4
*d)
```

Maple [A]
time = 0.22, size = 75, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-4\ln(\sin(dx+c))-\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{(1+\sin(dx+c))^2}-\frac{3}{1+\sin(dx+c)}+4\ln(1+\sin(dx+c))}{a^4d}$
default	$\frac{-\frac{1}{\sin(dx+c)}-4\ln(\sin(dx+c))-\frac{1}{3(1+\sin(dx+c))^3}-\frac{1}{(1+\sin(dx+c))^2}-\frac{3}{1+\sin(dx+c)}+4\ln(1+\sin(dx+c))}{a^4d}$
risch	$-\frac{8i\left(15i e^{6i(dx+c)}+3\,e^{7i(dx+c)}-36i e^{4i(dx+c)}-31\,e^{5i(dx+c)}+15i e^{2i(dx+c)}+31\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{3\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^6 d\,a^4}-\frac{4\ln\left(e^{2i(dx+c)}-1\right.}{a^4d}$
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{51\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{51\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{209\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{209\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{1159\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4/d*(-1/sin(d*x+c)-4*ln(sin(d*x+c))-1/3/(1+sin(d*x+c))^3-1/(1+sin(d*x+c
))^2-3/(1+sin(d*x+c))+4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 114, normalized size = 1.03

$$-\frac{\frac{12\,\sin(dx+c)^{3}+30\,\sin(dx+c)^{2}+22\,\sin(dx+c)+3}{a^{4}\,\sin(dx+c)^{4}+3\,a^{4}\,\sin(dx+c)^{3}+3\,a^{4}\,\sin(dx+c)^{2}+a^{4}\,\sin(dx+c)}-\frac{12\,\log(\sin(dx+c)+1)}{a^{4}}+\frac{12\,\log(\sin(dx+c))}{a^{4}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((12*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*si
n(d*x + c)^4 + 3*a^4*sin(d*x + c)^3 + 3*a^4*sin(d*x + c)^2 + a^4*sin(d*x +
c)) - 12*log(sin(d*x + c) + 1)/a^4 + 12*log(sin(d*x + c))/a^4)/d
```

Fricas [A]
time = 0.34, size = 201, normalized size = 1.81

$$\frac{30\,\cos\left(dx+c\right)^{2}-12\,\left(\cos\left(dx+c\right)^{4}-5\,\cos\left(dx+c\right)^{2}-\left(3\,\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)+4\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+12\,\left(\cos\left(dx+c\right)^{4}-5\,\cos\left(dx+c\right)^{2}-\left(3\,\cos\left(dx+c\right)^{2}-4\right)\sin\left(dx+c\right)+4\right)\log\left(\sin\left(dx+c\right)+1\right)+2\,\left(6\,\cos\left(dx+c\right)^{2}-17\right)\sin\left(dx+c\right)-33}{3\,\left(a^{4}d\cos\left(dx+c\right)^{4}-5\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d-\left(3\,a^{4}d\cos\left(dx+c\right)^{2}-4\,a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(30*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 5*cos(d*x + c)^2 - (3*cos(d*x
 + c)^2 - 4)*sin(d*x + c) + 4)*log(1/2*sin(d*x + c)) + 12*(cos(d*x + c)^4 -
 5*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 4)*sin(d*x + c) + 4)*log(sin(d*x +
c) + 1) + 2*(6*cos(d*x + c)^2 - 17)*sin(d*x + c) - 33)/(a^4*d*cos(d*x + c)^
4 - 5*a^4*d*cos(d*x + c)^2 + 4*a^4*d - (3*a^4*d*cos(d*x + c)^2 - 4*a^4*d)*s
in(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sin^{4}\left(c+dx\right)+4\sin^{3}\left(c+dx\right)+6\sin^{2}\left(c+dx\right)+4\sin\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.52, size = 87, normalized size = 0.78

$$\frac{\frac{12\,\log(|\sin(dx+c)+1|)}{a^{4}}-\frac{12\,\log(|\sin(dx+c)|)}{a^{4}}-\frac{12\,\sin(dx+c)^{3}+30\,\sin(dx+c)^{2}+22\,\sin(dx+c)+3}{a^{4}(\sin(dx+c)+1)^{3}\sin(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c)))/a^4 - (1
2*sin(d*x + c)^3 + 30*sin(d*x + c)^2 + 22*sin(d*x + c) + 3)/(a^4*(sin(d*x +
 c) + 1)^3*sin(d*x + c)))/d
```

Mupad [B]
time = 8.72, size = 251, normalized size = 2.26

$$\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+74\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{307\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+60\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1}{d\left(2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+30a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+40a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+30a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+12a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d}+\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (9*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + 60*tan(c/2 + (d*x)/2)^3 +
(307*tan(c/2 + (d*x)/2)^4)/3 + 74*tan(c/2 + (d*x)/2)^5 + 23*tan(c/2 + (d*x)
/2)^6 - 1)/(d*(12*a^4*tan(c/2 + (d*x)/2)^2 + 30*a^4*tan(c/2 + (d*x)/2)^3 +
40*a^4*tan(c/2 + (d*x)/2)^4 + 30*a^4*tan(c/2 + (d*x)/2)^5 + 12*a^4*tan(c/2
+ (d*x)/2)^6 + 2*a^4*tan(c/2 + (d*x)/2)^7 + 2*a^4*tan(c/2 + (d*x)/2))) - (4
*log(tan(c/2 + (d*x)/2)))/(a^4*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d)
 - tan(c/2 + (d*x)/2)/(2*a^4*d)
```

3.256 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=131

$$\frac{4\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(1+\sin(c+dx))}{a^4d}+\frac{1}{3ad(a+a\sin(c+dx))^3}+\frac{}{2d\,(a}$$

```
[Out] 4*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^4/d+10*ln(sin(d*x+c))/a^4/d-10*ln(1+s
in(d*x+c))/a^4/d+1/3/a/d/(a+a*sin(d*x+c))^3+3/2/d/(a^2+a^2*sin(d*x+c))^2+6/
d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$\frac{6}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{10\log(\sin(c+dx))}{a^4d}-\frac{10\log(\sin(c+dx)+1)}{a^4d}+\frac{3}{2d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{1}{3ad(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (10*Log[Sin[c + d*x]]
)/(a^4*d) - (10*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(3*a*d*(a + a*Sin[c + d*
x])^3) + 3/(2*d*(a^2 + a^2*Sin[c + d*x])^2) + 6/(d*(a^4 + a^4*Sin[c + d*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^3}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{1}{x^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{1}{a^4x^3} - \frac{4}{a^5x^2} + \frac{10}{a^6x} - \frac{1}{a^3(a+x)^4} - \frac{3}{a^4(a+x)^3} - \frac{6}{a^5(a+x)^2} - \frac{10}{a^6(a+x)}\right) dx}{d}$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{10\log(\sin(c+dx))}{a^4d} - \frac{10\log(1+\sin(c+dx}{a^4d}$$

Mathematica [A]
time = 2.55, size = 85, normalized size = 0.65

$$\frac{24\csc(c+dx) - 3\csc^2(c+dx) + 60\log(\sin(c+dx)) - 60\log(1+\sin(c+dx)) + \frac{2}{(1+\sin(c+dx))^3} + \frac{9}{(1+\sin(c+dx))^2} + \frac{36}{1+\sin(c+dx)}}{6a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (24*Csc[c + d*x] - 3*Csc[c + d*x]^2 + 60*Log[Sin[c + d*x]] - 60*Log[1 + Sin
[c + d*x]] + 2/(1 + Sin[c + d*x])^3 + 9/(1 + Sin[c + d*x])^2 + 36/(1 + Sin[
c + d*x]))/(6*a^4*d)
```

Maple [A]
time = 0.22, size = 85, normalized size = 0.65

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+10\ln(\sin(dx+c))+\frac{1}{3(1+\sin(dx+c))^3}+\frac{3}{2(1+\sin(dx+c))^2}+\frac{6}{1+\sin(dx+c)}-10\ln(1+\sin(dx+c))}{a^4d}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+10\ln(\sin(dx+c))+\frac{1}{3(1+\sin(dx+c))^3}+\frac{3}{2(1+\sin(dx+c))^2}+\frac{6}{1+\sin(dx+c)}-10\ln(1+\sin(dx+c))}{a^4d}$
risch	$\frac{4i(75i e^{8i(dx+c)}+15 e^{9i(dx+c)}-255i e^{6i(dx+c)}-170 e^{7i(dx+c)}+255i e^{4i(dx+c)}+298 e^{5i(dx+c)}-75i e^{2i(dx+c)}-170 e^{3i(dx+c)}+}{3(e^{2i(dx+c)}-1)^2(e^{i(dx+c)}+i)^6 d a^4}$
norman	$\frac{-\frac{60(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{ad}-\frac{60(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{ad}-\frac{1}{8ad}+\frac{9\tan(\frac{dx}{2}+\frac{c}{2})}{8ad}+\frac{9(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{8ad}-\frac{\tan^{11}(\frac{dx}{2}+\frac{c}{2})}{8ad}-\frac{1995(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{8ad}-\frac{199}{}}{\tan(\frac{dx}{2}+\frac{c}{2})^2 a^3 (\tan(\frac{dx}{2}+\frac{c}{2})+1)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4/d*(-1/2/sin(d*x+c)^2+4/sin(d*x+c)+10*ln(sin(d*x+c))+1/3/(1+sin(d*x+c)
)^3+3/2/(1+sin(d*x+c))^2+6/(1+sin(d*x+c))-10*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 126, normalized size = 0.96

$$\frac{\frac{60\sin(dx+c)^4+150\sin(dx+c)^3+110\sin(dx+c)^2+15\sin(dx+c)-3}{a^4\sin(dx+c)^5+3\,a^4\sin(dx+c)^4+3\,a^4\sin(dx+c)^3+a^4\sin(dx+c)^2}-\frac{60\log(\sin(dx+c)+1)}{a^4}+\frac{60\log(\sin(dx+c))}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*((60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + 3*a^4*sin(d*x + c)^4 + 3*a^4*sin(d*x +
c)^3 + a^4*sin(d*x + c)^2) - 60*log(sin(d*x + c) + 1)/a^4 + 60*log(sin(d*x
+ c))/a^4)/d
```

Fricas [A]
time = 0.37, size = 242, normalized size = 1.85

$$\frac{60\cos(dx+c)^4-230\cos(dx+c)^2+60\left(3\cos(dx+c)^4-7\cos(dx+c)^2+\left(\cos(dx+c)^4-5\cos(dx+c)^2+4\right)\sin(dx+c)+4\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-60\left(3\cos(dx+c)^4-7\cos(dx+c)^2+\left(\cos(dx+c)^4-5\cos(dx+c)^2+4\right)\sin(dx+c)+4\right)\log(\sin(dx+c)+1)-15\left(10\cos(dx+c)^2-11\right)\sin(dx+c)+167}{6\left(3\,a^4d\cos(dx+c)^4-7\,a^4d\cos(dx+c)^2+4\,a^4d+\left(a^4d\cos(dx+c)^4-5\,a^4d\cos(dx+c)^2+4\,a^4d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*cos(d*x + c)^4 - 230*cos(d*x + c)^2 + 60*(3*cos(d*x + c)^4 - 7*cos(
d*x + c)^2 + (cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(
1/2*sin(d*x + c)) - 60*(3*cos(d*x + c)^4 - 7*cos(d*x + c)^2 + (cos(d*x + c)
^4 - 5*cos(d*x + c)^2 + 4)*sin(d*x + c) + 4)*log(sin(d*x + c) + 1) - 15*(10
*cos(d*x + c)^2 - 11)*sin(d*x + c) + 167)/(3*a^4*d*cos(d*x + c)^4 - 7*a^4*d
*cos(d*x + c)^2 + 4*a^4*d + (a^4*d*cos(d*x + c)^4 - 5*a^4*d*cos(d*x + c)^2
+ 4*a^4*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos(c+dx)\csc^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3
+ 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.48, size = 97, normalized size = 0.74

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a^4}-\frac{60 \log(|\sin(dx+c)|)}{a^4}-\frac{60 \sin(dx+c)^4+150 \sin(dx+c)^3+110 \sin(dx+c)^2+15 \sin(dx+c)-3}{a^4(\sin(dx+c)+1)^3 \sin(dx+c)^2}}{6\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c)))/a^4 - (
60*sin(d*x + c)^4 + 150*sin(d*x + c)^3 + 110*sin(d*x + c)^2 + 15*sin(d*x +
c) - 3)/(a^4*(sin(d*x + c) + 1)^3*sin(d*x + c)^2))/d
```

Mupad [B]
time = 8.69, size = 286, normalized size = 2.18

$$\frac{10 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\, d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\, a^4\, d}-\frac{72 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{465 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2}+\frac{881 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+\frac{255 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}-30 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{81 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}-5 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{1}{2}}{d\left(4 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+24 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+60 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+80 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+60 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+24 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4 a^4 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{20 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4\, d}+\frac{2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (10*log(tan(c/2 + (d*x)/2)))/(a^4*d) - tan(c/2 + (d*x)/2)^2/(8*a^4*d) - ((2
55*tan(c/2 + (d*x)/2)^4)/2 - (81*tan(c/2 + (d*x)/2)^2)/2 - 30*tan(c/2 + (d*
x)/2)^3 - 5*tan(c/2 + (d*x)/2) + (881*tan(c/2 + (d*x)/2)^5)/3 + (465*tan(c/
2 + (d*x)/2)^6)/2 + 72*tan(c/2 + (d*x)/2)^7 + 1/2)/(d*(4*a^4*tan(c/2 + (d*x
)/2)^2 + 24*a^4*tan(c/2 + (d*x)/2)^3 + 60*a^4*tan(c/2 + (d*x)/2)^4 + 80*a^4
*tan(c/2 + (d*x)/2)^5 + 60*a^4*tan(c/2 + (d*x)/2)^6 + 24*a^4*tan(c/2 + (d*x
)/2)^7 + 4*a^4*tan(c/2 + (d*x)/2)^8)) - (20*log(tan(c/2 + (d*x)/2) + 1))/(a
^4*d) + (2*tan(c/2 + (d*x)/2))/(a^4*d)
```

3.257 $\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] -2*arctanh((a+a*sin(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d+2*(a+a*sin(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2786, 52, 65, 213}

$$\frac{2\sqrt{a\sin(c+dx)+a}}{d}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\sin(c+dx)+a}}{\sqrt{a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + a*Sin[c + d*x]]/Sqrt[a]])/d + (2*Sqrt[a + a*Si
n[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{2\sqrt{a+a\sin(c+dx)}}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+a\sin(c+d}\right.}{d} \\
&= -\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+a\sin(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2\sqrt{a+a\sin(c+dx)}}{d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 118 vs. 2(51) = 102.
time = 0.11, size = 118, normalized size = 2.31

$$\frac{\left(2\cos\left(\frac{1}{2}(c+dx)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{a(1+\sin(c+dx))}}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] + Log[1 - Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1
 + Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 2*Sin[(c + d*x)/2])*Sqrt[a*(1 + S
in[c + d*x])])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.82

method	result	size

derivativedivides	$\frac{2\sqrt{a+a\sin(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+a\sin(dx+c)}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+a\sin(dx+c)}}{\sqrt{a}}\right)}{d}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(a+a*sin(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+a*sin(d*x+c))^(1/2)/a^(1
/2)))
```

Maxima [A]
time = 0.51, size = 61, normalized size = 1.20

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{a\sin(dx+c)+a}-\sqrt{a}}{\sqrt{a\sin(dx+c)+a}+\sqrt{a}}\right)+2\sqrt{a\sin(dx+c)+a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima
")
```

```
[Out] (sqrt(a)*log((sqrt(a*sin(d*x + c) + a) - sqrt(a))/(sqrt(a*sin(d*x + c) + a)
 + sqrt(a))) + 2*sqrt(a*sin(d*x + c) + a))/d
```

Fricas [A]
time = 0.35, size = 86, normalized size = 1.69

$$\frac{\sqrt{a}\log\left(\frac{a\cos(dx+c)^2+4\sqrt{a\sin(dx+c)+a}\sqrt{a}(\sin(dx+c)+2)-8a\sin(dx+c)-9a}{\cos(dx+c)^2-1}\right)+4\sqrt{a\sin(dx+c)+a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*(sqrt(a)*log((a*cos(d*x + c)^2 + 4*sqrt(a*sin(d*x + c) + a)*sqrt(a)*(si
n(d*x + c) + 2) - 8*a*sin(d*x + c) - 9*a)/(cos(d*x + c)^2 - 1)) + 4*sqrt(a*
sin(d*x + c) + a))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a(\sin(c+dx)+1)}\cos(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)*csc(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(43) = 86$.
time = 0.47, size = 89, normalized size = 1.75

$$\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)+4\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(-1/4*pi + 1/2*d*x + 1/2*c))
/abs(2*sqrt(2) + 4*cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 4*cos(-1/4*pi + 1/2*d
*x + 1/2*c))*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))/d
```

Mupad [B]
time = 8.73, size = 43, normalized size = 0.84

$$\frac{2\sqrt{a+a\sin\left(c+d\,x\right)}}{d}-\frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+a\sin\left(c+d\,x\right)}}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] (2*(a + a*sin(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + a*sin(c + d*x))^(1
/2)/a^(1/2)))/d
```

3.258 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=114

$$\frac{a^4\sin^{1+n}(c+dx)}{d(1+n)}+\frac{4a^4\sin^{2+n}(c+dx)}{d(2+n)}+\frac{6a^4\sin^{3+n}(c+dx)}{d(3+n)}+\frac{4a^4\sin^{4+n}(c+dx)}{d(4+n)}+\frac{a^4\sin^{5+n}(c+dx)}{d(5+n)}$$

```
[Out] a^4*sin(d*x+c)^(1+n)/d/(1+n)+4*a^4*sin(d*x+c)^(2+n)/d/(2+n)+6*a^4*sin(d*x+c
)^(3+n)/d/(3+n)+4*a^4*sin(d*x+c)^(4+n)/d/(4+n)+a^4*sin(d*x+c)^(5+n)/d/(5+n)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 45}

$$\frac{a^4\sin^{n+1}(c+dx)}{d(n+1)}+\frac{4a^4\sin^{n+2}(c+dx)}{d(n+2)}+\frac{6a^4\sin^{n+3}(c+dx)}{d(n+3)}+\frac{4a^4\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a^4\sin^{n+5}(c+dx)}{d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (4*a^4*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (6*a^4*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (4*a^4*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (a^4*Sin[c + d*x]^(5 + n))/(d*(5 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^4\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n + 4a^4\left(\frac{x}{a}\right)^{1+n} + 6a^4\left(\frac{x}{a}\right)^{2+n} + 4a^4\left(\frac{x}{a}\right)^{3+}\right.\right.}{ad}$$
$$= \frac{a^4\sin^{1+n}(c+dx)}{d(1+n)} + \frac{4a^4\sin^{2+n}(c+dx)}{d(2+n)} + \frac{6a^4\sin^{3+n}(c+}{d(3+n)}$$

Mathematica [A]
time = 0.20, size = 80, normalized size = 0.70

$$\frac{a^4\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{4\sin(c+dx)}{2+n}+\frac{6\sin^2(c+dx)}{3+n}+\frac{4\sin^3(c+dx)}{4+n}+\frac{\sin^4(c+dx)}{5+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^4,x]
[Out] (a^4*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (4*Sin[c + d*x])/(2 + n) + (6*Sin
[c + d*x]^2)/(3 + n) + (4*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n))
)/d
```

Maple [F]
time = 0.73, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x)
```

Maxima [A]
time = 0.28, size = 103, normalized size = 0.90

$$\frac{\frac{a^4\sin(dx+c)^{n+5}}{n+5}+\frac{4\,a^4\sin(dx+c)^{n+4}}{n+4}+\frac{6\,a^4\sin(dx+c)^{n+3}}{n+3}+\frac{4\,a^4\sin(dx+c)^{n+2}}{n+2}+\frac{a^4\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] (a^4*sin(d*x + c)^(n + 5)/(n + 5) + 4*a^4*sin(d*x + c)^(n + 4)/(n + 4) + 6*
a^4*sin(d*x + c)^(n + 3)/(n + 3) + 4*a^4*sin(d*x + c)^(n + 2)/(n + 2) + a^4
*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(114) = 228$.
time = 0.38, size = 302, normalized size = 2.65

$$\frac{\left(8\,a^{4}n^{4}+96\,a^{4}n^{3}+400\,a^{4}n^{2}+672\,a^{4}n+4\,\left(a^{4}n^{4}+11\,a^{4}n^{3}+41\,a^{4}n^{2}+61\,a^{4}n+30\,a^{4}\right)\cos\left(dx+c\right)^{4}+360\,a^{4}-4\,\left(3\,a^{4}n^{4}+35\,a^{4}n^{3}+141\,a^{4}n^{2}+229\,a^{4}n+120\,a^{4}\right)\cos\left(dx+c\right)^{2}+\left(8\,a^{4}n^{4}+96\,a^{4}n^{3}+400\,a^{4}n^{2}+672\,a^{4}n+\left(a^{4}n^{4}+10\,a^{4}n^{3}+35\,a^{4}n^{2}+50\,a^{4}n+24\,a^{4}\right)\cos\left(dx+c\right)^{4}+384\,a^{4}-4\,\left(2\,a^{4}n^{4}+23\,a^{4}n^{3}+91\,a^{4}n^{2}+142\,a^{4}n+72\,a^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\sin\left(dx+c\right)^{n}}{dn^{5}+15\,dn^{4}+85\,dn^{3}+225\,dn^{2}+274\,dn+120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] (8*a^4*n^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + 4*(a^4*n^4 + 11*a^4*n^3
 + 41*a^4*n^2 + 61*a^4*n + 30*a^4)*cos(d*x + c)^4 + 360*a^4 - 4*(3*a^4*n^4
+ 35*a^4*n^3 + 141*a^4*n^2 + 229*a^4*n + 120*a^4)*cos(d*x + c)^2 + (8*a^4*n
^4 + 96*a^4*n^3 + 400*a^4*n^2 + 672*a^4*n + (a^4*n^4 + 10*a^4*n^3 + 35*a^4*
n^2 + 50*a^4*n + 24*a^4)*cos(d*x + c)^4 + 384*a^4 - 4*(2*a^4*n^4 + 23*a^4*n
^3 + 91*a^4*n^2 + 142*a^4*n + 72*a^4)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x
 + c)^n/(d*n^5 + 15*d*n^4 + 85*d*n^3 + 225*d*n^2 + 274*d*n + 120*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(97) = 194$.
time = 5.93, size = 1833, normalized size = 16.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**4*sin(c)**n*cos(c), Eq(d, 0)), (a**4*log(sin(c
 + d*x))/d - 4*a**4/(d*sin(c + d*x)) - 3*a**4/(d*sin(c + d*x)**2) - 4*a**4/
(3*d*sin(c + d*x)**3) - a**4/(4*d*sin(c + d*x)**4), Eq(n, -5)), (4*a**4*log
(sin(c + d*x))/d + a**4*sin(c + d*x)/d - 6*a**4/(d*sin(c + d*x)) - 2*a**4/(
d*sin(c + d*x)**2) - a**4/(3*d*sin(c + d*x)**3), Eq(n, -4)), (6*a**4*log(si
n(c + d*x))/d + a**4*sin(c + d*x)**2/(2*d) + 4*a**4*sin(c + d*x)/d - 4*a**4
/(d*sin(c + d*x)) - a**4/(2*d*sin(c + d*x)**2), Eq(n, -3)), (4*a**4*log(sin
(c + d*x))/d + a**4*sin(c + d*x)**3/(3*d) + 2*a**4*sin(c + d*x)**2/d + 6*a*
*4*sin(c + d*x)/d - a**4/(d*sin(c + d*x)), Eq(n, -2)), (a**4*log(sin(c + d*
x))/d + a**4*sin(c + d*x)**4/(4*d) + 4*a**4*sin(c + d*x)**3/(3*d) + 3*a**4*
sin(c + d*x)**2/d + 4*a**4*sin(c + d*x)/d, Eq(n, -1)), (a**4*n**4*sin(c + d
*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d
*n + 120*d) + 4*a**4*n**4*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n*
*4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 6*a**4*n**4*sin(c + d*x)**
3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n +
120*d) + 4*a**4*n**4*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 +
85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + a**4*n**4*sin(c + d*x)*sin(c +
d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 1
0*a**4*n**3*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3
```

```
 + 225*d*n**2 + 274*d*n + 120*d) + 44*a**4*n**3*sin(c + d*x)**4*sin(c + d*x
)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 72*a
**4*n**3*sin(c + d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 +
225*d*n**2 + 274*d*n + 120*d) + 52*a**4*n**3*sin(c + d*x)**2*sin(c + d*x)**
n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 14*a**4
*n**3*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*
n**2 + 274*d*n + 120*d) + 35*a**4*n**2*sin(c + d*x)**5*sin(c + d*x)**n/(d*n
**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 164*a**4*n**2
*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n*
*2 + 274*d*n + 120*d) + 294*a**4*n**2*sin(c + d*x)**3*sin(c + d*x)**n/(d*n*
*5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 236*a**4*n**2*
sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**
2 + 274*d*n + 120*d) + 71*a**4*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**5 +
15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 50*a**4*n*sin(c + d
*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d
*n + 120*d) + 244*a**4*n*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**5 + 15*d*n**
4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 468*a**4*n*sin(c + d*x)**3*
sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 12
0*d) + 428*a**4*n*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*
d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 154*a**4*n*sin(c + d*x)*sin(c + d*
x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 24*
a**4*sin(c + d*x)**5*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*
d*n**2 + 274*d*n + 120*d) + 120*a**4*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**
5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 240*a**4*sin(c
+ d*x)**3*sin(c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 27
4*d*n + 120*d) + 240*a**4*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**5 + 15*d*n*
*4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d) + 120*a**4*sin(c + d*x)*sin(
c + d*x)**n/(d*n**5 + 15*d*n**4 + 85*d*n**3 + 225*d*n**2 + 274*d*n + 120*d)
, True))
```

Giac [A]
time = 0.52, size = 127, normalized size = 1.11

$$\frac{\frac{a^4 \sin(dx+c)^n \sin(dx+c)^5}{n+5} + \frac{4\,a^4 \sin(dx+c)^n \sin(dx+c)^4}{n+4} + \frac{6\,a^4 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{4\,a^4 \sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{a^4 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] (a^4*sin(d*x + c)^n*sin(d*x + c)^5/(n + 5) + 4*a^4*sin(d*x + c)^n*sin(d*x +
 c)^4/(n + 4) + 6*a^4*sin(d*x + c)^n*sin(d*x + c)^3/(n + 3) + 4*a^4*sin(d*x
 + c)^n*sin(d*x + c)^2/(n + 2) + a^4*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Mupad [B]
time = 11.76, size = 370, normalized size = 3.25

$$\frac{a^4 \sin(c+d\,x)^n\,\left(4888\,n+5040\,\sin(c+d\,x)-2880\,\cos(2\,c+2\,d\,x)+240\,\cos(4\,c+4\,d\,x)-1080\,\sin(3\,c+3\,d\,x)+24\,\sin(5\,c+5\,d\,x)+8580\,n\,\sin(c+d\,x)-5376\,n\,\cos(2\,c+2\,d\,x)+488\,n\,\cos(4\,c+4\,d\,x)-2122\,n\,\sin(3\,c+3\,d\,x)+50\,n\,\sin(5\,c+5\,d\,x)+5014\,n^2\,\sin(c+d\,x)+1188\,n^3\,\sin(c+d\,x)+98\,n^4\,\sin(c+d\,x)+2872\,n^2+680\,n^3+56\,n^4-3200\,n^2\,\cos(2\,c+2\,d\,x)-768\,n^3\,\cos(2\,c+2\,d\,x)-64\,n^4\,\cos(2\,c+2\,d\,x)+328\,n^2\,\cos(4\,c+4\,d\,x)+88\,n^3\,\cos(4\,c+4\,d\,x)+8\,n^4\,\cos(4\,c+4\,d\,x)-1351\,n^2\,\sin(3\,c+3\,d\,x)-338\,n^3\,\sin(3\,c+3\,d\,x)-29\,n^4\,\sin(3\,c+3\,d\,x)+35\,n^2\,\sin(5\,c+5\,d\,x)+10\,n^3\,\sin(5\,c+5\,d\,x)+n^4\,\sin(5\,c+5\,d\,x)+2640\right)}{16\,d\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (a^4*sin(c + d*x)^n*(4888*n + 5040*sin(c + d*x) - 2880*cos(2*c + 2*d*x) + 2
40*cos(4*c + 4*d*x) - 1080*sin(3*c + 3*d*x) + 24*sin(5*c + 5*d*x) + 8580*n*
sin(c + d*x) - 5376*n*cos(2*c + 2*d*x) + 488*n*cos(4*c + 4*d*x) - 2122*n*si
n(3*c + 3*d*x) + 50*n*sin(5*c + 5*d*x) + 5014*n^2*sin(c + d*x) + 1188*n^3*s
in(c + d*x) + 98*n^4*sin(c + d*x) + 2872*n^2 + 680*n^3 + 56*n^4 - 3200*n^2*
cos(2*c + 2*d*x) - 768*n^3*cos(2*c + 2*d*x) - 64*n^4*cos(2*c + 2*d*x) + 328
*n^2*cos(4*c + 4*d*x) + 88*n^3*cos(4*c + 4*d*x) + 8*n^4*cos(4*c + 4*d*x) -
1351*n^2*sin(3*c + 3*d*x) - 338*n^3*sin(3*c + 3*d*x) - 29*n^4*sin(3*c + 3*d
*x) + 35*n^2*sin(5*c + 5*d*x) + 10*n^3*sin(5*c + 5*d*x) + n^4*sin(5*c + 5*d
*x) + 2640))/(16*d*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))
```

3.259 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}+\frac{3a^3\sin^{3+n}(c+dx)}{d(3+n)}+\frac{a^3\sin^{4+n}(c+dx)}{d(4+n)}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)+3*a^3*sin(d*x+c
)^(3+n)/d/(3+n)+a^3*sin(d*x+c)^(4+n)/d/(4+n)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 45}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{3a^3\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a^3\sin^{n+4}(c+dx)}{d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (3*a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) + (a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^3\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n + 3a^3\left(\frac{x}{a}\right)^{1+n} + 3a^3\left(\frac{x}{a}\right)^{2+n} + a^3\left(\frac{x}{a}\right)^{3\text{-}}\right.\right.}{ad}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} + \frac{3a^3\sin^{3+n}(c}{d(3+n}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.71

$$\frac{a^3\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{3\sin(c+dx)}{2+n}+\frac{3\sin^2(c+dx)}{3+n}+\frac{\sin^3(c+dx)}{4+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + (3*Sin
[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3/(4 + n)))/d
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.91

$$\frac{\frac{a^3\sin(dx+c)^{n+4}}{n+4}+\frac{3\,a^3\sin(dx+c)^{n+3}}{n+3}+\frac{3\,a^3\sin(dx+c)^{n+2}}{n+2}+\frac{a^3\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (a^3*sin(d*x + c)^(n + 4)/(n + 4) + 3*a^3*sin(d*x + c)^(n + 3)/(n + 3) + 3*
a^3*sin(d*x + c)^(n + 2)/(n + 2) + a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(91) = 182$.

time = 0.35, size = 210, normalized size = 2.31

$$\frac{\left(4 a^3 n^3 + 30 a^3 n^2 + \left(a^3 n^3 + 6 a^3 n^2 + 11 a^3 n + 6 a^3\right) \cos\left(dx+c\right)^4 + 68 a^3 n + 42 a^3 - \left(5 a^3 n^3 + 36 a^3 n^2 + 79 a^3 n + 48 a^3\right) \cos\left(dx+c\right)^2 + \left(4 a^3 n^3 + 30 a^3 n^2 + 68 a^3 n + 48 a^3 - 3\left(a^3 n^3 + 7 a^3 n^2 + 14 a^3 n + 8 a^3\right) \cos\left(dx+c\right)^2\right) \sin\left(dx+c\right)\right) \sin\left(dx+c\right)^n}{dn^4 + 10 dn^3 + 35 dn^2 + 50 dn + 24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (4*a^3*n^3 + 30*a^3*n^2 + (a^3*n^3 + 6*a^3*n^2 + 11*a^3*n + 6*a^3)*cos(d*x
+ c)^4 + 68*a^3*n + 42*a^3 - (5*a^3*n^3 + 36*a^3*n^2 + 79*a^3*n + 48*a^3)*c
os(d*x + c)^2 + (4*a^3*n^3 + 30*a^3*n^2 + 68*a^3*n + 48*a^3 - 3*(a^3*n^3 +
7*a^3*n^2 + 14*a^3*n + 8*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/
(d*n^4 + 10*d*n^3 + 35*d*n^2 + 50*d*n + 24*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1061 vs. $2(76) = 152$.

time = 2.77, size = 1061, normalized size = 11.66

$$\begin{cases} x\left(a\sin\left(c\right)+a\right)^3 \sin^n\left(c\right)\cos\left(c\right) & \text{for}\: d = 0 \\ \frac{a^3\log\left(\sin\left(c+dx\right)\right)}{d} - \frac{3a^3}{d\sin\left(c+dx\right)} - \frac{3a^3}{2d\sin^2\left(c+dx\right)} - \frac{a^3}{3d\sin^3\left(c+dx\right)} & \text{for}\: n = -4 \\ \frac{3a^3\log\left(\sin\left(c+dx\right)\right)}{d} + \frac{a^3\sin\left(c+dx\right)}{d} - \frac{3a^3}{d\sin\left(c+dx\right)} - \frac{a^3}{2d\sin^2\left(c+dx\right)} & \text{for}\: n = -3 \\ \frac{3a^3\log\left(\sin\left(c+dx\right)\right)}{d} + \frac{a^3\sin^2\left(c+dx\right)}{2d} + \frac{3a^3\sin\left(c+dx\right)}{d} - \frac{a^3}{d\sin\left(c+dx\right)} & \text{for}\: n = -2 \\ \frac{a^3\log\left(\sin\left(c+dx\right)\right)}{d} + \frac{a^3\sin^3\left(c+dx\right)}{3d} + \frac{3a^3\sin^2\left(c+dx\right)}{2d} + \frac{3a^3\sin\left(c+dx\right)}{d} & \text{for}\: n = -1 \\ \frac{a^3n^3\sin^4\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{3a^3n^3\sin^3\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{3a^3n^3\sin^2\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{a^3n^3\sin\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{6a^3n^2\sin^4\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{21a^3n^2\sin^3\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{24a^3n^2\sin^2\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{9a^3n^2\sin\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{11a^3n\sin^4\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{42a^3n\sin^3\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{57a^3n\sin^2\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{26a^3n\sin\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{6a^3\sin^4\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{24a^3\sin^3\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{36a^3\sin^2\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} + \frac{24a^3\sin\left(c+dx\right)\sin^n\left(c+dx\right)}{dn^4+10dn^3+35dn^2+50dn+24d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**3*sin(c)**n*cos(c), Eq(d, 0)), (a**3*log(sin(c
 + d*x))/d - 3*a**3/(d*sin(c + d*x)) - 3*a**3/(2*d*sin(c + d*x)**2) - a**3/
(3*d*sin(c + d*x)**3), Eq(n, -4)), (3*a**3*log(sin(c + d*x))/d + a**3*sin(c
 + d*x)/d - 3*a**3/(d*sin(c + d*x)) - a**3/(2*d*sin(c + d*x)**2), Eq(n, -3)
), (3*a**3*log(sin(c + d*x))/d + a**3*sin(c + d*x)**2/(2*d) + 3*a**3*sin(c
+ d*x)/d - a**3/(d*sin(c + d*x)), Eq(n, -2)), (a**3*log(sin(c + d*x))/d + a
**3*sin(c + d*x)**3/(3*d) + 3*a**3*sin(c + d*x)**2/(2*d) + 3*a**3*sin(c + d
*x)/d, Eq(n, -1)), (a**3*n**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 3*a**3*n**3*sin(c
 + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d)
 + a**3*n**3*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 +
 50*d*n + 24*d) + 6*a**3*n**2*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*
d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 21*a**3*n**2*sin(c + d*x)**3*sin(c +
d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*n**2*sin
(c + d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*
d) + 9*a**3*n**2*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n*
*2 + 50*d*n + 24*d) + 11*a**3*n*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 1
0*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 42*a**3*n*sin(c + d*x)**3*sin(c + d
*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 57*a**3*n*sin(c +
```

```
 d*x)**2*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) +
 26*a**3*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 5
0*d*n + 24*d) + 6*a**3*sin(c + d*x)**4*sin(c + d*x)**n/(d*n**4 + 10*d*n**3
+ 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(c + d*x)**3*sin(c + d*x)**n/(d*n
**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 36*a**3*sin(c + d*x)**2*sin(
c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d) + 24*a**3*sin(
c + d*x)*sin(c + d*x)**n/(d*n**4 + 10*d*n**3 + 35*d*n**2 + 50*d*n + 24*d),
True))
```

Giac [A]
time = 0.46, size = 101, normalized size = 1.11

$$\frac{\frac{a^3 \sin(dx+c)^n \sin(dx+c)^4}{n+4} + \frac{3\,a^3 \sin(dx+c)^n \sin(dx+c)^3}{n+3} + \frac{3\,a^3 \sin(dx+c)^n \sin(dx+c)^2}{n+2} + \frac{a^3 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (a^3*sin(d*x + c)^n*sin(d*x + c)^4/(n + 4) + 3*a^3*sin(d*x + c)^n*sin(d*x +
 c)^3/(n + 3) + 3*a^3*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) + a^3*sin(d*x +
 c)^(n + 1)/(n + 1))/d
```

Mupad [B]
time = 10.42, size = 242, normalized size = 2.66

$$\frac{a^3 \sin(c+d\,x)^n\,(261\,n + 336\,\sin(c+d\,x) - 168\,\cos(2\,c+2\,d\,x) + 6\,\cos(4\,c+4\,d\,x) - 48\,\sin(3\,c+3\,d\,x) + 460\,n\,\sin(c+d\,x) - 272\,n\,\cos(2\,c+2\,d\,x) + 11\,n\,\cos(4\,c+4\,d\,x) - 84\,n\,\sin(3\,c+3\,d\,x) + 198\,n^2\,\sin(c+d\,x) + 26\,n^3\,\sin(c+d\,x) + 114\,n^2 + 15\,n^3 - 120\,n^2\,\cos(2\,c+2\,d\,x) - 16\,n^3\,\cos(2\,c+2\,d\,x) + 6\,n^2\,\cos(4\,c+4\,d\,x) + n^3\,\cos(4\,c+4\,d\,x) - 42\,n^2\,\sin(3\,c+3\,d\,x) - 6\,n^3\,\sin(3\,c+3\,d\,x) + 162)}{8\,d\,(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x)^n*(261*n + 336*sin(c + d*x) - 168*cos(2*c + 2*d*x) + 6*co
s(4*c + 4*d*x) - 48*sin(3*c + 3*d*x) + 460*n*sin(c + d*x) - 272*n*cos(2*c +
 2*d*x) + 11*n*cos(4*c + 4*d*x) - 84*n*sin(3*c + 3*d*x) + 198*n^2*sin(c + d
*x) + 26*n^3*sin(c + d*x) + 114*n^2 + 15*n^3 - 120*n^2*cos(2*c + 2*d*x) - 1
6*n^3*cos(2*c + 2*d*x) + 6*n^2*cos(4*c + 4*d*x) + n^3*cos(4*c + 4*d*x) - 42
*n^2*sin(3*c + 3*d*x) - 6*n^3*sin(3*c + 3*d*x) + 162))/(8*d*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24))
```

3.260 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=68

$$\frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}+\frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)+a^2*sin(d*x+c)^
(3+n)/d/(3+n)
```

Rubi [A]

time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 45}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^2\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n + 2a^2\left(\frac{x}{a}\right)^{1+n} + a^2\left(\frac{x}{a}\right)^{2+n}\right) dx, x, a\sin\right.}{ad}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}+\frac{a^2\sin^{3+n}(c+}{d(3+n)}$$

Mathematica [**A**]
time = 0.15, size = 50, normalized size = 0.74

$$\frac{a^2 \sin^{1+n}(c+dx) \left(\frac{1}{1+n}+\frac{2 \sin(c+dx)}{2+n}+\frac{\sin^2(c+dx)}{3+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n)))/d
```

Maple [**F**]
time = 0.27, size = 0, normalized size = 0.00

$$\int \cos(dx+c) \left(\sin^n(dx+c)\right) (a+a\sin(dx+c))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [**A**]
time = 0.28, size = 63, normalized size = 0.93

$$\frac{\frac{a^2 \sin(dx+c)^{n+3}}{n+3}+\frac{2\, a^2 \sin(dx+c)^{n+2}}{n+2}+\frac{a^2 \sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (a^2*sin(d*x + c)^(n + 3)/(n + 3) + 2*a^2*sin(d*x + c)^(n + 2)/(n + 2) + a^
2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [**A**]
time = 0.37, size = 135, normalized size = 1.99

$$\frac{\left(2\,a^2n^2+8\,a^2n-2\,(a^2n^2+4\,a^2n+3\,a^2)\cos(dx+c)^2+6\,a^2+\left(2\,a^2n^2+8\,a^2n-(a^2n^2+3\,a^2n+2\,a^2)\cos(dx+c)^2+8\,a^2\right)\sin(dx+c)\right)\sin(dx+c)^n}{dn^3+6\,dn^2+11\,dn+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*n^2 + 8*a^2*n - 2*(a^2*n^2 + 4*a^2*n + 3*a^2)*cos(d*x + c)^2 + 6*a^2
 + (2*a^2*n^2 + 8*a^2*n - (a^2*n^2 + 3*a^2*n + 2*a^2)*cos(d*x + c)^2 + 8*a^
2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^3 + 6*d*n^2 + 11*d*n + 6*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(56) = 112$.
time = 1.54, size = 530, normalized size = 7.79

$$\begin{cases} x(a\sin{(c)}+a)^2\sin^n{(c)}\cos{(c)} & \text{for } d=0 \\ \frac{a^2\log{(\sin{(c+dx)})}}{d} - \frac{2a^2}{d\sin{(c+dx)}} - \frac{a^2}{2d\sin^2{(c+dx)}} & \text{for } n=-3 \\ \frac{2a^2\log{(\sin{(c+dx)})}}{d} + \frac{a^2\sin{(c+dx)}}{d} - \frac{a^2}{d\sin{(c+dx)}} & \text{for } n=-2 \\ \frac{a^2\log{(\sin{(c+dx)})}}{d} + \frac{a^2\sin^2{(c+dx)}}{2d} + \frac{2a^2\sin{(c+dx)}}{d} & \text{for } n=-1 \\ \frac{a^2n^2\sin^3{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{2a^2n^2\sin^2{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{a^2n^2\sin{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{3a^2n\sin^3{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{8a^2n\sin^2{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{5a^2n\sin{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{2a^2\sin^3{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{6a^2\sin^2{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} + \frac{6a^2\sin{(c+dx)}\sin^n{(c+dx)}}{dn^3+6dn^2+11dn+6d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**2*sin(c)**n*cos(c), Eq(d, 0)), (a**2*log(sin(c
 + d*x))/d - 2*a**2/(d*sin(c + d*x)) - a**2/(2*d*sin(c + d*x)**2), Eq(n, -3
)), (2*a**2*log(sin(c + d*x))/d + a**2*sin(c + d*x)/d - a**2/(d*sin(c + d*x
)), Eq(n, -2)), (a**2*log(sin(c + d*x))/d + a**2*sin(c + d*x)**2/(2*d) + 2*
a**2*sin(c + d*x)/d, Eq(n, -1)), (a**2*n**2*sin(c + d*x)**3*sin(c + d*x)**n
/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*n**2*sin(c + d*x)**2*sin(c + d
*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + a**2*n**2*sin(c + d*x)*sin(c +
d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 3*a**2*n*sin(c + d*x)**3*sin(c
 + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 8*a**2*n*sin(c + d*x)**2*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 5*a**2*n*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 2*a**2*sin(c + d*x)**3*s
in(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)**2*
sin(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d) + 6*a**2*sin(c + d*x)*si
n(c + d*x)**n/(d*n**3 + 6*d*n**2 + 11*d*n + 6*d), True))
```

Giac [A]
time = 0.43, size = 75, normalized size = 1.10

$$\frac{\frac{a^2\sin(dx+c)^n\sin(dx+c)^3}{n+3} + \frac{2\,a^2\sin(dx+c)^n\sin(dx+c)^2}{n+2} + \frac{a^2\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*sin(d*x + c)^n*sin(d*x + c)^3/(n + 3) + 2*a^2*sin(d*x + c)^n*sin(d*x +
 c)^2/(n + 2) + a^2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Mupad [B]
time = 9.64, size = 147, normalized size = 2.16

$$\frac{a^2\sin\left(c+d\,x\right)^n\,\left(16\,n+30\,\sin\left(c+d\,x\right)-2\,\sin\left(3\,c+3\,d\,x\right)+29\,n\,\sin\left(c+d\,x\right)+16\,n\,\left(2\sin\left(c+d\,x\right)^2-1\right)-3\,n\,\sin\left(3\,c+3\,d\,x\right)+7\,n^2\,\sin\left(c+d\,x\right)+4\,n^2\,\left(2\sin\left(c+d\,x\right)^2-1\right)+4\,n^2+24\sin\left(c+d\,x\right)^2-n^2\,\sin\left(3\,c+3\,d\,x\right)\right)}{4\,d\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(16*n + 30*sin(c + d*x) - 2*sin(3*c + 3*d*x) + 29*n*sin
(c + d*x) + 16*n*(2*sin(c + d*x)^2 - 1) - 3*n*sin(3*c + 3*d*x) + 7*n^2*sin(
c + d*x) + 4*n^2*(2*sin(c + d*x)^2 - 1) + 4*n^2 + 24*sin(c + d*x)^2 - n^2*s
in(3*c + 3*d*x)))/(4*d*(11*n + 6*n^2 + n^3 + 6))
```

3.261 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2912, 45}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n}\right)\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}\end{aligned}$$

Mathematica [**A**]
time = 0.23, size = 38, normalized size = 0.93

$$\frac{a\sin^{1+n}(c+dx)(2+n+(1+n)\sin(c+dx))}{d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
[Out] (a*Sin[c + d*x]^(1 + n)*(2 + n + (1 + n)*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

Maple [**F**]
time = 0.17, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [**A**]
time = 0.28, size = 39, normalized size = 0.95

$$\frac{\frac{a\sin(dx+c)^{n+2}}{n+2}+\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] (a*sin(d*x + c)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [**A**]
time = 0.34, size = 62, normalized size = 1.51

$$-\frac{\left((an+a)\cos\left(dx+c\right)^{2}-an-(an+2\,a)\sin\left(dx+c\right)-a\right)\sin\left(dx+c\right)^{n}}{dn^{2}+3\,dn+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -((a*n + a)*cos(d*x + c)^2 - a*n - (a*n + 2*a)*sin(d*x + c) - a)*sin(d*x +
c)^n/(d*n^2 + 3*d*n + 2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(32) = 64$.
time = 0.72, size = 190, normalized size = 4.63

$$\begin{cases} x(a\sin(c)+a)\sin^n(c)\cos(c) & \text{for } d=0 \\ \frac{a\log(\sin(c+dx))}{d} - \frac{a}{d\sin(c+dx)} & \text{for } n=-2 \\ \frac{a\log(\sin(c+dx))}{d} + \frac{a\sin(c+dx)}{d} & \text{for } n=-1 \\ \frac{an\sin^2(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d} + \frac{an\sin(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d} + \frac{a\sin^2(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d} + \frac{2a\sin(c+dx)\sin^n(c+dx)}{dn^2+3dn+2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c), Eq(d, 0)), (a*log(sin(c + d*x
))/d - a/(d*sin(c + d*x)), Eq(n, -2)), (a*log(sin(c + d*x))/d + a*sin(c + d
*x)/d, Eq(n, -1)), (a*n*sin(c + d*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2
*d) + a*n*sin(c + d*x)*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + a*sin(c + d
*x)**2*sin(c + d*x)**n/(d*n**2 + 3*d*n + 2*d) + 2*a*sin(c + d*x)*sin(c + d*
x)**n/(d*n**2 + 3*d*n + 2*d), True))
```

Giac [A]
time = 0.46, size = 45, normalized size = 1.10

$$\frac{\frac{a\sin(dx+c)^n\sin(dx+c)^2}{n+2} + \frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*sin(d*x + c)^n*sin(d*x + c)^2/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/
d
```

Mupad [B]
time = 9.18, size = 67, normalized size = 1.63

$$\frac{a\sin(c+d\,x)^n\,\left(n+4\,\sin(c+d\,x)+2\,n\,\sin(c+d\,x)+n\,\left(2\sin(c+d\,x)^2-1\right)+2\sin(c+d\,x)^2\right)}{2\,d\,\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(n + 4*sin(c + d*x) + 2*n*sin(c + d*x) + n*(2*sin(c + d*x
)^2 - 1) + 2*sin(c + d*x)^2))/(2*d*(3*n + n^2 + 2))
```

3.262 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{ad(1+n)}$$

```
[Out] hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a/d/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 66}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{ad(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{ad(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
*d*(1 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)(\sin^n(dx+c))}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^n}{a+a\,\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x)), x)
```

3.263 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=38

$$\frac{{}_2F_1(2,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^2d(1+n)}$$

```
[Out] hypergeom([2, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^2/d/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 66}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(2,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^2d(1+n)}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{{}_2F_1(2,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^2d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^2*d*(1 + n))
```

Maple [**F**]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x +
 c) - 2*a^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2, x)
```

3.264 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=38

$$\frac{{}_2F_1(3,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}$$

```
[Out] hypergeom([3, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^3/d/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 66}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(3,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{{}_2F_1(3,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^3*d*(1 + n))
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)(\sin^n(dx+c))}{(a+a\sin(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^3*
cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3, x)
```

3.265 $\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=38

$$\frac{{}_2F_1(4,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}$$

```
[Out] hypergeom([4, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^4/d/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2912, 66}

$$\frac{\sin^{n+1}(c+dx)\,{}_2F_1(4,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{{}_2F_1(4,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.00

$$\frac{{}_2F_1(4,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
[Out] (Hypergeometric2F1[4, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a
^4*d*(1 + n))
```

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
[Out] int(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
[Out] integral(sin(d*x + c)^n*cos(d*x + c)/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x +
c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
```

```
[Out] int((cos(c + d*x)*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

3.266 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{ax}{16}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)}{6d}$$

```
[Out] 1/16*a*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d-1/8*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2645, 14, 2648, 2715, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin^3(c+dx)\,dx + a\int \cos^2(c+dx)\sin^4$$
$$= -\frac{a\cos^3(c+dx)\sin^3(c+dx)}{6d} + \frac{1}{2}a\int \cos^2(c+dx)\sin^2($$
$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin^3(c+d$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c}{16d}$$
$$= \frac{ax}{16} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin}{16d}$$

Mathematica [A]
time = 0.16, size = 71, normalized size = 0.68

$$\frac{a(60dx - 120\cos(c+dx) - 20\cos(3(c+dx)) + 12\cos(5(c+dx)) - 15\sin(2(c+dx)) - 15\sin(4(c+dx)) + 5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*d*x - 120*Cos[c + d*x] - 20*Cos[3*(c + d*x)] + 12*Cos[5*(c + d*x)] -
 15*Sin[2*(c + d*x)] - 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.14, size = 95, normalized size = 0.90

method	result
risch	$\frac{ax}{16}-\frac{a\cos(dx+c)}{8d}+\frac{a\sin(6dx+6c)}{192d}+\frac{a\cos(5dx+5c)}{80d}-\frac{a\sin(4dx+4c)}{64d}-\frac{a\cos(3dx+3c)}{48d}-\frac{a\sin(2dx+2c)}{64d}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\sin(dx+c)\cos}{16}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\sin(dx+c)\cos}{16}\right.}{d}$
norman	$\frac{ax}{16}-\frac{4a}{15d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{17a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{19a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{19a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{17a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a\left(\tan^{11}\left(\frac{dx}{2}\right.\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a*(-1/6*sin(d*x+c
)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*sin(d*x+c)*cos(d*x+c)+1/1
6*d*x+1/16*c))
```

Maxima [A]

time = 0.28, size = 65, normalized size = 0.62

$$\frac{64\left(3\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^3\right)a-5\left(4\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\sin\left(4\,dx+4\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]

time = 0.36, size = 73, normalized size = 0.70

$$\frac{48\,a\cos\left(dx+c\right)^5-80\,a\cos\left(dx+c\right)^3+15\,adx+5\left(8\,a\cos\left(dx+c\right)^5-14\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*a*d*x + 5*(8*a*cos(d*
x + c)^5 - 14*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(92) = 184$.

time = 0.45, size = 192, normalized size = 1.83

$$\begin{cases}\frac{ax\sin^6\left(c+dx\right)}{16}+\frac{3ax\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)}{16}+\frac{3ax\sin^2\left(c+dx\right)\cos^4\left(c+dx\right)}{16}+\frac{ax\cos^6\left(c+dx\right)}{16}+\frac{a\sin^5\left(c+dx\right)\cos\left(c+dx\right)}{16d}-\frac{a\sin^3\left(c+dx\right)\cos^3\left(c+dx\right)}{6d}-\frac{a\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{a\sin\left(c+dx\right)\cos^5\left(c+dx\right)}{16d}-\frac{2a\cos^5\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin^3\left(c\right)\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) - a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*3*cos(c)**2, True))
```

Giac [A]
time = 0.48, size = 92, normalized size = 0.88

$$\frac{1}{16}\,ax+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d - 1/6
4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.13, size = 226, normalized size = 2.15

$$\frac{a\,x}{16}+\frac{\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{17\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,\left(225\,c+225\,d\,x-960\right)}{240}-\frac{15\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\left(\frac{a\,\left(300\,c+300\,d\,x-640\right)}{240}-\frac{5\,a\,\left(c+d\,x\right)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}-\frac{17\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\left(\frac{a\,\left(90\,c+90\,d\,x-384\right)}{240}-\frac{3\,a\,\left(c+d\,x\right)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{a\,\left(15\,c+15\,d\,x-64\right)}{240}-\frac{a\,\left(c+d\,x\right)}{16}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 + ((a*(15*c + 15*d*x - 64))/240 - (a*tan(c/2 + (d*x)/2))/8 - (a*(c
 + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(90*c + 90*d*x - 384))/240 - (3*a*(c
 + d*x))/8) + tan(c/2 + (d*x)/2)^6*((a*(300*c + 300*d*x - 640))/240 - (5*a*
(c + d*x))/4) + tan(c/2 + (d*x)/2)^8*((a*(225*c + 225*d*x - 960))/240 - (15
*a*(c + d*x))/16) - (17*a*tan(c/2 + (d*x)/2)^3)/24 + (19*a*tan(c/2 + (d*x)/
2)^5)/4 - (19*a*tan(c/2 + (d*x)/2)^7)/4 + (17*a*tan(c/2 + (d*x)/2)^9)/24 +
(a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.267 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{ax}{8}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*a*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/8*a*cos(d*x+c)*sin(d*x+
c)/d-1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 14}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a\int \cos^2(c+dx)\sin^3\\ &= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \\ &= \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \\ &= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin}{8d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.67

$$\frac{a(60c+60dx-60\cos(c+dx)-10\cos(3(c+dx))+6\cos(5(c+dx))-15\sin(4(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(60*c + 60*d*x - 60*Cos[c + d*x] - 10*Cos[3*(c + d*x)] + 6*Cos[5*(c + d*
x)] - 15*Sin[4*(c + d*x)]))/(480*d)
```

Maple [A]
time = 0.10, size = 77, normalized size = 0.95

method	result
risch	$\frac{ax}{8}-\frac{a\cos(dx+c)}{8d}+\frac{a\cos(5dx+5c)}{80d}-\frac{a\sin(4dx+4c)}{32d}-\frac{a\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{a\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)}{d}$
norman	$\frac{ax}{8}-\frac{4a}{15d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{5ax\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{4}$ ... $\left(1+\tan\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*
c)+a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

Maxima [A]
time = 0.27, size = 52, normalized size = 0.64

$$\frac{32\left(3\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^3\right)a+15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a + 15*(4*d*x + 4*c - sin(4
*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.77

$$\frac{24\,a\cos\left(dx+c\right)^5-40\,a\cos\left(dx+c\right)^3+15\,adx-15\left(2\,a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d
*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 0.27, size = 144, normalized size = 1.78

$$\begin{cases}\frac{ax\sin^4\left(c+dx\right)}{8}+\frac{ax\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{ax\cos^4\left(c+dx\right)}{8}+\frac{a\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{a\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{2a\cos^5\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin^2\left(c\right)\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)**2*cos(c + d*x)**3/(3*d) - a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**2, Tru
e))
```

Giac [A]
time = 0.48, size = 62, normalized size = 0.77

$$\frac{1}{8}\,ax+\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*cos
(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 11.97, size = 198, normalized size = 2.44

$$\frac{a\,x}{8}+\frac{\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\left(\frac{a\,\left(150\,c+150\,d\,x-480\right)}{120}-\frac{5\,a\,\left(c+d\,x\right)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{a\,\left(150\,c+150\,d\,x+160\right)}{120}-\frac{5\,a\,\left(c+d\,x\right)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\left(\frac{a\,\left(75\,c+75\,d\,x-160\right)}{120}-\frac{5\,a\,\left(c+d\,x\right)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a\,\left(15\,c+15\,d\,x-32\right)}{120}-\frac{a\,\left(c+d\,x\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 + ((a*(15*c + 15*d*x - 32))/120 - (a*tan(c/2 + (d*x)/2))/4 - (a*(c
+ d*x))/8 + tan(c/2 + (d*x)/2)^2*((a*(75*c + 75*d*x - 160))/120 - (5*a*(c +
 d*x))/8) + tan(c/2 + (d*x)/2)^4*((a*(150*c + 150*d*x + 160))/120 - (5*a*(c
 + d*x))/4) + tan(c/2 + (d*x)/2)^6*((a*(150*c + 150*d*x - 480))/120 - (5*a*
(c + d*x))/4) + (3*a*tan(c/2 + (d*x)/2)^3)/2 - (3*a*tan(c/2 + (d*x)/2)^7)/2
 + (a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.268 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{ax}{8}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*a*x-1/3*a*cos(d*x+c)^3/d+1/8*a*cos(d*x+c)*sin(d*x+c)/d-1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {2917, 2645, 30, 2648, 2715, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\sin(c+dx)\,dx + a\int \cos^2(c+dx)\sin^2(c$$
$$= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}a\int \cos^2(c+dx)\,dx - \frac{a}{}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3(c+}{}$$
$$= \frac{ax}{8} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos^3}{}$$

Mathematica [A]
time = 0.08, size = 42, normalized size = 0.65

$$-\frac{a(24\cos(c+dx)+8\cos(3(c+dx))+3(-4dx+\sin(4(c+dx))))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] -1/96*(a*(24*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(-4*d*x + Sin[4*(c + d*x
)])))/d
```

Maple [A]
time = 0.09, size = 57, normalized size = 0.88

method	result

risch	$\frac{ax}{8}-\frac{a\cos(dx+c)}{4d}-\frac{a\sin(4dx+4c)}{32d}-\frac{a\cos(3dx+3c)}{12d}$
derivativedivides	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+a\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+a\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
norman	$\frac{\frac{ax}{8}-\frac{2a}{3d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{7a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{7a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*cos(d*x+c)^3+a*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos
(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.60

$$-\frac{32\,a\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.35, size = 51, normalized size = 0.78

$$-\frac{8\,a\cos\left(dx+c\right)^3-3\,adx+3\left(2\,a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*a*d*x + 3*(2*a*cos(d*x + c)^3 - a*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(56) = 112$.
time = 0.20, size = 119, normalized size = 1.83

$$\begin{cases} \frac{ax\sin^4\left(c+dx\right)}{8}+\frac{ax\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{ax\cos^4\left(c+dx\right)}{8}+\frac{a\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{a\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{a\cos^3\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^2\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*sin(c)
 + a)*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.45, size = 47, normalized size = 0.72

$$\frac{1}{8}\,ax - \frac{a\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{a\cos\left(dx + c\right)}{4\,d} - \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*a*sin(4*d
*x + 4*c)/d
```

Mupad [B]
time = 12.33, size = 198, normalized size = 3.05

$$\frac{a\,x}{8}+\frac{\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\left(\frac{a\,\left(12\,c+12\,d\,x-48\right)}{24}-\frac{a\,\left(c+d\,x\right)}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\left(\frac{a\,\left(18\,c+18\,d\,x-48\right)}{24}-\frac{3\,a\,\left(c+d\,x\right)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\left(\frac{a\,\left(12\,c+12\,d\,x-16\right)}{24}-\frac{a\,\left(c+d\,x\right)}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{a\,\left(3\,c+3\,d\,x-16\right)}{24}-\frac{a\,\left(c+d\,x\right)}{8}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 + ((a*(3*c + 3*d*x - 16))/24 - (a*tan(c/2 + (d*x)/2))/4 - (a*(c + d
*x))/8 + tan(c/2 + (d*x)/2)^2*((a*(12*c + 12*d*x - 16))/24 - (a*(c + d*x))/
2) + tan(c/2 + (d*x)/2)^6*((a*(12*c + 12*d*x - 48))/24 - (a*(c + d*x))/2) +
 tan(c/2 + (d*x)/2)^4*((a*(18*c + 18*d*x - 48))/24 - (3*a*(c + d*x))/4) + (
7*a*tan(c/2 + (d*x)/2)^3)/4 - (7*a*tan(c/2 + (d*x)/2)^5)/4 + (a*tan(c/2 + (
d*x)/2)^7)/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.269 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{ax}{2}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2917, 2672, 327, 212, 2715, 8}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\,dx + a\int \cos(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx - \frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,d \right.}{}$$

$$= \frac{ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\text{Subst}}{}$$

$$= \frac{ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos(c+}{}$$

Mathematica [A]
time = 0.06, size = 74, normalized size = 1.45

$$\frac{a(c+dx)}{2d} + \frac{a\cos(c+dx)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.13, size = 55, normalized size = 1.08

method	result
derivativedivides	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{ax}{2}+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{ax}{2}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a*(1/2*sin(d*x+c)*cos(d*x+c)+
1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 57, normalized size = 1.12

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a+2\,a(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a + 2*a*(2*cos(d*x + c) - log(cos(d*x
 + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 60, normalized size = 1.18

$$\frac{adx+a\cos(dx+c)\sin(dx+c)+2\,a\cos(dx+c)-a\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+a\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + a*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos^2(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**2*csc(c + d*x), x))
```

Giac [A]
time = 0.46, size = 87, normalized size = 1.71

$$\frac{(dx+c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(a*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 8.72, size = 160, normalized size = 3.14

$$\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{a\,\mathrm{atan}\left(\frac{a^2}{2\,a^2-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (2*a + a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 - a*tan(c/2 + (d*x)/
2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (a*log(tan(
c/2 + (d*x)/2)))/d + (a*atan(a^2/(2*a^2 - a^2*tan(c/2 + (d*x)/2)) + (2*a^2*
tan(c/2 + (d*x)/2))/(2*a^2 - a^2*tan(c/2 + (d*x)/2))))/d
```

3.270 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-ax-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {2789, 2672, 327, 212, 3554, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot(c+dx)\,dx + a\int \cot^2(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d} - a\int 1\,dx - \frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+\right.}{d} \\
&= -ax - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 75, normalized size = 1.83

$$\frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

Maple [A]
time = 0.11, size = 49, normalized size = 1.20

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)+a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
default	$\frac{a(-\cot(dx+c)-dx-c)+a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
risch	$-ax+\frac{a\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{2ia}{d(\mathrm{e}^{2i(dx+c)}-1)}-\frac{a\ln(\mathrm{e}^{i(dx+c)}+1)}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}-1)}{d}$	91
norman	$\frac{-\frac{a}{2d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)+a*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.48, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.
time = 0.38, size = 84, normalized size = 2.05

$$-\frac{a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2\,a\cos(dx+c)+2\,(adx-a\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2(c+dx)\csc^2(c+dx)\,dx+\int\sin(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos
(c + d*x)**2*csc(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.
time = 0.45, size = 108, normalized size = 2.63

$$-\frac{6\,(dx+c)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

Mupad [B]
time = 8.80, size = 108, normalized size = 2.63

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)+a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{2}}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (2*a*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 -
pi/4 + (d*x)/2))) + a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*co
s(c + d*x) - (a*sin(2*c + 2*d*x))/2)/(d*sin(c + d*x))
```

3.271 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-ax+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -a*x+1/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d-1/2*a*cot(d*x+c)*csc(d*x+c)
/d
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2917, 2691, 3855, 3554, 8}

$$-\frac{a\cot(c+dx)}{d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc($$
$$= -ax + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot(c$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 109, normalized size = 2.10

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\cot(c+dx)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.12, size = 71, normalized size = 1.37

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a(-\cot(dx+c)-dx-c)}{d}$
risch	$-ax+\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}-2ie^{2i(dx+c)}+2i\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}$

norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot
(d*x+c)))+a*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 66, normalized size = 1.27

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 -
1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(48) = 96.
time = 0.36, size = 114, normalized size = 2.19

$$-\frac{4\,adx\cos\left(dx+c\right)^2-4\,adx-4\,a\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*d*x*cos(d*x + c)^2 - 4*a*d*x - 4*a*cos(d*x + c)*sin(d*x + c) - 2*
a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*co
s(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x)**3, x) + Integral(sin(c + d*x)*cos
(c + d*x)**2*csc(c + d*x)**3, x))
```

Giac [A]
time = 0.45, size = 95, normalized size = 1.83

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,(dx+c)a-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*a - 4*a*log(abs(tan(1/2*d*x + 1
/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(
1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 8.68, size = 145, normalized size = 2.79

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a*cot(c/2 + (d*x)/2))/(2*d) - (2*a*atan((2*
cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) - 2*sin(c/2 +
(d*x)/2))))/d - (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) - (a*c
ot(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d)
```

3.272 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/2*a*cot(d*x+c)*csc(d*x+c
)/d
```

Rubi [A]
time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2917, 2687, 30, 2691, 3855}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + a\int \cot^2(c+dx)\csc^2$$
$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{aS}{}$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d - (a*Csc[(c + d*x)/2]^2)/(8*d) + (a*Log[Cos[(c +
d*x)/2]])/(2*d) - (a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d)
```

Maple [A]
time = 0.14, size = 72, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$\frac{a\left(6i e^{4i(dx+c)}+3\,e^{5i(dx+c)}+2i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}$

norman	$\frac{-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} - \frac{a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a/sin(d*x+c)^3*cos(d*x+c)^3+a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2
*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 1.17

$$\frac{3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{4\,a}{\tan(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.
time = 0.37, size = 119, normalized size = 2.29

$$\frac{4\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)*sin(d*x + c) + 3*(a*cos(d*x + c
)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*cos(d*x + c)^2 - a
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^2\left(c+dx\right)\csc^4\left(c+dx\right)\,dx+\int\sin\left(c+dx\right)\cos^2\left(c+dx\right)\csc^4\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

[Out] a*(Integral(cos(c + d*x)**2*csc(c + d*x)**4, x) + Integral(sin(c + d*x)*cos(c + d*x)**2*csc(c + d*x)**4, x))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(46) = 92.
time = 0.46, size = 115, normalized size = 2.21

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d

Mupad [B]
time = 8.58, size = 111, normalized size = 2.13

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)

[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*log(tan(c/2 + (d*x)/2)))/(2*d) - (cot(c/2 + (d*x)/2)^3*(a/3 + a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(8*d)

3.273 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d+1/8*a*cot(d*x+c)*csc(d*x+c
)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 30}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + a\int \cot^2(c+dx)\csc$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx +$$
$$= -\frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c}{}$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)}{8d}$$

Mathematica [A]
time = 0.04, size = 135, normalized size = 1.82

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.17, size = 90, normalized size = 1.22

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$
default	$\frac{a\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$
risch	$-\frac{a(3\,e^{7i(dx+c)}+21\,e^{5i(dx+c)}-24ie^{6i(dx+c)}+21\,e^{3i(dx+c)}+24ie^{4i(dx+c)}+3\,e^{i(dx+c)}-8ie^{2i(dx+c)}+8i)}{12d(e^{2i(dx+c)}-1)^4}-\frac{a\ln(e^{i(dx+c)}}{8d}$
norman	$\frac{-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a(\mathrm{ta}}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*co
s(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*a/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.08

$$-\frac{3\,a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*a/tan(d*x
+ c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(66) = 132.
time = 0.38, size = 137, normalized size = 1.85

$$-\frac{16\,a\cos\left(dx+c\right)^3\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.49, size = 116, normalized size = 1.57

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*a*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*a*tan(1/2*d*x + 1/2*c)^3 - 8*a*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.59, size = 112, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a}{4}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2
 + (d*x)/2)^4)/(64*d) - (a*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x
)/2)^4*(a/4 + (2*a*tan(c/2 + (d*x)/2))/3 - 2*a*tan(c/2 + (d*x)/2)^3))/(16*d
)
```

3.274 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+}{4d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)^5/d+1/8*a
*cot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2687, 14, 2691, 3853, 3855}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + a\int \cot^2(c+dx)\csc$$

$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + $$

$$= \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx}{5d}$$

Mathematica [A]
time = 0.06, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} + \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
sc[c + d*x]^4)/(5*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*
x)/2]])/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*
d)
```

Maple [A]
time = 0.15, size = 110, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+a\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
default	$\frac{a\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+a\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
risch	$-\frac{a\left(15\,e^{9i(dx+c)}+240ie^{6i(dx+c)}+90\,e^{7i(dx+c)}+80ie^{4i(dx+c)}+80ie^{2i(dx+c)}-90\,e^{3i(dx+c)}-16i-15\,e^{i(dx+c)}\right)}{60d\left(e^{2i(dx+c)}-1\right)^{5}}-\frac{a\ln(e^{i}\cdots}{\cdots}$
norman	$\frac{-\frac{a}{160d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60d}-\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{5a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5a\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\cdots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+a*(-
1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d*x+c)-
1/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.02

$$-\frac{15\,a\left(\frac{2\left(\cos(dx+c)^{3}+\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\left(5\,\tan(dx+c)^{2}+3\right)a}{\tan(dx+c)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(80) = 160$.
time = 0.38, size = 169, normalized size = 1.88

$$\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-30\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos
(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.45, size = 144, normalized size = 1.60

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{274\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*a*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

Mupad [B]
time = 8.65, size = 143, normalized size = 1.59

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/
2 + (d*x)/2)^4)/(64*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) - (a*log(tan(c/2
+ (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + (a*tan(c/2 + (d*x)/2))/2
+ (a*tan(c/2 + (d*x)/2)^2)/3 - 2*a*tan(c/2 + (d*x)/2)^4))/(32*d)
```

3.275 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=135

$$\frac{a^2x}{8}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^2\cos^3(c+dx)\sin(}{4d}$$

```
[Out] 1/8*a^2*x-2/3*a^2*cos(d*x+c)^3/d+3/5*a^2*cos(d*x+c)^5/d-1/7*a^2*cos(d*x+c)^
7/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a
^2*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/8 - (2*a^2*Cos[c + d*x]^3)/(3*d) + (3*a^2*Cos[c + d*x]^5)/(5*d) - (
a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(4*d) - (a^2*Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cos^2(c+dx)\sin^3(c+dx)+2a^2\cos^2(c+dx)\sin^4\right.\\
&= a^2\int \cos^2(c+dx)\sin^3(c+dx)\,dx+a^2\int \cos^2(c+dx)\,\mathrm{s}\\
&= -\frac{a^2\cos^3(c+dx)\sin^3(c+dx)}{3d}+a^2\int \cos^2(c+dx)\sin^2\\
&= -\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a^2\cos^3(c+dx)\sin^3(c+}{3d}\\
&= -\frac{2a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^7(c+dx}{7d}\\
&= \frac{a^2x}{8}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^7(c}{7d}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 86, normalized size = 0.64

$$\frac{a^2(840c+840dx-1365\cos(c+dx)-175\cos(3(c+dx))+147\cos(5(c+dx))-15\cos(7(c+dx))-210\sin(2(c+dx))-210\sin(4(c+dx))+70\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Cos[
5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c +
d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A]

time = 0.21, size = 151, normalized size = 1.12

method	result
risch	$\frac{a^2x}{8}-\frac{13a^2\cos(dx+c)}{64d}-\frac{a^2\cos(7dx+7c)}{448d}+\frac{a^2\sin(6dx+6c)}{96d}+\frac{7a^2\cos(5dx+5c)}{320d}-\frac{a^2\sin(4dx+4c)}{32d}-\frac{5a^2\cos(3}{192}$
derivativedivides	$\frac{a^2\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+2a^2\left(-\frac{(\sin^3(dx+c))(\cos^3(dx+c))}{6}-\frac{(\cos^3(dx+c))\sin(dx+c)}{8}+\frac{\sin(dx+c)}{1}\right.}{d}$
default	$\frac{a^2\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos^3(dx+c))}{15}\right)+2a^2\left(-\frac{(\sin^3(dx+c))(\cos^3(dx+c))}{6}-\frac{(\cos^3(dx+c))\sin(dx+c)}{8}+\frac{\sin(dx+c)}{1}\right.}{d}$
norman	$\frac{\frac{a^2x}{8}-\frac{44a^2}{105d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{97a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{97a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{5a^2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{a^2(t}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a^2*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*sin(d*x+c)*cos(d*x+
c)+1/16*d*x+1/16*c)+a^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*c
os(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A]

time = 0.29, size = 105, normalized size = 0.78

$$-\frac{32\left(15\cos\left(dx+c\right)^{7}-42\cos\left(dx+c\right)^{5}+35\cos\left(dx+c\right)^{3}\right)a^{2}-224\left(3\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)a^{2}+35\left(4\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\sin\left(4\,dx+4\,c\right)\right)a^{2}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3360*(32*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^2
 - 224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 + 35*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]

time = 0.35, size = 98, normalized size = 0.73

$$-\frac{120\,a^{2}\cos\left(dx+c\right)^{7}-504\,a^{2}\cos\left(dx+c\right)^{5}+560\,a^{2}\cos\left(dx+c\right)^{3}-105\,a^{2}dx-35\left(8\,a^{2}\cos\left(dx+c\right)^{5}-14\,a^{2}\cos\left(dx+c\right)^{3}+3\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 504*a^2*cos(d*x + c)^5 + 560*a^2*cos(d*x +
 c)^3 - 105*a^2*d*x - 35*(8*a^2*cos(d*x + c)^5 - 14*a^2*cos(d*x + c)^3 + 3*
a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(121) = 242$.
time = 0.67, size = 275, normalized size = 2.04

$$\begin{cases} \frac{a^2x\sin^6(c+dx)}{8} + \frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{a^2x\cos^6(c+dx)}{8} + \frac{a^2\sin^5(c+dx)\cos(c+dx)}{8d} - \frac{a^2\sin^4(c+dx)\cos^3(c+dx)}{3d} - \frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{4a^2\sin^2(c+dx)\cos^5(c+dx)}{15d} - \frac{a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d} - \frac{8a^2\cos^7(c+dx)}{105d} - \frac{2a^2\cos^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2\sin^3(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a**2*sin(c + d*x)**4*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 4*a**2*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*
d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 8*a**2*cos(c + d*x)**7/(105*
d) - 2*a**2*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**2, True))
```

Giac [A]
time = 0.52, size = 123, normalized size = 0.91

$$\frac{1}{8}a^2x - \frac{a^2\cos(7\,dx+7\,c)}{448\,d} + \frac{7\,a^2\cos(5\,dx+5\,c)}{320\,d} - \frac{5\,a^2\cos(3\,dx+3\,c)}{192\,d} - \frac{13\,a^2\cos(dx+c)}{64\,d} + \frac{a^2\sin(6\,dx+6\,c)}{96\,d} - \frac{a^2\sin(4\,dx+4\,c)}{32\,d} - \frac{a^2\sin(2\,dx+2\,c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x - 1/448*a^2*cos(7*d*x + 7*c)/d + 7/320*a^2*cos(5*d*x + 5*c)/d - 5
/192*a^2*cos(3*d*x + 3*c)/d - 13/64*a^2*cos(d*x + c)/d + 1/96*a^2*sin(6*d*x
 + 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d - 1/32*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.31, size = 331, normalized size = 2.45

$$\frac{a^2\,x}{8} - \frac{\frac{a^2\,(c+d\,x)}{8} + \frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} - \frac{97\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{97\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{12} - \frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3} - \frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4} - \frac{a^2\,(105\,c+105\,d\,x-352)}{840} + {\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{7\,a^2\,(c+d\,x)}{8} - \frac{a^2\,(735\,c+735\,d\,x-2464)}{840}\right) + {\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{21\,a^2\,(c+d\,x)}{8} - \frac{a^2\,(2205\,c+2205\,d\,x-3360)}{840}\right) + {\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{21\,a^2\,(c+d\,x)}{8} - \frac{a^2\,(2205\,c+2205\,d\,x-4032)}{840}\right) + {\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{35\,a^2\,(c+d\,x)}{8} - \frac{a^2\,(3675\,c+3675\,d\,x+2240)}{840}\right) + {\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{35\,a^2\,(c+d\,x)}{8} - \frac{a^2\,(3675\,c+3675\,d\,x-14560)}{840}\right) + \frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 - ((a^2*(c + d*x))/8 + (5*a^2*tan(c/2 + (d*x)/2)^3)/3 - (97*a^2*t
an(c/2 + (d*x)/2)^5)/12 + (97*a^2*tan(c/2 + (d*x)/2)^9)/12 - (5*a^2*tan(c/2
 + (d*x)/2)^11)/3 - (a^2*tan(c/2 + (d*x)/2)^13)/4 - (a^2*(105*c + 105*d*x -
 352))/840 + tan(c/2 + (d*x)/2)^2*((7*a^2*(c + d*x))/8 - (a^2*(735*c + 735*
d*x - 2464))/840) + tan(c/2 + (d*x)/2)^10*((21*a^2*(c + d*x))/8 - (a^2*(220
5*c + 2205*d*x - 3360))/840) + tan(c/2 + (d*x)/2)^4*((21*a^2*(c + d*x))/8 -
 (a^2*(2205*c + 2205*d*x - 4032))/840) + tan(c/2 + (d*x)/2)^6*((35*a^2*(c +
 d*x))/8 - (a^2*(3675*c + 3675*d*x + 2240))/840) + tan(c/2 + (d*x)/2)^8*((3
5*a^2*(c + d*x))/8 - (a^2*(3675*c + 3675*d*x - 14560))/840) + (a^2*tan(c/2
+ (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.276 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{3a^2x}{16}-\frac{a^2\cos^5(c+dx)}{10d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{\cos^3(c+dx)(a+a\sin(c+$$

```
[Out] 3/16*a^2*x-1/10*a^2*cos(d*x+c)^5/d+3/16*a^2*cos(d*x+c)*sin(d*x+c)/d+1/8*a^2
*cos(d*x+c)^3*sin(d*x+c)/d-1/6*cos(d*x+c)^3*(a+a*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2949, 2748, 2715, 8}

$$-\frac{a^2\cos^5(c+dx)}{10d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3a^2x}{16}-\frac{\cos^3(c+dx)(a\sin(c+dx)+a)^3}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/16 - (a^2*Cos[c + d*x]^5)/(10*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*
x])/(16*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (Cos[c + d*x]^3*(a +
 a*Sin[c + d*x])^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
```

```
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad} + \frac{1}{2}a\int \cos^4(c+d$$
$$= -\frac{a^2\cos^5(c+dx)}{10d} - \frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6ad}$$
$$= -\frac{a^2\cos^5(c+dx)}{10d} + \frac{a^2\cos^3(c+dx)\sin(c+dx)}{8d} - \cos$$
$$= -\frac{a^2\cos^5(c+dx)}{10d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^2}{}$$
$$= \frac{3a^2x}{16} - \frac{a^2\cos^5(c+dx)}{10d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [A]
time = 0.32, size = 76, normalized size = 0.74

$$\frac{a^2(180c+180dx-240\cos(c+dx)-40\cos(3(c+dx))+24\cos(5(c+dx))-15\sin(2(c+dx))-45\sin(4(c+dx))+5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5*(
c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])
)/(960*d)
```

Maple [A]
time = 0.16, size = 142, normalized size = 1.38

method	result
risch	$\frac{3a^2x}{16} - \frac{a^2\cos(dx+c)}{4d} + \frac{a^2\sin(6dx+6c)}{192d} + \frac{a^2\cos(5dx+5c)}{40d} - \frac{3a^2\sin(4dx+4c)}{64d} - \frac{a^2\cos(3dx+3c)}{24d} - \frac{a^2\sin(2dx}{64d}$
derivativedivides	$\frac{a^2\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + 2a^2\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5} - \frac{2(\cos^3(dx+c))}{15}\right) + a^2\left(-\right.}{d}$
default	$\frac{a^2\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + 2a^2\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5} - \frac{2(\cos^3(dx+c))}{15}\right) + a^2\left(-\right.}{d}$
norman	$\frac{3a^2x}{16} - \frac{8a^2}{15d} - \frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d} + \frac{13a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d} + \frac{25a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} - \frac{25a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} - \frac{13a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d} + \frac{3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)+2*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+a^2*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*sin(d*x+c)*cos(d*x+
c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.31, size = 93, normalized size = 0.90

$$\frac{128\,\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)a^{2}-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a^{2}+30\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^{2}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 5*(4*sin(2*d*x + 2*c
)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2 + 30*(4*d*x + 4*c - sin(4*d*x
 + 4*c))*a^2)/d
```

Fricas [A]
time = 0.35, size = 85, normalized size = 0.83

$$\frac{96\,a^{2}\cos\left(dx+c\right)^{5}-160\,a^{2}\cos\left(dx+c\right)^{3}+45\,a^{2}dx+5\,\left(8\,a^{2}\cos\left(dx+c\right)^{5}-26\,a^{2}\cos\left(dx+c\right)^{3}+9\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*a^2*cos(d*x + c)^5 - 160*a^2*cos(d*x + c)^3 + 45*a^2*d*x + 5*(8*a
^2*cos(d*x + c)^5 - 26*a^2*cos(d*x + c)^3 + 9*a^2*cos(d*x + c))*sin(d*x + c
))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(92) = 184.
time = 0.83, size = 309, normalized size = 3.00

$$\begin{cases} \frac{a^{2}x\sin^{6}\left(c+dx\right)}{16}+\frac{3a^{2}x\sin^{4}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{16}+\frac{a^{2}x\sin^{4}\left(c+dx\right)}{8}+\frac{3a^{2}x\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{16}+\frac{a^{2}x\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{a^{2}x\cos^{6}\left(c+dx\right)}{16}+\frac{a^{2}x\cos^{4}\left(c+dx\right)}{8}+\frac{a^{2}\sin^{5}\left(c+dx\right)\cos\left(c+dx\right)}{16d}-\frac{a^{2}\sin^{3}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{6d}+\frac{a^{2}\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{2a^{2}\sin^{2}\left(c+dx\right)\cos^{3}\left(c+dx\right)}{3d}-\frac{a^{2}\sin\left(c+dx\right)\cos^{5}\left(c+dx\right)}{16d}-\frac{a^{2}\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{4a^{2}\cos^{5}\left(c+dx\right)}{15d} & \text{for}\: d \neq 0 \\ x\left(a\sin\left(c\right)+a\right)^{2}\sin^{2}\left(c\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)*
*4/16 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**2*x*cos(c + d*x)**6/1
6 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a
**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + a**2*sin(c + d*x)**3*cos(c + d*
x)/(8*d) - 2*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)
*cos(c + d*x)**5/(16*d) - a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 4*a**2*
cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**2
, True))
```

Giac [A]
time = 0.47, size = 106, normalized size = 1.03

$$\frac{3}{16}\,a^2x+\frac{a^2\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{24\,d}-\frac{a^2\cos\left(dx+c\right)}{4\,d}+\frac{a^2\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\,a^2\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{a^2\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/16*a^2*x + 1/40*a^2*cos(5*d*x + 5*c)/d - 1/24*a^2*cos(3*d*x + 3*c)/d - 1/
4*a^2*cos(d*x + c)/d + 1/192*a^2*sin(6*d*x + 6*c)/d - 3/64*a^2*sin(4*d*x +
4*c)/d - 1/64*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.16, size = 257, normalized size = 2.50

$$\frac{3\,a^2\,x}{16}-\frac{\frac{3\,a^2\,\left(c+d\,x\right)}{16}-\frac{13\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24}-\frac{25\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+\frac{25\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+\frac{13\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{24}-\frac{3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}-\frac{a^2\,\left(45\,c+45\,d\,x-128\right)}{240}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{9\,a^2\,\left(c+d\,x\right)}{8}-\frac{a^2\,\left(270\,c+270\,d\,x-768\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{15\,a^2\,\left(c+d\,x\right)}{4}-\frac{a^2\,\left(900\,c+900\,d\,x-1280\right)}{240}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{45\,a^2\,\left(c+d\,x\right)}{16}-\frac{a^2\,\left(675\,c+675\,d\,x-1920\right)}{240}\right)+\frac{3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/16 - ((3*a^2*(c + d*x))/16 - (13*a^2*tan(c/2 + (d*x)/2)^3)/24 - (
25*a^2*tan(c/2 + (d*x)/2)^5)/4 + (25*a^2*tan(c/2 + (d*x)/2)^7)/4 + (13*a^2*
tan(c/2 + (d*x)/2)^9)/24 - (3*a^2*tan(c/2 + (d*x)/2)^11)/8 - (a^2*(45*c + 4
5*d*x - 128))/240 + tan(c/2 + (d*x)/2)^2*((9*a^2*(c + d*x))/8 - (a^2*(270*c
 + 270*d*x - 768))/240) + tan(c/2 + (d*x)/2)^6*((15*a^2*(c + d*x))/4 - (a^2
*(900*c + 900*d*x - 1280))/240) + tan(c/2 + (d*x)/2)^8*((45*a^2*(c + d*x))/
16 - (a^2*(675*c + 675*d*x - 1920))/240) + (3*a^2*tan(c/2 + (d*x)/2))/8)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.277 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=91

$$\frac{a^2x}{4}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/4*a^2*x-2/3*a^2*cos(d*x+c)^3/d+1/5*a^2*cos(d*x+c)^5/d+1/4*a^2*cos(d*x+c)*
sin(d*x+c)/d-1/2*a^2*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{10d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{a^2x}{4}-\frac{\cos^3(c+dx)(a\sin(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/4 - (a^2*Cos[c + d*x]^3)/(6*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(4
*d) - (Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(5*d) - (Cos[c + d*x]^3*(a^2
+ a^2*Sin[c + d*x]))/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
```

```
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{2}{5}\int \cos^2(c+dx)\\
&= -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{\cos^3(c+dx)\,(a^2}{10}\\
&= -\frac{a^2\cos^3(c+dx)}{6d}-\frac{\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\\
&= -\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-\frac{\cos^3(}{}\\
&= \frac{a^2x}{4}-\frac{a^2\cos^3(c+dx)}{6d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{4d}-
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 57, normalized size = 0.63

$$\frac{a^2(-90\cos(c+dx)-25\cos(3(c+dx))+3(20c+20dx+\cos(5(c+dx))-5\sin(4(c+dx))))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*c + 20*d*x + Cos[5*(c
+ d*x)] - 5*Sin[4*(c + d*x)])))/(240*d)
```

Maple [A]

time = 0.13, size = 95, normalized size = 1.04

method	result

risch	$\frac{a^2x}{4}-\frac{3a^2\cos(dx+c)}{8d}+\frac{a^2\cos(5dx+5c)}{80d}-\frac{a^2\sin(4dx+4c)}{16d}-\frac{5a^2\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3}+2a^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3\right.}{}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3}+2a^2\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3\right.}{}\right.}{d}$
norman	$\frac{\frac{a^2x}{4}-\frac{14a^2}{15d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{5a^2x\left(\tan\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*cos(d*x+c)^3+2*a^2*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+
c)*cos(d*x+c)+1/8*d*x+1/8*c)+a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d
*x+c)^3))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.76

$$-\frac{80\,a^2\cos\left(dx+c\right)^3-16\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^2-15\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 16*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
2 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.35, size = 72, normalized size = 0.79

$$\frac{12\,a^2\cos\left(dx+c\right)^5-40\,a^2\cos\left(dx+c\right)^3+15\,a^2dx-15\,\left(2\,a^2\cos\left(dx+c\right)^3-a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^2*cos(d*x + c)^5 - 40*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 15*(2*a^
2*cos(d*x + c)^3 - a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(80) = 160$.
time = 0.31, size = 172, normalized size = 1.89

$$\begin{cases}\frac{a^2x\sin^4(c+dx)}{4}+\frac{a^2x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{a^2x\cos^4(c+dx)}{4}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{a^2\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{2a^2\cos^5(c+dx)}{15d}-\frac{a^2\cos^3(c+dx)}{3d} & \text{for}\ d\neq 0\\ x\left(a\sin(c)+a\right)^2\sin(c)\cos^2(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/4 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/2 + a**2*x*cos(c + d*x)**4/4 + a**2*sin(c + d*x)**3*cos(c + d*x)/(4*d) -
a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**2*sin(c + d*x)*cos(c + d*x)
**3/(4*d) - 2*a**2*cos(c + d*x)**5/(15*d) - a**2*cos(c + d*x)**3/(3*d), Ne(
d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.49, size = 72, normalized size = 0.79

$$\frac{1}{4}a^2x + \frac{a^2\cos(5\,dx + 5\,c)}{80\,d} - \frac{5\,a^2\cos(3\,dx + 3\,c)}{48\,d} - \frac{3\,a^2\cos(dx + c)}{8\,d} - \frac{a^2\sin(4\,dx + 4\,c)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*x + 1/80*a^2*cos(5*d*x + 5*c)/d - 5/48*a^2*cos(3*d*x + 3*c)/d - 3/8
*a^2*cos(d*x + c)/d - 1/16*a^2*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 12.13, size = 262, normalized size = 2.88

$$\frac{a^2\,x}{4}-\frac{\frac{a^2\,\left(c+d\,x\right)}{4}-3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{2}-\frac{a^2\,\left(15\,c+15\,d\,x-56\right)}{60}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{4}-\frac{a^2\,\left(75\,c+75\,d\,x-120\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{4}-\frac{a^2\,\left(75\,c+75\,d\,x-160\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(150\,c+150\,d\,x-80\right)}{60}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{5\,a^2\,\left(c+d\,x\right)}{2}-\frac{a^2\,\left(150\,c+150\,d\,x-480\right)}{60}\right)+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/4 - ((a^2*(c + d*x))/4 - 3*a^2*tan(c/2 + (d*x)/2)^3 + 3*a^2*tan(c/2
 + (d*x)/2)^7 - (a^2*tan(c/2 + (d*x)/2)^9)/2 - (a^2*(15*c + 15*d*x - 56))/6
0 + tan(c/2 + (d*x)/2)^8*((5*a^2*(c + d*x))/4 - (a^2*(75*c + 75*d*x - 120))
/60) + tan(c/2 + (d*x)/2)^2*((5*a^2*(c + d*x))/4 - (a^2*(75*c + 75*d*x - 16
0))/60) + tan(c/2 + (d*x)/2)^4*((5*a^2*(c + d*x))/2 - (a^2*(150*c + 150*d*x
 - 80))/60) + tan(c/2 + (d*x)/2)^6*((5*a^2*(c + d*x))/2 - (a^2*(150*c + 150
*d*x - 480))/60) + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1
)^5)
```

3.278 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=71

$$a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{d}$$

```
[Out] a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3/d+a^2
*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2952, 2715, 8, 2672, 327, 212, 2645, 30}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d - (a^2*Cos[c +
 d*x]^3)/(3*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\cos^2(c+dx)+a^2\cos(c+dx)\cot(c+dx)+a^2 \mathrm{c}\right.\\ &= a^2\int \cos(c+dx)\cot(c+dx)\,dx+a^2\int \cos^2(c+dx)\sin\\ &= \frac{a^2\cos(c+dx)\sin(c+dx)}{d}+a^2\int 1\,dx-\frac{a^2\mathrm{Subst}\left(\int x^2\right.}{}\\ &= a^2x+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx}{}\\ &= a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\mathrm{co}}{}\end{aligned}$$

Mathematica [A]

time = 0.27, size = 71, normalized size = 1.00

$$\frac{a^2\left(9\cos(c+dx)-\cos(3(c+dx))+6\left(2\left(c+dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin(2(c+dx))\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(2*(c + d*x - Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)])))/(12*d)
```

Maple [A]
time = 0.15, size = 73, normalized size = 1.03

method	result
derivativedivides	$\frac{a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+2a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{a^2\left(\cos^3(dx+c)\right)}{3}}{d}$
default	$\frac{a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+2a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{a^2\left(\cos^3(dx+c)\right)}{3}}{d}$
risch	$a^2x+\frac{3a^2e^{i(dx+c)}}{8d}+\frac{3a^2e^{-i(dx+c)}}{8d}+\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{a^2\cos(3dx+3c)}{12d}+\frac{a^2\sin(2dx+$
norman	$\frac{a^2x+a^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2x}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-1/3*a^2*cos(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 75, normalized size = 1.06

$$-\frac{2\,a^2\cos\left(dx+c\right)^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2-3\,a^2\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.43, size = 86, normalized size = 1.21

$$-\frac{2\,a^2\cos\left(dx+c\right)^3-6\,a^2dx-6\,a^2\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^2\cos\left(dx+c\right)+3\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^2*cos(d*x + c)^3 - 6*a^2*d*x - 6*a^2*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \cos^2(c+dx)\csc(c+dx)\,dx+\int 2\sin(c+dx)\cos^2(c+dx)\csc(c+dx)\,dx+\int \sin^2(c+dx)\cos^2(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(2*sin(c + d*x)*c
os(c + d*x)**2*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)**2*
csc(c + d*x), x))
```

Giac [A]
time = 0.48, size = 101, normalized size = 1.42

$$\frac{3\,(dx+c)a^2+3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a^2*tan(
1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2
*c) - 2*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 9.04, size = 188, normalized size = 2.65

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{-2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{4\,a^2}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{4\,a^4}{4\,a^4-4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^4-4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c
/2 + (d*x)/2)^5 + (4*a^2)/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*
x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a^2*atan
((4*a^4)/(4*a^4 - 4*a^4*tan(c/2 + (d*x)/2)) + (4*a^4*tan(c/2 + (d*x)/2))/(4
*a^4 - 4*a^4*tan(c/2 + (d*x)/2))))/d
```

3.279 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$-\frac{a^2x}{2}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -1/2*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d-a^2*cot(d*x+c)/d+
1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 2718, 2715}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(a^2*x) - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (
a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c\right.}{a^2}$$
$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(}{2d}$$
$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+}{d}$$

Mathematica [A]
time = 0.42, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\left(c+dx-4\cos(c+dx)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d
*x)] + 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c
 + d*x)/2]])*Sin[c + d*x]))/d
```

Maple [A]
time = 0.15, size = 80, normalized size = 1.08

method	result
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(-\cot(dx+c)-dx-c)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$

risch	$-\frac{a^2x}{2}-\frac{ia^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{a^2\mathrm{e}^{i(dx+c)}}{d}+\frac{a^2\mathrm{e}^{-i(dx+c)}}{d}+\frac{ia^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d(\mathrm{e}^{2i(dx+c)}-1)}-\frac{2a^2\ln(\mathrm{e}^{i(dx+c)}+1)}{d}-$
norman	$\frac{-\frac{a^2}{2d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a^2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-a^2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)+2*a^2*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a
^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 79, normalized size = 1.07

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2-4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2+4\,a^2(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A]
time = 0.40, size = 105, normalized size = 1.42

$$-\frac{a^2\cos(dx+c)^3+2\,a^2\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,a^2\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+a^2\cos(dx+c)+(a^2dx-4\,a^2\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos^2(c+dx)\csc^2(c+dx)\,dx+\int 2\sin(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx+\int\sin^2(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(2*sin(c + d*x
)*cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**2*cos(c + d*
x)**2*csc(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(70) = 140.
time = 0.46, size = 143, normalized size = 1.93

$$-\frac{(dx+c)a^2-4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 8.81, size = 201, normalized size = 2.72

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^2}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (8*a^2*tan(c/2 + (d*x)/2)^3 - 3*a^2*tan
(c/2 + (d*x)/2)^4 - a^2 + 8*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2
) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (a^2*atan(a^4/(4*a^
4 + a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + a^4*tan(c
/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

3.280 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$-2a^2x-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -2*a^2*x-1/2*a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-2*a^2*cot(d*x+c)/d-
1/2*a^2*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2951, 3852, 8, 3853, 3855, 2718}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x])/d - (2*a^
2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-2a^4+2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(\right.}{a^2}$$

$$= -2a^2x + a^2\int \csc^3(c+dx)\,dx - a^2\int \sin(c+dx)\,dx +$$

$$= -2a^2x + \frac{a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{2d} -$$

$$= -2a^2x - \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^2\cos(c+dx)}{d} - \frac{2}{}$$

Mathematica [A]
time = 0.46, size = 102, normalized size = 1.40

$$\frac{a^2\left(-16c-16dx+8\cos(c+dx)-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+8\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-16*c - 16*d*x + 8*Cos[c + d*x] - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/
2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]
^2 + 8*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.17, size = 103, normalized size = 1.41

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2(-\cot(dx+c)-dx-c)+a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot}{d}$

default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2(-\cot(dx+c)-dx-c)+a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$
risch	$-2a^2x+\frac{a^2e^{i(dx+c)}}{2d}+\frac{a^2e^{-i(dx+c)}}{2d}+\frac{a^2(e^{3i(dx+c)}+e^{i(dx+c)}-4ie^{2i(dx+c)}+4i)}{d(e^{2i(dx+c)}-1)^2}+\frac{a^2\ln(e^{i(dx+c)}-1)}{2d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{2d}$
norman	$\frac{\frac{a^2(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{d}+\frac{a^2(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{d}-\frac{a^2}{8d}-\frac{a^2\tan(\frac{dx}{2}+\frac{c}{2})}{d}-\frac{a^2(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{d}+\frac{a^2(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{8d}-2a^2x(\tan^2(\frac{dx}{2}+\frac{c}{2}))\ldots}{\tan(\frac{dx}{2}+\frac{c}{2})^2(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-c
ot(d*x+c)))+2*a^2*(-cot(d*x+c)-d*x-c)+a^2*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x
+c))))
```

Maxima [A]
time = 0.48, size = 104, normalized size = 1.42

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-2a^2(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a^2 - a^2*(2*cos(d*x + c)/(cos(d*x + c)^
2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*a^2*(2*cos(d*x
+ c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.
time = 0.40, size = 143, normalized size = 1.96

$$-\frac{8a^2dx\cos(dx+c)^2-4a^2\cos(dx+c)^3-8a^2dx-8a^2\cos(dx+c)\sin(dx+c)+2a^2\cos(dx+c)+(a^2\cos(dx+c)^2-a^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(a^2\cos(dx+c)^2-a^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{4(d\cos(dx+c)^2-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)^3 - 8*a^2*d*x - 8*a^2*c
os(d*x + c)*sin(d*x + c) + 2*a^2*cos(d*x + c) + (a^2*cos(d*x + c)^2 - a^2)*
log(1/2*cos(d*x + c) + 1/2) - (a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x +
 c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos^2(c+dx)\csc^3(c+dx)\,dx+\int 2\sin(c+dx)\cos^2(c+dx)\csc^3(c+dx)\,dx+\int\sin^2(c+dx)\cos^2(c+dx)\csc^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**2*csc(c + d*x)**3, x) + Integral(2*sin(c + d*x
)*cos(c + d*x)**2*csc(c + d*x)**3, x) + Integral(sin(c + d*x)**2*cos(c + d*
x)**2*csc(c + d*x)**3, x))
```

Giac [A]
time = 0.47, size = 128, normalized size = 1.75

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,(dx+c)a^2+4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{16\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a^2 + 4*a^2*log(abs(tan(1/2*
d*x + 1/2*c))) + 8*a^2*tan(1/2*d*x + 1/2*c) + 16*a^2/(tan(1/2*d*x + 1/2*c)^
2 + 1) - (6*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/
tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 8.78, size = 213, normalized size = 2.92

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{4\,a^2\,\mathrm{atan}\left(\frac{16\,a^4}{4\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (4*a^2*tan(c/2 + (d*x)/2)^3 - (15*a^2*ta
n(c/2 + (d*x)/2)^2)/2 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (
d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) + (a^2*log(tan(c/2 + (d*x)/2)))/(2*d)
+ (4*a^2*atan((16*a^4)/(4*a^4 + 16*a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2
 + (d*x)/2))/(4*a^4 + 16*a^4*tan(c/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)
/2))/d
```

3.281 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$-a^2x+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -a^2*x+a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-a^
2*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d - (a^2*Cot[
c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx) + 2a^2\cot^2(c+dx)\csc(c+dx) + a\right.$$

$$= a^2\int \cot^2(c+dx)\,dx + a^2\int \cot^2(c+dx)\csc^2(c+dx$$

$$= -\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{d} - a^2\int$$

$$= -a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} - \frac{a}{}$$

Mathematica [A]
time = 0.43, size = 140, normalized size = 1.92

$$-\frac{a^2\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)+6\csc^2\left(\frac{1}{2}(c+dx)\right)-24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+24\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\sec^2\left(\frac{1}{2}(c+dx)\right)-8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-8\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/24*(a^2*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 24*
Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 - 8
*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 -
8*Tan[(c + d*x)/2]))/d
```

Maple [A]
time = 0.19, size = 97, normalized size = 1.33

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^2(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a^2(-\cot(dx+c)-dx-c)}{d}$
risch	$-a^2x+\frac{2a^2\left(3\,e^{5i(dx+c)}+6ie^{2i(dx+c)}-2i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{24d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{5a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{5a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/sin(d*x+c)^3*cos(d*x+c)^3+2*a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 83, normalized size = 1.14

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2\,a^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*a^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(71) = 142.
time = 0.36, size = 166, normalized size = 2.27

$$-\frac{4\,a^2\cos\left(dx+c\right)^3-6\,a^2\cos\left(dx+c\right)-3\,\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+6\,\left(a^2dx\cos\left(dx+c\right)^2-a^2dx-a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*a^2*cos(d*x + c)^3 - 6*a^2*cos(d*x + c) - 3*(a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)
```

```
*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(a^2*d*x*cos(d*x + c)^2 - a^
2*d*x - a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c
))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 141, normalized size = 1.93

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)a^2-24\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{44\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*a^2 - 24*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 9*a^2*tan(1/2*d*x + 1/2*c
) + (44*a^2*tan(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*t
an(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [**B**]
time = 8.94, size = 193, normalized size = 2.64

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(24*d) - (a^2
*cot(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a^2*a
tan((cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) - sin(c/2
 + (d*x)/2))))/d - (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*
a^2*cot(c/2 + (d*x)/2))/(8*d) + (3*a^2*tan(c/2 + (d*x)/2))/(8*d)
```

3.282 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=82

$$\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 5/8*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-3/8*a^2*cot(d*x+c)*csc
(d*x+c)/d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2952, 2691, 3855, 2687, 30, 3853}

$$-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (3*a^2
*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^2(c+dx)\csc(c+dx)+2a^2\cot^2(c+dx)\csc^2\right.$$

$$= a^2\int \cot^2(c+dx)\csc(c+dx)\,dx + a^2\int \cot^2(c+dx)$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc^3(c+}{4d}$$

$$= \frac{a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\cot(}{}$$

$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{3a^2\mathrm{co}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 209 vs. 2(82) = 164.

time = 0.10, size = 209, normalized size = 2.55

$$a^2\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[(c
 + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2]^
2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))
```

Maple [A]
time = 0.22, size = 143, normalized size = 1.74

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{2a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{2a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
risch	$\frac{a^2\left(9\,e^{7i(dx+c)}-33\,e^{5i(dx+c)}+48i e^{6i(dx+c)}-33\,e^{3i(dx+c)}-48i e^{4i(dx+c)}+9\,e^{i(dx+c)}+16i e^{2i(dx+c)}-16i\right)}{12d\left(e^{2i(dx+c)}-1\right)^4}-\frac{5a^2\ln\left(e^{i(dx+c}\right.}{8d}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}-\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*
cos(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-2/3*a^2/sin(d*x+c)^3*cos(d*x+c)^3
+a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]
time = 0.28, size = 130, normalized size = 1.59

$$-\frac{3\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{32\,a^2}{\tan(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a^2/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.
time = 0.35, size = 155, normalized size = 1.89

$$-\frac{32\,a^2\cos\left(dx+c\right)^3\sin\left(dx+c\right)-18\,a^2\cos\left(dx+c\right)^3+30\,a^2\cos\left(dx+c\right)-15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a^2*cos(d*x + c)^3*sin(d*x + c) - 18*a^2*cos(d*x + c)^3 + 30*a^2*
cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2
*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(74) = 148.
time = 0.47, size = 164, normalized size = 2.00

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{250\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^
2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 48*a^2*
tan(1/2*d*x + 1/2*c) + (250*a^2*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*tan(1/2*d*x
 + 1/2*c)^3 - 24*a^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.62, size = 161, normalized size = 1.96

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a^2}{4}\right)}{16\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) + (a^2
*tan(c/2 + (d*x)/2)^4)/(64*d) - (5*a^2*log(tan(c/2 + (d*x)/2)))/(8*d) - (co
t(c/2 + (d*x)/2)^4*(2*a^2*tan(c/2 + (d*x)/2)^2 - 4*a^2*tan(c/2 + (d*x)/2)^3
 + a^2/4 + (4*a^2*tan(c/2 + (d*x)/2))/3))/(16*d) - (a^2*tan(c/2 + (d*x)/2))
/(4*d)
```

3.283 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}$$

```
[Out] 1/4*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-1/5*a^2*cot(d*x+c)^5/d
+1/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.241$, Rules used = {2952, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{a^2\cot(c+dx)\csc(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Cos[c + d*x]])/(4*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot
[c + d*x]^5)/(5*d) + (a^2*Cot[c + d*x]*Csc[c + d*x])/(4*d) - (a^2*Cot[c + d
*x]*Csc[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)\csc^2(c+dx)+2a^2\cot^2(c+dx)\,\text{csc}\right. \\
&= a^2\int \cot^2(c+dx)\csc^2(c+dx)\,dx+a^2\int \cot^2(c+dx \\
&= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}-\frac{1}{2}a^2\int \csc^3(c+dx)\,d \\
&= -\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)\csc(c+dx)}{4d}-\frac{a^2\,c}{} \\
&= \frac{a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(}{5}
\end{aligned}$$

Mathematica [A]
time = 0.70, size = 189, normalized size = 1.89

$$-\frac{a^2\csc^5(c+dx)\left(200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+dx))-150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+180\sin(2(c+dx))+75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+30\sin(4(c+dx))-15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(a^2*Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos
[5*(c + d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d
*x)/2]]*Sin[c + d*x] + 180*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[
3*(c + d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c + d*
x)] - 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*
Sin[5*(c + d*x)]))/d
```

Maple [A]
time = 0.21, size = 136, normalized size = 1.36

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+2a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^2\left(\cos^3(d\cdots\right.}{3\sin(dx\cdots}}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+2a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^2\left(\cos^3(d\cdots\right.}{3\sin(dx\cdots}}{d}$
risch	$-\frac{a^2\left(-60ie^{8i(dx+c)}+15\,e^{9i(dx+c)}+240ie^{6i(dx+c)}+90\,e^{7i(dx+c)}-40ie^{4i(dx+c)}+80ie^{2i(dx+c)}-90\,e^{3i(dx+c)}-28i-15\,e^{i(dx+\cdots}\right.}{30d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a^2}{160d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}-\frac{31a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}-\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{37a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{13a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{13a^2\left(\tan^8\left(\cdots\right.\right.}{96d}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+2*
a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d
*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*a^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.09

$$-\frac{15\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{40\,a^2}{\tan(dx+c)^3}+\frac{8\left(5\,\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*a^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(90) = 180.
time = 0.35, size = 189, normalized size = 1.89

$$\frac{56\,a^2\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3+15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-30\left(a^2\cos\left(dx+c\right)^3+a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 + 15*(a^2*cos(d*x + c)
^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 30*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c))*sin(d*x +
 c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.49, size = 164, normalized size = 1.64

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{274\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+90\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^
2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 90*a^2*
tan(1/2*d*x + 1/2*c) + (274*a^2*tan(1/2*d*x + 1/2*c)^5 + 90*a^2*tan(1/2*d*x
 + 1/2*c)^4 - 25*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) -
 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 8.63, size = 160, normalized size = 1.60

$$\frac{5\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^2\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(-6\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{5\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{5}\right)}{32\,d}-\frac{3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (5*a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) + (
a^2*tan(c/2 + (d*x)/2)^5)/(160*d) - (a^2*log(tan(c/2 + (d*x)/2)))/(4*d) - (
cot(c/2 + (d*x)/2)^5*((5*a^2*tan(c/2 + (d*x)/2)^2)/3 - 6*a^2*tan(c/2 + (d*x
)/2)^4 + a^2/5 + a^2*tan(c/2 + (d*x)/2)))/(32*d) - (3*a^2*tan(c/2 + (d*x)/2
))/(16*d)
```

3.284 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{16d}-\frac{5a^2\cot(c+dx)}{24d}$$

```
[Out] 3/16*a^2*arctanh(cos(d*x+c))/d-2/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/
d+3/16*a^2*cot(d*x+c)*csc(d*x+c)/d-5/24*a^2*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a
^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3853, 3855, 2687, 14}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^2\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^
2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^2
*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6
*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^2(c+dx)\csc^3(c+dx)+2a^2\cot^2(c+dx)\csc \cdots\right.\\ &= a^2\int \cot^2(c+dx)\csc^3(c+dx)\,dx+a^2\int \cot^2(c+dx \cdots\\ &= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^2\cot(c+dx)\csc^5(c \cdots}{6d}\\ &= \frac{a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{5a^2\cot(c+dx)\csc^3(c+ \cdots}{24d}\\ &= \frac{a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5 \cdots}{5 \cdots}\\ &= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\mathrm{co} \cdots}{\cdots}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 229, normalized size = 1.85

$$\frac{a^2\csc^6(c+dx)\left(-1500\cos(c+dx)+130\cos(3(c+dx))+90\cos(5(c+dx))+450\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-675\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+270\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-45\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-450\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+675\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-270\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+45\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-960\sin(2(c+dx))-384\sin(4(c+dx))+64\sin(6(c+dx))\right)}{7680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

[Out] (a^2*Csc[c + d*x]^6*(-1500*Cos[c + d*x] + 130*Cos[3*(c + d*x)] + 90*Cos[5*(c + d*x)] + 450*Log[Cos[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 45*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 450*Log[Sin[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 45*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c + d*x)] + 64*Sin[6*(c + d*x)]))/(7680*d)

Maple [A]
time = 0.25, size = 200, normalized size = 1.61

method	result
risch	$-\frac{a^2(45\,\mathrm{e}^{11i(dx+c)}+65\,\mathrm{e}^{9i(dx+c)}-750\,\mathrm{e}^{7i(dx+c)}+960i\mathrm{e}^{8i(dx+c)}-750\,\mathrm{e}^{5i(dx+c)}-640i\mathrm{e}^{6i(dx+c)}+65\,\mathrm{e}^{3i(dx+c)}+45\,\mathrm{e}^{i(dx+c)}}{120d(\mathrm{e}^{2i(dx+c)}-1)^6}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+2a^2\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2(\cos^3(dx+c)}{15\sin(dx+c)^3}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+2a^2\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2(\cos^3(dx+c)}{15\sin(dx+c)^3}\right.}{d}$
norman	$-\frac{a^2}{384d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}-\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{11a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{17a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}+\frac{5a^2\left(\tan^7\left(\frac{d}{2}\right.\right.}{48d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^3-1/8/sin(d*x+c)^4*cos(d*x+c)^3-1/16/sin(d*x+c)^2*cos(d*x+c)^3-1/16*cos(d*x+c)-1/16*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c))))

Maxima [A]
time = 0.30, size = 187, normalized size = 1.51

$$-\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log(\cos(dx+c)+1)+3\,\log(\cos(dx+c)-1)\right)+30\,a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+\frac{64\left(5\,\tan(dx+c)^2+3\right)a^2}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 30*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(112) = 224$.
time = 0.35, size = 227, normalized size = 1.83

$$-\frac{90\,a^{2}\cos\left(dx+c\right)^{5}-80\,a^{2}\cos\left(dx+c\right)^{3}-90\,a^{2}\cos\left(dx+c\right)-45\left(a^{2}\cos\left(dx+c\right)^{6}-3\,a^{2}\cos\left(dx+c\right)^{4}+3\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+45\left(a^{2}\cos\left(dx+c\right)^{6}-3\,a^{2}\cos\left(dx+c\right)^{4}+3\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+64\left(2\,a^{2}\cos\left(dx+c\right)^{5}-5\,a^{2}\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 90*a^2*cos(d*x + c)
 - 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a
^2)*log(1/2*cos(d*x + c) + 1/2) + 45*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x +
c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) + 64*(2*a^2
*cos(d*x + c)^5 - 5*a^2*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(112) = 224$.
time = 0.47, size = 228, normalized size = 1.84

$$\frac{5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+40\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-360\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-240\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{882\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+240\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-40\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-45\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*
d*x + 1/2*c)^2 - 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d
*x + 1/2*c) + (882*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a^2*tan(1/2*d*x + 1/2*c)^3 - 45*a^
2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*tan(1/2*d*x + 1/2*c) - 5*a^2)/tan(1/2*d*x
 + 1/2*c)^6)/d
```

Mupad [B]
time = 9.66, size = 339, normalized size = 2.73

$$-\frac{a^{2}\left(5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+360\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\right)}{1920\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] -(a^2*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -
 45*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 40*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.285 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$\frac{5a^3x}{16}-\frac{4a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{a^3\cos^7(c+dx)}{7d}+\frac{5a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{5a^3\cos^3(c+dx)\mathrm{s}}{8d}$$

```
[Out] 5/16*a^3*x-4/3*a^3*cos(d*x+c)^3/d+a^3*cos(d*x+c)^5/d-1/7*a^3*cos(d*x+c)^7/d
+5/16*a^3*cos(d*x+c)*sin(d*x+c)/d-5/8*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/2*a^3
*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2648, 2715, 8, 2645, 14, 276}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{d}-\frac{4a^3\cos^3(c+dx)}{3d}-\frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/16 - (4*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/d - (a^3
*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*
Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d*x]^3*Sin[c + d*x]^3)/(2
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^2(c+dx)\sin^2(c+dx)+3a^3\cos^2(c+dx)\sin^3\right.$$
$$= a^3\int \cos^2(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^2(c+dx)\,\mathrm{s}$$
$$= -\frac{a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^3(c+dx)\sin^3(c+}{2d}$$
$$= \frac{a^3\cos(c+dx)\sin(c+dx)}{8d} - \frac{5a^3\cos^3(c+dx)\sin(c+d}{8d}$$
$$= \frac{a^3x}{8} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c-}{7d}$$
$$= \frac{5a^3x}{16} - \frac{4a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{d} - \frac{a^3\cos^7(c}{7d}$$

Mathematica [A]
time = 0.48, size = 86, normalized size = 0.65

$$\frac{a^3(420c+420dx-609\cos(c+dx)-91\cos(3(c+dx))+63\cos(5(c+dx))-3\cos(7(c+dx))-63\sin(2(c+dx))-105\sin(4(c+dx))+21\sin(6(c+dx)))}{1344d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(420*c + 420*d*x - 609*Cos[c + d*x] - 91*Cos[3*(c + d*x)] + 63*Cos[5*(
c + d*x)] - 3*Cos[7*(c + d*x)] - 63*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)]
 + 21*Sin[6*(c + d*x)]))/(1344*d)
```

Maple [A]

time = 0.22, size = 194, normalized size = 1.47

method	result
risch	$\frac{5a^3x}{16} - \frac{29a^3\cos(dx+c)}{64d} - \frac{a^3\cos(7dx+7c)}{448d} + \frac{a^3\sin(6dx+6c)}{64d} + \frac{3a^3\cos(5dx+5c)}{64d} - \frac{5a^3\sin(4dx+4c)}{64d} - \frac{13a^3 c}{}$
derivativedivides	$a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + 3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5} - \frac{2\left(\cos^3(dx+c)\right)}{15}\right) + 3a^3\Big($
default	$a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4} + \frac{\sin(dx+c)\cos(dx+c)}{8} + \frac{dx}{8} + \frac{c}{8}\right) + 3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5} - \frac{2\left(\cos^3(dx+c)\right)}{15}\right) + 3a^3\Big($
norman	$\frac{5a^3x}{16} - \frac{20a^3}{21d} - \frac{5a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d} - \frac{3a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} + \frac{119a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} - \frac{119a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} + \frac{3a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)+3*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a^3*(-1/6*s
in(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*sin(d*x+c)*cos(d*
x+c)+1/16*d*x+1/16*c)+a^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2
*cos(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A]

time = 0.28, size = 129, normalized size = 0.98

$$-\frac{64\left(15\cos\left(dx+c\right)^7-42\cos\left(dx+c\right)^5+35\cos\left(dx+c\right)^3\right)a^3-1344\left(3\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^3\right)a^3+105\left(4\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\sin\left(4\,dx+4\,c\right)\right)a^3-210\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^3}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6720*(64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*a^3
 - 1344*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3 + 105*(4*sin(2*d*x + 2*c)
^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 210*(4*d*x + 4*c - sin(4*d*x
 + 4*c))*a^3)/d
```

Fricas [A]
time = 0.36, size = 98, normalized size = 0.74

$$-\frac{48\,a^3\cos(dx+c)^7 - 336\,a^3\cos(dx+c)^5 + 448\,a^3\cos(dx+c)^3 - 105\,a^3dx - 21\left(8\,a^3\cos(dx+c)^5 - 18\,a^3\cos(dx+c)^3 + 5\,a^3\cos(dx+c)\right)\sin(dx+c)}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/336*(48*a^3*cos(d*x + c)^7 - 336*a^3*cos(d*x + c)^5 + 448*a^3*cos(d*x +
c)^3 - 105*a^3*d*x - 21*(8*a^3*cos(d*x + c)^5 - 18*a^3*cos(d*x + c)^3 + 5*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(122) = 244.
time = 0.73, size = 379, normalized size = 2.87

$$\begin{cases} \frac{3a^3x\sin^6(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{a^3x\sin^4(c+dx)}{8} + \frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^6(c+dx)}{16} + \frac{a^3x\cos^4(c+dx)}{8} + \frac{3a^3\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{a^3\sin^4(c+dx)\cos^3(c+dx)}{3d} - \frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d} + \frac{a^3\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{4a^3\sin^2(c+dx)\cos^5(c+dx)}{15d} - \frac{a^3\sin^2(c+dx)\cos^3(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{a^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{8a^3\cos^7(c+dx)}{105d} - \frac{2a^3\cos^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\sin^2(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c + d
*x)**2/16 + a**3*x*sin(c + d*x)**4/8 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**3*x*cos(c + d*x)*
*6/16 + a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
d) - a**3*sin(c + d*x)**4*cos(c + d*x)**3/(3*d) - a**3*sin(c + d*x)**3*cos(
c + d*x)**3/(2*d) + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a**3*sin(c
+ d*x)**2*cos(c + d*x)**5/(15*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d -
 3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a**3*sin(c + d*x)*cos(c + d*x
)**3/(8*d) - 8*a**3*cos(c + d*x)**7/(105*d) - 2*a**3*cos(c + d*x)**5/(5*d),
 Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**2, True))
```

Giac [A]
time = 0.46, size = 123, normalized size = 0.93

$$\frac{5}{16}\,a^3x - \frac{a^3\cos(7\,dx+7\,c)}{448\,d} + \frac{3\,a^3\cos(5\,dx+5\,c)}{64\,d} - \frac{13\,a^3\cos(3\,dx+3\,c)}{192\,d} - \frac{29\,a^3\cos(dx+c)}{64\,d} + \frac{a^3\sin(6\,dx+6\,c)}{64\,d} - \frac{5\,a^3\sin(4\,dx+4\,c)}{64\,d} - \frac{3\,a^3\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/16*a^3*x - 1/448*a^3*cos(7*d*x + 7*c)/d + 3/64*a^3*cos(5*d*x + 5*c)/d - 1
3/192*a^3*cos(3*d*x + 3*c)/d - 29/64*a^3*cos(d*x + c)/d + 1/64*a^3*sin(6*d*
x + 6*c)/d - 5/64*a^3*sin(4*d*x + 4*c)/d - 3/64*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]

time = 12.16, size = 331, normalized size = 2.51

$$\frac{5\,a^3\,x}{16}-\frac{\frac{5\,a^3\,\left(c+d\,x\right)}{16}+\frac{3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{119\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{8}+\frac{119\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{8}-\frac{3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{2}-\frac{5\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{8}-\frac{a^3\,\left(105\,c+105\,d\,x-320\right)}{336}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{35\,a^3\,\left(c+d\,x\right)}{16}-\frac{a^3\,\left(735\,c+735\,d\,x-2240\right)}{336}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{105\,a^3\,\left(c+d\,x\right)}{16}-\frac{a^3\,\left(2205\,c+2205\,d\,x-2688\right)}{336}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{175\,a^3\,\left(c+d\,x\right)}{16}-\frac{a^3\,\left(3675\,c+3675\,d\,x-896\right)}{336}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{105\,a^3\,\left(c+d\,x\right)}{16}-\frac{a^3\,\left(2205\,c+2205\,d\,x-4032\right)}{336}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{175\,a^3\,\left(c+d\,x\right)}{16}-\frac{a^3\,\left(3675\,c+3675\,d\,x-10304\right)}{336}\right)+\frac{5\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*a^3*x)/16 - ((5*a^3*(c + d*x))/16 + (3*a^3*tan(c/2 + (d*x)/2)^3)/2 - (11
9*a^3*tan(c/2 + (d*x)/2)^5)/8 + (119*a^3*tan(c/2 + (d*x)/2)^9)/8 - (3*a^3*t
an(c/2 + (d*x)/2)^11)/2 - (5*a^3*tan(c/2 + (d*x)/2)^13)/8 - (a^3*(105*c + 1
05*d*x - 320))/336 + tan(c/2 + (d*x)/2)^2*((35*a^3*(c + d*x))/16 - (a^3*(73
5*c + 735*d*x - 2240))/336) + tan(c/2 + (d*x)/2)^4*((105*a^3*(c + d*x))/16
- (a^3*(2205*c + 2205*d*x - 2688))/336) + tan(c/2 + (d*x)/2)^6*((175*a^3*(c
 + d*x))/16 - (a^3*(3675*c + 3675*d*x - 896))/336) + tan(c/2 + (d*x)/2)^10*
((105*a^3*(c + d*x))/16 - (a^3*(2205*c + 2205*d*x - 4032))/336) + tan(c/2 +
 (d*x)/2)^8*((175*a^3*(c + d*x))/16 - (a^3*(3675*c + 3675*d*x - 10304))/336
) + (5*a^3*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.286 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=117

$$\frac{7a^3x}{16}-\frac{4a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{7a^3\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{a^3\cos}{}$$

```
[Out] 7/16*a^3*x-4/3*a^3*cos(d*x+c)^3/d+3/5*a^3*cos(d*x+c)^5/d+7/16*a^3*cos(d*x+c
)*sin(d*x+c)/d-7/8*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^3*sin(d
*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{7a^3\cos^3(c+dx)}{24d}-\frac{7\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{7a^3x}{16}-\frac{a\cos^3(c+dx)(a\sin(c+dx)+a)^2}{10d}-\frac{\cos^3(c+dx)(a\sin(c+dx)+a)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*x)/16 - (7*a^3*Cos[c + d*x]^3)/(24*d) + (7*a^3*Cos[c + d*x]*Sin[c +
d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(10*d) - (Cos[c +
d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (7*Cos[c + d*x]^3*(a^3 + a^3*Sin[c +
 d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}+\frac{1}{2}\int \cos^2(c+dx)$$
$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a}{}$$
$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a}{}$$
$$= -\frac{7a^3\cos^3(c+dx)}{24d}-\frac{a\cos^3(c+dx)(a+a\sin(c+dx))}{10d}$$
$$= -\frac{7a^3\cos^3(c+dx)}{24d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\,c}{}$$
$$= \frac{7a^3x}{16}-\frac{7a^3\cos^3(c+dx)}{24d}+\frac{7a^3\cos(c+dx)\sin(c+dx)}{16d}$$

Mathematica [A]
time = 0.35, size = 76, normalized size = 0.65

$$\frac{a^3(450c+420dx-600\cos(c+dx)-140\cos(3(c+dx))+36\cos(5(c+dx))-15\sin(2(c+dx))-105\sin(4(c+dx))+5\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(450*c + 420*d*x - 600*Cos[c + d*x] - 140*Cos[3*(c + d*x)] + 36*Cos[5*
(c + d*x)] - 15*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)
]))/(960*d)
```

Maple [A]
time = 0.17, size = 156, normalized size = 1.33

method	result
risch	$\frac{7a^3x}{16} - \frac{5a^3\cos(dx+c)}{8d} + \frac{a^3\sin(6dx+6c)}{192d} + \frac{3a^3\cos(5dx+5c)}{80d} - \frac{7a^3\sin(4dx+4c)}{64d} - \frac{7a^3\cos(3dx+3c)}{48d} - \frac{a^3\sin(2d}{64}$
derivativedivides	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^3\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^3\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos}{}\right.}{d}$
default	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^3\left(-\frac{(\cos^3(dx+c))\sin(dx+c)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^3\left(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(\cos}{}\right.}{d}$
norman	$\frac{7a^3x}{16}-\frac{22a^3}{15d}-\frac{7a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{73a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{37a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{37a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{73a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{7a^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*cos(d*x+c)^3+3*a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+
c)*cos(d*x+c)+1/8*d*x+1/8*c)+3*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos
(d*x+c)^3)+a^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+
1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.29, size = 106, normalized size = 0.91

$$-\frac{320\,a^3\cos\left(dx+c\right)^3-192\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3+5\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)a^3-90\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*
a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^3 - 9
0*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.34, size = 85, normalized size = 0.73

$$\frac{144\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\,a^3dx+5\left(8\,a^3\cos\left(dx+c\right)^5-50\,a^3\cos\left(dx+c\right)^3+21\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*a^3*d*x + 5*(8
*a^3*cos(d*x + c)^5 - 50*a^3*cos(d*x + c)^3 + 21*a^3*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(110) = 220$.
time = 0.49, size = 328, normalized size = 2.80

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3x\cos^6(c+dx)}{16} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{a^3\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a^3\sin^2(c+dx)\cos^3(c+dx)}{d} - \frac{a^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{3a^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{2a^3\cos^5(c+dx)}{5d} - \frac{a^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\sin(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + a**3*x*cos(c + d*x)*
*6/16 + 3*a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*
d) - a**3*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*a**3*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**3/d - a**3*sin(c + d*
x)*cos(c + d*x)**5/(16*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a
**3*cos(c + d*x)**5/(5*d) - a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*si
n(c) + a)**3*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.49, size = 106, normalized size = 0.91

$$\frac{7}{16}a^3x + \frac{3a^3\cos(5dx+5c)}{80d} - \frac{7a^3\cos(3dx+3c)}{48d} - \frac{5a^3\cos(dx+c)}{8d} + \frac{a^3\sin(6dx+6c)}{192d} - \frac{7a^3\sin(4dx+4c)}{64d} - \frac{a^3\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 7/16*a^3*x + 3/80*a^3*cos(5*d*x + 5*c)/d - 7/48*a^3*cos(3*d*x + 3*c)/d - 5/
8*a^3*cos(d*x + c)/d + 1/192*a^3*sin(6*d*x + 6*c)/d - 7/64*a^3*sin(4*d*x +
4*c)/d - 1/64*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.72, size = 349, normalized size = 2.98

$$\frac{7a^3x}{16} - \frac{\frac{37a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4} - \frac{37a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} - \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24} + \frac{73a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{24} - \frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{8} + \frac{a^3(105c+105dx)}{240} - \frac{a^3(105c+105dx-352)}{240} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{a^3(105c+105dx)}{40} - \frac{a^3(630c+630dx-480)}{240}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3(105c+105dx)}{40} - \frac{a^3(630c+630dx-1632)}{240}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3(105c+105dx)}{16} - \frac{a^3(1575c+1575dx-960)}{240}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{a^3(105c+105dx)}{16} - \frac{a^3(1575c+1575dx-4320)}{240}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^3(105c+105dx)}{12} - \frac{a^3(2100c+2100dx-3520)}{240}\right) + \frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (7*a^3*x)/16 - ((37*a^3*tan(c/2 + (d*x)/2)^7)/4 - (37*a^3*tan(c/2 + (d*x)/2
)^5)/4 - (73*a^3*tan(c/2 + (d*x)/2)^3)/24 + (73*a^3*tan(c/2 + (d*x)/2)^9)/2
4 - (7*a^3*tan(c/2 + (d*x)/2)^11)/8 + (a^3*(105*c + 105*d*x))/240 - (a^3*(1
05*c + 105*d*x - 352))/240 + tan(c/2 + (d*x)/2)^10*((a^3*(105*c + 105*d*x))
/40 - (a^3*(630*c + 630*d*x - 480))/240) + tan(c/2 + (d*x)/2)^2*((a^3*(105*
c + 105*d*x))/40 - (a^3*(630*c + 630*d*x - 1632))/240) + tan(c/2 + (d*x)/2)
```

```
^4*((a^3*(105*c + 105*d*x))/16 - (a^3*(1575*c + 1575*d*x - 960))/240) + tan
(c/2 + (d*x)/2)^8*((a^3*(105*c + 105*d*x))/16 - (a^3*(1575*c + 1575*d*x - 4
320))/240) + tan(c/2 + (d*x)/2)^6*((a^3*(105*c + 105*d*x))/12 - (a^3*(2100*
c + 2100*d*x - 3520))/240) + (7*a^3*tan(c/2 + (d*x)/2))/8)/(d*(tan(c/2 + (d
*x)/2)^2 + 1)^6)
```

3.287 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=99

$$\frac{13a^3x}{8}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{13a^3\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^3\cos^3(c+\cdots}{}$$

```
[Out] 13/8*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-a^3*cos(d*x+c)^3/d+13
/8*a^3*cos(d*x+c)*sin(d*x+c)/d-1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2952, 2715, 8, 2672, 327, 212, 2645, 30, 2648}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{13a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{13a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d - (a^3*
Cos[c + d*x]^3)/d + (13*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c +
 d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^2(c+dx) + a^3\cos(c+dx)\cot(c+dx) + 3a^3\right.$$
$$= a^3\int \cos(c+dx)\cot(c+dx)\,dx + a^3\int \cos^2(c+dx)\sin$$
$$= \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^3(c+dx)\sin(c+d}{4d}$$
$$= \frac{3a^3x}{2} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{d} + \frac{13a^3\cos(c}{}$$
$$= \frac{13a^3x}{8} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} - \frac{a^3}{}$$

Mathematica [A]
time = 0.50, size = 82, normalized size = 0.83

$$\frac{a^3\left(52c+52dx+8\cos(c+dx)-8\cos(3(c+dx))-32\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+24\sin(2(c+dx))-\sin(4(c+dx))\right)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(52*c + 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c +
 d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 24*Sin[2*(c + d*x)] - Sin[4*(c + d*x
)]))/(32*d)
```

Maple [A]
time = 0.18, size = 115, normalized size = 1.16

method	result
derivativedivides	$\frac{a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-a^3\left(\cos^3(dx+c)\right)+a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin}{4}\right.}{d}$
default	$\frac{a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-a^3\left(\cos^3(dx+c)\right)+a^3\left(-\frac{\left(\cos^3(dx+c)\right)\sin}{4}\right.}{d}$
risch	$\frac{13a^3x}{8}+\frac{a^3e^{i(dx+c)}}{8d}+\frac{a^3e^{-i(dx+c)}}{8d}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{a^3\sin(4dx+4c)}{32d}-\frac{a^3\cos(3dx}{4d}$
norman	$\frac{\frac{13a^3x}{8}+\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{19a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{19a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{11a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{13a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{39a^3}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-a^3*cos(d*x+c)^3+a^3*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8
*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A]
time = 0.28, size = 99, normalized size = 1.00

$$-\frac{32\,a^3\cos\left(dx+c\right)^3-\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^3-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3-16\,a^3\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a^3*cos(d*x + c)^3 - (4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 - 24*(2
*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d*x +
 c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 101, normalized size = 1.02

$$-\frac{8\,a^3\cos\left(dx+c\right)^3-13\,a^3dx-8\,a^3\cos\left(dx+c\right)+4\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-4\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(2\,a^3\cos\left(dx+c\right)^3-13\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^3*cos(d*x + c)^3 - 13*a^3*d*x - 8*a^3*cos(d*x + c) + 4*a^3*log(1/
2*cos(d*x + c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) + (2*a^3*cos(d*x
 + c)^3 - 13*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int 3\sin\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(3*sin(c + d*x)*c
os(c + d*x)**2*csc(c + d*x), x) + Integral(3*sin(c + d*x)**2*cos(c + d*x)**
2*csc(c + d*x), x) + Integral(sin(c + d*x)**3*cos(c + d*x)**2*csc(c + d*x),
 x))
```

Giac [A]
time = 0.48, size = 144, normalized size = 1.45

$$\frac{13\left(dx+c\right)a^3+8\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-11\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(13*(d*x + c)*a^3 + 8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(11*a^3*ta
n(1/2*d*x + 1/2*c)^7 + 16*a^3*tan(1/2*d*x + 1/2*c)^6 + 19*a^3*tan(1/2*d*x +
 1/2*c)^5 - 19*a^3*tan(1/2*d*x + 1/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 -
 11*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 10.36, size = 244, normalized size = 2.46

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{13\,a^3\,\mathrm{atan}\left(\frac{169\,a^6}{16\,\left(\frac{13\,a^6}{2}-\frac{169\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{13\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{13\,a^6}{2}-\frac{169\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{11\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}-4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-\frac{19\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+\frac{19\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{4}+4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (13*a^3*atan((169*a^6)/(16*((13*a^6)/2 -
(169*a^6*tan(c/2 + (d*x)/2))/16)) + (13*a^6*tan(c/2 + (d*x)/2))/(2*((13*a^6
)/2 - (169*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) + (4*a^3*tan(c/2 + (d*x)/2)
^2 + (19*a^3*tan(c/2 + (d*x)/2)^3)/4 - (19*a^3*tan(c/2 + (d*x)/2)^5)/4 - 4*
a^3*tan(c/2 + (d*x)/2)^6 - (11*a^3*tan(c/2 + (d*x)/2)^7)/4 + (11*a^3*tan(c/
2 + (d*x)/2))/4)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.288 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$\frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)
^3/d-a^3*cot(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 2718, 2715, 2713}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\right.}{a^2}\\ &= 2a^3x+a^3\int \csc^2(c+dx)\,dx-a^3\int \sin^3(c+dx)\,dx-\left(2a^3\right)\int \sin\\ &= 2a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+}{}\\ &= \frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+d}{3d}\end{aligned}$$

Mathematica [A]
time = 0.78, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)(15-66\sin(c+dx))-12\left(c+dx-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)+\cos(3(c+dx))(9+2\sin(c+dx))\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
[Out] -1/48*(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c +
 d*x]) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*S
in[c + d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/d
```

Maple [A]
time = 0.16, size = 94, normalized size = 1.02

method	result
derivativedivides	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{a^3\left(\cos^3(dx+c)\right.}{3}}{d}$
default	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{a^3\left(\cos^3(dx+c)\right.}{3}}{d}$
risch	$\frac{a^3x}{2}-\frac{3ia^3e^{2i(dx+c)}}{8d}+\frac{11a^3e^{i(dx+c)}}{8d}+\frac{11a^3e^{-i(dx+c)}}{8d}+\frac{3ia^3e^{-2i(dx+c)}}{8d}-\frac{2ia^3}{d(e^{2i(dx+c)}-1)}-\frac{3a^3\ln\left(e^{i(dx+c)}+\right.}{d}$
norman	$\frac{\frac{4a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{2d}+\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^3x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-cot(d*x+c)-d*x-c)+3*a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3
*a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-1/3*a^3*cos(d*x+c)^3)
```

Maxima [A]
time = 0.49, size = 93, normalized size = 1.01

$$-\frac{4\,a^3\cos\left(dx+c\right)^3-9\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-18\,a^3(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right))}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 121, normalized size = 1.32

$$-\frac{9\,a^3\cos\left(dx+c\right)^3+9\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,a^3\cos\left(dx+c\right)+\left(2\,a^3\cos\left(dx+c\right)^3-3\,a^3dx-18\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\cos^2(c+dx)\csc^2(c+dx)\,dx+\int 3\sin(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx+\int 3\sin^2(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx+\int\sin^3(c+dx)\cos^2(c+dx)\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x
)*cos(c + d*x)**2*csc(c + d*x)**2, x) + Integral(3*sin(c + d*x)**2*cos(c +
d*x)**2*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cos(c + d*x)**2*csc(
c + d*x)**2, x))
```

Giac [A]
time = 0.47, size = 162, normalized size = 1.76

$$\frac{3\,(dx+c)a^3+18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 8.77, size = 264, normalized size = 2.87

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^6}{6\,a^6-a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{-7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-a^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (a^3*atan(a^6/(6*a^6 - a^6*tan(c/2 + (d
*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - a^6*tan(c/2 + (d*x)/2))))/d +
 (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(
c/2 + (d*x)/2)^3 - 3*a^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*tan(c/2 + (d*x)/2)^5
- 7*a^3*tan(c/2 + (d*x)/2)^6 - a^3 + (32*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*t
an(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^5 + 2*tan
(c/2 + (d*x)/2)^7))
```

3.289 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{5a^3x}{2}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{a^3\cos(c+$$

```
[Out] -5/2*a^3*x-5/2*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c
)/d-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*a^3*x)/2 - (5*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (3*a^3*Cos[c + d*x])/d
 - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (a^3*Co
s[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\int \left(-2a^5+2a^5\csc(c+dx)+3a^5\csc^2(c+dx)+a^5\csc^3\right.}{a^2}\\ &= -2a^3x+a^3\int \csc^3(c+dx)\,dx-a^3\int \sin^2(c+dx)\,dx\\ &= -2a^3x-\frac{2a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\\ &= -\frac{5a^3x}{2}-\frac{5a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3a^3\cos(c+dx)}{d}-\end{aligned}$$

Mathematica [A]
time = 0.80, size = 112, normalized size = 1.14

$$\frac{a^3\left(-20c-20dx+24\cos(c+dx)-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-20\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+2\sin(2(c+dx))+12\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-20*c - 20*d*x + 24*Cos[c + d*x] - 12*Cot[(c + d*x)/2] - Csc[(c + d*x
)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x
)/2]^2 + 2*Sin[2*(c + d*x)] + 12*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.18, size = 130, normalized size = 1.33

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3(-\cot(dx+c)-dx-c)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3(-\cot(dx+c)-dx-c)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(}{d}$
risch	$-\frac{5a^3x}{2}-\frac{ia^3e^{2i(dx+c)}}{8d}+\frac{3a^3e^{i(dx+c)}}{2d}+\frac{3a^3e^{-i(dx+c)}}{2d}+\frac{ia^3e^{-2i(dx+c)}}{8d}+\frac{a^3\left(e^{3i(dx+c)}+e^{i(dx+c)}-6ie^{2i(dx+c)}+6\right.}{d\left(e^{2i(dx+c)}-1\right)^2}$
norman	$\frac{-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{2a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{5a^3x\left(\tan^2\left(\frac{dx}{2}\right.\right.}{2}}{\tan\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-c
ot(d*x+c)))+3*a^3*(-cot(d*x+c)-d*x-c)+3*a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d
*x+c)))+a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.48, size = 124, normalized size = 1.27

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^3-12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)+6\,a^3(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 - 12*(d*x + c + 1/tan(d*x + c))*a
^3 + a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log
(cos(d*x + c) - 1)) + 6*a^3*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(c
os(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 159, normalized size = 1.62

$$-\frac{10\,a^3dx\cos(dx+c)^2-12\,a^3\cos(dx+c)^3-10\,a^3dx+10\,a^3\cos(dx+c)+5\left(a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-5\left(a^3\cos(dx+c)^2-a^3\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(a^3\cos(dx+c)^3+5\,a^3\cos(dx+c)\right)\sin(dx+c)}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 12*a^3*cos(d*x + c)^3 - 10*a^3*d*x + 10*a
^3*cos(d*x + c) + 5*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)
- 5*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*cos(d*
x + c)^3 + 5*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(90) = 180.
time = 0.48, size = 184, normalized size = 1.88

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,(dx+c)a^3+20\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 20*(d*x + c)*a^3 + 20*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (10*a^3*tan(1/2*d*x + 1/2*c
)^6 + 20*a^3*tan(1/2*d*x + 1/2*c)^5 - 27*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*a^
3*tan(1/2*d*x + 1/2*c)^3 - 36*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d
*x + 1/2*c) + a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

Mupad [B]
time = 8.68, size = 259, normalized size = 2.64

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{47\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{5\,a^3\,\mathrm{atan}\left(\frac{25\,a^6}{25\,a^6+25\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{25\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\,a^6+25\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (5*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) -
(8*a^3*tan(c/2 + (d*x)/2)^3 - 23*a^3*tan(c/2 + (d*x)/2)^2 - (47*a^3*tan(c/2
 + (d*x)/2)^4)/2 + 10*a^3*tan(c/2 + (d*x)/2)^5 + a^3/2 + 6*a^3*tan(c/2 + (d
*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (
d*x)/2)^6)) + (5*a^3*atan((25*a^6)/(25*a^6 + 25*a^6*tan(c/2 + (d*x)/2)) - (
25*a^6*tan(c/2 + (d*x)/2))/(25*a^6 + 25*a^6*tan(c/2 + (d*x)/2))))/d + (3*a^
3*tan(c/2 + (d*x)/2))/(2*d)
```

3.290 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$-3a^3x+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)\csc(c+$$

```
[Out] -3*a^3*x+1/2*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c)/d-
1/3*a^3*cot(d*x+c)^3/d-3/2*a^3*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3855, 3852, 8, 3853, 2718}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x + (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d - (3*a^
3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c
+ d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-3a^5 - 2a^5\csc(c+dx) + 2a^5\csc^2(c+dx) + 3a^5 \mathrm{cs}\right.}{a^2}$$

$$= -3a^3x + a^3\int \csc^4(c+dx)\,dx - a^3\int \sin(c+dx)\,dx$$

$$= -3a^3x + \frac{2a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} -$$

$$= -3a^3x + \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} -$$

Mathematica [A]
time = 0.35, size = 148, normalized size = 1.63

$$\frac{a^3\left(-72c - 72dx + 24\cos(c+dx) - 32\cot\left(\frac{1}{2}(c+dx)\right) - 9\csc^2\left(\frac{1}{2}(c+dx)\right) + 12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 9\sec^2\left(\frac{1}{2}(c+dx)\right) + 8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) - \frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) + 32\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-72*c - 72*d*x + 24*Cos[c + d*x] - 32*Cot[(c + d*x)/2] - 9*Csc[(c + d
*x)/2]^2 + 12*Log[Cos[(c + d*x)/2]] - 12*Log[Sin[(c + d*x)/2]] + 9*Sec[(c +
 d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[
c + d*x])/2 + 32*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A]
time = 0.19, size = 125, normalized size = 1.37

method	result

derivativedivides	$\frac{-\frac{a^3(\cos^3(dx+c))}{3\sin(dx+c)^3}+3a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3(-\cot(dx+c)-dx-c)+a^3(\cos(dx+c)+}{d}$
default	$\frac{-\frac{a^3(\cos^3(dx+c))}{3\sin(dx+c)^3}+3a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3(-\cot(dx+c)-dx-c)+a^3(\cos(dx+c)+}{d}$
risch	$-3a^3x+\frac{a^3e^{i(dx+c)}}{2d}+\frac{a^3e^{-i(dx+c)}}{2d}+\frac{a^3(-12ie^{4i(dx+c)}+9e^{5i(dx+c)}+36ie^{2i(dx+c)}-16i-9e^{i(dx+c)})}{3d(e^{2i(dx+c)}-1)^3}+\frac{a^3\ln(e^{i(}}{2}$
norman	$\frac{-\frac{a^3}{24d}-\frac{3a^3\tan(\frac{dx}{2}+\frac{c}{2})}{8d}-\frac{3a^3(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{2d}-\frac{23a^3(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{8d}+\frac{23a^3(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{8d}+\frac{3a^3(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{2d}+\frac{3a^3(\tan^{11}(}{8d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/sin(d*x+c)^3*cos(d*x+c)^3+3*a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-cot(d*x+c)-d*x-c)+
a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 117, normalized size = 1.29

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-9a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-6a^3(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))+\frac{4a^3}{\tan(dx+c)^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a^3 - 9*a^3*(2*cos(d*x + c)/(cos(d*x +
 c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*a^3*(2*cos(
d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d*x +
 c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(85) = 170.
time = 0.37, size = 180, normalized size = 1.98

$$-\frac{32a^3\cos(dx+c)^3-36a^3\cos(dx+c)-3(a^3\cos(dx+c)^2-a^3)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3(a^3\cos(dx+c)^2-a^3)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+6(6a^3dx\cos(dx+c)^2-2a^3\cos(dx+c)^3-6a^3dx-a^3\cos(dx+c))\sin(dx+c)}{12(d\cos(dx+c)^2-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*a^3*cos(d*x + c)^3 - 36*a^3*cos(d*x + c) - 3*(a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^3*cos(d*x + c)^2 - a
```

```
^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(6*a^3*d*x*cos(d*x + c)^2
 - 2*a^3*cos(d*x + c)^3 - 6*a^3*d*x - a^3*cos(d*x + c))*sin(d*x + c))/((d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.48, size = 161, normalized size = 1.77

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)a^3-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{48\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{22\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-33\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x +
 c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 33*a^3*tan(1/2*d*x + 1/2*
c) + 48*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (22*a^3*tan(1/2*d*x + 1/2*c)^3 -
 33*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*
d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.68, size = 249, normalized size = 2.74

$$\frac{3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{6\,a^3\,\mathrm{atan}\left(\frac{36\,a^6}{6\,a^6-36\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-36\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{11\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-13\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{34\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{3}}{d\,\left(8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{11\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (a
^3*log(tan(c/2 + (d*x)/2)))/(2*d) - (6*a^3*atan((36*a^6)/(6*a^6 - 36*a^6*ta
n(c/2 + (d*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - 36*a^6*tan(c/2 + (d
*x)/2))))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/3 - 13*a^3*tan(c/2 + (d*x)/2)^
3 + 11*a^3*tan(c/2 + (d*x)/2)^4 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*t
an(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) + (11*a^3*tan(c/2 + (d*x)/2)
)/(8*d)
```

3.291 $\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$-a^3x+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}-\frac{11a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^3\cot(c+$$

```
[Out] -a^3*x+13/8*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d-a^3*cot(d*x+c)^3/d-1
1/8*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{11a^3\cot(c+dx)\csc(c+dx)}{8d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (13*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cot[c + d*x])/d - (a
^3*Cot[c + d*x]^3)/d - (11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[
c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^2(c+dx)+3a^3\cot^2(c+dx)\csc(c+dx)+3\right.$$

$$= a^3\int \cot^2(c+dx)\,dx + a^3\int \cot^2(c+dx)\csc^3(c+dx$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^3 c}{}$$

$$= -a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cot(c+dx)}{d} -$$

$$= -a^3x + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cot(c+dx)}{d} -$$

Mathematica [A]
time = 0.43, size = 133, normalized size = 1.33

$$\frac{a^3\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)-8\left(8c+8dx-13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+13\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)\right)-\csc^4\left(\frac{1}{2}(c+dx)\right)(1+4\sin(c+dx))\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 -
 8*(8*c + 8*d*x - 13*Log[Cos[(c + d*x)/2]] + 13*Log[Sin[(c + d*x)/2]] - 8*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4) - Csc[(c + d*x)/2]^4*(1 + 4*Sin[c + d*x])
))/(64*d)
```

Maple [A]

time = 0.22, size = 164, normalized size = 1.64

method	result
risch	$-a^3x+\frac{a^3\left(11\,\mathrm{e}^{7i(dx+c)}-19\,\mathrm{e}^{5i(dx+c)}+16i\mathrm{e}^{6i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}+11\,\mathrm{e}^{i(dx+c)}-16i\mathrm{e}^{2i(dx+c)}\right)}{4d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}+\frac{13a^3\ln\left(\mathrm{e}^{i(dx+c)}+1\right.}{8d}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^3\left(\cos^3(dx+c)\right)}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^3\left(\cos^3(dx+c)\right)}{\sin(dx+c)^3}+3a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
norman	$-\frac{a^3}{64d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{27a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{5a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{a^3\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*
cos(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-a^3/sin(d*x+c)^3*cos(d*x+c)^3+3*a
^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x
+c)))+a^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.48, size = 147, normalized size = 1.47

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a^3}{\tan(dx+c)^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*a^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - 12*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(co
s(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^3/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(94) = 188$.
time = 0.38, size = 190, normalized size = 1.90

$$-\frac{16 a^3 dx \cos(dx+c)^4 - 32 a^3 dx \cos(dx+c)^2 - 22 a^3 \cos(dx+c)^3 + 16 a^3 dx + 16 a^3 \cos(dx+c)\sin(dx+c) + 26 a^3 \cos(dx+c) - 13 \left(a^3 \cos(dx+c)^4 - 2 a^3 \cos(dx+c)^2 + a^3\right) \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 13 \left(a^3 \cos(dx+c)^4 - 2 a^3 \cos(dx+c)^2 + a^3\right) \log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{16 \left(d \cos(dx+c)^4 - 2 d \cos(dx+c)^2 + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*d*x*cos(d*x + c)^4 - 32*a^3*d*x*cos(d*x + c)^2 - 22*a^3*cos(d
*x + c)^3 + 16*a^3*d*x + 16*a^3*cos(d*x + c)*sin(d*x + c) + 26*a^3*cos(d*x
+ c) - 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 13*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/
2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.50, size = 174, normalized size = 1.74

$$\frac{3 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 24 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 72 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 192 (dx+c) a^3 - 312 a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + 24 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{650 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 24 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 72 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 24 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 3 a^3}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4}}{192 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^
3*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*a^3 - 312*a^3*log(abs(tan(1/2*d*x
+ 1/2*c))) + 24*a^3*tan(1/2*d*x + 1/2*c) + (650*a^3*tan(1/2*d*x + 1/2*c)^4
- 24*a^3*tan(1/2*d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^3*ta
n(1/2*d*x + 1/2*c) - 3*a^3)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.89, size = 237, normalized size = 2.37

$$\frac{3 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{8 d} - \frac{a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8 d} - \frac{a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{64 d} - \frac{3 a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{8 d} + \frac{a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{8 d} + \frac{a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4}{64 d} - \frac{2 a^3 \operatorname{atan}\left(\frac{8 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + 13 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{13 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) - 8 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} - \frac{13 a^3 \ln\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{8 d} - \frac{a^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)}{8 d} + \frac{a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^3*cot(c/2 + (d*x)/2)^3)/(8*d) - (a^
3*cot(c/2 + (d*x)/2)^4)/(64*d) - (3*a^3*cot(c/2 + (d*x)/2)^2)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (2*a^3*at
an((8*cos(c/2 + (d*x)/2) + 13*sin(c/2 + (d*x)/2))/(13*cos(c/2 + (d*x)/2) -
8*sin(c/2 + (d*x)/2))))/d - (13*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/(8*d) - (a^3*cot(c/2 + (d*x)/2))/(8*d) + (a^3*tan(c/2 + (d*x)/2))/(8*d
)
```

3.292 $\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=100

$$\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{3a^3\cot(c+dx)\text{c}}{4d}$$

```
[Out] 7/8*a^3*arctanh(cos(d*x+c))/d-4/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d
-1/8*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2691, 3855, 2687, 30, 3853, 14}

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (a^3*C
ot[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(\right.\\ &= a^3\int \cot^2(c+dx)\csc(c+dx)\,dx+a^3\int \cot^2(c+dx)\,\mathrm{cs}\\ &= -\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{3a^3\cot(c+dx)\csc^3(c+}{4d}\\ &= \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot(c+}{}\\ &= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(}{5}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(100) = 200.
time = 0.10, size = 267, normalized size = 2.67

$$a^3\left(\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{30d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{59\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{480d}-\frac{3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{160d}+\frac{7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{30d}+\frac{59\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{480d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{160d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((17*Cot[(c + d*x)/2])/(30*d) - Csc[(c + d*x)/2]^2/(32*d) - (59*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(480*d) - (3*Csc[(c + d*x)/2]^4)/(64*d) - (Co
t[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(160*d) + (7*Log[Cos[(c + d*x)/2]])/(8*d
) - (7*Log[Sin[(c + d*x)/2]])/(8*d) + Sec[(c + d*x)/2]^2/(32*d) + (3*Sec[(c
 + d*x)/2]^4)/(64*d) - (17*Tan[(c + d*x)/2])/(30*d) + (59*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(480*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(160*d)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(90) = 180$.
time = 0.24, size = 185, normalized size = 1.85

method	result
risch	$\frac{a^3(360ie^{8i(dx+c)}+15\,e^{9i(dx+c)}-960ie^{6i(dx+c)}-390\,e^{7i(dx+c)}+400ie^{4i(dx+c)}-320ie^{2i(dx+c)}+390\,e^{3i(dx+c)}+136i-15\,e}{60d\left(e^{2i(dx+c)}-1\right)^5}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+3a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^3\left(\cos^3\right.}{\sin(d}}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+3a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^3\left(\cos^3\right.}{\sin(d}}{d}$
norman	$-\frac{a^3}{160d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{37a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{17a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}+\frac{37a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{37a^3\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+3*
a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d
*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-a^3/sin(d*x+c)^3*cos(d*x+c)^3+a^3*(-1/
2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 155, normalized size = 1.55

$$-\frac{45\,a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{240\,a^3}{\tan(dx+c)^3}+\frac{16\left(5\,\tan(dx+c)^2+3\right)a^3}{\tan(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*a^3*(2
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x +
```

```
c) - 1)) + 240*a^3/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d*x +
 c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(90) = 180$.
time = 0.35, size = 190, normalized size = 1.90

$$\frac{272\,a^3\cos\left(dx+c\right)^5-320\,a^3\cos\left(dx+c\right)^3+105\,\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\,\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\,\left(a^3\cos\left(dx+c\right)^3-7\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*a^3*cos(d*x + c)^5 - 320*a^3*cos(d*x + c)^3 + 105*(a^3*cos(d*x +
 c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 105*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 30*(a^3*cos(d*x + c)^3 - 7*a^3*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(90) = 180$.
time = 0.52, size = 196, normalized size = 1.96

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+130\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-420\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1918\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+420\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-130\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 130*a
^3*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*tan(1/2*d*x + 1/2*c)^2 - 840*a^3*log(ab
s(tan(1/2*d*x + 1/2*c))) - 420*a^3*tan(1/2*d*x + 1/2*c) + (1918*a^3*tan(1/2
*d*x + 1/2*c)^5 + 420*a^3*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*tan(1/2*d*x + 1/
2*c)^3 - 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 45*a^3*tan(1/2*d*x + 1/2*c) - 6*a
^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 9.30, size = 291, normalized size = 2.91

$$-\frac{a^3\,\left(6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-45\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-130\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6-420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+130\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+840\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}{960\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(a^3*(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 - 45*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^9 + 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 1
30*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^7 + 420*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 420*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^3 + 130*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5
))/(960*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.293 $\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=124

$$\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{7a^3\cot(c+dx)\csc(c+dx)}{16d}-\frac{17a^3\cot(c+dx)}{24d}$$

```
[Out] 7/16*a^3*arctanh(cos(d*x+c))/d-4/3*a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/
d+7/16*a^3*cot(d*x+c)*csc(d*x+c)/d-17/24*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*
a^3*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^3(c+dx)}{3d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{17a^3\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{7a^3\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (7*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^
3*Cot[c + d*x]^5)/(5*d) + (7*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (17*a^
3*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(
6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\operatorname{csc}\right. \\
&= a^3\int \cot^2(c+dx)\csc^2(c+dx)\,dx+a^3\int \cot^2(c+dx \\
&= -\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{a^3\cot(c+dx)\csc^5(c}{6d} \\
&= -\frac{a^3\cot^3(c+dx)}{3d}+\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}-\frac{17}{} \\
&= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{3a^3\operatorname{co}}{} \\
&= \frac{7a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{3a^3\operatorname{co}}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 252 vs. 2(124) = 248.
time = 2.87, size = 252, normalized size = 2.03

$$
-\frac{a^3\left(\csc^6\left(\frac{1}{2}(c+dx)\right)(18+5\csc(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(34+90\csc(c+dx))-2\csc^2\left(\frac{1}{2}(c+dx)\right)(176+105\csc(c+dx))-840\csc(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(97+159\cos(c+dx)+44\cos(2(c+dx)))\sec^6\left(\frac{1}{2}(c+dx)\right)+840\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-1440\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-320\csc^7(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)(1+\sin(c+dx))^3}{1920d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(a^3*(Csc[(c + d*x)/2]^6*(18 + 5*Csc[c + d*x]) + Csc[(c + d*x)/2]^4
*(34 + 90*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^2*(176 + 105*Csc[c + d*x]) - 8
40*Csc[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (97 + 159
*Cos[c + d*x] + 44*Cos[2*(c + d*x)])*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^2 - 1440*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c +
 d*x]^7*Sin[(c + d*x)/2]^6)*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]

time = 0.23, size = 222, normalized size = 1.79

method	result
risch	$-\frac{a^3(105\,\mathrm{e}^{11i(dx+c)}+365\,\mathrm{e}^{9i(dx+c)}-240i\mathrm{e}^{10i(dx+c)}-1110\,\mathrm{e}^{7i(dx+c)}+2160i\mathrm{e}^{8i(dx+c)}-1110\,\mathrm{e}^{5i(dx+c)}-1760i\mathrm{e}^{6i(dx+c)}+36}{120d\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+3a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right.}{15\sin(dx+c)^3}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+3a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right.}{15\sin(dx+c)^3}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^3-1/8/sin(d*x+c)^4*cos(d*x+c)^3-1/16
/sin(d*x+c)^2*cos(d*x+c)^3-1/16*cos(d*x+c)-1/16*ln(csc(d*x+c)-cot(d*x+c)))+
3*a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+3*a^3
*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d*x+
c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*a^3/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]

time = 0.29, size = 200, normalized size = 1.61

$$-\frac{5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+90\,a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{160\,a^3}{\tan(dx+c)^3}+\frac{96\left(5\,\tan(dx+c)^2+3\right)a^3}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a^3*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
```

```
s(d*x + c) - 1)) + 160*a^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^3/t
an(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(112) = 224$.
time = 0.36, size = 227, normalized size = 1.83

$$-\frac{210 a^3 \cos(dx+c)^5 - 80 a^3 \cos(dx+c)^3 - 210 a^3 \cos(dx+c) - 105 \left(a^3 \cos(dx+c)^6 - 3 a^3 \cos(dx+c)^4 + 3 a^3 \cos(dx+c)^2 - a^3\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 105 \left(a^3 \cos(dx+c)^6 - 3 a^3 \cos(dx+c)^4 + 3 a^3 \cos(dx+c)^2 - a^3\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 32 \left(11 a^3 \cos(dx+c)^5 - 20 a^3 \cos(dx+c)^3\right) \sin(dx+c)}{480 \left(d \cos(dx+c)^6 - 3 d \cos(dx+c)^4 + 3 d \cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(210*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 210*a^3*cos(d*x +
c) - 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2
- a^3)*log(1/2*cos(d*x + c) + 1/2) + 105*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 32*(1
1*a^3*cos(d*x + c)^5 - 20*a^3*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)
^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(112) = 224$.
time = 0.50, size = 228, normalized size = 1.84

$$\frac{5 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 36 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 105 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 140 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 15 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 840 a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 600 a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{2058 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 + 600 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 + 15 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 - 140 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 105 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 36 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 5 a^3}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 105*
a^3*tan(1/2*d*x + 1/2*c)^4 + 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 840*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 600*a^3*tan(1/2
*d*x + 1/2*c) + (2058*a^3*tan(1/2*d*x + 1/2*c)^6 + 600*a^3*tan(1/2*d*x + 1/
2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 140*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
05*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) - 5*a^3)/tan(1/
2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.67, size = 339, normalized size = 2.73

$$-\frac{a^3\left(5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+600\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-600\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\right)}{1920\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] -(a^3*(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 - 36*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 36*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -
 105*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 140*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 600*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 600*cos(c/2 + (d*x)/2)^7*sin(c/2
 + (d*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2
+ (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)
```

3.294 $\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=137

$$\frac{21a^4x}{16}-\frac{7a^4\cos^3(c+dx)}{8d}+\frac{21a^4\cos(c+dx)\sin(c+dx)}{16d}-\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}-3\cos^3(c+dx)$$

```
[Out] 21/16*a^4*x-7/8*a^4*cos(d*x+c)^3/d+21/16*a^4*cos(d*x+c)*sin(d*x+c)/d-1/6*a*
cos(d*x+c)^3*(a+a*sin(d*x+c))^3/d-3/10*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c))^2/
d-21/40*cos(d*x+c)^3*(a^4+a^4*sin(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2757, 2748, 2715, 8}

$$-\frac{7a^4\cos^3(c+dx)}{8d}-\frac{21\cos^3(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{40d}+\frac{21a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{21a^4x}{16}-\frac{3\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{10d}-\frac{a\cos^3(c+dx)(a\sin(c+dx)+a)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (21*a^4*x)/16 - (7*a^4*Cos[c + d*x]^3)/(8*d) + (21*a^4*Cos[c + d*x]*Sin[c +
 d*x])/(16*d) - (a*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6*d) - (3*Cos[c
+ d*x]^3*(a^2 + a^2*Sin[c + d*x])^2)/(10*d) - (21*Cos[c + d*x]^3*(a^4 + a^4
*Sin[c + d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} + \frac{1}{2}(3a)\int \cos^2(c+dx)(a+a\sin$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c\right.}{10d}$$

$$= -\frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c+dx)\left(a^2+a^2\sin(c\right.}{10d}$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} - \frac{a\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d} - \frac{3\cos^3(c}{}$$

$$= -\frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(c+dx)(}{}$$

$$= \frac{21a^4x}{16} - \frac{7a^4\cos^3(c+dx)}{8d} + \frac{21a^4\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^3(}{}$$

Mathematica [A]
time = 0.31, size = 151, normalized size = 1.10

$$-\frac{a^4\cos^3(c+dx)\left(630\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}\,(448-373\sin(c+dx)-331\sin^2(c+dx)-94\sin^3(c+dx)+158\sin^4(c+dx)+152\sin^5(c+dx)+40\sin^6(c+dx))\right)}{240d(-1+\sin(c+dx))^2(1+\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/240*(a^4*Cos[c + d*x]^3*(630*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]]*Sqrt
[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(448 - 373*Sin[c + d*x] - 331*S
in[c + d*x]^2 - 94*Sin[c + d*x]^3 + 158*Sin[c + d*x]^4 + 152*Sin[c + d*x]^5
 + 40*Sin[c + d*x]^6)))/(d*(-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^(3/2))
```

Maple [A]
time = 0.20, size = 182, normalized size = 1.33

method	result
risch	$\frac{21a^4x}{16} - \frac{3a^4\cos(dx+c)}{2d} + \frac{a^4\sin(6dx+6c)}{192d} + \frac{a^4\cos(5dx+5c)}{20d} - \frac{13a^4\sin(4dx+4c)}{64d} - \frac{5a^4\cos(3dx+3c)}{12d} + \frac{15a^4\sin}{}$

derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\sin(dx+c)\cos(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+4a^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+\right.}{5}\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{8}+\frac{\sin(dx+c)\cos(dx+c)}{16}+\frac{dx}{16}+\frac{c}{16}\right)+4a^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+\right.}{5}\right.}{}$
norman	$\frac{\frac{21a^4x}{16}-\frac{56a^4}{15d}-\frac{5a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{235a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{63a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{63a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{235a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*cos(d*x+c)^3*sin(d*x+c)+1/16*s
in(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c)+4*a^4*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3
-2/15*cos(d*x+c)^3)+6*a^4*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(
d*x+c)+1/8*d*x+1/8*c)-4/3*cos(d*x+c)^3*a^4+a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1
/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 128, normalized size = 0.93

$$-\frac{1280\,a^4\cos\left(dx+c\right)^3-256\left(3\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^3\right)a^4+5\left(4\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\sin\left(4\,dx+4\,c\right)\right)a^4-180\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-240\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/960*(1280*a^4*cos(d*x + c)^3 - 256*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)
*a^4 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^4 -
180*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 240*(2*d*x + 2*c + sin(2*d*x + 2
*c))*a^4)/d
```

Fricas [A]
time = 0.35, size = 85, normalized size = 0.62

$$\frac{192\,a^4\cos\left(dx+c\right)^5-640\,a^4\cos\left(dx+c\right)^3+315\,a^4dx+5\left(8\,a^4\cos\left(dx+c\right)^5-86\,a^4\cos\left(dx+c\right)^3+63\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/240*(192*a^4*cos(d*x + c)^5 - 640*a^4*cos(d*x + c)^3 + 315*a^4*d*x + 5*(8
*a^4*cos(d*x + c)^5 - 86*a^4*cos(d*x + c)^3 + 63*a^4*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(128) = 256$.
time = 0.55, size = 381, normalized size = 2.78

$$\begin{cases}\frac{a^4x\sin^6(c+dx)}{16}+\frac{3a^4x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3a^4x\sin^4(c+dx)}{4}+\frac{3a^4x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{a^4x\sin^2(c+dx)}{2}+\frac{a^4x\cos^6(c+dx)}{16}+\frac{3a^4x\cos^4(c+dx)}{4}+\frac{a^4x\cos^2(c+dx)}{2}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{16d}-\frac{a^4\sin^3(c+dx)\cos^3(c+dx)}{6d}+\frac{3a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{4a^4\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{a^4\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{3a^4\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{8a^4\cos^5(c+dx)}{15d}-\frac{4a^4\cos^3(c+dx)}{3d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^4\cos^2(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x*sin(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**4*x*sin(c + d*x)**4/4 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x
)**4/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + a**4*x*sin(c + d*x)*
*2/2 + a**4*x*cos(c + d*x)**6/16 + 3*a**4*x*cos(c + d*x)**4/4 + a**4*x*cos(
c + d*x)**2/2 + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**4*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/(4*d) - 4*
a**4*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - a**4*sin(c + d*x)*cos(c + d*x)
**5/(16*d) - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a**4*sin(c + d*x)*
cos(c + d*x)/(2*d) - 8*a**4*cos(c + d*x)**5/(15*d) - 4*a**4*cos(c + d*x)**3
/(3*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*cos(c)**2, True))
```

Giac [A]
time = 0.47, size = 106, normalized size = 0.77

$$\frac{21}{16}a^4x+\frac{a^4\cos(5\,dx+5\,c)}{20\,d}-\frac{5\,a^4\cos(3\,dx+3\,c)}{12\,d}-\frac{3\,a^4\cos(dx+c)}{2\,d}+\frac{a^4\sin(6\,dx+6\,c)}{192\,d}-\frac{13\,a^4\sin(4\,dx+4\,c)}{64\,d}+\frac{15\,a^4\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 21/16*a^4*x + 1/20*a^4*cos(5*d*x + 5*c)/d - 5/12*a^4*cos(3*d*x + 3*c)/d - 3
/2*a^4*cos(d*x + c)/d + 1/192*a^4*sin(6*d*x + 6*c)/d - 13/64*a^4*sin(4*d*x
+ 4*c)/d + 15/64*a^4*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.73, size = 349, normalized size = 2.55

$$\frac{21\,a^4\,x}{16}-\frac{\frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{63\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\frac{235\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}-\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{a^4\,(315\,c+315\,d\,x)}{240}-\frac{a^4\,(315\,c+315\,d\,x-896)}{240}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(\frac{a^4\,(315\,c+315\,d\,x)}{40}-\frac{a^4\,(1890\,c+1890\,d\,x-1920)}{240}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^4\,(315\,c+315\,d\,x)}{40}-\frac{a^4\,(1890\,c+1890\,d\,x-3456)}{240}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^4\,(315\,c+315\,d\,x)}{16}-\frac{a^4\,(4725\,c+4725\,d\,x-3840)}{240}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{a^4\,(315\,c+315\,d\,x)}{16}-\frac{a^4\,(4725\,c+4725\,d\,x-9600)}{240}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^4\,(315\,c+315\,d\,x)}{12}-\frac{a^4\,(6300\,c+6300\,d\,x-8960)}{240}\right)+\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (21*a^4*x)/16 - ((63*a^4*tan(c/2 + (d*x)/2)^7)/4 - (63*a^4*tan(c/2 + (d*x)/
2)^5)/4 - (235*a^4*tan(c/2 + (d*x)/2)^3)/24 + (235*a^4*tan(c/2 + (d*x)/2)^9
)/24 - (5*a^4*tan(c/2 + (d*x)/2)^11)/8 + (a^4*(315*c + 315*d*x))/240 - (a^4
*(315*c + 315*d*x - 896))/240 + tan(c/2 + (d*x)/2)^10*((a^4*(315*c + 315*d*
x))/40 - (a^4*(1890*c + 1890*d*x - 1920))/240) + tan(c/2 + (d*x)/2)^2*((a^4
*(315*c + 315*d*x))/40 - (a^4*(1890*c + 1890*d*x - 3456))/240) + tan(c/2 +
(d*x)/2)^4*((a^4*(315*c + 315*d*x))/16 - (a^4*(4725*c + 4725*d*x - 3840))/2
40) + tan(c/2 + (d*x)/2)^8*((a^4*(315*c + 315*d*x))/16 - (a^4*(4725*c + 472
5*d*x - 9600))/240) + tan(c/2 + (d*x)/2)^6*((a^4*(315*c + 315*d*x))/12 - (a
^4*(6300*c + 6300*d*x - 8960))/240) + (5*a^4*tan(c/2 + (d*x)/2))/8)/(d*(tan
(c/2 + (d*x)/2)^2 + 1)^6)
```

3.295 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=117

$$\frac{5a^4x}{2}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^4\cos(c+dx)}{d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos^5(c+dx)}{5d}+\frac{5a^4\cos(c+dx)\sin(c+$$

```
[Out] 5/2*a^4*x-a^4*arctanh(cos(d*x+c))/d+a^4*cos(d*x+c)/d-7/3*a^4*cos(d*x+c)^3/d
+1/5*a^4*cos(d*x+c)^5/d+5/2*a^4*cos(d*x+c)*sin(d*x+c)/d-a^4*cos(d*x+c)^3*si
n(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2952, 2715, 8, 2672, 327, 212, 2645, 30, 2648, 14}

$$\frac{a^4\cos^5(c+dx)}{5d}-\frac{7a^4\cos^3(c+dx)}{3d}+\frac{a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos^3(c+dx)}{d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*x)/2 - (a^4*ArcTanh[Cos[c + d*x]])/d + (a^4*Cos[c + d*x])/d - (7*a^4
*Cos[c + d*x]^3)/(3*d) + (a^4*Cos[c + d*x]^5)/(5*d) + (5*a^4*Cos[c + d*x]*S
in[c + d*x])/(2*d) - (a^4*Cos[c + d*x]^3*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^4\,dx = \int \left(4a^4\cos^2(c+dx) + a^4\cos(c+dx)\cot(c+dx) + 6a^4\right.$$

$$= a^4\int \cos(c+dx)\cot(c+dx)\,dx + a^4\int \cos^2(c+dx)\sin$$

$$= \frac{2a^4\cos(c+dx)\sin(c+dx)}{d} - \frac{a^4\cos^3(c+dx)\sin(c+d}{d}$$

$$= 2a^4x + \frac{a^4\cos(c+dx)}{d} - \frac{2a^4\cos^3(c+dx)}{d} + \frac{5a^4\cos(c+}{}$$

$$= \frac{5a^4x}{2} - \frac{a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^4\cos(c+dx)}{d} - \frac{7a^4}{}$$

Mathematica [A]
time = 0.79, size = 95, normalized size = 0.81

$$\frac{a^4\left(-150\cos(c+dx) - 125\cos(3(c+dx)) + 3\cos(5(c+dx)) + 30\left(20c + 20dx - 8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 8\sin(2(c+dx)) - \sin(4(c+dx))\right)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(-150*Cos[c + d*x] - 125*Cos[3*(c + d*x)] + 3*Cos[5*(c + d*x)] + 30*(2
0*c + 20*d*x - 8*Log[Cos[(c + d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] + 8*Sin[2*
(c + d*x)] - Sin[4*(c + d*x)])))/(240*d)
```

Maple [A]
time = 0.20, size = 149, normalized size = 1.27

method	result
risch	$\frac{5a^4x}{2} - \frac{5a^4e^{i(dx+c)}}{16d} - \frac{5a^4e^{-i(dx+c)}}{16d} + \frac{a^4\ln\left(e^{i(dx+c)}-1\right)}{d} - \frac{a^4\ln\left(e^{i(dx+c)}+1\right)}{d} + \frac{a^4\cos(5dx+5c)}{80d} - \frac{a^4\sin(4a}{8d}$
derivativedivides	$\frac{a^4(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2\left(\cos^3(dx+c)\right)a^4+4a^4\left(-\frac{\left(\cos^3(dx+c)\right)}{4}\right.}{d}$
default	$\frac{a^4(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+4a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2\left(\cos^3(dx+c)\right)a^4+4a^4\left(-\frac{\left(\cos^3(dx+c)\right)}{4}\right.}{d}$
norman	$\frac{\frac{5a^4x}{2}-\frac{34a^4}{15d}+\frac{3a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{14a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{14a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{25a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+4*a^4*(1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-2*cos(d*x+c)^3*a^4+4*a^4*(-1/4*cos(d*x+c)^3*sin(d*x+c)
+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c)+a^4*(-1/5*sin(d*x+c)^2*cos(d*x+c)
^3-2/15*cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.07

$$-\frac{240\,a^4\cos\left(dx+c\right)^3-8\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^4-15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-120\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4-60\,a^4(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right))}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(240*a^4*cos(d*x + c)^3 - 8*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
4 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 120*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*a^4 - 60*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x
+ c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 115, normalized size = 0.98

$$\frac{6\,a^4\cos\left(dx+c\right)^5-70\,a^4\cos\left(dx+c\right)^3+75\,a^4dx+30\,a^4\cos\left(dx+c\right)-15\,a^4\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\,a^4\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(2\,a^4\cos\left(dx+c\right)^3-5\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/30*(6*a^4*cos(d*x + c)^5 - 70*a^4*cos(d*x + c)^3 + 75*a^4*d*x + 30*a^4*co
s(d*x + c) - 15*a^4*log(1/2*cos(d*x + c) + 1/2) + 15*a^4*log(-1/2*cos(d*x +
 c) + 1/2) - 15*(2*a^4*cos(d*x + c)^3 - 5*a^4*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int 4\sin\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(cos(c + d*x)**2*csc(c + d*x), x) + Integral(4*sin(c + d*x)*c
os(c + d*x)**2*csc(c + d*x), x) + Integral(6*sin(c + d*x)**2*cos(c + d*x)**
2*csc(c + d*x), x) + Integral(4*sin(c + d*x)**3*cos(c + d*x)**2*csc(c + d*x
), x) + Integral(sin(c + d*x)**4*cos(c + d*x)**2*csc(c + d*x), x))
```

Giac [A]
time = 0.49, size = 181, normalized size = 1.55

$$\frac{75\left(dx+c\right)a^4+30\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+150\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+210\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+300\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-210\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+34\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/30*(75*(d*x + c)*a^4 + 30*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(45*a^4*
tan(1/2*d*x + 1/2*c)^9 + 150*a^4*tan(1/2*d*x + 1/2*c)^8 + 210*a^4*tan(1/2*d
*x + 1/2*c)^7 + 300*a^4*tan(1/2*d*x + 1/2*c)^6 + 40*a^4*tan(1/2*d*x + 1/2*c
)^4 - 210*a^4*tan(1/2*d*x + 1/2*c)^3 + 20*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a
^4*tan(1/2*d*x + 1/2*c) + 34*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 10.42, size = 295, normalized size = 2.52

$$\frac{a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^4\,\mathrm{atan}\left(\frac{25\,a^8}{10\,a^8-25\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{10\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{10\,a^8-25\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+10\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+14\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+20\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{8\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-14\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{4\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-3\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{34\,a^4}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^4)/sin(c + d*x),x)
```

```
[Out] (a^4*log(tan(c/2 + (d*x)/2)))/d + (5*a^4*atan((25*a^8)/(10*a^8 - 25*a^8*tan
(c/2 + (d*x)/2)) + (10*a^8*tan(c/2 + (d*x)/2))/(10*a^8 - 25*a^8*tan(c/2 + (
d*x)/2))))/d - ((4*a^4*tan(c/2 + (d*x)/2)^2)/3 - 14*a^4*tan(c/2 + (d*x)/2)^
3 + (8*a^4*tan(c/2 + (d*x)/2)^4)/3 + 20*a^4*tan(c/2 + (d*x)/2)^6 + 14*a^4*t
an(c/2 + (d*x)/2)^7 + 10*a^4*tan(c/2 + (d*x)/2)^8 + 3*a^4*tan(c/2 + (d*x)/2
)^9 + (34*a^4)/15 - 3*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 +
10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8
+ tan(c/2 + (d*x)/2)^10 + 1))
```

3.296 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$\frac{17a^4x}{8}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot(c+dx)}{d}+\frac{23a^4\cos(c+dx)\sin(c+dx)}{8d}$$

```
[Out] 17/8*a^4*x-4*a^4*arctanh(cos(d*x+c))/d+4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c
)^3/d-a^4*cot(d*x+c)/d+23/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*
sin(d*x+c)^3/d
```

Rubi [A]

time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 2715, 2713}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{17a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a\right.}{a^2}$$

$$= 5a^4x + a^4\int \csc^2(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(4a^4\right)\int \csc$$

$$= 5a^4x - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^4\cos(c+dx)\sin(c+dx)}{2d} + $$

$$= \frac{5a^4x}{2} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+}{3d}$$

$$= \frac{17a^4x}{8} - \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c}{3d}$$

Mathematica [A]
time = 1.12, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-48\cos(c+dx)-147\cos(3(c+dx))+3\cos(5(c+dx))+408c\sin(c+dx)+408dx\sin(c+dx)-768\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+768\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

Maple [A]
time = 0.18, size = 136, normalized size = 1.17

method	result
derivativedivides	$\frac{a^4(-\cot(dx+c)-dx-c)+4a^4(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+6a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{4\left(\cos^3(dx+c)\right)}{3}}{d}$
default	$\frac{a^4(-\cot(dx+c)-dx-c)+4a^4(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+6a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{4\left(\cos^3(dx+c)\right)}{3}}{d}$
risch	$\frac{17a^4x}{8}-\frac{3ia^4e^{2i(dx+c)}}{4d}+\frac{3a^4e^{i(dx+c)}}{2d}+\frac{3a^4e^{-i(dx+c)}}{2d}+\frac{3ia^4e^{-2i(dx+c)}}{4d}-\frac{2ia^4}{d(e^{2i(dx+c)}-1)}-\frac{4a^4\ln(e^{i(dx+c)}+}{d}$
norman	$\frac{-\frac{a^4}{2d}+\frac{17a^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{27a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{27a^4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{17a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{17a^4x\,\mathrm{t}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-cot(d*x+c)-d*x-c)+4*a^4*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+6
*a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-4/3*cos(d*x+c)^3*a^4+a^4*(-1
/4*cos(d*x+c)^3*sin(d*x+c)+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A]

time = 0.48, size = 117, normalized size = 1.01

$$-\frac{128\,a^4\cos(dx+c)^3-3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^4-144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))a^4+96\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^4-192\,a^4(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 14
4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4
- 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A]

time = 0.36, size = 135, normalized size = 1.16

$$\frac{6\,a^4\cos(dx+c)^5-81\,a^4\cos(dx+c)^3-48\,a^4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+48\,a^4\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+51\,a^4\cos(dx+c)-(32\,a^4\cos(dx+c)^3-51\,a^4dx-96\,a^4\cos(dx+c))\sin(dx+c)}{24\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
```

```
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 194, normalized size = 1.67

$$\frac{51\,(dx+c)a^4+96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+93\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-192\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-93\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-256\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-69\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-64\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 8.83, size = 295, normalized size = 2.54

$$\frac{4\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^4\,\mathrm{atan}\left(\frac{289\,a^8}{16\,\left(34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{34\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{34\,a^8-\frac{289\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}+\frac{-\frac{25\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{2}-\frac{39\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{2}+32\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\frac{19\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{2}+\frac{128\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\frac{15\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{32\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-a^4}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^4)/sin(c + d*x)^2,x)
```

```
[Out] (4*a^4*log(tan(c/2 + (d*x)/2)))/d + (17*a^4*atan((289*a^8)/(16*(34*a^8 - (2
89*a^8*tan(c/2 + (d*x)/2))/16)) + (34*a^8*tan(c/2 + (d*x)/2))/(34*a^8 - (28
9*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + ((15*a^4*tan(c/2 + (d*x)/2)^2)/2 +
(128*a^4*tan(c/2 + (d*x)/2)^3)/3 + (19*a^4*tan(c/2 + (d*x)/2)^4)/2 + 32*a^4
*tan(c/2 + (d*x)/2)^5 - (39*a^4*tan(c/2 + (d*x)/2)^6)/2 - (25*a^4*tan(c/2 +
 (d*x)/2)^8)/2 - a^4 + (32*a^4*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 + (d*x)
/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/
2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^4*tan(c/2 + (d*x)/2))/(2*d)
```

3.297 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$\frac{3x}{8a}+\frac{\cos(c+dx)}{ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos(c+dx)\sin^3(c+dx)}{4ad}$$

```
[Out] 3/8*x/a+cos(d*x+c)/a/d-2/3*cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d-3/8*cos(d*
x+c)*sin(d*x+c)/a/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2918, 2715, 8, 2713}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + Cos[c + d*x]/(a*d) - (2*Cos[c + d*x]^3)/(3*a*d) + Cos[c + d*x
]^5/(5*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]*Sin[c +
 d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sin^4(c+dx)\,dx}{a} - \frac{\int \sin^5(c+dx)\,dx}{a} \\
&= -\frac{\cos(c+dx)\sin^3(c+dx)}{4ad} + \frac{3\int \sin^2(c+dx)\,dx}{4a} + \frac{\mathrm{Subst}\left(\int (1-2x^2+x \right.}{a} \\
&= \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c+dx)}{8ad} \\
&= \frac{3x}{8a} + \frac{\cos(c+dx)}{ad} - \frac{2\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{3\cos(c+dx)\sin(c}{8ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 281 vs. 2(104) = 208.
time = 3.73, size = 281, normalized size = 2.70

$$\frac{1}{480}\left(\frac{180x}{a}+\frac{300\cos(c)\cos(dx)}{ad}-\frac{50\cos(3c)\cos(3dx)}{ad}+\frac{6\cos(5c)\cos(5dx)}{ad}-\frac{120\cos(2dx)\sin(2c)}{ad}+\frac{15\cos(4dx)\sin(4c)}{ad}-\frac{300\sin(c)\sin(dx)}{ad}-\frac{120\cos(2c)\sin(2dx)}{ad}+\frac{50\sin(3c)\sin(3dx)}{ad}+\frac{15\cos(4c)\sin(4dx)}{ad}-\frac{6\sin(5c)\sin(5dx)}{ad}-\frac{60\sin\left(\frac{dx}{2}\right)}{ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{30\sin(c+dx)}{ad(1+\sin(c+dx))}+\frac{60\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a+a\sin(c+dx))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((180*x)/a + (300*Cos[c]*Cos[d*x])/(a*d) - (50*Cos[3*c]*Cos[3*d*x])/(a*d) +
 (6*Cos[5*c]*Cos[5*d*x])/(a*d) - (120*Cos[2*d*x]*Sin[2*c])/(a*d) + (15*Cos[
4*d*x]*Sin[4*c])/(a*d) - (300*Sin[c]*Sin[d*x])/(a*d) - (120*Cos[2*c]*Sin[2*
d*x])/(a*d) + (50*Sin[3*c]*Sin[3*d*x])/(a*d) + (15*Cos[4*c]*Sin[4*d*x])/(a*
d) - (6*Sin[5*c]*Sin[5*d*x])/(a*d) - (60*Sin[(d*x)/2])/(a*d*(Cos[c/2] + Sin
[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (30*Sin[c + d*x])/(a*d*(1 +
 Sin[c + d*x])) + (60*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x])))/480
```

Maple [A]
time = 0.21, size = 116, normalized size = 1.12

method	result
risch	$\frac{3x}{8a}+\frac{5\cos(dx+c)}{8ad}+\frac{\cos(5dx+5c)}{80ad}+\frac{\sin(4dx+4c)}{32ad}-\frac{5\cos(3dx+3c)}{48ad}-\frac{\sin(2dx+2c)}{4ad}$
derivativedivides	$\frac{\frac{32\left(\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}+\frac{1}{30}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{3\,\mathrm{arct}}{}}{ad}$

default	$\frac{\frac{32\left(\frac{3\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{7\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{128}+\frac{1}{30}\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}+\frac{3\arctan}{}}{ad}$
norman	$-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{3x}{8a}+\frac{19}{60ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{30ad}+\frac{3x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{3x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{9x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{45x\left(\tan^{4}\left(\frac{dx}{2}+\right.\right.}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*((3/128*tan(1/2*d*x+1/2*c)^9+7/64*tan(1/2*d*x+1/2*c)^7+1/3*tan(1/2*d
*x+1/2*c)^4-7/64*tan(1/2*d*x+1/2*c)^3+1/6*tan(1/2*d*x+1/2*c)^2-3/128*tan(1/
2*d*x+1/2*c)+1/30)/(1+tan(1/2*d*x+1/2*c)^2)^5+3/128*arctan(tan(1/2*d*x+1/2*
c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(94) = 188$.

time = 0.49, size = 258, normalized size = 2.48

$$-\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{320\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{210\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{640\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{210\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{45\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-64}{a+\frac{5a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{10a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 320*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 640*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 45*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 64)/(a + 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.35, size = 68, normalized size = 0.65

$$\frac{24\cos(dx+c)^{5}-80\cos(dx+c)^{3}+45\,dx+15\left(2\cos(dx+c)^{3}-5\cos(dx+c)\right)\sin(dx+c)+120\cos(dx+c)}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 80*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 - 5*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/(a*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(85) = 170$.

time = 17.00, size = 1360, normalized size = 13.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x
/2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a
*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d
*x/2)**2 + 120*a*d) + 450*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 450*d*x*ta
n(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)*
*8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*
tan(c/2 + d*x/2)**2 + 120*a*d) + 225*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c
/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**
6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) +
 45*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 90*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10
 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*ta
n(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 420*tan(c/2 +
d*x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 120
0*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
+ d*x/2)**2 + 120*a*d) + 1280*tan(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 420*tan(c/
2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) + 640*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x
/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200
*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 90*tan(
c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c
/2 + d*x/2)**2 + 120*a*d) + 128/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan
(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)
**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d), Ne(d, 0)), (x*sin(c)**4*cos(c
)**2/(a*sin(c) + a), True))
```

Giac [**A**]

time = 0.48, size = 114, normalized size = 1.10

$$\frac{\frac{45\,(dx+c)}{a}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(45*(d*x + c)/a + 2*(45*tan(1/2*d*x + 1/2*c)^9 + 210*tan(1/2*d*x + 1/
2*c)^7 + 640*tan(1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 + 320*tan(
1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^5*a))/d
```

Mupad [B]
time = 11.91, size = 107, normalized size = 1.03

$$\frac{3\,x}{8\,a}+\frac{\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{16}{15}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(8*a) + ((16*tan(c/2 + (d*x)/2)^2)/3 - (3*tan(c/2 + (d*x)/2))/4 - (7*
tan(c/2 + (d*x)/2)^3)/2 + (32*tan(c/2 + (d*x)/2)^4)/3 + (7*tan(c/2 + (d*x)/
2)^7)/2 + (3*tan(c/2 + (d*x)/2)^9)/4 + 16/15)/(a*d*(tan(c/2 + (d*x)/2)^2 +
1)^5)
```

3.298 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=87

$$-\frac{3x}{8a}-\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos(c+dx)\sin^3(c+dx)}{4ad}$$

```
[Out] -3/8*x/a-cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+
1/4*cos(d*x+c)*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2918, 2713, 2715, 8}

$$\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - Cos[c + d*x]/(a*d) + Cos[c + d*x]^3/(3*a*d) + (3*Cos[c + d*x
]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)\,dx}{a} - \frac{\int \sin^4(c+dx)\,dx}{a}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4ad} - \frac{3\int \sin^2(c+dx)\,dx}{4a} - \frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, c\right.}{ad}$$

$$= -\frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+dx)\,\text{si}}{4aa}$$

$$= -\frac{3x}{8a} - \frac{\cos(c+dx)}{ad} + \frac{\cos^3(c+dx)}{3ad} + \frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos(c+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 271 vs. 2(87) = 174.
time = 1.22, size = 271, normalized size = 3.11

$$\frac{24(c-3dx)\cos\left(\frac{c}{2}\right)-72\cos\left(\frac{c}{2}+dx\right)-72\cos\left(\frac{3c}{2}+dx\right)+24\cos\left(\frac{3c}{2}+2dx\right)-24\cos\left(\frac{5c}{2}+2dx\right)+8\cos\left(\frac{5c}{2}+3dx\right)+8\cos\left(\frac{7c}{2}+3dx\right)-3\cos\left(\frac{7c}{2}+4dx\right)+3\cos\left(\frac{9c}{2}+4dx\right)-48\sin\left(\frac{c}{2}\right)+24c\sin\left(\frac{c}{2}\right)-72dx\sin\left(\frac{c}{2}\right)+72\sin\left(\frac{c}{2}+dx\right)-72\sin\left(\frac{3c}{2}+dx\right)+24\sin\left(\frac{3c}{2}+2dx\right)+24\sin\left(\frac{5c}{2}+2dx\right)-8\sin\left(\frac{5c}{2}+3dx\right)+8\sin\left(\frac{7c}{2}+3dx\right)-3\sin\left(\frac{7c}{2}+4dx\right)-3\sin\left(\frac{9c}{2}+4dx\right)}{192ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (24*(c - 3*d*x)*Cos[c/2] - 72*Cos[c/2 + d*x] - 72*Cos[(3*c)/2 + d*x] + 24*C
os[(3*c)/2 + 2*d*x] - 24*Cos[(5*c)/2 + 2*d*x] + 8*Cos[(5*c)/2 + 3*d*x] + 8*
Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 4*d*x] - 48
*Sin[c/2] + 24*c*Sin[c/2] - 72*d*x*Sin[c/2] + 72*Sin[c/2 + d*x] - 72*Sin[(3
*c)/2 + d*x] + 24*Sin[(3*c)/2 + 2*d*x] + 24*Sin[(5*c)/2 + 2*d*x] - 8*Sin[(5
*c)/2 + 3*d*x] + 8*Sin[(7*c)/2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9
*c)/2 + 4*d*x])/(192*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.19, size = 116, normalized size = 1.33

method	result
risch	$-\frac{3x}{8a}-\frac{3\cos(dx+c)}{4ad}-\frac{\sin(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{12ad}+\frac{\sin(2dx+2c)}{4ad}$
derivativedivides	$\frac{\frac{16\left(-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}-\frac{1}{12}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{3\,\text{ar}}{}}{ad}$

default	$\frac{\frac{16\left(-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}-\frac{1}{12}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-3}{ad}$
norman	$\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{3x}{8a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{15x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{15x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{15x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{15x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*((-3/64*tan(1/2*d*x+1/2*c)^7-11/64*tan(1/2*d*x+1/2*c)^5-1/4*tan(1/2*
d*x+1/2*c)^4+11/64*tan(1/2*d*x+1/2*c)^3-1/3*tan(1/2*d*x+1/2*c)^2+3/64*tan(1
/2*d*x+1/2*c)-1/12)/(1+tan(1/2*d*x+1/2*c)^2)^4-3/64*arctan(tan(1/2*d*x+1/2*
c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(79) = 158$.

time = 0.49, size = 237, normalized size = 2.72

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{64\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 64*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 9*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 - 16)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*
a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 9*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.34, size = 58, normalized size = 0.67

$$\frac{8\cos\left(dx+c\right)^3-9\,dx-3\left(2\cos\left(dx+c\right)^3-5\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-24\cos\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(
d*x + c) - 24*cos(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(70) = 140$.
time = 11.26, size = 1049, normalized size = 12.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-9*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 36*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) - 54*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 36*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 9*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 18*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 66*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) - 96*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) + 66*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 128*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) + 18*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 32/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)
**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d), N
e(d, 0)), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a), True))
```

Giac [A]
time = 0.45, size = 114, normalized size = 1.31

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{2\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+48\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-33\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+64\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 33*tan(1/2*d*x + 1/2*c
)^5 + 48*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)^3 + 64*tan(1/2*d*
x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4
*a))/d
```

Mupad [B]
time = 8.61, size = 79, normalized size = 0.91

$$\frac{\cos(c+dx)^3}{3\,a\,d}-\frac{\cos(c+dx)}{a\,d}-\frac{3\,x}{8\,a}-\frac{\cos(c+dx)^3\,\sin(c+dx)}{4\,a\,d}+\frac{5\,\cos(c+dx)\,\sin(c+dx)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] cos(c + d*x)^3/(3*a*d) - cos(c + d*x)/(a*d) - (3*x)/(8*a) - (cos(c + d*x)^3
*sin(c + d*x))/(4*a*d) + (5*cos(c + d*x)*sin(c + d*x))/(8*a*d)
```

3.299 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{x}{2a}+\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] 1/2*x/a+cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2918, 2715, 8, 2713}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}+\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + Cos[c + d*x]/(a*d) - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c
 + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin^2(c+dx)\,dx}{a} - \frac{\int \sin^3(c+dx)\,dx}{a}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int 1\,dx}{2a} + \frac{\mathrm{Subst}\left(\int (1-x^2)\,dx, x, \cos(c+dx\right.}{ad}$$
$$= \frac{x}{2a} + \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 0.74

$$\frac{6c+6dx+9\cos(c+dx)-\cos(3(c+dx))-3\sin(2(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (6*c + 6*d*x + 9*Cos[c + d*x] - Cos[3*(c + d*x)] - 3*Sin[2*(c + d*x)])/(12*
a*d)
```

Maple [A]
time = 0.16, size = 77, normalized size = 1.24

method	result
risch	$\frac{x}{2a}+\frac{3\cos(dx+c)}{4ad}-\frac{\cos(3dx+3c)}{12ad}-\frac{\sin(2dx+2c)}{4ad}$
derivativedivides	$\frac{\frac{8\left(\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
default	$\frac{\frac{8\left(\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
norman	$\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{x}{2a}+\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*((1/8*tan(1/2*d*x+1/2*c)^5+1/2*tan(1/2*d*x+1/2*c)^2-1/8*tan(1/2*d*x+1
/2*c)+1/6)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(56) = 112$.
time = 0.49, size = 156, normalized size = 2.52

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-4}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4)/(a + 3*a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
)/d
```

Fricas [A]
time = 0.36, size = 45, normalized size = 0.73

$$-\frac{2\cos(dx+c)^3-3\,dx+3\cos(dx+c)\sin(dx+c)-6\cos(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(46) = 92$.
time = 4.83, size = 563, normalized size = 9.08

$$\begin{cases} \frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{9dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{3dx}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{24\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}-\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad}+\frac{8}{6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad} & \text{for}\: d \neq 0 \\ \frac{x\sin^2(c)\cos^2(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*ta
n(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d
*x/2)**4/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*t
an(c/2 + d*x/2)**2 + 6*a*d) + 9*d*x*tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*
x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d)
```

```
+ 3*d*x/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d) + 6*tan(c/2 + d*x/2)**5/(6*a*d*tan(c/2 + d*x/2)*
*6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) + 24*
tan(c/2 + d*x/2)**2/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4
 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a*d) - 6*tan(c/2 + d*x/2)/(6*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*tan(c/2 + d*x/2)**2 + 6*a
*d) + 8/(6*a*d*tan(c/2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 18*a*d*ta
n(c/2 + d*x/2)**2 + 6*a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a
), True))
```

Giac [A]
time = 0.45, size = 75, normalized size = 1.21

$$\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^
2 - 3*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 10.46, size = 66, normalized size = 1.06

$$\frac{x}{2\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4}{3}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(2*a) + (4*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)
^5 + 4/3)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.300 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$-\frac{x}{2a}-\frac{\cos(c+dx)}{ad}+\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] -1/2*x/a-cos(d*x+c)/a/d+1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {2918, 2718, 2715, 8}

$$-\frac{\cos(c+dx)}{ad}+\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*x/a - Cos[c + d*x]/(a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sin(c+dx)\,dx}{a} - \frac{\int \sin^2(c+dx)\,dx}{a}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a}$$
$$= -\frac{x}{2a} - \frac{\cos(c+dx)}{ad} + \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 161 vs. $2(45) = 90$.
time = 0.42, size = 161, normalized size = 3.58

$$\frac{2(c-2dx)\cos\left(\frac{c}{2}\right)-4\cos\left(\frac{c}{2}+dx\right)-4\cos\left(\frac{3c}{2}+dx\right)+\cos\left(\frac{3c}{2}+2dx\right)-\cos\left(\frac{5c}{2}+2dx\right)-4\sin\left(\frac{c}{2}\right)+2c\sin\left(\frac{c}{2}\right)-4dx\sin\left(\frac{c}{2}\right)+4\sin\left(\frac{c}{2}+dx\right)-4\sin\left(\frac{3c}{2}+dx\right)+\sin\left(\frac{3c}{2}+2dx\right)+\sin\left(\frac{5c}{2}+2dx\right)}{8ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(c - 2*d*x)*Cos[c/2] - 4*Cos[c/2 + d*x] - 4*Cos[(3*c)/2 + d*x] + Cos[(3*
c)/2 + 2*d*x] - Cos[(5*c)/2 + 2*d*x] - 4*Sin[c/2] + 2*c*Sin[c/2] - 4*d*x*Si
n[c/2] + 4*Sin[c/2 + d*x] - 4*Sin[(3*c)/2 + d*x] + Sin[(3*c)/2 + 2*d*x] + S
in[(5*c)/2 + 2*d*x])/(8*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.12, size = 77, normalized size = 1.71

method	result
risch	$-\frac{x}{2a}-\frac{\cos(dx+c)}{ad}+\frac{\sin(2dx+2c)}{4ad}$
derivativedivides	$\frac{\frac{4\left(-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{1}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
default	$\frac{\frac{4\left(-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}-\frac{1}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$
norman	$\frac{-\frac{1}{ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{2a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^4\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*((-1/4*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+1/4*tan(1/2*d*x+
1/2*c)-1/2)/(1+tan(1/2*d*x+1/2*c)^2)^2-1/4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(41) = 82$.

time = 0.48, size = 133, normalized size = 2.96

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - arctan(sin(d*x + c)/
(cos(d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.34, size = 34, normalized size = 0.76

$$-\frac{dx-\cos(dx+c)\sin(dx+c)+2\cos(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x - cos(d*x + c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(32) = 64$.

time = 2.50, size = 366, normalized size = 8.13

$$\begin{cases}-\frac{dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{dx}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad}-\frac{4}{2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+4ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad} & \text{for } d \neq 0 \\ \frac{x\sin(c)\cos^2(c)}{a\sin(c)+a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(
c/2 + d*x/2)**2 + 2*a*d) - 2*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - d*x/(2*a*d*tan(c/2 + d*x/2)**4
+ 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 2*tan(c/2 + d*x/2)**3/(2*a*d*tan(c/2
 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) - 4*tan(c/2 + d*x/2)**2/(
2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) + 2*tan(c/2
+ d*x/2)/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d) -
4/(2*a*d*tan(c/2 + d*x/2)**4 + 4*a*d*tan(c/2 + d*x/2)**2 + 2*a*d), Ne(d, 0)
), (x*sin(c)*cos(c)**2/(a*sin(c) + a), True))
```

Giac [A]
time = 0.44, size = 72, normalized size = 1.60

$$-\frac{\frac{dx+c}{a}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a + 2*(tan(1/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 -
tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B]
time = 8.69, size = 33, normalized size = 0.73

$$-\frac{x}{2\,a}-\frac{\cos\left(c+d\,x\right)-\frac{\sin\left(2\,c+2\,d\,x\right)}{4}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/(2*a) - (cos(c + d*x) - sin(2*c + 2*d*x)/4)/(a*d)
```

3.301 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=22

$$-\frac{x}{a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -x/a-arctanh(cos(d*x+c))/a/d
```

Rubi [A]

time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2918, 3855, 8}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int 1\,dx}{a}+\frac{\int \csc(c+dx)\,dx}{a}\\ &= -\frac{x}{a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 37, normalized size = 1.68

$$-\frac{c+dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] -((c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d))
```

Maple [A]
time = 0.14, size = 31, normalized size = 1.41

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	31
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	31
risch	$-\frac{x}{a}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}$	47
norman	$\frac{-\frac{x}{a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(ln(tan(1/2*d*x+1/2*c))-2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(22) = 44$.
time = 0.49, size = 52, normalized size = 2.36

$$-\frac{\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a)/d
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.68

$$-\frac{2\,dx+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*x + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(
a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.47, size = 31, normalized size = 1.41

$$-\frac{\frac{dx+c}{a} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/a - log(abs(tan(1/2*d*x + 1/2*c)))/a)/d
```

Mupad [B]
time = 8.77, size = 79, normalized size = 3.59

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)}{a\,d}+\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 - pi
/4 + (d*x)/2))))/(a*d) + log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d)
```

3.302 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)} \, dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2785, 3852, 8, 3855}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]

[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc(c+dx)\,dx}{a} + \frac{\int \csc^2(c+dx)\,dx}{a}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{ad}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.
time = 0.19, size = 69, normalized size = 2.38

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)
/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(a*d)
```

Maple [A]
time = 0.15, size = 44, normalized size = 1.52

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}$	63
norman	$\frac{\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{2ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(29) = 58.
time = 0.29, size = 70, normalized size = 2.41

$$-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.
time = 0.35, size = 62, normalized size = 2.14

$$\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2\left(c+dx\right)\csc^2\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.42, size = 65, normalized size = 2.24

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.60, size = 25, normalized size = 0.86

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\cot(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
[Out] -(log(tan(c/2 + (d*x)/2)) + cot(c + d*x))/(a*d)
```

3.303 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2918, 3853, 3855, 3852, 8}

$$\frac{\cot(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Csc[c
 + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc^2(c+dx)\,dx}{a}+\frac{\int \csc^3(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 94, normalized size = 1.77

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-\cos(c+dx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^2(c+dx)+\sin(2(c+dx))\right)}{8ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-Cos[c + d*x] + (-Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Sin[2*(c + d*x)]))/(8*a*d*
(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.20, size = 72, normalized size = 1.36

method	result	size
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	72
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$	72
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}+2ie^{2i(dx+c)}-2i}{ad\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}$	95

norman	$\frac{-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$	128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/2*
c)^2+2*ln(tan(1/2*d*x+1/2*c))+2/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(49) = 98$.
time = 0.29, size = 115, normalized size = 2.17

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*
x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.35, size = 88, normalized size = 1.66

$$-\frac{(\cos(dx+c)^2-1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)^2-1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+4\cos(dx+c)\sin(dx+c)-2\cos(dx+c)}{4\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 4*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x +
 c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 94, normalized size = 1.77

$$\frac{\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{2}}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x + 1/2*c)^2 - 4*a*t
an(1/2*d*x + 1/2*c))/a^2 - (6*tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*
c) + 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 8.65, size = 87, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}\right)}{4\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2 + (d*x)/2))/(2*a*d) - tan(c/2 +
(d*x)/2)/(2*a*d) + (cot(c/2 + (d*x)/2)^2*(2*tan(c/2 + (d*x)/2) - 1/2))/(4*a
*d)
```

3.304 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x
+c)*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2918, 3852, 3853, 3855}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
 + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \csc^3(c+dx)\,dx}{a}+\frac{\int \csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\int \csc(c+dx)\,dx}{2a}-\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(\ldots\right.}{ad}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+\ldots}{2ad}$$

Mathematica [A]
time = 0.48, size = 126, normalized size = 1.75

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-12\cos(c+dx)(-1+\sin(c+dx))-4\left(\cos(3(c+dx))+3\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)\right)\right)}{192ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)
/2])^2*(-12*Cos[c + d*x]*(-1 + Sin[c + d*x]) - 4*(Cos[3*(c + d*x)] + 3*(Log
[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3)))/(a*d*(1 + Sin
[c + d*x]))
```

Maple [A]
time = 0.22, size = 96, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8da}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8da}$
risch	$-\frac{3\,e^{5i(dx+c)}-12ie^{2i(dx+c)}+4i-3\,e^{i(dx+c)}}{3ad\left(e^{2i(dx+c)}-1\right)^3}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2+3*tan(1/2*d*x+1/2*c)
-3/tan(1/2*d*x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c))+1/tan(1/2*d*x+1/2*c)^2-1/3/t
an(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(66) = 132$.
time = 0.29, size = 153, normalized size = 2.12

$$\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} - \frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a} - \frac{12 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} - \frac{9 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)^3}{a \sin(dx+c)^3}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a + (3*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.34, size = 119, normalized size = 1.65

$$-\frac{8 \cos(dx+c)^3 - 3 \left(\cos(dx+c)^2 - 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 3 \left(\cos(dx+c)^2 - 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 6 \cos(dx+c) \sin(dx+c) - 12 \cos(dx+c)}{12 \left(ad \cos(dx+c)^2 - ad\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^
2 - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx) \csc^4(c+dx)}{\sin(c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.47, size = 128, normalized size = 1.78

$$-\frac{\frac{12 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} - \frac{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 3 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 9 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3} - \frac{22 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a - (a^2*tan(1/2*d*x + 1/2*c)^3 -
3*a^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/
2*d*x + 1/2*c)^3 - 9*tan(1/2*d*x + 1/2*c)^2 + 3*tan(1/2*d*x + 1/2*c) - 1)/(
a*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 8.62, size = 119, normalized size = 1.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - log(tan(c/2
+ (d*x)/2))/(2*a*d) + (3*tan(c/2 + (d*x)/2))/(8*a*d) - (cot(c/2 + (d*x)/2)^
3*(3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + 1/3))/(8*a*d)
```

3.305 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

```
[Out] -3/8*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d+1/3*cot(d*x+c)^3/a/d-3/8*cot(d*
x+c)*csc(d*x+c)/a/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2918, 3853, 3855, 3852}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(8*a*d) + Cot[c + d*x]/(a*d) + Cot[c + d*x]^3/(3
*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^
3)/(4*a*d)
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc^4(c+dx)\,dx}{a}+\frac{\int \csc^5(c+dx)\,dx}{a}\\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\int \csc^3(c+dx)\,dx}{4a}+\frac{\mathrm{Subst}\left(\int (1+x^2)\ dx, x\right.}{ad}\\
&= \frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc}{4ad}\\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{3\cot(c+dx)\csc}{8ad}
\end{aligned}
$$

Mathematica [A]
time = 0.80, size = 125, normalized size = 1.32

$$
-\frac{\csc^4(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(66\cos(c+dx)+72\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^4(c+dx)+2\cos(3(c+dx))(-9+16\sin(c+dx))-48\sin(2(c+dx))\right)}{192ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(66*Cos[c +
d*x] + 72*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 +
2*Cos[3*(c + d*x)]*(-9 + 16*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(a*d*(1 +
 Sin[c + d*x]))
```

Maple [A]
time = 0.26, size = 124, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+6}{16da}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+6}{16da}$
risch	$\frac{9\,e^{7i(dx+c)}-48ie^{4i(dx+c)}-33\,e^{5i(dx+c)}+64ie^{2i(dx+c)}-33\,e^{3i(dx+c)}-16i+9\,e^{i(dx+c)}}{12ad\left(e^{2i(dx+c)}-1\right)^4}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8ad}-\frac{3\ln\left(e^{i(dx+}\right.}{8ad}$

norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{3}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(1/4*tan(1/2*d*x+1/2*c)^4-2/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1
/2*c)^2-6*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4-2/tan(1/2*d*x+1/2*c)^
2+2/3/tan(1/2*d*x+1/2*c)^3+6*ln(tan(1/2*d*x+1/2*c))+6/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(87) = 174$.
time = 0.29, size = 195, normalized size = 2.05

$$-\frac{\frac{\frac{72\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{72\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*((72*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (8*sin(d*x
+ c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 72*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4)
)/d
```

Fricas [A]
time = 0.37, size = 143, normalized size = 1.51

$$\frac{18\cos(dx+c)^3-9\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+9\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-16\left(2\cos(dx+c)^3-3\cos(dx+c)\right)\sin(dx+c)-30\cos(dx+c)}{48\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(18*cos(d*x + c)^3 - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2
*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*c
os(d*x + c) + 1/2) - 16*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) -
30*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2(c+dx)\csc^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 157, normalized size = 1.65

$$\frac{\frac{72 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^3*tan(1/2*d*x + 1/2*c)^4
- 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 72*a^3*tan
(1/2*d*x + 1/2*c))/a^4 - (150*tan(1/2*d*x + 1/2*c)^4 - 72*tan(1/2*d*x + 1/2
*c)^3 + 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*
d*x + 1/2*c)^4))/d
```

Mupad [B]
time = 8.68, size = 151, normalized size = 1.59

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{1}{4}\right)}{16\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + tan(c/2 + (d
*x)/2)^4/(64*a*d) + (3*log(tan(c/2 + (d*x)/2)))/(8*a*d) - (3*tan(c/2 + (d*x
)/2))/(8*a*d) + (cot(c/2 + (d*x)/2)^4*((2*tan(c/2 + (d*x)/2))/3 - 2*tan(c/2
 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a*d)
```

3.306 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)}{ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)}{4}$$

[Out] 3/8*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-2/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+3/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*csc(d*x+c)^3/a/d

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2918, 3852, 3853, 3855}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]

[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) - (2*Cot[c + d*x]^3)/(3*a*d) - Cot[c + d*x]^5/(5*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc^5(c+dx)\,dx}{a}+\frac{\int \csc^6(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\int \csc^3(c+dx)\,dx}{4a}-\frac{\operatorname{Subst}\left(\int (1+2x^2+x^4)\right.}{ad}\\ &= -\frac{\cot(c+dx)}{ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{3\cot(c+dx)\csc(c+dx}{8ad}\\ &= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)}{ad}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{3}{}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(-640\cos(c+dx)+320\cos(3(c+dx))-64\cos(5(c+dx))+450\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-450\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+420\sin(2(c+dx))-225\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+225\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-90\sin(4(c+dx))+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-45\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{1920ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^5*(-640*Cos[c + d*x] + 320*Cos[3*(c + d*x)] - 64*Cos[5*(c + d
*x)] + 450*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] + 420*Sin[2*(c + d*x)] - 225*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d
*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 4
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(1920*a*d)
```

Maple [A]
time = 0.26, size = 150, normalized size = 1.32

method	result
risch	$-\frac{45\,e^{9i(dx+c)}-210\,e^{7i(dx+c)}+640i e^{4i(dx+c)}-320i e^{2i(dx+c)}+210\,e^{3i(dx+c)}+64i-45\,e^{i(dx+c)}}{60ad\left(e^{2i(dx+c)}-1\right)^5}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8ad}+\frac{3}{}$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-12\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{32da}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-12\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{32da}$

norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}-\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}-\frac{7\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad}+\frac{7(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a*(1/5*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4+5/3*tan(1/2*d*x
+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+10*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*
c)^5-12*ln(tan(1/2*d*x+1/2*c))-5/3/tan(1/2*d*x+1/2*c)^3-10/tan(1/2*d*x+1/2*
c)+1/2/tan(1/2*d*x+1/2*c)^4+4/tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.
time = 0.28, size = 234, normalized size = 2.05

$$\frac{\frac{\frac{300\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{300\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((300*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 360*log(sin(
d*x + c)/(cos(d*x + c) + 1))/a + (15*sin(d*x + c)/(cos(d*x + c) + 1) - 50*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 300*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*si
n(d*x + c)^5))/d
```

Fricas [A]
time = 0.36, size = 173, normalized size = 1.52

$$-\frac{128\cos\left(dx+c\right)^5-320\cos\left(dx+c\right)^3-45\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+45\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\left(3\cos\left(dx+c\right)^3-5\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+240\cos\left(dx+c\right)}{240\left(ad\cos\left(dx+c\right)^4-2\,ad\cos\left(dx+c\right)^2+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(128*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 45*(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 45*(cos(d*x +
c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30
*(3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((a*d
*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.46, size = 187, normalized size = 1.64

$$-\frac{\frac{360 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a}-\frac{6 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-15 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+50 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-120 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+300 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^5}-\frac{822 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-300 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+120 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-50 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-6}{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5}}{960 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a - (6*a^4*tan(1/2*d*x + 1/2*c)^
5 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 50*a^4*tan(1/2*d*x + 1/2*c)^3 - 120*a^4
*tan(1/2*d*x + 1/2*c)^2 + 300*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (822*tan(1/2*
d*x + 1/2*c)^5 - 300*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*d*x + 1/2*c)^3 -
50*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1
/2*c)^5))/d
```

Mupad [B]
time = 9.25, size = 291, normalized size = 2.55

$$-\frac{6 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^{10}-6 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^{10}+15 \cos\left(\frac{c}{2}+\frac{d x}{2}\right) \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^9-15 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^9 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)-50 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^2 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^8+120 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^3 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^7-300 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^4 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^6+300 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^6 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^4-120 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^7 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^3+50 \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^8 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^2+360 \ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right) \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^5 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^5}{960 a d \cos\left(\frac{c}{2}+\frac{d x}{2}\right)^5 \sin\left(\frac{c}{2}+\frac{d x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] -(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 + 15*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 50*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
 (d*x)/2)^7 - 300*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 300*cos(c/2 +
 (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)
/2)^3 + 50*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 360*log(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(960*
a*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.307 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=111

$$-\frac{27x}{8a^2}-\frac{4\cos(c+dx)}{a^2d}+\frac{2\cos^3(c+dx)}{3a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{2\cos(c+}{a^2d(1+\sin(c}$$

```
[Out] -27/8*x/a^2-4*cos(d*x+c)/a^2/d+2/3*cos(d*x+c)^3/a^2/d+11/8*cos(d*x+c)*sin(d
*x+c)/a^2/d+1/4*cos(d*x+c)*sin(d*x+c)^3/a^2/d-2*cos(d*x+c)/a^2/d/(1+sin(d*x
+c))
```

Rubi [A]

time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2953, 3045, 2718, 2715, 8, 2713, 2727}

$$\frac{2\cos^3(c+dx)}{3a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{27x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-27*x)/(8*a^2) - (4*Cos[c + d*x])/(a^2*d) + (2*Cos[c + d*x]^3)/(3*a^2*d) +
 (11*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(
4*a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\sin^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2} \\
&= \frac{\int \Big(-2+2\sin(c+dx)-2\sin^2(c+dx)+2\sin^3(c+dx)-\sin^4(c+dx)+ \cdots}{a^2} \\
&= -\frac{2x}{a^2}-\frac{\int \sin^4(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}-\frac{2\int \sin^2(c+dx)\,dx}{a^2}+ \cdots \\
&= -\frac{2x}{a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d} \cdots \\
&= -\frac{3x}{a^2}-\frac{4\cos(c+dx)}{a^2d}+\frac{2\cos^3(c+dx)}{3a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{8a^2d}+ \cdots \\
&= -\frac{27x}{8a^2}-\frac{4\cos(c+dx)}{a^2d}+\frac{2\cos^3(c+dx)}{3a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{8a^2d}+ \cdots
\end{aligned}
$$

Mathematica [A]
time = 1.09, size = 209, normalized size = 1.88

$$
\frac{(4-648dx)\cos\left(\frac{dx}{2}\right)-340\cos\left(c+\frac{dx}{2}\right)-264\cos\left(c+\frac{3dx}{2}\right)-56\cos\left(3c+\frac{5dx}{2}\right)+13\cos\left(3c+\frac{7dx}{2}\right)+3\cos\left(5c+\frac{9dx}{2}\right)+1100\sin\left(\frac{dx}{2}\right)+4\sin\left(c+\frac{dx}{2}\right)-648dx\sin\left(c+\frac{dx}{2}\right)-264\sin\left(2c+\frac{3dx}{2}\right)+56\sin\left(2c+\frac{5dx}{2}\right)+13\sin\left(4c+\frac{7dx}{2}\right)-3\sin\left(4c+\frac{9dx}{2}\right)}{192a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((4 - 648*d*x)*Cos[(d*x)/2] - 340*Cos[c + (d*x)/2] - 264*Cos[c + (3*d*x)/2]
 - 56*Cos[3*c + (5*d*x)/2] + 13*Cos[3*c + (7*d*x)/2] + 3*Cos[5*c + (9*d*x)/
2] + 1100*Sin[(d*x)/2] + 4*Sin[c + (d*x)/2] - 648*d*x*Sin[c + (d*x)/2] - 26
4*Sin[2*c + (3*d*x)/2] + 56*Sin[2*c + (5*d*x)/2] + 13*Sin[4*c + (7*d*x)/2]
- 3*Sin[4*c + (9*d*x)/2])/(192*a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.29, size = 143, normalized size = 1.29

method	result
risch	$-\frac{27x}{8a^2}-\frac{7\,\mathrm{e}^{i(dx+c)}}{4d\,a^2}-\frac{7\,\mathrm{e}^{-i(dx+c)}}{4d\,a^2}-\frac{4}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)}-\frac{\sin(4dx+4c)}{32a^2d}+\frac{\cos(3dx+3c)}{6d\,a^2}+\frac{3\sin(2dx+2c)}{4a^2d}$
derivativedivides	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4\left(\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{17\left(\tan^2\left(\right.\right.}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^2}$
default	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4\left(\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{17\left(\tan^2\left(\right.\right.}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^2}$
norman	$-\frac{81x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{841\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{27\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{27x}{8a}-\frac{32}{3ad}-\frac{101\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{513x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-243$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*(-1/8/(tan(1/2*d*x+1/2*c)+1)-1/8*(11/16*tan(1/2*d*x+1/2*c)^7+tan(1
/2*d*x+1/2*c)^6+19/16*tan(1/2*d*x+1/2*c)^5+5*tan(1/2*d*x+1/2*c)^4-19/16*tan
(1/2*d*x+1/2*c)^3+17/3*tan(1/2*d*x+1/2*c)^2-11/16*tan(1/2*d*x+1/2*c)+5/3)/(
1+tan(1/2*d*x+1/2*c)^2)^4-27/128*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(103) = 206$.
time = 0.51, size = 398, normalized size = 3.59

$$-\frac{\frac{\frac{47\sin(dx+c)}{\cos(dx+c)+1}+\frac{431\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{215\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{471\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{297\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{297\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{81\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{81\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+128}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{81\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((47*sin(d*x + c)/(cos(d*x + c) + 1) + 431*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 215*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 471*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 297*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 297*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 81*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 81*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8 + 128)/(a^2 + a^2*sin(d*x + c)/(cos(d*x
+ c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^2*
sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^8/(cos(
d*x + c) + 1)^8 + a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 81*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [**A**]
time = 0.35, size = 144, normalized size = 1.30

$$\frac{6 \cos(dx+c)^5 + 16 \cos(dx+c)^4 - 29 \cos(dx+c)^3 - 81 dx - 3(27 dx + 35) \cos(dx+c) - 96 \cos(dx+c)^2 - (6 \cos(dx+c)^4 - 10 \cos(dx+c)^3 + 81 dx - 39 \cos(dx+c)^2 + 57 \cos(dx+c) - 48) \sin(dx+c) - 48}{24 (a^2 d \cos(dx+c) + a^2 d \sin(dx+c) + a^2 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*cos(d*x + c)^5 + 16*cos(d*x + c)^4 - 29*cos(d*x + c)^3 - 81*d*x - 3
*(27*d*x + 35)*cos(d*x + c) - 96*cos(d*x + c)^2 - (6*cos(d*x + c)^4 - 10*co
s(d*x + c)^3 + 81*d*x - 39*cos(d*x + c)^2 + 57*cos(d*x + c) - 48)*sin(d*x +
 c) - 48)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(104) = 208$.
time = 32.13, size = 3580, normalized size = 32.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-81*d*x*tan(c/2 + d*x/2)**9/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*
a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(
c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x
/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24
*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)**8/(24*a**2
*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**
2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x
*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d
*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 +
```

144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d
*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 +
d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/
2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*
a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*ta
n(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x
/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 486*d*x*tan(c/2 + d*x/2)
5/(24*a2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**
2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/
2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)
3 + 96*a2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*
d) - 486*d*x*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d
*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 +
d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4
 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*
d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(c/2 + d*x/2)**3/(24*a**2*d*ta
n(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x
/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 1
44*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*t
an(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 324*d*x*tan(
c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)
8 + 96*a2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a
2*d*tan(c/2 + d*x/2)5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2
) + 24*a**2*d) - 81*d*x*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**9 + 2
4*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*ta
n(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d
*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 +
24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 81*d*x/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
2*d*tan(c/2 + d*x/2)6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 162*tan(c/2 + d*x/2)**8/(
24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) -
162*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6
 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**
2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2
 + d*x/2) + 24*a**2*d) - 594*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a
2*d*tan(c/2 + d*x/2)6 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan
(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/

```
2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) - 594*tan(c/2 + d*x/2)**5/(
24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d
*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**3 +
 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2 + d*x/2) + 24*a**2*d) -
942*tan(c/2 + d*x/2)**4/(24*a**2*d*tan(c/2 + d*x/2)**9 + 24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**7 + 96*a**2*d*tan(c/2 + d*x/2)**6
 + 144*a**2*d*tan(c/2 + d*x/2)**5 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**
2*d*tan(c/2 + d*x/2)**3 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d*tan(c/2
 + d*x/2) + 24*a**2*d) - 430*tan(c/2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2
)**9 + 24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*...
```

Giac [A]
time = 0.45, size = 145, normalized size = 1.31

$$-\frac{\frac{81\,(dx+c)}{a^2}+\frac{96}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-57\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+272\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+80\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(81*(d*x + c)/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(33*tan(1
/2*d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 + 57*tan(1/2*d*x + 1/2*c)^5 +
 240*tan(1/2*d*x + 1/2*c)^4 - 57*tan(1/2*d*x + 1/2*c)^3 + 272*tan(1/2*d*x +
 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 80)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a
^2))/d
```

Mupad [B]
time = 12.34, size = 147, normalized size = 1.32

$$-\frac{27\,x}{8\,a^2}-\frac{\frac{27\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}+\frac{27\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{99\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+\frac{99\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{157\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{215\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12}+\frac{431\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{12}+\frac{47\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{12}+\frac{32}{3}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (27*x)/(8*a^2) - ((47*tan(c/2 + (d*x)/2))/12 + (431*tan(c/2 + (d*x)/2)^2)
/12 + (215*tan(c/2 + (d*x)/2)^3)/12 + (157*tan(c/2 + (d*x)/2)^4)/4 + (99*ta
n(c/2 + (d*x)/2)^5)/4 + (99*tan(c/2 + (d*x)/2)^6)/4 + (27*tan(c/2 + (d*x)/2
)^7)/4 + (27*tan(c/2 + (d*x)/2)^8)/4 + 32/3)/(a^2*d*(tan(c/2 + (d*x)/2) + 1
)*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.308 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$\frac{3x}{a^2}+\frac{3\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

```
[Out] 3*x/a^2+3*cos(d*x+c)/a^2/d-1/3*cos(d*x+c)^3/a^2/d-cos(d*x+c)*sin(d*x+c)/a^2
/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2953, 3045, 2718, 2715, 8, 2713, 2727}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{3\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/a^2 + (3*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^3/(3*a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{\int \left(2-2\sin(c+dx)+2\sin^2(c+dx)-\sin^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\ &= \frac{2x}{a^2}-\frac{\int \sin^3(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \text{}}{}\\ &= \frac{2x}{a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}+\frac{\int}{}\\ &= \frac{3x}{a^2}+\frac{3\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}+\frac{2\cos(}{a^2d(1+\text{s}}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 165, normalized size = 1.99

$$-\frac{-72dx\cos\left(\frac{dx}{2}\right)-31\cos\left(c+\frac{dx}{2}\right)-27\cos\left(c+\frac{3dx}{2}\right)-5\cos\left(3c+\frac{5dx}{2}\right)+\cos\left(3c+\frac{7dx}{2}\right)+131\sin\left(\frac{dx}{2}\right)-72dx\sin\left(c+\frac{dx}{2}\right)-27\sin\left(2c+\frac{3dx}{2}\right)+5\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(4c+\frac{7dx}{2}\right)}{24a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]

[Out] -1/24*(-72*d*x*Cos[(d*x)/2] - 31*Cos[c + (d*x)/2] - 27*Cos[c + (3*d*x)/2] - 5*Cos[3*c + (5*d*x)/2] + Cos[3*c + (7*d*x)/2] + 131*Sin[(d*x)/2] - 72*d*x*Sin[c + (d*x)/2] - 27*Sin[2*c + (3*d*x)/2] + 5*Sin[2*c + (5*d*x)/2] + Sin[4*c + (7*d*x)/2])/(a^2*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

Maple [A]
time = 0.25, size = 104, normalized size = 1.25

method	result
risch	$\frac{3x}{a^2}+\frac{11\,e^{i(dx+c)}}{8d\,a^2}+\frac{11\,e^{-i(dx+c)}}{8d\,a^2}+\frac{4}{d\,a^2\left(e^{i(dx+c)}+i\right)}-\frac{\cos(3dx+3c)}{12d\,a^2}-\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{\frac{16}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4}+\frac{4\left(\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{4}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
default	$\frac{\frac{16}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4}+\frac{4\left(\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{4}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
norman	$\frac{46\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3x}{a}+\frac{28}{3ad}+\frac{22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{9x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{24x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{75x\left(\tan^4\left(\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 16/d/a^2*(1/4/(tan(1/2*d*x+1/2*c)+1)+1/4*(1/2*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4+3*tan(1/2*d*x+1/2*c)^2-1/2*tan(1/2*d*x+1/2*c)+4/3)/(1+tan(1/2*d*x+1/2*c)^2)^3+3/8*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(81) = 162.
time = 0.50, size = 312, normalized size = 3.76

$$\frac{2\left(\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{33\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{24\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{9\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+14}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 2/3*((5*sin(d*x + c)/(cos(d*x + c) + 1) + 33*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9*sin(d*x + c)^6/(co

```
s(d*x + c) + 1)^6 + 14)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^2*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
)/d
```

Fricas [A]
time = 0.37, size = 119, normalized size = 1.43

$$-\frac{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-9\,dx-3\,\left(3\,dx+4\right)\cos\left(dx+c\right)-9\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{3}-9\,dx+3\,\cos\left(dx+c\right)^{2}-6\,\cos\left(dx+c\right)+6\right)\sin\left(dx+c\right)-6}{3\,\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 9*d*x - 3*(3*d*x + 4)*cos(d*x + c
) - 9*cos(d*x + c)^2 + (cos(d*x + c)^3 - 9*d*x + 3*cos(d*x + c)^2 - 6*cos(d
*x + c) + 6)*sin(d*x + c) - 6)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a
^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2263 vs. $2(75) = 150$.
time = 19.36, size = 2263, normalized size = 27.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**7/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2
*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3
*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*
tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x
/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a*
*2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*t
an(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2
)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2
*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 +
d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)*
*7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*
x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 27*d*x*tan(c/2 + d*x/2)**
3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*t
an(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/
```

```
2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d
) + 27*d*x*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan
(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)
**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 +
 d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 +
9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(
c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 9*d*x/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 18*tan(c/
2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 18*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 48*tan(c/2 + d*x/2)**4/(3*a**2*d*t
an(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**
2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 36*tan(c/
2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6
+ 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2
) + 3*a**2*d) + 66*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**7 + 3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**5 + 9*a**2*d*tan(c/2
+ d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d*tan(c/2 + d*x/2)**2 +
 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 10*tan(c/2 + d*x/2)/(3*a**2*d*tan(
c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a**2*d
*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d) + 28/(3*a**2*d
*tan(c/2 + d*x/2)**7 + 3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*
x/2)**5 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**3 + 9*a
**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d*tan(c/2 + d*x/2) + 3*a**2*d), Ne(d, 0)
), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.45, size = 106, normalized size = 1.28

$$\frac{\frac{9\,(dx+c)}{a^2}+\frac{12}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^2 + 12/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(3*tan(1/2*d
*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^4 + 18*tan(1/2*d*x + 1/2*c)^2 - 3*ta
n(1/2*d*x + 1/2*c) + 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

Mupad [B]
time = 12.38, size = 120, normalized size = 1.45

$$\frac{3\,x}{a^2}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{28}{3}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*x)/a^2 + ((10*tan(c/2 + (d*x)/2))/3 + 22*tan(c/2 + (d*x)/2)^2 + 12*tan(c
/2 + (d*x)/2)^3 + 16*tan(c/2 + (d*x)/2)^4 + 6*tan(c/2 + (d*x)/2)^5 + 6*tan(
c/2 + (d*x)/2)^6 + 28/3)/(a^2*d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2
)^2 + 1)^3)
```

3.309 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$-\frac{5x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

```
[Out] -5/2*x/a^2-2*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/a^2/d-2*cos(d*x+c)/
a^2/d/(1+sin(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.241$, Rules used = {2953, 3029, 2788, 2718, 2715, 8, 2727}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) - (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3029

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\sin^2(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{\int (a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(-2+2\sin(c+dx)-\sin^2(c+dx)+\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\
&= -\frac{2x}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}\\
&= -\frac{2x}{a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}-\\
&= -\frac{5x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}
\end{aligned}
$$

Mathematica [A]

time = 0.12, size = 69, normalized size = 1.00

$$\frac{-10(c+dx)-8\cos(c+dx)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+\sin(2(c+dx))}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-10*(c + d*x) - 8*Cos[c + d*x] + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]) + Sin[2*(c + d*x)])/(4*a^2*d)
```

Maple [A]
time = 0.22, size = 91, normalized size = 1.32

method	result
risch	$-\frac{5x}{2a^2}-\frac{e^{i(dx+c)}}{d\,a^2}-\frac{e^{-i(dx+c)}}{d\,a^2}-\frac{4}{d\,a^2\left(e^{i(dx+c)}+i\right)}+\frac{\sin(2dx+2c)}{4a^2d}$
derivativedivides	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
default	$\frac{-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{4\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
norman	$-\frac{8}{ad}-\frac{19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{5\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{41\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{15\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{68\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{36\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*(-1/2/(tan(1/2*d*x+1/2*c)+1)-1/2*(1/4*tan(1/2*d*x+1/2*c)^3+tan(1/2*
d*x+1/2*c)^2-1/4*tan(1/2*d*x+1/2*c)+1)/(1+tan(1/2*d*x+1/2*c)^2)^2-5/8*arcta
n(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(65) = 130$.
time = 0.52, size = 226, normalized size = 3.28

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+8}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{5\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 8)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^2*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [**A**]
time = 0.35, size = 100, normalized size = 1.45

$$-\frac{\cos\left(dx+c\right)^{3}+5\,dx+\left(5\,dx+7\right)\cos\left(dx+c\right)+4\,\cos\left(dx+c\right)^{2}+\left(5\,dx-\cos\left(dx+c\right)^{2}+3\,\cos\left(dx+c\right)-4\right)\sin\left(dx+c\right)+4}{2\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + 5*d*x + (5*d*x + 7)*cos(d*x + c) + 4*cos(d*x + c)^2
+ (5*d*x - cos(d*x + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) + 4)/(a^2*d*co
s(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(63) = 126$.
time = 12.00, size = 1248, normalized size = 18.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-5*d*x*tan(c/2 + d*x/2)**5/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**
2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 +
 d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 5*d*x*tan(c/2 + d*x/2)
**4/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d
*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*
x/2) + 2*a**2*d) - 10*d*x*tan(c/2 + d*x/2)**3/(2*a**2*d*tan(c/2 + d*x/2)**5
 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*t
an(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 10*d*x*tan(c/2
 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 +
 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan
(c/2 + d*x/2) + 2*a**2*d) - 5*d*x*tan(c/2 + d*x/2)/(2*a**2*d*tan(c/2 + d*x/
2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**
2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 5*d*x/(2*
a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/
2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) +
2*a**2*d) - 10*tan(c/2 + d*x/2)**4/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d
```

```
*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*
x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 10*tan(c/2 + d*x/2)**3/(2
*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c
/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) +
 2*a**2*d) - 22*tan(c/2 + d*x/2)**2/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*
d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d
*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*a**2*d) - 6*tan(c/2 + d*x/2)/(2*a*
*2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 + 4*a**2*d*tan(c/2
+ d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan(c/2 + d*x/2) + 2*
a**2*d) - 16/(2*a**2*d*tan(c/2 + d*x/2)**5 + 2*a**2*d*tan(c/2 + d*x/2)**4 +
 4*a**2*d*tan(c/2 + d*x/2)**3 + 4*a**2*d*tan(c/2 + d*x/2)**2 + 2*a**2*d*tan
(c/2 + d*x/2) + 2*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**2/(a*sin(c) + a)
**2, True))
```

Giac [A]
time = 0.48, size = 91, normalized size = 1.32

$$-\frac{\frac{5\,(dx+c)}{a^2}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{8}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 + 2*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^
2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 8/(a^2
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B]
time = 10.79, size = 95, normalized size = 1.38

$$-\frac{5\,x}{2\,a^2}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (5*x)/(2*a^2) - (3*tan(c/2 + (d*x)/2) + 11*tan(c/2 + (d*x)/2)^2 + 5*tan(c
/2 + (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4 + 8)/(a^2*d*(tan(c/2 + (d*x)/2) +
1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.310 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 2*x/a^2+cos(d*x+c)/a^2/d+2*cos(d*x+c)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2936, 2718}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\,(a^2\sin(c+dx)+a^2)}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (2*Cos[c + d*x])/(d*(a^2 + a^2*Sin[c + d
*x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}-\frac{\int(-2a+a\sin(c+dx))\,dx}{a^3}\\
&= \frac{2x}{a^2}+\frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}-\frac{\int\sin(c+dx)\,dx}{a^2}\\
&= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{d\,(a^2+a^2\sin(c+dx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 117 vs. $2(47) = 94$.
time = 0.25, size = 117, normalized size = 2.49

$$\frac{12dx\cos\left(\frac{dx}{2}\right)+2\cos\left(c+\frac{dx}{2}\right)+3\cos\left(c+\frac{3dx}{2}\right)-28\sin\left(\frac{dx}{2}\right)+12dx\sin\left(c+\frac{dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)}{6a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (12*d*x*Cos[(d*x)/2] + 2*Cos[c + (d*x)/2] + 3*Cos[c + (3*d*x)/2] - 28*Sin[(
d*x)/2] + 12*d*x*Sin[c + (d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(6*a^2*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.21, size = 50, normalized size = 1.06

method	result
derivativedivides	$\frac{\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
default	$\frac{\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{4}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$
risch	$\frac{2x}{a^2}+\frac{e^{i(dx+c)}}{2d\,a^2}+\frac{e^{-i(dx+c)}}{2d\,a^2}+\frac{4}{d\,a^2\left(e^{i(dx+c)}+i\right)}$
norman	$\frac{6}{ad}+\frac{4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2x}{a}+\frac{6x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{12x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{20x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{24x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*(1/(tan(1/2*d*x+1/2*c)+1)+1/2/(1+tan(1/2*d*x+1/2*c)^2)+arctan(tan(1
/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.
time = 0.49, size = 139, normalized size = 2.96

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+3}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*((sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 3)/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1) + a^2*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 2*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.35, size = 77, normalized size = 1.64

$$\frac{2\,dx + (2\,dx + 3)\cos(dx + c) + \cos(dx + c)^2 + (2\,dx + \cos(dx + c) - 2)\sin(dx + c) + 2}{a^2d\cos(dx + c) + a^2d\sin(dx + c) + a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*d*x + (2*d*x + 3)*cos(d*x + c) + cos(d*x + c)^2 + (2*d*x + cos(d*x + c)
- 2)*sin(d*x + c) + 2)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(41) = 82$.
time = 6.91, size = 479, normalized size = 10.19

$$\begin{cases} \frac{2dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2dx}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{4\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{6}{a^2d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} & \text{for}\: d \neq 0 \\ \frac{x\sin(c)\cos^2(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*d*x*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*t
an(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*
x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 2*d*x*tan(c/2 + d*x/2)/(a**2*d*tan(c/2 + d*x/2)
**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*d*
x/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2
 + d*x/2) + a**2*d) + 4*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**3 + a
**2*d*tan(c/2 + d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d) + 2*tan(c/2 +
 d*x/2)/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2 + d*x/2)**2 + a**2*d*t
an(c/2 + d*x/2) + a**2*d) + 6/(a**2*d*tan(c/2 + d*x/2)**3 + a**2*d*tan(c/2
+ d*x/2)**2 + a**2*d*tan(c/2 + d*x/2) + a**2*d), Ne(d, 0)), (x*sin(c)*cos(c
)**2/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.43, size = 78, normalized size = 1.66

$$\frac{2\left(\frac{dx+c}{a^2} + \frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)/a^2 + (2*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 3)/((
tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) + 1)
*a^2))/d
```

Mupad [B]
time = 8.86, size = 68, normalized size = 1.45

$$\frac{2\,x}{a^2}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (2*x)/a^2 + (2*tan(c/2 + (d*x)/2) + 4*tan(c/2 + (d*x)/2)^2 + 6)/(a^2*d*(tan
(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.311 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2953, 3045, 3855, 2727}

$$\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x
]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\ &= \frac{\int \left(\csc(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\ &= \frac{\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c+dx)}\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 115 vs. 2(40) = 80.
time = 0.11, size = 115, normalized size = 2.88

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(4+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2]*(Log[Cos[(c +
d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (4 + Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]])*Sin[(c + d*x)/2]))/(a^2*d*(1 + Sin[c + d*x])^2))
```

Maple [A]
time = 0.27, size = 34, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$	34
default	$\frac{\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$	34
risch	$\frac{4}{d\,a^2\left(e^{i(dx+c)}+i\right)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$	63
norman	$\frac{\frac{4}{ad}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$	134

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(4/(tan(1/2*d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.38

$$\frac{\frac{4}{a^2+\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (4/(a^2 + a^2*sin(d*x + c)/(cos(d*x + c) + 1)) + log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(40) = 80.
time = 0.34, size = 103, normalized size = 2.58

$$-\frac{(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\cos(dx+c)+4\sin(dx+c)-4}{2\,(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(d*x + c) + sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(
d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c)
+ 4*sin(d*x + c) - 4)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^2(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.45, size = 38, normalized size = 0.95

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] (log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 4/(a^2*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B]
time = 8.68, size = 39, normalized size = 0.98

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + 4/(a^2*d*(tan(c/2 + (d*x)/2) + 1))
```

3.312 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-3*cot(d*x+c)/a^2/d+2*cot(d*x+c)/a^2/d/(1+sin(d*
x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2788, 3855, 3852, 8, 3862}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c + d*x])
/(a^2*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(2-2\csc(c+dx)+\csc^2(c+dx)-\frac{2}{1+\csc(c+dx)}\right)dx}{a^2}\\ &= \frac{2x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\csc(c+dx)}\,dx}{a^2}\\ &= \frac{2x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}+\frac{2\int -1\,dx}{a^2}-\frac{\operatorname{Subst}(\int 1\end{aligned}$$

$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d(1+\csc(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(54) = 108.
time = 0.57, size = 216, normalized size = 4.00

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{3}{2}(c+dx)\right)\left(5+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-3-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2\left(-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(1-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(Cos[(3*(c + d*x))/2]*(5 + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c + d*
x)/2]]) + Cos[(c + d*x)/2]*(-3 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d
*x)/2]]) + 2*(-2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Cos[c +
d*x]*(1 - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)
/2]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.26, size = 59, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	59

default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	59
risch	$-\frac{2\left(-3+i\,e^{i(dx+c)}+2\,e^{2i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)a^2d}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}-1\right)}{a^2d}$	102
norman	$\frac{-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{2ad}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{27\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-8/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)
-4*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.

time = 0.29, size = 116, normalized size = 2.15

$$-\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^2*sin(d*x + c)/(cos(d*x +
c) + 1) + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(54) = 108$.

time = 0.35, size = 160, normalized size = 2.96

$$\frac{3\cos(dx+c)^2+\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(3\cos(dx+c)+2)\sin(dx+c)+\cos(dx+c)-2}{a^2d\cos(dx+c)^2-a^2d-\left(a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (3*cos(d*x + c)^2 + (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*
log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x
+ c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + (3*cos(d*x + c) + 2)*sin(d*x + c)
+ cos(d*x + c) - 2)/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a
^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.47, size = 90, normalized size = 1.67

$$-\frac{\frac{4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a^2 - (2*
tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c))*a^2))/d
```

Mupad [B]
time = 8.69, size = 87, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (9*tan(c/2 + (d*x)/2) + 1)/(d*(2*a^2*tan(c/2
 + (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2)))/(a
^2*d)
```

3.313 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=78

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

```
[Out] -5/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^2/d+2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2953, 3045, 3855, 3852, 8, 3853, 2727}

$$\frac{2\cot(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d) + (2*Cos[c + d*x])/(a^2*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
```

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{\int \left(2\csc(c+dx)-2\csc^2(c+dx)+\csc^3(c+dx)-\frac{2}{1+\sin(c+dx)}\right)dx}{a^2}\\
&= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{2\int \csc(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \frac{1}{1+\sin(c}}{a^2}\\
&= -\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx)}\\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{}{a^2d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(78) = 156.
time = 0.58, size = 364, normalized size = 4.67

$$
-\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{d(a+a\sin(c+dx))^2}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d(a+a\sin(c+dx))^2}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{8d(a+a\sin(c+dx))^2}-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{2d(a+a\sin(c+dx))^2}+\frac{5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{2d(a+a\sin(c+dx))^2}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{8d(a+a\sin(c+dx))^2}-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\tan\left(\frac{1}{2}(c+dx)\right)}{d(a+a\sin(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a + a*Sin
[c + d*x])^2) + (Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/
(d*(a + a*Sin[c + d*x])^2) - (Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^4)/(8*d*(a + a*Sin[c + d*x])^2) - (5*Log[Cos[(c + d*x)/2]]*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[c + d*x])^2) + (5*Log[
Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(2*d*(a + a*Sin[
c + d*x])^2) + (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
/(8*d*(a + a*Sin[c + d*x])^2) - ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Ta
n[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.29, size = 87, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^2}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^2}$
risch	$\frac{5\,e^{4i(dx+c)}-11\,e^{2i(dx+c)}+5ie^{3i(dx+c)}+8-3ie^{i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)a^2d}-\frac{5\ln\left(e^{i(dx+c)}+1\right)}{2a^2d}+\frac{5\ln\left(e^{i(dx+c)}-1\right)}{2a^2d}$
norman	$\frac{\frac{10\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{37\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{71\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{5\ln(t$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x+1/2*c)+16/(tan(1/2*d*x+1/
2*c)+1)-1/2/tan(1/2*d*x+1/2*c)^2+4/tan(1/2*d*x+1/2*c)+10*ln(tan(1/2*d*x+1/2
*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(74) = 148.
time = 0.29, size = 161, normalized size = 2.06

$$\frac{\frac{\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{20\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((7*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 1)/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3) - (8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/a^2 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(74) = 148$.
time = 0.36, size = 246, normalized size = 3.15

$$\frac{16\cos(dx+c)^3+10\cos(dx+c)^2-5\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(8\cos(dx+c)^2+3\cos(dx+c)-4\right)\sin(dx+c)-14\cos(dx+c)-8}{4\left(a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-a^2d+\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*cos(d*x + c)^3 + 10*cos(d*x + c)^2 - 5*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(8*cos(d*x
 + c)^2 + 3*cos(d*x + c) - 4)*sin(d*x + c) - 14*cos(d*x + c) - 8)/(a^2*d*co
s(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*c
os(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.50, size = 116, normalized size = 1.49

$$\frac{\frac{20\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{32}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(20*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 32/(a^2*(tan(1/2*d*x + 1/2*c) + 1)) - (30
```

```
*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/
2*c)^2))/d
```

Mupad [B]
time = 8.66, size = 120, normalized size = 1.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}+\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\,\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (5*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) + ((
7*tan(c/2 + (d*x)/2))/2 + 20*tan(c/2 + (d*x)/2)^2 - 1/2)/(d*(4*a^2*tan(c/2
+ (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^3)) - tan(c/2 + (d*x)/2)/(a^2*d)
```

3.314 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^2/d-3*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+cot(d
*x+c)*csc(d*x+c)/a^2/d-2*cos(d*x+c)/a^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2953, 3045, 3855, 3852, 8, 3853, 2727}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^2*d) - (3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]
^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
```

```
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{\int \left(-2\csc(c+dx)+2\csc^2(c+dx)-2\csc^3(c+dx)+\csc^4(c+dx)+\frac{}{1+\sin \ldots}\right.}{a^2}$$

$$= \frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}+\frac{2\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c \ldots}{a \ldots}$$

$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d(1+\sin(c+dx))} \ldots$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c \ldots}{a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 472 vs. 2(91) = 182.
time = 0.92, size = 472, normalized size = 5.19

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-10\cos\left(\frac{3}{2}(c+dx)\right)+20\cos\left(\frac{5}{2}(c+dx)\right)-9\cos\left(\frac{5}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+9\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)\left(8+9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3\cos\left(\frac{3}{2}(c+dx)\right)\left(14+9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+9\cos\left(\frac{5}{2}(c+dx)\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-9\cos\left(\frac{7}{2}(c+dx)\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\sin\left(\frac{1}{2}(c+dx)\right)+27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)-27\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)-6\sin\left(\frac{3}{2}(c+dx)\right)+27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{3}{2}(c+dx)\right)-27\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{3}{2}(c+dx)\right)-2\sin\left(\frac{5}{2}(c+dx)\right)-9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{5}{2}(c+dx)\right)+9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{5}{2}(c+dx)\right)+8\sin\left(\frac{7}{2}(c+dx)\right)-9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)+9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{7}{2}(c+dx)\right)\right)}{192a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^3*(-10*Cos[(5*(c + d*x))/2] + 20*Cos
[(7*(c + d*x))/2] - 9*Cos[(5*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 9*Cos[(7
*(c + d*x))/2]*Log[Cos[(c + d*x)/2]] + 3*Cos[(c + d*x)/2]*(8 + 9*Log[Cos[(c
 + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) - 3*Cos[(3*(c + d*x))/2]*(14 + 9*Log
[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 9*Cos[(5*(c + d*x))/2]*Log[
Sin[(c + d*x)/2]] - 9*Cos[(7*(c + d*x))/2]*Log[Sin[(c + d*x)/2]] + 12*Sin[(
c + d*x)/2] + 27*Log[Cos[(c + d*x)/2]]*Sin[(c + d*x)/2] - 27*Log[Sin[(c + d
*x)/2]]*Sin[(c + d*x)/2] - 6*Sin[(3*(c + d*x))/2] + 27*Log[Cos[(c + d*x)/2]
]*Sin[(3*(c + d*x))/2] - 27*Log[Sin[(c + d*x)/2]]*Sin[(3*(c + d*x))/2] - 2*
Sin[(5*(c + d*x))/2] - 9*Log[Cos[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 9*Log
[Sin[(c + d*x)/2]]*Sin[(5*(c + d*x))/2] + 8*Sin[(7*(c + d*x))/2] - 9*Log[Co
s[(c + d*x)/2]]*Sin[(7*(c + d*x))/2] + 9*Log[Sin[(c + d*x)/2]]*Sin[(7*(c +
d*x))/2]))/(192*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.36, size = 113, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{32}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{11}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\,\mathrm{l}}{8d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{32}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{11}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-24\,\mathrm{l}}{8d\,a^2}$
risch	$-\frac{2\left(9i\,e^{5i(dx+c)}-24\,e^{4i(dx+c)}+9\,e^{6i(dx+c)}-18i\,e^{3i(dx+c)}+33\,e^{2i(dx+c)}+5i\,e^{i(dx+c)}-14\right)}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)a^2d}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{3\ln(}{}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{65\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{83\left(\tan^4\left(\frac{d}{}\right.\right.}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2+11*tan(1/2*d*x+1
/2*c)-32/(tan(1/2*d*x+1/2*c)+1)-1/3/tan(1/2*d*x+1/2*c)^3+2/tan(1/2*d*x+1/2*
c)^2-11/tan(1/2*d*x+1/2*c)-24*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(89) = 178$.
time = 0.28, size = 199, normalized size = 2.19

$$\frac{\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{129\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((5*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 129*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (33*sin(d*
x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 72*log(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(89) = 178$.
time = 0.37, size = 302, normalized size = 3.32

$$\frac{28 \cos(dx+c)^4 + 10 \cos(dx+c)^3 - 42 \cos(dx+c)^2 + 9\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(dx+c)^2 - \cos(dx+c) - 1\right)\sin(dx+c) + 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) - 9\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 - \left(\cos(dx+c)^3 + \cos(dx+c)^2 - \cos(dx+c) - 1\right)\sin(dx+c) + 1\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 2\left(14\cos(dx+c)^3 + 9\cos(dx+c)^2 - 12\cos(dx+c) - 6\right)\sin(dx+c) - 12\cos(dx+c) + 12}{6\left(a^2 d\cos(dx+c)^4 - 2a^2 d\cos(dx+c)^2 + a^2 d - \left(a^2 d\cos(dx+c)^3 + a^2 d\cos(dx+c)^2 - a^2 d\cos(dx+c) - a^2 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*cos(d*x + c)^4 + 10*cos(d*x + c)^3 - 42*cos(d*x + c)^2 + 9*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c
) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 9*(cos(d*x + c)^4 -
2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1)*sin
(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*cos(d*x + c)^3 + 9*cos(
d*x + c)^2 - 12*cos(d*x + c) - 6)*sin(d*x + c) - 12*cos(d*x + c) + 12)/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.50, size = 146, normalized size = 1.60

$$-\frac{\frac{72\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} + \frac{96}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)} - \frac{132\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3} - \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 6a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 33a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 96/(a^2*(tan(1/2*d*x + 1/2*c
) + 1)) - (132*tan(1/2*d*x + 1/2*c)^3 - 33*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1
/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*
c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 33*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 8.65, size = 153, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{43\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{3}}{d\,\left(8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) - (3*log(t
an(c/2 + (d*x)/2)))/(a^2*d) - (9*tan(c/2 + (d*x)/2)^2 - (5*tan(c/2 + (d*x)/
2))/3 + 43*tan(c/2 + (d*x)/2)^3 + 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 8*a
^2*tan(c/2 + (d*x)/2)^4)) + (11*tan(c/2 + (d*x)/2))/(8*a^2*d)
```

3.315 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$-\frac{11x}{2a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}-\frac{19\cos(c+dx)}{3a^3d(1+\sin(c+dx))}$$

```
[Out] -11/2*x/a^3-3*cos(d*x+c)/a^3/d+1/2*cos(d*x+c)*sin(d*x+c)/a^3/d+2/3*cos(d*x+
c)/a^3/d/(1+sin(d*x+c))^2-19/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2953, 3045, 2718, 2715, 8, 2729, 2727}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{19\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (3*Cos[c + d*x])/(a^3*d) + (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])^2) - (19*Cos[c + d*x
])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2}\\
&= \frac{\int \left(-\frac{5}{a}+\frac{3\sin(c+dx)}{a}-\frac{\sin^2(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}+\frac{7}{a(1+\sin(c+dx))}\right)dx}{a^2}\\
&= -\frac{5x}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}+\frac{7
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5x}{a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))
\\
&= -\frac{11x}{2a^3}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(97) = 194.
time = 0.81, size = 197, normalized size = 2.03

$$
-\frac{1980dx\cos\left(\frac{dx}{2}\right)-1326\cos\left(c+\frac{dx}{2}\right)+2012\cos\left(c+\frac{3dx}{2}\right)-660dx\cos\left(2c+\frac{3dx}{2}\right)-135\cos\left(3c+\frac{5dx}{2}\right)+15\cos\left(3c+\frac{7dx}{2}\right)-3216\sin\left(\frac{dx}{2}\right)+1980dx\sin\left(c+\frac{dx}{2}\right)+660dx\sin\left(c+\frac{3dx}{2}\right)+498\sin\left(2c+\frac{3dx}{2}\right)+135\sin\left(2c+\frac{5dx}{2}\right)+15\sin\left(4c+\frac{7dx}{2}\right)}{240a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}
$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]

[Out] -1/240*(1980*d*x*Cos[(d*x)/2] - 1326*Cos[c + (d*x)/2] + 2012*Cos[c + (3*d*x)/2] - 660*d*x*Cos[2*c + (3*d*x)/2] - 135*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c + (7*d*x)/2] - 3216*Sin[(d*x)/2] + 1980*d*x*Sin[c + (d*x)/2] + 660*d*x*Sin[c + (3*d*x)/2] + 498*Sin[2*c + (3*d*x)/2] + 135*Sin[2*c + (5*d*x)/2] + 15*Sin[4*c + (7*d*x)/2])/(a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)

Maple [A]
time = 0.34, size = 123, normalized size = 1.27

method	result
derivativedivides	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2\left(\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-11\,\mathrm{arcta}}{d\,a^{3}}$
default	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{2\left(\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3\right)}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-11\,\mathrm{arcta}}{d\,a^{3}}$
risch	$-\frac{11x}{2a^{3}}-\frac{i\mathrm{e}^{2i(dx+c)}}{8d\,a^{3}}-\frac{3\,\mathrm{e}^{i(dx+c)}}{2d\,a^{3}}-\frac{3\,\mathrm{e}^{-i(dx+c)}}{2d\,a^{3}}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8d\,a^{3}}-\frac{2\left(36i\mathrm{e}^{i(dx+c)}+21\,\mathrm{e}^{2i(dx+c)}-19\right)}{3d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}}$
norman	$\frac{-\frac{55x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{663\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{11\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{11x}{2a}-\frac{52}{3ad}-\frac{227\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{385x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{165x}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 16/d/a^3*(1/6/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/2*d*x+1/2*c)+1)^2-5/8/(tan(1/2*d*x+1/2*c)+1)-1/8*(1/2*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2-1/2*tan(1/2*d*x+1/2*c)+3)/(1+tan(1/2*d*x+1/2*c)^2)^2-11/16*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(89) = 178.
time = 0.49, size = 314, normalized size = 3.24

$$-\frac{\frac{\frac{123\sin(dx+c)}{\cos(dx+c)+1}+\frac{161\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{210\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{154\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{99\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{33\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+52}{a^{3}+\frac{3a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{7a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{7a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{5a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{3a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}+\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

```
[Out] -1/3*((123*sin(d*x + c)/(cos(d*x + c) + 1) + 161*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 210*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 154*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 99*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1
) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 7*a^3*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 7*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 3*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a
^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

Fricas [A]
time = 0.35, size = 163, normalized size = 1.68

$$\frac{3\cos(dx+c)^4-(33\,dx-53)\cos(dx+c)^2-12\cos(dx+c)^3+66\,dx+(33\,dx+64)\cos(dx+c)+\left(3\cos(dx+c)^3+66\,dx+(33\,dx+68)\cos(dx+c)+15\cos(dx+c)^2+4\right)\sin(dx+c)-4}{6\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2\,a^3d-\left(a^3d\cos(dx+c)+2\,a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^4 - (33*d*x - 53)*cos(d*x + c)^2 - 12*cos(d*x + c)^3 +
66*d*x + (33*d*x + 64)*cos(d*x + c) + (3*cos(d*x + c)^3 + 66*d*x + (33*d*x
+ 68)*cos(d*x + c) + 15*cos(d*x + c)^2 + 4)*sin(d*x + c) - 4)/(a^3*d*cos(d*
x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*si
n(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(90) = 180$.
time = 34.64, size = 2264, normalized size = 23.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-33*d*x*tan(c/2 + d*x/2)**7/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a
**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c
/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)
**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**6/(
6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*ta
n(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x
/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**
3*d) - 165*d*x*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*
d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 +
 d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2
+ 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 231*d*x*tan(c/2 + d*x/2)**4/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
```

```
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 231*d*x*tan(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*t
an(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*
x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 1
8*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 165*d*x*tan(c/2 + d*x/2)**2/(6*a**3
*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2
+ d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3
 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) -
 99*d*x*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2
+ d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4
 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*
d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*
a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(
c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2
)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2)**6/(6*a
**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c
/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)
**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d
) - 198*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c
/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)
**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a*
*3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 308*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c
/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)
**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a*
*3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 420*tan
(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)
**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a*
*3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/
2 + d*x/2) + 6*a**3*d) - 322*tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)
**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a*
*3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/
2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 246*tan(c/2 + d*x/
2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**6 + 30*a**3*
d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3*d*tan(c/2 +
 d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2 + d*x/2) + 6
*a**3*d) - 104/(6*a**3*d*tan(c/2 + d*x/2)**7 + 18*a**3*d*tan(c/2 + d*x/2)**
6 + 30*a**3*d*tan(c/2 + d*x/2)**5 + 42*a**3*d*tan(c/2 + d*x/2)**4 + 42*a**3
*d*tan(c/2 + d*x/2)**3 + 30*a**3*d*tan(c/2 + d*x/2)**2 + 18*a**3*d*tan(c/2
+ d*x/2) + 6*a**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**2/(a*sin(c) + a)**3,
True))
```

<u>Giac</u> [**A**]

time = 0.45, size = 117, normalized size = 1.21

$$-\frac{\frac{33\,(dx+c)}{a^3}+\frac{6\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3}+\frac{4\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 6*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)
^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 4*(15
*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 12.07, size = 121, normalized size = 1.25

$$-\frac{11\,x}{2\,a^3}-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+33\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{154\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+70\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{161\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+41\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{52}{3}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*x)/(2*a^3) - (41*tan(c/2 + (d*x)/2) + (161*tan(c/2 + (d*x)/2)^2)/3 +
70*tan(c/2 + (d*x)/2)^3 + (154*tan(c/2 + (d*x)/2)^4)/3 + 33*tan(c/2 + (d*x)
/2)^5 + 11*tan(c/2 + (d*x)/2)^6 + 52/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^3*(
tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.316 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{3x}{a^3}+\frac{3\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3}+\frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}$$

```
[Out] 3*x/a^3+3*cos(d*x+c)/a^3/d-1/3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^3+2*cos(d*x+
c)^3/a/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2950, 2759, 2761, 8}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3x}{a^3}+\frac{2\cos^3(c+dx)}{ad(a\sin(c+dx)+a)^2}-\frac{\cos^3(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(3*d*(a + a*Sin[c + d
*x])^3) + (2*Cos[c + d*x]^3)/(a*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2950

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*m)), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} - \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx \\ &= -\frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{3\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{a^2} \\ &= \frac{3\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2} + \frac{3\int 1}{a^3} \\ &= \frac{3x}{a^3} + \frac{3\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{3d(a+a\sin(c+dx))^3} + \frac{2\cos^3(c+dx)}{ad(a+a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]

time = 0.49, size = 96, normalized size = 1.26

$$\frac{9c+9dx+3\cos(c+dx)-\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(11+13\sin(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*c + 9*d*x + 3*Cos[c + d*x] - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 -
 (2*Sin[(c + d*x)/2]*(11 + 13*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3)/(3*a^3*d)
```

Maple [A]

time = 0.28, size = 84, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{8}{4+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
default	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{8}{4+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
risch	$\frac{3x}{a^3}+\frac{e^{i(dx+c)}}{2d\,a^3}+\frac{e^{-i(dx+c)}}{2d\,a^3}+\frac{-\frac{26}{3}+16i e^{i(dx+c)}+10\,e^{2i(dx+c)}}{d\,a^3\left(e^{i(dx+c)}+i\right)^3}$

norman	$\frac{\frac{238\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{3x}{a}+\frac{28}{3ad}+\frac{122\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{15x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{42x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{153x\left(\tan^{4}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^3*(-1/3/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(tan(1/2*d*x+1/2*c)+1)^2+3/4/(ta
n(1/2*d*x+1/2*c)+1)+1/4/(1+tan(1/2*d*x+1/2*c)^2)+3/4*arctan(tan(1/2*d*x+1/2
*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(74) = 148$.
time = 0.49, size = 228, normalized size = 3.00

$$\frac{2\left(\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{29\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/3*((33*sin(d*x + c)/(cos(d*x + c) + 1) + 29*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 4*a^3*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.34, size = 144, normalized size = 1.89

$$\frac{(9\,dx-16)\cos(dx+c)^2+3\cos(dx+c)^3-18\,dx-(9\,dx+17)\cos(dx+c)-(18\,dx+(9\,dx+19)\cos(dx+c)+3\cos(dx+c)^2+2)\sin(dx+c)+2}{3\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2\,a^3d-(a^3d\cos(dx+c)+2\,a^3d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/3*((9*d*x - 16)*cos(d*x + c)^2 + 3*cos(d*x + c)^3 - 18*d*x - (9*d*x + 17)
*cos(d*x + c) - (18*d*x + (9*d*x + 19)*cos(d*x + c) + 3*cos(d*x + c)^2 + 2)
*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (
a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. $2(68) = 136$.
time = 21.05, size = 1246, normalized size = 16.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**5/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3
*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2
+ d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/
2)**4/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**
3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2
+ d*x/2) + 3*a**3*d) + 36*d*x*tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2
)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a*
*3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 36*d*x*t
an(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2
)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a*
*3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 27*d*x*tan(c/2 + d*x/2)/(3*a**3*d*tan(c
/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)*
*3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d)
+ 9*d*x/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a
**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/
2 + d*x/2) + 3*a**3*d) + 18*tan(c/2 + d*x/2)**4/(3*a**3*d*tan(c/2 + d*x/2)*
*5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3
*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 54*tan(c/2
 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 +
 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*t
an(c/2 + d*x/2) + 3*a**3*d) + 58*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*
x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12
*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) + 66*ta
n(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d*tan(c/2 + d*x/2)**4
 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d
*tan(c/2 + d*x/2) + 3*a**3*d) + 28/(3*a**3*d*tan(c/2 + d*x/2)**5 + 9*a**3*d
*tan(c/2 + d*x/2)**4 + 12*a**3*d*tan(c/2 + d*x/2)**3 + 12*a**3*d*tan(c/2 +
d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*sin(c)**2*
cos(c)**2/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.45, size = 80, normalized size = 1.05

$$\frac{\frac{9\,(dx+c)}{a^3}+\frac{6}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)/a^3 + 6/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + 2*(9*tan(1/2*
d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(tan(1/2*d*x + 1/2*c) +
 1)^3))/d
```

Mupad [B]
time = 11.32, size = 94, normalized size = 1.24

$$\frac{3\,x}{a^3}+\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{58\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{28}{3}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*x)/a^3 + (22*tan(c/2 + (d*x)/2) + (58*tan(c/2 + (d*x)/2)^2)/3 + 18*tan(c
/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^4 + 28/3)/(a^3*d*(tan(c/2 + (d*x)/2)
 + 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.317 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$-\frac{x}{a^3}-\frac{7\cos(c+dx)}{3a^3d(1+\sin(c+dx))}+\frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2}$$

```
[Out] -x/a^3-7/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))+2/3*cos(d*x+c)/a/d/(a+a*sin(d*x+
c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2936, 2814, 2727}

$$-\frac{7\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}-\frac{x}{a^3}+\frac{2\cos(c+dx)}{3ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x])) + (2*Cos[c + d*x])
/(3*a*d*(a + a*Sin[c + d*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} - \frac{\int \frac{-4a+3a\sin(c+dx)}{a+a\sin(c+dx)}\,dx}{3a^3}$$
$$= -\frac{x}{a^3} + \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} + \frac{7\int \frac{1}{a+a\sin(c+dx)}\,dx}{3a^2}$$
$$= -\frac{x}{a^3} + \frac{2\cos(c+dx)}{3ad(a+a\sin(c+dx))^2} - \frac{7\cos(c+dx)}{3d\,(a^3+a^3\sin(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(61) = 122.
time = 0.32, size = 145, normalized size = 2.38

$$-\frac{180dx\cos\left(\frac{dx}{2}\right)-351\cos\left(c+\frac{dx}{2}\right)+277\cos\left(c+\frac{3dx}{2}\right)-60dx\cos\left(2c+\frac{3dx}{2}\right)-471\sin\left(\frac{dx}{2}\right)+180dx\sin\left(c+\frac{dx}{2}\right)+60dx\sin\left(c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{3dx}{2}\right)}{120a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/120*(180*d*x*Cos[(d*x)/2] - 351*Cos[c + (d*x)/2] + 277*Cos[c + (3*d*x)/2
] - 60*d*x*Cos[2*c + (3*d*x)/2] - 471*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/
2] + 60*d*x*Sin[c + (3*d*x)/2] + 3*Sin[2*c + (3*d*x)/2])/(a^3*d*(Cos[c/2] +
 Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 0.26, size = 67, normalized size = 1.10

method	result
risch	$-\frac{x}{a^3}-\frac{2\left(12i\mathrm{e}^{i(dx+c)}+9\,\mathrm{e}^{2i(dx+c)}-7\right)}{3d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)^3}$
derivativedivides	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
default	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
norman	$-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{12\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{56\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{x}{a}-\frac{5x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{13x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*(2/3/(tan(1/2*d*x+1/2*c)+1)^3-1/(tan(1/2*d*x+1/2*c)+1)^2-1/2/(tan(1
/2*d*x+1/2*c)+1)-1/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.
time = 0.50, size = 142, normalized size = 2.33

$$-\frac{2\left(\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+5}{a^3+\frac{3a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/3*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 5)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(57) = 114$.
time = 0.38, size = 124, normalized size = 2.03

$$-\frac{(3\,dx-7)\cos(dx+c)^2-6\,dx-(3\,dx+5)\cos(dx+c)-(6\,dx+(3\,dx+7)\cos(dx+c)+2)\sin(dx+c)+2}{3\left(a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-2\,a^3d-(a^3d\cos(dx+c)+2\,a^3d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*((3*d*x - 7)*cos(d*x + c)^2 - 6*d*x - (3*d*x + 5)*cos(d*x + c) - (6*d*
x + (3*d*x + 7)*cos(d*x + c) + 2)*sin(d*x + c) + 2)/(a^3*d*cos(d*x + c)^2 -
 a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)*sin(d*x + c)
)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(53) = 106$.
time = 10.80, size = 529, normalized size = 8.67

$$\begin{cases} -\frac{3dx\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{9dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{9dx\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{3dx}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{6\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} - \frac{10}{3a^3d\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^3d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^3d} & \text{for}\ d \neq 0 \\ \frac{x\sin(c)\cos^2(c)}{(a\sin(c)+a)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**3/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**
3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 9*d*x*tan
(c/2 + d*x/2)**2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)*
*2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 9*d*x*tan(c/2 + d*x/2)/(3*a**3
*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 +
```

```
d*x/2) + 3*a**3*d) - 3*d*x/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2
 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d) - 6*tan(c/2 + d*x/2)**
2/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*t
an(c/2 + d*x/2) + 3*a**3*d) - 24*tan(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2
)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3*d*tan(c/2 + d*x/2) + 3*a**3*d)
 - 10/(3*a**3*d*tan(c/2 + d*x/2)**3 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 9*a**3
*d*tan(c/2 + d*x/2) + 3*a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**2/(a*sin(c) +
 a)**3, True))
```

Giac [A]
time = 0.47, size = 60, normalized size = 0.98

$$-\frac{\frac{3\,(dx+c)}{a^3}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^3 + 2*(3*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*
c) + 5)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 8.77, size = 54, normalized size = 0.89

$$-\frac{x}{a^3}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{10}{3}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - x/a^3 - (8*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^2 + 10/3)/(a^3*d*(ta
n(c/2 + (d*x)/2) + 1)^3)
```

3.318 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{5\cos(c+dx)}{3a^3d(1+\sin(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+2/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+5/3*cos(d*
x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2953, 3045, 3855, 2729, 2727}

$$\frac{5\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d
*x])^2) + (5*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2} \\
&= \frac{\int \left(\frac{\csc(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}-\frac{1}{a(1+\sin(c+dx))}\right)dx}{a^2} \\
&= \frac{\int \csc(c+dx)\,dx}{a^3}-\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{\cos(c+dx)}{a^3d(1+\sin(c+dx))}- \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{5\cos(c+dx)}{3a^3d(1+\sin(c+dx))}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 185 vs. 2(68) = 136.
time = 0.28, size = 185, normalized size = 2.72

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-4\sin\left(\frac{1}{2}(c+dx)\right)+2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-10\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\right)}{3d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-4*Sin[(c + d*x)/2] + 2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]) - 10*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 - 3*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3 + 3*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/
(3*d*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.28, size = 64, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}$
default	$\frac{\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}}$
risch	$\frac{8i\mathrm{e}^{i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}-\frac{10}{3}}{d\,a^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d\,a^{3}}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d\,a^{3}}$
norman	$\frac{\frac{6\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{20\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{14}{3ad}+\frac{52\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}+\frac{94\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{98\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{112\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(8/3/(tan(1/2*d*x+1/2*c)+1)^3-4/(tan(1/2*d*x+1/2*c)+1)^2+6/(tan(1/2
*d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(64) = 128$.
time = 0.28, size = 143, normalized size = 2.10

$$\frac{\frac{2\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+7\right)}{a^{3}+\frac{3a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(2*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 7)/(a^3 + 3*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(64) = 128$.
time = 0.36, size = 194, normalized size = 2.85

$$-\frac{10\cos(dx+c)^{2}+3\left(\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\,(5\cos(dx+c)-2)\sin(dx+c)+14\cos(dx+c)+4}{6\left(a^{3}d\cos(dx+c)^{2}-a^{3}d\cos(dx+c)-2\,a^{3}d-\left(a^{3}d\cos(dx+c)+2\,a^{3}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
c) - cos(d*x + c) - 2)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (c
```

```
os(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2*cos(d*x + c) + 1
/2) + 2*(5*cos(d*x + c) - 2)*sin(d*x + c) + 14*cos(d*x + c) + 4)/(a^3*d*cos
(d*x + c)^2 - a^3*d*cos(d*x + c) - 2*a^3*d - (a^3*d*cos(d*x + c) + 2*a^3*d)
*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^2(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.48, size = 66, normalized size = 0.97

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(9*tan(1/2*d*x + 1/2*c)^2 + 1
2*tan(1/2*d*x + 1/2*c) + 7)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 8.90, size = 64, normalized size = 0.94

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{14}{3}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + (8*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)
/2)^2 + 14/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^3)
```

3.319 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{13\cot(c+dx)}{3a^3d(1+\csc(c+dx))}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-14/3*cot(d*x+c)/a^3/d+2/3*cot(d*x+c)/a^3/d/(1+s
in(d*x+c))^2+3*cot(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 3862, 4004, 3879}

$$-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13\cot(c+dx)}{3a^3d(\csc(c+dx)+1)}+\frac{2\cot(c+dx)}{3a^3d(\csc(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) + (2*Cot[c + d*x])
/(3*a^3*d*(1 + Csc[c + d*x])^2) - (13*Cot[c + d*x])/(3*a^3*d*(1 + Csc[c + d
*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \left(\frac{5}{a}-\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(1+\csc(c+dx))^2}-\frac{7}{a(1+\csc(c+dx))}\right)dx}{a^2}\\
&= \frac{5x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{2\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}-\\
&= \frac{5x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\frac{7\cot(c+dx)}{a^3d(1+\csc(c+dx))}\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-\\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{2\cot(c+dx)}{3a^3d(1+\csc(c+dx))^2}-
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(82) = 164.
time = 1.15, size = 255, normalized size = 3.11

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(c+dx)\right)-4\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+44\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-3\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d(a+a\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(8*Sin[(c + d*x)/2] - 4*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]) + 44*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2 - 3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3
 + 18*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 18*Lo
g[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^3*Tan[(c + d*x)/2]))/(6*d*(a + a*Sin[c + d*x])^3
)
```

Maple [A]
time = 0.30, size = 89, normalized size = 1.09

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{20}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{3}}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{20}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^{3}}$
risch	$-\frac{2\left(-29\,e^{2i(dx+c)}+27i e^{3i(dx+c)}+14-33i e^{i(dx+c)}+9\,e^{4i(dx+c)}\right)}{3\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^{3}a^{3}d}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{d\,a^{3}}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{d\,a^{3}}$
norman	$\frac{-\frac{18\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{41\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{151\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{117\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{469\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-16/3/(tan(1/2*d*x+1/2*c)+1)^3+8/(tan(1/2*d*x+
1/2*c)+1)^2-20/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x
+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(78) = 156.
time = 0.29, size = 202, normalized size = 2.46

$$-\frac{\frac{\frac{61\sin(dx+c)}{\cos(dx+c)+1}+\frac{105\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{63\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+3}{\frac{a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{18\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{3\sin(dx+c)}{a^{3}(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((61*sin(d*x + c)/(cos(d*x + c) + 1) + 105*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3)/(a^3*sin(d*x + c)/(c
os(d*x + c) + 1) + 3*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) +
18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x + c)/(a^3*(cos(d*x
+ c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(78) = 156$.
time = 0.34, size = 279, normalized size = 3.40

$$-\frac{28 \cos\left(dx+c\right)^{3}-10 \cos\left(dx+c\right)^{2}-9 \left(\cos\left(dx+c\right)^{3}+2 \cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right) \sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+9 \left(\cos\left(dx+c\right)^{3}+2 \cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right) \sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)-2 \left(14 \cos\left(dx+c\right)^{2}+19 \cos\left(dx+c\right)+2\right) \sin\left(dx+c\right)-34 \cos\left(dx+c\right)+4}{6 \left(a^{3}d\cos\left(dx+c\right)^{3}+2 a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2 a^{3}d+\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2 a^{3}d\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(28*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + 2*cos(d*x
 + c)^2 + (cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) -
 2)*log(1/2*cos(d*x + c) + 1/2) + 9*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + (c
os(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*log(-1/2
*cos(d*x + c) + 1/2) - 2*(14*cos(d*x + c)^2 + 19*cos(d*x + c) + 2)*sin(d*x
+ c) - 34*cos(d*x + c) + 4)/(a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2
- a^3*d*cos(d*x + c) - 2*a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c)
 - 2*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.47, size = 109, normalized size = 1.33

$$-\frac{\frac{18 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}-\frac{3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{3}}-\frac{3 \left(6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{3} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{4 \left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a^{3} \left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^3 -
3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(15*tan(1/2*d
*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3*(tan(1/2*d*x + 1/2*c) +
1)^3))/d
```

Mupad [B]
time = 8.81, size = 145, normalized size = 1.77

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{61\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1}{d\,\left(2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((61*tan(c/2 + (d*x)/2))/3 + 35*tan(c/2 + (d
*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^3 + 1)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 6*
a^3*tan(c/2 + (d*x)/2)^3 + 2*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*
x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d)
```

3.320 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx))^2}+\frac{17\cos(c+a}{3a^3d(1+\sin(c+}$$

```
[Out] -11/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c
)/a^3/d+2/3*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+17/3*cos(d*x+c)/a^3/d/(1+sin(
d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2953, 3045, 3855, 3852, 8, 3853, 2729, 2727}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{17\cos(c+dx)}{3a^3d(\sin(c+dx)+1)}+\frac{2\cos(c+dx)}{3a^3d(\sin(c+dx)+1)^2}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
 d*x]*Csc[c + d*x])/(2*a^3*d) + (2*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]
)^2) + (17*Cos[c + d*x])/(3*a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
```

```
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3045

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx}{a^2} \\
&= \frac{\int \left(\frac{5\csc(c+dx)}{a}-\frac{3\csc^2(c+dx)}{a}+\frac{\csc^3(c+dx)}{a}-\frac{2}{a(1+\sin(c+dx))^2}-\frac{5}{a(1+\sin(c+dx))}\right)dx}{a^2} \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}+\frac{5\int \csc(c
\\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{2\cos(c+dx)}{3a^3d(1+\sin(c+dx
\\
&= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{}{3a
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(106) = 212.

time = 4.40, size = 308, normalized size = 2.91

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-32\sin\left(\frac{1}{2}(c+dx)\right)-3\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^3\sin\left(\frac{1}{2}(c+dx)\right)+16\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-272\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2+36\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-132\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+132\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3-36\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\tan\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{1}{2}(c+dx)\right)\left(1+\tan\left(\frac{1}{2}(c+dx)\right)\right)^3\right)}{24a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-32*Sin[(c + d*x)/2] - 3*(1 + Cot
[(c + d*x)/2])^3*Sin[(c + d*x)/2] + 16*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
) - 272*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + 36*Cot[(
c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 132*Log[Cos[(c + d*x)
/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 132*Log[Sin[(c + d*x)/2]]*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 36*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^3*Tan[(c + d*x)/2] + 3*Cos[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])^3))/(2
4*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]

time = 0.37, size = 117, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{56}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{56}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+}{4d\,a^3}$
risch	$\frac{99i\,e^{5i(dx+c)}+33\,e^{6i(dx+c)}-210i\,e^{3i(dx+c)}-154\,e^{4i(dx+c)}+123i\,e^{i(dx+c)}+161\,e^{2i(dx+c)}-52}{3\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)^3a^3d}+\frac{11\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^3}-11$
norman	$\frac{\frac{33\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{7\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{215\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{151\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{865(\tan}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-6*tan(1/2*d*x+1/2*c)+32/3/(tan(1/2*d*x+
1/2*c)+1)^3-16/(tan(1/2*d*x+1/2*c)+1)^2+56/(tan(1/2*d*x+1/2*c)+1)-1/2/tan(1
/2*d*x+1/2*c)^2+6/tan(1/2*d*x+1/2*c)+22*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(98) = 196.

time = 0.30, size = 247, normalized size = 2.33

$$\frac{\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{403\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{681\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{372\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{3\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3}+\frac{132\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 403*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 681*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 372*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 3*(12*sin(d*x + c)/(cos(d*x
+ c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 132*log(sin(d*x + c)
/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(98) = 196$.
time = 0.35, size = 365, normalized size = 3.44

$$-\frac{104\,\cos\left(dx+c\right)^{4}+142\,\cos\left(dx+c\right)^{3}-90\,\cos\left(dx+c\right)^{2}+33\,\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)+2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-33\,\left(\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+2\,\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)+2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\left(52\,\cos\left(dx+c\right)^{3}-19\,\cos\left(dx+c\right)^{2}-64\,\cos\left(dx+c\right)+4\right)\sin\left(dx+c\right)-136\,\cos\left(dx+c\right)-8}{12\,\left(a^{3}d\cos\left(dx+c\right)^{4}-a^{3}d\cos\left(dx+c\right)^{3}-3\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\cos\left(dx+c\right)+2\,a^{3}d-\left(a^{3}d\cos\left(dx+c\right)^{3}+2\,a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-2\,a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*cos(d*x + c)^4 + 142*cos(d*x + c)^3 - 90*cos(d*x + c)^2 + 33*(co
s(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 - (cos(d*x + c)^3 + 2*cos(
d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos
(d*x + c) + 1/2) - 33*(cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 -
 (cos(d*x + c)^3 + 2*cos(d*x + c)^2 - cos(d*x + c) - 2)*sin(d*x + c) + cos(
d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 - 19*cos(
d*x + c)^2 - 64*cos(d*x + c) + 4)*sin(d*x + c) - 136*cos(d*x + c) - 8)/(a^3
*d*cos(d*x + c)^4 - a^3*d*cos(d*x + c)^3 - 3*a^3*d*cos(d*x + c)^2 + a^3*d*c
os(d*x + c) + 2*a^3*d - (a^3*d*cos(d*x + c)^3 + 2*a^3*d*cos(d*x + c)^2 - a^
3*d*cos(d*x + c) - 2*a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^{2}{\left(c+dx \right)}\csc^{3}{\left(c+dx \right)}}{\sin^{3}{\left(c+dx \right)}+3\sin^{2}{\left(c+dx \right)}+3\sin{\left(c+dx \right)}+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.48, size = 143, normalized size = 1.35

$$\frac{\frac{132\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\left(66\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{3\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6}+\frac{16\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*(66*tan(1/2*d*x + 1/2*c)^2
 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 3*(a^3*tan(1
/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(21*tan(1/2*d*x +
 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 19)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^3
))/d
```

Mupad [B]
time = 8.66, size = 178, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{62\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{227\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{403\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\,\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+12\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (11*log(tan(c/2 + (d*x)/2)))/(2*a^3*d) + (
(9*tan(c/2 + (d*x)/2))/2 + (403*tan(c/2 + (d*x)/2)^2)/6 + (227*tan(c/2 + (d
*x)/2)^3)/2 + 62*tan(c/2 + (d*x)/2)^4 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2
 + 12*a^3*tan(c/2 + (d*x)/2)^3 + 12*a^3*tan(c/2 + (d*x)/2)^4 + 4*a^3*tan(c/
2 + (d*x)/2)^5)) - (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

3.321 $\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=144

$$\frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4}+\frac{2\cos(e+fx)}{105f\left(a^2+a^2\sin(e+fx)\right)^3}+\frac{4\cos(e+fx)}{315f\left(a^3+a^3\sin(e+f\right.}$$

```
[Out] 2/9*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^5-19/63*cos(f*x+e)/a^2/f/(a+a*sin(f*x+e
))^4+2/105*cos(f*x+e)/f/(a^2+a^2*sin(f*x+e))^3+4/315*cos(f*x+e)/f/(a^3+a^3*
sin(f*x+e))^2+4/315*cos(f*x+e)/f/(a^6+a^6*sin(f*x+e))
```

Rubi [A]

time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2936, 2829, 2729, 2727}

$$\frac{4\cos(e+fx)}{315f\left(a^6\sin(e+fx)+a^6\right)}+\frac{4\cos(e+fx)}{315f\left(a^3\sin(e+fx)+a^3\right)^2}+\frac{2\cos(e+fx)}{105f\left(a^2\sin(e+fx)+a^2\right)^3}-\frac{19\cos(e+fx)}{63a^2f(a\sin(e+fx)+a)^4}+\frac{2\cos(e+fx)}{9af(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] (2*Cos[e + f*x])/(9*a*f*(a + a*Sin[e + f*x])^5) - (19*Cos[e + f*x])/(63*a^2
*f*(a + a*Sin[e + f*x])^4) + (2*Cos[e + f*x])/(105*f*(a^2 + a^2*Sin[e + f*x
])^3) + (4*Cos[e + f*x])/(315*f*(a^3 + a^3*Sin[e + f*x])^2) + (4*Cos[e + f*
x])/(315*f*(a^6 + a^6*Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
```

```
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2936

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[2*(b*c - a*d)*Cos
[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(2*m + 3))), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
 b^2, 0] && LtQ[m, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx &= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{\int \frac{-10a+9a\sin(e+fx)}{(a+a\sin(e+fx))^4}\,dx}{9a^3}\\ &= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4}-\frac{2\int \frac{1}{(a+a\sin(e+fx))^3}\,\cdots}{21a^3}\\ &= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4}+\frac{2\cos(e+f\cdots}{105f\left(a^2+a^2\sin(\cdots\right.}\\ &= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4}+\frac{2\cos(e+f\cdots}{105f\left(a^2+a^2\sin(\cdots\right.}\\ &= \frac{2\cos(e+fx)}{9af(a+a\sin(e+fx))^5}-\frac{19\cos(e+fx)}{63a^2f(a+a\sin(e+fx))^4}+\frac{2\cos(e+f\cdots}{105f\left(a^2+a^2\sin(\cdots\right.}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 171, normalized size = 1.19

$$-\frac{378\cos\left(e+\frac{fx}{2}\right)+210\cos\left(e+\frac{3fx}{2}\right)-108\cos\left(3e+\frac{5fx}{2}\right)+225\cos\left(3e+\frac{7fx}{2}\right)+3\cos\left(5e+\frac{9fx}{2}\right)+3150\sin\left(\frac{fx}{2}\right)+2562\sin\left(2e+\frac{3fx}{2}\right)-900\sin\left(2e+\frac{5fx}{2}\right)-27\sin\left(4e+\frac{7fx}{2}\right)+25\sin\left(4e+\frac{9fx}{2}\right)}{13860a^6f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] -1/13860*(378*Cos[e + (f*x)/2] + 210*Cos[e + (3*f*x)/2] - 108*Cos[3*e + (5*
f*x)/2] + 225*Cos[3*e + (7*f*x)/2] + 3*Cos[5*e + (9*f*x)/2] + 3150*Sin[(f*x
)/2] + 2562*Sin[2*e + (3*f*x)/2] - 900*Sin[2*e + (5*f*x)/2] - 27*Sin[4*e +
(7*f*x)/2] + 25*Sin[4*e + (9*f*x)/2])/(a^6*f*(Cos[e/2] + Sin[e/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^9)
```

Maple [A]
time = 0.42, size = 130, normalized size = 0.90

method	result
risch	$-\frac{8\left(105\,\mathrm{e}^{6i(fx+e)}+21i\mathrm{e}^{3i(fx+e)}-126\,\mathrm{e}^{4i(fx+e)}+9i\mathrm{e}^{i(fx+e)}+36\,\mathrm{e}^{2i(fx+e)}-1\right)}{315f\,a^6\left(\mathrm{e}^{i(fx+e)}+i\right)^9}$
derivativedivides	$\frac{-\frac{248}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^6}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^8}+\frac{336}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{464}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^7}-\frac{36}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{f\,a^6}$
default	$\frac{-\frac{248}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^6}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^8}+\frac{336}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{464}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^7}-\frac{36}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+\right.}}{f\,a^6}$
norman	$-\frac{22}{315af}-\frac{2\left(\tan^{15}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{6\left(\tan^{14}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{18\left(\tan^{13}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{fa}-\frac{242\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{315fa}-\frac{174\left(\tan^{12}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{5fa}-\frac{274\left(\tan^3\left(\frac{f}{}\right.\right.}{35fc}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
```

```
[Out] 4/f/a^6*(-62/3/(tan(1/2*f*x+1/2*e)+1)^6-1/2/(tan(1/2*f*x+1/2*e)+1)^2-8/(tan
(1/2*f*x+1/2*e)+1)^8+84/5/(tan(1/2*f*x+1/2*e)+1)^5+116/7/(tan(1/2*f*x+1/2*e
)+1)^7-9/(tan(1/2*f*x+1/2*e)+1)^4+16/9/(tan(1/2*f*x+1/2*e)+1)^9+3/(tan(1/2*
f*x+1/2*e)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(134) = 268$.
time = 0.33, size = 355, normalized size = 2.47

$$-\frac{2\left(\frac{99\sin(fx+e)}{\cos(fx+e)+1}+\frac{81\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{609\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{441\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{945\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{315\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{315\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+11\right)}{315\left(a^6+\frac{9a^6\sin(fx+e)}{\cos(fx+e)+1}+\frac{36a^6\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{84a^6\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126a^6\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{126a^6\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84a^6\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36a^6\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{9a^6\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{a^6\sin(fx+e)^9}{(\cos(fx+e)+1)^9}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")
```

```
[Out] -2/315*(99*sin(f*x + e)/(cos(f*x + e) + 1) + 81*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 + 609*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 945*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 315*sin(f*x +
 e)^6/(cos(f*x + e) + 1)^6 + 315*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 11)/
((a^6 + 9*a^6*sin(f*x + e)/(cos(f*x + e) + 1) + 36*a^6*sin(f*x + e)^2/(cos(
f*x + e) + 1)^2 + 84*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*a^6*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 84*a^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 36*a^6*sin(f*x + e)^7/(co
s(f*x + e) + 1)^7 + 9*a^6*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + a^6*sin(f*x
 + e)^9/(cos(f*x + e) + 1)^9)*f)
```

Fricas [A]
time = 0.37, size = 263, normalized size = 1.83

$$\frac{4\cos(fx+e)^5-16\cos(fx+e)^4-50\cos(fx+e)^3-65\cos(fx+e)^2-\left(4\cos(fx+e)^4+20\cos(fx+e)^3-30\cos(fx+e)^2+35\cos(fx+e)+70\right)\sin(fx+e)+35\cos(fx+e)+70}{315\left(a^6f\cos(fx+e)^5+5a^6f\cos(fx+e)^4-8a^6f\cos(fx+e)^3-20a^6f\cos(fx+e)^2+8a^6f\cos(fx+e)+16a^6f+\left(a^6f\cos(fx+e)^4-4a^6f\cos(fx+e)^3-12a^6f\cos(fx+e)^2+8a^6f\cos(fx+e)+16a^6f\right)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")

[Out] 1/315*(4*cos(f*x + e)^5 - 16*cos(f*x + e)^4 - 50*cos(f*x + e)^3 - 65*cos(f*x + e)^2 - (4*cos(f*x + e)^4 + 20*cos(f*x + e)^3 - 30*cos(f*x + e)^2 + 35*cos(f*x + e) + 70)*sin(f*x + e) + 35*cos(f*x + e) + 70)/(a^6*f*cos(f*x + e)^5 + 5*a^6*f*cos(f*x + e)^4 - 8*a^6*f*cos(f*x + e)^3 - 20*a^6*f*cos(f*x + e)^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f + (a^6*f*cos(f*x + e)^4 - 4*a^6*f*cos(f*x + e)^3 - 12*a^6*f*cos(f*x + e)^2 + 8*a^6*f*cos(f*x + e) + 16*a^6*f)*sin(f*x + e))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(131) = 262$.
time = 70.82, size = 1501, normalized size = 10.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**2*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)

[Out] Piecewise((-630*tan(e/2 + f*x/2)**7/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 630*tan(e/2 + f*x/2)**6/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 1890*tan(e/2 + f*x/2)**5/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 882*tan(e/2 + f*x/2)**4/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 1218*tan(e/2 + f*x/2)**3/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 162*tan(e/2 + f*x/2)**2/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 +

```
 26460*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 3969
0*a**6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**
6*f*tan(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 198*
tan(e/2 + f*x/2)/(315*a**6*f*tan(e/2 + f*x/2)**9 + 2835*a**6*f*tan(e/2 + f*
x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 26460*a**6*f*tan(e/2 + f*x/2)*
*6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**6*f*tan(e/2 + f*x/2)**4 +
26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*tan(e/2 + f*x/2)**2 + 2835*
a**6*f*tan(e/2 + f*x/2) + 315*a**6*f) - 22/(315*a**6*f*tan(e/2 + f*x/2)**9
+ 2835*a**6*f*tan(e/2 + f*x/2)**8 + 11340*a**6*f*tan(e/2 + f*x/2)**7 + 2646
0*a**6*f*tan(e/2 + f*x/2)**6 + 39690*a**6*f*tan(e/2 + f*x/2)**5 + 39690*a**
6*f*tan(e/2 + f*x/2)**4 + 26460*a**6*f*tan(e/2 + f*x/2)**3 + 11340*a**6*f*t
an(e/2 + f*x/2)**2 + 2835*a**6*f*tan(e/2 + f*x/2) + 315*a**6*f), Ne(f, 0)),
 (x*sin(e)*cos(e)**2/(a*sin(e) + a)**6, True))
```

Giac [A]
time = 0.54, size = 120, normalized size = 0.83

$$-\frac{2\left(315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{7}+315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{6}+945\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{5}+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{4}+609\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{3}+81\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^{2}+99\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+11\right)}{315\,a^{6}f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/315*(315*tan(1/2*f*x + 1/2*e)^7 + 315*tan(1/2*f*x + 1/2*e)^6 + 945*tan(1
/2*f*x + 1/2*e)^5 + 441*tan(1/2*f*x + 1/2*e)^4 + 609*tan(1/2*f*x + 1/2*e)^3
 + 81*tan(1/2*f*x + 1/2*e)^2 + 99*tan(1/2*f*x + 1/2*e) + 11)/(a^6*f*(tan(1/
2*f*x + 1/2*e) + 1)^9)
```

Mupad [B]
time = 9.06, size = 207, normalized size = 1.44

$$-\frac{2\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\left(11\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}+99\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+81\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}+609\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}+441\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{3}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{4}+945\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^{2}\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{5}+315\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{6}+315\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^{7}\right)}{315\,a^{6}f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*sin(e + f*x))/(a + a*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*(11*cos(e/2 + (f*x)/2)^7 + 315*sin(e/2 + (f*x)/2)^
7 + 315*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 + 99*cos(e/2 + (f*x)/2)^6*s
in(e/2 + (f*x)/2) + 945*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^5 + 441*cos
(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4 + 609*cos(e/2 + (f*x)/2)^4*sin(e/2 +
 (f*x)/2)^3 + 81*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2))/(315*a^6*f*(co
s(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^9)
```

3.322 $\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=193

$$-\frac{76a\cos(c+dx)}{495d\sqrt{a+a\sin(c+dx)}}-\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}}+\frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}+\frac{152\cos(c+dx)\sqrt{a}}{3465}$$

```
[Out] -76/1155*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-76/495*a*cos(d*x+c)/d/(a+a*s
in(d*x+c))^(1/2)-38/693*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+
2/99*a*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+152/3465*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d+2/11*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1/
2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2958, 3055, 3060, 2849, 2838, 2830, 2725}

$$\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}+\frac{2a\sin^4(c+dx)\cos(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\sin^3(c+dx)\cos(c+dx)}{693d\sqrt{a\sin(c+dx)+a}}-\frac{76\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{1155ad}+\frac{152\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3465d}-\frac{76a\cos(c+dx)}{495d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-76*a*Cos[c + d*x])/(495*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(693*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sin[c
+ d*x]^4)/(99*d*Sqrt[a + a*Sin[c + d*x]]) + (152*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(3465*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c + d
*x]])/(11*d) - (76*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(1155*a*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2958

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{2\int \mathrm{s}}{}$$

$$= \frac{2a\cos(c+dx)\sin^4(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+d}{11}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c}{99d\sqrt{a+a\sin(c+}}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c}{99d\sqrt{a+a\sin(c+}}$$

$$= -\frac{38a\cos(c+dx)\sin^3(c+dx)}{693d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sin^4(c}{99d\sqrt{a+a\sin(c+}}$$

$$= -\frac{76a\cos(c+dx)}{495d\sqrt{a+a\sin(c+dx)}} - \frac{38a\cos(c+dx)\sin^3(c+}{693d\sqrt{a+a\sin(c+d}}$$

Mathematica [A]
time = 0.86, size = 109, normalized size = 0.56

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(5657-3540\cos(2(c+dx))+315\cos(4(c+dx))+7638\sin(c+dx)-1330\sin(3(c+dx)))}{13860d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/13860*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])
]*(5657 - 3540*Cos[2*(c + d*x)] + 315*Cos[4*(c + d*x)] + 7638*Sin[c + d*x]
- 1330*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.64, size = 85, normalized size = 0.44

method	result	size
default	$-\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^2(315(\sin^4(dx+c))+665(\sin^3(dx+c))+570(\sin^2(dx+c))+456\sin(dx+c)+304)}{3465\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(315*sin(d*x+c)^4+665*sin(d*x+c)^
3+570*sin(d*x+c)^2+456*sin(d*x+c)+304)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

Fricas [A]
time = 0.35, size = 151, normalized size = 0.78

$$\frac{2\left(315\cos\left(dx+c\right)^{6}+350\cos\left(dx+c\right)^{5}-500\cos\left(dx+c\right)^{4}-586\cos\left(dx+c\right)^{3}+17\cos\left(dx+c\right)^{2}+\left(315\cos\left(dx+c\right)^{5}-35\cos\left(dx+c\right)^{4}-535\cos\left(dx+c\right)^{3}+51\cos\left(dx+c\right)^{2}+68\cos\left(dx+c\right)+136\right)\sin\left(dx+c\right)-68\cos\left(dx+c\right)-136\right)\sqrt{a\sin\left(dx+c\right)+a}}{3465\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 500*cos(d*x + c)^4 - 586*
cos(d*x + c)^3 + 17*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^
4 - 535*cos(d*x + c)^3 + 51*cos(d*x + c)^2 + 68*cos(d*x + c) + 136)*sin(d*x
 + c) - 68*cos(d*x + c) - 136)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 157, normalized size = 0.81

$$\frac{8\sqrt{2}\left(2520\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-7700\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+8910\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-4851\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1155\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)\sqrt{a}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 8/3465*sqrt(2)*(2520*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^11 - 7700*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^9 + 8910*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^7 - 4851*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^5 + 1155*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c+d\,x)^2\,\sin (c+d\,x)^3\,\sqrt {a+a\,\sin (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2), x)
```

3.323 $\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{8a^2\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}+\frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2\cos^3(c+dx)(\cdots}{}$$

```
[Out] -8/63*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^3*(a+a*sin(d
*x+c))^(3/2)/a/d-2/21*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+4/21*cos(d*x+
c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2957, 2935, 2753, 2752}

$$-\frac{8a^2\cos^3(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9ad}+\frac{4\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}-\frac{2a\cos^3(c+dx)}{21d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*
x]^3)/(21*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
+ d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9ad} + \frac{2\int \cos^2(c+d\ldots}{\ldots}$$

$$= \frac{4\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d} - \frac{2\cos^3(c+dx)(a\ldots}{9\ldots}$$

$$= -\frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos^3(c+dx)\sqrt{a+a\sin\ldots}}{21d}$$

$$= -\frac{8a^2\cos^3(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^3(c+dx)}{21d\sqrt{a+a\sin(c+dx)}}\ldots$$

Mathematica [A]
time = 0.41, size = 99, normalized size = 0.80

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(-62+30\cos(2(c+dx))-69\sin(c+dx)+7\sin(3(c+dx)))}{126d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-62 +
30*Cos[2*(c + d*x)] - 69*Sin[c + d*x] + 7*Sin[3*(c + d*x)]))/(126*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.36, size = 75, normalized size = 0.60

method	result	size
default	$-\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^2(7(\sin^3(dx+c))+15(\sin^2(dx+c))+12\sin(dx+c)+8)}{63\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/63*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(7*sin(d*x+c)^3+15*sin(d*x+c)^2+12*
sin(d*x+c)+8)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

Fricas [A]
time = 0.35, size = 130, normalized size = 1.05

$$\frac{2\left(7\cos\left(dx+c\right)^5-\cos\left(dx+c\right)^4-11\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2-\left(7\cos\left(dx+c\right)^4+8\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)^2-4\cos\left(dx+c\right)-8\right)\sin\left(dx+c\right)-4\cos\left(dx+c\right)-8\right)\sqrt{a\sin\left(dx+c\right)+a}}{63\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/63*(7*cos(d*x + c)^5 - cos(d*x + c)^4 - 11*cos(d*x + c)^3 + cos(d*x + c)^
2 - (7*cos(d*x + c)^4 + 8*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c
) - 8)*sin(d*x + c) - 4*cos(d*x + c) - 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d
*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2*cos(c + d*x)**2, x)

Giac [A]
time = 0.44, size = 128, normalized size = 1.03

$$-\frac{8\sqrt{2}\left(28\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^9-72\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7+63\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-21\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3\right)\sqrt{a}}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -8/63*sqrt(2)*(28*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)^9 - 72*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^7 + 63*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
 1/2*c)^5 - 21*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\sin\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)
```

3.324 $\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=92

$$-\frac{8a^2\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}$$

```
[Out] -8/105*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/35*a*cos(d*x+c)^3/d/(a+a
*sin(d*x+c))^(1/2)-2/7*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2935, 2753, 2752}

$$-\frac{8a^2\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-8*a^2*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d
*x]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c
 + d*x]])/(7*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
```

```
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7d}+\frac{1}{7}\int \cos^2(c+dx$$
$$= -\frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^3(c+dx)\sqrt{a+a\sin(}{7d}$$
$$= -\frac{8a^2\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.29, size = 89, normalized size = 0.97

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(59-15\cos(2(c+dx))+66\sin(c+dx))}{105d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
[Out] -1/105*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*
(59 - 15*Cos[2*(c + d*x)] + 66*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [A]
time = 4.90, size = 65, normalized size = 0.71

method	result	size
default	$-\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^2(15(\sin^2(dx+c))+33\sin(dx+c)+22)}{105\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/105*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+33*sin(d*x+c)+22)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*sin(d*x + c), x)
```

Fricas [A]
time = 0.35, size = 111, normalized size = 1.21

$$-\frac{2\,\left(15\,\cos\left(dx+c\right)^{4}+18\,\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}+\left(15\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)^{2}-4\,\cos\left(dx+c\right)-8\right)\sin\left(dx+c\right)+4\,\cos\left(dx+c\right)+8\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - cos(d*x + c)^2 + (15*cos(d*
x + c)^3 - 3*cos(d*x + c)^2 - 4*cos(d*x + c) - 8)*sin(d*x + c) + 4*cos(d*x
+ c) + 8)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.45, size = 99, normalized size = 1.08

$$\frac{8\,\sqrt{2}\,\left(30\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-63\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+35\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}\right)\sqrt{a}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] 8/105*sqrt(2)*(30*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)^7 - 63*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^5 + 35*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

3.325 $\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+2/3*a*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2953, 3055, 3060, 2852, 212}

$$\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}+\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}\,dx}{a^2} \\
&= \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}+\frac{2\int \csc(c+dx)\sqrt{a} \\
&= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a+a\sin(c+}{3d} \\
&= \frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a+a\sin(c+}{3d} \\
&= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{2a\cos(c}{3d\sqrt{a+a\sin}}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 143, normalized size = 1.54

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(3\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)-3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(3*Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2] - 3*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 3*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2]))/(3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.95, size = 103, normalized size = 1.11

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)+(a-a\sin(dx+c))^{\frac{3}{2}}-3a\sqrt{a-}\right.}{3a\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)-3*a*(a-a*sin(d*x+c))^(1/2))/a/
cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(79) = 158$.
time = 0.37, size = 250, normalized size = 2.69

$$\frac{3\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)+1\right)\sqrt{a\sin\left(dx+c\right)+a}}{6\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
*(cos(d*x + c)^2 + (cos(d*x + c) - 1)*sin(d*x + c) + 2*cos(d*x + c) + 1)*sq
rt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [A]
time = 0.45, size = 132, normalized size = 1.42

$$\frac{\sqrt{2}\left(8\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-12\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] 1/6*sqrt(2)*(8*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x +
1/2*c)^3 - 3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))
/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c)) - 12*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x), x)
```

3.326 $\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d}$$

```
[Out] -arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+3*a*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2795, 3060, 2852, 212}

$$\frac{3a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (
3*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*S
in[c + d*x]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}-\frac{3}{2}a\sin(c+\right.}{a}$$

$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} + \frac{1}{2}\int \mathrm{cs}$$

$$= \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{a\mathrm{Subst}}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{3a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(89) = 178.
time = 0.84, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-4\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)+4\sin\left(\frac{1}{2}(c+dx)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+2\sin\left(\frac{3}{2}(c+dx)\right)\right)}{d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-4*Cos[(c + d*x)/2] + 2*Cos
[(3*(c + d*x))/2] + 4*Sin[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[c + d*x] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 2*Sin[(3*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x
)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A]
time = 5.52, size = 125, normalized size = 1.40

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(\sin(dx+c)\left(2\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+\right.}}{\sqrt{a}}\right.\right.\right.}{\sin(dx+c)a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] (1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c))^(1
/2)*a^(3/2)-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)-(a-a*sin(d*x+c))^(
1/2)*a^(3/2))/sin(d*x+c)/a^(3/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(79) = 158$.
time = 0.35, size = 279, normalized size = 3.13

$$\frac{\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(2\cos\left(dx+c\right)^{2}+\left(2\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-3\right)\sqrt{a\sin\left(dx+c\right)+a}}{4\left(d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) - 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 3)*sin(d*x + c) -
 cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.46, size = 149, normalized size = 1.67

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{4\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1}\right)\sqrt{a}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
[Out] -1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)
)/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c)) + 8*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c) + 4*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1
/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2, x)
```

3.327 $\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d}$$

```
[Out] 5/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-1/4*a*cot(
d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^
(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2953, 3054, 3059, 2852, 212}

$$-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (5*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
- (a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
```

```
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2d} + \frac{\int \csc^2}{}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a}}{2d}$$

$$= -\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a}}{2d}$$

$$= \frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{a\cot(c+}{4d\sqrt{a+a\sin(}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(101) = 202.
time = 0.60, size = 249, normalized size = 2.47

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(2\cos\left(\frac{1}{2}(c+dx)\right)+6\cos\left(\frac{3}{2}(c+dx)\right)-5\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+5\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+5\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-5\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)+6\sin\left(\frac{3}{2}(c+dx)\right)\right)}{4d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(2*Cos[(c + d*x)/2] + 6
*Cos[(3*(c + d*x))/2] - 5*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*
Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 5*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 5*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2] + 6*Sin[(3*(c + d*x))/2]))/
(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [**A**]
time = 5.67, size = 126, normalized size = 1.25

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-5\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^2(\sin^2(dx+c))+5\sqrt{-a(}\right.}{4a^{\frac{3}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-5*arctanh((-a*(sin(d*x+c)-1
))^(1/2)/a^(1/2))*a^2*sin(d*x+c)^2+5*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)-3*(-
a*(sin(d*x+c)-1))^(3/2)*a^(1/2))/a^(3/2)/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d
*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(85) = 170$.

time = 0.35, size = 319, normalized size = 3.16

$$\frac{5\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\cos\left(dx+c\right)^{2}+\left(3\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(5*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^2
+ (3*cos(d*x + c) + 1)*sin(d*x + c) + 2*cos(d*x + c) - 1)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x
 + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^{2}\left(c+dx\right)\csc^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**2*csc(c + d*x)**3, x)
```

Giac [A]

time = 0.49, size = 155, normalized size = 1.53

$$\frac{\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{4\left(6\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/16*sqrt(2)*(5*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*
c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c)) - 4*(6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^3 - 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^2 \sqrt{a+a\sin(c+dx)}}{\sin(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3,x)
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3, x)
```

3.328 $\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}+\frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(}{}$$

```
[Out] 3/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+3/8*a*cot(
d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x
+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2953, 3054, 3059, 2851, 2852, 212}

$$\frac{3a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
+ (3*a*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c
 + d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqr
t[a + a*Sin[c + d*x]])/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{3/2}}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{\int \mathrm{cs}}{}$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{3a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} -$$

$$= \frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{3a\cot(c}{8d\sqrt{a+a\sin}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 285 vs. 2(137) = 274.
time = 1.07, size = 285, normalized size = 2.08

$$-\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-12\cos\left(\frac{1}{2}(c+dx)\right)+58\cos\left(\frac{3}{2}(c+dx)\right)+18\cos\left(\frac{5}{2}(c+dx)\right)+12\sin\left(\frac{1}{2}(c+dx)\right)-27\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+27\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+58\sin\left(\frac{3}{2}(c+dx)\right)-18\sin\left(\frac{5}{2}(c+dx)\right)+9\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-9\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/24*(Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-12*Cos[(c + d*x)/2]
 + 58*Cos[(3*(c + d*x))/2] + 18*Cos[(5*(c + d*x))/2] + 12*Sin[(c + d*x)/2]
- 27*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 27*Log[1 -
 Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 58*Sin[(3*(c + d*x))/2
] - 18*Sin[(5*(c + d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
]*Sin[3*(c + d*x)] - 9*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4
]^2)^3)
```

Maple [A]
time = 6.24, size = 144, normalized size = 1.05

method	result

default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(9(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+9\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{4}(\mathrm{si}}{24a^{\frac{7}{2}}\sin(dx+c)^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(9*(-a*(sin(d*x+c)-1)
)^(5/2)*a^(3/2)+9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)
^3-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-9*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))
/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(117) = 234$.
time = 0.36, size = 361, normalized size = 2.64

$$\frac{9\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(9\cos\left(dx+c\right)^{3}+19\cos\left(dx+c\right)^{2}-\left(9\cos\left(dx+c\right)^{2}-10\cos\left(dx+c\right)-11\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-11\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(d\cos\left(dx+c\right)^{4}-2d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(9*cos(d*x + c)^3 + 19*cos(d*x + c)^2 - (9*cos(d*x + c)^2 - 10*cos(d*x + c
) - 11)*sin(d*x + c) - cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(d*cos(
d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*
cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.48, size = 184, normalized size = 1.34

$$\frac{\sqrt{2}\left(9\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(36\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-16\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-9\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/96*sqrt(2)*(9*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*
c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c)) + 4*(36*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^5 - 16*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^3 - 9*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4, x)
```

3.329 $\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=233

$$-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a+a\sin(c+dx)}}-\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}}-\frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}+\frac{3448a\cos(c-$$

```
[Out] -1724/15015*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+2/13*cos(d*x+c)*sin(d*x+c)^
4*(a+a*sin(d*x+c))^(3/2)/d-1724/6435*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2
)-862/9009*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-38/1287*a^2
*cos(d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+3448/45045*a*cos(d*x+c)*(
a+a*sin(d*x+c))^(1/2)/d+6/143*a*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.47, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2958, 3055, 3060, 2849, 2838, 2830, 2725}

$$-\frac{38a^2\sin^4(c+dx)\cos(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{862a^2\sin^3(c+dx)\cos(c+dx)}{9009d\sqrt{a\sin(c+dx)+a}}-\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a\sin(c+dx)+a}}+\frac{2\sin^4(c+dx)\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{13d}+\frac{6a\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{1724\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{15015d}+\frac{3448a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{45045d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-1724*a^2*Cos[c + d*x])/(6435*d*Sqrt[a + a*Sin[c + d*x]]) - (862*a^2*Cos[c
 + d*x]*Sin[c + d*x]^3)/(9009*d*Sqrt[a + a*Sin[c + d*x]]) - (38*a^2*Cos[c +
 d*x]*Sin[c + d*x]^4)/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (3448*a*Cos[c + d
*x]*Sqrt[a + a*Sin[c + d*x]])/(45045*d) + (6*a*Cos[c + d*x]*Sin[c + d*x]^4*
Sqrt[a + a*Sin[c + d*x]])/(143*d) - (1724*Cos[c + d*x]*(a + a*Sin[c + d*x])
^(3/2))/(15015*d) + (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^(3/
2))/(13*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2958

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^{3/2}}{13d} + \frac{2}{}$$

$$= \frac{6a\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} + \frac{2}{}$$

$$= -\frac{38a^2\cos(c+dx)\sin^4(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{6a\cos(c+dx)\sin}{}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)}{1287d\sqrt{a+a\,s}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)}{1287d\sqrt{a+a\,s}}$$

$$= -\frac{862a^2\cos(c+dx)\sin^3(c+dx)}{9009d\sqrt{a+a\sin(c+dx)}} - \frac{38a^2\cos(c+dx)}{1287d\sqrt{a+a\,s}}$$

$$= -\frac{1724a^2\cos(c+dx)}{6435d\sqrt{a+a\sin(c+dx)}} - \frac{862a^2\cos(c+dx)\sin}{9009d\sqrt{a+a\sin(}}$$

Mathematica [A]
time = 2.82, size = 120, normalized size = 0.52

$$-\frac{a\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(281816-194160\cos(2(c+dx))+22680\cos(4(c+dx))+381174\sin(c+dx)-77665\sin(3(c+dx))+3465\sin(5(c+dx)))}{360360d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/360360*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*
x])]*(281816 - 194160*Cos[2*(c + d*x)] + 22680*Cos[4*(c + d*x)] + 381174*Si
n[c + d*x] - 77665*Sin[3*(c + d*x)] + 3465*Sin[5*(c + d*x)]))/(d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.88, size = 97, normalized size = 0.42

method	result

default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2(3465(\sin^5(dx+c))+11340(\sin^4(dx+c))+15085(\sin^3(dx+c))+12930(\sin^2(dx+c))+10344\sin(dx}{45045\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/45045*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(3465*sin(d*x+c)^5+11340*sin(d
*x+c)^4+15085*sin(d*x+c)^3+12930*sin(d*x+c)^2+10344*sin(d*x+c)+6896)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^3, x)
```

Fricas [A]
time = 0.34, size = 189, normalized size = 0.81

$$-\frac{2\left(3465\,a\cos\left(dx+c\right)^{7}-4410\,a\cos\left(dx+c\right)^{6}-14140\,a\cos\left(dx+c\right)^{5}+7330\,a\cos\left(dx+c\right)^{4}+15299\,a\cos\left(dx+c\right)^{3}-568\,a\cos\left(dx+c\right)^{2}+2272\,a\cos\left(dx+c\right)-\left(3465\,a\cos\left(dx+c\right)^{6}+7875\,a\cos\left(dx+c\right)^{5}-6265\,a\cos\left(dx+c\right)^{4}-13595\,a\cos\left(dx+c\right)^{3}+1704\,a\cos\left(dx+c\right)^{2}+2272\,a\cos\left(dx+c\right)+4544\,a\right)\sin\left(dx+c\right)+4544\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{45045\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/45045*(3465*a*cos(d*x + c)^7 - 4410*a*cos(d*x + c)^6 - 14140*a*cos(d*x +
 c)^5 + 7330*a*cos(d*x + c)^4 + 15299*a*cos(d*x + c)^3 - 568*a*cos(d*x + c)
^2 + 2272*a*cos(d*x + c) - (3465*a*cos(d*x + c)^6 + 7875*a*cos(d*x + c)^5 -
 6265*a*cos(d*x + c)^4 - 13595*a*cos(d*x + c)^3 + 1704*a*cos(d*x + c)^2 + 2
272*a*cos(d*x + c) + 4544*a)*sin(d*x + c) + 4544*a)*sqrt(a*sin(d*x + c) + a
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 192, normalized size = 0.82

$$-\frac{16\sqrt{2}\left(27720\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-114660\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+190190\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-160875\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+72072\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-15015\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)\sqrt{a}}{45045\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -16/45045*sqrt(2)*(27720*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^13 - 114660*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^11 + 190190*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 160875*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 72072*a*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 15015*a*sgn(cos(-1/4*pi + 1/2*d
*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^3\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2), x)
```

3.330 $\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{64a^3\cos^3(c+dx)}{385d(a+a\sin(c+dx))^{3/2}}-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}}-\frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{77d}+\frac{4\cos^3(c+dx}{}$$

```
[Out] -64/385*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+4/33*cos(d*x+c)^3*(a+a*si
n(d*x+c))^(3/2)/d-2/11*cos(d*x+c)^3*(a+a*sin(d*x+c))^(5/2)/a/d-48/385*a^2*c
os(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-6/77*a*cos(d*x+c)^3*(a+a*sin(d*x+c))^(
1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2957, 2935, 2753, 2752}

$$-\frac{64a^3\cos^3(c+dx)}{385d(a\sin(c+dx)+a)^{3/2}}-\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{5/2}}{11ad}+\frac{4\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{33d}-\frac{6a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(385*d*(a + a*Sin[c + d*x])^(3/2)) - (48*a^2*Cos[c
 + d*x]^3)/(385*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(77*d) + (4*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(33
*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(5/2))/(11*a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx &= -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{5/2}}{11ad}+\frac{2\int \cos^2(c-\\ &= \frac{4\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{33d}-\frac{2\cos^3(c+dx\\ &= -\frac{6a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{77d}+\frac{4\cos^3(c+\\ &= -\frac{48a^2\cos^3(c+dx)}{385d\sqrt{a+a\sin(c+dx)}}-\frac{6a\cos^3(c+dx)\sqrt{a+}}{77d}\\ &= -\frac{64a^3\cos^3(c+dx)}{385d(a+a\sin(c+dx))^{3/2}}-\frac{48a^2\cos^3(c+dx}{385d\sqrt{a+a\sin(c+}}\end{aligned}$$

Mathematica [A]
time = 1.30, size = 110, normalized size = 0.71

$$-\frac{a\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(4159-2280\cos(2(c+dx))+105\cos(4(c+dx))+5076\sin(c+dx)-700\sin(3(c+dx)))}{4620d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4620*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x]
)]*(4159 - 2280*Cos[2*(c + d*x)] + 105*Cos[4*(c + d*x)] + 5076*Sin[c + d*x]
 - 700*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.16, size = 87, normalized size = 0.56

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2(105(\sin^4(dx+c))+350(\sin^3(dx+c))+465(\sin^2(dx+c))+372\sin(dx+c)+248)}{1155\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/1155*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(105*sin(d*x+c)^4+350*sin(d*x+c
)^3+465*sin(d*x+c)^2+372*sin(d*x+c)+248)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c)^2, x)
```

Fricas [A]
time = 0.35, size = 166, normalized size = 1.06

$$\frac{2\left(105\,a\cos\left(dx+c\right)^{6}+245\,a\cos\left(dx+c\right)^{5}-185\,a\cos\left(dx+c\right)^{4}-397\,a\cos\left(dx+c\right)^{3}+24\,a\cos\left(dx+c\right)^{2}-96\,a\cos\left(dx+c\right)+\left(105\,a\cos\left(dx+c\right)^{5}-140\,a\cos\left(dx+c\right)^{4}-325\,a\cos\left(dx+c\right)^{3}+72\,a\cos\left(dx+c\right)^{2}+96\,a\cos\left(dx+c\right)+192\,a\right)\sin\left(dx+c\right)-192\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{1155\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/1155*(105*a*cos(d*x + c)^6 + 245*a*cos(d*x + c)^5 - 185*a*cos(d*x + c)^4
- 397*a*cos(d*x + c)^3 + 24*a*cos(d*x + c)^2 - 96*a*cos(d*x + c) + (105*a*c
os(d*x + c)^5 - 140*a*cos(d*x + c)^4 - 325*a*cos(d*x + c)^3 + 72*a*cos(d*x
+ c)^2 + 96*a*cos(d*x + c) + 192*a)*sin(d*x + c) - 192*a)*sqrt(a*sin(d*x +
c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^{\frac{3}{2}}\sin^2(c+dx)\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)**2*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.43, size = 162, normalized size = 1.04

$$\frac{16 \sqrt{2} \left(420 a \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 1540 a \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{9} + 2145 a \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{7} - 1386 a \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{5} + 385 a \operatorname{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right) \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{3}\right) \sqrt{a}}{1155 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 16/1155*sqrt(2)*(420*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^11 - 1540*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^9 + 2145*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 1/2*d*x + 1/2*c)^7 - 1386*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^5 + 385*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

3.331 $\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{64a^3\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2\cos^3(c+dx)}{}$$

```
[Out] -64/315*a^3*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^3*(a+a*sin
(d*x+c))^(3/2)/d-16/105*a^2*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/21*a*co
s(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2935, 2753, 2752}

$$-\frac{64a^3\cos^3(c+dx)}{315d(a\sin(c+dx)+a)^{3/2}}-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{9d}-\frac{2a\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^3)/(315*d*(a + a*Sin[c + d*x])^(3/2)) - (16*a^2*Cos[c
 + d*x]^3)/(105*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]^3*Sqrt[a +
a*Sin[c + d*x]])/(21*d) - (2*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2))/(9*
d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^3(c+dx)(a+a\sin(c+dx))^{3/2}}{9d}+\frac{1}{3}\int \cos^2(c-$$

$$= -\frac{2a\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{21d}-\frac{2\cos^3(c+d}{}$$

$$= -\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos^3(c+dx)\sqrt{a+}}{21d}$$

$$= -\frac{64a^3\cos^3(c+dx)}{315d(a+a\sin(c+dx))^{3/2}}-\frac{16a^2\cos^3(c+dx)}{105d\sqrt{a+a\sin(c+}}$$

Mathematica [A]
time = 1.06, size = 100, normalized size = 0.81

$$\frac{a\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}\,(-664+240\cos(2(c+dx))-741\sin(c+dx)+35\sin(3(c+dx)))}{630d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*Sqrt[a*(1 + Sin[c + d*x])]*(-664
 + 240*Cos[2*(c + d*x)] - 741*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/(630*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.38, size = 77, normalized size = 0.62

method	result	size
default	$-\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^2(35(\sin^3(dx+c))+120(\sin^2(dx+c))+159\sin(dx+c)+106)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+120*sin(d*x+c)^
2+159*sin(d*x+c)+106)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*sin(d*x + c), x)
```

Fricas [A]
time = 0.36, size = 145, normalized size = 1.17

$$\frac{2\left(35\,a\cos\left(dx+c\right)^{5}-50\,a\cos\left(dx+c\right)^{4}-109\,a\cos\left(dx+c\right)^{3}+8\,a\cos\left(dx+c\right)^{2}-32\,a\cos\left(dx+c\right)-\left(35\,a\cos\left(dx+c\right)^{4}+85\,a\cos\left(dx+c\right)^{3}-24\,a\cos\left(dx+c\right)^{2}-32\,a\cos\left(dx+c\right)-64\,a\right)\sin\left(dx+c\right)-64\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/315*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 109*a*cos(d*x + c)^3 + 8
*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*cos(d*x
 + c)^3 - 24*a*cos(d*x + c)^2 - 32*a*cos(d*x + c) - 64*a)*sin(d*x + c) - 64
*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**(3/2)*sin(c + d*x)*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.43, size = 132, normalized size = 1.06

$$-\frac{16\sqrt{2}\left(70\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-225\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+252\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-105\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)\sqrt{a}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -16/315*sqrt(2)*(70*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^9 - 225*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
```

```
/2*d*x + 1/2*c)^7 + 252*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^5 - 105*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^3)*sqrt(a)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2 \sin(c+dx) \left(a+a\sin(c+dx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

3.332 $\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=123

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}+\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}+\frac{2\cos$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+2/5*cos(d*x
+c)*(a+a*sin(d*x+c))^(3/2)/d-2/5*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+2/
5*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$,
Rules used = {2953, 3055, 3060, 2852, 212}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}+\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
2*a^2*Cos[c + d*x])/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*a*Cos[c + d*x]*Sqrt
[a + a*Sin[c + d*x]])/(5*d) + (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(
5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
```

```
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/2}\,d}{a^2}$$

$$= \frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} + \frac{2\int \csc(c+dx)(a}{}$$

$$= \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} + \frac{2\cos(c+dx)(a+}{5}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(}}{5d}$$

$$= -\frac{2a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2a\cos(c+dx)\sqrt{a+a\sin(}}{5d}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos(}{5d\sqrt{a+a\sin}}$$

Mathematica [A]
time = 0.20, size = 145, normalized size = 1.18

$$\frac{(a(1+\sin(c+dx)))^{3/2}\left(5\cos\left(\frac{3}{2}(c+dx)\right)-\cos\left(\frac{5}{2}(c+dx)\right)-10\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+10\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{10d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(5*Cos[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2
] - 10*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 10*Log[1 - Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + 5*Sin[(3*(c + d*x))/2] + Sin[(5*(c + d*x))/2])
)/(10*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 5.95, size = 123, normalized size = 1.00

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(5a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{5}{2}}+5a(a-a\sin\right.}{5a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(5*a^(5/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(5/2)+5*a*(a-a*sin(d*x+c))^(3/2)-5*a
^2*(a-a*sin(d*x+c))^(1/2))/a/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(105) = 210$.
time = 0.39, size = 282, normalized size = 2.29

$$\frac{5\,(a\cos(dx+c)+a\sin(dx+c)+a)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\,\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\,\left(a\cos(dx+c)^3-2\,a\cos(dx+c)^2-2\,a\cos(dx+c)-\left(a\cos(dx+c)^2+3\,a\cos(dx+c)+a\right)\sin(dx+c)+a\right)\sqrt{a\sin(dx+c)+a}}{10\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/10*(5*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3
 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
 - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c)
+ (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)
^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)
) - 4*(a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) - (a*cos(d*
x + c)^2 + 3*a*cos(d*x + c) + a)*sin(d*x + c) + a)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [A]
time = 0.49, size = 165, normalized size = 1.34

$$-\frac{\sqrt{2}\left(16\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+20\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/10*sqrt(2)*(16*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^5 - 40*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^3 + 5*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) + 20*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x), x)
```

3.333 $\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\text{cot(}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-cot(d*x+c)*
(a+a*sin(d*x+c))^(3/2)/d+11/3*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+5/3*a
*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2795, 3055, 3060, 2852, 212}

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{11a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{5a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
11*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (5*a*Cos[c + d*x]*Sqr
t[a + a*Sin[c + d*x]])/(3*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d}+\frac{\int \csc(c+dx)\left(\frac{3a}{2}-\frac{5}{2}a\sin(\ldots\right)}{\ldots}$$

$$=\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\frac{\cot(c+dx)(a+a\sin(c+d\ldots}{d}$$

$$=\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\ldots$$

$$=\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{5a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}-\ldots$$

$$=-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{11a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}\ldots$$

Mathematica [**A**]

time = 0.55, size = 233, normalized size = 1.93

$$-\frac{a \csc^4\left(\frac{1}{2}(c+dx)\right) \sqrt{a(1+\sin(c+dx))} \left(14 \cos\left(\frac{1}{2}(c+dx)\right) - 9 \cos\left(\frac{3}{2}(c+dx)\right) + \cos\left(\frac{5}{2}(c+dx)\right) - 14 \sin\left(\frac{1}{2}(c+dx)\right) + 9 \log\left(1+\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx) - 9 \log\left(1-\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx) - 9 \sin\left(\frac{3}{2}(c+dx)\right) - \sin\left(\frac{5}{2}(c+dx)\right)\right)}{3d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right) - \sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right) + \sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(14*Cos[(c + d*x)/2]
- 9*Cos[(3*(c + d*x))/2] + Cos[(5*(c + d*x))/2] - 14*Sin[(c + d*x)/2] + 9*L
og[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Sin[(3*(c + d*x))/2] - Sin[
(5*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d
*x)/4])*(Csc[(c + d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A]
time = 6.25, size = 144, normalized size = 1.19

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sin(dx+c)\left(2(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{a}-12\sqrt{a-a\sin\left(dx+c\right)}\,a^{\frac{3}{2}}+9a\right.\right.}{3\sin(dx+c)\sqrt{a}\,\cos(dx+c)\sqrt{a+a\sin\left(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin(d*x+c
))^(3/2)*a^(1/2)-12*(a-a*sin(d*x+c))^(1/2)*a^(3/2)+9*arctanh((a-a*sin(d*x+c
))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/a^(1/2)
/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(105) = 210$.
time = 0.36, size = 315, normalized size = 2.60

$$\frac{9\left(a\cos\left(dx+c\right)^2 - \left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right) - a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 - 4\left(\cos(dx+c)^2 + (\cos(dx+c)+3)\sin(dx+c) - 2\cos(dx+c) - 3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a} - 9a\cos(dx+c) + \left(a\cos(dx+c)^2 + 8a\cos(dx+c) - a\right)\sin(dx+c) - a}{\cos(dx+c)^3 + \cos(dx+c)^2 + \left(\cos(dx+c)^2 - 1\right)\sin(dx+c) - \cos(dx+c) - 1}\right) + 4\left(2a\cos\left(dx+c\right)^3 - 8a\cos\left(dx+c\right)^2 + a\cos\left(dx+c\right) - \left(2a\cos\left(dx+c\right)^2 + 10a\cos\left(dx+c\right) + 11a\right)\sin\left(dx+c\right) + 11a\right)\sqrt{a\sin\left(dx+c\right)+a}}{12\left(d\cos\left(dx+c\right)^2 - \left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right) - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(9*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
 c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a
) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x +
c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)) + 4*(2*a*cos(d*x + c)^3 - 8*a*cos(d*x + c)^2 + a*cos
(d*x + c) - (2*a*cos(d*x + c)^2 + 10*a*cos(d*x + c) + 11*a)*sin(d*x + c) +
11*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2 - (d*cos(d*x + c) + d)*si
n(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

Giac [A]
time = 1.29, size = 183, normalized size = 1.51

$$\frac{\sqrt{2}\left(16\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-48\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/12*sqrt(2)*(16*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c)^3 - 9*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)) - 48*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c) - 12*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2, x)
```

3.334 $\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}+\frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}-\frac{\cot(c+$$

[Out] 1/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d+13/4*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-3/4*a*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.34, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2953, 3054, 3060, 2852, 212}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}+\frac{13a^2\cos(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]

[Out] (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d) + (13*a^2*Cos[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(2*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
```

```
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))^{5/}}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}}{2d} + \frac{\int \mathrm{cs}}{}$$

$$= -\frac{3a\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} - \frac{\cot(c+dx)\,\mathrm{cs}}{}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c}}{4d}$$

$$= \frac{13a^2\cos(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\sqrt{a+a\sin(c}}{4d}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{13a^2\cos(c-}{4d\sqrt{a+a\sin(}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 271 vs. 2(131) = 262.
time = 0.49, size = 271, normalized size = 2.07

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-22\cos\left(\frac{1}{2}(c+dx)\right)+22\cos\left(\frac{3}{2}(c+dx)\right)+8\cos\left(\frac{5}{2}(c+dx)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+22\sin\left(\frac{1}{2}(c+dx)\right)+22\sin\left(\frac{3}{2}(c+dx)\right)-8\sin\left(\frac{5}{2}(c+dx)\right)\right)}{4d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/4*(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-22*Cos[(c + d*x)/2]
 + 22*Cos[(3*(c + d*x))/2] + 8*Cos[(5*(c + d*x))/2] - Log[1 + Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - Cos[2*(c + d*
x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 22*Sin[(c + d*x)/2] + 22
*Sin[(3*(c + d*x))/2] - 8*Sin[(5*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*
(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A]
time = 6.53, size = 151, normalized size = 1.15

method	result

default	$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(8\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{3}{2}}(\sin^2(dx+c))+\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(d}{\sqrt{}}\right.\right.}{4\sin(dx+c)^2\sqrt{a}\,\cos(dx+c)\sqrt{a+a\,\mathrm{s}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(8*(-a*(sin(d*x+c)-1))^(1/2)*a
^(3/2)*sin(d*x+c)^2+arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^2*sin(d*x+
c)^2+7*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)-9*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2
))/sin(d*x+c)^2/a^(1/2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(111) = 222$.
time = 0.38, size = 359, normalized size = 2.74

$$\frac{\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-7a\cos\left(dx+c\right)^{2}+4\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}+8a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)-a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1}\right)+4\left(8a\cos\left(dx+c\right)^{3}+15a\cos\left(dx+c\right)^{2}-6a\cos\left(dx+c\right)-\left(8a\cos\left(dx+c\right)^{2}-7a\cos\left(dx+c\right)-13a\right)\sin\left(dx+c\right)-13a\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x +
c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c
)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c)
- 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^
2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^
2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(d*x
 + c)^3 + 15*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - (8*a*cos(d*x + c)^2 - 7*
a*cos(d*x + c) - 13*a)*sin(d*x + c) - 13*a)*sqrt(a*sin(d*x + c) + a))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^2 - d)*s
in(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 185, normalized size = 1.41

$$\frac{\sqrt{2}\left(\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-32\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{4\left(14\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-9\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/16*sqrt(2)*(sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*
c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c)) - 32*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) - 4*(14*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^3 - 9*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3, x)
```

3.335 $\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=139

$$\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}+\frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{4d}$$

```
[Out] 13/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/3*cot(d
*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d+5/24*a^2*cot(d*x+c)/d/(a+a*sin(
d*x+c))^(1/2)-1/4*a*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.39, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2953, 3054, 3059, 2852, 212}

$$\frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}+\frac{5a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (13*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 + (5*a^2*Cot[c + d*x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*C
sc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d) - (Cot[c + d*x]*Csc[c + d*x]^2*
(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
```

```
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))(a+a\sin(c+dx))}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}}{3d} +$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} -$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{4d}$$

$$= \frac{5a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{4d}$$

$$= \frac{13a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} + \frac{5a^2\cos}{24d\sqrt{a+}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 286 vs. 2(139) = 278.
time = 0.66, size = 286, normalized size = 2.06

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(12\cos\left(\frac{1}{2}(c+dx)\right)+70\cos\left(\frac{3}{2}(c+dx)\right)-18\cos\left(\frac{5}{2}(c+dx)\right)-12\sin\left(\frac{1}{2}(c+dx)\right)-117\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+117\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+70\sin\left(\frac{3}{2}(c+dx)\right)+18\sin\left(\frac{5}{2}(c+dx)\right)+39\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-39\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(12*Cos[(c + d*x)/2
] + 70*Cos[(3*(c + d*x))/2] - 18*Cos[(5*(c + d*x))/2] - 12*Sin[(c + d*x)/2]
 - 117*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 117*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 70*Sin[(3*(c + d*x)
)/2] + 18*Sin[(5*(c + d*x))/2] + 39*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^3)
```

Maple [**A**]
time = 6.95, size = 144, normalized size = 1.04

method	result

default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-39\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3(\sin^3(dx+c))+9(-a(\sin(dx\right.}{24a^{\frac{3}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(-39*arctanh((-a*(si
n(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+9*(-a*(sin(d*x+c)-1))^(5/2)*a^
(1/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+39*(-a*(sin(d*x+c)-1))^(1/2)*a^(
5/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*csc(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. 2(119) = 238.
time = 0.36, size = 380, normalized size = 2.73

$$\frac{39\,(a\cos(dx+c)^4 - 2a\cos(dx+c)^2 - (a\cos(dx+c)^3 + a\cos(dx+c)^2 - a\cos(dx+c) - a)\sin(dx+c) + a)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3 - 7a\cos(dx+c)^2 + 4(\cos(dx+c)^2 + (\cos(dx+c)+3)\sin(dx+c) - 2\cos(dx+c) - 3)\sqrt{a\sin(dx+c)+a}\,\sqrt{a} - 9a\cos(dx+c) + (a\cos(dx+c)^2 + 8a\cos(dx+c) - a)\sin(dx+c) - a}{\cos(dx+c)^3 + \cos(dx+c)^2 + (\cos(dx+c)^2-1)\sin(dx+c) - \cos(dx+c) - 1}\right) + 4\,(9a\cos(dx+c)^3 - 13a\cos(dx+c)^2 - 17a\cos(dx+c) - (9a\cos(dx+c)^2 + 22a\cos(dx+c) + 5a)\sin(dx+c) + 5a)\sqrt{a\sin(dx+c)+a}}{96\,(d\cos(dx+c)^4 - 2d\cos(dx+c)^2 - (d\cos(dx+c)^3 + d\cos(dx+c)^2 - d\cos(dx+c) - d)\sin(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(39*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*cos
(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
+ c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*
x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x
 + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*
x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c
) - 1)) + 4*(9*a*cos(d*x + c)^3 - 13*a*cos(d*x + c)^2 - 17*a*cos(d*x + c) -
 (9*a*cos(d*x + c)^2 + 22*a*cos(d*x + c) + 5*a)*sin(d*x + c) + 5*a)*sqrt(a*
sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 - (d*cos(d*x + c)
^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.61, size = 188, normalized size = 1.35

$$\frac{\sqrt{2}\left(39\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)-\frac{4\left(36a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-80a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+39a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/96*sqrt(2)*(39*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)) - 4*(36*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^5 - 80*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 1/2*d*x + 1/2*c)^3 + 39*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)
/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^2*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4, x)
```

3.336 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=158

$$-\frac{4\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a+a\sin}}{315ad}$$

```
[Out] -4/105*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a^2/d-4/45*cos(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)-2/63*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+2/9*cos(
d*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+8/315*cos(d*x+c)*(a+a*sin(d*x+
c))^(1/2)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2958, 3060, 2849, 2838, 2830, 2725}

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105a^2d}+\frac{2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315ad}-\frac{4\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-4*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c +
 d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sin[c + d*x]^4)/
(9*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/
(315*a*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105*a^2*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
```

```
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2958

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{9a}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \sin^2(c+dx)}{}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)(}{}$$
$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{8\cos(c+dx)\sqrt{}}{3}$$
$$= -\frac{4\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin}{9d\sqrt{a+a\sin(}}$$

Mathematica [A]
time = 0.90, size = 97, normalized size = 0.61

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-124+60\cos(2(c+dx))-201\sin(c+dx)+35\sin(3(c+dx))\right)}{630d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])*(-124 + 60*Cos[2*(c + d*x)] - 201*Sin[c + d*x] + 35*Sin[3*(c + d*x)]))/
(630*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 5.10, size = 74, normalized size = 0.47

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^2(35(\sin^3(dx+c))+30(\sin^2(dx+c))+24\sin(dx+c)+16)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(35*sin(d*x+c)^3+30*sin(d*x+c)^2+24*
sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.34, size = 136, normalized size = 0.86

$$\frac{2\left(35\cos\left(dx+c\right)^{5}+40\cos\left(dx+c\right)^{4}-64\cos\left(dx+c\right)^{3}-82\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}-5\cos\left(dx+c\right)^{3}-69\cos\left(dx+c\right)^{2}+13\cos\left(dx+c\right)+26\right)\sin\left(dx+c\right)+13\cos\left(dx+c\right)+26\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 64*cos(d*x + c)^3 - 82*cos(d
*x + c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 69*cos(d*x + c)^2 + 13*
cos(d*x + c) + 26)*sin(d*x + c) + 13*cos(d*x + c) + 26)*sqrt(a*sin(d*x + c)
 + a)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 103, normalized size = 0.65

$$-\frac{4\sqrt{2}\left(280\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-540\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+378\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-105\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)}{315\,ad\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/315*sqrt(2)*(280*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 540*sqrt(a)*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 378*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*
```

```
c)^5 - 105*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3)/(a*d*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)^2\sin (c+d\,x)^3}{\sqrt{a+a\,\sin (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(1/2), x)
```

3.337 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$-\frac{22a\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}}+\frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad}$$

```
[Out] -22/105*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)+12/35*cos(d*x+c)^3/d/(a+a*s
in(d*x+c))^(1/2)-2/7*cos(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.23, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2956, 2935, 2753, 2752}

$$-\frac{2\cos^3(c+dx)\sqrt{a\sin(c+dx)+a}}{7ad}+\frac{12\cos^3(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{22a\cos^3(c+dx)}{105d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-22*a*Cos[c + d*x]^3)/(105*d*(a + a*Sin[c + d*x])^(3/2)) + (12*Cos[c + d*x
]^3)/(35*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + a*Sin[c +
 d*x]])/(7*a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\left(-\frac{a}{2}-2a\sin(c+dx)\right)\sqrt{a+a\sin
}{2a^2} \\
&= \frac{\cos^3(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11\int \cos^2(
}{} \\
&= \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{a+a\sin(c+dx)}}{7ad} + \frac{11}{35}\int \frac{}{\sqrt{
}} \\
&= -\frac{22a\cos^3(c+dx)}{105d(a+a\sin(c+dx))^{3/2}} + \frac{12\cos^3(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)\sqrt{
}{}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 87, normalized size = 0.95

$$
-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(31-15\cos(2(c+dx))+24\sin(c+dx))}{105d\sqrt{a(1+\sin(c+dx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/105*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(31 - 15*Cos[2*(c + d*x)] + 24*Sin[c + d*x]))/(d*Sqrt[a*(1 + Sin
[c + d*x])])
```

Maple [A]
time = 3.59, size = 64, normalized size = 0.70

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^2(15(\sin^2(dx+c))+12\sin(dx+c)+8)}{105d\cos(dx+c)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/105/d*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(15*sin(d*x+c)^2+12*sin(d*x+c)+8)/
cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.35, size = 115, normalized size = 1.25

$$-\frac{2\left(15\cos\left(dx+c\right)^{4}-3\cos\left(dx+c\right)^{3}-29\cos\left(dx+c\right)^{2}+\left(15\cos\left(dx+c\right)^{3}+18\cos\left(dx+c\right)^{2}-11\cos\left(dx+c\right)-22\right)\sin\left(dx+c\right)+11\cos\left(dx+c\right)+22\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/105*(15*cos(d*x + c)^4 - 3*cos(d*x + c)^3 - 29*cos(d*x + c)^2 + (15*cos(
d*x + c)^3 + 18*cos(d*x + c)^2 - 11*cos(d*x + c) - 22)*sin(d*x + c) + 11*co
s(d*x + c) + 22)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c) + a*d*sin(d*x +
 c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.51, size = 84, normalized size = 0.91

$$\frac{4\sqrt{2}\left(60\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-84\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+35\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}\right)}{105\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 4/105*sqrt(2)*(60*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 84*sqrt(a)*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 35*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3
)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2), x)
```

3.338 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=60

$$\frac{2a\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 2/15*a*cos(d*x+c)^3/d/(a+a*sin(d*x+c))^(3/2)-2/5*cos(d*x+c)^3/d/(a+a*sin(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2935, 2752}

$$\frac{2a\cos^3(c+dx)}{15d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^3(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (2*a*Cos[c + d*x]^3)/(15*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^3)
/(5*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{1}{5}\int \frac{\cos^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{2a\cos^3(c+dx)}{15d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^3(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.33, size = 77, normalized size = 1.28

$$-\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(2+3\sin(c+dx))}{15d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])*(2 + 3*Sin[c + d*x]))/(15*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 6.20, size = 54, normalized size = 0.90

method	result	size
default	$-\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^2(3\sin(dx+c)+2)}{15\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(1+sin(d*x+c))*(sin(d*x+c)-1)^2*(3*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.34, size = 96, normalized size = 1.60

$$-\frac{2\left(3\cos\left(dx+c\right)^3+4\cos\left(dx+c\right)^2-\left(3\cos\left(dx+c\right)^2-\cos\left(dx+c\right)-2\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-2\right)\sqrt{a\sin\left(dx+c\right)+a}}{15\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/15*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x +
c) - 2)*sin(d*x + c) - cos(d*x + c) - 2)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(
d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)*cos(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.56, size = 65, normalized size = 1.08

$$-\frac{4\sqrt{2}\left(6\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-5\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3\right)}{15\,ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -4/15*sqrt(2)*(6*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 5*sqrt(a)*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2), x)
```

3.339 $\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+2*cos(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2953, 3060, 2852, 212}

$$\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a}\\
&= \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 116, normalized size = 1.84

$$
\frac{\left(2\cos\left(\frac{1}{2}(c+dx)\right)-\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sin(c+dx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((2*Cos[(c + d*x)/2] - Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 5.42, size = 87, normalized size = 1.38

method	result
default	$\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sqrt{a-a\sin\left(dx+c\right)}-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)\right)}{a\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a*((a-a*sin(d*x+c))^(1/2)-a^(1/2
)*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(55) = 110.
time = 0.36, size = 236, normalized size = 3.75

$$\frac{\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\,\sqrt{a\sin\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{2\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a*d*cos(d*x + c
) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(c+dx\right)\csc\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.
time = 0.58, size = 112, normalized size = 1.78

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)/(a*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

3.340 $\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}$$

[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2795, 21, 2852, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]

[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) - Cot[c + d*x]/(d*Sqrt[a + a*Sin[c + d*x]])

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2795

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
```

```
2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}\\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a}\\
&= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.
time = 0.24, size = 138, normalized size = 2.23

$$\frac{\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(-2\cos\left(\frac{1}{2}(c+dx)\right)+2\sin\left(\frac{1}{2}(c+dx)\right)+\left(\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)\left(1+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-2*Cos[(c + d*x)/2] + 2*Sin[(c + d*x)/2
] + (Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]])*Sin[c + d*x])*(1 + Tan[(c + d*x)/2]))/(8*d*Sqrt[a*(1
 + Sin[c + d*x])])
```

Maple [A]
time = 4.99, size = 103, normalized size = 1.66

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)a\sin(dx+c)+\sqrt{a-a\sin(dx+c)}\right.}{a^{\frac{3}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-arctanh((a-a*sin(d*x+c))^(1/2)/
a^(1/2))*a*sin(d*x+c)+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/a^(3/2)/sin(d*x+c)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(54) = 108$.
time = 0.36, size = 263, normalized size = 4.24

$$\frac{\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{4\left(ad\cos(dx+c)^2-ad-(ad\cos(dx+c)+ad)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*((cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*
cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3
)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a
*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)
/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos
(d*x + c) - 1)) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) +
 1))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(54) = 108.
time = 0.61, size = 132, normalized size = 2.13

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c))) - 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((2*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)^2\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

3.341 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d}+\frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 1/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+1/4*cot(d*
x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^
(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2953, 3059, 2851, 2852, 212}

$$\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(4*Sqrt[a]*d) + Co
t[c + d*x]/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{4a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a}}{8a}$$

$$= \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x,\right.}{4}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{2d\sqrt{a+a\sin}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 272 vs. 2(100) = 200.
time = 1.30, size = 272, normalized size = 2.72

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-8+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+4\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+4\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 + 4*Cot[(c + d*x)/4] - Csc[(c +
d*x)/4]^2 + 4*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d*x)/4]
 - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Sin[(c +
d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (8*Sin[(c + d*x)/4])
/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*
(1 + Sin[c + d*x])])
```

Maple [A]
time = 7.29, size = 124, normalized size = 1.24

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left((-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^3(\sin^2$ ... $}{4a^{\frac{7}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*((-a*(sin(d*x+c)-1))^
(3/2)*a^(3/2)-arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+(
-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(84) = 168$.
time = 0.36, size = 320, normalized size = 3.20

$$\frac{\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-3\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}-ad\cos\left(dx+c\right)-ad+\left(ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*((cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
- cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(cos(d*x + c)^2 + (c
os(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a
))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(c+dx\right)\csc^{3}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.53, size = 149, normalized size = 1.49

$$\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/16*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) + 4*(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + sin(-1/
4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2*a*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sin\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

3.342 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d}+\frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 1/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+1/8*cot(d*
x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/12*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))
^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2953, 3059, 2851, 2852, 212}

$$\frac{\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(8*Sqrt[a]*d) + Co
t[c + d*x]/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x])/(12
*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))\sqrt{a+a\sin(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{6a}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc^2(c+dx)\sqrt{a}}{}$$

$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c-}{3d\sqrt{a+a\sin(c+}}$$

$$= \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c-}{3d\sqrt{a+a\sin(c+}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx}{12d\sqrt{a+a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(135) = 270.
time = 0.55, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-60\cos\left(\frac{1}{2}(c+dx)\right)+2\cos\left(\frac{3}{2}(c+dx)\right)-6\cos\left(\frac{5}{2}(c+dx)\right)+60\sin\left(\frac{1}{2}(c+dx)\right)+9\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-9\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+2\sin\left(\frac{3}{2}(c+dx)\right)+6\sin\left(\frac{5}{2}(c+dx)\right)-3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-60*Cos[(c + d*x
)/2] + 2*Cos[(3*(c + d*x))/2] - 6*Cos[(5*(c + d*x))/2] + 60*Sin[(c + d*x)/2
] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 9*Log[1 -
 Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 2*Sin[(3*(c + d*x))/2]
 + 6*Sin[(5*(c + d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
+ d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(1 + Sin
[c + d*x])])
```

Maple [A]
time = 6.03, size = 144, normalized size = 1.07

method	result

default	$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{5}{2}}-8(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{7}{2}}+3\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(}{\sqrt{}}\right.\right.}{24a^{\frac{11}{2}}\sin(dx+c)^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(3*(-a*(sin(d*x+c)-1
))^(5/2)*a^(5/2)-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+3*arctanh((-a*(sin(d*x
+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^3-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2)
)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(115) = 230$.
time = 0.35, size = 367, normalized size = 2.72

$$\frac{3\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(3\cos(dx+c)^3+\cos(dx+c)^2-\left(3\cos(dx+c)^2+2\cos(dx+c)+7\right)\sin(dx+c)+5\cos(dx+c)+7\right)\sqrt{a\sin(dx+c)+a}}{96\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad-\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(3*cos(d*x + c)^3 + cos(d*x + c)^2 - (3*cos(d*x + c)^2 + 2*cos(d*x + c) +
7)*sin(d*x + c) + 5*cos(d*x + c) + 7)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*
x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*cos(d*x +
 c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.58, size = 168, normalized size = 1.24

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{4\left(12\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-16\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/96*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*
x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) + 4*(12*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 16*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

3.343 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=184

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}}-\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c}{7ad\sqrt{}}$$

```
[Out] 2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^
(3/2)/d-344/105*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-16/35*cos(d*x+c)*sin(
d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+2/7*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+a*si
n(d*x+c))^(1/2)+76/105*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.39, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2958, 3062, 3047, 3102, 2830, 2728, 212}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{76\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105a^2d}+\frac{2\sin^3(c+dx)\cos(c+dx)}{7ad\sqrt{a\sin(c+dx)+a}}-\frac{16\sin^2(c+dx)\cos(c+dx)}{35ad\sqrt{a\sin(c+dx)+a}}-\frac{344\cos(c+dx)}{105ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)]])/(a^(3/2)*d) - (344*Cos[c + d*x])/(105*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
16*Cos[c + d*x]*Sin[c + d*x]^2)/(35*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[
c + d*x]*Sin[c + d*x]^3)/(7*a*d*Sqrt[a + a*Sin[c + d*x]]) + (76*Cos[c + d*x
]*Sqrt[a + a*Sin[c + d*x]])/(105*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2958

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^2(c+dx)(-3a^2+4a^2\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{7a^3}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{\sin(c+dx)(8a}{\sqrt{a+a}}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{4\int \frac{8a^3\sin(c+dx}{\sqrt{a+a}}$$

$$= -\frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sin^3(c+dx)}{7ad\sqrt{a+a\sin(c+dx)}} + \frac{76\cos(c+dx}{}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx}{7ad\sqrt{a+a}}$$

$$= -\frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} - \frac{16\cos(c+dx)\sin^2(c+dx)}{35ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx}{7ad\sqrt{a+a}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{344\cos(c+dx)}{105ad\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.34, size = 201, normalized size = 1.09

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((1680+1680i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)-1365\cos\left(\frac{1}{2}(c+dx)\right)+245\cos\left(\frac{3}{2}(c+dx)\right)+63\cos\left(\frac{5}{2}(c+dx)\right)-15\cos\left(\frac{7}{2}(c+dx)\right)+1365\sin\left(\frac{1}{2}(c+dx)\right)+245\sin\left(\frac{3}{2}(c+dx)\right)-63\sin\left(\frac{5}{2}(c+dx)\right)-15\sin\left(\frac{7}{2}(c+dx)\right)\right)}{420a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((1680 + 1680*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)
*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 1365*
Cos[(c + d*x)/2] + 245*Cos[(3*(c + d*x))/2] + 63*Cos[(5*(c + d*x))/2] - 15*
Cos[(7*(c + d*x))/2] + 1365*Sin[(c + d*x)/2] + 245*Sin[(3*(c + d*x))/2] - 6
3*Sin[(5*(c + d*x))/2] - 15*Sin[(7*(c + d*x))/2]))/(420*a^2*d*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.36, size = 148, normalized size = 0.80

method	result
default	$\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(105a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-15(a-a\sin(dx+c)\right.}{105a^{5}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*2^(1/2)*arctanh
(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+21*a
*(a-a*sin(d*x+c))^(5/2)-35*a^2*(a-a*sin(d*x+c))^(3/2)-105*a^3*(a-a*sin(d*x+
c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.35, size = 259, normalized size = 1.41

$$\frac{\frac{105\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(15\cos\left(dx+c\right)^{4}-24\cos\left(dx+c\right)^{3}-92\cos\left(dx+c\right)^{2}+\left(15\cos\left(dx+c\right)^{3}+39\cos\left(dx+c\right)^{2}-53\cos\left(dx+c\right)-211\right)\sin\left(dx+c\right)+158\cos\left(dx+c\right)+211\right)\sqrt{a\sin\left(dx+c\right)+a}}{105\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/105*(105*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(15*
cos(d*x + c)^4 - 24*cos(d*x + c)^3 - 92*cos(d*x + c)^2 + (15*cos(d*x + c)^3
 + 39*cos(d*x + c)^2 - 53*cos(d*x + c) - 211)*sin(d*x + c) + 158*cos(d*x +
```

```
c) + 211)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 182, normalized size = 0.99

$$-\frac{\frac{105\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{105\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(120a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-84a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+70a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+105a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{14}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/105*(105*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 105*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1
/2*c) + 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(120*a
^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 84*a^(25/2)*sin(-1/4*pi + 1/2*d*
x + 1/2*c)^5 + 70*a^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 105*a^(25/2)*
sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^14*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,{\sin\left(c+d\,x\right)}^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2), x)
```

3.344 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=140

$$-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}}-\frac{4\cos(c+$$

```
[Out] -2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a
^(3/2)/d+18/5*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)^3/a/d/(a
+a*sin(d*x+c))^(1/2)-4/5*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2957, 2937, 2830, 2728, 212}

$$-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{2\cos^3(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}+\frac{18\cos(c+dx)}{5ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(3/2)*d) + (18*Cos[c + d*x])/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*
Cos[c + d*x]^3)/(5*a*d*Sqrt[a + a*Sin[c + d*x]]) - (4*Cos[c + d*x]*Sqrt[a +
 a*Sin[c + d*x]])/(5*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2937

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\cos^2(c+dx)\left(-\frac{a}{2}-3a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{3/2}}\,dx}{5a}$$

$$= -\frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} - \frac{4\int \frac{-3}{\sqrt{a}}}{}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+}}{5a^2d}$$

$$= \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^3(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+}}{5a^2d}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{18\cos(c+dx)}{5ad\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 150, normalized size = 1.07

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left((40+40i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+30\cos\left(\frac{1}{2}(c+dx)\right)-5\cos\left(\frac{3}{2}(c+dx)\right)-\cos\left(\frac{5}{2}(c+dx)\right)-30\sin\left(\frac{1}{2}(c+dx)\right)-5\sin\left(\frac{3}{2}(c+dx)\right)+\sin\left(\frac{5}{2}(c+dx)\right)\right)}{10d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((40 + 40*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 30*Cos[(c + d*x)/2] - 5*Cos
[(3*(c + d*x))/2] - Cos[(5*(c + d*x))/2] - 30*Sin[(c + d*x)/2] - 5*Sin[(3*(
c + d*x))/2] + Sin[(5*(c + d*x))/2]))/(10*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 5.15, size = 114, normalized size = 0.81

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(5a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{5}{2}}\right.}{5d\,a^{4}\cos(dx+c)\sqrt{a\left(1+\sin\left(dx+c\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/5/d*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(5*a^(5/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(5/2)-5*a^2*(a
-a*sin(d*x+c))^(1/2))/a^4/cos(d*x+c)/(a*(1+sin(d*x+c)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.36, size = 236, normalized size = 1.69

$$\frac{\frac{5\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\,\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-2\left(\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-9\right)\sin\left(dx+c\right)-7\cos\left(dx+c\right)-9\right)\sqrt{a\sin\left(dx+c\right)+a}}{5\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/5*(5*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 2*(cos(d*x
 + c)^3 + 3*cos(d*x + c)^2 - (cos(d*x + c)^2 - 2*cos(d*x + c) - 9)*sin(d*x
+ c) - 7*cos(d*x + c) - 9)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) +
a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\cos^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.55, size = 144, normalized size = 1.03

$$\frac{\frac{5\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{5\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(4a^{\frac{17}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+5a^{\frac{17}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{10}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/5*(5*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c))) - 5*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c) +
 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(4*a^(17/2)*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 5*a^(17/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)
)/(a^10*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\sin(c+d\,x)^2}{(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2), x)
```

3.345 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d}$$

```
[Out] 2*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^
(3/2)/d-10/3*cos(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+2/3*cos(d*x+c)*(a+a*sin(
d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2937, 2830, 2728, 212}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) + (2*C
os[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2937

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[d*Cos[e + f*x]*((
a + b*Sin[e + f*x])^(m + 2)/(b^2*f*(m + 3))), x] - Dist[1/(b^2*(m + 3)), In
t[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) -
a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \frac{\frac{a}{2}-\frac{5}{2}a\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^2}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\int \ldots}{\sqrt{a \ldots}}$$

$$= -\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{4\text{Subst}\Big(\ldots}{\ldots}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{2\,\mathrm{c}\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.42, size = 149, normalized size = 1.38

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((12+12i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)-9\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)+9\sin\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((12 + 12*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1
)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 9*Cos[(c
+ d*x)/2] + Cos[(3*(c + d*x))/2] + 9*Sin[(c + d*x)/2] + Sin[(3*(c + d*x))/2
]))/(3*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.45, size = 110, normalized size = 1.02

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)+(a-a\sin(dx+c)\right.}{3a^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/a^3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*a^(3/2)*2^(1/2)*arcta
nh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+(a-a*sin(d*x+c))^(3/2)+3*a*(
a-a*sin(d*x+c))^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(91) = 182$.
time = 0.38, size = 215, normalized size = 1.99

$$\frac{\frac{3\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+2\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)-4\cos\left(dx+c\right)-5\right)\sqrt{a\sin\left(dx+c\right)+a}}{3\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 -
 (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(
d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
- (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 2*(cos(d*x
 + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sqrt(a*sin(
d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
[Out] Integral(sin(c + d*x)*cos(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.55, size = 144, normalized size = 1.33

$$-\frac{\frac{3\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{3\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(2a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+3a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{6}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
[Out] -1/3*(3*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(3/2)*sgn(cos(-1
/4*pi + 1/2*d*x + 1/2*c))) - 3*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c)
+ 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(2*a^(9/2)*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 3*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))
/(a^6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^2*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2), x)
```

3.346 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+2*arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2953, 3064, 2728, 212, 2852}

$$\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(a^(3/2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\csc(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{a^2} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} + \frac{4\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{\ldots}{\sqrt{a\ldots}}\right.}{ad}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx\ldots}}\right.}{a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 130, normalized size = 1.53

$$-\frac{\left((4+4i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}{d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((((4 + 4*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[1 - Cos[(c + d*x
```

```
)/2] + Sin[(c + d*x)/2]])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)/(d*(a*(1
 + Sin[c + d*x]))^(3/2)))
```

Maple [A]
time = 6.03, size = 97, normalized size = 1.14

method	result
default	$\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{\sqrt{}}\right.\right.}{a^{\frac{3}{2}}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(a-a*sin(d*
x+c))^(1/2)*2^(1/2)/a^(1/2))-arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2)))/a^(3/
2)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(70) = 140$.
time = 0.37, size = 291, normalized size = 3.42

$$\frac{2\sqrt{2}\sqrt{a}\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)+\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*sqrt(2)*sqrt(a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x +
c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/s
qrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x +
 c) - cos(d*x + c) - 2)) + sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
```

```
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(70) = 140.
time = 0.53, size = 152, normalized size = 1.79

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{2\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) + 2*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^
2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*log(-sin(-1/4*pi + 1/2*d*x + 1/2
*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

3.347 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-2*arctanh(1/
2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-cot(
d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2794, 3064, 2728, 212, 2852}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(3/2)*d) - Cot[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2794

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[(a + b*Sin[e + f*x])^(m + 1)*((b*m - a*(m + 1)*Sin[e + f*x])/
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
```

```
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{3\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^2} + \frac{2\int \frac{}{\sqrt{a}}$$

$$= -\frac{\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} + \frac{3\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{ad} - \frac{4S}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.57, size = 206, normalized size = 1.82

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left((16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+2\left(3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec\left(\frac{1}{2}(c+dx)\right)+\csc(c+dx)\sin^2\left(\frac{1}{4}(c+dx)\right)-\csc(c+dx)\sin\left(\frac{1}{4}(c+dx)\right)\sin\left(\frac{3}{4}(c+dx)\right)\right)\right)}{4d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 2*(3*Log
[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3*Log[1 - Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2] + Csc[c + d*x]*Sin[(c + d*x)/4]^2 - Csc[
c + d*x]*Sin[(c + d*x)/4]*Sin[(3*(c + d*x))/4])))/(4*d*(a*(1 + Sin[c + d*x]
))^(3/2))
```

Maple [A]
time = 7.03, size = 135, normalized size = 1.19

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-\sin(dx+c)a^2\left(-2\sqrt{2}\ \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\ \sqrt{2}}{2\sqrt{a}}\right)+3\,\mathrm{a}\right.\right.}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-sin(d*x+c)*a^2*(-2*2^
(1/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+3*arctanh((a-a*si
n(d*x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*csc(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(96) = 192.
time = 0.38, size = 421, normalized size = 3.73

$$\frac{3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{4\sqrt{2}\left(a\cos(dx+c)^2-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{4\left(a^2d\cos(dx+c)^2-a^2d-(a^2d\cos(dx+c)+a^2d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^2*d*cos(d*x + c)^2 - a^2*d - (a^2*d*cos(d*x + c) + a^2*d)*s
in(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(96) = 192.
time = 0.65, size = 205, normalized size = 1.81

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{4\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) + 4*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a
^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*log(-sin(-1/4*pi + 1/2*d*x + 1/
2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c)/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^2*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2}{\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

3.348 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-$$

```
[Out] -11/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+2*arctan
h(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d+
5/4*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+
a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2953, 3063, 3064, 2728, 212, 2852}

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}+\frac{5\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d
) + (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(3/2)*d) + (5*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (
Cot[c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{5a^2}{2}+\frac{3}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^3}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{11a^3}{4}-\frac{5}{4}a^3\sin\right.}{\sqrt{a+a\sin(c+}}}{2a^4}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{11\int \csc(c+dx)\sqrt{a}}{8a}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{11\text{Subst}\left(\int \frac{1}{a-x^2}\,dx,\right.}{}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+}}\right.}{a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.60, size = 309, normalized size = 2.02

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-24-(128+128i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)-44\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+44\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{24\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{24\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+12\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 - (128 + 128*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 12*Cot[(c + d*x)/4]
 - Csc[(c + d*x)/4]^2 - 44*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4
4*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 6.08, size = 164, normalized size = 1.07

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(3\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{7}{2}}-5(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{5}{2}}+8\sqrt{2}\,\operatorname{arcta}\right.}{4a^{\frac{11}{2}}\sin(dx+c)^{2}\cos(dx+c)\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-1))^(1/2)*a
^(7/2)-5*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+8*2^(1/2)*arctanh(1/2*(-a*(sin(d
*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*sin(d*x+c)^2-11*a^4*arctanh((-a*(sin(d
*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^2)/a^(11/2)/sin(d*x+c)^2/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(128) = 256$.
time = 0.37, size = 508, normalized size = 3.32

$$\frac{11\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{16\sqrt{2}\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}-a\cos(dx+c)+\left(a\cos(dx+c)^{2}-a\right)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(5\cos\left(dx+c\right)^{2}+\left(5\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)-7\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-a^{2}d+\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(11*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 16*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
```

```
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(5*cos(d*x + c)^2 + (5*cos(d*x + c) + 7)*sin(d*
x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^
3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d + (a^2*d*cos(d*x + c)
^2 - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.52, size = 224, normalized size = 1.46

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{11\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{16\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{16\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\left(10\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/16*sqrt(2)*sqrt(a)*(11*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) + 16*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1
)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 16*log(-sin(-1/4*pi + 1/2*d*x
 + 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(10*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^3 - 3*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^2 - 1)^2*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{\sin(c+d\,x)^3\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

3.349 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}+$$

```
[Out] 23/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-2*arctanh
(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(3/2)/d-9
/8*cot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+7/12*cot(d*x+c)*csc(d*x+c)/a/d/(a+
a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2953, 3063, 3064, 2728, 212, 2852}

$$\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{9\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}+\frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(3/2)*d)
 - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(3/2)*d) - (9*Cot[c + d*x])/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (7
*Cot[c + d*x]*Csc[c + d*x])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2953

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= \frac{\int \frac{\csc^4(c+dx)(a-a\sin(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{7a^2}{2}+\frac{5}{2}a^2\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a^3} \\
&= \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{27a^3}{4}-\right.}{\sqrt{a+a\sin}}}{6a} \\
&= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc}{3ad\sqrt{a+a\sin}} \\
&= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc}{3ad\sqrt{a+a\sin}} \\
&= -\frac{9\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{7\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc}{3ad\sqrt{a+a\sin}} \\
&= \frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+}}\right.}{a^{3/2}d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.61, size = 332, normalized size = 1.74

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left((768+768i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(228\cos\left(\frac{1}{2}(c+dx)\right)-110\cos\left(\frac{3}{2}(c+dx)\right)-54\cos\left(\frac{5}{2}(c+dx)\right)-228\sin\left(\frac{1}{2}(c+dx)\right)-207\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+207\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-110\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+dx)\right)+69\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-69\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}\right)}{192d(a(1+\sin(c+dx)))^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*((768 + 768*I)*(-1)^(3/4)*ArcTanh[
(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(22
8*Cos[(c + d*x)/2] - 110*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 2
28*Sin[(c + d*x)/2] - 207*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
c + d*x] + 207*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
110*Sin[(3*(c + d*x))/2] + 54*Sin[(5*(c + d*x))/2] + 69*Log[1 + Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 69*Log[1 - Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]
^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 6.87, size = 182, normalized size = 0.95

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-69a^{6}\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)(\sin^{3}(dx+c))+27(-a(\sin(}{24a^{\frac{15}{2}}\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/24/a^(15/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-69*a^6*arctanh((-
a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^3+27*(-a*(sin(d*x+c)-1))^(5/2)*
a^(7/2)-40*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)+48*2^(1/2)*arctanh(1/2*(-a*(si
n(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^6*sin(d*x+c)^3+21*(-a*(sin(d*x+c)-1))
^(1/2)*a^(11/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(162) = 324$.
time = 0.38, size = 564, normalized size = 2.95

$$\frac{69\left(\cos(dx+c)^4-2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{96\sqrt{2}\left(a\cos(dx+c)^4-2a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\log\left(\text{[illegible]}\right)}{\sqrt{a}}+4\left(27\cos(dx+c)^3+41\cos(dx+c)^2-\left(27\cos(dx+c)^2-14\cos(dx+c)-49\right)\sin(dx+c)-35\cos(dx+c)-49\right)\sqrt{a\sin(dx+c)+a}}{96\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d-\left(a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2-a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(69*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
```

```
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
96*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*c
os(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
- (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos
(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2
 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(27*cos
(d*x + c)^3 + 41*cos(d*x + c)^2 - (27*cos(d*x + c)^2 - 14*cos(d*x + c) - 49
)*sin(d*x + c) - 35*cos(d*x + c) - 49)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos
(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3 + a^2*
d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.55, size = 240, normalized size = 1.26

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{69\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{96\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{96\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\left(108\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-80\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+21\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/96*sqrt(2)*sqrt(a)*(69*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) + 96*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)
/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 96*log(-sin(-1/4*pi + 1/2*d*x
+ 1/2*c) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(108*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^5 - 80*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 21*sin(-1/4*
pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3*a^2*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2}{{\sin\left(c+d\,x\right)}^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

3.350 $\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] 1/4*a*sin(d*x+c)^4/d+1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6/d-1/7*a*sin(d*
x+c)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 76}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6
*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^3(a+x)^2}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^3(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^3+a^2x^4-ax^5-x^6\right)\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{a\sin^4(c+dx)}{4d} + \frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^6(c+dx)}{6d} - \frac{a\sin}{}$$

Mathematica [A]
time = 0.22, size = 51, normalized size = 0.78

$$\frac{a(-315\cos(2(c+dx))+35\cos(6(c+dx))+96(9+5\cos(2(c+dx)))\sin^5(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] (a*(-315*Cos[2*(c + d*x)] + 35*Cos[6*(c + d*x)] + 96*(9 + 5*Cos[2*(c + d*x)
])*Sin[c + d*x]^5))/(6720*d)
```

Maple [A]
time = 0.19, size = 92, normalized size = 1.42

method	result
risch	$\frac{3a\sin(dx+c)}{64d}+\frac{a\sin(7dx+7c)}{448d}+\frac{a\cos(6dx+6c)}{192d}-\frac{a\sin(5dx+5c)}{320d}-\frac{a\sin(3dx+3c)}{64d}-\frac{3a\cos(2dx+2c)}{64d}$
derivativedivides	$\frac{a\left(-\frac{(\sin^2(dx+c))(\cos^4(dx+c))}{6}-\frac{(\cos^4(dx+c))}{12}\right)+a\left(-\frac{(\sin^3(dx+c))(\cos^4(dx+c))}{7}-\frac{3\sin(dx+c)(\cos^4(dx+c))}{35}+\frac{(2+\cos^2(dx+c)}{35}\right.}{d}$
default	$\frac{a\left(-\frac{(\sin^2(dx+c))(\cos^4(dx+c))}{6}-\frac{(\cos^4(dx+c))}{12}\right)+a\left(-\frac{(\sin^3(dx+c))(\cos^4(dx+c))}{7}-\frac{3\sin(dx+c)(\cos^4(dx+c))}{35}+\frac{(2+\cos^2(dx+c)}{35}\right.}{d}$
norman	$\frac{\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{32a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{192a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{32a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{4a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+a*(-1/7*sin(d*x+c
)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x
+c)))
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.77

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-84\,a\sin\left(dx+c\right)^{5}-105\,a\sin\left(dx+c\right)^{4}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.35, size = 72, normalized size = 1.11

$$\frac{70\,a\cos\left(dx+c\right)^{6}-105\,a\cos\left(dx+c\right)^{4}+12\,\left(5\,a\cos\left(dx+c\right)^{6}-8\,a\cos\left(dx+c\right)^{4}+a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*cos(d*x + c)^6 - 105*a*cos(d*x + c)^4 + 12*(5*a*cos(d*x + c)^6
- 8*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.65, size = 90, normalized size = 1.38

$$\begin{cases} \frac{2a\sin^{7}\left(c+dx\right)}{35d}+\frac{a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{4d}-\frac{a\cos^{6}\left(c+dx\right)}{12d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin^{3}\left(c\right)\cos^{3}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**7/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**2/(
5*d) - a*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - a*cos(c + d*x)**6/(12*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**3, True))
```

Giac [A]
time = 0.58, size = 50, normalized size = 0.77

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-84\,a\sin\left(dx+c\right)^{5}-105\,a\sin\left(dx+c\right)^{4}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 1
05*a*sin(d*x + c)^4)/d

Mupad [B]
time = 0.07, size = 49, normalized size = 0.75

$$\frac{-\frac{a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{6}+\frac{a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^3*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)

[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/6 - (a*si
n(c + d*x)^7)/7)/d

3.351 $\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^6(c+dx)}{6d}$$

```
[Out] 1/3*a*sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*
x+c)^6/d
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 76}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5
*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^2(a+x)^2}{a^2}\,dx,x,a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^2(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^2+a^2x^3-ax^4-x^5\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin}{}$$

Mathematica [A]
time = 0.16, size = 51, normalized size = 0.78

$$\frac{a(-45\cos(2(c+dx))+5\cos(6(c+dx))+32(7+3\cos(2(c+dx)))\sin^3(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-45*Cos[2*(c + d*x)] + 5*Cos[6*(c + d*x)] + 32*(7 + 3*Cos[2*(c + d*x)])
*Sin[c + d*x]^3))/(960*d)
```

Maple [A]
time = 0.14, size = 74, normalized size = 1.14

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)}{d}$
risch	$\frac{a\sin(dx+c)}{8d}+\frac{a\cos(6dx+6c)}{192d}-\frac{a\sin(5dx+5c)}{80d}-\frac{a\sin(3dx+3c)}{48d}-\frac{3a\cos(2dx+2c)}{64d}$
norman	$\frac{\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8a(\mathrm{t}}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+a*(-
1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4))
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.77

$$-\frac{10\,a\sin\left(dx+c\right)^{6}+12\,a\sin\left(dx+c\right)^{5}-15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 62, normalized size = 0.95

$$\frac{10\,a\cos\left(dx+c\right)^{6}-15\,a\cos\left(dx+c\right)^{4}-4\,\left(3\,a\cos\left(dx+c\right)^{4}-a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 4*(3*a*cos(d*x + c)^4 - a
*cos(d*x + c)^2 - 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.45, size = 90, normalized size = 1.38

$$\begin{cases}\frac{2a\sin^{5}\left(c+dx\right)}{15d}+\frac{a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{4d}-\frac{a\cos^{6}\left(c+dx\right)}{12d} & \text{for}\: d\neq 0\\x\left(a\sin\left(c\right)+a\right)\sin^{2}\left(c\right)\cos^{3}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**5/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - a*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - a*cos(c + d*x)**6/(12*d),
Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**3, True))
```

Giac [A]
time = 0.51, size = 50, normalized size = 0.77

$$-\frac{10\,a\sin\left(dx+c\right)^{6}+12\,a\sin\left(dx+c\right)^{5}-15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 15*a*sin(d*x + c)^4 - 20
*a*sin(d*x + c)^3)/d
```

Mupad [B]
time = 0.06, size = 49, normalized size = 0.75

$$\frac{-\frac{a\sin(c+d\,x)^6}{6}-\frac{a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (a*sin(c + d*x)^5)/5 - (a*si
n(c + d*x)^6)/6)/d
```

3.352 $\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$-\frac{a\cos^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] -1/4*a*cos(d*x+c)^4/d+1/3*a*sin(d*x+c)^3/d-1/5*a*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2913, 2645, 30, 2644, 14}

$$-\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*Cos[c + d*x]^4)/d + (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^5)/(
5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^3(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\sin(c+dx)\,dx + a\int \cos^3(c+dx)\sin^2(c$$
$$= -\frac{a\text{Subst}\left(\int x^3\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int x^2(1-x^2)\right.}{d}$$
$$= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\text{Subst}\left(\int (x^2-x^4)\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{a\cos^4(c+dx)}{4d} + \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin^5(c+dx)}{5d}$$

Mathematica [A]
time = 0.08, size = 58, normalized size = 1.18

$$-\frac{a(45+60\cos(2(c+dx))+15\cos(4(c+dx))-60\sin(c+dx)+10\sin(3(c+dx))+6\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] -1/480*(a*(45 + 60*Cos[2*(c + d*x)] + 15*Cos[4*(c + d*x)] - 60*Sin[c + d*x]
 + 10*Sin[3*(c + d*x)] + 6*Sin[5*(c + d*x)]))/d
```

Maple [A]
time = 0.10, size = 54, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+a\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)}{d}$
risch	$\frac{a\sin(dx+c)}{8d}-\frac{a\sin(5dx+5c)}{80d}-\frac{a\cos(4dx+4c)}{32d}-\frac{a\sin(3dx+3c)}{48d}-\frac{a\cos(2dx+2c)}{8d}$

norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4*a*cos(d*x+c)^4+a*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)
^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 50, normalized size = 1.02

$$-\frac{12\,a\sin\left(dx+c\right)^5+15\,a\sin\left(dx+c\right)^4-20\,a\sin\left(dx+c\right)^3-30\,a\sin\left(dx+c\right)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.35, size = 51, normalized size = 1.04

$$-\frac{15\,a\cos\left(dx+c\right)^4+4\,\left(3\,a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2-2\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*a*cos(d*x + c)^4 + 4*(3*a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c))/d
```

Sympy [A]
time = 0.28, size = 66, normalized size = 1.35

$$\begin{cases}\frac{2a\sin^5\left(c+dx\right)}{15d}+\frac{a\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{3d}-\frac{a\cos^4\left(c+dx\right)}{4d} & \text{for } d\neq 0\\ x(a\sin\left(c\right)+a)\sin\left(c\right)\cos^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**5/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)*
*3, True))
```

Giac [A]
time = 0.55, size = 50, normalized size = 1.02

$$-\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30
*a*sin(d*x + c)^2)/d
```

Mupad [B]
time = 0.06, size = 49, normalized size = 1.00

$$\frac{-\frac{a\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (a*sin(c + d*x)^4)/4 - (a*si
n(c + d*x)^5)/5)/d
```

3.353 $\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=45

$$\frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^3/a^2/d-1/4*(a+a*sin(d*x+c))^4/a^3/d
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {2746, 45}

$$\frac{2(a\sin(c+dx)+a)^3}{3a^2d}-\frac{(a\sin(c+dx)+a)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - (a + a*Sin[c + d*x])^4/(4*a^3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+a\sin(c+dx))\,dx&=\frac{\text{Subst}\left(\int (a-x)(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{\text{Subst}\left(\int \left(2a(a+x)^2-(a+x)^3\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\&=\frac{2(a+a\sin(c+dx))^3}{3a^2d}-\frac{(a+a\sin(c+dx))^4}{4a^3d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.98

$$-\frac{a\cos^4(c+dx)}{4d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x]),x]
[Out] -1/4*(a*Cos[c + d*x]^4)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.11, size = 36, normalized size = 0.80

method	result	siz
derivativedivides	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
default	$\frac{-\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	36
risch	$\frac{3a\sin(dx+c)}{4d}-\frac{a\cos(4dx+4c)}{32d}+\frac{a\sin(3dx+3c)}{12d}-\frac{a\cos(2dx+2c)}{8d}$	59
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{10a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{10a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*a*cos(d*x+c)^4+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,a\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 39, normalized size = 0.87

$$-\frac{3\,a\cos\left(dx+c\right)^4-4\,\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 - 4*(a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.21, size = 60, normalized size = 1.33

$$\begin{cases} \frac{2a\sin^3(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a\cos^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - a
*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A]
time = 0.55, size = 48, normalized size = 1.07

$$-\frac{3\,a\sin\left(dx+c\right)^4+4\,a\sin\left(dx+c\right)^3-6\,a\sin\left(dx+c\right)^2-12\,a\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 6*a*sin(d*x + c)^2 - 12*a*
sin(d*x + c))/d
```

Mupad [B]
time = 0.06, size = 46, normalized size = 1.02

$$\frac{-\frac{a\sin(c+d\,x)^4}{4}-\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}+a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x) + (a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^3)/3 - (a*sin(c +
d*x)^4)/4)/d
```

3.354 $\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=56

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 76}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{a^3}{x}-ax-x^2\right)\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 1.00

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) - (
a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.12, size = 45, normalized size = 0.80

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{a\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$-iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{-2i(dx+c)}}{8d}-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{3a\sin(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.84

$$-\frac{2 a \sin (dx+c)^3+3 a \sin (dx+c)^2-6 a \log (\sin (dx+c))-6 a \sin (dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(sin(d*x + c)) - 6*a
*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.91

$$\frac{3 a \cos (dx+c)^2+6 a \log \left(\frac{1}{2} \sin (dx+c)\right)+2 \left(a \cos (dx+c)^2+2 a\right) \sin (dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/6*(3*a*cos(d*x + c)^2 + 6*a*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2 +
 2*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \cos^3(c+dx)\csc(c+dx)\,dx+\int \sin(c+dx)\cos^3(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)*(a+a*sin(d*x+c)),x)
[Out] a*(Integral(cos(c + d*x)**3*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**3*csc(c + d*x), x))
```

Giac [A]
time = 0.54, size = 48, normalized size = 0.86

$$-\frac{2 a \sin (dx+c)^3+3 a \sin (dx+c)^2-6 a \log (|\sin (dx+c)|)-6 a \sin (dx+c)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 6*a*log(abs(sin(d*x + c)))
- 6*a*sin(d*x + c))/d
```

Mupad [B]
time = 8.73, size = 92, normalized size = 1.64

$$\frac{a\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^2}{2\,d}+\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (2*a*sin(c + d*x))/(3*d) + (a*cos(c
+ d*x)^2*sin(c + d*x))/(3*d)
```

3.355 $\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=53

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 76}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^2}\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}+\frac{a^2}{x}-x\right)\,dx,x,a\sin(c+dx)\right)}{ad}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 1.00

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*S
in[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.11, size = 65, normalized size = 1.23

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
risch	$-iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{a\,e^{-2i(dx+c)}}{8d}-\frac{2iac}{d}-\frac{2ia\,e^{i(dx+c)}}{d(e^{2i(dx+c)}-1)}+\frac{a\ln(e^{2i(dx+c}}{d}$
norman	$\frac{-\frac{a}{2d}-\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{7a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(1+t\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+a*(1/2*cos(
d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.87

$$-\frac{a\sin\left(dx+c\right)^{2}-2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + 2*a/sin
(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 68, normalized size = 1.28

$$\frac{4\,a\cos\left(dx+c\right)^{2}+4\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(2\,a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)-8\,a}{4\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*cos(d*x + c)^2 + 4*a*log(1/2*sin(d*x + c))*sin(d*x + c) + (2*a*cos
(d*x + c)^2 - a)*sin(d*x + c) - 8*a)/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{3}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos
(c + d*x)**3*csc(c + d*x)**2, x))
```

Giac [A]
time = 0.55, size = 47, normalized size = 0.89

$$-\frac{a\sin\left(dx+c\right)^{2}-2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a}{\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*sin(d*x + c)^2 - 2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + 2*
a/sin(d*x + c))/d
```

Mupad [B]
time = 8.70, size = 140, normalized size = 2.64

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+6\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*ta
n(c/2 + (d*x)/2))/(2*d) - (a + 6*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*
x)/2)^3 + 5*a*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) + 4*tan(c/2 +
(d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5))
```

3.356 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 76}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 1.11

$$-\frac{a\csc(c+dx)}{d}-\frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d}-\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

Maple [A]
time = 0.14, size = 67, normalized size = 1.24

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(ie^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\ln\left(\tan\left(\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+a*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+
cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.83

$$-\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A]
time = 0.37, size = 69, normalized size = 1.28

$$-\frac{2\,{\left(a\cos\left(dx+c\right)^{2}-a\right)}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\,{\left(a\cos\left(dx+c\right)^{2}-2\,a\right)}\sin\left(dx+c\right)-a}{2\,{\left(d\cos\left(dx+c\right)^{2}-d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{3}\left(c+dx\right)\csc^{3}\left(c+dx\right)\,dx+\int\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)\csc^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**3*csc(c + d*x)**3, x) + Integral(sin(c + d*x)*cos
(c + d*x)**3*csc(c + d*x)**3, x))
```

Giac [A]
time = 0.52, size = 46, normalized size = 0.85

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin\left(dx+c\right)+a}{\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a
)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 8.74, size = 146, normalized size = 2.70

$$\frac{a\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{10\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+2\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{a\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 +
 2*a*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*a*tan(c/2 + (d*x)
/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/2 +
 (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.357 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2915, 12, 45}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int (a-x)x^2\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(ax^2-x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\sin^3(c+dx)}{3ad} - \frac{\sin^4(c+dx)}{4ad}$$

Mathematica [A]
time = 0.11, size = 28, normalized size = 0.76

$$\frac{(4-3\sin(c+dx))\sin^3(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] ((4 - 3*Sin[c + d*x])*Sin[c + d*x]^3)/(12*a*d)
```

Maple [A]
time = 0.12, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}}{ad}$	30
default	$-\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}}{ad}$	30
risch	$\frac{\sin(dx+c)}{4ad}-\frac{\cos(4dx+4c)}{32ad}-\frac{\sin(3dx+3c)}{12ad}+\frac{\cos(2dx+2c)}{8ad}$	67
norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*(1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3)
```

Maxima [A]
time = 0.30, size = 29, normalized size = 0.78

$$-\frac{3\,\sin\left(dx+c\right)^4-4\,\sin\left(dx+c\right)^3}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

Fricas [A]
time = 0.34, size = 47, normalized size = 1.27

$$-\frac{3\,\cos\left(dx+c\right)^{4}-6\,\cos\left(dx+c\right)^{2}+4\,\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(26) = 52$.
time = 8.68, size = 277, normalized size = 7.49

$$\begin{cases} \frac{8\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{12\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{8\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} & \text{for}\: d \neq 0 \\ \frac{x\sin^{2}\left(c\right)\cos^{3}\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/
2 + d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3
*a*d) - 12*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2
+ d*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a
*d) + 8*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**8 + 12*a*d*tan(c/2 + d
*x/2)**6 + 18*a*d*tan(c/2 + d*x/2)**4 + 12*a*d*tan(c/2 + d*x/2)**2 + 3*a*d)
, Ne(d, 0)), (x*sin(c)**2*cos(c)**3/(a*sin(c) + a), True))
```

Giac [A]
time = 0.61, size = 29, normalized size = 0.78

$$-\frac{3\,\sin\left(dx+c\right)^{4}-4\,\sin\left(dx+c\right)^{3}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*sin(d*x + c)^4 - 4*sin(d*x + c)^3)/(a*d)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.70

$$-\frac{\sin\left(c+d\,x\right)^3\,\left(3\,\sin\left(c+d\,x\right)-4\right)}{12\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
[Out] -(sin(c + d*x)^3*(3*sin(c + d*x) - 4))/(12*a*d)
```

3.358 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] 1/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2914, 2644, 30}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int x\,dx, x, \sin(c+dx))}{ad} - \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{ad}$$
$$= \frac{\sin^2(c+dx)}{2ad} - \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.07, size = 28, normalized size = 0.76

$$\frac{(3-2\sin(c+dx))\sin^2(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] ((3 - 2*Sin[c + d*x])*Sin[c + d*x]^2)/(6*a*d)
```

Maple [A]
time = 0.09, size = 30, normalized size = 0.81

method	result	size
derivativedivides	$-\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}}{ad}$	30
default	$-\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}}{ad}$	30
risch	$-\frac{\sin(dx+c)}{4ad}+\frac{\sin(3dx+3c)}{12ad}-\frac{\cos(2dx+2c)}{4ad}$	50
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.78

$$-\frac{2\,\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.00

$$-\frac{3 \cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 1\right)\sin(dx+c)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(26) = 52$.
time = 4.82, size = 224, normalized size = 6.05

$$\begin{cases} \frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{8\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{6\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} & \text{for}\ d \neq 0 \\ \frac{x\sin(c)\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+a*sin(d*x+c)),x)
[Out] Piecewise((6*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2
 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*tan(c/2 + d*x/2)**3/(
3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x
/2)**2 + 3*a*d) + 6*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*
tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*sin
(c)*cos(c)**3/(a*sin(c) + a), True))
```

Giac [A]
time = 0.59, size = 29, normalized size = 0.78

$$-\frac{2 \sin(dx+c)^3 - 3 \sin(dx+c)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/6*(2*sin(d*x + c)^3 - 3*sin(d*x + c)^2)/(a*d)
```

Mupad [B]
time = 8.54, size = 26, normalized size = 0.70

$$-\frac{\sin(c+d\,x)^2\,\left(2\,\sin(c+d\,x)-3\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x))/(a + a*sin(c + d*x)),x)
[Out] -(sin(c + d*x)^2*(2*sin(c + d*x) - 3))/(6*a*d)
```

3.359 $\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2746}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\,dx, x, a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.75

$$-\frac{(-2+\sin(c+dx))\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + a*Sin[c + d*x]),x]
[Out] -1/2*((-2 + Sin[c + d*x])*Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$	28
default	$-\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{ad}$	28
risch	$\frac{\sin(dx+c)}{ad}+\frac{\cos(2dx+2c)}{4ad}$	32
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/a/d*(1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2-2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.78

$$\frac{\cos(dx+c)^2+2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(cos(d*x + c)^2 + 2*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(22) = 44$.

time = 2.49, size = 158, normalized size = 4.94

$$\begin{cases} \frac{2\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} - \frac{2\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x\cos^3(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*tan(c/2 + d*x/2)**3/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*tan(c/2 +
 d*x/2)**2 + a*d) - 2*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**4 + 2*a*d*
tan(c/2 + d*x/2)**2 + a*d) + 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**4 +
2*a*d*tan(c/2 + d*x/2)**2 + a*d), Ne(d, 0)), (x*cos(c)**3/(a*sin(c) + a), T
rue))
```

Giac [A]
time = 0.50, size = 25, normalized size = 0.78

$$-\frac{\sin(dx+c)^2 - 2\sin(dx+c)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(sin(d*x + c)^2 - 2*sin(d*x + c))/(a*d)
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.69

$$-\frac{\sin(c+d\,x)\,\left(\sin(c+d\,x)-2\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)*(sin(c + d*x) - 2))/(2*a*d)
```

3.360 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 45}

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{a-x}{x}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad}$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 0.79

$$\frac{\log(\sin(c+dx)) - \sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Log[Sin[c + d*x]] - Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.10, size = 28, normalized size = 0.97

method	result	size
derivativedivides	$\frac{-\ln(\csc(dx+c))-\frac{1}{\csc(dx+c)}}{ad}$	28
default	$\frac{-\ln(\csc(dx+c))-\frac{1}{\csc(dx+c)}}{ad}$	28
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}-\frac{\sin(dx+c)}{ad}$	52
norman	$\frac{\frac{2}{ad}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	136

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-ln(csc(d*x+c))-1/csc(d*x+c))
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.93

$$\frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/a)/d
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.86

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - sin(d*x + c))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 28, normalized size = 0.97

$$\frac{\frac{\log\left(|\sin\left(dx+c\right)|\right)}{a}-\frac{\sin\left(dx+c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/a)/d
```

Mupad [B]
time = 8.76, size = 71, normalized size = 2.45

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - (2*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 +
(d*x)/2)^2)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.361 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=30

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 45}

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{a-x}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{1}{x}\right)\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.73

$$-\frac{\csc(c+dx)+\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] -((Csc[c + d*x] + Log[Sin[c + d*x]])/(a*d))
```

Maple [A]
time = 0.10, size = 24, normalized size = 0.80

method	result	size
derivativedivides	$\frac{-\csc(dx+c)+\ln(\csc(dx+c))}{ad}$	24
default	$\frac{-\csc(dx+c)+\ln(\csc(dx+c))}{ad}$	24
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2i\,e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$	70
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-csc(d*x+c)+ln(csc(d*x+c)))
```

Maxima [A]
time = 0.30, size = 29, normalized size = 0.97

$$-\frac{\frac{\log(\sin(dx+c))}{a}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c))/a + 1/(a*sin(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 34, normalized size = 1.13

$$-\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+1}{ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(1/2*sin(d*x + c))*sin(d*x + c) + 1)/(a*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 30, normalized size = 1.00

$$-\frac{\frac{\log\left(|\sin\left(dx+c\right)|\right)}{a}+\frac{1}{a\sin\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c)))/a + 1/(a*sin(d*x + c)))/d
```

Mupad [B]
time = 8.70, size = 59, normalized size = 1.97

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)+\frac{1}{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(log(tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)/2 - log(tan(c/2 + (d*x)/2)^2
 + 1) + 1/(2*tan(c/2 + (d*x)/2)))/(a*d)
```

3.362 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d
```

Rubi [A]
time = 0.05, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2785, 2686, 30, 8}

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{ad} - \frac{\text{Subst}(\int x\,dx, x, \csc(c+dx))}{ad}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad}$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 0.75

$$-\frac{(-2+\csc(c+dx))\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((-2 + Csc[c + d*x])*Csc[c + d*x])/(a*d)
```

Maple [A]
time = 0.09, size = 25, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\csc(dx+c)-\frac{\left(\csc^2(dx+c)\right)}{2}}{ad}$	25
default	$\frac{\csc(dx+c)-\frac{\left(\csc^2(dx+c)\right)}{2}}{ad}$	25
risch	$\frac{2i\left(-i\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}$	56
norman	$\frac{-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/d*(csc(d*x+c)-1/2*csc(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.81

$$\frac{2\,\sin\left(dx+c\right)-1}{2\,ad\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Fricas [A]
time = 0.33, size = 30, normalized size = 0.94

$$-\frac{2\sin(dx+c)-1}{2\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 26, normalized size = 0.81

$$\frac{2\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Mupad [B]
time = 8.63, size = 23, normalized size = 0.72

$$\frac{\sin(c+d\,x)-\frac{1}{2}}{a\,d\sin(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x) - 1/2)/(a*d*sin(c + d*x)^2)
```

3.363 $\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 45}

$$\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a\text{Subst}\left(\int \frac{a-x}{x^4}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{1}{x^3}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2ad} - \frac{\csc^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.76

$$\frac{\csc^3(c+dx)(-2+3\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^3*(-2 + 3*Sin[c + d*x]))/(6*a*d)
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.78

method	result	size
derivativedivides	$\frac{-\frac{\left(\csc^3(dx+c)\right)}{3}+\frac{\left(\csc^2(dx+c)\right)}{2}}{ad}$	29
default	$\frac{-\frac{\left(\csc^3(dx+c)\right)}{3}+\frac{\left(\csc^2(dx+c)\right)}{2}}{ad}$	29
risch	$-\frac{2\left(-4i e^{3i(dx+c)}+3\,e^{4i(dx+c)}-3\,e^{2i(dx+c)}\right)}{3ad\left(e^{2i(dx+c)}-1\right)^3}$	57
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-1/3*csc(d*x+c)^3+1/2*csc(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.70

$$\frac{3\,\sin(dx+c)-2}{6\,ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

Fricas [A]
time = 0.34, size = 38, normalized size = 1.03

$$-\frac{3\sin(dx+c)-2}{6\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(3*sin(d*x + c) - 2)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.56, size = 26, normalized size = 0.70

$$\frac{3\sin(dx+c)-2}{6ad\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/6*(3*sin(d*x + c) - 2)/(a*d*sin(d*x + c)^3)
```

Mupad [B]
time = 8.76, size = 36, normalized size = 0.97

$$\frac{\frac{5\sin(c+d\,x)}{16}+\frac{\sin(3\,c+3\,d\,x)}{16}-\frac{1}{3}}{a\,d\sin(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
[Out] ((5*sin(c + d*x))/16 + sin(3*c + 3*d*x)/16 - 1/3)/(a*d*sin(c + d*x)^3)
```

3.364 $\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] 1/3*csc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2915, 12, 45}

$$\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{a-x}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3ad} - \frac{\csc^4(c+dx)}{4ad}$$

Mathematica [A]
time = 0.04, size = 28, normalized size = 0.76

$$\frac{\csc^4(c+dx)(-3+4\sin(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^3*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^4*(-3 + 4*Sin[c + d*x]))/(12*a*d)
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.78

method	result	si
derivativedivides	$\frac{-\frac{\left(\csc^4(dx+c)\right)}{4}+\frac{\left(\csc^3(dx+c)\right)}{3}}{ad}$	2
default	$\frac{-\frac{\left(\csc^4(dx+c)\right)}{4}+\frac{\left(\csc^3(dx+c)\right)}{3}}{ad}$	2
risch	$-\frac{4i\left(-3i e^{4i(dx+c)}+2\,e^{5i(dx+c)}-2\,e^{3i(dx+c)}\right)}{3ad\left(e^{2i(dx+c)}-1\right)^4}$	5
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}+\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/a/d*(-1/4*csc(d*x+c)^4+1/3*csc(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 26, normalized size = 0.70

$$\frac{4\,\sin(dx+c)-3}{12\,ad\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.11

$$\frac{4 \sin (dx+c)-3}{12 \left(ad \cos (dx+c)^4-2 ad \cos (dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [A]
time = 0.57, size = 26, normalized size = 0.70

$$\frac{4 \sin (dx+c)-3}{12 \, ad \sin (dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/12*(4*sin(d*x + c) - 3)/(a*d*sin(d*x + c)^4)
```

Mupad [B]
time = 8.64, size = 25, normalized size = 0.68

$$\frac{\frac{\sin(c+d\,x)}{3}-\frac{1}{4}}{a\,d\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
[Out] (sin(c + d*x)/3 - 1/4)/(a*d*sin(c + d*x)^4)
```

3.365 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{3ax}{128}-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^9(c+dx)}{9d}+\frac{3a\cos(c+dx)\sin(c+dx)}{128d}+\frac{a\cos^3(c+dx)\sin(c+d}{64d}$$

```
[Out] 3/128*a*x-1/5*a*cos(d*x+c)^5/d+2/7*a*cos(d*x+c)^7/d-1/9*a*cos(d*x+c)^9/d+3/
128*a*cos(d*x+c)*sin(d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d
*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 276}

$$-\frac{a\cos^9(c+dx)}{9d}+\frac{2a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}+\frac{3ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d*x]^7)/(7*d) - (a*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a\int \cos^4(c+dx)\sin$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\,\mathrm{s}$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+}{8d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} +$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+dx)}{9d} +$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{2a\cos^7(c+dx)}{7d} - \frac{a\cos^9(c+}{9d}$$

Mathematica [A]
time = 0.21, size = 84, normalized size = 0.59

$$\frac{a(7560c+7560dx-7560\cos(c+dx)-1680\cos(3(c+dx))+1008\cos(5(c+dx))+180\cos(7(c+dx))-140\cos(9(c+dx))-2520\sin(4(c+dx))+315\sin(8(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*c + 7560*d*x - 7560*Cos[c + d*x] - 1680*Cos[3*(c + d*x)] + 1008*Co
s[5*(c + d*x)] + 180*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] - 2520*Sin[4*(
c + d*x)] + 315*Sin[8*(c + d*x)]))/(322560*d)
```

Maple [A]
time = 0.28, size = 124, normalized size = 0.87

method	result
risch	$\frac{3ax}{128}-\frac{3a\cos(dx+c)}{128d}-\frac{a\cos(9dx+9c)}{2304d}+\frac{a\sin(8dx+8c)}{1024d}+\frac{a\cos(7dx+7c)}{1792d}+\frac{a\cos(5dx+5c)}{320d}-\frac{a\sin(4dx+4c)}{128d}-$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+a\left(-\frac{\left(\sin^4(dx+\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+a\left(-\frac{\left(\sin^4(dx+\right.}{}\right.}{d}$
norman	$-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{63ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{13a\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{3ax}{128}-\frac{16a}{315d}+\frac{32a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{64a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{155a(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a*(-1/9*sin(d*x+c
)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

Maxima [A]
time = 0.29, size = 71, normalized size = 0.50

$$-\frac{1024\,\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(dx+c\right)^{5}\right)a-315\,\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)a}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a - 315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.37, size = 95, normalized size = 0.66

$$-\frac{4480\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+8064\,a\cos\left(dx+c\right)^{5}-945\,adx-315\,\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\left(dx+c\right)^{5}+2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 8064*a*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(131) = 262$.
time = 1.50, size = 272, normalized size = 1.90

$$\begin{cases} \frac{3ax\sin^8(c+dx)}{128} + \frac{3ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{9ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3ax\cos^8(c+dx)}{128} + \frac{3a\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11a\sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{a\sin^4(c+dx)\cos^5(c+dx)}{5d} - \frac{11a\sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{4a\sin^2(c+dx)\cos^7(c+dx)}{35d} - \frac{3a\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{8a\cos^9(c+dx)}{315d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^4(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - a*sin(c + d*x)
**4*cos(c + d*x)**5/(5*d) - 11*a*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) -
4*a*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*a*sin(c + d*x)*cos(c + d*x)*
*7/(128*d) - 8*a*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(
c)**4*cos(c)**4, True))
```

Giac [A]
time = 0.66, size = 107, normalized size = 0.75

$$\frac{3}{128}\,ax - \frac{a\cos(9\,dx+9\,c)}{2304\,d} + \frac{a\cos(7\,dx+7\,c)}{1792\,d} + \frac{a\cos(5\,dx+5\,c)}{320\,d} - \frac{a\cos(3\,dx+3\,c)}{192\,d} - \frac{3\,a\cos(dx+c)}{128\,d} + \frac{a\sin(8\,dx+8\,c)}{1024\,d} - \frac{a\sin(4\,dx+4\,c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*a*cos(9*d*x + 9*c)/d + 1/1792*a*cos(7*d*x + 7*c)/d + 1/3
20*a*cos(5*d*x + 5*c)/d - 1/192*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 12.34, size = 353, normalized size = 2.47

$$\frac{3\,a\,x}{128} + \frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64} + \frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32} - \frac{155\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32} + \left(\frac{a\,(79380\,c+79380\,d\,x-430080)}{40320} - \frac{63\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + \frac{169\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32} + \left(\frac{a\,(119070\,c+119070\,d\,x+645120)}{40320} - \frac{189\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \left(\frac{a\,(119070\,c+119070\,d\,x-903168)}{40320} - \frac{189\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - \frac{169\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32} + \left(\frac{a\,(79380\,c+79380\,d\,x+258048)}{40320} - \frac{63\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + \frac{155\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{32} + \left(\frac{a\,(34020\,c+34020\,d\,x-73728)}{40320} - \frac{27\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - \frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32} + \left(\frac{a\,(8505\,c+8505\,d\,x-18432)}{40320} - \frac{27\,a\,(c+d\,x)}{128}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} - \frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64} + \frac{a\,(945\,c+945\,d\,x-2048)}{40320} - \frac{3\,a\,(c+d\,x)}{128}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 + ((a*(945*c + 945*d*x - 2048))/40320 - (3*a*tan(c/2 + (d*x)/2)
)/64 - (3*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(8505*c + 8505*d*x -
18432))/40320 - (27*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(34020*c +
 34020*d*x - 73728))/40320 - (27*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^6*((
```

```
a*(79380*c + 79380*d*x + 258048))/40320 - (63*a*(c + d*x))/32) + tan(c/2 +
(d*x)/2)^12*((a*(79380*c + 79380*d*x - 430080))/40320 - (63*a*(c + d*x))/32
) + tan(c/2 + (d*x)/2)^10*((a*(119070*c + 119070*d*x + 645120))/40320 - (18
9*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^8*((a*(119070*c + 119070*d*x - 9031
68))/40320 - (189*a*(c + d*x))/64) - (13*a*tan(c/2 + (d*x)/2)^3)/32 + (155*
a*tan(c/2 + (d*x)/2)^5)/32 - (169*a*tan(c/2 + (d*x)/2)^7)/32 + (169*a*tan(c
/2 + (d*x)/2)^11)/32 - (155*a*tan(c/2 + (d*x)/2)^13)/32 + (13*a*tan(c/2 + (
d*x)/2)^15)/32 + (3*a*tan(c/2 + (d*x)/2)^17)/64)/(d*(tan(c/2 + (d*x)/2)^2 +
 1)^9)
```

3.366 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{3ax}{128}-\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^7(c+dx)}{7d}+\frac{3a\cos(c+dx)\sin(c+dx)}{128d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{a\cos^5(c+d}{1}$$

```
[Out] 3/128*a*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+3/128*a*cos(d*x+c)*sin(
d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d*x+c)^5*sin(d*x+c)/d-
1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2645, 14, 2648, 2715, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}+\frac{3ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a\int \cos^4(c+dx)\sin^4$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+d}{8d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^3(c+dx)\sin(c}{64d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)\sin(c}{128d}$$

$$= \frac{3ax}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3a\cos(c+dx)}{128}$$

Mathematica [A]
time = 0.16, size = 71, normalized size = 0.56

$$\frac{a(840dx - 1680\cos(c+dx) - 560\cos(3(c+dx)) + 112\cos(5(c+dx)) + 80\cos(7(c+dx)) - 280\sin(4(c+dx)) + 35\sin(8(c+dx)))}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

[Out] (a*(840*d*x - 1680*Cos[c + d*x] - 560*Cos[3*(c + d*x)] + 112*Cos[5*(c + d*x)] + 80*Cos[7*(c + d*x)] - 280*Sin[4*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35840*d)

Maple [A]
time = 0.21, size = 106, normalized size = 0.83

method	result
risch	$\frac{3ax}{128} - \frac{3a\cos(dx+c)}{64d} + \frac{a\sin(8dx+8c)}{1024d} + \frac{a\cos(7dx+7c)}{448d} + \frac{a\cos(5dx+5c)}{320d} - \frac{a\sin(4dx+4c)}{128d} - \frac{a\cos(3dx+3c)}{64d}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7} - \frac{2\left(\cos^5(dx+c)\right)}{35}\right) + a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8} - \frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16} + \frac{\left(\cos^3(dx+c)\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7} - \frac{2\left(\cos^5(dx+c)\right)}{35}\right) + a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8} - \frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16} + \frac{\left(\cos^3(dx+c)\right.}{}\right.}{d}$
norman	$-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d} - \frac{23a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d} + \frac{333a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d} + \frac{3ax}{128} - \frac{4a}{35d} - \frac{333a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d} + \frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16} + \frac{21ax}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))

Maxima [A]
time = 0.29, size = 61, normalized size = 0.48

$$\frac{1024\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a+35\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\sin\left(4\,dx+4\,c\right)\right)a}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a)/d

Fricas [A]
time = 0.37, size = 84, normalized size = 0.66

$$\frac{640\,a\cos\left(dx+c\right)^{7}-896\,a\cos\left(dx+c\right)^{5}+105\,adx+35\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\left(dx+c\right)^{5}+2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*a*d*x + 35*(16*a*
cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(116) = 232$.
time = 0.91, size = 248, normalized size = 1.95

$$\begin{cases} \frac{3ax\sin^{8}(c+dx)}{128} + \frac{3ax\sin^{6}(c+dx)\cos^{2}(c+dx)}{32} + \frac{9ax\sin^{4}(c+dx)\cos^{4}(c+dx)}{64} + \frac{3ax\sin^{2}(c+dx)\cos^{6}(c+dx)}{32} + \frac{3ax\cos^{8}(c+dx)}{128} + \frac{3a\sin^{7}(c+dx)\cos(c+dx)}{128d} + \frac{11a\sin^{5}(c+dx)\cos^{3}(c+dx)}{128d} - \frac{11a\sin^{3}(c+dx)\cos^{5}(c+dx)}{128d} - \frac{a\sin^{2}(c+dx)\cos^{5}(c+dx)}{5d} - \frac{3a\sin(c+dx)\cos^{7}(c+dx)}{128d} - \frac{2a\cos^{7}(c+dx)}{35d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^{3}(c)\cos^{4}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d
, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**4, True))
```

Giac [A]
time = 0.55, size = 92, normalized size = 0.72

$$\frac{3}{128}\,ax + \frac{a\cos(7\,dx+7\,c)}{448\,d} + \frac{a\cos(5\,dx+5\,c)}{320\,d} - \frac{a\cos(3\,dx+3\,c)}{64\,d} - \frac{3\,a\cos(dx+c)}{64\,d} + \frac{a\sin(8\,dx+8\,c)}{1024\,d} - \frac{a\sin(4\,dx+4\,c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*a*sin(8*d*x + 8*c)/d
- 1/128*a*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 12.48, size = 320, normalized size = 2.52

$$\frac{3\,a\,x}{128} + \frac{\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} + \frac{23\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64} + \left(\frac{a\,(2940\,c+2940\,d\,x-17920)}{4480} - \frac{21\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - \frac{333\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64} + \frac{671\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64} + \left(\frac{a\,(7350\,c+7350\,d\,x-17920)}{4480} - \frac{105\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - \frac{671\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{64} + \left(\frac{a\,(5880\,c+5880\,d\,x-28672)}{4480} - \frac{21\,a\,(c+d\,x)}{16}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + \frac{333\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{64} + \left(\frac{a\,(2940\,c+2940\,d\,x+3584)}{4480} - \frac{21\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - \frac{23\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{64} + \left(\frac{a\,(840\,c+840\,d\,x-4096)}{4480} - \frac{3\,a\,(c+d\,x)}{16}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} - \frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64} + \frac{a\,(105\,c+105\,d\,x-512)}{4480} - \frac{3\,a\,(c+d\,x)}{128}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 + ((a*(105*c + 105*d*x - 512))/4480 - (3*a*tan(c/2 + (d*x)/2))/
64 - (3*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(840*c + 840*d*x - 4096
))/4480 - (3*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2940*c + 2940*d*x
 + 3584))/4480 - (21*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^12*((a*(2940*c +
 2940*d*x - 17920))/4480 - (21*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^8*((a*
```

```
(7350*c + 7350*d*x - 17920))/4480 - (105*a*(c + d*x))/64) + tan(c/2 + (d*x)
/2)^6*((a*(5880*c + 5880*d*x - 28672))/4480 - (21*a*(c + d*x))/16) - (23*a*
tan(c/2 + (d*x)/2)^3)/64 + (333*a*tan(c/2 + (d*x)/2)^5)/64 - (671*a*tan(c/2
 + (d*x)/2)^7)/64 + (671*a*tan(c/2 + (d*x)/2)^9)/64 - (333*a*tan(c/2 + (d*x
)/2)^11)/64 + (23*a*tan(c/2 + (d*x)/2)^13)/64 + (3*a*tan(c/2 + (d*x)/2)^15)
/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.367 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{ax}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*a*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d+1/24*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 14}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a\int \cos^4(c+dx)\sin \\
&= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \\
&= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} \\
&= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)\sin(}{16d} \\
&= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos(c+dx)}{16d}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 81, normalized size = 0.79

$$\frac{a(420dx - 315\cos(c+dx) - 105\cos(3(c+dx)) + 21\cos(5(c+dx)) + 15\cos(7(c+dx)) + 105\sin(2(c+dx)) - 105\sin(4(c+dx)) - 35\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] (a*(420*d*x - 315*Cos[c + d*x] - 105*Cos[3*(c + d*x)] + 21*Cos[5*(c + d*x)]
 + 15*Cos[7*(c + d*x)] + 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] - 35*S
in[6*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.15, size = 88, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\Big(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2(\cos^5(dx+}{35}}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a\Big(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2(\cos^5(dx+}{35}}{d}$
risch	$\frac{ax}{16}-\frac{3a\cos(dx+c)}{64d}+\frac{a\cos(7dx+7c)}{448d}-\frac{a\sin(6dx+6c)}{192d}+\frac{a\cos(5dx+5c)}{320d}-\frac{a\sin(4dx+4c)}{64d}-\frac{a\cos(3dx+3c)}{64d}+$
norman	$\frac{4a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax}{16}-\frac{4a}{35d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{31a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{31a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{11a(\tan^{11}(\frac{dx}{2}}{6d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)+a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^
5))
```

Maxima [A]
time = 0.29, size = 65, normalized size = 0.63

$$\frac{192\left(5\cos\left(dx+c\right)^{7}-7\cos\left(dx+c\right)^{5}\right)a+35\left(4\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\sin\left(4\,dx+4\,c\right)\right)a}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*(192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(4*sin(2*d*x + 2*c
)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.37, size = 73, normalized size = 0.71

$$\frac{240\,a\cos\left(dx+c\right)^{7}-336\,a\cos\left(dx+c\right)^{5}+105\,adx-35\left(8\,a\cos\left(dx+c\right)^{5}-2\,a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*cos(d*x + c)^7 - 336*a*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.
time = 0.62, size = 192, normalized size = 1.86

$$\begin{cases}\frac{ax\sin^6(c+dx)}{16}+\frac{3ax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3ax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{ax\cos^6(c+dx)}{16}+\frac{a\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{2a\cos^7(c+dx)}{35d} & \text{for } d\neq 0\\ x(a\sin(c)+a)\sin^2(c)\cos^4(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a*sin(c + d*x)*cos(c + d*x)**
5/(16*d) - 2*a*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*
*2*cos(c)**4, True))
```

Giac [A]
time = 0.63, size = 107, normalized size = 1.04

$$\frac{1}{16}\,ax+\frac{a\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{a\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,a\cos\left(dx+c\right)}{64\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*a
*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.27, size = 292, normalized size = 2.83

$$\frac{a\,x}{16}+\frac{\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+\left(\frac{a\,\left(2205\,c+2205\,d\,x-6720\right)}{1680}-\frac{21\,a\,\left(c+d\,x\right)}{16}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\left(\frac{a\,\left(3675\,c+3675\,d\,x+6720\right)}{1680}-\frac{35\,a\,\left(c+d\,x\right)}{16}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+\left(\frac{a\,\left(3675\,c+3675\,d\,x-13440\right)}{1680}-\frac{35\,a\,\left(c+d\,x\right)}{16}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}-\frac{31\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{24}+\left(\frac{a\,\left(2205\,c+2205\,d\,x+2688\right)}{1680}-\frac{21\,a\,\left(c+d\,x\right)}{16}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6}+\left(\frac{a\,\left(735\,c+735\,d\,x-1344\right)}{1680}-\frac{7\,a\,\left(c+d\,x\right)}{16}\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{a\,\left(105\,c+105\,d\,x-192\right)}{1680}-\frac{a\,\left(c+d\,x\right)}{16}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 + ((a*(105*c + 105*d*x - 192))/1680 - (a*tan(c/2 + (d*x)/2))/8 - (
a*(c + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(735*c + 735*d*x - 1344))/1680 -
 (7*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(2205*c + 2205*d*x + 2688))
/1680 - (21*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^10*((a*(2205*c + 2205*d*x
 - 6720))/1680 - (21*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^8*((a*(3675*c +
3675*d*x + 6720))/1680 - (35*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^6*((a*(3
675*c + 3675*d*x - 13440))/1680 - (35*a*(c + d*x))/16) + (11*a*tan(c/2 + (d
*x)/2)^3)/6 - (31*a*tan(c/2 + (d*x)/2)^5)/24 + (31*a*tan(c/2 + (d*x)/2)^9)/
24 - (11*a*tan(c/2 + (d*x)/2)^11)/6 + (a*tan(c/2 + (d*x)/2)^13)/8)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^7)
```

3.368 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{ax}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*a*x-1/5*a*cos(d*x+c)^5/d+1/16*a*cos(d*x+c)*sin(d*x+c)/d+1/24*a*cos(d*x
+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2645, 30, 2648, 2715, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin(c+dx)\,dx + a\int \cos^4(c+dx)\sin^2 \\
&= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}a\int \cos^4(c+dx)\,dx - \\
&= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a\cos^5(\\
&= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos^3(c\\
&= \frac{ax}{16} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos(c+dx)\sin(c+dx)}{16d} + \frac{a\cos
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 71, normalized size = 0.82

$$
-\frac{a(-60dx+120\cos(c+dx)+60\cos(3(c+dx))+12\cos(5(c+dx))-15\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx)))}{960d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(a*(-60*d*x + 120*Cos[c + d*x] + 60*Cos[3*(c + d*x)] + 12*Cos[5*(c +
 d*x)] - 15*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/d
```

Maple [A]
time = 0.12, size = 68, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)}{d}$
risch	$\frac{ax}{16}-\frac{a\cos(dx+c)}{8d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\cos(5dx+5c)}{80d}-\frac{a\sin(4dx+4c)}{64d}-\frac{a\cos(3dx+3c)}{16d}+\frac{a\sin(2dx+2c)}{64d}$
norman	$\frac{ax}{16}-\frac{2a}{5d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{47a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{13a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{13a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{47a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.60

$$-\frac{192\,a\cos\left(dx+c\right)^5-5\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^5-15\,adx+5\left(8\,a\cos\left(dx+c\right)^5-2\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*a*d*x + 5*(8*a*cos(d*x + c)^5 - 2*a*cos(d*
x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(76) = 152.
time = 0.42, size = 167, normalized size = 1.92

$$\begin{cases} \frac{ax\sin^6(c+dx)}{16}+\frac{3ax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3ax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{ax\cos^6(c+dx)}{16}+\frac{a\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{a\cos^5(c+dx)}{5d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)\sin(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - a*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**4, True))
```

Giac [A]
time = 0.59, size = 92, normalized size = 1.06

$$\frac{1}{16}\,ax-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/64*a*sin(4*d*x + 4*c)/d + 1/6
4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.22, size = 292, normalized size = 3.36

$$\frac{a\,x}{16}+\frac{\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\left(\frac{a\,\left(90\,c+90\,d\,x-480\right)}{240}-\frac{3\,a\,\left(c+d\,x\right)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{47\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\left(\frac{a\,\left(225\,c+225\,d\,x-480\right)}{240}-\frac{15\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\left(\frac{a\,\left(300\,c+300\,d\,x-960\right)}{240}-\frac{5\,a\,\left(c+d\,x\right)}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\left(\frac{a\,\left(225\,c+225\,d\,x-960\right)}{240}-\frac{15\,a\,\left(c+d\,x\right)}{16}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{47\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\left(\frac{a\,\left(90\,c+90\,d\,x-96\right)}{240}-\frac{3\,a\,\left(c+d\,x\right)}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{a\,\left(15\,c+15\,d\,x-96\right)}{240}-\frac{a\,\left(c+d\,x\right)}{16}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 + ((a*(15*c + 15*d*x - 96))/240 - (a*tan(c/2 + (d*x)/2))/8 - (a*(c
 + d*x))/16 + tan(c/2 + (d*x)/2)^2*((a*(90*c + 90*d*x - 96))/240 - (3*a*(c
+ d*x))/8) + tan(c/2 + (d*x)/2)^10*((a*(90*c + 90*d*x - 480))/240 - (3*a*(c
 + d*x))/8) + tan(c/2 + (d*x)/2)^8*((a*(225*c + 225*d*x - 480))/240 - (15*a
*(c + d*x))/16) + tan(c/2 + (d*x)/2)^4*((a*(225*c + 225*d*x - 960))/240 - (
15*a*(c + d*x))/16) + tan(c/2 + (d*x)/2)^6*((a*(300*c + 300*d*x - 960))/240
 - (5*a*(c + d*x))/4) + (47*a*tan(c/2 + (d*x)/2)^3)/24 - (13*a*tan(c/2 + (d
*x)/2)^5)/4 + (13*a*tan(c/2 + (d*x)/2)^7)/4 - (47*a*tan(c/2 + (d*x)/2)^9)/2
4 + (a*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.369 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{3ax}{8}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(}{4d}$$

```
[Out] 3/8*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+3/8*a*c
os(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2672, 308, 212, 2715, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\,dx + a\int \cos^3(c+dx)\cot(c+dx)\,dx$$

$$= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3a)\int \cos^2(c+dx)\,dx -$$

$$= \frac{3a\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

$$= \frac{3ax}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3a\cos(c+dx)}{8d}$$

$$= \frac{3ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3}{}$$

Mathematica [A]
time = 0.12, size = 81, normalized size = 0.91

$$\frac{a\left(120\cos(c+dx)+8\cos(3(c+dx))+3\left(12c+12dx-32\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8\sin(2(c+dx))+\sin(4(c+dx))\right)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(12*c + 12*d*x - 32*Log[Cos[(
c + d*x)/2]] + 32*Log[Sin[(c + d*x)/2]] + 8*Sin[2*(c + d*x)] + Sin[4*(c + d
*x)])))/(96*d)
```

Maple [A]
time = 0.12, size = 76, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{5a\,e^{i(dx+c)}}{8d}+\frac{5a\,e^{-i(dx+c)}}{8d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\sin(4dx+4c)}{32d}+\frac{a\cos(3dx+3c)}{12d}+$
norman	$\frac{\frac{3ax}{8}+\frac{8a}{3d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a*(1/4*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.30, size = 81, normalized size = 0.91

$$\frac{16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a+3\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)a}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

Fricas [A]
time = 0.39, size = 88, normalized size = 0.99

$$\frac{8\,a\cos(dx+c)^3+9\,adx+24\,a\cos(dx+c)-12\,a\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(2\,a\cos(dx+c)^3+3\,a\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*a*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d
*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*a*cos(d*x + c)^3
+ 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^4(c+dx)\csc(c+dx)\,dx+\int\sin(c+dx)\cos^4(c+dx)\csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**4*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**4*csc(c + d*x), x))
```

Giac [A]
time = 0.65, size = 145, normalized size = 1.63

$$\frac{9\,(dx+c)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-32\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*a*tan(1/2
*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*a*tan(1/2*d*x + 1/2*c)^5
- 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*a*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d
*x + 1/2*c)^2 - 15*a*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

Mupad [B]
time = 10.19, size = 245, normalized size = 2.75

$$\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}+\frac{20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{8\,a}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{3\,a\,\mathrm{atan}\left(\frac{9\,a^2}{16\left(\frac{3\,a^2}{2}-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{3\,a^2}{2}-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] ((8*a)/3 + (5*a*tan(c/2 + (d*x)/2))/4 + (20*a*tan(c/2 + (d*x)/2)^2)/3 - (3*
a*tan(c/2 + (d*x)/2)^3)/4 + 8*a*tan(c/2 + (d*x)/2)^4 + (3*a*tan(c/2 + (d*x)
/2)^5)/4 + 4*a*tan(c/2 + (d*x)/2)^6 - (5*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(4*t
an(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan
(c/2 + (d*x)/2)^8 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)
/(16*((3*a^2)/2 - (9*a^2*tan(c/2 + (d*x)/2))/16)) + (3*a^2*tan(c/2 + (d*x)/
2))/(2*((3*a^2)/2 - (9*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

3.370 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{3ax}{2}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}$$

```
[Out] -3/2*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3/2*a*
cot(d*x+c)/d+1/2*a*cos(d*x+c)^2*cot(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2917, 2671, 294, 327, 209, 2672, 308, 212}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + a\int \cos^2(c+dx)\cot^2$$
$$= -\frac{a\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x^4}{(1+x^2)}\right.}{}$$
$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right.\right.}{d}$$
$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos}{}$$
$$= -\frac{3ax}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos}{}$$

Mathematica [A]
time = 0.32, size = 77, normalized size = 0.93

$$\frac{a\left(15\cos(c+dx)+\cos(3(c+dx))-3\left(6c+6dx+4\cot(c+dx)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin(2(c+dx))\right)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(15*Cos[c + d*x] + Cos[3*(c + d*x)] - 3*(6*c + 6*d*x + 4*Cot[c + d*x] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12
*d)
```

Maple [A]
time = 0.13, size = 94, normalized size = 1.13

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx\right.}{d}$
risch	$-\frac{3ax}{2}+\frac{ia\,e^{2i(dx+c)}}{8d}+\frac{5a\,e^{i(dx+c)}}{8d}+\frac{5a\,e^{-i(dx+c)}}{8d}-\frac{ia\,e^{-2i(dx+c)}}{8d}-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+$
norman	$\frac{\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{2d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{9a}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
-3/2*d*x-3/2*c)+a*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 90, normalized size = 1.08

$$\frac{\left(2\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a-3\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(c
os(d*x + c) - 1))*a - 3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)
^3 + tan(d*x + c)))*a)/d
```

Fricas [A]
time = 0.39, size = 107, normalized size = 1.29

$$\frac{3\,a\cos\left(dx+c\right)^{3}-3\,a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a\cos\left(dx+c\right)+\left(2\,a\cos\left(dx+c\right)^{3}-9\,adx+6\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*a*cos(d
*x + c)^3 - 9*a*d*x + 6*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{4}\left(c+dx\right)\csc^{2}\left(c+dx\right)\,dx+\int\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)\csc^{2}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**4*csc(c + d*x)**2, x) + Integral(sin(c + d*x)*cos
(c + d*x)**4*csc(c + d*x)**2, x))
```

Giac [A]
time = 0.54, size = 142, normalized size = 1.71

$$-\frac{9\left(dx+c\right)a-6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*a*tan(1/2*d*x + 1/2*c)^4 + 12*a*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 8.77, size = 244, normalized size = 2.94

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{16\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}-a}{d\,\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{3\,a\,\mathrm{atan}\left(\frac{9\,a^2}{6\,a^2+9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6\,a^2+9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((16*a*tan(c/2 + (d*x)/2))/3 - a - 5*a*tan(c/2 + (d*x)/2)^2 + 8*a*tan(c/2 +
 (d*x)/2)^3 - 3*a*tan(c/2 + (d*x)/2)^4 + 8*a*tan(c/2 + (d*x)/2)^5 + a*tan(c
/2 + (d*x)/2)^6)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(
c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (a*tan(c/2 + (d*x)/2))/(2*d)
+ (a*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)/(6*a^2 + 9*a^2*tan(c/2
+ (d*x)/2)) - (6*a^2*tan(c/2 + (d*x)/2))/(6*a^2 + 9*a^2*tan(c/2 + (d*x)/2))
))/d
```

3.371 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3ax}{2}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)}{2}$$

[Out] -3/2*a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d-3/2*a*cot(d*x+c)/d+1/2*a*cos(d*x+c)^2*cot(d*x+c)/d-1/2*a*cos(d*x+c)*cot(d*x+c)^2/d

Rubi [A]
time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2917, 2672, 294, 327, 212, 2671, 209}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + a\int \cos(c+dx)\cot^3($$

$$= -\frac{a\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x^4}{(1+x^{}}{}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d}$$

$$= -\frac{3a\cos(c+dx)}{2d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot(c}{2d}$$

$$= -\frac{3ax}{2} + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} - \frac{3a}{}$$

Mathematica [A]

time = 0.56, size = 94, normalized size = 1.00

$$-\frac{a\left(12c+12dx+8\cos(c+dx)+8\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)+2\sin(2(c+dx))\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*(12*c + 12*d*x + 8*Cos[c + d*x] + 8*Cot[c + d*x] + Csc[(c + d*x)/2]
^2 - 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]
^2 + 2*Sin[2*(c + d*x)]))/d
```

Maple [A]
time = 0.14, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right.\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right.\right.}{d}$
risch	$-\frac{3ax}{2}+\frac{ia\,e^{2i(dx+c)}}{8d}-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}-\frac{ia\,e^{-2i(dx+c)}}{8d}+\frac{a\left(e^{3i(dx+c)}+e^{i(dx+c)}-2ie^{2i(dx+c)}+2i\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+$
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-3}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*
ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.49, size = 101, normalized size = 1.07

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 139, normalized size = 1.48

$$-\frac{6\,adx\cos\left(dx+c\right)^{2}+4\,a\cos\left(dx+c\right)^{3}-6\,adx-6\,a\cos\left(dx+c\right)-3\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\left(a\cos\left(dx+c\right)^{3}-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*a*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 163, normalized size = 1.73

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,(dx+c)a-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*a - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*a*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*a*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*a*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

Mupad [B]
time = 8.71, size = 239, normalized size = 2.54

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{3\,a\,\mathrm{atan}\left(\frac{9\,a^2}{9\,a^2-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*a*tan(c/2 + (d*x)/2) + 9*a*tan(c/2
+ (d*x)/2)^2 + 8*a*tan(c/2 + (d*x)/2)^3 + (17*a*tan(c/2 + (d*x)/2)^4)/2 - 2
*a*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^
4 + 4*tan(c/2 + (d*x)/2)^6)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a*log(ta
n(c/2 + (d*x)/2)))/(2*d) - (3*a*atan((9*a^2)/(9*a^2 - 9*a^2*tan(c/2 + (d*x)
/2)) + (9*a^2*tan(c/2 + (d*x)/2))/(9*a^2 - 9*a^2*tan(c/2 + (d*x)/2))))/d
```

3.372 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$ax+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

```
[Out] a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*a*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2789, 2672, 294, 327, 212, 3554, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx\\ &= a\int \cos(c+dx)\cot^3(c+dx)\,dx+a\int \cot^4(c+dx)\,dx\\ &= -\frac{a\cot^3(c+dx)}{3d}-a\int \cot^2(c+dx)\,dx-\frac{a\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\text{c}\right.}{d}\\ &= \frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+a\int\\ &= ax-\frac{3a\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}\\ &= ax+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 125, normalized size = 1.52

$$-\frac{a\cos(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A]

time = 0.13, size = 86, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$ax-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}+\frac{a\left(12i e^{4i(dx+c)}+3\,e^{5i(dx+c)}-12i e^{2i(dx+c)}+8i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{2d}$
norman	$\frac{ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.50, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.
time = 0.36, size = 160, normalized size = 1.95

$$\frac{16\,a\cos\left(dx+c\right)^{3}+9\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-12\,a\cos\left(dx+c\right)+6\left(2\,adx\cos\left(dx+c\right)^{2}-2\,a\cos\left(dx+c\right)^{3}-2\,adx+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.67, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\frac{66\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.70, size = 228, normalized size = 2.78

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - (14*a*tan(c/
2 + (d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 - 5*a*tan(c/2 + (d*x)/2)^4)/(
d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x)
/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a^2*tan
(c/2 + (d*x)/2))/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.373 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$ax-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot^3(c+dx)\csc}{4d}$$

```
[Out] a*x-3/8*a*arctanh(cos(d*x+c))/d+a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d+3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2917, 2691, 3855, 3554, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (a*Cot[c + d*x])/d - (a*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\,dx + a\int \cot^4(c+dx)\csc(c+dx)\,dx\\ &= -\frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int\\ &= \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+}{8d}\\ &= ax - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c}{3d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 153, normalized size = 1.74

$$\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.16, size = 104, normalized size = 1.18

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+\right.}{d}$

risch	$ax - \frac{a(15\,\mathrm{e}^{7i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}-48i\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}+96i\mathrm{e}^{4i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}-80i\mathrm{e}^{2i(dx+c)}+32i)}{12d(\mathrm{e}^{2i(dx+c)}-1)^4} + \frac{3a\ln(\ldots}{}$
norman	$\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{7a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}\ldots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*co
s(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/3*cot(d*x+c)
^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 107, normalized size = 1.22

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a-3\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(80) = 160.
time = 0.36, size = 180, normalized size = 2.05

$$\frac{48\,adx\cos\left(dx+c\right)^4-96\,adx\cos\left(dx+c\right)^2-30\,a\cos\left(dx+c\right)^3+48\,adx+18\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-16\left(4\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*a*d*x*cos(d*x + c)^4 - 96*a*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*a*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*a*cos(d*x + c)^3 - 3*a*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.66, size = 153, normalized size = 1.74

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+192\,(dx+c)a+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*a + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*a*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*a*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.92, size = 217, normalized size = 2.47

$$\frac{2\,a\,\mathrm{atan}\left(\frac{8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}+\frac{5\,a\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{5\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{a\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}-\frac{a\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}-\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (2*a*atan((8*cos(c/2 + (d*x)/2) + 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/
2) - 8*sin(c/2 + (d*x)/2))))/d + (3*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/(8*d) + (5*a*cot(c/2 + (d*x)/2))/(8*d) - (5*a*tan(c/2 + (d*x)/2))/(8
*d) + (a*cot(c/2 + (d*x)/2)^2)/(8*d) - (a*cot(c/2 + (d*x)/2)^3)/(24*d) - (a
*cot(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2
 + (d*x)/2)^3)/(24*d) + (a*tan(c/2 + (d*x)/2)^4)/(64*d)
```

3.374 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d+3/8*a*cot(d*x+c)*csc(d*x+
c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2917, 2687, 30, 2691, 3855}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*a*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + a\int \cot^4(c+dx)\csc^2$$
$$= -\frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)\int \cot^2(c+dx)\csc$$
$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot^3(}{}$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3a\cot(c+}{}$$

Mathematica [A]

time = 0.03, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d + (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d
*x)/2]^4)/(64*d) - (3*a*Log[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(6
4*d)
```

Maple [A]

time = 0.16, size = 100, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$

risch	$-\frac{a\left(40i\,e^{8i(dx+c)}+25\,e^{9i(dx+c)}-10\,e^{7i(dx+c)}+80i\,e^{4i(dx+c)}+10\,e^{3i(dx+c)}+8i-25\,e^{i(dx+c)}\right)}{20d\left(e^{2i(dx+c)}-1\right)^{5}}-\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{8d}+3$
norman	$\frac{-\frac{a}{160d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{7a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{7a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a/sin(d*x+c)^5*cos(d*x+c)^5+a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8
/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c
)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.16

$$-\frac{5\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.
time = 0.36, size = 160, normalized size = 2.16

$$-\frac{16\,a\cos\left(dx+c\right)^5+15\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10\,\left(5\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a*cos(d*x + c)^4 - 2*a*cos(
d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*a*cos(d*x
 + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x
+ c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(66) = 132.

time = 0.64, size = 173, normalized size = 2.34

$$\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{274\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-40\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*a*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1
/2*d*x + 1/2*c)^3 - 40*a*tan(1/2*d*x + 1/2*c)^2 + 120*a*log(abs(tan(1/2*d*x
 + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*a*tan(1/2*d*x + 1/2*c)^5 + 2
0*a*tan(1/2*d*x + 1/2*c)^4 - 40*a*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x
 + 1/2*c)^2 + 5*a*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]

time = 10.43, size = 289, normalized size = 3.91

$$\frac{a\left(2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+20\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-20\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+10\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\right)}{320\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*(2*sin(c/2 + (d*x)/2)^10 - 2*cos(c/2 + (d*x)/2)^10 + 5*cos(c/2 + (d*x)/2
)*sin(c/2 + (d*x)/2)^9 - 5*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 10*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 40*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^7 + 20*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 20*cos(c/2 + (d
*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 40*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^
3 + 10*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5))/(320*d*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.375 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot^3(c+$$

```
[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/16*a*cot(d*x+c)*csc(d*
x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2917, 2691, 3853, 3855, 2687, 30}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/16*(a*ArcTanh[Cos[c + d*x]])/d - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a\int \cot^4(c+dx)\csc$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3($$

$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(}{}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a\cot(c-}{}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot(c+d}{}$$

Mathematica [A]
time = 0.03, size = 175, normalized size = 1.79

$$-\frac{a\cot^5(c+dx)}{5d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.20, size = 118, normalized size = 1.20

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{a\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{a\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}}{d}$
risch	$\frac{a(15\,e^{11i(dx+c)}+235\,e^{9i(dx+c)}-240ie^{10i(dx+c)}+390\,e^{7i(dx+c)}+240ie^{8i(dx+c)}+390\,e^{5i(dx+c)}-480ie^{6i(dx+c)}+235\,e^{3i(dx}}{120d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{a}{384d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{a(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x
+c)-cot(d*x+c)))-1/5*a/sin(d*x+c)^5*cos(d*x+c)^5)
```

Maxima [A]
time = 0.29, size = 106, normalized size = 1.08

$$\frac{5\,a\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,a}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d
*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
 1) + 3*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(88) = 176.
time = 0.36, size = 187, normalized size = 1.91

$$\frac{96\,a\cos\left(dx+c\right)^5\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)^5+80\,a\cos\left(dx+c\right)^3-30\,a\cos\left(dx+c\right)-15\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(96*a*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*
x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 +
3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6
- 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(88) = 176.
time = 0.64, size = 201, normalized size = 2.05

$$\frac{5a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+12a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-60a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+120a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+120a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{294a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+120a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-60a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{1920d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan
(1/2*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*
c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a*tan(1/2*d*x + 1/2*c) -
(294*a*tan(1/2*d*x + 1/2*c)^6 + 120*a*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2
*d*x + 1/2*c)^4 - 60*a*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2
 + 12*a*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.51, size = 337, normalized size = 3.44

$$\frac{a\left(5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-60\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+60\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6\right)}{1920d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (a*(5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 15
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 60*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 120*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^5 + 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 60*cos(c/2 + (d*x
)/2)^9*sin(c/2 + (d*x)/2)^3 + 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2
 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(
c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.376 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}$$

```
[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-1/1
6*a*cot(d*x+c)*csc(d*x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c
)^3*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2687, 14, 2691, 3853, 3855}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/16*(a*ArcTanh[Cos[c + d*x]])/d - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a\int \cot^4(c+dx)\csc$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\csc^3($$

$$= \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot(c+dx)\csc(c}{16d}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+}{7d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(114) = 228.
time = 0.06, size = 239, normalized size = 2.10

$$-\frac{2a\cot(c+dx)}{35d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{35d} + \frac{8a\cot(c+dx)\csc^4(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*
x)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
```

```
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (a*Log[Cos[(c + d*x)/2]])/(16*d) + (a*Log[Sin[(c +
 d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/
(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.21, size = 138, normalized size = 1.21

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(d}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(d}{}\right.}{d}$
risch	$\frac{a\left(105\,e^{13i(dx+c)}+3360ie^{10i(dx+c)}+1540\,e^{11i(dx+c)}+3360ie^{8i(dx+c)}+1085\,e^{9i(dx+c)}+6720ie^{6i(dx+c)}+1344ie^{4i(dx+c)}-\right.}{840d\left(e^{2i(dx+c)}-1\right)^7}$
norman	$-\frac{a}{896d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2240d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320d}+\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+a*(-
1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c
)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]
time = 0.29, size = 118, normalized size = 1.04

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(102) = 204$.
time = 0.40, size = 221, normalized size = 1.94

$$-\frac{192\,a\cos\left(dx+c\right)^7-672\,a\cos\left(dx+c\right)^5+105\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-70\left(3\,a\cos\left(dx+c\right)^5+8\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3360\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(a*cos(d*x + c)^
6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*a*cos(d*x + c)
^5 + 8*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(102) = 204.
time = 0.58, size = 229, normalized size = 2.01

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+840\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2178\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*a*tan(1/2*d*x + 1/2*c)^2 + 840*a*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*a*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*a*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*a*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
a*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 9.95, size = 385, normalized size = 3.38

$$\frac{a\left(15\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+840\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\right)}{13440\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (a*(15*sin(c/2 + (d*x)/2)^14 - 15*cos(c/2 + (d*x)/2)^14 + 35*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^13 - 35*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) -
21*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^11 - 105*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 105*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 315*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^8 - 315*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 105*cos(c/
2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (
d*x)/2)^4 + 105*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (
d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(13440*d*cos(c/2 + (d*x)/
2)^7*sin(c/2 + (d*x)/2)^7)
```

3.377 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot(c+dx)\csc(c+dx)}{128d}-\frac{a\cot(c+dx)\csc^3(c}{64d}$$

```
[Out] -3/128*a*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-3/
128*a*cot(d*x+c)*csc(d*x+c)/d-1/64*a*cot(d*x+c)*csc(d*x+c)^3/d+1/16*a*cot(d
*x+c)*csc(d*x+c)^5/d-1/8*a*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 14}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\csc^4(c+dx)\,dx+a\int \cot^4(c+dx)\,\mathrm{cs}\\ &= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{1}{8}(3a)\int \cot^2(c+dx)\,\mathrm{c}\\ &= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot^3(c+dx)\csc^5(c+d}{8d}\\ &= -\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot(c+dx)\csc^3}{64d}\\ &= -\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot(c+dx)\,\mathrm{csc}}{128d}\\ &= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c}{7d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 279 vs. 2(136) = 272.
time = 0.06, size = 279, normalized size = 2.05

$$-\frac{2a\cot(c+dx)}{35d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{35d}+\frac{8a\cot(c+dx)\csc^4(c+dx)}{35d}-\frac{a\cot(c+dx)\csc^6(c+dx)}{7d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

Maple [A]
time = 0.21, size = 156, normalized size = 1.15

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)+}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)+}{d}$
risch	$\frac{a\left(105\,e^{15i(dx+c)}-805\,e^{13i(dx+c)}-11655\,e^{11i(dx+c)}+8960ie^{12i(dx+c)}-23485\,e^{9i(dx+c)}-23485\,e^{7i(dx+c)}+8960ie^{8i(dx+c)}-\right.}{2240d\left(e^{2i(dx+c)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/64/
sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c)^
3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/7/sin(d*x+c)^7*co
s(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5))
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.01

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{256\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.36, size = 239, normalized size = 1.76

$$\frac{210\,a\cos\left(dx+c\right)^7-770\,a\cos\left(dx+c\right)^5-770\,a\cos\left(dx+c\right)^3+210\,a\cos\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^8-4\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+105\left(a\cos\left(dx+c\right)^8-4\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+256\left(2\,a\cos\left(dx+c\right)^7-7\,a\cos\left(dx+c\right)^5\right)\sin\left(dx+c\right)}{8960\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*a*cos(d*x + c)^7 - 7*a*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 201, normalized size = 1.48

$$\frac{35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-560\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1680\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+1680\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4566\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+1680\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-560\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+35\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{71680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*a*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*a*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 560
*a*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*a*tan(1/2*d*
x + 1/2*c)^3 + 80*a*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 10.22, size = 337, normalized size = 2.48

$$\frac{a\left(35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+80\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-80\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-112\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+112\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+1680\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8\right)}{71680\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] (a*(35*sin(c/2 + (d*x)/2)^16 - 35*cos(c/2 + (d*x)/2)^16 + 80*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^15 - 80*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) -
```

```
112*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 - 280*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^12 - 560*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 + 168
0*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 - 1680*cos(c/2 + (d*x)/2)^9*sin
(c/2 + (d*x)/2)^7 + 560*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 280*co
s(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 + 112*cos(c/2 + (d*x)/2)^13*sin(c/
2 + (d*x)/2)^3 + 1680*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 +
(d*x)/2)^8*sin(c/2 + (d*x)/2)^8))/(71680*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (
d*x)/2)^8)
```

3.378 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$\frac{9a^2x}{256}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{9a^2\cos(c+dx)\sin(c+dx)}{256d}+\frac{3a^2\cos^3(c+dx}{128}$$

```
[Out] 9/256*a^2*x-2/5*a^2*cos(d*x+c)^5/d+4/7*a^2*cos(d*x+c)^7/d-2/9*a^2*cos(d*x+c
)^9/d+9/256*a^2*cos(d*x+c)*sin(d*x+c)/d+3/128*a^2*cos(d*x+c)^3*sin(d*x+c)/d
-3/32*a^2*cos(d*x+c)^5*sin(d*x+c)/d-3/16*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d-1/
10*a^2*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 276}

$$-\frac{2a^2\cos^9(c+dx)}{9d}+\frac{4a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{3a^2\sin^3(c+dx)\cos^5(c+dx)}{16d}-\frac{3a^2\sin(c+dx)\cos^5(c+dx)}{32d}+\frac{3a^2\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{9a^2\sin(c+dx)\cos(c+dx)}{256d}+\frac{9a^2x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*a^2*x)/256 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Cos[c + d*x]^7)/(7*d)
 - (2*a^2*Cos[c + d*x]^9)/(9*d) + (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d)
 + (3*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (3*a^2*Cos[c + d*x]^5*Sin[
c + d*x])/(32*d) - (3*a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^2*Cos[
c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^4(c+dx)+2a^2\cos^4(c+dx)\sin^5\right.$$
$$= a^2\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^4(c+dx)\,\mathrm{s}$$
$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^2\cos^5(c+dx)\sin^5(c}{10d}$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{3a^2\cos^5(c+dx)\sin^3(c}{16d}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+d}{9d}$$
$$= -\frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9(c+d}{9d}$$
$$= \frac{3a^2x}{128} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9}{9}$$
$$= \frac{9a^2x}{256} - \frac{2a^2\cos^5(c+dx)}{5d} + \frac{4a^2\cos^7(c+dx)}{7d} - \frac{2a^2\cos^9}{9}$$

Mathematica [A]

time = 0.53, size = 116, normalized size = 0.63

$$\frac{a^2(22680c + 22680dx - 30240\cos(c+dx) - 6720\cos(3(c+dx)) + 4032\cos(5(c+dx)) + 720\cos(7(c+dx)) - 560\cos(9(c+dx)) - 1260\sin(2(c+dx)) - 7560\sin(4(c+dx)) + 630\sin(6(c+dx)) + 945\sin(8(c+dx)) - 126\sin(10(c+dx)))}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(22680*c + 22680*d*x - 30240*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 40
32*Cos[5*(c + d*x)] + 720*Cos[7*(c + d*x)] - 560*Cos[9*(c + d*x)] - 1260*Si
n[2*(c + d*x)] - 7560*Sin[4*(c + d*x)] + 630*Sin[6*(c + d*x)] + 945*Sin[8*(
c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

Maple [A]
time = 0.30, size = 218, normalized size = 1.18

method	result
risch	$\frac{9a^2x}{256} - \frac{3a^2\cos(dx+c)}{64d} - \frac{a^2\sin(10dx+10c)}{5120d} - \frac{a^2\cos(9dx+9c)}{1152d} + \frac{3a^2\sin(8dx+8c)}{2048d} + \frac{a^2\cos(7dx+7c)}{896d} + \frac{a^2\sin(}{10}$
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8} - \frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16} + \frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64} + \frac{3dx}{128} + \frac{3c}{128}\right) + 2a^2\left(-\frac{(\sin}{}\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8} - \frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16} + \frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64} + \frac{3dx}{128} + \frac{3c}{128}\right) + 2a^2\left(-\frac{(\sin}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+2*a^2*(-1/9*sin
(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+a
^2*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/32*sin
(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/256*d
*x+3/256*c))
```

Maxima [A]
time = 0.30, size = 123, normalized size = 0.66

$$-\frac{4096\,\left(35\,\cos\left(dx+c\right)^{9} - 90\,\cos\left(dx+c\right)^{7} + 63\,\cos\left(dx+c\right)^{5}\right)a^{2} + 63\,\left(32\,\sin\left(2\,dx+2\,c\right)^{5} - 120\,dx - 120\,c - 5\,\sin\left(8\,dx+8\,c\right) + 40\,\sin\left(4\,dx+4\,c\right)\right)a^{2} - 630\,\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right) - 8\,\sin\left(4\,dx+4\,c\right)\right)a^{2}}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/645120*(4096*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^2 + 63*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 4
```

```
0*sin(4*d*x + 4*c))*a^2 - 630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d
*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.36, size = 124, normalized size = 0.67

$$-\frac{17920\,a^2\cos\left(dx+c\right)^9-46080\,a^2\cos\left(dx+c\right)^7+32256\,a^2\cos\left(dx+c\right)^5-2835\,a^2dx+63\,\left(128\,a^2\cos\left(dx+c\right)^9-496\,a^2\cos\left(dx+c\right)^7+488\,a^2\cos\left(dx+c\right)^5-30\,a^2\cos\left(dx+c\right)^3-45\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/80640*(17920*a^2*cos(d*x + c)^9 - 46080*a^2*cos(d*x + c)^7 + 32256*a^2*c
os(d*x + c)^5 - 2835*a^2*d*x + 63*(128*a^2*cos(d*x + c)^9 - 496*a^2*cos(d*x
 + c)^7 + 488*a^2*cos(d*x + c)^5 - 30*a^2*cos(d*x + c)^3 - 45*a^2*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(177) = 354$.
time = 1.94, size = 554, normalized size = 2.99

$$\begin{cases} \frac{3a^2x\sin^{10}(c+dx)}{256}+\frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{3a^2x\sin^8(c+dx)}{128}+\frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{128}+\frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15a^2x\sin^4(c+dx)\cos^6(c+dx)}{128}+\frac{9a^2x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{15a^2x\sin^2(c+dx)\cos^8(c+dx)}{256}+\frac{3a^2x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{3a^2x\cos^{10}(c+dx)}{256}+\frac{3a^2x\cos^8(c+dx)}{128}+\frac{3a^2\sin^9(c+dx)\cos(c+dx)}{256d}+\frac{7a^2\sin^7(c+dx)\cos^3(c+dx)}{128d}+\frac{3a^2\sin^7(c+dx)\cos(c+dx)}{128d}-\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{10d}+\frac{11a^2\sin^5(c+dx)\cos^3(c+dx)}{128d}-\frac{2a^2\sin^4(c+dx)\cos^5(c+dx)}{5d}-\frac{7a^2\sin^3(c+dx)\cos^7(c+dx)}{128d}-\frac{11a^2\sin^3(c+dx)\cos^5(c+dx)}{128d}-\frac{8a^2\sin^2(c+dx)\cos^7(c+dx)}{35d}-\frac{3a^2\sin(c+dx)\cos^9(c+dx)}{256d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{128d}-\frac{16a^2\cos^9(c+dx)}{315d} & \text{for}\ d\neq 0 \\ x(a\sin(c)+a)^2\sin^4(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 3*a**2*x*cos(c
 + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(c
 + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**2*sin(c + d*x)**7*cos(c + d*x)/(1
28*d) - a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 11*a**2*sin(c + d*x)*
*5*cos(c + d*x)**3/(128*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) -
 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*a**2*sin(c + d*x)**3*c
os(c + d*x)**5/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 3*
a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**2*sin(c + d*x)*cos(c + d*x
)**7/(128*d) - 16*a**2*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*sin(c) + a
)**2*sin(c)**4*cos(c)**4, True))
```

Giac [A]
time = 0.70, size = 174, normalized size = 0.94

$$\frac{9}{256}\,a^2x-\frac{a^2\cos\left(9\,dx+9\,c\right)}{1152\,d}+\frac{a^2\cos\left(7\,dx+7\,c\right)}{896\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{96\,d}-\frac{3\,a^2\cos\left(dx+c\right)}{64\,d}-\frac{a^2\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{3\,a^2\sin\left(8\,dx+8\,c\right)}{2048\,d}+\frac{a^2\sin\left(6\,dx+6\,c\right)}{1024\,d}-\frac{3\,a^2\sin\left(4\,dx+4\,c\right)}{256\,d}-\frac{a^2\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 9/256*a^2*x - 1/1152*a^2*cos(9*d*x + 9*c)/d + 1/896*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/96*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos
(d*x + c)/d - 1/5120*a^2*sin(10*d*x + 10*c)/d + 3/2048*a^2*sin(8*d*x + 8*c)
/d + 1/1024*a^2*sin(6*d*x + 6*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d - 1/512*a
^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.21, size = 469, normalized size = 2.54

$$\frac{9\,a^2\,x}{256}-\frac{\frac{9\,a^2\,\left(c+d\,x\right)}{256}+\frac{87\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{128}-\frac{553\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160}-\frac{491\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{32}+\frac{2555\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{64}-\frac{2555\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{64}+\frac{491\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}+\frac{553\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{160}-\frac{87\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{128}-\frac{9\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{128}-\frac{a^2\,\left(2835\,c+2835\,d\,x-8192\right)}{80640}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{45\,a^2\,\left(c+d\,x\right)}{128}-\frac{a^2\,\left(28350\,c+28350\,d\,x-81920\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{405\,a^2\,\left(c+d\,x\right)}{256}-\frac{a^2\,\left(127575\,c+127575\,d\,x-368640\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{135\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(340200\,c+340200\,d\,x+737280\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{945\,a^2\,\left(c+d\,x\right)}{128}-\frac{a^2\,\left(595350\,c+595350\,d\,x+860160\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(\frac{135\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(340200\,c+340200\,d\,x-1720320\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{567\,a^2\,\left(c+d\,x\right)}{64}-\frac{a^2\,\left(714420\,c+714420\,d\,x-1032192\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{945\,a^2\,\left(c+d\,x\right)}{128}-\frac{a^2\,\left(595350\,c+595350\,d\,x-2580480\right)}{80640}\right)+\frac{9\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (9*a^2*x)/256 - ((9*a^2*(c + d*x))/256 + (87*a^2*tan(c/2 + (d*x)/2)^3)/128
- (553*a^2*tan(c/2 + (d*x)/2)^5)/160 - (491*a^2*tan(c/2 + (d*x)/2)^7)/32 +
(2555*a^2*tan(c/2 + (d*x)/2)^9)/64 - (2555*a^2*tan(c/2 + (d*x)/2)^11)/64 +
(491*a^2*tan(c/2 + (d*x)/2)^13)/32 + (553*a^2*tan(c/2 + (d*x)/2)^15)/160 -
(87*a^2*tan(c/2 + (d*x)/2)^17)/128 - (9*a^2*tan(c/2 + (d*x)/2)^19)/128 - (a
^2*(2835*c + 2835*d*x - 8192))/80640 + tan(c/2 + (d*x)/2)^2*((45*a^2*(c + d
*x))/128 - (a^2*(28350*c + 28350*d*x - 81920))/80640) + tan(c/2 + (d*x)/2)^
4*((405*a^2*(c + d*x))/256 - (a^2*(127575*c + 127575*d*x - 368640))/80640)
+ tan(c/2 + (d*x)/2)^6*((135*a^2*(c + d*x))/32 - (a^2*(340200*c + 340200*d*
x + 737280))/80640) + tan(c/2 + (d*x)/2)^12*((945*a^2*(c + d*x))/128 - (a^2
*(595350*c + 595350*d*x + 860160))/80640) + tan(c/2 + (d*x)/2)^14*((135*a^2
*(c + d*x))/32 - (a^2*(340200*c + 340200*d*x - 1720320))/80640) + tan(c/2 +
 (d*x)/2)^10*((567*a^2*(c + d*x))/64 - (a^2*(714420*c + 714420*d*x - 103219
2))/80640) + tan(c/2 + (d*x)/2)^8*((945*a^2*(c + d*x))/128 - (a^2*(595350*c
 + 595350*d*x - 2580480))/80640) + (9*a^2*tan(c/2 + (d*x)/2))/128)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^10)
```

3.379 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$\frac{3a^2x}{64}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{a^2\cos^3(c+dx)\sin}{32d}$$

```
[Out] 3/64*a^2*x-2/5*a^2*cos(d*x+c)^5/d+3/7*a^2*cos(d*x+c)^7/d-1/9*a^2*cos(d*x+c)
^9/d+3/64*a^2*cos(d*x+c)*sin(d*x+c)/d+1/32*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/
8*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{a^2\cos^9(c+dx)}{9d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{4d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{3a^2x}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/64 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]^7)/(7*d)
- (a^2*Cos[c + d*x]^9)/(9*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(64*d) + (
a^2*Cos[c + d*x]^3*Sin[c + d*x])/(32*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])
/(8*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)\sin^3(c+dx)+2a^2\cos^4(c+dx)\sin\right.$$

$$= a^2\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^4(c+dx)$$

$$= -\frac{a^2\cos^5(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\left(3a^2\right)\int \cos^4(c+d$$

$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{8d}-\frac{a^2\cos^5(c+dx)\sin^3(c}{4d}$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+d}{9d}$$

$$= -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9(c+d}{9d}$$

$$= \frac{3a^2x}{64}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos^7(c+dx)}{7d}-\frac{a^2\cos^9}{}$$

Mathematica [A]
time = 0.53, size = 86, normalized size = 0.54

$$\frac{a^2(7560c+7560dx-11340\cos(c+dx)-3360\cos(3(c+dx))+1008\cos(5(c+dx))+450\cos(7(c+dx))-70\cos(9(c+dx))-2520\sin(4(c+dx))+315\sin(8(c+dx)))}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7560*c + 7560*d*x - 11340*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 1008
*Cos[5*(c + d*x)] + 450*Cos[7*(c + d*x)] - 70*Cos[9*(c + d*x)] - 2520*Sin[4
*(c + d*x)] + 315*Sin[8*(c + d*x)]))/(161280*d)
```

Maple [A]
time = 0.30, size = 162, normalized size = 1.02

method	result
risch	$\frac{3a^2x}{64}-\frac{9a^2\cos(dx+c)}{128d}-\frac{a^2\cos(9dx+9c)}{2304d}+\frac{a^2\sin(8dx+8c)}{512d}+\frac{5a^2\cos(7dx+7c)}{1792d}+\frac{a^2\cos(5dx+5c)}{160d}-\frac{a^2\sin(4dx}{64d}$
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c\right.}{}\right.}{d}$
norman	$-\frac{68a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{155a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{3a^2x}{64}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}-\frac{52a^2}{315d}-\frac{4a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{164a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a^2*(-1/8*sin
(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)
^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

Maxima [A]
time = 0.29, size = 101, normalized size = 0.64

$$-\frac{512\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^2-4608\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2-315\left(24\,dx+24\,c+\sin(8\,dx+8\,c)-8\sin(4\,dx+4\,c)\right)a^2}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/161280*(512*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*
a^2 - 4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 - 315*(24*d*x + 24*c +
 sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.37, size = 111, normalized size = 0.70

$$-\frac{2240\,a^2\cos\left(dx+c\right)^9-8640\,a^2\cos\left(dx+c\right)^7+8064\,a^2\cos\left(dx+c\right)^5-945\,a^2dx-315\left(16\,a^2\cos\left(dx+c\right)^7-24\,a^2\cos\left(dx+c\right)^5+2\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*a^2*cos(d*x + c)^9 - 8640*a^2*cos(d*x + c)^7 + 8064*a^2*cos(
d*x + c)^5 - 945*a^2*d*x - 315*(16*a^2*cos(d*x + c)^7 - 24*a^2*cos(d*x + c)
^5 + 2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(146) = 292.
time = 1.31, size = 335, normalized size = 2.11

$$\begin{cases} \frac{3a^2x\sin^8(c+dx)}{64}+\frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{16}+\frac{9a^2x\sin^4(c+dx)\cos^4(c+dx)}{32}+\frac{3a^2x\sin^2(c+dx)\cos^6(c+dx)}{16}+\frac{3a^2x\cos^8(c+dx)}{64}+\frac{3a^2\sin^7(c+dx)\cos(c+dx)}{64d}+\frac{11a^2\sin^5(c+dx)\cos^3(c+dx)}{64d}-\frac{a^2\sin^4(c+dx)\cos^5(c+dx)}{5d}-\frac{11a^2\sin^3(c+dx)\cos^5(c+dx)}{64d}-\frac{4a^2\sin^2(c+dx)\cos^7(c+dx)}{35d}-\frac{a^2\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{64d}-\frac{8a^2\cos^9(c+dx)}{315d}-\frac{2a^2\cos^7(c+dx)}{35d} & \text{for}\ d\neq 0 \\ x(a\sin(c)+a)^2\sin^3(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/64 + 3*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/16 + 3*a**2*x*cos(c + d*x)**8/64 + 3*a**2*sin(c + d
*x)**7*cos(c + d*x)/(64*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(64*d)
 - a**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 11*a**2*sin(c + d*x)**3*cos
(c + d*x)**5/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - a**2*
sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7
/(64*d) - 8*a**2*cos(c + d*x)**9/(315*d) - 2*a**2*cos(c + d*x)**7/(35*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)**4, True))
```

Giac [A]
time = 0.66, size = 123, normalized size = 0.77

$$\frac{3}{64}\,a^2x-\frac{a^2\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{5\,a^2\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{160\,d}-\frac{a^2\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{9\,a^2\cos\left(dx+c\right)}{128\,d}+\frac{a^2\sin\left(8\,dx+8\,c\right)}{512\,d}-\frac{a^2\sin\left(4\,dx+4\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a^2*x - 1/2304*a^2*cos(9*d*x + 9*c)/d + 5/1792*a^2*cos(7*d*x + 7*c)/d
+ 1/160*a^2*cos(5*d*x + 5*c)/d - 1/48*a^2*cos(3*d*x + 3*c)/d - 9/128*a^2*co
s(d*x + c)/d + 1/512*a^2*sin(8*d*x + 8*c)/d - 1/64*a^2*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 12.15, size = 437, normalized size = 2.75

$$\frac{3\,a^2\,x}{64}-\frac{\frac{3\,a^2\,\left(c+d\,x\right)}{64}+\frac{13\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{16}-\frac{155\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{16}+\frac{169\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{16}-\frac{169\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{16}+\frac{155\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{16}-\frac{13\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{16}-\frac{3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{32}-\frac{a^2\,\left(945\,c+945\,d\,x-3328\right)}{20160}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{27\,a^2\,\left(c+d\,x\right)}{64}-\frac{a^2\,\left(8505\,c+8505\,d\,x-29952\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{27\,a^2\,\left(c+d\,x\right)}{16}-\frac{a^2\,\left(34020\,c+34020\,d\,x-39168\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(\frac{27\,a^2\,\left(c+d\,x\right)}{16}-\frac{a^2\,\left(34020\,c+34020\,d\,x-80640\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{63\,a^2\,\left(c+d\,x\right)}{16}-\frac{a^2\,\left(79380\,c+79380\,d\,x+16128\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{63\,a^2\,\left(c+d\,x\right)}{16}-\frac{a^2\,\left(79380\,c+79380\,d\,x-295680\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{189\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(119070\,c+119070\,d\,x+241920\right)}{20160}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{189\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(119070\,c+119070\,d\,x-661248\right)}{20160}\right)+\frac{3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/64 - ((3*a^2*(c + d*x))/64 + (13*a^2*tan(c/2 + (d*x)/2)^3)/16 - (
155*a^2*tan(c/2 + (d*x)/2)^5)/16 + (169*a^2*tan(c/2 + (d*x)/2)^7)/16 - (169
*a^2*tan(c/2 + (d*x)/2)^11)/16 + (155*a^2*tan(c/2 + (d*x)/2)^13)/16 - (13*a
^2*tan(c/2 + (d*x)/2)^15)/16 - (3*a^2*tan(c/2 + (d*x)/2)^17)/32 - (a^2*(945
*c + 945*d*x - 3328))/20160 + tan(c/2 + (d*x)/2)^2*((27*a^2*(c + d*x))/64 -
 (a^2*(8505*c + 8505*d*x - 29952))/20160) + tan(c/2 + (d*x)/2)^4*((27*a^2*(
c + d*x))/16 - (a^2*(34020*c + 34020*d*x - 39168))/20160) + tan(c/2 + (d*x)
/2)^14*((27*a^2*(c + d*x))/16 - (a^2*(34020*c + 34020*d*x - 80640))/20160)
+ tan(c/2 + (d*x)/2)^6*((63*a^2*(c + d*x))/16 - (a^2*(79380*c + 79380*d*x +
 16128))/20160) + tan(c/2 + (d*x)/2)^12*((63*a^2*(c + d*x))/16 - (a^2*(7938
0*c + 79380*d*x - 295680))/20160) + tan(c/2 + (d*x)/2)^10*((189*a^2*(c + d*
x))/32 - (a^2*(119070*c + 119070*d*x + 241920))/20160) + tan(c/2 + (d*x)/2)
^8*((189*a^2*(c + d*x))/32 - (a^2*(119070*c + 119070*d*x - 661248))/20160)
+ (3*a^2*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.380 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=141

$$\frac{11a^2x}{128}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^7(c+dx)}{7d}+\frac{11a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{11a^2\cos^3(c+dx)\sin(c+dx)}{192d}-$$

```
[Out] 11/128*a^2*x-2/5*a^2*cos(d*x+c)^5/d+2/7*a^2*cos(d*x+c)^7/d+11/128*a^2*cos(d
*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-11/48*a^2*cos(d*x+c
)^5*sin(d*x+c)/d-1/8*a^2*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 14}

$$\frac{2a^2\cos^7(c+dx)}{7d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{11a^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{11a^2\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{11a^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{11a^2x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*a^2*x)/128 - (2*a^2*Cos[c + d*x]^5)/(5*d) + (2*a^2*Cos[c + d*x]^7)/(7*d
) + (11*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (11*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a^2*Cos
[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cos^4(c+dx)\sin^2(c+dx)+2a^2\cos^4(c+dx)\sin^3\right.\\
&= a^2\int \cos^4(c+dx)\sin^2(c+dx)\,dx+a^2\int \cos^4(c+dx)\,\mathrm{s}\\
&= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{a^2\cos^5(c+dx)\sin^3(c+}{8d}\\
&= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{11a^2\cos^5(c+dx)\sin(c+}{48d}\\
&= -\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos(c+dx)}{16}\\
&= \frac{a^2x}{16}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^7(c+dx)}{7d}+\frac{11a^2\cos(}{}\\
&= \frac{11a^2x}{128}-\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^7(c+dx)}{7d}+\frac{11a^2\mathrm{c}}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 96, normalized size = 0.68

$$\frac{a^2(3360c+9240dx-10080\cos(c+dx)-3360\cos(3(c+dx))+672\cos(5(c+dx))+480\cos(7(c+dx))+1680\sin(2(c+dx))-2520\sin(4(c+dx))-560\sin(6(c+dx))+105\sin(8(c+dx)))}{107520d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]

[Out] (a^2*(3360*c + 9240*d*x - 10080*Cos[c + d*x] - 3360*Cos[3*(c + d*x)] + 672*Cos[5*(c + d*x)] + 480*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 2520*Sin[4*(c + d*x)] - 560*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]))/(107520*d)

Maple [A]
time = 0.24, size = 164, normalized size = 1.16

method	result
risch	$\frac{11a^2x}{128}-\frac{3a^2\cos(dx+c)}{32d}+\frac{a^2\sin(8dx+8c)}{1024d}+\frac{a^2\cos(7dx+7c)}{224d}-\frac{a^2\sin(6dx+6c)}{192d}+\frac{a^2\cos(5dx+5c)}{160d}-\frac{3a^2\sin(4}{128}$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5\right.}{}\right.}{d}$
norman	$\frac{\frac{11a^2x}{128}-\frac{8a^2}{35d}-\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{77a^2x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{385a^2x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{77a^2x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{77a^2x\left(\tan^{12}\left(\frac{dx}{2}\right.\right.}{32}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+2*a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))

Maxima [A]
time = 0.29, size = 102, normalized size = 0.72

$$\frac{6144\,(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5)a^2+560\,(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(4\,dx+4\,c))a^2+105\,(24\,dx+24\,c+\sin(8\,dx+8\,c)-8\,\sin(4\,dx+4\,c))a^2}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/107520*(6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2 + 105*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^2)/d

Fricas [A]
time = 0.37, size = 98, normalized size = 0.70

$$\frac{3840\,a^2\cos(dx+c)^7-5376\,a^2\cos(dx+c)^5+1155\,a^2dx+35\,(48\,a^2\cos(dx+c)^7-136\,a^2\cos(dx+c)^5+22\,a^2\cos(dx+c)^3+33\,a^2\cos(dx+c))\sin(dx+c)}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a^2*cos(d*x + c)^7 - 5376*a^2*cos(d*x + c)^5 + 1155*a^2*d*x +
 35*(48*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 + 22*a^2*cos(d*x + c)^3
 + 33*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(134) = 268$.
time = 0.94, size = 420, normalized size = 2.98

$$\begin{cases} \frac{3a^2x\sin^8(c+dx)}{128} + \frac{3a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{a^2x\sin^6(c+dx)}{16} + \frac{9a^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^2x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^2x\cos^8(c+dx)}{128} + \frac{a^2x\cos^6(c+dx)}{16} + \frac{3a^2\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11a^2\sin^5(c+dx)\cos^3(c+dx)}{128d} + \frac{a^2\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{11a^2\sin^3(c+dx)\cos^5(c+dx)}{128d} + \frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{2a^2\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{3a^2\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{a^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{4a^2\cos^7(c+dx)}{35d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^2\sin^2(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**8/128 + 3*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + a**2*x*sin(c + d*x)**6/16 + 9*a**2*x*sin(c + d*x)**4*cos(c + d
*x)**4/64 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**2*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
3*a**2*x*cos(c + d*x)**8/128 + a**2*x*cos(c + d*x)**6/16 + 3*a**2*sin(c + d
*x)**7*cos(c + d*x)/(128*d) + 11*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*
d) + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**2*sin(c + d*x)**3*cos
(c + d*x)**5/(128*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 2*a**2*
sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**7
/(128*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 4*a**2*cos(c + d*x)**
7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**4, True))
```

Giac [A]
time = 0.60, size = 140, normalized size = 0.99

$$\frac{11}{128}a^2x + \frac{a^2\cos(7dx+7c)}{224d} + \frac{a^2\cos(5dx+5c)}{160d} - \frac{a^2\cos(3dx+3c)}{32d} - \frac{3a^2\cos(dx+c)}{32d} + \frac{a^2\sin(8dx+8c)}{1024d} - \frac{a^2\sin(6dx+6c)}{192d} - \frac{3a^2\sin(4dx+4c)}{128d} + \frac{a^2\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 11/128*a^2*x + 1/224*a^2*cos(7*d*x + 7*c)/d + 1/160*a^2*cos(5*d*x + 5*c)/d
- 1/32*a^2*cos(3*d*x + 3*c)/d - 3/32*a^2*cos(d*x + c)/d + 1/1024*a^2*sin(8*
d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d - 3/128*a^2*sin(4*d*x + 4*c)/d
+ 1/64*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.35, size = 363, normalized size = 2.57

$$\frac{11a^2x}{128} - \frac{\frac{11a^2(c+dx)}{128} - \frac{259a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192} - \frac{1103a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{192} + \frac{2261a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{192} - \frac{2261a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{192} + \frac{1103a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192} + \frac{259a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192} - \frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} - \frac{a^2(1155c+1155dx-3072)}{13440} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{11a^2(c+dx)}{16} - \frac{a^2(9240c+9240dx-24576)}{13440}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{77a^2(c+dx)}{32} - \frac{a^2(32340c+32340dx+21504)}{13440}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{77a^2(c+dx)}{32} - \frac{a^2(32340c+32340dx-107520)}{13440}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{385a^2(c+dx)}{64} - \frac{a^2(80850c+80850dx-107520)}{13440}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{77a^2(c+dx)}{16} - \frac{a^2(64680c+64680dx-172032)}{13440}\right) + \frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)

[Out] (11*a^2*x)/128 - ((11*a^2*(c + d*x))/128 - (259*a^2*tan(c/2 + (d*x)/2)^3)/192 - (1103*a^2*tan(c/2 + (d*x)/2)^5)/192 + (2261*a^2*tan(c/2 + (d*x)/2)^7)/192 - (2261*a^2*tan(c/2 + (d*x)/2)^9)/192 + (1103*a^2*tan(c/2 + (d*x)/2)^11)/192 + (259*a^2*tan(c/2 + (d*x)/2)^13)/192 - (11*a^2*tan(c/2 + (d*x)/2)^15)/64 - (a^2*(1155*c + 1155*d*x - 3072))/13440 + tan(c/2 + (d*x)/2)^2*((11*a^2*(c + d*x))/16 - (a^2*(9240*c + 9240*d*x - 24576))/13440) + tan(c/2 + (d*x)/2)^4*((77*a^2*(c + d*x))/32 - (a^2*(32340*c + 32340*d*x + 21504))/13440) + tan(c/2 + (d*x)/2)^12*((77*a^2*(c + d*x))/32 - (a^2*(32340*c + 32340*d*x - 107520))/13440) + tan(c/2 + (d*x)/2)^8*((385*a^2*(c + d*x))/64 - (a^2*(80850*c + 80850*d*x - 107520))/13440) + tan(c/2 + (d*x)/2)^6*((77*a^2*(c + d*x))/16 - (a^2*(64680*c + 64680*d*x - 172032))/13440) + (11*a^2*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^8)

3.381 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{a^2x}{8}-\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{\cos^5(c+dx)(a+a\sin(c+d$$

```
[Out] 1/8*a^2*x-1/15*a^2*cos(d*x+c)^5/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/12*a^2*
cos(d*x+c)^3*sin(d*x+c)/d-1/7*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/21*cos(d*
x+c)^5*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{21d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2x}{8}-\frac{\cos^5(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*x)/8 - (a^2*Cos[c + d*x]^5)/(15*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(
8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (Cos[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(7*d) - (Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(21*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{2}{7}\int \cos^4(c+dx) \\ &= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{\cos^5(c+dx)\left(a^2\right.}{21} \\ &= -\frac{a^2\cos^5(c+dx)}{15d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d} \\ &= -\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{\cos^5}{} \\ &= -\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos}{} \\ &= \frac{a^2x}{8}-\frac{a^2\cos^5(c+dx)}{15d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{8d}+\end{aligned}$$

Mathematica [A]

time = 0.29, size = 86, normalized size = 0.67

$$\frac{a^2(840c+840dx-1155\cos(c+dx)-525\cos(3(c+dx))-63\cos(5(c+dx))+15\cos(7(c+dx))+210\sin(2(c+dx))-210\sin(4(c+dx))-70\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(840*c + 840*d*x - 1155*Cos[c + d*x] - 525*Cos[3*(c + d*x)] - 63*Cos[5
*(c + d*x)] + 15*Cos[7*(c + d*x)] + 210*Sin[2*(c + d*x)] - 210*Sin[4*(c + d
*x)] - 70*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A]

time = 0.18, size = 106, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5}+2a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos\right.}{7}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5}+2a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos\right.}{7}\right.}{d}$
risch	$\frac{a^2x}{8}-\frac{11a^2\cos(dx+c)}{64d}+\frac{a^2\cos(7dx+7c)}{448d}-\frac{a^2\sin(6dx+6c)}{96d}-\frac{3a^2\cos(5dx+5c)}{320d}-\frac{a^2\sin(4dx+4c)}{32d}-\frac{5a^2\cos(3d}{64d}$
norman	$\frac{a^2x}{8}-\frac{18a^2}{35d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{31a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{31a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{11a^2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{a^2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/5*a^2*cos(d*x+c)^5+2*a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))

Maxima [A]

time = 0.28, size = 82, normalized size = 0.64

$$-\frac{672\,a^2\cos\left(dx+c\right)^5-96\,\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^2-35\,\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2)/d

Fricas [A]

time = 0.36, size = 85, normalized size = 0.66

$$\frac{120\,a^2\cos\left(dx+c\right)^7-336\,a^2\cos\left(dx+c\right)^5+105\,a^2dx-35\,\left(8\,a^2\cos\left(dx+c\right)^5-2\,a^2\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/840*(120*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x + c)^5 + 105*a^2*d*x - 35*(8*a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/d

Sympy [A]

time = 0.62, size = 223, normalized size = 1.73

$$\begin{cases} \frac{a^2x\sin^6(c+dx)}{8}+\frac{3a^2x\sin^4(c+dx)\cos^2(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^4(c+dx)}{8}+\frac{a^2x\cos^6(c+dx)}{8}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{8d}+\frac{a^2\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{a^2\sin(c+dx)\cos^5(c+dx)}{8d}-\frac{2a^2\cos^7(c+dx)}{35d}-\frac{a^2\cos^5(c+dx)}{5d} & \text{for}\ d\neq 0\\ x(a\sin(c)+a)^2\sin(c)\cos^4(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/8 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x)
**2/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a**2*x*cos(c + d*x)**6
/8 + a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a**2*sin(c + d*x)**3*cos(c +
 d*x)**3/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - a**2*sin(c +
d*x)*cos(c + d*x)**5/(8*d) - 2*a**2*cos(c + d*x)**7/(35*d) - a**2*cos(c + d
*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**4, True))
```

Giac [A]
time = 0.66, size = 123, normalized size = 0.95

$$\frac{1}{8}a^2x+\frac{a^2\cos(7dx+7c)}{448d}-\frac{3a^2\cos(5dx+5c)}{320d}-\frac{5a^2\cos(3dx+3c)}{64d}-\frac{11a^2\cos(dx+c)}{64d}-\frac{a^2\sin(6dx+6c)}{96d}-\frac{a^2\sin(4dx+4c)}{32d}+\frac{a^2\sin(2dx+2c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x + 1/448*a^2*cos(7*d*x + 7*c)/d - 3/320*a^2*cos(5*d*x + 5*c)/d - 5
/64*a^2*cos(3*d*x + 3*c)/d - 11/64*a^2*cos(d*x + c)/d - 1/96*a^2*sin(6*d*x
+ 6*c)/d - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.75, size = 388, normalized size = 3.01

$$\frac{a^2x}{8}-\frac{\frac{31a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{12}-\frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-\frac{31a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{12}+\frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{3}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{4}+a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{c}{8}+\frac{dx}{8}-\frac{18}{35}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(7a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{7c}{8}+\frac{7dx}{8}-2\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(7a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{7c}{8}+\frac{7dx}{8}-\frac{8}{5}\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(21a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{21c}{8}+\frac{21dx}{8}-8\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(21a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{21c}{8}+\frac{21dx}{8}-\frac{14}{5}\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(35a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{35c}{8}+\frac{35dx}{8}-2\right)\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(35a^2\left(\frac{c}{8}+\frac{dx}{8}\right)-a^2\left(\frac{35c}{8}+\frac{35dx}{8}-16\right)\right)+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 - ((31*a^2*tan(c/2 + (d*x)/2)^5)/12 - (11*a^2*tan(c/2 + (d*x)/2)^
3)/3 - (31*a^2*tan(c/2 + (d*x)/2)^9)/12 + (11*a^2*tan(c/2 + (d*x)/2)^11)/3
- (a^2*tan(c/2 + (d*x)/2)^13)/4 + a^2*(c/8 + (d*x)/8) - a^2*(c/8 + (d*x)/8
- 18/35) + tan(c/2 + (d*x)/2)^12*(7*a^2*(c/8 + (d*x)/8) - a^2*((7*c)/8 + (7
*d*x)/8 - 2)) + tan(c/2 + (d*x)/2)^2*(7*a^2*(c/8 + (d*x)/8) - a^2*((7*c)/8
+ (7*d*x)/8 - 8/5)) + tan(c/2 + (d*x)/2)^10*(21*a^2*(c/8 + (d*x)/8) - a^2*(
(21*c)/8 + (21*d*x)/8 - 8)) + tan(c/2 + (d*x)/2)^4*(21*a^2*(c/8 + (d*x)/8)
- a^2*((21*c)/8 + (21*d*x)/8 - 14/5)) + tan(c/2 + (d*x)/2)^8*(35*a^2*(c/8 +
 (d*x)/8) - a^2*((35*c)/8 + (35*d*x)/8 - 2)) + tan(c/2 + (d*x)/2)^6*(35*a^2
*(c/8 + (d*x)/8) - a^2*((35*c)/8 + (35*d*x)/8 - 16)) + (a^2*tan(c/2 + (d*x)
/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.382 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{3a^2x}{4}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^5(c+dx)}{5d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/4*a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
-1/5*a^2*cos(d*x+c)^5/d+3/4*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)^
3*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2952, 2715, 8, 2672, 308, 212, 2645, 30}

$$-\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{2d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{4d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) - (a^2*Cos[c + d*x]^5)/(5*d) + (3*a^2*Cos[c + d*x]*Sin
[c + d*x])/(4*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\cot(c+dx)+a^2\right.$$

$$= a^2\int \cos^3(c+dx)\cot(c+dx)\,dx + a^2\int \cos^4(c+dx)\,\mathrm{s}$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}\left(3a^2\right)\int \cos^2(c+dx)\,d$$

$$= -\frac{a^2\cos^5(c+dx)}{5d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{4d} + \frac{a^2\,\mathrm{c}}{}$$

$$= \frac{3a^2x}{4} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} - \frac{a^2\cos^5(c+}{5d}$$

$$= \frac{3a^2x}{4} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2}{}$$

Mathematica [A]

time = 0.74, size = 96, normalized size = 0.81

$$\frac{a^2\left(270\cos(c+dx)+5\cos(3(c+dx))-3\cos(5(c+dx))+15\left(4\left(3c+3dx-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+8\sin(2(c+dx))+\sin(4(c+dx))\right)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*Cos[5*(c + d*x)] + 15*(4*(3
*c + 3*d*x - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(
c + d*x)] + Sin[4*(c + d*x)])))/(240*d)
```

Maple [A]

time = 0.20, size = 94, normalized size = 0.79

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a^2(c}{}}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a^2(c}{}}{d}$
risch	$\frac{3a^2x}{4}+\frac{9a^2e^{i(dx+c)}}{16d}+\frac{9a^2e^{-i(dx+c)}}{16d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}-\frac{a^2\cos(5dx+5c)}{80d}+\frac{a^2\sin(4dx}{16d}$
norman	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2x}{4}+\frac{34a^2}{15d}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+15a^2x(\tan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(1/4
*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5*a^2*cos(d*x+c)
^5)
```

Maxima [A]

time = 0.29, size = 98, normalized size = 0.82

$$-\frac{48\,a^2\cos\left(dx+c\right)^5-40\left(2\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^2-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)a^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2)/d
```

Fricas [A]
time = 0.36, size = 115, normalized size = 0.97

$$-\frac{12\,a^2\cos\left(dx+c\right)^5-20\,a^2\cos\left(dx+c\right)^3-45\,a^2dx-60\,a^2\cos\left(dx+c\right)+30\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-30\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\,\left(2\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a^2*d*x - 60*a^2*
cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x
 + c) + 1/2) - 15*(2*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\cos^4\left(c+dx\right)\csc\left(c+dx\right)\,dx+\int 2\sin\left(c+dx\right)\cos^4\left(c+dx\right)\csc\left(c+dx\right)\,dx+\int\sin^2\left(c+dx\right)\cos^4\left(c+dx\right)\csc\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(cos(c + d*x)**4*csc(c + d*x), x) + Integral(2*sin(c + d*x)*c
os(c + d*x)**4*csc(c + d*x), x) + Integral(sin(c + d*x)**2*cos(c + d*x)**4*
csc(c + d*x), x))
```

Giac [A]
time = 0.51, size = 181, normalized size = 1.52

$$\frac{45\,(dx+c)a^2+60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,\left(75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-320\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-75\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-68\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a^2 + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a^2*
tan(1/2*d*x + 1/2*c)^9 - 60*a^2*tan(1/2*d*x + 1/2*c)^8 + 30*a^2*tan(1/2*d*x
 + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)^6 - 320*a^2*tan(1/2*d*x + 1/2*c)
^4 - 30*a^2*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a^
2*tan(1/2*d*x + 1/2*c) - 68*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 10.22, size = 293, normalized size = 2.46

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{3\,a^2\,\mathrm{atan}\left(\frac{9\,a^4}{4\,\left(3\,a^4-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)}+\frac{3\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a^4-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}\right)}{2\,d}+\frac{-\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2}+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+12\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{32\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{28\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{34\,a^2}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (3*a^2*atan((9*a^4)/(4*(3*a^4 - (9*a^4*ta
n(c/2 + (d*x)/2))/4)) + (3*a^4*tan(c/2 + (d*x)/2))/(3*a^4 - (9*a^4*tan(c/2
+ (d*x)/2))/4)))/(2*d) + ((28*a^2*tan(c/2 + (d*x)/2)^2)/3 + a^2*tan(c/2 + (
d*x)/2)^3 + (32*a^2*tan(c/2 + (d*x)/2)^4)/3 + 12*a^2*tan(c/2 + (d*x)/2)^6 -
 a^2*tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)^8 - (5*a^2*tan(c/2 + (
d*x)/2)^9)/2 + (34*a^2)/15 + (5*a^2*tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 +
(d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2
+ (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
```

3.383 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{9a^2x}{8}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c}{8d}$$

```
[Out] -9/8*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+2/3*a^2*cos(d*x+c
)^3/d-a^2*cot(d*x+c)/d+1/8*a^2*cos(d*x+c)*sin(d*x+c)/d-1/4*a^2*cos(d*x+c)*s
in(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 2718, 2715, 2713}

$$\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}-\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{9a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a^2*x)/8 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d + (
2*a^2*Cos[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) - (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6+2a^6\csc(c+dx)+a^6\csc^2(c+dx)-4a^6\sin(c\right.}{}$$

$$= -a^2x+a^2\int \csc^2(c+dx)\,dx-a^2\int \sin^2(c+dx)\,dx+$$

$$= -a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^2\cos(c+dx)}{d}+$$

$$= -\frac{3a^2x}{2}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+$$

$$= -\frac{9a^2x}{8}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+$$

Mathematica [A]
time = 0.36, size = 83, normalized size = 0.72

$$\frac{a^2\left(240\cos(c+dx)+16\cos(3(c+dx))-3\left(36c+36dx+32\cot(c+dx)+64\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-64\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sin(4(c+dx))\right)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(240*Cos[c + d*x] + 16*Cos[3*(c + d*x)] - 3*(36*c + 36*d*x + 32*Cot[c
+ d*x] + 64*Log[Cos[(c + d*x)/2]] - 64*Log[Sin[(c + d*x)/2]] - Sin[4*(c + d
*x)]))))/(96*d)
```

Maple [A]
time = 0.17, size = 136, normalized size = 1.17

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\text{c}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\text{c}\right.}{d}$
risch	$-\frac{9a^2x}{8}+\frac{5a^2e^{i(dx+c)}}{4d}+\frac{5a^2e^{-i(dx+c)}}{4d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}-1\right)}{d}+a^2\text{s}$
norman	$-\frac{a^2}{2d}-\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{11a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{11a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{9a^2x\tan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)-3/2*d*x-3/2*c)+2*a^2*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+
c)))+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 128, normalized size = 1.10

$$\frac{32\left(2\cos\left(dx+c\right)^{3}+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^{2}+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)a^{2}-48\left(3\,dx+3\,c+\frac{3\tan(dx+c)^{2}+2}{\tan(dx+c)^{3}+\tan(dx+c)}\right)a^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/96*(32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^2 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^2 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 +
 tan(d*x + c)))*a^2)/d
```

Fricas [A]
time = 0.38, size = 135, normalized size = 1.16

$$-\frac{6\,a^{2}\cos\left(dx+c\right)^{5}-9\,a^{2}\cos\left(dx+c\right)^{3}+24\,a^{2}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-24\,a^{2}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+27\,a^{2}\cos\left(dx+c\right)-\left(16\,a^{2}\cos\left(dx+c\right)^{3}-27\,a^{2}dx+48\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^2*cos(d*x + c)^5 - 9*a^2*cos(d*x + c)^3 + 24*a^2*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) - 24*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 27*a^2*cos(d*x + c) - (16*a^2*cos(d*x + c)^3 - 27*a^2*d*x + 48*a^2*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 210, normalized size = 1.81

$$-\frac{27\,(dx+c)a^2-48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,\left(4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-96\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-192\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-64\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 12*a^2*ta
n(1/2*d*x + 1/2*c) + 12*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/
2*c) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^7 - 96*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 192*a^2*tan(1/2*d*x + 1/2*c)^4 + 21*a^2*tan(1
/2*d*x + 1/2*c)^3 - 160*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/
2*c) - 64*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 8.83, size = 310, normalized size = 2.67

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{9\,a^2\,\mathrm{atan}\left(\frac{81\,a^4}{16\,\left(9\,a^4+\frac{81\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{9\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^4+\frac{81\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}-\frac{\frac{3\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{2}-16\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\frac{a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{2}-32\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\frac{19\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{2}-\frac{80\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\frac{7\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}-\frac{32\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a^2}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (9*a^2*atan((81*a^4)/(16*(9*a^4 + (81*a
^4*tan(c/2 + (d*x)/2))/16)) - (9*a^4*tan(c/2 + (d*x)/2))/(9*a^4 + (81*a^4*t
an(c/2 + (d*x)/2))/16)))/(4*d) - ((7*a^2*tan(c/2 + (d*x)/2)^2)/2 - (80*a^2*
tan(c/2 + (d*x)/2)^3)/3 + (19*a^2*tan(c/2 + (d*x)/2)^4)/2 - 32*a^2*tan(c/2
```

```
+ (d*x)/2)^5 + (a^2*tan(c/2 + (d*x)/2)^6)/2 - 16*a^2*tan(c/2 + (d*x)/2)^7 +
 (3*a^2*tan(c/2 + (d*x)/2)^8)/2 + a^2 - (32*a^2*tan(c/2 + (d*x)/2))/3)/(d*(
2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8
*tan(c/2 + (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^2*tan(c/2 + (d*x)/2))
/(2*d)
```

3.384 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-3a^2x+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{a^2\cos(c+d$$

```
[Out] -3*a^2*x+1/2*a^2*arctanh(cos(d*x+c))/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*cot(d*x
+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d-a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-3a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -3*a^2*x + (a^2*ArcTanh[Cos[c + d*x]])/(2*d) + (a^2*Cos[c + d*x]^3)/(3*d) -
 (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (a^2*Cos[
c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int \left(-4a^6-a^6\csc(c+dx)+2a^6\csc^2(c+dx)+a^6\csc^3(\right.}{} \\ &= -4a^2x-a^2\int \csc(c+dx)\,dx+a^2\int \csc^3(c+dx)\,dx \\ &= -4a^2x+\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{a}{} \\ &= -3a^2x+\frac{a^2\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^2\cos^3(c+dx)}{3d}-\end{aligned}$$

Mathematica [A]
time = 1.44, size = 158, normalized size = 1.61

$$\frac{a^2(1+\sin(c+dx))^2\left(6\cos(c+dx)+2\cos(3(c+dx))+3\left(-24c-24dx-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)-4\sin(2(c+dx))+8\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x)] + 3*(-24*c -
 24*d*x - 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]]
 - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 4*Sin[2*(c + d*x)] + 8*Ta
n[(c + d*x)/2])))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.20, size = 158, normalized size = 1.61

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c}{2}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c}{2}\right.\right.}{d}$
risch	$-3a^2x+\frac{a^2e^{3i(dx+c)}}{24d}+\frac{ia^2e^{2i(dx+c)}}{4d}+\frac{a^2e^{i(dx+c)}}{8d}+\frac{a^2e^{-i(dx+c)}}{8d}-\frac{ia^2e^{-2i(dx+c)}}{4d}+\frac{a^2e^{-3i(dx+c)}}{24d}+\frac{a^2(e^3}{}$
norman	$\frac{\frac{a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{8d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{\left(1+\tan^2\right(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/
2*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+a^2*(1/3*cos(d*x+c)^3+cos(d*x+c
)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 151, normalized size = 1.54

$$\frac{2\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^2-12\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2+3a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-4\cos(dx+c)+3\log(\cos(dx+c)+1)-3\log(\cos(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^2 - 12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^2 + 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
- 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.36, size = 172, normalized size = 1.76

$$\frac{4a^2\cos(dx+c)^5-36a^2dx\cos(dx+c)^2-4a^2\cos(dx+c)^3+36a^2dx+6a^2\cos(dx+c)+3\left(a^2\cos(dx+c)^2-a^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(a^2\cos(dx+c)^2-a^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-12\left(a^2\cos(dx+c)^3-3a^2\cos(dx+c)\right)\sin(dx+c)}{12\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*a^2*cos(d*x + c)^5 - 36*a^2*d*x*cos(d*x + c)^2 - 4*a^2*cos(d*x + c)
^3 + 36*a^2*d*x + 6*a^2*cos(d*x + c) + 3*(a^2*cos(d*x + c)^2 - a^2)*log(1/2
*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
 1/2) - 12*(a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.63, size = 178, normalized size = 1.82

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)a^2-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{16\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a^2 - 12*a^2*log(abs(tan(
1/2*d*x + 1/2*c))) + 24*a^2*tan(1/2*d*x + 1/2*c) + 3*(6*a^2*tan(1/2*d*x + 1
/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*
a^2*tan(1/2*d*x + 1/2*c)^5 + 3*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*a^2*tan(1/2*d
*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 8.91, size = 303, normalized size = 3.09

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{6\,a^2\,\mathrm{atan}\left(\frac{36\,a^4}{6\,a^4-36\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^4-36\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{-4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (a^2*log(tan(c/2 + (d*x)/2)))/(2*d) - (6
*a^2*atan((36*a^4)/(6*a^4 - 36*a^4*tan(c/2 + (d*x)/2)) + (6*a^4*tan(c/2 + (
d*x)/2))/(6*a^4 - 36*a^4*tan(c/2 + (d*x)/2))))/d - (20*a^2*tan(c/2 + (d*x)/
```

```
2)^3 - (7*a^2*tan(c/2 + (d*x)/2)^2)/6 + (3*a^2*tan(c/2 + (d*x)/2)^4)/2 + 12
*a^2*tan(c/2 + (d*x)/2)^5 - (15*a^2*tan(c/2 + (d*x)/2)^6)/2 - 4*a^2*tan(c/2
 + (d*x)/2)^7 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^
2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2
)^8)) + (a^2*tan(c/2 + (d*x)/2))/d
```

3.385 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-\frac{a^2\cos(c+$$

[Out] -1/2*a^2*x+3*a^2*arctanh(cos(d*x+c))/d-2*a^2*cos(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 3853, 2718, 2715}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]

[Out] -1/2*(a^2*x) + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int\left(-a^6-4a^6\csc(c+dx)-a^6\csc^2(c+dx)+2a^6\csc^3(c+dx)+a^6\right.}{a^4}\\ &= -a^2x-a^2\int\csc^2(c+dx)\,dx+a^2\int\csc^4(c+dx)\,dx+a^2\int\sin^2(c\\ &= -a^2x+\frac{4a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx}{d}\\ &= -\frac{a^2x}{2}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+}{3d}\end{aligned}$$

Mathematica [A]
time = 4.23, size = 191, normalized size = 1.95

$$\frac{a^2(1+\sin(c+dx))^2\left(-12(c+dx)-48\cos(c+dx)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\csc^2\left(\frac{1}{2}(c+dx)\right)+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-72\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\sec^2\left(\frac{1}{2}(c+dx)\right)+8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-\frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-6\sin(2(c+dx))-4\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
```

```
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.19, size = 146, normalized size = 1.49

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-}{d}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{a^2e^{i(dx+c)}}{d}-\frac{a^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2a^2\left(3ie^{4i(dx+c)}+3\,e^{5i(dx+c)}+i-3\,e^{i(dx}\right.}{3d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{\frac{5a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{24d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{11a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{11a^2\left(\tan^6\left(\frac{dx}{2}\right.\right.}{12d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a^2*(-1/2/sin(d*x+c)^2*cos(
d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2
*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d
*x-3/2*c))
```

Maxima [A]
time = 0.49, size = 139, normalized size = 1.42

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(92) = 184$.
time = 0.37, size = 192, normalized size = 1.96

$$\frac{3\,a^2\cos\left(dx+c\right)^5-4\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)+9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(a^2dx\cos\left(dx+c\right)^2+4\,a^2\cos\left(dx+c\right)^3-a^2dx-6\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(92) = 184.
time = 0.59, size = 209, normalized size = 2.13

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)a^2-72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{24\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{132\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.81, size = 293, normalized size = 2.99

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,\mathrm{atan}\left(\frac{a^4}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^4-a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+34\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{19\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+36\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*atan(a^4/(6*a^4 - a^4*tan(c/2 + (d*x)/
```

```
2)) + (6*a^4*tan(c/2 + (d*x)/2))/(6*a^4 - a^4*tan(c/2 + (d*x)/2))))/d - (36
*a^2*tan(c/2 + (d*x)/2)^3 - (a^2*tan(c/2 + (d*x)/2)^2)/3 + (19*a^2*tan(c/2
+ (d*x)/2)^4)/3 + 34*a^2*tan(c/2 + (d*x)/2)^5 - 9*a^2*tan(c/2 + (d*x)/2)^6
+ a^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2
 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2))/(8*d)
```

3.386 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$2a^2x+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 2*a^2*x+9/8*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d+2*a^2*cot(d*x+c)/d-2
/3*a^2*cot(d*x+c)^3/d+1/8*a^2*cot(d*x+c)*csc(d*x+c)/d-1/4*a^2*cot(d*x+c)*cs
c(d*x+c)^3/d
```

Rubi [A]

time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2951, 3855, 3852, 8, 3853, 2718}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d}+\frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a^2\cot(c+dx)\csc(c+dx)}{8d}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (9*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d + (2*a
^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x]*Csc[c
 + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^6 - a^6\csc(c+dx) - 4a^6\csc^2(c+dx) - a^6\csc^3(c\right.}{}$$

$$= 2a^2x - a^2\int \csc(c+dx)\,dx - a^2\int \csc^3(c+dx)\,dx + a$$

$$= 2a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} + \frac{a^2}{}$$

$$= 2a^2x + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a}{}$$

$$= 2a^2x + \frac{9a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} + \frac{2a}{}$$

Mathematica [A]
time = 0.92, size = 215, normalized size = 1.85

$$-\frac{a^2\left(192\cot(c+dx)+\csc^4\left(\frac{1}{2}(c+dx)\right)(8+3\csc(c+dx))-2\csc^2\left(\frac{1}{2}(c+dx)\right)(64+3\csc(c+dx))-24\csc(c+dx)\left(16(c+dx)+9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+8(7+8\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)+24\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-48\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)(1+\sin(c+dx))^2}{192d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/192*(a^2*(192*Cot[c + d*x] + Csc[(c + d*x)/2]^4*(8 + 3*Csc[c + d*x]) - 2
*Csc[(c + d*x)/2]^2*(64 + 3*Csc[c + d*x]) - 24*Csc[c + d*x]*(16*(c + d*x) +
 9*Log[Cos[(c + d*x)/2]] - 9*Log[Sin[(c + d*x)/2]]) + 8*(7 + 8*Cos[c + d*x]
)*Sec[(c + d*x)/2]^4 + 24*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 48*Csc[c + d*
x]^5*Sin[(c + d*x)/2]^4)*Sin[c + d*x]*(1 + Sin[c + d*x])^2)/(d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.22, size = 168, normalized size = 1.45

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{d}$
risch	$2a^2x-\frac{a^2e^{i(dx+c)}}{2d}-\frac{a^2e^{-i(dx+c)}}{2d}-\frac{a^2\left(3\,e^{7i(dx+c)}+21\,e^{5i(dx+c)}-96ie^{6i(dx+c)}+21\,e^{3i(dx+c)}+192ie^{4i(dx+c)}+3\,e^{i(\right.}{12d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}-\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{13a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{7a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{7a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{13a^2\left(\tan^9\left(\frac{dx}{2}\right.\right.}{12d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*
cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/3*cot(
d*x+c)^3+cot(d*x+c)+d*x+c)+a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+
c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 167, normalized size = 1.44

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(108) = 216.
time = 0.36, size = 219, normalized size = 1.89

$$\frac{96\,a^2dx\cos\left(dx+c\right)^4-48\,a^2\cos\left(dx+c\right)^5-192\,a^2dx\cos\left(dx+c\right)^2+90\,a^2\cos\left(dx+c\right)^3+96\,a^2dx-54\,a^2\cos\left(dx+c\right)+27\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-27\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-32\left(4\,a^2\cos\left(dx+c\right)^3-3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a^2*d*x*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^5 - 192*a^2*d*x*cos(d
*x + c)^2 + 90*a^2*cos(d*x + c)^3 + 96*a^2*d*x - 54*a^2*cos(d*x + c) + 27*(
a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2
) - 27*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x +
 c) + 1/2) - 32*(4*a^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c))*sin(d*x + c))/(
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.69, size = 162, normalized size = 1.40

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+384\,(dx+c)a^2-216\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{384\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 + 384*(
d*x + c)*a^2 - 216*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 240*a^2*tan(1/2*d*x
 + 1/2*c) - 384*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (450*a^2*tan(1/2*d*x + 1
/2*c)^4 + 240*a^2*tan(1/2*d*x + 1/2*c)^3 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*
a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.77, size = 265, normalized size = 2.28

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{9\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{4\,a^2\,\mathrm{atan}\left(\frac{16\,a^4}{9\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{9\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^4+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}-\frac{-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{56\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a^2}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (9*
a^2*log(tan(c/2 + (d*x)/2)))/(8*d) - (4*a^2*atan((16*a^4)/(9*a^4 + 16*a^4*t
an(c/2 + (d*x)/2)) - (9*a^4*tan(c/2 + (d*x)/2))/(9*a^4 + 16*a^4*tan(c/2 + (
d*x)/2))))/d - (5*a^2*tan(c/2 + (d*x)/2))/(4*d) - ((a^2*tan(c/2 + (d*x)/2)^
2)/4 - (56*a^2*tan(c/2 + (d*x)/2)^3)/3 + 32*a^2*tan(c/2 + (d*x)/2)^4 - 20*a
^2*tan(c/2 + (d*x)/2)^5 + a^2/4 + (4*a^2*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(
c/2 + (d*x)/2)^4 + 16*tan(c/2 + (d*x)/2)^6))
```

3.387 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=118

$$a^2x-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{4d}$$

```
[Out] a^2*x-3/4*a^2*arctanh(cos(d*x+c))/d+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d
-1/5*a^2*cot(d*x+c)^5/d+3/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)^
3*csc(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{4d}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (a^2*Cot[c + d*x])/d - (a^2*C
ot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) + (3*a^2*Cot[c + d*x]*Csc
[c + d*x])/(4*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)+2a^2\cot^4(c+dx)\csc(c+dx)+a\right.$$

$$= a^2\int \cot^4(c+dx)\,dx + a^2\int \cot^4(c+dx)\csc^2(c+dx$$

$$= -\frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^3(c+dx)\csc(c+dx)}{2d} - a^2$$

$$= \frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} +$$

$$= a^2x - \frac{3a^2\tanh^{-1}(\cos(c+dx))}{4d} + \frac{a^2\cot(c+dx)}{d} - \frac{a^2}{}$$

Mathematica [A]
time = 0.40, size = 200, normalized size = 1.69

$$\frac{a^2\left(480c+480dx+272\cot\left(\frac{1}{2}(c+dx)\right)+150\csc^2\left(\frac{1}{2}(c+dx)\right)-360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-150\sec^2\left(\frac{1}{2}(c+dx)\right)+15\sec^4\left(\frac{1}{2}(c+dx)\right)-8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+96\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)(-30+\sin(c+dx))-\frac{3}{2}\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-272\tan\left(\frac{1}{2}(c+dx)\right)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(480*c + 480*d*x + 272*Cot[(c + d*x)/2] + 150*Csc[(c + d*x)/2]^2 - 360
*Log[Cos[(c + d*x)/2]] + 360*Log[Sin[(c + d*x)/2]] - 150*Sec[(c + d*x)/2]^2
```

```
+ 15*Sec[(c + d*x)/2]^4 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 96*Csc[c +
d*x]^5*Sin[(c + d*x)/2]^6 + (Csc[(c + d*x)/2]^4*(-30 + Sin[c + d*x]))/2 -
(3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 - 272*Tan[(c + d*x)/2]))/(480*d)
```

Maple [A]
time = 0.25, size = 130, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^2\left(-\frac{(\cot \cdots}{}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a^2\left(-\frac{(\cot \cdots}{}\right.}{d}$
risch	$a^2x-\frac{a^2\left(-60ie^{8i(dx+c)}+75\,e^{9i(dx+c)}+360ie^{6i(dx+c)}-30\,e^{7i(dx+c)}-320ie^{4i(dx+c)}+280ie^{2i(dx+c)}+30\,e^{3i(dx+c)}-68i-\cdots\right.}{30d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{a^2x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2}{160d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}-\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{257a^2\left(\tan^4 \cdots\right.}{480 \cdots}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^2/sin(d*x+c)^5*cos(d*x+c)^5+2*a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)
^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc
(d*x+c)-cot(d*x+c)))+a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 124, normalized size = 1.05

$$\frac{40\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-15\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{24\,a^2}{\tan(dx+c)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 15*a^
2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(108) = 216.
time = 0.37, size = 239, normalized size = 2.03

$$\frac{136\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3+120\,a^2\cos\left(dx+c\right)-45\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+45\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\left(4\,a^2dx\cos\left(dx+c\right)^4-8\,a^2dx\cos\left(dx+c\right)^2-5\,a^2\cos\left(dx+c\right)^3+4\,a^2dx+3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x + c)^3 + 120*a^2*cos(d*x +
c) - 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 45*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a
^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(4*a^2*d*x*cos(d*x + c)^
4 - 8*a^2*d*x*cos(d*x + c)^2 - 5*a^2*cos(d*x + c)^3 + 4*a^2*d*x + 3*a^2*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.64, size = 207, normalized size = 1.75

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+480\,(dx+c)a^2+360\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{822\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a^
2 + 360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 270*a^2*tan(1/2*d*x + 1/2*c) -
 (822*a^2*tan(1/2*d*x + 1/2*c)^5 - 270*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2
*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*tan(1/2*d*x
 + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 9.16, size = 275, normalized size = 2.33

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{9\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^2)/(4*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(96*d) - (a^2
*cot(c/2 + (d*x)/2)^4)/(32*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(160*d) - (a^2*t
an(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (a^2*tan(c
/2 + (d*x)/2)^4)/(32*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (2*a^2*atan(
(4*cos(c/2 + (d*x)/2) + 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/2) - 4*sin
(c/2 + (d*x)/2))))/d + (3*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(
4*d) + (9*a^2*cot(c/2 + (d*x)/2))/(16*d) - (9*a^2*tan(c/2 + (d*x)/2))/(16*d
)
```

3.388 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{5a^2\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2}{}$$

```
[Out] -7/16*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d+5/16*a^2*cot(d*x+c)*
csc(d*x+c)/d-1/4*a^2*cot(d*x+c)^3*csc(d*x+c)/d+1/8*a^2*cot(d*x+c)*csc(d*x+c
)^3/d-1/6*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3855, 2687, 30, 3853}

$$-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{a^2\cot(c+dx)\csc^3(c+dx)}{8d}+\frac{5a^2\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(16*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) + (5*a
^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x])/(4
*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c +
 d*x]^3)/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^4(c+dx)\csc(c+dx)+2a^2\cot^4(c+dx)\csc^2(\right.\\
&= a^2\int \cot^4(c+dx)\csc(c+dx)\,dx+a^2\int \cot^4(c+dx)\,\mathrm{cs}\\
&= -\frac{a^2\cot^3(c+dx)\csc(c+dx)}{4d}-\frac{a^2\cot^3(c+dx)\csc^3(c-}{6d}\\
&= -\frac{2a^2\cot^5(c+dx)}{5d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2}{}\\
&= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{5a^2\,\mathrm{co}}{}\\
&= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{5a^2\,\mathrm{co}}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(132) = 264.
time = 0.08, size = 267, normalized size = 2.02

$$a^2\left(-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{5d}+\frac{9\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{7\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{80d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{80d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{7\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}+\frac{7\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{9\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{5d}-\frac{7\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{80d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(-1/5*Cot[(c + d*x)/2]/d + (9*Csc[(c + d*x)/2]^2)/(64*d) + (7*Cot[(c +
d*x)/2]*Csc[(c + d*x)/2]^2)/(80*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/
(80*d) - Csc[(c + d*x)/2]^6/(384*d) - (7*Log[Cos[(c + d*x)/2]])/(16*d) + (7
*Log[Sin[(c + d*x)/2]])/(16*d) - (9*Sec[(c + d*x)/2]^2)/(64*d) + Sec[(c + d
*x)/2]^6/(384*d) + Tan[(c + d*x)/2]/(5*d) - (7*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(80*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))
```

Maple [A]
time = 0.26, size = 199, normalized size = 1.51

method	result
risch	$-\frac{a^2\left(135\,e^{11i(dx+c)}-445\,e^{9i(dx+c)}+480ie^{10i(dx+c)}-330\,e^{7i(dx+c)}-480ie^{8i(dx+c)}-330\,e^{5i(dx+c)}+960ie^{6i(dx+c)}-445\,e\right.}{120d\left(e^{2i(dx+c)}-1\right)^6}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{2a^2\left(\cos^5(dx+c)\right.}{5\sin(dx+c)^5}}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{2a^2\left(\cos^5(dx+c)\right.}{5\sin(dx+c)^5}}{d}$
norman	$-\frac{a^2}{384d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}-\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{3a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}+\frac{11a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{a^2\left(\tan^7\left(\frac{dx}{2}\right.\right.}{16d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/4
8/sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d
*x+c)-cot(d*x+c)))-2/5*a^2/sin(d*x+c)^5*cos(d*x+c)^5+a^2*(-1/4/sin(d*x+c)^4
*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)
+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 181, normalized size = 1.37

$$\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-30\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{192\,a^2}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*a^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a^2/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.37, size = 211, normalized size = 1.60

$$\frac{192 a^{2} \cos \left(d x+c\right)^{5} \sin \left(d x+c\right)-270 a^{2} \cos \left(d x+c\right)^{5}+560 a^{2} \cos \left(d x+c\right)^{3}-210 a^{2} \cos \left(d x+c\right)-105 \left(a^{2} \cos \left(d x+c\right)^{6}-3 a^{2} \cos \left(d x+c\right)^{4}+3 a^{2} \cos \left(d x+c\right)^{2}-a^{2}\right) \log \left(\frac{1}{2} \cos \left(d x+c\right)+\frac{1}{2}\right)+105 \left(a^{2} \cos \left(d x+c\right)^{6}-3 a^{2} \cos \left(d x+c\right)^{4}+3 a^{2} \cos \left(d x+c\right)^{2}-a^{2}\right) \log \left(-\frac{1}{2} \cos \left(d x+c\right)+\frac{1}{2}\right)}{480 \left(d \cos \left(d x+c\right)^{6}-3 d \cos \left(d x+c\right)^{4}+3 d \cos \left(d x+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a^2*cos(d*x + c)^5*sin(d*x + c) - 270*a^2*cos(d*x + c)^5 + 560*a
^2*cos(d*x + c)^3 - 210*a^2*cos(d*x + c) - 105*(a^2*cos(d*x + c)^6 - 3*a^2*
cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) +
105*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2
)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*
d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 229, normalized size = 1.73

$$\frac{5 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}+24 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+15 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4}-120 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}-255 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}+840 a^{2} \log \left(\left|\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)\right|\right)+240 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)-\frac{2058 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}+240 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}-255 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4}-120 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+15 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}+24 a^{2} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)+5 a^{2}}{\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 240*a^2*tan(1/2
*d*x + 1/2*c) - (2058*a^2*tan(1/2*d*x + 1/2*c)^6 + 240*a^2*tan(1/2*d*x + 1/
2*c)^5 - 255*a^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*tan(1/2*d*x + 1/2*c)^3 +
15*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/
2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.75, size = 339, normalized size = 2.57

$$\frac{a^{2} \left(5 \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{12}-5 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{12}+24 \cos \left(\frac{c}{2}+\frac{d x}{2}\right) \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{11}-24 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{11} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)+15 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{2} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{10}-120 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{3} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{9}-255 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{4} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{8}+240 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{5} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{7}-240 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{7} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{5}+255 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{8} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{4}+120 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{9} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{3}-15 \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{10} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{2}+840 \ln \left(\frac{\sin \left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos \left(\frac{c}{2}+\frac{d x}{2}\right)}\right) \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{6} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{6}\right)}{1920 d \cos \left(\frac{c}{2}+\frac{d x}{2}\right)^{6} \sin \left(\frac{c}{2}+\frac{d x}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (a^2*(5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 + 24*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^11 - 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) +
15*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 120*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^9 - 255*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 240*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 240*cos(c/2 + (d*x)/2)^7*sin(c/2
+ (d*x)/2)^5 + 255*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2
+ (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^
6*sin(c/2 + (d*x)/2)^6))/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.389 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=176

$$-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{128d}+\frac{7a^2\cot(c+$$

```
[Out] -11/128*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)
^7/d-11/128*a^2*cot(d*x+c)*csc(d*x+c)/d+7/64*a^2*cot(d*x+c)*csc(d*x+c)^3/d-
1/6*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d+1/16*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/8*
a^2*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.22, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3853, 3855, 2687, 14}

$$-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a^2\cot(c+dx)\csc^5(c+dx)}{16d}+\frac{7a^2\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{11a^2\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2
*a^2*Cot[c + d*x]^7)/(7*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
7*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^3
*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^3(c+dx)+2a^2\cot^4(c+dx)\operatorname{csc}\right.$$

$$= a^2\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^4(c+dx$$

$$= -\frac{a^2\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{a^2\cot^3(c+dx)\csc^5(}{8d}$$

$$= \frac{a^2\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^3(c+}{6d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+d}{1}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\operatorname{co}}{}$$

$$= -\frac{11a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2}{}$$

Mathematica [A]
time = 0.84, size = 291, normalized size = 1.65

$$-\frac{a^2\csc^8(c+dx)\left(158270\cos(c+dx)+77210\cos(3(c+dx))-18130\cos(5(c+dx))-2310\cos(7(c+dx))+40425\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-64680\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+32340\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-9240\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1155\cos(8(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-40425\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+64680\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-32340\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+9240\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-1155\cos(8(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+86016\sin(2(c+dx))+64512\sin(4(c+dx))+12288\sin(6(c+dx))-1536\sin(8(c+dx))\right)}{1720320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1720320*(a^2*Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)
] - 18130*Cos[5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x
)/2]] - 64680*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x
)]*Log[Cos[(c + d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 11
55*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 6
4680*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Si
n[(c + d*x)/2]] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*
(c + d*x)]*Log[Sin[(c + d*x)/2]] + 86016*Sin[2*(c + d*x)] + 64512*Sin[4*(c
+ d*x)] + 12288*Sin[6*(c + d*x)] - 1536*Sin[8*(c + d*x)]))/d
```

Maple [A]
time = 0.27, size = 256, normalized size = 1.45

method	result
risch	$\frac{a^2\left(1155\,\mathrm{e}^{15i(dx+c)}+9065\,\mathrm{e}^{13i(dx+c)}-38605\,\mathrm{e}^{11i(dx+c)}+53760i\mathrm{e}^{12i(dx+c)}-79135\,\mathrm{e}^{9i(dx+c)}-79135\,\mathrm{e}^{7i(dx+c)}+53760i\mathrm{e}^{8i(d}\right.}{6720d\left(\mathrm{e}^{2i(dx+\right.}}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)-}{}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/6
4/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c
)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/7/sin(d*x+c
)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+a^2*(-1/6/sin(d*x+c)^6*cos
(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c)^2*cos(d*x+c)^5+1/4
8*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 233, normalized size = 1.32

$$\frac{105\,a^2\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+280\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{1536\left(7\,\tan(dx+c)^2+5\right)a^2}{\tan(dx+c)^7}}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/26880*(105*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 280*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x +
 c)^7)/d
```

Fricas [**A**]
time = 0.35, size = 271, normalized size = 1.54

$$\frac{2310\,a^2\cos\left(dx+c\right)^7+490\,a^2\cos\left(dx+c\right)^5-8470\,a^2\cos\left(dx+c\right)^3+2310\,a^2\cos\left(dx+c\right)-1155\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+1155\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+1536\,\left(2\,a^2\cos\left(dx+c\right)^7-7\,a^2\cos\left(dx+c\right)^5\right)\sin\left(dx+c\right)}{26880\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*a^2*cos(d*x + c)^7 + 490*a^2*cos(d*x + c)^5 - 8470*a^2*cos(d*
x + c)^3 + 2310*a^2*cos(d*x + c) - 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x
 + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*
x + c) + 1/2) + 1155*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos
(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) + 15
36*(2*a^2*cos(d*x + c)^7 - 7*a^2*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
 c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 293, normalized size = 1.66

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1680\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+18480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+10080\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{50226\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+10080\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1680\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^2*tan(1/2*d*x + 1/2*c)^8 + 480*a^2*tan(1/2*d*x + 1/2*c)^7 +
 560*a^2*tan(1/2*d*x + 1/2*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 - 1680*a^2*tan(1/
```

```
2*d*x + 1/2*c)^2 + 18480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 10080*a^2*tan
(1/2*d*x + 1/2*c) - (50226*a^2*tan(1/2*d*x + 1/2*c)^8 + 10080*a^2*tan(1/2*d
*x + 1/2*c)^7 - 1680*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*a^2*tan(1/2*d*x + 1/
2*c)^5 - 2520*a^2*tan(1/2*d*x + 1/2*c)^4 - 672*a^2*tan(1/2*d*x + 1/2*c)^3 +
 560*a^2*tan(1/2*d*x + 1/2*c)^2 + 480*a^2*tan(1/2*d*x + 1/2*c) + 105*a^2)/t
an(1/2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 9.12, size = 319, normalized size = 1.81

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64\,d}+\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{320\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{448\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64\,d}-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{320\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{448\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}+\frac{11\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,d}-\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,d}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^2)/(128*d) + (a^2*cot(c/2 + (d*x)/2)^3)/(64*d) + (3
*a^2*cot(c/2 + (d*x)/2)^4)/(256*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(320*d) - (
a^2*cot(c/2 + (d*x)/2)^6)/(384*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(448*d) - (a
^2*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(128*d) - (a
^2*tan(c/2 + (d*x)/2)^3)/(64*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(256*d) - (a
^2*tan(c/2 + (d*x)/2)^5)/(320*d) + (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) + (a^
2*tan(c/2 + (d*x)/2)^7)/(448*d) + (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) + (11
*a^2*log(tan(c/2 + (d*x)/2)))/(128*d) - (3*a^2*cot(c/2 + (d*x)/2))/(64*d) +
 (3*a^2*tan(c/2 + (d*x)/2))/(64*d)
```

3.390 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot(c+dx)\csc(c+a}{64d}$$

```
[Out] -3/64*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-3/7*a^2*cot(d*x+c)^7
/d-1/9*a^2*cot(d*x+c)^9/d-3/64*a^2*cot(d*x+c)*csc(d*x+c)/d-1/32*a^2*cot(d*x
+c)*csc(d*x+c)^3/d+1/8*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/4*a^2*cot(d*x+c)^3*c
sc(d*x+c)^5/d
```

Rubi [A]
time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 14, 2691, 3853, 3855, 276}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{3a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d}+\frac{a^2\cot(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{32d}-\frac{3a^2\cot(c+dx)\csc(c+dx)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (3*a
^2*Cot[c + d*x]^7)/(7*d) - (a^2*Cot[c + d*x]^9)/(9*d) - (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(64*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (a^2*Cot
[c + d*x]*Csc[c + d*x]^5)/(8*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^4(c+dx) + 2a^2\cot^4(c+dx)\csc^5\right.$$

$$= a^2\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^4(c+dx)$$

$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{4d} - \frac{1}{4}\left(3a^2\right)\int \cot^2(c+dx$$

$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^5(c+}{4d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx}{9d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx}{9d}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{3a^2\text{co}}{}$$

Mathematica [A]

time = 0.94, size = 313, normalized size = 1.86

$$-\frac{a^2\csc^9(c+dx)\left(451584\cos(c+dx)+155904\cos(3(c+dx))-20736\cos(5(c+dx))-14976\cos(7(c+dx))+1664\cos(9(c+dx))+119070\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-119070\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+212940\sin(2(c+dx))-79380\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+79380\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+195300\sin(4(c+dx))+34020\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-34020\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+16380\sin(6(c+dx))-8505\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))+8505\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))-1890\sin(8(c+dx))+945\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(9(c+dx))-945\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(9(c+dx))\right)}{5160960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/5160960*(a^2*Csc[c + d*x]^9*(451584*Cos[c + d*x] + 155904*Cos[3*(c + d*x
)] - 20736*Cos[5*(c + d*x)] - 14976*Cos[7*(c + d*x)] + 1664*Cos[9*(c + d*x)
] + 119070*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[
3*(c + d*x)] + 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*
(c + d*x)] + 34020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(
c + d*x)/2]]*Sin[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c +
d*x)/2]]*Sin[7*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1
890*Sin[8*(c + d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log
[Sin[(c + d*x)/2]]*Sin[9*(c + d*x)]))/d
```

Maple [A]

time = 0.28, size = 220, normalized size = 1.31

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{9\sin(dx+c)^9}-\frac{4\left(\cos^5(dx+c)\right)}{63\sin(dx+c)^7}-\frac{8\left(\cos^5(dx+c)\right)}{315\sin(dx+c)^5}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{9\sin(dx+c)^9}-\frac{4\left(\cos^5(dx+c)\right)}{63\sin(dx+c)^7}-\frac{8\left(\cos^5(dx+c)\right)}{315\sin(dx+c)^5}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}\right.}{d}$
risch	$a^2\left(945\,e^{17i(dx+c)}-19584ie^{4i(dx+c)}-8190\,e^{15i(dx+c)}-8064ie^{6i(dx+c)}-97650\,e^{13i(dx+c)}+14976ie^{2i(dx+c)}-106470\,e^{11i\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^5-4/63/sin(d*x+c)^7*cos(d*x+c)^5-8/3
15/sin(d*x+c)^5*cos(d*x+c)^5)+2*a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/si
n(d*x+c)^6*cos(d*x+c)^5-1/64/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*c
os(d*x+c)^5+1/128*cos(d*x+c)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x
+c)))+a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5))
```

Maxima [A]

time = 0.29, size = 177, normalized size = 1.05

$$\frac{315\,a^2\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{1152\left(7\tan(dx+c)^2+5\right)a^2}{\tan(dx+c)^7}-\frac{128\left(63\tan(dx+c)^4+90\tan(dx+c)^2+35\right)a^2}{\tan(dx+c)^9}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/40320*(315*a^2*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) - 1152*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7 - 128*(63*tan(d*x + c)^4
 + 90*tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [A]
time = 0.36, size = 304, normalized size = 1.81

$$-\frac{3328\,a^2\cos\left(dx+c\right)^9-14976\,a^2\cos\left(dx+c\right)^7+16128\,a^2\cos\left(dx+c\right)^5+945\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-945\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-630\,\left(3\,a^2\cos\left(dx+c\right)^7-11\,a^2\cos\left(dx+c\right)^5-11\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*a^2*cos(d*x + c)^9 - 14976*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d
*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 945*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)
^4 - 4*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
- 630*(3*a^2*cos(d*x + c)^7 - 11*a^2*cos(d*x + c)^5 - 11*a^2*cos(d*x + c)^3
 + 3*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^
6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 261, normalized size = 1.55

$$\frac{70\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1008\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+11340\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{42774\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+11340\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-3360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1008\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+450\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+70\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/322560*(70*a^2*tan(1/2*d*x + 1/2*c)^9 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 +
450*a^2*tan(1/2*d*x + 1/2*c)^7 - 1008*a^2*tan(1/2*d*x + 1/2*c)^5 - 2520*a^2
*tan(1/2*d*x + 1/2*c)^4 - 3360*a^2*tan(1/2*d*x + 1/2*c)^3 + 15120*a^2*log(a
bs(tan(1/2*d*x + 1/2*c))) + 11340*a^2*tan(1/2*d*x + 1/2*c) - (42774*a^2*tan
(1/2*d*x + 1/2*c)^9 + 11340*a^2*tan(1/2*d*x + 1/2*c)^8 - 3360*a^2*tan(1/2*d
*x + 1/2*c)^6 - 2520*a^2*tan(1/2*d*x + 1/2*c)^5 - 1008*a^2*tan(1/2*d*x + 1/
2*c)^4 + 450*a^2*tan(1/2*d*x + 1/2*c)^2 + 315*a^2*tan(1/2*d*x + 1/2*c) + 70
*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 12.07, size = 387, normalized size = 2.30

$$\frac{a^2\left(70\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-70\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+11340\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-11340\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+15120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\right)}{322560\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*(70*sin(c/2 + (d*x)/2)^18 - 70*cos(c/2 + (d*x)/2)^18 + 315*cos(c/2 + (
d*x)/2)*sin(c/2 + (d*x)/2)^17 - 315*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2
) + 450*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 - 1008*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^14 - 2520*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13
 - 3360*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 + 11340*cos(c/2 + (d*x)/
2)^8*sin(c/2 + (d*x)/2)^10 - 11340*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^8 + 3360*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 2520*cos(c/2 + (d*x)
/2)^13*sin(c/2 + (d*x)/2)^5 + 1008*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)
^4 - 450*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9))/(32
2560*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

3.391 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=218

$$-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot(c+dx)\csc(c+d}{256d}$$

```
[Out] -9/256*a^2*arctanh(cos(d*x+c))/d-2/5*a^2*cot(d*x+c)^5/d-4/7*a^2*cot(d*x+c)^
7/d-2/9*a^2*cot(d*x+c)^9/d-9/256*a^2*cot(d*x+c)*csc(d*x+c)/d-3/128*a^2*cot(
d*x+c)*csc(d*x+c)^3/d+9/160*a^2*cot(d*x+c)*csc(d*x+c)^5/d-1/8*a^2*cot(d*x+c
)^3*csc(d*x+c)^5/d+3/80*a^2*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^2*cot(d*x+c)^3
*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.25, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3853, 3855, 2687, 276}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{4a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{10d}-\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{3a^2\cot(c+dx)\csc^7(c+dx)}{80d}+\frac{9a^2\cot(c+dx)\csc^5(c+dx)}{160d}-\frac{3a^2\cot(c+dx)\csc^3(c+dx)}{128d}-\frac{9a^2\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (4*
a^2*Cot[c + d*x]^7)/(7*d) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (9*a^2*Cot[c + d
*x]*Csc[c + d*x])/(256*d) - (3*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (
9*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (a^2*Cot[c + d*x]^3*Csc[c + d*
x]^5)/(8*d) + (3*a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x
]^3*Csc[c + d*x]^7)/(10*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^4(c+dx)\csc^5(c+dx)+2a^2\cot^4(c+dx)\csc\right.$$

$$= a^2\int \cot^4(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^4(c+dx$$

$$= -\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^3(c+dx)\csc^7(}{10d}$$

$$= \frac{a^2\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^3(c+dx)\csc^5(c+}{8d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+}{9d}$$

$$= -\frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+}{9d}$$

$$= -\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\,c}{}$$

$$= -\frac{9a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{4a^2\,c}{}$$

Mathematica [A]
time = 0.82, size = 353, normalized size = 1.62

$$-\frac{a^2\csc^{10}(c+dx)\left(3219300\cos(c+dx)+1237320\cos(3(c+dx))-278712\cos(5(c+dx))-54810\cos(7(c+dx))+5670\cos(9(c+dx))+357210\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-595350\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+340200\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-127575\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+28350\cos(8(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2835\cos(10(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-357210\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+595350\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-340200\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+127575\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-28350\cos(8(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2835\cos(10(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+1720320\sin(2(c+dx))+1228800\sin(4(c+dx))+184320\sin(6(c+dx))-40960\sin(8(c+dx))+4096\sin(10(c+dx))\right)}{41287680d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/41287680*(a^2*Csc[c + d*x]^10*(3219300*Cos[c + d*x] + 1237320*Cos[3*(c +
 d*x)] - 278712*Cos[5*(c + d*x)] - 54810*Cos[7*(c + d*x)] + 5670*Cos[9*(c +
 d*x)] + 357210*Log[Cos[(c + d*x)/2]] - 595350*Cos[2*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] + 340200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 127575*Cos[6*(
c + d*x)]*Log[Cos[(c + d*x)/2]] + 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/
2]] - 2835*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 357210*Log[Sin[(c + d*
x)/2]] + 595350*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 340200*Cos[4*(c +
d*x)]*Log[Sin[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]]
 - 28350*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2835*Cos[10*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] +
 184320*Sin[6*(c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)])
)/d
```

Maple [A]
time = 0.28, size = 310, normalized size = 1.42

method	result
risch	$\frac{a^2\left(2835\,\mathrm{e}^{19i(dx+c)}-27405\,\mathrm{e}^{17i(dx+c)}-139356\,\mathrm{e}^{15i(dx+c)}+618660\,\mathrm{e}^{13i(dx+c)}+184320i\mathrm{e}^{4i(dx+c)}+1609650\,\mathrm{e}^{11i(dx+c)}+129\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{32\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{128\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{256\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{256}+\frac{3\cos(dx+c)}{256}+\frac{3\ln(\csc(dx\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{32\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{128\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{256\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{256}+\frac{3\cos(dx+c)}{256}+\frac{3\ln(\csc(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/10/sin(d*x+c)^10*cos(d*x+c)^5-1/16/sin(d*x+c)^8*cos(d*x+c)^5-1
/32/sin(d*x+c)^6*cos(d*x+c)^5-1/128/sin(d*x+c)^4*cos(d*x+c)^5+1/256/sin(d*x
+c)^2*cos(d*x+c)^5+1/256*cos(d*x+c)^3+3/256*cos(d*x+c)+3/256*ln(csc(d*x+c)-
cot(d*x+c)))+2*a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^5-4/63/sin(d*x+c)^7*cos(d*
x+c)^5-8/315/sin(d*x+c)^5*cos(d*x+c)^5)+a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5
-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/64/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*
x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)
-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 283, normalized size = 1.30

$$\frac{63\,a^{2}\left(\frac{2\,\left(15\,\cos\left(dx+c\right)^{9}-70\,\cos\left(dx+c\right)^{7}+128\,\cos\left(dx+c\right)^{5}+70\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+630\,a^{2}\left(\frac{2\,\left(3\,\cos\left(dx+c\right)^{7}-11\,\cos\left(dx+c\right)^{5}-11\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{1024\,\left(63\,\tan\left(dx+c\right)^{4}+90\,\tan\left(dx+c\right)^{2}+35\right)a^{2}}{\tan\left(dx+c\right)^{9}}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^2*(2*(3*cos(d*x +
c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c
)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(c
os(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1024*(63*tan(d*x + c)^4 + 90*
tan(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [A]
time = 0.39, size = 340, normalized size = 1.56

$$\frac{5670\,a^{2}\cos\left(dx+c\right)^{9}-26460\,a^{2}\cos\left(dx+c\right)^{7}+16128\,a^{2}\cos\left(dx+c\right)^{5}+26460\,a^{2}\cos\left(dx+c\right)^{3}-5670\,a^{2}\cos\left(dx+c\right)-2835\,\left(a^{2}\cos\left(dx+c\right)^{10}-5\,a^{2}\cos\left(dx+c\right)^{8}+10\,a^{2}\cos\left(dx+c\right)^{6}-10\,a^{2}\cos\left(dx+c\right)^{4}+5\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2835\,\left(a^{2}\cos\left(dx+c\right)^{10}-5\,a^{2}\cos\left(dx+c\right)^{8}+10\,a^{2}\cos\left(dx+c\right)^{6}-10\,a^{2}\cos\left(dx+c\right)^{4}+5\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+1024\,\left(8\,a^{2}\cos\left(dx+c\right)^{9}-36\,a^{2}\cos\left(dx+c\right)^{7}+63\,a^{2}\cos\left(dx+c\right)^{5}\right)\sin\left(dx+c\right)}{161280\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*a^2*cos(d*x + c)^9 - 26460*a^2*cos(d*x + c)^7 + 16128*a^2*co
s(d*x + c)^5 + 26460*a^2*cos(d*x + c)^3 - 5670*a^2*cos(d*x + c) - 2835*(a^2
*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*co
s(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 28
35*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10
*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1
/2) + 1024*(8*a^2*cos(d*x + c)^9 - 36*a^2*cos(d*x + c)^7 + 63*a^2*cos(d*x +
 c)^5)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x
 + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 357, normalized size = 1.64

$$\frac{126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-4032\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-7560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-6720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1260\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+45360\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+30240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{132858\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+30240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1260\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-6720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-7560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-4032\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+945\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+560\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+126\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 945*a^2*tan(1/2*d*x + 1/2*c)^8 + 720*a^2*tan(1/2*d*x + 1/2*c)^7 - 630*a^
2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 7560*a^2*tan(1
/2*d*x + 1/2*c)^4 - 6720*a^2*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x
+ 1/2*c)^2 + 45360*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 30240*a^2*tan(1/2*d
*x + 1/2*c) - (132858*a^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x +
 1/2*c)^9 + 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 6720*a^2*tan(1/2*d*x + 1/2*c)
^7 - 7560*a^2*tan(1/2*d*x + 1/2*c)^6 - 4032*a^2*tan(1/2*d*x + 1/2*c)^5 - 63
0*a^2*tan(1/2*d*x + 1/2*c)^4 + 720*a^2*tan(1/2*d*x + 1/2*c)^3 + 945*a^2*tan
(1/2*d*x + 1/2*c)^2 + 560*a^2*tan(1/2*d*x + 1/2*c) + 126*a^2)/tan(1/2*d*x +
 1/2*c)^10)/d
```

Mupad [B]
time = 9.52, size = 395, normalized size = 1.81

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}+\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{320\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{1792\,d}-\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2304\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192\,d}-\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{320\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{1792\,d}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2304\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}+\frac{9\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{256\,d}-\frac{3\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^2)/sin(c + d*x)^11,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^3)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(1024*d) +
(3*a^2*cot(c/2 + (d*x)/2)^4)/(512*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(320*d) +
 (a^2*cot(c/2 + (d*x)/2)^6)/(2048*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(1792*d)
- (3*a^2*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(2304*
d) - (a^2*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a^2*tan(c/2 + (d*x)/2)^2)/(10
24*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(192*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(
512*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(320*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(2
048*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(1792*d) + (3*a^2*tan(c/2 + (d*x)/2)^8)
/(4096*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(2304*d) + (a^2*tan(c/2 + (d*x)/2)^1
0)/(10240*d) + (9*a^2*log(tan(c/2 + (d*x)/2)))/(256*d) - (3*a^2*cot(c/2 + (
d*x)/2))/(128*d) + (3*a^2*tan(c/2 + (d*x)/2))/(128*d)
```

3.392 $\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=203

$$\frac{15a^3x}{256}-\frac{4a^3\cos^5(c+dx)}{5d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{15a^3\cos(c+dx)\sin(c+\ldots}{256d}$$

```
[Out] 15/256*a^3*x-4/5*a^3*cos(d*x+c)^5/d+9/7*a^3*cos(d*x+c)^7/d-2/3*a^3*cos(d*x+
c)^9/d+1/11*a^3*cos(d*x+c)^11/d+15/256*a^3*cos(d*x+c)*sin(d*x+c)/d+5/128*a^
3*cos(d*x+c)^3*sin(d*x+c)/d-5/32*a^3*cos(d*x+c)^5*sin(d*x+c)/d-5/16*a^3*cos
(d*x+c)^5*sin(d*x+c)^3/d-3/10*a^3*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.28, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 276}

$$\frac{a^3\cos^{11}(c+dx)}{11d}-\frac{2a^3\cos^9(c+dx)}{3d}+\frac{9a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{5a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}-\frac{5a^3\sin(c+dx)\cos^5(c+dx)}{32d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{256d}+\frac{15a^3x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (9*a^3*Cos[c + d*x]^7)/(7*d
) - (2*a^3*Cos[c + d*x]^9)/(3*d) + (a^3*Cos[c + d*x]^11)/(11*d) + (15*a^3*C
os[c + d*x]*Sin[c + d*x])/(256*d) + (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(12
8*d) - (5*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) - (5*a^3*Cos[c + d*x]^5*S
in[c + d*x]^3)/(16*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cos^4(c+dx)\sin^4(c+dx)+3a^3\cos^4(c+dx)\sin^5\right.\\
&= a^3\int \cos^4(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^4(c+dx)\,\mathrm{s}\\
&= -\frac{a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{3a^3\cos^5(c+dx)\sin^5(c}{10d}\\
&= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{5a^3\cos^5(c+dx)\sin^3(c}{16d}\\
&= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+d}{3d}\\
&= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9(c+d}{3d}\\
&= \frac{3a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos^9}{3}\\
&= \frac{15a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos^7(c+dx)}{7d} - \frac{2a^3\cos}{}
\end{aligned}
$$

Mathematica [A]

time = 0.69, size = 126, normalized size = 0.62

$$\frac{a^3(138600c + 138600dx - 198660\cos(c + dx) - 41580\cos(3(c + dx)) + 27258\cos(5(c + dx)) + 3630\cos(7(c + dx)) - 3850\cos(9(c + dx)) + 210\cos(11(c + dx)) - 13860\sin(2(c + dx)) - 46200\sin(4(c + dx)) + 6930\sin(6(c + dx)) + 5775\sin(8(c + dx)) - 1386\sin(10(c + dx)))}{2365440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(138600*c + 138600*d*x - 198660*Cos[c + d*x] - 41580*Cos[3*(c + d*x)]
+ 27258*Cos[5*(c + d*x)] + 3630*Cos[7*(c + d*x)] - 3850*Cos[9*(c + d*x)] +
210*Cos[11*(c + d*x)] - 13860*Sin[2*(c + d*x)] - 46200*Sin[4*(c + d*x)] + 6
930*Sin[6*(c + d*x)] + 5775*Sin[8*(c + d*x)] - 1386*Sin[10*(c + d*x)]))/(23
65440*d)
```

Maple [A]
time = 0.43, size = 288, normalized size = 1.42

method	result
risch	$-\frac{43a^3\cos(dx+c)}{512d}+\frac{15a^3x}{256}+\frac{a^3\cos(11dx+11c)}{11264d}-\frac{3a^3\sin(10dx+10c)}{5120d}-\frac{5a^3\cos(9dx+9c)}{3072d}+\frac{5a^3\sin(8dx+8c)}{2048d}$
derivativedivides	$a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+3a^3\Big(-$
default	$a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+3a^3\Big(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*(-1/9*sin
(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+3
*a^3*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/32*s
in(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/256
*d*x+3/256*c)+a^3*(-1/11*sin(d*x+c)^6*cos(d*x+c)^5-2/33*sin(d*x+c)^4*cos(d*
x+c)^5-8/231*sin(d*x+c)^2*cos(d*x+c)^5-16/1155*cos(d*x+c)^5))
```

Maxima [A]
time = 0.29, size = 169, normalized size = 0.83

$$\frac{2048\left(105\cos\left(dx+c\right)^{11}-385\cos\left(dx+c\right)^{9}+495\cos\left(dx+c\right)^{7}-231\cos\left(dx+c\right)^{5}\right)a^3-22528\left(35\cos\left(dx+c\right)^{9}-90\cos\left(dx+c\right)^{7}+63\cos\left(dx+c\right)^{5}\right)a^3-693\left(32\sin\left(2dx+2c\right)^{5}-120dx-120c-5\sin\left(8dx+8c\right)+40\sin\left(4dx+4c\right)\right)a^3+2310\left(24dx+24c+\sin\left(8dx+8c\right)-8\sin\left(4dx+4c\right)\right)a^3}{2365440d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2365440*(2048*(105*cos(d*x + c)^11 - 385*cos(d*x + c)^9 + 495*cos(d*x + c
)^7 - 231*cos(d*x + c)^5)*a^3 - 22528*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^
7 + 63*cos(d*x + c)^5)*a^3 - 693*(32*sin(2*d*x + 2*c)^5 - 120*d*x - 120*c -
 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 + 2310*(24*d*x + 24*c + sin(
8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.38, size = 137, normalized size = 0.67

$$\frac{26880\,a^3\cos(dx+c)^{11}-197120\,a^3\cos(dx+c)^9+380160\,a^3\cos(dx+c)^7-236544\,a^3\cos(dx+c)^5+17325\,a^3dx-231\,(384\,a^3\cos(dx+c)^9-1168\,a^3\cos(dx+c)^7+984\,a^3\cos(dx+c)^5-50\,a^3\cos(dx+c)^3-75\,a^3\cos(dx+c))\sin(dx+c)}{295680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/295680*(26880*a^3*cos(d*x + c)^11 - 197120*a^3*cos(d*x + c)^9 + 380160*a^
3*cos(d*x + c)^7 - 236544*a^3*cos(d*x + c)^5 + 17325*a^3*d*x - 231*(384*a^3
*cos(d*x + c)^9 - 1168*a^3*cos(d*x + c)^7 + 984*a^3*cos(d*x + c)^5 - 50*a^3
*cos(d*x + c)^3 - 75*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(194) = 388.
time = 4.70, size = 648, normalized size = 3.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 3*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 3*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a**3*x*sin(c +
d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 3*a**3*x*cos(c
 + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 3*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**6*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d)
- 6*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(35*d) - 3*a**3*sin(c + d*x)**4*co
s(c + d*x)**5/(5*d) - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 11*
a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c
 + d*x)**9/(105*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 9*a**
3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**
7/(128*d) - 16*a**3*cos(c + d*x)**11/(1155*d) - 8*a**3*cos(c + d*x)**9/(105
*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**4, True))
```

Giac [A]
time = 0.72, size = 191, normalized size = 0.94

$$\frac{15}{256}\,a^3x+\frac{a^3\cos\left(11\,dx+11\,c\right)}{11264\,d}-\frac{5\,a^3\cos\left(9\,dx+9\,c\right)}{3072\,d}+\frac{11\,a^3\cos\left(7\,dx+7\,c\right)}{7168\,d}+\frac{59\,a^3\cos\left(5\,dx+5\,c\right)}{5120\,d}-\frac{9\,a^3\cos\left(3\,dx+3\,c\right)}{512\,d}-\frac{43\,a^3\cos\left(dx+c\right)}{512\,d}-\frac{3\,a^3\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a^3\sin\left(8\,dx+8\,c\right)}{2048\,d}+\frac{3\,a^3\sin\left(6\,dx+6\,c\right)}{1024\,d}-\frac{5\,a^3\sin\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 15/256*a^3*x + 1/11264*a^3*cos(11*d*x + 11*c)/d - 5/3072*a^3*cos(9*d*x + 9*
c)/d + 11/7168*a^3*cos(7*d*x + 7*c)/d + 59/5120*a^3*cos(5*d*x + 5*c)/d - 9/
512*a^3*cos(3*d*x + 3*c)/d - 43/512*a^3*cos(d*x + c)/d - 3/5120*a^3*sin(10*
d*x + 10*c)/d + 5/2048*a^3*sin(8*d*x + 8*c)/d + 3/1024*a^3*sin(6*d*x + 6*c)
/d - 5/256*a^3*sin(4*d*x + 4*c)/d - 3/512*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 11.72, size = 506, normalized size = 2.49

$$\frac{15\,a^3x}{256}-\frac{\frac{15\,a^3\,(c+d\,x)}{256}+\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{231\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640}-\frac{242\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+\frac{3987\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{64}-\frac{3987\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{242\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{5}+\frac{231\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{640}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{4}-\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}}{128}-\frac{a^3\,(17325\,c+17325\,d\,x-53248)}{295680}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{165\,a^3\,(c+d\,x)}{256}-\frac{a^3\,(190575\,c+190575\,d\,x-585728)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{825\,a^3\,(c+d\,x)}{256}-\frac{a^3\,(952875\,c+952875\,d\,x-2928640)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{2475\,a^3\,(c+d\,x)}{256}-\frac{a^3\,(2858625\,c+2858625\,d\,x+675840)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{2475\,a^3\,(c+d\,x)}{128}-\frac{a^3\,(5717250\,c+5717250\,d\,x-3379200)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\left(\frac{2475\,a^3\,(c+d\,x)}{256}-\frac{a^3\,(2858625\,c+2858625\,d\,x-9461760)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\left(\frac{2475\,a^3\,(c+d\,x)}{128}-\frac{a^3\,(5717250\,c+5717250\,d\,x-14192640)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\left(\frac{3465\,a^3\,(c+d\,x)}{128}-\frac{a^3\,(8004150\,c+8004150\,d\,x+16084992)}{295680}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(\frac{3465\,a^3\,(c+d\,x)}{128}-\frac{a^3\,(8004150\,c+8004150\,d\,x-40685568)}{295680}\right)+\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (15*a^3*x)/256 - ((15*a^3*(c + d*x))/256 + (5*a^3*tan(c/2 + (d*x)/2)^3)/4 -
 (231*a^3*tan(c/2 + (d*x)/2)^5)/640 - (242*a^3*tan(c/2 + (d*x)/2)^7)/5 + (3
987*a^3*tan(c/2 + (d*x)/2)^9)/64 - (3987*a^3*tan(c/2 + (d*x)/2)^13)/64 + (2
42*a^3*tan(c/2 + (d*x)/2)^15)/5 + (231*a^3*tan(c/2 + (d*x)/2)^17)/640 - (5*
a^3*tan(c/2 + (d*x)/2)^19)/4 - (15*a^3*tan(c/2 + (d*x)/2)^21)/128 - (a^3*(1
7325*c + 17325*d*x - 53248))/295680 + tan(c/2 + (d*x)/2)^2*((165*a^3*(c + d
*x))/256 - (a^3*(190575*c + 190575*d*x - 585728))/295680) + tan(c/2 + (d*x)
/2)^4*((825*a^3*(c + d*x))/256 - (a^3*(952875*c + 952875*d*x - 2928640))/29
5680) + tan(c/2 + (d*x)/2)^6*((2475*a^3*(c + d*x))/256 - (a^3*(2858625*c +
2858625*d*x + 675840))/295680) + tan(c/2 + (d*x)/2)^8*((2475*a^3*(c + d*x))
/128 - (a^3*(5717250*c + 5717250*d*x - 3379200))/295680) + tan(c/2 + (d*x)/
2)^16*((2475*a^3*(c + d*x))/256 - (a^3*(2858625*c + 2858625*d*x - 9461760))
/295680) + tan(c/2 + (d*x)/2)^14*((2475*a^3*(c + d*x))/128 - (a^3*(5717250*
c + 5717250*d*x - 14192640))/295680) + tan(c/2 + (d*x)/2)^12*((3465*a^3*(c
+ d*x))/128 - (a^3*(8004150*c + 8004150*d*x + 16084992))/295680) + tan(c/2
+ (d*x)/2)^10*((3465*a^3*(c + d*x))/128 - (a^3*(8004150*c + 8004150*d*x - 4
0685568))/295680) + (15*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)
^2 + 1)^11)
```

3.393 $\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$\frac{21a^3x}{256}-\frac{4a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{a^3\cos^9(c+dx)}{3d}+\frac{21a^3\cos(c+dx)\sin(c+dx)}{256d}+\frac{7a^3\cos^3(c+dx)}{128d}$$

```
[Out] 21/256*a^3*x-4/5*a^3*cos(d*x+c)^5/d+a^3*cos(d*x+c)^7/d-1/3*a^3*cos(d*x+c)^9
/d+21/256*a^3*cos(d*x+c)*sin(d*x+c)/d+7/128*a^3*cos(d*x+c)^3*sin(d*x+c)/d-7
/32*a^3*cos(d*x+c)^5*sin(d*x+c)/d-7/16*a^3*cos(d*x+c)^5*sin(d*x+c)^3/d-1/10
*a^3*cos(d*x+c)^5*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.26, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{a^3\cos^9(c+dx)}{3d}+\frac{a^3\cos^7(c+dx)}{d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}-\frac{7a^3\sin^3(c+dx)\cos^5(c+dx)}{16d}-\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{32d}+\frac{7a^3\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{21a^3\sin(c+dx)\cos(c+dx)}{256d}+\frac{21a^3x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (21*a^3*x)/256 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (a^3*Cos[c + d*x]^7)/d - (a
^3*Cos[c + d*x]^9)/(3*d) + (21*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (7*
a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*
x])/(32*d) - (7*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*d) - (a^3*Cos[c + d*
x]^5*Sin[c + d*x]^5)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^3(c+dx) + 3a^3\cos^4(c+dx)\sin^4\right.$$

$$= a^3\int \cos^4(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^4(c+dx)\,\mathrm{s}$$

$$= -\frac{3a^3\cos^5(c+dx)\sin^3(c+dx)}{8d} - \frac{a^3\cos^5(c+dx)\sin^5(}{10d}$$

$$= -\frac{3a^3\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{7a^3\cos^5(c+dx)\sin^3(}{16d}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c+dx)}{3d}$$

$$= \frac{9a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(c}{3d}$$

$$= \frac{21a^3x}{256} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{a^3\cos^7(c+dx)}{d} - \frac{a^3\cos^9(}{3}$$

Mathematica [A]
time = 0.79, size = 116, normalized size = 0.64

$$\frac{a^3(2700c+2520dx-3600\cos(c+dx)-960\cos(3(c+dx))+384\cos(5(c+dx))+120\cos(7(c+dx))-40\cos(9(c+dx))-60\sin(2(c+dx))-840\sin(4(c+dx))+30\sin(6(c+dx))+105\sin(8(c+dx))-6\sin(10(c+dx)))}{30720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(2700*c + 2520*d*x - 3600*Cos[c + d*x] - 960*Cos[3*(c + d*x)] + 384*Co
s[5*(c + d*x)] + 120*Cos[7*(c + d*x)] - 40*Cos[9*(c + d*x)] - 60*Sin[2*(c +
 d*x)] - 840*Sin[4*(c + d*x)] + 30*Sin[6*(c + d*x)] + 105*Sin[8*(c + d*x)]
- 6*Sin[10*(c + d*x)]))/(30720*d)
```

Maple [A]
time = 0.34, size = 252, normalized size = 1.38

method	result
risch	$\frac{21a^3x}{256} - \frac{15a^3\cos(dx+c)}{128d} - \frac{a^3\sin(10dx+10c)}{5120d} - \frac{a^3\cos(9dx+9c)}{768d} + \frac{7a^3\sin(8dx+8c)}{2048d} + \frac{a^3\cos(7dx+7c)}{256d} + \frac{a^3\sin(}{1}$
derivativedivides	$a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7} - \frac{2\left(\cos^5(dx+c)\right)}{35}\right) + 3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8} - \frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16} + \frac{\left(\cos^3(dx+c\right.}{}\right.$

default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a^3*(-1/8*sin
(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+3*a^3*(-1/9*sin(d*x+c)^4*cos(d*x+
c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5)+a^3*(-1/10*sin(d*x+
c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/32*sin(d*x+c)*cos(d*x+c)
^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.29, size = 149, normalized size = 0.82

$$-\frac{2048\left(35\cos\left(dx+c\right)^9-90\cos\left(dx+c\right)^7+63\cos\left(dx+c\right)^5\right)a^3-6144\left(5\cos\left(dx+c\right)^7-7\cos\left(dx+c\right)^5\right)a^3+21\left(32\sin\left(2dx+2c\right)^5-120dx-120c-5\sin\left(8dx+8c\right)+40\sin\left(4dx+4c\right)\right)a^3-630\left(24dx+24c+\sin\left(8dx+8c\right)-8\sin\left(4dx+4c\right)\right)a^3}{215040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/215040*(2048*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 + 21*(32*sin(2*d*x +
2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^3 -
630*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.39, size = 124, normalized size = 0.68

$$-\frac{1280\,a^3\cos\left(dx+c\right)^9-3840\,a^3\cos\left(dx+c\right)^7+3072\,a^3\cos\left(dx+c\right)^5-315\,a^3dx+3\left(128\,a^3\cos\left(dx+c\right)^9-816\,a^3\cos\left(dx+c\right)^7+968\,a^3\cos\left(dx+c\right)^5-70\,a^3\cos\left(dx+c\right)^3-105\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3840*(1280*a^3*cos(d*x + c)^9 - 3840*a^3*cos(d*x + c)^7 + 3072*a^3*cos(d
*x + c)^5 - 315*a^3*d*x + 3*(128*a^3*cos(d*x + c)^9 - 816*a^3*cos(d*x + c)^
7 + 968*a^3*cos(d*x + c)^5 - 70*a^3*cos(d*x + c)^3 - 105*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(172) = 344.
time = 3.01, size = 595, normalized size = 3.27

$$\begin{cases} \frac{3a^3x\sin^{10}(c+dx)}{256}+\frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256}+\frac{9a^3x\sin^8(c+dx)}{128}+\frac{15a^3x\sin^6(c+dx)\cos^4(c+dx)}{128}+\frac{9a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{15a^3x\sin^4(c+dx)\cos^6(c+dx)}{128}+\frac{27a^3x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{15a^3x\sin^2(c+dx)\cos^8(c+dx)}{256}+\frac{9a^3x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{3a^3x\cos^{10}(c+dx)}{256}+\frac{9a^3x\cos^8(c+dx)}{128}+\frac{3a^3\sin^9(c+dx)\cos(c+dx)}{256d}+\frac{7a^3\sin^7(c+dx)\cos^3(c+dx)}{128d}+\frac{9a^3\sin^7(c+dx)\cos(c+dx)}{128d}-\frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d}+\frac{33a^3\sin^5(c+dx)\cos^3(c+dx)}{128d}-\frac{3a^3\sin^4(c+dx)\cos^5(c+dx)}{5d}-\frac{7a^3\sin^3(c+dx)\cos^7(c+dx)}{128d}-\frac{33a^3\sin^3(c+dx)\cos^5(c+dx)}{128d}-\frac{12a^3\sin^2(c+dx)\cos^7(c+dx)}{35d}-\frac{a^3\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{3a^3\sin(c+dx)\cos^9(c+dx)}{256d}-\frac{9a^3\sin(c+dx)\cos^7(c+dx)}{128d}-\frac{8a^3\cos^9(c+dx)}{105d}-\frac{2a^3\cos^7(c+dx)}{35d} & \text{for}\ d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^3\sin^3\left(c\right)\cos^4\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 9*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 9*a**3*x*cos(
c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**3*sin(c + d*x)**7*cos(c + d*x)/(
128*d) - a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**3*sin(c + d*x)
**5*cos(c + d*x)**3/(128*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d)
- 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 33*a**3*sin(c + d*x)**3*
cos(c + d*x)**5/(128*d) - 12*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) -
a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d*
x)**9/(256*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos(c
+ d*x)**9/(105*d) - 2*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)
+ a)**3*sin(c)**3*cos(c)**4, True))
```

Giac [A]
time = 0.74, size = 174, normalized size = 0.96

$$\frac{21}{256}a^3x-\frac{a^3\cos(9\,dx+9\,c)}{768\,d}+\frac{a^3\cos(7\,dx+7\,c)}{256\,d}+\frac{a^3\cos(5\,dx+5\,c)}{80\,d}-\frac{a^3\cos(3\,dx+3\,c)}{32\,d}-\frac{15\,a^3\cos(dx+c)}{128\,d}-\frac{a^3\sin(10\,dx+10\,c)}{5120\,d}+\frac{7\,a^3\sin(8\,dx+8\,c)}{2048\,d}+\frac{a^3\sin(6\,dx+6\,c)}{1024\,d}-\frac{7\,a^3\sin(4\,dx+4\,c)}{256\,d}-\frac{a^3\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 21/256*a^3*x - 1/768*a^3*cos(9*d*x + 9*c)/d + 1/256*a^3*cos(7*d*x + 7*c)/d
+ 1/80*a^3*cos(5*d*x + 5*c)/d - 1/32*a^3*cos(3*d*x + 3*c)/d - 15/128*a^3*co
s(d*x + c)/d - 1/5120*a^3*sin(10*d*x + 10*c)/d + 7/2048*a^3*sin(8*d*x + 8*c
)/d + 1/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d - 1/512*
a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.81, size = 572, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (21*a^3*x)/256 - ((203*a^3*tan(c/2 + (d*x)/2)^3)/128 - (1973*a^3*tan(c/2 +
(d*x)/2)^5)/160 - (463*a^3*tan(c/2 + (d*x)/2)^7)/32 + (3231*a^3*tan(c/2 + (
d*x)/2)^9)/64 - (3231*a^3*tan(c/2 + (d*x)/2)^11)/64 + (463*a^3*tan(c/2 + (d
```

*x)/2)^13)/32 + (1973*a^3*tan(c/2 + (d*x)/2)^15)/160 - (203*a^3*tan(c/2 + (d*x)/2)^17)/128 - (21*a^3*tan(c/2 + (d*x)/2)^19)/128 + (a^3*(315*c + 315*d*x))/3840 - (a^3*(315*c + 315*d*x - 1024))/3840 + tan(c/2 + (d*x)/2)^18*((a^3*(315*c + 315*d*x))/384 - (a^3*(3150*c + 3150*d*x))/3840) + tan(c/2 + (d*x)/2)^2*((a^3*(315*c + 315*d*x))/384 - (a^3*(3150*c + 3150*d*x - 10240))/3840) + tan(c/2 + (d*x)/2)^16*((3*a^3*(315*c + 315*d*x))/256 - (a^3*(14175*c + 14175*d*x - 15360))/3840) + tan(c/2 + (d*x)/2)^4*((3*a^3*(315*c + 315*d*x))/256 - (a^3*(14175*c + 14175*d*x - 30720))/3840) + tan(c/2 + (d*x)/2)^6*((a^3*(315*c + 315*d*x))/32 - (a^3*(37800*c + 37800*d*x + 30720))/3840) + tan(c/2 + (d*x)/2)^12*((7*a^3*(315*c + 315*d*x))/128 - (a^3*(66150*c + 66150*d*x + 30720))/3840) + tan(c/2 + (d*x)/2)^14*((a^3*(315*c + 315*d*x))/32 - (a^3*(37800*c + 37800*d*x - 153600))/3840) + tan(c/2 + (d*x)/2)^10*((21*a^3*(315*c + 315*d*x))/320 - (a^3*(79380*c + 79380*d*x - 129024))/3840) + tan(c/2 + (d*x)/2)^8*((7*a^3*(315*c + 315*d*x))/128 - (a^3*(66150*c + 66150*d*x - 245760))/3840) + (21*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)

3.394 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=159

$$\frac{17a^3x}{128}-\frac{4a^3\cos^5(c+dx)}{5d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{a^3\cos^9(c+dx)}{9d}+\frac{17a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{17a^3\cos^3(c+d}{192}$$

```
[Out] 17/128*a^3*x-4/5*a^3*cos(d*x+c)^5/d+5/7*a^3*cos(d*x+c)^7/d-1/9*a^3*cos(d*x+
c)^9/d+17/128*a^3*cos(d*x+c)*sin(d*x+c)/d+17/192*a^3*cos(d*x+c)^3*sin(d*x+c
)/d-17/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d-3/8*a^3*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2648, 2715, 8, 2645, 14, 276}

$$-\frac{a^3\cos^9(c+dx)}{9d}+\frac{5a^3\cos^7(c+dx)}{7d}-\frac{4a^3\cos^5(c+dx)}{5d}-\frac{3a^3\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{17a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{17a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{17a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{17a^3x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (17*a^3*x)/128 - (4*a^3*Cos[c + d*x]^5)/(5*d) + (5*a^3*Cos[c + d*x]^7)/(7*d
) - (a^3*Cos[c + d*x]^9)/(9*d) + (17*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d)
 + (17*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (17*a^3*Cos[c + d*x]^5*Si
n[c + d*x])/(48*d) - (3*a^3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^4(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin\right.$$

$$= a^3\int \cos^4(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^4(c+dx)$$

$$= -\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{3a^3\cos^5(c+dx)\sin^3(}{8d}$$

$$= \frac{a^3\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{17a^3\cos^5(c+dx)\sin(c}{48d}$$

$$= -\frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(c+a}{9d}$$

$$= \frac{a^3x}{16} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\cos^9(}{9}$$

$$= \frac{17a^3x}{128} - \frac{4a^3\cos^5(c+dx)}{5d} + \frac{5a^3\cos^7(c+dx)}{7d} - \frac{a^3\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.65, size = 106, normalized size = 0.67

$$\frac{a^3(30240c+42840dx-52920\cos(c+dx)-16800\cos(3(c+dx))+4032\cos(5(c+dx))+2340\cos(7(c+dx))-140\cos(9(c+dx))+5040\sin(2(c+dx))-12600\sin(4(c+dx))-1680\sin(6(c+dx))+945\sin(8(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(30240*c + 42840*d*x - 52920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] + 4
032*Cos[5*(c + d*x)] + 2340*Cos[7*(c + d*x)] - 140*Cos[9*(c + d*x)] + 5040*
Sin[2*(c + d*x)] - 12600*Sin[4*(c + d*x)] - 1680*Sin[6*(c + d*x)] + 945*Sin
[8*(c + d*x)]))/(322560*d)
```

Maple [A]
time = 0.32, size = 216, normalized size = 1.36

method	result
risch	$\frac{17a^3x}{128}-\frac{21a^3\cos(dx+c)}{128d}-\frac{a^3\cos(9dx+9c)}{2304d}+\frac{3a^3\sin(8dx+8c)}{1024d}+\frac{13a^3\cos(7dx+7c)}{1792d}-\frac{a^3\sin(6dx+6c)}{192d}+\frac{a^3\cos}{}$
derivativedivides	$a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(\right.}{35}\right.$
default	$a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(\right.}{35}\right.$
norman	$-\frac{35a^3\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{357a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{4a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{1071a^3x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{1071a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d
*x+c)^5)+3*a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5
+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+a^3*(-1/9
*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^
5))
```

Maxima [A]
time = 0.29, size = 138, normalized size = 0.87

$$-\frac{1024\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^3-27648\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^3-1680\left(4\sin(2dx+2c)^3+12dx+12c-3\sin(4dx+4c)\right)a^3-945\left(24dx+24c+\sin(8dx+8c)-8\sin(4dx+4c)\right)a^3}{322560d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/322560*(1024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^3 - 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^3 - 1680*(4*sin(2*d*x
+ 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 945*(24*d*x + 24*c + s
in(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [**A**]
time = 0.39, size = 111, normalized size = 0.70

$$-\frac{4480\,a^{3}\cos\left(dx+c\right)^{9}-28800\,a^{3}\cos\left(dx+c\right)^{7}+32256\,a^{3}\cos\left(dx+c\right)^{5}-5355\,a^{3}dx-105\,\left(144\,a^{3}\cos\left(dx+c\right)^{7}-280\,a^{3}\cos\left(dx+c\right)^{5}+34\,a^{3}\cos\left(dx+c\right)^{3}+51\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*a^3*cos(d*x + c)^9 - 28800*a^3*cos(d*x + c)^7 + 32256*a^3*co
s(d*x + c)^5 - 5355*a^3*d*x - 105*(144*a^3*cos(d*x + c)^7 - 280*a^3*cos(d*x
 + c)^5 + 34*a^3*cos(d*x + c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(151) = 302$.
time = 2.09, size = 486, normalized size = 3.06

$$\begin{cases} \frac{9a^{3}x\sin^{8}{\left(c+dx\right)}}{128}+\frac{9a^{3}x\sin^{6}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{32}+\frac{a^{3}x\sin^{6}{\left(c+dx\right)}}{16}+\frac{27a^{3}x\sin^{4}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{64}+\frac{3a^{3}x\sin^{4}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{16}+\frac{9a^{3}x\sin^{2}{\left(c+dx\right)}\cos^{6}{\left(c+dx\right)}}{32}+\frac{3a^{3}x\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{16}+\frac{9a^{3}x\cos^{8}{\left(c+dx\right)}}{128}+\frac{a^{3}x\cos^{6}{\left(c+dx\right)}}{16}+\frac{9a^{3}\sin^{7}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{128d}+\frac{33a^{3}\sin^{5}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{128d}+\frac{a^{3}\sin^{5}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{16d}-\frac{a^{3}\sin^{4}{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{5d}-\frac{33a^{3}\sin^{3}{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{128d}+\frac{a^{3}\sin^{3}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{6d}-\frac{4a^{3}\sin^{2}{\left(c+dx\right)}\cos^{7}{\left(c+dx\right)}}{35d}-\frac{3a^{3}\sin^{2}{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{5d}-\frac{9a^{3}\sin{\left(c+dx\right)}\cos^{7}{\left(c+dx\right)}}{128d}-\frac{a^{3}\sin{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{16d}-\frac{8a^{3}\cos^{9}{\left(c+dx\right)}}{315d}-\frac{6a^{3}\cos^{7}{\left(c+dx\right)}}{35d} & \text{for}\: d\neq 0 \\ x\left(a\sin{\left(c\right)}+a\right)^{3}\sin^{2}{\left(c\right)}\cos^{4}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**8/128 + 9*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + a**3*x*sin(c + d*x)**6/16 + 27*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**4/64 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 9*a**3*x*cos(c + d*x)**8/128 + a**3*x*cos(c + d*x)**6/16 + 9*a**3*sin(c +
d*x)**7*cos(c + d*x)/(128*d) + 33*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128
*d) + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - a**3*sin(c + d*x)**4*cos(c
 + d*x)**5/(5*d) - 33*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + a**3*s
in(c + d*x)**3*cos(c + d*x)**3/(6*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)*
*7/(35*d) - 3*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 9*a**3*sin(c + d
*x)*cos(c + d*x)**7/(128*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 8*
a**3*cos(c + d*x)**9/(315*d) - 6*a**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x
*(a*sin(c) + a)**3*sin(c)**2*cos(c)**4, True))
```

Giac [**A**]
time = 0.59, size = 157, normalized size = 0.99

$$\frac{17}{128}\,a^{3}x-\frac{a^{3}\cos\left(9\,dx+9\,c\right)}{2304\,d}+\frac{13\,a^{3}\cos\left(7\,dx+7\,c\right)}{1792\,d}+\frac{a^{3}\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,a^{3}\cos\left(3\,dx+3\,c\right)}{96\,d}-\frac{21\,a^{3}\cos\left(dx+c\right)}{128\,d}+\frac{3\,a^{3}\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a^{3}\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{5\,a^{3}\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{a^{3}\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 17/128*a^3*x - 1/2304*a^3*cos(9*d*x + 9*c)/d + 13/1792*a^3*cos(7*d*x + 7*c)/d + 1/80*a^3*cos(5*d*x + 5*c)/d - 5/96*a^3*cos(3*d*x + 3*c)/d - 21/128*a^3*cos(d*x + c)/d + 3/1024*a^3*sin(8*d*x + 8*c)/d - 1/192*a^3*sin(6*d*x + 6*c)/d - 5/128*a^3*sin(4*d*x + 4*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d

Mupad [B]
time = 12.28, size = 437, normalized size = 2.75

$$\frac{17a^3x}{128}-\frac{\frac{17a^3(c+dx)}{128}-\frac{35a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96}-\frac{537a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{32}+\frac{531a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{32}-\frac{531a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{32}+\frac{537a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{32}+\frac{35a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96}-\frac{17a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{64}-\frac{a^3(5355c+5355dx-15872)}{40320}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{153a^3(c+dx)}{128}-\frac{a^3(48195c+48195dx-142848)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{153a^3(c+dx)}{32}-\frac{a^3(192780c+192780dx-87552)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{153a^3(c+dx)}{32}-\frac{a^3(192780c+192780dx-483840)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{357a^3(c+dx)}{32}-\frac{a^3(449820c+449820dx-419328)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{1071a^3(c+dx)}{64}-\frac{a^3(674730c+674730dx+161280)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{357a^3(c+dx)}{32}-\frac{a^3(449820c+449820dx-913920)}{40320}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{1071a^3(c+dx)}{64}-\frac{a^3(674730c+674730dx-2161152)}{40320}\right)+\frac{17a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)

[Out] (17*a^3*x)/128 - ((17*a^3*(c + d*x))/128 - (35*a^3*tan(c/2 + (d*x)/2)^3)/96 - (537*a^3*tan(c/2 + (d*x)/2)^5)/32 + (531*a^3*tan(c/2 + (d*x)/2)^7)/32 - (531*a^3*tan(c/2 + (d*x)/2)^11)/32 + (537*a^3*tan(c/2 + (d*x)/2)^13)/32 + (35*a^3*tan(c/2 + (d*x)/2)^15)/96 - (17*a^3*tan(c/2 + (d*x)/2)^17)/64 - (a^3*(5355*c + 5355*d*x - 15872))/40320 + tan(c/2 + (d*x)/2)^2*((153*a^3*(c + d*x))/128 - (a^3*(48195*c + 48195*d*x - 142848))/40320) + tan(c/2 + (d*x)/2)^4*((153*a^3*(c + d*x))/32 - (a^3*(192780*c + 192780*d*x - 87552))/40320) + tan(c/2 + (d*x)/2)^14*((153*a^3*(c + d*x))/32 - (a^3*(192780*c + 192780*d*x - 483840))/40320) + tan(c/2 + (d*x)/2)^6*((357*a^3*(c + d*x))/32 - (a^3*(449820*c + 449820*d*x - 419328))/40320) + tan(c/2 + (d*x)/2)^10*((1071*a^3*(c + d*x))/64 - (a^3*(674730*c + 674730*d*x + 161280))/40320) + tan(c/2 + (d*x)/2)^12*((357*a^3*(c + d*x))/32 - (a^3*(449820*c + 449820*d*x - 913920))/40320) + tan(c/2 + (d*x)/2)^8*((1071*a^3*(c + d*x))/64 - (a^3*(674730*c + 674730*d*x - 2161152))/40320) + (17*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)

3.395 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=157

$$\frac{27a^3x}{128}-\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{9a^3\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{3a\cos^5(c+dx)(a+}{56d}$$

```
[Out] 27/128*a^3*x-9/80*a^3*cos(d*x+c)^5/d+27/128*a^3*cos(d*x+c)*sin(d*x+c)/d+9/6
4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-3/56*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/
8*cos(d*x+c)^5*(a+a*sin(d*x+c))^3/d-9/112*cos(d*x+c)^5*(a^3+a^3*sin(d*x+c))
/d
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{9a^3\cos^5(c+dx)}{80d}-\frac{9\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{27a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{27a^3x}{128}-\frac{\cos^5(c+dx)(a\sin(c+dx)+a)^3}{8d}-\frac{3a\cos^5(c+dx)(a\sin(c+dx)+a)^2}{56d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*x)/128 - (9*a^3*Cos[c + d*x]^5)/(80*d) + (27*a^3*Cos[c + d*x]*Sin[c
 + d*x])/(128*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) - (3*a*Cos[c
+ d*x]^5*(a + a*Sin[c + d*x])^2)/(56*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*
x])^3)/(8*d) - (9*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{3}{8}\int \cos^4(c+dx)($$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a}{8}$$

$$= -\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a}{8}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}-\frac{3a\cos^5(c+dx)(a+a\sin(c+dx))}{56d}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{9a^3\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{3a}{}$$

$$= -\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{9a^3}{}$$

$$= \frac{27a^3x}{128}-\frac{9a^3\cos^5(c+dx)}{80d}+\frac{27a^3\cos(c+dx)\sin(c+dx}{128d}$$

Mathematica [A]
time = 0.36, size = 96, normalized size = 0.61

$$\frac{a^3(8400c+7560dx-9520\cos(c+dx)-3920\cos(3(c+dx))-112\cos(5(c+dx))+240\cos(7(c+dx))+1680\sin(2(c+dx))-1960\sin(4(c+dx))-560\sin(6(c+dx))+35\sin(8(c+dx)))}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(8400*c + 7560*d*x - 9520*Cos[c + d*x] - 3920*Cos[3*(c + d*x)] - 112*C
os[5*(c + d*x)] + 240*Cos[7*(c + d*x)] + 1680*Sin[2*(c + d*x)] - 1960*Sin[4
*(c + d*x)] - 560*Sin[6*(c + d*x)] + 35*Sin[8*(c + d*x)]))/(35840*d)
```

Maple [A]
time = 0.24, size = 178, normalized size = 1.13

method	result
risch	$\frac{27a^3x}{128} - \frac{17a^3\cos(dx+c)}{64d} + \frac{a^3\sin(8dx+8c)}{1024d} + \frac{3a^3\cos(7dx+7c)}{448d} - \frac{a^3\sin(6dx+6c)}{64d} - \frac{a^3\cos(5dx+5c)}{320d} - \frac{7a^3\sin}{}$
derivativedivides	$\frac{-\frac{a^3\left(\cos^5(dx+c)\right)}{5}+3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{}\right.}{}$
default	$\frac{-\frac{a^3\left(\cos^5(dx+c)\right)}{5}+3a^3\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{}\right.}{}$
norman	$\frac{\frac{27a^3\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{189a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{10a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{189a^3x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{945a^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^3*cos(d*x+c)^5+3*a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*(-1/7*sin(d*x+c)^2
*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+a^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*s
in(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*
d*x+3/128*c))
```

Maxima [A]
time = 0.28, size = 115, normalized size = 0.73

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5-3072\,\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)a^3-560\,\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^3-35\,\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)a^3}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)
^5)*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.35, size = 98, normalized size = 0.62

$$\frac{1920\,a^3\cos\left(dx+c\right)^7-3584\,a^3\cos\left(dx+c\right)^5+945\,a^3dx+35\,\left(16\,a^3\cos\left(dx+c\right)^7-88\,a^3\cos\left(dx+c\right)^5+18\,a^3\cos\left(dx+c\right)^3+27\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a^3*cos(d*x + c)^7 - 3584*a^3*cos(d*x + c)^5 + 945*a^3*d*x + 3
5*(16*a^3*cos(d*x + c)^7 - 88*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^3 +
27*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(148) = 296$.
time = 1.03, size = 440, normalized size = 2.80

$$\begin{cases} \frac{3a^3x\sin^8(c+dx)}{128} + \frac{3a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{3a^3x\sin^6(c+dx)}{16} + \frac{9a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{9a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{9a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^3x\cos^8(c+dx)}{128} + \frac{3a^3x\cos^6(c+dx)}{16} + \frac{3a^3\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11a^3\sin^5(c+dx)\cos^3(c+dx)}{128d} + \frac{3a^3\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{11a^3\sin^3(c+dx)\cos^5(c+dx)}{128d} + \frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{2d} - \frac{3a^3\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{3a^3\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{3a^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{6a^3\cos^7(c+dx)}{35d} - \frac{a^3\cos^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\sin(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**8/128 + 3*a**3*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 3*a**3*x*sin(c + d*x)**6/16 + 9*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**4/64 + 9*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*a**3*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 3*a**3*x*cos(c + d*x)**8/128 + 3*a**3*x*cos(c + d*x)**6/16 + 3*a**3*sin(c
 + d*x)**7*cos(c + d*x)/(128*d) + 11*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 11*a**3*sin(c + d*x)*
*3*cos(c + d*x)**5/(128*d) + a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3
*a**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**3*sin(c + d*x)*cos(c + d
*x)**7/(128*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 6*a**3*cos(c
+ d*x)**7/(35*d) - a**3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*sin(c) + a)
**3*sin(c)*cos(c)**4, True))
```

Giac [A]
time = 0.59, size = 140, normalized size = 0.89

$$\frac{27}{128}a^3x + \frac{3a^3\cos(7dx+7c)}{448d} - \frac{a^3\cos(5dx+5c)}{320d} - \frac{7a^3\cos(3dx+3c)}{64d} - \frac{17a^3\cos(dx+c)}{64d} + \frac{a^3\sin(8dx+8c)}{1024d} - \frac{a^3\sin(6dx+6c)}{64d} - \frac{7a^3\sin(4dx+4c)}{128d} + \frac{3a^3\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/128*a^3*x + 3/448*a^3*cos(7*d*x + 7*c)/d - 1/320*a^3*cos(5*d*x + 5*c)/d
- 7/64*a^3*cos(3*d*x + 3*c)/d - 17/64*a^3*cos(d*x + c)/d + 1/1024*a^3*sin(8
*d*x + 8*c)/d - 1/64*a^3*sin(6*d*x + 6*c)/d - 7/128*a^3*sin(4*d*x + 4*c)/d
+ 3/64*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.76, size = 461, normalized size = 2.94

$$\frac{27a^3x}{128} - \frac{\frac{919a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{64} - \frac{437a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{64} - \frac{305a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64} - \frac{919a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{64} + \frac{437a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{64} + \frac{305a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64} - \frac{27a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64} + \frac{a^3(945c+945dx)}{4480} - \frac{a^3(945c+945dx-3328)}{4480} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{a^3(945c+945dx)}{560} - \frac{a^3(7560c+7560dx-8960)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3(945c+945dx)}{560} - \frac{a^3(7560c+7560dx-17664)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3(945c+945dx)}{160} - \frac{a^3(26460c+26460dx-12544)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{a^3(945c+945dx)}{160} - \frac{a^3(26460c+26460dx-80640)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{a^3(945c+945dx)}{80} - \frac{a^3(52920c+52920dx-44800)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^3(945c+945dx)}{80} - \frac{a^3(52920c+52920dx-141568)}{4480}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{a^3(945c+945dx)}{64} - \frac{a^3(66150c+66150dx-116480)}{4480}\right) + \frac{27a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (27*a^3*x)/128 - ((919*a^3*tan(c/2 + (d*x)/2)^7)/64 - (437*a^3*tan(c/2 + (d
*x)/2)^5)/64 - (305*a^3*tan(c/2 + (d*x)/2)^3)/64 - (919*a^3*tan(c/2 + (d*x)
/2)^9)/64 + (437*a^3*tan(c/2 + (d*x)/2)^11)/64 + (305*a^3*tan(c/2 + (d*x)/2
)^13)/64 - (27*a^3*tan(c/2 + (d*x)/2)^15)/64 + (a^3*(945*c + 945*d*x))/4480
 - (a^3*(945*c + 945*d*x - 3328))/4480 + tan(c/2 + (d*x)/2)^14*((a^3*(945*c
 + 945*d*x))/560 - (a^3*(7560*c + 7560*d*x - 8960))/4480) + tan(c/2 + (d*x)
/2)^2*((a^3*(945*c + 945*d*x))/560 - (a^3*(7560*c + 7560*d*x - 17664))/4480
) + tan(c/2 + (d*x)/2)^4*((a^3*(945*c + 945*d*x))/160 - (a^3*(26460*c + 264
60*d*x - 12544))/4480) + tan(c/2 + (d*x)/2)^12*((a^3*(945*c + 945*d*x))/160
 - (a^3*(26460*c + 26460*d*x - 80640))/4480) + tan(c/2 + (d*x)/2)^10*((a^3*
(945*c + 945*d*x))/80 - (a^3*(52920*c + 52920*d*x - 44800))/4480) + tan(c/2
 + (d*x)/2)^6*((a^3*(945*c + 945*d*x))/80 - (a^3*(52920*c + 52920*d*x - 141
568))/4480) + tan(c/2 + (d*x)/2)^8*((a^3*(945*c + 945*d*x))/64 - (a^3*(6615
0*c + 66150*d*x - 116480))/4480) + (27*a^3*tan(c/2 + (d*x)/2))/64)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^8)
```

3.396 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=143

$$\frac{19a^3x}{16}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{19a^3\cos(c+dx)\sin(c-}{16d}$$

```
[Out] 19/16*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3
/d-3/5*a^3*cos(d*x+c)^5/d+19/16*a^3*cos(d*x+c)*sin(d*x+c)/d+19/24*a^3*cos(d
*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2952, 2715, 8, 2672, 308, 212, 2645, 30, 2648}

$$-\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{16d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{19a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a^3
*Cos[c + d*x]^3)/(3*d) - (3*a^3*Cos[c + d*x]^5)/(5*d) + (19*a^3*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(3a^3\cos^4(c+dx)+a^3\cos^3(c+dx)\cot(c+dx)+3a^3 \right. \\
&= a^3\int \cos^3(c+dx)\cot(c+dx)\,dx + a^3\int \cos^4(c+dx)\sin \\
&= \frac{3a^3\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{a^3\cos^5(c+dx)\sin(c+d}{6d} \\
&= -\frac{3a^3\cos^5(c+dx)}{5d} + \frac{9a^3\cos(c+dx)\sin(c+dx)}{8d} + \frac{19a^3}{} \\
&= \frac{9a^3x}{8} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos^5(c+}{5d} \\
&= \frac{19a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a^3}{}
\end{aligned}
$$

Mathematica [A]

time = 0.71, size = 102, normalized size = 0.71

$$\frac{a^3\left(1140c+1140dx+840\cos(c+dx)-100\cos(3(c+dx))-36\cos(5(c+dx))-960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+735\sin(2(c+dx))+75\sin(4(c+dx))-5\sin(6(c+dx))\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1140*c + 1140*d*x + 840*Cos[c + d*x] - 100*Cos[3*(c + d*x)] - 36*Cos[
5*(c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 735*
Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]

time = 0.22, size = 147, normalized size = 1.03

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{3a^3(}{}}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{3a^3(}{}}{d}$
risch	$\frac{19a^3x}{16}+\frac{7a^3e^{i(dx+c)}}{16d}+\frac{7a^3e^{-i(dx+c)}}{16d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^3\sin(6dx+6c)}{192d}-\frac{3a^3\cos(5c}{80d}$
norman	$\frac{\frac{19a^3x}{16}+\frac{22a^3}{15d}+\frac{29a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{173a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{7a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{7a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{173a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{2}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(1/4
*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-3/5*a^3*cos(d*x+c)
^5+a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.29, size = 135, normalized size = 0.94

$$-\frac{576\,a^3\cos\left(dx+c\right)^5-160\,\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-5\,\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^3-90\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a^3*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*
c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3 - 90*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3)/d
```

Fricas [A]
time = 0.38, size = 128, normalized size = 0.90

$$-\frac{144\,a^3\cos\left(dx+c\right)^5-80\,a^3\cos\left(dx+c\right)^3-285\,a^3dx-240\,a^3\cos\left(dx+c\right)+120\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-120\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+5\,\left(8\,a^3\cos\left(dx+c\right)^5-38\,a^3\cos\left(dx+c\right)^3-57\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 285*a^3*d*x - 240*
a^3*cos(d*x + c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*c
os(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^5 - 38*a^3*cos(d*x + c)^3 - 57*a
^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 229, normalized size = 1.60

$$\frac{285\,(dx+c)a^3+240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(435\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+865\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1200\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-210\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1760\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+210\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1440\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-865\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1296\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-435\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-176\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(285*(d*x + c)*a^3 + 240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(435*
a^3*tan(1/2*d*x + 1/2*c)^11 + 240*a^3*tan(1/2*d*x + 1/2*c)^10 + 865*a^3*tan
(1/2*d*x + 1/2*c)^9 - 1200*a^3*tan(1/2*d*x + 1/2*c)^8 - 210*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 1760*a^3*tan(1/2*d*x + 1/2*c)^6 + 210*a^3*tan(1/2*d*x + 1/2*c
)^5 - 1440*a^3*tan(1/2*d*x + 1/2*c)^4 - 865*a^3*tan(1/2*d*x + 1/2*c)^3 - 12
96*a^3*tan(1/2*d*x + 1/2*c)^2 - 435*a^3*tan(1/2*d*x + 1/2*c) - 176*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 10.51, size = 355, normalized size = 2.48

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{19\,a^3\,\mathrm{atan}\left(\frac{361\,a^6}{64\,\left(\frac{19\,a^6}{4}-\frac{361\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}+\frac{19\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{19\,a^6}{4}-\frac{361\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{-\frac{29\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}-2\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-\frac{173\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{24}+10\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+\frac{7\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{4}+\frac{44\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{3}-\frac{7\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{4}+12\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+\frac{173\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{24}+\frac{54\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{5}+\frac{29\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{22\,a^3}{15}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (19*a^3*atan((361*a^6)/(64*((19*a^6)/4 -
(361*a^6*tan(c/2 + (d*x)/2))/64)) + (19*a^6*tan(c/2 + (d*x)/2))/(4*((19*a^6
)/4 - (361*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + ((54*a^3*tan(c/2 + (d*x)/
2)^2)/5 + (173*a^3*tan(c/2 + (d*x)/2)^3)/24 + 12*a^3*tan(c/2 + (d*x)/2)^4 -
 (7*a^3*tan(c/2 + (d*x)/2)^5)/4 + (44*a^3*tan(c/2 + (d*x)/2)^6)/3 + (7*a^3*
tan(c/2 + (d*x)/2)^7)/4 + 10*a^3*tan(c/2 + (d*x)/2)^8 - (173*a^3*tan(c/2 +
(d*x)/2)^9)/24 - 2*a^3*tan(c/2 + (d*x)/2)^10 - (29*a^3*tan(c/2 + (d*x)/2)^1
1)/8 + (22*a^3)/15 + (29*a^3*tan(c/2 + (d*x)/2))/8)/(d*(6*tan(c/2 + (d*x)/2
)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x
)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.397 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{3a^3x}{8}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{d}-\frac{a^3\cos^5(c+dx)}{5d}-\frac{a^3\cot(c+dx)}{d}+\frac{11a}{}$$

```
[Out] -3/8*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)^3/
d-1/5*a^3*cos(d*x+c)^5/d-a^3*cot(d*x+c)/d+11/8*a^3*cos(d*x+c)*sin(d*x+c)/d-
3/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 2718, 2715, 2713}

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{11a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d + (
a^3*Cos[c + d*x]^3)/d - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c + d*x])/d +
 (11*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (3*a^3*Cos[c + d*x]*Sin[c + d*x
]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7+3a^7\csc(c+dx)+a^7\csc^2(c+dx)-5a^7\sin(c+\right.}{}$$

$$= a^3x + a^3\int \csc^2(c+dx)\,dx + a^3\int \sin^3(c+dx)\,dx + a$$

$$= a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{5a^3\cos(c+dx)}{d} + \frac{5a}{}$$

$$= -\frac{3a^3x}{2} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} +$$

$$= -\frac{3a^3x}{8} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} +$$

Mathematica [A]

time = 1.64, size = 148, normalized size = 1.13

$$\frac{(a+a\sin(c+dx))^3\left(-60(c+dx)+580\cos(c+dx)+30\cos(3(c+dx))-2\cos(5(c+dx))-80\cot\left(\frac{1}{2}(c+dx)\right)-480\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+80\sin(2(c+dx))+15\sin(4(c+dx))+80\tan\left(\frac{1}{2}(c+dx)\right)\right)}{160d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-60*(c + d*x) + 580*Cos[c + d*x] + 30*Cos[3*(c + d
*x)] - 2*Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 480*Log[Cos[(c + d*x)/2]]
```

```
 + 480*Log[Sin[(c + d*x)/2]] + 80*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] +
80*Tan[(c + d*x)/2]))/(160*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.20, size = 150, normalized size = 1.15

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-c\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-c\right.}{d}$
risch	$-\frac{3a^3x}{8}-\frac{ia^3e^{2i(dx+c)}}{4d}+\frac{29a^3e^{i(dx+c)}}{16d}+\frac{29a^3e^{-i(dx+c)}}{16d}+\frac{ia^3e^{-2i(dx+c)}}{4d}-\frac{2ia^3}{d\left(e^{2i(dx+c)}-1\right)}+\frac{3a^3\ln\left(e^{i(dx+\right.}}{d}$
norman	$\frac{\frac{28a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{2d}+\frac{3a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{3a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^3\left(\tan^{12}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)-3/2*d*x-3/2*c)+3*a^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+
c)))+3*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5
*a^3*cos(d*x+c)^5)
```

Maxima [A]
time = 0.50, size = 141, normalized size = 1.08

$$-\frac{32\,a^3\cos\left(dx+c\right)^5-80\,\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-15\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3+80\,\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2+2}{\tan\left(dx+c\right)^3+\tan\left(dx+c\right)}\right)a^3}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*a^3*cos(d*x + c)^5 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 + 80*(3*d*x + 3*c + (3*tan(d*x +
c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A]
time = 0.37, size = 147, normalized size = 1.12

$$-\frac{30\,a^3\cos\left(dx+c\right)^5-5\,a^3\cos\left(dx+c\right)^3+60\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\,a^3\cos\left(dx+c\right)+\left(8\,a^3\cos\left(dx+c\right)^5-40\,a^3\cos\left(dx+c\right)^3+15\,a^3dx-120\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^3 + 60*a^3*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 60*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) + 15*a^3*cos(d*x + c) + (8*a^3*cos(d*x + c)^5 - 40*a^3*cos(d*x + c)^3 +
15*a^3*d*x - 120*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.60, size = 226, normalized size = 1.73

$$-\frac{15\,(dx+c)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{20\,\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-200\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-55\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-152\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/40*(15*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 20*a^3*t
an(1/2*d*x + 1/2*c) + 20*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1
/2*c) + 2*(55*a^3*tan(1/2*d*x + 1/2*c)^9 - 200*a^3*tan(1/2*d*x + 1/2*c)^8 -
 10*a^3*tan(1/2*d*x + 1/2*c)^7 - 720*a^3*tan(1/2*d*x + 1/2*c)^6 - 800*a^3*t
an(1/2*d*x + 1/2*c)^4 + 10*a^3*tan(1/2*d*x + 1/2*c)^3 - 560*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55*a^3*tan(1/2*d*x + 1/2*c) - 152*a^3)/(tan(1/2*d*x + 1/2*c)^
2 + 1)^5)/d
```

Mupad [B]
time = 8.81, size = 356, normalized size = 2.72

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{-\frac{13\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{2}+20\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9-4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+72\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-10\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+80\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-11\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+56\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{76\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}-a^3}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{3\,a^3\,\mathrm{atan}\left(\frac{9\,a^6}{16\,\left(\frac{9\,a^6}{2}+\frac{9\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{9\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{9\,a^6}{2}+\frac{9\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + ((a^3*tan(c/2 + (d*x)/2)^2)/2 + 56*a^3*
tan(c/2 + (d*x)/2)^3 - 11*a^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*tan(c/2 + (d*x)
/2)^5 - 10*a^3*tan(c/2 + (d*x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^7 - 4*a^3*t
an(c/2 + (d*x)/2)^8 + 20*a^3*tan(c/2 + (d*x)/2)^9 - (13*a^3*tan(c/2 + (d*x)
```

```
/2)^10)/2 - a^3 + (76*a^3*tan(c/2 + (d*x)/2))/5)/(d*(2*tan(c/2 + (d*x)/2) +
 10*tan(c/2 + (d*x)/2)^3 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2)^
7 + 10*tan(c/2 + (d*x)/2)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (3*a^3*atan((9*a^
6)/(16*((9*a^6)/2 + (9*a^6*tan(c/2 + (d*x)/2))/16)) - (9*a^6*tan(c/2 + (d*x
)/2))/(2*((9*a^6)/2 + (9*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) + (a^3*tan(c/
2 + (d*x)/2))/(2*d)
```

3.398 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$-\frac{33a^3x}{8}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c}{2d}$$

```
[Out] -33/8*a^3*x-3/2*a^3*arctanh(cos(d*x+c))/d+2*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)
^3/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*x+c)/d-7/8*a^3*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{a^3\cos^3(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{7a^3\sin(c+dx)\cos(c+dx)}{8d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{33a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-33*a^3*x)/8 - (3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (2*a^3*Cos[c + d*x])/
d + (a^3*Cos[c + d*x]^3)/d - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]*Csc
[c + d*x])/(2*d) - (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^3*Cos[c + d
*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx &= \frac{\int \left(-5a^7+a^7\csc(c+dx)+3a^7\csc^2(c+dx)+a^7\csc^3(\right.}{} \\
&= -5a^3x+a^3\int \csc(c+dx)\,dx+a^3\int \csc^3(c+dx)\,dx+ \\
&= -5a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^3\cos(c+dx)}{d}- \\
&= -\frac{9a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2a^3\cos(c+dx)}{d} \\
&= -\frac{33a^3x}{8}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{2a^3\cos(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]

time = 1.96, size = 164, normalized size = 1.20

$$
\frac{(a+a\sin(c+dx))^3\left(-132(c+dx)+88\cos(c+dx)+8\cos(3(c+dx))-48\cot\left(\frac{1}{2}(c+dx)\right)-4\csc^2\left(\frac{1}{2}(c+dx)\right)-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+48\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\sec^2\left(\frac{1}{2}(c+dx)\right)-16\sin(2(c+dx))+\sin(4(c+dx))+48\tan\left(\frac{1}{2}(c+dx)\right)\right)}{32d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-132*(c + d*x) + 88*Cos[c + d*x] + 8*Cos[3*(c + d*
x)] - 48*Cot[(c + d*x)/2] - 4*Csc[(c + d*x)/2]^2 - 48*Log[Cos[(c + d*x)/2]]
 + 48*Log[Sin[(c + d*x)/2]] + 4*Sec[(c + d*x)/2]^2 - 16*Sin[2*(c + d*x)] +
Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^6)
```

Maple [A]
time = 0.20, size = 196, normalized size = 1.43

method	result
derivativedivides	$a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c}{2}\right.\right.$
default	$a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c}{2}\right.\right.$
risch	$-\frac{33a^3x}{8}+\frac{a^3e^{3i(dx+c)}}{8d}+\frac{ia^3e^{2i(dx+c)}}{4d}+\frac{11a^3e^{i(dx+c)}}{8d}+\frac{11a^3e^{-i(dx+c)}}{8d}-\frac{ia^3e^{-2i(dx+c)}}{4d}+\frac{a^3e^{-3i(dx+c)}}{8d}+$
norman	$\frac{5a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{25a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{27a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{27a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{25a^3\left(\tan^9\left(\frac{dx}{2}\right.\right.}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/
2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+3*a^3*(1/3*cos(d*x+c)^3+cos(d*x
+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d
*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 183, normalized size = 1.34

$$\frac{16\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^3+(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^3-48\left(3dx+3c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3+8a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-4\cos(dx+c)+3\log(\cos(dx+c)+1)-3\log(\cos(dx+c)-1)\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/32*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^3 + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
```

```
 + 2*c))*a^3 - 48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + t
an(d*x + c)))*a^3 + 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x
+ c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 185, normalized size = 1.35

$$\frac{8\,a^3\cos\left(dx+c\right)^5-33\,a^3dx\cos\left(dx+c\right)^2+8\,a^3\cos\left(dx+c\right)^3+33\,a^3dx-12\,a^3\cos\left(dx+c\right)-6\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+6\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(2\,a^3\cos\left(dx+c\right)^5-11\,a^3\cos\left(dx+c\right)^3+33\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a^3*cos(d*x + c)^5 - 33*a^3*d*x*cos(d*x + c)^2 + 8*a^3*cos(d*x + c)^
3 + 33*a^3*d*x - 12*a^3*cos(d*x + c) - 6*(a^3*cos(d*x + c)^2 - a^3)*log(1/2
*cos(d*x + c) + 1/2) + 6*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) +
 1/2) + (2*a^3*cos(d*x + c)^5 - 11*a^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.67, size = 241, normalized size = 1.76

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-33\left(dx+c\right)a^3+12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{18\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{2\left(7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+56\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 - 33*(d*x + c)*a^3 + 12*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) - (18*a^3*tan(1/2*d*x + 1/2*c
)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(7*a^3*
tan(1/2*d*x + 1/2*c)^7 + 40*a^3*tan(1/2*d*x + 1/2*c)^6 + 15*a^3*tan(1/2*d*x
 + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 15*a^3*tan(1/2*d*x + 1/2*c)^3
 + 56*a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^3*tan(1/2*d*x + 1/2*c) + 24*a^3)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]

time = 8.81, size = 347, normalized size = 2.53

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{3\,a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{33\,a^3\,\mathrm{atan}\left(\frac{1089\,a^6}{16\,\left(\frac{99\,a^6}{4}+\frac{1089\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{99\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{99\,a^6}{4}+\frac{1089\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{79\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-9\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+70\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+53\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-31\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a^3}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) +
(33*a^3*atan((1089*a^6)/(16*((99*a^6)/4 + (1089*a^6*tan(c/2 + (d*x)/2))/16)
) - (99*a^6*tan(c/2 + (d*x)/2))/(4*((99*a^6)/4 + (1089*a^6*tan(c/2 + (d*x)/
2))/16)))))/(4*d) + (22*a^3*tan(c/2 + (d*x)/2)^2 - 31*a^3*tan(c/2 + (d*x)/2)
^3 + 53*a^3*tan(c/2 + (d*x)/2)^4 - 51*a^3*tan(c/2 + (d*x)/2)^5 + 70*a^3*tan
(c/2 + (d*x)/2)^6 - 9*a^3*tan(c/2 + (d*x)/2)^7 + (79*a^3*tan(c/2 + (d*x)/2)
^8)/2 + a^3*tan(c/2 + (d*x)/2)^9 - a^3/2 - 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*
tan(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 +
16*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) + (3*a^3*tan(c/2 + (d*x
)/2))/(2*d)
```

3.399 $\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=134

$$-\frac{7a^3x}{2}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-3a$$

```
[Out] -7/2*a^3*x+7/2*a^3*arctanh(cos(d*x+c))/d-2*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x
+c)^3/d-2*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3/2*a^3*cot(d*x+c)*csc(d*
x+c)/d-3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{7a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*a^3*x)/2 + (7*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^3*Cos[c + d*x])/d
 + (a^3*Cos[c + d*x]^3)/(3*d) - (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^
3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-5a^7 - 5a^7\csc(c+dx) + a^7\csc^2(c+dx) + 3a^7\csc^3(c+dx) + a^7 \cdots\right.}{\cdots}$$

$$= -5a^3x + a^3\int \csc^2(c+dx)\,dx + a^3\int \csc^4(c+dx)\,dx + a^3\int \sin(c \cdots$$

$$= -5a^3x + \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+d \cdots}{2 \cdots}$$

$$= -\frac{7a^3x}{2} + \frac{7a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+ \cdots}{3d}$$

Mathematica [A]
time = 4.94, size = 201, normalized size = 1.50

$$\frac{a^3(1+\sin(c+dx))^3\left(-84(c+dx) - 42\cos(c+dx) + 2\cos(3(c+dx)) - 20\cot\left(\frac{1}{2}(c+dx)\right) - 9\csc^2\left(\frac{1}{2}(c+dx)\right) + 84\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 84\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 9\sec^2\left(\frac{1}{2}(c+dx)\right) + 8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) - \frac{1}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) - 18\sin(2(c+dx)) + 20\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-84*(c + d*x) - 42*Cos[c + d*x] + 2*Cos[3*(c + d
*x)] - 20*Cot[(c + d*x)/2] - 9*Csc[(c + d*x)/2]^2 + 84*Log[Cos[(c + d*x)/2]
] - 84*Log[Sin[(c + d*x)/2]] + 9*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[
(c + d*x)/2]^4 - (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 18*Sin[2*(c + d*x)]
+ 20*Tan[(c + d*x)/2]))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.21, size = 184, normalized size = 1.37

method	result
derivativedivides	$a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)$
default	$a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)$
risch	$-\frac{7a^3x}{2}+\frac{3ia^3\mathrm{e}^{2i(dx+c)}}{8d}-\frac{7a^3\mathrm{e}^{i(dx+c)}}{8d}-\frac{7a^3\mathrm{e}^{-i(dx+c)}}{8d}-\frac{3ia^3\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{a^3\left(-6i\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}+24\right.}{3d\left(\mathrm{e}^{2i(dx+c)}\right.}$
norman	$\frac{a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{24d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{39a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{39a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{3a^3\left(\tan^{11}\left(\right.\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/2/sin(d*x+c)^2*cos(
d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a
^3*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2
*d*x-3/2*c)+a^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 185, normalized size = 1.38

$$\frac{2\left(2\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-18\left(3\,dx+3\,c+\frac{3\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3+4\left(3\,dx+3\,c+\frac{3\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+9\,a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-4\cos\left(dx+c\right)+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*a^3 - 18*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*
x + c)^3 + tan(d*x + c)))*a^3 + 4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan
(d*x + c)^3)*a^3 + 9*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x +
 c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 206, normalized size = 1.54

$$\frac{18\,a^3\cos\left(dx+c\right)^5-56\,a^3\cos\left(dx+c\right)^3+42\,a^3\cos\left(dx+c\right)+21\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-21\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,\left(2\,a^3\cos\left(dx+c\right)^5-21\,a^3dx\cos\left(dx+c\right)^2-14\,a^3\cos\left(dx+c\right)^3+21\,a^3dx+21\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^5 - 56*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c) +
 21*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
1*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*
(2*a^3*cos(d*x + c)^5 - 21*a^3*d*x*cos(d*x + c)^2 - 14*a^3*cos(d*x + c)^3 +
 21*a^3*d*x + 21*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(122) = 244.
time = 0.57, size = 250, normalized size = 1.87

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-252\,(dx+c)a^3-252\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+63\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{154\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+153\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+291\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-192\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-414\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-167\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3}}{72\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^3*tan(1/2*d*x + 1/2*c)^2 - 252*(d
*x + c)*a^3 - 252*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 63*a^3*tan(1/2*d*x +
 1/2*c) + (154*a^3*tan(1/2*d*x + 1/2*c)^9 + 153*a^3*tan(1/2*d*x + 1/2*c)^8
+ 291*a^3*tan(1/2*d*x + 1/2*c)^7 - 192*a^3*tan(1/2*d*x + 1/2*c)^6 - 195*a^3
*tan(1/2*d*x + 1/2*c)^5 - 414*a^3*tan(1/2*d*x + 1/2*c)^4 - 167*a^3*tan(1/2*
d*x + 1/2*c)^3 - 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 27*a^3*tan(1/2*d*x + 1/2*c
) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^3)/d
```

Mupad [B]
time = 8.78, size = 339, normalized size = 2.53

$$\frac{3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{7\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{7\,a^3\,\mathrm{atan}\left(\frac{49\,a^6}{49\,a^6-49\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{49\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{49\,a^6-49\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{-17\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+19\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{64\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+73\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+46\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{107\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+8\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{3}}{d\,\left(8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)

[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (7*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) - (7*a^3*atan((49*a^6)/(49*a^6 - 49*a^6*tan(c/2 + (d*x)/2)) + (49*a^6*tan(c/2 + (d*x)/2))/(49*a^6 - 49*a^6*tan(c/2 + (d*x)/2))))/d + (7*a^3*tan(c/2 + (d*x)/2))/(8*d) - (8*a^3*tan(c/2 + (d*x)/2)^2 + (107*a^3*tan(c/2 + (d*x)/2)^3)/3 + 46*a^3*tan(c/2 + (d*x)/2)^4 + 73*a^3*tan(c/2 + (d*x)/2)^5 + (64*a^3*tan(c/2 + (d*x)/2)^6)/3 + 19*a^3*tan(c/2 + (d*x)/2)^7 - 17*a^3*tan(c/2 + (d*x)/2)^8 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9))

3.400 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$\frac{3a^3x}{2}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}-\frac{7a^3\cot(c+dx)\csc(c+dx)}{8d}$$

```
[Out] 3/2*a^3*x+33/8*a^3*arctanh(cos(d*x+c))/d-3*a^3*cos(d*x+c)/d+2*a^3*cot(d*x+c
)/d-a^3*cot(d*x+c)^3/d-7/8*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)*c
sc(d*x+c)^3/d-1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{2a^3\cot(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{7a^3\cot(c+dx)\csc(c+dx)}{8d}+\frac{3a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*x)/2 + (33*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (3*a^3*Cos[c + d*x])/d
 + (2*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (7*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) - (a^3*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
```

```
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(a^7-5a^7\csc(c+dx)-5a^7\csc^2(c+dx)+a^7\csc^3(c\right.}{}$$

$$= a^3x + a^3\int \csc^3(c+dx)\,dx + a^3\int \csc^5(c+dx)\,dx + a$$

$$= a^3x + \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{3a^3\cos(c+dx)}{d} - \frac{a^3}{}$$

$$= \frac{3a^3x}{2} + \frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a^3\cos(c+dx)}{d} +$$

$$= \frac{3a^3x}{2} + \frac{33a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3a^3\cos(c+dx)}{d} +$$

Mathematica [A]
time = 0.93, size = 215, normalized size = 1.56

$$\frac{a^3(1+\sin(c+dx))^3\left(96(c+dx)-192\cos(c+dx)+96\cot\left(\frac{1}{2}(c+dx)\right)-14\csc^2\left(\frac{1}{2}(c+dx)\right)-\csc^4\left(\frac{1}{2}(c+dx)\right)+264\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-264\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+14\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+64\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-4\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-16\sin(2(c+dx))-96\tan\left(\frac{1}{2}(c+dx)\right)\right)}{64d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(96*(c + d*x) - 192*Cos[c + d*x] + 96*Cot[(c + d*
x)/2] - 14*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 264*Log[Cos[(c + d*x)/
2]] - 264*Log[Sin[(c + d*x)/2]] + 14*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^
4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c + d*x)/2]^4*Sin[c + d*x
] - 16*Sin[2*(c + d*x)] - 96*Tan[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)
```

Maple [A]

time = 0.24, size = 224, normalized size = 1.62

method	result
risch	$\frac{3a^3x}{2}+\frac{ia^3 e^{2i(dx+c)}}{8d}-\frac{3a^3 e^{i(dx+c)}}{2d}-\frac{3a^3 e^{-i(dx+c)}}{2d}-\frac{ia^3 e^{-2i(dx+c)}}{8d}+\frac{a^3\left(7\,e^{7i(dx+c)}-15\,e^{5i(dx+c)}+40ie^{6i(dx+}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{}$
norman	$\frac{\frac{a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{64d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{19a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{11a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{11a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a^3\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*
cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/3*cot(
d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*
x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-1/sin(d*x+c)*cos
(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]

time = 0.49, size = 209, normalized size = 1.51

$$-\frac{8\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^3-16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
c)))*a^3 - 16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + a
^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^
2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 12*a^3*(2*cos
(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 231, normalized size = 1.67

$$\frac{24\,a^{3}dx\cos\left(dx+c\right)^{4}-48\,a^{3}\cos\left(dx+c\right)^{5}-48\,a^{3}dx\cos\left(dx+c\right)^{2}+110\,a^{3}\cos\left(dx+c\right)^{3}+24\,a^{3}dx-66\,a^{3}\cos\left(dx+c\right)+33\,\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-33\,\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-8\,\left(a^{3}\cos\left(dx+c\right)^{5}+4\,a^{3}\cos\left(dx+c\right)^{3}-3\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*a^3*d*x*cos(d*x + c)^4 - 48*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*
x + c)^2 + 110*a^3*cos(d*x + c)^3 + 24*a^3*d*x - 66*a^3*cos(d*x + c) + 33*(
a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2
) - 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x +
 c) + 1/2) - 8*(a^3*cos(d*x + c)^5 + 4*a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.63, size = 241, normalized size = 1.75

$$\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+16\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+96\,(dx+c)a^{3}-264\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-88\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{64\,\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}+\frac{550\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+88\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 16*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 96*(d*x + c)*a^3 - 264*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 88*a^3*tan(1/2*d*x + 1/2*c) + 64*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^
3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2 + (550*a^3*tan(1/2*d*x + 1/2*c)^4 + 88*a^3*tan(1/2*d*x + 1
/2*c)^3 - 16*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)
/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.78, size = 329, normalized size = 2.38

$$\frac{a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{4\,d}+\frac{a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{8\,d}+\frac{a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64\,d}-\frac{33\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}-\frac{3\,a^{3}\,\mathrm{atan}\left(\frac{9\,a^{6}}{\frac{99\,a^{6}}{4}+9\,a^{6}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{99\,a^{6}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,\left(\frac{99\,a^{6}}{4}+9\,a^{6}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d}-\frac{11\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}-\frac{-38\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+100\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-26\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\frac{417\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{4}-18\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\frac{9\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{2}+2\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a^{3}}{4}}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+32\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+16\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^4)/(64*d) - (33*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) - (3*
a^3*atan((9*a^6)/((99*a^6)/4 + 9*a^6*tan(c/2 + (d*x)/2)) - (99*a^6*tan(c/2
+ (d*x)/2))/(4*((99*a^6)/4 + 9*a^6*tan(c/2 + (d*x)/2)))))/d - (11*a^3*tan(c
/2 + (d*x)/2))/(8*d) - ((9*a^3*tan(c/2 + (d*x)/2)^2)/2 - 18*a^3*tan(c/2 + (
d*x)/2)^3 + (417*a^3*tan(c/2 + (d*x)/2)^4)/4 - 26*a^3*tan(c/2 + (d*x)/2)^5
+ 100*a^3*tan(c/2 + (d*x)/2)^6 - 38*a^3*tan(c/2 + (d*x)/2)^7 + a^3/4 + 2*a^
3*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6
 + 16*tan(c/2 + (d*x)/2)^8))
```

3.401 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$3a^3x+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cos(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{11a^3 \cot}{}$$

```
[Out] 3*a^3*x+3/8*a^3*arctanh(cos(d*x+c))/d-a^3*cos(d*x+c)/d+3*a^3*cot(d*x+c)/d-a
^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+11/8*a^3*cot(d*x+c)*csc(d*x+c)/d-3
/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3855, 3852, 8, 3853, 2718}

$$-\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{3a^3\cot(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{11a^3\cot(c+dx)\csc(c+dx)}{8d}+3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] 3*a^3*x + (3*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (a^3*Cos[c + d*x])/d + (3*a
^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) +
(11*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int (3a^7+a^7\csc(c+dx)-5a^7\csc^2(c+dx)-5a^7\csc^3(c$$

$$= 3a^3x + a^3\int \csc(c+dx)\,dx + a^3\int \csc^4(c+dx)\,dx + a$$

$$= 3a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{5a^3}{}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a}{}$$

$$= 3a^3x + \frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3\cos(c+dx)}{d} + \frac{3a}{}$$

Mathematica [A]
time = 0.39, size = 216, normalized size = 1.64

$$\frac{a^3\left(960c+960dx-320\cos(c+dx)+608\cot\left(\frac{1}{2}(c+dx)\right)+110\csc^2\left(\frac{1}{2}(c+dx)\right)-15\csc^4\left(\frac{1}{2}(c+dx)\right)+120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-110\sec^2\left(\frac{1}{2}(c+dx)\right)+15\sec^4\left(\frac{1}{2}(c+dx)\right)+208\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+64\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)-13\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-608\tan\left(\frac{1}{2}(c+dx)\right)\right)}{320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(960*c + 960*d*x - 320*Cos[c + d*x] + 608*Cot[(c + d*x)/2] + 110*Csc[(
c + d*x)/2]^2 - 15*Csc[(c + d*x)/2]^4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log
[Sin[(c + d*x)/2]] - 110*Sec[(c + d*x)/2]^2 + 15*Sec[(c + d*x)/2]^4 + 208*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 1
3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - Csc[(c + d*x)/2]^6*Sin[c + d*x] - 608*T
an[(c + d*x)/2]))/(320*d)
```

Maple [A]
time = 0.24, size = 190, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{a^{3}\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+3a^{3}\left(-\frac{\cos^{5}(dx+c)}{4\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{8\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^{3}\left(-\right.}{d}$
default	$\frac{-\frac{a^{3}\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+3a^{3}\left(-\frac{\cos^{5}(dx+c)}{4\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{8\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a^{3}\left(-\right.}{d}$
risch	$3a^{3}x-\frac{a^{3}e^{i(dx+c)}}{2d}-\frac{a^{3}e^{-i(dx+c)}}{2d}-\frac{a^{3}\left(-200ie^{8i(dx+c)}+55e^{9i(dx+c)}+720ie^{6i(dx+c)}+10e^{7i(dx+c)}-800ie^{4i(dx+c)}\right.}{20d\left(e^{2i(dx+c)}-1\right)^{5}}$
norman	$\frac{-\frac{a^{3}}{160d}-\frac{3a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{9a^{3}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}+\frac{7a^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{121a^{3}\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}+\frac{267a^{3}\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^3/sin(d*x+c)^5*cos(d*x+c)^5+3*a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)
^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc
(d*x+c)-cot(d*x+c)))+3*a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+a^3*(-1/2/s
in(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-
cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 180, normalized size = 1.36

$$\frac{80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^{2}-1}{\tan(dx+c)^{3}}\right)a^{3}-15\,a^{3}\left(\frac{2\left(5\cos(dx+c)^{3}-3\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)+20\,a^{3}\left(\frac{2\cos(dx+c)}{\cos(dx+c)^{2}-1}-4\cos\left(dx+c\right)+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{16\,a^{3}}{\tan(dx+c)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 15*a^3
*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
+ 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*a^3*(2*cos(d
*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(124) = 248.
time = 0.38, size = 252, normalized size = 1.91

$$\frac{304\,a^{3}\cos\left(dx+c\right)^{5}-560\,a^{3}\cos\left(dx+c\right)^{3}+240\,a^{3}\cos\left(dx+c\right)+15\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10\left(24\,a^{3}dx\cos\left(dx+c\right)^{4}-8\,a^{3}\cos\left(dx+c\right)^{5}-48\,a^{3}dx\cos\left(dx+c\right)^{2}+5\,a^{3}\cos\left(dx+c\right)^{3}+24\,a^{3}dx-3\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] 1/80*(304*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 240*a^3*cos(d*x + c
) + 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 15*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^
3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(24*a^3*d*x*cos(d*x + c)^
4 - 8*a^3*cos(d*x + c)^5 - 48*a^3*d*x*cos(d*x + c)^2 + 5*a^3*cos(d*x + c)^3
 + 24*a^3*d*x - 3*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)*sin(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A]
time = 0.68, size = 226, normalized size = 1.71

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-580\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{640\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{274\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+580\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+80\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 + 30*a^
3*tan(1/2*d*x + 1/2*c)^3 - 80*a^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^
3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 580*a^3*tan(1/2*d*x + 1/2*c) -
 640*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + (274*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
80*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 - 30*a^3*tan(
1/2*d*x + 1/2*c)^2 - 15*a^3*tan(1/2*d*x + 1/2*c) - 2*a^3)/tan(1/2*d*x + 1/2
*c)^5)/d

Mupad [B]
time = 10.11, size = 554, normalized size = 4.20

$$-\frac{a^3\left(2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+65\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+550\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+80\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+560\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-550\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-65\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+1920\,\mathrm{atan}\left(\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+1920\,\mathrm{atan}\left(\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}{320\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)

[Out] -(a^3*(2*cos(c/2 + (d*x)/2)^12 - 2*sin(c/2 + (d*x)/2)^12 - 15*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^11 + 15*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) -

```
 32*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 65*cos(c/2 + (d*x)/2)^3*si
n(c/2 + (d*x)/2)^9 + 550*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 80*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 560*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^5 - 550*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 65*cos(c/2 +
(d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 32*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/
2)^2 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*
sin(c/2 + (d*x)/2)^7 + 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 + 1920*atan((8*cos(c/2 + (d*x)/2) - si
n(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d
*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1920*atan((8*cos(c/2 + (d*x)/2) - sin(c/2 +
 (d*x)/2))/(cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^
7*sin(c/2 + (d*x)/2)^5))/(320*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5*(
cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))
```

3.402 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=168

$$a^3x - \frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d} + \frac{17a^3\cot(c+dx)\csc(c+}{16d}$$

```
[Out] a^3*x-19/16*a^3*arctanh(cos(d*x+c))/d+a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3
/d-3/5*a^3*cot(d*x+c)^5/d+17/16*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x
+c)^3*csc(d*x+c)/d+1/8*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)^3*c
sc(d*x+c)^3/d
```

Rubi [A]
time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{3a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^3(c+dx)}{3d} + \frac{a^3\cot(c+dx)}{d} - \frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{3a^3\cot^3(c+dx)\csc(c+dx)}{4d} + \frac{a^3\cot(c+dx)\csc^3(c+dx)}{8d} + \frac{17a^3\cot(c+dx)\csc(c+dx)}{16d} + a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (19*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cot[c + d*x])/d - (a^3
*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]^5)/(5*d) + (17*a^3*Cot[c + d*x
]*Csc[c + d*x])/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x])/(4*d) + (a^3*C
ot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)+3a^3\cot^4(c+dx)\csc(c+dx)+3\right.$$

$$= a^3\int \cot^4(c+dx)\,dx + a^3\int \cot^4(c+dx)\csc^3(c+dx$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a}{}$$

$$= \frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{3d} - \frac{3a^3\cot^5(c+dx)}{5d}$$

$$= a^3x - \frac{9a^3\tanh^{-1}(\cos(c+dx))}{8d} + \frac{a^3\cot(c+dx)}{d} - \frac{a}{}$$

$$= a^3x - \frac{19a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^3\cot(c+dx)}{d} - $$

Mathematica [A]
time = 0.75, size = 217, normalized size = 1.29

$$\frac{a^3\left(1920c+1920dx+704\cot\left(\frac{1}{2}(c+dx)\right)+870\csc^2\left(\frac{1}{2}(c+dx)\right)-2280\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2280\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-870\sec^2\left(\frac{1}{2}(c+dx)\right)+60\sec^4\left(\frac{1}{2}(c+dx)\right)+5\sec^6\left(\frac{1}{2}(c+dx)\right)-1376\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-\csc^6\left(\frac{1}{2}(c+dx)\right)(5+18\sin(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(-60+86\sin(c+dx))-704\tan\left(\frac{1}{2}(c+dx)\right)+36\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1920*c + 1920*d*x + 704*Cot[(c + d*x)/2] + 870*Csc[(c + d*x)/2]^2 - 2
280*Log[Cos[(c + d*x)/2]] + 2280*Log[Sin[(c + d*x)/2]] - 870*Sec[(c + d*x)/
2]^2 + 60*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 1376*Csc[c + d*x]^3*S
in[(c + d*x)/2]^4 - Csc[(c + d*x)/2]^6*(5 + 18*Sin[c + d*x]) + Csc[(c + d*x
)/2]^4*(-60 + 86*Sin[c + d*x]) - 704*Tan[(c + d*x)/2] + 36*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2]))/(1920*d)
```

Maple [A]
time = 0.22, size = 225, normalized size = 1.34

method	result
risch	$a^3x-\frac{a^3\left(435\,e^{11i(dx+c)}-865\,e^{9i(dx+c)}+240ie^{10i(dx+c)}-210\,e^{7i(dx+c)}+1200ie^{8i(dx+c)}-210\,e^{5i(dx+c)}-1760ie^{6i(dx+c)}\right.}{120d\left(e^{2i(dx+c)}-1\right)^6}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{3a^3\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{3a^3\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/4
8/sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d
*x+c)-cot(d*x+c)))-3/5*a^3/sin(d*x+c)^5*cos(d*x+c)^5+3*a^3*(-1/4/sin(d*x+c)
^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+
c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.49, size = 215, normalized size = 1.28

$$\frac{160\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3+5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-90\,a^3\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{288\,a^3}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*
x + c) - 1)) - 288*a^3/tan(d*x + c)^5)/d
```

Fricas [**A**]
time = 0.38, size = 290, normalized size = 1.73

$$\frac{480\,a^3dx\cos\left(dx+c\right)^6-1440\,a^3dx\cos\left(dx+c\right)^4-870\,a^3\cos\left(dx+c\right)^5+1440\,a^3dx\cos\left(dx+c\right)^2+1520\,a^3\cos\left(dx+c\right)^3-480\,a^3dx-570\,a^3\cos\left(dx+c\right)-285\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+285\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-32\,\left(11\,a^3\cos\left(dx+c\right)^5-35\,a^3\cos\left(dx+c\right)^3+15\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*d*x*cos(d*x + c)^4 - 870*a^3*c
os(d*x + c)^5 + 1440*a^3*d*x*cos(d*x + c)^2 + 1520*a^3*cos(d*x + c)^3 - 480
*a^3*d*x - 570*a^3*cos(d*x + c) - 285*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x +
 c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 285*(a^3*
cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/
2*cos(d*x + c) + 1/2) - 32*(11*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 +
 15*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 239, normalized size = 1.42

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+75\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-100\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-735\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1920\,(dx+c)a^3+2280\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-840\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{5586\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-840\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-735\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-100\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 75*a
^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan(1/2*d*x + 1/2*c)^3 - 735*a^3*tan(1/
2*d*x + 1/2*c)^2 + 1920*(d*x + c)*a^3 + 2280*a^3*log(abs(tan(1/2*d*x + 1/2*
```

```
c))) - 840*a^3*tan(1/2*d*x + 1/2*c) - (5586*a^3*tan(1/2*d*x + 1/2*c)^6 - 84
0*a^3*tan(1/2*d*x + 1/2*c)^5 - 735*a^3*tan(1/2*d*x + 1/2*c)^4 - 100*a^3*tan
(1/2*d*x + 1/2*c)^3 + 75*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x +
1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.38, size = 313, normalized size = 1.86

$$\frac{49\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{5\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{5\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{49\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{16\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+19\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{19\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-16\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{19\,a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{16\,d}+\frac{7\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (49*a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (5*a^3*cot(c/2 + (d*x)/2)^3)/(96*d)
 - (5*a^3*cot(c/2 + (d*x)/2)^4)/(128*d) - (3*a^3*cot(c/2 + (d*x)/2)^5)/(160
*d) - (a^3*cot(c/2 + (d*x)/2)^6)/(384*d) - (49*a^3*tan(c/2 + (d*x)/2)^2)/(1
28*d) - (5*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) + (5*a^3*tan(c/2 + (d*x)/2)^4)/
(128*d) + (3*a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (a^3*tan(c/2 + (d*x)/2)^6)
/(384*d) + (2*a^3*atan((16*cos(c/2 + (d*x)/2) + 19*sin(c/2 + (d*x)/2))/(19*
cos(c/2 + (d*x)/2) - 16*sin(c/2 + (d*x)/2))))/d + (19*a^3*log(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)))/(16*d) + (7*a^3*cot(c/2 + (d*x)/2))/(16*d) - (7*
a^3*tan(c/2 + (d*x)/2))/(16*d)
```

3.403 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=150

$$-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{7d}+\frac{3a^3\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^3\cot^3(c+dx}{4}$$

```
[Out] -9/16*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-1/7*a^3*cot(d*x+c)^7
/d+3/16*a^3*cot(d*x+c)*csc(d*x+c)/d-1/4*a^3*cot(d*x+c)^3*csc(d*x+c)/d+3/8*a
^3*cot(d*x+c)*csc(d*x+c)^3/d-1/2*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.21, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2691, 3855, 2687, 30, 3853, 14}

$$-\frac{a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{8d}+\frac{3a^3\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(16*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a^3
*Cot[c + d*x]^7)/(7*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (a^3*Co
t[c + d*x]^3*Csc[c + d*x])/(4*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d
) - (a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cot^4(c+dx)\csc(c+dx)+3a^3\cot^4(c+dx)\csc^2(\right.\\
&= a^3\int \cot^4(c+dx)\csc(c+dx)\,dx + a^3\int \cot^4(c+dx)\,c\\
&= -\frac{a^3\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{a^3\cot^3(c+dx)\csc^3(c-}{2d}\\
&= -\frac{3a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc(c+dx)}{8d} - \frac{a^3\,}{}\\
&= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot}{}\\
&= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 363 vs. 2(150) = 300.
time = 0.09, size = 363, normalized size = 2.42

$$a^3\left(-\frac{23\cot\left(\frac{1}{2}(c+dx)\right)}{70d}+\frac{7\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{297\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{2240d}+\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{31\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{2240d}-\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{128d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^6\left(\frac{1}{2}(c+dx)\right)}{896d}-\frac{9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}+\frac{9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}-\frac{7\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{128d}+\frac{23\tan\left(\frac{1}{2}(c+dx)\right)}{70d}-\frac{297\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{2240d}+\frac{31\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{2240d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{896d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-23*Cot[(c + d*x)/2])/(70*d) + (7*Csc[(c + d*x)/2]^2)/(64*d) + (297*C
ot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2240*d) + Csc[(c + d*x)/2]^4/(32*d) -
(31*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(2240*d) - Csc[(c + d*x)/2]^6/(128
*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(896*d) - (9*Log[Cos[(c + d*x)/
2]])/(16*d) + (9*Log[Sin[(c + d*x)/2]])/(16*d) - (7*Sec[(c + d*x)/2]^2)/(64
*d) - Sec[(c + d*x)/2]^4/(32*d) + Sec[(c + d*x)/2]^6/(128*d) + (23*Tan[(c +
 d*x)/2])/(70*d) - (297*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(2240*d) + (31
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(2240*d) + (Sec[(c + d*x)/2]^6*Tan[(c
 + d*x)/2])/(896*d))
```

Maple [A]

time = 0.25, size = 241, normalized size = 1.61

method	result
risch	$-\frac{a^{3}\left(1680 i \mathrm{e}^{12 i (d x +c)}+245 \,\mathrm{e}^{13 i (d x +c)}-4480 i \mathrm{e}^{10 i (d x +c)}-2380 \,\mathrm{e}^{11 i (d x +c)}+3920 i \mathrm{e}^{8 i (d x +c)}-455 \,\mathrm{e}^{9 i (d x +c)}-8960 i \mathrm{e}^{6 i (d x +}\right.}{280 d \left(\mathrm{e}^{2 i (d x +c)}-1\right)^{7}}$
derivativedivides	$\frac{a^{3} \left(-\frac{\cos^{5}\left(d x +c \right)}{7 \sin \left(d x +c \right)^{7}}-\frac{2 \left(\cos^{5}\left(d x +c \right)\right)}{35 \sin \left(d x +c \right)^{5}}\right)+3 a^{3} \left(-\frac{\cos^{5}\left(d x +c \right)}{6 \sin \left(d x +c \right)^{6}}-\frac{\cos^{5}\left(d x +c \right)}{24 \sin \left(d x +c \right)^{4}}+\frac{\cos^{5}\left(d x +c \right)}{48 \sin \left(d x +c \right)^{2}}+\frac{\left(\cos^{3}\left(d x +c \right)\right)}{48}+\frac{\cos \left(d x +c \right)}{16}+\frac{\ln (c}{}\right.}{}$
default	$\frac{a^{3} \left(-\frac{\cos^{5}\left(d x +c \right)}{7 \sin \left(d x +c \right)^{7}}-\frac{2 \left(\cos^{5}\left(d x +c \right)\right)}{35 \sin \left(d x +c \right)^{5}}\right)+3 a^{3} \left(-\frac{\cos^{5}\left(d x +c \right)}{6 \sin \left(d x +c \right)^{6}}-\frac{\cos^{5}\left(d x +c \right)}{24 \sin \left(d x +c \right)^{4}}+\frac{\cos^{5}\left(d x +c \right)}{48 \sin \left(d x +c \right)^{2}}+\frac{\left(\cos^{3}\left(d x +c \right)\right)}{48}+\frac{\cos \left(d x +c \right)}{16}+\frac{\ln (c}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+3*
a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin
(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)
-cot(d*x+c)))-3/5*a^3/sin(d*x+c)^5*cos(d*x+c)^5+a^3*(-1/4/sin(d*x+c)^4*cos(
d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*
ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.29, size = 206, normalized size = 1.37

$$\frac{35 \, a^{3} {\left(\frac{2 \, {\left(3 \, \cos\left(d x + c\right)^{5} + 8 \, \cos\left(d x + c\right)^{3} - 3 \, \cos\left(d x + c\right)\right)}}{\cos\left(d x + c\right)^{6} - 3 \, \cos\left(d x + c\right)^{4} + 3 \, \cos\left(d x + c\right)^{2} - 1} - 3 \, \log\left(\cos\left(d x + c\right) + 1\right) + 3 \, \log\left(\cos\left(d x + c\right) - 1\right)\right)} - 70 \, a^{3} {\left(\frac{2 \, {\left(5 \, \cos\left(d x + c\right)^{3} - 3 \, \cos\left(d x + c\right)\right)}}{\cos\left(d x + c\right)^{4} - 2 \, \cos\left(d x + c\right)^{2} + 1} + 3 \, \log\left(\cos\left(d x + c\right) + 1\right) - 3 \, \log\left(\cos\left(d x + c\right) - 1\right)\right)} - \frac{672 \, a^{3}}{\tan\left(d x + c\right)^{5}} - \frac{32 \, {\left(7 \, \tan\left(d x + c\right)^{2} + 5\right)} a^{3}}{\tan\left(d x + c\right)^{7}}}{1120 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(c
os(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x +
c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) -
 3*log(cos(d*x + c) - 1)) - 672*a^3/tan(d*x + c)^5 - 32*(7*tan(d*x + c)^2 +
 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.36, size = 247, normalized size = 1.65

$$-\frac{736\,a^3\cos\left(dx+c\right)^7-896\,a^3\cos\left(dx+c\right)^5+315\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-315\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+70\,\left(7\,a^3\cos\left(dx+c\right)^5-24\,a^3\cos\left(dx+c\right)^3+9\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1120\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(736*a^3*cos(d*x + c)^7 - 896*a^3*cos(d*x + c)^5 + 315*(a^3*cos(d*x
 + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 315*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4
 + 3*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
70*(7*a^3*cos(d*x + c)^5 - 24*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c))*sin(
d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 261, normalized size = 1.74

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+77\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-455\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-665\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2520\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{6534\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-665\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-455\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+77\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 77*a
^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(1/2*d*x + 1/2*c)^4 - 455*a^3*tan(1/2
*d*x + 1/2*c)^3 - 665*a^3*tan(1/2*d*x + 1/2*c)^2 + 2520*a^3*log(abs(tan(1/2
*d*x + 1/2*c))) + 945*a^3*tan(1/2*d*x + 1/2*c) - (6534*a^3*tan(1/2*d*x + 1/
```

```
2*c)^7 + 945*a^3*tan(1/2*d*x + 1/2*c)^6 - 665*a^3*tan(1/2*d*x + 1/2*c)^5 -
455*a^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x + 1/2*c)^3 + 77*a^3*tan
(1/2*d*x + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/
2*c)^7)/d
```

Mupad [B]
time = 10.32, size = 387, normalized size = 2.58

$$\frac{a^3\left(5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+77\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-455\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-665\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+665\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+455\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3-77\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2520\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}{4480\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (a^3*(5*sin(c/2 + (d*x)/2)^14 - 5*cos(c/2 + (d*x)/2)^14 + 35*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^13 - 35*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) +
77*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 35*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^11 - 455*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 665*c
os(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 945*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2)^8 - 945*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 665*cos(c/2
 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 455*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d
*x)/2)^4 + 35*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 77*cos(c/2 + (d*
x)/2)^12*sin(c/2 + (d*x)/2)^2 + 2520*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(4480*d*cos(c/2 + (d*x)/2)
^7*sin(c/2 + (d*x)/2)^7)
```

3.404 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{27a^3\cot(c+dx)\csc(c+dx)}{128d}+\frac{23a^3\cot(c+\cdots}{}$$

```
[Out] -27/128*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-3/7*a^3*cot(d*x+c)
^7/d-27/128*a^3*cot(d*x+c)*csc(d*x+c)/d+23/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d
-1/2*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d+1/16*a^3*cot(d*x+c)*csc(d*x+c)^5/d-1/8
*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.24, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2952, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d}+\frac{a^3\cot(c+dx)\csc^5(c+dx)}{16d}+\frac{23a^3\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{27a^3\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-27*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (3
*a^3*Cot[c + d*x]^7)/(7*d) - (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (
23*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(2*d) + (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^
3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^2(c+dx)+3a^3\cot^4(c+dx)\csc\right.$$

$$= a^3\int \cot^4(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^4(c+dx$$

$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{2d} - \frac{a^3\cot^3(c+dx)\csc^5(}{8d}$$

$$= -\frac{a^3\cot^5(c+dx)}{5d} + \frac{3a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3}{}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d} - \frac{3a^3\cot(c+}{}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3}{}$$

$$= -\frac{27a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{3a^3}{}$$

Mathematica [A]
time = 4.36, size = 313, normalized size = 1.78

$$-\frac{a^3\left(10\csc^8\left(\frac{1}{2}(c+dx)\right)(24+7\csc(c+dx))+8\csc^6\left(\frac{1}{2}(c+dx)\right)(-76+105\csc(c+dx))+8\csc^2\left(\frac{1}{2}(c+dx)\right)(1664+945\csc(c+dx))-4\csc^4\left(\frac{1}{2}(c+dx)\right)(856+1715\csc(c+dx))-4\left(-7560\csc(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(703+1056\cos(c+dx)+517\cos(2(c+dx))+104\cos(3(c+dx)))\sec^8\left(\frac{1}{2}(c+dx)\right)+7560\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-27440\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+13440\csc^7(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)+4480\csc^9(c+dx)\sin^8\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)(1+\sin(c+dx))^3}{143360d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/143360*(a^3*(10*Csc[(c + d*x)/2]^8*(24 + 7*Csc[c + d*x]) + 8*Csc[(c + d*
x)/2]^6*(-76 + 105*Csc[c + d*x]) + 8*Csc[(c + d*x)/2]^2*(1664 + 945*Csc[c +
 d*x]) - 4*Csc[(c + d*x)/2]^4*(856 + 1715*Csc[c + d*x]) - 4*(-7560*Csc[c +
d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + (703 + 1056*Cos[c +
d*x] + 517*Cos[2*(c + d*x)] + 104*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^8 + 75
60*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 27440*Csc[c + d*x]^5*Sin[(c + d*x)/2
]^4 + 13440*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6 + 4480*Csc[c + d*x]^9*Sin[(c
+ d*x)/2]^8))*Sin[c + d*x]*(1 + Sin[c + d*x])^3)/(d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^6)
```

Maple [A]
time = 0.26, size = 278, normalized size = 1.58

method	result
risch	$a^3(945\,e^{15i(dx+c)}+10675\,e^{13i(dx+c)}+6272ie^{4i(dx+c)}-15295\,e^{11i(dx+c)}-70784ie^{6i(dx+c)}-32165\,e^{9i(dx+c)}-8832ie^{2i(dx+$
derivativedivides	$a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)$
default	$a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/6
4/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c
)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/7/sin(d*x+c
)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+3*a^3*(-1/6/sin(d*x+c)^6*c
os(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c)^2*cos(d*x+c)^5+1
/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d*x+c)))-1/5*a^3/si
n(d*x+c)^5*cos(d*x+c)^5)
```

Maxima [A]
time = 0.29, size = 246, normalized size = 1.40

$$\frac{35\,a^3\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+280\,a^3\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-\frac{1792\,a^3}{\tan(dx+c)^5}-\frac{768\left(7\tan(dx+c)^2+5\right)a^3}{\tan(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1)) - 1792*a^3/tan(d*x + c)^5 - 768*(7*tan(d*x +
c)^2 + 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.36, size = 271, normalized size = 1.54

$$\frac{1890\,a^3\cos\left(dx+c\right)^7+2030\,a^3\cos\left(dx+c\right)^5-6930\,a^3\cos\left(dx+c\right)^3+1890\,a^3\cos\left(dx+c\right)-945\,\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+945\,\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+256\,\left(13\,a^3\cos\left(dx+c\right)^7-28\,a^3\cos\left(dx+c\right)^5\right)\sin\left(dx+c\right)}{8960\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(1890*a^3*cos(d*x + c)^7 + 2030*a^3*cos(d*x + c)^5 - 6930*a^3*cos(d*
x + c)^3 + 1890*a^3*cos(d*x + c) - 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x
+ c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x
 + c) + 1/2) + 945*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d
*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 256*
(13*a^3*cos(d*x + c)^7 - 28*a^3*cos(d*x + c)^5)*sin(d*x + c))/(d*cos(d*x +
c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 293, normalized size = 1.66

$$\frac{35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{41094\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+9520\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{71680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*tan(1/2*d*x + 1/2*c)^7 + 5
60*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^5 - 1960*a^3*t
an(1/2*d*x + 1/2*c)^4 - 3920*a^3*tan(1/2*d*x + 1/2*c)^3 - 1680*a^3*tan(1/2*
d*x + 1/2*c)^2 + 15120*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9520*a^3*tan(1/
2*d*x + 1/2*c) - (41094*a^3*tan(1/2*d*x + 1/2*c)^8 + 9520*a^3*tan(1/2*d*x +
 1/2*c)^7 - 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 3920*a^3*tan(1/2*d*x + 1/2*c)
^5 - 1960*a^3*tan(1/2*d*x + 1/2*c)^4 + 112*a^3*tan(1/2*d*x + 1/2*c)^3 + 560
*a^3*tan(1/2*d*x + 1/2*c)^2 + 240*a^3*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/
2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 9.22, size = 319, normalized size = 1.81

$$\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{7\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}+\frac{7\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}-\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{7\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}+\frac{27\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,d}-\frac{17\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{17\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (3*a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (7*a^3*cot(c/2 + (d*x)/2)^3)/(128*d)
 + (7*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) - (a^3*cot(c/2 + (d*x)/2)^5)/(640*d
) - (a^3*cot(c/2 + (d*x)/2)^6)/(128*d) - (3*a^3*cot(c/2 + (d*x)/2)^7)/(896*
d) - (a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (3*a^3*tan(c/2 + (d*x)/2)^2)/(12
8*d) - (7*a^3*tan(c/2 + (d*x)/2)^3)/(128*d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/
(256*d) + (a^3*tan(c/2 + (d*x)/2)^5)/(640*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(
128*d) + (3*a^3*tan(c/2 + (d*x)/2)^7)/(896*d) + (a^3*tan(c/2 + (d*x)/2)^8)/
(2048*d) + (27*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) - (17*a^3*cot(c/2 + (d*
x)/2))/(128*d) + (17*a^3*tan(c/2 + (d*x)/2))/(128*d)
```

3.405 $\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^9(c+dx)}{9d}-\frac{17a^3\cot(c+dx)\csc(c-}{128d}$$

```
[Out] -17/128*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-5/7*a^3*cot(d*x+c)
^7/d-1/9*a^3*cot(d*x+c)^9/d-17/128*a^3*cot(d*x+c)*csc(d*x+c)/d+5/64*a^3*cot
(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d+3/16*a^3*cot(d*x
+c)*csc(d*x+c)^5/d-3/8*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.24, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2691, 3853, 3855, 2687, 14, 276}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{5a^3\cot^7(c+dx)}{7d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{3a^3\cot(c+dx)\csc^5(c+dx)}{16d}+\frac{5a^3\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{17a^3\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (5
*a^3*Cot[c + d*x]^7)/(7*d) - (a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Cot[c + d
*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a
^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^
5)/(16*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^3(c+dx)+3a^3\cot^4(c+dx)\csc^4\right.$$

$$= a^3\int \cot^4(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^4(c+dx)$$

$$= -\frac{a^3\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{3a^3\cot^3(c+dx)\csc^5(}{8d}$$

$$= \frac{a^3\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^3(c+}{6d}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx}{9d}$$

$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3\cot}{}$$

$$= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{5a^3 c}{}$$

Mathematica [A]
time = 0.94, size = 313, normalized size = 1.61

$$-\frac{a^3 \csc^9(c+dx) \left(1161216 \cos(c+dx)+247296 \cos(3(c+dx))-198144 \cos(5(c+dx))-71424 \cos(7(c+dx))+7936 \cos(9(c+dx))+674730 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx)-674730 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx)+669060 \sin(2(c+dx))-449820 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(3(c+dx))+449820 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(3(c+dx))+676620 \sin(4(c+dx))+192780 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(5(c+dx))-192780 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(5(c+dx))-14700 \sin(6(c+dx))-48195 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(7(c+dx))+48195 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(7(c+dx))-10710 \sin(8(c+dx))+5355 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(9(c+dx))-5355 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(9(c+dx))\right)}{10321920 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/10321920*(a^3*Csc[c + d*x]^9*(1161216*Cos[c + d*x] + 247296*Cos[3*(c + d
*x)] - 198144*Cos[5*(c + d*x)] - 71424*Cos[7*(c + d*x)] + 7936*Cos[9*(c + d
*x)] + 674730*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 674730*Log[Sin[(c + d*x)
/2]]*Sin[c + d*x] + 669060*Sin[2*(c + d*x)] - 449820*Log[Cos[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 449820*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 676620*S
in[4*(c + d*x)] + 192780*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 192780*Lo
g[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 14700*Sin[6*(c + d*x)] - 48195*Log[C
os[(c + d*x)/2]]*Sin[7*(c + d*x)] + 48195*Log[Sin[(c + d*x)/2]]*Sin[7*(c +
d*x)] - 10710*Sin[8*(c + d*x)] + 5355*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)
] - 5355*Log[Sin[(c + d*x)/2]]*Sin[9*(c + d*x)]))/d
```

Maple [A]
time = 0.27, size = 316, normalized size = 1.63

method	result
risch	$\frac{a^3\left(5355\,e^{17i(dx+c)}-43776ie^{4i(dx+c)}+7350\,e^{15i(dx+c)}+209664ie^{6i(dx+c)}-338310\,e^{13i(dx+c)}+71424ie^{2i(dx+c)}-334530\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{9\sin(dx+c)^9}-\frac{4\left(\cos^5(dx+c)\right)}{63\sin(dx+c)^7}-\frac{8\left(\cos^5(dx+c)\right)}{315\sin(dx+c)^5}\right)+3a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{9\sin(dx+c)^9}-\frac{4\left(\cos^5(dx+c)\right)}{63\sin(dx+c)^7}-\frac{8\left(\cos^5(dx+c)\right)}{315\sin(dx+c)^5}\right)+3a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^5-4/63/sin(d*x+c)^7*cos(d*x+c)^5-8/3
15/sin(d*x+c)^5*cos(d*x+c)^5)+3*a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/si
n(d*x+c)^6*cos(d*x+c)^5-1/64/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*c
os(d*x+c)^5+1/128*cos(d*x+c)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x
+c)))+3*a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)
+a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/si
n(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c
)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 268, normalized size = 1.38

$$\frac{945\,a^3\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+840\,a^3\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{6912\left(7\tan(dx+c)^2+5\right)a^3}{\tan(dx+c)^7}-\frac{256\left(63\tan(dx+c)^4+90\tan(dx+c)^2+35\right)a^3}{\tan(dx+c)^9}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/80640*(945*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)
^3 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)
) + 840*a^3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 6912*(7*tan(d*x + c)^2 + 5)*a^3/tan(d*x +
 c)^7 - 256*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan(d*x + c)^9
)/d
```

Fricas [A]
time = 0.37, size = 304, normalized size = 1.57

$$-\frac{15872\,a^3\cos\left(dx+c\right)^9-71424\,a^3\cos\left(dx+c\right)^7+64512\,a^3\cos\left(dx+c\right)^5+5355\,\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-5355\,\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-210\,\left(51\,a^3\cos\left(dx+c\right)^7-59\,a^3\cos\left(dx+c\right)^5-187\,a^3\cos\left(dx+c\right)^3+51\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/80640*(15872*a^3*cos(d*x + c)^9 - 71424*a^3*cos(d*x + c)^7 + 64512*a^3*c
os(d*x + c)^5 + 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos
(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 5355*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
 c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
c) - 210*(51*a^3*cos(d*x + c)^7 - 59*a^3*cos(d*x + c)^5 - 187*a^3*cos(d*x +
 c)^3 + 51*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 325, normalized size = 1.68

$$\frac{140\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+2340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-4032\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5040\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+85680\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+52920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{242386\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+52920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-5040\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-16800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-12600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4032\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+140\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/645120*(140*a^3*tan(1/2*d*x + 1/2*c)^9 + 945*a^3*tan(1/2*d*x + 1/2*c)^8 +
 2340*a^3*tan(1/2*d*x + 1/2*c)^7 + 1680*a^3*tan(1/2*d*x + 1/2*c)^6 - 4032*a
^3*tan(1/2*d*x + 1/2*c)^5 - 12600*a^3*tan(1/2*d*x + 1/2*c)^4 - 16800*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 5040*a^3*tan(1/2*d*x + 1/2*c)^2 + 85680*a^3*log(abs(
tan(1/2*d*x + 1/2*c))) + 52920*a^3*tan(1/2*d*x + 1/2*c) - (242386*a^3*tan(1
/2*d*x + 1/2*c)^9 + 52920*a^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 16800*a^3*tan(1/2*d*x + 1/2*c)^6 - 12600*a^3*tan(1/2*d*x + 1/
2*c)^5 - 4032*a^3*tan(1/2*d*x + 1/2*c)^4 + 1680*a^3*tan(1/2*d*x + 1/2*c)^3
+ 2340*a^3*tan(1/2*d*x + 1/2*c)^2 + 945*a^3*tan(1/2*d*x + 1/2*c) + 140*a^3)
/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 9.32, size = 357, normalized size = 1.84

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,d}-\frac{13\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4608\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{192\,d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,d}+\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4608\,d}+\frac{17\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,d}-\frac{21\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}+\frac{21\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^10,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^2)/(128*d) + (5*a^3*cot(c/2 + (d*x)/2)^3)/(192*d) +
 (5*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) + (a^3*cot(c/2 + (d*x)/2)^5)/(160*d)
- (a^3*cot(c/2 + (d*x)/2)^6)/(384*d) - (13*a^3*cot(c/2 + (d*x)/2)^7)/(3584*
d) - (3*a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(46
08*d) - (a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (5*a^3*tan(c/2 + (d*x)/2)^3)/(
192*d) - (5*a^3*tan(c/2 + (d*x)/2)^4)/(256*d) - (a^3*tan(c/2 + (d*x)/2)^5)/
(160*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (13*a^3*tan(c/2 + (d*x)/2)^7
)/(3584*d) + (3*a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^3*tan(c/2 + (d*x)/2
)^9)/(4608*d) + (17*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) - (21*a^3*cot(c/2
+ (d*x)/2))/(256*d) + (21*a^3*tan(c/2 + (d*x)/2))/(256*d)
```

3.406 $\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{a^3\cot^9(c+dx)}{3d}-\frac{21a^3\cot(c+dx)\csc(c+d}{256d}$$

```
[Out] -21/256*a^3*arctanh(cos(d*x+c))/d-4/5*a^3*cot(d*x+c)^5/d-a^3*cot(d*x+c)^7/d
-1/3*a^3*cot(d*x+c)^9/d-21/256*a^3*cot(d*x+c)*csc(d*x+c)/d-7/128*a^3*cot(d*
x+c)*csc(d*x+c)^3/d+29/160*a^3*cot(d*x+c)*csc(d*x+c)^5/d-3/8*a^3*cot(d*x+c)
^3*csc(d*x+c)^5/d+3/80*a^3*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^3*cot(d*x+c)^3*
csc(d*x+c)^7/d
```

Rubi [A]
time = 0.28, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 14, 2691, 3853, 3855, 276}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{a^3\cot^7(c+dx)}{d}-\frac{4a^3\cot^5(c+dx)}{5d}-\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{10d}-\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{3a^3\cot(c+dx)\csc^7(c+dx)}{80d}+\frac{29a^3\cot(c+dx)\csc^5(c+dx)}{160d}-\frac{7a^3\cot(c+dx)\csc^3(c+dx)}{128d}-\frac{21a^3\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^5)/(5*d) - (a
^3*Cot[c + d*x]^7)/d - (a^3*Cot[c + d*x]^9)/(3*d) - (21*a^3*Cot[c + d*x]*Cs
c[c + d*x])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (29*a^3
*Cot[c + d*x]*Csc[c + d*x]^5)/(160*d) - (3*a^3*Cot[c + d*x]^3*Csc[c + d*x]^
5)/(8*d) + (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^3*Cot[c + d*x]^3
*Csc[c + d*x]^7)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^4(c+dx)\csc^4(c+dx) + 3a^3\cot^4(c+dx)\csc^5\right.$$

$$= a^3\int \cot^4(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^4(c+dx)$$

$$= -\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^3\cot^3(c+dx)\csc^7(}{10d}$$

$$= \frac{3a^3\cot(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^3(c+dx)\csc^5(c}{8d}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d}$$

$$= -\frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{d} - \frac{a^3\cot^9(c+dx)}{3d}$$

$$= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot}{}$$

$$= -\frac{21a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot}{}$$

Mathematica [**A**]
time = 1.78, size = 366, normalized size = 1.69

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-4096*Cot[(c + d*x)/2] - 1260*Csc[(c + d*x)/2]^2
 - 5040*Log[Cos[(c + d*x)/2]] + 5040*Log[Sin[(c + d*x)/2]] + 1260*Sec[(c +
d*x)/2]^2 - 180*Sec[(c + d*x)/2]^4 - 390*Sec[(c + d*x)/2]^6 + 75*Sec[(c + d
*x)/2]^8 + 6*Sec[(c + d*x)/2]^10 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4
*Csc[(c + d*x)/2]^4*(-45 + Sin[c + d*x]) + 5*Csc[(c + d*x)/2]^8*(-15 + 4*Si
n[c + d*x]) - 2*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 6*Csc[(c + d*x)
/2]^6*(65 + 42*Sin[c + d*x]) + 4096*Tan[(c + d*x)/2] - 504*Sec[(c + d*x)/2]
^4*Tan[(c + d*x)/2] - 40*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2] + 40*Sec[(c +
d*x)/2]^8*Tan[(c + d*x)/2]))/(61440*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6)
```

Maple [**A**]
time = 0.29, size = 352, normalized size = 1.63

method	result

risch	$\frac{a^3(315\,e^{19i(dx+c)}-3045\,e^{17i(dx+c)}-23676\,e^{15i(dx+c)}+15360ie^{4i(dx+c)}+27780\,e^{13i(dx+c)}+122880ie^{8i(dx+c)}+96930\,e^{1}}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{32\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{128\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{256\sin(dx+c)^2}+\frac{(\cos^3(dx+c))}{256}+\frac{3\cos(dx+c)}{256}+\frac{3\ln(\csc(}{}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{32\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{128\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{256\sin(dx+c)^2}+\frac{(\cos^3(dx+c))}{256}+\frac{3\cos(dx+c)}{256}+\frac{3\ln(\csc(}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/10/sin(d*x+c)^10*cos(d*x+c)^5-1/16/sin(d*x+c)^8*cos(d*x+c)^5-1
/32/sin(d*x+c)^6*cos(d*x+c)^5-1/128/sin(d*x+c)^4*cos(d*x+c)^5+1/256/sin(d*x
+c)^2*cos(d*x+c)^5+1/256*cos(d*x+c)^3+3/256*cos(d*x+c)+3/256*ln(csc(d*x+c)-
cot(d*x+c)))+3*a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^5-4/63/sin(d*x+c)^7*cos(d*
x+c)^5-8/315/sin(d*x+c)^5*cos(d*x+c)^5)+3*a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)
^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/64/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(
d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+
c)-cot(d*x+c)))+a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d
*x+c)^5))
```

Maxima [A]
time = 0.29, size = 308, normalized size = 1.43

$$\frac{21\,a^3\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7+128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)+630\,a^3\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-\frac{1536\left(7\tan(dx+c)^2+5\right)a^3}{\tan(dx+c)^7}-\frac{512\left(63\tan(dx+c)^4+90\tan(dx+c)^2+35\right)a^3}{\tan(dx+c)^9}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] 1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 + 128*cos(d*x + c
)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c
)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 630*a^3*(2*(3*cos(d*x + c
)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 + 3*cos(d*x + c))/(cos(d*x + c)
^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 1536*(7*tan(d*x + c)^2 + 5)*a^
3/tan(d*x + c)^7 - 512*(63*tan(d*x + c)^4 + 90*tan(d*x + c)^2 + 35)*a^3/tan
(d*x + c)^9)/d
```

Fricas [A]
time = 0.39, size = 340, normalized size = 1.57

$$\frac{630\,a^3\cos(dx+c)^9-2940\,a^3\cos(dx+c)^7+768\,a^3\cos(dx+c)^5+2940\,a^3\cos(dx+c)^3-630\,a^3\cos(dx+c)-315\left(a^3\cos(dx+c)^{10}-5\,a^3\cos(dx+c)^8+10\,a^3\cos(dx+c)^6-10\,a^3\cos(dx+c)^4+5\,a^3\cos(dx+c)^2-a^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+315\left(a^3\cos(dx+c)^{10}-5\,a^3\cos(dx+c)^8+10\,a^3\cos(dx+c)^6-10\,a^3\cos(dx+c)^4+5\,a^3\cos(dx+c)^2-a^3\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+512\left(2\,a^3\cos(dx+c)^9-9\,a^3\cos(dx+c)^7+12\,a^3\cos(dx+c)^5\right)\sin(dx+c)}{7680\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/7680*(630*a^3*cos(d*x + c)^9 - 2940*a^3*cos(d*x + c)^7 + 768*a^3*cos(d*x
+ c)^5 + 2940*a^3*cos(d*x + c)^3 - 630*a^3*cos(d*x + c) - 315*(a^3*cos(d*x
+ c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c
)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^3*co
s(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d
*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 512*
(2*a^3*cos(d*x + c)^9 - 9*a^3*cos(d*x + c)^7 + 12*a^3*cos(d*x + c)^5)*sin(d
*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10
*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.70, size = 357, normalized size = 1.65

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-384\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-840\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5040\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{14762\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+3600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-960\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-840\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-384\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+40\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{61440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/61440*(6*a^3*tan(1/2*d*x + 1/2*c)^10 + 40*a^3*tan(1/2*d*x + 1/2*c)^9 + 10
5*a^3*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a^3*tan(
1/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 840*a^3*tan(1/2*d*x +
 1/2*c)^4 - 960*a^3*tan(1/2*d*x + 1/2*c)^3 + 60*a^3*tan(1/2*d*x + 1/2*c)^2
+ 5040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3600*a^3*tan(1/2*d*x + 1/2*c) -
 (14762*a^3*tan(1/2*d*x + 1/2*c)^10 + 3600*a^3*tan(1/2*d*x + 1/2*c)^9 + 60*
a^3*tan(1/2*d*x + 1/2*c)^8 - 960*a^3*tan(1/2*d*x + 1/2*c)^7 - 840*a^3*tan(1
/2*d*x + 1/2*c)^6 - 384*a^3*tan(1/2*d*x + 1/2*c)^5 - 30*a^3*tan(1/2*d*x + 1
/2*c)^4 + 120*a^3*tan(1/2*d*x + 1/2*c)^3 + 105*a^3*tan(1/2*d*x + 1/2*c)^2 +
 40*a^3*tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^10)/d
```

Mupad [**B**]
time = 9.55, size = 395, normalized size = 1.83

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}+\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{512\,d}-\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{1536\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64\,d}-\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{512\,d}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{1536\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}+\frac{21\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{256\,d}-\frac{15\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}+\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^3)/sin(c + d*x)^11,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(64*d) - (a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (
7*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) + (a^3*cot(c/2 + (d*x)/2)^5)/(160*d) +
(a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^7)/(512*d) -
(7*a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(1536*d)
 - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a^3*tan(c/2 + (d*x)/2)^2)/(1024
*d) - (a^3*tan(c/2 + (d*x)/2)^3)/(64*d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/(512
*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) - (a^3*tan(c/2 + (d*x)/2)^6)/(2048
*d) + (a^3*tan(c/2 + (d*x)/2)^7)/(512*d) + (7*a^3*tan(c/2 + (d*x)/2)^8)/(40
96*d) + (a^3*tan(c/2 + (d*x)/2)^9)/(1536*d) + (a^3*tan(c/2 + (d*x)/2)^10)/(
10240*d) + (21*a^3*log(tan(c/2 + (d*x)/2)))/(256*d) - (15*a^3*cot(c/2 + (d*
x)/2))/(256*d) + (15*a^3*tan(c/2 + (d*x)/2))/(256*d)
```

3.407 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=187

$$\frac{55a^4x}{256}-\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}+\frac{55a^4\cos^3(c+dx)\sin(c+dx)}{384d}+\frac{11a^4\cos^5(c+dx)\,\mathrm{s}}{96d}$$

```
[Out] 55/256*a^4*x-11/112*a^4*cos(d*x+c)^7/d+55/256*a^4*cos(d*x+c)*sin(d*x+c)/d+5
5/384*a^4*cos(d*x+c)^3*sin(d*x+c)/d+11/96*a^4*cos(d*x+c)^5*sin(d*x+c)/d-1/1
0*cos(d*x+c)^5*(a+a*sin(d*x+c))^5/a/d-1/18*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c)
)^2/d-11/144*cos(d*x+c)^7*(a^4+a^4*sin(d*x+c))/d
```

Rubi [A]
time = 0.17, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2949, 2757, 2748, 2715, 8}

$$-\frac{11a^4\cos^7(c+dx)}{112d}-\frac{11\cos^7(c+dx)\left(a^4\sin(c+dx)+a^4\right)}{144d}+\frac{11a^4\sin(c+dx)\cos^5(c+dx)}{96d}+\frac{55a^4\sin(c+dx)\cos^3(c+dx)}{384d}+\frac{55a^4\sin(c+dx)\cos(c+dx)}{256d}+\frac{55a^4x}{256}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)^2}{18d}-\frac{\cos^5(c+dx)(a\sin(c+dx)+a)^5}{10ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (55*a^4*x)/256 - (11*a^4*Cos[c + d*x]^7)/(112*d) + (55*a^4*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (55*a^4*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^4
*Cos[c + d*x]^5*Sin[c + d*x])/(96*d) - (Cos[c + d*x]^5*(a + a*Sin[c + d*x])
^5)/(10*a*d) - (Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x])^2)/(18*d) - (11*Cos
[c + d*x]^7*(a^4 + a^4*Sin[c + d*x]))/(144*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^4\,dx &= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}+\frac{1}{2}a\int \cos^6(c+d \\
&= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\,(a^2 \\
&= -\frac{\cos^5(c+dx)(a+a\sin(c+dx))^5}{10ad}-\frac{\cos^7(c+dx)\,(a^2 \\
&= -\frac{11a^4\cos^7(c+dx)}{112d}-\frac{\cos^5(c+dx)(a+a\sin(c+dx)) \\
&= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{11a^4\cos^5(c+dx)\sin(c+dx)}{96d}- \\
&= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos^3(c+dx)\sin(c+dx)}{384d}+ \\
&= -\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+dx)}{256d}+ \\
&= \frac{55a^4x}{256}-\frac{11a^4\cos^7(c+dx)}{112d}+\frac{55a^4\cos(c+dx)\sin(c+}{256d}
\end{aligned}
$$

Mathematica [A]

time = 0.89, size = 116, normalized size = 0.62

$$
\frac{a^4(136080c+138600dx-181440\cos(c+dx)-53760\cos(3(c+dx))+16128\cos(5(c+dx))+7200\cos(7(c+dx))-1120\cos(9(c+dx))+8820\sin(2(c+dx))-42840\sin(4(c+dx))-2730\sin(6(c+dx))+4095\sin(8(c+dx))-126\sin(10(c+dx)))}{645120d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(136080*c + 138600*d*x - 181440*Cos[c + d*x] - 53760*Cos[3*(c + d*x)]
+ 16128*Cos[5*(c + d*x)] + 7200*Cos[7*(c + d*x)] - 1120*Cos[9*(c + d*x)] +
8820*Sin[2*(c + d*x)] - 42840*Sin[4*(c + d*x)] - 2730*Sin[6*(c + d*x)] + 40
95*Sin[8*(c + d*x)] - 126*Sin[10*(c + d*x)]))/(645120*d)
```

Maple [A]
time = 0.36, size = 306, normalized size = 1.64

method	result
risch	$\frac{55a^4x}{256} - \frac{9a^4\cos(dx+c)}{32d} - \frac{a^4\sin(10dx+10c)}{5120d} - \frac{a^4\cos(9dx+9c)}{576d} + \frac{13a^4\sin(8dx+8c)}{2048d} + \frac{5a^4\cos(7dx+7c)}{448d} - \frac{13a^4}{}$
derivativedivides	$\frac{a^4\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+4a^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(\right.}{35}\right.}{}$
default	$\frac{a^4\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+4a^4\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(\right.}{35}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+4*a^4*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d
*x+c)^5)+6*a^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5
+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+4*a^4*(-1
/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c
)^5)+a^4*(-1/10*sin(d*x+c)^5*cos(d*x+c)^5-1/16*sin(d*x+c)^3*cos(d*x+c)^5-1/
32*sin(d*x+c)*cos(d*x+c)^5+1/128*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/256*d*x+3/256*c))
```

Maxima [A]
time = 0.29, size = 186, normalized size = 0.99

$$-\frac{8192\left(35\cos(dx+c)^9-90\cos(dx+c)^7+63\cos(dx+c)^5\right)a^4-73728\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^4+63\left(32\sin(2dx+2c)^5-120dx-120c-5\sin(8dx+8c)+40\sin(4dx+4c)\right)a^4-3360\left(4\sin(2dx+2c)^3+12dx+12c-3\sin(4dx+4c)\right)a^4-3780\left(24dx+24c+\sin(8dx+8c)-8\sin(4dx+4c)\right)a^4}{645120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/645120*(8192*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)
*a^4 - 73728*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^4 + 63*(32*sin(2*d*x +
 2*c)^5 - 120*d*x - 120*c - 5*sin(8*d*x + 8*c) + 40*sin(4*d*x + 4*c))*a^4 -
```

3360*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4 - 378
0*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a^4)/d

Fricas [A]
time = 0.38, size = 124, normalized size = 0.66

$$-\frac{35840\,a^4\cos\left(dx+c\right)^9-138240\,a^4\cos\left(dx+c\right)^7+129024\,a^4\cos\left(dx+c\right)^5-17325\,a^4dx+21\,\left(384\,a^4\cos\left(dx+c\right)^9-3888\,a^4\cos\left(dx+c\right)^7+5704\,a^4\cos\left(dx+c\right)^5-550\,a^4\cos\left(dx+c\right)^3-825\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/80640*(35840*a^4*cos(d*x + c)^9 - 138240*a^4*cos(d*x + c)^7 + 129024*a^4
*cos(d*x + c)^5 - 17325*a^4*d*x + 21*(384*a^4*cos(d*x + c)^9 - 3888*a^4*cos
(d*x + c)^7 + 5704*a^4*cos(d*x + c)^5 - 550*a^4*cos(d*x + c)^3 - 825*a^4*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(173) = 346$.
time = 2.88, size = 746, normalized size = 3.99

$$\begin{cases} \frac{3a^4x\sin^{10}(c+dx)}{256} + \frac{15a^4x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{9a^4x\sin^8(c+dx)}{64} + \frac{15a^4x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{9a^4x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{a^4x\sin^6(c+dx)}{16} + \frac{15a^4x\sin^4(c+dx)\cos^6(c+dx)}{128} + \frac{27a^4x\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{3a^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^4x\sin^2(c+dx)\cos^8(c+dx)}{256} + \frac{9a^4x\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{3a^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3a^4x\cos^{10}(c+dx)}{256} + \frac{9a^4x\cos^8(c+dx)}{64} + \frac{a^4x\cos^6(c+dx)}{16} + \frac{3a^4\sin^9(c+dx)\cos(c+dx)}{256d} + \frac{7a^4\sin^7(c+dx)\cos^3(c+dx)}{128d} + \cdots & \text{for}\: d \neq 0 \\ x(a\sin(c)+a)^4\sin^2(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**10/256 + 15*a**4*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 9*a**4*x*sin(c + d*x)**8/64 + 15*a**4*x*sin(c + d*x)**6*cos
(c + d*x)**4/128 + 9*a**4*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + a**4*x*sin
(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 27*a**4*x
*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a**4*x*sin(c + d*x)**4*cos(c + d*x)
**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 9*a**4*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3
*a**4*x*cos(c + d*x)**10/256 + 9*a**4*x*cos(c + d*x)**8/64 + a**4*x*cos(c +
 d*x)**6/16 + 3*a**4*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**4*sin(c +
d*x)**7*cos(c + d*x)**3/(128*d) + 9*a**4*sin(c + d*x)**7*cos(c + d*x)/(64*d
) - a**4*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 33*a**4*sin(c + d*x)**5*c
os(c + d*x)**3/(64*d) + a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 4*a**4*s
in(c + d*x)**4*cos(c + d*x)**5/(5*d) - 7*a**4*sin(c + d*x)**3*cos(c + d*x)*
*7/(128*d) - 33*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) + a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 16*a**4*sin(c + d*x)**2*cos(c + d*x)**7/(35
*d) - 4*a**4*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 3*a**4*sin(c + d*x)*co
s(c + d*x)**9/(256*d) - 9*a**4*sin(c + d*x)*cos(c + d*x)**7/(64*d) - a**4*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 32*a**4*cos(c + d*x)**9/(315*d) - 8*a*
*4*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c) + a)**4*sin(c)**2*cos(c)
**4, True))
```

Giac [A]
time = 0.65, size = 174, normalized size = 0.93

$$\frac{55}{256}a^4x - \frac{a^4\cos(9dx+9c)}{576d} + \frac{5a^4\cos(7dx+7c)}{448d} + \frac{a^4\cos(5dx+5c)}{40d} - \frac{a^4\cos(3dx+3c)}{12d} - \frac{9a^4\cos(dx+c)}{32d} - \frac{a^4\sin(10dx+10c)}{5120d} + \frac{13a^4\sin(8dx+8c)}{2048d} - \frac{13a^4\sin(6dx+6c)}{3072d} - \frac{17a^4\sin(4dx+4c)}{256d} + \frac{7a^4\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 55/256*a^4*x - 1/576*a^4*cos(9*d*x + 9*c)/d + 5/448*a^4*cos(7*d*x + 7*c)/d
+ 1/40*a^4*cos(5*d*x + 5*c)/d - 1/12*a^4*cos(3*d*x + 3*c)/d - 9/32*a^4*cos(
d*x + c)/d - 1/5120*a^4*sin(10*d*x + 10*c)/d + 13/2048*a^4*sin(8*d*x + 8*c)
/d - 13/3072*a^4*sin(6*d*x + 6*c)/d - 17/256*a^4*sin(4*d*x + 4*c)/d + 7/512
*a^4*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.84, size = 572, normalized size = 3.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

```
[Out] (55*a^4*x)/256 - ((571*a^4*tan(c/2 + (d*x)/2)^3)/384 - (14149*a^4*tan(c/2 +
 (d*x)/2)^5)/480 - (469*a^4*tan(c/2 + (d*x)/2)^7)/32 + (4293*a^4*tan(c/2 +
(d*x)/2)^9)/64 - (4293*a^4*tan(c/2 + (d*x)/2)^11)/64 + (469*a^4*tan(c/2 + (
d*x)/2)^13)/32 + (14149*a^4*tan(c/2 + (d*x)/2)^15)/480 - (571*a^4*tan(c/2 +
 (d*x)/2)^17)/384 - (55*a^4*tan(c/2 + (d*x)/2)^19)/128 + (a^4*(17325*c + 17
325*d*x))/80640 - (a^4*(17325*c + 17325*d*x - 53248))/80640 + tan(c/2 + (d*
x)/2)^18*((a^4*(17325*c + 17325*d*x))/8064 - (a^4*(173250*c + 173250*d*x))/
80640) + tan(c/2 + (d*x)/2)^2*((a^4*(17325*c + 17325*d*x))/8064 - (a^4*(173
250*c + 173250*d*x - 532480))/80640) + tan(c/2 + (d*x)/2)^4*((a^4*(17325*c
+ 17325*d*x))/1792 - (a^4*(779625*c + 779625*d*x - 1105920))/80640) + tan(c
/2 + (d*x)/2)^16*((a^4*(17325*c + 17325*d*x))/1792 - (a^4*(779625*c + 77962
5*d*x - 1290240))/80640) + tan(c/2 + (d*x)/2)^6*((a^4*(17325*c + 17325*d*x)
)/672 - (a^4*(2079000*c + 2079000*d*x - 368640))/80640) + tan(c/2 + (d*x)/2
)^12*((a^4*(17325*c + 17325*d*x))/384 - (a^4*(3638250*c + 3638250*d*x - 860
160))/80640) + tan(c/2 + (d*x)/2)^14*((a^4*(17325*c + 17325*d*x))/672 - (a^
4*(2079000*c + 2079000*d*x - 6021120))/80640) + tan(c/2 + (d*x)/2)^10*((a^4
*(17325*c + 17325*d*x))/320 - (a^4*(4365900*c + 4365900*d*x - 6709248))/806
40) + tan(c/2 + (d*x)/2)^8*((a^4*(17325*c + 17325*d*x))/384 - (a^4*(3638250
*c + 3638250*d*x - 10321920))/80640) + (55*a^4*tan(c/2 + (d*x)/2))/128)/(d*
(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.408 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$-\frac{61a^4x}{8}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^4\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -61/8*a^4*x+2*a^4*arctanh(cos(d*x+c))/d+4/3*a^4*cos(d*x+c)^3/d-5*a^4*cot(d*
x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x+c)/d-19/8*a^4*cos(d*
x+c)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2788, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^4\cot(c+dx)\csc(c+dx)}{d}-\frac{61a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-10a^8 - 4a^8\csc(c+dx) + 4a^8\csc^2(c+dx) + 4a^8\csc^3(c+dx) + \right.}{}$$

$$= -10a^4x + a^4\int \csc^4(c+dx)\,dx + a^4\int \sin^4(c+dx)\,dx - \left(4a^4\right)\int \mathrm{c}$$

$$= -10a^4x + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{2a^4\cot(c+}{}$$

$$= -8a^4x + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c+}{d}$$

$$= -\frac{61a^4x}{8} + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 685 vs. 2(140) = 280.
time = 6.31, size = 685, normalized size = 4.89

$$-\frac{61(c+dx)(a+a\sin(c+dx))^4}{8d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\cos(c+dx)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\cos(3(c+dx))(a+a\sin(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{7\cot\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+a\sin(c+dx))^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}-\frac{5(a+a\sin(c+dx))^4\sin(2(c+dx))}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{(a+a\sin(c+dx))^4\sin(4(c+dx))}{32d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{7(a+a\sin(c+dx))^4\tan\left(\frac{1}{2}(c+dx)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a+a\sin(c+dx))^4\tan\left(\frac{1}{2}(c+dx)\right)}{24d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*(c + d*x)*(a + a*Sin[c + d*x])^4)/(8*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^8) + (Cos[c + d*x]*(a + a*Sin[c + d*x])^4)/(d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^8) + (Cos[3*(c + d*x)]*(a + a*Sin[c + d*x])^4)/(3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^8) - (7*Cot[(c + d*x)/2]*(a + a*Sin[c + d*x]
)^4)/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (Csc[(c + d*x)/2]^2*(a
 + a*Sin[c + d*x])^4)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2*(a + a*Sin[c + d*x])^4)/(24*d*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^8) + (2*Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])
^4)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) - (2*Log[Sin[(c + d*x)/2]]*
(a + a*Sin[c + d*x])^4)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (Sec[
(c + d*x)/2]^2*(a + a*Sin[c + d*x])^4)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^8) - (5*(a + a*Sin[c + d*x])^4*Sin[2*(c + d*x)])/(4*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^8) + ((a + a*Sin[c + d*x])^4*Sin[4*(c + d*x)])/(32*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (7*(a + a*Sin[c + d*x])^4*Tan[
(c + d*x)/2])/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8) + (Sec[(c + d*x
)/2]^2*(a + a*Sin[c + d*x])^4*Tan[(c + d*x)/2])/(24*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^8)
```

Maple [A]

time = 0.24, size = 222, normalized size = 1.59

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4a^4\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-}{}$
default	$\frac{a^4\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+4a^4\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)-}{}$
risch	$-\frac{61a^4x}{8}-\frac{ia^4e^{4i(dx+c)}}{64d}+\frac{5ia^4e^{2i(dx+c)}}{8d}+\frac{a^4e^{i(dx+c)}}{2d}+\frac{a^4e^{-i(dx+c)}}{2d}-\frac{5ia^4e^{-2i(dx+c)}}{8d}+\frac{ia^4e^{-4i(dx+c)}}{64d}+$
norman	$\frac{-\frac{a^4}{24d}-\frac{a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{61a^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{97a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{93a^4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{93a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{97a^4\left(\tan\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+4*a^4*(-1/2/sin(d*x+c)^2*cos(
d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+6*a
^4*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2
*d*x-3/2*c)+4*a^4*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a
^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.49, size = 218, normalized size = 1.56

$$\frac{64\,\left(2\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^{4}+3\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{4}-288\,\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a^{4}+32\,\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}-1}{\tan\left(dx+c\right)^{3}}\right)a^{4}+96\,a^{4}\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.56

$$-\frac{6\,a^{4}\cos\left(dx+c\right)^{7}-75\,a^{4}\cos\left(dx+c\right)^{5}+244\,a^{4}\cos\left(dx+c\right)^{3}-183\,a^{4}\cos\left(dx+c\right)-24\,\left(a^{4}\cos\left(dx+c\right)^{2}-a^{4}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+24\,\left(a^{4}\cos\left(dx+c\right)^{2}-a^{4}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\left(32\,a^{4}\cos\left(dx+c\right)^{5}-183\,a^{4}dx\cos\left(dx+c\right)^{2}-32\,a^{4}\cos\left(dx+c\right)^{3}+183\,a^{4}dx+48\,a^{4}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(130) = 260.
time = 0.59, size = 274, normalized size = 1.96

$$\frac{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-183\,\left(dx+c\right)a^{4}-48\,a^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{88\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-57\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{4}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}+\frac{2\,\left(57\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+96\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+81\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+96\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-81\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+32\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-57\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\,a^{4}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 8.81, size = 384, normalized size = 2.74

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{2\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{61\,a^4\,\mathrm{atan}\left(\frac{3721\,a^8}{16\,\left(61\,a^8-\frac{3721\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{61\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{61\,a^8-\frac{3721\,a^8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}\right)}{4\,d}+\frac{19\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{-19\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-60\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{67\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}-48\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{508\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+116\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{61\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^4}{3}}{d\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a
^4*log(tan(c/2 + (d*x)/2)))/d - (61*a^4*atan((3721*a^8)/(16*(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)) + (61*a^8*tan(c/2 + (d*x)/2))/(61*a^8 - (3721
*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + (19*a^4*tan(c/2 + (d*x)/2))/(8*d) -
((61*a^4*tan(c/2 + (d*x)/2)^2)/3 - (16*a^4*tan(c/2 + (d*x)/2)^3)/3 + 116*a^
4*tan(c/2 + (d*x)/2)^4 + (8*a^4*tan(c/2 + (d*x)/2)^5)/3 + (508*a^4*tan(c/2
+ (d*x)/2)^6)/3 - 48*a^4*tan(c/2 + (d*x)/2)^7 + (67*a^4*tan(c/2 + (d*x)/2)^
8)/3 - 60*a^4*tan(c/2 + (d*x)/2)^9 - 19*a^4*tan(c/2 + (d*x)/2)^10 + a^4/3 +
 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/
2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x
)/2)^11))
```

3.409 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=135

$$\frac{x}{16a}+\frac{\cos^3(c+dx)}{3ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}+\frac{\cos(c+dx)\sin(c+dx)}{16ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(}{}$$

```
[Out] 1/16*x/a+1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/1
6*cos(d*x+c)*sin(d*x+c)/a/d-1/8*cos(d*x+c)^3*sin(d*x+c)/a/d-1/6*cos(d*x+c)^
3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2648, 2715, 8, 2645, 276}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*
x]^7/(7*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) - (Cos[c + d*x]^3*Sin[c
 + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a}\\ &= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\big(\int\\ &= -\frac{\cos^3(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)}{8a}\\ &= \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad}\\ &= \frac{x}{16a} + \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c}{16ad}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 86, normalized size = 0.64

$$\frac{420c+420dx+525\cos(c+dx)+35\cos(3(c+dx))-63\cos(5(c+dx))+15\cos(7(c+dx))-105\sin(2(c+dx))-105\sin(4(c+dx))+35\sin(6(c+dx))}{6720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (420*c + 420*d*x + 525*Cos[c + d*x] + 35*Cos[3*(c + d*x)] - 63*Cos[5*(c + d
*x)] + 15*Cos[7*(c + d*x)] - 105*Sin[2*(c + d*x)] - 105*Sin[4*(c + d*x)] +
35*Sin[6*(c + d*x)])/(6720*a*d)
```

Maple [A]
time = 0.23, size = 168, normalized size = 1.24

method	result
risch	$\frac{x}{16a}+\frac{5\cos(dx+c)}{64ad}+\frac{\cos(7dx+7c)}{448ad}+\frac{\sin(6dx+6c)}{192ad}-\frac{3\cos(5dx+5c)}{320ad}-\frac{\sin(4dx+4c)}{64ad}+\frac{\cos(3dx+3c)}{192ad}-\frac{\sin(2dx\cdots}{64a\cdots}$
derivativedivides	$\frac{32\left(\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{97\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{\left(\tan^{4}\left(\cdots\right.\right.}{1\cdots}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}}{ad}$
default	$\frac{32\left(\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{97\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{\left(\tan^{4}\left(\cdots\right.\right.}{1\cdots}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}}{ad}$
norman	$\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{x}{16a}+\frac{23}{840ad}-\frac{41\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{420ad}-\frac{7\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}+\frac{x\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*((1/256*tan(1/2*d*x+1/2*c)^13+5/192*tan(1/2*d*x+1/2*c)^11-97/768*tan
(1/2*d*x+1/2*c)^9+1/3*tan(1/2*d*x+1/2*c)^8-1/6*tan(1/2*d*x+1/2*c)^6+97/768*
tan(1/2*d*x+1/2*c)^5+1/10*tan(1/2*d*x+1/2*c)^4-5/192*tan(1/2*d*x+1/2*c)^3+1
/30*tan(1/2*d*x+1/2*c)^2-1/256*tan(1/2*d*x+1/2*c)+1/210)/(1+tan(1/2*d*x+1/2
*c)^2)^7+1/256*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(121) = 242$.
time = 0.49, size = 380, normalized size = 2.81

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{896\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{700\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2688\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3395\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4480\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8960\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3395\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{700\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{105\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-128}{a+\frac{7a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 896*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2688*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4480*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 700*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 128)/(a + 7*a*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 35*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + 21*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c
```

```
)^12/(cos(d*x + c) + 1)^12 + a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [**A**]
time = 0.36, size = 80, normalized size = 0.59

$$\frac{240\,\cos\left(dx+c\right)^{7}-672\,\cos\left(dx+c\right)^{5}+560\,\cos\left(dx+c\right)^{3}+105\,dx+35\,\left(8\,\cos\left(dx+c\right)^{5}-14\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2635 vs. 2(107) = 214.
time = 46.49, size = 2635, normalized size = 19.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**14/(1680*a*d*tan(c/2 + d*x/2)**14 + 11
760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*t
an(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*
x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x
/2)**12/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 3
5280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*t
an(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) + 2205*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x
/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*
tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*
tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*tan(c/2 + d*x/2)**6/(1680*a*d*ta
n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d
*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 +
 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
+ 2205*d*x*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*t
an(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 +
```

11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x/2)**2/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
1680*a*d) + 105*d*x/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)**13/(1680*a*d*tan(c/
2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2
)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 352
80*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 14
00*tan(c/2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2
+ d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)*
*8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*
a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 6790*tan(c/2 + d*x/2)**9/(1680*a*d*ta
n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d
*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 +
35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
+ 17920*tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(
c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x
/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11
760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 8960*tan(c/2 + d*x/2)**6/(1680*a*
d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2
+ d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)*
*6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a
*d) + 6790*tan(c/2 + d*x/2)**5/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*t
an(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 +
11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 5376*tan(c/2 + d*x/2)**4/(1680
*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(
c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/
2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 168
0*a*d) - 1400*tan(c/2 + d*x/2)**3/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*
d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2
+ d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**
4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 1792*tan(c/2 + d*x/2)**2/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
1680*a*d) - 210*tan(c/2 + d*x/2)/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d
*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2
+ d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4
+ 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 256/(1680*a*d*tan(c/2 + d*x/
2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*t

```
an(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)...
```

Giac [A]
time = 0.56, size = 166, normalized size = 1.23

$$\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+700\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-3395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+8960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-700\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+896\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+128\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*x
+ 1/2*c)^11 - 3395*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 4
480*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*d*x + 1/2*c)^5 + 2688*tan(1/2*d*x
 + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 + 896*tan(1/2*d*x + 1/2*c)^2 - 105
*tan(1/2*d*x + 1/2*c) + 128)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a))/d
```

Mupad [B]
time = 11.38, size = 159, normalized size = 1.18

$$\frac{x}{16\,a}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{8}+\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{6}-\frac{97\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{24}+\frac{32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{3}-\frac{16\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{3}+\frac{97\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{24}+\frac{16\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{5}-\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{6}+\frac{16\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{15}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{16}{105}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(16*a) + ((16*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/8 - (5*tan(c/
2 + (d*x)/2)^3)/6 + (16*tan(c/2 + (d*x)/2)^4)/5 + (97*tan(c/2 + (d*x)/2)^5)
/24 - (16*tan(c/2 + (d*x)/2)^6)/3 + (32*tan(c/2 + (d*x)/2)^8)/3 - (97*tan(c
/2 + (d*x)/2)^9)/24 + (5*tan(c/2 + (d*x)/2)^11)/6 + tan(c/2 + (d*x)/2)^13/8
 + 16/105)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.410 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{x}{16a}-\frac{\cos^3(c+dx)}{3ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\cos(c+dx)\sin(c+dx)}{16ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad}$$

```
[Out] -1/16*x/a-1/3*cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d-1/16*cos(d*x+c)*sin(d*x
+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2645, 14, 2648, 2715, 8}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/16*x/a - Cos[c + d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a*d) + (Cos[c +
d*x]^3*Sin[c + d*x]^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} - \frac{\operatorname{Subst}\left(\int x^2\right.}{}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\,dx}{8a}\\ &= -\frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)}{8a}\\ &= -\frac{x}{16a} - \frac{\cos^3(c+dx)}{3ad} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c}{}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 377 vs. 2(117) = 234.
time = 3.53, size = 377, normalized size = 3.22

$$\frac{30(3c-4dx)\cos\left(\frac{c}{2}\right)-120\cos\left(\frac{c}{2}+dx\right)-120\cos\left(\frac{3c}{2}+dx\right)+15\cos\left(\frac{3c}{2}+2dx\right)-15\cos\left(\frac{5c}{2}+2dx\right)-20\cos\left(\frac{5c}{2}+3dx\right)-20\cos\left(\frac{7c}{2}+3dx\right)+15\cos\left(\frac{7c}{2}+4dx\right)-15\cos\left(\frac{9c}{2}+4dx\right)+12\cos\left(\frac{9c}{2}+5dx\right)+12\cos\left(\frac{11c}{2}+5dx\right)-5\cos\left(\frac{11c}{2}+6dx\right)+5\cos\left(\frac{13c}{2}+6dx\right)-180\sin\left(\frac{c}{2}\right)+90c\sin\left(\frac{c}{2}\right)-120dx\sin\left(\frac{c}{2}\right)+120\sin\left(\frac{c}{2}+dx\right)-120\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+20\sin\left(\frac{5c}{2}+3dx\right)-20\sin\left(\frac{7c}{2}+3dx\right)+15\sin\left(\frac{7c}{2}+4dx\right)+15\sin\left(\frac{9c}{2}+4dx\right)-12\sin\left(\frac{9c}{2}+5dx\right)+12\sin\left(\frac{11c}{2}+5dx\right)-5\sin\left(\frac{11c}{2}+6dx\right)-5\sin\left(\frac{13c}{2}+6dx\right)}{1920ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (30*(3*c - 4*d*x)*Cos[c/2] - 120*Cos[c/2 + d*x] - 120*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 20*Cos[(5*c)/2 + 3*d*x]
```

```
 - 20*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] - 15*Cos[(9*c)/2 + 4*d
*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)/2 +
 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] - 180*Sin[c/2] + 90*c*Sin[c/2] - 120*d*x*
Sin[c/2] + 120*Sin[c/2 + d*x] - 120*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2
*d*x] + 15*Sin[(5*c)/2 + 2*d*x] + 20*Sin[(5*c)/2 + 3*d*x] - 20*Sin[(7*c)/2
+ 3*d*x] + 15*Sin[(7*c)/2 + 4*d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 12*Sin[(9*c)
/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(1
3*c)/2 + 6*d*x])/(1920*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.19, size = 155, normalized size = 1.32

method	result
risch	$-\frac{x}{16a}-\frac{\cos(dx+c)}{8ad}-\frac{\sin(6dx+6c)}{192ad}+\frac{\cos(5dx+5c)}{80ad}+\frac{\sin(4dx+4c)}{64ad}-\frac{\cos(3dx+3c)}{48ad}+\frac{\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{\frac{16\left(-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{19\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{19\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+17(\tan}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{ad}$
default	$\frac{\frac{16\left(-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{17\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{19\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{19\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+17(\tan}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{ad}$
norman	$-\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}+\frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{x}{16a}-\frac{17}{120ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{60ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{7x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}-\frac{7x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*((-1/128*tan(1/2*d*x+1/2*c)^11-17/384*tan(1/2*d*x+1/2*c)^9-1/4*tan(1
/2*d*x+1/2*c)^8+19/64*tan(1/2*d*x+1/2*c)^7-1/6*tan(1/2*d*x+1/2*c)^6-19/64*t
an(1/2*d*x+1/2*c)^5+17/384*tan(1/2*d*x+1/2*c)^3-1/10*tan(1/2*d*x+1/2*c)^2+1
/128*tan(1/2*d*x+1/2*c)-1/60)/(1+tan(1/2*d*x+1/2*c)^2)^6-1/128*arctan(tan(1
/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(105) = 210.
time = 0.50, size = 339, normalized size = 2.90

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{192\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{570\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{320\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{570\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-32}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 192*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 85*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 570*sin(d*x + c)^5/(co
```

```
s(d*x + c) + 1)^5 - 320*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 570*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 85*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)^
11 - 32)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^1
0 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

Fricas [**A**]
time = 0.34, size = 70, normalized size = 0.60

$$\frac{48 \cos(dx+c)^5 - 80 \cos(dx+c)^3 - 15\,dx - 5\left(8 \cos(dx+c)^5 - 14 \cos(dx+c)^3 + 3 \cos(dx+c)\right)\sin(dx+c)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*d*x - 5*(8*cos(d*x + c)^5
 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2067 vs. 2(92) = 184.
time = 28.69, size = 2067, normalized size = 17.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**12/(240*a*d*tan(c/2 + d*x/2)**12 + 144
0*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/
2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2
 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1
440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(
c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)*
*2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 +
 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*ta
n(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2
)**2 + 240*a*d) - 300*d*x*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12
 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*
tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x
/2)**2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**
12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*
d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d
*x/2)**2 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)*
```

```
*12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a
*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 +
d*x/2)**2 + 240*a*d) - 15*d*x/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(
c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)
**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d
) - 30*tan(c/2 + d*x/2)**11/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/
2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**
6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d)
- 170*tan(c/2 + d*x/2)**9/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2
+ d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6
+ 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) -
960*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 +
d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 +
3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 11
40*tan(c/2 + d*x/2)**7/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d
*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3
600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 640
*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x
/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 360
0*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 1140*
tan(c/2 + d*x/2)**5/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/
2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600
*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 170*ta
n(c/2 + d*x/2)**3/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)
**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a
*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 384*tan(
c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**
10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d
*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 30*tan(c/2
 + d*x/2)/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3
600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c
/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 64/(240*a*d*tan(
c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2
)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a
*d*tan(c/2 + d*x/2)**2 + 240*a*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**4/(a*sin
(c) + a), True))
```

Giac [A]

time = 0.49, size = 153, normalized size = 1.31

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-570\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+570\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+192\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 - 570*tan(1/2*d*x + 1/2*c)^7 + 320*tan(1/2*d*x + 1/2*c)^6 + 570*tan(1/2*d*x + 1/2*c)^5 - 85*tan(1/2*d*x + 1/2*c)^3 + 192*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a))/d

Mupad [B]
time = 11.31, size = 147, normalized size = 1.26

$$-\frac{x}{16\,a}-\frac{\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{8}+\frac{17\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{24}+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-\frac{19\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{4}+\frac{8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{3}+\frac{19\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}-\frac{17\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24}+\frac{8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{5}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{4}{15}}{a\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)

[Out] - x/(16*a) - ((8*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 - (17*tan(c/2 + (d*x)/2)^3)/24 + (19*tan(c/2 + (d*x)/2)^5)/4 + (8*tan(c/2 + (d*x)/2)^6)/3 - (19*tan(c/2 + (d*x)/2)^7)/4 + 4*tan(c/2 + (d*x)/2)^8 + (17*tan(c/2 + (d*x)/2)^9)/24 + tan(c/2 + (d*x)/2)^11/8 + 4/15)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)

3.411 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{x}{8a}+\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

```
[Out] 1/8*x/a+1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/8*cos(d*x+c)*sin(d*x+c)
/a/d-1/4*cos(d*x+c)^3*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2648, 2715, 8, 2645, 14}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{\sin(c+dx)\cos(c+dx)}{8ad}+\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(8*a) + Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + (Cos[c + d*x]*S
in[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2(1-x^2)\,dx \right.}{ad}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\int 1\,dx}{8a} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{x}{8a} + \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos(c+dx)\sin(c+dx)}{8ad} - \frac{\cos^3(c+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(91) = 182.
time = 1.74, size = 258, normalized size = 2.84

$$\frac{120dx\cos\left(\frac{c}{2}\right)+60\cos\left(\frac{c}{2}+dx\right)+60\cos\left(\frac{3c}{2}+dx\right)+10\cos\left(\frac{5c}{2}+3dx\right)+10\cos\left(\frac{7c}{2}+3dx\right)-15\cos\left(\frac{7c}{2}+4dx\right)+15\cos\left(\frac{9c}{2}+4dx\right)-6\cos\left(\frac{9c}{2}+5dx\right)-6\cos\left(\frac{11c}{2}+5dx\right)+120\sin\left(\frac{c}{2}\right)+120dx\sin\left(\frac{c}{2}\right)-60\sin\left(\frac{c}{2}+dx\right)+60\sin\left(\frac{3c}{2}+dx\right)-10\sin\left(\frac{5c}{2}+3dx\right)+10\sin\left(\frac{7c}{2}+3dx\right)-15\sin\left(\frac{7c}{2}+4dx\right)-15\sin\left(\frac{9c}{2}+4dx\right)+6\sin\left(\frac{9c}{2}+5dx\right)-6\sin\left(\frac{11c}{2}+5dx\right)}{960ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*d*x*Cos[c/2] + 60*Cos[c/2 + d*x] + 60*Cos[(3*c)/2 + d*x] + 10*Cos[(5*c
)/2 + 3*d*x] + 10*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(
9*c)/2 + 4*d*x] - 6*Cos[(9*c)/2 + 5*d*x] - 6*Cos[(11*c)/2 + 5*d*x] + 120*Si
n[c/2] + 120*d*x*Sin[c/2] - 60*Sin[c/2 + d*x] + 60*Sin[(3*c)/2 + d*x] - 10*
Sin[(5*c)/2 + 3*d*x] + 10*Sin[(7*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] -
```

```
15*Sin[(9*c)/2 + 4*d*x] + 6*Sin[(9*c)/2 + 5*d*x] - 6*Sin[(11*c)/2 + 5*d*x])
/(960*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.17, size = 129, normalized size = 1.42

method	result
risch	$\frac{x}{8a}+\frac{\cos(dx+c)}{8ad}-\frac{\cos(5dx+5c)}{80ad}-\frac{\sin(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{48ad}$
derivativedivides	$\frac{\frac{8\left(\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{ad}$
default	$\frac{\frac{8\left(\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{ad}$
norman	$-\frac{11\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{x}{8a}+\frac{1}{60ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{30ad}+\frac{x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}+\frac{15x\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{8a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*((1/32*tan(1/2*d*x+1/2*c)^9-3/16*tan(1/2*d*x+1/2*c)^7+1/2*tan(1/2*d*x
+1/2*c)^6-1/6*tan(1/2*d*x+1/2*c)^4+3/16*tan(1/2*d*x+1/2*c)^3+1/6*tan(1/2*d*
x+1/2*c)^2-1/32*tan(1/2*d*x+1/2*c)+1/30)/(1+tan(1/2*d*x+1/2*c)^2)^5+1/32*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(81) = 162.
time = 0.49, size = 278, normalized size = 3.05

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-16}{a+\frac{5a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 16)/(a +
 5*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 10*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.35, size = 60, normalized size = 0.66

$$-\frac{24 \cos(dx+c)^5 - 40 \cos(dx+c)^3 - 15\,dx + 15\,(2 \cos(dx+c)^3 - \cos(dx+c)) \sin(dx+c)}{120\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120*(24*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)
^3 - cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1464 vs. $2(70) = 140$.
time = 17.52, size = 1464, normalized size = 16.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(120*a*d*tan(c/2 + d*x/2)**10 + 600*
a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 +
 d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/
2)**8/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 150*d*x*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 150*d*x*tan
(c/2 + d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**
8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*t
an(c/2 + d*x/2)**2 + 120*a*d) + 75*d*x*tan(c/2 + d*x/2)**2/(120*a*d*tan(c/2
 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6
+ 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 1
5*d*x/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*
d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*
x/2)**2 + 120*a*d) + 30*tan(c/2 + d*x/2)**9/(120*a*d*tan(c/2 + d*x/2)**10 +
 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(
c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 180*tan(c/2 + d*
x/2)**7/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*
a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 +
d*x/2)**2 + 120*a*d) + 480*tan(c/2 + d*x/2)**6/(120*a*d*tan(c/2 + d*x/2)**1
0 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*t
an(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) - 160*tan(c/2 +
 d*x/2)**4/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 12
```

```
00*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2
 + d*x/2)**2 + 120*a*d) + 180*tan(c/2 + d*x/2)**3/(120*a*d*tan(c/2 + d*x/2)
**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*
d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 160*tan(c/
2 + d*x/2)**2/(120*a*d*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 +
 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(
c/2 + d*x/2)**2 + 120*a*d) - 30*tan(c/2 + d*x/2)/(120*a*d*tan(c/2 + d*x/2)*
*10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*x/2)**6 + 1200*a*d
*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*a*d) + 32/(120*a*d
*tan(c/2 + d*x/2)**10 + 600*a*d*tan(c/2 + d*x/2)**8 + 1200*a*d*tan(c/2 + d*
x/2)**6 + 1200*a*d*tan(c/2 + d*x/2)**4 + 600*a*d*tan(c/2 + d*x/2)**2 + 120*
a*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A]
time = 0.59, size = 127, normalized size = 1.40

$$\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^9 - 90*tan(1/2*d*x + 1/2
*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 - 80*tan(1/2*d*x + 1/2*c)^4 + 90*tan(1/2
*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 16)
/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

Mupad [B]
time = 12.01, size = 120, normalized size = 1.32

$$\frac{x}{8\,a}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4}-\frac{3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{3}+\frac{3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{4}{15}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(8*a) + ((4*tan(c/2 + (d*x)/2)^2)/3 - tan(c/2 + (d*x)/2)/4 + (3*tan(c/2 +
 (d*x)/2)^3)/2 - (4*tan(c/2 + (d*x)/2)^4)/3 + 4*tan(c/2 + (d*x)/2)^6 - (3*t
an(c/2 + (d*x)/2)^7)/2 + tan(c/2 + (d*x)/2)^9/4 + 4/15)/(a*d*(tan(c/2 + (d*
x)/2)^2 + 1)^5)
```

3.412 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{x}{8a}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

```
[Out] -1/8*x/a-1/3*cos(d*x+c)^3/a/d-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^
3*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2918, 2645, 30, 2648, 2715, 8}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*x/a - Cos[c + d*x]^3/(3*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (
Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \cos^2(c+dx)\,dx}{4a} - \frac{\operatorname{Subst}\left(\int x^2\,dx, x, \cos(c+d\right.}{ad}\\ &= -\frac{\cos^3(c+dx)}{3ad} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int 1}{8}\\ &= -\frac{x}{8a} - \frac{\cos^3(c+dx)}{3ad} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 219 vs. 2(73) = 146.
time = 1.13, size = 219, normalized size = 3.00

$$-\frac{-24(c-dx)\cos\left(\frac{c}{2}\right)+24\cos\left(\frac{c}{2}+dx\right)+24\cos\left(\frac{3c}{2}+dx\right)+8\cos\left(\frac{5c}{2}+3dx\right)+8\cos\left(\frac{7c}{2}+3dx\right)-3\cos\left(\frac{7c}{2}+4dx\right)+3\cos\left(\frac{9c}{2}+4dx\right)+48\sin\left(\frac{c}{2}\right)-24c\sin\left(\frac{c}{2}\right)+24dx\sin\left(\frac{c}{2}\right)-24\sin\left(\frac{c}{2}+dx\right)+24\sin\left(\frac{3c}{2}+dx\right)-8\sin\left(\frac{5c}{2}+3dx\right)+8\sin\left(\frac{7c}{2}+3dx\right)-3\sin\left(\frac{7c}{2}+4dx\right)-3\sin\left(\frac{9c}{2}+4dx\right)}{192ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(-24*(c - d*x)*Cos[c/2] + 24*Cos[c/2 + d*x] + 24*Cos[(3*c)/2 + d*x]
+ 8*Cos[(5*c)/2 + 3*d*x] + 8*Cos[(7*c)/2 + 3*d*x] - 3*Cos[(7*c)/2 + 4*d*x]
+ 3*Cos[(9*c)/2 + 4*d*x] + 48*Sin[c/2] - 24*c*Sin[c/2] + 24*d*x*Sin[c/2] -
24*Sin[c/2 + d*x] + 24*Sin[(3*c)/2 + d*x] - 8*Sin[(5*c)/2 + 3*d*x] + 8*Sin[
```

```
(7*c)/2 + 3*d*x] - 3*Sin[(7*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 4*d*x])/(a*d*(C
os[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.14, size = 129, normalized size = 1.77

method	result
risch	$-\frac{x}{8a}-\frac{\cos(dx+c)}{4ad}+\frac{\sin(4dx+4c)}{32ad}-\frac{\cos(3dx+3c)}{12ad}$
derivativedivides	$\frac{\frac{4\left(-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\cdots\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}}{ad}$
default	$\frac{\frac{4\left(-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16}\cdots\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}}{ad}$
norman	$\frac{-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{8a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{5x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{5x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{5x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{5x\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*((-1/16*tan(1/2*d*x+1/2*c)^7-1/2*tan(1/2*d*x+1/2*c)^6+7/16*tan(1/2*d*
x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4-7/16*tan(1/2*d*x+1/2*c)^3-1/6*tan(1/2*d
*x+1/2*c)^2+1/16*tan(1/2*d*x+1/2*c)-1/6)/(1+tan(1/2*d*x+1/2*c)^2)^4-1/16*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.
time = 0.49, size = 257, normalized size = 3.52

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{21\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{24\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-8}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 21*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 24*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 8)/(a + 4*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.35, size = 50, normalized size = 0.68

$$-\frac{8\cos\left(dx+c\right)^{3}+3\,dx-3\left(2\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^3 + 3*d*x - 3*(2*cos(d*x + c)^3 - cos(d*x + c))*sin(d
*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(56) = 112.
time = 9.10, size = 1134, normalized size = 15.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**8/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) - 12*d*x*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96
*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d
*x/2)**2 + 24*a*d) - 18*d*x*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8
 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/
2 + d*x/2)**2 + 24*a*d) - 12*d*x*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/
2)**8 + 96*a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*t
an(c/2 + d*x/2)**2 + 24*a*d) - 3*d*x/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 6*tan(c/2 + d*x/2)**7/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*t
an(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)*
*2 + 24*a*d) - 48*tan(c/2 + d*x/2)**6/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*
tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)
**2 + 24*a*d) + 42*tan(c/2 + d*x/2)**5/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
)**2 + 24*a*d) - 48*tan(c/2 + d*x/2)**4/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*
d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/
2)**2 + 24*a*d) - 42*tan(c/2 + d*x/2)**3/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a
*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x
/2)**2 + 24*a*d) - 16*tan(c/2 + d*x/2)**2/(24*a*d*tan(c/2 + d*x/2)**8 + 96*
a*d*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*
x/2)**2 + 24*a*d) + 6*tan(c/2 + d*x/2)/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d
*tan(c/2 + d*x/2)**6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2
```

```
)**2 + 24*a*d) - 16/(24*a*d*tan(c/2 + d*x/2)**8 + 96*a*d*tan(c/2 + d*x/2)**
6 + 144*a*d*tan(c/2 + d*x/2)**4 + 96*a*d*tan(c/2 + d*x/2)**2 + 24*a*d), Ne(
d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a), True))
```

Giac [A]
time = 0.54, size = 127, normalized size = 1.74

$$-\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 24*tan(1/2*d*x + 1/2*c
)^6 - 21*tan(1/2*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*
x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 8)/((tan
(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B]
time = 8.80, size = 43, normalized size = 0.59

$$-\frac{6\,\cos\left(c+d\,x\right)+2\,\cos\left(3\,c+3\,d\,x\right)-\frac{3\,\sin\left(4\,c+4\,d\,x\right)}{4}+3\,d\,x}{24\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] -(6*cos(c + d*x) + 2*cos(3*c + 3*d*x) - (3*sin(4*c + 4*d*x))/4 + 3*d*x)/(24
*a*d)
```

3.413 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$-\frac{x}{2a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

```
[Out] -1/2*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a
/d
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2672, 327, 212, 2715, 8}

$$\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*x/a - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) - (Cos[c + d*x]
*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^2(c+dx)\,dx}{a}+\frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\int 1\,dx}{2a}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{x}{2a}+\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\ &= -\frac{x}{2a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}-\frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 60, normalized size = 1.02

$$-\frac{-4\cos(c+dx)+2\left(c+dx+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-4*Cos[c + d*x] + 2*(c + d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c
 + d*x)/2]]) + Sin[2*(c + d*x)])/(a*d)
```

Maple [A]
time = 0.17, size = 87, normalized size = 1.47

method	result
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-1\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
risch	$-\frac{x}{2a}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}-\frac{\sin(2dx+2c)}{4ad}$
norman	$\frac{\frac{1}{ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x}{2a}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^4\left(\frac{dx}{2}\right.\right.}{2a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(tan(1/2*d*x+1/2*c))-2*(-1/2*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*
c)^2+1/2*tan(1/2*d*x+1/2*c)-1)/(1+tan(1/2*d*x+1/2*c)^2)^2-arctan(tan(1/2*d*
x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(55) = 110$.

time = 0.49, size = 156, normalized size = 2.64

$$-\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-2}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2)/(a + 2*a*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a - log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.36, size = 57, normalized size = 0.97

$$-\frac{dx+\cos(dx+c)\sin(dx+c)-2\cos(dx+c)+\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c) + log(1/2*cos(d*x +
c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.51, size = 88, normalized size = 1.49

$$-\frac{\frac{dx+c}{a} - \frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(tan(1/2*d*x + 1
/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^2*a))/d
```

Mupad [B]
time = 8.85, size = 136, normalized size = 2.31

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}+\frac{\mathrm{atan}\left(\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2}\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + (2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2
) + tan(c/2 + (d*x)/2)^3 + 2)/(d*(a + 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2
+ (d*x)/2)^4)) + atan(1/(tan(c/2 + (d*x)/2) + 2) - (2*tan(c/2 + (d*x)/2))/(
tan(c/2 + (d*x)/2) + 2))/(a*d)
```

3.414 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=49

$$-\frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-cot(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2918, 3554, 8, 2672, 327, 212}

$$-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cot[c + d*x]/(a
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\,dx}{a}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{\int 1\,dx}{a}+\frac{\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}
\end{aligned}
$$

Mathematica [A]
time = 0.32, size = 93, normalized size = 1.90

$$
-\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(c+dx)+\left(c+dx+\cos(c+dx)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{2ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*((1 + Cot[(c + d*x)/2])^2*(Cos[c + d*x] + (c + d*x + Cos[c + d*x] - Lo
g[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x])*Tan[(c + d*x)/2]
)/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.18, size = 73, normalized size = 1.49

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$
risch	$-\frac{x}{a}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}$
norman	$\frac{-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2x\left(\tan^4\left(\frac{dx}{2}\right.\right.}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-4
/(1+tan(1/2*d*x+1/2*c)^2)-4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(49) = 98$.

time = 0.48, size = 154, normalized size = 3.14

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3) + 4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 2*log(sin(d*x + c
)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]

time = 0.38, size = 80, normalized size = 1.63

$$-\frac{2\left(dx+\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(d*x + cos(d*x + c))*sin(d*x + c) - log(1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*cos(d*x + c))/(a*
d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(49) = 98.
time = 0.57, size = 113, normalized size = 2.31

$$-\frac{\frac{6(dx+c)}{a}+\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - (2*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 10*tan(1/
2*d*x + 1/2*c) - 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a))/d
```

Mupad [B]
time = 8.72, size = 147, normalized size = 3.00

$$\frac{2\,\mathrm{atan}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}+\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}\right)}{a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*atan((4*tan(c/2 + (d*x)/2))/(4*tan(c/2 + (d*x)/2) - 4) + 4/(4*tan(c/2 +
(d*x)/2) - 4)))/(a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - (4*tan(c/2 + (d*x)/
2) + tan(c/2 + (d*x)/2)^2 + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (
d*x)/2)^3)) + tan(c/2 + (d*x)/2)/(2*a*d)
```

3.415 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] x/a+1/2*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/
d
```

Rubi [A]
time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2918, 2691, 3855, 3554, 8}

$$\frac{\cot(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + ArcTanh[Cos[c + d*x]]/(2*a*d) + Cot[c + d*x]/(a*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}\\
&= \frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\int 1\,dx}{a}\\
&= \frac{x}{a}+\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A]
time = 0.35, size = 102, normalized size = 1.76

$$
\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\left(2c+2dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^2(c+dx)+\cos(c+dx)(-1+2\sin(c+dx))\right)}{8ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*((2*c + 2*d*x + Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^2 + Cos[c + d*x]*(-1 + 2*Sin[c +
d*x])))/(8*a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.21, size = 84, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}$
risch	$\frac{x}{a}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+2ie^{2i(dx+c)}-2i}{ad\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}$

norman	$\frac{\frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^7\left(\frac{dx}{2}\right.}{8ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/2*
c)^2+2/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))+8*arctan(tan(1/2*d*x+1/2
*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(54) = 108$.
time = 0.49, size = 138, normalized size = 2.38

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 4*log(sin(d*x + c)/
(cos(d*x + c) + 1))/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(cos(d*x +
c) + 1)^2/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.79

$$\frac{4\,dx\cos(dx+c)^2-4\,dx+\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\cos(dx+c)\sin(dx+c)+2\cos(dx+c)}{4\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + (cos(d*x + c)^2 - 1)*log(1/2*cos(d*x +
c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 4*cos(d*x +
 c)*sin(d*x + c) + 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.56, size = 103, normalized size = 1.78

$$\frac{\frac{8(dx+c)}{a}-\frac{4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*(d*x + c)/a - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x +
1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^2 + 4*ta
n(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 8.75, size = 159, normalized size = 2.74

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{2\,\mathrm{atan}\left(\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{ad}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2ad}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*atan((2*cos(c/2 + (d*x)/2) - sin(c/2 + (d
*x)/2))/(cos(c/2 + (d*x)/2) + 2*sin(c/2 + (d*x)/2))))/(a*d) - cot(c/2 + (d*
x)/2)^2/(8*a*d) - log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(2*a*d) + cot(
c/2 + (d*x)/2)/(2*a*d) - tan(c/2 + (d*x)/2)/(2*a*d)
```

3.416 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x+c)*csc(d*x+c)
/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*C
sc[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\operatorname{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(58) = 116.
time = 0.37, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+\cos(c+dx)(3-6\sin(c+dx))+6\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)\right)}{96ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(a*d*(1 + Sin[c + d*x
]))
```

Maple [A]
time = 0.18, size = 94, normalized size = 1.62

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8da}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8da}$

risch	$-\frac{-6i e^{4i(dx+c)}+3 e^{5i(dx+c)}-2i-3 e^{i(dx+c)}}{3ad\left(e^{2i(dx+c)}-1\right)^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2ad}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)+1
/tan(1/2*d*x+1/2*c)+4*ln(tan(1/2*d*x+1/2*c))+1/tan(1/2*d*x+1/2*c)^2-1/3/tan
(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(52) = 104$.
time = 0.28, size = 155, normalized size = 2.67

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(52) = 104$.
time = 0.35, size = 111, normalized size = 1.91

$$\frac{4\cos(dx+c)^3-3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-6\cos(dx+c)\sin(dx+c)}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(52) = 104.
time = 0.54, size = 127, normalized size = 2.19

$$\frac{\frac{12 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a}+\frac{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-3 a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-3 a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^3}-\frac{22 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 8.67, size = 115, normalized size = 1.98

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2
+ (d*x)/2))/(2*a*d) - tan(c/2 + (d*x)/2)/(8*a*d) + (cot(c/2 + (d*x)/2)^3*(t
an(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2 - 1/3))/(8*a*d)
```

3.417 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

```
[Out] 1/8*arctanh(cos(d*x+c))/a/d+1/3*cot(d*x+c)^3/a/d+1/8*cot(d*x+c)*csc(d*x+c)/
a/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2691, 3853, 3855, 2687, 30}

$$\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a*d) + Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\int \csc^3(c+dx)\,dx}{4a}-\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot\right.}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\int \text{cs}}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+}{}\end{aligned}$$

Mathematica [A]
time = 0.82, size = 125, normalized size = 1.52

$$\frac{\csc^4(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-42\cos(c+dx)+2\cos(3(c+dx))(-3+8\sin(c+dx))+24\left(\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^4(c+dx)+\sin(2(c+dx))\right)\right)}{192ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-42*Cos[c + d*x] +
 2*Cos[3*(c + d*x)]*(-3 + 8*Sin[c + d*x]) + 24*((Log[Cos[(c + d*x)/2]] - Lo
g[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Sin[2*(c + d*x)])))/(192*a*d*(1 + Sin
[c + d*x]))
```

Maple [A]
time = 0.21, size = 98, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}{16da}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}{16da}$
risch	$-\frac{24ie^{6i(dx+c)}+3\,e^{7i(dx+c)}-24ie^{4i(dx+c)}+21\,e^{5i(dx+c)}+8ie^{2i(dx+c)}+21\,e^{3i(dx+c)}-8i+3\,e^{i(dx+c)}}{12da\left(e^{2i(dx+c)}-1\right)^4}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{8ad}-$
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\tan}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(1/4*tan(1/2*d*x+1/2*c)^4-2/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1
/2*c)-2/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))+2/3/tan(1/2*d*x+1/2*c)^
3-1/4/tan(1/2*d*x+1/2*c)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(74) = 148.
time = 0.28, size = 154, normalized size = 1.88

$$\frac{\frac{\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((24*sin(d*x + c)/(cos(d*x + c) + 1) - 8*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a - 24*log(sin(d*x + c)/(c
os(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

Fricas [A]
time = 0.34, size = 132, normalized size = 1.61

$$\frac{16\cos(dx+c)^3\sin(dx+c)-6\cos(dx+c)^3+3\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-6\cos(dx+c)}{48\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] 1/48*(16*cos(d*x + c)^3*sin(d*x + c) - 6*cos(d*x + c)^3 + 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep

Giac [**A**]
time = 0.54, size = 129, normalized size = 1.57

$$-\frac{\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/192*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*tan(1/2*d*x + 1/2*c))/a^4 - (50*tan(1/2*d*x + 1/2*c)^4 - 24*tan(1/2*d*x + 1/2*c)^3 + 8*tan(1/2*d*x + 1/2*c) - 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d

Mupad [**B**]
time = 8.68, size = 119, normalized size = 1.45

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{1}{4}\right)}{16\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - log(tan(c/2 + (d*x)/2))/(8*a*d) + tan(c/2 + (d*x)/2)/(8*a*d) - (cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2)^3 - (2*tan(c/2 + (d*x)/2))/3 + 1/4))/(16*a*d)

3.418 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d-1/8*
cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2687, 14, 2691, 3853, 3855}

$$-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]^3/(3*a*d) - Cot[c + d*x]^5/
(5*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]*Csc[c + d*x]^
3)/(4*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}\\
&= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc^3(c+dx)\,dx}{4a}+\frac{\text{Subst}\left(\int x^2(1+x^2)\,dx,\right.}{ad}\\
&= -\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc(c+dx)\,dx}{8a}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot(c+dx)\csc}{8ad}
\end{aligned}$$

Mathematica [A]

time = 0.44, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(320\cos(c+dx)+80\cos(3(c+dx))-16\cos(5(c+dx))+150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-180\sin(2(c+dx))-75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-30\sin(4(c+dx))+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{1920ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(320*Cos[c + d*x] + 80*Cos[3*(c + d*x)] - 16*Cos[5*
(c + d*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)
/2]]*Sin[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(
```

```
c + d*x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)]
 + 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin
[5*(c + d*x)]))/(a*d)
```

Maple [A]
time = 0.22, size = 124, normalized size = 1.24

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+4\ln\left(\mathrm{ta}\cdots\right.}{32da}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+4\ln\left(\mathrm{ta}\cdots\right.}{32da}$
risch	$\frac{15\,\mathrm{e}^{9i(dx+c)}-240i\mathrm{e}^{6i(dx+c)}+90\,\mathrm{e}^{7i(dx+c)}-80i\mathrm{e}^{4i(dx+c)}-80i\mathrm{e}^{2i(dx+c)}-90\,\mathrm{e}^{3i(dx+c)}+16i-15\,\mathrm{e}^{i(dx+c)}}{60ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)\cdots}{8ad}$
norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{3\left(\tan\cdots\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a*(1/5*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4+1/3*tan(1/2*d*x
+1/2*c)^3-2*tan(1/2*d*x+1/2*c)+2/tan(1/2*d*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^
3-1/5/tan(1/2*d*x+1/2*c)^5+4*ln(tan(1/2*d*x+1/2*c))+1/2/tan(1/2*d*x+1/2*c)^
4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(90) = 180$.
time = 0.29, size = 195, normalized size = 1.95

$$-\frac{\frac{\frac{60\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*((60*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (15*sin(d
*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^5/(a*sin(d*x + c)
^5))/d
```

Fricas [A]
time = 0.37, size = 161, normalized size = 1.61

$$\frac{32\,\cos\left(dx+c\right)^5-80\,\cos\left(dx+c\right)^3-15\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(ad\cos\left(dx+c\right)^4-2\,ad\cos\left(dx+c\right)^2+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.79, size = 157, normalized size = 1.57

$$\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-60\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-60\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a + (6*a^4*tan(1/2*d*x + 1/2*c)^5
 - 15*a^4*tan(1/2*d*x + 1/2*c)^4 + 10*a^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^4*t
an(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x + 1/2*c)^5 - 60*tan(1/2*d*x + 1
/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 6)/(a*tan(1
/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 8.75, size = 151, normalized size = 1.51

$$\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}+\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{5}\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(96*a*d) - tan(c/2 + (d*x)/2)^4/(64*a*d) + tan(c/2 + (
d*x)/2)^5/(160*a*d) + log(tan(c/2 + (d*x)/2))/(8*a*d) - tan(c/2 + (d*x)/2)/
(16*a*d) + (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)/2)/2 - tan(c/2 + (d*x)/2)
^2/3 + 2*tan(c/2 + (d*x)/2)^4 - 1/5))/(32*a*d)
```

3.419 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c}{}$$

```
[Out] 1/16*arctanh(cos(d*x+c))/a/d+1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d+1/16
*cot(d*x+c)*csc(d*x+c)/a/d+1/24*cot(d*x+c)*csc(d*x+c)^3/a/d-1/6*cot(d*x+c)*
csc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2691, 3853, 3855, 2687, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(16*a*d) + Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5
*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) + (Cot[c + d*x]*Csc[c + d*x]^3
)/(24*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a}\\ &= -\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\int \csc^5(c+dx)\,dx}{6a}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx\right.}{ad}\\ &= \frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\int \csc^3(c+dx)\,dx}{8a}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{\cot(c+dx)\csc(c+dx)}{16ad}+\frac{\cot(c+dx)\csc}{24ad}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{\cot(c+dx)\csc(c}{16ad}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(1140\cos(c+dx)+170\cos(3(c+dx))-30\cos(5(c+dx))-150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+225\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-90\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+15\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-225\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+90\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-15\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-480\sin(2(c+dx))-192\sin(4(c+dx))+32\sin(6(c+dx))\right)}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(1140*Cos[c + d*x] + 170*Cos[3*(c + d*x)] - 30*Cos[
5*(c + d*x)] - 150*Log[Cos[(c + d*x)/2]] + 225*Cos[2*(c + d*x)]*Log[Cos[(c
```

```
+ d*x)/2]] - 90*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 15*Cos[6*(c + d*x)
]*Log[Cos[(c + d*x)/2]] + 150*Log[Sin[(c + d*x)/2]] - 225*Cos[2*(c + d*x)]*
Log[Sin[(c + d*x)/2]] + 90*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 15*Cos[
6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 480*Sin[2*(c + d*x)] - 192*Sin[4*(c +
d*x)] + 32*Sin[6*(c + d*x)]))/(a*d)
```

Maple [A]
time = 0.27, size = 176, normalized size = 1.42

method	result
risch	$-\frac{15\,\mathrm{e}^{11i(dx+c)}-480i\mathrm{e}^{8i(dx+c)}-85\,\mathrm{e}^{9i(dx+c)}+320i\mathrm{e}^{6i(dx+c)}-570\,\mathrm{e}^{7i(dx+c)}-570\,\mathrm{e}^{5i(dx+c)}+192i\mathrm{e}^{2i(dx+c)}-85\,\mathrm{e}^{3i(dx+c)}-}{120da\left(\mathrm{e}^{2i(dx+c)}-1\right)^{6}}$
derivativedivides	$\frac{\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{64da}$
default	$\frac{\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-}{64da}$
norman	$\frac{-\frac{1}{384ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1920ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{384ad}+\frac{7\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}-\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{7\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{7\left(\tan^{9}\left(\frac{dx}{2}\right.\right.}{384ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a*(1/6*tan(1/2*d*x+1/2*c)^6-2/5*tan(1/2*d*x+1/2*c)^5+1/2*tan(1/2*d*x
+1/2*c)^4-2/3*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+4*tan(1/2*d*x+1
/2*c)+2/3/tan(1/2*d*x+1/2*c)^3-4/tan(1/2*d*x+1/2*c)+2/5/tan(1/2*d*x+1/2*c)^
5-1/2/tan(1/2*d*x+1/2*c)^4-4*ln(tan(1/2*d*x+1/2*c))+1/2/tan(1/2*d*x+1/2*c)^
2-1/6/tan(1/2*d*x+1/2*c)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(112) = 224$.
time = 0.29, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{120\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{12\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{120\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-5\right)(\cos(dx+c)+1)^{6}}{a\sin(dx+c)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1920*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (
12*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c)
```

```
 + 1)^4 - 120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6
/(a*sin(d*x + c)^6))/d
```

Fricas [A]
time = 0.36, size = 188, normalized size = 1.52

$$-\frac{30\cos\left(dx+c\right)^{5}-80\cos\left(dx+c\right)^{3}-15\left(\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-32\left(2\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)-30\cos\left(dx+c\right)}{480\left(ad\cos\left(dx+c\right)^{6}-3\,ad\cos\left(dx+c\right)^{4}+3\,ad\cos\left(dx+c\right)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(30*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 32*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 30*cos(d*x
+ c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a
*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.66, size = 216, normalized size = 1.74

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{5a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-12a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+15a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-20a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-15a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+120a^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{6}}-\frac{294\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1920*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (5*a^5*tan(1/2*d*x + 1/2*c)
^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 + 15*a^5*tan(1/2*d*x + 1/2*c)^4 - 20*a^5
*tan(1/2*d*x + 1/2*c)^3 - 15*a^5*tan(1/2*d*x + 1/2*c)^2 + 120*a^5*tan(1/2*d
*x + 1/2*c))/a^6 - (294*tan(1/2*d*x + 1/2*c)^6 - 120*tan(1/2*d*x + 1/2*c)^5
 + 15*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x +
 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/2*d*x + 1/2*c)^6))/d
```

Mupad [B]
time = 9.68, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+20\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-20\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{1920\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 15*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 20*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 20*cos(c/2 + (d*x)/
2)^9*sin(c/2 + (d*x)/2)^3 + 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^6)/(1920*a*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.420 $\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$-\frac{5x}{8a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{5\cos(c+$$

```
[Out] -5/8*x/a^2-2*cos(d*x+c)/a^2/d+5/3*cos(d*x+c)^3/a^2/d-4/5*cos(d*x+c)^5/a^2/d
+1/7*cos(d*x+c)^7/a^2/d+5/8*cos(d*x+c)*sin(d*x+c)/a^2/d+5/12*cos(d*x+c)*sin
(d*x+c)^3/a^2/d+1/3*cos(d*x+c)*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2713, 2715, 8}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^5(c+dx)\cos(c+dx)}{3a^2d}+\frac{5\sin^3(c+dx)\cos(c+dx)}{12a^2d}+\frac{5\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{5x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*x)/(8*a^2) - (2*Cos[c + d*x])/(a^2*d) + (5*Cos[c + d*x]^3)/(3*a^2*d) -
(4*Cos[c + d*x]^5)/(5*a^2*d) + Cos[c + d*x]^7/(7*a^2*d) + (5*Cos[c + d*x]*S
in[c + d*x])/(8*a^2*d) + (5*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*d) + (Cos[
c + d*x]*Sin[c + d*x]^5)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\sin^5(c+dx)-2a^2\sin^6(c+dx)+a^2\sin^7(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \sin^5(c+dx)\,dx}{a^2}+\frac{\int \sin^7(c+dx)\,dx}{a^2}-\frac{2\int \sin^6(c+dx)\,dx}{a^2}\\
&= \frac{\cos(c+dx)\sin^5(c+dx)}{3a^2d}-\frac{5\int \sin^4(c+dx)\,dx}{3a^2}-\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\right.}{a^2d}\\
&= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(}{}\\
&= -\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{5\cos(}{}\\
&= -\frac{5x}{8a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{5\cos^3(c+dx)}{3a^2d}-\frac{4\cos^5(c+dx)}{5a^2d}+\frac{\cos^7(c+dx)}{7a^2d}+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(147) = 294.
time = 3.59, size = 418, normalized size = 2.84

$$\frac{-210(1+40dx)\cos\left(\frac{c}{2}\right)-7875\cos\left(\frac{c}{2}+dx\right)-7875\cos\left(\frac{3c}{2}+dx\right)+3150\cos\left(\frac{3c}{2}+2dx\right)-3150\cos\left(\frac{5c}{2}+2dx\right)+1435\cos\left(\frac{5c}{2}+3dx\right)+1435\cos\left(\frac{7c}{2}+3dx\right)-630\cos\left(\frac{7c}{2}+4dx\right)+630\cos\left(\frac{9c}{2}+4dx\right)-231\cos\left(\frac{9c}{2}+5dx\right)-231\cos\left(\frac{11c}{2}+5dx\right)+70\cos\left(\frac{11c}{2}+6dx\right)-70\cos\left(\frac{13c}{2}+6dx\right)+15\cos\left(\frac{13c}{2}+7dx\right)+15\cos\left(\frac{15c}{2}+7dx\right)+210\sin\left(\frac{c}{2}\right)-8400dx\sin\left(\frac{c}{2}\right)+7875\sin\left(\frac{c}{2}+dx\right)-7875\sin\left(\frac{3c}{2}+dx\right)+3150\sin\left(\frac{3c}{2}+2dx\right)+3150\sin\left(\frac{5c}{2}+2dx\right)-1435\sin\left(\frac{5c}{2}+3dx\right)+1435\sin\left(\frac{7c}{2}+3dx\right)-630\sin\left(\frac{7c}{2}+4dx\right)-630\sin\left(\frac{9c}{2}+4dx\right)+231\sin\left(\frac{9c}{2}+5dx\right)-231\sin\left(\frac{11c}{2}+5dx\right)+70\sin\left(\frac{11c}{2}+6dx\right)+70\sin\left(\frac{13c}{2}+6dx\right)-15\sin\left(\frac{13c}{2}+7dx\right)+15\sin\left(\frac{15c}{2}+7dx\right)}{13440a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-210*(1 + 40*d*x)*Cos[c/2] - 7875*Cos[c/2 + d*x] - 7875*Cos[(3*c)/2 + d*x]
 + 3150*Cos[(3*c)/2 + 2*d*x] - 3150*Cos[(5*c)/2 + 2*d*x] + 1435*Cos[(5*c)/2
 + 3*d*x] + 1435*Cos[(7*c)/2 + 3*d*x] - 630*Cos[(7*c)/2 + 4*d*x] + 630*Cos[
(9*c)/2 + 4*d*x] - 231*Cos[(9*c)/2 + 5*d*x] - 231*Cos[(11*c)/2 + 5*d*x] + 7
0*Cos[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7*d*
```

```
x] + 15*Cos[(15*c)/2 + 7*d*x] + 210*Sin[c/2] - 8400*d*x*Sin[c/2] + 7875*Sin
[c/2 + d*x] - 7875*Sin[(3*c)/2 + d*x] + 3150*Sin[(3*c)/2 + 2*d*x] + 3150*Si
n[(5*c)/2 + 2*d*x] - 1435*Sin[(5*c)/2 + 3*d*x] + 1435*Sin[(7*c)/2 + 3*d*x]
- 630*Sin[(7*c)/2 + 4*d*x] - 630*Sin[(9*c)/2 + 4*d*x] + 231*Sin[(9*c)/2 + 5
*d*x] - 231*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c
)/2 + 6*d*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(13440*
a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.21, size = 168, normalized size = 1.14

method	result
risch	$-\frac{5x}{8a^2}-\frac{75\cos(dx+c)}{64a^2d}+\frac{\cos(7dx+7c)}{448d\,a^2}+\frac{\sin(6dx+6c)}{96a^2d}-\frac{11\cos(5dx+5c)}{320d\,a^2}-\frac{3\sin(4dx+4c)}{32a^2d}+\frac{41\cos(3dx+3c)}{192d\,a^2}$
derivativedivides	$\frac{64\left(\frac{-\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{25\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{283\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{11\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}+\frac{283\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}\right.}{a^2d}$
default	$\frac{64\left(\frac{-\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{25\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{283\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{11\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}+\frac{283\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^2*((-5/256*tan(1/2*d*x+1/2*c)^13-25/192*tan(1/2*d*x+1/2*c)^11-283/76
8*tan(1/2*d*x+1/2*c)^9-1/6*tan(1/2*d*x+1/2*c)^8-11/12*tan(1/2*d*x+1/2*c)^6+
283/768*tan(1/2*d*x+1/2*c)^5-13/20*tan(1/2*d*x+1/2*c)^4+25/192*tan(1/2*d*x+
1/2*c)^3-13/60*tan(1/2*d*x+1/2*c)^2+5/256*tan(1/2*d*x+1/2*c)-13/420)/(1+tan
(1/2*d*x+1/2*c)^2)^7-5/256*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(133) = 266$.
time = 0.50, size = 396, normalized size = 2.69

$$\frac{\frac{\frac{525\sin(dx+c)}{\cos(dx+c)+1}-\frac{5824\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3500\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{17472\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9905\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{24640\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{9905\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3500\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{525\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-832}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{525\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((525*sin(d*x + c)/(cos(d*x + c) + 1) - 5824*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 3500*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17472*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 9905*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 24640*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4480*sin(d*x + c)^8/(cos(d*x + c) + 1
```

```
)^8 - 9905*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3500*sin(d*x + c)^11/(cos(
d*x + c) + 1)^11 - 525*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 832)/(a^2 +
7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x
 + c) + 1)^14) - 525*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 88, normalized size = 0.60

$$\frac{120 \cos(dx+c)^7 - 672 \cos(dx+c)^5 + 1400 \cos(dx+c)^3 - 525\,dx + 35\left(8 \cos(dx+c)^5 - 26 \cos(dx+c)^3 + 33 \cos(dx+c)\right) \sin(dx+c) - 1680 \cos(dx+c)}{840\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 672*cos(d*x + c)^5 + 1400*cos(d*x + c)^3 - 525*
d*x + 35*(8*cos(d*x + c)^5 - 26*cos(d*x + c)^3 + 33*cos(d*x + c))*sin(d*x +
 c) - 1680*cos(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2895 vs. $2(138) = 276$.
time = 126.80, size = 2895, normalized size = 19.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-525*d*x*tan(c/2 + d*x/2)**14/(840*a**2*d*tan(c/2 + d*x/2)**14 +
 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 294
00*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a*
*2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) -
3675*d*x*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*
d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*t
an(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/
2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 11025*d*x*t
an(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2
+ d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d
*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)
**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 18375*d*x*tan(c/2 + d
*x/2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**1
2 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 +
29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*
a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 18375*d*x*tan(c/2 + d*x/2)**6/(8
```

40*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a
2*d*tan(c/2 + d*x/2)10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*
d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(
c/2 + d*x/2)**2 + 840*a**2*d) - 11025*d*x*tan(c/2 + d*x/2)**4/(840*a**2*d*t
an(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c
/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 +
 d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2
)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/2)**2/(840*a**2*d*tan(c/2 + d*x
/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)*
*10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 +
 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a
2*d) - 525*d*x/(840*a2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d
*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/
2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4
 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 1050*tan(c/2 + d*x/2)**1
3/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 176
40*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a
2*d*tan(c/2 + d*x/2)6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*
tan(c/2 + d*x/2)**2 + 840*a**2*d) - 7000*tan(c/2 + d*x/2)**11/(840*a**2*d*t
an(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c
/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 +
 d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2
)**2 + 840*a**2*d) - 19810*tan(c/2 + d*x/2)**9/(840*a**2*d*tan(c/2 + d*x/2)
14 + 5880*a2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10
 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17
640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2
*d) - 8960*tan(c/2 + d*x/2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2
*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*
tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c
/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 49280*tan(
c/2 + d*x/2)**6/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*
x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2
)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4
+ 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) + 19810*tan(c/2 + d*x/2)**5
/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 1764
0*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a*
*2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*t
an(c/2 + d*x/2)**2 + 840*a**2*d) - 34944*tan(c/2 + d*x/2)**4/(840*a**2*d*ta
n(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/
2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 +
d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)
2 + 840*a2*d) + 7000*tan(c/2 + d*x/2)**3/(840*a**2*d*tan(c/2 + d*x/2)**
14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 +
 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 1764
0*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d

```
) - 11648*tan(c/2 + d*x/2)**2/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*
d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*t
an(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/
2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) + 1050*tan(c/
2 + d*x/2)/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)*
*12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400...
```

Giac [A]
time = 0.62, size = 166, normalized size = 1.13

$$-\frac{\frac{525\,(dx+c)}{a^2}+\frac{2\left(525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+3500\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+9905\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+24640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9905\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+17472\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3500\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+5824\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+832\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(525*(d*x + c)/a^2 + 2*(525*tan(1/2*d*x + 1/2*c)^13 + 3500*tan(1/2*d
*x + 1/2*c)^11 + 9905*tan(1/2*d*x + 1/2*c)^9 + 4480*tan(1/2*d*x + 1/2*c)^8
+ 24640*tan(1/2*d*x + 1/2*c)^6 - 9905*tan(1/2*d*x + 1/2*c)^5 + 17472*tan(1/
2*d*x + 1/2*c)^4 - 3500*tan(1/2*d*x + 1/2*c)^3 + 5824*tan(1/2*d*x + 1/2*c)^
2 - 525*tan(1/2*d*x + 1/2*c) + 832)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

Mupad [B]
time = 12.22, size = 160, normalized size = 1.09

$$-\frac{5\,x}{8\,a^2}-\frac{\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+\frac{283\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{12}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\frac{176\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{283\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}+\frac{208\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}-\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{208\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{15}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{208}{105}}{a^2\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^5)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (5*x)/(8*a^2) - ((208*tan(c/2 + (d*x)/2)^2)/15 - (5*tan(c/2 + (d*x)/2))/4
 - (25*tan(c/2 + (d*x)/2)^3)/3 + (208*tan(c/2 + (d*x)/2)^4)/5 - (283*tan(c/
2 + (d*x)/2)^5)/12 + (176*tan(c/2 + (d*x)/2)^6)/3 + (32*tan(c/2 + (d*x)/2)^
8)/3 + (283*tan(c/2 + (d*x)/2)^9)/12 + (25*tan(c/2 + (d*x)/2)^11)/3 + (5*ta
n(c/2 + (d*x)/2)^13)/4 + 208/105)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.421 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=129

$$\frac{11x}{16a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\sin(c+dx)}{16a^2d}-\frac{11\cos(c+dx)\sin^3(c+dx)}{24a^2d}$$

```
[Out] 11/16*x/a^2+2*cos(d*x+c)/a^2/d-4/3*cos(d*x+c)^3/a^2/d+2/5*cos(d*x+c)^5/a^2/
d-11/16*cos(d*x+c)*sin(d*x+c)/a^2/d-11/24*cos(d*x+c)*sin(d*x+c)^3/a^2/d-1/6
*cos(d*x+c)*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2715, 8, 2713}

$$\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6a^2d}-\frac{11\sin^3(c+dx)\cos(c+dx)}{24a^2d}-\frac{11\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{11x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*x)/(16*a^2) + (2*Cos[c + d*x])/(a^2*d) - (4*Cos[c + d*x]^3)/(3*a^2*d) +
 (2*Cos[c + d*x]^5)/(5*a^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) -
 (11*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5
)/(6*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(a^2\sin^4(c+dx)-2a^2\sin^5(c+dx)+a^2\sin^6(c+dx)\right)\,dx}{a^4}\\
&= \frac{\int \sin^4(c+dx)\,dx}{a^2}+\frac{\int \sin^6(c+dx)\,dx}{a^2}-\frac{2\int \sin^5(c+dx)\,dx}{a^2}\\
&= -\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}-\frac{\cos(c+dx)\sin^5(c+dx)}{6a^2d}+\frac{3\int \sin^2(c+dx)\,d}{4a^2}\\
&= \frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+d}{8a^2d}\\
&= \frac{3x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)\,\mathrm{si}}{16a^2d}\\
&= \frac{11x}{16a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{4\cos^3(c+dx)}{3a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{11\cos(c+dx)}{16a^2}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.59

$$\frac{660c+660dx+1200\cos(c+dx)-200\cos(3(c+dx))+24\cos(5(c+dx))-465\sin(2(c+dx))+75\sin(4(c+dx))-5\sin(6(c+dx))}{960a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (660*c + 660*d*x + 1200*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 24*Cos[5*(c +
 d*x)] - 465*Sin[2*(c + d*x)] + 75*Sin[4*(c + d*x)] - 5*Sin[6*(c + d*x)])/(
960*a^2*d)
```

Maple [A]
time = 0.16, size = 153, normalized size = 1.19

method	result
risch	$\frac{11x}{16a^2}+\frac{5\cos(dx+c)}{4a^2d}-\frac{\sin(6dx+6c)}{192a^2d}+\frac{\cos(5dx+5c)}{40d\,a^2}+\frac{5\sin(4dx+4c)}{64a^2d}-\frac{5\cos(3dx+3c)}{24d\,a^2}-\frac{31\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{32\left(\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{187\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{47\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{47\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{187(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^2d}$
default	$\frac{32\left(\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{187\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}+\frac{47\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{47\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{187(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*((11/256*tan(1/2*d*x+1/2*c)^11+187/768*tan(1/2*d*x+1/2*c)^9+47/128
*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*d*x+1/2*c)^6-47/128*tan(1/2*d*x+1/2*c)^5+
tan(1/2*d*x+1/2*c)^4-187/768*tan(1/2*d*x+1/2*c)^3+2/5*tan(1/2*d*x+1/2*c)^2-
11/256*tan(1/2*d*x+1/2*c)+1/15)/(1+tan(1/2*d*x+1/2*c)^2)^6+11/256*arctan(ta
n(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(117) = 234$.

time = 0.49, size = 353, normalized size = 2.74

$$-\frac{\frac{\frac{165\sin(dx+c)}{\cos(dx+c)+1}-\frac{1536\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{935\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3840\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1410\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2560\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1410\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{935\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{165\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-256}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{165\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1536*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 935*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3840*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1410*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2560*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1410*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 935*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 165*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 256)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a
^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^
10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 165
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]

time = 0.35, size = 78, normalized size = 0.60

$$\frac{96\cos(dx+c)^5-320\cos(dx+c)^3+165\,dx-5\left(8\cos(dx+c)^5-38\cos(dx+c)^3+63\cos(dx+c)\right)\sin(dx+c)+480\cos(dx+c)}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*cos(d*x + c)^5 - 320*cos(d*x + c)^3 + 165*d*x - 5*(8*cos(d*x + c)
^5 - 38*cos(d*x + c)^3 + 63*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/
(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2271 vs. $2(122) = 244$.
time = 84.73, size = 2271, normalized size = 17.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((165*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 +
1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a
**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*t
an(c/2 + d*x/2)**2 + 240*a**2*d) + 990*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d
*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(
c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d
*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 2475*d*x*tan(c/2
 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) + 3300*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 2475*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*ta
n(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2
 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/
2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 990*d*x*tan(c/2 + d
*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**1
0 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 360
0*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d
) + 165*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)
**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**
2*d) + 330*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**
2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*ta
n(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 +
 d*x/2)**2 + 240*a**2*d) + 1870*tan(c/2 + d*x/2)**9/(240*a**2*d*tan(c/2 + d
*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)
```

```
**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1
440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 2820*tan(c/2 + d*x/2)**7/(24
0*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**
2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan
(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 5120*tan
(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d
*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)*
*6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 24
0*a**2*d) - 2820*tan(c/2 + d*x/2)**5/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d) + 7680*tan(c/2 + d*x/2)**4/(240*a**2*d*tan(c/
2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d
*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**
4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 1870*tan(c/2 + d*x/2)**
3/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 360
0*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*
d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 307
2*tan(c/2 + d*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/
2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*
x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2
 + 240*a**2*d) - 330*tan(c/2 + d*x/2)/(240*a**2*d*tan(c/2 + d*x/2)**12 + 14
40*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**
2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan
(c/2 + d*x/2)**2 + 240*a**2*d) + 512/(240*a**2*d*tan(c/2 + d*x/2)**12 + 144
0*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2
*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(
c/2 + d*x/2)**2 + 240*a**2*d), Ne(d, 0)), (x*sin(c)**4*cos(c)**4/(a*sin(c)
+ a)**2, True))
```

Giac [A]

time = 0.61, size = 153, normalized size = 1.19

$$\frac{\frac{165\,(dx+c)}{a^2}+\frac{2\left(165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+935\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+1410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+2560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-935\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1536\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+256\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(165*(d*x + c)/a^2 + 2*(165*tan(1/2*d*x + 1/2*c)^11 + 935*tan(1/2*d*x
 + 1/2*c)^9 + 1410*tan(1/2*d*x + 1/2*c)^7 + 2560*tan(1/2*d*x + 1/2*c)^6 - 1
410*tan(1/2*d*x + 1/2*c)^5 + 3840*tan(1/2*d*x + 1/2*c)^4 - 935*tan(1/2*d*x
+ 1/2*c)^3 + 1536*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*x + 1/2*c) + 256)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^2))/d
```

Mupad [B]

time = 11.24, size = 146, normalized size = 1.13

$$\frac{11\,x}{16\,a^2}+\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{187\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{187\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{32}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (11*x)/(16*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/5 - (11*tan(c/2 + (d*x)/2))/8
- (187*tan(c/2 + (d*x)/2)^3)/24 + 32*tan(c/2 + (d*x)/2)^4 - (47*tan(c/2 + (
d*x)/2)^5)/4 + (64*tan(c/2 + (d*x)/2)^6)/3 + (47*tan(c/2 + (d*x)/2)^7)/4 +
(187*tan(c/2 + (d*x)/2)^9)/24 + (11*tan(c/2 + (d*x)/2)^11)/8 + 32/15)/(a^2*
d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.422 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{3x}{4a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}+\frac{\cos(c+dx)\sin^3(c+dx)}{2a^2d}$$

```
[Out] -3/4*x/a^2-2*cos(d*x+c)/a^2/d+cos(d*x+c)^3/a^2/d-1/5*cos(d*x+c)^5/a^2/d+3/4
*cos(d*x+c)*sin(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2713, 2715, 8}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{2a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{3x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - (2*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(a^2*d) - Cos[c
+ d*x]^5/(5*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x
]*Sin[c + d*x]^3)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\sin^3(c+dx)-2a^2\sin^4(c+dx)+a^2\sin^5(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \sin^3(c+dx)\,dx}{a^2}+\frac{\int \sin^5(c+dx)\,dx}{a^2}-\frac{2\int \sin^4(c+dx)\,dx}{a^2} \\
&= \frac{\cos(c+dx)\sin^3(c+dx)}{2a^2d}-\frac{3\int \sin^2(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,c\right.}{a^2d} \\
&= -\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d} \\
&= -\frac{3x}{4a^2}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{3\cos(c+dx)\sin(}{4a^2d}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 308 vs. 2(102) = 204.
time = 0.95, size = 308, normalized size = 3.02

$$-\frac{5(1+24dx)\cos\left(\frac{c}{2}\right)+110\cos\left(\frac{c}{2}+dx\right)+110\cos\left(\frac{3c}{2}+dx\right)-40\cos\left(\frac{3c}{2}+2dx\right)+40\cos\left(\frac{5c}{2}+2dx\right)-15\cos\left(\frac{5c}{2}+3dx\right)-15\cos\left(\frac{7c}{2}+3dx\right)+5\cos\left(\frac{7c}{2}+4dx\right)-5\cos\left(\frac{9c}{2}+4dx\right)+\cos\left(\frac{9c}{2}+5dx\right)+\cos\left(\frac{11c}{2}+5dx\right)-5\sin\left(\frac{c}{2}\right)+120dx\sin\left(\frac{c}{2}\right)-110\sin\left(\frac{c}{2}+dx\right)+110\sin\left(\frac{3c}{2}+dx\right)-40\sin\left(\frac{3c}{2}+2dx\right)-40\sin\left(\frac{5c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+3dx\right)-15\sin\left(\frac{7c}{2}+3dx\right)+5\sin\left(\frac{7c}{2}+4dx\right)+5\sin\left(\frac{9c}{2}+4dx\right)-\sin\left(\frac{9c}{2}+5dx\right)+\sin\left(\frac{11c}{2}+5dx\right)}{160a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/160*(5*(1 + 24*d*x)*Cos[c/2] + 110*Cos[c/2 + d*x] + 110*Cos[(3*c)/2 + d*
x] - 40*Cos[(3*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 3
*d*x] - 15*Cos[(7*c)/2 + 3*d*x] + 5*Cos[(7*c)/2 + 4*d*x] - 5*Cos[(9*c)/2 +
4*d*x] + Cos[(9*c)/2 + 5*d*x] + Cos[(11*c)/2 + 5*d*x] - 5*Sin[c/2] + 120*d*
x*Sin[c/2] - 110*Sin[c/2 + d*x] + 110*Sin[(3*c)/2 + d*x] - 40*Sin[(3*c)/2 +
 2*d*x] - 40*Sin[(5*c)/2 + 2*d*x] + 15*Sin[(5*c)/2 + 3*d*x] - 15*Sin[(7*c)/
2 + 3*d*x] + 5*Sin[(7*c)/2 + 4*d*x] + 5*Sin[(9*c)/2 + 4*d*x] - Sin[(9*c)/2
+ 5*d*x] + Sin[(11*c)/2 + 5*d*x])/(a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.29, size = 129, normalized size = 1.26

method	result
risch	$-\frac{3x}{4a^2}-\frac{11\cos(dx+c)}{8a^2d}-\frac{\cos(5dx+5c)}{80d\,a^2}-\frac{\sin(4dx+4c)}{16a^2d}+\frac{3\cos(3dx+3c)}{16d\,a^2}+\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{16\left(\frac{-\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3\tan(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^2d}$
default	$\frac{16\left(\frac{-\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3\tan(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{a^2d}$
norman	$\frac{-\frac{33x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{269\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{29\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{3x}{4a}-\frac{12}{5ad}-\frac{57\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{10ad}-\frac{9\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{3\left(\tan^{16}\right.}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^2*((-3/32*tan(1/2*d*x+1/2*c)^9-7/16*tan(1/2*d*x+1/2*c)^7-1/4*tan(1/2
*d*x+1/2*c)^6-5/4*tan(1/2*d*x+1/2*c)^4+7/16*tan(1/2*d*x+1/2*c)^3-3/4*tan(1/
2*d*x+1/2*c)^2+3/32*tan(1/2*d*x+1/2*c)-3/20)/(1+tan(1/2*d*x+1/2*c)^2)^5-3/3
2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(94) = 188$.
time = 0.49, size = 290, normalized size = 2.84

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{200\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{40\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{70\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-24}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 200*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 70*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(a^2
 + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 10*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x
 + c)^8/(cos(d*x + c) + 1)^8 + a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) -
 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 68, normalized size = 0.67

$$-\frac{4\cos(dx+c)^5 - 20\cos(dx+c)^3 + 15\,dx + 5\,\left(2\cos(dx+c)^3 - 5\cos(dx+c)\right)\sin(dx+c) + 40\cos(dx+c)}{20\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(4*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*d*x + 5*(2*cos(d*x + c)^3
- 5*cos(d*x + c))*sin(d*x + c) + 40*cos(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. 2(94) = 188.
time = 54.13, size = 1608, normalized size = 15.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(20*a**2*d*tan(c/2 + d*x/2)**10 + 1
00*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d
*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 75*d*x
*tan(c/2 + d*x/2)**8/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 +
 d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**
4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 150*d*x*tan(c/2 + d*x/2)*
*6/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a
**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan
(c/2 + d*x/2)**2 + 20*a**2*d) - 150*d*x*tan(c/2 + d*x/2)**4/(20*a**2*d*tan(
c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*
x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 +
 20*a**2*d) - 75*d*x*tan(c/2 + d*x/2)**2/(20*a**2*d*tan(c/2 + d*x/2)**10 +
100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*
d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 15*d*
x/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a*
*2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(
c/2 + d*x/2)**2 + 20*a**2*d) - 30*tan(c/2 + d*x/2)**9/(20*a**2*d*tan(c/2 +
d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**
6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a*
*2*d) - 140*tan(c/2 + d*x/2)**7/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*
d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2
 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 80*tan(c/2 + d
*x/2)**6/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 +
 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2
```

```
*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 400*tan(c/2 + d*x/2)**4/(20*a**2*d*ta
n(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 +
d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2
 + 20*a**2*d) + 140*tan(c/2 + d*x/2)**3/(20*a**2*d*tan(c/2 + d*x/2)**10 + 1
00*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d
*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) - 240*ta
n(c/2 + d*x/2)**2/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*
x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 +
 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d) + 30*tan(c/2 + d*x/2)/(20*a**2
*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/2 + d*x/2)**8 + 200*a**2*d*tan(c
/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2)**4 + 100*a**2*d*tan(c/2 + d*x/
2)**2 + 20*a**2*d) - 48/(20*a**2*d*tan(c/2 + d*x/2)**10 + 100*a**2*d*tan(c/
2 + d*x/2)**8 + 200*a**2*d*tan(c/2 + d*x/2)**6 + 200*a**2*d*tan(c/2 + d*x/2
)**4 + 100*a**2*d*tan(c/2 + d*x/2)**2 + 20*a**2*d), Ne(d, 0)), (x*sin(c)**3
*cos(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.66, size = 127, normalized size = 1.25

$$-\frac{\frac{15\,(dx+c)}{a^2}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+200\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 70*tan(1/2*d*x + 1
/2*c)^7 + 40*tan(1/2*d*x + 1/2*c)^6 + 200*tan(1/2*d*x + 1/2*c)^4 - 70*tan(1
/2*d*x + 1/2*c)^3 + 120*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) +
24)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

Mupad [B]
time = 8.72, size = 89, normalized size = 0.87

$$\frac{3\,\cos\left(3\,c+3\,d\,x\right)}{16\,a^2\,d}-\frac{11\,\cos\left(c+d\,x\right)}{8\,a^2\,d}-\frac{3\,x}{4\,a^2}-\frac{\cos\left(5\,c+5\,d\,x\right)}{80\,a^2\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{2\,a^2\,d}-\frac{\sin\left(4\,c+4\,d\,x\right)}{16\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*cos(3*c + 3*d*x))/(16*a^2*d) - (11*cos(c + d*x))/(8*a^2*d) - (3*x)/(4*a^
2) - cos(5*c + 5*d*x)/(80*a^2*d) + sin(2*c + 2*d*x)/(2*a^2*d) - sin(4*c + 4
*d*x)/(16*a^2*d)
```

3.423 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{7x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}$$

```
[Out] 7/8*x/a^2+2*cos(d*x+c)/a^2/d-2/3*cos(d*x+c)^3/a^2/d-7/8*cos(d*x+c)*sin(d*x+
c)/a^2/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2715, 8, 2713}

$$-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}-\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) + (2*Cos[c + d*x])/(a^2*d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (
7*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\sin^2(c+dx)-2a^2\sin^3(c+dx)+a^2\sin^4(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{\int \sin^4(c+dx)\,dx}{a^2}-\frac{2\int \sin^3(c+dx)\,dx}{a^2} \\
&= -\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^2d}+\frac{\int 1\,dx}{2a^2}+\frac{3\int \sin^2
\end{aligned}$$

$$\begin{aligned}
&= \frac{x}{2a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}-\cos(\\
&= \frac{7x}{8a^2}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}-\cos(
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 258 vs. 2(87) = 174.
time = 1.19, size = 258, normalized size = 2.97

$$\frac{168dx\cos\left(\frac{c}{2}\right)+144\cos\left(\frac{c}{2}+dx\right)+144\cos\left(\frac{3c}{2}+dx\right)-48\cos\left(\frac{3c}{2}+2dx\right)+48\cos\left(\frac{5c}{2}+2dx\right)-16\cos\left(\frac{5c}{2}+3dx\right)-16\cos\left(\frac{7c}{2}+3dx\right)+3\cos\left(\frac{7c}{2}+4dx\right)-3\cos\left(\frac{9c}{2}+4dx\right)+8\sin\left(\frac{c}{2}\right)+168dx\sin\left(\frac{c}{2}\right)-144\sin\left(\frac{c}{2}+dx\right)+144\sin\left(\frac{3c}{2}+dx\right)-48\sin\left(\frac{3c}{2}+2dx\right)-48\sin\left(\frac{5c}{2}+2dx\right)+16\sin\left(\frac{5c}{2}+3dx\right)-16\sin\left(\frac{7c}{2}+3dx\right)+3\sin\left(\frac{7c}{2}+4dx\right)+3\sin\left(\frac{9c}{2}+4dx\right)}{192a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (168*d*x*Cos[c/2] + 144*Cos[c/2 + d*x] + 144*Cos[(3*c)/2 + d*x] - 48*Cos[(3
*c)/2 + 2*d*x] + 48*Cos[(5*c)/2 + 2*d*x] - 16*Cos[(5*c)/2 + 3*d*x] - 16*Cos
[(7*c)/2 + 3*d*x] + 3*Cos[(7*c)/2 + 4*d*x] - 3*Cos[(9*c)/2 + 4*d*x] + 8*Sin
[c/2] + 168*d*x*Sin[c/2] - 144*Sin[c/2 + d*x] + 144*Sin[(3*c)/2 + d*x] - 48
*Sin[(3*c)/2 + 2*d*x] - 48*Sin[(5*c)/2 + 2*d*x] + 16*Sin[(5*c)/2 + 3*d*x] -
 16*Sin[(7*c)/2 + 3*d*x] + 3*Sin[(7*c)/2 + 4*d*x] + 3*Sin[(9*c)/2 + 4*d*x])
/(192*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [**A**]
time = 0.24, size = 114, normalized size = 1.31

method	result
risch	$\frac{7x}{8a^2}+\frac{3\cos(dx+c)}{2a^2d}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{\cos(3dx+3c)}{6d\,a^2}-\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{\frac{8\left(\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{15\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}+\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\arctan(\mathrm{t}}{}}{d\,a^2}$
default	$\frac{\frac{8\left(\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{15\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}+\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{7\arctan(\mathrm{t}}{}}{d\,a^2}$
norman	$\frac{21x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{205\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{7\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}+\frac{7x}{8a}+\frac{8}{3ad}+\frac{25\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{133x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{63x(\tan^{13}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*((7/32*tan(1/2*d*x+1/2*c)^7+15/32*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+
1/2*c)^4-15/32*tan(1/2*d*x+1/2*c)^3+4/3*tan(1/2*d*x+1/2*c)^2-7/32*tan(1/2*d
*x+1/2*c)+1/3)/(1+tan(1/2*d*x+1/2*c)^2)^4+7/32*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(79) = 158$.

time = 0.49, size = 247, normalized size = 2.84

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{128\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{45\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{21\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 128*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 45*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 45*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 21*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 - 32)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 6*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin
(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]

time = 0.34, size = 58, normalized size = 0.67

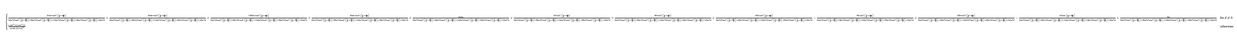
$$-\frac{16\cos(dx+c)^3-21\,dx-3\left(2\cos(dx+c)^3-9\cos(dx+c)\right)\sin(dx+c)-48\cos(dx+c)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(16*cos(d*x + c)^3 - 21*d*x - 3*(2*cos(d*x + c)^3 - 9*cos(d*x + c))*s
in(d*x + c) - 48*cos(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(82) = 164$.
time = 32.00, size = 1153, normalized size = 13.25



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((21*d*x*tan(c/2 + d*x/2)**8/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a
**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(
c/2 + d*x/2)**2 + 24*a**2*d) + 84*d*x*tan(c/2 + d*x/2)**6/(24*a**2*d*tan(c/
2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)
**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 126*d*x*tan(c/2 + d*x/2)
**4/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a*
*2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 84*
d*x*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
2 + 24*a**2*d) + 21*d*x/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2
+ d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**
2 + 24*a**2*d) + 42*tan(c/2 + d*x/2)**7/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d) + 90*tan(c/2 + d*x/2)**5/(24*a**2*d*tan(c/2
+ d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**
4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 192*tan(c/2 + d*x/2)**4/(2
4*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*t
an(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**2*d) - 90*tan(c/
2 + d*x/2)**3/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*tan(c/2 + d*x/2)**
6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d*x/2)**2 + 24*a**
2*d) + 256*tan(c/2 + d*x/2)**2/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96*a**2*d*t
an(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*tan(c/2 + d
*x/2)**2 + 24*a**2*d) - 42*tan(c/2 + d*x/2)/(24*a**2*d*tan(c/2 + d*x/2)**8
+ 96*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*
d*tan(c/2 + d*x/2)**2 + 24*a**2*d) + 64/(24*a**2*d*tan(c/2 + d*x/2)**8 + 96
*a**2*d*tan(c/2 + d*x/2)**6 + 144*a**2*d*tan(c/2 + d*x/2)**4 + 96*a**2*d*ta
n(c/2 + d*x/2)**2 + 24*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**4/(a*sin(c)
 + a)**2, True))
```

Giac [A]
time = 0.51, size = 114, normalized size = 1.31

$$\frac{\frac{21\,(dx+c)}{a^2}+\frac{2\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+128\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 + 2*(21*tan(1/2*d*x + 1/2*c)^7 + 45*tan(1/2*d*x + 1/
2*c)^5 + 96*tan(1/2*d*x + 1/2*c)^4 - 45*tan(1/2*d*x + 1/2*c)^3 + 128*tan(1/
2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 +
 1)^4*a^2))/d
```

Mupad [B]
time = 8.66, size = 79, normalized size = 0.91

$$\frac{7\,x}{8\,a^2}+\frac{2\,\cos\left(c+d\,x\right)}{a^2\,d}-\frac{2\cos\left(c+d\,x\right)^3}{3\,a^2\,d}+\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^2\,d}-\frac{9\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (7*x)/(8*a^2) + (2*cos(c + d*x))/(a^2*d) - (2*cos(c + d*x)^3)/(3*a^2*d) + (
cos(c + d*x)^3*sin(c + d*x))/(4*a^2*d) - (9*cos(c + d*x)*sin(c + d*x))/(8*a
^2*d)
```

3.424 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{x}{a^2}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}-\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}$$

```
[Out] -x/a^2-2/3*cos(d*x+c)^3/a^2/d-cos(d*x+c)*sin(d*x+c)/a^2/d-cos(d*x+c)^5/d/(a
+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2938, 2761, 2715, 8}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{x}{a^2}-\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Cos[c + d*x]^3)/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*
d) - Cos[c + d*x]^5/(d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{a} \\
&= -\frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{2\int \cos^2(c+dx)\,dx}{a^2} \\
&= -\frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2} - \frac{\int 1\,d}{a^2} \\
&= -\frac{x}{a^2} - \frac{2\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^2}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 204 vs. 2(70) = 140.
time = 0.63, size = 204, normalized size = 2.91

$$\frac{-2(1+12dx)\cos\left(\frac{c}{2}\right)-21\cos\left(\frac{c}{2}+dx\right)-21\cos\left(\frac{3c}{2}+dx\right)+6\cos\left(\frac{3c}{2}+2dx\right)-6\cos\left(\frac{5c}{2}+2dx\right)+\cos\left(\frac{5c}{2}+3dx\right)+\cos\left(\frac{7c}{2}+3dx\right)+2\sin\left(\frac{c}{2}\right)-24dx\sin\left(\frac{c}{2}\right)+21\sin\left(\frac{c}{2}+dx\right)-21\sin\left(\frac{3c}{2}+dx\right)+6\sin\left(\frac{3c}{2}+2dx\right)+6\sin\left(\frac{5c}{2}+2dx\right)-\sin\left(\frac{5c}{2}+3dx\right)+\sin\left(\frac{7c}{2}+3dx\right)}{24a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(1 + 12*d*x)*Cos[c/2] - 21*Cos[c/2 + d*x] - 21*Cos[(3*c)/2 + d*x] + 6*C
os[(3*c)/2 + 2*d*x] - 6*Cos[(5*c)/2 + 2*d*x] + Cos[(5*c)/2 + 3*d*x] + Cos[(
7*c)/2 + 3*d*x] + 2*Sin[c/2] - 24*d*x*Sin[c/2] + 21*Sin[c/2 + d*x] - 21*Sin
[(3*c)/2 + d*x] + 6*Sin[(3*c)/2 + 2*d*x] + 6*Sin[(5*c)/2 + 2*d*x] - Sin[(5*
c)/2 + 3*d*x] + Sin[(7*c)/2 + 3*d*x])/(24*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.20, size = 90, normalized size = 1.29

method	result
risch	$-\frac{x}{a^2}-\frac{7\cos(dx+c)}{4a^2d}+\frac{\cos(3dx+3c)}{12d\,a^2}+\frac{\sin(2dx+2c)}{2a^2d}$
derivativedivides	$\frac{\frac{4\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{5}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$

default	$$\frac{\frac{4\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{5}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$
norman	$$-\frac{16\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{x}{a}-\frac{10}{3ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{8x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25x\left(\tan^4\left(\frac{dx}{2}\right.\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*((-1/2*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4-2*tan(1/2*d*x+
1/2*c)^2+1/2*tan(1/2*d*x+1/2*c)-5/6)/(1+tan(1/2*d*x+1/2*c)^2)^3-1/2*arctan(
tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(68) = 136$.
time = 0.49, size = 184, normalized size = 2.63

$$\frac{2\left(\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/3*((3*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 5)/(a^2 + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 43, normalized size = 0.61

$$\frac{\cos(dx+c)^3-3\,dx+3\,\cos(dx+c)\sin(dx+c)-6\,\cos(dx+c)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*d*x + 3*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x + c))
/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(63) = 126$.
time = 18.35, size = 694, normalized size = 9.91

$$\begin{cases} -\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{9dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{3dx}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{24\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}+\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d}-\frac{10}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^2d} & \text{for}\ d\neq 0 \\ \frac{x\sin(c)\cos^4(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*
tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/
2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 9*d*x*tan(c/2 + d*x/2)**
2/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*t
an(c/2 + d*x/2)**2 + 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a*
*2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan
(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)*
*4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 6*tan(c/2 + d*x/2)**4/(3*a*
*2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 + 3*a**2*d) - 24*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)
**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6 + 9*a**2*d*tan(c/2 +
d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 10/(3*a**2*d*tan(c/2
 + d*x/2)**6 + 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2
+ 3*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.75, size = 88, normalized size = 1.26

$$-\frac{\frac{3\,(dx+c)}{a^2}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 3*tan(1/2*d*x + 1/2*c
)^4 + 12*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 5)/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^3*a^2))/d
```

Mupad [B]
time = 8.64, size = 55, normalized size = 0.79

$$\frac{\cos\left(3\,c+3\,d\,x\right)}{12\,a^2\,d}-\frac{7\,\cos\left(c+d\,x\right)}{4\,a^2\,d}-\frac{x}{a^2}+\frac{\sin\left(2\,c+2\,d\,x\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] cos(3*c + 3*d*x)/(12*a^2*d) - (7*cos(c + d*x))/(4*a^2*d) - x/a^2 + sin(2*c
+ 2*d*x)/(2*a^2*d)
```

3.425 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=36

$$-\frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{a^2d}$$

```
[Out] -2*x/a^2-arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/a^2/d
```

Rubi [A]

time = 0.10, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2948, 2825, 2814, 3855}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c + d*x]/(a^2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= -\frac{\cos(c+dx)}{a^2d}+\frac{\int \csc(c+dx)\left(a^2-2a^2\sin(c+dx)\right)\,dx}{a^4} \\
&= -\frac{2x}{a^2}-\frac{\cos(c+dx)}{a^2d}+\frac{\int \csc(c+dx)\,dx}{a^2} \\
&= -\frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 46, normalized size = 1.28

$$
-\frac{2c+2dx+\cos(c+dx)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((2*c + 2*d*x + Cos[c + d*x] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2
]])/(a^2*d))
```

Maple [A]
time = 0.25, size = 48, normalized size = 1.33

method	result
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
risch	$-\frac{2x}{a^2}-\frac{e^{i(dx+c)}}{2d\,a^2}-\frac{e^{-i(dx+c)}}{2d\,a^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}$
norman	$-\frac{2}{ad}-\frac{2x}{a}-\frac{6x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{12x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{20x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{24x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{24x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{20x\left(\tan^6\left(\frac{d}{}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(ln(tan(1/2*d*x+1/2*c))-2/(1+tan(1/2*d*x+1/2*c)^2)-4*arctan(tan(1/2
*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(36) = 72$.
time = 0.49, size = 82, normalized size = 2.28

$$-\frac{\frac{2}{a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2/(a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 4*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 45, normalized size = 1.25

$$-\frac{4\,dx+2\,\cos\left(dx+c\right)+\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d*x + 2*cos(d*x + c) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4\left(c+dx\right)\csc\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.57, size = 52, normalized size = 1.44

$$-\frac{\frac{2\,(dx+c)}{a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(d*x + c)/a^2 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 2/((tan(1/2*d*x +
1/2*c)^2 + 1)*a^2))/d
```

Mupad [B]
time = 8.72, size = 97, normalized size = 2.69

$$\frac{4\,\mathrm{atan}\left(\frac{16}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8}\right)}{a^2\,d}-\frac{2}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (4*atan(16/(16*tan(c/2 + (d*x)/2) + 8) - (8*tan(c/2 + (d*x)/2))/(16*tan(c/2
 + (d*x)/2) + 8)))/(a^2*d) - 2/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + log(t
an(c/2 + (d*x)/2))/(a^2*d)
```

3.426 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=35

$$\frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] x/a^2+2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 3855, 3852, 8}

$$-\frac{\cot(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2-2a^2\csc(c+dx)+a^2\csc^2(c+dx)\right)\,dx}{a^4}\\ &= \frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}\\ &= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}\\ &= \frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 98 vs. 2(35) = 70.
time = 0.28, size = 98, normalized size = 2.80

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2(c+dx)-\cot\left(\frac{1}{2}(c+dx)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2*(c + d*x) - Cot[(c + d*x)/2] +
4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Tan[(c + d*x)/2]))/(2*d
*(a + a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.24, size = 56, normalized size = 1.60

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$
risch	$\frac{x}{a^2}-\frac{2i}{a^2d\left(e^{2i(dx+c)}-1\right)}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{2\ln\left(e^{i(dx+c)}-1\right)}{a^2d}$

norman	$\frac{x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{1}{2ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{5x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{7x\left(\tan^4\left(\frac{d}{}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c))
+4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(35) = 70$.
time = 0.49, size = 93, normalized size = 2.66

$$\frac{\frac{4\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\cos(dx+c)+1}{a^2\sin(dx+c)}+\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 4*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 - (cos(d*x + c) + 1)/(a^2*sin(d*x + c)) + sin(d*x + c)
/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.36, size = 70, normalized size = 2.00

$$\frac{dx\sin(dx+c)+\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-\cos(dx+c)}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (d*x*sin(d*x + c) + log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos
(d*x + c) + 1/2)*sin(d*x + c) - cos(d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.77, size = 73, normalized size = 2.09

$$\frac{\frac{2\,(dx+c)}{a^2} - \frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} + \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2} + \frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)/a^2 - 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + tan(1/2*d*x +
 1/2*c)/a^2 + (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.83, size = 95, normalized size = 2.71

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{5}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{5\,\cos\left(\frac{c}{2}-\mathrm{atan}\left(\frac{1}{2}\right)+\frac{d\,x}{2}\right)}\right)}{a^2\,d} - \frac{\cot(c+d\,x)}{a^2\,d} - \frac{2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] - (2*atan((5^(1/2)*(cos(c/2 + (d*x)/2) - 2*sin(c/2 + (d*x)/2)))/(5*cos(c/2
- atan(1/2) + (d*x)/2))))/(a^2*d) - cot(c + d*x)/(a^2*d) - (2*log(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d)
```

3.427 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=54

$$-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] -3/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2948, 2836, 3855, 3852, 8, 3853}

$$\frac{2\cot(c+dx)}{a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\csc(c+dx)-2a^2\csc^2(c+dx)+a^2\csc^3(c+dx)\right)\,dx}{a^4}\\ &= \frac{\int \csc(c+dx)\,dx}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\mathrm{Su}}{}\\ &= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 86, normalized size = 1.59

$$-\frac{\left(\cot(c+dx)(-4+\csc(c+dx))+3\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{2a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((Cot[c + d*x]*(-4 + Csc[c + d*x]) + 3*(Log[Cos[(c + d*x)/2]] - Log[Si
n[(c + d*x)/2]]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(a^2*d*(1 + Sin[
c + d*x])^2)
```

Maple [A]
time = 0.26, size = 72, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{4d\,a^2}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{4d\,a^2}$
risch	$\frac{e^{3i(dx+c)}+e^{i(dx+c)}+4ie^{2i(dx+c)}-4i}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{3\ln\left(e^{i(dx+c)}-1\right)}{2a^2d}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2a^2d}$
norman	$\frac{\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{37\left(\tan^5\left(\right.\right.}{4c}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x+1/2*c)+4/tan(1/2*d*x+1/2*
c)+6*ln(tan(1/2*d*x+1/2*c))-1/2/tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(50) = 100$.

time = 0.29, size = 115, normalized size = 2.13

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^2\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (8*sin(d*x + c)/(c
os(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)^2/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]

time = 0.36, size = 93, normalized size = 1.72

$$-\frac{3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+8\cos(dx+c)\sin(dx+c)-2\cos(dx+c)}{4\left(a^2d\cos(dx+c)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d
*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.62, size = 98, normalized size = 1.81

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{18\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 -
8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - (18*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*
x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 8.69, size = 84, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{8}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - ta
n(c/2 + (d*x)/2)/(a^2*d) + (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8)
)/(a^2*d)
```

3.428 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] arctanh(cos(d*x+c))/a^2/d-2*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+cot(d*x
+c)*csc(d*x+c)/a^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2787, 2836, 3852, 8, 3853, 3855}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(
3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(a^2\csc^2(c+dx)-2a^2\csc^3(c+dx)+a^2\csc^4(c+dx)\right)\,dx}{a^4}\\ &= \frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}-\frac{2\int \csc^3(c+dx)\,dx}{a^2}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^2d}-\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 121, normalized size = 1.83

$$\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-9\cos(c+dx)+5\cos(3(c+dx))+6\left(2\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)+\sin(2(c+dx))\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{96a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(-9*Cos[c + d*x] + 5*Cos[3*(c
+ d*x)] + 6*(2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]
^3 + Sin[2*(c + d*x)]))*Tan[(c + d*x)/2])/(96*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 98, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{8d\,a^{2}}$
default	$\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{8d\,a^{2}}$
risch	$-\frac{2\left(3i\,e^{4i(dx+c)}+3\,e^{5i(dx+c)}-12i\,e^{2i(dx+c)}+5i-3\,e^{i(dx+c)}\right)}{3a^{2}d\left(e^{2i(dx+c)}-1\right)^{3}}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^{2}d}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^{2}d}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{23\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{29\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2+7*tan(1/2*d*x+1/
2*c)-7/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c))+2/tan(1/2*d*x+1/2*c)^2-1
/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.
time = 0.28, size = 153, normalized size = 2.32

$$\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}+\frac{\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{21\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)(\cos(dx+c)+1)^{3}}{a^{2}\sin(dx+c)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((21*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2 + (6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.38, size = 123, normalized size = 1.86

$$-\frac{10\cos(dx+c)^{3}-3\left(\cos(dx+c)^{2}-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^{2}-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+6\cos(dx+c)\sin(dx+c)-12\cos(dx+c)}{6\left(a^{2}d\cos(dx+c)^{2}-a^{2}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(10*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*cos(d*x + c)*sin(d*x + c) - 12*cos(d*x + c))/((a^2*d*cos(d*x + c
)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.80, size = 128, normalized size = 1.94

$$-\frac{\frac{24 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2}-\frac{44 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-21 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+6 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3}-\frac{a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-6 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+21 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^6}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (44*tan(1/2*d*x + 1/2*c)^3 -
 21*tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x +
 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 + 2
1*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 8.67, size = 119, normalized size = 1.80

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) - log(tan(
c/2 + (d*x)/2))/(a^2*d) + (7*tan(c/2 + (d*x)/2))/(8*a^2*d) - (cot(c/2 + (d*
x)/2)^3*(7*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 1/3))/(8*a^2*d)
```

3.429 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2} \, dx$

Optimal. Leaf size=96

$$-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+d}{4a^2d}$$

```
[Out] -7/8*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+2/3*cot(d*x+c)^3/a^2/d-7/
8*cot(d*x+c)*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2948, 2836, 3853, 3855, 3852}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{7\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (2*Cot[c
+ d*x]^3)/(3*a^2*d) - (7*Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\csc^3(c+dx)-2a^2\csc^4(c+dx)+a^2\csc^5(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{\int \csc^5(c+dx)\,dx}{a^2}-\frac{2\int \csc^4(c+dx)\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+ \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\operatorname{cs}}{8a^2d} \\
&= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{7\cot(c+dx)}{8a^2}
\end{aligned}$$

Mathematica [A]
time = 1.02, size = 116, normalized size = 1.21

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(45\cos(c+dx)+84\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^4(c+dx)+\cos(3(c+dx))(-21+32\sin(c+dx))-48\sin(2(c+dx))\right)}{1536a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1536*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(45*Cos[c + d*x] + 84*(Log
[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x
)]*(-21 + 32*Sin[c + d*x]) - 48*Sin[2*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x]
)^2)
```

Maple [A]
time = 0.32, size = 124, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{12}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\right.}}{16d\,a^2}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{12}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\right.}}{16d\,a^2}$
risch	$\frac{21\,e^{7i(dx+c)}-96i e^{4i(dx+c)}-45\,e^{5i(dx+c)}+128i e^{2i(dx+c)}-45\,e^{3i(dx+c)}-32i+21\,e^{i(dx+c)}}{12d\,a^2\left(e^{2i(dx+c)}-1\right)^4}-\frac{7\ln\left(e^{i(dx+c)}+1\right)}{8a^2d}+\frac{7\ln(e}{}$
norman	$\frac{-\frac{1}{64ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{15\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{15\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^2*(1/4*tan(1/2*d*x+1/2*c)^4-4/3*tan(1/2*d*x+1/2*c)^3+4*tan(1/2*d*x
+1/2*c)^2-12*tan(1/2*d*x+1/2*c)+12/tan(1/2*d*x+1/2*c)+14*ln(tan(1/2*d*x+1/2
*c))-1/4/tan(1/2*d*x+1/2*c)^4-4/tan(1/2*d*x+1/2*c)^2+4/3/tan(1/2*d*x+1/2*c)
^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(88) = 176$.
time = 0.28, size = 195, normalized size = 2.03

$$-\frac{\frac{\frac{144\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{144\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/192*((144*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4)/a^2 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (16*
sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
144*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(
d*x + c)^4))/d
```

Fricas [A]
time = 0.37, size = 149, normalized size = 1.55

$$\frac{42\cos(dx+c)^3-21\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+21\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-32\left(2\cos(dx+c)^3-3\cos(dx+c)\right)\sin(dx+c)-54\cos(dx+c)}{48\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(42*cos(d*x + c)^3 - 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 32*(2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c)
- 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.70, size = 157, normalized size = 1.64

$$\frac{\frac{168 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{350 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-144 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+48 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}+\frac{3\,a^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-16\,a^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+48\,a^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-144\,a^6 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^8}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(168*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (350*tan(1/2*d*x + 1/2*c)^4
 - 144*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x
+ 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) + (3*a^6*tan(1/2*d*x + 1/2*c)^4
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 48*a^6*tan(1/2*d*x + 1/2*c)^2 - 144*a^6*t
an(1/2*d*x + 1/2*c))/a^8)/d
```

Mupad [B]
time = 8.72, size = 151, normalized size = 1.57

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}+\frac{7\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{1}{4}\right)}{16\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + tan(c/2
+ (d*x)/2)^4/(64*a^2*d) + (7*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) - (3*tan(c/
2 + (d*x)/2))/(4*a^2*d) + (cot(c/2 + (d*x)/2)^4*((4*tan(c/2 + (d*x)/2))/3 -
 4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a^2*d)
```

3.430 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=112

$$\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)}{2}$$

```
[Out] 3/4*arctanh(cos(d*x+c))/a^2/d-2*cot(d*x+c)/a^2/d-cot(d*x+c)^3/a^2/d-1/5*cot
(d*x+c)^5/a^2/d+3/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)*csc(d*x+c)^3
/a^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 3852, 3853, 3855}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) - (2*Cot[c + d*x])/(a^2*d) - Cot[c + d*
x]^3/(a^2*d) - Cot[c + d*x]^5/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(4*
a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(2*a^2*d)
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\csc^4(c+dx)-2a^2\csc^5(c+dx)+a^2\csc^6(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \csc^6(c+dx)\,dx}{a^2}-\frac{2\int \csc^5(c+dx)\,dx}{a^2} \\
&= \frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{3\int \csc^3(c+dx)\,dx}{2a^2}-\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cdots\right.}{a^2d} \\
&= -\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx\cdots}{4a^2d} \\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5a^2d}+\cdots
\end{aligned}$$

Mathematica [A]
time = 0.56, size = 189, normalized size = 1.69

$$\frac{\csc^5(c+dx)\left(-160\cos(c+dx)+120\cos(3(c+dx))-24\cos(5(c+dx))+150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+140\sin(2(c+dx))-75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-30\sin(4(c+dx))+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{320a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-160*Cos[c + d*x] + 120*Cos[3*(c + d*x)] - 24*Cos[5*(c + d
*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] + 140*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*
x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)] + 15*
Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c
+ d*x)]))/(320*a^2*d)
```

Maple [A]
time = 0.33, size = 148, normalized size = 1.32

method	result
risch	$-\frac{15\,\mathrm{e}^{9i(dx+c)}-40i\mathrm{e}^{6i(dx+c)}-70\,\mathrm{e}^{7i(dx+c)}+200i\mathrm{e}^{4i(dx+c)}-120i\mathrm{e}^{2i(dx+c)}+70\,\mathrm{e}^{3i(dx+c)}+24i-15\,\mathrm{e}^{i(dx+c)}}{10a^2d\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}+\frac{3\ln\left(\mathrm{e}^{i(dx\right.}}{4a^2}$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{22}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^2}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{22}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^2}$
norman	$\frac{-\frac{1}{160ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160ad}+\frac{9\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160ad}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}-\frac{9\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160ad}+\frac{3(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a\left(\tan\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4+3*tan(1/2*d*x+1/2
*c)^3-8*tan(1/2*d*x+1/2*c)^2+22*tan(1/2*d*x+1/2*c)-22/tan(1/2*d*x+1/2*c)-1/
5/tan(1/2*d*x+1/2*c)^5+1/tan(1/2*d*x+1/2*c)^4-3/tan(1/2*d*x+1/2*c)^3-24*ln(
tan(1/2*d*x+1/2*c))+8/tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(104) = 208$.
time = 0.29, size = 233, normalized size = 2.08

$$\frac{\frac{\frac{110\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{110\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/160*((110*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 15*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 120*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a^2 + (5*sin(d*x + c)/(cos(d*x + c) + 1) - 15*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- 110*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)*(cos(d*x + c) + 1)^5/(a^2*si
n(d*x + c)^5))/d
```

Fricas [A]
time = 0.36, size = 179, normalized size = 1.60

$$-\frac{48\cos\left(dx+c\right)^5-120\cos\left(dx+c\right)^3-15\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10\left(3\cos\left(dx+c\right)^3-5\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+80\cos\left(dx+c\right)}{40\left(a^2d\cos\left(dx+c\right)^4-2a^2d\cos\left(dx+c\right)^2+a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(48*cos(d*x + c)^5 - 120*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(
3*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c) + 80*cos(d*x + c))/((a^2*d*
cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.67, size = 186, normalized size = 1.66

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{274\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-5a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+15a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-40a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+110a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{10}}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/160*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^
5 - 110*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x
 + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^5) - (a
^8*tan(1/2*d*x + 1/2*c)^5 - 5*a^8*tan(1/2*d*x + 1/2*c)^4 + 15*a^8*tan(1/2*d
*x + 1/2*c)^3 - 40*a^8*tan(1/2*d*x + 1/2*c)^2 + 110*a^8*tan(1/2*d*x + 1/2*c
))/a^10)/d
```

Mupad [B]
time = 9.30, size = 289, normalized size = 2.58

$$-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7-110\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+110\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(cos(c/2 + (d*x)/2)^10 - sin(c/2 + (d*x)/2)^10 + 5*cos(c/2 + (d*x)/2)*sin(
c/2 + (d*x)/2)^9 - 5*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 15*cos(c/2 +
 (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 40*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/
2)^7 - 110*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 110*cos(c/2 + (d*x)/
2)^6*sin(c/2 + (d*x)/2)^4 - 40*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3 +
15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 120*log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(160*a^2*d*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.431 $\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\cot(c+dx)\csc(c+dx)}{16a^2d}-\frac{11 c}{}$$

[Out] -11/16*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+4/3*cot(d*x+c)^3/a^2/d+2/5*cot(d*x+c)^5/a^2/d-11/16*cot(d*x+c)*csc(d*x+c)/a^2/d-11/24*cot(d*x+c)*csc(d*x+c)^3/a^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5/a^2/d

Rubi [A]
time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 3853, 3855, 3852}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{11\cot(c+dx)\csc^3(c+dx)}{24a^2d}-\frac{11\cot(c+dx)\csc(c+dx)}{16a^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]

[Out] (-11*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (4*Cot[c + d*x]^3)/(3*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2*d)

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\csc^5(c+dx)-2a^2\csc^6(c+dx)+a^2\csc^7(c+dx)\right)\,dx}{a^4} \\
&= \frac{\int \csc^5(c+dx)\,dx}{a^2}+\frac{\int \csc^7(c+dx)\,dx}{a^2}-\frac{2\int \csc^6(c+dx)\,dx}{a^2} \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}+\frac{3\int \csc^3(c+dx)\,c}{4a^2} \\
&= \frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+d}{8a^2d} \\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx}{5a^2d} \\
&= -\frac{11\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{4\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+d}{5a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.59, size = 229, normalized size = 1.66

$$\frac{\csc^6(c+dx)\left(-2820\cos(c+dx)+1870\cos(3(c+dx))-330\cos(5(c+dx))-1650\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2475\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-990\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+165\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1650\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2475\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+990\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-165\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3840\sin(2(c+dx))-1536\sin(4(c+dx))+256\sin(6(c+dx))\right)}{7680a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(-2820*Cos[c + d*x] + 1870*Cos[3*(c + d*x)] - 330*Cos[5*(c
+ d*x)] - 1650*Log[Cos[(c + d*x)/2]] + 2475*Cos[2*(c + d*x)]*Log[Cos[(c + d
*x)/2]] - 990*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 165*Cos[6*(c + d*x)]
*Log[Cos[(c + d*x)/2]] + 1650*Log[Sin[(c + d*x)/2]] - 2475*Cos[2*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 990*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 165*C
```

```
os[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3840*Sin[2*(c + d*x)] - 1536*Sin[4*
(c + d*x)] + 256*Sin[6*(c + d*x)]))/(7680*a^2*d)
```

Maple [A]
time = 0.38, size = 176, normalized size = 1.28

method	result
risch	$\frac{165\,\mathrm{e}^{11i(dx+c)}-935\,\mathrm{e}^{9i(dx+c)}+2560i\mathrm{e}^{6i(dx+c)}+1410\,\mathrm{e}^{7i(dx+c)}-3840i\mathrm{e}^{4i(dx+c)}+1410\,\mathrm{e}^{5i(dx+c)}+1536i\mathrm{e}^{2i(dx+c)}-935\,\mathrm{e}^{3}}{120d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$
derivativedivides	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{31\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{40}{\tan\left(\frac{dx}{2}+\right.}}{64d\,a^2}$
default	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{31\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{40}{\tan\left(\frac{dx}{2}+\right.}}{64d\,a^2}$
norman	$\frac{-\frac{1}{384ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}-\frac{11\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}+\frac{11\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{11\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^2*(1/6*tan(1/2*d*x+1/2*c)^6-4/5*tan(1/2*d*x+1/2*c)^5+5/2*tan(1/2*d
*x+1/2*c)^4-20/3*tan(1/2*d*x+1/2*c)^3+31/2*tan(1/2*d*x+1/2*c)^2-40*tan(1/2*
d*x+1/2*c)+40/tan(1/2*d*x+1/2*c)-1/6/tan(1/2*d*x+1/2*c)^6-31/2/tan(1/2*d*x+
1/2*c)^2+44*ln(tan(1/2*d*x+1/2*c))+20/3/tan(1/2*d*x+1/2*c)^3-5/2/tan(1/2*d*
x+1/2*c)^4+4/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(126) = 252$.
time = 0.29, size = 275, normalized size = 1.99

$$-\frac{\frac{\frac{1200\sin(dx+c)}{\cos(dx+c)+1}-\frac{465\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{1320\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{75\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{465\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5\right)(\cos(dx+c)+1)^6}{a^2\sin(dx+c)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((1200*sin(d*x + c)/(cos(d*x + c) + 1) - 465*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 75*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a^2 - 1320*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 75*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 465*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 1200*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x
 + c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [A]
time = 0.37, size = 204, normalized size = 1.48

$$\frac{330 \cos\left(dx+c\right)^{5}-880 \cos\left(dx+c\right)^{3}-165 \left(\cos\left(dx+c\right)^{6}-3 \cos\left(dx+c\right)^{4}+3 \cos\left(dx+c\right)^{2}-1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+165 \left(\cos\left(dx+c\right)^{6}-3 \cos\left(dx+c\right)^{4}+3 \cos\left(dx+c\right)^{2}-1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)-64 \left(8 \cos\left(dx+c\right)^{5}-20 \cos\left(dx+c\right)^{3}+15 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)+630 \cos\left(dx+c\right)}{480 \left(a^{2} d \cos\left(dx+c\right)^{6}-3 a^{2} d \cos\left(dx+c\right)^{4}+3 a^{2} d \cos\left(dx+c\right)^{2}-a^{2} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(330*cos(d*x + c)^5 - 880*cos(d*x + c)^3 - 165*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 165*(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 64*(8*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(
d*x + c) + 630*cos(d*x + c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.54, size = 215, normalized size = 1.56

$$\frac{\frac{1320 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{2}}-\frac{3234 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-1200 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+465 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-200 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+75 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-24 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5}{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}}+\frac{5 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-24 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+75 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-200 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+465 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-1200 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{12}}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(1320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (3234*tan(1/2*d*x + 1/2*c
)^6 - 1200*tan(1/2*d*x + 1/2*c)^5 + 465*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/
2*d*x + 1/2*c)^3 + 75*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)
/(a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*ta
n(1/2*d*x + 1/2*c)^5 + 75*a^10*tan(1/2*d*x + 1/2*c)^4 - 200*a^10*tan(1/2*d*
x + 1/2*c)^3 + 465*a^10*tan(1/2*d*x + 1/2*c)^2 - 1200*a^10*tan(1/2*d*x + 1/
2*c))/a^12)/d
```

Mupad [B]
time = 9.74, size = 339, normalized size = 2.46

$$\frac{5 \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-24 \cos\left(\frac{c}{2}+\frac{dx}{2}\right) \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+24 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)+75 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-200 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+465 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-1200 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+1200 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-465 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+200 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-75 \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1320 \ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right) \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{1920 a^{2} d \cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} \sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 75*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 200*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 465*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 1200*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1200*cos(c/2 + (d*x)/2)^7*sin(c/2 + (
d*x)/2)^5 - 465*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 200*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 75*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 1320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*
sin(c/2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)
```

3.432 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$\frac{51x}{8a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

```
[Out] 51/8*x/a^3+7*cos(d*x+c)/a^3/d-cos(d*x+c)^3/a^3/d-19/8*cos(d*x+c)*sin(d*x+c)
/a^3/d-1/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2951, 2718, 2715, 8, 2713, 2727}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{7\cos(c+dx)}{a^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) + (7*Cos[c + d*x])/(a^3*d) - Cos[c + d*x]^3/(a^3*d) - (19*Co
s[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d
) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sin(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$

$$= \frac{\int \Big(4a-4a\sin(c+dx)+4a\sin^2(c+dx)-3a\sin^3(c+dx)+a\sin^4(c+}{a^4}$$

$$= \frac{4x}{a^3}+\frac{\int \sin^4(c+dx)\,dx}{a^3}-\frac{3\int \sin^3(c+dx)\,dx}{a^3}-\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4\int}{}$$

$$= \frac{4x}{a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin^3(c+d}{4a^3d}$$

$$= \frac{6x}{a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(c}{}$$

$$= \frac{51x}{8a^3}+\frac{7\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos(}{}$$

Mathematica [A]
time = 1.02, size = 195, normalized size = 1.79

$$\frac{2040dx\cos\left(\frac{dx}{2}\right)+997\cos\left(c+\frac{dx}{2}\right)+800\cos\left(c+\frac{3dx}{2}\right)+160\cos\left(3c+\frac{5dx}{2}\right)-35\cos\left(3c+\frac{7dx}{2}\right)-5\cos\left(5c+\frac{9dx}{2}\right)-3563\sin\left(\frac{dx}{2}\right)+2040dx\sin\left(c+\frac{dx}{2}\right)+800\sin\left(2c+\frac{3dx}{2}\right)-160\sin\left(2c+\frac{5dx}{2}\right)-35\sin\left(4c+\frac{7dx}{2}\right)+5\sin\left(4c+\frac{9dx}{2}\right)}{320a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2040*d*x*Cos[(d*x)/2] + 997*Cos[c + (d*x)/2] + 800*Cos[c + (3*d*x)/2] + 16
0*Cos[3*c + (5*d*x)/2] - 35*Cos[3*c + (7*d*x)/2] - 5*Cos[5*c + (9*d*x)/2] -
 3563*Sin[(d*x)/2] + 2040*d*x*Sin[c + (d*x)/2] + 800*Sin[2*c + (3*d*x)/2] -
 160*Sin[2*c + (5*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)/2
])/(320*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]

time = 0.17, size = 143, normalized size = 1.31

method	result
risch	$\frac{51x}{8a^3}+\frac{25\,\mathrm{e}^{i(dx+c)}}{8d\,a^3}+\frac{25\,\mathrm{e}^{-i(dx+c)}}{8d\,a^3}+\frac{8}{d\,a^3\left(\mathrm{e}^{i(dx+c)}+i\right)}+\frac{\sin(4dx+4c)}{32d\,a^3}-\frac{\cos(3dx+3c)}{4d\,a^3}-\frac{5\sin(2dx+2c)}{4d\,a^3}$
derivativedivides	$\frac{\frac{8\left(\frac{19\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{27\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{27\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\tan\left(\frac{dx}{2}+\cdots\right.}{32}\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\cdots}{d\,a^3}$
default	$\frac{\frac{8\left(\frac{19\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{27\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{27\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\tan\left(\frac{dx}{2}+\cdots\right.}{32}\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}\cdots}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*(1/2*(19/32*tan(1/2*d*x+1/2*c)^7+tan(1/2*d*x+1/2*c)^6+27/32*tan(1/
2*d*x+1/2*c)^5+9/2*tan(1/2*d*x+1/2*c)^4-27/32*tan(1/2*d*x+1/2*c)^3+5*tan(1/
2*d*x+1/2*c)^2-19/32*tan(1/2*d*x+1/2*c)+3/2)/(1+tan(1/2*d*x+1/2*c)^2)^4+51/
64*arctan(tan(1/2*d*x+1/2*c))+1/2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(103) = 206.

time = 0.50, size = 398, normalized size = 3.65

$$\frac{\frac{\frac{29\sin(dx+c)}{\cos(dx+c)+1}+\frac{269\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{133\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{309\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{171\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{187\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{51\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{51\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+80}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{51\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((29*sin(d*x + c)/(cos(d*x + c) + 1) + 269*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 133*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 309*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 171*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 187*sin(d*x +
```

```
c)^6/(cos(d*x + c) + 1)^6 + 51*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 51*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 80)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c
) + 1) + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a^3*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 4*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8 + a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 51*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [**A**]
time = 0.35, size = 144, normalized size = 1.32

$$-\frac{2\cos(dx+c)^5+8\cos(dx+c)^4-15\cos(dx+c)^3-51\,dx-(51\,dx+67)\cos(dx+c)-56\cos(dx+c)^2-(2\cos(dx+c)^4-6\cos(dx+c)^3+51\,dx-21\cos(dx+c)^2+35\cos(dx+c)-32)\sin(dx+c)-32}{8\,(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*cos(d*x + c)^5 + 8*cos(d*x + c)^4 - 15*cos(d*x + c)^3 - 51*d*x - (5
1*d*x + 67)*cos(d*x + c) - 56*cos(d*x + c)^2 - (2*cos(d*x + c)^4 - 6*cos(d*
x + c)^3 + 51*d*x - 21*cos(d*x + c)^2 + 35*cos(d*x + c) - 32)*sin(d*x + c)
- 32)/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3578 vs. $2(100) = 200$.
time = 87.11, size = 3578, normalized size = 32.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((51*d*x*tan(c/2 + d*x/2)**9/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**
3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2
 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**
4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*
d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x*tan(c/2 + d*x/2)**8/(8*a**3*d*tan(c
/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)*
*7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**
3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2
 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 204*d*x*tan(c/2 + d*
x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a
**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c
/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)
**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d)
 + 204*d*x*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan
```

```
(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/
2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*
a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c
/2 + d*x/2) + 8*a**3*d) + 306*d*x*tan(c/2 + d*x/2)**5/(8*a**3*d*tan(c/2 + d
*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 3
2*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*ta
n(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x
/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 306*d*x*tan(c/2 + d*x/2)**
4/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*
tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d
*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 +
32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 204
*d*x*tan(c/2 + d*x/2)**3/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 +
 d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6
+ 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d
*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d
*x/2) + 8*a**3*d) + 204*d*x*tan(c/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)*
*9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3
*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2
+ d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2
 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x*tan(c/2 + d*x/2)/(8*a**3*
d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 +
d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 +
 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*
tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 51*d*x/(8*a**
3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
+ d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5
 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*
d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 102*tan(c/2
 + d*x/2)**8/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 +
 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*
tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d
*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a*
*3*d) + 102*tan(c/2 + d*x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*ta
n(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x
/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32
*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(
c/2 + d*x/2) + 8*a**3*d) + 374*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/
2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a
**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c
/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)
**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 342*tan(c/2 + d*x/2)**5/(8*a*
*3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**
5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**3 + 32*a**3
```

```
*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a**3*d) + 618*tan(c/
2 + d*x/2)**4/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*tan(c/2 + d*x/2)**8
+ 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d
*tan(c/2 + d*x/2)**5 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 +
d*x/2)**3 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d*tan(c/2 + d*x/2) + 8*a
**3*d) + 266*tan(c/2 + d*x/2)**3/(8*a**3*d*tan(c/2 + d*x/2)**9 + 8*a**3*d*t
an(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**7 + 32*a**3*d*tan(c/2 + d*
x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**5 + 48*a*...
```

Giac [A]
time = 0.45, size = 145, normalized size = 1.33

$$\frac{\frac{51\,(dx+c)}{a^3}+\frac{64}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+144\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(19*tan(1/2
*d*x + 1/2*c)^7 + 32*tan(1/2*d*x + 1/2*c)^6 + 27*tan(1/2*d*x + 1/2*c)^5 + 1
44*tan(1/2*d*x + 1/2*c)^4 - 27*tan(1/2*d*x + 1/2*c)^3 + 160*tan(1/2*d*x + 1
/2*c)^2 - 19*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3
))/d
```

Mupad [B]
time = 12.35, size = 146, normalized size = 1.34

$$\frac{51\,x}{8\,a^3}+\frac{\frac{51\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}+\frac{51\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{187\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+\frac{171\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{309\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{133\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{269\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{29\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+20}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (51*x)/(8*a^3) + ((29*tan(c/2 + (d*x)/2))/4 + (269*tan(c/2 + (d*x)/2)^2)/4
+ (133*tan(c/2 + (d*x)/2)^3)/4 + (309*tan(c/2 + (d*x)/2)^4)/4 + (171*tan(c/
2 + (d*x)/2)^5)/4 + (187*tan(c/2 + (d*x)/2)^6)/4 + (51*tan(c/2 + (d*x)/2)^7
)/4 + (51*tan(c/2 + (d*x)/2)^8)/4 + 20)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)*(ta
n(c/2 + (d*x)/2)^2 + 1)^4)
```

3.433 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=87

$$-\frac{11x}{2a^3}-\frac{5\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

```
[Out] -11/2*x/a^3-5*cos(d*x+c)/a^3/d+1/3*cos(d*x+c)^3/a^3/d+3/2*cos(d*x+c)*sin(d*
x+c)/a^3/d-4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2788, 2718, 2715, 8, 2713, 2727}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{5\cos(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) - (5*Cos[c + d*x])/(a^3*d) + Cos[c + d*x]^3/(3*a^3*d) + (3*
Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c +
 d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int (a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(-4a+4a\sin(c+dx)-3a\sin^2(c+dx)+a\sin^3(c+dx)+\frac{4a}{1+\sin(c+dx)}\right.}{a^4}$$

$$= -\frac{4x}{a^3}+\frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{3\int \sin^2(c+dx)\,dx}{a^3}+\frac{4\int \sin(c+dx)\,dx}{a^3}+\frac{4}{}$$

$$= -\frac{4x}{a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

$$= -\frac{11x}{2a^3}-\frac{5\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{}{a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(87) = 174.
time = 0.97, size = 181, normalized size = 2.08

$$\frac{(1-660dx)\cos\left(\frac{dx}{2}\right)-286\cos\left(c+\frac{dx}{2}\right)-240\cos\left(c+\frac{3dx}{2}\right)-40\cos\left(3c+\frac{5dx}{2}\right)+5\cos\left(3c+\frac{7dx}{2}\right)+1244\sin\left(\frac{dx}{2}\right)+\sin\left(c+\frac{dx}{2}\right)-660dx\sin\left(c+\frac{dx}{2}\right)-240\sin\left(2c+\frac{3dx}{2}\right)+40\sin\left(2c+\frac{5dx}{2}\right)+5\sin\left(4c+\frac{7dx}{2}\right)}{120a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]

[Out] ((1 - 660*d*x)*Cos[(d*x)/2] - 286*Cos[c + (d*x)/2] - 240*Cos[c + (3*d*x)/2] - 40*Cos[3*c + (5*d*x)/2] + 5*Cos[3*c + (7*d*x)/2] + 1244*Sin[(d*x)/2] + Sin[c + (d*x)/2] - 660*d*x*Sin[c + (d*x)/2] - 240*Sin[2*c + (3*d*x)/2] + 40*Sin[2*c + (5*d*x)/2] + 5*Sin[4*c + (7*d*x)/2])/(120*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

Maple [A]
time = 0.33, size = 104, normalized size = 1.20

method	result
risch	$-\frac{11x}{2a^3}-\frac{19\,e^{i(dx+c)}}{8d\,a^3}-\frac{19\,e^{-i(dx+c)}}{8d\,a^3}-\frac{8}{d\,a^3\left(e^{i(dx+c)}+i\right)}+\frac{\cos(3dx+3c)}{12d\,a^3}+\frac{3\sin(2dx+2c)}{4d\,a^3}$
derivativedivides	$\frac{-\frac{8\left(\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{7}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
default	$\frac{-\frac{8\left(\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{7}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
norman	$\frac{-\frac{220x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{5825\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{175\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{11x}{2a}-\frac{52}{3ad}-\frac{227\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{55\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{11(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 8/d/a^3*(-(3/8*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4+5/2*tan(1/2*d*x+1/2*c)^2-3/8*tan(1/2*d*x+1/2*c)+7/6)/(1+tan(1/2*d*x+1/2*c)^2)^3-11/8*arctan(tan(1/2*d*x+1/2*c))-1/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(81) = 162.
time = 0.49, size = 312, normalized size = 3.59

$$-\frac{\frac{\frac{19\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{60\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{33\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+52}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{33\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/3*((19*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 60*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 96*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 33*sin(d*x + c)^

```
6/(cos(d*x + c) + 1)^6 + 52)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*a^3*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(
d*x + c)^7/(cos(d*x + c) + 1)^7) + 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a^3)/d
```

Fricas [A]
time = 0.35, size = 123, normalized size = 1.41

$$\frac{2 \cos(dx+c)^4 - 7 \cos(dx+c)^3 - 33\,dx - 3\,(11\,dx+15)\cos(dx+c) - 30\cos(dx+c)^2 + \left(2\cos(dx+c)^3 - 33\,dx + 9\cos(dx+c)^2 - 21\cos(dx+c) + 24\right)\sin(dx+c) - 24}{6\,(a^3 d\cos(dx+c) + a^3 d\sin(dx+c) + a^3 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*cos(d*x + c)^4 - 7*cos(d*x + c)^3 - 33*d*x - 3*(11*d*x + 15)*cos(d*x
 + c) - 30*cos(d*x + c)^2 + (2*cos(d*x + c)^3 - 33*d*x + 9*cos(d*x + c)^2 -
 21*cos(d*x + c) + 24)*sin(d*x + c) - 24)/(a^3*d*cos(d*x + c) + a^3*d*sin(d
*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2264 vs. $2(80) = 160$.
time = 56.68, size = 2264, normalized size = 26.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-33*d*x*tan(c/2 + d*x/2)**7/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a*
*3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/
2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)*
*2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x*tan(c/2 + d*x/2)**6/(6*
a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c
/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)
**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d)
 - 99*d*x*tan(c/2 + d*x/2)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(
c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2
)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a*
*3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**4/(6*a**3*d*ta
n(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/
2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*
a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 99*d*x
*tan(c/2 + d*x/2)**3/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x
/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18
```

```
*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(
c/2 + d*x/2) + 6*a**3*d) - 99*d*x*tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d
*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 1
8*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*ta
n(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 33*d*x*tan(c/2
+ d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*
a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(
c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2)
 + 6*a**3*d) - 33*d*x/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*
x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 1
8*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan
(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2)**6/(6*a**3*d*tan(c/2 + d*x/
2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a
**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c
/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 66*tan(c/2 + d*x/2
)**5/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3
*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2
+ d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6
*a**3*d) - 192*tan(c/2 + d*x/2)**4/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d
*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 +
d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 +
 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 120*tan(c/2 + d*x/2)**3/(6*a**3*d*
tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*
x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 1
8*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 246*
tan(c/2 + d*x/2)**2/(6*a**3*d*tan(c/2 + d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/
2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*
a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c
/2 + d*x/2) + 6*a**3*d) - 38*tan(c/2 + d*x/2)/(6*a**3*d*tan(c/2 + d*x/2)**7
 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 + 18*a**3*d
*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*tan(c/2 +
d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d) - 104/(6*a**3*d*tan(c/2 +
 d*x/2)**7 + 6*a**3*d*tan(c/2 + d*x/2)**6 + 18*a**3*d*tan(c/2 + d*x/2)**5 +
 18*a**3*d*tan(c/2 + d*x/2)**4 + 18*a**3*d*tan(c/2 + d*x/2)**3 + 18*a**3*d*
tan(c/2 + d*x/2)**2 + 6*a**3*d*tan(c/2 + d*x/2) + 6*a**3*d), Ne(d, 0)), (x*
sin(c)**2*cos(c)**4/(a*sin(c) + a)**3, True))
```

Giac [A]

time = 0.48, size = 106, normalized size = 1.22

$$-\frac{\frac{33\,(dx+c)}{a^3}+\frac{48}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+28\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 + 48/(a^3*(tan(1/2*d*x + 1/2*c) + 1)) + 2*(9*tan(1/2
*d*x + 1/2*c)^5 + 24*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2*c)^2 - 9
*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [B]
time = 12.35, size = 121, normalized size = 1.39

$$-\frac{11\,x}{2\,a^3}-\frac{11\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+11\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+32\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+41\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{19\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{52}{3}}{a^3\,d\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (11*x)/(2*a^3) - ((19*tan(c/2 + (d*x)/2))/3 + 41*tan(c/2 + (d*x)/2)^2 + 2
0*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/2)^4 + 11*tan(c/2 + (d*x)/2)^5
+ 11*tan(c/2 + (d*x)/2)^6 + 52/3)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2
+ (d*x)/2)^2 + 1)^3)
```

3.434 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=80

$$\frac{9x}{2a^3}+\frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\,(a^3+a^3\sin(c+dx))}$$

```
[Out] 9/2*x/a^3+9/2*cos(d*x+c)/a^3/d+cos(d*x+c)^5/d/(a+a*sin(d*x+c))^3+3/2*cos(d*
x+c)^3/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2938, 2758, 2761, 8}

$$\frac{9\cos(c+dx)}{2a^3d}+\frac{3\cos^3(c+dx)}{2d\,(a^3\sin(c+dx)+a^3)}+\frac{9x}{2a^3}+\frac{\cos^5(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (9*x)/(2*a^3) + (9*Cos[c + d*x])/(2*a^3*d) + Cos[c + d*x]^5/(d*(a + a*Sin[c
 + d*x])^3) + (3*Cos[c + d*x]^3)/(2*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}\\ &= \frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int \frac{\cos^2(c+dx)}{a+a\sin(c+dx)}\,dx}{2a^2}\\ &= \frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}+\frac{9\int 1\,d}{2a^3}\\ &= \frac{9x}{2a^3}+\frac{9\cos(c+dx)}{2a^3d}+\frac{\cos^5(c+dx)}{d(a+a\sin(c+dx))^3}+\frac{3\cos^3(c+dx)}{2d\left(a^3+a^3\sin(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 143, normalized size = 1.79

$$\frac{180dx\cos\left(\frac{dx}{2}\right)+59\cos\left(c+\frac{dx}{2}\right)+55\cos\left(c+\frac{3dx}{2}\right)+5\cos\left(3c+\frac{5dx}{2}\right)-381\sin\left(\frac{dx}{2}\right)+180dx\sin\left(c+\frac{dx}{2}\right)+55\sin\left(2c+\frac{3dx}{2}\right)-5\sin\left(2c+\frac{5dx}{2}\right)}{40a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (180*d*x*Cos[(d*x)/2] + 59*Cos[c + (d*x)/2] + 55*Cos[c + (3*d*x)/2] + 5*Cos
[3*c + (5*d*x)/2] - 381*Sin[(d*x)/2] + 180*d*x*Sin[c + (d*x)/2] + 55*Sin[2*
c + (3*d*x)/2] - 5*Sin[2*c + (5*d*x)/2])/(40*a^3*d*(Cos[c/2] + Sin[c/2])*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.27, size = 92, normalized size = 1.15

method	result
risch	$\frac{9x}{2a^3}+\frac{3\,e^{i(dx+c)}}{2d\,a^3}+\frac{3\,e^{-i(dx+c)}}{2d\,a^3}+\frac{8}{d\,a^3\left(e^{i(dx+c)}+i\right)}-\frac{\sin(2dx+2c)}{4d\,a^3}$

derivativedivides	$\frac{\frac{4\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+9\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
default	$\frac{\frac{4\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+9\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
norman	$\frac{45x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{533\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{9\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{9x}{2a}+\frac{14}{ad}+\frac{61\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{315x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{135x\left(\tan^{13}\right.}{2\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*((1/4*tan(1/2*d*x+1/2*c)^3+3/2*tan(1/2*d*x+1/2*c)^2-1/4*tan(1/2*d*x
+1/2*c)+3/2)/(1+tan(1/2*d*x+1/2*c)^2)^2+9/4*arctan(tan(1/2*d*x+1/2*c))+2/(t
an(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(74) = 148$.
time = 0.50, size = 225, normalized size = 2.81

$$\frac{\frac{\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+14}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((5*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 14)/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.36, size = 100, normalized size = 1.25

$$\frac{\cos(dx+c)^3+9\,dx+(9\,dx+13)\cos(dx+c)+6\cos(dx+c)^2+\left(9\,dx-\cos(dx+c)^2+5\cos(dx+c)-8\right)\sin(dx+c)+8}{2\left(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(d*x + c)^3 + 9*d*x + (9*d*x + 13)*cos(d*x + c) + 6*cos(d*x + c)^2
+ (9*d*x - cos(d*x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) + 8)/(a^3*d*co
s(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(71) = 142$.
time = 32.65, size = 1244, normalized size = 15.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*d*x*tan(c/2 + d*x/2)**5/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3
*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 +
d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 9*d*x*tan(c/2 + d*x/2)*
*4/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*
tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x
/2) + 2*a**3*d) + 18*d*x*tan(c/2 + d*x/2)**3/(2*a**3*d*tan(c/2 + d*x/2)**5
+ 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*ta
n(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 18*d*x*tan(c/2
+ d*x/2)**2/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 +
4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(
c/2 + d*x/2) + 2*a**3*d) + 9*d*x*tan(c/2 + d*x/2)/(2*a**3*d*tan(c/2 + d*x/2
)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3
*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 9*d*x/(2*a
**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2
*a**3*d) + 18*tan(c/2 + d*x/2)**4/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x
/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 14*tan(c/2 + d*x/2)**3/(2*
a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/
2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) +
2*a**3*d) + 42*tan(c/2 + d*x/2)**2/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d
*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*
x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*a**3*d) + 10*tan(c/2 + d*x/2)/(2*a*
*3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
+ d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan(c/2 + d*x/2) + 2*
a**3*d) + 28/(2*a**3*d*tan(c/2 + d*x/2)**5 + 2*a**3*d*tan(c/2 + d*x/2)**4 +
 4*a**3*d*tan(c/2 + d*x/2)**3 + 4*a**3*d*tan(c/2 + d*x/2)**2 + 2*a**3*d*tan
(c/2 + d*x/2) + 2*a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**4/(a*sin(c) + a)**3
, True))
```

Giac [A]

time = 0.49, size = 91, normalized size = 1.14

$$\frac{\frac{9\,(dx+c)}{a^3}+\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{16}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*(d*x + c)/a^3 + 2*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 16/(a^3
*(tan(1/2*d*x + 1/2*c) + 1)))/d
```

Mupad [B]
time = 10.80, size = 94, normalized size = 1.18

$$\frac{9\,x}{2\,a^3}+\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+14}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (9*x)/(2*a^3) + (5*tan(c/2 + (d*x)/2) + 21*tan(c/2 + (d*x)/2)^2 + 7*tan(c/2
 + (d*x)/2)^3 + 9*tan(c/2 + (d*x)/2)^4 + 14)/(a^3*d*(tan(c/2 + (d*x)/2) + 1
)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.435 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

```
[Out] x/a^3-arctanh(cos(d*x+c))/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {2954, 2951, 3855, 2727}

$$\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 - ArcTanh[Cos[c + d*x]]/(a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c
+ d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(a+a\csc(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}\\ &= \frac{x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3}\\ &= \frac{x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 122 vs. 2(45) = 90.
time = 0.21, size = 122, normalized size = 2.71

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(c+dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(-8+c+dx-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2]*(c + d*x - Log[C
os[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + (-8 + c + d*x - Log[Cos[(c + d*
x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[(c + d*x)/2]))/(a^3*d*(1 + Sin[c + d*x]
)^3)
```

Maple [A]
time = 0.30, size = 46, normalized size = 1.02

method	result
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^3}$
risch	$\frac{x}{a^3}+\frac{8}{d\,a^3\left(e^{i(dx+c)}+i\right)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{d\,a^3}$

norman	$\frac{x}{a}+\frac{x\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{8}{ad}+\frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{32\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{72\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{72\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(ln(tan(1/2*d*x+1/2*c))+2*arctan(tan(1/2*d*x+1/2*c))+8/(tan(1/2*d*x
+1/2*c)+1))
```

Maxima [A]
time = 0.50, size = 78, normalized size = 1.73

$$\frac{\frac{8}{a^3+\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (8/(a^3 + a^3*sin(d*x + c)/(cos(d*x + c) + 1)) + 2*arctan(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(45) = 90$.
time = 0.37, size = 117, normalized size = 2.60

$$\frac{2\,dx+2\,(dx+4)\cos(dx+c)-(\cos(dx+c)+\sin(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(\cos(dx+c)+\sin(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\,(dx-4)\sin(dx+c)+8}{2\,(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*d*x + 2*(d*x + 4)*cos(d*x + c) - (cos(d*x + c) + sin(d*x + c) + 1)*l
og(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c) + sin(d*x + c) + 1)*log(-1/2*cos
(d*x + c) + 1/2) + 2*(d*x - 4)*sin(d*x + c) + 8)/(a^3*d*cos(d*x + c) + a^3*
d*sin(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\cos^4(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.46, size = 47, normalized size = 1.04

$$\frac{\frac{dx+c}{a^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{8}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 8/(a^3*(tan(1/2*d*x +
 1/2*c) + 1)))/d
```

Mupad [B]
time = 8.73, size = 115, normalized size = 2.56

$$\frac{8}{d\,\left(a^3+a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{2\,\mathrm{atan}\left(\frac{4\,a^3}{4\,a^3-4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3-4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] 8/(d*(a^3 + a^3*tan(c/2 + (d*x)/2))) + log(tan(c/2 + (d*x)/2))/(a^3*d) + (2
*atan((4*a^3)/(4*a^3 - 4*a^3*tan(c/2 + (d*x)/2)) + (4*a^3*tan(c/2 + (d*x)/2
))/(4*a^3 - 4*a^3*tan(c/2 + (d*x)/2))))/(a^3*d)
```

3.436 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=54

$$\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-4*cos(d*x+c)/a^3/d/(1+sin(d*x+
c))
```

Rubi [A]
time = 0.16, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.207$, Rules used = {2954, 2951, 3855, 3852, 8, 2727}

$$-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (4*Cos[c + d*x])
/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-3a\csc(c+dx)+a\csc^2(c+dx)+\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4} \\
&= \frac{\int \csc^2(c+dx)\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{1+\sin(c+dx)}\,dx}{a^3} \\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+
\end{aligned}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 156 vs. 2(54) = 108.
time = 0.48, size = 156, normalized size = 2.89

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(-17+\cot^2\left(\frac{1}{2}(c+dx)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cot\left(\frac{1}{2}(c+dx)\right)\left(1-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\tan\left(\frac{1}{2}(c+dx)\right)}{2a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((Cos[(c + d*x)/2]*(-17 + Cot[(c + d*x)/2]^2 - 6*Log[Cos[(c + d*x)/2]]
 + 6*Log[Sin[(c + d*x)/2]] + Cot[(c + d*x)/2]*(1 - 6*Log[Cos[(c + d*x)/2]]
+ 6*Log[Sin[(c + d*x)/2]])) - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^5*Tan[(c + d*x)/2])/(a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.32, size = 59, normalized size = 1.09

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{2d\,a^3}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{2d\,a^3}$
risch	$-\frac{2\left(-5+i\mathrm{e}^{i(dx+c)}+4\,\mathrm{e}^{2i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)a^3d}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d\,a^3}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{153\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{107\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{50\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{15\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\left(\tan\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2*c))
-16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(54) = 108$.
time = 0.30, size = 116, normalized size = 2.15

$$-\frac{\frac{\frac{17\sin(dx+c)}{\cos(dx+c)+1}+1}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*((17*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/(a^3*sin(d*x + c)/(cos(d*x +
 c) + 1) + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) + 6*log(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 - sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(54) = 108$.
time = 0.35, size = 165, normalized size = 3.06

$$\frac{10\cos(dx+c)^2+3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(5\cos(dx+c)+4\right)\sin(dx+c)+2\cos(dx+c)-8}{2\left(a^3d\cos(dx+c)^2-a^3d-\left(a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c
) - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)
*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(5*cos(d*x + c) + 4)*si
n(d*x + c) + 2*cos(d*x + c) - 8)/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos
(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.51, size = 90, normalized size = 1.67

$$-\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-14\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - tan(1/2*d*x + 1/2*c)/a^3 - (3*
tan(1/2*d*x + 1/2*c)^2 - 14*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))*a^3))/d
```

Mupad [B]
time = 8.67, size = 87, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (17*tan(c/2 + (d*x)/2) + 1)/(d*(2*a^3*tan(c/
2 + (d*x)/2)^2 + 2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(
a^3*d)
```

3.437 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=78

$$-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

```
[Out] -9/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c)
/a^3/d+4*cos(d*x+c)/a^3/d/(1+sin(d*x+c))
```

Rubi [A]

time = 0.17, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2727}

$$\frac{3\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c +
d*x]*Csc[c + d*x])/(2*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^3(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(4a\csc(c+dx)-3a\csc^2(c+dx)+a\csc^3(c+dx)-\frac{4a}{1+\sin(c+dx)}\right)dx}{a^4}$$

$$= \frac{\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}+\frac{4\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \frac{1}{1+\sin(c+}}{a^3}$$

$$= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4\cos(c+dx)}{a^3d(1+\sin(c+dx))}$$

$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{4}{a^3d(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(78) = 156.
time = 4.41, size = 213, normalized size = 2.73

$$-\frac{\left(\csc^2\left(\frac{1}{2}(c+dx)\right)+2\csc(c+dx)\right)^5\left(\csc^6\left(\frac{1}{2}(c+dx)\right)(-6+\csc(c+dx))-8(-6+\csc(c+dx))\csc^3(c+dx)+2\csc^4\left(\frac{1}{2}(c+dx)\right)\csc(c+dx)\left(-6+\csc(c+dx)+18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-4\csc^2\left(\frac{1}{2}(c+dx)\right)\csc^2(c+dx)\left(-38+\csc(c+dx)-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)\sin^8\left(\frac{1}{2}(c+dx)\right)\sin^7(c+dx)}{512a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/512*((Csc[(c + d*x)/2]^2 + 2*Csc[c + d*x])^5*(Csc[(c + d*x)/2]^6*(-6 + C
sc[c + d*x]) - 8*(-6 + Csc[c + d*x])*Csc[c + d*x]^3 + 2*Csc[(c + d*x)/2]^4*
Csc[c + d*x]*(-6 + Csc[c + d*x] + 18*Log[Cos[(c + d*x)/2]] - 18*Log[Sin[(c
+ d*x)/2]]) - 4*Csc[(c + d*x)/2]^2*Csc[c + d*x]^2*(-38 + Csc[c + d*x] - 18*
Log[Cos[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)/2]]))*Sin[(c + d*x)/2]^8*Sin[c
 + d*x]^7)/(a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 87, normalized size = 1.12

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+18\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{32}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+18\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{32}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{4d\,a^3}$
risch	$\frac{9\,e^{4i(dx+c)}-21\,e^{2i(dx+c)}+7i e^{3i(dx+c)}+14-5i e^{i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}+i\right)a^3 d}-\frac{9\ln\left(e^{i(dx+c)}+1\right)}{2d\,a^3}+\frac{9\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^3}$
norman	$\frac{-\frac{1}{8ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{81\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{51\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{303\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a^2\left(\tan\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-6*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/
2*c)^2+6/tan(1/2*d*x+1/2*c)+18*ln(tan(1/2*d*x+1/2*c))+32/(tan(1/2*d*x+1/2*c
)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(74) = 148.
time = 0.29, size = 161, normalized size = 2.06

$$\frac{\frac{\frac{11\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{36\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((11*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 1)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^3/(
```

```
cos(d*x + c) + 1)^3) - (12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)/a^3 + 36*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(74) = 148$.
time = 0.36, size = 246, normalized size = 3.15

$$\frac{28\cos(dx+c)^3+18\cos(dx+c)^2-9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(14\cos(dx+c)^2+5\cos(dx+c)-8\right)\sin(dx+c)-26\cos(dx+c)-16}{4\left(a^3d\cos(dx+c)^3+a^3d\cos(dx+c)^2-a^3d\cos(dx+c)-a^3d+\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*cos(d*x + c)^3 + 18*cos(d*x + c)^2 - 9*(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*log(1/2*cos(d*
x + c) + 1/2) + 9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*s
in(d*x + c) - cos(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2*(14*cos(d*
x + c)^2 + 5*cos(d*x + c) - 8)*sin(d*x + c) - 26*cos(d*x + c) - 16)/(a^3*d*
cos(d*x + c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d
*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.48, size = 116, normalized size = 1.49

$$\frac{\frac{36\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{64}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{54\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(36*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 64/(a^3*(tan(1/2*d*x + 1/2*c)
+ 1)) - (54*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(
1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/
2*c))/a^6)/d
```

Mupad [B]
time = 8.69, size = 120, normalized size = 1.54

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{9\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{2}}{d\,\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (9*log(tan(c/2 + (d*x)/2)))/(2*a^3*d) + ((
11*tan(c/2 + (d*x)/2))/2 + 38*tan(c/2 + (d*x)/2)^2 - 1/2)/(d*(4*a^3*tan(c/2
 + (d*x)/2)^2 + 4*a^3*tan(c/2 + (d*x)/2)^3)) - (3*tan(c/2 + (d*x)/2))/(2*a^
3*d)
```

3.438 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{5\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(1+\csc(c+dx))}$$

```
[Out] 11/2*arctanh(cos(d*x+c))/a^3/d-13*cot(d*x+c)/a^3/d-13/3*cot(d*x+c)^3/a^3/d+
11/2*cot(d*x+c)*csc(d*x+c)/a^3/d+4*cot(d*x+c)*csc(d*x+c)^2/a^3/d/(1+sin(d*x
+c))
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2788, 3855, 3852, 8, 3853, 3862}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{5\cot(c+dx)}{a^3d}+\frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{4\cot(c+dx)}{a^3d(\csc(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (11*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (5*Cot[c + d*x])/(a^3*d) - Cot[c + d
*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (4*Cot[c + d*x]
)/(a^3*d*(1 + Csc[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \Big(4a - 4a\csc(c+dx) + 4a\csc^2(c+dx) - 3a\csc^3(c+dx) + a\csc^4(c+dx) -}{a^4}$$
$$= \frac{4x}{a^3} + \frac{\int \csc^4(c+dx)\,dx}{a^3} - \frac{3\int \csc^3(c+dx)\,dx}{a^3} - \frac{4\int \csc(c+dx)\,dx}{a^3} + \frac{4\int \csc^2}{}$$
$$= \frac{4x}{a^3} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{3\cot(c+dx)\csc(c+dx)}{2a^3d} - \frac{4\cot(c+dx)}{a^3d(1+\csc(c+dx}$$
$$= \frac{11\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{5\cot(c+dx)}{a^3d} - \frac{\cot^3(c+dx)}{3a^3d} + \frac{3\cot(c+dx)\csc(c}{2a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 251 vs. 2(96) = 192.
time = 3.93, size = 251, normalized size = 2.61

$$-\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^5\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)\left(-4\sin^8\left(\frac{1}{2}(c+dx)\right)-8\sin^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)(-2+7\sin(c+dx))+\frac{1}{4}\sin^4(c+dx)\left(-8+\cot\left(\frac{1}{2}(c+dx)\right)+28\sin(c+dx)\right)-\frac{1}{2}\sin^2\left(\frac{1}{2}(c+dx)\right)\sin^3(c+dx)\left(9+\left(-28+66\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-66\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)+\sin^4\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)\left(9-2\left(62+33\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-33\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)\right)}{12a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/12*((1 + Cot[(c + d*x)/2])^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2*(-4*Sin[(
c + d*x)/2]^8 - 8*Sin[(c + d*x)/2]^6*Sin[c + d*x]*(-2 + 7*Sin[c + d*x]) + (
Sin[c + d*x]^4*(-8 + Cot[(c + d*x)/2] + 28*Sin[c + d*x]))/4 - (Sin[(c + d*x
```

```
)/2]^2*Sin[c + d*x]^3*(9 + (-28 + 66*Log[Cos[(c + d*x)/2]] - 66*Log[Sin[(c
+ d*x)/2]])*Sin[c + d*x]))/2 + Sin[(c + d*x)/2]^4*Sin[c + d*x]^2*(9 - 2*(62
 + 33*Log[Cos[(c + d*x)/2]] - 33*Log[Sin[(c + d*x)/2]])*Sin[c + d*x])))/(a^
3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]

time = 0.33, size = 113, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{19}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-44\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^3}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+19\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{19}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-44\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^3}$
risch	$-\frac{33i e^{5i(dx+c)}-96\,e^{4i(dx+c)}+33\,e^{6i(dx+c)}-60i e^{3i(dx+c)}+123\,e^{2i(dx+c)}+19i e^{i(dx+c)}-52}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)a^3d}+\frac{11\ln\left(e^{i(dx+c)}+1\right)}{2d\,a^3}-\frac{11\ln}{}$
norman	$\frac{-\frac{107\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{11\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{301(\tan}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)^2+19*tan(1/2*d*x+1
/2*c)-1/3/tan(1/2*d*x+1/2*c)^3+3/tan(1/2*d*x+1/2*c)^2-19/tan(1/2*d*x+1/2*c)
-44*ln(tan(1/2*d*x+1/2*c))-64/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(98) = 196$.

time = 0.29, size = 199, normalized size = 2.07

$$\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{249\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{57\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{132\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^3*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (57*sin(d*
x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 132*log(sin(d*x + c)/(cos(d*x + c) + 1)
)/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(98) = 196$.

time = 0.36, size = 302, normalized size = 3.15

$$\frac{104 \cos (dx+c)^4+38 \cos (dx+c)^3-156 \cos (dx+c)^2+33 \left(\cos (dx+c)^4-2 \cos (dx+c)^2-\left(\cos (dx+c)^3+\cos (dx+c)^2-\cos (dx+c)-1\right) \sin (dx+c)+1\right) \log \left(\frac{1}{2} \cos (dx+c)+\frac{1}{2}\right)-33 \left(\cos (dx+c)^4-2 \cos (dx+c)^2-\left(\cos (dx+c)^3+\cos (dx+c)^2-\cos (dx+c)-1\right) \sin (dx+c)+1\right) \log \left(-\frac{1}{2} \cos (dx+c)+\frac{1}{2}\right)+2 \left(52 \cos (dx+c)^3+33 \cos (dx+c)^2-45 \cos (dx+c)-24\right) \sin (dx+c)-42 \cos (dx+c)+48}{12 \left(a^3 d \cos (dx+c)^4-2 a^3 d \cos (dx+c)^2+a^3 d-\left(a^3 d \cos (dx+c)^3+a^3 d \cos (dx+c)^2-a^3 d \cos (dx+c)-a^3 d\right) \sin (dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(104*cos(d*x + c)^4 + 38*cos(d*x + c)^3 - 156*cos(d*x + c)^2 + 33*(cos
(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*sin(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 33*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) - 1
)*sin(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(52*cos(d*x + c)^3 + 3
3*cos(d*x + c)^2 - 45*cos(d*x + c) - 24)*sin(d*x + c) - 42*cos(d*x + c) + 4
8)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x
+ c)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.51, size = 146, normalized size = 1.52

$$-\frac{\frac{132 \log \left(\left|\tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3}+\frac{192}{a^3 \left(\tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}-\frac{242 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-57 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+9 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)-1}{a^3 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^3}-\frac{a^6 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-9 a^6 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+57 a^6 \tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^9}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(132*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 192/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)) - (242*tan(1/2*d*x + 1/2*c)^3 - 57*tan(1/2*d*x + 1/2*c)^2 + 9*tan
(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/
2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 57*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

Mupad [B]

time = 8.69, size = 153, normalized size = 1.59

$$\frac{\tan \left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24 a^3 d}-\frac{3 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8 a^3 d}-\frac{11 \ln \left(\tan \left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2 a^3 d}-\frac{83 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)^3+16 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{8 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{1}{3}}{d \left(8 a^3 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)^4+8 a^3 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}+\frac{19 \tan \left(\frac{c}{2}+\frac{dx}{2}\right)}{8 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - (11*
log(tan(c/2 + (d*x)/2)))/(2*a^3*d) - (16*tan(c/2 + (d*x)/2)^2 - (8*tan(c/2
+ (d*x)/2))/3 + 83*tan(c/2 + (d*x)/2)^3 + 1/3)/(d*(8*a^3*tan(c/2 + (d*x)/2)
^3 + 8*a^3*tan(c/2 + (d*x)/2)^4)) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d)
```

3.439 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{19\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}$$

```
[Out] -51/8*arctanh(cos(d*x+c))/a^3/d+7*cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-19/8*
cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d+4*cos(d*x+c)/
a^3/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2727}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{4\cos(c+dx)}{a^3d(\sin(c+dx)+1)}-\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{19\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-51*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (7*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (19*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d) + (4*Cos[c + d*x])/(a^3*d*(1 + Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^5(c+dx)\sec^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \Big(4a\csc(c+dx)-4a\csc^2(c+dx)+4a\csc^3(c+dx)-3a\csc^4(c+dx)+}{a^4} \\
&= \frac{\int \csc^5(c+dx)\,dx}{a^3}-\frac{3\int \csc^4(c+dx)\,dx}{a^3}+\frac{4\int \csc(c+dx)\,dx}{a^3}-\frac{4\int \csc^2(c}{a^3} \\
&= -\frac{4\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc^3(c+}{4a^3d} \\
&= -\frac{6\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{19\cot(c+dx)\,c}{8a^3d} \\
&= -\frac{51\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{7\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{19\cot(c+dx)}{8a^3}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 601 vs. 2(117) = 234.

time = 6.14, size = 601, normalized size = 5.14

$$-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{d(a+a\sin(c+dx))^3}+\frac{3\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{d(a+a\sin(c+dx))^3}-\frac{19\csc^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{32d(a+a\sin(c+dx))^3}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{8d(a+a\sin(c+dx))^3}-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{64d(a+a\sin(c+dx))^3}-\frac{51\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{8d(a+a\sin(c+dx))^3}+\frac{51\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{8d(a+a\sin(c+dx))^3}+\frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{32d(a+a\sin(c+dx))^3}+\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{64d(a+a\sin(c+dx))^3}-\frac{3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\tan\left(\frac{1}{2}(c+dx)\right)}{d(a+a\sin(c+dx))^3}-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\tan\left(\frac{1}{2}(c+dx)\right)}{8d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a + a*Sin
[c + d*x])^3) + (3*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6
)/(d*(a + a*Sin[c + d*x])^3) - (19*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^6)/(32*d*(a + a*Sin[c + d*x])^3) + (Cot[(c + d*x)/2]*Csc[(
c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c +
d*x])^3) - (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(64
*d*(a + a*Sin[c + d*x])^3) - (51*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) + (51*Log[Sin[(c + d*x)/2
]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(8*d*(a + a*Sin[c + d*x])^3) +
(19*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(32*d*(a +
a*Sin[c + d*x])^3) + (Sec[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^6)/(64*d*(a + a*Sin[c + d*x])^3) - (3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6*Tan[(c + d*x)/2])/(d*(a + a*Sin[c + d*x])^3) - (Sec[(c + d*x)/2]^2*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*Tan[(c + d*x)/2])/(8*d*(a + a*Sin[c
+ d*x])^3)
```

Maple [A]

time = 0.38, size = 139, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{16d\,a^3}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-50\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{10}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{16d\,a^3}$
risch	$\frac{51\,\mathrm{e}^{8i(dx+c)}-187\,\mathrm{e}^{6i(dx+c)}+51i\mathrm{e}^{7i(dx+c)}+309\,\mathrm{e}^{4i(dx+c)}-171i\mathrm{e}^{5i(dx+c)}-269\,\mathrm{e}^{2i(dx+c)}+133i\mathrm{e}^{3i(dx+c)}+80-29i\mathrm{e}^{i(dx+c)}}{4\left(\mathrm{e}^{2i(dx+c)}-1\right)^4\left(\mathrm{e}^{i(dx+c)}+i\right)a^3d}$
norman	$\frac{-\frac{1}{64ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{35\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{35\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^3*(1/4*tan(1/2*d*x+1/2*c)^4-2*tan(1/2*d*x+1/2*c)^3+10*tan(1/2*d*x+
1/2*c)^2-50*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4+2/tan(1/2*d*x+1/2*c
)^3-10/tan(1/2*d*x+1/2*c)^2+50/tan(1/2*d*x+1/2*c)+102*ln(tan(1/2*d*x+1/2*c)
)+128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(111) = 222$.
time = 0.29, size = 241, normalized size = 2.06

$$\frac{\frac{\frac{7 \sin(dx+c)}{\cos(dx+c)+1} - \frac{32 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{160 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{712 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - 1}{\frac{a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{\frac{200 \sin(dx+c)}{\cos(dx+c)+1} - \frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{8 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3} + \frac{408 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{64\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/64*((7*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 712*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5) - (200*sin(d*x + c)/(cos(d*x + c) + 1) - 40*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^3 + 408*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(111) = 222$.
time = 0.36, size = 381, normalized size = 3.26

$$\frac{160 \cos(dx+c)^5 + 102 \cos(dx+c)^4 - 298 \cos(dx+c)^3 - 170 \cos(dx+c)^2 - 51 \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2 \cos(dx+c)^3 - 2 \cos(dx+c)^2 + \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \sin(dx+c) + \cos(dx+c) + 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 51 \left(\cos(dx+c)^5 + \cos(dx+c)^4 - 2 \cos(dx+c)^3 - 2 \cos(dx+c)^2 + \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \sin(dx+c) + \cos(dx+c) + 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 2 \left(80 \cos(dx+c)^4 + 29 \cos(dx+c)^3 - 120 \cos(dx+c)^2 - 35 \cos(dx+c) + 32\right) \sin(dx+c) + 134 \cos(dx+c) + 64}{16 \left(a^3 d \cos(dx+c)^5 + a^3 d \cos(dx+c)^4 - 2 a^3 d \cos(dx+c)^3 - 2 a^3 d \cos(dx+c)^2 + a^3 d \cos(dx+c) + a^3 d + \left(a^3 d \cos(dx+c)^4 - 2 a^3 d \cos(dx+c)^2 + a^3 d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(160*cos(d*x + c)^5 + 102*cos(d*x + c)^4 - 298*cos(d*x + c)^3 - 170*co
s(d*x + c)^2 - 51*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*c
os(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(
d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 51*(cos(d*x + c)^5 + cos(d*x +
c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x +
c)^2 + 1)*sin(d*x + c) + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) - 2
*(80*cos(d*x + c)^4 + 29*cos(d*x + c)^3 - 120*cos(d*x + c)^2 - 35*cos(d*x +
 c) + 32)*sin(d*x + c) + 134*cos(d*x + c) + 64)/(a^3*d*cos(d*x + c)^5 + a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3*d
*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^
3*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.55, size = 174, normalized size = 1.49

$$\frac{\frac{408 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3}+\frac{512}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}-\frac{850 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-200 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+40 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4}+\frac{a^9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-8 a^9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+40 a^9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-200 a^9 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{12}}}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(408*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 512/(a^3*(tan(1/2*d*x + 1/2*
c) + 1)) - (850*tan(1/2*d*x + 1/2*c)^4 - 200*tan(1/2*d*x + 1/2*c)^3 + 40*ta
n(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c
)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 40*a^9*
tan(1/2*d*x + 1/2*c)^2 - 200*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

Mupad [B]
time = 9.13, size = 176, normalized size = 1.50

$$\frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^2}{8 a^3 d}-\frac{\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{8 a^3 d}+\frac{\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^4}{64 a^3 d}+\frac{51 \ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)\right)}{8 a^3 d}-\frac{25 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{8 a^3 d}+\frac{\cot\left(\frac{c}{2}+\frac{d x}{2}\right)^4 \left(\frac{89 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^4}{8}+\frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^2}{2}+\frac{7 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{64}-\frac{1}{64}\right)}{a^3 d \left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(8*a^3*d) + tan(c
/2 + (d*x)/2)^4/(64*a^3*d) + (51*log(tan(c/2 + (d*x)/2)))/(8*a^3*d) - (25*t
an(c/2 + (d*x)/2))/(8*a^3*d) + (cot(c/2 + (d*x)/2)^4*((7*tan(c/2 + (d*x)/2)
)/64 - tan(c/2 + (d*x)/2)^2/2 + (5*tan(c/2 + (d*x)/2)^3)/2 + (89*tan(c/2 +
(d*x)/2)^4)/8 - 1/64))/(a^3*d*(tan(c/2 + (d*x)/2) + 1))
```

3.440 $\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx$

Optimal. Leaf size=58

$$\frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6}-\frac{6\cos^5(e+fx)}{35af(a+a\sin(e+fx))^5}$$

```
[Out] 1/7*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^6-6/35*cos(f*x+e)^5/a/f/(a+a*sin(f*x+e)
)^5
```

Rubi [A]
time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2938, 2750}

$$\frac{\cos^5(e+fx)}{7f(a\sin(e+fx)+a)^6}-\frac{6\cos^5(e+fx)}{35af(a\sin(e+fx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
```

```
[Out] Cos[e + f*x]^5/(7*f*(a + a*Sin[e + f*x])^6) - (6*Cos[e + f*x]^5)/(35*a*f*(a
 + a*Sin[e + f*x])^5)
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin(e+fx)}{(a+a\sin(e+fx))^6}\,dx = \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6} + \frac{6\int \frac{\cos^4(e+fx)}{(a+a\sin(e+fx))^5}\,dx}{7a}$$
$$= \frac{\cos^5(e+fx)}{7f(a+a\sin(e+fx))^6} - \frac{6\cos^5(e+fx)}{35af(a+a\sin(e+fx))^5}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(58) = 116$.
time = 0.89, size = 143, normalized size = 2.47

$$\frac{4585\cos\left(e+\frac{fx}{2}\right) - 2982\cos\left(e+\frac{3fx}{2}\right) - 1148\cos\left(3e+\frac{5fx}{2}\right) + 197\cos\left(3e+\frac{7fx}{2}\right) + 2275\sin\left(\frac{fx}{2}\right) + 1134\sin\left(2e+\frac{3fx}{2}\right) - 224\sin\left(2e+\frac{5fx}{2}\right) + \sin\left(4e+\frac{7fx}{2}\right)}{4620a^6f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x])/(a + a*Sin[e + f*x])^6,x]
[Out] (4585*Cos[e + (f*x)/2] - 2982*Cos[e + (3*f*x)/2] - 1148*Cos[3*e + (5*f*x)/2
] + 197*Cos[3*e + (7*f*x)/2] + 2275*Sin[(f*x)/2] + 1134*Sin[2*e + (3*f*x)/2
] - 224*Sin[2*e + (5*f*x)/2] + Sin[4*e + (7*f*x)/2])/(4620*a^6*f*(Cos[e/2]
+ Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)
```

Maple [A]
time = 0.23, size = 100, normalized size = 1.72

method	result
risch	$-\frac{2\left(35i\,e^{5i(fx+e)}+35\,e^{6i(fx+e)}-70i\,e^{3i(fx+e)}-140\,e^{4i(fx+e)}+7i\,e^{i(fx+e)}+91\,e^{2i(fx+e)}-6\right)}{35f\,a^6\left(e^{i(fx+e)}+i\right)^7}$
derivativedivides	$\frac{-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^6}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{224}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}}{f\,a^6}$
default	$\frac{-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^6}-\frac{32}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}+\frac{12}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}+\frac{224}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5}+\frac{64}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right.}}{f\,a^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x,method=_RETURNVERBOSE)
[Out] 4/f/a^6*(-8/(tan(1/2*f*x+1/2*e)+1)^6-8/(tan(1/2*f*x+1/2*e)+1)^4+3/(tan(1/2*
f*x+1/2*e)+1)^3+56/5/(tan(1/2*f*x+1/2*e)+1)^5+16/7/(tan(1/2*f*x+1/2*e)+1)^7
-1/2/(tan(1/2*f*x+1/2*e)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(54) = 108$.
time = 0.29, size = 269, normalized size = 4.64

$$-\frac{2\left(\frac{7\sin(fx+e)}{\cos(fx+e)+1} - \frac{14\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{70\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{35\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{35\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + 1\right)}{35\left(a^6 + \frac{7a^6\sin(fx+e)}{\cos(fx+e)+1} + \frac{21a^6\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{35a^6\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{35a^6\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{21a^6\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{7a^6\sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{a^6\sin(fx+e)^7}{(\cos(fx+e)+1)^7}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="maxima")

[Out] -2/35*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 14*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 70*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 35*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1)/((a^6 + 7*a^6*sin(f*x + e)/(cos(f*x + e) + 1) + 21*a^6*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*a^6*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*a^6*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 21*a^6*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*a^6*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + a^6*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)*f)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(58) = 116$.
time = 0.34, size = 211, normalized size = 3.64

$$\frac{6\cos\left(fx+e\right)^{4}-11\cos\left(fx+e\right)^{3}-27\cos\left(fx+e\right)^{2}+\left(6\cos\left(fx+e\right)^{3}+17\cos\left(fx+e\right)^{2}-10\cos\left(fx+e\right)-20\right)\sin\left(fx+e\right)+10\cos\left(fx+e\right)+20}{35\left(a^{6}f\cos\left(fx+e\right)^{4}-3a^{6}f\cos\left(fx+e\right)^{3}-8a^{6}f\cos\left(fx+e\right)^{2}+4a^{6}f\cos\left(fx+e\right)+8a^{6}f-\left(a^{6}f\cos\left(fx+e\right)^{3}+4a^{6}f\cos\left(fx+e\right)^{2}-4a^{6}f\cos\left(fx+e\right)-8a^{6}f\right)\sin\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="fricas")

[Out] 1/35*(6*cos(f*x + e)^4 - 11*cos(f*x + e)^3 - 27*cos(f*x + e)^2 + (6*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - 10*cos(f*x + e) - 20)*sin(f*x + e) + 10*cos(f*x + e) + 20)/(a^6*f*cos(f*x + e)^4 - 3*a^6*f*cos(f*x + e)^3 - 8*a^6*f*cos(f*x + e)^2 + 4*a^6*f*cos(f*x + e) + 8*a^6*f - (a^6*f*cos(f*x + e)^3 + 4*a^6*f*cos(f*x + e)^2 - 4*a^6*f*cos(f*x + e) - 8*a^6*f)*sin(f*x + e))

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 901 vs. $2(49) = 98$.
time = 131.46, size = 901, normalized size = 15.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**4*sin(f*x+e)/(a+a*sin(f*x+e))**6,x)

[Out] Piecewise((-70*tan(e/2 + f*x/2)**5/(35*a**6*f*tan(e/2 + f*x/2)**7 + 245*a**6*f*tan(e/2 + f*x/2)**6 + 735*a**6*f*tan(e/2 + f*x/2)**5 + 1225*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**6*f*tan(e/2 + f*x/2)**3 + 735*a**6*f*tan(e/2 + f*x/2)**2 + 245*a**6*f*tan(e/2 + f*x/2) + 35*a**6*f) + 70*tan(e/2 + f*x/2)**4/(35*a**6*f*tan(e/2 + f*x/2)**7 + 245*a**6*f*tan(e/2 + f*x/2)**6 + 735*a**6*f*tan(e/2 + f*x/2)**5 + 1225*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**6*f*tan(e/2 + f*x/2)**3 + 735*a**6*f*tan(e/2 + f*x/2)**2 + 245*a**6*f*tan(e/2 + f*x/2) + 35*a**6*f) - 140*tan(e/2 + f*x/2)**3/(35*a**6*f*tan(e/2 + f*x/2)**7 + 245*a**6*f*tan(e/2 + f*x/2)**6 + 735*a**6*f*tan(e/2 + f*x/2)**5 + 1225*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**6*f*tan(e/2 + f*x/2)**3 + 735*a**6*f*tan(

```
e/2 + f*x/2)**2 + 245*a**6*f*tan(e/2 + f*x/2) + 35*a**6*f) + 28*tan(e/2 + f
*x/2)**2/(35*a**6*f*tan(e/2 + f*x/2)**7 + 245*a**6*f*tan(e/2 + f*x/2)**6 +
735*a**6*f*tan(e/2 + f*x/2)**5 + 1225*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**
6*f*tan(e/2 + f*x/2)**3 + 735*a**6*f*tan(e/2 + f*x/2)**2 + 245*a**6*f*tan(e
/2 + f*x/2) + 35*a**6*f) - 14*tan(e/2 + f*x/2)/(35*a**6*f*tan(e/2 + f*x/2)*
*7 + 245*a**6*f*tan(e/2 + f*x/2)**6 + 735*a**6*f*tan(e/2 + f*x/2)**5 + 1225
*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**6*f*tan(e/2 + f*x/2)**3 + 735*a**6*f*
tan(e/2 + f*x/2)**2 + 245*a**6*f*tan(e/2 + f*x/2) + 35*a**6*f) - 2/(35*a**6
*f*tan(e/2 + f*x/2)**7 + 245*a**6*f*tan(e/2 + f*x/2)**6 + 735*a**6*f*tan(e/
2 + f*x/2)**5 + 1225*a**6*f*tan(e/2 + f*x/2)**4 + 1225*a**6*f*tan(e/2 + f*x
/2)**3 + 735*a**6*f*tan(e/2 + f*x/2)**2 + 245*a**6*f*tan(e/2 + f*x/2) + 35*
a**6*f), Ne(f, 0)), (x*sin(e)*cos(e)**4/(a*sin(e) + a)**6, True))
```

Giac [A]
time = 0.53, size = 92, normalized size = 1.59

$$-\frac{2 \left(35 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 - 35 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 70 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 14 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 7 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)}{35 a^6 f \left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)/(a+a*sin(f*x+e))^6,x, algorithm="giac")
```

```
[Out] -2/35*(35*tan(1/2*f*x + 1/2*e)^5 - 35*tan(1/2*f*x + 1/2*e)^4 + 70*tan(1/2*f
*x + 1/2*e)^3 - 14*tan(1/2*f*x + 1/2*e)^2 + 7*tan(1/2*f*x + 1/2*e) + 1)/(a^
6*f*(tan(1/2*f*x + 1/2*e) + 1)^7)
```

Mupad [B]
time = 8.89, size = 157, normalized size = 2.71

$$-\frac{2 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^2 \left(\cos\left(\frac{e}{2} + \frac{f x}{2}\right)^5 + 7 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^4 \sin\left(\frac{e}{2} + \frac{f x}{2}\right) - 14 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)^2 + 70 \cos\left(\frac{e}{2} + \frac{f x}{2}\right)^2 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)^3 - 35 \cos\left(\frac{e}{2} + \frac{f x}{2}\right) \sin\left(\frac{e}{2} + \frac{f x}{2}\right)^4 + 35 \sin\left(\frac{e}{2} + \frac{f x}{2}\right)^5\right)}{35 a^6 f \left(\cos\left(\frac{e}{2} + \frac{f x}{2}\right) + \sin\left(\frac{e}{2} + \frac{f x}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x))/(a + a*sin(e + f*x))^6,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)^2*(cos(e/2 + (f*x)/2)^5 + 35*sin(e/2 + (f*x)/2)^5 -
35*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^4 + 7*cos(e/2 + (f*x)/2)^4*sin(e/2
 + (f*x)/2) + 70*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^3 - 14*cos(e/2 + (
f*x)/2)^3*sin(e/2 + (f*x)/2)^2))/(35*a^6*f*(cos(e/2 + (f*x)/2) + sin(e/2 +
(f*x)/2))^7)
```

3.441 $\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx$

Optimal. Leaf size=89

$$-\frac{a\cos^7(e+fx)}{18f(a+a\sin(e+fx))^8}+\frac{25\cos^5(e+fx)}{126af(a+a\sin(e+fx))^6}-\frac{47\cos^5(e+fx)}{315a^2f(a+a\sin(e+fx))^5}$$

```
[Out] -1/18*a*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^8+25/126*cos(f*x+e)^5/a/f/(a+a*sin(
f*x+e))^6-47/315*cos(f*x+e)^5/a^2/f/(a+a*sin(f*x+e))^5
```

Rubi [A]
time = 0.32, antiderivative size = 131, normalized size of antiderivative = 1.47, number of steps used = 18, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2954, 2951, 2729, 2727}

$$-\frac{47\cos(e+fx)}{315a^7f(\sin(e+fx)+1)}+\frac{268\cos(e+fx)}{315a^7f(\sin(e+fx)+1)^2}-\frac{181\cos(e+fx)}{105a^7f(\sin(e+fx)+1)^3}+\frac{92\cos(e+fx)}{63a^7f(\sin(e+fx)+1)^4}-\frac{4\cos(e+fx)}{9a^7f(\sin(e+fx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (-4*Cos[e + f*x])/(9*a^7*f*(1 + Sin[e + f*x])^5) + (92*Cos[e + f*x])/(63*a^
7*f*(1 + Sin[e + f*x])^4) - (181*Cos[e + f*x])/(105*a^7*f*(1 + Sin[e + f*x]
)^3) + (268*Cos[e + f*x])/(315*a^7*f*(1 + Sin[e + f*x])^2) - (47*Cos[e + f*
x])/(315*a^7*f*(1 + Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^2(e+fx)}{(a+a\sin(e+fx))^7}\,dx = \frac{\int \sec^8(e+fx)(a-a\sin(e+fx))^7\tan^2(e+fx)\,dx}{a^{14}}$$

$$= \frac{\int \left(\frac{4}{a^3(1+\sin(e+fx))^5} - \frac{12}{a^3(1+\sin(e+fx))^4} + \frac{13}{a^3(1+\sin(e+fx))^3} - \frac{6}{a^3(1+\sin(e+fx))^2} + \cdots\right.}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^7} + \frac{4\int \frac{1}{(1+\sin(e+fx))^5}\,dx}{a^7} - \frac{6\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^7} - \frac{12\int \frac{1}{(1+\sin \cdots}}{}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{12\cos(e+fx)}{7a^7f(1+\sin(e+fx))^4} - \frac{13\cos(e+f\cdots}{5a^7f(1+\sin(e+\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{11\cos(e+\cdots}{7a^7f(1+\sin(e\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+\cdots}{105a^7f(1+\sin(\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+\cdots}{105a^7f(1+\sin(\cdots}$$

$$= -\frac{4\cos(e+fx)}{9a^7f(1+\sin(e+fx))^5} + \frac{92\cos(e+fx)}{63a^7f(1+\sin(e+fx))^4} - \frac{181\cos(e+\cdots}{105a^7f(1+\sin(\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(89) = 178.
time = 1.85, size = 293, normalized size = 3.29

$$\frac{1890\cos\left(\frac{fx}{2}\right)+718830\cos\left(e+\frac{fx}{2}\right)-467208\cos\left(e+\frac{3fx}{2}\right)-1260\cos\left(2e+\frac{3fx}{2}\right)-540\cos\left(2e+\frac{5fx}{2}\right)-179640\cos\left(3e+\frac{5fx}{2}\right)+30753\cos\left(3e+\frac{7fx}{2}\right)+135\cos\left(4e+\frac{7fx}{2}\right)+15\cos\left(4e+\frac{9fx}{2}\right)-15\cos\left(5e+\frac{9fx}{2}\right)+971082\sin\left(\frac{fx}{2}\right)+1890\sin\left(e+\frac{fx}{2}\right)+1260\sin\left(e+\frac{3fx}{2}\right)+659400\sin\left(2e+\frac{3fx}{2}\right)-303192\sin\left(2e+\frac{5fx}{2}\right)-540\sin\left(3e+\frac{5fx}{2}\right)-135\sin\left(3e+\frac{7fx}{2}\right)-89955\sin\left(4e+\frac{7fx}{2}\right)+13427\sin\left(4e+\frac{9fx}{2}\right)+15\sin\left(5e+\frac{9fx}{2}\right)}{720720a^7f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^2)/(a + a*Sin[e + f*x])^7,x]
```

```
[Out] (1890*Cos[(f*x)/2] + 718830*Cos[e + (f*x)/2] - 467208*Cos[e + (3*f*x)/2] -
1260*Cos[2*e + (3*f*x)/2] - 540*Cos[2*e + (5*f*x)/2] - 179640*Cos[3*e + (5*
```

f*x)/2] + 30753*Cos[3*e + (7*f*x)/2] + 135*Cos[4*e + (7*f*x)/2] + 15*Cos[4*e + (9*f*x)/2] - 15*Cos[5*e + (9*f*x)/2] + 971082*Sin[(f*x)/2] + 1890*Sin[e + (f*x)/2] + 1260*Sin[e + (3*f*x)/2] + 659400*Sin[2*e + (3*f*x)/2] - 303192*Sin[2*e + (5*f*x)/2] - 540*Sin[3*e + (5*f*x)/2] - 135*Sin[3*e + (7*f*x)/2] - 89955*Sin[4*e + (7*f*x)/2] + 13427*Sin[4*e + (9*f*x)/2] + 15*Sin[5*e + (9*f*x)/2])/(720720*a^7*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^9)

Maple [A]
time = 0.23, size = 115, normalized size = 1.29

method	result
derivativedivides	$\frac{\frac{352}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}-\frac{328}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}+\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{128}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{9}}+\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{f\,a^{7}}$
default	$\frac{\frac{352}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}-\frac{328}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}+\frac{64}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}-\frac{8}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{3}}-\frac{128}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{9}}+\frac{20}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}}{f\,a^{7}}$
risch	$-\frac{2\left(-2520i\,e^{5i(fx+e)}-2310\,e^{6i(fx+e)}+3402\,e^{4i(fx+e)}+630i\,e^{7i(fx+e)}+315\,e^{8i(fx+e)}+1638i\,e^{3i(fx+e)}-1062\,e^{2i(fx+e)}-1\right.}{315f\,a^{7}\left(e^{i(fx+e)}+i\right)^{9}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x,method=_RETURNVERBOSE)

[Out] 8/f/a^7*(44/3/(tan(1/2*f*x+1/2*e)+1)^6-41/5/(tan(1/2*f*x+1/2*e)+1)^5+8/(tan(1/2*f*x+1/2*e)+1)^8-1/3/(tan(1/2*f*x+1/2*e)+1)^3-16/9/(tan(1/2*f*x+1/2*e)+1)^9+5/2/(tan(1/2*f*x+1/2*e)+1)^4-104/7/(tan(1/2*f*x+1/2*e)+1)^7)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(83) = 166.
time = 0.31, size = 335, normalized size = 3.76

$$-\frac{4\left(\frac{9\sin(fx+e)}{\cos(fx+e)+1}+\frac{36\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{126\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{441\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{315\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{210\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+1\right)}{315\left(a^7+\frac{9a^7\sin(fx+e)}{\cos(fx+e)+1}+\frac{36a^7\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{84a^7\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{126a^7\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{126a^7\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{84a^7\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{36a^7\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{9a^7\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{a^7\sin(fx+e)^9}{(\cos(fx+e)+1)^9}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="maxima")

[Out] -4/315*(9*sin(f*x + e)/(cos(f*x + e) + 1) + 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 126*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 441*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 210*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1)/((a^7 + 9*a^7*sin(f*x + e)/(cos(f*x + e) + 1) + 36*a^7*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*a^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*a^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 126*a^7*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*a^7*sin(f*x + e)^6/(cos(f*x + e) + 1

```
)^6 + 36*a^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*a^7*sin(f*x + e)^8/(co
s(f*x + e) + 1)^8 + a^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(89) = 178$.
time = 0.36, size = 263, normalized size = 2.96

$$-\frac{47\cos(fx+e)^5+127\cos(fx+e)^4-115\cos(fx+e)^3-265\cos(fx+e)^2-(47\cos(fx+e)^4-80\cos(fx+e)^3-195\cos(fx+e)^2+70\cos(fx+e)+140)\sin(fx+e)+70\cos(fx+e)+140}{315\left(a^7f\cos(fx+e)^5+5a^7f\cos(fx+e)^4-8a^7f\cos(fx+e)^3-20a^7f\cos(fx+e)^2+8a^7f\cos(fx+e)+16a^7f+(a^7f\cos(fx+e)^4-4a^7f\cos(fx+e)^3-12a^7f\cos(fx+e)^2+8a^7f\cos(fx+e)+16a^7f)\sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="fricas
")
```

```
[Out] -1/315*(47*cos(f*x + e)^5 + 127*cos(f*x + e)^4 - 115*cos(f*x + e)^3 - 265*c
os(f*x + e)^2 - (47*cos(f*x + e)^4 - 80*cos(f*x + e)^3 - 195*cos(f*x + e)^2
 + 70*cos(f*x + e) + 140)*sin(f*x + e) + 70*cos(f*x + e) + 140)/(a^7*f*cos(
f*x + e)^5 + 5*a^7*f*cos(f*x + e)^4 - 8*a^7*f*cos(f*x + e)^3 - 20*a^7*f*cos
(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a^7*f + (a^7*f*cos(f*x + e)^4 - 4*a
^7*f*cos(f*x + e)^3 - 12*a^7*f*cos(f*x + e)^2 + 8*a^7*f*cos(f*x + e) + 16*a
^7*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**2/(a+a*sin(f*x+e))**7,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 106, normalized size = 1.19

$$-\frac{4\left(210\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6-315\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+441\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-126\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+36\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+9\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{315a^7f\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^2/(a+a*sin(f*x+e))^7,x, algorithm="giac")
```

```
[Out] -4/315*(210*tan(1/2*f*x + 1/2*e)^6 - 315*tan(1/2*f*x + 1/2*e)^5 + 441*tan(1
/2*f*x + 1/2*e)^4 - 126*tan(1/2*f*x + 1/2*e)^3 + 36*tan(1/2*f*x + 1/2*e)^2
+ 9*tan(1/2*f*x + 1/2*e) + 1)/(a^7*f*(tan(1/2*f*x + 1/2*e) + 1)^9)
```

Mupad [B]
time = 9.05, size = 181, normalized size = 2.03

$$-\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6+9\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+36\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-126\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^3+441\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4-315\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^5+210\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6\right)}{315a^7f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x)^2)/(a + a*sin(e + f*x))^7,x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^3*(cos(e/2 + (f*x)/2)^6 + 210*sin(e/2 + (f*x)/2)^6 -
 315*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^5 + 9*cos(e/2 + (f*x)/2)^5*sin(e
/2 + (f*x)/2) + 441*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^4 - 126*cos(e/2
 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^3 + 36*cos(e/2 + (f*x)/2)^4*sin(e/2 + (f*x
)/2)^2))/(315*a^7*f*(cos(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^9)
```

3.442 $\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx$

Optimal. Leaf size=157

$$\frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+fx)}{231a^8f(1+\sin(e+fx))^4}-\frac{846\cos(e+fx)}{385a^8f(1+\sin(e+fx))^3}$$

```
[Out] 4/11*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^6-52/33*cos(f*x+e)/a^8/f/(1+sin(f*x+e)
)^5+617/231*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^4-846/385*cos(f*x+e)/a^8/f/(1+s
in(f*x+e))^3+1003/1155*cos(f*x+e)/a^8/f/(1+sin(f*x+e))^2-152/1155*cos(f*x+e
)/a^8/f/(1+sin(f*x+e))
```

Rubi [A]
time = 0.38, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2954, 2951, 2729, 2727}

$$-\frac{152\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)}+\frac{1003\cos(e+fx)}{1155a^8f(\sin(e+fx)+1)^2}-\frac{846\cos(e+fx)}{385a^8f(\sin(e+fx)+1)^3}+\frac{617\cos(e+fx)}{231a^8f(\sin(e+fx)+1)^4}-\frac{52\cos(e+fx)}{33a^8f(\sin(e+fx)+1)^5}+\frac{4\cos(e+fx)}{11a^8f(\sin(e+fx)+1)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] (4*Cos[e + f*x])/(11*a^8*f*(1 + Sin[e + f*x])^6) - (52*Cos[e + f*x])/(33*a^
8*f*(1 + Sin[e + f*x])^5) + (617*Cos[e + f*x])/(231*a^8*f*(1 + Sin[e + f*x]
)^4) - (846*Cos[e + f*x])/(385*a^8*f*(1 + Sin[e + f*x])^3) + (1003*Cos[e +
f*x])/(1155*a^8*f*(1 + Sin[e + f*x])^2) - (152*Cos[e + f*x])/(1155*a^8*f*(1
 + Sin[e + f*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
```

```
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)\sin^3(e+fx)}{(a+a\sin(e+fx))^8}\,dx = \frac{\int \sec^9(e+fx)(a-a\sin(e+fx))^8\tan^3(e+fx)\,dx}{a^{16}}$$

$$= \frac{\int \left(-\frac{4}{a^4(1+\sin(e+fx))^6}+\frac{16}{a^4(1+\sin(e+fx))^5}-\frac{25}{a^4(1+\sin(e+fx))^4}+\frac{19}{a^4(1+\sin(e+fx))^3}-\right.}{a^4}$$

$$= \frac{\int \frac{1}{1+\sin(e+fx)}\,dx}{a^8}-\frac{4\int \frac{1}{(1+\sin(e+fx))^6}\,dx}{a^8}-\frac{7\int \frac{1}{(1+\sin(e+fx))^2}\,dx}{a^8}+\frac{16\int \frac{1}{(1+\sin(}}{a^8}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{16\cos(e+fx)}{9a^8f(1+\sin(e+fx))^5}+\frac{25\cos(e+fx)}{7a^8f(1+\sin(e+f}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{23\cos(e+fx)}{9a^8f(1+\sin(e+}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+f}{231a^8f(1+\sin(e}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+f}{231a^8f(1+\sin(e}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+f}{231a^8f(1+\sin(e}$$

$$= \frac{4\cos(e+fx)}{11a^8f(1+\sin(e+fx))^6}-\frac{52\cos(e+fx)}{33a^8f(1+\sin(e+fx))^5}+\frac{617\cos(e+f}{231a^8f(1+\sin(e}$$

Mathematica [A]
time = 2.38, size = 195, normalized size = 1.24

$$-\frac{-486024\cos\left(e+\frac{fx}{2}\right)+351450\cos\left(e+\frac{3fx}{2}\right)+180015\cos\left(3e+\frac{5fx}{2}\right)-63580\cos\left(3e+\frac{7fx}{2}\right)-15004\cos\left(5e+\frac{9fx}{2}\right)+1975\cos\left(5e+\frac{11fx}{2}\right)-425964\sin\left(\frac{fx}{2}\right)-299970\sin\left(2e+\frac{3fx}{2}\right)+145695\sin\left(2e+\frac{5fx}{2}\right)+44990\sin\left(4e+\frac{7fx}{2}\right)-6710\sin\left(4e+\frac{9fx}{2}\right)+\sin\left(6e+\frac{11fx}{2}\right)}{240240a^8f\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^4*Sin[e + f*x]^3)/(a + a*Sin[e + f*x])^8,x]
```

```
[Out] -1/240240*(-486024*Cos[e + (f*x)/2] + 351450*Cos[e + (3*f*x)/2] + 180015*Co
s[3*e + (5*f*x)/2] - 63580*Cos[3*e + (7*f*x)/2] - 15004*Cos[5*e + (9*f*x)/2
] + 1975*Cos[5*e + (11*f*x)/2] - 425964*Sin[(f*x)/2] - 299970*Sin[2*e + (3*
f*x)/2] + 145695*Sin[2*e + (5*f*x)/2] + 44990*Sin[4*e + (7*f*x)/2] - 6710*S
in[4*e + (9*f*x)/2] + Sin[6*e + (11*f*x)/2])/(a^8*f*(Cos[e/2] + Sin[e/2])*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^11)
```

Maple [A]
time = 0.26, size = 130, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{128}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{10}}+\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{11}}-\frac{136}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}+\frac{176}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{384}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}+\frac{896}{3\left(\tan\left(\frac{fx}{2}+\ldots\right.\right.}}{f\,a^{8}}$
default	$\frac{-\frac{128}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{10}}+\frac{256}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{11}}-\frac{136}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{6}}+\frac{176}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{5}}-\frac{384}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^{8}}+\frac{896}{3\left(\tan\left(\frac{fx}{2}+\ldots\right.\right.}}{f\,a^{8}}$
risch	$-\frac{2\left(3465i\,e^{9i(fx+e)}+1155\,e^{10i(fx+e)}-23100i\,e^{7i(fx+e)}-13860\,e^{8i(fx+e)}+32802i\,e^{5i(fx+e)}+37422\,e^{6i(fx+e)}-11220i\,e^{3\ldots}\right.}{1155\,f\,a^{8}\left(e^{i(fx+e)}+i\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 16/f/a^8*(-8/(tan(1/2*f*x+1/2*e)+1)^10+16/11/(tan(1/2*f*x+1/2*e)+1)^11-17/2
/(tan(1/2*f*x+1/2*e)+1)^6+11/5/(tan(1/2*f*x+1/2*e)+1)^5-24/(tan(1/2*f*x+1/2
*e)+1)^8+56/3/(tan(1/2*f*x+1/2*e)+1)^9+129/7/(tan(1/2*f*x+1/2*e)+1)^7-1/4/(
tan(1/2*f*x+1/2*e)+1)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(145) = 290$.
time = 0.30, size = 401, normalized size = 2.55

$$-\frac{4\left(\frac{11\sin(fx+e)}{\cos(fx+e)+1}+\frac{55\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{165\sin(fx+e)^3}{(\cos(fx+e)+1)^3}-\frac{825\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{2541\sin(fx+e)^5}{(\cos(fx+e)+1)^5}-\frac{2079\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{1155\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+1\right)}{1155\left(a^8+\frac{11a^8\sin(fx+e)}{\cos(fx+e)+1}+\frac{55a^8\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{165a^8\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{330a^8\sin(fx+e)^4}{(\cos(fx+e)+1)^4}+\frac{462a^8\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{462a^8\sin(fx+e)^6}{(\cos(fx+e)+1)^6}+\frac{330a^8\sin(fx+e)^7}{(\cos(fx+e)+1)^7}+\frac{165a^8\sin(fx+e)^8}{(\cos(fx+e)+1)^8}+\frac{55a^8\sin(fx+e)^9}{(\cos(fx+e)+1)^9}+\frac{11a^8\sin(fx+e)^{10}}{(\cos(fx+e)+1)^{10}}+\frac{a^8\sin(fx+e)^{11}}{(\cos(fx+e)+1)^{11}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="maxima
")
```

```
[Out] -4/1155*(11*sin(f*x + e)/(cos(f*x + e) + 1) + 55*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 165*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 825*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 2541*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2079*sin(f*
x + e)^6/(cos(f*x + e) + 1)^6 + 1155*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 +
1)/((a^8 + 11*a^8*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^8*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 165*a^8*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*a^8
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 462*a^8*sin(f*x + e)^5/(cos(f*x + e)
```

```
 + 1)^5 + 462*a^8*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 330*a^8*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 165*a^8*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 55
*a^8*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*a^8*sin(f*x + e)^10/(cos(f*x
+ e) + 1)^10 + a^8*sin(f*x + e)^11/(cos(f*x + e) + 1)^11)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(157) = 314$.
time = 0.35, size = 315, normalized size = 2.01

$$\frac{152 \cos (fx+e)^6 - 243 \cos (fx+e)^5 - 745 \cos (fx+e)^4 + 455 \cos (fx+e)^3 + 1015 \cos (fx+e)^2 + (152 \cos (fx+e)^5 + 395 \cos (fx+e)^4 - 350 \cos (fx+e)^3 - 805 \cos (fx+e)^2 + 210 \cos (fx+e) + 420) \sin (fx+e) - 210 \cos (fx+e) - 420}{1155 \left(a^8 f \cos (fx+e)^6 - 5 a^8 f \cos (fx+e)^5 - 18 a^8 f \cos (fx+e)^4 + 20 a^8 f \cos (fx+e)^3 + 48 a^8 f \cos (fx+e)^2 - 16 a^8 f \cos (fx+e) - 32 a^8 f - (a^8 f \cos (fx+e)^5 + 6 a^8 f \cos (fx+e)^4 - 12 a^8 f \cos (fx+e)^3 - 32 a^8 f \cos (fx+e)^2 + 16 a^8 f \cos (fx+e) + 32 a^8 f) \sin (fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="fricas
")
```

```
[Out] 1/1155*(152*cos(f*x + e)^6 - 243*cos(f*x + e)^5 - 745*cos(f*x + e)^4 + 455*
cos(f*x + e)^3 + 1015*cos(f*x + e)^2 + (152*cos(f*x + e)^5 + 395*cos(f*x +
e)^4 - 350*cos(f*x + e)^3 - 805*cos(f*x + e)^2 + 210*cos(f*x + e) + 420)*si
n(f*x + e) - 210*cos(f*x + e) - 420)/(a^8*f*cos(f*x + e)^6 - 5*a^8*f*cos(f*
x + e)^5 - 18*a^8*f*cos(f*x + e)^4 + 20*a^8*f*cos(f*x + e)^3 + 48*a^8*f*cos
(f*x + e)^2 - 16*a^8*f*cos(f*x + e) - 32*a^8*f - (a^8*f*cos(f*x + e)^5 + 6*
a^8*f*cos(f*x + e)^4 - 12*a^8*f*cos(f*x + e)^3 - 32*a^8*f*cos(f*x + e)^2 +
16*a^8*f*cos(f*x + e) + 32*a^8*f)*sin(f*x + e))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*sin(f*x+e)**3/(a+a*sin(f*x+e))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.68, size = 120, normalized size = 0.76

$$-\frac{4 \left(1155 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^7 - 2079 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^6 + 2541 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 - 825 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 165 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + 55 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 11 \tan \left(\frac{1}{2} fx + \frac{1}{2} e\right) + 1\right)}{1155 a^8 f \left(\tan \left(\frac{1}{2} fx + \frac{1}{2} e\right) + 1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*sin(f*x+e)^3/(a+a*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -4/1155*(1155*tan(1/2*f*x + 1/2*e)^7 - 2079*tan(1/2*f*x + 1/2*e)^6 + 2541*t
an(1/2*f*x + 1/2*e)^5 - 825*tan(1/2*f*x + 1/2*e)^4 + 165*tan(1/2*f*x + 1/2*
e)^3 + 55*tan(1/2*f*x + 1/2*e)^2 + 11*tan(1/2*f*x + 1/2*e) + 1)/(a^8*f*(tan
(1/2*f*x + 1/2*e) + 1)^11)
```

Mupad [B]
time = 9.41, size = 205, normalized size = 1.31

$$-\frac{4\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^7+11\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^6\sin\left(\frac{e}{2}+\frac{fx}{2}\right)+55\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^5\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2+165\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^4\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^3-825\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^3\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^4+2541\cos\left(\frac{e}{2}+\frac{fx}{2}\right)^2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^5-2079\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^6+1155\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^7\right)}{1155\,a^8\,f\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)+\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*sin(e + f*x)^3)/(a + a*sin(e + f*x))^8,x)
```

```
[Out] -(4*cos(e/2 + (f*x)/2)^4*(cos(e/2 + (f*x)/2)^7 + 1155*sin(e/2 + (f*x)/2)^7
- 2079*cos(e/2 + (f*x)/2)*sin(e/2 + (f*x)/2)^6 + 11*cos(e/2 + (f*x)/2)^6*si
n(e/2 + (f*x)/2) + 2541*cos(e/2 + (f*x)/2)^2*sin(e/2 + (f*x)/2)^5 - 825*cos
(e/2 + (f*x)/2)^3*sin(e/2 + (f*x)/2)^4 + 165*cos(e/2 + (f*x)/2)^4*sin(e/2 +
 (f*x)/2)^3 + 55*cos(e/2 + (f*x)/2)^5*sin(e/2 + (f*x)/2)^2))/(1155*a^8*f*(c
os(e/2 + (f*x)/2) + sin(e/2 + (f*x)/2))^11)
```

3.443 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{1472a^3\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}}-\frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}}-\frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}+\frac{20\cos^5(c+dx)\sqrt{}}{14}$$

```
[Out] -1472/45045*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-368/9009*a^2*cos(d*x+
c)^5/d/(a+a*sin(d*x+c))^(3/2)-2/13*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/a/d-
46/1287*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)+20/143*cos(d*x+c)^5*(a+a*si
n(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2957, 2935, 2753, 2752}

$$-\frac{1472a^3\cos^5(c+dx)}{45045d(a\sin(c+dx)+a)^{5/2}}-\frac{368a^2\cos^5(c+dx)}{9009d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13ad}+\frac{20\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}-\frac{46a\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-1472*a^3*Cos[c + d*x]^5)/(45045*d*(a + a*Sin[c + d*x])^(5/2)) - (368*a^2*
Cos[c + d*x]^5)/(9009*d*(a + a*Sin[c + d*x])^(3/2)) - (46*a*Cos[c + d*x]^5)
/(1287*d*Sqrt[a + a*Sin[c + d*x]]) + (20*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
d*x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13ad} + \frac{2\int \cos^4(c+ \cdots}{\cdots}$$

$$= \frac{20\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d} - \frac{2\cos^5(c+dx) \cdots}{\cdots}$$

$$= -\frac{46a\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} + \frac{20\cos^5(c+dx)\sqrt{a+ \cdots}}{143d}$$

$$= -\frac{368a^2\cos^5(c+dx)}{9009d(a+a\sin(c+dx))^{3/2}} - \frac{46a\cos^5(c+dx \cdots}{1287d\sqrt{a+a\sin(c \cdots}}$$

$$= -\frac{1472a^3\cos^5(c+dx)}{45045d(a+a\sin(c+dx))^{5/2}} - \frac{368a^2\cos^5(c+ \cdots}{9009d(a+a\sin(c \cdots}}$$

Mathematica [A]
time = 2.55, size = 109, normalized size = 0.70

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(81183-62440\cos(2(c+dx))+3465\cos(4(c+dx))+119780\sin(c+dx)-21420\sin(3(c+dx)))}{180180d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/180180*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x]
)]*(81183 - 62440*Cos[2*(c + d*x)] + 3465*Cos[4*(c + d*x)] + 119780*Sin[c +
 d*x] - 21420*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.00, size = 85, normalized size = 0.54

method	result	size
default	$\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^3(3465(\sin^4(dx+c))+10710(\sin^3(dx+c))+12145(\sin^2(dx+c))+6940\sin(dx+c)+2776)}{45045\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/45045*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(3465*sin(d*x+c)^4+10710*sin(d*x+
c)^3+12145*sin(d*x+c)^2+6940*sin(d*x+c)+2776)/cos(d*x+c)/(a+a*sin(d*x+c))^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [A]
time = 0.35, size = 172, normalized size = 1.10

$$\frac{2\,\left(3465\,\cos\left(dx+c\right)^{7}-315\,\cos\left(dx+c\right)^{6}-4585\,\cos\left(dx+c\right)^{5}+115\,\cos\left(dx+c\right)^{4}-184\,\cos\left(dx+c\right)^{3}+368\,\cos\left(dx+c\right)^{2}-\left(3465\,\cos\left(dx+c\right)^{6}+3780\,\cos\left(dx+c\right)^{5}-805\,\cos\left(dx+c\right)^{4}-920\,\cos\left(dx+c\right)^{3}-1104\,\cos\left(dx+c\right)^{2}-1472\,\cos\left(dx+c\right)-2944\right)\sin\left(dx+c\right)-1472\,\cos\left(dx+c\right)-2944\right)\sqrt{a\sin\left(dx+c\right)+a}}{45045\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/45045*(3465*cos(d*x + c)^7 - 315*cos(d*x + c)^6 - 4585*cos(d*x + c)^5 + 1
15*cos(d*x + c)^4 - 184*cos(d*x + c)^3 + 368*cos(d*x + c)^2 - (3465*cos(d*x
 + c)^6 + 3780*cos(d*x + c)^5 - 805*cos(d*x + c)^4 - 920*cos(d*x + c)^3 - 1
104*cos(d*x + c)^2 - 1472*cos(d*x + c) - 2944)*sin(d*x + c) - 1472*cos(d*x
+ c) - 2944)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)

[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*sin(c + d*x)**2*cos(c + d*x)**4, x)

Giac [A]
time = 0.45, size = 157, normalized size = 1.01

$$\frac{32 \sqrt{2} \left(13860 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^{13} - 49140 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^{11} + 65065 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^{9} - 38610 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^{7} + 9009 \operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)\right) \sin\left(-\frac{1}{4}\pi + \frac{1}{2}dx + \frac{1}{2}c\right)^{5}\right) \sqrt{a}}{45045 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac")

[Out] 32/45045*sqrt(2)*(13860*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 - 49140*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 + 65065*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 38610*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 9009*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c + d\,x\right)^4 \sin\left(c + d\,x\right)^2 \sqrt{a + a\,\sin\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2),x)

[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2), x)

3.444 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{64a^3\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}}-\frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^5(c+dx)\sqrt{a+a}}{11d}$$

```
[Out] -64/3465*a^3*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-16/693*a^2*cos(d*x+c)^5/
d/(a+a*sin(d*x+c))^(3/2)-2/99*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/11*
cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2935, 2753, 2752}

$$-\frac{64a^3\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{16a^2\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11d}-\frac{2a\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-64*a^3*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (16*a^2*Cos[
c + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) - (2*a*Cos[c + d*x]^5)/(99*d
*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(1
1*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11d} + \frac{1}{11}\int \cos^4(c+$$

$$= -\frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin}}{11d}$$

$$= -\frac{16a^2\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} - \frac{2a\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{64a^3\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}} - \frac{16a^2\cos^5(c+dx}{693d(a+a\sin(c+dx}$$

Mathematica [A]
time = 1.61, size = 99, normalized size = 0.80

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(-3648+1960\cos(2(c+dx))-5165\sin(c+dx)+315\sin(3(c+dx)))}{6930d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-3648
+ 1960*Cos[2*(c + d*x)] - 5165*Sin[c + d*x] + 315*Sin[3*(c + d*x)]))/(6930*
d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.91, size = 75, normalized size = 0.60

method	result	size
default	$\frac{2(1+\sin(dx+c))a(\sin(dx+c)-1)^3(315(\sin^3(dx+c))+980(\sin^2(dx+c))+1055\sin(dx+c)+422)}{3465\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465*(1+sin(d*x+c))*a*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+980*sin(d*x+c)^2
+1055*sin(d*x+c)+422)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [A]
time = 0.35, size = 151, normalized size = 1.22

$$-\frac{2\left(315\cos\left(dx+c\right)^{6}+350\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{3}-16\cos\left(dx+c\right)^{2}+\left(315\cos\left(dx+c\right)^{5}-35\cos\left(dx+c\right)^{4}-40\cos\left(dx+c\right)^{3}-48\cos\left(dx+c\right)^{2}-64\cos\left(dx+c\right)-128\right)\sin\left(dx+c\right)+64\cos\left(dx+c\right)+128\right)\sqrt{a\sin\left(dx+c\right)+a}}{3465\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 + 350*cos(d*x + c)^5 - 5*cos(d*x + c)^4 + 8*cos
(d*x + c)^3 - 16*cos(d*x + c)^2 + (315*cos(d*x + c)^5 - 35*cos(d*x + c)^4 -
 40*cos(d*x + c)^3 - 48*cos(d*x + c)^2 - 64*cos(d*x + c) - 128)*sin(d*x + c
) + 64*cos(d*x + c) + 128)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin
(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 128, normalized size = 1.03

$$-\frac{32\sqrt{2}\left(630\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-1925\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+1980\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-693\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)\sqrt{a}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -32/3465*sqrt(2)*(630*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^11 - 1925*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^9 + 1980*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^7 - 693*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos (c+d\,x)^4\,\sin (c+d\,x)\,\sqrt{a+a\,\sin (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(1/2), x)
```

3.445 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{8a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}}+\frac{164\cos(c+$$

```
[Out] -12/35*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-2*arctanh(cos(d*x+c)*a^(1/2)/(
a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+8/15*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
-2/7*a*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+164/105*cos(d*x+c)*
(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.31, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2960, 2849, 2838, 2830, 2725, 3125, 3060, 2852, 212}

$$-\frac{2a\sin^3(c+dx)\cos(c+dx)}{7d\sqrt{a\sin(c+dx)+a}}-\frac{12\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35ad}+\frac{164\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{105d}+\frac{8a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
8*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sin[c
 + d*x]^3)/(7*d*Sqrt[a + a*Sin[c + d*x]]) + (164*Cos[c + d*x]*Sqrt[a + a*Si
n[c + d*x]])/(105*d) - (12*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(35*a*d
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc(c+dx)$$

$$= -\frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\,\mathrm{si}}}{3d}$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} +$$

$$= \frac{4a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sin^3(c+dx)}{7d\sqrt{a+a\sin(c+dx)}} +$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{8a\cos(}{15d\sqrt{a+a}}$$

Mathematica [A]
time = 0.25, size = 195, normalized size = 1.23

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(525\cos\left(\frac{1}{2}(c+dx)\right)+175\cos\left(\frac{3}{2}(c+dx)\right)+21\cos\left(\frac{5}{2}(c+dx)\right)+15\cos\left(\frac{7}{2}(c+dx)\right)-420\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+420\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-525\sin\left(\frac{1}{2}(c+dx)\right)+175\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)+15\sin\left(\frac{7}{2}(c+dx)\right)\right)}{420d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(525*Cos[(c + d*x)/2] + 175*Cos[(3*(c + d*x))/2
] + 21*Cos[(5*(c + d*x))/2] + 15*Cos[(7*(c + d*x))/2] - 420*Log[1 + Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 420*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] - 525*Sin[(c + d*x)/2] + 175*Sin[(3*(c + d*x))/2] - 21*Sin[(5*(c + d*
x))/2] + 15*Sin[(7*(c + d*x))/2]))/(420*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))
```

Maple [A]
time = 5.51, size = 141, normalized size = 0.89

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(105a^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)-15(a-a\sin(dx+c))^{\frac{7}{2}}+63a(}{105a^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\;d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(105*a^(7/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))-15*(a-a*sin(d*x+c))^(7/2)+63*a*(a-a*sin(d*x+c))^(
5/2)-35*a^2*(a-a*sin(d*x+c))^(3/2)-105*a^3*(a-a*sin(d*x+c))^(1/2))/a^3/cos(
d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(137) = 274$.
time = 0.36, size = 294, normalized size = 1.85

$$\frac{105\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(15\cos\left(dx+c\right)^{4}+18\cos\left(dx+c\right)^{3}+34\cos\left(dx+c\right)^{2}+\left(15\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}+31\cos\left(dx+c\right)-43\right)\sin\left(dx+c\right)+74\cos\left(dx+c\right)+43\right)\sqrt{a\sin\left(dx+c\right)+a}}{210\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/210*(105*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3
- 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c)
- 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) +
 (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^
3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1))
 + 4*(15*cos(d*x + c)^4 + 18*cos(d*x + c)^3 + 34*cos(d*x + c)^2 + (15*cos(d
*x + c)^3 - 3*cos(d*x + c)^2 + 31*cos(d*x + c) - 43)*sin(d*x + c) + 74*cos(
```

```
d*x + c) + 43)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) +
 d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sin\left(c+dx\right)+1\right)}\,\cos^4\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(c + d*x) + 1))*cos(c + d*x)**4*csc(c + d*x), x)
```

Giac [A]
time = 0.56, size = 190, normalized size = 1.19

$$-\frac{\sqrt{2}\left(480\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-1008\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+280\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+105\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+420\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/210*sqrt(2)*(480*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^7 - 1008*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^5 + 280*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^3 + 105*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) + 420*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x), x)
```

3.446 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}+\frac{4\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{15d}-\cot($$

[Out] -2/5*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a/d-arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+61/15*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+4/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]

time = 0.31, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2960, 2838, 2830, 2725, 3123, 3060, 2852, 212}

$$-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5ad}+\frac{4\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15d}+\frac{61a\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]

[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d) + (61*a*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(15*d) - (Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/d - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*a*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
```

```
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^2(c+$$
$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{d} - \frac{2\cos(c+dx)(a}{}$$
$$= \frac{5a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(c-}}{15d}$$
$$= \frac{61a\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)\sqrt{a+a\sin(}}{15d}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cos}{15d\sqrt{a+a}}$$

Mathematica [A]
time = 0.58, size = 258, normalized size = 1.74

$$\frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-155\cos\left(\frac{1}{2}(c+dx)\right)+87\cos\left(\frac{3}{2}(c+dx)\right)+5\cos\left(\frac{5}{2}(c+dx)\right)+3\cos\left(\frac{7}{2}(c+dx)\right)+155\sin\left(\frac{1}{2}(c+dx)\right)-30\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+30\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+87\sin\left(\frac{3}{2}(c+dx)\right)-5\sin\left(\frac{5}{2}(c+dx)\right)+3\sin\left(\frac{7}{2}(c+dx)\right)\right)}{30d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(-155*Cos[(c + d*x)/2] + 87*
Cos[(3*(c + d*x))/2] + 5*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))/2] + 15
5*Sin[(c + d*x)/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c
+ d*x] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 87*
Sin[(3*(c + d*x))/2] - 5*Sin[(5*(c + d*x))/2] + 3*Sin[(7*(c + d*x))/2]))/(3
0*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c +
d*x)/4] + Sec[(c + d*x)/4]))
```

Maple [A]
time = 5.27, size = 162, normalized size = 1.09

method	result

default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(\sin(dx+c)\left(6(a-a\sin(dx+c))^{\frac{5}{2}}a^{\frac{3}{2}}-20(a-a\sin(dx+c))^{\frac{3}{2}}a^{\frac{5}{2}}-30\sqrt{a-a\sin\right.\right.}{15a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(sin(d*x+c)*(6*(a-a*
sin(d*x+c))^(5/2)*a^(3/2)-20*(a-a*sin(d*x+c))^(3/2)*a^(5/2)-30*(a-a*sin(d*x
+c))^(1/2)*a^(7/2)+15*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^4)+15*(a-a*
sin(d*x+c))^(1/2)*a^(7/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(128) = 256$.
time = 0.36, size = 320, normalized size = 2.16

$$\frac{15\left(\cos(dx+c)^2-(\cos(dx+c)+1)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(6\cos(dx+c)^4+8\cos(dx+c)^3+40\cos(dx+c)^2+\left(6\cos(dx+c)^3-2\cos(dx+c)^2+38\cos(dx+c)+61\right)\sin(dx+c)-23\cos(dx+c)-61\right)\sqrt{a\sin(dx+c)+a}}{60\left(d\cos(dx+c)^2-(d\cos(dx+c)+d)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/60*(15*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c)
 + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c)
- a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
 cos(d*x + c) - 1)) - 4*(6*cos(d*x + c)^4 + 8*cos(d*x + c)^3 + 40*cos(d*x +
 c)^2 + (6*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + 38*cos(d*x + c) + 61)*sin(d*
x + c) - 23*cos(d*x + c) - 61)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**A**]
time = 0.49, size = 208, normalized size = 1.41

$$\frac{\sqrt{2}\left(96\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-160\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-120\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{60\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/60*sqrt(2)*(96*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^5 - 160*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^3 - 15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*
c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c)) - 120*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c) - 60*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*
x + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^2, x)
```

3.447 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$\frac{13\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}-\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a\cdots}}{3\cdots}$$

[Out] 13/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-2/3*a*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/4*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]

time = 0.26, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2960, 2830, 2725, 3123, 3059, 2852, 212}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{2a\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{a\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{13\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]

[Out] (13*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d) - (2*a*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x])/(4*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sin(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^3(c+dx)$$

$$= -\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc(}{}$$

$$= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - $$

$$= -\frac{2a\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - $$

$$= \frac{13\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{2a\cos(c}{3d\sqrt{a+a\sin}}$$

Mathematica [A]
time = 0.74, size = 297, normalized size = 1.90

$$\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-26\cos\left(\frac{1}{2}(c+dx)\right)-14\cos\left(\frac{3}{2}(c+dx)\right)+12\cos\left(\frac{5}{2}(c+dx)\right)+4\cos\left(\frac{7}{2}(c+dx)\right)+39\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-39\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-39\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+39\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+26\sin\left(\frac{1}{2}(c+dx)\right)-14\sin\left(\frac{3}{2}(c+dx)\right)-12\sin\left(\frac{5}{2}(c+dx)\right)+4\sin\left(\frac{7}{2}(c+dx)\right)\right)}{12d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-26*Cos[(c + d*x)/2] - 14*C
os[(3*(c + d*x))/2] + 12*Cos[(5*(c + d*x))/2] + 4*Cos[(7*(c + d*x))/2] + 39
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Cos[2*(c + d*x)]*Log[1 +
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 39*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] + 26*Sin[(c + d*x)/2] - 14*Sin[(3*(c + d*x))/2] - 12*Sin[(5*(c + d*x))
/2] + 4*Sin[(7*(c + d*x))/2]))/(12*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/
4]^2 - Sec[(c + d*x)/4]^2)^2)
```

Maple [A]
time = 5.88, size = 178, normalized size = 1.14

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(8(-a(\sin(dx+c)-1))^{\frac{3}{2}}(\sin^2(dx+c))\sqrt{a}-24\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{3}{2}}\right.}{12a^{\frac{3}{2}}\sin(dx+c)^2 c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/12*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(8*(-a*(sin(d*x+c)-1)
)^(3/2)*sin(d*x+c)^2*a^(1/2)-24*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2)*sin(d*x+c
)^2+39*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^2*sin(d*x+c)^2+9*(-a*(s
in(d*x+c)-1))^(3/2)*a^(1/2)-15*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c
)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(132) = 264$.
time = 0.38, size = 359, normalized size = 2.30

$$\frac{39\left(\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^2-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(8\cos\left(dx+c\right)^4+16\cos\left(dx+c\right)^3-9\cos\left(dx+c\right)^2+\left(8\cos\left(dx+c\right)^3-8\cos\left(dx+c\right)^2-17\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)-22\cos\left(dx+c\right)-5\right)\sqrt{a\sin\left(dx+c\right)+a}}{48\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(39*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^4
 + 16*cos(d*x + c)^3 - 9*cos(d*x + c)^2 + (8*cos(d*x + c)^3 - 8*cos(d*x + c
)^2 - 17*cos(d*x + c) + 5)*sin(d*x + c) - 22*cos(d*x + c) - 5)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [A]
time = 0.47, size = 211, normalized size = 1.35

$$-\frac{\sqrt{2}\left(64\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-39\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-96\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,\left(6\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/48*sqrt(2)*(64*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x
 + 1/2*c)^3 - 39*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) - 96*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c) + 12*(6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^3 - 5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^3, x)
```

3.448 $\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 11/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-2*a*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+11/8*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1
/12*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x
+c)^2*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2797, 2725, 3123, 3059, 2851, 2852, 212}

$$-\frac{2a\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{11a\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (2*a*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (11*a*Cot[c + d*x])/(8
*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x])/(12*d*Sqrt[a +
 a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]])/
(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
```

```
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \sqrt{a+a\sin(c+dx)}\,dx + \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,($$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c-}}{3d}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+da}{}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx}{12d\sqrt{a+a}}$$

$$= -\frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{11a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx}{12d\sqrt{a+a}}$$

$$= \frac{11\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{2a\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} -$$

Mathematica [A]
time = 1.28, size = 309, normalized size = 1.90

$$\frac{\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(252\cos\left(\frac{1}{2}(c+dx)\right)-250\cos\left(\frac{3}{2}(c+dx)\right)-114\cos\left(\frac{5}{2}(c+dx)\right)+48\cos\left(\frac{7}{2}(c+dx)\right)-252\sin\left(\frac{1}{2}(c+dx)\right)+99\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-99\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-250\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)-33\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+33\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+48\sin\left(\frac{7}{2}(c+dx)\right)\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(252*Cos[(c + d*x)/2] - 250
*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2] + 48*Cos[(7*(c + d*x))/2]
- 252*Sin[(c + d*x)/2] + 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[c + d*x] - 99*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 250*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] - 33*Log[1 + Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 33*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 48*Sin[(7*(c + d*x))/2]))/(24*d*(1 +
 Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

Maple [A]
time = 6.56, size = 170, normalized size = 1.04

method	result

default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(-48\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{7}{2}}\left(\sin^{3}(dx+c)\right)+33(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}\right.}{24a^{\frac{7}{2}}\sin(dx+c)^{3}\cos(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(-48*(-a*(sin(d*x+c)-
1))^(1/2)*a^(7/2)*sin(d*x+c)^3+33*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)+33*arct
anh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^3-56*(-a*(sin(d*x+c)-
1))^(3/2)*a^(5/2)+15*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^3/cos(d*
x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(141) = 282$.
time = 0.37, size = 380, normalized size = 2.33

$$\frac{33\left(\cos(dx+c)^{4}-2\cos(dx+c)^{2}-\left(\cos(dx+c)^{3}+\cos(dx+c)^{2}-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(48\cos(dx+c)^{4}-33\cos(dx+c)^{3}-139\cos(dx+c)^{2}+\left(48\cos(dx+c)^{3}+81\cos(dx+c)^{2}-58\cos(dx+c)-83\right)\sin(dx+c)+25\cos(dx+c)+83\right)\sqrt{a\sin(dx+c)+a}}{96\left(d\cos(dx+c)^{4}-2d\cos(dx+c)^{2}-\left(d\cos(dx+c)^{3}+d\cos(dx+c)^{2}-d\cos(dx+c)-d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(33*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
4*(48*cos(d*x + c)^4 - 33*cos(d*x + c)^3 - 139*cos(d*x + c)^2 + (48*cos(d*x
 + c)^3 + 81*cos(d*x + c)^2 - 58*cos(d*x + c) - 83)*sin(d*x + c) + 25*cos(d
*x + c) + 83)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c
```

```
)^2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8008 deep
```

Giac [A]
time = 0.54, size = 211, normalized size = 1.29

$$\frac{\sqrt{2}\left(33\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+192\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{4\left(132\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-112\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+15\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
[Out] 1/96*sqrt(2)*(33*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c)) + 192*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c) + 4*(132*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^5 - 112*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^3 + 15*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^4, x)
```

3.449 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=173

$$-\frac{67\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d}+\frac{61a\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}+\frac{61a\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}}-\frac{a\cot(c+\cdots}{24d\sqrt{a+\cdots}}$$

```
[Out] -67/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d+61/64*a
*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+61/96*a*cot(d*x+c)*csc(d*x+c)/d/(a+a*s
in(d*x+c))^(1/2)-1/24*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-1/
4*cot(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.34, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2960, 2852, 212, 3123, 3059, 2851}

$$\frac{61a\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{67\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{61a\cot(c+dx)\csc(c+dx)}{96d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-67*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*
d) + (61*a*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (61*a*Cot[c + d*
x]*Csc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt
[a + a*Sin[c + d*x]])/(4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^5(c+dx)$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4d} + \frac{\int \csc}{}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c+dx}{24d\sqrt{a+a}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(c+}{96d\sqrt{a+}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(}{64d\sqrt{a+a}}$$

$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{61a\cot(}{64d\sqrt{a+a}}$$

$$= -\frac{67\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} + \frac{61a\cot}{64d\sqrt{a+a}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 367 vs. 2(173) = 346.
time = 1.93, size = 367, normalized size = 2.12

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/192*(Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(442*Cos[(c + d*x)/2
] - 162*Cos[(3*(c + d*x))/2] + 122*Cos[(5*(c + d*x))/2] + 366*Cos[(7*(c + d
*x))/2] + 603*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 804*Cos[2*(c +
 d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 201*Cos[4*(c + d*x)]*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 603*Log[1 - Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]] + 804*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 201*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] - 442*Sin[(c + d*x)/2] - 162*Sin[(3*(c + d*x))/2] - 122*Sin[(5*(c +
```

```
d*x))/2] + 366*Sin[(7*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d
*x)/4]^2 - Sec[(c + d*x)/4]^2)^4)
```

Maple [A]
time = 6.87, size = 162, normalized size = 0.94

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(201\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{4}\left(\sin^{4}(dx+c)\right)+183(-a(\sin$ … over $192a^{\frac{7}{2}}\sin(dx+c)^{4}\cos(dx+c)\sqrt{}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(201*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+c)^4+183*(-a*(sin(d*x+c)-1))^(7/2)
*a^(1/2)-671*(-a*(sin(d*x+c)-1))^(5/2)*a^(3/2)+737*(-a*(sin(d*x+c)-1))^(3/2
)*a^(5/2)-201*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2))/sin(d*x+c)^4/cos(d*x+c)/(a
+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(149) = 298$.
time = 0.39, size = 415, normalized size = 2.40

$$\frac{201\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)+1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(183\,\cos\left(dx+c\right)^{4}+122\,\cos\left(dx+c\right)^{3}-188\,\cos\left(dx+c\right)^{2}+\left(183\,\cos\left(dx+c\right)^{3}+61\,\cos\left(dx+c\right)^{2}-127\,\cos\left(dx+c\right)-53\right)\sin\left(dx+c\right)-74\,\cos\left(dx+c\right)+53\right)\sqrt{a\sin\left(dx+c\right)+a}}{768\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}-2\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(201*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
+ c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c
) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)
```

```
^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
 c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
- a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(183*cos(d*x + c)^4 + 122*cos(d*x
 + c)^3 - 188*cos(d*x + c)^2 + (183*cos(d*x + c)^3 + 61*cos(d*x + c)^2 - 12
7*cos(d*x + c) - 53)*sin(d*x + c) - 74*cos(d*x + c) + 53)*sqrt(a*sin(d*x +
c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*co
s(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 213, normalized size = 1.23

$$-\frac{\sqrt{2}\left(201\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{4\left(1464\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2684\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1474\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-201\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}\right)\sqrt{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/768*sqrt(2)*(201*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) - 4*(1464*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 1/2*d*x + 1/2*c)^7 - 2684*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^5 + 1474*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^3 - 201*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^4)*sqrt(a
)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^5,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^5, x)
```

3.450 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d}-\frac{31a\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}+\frac{97a\cot(c+dx)\csc(c+dx)}{192d\sqrt{a+a\sin(c+dx)}}+\frac{97a\cot}{240d}$$

```
[Out] -31/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-31/128
*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+97/192*a*cot(d*x+c)*csc(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+97/240*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/
2)-1/40*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*c
sc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.44, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2960, 2851, 2852, 212, 3123, 3059}

$$-\frac{31a\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{97a\cot(c+dx)\csc^2(c+dx)}{240d\sqrt{a\sin(c+dx)+a}}+\frac{97a\cot(c+dx)\csc(c+dx)}{192d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-31*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128
*d) - (31*a*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c +
d*x]*Csc[c + d*x])/(192*d*Sqrt[a + a*Sin[c + d*x]]) + (97*a*Cot[c + d*x]*Cs
c[c + d*x]^2)/(240*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*
x]^3)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a
 + a*Sin[c + d*x]])/(5*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^6(c+$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{}}{5d}$$

$$= -\frac{a\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c}{d\sqrt{a+a\sin}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a\cot(c}{d\sqrt{a+a\sin}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cos}{128d\sqrt{a+}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{31a\cos}{128d\sqrt{a+}}$$

$$= -\frac{31\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} - \frac{31a}{128d\sqrt{a}}$$

Mathematica [A]
time = 3.24, size = 403, normalized size = 1.93

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/1920*(Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(10180*Cos[(c + d*x
)/2] - 2240*Cos[(3*(c + d*x))/2] - 1392*Cos[(5*(c + d*x))/2] + 4810*Cos[(7*
(c + d*x))/2] + 930*Cos[(9*(c + d*x))/2] - 10180*Sin[(c + d*x)/2] + 4650*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 4650*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2240*Sin[(3*(c + d*x))/2] +
 1392*Sin[(5*(c + d*x))/2] - 2325*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[3*(c + d*x)] + 2325*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
```

n[3*(c + d*x)] + 4810*Sin[(7*(c + d*x))/2] - 930*Sin[(9*(c + d*x))/2] + 465
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 465*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(1 + Cot[(c +
d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)

Maple [A]
time = 6.33, size = 180, normalized size = 0.86

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(465(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{3}{2}}+465\operatorname{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)\right.}{1920a^{\frac{11}{2}}\sin(dx+c)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/1920*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(11/2)*(465*(-a*(sin(d*x
+c)-1))^(9/2)*a^(3/2)+465*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^6*si
n(d*x+c)^5-890*(-a*(sin(d*x+c)-1))^(7/2)*a^(5/2)-896*(-a*(sin(d*x+c)-1))^(5
/2)*a^(7/2)+2170*(-a*(sin(d*x+c)-1))^(3/2)*a^(9/2)-465*(-a*(sin(d*x+c)-1))^
(1/2)*a^(11/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(181) = 362.
time = 0.38, size = 461, normalized size = 2.21

$$\frac{465\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(465\cos(dx+c)^5+1435\cos(dx+c)^4-154\cos(dx+c)^3-1662\cos(dx+c)^2-\left(465\cos(dx+c)^4-970\cos(dx+c)^3-1124\cos(dx+c)^2+538\cos(dx+c)+611\right)\sin(dx+c)+73\cos(dx+c)+611\right)\sqrt{a\sin(dx+c)+a}}{7680\left(d\cos(dx+c)^6-3d\cos(dx+c)^4+3d\cos(dx+c)^2-\left(d\cos(dx+c)^5+d\cos(dx+c)^4-2d\cos(dx+c)^3-2d\cos(dx+c)^2+d\cos(dx+c)+d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/7680*(465*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*
x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
```

```
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)
^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(465*cos(d*x
+ c)^5 + 1435*cos(d*x + c)^4 - 154*cos(d*x + c)^3 - 1662*cos(d*x + c)^2 - (
465*cos(d*x + c)^4 - 970*cos(d*x + c)^3 - 1124*cos(d*x + c)^2 + 538*cos(d*x
 + c) + 611)*sin(d*x + c) + 73*cos(d*x + c) + 611)*sqrt(a*sin(d*x + c) + a)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - (d*cos(d*x
+ c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos
(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 242, normalized size = 1.16

$$-\frac{\sqrt{2}\left(465\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(7440\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-7120\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-3584\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+4340\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-465\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{5}}\right)\sqrt{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/7680*sqrt(2)*(465*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
 + 1/2*d*x + 1/2*c)) + 4*(7440*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^9 - 7120*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^7 - 3584*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^5 + 4340*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3 - 465*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^5)*sqrt
(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}{{\sin\left(c+d\,x\right)}^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^6,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^6, x)
```

3.451 $\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=245

$$-\frac{55\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d}-\frac{55a\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}}-\frac{55a\cot(c+dx)\csc(c+dx)}{768d\sqrt{a+a\sin(c+dx)}}+\frac{329a\,\mathrm{cc}}{960d}$$

```
[Out] -55/512*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-55/512
*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-55/768*a*cot(d*x+c)*csc(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+329/960*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1
/2)+47/160*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/60*a*cot(d*
x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/6*cot(d*x+c)*csc(d*x+c)^5*(a+a
*sin(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.52, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2960, 2851, 2852, 212, 3123, 3059}

$$-\frac{55a\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{55\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{6d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{60d\sqrt{a\sin(c+dx)+a}}+\frac{47a\cot(c+dx)\csc^3(c+dx)}{160d\sqrt{a\sin(c+dx)+a}}+\frac{329a\cot(c+dx)\csc^2(c+dx)}{960d\sqrt{a\sin(c+dx)+a}}-\frac{55a\cot(c+dx)\csc(c+dx)}{768d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-55*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(512
*d) - (55*a*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) - (55*a*Cot[c +
d*x]*Csc[c + d*x])/(768*d*Sqrt[a + a*Sin[c + d*x]]) + (329*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(960*d*Sqrt[a + a*Sin[c + d*x]]) + (47*a*Cot[c + d*x]*Csc[c
+ d*x]^3)/(160*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^4
)/(60*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^5*Sqrt[a + a
*Sin[c + d*x]])/(6*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^7(c+$$

$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+dx}{6}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3a\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot}{4d\sqrt{a+a}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{3a\cot}{4d\sqrt{a+a}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\,c}{512d\sqrt{a-}}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{55a\,c}{512d\sqrt{a-}}$$

$$= -\frac{55\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{55a}{512d\sqrt{a}}$$

Mathematica [A]
time = 5.80, size = 485, normalized size = 1.98

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(-24540*Cos[(c + d*x)/2] -
25684*Cos[(3*(c + d*x))/2] - 14490*Cos[(5*(c + d*x))/2] - 15006*Cos[(7*(c +
 d*x))/2] - 550*Cos[(9*(c + d*x))/2] - 1650*Cos[(11*(c + d*x))/2] - 8250*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12375*Cos[2*(c + d*x)]*Log[1 +
```

```
 Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4950*Cos[4*(c + d*x)]*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 825*Cos[6*(c + d*x)]*Log[1 + Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]] + 8250*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 12375*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4
950*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 825*Cos
[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24540*Sin[(c +
 d*x)/2] - 25684*Sin[(3*(c + d*x))/2] + 14490*Sin[(5*(c + d*x))/2] - 15006*
Sin[(7*(c + d*x))/2] + 550*Sin[(9*(c + d*x))/2] - 1650*Sin[(11*(c + d*x))/2
]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^6)
```

Maple [A]
time = 7.58, size = 198, normalized size = 0.81

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-825(-a(\sin(dx+c)-1))^{\frac{11}{2}}a^{\frac{5}{2}}+4675(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{7}{2}}-7818(-a(\sin(dx\right.}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-825*(-a*(sin(d*x+c)-1))^
(11/2)*a^(5/2)+4675*(-a*(sin(d*x+c)-1))^(9/2)*a^(7/2)-7818*(-a*(sin(d*x+c)-
1))^(7/2)*a^(9/2)+1398*(-a*(sin(d*x+c)-1))^(5/2)*a^(11/2)+4675*(-a*(sin(d*x
+c)-1))^(3/2)*a^(13/2)-825*(-a*(sin(d*x+c)-1))^(1/2)*a^(15/2)+825*arctanh((
-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(d*x+c)^6)/a^(15/2)/sin(d*x+c)^6/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(213) = 426$.
time = 0.38, size = 525, normalized size = 2.14

$$\frac{825\left(\cos(dx+c)^7+\cos(dx+c)^6-3\cos(dx+c)^5-3\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2+\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(825\cos(dx+c)^6+550\cos(dx+c)^5+707\cos(dx+c)^4+1156\cos(dx+c)^3-225\cos(dx+c)^2+\left(825\cos(dx+c)^5+275\cos(dx+c)^4+982\cos(dx+c)^3-174\cos(dx+c)^2-399\cos(dx+c)+27\right)\sin(dx+c)-426\cos(dx+c)-27\right)\sqrt{a\sin(dx+c)+a}}{30720\left(d\cos(dx+c)^7+d\cos(dx+c)^6-3d\cos(dx+c)^5-3d\cos(dx+c)^4+3d\cos(dx+c)^3+3d\cos(dx+c)^2-d\cos(dx+c)+\left(d\cos(dx+c)^6-3d\cos(dx+c)^4+3d\cos(dx+c)^2-d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(825*(cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 + (cos(d*x + c)^6 - 3*cos(d*
x + c)^4 + 3*cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)*sqrt(a)*l
og((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x +
c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a)
 - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c
) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c)
 - cos(d*x + c) - 1)) + 4*(825*cos(d*x + c)^6 + 550*cos(d*x + c)^5 + 707*co
s(d*x + c)^4 + 1156*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + (825*cos(d*x + c)
^5 + 275*cos(d*x + c)^4 + 982*cos(d*x + c)^3 - 174*cos(d*x + c)^2 - 399*cos
(d*x + c) + 27)*sin(d*x + c) - 426*cos(d*x + c) - 27)*sqrt(a*sin(d*x + c) +
 a))/(d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*
x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*co
s(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) -
d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 271, normalized size = 1.11

$$-\frac{\sqrt{2}\left(825\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(26400\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-74800\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+62544\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-5592\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-9350\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+825\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{6}}\right)\sqrt{a}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/30720*sqrt(2)*(825*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*p
i + 1/2*d*x + 1/2*c)) + 4*(26400*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^11 - 74800*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 62544*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 5592*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
```

```
sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 9350*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 825*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))
*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^6
)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^4 \sqrt{a+a\sin(c+dx)}}{\sin(c+dx)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^7,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^7, x)
```

3.452 $\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d}-\frac{61a\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}}-\frac{61a\cot(c+dx)\csc(c+dx)}{1536d\sqrt{a+a\sin(c+dx)}}-\frac{61a\cot(c+dx)\csc^2(c+dx)}{1920d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -61/1024*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))*a^(1/2)/d-61/10
24*a*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-61/1536*a*cot(d*x+c)*csc(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)-61/1920*a*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))
^(1/2)+579/2240*a*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+193/840*
a*cot(d*x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/84*a*cot(d*x+c)*csc(d*
x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c)
)^(1/2)/d
```

Rubi [A]
time = 0.59, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2960, 2851, 2852, 212, 3123, 3059}

$$-\frac{61a\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}-\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{7d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{84d\sqrt{a\sin(c+dx)+a}}+\frac{193a\cot(c+dx)\csc^4(c+dx)}{840d\sqrt{a\sin(c+dx)+a}}+\frac{579a\cot(c+dx)\csc^3(c+dx)}{2240d\sqrt{a\sin(c+dx)+a}}-\frac{61a\cot(c+dx)\csc^2(c+dx)}{1920d\sqrt{a\sin(c+dx)+a}}-\frac{61a\cot(c+dx)\csc(c+dx)}{1536d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-61*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(102
4*d) - (61*a*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c
+ d*x]*Csc[c + d*x])/(1536*d*Sqrt[a + a*Sin[c + d*x]]) - (61*a*Cot[c + d*x]
*Csc[c + d*x]^2)/(1920*d*Sqrt[a + a*Sin[c + d*x]]) + (579*a*Cot[c + d*x]*Cs
c[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (193*a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(840*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^
5)/(84*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^6*Sqrt[a +
a*Sin[c + d*x]])/(7*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
```

```
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx = \int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,dx + \int \csc^8(c+$$

$$= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+a}{7}$$

$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+}{3d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{5a\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{5a\cot(c+dx)\csc(c+dx}{12d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot}{8d\sqrt{a+a}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{5a\cot}{8d\sqrt{a+a}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\,c}{1024d\sqrt{a}}$$

$$= -\frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{61a\,c}{1024d\sqrt{a}}$$

$$= -\frac{61\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{61a}{1024d\sqrt{}}$$

Mathematica [A]
time = 1.35, size = 191, normalized size = 0.68

$$\frac{\sqrt{a(1+\sin(c+dx))}\left(-102480\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+102480\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\csc^7(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(-201298-244533\cos(2(c+dx))-52094\cos(4(c+dx))+6405\cos(6(c+dx))+49128\sin(c+dx)-179636\sin(3(c+dx))-8540\sin(5(c+dx)))\right)}{3440640d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*(-102480*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 102480*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Csc[c + d*x]
```

```
^7*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(-201298 - 244533*Cos[2*(c + d*x)]
 - 52094*Cos[4*(c + d*x)] + 6405*Cos[6*(c + d*x)] + 49128*Sin[c + d*x] - 17
9636*Sin[3*(c + d*x)] - 8540*Sin[5*(c + d*x)])))/(3440640*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 7.42, size = 216, normalized size = 0.77

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(6405(-a(\sin(dx+c)-1))^{\frac{13}{2}}a^{\frac{7}{2}}-42700(-a(\sin(dx+c)-1))^{\frac{11}{2}}a^{\frac{9}{2}}+120841(-a(\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/107520*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(19/2)*(6405*(-a*(sin(
d*x+c)-1))^(13/2)*a^(7/2)-42700*(-a*(sin(d*x+c)-1))^(11/2)*a^(9/2)+120841*(
-a*(sin(d*x+c)-1))^(9/2)*a^(11/2)+6405*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^
(1/2))*a^10*sin(d*x+c)^7-156672*(-a*(sin(d*x+c)-1))^(7/2)*a^(13/2)+51191*(-
a*(sin(d*x+c)-1))^(5/2)*a^(15/2)+42700*(-a*(sin(d*x+c)-1))^(3/2)*a^(17/2)-6
405*(-a*(sin(d*x+c)-1))^(1/2)*a^(19/2))/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(a*sin(d*x + c) + a)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(245) = 490$.
time = 0.36, size = 567, normalized size = 2.02

$$\frac{6405\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2-\left(\cos(dx+c)^7+\cos(dx+c)^6-3\cos(dx+c)^5-3\cos(dx+c)^4+3\cos(dx+c)^3+3\cos(dx+c)^2-\cos(dx+c)-1\right)\sin(dx+c)+1\right)\sqrt{a}\log\left(\text{[illegible]}\right)+4\left(6405\cos(dx+c)^7+2135\cos(dx+c)^6-22631\cos(dx+c)^5-37613\cos(dx+c)^4+1343\cos(dx+c)^3+27477\cos(dx+c)^2-\left(6405\cos(dx+c)^6+4270\cos(dx+c)^5-18361\cos(dx+c)^4+19252\cos(dx+c)^3+20595\cos(dx+c)^2-6882\cos(dx+c)-7359\right)\sin(dx+c)-477\cos(dx+c)-7359\right)\sqrt{a\sin(dx+c)+a}}{430080\left(d\cos(dx+c)^8-4d\cos(dx+c)^6+6d\cos(dx+c)^4-4d\cos(dx+c)^2-\left(d\cos(dx+c)^7+d\cos(dx+c)^6-3d\cos(dx+c)^5-3d\cos(dx+c)^4+3d\cos(dx+c)^3+3d\cos(dx+c)^2-d\cos(dx+c)-d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/430080*(6405*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*co
s(d*x + c)^2 - (cos(d*x + c)^7 + cos(d*x + c)^6 - 3*cos(d*x + c)^5 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^3 + 3*cos(d*x + c)^2 - cos(d*x + c) - 1)*sin(d*
x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(6405*cos(d*x + c)^7 + 2135*
cos(d*x + c)^6 - 22631*cos(d*x + c)^5 - 37613*cos(d*x + c)^4 + 1343*cos(d*x
 + c)^3 + 27477*cos(d*x + c)^2 - (6405*cos(d*x + c)^6 + 4270*cos(d*x + c)^5
 - 18361*cos(d*x + c)^4 + 19252*cos(d*x + c)^3 + 20595*cos(d*x + c)^2 - 688
2*cos(d*x + c) - 7359)*sin(d*x + c) - 477*cos(d*x + c) - 7359)*sqrt(a*sin(d
*x + c) + a))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 -
 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos
(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 300, normalized size = 1.07

$$-\frac{\sqrt{2}\left(6405\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(409920\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1366400\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1933456\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1253376\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+204764\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+85400\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6405\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{7}}\right)\sqrt{a}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/430080*sqrt(2)*(6405*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*
x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c)) + 4*(409920*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c)^13 - 1366400*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 + 1933456*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 1253376*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 204764*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 85400*sgn(cos(-1/4*pi + 1/
2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 6405*sgn(cos(-1/4*pi + 1
```

```
/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x +
 1/2*c)^2 - 1)^7)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^4\,\sqrt{a+a\,\sin (c+d\,x)}}{\sin (c+d\,x)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^8,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(1/2))/sin(c + d*x)^8, x)
```

3.453 $\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=188

$$-\frac{256a^4\cos^5(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/2}}-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}}-\frac{14a\cos^5(c+dx)}{4}$$

```
[Out] -256/6435*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/1287*a^3*cos(d*x+c)^
5/d/(a+a*sin(d*x+c))^(3/2)+4/39*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-2/15*
cos(d*x+c)^5*(a+a*sin(d*x+c))^(5/2)/a/d-56/1287*a^2*cos(d*x+c)^5/d/(a+a*sin
(d*x+c))^(1/2)-14/429*a*cos(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2957, 2935, 2753, 2752}

$$-\frac{256a^4\cos^5(c+dx)}{6435d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1287d(a\sin(c+dx)+a)^{3/2}}-\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{5/2}}{15ad}+\frac{4\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{39d}-\frac{14a\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{429d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(6435*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1287*d*(a + a*Sin[c + d*x])^(3/2)) - (56*a^2*Cos[c + d*x]^5)/
(1287*d*Sqrt[a + a*Sin[c + d*x]]) - (14*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c +
 d*x]])/(429*d) + (4*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(39*d) - (2
*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(5/2))/(15*a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{5/2}}{15ad} + \frac{2\int \cos^4(c+$$

$$= \frac{4\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{39d} - \frac{2\cos^5(c+dx)$$

$$= -\frac{14a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{429d} + \frac{4\cos^5(c+$$

$$= -\frac{56a^2\cos^5(c+dx)}{1287d\sqrt{a+a\sin(c+dx)}} - \frac{14a\cos^5(c+dx)\sqrt{a-}}{429d}$$

$$= -\frac{64a^3\cos^5(c+dx)}{1287d(a+a\sin(c+dx))^{3/2}} - \frac{56a^2\cos^5(c+dx}{1287d\sqrt{a+a\sin(c-}}$$

$$= -\frac{256a^4\cos^5(c+dx)}{6435d(a+a\sin(c+dx))^{5/2}} - \frac{64a^3\cos^5(c+d}{1287d(a+a\sin(c+}$$

Mathematica [A]
time = 6.57, size = 120, normalized size = 0.64

$$-\frac{a\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(43122-36640\cos(2(c+dx))+3630\cos(4(c+dx))+66470\sin(c+dx)-14445\sin(3(c+dx))+429\sin(5(c+dx)))}{51480d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/51480*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x
])]*(43122 - 36640*Cos[2*(c + d*x)] + 3630*Cos[4*(c + d*x)] + 66470*Sin[c +
 d*x] - 14445*Sin[3*(c + d*x)] + 429*Sin[5*(c + d*x)]))/(d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.78, size = 97, normalized size = 0.52

method	result
default	$\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^3(429(\sin^5(dx+c))+1815(\sin^4(dx+c))+3075(\sin^3(dx+c))+2765(\sin^2(dx+c))+1580\sin(dx+c)+6}{6435\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/6435*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(429*sin(d*x+c)^5+1815*sin(d*x+c
)^4+3075*sin(d*x+c)^3+2765*sin(d*x+c)^2+1580*sin(d*x+c)+632)/cos(d*x+c)/(a+
a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [A]
time = 0.35, size = 210, normalized size = 1.12

$$\frac{2\,(429\,a\cos(dx+c)^8+957\,a\cos(dx+c)^7-633\,a\cos(dx+c)^6-1301\,a\cos(dx+c)^5+20\,a\cos(dx+c)^4-32\,a\cos(dx+c)^3+64\,a\cos(dx+c)^2-256\,a\cos(dx+c)+(429\,a\cos(dx+c)^7-528\,a\cos(dx+c)^6-1161\,a\cos(dx+c)^5+140\,a\cos(dx+c)^4+160\,a\cos(dx+c)^3+192\,a\cos(dx+c)^2+256\,a\cos(dx+c)+512\,a)\sin(dx+c)-512\,a)\sqrt{a\sin(dx+c)+a}}{6435\,(d\cos(dx+c)+d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/6435*(429*a*cos(d*x + c)^8 + 957*a*cos(d*x + c)^7 - 633*a*cos(d*x + c)^6
- 1301*a*cos(d*x + c)^5 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*
cos(d*x + c)^2 - 256*a*cos(d*x + c) + (429*a*cos(d*x + c)^7 - 528*a*cos(d*x
 + c)^6 - 1161*a*cos(d*x + c)^5 + 140*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)
^3 + 192*a*cos(d*x + c)^2 + 256*a*cos(d*x + c) + 512*a)*sin(d*x + c) - 512*
a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 192, normalized size = 1.02

$$-\frac{64\sqrt{2}\left(1716\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{15}-7920\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+14625\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-13585\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+6435\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-1287\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)\sqrt{a}}{6435\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -64/6435*sqrt(2)*(1716*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^15 - 7920*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^13 + 14625*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^11 - 13585*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 6435*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c
))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 1287*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/
2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2), x)
```

3.454 $\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=156

$$-\frac{256a^4\cos^5(c+dx)}{5005d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}}-\frac{6a\cos^5(c+dx)\sqrt{a}}{143}$$

```
[Out] -256/5005*a^4*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-64/1001*a^3*cos(d*x+c)^
5/d/(a+a*sin(d*x+c))^(3/2)-2/13*cos(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-8/143
*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-6/143*a*cos(d*x+c)^5*(a+a*sin(d*
x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2935, 2753, 2752}

$$-\frac{256a^4\cos^5(c+dx)}{5005d(a\sin(c+dx)+a)^{5/2}}-\frac{64a^3\cos^5(c+dx)}{1001d(a\sin(c+dx)+a)^{3/2}}-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{13d}-\frac{6a\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-256*a^4*Cos[c + d*x]^5)/(5005*d*(a + a*Sin[c + d*x])^(5/2)) - (64*a^3*Cos
[c + d*x]^5)/(1001*d*(a + a*Sin[c + d*x])^(3/2)) - (8*a^2*Cos[c + d*x]^5)/(
143*d*Sqrt[a + a*Sin[c + d*x]]) - (6*a*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*
x]])/(143*d) - (2*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(13*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+a\sin(c+dx))^{3/2}}{13d}+\frac{3}{13}\int \cos^4(c+$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx}{}$$

$$= -\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+dx)}}-\frac{6a\cos^5(c+dx)\sqrt{a+a}}{143d}$$

$$= -\frac{64a^3\cos^5(c+dx)}{1001d(a+a\sin(c+dx))^{3/2}}-\frac{8a^2\cos^5(c+dx)}{143d\sqrt{a+a\sin(c+c}}$$

$$= -\frac{256a^4\cos^5(c+dx)}{5005d(a+a\sin(c+dx))^{5/2}}-\frac{64a^3\cos^5(c+dx}{1001d(a+a\sin(c+a}}$$

Mathematica [A]
time = 3.52, size = 110, normalized size = 0.71

$$-\frac{a\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(19559-12600\cos(2(c+dx))+385\cos(4(c+dx))+28230\sin(c+dx)-3290\sin(3(c+dx)))}{20020d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
[Out] -1/20020*(a*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x
])]*(19559 - 12600*Cos[2*(c + d*x)] + 385*Cos[4*(c + d*x)] + 28230*Sin[c +
d*x] - 3290*Sin[3*(c + d*x)]))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.07, size = 87, normalized size = 0.56

method	result	size
default	$\frac{2(1+\sin(dx+c))a^2(\sin(dx+c)-1)^3(385(\sin^4(dx+c))+1645(\sin^3(dx+c))+2765(\sin^2(dx+c))+2295\sin(dx+c)+918)}{5005\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5005*(1+sin(d*x+c))*a^2*(sin(d*x+c)-1)^3*(385*sin(d*x+c)^4+1645*sin(d*x+c
)^3+2765*sin(d*x+c)^2+2295*sin(d*x+c)+918)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [A]
time = 0.35, size = 189, normalized size = 1.21

$$\frac{2\,\left(385\,a\cos\left(dx+c\right)^{7}-490\,a\cos\left(dx+c\right)^{6}-1015\,a\cos\left(dx+c\right)^{5}+20\,a\cos\left(dx+c\right)^{4}-32\,a\cos\left(dx+c\right)^{3}+64\,a\cos\left(dx+c\right)^{2}-256\,a\cos\left(dx+c\right)-\left(385\,a\cos\left(dx+c\right)^{6}+875\,a\cos\left(dx+c\right)^{5}-140\,a\cos\left(dx+c\right)^{4}-160\,a\cos\left(dx+c\right)^{3}-192\,a\cos\left(dx+c\right)^{2}-256\,a\cos\left(dx+c\right)-512\,a\right)\sin\left(dx+c\right)-512\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{5005\,\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/5005*(385*a*cos(d*x + c)^7 - 490*a*cos(d*x + c)^6 - 1015*a*cos(d*x + c)^5
 + 20*a*cos(d*x + c)^4 - 32*a*cos(d*x + c)^3 + 64*a*cos(d*x + c)^2 - 256*a*
cos(d*x + c) - (385*a*cos(d*x + c)^6 + 875*a*cos(d*x + c)^5 - 140*a*cos(d*x
 + c)^4 - 160*a*cos(d*x + c)^3 - 192*a*cos(d*x + c)^2 - 256*a*cos(d*x + c)
- 512*a)*sin(d*x + c) - 512*a)*sqrt(a*sin(d*x + c) + a)/(d*cos(d*x + c) + d
*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 162, normalized size = 1.04

$$\frac{64\,\sqrt{2}\left(770\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-3185\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+5005\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3575\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1001\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right)\sqrt{a}}{5005\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] 64/5005*sqrt(2)*(770*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/
2*d*x + 1/2*c)^13 - 3185*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^11 + 5005*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c)^9 - 3575*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^7 + 1001*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 1/2*d*x + 1/2*c)^5)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^4 \sin(c+dx) \left(a+a\sin(c+dx)\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^(3/2), x)
```

3.455 $\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=199

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a+a\sin(c+dx)}}-\frac{34a^2\cos(c+dx)\sin^3(c+dx)}{63d\sqrt{a+a\sin(c+dx)}}-\frac{2a^2\cos(}{9d\sqrt{a}}$$

```
[Out] -2*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+16/105*cos(
d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-14/45*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/
2)-34/63*a^2*cos(d*x+c)*sin(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-2/9*a^2*cos(d
*x+c)*sin(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+388/315*a*cos(d*x+c)*(a+a*sin(d
*x+c))^(1/2)/d
```

Rubi [A]

time = 0.46, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2960, 2842, 21, 2849, 2838, 2830, 2725, 3125, 3055, 3060, 2852, 212}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}-\frac{2a^2\sin^4(c+dx)\cos(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}-\frac{34a^2\sin^3(c+dx)\cos(c+dx)}{63d\sqrt{a\sin(c+dx)+a}}-\frac{14a^2\cos(c+dx)}{45d\sqrt{a\sin(c+dx)+a}}+\frac{16\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{105d}+\frac{388a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d - (
14*a^2*Cos[c + d*x])/(45*d*Sqrt[a + a*Sin[c + d*x]]) - (34*a^2*Cos[c + d*x]
*Sin[c + d*x]^3)/(63*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[
c + d*x]^4)/(9*d*Sqrt[a + a*Sin[c + d*x]]) + (388*a*Cos[c + d*x]*Sqrt[a + a
*Sin[c + d*x]])/(315*d) + (16*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(105
*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc(c+d$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4\cos(c+dx)(a+a\,\mathrm{s}}{5d}$$

$$= -\frac{2a^2\cos(c+dx)\sin^4(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} + \frac{4a\cos(c+dx)\sqrt{a+}}{5d}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx}{63d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{6a^2\cos(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{34a^2\cos(c+dx)\sin^3(c+dx}{63d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{6a^2\cos(}{5d\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{14a^2\cos}{45d\sqrt{a+a}}$$

Mathematica [A]
time = 0.51, size = 219, normalized size = 1.10

$$\frac{(a(1+\sin(c+dx)))^{3/2}\left(1260\cos\left(\frac{1}{2}(c+dx)\right)+1470\cos\left(\frac{3}{2}(c+dx)\right)-126\cos\left(\frac{5}{2}(c+dx)\right)+135\cos\left(\frac{7}{2}(c+dx)\right)-35\cos\left(\frac{9}{2}(c+dx)\right)-2520\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2520\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-1260\sin\left(\frac{1}{2}(c+dx)\right)+1470\sin\left(\frac{3}{2}(c+dx)\right)+126\sin\left(\frac{5}{2}(c+dx)\right)+135\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(3/2)*(1260*Cos[(c + d*x)/2] + 1470*Cos[(3*(c + d*x
))/2] - 126*Cos[(5*(c + d*x))/2] + 135*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c
+ d*x))/2] - 2520*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2520*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1260*Sin[(c + d*x)/2] + 1470*Sin[
(3*(c + d*x))/2] + 126*Sin[(5*(c + d*x))/2] + 135*Sin[(7*(c + d*x))/2] + 35
*Sin[(9*(c + d*x))/2]))/(2520*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 7.03, size = 159, normalized size = 0.80

method	result

default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(315a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)+35(a-a\sin(dx+c))^{\frac{9}{2}}-225a\right.}{315a^{3}\cos(dx+c)\sqrt{a+a\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/315*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(315*a^(9/2)*arctanh((a-a*s
in(d*x+c))^(1/2)/a^(1/2))+35*(a-a*sin(d*x+c))^(9/2)-225*a*(a-a*sin(d*x+c))^
(7/2)+441*a^2*(a-a*sin(d*x+c))^(5/2)-105*a^3*(a-a*sin(d*x+c))^(3/2)-315*a^4
*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c), x)
```

Fricas [A]
time = 0.38, size = 332, normalized size = 1.67

$$\frac{315\left(a\cos\left(dx+c\right)+a\sin\left(dx+c\right)+a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(35a\cos\left(dx+c\right)^{5}-50a\cos\left(dx+c\right)^{4}-46a\cos\left(dx+c\right)^{3}-118a\cos\left(dx+c\right)^{2}-158a\cos\left(dx+c\right)-\left(35a\cos\left(dx+c\right)^{4}+85a\cos\left(dx+c\right)^{3}+39a\cos\left(dx+c\right)^{2}+157a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)-a\right)\sqrt{a\sin\left(dx+c\right)+a}}{630\left(d\cos\left(dx+c\right)+d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/630*(315*(a*cos(d*x + c) + a*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c
)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x +
 c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x +
c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) -
 1)) - 4*(35*a*cos(d*x + c)^5 - 50*a*cos(d*x + c)^4 - 46*a*cos(d*x + c)^3 -
 118*a*cos(d*x + c)^2 - 158*a*cos(d*x + c) - (35*a*cos(d*x + c)^4 + 85*a*co
s(d*x + c)^3 + 39*a*cos(d*x + c)^2 + 157*a*cos(d*x + c) - a)*sin(d*x + c) -
 a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c) + d*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 225, normalized size = 1.13

$$\frac{\sqrt{2}\left(2240\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-7200\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7056\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-840\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-315\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-1260\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] 1/630*sqrt(2)*(2240*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^9 - 7200*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
1/2*d*x + 1/2*c)^7 + 7056*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c)^5 - 840*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*
pi + 1/2*d*x + 1/2*c)^3 - 315*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi
+ 1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(
cos(-1/4*pi + 1/2*d*x + 1/2*c)) - 1260*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)
)*sin(-1/4*pi + 1/2*d*x + 1/2*c))*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x), x)
```

3.456 $\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=178

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d}+\frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}}+\frac{69a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{35d}+\frac{4}{}$$

```
[Out] -3*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d+4/35*cos(d*
x+c)*(a+a*sin(d*x+c))^(3/2)/d-cot(d*x+c)*(a+a*sin(d*x+c))^(3/2)/d-2/7*cos(d
*x+c)*(a+a*sin(d*x+c))^(5/2)/a/d+171/35*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(
1/2)+69/35*a*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.42, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2960, 2838, 2830, 2726, 2725, 3123, 3055, 3060, 2852, 212}

$$-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{d}+\frac{171a^2\cos(c+dx)}{35d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{5/2}}{7ad}+\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{35d}+\frac{69a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{35d}-\frac{\cot(c+dx)(a\sin(c+dx)+a)^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/d + (
171*a^2*Cos[c + d*x])/(35*d*Sqrt[a + a*Sin[c + d*x]]) + (69*a*Cos[c + d*x]*
Sqrt[a + a*Sin[c + d*x]])/(35*d) + (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/
2))/(35*d) - (Cot[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/d - (2*Cos[c + d*x]*
(a + a*Sin[c + d*x])^(5/2))/(7*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
```

```
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^2(c$$

$$= -\frac{\cot(c+dx)(a+a\sin(c+dx))^{3/2}}{d} - \frac{2\cos(c+dx)}{}$$

$$= \frac{7a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} + \frac{4\cos(c+dx)}{}$$

$$= \frac{19a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a\,\mathrm{si}}}{35d}$$

$$= \frac{171a^2\cos(c+dx)}{35d\sqrt{a+a\sin(c+dx)}} + \frac{69a\cos(c+dx)\sqrt{a+a}}{35d}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{171a^2}{35d\sqrt{a+}}$$

Mathematica [A]
time = 0.93, size = 283, normalized size = 1.59

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(840\cos\left(\frac{1}{2}(c+dx)\right)-574\cos\left(\frac{3}{2}(c+dx)\right)+30\cos\left(\frac{5}{2}(c+dx)\right)-21\cos\left(\frac{7}{2}(c+dx)\right)+5\cos\left(\frac{9}{2}(c+dx)\right)-840\sin\left(\frac{1}{2}(c+dx)\right)+420\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-420\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-574\sin\left(\frac{3}{2}(c+dx)\right)-30\sin\left(\frac{5}{2}(c+dx)\right)-21\sin\left(\frac{7}{2}(c+dx)\right)-5\sin\left(\frac{9}{2}(c+dx)\right)\right)}{140d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)-\sec\left(\frac{1}{4}(c+dx)\right)\right)\left(\csc\left(\frac{1}{4}(c+dx)\right)+\sec\left(\frac{1}{4}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/140*(a*Csc[(c + d*x)/2]^4*Sqrt[a*(1 + Sin[c + d*x])]*(840*Cos[(c + d*x)/
2] - 574*Cos[(3*(c + d*x))/2] + 30*Cos[(5*(c + d*x))/2] - 21*Cos[(7*(c + d*
x))/2] + 5*Cos[(9*(c + d*x))/2] - 840*Sin[(c + d*x)/2] + 420*Log[1 + Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 420*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*Sin[c + d*x] - 574*Sin[(3*(c + d*x))/2] - 30*Sin[(5*(c
+ d*x))/2] - 21*Sin[(7*(c + d*x))/2] - 5*Sin[(9*(c + d*x))/2]))/(d*(1 + Cot
[(c + d*x)/2])*(Csc[(c + d*x)/4] - Sec[(c + d*x)/4])*(Csc[(c + d*x)/4] + Se
c[(c + d*x)/4]))
```

Maple [**A**]
time = 5.34, size = 180, normalized size = 1.01

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sin(dx+c)\left(10(a-a\sin(dx+c))^{\frac{7}{2}}\sqrt{a}-56(a-a\sin(dx+c))^{\frac{5}{2}}a^{\frac{3}{2}}+70(a-a\sin(dx+\right.\right.}{35a^{\frac{5}{2}}\sin(dx+c)\cos(dx+c)\ \sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/35*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(sin(d*x+c)*(10*(a-a*
sin(d*x+c))^(7/2)*a^(1/2)-56*(a-a*sin(d*x+c))^(5/2)*a^(3/2)+70*(a-a*sin(d*x
+c))^(3/2)*a^(5/2)+140*(a-a*sin(d*x+c))^(1/2)*a^(7/2)-105*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2))*a^4)-35*(a-a*sin(d*x+c))^(1/2)*a^(7/2))/sin(d*x+c)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(154) = 308$.
time = 0.37, size = 360, normalized size = 2.02

$$\frac{105\left(a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(10\,a\cos\left(dx+c\right)^{5}-16\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{3}-120\,a\cos\left(dx+c\right)^{2}+33\,a\cos\left(dx+c\right)-\left(10\,a\cos\left(dx+c\right)^{4}+26\,a\cos\left(dx+c\right)^{3}+18\,a\cos\left(dx+c\right)^{2}+138\,a\cos\left(dx+c\right)+171\,a\right)\sin\left(dx+c\right)+171\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{140\left(d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/140*(105*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(
a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*
x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqr
t(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x
 + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - cos(d*x + c) - 1)) + 4*(10*a*cos(d*x + c)^5 - 16*a*cos(d*x + c)^4 -
8*a*cos(d*x + c)^3 - 120*a*cos(d*x + c)^2 + 33*a*cos(d*x + c) - (10*a*cos(d
*x + c)^4 + 26*a*cos(d*x + c)^3 + 18*a*cos(d*x + c)^2 + 138*a*cos(d*x + c)
+ 171*a)*sin(d*x + c) + 171*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^2
- (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 243, normalized size = 1.37

$$-\frac{\sqrt{2}\left(320\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-896\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+560\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+560\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{140\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}\right)\sqrt{a}}{140\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/140*sqrt(2)*(320*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^7 - 896*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1
/2*d*x + 1/2*c)^5 + 560*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi +
 1/2*d*x + 1/2*c)^3 + 105*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/
2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c)) + 560*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c) + 140*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin
(-1/4*pi + 1/2*d*x + 1/2*c)/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1))*sqrt(
a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\sin\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^2, x)
```

3.457 $\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=186

$$\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d}+\frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d}-\frac{3a\,\mathrm{co}}{}$$

```
[Out] 9/4*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-2/5*cos(d*
x+c)*(a+a*sin(d*x+c))^(3/2)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(3
/2)/d+73/20*a^2*cos(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*(a+a*s
in(d*x+c))^(1/2)/d-3/4*a*cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2960, 2830, 2726, 2725, 3123, 3054, 3060, 2852, 212}

$$\frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4d}+\frac{73a^2\cos(c+dx)}{20d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{5d}-\frac{2\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}-\frac{3a\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}-\frac{\cot(c+dx)\csc(c+dx)(a\sin(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (9*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*d)
+ (73*a^2*Cos[c + d*x])/(20*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]
*Sqrt[a + a*Sin[c + d*x]])/(5*d) - (3*a*Cot[c + d*x]*Sqrt[a + a*Sin[c + d*x
]])/(4*d) - (2*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(5*d) - (Cot[c + d*
x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
```

```
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \sin(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^3(c+$$

$$= -\frac{2\cos(c+dx)(a+a\sin(c+dx))^{3/2}}{5d} - \frac{\cot(c+dx)\,c}{}$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{5d} - \frac{3a\cot(c+d}{}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin}}{5d}$$

$$= \frac{73a^2\cos(c+dx)}{20d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin}}{5d}$$

$$= \frac{9a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} + \frac{73a^2\cos}{20d\sqrt{a+a}}$$

Mathematica [A]
time = 0.77, size = 322, normalized size = 1.73

$$-\frac{a\csc^7\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-118\cos\left(\frac{1}{2}(c+dx)\right)+130\cos\left(\frac{3}{2}(c+dx)\right)+36\cos\left(\frac{5}{2}(c+dx)\right)-10\cos\left(\frac{7}{2}(c+dx)\right)+2\cos\left(\frac{9}{2}(c+dx)\right)-45\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+45\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+45\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-45\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+118\sin\left(\frac{1}{2}(c+dx)\right)+130\sin\left(\frac{3}{2}(c+dx)\right)-36\sin\left(\frac{5}{2}(c+dx)\right)-10\sin\left(\frac{7}{2}(c+dx)\right)-2\sin\left(\frac{9}{2}(c+dx)\right)\right)}{20d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/20*(a*Csc[(c + d*x)/2]^7*Sqrt[a*(1 + Sin[c + d*x])]*(-118*Cos[(c + d*x)/
2] + 130*Cos[(3*(c + d*x))/2] + 36*Cos[(5*(c + d*x))/2] - 10*Cos[(7*(c + d*
x))/2] + 2*Cos[(9*(c + d*x))/2] - 45*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 45*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 45*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 45*Cos[2*(c + d*x)]*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 118*Sin[(c + d*x)/2] + 130*Sin[(
3*(c + d*x))/2] - 36*Sin[(5*(c + d*x))/2] - 10*Sin[(7*(c + d*x))/2] - 2*Sin
[(9*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c
+ d*x)/4]^2)^2)
```

Maple [A]
time = 5.41, size = 178, normalized size = 0.96

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-8(-a(\sin(dx+c)-1))^{\frac{5}{2}}(\sin^2(dx+c))\sqrt{a}+40a^{\frac{3}{2}}(-a(\sin(dx+c)-1))^{\frac{3}{2}}(\sin^2(dx+\right.}{20a^{\frac{3}{2}}\sin(dx+c)^2\cos(dx-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/20*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-8*(-a*(sin(d*x+c)-1))^(5/2)
*sin(d*x+c)^2*a^(1/2)+40*a^(3/2)*(-a*(sin(d*x+c)-1))^(3/2)*sin(d*x+c)^2+45*
arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+35*(-a*(sin(d*x
+c)-1))^(3/2)*a^(3/2)-45*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/a^(3/2)/sin(d*x
+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(158) = 316$.
time = 0.36, size = 404, normalized size = 2.17

$$\frac{45\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(8a\cos(dx+c)^5-16a\cos(dx+c)^4+16a\cos(dx+c)^3+99a\cos(dx+c)^2-14a\cos(dx+c)-\left(8a\cos(dx+c)^4+24a\cos(dx+c)^3+40a\cos(dx+c)^2-59a\cos(dx+c)-73a\right)\sin(dx+c)-73a\right)\sqrt{a\sin(dx+c)+a}}{80\left(d\cos(dx+c)^3+d\cos(dx+c)^2-d\cos(dx+c)+\left(d\cos(dx+c)^2-d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/80*(45*(a*cos(d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x
 + c)^2 - a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x
+ c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x +
c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x +
c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x +
c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(8*a*cos(
d*x + c)^5 - 16*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^3 + 99*a*cos(d*x + c)^
2 - 14*a*cos(d*x + c) - (8*a*cos(d*x + c)^4 + 24*a*cos(d*x + c)^3 + 40*a*co
s(d*x + c)^2 - 59*a*cos(d*x + c) - 73*a)*sin(d*x + c) - 73*a)*sqrt(a*sin(d*
x + c) + a))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos
(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 218, normalized size = 1.17

$$\frac{\sqrt{2}\left(128\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-320\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{20\left(14\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,a\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}\right)\sqrt{a}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/80*sqrt(2)*(128*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^5 - 320*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^3 + 45*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c)) - 20*(14*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3 - 9*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^2)*sqrt
(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{{\sin\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^3, x)
```

3.458 $\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d}-\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}}-\frac{2a\cos(c+dx)}{}$$

```
[Out] 37/8*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/3*cot(d
*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(3/2)/d-8/3*a^2*cos(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)+29/24*a^2*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*a*cos(d*x+c)
*(a+a*sin(d*x+c))^(1/2)/d-1/4*a*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.32, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2797, 2726, 2725, 3123, 3054, 3059, 2852, 212}

$$\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8d}-\frac{8a^2\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{29a^2\cot(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}-\frac{2a\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a\sin(c+dx)+a)^{3/2}}{3d}-\frac{a\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*d)
 - (8*a^2*Cos[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) + (29*a^2*Cot[c + d*
x])/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (2*a*Cos[c + d*x]*Sqrt[a + a*Sin[c +
d*x]])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(4*d)
 - (Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2))/(3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
```

```
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
```

```
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int (a+a\sin(c+dx))^{3/2}\,dx + \int \csc^4(c+dx)(a+a\sin(c+dx))^{3}$$

$$= -\frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3d}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c}{}$$

$$= -\frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{29a^2\cot(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{2a\cos(c}{}$$

$$= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{8a^2\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}$$

Mathematica [A]
time = 0.92, size = 334, normalized size = 1.70

$$-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(-276\cos\left(\frac{1}{2}(c+dx)\right)+326\cos\left(\frac{3}{2}(c+dx)\right)+78\cos\left(\frac{5}{2}(c+dx)\right)-72\cos\left(\frac{7}{2}(c+dx)\right)+8\cos\left(\frac{9}{2}(c+dx)\right)+276\sin\left(\frac{1}{2}(c+dx)\right)-333\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+333\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+326\sin\left(\frac{3}{2}(c+dx)\right)-78\sin\left(\frac{5}{2}(c+dx)\right)+111\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-111\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-72\sin\left(\frac{7}{2}(c+dx)\right)-8\sin\left(\frac{9}{2}(c+dx)\right)\right)}{24d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/24*(a*Csc[(c + d*x)/2]^10*Sqrt[a*(1 + Sin[c + d*x])]*(-276*Cos[(c + d*x)
/2] + 326*Cos[(3*(c + d*x))/2] + 78*Cos[(5*(c + d*x))/2] - 72*Cos[(7*(c + d
*x))/2] + 8*Cos[(9*(c + d*x))/2] + 276*Sin[(c + d*x)/2] - 333*Log[1 + Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 333*Log[1 - Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*Sin[c + d*x] + 326*Sin[(3*(c + d*x))/2] - 78*Sin[(5*(c
 + d*x))/2] + 111*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d
*x)] - 111*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] -
```

```
72*Sin[(7*(c + d*x))/2] - 8*Sin[(9*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2]
)*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3)
```

Maple [A]
time = 6.11, size = 196, normalized size = 0.99

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(16(-a(\sin(dx+c)-1))^{\frac{3}{2}}a^{\frac{3}{2}}(\sin^3(dx+c))-96a^{\frac{5}{2}}\sqrt{-a(\sin(dx+c)-1)}\,(\right.}{24a^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(3/2)*(16*(-a*(sin(d*x+c)-1
))^(3/2)*a^(3/2)*sin(d*x+c)^3-96*a^(5/2)*(-a*(sin(d*x+c)-1))^(1/2)*sin(d*x+
c)^3+111*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+15*(-a
*(sin(d*x+c)-1))^(5/2)*a^(1/2)-8*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)-15*(-a*(
sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^4, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(169) = 338$.
time = 0.36, size = 424, normalized size = 2.15

$$\frac{111\left(a\cos(dx+c)^4-2a\cos(dx+c)^2-\left(a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(16a\cos(dx+c)^5-64a\cos(dx+c)^4-17a\cos(dx+c)^3+165a\cos(dx+c)^2+9a\cos(dx+c)-\left(16a\cos(dx+c)^4+80a\cos(dx+c)^3+63a\cos(dx+c)^2-102a\cos(dx+c)-93a\right)\sin(dx+c)-93a\right)\sqrt{a\sin(dx+c)+a}}{96\left(d\cos(dx+c)^4-2d\cos(dx+c)^2-\left(d\cos(dx+c)^3+d\cos(dx+c)^2-d\cos(dx+c)-d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(111*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a*co
s(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x
```

```
 + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d
*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*
x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d
*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x +
c) - 1)) - 4*(16*a*cos(d*x + c)^5 - 64*a*cos(d*x + c)^4 - 17*a*cos(d*x + c)
^3 + 165*a*cos(d*x + c)^2 + 9*a*cos(d*x + c) - (16*a*cos(d*x + c)^4 + 80*a*
cos(d*x + c)^3 + 63*a*cos(d*x + c)^2 - 102*a*cos(d*x + c) - 93*a)*sin(d*x +
 c) - 93*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c)
 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 246, normalized size = 1.25

$$-\frac{\sqrt{2}\left(128\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-111\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-384\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4\left(60\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-16\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}\right)\sqrt{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/96*sqrt(2)*(128*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*
d*x + 1/2*c)^3 - 111*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c)) - 384*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4
*pi + 1/2*d*x + 1/2*c) - 4*(60*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1
/4*pi + 1/2*d*x + 1/2*c)^5 - 16*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^3 - 15*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3)*sqr
t(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^4, x)
```

3.459 $\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}+\frac{19a^2\cot(c+dx)\,\mathrm{c}}{32d\sqrt{a+a\sin(}}$$

```
[Out] 21/64*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/4*cot(
d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(3/2)/d-2*a^2*cos(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+149/64*a^2*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+19/32*a^2*cot(d*
x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/8*a*cot(d*x+c)*csc(d*x+c)^2*(a+a
*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.46, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2960, 2842, 21, 2852, 212, 3123, 3054, 3059, 2851}

$$\frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64d}-\frac{2a^2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{149a^2\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}+\frac{19a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^3(c+dx)(a\sin(c+dx)+a)^{3/2}}{4d}-\frac{a\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (21*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*d
) - (2*a^2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + (149*a^2*Cot[c + d*
x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (19*a^2*Cot[c + d*x]*Csc[c + d*x])/(3
2*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + a*S
in[c + d*x]])/(8*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/
2))/(4*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
```

)

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^5(c+$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)(}{4d}$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^2(c+dx)}{8d}$$

$$= -\frac{2a^2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{19a^2\cot(c+dx)\csc(c+d}{32d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos}{d\sqrt{a+a\sin}}$$

$$= -\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{2a^2\cos}{d\sqrt{a+a\sin}}$$

$$= \frac{21a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{2a^2\cos(}{d\sqrt{a+a\sin}}$$

Mathematica [A]
time = 0.97, size = 392, normalized size = 1.91

$$-\frac{a\csc^{13}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sin(c+dx))}\left(1486\cos\left(\frac{1}{2}(c+dx)\right)-1030\cos\left(\frac{3}{2}(c+dx)\right)-754\cos\left(\frac{5}{2}(c+dx)\right)+426\cos\left(\frac{7}{2}(c+dx)\right)+128\cos\left(\frac{9}{2}(c+dx)\right)-63\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+84\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-21\cos(4(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+63\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-84\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+21\cos(4(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-1486\sin\left(\frac{1}{2}(c+dx)\right)-1030\sin\left(\frac{3}{2}(c+dx)\right)+754\sin\left(\frac{5}{2}(c+dx)\right)+426\sin\left(\frac{7}{2}(c+dx)\right)-128\sin\left(\frac{9}{2}(c+dx)\right)\right)}{64d\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/64*(a*Csc[(c + d*x)/2]^13*Sqrt[a*(1 + Sin[c + d*x])]*(1486*Cos[(c + d*x)
/2] - 1030*Cos[(3*(c + d*x))/2] - 754*Cos[(5*(c + d*x))/2] + 426*Cos[(7*(c
+ d*x))/2] + 128*Cos[(9*(c + d*x))/2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] + 84*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 21*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6
3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 84*Cos[2*(c + d*x)]*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 21*Cos[4*(c + d*x)]*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] - 1486*Sin[(c + d*x)/2] - 1030*Sin[(3*(c + d
*x))/2] + 754*Sin[(5*(c + d*x))/2] + 426*Sin[(7*(c + d*x))/2] - 128*Sin[(9*
(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*
x)/4]^2)^4)
```

Maple [A]
time = 6.88, size = 188, normalized size = 0.92

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-128\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,a^{\frac{7}{2}}\left(\sin^{4}(dx+c)\right)+21\,\mathrm{arctanh}\left(\frac{\sqrt{-a\,(\mathrm{si}}{}\right.\right.}{64a^{\frac{5}{2}}\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/64*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-128*(-a*(sin(d*x+c)-1))^(1/
2)*a^(7/2)*sin(d*x+c)^4+21*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*s
in(d*x+c)^4-149*(-a*(sin(d*x+c)-1))^(7/2)*a^(1/2)+461*(-a*(sin(d*x+c)-1))^(
5/2)*a^(3/2)-435*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)+107*(-a*(sin(d*x+c)-1))^
(1/2)*a^(7/2))/a^(5/2)/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^5, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 460 vs. 2(179) = 358.
time = 0.38, size = 460, normalized size = 2.24

$$\frac{21\left(a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}-2\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+\left(\cos(dx+c)+3\right)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(128\,a\cos\left(dx+c\right)^{5}+277\,a\cos\left(dx+c\right)^{4}-242\,a\cos\left(dx+c\right)^{3}-500\,a\cos\left(dx+c\right)^{2}+130\,a\cos\left(dx+c\right)-\left(128\,a\cos\left(dx+c\right)^{4}-149\,a\cos\left(dx+c\right)^{3}-391\,a\cos\left(dx+c\right)^{2}+109\,a\cos\left(dx+c\right)+239\,a\right)\sin\left(dx+c\right)+239\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{256\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}-2\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(21*(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*c
os(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a
)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*
(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqr
t(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*
```

```
cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos
(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(128*a*cos(d*x + c)^
5 + 277*a*cos(d*x + c)^4 - 242*a*cos(d*x + c)^3 - 500*a*cos(d*x + c)^2 + 13
0*a*cos(d*x + c) - (128*a*cos(d*x + c)^4 - 149*a*cos(d*x + c)^3 - 391*a*cos
(d*x + c)^2 + 109*a*cos(d*x + c) + 239*a)*sin(d*x + c) + 239*a)*sqrt(a*sin(
d*x + c) + a))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 -
2*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 246, normalized size = 1.20

$$\frac{\sqrt{2}\left(21\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+512\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{4\left(1192\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-1844\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+870\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-107\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{4}}\right)\sqrt{a}}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/256*sqrt(2)*(21*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x +
1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c)) + 512*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi
 + 1/2*d*x + 1/2*c) + 4*(1192*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^7 - 1844*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^5 + 870*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*si
n(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 107*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^4)
*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^5,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^5, x)
```

3.460 $\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=215

$$-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d}+\frac{91a^2\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}+\frac{73a^2\cot(c+dx)\csc(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}+\frac{31a^2\,\mathrm{co}}{80d}$$

```
[Out] -165/128*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/5*c
ot(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^(3/2)/d+91/128*a^2*cot(d*x+c)/d/(a+
a*sin(d*x+c))^(1/2)+73/64*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2
)+31/80*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-3/40*a*cot(d*x
+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.53, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2960, 2841, 21, 2852, 212, 3123, 3054, 3059, 2851}

$$-\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128d}+\frac{91a^2\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}+\frac{31a^2\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}+\frac{73a^2\cot(c+dx)\csc(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^4(c+dx)(a\sin(c+dx)+a)^{3/2}}{5d}-\frac{3a\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-165*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(12
8*d) + (91*a^2*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) + (73*a^2*Cot
[c + d*x]*Csc[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (31*a^2*Cot[c + d
*x]*Csc[c + d*x]^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[c + d*x]*Csc
[c + d*x]^3*Sqrt[a + a*Sin[c + d*x]])/(40*d) - (Cot[c + d*x]*Csc[c + d*x]^4
*(a + a*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
```

```
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^2(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^6(c$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)(}{5d}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^3(c+d}{40}$$

$$= -\frac{a^2\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{31a^2\cot(c+dx)\csc^2(c+}{80d\sqrt{a+a\sin(c+dx}}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} - \frac{a^2\cot}{d\sqrt{a+a}}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2}{128d\sqrt{a}}$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{d} + \frac{91a^2}{128d\sqrt{a}}$$

$$= -\frac{165a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128d} + \frac{91}{128d\sqrt{}}$$

Mathematica [A]
time = 1.26, size = 404, normalized size = 1.88

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/640*(a*Csc[(c + d*x)/2]^16*Sqrt[a*(1 + Sin[c + d*x])]*(1380*Cos[(c + d*x
)/2] + 320*Cos[(3*(c + d*x))/2] + 1296*Cos[(5*(c + d*x))/2] + 2010*Cos[(7*(
c + d*x))/2] - 910*Cos[(9*(c + d*x))/2] - 1380*Sin[(c + d*x)/2] + 8250*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 8250*Log[1 - Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 320*Sin[(3*(c + d*x))/2] - 12
96*Sin[(5*(c + d*x))/2] - 4125*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[3*(c + d*x)] + 4125*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3
```

```
*(c + d*x)] + 2010*Sin[(7*(c + d*x))/2] + 910*Sin[(9*(c + d*x))/2] + 825*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 825*Log[1 - C
os[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(1 + Cot[(c + d*x
)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^5)
```

Maple [A]
time = 6.42, size = 180, normalized size = 0.84

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-825\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{5}\left(\sin^{5}(dx+c)\right)+455(-a(\sin(a}{640a^{\frac{7}{2}}\sin(dx+c)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(7/2)*(-825*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^5+455*(-a*(sin(d*x+c)-1))^(9/2)
*a^(1/2)-2550*(-a*(sin(d*x+c)-1))^(7/2)*a^(3/2)+4992*(-a*(sin(d*x+c)-1))^(5
/2)*a^(5/2)-3850*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)+825*(-a*(sin(d*x+c)-1))^
(1/2)*a^(9/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^6, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. 2(187) = 374.
time = 0.37, size = 488, normalized size = 2.27

$$\frac{825\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{5}+a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{3}-2\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(455\,a\cos\left(dx+c\right)^{5}-275\,a\cos\left(dx+c\right)^{4}-982\,a\cos\left(dx+c\right)^{3}+174\,a\cos\left(dx+c\right)^{2}+399\,a\cos\left(dx+c\right)-\left(455\,a\cos\left(dx+c\right)^{4}+730\,a\cos\left(dx+c\right)^{3}-252\,a\cos\left(dx+c\right)^{2}-426\,a\cos\left(dx+c\right)-27\,a\right)\sin\left(dx+c\right)-27\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{2560\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{3}-2\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(825*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - (
a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)
```

```
^2 + a*cos(d*x + c) + a)*sin(d*x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
 4*(455*a*cos(d*x + c)^5 - 275*a*cos(d*x + c)^4 - 982*a*cos(d*x + c)^3 + 17
4*a*cos(d*x + c)^2 + 399*a*cos(d*x + c) - (455*a*cos(d*x + c)^4 + 730*a*cos
(d*x + c)^3 - 252*a*cos(d*x + c)^2 - 426*a*cos(d*x + c) - 27*a)*sin(d*x + c
) - 27*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^5 + d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 248, normalized size = 1.15

$$-\frac{\sqrt{2}\left(825\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-\frac{4\left(7280\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-20400\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+19968\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-7700\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+825\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}}\right)\sqrt{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/2560*sqrt(2)*(825*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*
pi + 1/2*d*x + 1/2*c)) - 4*(7280*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^9 - 20400*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 19968*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 7700*a*sgn(cos(-1/4*pi + 1/2*d*x + 1
/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 825*a*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^
2 - 1)^5)*sqrt(a)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^6,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^6, x)
```

3.461 $\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=253

$$-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d}-\frac{179a^2\cot(c+dx)}{512d\sqrt{a+a\sin(c+dx)}}+\frac{111a^2\cot(c+dx)\csc(c+dx)}{256d\sqrt{a+a\sin(c+dx)}}+\frac{239a}{32}$$

[Out] -179/512*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/6*cot(d*x+c)*csc(d*x+c)^5*(a+a*sin(d*x+c))^(3/2)/d-179/512*a^2*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+111/256*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+239/320*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+137/480*a^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/20*a*cot(d*x+c)*csc(d*x+c)^4*(a+a*sin(d*x+c))^(1/2)/d

Rubi [A]
time = 0.61, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2960, 2841, 21, 2851, 2852, 212, 3123, 3054, 3059}

$$-\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{512d}-\frac{179a^2\cot(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}+\frac{137a^2\cot(c+dx)\csc^3(c+dx)}{480d\sqrt{a\sin(c+dx)+a}}+\frac{239a^2\cot(c+dx)\csc^2(c+dx)}{320d\sqrt{a\sin(c+dx)+a}}+\frac{111a^2\cot(c+dx)\csc(c+dx)}{256d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^5(c+dx)(a\sin(c+dx)+a)^{3/2}}{6d}-\frac{a\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{20d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]

[Out] (-179*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(512*d) - (179*a^2*Cot[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (111*a^2*Cot[c + d*x]*Csc[c + d*x])/(256*d*Sqrt[a + a*Sin[c + d*x]]) + (239*a^2*Cot[c + d*x]*Csc[c + d*x]^2)/(320*d*Sqrt[a + a*Sin[c + d*x]]) + (137*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(480*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a + a*Sin[c + d*x]])/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2))/(6*d)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^3(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^7(c+$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^5(c+d}{}$$

$$= -\frac{a^2\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^4(c+}{2}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx}{2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7a^2\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc(c+dx}{2d\sqrt{a+a\sin(c+dx)}}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{7a^2\cot}{4d\sqrt{a+a}}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2}{512d\sqrt{a+}}$$

$$= -\frac{7a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4d} - \frac{179a^2}{512d\sqrt{a+}}$$

$$= -\frac{179a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{512d} - \frac{179a}{512d\sqrt{a}}$$

Mathematica [**A**]
time = 1.91, size = 486, normalized size = 1.92

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^19*Sqrt[a*(1 + Sin[c + d*x])]*(25140*Cos[(c + d*x)/2] -
 71972*Cos[(3*(c + d*x))/2] - 42690*Cos[(5*(c + d*x))/2] - 5718*Cos[(7*(c +
 d*x))/2] + 18690*Cos[(9*(c + d*x))/2] - 5370*Cos[(11*(c + d*x))/2] - 26850
*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 40275*Cos[2*(c + d*x)]*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 16110*Cos[4*(c + d*x)]*Log[1 + C
```

```
os[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2685*Cos[6*(c + d*x)]*Log[1 + Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] + 26850*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]] - 40275*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] + 16110*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] -
2685*Cos[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 25140*
Sin[(c + d*x)/2] - 71972*Sin[(3*(c + d*x))/2] + 42690*Sin[(5*(c + d*x))/2]
- 5718*Sin[(7*(c + d*x))/2] - 18690*Sin[(9*(c + d*x))/2] - 5370*Sin[(11*(c
+ d*x))/2]))/(7680*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c +
d*x)/4]^2)^6)
```

Maple [A]
time = 7.29, size = 198, normalized size = 0.78

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(2685\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^7(\sin^6(dx+c))-2685(-a(\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/7680*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2685*arctanh((-a*(sin(d*x
+c)-1))^(1/2)/a^(1/2))*a^7*sin(d*x+c)^6-2685*(-a*(sin(d*x+c)-1))^(11/2)*a^(
3/2)+10095*(-a*(sin(d*x+c)-1))^(9/2)*a^(5/2)-7794*(-a*(sin(d*x+c)-1))^(7/2)
*a^(7/2)-10866*(-a*(sin(d*x+c)-1))^(5/2)*a^(9/2)+15215*(-a*(sin(d*x+c)-1))^
(3/2)*a^(11/2)-2685*(-a*(sin(d*x+c)-1))^(1/2)*a^(13/2))/a^(11/2)/sin(d*x+c)
^6/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^7, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(221) = 442$.
time = 0.37, size = 557, normalized size = 2.20

$$\frac{2685\left(a\cos(dx+c)^7+a\cos(dx+c)^6-3a\cos(dx+c)^5-3a\cos(dx+c)^4+3a\cos(dx+c)^3+3a\cos(dx+c)^2-a\cos(dx+c)+\left(a\cos(dx+c)^6-3a\cos(dx+c)^4+3a\cos(dx+c)^2-a\right)\sin(dx+c)-a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(2685a\cos(dx+c)^6-3330a\cos(dx+c)^5-5649a\cos(dx+c)^4+7188a\cos(dx+c)^3+6715a\cos(dx+c)^2-2578a\cos(dx+c)+\left(2685a\cos(dx+c)^5+6015a\cos(dx+c)^4+366a\cos(dx+c)^3-6822a\cos(dx+c)^2-107a\cos(dx+c)+2471a\right)\sin(dx+c)-2471a\right)\sqrt{a\sin(dx+c)+a}}{30720\left(d\cos(dx+c)^7+d\cos(dx+c)^6-3d\cos(dx+c)^5-3d\cos(dx+c)^4+3d\cos(dx+c)^3+3d\cos(dx+c)^2-d\cos(dx+c)+\left(d\cos(dx+c)^6-3d\cos(dx+c)^4+3d\cos(dx+c)^2-d\right)\sin(dx+c)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/30720*(2685*(a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*cos(d*x + c
) + (a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*sin(d*
x + c) - a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(
d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x
+ c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c
)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(2685*a*cos(d*x + c)^6 - 333
0*a*cos(d*x + c)^5 - 5649*a*cos(d*x + c)^4 + 7188*a*cos(d*x + c)^3 + 6715*a
*cos(d*x + c)^2 - 2578*a*cos(d*x + c) + (2685*a*cos(d*x + c)^5 + 6015*a*cos
(d*x + c)^4 + 366*a*cos(d*x + c)^3 - 6822*a*cos(d*x + c)^2 - 107*a*cos(d*x
+ c) + 2471*a)*sin(d*x + c) - 2471*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)^6 - 3
*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 278, normalized size = 1.10

$$-\frac{\sqrt{2}\left(2685\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\left(85920\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-161520\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+62352\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+43464\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-30430\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2685\,a\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{6}}\right)\sqrt{a}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/30720*sqrt(2)*(2685*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c)) + 4*(85920*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 161520*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 62352*a*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 43464*a*sgn(cos(-1/4*pi + 1/2*d
```

```
*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 30430*a*sgn(cos(-1/4*pi + 1
/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 2685*a*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*
x + 1/2*c)^2 - 1)^6)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}{\sin\left(c+d\,x\right)^7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^7,x)
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^7, x)
```

3.462 $\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=291

$$-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d}-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a+a\sin(c+dx)}}-\frac{57a^2\cot(c+dx)\csc(c+dx)}{512d\sqrt{a+a\sin(c+dx)}}+\frac{199a^2}{640}$$

```
[Out] -171/1024*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/7*
cot(d*x+c)*csc(d*x+c)^6*(a+a*sin(d*x+c))^(3/2)/d-171/1024*a^2*cot(d*x+c)/d/
(a+a*sin(d*x+c))^(1/2)-57/512*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^
(1/2)+199/640*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1237/224
0*a^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+9/40*a^2*cot(d*x+c)*
csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)-1/28*a*cot(d*x+c)*csc(d*x+c)^5*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.70, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2960, 2841, 21, 2851, 2852, 212, 3123, 3054, 3059}

$$-\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{1024d}-\frac{171a^2\cot(c+dx)}{1024d\sqrt{a\sin(c+dx)+a}}+\frac{9a^2\cot(c+dx)\csc^4(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{1237a^2\cot(c+dx)\csc^3(c+dx)}{2240d\sqrt{a\sin(c+dx)+a}}+\frac{199a^2\cot(c+dx)\csc^2(c+dx)}{640d\sqrt{a\sin(c+dx)+a}}-\frac{57a^2\cot(c+dx)\csc(c+dx)}{512d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^6(c+dx)(a\sin(c+dx)+a)^{3/2}}{7d}-\frac{a\cot(c+dx)\csc^5(c+dx)\sqrt{a\sin(c+dx)+a}}{28d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-171*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(10
24*d) - (171*a^2*Cot[c + d*x])/(1024*d*Sqrt[a + a*Sin[c + d*x]]) - (57*a^2*
Cot[c + d*x]*Csc[c + d*x])/(512*d*Sqrt[a + a*Sin[c + d*x]]) + (199*a^2*Cot[
c + d*x]*Csc[c + d*x]^2)/(640*d*Sqrt[a + a*Sin[c + d*x]]) + (1237*a^2*Cot[c
 + d*x]*Csc[c + d*x]^3)/(2240*d*Sqrt[a + a*Sin[c + d*x]]) + (9*a^2*Cot[c +
d*x]*Csc[c + d*x]^4)/(40*d*Sqrt[a + a*Sin[c + d*x]]) - (a*Cot[c + d*x]*Csc[
c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(28*d) - (Cot[c + d*x]*Csc[c + d*x]^6*
(a + a*Sin[c + d*x])^(3/2))/(7*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
```

```
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^4(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^8(c$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^6(c+}{}$$

$$= -\frac{a^2\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{a\cot(c+dx)\csc^5(c}{}$$

$$= -\frac{11a^2\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^2}{3d\sqrt{a+a\sin(c}}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+}{12d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{11a^2\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{11a^2\cot(c+dx)\csc(c+}{12d\sqrt{a+a\sin(c+d}}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{11a^2}{8d\sqrt{a+}}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171}{1024d\sqrt{}}$$

$$= -\frac{11a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8d} - \frac{171}{1024d\sqrt{}}$$

$$= -\frac{171a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{1024d} - \frac{17}{1024d}$$

Mathematica [A]
time = 3.46, size = 522, normalized size = 1.79

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*Csc[(c + d*x)/2]^22*Sqrt[a*(1 + Sin[c + d*x])]*(-306488*Cos[(c + d*x)/2]
 - 177170*Cos[(3*(c + d*x))/2] + 6566*Cos[(5*(c + d*x))/2] - 219540*Cos[(7*
```

(c + d*x))/2] + 33292*Cos[(9*(c + d*x))/2] - 3990*Cos[(11*(c + d*x))/2] + 11970*Cos[(13*(c + d*x))/2] + 306488*Sin[(c + d*x)/2] - 209475*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 209475*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 177170*Sin[(3*(c + d*x))/2] - 6566*Sin[(5*(c + d*x))/2] + 125685*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 125685*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 219540*Sin[(7*(c + d*x))/2] - 33292*Sin[(9*(c + d*x))/2] - 41895*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 41895*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 3990*Sin[(11*(c + d*x))/2] - 11970*Sin[(13*(c + d*x))/2] + 5985*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 5985*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(35840*d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^7)

Maple [A]
time = 7.15, size = 216, normalized size = 0.74

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(5985(-a(\sin(dx+c)-1))^{\frac{13}{2}}a^{\frac{5}{2}}-39900(-a(\sin(dx+c)-1))^{\frac{11}{2}}a^{\frac{7}{2}}+5985\,\mathrm{arctanh}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/35840*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(15/2)*(5985*(-a*(sin(d*x+c)-1))^(13/2)*a^(5/2)-39900*(-a*(sin(d*x+c)-1))^(11/2)*a^(7/2)+5985*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^9*sin(d*x+c)^7+98581*(-a*(sin(d*x+c)-1))^(9/2)*a^(9/2)-95232*(-a*(sin(d*x+c)-1))^(7/2)*a^(11/2)+1771*(-a*(sin(d*x+c)-1))^(5/2)*a^(13/2)+39900*(-a*(sin(d*x+c)-1))^(3/2)*a^(15/2)-5985*(-a*(sin(d*x+c)-1))^(1/2)*a^(17/2))/sin(d*x+c)^7/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^8, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(255) = 510$.
time = 0.40, size = 600, normalized size = 2.06

$$\frac{5985\left(a\cos(dx+c)^8-4a\cos(dx+c)^6+6a\cos(dx+c)^4-4a\cos(dx+c)^2-\left(a\cos(dx+c)^7+a\cos(dx+c)^6-3a\cos(dx+c)^5-3a\cos(dx+c)^4+3a\cos(dx+c)^3+3a\cos(dx+c)^2-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(5985a\cos(dx+c)^7+1995a\cos(dx+c)^6-6811a\cos(dx+c)^5-14633a\cos(dx+c)^4-5997a\cos(dx+c)^3+10097a\cos(dx+c)^2+1703a\cos(dx+c)-\left(5985a\cos(dx+c)^6+3990a\cos(dx+c)^5-2821a\cos(dx+c)^4+11812a\cos(dx+c)^3+5815a\cos(dx+c)^2-4282a\cos(dx+c)-2579a\right)\sin(dx+c)-2579a\right)\sqrt{a\sin(dx+c)+a}}{143360\left(d\cos(dx+c)^8-4d\cos(dx+c)^6+6d\cos(dx+c)^4-4d\cos(dx+c)^2-\left(d\cos(dx+c)^7+d\cos(dx+c)^6-3d\cos(dx+c)^5-3d\cos(dx+c)^4+3d\cos(dx+c)^3+3d\cos(dx+c)^2-d\cos(dx+c)-d\right)\sin(dx+c)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/143360*(5985*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4
- 4*a*cos(d*x + c)^2 - (a*cos(d*x + c)^7 + a*cos(d*x + c)^6 - 3*a*cos(d*x +
 c)^5 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - a*co
s(d*x + c) - a)*sin(d*x + c) + a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d
*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x
 + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x
 + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x
 + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(5985*
a*cos(d*x + c)^7 + 1995*a*cos(d*x + c)^6 - 6811*a*cos(d*x + c)^5 - 14633*a*
cos(d*x + c)^4 - 5997*a*cos(d*x + c)^3 + 10097*a*cos(d*x + c)^2 + 1703*a*co
s(d*x + c) - (5985*a*cos(d*x + c)^6 + 3990*a*cos(d*x + c)^5 - 2821*a*cos(d*
x + c)^4 + 11812*a*cos(d*x + c)^3 + 5815*a*cos(d*x + c)^2 - 4282*a*cos(d*x
+ c) - 2579*a)*sin(d*x + c) - 2579*a)*sqrt(a*sin(d*x + c) + a))/(d*cos(d*x
+ c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*
cos(d*x + c)^7 + d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4
 + 3*d*cos(d*x + c)^3 + 3*d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x +
c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 308, normalized size = 1.06

$$-\frac{\sqrt{2}\left(5985\sqrt{2}\,a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)+\frac{4\left(383040a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{13}-1276800a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1577296a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-761856a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+7084a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+79800a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-5985a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^7}\right)\sqrt{a}}{143360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

[Out] -1/143360*sqrt(2)*(5985*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)) + 4*(383040*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 - 1276800*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 + 1577296*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 761856*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 7084*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 79800*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 5985*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^7)*sqrt(a)/d

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^4 (a+a\sin(c+dx))^{3/2}}{\sin(c+dx)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^8,x)

[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^8, x)

3.463 $\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=329

$$-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{16384d}-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a+a\sin(c+dx)}}-\frac{529a^2\cot(c+dx)\csc(c+dx)}{8192d\sqrt{a+a\sin(c+dx)}}-5$$

```
[Out] -1587/16384*a^(3/2)*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d-1/
8*cot(d*x+c)*csc(d*x+c)^7*(a+a*sin(d*x+c))^(3/2)/d-1587/16384*a^2*cot(d*x+c
)/d/(a+a*sin(d*x+c))^(1/2)-529/8192*a^2*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)-529/10240*a^2*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+
8653/35840*a^2*cot(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)+1957/4480*a
^2*cot(d*x+c)*csc(d*x+c)^4/d/(a+a*sin(d*x+c))^(1/2)+83/448*a^2*cot(d*x+c)*c
sc(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-3/112*a*cot(d*x+c)*csc(d*x+c)^6*(a+a*s
in(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.79, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2960, 2841, 21, 2851, 2852, 212, 3123, 3054, 3059}

$$-\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{16384d}-\frac{1587a^2\cot(c+dx)}{16384d\sqrt{a\sin(c+dx)+a}}+\frac{83a^2\cot(c+dx)\csc^5(c+dx)}{448d\sqrt{a\sin(c+dx)+a}}+\frac{1957a^2\cot(c+dx)\csc^4(c+dx)}{4480d\sqrt{a\sin(c+dx)+a}}+\frac{8653a^2\cot(c+dx)\csc^3(c+dx)}{35840d\sqrt{a\sin(c+dx)+a}}-\frac{529a^2\cot(c+dx)\csc^2(c+dx)}{10240d\sqrt{a\sin(c+dx)+a}}-\frac{529a^2\cot(c+dx)\csc(c+dx)}{8192d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^7(c+dx)(a\sin(c+dx)+a)^{3/2}}{8d}-\frac{3a\cot(c+dx)\csc^6(c+dx)\sqrt{a\sin(c+dx)+a}}{112d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-1587*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(1
6384*d) - (1587*a^2*Cot[c + d*x])/(16384*d*Sqrt[a + a*Sin[c + d*x]]) - (529
*a^2*Cot[c + d*x]*Csc[c + d*x])/(8192*d*Sqrt[a + a*Sin[c + d*x]]) - (529*a^
2*Cot[c + d*x]*Csc[c + d*x]^2)/(10240*d*Sqrt[a + a*Sin[c + d*x]]) + (8653*a
^2*Cot[c + d*x]*Csc[c + d*x]^3)/(35840*d*Sqrt[a + a*Sin[c + d*x]]) + (1957*
a^2*Cot[c + d*x]*Csc[c + d*x]^4)/(4480*d*Sqrt[a + a*Sin[c + d*x]]) + (83*a^
2*Cot[c + d*x]*Csc[c + d*x]^5)/(448*d*Sqrt[a + a*Sin[c + d*x]]) - (3*a*Cot[
c + d*x]*Csc[c + d*x]^6*Sqrt[a + a*Sin[c + d*x]])/(112*d) - (Cot[c + d*x]*C
sc[c + d*x]^7*(a + a*Sin[c + d*x])^(3/2))/(8*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2841

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*(b*c - a*d)*Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*
d))), x] + Dist[b^2/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m -
2)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*c*(m - 2) - b*d*(m - 2*n - 4) - (b*c
*(m - 1) - a*d*(m + 2*n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] ||
(IntegerQ[m] && EqQ[c, 0]))
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
```

```
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx = \int \csc^5(c+dx)(a+a\sin(c+dx))^{3/2}\,dx + \int \csc^9(c+$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^7(c+$$

$$= -\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{3a\cot(c+dx)\csc^6(c$$

$$= -\frac{5a^2\cot(c+dx)\csc^2(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} - \frac{a^2\cot(c+dx)\csc^3(c}{4d\sqrt{a+a\sin(c+}$$

$$= -\frac{25a^2\cot(c+dx)\csc(c+dx)}{32d\sqrt{a+a\sin(c+dx)}} - \frac{5a^2\cot(c+dx)\csc^2}{8d\sqrt{a+a\sin(c+}$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+}{32d\sqrt{a+a\sin(c+dx}$$

$$= -\frac{75a^2\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} - \frac{25a^2\cot(c+dx)\csc(c+}{32d\sqrt{a+a\sin(c+dx}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{75a^2 c}{64d\sqrt{a+}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587}{16384d\sqrt{}}$$

$$= -\frac{75a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64d} - \frac{1587}{16384d\sqrt{}}$$

$$= -\frac{1587a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{16384d} - \frac{15}{16384d}$$

Mathematica [A]
time = 4.52, size = 604, normalized size = 1.84

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/573440*(a*Csc[(c + d*x)/2]^25*Sqrt[a*(1 + Sin[c + d*x])]*(3037258*Cos[(c
 + d*x)/2] + 10394286*Cos[(3*(c + d*x))/2] + 3369650*Cos[(5*(c + d*x))/2] +
 3171574*Cos[(7*(c + d*x))/2] - 2341070*Cos[(9*(c + d*x))/2] + 866502*Cos[(
11*(c + d*x))/2] - 37030*Cos[(13*(c + d*x))/2] - 111090*Cos[(15*(c + d*x))/
2] + 1944075*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3110520*Cos[2*(
c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 1555260*Cos[4*(c +
 d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 444360*Cos[6*(c + d*x
)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 55545*Cos[8*(c + d*x)]*Lo
g[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 1944075*Log[1 - Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 3110520*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] - 1555260*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 444360*Cos[6*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 55545*Cos[8*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d
*x)/2]] - 3037258*Sin[(c + d*x)/2] + 10394286*Sin[(3*(c + d*x))/2] - 336965
0*Sin[(5*(c + d*x))/2] + 3171574*Sin[(7*(c + d*x))/2] + 2341070*Sin[(9*(c +
 d*x))/2] + 866502*Sin[(11*(c + d*x))/2] + 37030*Sin[(13*(c + d*x))/2] - 11
1090*Sin[(15*(c + d*x))/2]))/(d*(1 + Cot[(c + d*x)/2])*(Csc[(c + d*x)/4]^2
- Sec[(c + d*x)/4]^2)^8)
```

Maple [A]
time = 7.01, size = 234, normalized size = 0.71

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-55545(-a(\sin(dx+c)-1))^{\frac{15}{2}}a^{\frac{7}{2}}+425845(-a(\sin(dx+c)-1))^{\frac{13}{2}}a^{\frac{9}{2}}-1418249\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/573440*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-55545*(-a*(sin(d*x+c)-
1))^(15/2)*a^(7/2)+425845*(-a*(sin(d*x+c)-1))^(13/2)*a^(9/2)-1418249*(-a*(s
in(d*x+c)-1))^(11/2)*a^(11/2)+2509197*(-a*(sin(d*x+c)-1))^(9/2)*a^(13/2)-21
76627*(-a*(sin(d*x+c)-1))^(7/2)*a^(15/2)+416759*(-a*(sin(d*x+c)-1))^(5/2)*a
^(17/2)+425845*(-a*(sin(d*x+c)-1))^(3/2)*a^(19/2)-55545*(-a*(sin(d*x+c)-1))
^(1/2)*a^(21/2)+55545*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^11*sin(d
*x+c)^8)/a^(19/2)/sin(d*x+c)^8/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")

[Out] integrate((a*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*csc(d*x + c)^9, x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. $2(289) = 578$.
time = 0.40, size = 657, normalized size = 2.00

$$\frac{55545\,\left(a\cos\left(dx+c\right)^{9}+a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{7}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{5}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{3}-4\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+a\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-4\,\left(\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)+3\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^{2}+8\,a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)-a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1}\right)+4\,\left(55545\,a\cos\left(dx+c\right)^{8}+37030\,a\cos\left(dx+c\right)^{7}-214774\,a\cos\left(dx+c\right)^{6}+27358\,a\cos\left(dx+c\right)^{5}+199004\,a\cos\left(dx+c\right)^{4}-185006\,a\cos\left(dx+c\right)^{3}-153786\,a\cos\left(dx+c\right)^{2}+48938\,a\cos\left(dx+c\right)+\left(55545\,a\cos\left(dx+c\right)^{7}+18515\,a\cos\left(dx+c\right)^{6}-196259\,a\cos\left(dx+c\right)^{5}-223617\,a\cos\left(dx+c\right)^{4}-24613\,a\cos\left(dx+c\right)^{3}+160393\,a\cos\left(dx+c\right)^{2}+6607\,a\cos\left(dx+c\right)-42331\,a\right)\sin\left(dx+c\right)+42331\,a\right)\sqrt{a\sin\left(dx+c\right)+a}}{2293760\,\left(d\cos\left(dx+c\right)^{9}+d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{7}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{5}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")

[Out] 1/2293760*(55545*(a*cos(d*x + c)^9 + a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^7
 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^5 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*
x + c)^3 - 4*a*cos(d*x + c)^2 + a*cos(d*x + c) + (a*cos(d*x + c)^8 - 4*a*co
s(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) +
a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 +
 (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c)
+ a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)
*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*
sin(d*x + c) - cos(d*x + c) - 1)) + 4*(55545*a*cos(d*x + c)^8 + 37030*a*cos
(d*x + c)^7 - 214774*a*cos(d*x + c)^6 + 27358*a*cos(d*x + c)^5 + 199004*a*c
os(d*x + c)^4 - 185006*a*cos(d*x + c)^3 - 153786*a*cos(d*x + c)^2 + 48938*a
*cos(d*x + c) + (55545*a*cos(d*x + c)^7 + 18515*a*cos(d*x + c)^6 - 196259*a
*cos(d*x + c)^5 - 223617*a*cos(d*x + c)^4 - 24613*a*cos(d*x + c)^3 + 160393
*a*cos(d*x + c)^2 + 6607*a*cos(d*x + c) - 42331*a)*sin(d*x + c) + 42331*a)*
sqrt(a*sin(d*x + c) + a))/(d*cos(d*x + c)^9 + d*cos(d*x + c)^8 - 4*d*cos(d*
x + c)^7 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^5 + 6*d*cos(d*x + c)^4 - 4
*d*cos(d*x + c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^8
 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*
x + c) + d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+a*sin(d*x+c))**(3/2),x)

[Out] Timed out

Giac [A]
time = 0.55, size = 338, normalized size = 1.03

$$-\frac{\sqrt{2}\left(55545\,\sqrt{2}\,a\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)+\frac{4\,\left(7109760\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-27254080\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+45383968\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-40147152\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+17413016\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1667036\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-851690\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+55545\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{8}}\right)\sqrt{a}}{2293760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/2293760*sqrt(2)*(55545*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/
2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c)) + 4*(7109760*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*
c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^15 - 27254080*a*sgn(cos(-1/4*pi + 1/2*d*
x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^13 + 45383968*a*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 40147152*a*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 17413016*a*s
gn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 16670
36*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 -
 851690*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1/2*c
)^3 + 55545*a*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/(2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^8)*sqrt(a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,(a+a\,\sin(c+d\,x))^{3/2}}{\sin(c+d\,x)^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^9,x)
```

```
[Out] int((cos(c + d*x)^4*(a + a*sin(c + d*x))^(3/2))/sin(c + d*x)^9, x)
```

3.464 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=124

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}}-\frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}}+\frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos^5(c+dx)\sqrt{a+a}}{11ad}$$

```
[Out] -152/3465*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-38/693*a*cos(d*x+c)^5/d
/(a+a*sin(d*x+c))^(3/2)+20/99*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)-2/11*co
s(d*x+c)^5*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2956, 2935, 2753, 2752}

$$-\frac{152a^2\cos^5(c+dx)}{3465d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)\sqrt{a\sin(c+dx)+a}}{11ad}+\frac{20\cos^5(c+dx)}{99d\sqrt{a\sin(c+dx)+a}}-\frac{38a\cos^5(c+dx)}{693d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-152*a^2*Cos[c + d*x]^5)/(3465*d*(a + a*Sin[c + d*x])^(5/2)) - (38*a*Cos[c
 + d*x]^5)/(693*d*(a + a*Sin[c + d*x])^(3/2)) + (20*Cos[c + d*x]^5)/(99*d*S
qrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]^5*Sqrt[a + a*Sin[c + d*x]])/(11*
a*d)
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\int \cos^4(c+dx)\left(-\frac{a}{2}-4a\sin(c+dx)\right)\sqrt{a+a\,\text{s}}}{4a^2}$$

$$= \frac{\cos^5(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19\int \cos^4}{}$$

$$= \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)\sqrt{a+a\sin(c+dx)}}{11ad} + \frac{19}{99}\int -$$

$$= -\frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos^5(c+dx)}{99d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos^5(c+dx)}{}$$

$$= -\frac{152a^2\cos^5(c+dx)}{3465d(a+a\sin(c+dx))^{5/2}} - \frac{38a\cos^5(c+dx)}{693d(a+a\sin(c+dx))^{3/2}} + \frac{20\cos}{99d\sqrt{a+}}$$

Mathematica [A]

time = 1.21, size = 143, normalized size = 1.15

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(5773\cos\left(\frac{1}{2}(c+dx)\right)-3495\cos\left(\frac{3}{2}(c+dx)\right)-1505\cos\left(\frac{5}{2}(c+dx)\right)+315\cos\left(\frac{7}{2}(c+dx)\right)+5773\sin\left(\frac{1}{2}(c+dx)\right)+3495\sin\left(\frac{3}{2}(c+dx)\right)-1505\sin\left(\frac{5}{2}(c+dx)\right)-315\sin\left(\frac{7}{2}(c+dx)\right)\right)}{13860d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/13860*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(5773*Cos[(c + d*x)/2] -
3495*Cos[(3*(c + d*x))/2] - 1505*Cos[(5*(c + d*x))/2] + 315*Cos[(7*(c + d*x
```

```
))/2] + 5773*Sin[(c + d*x)/2] + 3495*Sin[(3*(c + d*x))/2] - 1505*Sin[(5*(c
+ d*x))/2] - 315*Sin[(7*(c + d*x))/2]))/(d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 4.83, size = 74, normalized size = 0.60

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^{3}(315(\sin^{3}(dx+c))+595(\sin^{2}(dx+c))+340\sin(dx+c)+136)}{3465\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\, d}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/3465*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(315*sin(d*x+c)^3+595*sin(d*x+c)^2+3
40*sin(d*x+c)+136)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.36, size = 155, normalized size = 1.25

$$-\frac{2\,\left(315\,\cos\left(dx+c\right)^{6}-35\,\cos\left(dx+c\right)^{5}-445\,\cos\left(dx+c\right)^{4}+19\,\cos\left(dx+c\right)^{3}-38\,\cos\left(dx+c\right)^{2}+\left(315\,\cos\left(dx+c\right)^{5}+350\,\cos\left(dx+c\right)^{4}-95\,\cos\left(dx+c\right)^{3}-114\,\cos\left(dx+c\right)^{2}-152\,\cos\left(dx+c\right)-304\right)\sin\left(dx+c\right)+152\,\cos\left(dx+c\right)+304\right)\sqrt{a\sin\left(dx+c\right)+a}}{3465\,\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/3465*(315*cos(d*x + c)^6 - 35*cos(d*x + c)^5 - 445*cos(d*x + c)^4 + 19*c
os(d*x + c)^3 - 38*cos(d*x + c)^2 + (315*cos(d*x + c)^5 + 350*cos(d*x + c)^
4 - 95*cos(d*x + c)^3 - 114*cos(d*x + c)^2 - 152*cos(d*x + c) - 304)*sin(d*
x + c) + 152*cos(d*x + c) + 304)*sqrt(a*sin(d*x + c) + a)/(a*d*cos(d*x + c)
 + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**4/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.42, size = 103, normalized size = 0.83

$$-\frac{16 \sqrt{2} \left(1260 \sqrt{a} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{11} - 3080 \sqrt{a} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{9} + 2475 \sqrt{a} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{7} - 693 \sqrt{a} \sin\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)^{5}\right)}{3465 \, ad\mathrm{sgn}\left(\cos\left(-\frac{1}{4} \pi + \frac{1}{2} dx + \frac{1}{2} c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -16/3465*sqrt(2)*(1260*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 3080*sqr
t(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 2475*sqrt(a)*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^7 - 693*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)/(a*d*sgn(cos(-1/
4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{\sqrt{a+a\,\sin\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(1/2), x)
```

3.465 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=92

$$\frac{8a^2\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}+\frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 8/315*a^2*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+2/63*a*cos(d*x+c)^5/d/(a+a*
sin(d*x+c))^(3/2)-2/9*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2935, 2753, 2752}

$$\frac{8a^2\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{9d\sqrt{a\sin(c+dx)+a}}+\frac{2a\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (8*a^2*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (2*a*Cos[c + d*
x]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*d*Sqrt[a +
a*Sin[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = -\frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{9}\int \frac{\cos^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$$
$$= \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9d\sqrt{a+a\sin(c+dx)}} - \frac{1}{63}(4a)\int \frac{\cos^4}{(a+a\sin}$$
$$= \frac{8a^2\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}} + \frac{2a\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+}{9d\sqrt{a+a\sin(c}}$$

Mathematica [A]
time = 1.10, size = 87, normalized size = 0.95

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(87-35\cos(2(c+dx))+130\sin(c+dx))}{315d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(87 - 35*Cos[2*(c + d*x)] + 130*Sin[c + d*x]))/(d*Sqrt[a*(1 + Si
n[c + d*x])])
```

Maple [A]
time = 5.21, size = 64, normalized size = 0.70

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^3\left(35\left(\sin^2(dx+c)\right)+65\sin(dx+c)+26\right)}{315\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+65*sin(d*x+c)+26)/co
s(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.34, size = 136, normalized size = 1.48

$$-\frac{2 \left(35 \cos \left(dx+c\right)^{5}+40 \cos \left(dx+c\right)^{4}-\cos \left(dx+c\right)^{3}+2 \cos \left(dx+c\right)^{2}-\left(35 \cos \left(dx+c\right)^{4}-5 \cos \left(dx+c\right)^{3}-6 \cos \left(dx+c\right)^{2}-8 \cos \left(dx+c\right)-16\right) \sin \left(dx+c\right)-8 \cos \left(dx+c\right)-16\right) \sqrt{a \sin \left(dx+c\right)+a}}{315 \left(ad \cos \left(dx+c\right)+ad \sin \left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - cos(d*x + c)^3 + 2*cos(d*x
+ c)^2 - (35*cos(d*x + c)^4 - 5*cos(d*x + c)^3 - 6*cos(d*x + c)^2 - 8*cos(d
*x + c) - 16)*sin(d*x + c) - 8*cos(d*x + c) - 16)*sqrt(a*sin(d*x + c) + a)/
(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 84, normalized size = 0.91

$$\frac{16 \sqrt{2} \left(70 \sqrt{a} \sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{9}-135 \sqrt{a} \sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{7}+63 \sqrt{a} \sin \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)^{5}\right)}{315 ad \mathrm{sgn}\left(\cos \left(-\frac{1}{4} \pi+\frac{1}{2} dx+\frac{1}{2} c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] 16/315*sqrt(2)*(70*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 135*sqrt(a)*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 63*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)
^5)/(a*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(c+d\,x\right)^4\,\sin \left(c+d\,x\right)}{\sqrt{a+a\,\sin \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(1/2), x)
```

3.466 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=130

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}}+\frac{2\cos(c+dx)\sqrt{a}}{15}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+32/15*cos(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)^2/d/(a+a*sin(d*x+c
))^(1/2)+2/15*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.39, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2960, 2857, 3047, 3102, 2830, 2728, 212, 3125, 3064, 2852}

$$-\frac{2\sin^2(c+dx)\cos(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{15ad}+\frac{32\cos(c+dx)}{15d\sqrt{a\sin(c+dx)+a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(Sqrt[a]*d) +
 (32*Cos[c + d*x])/(15*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c
+ d*x]^2)/(5*d*Sqrt[a + a*Sin[c + d*x]]) + (2*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(15*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{\sin(c+dx)(-4a+a\sin(c+}{\sqrt{a+a\sin(c+dx}}}{5a}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} - \frac{\int \frac{-4a\sin(c+dx)+a\sin^2(c+}{\sqrt{a+a\sin(c+dx}}}{5a}$$

$$= \frac{4\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^2(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{2\cos(c+dx)\sqrt{a+a}}{15ad}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx}}\right.}{\sqrt{a}\,d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx}}\right.}{\sqrt{a}\,d}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{32\cos(c+dx)}{15d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+}{5d\sqrt{a-}}$$

Mathematica [A]
time = 0.18, size = 169, normalized size = 1.30

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(60\cos\left(\frac{1}{2}(c+dx)\right)+5\cos\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{5}{2}(c+dx)\right)-30\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+30\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-60\sin\left(\frac{1}{2}(c+dx)\right)+5\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(60*Cos[(c + d*x)/2] + 5*Cos[(3*(c +
 d*x))/2] + 3*Cos[(5*(c + d*x))/2] - 30*Log[1 + Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 30*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 60*Sin[(c + d
*x)/2] + 5*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*d*Sqrt[a*(1
+ Sin[c + d*x])])
```

Maple [A]
time = 6.23, size = 123, normalized size = 0.95

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(15a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)+3(a-a\sin(dx+c))^{\frac{5}{2}}-5a(a-}{15a^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*a^(5/2)*arctanh((a-a*sin
(d*x+c))^(1/2)/a^(1/2))+3*(a-a*sin(d*x+c))^(5/2)-5*a*(a-a*sin(d*x+c))^(3/2)
-15*a^2*(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(112) = 224$.
time = 0.35, size = 279, normalized size = 2.15

$$\frac{15\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}-\left(3\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+13\right)\sin\left(dx+c\right)+14\cos\left(dx+c\right)+13\right)\sqrt{a\sin\left(dx+c\right)+a}}{30\left(ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/30*(15*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 4*(3*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - cos(d*x + c)
+ 13)*sin(d*x + c) + 14*cos(d*x + c) + 13)*sqrt(a*sin(d*x + c) + a))/(a*d*c
os(d*x + c) + a*d*sin(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{\sqrt{a(\sin(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.48, size = 157, normalized size = 1.21

$$-\frac{\frac{15\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\left(12\sqrt{2}a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-10\sqrt{2}a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-15\sqrt{2}a^{\frac{9}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^5\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] -1/15*(15*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/abs(2*sqrt
(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x
 + 1/2*c))) - 2*(12*sqrt(2)*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 10*s
qrt(2)*a^(9/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 15*sqrt(2)*a^(9/2)*sin(-1
/4*pi + 1/2*d*x + 1/2*c))/(a^5*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^4}{\sin\left(c+d\,x\right)\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(1/2)), x)
```

3.467 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=119

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad}$$

```
[Out] arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+4/3*cos(d*x+c)
/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c
)*(a+a*sin(d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.34, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2960, 2838, 2830, 2728, 212, 3123, 3064, 2852}

$$-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3ad}+\frac{4\cos(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(Sqrt[a]*d) + (4*C
os[c + d*x])/(3*d*Sqrt[a + a*Sin[c + d*x]]) - Cot[c + d*x]/(d*Sqrt[a + a*Si
n[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(3*a*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
```

```
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
```

```
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^2(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3ad} + \frac{2\int \frac{\frac{a}{2}}{\sqrt{a+}}}{}$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\,\mathrm{si}}}{3ad}$$

$$= \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\,\mathrm{si}}}{3ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4\cos(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+}{d\sqrt{a+a\sin}}$$

Mathematica [A]
time = 0.34, size = 190, normalized size = 1.60

$$\frac{\csc\left(\frac{1}{4}(c+dx)\right)\sec\left(\frac{1}{4}(c+dx)\right)\left(-10\cos\left(\frac{1}{2}(c+dx)\right)+3\cos\left(\frac{3}{2}(c+dx)\right)+\cos\left(\frac{5}{2}(c+dx)\right)+10\sin\left(\frac{1}{2}(c+dx)\right)+3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+3\sin\left(\frac{3}{2}(c+dx)\right)-\sin\left(\frac{5}{2}(c+dx)\right)\right)\left(1+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/4]*Sec[(c + d*x)/4]*(-10*Cos[(c + d*x)/2] + 3*Cos[(3*(c + d*
x))/2] + Cos[(5*(c + d*x))/2] + 10*Sin[(c + d*x)/2] + 3*Log[1 + Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]]*Sin[c + d*x] + 3*Sin[(3*(c + d*x))/2] - Sin[(5*(c + d*x))/2])*
(1 + Tan[(c + d*x)/2]))/(24*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 5.25, size = 126, normalized size = 1.06

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sin(dx+c)\left(2(a-a\sin(dx+c))^{\frac{3}{2}}\sqrt{a}+3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}}{\sqrt{a}}\right)\right.\right.}{3a^{\frac{5}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(sin(d*x+c)*(2*(a-a*si
n(d*x+c))^(3/2)*a^(1/2)+3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a^2)-3*(a
-a*sin(d*x+c))^(1/2)*a^(3/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^2/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(103) = 206$.
time = 0.36, size = 306, normalized size = 2.57

$$\frac{3\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(2\cos\left(dx+c\right)^{3}+4\cos\left(dx+c\right)^{2}-\left(2\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)-7\right)\sin\left(dx+c\right)-5\cos\left(dx+c\right)-7\right)\sqrt{a\sin\left(dx+c\right)+a}}{12\left(ad\cos\left(dx+c\right)^{2}-ad-\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c)
+ 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) -
9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
 a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) -
cos(d*x + c) - 1)) - 4*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - (2*cos(d*x +
c)^2 - 2*cos(d*x + c) - 7)*sin(d*x + c) - 5*cos(d*x + c) - 7)*sqrt(a*sin(d*
x + c) + a))/(a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d*x + c) + a*d)*sin(d*x +
 c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{\sqrt{a\left(\sin\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/sqrt(a*(sin(c + d*x) + 1)), x)
```

Giac [A]
time = 0.46, size = 178, normalized size = 1.50

$$-\frac{\frac{8\,\sqrt{2}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{3\,\log\left(\left|\frac{1}{2}\,\sqrt{2}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{3\,\log\left(\left|-\frac{1}{2}\,\sqrt{2}+\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{6\,\sqrt{2}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/6*(8*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3/(sqrt(a)*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))) + 3*log(abs(1/2*sqrt(2) + sin(-1/4*pi + 1/2*d*x + 1/2*c
)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 3*log(abs(-1/2*sqrt(2)
+ sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))) + 6*sqrt(2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((2*sin(-1/4*pi + 1/2*d*x
+ 1/2*c)^2 - 1)*sqrt(a)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^2\,\sqrt{a+a\,\sin\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(1/2)), x)
```

3.468 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}}+\frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 9/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-2*cos(d*x+
c)/d/(a+a*sin(d*x+c))^(1/2)+1/4*cot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot
(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2960, 2830, 2728, 212, 3123, 3063, 3064, 2852}

$$-\frac{2\cos(c+dx)}{d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*Sqrt[a]*d)
- (2*Cos[c + d*x])/(d*Sqrt[a + a*Sin[c + d*x]]) + Cot[c + d*x]/(4*d*Sqrt[a
+ a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x])/(2*d*Sqrt[a + a*Sin[c + d*
x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
```

```
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
```

```
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\sin(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^3(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}-\frac{5}{2}a\sin\right.}{\sqrt{a+a\sin(c+}}}{2a}$$

$$= -\frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+}{2d\sqrt{a+a\sin(c+d}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{}{4d\sqrt{}}$$

$$= \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{}{4d\sqrt{}}$$

$$= \frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{2\cos(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c}{4d\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 296 vs. 2(125) = 250.
time = 2.48, size = 296, normalized size = 2.37

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-8-64\cos\left(\frac{1}{2}(c+dx)\right)+4\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)+36\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-36\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{8\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+64\sin\left(\frac{1}{2}(c+dx)\right)+4\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-8 - 64*Cos[(c + d*x)/2] + 4*Cot[(c
 + d*x)/4] - Csc[(c + d*x)/4]^2 + 36*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 36*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]
^2 + 2/(Cos[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (8*Sin[(c + d*x)/4])/(Cos[
```

```
(c + d*x)/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^
2 + (8*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 64*Sin[(c
+ d*x)/2] + 4*Tan[(c + d*x)/4]))/(32*d*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 5.93, size = 150, normalized size = 1.20

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(8\sqrt{-a(\sin(dx+c)-1)}\,a^{\frac{3}{2}}(\sin^2(dx+c))-9\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(}{}\right.\right.}{4a^{\frac{5}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\,\mathrm{si}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(5/2)*(8*(-a*(sin(d*x+c)-1)
)^(1/2)*a^(3/2)*sin(d*x+c)^2-9*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a
^2*sin(d*x+c)^2+(-a*(sin(d*x+c)-1))^(3/2)*a^(1/2)+(-a*(sin(d*x+c)-1))^(1/2)
*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^3/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(107) = 214$.
time = 0.37, size = 346, normalized size = 2.77

$$\frac{9\left(\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(8\,\cos(dx+c)^3+9\,\cos(dx+c)^2-\left(8\,\cos(dx+c)^2-\cos(dx+c)-11\right)\sin(dx+c)-10\,\cos(dx+c)-11\right)\sqrt{a\sin(dx+c)+a}}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad+\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(9*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 +
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
```

```
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(8*cos(d*x + c)^3
+ 9*cos(d*x + c)^2 - (8*cos(d*x + c)^2 - cos(d*x + c) - 11)*sin(d*x + c) -
10*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(d*x + c)^3 + a*d*c
os(d*x + c)^2 - a*d*cos(d*x + c) - a*d + (a*d*cos(d*x + c)^2 - a*d)*sin(d*x
 + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^3\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(1/2)), x)
```

3.469 $\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=135

$$-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d}+\frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -7/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)+9/8*cot(d
*x+c)/d/(a+a*sin(d*x+c))^(1/2)+1/12*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*x+c)
)^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2797, 2728, 212, 3123, 3063, 3064, 2852}

$$\frac{9\cot(c+dx)}{8d\sqrt{a\sin(c+dx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*Sqrt[a]*d)
 + (9*Cot[c + d*x])/(8*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c +
d*x])/(12*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d*
Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2797

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
```

```
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^4(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{3a} - \frac{2\text{Subst}\left(\int \frac{1}{2a-x^2} \right.}{}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\cot(c+dx)\csc(c+dx)}{12d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{3d\sqrt{}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c}{12d\sqrt{a}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c}{12d\sqrt{a}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c}{12d\sqrt{a}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{9\cot(c+dx)}{8d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\,\mathrm{cs}}{12d\sqrt{a+a\sin}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(135) = 270.
time = 0.45, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(36\cos\left(\frac{1}{2}(c+dx)\right)-46\cos\left(\frac{3}{2}(c+dx)\right)-54\cos\left(\frac{5}{2}(c+dx)\right)-36\sin\left(\frac{1}{2}(c+dx)\right)-63\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+63\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-46\sin\left(\frac{3}{2}(c+dx)\right)+54\sin\left(\frac{5}{2}(c+dx)\right)+21\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-21\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(36*Cos[(c + d*x)
/2] - 46*Cos[(3*(c + d*x))/2] - 54*Cos[(5*(c + d*x))/2] - 36*Sin[(c + d*x)/
2] - 63*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 63*Log[
1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 46*Sin[(3*(c + d*x)
)/2] + 54*Sin[(5*(c + d*x))/2] + 21*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 21*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*Sqrt[a*(
1 + Sin[c + d*x])])
```

Maple [A]
time = 5.20, size = 144, normalized size = 1.07

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-21\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{3}\left(\sin^{3}(dx+c)\right)+27(-a(\sin(d\right.}{24a^{\frac{7}{2}}\sin(dx+c)^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-21*arctanh((-a*(sin(d*x+c)-
1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^3+27*(-a*(sin(d*x+c)-1))^(5/2)*a^(1/2)-56
*(-a*(sin(d*x+c)-1))^(3/2)*a^(3/2)+21*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/a^
(7/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^4/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(115) = 230$.
time = 0.36, size = 369, normalized size = 2.73

$$\frac{21\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(27\cos\left(dx+c\right)^{3}+25\cos\left(dx+c\right)^{2}-\left(27\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)-17\right)\sin\left(dx+c\right)-19\cos\left(dx+c\right)-17\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(ad\cos\left(dx+c\right)^{4}-2ad\cos\left(dx+c\right)^{2}+ad-\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}-ad\cos\left(dx+c\right)-ad\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(21*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c
)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2
*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a
*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
 cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) -
```

```
4*(27*cos(d*x + c)^3 + 25*cos(d*x + c)^2 - (27*cos(d*x + c)^2 + 2*cos(d*x +
 c) - 17)*sin(d*x + c) - 19*cos(d*x + c) - 17)*sqrt(a*sin(d*x + c) + a))/(a
*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d - (a*d*cos(d*x + c)^3 + a*d*
cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^4\,\sqrt{a+a\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(1/2)), x)
```

3.470 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=170

$$-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{a}\,d}-\frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}+\frac{53\cot(c+dx)\csc(c+dx)}{96d\sqrt{a+a\sin(c+dx)}}+\frac{\cot(c+dx)\,\mathrm{cs}}{24d\sqrt{a+a\,\mathrm{si}}}$$

```
[Out] -11/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-11/64*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)+53/96*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d
*x+c))^(1/2)+1/24*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)-1/4*cot(
d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.58, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$,
Rules used = {2960, 2859, 2728, 212, 2852, 3123, 3063, 3064}

$$-\frac{11\cot(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a\sin(c+dx)+a}}+\frac{53\cot(c+dx)\csc(c+dx)}{96d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*Sqrt[a]*
d) - (11*Cot[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (53*Cot[c + d*x]*C
sc[c + d*x])/(96*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^2
)/(24*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*d*Sqrt
[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2859

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
```

```
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx &= \int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^5(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}-\frac{9}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{4a} + \frac{\int \csc(c+dx \ldots}{\ldots} \\
&= \frac{\cot(c+dx)\csc^2(c+dx)}{24d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{53a^2}{4}-\ldots\right)}{\sqrt{a+a\sin(c \ldots}}}{12a^2 \ldots} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+ \ldots}}\right)}{\sqrt{a}\,d} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+ \ldots}}\right)}{\sqrt{a}\,d} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+ \ldots}}\right)}{\sqrt{a}\,d} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+ \ldots}}\right)}{\sqrt{a}\,d} \\
&= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64\sqrt{a}\,d} - \frac{11\cot(c+dx)}{64d\sqrt{a+a\sin(c+dx)}} + \frac{53\cot(\ldots}{96d\sqrt{\ldots}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 374 vs. 2(170) = 340.
time = 0.68, size = 374, normalized size = 2.20

$$
\frac{\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(214\cos\left(\frac{1}{2}(c+dx)\right)-558\cos\left(\frac{3}{2}(c+dx)\right)-490\cos\left(\frac{5}{2}(c+dx)\right)+66\cos\left(\frac{7}{2}(c+dx)\right)-99\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+132\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-33\cos(4(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+99\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-132\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+33\cos(4(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-214\sin\left(\frac{1}{2}(c+dx)\right)-558\sin\left(\frac{3}{2}(c+dx)\right)+490\sin\left(\frac{5}{2}(c+dx)\right)+66\sin\left(\frac{7}{2}(c+dx)\right)\right)}{192d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^4\sqrt{a(1+\sin(c+dx))}}
$$

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + a*Sin[c + d*x]],x]

[Out] (Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(214*Cos[(c + d*x)/2] - 558*Cos[(3*(c + d*x))/2] - 490*Cos[(5*(c + d*x))/2] + 66*Cos[(7*(c + d*x))/2] - 99*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 132*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 33*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 99*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 132*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 33*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 214*Sin[(c + d*x)/2] - 558*Sin[(3*(c + d*x))/2] + 490*Sin[(5*(c + d*x))/2] + 66*Sin[(7*(c + d*x))/2]))/(192*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4*Sqrt[a*(1 + Sin[c + d*x])])

Maple [A]
time = 6.17, size = 162, normalized size = 0.95

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(33\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^5(\sin^4(dx+c))-33(-a(\sin(dx\right.}{192a^{\frac{11}{2}}\sin(dx+c)^4\cos(dx+c)\sqrt{a+}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/192*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(33*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x+c)^4-33*(-a*(sin(d*x+c)-1))^(7/2)*a^(3/2)-7*(-a*(sin(d*x+c)-1))^(5/2)*a^(5/2)+121*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)-33*(-a*(sin(d*x+c)-1))^(1/2)*a^(9/2))/a^(11/2)/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^5/sqrt(a*sin(d*x + c) + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(146) = 292.
time = 0.39, size = 426, normalized size = 2.51

$$\frac{33\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\sin(dx+c)+\cos(dx+c)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(33\cos(dx+c)^4-106\cos(dx+c)^3-164\cos(dx+c)^2+\left(33\cos(dx+c)^3+139\cos(dx+c)^2-25\cos(dx+c)-83\right)\sin(dx+c)+58\cos(dx+c)+83\right)\sqrt{a\sin(dx+c)+a}}{768\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2ad\cos(dx+c)^3-2ad\cos(dx+c)^2+ad\cos(dx+c)+ad+\left(ad\cos(dx+c)^4-2ad\cos(dx+c)^2+ad\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(33*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
 c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
 + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^
2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x +
c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
 a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 -
1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(33*cos(d*x + c)^4 - 106*cos(d*x +
 c)^3 - 164*cos(d*x + c)^2 + (33*cos(d*x + c)^3 + 139*cos(d*x + c)^2 - 25*c
os(d*x + c) - 83)*sin(d*x + c) + 58*cos(d*x + c) + 83)*sqrt(a*sin(d*x + c)
+ a))/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a
*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d + (a*d*cos(d*x + c)^4 - 2*a*d*co
s(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^5\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(1/2)), x)
```

3.471 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx$

Optimal. Leaf size=205

$$-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{a}\,d}-\frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}}-\frac{3\cot(c+dx)\csc(c+dx)}{64d\sqrt{a+a\sin(c+dx)}}+\frac{29\cot(c+dx)\,c}{80d\sqrt{a+a\,\mathrm{si}}}$$

```
[Out] -9/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/d/a^(1/2)-9/128*c
ot(d*x+c)/d/(a+a*sin(d*x+c))^(1/2)-3/64*cot(d*x+c)*csc(d*x+c)/d/(a+a*sin(d*
x+c))^(1/2)+29/80*cot(d*x+c)*csc(d*x+c)^2/d/(a+a*sin(d*x+c))^(1/2)+1/40*cot
(d*x+c)*csc(d*x+c)^3/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*csc(d*x+c)^4/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2960, 2858, 3064, 2728, 212, 2852, 3123, 3063}

$$-\frac{9\cot(c+dx)}{128d\sqrt{a\sin(c+dx)+a}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128\sqrt{a}\,d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a\sin(c+dx)+a}}+\frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a\sin(c+dx)+a}}+\frac{29\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a\sin(c+dx)+a}}-\frac{3\cot(c+dx)\csc(c+dx)}{64d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] (-9*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*Sqrt[a]*
d) - (9*Cot[c + d*x])/(128*d*Sqrt[a + a*Sin[c + d*x]]) - (3*Cot[c + d*x]*Cs
c[c + d*x])/(64*d*Sqrt[a + a*Sin[c + d*x]]) + (29*Cot[c + d*x]*Csc[c + d*x]
^2)/(80*d*Sqrt[a + a*Sin[c + d*x]]) + (Cot[c + d*x]*Csc[c + d*x]^3)/(40*d*S
qrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*d*Sqrt[a + a*Si
n[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2960

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx = \int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx + \int \frac{\csc^6(c+dx)\left(1-2\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)}{5d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^5(c+dx)\left(-\frac{a}{2}-\frac{11}{2}a\right.}{\sqrt{a+a\sin(c}}}{5a}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3(c+dx)}{40d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c}{5d\sqrt{a+a\sin(c+}}$$

$$= -\frac{\cot(c+dx)}{d\sqrt{a+a\sin(c+dx)}} + \frac{29\cot(c+dx)\csc^2(c+dx)}{80d\sqrt{a+a\sin(c+dx)}} + \frac{\cot(c+dx)\csc^3}{40d\sqrt{a+a\sin(}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{\sqrt{a}\,d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{\sqrt{a}\,d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{\sqrt{a}\,d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{\sqrt{a}\,d}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128\sqrt{a}\,d} - \frac{9\cot(c+dx)}{128d\sqrt{a+a\sin(c+dx)}} - \frac{3\cot(}{64d\sqrt{}}$$

Mathematica [A]
time = 0.76, size = 410, normalized size = 2.00

$$-\frac{\csc^{15}\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(820\cos\left(\frac{1}{2}(c+dx)\right)+1600\cos\left(\frac{3}{2}(c+dx)\right)+1616\cos\left(\frac{5}{2}(c+dx)\right)-30\cos\left(\frac{7}{2}(c+dx)\right)+90\cos\left(\frac{9}{2}(c+dx)\right)-820\sin\left(\frac{1}{2}(c+dx)\right)+450\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-450\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+1600\sin\left(\frac{3}{2}(c+dx)\right)-1616\sin\left(\frac{5}{2}(c+dx)\right)-225\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+225\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-30\sin\left(\frac{7}{2}(c+dx)\right)-90\sin\left(\frac{9}{2}(c+dx)\right)+45\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-45\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{640d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/Sqrt[a + a*Sin[c + d*x]],x]
```

```
[Out] -1/640*(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(820*Cos[
(c + d*x)/2] + 1600*Cos[(3*(c + d*x))/2] + 1616*Cos[(5*(c + d*x))/2] - 30*C
os[(7*(c + d*x))/2] + 90*Cos[(9*(c + d*x))/2] - 820*Sin[(c + d*x)/2] + 450*
Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 450*Log[1 - Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 1600*Sin[(3*(c + d*x))/2]
- 1616*Sin[(5*(c + d*x))/2] - 225*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[3*(c + d*x)] + 225*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 30*Sin[(7*(c + d*x))/2] - 90*Sin[(9*(c + d*x))/2] + 45*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[1 - Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(Csc[(c + d*x)/4]^2
- Sec[(c + d*x)/4]^2)^5*Sqrt[a*(1 + Sin[c + d*x])])
```

Maple [A]
time = 6.52, size = 180, normalized size = 0.88

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(45(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{5}{2}}-210(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{7}{2}}+45\,\mathrm{arctanh}\left(\frac{\sqrt{-a}}{}\right.\right.}{640a^{\frac{15}{2}}\sin(dx+c)^5\,\mathrm{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/640*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)/a^(15/2)*(45*(-a*(sin(d*x+c
)-1))^(9/2)*a^(5/2)-210*(-a*(sin(d*x+c)-1))^(7/2)*a^(7/2)+45*arctanh((-a*(s
in(d*x+c)-1))^(1/2)/a^(1/2))*a^7*sin(d*x+c)^5+128*(-a*(sin(d*x+c)-1))^(5/2)
*a^(9/2)+210*(-a*(sin(d*x+c)-1))^(3/2)*a^(11/2)-45*(-a*(sin(d*x+c)-1))^(1/2
)*a^(13/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^6/sqrt(a*sin(d*x + c) + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(177) = 354$.
time = 0.38, size = 472, normalized size = 2.30

$$\frac{45\left(\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(45\,\cos\left(dx+c\right)^{5}+15\,\cos\left(dx+c\right)^{4}+142\,\cos\left(dx+c\right)^{3}+186\,\cos\left(dx+c\right)^{2}-\left(45\,\cos\left(dx+c\right)^{4}+30\,\cos\left(dx+c\right)^{3}+172\,\cos\left(dx+c\right)^{2}-14\,\cos\left(dx+c\right)-73\right)\sin\left(dx+c\right)-59\,\cos\left(dx+c\right)-73\right)\sqrt{a\sin\left(dx+c\right)+a}}{2560\left(ad\cos\left(dx+c\right)^{6}-3\,ad\cos\left(dx+c\right)^{4}+3\,ad\cos\left(dx+c\right)^{2}-ad-\left(ad\cos\left(dx+c\right)^{5}+ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{3}-2\,ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(45*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(45*cos(d*x +
c)^5 + 15*cos(d*x + c)^4 + 142*cos(d*x + c)^3 + 186*cos(d*x + c)^2 - (45*co
s(d*x + c)^4 + 30*cos(d*x + c)^3 + 172*cos(d*x + c)^2 - 14*cos(d*x + c) - 7
3)*sin(d*x + c) - 59*cos(d*x + c) - 73)*sqrt(a*sin(d*x + c) + a))/(a*d*cos(
d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d - (a*d*cos(d
*x + c)^5 + a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^
2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^6\,\sqrt{a+a\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(1/2)), x)
```

3.472 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=205

$$-\frac{4\cos(c+dx)}{165ad\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}}+\frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}}+\frac{8\cos(c+dx)\sqrt{a+}}{1155a}$$

```
[Out] -4/385*cos(d*x+c)*(a+a*sin(d*x+c))^(3/2)/a^3/d-4/165*cos(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)-2/231*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)+14
/33*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+a*sin(d*x+c))^(1/2)+8/1155*cos(d*x+c)*(a
+a*sin(d*x+c))^(1/2)/a^2/d-2/11*cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^(1
/2)/a^2/d
```

Rubi [A]
time = 0.51, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2959, 2849, 2838, 2830, 2725, 3125, 3060}

$$-\frac{4\cos(c+dx)(a\sin(c+dx)+a)^{3/2}}{385a^3d}-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{11a^2d}+\frac{8\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{1155a^2d}+\frac{14\sin^4(c+dx)\cos(c+dx)}{33ad\sqrt{a\sin(c+dx)+a}}-\frac{2\sin^3(c+dx)\cos(c+dx)}{231ad\sqrt{a\sin(c+dx)+a}}-\frac{4\cos(c+dx)}{165ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*Cos[c + d*x])/(165*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[
c + d*x]^3)/(231*a*d*Sqrt[a + a*Sin[c + d*x]]) + (14*Cos[c + d*x]*Sin[c + d
*x]^4)/(33*a*d*Sqrt[a + a*Sin[c + d*x]]) + (8*Cos[c + d*x]*Sqrt[a + a*Sin[c
 + d*x]])/(1155*a^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + a*Sin[c +
d*x]])/(11*a^2*d) - (4*Cos[c + d*x]*(a + a*Sin[c + d*x])^(3/2))/(385*a^3*d)
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2838

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !L
tQ[m, -2^(-1)]
```

Rule 2849

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[-2*b*Cos[e + f*x]*((c + d*Sin[e + f*x])
^n/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[2*n*((b*c + a*d)/(b*(
2*n + 1))), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \sin^3(c+dx)\sqrt{a+a\sin(c+dx)}\,(1+\sin^2(c+dx))\,dx}{a^2} - \frac{2\int \sin^4(c+d}{}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+a\sin(c+dx}}{11a^2d}$$

$$= \frac{32\cos(c+dx)\sin^3(c+dx)}{63ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx}{}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+d}{}$$

$$= -\frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)\sin^4(c+dx)}{33ad\sqrt{a+a\sin(c+dx)}} - \frac{128\cos(c+}{}$$

$$= \frac{64\cos(c+dx)}{45ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+dx)}{33ad\sqrt{a+a\,s}}$$

$$= -\frac{4\cos(c+dx)}{165ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)}{231ad\sqrt{a+a\sin(c+dx)}} + \frac{14\cos(c+}{33ad\sqrt{a+}}$$

Mathematica [A]
time = 3.73, size = 102, normalized size = 0.50

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(-204+140\cos(2(c+dx))-475\sin(c+dx)+105\sin(3(c+dx)))}{2310a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
[Out] ((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(-204 +
 140*Cos[2*(c + d*x)] - 475*Sin[c + d*x] + 105*Sin[3*(c + d*x)]))/(2310*a^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.35, size = 77, normalized size = 0.38

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^3\left(105\left(\sin^3(dx+c)\right)+70\left(\sin^2(dx+c)\right)+40\sin(dx+c)+16\right)}{1155a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/1155/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(105*sin(d*x+c)^3+70*sin(d*x+c)^2+
40*sin(d*x+c)+16)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.35, size = 161, normalized size = 0.79

$$-\frac{2\,\left(105\,\cos\left(dx+c\right)^{6}-140\,\cos\left(dx+c\right)^{5}-460\,\cos\left(dx+c\right)^{4}+274\,\cos\left(dx+c\right)^{3}+607\,\cos\left(dx+c\right)^{2}+\left(105\,\cos\left(dx+c\right)^{5}+245\,\cos\left(dx+c\right)^{4}-215\,\cos\left(dx+c\right)^{3}-489\,\cos\left(dx+c\right)^{2}+118\,\cos\left(dx+c\right)+236\right)\sin\left(dx+c\right)-118\,\cos\left(dx+c\right)-236\right)\sqrt{a\sin\left(dx+c\right)+a}}{1155\,\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/1155*(105*cos(d*x + c)^6 - 140*cos(d*x + c)^5 - 460*cos(d*x + c)^4 + 274
*cos(d*x + c)^3 + 607*cos(d*x + c)^2 + (105*cos(d*x + c)^5 + 245*cos(d*x +
c)^4 - 215*cos(d*x + c)^3 - 489*cos(d*x + c)^2 + 118*cos(d*x + c) + 236)*si
n(d*x + c) - 118*cos(d*x + c) - 236)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d*
x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 103, normalized size = 0.50

$$-\frac{8\,\sqrt{2}\left(840\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1540\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+990\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-231\,\sqrt{a}\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\right)}{1155\,a^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -8/1155*sqrt(2)*(840*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 1540*sqrt(
a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 + 990*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1
/2*c)^7 - 231*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5)/(a^2*d*sgn(cos(-1/4
*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(3/2), x)
```

3.473 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=92

$$-\frac{46a\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}}+\frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}}-\frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -46/315*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+20/63*cos(d*x+c)^5/d/(a+a*s
in(d*x+c))^(3/2)-2/9*cos(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2956, 2935, 2753, 2752}

$$-\frac{2\cos^5(c+dx)}{9ad\sqrt{a\sin(c+dx)+a}}+\frac{20\cos^5(c+dx)}{63d(a\sin(c+dx)+a)^{3/2}}-\frac{46a\cos^5(c+dx)}{315d(a\sin(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-46*a*Cos[c + d*x]^5)/(315*d*(a + a*Sin[c + d*x])^(5/2)) + (20*Cos[c + d*x
]^5)/(63*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(9*a*d*Sqrt[a +
 a*Sin[c + d*x]])
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2753

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && N
eQ[m + p, 0]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
```

```
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rule 2956

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1))), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-2a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{2a^2}$$

$$= \frac{\cos^5(c+dx)}{2d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}} + \frac{23\int \frac{\cos^4(c+d}{\sqrt{a+a\sin(c}}}{36a}$$

$$= \frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+dx)}{9ad\sqrt{a+a\sin(c+dx)}} + \frac{23}{63}\int \frac{\cos^4(c}{(a+a\sin(}$$

$$= -\frac{46a\cos^5(c+dx)}{315d(a+a\sin(c+dx))^{5/2}} + \frac{20\cos^5(c+dx)}{63d(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c}{9ad\sqrt{a+a\,\mathrm{s}}}$$

Mathematica [A]
time = 2.76, size = 92, normalized size = 1.00

$$-\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(51-35\cos(2(c+dx))+40\sin(c+dx))}{315a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/315*((Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*
(51 - 35*Cos[2*(c + d*x)] + 40*Sin[c + d*x]))/(a^2*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]))
```

Maple [A]
time = 4.85, size = 67, normalized size = 0.73

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^{3}(35(\sin^{2}(dx+c))+20\sin(dx+c)+8)}{315a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/315/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(35*sin(d*x+c)^2+20*sin(d*x+c)+8)/c
os(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 0.36, size = 142, normalized size = 1.54

$$-\frac{2\left(35\cos\left(dx+c\right)^{5}+85\cos\left(dx+c\right)^{4}-73\cos\left(dx+c\right)^{3}-169\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}-50\cos\left(dx+c\right)^{3}-123\cos\left(dx+c\right)^{2}+46\cos\left(dx+c\right)+92\right)\sin\left(dx+c\right)+46\cos\left(dx+c\right)+92\right)\sqrt{a\sin\left(dx+c\right)+a}}{315\left(a^{2}d\cos\left(dx+c\right)+a^{2}d\sin\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/315*(35*cos(d*x + c)^5 + 85*cos(d*x + c)^4 - 73*cos(d*x + c)^3 - 169*cos
(d*x + c)^2 - (35*cos(d*x + c)^4 - 50*cos(d*x + c)^3 - 123*cos(d*x + c)^2 +
 46*cos(d*x + c) + 92)*sin(d*x + c) + 46*cos(d*x + c) + 92)*sqrt(a*sin(d*x
+ c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 84, normalized size = 0.91

$$\frac{8\sqrt{2}\left(140\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-180\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+63\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)}{315\,a^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 8/315*sqrt(2)*(140*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 180*sqrt(a)*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 63*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)
^5)/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(3/2), x)
```

3.474 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6a\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}$$

```
[Out] 6/35*a*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/7*cos(d*x+c)^5/d/(a+a*sin(d*
x+c))^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2935, 2752}

$$\frac{6a\cos^5(c+dx)}{35d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (6*a*Cos[c + d*x]^5)/(35*d*(a + a*Sin[c + d*x])^(5/2)) - (2*Cos[c + d*x]^5)
/(7*d*(a + a*Sin[c + d*x])^(3/2))
```

Rule 2752

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^(m - 1)/(f*g*(m - 1))), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2935

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx &= -\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}-\frac{3}{7}\int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx\\ &= \frac{6a\cos^5(c+dx)}{35d(a+a\sin(c+dx))^{5/2}}-\frac{2\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.31, size = 82, normalized size = 1.37

$$-\frac{2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\sqrt{a(1+\sin(c+dx))}\,(2+5\sin(c+dx))}{35a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*Sqrt[a*(1 + Sin[c + d*x])]*(2 +
 5*Sin[c + d*x]))/(35*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 3.99, size = 57, normalized size = 0.95

method	result	size
default	$\frac{2(1+\sin(dx+c))(\sin(dx+c)-1)^3(5\sin(dx+c)+2)}{35a\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35/a*(1+sin(d*x+c))*(sin(d*x+c)-1)^3*(5*sin(d*x+c)+2)/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.
time = 0.36, size = 121, normalized size = 2.02

$$\frac{2\left(5\cos(dx+c)^4-8\cos(dx+c)^3-19\cos(dx+c)^2+\left(5\cos(dx+c)^3+13\cos(dx+c)^2-6\cos(dx+c)-12\right)\sin(dx+c)+6\cos(dx+c)+12\right)\sqrt{a\sin(dx+c)+a}}{35\left(a^2d\cos(dx+c)+a^2d\sin(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/35*(5*cos(d*x + c)^4 - 8*cos(d*x + c)^3 - 19*cos(d*x + c)^2 + (5*cos(d*x
+ c)^3 + 13*cos(d*x + c)^2 - 6*cos(d*x + c) - 12)*sin(d*x + c) + 6*cos(d*x
+ c) + 12)*sqrt(a*sin(d*x + c) + a)/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c
) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 65, normalized size = 1.08

$$-\frac{8\sqrt{2}\left(10\sqrt{a}\,\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-7\sqrt{a}\,\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}\right)}{35\,a^{2}d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -8/35*sqrt(2)*(10*sqrt(a)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 7*sqrt(a)*sin(
-1/4*pi + 1/2*d*x + 1/2*c)^5)/(a^2*d*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(3/2), x)
```

3.475 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}+\frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+10/3*cos(d*
x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.24, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2959, 2725, 3125, 3060, 2852, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{2\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}+\frac{10\cos(c+dx)}{3ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) +
 (10*Cos[c + d*x])/(3*a*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sqrt[
a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3060

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,(1+\sin^2(c+dx))\,dx}{a^2} - \frac{2\int \sqrt{a+a\sin}}{a^2}$$

$$= \frac{4\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{2\int \csc(c+}{}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \csc(c+}{}$$

$$= \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} - \frac{2\text{Subst}\Big(}{}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{10\cos(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c}{}$$

Mathematica [A]

time = 0.20, size = 147, normalized size = 1.50

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(9\cos\left(\frac{1}{2}(c+dx)\right)-\cos\left(\frac{3}{2}(c+dx)\right)-3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-9\sin\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{3}{2}(c+dx)\right)\right)}{3d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(9*Cos[(c + d*x)/2] - Cos[(3*(c +
d*x))/2] - 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*Sin[(c + d*x)/2] - Sin[(3*(c + d*x))/2]
))/(3*d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]

time = 4.64, size = 105, normalized size = 1.07

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}}{\sqrt{a}}\right)-(a-a\sin(dx+c))^{\frac{3}{2}}-3a\sqrt{a-}\right.}{3a^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*a^(3/2)*arctanh((a-a*sin(d
*x+c))^(1/2)/a^(1/2))-(a-a*sin(d*x+c))^(3/2)-3*a*(a-a*sin(d*x+c))^(1/2))/a^
3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/(a*sin(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(84) = 168$.
time = 0.37, size = 260, normalized size = 2.65

$$\frac{3\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)+5\right)\sin\left(dx+c\right)-4\,\cos\left(dx+c\right)-5\right)\sqrt{a\sin\left(dx+c\right)+a}}{6\left(a^2d\cos\left(dx+c\right)+a^2d\sin\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4
*(cos(d*x + c)^2 + (cos(d*x + c) + 5)*sin(d*x + c) - 4*cos(d*x + c) - 5)*sq
rt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c) + a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)}{\left(a\left(\sin\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.49, size = 138, normalized size = 1.41

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\sqrt{2}\,\log\left(\frac{\left|-2\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\left(2\,a^4\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^4\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^6\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] -1/6*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*
x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))) + 4*(2*a^4*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 +
 3*a^4*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^6*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(3/2)), x)
```

3.476 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d}-\frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2d}$$

```
[Out] 3*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-cos(d*x+c)/a
/d/(a+a*sin(d*x+c))^(1/2)-cot(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2959, 2852, 212, 3123, 21, 2842}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)\sqrt{a\sin(c+dx)+a}}{a^2d}-\frac{\cos(c+dx)}{ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(3/2)*d) -
Cos[c + d*x]/(a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Sqrt[a + a*Sin[
c + d*x]])/(a^2*d)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
```

```
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^2(c+dx)\sqrt{a+a\sin(c+dx)}\,\left(1+\sin^2(c+dx)\right)\,dx}{a^2} - \frac{2\int \csc(c+dx}{}$$

$$= -\frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\int \csc(c+dx)\left(\frac{a}{2}+\frac{1}{2}a\sin(c+dx)\right)\sqrt{}}{a^3}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cot(c+dx)\sqrt{a+a\sin(c+dx)}}{a^2d} + \frac{\int}{}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx}{}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx}{}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} - \frac{\cos(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 220 vs. 2(94) = 188.
time = 0.50, size = 220, normalized size = 2.34

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(2-8\cos\left(\frac{1}{2}(c+dx)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+6\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+8\sin\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{4}(c+dx)\right)\right)}{4d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(2 - 8*Cos[(c + d*x)/2] - Cot[(c +
 d*x)/4] + 6*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*Log[1 - Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] + (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] -
Sin[(c + d*x)/4]) - (2*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/
4]) + 8*Sin[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c + d*x]))^(
3/2))
```

Maple [A]
time = 4.90, size = 123, normalized size = 1.31

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(\sin(dx+c)\left(2\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{a}-3\,\mathrm{arctanh}\left(\frac{\sqrt{a-a\sin(}{\sqrt{a}}\right.\right.\right.}{a^{\frac{5}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/a^(5/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(sin(d*x+c)*(2*(a-a*sin
(d*x+c))^(1/2)*a^(1/2)-3*arctanh((a-a*sin(d*x+c))^(1/2)/a^(1/2))*a)+(a-a*si
n(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(84) = 168$.
time = 0.38, size = 291, normalized size = 3.10

$$\frac{3\left(\cos\left(dx+c\right)^2-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2+4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(2\,\cos\left(dx+c\right)^2+\left(2\,\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)-1\right)\sqrt{a\sin\left(dx+c\right)+a}}{4\left(a^2d\cos\left(dx+c\right)^2-a^2d-\left(a^2d\cos\left(dx+c\right)+a^2d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(3*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 4*(2*cos(d*x + c)^2 + (2*cos(d*x + c) + 1)*sin(d*x + c)
 + cos(d*x + c) - 1)*sqrt(a*sin(d*x + c) + a))/(a^2*d*cos(d*x + c)^2 - a^2*
d - (a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{(a(\sin(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a*(sin(c + d*x) + 1))**(3/2), x)
```

Giac [A]
time = 0.46, size = 165, normalized size = 1.76

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{8\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) + 8*sin(-1/4*pi + 1/2*d*x + 1/2*c)/(a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)/((2*sin(-
1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)^2\,(a+a\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(3/2)), x)
```

3.477 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d}+\frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d}$$

```
[Out] -3/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d+7/4*cot(d
*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))
^(1/2)/a^2/d
```

Rubi [A]
time = 0.33, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2959, 2851, 2852, 212, 3123, 3059}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a\sin(c+dx)+a}}{2a^2d}+\frac{7\cot(c+dx)}{4ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(3/2)*d)
 + (7*Cot[c + d*x])/(4*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[a + a*Sin[c + d*x]])/(2*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^3(c+dx)\sqrt{a+a\sin(c+dx)}\,(1+\sin^2(c+dx))\,dx}{a^2} - \frac{2\int \csc^2(c+d\ldots}{}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \ldots$$

$$= \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+a\sin(c+dx)}}{2a^2d} + \ldots$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+d\ldots}{}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} + \frac{7\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+\ldots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 274 vs. 2(106) = 212.

time = 1.57, size = 274, normalized size = 2.58

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-24+12\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)-12\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{24\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{24\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+12\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-24 + 12*Cot[(c + d*x)/4] - Csc[(
c + d*x)/4]^2 - 12*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*Log[1
- Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Cos[(c + d
*x)/4] - Sin[(c + d*x)/4])^2 - (24*Sin[(c + d*x)/4])/(Cos[(c + d*x)/4] - Si
n[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (24*Sin[(c +
d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 12*Tan[(c + d*x)/4]))/(32*
d*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]

time = 5.73, size = 126, normalized size = 1.19

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(3\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^2(\sin^2(dx+c))+5(-a(\sin(dx\ldots\right.}{4a^{\frac{7}{2}}\sin(dx+c)^2\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*arctanh((-a*(sin(d
*x+c)-1))^(1/2)/a^(1/2))*a^2*sin(d*x+c)^2+5*(-a*(sin(d*x+c)-1))^(3/2)*a^(1/
2)-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(3/2))/sin(d*x+c)^2/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(90) = 180$.
time = 0.36, size = 337, normalized size = 3.18

$$\frac{3\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7\,a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)-4\left(5\,\cos\left(dx+c\right)^{2}+\left(5\,\cos\left(dx+c\right)+7\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)-7\right)\sqrt{a\sin\left(dx+c\right)+a}}{16\left(a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-a^{2}d+\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(3*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c
) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*s
qrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*
a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (c
os(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) - 4*(5*cos(d*x + c)^2
+ (5*cos(d*x + c) + 7)*sin(d*x + c) - 2*cos(d*x + c) - 7)*sqrt(a*sin(d*x +
c) + a))/(a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c)
- a^2*d + (a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.49, size = 152, normalized size = 1.43

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{4\left(10\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/16*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(10*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3
*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^
2*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^3\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(3/2)), x)
```

3.478 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}}+\frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c$$

```
[Out] -1/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-1/8*cot(d
*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)+11/12*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(d
*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.37, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2796, 2851, 2852, 212, 3123, 3059}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{3/2}d}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a\sin(c+dx)+a}}{3a^2d}-\frac{\cot(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}+\frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/8*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]]/(a^(3/2)*d) -
 Cot[c + d*x]/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) + (11*Cot[c + d*x]*Csc[c + d
*x])/(12*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[
a + a*Sin[c + d*x]])/(3*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2796

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x]^
2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
```

```
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^4(c+dx)\sqrt{a+a\sin(c+dx)}\,(1+\sin^2(c+dx))\,dx}{a^2} - \frac{2\int \csc^3(c+dx)\sqrt{\cdots}}{\cdots}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+a\sin(c+dx)}}{3a^2d} + \frac{\int \mathrm{c}\cdots}{\cdots}$$

$$= \frac{3\cot(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+\cdots}{3\cdots}$$

$$= -\frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx)\csc(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c\cdots}{\cdots}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx\cdots}{12ad\sqrt{a+a\cdots}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{3/2}d} - \frac{\cot(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} + \frac{11\cot(c+dx\cdots}{12ad\sqrt{a+a\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(144) = 288.
time = 0.60, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(-132\cos\left(\frac{1}{2}(c+dx)\right)+62\cos\left(\frac{3}{2}(c+dx)\right)+6\cos\left(\frac{5}{2}(c+dx)\right)+132\sin\left(\frac{1}{2}(c+dx)\right)-9\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+9\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+62\sin\left(\frac{3}{2}(c+dx)\right)-6\sin\left(\frac{5}{2}(c+dx)\right)+3\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-3\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{24d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Csc[(c + d*x)/2]^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(-132*Cos[(c +
d*x)/2] + 62*Cos[(3*(c + d*x))/2] + 6*Cos[(5*(c + d*x))/2] + 132*Sin[(c + d
*x)/2] - 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 9*Lo
g[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 62*Sin[(3*(c + d*
x))/2] - 6*Sin[(5*(c + d*x))/2] + 3*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] - 3*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)]))/(24*d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^3*(a*(1 + S
in[c + d*x]))^(3/2))
```

Maple [A]
time = 6.61, size = 144, normalized size = 1.00

method	result

default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(3(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+3\,\mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}}{\sqrt{a}}\right)a^{4}}{24a^{\frac{11}{2}}\sin(dx+c)^{3}\cos(dx+c)\sqrt{a+a\sin\left(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/24/a^(11/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(3*(-a*(sin(d*x+c)-
1))^(5/2)*a^(3/2)+3*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^4*sin(d*x+
c)^3+8*(-a*(sin(d*x+c)-1))^(3/2)*a^(5/2)-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(7/2
))/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(124) = 248$.
time = 0.39, size = 383, normalized size = 2.66

$$\frac{3\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\cos\left(dx+c\right)^{3}+17\cos\left(dx+c\right)^{2}-\left(3\cos\left(dx+c\right)^{2}-14\cos\left(dx+c\right)-25\right)\sin\left(dx+c\right)-11\cos\left(dx+c\right)-25\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(a^{2}d\cos\left(dx+c\right)^{4}-2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d-\left(a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\cos\left(dx+c\right)-a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)
^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*
cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*
cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 +
cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4
*(3*cos(d*x + c)^3 + 17*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 14*cos(d*x + c
) - 25)*sin(d*x + c) - 11*cos(d*x + c) - 25)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d - (a^2*d*cos(d*x + c)^3
+ a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [A]
time = 0.48, size = 168, normalized size = 1.17

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\left(12\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/96*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) + 4*(12*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 1
6*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 3*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*
sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2
*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^4\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(3/2)), x)
```

3.479 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{3/2}d}-\frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}}+\frac{5\cot(c+dx)\,\mathrm{c}}{8ad\sqrt{a+a\,\mathrm{si}}}$$

```
[Out] -3/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-3/64*cot
(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/32*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*sin(
d*x+c))^(1/2)+5/8*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)-1/4*co
t(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]

time = 0.50, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2959, 2851, 2852, 212, 3123, 3059}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{3/2}d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a\sin(c+dx)+a}}{4a^2d}-\frac{3\cot(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}+\frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(3/2)*d
) - (3*Cot[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[
c + d*x])/(32*a*d*Sqrt[a + a*Sin[c + d*x]]) + (5*Cot[c + d*x]*Csc[c + d*x]^
2)/(8*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a +
 a*Sin[c + d*x]])/(4*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^5(c+dx)\sqrt{a+a\sin(c+dx)}\,(1+\sin^2(c+dx))\,dx}{a^2} - \frac{2\int \csc^4(c+d\ldots}{}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+a\sin(c+dx)}}{4a^2d}\ldots$$

$$= \frac{5\cot(c+dx)\csc(c+dx)}{6ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2(c+dx)}{8ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(\ldots}{}$$

$$= \frac{5\cot(c+dx)}{4ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\csc^2\ldots}{8ad\sqrt{a+a\sin(c\ldots}}$$

$$= -\frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} + \frac{5\cot(c+dx)\,c\ldots}{8ad\sqrt{a+a\,\mathrm{si}\ldots}}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+\ldots}{32ad\sqrt{a\ldots}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{3/2}d} - \frac{3\cot(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c\ldots}{32ad\sqrt{\ldots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 376 vs. $2(182) = 364$.

time = 0.76, size = 376, normalized size = 2.07

$$-\frac{\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(446\cos\left(\frac{1}{2}(c+dx)\right)-182\cos\left(\frac{3}{2}(c+dx)\right)-2\cos\left(\frac{5}{2}(c+dx)\right)-6\cos\left(\frac{7}{2}(c+dx)\right)+9\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-12\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(4(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-9\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+12\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(4(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-446\sin\left(\frac{1}{2}(c+dx)\right)-182\sin\left(\frac{3}{2}(c+dx)\right)+2\sin\left(\frac{5}{2}(c+dx)\right)-6\sin\left(\frac{7}{2}(c+dx)\right)\right)}{64d\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^4(a(1+\sin(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/64*(Csc[(c + d*x)/2]^12*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(446*Cos
[(c + d*x)/2] - 182*Cos[(3*(c + d*x))/2] - 2*Cos[(5*(c + d*x))/2] - 6*Cos[(
7*(c + d*x))/2] + 9*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 12*Cos[2
*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3*Cos[4*(c + d*x
)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 9*Log[1 - Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 3*Cos[4*(c + d*x)]*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]] - 446*Sin[(c + d*x)/2] - 182*Sin[(3*(c + d*x))/2] + 2*Sin[(5*(c + d*x)
)/2] - 6*Sin[(7*(c + d*x))/2]))/(d*(Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2
)^4*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 6.81, size = 162, normalized size = 0.89

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-3(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{5}{2}}+11(-a(\sin(dx+c)-1))^{\frac{5}{2}}a^{\frac{7}{2}}+11(-a(\sin(dx+c)-1)\right.}{64a^{\frac{15}{2}}\sin(dx+c)^{4}\cos(dx+c)\sqrt{a+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/64*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-3*(-a*(sin(d*x+c)-1))^(7/2
)*a^(5/2)+11*(-a*(sin(d*x+c)-1))^(5/2)*a^(7/2)+11*(-a*(sin(d*x+c)-1))^(3/2)
*a^(9/2)-3*(-a*(sin(d*x+c)-1))^(1/2)*a^(11/2)+3*arctanh((-a*(sin(d*x+c)-1))
^(1/2)/a^(1/2))*a^6*sin(d*x+c)^4)/a^(15/2)/sin(d*x+c)^4/cos(d*x+c)/(a+a*sin
(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(158) = 316.
time = 0.36, size = 442, normalized size = 2.43

$$\frac{3\left(\cos\left(dx+c\right)^{5}+\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{3}-2\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(3\cos\left(dx+c\right)^{4}+2\cos\left(dx+c\right)^{3}+20\cos\left(dx+c\right)^{2}+\left(3\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}+21\cos\left(dx+c\right)+39\right)\sin\left(dx+c\right)-18\cos\left(dx+c\right)-39\right)\sqrt{a\sin\left(dx+c\right)+a}}{256\left(a^{2}d\cos\left(dx+c\right)^{5}+a^{2}d\cos\left(dx+c\right)^{4}-2a^{2}d\cos\left(dx+c\right)^{3}-2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)+a^{2}d+\left(a^{2}d\cos\left(dx+c\right)^{4}-2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/256*(3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x +
c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x + c)
+ 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c
) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) -
```

```
a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1
)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(3*cos(d*x + c)^4 + 2*cos(d*x + c)^
3 + 20*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + cos(d*x + c)^2 + 21*cos(d*x + c
) + 39)*sin(d*x + c) - 18*cos(d*x + c) - 39)*sqrt(a*sin(d*x + c) + a))/(a^2
*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^2*d
*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d + (a^2*d*cos(d*x + c)^4 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [A]
time = 0.51, size = 184, normalized size = 1.01

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{3\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\left(24\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-44\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-22\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/256*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) + 4*(24*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 -
44*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 - 22*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 +
 3*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1
)^4*a^2*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^5\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(3/2)), x)
```

3.480 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128a^{3/2}d}-\frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}}-\frac{\cot(c+dx)\csc}{80ad\sqrt{a+a\sin}}$$

```
[Out] -3/128*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(3/2)/d-3/128*c
ot(d*x+c)/a/d/(a+a*sin(d*x+c))^(1/2)-1/64*cot(d*x+c)*csc(d*x+c)/a/d/(a+a*si
n(d*x+c))^(1/2)-1/80*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+a*sin(d*x+c))^(1/2)+19/
40*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+a*sin(d*x+c))^(1/2)-1/5*cot(d*x+c)*csc(d*
x+c)^4*(a+a*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A]
time = 0.59, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2959, 2851, 2852, 212, 3123, 3059}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\ \cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{128a^{3/2}d}-\frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a\sin(c+dx)+a}}{5a^2d}-\frac{3\cot(c+dx)}{128ad\sqrt{a\sin(c+dx)+a}}+\frac{19\cot(c+dx)\csc^3(c+dx)}{40ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-3*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(128*a^(3/2)*
d) - (3*Cot[c + d*x])/(128*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Cs
c[c + d*x])/(64*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^
2)/(80*a*d*Sqrt[a + a*Sin[c + d*x]]) + (19*Cot[c + d*x]*Csc[c + d*x]^3)/(40
*a*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[a + a*Si
n[c + d*x]])/(5*a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2851

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Dis
t[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2))), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
```

```
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3059

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Dist[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]
*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^{3/2}}\,dx = \frac{\int \csc^6(c+dx)\sqrt{a+a\sin(c+dx)}\,\left(1+\sin^2(c+dx)\right)\,dx}{a^2} - \frac{2\int \csc^5(c+dx}{}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{2ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^4(c+dx)\sqrt{a+a\sin(c+dx)}}{5a^2d} +$$

$$= \frac{7\cot(c+dx)\csc^2(c+dx)}{12ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\csc^3(c+dx)}{40ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc}{}$$

$$= \frac{35\cot(c+dx)\csc(c+dx)}{48ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{80ad\sqrt{a+a\sin(c+dx)}} + \frac{19\cot(c+dx)\mathrm{cs}}{40ad\sqrt{a+a\sin}}$$

$$= \frac{35\cot(c+dx)}{32ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(}{80ad\sqrt{a+a\sin(}}$$

$$= -\frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{64ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\mathrm{c}}{80ad\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{35\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{32a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c}{64ad\sqrt{}}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{128a^{3/2}d} - \frac{3\cot(c+dx)}{128ad\sqrt{a+a\sin(c+dx)}} - \frac{\cot(}{64ad}$$

Mathematica [A]
time = 1.00, size = 412, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/640*(Csc[(c + d*x)/2]^15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(7100*C
os[(c + d*x)/2] - 2880*Cos[(3*(c + d*x))/2] - 144*Cos[(5*(c + d*x))/2] - 10
*Cos[(7*(c + d*x))/2] + 30*Cos[(9*(c + d*x))/2] - 7100*Sin[(c + d*x)/2] + 1
50*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[1 -
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 2880*Sin[(3*(c + d*x))/
2] + 144*Sin[(5*(c + d*x))/2] - 75*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*Sin[3*(c + d*x)] + 75*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 10*Sin[(7*(c + d*x))/2] - 30*Sin[(9*(c + d*x))/2] + 15*Log[
1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[1 - Cos[
```

```
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(Csc[(c + d*x)/4]^2
- Sec[(c + d*x)/4]^2)^5*(a*(1 + Sin[c + d*x]))^(3/2))
```

Maple [A]
time = 7.68, size = 180, normalized size = 0.82

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(15(-a(\sin(dx+c)-1))^{\frac{9}{2}}a^{\frac{7}{2}}-70(-a(\sin(dx+c)-1))^{\frac{7}{2}}a^{\frac{9}{2}}+128(-a(\sin(dx+c)-\right.}{640a^{\frac{19}{2}}\sin(dx+c)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/640/a^(19/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(15*(-a*(sin(d*x+c
)-1))^(9/2)*a^(7/2)-70*(-a*(sin(d*x+c)-1))^(7/2)*a^(9/2)+128*(-a*(sin(d*x+c
)-1))^(5/2)*a^(11/2)+15*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^8*sin(
d*x+c)^5+70*(-a*(sin(d*x+c)-1))^(3/2)*a^(13/2)-15*(-a*(sin(d*x+c)-1))^(1/2)
*a^(15/2))/sin(d*x+c)^5/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. 2(192) = 384.
time = 0.36, size = 492, normalized size = 2.24

$$\frac{15\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)+1\right)\sin(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+4\left(15\cos(dx+c)^5+5\cos(dx+c)^4-38\cos(dx+c)^3-194\cos(dx+c)^2-\left(15\cos(dx+c)^4+10\cos(dx+c)^3-28\cos(dx+c)^2+166\cos(dx+c)+317\right)\sin(dx+c)+151\cos(dx+c)+317\right)\sqrt{a\sin(dx+c)+a}}{2560\left(a^2d\cos(dx+c)^6-3a^2d\cos(dx+c)^4+3a^2d\cos(dx+c)^2-a^2d-\left(a^2d\cos(dx+c)^5+a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 1/2560*(15*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x
 + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x +
c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^
```

```
2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) -
3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2
+ 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2
+ (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*(15*cos(d*x +
c)^5 + 5*cos(d*x + c)^4 - 38*cos(d*x + c)^3 - 194*cos(d*x + c)^2 - (15*cos(
d*x + c)^4 + 10*cos(d*x + c)^3 - 28*cos(d*x + c)^2 + 166*cos(d*x + c) + 317
)*sin(d*x + c) + 151*cos(d*x + c) + 317)*sqrt(a*sin(d*x + c) + a))/(a^2*d*c
os(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d - (
a^2*d*cos(d*x + c)^5 + a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^3 - 2*a^
2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+a*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 200, normalized size = 0.91

$$-\frac{\sqrt{2}\,\sqrt{a}\left(\frac{15\,\sqrt{2}\,\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{4\left(240\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-560\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+512\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+140\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(2\,\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}\right)}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+a*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] -1/2560*sqrt(2)*sqrt(a)*(15*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/
2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^2*sgn
(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 4*(240*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9
 - 560*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 512*sin(-1/4*pi + 1/2*d*x + 1/2*c
)^5 + 140*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 15*sin(-1/4*pi + 1/2*d*x + 1/2
*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^5*a^2*sgn(cos(-1/4*pi + 1/2*
d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^6\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + a*sin(c + d*x))^(3/2)), x)
```

3.481 $\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}}+\frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}}-\frac{424}{69}$$

```
[Out] -4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a
^(5/2)/d+4496/693*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+200/231*cos(d*x+c
)*sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)-424/693*cos(d*x+c)*sin(d*x+c)^3
/a^2/d/(a+a*sin(d*x+c))^(1/2)+46/99*cos(d*x+c)*sin(d*x+c)^4/a^2/d/(a+a*sin(
d*x+c))^(1/2)-2/11*cos(d*x+c)*sin(d*x+c)^5/a^2/d/(a+a*sin(d*x+c))^(1/2)-104
8/693*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d
```

Rubi [A]
time = 0.90, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2959, 2857, 3062, 3047, 3102, 2830, 2728, 212, 3125}

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{1048\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{693a^3d}-\frac{2\sin^5(c+dx)\cos(c+dx)}{11a^2d\sqrt{a\sin(c+dx)+a}}+\frac{46\sin^4(c+dx)\cos(c+dx)}{99a^2d\sqrt{a\sin(c+dx)+a}}-\frac{424\sin^3(c+dx)\cos(c+dx)}{693a^2d\sqrt{a\sin(c+dx)+a}}+\frac{200\sin^2(c+dx)\cos(c+dx)}{231a^2d\sqrt{a\sin(c+dx)+a}}+\frac{4496\cos(c+dx)}{693a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
])])/(a^(5/2)*d) + (4496*Cos[c + d*x])/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]])
 + (200*Cos[c + d*x]*Sin[c + d*x]^2)/(231*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (424*Cos[c + d*x]*Sin[c + d*x]^3)/(693*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (
46*Cos[c + d*x]*Sin[c + d*x]^4)/(99*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Co
s[c + d*x]*Sin[c + d*x]^5)/(11*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (1048*Cos[
c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(693*a^3*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^4(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\sin^5(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^5(c+dx)}{11a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^4(c+dx)(\frac{2}{\cdots}}{\sqrt{a+a\,\mathrm{s}\cdots}}}{1\cdots}$$

$$= -\frac{4\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+dx)\sin^4(c+dx)}{99a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+d\cdots}{11a^2d\sqrt{a+\cdots}}$$

$$= \frac{76\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+\cdots}{99a^2d\sqrt{a\cdots}}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+\cdots}{99a^2d\sqrt{a\cdots}}$$

$$= \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+dx)\sin^3(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{46\cos(c+\cdots}{99a^2d\sqrt{a\cdots}}$$

$$= \frac{1144\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+\cdots}{693a^2d\sqrt{a\cdots}}$$

$$= \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx)}} + \frac{200\cos(c+dx)\sin^2(c+dx)}{231a^2d\sqrt{a+a\sin(c+dx)}} - \frac{424\cos(c+\cdots}{693a^2d\sqrt{a\cdots}}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx\cdots}}$$

$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4496\cos(c+dx)}{693a^2d\sqrt{a+a\sin(c+dx\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.97, size = 224, normalized size = 0.86

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left((88704+88704i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+73458\cos\left(\frac{1}{2}(c+dx)\right)-15246\cos\left(\frac{3}{2}(c+dx)\right)-4851\cos\left(\frac{5}{2}(c+dx)\right)+1485\cos\left(\frac{7}{2}(c+dx)\right)+385\cos\left(\frac{9}{2}(c+dx)\right)-63\cos\left(\frac{11}{2}(c+dx)\right)-73458\sin\left(\frac{1}{2}(c+dx)\right)-15246\sin\left(\frac{3}{2}(c+dx)\right)+4851\sin\left(\frac{5}{2}(c+dx)\right)+1485\sin\left(\frac{7}{2}(c+dx)\right)-385\sin\left(\frac{9}{2}(c+dx)\right)-63\sin\left(\frac{11}{2}(c+dx)\right)\right)}{11088d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((88704 + 88704*I)*(-1)^(3/4)*ArcT
anh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 73458*Cos[(c + d*x)/2
] - 15246*Cos[(3*(c + d*x))/2] - 4851*Cos[(5*(c + d*x))/2] + 1485*Cos[(7*(c
 + d*x))/2] + 385*Cos[(9*(c + d*x))/2] - 63*Cos[(11*(c + d*x))/2] - 73458*S
in[(c + d*x)/2] - 15246*Sin[(3*(c + d*x))/2] + 4851*Sin[(5*(c + d*x))/2] +
1485*Sin[(7*(c + d*x))/2] - 385*Sin[(9*(c + d*x))/2] - 63*Sin[(11*(c + d*x)
)/2]))/(11088*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 6.38, size = 166, normalized size = 0.64

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\,\left(1386a^{\frac{11}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-63(a-a\sin(dx\right.}{693a^{8}\cos(dx+c)\sqrt{a+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/693*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1386*a^(11/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-63*(a-a*sin(d*x+c))^(11/2)+
154*a*(a-a*sin(d*x+c))^(9/2)-198*a^2*(a-a*sin(d*x+c))^(7/2)-231*a^4*(a-a*si
n(d*x+c))^(3/2)-1386*a^5*(a-a*sin(d*x+c))^(1/2))/a^8/cos(d*x+c)/(a+a*sin(d*
x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^4/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.37, size = 299, normalized size = 1.15

$$\frac{2\left(\frac{693\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(63\cos\left(dx+c\right)^6-161\cos\left(dx+c\right)^5-562\cos\left(dx+c\right)^4+622\cos\left(dx+c\right)^3+1759\cos\left(dx+c\right)^2+\left(63\cos\left(dx+c\right)^5+224\cos\left(dx+c\right)^4-338\cos\left(dx+c\right)^3-960\cos\left(dx+c\right)^2+799\cos\left(dx+c\right)+2984\right)\sin\left(dx+c\right)-2185\cos\left(dx+c\right)-2984\right)\sqrt{a\sin\left(dx+c\right)+a}\right)}{693\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/693*(693*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (63*co
s(d*x + c)^6 - 161*cos(d*x + c)^5 - 562*cos(d*x + c)^4 + 622*cos(d*x + c)^3
 + 1759*cos(d*x + c)^2 + (63*cos(d*x + c)^5 + 224*cos(d*x + c)^4 - 338*cos(
d*x + c)^3 - 960*cos(d*x + c)^2 + 799*cos(d*x + c) + 2984)*sin(d*x + c) - 2
185*cos(d*x + c) - 2984)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^
3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3278 deep
```

Giac [A]

time = 0.49, size = 201, normalized size = 0.77

$$\frac{2\left(\frac{693\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{693\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(1008a^{\frac{61}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-1232a^{\frac{61}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+792a^{\frac{61}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+231a^{\frac{61}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+693a^{\frac{61}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{33}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{693d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 2/693*(693*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos
(-1/4*pi + 1/2*d*x + 1/2*c))) - 693*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/
2*c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(1008*a
^(61/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^11 - 1232*a^(61/2)*sin(-1/4*pi + 1/2
*d*x + 1/2*c)^9 + 792*a^(61/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 231*a^(61
/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 693*a^(61/2)*sin(-1/4*pi + 1/2*d*x +
 1/2*c))/(a^33*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^4}{\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^4)/(a + a*sin(c + d*x))^(5/2), x)
```

3.482 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=222

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}}-\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}}+\frac{38\cos\cdots}{63a^2\cdots}$$

[Out] 4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-2048/315*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-92/105*cos(d*x+c)*sin(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)+38/63*cos(d*x+c)*sin(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^(1/2)-2/9*cos(d*x+c)*sin(d*x+c)^4/a^2/d/(a+a*sin(d*x+c))^(1/2)+472/315*cos(d*x+c)*(a+a*sin(d*x+c))^(1/2)/a^3/d

Rubi [A]

time = 0.74, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2959, 2857, 3062, 3047, 3102, 2830, 2728, 212, 3125}

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{472\cos(c+dx)\sqrt{a\sin(c+dx)+a}}{315a^3d}-\frac{2\sin^4(c+dx)\cos(c+dx)}{9a^2d\sqrt{a\sin(c+dx)+a}}+\frac{38\sin^3(c+dx)\cos(c+dx)}{63a^2d\sqrt{a\sin(c+dx)+a}}-\frac{92\sin^2(c+dx)\cos(c+dx)}{105a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2048\cos(c+dx)}{315a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]

[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(a^(5/2)*d) - (2048*Cos[c + d*x])/(315*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (92*Cos[c + d*x]*Sin[c + d*x]^2)/(105*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (38*Cos[c + d*x]*Sin[c + d*x]^3)/(63*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (2*Cos[c + d*x]*Sin[c + d*x]^4)/(9*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (472*Cos[c + d*x]*Sqrt[a + a*Sin[c + d*x]])/(315*a^3*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 2857

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[-2*d*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n - 1)/(f*(2*n - 1)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(b*(2*n - 1))
, Int[((c + d*Sin[e + f*x])^(n - 2)/Sqrt[a + b*Sin[e + f*x]])*Simp[a*c*d -
b*(2*d^2*(n - 1) + c^2*(2*n - 1)) + d*(a*d - b*c*(4*n - 3))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3062

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(f*(m +
n + 1))), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Si
n[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m
+ n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\sin^3(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\sin^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{4\cos(c+dx)\sin^3(c+dx)}{7a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)}{9a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\sin^3(c+dx)\left(\frac{17}{2}\ldots\right.}{\sqrt{a+a\sin\ldots}}}{9\ldots}$$

$$= -\frac{4\cos(c+dx)\sin^2(c+dx)}{35a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+d\ldots}{9a^2d\sqrt{a+\ldots}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+\ldots}{9a^2d\sqrt{a\ldots}}$$

$$= -\frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c+dx)\sin^3(c+dx)}{63a^2d\sqrt{a+a\sin(c+dx)}} - \frac{2\cos(c+\ldots}{9a^2d\sqrt{a\ldots}}$$

$$= -\frac{296\cos(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c\ldots}{63a^2d\sqrt{a\ldots}}$$

$$= -\frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}} - \frac{92\cos(c+dx)\sin^2(c+dx)}{105a^2d\sqrt{a+a\sin(c+dx)}} + \frac{38\cos(c\ldots}{63a^2d\sqrt{a\ldots}}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}}$$

$$= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2048\cos(c+dx)}{315a^2d\sqrt{a+a\sin(c+dx)}}$$

<u>Mathematica</u> **[C]** Result contains complex when optimal does not.
time = 2.69, size = 225, normalized size = 1.01

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((20160+20160i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)-16380\cos\left(\frac{1}{2}(c+dx)\right)+3150\cos\left(\frac{3}{2}(c+dx)\right)+882\cos\left(\frac{5}{2}(c+dx)\right)-225\cos\left(\frac{7}{2}(c+dx)\right)-35\cos\left(\frac{9}{2}(c+dx)\right)+16380\sin\left(\frac{1}{2}(c+dx)\right)+3150\sin\left(\frac{3}{2}(c+dx)\right)-882\sin\left(\frac{5}{2}(c+dx)\right)-225\sin\left(\frac{7}{2}(c+dx)\right)+35\sin\left(\frac{9}{2}(c+dx)\right)\right)}{2520a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((20160 + 20160*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/
2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 163
```

```
80*Cos[(c + d*x)/2] + 3150*Cos[(3*(c + d*x))/2] + 882*Cos[(5*(c + d*x))/2]
- 225*Cos[(7*(c + d*x))/2] - 35*Cos[(9*(c + d*x))/2] + 16380*Sin[(c + d*x)/
2] + 3150*Sin[(3*(c + d*x))/2] - 882*Sin[(5*(c + d*x))/2] - 225*Sin[(7*(c +
 d*x))/2] + 35*Sin[(9*(c + d*x))/2]))/(2520*a^3*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]))
```

Maple [A]
time = 6.12, size = 166, normalized size = 0.75

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-630a^{\frac{9}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)+35(a-a\sin(dx+\right.}{315a^{7}\cos(dx+c)\sqrt{a+a\,\mathrm{s}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/315/a^7*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-630*a^(9/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+35*(a-a*sin(d*x+c))^(9/2
)-45*a*(a-a*sin(d*x+c))^(7/2)+63*a^2*(a-a*sin(d*x+c))^(5/2)+105*a^3*(a-a*si
n(d*x+c))^(3/2)+630*a^4*(a-a*sin(d*x+c))^(1/2))/cos(d*x+c)/(a+a*sin(d*x+c))
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.36, size = 280, normalized size = 1.26

$$\frac{2\left(\frac{315\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}-\left(\cos(dx+c)-2\right)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\left(\cos(dx+c)-\sin(dx+c)+1\right)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-\left(\cos(dx+c)+2\right)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-\left(35\cos\left(dx+c\right)^{5}+130\cos\left(dx+c\right)^{4}-208\cos\left(dx+c\right)^{3}-634\cos\left(dx+c\right)^{2}-\left(35\cos\left(dx+c\right)^{4}-95\cos\left(dx+c\right)^{3}-303\cos\left(dx+c\right)^{2}+331\cos\left(dx+c\right)+1292\right)\sin\left(dx+c\right)+961\cos\left(dx+c\right)+1292\right)\sqrt{a\sin\left(dx+c\right)+a}\right)}{315\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/315*(315*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)
^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(
cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - (35*co
s(d*x + c)^5 + 130*cos(d*x + c)^4 - 208*cos(d*x + c)^3 - 634*cos(d*x + c)^2
 - (35*cos(d*x + c)^4 - 95*cos(d*x + c)^3 - 303*cos(d*x + c)^2 + 331*cos(d*
x + c) + 1292)*sin(d*x + c) + 961*cos(d*x + c) + 1292)*sqrt(a*sin(d*x + c)
+ a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 201, normalized size = 0.91

$$-\frac{2\left(\frac{315\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{315\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(280a^{\frac{49}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-180a^{\frac{49}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+126a^{\frac{49}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+105a^{\frac{49}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+315a^{\frac{49}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{27}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -2/315*(315*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(co
s(-1/4*pi + 1/2*d*x + 1/2*c))) - 315*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1
/2*c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(280*a
^(49/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^9 - 180*a^(49/2)*sin(-1/4*pi + 1/2*d
*x + 1/2*c)^7 + 126*a^(49/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 105*a^(49/2
)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 315*a^(49/2)*sin(-1/4*pi + 1/2*d*x + 1
/2*c))/(a^27*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^3}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + a*sin(c + d*x))^(5/2), x)
```

3.483 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}}+\frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}-\frac{2\,\mathrm{co}}{7ad(a+}$$

```
[Out] 4/7*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)+2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*x
+c))^(3/2)-2/7*cos(d*x+c)^5/a/d/(a+a*sin(d*x+c))^(3/2)-4*arctanh(1/2*cos(d*
x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+4*cos(d*x+c)
/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2957, 2939, 2758, 2728, 212}

$$-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)}{7ad(a\sin(c+dx)+a)^{3/2}}+\frac{4\cos^5(c+dx)}{7d(a\sin(c+dx)+a)^{5/2}}+\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(a^(5/2)*d) + (4*Cos[c + d*x]^5)/(7*d*(a + a*Sin[c + d*x])^(5/2)) + (2
*Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (2*Cos[c + d*x]^5)/(7
*a*d*(a + a*Sin[c + d*x])^(3/2)) + (4*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c
 + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(b*f*g*(m + p + 2))), x] + Dist[1/(b*
(m + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*
(p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = -\frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \frac{2\int \frac{\cos^4(c+dx)\left(-\frac{3a}{2}-5a\sin(c+dx)\right)}{(a+a\sin(c+dx))^{5/2}}\,dx}{7a}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^5(c+dx)}{7ad(a+a\sin(c+dx))^{3/2}} + \int \frac{\cos^4(c+}{(a+a\sin(c+}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+}{7ad(a+a\sin(c+}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+}{7ad(a+a\sin(c+}$$

$$= \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} + \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\cos^5(c+}{7ad(a+a\sin(c+}$$

$$= -\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\cos^5(c+dx)}{7d(a+a\sin(c+dx))^{5/2}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.11, size = 201, normalized size = 1.19

$$-\frac{\sqrt{a(1+\sin(c+dx))}\left((672+672i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)-525\cos\left(\frac{1}{2}(c+dx)\right)+91\cos\left(\frac{3}{2}(c+dx)\right)+21\cos\left(\frac{5}{2}(c+dx)\right)-3\cos\left(\frac{7}{2}(c+dx)\right)+525\sin\left(\frac{1}{2}(c+dx)\right)+91\sin\left(\frac{3}{2}(c+dx)\right)-21\sin\left(\frac{5}{2}(c+dx)\right)-3\sin\left(\frac{7}{2}(c+dx)\right)\right)}{84a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/84*(Sqrt[a*(1 + Sin[c + d*x])]*((672 + 672*I)*(-1)^(3/4)*ArcTanh[(1/2 +
I/2)*(-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 5
25*Cos[(c + d*x)/2] + 91*Cos[(3*(c + d*x))/2] + 21*Cos[(5*(c + d*x))/2] - 3
*Cos[(7*(c + d*x))/2] + 525*Sin[(c + d*x)/2] + 91*Sin[(3*(c + d*x))/2] - 21
*Sin[(5*(c + d*x))/2] - 3*Sin[(7*(c + d*x))/2]))/(a^3*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]))
```

Maple [A]
time = 4.98, size = 132, normalized size = 0.78

method	result
default	$-\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(42a^{\frac{7}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-3(a-a\sin(dx+c))\right.}{21a^6\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/21*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(42*a^(7/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(7/2)-7*a^2*
(a-a*sin(d*x+c))^(3/2)-42*a^3*(a-a*sin(d*x+c))^(1/2))/a^6/cos(d*x+c)/(a+a*s
in(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 0.37, size = 258, normalized size = 1.53

$$\frac{2\left(\frac{21\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\,\sqrt{a\sin(dx+c)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos(dx+c)^4-9\cos(dx+c)^3-31\cos(dx+c)^2+\left(3\cos(dx+c)^3+12\cos(dx+c)^2-19\cos(dx+c)-80\right)\sin(dx+c)+61\cos(dx+c)+80\right)\sqrt{a\sin(dx+c)+a}\right)}{21\,(a^3d\cos(dx+c)+a^3d\sin(dx+c)+a^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/21*(21*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^4 - 9*cos(d*x + c)^3 - 31*cos(d*x + c)^2 + (3*cos(d*x + c)^3 + 12*c
os(d*x + c)^2 - 19*cos(d*x + c) - 80)*sin(d*x + c) + 61*cos(d*x + c) + 80)*
sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 163, normalized size = 0.96

$$\frac{2\left(\frac{21\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{21\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(12a^{\frac{37}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7+7a^{\frac{37}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+21a^{\frac{37}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{21}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 2/21*(21*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) - 21*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*c
) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(12*a^(37/
2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^7 + 7*a^(37/2)*sin(-1/4*pi + 1/2*d*x + 1/
2*c)^3 + 21*a^(37/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^21*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + a*sin(c + d*x))^(5/2), x)
```

3.484 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=137

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}}-\frac{4\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2/5*cos(d*x+c)^5/d/(a+a*sin(d*x+c))^(5/2)-2/3*cos(d*x+c)^3/a/d/(a+a*sin(d*
x+c))^(3/2)+4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2)
)*2^(1/2)/a^(5/2)/d-4*cos(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2939, 2758, 2728, 212}

$$\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}-\frac{2\cos^5(c+dx)}{5d(a\sin(c+dx)+a)^{5/2}}-\frac{2\cos^3(c+dx)}{3ad(a\sin(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - (2*Cos[c + d*x]^5)/(5*d*(a + a*Sin[c + d*x])^(5/2)) - (2*
Cos[c + d*x]^3)/(3*a*d*(a + a*Sin[c + d*x])^(3/2)) - (4*Cos[c + d*x])/(a^2*
d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
```

```
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx &= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \int \frac{\cos^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx \\
&= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{2\int \frac{\cos^2(c+dx)}{(a+a\sin(c+dx))^{3/}}}{a} \\
&= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx}{a^2d\sqrt{a+a\sin(c}} \\
&= -\frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{2\cos^3(c+dx)}{3ad(a+a\sin(c+dx))^{3/2}} - \frac{4\cos(c+dx}{a^2d\sqrt{a+a\sin(c}} \\
&= \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos^5(c+dx)}{5d(a+a\sin(c+dx))^{5/2}} - \frac{}{3a}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 177, normalized size = 1.29

$$\frac{\sqrt{a(1+\sin(c+dx))}\left((240+240i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{dx}{4}\right)\left(\cos\left(\frac{1}{4}(2c+dx)\right)-\sin\left(\frac{1}{4}(2c+dx)\right)\right)\right)-180\cos\left(\frac{1}{2}(c+dx)\right)+25\cos\left(\frac{3}{2}(c+dx)\right)+3\cos\left(\frac{5}{2}(c+dx)\right)+180\sin\left(\frac{1}{2}(c+dx)\right)+25\sin\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{5}{2}(c+dx)\right)\right)}{30a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a*(1 + Sin[c + d*x])]*((240 + 240*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(
-1)^(3/4)*Sec[(d*x)/4]*(Cos[(2*c + d*x)/4] - Sin[(2*c + d*x)/4])] - 180*Cos
```

```
[(c + d*x)/2] + 25*Cos[(3*(c + d*x))/2] + 3*Cos[(5*(c + d*x))/2] + 180*Sin[
(c + d*x)/2] + 25*Sin[(3*(c + d*x))/2] - 3*Sin[(5*(c + d*x))/2]))/(30*a^3*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 5.83, size = 130, normalized size = 0.95

method	result
default	$\frac{2(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(30a^{\frac{5}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin\left(dx+c\right)}\,\sqrt{2}}{2\sqrt{a}}\right)-3(a-a\sin(dx+c))^{\frac{5}{2}}\right.}{15a^{5}\cos(dx+c)\sqrt{a+a\sin\left(dx+c\right)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(30*a^(5/2)*2^(1/2)*arctanh(1
/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-3*(a-a*sin(d*x+c))^(5/2)-5*a*(a-
a*sin(d*x+c))^(3/2)-30*a^2*(a-a*sin(d*x+c))^(1/2))/a^5/cos(d*x+c)/(a+a*sin(
d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(118) = 236.
time = 0.38, size = 239, normalized size = 1.74

$$\frac{2\left(\frac{15\sqrt{2}\left(a\cos(dx+c)+a\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+\left(3\cos\left(dx+c\right)^{3}+14\cos\left(dx+c\right)^{2}-\left(3\cos\left(dx+c\right)^{2}-11\cos\left(dx+c\right)-52\right)\sin\left(dx+c\right)-41\cos\left(dx+c\right)-52\right)\sqrt{a\sin\left(dx+c\right)+a}\right)}{15\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\sin\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(15*sqrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2
 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(co
```

```
s(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^
2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + (3*cos(d
*x + c)^3 + 14*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 11*cos(d*x + c) - 52)*s
in(d*x + c) - 41*cos(d*x + c) - 52)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*
x + c) + a^3*d*sin(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 163, normalized size = 1.19

$$-\frac{2\left(\frac{15\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{15\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{2\sqrt{2}\left(6a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+5a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^{\frac{25}{2}}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{15}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] -2/15*(15*sqrt(2)*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^(5/2)*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) - 15*sqrt(2)*log(-sin(-1/4*pi + 1/2*d*x + 1/2*
c) + 1)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 2*sqrt(2)*(6*a^(25/
2)*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 5*a^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/
2*c)^3 + 15*a^(25/2)*sin(-1/4*pi + 1/2*d*x + 1/2*c))/(a^15*sgn(cos(-1/4*pi
+ 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + a*sin(c + d*x))^(5/2), x)
```

3.485 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -2*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arctanh(1
/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-2*c
os(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2959, 2728, 212, 3125, 3064, 2852}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{2\cos(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) +
 (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]
]])])/(a^(5/2)*d) - (2*Cos[c + d*x])/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3125

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1
)/(d*f*(m + n + 2))), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x
])^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1)
) + C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{1}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(\frac{a}{2}-\frac{1}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} + \frac{4\text{Subst}\left(\int \frac{}{2a}\right.}{}$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \int$$

$$= \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\cos(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - 2$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+\right.}}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 154, normalized size = 1.36

$$-\frac{\left((8+8i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+2\cos\left(\frac{1}{2}(c+dx)\right)+\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}{d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((((8 + 8*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)
/4])] + 2*Cos[(c + d*x)/2] + Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*Sin[(c + d*x)/2])*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^5)/(d*(a*(1 + Sin[c + d*x]))^(5/2)))
```

Maple [A]
time = 5.30, size = 119, normalized size = 1.05

method	result
default	$\frac{2(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(2\sqrt{a}\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)-\sqrt{a}\,\text{arctanh}\left(\right.\right.}{a^3\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(2*a^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))-a^(1/2)*arctanh((a-a*sin(d*x+c))^(1/
2)/a^(1/2))-(a-a*sin(d*x+c))^(1/2))/a^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(96) = 192$.
time = 0.40, size = 377, normalized size = 3.34

$$\frac{\sqrt{a}\left(\cos\left(dx+c\right)+\sin\left(dx+c\right)+1\right)\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\,\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9\,a\cos(dx+c)+\left(a\cos(dx+c)^2+8\,a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{4\sqrt{2}\,(a\cos(dx+c)+a\sin(dx+c)+a)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\,(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\,\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\sqrt{a\sin\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{2\left(a^3d\cos\left(dx+c\right)+a^3d\sin\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] 1/2*(sqrt(a)*(cos(d*x + c) + sin(d*x + c) + 1)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*co
s(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*co
s(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + co
s(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 4*s
qrt(2)*(a*cos(d*x + c) + a*sin(d*x + c) + a)*log(-(cos(d*x + c)^2 - (cos(d*
x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c)
 - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d
*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) - 4*sqrt(a*sin(d*x +
 c) + a)*(cos(d*x + c) - sin(d*x + c) + 1))/(a^3*d*cos(d*x + c) + a^3*d*sin
(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+a*sin(d*x+c))**(5/2),x)
```

[Out] Timed out

Giac [A]
time = 0.48, size = 184, normalized size = 1.63

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{4\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+a*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] -1/2*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
+ 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^3*sgn(cos(-
1/4*pi + 1/2*d*x + 1/2*c))) + 4*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a^
3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*log(-sin(-1/4*pi + 1/2*d*x + 1/2
*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sin(-1/4*pi + 1/2*d*
x + 1/2*c)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^4}{\sin(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)*(a + a*sin(c + d*x))^(5/2)), x)
```

3.486 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] 5*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d-4*arctanh(1/
2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-cot(
d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2959, 2859, 2728, 212, 2852, 3123, 3064}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{\cot(c+dx)}{a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(a^(5/2)*d) -
(4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]]
)])/(a^(5/2)*d) - Cot[c + d*x]/(a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
```

```
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2859

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e +
f*x]], x], x] - Dist[d/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin
[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^2(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(-\frac{a}{2}+\frac{3}{2}a\sin(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^3} - \frac{2\int \csc(c+dx}{}$$

$$= -\frac{\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\int \csc(c+dx)\sqrt{a+a\sin(c+dx)}\,dx}{2a^3} + \frac{2\int -}{}$$

$$= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx}}\right.}{a^{5/2}d}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx}}\right.}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.24, size = 170, normalized size = 1.50

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left((32+32i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\cot\left(\frac{1}{4}(c+dx)\right)+10\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-10\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec\left(\frac{1}{2}(c+dx)\right)-\tan\left(\frac{1}{4}(c+dx)\right)\right)}{4d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((32 + 32*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - Cot[(c + d*x)/4] + 10*Log[1
 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 10*Log[1 - Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] + 2*Sec[(c + d*x)/2] - Tan[(c + d*x)/4]))/(4*d*(a*(1 + Sin[c
 + d*x]))^(5/2))
```

Maple [A]
time = 6.40, size = 133, normalized size = 1.18

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(-\sin(dx+c)a\left(-4\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(dx+c)}\sqrt{2}}{2\sqrt{a}}\right)+5\,\operatorname{arcta}\right.\right.}{a^{\frac{7}{2}}\sin(dx+c)\cos(dx+c)\sqrt{a+a\sin(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/a^(7/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-sin(d*x+c)*a*(-4*2^(1
/2)*arctanh(1/2*(a-a*sin(d*x+c))^(1/2)*2^(1/2)/a^(1/2))+5*arctanh((a-a*sin(
d*x+c))^(1/2)/a^(1/2)))+(a-a*sin(d*x+c))^(1/2)*a^(1/2))/sin(d*x+c)/cos(d*x+
c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*csc(d*x + c)^2/(a*sin(d*x + c) + a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(96) = 192$.
time = 0.38, size = 421, normalized size = 3.73

$$\frac{5\left(\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)+1\right)\sin\left(dx+c\right)-1\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\,\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{8\sqrt{2}\left(a\cos(dx+c)^{2}-(a\cos(dx+c)+a)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\sqrt{a\sin\left(dx+c\right)+a}\left(\cos\left(dx+c\right)-\sin\left(dx+c\right)+1\right)}{4\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d-\left(a^{3}d\cos\left(dx+c\right)+a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/4*(5*(cos(d*x + c)^2 - (cos(d*x + c) + 1)*sin(d*x + c) - 1)*sqrt(a)*log((
a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) +
 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9
*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) -
a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - c
os(d*x + c) - 1)) + 8*sqrt(2)*(a*cos(d*x + c)^2 - (a*cos(d*x + c) + a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) + 4*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*
x + c) + 1))/(a^3*d*cos(d*x + c)^2 - a^3*d - (a^3*d*cos(d*x + c) + a^3*d)*s
in(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(96) = 192.

time = 0.46, size = 205, normalized size = 1.81

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{5\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{8\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{8\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(2)*sqrt(a)*(5*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x
 + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^3*sgn(cos(
-1/4*pi + 1/2*d*x + 1/2*c))) + 8*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1)/(a
^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 8*log(-sin(-1/4*pi + 1/2*d*x + 1/
2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*sin(-1/4*pi + 1/2*d
*x + 1/2*c)/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)*a^3*sgn(cos(-1/4*pi +
 1/2*d*x + 1/2*c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)^2\,(a+a\,\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + a*sin(c + d*x))^(5/2)), x)
```

3.487 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}}$$

```
[Out] -23/4*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arctan
h(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+
9/4*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d
/(a+a*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2959, 2858, 3064, 2728, 212, 2852, 3123, 3063}

$$-\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{4a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{9\cot(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-23*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(4*a^(5/2)*d
) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d
*x]])])/(a^(5/2)*d) + (9*Cot[c + d*x])/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]]) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
```

```
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^3(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^2(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(-\frac{a}{2}+\frac{7}{2}a \cdots\right.}{\sqrt{a+a\sin(c \cdots}}}{2a^3}$$

$$= \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc(c+dx)\left(\frac{15a^2}{4}-\frac{1}{4}\cdots\right.}{\sqrt{a+a\sin(\cdots}}}{2a^4 \cdots}$$

$$= \frac{9\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{15\int \csc(c+dx)\cdots}{\cdots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+\cdots}}\right.}{a^{5/2}d}$$

$$= -\frac{23\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c-\cdots}}\right.}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.88, size = 309, normalized size = 2.02

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-40-(256+256i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)+20\cot\left(\frac{1}{4}(c+dx)\right)-\csc^2\left(\frac{1}{4}(c+dx)\right)-92\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+92\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{4}(c+dx)\right)+\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}-\frac{40\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2}+\frac{40\sin\left(\frac{1}{4}(c+dx)\right)}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}+20\tan\left(\frac{1}{4}(c+dx)\right)\right)}{32d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(-40 - (256 + 256*I)*(-1)^(3/4)*Ar
cTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] + 20*Cot[(c + d*x)/4]
```

```
- Csc[(c + d*x)/4]^2 - 92*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9
2*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + Sec[(c + d*x)/4]^2 + 2/(Co
s[(c + d*x)/4] - Sin[(c + d*x)/4])^2 - (40*Sin[(c + d*x)/4])/(Cos[(c + d*x)
/4] - Sin[(c + d*x)/4]) - 2/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4])^2 + (40*S
in[(c + d*x)/4])/(Cos[(c + d*x)/4] + Sin[(c + d*x)/4]) + 20*Tan[(c + d*x)/4
]))/(32*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 7.48, size = 164, normalized size = 1.07

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(23\,\mathrm{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)a^{3}(\sin^{2}(dx+c))-16\sqrt{2}\ \mathrm{arcta}}{4a^{\frac{11}{2}}\sin(dx+c)^{2}\cos(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/4/a^(11/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(23*arctanh((-a*(sin
(d*x+c)-1))^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2-16*2^(1/2)*arctanh(1/2*(-a*(sin
(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^3*sin(d*x+c)^2+9*(-a*(sin(d*x+c)-1))^(
3/2)*a^(3/2)-7*(-a*(sin(d*x+c)-1))^(1/2)*a^(5/2))/sin(d*x+c)^2/cos(d*x+c)/(
a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(128) = 256.
time = 0.38, size = 508, normalized size = 3.32

$$\frac{23\left(\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin(dx+c)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{32\sqrt{2}\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}-a\cos(dx+c)+\left(a\cos(dx+c)^{2}-a\right)\sin(dx+c)-a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin(dx+c)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(9\cos(dx+c)^{2}+(9\cos(dx+c)+11)\sin(dx+c)-2\cos(dx+c)-11\right)\sqrt{a\sin(dx+c)+a}}{16\left(a^{3}d\cos(dx+c)^{3}+a^{3}d\cos(dx+c)^{2}-a^{3}d\cos(dx+c)-a^{3}d+\left(a^{3}d\cos(dx+c)^{2}-a^{3}d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/16*(23*(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x +
c) - cos(d*x + c) - 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*
sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8
*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (
cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 32*sqrt(2)*(a*cos(d
*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) + (a*cos(d*x + c)^2 - a)*sin(
d*x + c) - a)*log(-(cos(d*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sq
rt(2)*sqrt(a*sin(d*x + c) + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) +
3*cos(d*x + c) + 2)/(cos(d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos
(d*x + c) - 2))/sqrt(a) - 4*(9*cos(d*x + c)^2 + (9*cos(d*x + c) + 11)*sin(d
*x + c) - 2*cos(d*x + c) - 11)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c
)^3 + a^3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d + (a^3*d*cos(d*x +
c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5006 deep
```

Giac [**A**]
time = 0.47, size = 224, normalized size = 1.46

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{23\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{32\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{32\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\left(18\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-7\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -1/16*sqrt(2)*sqrt(a)*(23*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^3*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) + 32*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) + 1
)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 32*log(-sin(-1/4*pi + 1/2*d*x
 + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(18*sin(-1/4*p
i + 1/2*d*x + 1/2*c)^3 - 7*sin(-1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi
+ 1/2*d*x + 1/2*c)^2 - 1)^2*a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)^3 (a+a\sin(c+dx))^{5/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + a*sin(c + d*x))^(5/2)), x)
```

3.488 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}-\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}}+$$

```
[Out] 45/8*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d-4*arctanh
(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d-1
9/8*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+13/12*cot(d*x+c)*csc(d*x+c)/a^2
/d/(a+a*sin(d*x+c))^(1/2)-1/3*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+a*sin(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2796, 2858, 3063, 3064, 2728, 212, 2852, 3123}

$$\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{8a^{5/2}d}-\frac{4\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(c+dx)}{\sqrt{2}\,\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}-\frac{19\cot(c+dx)}{8a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a\sin(c+dx)+a}}+\frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(8*a^(5/2)*d)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*
x]])])/(a^(5/2)*d) - (19*Cot[c + d*x])/(8*a^2*d*Sqrt[a + a*Sin[c + d*x]]) +
 (13*Cot[c + d*x]*Csc[c + d*x])/(12*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a^2*d*Sqrt[a + a*Sin[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2796

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x]^
2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(c+dx)\left(1+\sin^2(c+dx)\right)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^3(c+dx)\left(-\frac{a}{2}+\frac{11}{2}a\sin\right.}{\sqrt{a+a\sin(c+}}}{3a^3}$$

$$= -\frac{\cot(c+dx)}{2a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2}{3a^2d\sqrt{a+a\sin(}}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2}{3a^2d\sqrt{a+a\sin(}}$$

$$= -\frac{19\cot(c+dx)}{8a^2d\sqrt{a+a\sin(c+dx)}} + \frac{13\cot(c+dx)\csc(c+dx)}{12a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2}{3a^2d\sqrt{a+a\sin(}}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{2a^{5/2}d} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right.}{a^{5/2}d}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{8a^{5/2}d} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx}}\right.}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.69, size = 332, normalized size = 1.74

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left((1536+1536i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\frac{8\csc^9\left(\frac{1}{2}(c+dx)\right)\left(396\cos\left(\frac{1}{2}(c+dx)\right)-218\cos\left(\frac{3}{2}(c+dx)\right)-114\cos\left(\frac{5}{2}(c+dx)\right)-396\sin\left(\frac{1}{2}(c+dx)\right)-405\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+405\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-218\sin\left(\frac{3}{2}(c+dx)\right)+114\sin\left(\frac{5}{2}(c+dx)\right)+135\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-135\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))\right)}{\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^3}\right)}{192d(a(1+\sin(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (8*Csc[(c + d*x)/2]^9*(
396*Cos[(c + d*x)/2] - 218*Cos[(3*(c + d*x))/2] - 114*Cos[(5*(c + d*x))/2]
- 396*Sin[(c + d*x)/2] - 405*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*S
in[c + d*x] + 405*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 218*Sin[(3*(c + d*x))/2] + 114*Sin[(5*(c + d*x))/2] + 135*Log[1 + Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 135*Log[1 - Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]))/(Csc[(c + d*x)/4]^2 - Sec[(c + d
*x)/4]^2)^3))/(192*d*(a*(1 + Sin[c + d*x]))^(5/2))
```

Maple [A]
time = 6.55, size = 182, normalized size = 0.95

method	result
default	$\frac{(1+\sin(dx+c))\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\left(-96\sqrt{2}\ \mathrm{arctanh}\left(\frac{\sqrt{-a\left(\sin\left(dx+c\right)-1\right)}\ \sqrt{2}}{2\sqrt{a}}\right)a^{5}\left(\sin^{3}(dx+c)\right)-57\right.}{24a^{\frac{15}{2}}\sin(dx-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/24*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(-96*2^(1/2)*arctanh(1/2*(-a*
(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^5*sin(d*x+c)^3-57*(-a*(sin(d*x+c)-
1))^(5/2)*a^(5/2)+88*(-a*(sin(d*x+c)-1))^(3/2)*a^(7/2)-39*(-a*(sin(d*x+c)-1
))^(1/2)*a^(9/2)+135*arctanh((-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*a^5*sin(d*x
+c)^3)/a^(15/2)/sin(d*x+c)^3/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(162) = 324.
time = 0.39, size = 564, normalized size = 2.95

$$\frac{135\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}-\left(\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-7a\cos(dx+c)^{2}+4\left(\cos(dx+c)^{2}+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^{2}+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}+\left(\cos(dx+c)^{2}-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{192\sqrt{2}\left(a\cos(dx+c)^{4}-2a\cos(dx+c)^{2}-\left(a\cos(dx+c)^{3}+a\cos(dx+c)^{2}-a\cos(dx+c)-a\right)\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^{2}-(\cos(dx+c)-2)\sin(dx+c)-\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}\left(\cos(dx+c)-\sin(dx+c)+1\right)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^{2}-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}+4\left(57\cos\left(dx+c\right)^{3}+83\cos\left(dx+c\right)^{2}-\left(57\cos\left(dx+c\right)^{2}-26\cos\left(dx+c\right)-91\right)\sin\left(dx+c\right)-65\cos\left(dx+c\right)-91\right)\sqrt{a\sin\left(dx+c\right)+a}}{96\left(a^{3}d\cos\left(dx+c\right)^{4}-2a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d-\left(a^{3}d\cos\left(dx+c\right)^{3}+a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\cos\left(dx+c\right)-a^{3}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/96*(135*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x +
c)^2 - cos(d*x + c) - 1)*sin(d*x + c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 -
7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 + (cos(d*x + c) + 3)*sin(d*x + c) -
2*cos(d*x + c) - 3)*sqrt(a*sin(d*x + c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (
a*cos(d*x + c)^2 + 8*a*cos(d*x + c) - a)*sin(d*x + c) - a)/(cos(d*x + c)^3
+ cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x + c) - cos(d*x + c) - 1)) +
 192*sqrt(2)*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^3 + a
*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-(cos(d*x + c)^
2 - (cos(d*x + c) - 2)*sin(d*x + c) - 2*sqrt(2)*sqrt(a*sin(d*x + c) + a)*(c
os(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(d*x + c)
^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) + 4*(57*c
os(d*x + c)^3 + 83*cos(d*x + c)^2 - (57*cos(d*x + c)^2 - 26*cos(d*x + c) -
91)*sin(d*x + c) - 65*cos(d*x + c) - 91)*sqrt(a*sin(d*x + c) + a))/(a^3*d*c
os(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d - (a^3*d*cos(d*x + c)^3 + a^
3*d*cos(d*x + c)^2 - a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8009 deep
```

Giac [A]
time = 0.47, size = 240, normalized size = 1.26

$$\frac{\sqrt{2}\sqrt{a}\left(\frac{135\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{192\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{192\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\left(228\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5-176\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3+39\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] 1/96*sqrt(2)*sqrt(a)*(135*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^3*sgn(c
os(-1/4*pi + 1/2*d*x + 1/2*c))) + 192*log(sin(-1/4*pi + 1/2*d*x + 1/2*c) +
1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 192*log(-sin(-1/4*pi + 1/2*d
```

```
*x + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(228*sin(-1/
4*pi + 1/2*d*x + 1/2*c)^5 - 176*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 + 39*sin(-
1/4*pi + 1/2*d*x + 1/2*c))/((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^3*a^3*
sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+dx)^4}{\sin(c+dx)^4\,(a+a\,\sin(c+dx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)),x)
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + a*sin(c + d*x))^(5/2)), x)
```

3.489 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=229

$$-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+dx)}}\right)}{a^{5/2}d}+\frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+d}}$$

[Out] -363/64*arctanh(cos(d*x+c)*a^(1/2)/(a+a*sin(d*x+c))^(1/2))/a^(5/2)/d+4*arctanh(1/2*cos(d*x+c)*a^(1/2)*2^(1/2)/(a+a*sin(d*x+c))^(1/2))*2^(1/2)/a^(5/2)/d+149/64*cot(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)-107/96*cot(d*x+c)*csc(d*x+c)/a^2/d/(a+a*sin(d*x+c))^(1/2)+17/24*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+a*sin(d*x+c))^(1/2)-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d/(a+a*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.86, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2959, 2858, 3063, 3064, 2728, 212, 2852, 3123}

$$-\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a\sin(c+dx)+a}}\right)}{64a^{5/2}d}+\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a\sin(c+dx)+a}}\right)}{a^{5/2}d}+\frac{149\cot(c+dx)}{64a^2d\sqrt{a\sin(c+dx)+a}}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a\sin(c+dx)+a}}+\frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a\sin(c+dx)+a}}-\frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a\sin(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]

[Out] (-363*ArcTanh[(Sqrt[a]*Cos[c + d*x])/Sqrt[a + a*Sin[c + d*x]]])/(64*a^(5/2)*d) + (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sin[c + d*x]])])/(a^(5/2)*d) + (149*Cot[c + d*x])/(64*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (107*Cot[c + d*x]*Csc[c + d*x])/(96*a^2*d*Sqrt[a + a*Sin[c + d*x]]) + (17*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2*d*Sqrt[a + a*Sin[c + d*x]]) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d*Sqrt[a + a*Sin[c + d*x]])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2858

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Dist[1/(2*
b*(n + 1)*(c^2 - d^2)), Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c*
(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2959

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[-2/(a*b*d), Int[(d*S
in[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 2), x], x] + Dist[1/a^2, Int
[(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^2), x],
x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3063

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3123

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(b*d*(n + 1)*(c^2 -
d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*
d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(
c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^{5/2}}\,dx = \frac{\int \frac{\csc^5(c+dx)(1+\sin^2(c+dx))}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^4(c+dx)}{\sqrt{a+a\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2\cot(c+dx)\csc^2(c+dx)}{3a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} + \frac{\int \frac{\csc^4(c+dx)\left(-\frac{a}{2}+\frac{1}{\cdots}\right.}{\sqrt{a+a\sin(\cdots}}}{4a^{3}\cdots}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{6a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\csc^2(c+dx)}{24a^2d\sqrt{a+a\sin(c+dx)}} - \frac{\cot(c+dx)\,\mathrm{c}\cdots}{4a^2d\sqrt{a+a\cdots}}$$

$$= \frac{7\cot(c+dx)}{4a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx)\cdots}{24a^2d\sqrt{a+a\cdots}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx\cdots}{24a^2d\sqrt{a+a\cdots}}$$

$$= \frac{149\cot(c+dx)}{64a^2d\sqrt{a+a\sin(c+dx)}} - \frac{107\cot(c+dx)\csc(c+dx)}{96a^2d\sqrt{a+a\sin(c+dx)}} + \frac{17\cot(c+dx\cdots}{24a^2d\sqrt{a+a\cdots}}$$

$$= -\frac{9\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{4a^{5/2}d} + \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{2}\sqrt{a+a\sin(c+\cdots}}\right.}{a^{5/2}d}$$

$$= -\frac{363\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx)}{\sqrt{a+a\sin(c+dx)}}\right)}{64a^{5/2}d} + \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(c+dx\cdots}{\sqrt{2}\sqrt{a+a\sin(c\cdots}}\right.}{a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.13, size = 414, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left((-24576-24576i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(c+dx)\right)\right)\right)-\frac{16\csc^{12}\left(\frac{1}{2}(c+dx)\right)\left(6250\cos\left(\frac{1}{2}(c+dx)\right)-4626\cos\left(\frac{3}{2}(c+dx)\right)-1750\cos\left(\frac{5}{2}(c+dx)\right)+894\cos\left(\frac{7}{2}(c+dx)\right)+3267\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-4356\cos(2(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+1089\cos(4(c+dx))\log\left(1+\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-3267\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4356\cos(2(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-1089\cos(4(c+dx))\log\left(1-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-6250\sin\left(\frac{1}{2}(c+dx)\right)-4626\sin\left(\frac{3}{2}(c+dx)\right)+1750\sin\left(\frac{5}{2}(c+dx)\right)+894\sin\left(\frac{7}{2}(c+dx)\right)\right)}{\left(\csc^2\left(\frac{1}{4}(c+dx)\right)-\sec^2\left(\frac{1}{4}(c+dx)\right)\right)^4}\right)}{3072d(a(1+\sin(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + a*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*((-24576 - 24576*I)*(-1)^(3/4)*Arc
Tanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(c + d*x)/4])] - (16*Csc[(c + d*x)/2]
^12*(6250*Cos[(c + d*x)/2] - 4626*Cos[(3*(c + d*x))/2] - 1750*Cos[(5*(c + d
*x))/2] + 894*Cos[(7*(c + d*x))/2] + 3267*Log[1 + Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - 4356*Cos[2*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 1089*Cos[4*(c + d*x)]*Log[1 + Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- 3267*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4356*Cos[2*(c + d*x)]
*Log[1 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 1089*Cos[4*(c + d*x)]*Log[1
 - Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6250*Sin[(c + d*x)/2] - 4626*Sin[
(3*(c + d*x))/2] + 1750*Sin[(5*(c + d*x))/2] + 894*Sin[(7*(c + d*x))/2]))/(
Csc[(c + d*x)/4]^2 - Sec[(c + d*x)/4]^2)^4))/(3072*d*(a*(1 + Sin[c + d*x]))
^(5/2))
```

Maple [A]
time = 7.19, size = 200, normalized size = 0.87

method	result
default	$-\frac{(1+\sin(dx+c))\sqrt{-a(\sin(dx+c)-1)}\left(1089a^7\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(dx+c)-1)}}{\sqrt{a}}\right)(\sin^4(dx+c))+447(-a(\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/192/a^(19/2)*(1+sin(d*x+c))*(-a*(sin(d*x+c)-1))^(1/2)*(1089*a^7*arctanh(
(-a*(sin(d*x+c)-1))^(1/2)/a^(1/2))*sin(d*x+c)^4+447*(-a*(sin(d*x+c)-1))^(7/
2)*a^(7/2)-1127*(-a*(sin(d*x+c)-1))^(5/2)*a^(9/2)-768*2^(1/2)*arctanh(1/2*(
-a*(sin(d*x+c)-1))^(1/2)*2^(1/2)/a^(1/2))*a^7*sin(d*x+c)^4+1049*(-a*(sin(d*
x+c)-1))^(3/2)*a^(11/2)-321*(-a*(sin(d*x+c)-1))^(1/2)*a^(13/2))/sin(d*x+c)^
4/cos(d*x+c)/(a+a*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(196) = 392$.
time = 0.39, size = 643, normalized size = 2.81

$$\frac{1089\left(\cos\left(dx+c\right)^5+\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^3-2\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-2\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-4\left(\cos(dx+c)^2+(\cos(dx+c)+3)\sin(dx+c)-2\cos(dx+c)-3\right)\sqrt{a\sin\left(dx+c\right)+a}\sqrt{a}-9a\cos(dx+c)+\left(a\cos(dx+c)^2+8a\cos(dx+c)-a\right)\sin(dx+c)-a}{\cos(dx+c)^3+\cos(dx+c)^2+\left(\cos(dx+c)^2-1\right)\sin(dx+c)-\cos(dx+c)-1}\right)+\frac{1536\sqrt{2}\left(a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+\left(a\cos(dx+c)^4-2a\cos(dx+c)^2+a\right)\sin(dx+c)+a\right)\log\left(-\frac{\cos(dx+c)^2-(\cos(dx+c)-2)\sin(dx+c)+\frac{2\sqrt{2}\sqrt{a\sin\left(dx+c\right)+a}(\cos(dx+c)-\sin(dx+c)+1)}{\sqrt{a}}+3\cos(dx+c)+2}{\cos(dx+c)^2-(\cos(dx+c)+2)\sin(dx+c)-\cos(dx+c)-2}\right)}{\sqrt{a}}-4\left(447\cos\left(dx+c\right)^4-214\cos\left(dx+c\right)^3-1244\cos\left(dx+c\right)^2+\left(447\cos\left(dx+c\right)^3+661\cos\left(dx+c\right)^2-583\cos\left(dx+c\right)-845\right)\sin\left(dx+c\right)+262\cos\left(dx+c\right)+845\right)\sqrt{a\sin\left(dx+c\right)+a}}{768\left(a^3d\cos\left(dx+c\right)^5+a^3d\cos\left(dx+c\right)^4-2a^3d\cos\left(dx+c\right)^3-2a^3d\cos\left(dx+c\right)^2+a^3d\cos\left(dx+c\right)+a^3d+\left(a^3d\cos\left(dx+c\right)^4-2a^3d\cos\left(dx+c\right)^2+a^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 1/768*(1089*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x
 + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + cos(d*x +
c) + 1)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c
)^2 + (cos(d*x + c) + 3)*sin(d*x + c) - 2*cos(d*x + c) - 3)*sqrt(a*sin(d*x
+ c) + a)*sqrt(a) - 9*a*cos(d*x + c) + (a*cos(d*x + c)^2 + 8*a*cos(d*x + c)
 - a)*sin(d*x + c) - a)/(cos(d*x + c)^3 + cos(d*x + c)^2 + (cos(d*x + c)^2
- 1)*sin(d*x + c) - cos(d*x + c) - 1)) + 1536*sqrt(2)*(a*cos(d*x + c)^5 + a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
+ (a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*sin(d*x + c) + a)*log(-(cos(d
*x + c)^2 - (cos(d*x + c) - 2)*sin(d*x + c) + 2*sqrt(2)*sqrt(a*sin(d*x + c)
 + a)*(cos(d*x + c) - sin(d*x + c) + 1)/sqrt(a) + 3*cos(d*x + c) + 2)/(cos(
d*x + c)^2 - (cos(d*x + c) + 2)*sin(d*x + c) - cos(d*x + c) - 2))/sqrt(a) -
 4*(447*cos(d*x + c)^4 - 214*cos(d*x + c)^3 - 1244*cos(d*x + c)^2 + (447*co
s(d*x + c)^3 + 661*cos(d*x + c)^2 - 583*cos(d*x + c) - 845)*sin(d*x + c) +
262*cos(d*x + c) + 845)*sqrt(a*sin(d*x + c) + a))/(a^3*d*cos(d*x + c)^5 + a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 + a^3
*d*cos(d*x + c) + a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 +
a^3*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+a*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 256, normalized size = 1.12

$$-\frac{\sqrt{2}\sqrt{a}\left(\frac{1089\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}+\frac{1536\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{1536\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}-\frac{4\left(3576\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^7-4508\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^5+2098\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^3-321\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}dx+\frac{1}{2}c\right)\right)}\right)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+a*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] -1/768*sqrt(2)*sqrt(a)*(1089*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1
/2*d*x + 1/2*c))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*d*x + 1/2*c)))/(a^3*sg
n(cos(-1/4*pi + 1/2*d*x + 1/2*c))) + 1536*log(sin(-1/4*pi + 1/2*d*x + 1/2*c
) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 1536*log(-sin(-1/4*pi +
1/2*d*x + 1/2*c) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*d*x + 1/2*c))) - 4*(3576*s
in(-1/4*pi + 1/2*d*x + 1/2*c)^7 - 4508*sin(-1/4*pi + 1/2*d*x + 1/2*c)^5 + 2
098*sin(-1/4*pi + 1/2*d*x + 1/2*c)^3 - 321*sin(-1/4*pi + 1/2*d*x + 1/2*c))/
((2*sin(-1/4*pi + 1/2*d*x + 1/2*c)^2 - 1)^4*a^3*sgn(cos(-1/4*pi + 1/2*d*x +
 1/2*c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sin\left(c+d\,x\right)^5\,\left(a+a\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + a*sin(c + d*x))^(5/2)), x)
```

3.490 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+d\right.}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^2*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+2*a^2*cos(d*x+c)*hypergeom([-3/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+a^2*cos(d*x+c)*hypergeom([-3/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*sin
(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2952, 2657}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}+\frac{a^2\cos(c+dx)\sin^{n+3}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+3}{2};\frac{n+5}{2};\sin^2(c+dx)\right)}{d(n+3)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cos^4(c+dx)\sin^n(c+dx)+2a^2\cos^4(c+dx)\sin^1\right. \\
&= a^2\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^4(c+dx) \\
&= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c}{d(1+n)\sqrt{\cos^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 164, normalized size = 0.82

$$
\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{1+n}(c+dx)\left((6+5n+n^2)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+(1+n)\sin(c+dx)\left(2(3+n)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)+(2+n)\,{}_2F_1\left(-\frac{3}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(c+dx)\right)\sin(c+dx)\right)\right)}{d(1+n)(2+n)(3+n)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$
\int \left(\cos^4\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)\left(a+a\sin\left(dx+c\right)\right)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^6 - 2*a^2*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*c
os(d*x + c)^4)*sin(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x))^2, x)
```

3.491 $\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)\sin^{2+n}(c+dx)}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+a*cos(d*x+c)*hypergeom([-3/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2917, 2657}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^n(c+dx)\,dx + a\int \cos^4(c+dx)\,\mathrm{si}$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + a*sin(c + d*x)), x)
```

3.492 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{ad(1+n)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)\sin}{ad(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([-1/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)
^(1+n)/a/d/(1+n)/(cos(d*x+c)^2)^(1/2)-cos(d*x+c)*hypergeom([-1/2, 1+1/2*n],
[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/a/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2918, 2657}

$$\frac{\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{ad(n+1)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{ad(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x]^(1 + n))/(a*d*(1 + n)*Sqrt[Cos[c + d*x]^2]) - (Cos[c + d*x]*H
ypergeometric2F1[-1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(
2 + n))/(a*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^2(c+dx)\sin^n(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^{1+n}(c+dx)\,dx}{a}$$
$$= \frac{\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{ad(1+n)\sqrt{\cos^2(c+dx)}} - \frac{\cos(c+dx)\,{}_2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 441 vs. 2(134) = 268.
time = 11.10, size = 441, normalized size = 3.29

$$\frac{2^{1+n}\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{1+\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\frac{{}_2F_1\left(\frac{1+n}{2},4+n;\frac{3+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{1+n}+\tan\left(\frac{1}{2}(c+dx)\right)\left(-\frac{2\,{}_2F_1\left(\frac{2+n}{2},4+n;\frac{4+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{2+n}+\tan\left(\frac{1}{2}(c+dx)\right)\left(-\frac{{}_2F_1\left(\frac{3+n}{2},4+n;\frac{5+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{3+n}+\frac{4\,{}_2F_1\left(\frac{4+n}{2},4+n;\frac{6+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4+n}-\frac{{}_2F_1\left(4+n,\frac{5+n}{2};\frac{7+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)}{5+n}-\frac{2\,{}_2F_1\left(4+n,\frac{6+n}{2};\frac{8+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3\left(\frac{1}{2}(c+dx)\right)}{6+n}+\frac{{}_2F_1\left(4+n,\frac{7+n}{2};\frac{9+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^4\left(\frac{1}{2}(c+dx)\right)}{7+n}\right)\right)\right)}{d(a+a\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2^(1 + n)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*Tan[(c + d*x)/2]*(Tan[(c
 + d*x)/2]/(1 + Tan[(c + d*x)/2]^2))^n*(1 + Tan[(c + d*x)/2]^2)^n*(Hypergeo
metric2F1[(1 + n)/2, 4 + n, (3 + n)/2, -Tan[(c + d*x)/2]^2]/(1 + n) + Tan[(
c + d*x)/2]*((-2*Hypergeometric2F1[(2 + n)/2, 4 + n, (4 + n)/2, -Tan[(c + d
*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*(-(Hypergeometric2F1[(3 + n)/2, 4 + n
, (5 + n)/2, -Tan[(c + d*x)/2]^2]/(3 + n)) + (4*Hypergeometric2F1[(4 + n)/2
, 4 + n, (6 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2])/(4 + n) - (Hyper
geometric2F1[4 + n, (5 + n)/2, (7 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x
)/2]^2)/(5 + n) - (2*Hypergeometric2F1[4 + n, (6 + n)/2, (8 + n)/2, -Tan[(c
 + d*x)/2]^2]*Tan[(c + d*x)/2]^3)/(6 + n) + (Hypergeometric2F1[4 + n, (7 +
n)/2, (9 + n)/2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^4)/(7 + n)))))/(d*(a
 + a*Sin[c + d*x]))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4\left(dx+c\right)\right)\left(\sin^n\left(dx+c\right)\right)}{a+a\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n}{a+a\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x)), x)
```

3.493 $\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=173

$$-\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)}+\frac{(3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{a^2d(1+n)(2+n)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\ldots}{}$$

```
[Out] -cos(d*x+c)*sin(d*x+c)^(1+n)/a^2/d/(2+n)+(3+2*n)*cos(d*x+c)*hypergeom([1/2,
 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/a^2/d/(1+n)/(2+n)/(c
os(d*x+c)^2)^(1/2)-2*cos(d*x+c)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(d*x+
c)^2)*sin(d*x+c)^(2+n)/a^2/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2948, 2842, 2827, 2722}

$$\frac{(2n+3)\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{a^2d(n+1)(n+2)\sqrt{\cos^2(c+dx)}}-\frac{2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{a^2d(n+2)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+1}(c+dx)}{a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(a^2*d*(2 + n))) + ((3 + 2*n)*Cos[c +
 d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c +
d*x]^(1 + n))/(a^2*d*(1 + n)*(2 + n)*Sqrt[Cos[c + d*x]^2]) - (2*Cos[c + d*x
]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(a^2*d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2842

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*x
```

```
])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
+ n)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a*b*c*(
m - 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n -
2))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c
- a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !LtQ[n
, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c
, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sin^n(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{\int \sin^n(c+dx)\left(a^2(3+2n)-2a^2(2+n)\,\mathrm{s}\right.}{a^4(2+n)}\\ &= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} - \frac{2\int \sin^{1+n}(c+dx)\,dx}{a^2} + \frac{(3+2n)\int \sin^n(\mathrm{c}}{a^2(2+\mathrm{r}}\\ &= -\frac{\cos(c+dx)\sin^{1+n}(c+dx)}{a^2d(2+n)} + \frac{(3+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2\right.}{a^2d(1+n)(2+n)\sqrt{\cos^2(}}\end{aligned}$$

Mathematica [A]
time = 3.85, size = 312, normalized size = 1.80

$$\frac{2\sec^2\left(\frac{1}{2}(c+dx)\right)^n\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\sin^n(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{{}_2F_1\left(\frac{1+n}{2},3+n;\frac{3+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{1+n}+\tan\left(\frac{1}{2}(c+dx)\right)\left(-\frac{4\,{}_2F_1\left(\frac{2+n}{2},3+n;\frac{4+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{2+n}+\tan\left(\frac{1}{2}(c+dx)\right)\left(\frac{6\,{}_2F_1\left(\frac{3+n}{2},3+n;\frac{5+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}{3+n}-\frac{4\,{}_2F_1\left(3+n,\frac{4+n}{2};\frac{6+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4+n}+\frac{{}_2F_1\left(3+n,\frac{5+n}{2};\frac{7+n}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^2\left(\frac{1}{2}(c+dx)\right)}{5+n}\right)\right)\right)}{d(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*Sin[c + d
*x]^n*Tan[(c + d*x)/2]*(Hypergeometric2F1[(1 + n)/2, 3 + n, (3 + n)/2, -Tan
[(c + d*x)/2]^2]/(1 + n) + Tan[(c + d*x)/2]*((-4*Hypergeometric2F1[(2 + n)/
2, 3 + n, (4 + n)/2, -Tan[(c + d*x)/2]^2])/(2 + n) + Tan[(c + d*x)/2]*((6*H
ypergeometric2F1[(3 + n)/2, 3 + n, (5 + n)/2, -Tan[(c + d*x)/2]^2])/(3 + n)
 - (4*Hypergeometric2F1[3 + n, (4 + n)/2, (6 + n)/2, -Tan[(c + d*x)/2]^2]*T
an[(c + d*x)/2])/(4 + n) + (Hypergeometric2F1[3 + n, (5 + n)/2, (7 + n)/2,
```

```
-Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(5 + n)))))/(d*(a + a*Sin[c + d*x]
)^2)
```

Maple [F]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(\cos^4(dx+c))\,(\sin^n(dx+c))}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^4/(a^2*cos(d*x + c)^2 - 2*a^2*sin(d*x
 + c) - 2*a^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^4/(a*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2, x)
```

3.494 $\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^9(c+dx)}{9d}+\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^{11}(c+dx)}{11d}$$

```
[Out] 1/6*a*sin(d*x+c)^6/d+1/7*a*sin(d*x+c)^7/d-1/4*a*sin(d*x+c)^8/d-2/9*a*sin(d*
x+c)^9/d+1/10*a*sin(d*x+c)^10/d+1/11*a*sin(d*x+c)^11/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,
Rules used = {2915, 12, 90}

$$\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{10}(c+dx)}{10d}-\frac{2a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (a*Sin[c +
d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^5(a+x)^3}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^5(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^{10}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5+a^4x^6-2a^3x^7-2a^2x^8+ax^9+x^{10}\right)\,d\right.}{a^{10}d}$$
$$= \frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a}{}$$

Mathematica [A]
time = 0.32, size = 97, normalized size = 1.00

$$-\frac{a(34650\cos(2(c+dx))-5775\cos(6(c+dx))+693\cos(10(c+dx))-34650\sin(c+dx)+11550\sin(3(c+dx))+3465\sin(5(c+dx))-2475\sin(7(c+dx))-385\sin(9(c+dx))+315\sin(11(c+dx)))}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3548160*(a*(34650*Cos[2*(c + d*x)] - 5775*Cos[6*(c + d*x)] + 693*Cos[10*
(c + d*x)] - 34650*Sin[c + d*x] + 11550*Sin[3*(c + d*x)] + 3465*Sin[5*(c +
d*x)] - 2475*Sin[7*(c + d*x)] - 385*Sin[9*(c + d*x)] + 315*Sin[11*(c + d*x)
]))/d
```

Maple [A]
time = 0.33, size = 138, normalized size = 1.42

method	result
risch	$\frac{5a\sin(dx+c)}{512d}-\frac{a\sin(11dx+11c)}{11264d}-\frac{a\cos(10dx+10c)}{5120d}+\frac{a\sin(9dx+9c)}{9216d}+\frac{5a\sin(7dx+7c)}{7168d}+\frac{5a\cos(6dx+6c)}{3072d}-$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+a\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{11}-\frac{5\left(\sin^3(dx+\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+a\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{11}-\frac{5\left(\sin^3(dx+\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6)+a*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x
```

```
+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.74

$$\frac{1260\,a\sin\left(dx+c\right)^{11}+1386\,a\sin\left(dx+c\right)^{10}-3080\,a\sin\left(dx+c\right)^{9}-3465\,a\sin\left(dx+c\right)^{8}+1980\,a\sin\left(dx+c\right)^{7}+2310\,a\sin\left(dx+c\right)^{6}}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/13860*(1260*a*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*a*sin(d*x +
 c)^9 - 3465*a*sin(d*x + c)^8 + 1980*a*sin(d*x + c)^7 + 2310*a*sin(d*x + c)
^6)/d
```

Fricas [A]
time = 0.37, size = 106, normalized size = 1.09

$$-\frac{1386\,a\cos\left(dx+c\right)^{10}-3465\,a\cos\left(dx+c\right)^{8}+2310\,a\cos\left(dx+c\right)^{6}+20\,\left(63\,a\cos\left(dx+c\right)^{10}-161\,a\cos\left(dx+c\right)^{8}+113\,a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}-8\,a\right)\sin\left(dx+c\right)}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*a*cos(d*x + c)^10 - 161*a*cos(d*x + c)^8 + 113*a*cos(d*x + c
)^6 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A]
time = 2.91, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^{11}{\left(c+dx\right)}}{693d}+\frac{4a\sin^{9}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{63d}+\frac{a\sin^{7}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{7d}-\frac{a\sin^{4}{\left(c+dx\right)}\cos^{6}{\left(c+dx\right)}}{6d}-\frac{a\sin^{2}{\left(c+dx\right)}\cos^{8}{\left(c+dx\right)}}{12d}-\frac{a\cos^{10}{\left(c+dx\right)}}{60d} & \text{for}\: d\neq 0 \\ x\left(a\sin{\left(c\right)}+a\right)\sin^{5}{\left(c\right)}\cos^{5}{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
[Out] Piecewise((8*a*sin(c + d*x)**11/(693*d) + 4*a*sin(c + d*x)**9*cos(c + d*x)*
*2/(63*d) + a*sin(c + d*x)**7*cos(c + d*x)**4/(7*d) - a*sin(c + d*x)**4*cos
(c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d
*x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**5*cos(c)**5, True))
```

Giac [A]
time = 0.53, size = 133, normalized size = 1.37

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{a\sin\left(11\,dx+11\,c\right)}{11264\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{9216\,d}+\frac{5\,a\sin\left(7\,dx+7\,c\right)}{7168\,d}-\frac{a\sin\left(5\,dx+5\,c\right)}{1024\,d}-\frac{5\,a\sin\left(3\,dx+3\,c\right)}{1536\,d}+\frac{5\,a\sin\left(dx+c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d - 1/11264*a*sin(11*d*x + 11*c)/d + 1/9216*a*sin(9*d*x + 9*c)
/d + 5/7168*a*sin(7*d*x + 7*c)/d - 1/1024*a*sin(5*d*x + 5*c)/d - 5/1536*a*s
in(3*d*x + 3*c)/d + 5/512*a*sin(d*x + c)/d
```

Mupad [B]
time = 8.72, size = 71, normalized size = 0.73

$$\frac{\frac{a\sin(c+dx)^{11}}{11}+\frac{a\sin(c+dx)^{10}}{10}-\frac{2a\sin(c+dx)^9}{9}-\frac{a\sin(c+dx)^8}{4}+\frac{a\sin(c+dx)^7}{7}+\frac{a\sin(c+dx)^6}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7 - (a*sin(c + d*x)^8)/4 - (2*a*
sin(c + d*x)^9)/9 + (a*sin(c + d*x)^10)/10 + (a*sin(c + d*x)^11)/11)/d
```

3.495 $\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^9(c+dx)}{9d}+\frac{a\sin^{10}(c+dx)}{10d}$$

```
[Out] 1/5*a*sin(d*x+c)^5/d+1/6*a*sin(d*x+c)^6/d-2/7*a*sin(d*x+c)^7/d-1/4*a*sin(d*
x+c)^8/d+1/9*a*sin(d*x+c)^9/d+1/10*a*sin(d*x+c)^10/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a\sin^{10}(c+dx)}{10d}+\frac{a\sin^9(c+dx)}{9d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^6(c+dx)}{6d}+\frac{a\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (a*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*
x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^4(a+x)^3}{a^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^4(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int (a^5x^4+a^4x^5-2a^3x^6-2a^2x^7+ax^8+x^9)\,dx\right.}{a^9d}$$
$$= \frac{a\sin^5(c+dx)}{5d} + \frac{a\sin^6(c+dx)}{6d} - \frac{2a\sin^7(c+dx)}{7d} - \frac{a}{}$$

Mathematica [A]
time = 0.24, size = 87, normalized size = 0.90

$$-\frac{a(3150\cos(2(c+dx)) - 525\cos(6(c+dx)) + 63\cos(10(c+dx)) - 7560\sin(c+dx) + 1680\sin(3(c+dx)) + 1008\sin(5(c+dx)) - 180\sin(7(c+dx)) - 140\sin(9(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
[Out] -1/322560*(a*(3150*Cos[2*(c + d*x)] - 525*Cos[6*(c + d*x)] + 63*Cos[10*(c +
 d*x)] - 7560*Sin[c + d*x] + 1680*Sin[3*(c + d*x)] + 1008*Sin[5*(c + d*x)]
- 180*Sin[7*(c + d*x)] - 140*Sin[9*(c + d*x)]))/d
```

Maple [A]
time = 0.25, size = 120, normalized size = 1.24

method	result
risch	$\frac{3a\sin(dx+c)}{128d} - \frac{a\cos(10dx+10c)}{5120d} + \frac{a\sin(9dx+9c)}{2304d} + \frac{a\sin(7dx+7c)}{1792d} + \frac{5a\cos(6dx+6c)}{3072d} - \frac{a\sin(5dx+5c)}{320d} - \frac{a\,\mathrm{si}}{}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9} - \frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right) + a\left(-\frac{\left(\sin^4(dx+c)\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9} - \frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21} + \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right) + a\left(-\frac{\left(\sin^4(dx+c)\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10*sin(d*x+c)^4*cos(d*
x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.74

$$\frac{126\,a\sin\left(dx+c\right)^{10}+140\,a\sin\left(dx+c\right)^{9}-315\,a\sin\left(dx+c\right)^{8}-360\,a\sin\left(dx+c\right)^{7}+210\,a\sin\left(dx+c\right)^{6}+252\,a\sin\left(dx+c\right)^{5}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1260*(126*a*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*a*sin(d*x + c)^8
 - 360*a*sin(d*x + c)^7 + 210*a*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.98

$$-\frac{126\,a\cos\left(dx+c\right)^{10}-315\,a\cos\left(dx+c\right)^{8}+210\,a\cos\left(dx+c\right)^{6}-4\,\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 + 210*a*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [A]
time = 1.77, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^{9}\left(c+dx\right)}{315d}+\frac{4a\sin^{7}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{35d}+\frac{a\sin^{5}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{5d}-\frac{a\sin^{4}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{6d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{8}\left(c+dx\right)}{12d}-\frac{a\cos^{10}\left(c+dx\right)}{60d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)\sin^{4}\left(c\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a*sin(c + d*x)**4*cos(
c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a*cos(c + d*
x)**10/(60*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**4*cos(c)**5, True))
```

Giac [A]
time = 0.51, size = 118, normalized size = 1.22

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{3\,a\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d
- 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x
 + c)/d
```

Mupad [B]
time = 8.70, size = 71, normalized size = 0.73

$$\frac{\frac{a\sin(c+dx)^{10}}{10}+\frac{a\sin(c+dx)^9}{9}-\frac{a\sin(c+dx)^8}{4}-\frac{2\,a\sin(c+dx)^7}{7}+\frac{a\sin(c+dx)^6}{6}+\frac{a\sin(c+dx)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 - (2*a*sin(c + d*x)^7)/7 - (a*
sin(c + d*x)^8)/4 + (a*sin(c + d*x)^9)/9 + (a*sin(c + d*x)^10)/10)/d
```

3.496 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^8(c+dx)}{8d}+\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^9(c+dx)}{9d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/5*a*sin(d*x+c)^5/d-2/7*a*sin(d
*x+c)^7/d+1/9*a*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2645, 14, 2644, 276}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^5)/(
5*d) - (2*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + a\int \cos^5(c+dx)\sin$$

$$= -\frac{a\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\Big(}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^5-x^7)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\big(\int}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^5(c+dx)}{5d} - $$

Mathematica [A]
time = 0.26, size = 97, normalized size = 1.20

$$\frac{a(-7560\cos(2(c+dx))-1260\cos(4(c+dx))+840\cos(6(c+dx))+315\cos(8(c+dx))+7560\sin(c+dx)-1680\sin(3(c+dx))-1008\sin(5(c+dx))+180\sin(7(c+dx))+140\sin(9(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-7560*Cos[2*(c + d*x)] - 1260*Cos[4*(c + d*x)] + 840*Cos[6*(c + d*x)] +
 315*Cos[8*(c + d*x)] + 7560*Sin[c + d*x] - 1680*Sin[3*(c + d*x)] - 1008*Si
n[5*(c + d*x)] + 180*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/(322560*d)
```

Maple [A]
time = 0.26, size = 102, normalized size = 1.26

method	result

derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)\right.}{}\right.}{d}$
risch	$\frac{3a\sin(dx+c)}{128d}+\frac{a\sin(9dx+9c)}{2304d}+\frac{a\cos(8dx+8c)}{1024d}+\frac{a\sin(7dx+7c)}{1792d}+\frac{a\cos(6dx+6c)}{384d}-\frac{a\sin(5dx+5c)}{320d}-\frac{a\cos(4dx}{256d}$
norman	$\frac{\frac{32a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{384a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{6976a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d}-\frac{384a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{32a\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{4a\left(\tan^6\left(\frac{dx}{2}\right.\right.}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a*(-1/9*sin(d*x+c
)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]

time = 0.28, size = 72, normalized size = 0.89

$$\frac{280\,a\sin\left(dx+c\right)^{9}+315\,a\sin\left(dx+c\right)^{8}-720\,a\sin\left(dx+c\right)^{7}-840\,a\sin\left(dx+c\right)^{6}+504\,a\sin\left(dx+c\right)^{5}+630\,a\sin\left(dx+c\right)^{4}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*a*sin(d*x + c)^7
- 840*a*sin(d*x + c)^6 + 504*a*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d
```

Fricas [A]

time = 0.35, size = 84, normalized size = 1.04

$$\frac{315\,a\cos\left(dx+c\right)^{8}-420\,a\cos\left(dx+c\right)^{6}+8\,\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*a*cos(d*x + c)^
8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*si
n(d*x + c))/d
```

Sympy [A]
time = 1.64, size = 114, normalized size = 1.41

$$\begin{cases} \frac{8a\sin^9(c+dx)}{315d} + \frac{4a\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{a\sin^5(c+dx)\cos^4(c+dx)}{5d} - \frac{a\sin^2(c+dx)\cos^6(c+dx)}{6d} - \frac{a\cos^8(c+dx)}{24d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^3(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a*sin(c + d*x)**2*cos(
c + d*x)**6/(6*d) - a*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)*
sin(c)**3*cos(c)**5, True))
```

Giac [A]
time = 0.49, size = 133, normalized size = 1.64

$$\frac{a\cos(8\,dx+8\,c)}{1024\,d} + \frac{a\cos(6\,dx+6\,c)}{384\,d} - \frac{a\cos(4\,dx+4\,c)}{256\,d} - \frac{3\,a\cos(2\,dx+2\,c)}{128\,d} + \frac{a\sin(9\,dx+9\,c)}{2304\,d} + \frac{a\sin(7\,dx+7\,c)}{1792\,d} - \frac{a\sin(5\,dx+5\,c)}{320\,d} - \frac{a\sin(3\,dx+3\,c)}{192\,d} + \frac{3\,a\sin(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1
792*a*sin(7*d*x + 7*c)/d - 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x +
 3*c)/d + 3/128*a*sin(d*x + c)/d
```

Mupad [B]
time = 0.05, size = 71, normalized size = 0.88

$$\frac{\frac{a\sin(c+d\,x)^9}{9} + \frac{a\sin(c+d\,x)^8}{8} - \frac{2\,a\sin(c+d\,x)^7}{7} - \frac{a\sin(c+d\,x)^6}{3} + \frac{a\sin(c+d\,x)^5}{5} + \frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/3 - (2*a*
sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/9)/d
```

3.497 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^8(c+dx)}{8d}+\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d
*x+c)^5/d+1/7*a*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2644, 276, 2645, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (a*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + a\int \cos^5(c+dx)\sin$$

$$= -\frac{a\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\right.}{}$$

$$= \frac{a\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sin(c+dx)\right)}{d} - \frac{a\text{Subs}}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - $$

Mathematica [A]
time = 0.20, size = 87, normalized size = 1.07

$$-\frac{a(2520\cos(2(c+dx))+420\cos(4(c+dx))-280\cos(6(c+dx))-105\cos(8(c+dx))-8400\sin(c+dx)+560\sin(3(c+dx))+1008\sin(5(c+dx))+240\sin(7(c+dx)))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/107520*(a*(2520*Cos[2*(c + d*x)] + 420*Cos[4*(c + d*x)] - 280*Cos[6*(c +
 d*x)] - 105*Cos[8*(c + d*x)] - 8400*Sin[c + d*x] + 560*Sin[3*(c + d*x)] +
1008*Sin[5*(c + d*x)] + 240*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.19, size = 84, normalized size = 1.04

method	result

derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right.}{d}$
risch	$\frac{5a\sin(dx+c)}{64d}+\frac{a\cos(8dx+8c)}{1024d}-\frac{a\sin(7dx+7c)}{448d}+\frac{a\cos(6dx+6c)}{384d}-\frac{3a\sin(5dx+5c)}{320d}-\frac{a\cos(4dx+4c)}{256d}-\frac{a\sin(3c}{19}$
norman	$\frac{\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{688a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{688a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{8a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A]
time = 0.30, size = 72, normalized size = 0.89

$$\frac{105\,a\sin\left(dx+c\right)^{8}+120\,a\sin\left(dx+c\right)^{7}-280\,a\sin\left(dx+c\right)^{6}-336\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}+280\,a\sin\left(dx+c\right)^{3}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*a*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*a*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 73, normalized size = 0.90

$$\frac{105\,a\cos\left(dx+c\right)^{8}-140\,a\cos\left(dx+c\right)^{6}-8\left(15\,a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}-8\,a\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*a*cos(d*x + c)^8 - 140*a*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A]
time = 1.44, size = 114, normalized size = 1.41

$$\begin{cases}\frac{8a\sin^{7}\left(c+dx\right)}{105d}+\frac{4a\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{15d}+\frac{a\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{3d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{6d}-\frac{a\cos^{8}\left(c+dx\right)}{24d} & \text{for } d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin^{2}\left(c\right)\cos^{5}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*sin(c + d*x)**2*cos(
c + d*x)**6/(6*d) - a*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)*
sin(c)**2*cos(c)**5, True))
```

Giac [A]
time = 0.50, size = 118, normalized size = 1.46

$$\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{384\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{3\,a\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5\,c)}{320\,d}-\frac{a\sin(3\,dx+3\,c)}{192\,d}+\frac{5\,a\sin(dx+c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

Mupad [B]
time = 8.69, size = 71, normalized size = 0.88

$$\frac{\frac{a\sin(c+d\,x)^8}{8}+\frac{a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{3}-\frac{2\,a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (2*a*sin(c + d*x)^5)/5 - (a*
sin(c + d*x)^6)/3 + (a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8)/d
```

3.498 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d*x+c)^5/d+1/7*a*sin(d
*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 2645, 30, 2644, 276}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (a*Sin[c + d*x]^3)/(3*d) - (2*a*Sin[c + d*x]^5)
/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin(c+dx)\,dx + a\int \cos^5(c+dx)\sin^2$$

$$= -\frac{a\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int x^2(1-x\right.}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \sin(\right.}{d}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2a\sin^5(c+dx)}{5d} + $$

Mathematica [A]
time = 0.12, size = 78, normalized size = 1.20

$$-\frac{a(350+525\cos(2(c+dx))+210\cos(4(c+dx))+35\cos(6(c+dx))-525\sin(c+dx)+35\sin(3(c+dx))+63\sin(5(c+dx))+15\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(a*(350 + 525*Cos[2*(c + d*x)] + 210*Cos[4*(c + d*x)] + 35*Cos[6*(c
 + d*x)] - 525*Sin[c + d*x] + 35*Sin[3*(c + d*x)] + 63*Sin[5*(c + d*x)] + 1
5*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.13, size = 64, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6}+a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$

default	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6}+a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$
risch	$\frac{5a\sin(dx+c)}{64d}-\frac{a\sin(7dx+7c)}{448d}-\frac{a\cos(6dx+6c)}{192d}-\frac{3a\sin(5dx+5c)}{320d}-\frac{a\cos(4dx+4c)}{32d}-\frac{a\sin(3dx+3c)}{192d}-\frac{5a\cos(2}{6}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{32a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{3}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a*cos(d*x+c)^6+a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+
c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.11

$$\frac{30\,a\sin\left(dx+c\right)^7+35\,a\sin\left(dx+c\right)^6-84\,a\sin\left(dx+c\right)^5-105\,a\sin\left(dx+c\right)^4+70\,a\sin\left(dx+c\right)^3+105\,a\sin\left(dx+c\right)^2}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*a*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*a*sin(d*x + c)^5 - 10
5*a*sin(d*x + c)^4 + 70*a*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.95

$$-\frac{35\,a\cos\left(dx+c\right)^6+2\left(15\,a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2-8\,a\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 -
 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.59, size = 90, normalized size = 1.38

$$\begin{cases}\frac{8a\sin^7\left(c+dx\right)}{105d}+\frac{4a\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{a\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d}-\frac{a\cos^6\left(c+dx\right)}{6d} & \text{for } d\neq 0\\ x\left(a\sin\left(c\right)+a\right)\sin\left(c\right)\cos^5\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**5, True))
```

Giac [A]
time = 0.53, size = 103, normalized size = 1.58

$$-\frac{a\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{a\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{64\,d}-\frac{a\sin\left(7\,dx+7\,c\right)}{448\,d}-\frac{3\,a\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\,a\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/320*a*sin(5*d*x + 5*c)/d - 1/192*
a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

Mupad [B]
time = 8.70, size = 71, normalized size = 1.09

$$\frac{\frac{a\,{\sin\left(c+d\,x\right)}^7}{7}+\frac{a\,{\sin\left(c+d\,x\right)}^6}{6}-\frac{2\,a\,{\sin\left(c+d\,x\right)}^5}{5}-\frac{a\,{\sin\left(c+d\,x\right)}^4}{2}+\frac{a\,{\sin\left(c+d\,x\right)}^3}{3}+\frac{a\,{\sin\left(c+d\,x\right)}^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (a*sin(c + d*x)^4)/2 - (2*a*
sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7)/d
```

3.499 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-a*sin(d*x+c)^2/d-2/3*a*sin(d*x+c)^3/d+1/4
*a*sin(d*x+c)^4/d+1/5*a*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$\frac{a\sin^5(c+dx)}{5d}+\frac{a\sin^4(c+dx)}{4d}-\frac{2a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x}\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4+\frac{a^5}{x}-2a^3x-2a^2x^2+ax^3+x^4\right)\,dx, x, a\right.}{a^4d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 1.00

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*a*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.13, size = 65, normalized size = 0.76

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$-iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{3a\,e^{-2i(dx+c)}}{16d}-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{5a\sin(dx+c)}{8d}+\frac{a\sin(5dx+5c)}{80d}+a\cos$
norman	$\frac{\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{116a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-8a(}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/5*a*(8/3+cos(d*
x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.80

$$\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,a\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 74, normalized size = 0.86

$$\frac{15\,a\cos\left(dx+c\right)^{4}+30\,a\cos\left(dx+c\right)^{2}+60\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\cos^{5}\left(c+dx\right)\csc\left(c+dx\right)\,dx+\int\sin\left(c+dx\right)\cos^{5}\left(c+dx\right)\csc\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(cos(c + d*x)**5*csc(c + d*x), x) + Integral(sin(c + d*x)*cos(c
+ d*x)**5*csc(c + d*x), x))
```

Giac [A]
time = 0.52, size = 70, normalized size = 0.81

$$\frac{12\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,a\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*a*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c))/d
```

Mupad [B]
time = 8.87, size = 126, normalized size = 1.47

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a\,{\cos\left(c+d\,x\right)}^2}{2\,d}+\frac{a\,{\cos\left(c+d\,x\right)}^4}{4\,d}+\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}+\frac{4\,a\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{a\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (8*a*sin(
c + d*x))/(15*d) + (4*a*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (a*cos(c + d*
x)^4*sin(c + d*x))/(5*d)
```

3.500 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d-a*sin(d*x+c)^2/d+1/3*a*
sin(d*x+c)^3/d+1/4*a*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2915, 12, 90}

$$\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}+\frac{a^4}{x}-2a^2x+ax^2+x^3\right)\,dx, x, \right.}{a^3 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 1.00

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.12, size = 85, normalized size = 1.02

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
risch	$-iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{7ia\,e^{i(dx+c)}}{8d}-\frac{7ia\,e^{-i(dx+c)}}{8d}+\frac{3a\,e^{-2i(dx+c)}}{16d}-\frac{2iac}{d}-\frac{2ia\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)}+$
norman	$\frac{-\frac{a}{2d}-\frac{13a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{43a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{43a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{13a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(
d*x+c))+a*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.83

$$\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,a\sin\left(dx+c\right)^{2}+12\,a\log\left(\sin\left(dx+c\right)\right)-24\,a\sin\left(dx+c\right)-\frac{12\,a}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*
log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 1.10

$$\frac{32\,a\cos\left(dx+c\right)^{4}+128\,a\cos\left(dx+c\right)^{2}+96\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\left(8\,a\cos\left(dx+c\right)^{4}+16\,a\cos\left(dx+c\right)^{2}-11\,a\right)\sin\left(dx+c\right)-256\,a}{96\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*a*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*a*cos(d*x + c)^4 + 16*a*cos(d*x + c)^2 - 11*a)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 79, normalized size = 0.95

$$\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,a\sin\left(dx+c\right)^{2}+12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-24\,a\sin\left(dx+c\right)-\frac{12\,(a\sin(dx+c)+a)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 12*a*
log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(a*sin(d*x + c) + a)/sin(d*
x + c))/d
```

Mupad [B]
time = 8.88, size = 250, normalized size = 3.01

$$\frac{a\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{a\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{d}-\frac{4\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d}+\frac{8\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}-\frac{8\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{d}+\frac{4\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d}-\frac{9\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{20\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,d\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{16\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3\,d\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{8\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3\,d\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d - (4*a*cos(c/2 + (d*x)/2)^2)/d + (8*a*cos(c/2 + (d*x)/2)^4)/d - (
8*a*cos(c/2 + (d*x)/2)^6)/d + (4*a*cos(c/2 + (d*x)/2)^8)/d - (9*a*cos(c/2 +
 (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 +
 (d*x)/2)) + (20*a*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a*c
os(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a*cos(c/2 + (d*x)/2)^7)/
(3*d*sin(c/2 + (d*x)/2))
```

3.501 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d+
1/2*a*sin(d*x+c)^2/d+1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin^2(c+dx)}{2d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/(3
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{a^4}{x^2}-\frac{2a^3}{x}+ax+x^2\right)dx, x, a\sin\right.}{a^2 d}$$
$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.90

$$-\frac{a\csc(c+dx)}{d}-\frac{2a\sin(c+dx)}{d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a(\csc^2(c+dx)+4\log(\sin(c+dx))-\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (2*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.13, size = 105, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$2iax+\frac{ia\,e^{3i(dx+c)}}{24d}-\frac{a\,e^{2i(dx+c)}}{8d}+\frac{7ia\,e^{i(dx+c)}}{8d}-\frac{7ia\,e^{-i(dx+c)}}{8d}-\frac{a\,e^{-2i(dx+c)}}{8d}-\frac{ia\,e^{-3i(dx+c)}}{24d}+\frac{4iac}{d}$
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{6a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{25a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{6a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(s
in(d*x+c)))+a*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 68, normalized size = 0.79

$$\frac{2\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\sin\left(dx+c\right)\right)-12\,a\sin\left(dx+c\right)-\frac{3\,(2\,a\sin(dx+c)+a)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
a*sin(d*x + c) - 3*(2*a*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 102, normalized size = 1.19

$$-\frac{6\,a\cos\left(dx+c\right)^{4}-9\,a\cos\left(dx+c\right)^{2}+24\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}-8\,a\right)\sin\left(dx+c\right)-3\,a}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - 8*a)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.46, size = 82, normalized size = 0.95

$$\frac{2\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,a\sin\left(dx+c\right)+\frac{3\,\left(6\,a\sin(dx+c)^{2}-2\,a\sin(dx+c)-a\right)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*a*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x
 + c)^2)/d
```

Mupad [B]
time = 8.86, size = 229, normalized size = 2.66

$$\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{18\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{82\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+22\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (3*
a*tan(c/2 + (d*x)/2)^2)/2 + 22*a*tan(c/2 + (d*x)/2)^3 - (13*a*tan(c/2 + (d*
x)/2)^4)/2 + (82*a*tan(c/2 + (d*x)/2)^5)/3 - (15*a*tan(c/2 + (d*x)/2)^6)/2
+ 18*a*tan(c/2 + (d*x)/2)^7)/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x
)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (a*tan(c/2 +
(d*x)/2))/(2*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a*log(tan(c/2 + (d*x)
/2)))/d
```

3.502 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{2a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}$$

```
[Out] 2*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-2*a*ln(sin(d*x+c
))/d+a*sin(d*x+c)/d+1/2*a*sin(d*x+c)^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$\frac{a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{2a\csc(c+dx)}{d}-\frac{2a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}+\frac{a^4}{x^3}-\frac{2a^3}{x^2}-\frac{2a^2}{x}+x\right)dx, x, a\sin(c+\right.}{ad}$$
$$= \frac{2a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2a}{}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 0.89

$$\frac{2a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a(\csc^2(c+dx)+4\log(\sin(c+dx))-\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.15, size = 121, normalized size = 1.42

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\cos\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\cos\right.}{d}$
risch	$2iax-\frac{a\,e^{2i(dx+c)}}{8d}-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{4iac}{d}+\frac{2ia\left(6\,e^{5i(dx+c)}-8\,e^{3i(dx+c)}-3ie^{4i}\right.}{3d\left(e^{2i(dx+\right.}}$
norman	$\frac{-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{19a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{55a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{55a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{19a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d
*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2
*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.81

$$\frac{3\,a\sin\left(dx+c\right)^2-12\,a\log\left(\sin\left(dx+c\right)\right)+6\,a\sin\left(dx+c\right)+\frac{12\,a\sin(dx+c)^2-3\,a\sin(dx+c)-2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.36, size = 117, normalized size = 1.38

$$-\frac{12\,a\cos\left(dx+c\right)^4-48\,a\cos\left(dx+c\right)^2+24\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\left(2\,a\cos\left(dx+c\right)^4-3\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)+32\,a}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*a*cos(d*x + c)^4 - 3*a*cos(d*x
+ c)^2 - a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.45, size = 81, normalized size = 0.95

$$\frac{3\,a\sin\left(dx+c\right)^2-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+6\,a\sin\left(dx+c\right)+\frac{22\,a\sin(dx+c)^3+12\,a\sin(dx+c)^2-3\,a\sin(dx+c)-2\,a}{\sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*a*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

Mupad [B]
time = 8.79, size = 218, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}-\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{23\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\frac{89\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{3}-2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{19\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a}{3}}{d\,\left(8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+16\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a
*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*a*log(t
an(c/2 + (d*x)/2)))/d + ((19*a*tan(c/2 + (d*x)/2)^2)/3 - a*tan(c/2 + (d*x)/
2) - a/3 - 2*a*tan(c/2 + (d*x)/2)^3 + (89*a*tan(c/2 + (d*x)/2)^4)/3 + 15*a*
tan(c/2 + (d*x)/2)^5 + 23*a*tan(c/2 + (d*x)/2)^6)/(d*(8*tan(c/2 + (d*x)/2)^
3 + 16*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7))
```

3.503 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] 2*a*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 90}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\end{aligned}$$

Mathematica [A]
time = 0.15, size = 87, normalized size = 1.07

$$\frac{2a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}+\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

Maple [A]
time = 0.15, size = 101, normalized size = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\right.}{d}$
risch	$-iax-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4ia\left(3ie^{6i(dx+c)}+3\,e^{7i(dx+c)}-3ie^{4i(dx+c)}-7\,e^{5i(dx+c)}+3ie^{2i(dx+c)}\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{15a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{11a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{64d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+a*(-1/3/sin(d*x+
c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,a\sin\left(dx+c\right)+\frac{24\,a\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,a\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.37, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos\left(dx+c\right)^2-12\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,\left(3\,a\cos\left(dx+c\right)^4-12\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)-9\,a}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.55, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

Mupad [B]
time = 9.03, size = 207, normalized size = 2.56

$$\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan(
c/2 + (d*x)/2))/3 - a/4 + (40*a*tan(c/2 + (d*x)/2)^3)/3 + 3*a*tan(c/2 + (d*
x)/2)^4 + 46*a*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^
4)/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.504 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+a*csc(d*x+c)^2/d+2/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d-
1/5*a*csc(d*x+c)^5/d+a*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{2a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (a*Log[Sin[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a^5}{x^6}+\frac{a^4}{x^5}-\frac{2a^3}{x^4}-\frac{2a^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\,\text{c}}{}$$

Mathematica [A]

time = 0.12, size = 92, normalized size = 1.07

$$-\frac{a\csc(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (a*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

Maple [A]

time = 0.16, size = 121, normalized size = 1.41

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx\right.}{2}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx\right.}{2}\right.}{d}$
risch	$-iax-\frac{2iac}{d}-\frac{2ia\left(15\,e^{9i(dx+c)}-20\,e^{7i(dx+c)}-30ie^{8i(dx+c)}+58\,e^{5i(dx+c)}+60ie^{6i(dx+c)}-20\,e^{3i(dx+c)}-60ie^{4i(dx+c)}\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a}{160d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}+\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{25a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{25a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{96d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+
a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.84

$$\frac{60\,a\log\left(\sin\left(dx+c\right)\right)-\frac{60\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}-40\,a\sin\left(dx+c\right)^{2}+15\,a\sin\left(dx+c\right)+12\,a}{\sin\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*a*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.36, size = 124, normalized size = 1.44

$$-\frac{60\,a\cos\left(dx+c\right)^{4}-80\,a\cos\left(dx+c\right)^{2}-60\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+15\,\left(4\,a\cos\left(dx+c\right)^{2}-3\,a\right)\sin\left(dx+c\right)+32\,a}{60\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^4 - 2
*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*a*cos(d*x
 + c)^2 - 3*a)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.49, size = 84, normalized size = 0.98

$$\frac{60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{137\,a\sin\left(dx+c\right)^{5}+60\,a\sin\left(dx+c\right)^{4}-60\,a\sin\left(dx+c\right)^{3}-40\,a\sin\left(dx+c\right)^{2}+15\,a\sin\left(dx+c\right)+12\,a}{\sin\left(dx+c\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(sin(d*x + c))) - (137*a*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*a*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*a*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 8.90, size = 193, normalized size = 2.24

$$\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(10\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-\frac{5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
5*a*tan(c/2 + (d*x)/2))/(16*d) + (5*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan
(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (a*log(tan(c
/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + (a*tan(c/2 + (d*x)/2))/2 -
 (5*a*tan(c/2 + (d*x)/2)^2)/3 - 6*a*tan(c/2 + (d*x)/2)^3 + 10*a*tan(c/2 + (
d*x)/2)^4))/(32*d)
```

3.505 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-a*csc(d*x+c)/d+2/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^
5/d
```

Rubi [A]
time = 0.07, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 30, 2686, 200}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc(c+dx)\,dx + a\int \cot^5(c+dx)\csc^2$$

$$= -\frac{a\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (-1+\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c\right.}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a}{}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc(c+dx)}{d} + \frac{2a\csc^3(c+dx)}{3d} - \frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x])/d + (2*a*Csc[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.18, size = 110, normalized size = 1.80

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)}{d}$

default	$\dfrac{-\frac{a\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)}{d}$
risch	$-\dfrac{2ia\left(15i e^{10i(dx+c)}+15\,e^{11i(dx+c)}-35\,e^{9i(dx+c)}+50i e^{6i(dx+c)}+78\,e^{7i(dx+c)}-78\,e^{5i(dx+c)}+15i e^{2i(dx+c)}+35\,e^{3i(dx+c)}\right.}{15d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\dfrac{-\frac{a}{384d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d}+\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{25a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/1
5/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.15

$$-\frac{30\,a\sin\left(dx+c\right)^5+15\,a\sin\left(dx+c\right)^4-20\,a\sin\left(dx+c\right)^3-15\,a\sin\left(dx+c\right)^2+6\,a\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.36, size = 100, normalized size = 1.64

$$\frac{15\,a\cos\left(dx+c\right)^4-15\,a\cos\left(dx+c\right)^2+2\,\left(15\,a\cos\left(dx+c\right)^4-20\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)+5\,a}{30\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*a*cos(d*x + c)^4 -
20*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 70, normalized size = 1.15

$$-\frac{30\,a\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,a\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*a*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*a*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Mupad [B]
time = 8.88, size = 69, normalized size = 1.13

$$-\frac{a\sin\left(c+d\,x\right)^5+\frac{a\sin\left(c+d\,x\right)^4}{2}-\frac{2\,a\sin\left(c+d\,x\right)^3}{3}-\frac{a\sin\left(c+d\,x\right)^2}{2}+\frac{a\,\sin\left(c+d\,x\right)}{5}+\frac{a}{6}}{d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] -(a/6 + (a*sin(c + d*x))/5 - (a*sin(c + d*x)^2)/2 - (2*a*sin(c + d*x)^3)/3
+ (a*sin(c + d*x)^4)/2 + a*sin(c + d*x)^5)/(d*sin(c + d*x)^6)
```

3.506 $\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d*x+c)^5/d-1/7*a*csc(d
*x+c)^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2686, 276, 2687, 30}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + a\int \cot^5(c+dx)\operatorname{csc}\\ &= -\frac{a\mathrm{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\mathrm{Subst}\left(\int x^2(-1\right.}{}\\ &= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(\right.}{d}\\ &= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(57) = 114$.
time = 0.18, size = 128, normalized size = 1.97

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{a\left(\cos^6(dx\right.}{6\sin(dx+}}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{a\left(\cos^6(dx\right.}{6\sin(dx+}}{d}$
risch	$\frac{2a\left(140ie^{11i(dx+c)}+105\,\mathrm{e}^{12i(dx+c)}+112ie^{9i(dx+c)}-105\,\mathrm{e}^{10i(dx+c)}+456ie^{7i(dx+c)}+350\,\mathrm{e}^{8i(dx+c)}+112ie^{5i(dx+c)}-350\,\mathrm{e}^{6\right.}{105d\left(\mathrm{e}^{2i(dx+c)}-1\right)^7}$
norman	$-\frac{a}{896d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280d}+\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}-\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.08

$$-\frac{105\,a\sin\left(dx+c\right)^5+70\,a\sin\left(dx+c\right)^4-105\,a\sin\left(dx+c\right)^3-84\,a\sin\left(dx+c\right)^2+35\,a\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Fricas [A]
time = 0.35, size = 106, normalized size = 1.63

$$\frac{70\,a\cos\left(dx+c\right)^4-56\,a\cos\left(dx+c\right)^2+35\left(3\,a\cos\left(dx+c\right)^4-3\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)+16\,a}{210\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*a*cos(d*x + c)^4 -
 3*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 70, normalized size = 1.08

$$-\frac{105\,a\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,a\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,a\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(105*a*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*a*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*a*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Mupad [B]
time = 8.83, size = 70, normalized size = 1.08

$$-\frac{105\,a\sin\left(c+d\,x\right)^{5}+70\,a\sin\left(c+d\,x\right)^{4}-105\,a\sin\left(c+d\,x\right)^{3}-84\,a\sin\left(c+d\,x\right)^{2}+35\,a\,\sin\left(c+d\,x\right)+30\,a}{210\,d\sin\left(c+d\,x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] -(30*a + 35*a*sin(c + d*x) - 84*a*sin(c + d*x)^2 - 105*a*sin(c + d*x)^3 + 7
0*a*sin(c + d*x)^4 + 105*a*sin(c + d*x)^5)/(210*d*sin(c + d*x)^7)
```

3.507 $\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d
*x+c)^5/d-1/7*a*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 14, 2686, 276}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^3)/(
3*d) + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + a\int \cot^5(c+dx)\csc$$

$$= -\frac{a\text{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Sub}}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a}{}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 1.09

$$-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a(6\csc^4(c+dx)-8\csc^6(c+dx)+3\csc^8(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3)/d + (2*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(71) = 142$.
time = 0.20, size = 148, normalized size = 1.83

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(c}{}\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(c}{}\right.}{}\right.}{d}$
risch	$\frac{4ia(105i e^{12i(dx+c)}+70\,e^{13i(dx+c)}+140i e^{10i(dx+c)}-14\,e^{11i(dx+c)}+350i e^{8i(dx+c)}+172\,e^{9i(dx+c)}+140i e^{6i(dx+c)}-172\,e^{7i}}{105d\left(e^{2i(dx+c)}-1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-
1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+
c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin\left(dx+c\right)^5+210\,a\sin\left(dx+c\right)^4-336\,a\sin\left(dx+c\right)^3-280\,a\sin\left(dx+c\right)^2+120\,a\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.35, size = 109, normalized size = 1.35

$$-\frac{210\,a\cos\left(dx+c\right)^4-140\,a\cos\left(dx+c\right)^2+8\left(35\,a\cos\left(dx+c\right)^4-28\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)+35\,a}{840\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*a*cos(d*x + c)^
4 - 28*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-336\,a\sin\left(dx+c\right)^{3}-280\,a\sin\left(dx+c\right)^{2}+120\,a\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*a*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*a*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Mupad [B]
time = 8.84, size = 70, normalized size = 0.86

$$-\frac{280\,a\sin\left(c+d\,x\right)^{5}+210\,a\sin\left(c+d\,x\right)^{4}-336\,a\sin\left(c+d\,x\right)^{3}-280\,a\sin\left(c+d\,x\right)^{2}+120\,a\,\sin\left(c+d\,x\right)+105\,a}{840\,d\sin\left(c+d\,x\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(105*a + 120*a*sin(c + d*x) - 280*a*sin(c + d*x)^2 - 336*a*sin(c + d*x)^3
+ 210*a*sin(c + d*x)^4 + 280*a*sin(c + d*x)^5)/(840*d*sin(c + d*x)^8)
```

3.508 $\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/5*a*csc(d*x+c)^5/d+2/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2913, 2686, 276, 2687, 14}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + a\int \cot^5(c+dx)\operatorname{csc}$$

$$= -\frac{a\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}}{}$$

$$= -\frac{a\text{Subst}\left(\int \left(x^5+x^7\right)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\text{Subst}(\int}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a}{}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 1.09

$$-\frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d} - \frac{a(6\csc^4(c+dx) - 8\csc^6(c+dx) + 3\csc^8(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(71) = 142$.
time = 0.20, size = 166, normalized size = 2.05

method	result

risch	$-\frac{4a\left(504i\,e^{13i(dx+c)}+315\,e^{14i(dx+c)}+864i\,e^{11i(dx+c)}+105\,e^{12i(dx+c)}+1744i\,e^{9i(dx+c)}+630\,e^{10i(dx+c)}+864i\,e^{7i(dx+c)}-\right.}{315d\left(e^{2i(dx+c)}-1\right)^{9}}$
derivativedivides	$\frac{a\left(-\frac{\cos^{6}(dx+c)}{9\sin(dx+c)^{9}}-\frac{\cos^{6}(dx+c)}{21\sin(dx+c)^{7}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{315\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx}{105}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^{6}(dx+c)}{9\sin(dx+c)^{9}}-\frac{\cos^{6}(dx+c)}{21\sin(dx+c)^{7}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)^{5}}+\frac{\cos^{6}(dx+c)}{315\sin(dx+c)^{3}}-\frac{\cos^{6}(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx}{105}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/8
/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.86

$$-\frac{630\,a\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,a\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,a\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A]
time = 0.41, size = 115, normalized size = 1.42

$$-\frac{504\,a\cos\left(dx+c\right)^{4}-288\,a\cos\left(dx+c\right)^{2}+105\left(6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+64\,a}{2520\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*a*cos(d*x + c
)^4 - 4*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
```

```
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.53, size = 70, normalized size = 0.86

$$-\frac{630\,a\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,a\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,a\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2520*(630*a*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*a*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Mupad [B]
time = 8.88, size = 70, normalized size = 0.86

$$-\frac{\frac{a\sin(c+d\,x)^5}{4}+\frac{a\sin(c+d\,x)^4}{5}-\frac{a\sin(c+d\,x)^3}{3}-\frac{2\,a\sin(c+d\,x)^2}{7}+\frac{a\,\sin(c+d\,x)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
[Out] -(a/9 + (a*sin(c + d*x))/8 - (2*a*sin(c + d*x)^2)/7 - (a*sin(c + d*x)^3)/3
+ (a*sin(c + d*x)^4)/5 + (a*sin(c + d*x)^5)/4)/(d*sin(c + d*x)^9)
```

3.509 $\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^{10}(c+dx)}{10d}$$

```
[Out] -1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d+2/7*a*csc(d*x+c)^7/d+1/4*a*csc(d
*x+c)^8/d-1/9*a*csc(d*x+c)^9/d-1/10*a*csc(d*x+c)^10/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d - (a*Csc[c + d*x]^6)/(6*d) + (2*a*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{a^6\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^6\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}+\frac{a^4}{x^{10}}-\frac{2a^3}{x^9}-\frac{2a^2}{x^8}+\frac{a}{x^7}+\frac{1}{x^6}\right)\,dx, x, a\,\text{si}\right.}{d}$$

$$= -\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2a\csc^7(c+dx)}{7d}+$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.91

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a(10\csc^6(c+dx)-15\csc^8(c+dx)+6\csc^{10}(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.
time = 0.24, size = 184, normalized size = 1.90

method	result
risch	$-\frac{32ia\left(105i\,\mathrm{e}^{14i(dx+c)}+63\,\mathrm{e}^{15i(dx+c)}+210i\,\mathrm{e}^{12i(dx+c)}+45\,\mathrm{e}^{13i(dx+c)}+378i\,\mathrm{e}^{10i(dx+c)}+110\,\mathrm{e}^{11i(dx+c)}+210i\,\mathrm{e}^{8i(dx+c)}-11\right.}{315d\left(\mathrm{e}^{2i(dx+c)}-1\right)^{10}}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^6}\right)+a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\text{co}}{105}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^6}\right)+a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\text{co}}{105}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x
+c)^7*cos(d*x+c)^6-1/105/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d
*x+c)^6-1/105/sin(d*x+c)*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,a\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,a\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Fricas [A]
time = 0.39, size = 122, normalized size = 1.26

$$\frac{210\,a\cos\left(dx+c\right)^{4}-105\,a\cos\left(dx+c\right)^{2}+4\left(63\,a\cos\left(dx+c\right)^{4}-36\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)+21\,a}{1260\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*a*cos(d*x + c)^
4 - 36*a*cos(d*x + c)^2 + 8*a)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,a\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,a\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*a*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*a*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*a*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Mupad [B]
time = 8.86, size = 70, normalized size = 0.72

$$-\frac{252\,a\sin\left(c+d\,x\right)^5+210\,a\sin\left(c+d\,x\right)^4-360\,a\sin\left(c+d\,x\right)^3-315\,a\sin\left(c+d\,x\right)^2+140\,a\,\sin\left(c+d\,x\right)+126\,a}{1260\,d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] -(126*a + 140*a*sin(c + d*x) - 315*a*sin(c + d*x)^2 - 360*a*sin(c + d*x)^3
+ 210*a*sin(c + d*x)^4 + 252*a*sin(c + d*x)^5)/(1260*d*sin(c + d*x)^10)
```

3.510 $\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^{10}(c+dx)}{10d}-\frac{a\csc^{11}(c+dx)}{11d}$$

```
[Out] -1/6*a*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d+1/4*a*csc(d*x+c)^8/d+2/9*a*csc(d
*x+c)^9/d-1/10*a*csc(d*x+c)^10/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a\csc^{10}(c+dx)}{10d}+\frac{2a\csc^9(c+dx)}{9d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Csc[c + d*x]^6)/d - (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^2(a+x)^3}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^7\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{a^5}{x^{12}}+\frac{a^4}{x^{11}}-\frac{2a^3}{x^{10}}-\frac{2a^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)\,dx, x, a\,\text{si}\right.}{d}$$
$$= -\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}+\frac{2a}{}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 0.91

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^{11}(c+dx)}{11d}-\frac{a(10\csc^6(c+dx)-15\csc^8(c+dx)+6\csc^{10}(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
[Out] -1/7*(a*Csc[c + d*x]^7)/d + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(85) = 170$.
time = 0.23, size = 202, normalized size = 2.08

method	result
risch	$\frac{32a\left(1980ie^{15i(dx+c)}+1155\,e^{16i(dx+c)}+4400ie^{13i(dx+c)}+1155\,e^{14i(dx+c)}+7400ie^{11i(dx+c)}+1848\,e^{12i(dx+c)}+4400ie^{9i(dx+c)}\right.}{3465d\left(e^{2i(dx+c)}-1\right)^{11}}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^9}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{231\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos}{}\right.}{23}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^9}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{231\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos}{}\right.}{23}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d*x+c)^9*cos(d*x+c)^6-5/2
31/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos(d*x+c)^6+1/693/sin(d*x+
c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/231*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x
+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.72

$$-\frac{2310\,a\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,a\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,a\sin\left(dx+c\right)+1260\,a}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

Fricas [A]
time = 0.36, size = 128, normalized size = 1.32

$$\frac{1980\,a\cos\left(dx+c\right)^{4}-880\,a\cos\left(dx+c\right)^{2}+231\left(10\,a\cos\left(dx+c\right)^{4}-5\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)+160\,a}{13860\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*a*cos(d*x +
 c)^4 - 5*a*cos(d*x + c)^2 + a)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 70, normalized size = 0.72

$$-\frac{2310\,a\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,a\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,a\sin\left(dx+c\right)+1260\,a}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*a*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*a*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*a*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

Mupad [B]
time = 8.98, size = 70, normalized size = 0.72

$$-\frac{\frac{a\sin(c+d\,x)^5}{6}+\frac{a\sin(c+d\,x)^4}{7}-\frac{a\sin(c+d\,x)^3}{4}-\frac{2\,a\sin(c+d\,x)^2}{9}+\frac{a\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (a*sin(c + d*x))/10 - (2*a*sin(c + d*x)^2)/9 - (a*sin(c + d*x)^3)/
4 + (a*sin(c + d*x)^4)/7 + (a*sin(c + d*x)^5)/6)/(d*sin(c + d*x)^11)
```

3.511 $\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^6(c+dx)}{6d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^8(c+dx)}{8d}+\frac{2a^2\sin^9(c+dx)}{9d}+a$$

```
[Out] 1/4*a^2*sin(d*x+c)^4/d+2/5*a^2*sin(d*x+c)^5/d-1/6*a^2*sin(d*x+c)^6/d-4/7*a^
2*sin(d*x+c)^7/d-1/8*a^2*sin(d*x+c)^8/d+2/9*a^2*sin(d*x+c)^9/d+1/10*a^2*sin
(d*x+c)^10/d
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin^{10}(c+dx)}{10d}+\frac{2a^2\sin^9(c+dx)}{9d}-\frac{a^2\sin^8(c+dx)}{8d}-\frac{4a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*Sin[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*
x]^6)/(6*d) - (4*a^2*Sin[c + d*x]^7)/(7*d) - (a^2*Sin[c + d*x]^8)/(8*d) + (
2*a^2*Sin[c + d*x]^9)/(9*d) + (a^2*Sin[c + d*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)^4}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int (a^6x^3+2a^5x^4-a^4x^5-4a^3x^6-a^2x^7+2ax^8\right.}{a^8d}$$
$$= \frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^6(c+dx)}{6d}-$$

Mathematica [A]
time = 0.57, size = 110, normalized size = 0.87

$$-\frac{a^2(-2625+10710\cos(2(c+dx))+1260\cos(4(c+dx))-1365\cos(6(c+dx))-315\cos(8(c+dx))+63\cos(10(c+dx))-15120\sin(c+dx)+3360\sin(3(c+dx))+2016\sin(5(c+dx))-360\sin(7(c+dx))-280\sin(9(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/322560*(a^2*(-2625 + 10710*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 13
65*Cos[6*(c + d*x)] - 315*Cos[8*(c + d*x)] + 63*Cos[10*(c + d*x)] - 15120*S
in[c + d*x] + 3360*Sin[3*(c + d*x)] + 2016*Sin[5*(c + d*x)] - 360*Sin[7*(c
+ d*x)] - 280*Sin[9*(c + d*x)]))/d
```

Maple [A]
time = 0.32, size = 158, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx\right.}{}\right.}{d}$
risch	$\frac{3a^2\sin(dx+c)}{64d}-\frac{a^2\cos(10dx+10c)}{5120d}+\frac{a^2\sin(9dx+9c)}{1152d}+\frac{a^2\cos(8dx+8c)}{1024d}+\frac{a^2\sin(7dx+7c)}{896d}+\frac{13a^2\cos(6dx+6c)}{3072d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+2*a^2*(-1/9*sin
(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4
```

```
+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*si
n(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))
```

Maxima [A]
time = 0.29, size = 97, normalized size = 0.76

$$\frac{252\,a^2\sin\left(dx+c\right)^{10}+560\,a^2\sin\left(dx+c\right)^{9}-315\,a^2\sin\left(dx+c\right)^{8}-1440\,a^2\sin\left(dx+c\right)^{7}-420\,a^2\sin\left(dx+c\right)^{6}+1008\,a^2\sin\left(dx+c\right)^{5}+630\,a^2\sin\left(dx+c\right)^{4}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2520*(252*a^2*sin(d*x + c)^10 + 560*a^2*sin(d*x + c)^9 - 315*a^2*sin(d*x
+ c)^8 - 1440*a^2*sin(d*x + c)^7 - 420*a^2*sin(d*x + c)^6 + 1008*a^2*sin(d*
x + c)^5 + 630*a^2*sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.39, size = 111, normalized size = 0.87

$$-\frac{252\,a^2\cos\left(dx+c\right)^{10}-945\,a^2\cos\left(dx+c\right)^{8}+840\,a^2\cos\left(dx+c\right)^{6}-16\,\left(35\,a^2\cos\left(dx+c\right)^{8}-50\,a^2\cos\left(dx+c\right)^{6}+3\,a^2\cos\left(dx+c\right)^{4}+4\,a^2\cos\left(dx+c\right)^{2}+8\,a^2\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2520*(252*a^2*cos(d*x + c)^10 - 945*a^2*cos(d*x + c)^8 + 840*a^2*cos(d*x
 + c)^6 - 16*(35*a^2*cos(d*x + c)^8 - 50*a^2*cos(d*x + c)^6 + 3*a^2*cos(d*x
 + c)^4 + 4*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d
```

Sympy [A]
time = 1.77, size = 189, normalized size = 1.49

$$\begin{cases} \frac{16a^2\sin^9(c+dx)}{315d}+\frac{8a^2\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{2a^2\sin^5(c+dx)\cos^4(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)\cos^6(c+dx)}{6d}-\frac{a^2\sin^2(c+dx)\cos^8(c+dx)}{12d}-\frac{a^2\sin^2(c+dx)\cos^6(c+dx)}{6d}-\frac{a^2\cos^{10}(c+dx)}{60d}-\frac{a^2\cos^8(c+dx)}{24d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^2\sin^3(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**9/(315*d) + 8*a**2*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 2*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - a**2*sin(c
 + d*x)**4*cos(c + d*x)**6/(6*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**8/(12
*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - a**2*cos(c + d*x)**10/(6
0*d) - a**2*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*
*3*cos(c)**5, True))
```

Giac [A]
time = 0.54, size = 168, normalized size = 1.32

$$-\frac{a^2\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{a^2\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{13\,a^2\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{a^2\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{17\,a^2\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a^2\sin\left(9\,dx+9\,c\right)}{1152\,d}+\frac{a^2\sin\left(7\,dx+7\,c\right)}{896\,d}-\frac{a^2\sin\left(5\,dx+5\,c\right)}{160\,d}-\frac{a^2\sin\left(3\,dx+3\,c\right)}{96\,d}+\frac{3\,a^2\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/5120*a^2*cos(10*d*x + 10*c)/d + 1/1024*a^2*cos(8*d*x + 8*c)/d + 13/3072*
a^2*cos(6*d*x + 6*c)/d - 1/256*a^2*cos(4*d*x + 4*c)/d - 17/512*a^2*cos(2*d*
x + 2*c)/d + 1/1152*a^2*sin(9*d*x + 9*c)/d + 1/896*a^2*sin(7*d*x + 7*c)/d -
 1/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 3/64*a^2*sin(
d*x + c)/d
```

Mupad [B]
time = 8.74, size = 96, normalized size = 0.76

$$\frac{\frac{a^2\sin(c+d\,x)^{10}}{10}+\frac{2\,a^2\sin(c+d\,x)^9}{9}-\frac{a^2\sin(c+d\,x)^8}{8}-\frac{4\,a^2\sin(c+d\,x)^7}{7}-\frac{a^2\sin(c+d\,x)^6}{6}+\frac{2\,a^2\sin(c+d\,x)^5}{5}+\frac{a^2\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^4)/4 + (2*a^2*sin(c + d*x)^5)/5 - (a^2*sin(c + d*x)^6)/6
 - (4*a^2*sin(c + d*x)^7)/7 - (a^2*sin(c + d*x)^8)/8 + (2*a^2*sin(c + d*x)^
9)/9 + (a^2*sin(c + d*x)^10)/10)/d
```

3.512 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=109

$$\frac{4(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(a+a\sin(c+dx))^6}{a^4d}+\frac{13(a+a\sin(c+dx))^7}{7a^5d}-\frac{3(a+a\sin(c+dx))^8}{4a^6d}+\frac{(a+a\sin(c+dx))^9}{9a^7d}$$

```
[Out] 4/5*(a+a*sin(d*x+c))^5/a^3/d-2*(a+a*sin(d*x+c))^6/a^4/d+13/7*(a+a*sin(d*x+c
))^7/a^5/d-3/4*(a+a*sin(d*x+c))^8/a^6/d+1/9*(a+a*sin(d*x+c))^9/a^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{(a\sin(c+dx)+a)^9}{9a^7d}-\frac{3(a\sin(c+dx)+a)^8}{4a^6d}+\frac{13(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(a\sin(c+dx)+a)^6}{a^4d}+\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(a + a*Sin[c + d*x])^6)/(a^4*d) +
 (13*(a + a*Sin[c + d*x])^7)/(7*a^5*d) - (3*(a + a*Sin[c + d*x])^8)/(4*a^6*
d) + (a + a*Sin[c + d*x])^9/(9*a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^4}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^4-12a^3(a+x)^5+13a^2(a+x)^6-\right.\right.}{a^7d}$$
$$= \frac{4(a+a\sin(c+dx))^5}{5a^3d} - \frac{2(a+a\sin(c+dx))^6}{a^4d} + \frac{13(a+}{}$$

Mathematica [A]

time = 0.53, size = 99, normalized size = 0.91

$$-\frac{a^2(7560\cos(2(c+dx))+1260\cos(4(c+dx))-840\cos(6(c+dx))-315\cos(8(c+dx))-16380\sin(c+dx)+1680\sin(3(c+dx))+2016\sin(5(c+dx))+270\sin(7(c+dx))-70\sin(9(c+dx)))}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
[Out] -1/161280*(a^2*(7560*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 840*Cos[6*(
c + d*x)] - 315*Cos[8*(c + d*x)] - 16380*Sin[c + d*x] + 1680*Sin[3*(c + d*x
)] + 2016*Sin[5*(c + d*x)] + 270*Sin[7*(c + d*x)] - 70*Sin[9*(c + d*x)]))/d
```

Maple [A]

time = 0.28, size = 156, normalized size = 1.43

method	result
risch	$\frac{13a^2\sin(dx+c)}{128d}+\frac{a^2\sin(9dx+9c)}{2304d}+\frac{a^2\cos(8dx+8c)}{512d}-\frac{3a^2\sin(7dx+7c)}{1792d}+\frac{a^2\cos(6dx+6c)}{192d}-\frac{a^2\sin(5dx+5c)}{80d}-$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{d}$
norman	$\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{48a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{136a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{11104a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d}-\frac{136a^2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{48a^2\left(\text{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))+2*a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a
^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.32, size = 97, normalized size = 0.89

$$\frac{140\,a^2\sin\left(dx+c\right)^9+315\,a^2\sin\left(dx+c\right)^8-180\,a^2\sin\left(dx+c\right)^7-840\,a^2\sin\left(dx+c\right)^6-252\,a^2\sin\left(dx+c\right)^5+630\,a^2\sin\left(dx+c\right)^4+420\,a^2\sin\left(dx+c\right)^3}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*a^2*sin(d*x + c)^9 + 315*a^2*sin(d*x + c)^8 - 180*a^2*sin(d*x +
 c)^7 - 840*a^2*sin(d*x + c)^6 - 252*a^2*sin(d*x + c)^5 + 630*a^2*sin(d*x +
 c)^4 + 420*a^2*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 98, normalized size = 0.90

$$\frac{315\,a^2\cos\left(dx+c\right)^8-420\,a^2\cos\left(dx+c\right)^6+4\,\left(35\,a^2\cos\left(dx+c\right)^8-95\,a^2\cos\left(dx+c\right)^6+12\,a^2\cos\left(dx+c\right)^4+16\,a^2\cos\left(dx+c\right)^2+32\,a^2\right)\sin\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a^2*cos(d*x + c)^8 - 420*a^2*cos(d*x + c)^6 + 4*(35*a^2*cos(d*x
 + c)^8 - 95*a^2*cos(d*x + c)^6 + 12*a^2*cos(d*x + c)^4 + 16*a^2*cos(d*x +
c)^2 + 32*a^2)*sin(d*x + c))/d
```

Sympy [A]
time = 1.30, size = 190, normalized size = 1.74

$$\begin{cases} \frac{8a^2\sin^9\left(c+dx\right)}{315d}+\frac{4a^2\sin^7\left(c+dx\right)\cos^2\left(c+dx\right)}{35d}+\frac{8a^2\sin^7\left(c+dx\right)}{105d}+\frac{a^2\sin^5\left(c+dx\right)\cos^4\left(c+dx\right)}{5d}+\frac{4a^2\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{a^2\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d}-\frac{a^2\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{3d}-\frac{a^2\cos^8\left(c+dx\right)}{12d} & \text{for}\: d\neq 0 \\ x\left(a\sin\left(c\right)+a\right)^2\sin^2\left(c\right)\cos^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**9/(315*d) + 4*a**2*sin(c + d*x)**7*cos(c +
d*x)**2/(35*d) + 8*a**2*sin(c + d*x)**7/(105*d) + a**2*sin(c + d*x)**5*cos(
c + d*x)**4/(5*d) + 4*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + a**2*si
n(c + d*x)**3*cos(c + d*x)**4/(3*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**6/
(3*d) - a**2*cos(c + d*x)**8/(12*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)
**2*cos(c)**5, True))
```

Giac [A]
time = 0.51, size = 151, normalized size = 1.39

$$\frac{a^2\cos(8dx+8c)}{512d}+\frac{a^2\cos(6dx+6c)}{192d}-\frac{a^2\cos(4dx+4c)}{128d}-\frac{3a^2\cos(2dx+2c)}{64d}+\frac{a^2\sin(9dx+9c)}{2304d}-\frac{3a^2\sin(7dx+7c)}{1792d}-\frac{a^2\sin(5dx+5c)}{80d}-\frac{a^2\sin(3dx+3c)}{96d}+\frac{13a^2\sin(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a^2*cos(8*d*x + 8*c)/d + 1/192*a^2*cos(6*d*x + 6*c)/d - 1/128*a^2*cos
(4*d*x + 4*c)/d - 3/64*a^2*cos(2*d*x + 2*c)/d + 1/2304*a^2*sin(9*d*x + 9*c)
/d - 3/1792*a^2*sin(7*d*x + 7*c)/d - 1/80*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2
*sin(3*d*x + 3*c)/d + 13/128*a^2*sin(d*x + c)/d
```

Mupad [B]
time = 8.76, size = 96, normalized size = 0.88

$$\frac{\frac{a^2\sin(c+dx)^9}{9}+\frac{a^2\sin(c+dx)^8}{4}-\frac{a^2\sin(c+dx)^7}{7}-\frac{2a^2\sin(c+dx)^6}{3}-\frac{a^2\sin(c+dx)^5}{5}+\frac{a^2\sin(c+dx)^4}{2}+\frac{a^2\sin(c+dx)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^3)/3 + (a^2*sin(c + d*x)^4)/2 - (a^2*sin(c + d*x)^5)/5 -
 (2*a^2*sin(c + d*x)^6)/3 - (a^2*sin(c + d*x)^7)/7 + (a^2*sin(c + d*x)^8)/4
 + (a^2*sin(c + d*x)^9)/9)/d
```

3.513 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$-\frac{4(a+a\sin(c+dx))^5}{5a^3d}+\frac{4(a+a\sin(c+dx))^6}{3a^4d}-\frac{5(a+a\sin(c+dx))^7}{7a^5d}+\frac{(a+a\sin(c+dx))^8}{8a^6d}$$

```
[Out] -4/5*(a+a*sin(d*x+c))^5/a^3/d+4/3*(a+a*sin(d*x+c))^6/a^4/d-5/7*(a+a*sin(d*x
+c))^7/a^5/d+1/8*(a+a*sin(d*x+c))^8/a^6/d
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 78}

$$\frac{(a\sin(c+dx)+a)^8}{8a^6d}-\frac{5(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(a\sin(c+dx)+a)^6}{3a^4d}-\frac{4(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a + a*Sin[c + d*x])^5)/(5*a^3*d) + (4*(a + a*Sin[c + d*x])^6)/(3*a^4*d
) - (5*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (a + a*Sin[c + d*x])^8/(8*a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^4}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^4+8a^2(a+x)^5-5a(a+x)^6+(\right.\right.}{a^6d}$$
$$= -\frac{4(a+a\sin(c+dx))^5}{5a^3d}+\frac{4(a+a\sin(c+dx))^6}{3a^4d}-\frac{5(a+}{}$$

Mathematica [A]
time = 0.22, size = 90, normalized size = 1.01

$$-\frac{a^2(-2590+10920\cos(2(c+dx))+3780\cos(4(c+dx))+280\cos(6(c+dx))-105\cos(8(c+dx))-16800\sin(c+dx)+1120\sin(3(c+dx))+2016\sin(5(c+dx))+480\sin(7(c+dx)))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] -1/107520*(a^2*(-2590 + 10920*Cos[2*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 28
0*Cos[6*(c + d*x)] - 105*Cos[8*(c + d*x)] - 16800*Sin[c + d*x] + 1120*Sin[3
*(c + d*x)] + 2016*Sin[5*(c + d*x)] + 480*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.22, size = 102, normalized size = 1.15

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6}+2a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(\right.}{8}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6}+2a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(\right.}{8}\right.}{d}$
risch	$\frac{5a^2\sin(dx+c)}{32d}+\frac{a^2\cos(8dx+8c)}{1024d}-\frac{a^2\sin(7dx+7c)}{224d}-\frac{a^2\cos(6dx+6c)}{384d}-\frac{3a^2\sin(5dx+5c)}{160d}-\frac{9a^2\cos(4dx+4c)}{256d}-$
norman	$\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{16a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{1376a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{1376a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{16a^2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{16a^2\left(\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^2*cos(d*x+c)^6+2*a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^
6-1/24*cos(d*x+c)^6))
```

Maxima [A]
time = 0.29, size = 97, normalized size = 1.09

$$\frac{105\,a^2\sin(dx+c)^8+240\,a^2\sin(dx+c)^7-140\,a^2\sin(dx+c)^6-672\,a^2\sin(dx+c)^5-210\,a^2\sin(dx+c)^4+560\,a^2\sin(dx+c)^3+420\,a^2\sin(dx+c)^2}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/840*(105*a^2*sin(d*x + c)^8 + 240*a^2*sin(d*x + c)^7 - 140*a^2*sin(d*x +
c)^6 - 672*a^2*sin(d*x + c)^5 - 210*a^2*sin(d*x + c)^4 + 560*a^2*sin(d*x +
c)^3 + 420*a^2*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 85, normalized size = 0.96

$$\frac{105\,a^2\cos(dx+c)^8-280\,a^2\cos(dx+c)^6-16\left(15\,a^2\cos(dx+c)^6-3\,a^2\cos(dx+c)^4-4\,a^2\cos(dx+c)^2-8\,a^2\right)\sin(dx+c)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*a^2*cos(d*x + c)^8 - 280*a^2*cos(d*x + c)^6 - 16*(15*a^2*cos(d*x
 + c)^6 - 3*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c)
)/d
```

Sympy [A]
time = 0.87, size = 139, normalized size = 1.56

$$\begin{cases} \frac{16a^2\sin^7(c+dx)}{105d}+\frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)\cos^6(c+dx)}{6d}-\frac{a^2\cos^8(c+dx)}{24d}-\frac{a^2\cos^6(c+dx)}{6d} & \text{for } d\neq 0\\ x(a\sin(c)+a)^2\sin(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**7/(105*d) + 8*a**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(15*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a**2*sin(c
 + d*x)**2*cos(c + d*x)**6/(6*d) - a**2*cos(c + d*x)**8/(24*d) - a**2*cos(c
 + d*x)**6/(6*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)*cos(c)**5, True))
```

Giac [A]
time = 0.52, size = 134, normalized size = 1.51

$$\frac{a^2\cos(8\,dx+8\,c)}{1024\,d}-\frac{a^2\cos(6\,dx+6\,c)}{384\,d}-\frac{9\,a^2\cos(4\,dx+4\,c)}{256\,d}-\frac{13\,a^2\cos(2\,dx+2\,c)}{128\,d}-\frac{a^2\sin(7\,dx+7\,c)}{224\,d}-\frac{3\,a^2\sin(5\,dx+5\,c)}{160\,d}-\frac{a^2\sin(3\,dx+3\,c)}{96\,d}+\frac{5\,a^2\sin(dx+c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*a^2*cos(8*d*x + 8*c)/d - 1/384*a^2*cos(6*d*x + 6*c)/d - 9/256*a^2*co
s(4*d*x + 4*c)/d - 13/128*a^2*cos(2*d*x + 2*c)/d - 1/224*a^2*sin(7*d*x + 7*
c)/d - 3/160*a^2*sin(5*d*x + 5*c)/d - 1/96*a^2*sin(3*d*x + 3*c)/d + 5/32*a^
2*sin(d*x + c)/d
```

Mupad [B]
time = 8.81, size = 96, normalized size = 1.08

$$\frac{\frac{a^2\sin(c+dx)^8}{8}+\frac{2a^2\sin(c+dx)^7}{7}-\frac{a^2\sin(c+dx)^6}{6}-\frac{4a^2\sin(c+dx)^5}{5}-\frac{a^2\sin(c+dx)^4}{4}+\frac{2a^2\sin(c+dx)^3}{3}+\frac{a^2\sin(c+dx)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2)/2 + (2*a^2*sin(c + d*x)^3)/3 - (a^2*sin(c + d*x)^4)/4
 - (4*a^2*sin(c + d*x)^5)/5 - (a^2*sin(c + d*x)^6)/6 + (2*a^2*sin(c + d*x)^
7)/7 + (a^2*sin(c + d*x)^8)/8)/d
```

3.514 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^5(c+dx)}{5d}$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d-4/3*a^2*sin(
d*x+c)^3/d-1/4*a^2*sin(d*x+c)^4/d+2/5*a^2*sin(d*x+c)^5/d+1/6*a^2*sin(d*x+c)
^6/d
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin^6(c+dx)}{6d}+\frac{2a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)}{4d}-\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(
2*d) - (4*a^2*Sin[c + d*x]^3)/(3*d) - (a^2*Sin[c + d*x]^4)/(4*d) + (2*a^2*S
in[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(2a^5+\frac{a^6}{x}-a^4x-4a^3x^2-a^2x^3+2ax^4+x^5\right)\right.}{a^4 d}$$

$$= \frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.06, size = 78, normalized size = 0.66

$$\frac{a^2(60\log(\sin(c+dx))+120\sin(c+dx)-30\sin^2(c+dx)-80\sin^3(c+dx)-15\sin^4(c+dx)+24\sin^5(c+dx)+10\sin^6(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(60*Log[Sin[c + d*x]] + 120*Sin[c + d*x] - 30*Sin[c + d*x]^2 - 80*Sin[
c + d*x]^3 - 15*Sin[c + d*x]^4 + 24*Sin[c + d*x]^5 + 10*Sin[c + d*x]^6))/(6
0*d)
```

Maple [A]
time = 0.18, size = 82, normalized size = 0.69

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{2a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{a^2\left(\cos^6(dx+c)\right)}{6}}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{2a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{a^2\left(\cos^6(dx+c)\right)}{6}}{d}$
risch	$-ia^2x+\frac{19a^2\mathrm{e}^{2i(dx+c)}}{128d}+\frac{19a^2\mathrm{e}^{-2i(dx+c)}}{128d}-\frac{2ia^2c}{d}+\frac{a^2\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}+\frac{5a^2\sin(dx+c)}{4d}-\frac{a^2\cos(6dx+6c)}{192d}$
norman	$\frac{\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{28a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{104a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{104a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{28a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4a^2\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a^2*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+2/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*a^2*cos(d*x+c)^6)

Maxima [A]
time = 0.29, size = 94, normalized size = 0.79

$$\frac{10\,a^2\sin\left(dx+c\right)^6+24\,a^2\sin\left(dx+c\right)^5-15\,a^2\sin\left(dx+c\right)^4-80\,a^2\sin\left(dx+c\right)^3-30\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\sin\left(dx+c\right)\right)+120\,a^2\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c)) + 120*a^2*sin(d*x + c))/d

Fricas [A]
time = 0.37, size = 99, normalized size = 0.83

$$-\frac{10\,a^2\cos\left(dx+c\right)^6-15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2-60\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(3\,a^2\cos\left(dx+c\right)^4+4\,a^2\cos\left(dx+c\right)^2+8\,a^2\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/60*(10*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 + 4*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d*x + c))/d

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A]
time = 0.51, size = 95, normalized size = 0.80

$$\frac{10\,a^2\sin\left(dx+c\right)^6+24\,a^2\sin\left(dx+c\right)^5-15\,a^2\sin\left(dx+c\right)^4-80\,a^2\sin\left(dx+c\right)^3-30\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+120\,a^2\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/60*(10*a^2*sin(d*x + c)^6 + 24*a^2*sin(d*x + c)^5 - 15*a^2*sin(d*x + c)^4 - 80*a^2*sin(d*x + c)^3 - 30*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) + 120*a^2*sin(d*x + c))/d

Mupad [B]
time = 9.07, size = 132, normalized size = 1.11

$$\frac{5\,a^2\,\sin\left(c+d\,x\right)}{4\,d}-\frac{a^2\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{19\,a^2\,\cos\left(2\,c+2\,d\,x\right)}{64\,d}-\frac{a^2\,\cos\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{5\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{24\,d}+\frac{a^2\,\sin\left(5\,c+5\,d\,x\right)}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (5*a^2*sin(c + d*x))/(4*d) - (a^2*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^2*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (19*a^2*cos(2*c + 2*d*x))/(64*
d) - (a^2*cos(6*c + 6*d*x))/(192*d) + (5*a^2*sin(3*c + 3*d*x))/(24*d) + (a^
2*sin(5*c + 5*d*x))/(40*d)
```

3.515 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=114

$$-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{2d}$$

```
[Out] -a^2*csc(d*x+c)/d+2*a^2*ln(sin(d*x+c))/d-a^2*sin(d*x+c)/d-2*a^2*sin(d*x+c)^
2/d-1/3*a^2*sin(d*x+c)^3/d+1/2*a^2*sin(d*x+c)^4/d+1/5*a^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^4(c+dx)}{2d}-\frac{a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}+\frac{2a^5}{x}-4a^3x-a^2x^2+2ax^3+x^4\right)\right.}{a^3d}$$
$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+d}{d}$$

Mathematica [A]
time = 0.04, size = 114, normalized size = 1.00

$$-\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^2(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^4(c+dx)}{2d}+\frac{a^2\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/
d - (2*a^2*Sin[c + d*x]^2)/d - (a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*
x]^4)/(2*d) + (a^2*Sin[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.17, size = 121, normalized size = 1.06

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+2a^2\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+2a^2\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+}{d}$
risch	$-2ia^2x+\frac{3a^2e^{2i(dx+c)}}{8d}+\frac{9ia^2e^{i(dx+c)}}{16d}-\frac{9ia^2e^{-i(dx+c)}}{16d}+\frac{3a^2e^{-2i(dx+c)}}{8d}-\frac{4ia^2c}{d}-\frac{2ia^2e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)}+\frac{2a^2}{}$
norman	$\frac{-\frac{a^2}{2d}-\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{109a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{314a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{109a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{5a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\tan\left(\frac{d}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*si
n(d*x+c))+2*a^2*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/5*a^2*
(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 94, normalized size = 0.82

$$\frac{6\,a^2\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^3-60\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\sin\left(dx+c\right)\right)-30\,a^2\sin\left(dx+c\right)-\frac{30\,a^2}{\sin(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(sin(d*x + c)) - 30*a^2*sin(d*x + c) -
30*a^2/sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.04

$$-\frac{48\,a^2\cos\left(dx+c\right)^6-64\,a^2\cos\left(dx+c\right)^4-256\,a^2\cos\left(dx+c\right)^2-480\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+512\,a^2-15\left(8\,a^2\cos\left(dx+c\right)^4+16\,a^2\cos\left(dx+c\right)^2-11\,a^2\right)\sin\left(dx+c\right)}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 - 64*a^2*cos(d*x + c)^4 - 256*a^2*cos(d*x + c
)^2 - 480*a^2*log(1/2*sin(d*x + c))*sin(d*x + c) + 512*a^2 - 15*(8*a^2*cos(
d*x + c)^4 + 16*a^2*cos(d*x + c)^2 - 11*a^2)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.47, size = 107, normalized size = 0.94

$$\frac{6\,a^2\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4-10\,a^2\sin\left(dx+c\right)^3-60\,a^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-30\,a^2\sin\left(dx+c\right)-\frac{30\,(2\,a^2\sin(dx+c)+a^2)}{\sin(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*a^2*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 10*a^2*sin(d*x + c)^3
- 60*a^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) - 30*a^2*sin(d*x +
c) - 30*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c))/d
```

Mupad [B]
time = 8.95, size = 333, normalized size = 2.92

$$\frac{16\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{8\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}-\frac{16\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{8\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}-\frac{2\,a^2\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{2\,a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{176\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{328\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{96\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{32\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{5\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{a^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (16*a^2*cos(c/2 + (d*x)/2)^4)/d - (8*a^2*cos(c/2 + (d*x)/2)^2)/d - (16*a^2*
cos(c/2 + (d*x)/2)^6)/d + (8*a^2*cos(c/2 + (d*x)/2)^8)/d - (2*a^2*log(1/cos
(c/2 + (d*x)/2)^2))/d + (2*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d - (2*a^2*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (176*a^2*cos(c/
2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (328*a^2*cos(c/2 + (d*x)/2)^7)/
(15*d*sin(c/2 + (d*x)/2)) + (96*a^2*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 + (d
*x)/2)) - (32*a^2*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) - (5*a^2*
cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^2*sin(c/2 + (d*x)/2))/(2*
d*cos(c/2 + (d*x)/2))
```

3.516 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{a^2\log(\sin(c+dx))}{d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin^3(c+dx}{3d}$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-a^2*ln(sin(d*x+c))/d-4*a^2*sin(d
*x+c)/d-1/2*a^2*sin(d*x+c)^2/d+2/3*a^2*sin(d*x+c)^3/d+1/4*a^2*sin(d*x+c)^4/
d
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin^4(c+dx)}{4d}+\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (a^2*Log[Sin[c + d*x
]])/d - (4*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + (2*a^2*Sin[c
+ d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+\frac{a^6}{x^3}+\frac{2a^5}{x^2}-\frac{a^4}{x}-a^2x+2ax^2+x^3\right) d\right.}{a^2 d}$$
$$= -\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{a^2\log(\sin(c+d}{d}$$

Mathematica [A]

time = 0.13, size = 76, normalized size = 0.66

$$-\frac{a^2(24\csc(c+dx)+6\csc^2(c+dx)+12\log(\sin(c+dx))+48\sin(c+dx)+6\sin^2(c+dx)-8\sin^3(c+dx)-3\sin^4(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
[Out] -1/12*(a^2*(24*Csc[c + d*x] + 6*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] + 48*
Sin[c + d*x] + 6*Sin[c + d*x]^2 - 8*Sin[c + d*x]^3 - 3*Sin[c + d*x]^4))/d
```

Maple [A]

time = 0.19, size = 142, normalized size = 1.22

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
risch	$ia^2x+\frac{ia^2e^{3i(dx+c)}}{12d}+\frac{a^2e^{2i(dx+c)}}{16d}+\frac{7ia^2e^{i(dx+c)}}{4d}-\frac{7ia^2e^{-i(dx+c)}}{4d}+\frac{a^2e^{-2i(dx+c)}}{16d}-\frac{ia^2e^{-3i(dx+c)}}{12d}+\frac{2ia^2}{d}$
norman	$\frac{-\frac{a^2}{8d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{13a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{86a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{86a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{13a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2\left(\tan^{11}\left(\frac{d}{}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln
(sin(d*x+c)))+2*a^2*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d
```

```
*x+c)^2)*sin(d*x+c))+a^2*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))
)
```

Maxima [A]
time = 0.28, size = 93, normalized size = 0.80

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3-6\,a^2\sin\left(dx+c\right)^2-12\,a^2\log\left(\sin\left(dx+c\right)\right)-48\,a^2\sin\left(dx+c\right)-\frac{6\,\left(4\,a^2\sin(dx+c)+a^2\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(sin(d*x + c)) - 48*a^2*sin(d*x + c) - 6*(4*a^2*sin(d*x + c) + a^
2)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.37, size = 131, normalized size = 1.13

$$\frac{24\,a^2\cos\left(dx+c\right)^6-24\,a^2\cos\left(dx+c\right)^4-9\,a^2\cos\left(dx+c\right)^2+57\,a^2-96\,\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-64\,\left(a^2\cos\left(dx+c\right)^4+4\,a^2\cos\left(dx+c\right)^2-8\,a^2\right)\sin\left(dx+c\right)}{96\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^2*cos(d*x + c)^6 - 24*a^2*cos(d*x + c)^4 - 9*a^2*cos(d*x + c)^2
+ 57*a^2 - 96*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 64*(a^2*co
s(d*x + c)^4 + 4*a^2*cos(d*x + c)^2 - 8*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.50, size = 109, normalized size = 0.94

$$\frac{3\,a^2\sin\left(dx+c\right)^4+8\,a^2\sin\left(dx+c\right)^3-6\,a^2\sin\left(dx+c\right)^2-12\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-48\,a^2\sin\left(dx+c\right)+\frac{6\,\left(3\,a^2\sin(dx+c)^2-4\,a^2\sin(dx+c)-a^2\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^2*sin(d*x + c)^4 + 8*a^2*sin(d*x + c)^3 - 6*a^2*sin(d*x + c)^2 -
12*a^2*log(abs(sin(d*x + c))) - 48*a^2*sin(d*x + c) + 6*(3*a^2*sin(d*x + c)
^2 - 4*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 8.94, size = 297, normalized size = 2.56

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{36\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{17\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+\frac{272\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{296\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+48\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2)))/d - (
2*a^2*tan(c/2 + (d*x)/2)^2 + 48*a^2*tan(c/2 + (d*x)/2)^3 + 11*a^2*tan(c/2 +
 (d*x)/2)^4 + (296*a^2*tan(c/2 + (d*x)/2)^5)/3 + 2*a^2*tan(c/2 + (d*x)/2)^6
 + (272*a^2*tan(c/2 + (d*x)/2)^7)/3 + (17*a^2*tan(c/2 + (d*x)/2)^8)/2 + 36*
a^2*tan(c/2 + (d*x)/2)^9 + a^2/2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2
+ (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c
/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (a^2*tan(c/2 + (d*x)/2))/d -
(a^2*tan(c/2 + (d*x)/2)^2)/(8*d)
```

3.517 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=110

$$\frac{a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{4a^2\log(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{d}+\frac{a}{}$$

[Out] a^2*csc(d*x+c)/d-a^2*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-4*a^2*ln(sin(d*x+c))/d-a^2*sin(d*x+c)/d+a^2*sin(d*x+c)^2/d+1/3*a^2*sin(d*x+c)^3/d

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^2(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}-\frac{4a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]

[Out] (a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*d) - (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3)/(3*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^6}{x^4}+\frac{2a^5}{x^3}-\frac{a^4}{x^2}-\frac{4a^3}{x}+2ax+x^2\right)\,dx, x$$
$$= \frac{a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{4a$$

Mathematica [A]

time = 0.13, size = 74, normalized size = 0.67

$$\frac{a^2(3\csc(c+dx)-3\csc^2(c+dx)-\csc^3(c+dx)-12\log(\sin(c+dx))-3\sin(c+dx)+3\sin^2(c+dx)+\sin^3(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(3*Csc[c + d*x] - 3*Csc[c + d*x]^2 - Csc[c + d*x]^3 - 12*Log[Sin[c + d
*x]] - 3*Sin[c + d*x] + 3*Sin[c + d*x]^2 + Sin[c + d*x]^3))/(3*d)
```

Maple [A]

time = 0.19, size = 177, normalized size = 1.61

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+2a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\text{co}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+2a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\text{co}\right.}{d}$
risch	$4ia^2x+\frac{ia^2e^{3i(dx+c)}}{24d}-\frac{a^2e^{2i(dx+c)}}{4d}+\frac{3ia^2e^{i(dx+c)}}{8d}-\frac{3ia^2e^{-i(dx+c)}}{8d}-\frac{a^2e^{-2i(dx+c)}}{4d}-\frac{ia^2e^{-3i(dx+c)}}{24d}+\frac{8ia}{}$
norman	$\frac{-\frac{a^2}{24d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{5a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{5a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)
```

```
^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+a^2*(-1/sin(d*x+c)*cos(d
*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 93, normalized size = 0.85

$$\frac{a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-12\,a^2\log(\sin(dx+c))-3\,a^2\sin(dx+c)+\frac{3\,a^2\sin(dx+c)^2-3\,a^2\sin(dx+c)-a^2}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) -
 3*a^2*sin(d*x + c) + (3*a^2*sin(d*x + c)^2 - 3*a^2*sin(d*x + c) - a^2)/sin
(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 115, normalized size = 1.05

$$\frac{2\,a^2\cos(dx+c)^6-24\,\left(a^2\cos(dx+c)^2-a^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-3\,\left(2\,a^2\cos(dx+c)^4-3\,a^2\cos(dx+c)^2-a^2\right)\sin(dx+c)}{6\,\left(d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^6 - 24*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x +
 c))*sin(d*x + c) - 3*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 - a^2)*s
in(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.49, size = 107, normalized size = 0.97

$$\frac{a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-12\,a^2\log(|\sin(dx+c)|)-3\,a^2\sin(dx+c)+\frac{22\,a^2\sin(dx+c)^3+3\,a^2\sin(dx+c)^2-3\,a^2\sin(dx+c)-a^2}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c
))) - 3*a^2*sin(d*x + c) + (22*a^2*sin(d*x + c)^3 + 3*a^2*sin(d*x + c)^2 -
3*a^2*sin(d*x + c) - a^2)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 8.92, size = 288, normalized size = 2.62

$$\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-30\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-26\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (3*a^2*tan(c/2 + (d*x)/2))/(8*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (4*a
^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) - (6*a^2*t
an(c/2 + (d*x)/2)^3 - 2*a^2*tan(c/2 + (d*x)/2)^2 + 8*a^2*tan(c/2 + (d*x)/2)
^4 - 26*a^2*tan(c/2 + (d*x)/2)^5 + 2*a^2*tan(c/2 + (d*x)/2)^6 - 30*a^2*tan(
c/2 + (d*x)/2)^7 + 13*a^2*tan(c/2 + (d*x)/2)^8 + a^2/3 + 2*a^2*tan(c/2 + (d
*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 +
 (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (4*a^2*log(tan(c/2 + (d*x)/2)^2 +
1))/d
```

3.518 $\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{4a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{a^2\log(\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}$$

```
[Out] 4*a^2*csc(d*x+c)/d+1/2*a^2*csc(d*x+c)^2/d-2/3*a^2*csc(d*x+c)^3/d-1/4*a^2*cs
c(d*x+c)^4/d-a^2*ln(sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) - (2*a^2*Csc[c + d*x]^3
)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) - (a^2*Log[Sin[c + d*x]])/d + (2*a^2*S
in[c + d*x])/d + (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(2a+\frac{a^6}{x^5}+\frac{2a^5}{x^4}-\frac{a^4}{x^3}-\frac{4a^3}{x^2}-\frac{a^2}{x}+x\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{4a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 76, normalized size = 0.66

$$\frac{a^2(48\csc(c+dx)+6\csc^2(c+dx)-8\csc^3(c+dx)-3\csc^4(c+dx)-12\log(\sin(c+dx))+24\sin(c+dx)+6\sin^2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(48*Csc[c + d*x] + 6*Csc[c + d*x]^2 - 8*Csc[c + d*x]^3 - 3*Csc[c + d*x
]^4 - 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] + 6*Sin[c + d*x]^2))/(12*d)
```

Maple [A]
time = 0.21, size = 158, normalized size = 1.36

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$ia^2x-\frac{a^2e^{2i(dx+c)}}{8d}-\frac{ia^2e^{i(dx+c)}}{d}+\frac{ia^2e^{-i(dx+c)}}{d}-\frac{a^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2c}{d}+\frac{2ia^2\left(3ie^{6i(dx+c)}+12e^{7i(dx+c)}-\right.}{}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{19a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{55a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{55a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{19a^2\left(\tan^9\left(\right.\right.}{12d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a^2*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))+a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4
-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.81

$$\frac{6\,a^2\sin(dx+c)^2-12\,a^2\log(\sin(dx+c))+24\,a^2\sin(dx+c)+\frac{48\,a^2\sin(dx+c)^3+6\,a^2\sin(dx+c)^2-8\,a^2\sin(dx+c)-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(sin(d*x + c)) + 24*a^2*sin(d*x + c)
 + (48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 - 8*a^2*sin(d*x + c) - 3*a
^2)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.37, size = 152, normalized size = 1.31

$$-\frac{6\,a^2\cos\left(dx+c\right)^6-15\,a^2\cos\left(dx+c\right)^4+18\,a^2\cos\left(dx+c\right)^2-6\,a^2+12\,\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(3\,a^2\cos\left(dx+c\right)^4-12\,a^2\cos\left(dx+c\right)^2+8\,a^2\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*a^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 + 18*a^2*cos(d*x + c)^2
 - 6*a^2 + 12*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin
(d*x + c)) - 8*(3*a^2*cos(d*x + c)^4 - 12*a^2*cos(d*x + c)^2 + 8*a^2)*sin(d
*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.49, size = 108, normalized size = 0.93

$$\frac{6\,a^2\sin\left(dx+c\right)^2-12\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+24\,a^2\sin\left(dx+c\right)+\frac{25\,a^2\sin(dx+c)^4+48\,a^2\sin(dx+c)^3+6\,a^2\sin(dx+c)^2-8\,a^2\sin(dx+c)-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*sin(d*x + c)^2 - 12*a^2*log(abs(sin(d*x + c))) + 24*a^2*sin(d*x
 + c) + (25*a^2*sin(d*x + c)^4 + 48*a^2*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)
^2 - 8*a^2*sin(d*x + c) - 3*a^2)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 8.80, size = 276, normalized size = 2.38

$$\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}-\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^2\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{7\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}+\frac{a^2\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}+\frac{92\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+33\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{356\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{7\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+\frac{76\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{4\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a^2}{4}}{d\,\left(16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(16*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) - (a^
2*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^2*log(tan(c/2 + (d*x)/2)))/d + (7*a^2*t
an(c/2 + (d*x)/2))/(4*d) + (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d + ((a^2*ta
n(c/2 + (d*x)/2)^2)/2 + (76*a^2*tan(c/2 + (d*x)/2)^3)/3 + (7*a^2*tan(c/2 +
(d*x)/2)^4)/4 + (356*a^2*tan(c/2 + (d*x)/2)^5)/3 + 33*a^2*tan(c/2 + (d*x)/2
)^6 + 92*a^2*tan(c/2 + (d*x)/2)^7 - a^2/4 - (4*a^2*tan(c/2 + (d*x)/2))/3)/(
d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2
)^8))
```

3.519 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=112

$$\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{2d}-\frac{a^2\csc^5(c+dx)}{5d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

```
[Out] a^2*csc(d*x+c)/d+2*a^2*csc(d*x+c)^2/d+1/3*a^2*csc(d*x+c)^3/d-1/2*a^2*csc(d*
x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d+2*a^2*ln(sin(d*x+c))/d+a^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^2\sin(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^4(c+dx)}{2d}+\frac{a^2\csc^3(c+dx)}{3d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc(c+dx)}{d}+\frac{2a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^2)/d + (a^2*Csc[c + d*x]^3)/(3*d
) - (a^2*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5)/(5*d) + (2*a^2*Log[Si
n[c + d*x]])/d + (a^2*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(1+\frac{a^6}{x^6}+\frac{2a^5}{x^5}-\frac{a^4}{x^4}-\frac{4a^3}{x^3}-\frac{a^2}{x^2}+\frac{2a}{x}\right)\,dx, x, a$$
$$= \frac{a^2\csc(c+dx)}{d}+\frac{2a^2\csc^2(c+dx)}{d}+\frac{a^2\csc^3(c+dx)}{3d}-$$

Mathematica [A]

time = 0.10, size = 76, normalized size = 0.68

$$\frac{a^2(30\csc(c+dx)+60\csc^2(c+dx)+10\csc^3(c+dx)-15\csc^4(c+dx)-6\csc^5(c+dx)+60\log(\sin(c+dx))+30\sin(c+dx))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(30*Csc[c + d*x] + 60*Csc[c + d*x]^2 + 10*Csc[c + d*x]^3 - 15*Csc[c +
d*x]^4 - 6*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 30*Sin[c + d*x]))/(30*d)
```

Maple [A]

time = 0.21, size = 193, normalized size = 1.72

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+2a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+2a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{d}$
risch	$-2ia^2x-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{ia^2e^{-i(dx+c)}}{2d}-\frac{4ia^2c}{d}+\frac{2ia^2\left(15\,e^{9i(dx+c)}-80\,e^{7i(dx+c)}+60ie^{8i(dx+c)}+82\,e^{5i(dx+c)}-1\right.}{15d(e^2}$
norman	$\frac{-\frac{a^2}{160d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}-\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{277a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{355a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{355a^2\left(\tan^8\right.}{96}}{\tan\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5
/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)+2*a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+a^2*(-1/3/sin(d
*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*
x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 94, normalized size = 0.84

$$\frac{60\,a^2\log\left(\sin\left(dx+c\right)\right)+30\,a^2\sin\left(dx+c\right)+\frac{30\,a^2\sin(dx+c)^4+60\,a^2\sin(dx+c)^3+10\,a^2\sin(dx+c)^2-15\,a^2\sin(dx+c)-6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a^2*log(sin(d*x + c)) + 30*a^2*sin(d*x + c) + (30*a^2*sin(d*x + c)
^4 + 60*a^2*sin(d*x + c)^3 + 10*a^2*sin(d*x + c)^2 - 15*a^2*sin(d*x + c) -
6*a^2)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.37, size = 153, normalized size = 1.37

$$-\frac{30\,a^2\cos\left(dx+c\right)^6-120\,a^2\cos\left(dx+c\right)^4+160\,a^2\cos\left(dx+c\right)^2-60\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-64\,a^2+15\left(4\,a^2\cos\left(dx+c\right)^2-3\,a^2\right)\sin\left(dx+c\right)}{30\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*a^2*cos(d*x + c)^6 - 120*a^2*cos(d*x + c)^4 + 160*a^2*cos(d*x + c
)^2 - 60*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)*log(1/2*sin(d*x
+ c))*sin(d*x + c) - 64*a^2 + 15*(4*a^2*cos(d*x + c)^2 - 3*a^2)*sin(d*x + c
))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 109, normalized size = 0.97

$$\frac{60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+30\,a^2\sin\left(dx+c\right)-\frac{137\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-60\,a^2\sin(dx+c)^3-10\,a^2\sin(dx+c)^2+15\,a^2\sin(dx+c)+6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(sin(d*x + c))) + 30*a^2*sin(d*x + c) - (137*a^2*sin(d*
x + c)^5 - 30*a^2*sin(d*x + c)^4 - 60*a^2*sin(d*x + c)^3 - 10*a^2*sin(d*x +
 c)^2 + 15*a^2*sin(d*x + c) + 6*a^2)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 8.79, size = 267, normalized size = 2.38

$$\frac{82\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{55\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a^2}{5}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] ((2*a^2*tan(c/2 + (d*x)/2)^2)/15 + 11*a^2*tan(c/2 + (d*x)/2)^3 + (55*a^2*ta
n(c/2 + (d*x)/2)^4)/3 + 12*a^2*tan(c/2 + (d*x)/2)^5 + 82*a^2*tan(c/2 + (d*x
)/2)^6 - a^2/5 - a^2*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 32*t
an(c/2 + (d*x)/2)^7)) + (3*a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2*tan(c/2 +
 (d*x)/2)^3)/(96*d) - (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) - (a^2*tan(c/2 + (d
*x)/2)^5)/(160*d) + (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (9*a^2*tan(c/2 + (d
*x)/2))/(16*d) - (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.520 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=119

$$-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^4(c+dx)}{4d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2\csc^6(c+dx)}{6d}+$$

```
[Out] -2*a^2*csc(d*x+c)/d+1/2*a^2*csc(d*x+c)^2/d+4/3*a^2*csc(d*x+c)^3/d+1/4*a^2*c
sc(d*x+c)^4/d-2/5*a^2*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d+a^2*ln(sin(d*x+
c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{4d}+\frac{4a^2\csc^3(c+dx)}{3d}+\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/(2*d) + (4*a^2*Csc[c + d*x]^
3)/(3*d) + (a^2*Csc[c + d*x]^4)/(4*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2
*Csc[c + d*x]^6)/(6*d) + (a^2*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a-x)^2(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a^6}{x^7}+\frac{2a^5}{x^6}-\frac{a^4}{x^5}-\frac{4a^3}{x^4}-\frac{a^2}{x^3}+\frac{2a}{x^2}+\frac{1}{x}\right)\,dx, x,\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d}+\frac{a^2\csc^2(c+dx)}{2d}+\frac{4a^2\csc^3(c+dx)}{3d}$$

Mathematica [A]

time = 0.03, size = 102, normalized size = 0.86

$$a^2\left(-\frac{2\csc(c+dx)}{d}+\frac{\csc^2(c+dx)}{2d}+\frac{4\csc^3(c+dx)}{3d}+\frac{\csc^4(c+dx)}{4d}-\frac{2\csc^5(c+dx)}{5d}-\frac{\csc^6(c+dx)}{6d}+\frac{\log(\sin(c+dx))}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*((-2*Csc[c + d*x])/d + Csc[c + d*x]^2/(2*d) + (4*Csc[c + d*x]^3)/(3*d)
+ Csc[c + d*x]^4/(4*d) - (2*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d) +
Log[Sin[c + d*x]]/d)
```

Maple [A]

time = 0.25, size = 147, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+2a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+2a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+a^2\left(-\right.}{d}$
risch	$-ia^2x-\frac{2ia^2c}{d}-\frac{2ia^2\left(-15ie^{10i(dx+c)}+30\,e^{11i(dx+c)}+90ie^{8i(dx+c)}-70\,e^{9i(dx+c)}-70ie^{6i(dx+c)}+156\,e^{7i(dx+c)}+90ie\right.}{15d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{a^2}{384d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}-\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{19a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}+\frac{7a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}-\frac{103a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{85a^2\left(\tan^7\left(\right.\right.}{48d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^2/sin(d*x+c)^6*cos(d*x+c)^6+2*a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)
^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c
)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.29, size = 97, normalized size = 0.82

$$\frac{60\,a^2\log\left(\sin\left(dx+c\right)\right)-\frac{120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*a^2*log(sin(d*x + c)) - (120*a^2*sin(d*x + c)^5 - 30*a^2*sin(d*x +
 c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^2 + 24*a^2*sin(d*x + c)
 + 10*a^2)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.40

$$-\frac{30\,a^2\cos\left(dx+c\right)^4-75\,a^2\cos\left(dx+c\right)^2+35\,a^2-60\,\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(15\,a^2\cos\left(dx+c\right)^4-20\,a^2\cos\left(dx+c\right)^2+8\,a^2\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(30*a^2*cos(d*x + c)^4 - 75*a^2*cos(d*x + c)^2 + 35*a^2 - 60*(a^2*cos
(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*si
n(d*x + c)) - 8*(15*a^2*cos(d*x + c)^4 - 20*a^2*cos(d*x + c)^2 + 8*a^2)*sin
(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 111, normalized size = 0.93

$$\frac{60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{147\,a^2\sin(dx+c)^6+120\,a^2\sin(dx+c)^5-30\,a^2\sin(dx+c)^4-80\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+24\,a^2\sin(dx+c)+10\,a^2}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^2*log(abs(sin(d*x + c))) - (147*a^2*sin(d*x + c)^6 + 120*a^2*sin
(d*x + c)^5 - 30*a^2*sin(d*x + c)^4 - 80*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*
x + c)^2 + 24*a^2*sin(d*x + c) + 10*a^2)/sin(d*x + c)^6)/d
```

Mupad [B]
time = 8.98, size = 217, normalized size = 1.82

$$\frac{19\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{48\,d}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{80\,d}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(40\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{19\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{20\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+\frac{a^2}{6}\right)}{64\,d}-\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (19*a^2*tan(c/2 + (d*x)/2)^2)/(128*d) + (5*a^2*tan(c/2 + (d*x)/2)^3)/(48*d)
 - (a^2*tan(c/2 + (d*x)/2)^5)/(80*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) +
 (a^2*log(tan(c/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^6*(40*a^2*tan(c/2 +
(d*x)/2)^5 - (19*a^2*tan(c/2 + (d*x)/2)^4)/2 - (20*a^2*tan(c/2 + (d*x)/2)^3
)/3 + a^2/6 + (4*a^2*tan(c/2 + (d*x)/2))/5))/(64*d) - (5*a^2*tan(c/2 + (d*x
)/2))/(8*d) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.521 $\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{2(a+a\sin(c+dx))^6}{3a^3d}-\frac{12(a+a\sin(c+dx))^7}{7a^4d}+\frac{13(a+a\sin(c+dx))^8}{8a^5d}-\frac{2(a+a\sin(c+dx))^9}{3a^6d}+\frac{(a+a\sin(c+dx))^{10}}{10a^7d}$$

```
[Out] 2/3*(a+a*sin(d*x+c))^6/a^3/d-12/7*(a+a*sin(d*x+c))^7/a^4/d+13/8*(a+a*sin(d*
x+c))^8/a^5/d-2/3*(a+a*sin(d*x+c))^9/a^6/d+1/10*(a+a*sin(d*x+c))^10/a^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{(a\sin(c+dx)+a)^{10}}{10a^7d}-\frac{2(a\sin(c+dx)+a)^9}{3a^6d}+\frac{13(a\sin(c+dx)+a)^8}{8a^5d}-\frac{12(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (12*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) + (13*(a + a*Sin[c + d*x])^8)/(8*a^5*d) - (2*(a + a*Sin[c + d*x])^9)/(3*a
^6*d) + (a + a*Sin[c + d*x])^10/(10*a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2(a+x)^5}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(a+x)^5-12a^3(a+x)^6+13a^2(a+x)^7-\right.\right.}{a^7d}$$
$$= \frac{2(a+a\sin(c+dx))^6}{3a^3d} - \frac{12(a+a\sin(c+dx))^7}{7a^4d} + \frac{13(a}{}$$

Mathematica [A]
time = 0.66, size = 110, normalized size = 0.99

$$-\frac{a^3(-2835+34440\cos(2(c+dx))+5040\cos(4(c+dx))-4060\cos(6(c+dx))-1260\cos(8(c+dx))+84\cos(10(c+dx))-63840\sin(c+dx)+8960\sin(3(c+dx))+8064\sin(5(c+dx))+240\sin(7(c+dx))-560\sin(9(c+dx)))}{430080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/430080*(a^3*(-2835 + 34440*Cos[2*(c + d*x)] + 5040*Cos[4*(c + d*x)] - 40
60*Cos[6*(c + d*x)] - 1260*Cos[8*(c + d*x)] + 84*Cos[10*(c + d*x)] - 63840*
Sin[c + d*x] + 8960*Sin[3*(c + d*x)] + 8064*Sin[5*(c + d*x)] + 240*Sin[7*(c
 + d*x)] - 560*Sin[9*(c + d*x)]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(101) = 202$.
time = 0.35, size = 208, normalized size = 1.87

method	result
risch	$\frac{19a^3\sin(dx+c)}{128d} - \frac{a^3\cos(10dx+10c)}{5120d} + \frac{a^3\sin(9dx+9c)}{768d} + \frac{3a^3\cos(8dx+8c)}{1024d} - \frac{a^3\sin(7dx+7c)}{1792d} + \frac{29a^3\cos(6dx+6c}{3072d}$
derivativedivides	$\frac{a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{}$
default	$\frac{a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))+3*a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3
*a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/
3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^3*(-1/10*sin(d*x+c)^4*cos(d*
x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))
```

Maxima [A]
time = 0.36, size = 110, normalized size = 0.99

$$\frac{84\,a^3\sin\left(dx+c\right)^{10}+280\,a^3\sin\left(dx+c\right)^9+105\,a^3\sin\left(dx+c\right)^8-600\,a^3\sin\left(dx+c\right)^7-700\,a^3\sin\left(dx+c\right)^6+168\,a^3\sin\left(dx+c\right)^5+630\,a^3\sin\left(dx+c\right)^4+280\,a^3\sin\left(dx+c\right)^3}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/840*(84*a^3*sin(d*x + c)^10 + 280*a^3*sin(d*x + c)^9 + 105*a^3*sin(d*x +
c)^8 - 600*a^3*sin(d*x + c)^7 - 700*a^3*sin(d*x + c)^6 + 168*a^3*sin(d*x +
c)^5 + 630*a^3*sin(d*x + c)^4 + 280*a^3*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 111, normalized size = 1.00

$$-\frac{84\,a^3\cos\left(dx+c\right)^{10}-525\,a^3\cos\left(dx+c\right)^8+560\,a^3\cos\left(dx+c\right)^6-8\left(35\,a^3\cos\left(dx+c\right)^8-65\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4+8\,a^3\cos\left(dx+c\right)^2+16\,a^3\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/840*(84*a^3*cos(d*x + c)^10 - 525*a^3*cos(d*x + c)^8 + 560*a^3*cos(d*x +
 c)^6 - 8*(35*a^3*cos(d*x + c)^8 - 65*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x +
c)^4 + 8*a^3*cos(d*x + c)^2 + 16*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(99) = 198$.
time = 2.28, size = 255, normalized size = 2.30

$$\begin{cases} \frac{8a^3\sin^9(c+dx)}{105d} + \frac{12a^3\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{8a^3\sin^7(c+dx)}{105d} + \frac{3a^3\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{4a^3\sin^5(c+dx)\cos^2(c+dx)}{15d} - \frac{a^3\sin^4(c+dx)\cos^6(c+dx)}{6d} + \frac{a^3\sin^3(c+dx)\cos^4(c+dx)}{3d} - \frac{a^3\sin^2(c+dx)\cos^8(c+dx)}{12d} - \frac{a^3\sin^2(c+dx)\cos^6(c+dx)}{2d} - \frac{a^3\cos^{10}(c+dx)}{60d} - \frac{a^3\cos^8(c+dx)}{8d} & \text{for}\ d \neq 0 \\ x(a\sin(c)+a)^3\sin^2(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(105*d) + 12*a**3*sin(c + d*x)**7*cos(c +
 d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(105*d) + 3*a**3*sin(c + d*x)**5*c
os(c + d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) - a**3
*sin(c + d*x)**4*cos(c + d*x)**6/(6*d) + a**3*sin(c + d*x)**3*cos(c + d*x)*
*4/(3*d) - a**3*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - a**3*sin(c + d*x)*
```

```
*2*cos(c + d*x)**6/(2*d) - a**3*cos(c + d*x)**10/(60*d) - a**3*cos(c + d*x)
**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**2*cos(c)**5, True))
```

Giac [A]
time = 0.65, size = 168, normalized size = 1.51

$$-\frac{a^{3} \cos \left(10 d x+10 c\right)}{5120 d}+\frac{3 a^{3} \cos \left(8 d x+8 c\right)}{1024 d}+\frac{29 a^{3} \cos \left(6 d x+6 c\right)}{3072 d}-\frac{3 a^{3} \cos \left(4 d x+4 c\right)}{256 d}-\frac{41 a^{3} \cos \left(2 d x+2 c\right)}{512 d}+\frac{a^{3} \sin \left(9 d x+9 c\right)}{768 d}-\frac{a^{3} \sin \left(7 d x+7 c\right)}{1792 d}-\frac{3 a^{3} \sin \left(5 d x+5 c\right)}{160 d}-\frac{a^{3} \sin \left(3 d x+3 c\right)}{48 d}+\frac{19 a^{3} \sin \left(d x+c\right)}{128 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/5120*a^3*cos(10*d*x + 10*c)/d + 3/1024*a^3*cos(8*d*x + 8*c)/d + 29/3072*
a^3*cos(6*d*x + 6*c)/d - 3/256*a^3*cos(4*d*x + 4*c)/d - 41/512*a^3*cos(2*d*
x + 2*c)/d + 1/768*a^3*sin(9*d*x + 9*c)/d - 1/1792*a^3*sin(7*d*x + 7*c)/d -
 3/160*a^3*sin(5*d*x + 5*c)/d - 1/48*a^3*sin(3*d*x + 3*c)/d + 19/128*a^3*si
n(d*x + c)/d
```

Mupad [B]
time = 8.71, size = 109, normalized size = 0.98

$$\frac{\frac{a^3\,{\sin\left(c+d\,x\right)}^{10}}{10}+\frac{a^3\,{\sin\left(c+d\,x\right)}^9}{3}+\frac{a^3\,{\sin\left(c+d\,x\right)}^8}{8}-\frac{5\,a^3\,{\sin\left(c+d\,x\right)}^7}{7}-\frac{5\,a^3\,{\sin\left(c+d\,x\right)}^6}{6}+\frac{a^3\,{\sin\left(c+d\,x\right)}^5}{5}+\frac{3\,a^3\,{\sin\left(c+d\,x\right)}^4}{4}+\frac{a^3\,{\sin\left(c+d\,x\right)}^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^3)/3 + (3*a^3*sin(c + d*x)^4)/4 + (a^3*sin(c + d*x)^5)/5
 - (5*a^3*sin(c + d*x)^6)/6 - (5*a^3*sin(c + d*x)^7)/7 + (a^3*sin(c + d*x)^
8)/8 + (a^3*sin(c + d*x)^9)/3 + (a^3*sin(c + d*x)^10)/10)/d
```

3.522 $\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=89

$$-\frac{2(a+a\sin(c+dx))^6}{3a^3d}+\frac{8(a+a\sin(c+dx))^7}{7a^4d}-\frac{5(a+a\sin(c+dx))^8}{8a^5d}+\frac{(a+a\sin(c+dx))^9}{9a^6d}$$

```
[Out] -2/3*(a+a*sin(d*x+c))^6/a^3/d+8/7*(a+a*sin(d*x+c))^7/a^4/d-5/8*(a+a*sin(d*x
+c))^8/a^5/d+1/9*(a+a*sin(d*x+c))^9/a^6/d
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 78}

$$\frac{(a\sin(c+dx)+a)^9}{9a^6d}-\frac{5(a\sin(c+dx)+a)^8}{8a^5d}+\frac{8(a\sin(c+dx)+a)^7}{7a^4d}-\frac{2(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-2*(a + a*Sin[c + d*x])^6)/(3*a^3*d) + (8*(a + a*Sin[c + d*x])^7)/(7*a^4*d
) - (5*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (a + a*Sin[c + d*x])^9/(9*a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x(a+x)^5}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3(a+x)^5+8a^2(a+x)^6-5a(a+x)^7+(\right.\right.}{a^6d}$$
$$= -\frac{2(a+a\sin(c+dx))^6}{3a^3d}+\frac{8(a+a\sin(c+dx))^7}{7a^4d}-\frac{5(a+}{}$$

Mathematica [A]
time = 0.47, size = 100, normalized size = 1.12

$$\frac{a^3(4662-9576\cos(2(c+dx))-2772\cos(4(c+dx))+168\cos(6(c+dx))+189\cos(8(c+dx))+16632\sin(c+dx)-1344\sin(3(c+dx))-2016\sin(5(c+dx))-396\sin(7(c+dx))+28\sin(9(c+dx)))}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(4662 - 9576*Cos[2*(c + d*x)] - 2772*Cos[4*(c + d*x)] + 168*Cos[6*(c +
 d*x)] + 189*Cos[8*(c + d*x)] + 16632*Sin[c + d*x] - 1344*Sin[3*(c + d*x)]
- 2016*Sin[5*(c + d*x)] - 396*Sin[7*(c + d*x)] + 28*Sin[9*(c + d*x)]))/(645
12*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(81) = 162.
time = 0.31, size = 170, normalized size = 1.91

method	result
risch	$\frac{33a^3\sin(dx+c)}{128d}+\frac{a^3\sin(9dx+9c)}{2304d}+\frac{3a^3\cos(8dx+8c)}{1024d}-\frac{11a^3\sin(7dx+7c)}{1792d}+\frac{a^3\cos(6dx+6c)}{384d}-\frac{a^3\sin(5dx+5c)}{32d}$
derivativedivides	$\frac{-\frac{a^3\left(\cos^6(dx+c)\right)}{6}+3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6\right.}{8}\right.}{}$
default	$\frac{-\frac{a^3\left(\cos^6(dx+c)\right)}{6}+3a^3\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6\right.}{8}\right.}{}$
norman	$\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{16a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{72a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7d}+\frac{3872a^3\left(\tan^9\left(\frac{d}{2}\right.\right.}{63d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^3*cos(d*x+c)^6+3*a^3*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/8*sin(d*x+c)^2*cos(d*x+c
)^6-1/24*cos(d*x+c)^6)+a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*
cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 110, normalized size = 1.24

$$\frac{56\,a^3\sin\left(dx+c\right)^9+189\,a^3\sin\left(dx+c\right)^8+72\,a^3\sin\left(dx+c\right)^7-420\,a^3\sin\left(dx+c\right)^6-504\,a^3\sin\left(dx+c\right)^5+126\,a^3\sin\left(dx+c\right)^4+504\,a^3\sin\left(dx+c\right)^3+252\,a^3\sin\left(dx+c\right)^2}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/504*(56*a^3*sin(d*x + c)^9 + 189*a^3*sin(d*x + c)^8 + 72*a^3*sin(d*x + c)
^7 - 420*a^3*sin(d*x + c)^6 - 504*a^3*sin(d*x + c)^5 + 126*a^3*sin(d*x + c)
^4 + 504*a^3*sin(d*x + c)^3 + 252*a^3*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.37, size = 98, normalized size = 1.10

$$\frac{189\,a^3\cos\left(dx+c\right)^8-336\,a^3\cos\left(dx+c\right)^6+8\,\left(7\,a^3\cos\left(dx+c\right)^8-37\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4+8\,a^3\cos\left(dx+c\right)^2+16\,a^3\right)\sin\left(dx+c\right)}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/504*(189*a^3*cos(d*x + c)^8 - 336*a^3*cos(d*x + c)^6 + 8*(7*a^3*cos(d*x +
 c)^8 - 37*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 + 8*a^3*cos(d*x + c)^2
 + 16*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(78) = 156$.
time = 1.29, size = 202, normalized size = 2.27

$$\begin{cases} \frac{8a^3\sin^9(c+dx)}{315d}+\frac{4a^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{8a^3\sin^7(c+dx)}{35d}+\frac{a^3\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{4a^3\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{a^3\sin^3(c+dx)\cos^4(c+dx)}{d}-\frac{a^3\sin^2(c+dx)\cos^6(c+dx)}{2d}-\frac{a^3\cos^8(c+dx)}{8d}-\frac{a^3\cos^6(c+dx)}{6d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^3\sin(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**9/(315*d) + 4*a**3*sin(c + d*x)**7*cos(c +
d*x)**2/(35*d) + 8*a**3*sin(c + d*x)**7/(35*d) + a**3*sin(c + d*x)**5*cos(c
 + d*x)**4/(5*d) + 4*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a**3*sin(
c + d*x)**3*cos(c + d*x)**4/d - a**3*sin(c + d*x)**2*cos(c + d*x)**6/(2*d)
- a**3*cos(c + d*x)**8/(8*d) - a**3*cos(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a
*sin(c) + a)**3*sin(c)*cos(c)**5, True))
```

Giac [A]
time = 0.52, size = 151, normalized size = 1.70

$$\frac{3\,a^3\cos\left(8\,dx+8\,c\right)}{1024\,d}+\frac{a^3\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{11\,a^3\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{19\,a^3\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{a^3\sin\left(9\,dx+9\,c\right)}{2304\,d}-\frac{11\,a^3\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a^3\sin\left(5\,dx+5\,c\right)}{32\,d}-\frac{a^3\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{33\,a^3\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/1024*a^3*cos(8*d*x + 8*c)/d + 1/384*a^3*cos(6*d*x + 6*c)/d - 11/256*a^3*c
os(4*d*x + 4*c)/d - 19/128*a^3*cos(2*d*x + 2*c)/d + 1/2304*a^3*sin(9*d*x +
9*c)/d - 11/1792*a^3*sin(7*d*x + 7*c)/d - 1/32*a^3*sin(5*d*x + 5*c)/d - 1/4
8*a^3*sin(3*d*x + 3*c)/d + 33/128*a^3*sin(d*x + c)/d
```

Mupad [B]
time = 0.07, size = 108, normalized size = 1.21

$$\frac{\frac{a^3\sin(c+d\,x)^9}{9}+\frac{3\,a^3\sin(c+d\,x)^8}{8}+\frac{a^3\sin(c+d\,x)^7}{7}-\frac{5\,a^3\sin(c+d\,x)^6}{6}-a^3\sin\left(c+d\,x\right)^5+\frac{a^3\sin(c+d\,x)^4}{4}+a^3\sin\left(c+d\,x\right)^3+\frac{a^3\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2)/2 + a^3*sin(c + d*x)^3 + (a^3*sin(c + d*x)^4)/4 - a^3
*sin(c + d*x)^5 - (5*a^3*sin(c + d*x)^6)/6 + (a^3*sin(c + d*x)^7)/7 + (3*a^
3*sin(c + d*x)^8)/8 + (a^3*sin(c + d*x)^9)/9)/d
```

3.523 $\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=137

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^5(c+dx)}{5d}$$

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x+c)^2/d-5/3*a^3*sin(
d*x+c)^3/d-5/4*a^3*sin(d*x+c)^4/d+1/5*a^3*sin(d*x+c)^5/d+1/2*a^3*sin(d*x+c)
^6/d+1/7*a^3*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin^7(c+dx)}{7d}+\frac{a^3\sin^6(c+dx)}{2d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{5a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(
2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) - (5*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*S
in[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(2*d) + (a^3*Sin[c + d*x]^7)/(7
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^6 + \frac{a^7}{x} + a^5x - 5a^4x^2 - 5a^3x^3 + a^2x^4 + 3a\right.\right.}{a^4 d}$$
$$= \frac{a^3\log(\sin(c+dx))}{d} + \frac{3a^3\sin(c+dx)}{d} + \frac{a^3\sin^2(c+dx)}{2d}$$

Mathematica [A]

time = 0.08, size = 88, normalized size = 0.64

$$\frac{a^3(420\log(\sin(c+dx)) + 1260\sin(c+dx) + 210\sin^2(c+dx) - 700\sin^3(c+dx) - 525\sin^4(c+dx) + 84\sin^5(c+dx) + 210\sin^6(c+dx) + 60\sin^7(c+dx))}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(420*Log[Sin[c + d*x]] + 1260*Sin[c + d*x] + 210*Sin[c + d*x]^2 - 700*
Sin[c + d*x]^3 - 525*Sin[c + d*x]^4 + 84*Sin[c + d*x]^5 + 210*Sin[c + d*x]^
6 + 60*Sin[c + d*x]^7))/(420*d)
```

Maple [A]

time = 0.22, size = 131, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\left(\frac{(\cos^4(dx+c))}{4} + \frac{(\cos^2(dx+c))}{2} + \ln(\sin(dx+c))\right) + \frac{3a^3\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} - \frac{a^3(\cos^6(dx+c))}{2} + a^3}{d}$
default	$\frac{a^3\left(\frac{(\cos^4(dx+c))}{4} + \frac{(\cos^2(dx+c))}{2} + \ln(\sin(dx+c))\right) + \frac{3a^3\left(\frac{8}{3} + \cos^4(dx+c) + \frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5} - \frac{a^3(\cos^6(dx+c))}{2} + a^3}{d}$
risch	$\frac{9a^3 e^{2i(dx+c)}}{128d} - ia^3x + \frac{9a^3 e^{-2i(dx+c)}}{128d} - \frac{2ia^3c}{d} + \frac{a^3\ln\left(e^{2i(dx+c)}-1\right)}{d} + \frac{125a^3\sin(dx+c)}{64d} - \frac{a^3\sin(7dx+7c)}{448d} -$
norman	$\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{2a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{68a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} + \frac{646a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d} + \frac{2488a^3\left(\tan^7\left(\frac{dx}{2}+\right.\right.}{35d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+3/5*a^3*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/2*a^3*cos(d*x+c)^6+a^3*(-1/7*sin(
d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.78

$$\frac{60\,a^3\sin\left(dx+c\right)^7+210\,a^3\sin\left(dx+c\right)^6+84\,a^3\sin\left(dx+c\right)^5-525\,a^3\sin\left(dx+c\right)^4-700\,a^3\sin\left(dx+c\right)^3+210\,a^3\sin\left(dx+c\right)^2+420\,a^3\log\left(\sin\left(dx+c\right)\right)+1260\,a^3\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(sin(d*x + c)) + 1260*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 112, normalized size = 0.82

$$-\frac{210\,a^3\cos\left(dx+c\right)^6-105\,a^3\cos\left(dx+c\right)^4-210\,a^3\cos\left(dx+c\right)^2-420\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(15\,a^3\cos\left(dx+c\right)^6-66\,a^3\cos\left(dx+c\right)^4-88\,a^3\cos\left(dx+c\right)^2-176\,a^3\right)\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/420*(210*a^3*cos(d*x + c)^6 - 105*a^3*cos(d*x + c)^4 - 210*a^3*cos(d*x +
 c)^2 - 420*a^3*log(1/2*sin(d*x + c)) + 4*(15*a^3*cos(d*x + c)^6 - 66*a^3*c
os(d*x + c)^4 - 88*a^3*cos(d*x + c)^2 - 176*a^3)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.50, size = 108, normalized size = 0.79

$$\frac{60\,a^3\sin\left(dx+c\right)^7+210\,a^3\sin\left(dx+c\right)^6+84\,a^3\sin\left(dx+c\right)^5-525\,a^3\sin\left(dx+c\right)^4-700\,a^3\sin\left(dx+c\right)^3+210\,a^3\sin\left(dx+c\right)^2+420\,a^3\log\left(|\sin\left(dx+c\right)|\right)+1260\,a^3\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/420*(60*a^3*sin(d*x + c)^7 + 210*a^3*sin(d*x + c)^6 + 84*a^3*sin(d*x + c)
^5 - 525*a^3*sin(d*x + c)^4 - 700*a^3*sin(d*x + c)^3 + 210*a^3*sin(d*x + c)
^2 + 420*a^3*log(abs(sin(d*x + c))) + 1260*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 8.99, size = 178, normalized size = 1.30

$$\frac{176\,a^3\,\sin\left(c+d\,x\right)}{105\,d}-\frac{a^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\,{\cos\left(c+d\,x\right)}^2}{2\,d}+\frac{a^3\,{\cos\left(c+d\,x\right)}^4}{4\,d}-\frac{a^3\,{\cos\left(c+d\,x\right)}^6}{2\,d}+\frac{88\,a^3\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{22\,a^3\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{a^3\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (176*a^3*sin(c + d*x))/(105*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3
*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos(c + d*x)^2)/(2*d)
 + (a^3*cos(c + d*x)^4)/(4*d) - (a^3*cos(c + d*x)^6)/(2*d) + (88*a^3*cos(c
+ d*x)^2*sin(c + d*x))/(105*d) + (22*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d
) - (a^3*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

3.524 $\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^4(c+dx)}{4d}$$

```
[Out] -a^3*csc(d*x+c)/d+3*a^3*ln(sin(d*x+c))/d+a^3*sin(d*x+c)/d-5/2*a^3*sin(d*x+c
)^2/d-5/3*a^3*sin(d*x+c)^3/d+1/4*a^3*sin(d*x+c)^4/d+3/5*a^3*sin(d*x+c)^5/d+
1/6*a^3*sin(d*x+c)^6/d
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin^6(c+dx)}{6d}+\frac{3a^3\sin^5(c+dx)}{5d}+\frac{a^3\sin^4(c+dx)}{4d}-\frac{5a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])/
d - (5*a^3*Sin[c + d*x]^2)/(2*d) - (5*a^3*Sin[c + d*x]^3)/(3*d) + (a^3*Sin[
c + d*x]^4)/(4*d) + (3*a^3*Sin[c + d*x]^5)/(5*d) + (a^3*Sin[c + d*x]^6)/(6*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5 + \frac{a^7}{x^2} + \frac{3a^6}{x} - 5a^4x - 5a^3x^2 + a^2x^3 + 3ax^{\cdot}\right.\right.}{a^3 d}$$
$$= -\frac{a^3\csc(c+dx)}{d} + \frac{3a^3\log(\sin(c+dx))}{d} + \frac{a^3\sin(c+dx}{d}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.65

$$-\frac{a^3(60\csc(c+dx) - 180\log(\sin(c+dx)) - 60\sin(c+dx) + 150\sin^2(c+dx) + 100\sin^3(c+dx) - 15\sin^4(c+dx) - 36\sin^5(c+dx) - 10\sin^6(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(a^3*(60*Csc[c + d*x] - 180*Log[Sin[c + d*x]] - 60*Sin[c + d*x] + 150
*Sin[c + d*x]^2 + 100*Sin[c + d*x]^3 - 15*Sin[c + d*x]^4 - 36*Sin[c + d*x]^
5 - 10*Sin[c + d*x]^6))/d
```

Maple [A]
time = 0.20, size = 134, normalized size = 1.01

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)} - \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right) + 3a^3\left(\frac{\left(\cos^4(dx+c)\right)}{4} + \frac{\left(\cos^2(dx+c)\right)}{2} + \ln(\sin(dx+c))\right) +}{d}$
default	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)} - \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right) + 3a^3\left(\frac{\left(\cos^4(dx+c)\right)}{4} + \frac{\left(\cos^2(dx+c)\right)}{2} + \ln(\sin(dx+c))\right) +}{d}$
risch	$-3ia^3x + \frac{67a^3 e^{2i(dx+c)}}{128d} - \frac{6ia^3c}{d} + \frac{67a^3 e^{-2i(dx+c)}}{128d} - \frac{2ia^3 e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)} - \frac{ia^3 e^{i(dx+c)}}{16d} + \frac{ia^3 e^{-i(dx+c)}}{16d} + \frac{3a^3}{}$
norman	$\frac{-\frac{a^3}{2d} - \frac{3a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} - \frac{83a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d} - \frac{183a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10d} - \frac{183a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10d} - \frac{83a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d} - \frac{3a^3(}{}}{\left(1+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*si
n(d*x+c))+3*a^3*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+3/5*a^3*
(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*a^3*cos(d*x+c)^6)
```

Maxima [A]
time = 0.31, size = 107, normalized size = 0.80

$$\frac{10\,a^3\sin\left(dx+c\right)^6+36\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4-100\,a^3\sin\left(dx+c\right)^3-150\,a^3\sin\left(dx+c\right)^2+180\,a^3\log\left(\sin\left(dx+c\right)\right)+60\,a^3\sin\left(dx+c\right)-\frac{60\,a^3}{\sin(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(sin(d*x +
c)) + 60*a^3*sin(d*x + c) - 60*a^3/sin(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 131, normalized size = 0.98

$$-\frac{144\,a^3\cos\left(dx+c\right)^6-32\,a^3\cos\left(dx+c\right)^4-128\,a^3\cos\left(dx+c\right)^2-720\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+256\,a^3+5\,\left(8\,a^3\cos\left(dx+c\right)^6-36\,a^3\cos\left(dx+c\right)^4-72\,a^3\cos\left(dx+c\right)^2+47\,a^3\right)\sin\left(dx+c\right)}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*a^3*cos(d*x + c)^6 - 32*a^3*cos(d*x + c)^4 - 128*a^3*cos(d*x +
c)^2 - 720*a^3*log(1/2*sin(d*x + c))*sin(d*x + c) + 256*a^3 + 5*(8*a^3*cos(
d*x + c)^6 - 36*a^3*cos(d*x + c)^4 - 72*a^3*cos(d*x + c)^2 + 47*a^3)*sin(d*
x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.62, size = 120, normalized size = 0.90

$$\frac{10\,a^3\sin\left(dx+c\right)^6+36\,a^3\sin\left(dx+c\right)^5+15\,a^3\sin\left(dx+c\right)^4-100\,a^3\sin\left(dx+c\right)^3-150\,a^3\sin\left(dx+c\right)^2+180\,a^3\log\left(|\sin\left(dx+c\right)|\right)+60\,a^3\sin\left(dx+c\right)-\frac{60\,(3\,a^3\sin(dx+c)+a^3)}{\sin(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*a^3*sin(d*x + c)^6 + 36*a^3*sin(d*x + c)^5 + 15*a^3*sin(d*x + c)^4
 - 100*a^3*sin(d*x + c)^3 - 150*a^3*sin(d*x + c)^2 + 180*a^3*log(abs(sin(d*
x + c))) + 60*a^3*sin(d*x + c) - 60*(3*a^3*sin(d*x + c) + a^3)/sin(d*x + c)
)/d
```

Mupad [B]
time = 9.18, size = 371, normalized size = 2.79

$$\frac{14\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{10\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{8\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3\,d}-\frac{28\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}+\frac{32\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{d}-\frac{32\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3\,d}-\frac{3\,a^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{3\,a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{46\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{688\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{1064\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{288\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{96\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{3\,a^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{a^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (14*a^3*cos(c/2 + (d*x)/2)^4)/d - (10*a^3*cos(c/2 + (d*x)/2)^2)/d + (8*a^3*
cos(c/2 + (d*x)/2)^6)/(3*d) - (28*a^3*cos(c/2 + (d*x)/2)^8)/d + (32*a^3*cos
(c/2 + (d*x)/2)^10)/d - (32*a^3*cos(c/2 + (d*x)/2)^12)/(3*d) - (3*a^3*log(1
/cos(c/2 + (d*x)/2)^2))/d + (3*a^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (46*a^3*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (688*a^3*c
os(c/2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (1064*a^3*cos(c/2 + (d*x)/
2)^7)/(15*d*sin(c/2 + (d*x)/2)) + (288*a^3*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c
/2 + (d*x)/2)) - (96*a^3*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) +
(3*a^3*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^3*sin(c/2 + (d*x)/
2))/(2*d*cos(c/2 + (d*x)/2))
```

3.525 $\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{a^3\log(\sin(c+dx))}{d}-\frac{5a^3\sin(c+dx)}{d}-\frac{5a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx}{3d}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+a^3*ln(sin(d*x+c))/d-5*a^3*sin(d
*x+c)/d-5/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(d*x+c)^3/d+3/4*a^3*sin(d*x+c)^4/
d+1/5*a^3*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin^5(c+dx)}{5d}+\frac{3a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{3d}-\frac{5a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (a^3*Log[Sin[c + d*x
]])/d - (5*a^3*Sin[c + d*x])/d - (5*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c
+ d*x]^3)/(3*d) + (3*a^3*Sin[c + d*x]^4)/(4*d) + (a^3*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4+\frac{a^7}{x^3}+\frac{3a^6}{x^2}+\frac{a^5}{x}-5a^3x+a^2x^2+3ax^3\right.\right.}{a^2 d}$$
$$= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{a^3\log(\sin(c+d}{d}$$

Mathematica [A]

time = 0.11, size = 86, normalized size = 0.65

$$-\frac{a^3(180\csc(c+dx)+30\csc^2(c+dx)-60\log(\sin(c+dx))+300\sin(c+dx)+150\sin^2(c+dx)-20\sin^3(c+dx)-45\sin^4(c+dx)-12\sin^5(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(a^3*(180*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 60*Log[Sin[c + d*x]] + 3
00*Sin[c + d*x] + 150*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 - 45*Sin[c + d*x]^
4 - 12*Sin[c + d*x]^5))/d
```

Maple [A]

time = 0.20, size = 174, normalized size = 1.31

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$-ia^3x+\frac{7ia^3e^{3i(dx+c)}}{96d}+\frac{7a^3e^{2i(dx+c)}}{16d}+\frac{37ia^3e^{i(dx+c)}}{16d}-\frac{37ia^3e^{-i(dx+c)}}{16d}+\frac{7a^3e^{-2i(dx+c)}}{16d}-\frac{7ia^3e^{-3i(dx+c)}}{96d}$
norman	$\frac{-\frac{a^3}{8d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{19a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{359a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{1174a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{359a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{19a^3\left(\right.}{}}{\left(1+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln
(sin(d*x+c)))+3*a^3*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c))+3*a^3*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)
))+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 106, normalized size = 0.80

$$\frac{12\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3-150\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\sin\left(dx+c\right)\right)-300\,a^3\sin\left(dx+c\right)-\frac{30\,\left(6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) - 300*a^3*sin(d*x + c)
 - 30*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.40, size = 145, normalized size = 1.09

$$\frac{360\,a^3\cos\left(dx+c\right)^6+120\,a^3\cos\left(dx+c\right)^4-855\,a^3\cos\left(dx+c\right)^2+615\,a^3+480\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+32\,\left(3\,a^3\cos\left(dx+c\right)^6-14\,a^3\cos\left(dx+c\right)^4-56\,a^3\cos\left(dx+c\right)^2+112\,a^3\right)\sin\left(dx+c\right)}{480\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(360*a^3*cos(d*x + c)^6 + 120*a^3*cos(d*x + c)^4 - 855*a^3*cos(d*x +
c)^2 + 615*a^3 + 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 32*
(3*a^3*cos(d*x + c)^6 - 14*a^3*cos(d*x + c)^4 - 56*a^3*cos(d*x + c)^2 + 112
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.54, size = 120, normalized size = 0.90

$$\frac{12\,a^3\sin\left(dx+c\right)^5+45\,a^3\sin\left(dx+c\right)^4+20\,a^3\sin\left(dx+c\right)^3-150\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)-300\,a^3\sin\left(dx+c\right)-\frac{30\,\left(3\,a^3\sin(dx+c)^2+6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(12*a^3*sin(d*x + c)^5 + 45*a^3*sin(d*x + c)^4 + 20*a^3*sin(d*x + c)^3
 - 150*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) - 300*a^3*sin(d*x
 + c) - 30*(3*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2
)/d
```

Mupad [B]
time = 9.01, size = 342, normalized size = 2.57

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{46\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\frac{81\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{538\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}+\frac{149\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+\frac{3796\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15}+77\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{628\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+45\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+70\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a^3
*tan(c/2 + (d*x)/2))/(2*d) - ((5*a^3*tan(c/2 + (d*x)/2)^2)/2 + 70*a^3*tan(c
/2 + (d*x)/2)^3 + 45*a^3*tan(c/2 + (d*x)/2)^4 + (628*a^3*tan(c/2 + (d*x)/2)
^5)/3 + 77*a^3*tan(c/2 + (d*x)/2)^6 + (3796*a^3*tan(c/2 + (d*x)/2)^7)/15 +
(149*a^3*tan(c/2 + (d*x)/2)^8)/2 + (538*a^3*tan(c/2 + (d*x)/2)^9)/3 + (81*a
^3*tan(c/2 + (d*x)/2)^10)/2 + 46*a^3*tan(c/2 + (d*x)/2)^11 + a^3/2 + 6*a^3*
tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 +
40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^1
0 + 4*tan(c/2 + (d*x)/2)^12)) - (a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.526 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{5a^3\log(\sin(c+dx))}{d}-\frac{5a^3\sin(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}$$

```
[Out] -a^3*csc(d*x+c)/d-3/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d-5*a^3*ln(si
n(d*x+c))/d-5*a^3*sin(d*x+c)/d+1/2*a^3*sin(d*x+c)^2/d+a^3*sin(d*x+c)^3/d+1/
4*a^3*sin(d*x+c)^4/d
```

Rubi [A]

time = 0.08, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin^4(c+dx)}{4d}+\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin^2(c+dx)}{2d}-\frac{5a^3\sin(c+dx)}{d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c+dx)}{d}-\frac{5a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^
3)/(3*d) - (5*a^3*Log[Sin[c + d*x]])/d - (5*a^3*Sin[c + d*x])/d + (a^3*Sin[
c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^5}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^3+\frac{a^7}{x^4}+\frac{3a^6}{x^3}+\frac{a^5}{x^2}-\frac{5a^4}{x}+a^2x+3ax^2+\right.\right.}{ad}$$
$$= -\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-$$

Mathematica [A]
time = 0.18, size = 86, normalized size = 0.66

$$-\frac{a^3(12\csc(c+dx)+18\csc^2(c+dx)+4\csc^3(c+dx)+60\log(\sin(c+dx))+60\sin(c+dx)-6\sin^2(c+dx)-12\sin^3(c+dx)-3\sin^4(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/12*(a^3*(12*Csc[c + d*x] + 18*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 + 60*Log
[Sin[c + d*x]] + 60*Sin[c + d*x] - 6*Sin[c + d*x]^2 - 12*Sin[c + d*x]^3 - 3
*Sin[c + d*x]^4))/d
```

Maple [A]
time = 0.21, size = 210, normalized size = 1.60

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+3a^3\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\mathrm{co}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+3a^3\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\mathrm{co}\right.}{}$
risch	$5ia^3x+\frac{a^3\mathrm{e}^{4i(dx+c)}}{64d}+\frac{ia^3\mathrm{e}^{3i(dx+c)}}{8d}-\frac{3a^3\mathrm{e}^{2i(dx+c)}}{16d}+\frac{17ia^3\mathrm{e}^{i(dx+c)}}{8d}-\frac{17ia^3\mathrm{e}^{-i(dx+c)}}{8d}-\frac{3a^3\mathrm{e}^{-2i(dx+c)}}{16d}-$
norman	$\frac{-\frac{a^3}{24d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{19a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{107a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{683a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{683a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{107a^3(}{}}{\left(1+\mathrm{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)
```

```
^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+3*a^3*(-1/sin(d*x+c)*cos
(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^3*(1/4*cos(d*x+
c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 0.82

$$\frac{3\,a^3 \sin\left(dx+c\right)^4 + 12\,a^3 \sin\left(dx+c\right)^3 + 6\,a^3 \sin\left(dx+c\right)^2 - 60\,a^3 \log\left(\sin\left(dx+c\right)\right) - 60\,a^3 \sin\left(dx+c\right) - \frac{2\left(6\,a^3 \sin(dx+c)^2 + 9\,a^3 \sin(dx+c) + 2\,a^3\right)}{\sin(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(sin(d*x + c)) - 60*a^3*sin(d*x + c) - 2*(6*a^3*sin(d*x + c)^2 +
 9*a^3*sin(d*x + c) + 2*a^3)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 159, normalized size = 1.21

$$\frac{96\,a^3 \cos\left(dx+c\right)^6 + 192\,a^3 \cos\left(dx+c\right)^4 - 768\,a^3 \cos\left(dx+c\right)^2 + 512\,a^3 - 480\left(a^3 \cos\left(dx+c\right)^2 - a^3\right) \log\left(\frac{1}{2} \sin\left(dx+c\right)\right) \sin\left(dx+c\right) + 3\left(8\,a^3 \cos\left(dx+c\right)^6 - 40\,a^3 \cos\left(dx+c\right)^4 + 45\,a^3 \cos\left(dx+c\right)^2 + 35\,a^3\right) \sin\left(dx+c\right)}{96\left(d \cos\left(dx+c\right)^2 - d\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(96*a^3*cos(d*x + c)^6 + 192*a^3*cos(d*x + c)^4 - 768*a^3*cos(d*x + c)
^2 + 512*a^3 - 480*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 3*(8*a^3*cos(d*x + c)^6 - 40*a^3*cos(d*x + c)^4 + 45*a^3*cos(d*x +
c)^2 + 35*a^3)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.50, size = 122, normalized size = 0.93

$$\frac{3\,a^3 \sin\left(dx+c\right)^4 + 12\,a^3 \sin\left(dx+c\right)^3 + 6\,a^3 \sin\left(dx+c\right)^2 - 60\,a^3 \log\left(\left|\sin\left(dx+c\right)\right|\right) - 60\,a^3 \sin\left(dx+c\right) + \frac{2\left(55\,a^3 \sin(dx+c)^3 - 6\,a^3 \sin(dx+c)^2 - 9\,a^3 \sin(dx+c) - 2\,a^3\right)}{\sin(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^3*sin(d*x + c)^4 + 12*a^3*sin(d*x + c)^3 + 6*a^3*sin(d*x + c)^2 -
 60*a^3*log(abs(sin(d*x + c))) - 60*a^3*sin(d*x + c) + 2*(55*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 9*a^3*sin(d*x + c) - 2*a^3)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 8.98, size = 333, normalized size = 2.54

$$\frac{5\,a^3\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}-\frac{5\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}-\frac{85\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-13\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\frac{589\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{3}-52\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\frac{622\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{3}+2\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+102\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+12\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\frac{19\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}+3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{3}}{d\,\left(8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+48\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (5*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(24*d)
 - (5*a^3*log(tan(c/2 + (d*x)/2)))/d - (5*a^3*tan(c/2 + (d*x)/2))/(8*d) - (
3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - ((19*a^3*tan(c/2 + (d*x)/2)^2)/3 + 12*a
^3*tan(c/2 + (d*x)/2)^3 + 102*a^3*tan(c/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d
*x)/2)^5 + (622*a^3*tan(c/2 + (d*x)/2)^6)/3 - 52*a^3*tan(c/2 + (d*x)/2)^7 +
 (589*a^3*tan(c/2 + (d*x)/2)^8)/3 - 13*a^3*tan(c/2 + (d*x)/2)^9 + 85*a^3*ta
n(c/2 + (d*x)/2)^10 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*
x)/2)^3 + 32*tan(c/2 + (d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 +
(d*x)/2)^9 + 8*tan(c/2 + (d*x)/2)^11))
```

3.527 $\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=131

$$\frac{5a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{5a^3\log(\sin(c+dx))}{d}+\frac{a^3\sin(c+dx)}{d}+$$

```
[Out] 5*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-a^3*csc(d*x+c)^3/d-1/4*a^3*csc(d*
x+c)^4/d-5*a^3*ln(sin(d*x+c))/d+a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3
*a^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c+dx)}{d}-\frac{5a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3)/
d - (a^3*Csc[c + d*x]^4)/(4*d) - (5*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c +
 d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2+\frac{a^7}{x^5}+\frac{3a^6}{x^4}+\frac{a^5}{x^3}-\frac{5a^4}{x^2}-\frac{5a^3}{x}+3ax+x^2\right)dx, x, a\sin(c\right.}{d}$$
$$= \frac{5a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+d}{4d}$$

Mathematica [A]
time = 0.32, size = 86, normalized size = 0.66

$$\frac{a^3(60\csc(c+dx)-6\csc^2(c+dx)-12\csc^3(c+dx)-3\csc^4(c+dx)-60\log(\sin(c+dx))+12\sin(c+dx)+18\sin^2(c+dx)+4\sin^3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(60*Csc[c + d*x] - 6*Csc[c + d*x]^2 - 12*Csc[c + d*x]^3 - 3*Csc[c + d*
x]^4 - 60*Log[Sin[c + d*x]] + 12*Sin[c + d*x] + 18*Sin[c + d*x]^2 + 4*Sin[c
 + d*x]^3))/(12*d)
```

Maple [A]
time = 0.22, size = 210, normalized size = 1.60

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{}$
risch	$5ia^3x+\frac{ia^3e^{3i(dx+c)}}{24d}-\frac{3a^3e^{2i(dx+c)}}{8d}-\frac{5ia^3e^{i(dx+c)}}{8d}+\frac{5ia^3e^{-i(dx+c)}}{8d}-\frac{3a^3e^{-2i(dx+c)}}{8d}-\frac{ia^3e^{-3i(dx+c)}}{24d}+$
norman	$\frac{-\frac{a^3}{64d}-\frac{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{15a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{7a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{81a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{115a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{81a^3\left(\tan^9\right.}{8d}}{\tan\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*a^3*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)
```

```
^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+a^3*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.82

$$\frac{4\,a^3\sin\left(dx+c\right)^3+18\,a^3\sin\left(dx+c\right)^2-60\,a^3\log\left(\sin\left(dx+c\right)\right)+12\,a^3\sin\left(dx+c\right)+\frac{3\left(20\,a^3\sin(dx+c)^3-2\,a^3\sin(dx+c)^2-4\,a^3\sin(dx+c)-a^3\right)}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(sin(d*x + c
)) + 12*a^3*sin(d*x + c) + 3*(20*a^3*sin(d*x + c)^3 - 2*a^3*sin(d*x + c)^2
- 4*a^3*sin(d*x + c) - a^3)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.40, size = 159, normalized size = 1.21

$$-\frac{18\,a^3\cos\left(dx+c\right)^6-45\,a^3\cos\left(dx+c\right)^4+30\,a^3\cos\left(dx+c\right)^2+60\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+4\left(a^3\cos\left(dx+c\right)^6-6\,a^3\cos\left(dx+c\right)^4+24\,a^3\cos\left(dx+c\right)^2-16\,a^3\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
[Out] -1/12*(18*a^3*cos(d*x + c)^6 - 45*a^3*cos(d*x + c)^4 + 30*a^3*cos(d*x + c)^
2 + 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(1/2*sin(d*x +
c)) + 4*(a^3*cos(d*x + c)^6 - 6*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^2
- 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
[Out] Timed out
```

Giac [A]
time = 0.55, size = 121, normalized size = 0.92

$$\frac{4\,a^3\sin\left(dx+c\right)^3+18\,a^3\sin\left(dx+c\right)^2-60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a^3\sin\left(dx+c\right)+\frac{125\,a^3\sin(dx+c)^4+60\,a^3\sin(dx+c)^3-6\,a^3\sin(dx+c)^2-12\,a^3\sin(dx+c)-3\,a^3}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*a^3*sin(d*x + c)^3 + 18*a^3*sin(d*x + c)^2 - 60*a^3*log(abs(sin(d*x
 + c))) + 12*a^3*sin(d*x + c) + (125*a^3*sin(d*x + c)^4 + 60*a^3*sin(d*x +
c)^3 - 6*a^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) - 3*a^3)/sin(d*x + c)^4)/
d
```

Mupad [B]
time = 8.94, size = 322, normalized size = 2.46

$$\frac{66\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+93\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{620\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{347\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{39\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+28\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{15\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a^3}{4}}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{5\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (28*a^3*tan(c/2 + (d*x)/2)^3 - (15*a^3*tan(c/2 + (d*x)/2)^2)/4 - (39*a^3*ta
n(c/2 + (d*x)/2)^4)/4 + 128*a^3*tan(c/2 + (d*x)/2)^5 + (347*a^3*tan(c/2 + (
d*x)/2)^6)/4 + (620*a^3*tan(c/2 + (d*x)/2)^7)/3 + 93*a^3*tan(c/2 + (d*x)/2)
^8 + 66*a^3*tan(c/2 + (d*x)/2)^9 - a^3/4 - 2*a^3*tan(c/2 + (d*x)/2))/(d*(16
*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 +
 16*tan(c/2 + (d*x)/2)^10)) - (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) - (a^3*tan(c
/2 + (d*x)/2)^4)/(64*d) - (3*a^3*tan(c/2 + (d*x)/2)^2)/(16*d) - (5*a^3*log(
tan(c/2 + (d*x)/2)))/d + (17*a^3*tan(c/2 + (d*x)/2))/(8*d) + (5*a^3*log(tan
(c/2 + (d*x)/2)^2 + 1))/d
```

3.528 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$\frac{5a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^5(c+dx)}{5d}+\frac{a^3\log(\sin(c+dx))}{d}$$

```
[Out] 5*a^3*csc(d*x+c)/d+5/2*a^3*csc(d*x+c)^2/d-1/3*a^3*csc(d*x+c)^3/d-3/4*a^3*cs
c(d*x+c)^4/d-1/5*a^3*csc(d*x+c)^5/d+a^3*ln(sin(d*x+c))/d+3*a^3*sin(d*x+c)/d
+1/2*a^3*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin^2(c+dx)}{2d}+\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\csc^5(c+dx)}{5d}-\frac{3a^3\csc^4(c+dx)}{4d}-\frac{a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*Csc[c + d*x])/d + (5*a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Csc[c + d*x]^3
)/(3*d) - (3*a^3*Csc[c + d*x]^4)/(4*d) - (a^3*Csc[c + d*x]^5)/(5*d) + (a^3*
Log[Sin[c + d*x]])/d + (3*a^3*Sin[c + d*x])/d + (a^3*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(3a+\frac{a^7}{x^6}+\frac{3a^6}{x^5}+\frac{a^5}{x^4}-\frac{5a^4}{x^3}-\frac{5a^3}{x^2}+\frac{a^2}{x}+x\right) d\right.}{d}$$
$$= \frac{5a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc^3(c+dx)}{3d}-$$

Mathematica [A]

time = 0.12, size = 86, normalized size = 0.65

$$\frac{a^3(300\csc(c+dx)+150\csc^2(c+dx)-20\csc^3(c+dx)-45\csc^4(c+dx)-12\csc^5(c+dx)+60\log(\sin(c+dx))+180\sin(c+dx)+30\sin^2(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(300*Csc[c + d*x] + 150*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 45*Csc[c
+ d*x]^4 - 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]] + 180*Sin[c + d*x] + 30
*Sin[c + d*x]^2))/(60*d)
```

Maple [A]

time = 0.26, size = 246, normalized size = 1.85

method	result
risch	$-ia^3x-\frac{a^3 e^{2i(dx+c)}}{8d}-\frac{3ia^3 e^{i(dx+c)}}{2d}+\frac{3ia^3 e^{-i(dx+c)}}{2d}-\frac{a^3 e^{-2i(dx+c)}}{8d}-\frac{2ia^3c}{d}+\frac{2ia^3\left(75 e^{9i(dx+c)}-280 e^{7i(}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+3a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+3a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{}$
norman	$\frac{-\frac{a^3}{160d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-\frac{11a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120d}+\frac{19a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{83a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{601a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{1235a^3\left(\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5
/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)+3*a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*a^3*(-1/3/sin
(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(
d*x+c)^2)*sin(d*x+c))+a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-
cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.80

$$\frac{30\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\sin\left(dx+c\right)\right)+180\,a^3\sin\left(dx+c\right)+\frac{300\,a^3\sin(dx+c)^4+150\,a^3\sin(dx+c)^3-20\,a^3\sin(dx+c)^2-45\,a^3\sin(dx+c)-12\,a^3}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)) + 180*a^3*sin(d*x +
c) + (300*a^3*sin(d*x + c)^4 + 150*a^3*sin(d*x + c)^3 - 20*a^3*sin(d*x + c)
^2 - 45*a^3*sin(d*x + c) - 12*a^3)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.40, size = 179, normalized size = 1.35

$$-\frac{180\,a^3\cos\left(dx+c\right)^6-840\,a^3\cos\left(dx+c\right)^4+1120\,a^3\cos\left(dx+c\right)^2-448\,a^3-60\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+15\left(2\,a^3\cos\left(dx+c\right)^6-5\,a^3\cos\left(dx+c\right)^4+14\,a^3\cos\left(dx+c\right)^2-8\,a^3\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(180*a^3*cos(d*x + c)^6 - 840*a^3*cos(d*x + c)^4 + 1120*a^3*cos(d*x +
 c)^2 - 448*a^3 - 60*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(
1/2*sin(d*x + c))*sin(d*x + c) + 15*(2*a^3*cos(d*x + c)^6 - 5*a^3*cos(d*x +
 c)^4 + 14*a^3*cos(d*x + c)^2 - 8*a^3)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 122, normalized size = 0.92

$$\frac{30\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+180\,a^3\sin\left(dx+c\right)-\frac{137\,a^3\sin(dx+c)^5-300\,a^3\sin(dx+c)^4-150\,a^3\sin(dx+c)^3+20\,a^3\sin(dx+c)^2+45\,a^3\sin(dx+c)+12\,a^3}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(30*a^3*sin(d*x + c)^2 + 60*a^3*log(abs(sin(d*x + c))) + 180*a^3*sin(d
*x + c) - (137*a^3*sin(d*x + c)^5 - 300*a^3*sin(d*x + c)^4 - 150*a^3*sin(d*
x + c)^3 + 20*a^3*sin(d*x + c)^2 + 45*a^3*sin(d*x + c) + 12*a^3)/sin(d*x +
c)^5)/d
```

Mupad [B]
time = 8.95, size = 311, normalized size = 2.34

$$\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,d}-\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{266\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+78\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{1013\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{53\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2}+\frac{1037\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}+11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{41\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}-\frac{a^3}{5}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+64\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}+\frac{37\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}-\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (7*a^3*tan(c/2 + (d*x)/2)^2)/(16*d) - (7*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) -
 (3*a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) +
 (a^3*log(tan(c/2 + (d*x)/2)))/d + (11*a^3*tan(c/2 + (d*x)/2)^3 - (41*a^3*t
an(c/2 + (d*x)/2)^2)/15 + (1037*a^3*tan(c/2 + (d*x)/2)^4)/15 + (53*a^3*tan(
c/2 + (d*x)/2)^5)/2 + (1013*a^3*tan(c/2 + (d*x)/2)^6)/3 + 78*a^3*tan(c/2 +
(d*x)/2)^7 + 266*a^3*tan(c/2 + (d*x)/2)^8 - a^3/5 - (3*a^3*tan(c/2 + (d*x)/
2))/2)/(d*(32*tan(c/2 + (d*x)/2)^5 + 64*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 +
 (d*x)/2)^9)) + (37*a^3*tan(c/2 + (d*x)/2))/(16*d) - (a^3*log(tan(c/2 + (d*
x)/2)^2 + 1))/d
```

3.529 $\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc^3(c+dx)}{3d}-\frac{a^3\csc^4(c+dx)}{4d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^6(c+dx)}{6d}+$$

```
[Out] -a^3*csc(d*x+c)/d+5/2*a^3*csc(d*x+c)^2/d+5/3*a^3*csc(d*x+c)^3/d-1/4*a^3*csc
(d*x+c)^4/d-3/5*a^3*csc(d*x+c)^5/d-1/6*a^3*csc(d*x+c)^6/d+3*a^3*ln(sin(d*x+
c))/d+a^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3\sin(c+dx)}{d}-\frac{a^3\csc^6(c+dx)}{6d}-\frac{3a^3\csc^5(c+dx)}{5d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{5a^3\csc^3(c+dx)}{3d}+\frac{5a^3\csc^2(c+dx)}{2d}-\frac{a^3\csc(c+dx)}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) + (5*a^3*Csc[c + d*x]^2)/(2*d) + (5*a^3*Csc[c + d*x
]^3)/(3*d) - (a^3*Csc[c + d*x]^4)/(4*d) - (3*a^3*Csc[c + d*x]^5)/(5*d) - (a
^3*Csc[c + d*x]^6)/(6*d) + (3*a^3*Log[Sin[c + d*x]])/d + (a^3*Sin[c + d*x])
/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)^5}{x^7}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a-x)^2(a+x)^5}{x^7}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(1+\frac{a^7}{x^7}+\frac{3a^6}{x^6}+\frac{a^5}{x^5}-\frac{5a^4}{x^4}-\frac{5a^3}{x^3}+\frac{a^2}{x^2}+\frac{3a}{x}\right)\right.}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d}+\frac{5a^3\csc^2(c+dx)}{2d}+\frac{5a^3\csc^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.03, size = 113, normalized size = 0.85

$$a^3\left(-\frac{\csc(c+dx)}{d}+\frac{5\csc^2(c+dx)}{2d}+\frac{5\csc^3(c+dx)}{3d}-\frac{\csc^4(c+dx)}{4d}-\frac{3\csc^5(c+dx)}{5d}-\frac{\csc^6(c+dx)}{6d}+\frac{3\log(\sin(c+dx))}{d}+\frac{\sin(c+dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*(-(Csc[c + d*x]/d) + (5*Csc[c + d*x]^2)/(2*d) + (5*Csc[c + d*x]^3)/(3*d
) - Csc[c + d*x]^4/(4*d) - (3*Csc[c + d*x]^5)/(5*d) - Csc[c + d*x]^6/(6*d)
+ (3*Log[Sin[c + d*x]])/d + Sin[c + d*x]/d)
```

Maple [A]
time = 0.24, size = 215, normalized size = 1.62

method	result
derivativedivides	$\frac{-\frac{a^3\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+3a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+3a^3\left(-\right.}{}$
default	$\frac{-\frac{a^3\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+3a^3\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+3a^3\left(-\right.}{}$
risch	$-3ia^3x-\frac{ia^3e^{i(dx+c)}}{2d}+\frac{ia^3e^{-i(dx+c)}}{2d}-\frac{6ia^3c}{d}-\frac{2ia^3\left(-75ie^{10i(dx+c)}+15\,e^{11i(dx+c)}+270ie^{8i(dx+c)}+25\,e^{9i(dx+\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^3/sin(d*x+c)^6*cos(d*x+c)^6+3*a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)
^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(
```

```
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x
+c)^2+ln(sin(d*x+c)))+a^3*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(
d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.34, size = 108, normalized size = 0.81

$$\frac{180\,a^3\log\left(\sin\left(dx+c\right)\right)+60\,a^3\sin\left(dx+c\right)-\frac{60\,a^3\sin(dx+c)^5-150\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3+15\,a^3\sin(dx+c)^2+36\,a^3\sin(dx+c)+10\,a^3}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(180*a^3*log(sin(d*x + c)) + 60*a^3*sin(d*x + c) - (60*a^3*sin(d*x + c
)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*x + c)^3 + 15*a^3*sin(d*x + c)
^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.41, size = 180, normalized size = 1.35

$$-\frac{150\,a^3\cos\left(dx+c\right)^4-285\,a^3\cos\left(dx+c\right)^2+125\,a^3-180\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,\left(15\,a^3\cos\left(dx+c\right)^6-30\,a^3\cos\left(dx+c\right)^4+40\,a^3\cos\left(dx+c\right)^2-16\,a^3\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(150*a^3*cos(d*x + c)^4 - 285*a^3*cos(d*x + c)^2 + 125*a^3 - 180*(a^3
*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/
2*sin(d*x + c)) - 4*(15*a^3*cos(d*x + c)^6 - 30*a^3*cos(d*x + c)^4 + 40*a^3
*cos(d*x + c)^2 - 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 122, normalized size = 0.92

$$\frac{180\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a^3\sin\left(dx+c\right)-\frac{441\,a^3\sin(dx+c)^6+60\,a^3\sin(dx+c)^5-150\,a^3\sin(dx+c)^4-100\,a^3\sin(dx+c)^3+15\,a^3\sin(dx+c)^2+36\,a^3\sin(dx+c)+10\,a^3}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(180*a^3*log(abs(sin(d*x + c))) + 60*a^3*sin(d*x + c) - (441*a^3*sin(d
*x + c)^6 + 60*a^3*sin(d*x + c)^5 - 150*a^3*sin(d*x + c)^4 - 100*a^3*sin(d*
x + c)^3 + 15*a^3*sin(d*x + c)^2 + 36*a^3*sin(d*x + c) + 10*a^3)/sin(d*x +
c)^6)/d
```

Mupad [B]
time = 9.84, size = 296, normalized size = 2.23

$$\frac{67\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{a^3\left(5760\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-5760\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{1920\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{31\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{16}+\frac{67\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128}+\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{96}+\frac{63\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128}+\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{240}-\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{384}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{160}-\frac{a^3}{384}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (67*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) + (11*a^3*tan(c/2 + (d*x)/2)^3)/(96*d
) - (a^3*tan(c/2 + (d*x)/2)^4)/(32*d) - (3*a^3*tan(c/2 + (d*x)/2)^5)/(160*d
) - (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (a^3*(5760*log(tan(c/2 + (d*x)/2))
 - 5760*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (a^3*tan(c/2 + (d*x)/2))
/(16*d) + (cot(c/2 + (d*x)/2)^6*((23*a^3*tan(c/2 + (d*x)/2)^3)/240 - (13*a^
3*tan(c/2 + (d*x)/2)^2)/384 + (63*a^3*tan(c/2 + (d*x)/2)^4)/128 + (5*a^3*ta
n(c/2 + (d*x)/2)^5)/96 + (67*a^3*tan(c/2 + (d*x)/2)^6)/128 + (31*a^3*tan(c/
2 + (d*x)/2)^7)/16 - a^3/384 - (3*a^3*tan(c/2 + (d*x)/2))/160))/(d*(tan(c/2
 + (d*x)/2)^2 + 1))
```

3.530 $\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=145

$$-\frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{4a^4\log(\sin(c+dx))}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{2a^4\sin^2(c+}{d}$$

```
[Out] -4*a^4*csc(d*x+c)/d-2*a^4*csc(d*x+c)^2/d-1/3*a^4*csc(d*x+c)^3/d-4*a^4*ln(si
n(d*x+c))/d-10*a^4*sin(d*x+c)/d-2*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*x+c)^3/d
+a^4*sin(d*x+c)^4/d+1/5*a^4*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^4\sin^5(c+dx)}{5d}+\frac{a^4\sin^4(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}-\frac{2a^4\sin^2(c+dx)}{d}-\frac{10a^4\sin(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc(c+dx)}{d}-\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (a^4*Csc[c + d*x]^3)/(
3*d) - (4*a^4*Log[Sin[c + d*x]])/d - (10*a^4*Sin[c + d*x])/d - (2*a^4*Sin[c
 + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/d + (a^4
*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-10a^4+\frac{a^8}{x^4}+\frac{4a^7}{x^3}+\frac{4a^6}{x^2}-\frac{4a^5}{x}-4a^3x+4a^2x\right.\right.}{ad}$$
$$= -\frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{a^4\csc^3(c+dx)}{3d}$$

Mathematica [A]

time = 0.12, size = 96, normalized size = 0.66

$$-\frac{a^4(60\csc(c+dx)+30\csc^2(c+dx)+5\csc^3(c+dx)+60\log(\sin(c+dx))+150\sin(c+dx)+30\sin^2(c+dx)-20\sin^3(c+dx)-15\sin^4(c+dx)-3\sin^5(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(a^4*(60*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 5*Csc[c + d*x]^3 + 60*Log
[Sin[c + d*x]] + 150*Sin[c + d*x] + 30*Sin[c + d*x]^2 - 20*Sin[c + d*x]^3 -
 15*Sin[c + d*x]^4 - 3*Sin[c + d*x]^5))/d
```

Maple [A]

time = 0.22, size = 242, normalized size = 1.67

method	result
derivativedivides	$\frac{a^4\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+4a^4\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\text{co}\right.}{}$
default	$\frac{a^4\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+4a^4\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\text{co}\right.}{}$
risch	$4ia^4x-\frac{ia^4e^{5i(dx+c)}}{160d}+\frac{a^4e^{4i(dx+c)}}{16d}+\frac{19ia^4e^{3i(dx+c)}}{96d}+\frac{a^4e^{2i(dx+c)}}{4d}+\frac{71ia^4e^{i(dx+c)}}{16d}-\frac{71ia^4e^{-i(dx+c)}}{16d}+a$
norman	$\frac{-\frac{a^4}{24d}-\frac{a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{7a^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{199a^4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{305a^4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8111a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60d}-\frac{305a^4(\text{t}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+4*a^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+6*a^4*(-1/sin(d*x+c)*cos
(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+4*a^4*(1/4*cos(d*
x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 119, normalized size = 0.82

$$\frac{3\,a^4\sin\left(dx+c\right)^5+15\,a^4\sin\left(dx+c\right)^4+20\,a^4\sin\left(dx+c\right)^3-30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\sin\left(dx+c\right)\right)-150\,a^4\sin\left(dx+c\right)-\frac{5\left(12\,a^4\sin(dx+c)^2+6\,a^4\sin(dx+c)+a^4\right)}{\sin(dx+c)^3}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c)) - 150*a^4*sin(d*x + c) -
 5*(12*a^4*sin(d*x + c)^2 + 6*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.40, size = 172, normalized size = 1.19

$$-\frac{24\,a^4\cos\left(dx+c\right)^8-256\,a^4\cos\left(dx+c\right)^6-576\,a^4\cos\left(dx+c\right)^4+2304\,a^4\cos\left(dx+c\right)^2-1536\,a^4+480\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-15\,\left(8\,a^4\cos\left(dx+c\right)^6-8\,a^4\cos\left(dx+c\right)^4-3\,a^4\cos\left(dx+c\right)^2+19\,a^4\right)\sin\left(dx+c\right)}{120\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/120*(24*a^4*cos(d*x + c)^8 - 256*a^4*cos(d*x + c)^6 - 576*a^4*cos(d*x +
c)^4 + 2304*a^4*cos(d*x + c)^2 - 1536*a^4 + 480*(a^4*cos(d*x + c)^2 - a^4)*
log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(8*a^4*cos(d*x + c)^6 - 8*a^4*cos(d
*x + c)^4 - 3*a^4*cos(d*x + c)^2 + 19*a^4)*sin(d*x + c))/((d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 135, normalized size = 0.93

$$\frac{3\,a^4\sin\left(dx+c\right)^5+15\,a^4\sin\left(dx+c\right)^4+20\,a^4\sin\left(dx+c\right)^3-30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)-150\,a^4\sin\left(dx+c\right)+\frac{5\left(22\,a^4\sin(dx+c)^3-12\,a^4\sin(dx+c)^2-6\,a^4\sin(dx+c)-a^4\right)}{\sin(dx+c)^3}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^4*sin(d*x + c)^5 + 15*a^4*sin(d*x + c)^4 + 20*a^4*sin(d*x + c)^3
- 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x + c))) - 150*a^4*sin(d*x +
 c) + 5*(22*a^4*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 - 6*a^4*sin(d*x + c)
 - a^4)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 8.97, size = 378, normalized size = 2.61

$$\frac{4\,a^4\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{4\,a^4\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{177\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+68\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+640\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+84\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{4549\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{5}+104\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+728\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+104\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{745\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+20\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{56\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^4}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{17\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (4*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^4*tan(c/2 + (d*x)/2)^3)/(24*d)
 - (4*a^4*log(tan(c/2 + (d*x)/2)))/d - ((56*a^4*tan(c/2 + (d*x)/2)^2)/3 + 2
0*a^4*tan(c/2 + (d*x)/2)^3 + (745*a^4*tan(c/2 + (d*x)/2)^4)/3 + 104*a^4*tan
(c/2 + (d*x)/2)^5 + 728*a^4*tan(c/2 + (d*x)/2)^6 + 104*a^4*tan(c/2 + (d*x)/
2)^7 + (4549*a^4*tan(c/2 + (d*x)/2)^8)/5 + 84*a^4*tan(c/2 + (d*x)/2)^9 + 64
0*a^4*tan(c/2 + (d*x)/2)^10 + 68*a^4*tan(c/2 + (d*x)/2)^11 + 177*a^4*tan(c/
2 + (d*x)/2)^12 + a^4/3 + 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2
)^3 + 40*tan(c/2 + (d*x)/2)^5 + 80*tan(c/2 + (d*x)/2)^7 + 80*tan(c/2 + (d*x
)/2)^9 + 40*tan(c/2 + (d*x)/2)^11 + 8*tan(c/2 + (d*x)/2)^13)) - (17*a^4*tan
(c/2 + (d*x)/2))/(8*d) - (a^4*tan(c/2 + (d*x)/2)^2)/(2*d)
```

3.531 $\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=148

$$\frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{10a^4\log(\sin(c+dx))}{d}-\frac{4a^4\sin(c+}{d}$$

[Out] 4*a^4*csc(d*x+c)/d-2*a^4*csc(d*x+c)^2/d-4/3*a^4*csc(d*x+c)^3/d-1/4*a^4*csc(d*x+c)^4/d-10*a^4*ln(sin(d*x+c))/d-4*a^4*sin(d*x+c)/d+2*a^4*sin(d*x+c)^2/d+4/3*a^4*sin(d*x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d

Rubi [A]
time = 0.06, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}-\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^4(c+dx)}{4d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{2a^4\csc^2(c+dx)}{d}+\frac{4a^4\csc(c+dx)}{d}-\frac{10a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]

[Out] (4*a^4*Csc[c + d*x])/d - (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)/(3*d) - (a^4*Csc[c + d*x]^4)/(4*d) - (10*a^4*Log[Sin[c + d*x]])/d - (4*a^4*Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d)

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+\frac{a^8}{x^5}+\frac{4a^7}{x^4}+\frac{4a^6}{x^3}-\frac{4a^5}{x^2}-\frac{10a^4}{x}+4a^2x+4ax^2+x^3\right)\right.}{d}$$
$$= \frac{4a^4\csc(c+dx)}{d}-\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4(c-}{4d}$$

Mathematica [A]
time = 0.11, size = 96, normalized size = 0.65

$$\frac{a^4(48\csc(c+dx)-24\csc^2(c+dx)-16\csc^3(c+dx)-3\csc^4(c+dx)-120\log(\sin(c+dx))-48\sin(c+dx)+24\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(48*Csc[c + d*x] - 24*Csc[c + d*x]^2 - 16*Csc[c + d*x]^3 - 3*Csc[c + d
*x]^4 - 120*Log[Sin[c + d*x]] - 48*Sin[c + d*x] + 24*Sin[c + d*x]^2 + 16*Si
n[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

Maple [A]
time = 0.23, size = 243, normalized size = 1.64

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{}$
default	$\frac{a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{}$
risch	$10ia^4x+\frac{a^4e^{4i(dx+c)}}{64d}+\frac{ia^4e^{3i(dx+c)}}{6d}-\frac{9a^4e^{2i(dx+c)}}{16d}+\frac{3ia^4e^{i(dx+c)}}{2d}-\frac{3ia^4e^{-i(dx+c)}}{2d}-\frac{9a^4e^{-2i(dx+c)}}{16d}-$
norman	$\frac{-\frac{a^4}{64d}-\frac{a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d}-\frac{5a^4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{5a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{3a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{5a^4\left(\tan^9\left(\frac{dx}{2}+\right.\right.}{6d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+4*a^4*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))+6*a^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)
^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+4*a^4*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+co
```

```
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^4*(1/4*cos(d*x+c)^4+1/2*cos(d*x+
c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.32, size = 120, normalized size = 0.81

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+24\,a^4\sin\left(dx+c\right)^2-120\,a^4\log\left(\sin\left(dx+c\right)\right)-48\,a^4\sin\left(dx+c\right)+\frac{48\,a^4\sin(dx+c)^3-24\,a^4\sin(dx+c)^2-16\,a^4\sin(dx+c)-3\,a^4}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(sin(d*x + c)) - 48*a^4*sin(d*x + c) + (48*a^4*sin(d*x + c)^3
- 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.41, size = 144, normalized size = 0.97

$$\frac{24\,a^4\cos\left(dx+c\right)^8-128\,a^4\cos\left(dx+c\right)^6\sin\left(dx+c\right)-288\,a^4\cos\left(dx+c\right)^6+615\,a^4\cos\left(dx+c\right)^4-270\,a^4\cos\left(dx+c\right)^2-105\,a^4-960\,\left(a^4\cos\left(dx+c\right)^4-2\,a^4\cos\left(dx+c\right)^2+a^4\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{96\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*a^4*cos(d*x + c)^8 - 128*a^4*cos(d*x + c)^6*sin(d*x + c) - 288*a^4
*cos(d*x + c)^6 + 615*a^4*cos(d*x + c)^4 - 270*a^4*cos(d*x + c)^2 - 105*a^4
 - 960*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*sin(d*x +
c)))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.69, size = 134, normalized size = 0.91

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+24\,a^4\sin\left(dx+c\right)^2-120\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)-48\,a^4\sin\left(dx+c\right)+\frac{250\,a^4\sin(dx+c)^4+48\,a^4\sin(dx+c)^3-24\,a^4\sin(dx+c)^2-16\,a^4\sin(dx+c)-3\,a^4}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 24*a^4*sin(d*x + c)^2
- 120*a^4*log(abs(sin(d*x + c))) - 48*a^4*sin(d*x + c) + (250*a^4*sin(d*x +
 c)^4 + 48*a^4*sin(d*x + c)^3 - 24*a^4*sin(d*x + c)^2 - 16*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 9.07, size = 368, normalized size = 2.49

$$\frac{3\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,d}-\frac{a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{10\,a^4\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{104\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-119\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+120\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{1135\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}+80\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-73\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+48\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{75\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-\frac{40\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+10\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{8\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a^4}{4}}{d\,\left(16\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+64\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+96\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+64\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{9\,a^4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}+\frac{10\,a^4\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^5,x)
```

```
[Out] (3*a^4*tan(c/2 + (d*x)/2))/(2*d) - (a^4*tan(c/2 + (d*x)/2)^3)/(6*d) - (a^4*
tan(c/2 + (d*x)/2)^4)/(64*d) - (10*a^4*log(tan(c/2 + (d*x)/2)))/d - (10*a^4
*tan(c/2 + (d*x)/2)^2 - (40*a^4*tan(c/2 + (d*x)/2)^3)/3 + (75*a^4*tan(c/2 +
 (d*x)/2)^4)/2 + 48*a^4*tan(c/2 + (d*x)/2)^5 - 73*a^4*tan(c/2 + (d*x)/2)^6
+ 80*a^4*tan(c/2 + (d*x)/2)^7 - (1135*a^4*tan(c/2 + (d*x)/2)^8)/4 + 120*a^4
*tan(c/2 + (d*x)/2)^9 - 119*a^4*tan(c/2 + (d*x)/2)^10 + 104*a^4*tan(c/2 + (
d*x)/2)^11 + a^4/4 + (8*a^4*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2
)^4 + 64*tan(c/2 + (d*x)/2)^6 + 96*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2 + (d*x
)/2)^10 + 16*tan(c/2 + (d*x)/2)^12)) - (9*a^4*tan(c/2 + (d*x)/2)^2)/(16*d)
+ (10*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.532 $\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=146

$$\frac{10a^4\csc(c+dx)}{d}+\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{4a^4\log(\sin(c+d}{d}$$

```
[Out] 10*a^4*csc(d*x+c)/d+2*a^4*csc(d*x+c)^2/d-4/3*a^4*csc(d*x+c)^3/d-a^4*csc(d*x
+c)^4/d-1/5*a^4*csc(d*x+c)^5/d-4*a^4*ln(sin(d*x+c))/d+4*a^4*sin(d*x+c)/d+2*
a^4*sin(d*x+c)^2/d+1/3*a^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin(c+dx)}{d}-\frac{a^4\csc^5(c+dx)}{5d}-\frac{a^4\csc^4(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}+\frac{2a^4\csc^2(c+dx)}{d}+\frac{10a^4\csc(c+dx)}{d}-\frac{4a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (10*a^4*Csc[c + d*x])/d + (2*a^4*Csc[c + d*x]^2)/d - (4*a^4*Csc[c + d*x]^3)
/(3*d) - (a^4*Csc[c + d*x]^4)/d - (a^4*Csc[c + d*x]^5)/(5*d) - (4*a^4*Log[S
in[c + d*x]])/d + (4*a^4*Sin[c + d*x])/d + (2*a^4*Sin[c + d*x]^2)/d + (a^4*
Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2(a+x)^6}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(4a^2+\frac{a^8}{x^6}+\frac{4a^7}{x^5}+\frac{4a^6}{x^4}-\frac{4a^5}{x^3}-\frac{10a^4}{x^2}-\frac{4a^3}{x}+4a\right.\right.}{d}$$
$$= \frac{10a^4\csc(c+dx)}{d}+\frac{2a^4\csc^2(c+dx)}{d}-\frac{4a^4\csc^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.13, size = 96, normalized size = 0.66

$$\frac{a^4(150\csc(c+dx)+30\csc^2(c+dx)-20\csc^3(c+dx)-15\csc^4(c+dx)-3\csc^5(c+dx)-60\log(\sin(c+dx))+60\sin(c+dx)+30\sin^2(c+dx)+5\sin^3(c+dx))}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(150*Csc[c + d*x] + 30*Csc[c + d*x]^2 - 20*Csc[c + d*x]^3 - 15*Csc[c +
 d*x]^4 - 3*Csc[c + d*x]^5 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] + 30*Si
n[c + d*x]^2 + 5*Sin[c + d*x]^3))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(140) = 280.
time = 0.24, size = 298, normalized size = 2.04

method	result
risch	$4ia^4x+\frac{ia^4e^{3i(dx+c)}}{24d}-\frac{a^4e^{2i(dx+c)}}{2d}-\frac{17ia^4e^{i(dx+c)}}{8d}+\frac{17ia^4e^{-i(dx+c)}}{8d}-\frac{a^4e^{-2i(dx+c)}}{2d}-\frac{ia^4e^{-3i(dx+c)}}{24d}+$
derivativedivides	$\frac{a^4\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+4a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\right.}{}$
default	$\frac{a^4\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+4a^4\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5
/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)+4*a^4*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+6*a^4*(-1/3/sin
(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(
d*x+c)^2)*sin(d*x+c))+4*a^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^
4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+a^4*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d
*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.38, size = 120, normalized size = 0.82

$$\frac{5\,a^4\sin\left(dx+c\right)^3+30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\sin\left(dx+c\right)\right)+60\,a^4\sin\left(dx+c\right)+\frac{150\,a^4\sin(dx+c)^4+30\,a^4\sin(dx+c)^3-20\,a^4\sin(dx+c)^2-15\,a^4\sin(dx+c)-3\,a^4}{\sin(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(sin(d*x + c
)) + 60*a^4*sin(d*x + c) + (150*a^4*sin(d*x + c)^4 + 30*a^4*sin(d*x + c)^3
- 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c) - 3*a^4)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.40, size = 192, normalized size = 1.32

$$\frac{5\,a^4\cos\left(dx+c\right)^8-80\,a^4\cos\left(dx+c\right)^6+360\,a^4\cos\left(dx+c\right)^4-480\,a^4\cos\left(dx+c\right)^2+192\,a^4-60\,\left(a^4\cos\left(dx+c\right)^4-2\,a^4\cos\left(dx+c\right)^2+a^4\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-15\,\left(2\,a^4\cos\left(dx+c\right)^6-5\,a^4\cos\left(dx+c\right)^4+6\,a^4\cos\left(dx+c\right)^2-2\,a^4\right)\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(5*a^4*cos(d*x + c)^8 - 80*a^4*cos(d*x + c)^6 + 360*a^4*cos(d*x + c)^4
 - 480*a^4*cos(d*x + c)^2 + 192*a^4 - 60*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*
x + c)^2 + a^4)*log(1/2*sin(d*x + c))*sin(d*x + c) - 15*(2*a^4*cos(d*x + c)
^6 - 5*a^4*cos(d*x + c)^4 + 6*a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 134, normalized size = 0.92

$$\frac{5\,a^4\sin\left(dx+c\right)^3+30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a^4\sin\left(dx+c\right)+\frac{137\,a^4\sin(dx+c)^5+150\,a^4\sin(dx+c)^4+30\,a^4\sin(dx+c)^3-20\,a^4\sin(dx+c)^2-15\,a^4\sin(dx+c)-3\,a^4}{\sin(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/15*(5*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2 - 60*a^4*log(abs(sin(d*x
 + c))) + 60*a^4*sin(d*x + c) + (137*a^4*sin(d*x + c)^5 + 150*a^4*sin(d*x +
 c)^4 + 30*a^4*sin(d*x + c)^3 - 20*a^4*sin(d*x + c)^2 - 15*a^4*sin(d*x + c)
 - 3*a^4)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 9.09, size = 357, normalized size = 2.45

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4\,d}-\frac{19\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16\,d}-\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{4\,a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{398\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+264\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+1017\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+278\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{3314\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{5}+18\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{612\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{104\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}-2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{a^4}{5}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+96\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+96\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}+\frac{71\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}+\frac{4\,a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + a*sin(c + d*x))^4)/sin(c + d*x)^6,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(4*d) - (19*a^4*tan(c/2 + (d*x)/2)^3)/(96*d) - (
a^4*tan(c/2 + (d*x)/2)^4)/(16*d) - (a^4*tan(c/2 + (d*x)/2)^5)/(160*d) - (4*
a^4*log(tan(c/2 + (d*x)/2)))/d + (2*a^4*tan(c/2 + (d*x)/2)^3 - (104*a^4*tan
(c/2 + (d*x)/2)^2)/15 + (612*a^4*tan(c/2 + (d*x)/2)^4)/5 + 18*a^4*tan(c/2 +
 (d*x)/2)^5 + (3314*a^4*tan(c/2 + (d*x)/2)^6)/5 + 278*a^4*tan(c/2 + (d*x)/2
)^7 + 1017*a^4*tan(c/2 + (d*x)/2)^8 + 264*a^4*tan(c/2 + (d*x)/2)^9 + 398*a^
4*tan(c/2 + (d*x)/2)^10 - a^4/5 - 2*a^4*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2
+ (d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c/2 + (d*x)/2)^9 + 32*tan(c
/2 + (d*x)/2)^11)) + (71*a^4*tan(c/2 + (d*x)/2))/(16*d) + (4*a^4*log(tan(c/
2 + (d*x)/2)^2 + 1))/d
```

3.533 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] 1/4*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d-1/6*sin(d*x+c)^6/a/d+1/7*sin(d*x+
c)^7/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 76}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) +
Sin[c + d*x]^7/(7*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3(a+x)}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3(a+x)\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x^3-a^2x^4-ax^5+x^6\right)\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\sin^4(c+dx)}{4ad} - \frac{\sin^5(c+dx)}{5ad} - \frac{\sin^6(c+dx)}{6ad} + \frac{\sin^7(c+dx)}{7ad}$$

Mathematica [A]
time = 0.25, size = 48, normalized size = 0.66

$$\frac{\sin^4(c+dx)\left(105-84\sin(c+dx)-70\sin^2(c+dx)+60\sin^3(c+dx)\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^4*(105 - 84*Sin[c + d*x] - 70*Sin[c + d*x]^2 + 60*Sin[c + d*x
]^3))/(420*a*d)
```

Maple [A]
time = 0.26, size = 49, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{da}$
default	$\frac{\frac{\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{da}$
risch	$-\frac{3\sin(dx+c)}{64ad}-\frac{\sin(7dx+7c)}{448ad}+\frac{\cos(6dx+6c)}{192ad}+\frac{\sin(5dx+5c)}{320ad}+\frac{\sin(3dx+3c)}{64ad}-\frac{3\cos(2dx+2c)}{64ad}$
norman	$\frac{\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{12\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{12\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{16\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{16\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+464}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/7*sin(d*x+c)^7-1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A]
time = 0.30, size = 49, normalized size = 0.67

$$\frac{60\,\sin\left(dx+c\right)^7-70\,\sin\left(dx+c\right)^6-84\,\sin\left(dx+c\right)^5+105\,\sin\left(dx+c\right)^4}{420\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)

Fricas [A]
time = 0.38, size = 67, normalized size = 0.92

$$\frac{70 \cos(dx+c)^6 - 105 \cos(dx+c)^4 - 12\left(5 \cos(dx+c)^6 - 8 \cos(dx+c)^4 + \cos(dx+c)^2 + 2\right)\sin(dx+c)}{420\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/420*(70*cos(d*x + c)^6 - 105*cos(d*x + c)^4 - 12*(5*cos(d*x + c)^6 - 8*co
s(d*x + c)^4 + cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(53) = 106$.
time = 43.52, size = 981, normalized size = 13.44

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Piecewise((420*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d
*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 7
35*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 672*tan(c/2 + d*x/2)**9/(105*a*d*ta
n(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/
2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205
*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 140*tan
(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**
12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*
d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*
x/2)**2 + 105*a*d) + 576*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**14
+ 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*t
an(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/
2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 140*tan(c/2 + d*x/2)**6/(1
05*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c
/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)*
*6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
- 672*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
 d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8

```
+ 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan
(c/2 + d*x/2)**2 + 105*a*d) + 420*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*
x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3
675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c
/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne(d, 0)), (x*sin(
c)**3*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A]
time = 0.48, size = 49, normalized size = 0.67

$$\frac{60 \sin (dx+c)^{7}-70 \sin (dx+c)^{6}-84 \sin (dx+c)^{5}+105 \sin (dx+c)^{4}}{420\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(60*sin(d*x + c)^7 - 70*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin(
d*x + c)^4)/(a*d)
```

Mupad [B]
time = 0.07, size = 57, normalized size = 0.78

$$\frac{\frac{\sin(c+d\,x)^4}{4\,a}-\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{6\,a}+\frac{\sin(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a) + sin(c
 + d*x)^7/(7*a))/d
```

3.534 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^6(c+dx)}{6ad}$$

```
[Out] 1/3*sin(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d+1/6*sin(d*x+
c)^6/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2914, 2644, 14}

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d) +
Sin[c + d*x]^6/(6*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^3(c+dx)\sin^3(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^3(1-x^2)\,dx, x, \sin(c-\right.}{ad}$$
$$= \frac{\text{Subst}\left(\int (x^2-x^4)\,dx, x, \sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int (x^3-x^5)\,dx, x, \sin(c+\right.}{ad}$$
$$= \frac{\sin^3(c+dx)}{3ad} - \frac{\sin^4(c+dx)}{4ad} - \frac{\sin^5(c+dx)}{5ad} + \frac{\sin^6(c+dx)}{6ad}$$

Mathematica [A]
time = 0.17, size = 48, normalized size = 0.66

$$\frac{\sin^3(c+dx)\left(20-15\sin(c+dx)-12\sin^2(c+dx)+10\sin^3(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^3*(20 - 15*Sin[c + d*x] - 12*Sin[c + d*x]^2 + 10*Sin[c + d*x]
^3))/(60*a*d)
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}}{da}$
default	$\frac{\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}}{da}$
risch	$\frac{\sin(dx+c)}{8ad}-\frac{\cos(6dx+6c)}{192ad}-\frac{\sin(5dx+5c)}{80ad}-\frac{\sin(3dx+3c)}{48ad}+\frac{3\cos(2dx+2c)}{64ad}$
norman	$\frac{\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{28\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{28\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{4\left(\tan^5\right.}{1}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/6*sin(d*x+c)^6-1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 49, normalized size = 0.67

$$\frac{10 \sin(dx+c)^6 - 12 \sin(dx+c)^5 - 15 \sin(dx+c)^4 + 20 \sin(dx+c)^3}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

Fricas [A]
time = 0.40, size = 59, normalized size = 0.81

$$-\frac{10 \cos(dx+c)^6 - 15 \cos(dx+c)^4 + 4\left(3 \cos(dx+c)^4 - \cos(dx+c)^2 - 2\right)\sin(dx+c)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*cos(d*x + c)^6 - 15*cos(d*x + c)^4 + 4*(3*cos(d*x + c)^4 - cos(d*
x + c)^2 - 2)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(53) = 106$.
time = 25.96, size = 862, normalized size = 11.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan
(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)*
*6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 6
0*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2
)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d
*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 +
d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*
a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d
*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 40*tan(c/2 + d*x/2)**6/(1
5*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2
+ d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 9
0*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 24*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/
2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8
```

```
+ 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/
2 + d*x/2)**2 + 15*a*d) - 60*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**
12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*ta
n(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**
2 + 15*a*d) + 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*
tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/
2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d),
 Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A]
time = 0.46, size = 49, normalized size = 0.67

$$\frac{10 \sin \left(dx+c\right)^{6}-12 \sin \left(dx+c\right)^{5}-15 \sin \left(dx+c\right)^{4}+20 \sin \left(dx+c\right)^{3}}{60\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*
x + c)^3)/(a*d)
```

Mupad [B]
time = 0.06, size = 57, normalized size = 0.78

$$\frac{\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{4\,a}-\frac{\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{6\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) + sin(c
 + d*x)^6/(6*a))/d
```

3.535 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] -1/4*cos(d*x+c)^4/a/d-1/3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2914, 2645, 30, 2644, 14}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*Cos[c + d*x]^4/(a*d) - Sin[c + d*x]^3/(3*a*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^3(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^4(c+dx)}{4ad}-\frac{\operatorname{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^4(c+dx)}{4ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.87

$$\frac{\sin^2(c+dx)\left(30-20\sin(c+dx)-15\sin^2(c+dx)+12\sin^3(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^2*(30 - 20*Sin[c + d*x] - 15*Sin[c + d*x]^2 + 12*Sin[c + d*x]
^3))/(60*a*d)
```

Maple [A]
time = 0.18, size = 49, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{da}$
default	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{da}$
risch	$-\frac{\sin(dx+c)}{8ad}+\frac{\sin(5dx+5c)}{80ad}-\frac{\cos(4dx+4c)}{32ad}+\frac{\sin(3dx+3c)}{48ad}-\frac{\cos(2dx+2c)}{8ad}$

norman	$\frac{\frac{2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{12\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{12\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{12(\mathrm{ta}}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/5*sin(d*x+c)^5-1/4*sin(d*x+c)^4-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A]
time = 0.31, size = 49, normalized size = 0.89

$$\frac{12\sin\left(dx+c\right)^{5}-15\sin\left(dx+c\right)^{4}-20\sin\left(dx+c\right)^{3}+30\sin\left(dx+c\right)^{2}}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.89

$$-\frac{15\cos\left(dx+c\right)^{4}-4\left(3\cos\left(dx+c\right)^{4}-\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(15*cos(d*x + c)^4 - 4*(3*cos(d*x + c)^4 - cos(d*x + c)^2 - 2)*sin(d*
x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(39) = 78$.
time = 15.60, size = 741, normalized size = 13.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan
(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**
4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**7/(15*a*d*t
an(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)
```

```
**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
30*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) + 16*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*
x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a
*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2
+ d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150
*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d
*x/2)**2 + 15*a*d) - 40*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 +
75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2
+ d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*tan(c/2 + d*x/2)**2
/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/
2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 +
15*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a), True))
```

Giac [A]
time = 0.44, size = 49, normalized size = 0.89

$$\frac{12 \sin \left(dx+c\right)^{5}-15 \sin \left(dx+c\right)^{4}-20 \sin \left(dx+c\right)^{3}+30 \sin \left(dx+c\right)^{2}}{60 \, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 30*sin(d*
x + c)^2)/(a*d)
```

Mupad [B]
time = 0.06, size = 57, normalized size = 1.04

$$\frac{\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{4\,a}+\frac{\sin(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) + sin(c
 + d*x)^5/(5*a))/d
```

3.536 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=65

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 76}

$$\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin
[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2+\frac{a^3}{x}-ax+x^2\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{2ad} + \frac{\sin^3(c+dx)}{3ad}$$

Mathematica [A]
time = 0.04, size = 49, normalized size = 0.75

$$\frac{-2+6\log(\sin(c+dx))-6\sin(c+dx)-3\sin^2(c+dx)+2\sin^3(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (-2 + 6*Log[Sin[c + d*x]] - 6*Sin[c + d*x] - 3*Sin[c + d*x]^2 + 2*Sin[c + d
*x]^3)/(6*a*d)
```

Maple [A]
time = 0.18, size = 44, normalized size = 0.68

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(\sin(dx+c))}{da}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)+\ln(\sin(dx+c))}{da}$
risch	$-\frac{ix}{a}+\frac{e^{2i(dx+c)}}{8ad}+\frac{e^{-2i(dx+c)}}{8ad}-\frac{2ic}{ad}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}-\frac{3\sin(dx+c)}{4ad}-\frac{\sin(3dx+3c)}{12ad}$
norman	$\frac{\frac{2}{ad}+\frac{2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{14\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+14(\tan}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2-sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 51, normalized size = 0.78

$$\frac{\frac{2\sin(dx+c)^3-3\sin(dx+c)^2-6\sin(dx+c)}{a}+\frac{6\log(\sin(dx+c))}{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 6*sin(d*x + c))/a + 6*log(sin(d
*x + c))/a)/d
```

Fricas [A]
time = 0.41, size = 48, normalized size = 0.74

$$\frac{3\cos(dx+c)^2-2\left(\cos(dx+c)^2+2\right)\sin(dx+c)+6\log\left(\frac{1}{2}\sin(dx+c)\right)}{6ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 2)*sin(d*x + c) + 6*log(1/2*sin
(d*x + c)))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 61, normalized size = 0.94

$$\frac{\frac{6\log(|\sin(dx+c)|)}{a}+\frac{2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2-6a^2\sin(dx+c)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c
)^2 - 6*a^2*sin(d*x + c))/a^3)/d
```

Mupad [B]
time = 9.05, size = 102, normalized size = 1.57

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{a\,d}-\frac{2\,\sin\left(c+d\,x\right)}{3\,a\,d}+\frac{\cos\left(c+d\,x\right)^2}{2\,a\,d}-\frac{\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - log(1/cos(c/2 + (d*x)/2)
^2)/(a*d) - (2*sin(c + d*x))/(3*a*d) + cos(c + d*x)^2/(2*a*d) - (cos(c + d*
x)^2*sin(c + d*x))/(3*a*d)
```

3.537 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=62

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 76}

$$\frac{\sin^2(c+dx)}{2ad}-\frac{\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) + Sin[
c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^3}{x^2}-\frac{a^2}{x}+x\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} + \frac{\sin^2(c+dx)}{2ad}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.73

$$\frac{6-2\csc(c+dx)-2\log(\sin(c+dx))-2\sin(c+dx)+\sin^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
[Out] (6 - 2*Csc[c + d*x] - 2*Log[Sin[c + d*x]] - 2*Sin[c + d*x] + Sin[c + d*x]^2
)/(2*a*d)
```

Maple [A]
time = 0.20, size = 46, normalized size = 0.74

method	result
derivativedivides	$\frac{-\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}+\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{da}$
default	$\frac{-\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}+\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)}{da}$
risch	$\frac{ix}{a}-\frac{e^{2i(dx+c)}}{8ad}+\frac{ie^{i(dx+c)}}{2ad}-\frac{ie^{-i(dx+c)}}{2ad}-\frac{e^{-2i(dx+c)}}{8ad}+\frac{2ic}{ad}-\frac{2ie^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{7\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^6\left(\frac{dx}{2}+\right.}{2ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-ln(sin(d*x+c))-1/sin(d*x+c)+1/2*sin(d*x+c)^2-sin(d*x+c))
```

Maxima [**A**]
time = 0.30, size = 52, normalized size = 0.84

$$\frac{\frac{\sin(dx+c)^2-2\sin(dx+c)}{a}-\frac{2\log(\sin(dx+c))}{a}-\frac{2}{a\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*((sin(d*x + c)^2 - 2*sin(d*x + c))/a - 2*log(sin(d*x + c))/a - 2/(a*sin
(d*x + c)))/d
```

Fricas [**A**]
time = 0.41, size = 65, normalized size = 1.05

$$\frac{4\cos(dx+c)^2-(2\cos(dx+c)^2-1)\sin(dx+c)-4\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-8}{4ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(4*cos(d*x + c)^2 - (2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 4*log(1/2*sin
(d*x + c))*sin(d*x + c) - 8)/(a*d*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**A**]
time = 0.41, size = 65, normalized size = 1.05

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a}-\frac{a\sin(dx+c)^2-2a\sin(dx+c)}{a^2}-\frac{2(\sin(dx+c)-1)}{a\sin(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - (a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^
2 - 2*(sin(d*x + c) - 1)/(a*sin(d*x + c)))/d
```

Mupad [B]
time = 8.80, size = 146, normalized size = 2.35

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) - (6*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 +
 (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4 + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*
tan(c/2 + (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)/(2*a
*d) - log(tan(c/2 + (d*x)/2))/(a*d)
```

3.538 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-ln(sin(d*x+c))/a/d+sin(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 76}

$$\frac{\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - Log[Sin[c + d*x]]/(a*d) + Sin
[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^3}{x^3}-\frac{a^2}{x^2}-\frac{a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{\log(\sin(c+dx))}{ad} + \frac{\sin(c+dx)}{ad}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.75

$$-\frac{3-2\csc(c+dx)+\csc^2(c+dx)+2\log(\sin(c+dx))-2\sin(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2*(3 - 2*Csc[c + d*x] + Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]] - 2*Sin[c +
 d*x])/(a*d)
```

Maple [A]
time = 0.20, size = 42, normalized size = 0.70

method	result
derivativedivides	$\frac{\sin(dx+c)-\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{da}$
default	$\frac{\sin(dx+c)-\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{da}$
risch	$\frac{ix}{a}-\frac{i e^{i(dx+c)}}{2ad}+\frac{i e^{-i(dx+c)}}{2ad}+\frac{2ic}{ad}+\frac{2i\left(-i e^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{ad\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$
norman	$\frac{\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{11\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{11\left(\tan^5\left(\frac{dx}{2}\cdots\right.\right.}{4ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(sin(d*x+c)-ln(sin(d*x+c))+1/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.31, size = 52, normalized size = 0.87

$$-\frac{\frac{2\log(\sin(dx+c))}{a}-\frac{2\sin(dx+c)}{a}-\frac{2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*log(sin(d*x + c))/a - 2*sin(d*x + c)/a - (2*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d

Fricas [A]
time = 0.41, size = 61, normalized size = 1.02

$$-\frac{2\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)-1}{2\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 2*(cos(d*x + c)^2 - 2)*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.48, size = 63, normalized size = 1.05

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a}-\frac{2\sin(dx+c)}{a}-\frac{3\sin(dx+c)^2+2\sin(dx+c)-1}{a\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/2*(2*log(abs(sin(d*x + c)))/a - 2*sin(d*x + c)/a - (3*sin(d*x + c)^2 + 2*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d

Mupad [B]
time = 8.93, size = 150, normalized size = 2.50

$$\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{2}}{d\left(4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2/2 + 10*tan(c/2 + (d*x)/2)^3 -
1/2)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 4*a*tan(c/2 + (d*x)/2)^4)) - log(tan(c/
2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)/(2*
a*d) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.539 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d+ln(sin(d*x+c))/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 76}

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) + Log[
Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4}-\frac{a^2}{x^3}-\frac{a}{x^2}+\frac{1}{x}\right)\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\log(\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.75

$$\frac{6\csc(c+dx)+3\csc^2(c+dx)-2\csc^3(c+dx)+6\log(\sin(c+dx))}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 6*Log[Sin[c + d*x]]
)/(6*a*d)
```

Maple [A]
time = 0.22, size = 44, normalized size = 0.69

method	result
derivativedivides	$\frac{\ln(\sin(dx+c))-\frac{1}{3\sin(dx+c)^3}+\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^2}}{da}$
default	$\frac{\ln(\sin(dx+c))-\frac{1}{3\sin(dx+c)^3}+\frac{1}{\sin(dx+c)}+\frac{1}{2\sin(dx+c)^2}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{2i\left(3\,\mathrm{e}^{5i(dx+c)}-2\,\mathrm{e}^{3i(dx+c)}+3i\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{i(dx+c)}-3i\mathrm{e}^{2i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{ad}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}+\frac{5\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{12ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(sin(d*x+c))-1/3/sin(d*x+c)^3+1/sin(d*x+c)+1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 50, normalized size = 0.78

$$\frac{\frac{6\,\log\left(\sin\left(dx+c\right)\right)}{a}+\frac{6\,\sin\left(dx+c\right)^{2}+3\,\sin\left(dx+c\right)-2}{a\sin\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (6*sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin
(d*x + c)^3))/d
```

Fricas [A]
time = 0.39, size = 75, normalized size = 1.17

$$\frac{6\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+6\,\cos\left(dx+c\right)^{2}-3\,\sin\left(dx+c\right)-4}{6\,\left(ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) + 6*cos(d*x
+ c)^2 - 3*sin(d*x + c) - 4)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.43, size = 62, normalized size = 0.97

$$\frac{\frac{6\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}-\frac{11\,\sin\left(dx+c\right)^{3}-6\,\sin\left(dx+c\right)^{2}-3\,\sin\left(dx+c\right)+2}{a\sin\left(dx+c\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a - (11*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 3
*sin(d*x + c) + 2)/(a*sin(d*x + c)^3))/d
```

Mupad [B]
time = 8.95, size = 138, normalized size = 2.16

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + log(tan(c/2
+ (d*x)/2))/(a*d) + (3*tan(c/2 + (d*x)/2))/(8*a*d) - log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) + (cot(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*
x)/2)^2 - 1/3))/(8*a*d)
```

3.540 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] -1/4*cot(d*x+c)^4/a/d-csc(d*x+c)/a/d+1/3*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2785, 2687, 30, 2686}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*Cot[c + d*x]^4/(a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.59

$$-\frac{(-1+\csc(c+dx))^3(5+3\csc(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
[Out] -1/12*((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(a*d)
```

Maple [A]
time = 0.20, size = 49, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{2\sin(dx+c)^2}+\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}}{da}$
default	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{2\sin(dx+c)^2}+\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}}{da}$
risch	$-\frac{2i\left(-3i\mathrm{e}^{6i(dx+c)}+3\,\mathrm{e}^{7i(dx+c)}-5\,\mathrm{e}^{5i(dx+c)}-3i\mathrm{e}^{2i(dx+c)}+5\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}-\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{\tan^9(\ldots}{6\ldots}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/4/sin(d*x+c)^4+1/2/sin(d*x+c)^2+1/3/sin(d*x+c)^3-1/sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.90

$$-\frac{12\,\sin\left(dx+c\right)^3-6\,\sin\left(dx+c\right)^2-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Fricas [A]
time = 0.36, size = 63, normalized size = 1.24

$$-\frac{6\cos(dx+c)^2-4\left(3\cos(dx+c)^2-2\right)\sin(dx+c)-3}{12\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.51, size = 46, normalized size = 0.90

$$-\frac{12\sin(dx+c)^3-6\sin(dx+c)^2-4\sin(dx+c)+3}{12\,ad\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Mupad [B]
time = 8.93, size = 45, normalized size = 0.88

$$\frac{-\sin(c+d\,x)^3+\frac{\sin(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)}{3}-\frac{1}{4}}{a\,d\sin(c+d\,x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/3 + sin(c + d*x)^2/2 - sin(c + d*x)^3 - 1/4)/(a*d*sin(c + d*x
)^4)
```

3.541 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/4*cot(d*x+c)^4/a/d+1/3*csc(d*x+c)^3/a/d-1/5*csc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2914, 2686, 14, 2687, 30}

$$\frac{\cot^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^4/(4*a*d) + Csc[c + d*x]^3/(3*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^3(c+dx)\,dx}{a}\\
&= \frac{\mathrm{Subst}\left(\int x^3\,dx, x, -\cot(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \csc(c+dx\right.}{ad}\\
&= \frac{\cot^4(c+dx)}{4ad}-\frac{\mathrm{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= \frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 0.87

$$\frac{\csc^2(c+dx)\left(-30+20\csc(c+dx)+15\csc^2(c+dx)-12\csc^3(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^2*(-30 + 20*Csc[c + d*x] + 15*Csc[c + d*x]^2 - 12*Csc[c + d*x
]^3))/(60*a*d)
```

Maple [A]
time = 0.23, size = 49, normalized size = 0.89

method	result
derivativedivides	$\frac{\frac{1}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}}{da}$
default	$\frac{\frac{1}{3\sin(dx+c)^3}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}}{da}$
risch	$\frac{-\frac{8i e^{7i(dx+c)}}{3}+2\,e^{8i(dx+c)}-\frac{16i e^{5i(dx+c)}}{15}-2\,e^{6i(dx+c)}-\frac{8i e^{3i(dx+c)}}{3}+2\,e^{4i(dx+c)}-2\,e^{2i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)^5}$

norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}+\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96ad}+\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}-\tan}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2-1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.84

$$-\frac{30\sin(dx+c)^3-20\sin(dx+c)^2-15\sin(dx+c)+12}{60\,ad\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

Fricas [A]
time = 0.38, size = 71, normalized size = 1.29

$$-\frac{20\cos(dx+c)^2-15\left(2\cos(dx+c)^2-1\right)\sin(dx+c)-8}{60\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(20*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^2 - 1)*sin(d*x + c) - 8)/((a*
d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.46, size = 46, normalized size = 0.84

$$-\frac{30\sin(dx+c)^3-20\sin(dx+c)^2-15\sin(dx+c)+12}{60\,ad\sin(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(30*sin(d*x + c)^3 - 20*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*d*s
in(d*x + c)^5)
```

Mupad [B]
time = 8.96, size = 46, normalized size = 0.84

$$\frac{-30\sin\left(c+d\,x\right)^3+20\sin\left(c+d\,x\right)^2+15\,\sin\left(c+d\,x\right)-12}{60\,a\,d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] (15*sin(c + d*x) + 20*sin(c + d*x)^2 - 30*sin(c + d*x)^3 - 12)/(60*a*d*sin(
c + d*x)^5)
```

3.542 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}$$

```
[Out] -1/3*csc(d*x+c)^3/a/d+1/4*csc(d*x+c)^4/a/d+1/5*csc(d*x+c)^5/a/d-1/6*csc(d*x
+c)^6/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 76}

$$-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*Csc[c + d*x]^3/(a*d) + Csc[c + d*x]^4/(4*a*d) + Csc[c + d*x]^5/(5*a*d)
 - Csc[c + d*x]^6/(6*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a^3}{x^7}-\frac{a^2}{x^6}-\frac{a}{x^5}+\frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(-20+15\csc(c+dx)+12\csc^2(c+dx)-10\csc^3(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(-20 + 15*Csc[c + d*x] + 12*Csc[c + d*x]^2 - 10*Csc[c + d*x
]^3))/(60*a*d)
```

Maple [A]
time = 0.26, size = 49, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}}{da}$
default	$\frac{-\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}}{da}$
risch	$\frac{4i\left(-15i e^{8i(dx+c)}+10\,e^{9i(dx+c)}-10i e^{6i(dx+c)}-6\,e^{7i(dx+c)}-15i e^{4i(dx+c)}+6\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}\right)}{15ad\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{1}{384ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1920ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}-\frac{5\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4-1/3/sin(d*x+c)^3
)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.63

$$-\frac{20 \sin(dx+c)^3 - 15 \sin(dx+c)^2 - 12 \sin(dx+c) + 10}{60 \, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

Fricas [A]
time = 0.36, size = 76, normalized size = 1.04

$$\frac{15 \cos(dx+c)^2 - 4\left(5 \cos(dx+c)^2 - 2\right)\sin(dx+c) - 5}{60\left(ad\cos(dx+c)^6 - 3\,ad\cos(dx+c)^4 + 3\,ad\cos(dx+c)^2 - ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 2)*sin(d*x + c) - 5)/(a*d*c
os(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.49, size = 46, normalized size = 0.63

$$-\frac{20 \sin(dx+c)^3 - 15 \sin(dx+c)^2 - 12 \sin(dx+c) + 10}{60 \, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(20*sin(d*x + c)^3 - 15*sin(d*x + c)^2 - 12*sin(d*x + c) + 10)/(a*d*s
in(d*x + c)^6)
```

Mupad [B]
time = 8.94, size = 46, normalized size = 0.63

$$\frac{-20 \sin\left(c+d\,x\right)^3+15 \sin\left(c+d\,x\right)^2+12\,\sin\left(c+d\,x\right)-10}{60\,a\,d \sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
[Out] (12*sin(c + d*x) + 15*sin(c + d*x)^2 - 20*sin(c + d*x)^3 - 10)/(60*a*d*sin(
c + d*x)^6)
```

3.543 $\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] -1/4*csc(d*x+c)^4/a/d+1/5*csc(d*x+c)^5/a/d+1/6*csc(d*x+c)^6/a/d-1/7*csc(d*x
+c)^7/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 76}

$$-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*Csc[c + d*x]^4/(a*d) + Csc[c + d*x]^5/(5*a*d) + Csc[c + d*x]^6/(6*a*d)
 - Csc[c + d*x]^7/(7*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^2(a+x)}{x^8}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{(a-x)^2(a+x)}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{a^3}{x^8}-\frac{a^2}{x^7}-\frac{a}{x^6}+\frac{1}{x^5}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 0.66

$$\frac{\csc^4(c+dx)\left(-105+84\csc(c+dx)+70\csc^2(c+dx)-60\csc^3(c+dx)\right)}{420ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^4*(-105 + 84*Csc[c + d*x] + 70*Csc[c + d*x]^2 - 60*Csc[c + d*
x]^3))/(420*a*d)
```

Maple [A]
time = 0.28, size = 49, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{7\sin(dx+c)^7}+\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{4\sin(dx+c)^4}}{da}$
default	$\frac{-\frac{1}{7\sin(dx+c)^7}+\frac{1}{6\sin(dx+c)^6}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{4\sin(dx+c)^4}}{da}$
risch	$-\frac{4\left(-168i e^{9i(dx+c)}+105\,e^{10i(dx+c)}-144i e^{7i(dx+c)}-35\,e^{8i(dx+c)}-168i e^{5i(dx+c)}+35\,e^{6i(dx+c)}-105\,e^{4i(dx+c)}\right)}{105da\left(e^{2i(dx+c)}-1\right)^7}$
norman	$\frac{-\frac{1}{896ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{672ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{960ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{128ad}-\tan}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/7/sin(d*x+c)^7+1/6/sin(d*x+c)^6+1/5/sin(d*x+c)^5-1/4/sin(d*x+c)^4
)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.63

$$-\frac{105 \sin (dx+c)^{3}-84 \sin (dx+c)^{2}-70 \sin (dx+c)+60}{420 \, ad \sin (dx+c)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

Fricas [A]
time = 0.36, size = 84, normalized size = 1.15

$$\frac{84 \cos (dx+c)^{2}-35 \left(3 \cos (dx+c)^{2}-1\right) \sin (dx+c)-24}{420 \left(ad \cos (dx+c)^{6}-3 \, ad \cos (dx+c)^{4}+3 \, ad \cos (dx+c)^{2}-ad\right) \sin (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(84*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^2 - 1)*sin(d*x + c) - 24)/((a
*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(
d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 46, normalized size = 0.63

$$-\frac{105 \sin (dx+c)^{3}-84 \sin (dx+c)^{2}-70 \sin (dx+c)+60}{420 \, ad \sin (dx+c)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(105*sin(d*x + c)^3 - 84*sin(d*x + c)^2 - 70*sin(d*x + c) + 60)/(a*d
*sin(d*x + c)^7)
```

Mupad [B]
time = 8.96, size = 46, normalized size = 0.63

$$\frac{-105\sin\left(c+d\,x\right)^3+84\sin\left(c+d\,x\right)^2+70\,\sin\left(c+d\,x\right)-60}{420\,a\,d\sin\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
[Out] (70*sin(c + d*x) + 84*sin(c + d*x)^2 - 105*sin(c + d*x)^3 - 60)/(420*a*d*si
n(c + d*x)^7)
```

3.544 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}+\frac{\sin^6(c+dx)}{6a^2d}$$

```
[Out] 1/4*sin(d*x+c)^4/a^2/d-2/5*sin(d*x+c)^5/a^2/d+1/6*sin(d*x+c)^6/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 45}

$$\frac{\sin^6(c+dx)}{6a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}+\frac{\sin^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^4/(4*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + Sin[c + d*x]^6/(6
*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a^3}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^3\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^3-2ax^4+x^5\right)\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\sin^4(c+dx)}{4a^2d} - \frac{2\sin^5(c+dx)}{5a^2d} + \frac{\sin^6(c+dx)}{6a^2d}$$

Mathematica [A]
time = 0.39, size = 38, normalized size = 0.69

$$\frac{\sin^4(c+dx)\left(15-24\sin(c+dx)+10\sin^2(c+dx)\right)}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
[Out] (Sin[c + d*x]^4*(15 - 24*Sin[c + d*x] + 10*Sin[c + d*x]^2))/(60*a^2*d)
```

Maple [A]
time = 0.14, size = 39, normalized size = 0.71

method	result	size
derivativedivides	$\frac{\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{2\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{d\,a^2}$	39
default	$\frac{\frac{\left(\sin^6(dx+c)\right)}{6}-\frac{2\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{d\,a^2}$	39
risch	$-\frac{\sin(dx+c)}{4a^2d}-\frac{\cos(6dx+6c)}{192d\,a^2}-\frac{\sin(5dx+5c)}{40a^2d}+\frac{\cos(4dx+4c)}{16d\,a^2}+\frac{\sin(3dx+3c)}{8d\,a^2}-\frac{13\cos(2dx+2c)}{64d\,a^2}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/6*sin(d*x+c)^6-2/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.71

$$\frac{10\,\sin\left(dx+c\right)^6-24\,\sin\left(dx+c\right)^5+15\,\sin\left(dx+c\right)^4}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)

Fricas [**A**]
time = 0.39, size = 67, normalized size = 1.22

$$-\frac{10\cos\left(dx+c\right)^{6}-45\cos\left(dx+c\right)^{4}+60\cos\left(dx+c\right)^{2}+24\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{60\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")

[Out] -1/60*(10*cos(d*x + c)^6 - 45*cos(d*x + c)^4 + 60*cos(d*x + c)^2 + 24*(cos(
d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c))/(a^2*d)

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(46) = 92$.
time = 68.19, size = 682, normalized size = 12.40

$$\begin{cases} \frac{60\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{192\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} + \frac{280\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{192\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} + \frac{60\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+300a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+225a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+90a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} & \text{for}\: d \neq 0 \\ \frac{x\sin^{3}\left(c\right)\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)

[Out] Piecewise((60*tan(c/2 + d*x/2)**8/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2
*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c
/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2
)**2 + 15*a**2*d) - 192*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**12
 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a*
*2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c
/2 + d*x/2)**2 + 15*a**2*d) + 280*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c/2 +
d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**
8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a*
*2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 192*tan(c/2 + d*x/2)**5/(15*a**2*d*
tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2
+ d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)*
*4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 60*tan(c/2 + d*x/2)**4/(1
5*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d
*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2
+ d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d), Ne(d, 0)), (x*sin
(c)**3*cos(c)**5/(a*sin(c) + a)**2, True))

Giac [**A**]
time = 0.46, size = 39, normalized size = 0.71

$$\frac{10\sin\left(dx+c\right)^{6}-24\sin\left(dx+c\right)^{5}+15\sin\left(dx+c\right)^{4}}{60\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*sin(d*x + c)^6 - 24*sin(d*x + c)^5 + 15*sin(d*x + c)^4)/(a^2*d)
```

Mupad [B]
time = 0.06, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^4\,\left(10\sin\left(c+d\,x\right)^2-24\,\sin\left(c+d\,x\right)+15\right)}{60\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^4*(10*sin(c + d*x)^2 - 24*sin(c + d*x) + 15))/(60*a^2*d)
```

3.545 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^3(c+dx)}{3a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^5(c+dx)}{5a^2d}$$

```
[Out] 1/3*sin(d*x+c)^3/a^2/d-1/2*sin(d*x+c)^4/a^2/d+1/5*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 45}

$$\frac{\sin^5(c+dx)}{5a^2d}-\frac{\sin^4(c+dx)}{2a^2d}+\frac{\sin^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^3/(3*a^2*d) - Sin[c + d*x]^4/(2*a^2*d) + Sin[c + d*x]^5/(5*a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^2x^2\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^2-2ax^3+x^4\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\sin^3(c+dx)}{3a^2d} - \frac{\sin^4(c+dx)}{2a^2d} + \frac{\sin^5(c+dx)}{5a^2d}$$

Mathematica [A]
time = 0.49, size = 38, normalized size = 0.69

$$-\frac{\sin^3(c+dx)(-13+3\cos(2(c+dx))+15\sin(c+dx))}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*(Sin[c + d*x]^3*(-13 + 3*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.26, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{2}+\frac{\left(\sin^3(dx+c)\right)}{3}}{d\,a^2}$
default	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{2}+\frac{\left(\sin^3(dx+c)\right)}{3}}{d\,a^2}$
risch	$\frac{3\sin(dx+c)}{8a^2d}+\frac{\sin(5dx+5c)}{80a^2d}-\frac{\cos(4dx+4c)}{16d\,a^2}-\frac{7\sin(3dx+3c)}{48d\,a^2}+\frac{\cos(2dx+2c)}{4d\,a^2}$
norman	$\frac{\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{8\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{112\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{112\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{16\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.71

$$\frac{6\,\sin\left(dx+c\right)^5-15\,\sin\left(dx+c\right)^4+10\,\sin\left(dx+c\right)^3}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.07

$$-\frac{15\,\cos\left(dx+c\right)^{4}-30\,\cos\left(dx+c\right)^{2}-2\,\left(3\,\cos\left(dx+c\right)^{4}-11\,\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 - 11*cos
(d*x + c)^2 + 8)*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(44) = 88$.
time = 44.62, size = 588, normalized size = 10.69

$$\begin{cases} \frac{40\tan^{7}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{120\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} + \frac{176\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} - \frac{120\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} + \frac{40\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{15a^{2}d\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+150a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+75a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+15a^{2}d} & \text{for}\: d \neq 0 \\ \frac{x\sin^{2}\left(c\right)\cos^{5}\left(c\right)}{\left(a\sin\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((40*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2
*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/
2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*tan(c/2 +
d*x/2)**6/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 +
 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*
d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 176*tan(c/2 + d*x/2)**5/(15*a**2*d*tan
(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*
x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 +
15*a**2*d) - 120*tan(c/2 + d*x/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a
**2*d*tan(c/2 + d*x/2)**8 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan
(c/2 + d*x/2)**4 + 75*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 40*tan(c/2
+ d*x/2)**3/(15*a**2*d*tan(c/2 + d*x/2)**10 + 75*a**2*d*tan(c/2 + d*x/2)**8
 + 150*a**2*d*tan(c/2 + d*x/2)**6 + 150*a**2*d*tan(c/2 + d*x/2)**4 + 75*a**
2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*
sin(c) + a)**2, True))
```

Giac [A]
time = 0.43, size = 39, normalized size = 0.71

$$\frac{6\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}+10\,\sin\left(dx+c\right)^{3}}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/30*(6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 10*sin(d*x + c)^3)/(a^2*d)
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^3\,\left(6\sin\left(c+d\,x\right)^2-15\,\sin\left(c+d\,x\right)+10\right)}{30\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
[Out] (sin(c + d*x)^3*(6*sin(c + d*x)^2 - 15*sin(c + d*x) + 10))/(30*a^2*d)
```

3.546 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^4(c+dx)}{4a^2d}$$

```
[Out] 1/2*sin(d*x+c)^2/a^2/d-2/3*sin(d*x+c)^3/a^2/d+1/4*sin(d*x+c)^4/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 45}

$$\frac{\sin^4(c+dx)}{4a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^2/(2*a^2*d) - (2*Sin[c + d*x]^3)/(3*a^2*d) + Sin[c + d*x]^4/(4
*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a}\,dx, x, a\sin(c+dx)\right)}{a^5d} \\
&= \frac{\text{Subst}\left(\int (a-x)^2x\,dx, x, a\sin(c+dx)\right)}{a^6d} \\
&= \frac{\text{Subst}\left(\int \left(a^2x-2ax^2+x^3\right)\,dx, x, a\sin(c+dx)\right)}{a^6d} \\
&= \frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^4(c+dx)}{4a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 38, normalized size = 0.69

$$
\frac{\sin^2(c+dx)\left(6-8\sin(c+dx)+3\sin^2(c+dx)\right)}{12a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (Sin[c + d*x]^2*(6 - 8*Sin[c + d*x] + 3*Sin[c + d*x]^2))/(12*a^2*d)
```

Maple [A]
time = 0.24, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{2\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{d\,a^2}$
default	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{2\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{d\,a^2}$
risch	$-\frac{\sin(dx+c)}{2a^2d}+\frac{\cos(4dx+4c)}{32d\,a^2}+\frac{\sin(3dx+3c)}{6d\,a^2}-\frac{3\cos(2dx+2c)}{8d\,a^2}$
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6(\tan^5 \cdots}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6 a(\tan(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/4*sin(d*x+c)^4-2/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.71

$$
\frac{3\,\sin\left(dx+c\right)^4-8\,\sin\left(dx+c\right)^3+6\,\sin\left(dx+c\right)^2}{12\,a^2d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

Fricas [A]
time = 0.37, size = 47, normalized size = 0.85

$$\frac{3\cos(dx+c)^4-12\cos(dx+c)^2+8\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8*(cos(d*x + c)^2 - 1)*sin(d*x
 + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(46) = 92$.
time = 27.75, size = 493, normalized size = 8.96

$$\begin{cases} \frac{6\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} - \frac{16\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} + \frac{24\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} - \frac{16\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} + \frac{6\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+18a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+12a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3a^{2}d} & \text{for}\: d \neq 0 \\ \frac{x\sin(c)\cos^{5}(c)}{(a\sin(c)+a)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*
tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d
*x/2)**2 + 3*a**2*d) - 16*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**8
 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*
d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 24*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/
2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)*
*4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 16*tan(c/2 + d*x/2)**3/(3*
a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(
c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 6*tan(c/2 + d
*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18
*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne
(d, 0)), (x*sin(c)*cos(c)**5/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.47, size = 39, normalized size = 0.71

$$\frac{3\sin(dx+c)^4-8\sin(dx+c)^3+6\sin(dx+c)^2}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(d*x + c)^4 - 8*sin(d*x + c)^3 + 6*sin(d*x + c)^2)/(a^2*d)
```

Mupad [B]
time = 8.83, size = 36, normalized size = 0.65

$$\frac{\sin\left(c+d\,x\right)^2\,\left(3\sin\left(c+d\,x\right)^2-8\,\sin\left(c+d\,x\right)+6\right)}{12\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*(3*sin(c + d*x)^2 - 8*sin(c + d*x) + 6))/(12*a^2*d)
```

3.547 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{\log(\sin(c+dx))}{a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin^2(c+dx)}{2a^2d}$$

```
[Out] ln(sin(d*x+c))/a^2/d-2*sin(d*x+c)/a^2/d+1/2*sin(d*x+c)^2/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 45}

$$\frac{\sin^2(c+dx)}{2a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - (2*Sin[c + d*x])/(a^2*d) + Sin[c + d*x]^2/(2*a^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2}{x}+x\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\log(\sin(c+dx))}{a^2 d} - \frac{2\sin(c+dx)}{a^2 d} + \frac{\sin^2(c+dx)}{2a^2 d}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.77

$$\frac{2\log(\sin(c+dx)) - 4\sin(c+dx) + \sin^2(c+dx)}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (2*Log[Sin[c + d*x]] - 4*Sin[c + d*x] + Sin[c + d*x]^2)/(2*a^2*d)
```

Maple [A]
time = 0.25, size = 34, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-2\sin(dx+c)+\ln(\sin(dx+c))}{d\,a^2}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-2\sin(dx+c)+\ln(\sin(dx+c))}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{e^{2i(dx+c)}}{8d\,a^2}-\frac{e^{-2i(dx+c)}}{8d\,a^2}-\frac{2ic}{a^2 d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^2 d}-\frac{2\sin(dx+c)}{a^2 d}$
norman	$\frac{-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{10\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{10\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{18\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{18\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{30(\text{ta}\cdots}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(1/2*sin(d*x+c)^2-2*sin(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 39, normalized size = 0.83

$$\frac{\frac{\sin(dx+c)^2-4\,\sin(dx+c)}{a^2}+\frac{2\,\log(\sin(dx+c))}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
[Out] 1/2*((sin(d*x + c)^2 - 4*sin(d*x + c))/a^2 + 2*log(sin(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.39, size = 36, normalized size = 0.77

$$-\frac{\cos\left(dx+c\right)^{2}-2\,\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\sin\left(dx+c\right)}{2\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
[Out] -1/2*(cos(d*x + c)^2 - 2*log(1/2*sin(d*x + c)) + 4*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.00

$$\frac{\frac{2\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{2}}+\frac{a^{2}\sin\left(dx+c\right)^{2}-4\,a^{2}\sin\left(dx+c\right)}{a^{4}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (a^2*sin(d*x + c)^2 - 4*a^2*sin(d*x + c
))/a^4)/d
```

Mupad [B]
time = 9.11, size = 120, normalized size = 2.55

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) - (
4*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*
(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 + a^2))
```

3.548 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=43

$$-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}+\frac{\sin(c+dx)}{a^2d}$$

```
[Out] -csc(d*x+c)/a^2/d-2*ln(sin(d*x+c))/a^2/d+sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 45}

$$\frac{\sin(c+dx)}{a^2d}-\frac{\csc(c+dx)}{a^2d}-\frac{2\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*Log[Sin[c + d*x]])/(a^2*d) + Sin[c + d*x]/(a^2
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{a^2 d} - \frac{2\log(\sin(c+dx))}{a^2 d} + \frac{\sin(c+dx)}{a^2 d}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.74

$$-\frac{\csc(c+dx)+2\log(\sin(c+dx))-\sin(c+dx)}{a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
[Out] -((Csc[c + d*x] + 2*Log[Sin[c + d*x]] - Sin[c + d*x])/(a^2*d))
```

Maple [A]
time = 0.26, size = 34, normalized size = 0.79

method	result
derivativedivides	$\frac{\sin(dx+c)-2\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}}{d\,a^2}$
default	$\frac{\sin(dx+c)-2\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}}{d\,a^2}$
risch	$\frac{2ix}{a^2}-\frac{ie^{i(dx+c)}}{2d\,a^2}+\frac{ie^{-i(dx+c)}}{2d\,a^2}+\frac{4ic}{a^2 d}-\frac{2ie^{i(dx+c)}}{d\,a^2\left(e^{2i(dx+c)}-1\right)}-\frac{2\ln\left(e^{2i(dx+c)}-1\right)}{a^2 d}$
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{21\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{21\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{16\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+16}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(sin(d*x+c)-2*ln(sin(d*x+c))-1/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.95

$$-\frac{\frac{2\,\log(\sin(dx+c))}{a^2}-\frac{\sin(dx+c)}{a^2}+\frac{1}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -(2*log(sin(d*x + c))/a^2 - sin(d*x + c)/a^2 + 1/(a^2*sin(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 42, normalized size = 0.98

$$-\frac{\cos(dx+c)^2 + 2\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)}{a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(cos(d*x + c)^2 + 2*log(1/2*sin(d*x + c))*sin(d*x + c))/(a^2*d*sin(d*x + c
))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 53, normalized size = 1.23

$$-\frac{\frac{2\log(|\sin(dx+c)|)}{a^2} - \frac{\sin(dx+c)}{a^2} - \frac{2\sin(dx+c)-1}{a^2\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*log(abs(sin(d*x + c)))/a^2 - sin(d*x + c)/a^2 - (2*sin(d*x + c) - 1)/(a
^2*sin(d*x + c)))/d
```

Mupad [B]
time = 8.92, size = 110, normalized size = 2.56

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}{d\,\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}+\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2 - 1)/(d*(2*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan(c/2
 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)/(2
*a^2*d) + (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a^2*d)
```

3.549 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=47

$$\frac{2\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

[Out] 2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+ln(sin(d*x+c))/a^2/d

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 45}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]

[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + Log[Sin[c + d*x]]/(a^2*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^3}-\frac{2a}{x^2}+\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{2\csc(c+dx)}{a^2 d} - \frac{\csc^2(c+dx)}{2a^2 d} + \frac{\log(\sin(c+dx))}{a^2 d}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.81

$$\frac{4\csc(c+dx)-\csc^2(c+dx)+2\log(\sin(c+dx))}{2a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 2*Log[Sin[c + d*x]])/(2*a^2*d)
```

Maple [A]
time = 0.27, size = 36, normalized size = 0.77

method	result
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{2}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
default	$\frac{\ln(\sin(dx+c))+\frac{2}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
risch	$-\frac{ix}{a^2}-\frac{2ic}{a^2 d}+\frac{2i\left(-i e^{2i(dx+c)}+2 e^{3i(dx+c)}-2 e^{i(dx+c)}\right)}{a^2 d\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^2 d}$
norman	$\frac{-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{5\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{23\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{23(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(ln(sin(d*x+c))+2/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.85

$$\frac{\frac{2\log(\sin(dx+c))}{a^2}+\frac{4\sin(dx+c)-1}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*log(sin(d*x + c))/a^2 + (4*sin(d*x + c) - 1)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.39, size = 55, normalized size = 1.17

$$\frac{2\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\sin\left(dx+c\right)+1}{2\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*sin(d*x + c) + 1)/(a^
2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 52, normalized size = 1.11

$$\frac{\frac{2\log\left(|\sin\left(dx+c\right)|\right)}{a^{2}}-\frac{3\sin\left(dx+c\right)^{2}-4\sin\left(dx+c\right)+1}{a^{2}\sin\left(dx+c\right)^{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 - (3*sin(d*x + c)^2 - 4*sin(d*x + c) + 1)
/(a^2*sin(d*x + c)^2))/d
```

Mupad [B]
time = 8.89, size = 104, normalized size = 2.21

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^{2}d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{8a^{2}d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^{2}d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{a^{2}d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{8}\right)}{a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) + tan(c/2
+ (d*x)/2)/(a^2*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a^2*d) + (cot(c/2 + (d*
x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8))/(a^2*d)
```

3.550 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

[Out] -1/3*csc(d*x+c)^3*(a-a*sin(d*x+c))^3/a^5/d

Rubi [A]
time = 0.06, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 37}

$$-\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]

[Out] -1/3*(Csc[c + d*x]^3*(a - a*Sin[c + d*x])^3)/(a^5*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc^3(c+dx)(a-a\sin(c+dx))^3}{3a^5d}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 0.65

$$-\frac{(-1+\csc(c+dx))^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*(-1 + Csc[c + d*x])^3/(a^2*d)
```

Maple [A]
time = 0.26, size = 37, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{1}{\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}}{d\,a^2}$
default	$\frac{\frac{1}{\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}-\frac{1}{\sin(dx+c)}}{d\,a^2}$
risch	$-\frac{2i\left(3\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}-6i e^{4i(dx+c)}+3\,e^{i(dx+c)}+6i e^{2i(dx+c)}\right)}{3a^2d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(1/sin(d*x+c)^2-1/3/sin(d*x+c)^3-1/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 36, normalized size = 1.16

$$-\frac{3\,\sin(dx+c)^2-3\,\sin(dx+c)+1}{3\,a^2d\sin(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

Fricas [A]
time = 0.37, size = 52, normalized size = 1.68

$$-\frac{3 \cos\left(dx+c\right)^{2}+3 \sin\left(dx+c\right)-4}{3 \left(a^{2} d \cos\left(dx+c\right)^{2}-a^{2} d\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*cos(d*x + c)^2 + 3*sin(d*x + c) - 4)/((a^2*d*cos(d*x + c)^2 - a^2*d
)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.48, size = 36, normalized size = 1.16

$$-\frac{3 \sin\left(dx+c\right)^{2}-3 \sin\left(dx+c\right)+1}{3 a^{2} d \sin\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(d*x + c)^2 - 3*sin(d*x + c) + 1)/(a^2*d*sin(d*x + c)^3)
```

Mupad [B]
time = 8.93, size = 34, normalized size = 1.10

$$-\frac{\sin\left(c+d\,x\right)^2-\sin\left(c+d\,x\right)+\frac{1}{3}}{a^2\,d\,\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2 - sin(c + d*x) + 1/3)/(a^2*d*sin(c + d*x)^3)
```

3.551 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

```
[Out] -1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 45}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*Csc[c + d*x]^2/(a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4
/(4*a^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(-6+3\cos(2(c+dx))+8\sin(c+dx))}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A]
time = 0.28, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
default	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
risch	$\frac{2\,e^{6i(dx+c)}-8\,e^{4i(dx+c)}-\frac{16i e^{5i(dx+c)}}{3}+2\,e^{2i(dx+c)}+\frac{16i e^{3i(dx+c)}}{3}}{a^2d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{1}{64ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{5\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{\tan^{11}\left(\frac{dx}{2}\right.}{64ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(2/3/sin(d*x+c)^3-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.65

$$-\frac{6\,\sin\left(dx+c\right)^2-8\,\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Fricas [A]
time = 0.39, size = 57, normalized size = 1.04

$$\frac{6\,\cos\left(dx+c\right)^2+8\,\sin\left(dx+c\right)-9}{12\,\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+c\right)^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.48, size = 36, normalized size = 0.65

$$-\frac{6\,\sin\left(dx+c\right)^{2}-8\,\sin\left(dx+c\right)+3}{12\,a^{2}d\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Mupad [B]
time = 8.92, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin\left(c+d\,x\right)^2}{2}-\frac{2\,\sin\left(c+d\,x\right)}{3}+\frac{1}{4}}{a^2\,d\,\sin\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2/2 - (2*sin(c + d*x))/3 + 1/4)/(a^2*d*sin(c + d*x)^4)
```

3.552 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^3(c+dx)}{3a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*csc(d*x+c)^3/a^2/d+1/2*csc(d*x+c)^4/a^2/d-1/5*csc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 45}

$$-\frac{\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*Csc[c + d*x]^3/(a^2*d) + Csc[c + d*x]^4/(2*a^2*d) - Csc[c + d*x]^5/(5*
a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a^2}{x^6} - \frac{2a}{x^5} + \frac{1}{x^4}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{2a^2d} - \frac{\csc^5(c+dx)}{5a^2d}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.69

$$\frac{\csc^5(c+dx)(-11+5\cos(2(c+dx))+15\sin(c+dx))}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
[Out] (Csc[c + d*x]^5*(-11 + 5*Cos[2*(c + d*x)] + 15*Sin[c + d*x]))/(30*a^2*d)
```

Maple [A]
time = 0.28, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}}{d\,a^2}$
default	$\frac{-\frac{1}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}}{d\,a^2}$
risch	$\frac{8i\left(5\,e^{7i(dx+c)}-22\,e^{5i(dx+c)}-15i e^{6i(dx+c)}+5\,e^{3i(dx+c)}+15i e^{4i(dx+c)}\right)}{15a^2d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{160ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{480ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{480ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160da}+5(\text{t}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-1/5/sin(d*x+c)^5-1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.65

$$-\frac{10\,\sin\left(dx+c\right)^2 - 15\,\sin\left(dx+c\right) + 6}{30\,a^2 d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

Fricas [A]
time = 0.37, size = 65, normalized size = 1.18

$$\frac{10\cos\left(dx+c\right)^2+15\sin\left(dx+c\right)-16}{30\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+c\right)^2+a^2d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(10*cos(d*x + c)^2 + 15*sin(d*x + c) - 16)/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.50, size = 36, normalized size = 0.65

$$-\frac{10\sin\left(dx+c\right)^2-15\sin\left(dx+c\right)+6}{30\,a^2d\sin\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(10*sin(d*x + c)^2 - 15*sin(d*x + c) + 6)/(a^2*d*sin(d*x + c)^5)
```

Mupad [B]
time = 8.94, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin\left(c+d\,x\right)^2}{3}-\frac{\sin\left(c+d\,x\right)}{2}+\frac{1}{5}}{a^2\,d\sin\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2/3 - sin(c + d*x)/2 + 1/5)/(a^2*d*sin(c + d*x)^5)
```

3.553 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{6a^2d}$$

```
[Out] -1/4*csc(d*x+c)^4/a^2/d+2/5*csc(d*x+c)^5/a^2/d-1/6*csc(d*x+c)^6/a^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 45}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*Csc[c + d*x]^4/(a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6
/(6*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{a^7(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a-x)^2}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a^2}{x^7}-\frac{2a}{x^6}+\frac{1}{x^5}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.69

$$-\frac{\csc^4(c+dx)\,(15-24\csc(c+dx)+10\csc^2(c+dx))}{60a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
[Out] -1/60*(Csc[c + d*x]^4*(15 - 24*Csc[c + d*x] + 10*Csc[c + d*x]^2))/(a^2*d)
```

Maple [A]
time = 0.31, size = 39, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{2}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
default	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{2}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
risch	$-\frac{4\left(15\,e^{8i(dx+c)}-70\,e^{6i(dx+c)}-48i e^{7i(dx+c)}+15\,e^{4i(dx+c)}+48i e^{5i(dx+c)}\right)}{15a^2d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{1}{384ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1920ad}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}-\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+3(\tan}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(-1/4/sin(d*x+c)^4+2/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.65

$$-\frac{15\,\sin\left(dx+c\right)^2-24\,\sin\left(dx+c\right)+10}{60\,a^2d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.37, size = 72, normalized size = 1.31

$$-\frac{15 \cos (dx+c)^2+24 \sin (dx+c)-25}{60 \left(a^2 d \cos (dx+c)^6-3 a^2 d \cos (dx+c)^4+3 a^2 d \cos (dx+c)^2-a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/60*(15*cos(d*x + c)^2 + 24*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^6 - 3*
a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.51, size = 36, normalized size = 0.65

$$-\frac{15 \sin (dx+c)^2-24 \sin (dx+c)+10}{60 a^2 d \sin (dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*sin(d*x + c)^2 - 24*sin(d*x + c) + 10)/(a^2*d*sin(d*x + c)^6)
```

Mupad [B]
time = 8.95, size = 36, normalized size = 0.65

$$-\frac{\frac{\sin(c+d\,x)^2}{4}-\frac{2\,\sin(c+d\,x)}{5}+\frac{1}{6}}{a^2\,d\,\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(sin(c + d*x)^2/4 - (2*sin(c + d*x))/5 + 1/6)/(a^2*d*sin(c + d*x)^6)
```

3.554 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{\sin^5(c+dx)}{5a^3d}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-2*sin(d*x+c)^2/a^3/d+4/3*sin(d
*x+c)^3/a^3/d-3/4*sin(d*x+c)^4/a^3/d+1/5*sin(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{\sin^5(c+dx)}{5a^3d}-\frac{3\sin^4(c+dx)}{4a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (2*Sin[c +
d*x]^2)/(a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3*d) - (3*Sin[c + d*x]^4)/(4*a^3*
d) + Sin[c + d*x]^5/(5*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^3}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4-4a^3x+4a^2x^2-3ax^3+x^4-\frac{4a^5}{a+x}\right)dx, x, a\sin(c+dx)\right)}{a^8d}$$
$$= -\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{2\sin^2(c+dx)}{a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}$$

Mathematica [A]
time = 0.68, size = 71, normalized size = 0.70

$$\frac{45-3840\log(1+\sin(c+dx))+3840\sin(c+dx)-1920\sin^2(c+dx)+1280\sin^3(c+dx)-720\sin^4(c+dx)+192\sin^5(c+dx)}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
[Out] (45 - 3840*Log[1 + Sin[c + d*x]] + 3840*Sin[c + d*x] - 1920*Sin[c + d*x]^2
+ 1280*Sin[c + d*x]^3 - 720*Sin[c + d*x]^4 + 192*Sin[c + d*x]^5)/(960*a^3*d
)
```

Maple [A]
time = 0.18, size = 68, normalized size = 0.67

method	result
derivativedivides	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{4\left(\sin^3(dx+c)\right)}{3}-2(\sin^2(dx+c))+4\sin(dx+c)-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{\frac{\left(\sin^5(dx+c)\right)}{5}-\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{4\left(\sin^3(dx+c)\right)}{3}-2(\sin^2(dx+c))+4\sin(dx+c)-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{4ix}{a^3}-\frac{41ie^{i(dx+c)}}{16d\,a^3}+\frac{41ie^{-i(dx+c)}}{16d\,a^3}+\frac{8ic}{d\,a^3}-\frac{8\ln\left(e^{i(dx+c)}+i\right)}{d\,a^3}+\frac{\sin(5dx+5c)}{80d\,a^3}-\frac{3\cos(4dx+4c)}{32d\,a^3}-\frac{19\sin(3dx+3}{48d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(1/5*sin(d*x+c)^5-3/4*sin(d*x+c)^4+4/3*sin(d*x+c)^3-2*sin(d*x+c)^2+
4*sin(d*x+c)-4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.72

$$\frac{\frac{12\sin(dx+c)^5-45\sin(dx+c)^4+80\sin(dx+c)^3-120\sin(dx+c)^2+240\sin(dx+c)}{a^3}-\frac{240\log(\sin(dx+c)+1)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*((12*sin(d*x + c)^5 - 45*sin(d*x + c)^4 + 80*sin(d*x + c)^3 - 120*sin(
d*x + c)^2 + 240*sin(d*x + c))/a^3 - 240*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.39, size = 70, normalized size = 0.69

$$-\frac{45 \cos\left(dx+c\right)^{4}-210 \cos\left(dx+c\right)^{2}-4 \left(3 \cos\left(dx+c\right)^{4}-26 \cos\left(dx+c\right)^{2}+83\right) \sin\left(dx+c\right)+240 \log\left(\sin\left(dx+c\right)+1\right)}{60\,a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(45*cos(d*x + c)^4 - 210*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 - 26*co
s(d*x + c)^2 + 83)*sin(d*x + c) + 240*log(sin(d*x + c) + 1))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2558 vs. $2(94) = 188$.
time = 116.36, size = 2558, normalized size = 25.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-120*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**10/(15*a**3*d*t
an(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 +
d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2
+ 15*a**3*d) - 600*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**8/(15*a**3*d
*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2
+ d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**
2 + 15*a**3*d) - 1200*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(15*a**
3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c
/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2
)**2 + 15*a**3*d) - 1200*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(15*
a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*ta
n(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*
x/2)**2 + 15*a**3*d) - 600*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(1
5*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*
tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 +
d*x/2)**2 + 15*a**3*d) - 120*log(tan(c/2 + d*x/2) + 1)/(15*a**3*d*tan(c/2 +
 d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**
```

```
6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**
3*d) + 60*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**10/(15*a**3*d*tan(
c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x
/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 1
5*a**3*d) + 300*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(15*a**3*d
*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2
+ d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**
2 + 15*a**3*d) + 600*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(15*a
**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan
(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x
/2)**2 + 15*a**3*d) + 600*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/
(15*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*
d*tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2
+ d*x/2)**2 + 15*a**3*d) + 300*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
)**2/(15*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*
a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan
(c/2 + d*x/2)**2 + 15*a**3*d) + 60*log(tan(c/2 + d*x/2)**2 + 1)/(15*a**3*d*
tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 +
 d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2
 + 15*a**3*d) + 120*tan(c/2 + d*x/2)**9/(15*a**3*d*tan(c/2 + d*x/2)**10 + 7
5*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3*d*
tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) - 120*tan(
c/2 + d*x/2)**8/(15*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2
)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75
*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) + 640*tan(c/2 + d*x/2)**7/(15*a**3
*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/
2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)
**2 + 15*a**3*d) - 540*tan(c/2 + d*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**10
+ 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3
*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) + 1136*
tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d
*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4
+ 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) - 540*tan(c/2 + d*x/2)**4/(15*
a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*ta
n(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*
x/2)**2 + 15*a**3*d) + 640*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)*
*10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*
a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) - 1
20*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2
+ d*x/2)**8 + 150*a**3*d*tan(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)*
*4 + 75*a**3*d*tan(c/2 + d*x/2)**2 + 15*a**3*d) + 120*tan(c/2 + d*x/2)/(15*
a**3*d*tan(c/2 + d*x/2)**10 + 75*a**3*d*tan(c/2 + d*x/2)**8 + 150*a**3*d*ta
n(c/2 + d*x/2)**6 + 150*a**3*d*tan(c/2 + d*x/2)**4 + 75*a**3*d*tan(c/2 + d*
x/2)**2 + 15*a**3*d), Ne(d, 0)), (x*sin(c)**3*cos(c)**5/(a*sin(c) + a)**3,
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(96) = 192.
time = 0.51, size = 193, normalized size = 1.89

$$\frac{\frac{60 \log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{3}} - \frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}} - \frac{137 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-120 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+805 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-640 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1910 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1136 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1910 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-640 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+805 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-120 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+137}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5} a^{3}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15*(60*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 120*log(abs(tan(1/2*d*x + 1/
2*c) + 1))/a^3 - (137*tan(1/2*d*x + 1/2*c)^10 - 120*tan(1/2*d*x + 1/2*c)^9
+ 805*tan(1/2*d*x + 1/2*c)^8 - 640*tan(1/2*d*x + 1/2*c)^7 + 1910*tan(1/2*d*
x + 1/2*c)^6 - 1136*tan(1/2*d*x + 1/2*c)^5 + 1910*tan(1/2*d*x + 1/2*c)^4 -
640*tan(1/2*d*x + 1/2*c)^3 + 805*tan(1/2*d*x + 1/2*c)^2 - 120*tan(1/2*d*x +
 1/2*c) + 137)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

Mupad [B]
time = 0.06, size = 83, normalized size = 0.81

$$-\frac{\frac{4\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3}-\frac{4\,\sin\left(c+d\,x\right)}{a^3}+\frac{2\,{\sin\left(c+d\,x\right)}^2}{a^3}-\frac{4\,{\sin\left(c+d\,x\right)}^3}{3\,a^3}+\frac{3\,{\sin\left(c+d\,x\right)}^4}{4\,a^3}-\frac{{\sin\left(c+d\,x\right)}^5}{5\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (2*sin(c + d*x)^2)
/a^3 - (4*sin(c + d*x)^3)/(3*a^3) + (3*sin(c + d*x)^4)/(4*a^3) - sin(c + d*
x)^5/(5*a^3))/d
```

3.555 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$\frac{4\log(1+\sin(c+dx))}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{\sin^4(c+dx)}{4a^3d}$$

```
[Out] 4*ln(1+sin(d*x+c))/a^3/d-4*sin(d*x+c)/a^3/d+2*sin(d*x+c)^2/a^3/d-sin(d*x+c)
^3/a^3/d+1/4*sin(d*x+c)^4/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{\sin^4(c+dx)}{4a^3d}-\frac{\sin^3(c+dx)}{a^3d}+\frac{2\sin^2(c+dx)}{a^3d}-\frac{4\sin(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Log[1 + Sin[c + d*x]])/(a^3*d) - (4*Sin[c + d*x])/(a^3*d) + (2*Sin[c + d
*x]^2)/(a^3*d) - Sin[c + d*x]^3/(a^3*d) + Sin[c + d*x]^4/(4*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2x^2}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^3+4a^2x-3ax^2+x^3+\frac{4a^4}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{4\log(1+\sin(c+dx))}{a^3d} - \frac{4\sin(c+dx)}{a^3d} + \frac{2\sin^2(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} +$$

Mathematica [A]
time = 0.68, size = 59, normalized size = 0.72

$$\frac{35-36\cos(2(c+dx))+\cos(4(c+dx))+128\log(1+\sin(c+dx))-152\sin(c+dx)+8\sin(3(c+dx))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (35 - 36*Cos[2*(c + d*x)] + Cos[4*(c + d*x)] + 128*Log[1 + Sin[c + d*x]] -
152*Sin[c + d*x] + 8*Sin[3*(c + d*x)])/(32*a^3*d)
```

Maple [A]
time = 0.16, size = 58, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\left(\sin^3(dx+c)\right)+2\left(\sin^2(dx+c)\right)-4\sin(dx+c)+4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{\frac{\left(\sin^4(dx+c)\right)}{4}-\left(\sin^3(dx+c)\right)+2\left(\sin^2(dx+c)\right)-4\sin(dx+c)+4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$-\frac{4ix}{a^3}+\frac{19i\mathrm{e}^{i(dx+c)}}{8d\,a^3}-\frac{19i\mathrm{e}^{-i(dx+c)}}{8d\,a^3}-\frac{8ic}{d\,a^3}+\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}+\frac{\cos(4dx+4c)}{32d\,a^3}+\frac{\sin(3dx+3c)}{4d\,a^3}-\frac{9\cos(2dx-}{8d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(1/4*sin(d*x+c)^4-sin(d*x+c)^3+2*sin(d*x+c)^2-4*sin(d*x+c)+4*ln(1+s
in(d*x+c)))
```

Maxima [A]
time = 0.30, size = 61, normalized size = 0.74

$$\frac{\frac{\sin(dx+c)^4-4\,\sin(dx+c)^3+8\,\sin(dx+c)^2-16\,\sin(dx+c)}{a^3}+\frac{16\,\log(\sin(dx+c)+1)}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*((sin(d*x + c)^4 - 4*sin(d*x + c)^3 + 8*sin(d*x + c)^2 - 16*sin(d*x + c
))/a^3 + 16*log(sin(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.37, size = 56, normalized size = 0.68

$$\frac{\cos(dx+c)^4 - 10\cos(dx+c)^2 + 4\left(\cos(dx+c)^2 - 5\right)\sin(dx+c) + 16\log(\sin(dx+c)+1)}{4a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 5)*sin(d*x +
c) + 16*log(sin(d*x + c) + 1))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1698 vs. $2(73) = 146$.
time = 73.43, size = 1698, normalized size = 20.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**8/(a**3*d*tan(c/2
+ d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 +
 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 32*log(tan(c/2 + d*x/2) + 1)*tan(
c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6
+ 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 4
8*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8
 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*t
an(c/2 + d*x/2)**2 + a**3*d) + 32*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2
)**2/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) + 8*log(tan(c/
2 + d*x/2) + 1)/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6
+ 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4
*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(a**3*d*tan(c/2 + d*x/2)*
*8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d
*tan(c/2 + d*x/2)**2 + a**3*d) - 16*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a*
```

```
*3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 24*log(
tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8 +
4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(
c/2 + d*x/2)**2 + a**3*d) - 16*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
)**2/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*
tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d) - 4*log(tan(c/
2 + d*x/2)**2 + 1)/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)*
*6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**3*d)
- 8*tan(c/2 + d*x/2)**7/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2 + d*
x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2 + a**
3*d) + 8*tan(c/2 + d*x/2)**6/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*tan(c/2
 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/2)**2
+ a**3*d) - 32*tan(c/2 + d*x/2)**5/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a**3*d*t
an(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2 + d*x/
2)**2 + a**3*d) + 20*tan(c/2 + d*x/2)**4/(a**3*d*tan(c/2 + d*x/2)**8 + 4*a*
*3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*tan(c/2
+ d*x/2)**2 + a**3*d) - 32*tan(c/2 + d*x/2)**3/(a**3*d*tan(c/2 + d*x/2)**8
+ 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3*d*ta
n(c/2 + d*x/2)**2 + a**3*d) + 8*tan(c/2 + d*x/2)**2/(a**3*d*tan(c/2 + d*x/2
)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a**3
*d*tan(c/2 + d*x/2)**2 + a**3*d) - 8*tan(c/2 + d*x/2)/(a**3*d*tan(c/2 + d*x
/2)**8 + 4*a**3*d*tan(c/2 + d*x/2)**6 + 6*a**3*d*tan(c/2 + d*x/2)**4 + 4*a*
*3*d*tan(c/2 + d*x/2)**2 + a**3*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**5/(a*si
n(c) + a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(80) = 160.
time = 0.48, size = 167, normalized size = 2.04

$$-\frac{\frac{12\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+124\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-96\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-96\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+124\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+25}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/3*(12*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 24*log(abs(tan(1/2*d*x + 1/2
*c) + 1))/a^3 - (25*tan(1/2*d*x + 1/2*c)^8 - 24*tan(1/2*d*x + 1/2*c)^7 + 12
4*tan(1/2*d*x + 1/2*c)^6 - 96*tan(1/2*d*x + 1/2*c)^5 + 210*tan(1/2*d*x + 1/
2*c)^4 - 96*tan(1/2*d*x + 1/2*c)^3 + 124*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/
2*d*x + 1/2*c) + 25)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

Mupad [B]
time = 8.82, size = 69, normalized size = 0.84

$$\frac{\frac{4\ln(\sin(c+d\,x)+1)}{a^3}-\frac{4\sin(c+d\,x)}{a^3}+\frac{2\sin(c+d\,x)^2}{a^3}-\frac{\sin(c+d\,x)^3}{a^3}+\frac{\sin(c+d\,x)^4}{4\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (2*sin(c + d*x)^2)/
a^3 - sin(c + d*x)^3/a^3 + sin(c + d*x)^4/(4*a^3))/d
```

3.556 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=68

$$-\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{\sin^3(c+dx)}{3a^3d}$$

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d+4*sin(d*x+c)/a^3/d-3/2*sin(d*x+c)^2/a^3/d+1/3*sin
(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 78}

$$\frac{\sin^3(c+dx)}{3a^3d}-\frac{3\sin^2(c+dx)}{2a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Log[1 + Sin[c + d*x]])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d) - (3*Sin[c +
d*x]^2)/(2*a^3*d) + Sin[c + d*x]^3/(3*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d} \\
&= \frac{\text{Subst}\left(\int \frac{(a-x)^2x}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^6 d} \\
&= \frac{\text{Subst}\left(\int \left(4a^2-3ax+x^2-\frac{4a^3}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^6 d} \\
&= -\frac{4\log(1+\sin(c+dx))}{a^3 d}+\frac{4\sin(c+dx)}{a^3 d}-\frac{3\sin^2(c+dx)}{2a^3 d}+\frac{\sin^3(c+dx)}{3a^3 d}
\end{aligned}
$$

Mathematica [A]
time = 0.25, size = 51, normalized size = 0.75

$$
\frac{15-384\log(1+\sin(c+dx))+384\sin(c+dx)-144\sin^2(c+dx)+32\sin^3(c+dx)}{96a^3 d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
[Out] (15 - 384*Log[1 + Sin[c + d*x]] + 384*Sin[c + d*x] - 144*Sin[c + d*x]^2 + 3
2*Sin[c + d*x]^3)/(96*a^3*d)
```

Maple [A]
time = 0.32, size = 48, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{3\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3}-\frac{3\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{4ix}{a^3}-\frac{17i\mathrm{e}^{i(dx+c)}}{8d\,a^3}+\frac{17i\mathrm{e}^{-i(dx+c)}}{8d\,a^3}+\frac{8ic}{d\,a^3}-\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}-\frac{\sin(3dx+3c)}{12d\,a^3}+\frac{3\cos(2dx+2c)}{4d\,a^3}$
norman	$\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{34\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{34\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{278\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{278\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+628($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(1/3*sin(d*x+c)^3-3/2*sin(d*x+c)^2+4*sin(d*x+c)-4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.78

$$\frac{\frac{2 \sin(dx+c)^3-9 \sin(dx+c)^2+24 \sin(dx+c)}{a^3}-\frac{24 \log(\sin(dx+c)+1)}{a^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*((2*sin(d*x + c)^3 - 9*sin(d*x + c)^2 + 24*sin(d*x + c))/a^3 - 24*log(s
in(d*x + c) + 1)/a^3)/d
```

Fricas [A]
time = 0.40, size = 48, normalized size = 0.71

$$\frac{9 \cos(dx+c)^2-2 \left(\cos(dx+c)^2-13\right) \sin(dx+c)-24 \log(\sin(dx+c)+1)}{6 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(9*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 13)*sin(d*x + c) - 24*log(sin(d
*x + c) + 1))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. 2(61) = 122.
time = 45.14, size = 1102, normalized size = 16.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-24*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**6/(3*a**3*d*tan(
c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)*
*2 + 3*a**3*d) - 72*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(3*a**3*d
*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*
x/2)**2 + 3*a**3*d) - 72*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**2/(3*a
**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2
 + d*x/2)**2 + 3*a**3*d) - 24*log(tan(c/2 + d*x/2) + 1)/(3*a**3*d*tan(c/2 +
 d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 +
3*a**3*d) + 12*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(3*a**3*d*t
an(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/
2)**2 + 3*a**3*d) + 36*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(3*
a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/
2 + d*x/2)**2 + 3*a**3*d) + 36*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
```

```
)**2/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*
d*tan(c/2 + d*x/2)**2 + 3*a**3*d) + 12*log(tan(c/2 + d*x/2)**2 + 1)/(3*a**3
*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 +
d*x/2)**2 + 3*a**3*d) + 24*tan(c/2 + d*x/2)**5/(3*a**3*d*tan(c/2 + d*x/2)**
6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d)
 - 18*tan(c/2 + d*x/2)**4/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2
+ d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d) + 56*tan(c/2 + d*x/2
)**3/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*
d*tan(c/2 + d*x/2)**2 + 3*a**3*d) - 18*tan(c/2 + d*x/2)**2/(3*a**3*d*tan(c/
2 + d*x/2)**6 + 9*a**3*d*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2
 + 3*a**3*d) + 24*tan(c/2 + d*x/2)/(3*a**3*d*tan(c/2 + d*x/2)**6 + 9*a**3*d
*tan(c/2 + d*x/2)**4 + 9*a**3*d*tan(c/2 + d*x/2)**2 + 3*a**3*d), Ne(d, 0)),
 (x*sin(c)*cos(c)**5/(a*sin(c) + a)**3, True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(64) = 128.
time = 0.47, size = 141, normalized size = 2.07

$$\frac{2\left(\frac{6\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{3}}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{3}}-\frac{11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-28\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(6*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^3 - (11*tan(1/2*d*x + 1/2*c)^6 - 12*tan(1/2*d*x + 1/2*c)^5 + 42*t
an(1/2*d*x + 1/2*c)^4 - 28*tan(1/2*d*x + 1/2*c)^3 + 42*tan(1/2*d*x + 1/2*c)
^2 - 12*tan(1/2*d*x + 1/2*c) + 11)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [**B**]
time = 0.05, size = 57, normalized size = 0.84

$$-\frac{\frac{4\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^3}-\frac{4\,\sin\left(c+d\,x\right)}{a^3}+\frac{3\,{\sin\left(c+d\,x\right)}^2}{2\,a^3}-\frac{{\sin\left(c+d\,x\right)}^3}{3\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((4*log(sin(c + d*x) + 1))/a^3 - (4*sin(c + d*x))/a^3 + (3*sin(c + d*x)^2)
/(2*a^3) - sin(c + d*x)^3/(3*a^3))/d
```

3.557 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=45

$$\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(1+\sin(c+dx))}{a^3d}+\frac{\sin(c+dx)}{a^3d}$$

```
[Out] ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d+sin(d*x+c)/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 84}

$$\frac{\sin(c+dx)}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d) + Sin[c + d*x
]/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^2}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}-\frac{4a}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^3 d} - \frac{4\log(1+\sin(c+dx))}{a^3 d} + \frac{\sin(c+dx)}{a^3 d}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.71

$$\frac{\log(\sin(c+dx)) - 4\log(1+\sin(c+dx)) + \sin(c+dx)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
[Out] (Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]] + Sin[c + d*x])/(a^3*d)
```

Maple [A]
time = 0.32, size = 33, normalized size = 0.73

method	result
derivativedivides	$\frac{\sin(dx+c)+\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{\sin(dx+c)+\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{3ix}{a^3}-\frac{i\mathrm{e}^{i(dx+c)}}{2d\,a^3}+\frac{i\mathrm{e}^{-i(dx+c)}}{2d\,a^3}+\frac{6ic}{d\,a^3}-\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{50\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{50\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{76\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{76\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{10\left(\tan^{2}\left(\right.\right.}{a}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a^2\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(sin(d*x+c)+ln(sin(d*x+c))-4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.96

$$-\frac{\frac{4\,\log(\sin(dx+c)+1)}{a^3}-\frac{\log(\sin(dx+c))}{a^3}-\frac{\sin(dx+c)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(4*log(sin(d*x + c) + 1)/a^3 - log(sin(d*x + c))/a^3 - sin(d*x + c)/a^3)/d
```

Fricas [A]
time = 0.42, size = 34, normalized size = 0.76

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\log\left(\sin\left(dx+c\right)+1\right)+\sin\left(dx+c\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - 4*log(sin(d*x + c) + 1) + sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(45) = 90.
time = 0.50, size = 103, normalized size = 2.29

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{8\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/a^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3*tan(1/2*d*x + 1/2*c)^2 - 2
*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

Mupad [B]
time = 8.92, size = 95, normalized size = 2.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) +
 (2*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^2 + a^3)) + (3*log(tan(c
/2 + (d*x)/2)^2 + 1))/(a^3*d)
```

3.558 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=47

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(1+\sin(c+dx))}{a^3d}$$

```
[Out] -csc(d*x+c)/a^3/d-3*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\csc(c+dx)}{a^3d}-\frac{3\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]/(a^3*d)) - (3*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c
+ d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{3}{x}+\frac{4}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{\csc(c+dx)}{a^3 d} - \frac{3\log(\sin(c+dx))}{a^3 d} + \frac{4\log(1+\sin(c+dx))}{a^3 d}$$

Mathematica [A]
time = 0.04, size = 35, normalized size = 0.74

$$-\frac{\csc(c+dx)+3\log(\sin(c+dx))-4\log(1+\sin(c+dx))}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
[Out] -((Csc[c + d*x] + 3*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c + d*x]])/(a^3*d))
```

Maple [A]
time = 0.34, size = 39, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-3\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{\sin(dx+c)}-3\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$-\frac{ix}{a^3}-\frac{2ic}{d\,a^3}-\frac{2i\,e^{i(dx+c)}}{d\,a^3\left(e^{2i(dx+c)}-1\right)}+\frac{8\ln\left(e^{i(dx+c)}+i\right)}{d\,a^3}-\frac{3\ln\left(e^{2i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{\frac{37\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{37\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{73\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{73\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{11}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^2\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(-1/sin(d*x+c)-3*ln(sin(d*x+c))+4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.94

$$\frac{\frac{4\log(\sin(dx+c)+1)}{a^3}-\frac{3\log(\sin(dx+c))}{a^3}-\frac{1}{a^3\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (4*log(sin(d*x + c) + 1)/a^3 - 3*log(sin(d*x + c))/a^3 - 1/(a^3*sin(d*x + c
)))/d
```

Fricas [A]
time = 0.39, size = 52, normalized size = 1.11

$$-\frac{3 \log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+1}{a^3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -(3*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*log(sin(d*x + c) + 1)*sin(d*x +
c) + 1)/(a^3*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(47) = 94.
time = 0.49, size = 101, normalized size = 2.15

$$-\frac{\frac{2\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(tan(1/2*d*x + 1/2*c)^2 + 1)/a^3 - 16*log(abs(tan(1/2*d*x + 1/2*
c) + 1))/a^3 + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x + 1/2*c)/
a^3 - (6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.98, size = 71, normalized size = 1.51

$$-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(3*log(tan(c/2 + (d*x)/2)) - 8*log(tan(c/2 + (d*x)/2) + 1) + cot(c/2 + (d*
x)/2)/2 + tan(c/2 + (d*x)/2)/2 + log(tan(c/2 + (d*x)/2)^2 + 1))/(a^3*d)
```

3.559 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=65

$$\frac{3\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(1+\sin(c+dx))}{a^3d}$$

```
[Out] 3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin
(d*x+c))/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (4*Log[Sin[c + d*x]])
/(a^3*d) - (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^3}-\frac{3}{x^2}+\frac{4}{ax}-\frac{4}{a(a+x)}\right)dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{3\csc(c+dx)}{a^3 d} - \frac{\csc^2(c+dx)}{2a^3 d} + \frac{4\log(\sin(c+dx))}{a^3 d} - \frac{4\log(1+\sin(c+dx)}{a^3 d}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.75

$$\frac{6\csc(c+dx)-\csc^2(c+dx)+8\log(\sin(c+dx))-8\log(1+\sin(c+dx))}{2a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 8*Log[Sin[c + d*x]] - 8*Log[1 + Sin[c +
d*x]])/(2*a^3*d)
```

Maple [A]
time = 0.35, size = 49, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{2i\left(-i\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{a^3 d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}+\frac{4\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{13\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{7\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{91\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{91(}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a^2 (\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/2/sin(d*x+c)^2+3/sin(d*x+c)+4*ln(sin(d*x+c))-4*ln(1+sin(d*x+c))
)
```

Maxima [A]
time = 0.28, size = 55, normalized size = 0.85

$$-\frac{\frac{8\,\log\left(\sin\left(dx+c\right)+1\right)}{a^3}-\frac{8\,\log\left(\sin\left(dx+c\right)\right)}{a^3}-\frac{6\,\sin\left(dx+c\right)-1}{a^3\,\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(8*log(sin(d*x + c) + 1)/a^3 - 8*log(sin(d*x + c))/a^3 - (6*sin(d*x +
c) - 1)/(a^3*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.38, size = 76, normalized size = 1.17

$$\frac{8\,\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-6\,\sin\left(dx+c\right)+1}{2\,\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(8*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 8*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 6*sin(d*x + c) + 1)/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 115, normalized size = 1.77

$$-\frac{\frac{64\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{32\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(64*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 32*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 + (48*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a
^3*tan(1/2*d*x + 1/2*c)^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d
*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 8.88, size = 107, normalized size = 1.65

$$\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}\right)}{4\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (8*l
og(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (3*tan(c/2 + (d*x)/2))/(2*a^3*d) + (c
ot(c/2 + (d*x)/2)^2*(6*tan(c/2 + (d*x)/2) - 1/2))/(4*a^3*d)
```

3.560 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{4\csc(c+dx)}{a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{\csc^3(c+dx)}{3a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(1+\sin(c+dx))}{a^3d}$$

```
[Out] -4*csc(d*x+c)/a^3/d+3/2*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^3/d-4*ln(sin(
d*x+c))/a^3/d+4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^2(c+dx)}{2a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (3*Csc[c + d*x]^2)/(2*a^3*d) - Csc[c + d*x]^3/(
3*a^3*d) - (4*Log[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\text{Subst}\left(\int \frac{a^4(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5 d} \\ &= \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{ad} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a}{x^4}-\frac{3}{x^3}+\frac{4}{ax^2}-\frac{4}{a^2x}+\frac{4}{a^2(a+x)}\right)\,dx, x, a\sin(c+dx)\right)}{ad} \\ &= -\frac{4\csc(c+dx)}{a^3 d}+\frac{3\csc^2(c+dx)}{2a^3 d}-\frac{\csc^3(c+dx)}{3a^3 d}-\frac{4\log(\sin(c+dx))}{a^3 d}+\frac{4\,\mathrm{l}}{}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.71

$$-\frac{24\csc(c+dx)-9\csc^2(c+dx)+2\csc^3(c+dx)+24\log(\sin(c+dx))-24\log(1+\sin(c+dx))}{6a^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6*(24*Csc[c + d*x] - 9*Csc[c + d*x]^2 + 2*Csc[c + d*x]^3 + 24*Log[Sin[c
+ d*x]] - 24*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Maple [A]
time = 0.35, size = 59, normalized size = 0.71

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{3}{2\sin(dx+c)^2}-\frac{4}{\sin(dx+c)}-4\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3}+\frac{3}{2\sin(dx+c)^2}-\frac{4}{\sin(dx+c)}-4\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$-\frac{2i\left(12\,\mathrm{e}^{5i(dx+c)}-28\,\mathrm{e}^{3i(dx+c)}-9i\mathrm{e}^{4i(dx+c)}+12\,\mathrm{e}^{i(dx+c)}+9i\mathrm{e}^{2i(dx+c)}\right)}{3a^3 d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}+\frac{8\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d\,a^3}-\frac{4\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{17\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{121\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{121\left(\tan\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a^2\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/3/sin(d*x+c)^3+3/2/sin(d*x+c)^2-4/sin(d*x+c)-4*ln(sin(d*x+c))+4
*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.78

$$\frac{\frac{24 \log(\sin(dx+c)+1)}{a^3} - \frac{24 \log(\sin(dx+c))}{a^3} - \frac{24 \sin(dx+c)^2 - 9 \sin(dx+c) + 2}{a^3 \sin(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(24*log(sin(d*x + c) + 1)/a^3 - 24*log(sin(d*x + c))/a^3 - (24*sin(d*x
+ c)^2 - 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.39, size = 106, normalized size = 1.28

$$-\frac{24 \left(\cos\left(dx+c\right)^2-1\right) \log\left(\frac{1}{2} \sin\left(dx+c\right)\right) \sin\left(dx+c\right) - 24 \left(\cos\left(dx+c\right)^2-1\right) \log\left(\sin\left(dx+c\right)+1\right) \sin\left(dx+c\right) + 24 \cos\left(dx+c\right)^2 + 9 \sin\left(dx+c\right) - 26}{6 \left(a^3 d \cos\left(dx+c\right)^2 - a^3 d\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*sin(d*x + c) - 24*(cos(
d*x + c)^2 - 1)*log(sin(d*x + c) + 1)*sin(d*x + c) + 24*cos(d*x + c)^2 + 9*
sin(d*x + c) - 26)/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.54, size = 145, normalized size = 1.75

$$\frac{\frac{192 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right)}{a^3} - \frac{96 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} + \frac{176 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 51 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3} - \frac{a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 9 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 51 a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^9}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(192*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 96*log(abs(tan(1/2*d*x +
 1/2*c)))/a^3 + (176*tan(1/2*d*x + 1/2*c)^3 - 51*tan(1/2*d*x + 1/2*c)^2 + 9
*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^6*tan(1/2*d*x
+ 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 51*a^6*tan(1/2*d*x + 1/2*c))/a^
9)/d
```

Mupad [B]
time = 8.99, size = 139, normalized size = 1.67

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (4*l
og(tan(c/2 + (d*x)/2)))/(a^3*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) -
 (17*tan(c/2 + (d*x)/2))/(8*a^3*d) - (cot(c/2 + (d*x)/2)^3*(17*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2) + 1/3))/(8*a^3*d)
```

3.561 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$\frac{4\csc(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(1+\sin(c+dx))}{a^3d}$$

```
[Out] 4*csc(d*x+c)/a^3/d-2*csc(d*x+c)^2/a^3/d+csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4
/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5}-\frac{3}{x^4}+\frac{4}{ax^3}-\frac{4}{a^2x^2}+\frac{4}{a^3x}-\frac{4}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{\csc^4(c+dx)}{4a^3d}+\frac{4\log(\sin(c+d}{a^3d}$$

Mathematica [A]
time = 0.21, size = 69, normalized size = 0.72

$$\frac{16\csc(c+dx)-8\csc^2(c+dx)+4\csc^3(c+dx)-\csc^4(c+dx)+16\log(\sin(c+dx))-16\log(1+\sin(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

Maple [A]
time = 0.37, size = 67, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))-4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$\frac{4i\left(-2ie^{6i(dx+c)}+2\,e^{7i(dx+c)}+5ie^{4i(dx+c)}-8\,e^{5i(dx+c)}-2ie^{2i(dx+c)}+8\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{a^3d\left(e^{2i(dx+c)}-1\right)^4}-\frac{8\ln\left(e^{i(dx+c)}+i\right)}{d\,a^3}+\frac{4\,l}{}$
norman	$\frac{-\frac{16\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{16\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{64ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{21\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{21\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d/a^3*(-1/4/sin(d*x+c)^4+1/sin(d*x+c)^3-2/sin(d*x+c)^2+4/sin(d*x+c)+4*ln(
sin(d*x+c))-4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 75, normalized size = 0.78

$$-\frac{\frac{16\log(\sin(dx+c)+1)}{a^3}-\frac{16\log(\sin(dx+c))}{a^3}-\frac{16\sin(dx+c)^3-8\sin(dx+c)^2+4\sin(dx+c)-1}{a^3\sin(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

Fricas [A]
time = 0.40, size = 131, normalized size = 1.36

$$\frac{8\cos(dx+c)^2+16\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-16\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\sin(dx+c)+1\right)-4\left(4\cos(dx+c)^2-5\right)\sin(dx+c)-9}{4\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.51, size = 174, normalized size = 1.81

$$-\frac{\frac{1536\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{768\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{1600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-456\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}+\frac{3\left(a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-152a^9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^{12}}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

Mupad [B]
time = 8.88, size = 171, normalized size = 1.78

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^3\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}+\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{4}\right)}{16\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(8*a^3*d) - (9*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - tan(
c/2 + (d*x)/2)^4/(64*a^3*d) + (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (8*log(
tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d) + (cot
(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^2 + 38*tan(c
/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

3.562 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=117

$$-\frac{4\csc(c+dx)}{a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^5(c+dx)}{5a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(1$$

```
[Out] -4*csc(d*x+c)/a^3/d+2*csc(d*x+c)^2/a^3/d-4/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x
+c)^4/a^3/d-1/5*csc(d*x+c)^5/a^3/d-4*ln(sin(d*x+c))/a^3/d+4*ln(1+sin(d*x+c)
)/a^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^3(c+dx)}{3a^3d}+\frac{2\csc^2(c+dx)}{a^3d}-\frac{4\csc(c+dx)}{a^3d}-\frac{4\log(\sin(c+dx))}{a^3d}+\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-4*Csc[c + d*x])/(a^3*d) + (2*Csc[c + d*x]^2)/(a^3*d) - (4*Csc[c + d*x]^3)
/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - Csc[c + d*x]^5/(5*a^3*d) - (4*L
og[Sin[c + d*x]])/(a^3*d) + (4*Log[1 + Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a}{x^6} - \frac{3}{x^5} + \frac{4}{ax^4} - \frac{4}{a^2x^3} + \frac{4}{a^3x^2} - \frac{4}{a^4x} + \frac{4}{a^4(a+x)}\right)\,dx, x, a\sin(c+dx\right.}{d}$$
$$= -\frac{4\csc(c+dx)}{a^3d} + \frac{2\csc^2(c+dx)}{a^3d} - \frac{4\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{\csc^5(}{5}$$

Mathematica [A]
time = 0.09, size = 79, normalized size = 0.68

$$-\frac{240\csc(c+dx) - 120\csc^2(c+dx) + 80\csc^3(c+dx) - 45\csc^4(c+dx) + 12\csc^5(c+dx) + 240\log(\sin(c+dx)) - 240\log(1+\sin(c+dx))}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/60*(240*Csc[c + d*x] - 120*Csc[c + d*x]^2 + 80*Csc[c + d*x]^3 - 45*Csc[c
 + d*x]^4 + 12*Csc[c + d*x]^5 + 240*Log[Sin[c + d*x]] - 240*Log[1 + Sin[c +
 d*x]])/(a^3*d)
```

Maple [A]
time = 0.38, size = 79, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{1}{5\sin(dx+c)^5}+\frac{3}{4\sin(dx+c)^4}-\frac{4}{3\sin(dx+c)^3}+\frac{2}{\sin(dx+c)^2}-\frac{4}{\sin(dx+c)}-4\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
default	$\frac{-\frac{1}{5\sin(dx+c)^5}+\frac{3}{4\sin(dx+c)^4}-\frac{4}{3\sin(dx+c)^3}+\frac{2}{\sin(dx+c)^2}-\frac{4}{\sin(dx+c)}-4\ln(\sin(dx+c))+4\ln(1+\sin(dx+c))}{d\,a^3}$
risch	$-\frac{4i\left(30\,e^{9i(dx+c)}-160\,e^{7i(dx+c)}-30ie^{8i(dx+c)}+284\,e^{5i(dx+c)}+135ie^{6i(dx+c)}-160\,e^{3i(dx+c)}-135ie^{4i(dx+c)}+30\,e^{i(dx+c)}\right.}{15a^3d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{160ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{5\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{5\left(\mathrm{ta}\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/5/sin(d*x+c)^5+3/4/sin(d*x+c)^4-4/3/sin(d*x+c)^3+2/sin(d*x+c)^2
-4/sin(d*x+c)-4*ln(sin(d*x+c))+4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.73

$$\frac{\frac{240 \log(\sin(dx+c)+1)}{a^3} - \frac{240 \log(\sin(dx+c))}{a^3} - \frac{240 \sin(dx+c)^4 - 120 \sin(dx+c)^3 + 80 \sin(dx+c)^2 - 45 \sin(dx+c) + 12}{a^3 \sin(dx+c)^5}}{60 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(240*log(sin(d*x + c) + 1)/a^3 - 240*log(sin(d*x + c))/a^3 - (240*sin(
d*x + c)^4 - 120*sin(d*x + c)^3 + 80*sin(d*x + c)^2 - 45*sin(d*x + c) + 12)
/(a^3*sin(d*x + c)^5))/d
```

Fricas [A]
time = 0.40, size = 161, normalized size = 1.38

$$-\frac{240 \cos(dx+c)^4 + 240 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \sin(dx+c)\right) \sin(dx+c) - 240 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \log(\sin(dx+c)+1) \sin(dx+c) - 560 \cos(dx+c)^2 + 15 \left(8 \cos(dx+c)^2 - 11\right) \sin(dx+c) + 332}{60 \left(a^3 d \cos(dx+c)^4 - 2 a^3 d \cos(dx+c)^2 + a^3 d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/60*(240*cos(d*x + c)^4 + 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log
(1/2*sin(d*x + c))*sin(d*x + c) - 240*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
1)*log(sin(d*x + c) + 1)*sin(d*x + c) - 560*cos(d*x + c)^2 + 15*(8*cos(d*x
+ c)^2 - 11)*sin(d*x + c) + 332)/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x +
 c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.58, size = 204, normalized size = 1.74

$$\frac{\frac{7680 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^3} - \frac{3840 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3} + \frac{8768 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 - 2460 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 660 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 190 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 45 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 6}{a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5} - \frac{6 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 - 45 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 190 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 660 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 2460 a^{12} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{15}}}{960 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(7680*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 3840*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (8768*tan(1/2*d*x + 1/2*c)^5 - 2460*tan(1/2*d*x + 1/2*c
)^4 + 660*tan(1/2*d*x + 1/2*c)^3 - 190*tan(1/2*d*x + 1/2*c)^2 + 45*tan(1/2*
d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) - (6*a^12*tan(1/2*d*x + 1/2*
c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 190*a^12*tan(1/2*d*x + 1/2*c)^3 - 6
60*a^12*tan(1/2*d*x + 1/2*c)^2 + 2460*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]

time = 8.96, size = 203, normalized size = 1.74

$$\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a^3\,d}-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a^3\,d}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a^3\,d}-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{8\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}-\frac{41\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(82\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{5}\right)}{32\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (11*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - (19*tan(c/2 + (d*x)/2)^3)/(96*a^3*d)
 + (3*tan(c/2 + (d*x)/2)^4)/(64*a^3*d) - tan(c/2 + (d*x)/2)^5/(160*a^3*d) -
 (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (8*log(tan(c/2 + (d*x)/2) + 1))/(a^3
*d) - (41*tan(c/2 + (d*x)/2))/(16*a^3*d) - (cot(c/2 + (d*x)/2)^5*((19*tan(c
/2 + (d*x)/2)^2)/3 - (3*tan(c/2 + (d*x)/2))/2 - 22*tan(c/2 + (d*x)/2)^3 + 8
2*tan(c/2 + (d*x)/2)^4 + 1/5))/(32*a^3*d)
```

3.563 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$\frac{12\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\log(1+\sin(c+dx))}{a^4d}$$

```
[Out] 12*csc(d*x+c)/a^4/d-4*csc(d*x+c)^2/a^4/d+4/3*csc(d*x+c)^3/a^4/d-1/4*csc(d*x
+c)^4/a^4/d+16*ln(sin(d*x+c))/a^4/d-16*ln(1+sin(d*x+c))/a^4/d+4/d/(a^4+a^4*
sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{4}{d(a^4\sin(c+dx)+a^4)}-\frac{\csc^4(c+dx)}{4a^4d}+\frac{4\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{12\csc(c+dx)}{a^4d}+\frac{16\log(\sin(c+dx))}{a^4d}-\frac{16\log(\sin(c+dx)+1)}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (12*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (4*Csc[c + d*x]^3)
/(3*a^4*d) - Csc[c + d*x]^4/(4*a^4*d) + (16*Log[Sin[c + d*x]])/(a^4*d) - (1
6*Log[1 + Sin[c + d*x]])/(a^4*d) + 4/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^5} - \frac{4}{ax^4} + \frac{8}{a^2x^3} - \frac{12}{a^3x^2} + \frac{16}{a^4x} - \frac{4}{a^3(a+x)^2} - \frac{16}{a^4(a+x)}\right)dx, x, a\sin(c+dx\right.}{d}$$
$$= \frac{12\csc(c+dx)}{a^4d} - \frac{4\csc^2(c+dx)}{a^4d} + \frac{4\csc^3(c+dx)}{3a^4d} - \frac{\csc^4(c+dx)}{4a^4d} + \frac{16\log(\sin(c}{a^4d}$$

Mathematica [A]
time = 0.48, size = 81, normalized size = 0.68

$$\frac{144\csc(c+dx) - 48\csc^2(c+dx) + 16\csc^3(c+dx) - 3\csc^4(c+dx) + 192\log(\sin(c+dx)) - 192\log(1+\sin(c+dx)) + \frac{48}{1+\sin(c+dx)}}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
[Out] (144*Csc[c + d*x] - 48*Csc[c + d*x]^2 + 16*Csc[c + d*x]^3 - 3*Csc[c + d*x]^
4 + 192*Log[Sin[c + d*x]] - 192*Log[1 + Sin[c + d*x]] + 48/(1 + Sin[c + d*x
]))/(12*a^4*d)
```

Maple [A]
time = 0.42, size = 81, normalized size = 0.68

method	result
derivativedivides	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{4}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{12}{\sin(dx+c)}+16\ln(\sin(dx+c))+\frac{4}{1+\sin(dx+c)}-16\ln(1+\sin(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{4\sin(dx+c)^4}+\frac{4}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{12}{\sin(dx+c)}+16\ln(\sin(dx+c))+\frac{4}{1+\sin(dx+c)}-16\ln(1+\sin(dx+c))}{d\,a^4}$
risch	$\frac{4i(24ie^{8i(dx+c)}+24\,e^{9i(dx+c)}-85ie^{6i(dx+c)}-80\,e^{7i(dx+c)}+85ie^{4i(dx+c)}+106\,e^{5i(dx+c)}-24ie^{2i(dx+c)}-80\,e^{3i(dx+c)}+24\,e^{i}}{3\left(e^{2i(dx+c)}-1\right)^4\left(e^{i(dx+c)}+i\right)^2a^4d}$
norman	$\frac{-\frac{96\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{96\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{64ad}+\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}-\frac{43\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{129\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{129\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{64ad}}{\tan\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d/a^4*(-1/4/sin(d*x+c)^4+4/3/sin(d*x+c)^3-4/sin(d*x+c)^2+12/sin(d*x+c)+16
*ln(sin(d*x+c))+4/(1+sin(d*x+c))-16*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 100, normalized size = 0.83

$$\frac{\frac{192\sin(dx+c)^4+96\sin(dx+c)^3-32\sin(dx+c)^2+13\sin(dx+c)-3}{a^4\sin(dx+c)^5+a^4\sin(dx+c)^4} - \frac{192\log(\sin(dx+c)+1)}{a^4} + \frac{192\log(\sin(dx+c))}{a^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/12*((192*sin(d*x + c)^4 + 96*sin(d*x + c)^3 - 32*sin(d*x + c)^2 + 13*sin(
d*x + c) - 3)/(a^4*sin(d*x + c)^5 + a^4*sin(d*x + c)^4) - 192*log(sin(d*x +
 c) + 1)/a^4 + 192*log(sin(d*x + c))/a^4)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(116) = 232$.
time = 0.39, size = 235, normalized size = 1.96

$$\frac{192\,\cos\left(dx+c\right)^{4}-352\,\cos\left(dx+c\right)^{2}+192\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-192\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+1\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(96\,\cos\left(dx+c\right)^{2}-109\right)\sin\left(dx+c\right)+157}{12\,\left(a^{4}d\cos\left(dx+c\right)^{4}-2\,a^{4}d\cos\left(dx+c\right)^{2}+a^{4}d+\left(a^{4}d\cos\left(dx+c\right)^{4}-2\,a^{4}d\cos\left(dx+c\right)^{2}+a^{4}d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/12*(192*cos(d*x + c)^4 - 352*cos(d*x + c)^2 + 192*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log
(1/2*sin(d*x + c)) - 192*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (96*co
s(d*x + c)^2 - 109)*sin(d*x + c) + 157)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos
(d*x + c)^2 + a^4*d + (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^2 + a^4*
d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**A**]
time = 0.51, size = 218, normalized size = 1.82

$$-\frac{\frac{6144\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{3072\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{4}}-\frac{1536\,\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\right)}{a^{4}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{2}}+\frac{6400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1248\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+204\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}+\frac{3\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-32\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+204\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1248\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{16}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/192*(6144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 3072*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1536*(6*tan(1/2*d*x + 1/2*c)^2 + 11*tan(1/2*d*x + 1/2*
```

```
c) + 6)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (6400*tan(1/2*d*x + 1/2*c)^4 -
 1248*tan(1/2*d*x + 1/2*c)^3 + 204*tan(1/2*d*x + 1/2*c)^2 - 32*tan(1/2*d*x
+ 1/2*c) + 3)/(a^4*tan(1/2*d*x + 1/2*c)^4) + (3*a^12*tan(1/2*d*x + 1/2*c)^4
 - 32*a^12*tan(1/2*d*x + 1/2*c)^3 + 204*a^12*tan(1/2*d*x + 1/2*c)^2 - 1248*
a^12*tan(1/2*d*x + 1/2*c))/a^16)/d
```

Mupad [B]
time = 8.89, size = 233, normalized size = 1.94

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6\,a^4\,d}-\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^4\,d}+\frac{16\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{32\ \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^4\,d}+\frac{-24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+191\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{218\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{143\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{12}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6}-\frac{1}{4}}{d\,\left(16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+32\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+16\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + a*sin(c + d*x))^4),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(6*a^4*d) - (17*tan(c/2 + (d*x)/2)^2)/(16*a^4*d) - tan
(c/2 + (d*x)/2)^4/(64*a^4*d) + (16*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (32*l
og(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + ((13*tan(c/2 + (d*x)/2))/6 - (143*tan
(c/2 + (d*x)/2)^2)/12 + (218*tan(c/2 + (d*x)/2)^3)/3 + 191*tan(c/2 + (d*x)/
2)^4 - 24*tan(c/2 + (d*x)/2)^5 - 1/4)/(d*(16*a^4*tan(c/2 + (d*x)/2)^4 + 32*
a^4*tan(c/2 + (d*x)/2)^5 + 16*a^4*tan(c/2 + (d*x)/2)^6)) + (13*tan(c/2 + (d
*x)/2))/(2*a^4*d)
```

3.564 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$-\frac{16\csc(c+dx)}{a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{\csc^5(c+dx)}{5a^4d}-\frac{20\log(\sin(c+dx))}{a^4d}+\frac{20\log}{}$$

```
[Out] -16*csc(d*x+c)/a^4/d+6*csc(d*x+c)^2/a^4/d-8/3*csc(d*x+c)^3/a^4/d+csc(d*x+c)
^4/a^4/d-1/5*csc(d*x+c)^5/a^4/d-20*ln(sin(d*x+c))/a^4/d+20*ln(1+sin(d*x+c))
/a^4/d-4/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{4}{d\left(a^4\sin(c+dx)+a^4\right)}-\frac{\csc^5(c+dx)}{5a^4d}+\frac{\csc^4(c+dx)}{a^4d}-\frac{8\csc^3(c+dx)}{3a^4d}+\frac{6\csc^2(c+dx)}{a^4d}-\frac{16\csc(c+dx)}{a^4d}-\frac{20\log(\sin(c+dx))}{a^4d}+\frac{20\log(\sin(c+dx)+1)}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-16*Csc[c + d*x])/(a^4*d) + (6*Csc[c + d*x]^2)/(a^4*d) - (8*Csc[c + d*x]^3
)/(3*a^4*d) + Csc[c + d*x]^4/(a^4*d) - Csc[c + d*x]^5/(5*a^4*d) - (20*Log[S
in[c + d*x]])/(a^4*d) + (20*Log[1 + Sin[c + d*x]])/(a^4*d) - 4/(d*(a^4 + a^
4*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^2}{x^6(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{1}{x^6}-\frac{4}{ax^5}+\frac{8}{a^2x^4}-\frac{12}{a^3x^3}+\frac{16}{a^4x^2}-\frac{20}{a^5x}+\frac{4}{a^4(a+x)^2}+\frac{20}{a^5(a+x)}\right)dx, x,\right.}{d}$$
$$= -\frac{16\csc(c+dx)}{a^4 d}+\frac{6\csc^2(c+dx)}{a^4 d}-\frac{8\csc^3(c+dx)}{3a^4 d}+\frac{\csc^4(c+dx)}{a^4 d}-\frac{\csc^5(}{5}$$

Mathematica [A]
time = 0.21, size = 91, normalized size = 0.67

$$-\frac{240\csc(c+dx)-90\csc^2(c+dx)+40\csc^3(c+dx)-15\csc^4(c+dx)+3\csc^5(c+dx)+300\log(\sin(c+dx))-300\log(1+\sin(c+dx))+\frac{60}{1+\sin(c+dx)}}{15a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/15*(240*Csc[c + d*x] - 90*Csc[c + d*x]^2 + 40*Csc[c + d*x]^3 - 15*Csc[c
+ d*x]^4 + 3*Csc[c + d*x]^5 + 300*Log[Sin[c + d*x]] - 300*Log[1 + Sin[c + d
*x]] + 60/(1 + Sin[c + d*x]))/(a^4*d)
```

Maple [A]
time = 0.34, size = 89, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{1}{5\sin(dx+c)^5}+\frac{1}{\sin(dx+c)^4}-\frac{8}{3\sin(dx+c)^3}+\frac{6}{\sin(dx+c)^2}-\frac{16}{\sin(dx+c)}-20\ln(\sin(dx+c))-\frac{4}{1+\sin(dx+c)}+20\ln(1+\sin(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{5\sin(dx+c)^5}+\frac{1}{\sin(dx+c)^4}-\frac{8}{3\sin(dx+c)^3}+\frac{6}{\sin(dx+c)^2}-\frac{16}{\sin(dx+c)}-20\ln(\sin(dx+c))-\frac{4}{1+\sin(dx+c)}+20\ln(1+\sin(dx+c))}{d\,a^4}$
risch	$-\frac{8i(75i\mathrm{e}^{10i(dx+c)}+75\,\mathrm{e}^{11i(dx+c)}-350i\mathrm{e}^{8i(dx+c)}-325\,\mathrm{e}^{9i(dx+c)}+574i\mathrm{e}^{6i(dx+c)}+552\,\mathrm{e}^{7i(dx+c)}-350i\mathrm{e}^{4i(dx+c)}-552\,\mathrm{e}^{5i(d}}{15(\mathrm{e}^{2i(dx+c)}-1)^5(\mathrm{e}^{i(dx+c)}+i)^2a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^4*(-1/5/sin(d*x+c)^5+1/sin(d*x+c)^4-8/3/sin(d*x+c)^3+6/sin(d*x+c)^2-1
6/sin(d*x+c)-20*ln(sin(d*x+c))-4/(1+sin(d*x+c))+20*ln(1+sin(d*x+c)))
```

Maxima [A]

time = 0.31, size = 110, normalized size = 0.81

$$-\frac{\frac{300 \sin(dx+c)^5+150 \sin(dx+c)^4-50 \sin(dx+c)^3+25 \sin(dx+c)^2-12 \sin(dx+c)+3}{a^4 \sin(dx+c)^6+a^4 \sin(dx+c)^5}-\frac{300 \log(\sin(dx+c)+1)}{a^4}+\frac{300 \log(\sin(dx+c))}{a^4}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] -1/15*((300*sin(d*x + c)^5 + 150*sin(d*x + c)^4 - 50*sin(d*x + c)^3 + 25*si
n(d*x + c)^2 - 12*sin(d*x + c) + 3)/(a^4*sin(d*x + c)^6 + a^4*sin(d*x + c)^
5) - 300*log(sin(d*x + c) + 1)/a^4 + 300*log(sin(d*x + c))/a^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(131) = 262.

time = 0.43, size = 283, normalized size = 2.10

$$\frac{150 \cos(dx+c)^4-325 \cos(dx+c)^2-300 \left(\cos(dx+c)^6-3 \cos(dx+c)^4+3 \cos(dx+c)^2-\left(\cos(dx+c)^4-2 \cos(dx+c)^2+1\right)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+300 \left(\cos(dx+c)^6-3 \cos(dx+c)^4+3 \cos(dx+c)^2-\left(\cos(dx+c)^4-2 \cos(dx+c)^2+1\right)\sin(dx+c)-1\right)\log(\sin(dx+c)+1)+2 \left(150 \cos(dx+c)^4-275 \cos(dx+c)^2+119\right)\sin(dx+c)+178}{15 \left(a^4 d\cos(dx+c)^6-3 a^4 d\cos(dx+c)^4+3 a^4 d\cos(dx+c)^2-a^4 d-\left(a^4 d\cos(dx+c)^4-2 a^4 d\cos(dx+c)^2+a^4 d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/15*(150*cos(d*x + c)^4 - 325*cos(d*x + c)^2 - 300*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*si
n(d*x + c) - 1)*log(1/2*sin(d*x + c)) + 300*(cos(d*x + c)^6 - 3*cos(d*x + c
)^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x +
c) - 1)*log(sin(d*x + c) + 1) + 2*(150*cos(d*x + c)^4 - 275*cos(d*x + c)^2
+ 119)*sin(d*x + c) + 178)/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 +
 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x +
 c)^2 + a^4*d)*sin(d*x + c))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]

time = 0.50, size = 248, normalized size = 1.84

$$\frac{\frac{19200 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{9600 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{1920 \left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+28 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\right)}{a^4 \left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^2}+\frac{21920 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4350 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+840 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-175 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^4 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{3\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+175\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-840\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4350\,a^{16} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{20}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(19200*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 9600*log(abs(tan(1/2*
d*x + 1/2*c)))/a^4 - 1920*(15*tan(1/2*d*x + 1/2*c)^2 + 28*tan(1/2*d*x + 1/2
*c) + 15)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^2) + (21920*tan(1/2*d*x + 1/2*c)^
5 - 4350*tan(1/2*d*x + 1/2*c)^4 + 840*tan(1/2*d*x + 1/2*c)^3 - 175*tan(1/2*
d*x + 1/2*c)^2 + 30*tan(1/2*d*x + 1/2*c) - 3)/(a^4*tan(1/2*d*x + 1/2*c)^5)
- (3*a^16*tan(1/2*d*x + 1/2*c)^5 - 30*a^16*tan(1/2*d*x + 1/2*c)^4 + 175*a^1
6*tan(1/2*d*x + 1/2*c)^3 - 840*a^16*tan(1/2*d*x + 1/2*c)^2 + 4350*a^16*tan(
1/2*d*x + 1/2*c))/a^20)/d
```

Mupad [B]
time = 8.91, size = 266, normalized size = 1.97

$$\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^4\,d}-\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a^4\,d}-\frac{20\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^4\,d}-\frac{34\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+524\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{569\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{104\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{118\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+\frac{1}{5}}{d\,\left(32\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+32\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{40\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^4\,d}-\frac{145\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + a*sin(c + d*x))^4),x)
```

```
[Out] (7*tan(c/2 + (d*x)/2)^2)/(4*a^4*d) - (35*tan(c/2 + (d*x)/2)^3)/(96*a^4*d) +
 tan(c/2 + (d*x)/2)^4/(16*a^4*d) - tan(c/2 + (d*x)/2)^5/(160*a^4*d) - (20*l
og(tan(c/2 + (d*x)/2)))/(a^4*d) - ((118*tan(c/2 + (d*x)/2)^2)/15 - (8*tan(c
/2 + (d*x)/2))/5 - (104*tan(c/2 + (d*x)/2)^3)/3 + (569*tan(c/2 + (d*x)/2)^4
)/3 + 524*tan(c/2 + (d*x)/2)^5 + 34*tan(c/2 + (d*x)/2)^6 + 1/5)/(d*(32*a^4*
tan(c/2 + (d*x)/2)^5 + 64*a^4*tan(c/2 + (d*x)/2)^6 + 32*a^4*tan(c/2 + (d*x)
/2)^7)) + (40*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d) - (145*tan(c/2 + (d*x)/2
))/(16*a^4*d)
```

3.565 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}+\frac{a^3\sin^{3+n}(c+dx)}{d(3+n)}-\frac{5a^3\sin^{4+n}(c+dx)}{d(4+n)}-\frac{5a^3\sin^{5+n}(c+dx)}{d(5+n)}+\frac{a^3\sin^6}{d(}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)+a^3*sin(d*x+c)^
(3+n)/d/(3+n)-5*a^3*sin(d*x+c)^(4+n)/d/(4+n)-5*a^3*sin(d*x+c)^(5+n)/d/(5+n)
+a^3*sin(d*x+c)^(6+n)/d/(6+n)+3*a^3*sin(d*x+c)^(7+n)/d/(7+n)+a^3*sin(d*x+c)
^(8+n)/d/(8+n)
```

Rubi [A]
time = 0.13, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 90}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}+\frac{a^3\sin^{n+3}(c+dx)}{d(n+3)}-\frac{5a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{5a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{3a^3\sin^{n+7}(c+dx)}{d(n+7)}+\frac{a^3\sin^{n+8}(c+dx)}{d(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) + (a^3*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (5*a^3*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (5*a^3*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a^3*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (3*a^3*Sin[c + d*x]^(7 + n))/(d*(7 + n)) + (a^3*
Sin[c + d*x]^(8 + n))/(d*(8 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n + 3a^7\left(\frac{x}{a}\right)^{1+n} + a^7\left(\frac{x}{a}\right)^{2+n} - 5a^7\left(\frac{x}{a}\right)^{3+\cdots}\right.\right.}{}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} + \frac{a^3\sin^{3+n}(c+\cdots}{d(3+n)\cdots}$$

Mathematica [A]
time = 0.43, size = 123, normalized size = 0.68

$$\frac{a^3\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{3\sin(c+dx)}{2+n}+\frac{\sin^2(c+dx)}{3+n}-\frac{5\sin^3(c+dx)}{4+n}-\frac{5\sin^4(c+dx)}{5+n}+\frac{\sin^5(c+dx)}{6+n}+\frac{3\sin^6(c+dx)}{7+n}+\frac{\sin^7(c+dx)}{8+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) + Sin[c
+ d*x]^2/(3 + n) - (5*Sin[c + d*x]^3)/(4 + n) - (5*Sin[c + d*x]^4)/(5 + n)
+ Sin[c + d*x]^5/(6 + n) + (3*Sin[c + d*x]^6)/(7 + n) + Sin[c + d*x]^7/(8 +
 n)))/d
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [A]
time = 0.29, size = 161, normalized size = 0.89

$$\frac{\frac{a^3\sin(dx+c)^{n+8}}{n+8}+\frac{3a^3\sin(dx+c)^{n+7}}{n+7}+\frac{a^3\sin(dx+c)^{n+6}}{n+6}-\frac{5a^3\sin(dx+c)^{n+5}}{n+5}-\frac{5a^3\sin(dx+c)^{n+4}}{n+4}+\frac{a^3\sin(dx+c)^{n+3}}{n+3}+\frac{3a^3\sin(dx+c)^{n+2}}{n+2}+\frac{a^3\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (a^3*sin(d*x + c)^(n + 8)/(n + 8) + 3*a^3*sin(d*x + c)^(n + 7)/(n + 7) + a^
3*sin(d*x + c)^(n + 6)/(n + 6) - 5*a^3*sin(d*x + c)^(n + 5)/(n + 5) - 5*a^3
*sin(d*x + c)^(n + 4)/(n + 4) + a^3*sin(d*x + c)^(n + 3)/(n + 3) + 3*a^3*si
n(d*x + c)^(n + 2)/(n + 2) + a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(181) = 362$.
time = 0.44, size = 616, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 28*a^3*n^6 + 322*a^3*n^5 + 1960*a^3*n^4 + 6769*a^3*n^3 + 13132*
a^3*n^2 + 13068*a^3*n + 5040*a^3)*cos(d*x + c)^8 + 32*a^3*n^5 + 720*a^3*n^4
 - (5*a^3*n^7 + 142*a^3*n^6 + 1654*a^3*n^5 + 10180*a^3*n^4 + 35485*a^3*n^3
+ 69358*a^3*n^2 + 69416*a^3*n + 26880*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 +
23520*a^3*n^2 + 2*(2*a^3*n^7 + 49*a^3*n^6 + 470*a^3*n^5 + 2230*a^3*n^4 + 54
38*a^3*n^3 + 6361*a^3*n^2 + 2730*a^3*n)*cos(d*x + c)^4 + 39968*a^3*n + 2184
0*a^3 + 8*(2*a^3*n^6 + 45*a^3*n^5 + 380*a^3*n^4 + 1470*a^3*n^3 + 2498*a^3*n
^2 + 1365*a^3*n)*cos(d*x + c)^2 + (32*a^3*n^5 + 720*a^3*n^4 - 3*(a^3*n^7 +
29*a^3*n^6 + 343*a^3*n^5 + 2135*a^3*n^4 + 7504*a^3*n^3 + 14756*a^3*n^2 + 14
832*a^3*n + 5760*a^3)*cos(d*x + c)^6 + 6080*a^3*n^3 + 24000*a^3*n^2 + 2*(2*
a^3*n^7 + 53*a^3*n^6 + 566*a^3*n^5 + 3155*a^3*n^4 + 9908*a^3*n^3 + 17492*a^
3*n^2 + 15984*a^3*n + 5760*a^3)*cos(d*x + c)^4 + 44288*a^3*n + 30720*a^3 +
8*(2*a^3*n^6 + 47*a^3*n^5 + 425*a^3*n^4 + 1880*a^3*n^3 + 4268*a^3*n^2 + 468
8*a^3*n + 1920*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 3
6*d*n^7 + 546*d*n^6 + 4536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2
 + 109584*d*n + 40320*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 23312 vs. $2(155) = 310$.
time = 43.46, size = 23312, normalized size = 128.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**3*sin(c)**n*cos(c)**5, Eq(d, 0)), (a**3*log(si
n(c + d*x))/d - 8*a**3/(5*d*sin(c + d*x)) + a**3*cos(c + d*x)**2/(2*d*sin(c
 + d*x)**2) - a**3/(2*d*sin(c + d*x)**2) + 4*a**3*cos(c + d*x)**2/(5*d*sin(
c + d*x)**3) - 8*a**3/(105*d*sin(c + d*x)**3) - a**3*cos(c + d*x)**4/(4*d*s
in(c + d*x)**4) + a**3*cos(c + d*x)**2/(2*d*sin(c + d*x)**4) - 3*a**3*cos(c
```

+ d*x)**4/(5*d*sin(c + d*x)**5) + 4*a**3*cos(c + d*x)**2/(35*d*sin(c + d*x)**5) - a**3*cos(c + d*x)**4/(2*d*sin(c + d*x)**6) - a**3*cos(c + d*x)**4/(7*d*sin(c + d*x)**7), Eq(n, -8)), (3*a**3*log(sin(c + d*x))/d + 8*a**3*sin(c + d*x)/(3*d) + 4*a**3*cos(c + d*x)**2/(3*d*sin(c + d*x)) - 8*a**3/(5*d*sin(c + d*x)) + 3*a**3*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) - a**3/(6*d*sin(c + d*x)**2) - a**3*cos(c + d*x)**4/(3*d*sin(c + d*x)**3) + 4*a**3*cos(c + d*x)**2/(5*d*sin(c + d*x)**3) - 3*a**3*cos(c + d*x)**4/(4*d*sin(c + d*x)**4) + a**3*cos(c + d*x)**2/(6*d*sin(c + d*x)**4) - 3*a**3*cos(c + d*x)**4/(5*d*sin(c + d*x)**5) - a**3*cos(c + d*x)**4/(6*d*sin(c + d*x)**6), Eq(n, -7)), (-960*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**9/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 1920*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 960*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 960*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**9/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 1920*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**7/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 960*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 6*a**3*tan(c/2 + d*x/2)**14/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 45*a**3*tan(c/2 + d*x/2)**13/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 82*a**3*tan(c/2 + d*x/2)**12/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 330*a**3*tan(c/2 + d*x/2)**11/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 2074*a**3*tan(c/2 + d*x/2)**10/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 330*a**3*tan(c/2 + d*x/2)**9/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 12350*a**3*tan(c/2 + d*x/2)**8/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 510*a**3*tan(c/2 + d*x/2)**7/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 12350*a**3*tan(c/2 + d*x/2)**6/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 330*a**3*tan(c/2 + d*x/2)**5/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 2074*a**3*tan(c/2 + d*x/2)**4/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) + 330*a**3*tan(c/2 + d*x/2)**3/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 82*a**3*tan(c/2 + d*x/2)**2/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 45*a**3*tan(c/2 + d*x/2)/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5) - 6*a**3/(960*d*tan(c/2 + d*x/2)**9 + 1920*d*tan(c/2 + d*x/2)**7 + 960*d*tan(c/2 + d*x/2)**5), Eq(n, -6)), (960*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**

```
8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 2880*a**3*log(
tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**10 +
576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x
/2)**4) + 2880*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(192*d
*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)*
*6 + 192*d*tan(c/2 + d*x/2)**4) + 960*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan
(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 +
576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 960*a**3*log(tan(c
/2 + d*x/2))*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c
/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 2
880*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)*
*10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2
 + d*x/2)**4) - 2880*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(192*d*
tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**
6 + 192*d*tan(c/2 + d*x/2)**4) - 960*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d
*x/2)**4/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*ta
n(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 769 vs. 2(181) = 362.

time = 0.58, size = 769, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^8 + 10*n*sin(d*x + c)^n*sin(d*x + c)^8 -
2*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 24*sin(d*x + c)^n*sin(d*x + c)^8 - 24
*n*sin(d*x + c)^n*sin(d*x + c)^6 + n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 64*s
in(d*x + c)^n*sin(d*x + c)^6 + 14*n*sin(d*x + c)^n*sin(d*x + c)^4 + 48*sin(
d*x + c)^n*sin(d*x + c)^4)*a^3/(n^3 + 18*n^2 + 104*n + 192) + 3*(n^2*sin(d*
x + c)^n*sin(d*x + c)^7 + 8*n*sin(d*x + c)^n*sin(d*x + c)^7 - 2*n^2*sin(d*x
 + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 - 20*n*sin(d*x +
c)^n*sin(d*x + c)^5 + n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 42*sin(d*x + c)^n
*sin(d*x + c)^5 + 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + 35*sin(d*x + c)^n*si
n(d*x + c)^3)*a^3/(n^3 + 15*n^2 + 71*n + 105) + 3*(n^2*sin(d*x + c)^n*sin(d
*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(d*
x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x +
c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)^4
 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)*a
^3/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*si
n(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d
*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x
 + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^
```

```
n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a^3/(n^3 + 9*n^2 + 23*n +
15))/d
```

Mupad [B]
time = 15.45, size = 923, normalized size = 5.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (a^3*sin(c + d*x)^n*(3757604*n + 2585492*n^2 + 870443*n^3 + 162200*n^4 + 17
366*n^5 + 1028*n^6 + 27*n^7 + 1896720))/(128*d*(109584*n + 118124*n^2 + 672
84*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*sin
(c + d*x)^n*cos(8*c + 8*d*x)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 3
22*n^5 + 28*n^6 + n^7 + 5040))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 +
22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*sin(c + d*x)
*sin(c + d*x)^n*(n*3467760i + n^2*2140836i + n^3*675728i + n^4*118935i + n^
5*11975i + n^6*669i + n^7*17i + 2217600i)*1i)/(64*d*(109584*n + 118124*n^2
+ 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^
3*sin(c + d*x)^n*cos(6*c + 6*d*x)*(43280*n + 43094*n^2 + 21947*n^3 + 6260*n
^4 + 1010*n^5 + 86*n^6 + 3*n^7 + 16800))/(32*d*(109584*n + 118124*n^2 + 672
84*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*sin
(c + d*x)^n*cos(4*c + 4*d*x)*(303180*n + 273336*n^2 + 122023*n^3 + 29520*n^
4 + 3910*n^5 + 264*n^6 + 7*n^7 + 126000))/(32*d*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a^3*si
n(c + d*x)^n*cos(2*c + 2*d*x)*(596208*n + 333226*n^2 + 75333*n^3 + 5260*n^4
 - 498*n^5 - 86*n^6 - 3*n^7 + 332640))/(32*d*(109584*n + 118124*n^2 + 67284
*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*sin(c
 + d*x)^n*sin(7*c + 7*d*x)*(n*14832i + n^2*14756i + n^3*7504i + n^4*2135i +
 n^5*343i + n^6*29i + n^7*1i + 5760i)*3i)/(64*d*(109584*n + 118124*n^2 + 67
284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a^3*si
n(c + d*x)^n*sin(5*c + 5*d*x)*(n*94608i + n^2*81404i + n^3*33296i + n^4*678
5i + n^5*617i + n^6*11i - n^7*1i + 40320i)*1i)/(64*d*(109584*n + 118124*n^2
 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a
^3*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*583216i + n^2*567700i + n^3*275824i +
 n^4*72475i + n^5*10339i + n^6*745i + n^7*21i + 228480i)*1i)/(64*d*(109584*
n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
+ 40320))
```

3.566 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)}-\frac{a^2\sin^{3+n}(c+dx)}{d(3+n)}-\frac{4a^2\sin^{4+n}(c+dx)}{d(4+n)}-\frac{a^2\sin^{5+n}(c+dx)}{d(5+n)}+\frac{2a^2\sin^{6}}{d(6}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)-a^2*sin(d*x+c)^
(3+n)/d/(3+n)-4*a^2*sin(d*x+c)^(4+n)/d/(4+n)-a^2*sin(d*x+c)^(5+n)/d/(5+n)+2
*a^2*sin(d*x+c)^(6+n)/d/(6+n)+a^2*sin(d*x+c)^(7+n)/d/(7+n)
```

Rubi [A]

time = 0.12, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 90}

$$\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2\sin^{n+2}(c+dx)}{d(n+2)}-\frac{a^2\sin^{n+3}(c+dx)}{d(n+3)}-\frac{4a^2\sin^{n+4}(c+dx)}{d(n+4)}-\frac{a^2\sin^{n+5}(c+dx)}{d(n+5)}+\frac{2a^2\sin^{n+6}(c+dx)}{d(n+6)}+\frac{a^2\sin^{n+7}(c+dx)}{d(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a^2*Sin[c + d*x]^(4 +
n))/(d*(4 + n)) - (a^2*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (2*a^2*Sin[c + d
*x]^(6 + n))/(d*(6 + n)) + (a^2*Sin[c + d*x]^(7 + n))/(d*(7 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n (a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6\left(\frac{x}{a}\right)^n + 2a^6\left(\frac{x}{a}\right)^{1+n} - a^6\left(\frac{x}{a}\right)^{2+n} - 4a^6\left(\frac{x}{a}\right)^{3+}\right.\right.}{}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)} - \frac{a^2\sin^{3+n}(c+}{d(3+n)}$$

Mathematica [A]
time = 0.28, size = 110, normalized size = 0.69

$$\frac{a^2\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{2\sin(c+dx)}{2+n}-\frac{\sin^2(c+dx)}{3+n}-\frac{4\sin^3(c+dx)}{4+n}-\frac{\sin^4(c+dx)}{5+n}+\frac{2\sin^5(c+dx)}{6+n}+\frac{\sin^6(c+dx)}{7+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - Sin[c
+ d*x]^2/(3 + n) - (4*Sin[c + d*x]^3)/(4 + n) - Sin[c + d*x]^4/(5 + n) + (2
*Sin[c + d*x]^5)/(6 + n) + Sin[c + d*x]^6/(7 + n)))/d
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [A]
time = 0.28, size = 143, normalized size = 0.89

$$\frac{\frac{a^2\sin(dx+c)^{n+7}}{n+7}+\frac{2\,a^2\sin(dx+c)^{n+6}}{n+6}-\frac{a^2\sin(dx+c)^{n+5}}{n+5}-\frac{4\,a^2\sin(dx+c)^{n+4}}{n+4}-\frac{a^2\sin(dx+c)^{n+3}}{n+3}+\frac{2\,a^2\sin(dx+c)^{n+2}}{n+2}+\frac{a^2\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a^2*sin(d*x + c)^(n + 7)/(n + 7) + 2*a^2*sin(d*x + c)^(n + 6)/(n + 6) - a^
2*sin(d*x + c)^(n + 5)/(n + 5) - 4*a^2*sin(d*x + c)^(n + 4)/(n + 4) - a^2*s
```

```
in(d*x + c)^(n + 3)/(n + 3) + 2*a^2*sin(d*x + c)^(n + 2)/(n + 2) + a^2*sin(
d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(160) = 320$.
time = 0.43, size = 473, normalized size = 2.96

$$-\frac{\left(2\left(a^{2}n^{6}+22\,a^{2}n^{5}+190\,a^{2}n^{4}+820\,a^{2}n^{3}+1849\,a^{2}n^{2}+2038\,a^{2}n+840\,a^{2}\right)\cos\left(dx+c\right)^{6}-16\,a^{2}n^{4}-256\,a^{2}n^{3}-2\left(a^{2}n^{6}+18\,a^{2}n^{5}+118\,a^{2}n^{4}+348\,a^{2}n^{3}+457\,a^{2}n^{2}+210\,a^{2}n\right)\cos\left(dx+c\right)^{4}-1376\,a^{2}n^{2}-2816\,a^{2}n-8\left(a^{2}n^{5}+16\,a^{2}n^{4}+86\,a^{2}n^{3}+176\,a^{2}n^{2}+105\,a^{2}n\right)\cos\left(dx+c\right)^{2}-1680\,a^{2}+\left(\left(a^{2}n^{6}+21\,a^{2}n^{5}+175\,a^{2}n^{4}+735\,a^{2}n^{3}+1624\,a^{2}n^{2}+1764\,a^{2}n+720\,a^{2}\right)\cos\left(dx+c\right)^{6}-16\,a^{2}n^{4}-256\,a^{2}n^{3}-2\left(a^{2}n^{6}+20\,a^{2}n^{5}+159\,a^{2}n^{4}+640\,a^{2}n^{3}+1364\,a^{2}n^{2}+1440\,a^{2}n+576\,a^{2}\right)\cos\left(dx+c\right)^{4}-1472\,a^{2}n^{2}-3584\,a^{2}n-8\left(a^{2}n^{5}+17\,a^{2}n^{4}+108\,a^{2}n^{3}+316\,a^{2}n^{2}+416\,a^{2}n+192\,a^{2}\right)\cos\left(dx+c\right)^{2}-3072\,a^{2}\right)\sin\left(dx+c\right)\right)\sin\left(dx+c\right)^{n}}{dn^{7}+28\,dn^{6}+322\,dn^{5}+1960\,dn^{4}+6769\,dn^{3}+13132\,dn^{2}+13068\,dn+5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^6 + 22*a^2*n^5 + 190*a^2*n^4 + 820*a^2*n^3 + 1849*a^2*n^2 + 2038
*a^2*n + 840*a^2)*cos(d*x + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 +
18*a^2*n^5 + 118*a^2*n^4 + 348*a^2*n^3 + 457*a^2*n^2 + 210*a^2*n)*cos(d*x +
 c)^4 - 1376*a^2*n^2 - 2816*a^2*n - 8*(a^2*n^5 + 16*a^2*n^4 + 86*a^2*n^3 +
176*a^2*n^2 + 105*a^2*n)*cos(d*x + c)^2 - 1680*a^2 + ((a^2*n^6 + 21*a^2*n^5
 + 175*a^2*n^4 + 735*a^2*n^3 + 1624*a^2*n^2 + 1764*a^2*n + 720*a^2)*cos(d*x
 + c)^6 - 16*a^2*n^4 - 256*a^2*n^3 - 2*(a^2*n^6 + 20*a^2*n^5 + 159*a^2*n^4
+ 640*a^2*n^3 + 1364*a^2*n^2 + 1440*a^2*n + 576*a^2)*cos(d*x + c)^4 - 1472*
a^2*n^2 - 3584*a^2*n - 8*(a^2*n^5 + 17*a^2*n^4 + 108*a^2*n^3 + 316*a^2*n^2
+ 416*a^2*n + 192*a^2)*cos(d*x + c)^2 - 3072*a^2)*sin(d*x + c))*sin(d*x + c
)^n/(d*n^7 + 28*d*n^6 + 322*d*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 +
 13068*d*n + 5040*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 14997 vs. $2(134) = 268$.
time = 21.57, size = 14997, normalized size = 93.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**2*sin(c)**n*cos(c)**5, Eq(d, 0)), (a**2*log(si
n(c + d*x))/d - 16*a**2/(15*d*sin(c + d*x)) + a**2*cos(c + d*x)**2/(2*d*sin
(c + d*x)**2) - a**2/(6*d*sin(c + d*x)**2) + 8*a**2*cos(c + d*x)**2/(15*d*s
in(c + d*x)**3) - a**2*cos(c + d*x)**4/(4*d*sin(c + d*x)**4) + a**2*cos(c +
 d*x)**2/(6*d*sin(c + d*x)**4) - 2*a**2*cos(c + d*x)**4/(5*d*sin(c + d*x)**
5) - a**2*cos(c + d*x)**4/(6*d*sin(c + d*x)**6), Eq(n, -7)), (2*a**2*log(si
n(c + d*x))/d + 8*a**2*sin(c + d*x)/(3*d) + 4*a**2*cos(c + d*x)**2/(3*d*sin
(c + d*x)) - 8*a**2/(15*d*sin(c + d*x)) + a**2*cos(c + d*x)**2/(d*sin(c + d
*x)**2) - a**2*cos(c + d*x)**4/(3*d*sin(c + d*x)**3) + 4*a**2*cos(c + d*x)*
*2/(15*d*sin(c + d*x)**3) - a**2*cos(c + d*x)**4/(2*d*sin(c + d*x)**4) - a*
*2*cos(c + d*x)**4/(5*d*sin(c + d*x)**5), Eq(n, -6)), (192*a**2*log(tan(c/2
```

+ d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*ta
n(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 384*a**2*log(tan(c/2 + d*x
/2)**2 + 1)*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2
+ d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 192*a**2*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/
2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 192*a**2*log(tan(c/2 + d*x/2))*tan(c/2
+ d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d
*tan(c/2 + d*x/2)**4) - 384*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/
(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*
x/2)**4) - 192*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(192*d*tan(c/
2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 3*
a**2*tan(c/2 + d*x/2)**12/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/
2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 16*a**2*tan(c/2 + d*x/2)**11/(192*d*ta
n(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4)
+ 6*a**2*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 +
d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 304*a**2*tan(c/2 + d*x/2)**9/(192*
d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)*
*4) + 1760*a**2*tan(c/2 + d*x/2)**7/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(
c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 366*a**2*tan(c/2 + d*x/2)**6
/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d
*x/2)**4) + 1760*a**2*tan(c/2 + d*x/2)**5/(192*d*tan(c/2 + d*x/2)**8 + 384*
d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 304*a**2*tan(c/2 + d*x
/2)**3/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c
/2 + d*x/2)**4) + 6*a**2*tan(c/2 + d*x/2)**2/(192*d*tan(c/2 + d*x/2)**8 + 3
84*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 16*a**2*tan(c/2 + d
*x/2)/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/
2 + d*x/2)**4) - 3*a**2/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)
6 + 192*d*tan(c/2 + d*x/2)4), Eq(n, -5)), (96*a**2*log(tan(c/2 + d*x/2)
2 + 1)*tan(c/2 + d*x/2)9/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x
/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 288*a**2*lo
g(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**9 +
72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)
3) + 288*a2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(24*d*tan(
c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*
d*tan(c/2 + d*x/2)**3) + 96*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x
/2)**3/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2
+ d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 96*a**2*log(tan(c/2 + d*x/2))*tan
(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*
d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 288*a**2*log(tan(c/2 +
d*x/2))*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/
2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 288*a**2*log
(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan
(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 9
6*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**9
+ 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/

```
2)**3) - a**2*tan(c/2 + d*x/2)**12/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2
 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 6*a**
2*tan(c/2 + d*x/2)**11/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7
 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 6*a**2*tan(c/2 +
d*x/2)**10/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(
c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 15*a**2*tan(c/2 + d*x/2)**8/(
24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)
**5 + 24*d*tan(c/2 + d*x/2)**3) + 126*a**2*tan(c/2 + d*x/2)**7/(24*d*tan(c/
2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*
tan(c/2 + d*x/2)**3) + 20*a**2*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**
9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 576 vs. 2(160) = 320.
time = 0.57, size = 576, normalized size = 3.60

$$\frac{\frac{\left(n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{7}+8 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{7}-2 n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}+15 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{7}-20 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}+n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}-42 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}+12 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}+35 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}\right) a^{2}}{n^{3}+15 n^{2}+71 n+105}+\frac{2 \left(n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{6}+6 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{6}-2 n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{4}+8 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{6}-16 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{4}+n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{2}-24 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{4}+10 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{2}+24 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{2}\right) a^{2}}{n^{3}+12 n^{2}+44 n+48}+\frac{\left(n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}+4 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}-2 n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}+3 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{5}-12 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}+n^{2} \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)-10 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)^{3}+8 n \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)+15 \sin\left(dx+c\right)^{n} \sin\left(dx+c\right)\right) a^{2}}{n^{3}+9 n^{2}+23 n+15}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 8*n*sin(d*x + c)^n*sin(d*x + c)^7 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 - 20*
n*sin(d*x + c)^n*sin(d*x + c)^5 + n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 42*si
n(d*x + c)^n*sin(d*x + c)^5 + 12*n*sin(d*x + c)^n*sin(d*x + c)^3 + 35*sin(d
*x + c)^n*sin(d*x + c)^3)*a^2/(n^3 + 15*n^2 + 71*n + 105) + 2*(n^2*sin(d*x
+ c)^n*sin(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x +
 c)^n*sin(d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^
n*sin(d*x + c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*si
n(d*x + c)^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d
*x + c)^2)*a^2/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c
)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)
^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(d*x + c)^3 +
 n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x + c)^3 + 8*n*s
in(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a^2/(n^3 + 9*n
^2 + 23*n + 15))/d
```

Mupad [B]
time = 14.66, size = 819, normalized size = 5.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(n*16958i + n^2*10137i + n^3*2788i + n^4*398i + n^5*30i
 + n^6*1i + 9240i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n
^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*sin(7*c + 7*d*x)
*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)*1i)/(64*d*(n*
13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i +
 5040i)) + (a^2*sin(c + d*x)*sin(c + d*x)^n*(296844*n + 148360*n^2 + 36773*
n^3 + 4869*n^4 + 343*n^5 + 11*n^6 + 226800)*1i)/(64*d*(n*13068i + n^2*13132
i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*si
n(c + d*x)^n*cos(6*c + 6*d*x)*(n*2038i + n^2*1849i + n^3*820i + n^4*190i +
n^5*22i + n^6*1i + 840i))/(16*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*19
60i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*cos(4*c +
 4*d*x)*(n*5694i + n^2*4633i + n^3*1764i + n^4*334i + n^5*30i + n^6*1i + 25
20i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*
28i + n^7*1i + 5040i)) - (a^2*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n*20490i + n
^2*9159i + n^3*1228i - n^4*62i - n^5*22i - n^6*1i + 12600i))/(16*d*(n*13068
i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040
i)) + (a^2*sin(c + d*x)^n*sin(5*c + 5*d*x)*(2700*n + 2792*n^2 + 1445*n^3 +
397*n^4 + 55*n^5 + 3*n^6 + 1008)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*676
9i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (a^2*sin(c + d*x)^
n*sin(3*c + 3*d*x)*(71932*n + 58568*n^2 + 22569*n^3 + 4417*n^4 + 419*n^5 +
15*n^6 + 31920)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i +
n^5*322i + n^6*28i + n^7*1i + 5040i))
```

3.567 $\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{2a\sin^{4+n}(c+dx)}{d(4+n)}+\frac{a\sin^{5+n}(c+dx)}{d(5+n)}+\frac{a\sin^{6+n}(c+}{d(6+n)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)-2*a*sin(d*x+c)^(3+n)/
d/(3+n)-2*a*sin(d*x+c)^(4+n)/d/(4+n)+a*sin(d*x+c)^(5+n)/d/(5+n)+a*sin(d*x+c
)^(6+n)/d/(6+n)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2915, 90}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{2a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (a*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)^3\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n + a^5\left(\frac{x}{a}\right)^{1+n} - 2a^5\left(\frac{x}{a}\right)^{2+n} - 2a^5\left(\frac{x}{a}\right)^{3+n}\right.\right.}{a^5 d}$$
$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{a\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a\sin^{3+n}(c+dx}{d(3+n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 345 vs. 2(123) = 246.
time = 0.96, size = 345, normalized size = 2.80

$$\frac{a\sin^{1+n}(c+dx)\left(8544+10520n+4888n^2+1114n^3+128n^4+6n^5+8(336+692n+484n^2+147n^3+20n^4+n^5)\cos(2(c+dx))+2(144+324n+260n^2+95n^3+16n^4+n^5)\cos(4(c+dx))+2640\sin(c+dx)+4468n\sin(c+dx)+2258n^2\sin(c+dx)+474n^3\sin(c+dx)+46n^4\sin(c+dx)+2n^5\sin(c+dx)+840\sin(3(c+dx))+1798n\sin(3(c+dx))+1331n^2\sin(3(c+dx))+431n^3\sin(3(c+dx))+61n^4\sin(3(c+dx))+3n^5\sin(3(c+dx))+120\sin(5(c+dx))+274n\sin(5(c+dx))+225n^2\sin(5(c+dx))+85n^3\sin(5(c+dx))+15n^4\sin(5(c+dx))+n^5\sin(5(c+dx))\right)}{16d(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(8544 + 10520*n + 4888*n^2 + 1114*n^3 + 128*n^4 + 6
*n^5 + 8*(336 + 692*n + 484*n^2 + 147*n^3 + 20*n^4 + n^5)*Cos[2*(c + d*x)]
+ 2*(144 + 324*n + 260*n^2 + 95*n^3 + 16*n^4 + n^5)*Cos[4*(c + d*x)] + 2640
*Sin[c + d*x] + 4468*n*Sin[c + d*x] + 2258*n^2*Sin[c + d*x] + 474*n^3*Sin[c
 + d*x] + 46*n^4*Sin[c + d*x] + 2*n^5*Sin[c + d*x] + 840*Sin[3*(c + d*x)] +
 1798*n*Sin[3*(c + d*x)] + 1331*n^2*Sin[3*(c + d*x)] + 431*n^3*Sin[3*(c + d
*x)] + 61*n^4*Sin[3*(c + d*x)] + 3*n^5*Sin[3*(c + d*x)] + 120*Sin[5*(c + d*
x)] + 274*n*Sin[5*(c + d*x)] + 225*n^2*Sin[5*(c + d*x)] + 85*n^3*Sin[5*(c +
 d*x)] + 15*n^4*Sin[5*(c + d*x)] + n^5*Sin[5*(c + d*x)]))/(16*d*(1 + n)*(2
+ n)*(3 + n)*(4 + n)*(5 + n)*(6 + n))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [A]
time = 0.28, size = 109, normalized size = 0.89

$$\frac{\frac{a\sin(dx+c)^{n+6}}{n+6} + \frac{a\sin(dx+c)^{n+5}}{n+5} - \frac{2\,a\sin(dx+c)^{n+4}}{n+4} - \frac{2\,a\sin(dx+c)^{n+3}}{n+3} + \frac{a\sin(dx+c)^{n+2}}{n+2} + \frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*sin(d*x + c)^(n + 6)/(n + 6) + a*sin(d*x + c)^(n + 5)/(n + 5) - 2*a*sin(
d*x + c)^(n + 4)/(n + 4) - 2*a*sin(d*x + c)^(n + 3)/(n + 3) + a*sin(d*x + c
)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(123) = 246$.
time = 0.40, size = 282, normalized size = 2.29

$$-\frac{\left((an^5+15\,an^4+85\,an^3+225\,an^2+274\,an+120\,a)\cos\left(dx+c\right)^6-(an^5+11\,an^4+41\,an^3+61\,an^2+30\,an)\cos\left(dx+c\right)^4-8\,an^3-72\,an^2-4\,(an^4+9\,an^3+23\,an^2+15\,an)\cos\left(dx+c\right)^2-184\,an-\left((an^5+16\,an^4+95\,an^3+260\,an^2+324\,an+144\,a)\cos\left(dx+c\right)^4+8\,an^3+96\,an^2+4\,(an^4+13\,an^3+56\,an^2+92\,an+48\,a)\cos\left(dx+c\right)^2+352\,an+384\,a\right)\sin\left(dx+c\right)-120\,a\right)\sin\left(dx+c\right)^n}{dn^6+21\,dn^5+175\,dn^4+735\,dn^3+1624\,dn^2+1764\,dn+720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^5 + 15*a*n^4 + 85*a*n^3 + 225*a*n^2 + 274*a*n + 120*a)*cos(d*x + c)^
6 - (a*n^5 + 11*a*n^4 + 41*a*n^3 + 61*a*n^2 + 30*a*n)*cos(d*x + c)^4 - 8*a*
n^3 - 72*a*n^2 - 4*(a*n^4 + 9*a*n^3 + 23*a*n^2 + 15*a*n)*cos(d*x + c)^2 - 1
84*a*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*a)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8675 vs. $2(104) = 208$.
time = 13.32, size = 8675, normalized size = 70.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c)**5, Eq(d, 0)), (a*log(sin(c +
d*x))/d - 8*a/(15*d*sin(c + d*x)) + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2)
 + 4*a*cos(c + d*x)**2/(15*d*sin(c + d*x)**3) - a*cos(c + d*x)**4/(4*d*sin(
c + d*x)**4) - a*cos(c + d*x)**4/(5*d*sin(c + d*x)**5), Eq(n, -6)), (a*log(
sin(c + d*x))/d + 8*a*sin(c + d*x)/(3*d) + 4*a*cos(c + d*x)**2/(3*d*sin(c +
 d*x)) + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) - a*cos(c + d*x)**4/(3*d*s
in(c + d*x)**3) - a*cos(c + d*x)**4/(4*d*sin(c + d*x)**4), Eq(n, -5)), (48*
a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**
7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 96*a*log(tan(c/2
 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(
c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 48*a*log(tan(c/2 + d*x/2)**2
```

```
+ 1)*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)*
*5 + 24*d*tan(c/2 + d*x/2)**3) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2
)**7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 +
d*x/2)**3) - 96*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 +
 d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 48*a*lo
g(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**7 + 48*d*ta
n(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - a*tan(c/2 + d*x/2)**10/(24*
d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3
) - 3*a*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/
2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)**8/(24*d*tan(c/2
+ d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 110*a*
tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 +
24*d*tan(c/2 + d*x/2)**3) + 54*a*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)
**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 110*a*tan(c/2
+ d*x/2)**4/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan
(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2)**7 + 48
*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 3*a*tan(c/2 + d*x/2)/(
24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)
**3) - a/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/
2 + d*x/2)**3), Eq(n, -4)), (48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*
x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2
 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c/2 + d*x/2)**2 +
1)*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6
 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c/2
 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(
c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 48
*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2)*
*8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d
*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 +
d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(
c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(24*d*ta
n(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 2
4*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/
(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2
)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x
/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2
+ d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a*tan(c/2 + d*x/2)**10/(24*d*ta
n(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 2
4*d*tan(c/2 + d*x/2)**2) - 12*a*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)*
*8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d
*x/2)**2) - 144*a*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(
c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63
*a*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6
 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 200*a*tan(c/2 + d
*x/2)**5/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/
```

```
2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**4/(24*d*
tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 +
 24*d*tan(c/2 + d*x/2)**2) - 144*a*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/
2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2
+ d*x/2)**2) - 12*a*tan(c/2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c
/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a
/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/
2)**4 + 24*d*tan(c/2 + d*x/2)**2), Eq(n, -3)), (-6*a*log(tan(c/2 + d*x/2)**
2 + 1)*tan(c/2 + d*x/2)**9/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)
**7 + 36*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d
*x/2)) - 24*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(123) = 246$.

time = 0.46, size = 379, normalized size = 3.08

$$\frac{\frac{\left(n^2\sin(dx+c)^n\sin(dx+c)^6+6\,n\sin(dx+c)^n\sin(dx+c)^6-2\,n^2\sin(dx+c)^n\sin(dx+c)^4+8\sin(dx+c)^n\sin(dx+c)^6-16\,n\sin(dx+c)^n\sin(dx+c)^4+n^2\sin(dx+c)^n\sin(dx+c)^2-24\sin(dx+c)^n\sin(dx+c)^4+10\,n\sin(dx+c)^n\sin(dx+c)^2+24\sin(dx+c)^n\sin(dx+c)^2\right)a}{n^3+12\,n^2+44\,n+48}+\frac{\left(n^2\sin(dx+c)^n\sin(dx+c)^5+4\,n\sin(dx+c)^n\sin(dx+c)^5-2\,n^2\sin(dx+c)^n\sin(dx+c)^3+3\sin(dx+c)^n\sin(dx+c)^5-12\,n\sin(dx+c)^n\sin(dx+c)^3+n^2\sin(dx+c)^n\sin(dx+c)-10\sin(dx+c)^n\sin(dx+c)^3+8\,n\sin(dx+c)^n\sin(dx+c)+15\sin(dx+c)^n\sin(dx+c)\right)a}{n^3+9\,n^2+23\,n+15}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n
*sin(d*x + c)^n*sin(d*x + c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin
(d*x + c)^n*sin(d*x + c)^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*
x + c)^n*sin(d*x + c)^2)*a/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n
*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2*n^2*sin(d*x + c)^n*
sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n*sin(d*x + c)^n*sin(
d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d*x + c)^n*sin(d*x +
c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c)^n*sin(d*x + c))*a/
(n^3 + 9*n^2 + 23*n + 15))/d
```

Mupad [**B**]

time = 13.56, size = 550, normalized size = 4.47

$$\frac{a\sin\left(c+d\,x\right)^n\,\left(n^5+23\,n^4+237\,n^3+1129\,n^2+2234\,n+1320\right)}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\cos\left(6\,c+6\,d\,x\right)\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{32\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\cos\left(4\,c+4\,d\,x\right)\,\left(n^5+23\,n^4+173\,n^3+553\,n^2+762\,n+360\right)}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\,\sin\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^n\,\left(n^5\,1{}\mathrm{i}+n^4\,24{}\mathrm{i}+n^3\,263{}\mathrm{i}+n^2\,1476{}\mathrm{i}+n\,3876{}\mathrm{i}+3600{}\mathrm{i}\right)\,1{}\mathrm{i}}{8\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\cos\left(2\,c+2\,d\,x\right)\,\left(-n^5-15\,n^4+43\,n^3+927\,n^2+2670\,n+1800\right)}{32\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\sin\left(5\,c+5\,d\,x\right)\,\left(n^5\,1{}\mathrm{i}+n^4\,16{}\mathrm{i}+n^3\,95{}\mathrm{i}+n^2\,260{}\mathrm{i}+n\,324{}\mathrm{i}+144{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{a\sin\left(c+d\,x\right)^n\,\sin\left(3\,c+3\,d\,x\right)\,\left(n^5\,3{}\mathrm{i}+n^4\,64{}\mathrm{i}+n^3\,493{}\mathrm{i}+n^2\,1676{}\mathrm{i}+n\,2444{}\mathrm{i}+1200{}\mathrm{i}\right)\,1{}\mathrm{i}}{16\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(2234*n + 1129*n^2 + 237*n^3 + 23*n^4 + n^5 + 1320))/(16*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c
+ d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (a*si
n(c + d*x)^n*cos(4*c + 4*d*x)*(762*n + 553*n^2 + 173*n^3 + 23*n^4 + n^5 + 3
60))/(16*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) -
```

```
(a*sin(c + d*x)*sin(c + d*x)^n*(n*3876i + n^2*1476i + n^3*263i + n^4*24i +
n^5*1i + 3600i)*1i)/(8*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
n^6 + 720)) - (a*sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2 + 43*n^3
 - 15*n^4 - n^5 + 1800))/(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*324i + n^2*260i +
 n^3*95i + n^4*16i + n^5*1i + 144i)*1i)/(16*d*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*24
44i + n^2*1676i + n^3*493i + n^4*64i + n^5*3i + 1200i)*1i)/(16*d*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

3.568 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^{1+n}(c+dx)}{ad(1+n)}-\frac{\sin^{2+n}(c+dx)}{ad(2+n)}-\frac{\sin^{3+n}(c+dx)}{ad(3+n)}+\frac{\sin^{4+n}(c+dx)}{ad(4+n)}$$

```
[Out] sin(d*x+c)^(1+n)/a/d/(1+n)-sin(d*x+c)^(2+n)/a/d/(2+n)-sin(d*x+c)^(3+n)/a/d/
(3+n)+sin(d*x+c)^(4+n)/a/d/(4+n)
```

Rubi [A]
time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 76}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{\sin^{n+4}(c+dx)}{ad(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - S
in[c + d*x]^(3 + n)/(a*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(a*d*(4 + n))
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n(a+x)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n - a^3\left(\frac{x}{a}\right)^{1+n} - a^3\left(\frac{x}{a}\right)^{2+n} + a^3\left(\frac{x}{a}\right)^{3+n}\right)dx, x, a\sin(c+dx\right.}{a^5d}$$
$$= \frac{\sin^{1+n}(c+dx)}{ad(1+n)} - \frac{\sin^{2+n}(c+dx)}{ad(2+n)} - \frac{\sin^{3+n}(c+dx)}{ad(3+n)} + \frac{\sin^{4+n}(c+dx)}{ad(4+n)}$$

Mathematica [A]
time = 0.52, size = 74, normalized size = 0.81

$$\frac{\sin^{1+n}(c+dx)\left(\frac{4+n}{1+n} - \frac{(4+n)\sin(c+dx)}{2+n} - \frac{(4+n)\sin^2(c+dx)}{3+n} + \sin^3(c+dx)\right)}{ad(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((4 + n)/(1 + n) - ((4 + n)*Sin[c + d*x])/(2 + n) - (
(4 + n)*Sin[c + d*x]^2)/(3 + n) + Sin[c + d*x]^3))/(a*d*(4 + n))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(\cos^5(dx+c))(\sin^n(dx+c))}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [A]
time = 0.32, size = 124, normalized size = 1.36

$$\frac{((n^3+6n^2+11n+6)\sin(dx+c)^4 - (n^3+7n^2+14n+8)\sin(dx+c)^3 - (n^3+8n^2+19n+12)\sin(dx+c)^2 + (n^3+9n^2+26n+24)\sin(dx+c))\sin(dx+c)^n}{(n^4+10n^3+35n^2+50n+24)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - (n^3 + 7*n^2 + 14*n + 8)*sin(d*x
 + c)^3 - (n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 + (n^3 + 9*n^2 + 26*n +
24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*a*d)
```

Fricas [A]
time = 0.42, size = 134, normalized size = 1.47

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)\cos\left(dx+c\right)^4-\left(n^3+4\,n^2+3\,n\right)\cos\left(dx+c\right)^2-2\,n^2+\left(\left(n^3+7\,n^2+14\,n+8\right)\cos\left(dx+c\right)^2+2\,n^2+12\,n+16\right)\sin\left(dx+c\right)-8\,n-6\right)\sin\left(dx+c\right)^n}{adn^4+10\,adn^3+35\,adn^2+50\,adn+24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 - (n^3 + 4*n^2 + 3*n)*cos(d*x + c)
^2 - 2*n^2 + ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^2 + 2*n^2 + 12*n + 16)*
sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a*d*n^4 + 10*a*d*n^3 + 35*a*d*n^2 +
 50*a*d*n + 24*a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,3,0,0]%%%}+%%%{-1,[0,1,2,0,0]%%%}+%%%{-1,[0,1,1,0,0]%%
%}+%%%
```

Mupad [B]
time = 10.21, size = 228, normalized size = 2.51

$$\frac{{\sin\left(c+d\,x\right)}^n\,\left(144\,\sin\left(c+d\,x\right)-43\,n+24\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+16\,\sin\left(3\,c+3\,d\,x\right)+124\,n\,\sin\left(c+d\,x\right)+32\,n\,\cos\left(2\,c+2\,d\,x\right)+11\,n\,\cos\left(4\,c+4\,d\,x\right)+28\,n\,\sin\left(3\,c+3\,d\,x\right)+30\,n^2\,\sin\left(c+d\,x\right)+2\,n^3\,\sin\left(c+d\,x\right)-14\,n^2-n^3+8\,n^2\,\cos\left(2\,c+2\,d\,x\right)+6\,n^2\,\cos\left(4\,c+4\,d\,x\right)+n^3\,\cos\left(4\,c+4\,d\,x\right)+14\,n^2\,\sin\left(3\,c+3\,d\,x\right)+2\,n^3\,\sin\left(3\,c+3\,d\,x\right)-30\right)}{8\,a\,d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^n*(144*sin(c + d*x) - 43*n + 24*cos(2*c + 2*d*x) + 6*cos(4*c
+ 4*d*x) + 16*sin(3*c + 3*d*x) + 124*n*sin(c + d*x) + 32*n*cos(2*c + 2*d*x)
 + 11*n*cos(4*c + 4*d*x) + 28*n*sin(3*c + 3*d*x) + 30*n^2*sin(c + d*x) + 2*
n^3*sin(c + d*x) - 14*n^2 - n^3 + 8*n^2*cos(2*c + 2*d*x) + 6*n^2*cos(4*c +
4*d*x) + n^3*cos(4*c + 4*d*x) + 14*n^2*sin(3*c + 3*d*x) + 2*n^3*sin(3*c + 3
*d*x) - 30))/(8*a*d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

3.569 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{1+n}(c+dx)}{a^2d(1+n)}-\frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)}+\frac{\sin^{3+n}(c+dx)}{a^2d(3+n)}$$

```
[Out] sin(d*x+c)^(1+n)/a^2/d/(1+n)-2*sin(d*x+c)^(2+n)/a^2/d/(2+n)+sin(d*x+c)^(3+n
)/a^2/d/(3+n)
```

Rubi [A]
time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2915, 45}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{\sin^{n+3}(c+dx)}{a^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + Sin[c + d*x]^(3 + n)/(a^2*d*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(\frac{x}{a}\right)^n - 2a^2\left(\frac{x}{a}\right)^{1+n} + a^2\left(\frac{x}{a}\right)^{2+n}\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^2d(1+n)} - \frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)} + \frac{\sin^{3+n}(c+dx)}{a^2d(3+n)}$$

Mathematica [A]
time = 0.08, size = 50, normalized size = 0.74

$$\frac{\sin^{1+n}(c+dx)\left(\frac{1}{1+n} - \frac{2\sin(c+dx)}{2+n} + \frac{\sin^2(c+dx)}{3+n}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (2*Sin[c + d*x])/(2 + n) + Sin[c + d*
x]^2/(3 + n)))/(a^2*d)
```

Maple [F]
time = 2.08, size = 0, normalized size = 0.00

$$\int \frac{(\cos^5(dx+c))(\sin^n(dx+c))}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [A]
time = 0.33, size = 81, normalized size = 1.19

$$\frac{\left((n^2+3n+2)\sin(dx+c)^3 - 2(n^2+4n+3)\sin(dx+c)^2 + (n^2+5n+6)\sin(dx+c)\right)\sin(dx+c)^n}{(n^3+6n^2+11n+6)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] ((n^2 + 3*n + 2)*sin(d*x + c)^3 - 2*(n^2 + 4*n + 3)*sin(d*x + c)^2 + (n^2 +
 5*n + 6)*sin(d*x + c))*sin(d*x + c)^n/((n^3 + 6*n^2 + 11*n + 6)*a^2*d)
```

Fricas [A]
time = 0.39, size = 105, normalized size = 1.54

$$\frac{\left(2\left(n^2+4n+3\right)\cos\left(dx+c\right)^2-2n^2-\left(\left(n^2+3n+2\right)\cos\left(dx+c\right)^2-2n^2-8n-8\right)\sin\left(dx+c\right)-8n-6\right)\sin\left(dx+c\right)^n}{a^2dn^3+6a^2dn^2+11a^2dn+6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^2 + 4*n + 3)*cos(d*x + c)^2 - 2*n^2 - ((n^2 + 3*n + 2)*cos(d*x + c)^2
 - 2*n^2 - 8*n - 8)*sin(d*x + c) - 8*n - 6)*sin(d*x + c)^n/(a^2*d*n^3 + 6*a
^2*d*n^2 + 11*a^2*d*n + 6*a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[0,1,2,0,0]%%%}+%%%{-2,[0,1,1,0,0]%%%}+%%%{1,[0,1,0,0,0]%%%
} / %%
```

Mupad [B]
time = 9.54, size = 146, normalized size = 2.15

$$-\frac{\frac{\sin(c+d\,x)^n\,\left(24\sin(c+d\,x)^2-30\,\sin(c+d\,x)+2\,\sin(3\,c+3\,d\,x)\right)}{4}+\frac{n\sin(c+d\,x)^n\,\left(32\sin(c+d\,x)^2-29\,\sin(c+d\,x)+3\,\sin(3\,c+3\,d\,x)\right)}{4}+\frac{n^2\sin(c+d\,x)^n\,\left(8\sin(c+d\,x)^2-7\,\sin(c+d\,x)+\sin(3\,c+3\,d\,x)\right)}{4}}{a^2\,d\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((sin(c + d*x)^n*(2*sin(3*c + 3*d*x) - 30*sin(c + d*x) + 24*sin(c + d*x)^2
))/4 + (n*sin(c + d*x)^n*(3*sin(3*c + 3*d*x) - 29*sin(c + d*x) + 32*sin(c +
 d*x)^2))/4 + (n^2*sin(c + d*x)^n*(sin(3*c + 3*d*x) - 7*sin(c + d*x) + 8*si
n(c + d*x)^2))/4)/(a^2*d*(11*n + 6*n^2 + n^3 + 6))
```

3.570 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=85

$$-\frac{3\sin^{1+n}(c+dx)}{a^3d(1+n)}+\frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3d(1+n)}+\frac{\sin^{2+n}(c+dx)}{a^3d(2+n)}$$

```
[Out] -3*sin(d*x+c)^(1+n)/a^3/d/(1+n)+4*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin
(d*x+c)^(1+n)/a^3/d/(1+n)+sin(d*x+c)^(2+n)/a^3/d/(2+n)
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 90, 66}

$$\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^3d(n+1)}-\frac{3\sin^{n+1}(c+dx)}{a^3d(n+1)}+\frac{\sin^{n+2}(c+dx)}{a^3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + (4*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^3*d*(1 + n)) + Sin[c + d*x]^
(2 + n)/(a^3*d*(2 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a\left(\frac{x}{a}\right)^n + a\left(\frac{x}{a}\right)^{1+n} + \frac{4a^2\left(\frac{x}{a}\right)^n}{a+x}\right) dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{\sin^{2+n}(c+dx)}{a^3 d(2+n)} + \frac{4\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$
$$= -\frac{3\sin^{1+n}(c+dx)}{a^3 d(1+n)} + \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^3 d(1+n)} +$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.75

$$\frac{\sin^{1+n}(c+dx)\left(-3(2+n)+4(2+n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))+(1+n)\sin(c+dx)\right)}{a^3 d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
[Out] (Sin[c + d*x]^(1 + n)*(-3*(2 + n) + 4*(2 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, -Sin[c + d*x]] + (1 + n)*Sin[c + d*x]))/(a^3*d*(1 + n)*(2 + n))
```

Maple [F]
time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)}{\left(a+a\sin(dx+c)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(3*a^3*cos(d*x + c)^2 - 4*a^3 + (a^
3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3, x)
```

3.571 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=88

$$\frac{\sin^{1+n}(c+dx)}{a^4d(1+n)}-\frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d}+\frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)}$$

```
[Out] sin(d*x+c)^(1+n)/a^4/d/(1+n)-4*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*
x+c)^(1+n)/a^4/d+4*sin(d*x+c)^(1+n)/d/(a^4+a^4*sin(d*x+c))
```

Rubi [A]

time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2915, 91, 81, 66}

$$-\frac{4\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d}+\frac{\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+1}(c+dx)}{d\left(a^4\sin(c+dx)+a^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^4*d*(1 + n)) - (4*Hypergeometric2F1[1, 1 + n, 2 + n
, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d) + (4*Sin[c + d*x]^(1 + n))/(
d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{(a(3+4n)-x)\left(\frac{x}{a}\right)^n}{a+x}\,dx,x,a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^4d(1+n)} + \frac{4\sin^{1+n}(c+dx)}{d\left(a^4+a^4\sin(c+dx)\right)} - \frac{(4(1+n))\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^n}{a+x}\,dx,x\right.}{a^4d}$$
$$= \frac{\sin^{1+n}(c+dx)}{a^4d(1+n)} - \frac{4\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d} + \cdots$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 0.82

$$\frac{\sin^{1+n}(c+dx)(5+4n+\sin(c+dx)-4(1+n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))(1+\sin(c+dx)))}{a^4d(1+n)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
[Out] (Sin[c + d*x]^(1 + n)*(5 + 4*n + Sin[c + d*x] - 4*(1 + n)*Hypergeometric2F1
[1, 1 + n, 2 + n, -Sin[c + d*x]]*(1 + Sin[c + d*x])))/(a^4*d*(1 + n)*(1 + S
in[c + d*x]))
```

Maple [F]
time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)}{\left(a+a\sin(dx+c)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(a*sin(d*x + c) + a)^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

3.572 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{3ax}{256}-\frac{a\cos^7(c+dx)}{7d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^{11}(c+dx)}{11d}+\frac{3a\cos(c+dx)\sin(c+dx)}{256d}+\frac{a\cos^3(c+dx)\sin(c+ }{128d}$$

```
[Out] 3/256*a*x-1/7*a*cos(d*x+c)^7/d+2/9*a*cos(d*x+c)^9/d-1/11*a*cos(d*x+c)^11/d+
3/256*a*cos(d*x+c)*sin(d*x+c)/d+1/128*a*cos(d*x+c)^3*sin(d*x+c)/d+1/160*a*c
os(d*x+c)^5*sin(d*x+c)/d-3/80*a*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a*cos(d*x+c)
^7*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 276}

$$-\frac{a\cos^{11}(c+dx)}{11d}+\frac{2a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{3a\sin(c+dx)\cos(c+dx)}{256d}+\frac{3ax}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (2*a*Cos[c + d*x]^9)/(9*d) - (a*Co
s[c + d*x]^11)/(11*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c
+ d*x]^3*Sin[c + d*x])/(128*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) -
(3*a*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3
)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a\int \cos^6(c+dx)\sin$$

$$= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)$$

$$= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+}{10d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d}$$

$$= -\frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+dx)}{11d}$$

$$= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{2a\cos^9(c+dx)}{9d} - \frac{a\cos^{11}(c+}{11d}$$

Mathematica [A]
time = 0.39, size = 121, normalized size = 0.73

$$\frac{a(83160dx - 69300\cos(c+dx) - 23100\cos(3(c+dx)) + 6930\cos(5(c+dx)) + 4950\cos(7(c+dx)) - 770\cos(9(c+dx)) - 630\cos(11(c+dx)) + 13860\sin(2(c+dx)) - 27720\sin(4(c+dx)) - 6930\sin(6(c+dx)) + 3465\sin(8(c+dx)) + 1386\sin(10(c+dx)))}{7096320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(83160*d*x - 69300*Cos[c + d*x] - 23100*Cos[3*(c + d*x)] + 6930*Cos[5*(c
 + d*x)] + 4950*Cos[7*(c + d*x)] - 770*Cos[9*(c + d*x)] - 630*Cos[11*(c + d
*x)] + 13860*Sin[2*(c + d*x)] - 27720*Sin[4*(c + d*x)] - 6930*Sin[6*(c + d*
x)] + 3465*Sin[8*(c + d*x)] + 1386*Sin[10*(c + d*x)]))/(7096320*d)
```

Maple [A]

time = 0.34, size = 134, normalized size = 0.81

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3c}{256}$
risch	$-\frac{5a\cos(dx+c)}{512d}+\frac{3ax}{256}-\frac{a\cos(11dx+11c)}{11264d}+\frac{a\sin(10dx+10c)}{5120d}-\frac{a\cos(9dx+9c)}{9216d}+\frac{a\sin(8dx+8c)}{2048d}+\frac{5a\cos(7dx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*
(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*
c)+a*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*
cos(d*x+c)^7))
```

Maxima [A]

time = 0.28, size = 86, normalized size = 0.52

$$-\frac{10240\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a-693\left(32\sin(2dx+2c)^{5}+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)a}{7096320d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c
)^7)*a - 693*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c)
- 40*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]

time = 0.43, size = 106, normalized size = 0.64

$$-\frac{80640a\cos(dx+c)^{11}-197120a\cos(dx+c)^{9}+126720a\cos(dx+c)^{7}-10395adx-693\left(128a\cos(dx+c)^{9}-176a\cos(dx+c)^{7}+8a\cos(dx+c)^{5}+10a\cos(dx+c)^{3}+15a\cos(dx+c)\right)\sin(dx+c)}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/887040*(80640*a*cos(d*x + c)^11 - 197120*a*cos(d*x + c)^9 + 126720*a*cos(d*x + c)^7 - 10395*a*d*x - 693*(128*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^7 + 8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(153) = 306$.
time = 2.56, size = 318, normalized size = 1.93

$$\begin{cases} \frac{3ax\sin^{10}(c+dx)}{256} + \frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256} + \frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128} + \frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128} + \frac{15ax\sin^{2}(c+dx)\cos^{8}(c+dx)}{256} + \frac{3ax\cos^{10}(c+dx)}{256} + \frac{3a\sin^{9}(c+dx)\cos(c+dx)}{256d} + \frac{7a\sin^{7}(c+dx)\cos^{3}(c+dx)}{128d} + \frac{a\sin^{5}(c+dx)\cos^{5}(c+dx)}{10d} - \frac{a\sin^{4}(c+dx)\cos^{7}(c+dx)}{7d} - \frac{7a\sin^{3}(c+dx)\cos^{7}(c+dx)}{128d} - \frac{4a\sin^{2}(c+dx)\cos^{9}(c+dx)}{63d} - \frac{3a\sin(c+dx)\cos^{9}(c+dx)}{256d} - \frac{8a\cos^{11}(c+dx)}{693d} & \text{for}\ d \neq 0 \\ x(a\sin(c)+a)\sin^{4}(c)\cos^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)

[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - a*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7*a*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - 4*a*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 3*a*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 8*a*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**4*cos(c)**6, True))

Giac [A]
time = 0.52, size = 167, normalized size = 1.01

$$\frac{3}{256}ax - \frac{a\cos(11dx+11c)}{11264d} - \frac{a\cos(9dx+9c)}{9216d} + \frac{5a\cos(7dx+7c)}{7168d} + \frac{a\cos(5dx+5c)}{1024d} - \frac{5a\cos(3dx+3c)}{1536d} - \frac{5a\cos(dx+c)}{512d} + \frac{a\sin(10dx+10c)}{5120d} + \frac{a\sin(8dx+8c)}{2048d} - \frac{a\sin(6dx+6c)}{1024d} - \frac{a\sin(4dx+4c)}{256d} + \frac{a\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 3/256*a*x - 1/11264*a*cos(11*d*x + 11*c)/d - 1/9216*a*cos(9*d*x + 9*c)/d + 5/7168*a*cos(7*d*x + 7*c)/d + 1/1024*a*cos(5*d*x + 5*c)/d - 5/1536*a*cos(3*d*x + 3*c)/d - 5/512*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*c)/d + 1/2048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*sin(4*d*x + 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d

Mupad [B]
time = 11.87, size = 447, normalized size = 2.71

$$\frac{3ax}{256} + \frac{\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{128} + \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{4} - \frac{3323a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{640} + \left(\frac{a(1715175c+1715175dx-9461760)}{887040} - \frac{495a(c+dx)}{256}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + \frac{54a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{5} + \left(\frac{a(3430350c+3430350dx+23654400)}{887040} - \frac{495a(c+dx)}{128}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} - \frac{841a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64} + \left(\frac{a(4802490c+4802490dx-52039680)}{887040} - \frac{693a(c+dx)}{128}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + \left(\frac{a(4802490c+4802490dx+42577920)}{887040} - \frac{693a(c+dx)}{128}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \frac{841a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64} + \left(\frac{a(3430350c+3430350dx-30412800)}{887040} - \frac{495a(c+dx)}{128}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - \frac{54a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{5} + \left(\frac{a(1715175c+1715175dx+6082560)}{887040} - \frac{495a(c+dx)}{256}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + \frac{3323a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{640} + \left(\frac{a(571725c+571725dx-1126400)}{887040} - \frac{165a(c+dx)}{256}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{4} + \left(\frac{a(114345c+114345dx-225280)}{887040} - \frac{33a(c+dx)}{256}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} - \frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128} + \frac{a(10395c+10395dx-20480)}{887040} - \frac{3a(c+dx)}{256}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/256 + ((a*(10395*c + 10395*d*x - 20480))/887040 - (3*a*tan(c/2 + (d
*x)/2))/128 - (3*a*(c + d*x))/256 + tan(c/2 + (d*x)/2)^2*((a*(114345*c + 11
4345*d*x - 225280))/887040 - (33*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^4*(
(a*(571725*c + 571725*d*x - 1126400))/887040 - (165*a*(c + d*x))/256) + tan
(c/2 + (d*x)/2)^6*((a*(1715175*c + 1715175*d*x + 6082560))/887040 - (495*a*
(c + d*x))/256) + tan(c/2 + (d*x)/2)^16*((a*(1715175*c + 1715175*d*x - 9461
760))/887040 - (495*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^14*((a*(3430350*
c + 3430350*d*x + 23654400))/887040 - (495*a*(c + d*x))/128) + tan(c/2 + (d
*x)/2)^8*((a*(3430350*c + 3430350*d*x - 30412800))/887040 - (495*a*(c + d*x
))/128) + tan(c/2 + (d*x)/2)^10*((a*(4802490*c + 4802490*d*x + 42577920))/8
87040 - (693*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^12*((a*(4802490*c + 480
2490*d*x - 52039680))/887040 - (693*a*(c + d*x))/128) - (a*tan(c/2 + (d*x)/
2)^3)/4 + (3323*a*tan(c/2 + (d*x)/2)^5)/640 - (54*a*tan(c/2 + (d*x)/2)^7)/5
 + (841*a*tan(c/2 + (d*x)/2)^9)/64 - (841*a*tan(c/2 + (d*x)/2)^13)/64 + (54
*a*tan(c/2 + (d*x)/2)^15)/5 - (3323*a*tan(c/2 + (d*x)/2)^17)/640 + (a*tan(c
/2 + (d*x)/2)^19)/4 + (3*a*tan(c/2 + (d*x)/2)^21)/128)/(d*(tan(c/2 + (d*x)/
2)^2 + 1)^11)
```

3.573 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=149

$$\frac{3ax}{256}-\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^9(c+dx)}{9d}+\frac{3a\cos(c+dx)\sin(c+dx)}{256d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{128d}+\frac{a\cos^5(c+d}{1(}$$

```
[Out] 3/256*a*x-1/7*a*cos(d*x+c)^7/d+1/9*a*cos(d*x+c)^9/d+3/256*a*cos(d*x+c)*sin(
d*x+c)/d+1/128*a*cos(d*x+c)^3*sin(d*x+c)/d+1/160*a*cos(d*x+c)^5*sin(d*x+c)/
d-3/80*a*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2645, 14, 2648, 2715, 8}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{3a\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{3a\sin(c+dx)\cos(c+dx)}{256d}+\frac{3ax}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/256 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (3*a*Co
s[c + d*x]*Sin[c + d*x])/(256*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(128*d)
+ (a*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (3*a*Cos[c + d*x]^7*Sin[c + d*x
])/(80*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a\int \cos^6(c+dx)\sin^4\\ &= -\frac{a\cos^7(c+dx)\sin^3(c+dx)}{10d} + \frac{1}{10}(3a)\int \cos^6(c+dx)\,\mathrm{s}\\ &= -\frac{3a\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{a\cos^7(c+dx)\sin^3(c+}{10d}\\ &= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^5(c+dx)\sin(c}{160d}\\ &= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{a\cos^3(c+dx)\sin(c}{128d}\\ &= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx)\sin(c}{256d}\\ &= \frac{3ax}{256} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{3a\cos(c+dx}{256}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 101, normalized size = 0.68

$$\frac{a(7560dx - 15120\cos(c+dx) - 6720\cos(3(c+dx)) + 1080\cos(7(c+dx)) + 280\cos(9(c+dx)) + 1260\sin(2(c+dx)) - 2520\sin(4(c+dx)) - 630\sin(6(c+dx)) + 315\sin(8(c+dx)) + 126\sin(10(c+dx)))}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(7560*d*x - 15120*Cos[c + d*x] - 6720*Cos[3*(c + d*x)] + 1080*Cos[7*(c +
 d*x)] + 280*Cos[9*(c + d*x)] + 1260*Sin[2*(c + d*x)] - 2520*Sin[4*(c + d*x
)] - 630*Sin[6*(c + d*x)] + 315*Sin[8*(c + d*x)] + 126*Sin[10*(c + d*x)]))/
(645120*d)
```

Maple [A]
time = 0.24, size = 116, normalized size = 0.78

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{9}-\frac{2\left(\cos^{7}(dx+c)\right)}{63}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{10}-\frac{3\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^{5}(dx+c\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{9}-\frac{2\left(\cos^{7}(dx+c)\right)}{63}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{7}(dx+c)\right)}{10}-\frac{3\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^{5}(dx+c\right.}{}\right.}{d}$
risch	$\frac{3ax}{256}-\frac{3a\cos(dx+c)}{128d}+\frac{a\sin(10dx+10c)}{5120d}+\frac{a\cos(9dx+9c)}{2304d}+\frac{a\sin(8dx+8c)}{2048d}+\frac{3a\cos(7dx+7c)}{1792d}-\frac{a\sin(6dx+6c}{1024d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a*(-1/10*sin(d*x+
c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(
d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.29, size = 76, normalized size = 0.51

$$\frac{10240\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a+63\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\,dx+4\,c\right)\right)a}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/645120*(10240*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 63*(32*sin(2*d*x
+ 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.39, size = 95, normalized size = 0.64

$$\frac{8960\,a\cos\left(dx+c\right)^{9}-11520\,a\cos\left(dx+c\right)^{7}+945\,adx+63\left(128\,a\cos\left(dx+c\right)^{9}-176\,a\cos\left(dx+c\right)^{7}+8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*a*cos(d*x + c)^9 - 11520*a*cos(d*x + c)^7 + 945*a*d*x + 63*(1
28*a*cos(d*x + c)^9 - 176*a*cos(d*x + c)^7 + 8*a*cos(d*x + c)^5 + 10*a*cos(
d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(138) = 276$.
time = 1.80, size = 294, normalized size = 1.97

$$\begin{cases} \frac{3ax\sin^{10}(c+dx)}{256} + \frac{15ax\sin^{8}(c+dx)\cos^{2}(c+dx)}{256} + \frac{15ax\sin^{6}(c+dx)\cos^{4}(c+dx)}{128} + \frac{15ax\sin^{4}(c+dx)\cos^{6}(c+dx)}{128} + \frac{15ax\sin^{2}(c+dx)\cos^{8}(c+dx)}{256} + \frac{3ax\cos^{10}(c+dx)}{256} + \frac{3a\sin^{9}(c+dx)\cos(c+dx)}{256d} + \frac{7a\sin^{7}(c+dx)\cos^{3}(c+dx)}{128d} + \frac{a\sin^{5}(c+dx)\cos^{5}(c+dx)}{10d} - \frac{7a\sin^{3}(c+dx)\cos^{7}(c+dx)}{128d} - \frac{a\sin^{2}(c+dx)\cos^{7}(c+dx)}{7d} - \frac{3a\sin(c+dx)\cos^{9}(c+dx)}{256d} - \frac{2a\cos^{9}(c+dx)}{63d} & \text{for}\ d \neq 0 \\ x(a\sin(c)+a)\sin^{3}(c)\cos^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**10/256 + 15*a*x*sin(c + d*x)**8*cos(c + d*x)
**2/256 + 15*a*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a*x*sin(c + d*x)*
*4*cos(c + d*x)**6/128 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 3*a*x
*cos(c + d*x)**10/256 + 3*a*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + a*sin(c + d*x)**5*cos(c + d*x)**5/(10
*d) - 7*a*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) - a*sin(c + d*x)**2*cos(c
 + d*x)**7/(7*d) - 3*a*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*cos(c + d
*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**6, True))
```

Giac [A]
time = 0.55, size = 137, normalized size = 0.92

$$\frac{3}{256}\,ax + \frac{a\cos(9\,dx+9\,c)}{2304\,d} + \frac{3\,a\cos(7\,dx+7\,c)}{1792\,d} - \frac{a\cos(3\,dx+3\,c)}{96\,d} - \frac{3\,a\cos(dx+c)}{128\,d} + \frac{a\sin(10\,dx+10\,c)}{5120\,d} + \frac{a\sin(8\,dx+8\,c)}{2048\,d} - \frac{a\sin(6\,dx+6\,c)}{1024\,d} - \frac{a\sin(4\,dx+4\,c)}{256\,d} + \frac{a\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/256*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d + 1/5120*a*sin(10*d*x + 10*
c)/d + 1/2048*a*sin(8*d*x + 8*c)/d - 1/1024*a*sin(6*d*x + 6*c)/d - 1/256*a*
sin(4*d*x + 4*c)/d + 1/512*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.19, size = 414, normalized size = 2.78

$$\frac{3\,a\,x}{256} + \frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{128} + \frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128} + \left(\frac{a\,(42525\,c+42525\,d\,x-322560)}{80640} - \frac{135\,a\,(c+d\,x)}{256}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} - \frac{867\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{160} + \left(\frac{a\,(113400\,c+113400\,d\,x+215040)}{80640} - \frac{45\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + \frac{519\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32} + \left(\frac{a\,(198450\,c+198450\,d\,x-1075200)}{80640} - \frac{315\,a\,(c+d\,x)}{128}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - \frac{1879\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64} + \left(\frac{a\,(238140\,c+238140\,d\,x-645120)}{80640} - \frac{189\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \frac{1879\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64} - \frac{519\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32} + \left(\frac{a\,(113400\,c+113400\,d\,x-829440)}{80640} - \frac{45\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + \frac{867\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160} + \left(\frac{a\,(42525\,c+42525\,d\,x+92160)}{80640} - \frac{135\,a\,(c+d\,x)}{256}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - \frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{128} + \left(\frac{a\,(9450\,c+9450\,d\,x-51200)}{80640} - \frac{15\,a\,(c+d\,x)}{128}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} - \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128} + \frac{a\,(945\,c+945\,d\,x-5120)}{80640} - \frac{3\,a\,(c+d\,x)}{256}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/256 + ((a*(945*c + 945*d*x - 5120))/80640 - (3*a*tan(c/2 + (d*x)/2)
)/128 - (3*a*(c + d*x))/256 + tan(c/2 + (d*x)/2)^2*((a*(9450*c + 9450*d*x -
```

```
 51200))/80640 - (15*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(42525*c
+ 42525*d*x + 92160))/80640 - (135*a*(c + d*x))/256) + tan(c/2 + (d*x)/2)^1
6*((a*(42525*c + 42525*d*x - 322560))/80640 - (135*a*(c + d*x))/256) + tan(
c/2 + (d*x)/2)^14*((a*(113400*c + 113400*d*x + 215040))/80640 - (45*a*(c +
d*x))/32) + tan(c/2 + (d*x)/2)^6*((a*(113400*c + 113400*d*x - 829440))/8064
0 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^10*((a*(238140*c + 238140*d*x
 - 645120))/80640 - (189*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^12*((a*(1984
50*c + 198450*d*x - 1075200))/80640 - (315*a*(c + d*x))/128) - (29*a*tan(c/
2 + (d*x)/2)^3)/128 + (867*a*tan(c/2 + (d*x)/2)^5)/160 - (519*a*tan(c/2 + (
d*x)/2)^7)/32 + (1879*a*tan(c/2 + (d*x)/2)^9)/64 - (1879*a*tan(c/2 + (d*x)/
2)^11)/64 + (519*a*tan(c/2 + (d*x)/2)^13)/32 - (867*a*tan(c/2 + (d*x)/2)^15
)/160 + (29*a*tan(c/2 + (d*x)/2)^17)/128 + (3*a*tan(c/2 + (d*x)/2)^19)/128)
/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.574 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{5ax}{128}-\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^9(c+dx)}{9d}+\frac{5a\cos(c+dx)\sin(c+dx)}{128d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{a\cos^5(c+dx)}{4}$$

```
[Out] 5/128*a*x-1/7*a*cos(d*x+c)^7/d+1/9*a*cos(d*x+c)^9/d+5/128*a*cos(d*x+c)*sin(
d*x+c)/d+5/192*a*cos(d*x+c)^3*sin(d*x+c)/d+1/48*a*cos(d*x+c)^5*sin(d*x+c)/d
-1/8*a*cos(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A]

time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 14}

$$\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+dx)\cos(c+dx)}{128d}+\frac{5ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (a*Cos[c + d*x]^9)/(9*d) + (5*a*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d
) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x]
)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a\int \cos^6(c+dx)\sin\\ &= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx -\\ &= \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c+dx)\sin(c+dx)}{8d}\\ &= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos^3(c+dx)\sin}{192d}\\ &= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+dx)\sin}{128d}\\ &= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^9(c+dx)}{9d} + \frac{5a\cos(c+d}{12}\end{aligned}$$

Mathematica [A]

time = 0.20, size = 91, normalized size = 0.73

$$\frac{a(2520dx - 1512\cos(c+dx) - 672\cos(3(c+dx)) + 108\cos(7(c+dx)) + 28\cos(9(c+dx)) + 1008\sin(2(c+dx)) - 504\sin(4(c+dx)) - 336\sin(6(c+dx)) - 63\sin(8(c+dx)))}{64512d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(2520*d*x - 1512*Cos[c + d*x] - 672*Cos[3*(c + d*x)] + 108*Cos[7*(c + d*
x)] + 28*Cos[9*(c + d*x)] + 1008*Sin[2*(c + d*x)] - 504*Sin[4*(c + d*x)] -
336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)]))/(64512*d)
```

Maple [A]
time = 0.23, size = 98, normalized size = 0.78

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(\right.}{9}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cos^{7}(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+a\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{7}(\right.}{9}\right.}{d}$
risch	$\frac{5ax}{128}-\frac{3a\cos(dx+c)}{128d}+\frac{a\cos(9dx+9c)}{2304d}-\frac{a\sin(8dx+8c)}{1024d}+\frac{3a\cos(7dx+7c)}{1792d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\sin(4dx+4c)}{128d}-$
norman	$-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{105ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{5ax}{128}-\frac{4a}{63d}+\frac{45ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{45ax\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{105ax\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{4a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15
/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+a*(-1/9*sin(d*x+c)^2*cos(d*x+c
)^7-2/63*cos(d*x+c)^7))
```

Maxima [A]
time = 0.28, size = 76, normalized size = 0.61

$$\frac{1024\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a+21\left(64\sin\left(2dx+2c\right)^{3}+120dx+120c-3\sin\left(8dx+8c\right)-24\sin\left(4dx+4c\right)\right)a}{64512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/64512*(1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a + 21*(64*sin(2*d*x +
2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.39, size = 84, normalized size = 0.67

$$\frac{896\,a\cos\left(dx+c\right)^{9}-1152\,a\cos\left(dx+c\right)^{7}+315\,adx-21\left(48\,a\cos\left(dx+c\right)^{7}-8\,a\cos\left(dx+c\right)^{5}-10\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8064*(896*a*cos(d*x + c)^9 - 1152*a*cos(d*x + c)^7 + 315*a*d*x - 21*(48*a
*cos(d*x + c)^7 - 8*a*cos(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x +
 c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(116) = 232$.
time = 1.30, size = 248, normalized size = 1.98

$$\begin{cases} \frac{5ax\sin^8(c+dx)}{128} + \frac{5ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{5ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{5ax\cos^8(c+dx)}{128} + \frac{5a\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55a\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{73a\sin^3(c+dx)\cos^5(c+dx)}{384d} - \frac{a\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{5a\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{2a\cos^9(c+dx)}{63d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^2(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - a*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a*cos(c + d*x)**9/(63*d), Ne(
d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**6, True))
```

Giac [A]
time = 0.54, size = 122, normalized size = 0.98

$$\frac{5}{128}\,ax + \frac{a\cos(9\,dx+9\,c)}{2304\,d} + \frac{3\,a\cos(7\,dx+7\,c)}{1792\,d} - \frac{a\cos(3\,dx+3\,c)}{96\,d} - \frac{3\,a\cos(dx+c)}{128\,d} - \frac{a\sin(8\,dx+8\,c)}{1024\,d} - \frac{a\sin(6\,dx+6\,c)}{192\,d} - \frac{a\sin(4\,dx+4\,c)}{128\,d} + \frac{a\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/128*a*x + 1/2304*a*cos(9*d*x + 9*c)/d + 3/1792*a*cos(7*d*x + 7*c)/d - 1/9
6*a*cos(3*d*x + 3*c)/d - 3/128*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)
/d - 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2
*d*x + 2*c)/d
```

Mupad [B]
time = 12.36, size = 386, normalized size = 3.09

$$\frac{5\,a\,x}{128} + \frac{\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64} - \frac{191\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{96} + \left(\frac{a\,(11340\,c+11340\,d\,x-32256)}{8064} - \frac{45\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + \frac{83\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32} + \left(\frac{a\,(26460\,c+26460\,d\,x+53760)}{8064} - \frac{105\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - \frac{145\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32} + \left(\frac{a\,(39690\,c+39690\,d\,x-161280)}{8064} - \frac{315\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \left(\frac{a\,(39690\,c+39690\,d\,x+96768)}{8064} - \frac{315\,a\,(c+d\,x)}{64}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + \frac{145\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32} + \left(\frac{a\,(26460\,c+26460\,d\,x-96768)}{8064} - \frac{105\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} - \frac{83\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{32} + \left(\frac{a\,(11340\,c+11340\,d\,x+13824)}{8064} - \frac{45\,a\,(c+d\,x)}{32}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} + \frac{191\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96} + \left(\frac{a\,(2835\,c+2835\,d\,x-4608)}{8064} - \frac{45\,a\,(c+d\,x)}{128}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} - \frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64} + \frac{a\,(315\,c+315\,d\,x-512)}{8064} - \frac{5\,a\,(c+d\,x)}{128}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/128 + ((a*(315*c + 315*d*x - 512))/8064 - (5*a*tan(c/2 + (d*x)/2))/
64 - (5*a*(c + d*x))/128 + tan(c/2 + (d*x)/2)^2*((a*(2835*c + 2835*d*x - 46
```

08))/8064 - (45*a*(c + d*x))/128) + tan(c/2 + (d*x)/2)^4*((a*(11340*c + 113
40*d*x + 13824))/8064 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/2)^14*((a*(1
1340*c + 11340*d*x - 32256))/8064 - (45*a*(c + d*x))/32) + tan(c/2 + (d*x)/
2)^12*((a*(26460*c + 26460*d*x + 53760))/8064 - (105*a*(c + d*x))/32) + tan
(c/2 + (d*x)/2)^6*((a*(26460*c + 26460*d*x - 96768))/8064 - (105*a*(c + d*x
))/32) + tan(c/2 + (d*x)/2)^8*((a*(39690*c + 39690*d*x + 96768))/8064 - (31
5*a*(c + d*x))/64) + tan(c/2 + (d*x)/2)^10*((a*(39690*c + 39690*d*x - 16128
0))/8064 - (315*a*(c + d*x))/64) + (191*a*tan(c/2 + (d*x)/2)^3)/96 - (83*a*
tan(c/2 + (d*x)/2)^5)/32 + (145*a*tan(c/2 + (d*x)/2)^7)/32 - (145*a*tan(c/2
 + (d*x)/2)^11)/32 + (83*a*tan(c/2 + (d*x)/2)^13)/32 - (191*a*tan(c/2 + (d*
x)/2)^15)/96 + (5*a*tan(c/2 + (d*x)/2)^17)/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1
)^9)

3.575 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=109

$$\frac{5ax}{128}-\frac{a\cos^7(c+dx)}{7d}+\frac{5a\cos(c+dx)\sin(c+dx)}{128d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{48d}-$$

```
[Out] 5/128*a*x-1/7*a*cos(d*x+c)^7/d+5/128*a*cos(d*x+c)*sin(d*x+c)/d+5/192*a*cos(
d*x+c)^3*sin(d*x+c)/d+1/48*a*cos(d*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^7*s
in(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2645, 30, 2648, 2715, 8}

$$-\frac{a\cos^7(c+dx)}{7d}-\frac{a\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a\sin(c+dx)\cos(c+dx)}{128d}+\frac{5ax}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/128 - (a*Cos[c + d*x]^7)/(7*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(1
28*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (a*Cos[c + d*x]^5*Sin[c
 + d*x])/(48*d) - (a*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^6(c+dx)\sin(c+dx)\,dx + a\int \cos^6(c+dx)\sin^2(\\
&= -\frac{a\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}a\int \cos^6(c+dx)\,dx - \frac{a}{}\\
&= -\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{a\cos^7(c}{}\\
&= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos^3(c+dx)\sin(c+dx)}{192d} + \frac{a\cos^5(}{}\\
&= -\frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a\cos^3(}{}\\
&= \frac{5ax}{128} - \frac{a\cos^7(c+dx)}{7d} + \frac{5a\cos(c+dx)\sin(c+dx)}{128d} + \frac{5a}{}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 91, normalized size = 0.83

$$-\frac{a(-840dx+1680\cos(c+dx)+1008\cos(3(c+dx))+336\cos(5(c+dx))+48\cos(7(c+dx))-336\sin(2(c+dx))+168\sin(4(c+dx))+112\sin(6(c+dx))+21\sin(8(c+dx)))}{21504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/21504*(a*(-840*d*x + 1680*Cos[c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos
[5*(c + d*x)] + 48*Cos[7*(c + d*x)] - 336*Sin[2*(c + d*x)] + 168*Sin[4*(c +
 d*x)] + 112*Sin[6*(c + d*x)] + 21*Sin[8*(c + d*x)]))/d
```

Maple [A]
time = 0.18, size = 78, normalized size = 0.72

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7}+a\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7}+a\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)}{d}$
risch	$\frac{5ax}{128}-\frac{5a\cos(dx+c)}{64d}-\frac{a\sin(8dx+8c)}{1024d}-\frac{a\cos(7dx+7c)}{448d}-\frac{a\sin(6dx+6c)}{192d}-\frac{a\cos(5dx+5c)}{64d}-\frac{a\sin(4dx+4c)}{128d}-$
norman	$-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{35ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{5ax}{128}-\frac{2a}{7d}+\frac{5ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{35ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{35ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{2a(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a*cos(d*x+c)^7+a*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5
+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c))
```

Maxima [A]
time = 0.28, size = 63, normalized size = 0.58

$$-\frac{3072\,a\cos\left(dx+c\right)^{7}-7\,\left(64\,\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/21504*(3072*a*cos(d*x + c)^7 - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a)/d
```

Fricas [A]
time = 0.41, size = 73, normalized size = 0.67

$$-\frac{384\,a\cos\left(dx+c\right)^{7}-105\,adx+7\,\left(48\,a\cos\left(dx+c\right)^{7}-8\,a\cos\left(dx+c\right)^{5}-10\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(384*a*cos(d*x + c)^7 - 105*a*d*x + 7*(48*a*cos(d*x + c)^7 - 8*a*co
s(d*x + c)^5 - 10*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(102) = 204$.
time = 0.89, size = 223, normalized size = 2.05

$$\begin{cases} \frac{5ax\sin^8(c+dx)}{128} + \frac{5ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{5ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{5ax\cos^8(c+dx)}{128} + \frac{5a\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55a\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{73a\sin^3(c+dx)\cos^5(c+dx)}{384d} - \frac{5a\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{a\cos^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**8/128 + 5*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a*x*sin(c + d*x)**2*co
s(c + d*x)**6/32 + 5*a*x*cos(c + d*x)**8/128 + 5*a*sin(c + d*x)**7*cos(c +
d*x)/(128*d) + 55*a*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 73*a*sin(c +
d*x)**3*cos(c + d*x)**5/(384*d) - 5*a*sin(c + d*x)*cos(c + d*x)**7/(128*d)
- a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)*cos(c)**6, T
rue))
```

Giac [A]
time = 0.46, size = 122, normalized size = 1.12

$$\frac{5}{128}\,ax - \frac{a\cos(7\,dx+7\,c)}{448\,d} - \frac{a\cos(5\,dx+5\,c)}{64\,d} - \frac{3\,a\cos(3\,dx+3\,c)}{64\,d} - \frac{5\,a\cos(dx+c)}{64\,d} - \frac{a\sin(8\,dx+8\,c)}{1024\,d} - \frac{a\sin(6\,dx+6\,c)}{192\,d} - \frac{a\sin(4\,dx+4\,c)}{128\,d} + \frac{a\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/128*a*x - 1/448*a*cos(7*d*x + 7*c)/d - 1/64*a*cos(5*d*x + 5*c)/d - 3/64*a
*cos(3*d*x + 3*c)/d - 5/64*a*cos(d*x + c)/d - 1/1024*a*sin(8*d*x + 8*c)/d -
 1/192*a*sin(6*d*x + 6*c)/d - 1/128*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x
 + 2*c)/d
```

Mupad [B]
time = 9.39, size = 96, normalized size = 0.88

$$-\frac{a\left(210\,\cos(c+d\,x)+126\,\cos(3\,c+3\,d\,x)+42\,\cos(5\,c+5\,d\,x)+6\,\cos(7\,c+7\,d\,x)-42\,\sin(2\,c+2\,d\,x)+21\,\sin(4\,c+4\,d\,x)+14\,\sin(6\,c+6\,d\,x)+\frac{21\,\sin(8\,c+8\,d\,x)}{8}-105\,d\,x\right)}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] -(a*(210*cos(c + d*x) + 126*cos(3*c + 3*d*x) + 42*cos(5*c + 5*d*x) + 6*cos(
7*c + 7*d*x) - 42*sin(2*c + 2*d*x) + 21*sin(4*c + 4*d*x) + 14*sin(6*c + 6*d
*x) + (21*sin(8*c + 8*d*x))/8 - 105*d*x))/(2688*d)
```

3.576 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{5ax}{16}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+$$

[Out] 5/16*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d

Rubi [A]
time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2672, 308, 212, 2715, 8}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}+\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{5ax}{16}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]

[Out] (5*a*x)/16 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(6*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^6(c+dx)\,dx + a\int \cos^5(c+dx)\cot(c+dx)\,dx\\
&= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d} + \frac{1}{6}(5a)\int \cos^4(c+dx)\,dx -\\
&= \frac{5a\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}\\
&= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a\cos}{}\\
&= \frac{5ax}{16} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c}{3d}\end{aligned}$$

Mathematica [A]

time = 0.09, size = 100, normalized size = 0.79

$$\frac{a\left(300c+300dx+1320\cos(c+dx)+140\cos(3(c+dx))+12\cos(5(c+dx))-960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+225\sin(2(c+dx))+45\sin(4(c+dx))+5\sin(6(c+dx))\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(300*c + 300*d*x + 1320*Cos[c + d*x] + 140*Cos[3*(c + d*x)] + 12*Cos[5*(
c + d*x)] - 960*Log[Cos[(c + d*x)/2]] + 960*Log[Sin[(c + d*x)/2]] + 225*Sin
[2*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]

time = 0.14, size = 96, normalized size = 0.76

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^{5}(dx+c)\right)}{5}+\frac{\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c}{8}\right.}{6}\right.}{d}$
default	$\frac{a\left(\frac{\left(\cos^{5}(dx+c)\right)}{5}+\frac{\left(\cos^{3}(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(dx+c)\right)}{4}+\frac{15\cos(dx+c}{8}\right.}{6}\right.}{d}$
risch	$\frac{5ax}{16}+\frac{11a\,e^{i(dx+c)}}{16d}+\frac{11a\,e^{-i(dx+c)}}{16d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}-1\right)}{d}+\frac{a\sin(6dx+6c)}{192d}+\frac{a\cos(5dx+5c}{80d}$
norman	$\frac{\frac{28a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5ax}{16}+\frac{46a}{15d}+\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{5a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{15a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{15a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^{9}\left(\right.\right.}{24}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+
c)))+a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c))
```

Maxima [A]

time = 0.28, size = 106, normalized size = 0.83

$$\frac{32\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a-5\left(4\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(32*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 5*(4*sin(2*d*x + 2*c)^3 -
 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]

time = 0.40, size = 110, normalized size = 0.87

$$\frac{48\,a\cos\left(dx+c\right)^{5}+80\,a\cos\left(dx+c\right)^{3}+75\,adx+240\,a\cos\left(dx+c\right)-120\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+120\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 + 80*a*cos(d*x + c)^3 + 75*a*d*x + 240*a*cos(d*x
 + c) - 120*a*log(1/2*cos(d*x + c) + 1/2) + 120*a*log(-1/2*cos(d*x + c) + 1
/2) + 5*(8*a*cos(d*x + c)^5 + 10*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 201, normalized size = 1.58

$$\frac{75\,(dx+c)a+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-2160\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+450\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3680\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-450\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3360\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1488\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-368\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(165*a*tan
(1/2*d*x + 1/2*c)^11 - 720*a*tan(1/2*d*x + 1/2*c)^10 - 25*a*tan(1/2*d*x + 1
/2*c)^9 - 2160*a*tan(1/2*d*x + 1/2*c)^8 + 450*a*tan(1/2*d*x + 1/2*c)^7 - 36
80*a*tan(1/2*d*x + 1/2*c)^6 - 450*a*tan(1/2*d*x + 1/2*c)^5 - 3360*a*tan(1/2
*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^3 - 1488*a*tan(1/2*d*x + 1/2*c)
^2 - 165*a*tan(1/2*d*x + 1/2*c) - 368*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 10.71, size = 327, normalized size = 2.57

$$\frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+18\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{92\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+28\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{62\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{46\,a}{15}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a\,\mathrm{atan}\left(\frac{25\,a^2}{64\,\left(\frac{5\,a^2}{4}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{5\,a^2}{4}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] ((46*a)/15 + (11*a*tan(c/2 + (d*x)/2))/8 + (62*a*tan(c/2 + (d*x)/2)^2)/5 -
(5*a*tan(c/2 + (d*x)/2)^3)/24 + 28*a*tan(c/2 + (d*x)/2)^4 + (15*a*tan(c/2 +
 (d*x)/2)^5)/4 + (92*a*tan(c/2 + (d*x)/2)^6)/3 - (15*a*tan(c/2 + (d*x)/2)^7
)/4 + 18*a*tan(c/2 + (d*x)/2)^8 + (5*a*tan(c/2 + (d*x)/2)^9)/24 + 6*a*tan(c
/2 + (d*x)/2)^10 - (11*a*tan(c/2 + (d*x)/2)^11)/8)/(d*(6*tan(c/2 + (d*x)/2)
^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)
/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a*log(tan(
c/2 + (d*x)/2)))/d + (5*a*atan((25*a^2)/(64*((5*a^2)/4 - (25*a^2*tan(c/2 +
(d*x)/2))/64)) + (5*a^2*tan(c/2 + (d*x)/2))/(4*((5*a^2)/4 - (25*a^2*tan(c/2
 + (d*x)/2))/64))))/(8*d)
```

3.577 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=121

$$-\frac{15ax}{8}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}-\frac{15a\cot(c+dx)}{8d}+\frac{5a\cos^2(}{}$$

```
[Out] -15/8*a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+1/5*a
*cos(d*x+c)^5/d-15/8*a*cot(d*x+c)/d+5/8*a*cos(d*x+c)^2*cot(d*x+c)/d+1/4*a*c
os(d*x+c)^4*cot(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2917, 2671, 294, 327, 209, 2672, 308, 212}

$$\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}+\frac{5a\cos^2(c+dx)\cot(c+dx)}{8d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{15ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*a*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c +
 d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) - (15*a*Cot[c + d*x])/(8*d) + (5*
a*Cos[c + d*x]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^5(c+dx)\cot(c+dx)\,dx + a\int \cos^4(c+dx)\cot$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x^6}{(1+x}\right.}{}$$

$$= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\text{Subst}\left(\int \left(-1-x^2-x^4\right.\right.}{}$$

$$= \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{a\cos^5(c+dx)}{5d} + \frac{5a}{}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+}{3d}$$

$$= -\frac{15ax}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a}{}$$

Mathematica [A]

time = 0.20, size = 98, normalized size = 0.81

$$-\frac{a\left(900c+900dx-660\cos(c+dx)-70\cos(3(c+dx))-6\cos(5(c+dx))+480\cot(c+dx)+480\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+240\sin(2(c+dx))+15\sin(4(c+dx))\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a*(900*c + 900*d*x - 660*Cos[c + d*x] - 70*Cos[3*(c + d*x)] - 6*Cos
[5*(c + d*x)] + 480*Cot[c + d*x] + 480*Log[Cos[(c + d*x)/2]] - 480*Log[Sin[
(c + d*x)/2]] + 240*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)]))/d
```

Maple [A]

time = 0.13, size = 114, normalized size = 0.94

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+a\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+a\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}\right.}{d}$
risch	$-\frac{15ax}{8}+\frac{ia\,e^{2i(dx+c)}}{4d}+\frac{11a\,e^{i(dx+c)}}{16d}+\frac{11a\,e^{-i(dx+c)}}{16d}-\frac{ia\,e^{-2i(dx+c)}}{4d}-\frac{2ia}{d\left(e^{2i(dx+c)}-1\right)}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{6a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{2d}-\frac{17a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{17a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\right.\right.}{2d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+a*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))

Maxima [A]
time = 0.50, size = 121, normalized size = 1.00

$$\frac{4\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a-15\left(15\,dx+15\,c+\frac{15\tan\left(dx+c\right)^{4}+25\tan\left(dx+c\right)^{2}+8}{\tan\left(dx+c\right)^{5}+2\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 15*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a)/d

Fricas [A]
time = 0.43, size = 129, normalized size = 1.07

$$\frac{30\,a\cos\left(dx+c\right)^{5}+75\,a\cos\left(dx+c\right)^{3}-60\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+60\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-225\,a\cos\left(dx+c\right)+\left(24\,a\cos\left(dx+c\right)^{5}+40\,a\cos\left(dx+c\right)^{3}-225\,adx+120\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(30*a*cos(d*x + c)^5 + 75*a*cos(d*x + c)^3 - 60*a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 60*a*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*a*cos(d*x + c) + (24*a*cos(d*x + c)^5 + 40*a*cos(d*x + c)^3 - 225*a*d*x + 120*a*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep

Giac [A]
time = 0.56, size = 198, normalized size = 1.64

$$-\frac{225\,(dx+c)a-120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{60\,\left(2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(135\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+360\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+560\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-135\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+184\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)*a - 120*a*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1
/2*d*x + 1/2*c) + 60*(2*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) -
2*(135*a*tan(1/2*d*x + 1/2*c)^9 + 360*a*tan(1/2*d*x + 1/2*c)^8 + 150*a*tan(
1/2*d*x + 1/2*c)^7 + 720*a*tan(1/2*d*x + 1/2*c)^6 + 1120*a*tan(1/2*d*x + 1/
2*c)^4 - 150*a*tan(1/2*d*x + 1/2*c)^3 + 560*a*tan(1/2*d*x + 1/2*c)^2 - 135*
a*tan(1/2*d*x + 1/2*c) + 184*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 8.92, size = 313, normalized size = 2.59

$$\frac{\frac{7\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{2}+12\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+24\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7-10\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\frac{112\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{3}-15\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{56\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}-\frac{19\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{92\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}-a}{d\,\left(2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+20\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+20\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{15\,a\,\mathrm{atan}\left(\frac{225\,a^2}{16\,\left(\frac{15\,a^2}{2}+\frac{225\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{15\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{15\,a^2}{2}+\frac{225\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((92*a*tan(c/2 + (d*x)/2))/15 - a - (19*a*tan(c/2 + (d*x)/2)^2)/2 + (56*a*t
an(c/2 + (d*x)/2)^3)/3 - 15*a*tan(c/2 + (d*x)/2)^4 + (112*a*tan(c/2 + (d*x)
/2)^5)/3 - 10*a*tan(c/2 + (d*x)/2)^6 + 24*a*tan(c/2 + (d*x)/2)^7 + 12*a*tan
(c/2 + (d*x)/2)^9 + (7*a*tan(c/2 + (d*x)/2)^10)/2)/(d*(2*tan(c/2 + (d*x)/2)
 + 10*tan(c/2 + (d*x)/2)^3 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2
)^7 + 10*tan(c/2 + (d*x)/2)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (a*tan(c/2 + (d
*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d + (15*a*atan((225*a^2)/(16*((
15*a^2)/2 + (225*a^2*tan(c/2 + (d*x)/2))/16)) - (15*a^2*tan(c/2 + (d*x)/2))
/(2*((15*a^2)/2 + (225*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

3.578 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$-\frac{15ax}{8}+\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cos^3(c+dx)}{6d}-\frac{15a\cot(c+dx)}{8d}+\frac{5a\cos^2(c+dx)\cot(c-}{8d}$$

```
[Out] -15/8*a*x+5/2*a*arctanh(cos(d*x+c))/d-5/2*a*cos(d*x+c)/d-5/6*a*cos(d*x+c)^3
/d-15/8*a*cot(d*x+c)/d+5/8*a*cos(d*x+c)^2*cot(d*x+c)/d+1/4*a*cos(d*x+c)^4*c
ot(d*x+c)/d-1/2*a*cos(d*x+c)^3*cot(d*x+c)^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2917, 2672, 294, 308, 212, 2671, 327, 209}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{15a\cot(c+dx)}{8d}+\frac{a\cos^4(c+dx)\cot(c+dx)}{4d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{5a\cos^2(c+dx)\cot(c+dx)}{8d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{15ax}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*a*x)/8 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d)
- (5*a*Cos[c + d*x]^3)/(6*d) - (15*a*Cot[c + d*x])/(8*d) + (5*a*Cos[c + d*x
]^2*Cot[c + d*x])/(8*d) + (a*Cos[c + d*x]^4*Cot[c + d*x])/(4*d) - (a*Cos[c
+ d*x]^3*Cot[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^4(c+dx)\cot^2(c+dx)\,dx + a\int \cos^3(c+dx)\cot$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x}{(1+}\right.}{}$$

$$= \frac{a\cos^4(c+dx)\cot(c+dx)}{4d} - \frac{a\cos^3(c+dx)\cot^2(c+dx}{2d}$$

$$= \frac{5a\cos^2(c+dx)\cot(c+dx)}{8d} + \frac{a\cos^4(c+dx)\cot(c+dx}{4d}$$

$$= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} - \frac{15a\cot(c+dx)}{8d} +$$

$$= -\frac{15ax}{8} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5}{}$$

Mathematica [A]

time = 1.86, size = 117, normalized size = 0.87

$$-\frac{a\left(216\cos(c+dx)+8\cos(3(c+dx))+3\left(60c+60dx+32\cot(c+dx)+4\csc^2\left(\frac{1}{2}(c+dx)\right)-80\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+80\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\sec^2\left(\frac{1}{2}(c+dx)\right)+16\sin(2(c+dx))+\sin(4(c+dx))\right)\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(a*(216*Cos[c + d*x] + 8*Cos[3*(c + d*x)] + 3*(60*c + 60*d*x + 32*Cot
[c + d*x] + 4*Csc[(c + d*x)/2]^2 - 80*Log[Cos[(c + d*x)/2]] + 80*Log[Sin[(c
 + d*x)/2]] - 4*Sec[(c + d*x)/2]^2 + 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
)))/d
```

Maple [A]

time = 0.16, size = 136, normalized size = 1.01

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c\right.\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c\right.\right.}{d}$
risch	$-\frac{15ax}{8}-\frac{a\,e^{3i(dx+c)}}{24d}+\frac{ia\,e^{2i(dx+c)}}{4d}-\frac{9a\,e^{i(dx+c)}}{8d}-\frac{9a\,e^{-i(dx+c)}}{8d}-\frac{ia\,e^{-2i(dx+c)}}{4d}-\frac{a\,e^{-3i(dx+c)}}{24d}+\frac{a\left(e^{3i(dx\right.}{}$

norman	$-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{15a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{15a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/
2*cos(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/sin(d*x+c)*cos(d*x+c)^7-(
cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.50, size = 131, normalized size = 0.98

$$-\frac{2\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a+3\left(15\,dx+15\,c+\frac{15\tan\left(dx+c\right)^{4}+25\tan\left(dx+c\right)^{2}+8}{\tan\left(dx+c\right)^{5}+2\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*(2*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a + 3*(15*d*
x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*
tan(d*x + c)^3 + tan(d*x + c)))*a)/d
```

Fricas [A]
time = 0.38, size = 162, normalized size = 1.21

$$-\frac{8\,a\cos\left(dx+c\right)^{5}+45\,adx\cos\left(dx+c\right)^{2}+40\,a\cos\left(dx+c\right)^{3}-45\,adx-60\,a\cos\left(dx+c\right)-30\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+30\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(2\,a\cos\left(dx+c\right)^{5}+5\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^5 + 45*a*d*x*cos(d*x + c)^2 + 40*a*cos(d*x + c)^3 -
 45*a*d*x - 60*a*cos(d*x + c) - 30*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x +
 c) + 1/2) + 30*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*
a*cos(d*x + c)^5 + 5*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.45, size = 214, normalized size = 1.60

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,(dx+c)a-60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}+\frac{2\left(27\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-72\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-152\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-27\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-56\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 - 45*(d*x + c)*a - 60*a*log(abs(tan(1/2*d*
x + 1/2*c))) + 12*a*tan(1/2*d*x + 1/2*c) + 3*(30*a*tan(1/2*d*x + 1/2*c)^2 -
 4*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2 + 2*(27*a*tan(1/2*d*x
 + 1/2*c)^7 - 72*a*tan(1/2*d*x + 1/2*c)^6 + 3*a*tan(1/2*d*x + 1/2*c)^5 - 16
8*a*tan(1/2*d*x + 1/2*c)^4 - 3*a*tan(1/2*d*x + 1/2*c)^3 - 152*a*tan(1/2*d*x
 + 1/2*c)^2 - 27*a*tan(1/2*d*x + 1/2*c) - 56*a)/(tan(1/2*d*x + 1/2*c)^2 + 1
)^4)/d
```

Mupad [B]
time = 8.86, size = 321, normalized size = 2.40

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\frac{49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{2}+7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+58\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\frac{161\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}+17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{62\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\right)}-\frac{15\,a\,\mathrm{atan}\left(\frac{225\,a^{2}}{16\left(\frac{75\,a^{2}}{4}-\frac{225\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{75\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{75\,a^{2}}{4}-\frac{225\,a^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (5*a*log(ta
n(c/2 + (d*x)/2)))/(2*d) - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (62*a*tan(c/2 +
(d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 + (161*a*tan(c/2 + (d*x)/2)^4)/3
+ 13*a*tan(c/2 + (d*x)/2)^5 + 58*a*tan(c/2 + (d*x)/2)^6 + 7*a*tan(c/2 + (d*
x)/2)^7 + (49*a*tan(c/2 + (d*x)/2)^8)/2 - 7*a*tan(c/2 + (d*x)/2)^9)/(d*(4*t
an(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 1
6*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (15*a*atan((225*a^2)/(
16*((75*a^2)/4 - (225*a^2*tan(c/2 + (d*x)/2))/16)) + (75*a^2*tan(c/2 + (d*x
)/2))/(4*((75*a^2)/4 - (225*a^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

3.579 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=130

$$\frac{5ax}{2}+\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cos^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+d}{2d}$$

```
[Out] 5/2*a*x+5/2*a*arctanh(cos(d*x+c))/d-5/2*a*cos(d*x+c)/d-5/6*a*cos(d*x+c)^3/d
+5/2*a*cot(d*x+c)/d-1/2*a*cos(d*x+c)^3*cot(d*x+c)^2/d-5/6*a*cot(d*x+c)^3/d+
1/2*a*cos(d*x+c)^2*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2671, 294, 308, 209, 2672, 212}

$$-\frac{5a\cos^3(c+dx)}{6d}-\frac{5a\cos(c+dx)}{2d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}-\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}+\frac{5ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/2 + (5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a*Cos[c + d*x])/(2*d) -
(5*a*Cos[c + d*x]^3)/(6*d) + (5*a*Cot[c + d*x])/(2*d) - (a*Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*d) - (5*a*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot^3(c+dx)\,dx + a\int \cos^2(c+dx)\,\mathrm{cot}$$

$$= -\frac{a\mathrm{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\mathrm{Subst}\left(\int \frac{x}{(1+}\right.}{}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+}{2d}$$

$$= -\frac{a\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a\cos^2(c+dx)\cot^3(c+}{2d}$$

$$= -\frac{5a\cos(c+dx)}{2d} - \frac{5a\cos^3(c+dx)}{6d} + \frac{5a\cot(c+dx)}{2d} -$$

$$= \frac{5ax}{2} + \frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a\cos(c+dx)}{2d} - \frac{5a\,\mathrm{c}}{}$$

Mathematica [A]

time = 6.13, size = 174, normalized size = 1.34

$$\frac{5a(c+dx)}{2d}-\frac{9a\cos(c+dx)}{4d}-\frac{a\cos(3(c+dx))}{12d}+\frac{7a\cot(c+dx)}{3d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}+\frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*(c + d*x))/(2*d) - (9*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(12
*d) + (7*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^2)/(3*d) + (5*a*Log[Cos[(c + d*x)/2]])/(2*d) - (5*a*Log[S
in[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[2*(c + d*x)
])/(4*d)
```

Maple [A]
time = 0.16, size = 154, normalized size = 1.18

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+a\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+a\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}\right.}{d}$
risch	$\frac{5ax}{2}-\frac{ia\,e^{2i(dx+c)}}{8d}-\frac{9a\,e^{i(dx+c)}}{8d}-\frac{9a\,e^{-i(dx+c)}}{8d}+\frac{ia\,e^{-2i(dx+c)}}{8d}+\frac{a\left(18ie^{4i(dx+c)}+3\,e^{5i(dx+c)}-24ie^{2i(dx+c)}+\right.}{3d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{25a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{25a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos
(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+a*(-1
/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(d*x+
c)-5/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.51, size = 122, normalized size = 0.94

$$-\frac{\left(4\cos\left(dx+c\right)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a-2\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x
 + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a - 2*(15*d*x
+ 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(
d*x + c)^3))*a)/d
```

Fricas [A]
time = 0.39, size = 182, normalized size = 1.40

$$-\frac{6\,a\cos\left(dx+c\right)^{5}-40\,a\cos\left(dx+c\right)^{3}-15\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)+2\,\left(2\,a\cos\left(dx+c\right)^{5}-15\,adx\cos\left(dx+c\right)^{2}+10\,a\cos\left(dx+c\right)^{3}+15\,adx-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3 - 15*(a*cos(d*x + c)^2 - a)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(a*cos(d*x + c)^2 - a)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*a*cos(d*x + c) + 2*(2*a*cos(d*x +
 c)^5 - 15*a*d*x*cos(d*x + c)^2 + 10*a*cos(d*x + c)^3 + 15*a*d*x - 15*a*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.50, size = 220, normalized size = 1.69

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+180\,(dx+c)a-180\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-81\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{110\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-111\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-273\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+306\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-253\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+72\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{3}}}{72\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(3*a*tan(1/2*d*x + 1/2*c)^3 + 9*a*tan(1/2*d*x + 1/2*c)^2 + 180*(d*x +
c)*a - 180*a*log(abs(tan(1/2*d*x + 1/2*c))) - 81*a*tan(1/2*d*x + 1/2*c) + (
110*a*tan(1/2*d*x + 1/2*c)^9 + 9*a*tan(1/2*d*x + 1/2*c)^8 - 111*a*tan(1/2*d
*x + 1/2*c)^7 + 240*a*tan(1/2*d*x + 1/2*c)^6 - 273*a*tan(1/2*d*x + 1/2*c)^5
 + 306*a*tan(1/2*d*x + 1/2*c)^4 - 253*a*tan(1/2*d*x + 1/2*c)^3 + 72*a*tan(1
/2*d*x + 1/2*c)^2 - 9*a*tan(1/2*d*x + 1/2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

Mupad [B]
time = 8.97, size = 310, normalized size = 2.38

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+49\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{80\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+67\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-34\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{121\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{5\,a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{5\,a\,\mathrm{atan}\left(\frac{25\,a^2}{25\,a^2+25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\,a^2+25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - 8*a*tan(c/2
+ (d*x)/2)^2 + (121*a*tan(c/2 + (d*x)/2)^3)/3 - 34*a*tan(c/2 + (d*x)/2)^4 +
 67*a*tan(c/2 + (d*x)/2)^5 - (80*a*tan(c/2 + (d*x)/2)^6)/3 + 49*a*tan(c/2 +
 (d*x)/2)^7 - a*tan(c/2 + (d*x)/2)^8)/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c
/2 + (d*x)/2)^5 + 24*tan(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) - (9*a
*tan(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (5*a*log(tan
(c/2 + (d*x)/2)))/(2*d) - (5*a*atan((25*a^2)/(25*a^2 + 25*a^2*tan(c/2 + (d*
x)/2)) - (25*a^2*tan(c/2 + (d*x)/2))/(25*a^2 + 25*a^2*tan(c/2 + (d*x)/2))))
/d
```

3.580 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{5ax}{2}-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{15a\cos(c+dx)}{8d}+\frac{5a\cot(c+dx)}{2d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{5a\cot^3(c+\ldots}{6d}$$

```
[Out] 5/2*a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d+5/2*a*cot(d*x+c)/d
+5/8*a*cos(d*x+c)*cot(d*x+c)^2/d-5/6*a*cot(d*x+c)^3/d+1/2*a*cos(d*x+c)^2*co
t(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2917, 2672, 294, 327, 212, 2671, 308, 209}

$$\frac{15a\cos(c+dx)}{8d}-\frac{5a\cot^3(c+dx)}{6d}+\frac{5a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{5ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/2 - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d)
+ (5*a*Cot[c + d*x])/(2*d) + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) - (5*a
*Cot[c + d*x]^3)/(6*d) + (a*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*Cos[c
 + d*x]*Cot[c + d*x]^4)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^4(c+dx)\,dx + a\int \cos(c+dx)\cot^5($$

$$= -\frac{a\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int \frac{x^6}{(1+x^}$$

$$= \frac{a\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d}$$

$$= \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cos^2(c+dx)\cot^3(c+dx}{2d}$$

$$= \frac{15a\cos(c+dx)}{8d} + \frac{5a\cot(c+dx)}{2d} + \frac{5a\cos(c+dx)\cot^2(}{8d}$$

$$= \frac{5ax}{2} - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} + \frac{5a}{}$$

Mathematica [A]

time = 1.37, size = 138, normalized size = 1.03

$$\frac{a\left(192\cos(c+dx)-64\cot(c+dx)\left(-7+\csc^2(c+dx)\right)+3\left(18\csc^2\left(\frac{1}{2}(c+dx)\right)-\csc^4\left(\frac{1}{2}(c+dx)\right)+40\left(4c+4dx-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-18\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+16\sin(2(c+dx))\right)\right)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(192*Cos[c + d*x] - 64*Cot[c + d*x]*(-7 + Csc[c + d*x]^2) + 3*(18*Csc[(c
 + d*x)/2]^2 - Csc[(c + d*x)/2]^4 + 40*(4*c + 4*d*x - 3*Log[Cos[(c + d*x)/2
]] + 3*Log[Sin[(c + d*x)/2]]) - 18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4
+ 16*Sin[2*(c + d*x)])))/(192*d)
```

Maple [A]

time = 0.16, size = 172, normalized size = 1.28

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\cos^7(}{3\sin(}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{\cos^7(}{3\sin(}\right.}{d}$
risch	$\frac{5ax}{2}-\frac{ia\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{ia\,e^{-2i(dx+c)}}{8d}-\frac{a\left(27\,e^{7i(dx+c)}-3\,e^{5i(dx+c)}-72i e^{6i(dx+c)}-3\,e^{3i}\right.}{12d(e}$

norman	$\frac{-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{25a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{25a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{25a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{25a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*co
s(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))
+a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x
+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.50, size = 136, normalized size = 1.01

$$\frac{8\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^4+10\,\tan\left(dx+c\right)^2-2}{\tan\left(dx+c\right)^5+\tan\left(dx+c\right)^3}\right)a-3\,a\left(\frac{2\left(9\,\cos\left(dx+c\right)^3-7\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a - 3*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))
/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x
 + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.41, size = 202, normalized size = 1.51

$$\frac{120\,adx\cos\left(dx+c\right)^4+48\,a\cos\left(dx+c\right)^5-240\,adx\cos\left(dx+c\right)^2-150\,a\cos\left(dx+c\right)^3+120\,adx+90\,a\cos\left(dx+c\right)-45\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+45\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+8\left(3\,a\cos\left(dx+c\right)^5-20\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(120*a*d*x*cos(d*x + c)^4 + 48*a*cos(d*x + c)^5 - 240*a*d*x*cos(d*x +
c)^2 - 150*a*cos(d*x + c)^3 + 120*a*d*x + 90*a*cos(d*x + c) - 45*(a*cos(d*x
 + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 45*(a*cos(d
*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 8*(3*a*c
os(d*x + c)^5 - 20*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d*c
os(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.48, size = 213, normalized size = 1.59

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+480\,(dx+c)a+360\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-216\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{192\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}-\frac{750\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-216\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*tan(1
/2*d*x + 1/2*c)^2 + 480*(d*x + c)*a + 360*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 216*a*tan(1/2*d*x + 1/2*c) - 192*(a*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*
d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2 - (750*a*tan(1/2*d*x + 1/2*c)^4 - 216*a*tan(1/2*d*x + 1/2*c)^3 - 48*a*ta
n(1/2*d*x + 1/2*c)^2 + 8*a*tan(1/2*d*x + 1/2*c) + 3*a)/tan(1/2*d*x + 1/2*c)
^4)/d
```

Mupad [B]
time = 8.82, size = 300, normalized size = 2.24

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{4\,d}-\frac{9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64\,d}+\frac{15\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+36\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+\frac{154\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{3}+\frac{159\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{4}+\frac{50\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{3}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\right)}+\frac{5\,a\,\mathrm{atan}\left(\frac{25\,a^2}{\frac{75\,a^2}{4}-25\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{75\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,\left(\frac{75\,a^2}{4}-25\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^2)/(4*d) - (9*a*tan
(c/2 + (d*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^4)/(64*d) + (15*a*log(tan(c/
2 + (d*x)/2)))/(8*d) + ((7*a*tan(c/2 + (d*x)/2)^2)/2 - (2*a*tan(c/2 + (d*x)
/2))/3 - a/4 + (50*a*tan(c/2 + (d*x)/2)^3)/3 + (159*a*tan(c/2 + (d*x)/2)^4)
/4 + (154*a*tan(c/2 + (d*x)/2)^5)/3 + 36*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan(c
/2 + (d*x)/2)^7)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16
*tan(c/2 + (d*x)/2)^8)) + (5*a*atan((25*a^2)/((75*a^2)/4 - 25*a^2*tan(c/2 +
 (d*x)/2)) + (75*a^2*tan(c/2 + (d*x)/2))/(4*((75*a^2)/4 - 25*a^2*tan(c/2 +
(d*x)/2)))))/d
```

3.581 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-ax-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{15a\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}+\frac{a\cot^3(c+d}{3d}$$

```
[Out] -a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*a*
cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c)^
4/d-1/5*a*cot(d*x+c)^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2789, 2672, 294, 327, 212, 3554, 8}

$$\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{15a\tanh^{-1}(\cos(c+dx))}{8d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx \\
&= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos\right.}{d} \\
&= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \\
&= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a}{} \\
&= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx}{8d} \\
&= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 164, normalized size = 1.34

$$\frac{a\cos(c+dx)}{d}+\frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d}-\frac{15a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{15a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{9a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A]
time = 0.16, size = 129, normalized size = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c\right.}{8}\cdots}{d}$
default	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c\right.}{8}\cdots}{d}$
risch	$-ax+\frac{a\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{a\left(360i\mathrm{e}^{8i(dx+c)}+135\,\mathrm{e}^{9i(dx+c)}-720i\mathrm{e}^{6i(dx+c)}-150\,\mathrm{e}^{7i(dx+c)}+1120i\mathrm{e}^{4i(dx+c)}\cdots\right.}{60d\left(\mathrm{e}^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a}{160d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{15a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{59a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{59a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{15a\left(\tan^9\left(\frac{dx}{2}+\cdots\right.\right.}{64d}\cdots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+a*(-1/4/sin(d*
x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(
d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.50, size = 125, normalized size = 1.02

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.
time = 0.43, size = 222, normalized size = 1.82

$$-\frac{368\,a\cos\left(dx+c\right)^5-560\,a\cos\left(dx+c\right)^3+225\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-225\,\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+240\,a\cos\left(dx+c\right)+30\,\left(8\,adx\cos\left(dx+c\right)^4-8\,a\cos\left(dx+c\right)^5-16\,adx\cos\left(dx+c\right)^2+25\,a\cos\left(dx+c\right)^3+8\,adx-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 199, normalized size = 1.63

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-960\,(dx+c)a+1800\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1920\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{4110\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+660\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]

time = 8.87, size = 291, normalized size = 2.39

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{22\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-72\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\frac{59\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{3}-\frac{15\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{32\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{15}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}}{d\,\left(32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4\,d}-\frac{7\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}+\frac{15\,a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (32*a*
tan(c/2 + (d*x)/2)^2)/15 - (15*a*tan(c/2 + (d*x)/2)^3)/2 + (59*a*tan(c/2 +
(d*x)/2)^4)/3 - 72*a*tan(c/2 + (d*x)/2)^5 + 22*a*tan(c/2 + (d*x)/2)^6)/(d*(
32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (a*tan(c/2 + (d*x)/2)
^2)/(4*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^4)/(6
4*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a*log(tan(c/2 + (d*x)/2)))/(8
*d) + (2*a*atan((4*a^2)/((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (15*a^2*t
an(c/2 + (d*x)/2))/(2*((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

3.582 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=128

$$-ax+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)}{d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}+$$

```
[Out] -a*x+5/16*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+1/3*a*cot(d*x+c)^3/d-1/5*a
*cot(d*x+c)^5/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d+5/24*a*cot(d*x+c)^3*csc(d*x+
c)/d-1/6*a*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2917, 2691, 3855, 3554, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) + (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x])/d + (a*Cot[c
 + d*x]^3)/(3*d) - (a*Cot[c + d*x]^5)/(5*d) - (5*a*Cot[c + d*x]*Csc[c + d*x
])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a*Cot[c + d*x]^5*Cs
c[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\,dx + a\int \cot^6(c+dx)\csc(c+dx)\,dx$$

$$= -\frac{a\cot^5(c+dx)}{5d} - \frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)$$

$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} + \frac{5a\cot^3(c+dx)\csc(c}{24d}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - 5a$$

$$= -ax + \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot(c+dx)}{d} + a\cot$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 193, normalized size = 1.51

$$-\frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{11a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x)/2]^4)/(32*d) - (a*Csc[
(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2
, -Tan[c + d*x]^2])/(5*d) + (5*a*Log[Cos[(c + d*x)/2]])/(16*d) - (5*a*Log[S
in[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^2)/(64*d) - (a*Sec[(c + d
*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.19, size = 147, normalized size = 1.15

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+}{d}$
risch	$-ax+\frac{a\left(165\,\mathrm{e}^{11i(dx+c)}+25\,\mathrm{e}^{9i(dx+c)}-720i\mathrm{e}^{10i(dx+c)}+450\,\mathrm{e}^{7i(dx+c)}+2160i\mathrm{e}^{8i(dx+c)}+450\,\mathrm{e}^{5i(dx+c)}-3680i\mathrm{e}^{6i(dx+c)}-\right.}{120d\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$
norman	$-\frac{a}{384d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{59a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{59a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/
sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+
c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-co
t(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.53, size = 137, normalized size = 1.07

$$-\frac{32\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a-5\,a\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/480*(32*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a - 5*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x
+ c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(c
os(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(116) = 232.
time = 0.41, size = 254, normalized size = 1.98

$$-\frac{480\,adx\cos\left(dx+c\right)^6-1440\,adx\cos\left(dx+c\right)^4-330\,a\cos\left(dx+c\right)^5+1440\,adx\cos\left(dx+c\right)^2+400\,a\cos\left(dx+c\right)^3-480\,adx-150\,a\cos\left(dx+c\right)-75\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+75\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-32\left(23\,a\cos\left(dx+c\right)^5-35\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/480*(480*a*d*x*cos(d*x + c)^6 - 1440*a*d*x*cos(d*x + c)^4 - 330*a*cos(d*
x + c)^5 + 1440*a*d*x*cos(d*x + c)^2 + 400*a*cos(d*x + c)^3 - 480*a*d*x - 1
```

```
50*a*cos(d*x + c) - 75*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x
 + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 75*(a*cos(d*x + c)^6 - 3*a*cos(d
*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) - 32*(23*a
*cos(d*x + c)^5 - 35*a*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 208, normalized size = 1.62

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-140\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+225\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1920\,(dx+c)a-600\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+1320\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{1470\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1320\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-225\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+140\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*a*tan(1/2*d*x + 1/2*c)^5 - 45*a*tan
(1/2*d*x + 1/2*c)^4 - 140*a*tan(1/2*d*x + 1/2*c)^3 + 225*a*tan(1/2*d*x + 1/
2*c)^2 - 1920*(d*x + c)*a - 600*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1320*a*t
an(1/2*d*x + 1/2*c) + (1470*a*tan(1/2*d*x + 1/2*c)^6 - 1320*a*tan(1/2*d*x +
 1/2*c)^5 - 225*a*tan(1/2*d*x + 1/2*c)^4 + 140*a*tan(1/2*d*x + 1/2*c)^3 + 4
5*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*tan(1/2*d*x + 1/2*c) - 5*a)/tan(1/2*d*x +
 1/2*c)^6)/d
```

Mupad [**B**]
time = 9.46, size = 285, normalized size = 2.23

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}-\frac{5\,a\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{16\,d}-\frac{11\,a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-16\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{15\,a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{128\,d}+\frac{7\,a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96\,d}+\frac{3\,a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{128\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{384\,d}+\frac{15\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{128\,d}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96\,d}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{128\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(16*d) - (5*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(16*d) - (11*a*cot(c/2 + (d*x)/2))/(16*d) - (2*a*atan((16*cos(c/2
+ (d*x)/2) + 5*sin(c/2 + (d*x)/2))/(5*cos(c/2 + (d*x)/2) - 16*sin(c/2 + (d*
x)/2))))/d - (15*a*cot(c/2 + (d*x)/2)^2)/(128*d) + (7*a*cot(c/2 + (d*x)/2)^
3)/(96*d) + (3*a*cot(c/2 + (d*x)/2)^4)/(128*d) - (a*cot(c/2 + (d*x)/2)^5)/(
160*d) - (a*cot(c/2 + (d*x)/2)^6)/(384*d) + (15*a*tan(c/2 + (d*x)/2)^2)/(12
8*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (3*a*tan(c/2 + (d*x)/2)^4)/(128*
d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (a*tan(c/2 + (d*x)/2)^6)/(384*d)
```

3.583 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^7(c+dx)}{7d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{a\cot^5(c-$$

```
[Out] 5/16*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-5/16*a*cot(d*x+c)*csc(d*x
+c)/d+5/24*a*cot(d*x+c)^3*csc(d*x+c)/d-1/6*a*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2917, 2687, 30, 2691, 3855}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (a*Cot[c + d*x]^7)/(7*d) - (5*a*Cot[c
+ d*x]*Csc[c + d*x])/(16*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a
*Cot[c + d*x]^5*Csc[c + d*x])/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc(c+dx)\,dx + a\int \cot^6(c+dx)\,\mathrm{csc}$$

$$= -\frac{a\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{1}{6}(5a)\int \cot^4(c+dx)\,\mathrm{cs}$$

$$= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a\cot}{}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc(c+dx)}{16d} + \frac{5a\cot}{}$$

$$= \frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+}{}$$

Mathematica [A]
time = 0.03, size = 175, normalized size = 1.82

$$-\frac{a\cot^7(c+dx)}{7d} - \frac{11a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} - \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{11a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Cot[c + d*x]^7)/d - (11*a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c +
d*x)/2]^4)/(32*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) + (5*a*Log[Cos[(c + d*x)
/2]])/(16*d) - (5*a*Log[Sin[(c + d*x)/2]])/(16*d) + (11*a*Sec[(c + d*x)/2]^
2)/(64*d) - (a*Sec[(c + d*x)/2]^4)/(32*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.20, size = 128, normalized size = 1.33

method	result

derivativedivides	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+a\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx-}{}\right.}{d}$
default	$\frac{-\frac{a\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+a\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx-}{}\right.}{d}$
risch	$\frac{a\left(336i\,e^{12i(dx+c)}+231\,e^{13i(dx+c)}-196\,e^{11i(dx+c)}+1680i\,e^{8i(dx+c)}+595\,e^{9i(dx+c)}+1008i\,e^{4i(dx+c)}-595\,e^{5i(dx+c)}+196\,e^{3i\right.}{168d\left(e^{2i(dx+c)}-1\right)^7}$
norman	$-\frac{a}{896d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d}+\frac{3a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{448d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}-\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a(\mathrm{t}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a/sin(d*x+c)^7*cos(d*x+c)^7+a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/2
4/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^
5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.30, size = 106, normalized size = 1.10

$$\frac{7\,a\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,a}{\tan(dx+c)^7}}{672\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/672*(7*a*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x +
c) + 1) - 15*log(cos(d*x + c) - 1)) - 96*a/tan(d*x + c)^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(86) = 172.
time = 0.40, size = 210, normalized size = 2.19

$$\frac{96\,a\cos\left(dx+c\right)^7+105\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+14\left(33\,a\cos\left(dx+c\right)^5-40\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{672\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*a*cos(d*x + c)^7 + 105*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3
*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*co
s(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 14*(33*a*cos(d*x + c)^5 - 40*a*cos(d*x + c)^3
```

```
+ 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4
+ 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(86) = 172.
time = 0.60, size = 228, normalized size = 2.38

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-840\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2178\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+63\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2688*(3*a*tan(1/2*d*x + 1/2*c)^7 + 7*a*tan(1/2*d*x + 1/2*c)^6 - 21*a*tan(
1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)^4 + 63*a*tan(1/2*d*x + 1/2*c
)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 840*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 105*a*tan(1/2*d*x + 1/2*c) + (2178*a*tan(1/2*d*x + 1/2*c)^7 + 105*a*tan(1/
2*d*x + 1/2*c)^6 - 315*a*tan(1/2*d*x + 1/2*c)^5 - 63*a*tan(1/2*d*x + 1/2*c)
^4 + 63*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 - 7*a*tan(1/
2*d*x + 1/2*c) - 3*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [**B**]
time = 9.94, size = 385, normalized size = 4.01

$$-\frac{a\left(3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\right)}{2688\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
[Out] -(a*(3*cos(c/2 + (d*x)/2)^14 - 3*sin(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 21*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^11 - 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 + 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2)^8 - 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (
d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^4 - 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 21*cos(c/2 + (d*x)/2)
^12*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7))/(2688*d*cos(c/2 + (d*x)/2)^7*sin
(c/2 + (d*x)/2)^7)
```

3.584 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\cot(c+dx)\csc(c+dx)}{128d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{64d}+\frac{5a\cot^3(c$$

```
[Out] 5/128*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d+5/128*a*cot(d*x+c)*csc(d
*x+c)/d-5/64*a*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a*cot(d*x+c)^3*csc(d*x+c)^3/d
-1/8*a*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 30}

$$-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{64d}+\frac{5a\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) + (5*a*Cot[c
 + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) +
(5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]
^3)/(8*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a\int \cot^6(c+dx)\,\mathrm{cs} \\
&= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\,\mathrm{c} \\
&= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\,\mathrm{co}}{} \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{64d} + \frac{5a\,\mathrm{co}}{} \\
&= -\frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+dx)\csc(c+dx)}{128d} - \frac{5a\,\mathrm{cot}}{} \\
&= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} + \frac{5a\cot(c+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 215, normalized size = 1.76

$$
-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d}-\frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d}-\frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d}-\frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{15a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{7a\sec^6\left(\frac{1}{2}(c+dx)\right)}{1536d}+\frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]

[Out] -1/7*(a*Cot[c + d*x]^7)/d + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c
 + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*
```

```
x)/2]^8)/(2048*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) - (5*a*Log[Sin[(c +
 d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (15*a*Sec[(c + d*x)
/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*Sec[(c + d*x)/2]^8
)/(2048*d)
```

Maple [A]
time = 0.23, size = 146, normalized size = 1.20

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(dx+ \ldots}{\ldots}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(dx+ \ldots}{\ldots}\right.}{d}$
risch	$-\frac{a(105\,e^{15i(dx+c)}+2779\,e^{13i(dx+c)}+8064ie^{4i(dx+c)}+6265\,e^{11i(dx+c)}-8064ie^{6i(dx+c)}+12355\,e^{9i(dx+c)}+2688ie^{12i(dx+c \ldots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192
/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)
^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-1/7
*a/sin(d*x+c)^7*cos(d*x+c)^7)
```

Maxima [A]
time = 0.28, size = 126, normalized size = 1.03

$$-\frac{7\,a\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{768\,a}{\tan(dx+c)^7}}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/5376*(7*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3
+ 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)
) + 768*a/tan(d*x + c)^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(110) = 220.
time = 0.39, size = 225, normalized size = 1.84

$$-\frac{768\,a\cos\left(dx+c\right)^7\sin\left(dx+c\right)+210\,a\cos\left(dx+c\right)^7+1022\,a\cos\left(dx+c\right)^5-770\,a\cos\left(dx+c\right)^3+210\,a\cos\left(dx+c\right)-105\left(a\cos\left(dx+c\right)^8-4\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+105\left(a\cos\left(dx+c\right)^8-4\,a\cos\left(dx+c\right)^6+6\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{5376\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/5376*(768*a*cos(d*x + c)^7*sin(d*x + c) + 210*a*cos(d*x + c)^7 + 1022*a*
cos(d*x + c)^5 - 770*a*cos(d*x + c)^3 + 210*a*cos(d*x + c) - 105*(a*cos(d*x
 + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)
*log(1/2*cos(d*x + c) + 1/2) + 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 +
 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2))
/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x
+ c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(110) = 220.
time = 0.52, size = 256, normalized size = 2.10

$$\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1008\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1680\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-1680\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4566\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1680\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1008\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-168\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+336\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+112\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}}}{43008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/43008*(21*a*tan(1/2*d*x + 1/2*c)^8 + 48*a*tan(1/2*d*x + 1/2*c)^7 - 112*a*
tan(1/2*d*x + 1/2*c)^6 - 336*a*tan(1/2*d*x + 1/2*c)^5 + 168*a*tan(1/2*d*x +
 1/2*c)^4 + 1008*a*tan(1/2*d*x + 1/2*c)^3 + 336*a*tan(1/2*d*x + 1/2*c)^2 -
1680*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1680*a*tan(1/2*d*x + 1/2*c) + (4566
*a*tan(1/2*d*x + 1/2*c)^8 + 1680*a*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d
*x + 1/2*c)^6 - 1008*a*tan(1/2*d*x + 1/2*c)^5 - 168*a*tan(1/2*d*x + 1/2*c)^
4 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 112*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(
1/2*d*x + 1/2*c) - 21*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [**B**]
time = 9.15, size = 285, normalized size = 2.34

$$\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{3\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] (5*a*cot(c/2 + (d*x)/2))/(128*d) - (5*a*tan(c/2 + (d*x)/2))/(128*d) - (a*co
t(c/2 + (d*x)/2)^2)/(128*d) - (3*a*cot(c/2 + (d*x)/2)^3)/(128*d) - (a*cot(c
/2 + (d*x)/2)^4)/(256*d) + (a*cot(c/2 + (d*x)/2)^5)/(128*d) + (a*cot(c/2 +
(d*x)/2)^6)/(384*d) - (a*cot(c/2 + (d*x)/2)^7)/(896*d) - (a*cot(c/2 + (d*x)
/2)^8)/(2048*d) + (a*tan(c/2 + (d*x)/2)^2)/(128*d) + (3*a*tan(c/2 + (d*x)/2
)^3)/(128*d) + (a*tan(c/2 + (d*x)/2)^4)/(256*d) - (a*tan(c/2 + (d*x)/2)^5)/
(128*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) + (a*tan(c/2 + (d*x)/2)^7)/(896*
d) + (a*tan(c/2 + (d*x)/2)^8)/(2048*d) - (5*a*log(tan(c/2 + (d*x)/2)))/(128
*d)
```

3.585 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^9(c+dx)}{9d}+\frac{5a\cot(c+dx)\csc(c+dx)}{128d}-\frac{5a\cot(c+dx)\csc^3(c}{64d}$$

```
[Out] 5/128*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d+5/1
28*a*cot(d*x+c)*csc(d*x+c)/d-5/64*a*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a*cot(d*
x+c)^3*csc(d*x+c)^3/d-1/8*a*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2917, 2687, 14, 2691, 3853, 3855}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{5a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{64d}+\frac{5a\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*a*ArcTanh[Cos[c + d*x]])/(128*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (5*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (5*a*Cot[c + d*x
]*Csc[c + d*x]^3)/(64*d) + (5*a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a*
Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a\int \cot^6(c+dx)\operatorname{csc} \\
&= -\frac{a\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{1}{8}(5a)\int \cot^4(c+dx)\operatorname{cs} \\
&= \frac{5a\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a\cot^5(c+dx)\csc^3(c+d}{8d} \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{5a\cot(c+dx)\csc^3}{64d} \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{5a\cot(c+dx)\csc(}{128d} \\
&= \frac{5a\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+d}{9d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(138) = 276.
time = 0.07, size = 301, normalized size = 2.18

$$\frac{2a\cot(c+dx)}{63d} + \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d} - \frac{15a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d} + \frac{7a\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d} - \frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d} + \frac{a\cot(c+dx)\csc^2(c+dx)}{63d} - \frac{5a\cot(c+dx)\csc^4(c+dx)}{21d} + \frac{19a\cot(c+dx)\csc^6(c+dx)}{63d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d} + \frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d} - \frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d} + \frac{15a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d} - \frac{7a\sec^6\left(\frac{1}{2}(c+dx)\right)}{1536d} + \frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

[Out] (2*a*Cot[c + d*x])/(63*d) + (5*a*Csc[(c + d*x)/2]^2)/(512*d) - (15*a*Csc[(c + d*x)/2]^4)/(1024*d) + (7*a*Csc[(c + d*x)/2]^6)/(1536*d) - (a*Csc[(c + d*x)/2]^8)/(2048*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(63*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[c + d*x]*Csc[c + d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (5*a*Log[Cos[(c + d*x)/2]])/(128*d) - (5*a*Log[Sin[(c + d*x)/2]])/(128*d) - (5*a*Sec[(c + d*x)/2]^2)/(512*d) + (15*a*Sec[(c + d*x)/2]^4)/(1024*d) - (7*a*Sec[(c + d*x)/2]^6)/(1536*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)

Maple [A]
time = 0.22, size = 166, normalized size = 1.20

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+a\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-5\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+a\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-5\right.}{d}$
risch	$-\frac{a\left(315\,e^{17i(dx+c)}+6912ie^{4i(dx+c)}+8022\,e^{15i(dx+c)}+48384ie^{6i(dx+c)}+10458\,e^{13i(dx+c)}+26880ie^{12i(dx+c)}+18270\,e^{11}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7)+a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c))))

Maxima [A]
time = 0.33, size = 138, normalized size = 1.00

$$-\frac{21\,a\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{256\left(9\,\tan(dx+c)^2+7\right)a}{\tan(dx+c)^9}}{16128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/16128*(21*a*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 256*(9*tan(d*x + c)^2 + 7)*a/tan(d*x + c)^9)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(124) = 248$.

time = 0.38, size = 259, normalized size = 1.88

$$\frac{512 a \cos(dx+c)^9 - 2304 a \cos(dx+c)^7 + 315 \left(a \cos(dx+c)^8 - 4 a \cos(dx+c)^6 + 6 a \cos(dx+c)^4 - 4 a \cos(dx+c)^2 + a\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 315 \left(a \cos(dx+c)^8 - 4 a \cos(dx+c)^6 + 6 a \cos(dx+c)^4 - 4 a \cos(dx+c)^2 + a\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 42 \left(15 a \cos(dx+c)^7 + 73 a \cos(dx+c)^5 - 55 a \cos(dx+c)^3 + 15 a \cos(dx+c)\right) \sin(dx+c)}{16128 \left(d \cos(dx+c)^8 - 4 d \cos(dx+c)^6 + 6 d \cos(dx+c)^4 - 4 d \cos(dx+c)^2 + d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*a*cos(d*x + c)^9 - 2304*a*cos(d*x + c)^7 + 315*(a*cos(d*x + c)
^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a*cos(d*x + c)^8 - 4*a*cos(d*x
+ c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 42*(15*a*cos(d*x + c)^7 + 73*a*cos(d*x + c)^5 - 55*a
*cos(d*x + c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(124) = 248.
time = 0.56, size = 256, normalized size = 1.86

$$\frac{28 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^9 + 63 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 - 108 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 336 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 504 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 672 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 1008 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 5040 a \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - 1512 a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{14258 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9 + 1512 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8 - 1008 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 - 672 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 - 504 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 + 336 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 + 108 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 63 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 28 a}{\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9}}{129024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(28*a*tan(1/2*d*x + 1/2*c)^9 + 63*a*tan(1/2*d*x + 1/2*c)^8 - 108*a
*tan(1/2*d*x + 1/2*c)^7 - 336*a*tan(1/2*d*x + 1/2*c)^6 + 504*a*tan(1/2*d*x
+ 1/2*c)^4 + 672*a*tan(1/2*d*x + 1/2*c)^3 + 1008*a*tan(1/2*d*x + 1/2*c)^2 -
 5040*a*log(abs(tan(1/2*d*x + 1/2*c))) - 1512*a*tan(1/2*d*x + 1/2*c) + (142
58*a*tan(1/2*d*x + 1/2*c)^9 + 1512*a*tan(1/2*d*x + 1/2*c)^8 - 1008*a*tan(1/
2*d*x + 1/2*c)^7 - 672*a*tan(1/2*d*x + 1/2*c)^6 - 504*a*tan(1/2*d*x + 1/2*c
)^5 + 336*a*tan(1/2*d*x + 1/2*c)^3 + 108*a*tan(1/2*d*x + 1/2*c)^2 - 63*a*ta
n(1/2*d*x + 1/2*c) - 28*a)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 9.19, size = 285, normalized size = 2.07

$$\frac{3\,a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{3\,a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3584\,d}-\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4608\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3584\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4608\,d}-\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
```

```
[Out] (3*a*cot(c/2 + (d*x)/2))/(256*d) - (3*a*tan(c/2 + (d*x)/2))/(256*d) - (a*co
t(c/2 + (d*x)/2)^2)/(128*d) - (a*cot(c/2 + (d*x)/2)^3)/(192*d) - (a*cot(c/2
 + (d*x)/2)^4)/(256*d) + (a*cot(c/2 + (d*x)/2)^6)/(384*d) + (3*a*cot(c/2 +
(d*x)/2)^7)/(3584*d) - (a*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a*cot(c/2 + (d*
x)/2)^9)/(4608*d) + (a*tan(c/2 + (d*x)/2)^2)/(128*d) + (a*tan(c/2 + (d*x)/2
)^3)/(192*d) + (a*tan(c/2 + (d*x)/2)^4)/(256*d) - (a*tan(c/2 + (d*x)/2)^6)/
(384*d) - (3*a*tan(c/2 + (d*x)/2)^7)/(3584*d) + (a*tan(c/2 + (d*x)/2)^8)/(2
048*d) + (a*tan(c/2 + (d*x)/2)^9)/(4608*d) - (5*a*log(tan(c/2 + (d*x)/2)))/
(128*d)
```

3.586 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^9(c+dx)}{9d}+\frac{3a\cot(c+dx)\csc(c+dx)}{256d}+\frac{a\cot(c+dx)\csc^3(c+}{128d}$$

```
[Out] 3/256*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d+3/2
56*a*cot(d*x+c)*csc(d*x+c)/d+1/128*a*cot(d*x+c)*csc(d*x+c)^3/d-1/32*a*cot(d
*x+c)*csc(d*x+c)^5/d+1/16*a*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a*cot(d*x+c)^5
*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 14}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{32d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{128d}+\frac{3a\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c +
 d*x]^9)/(9*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(256*d) + (a*Cot[c + d*x]*
Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a\int \cot^6(c+dx)\,\mathrm{cs} \\
&= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\,\mathrm{csc}^5 \\
&= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+ \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} - \frac{a\cot(c+dx)\csc^5}{32d} \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{a\cot(c+dx)\csc^3}{128d} \\
&= -\frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+dx)}{9d} + \frac{3a\cot(c+dx)\,\mathrm{csc}}{256d} \\
&= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{a\cot^9(c+}{9d}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(160) = 320.
time = 0.06, size = 341, normalized size = 2.13

$$\frac{2a\cot(c+dx)}{63d}+\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{3a\csc^6\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{3a\csc^8\left(\frac{1}{2}(c+dx)\right)}{4096d}-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{63d}-\frac{5a\cot(c+dx)\csc^4(c+dx)}{21d}+\frac{19a\cot(c+dx)\csc^6(c+dx)}{63d}-\frac{a\cot(c+dx)\csc^8(c+dx)}{9d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{256d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256d}-\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{3a\sec^6\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{3a\sec^8\left(\frac{1}{2}(c+dx)\right)}{4096d}+\frac{a\sec^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(63*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c +
 d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d*
x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^2)/(63*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^4)/(21*d) + (19*a*Cot[
c + d*x]*Csc[c + d*x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) + (
3*a*Log[Cos[(c + d*x)/2]])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) -
(3*a*Sec[(c + d*x)/2]^2)/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*
Sec[(c + d*x)/2]^6)/(2048*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(
c + d*x)/2]^10)/(10240*d)
```

Maple [A]

time = 0.25, size = 184, normalized size = 1.15

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\frac{3\cos}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\frac{3\cos}{}\right.}{d}$
risch	$-\frac{a\left(945\,e^{19i(dx+c)}-9135\,e^{17i(dx+c)}-218484\,e^{15i(dx+c)}-46080ie^{4i(dx+c)}-653940\,e^{13i(dx+c)}-414720ie^{6i(dx+c)}-118377\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x+c)^8*cos(d*x+c)^7-1/1
60/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/sin(d*x
+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+c)^3-3/256*cos(d*x+c)-
3/256*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin
(d*x+c)^7*cos(d*x+c)^7))
```

Maxima [A]

time = 0.33, size = 158, normalized size = 0.99

$$-\frac{63\,a\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2560\left(9\,\tan(dx+c)^2+7\right)a}{\tan(dx+c)^9}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/161280*(63*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c
)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x + c
```

```
)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(9*tan(d*x + c)^2 +
7)*a/tan(d*x + c)^9)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(144) = 288$.
time = 0.41, size = 289, normalized size = 1.81

$$-\frac{1890\,a\cos\left(dx+c\right)^{9}-8820\,a\cos\left(dx+c\right)^{7}-16128\,a\cos\left(dx+c\right)^{5}+8820\,a\cos\left(dx+c\right)^{3}-1890\,a\cos\left(dx+c\right)-945\,\left(a\cos\left(dx+c\right)^{10}-5\,a\cos\left(dx+c\right)^{8}+10\,a\cos\left(dx+c\right)^{6}-10\,a\cos\left(dx+c\right)^{4}+5\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+945\,\left(a\cos\left(dx+c\right)^{10}-5\,a\cos\left(dx+c\right)^{8}+10\,a\cos\left(dx+c\right)^{6}-10\,a\cos\left(dx+c\right)^{4}+5\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2560\,\left(2\,a\cos\left(dx+c\right)^{9}-9\,a\cos\left(dx+c\right)^{7}\right)\sin\left(dx+c\right)}{161280\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/161280*(1890*a*cos(d*x + c)^9 - 8820*a*cos(d*x + c)^7 - 16128*a*cos(d*x
+ c)^5 + 8820*a*cos(d*x + c)^3 - 1890*a*cos(d*x + c) - 945*(a*cos(d*x + c)^
10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*c
os(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 945*(a*cos(d*x + c)^10 - 5
*a*cos(d*x + c)^8 + 10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x
 + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2560*(2*a*cos(d*x + c)^9 - 9*a*
cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*
d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 284, normalized size = 1.78

$$\frac{126\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1080\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2520\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1260\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{44286\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+15120\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1260\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-6720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2520\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1080\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-126\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1290240*(126*a*tan(1/2*d*x + 1/2*c)^10 + 280*a*tan(1/2*d*x + 1/2*c)^9 - 3
15*a*tan(1/2*d*x + 1/2*c)^8 - 1080*a*tan(1/2*d*x + 1/2*c)^7 - 630*a*tan(1/2
*d*x + 1/2*c)^6 + 2520*a*tan(1/2*d*x + 1/2*c)^4 + 6720*a*tan(1/2*d*x + 1/2*
c)^3 + 1260*a*tan(1/2*d*x + 1/2*c)^2 - 15120*a*log(abs(tan(1/2*d*x + 1/2*c)
)) - 15120*a*tan(1/2*d*x + 1/2*c) + (44286*a*tan(1/2*d*x + 1/2*c)^10 + 1512
```

```
0*a*tan(1/2*d*x + 1/2*c)^9 - 1260*a*tan(1/2*d*x + 1/2*c)^8 - 6720*a*tan(1/2
*d*x + 1/2*c)^7 - 2520*a*tan(1/2*d*x + 1/2*c)^6 + 630*a*tan(1/2*d*x + 1/2*c
)^4 + 1080*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c)^2 - 280*a*
tan(1/2*d*x + 1/2*c) - 126*a)/tan(1/2*d*x + 1/2*c)^10)/d
```

Mupad [B]
time = 9.40, size = 319, normalized size = 1.99

$$\frac{3\,a\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{1024\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{192\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{512\,d}+\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{2048\,d}+\frac{3\,a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{3584\,d}+\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{4096\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4608\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{10240\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{1024\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{192\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{512\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{2048\,d}-\frac{3\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{3584\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{4096\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4608\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{10240\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] (3*a*cot(c/2 + (d*x)/2))/(256*d) - (3*a*tan(c/2 + (d*x)/2))/(256*d) - (a*co
t(c/2 + (d*x)/2)^2)/(1024*d) - (a*cot(c/2 + (d*x)/2)^3)/(192*d) - (a*cot(c/
2 + (d*x)/2)^4)/(512*d) + (a*cot(c/2 + (d*x)/2)^6)/(2048*d) + (3*a*cot(c/2
+ (d*x)/2)^7)/(3584*d) + (a*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a*cot(c/2 + (
d*x)/2)^9)/(4608*d) - (a*cot(c/2 + (d*x)/2)^10)/(10240*d) + (a*tan(c/2 + (d
*x)/2)^2)/(1024*d) + (a*tan(c/2 + (d*x)/2)^3)/(192*d) + (a*tan(c/2 + (d*x)/
2)^4)/(512*d) - (a*tan(c/2 + (d*x)/2)^6)/(2048*d) - (3*a*tan(c/2 + (d*x)/2)
^7)/(3584*d) - (a*tan(c/2 + (d*x)/2)^8)/(4096*d) + (a*tan(c/2 + (d*x)/2)^9)
/(4608*d) + (a*tan(c/2 + (d*x)/2)^10)/(10240*d) - (3*a*log(tan(c/2 + (d*x)/
2)))/(256*d)
```

3.587 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^7(c+dx)}{7d}-\frac{2a\cot^9(c+dx)}{9d}-\frac{a\cot^{11}(c+dx)}{11d}+\frac{3a\cot(c+dx)\csc(c+dx)}{256d}+a$$

```
[Out] 3/256*a*arctanh(cos(d*x+c))/d-1/7*a*cot(d*x+c)^7/d-2/9*a*cot(d*x+c)^9/d-1/1
1*a*cot(d*x+c)^11/d+3/256*a*cot(d*x+c)*csc(d*x+c)/d+1/128*a*cot(d*x+c)*csc(
d*x+c)^3/d-1/32*a*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a*cot(d*x+c)^3*csc(d*x+c)^
5/d-1/10*a*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2687, 276, 2691, 3853, 3855}

$$-\frac{a\cot^{11}(c+dx)}{11d}-\frac{2a\cot^9(c+dx)}{9d}-\frac{a\cot^7(c+dx)}{7d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{32d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{128d}+\frac{3a\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Cos[c + d*x]])/(256*d) - (a*Cot[c + d*x]^7)/(7*d) - (2*a*Cot[c
 + d*x]^9)/(9*d) - (a*Cot[c + d*x]^11)/(11*d) + (3*a*Cot[c + d*x]*Csc[c + d
*x])/(256*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(32*d) + (a*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
+ d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a\int \cot^6(c+dx)\csc$$

$$= -\frac{a\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{1}{2}a\int \cot^4(c+dx)\csc^5($$

$$= \frac{a\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a\cot^5(c+dx)\csc^5(c+dx}{10d}$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} -$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} +$$

$$= -\frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+dx)}{9d} - \frac{a\cot^{11}(c+dx)}{11d} +$$

$$= \frac{3a\tanh^{-1}(\cos(c+dx))}{256d} - \frac{a\cot^7(c+dx)}{7d} - \frac{2a\cot^9(c+}{9d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 363 vs. 2(176) = 352.
time = 0.07, size = 363, normalized size = 2.06

$$\frac{8a\cot(c+dx)}{693d}+\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{3a\csc^6\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{3a\csc^8\left(\frac{1}{2}(c+dx)\right)}{4096d}-\frac{a\csc^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}+\frac{4a\cot(c+dx)\csc^2(c+dx)}{693d}+\frac{a\cot(c+dx)\csc^4(c+dx)}{231d}-\frac{113a\cot(c+dx)\csc^6(c+dx)}{693d}+\frac{23a\cot(c+dx)\csc^8(c+dx)}{99d}-\frac{a\cot(c+dx)\csc^{10}(c+dx)}{11d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{256d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{256d}-\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{3a\sec^6\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{3a\sec^8\left(\frac{1}{2}(c+dx)\right)}{4096d}+\frac{a\sec^{10}\left(\frac{1}{2}(c+dx)\right)}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (8*a*Cot[c + d*x])/(693*d) + (3*a*Csc[(c + d*x)/2]^2)/(1024*d) - (a*Csc[(c
+ d*x)/2]^4)/(1024*d) - (3*a*Csc[(c + d*x)/2]^6)/(2048*d) + (3*a*Csc[(c + d
*x)/2]^8)/(4096*d) - (a*Csc[(c + d*x)/2]^10)/(10240*d) + (4*a*Cot[c + d*x]*
Csc[c + d*x]^2)/(693*d) + (a*Cot[c + d*x]*Csc[c + d*x]^4)/(231*d) - (113*a*
Cot[c + d*x]*Csc[c + d*x]^6)/(693*d) + (23*a*Cot[c + d*x]*Csc[c + d*x]^8)/(
99*d) - (a*Cot[c + d*x]*Csc[c + d*x]^10)/(11*d) + (3*a*Log[Cos[(c + d*x)/2]
])/(256*d) - (3*a*Log[Sin[(c + d*x)/2]])/(256*d) - (3*a*Sec[(c + d*x)/2]^2)
/(1024*d) + (a*Sec[(c + d*x)/2]^4)/(1024*d) + (3*a*Sec[(c + d*x)/2]^6)/(204
8*d) - (3*a*Sec[(c + d*x)/2]^8)/(4096*d) + (a*Sec[(c + d*x)/2]^10)/(10240*d
)
```

Maple [A]

time = 0.26, size = 202, normalized size = 1.15

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+a\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c}{640\sin(dx+c}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+a\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c}{640\sin(dx+c}\right.}{d}$
risch	$-\frac{a\left(10395\,e^{21i(dx+c)}-110880\,e^{19i(dx+c)}-563200ie^{4i(dx+c)}-2302839\,e^{17i(dx+c)}-15206400ie^{8i(dx+c)}-4790016\,e^{15i(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8/6
93/sin(d*x+c)^7*cos(d*x+c)^7)+a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(
d*x+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*co
s(d*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d
*x+c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.30, size = 168, normalized size = 0.95

$$-\frac{693\,a\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2560\left(99\tan(dx+c)^4+154\tan(dx+c)^2+63\right)a}{\tan(dx+c)^{11}}}{1774080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1774080*(693*a*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2560*(99*tan(d*x + c)^4
 + 154*tan(d*x + c)^2 + 63)*a/tan(d*x + c)^11)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(158) = 316$.
time = 0.41, size = 320, normalized size = 1.82

$$\frac{20480\,a\cos\left(dx+c\right)^{11}-112640\,a\cos\left(dx+c\right)^{9}+253440\,a\cos\left(dx+c\right)^{7}+10395\,\left(a\cos\left(dx+c\right)^{10}-5\,a\cos\left(dx+c\right)^{8}+10\,a\cos\left(dx+c\right)^{6}-10\,a\cos\left(dx+c\right)^{4}+5\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-10395\,\left(a\cos\left(dx+c\right)^{10}-5\,a\cos\left(dx+c\right)^{8}+10\,a\cos\left(dx+c\right)^{6}-10\,a\cos\left(dx+c\right)^{4}+5\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-1386\,\left(15\,a\cos\left(dx+c\right)^{9}-70\,a\cos\left(dx+c\right)^{7}-128\,a\cos\left(dx+c\right)^{5}+70\,a\cos\left(dx+c\right)^{3}-15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1774080\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*a*cos(d*x + c)^11 - 112640*a*cos(d*x + c)^9 + 253440*a*cos
(d*x + c)^7 + 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a*cos(d*x
+ c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 10395*(a*cos(d*x + c)^10 - 5*a*cos(d*x + c)^8 + 10*a
*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a)*log(-1/2*co
s(d*x + c) + 1/2)*sin(d*x + c) - 1386*(15*a*cos(d*x + c)^9 - 70*a*cos(d*x +
 c)^7 - 128*a*cos(d*x + c)^5 + 70*a*cos(d*x + c)^3 - 15*a*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 -
 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(158) = 316$.
time = 0.58, size = 340, normalized size = 1.93

$$\frac{630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1386\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-770\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4950\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-6930\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6930\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+27720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+23100\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+13860\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-166320\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-69300\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{502266\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+69300\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-13860\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-23100\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-27720\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-6930\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6930\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4950\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3465\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+770\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1386\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-630\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{14192640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(630*a*tan(1/2*d*x + 1/2*c)^11 + 1386*a*tan(1/2*d*x + 1/2*c)^10
- 770*a*tan(1/2*d*x + 1/2*c)^9 - 3465*a*tan(1/2*d*x + 1/2*c)^8 - 4950*a*tan
```

```
(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/2*c)^6 + 6930*a*tan(1/2*d*x +
1/2*c)^5 + 27720*a*tan(1/2*d*x + 1/2*c)^4 + 23100*a*tan(1/2*d*x + 1/2*c)^3
+ 13860*a*tan(1/2*d*x + 1/2*c)^2 - 166320*a*log(abs(tan(1/2*d*x + 1/2*c)))
- 69300*a*tan(1/2*d*x + 1/2*c) + (502266*a*tan(1/2*d*x + 1/2*c)^11 + 69300*
a*tan(1/2*d*x + 1/2*c)^10 - 13860*a*tan(1/2*d*x + 1/2*c)^9 - 23100*a*tan(1/
2*d*x + 1/2*c)^8 - 27720*a*tan(1/2*d*x + 1/2*c)^7 - 6930*a*tan(1/2*d*x + 1/
2*c)^6 + 6930*a*tan(1/2*d*x + 1/2*c)^5 + 4950*a*tan(1/2*d*x + 1/2*c)^4 + 34
65*a*tan(1/2*d*x + 1/2*c)^3 + 770*a*tan(1/2*d*x + 1/2*c)^2 - 1386*a*tan(1/2
*d*x + 1/2*c) - 630*a)/tan(1/2*d*x + 1/2*c)^11)/d
```

Mupad [B]
time = 9.98, size = 387, normalized size = 2.20

$$\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}-\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}+\frac{5\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14336\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4096\,d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{18432\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{22528\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14336\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4096\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{18432\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{22528\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] (5*a*cot(c/2 + (d*x)/2))/(1024*d) - (5*a*tan(c/2 + (d*x)/2))/(1024*d) - (a*
cot(c/2 + (d*x)/2)^2)/(1024*d) - (5*a*cot(c/2 + (d*x)/2)^3)/(3072*d) - (a*c
ot(c/2 + (d*x)/2)^4)/(512*d) - (a*cot(c/2 + (d*x)/2)^5)/(2048*d) + (a*cot(c
/2 + (d*x)/2)^6)/(2048*d) + (5*a*cot(c/2 + (d*x)/2)^7)/(14336*d) + (a*cot(c
/2 + (d*x)/2)^8)/(4096*d) + (a*cot(c/2 + (d*x)/2)^9)/(18432*d) - (a*cot(c/2
 + (d*x)/2)^10)/(10240*d) - (a*cot(c/2 + (d*x)/2)^11)/(22528*d) + (a*tan(c/
2 + (d*x)/2)^2)/(1024*d) + (5*a*tan(c/2 + (d*x)/2)^3)/(3072*d) + (a*tan(c/2
 + (d*x)/2)^4)/(512*d) + (a*tan(c/2 + (d*x)/2)^5)/(2048*d) - (a*tan(c/2 + (
d*x)/2)^6)/(2048*d) - (5*a*tan(c/2 + (d*x)/2)^7)/(14336*d) - (a*tan(c/2 + (
d*x)/2)^8)/(4096*d) - (a*tan(c/2 + (d*x)/2)^9)/(18432*d) + (a*tan(c/2 + (d*
x)/2)^10)/(10240*d) + (a*tan(c/2 + (d*x)/2)^11)/(22528*d) - (3*a*log(tan(c/
2 + (d*x)/2)))/(256*d)
```

3.588 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$\frac{17a^2x}{1024}-\frac{2a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{17a^2\cos(c+dx)\sin(c+dx)}{1024d}+\frac{17a^2\cos^3(c+$$

```
[Out] 17/1024*a^2*x-2/7*a^2*cos(d*x+c)^7/d+4/9*a^2*cos(d*x+c)^9/d-2/11*a^2*cos(d*
x+c)^11/d+17/1024*a^2*cos(d*x+c)*sin(d*x+c)/d+17/1536*a^2*cos(d*x+c)^3*sin(
d*x+c)/d+17/1920*a^2*cos(d*x+c)^5*sin(d*x+c)/d-17/320*a^2*cos(d*x+c)^7*sin(
d*x+c)/d-17/120*a^2*cos(d*x+c)^7*sin(d*x+c)^3/d-1/12*a^2*cos(d*x+c)^7*sin(d
*x+c)^5/d
```

Rubi [A]
time = 0.27, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 276}

$$-\frac{2a^2\cos^{11}(c+dx)}{11d}+\frac{4a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{17a^2\sin^3(c+dx)\cos^7(c+dx)}{120d}-\frac{17a^2\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{17a^2\sin(c+dx)\cos^5(c+dx)}{1920d}+\frac{17a^2\sin(c+dx)\cos^3(c+dx)}{1536d}+\frac{17a^2\sin(c+dx)\cos(c+dx)}{1024d}+\frac{17a^2x}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (17*a^2*x)/1024 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (4*a^2*Cos[c + d*x]^9)/(9*
d) - (2*a^2*Cos[c + d*x]^11)/(11*d) + (17*a^2*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (17*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (17*a^2*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (17*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (17*a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^2*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^4(c+dx)+2a^2\cos^6(c+dx)\sin\right.$$

$$= a^2\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^2\int \cos^6(c+dx)$$

$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^2\cos^7(c+dx)\sin^5(}{12d}$$

$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{17a^2\cos^7(c+dx)\sin}{120d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+}{11d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+}{11d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\cos^{11}(c+}{11d}$$

$$= \frac{3a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\mathrm{co}}{}$$

$$= \frac{17a^2x}{1024} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{4a^2\cos^9(c+dx)}{9d} - \frac{2a^2\mathrm{c}}{}$$

Mathematica [A]

time = 0.96, size = 136, normalized size = 0.65

$$\frac{a^2(166320c+471240dx-554400\cos(c+dx)-184800\cos(3(c+dx))+55440\cos(5(c+dx))+39600\cos(7(c+dx))-6160\cos(9(c+dx))-5040\cos(11(c+dx))+55440\sin(2(c+dx))-162855\sin(4(c+dx))-27720\sin(6(c+dx))+24255\sin(8(c+dx))+5544\sin(10(c+dx))-1155\sin(12(c+dx)))}{28385280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(166320*c + 471240*d*x - 554400*Cos[c + d*x] - 184800*Cos[3*(c + d*x)]
 + 55440*Cos[5*(c + d*x)] + 39600*Cos[7*(c + d*x)] - 6160*Cos[9*(c + d*x)]
- 5040*Cos[11*(c + d*x)] + 55440*Sin[2*(c + d*x)] - 162855*Sin[4*(c + d*x)]
 - 27720*Sin[6*(c + d*x)] + 24255*Sin[8*(c + d*x)] + 5544*Sin[10*(c + d*x)]
 - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

Maple [A]

time = 0.48, size = 238, normalized size = 1.14

method	result
risch	$-\frac{a^2\sin(12dx+12c)}{24576d}-\frac{5a^2\cos(dx+c)}{256d}+\frac{17a^2x}{1024}+\frac{a^2\sin(10dx+10c)}{5120d}-\frac{a^2\cos(11dx+11c)}{5632d}-\frac{a^2\cos(9dx+9c)}{4608d}+$ 7
derivativedivides	$a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.$
default	$a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+a^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c))
```

Maxima [A]

time = 0.31, size = 138, normalized size = 0.66

$$-\frac{81920\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^{9}+99\cos\left(dx+c\right)^{7}\right)a^2-2772\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\,dx+4\,c\right)\right)a^2-1155\left(4\sin\left(4\,dx+4\,c\right)^{3}+120\,dx+120\,c+9\sin\left(8\,dx+8\,c\right)-48\sin\left(4\,dx+4\,c\right)\right)a^2}{28385280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(81920*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120
*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.42, size = 137, normalized size = 0.66

$$-\frac{645120\,a^2\cos\left(dx+c\right)^{11}-1576960\,a^2\cos\left(dx+c\right)^{9}+1013760\,a^2\cos\left(dx+c\right)^{7}-58905\,a^2dx+231\,\left(1280\,a^2\cos\left(dx+c\right)^{11}-4736\,a^2\cos\left(dx+c\right)^{9}+4272\,a^2\cos\left(dx+c\right)^{7}-136\,a^2\cos\left(dx+c\right)^{5}-170\,a^2\cos\left(dx+c\right)^{3}-255\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(645120*a^2*cos(d*x + c)^11 - 1576960*a^2*cos(d*x + c)^9 + 10137
60*a^2*cos(d*x + c)^7 - 58905*a^2*d*x + 231*(1280*a^2*cos(d*x + c)^11 - 473
6*a^2*cos(d*x + c)^9 + 4272*a^2*cos(d*x + c)^7 - 136*a^2*cos(d*x + c)^5 - 1
70*a^2*cos(d*x + c)^3 - 255*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(201) = 402$.
time = 3.53, size = 656, normalized size = 3.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**12/1024 + 15*a**2*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 3*a**2*x*sin(c + d*x)**10/256 + 75*a**2*x*sin(c + d*x)**8
*cos(c + d*x)**4/1024 + 15*a**2*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**2*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a**2*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*cos(
c + d*x)**12/1024 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**2*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*a
**2*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + a**2*sin(c + d*x)**5*cos(c +
d*x)**5/(10*d) - 2*a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**2*sin
(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)
**7/(128*d) - 8*a**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 5*a**2*sin(c
```

```
+ d*x)*cos(c + d*x)**11/(1024*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256
*d) - 16*a**2*cos(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin
(c)**4*cos(c)**6, True))
```

Giac [A]
time = 0.60, size = 208, normalized size = 1.00

$$\frac{17}{1024}\,a^2x-\frac{a^2\cos\left(11\,dx+11\,c\right)}{5632\,d}-\frac{a^2\cos\left(9\,dx+9\,c\right)}{4608\,d}+\frac{5\,a^2\cos\left(7\,dx+7\,c\right)}{3584\,d}+\frac{a^2\cos\left(5\,dx+5\,c\right)}{512\,d}-\frac{5\,a^2\cos\left(3\,dx+3\,c\right)}{768\,d}-\frac{5\,a^2\cos\left(dx+c\right)}{256\,d}-\frac{a^2\sin\left(12\,dx+12\,c\right)}{24576\,d}+\frac{a^2\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{7\,a^2\sin\left(8\,dx+8\,c\right)}{8192\,d}-\frac{a^2\sin\left(6\,dx+6\,c\right)}{1024\,d}-\frac{47\,a^2\sin\left(4\,dx+4\,c\right)}{8192\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 17/1024*a^2*x - 1/5632*a^2*cos(11*d*x + 11*c)/d - 1/4608*a^2*cos(9*d*x + 9*
c)/d + 5/3584*a^2*cos(7*d*x + 7*c)/d + 1/512*a^2*cos(5*d*x + 5*c)/d - 5/768
*a^2*cos(3*d*x + 3*c)/d - 5/256*a^2*cos(d*x + c)/d - 1/24576*a^2*sin(12*d*x
 + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d + 7/8192*a^2*sin(8*d*x + 8*c)/
d - 1/1024*a^2*sin(6*d*x + 6*c)/d - 47/8192*a^2*sin(4*d*x + 4*c)/d + 1/512*
a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 11.12, size = 518, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*((17*c)/1024 - (17*tan(c/2 + (d*x)/2))/512 - (128*tan(c/2 + (d*x)/2)^2
)/231 - (595*tan(c/2 + (d*x)/2)^3)/1536 - (64*tan(c/2 + (d*x)/2)^4)/21 + (1
1097*tan(c/2 + (d*x)/2)^5)/2560 + (704*tan(c/2 + (d*x)/2)^6)/63 + (27449*ta
n(c/2 + (d*x)/2)^7)/2560 - (384*tan(c/2 + (d*x)/2)^8)/7 - (202307*tan(c/2 +
 (d*x)/2)^9)/3840 + (192*tan(c/2 + (d*x)/2)^10)/7 + (28659*tan(c/2 + (d*x)/
2)^11)/256 - (64*tan(c/2 + (d*x)/2)^12)/3 - (28659*tan(c/2 + (d*x)/2)^13)/2
56 - 64*tan(c/2 + (d*x)/2)^14 + (202307*tan(c/2 + (d*x)/2)^15)/3840 + 32*ta
n(c/2 + (d*x)/2)^16 - (27449*tan(c/2 + (d*x)/2)^17)/2560 - (64*tan(c/2 + (d
*x)/2)^18)/3 - (11097*tan(c/2 + (d*x)/2)^19)/2560 + (595*tan(c/2 + (d*x)/2)
^21)/1536 + (17*tan(c/2 + (d*x)/2)^23)/512 + (17*d*x)/1024 + (51*tan(c/2 +
(d*x)/2)^2*(c + d*x))/256 + (561*tan(c/2 + (d*x)/2)^4*(c + d*x))/512 + (935
*tan(c/2 + (d*x)/2)^6*(c + d*x))/256 + (8415*tan(c/2 + (d*x)/2)^8*(c + d*x)
)/1024 + (1683*tan(c/2 + (d*x)/2)^10*(c + d*x))/128 + (3927*tan(c/2 + (d*x)
/2)^12*(c + d*x))/256 + (1683*tan(c/2 + (d*x)/2)^14*(c + d*x))/128 + (8415*
tan(c/2 + (d*x)/2)^16*(c + d*x))/1024 + (935*tan(c/2 + (d*x)/2)^18*(c + d*x
))/256 + (561*tan(c/2 + (d*x)/2)^20*(c + d*x))/512 + (51*tan(c/2 + (d*x)/2)
^22*(c + d*x))/256 + (17*tan(c/2 + (d*x)/2)^24*(c + d*x))/1024 - 32/693))/(
d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

3.589 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=183

$$\frac{3a^2x}{128}-\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{128d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{64d}$$

[Out] 3/128*a^2*x-2/7*a^2*cos(d*x+c)^7/d+1/3*a^2*cos(d*x+c)^9/d-1/11*a^2*cos(d*x+c)^11/d+3/128*a^2*cos(d*x+c)*sin(d*x+c)/d+1/64*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/80*a^2*cos(d*x+c)^5*sin(d*x+c)/d-3/40*a^2*cos(d*x+c)^7*sin(d*x+c)/d-1/5*a^2*cos(d*x+c)^7*sin(d*x+c)^3/d

Rubi [A]
time = 0.20, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{a^2\cos^{11}(c+dx)}{11d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3a^2\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{80d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{3a^2x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/128 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (a^2*Cos[c + d*x]^9)/(3*d) - (a^2*Cos[c + d*x]^11)/(11*d) + (3*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(80*d) - (3*a^2*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^3(c+dx)+2a^2\cos^6(c+dx)\sin\right.$$
$$= a^2\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^2\int \cos^6(c+dx)$$
$$= -\frac{a^2\cos^7(c+dx)\sin^3(c+dx)}{5d}+\frac{1}{5}\left(3a^2\right)\int \cos^6(c+d$$
$$= -\frac{3a^2\cos^7(c+dx)\sin(c+dx)}{40d}-\frac{a^2\cos^7(c+dx)\sin^3(}{5d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{a^2\cos^{11}(c+d}{11d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{a^2\cos^{11}(c+d}{11d}$$
$$= -\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{a^2\cos^{11}(c+d}{11d}$$
$$= \frac{3a^2x}{128}-\frac{2a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^9(c+dx)}{3d}-\frac{a^2\cos^{11}}{1}$$

Mathematica [A]
time = 0.70, size = 126, normalized size = 0.69

$$\frac{a^2(27720c+27720dx-39270\cos(c+dx)-16170\cos(3(c+dx))+1155\cos(5(c+dx))+2805\cos(7(c+dx))+385\cos(9(c+dx))-105\cos(11(c+dx))+4620\sin(2(c+dx))-9240\sin(4(c+dx))-2310\sin(6(c+dx))+1155\sin(8(c+dx))+462\sin(10(c+dx)))}{1182720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
[Out] (a^2*(27720*c + 27720*d*x - 39270*Cos[c + d*x] - 16170*Cos[3*(c + d*x)] + 1
155*Cos[5*(c + d*x)] + 2805*Cos[7*(c + d*x)] + 385*Cos[9*(c + d*x)] - 105*C
os[11*(c + d*x)] + 4620*Sin[2*(c + d*x)] - 9240*Sin[4*(c + d*x)] - 2310*Sin
[6*(c + d*x)] + 1155*Sin[8*(c + d*x)] + 462*Sin[10*(c + d*x)]))/(1182720*d)
```

Maple [A]
time = 0.36, size = 172, normalized size = 0.94

method	result
derivativedivides	$a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(d\right.}{}\right.$

default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+\right.}{}\right.}{}$
risch	$-\frac{17a^2\cos(dx+c)}{512d}+\frac{3a^2x}{128}-\frac{a^2\cos(11dx+11c)}{11264d}+\frac{a^2\sin(10dx+10c)}{2560d}+\frac{a^2\cos(9dx+9c)}{3072d}+\frac{a^2\sin(8dx+8c)}{1024d}+\frac{17a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a^2*(-1/10*si
n(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+a^2*(-1/11*si
n(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7))
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.63

$$-\frac{5120\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^{9}+99\cos\left(dx+c\right)^{7}\right)a^{2}-56320\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a^{2}-693\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\sin\left(8\,dx+8\,c\right)-40\sin\left(4\,dx+4\,c\right)\right)a^{2}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3548160*(5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)
^7)*a^2 - 56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 - 693*(32*sin(2*d
*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a
^2)/d
```

Fricas [A]
time = 0.45, size = 124, normalized size = 0.68

$$-\frac{13440\,a^{2}\cos\left(dx+c\right)^{11}-49280\,a^{2}\cos\left(dx+c\right)^{9}+42240\,a^{2}\cos\left(dx+c\right)^{7}-3465\,a^{2}dx-231\left(128\,a^{2}\cos\left(dx+c\right)^{9}-176\,a^{2}\cos\left(dx+c\right)^{7}+8\,a^{2}\cos\left(dx+c\right)^{5}+10\,a^{2}\cos\left(dx+c\right)^{3}+15\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{147840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/147840*(13440*a^2*cos(d*x + c)^11 - 49280*a^2*cos(d*x + c)^9 + 42240*a^2
*cos(d*x + c)^7 - 3465*a^2*d*x - 231*(128*a^2*cos(d*x + c)^9 - 176*a^2*cos(
d*x + c)^7 + 8*a^2*cos(d*x + c)^5 + 10*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.
time = 2.58, size = 384, normalized size = 2.10

$$\begin{cases}\frac{3a^2x\sin^{10}(c+dx)}{128}+\frac{15a^2x\sin^8(c+dx)\cos^2(c+dx)}{128}+\frac{15a^2x\sin^6(c+dx)\cos^4(c+dx)}{64}+\frac{15a^2x\sin^4(c+dx)\cos^6(c+dx)}{64}+\frac{15a^2x\sin^2(c+dx)\cos^8(c+dx)}{128}+\frac{3a^2x\cos^{10}(c+dx)}{128}+\frac{3a^2\sin^9(c+dx)\cos(c+dx)}{128d}+\frac{7a^2\sin^7(c+dx)\cos^3(c+dx)}{64d}+\frac{a^2\sin^5(c+dx)\cos^5(c+dx)}{5d}-\frac{a^2\sin^4(c+dx)\cos^7(c+dx)}{7d}-\frac{7a^2\sin^3(c+dx)\cos^7(c+dx)}{64d}-\frac{4a^2\sin^2(c+dx)\cos^9(c+dx)}{63d}-\frac{a^2\sin^2(c+dx)\cos^7(c+dx)}{7d}-\frac{3a^2\sin(c+dx)\cos^9(c+dx)}{128d}-\frac{8a^2\cos^{11}(c+dx)}{693d}-\frac{2a^2\cos^9(c+dx)}{63d} & \text{for}\ d\neq 0\\ x\left(a\sin(c)+a\right)^2\sin^3(c)\cos^6(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/128 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/128 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a**2*x*si
n(c + d*x)**4*cos(c + d*x)**6/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**
8/128 + 3*a**2*x*cos(c + d*x)**10/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)
/(128*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a**2*sin(c + d*x
)**5*cos(c + d*x)**5/(5*d) - a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 7
*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 4*a**2*sin(c + d*x)**2*cos(c
 + d*x)**9/(63*d) - a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 3*a**2*sin
(c + d*x)*cos(c + d*x)**9/(128*d) - 8*a**2*cos(c + d*x)**11/(693*d) - 2*a**
2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**3*cos(c)*
*6, True))
```

Giac [**A**]
time = 0.60, size = 191, normalized size = 1.04

$$\frac{3}{128}a^2x-\frac{a^2\cos(11\,dx+11\,c)}{11264\,d}+\frac{a^2\cos(9\,dx+9\,c)}{3072\,d}+\frac{17\,a^2\cos(7\,dx+7\,c)}{7168\,d}+\frac{a^2\cos(5\,dx+5\,c)}{1024\,d}-\frac{7\,a^2\cos(3\,dx+3\,c)}{512\,d}-\frac{17\,a^2\cos(dx+c)}{512\,d}+\frac{a^2\sin(10\,dx+10\,c)}{2560\,d}+\frac{a^2\sin(8\,dx+8\,c)}{1024\,d}-\frac{a^2\sin(6\,dx+6\,c)}{512\,d}-\frac{a^2\sin(4\,dx+4\,c)}{128\,d}+\frac{a^2\sin(2\,dx+2\,c)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/128*a^2*x - 1/11264*a^2*cos(11*d*x + 11*c)/d + 1/3072*a^2*cos(9*d*x + 9*c
)/d + 17/7168*a^2*cos(7*d*x + 7*c)/d + 1/1024*a^2*cos(5*d*x + 5*c)/d - 7/51
2*a^2*cos(3*d*x + 3*c)/d - 17/512*a^2*cos(d*x + c)/d + 1/2560*a^2*sin(10*d*
x + 10*c)/d + 1/1024*a^2*sin(8*d*x + 8*c)/d - 1/512*a^2*sin(6*d*x + 6*c)/d
- 1/128*a^2*sin(4*d*x + 4*c)/d + 1/256*a^2*sin(2*d*x + 2*c)/d
```

Mupad [**B**]
time = 12.05, size = 543, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*a^2*x)/128 - ((3*a^2*(c + d*x))/128 + (a^2*tan(c/2 + (d*x)/2)^3)/2 - (33
23*a^2*tan(c/2 + (d*x)/2)^5)/320 + (108*a^2*tan(c/2 + (d*x)/2)^7)/5 - (841*
a^2*tan(c/2 + (d*x)/2)^9)/32 + (841*a^2*tan(c/2 + (d*x)/2)^13)/32 - (108*a^
2*tan(c/2 + (d*x)/2)^15)/5 + (3323*a^2*tan(c/2 + (d*x)/2)^17)/320 - (a^2*ta
n(c/2 + (d*x)/2)^19)/2 - (3*a^2*tan(c/2 + (d*x)/2)^21)/64 - a^2*((3*c)/128
+ (3*d*x)/128 - 20/231) + tan(c/2 + (d*x)/2)^2*((33*a^2*(c + d*x))/128 - a^
2*((33*c)/128 + (33*d*x)/128 - 20/21)) + tan(c/2 + (d*x)/2)^18*((165*a^2*(c
 + d*x))/128 - a^2*((165*c)/128 + (165*d*x)/128 - 4)) + tan(c/2 + (d*x)/2)^
```

```
4*((165*a^2*(c + d*x))/128 - a^2*((165*c)/128 + (165*d*x)/128 - 16/21)) + t
an(c/2 + (d*x)/2)^14*((495*a^2*(c + d*x))/64 - a^2*((495*c)/64 + (495*d*x)/
64 + 16)) + tan(c/2 + (d*x)/2)^16*((495*a^2*(c + d*x))/128 - a^2*((495*c)/1
28 + (495*d*x)/128 - 12)) + tan(c/2 + (d*x)/2)^6*((495*a^2*(c + d*x))/128 -
 a^2*((495*c)/128 + (495*d*x)/128 - 16/7)) + tan(c/2 + (d*x)/2)^8*((495*a^2
*(c + d*x))/64 - a^2*((495*c)/64 + (495*d*x)/64 - 312/7)) + tan(c/2 + (d*x)
/2)^10*((693*a^2*(c + d*x))/64 - a^2*((693*c)/64 + (693*d*x)/64 + 40)) + ta
n(c/2 + (d*x)/2)^12*((693*a^2*(c + d*x))/64 - a^2*((693*c)/64 + (693*d*x)/6
4 - 80)) + (3*a^2*tan(c/2 + (d*x)/2))/64)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^11)
```

3.590 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=165

$$\frac{13a^2x}{256}-\frac{2a^2\cos^7(c+dx)}{7d}+\frac{2a^2\cos^9(c+dx)}{9d}+\frac{13a^2\cos(c+dx)\sin(c+dx)}{256d}+\frac{13a^2\cos^3(c+dx)\sin(c+dx)}{384d}+\cdots$$

```
[Out] 13/256*a^2*x-2/7*a^2*cos(d*x+c)^7/d+2/9*a^2*cos(d*x+c)^9/d+13/256*a^2*cos(d
*x+c)*sin(d*x+c)/d+13/384*a^2*cos(d*x+c)^3*sin(d*x+c)/d+13/480*a^2*cos(d*x+
c)^5*sin(d*x+c)/d-13/80*a^2*cos(d*x+c)^7*sin(d*x+c)/d-1/10*a^2*cos(d*x+c)^7
*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 14}

$$\frac{2a^2\cos^9(c+dx)}{9d}-\frac{2a^2\cos^7(c+dx)}{7d}-\frac{a^2\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{13a^2\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{13a^2\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{13a^2\sin(c+dx)\cos^3(c+dx)}{384d}+\frac{13a^2\sin(c+dx)\cos(c+dx)}{256d}+\frac{13a^2x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*x)/256 - (2*a^2*Cos[c + d*x]^7)/(7*d) + (2*a^2*Cos[c + d*x]^9)/(9*d
) + (13*a^2*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (13*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(384*d) + (13*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(480*d) - (13*a^2
*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (a^2*Cos[c + d*x]^7*Sin[c + d*x]^3)/
(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)\sin^2(c+dx)+2a^2\cos^6(c+dx)\sin^3\right.$$

$$= a^2\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^2\int \cos^6(c+dx)\,\mathrm{s}$$

$$= -\frac{a^2\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{a^2\cos^7(c+dx)\sin^3(c+}{10d}$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{13a^2\cos^7(c+dx)\sin(c+}{80d}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos^3(c+d}{19}$$

$$= -\frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{5a^2\cos(c+dx}{128}$$

$$= \frac{5a^2x}{128} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\cos}{}$$

$$= \frac{13a^2x}{256} - \frac{2a^2\cos^7(c+dx)}{7d} + \frac{2a^2\cos^9(c+dx)}{9d} + \frac{13a^2\mathrm{c}}{}$$

Mathematica [A]

time = 0.46, size = 106, normalized size = 0.64

$$\frac{a^2(12600c+32760dx-30240\cos(c+dx)-13440\cos(3(c+dx))+2160\cos(7(c+dx))+560\cos(9(c+dx))+11340\sin(2(c+dx))-7560\sin(4(c+dx))-3990\sin(6(c+dx))-315\sin(8(c+dx))+126\sin(10(c+dx)))}{645120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(12600*c + 32760*d*x - 30240*Cos[c + d*x] - 13440*Cos[3*(c + d*x)] + 2
160*Cos[7*(c + d*x)] + 560*Cos[9*(c + d*x)] + 11340*Sin[2*(c + d*x)] - 7560
*Sin[4*(c + d*x)] - 3990*Sin[6*(c + d*x)] - 315*Sin[8*(c + d*x)] + 126*Sin[
10*(c + d*x)]))/(645120*d)
```

Maple [A]
time = 0.30, size = 184, normalized size = 1.12

method	result
risch	$\frac{13a^2x}{256}-\frac{3a^2\cos(dx+c)}{64d}+\frac{a^2\sin(10dx+10c)}{5120d}+\frac{a^2\cos(9dx+9c)}{1152d}-\frac{a^2\sin(8dx+8c)}{2048d}+\frac{3a^2\cos(7dx+7c)}{896d}-\frac{19a^2}{}$
derivativedivides	$\frac{a^2\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)(}{9}\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+2a^2\left(-\frac{\left(\sin^2(dx+c)\right)(}{9}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+2*a^2*(-1/9*sin(d*x+c)^2*cos
(d*x+c)^7-2/63*cos(d*x+c)^7)+a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(
d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.30, size = 128, normalized size = 0.78

$$\frac{20480\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)a^2+63\left(32\sin(2dx+2c)^5+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)a^2+210\left(64\sin(2dx+2c)^3+120dx+120c-3\sin(8dx+8c)-24\sin(4dx+4c)\right)a^2}{645120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 63*(32*sin(2*d*
x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^
2 + 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*
sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.38, size = 111, normalized size = 0.67

$$\frac{17920\,a^{2}\cos\left(dx+c\right)^{9}-23040\,a^{2}\cos\left(dx+c\right)^{7}+4095\,a^{2}dx+21\,\left(384\,a^{2}\cos\left(dx+c\right)^{9}-1008\,a^{2}\cos\left(dx+c\right)^{7}+104\,a^{2}\cos\left(dx+c\right)^{5}+130\,a^{2}\cos\left(dx+c\right)^{3}+195\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a^2*cos(d*x + c)^9 - 23040*a^2*cos(d*x + c)^7 + 4095*a^2*d*x
 + 21*(384*a^2*cos(d*x + c)^9 - 1008*a^2*cos(d*x + c)^7 + 104*a^2*cos(d*x +
 c)^5 + 130*a^2*cos(d*x + c)^3 + 195*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(158) = 316.
time = 1.86, size = 529, normalized size = 3.21

$$\begin{cases} \frac{3a^{2}x\sin^{10}{(c+dx)}}{256}+\frac{15a^{2}x\sin^{8}{(c+dx)}\cos^{2}{(c+dx)}}{256}+\frac{5a^{2}x\sin^{8}{(c+dx)}}{128}+\frac{15a^{2}x\sin^{6}{(c+dx)}\cos^{4}{(c+dx)}}{128}+\frac{5a^{2}x\sin^{6}{(c+dx)}\cos^{2}{(c+dx)}}{32}+\frac{15a^{2}x\sin^{4}{(c+dx)}\cos^{6}{(c+dx)}}{128}+\frac{15a^{2}x\sin^{4}{(c+dx)}\cos^{4}{(c+dx)}}{64}+\frac{15a^{2}x\sin^{2}{(c+dx)}\cos^{8}{(c+dx)}}{256}+\frac{5a^{2}x\sin^{2}{(c+dx)}\cos^{6}{(c+dx)}}{32}+\frac{3a^{2}x\cos^{10}{(c+dx)}}{256}+\frac{5a^{2}x\cos^{8}{(c+dx)}}{128}+\frac{3a^{2}\sin^{9}{(c+dx)}\cos{(c+dx)}}{256d}+\frac{7a^{2}\sin^{7}{(c+dx)}\cos^{3}{(c+dx)}}{128d}+\frac{5a^{2}\sin^{7}{(c+dx)}\cos{(c+dx)}}{128d}+\frac{a^{2}\sin^{5}{(c+dx)}\cos^{5}{(c+dx)}}{10d}+\frac{55a^{2}\sin^{5}{(c+dx)}\cos^{3}{(c+dx)}}{384d}-\frac{7a^{2}\sin^{3}{(c+dx)}\cos^{7}{(c+dx)}}{128d}+\frac{73a^{2}\sin^{3}{(c+dx)}\cos^{5}{(c+dx)}}{384d}-\frac{2a^{2}\sin^{2}{(c+dx)}\cos^{7}{(c+dx)}}{7d}-\frac{3a^{2}\sin{(c+dx)}\cos^{9}{(c+dx)}}{256d}-\frac{5a^{2}\sin{(c+dx)}\cos^{7}{(c+dx)}}{128d}-\frac{4a^{2}\cos^{9}{(c+dx)}}{63d} & \text{for}\: d \neq 0 \\ x\left(a\sin{(c)}+a\right)^{2}\sin^{2}{(c)}\cos^{6}{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**2*x*sin(c + d*x)**8/128 + 15*a**2*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 5*a**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**2*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 3*a**2*x*cos(c + d*x)**10/256 + 5*a**2*x*cos(
c + d*x)**8/128 + 3*a**2*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**2*sin(
c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**2*sin(c + d*x)**7*cos(c + d*x)/(
128*d) + a**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**2*sin(c + d*x)
**5*cos(c + d*x)**3/(384*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(128*d
) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 2*a**2*sin(c + d*x)**
2*cos(c + d*x)**7/(7*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 5*a
**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 4*a**2*cos(c + d*x)**9/(63*d), N
e(d, 0)), (x*(a*sin(c) + a)**2*sin(c)**2*cos(c)**6, True))
```

Giac [A]
time = 0.53, size = 157, normalized size = 0.95

$$\frac{13}{256}\,a^{2}x+\frac{a^{2}\cos\left(9\,dx+9\,c\right)}{1152\,d}+\frac{3\,a^{2}\cos\left(7\,dx+7\,c\right)}{896\,d}-\frac{a^{2}\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{3\,a^{2}\cos\left(dx+c\right)}{64\,d}+\frac{a^{2}\sin\left(10\,dx+10\,c\right)}{5120\,d}-\frac{a^{2}\sin\left(8\,dx+8\,c\right)}{2048\,d}-\frac{19\,a^{2}\sin\left(6\,dx+6\,c\right)}{3072\,d}-\frac{3\,a^{2}\sin\left(4\,dx+4\,c\right)}{256\,d}+\frac{9\,a^{2}\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 13/256*a^2*x + 1/1152*a^2*cos(9*d*x + 9*c)/d + 3/896*a^2*cos(7*d*x + 7*c)/d
 - 1/48*a^2*cos(3*d*x + 3*c)/d - 3/64*a^2*cos(d*x + c)/d + 1/5120*a^2*sin(1
0*d*x + 10*c)/d - 1/2048*a^2*sin(8*d*x + 8*c)/d - 19/3072*a^2*sin(6*d*x + 6
*c)/d - 3/256*a^2*sin(4*d*x + 4*c)/d + 9/512*a^2*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.12, size = 469, normalized size = 2.84

$$\frac{13\,a^2\,x}{256}-\frac{\frac{13\,a^2\,\left(c+d\,x\right)}{256}-\frac{647\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{384}-\frac{2311\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{480}+\frac{457\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{32}-\frac{2169\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{64}+\frac{2169\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{64}-\frac{457\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}+\frac{2311\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{480}+\frac{647\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{384}-\frac{13\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{128}-\frac{a^2\,\left(4095\,c+4095\,d\,x-10240\right)}{80640}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{65\,a^2\,\left(c+d\,x\right)}{128}-\frac{a^2\,\left(40950\,c+40950\,d\,x-102400\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{585\,a^2\,\left(c+d\,x\right)}{256}-\frac{a^2\,\left(184275\,c+184275\,d\,x+184320\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}\,\left(\frac{585\,a^2\,\left(c+d\,x\right)}{256}-\frac{a^2\,\left(184275\,c+184275\,d\,x-645120\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(\frac{195\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(491400\,c+491400\,d\,x+430080\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{195\,a^2\,\left(c+d\,x\right)}{32}-\frac{a^2\,\left(491400\,c+491400\,d\,x-1658880\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{819\,a^2\,\left(c+d\,x\right)}{64}-\frac{a^2\,\left(1031940\,c+1031940\,d\,x-1290240\right)}{80640}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{1365\,a^2\,\left(c+d\,x\right)}{128}-\frac{a^2\,\left(859950\,c+859950\,d\,x-2150400\right)}{80640}\right)+\frac{13\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (13*a^2*x)/256 - ((13*a^2*(c + d*x))/256 - (647*a^2*tan(c/2 + (d*x)/2)^3)/3
84 - (2311*a^2*tan(c/2 + (d*x)/2)^5)/480 + (457*a^2*tan(c/2 + (d*x)/2)^7)/3
2 - (2169*a^2*tan(c/2 + (d*x)/2)^9)/64 + (2169*a^2*tan(c/2 + (d*x)/2)^11)/6
4 - (457*a^2*tan(c/2 + (d*x)/2)^13)/32 + (2311*a^2*tan(c/2 + (d*x)/2)^15)/4
80 + (647*a^2*tan(c/2 + (d*x)/2)^17)/384 - (13*a^2*tan(c/2 + (d*x)/2)^19)/1
28 - (a^2*(4095*c + 4095*d*x - 10240))/80640 + tan(c/2 + (d*x)/2)^2*((65*a^
2*(c + d*x))/128 - (a^2*(40950*c + 40950*d*x - 102400))/80640) + tan(c/2 +
(d*x)/2)^4*((585*a^2*(c + d*x))/256 - (a^2*(184275*c + 184275*d*x + 184320)
)/80640) + tan(c/2 + (d*x)/2)^16*((585*a^2*(c + d*x))/256 - (a^2*(184275*c
+ 184275*d*x - 645120))/80640) + tan(c/2 + (d*x)/2)^14*((195*a^2*(c + d*x))
/32 - (a^2*(491400*c + 491400*d*x + 430080))/80640) + tan(c/2 + (d*x)/2)^6*
((195*a^2*(c + d*x))/32 - (a^2*(491400*c + 491400*d*x - 1658880))/80640) +
tan(c/2 + (d*x)/2)^10*((819*a^2*(c + d*x))/64 - (a^2*(1031940*c + 1031940*d
*x - 1290240))/80640) + tan(c/2 + (d*x)/2)^12*((1365*a^2*(c + d*x))/128 - (
a^2*(859950*c + 859950*d*x - 2150400))/80640) + (13*a^2*tan(c/2 + (d*x)/2))
/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.591 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$\frac{5a^2x}{64}-\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{96d}+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{24d}$$

```
[Out] 5/64*a^2*x-1/28*a^2*cos(d*x+c)^7/d+5/64*a^2*cos(d*x+c)*sin(d*x+c)/d+5/96*a^
2*cos(d*x+c)^3*sin(d*x+c)/d+1/24*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/9*cos(d*x+
c)^7*(a+a*sin(d*x+c))^2/d-1/36*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{36d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{5a^2x}{64}-\frac{\cos^7(c+dx)(a\sin(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/64 - (a^2*Cos[c + d*x]^7)/(28*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*
x])/(64*d) + (5*a^2*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a^2*Cos[c + d*x]
^5*Sin[c + d*x])/(24*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - (
Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(36*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
```

```
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^2\,dx &= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{2}{9}\int \cos^6(c+dx) \\
&= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{\cos^7(c+dx)\left(a^2\right.}{36} \\
&= -\frac{a^2\cos^7(c+dx)}{28d}-\frac{\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d} \\
&= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{24d}-\frac{\cos^7}{} \\
&= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{96d}+\frac{a^2}{} \\
&= -\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{5a^2}{} \\
&= \frac{5a^2x}{64}-\frac{a^2\cos^7(c+dx)}{28d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{64d}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 106, normalized size = 0.69

$$\frac{a^2(2520c+2520dx-3276\cos(c+dx)-1848\cos(3(c+dx))-504\cos(5(c+dx))-18\cos(7(c+dx))+14\cos(9(c+dx))+1008\sin(2(c+dx))-504\sin(4(c+dx))-336\sin(6(c+dx))-63\sin(8(c+dx)))}{32256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(2520*c + 2520*d*x - 3276*Cos[c + d*x] - 1848*Cos[3*(c + d*x)] - 504*C
os[5*(c + d*x)] - 18*Cos[7*(c + d*x)] + 14*Cos[9*(c + d*x)] + 1008*Sin[2*(c
```

```
 + d*x)] - 504*Sin[4*(c + d*x)] - 336*Sin[6*(c + d*x)] - 63*Sin[8*(c + d*x)
]))/(32256*d)
```

Maple [A]
time = 0.26, size = 116, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{a^2(\cos^7(dx+c))}{7}+2a^2\left(-\frac{(\cos^7(dx+c))\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+a^2}{d}$
default	$\frac{-\frac{a^2(\cos^7(dx+c))}{7}+2a^2\left(-\frac{(\cos^7(dx+c))\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5(\cos^3(dx+c))}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+a^2}{d}$
risch	$\frac{5a^2x}{64}-\frac{13a^2\cos(dx+c)}{128d}+\frac{a^2\cos(9dx+9c)}{2304d}-\frac{a^2\sin(8dx+8c)}{512d}-\frac{a^2\cos(7dx+7c)}{1792d}-\frac{a^2\sin(6dx+6c)}{96d}-\frac{a^2\cos(5dx}{64d}$
norman	$\frac{\frac{45a^2x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{45a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{32a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7d}-\frac{83a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{2a^2\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{191a^2(\tan}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^2*cos(d*x+c)^7+2*a^2*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*
x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+a^2*
(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A]
time = 0.28, size = 93, normalized size = 0.61

$$-\frac{4608\,a^2\cos\left(dx+c\right)^7-512\,\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)a^2-21\,\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a^2}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 512*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^
7)*a^2 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
 24*sin(4*d*x + 4*c))*a^2)/d
```

Fricas [A]
time = 0.38, size = 98, normalized size = 0.64

$$\frac{448\,a^2\cos\left(dx+c\right)^9-1152\,a^2\cos\left(dx+c\right)^7+315\,a^2dx-21\,\left(48\,a^2\cos\left(dx+c\right)^7-8\,a^2\cos\left(dx+c\right)^5-10\,a^2\cos\left(dx+c\right)^3-15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*a^2*cos(d*x + c)^9 - 1152*a^2*cos(d*x + c)^7 + 315*a^2*d*x - 21
*(48*a^2*cos(d*x + c)^7 - 8*a^2*cos(d*x + c)^5 - 10*a^2*cos(d*x + c)^3 - 15
*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(139) = 278$.
time = 1.30, size = 282, normalized size = 1.84

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{64} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{5a^2x\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{5a^2x\cos^8(c+dx)}{64} + \frac{5a^2\sin^7(c+dx)\cos(c+dx)}{64d} + \frac{55a^2\sin^5(c+dx)\cos^3(c+dx)}{192d} + \frac{73a^2\sin^3(c+dx)\cos^5(c+dx)}{192d} - \frac{a^2\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{5a^2\sin(c+dx)\cos^7(c+dx)}{64d} - \frac{2a^2\cos^9(c+dx)}{63d} - \frac{a^2\cos^7(c+dx)}{7d} & \text{for}\: d \neq 0 \\ x(a\sin(c)+a)^2\sin(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/64 + 5*a**2*x*sin(c + d*x)**6*cos(c + d
*x)**2/16 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*a**2*x*sin(c +
 d*x)**2*cos(c + d*x)**6/16 + 5*a**2*x*cos(c + d*x)**8/64 + 5*a**2*sin(c +
d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(192*
d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - a**2*sin(c + d*x)**2
*cos(c + d*x)**7/(7*d) - 5*a**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 2*a**
2*cos(c + d*x)**9/(63*d) - a**2*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin
(c) + a)**2*sin(c)*cos(c)**6, True))
```

Giac [**A**]
time = 0.49, size = 157, normalized size = 1.03

$$\frac{5}{64}a^2x + \frac{a^2\cos(9dx+9c)}{2304d} - \frac{a^2\cos(7dx+7c)}{1792d} - \frac{a^2\cos(5dx+5c)}{64d} - \frac{11a^2\cos(3dx+3c)}{192d} - \frac{13a^2\cos(dx+c)}{128d} - \frac{a^2\sin(8dx+8c)}{512d} - \frac{a^2\sin(6dx+6c)}{96d} - \frac{a^2\sin(4dx+4c)}{64d} + \frac{a^2\sin(2dx+2c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a^2*x + 1/2304*a^2*cos(9*d*x + 9*c)/d - 1/1792*a^2*cos(7*d*x + 7*c)/d
- 1/64*a^2*cos(5*d*x + 5*c)/d - 11/192*a^2*cos(3*d*x + 3*c)/d - 13/128*a^2*
cos(d*x + c)/d - 1/512*a^2*sin(8*d*x + 8*c)/d - 1/96*a^2*sin(6*d*x + 6*c)/d
 - 1/64*a^2*sin(4*d*x + 4*c)/d + 1/32*a^2*sin(2*d*x + 2*c)/d
```

Mupad [**B**]
time = 10.85, size = 501, normalized size = 3.27

$$\frac{5a^2x}{64} - \frac{\frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{16} - \frac{191a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{48} - \frac{145a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{16} + \frac{145a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{16} - \frac{83a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{16} + \frac{191a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{48} - \frac{5a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{32} + \frac{a^2(315c+315dx)}{4032} - \frac{a^2(315c+315dx-1408)}{4032} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2(315c+315dx)}{448} - \frac{a^2(2835c+2835dx-4608)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\left(\frac{a^2(315c+315dx)}{448} - \frac{a^2(2835c+2835dx-8064)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^2(315c+315dx)}{112} - \frac{a^2(11340c+11340dx-18432)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{a^2(315c+315dx)}{112} - \frac{a^2(11340c+11340dx-32256)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{a^2(315c+315dx)}{48} - \frac{a^2(26460c+26460dx-21504)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{a^2(315c+315dx)}{32} - \frac{a^2(39690c+39690dx-16128)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^2(315c+315dx)}{48} - \frac{a^2(26460c+26460dx-96768)}{4032}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{a^2(315c+315dx)}{32} - \frac{a^2(39690c+39690dx-161280)}{4032}\right) + \frac{5a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/64 - ((83*a^2*tan(c/2 + (d*x)/2)^5)/16 - (191*a^2*tan(c/2 + (d*x)
/2)^3)/48 - (145*a^2*tan(c/2 + (d*x)/2)^7)/16 + (145*a^2*tan(c/2 + (d*x)/2)
```

```
^11)/16 - (83*a^2*tan(c/2 + (d*x)/2)^13)/16 + (191*a^2*tan(c/2 + (d*x)/2)^1
5)/48 - (5*a^2*tan(c/2 + (d*x)/2)^17)/32 + (a^2*(315*c + 315*d*x))/4032 - (
a^2*(315*c + 315*d*x - 1408))/4032 + tan(c/2 + (d*x)/2)^2*((a^2*(315*c + 31
5*d*x))/448 - (a^2*(2835*c + 2835*d*x - 4608))/4032) + tan(c/2 + (d*x)/2)^1
6*((a^2*(315*c + 315*d*x))/448 - (a^2*(2835*c + 2835*d*x - 8064))/4032) + t
an(c/2 + (d*x)/2)^4*((a^2*(315*c + 315*d*x))/112 - (a^2*(11340*c + 11340*d*
x - 18432))/4032) + tan(c/2 + (d*x)/2)^14*((a^2*(315*c + 315*d*x))/112 - (a
^2*(11340*c + 11340*d*x - 32256))/4032) + tan(c/2 + (d*x)/2)^12*((a^2*(315*
c + 315*d*x))/48 - (a^2*(26460*c + 26460*d*x - 21504))/4032) + tan(c/2 + (d
*x)/2)^8*((a^2*(315*c + 315*d*x))/32 - (a^2*(39690*c + 39690*d*x - 16128))/
4032) + tan(c/2 + (d*x)/2)^6*((a^2*(315*c + 315*d*x))/48 - (a^2*(26460*c +
26460*d*x - 96768))/4032) + tan(c/2 + (d*x)/2)^10*((a^2*(315*c + 315*d*x))/
32 - (a^2*(39690*c + 39690*d*x - 161280))/4032) + (5*a^2*tan(c/2 + (d*x)/2)
)/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.592 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=161

$$\frac{5a^2x}{8}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^7(c+dx)}{7d}+5a^2\cos$$

```
[Out] 5/8*a^2*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
+1/5*a^2*cos(d*x+c)^5/d-1/7*a^2*cos(d*x+c)^7/d+5/8*a^2*cos(d*x+c)*sin(d*x+c
)/d+5/12*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/3*a^2*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2952, 2715, 8, 2672, 308, 212, 2645, 30}

$$-\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cos[c + d*x]^7)/(7
*d) + (5*a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a^2*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\cos^6(c+dx)+a^2\cos^5(c+dx)\cot(c+dx)+a^2 \right.\\ &= a^2\int \cos^5(c+dx)\cot(c+dx)\,dx+a^2\int \cos^6(c+dx)\sin\\ &= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{3d}+\frac{1}{3}\left(5a^2\right)\int \cos^4(c+dx)\,dx\\ &= -\frac{a^2\cos^7(c+dx)}{7d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{12d}+\frac{a^2}{}\\ &= \frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^5(c+dx)}{5d}-\\ &= \frac{5a^2x}{8}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 112, normalized size = 0.70

$$\frac{a^2\left(4200c+4200dx+8715\cos(c+dx)+665\cos(3(c+dx))-21\cos(5(c+dx))-15\cos(7(c+dx))-6720\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6720\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3150\sin(2(c+dx))+630\sin(4(c+dx))+70\sin(6(c+dx))\right)}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(4200*c + 4200*d*x + 8715*Cos[c + d*x] + 665*Cos[3*(c + d*x)] - 21*Cos
[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log
[Sin[(c + d*x)/2]] + 3150*Sin[2*(c + d*x)] + 630*Sin[4*(c + d*x)] + 70*Sin[
6*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.23, size = 114, normalized size = 0.71

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^2\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{d}$
risch	$\frac{5a^2x}{8}+\frac{83a^2e^{i(dx+c)}}{128d}+\frac{83a^2e^{-i(dx+c)}}{128d}+\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}-\frac{a^2\cos(7dx+7c)}{448d}+\frac{a^2\sin(}{9}$
norman	$\frac{\frac{4a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^2x}{8}+\frac{292a^2}{105d}+\frac{11a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{7a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{85a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{85a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*
x+c)))+2*a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+5/16*d*x+5/16*c)-1/7*a^2*cos(d*x+c)^7)
```

Maxima [A]
time = 0.28, size = 123, normalized size = 0.76

$$-\frac{480\,a^2\cos\left(dx+c\right)^7-112\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^2+35\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^2}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*a^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^2)/d
```

Fricas [A]
time = 0.42, size = 141, normalized size = 0.88

$$-\frac{120\,a^2\cos\left(dx+c\right)^7-168\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3-525\,a^2dx-840\,a^2\cos\left(dx+c\right)+420\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-420\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-35\,\left(8\,a^2\cos\left(dx+c\right)^5+10\,a^2\cos\left(dx+c\right)^3+15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*a^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a^2*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a^2*cos(d*x + c)^5 + 1
0*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.49, size = 245, normalized size = 1.52

$$\frac{525\,(dx+c)a^2+840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1680\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+980\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-10080\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+2975\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-16240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-24640\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2975\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-14448\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-980\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6496\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1168\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a^2 + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*tan(1/2*d*x + 1/2*c)^12 + 980*a^2*t
an(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/2*d*x + 1/2*c)^10 + 2975*a^2*tan(1
/2*d*x + 1/2*c)^9 - 16240*a^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*
x + 1/2*c)^6 - 2975*a^2*tan(1/2*d*x + 1/2*c)^5 - 14448*a^2*tan(1/2*d*x + 1/
2*c)^4 - 980*a^2*tan(1/2*d*x + 1/2*c)^3 - 6496*a^2*tan(1/2*d*x + 1/2*c)^2 -
 1155*a^2*tan(1/2*d*x + 1/2*c) - 1168*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/
d
```

Mupad [B]
time = 10.84, size = 384, normalized size = 2.39

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{5\,a^2\,\mathrm{atan}\left(\frac{25\,a^4}{16\,\left(\frac{5\,a^4}{2}-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{5\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{5\,a^4}{2}-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{11\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{4}+4\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-\frac{7\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{3}+24\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-\frac{85\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{12}+\frac{116\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{3}+\frac{176\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{3}+\frac{85\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{12}+\frac{172\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{5}+\frac{7\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{3}+\frac{232\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{15}+\frac{11\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{292\,a^2}{105}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (5*a^2*atan((25*a^4)/(16*((5*a^4)/2 - (25
*a^4*tan(c/2 + (d*x)/2))/16)) + (5*a^4*tan(c/2 + (d*x)/2))/(2*((5*a^4)/2 -
(25*a^4*tan(c/2 + (d*x)/2))/16))))/(4*d) + ((232*a^2*tan(c/2 + (d*x)/2)^2)/
15 + (7*a^2*tan(c/2 + (d*x)/2)^3)/3 + (172*a^2*tan(c/2 + (d*x)/2)^4)/5 + (8
5*a^2*tan(c/2 + (d*x)/2)^5)/12 + (176*a^2*tan(c/2 + (d*x)/2)^6)/3 + (116*a^
2*tan(c/2 + (d*x)/2)^8)/3 - (85*a^2*tan(c/2 + (d*x)/2)^9)/12 + 24*a^2*tan(c
/2 + (d*x)/2)^10 - (7*a^2*tan(c/2 + (d*x)/2)^11)/3 + 4*a^2*tan(c/2 + (d*x)/
2)^12 - (11*a^2*tan(c/2 + (d*x)/2)^13)/4 + (292*a^2)/105 + (11*a^2*tan(c/2
+ (d*x)/2))/4)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*ta
n(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7
*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1))
```

3.593 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$-\frac{25a^2x}{16}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^5(c+dx)}{5d}-\frac{a^2\cot(c+dx)}{d}-$$

```
[Out] -25/16*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+2/3*a^2*cos(d*x
+c)^3/d+2/5*a^2*cos(d*x+c)^5/d-a^2*cot(d*x+c)/d-7/16*a^2*cos(d*x+c)*sin(d*x
+c)/d-7/24*a^2*cos(d*x+c)*sin(d*x+c)^3/d+1/6*a^2*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 2718, 2713, 2715}

$$\frac{2a^2\cos^5(c+dx)}{5d}+\frac{2a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{7a^2\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{16d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{25a^2x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-25*a^2*x)/16 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d +
 (2*a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Cos[c + d*x]^5)/(5*d) - (a^2*Cot[c +
 d*x])/d - (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (7*a^2*Cos[c + d*x]*S
in[c + d*x]^3)/(24*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int \left(-2a^8+2a^8\csc(c+dx)+a^8\csc^2(c+dx)-6a^8\sin\right.}{} \\
&= -2a^2x+a^2\int\csc^2(c+dx)\,dx-a^2\int\sin^6(c+dx)\,dx \\
&= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{6a^2\cos(c+dx)}{d} \\
&= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d} \\
&= -\frac{5a^2x}{4}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d} \\
&= -\frac{25a^2x}{16}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 110, normalized size = 0.70

$$
\frac{a^2\left(-1500c-1500dx+2640\cos(c+dx)+280\cos(3(c+dx))+24\cos(5(c+dx))-960\cot(c+dx)-1920\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+1920\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-255\sin(2(c+dx))+15\sin(4(c+dx))+5\sin(6(c+dx))\right)}{960d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

[Out] (a^2*(-1500*c - 1500*d*x + 2640*Cos[c + d*x] + 280*Cos[3*(c + d*x)] + 24*Co
s[5*(c + d*x)] - 960*Cot[c + d*x] - 1920*Log[Cos[(c + d*x)/2]] + 1920*Log[S
in[(c + d*x)/2]] - 255*Sin[2*(c + d*x)] + 15*Sin[4*(c + d*x)] + 5*Sin[6*(c
+ d*x)]))/(960*d)

Maple [A]
time = 0.20, size = 166, normalized size = 1.05

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+\right.}{3}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+\right.}{3}\right.}{d}$
risch	$-\frac{25a^2x}{16}+\frac{11a^2e^{i(dx+c)}}{8d}+\frac{11a^2e^{-i(dx+c)}}{8d}+\frac{17ia^2e^{2i(dx+c)}}{128d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}-1\right)}-\frac{17ia^2e^{-2i(dx+c)}}{128d}-\frac{2a^2\ln\left(e^{i(\right.}}{c}$
norman	$\frac{56a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{2d}-\frac{27a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{227a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{5a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{227a^2\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*co
s(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+2*a^2*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c
)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*
x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

Maxima [A]
time = 0.54, size = 173, normalized size = 1.09

$$\frac{64\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^2-120\left(15dx+15c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^2}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] 1/960*(64*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*sin(2*d*x + 2*c)^3
 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - 120*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2)/d

Fricas [A]
time = 0.41, size = 161, normalized size = 1.02

$$-\frac{40\,a^{2}\cos\left(dx+c\right)^{7}-50\,a^{2}\cos\left(dx+c\right)^{5}-125\,a^{2}\cos\left(dx+c\right)^{3}+240\,a^{2}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-240\,a^{2}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+375\,a^{2}\cos\left(dx+c\right)-\left(96\,a^{2}\cos\left(dx+c\right)^{5}+160\,a^{2}\cos\left(dx+c\right)^{3}-375\,a^{2}dx+480\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^2*cos(d*x + c)^7 - 50*a^2*cos(d*x + c)^5 - 125*a^2*cos(d*x + c
)^3 + 240*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 240*a^2*log(-1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 375*a^2*cos(d*x + c) - (96*a^2*cos(d*x +
c)^5 + 160*a^2*cos(d*x + c)^3 - 375*a^2*d*x + 480*a^2*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.49, size = 274, normalized size = 1.73

$$-\frac{375\,(dx+c)a^{2}-480\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{120\,\left(4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1440\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+595\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4320\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-150\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7360\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+150\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6720\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-595\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2976\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+736\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 120*a^
2*tan(1/2*d*x + 1/2*c) + 120*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
 + 1/2*c) - 2*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 1440*a^2*tan(1/2*d*x + 1/2
*c)^10 + 595*a^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a^2*tan(1/2*d*x + 1/2*c)^8 -
 150*a^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a^2*tan(1/2*d*x + 1/2*c)^6 + 150*a^2
*tan(1/2*d*x + 1/2*c)^5 + 6720*a^2*tan(1/2*d*x + 1/2*c)^4 - 595*a^2*tan(1/2
*d*x + 1/2*c)^3 + 2976*a^2*tan(1/2*d*x + 1/2*c)^2 - 105*a^2*tan(1/2*d*x + 1
/2*c) + 736*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 8.97, size = 401, normalized size = 2.54

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{25\,a^2\,\mathrm{atan}\left(\frac{625\,a^4}{64\,\left(\frac{25\,a^4}{2}+\frac{625\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}-\frac{25\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{25\,a^4}{2}+\frac{625\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{\frac{3\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{4}+24\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+\frac{47\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{12}+72\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9-\frac{35\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{2}+\frac{368\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{3}-\frac{35\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{2}+112\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-\frac{299\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{12}+\frac{248\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{5}-\frac{31\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}+\frac{184\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}-a^2}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+30\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+40\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+30\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+12\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^2,x)

[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d + (25*a^2*atan((625*a^4)/(64*((25*a^4)/2 + (625*a^4*tan(c/2 + (d*x)/2))/64)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a^4)/2 + (625*a^4*tan(c/2 + (d*x)/2))/64))))/(8*d) + ((248*a^2*tan(c/2 + (d*x)/2)^3)/5 - (31*a^2*tan(c/2 + (d*x)/2)^2)/4 - (299*a^2*tan(c/2 + (d*x)/2)^4)/12 + 112*a^2*tan(c/2 + (d*x)/2)^5 - (35*a^2*tan(c/2 + (d*x)/2)^6)/2 + (368*a^2*tan(c/2 + (d*x)/2)^7)/3 - (35*a^2*tan(c/2 + (d*x)/2)^8)/2 + 72*a^2*tan(c/2 + (d*x)/2)^9 + (47*a^2*tan(c/2 + (d*x)/2)^10)/12 + 24*a^2*tan(c/2 + (d*x)/2)^11 + (3*a^2*tan(c/2 + (d*x)/2)^12)/4 - a^2 + (184*a^2*tan(c/2 + (d*x)/2))/15)/(d*(2*tan(c/2 + (d*x)/2) + 12*tan(c/2 + (d*x)/2)^3 + 30*tan(c/2 + (d*x)/2)^5 + 40*tan(c/2 + (d*x)/2)^7 + 30*tan(c/2 + (d*x)/2)^9 + 12*tan(c/2 + (d*x)/2)^11 + 2*tan(c/2 + (d*x)/2)^13)) + (a^2*tan(c/2 + (d*x)/2))/(2*d)

3.594 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=140

$$-\frac{15a^2x}{4}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^5(c+dx)}{5d}-\frac{2a^2\cot(c+dx)}{d}-\frac{a^2\cot(c+dx)\csc(c}{2d}$$

```
[Out] -15/4*a^2*x+3/2*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d+1/5*a^2*cos(d*x+
c)^5/d-2*a^2*cot(d*x+c)/d-1/2*a^2*cot(d*x+c)*csc(d*x+c)/d-9/4*a^2*cos(d*x+c
)*sin(d*x+c)/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 3853, 2715, 2713}

$$\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{9a^2\sin(c+dx)\cos(c+dx)}{4d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{15a^2x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*x)/4 + (3*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (a^2*Cos[c + d*x])/d
+ (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]*C
sc[c + d*x])/(2*d) - (9*a^2*Cos[c + d*x]*Sin[c + d*x])/(4*d) + (a^2*Cos[c +
 d*x]*Sin[c + d*x]^3)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int \left(-6a^8-2a^8\csc(c+dx)+2a^8\csc^2(c+dx)+a^8\csc^3\right.}{}\\ &= -6a^2x+a^2\int \csc^3(c+dx)\,dx-a^2\int \sin^5(c+dx)\,dx\\ &= -6a^2x+\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc}{2d}\\ &= -3a^2x+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cos(c+dx)}{d}+\\ &= -\frac{15a^2x}{4}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cos(c+dx)}{d}+\end{aligned}$$

Mathematica [A]
time = 3.60, size = 174, normalized size = 1.24

$$\frac{(a+a\sin(c+dx))^2\left(-300(c+dx)-70\cos(c+dx)+5\cos(3(c+dx))+\cos(5(c+dx))-80\cot\left(\frac{1}{2}(c+dx)\right)-10\csc^2\left(\frac{1}{2}(c+dx)\right)+120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+10\sec^2\left(\frac{1}{2}(c+dx)\right)-80\sin(2(c+dx))-5\sin(4(c+dx))+80\tan\left(\frac{1}{2}(c+dx)\right)\right)}{80d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^2*(-300*(c + d*x) - 70*Cos[c + d*x] + 5*Cos[3*(c + d*
x)] + Cos[5*(c + d*x)] - 80*Cot[(c + d*x)/2] - 10*Csc[(c + d*x)/2]^2 + 120*
Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] + 10*Sec[(c + d*x)/2]^2 -
 80*Sin[2*(c + d*x)] - 5*Sin[4*(c + d*x)] + 80*Tan[(c + d*x)/2]))/(80*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.22, size = 188, normalized size = 1.34

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5\right.\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5\right.\right.}{}$
risch	$-\frac{15a^2x}{4}+\frac{a^2e^{3i(dx+c)}}{32d}+\frac{ia^2e^{2i(dx+c)}}{2d}-\frac{7a^2e^{i(dx+c)}}{16d}-\frac{7a^2e^{-i(dx+c)}}{16d}-\frac{ia^2e^{-2i(dx+c)}}{2d}+\frac{a^2e^{-3i(dx+c)}}{32d}+$
norman	$\frac{\frac{a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{8d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{17a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{10a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{10a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{17a^2\left(\tan^{11}\right.}{2d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-
5/2*cos(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/sin(d*x+c)*cos(d*x+
c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15
/8*c)+a^2*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]
time = 0.51, size = 191, normalized size = 1.36

$$\frac{2\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2-5\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2-15\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(2*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(co
s(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 5*(4*cos(d*x + c)^3 - 6*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) +
1) + 15*log(cos(d*x + c) - 1))*a^2 - 15*(15*d*x + 15*c + (15*tan(d*x + c)^4
 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)
))*a^2)/d
```

Fricas [A]
time = 0.40, size = 199, normalized size = 1.42

$$\frac{4\,a^2\cos\left(dx+c\right)^7-4\,a^2\cos\left(dx+c\right)^5-75\,a^2dx\cos\left(dx+c\right)^2-20\,a^2\cos\left(dx+c\right)^3+75\,a^2dx+30\,a^2\cos\left(dx+c\right)+15\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-5\left(2\,a^2\cos\left(dx+c\right)^5+5\,a^2\cos\left(dx+c\right)^3-15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{20\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/20*(4*a^2*cos(d*x + c)^7 - 4*a^2*cos(d*x + c)^5 - 75*a^2*d*x*cos(d*x + c)
^2 - 20*a^2*cos(d*x + c)^3 + 75*a^2*d*x + 30*a^2*cos(d*x + c) + 15*(a^2*cos
(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) - 15*(a^2*cos(d*x + c)^2 - a
^2)*log(-1/2*cos(d*x + c) + 1/2) - 5*(2*a^2*cos(d*x + c)^5 + 5*a^2*cos(d*x
+ c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.53, size = 244, normalized size = 1.74

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-150\,(dx+c)a^2-60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+40\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\left(18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{4\left(45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+50\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-50\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(5*a^2*tan(1/2*d*x + 1/2*c)^2 - 150*(d*x + c)*a^2 - 60*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) + 40*a^2*tan(1/2*d*x + 1/2*c) + 5*(18*a^2*tan(1/2*d*x +
 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(4
5*a^2*tan(1/2*d*x + 1/2*c)^9 + 50*a^2*tan(1/2*d*x + 1/2*c)^7 - 80*a^2*tan(1
/2*d*x + 1/2*c)^6 - 80*a^2*tan(1/2*d*x + 1/2*c)^4 - 50*a^2*tan(1/2*d*x + 1/
2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 - 45*a^2*tan(1/2*d*x + 1/2*c) - 16*a
^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 8.94, size = 377, normalized size = 2.69

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{15\,a^2\,\mathrm{atan}\left(\frac{225\,a^4}{4\left(\frac{45\,a^4}{2}-\frac{225\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)}+\frac{45\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{45\,a^4}{2}-\frac{225\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)}\right)}{2\,d}-\frac{-14\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{69\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+40\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+37\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+60\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+37\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+38\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{89\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{10}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^2}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) -
(15*a^2*atan((225*a^4)/(4*((45*a^4)/2 - (225*a^4*tan(c/2 + (d*x)/2))/4)) +
(45*a^4*tan(c/2 + (d*x)/2))/(2*((45*a^4)/2 - (225*a^4*tan(c/2 + (d*x)/2))/4
))))/(2*d) - ((89*a^2*tan(c/2 + (d*x)/2)^2)/10 + 38*a^2*tan(c/2 + (d*x)/2)^
3 + 37*a^2*tan(c/2 + (d*x)/2)^4 + 60*a^2*tan(c/2 + (d*x)/2)^5 + 37*a^2*tan(
c/2 + (d*x)/2)^6 + 40*a^2*tan(c/2 + (d*x)/2)^7 + (69*a^2*tan(c/2 + (d*x)/2)
^8)/2 + (a^2*tan(c/2 + (d*x)/2)^10)/2 - 14*a^2*tan(c/2 + (d*x)/2)^11 + a^2/
2 + 4*a^2*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*
x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 +
(d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12)) + (a^2*tan(c/2 + (d*x)/2))/d
```

3.595 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$\frac{5a^2x}{8}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{4a^2\cos(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-a^2\cot$$

```
[Out] 5/8*a^2*x+5*a^2*arctanh(cos(d*x+c))/d-4*a^2*cos(d*x+c)/d-2/3*a^2*cos(d*x+c)
^3/d+a^2*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-5/
8*a^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{4a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{5a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}+\frac{5a^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/8 + (5*a^2*ArcTanh[Cos[c + d*x]])/d - (4*a^2*Cos[c + d*x])/d - (2
*a^2*Cos[c + d*x]^3)/(3*d) + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^3)/(3
*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (5*a^2*Cos[c + d*x]*Sin[c + d*x])
/(8*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-6a^8\csc(c+dx)-2a^8\csc^2(c+dx)+2a^8\csc^3(c+\right.}{}$$

$$= a^2\int \csc^4(c+dx)\,dx - a^2\int \sin^4(c+dx)\,dx - \left(2a^2\right)$$

$$= \frac{6a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{6a^2\cos(c+dx)}{d} - \frac{a^2\cot(}{}$$

$$= a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} - \frac{2}{}$$

$$= \frac{5a^2x}{8} + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^2\cos(c+dx)}{d} -$$

Mathematica [A]
time = 5.41, size = 209, normalized size = 1.37

$$\frac{a^2(1+\sin(c+dx))^2\left(60(c+dx)-432\cos(c+dx)-16\cos(3(c+dx))+64\cot\left(\frac{1}{2}(c+dx)\right)-24\csc^2\left(\frac{1}{2}(c+dx)\right)+480\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+24\sec^2\left(\frac{1}{2}(c+dx)\right)+32\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-24\sin(2(c+dx))-3\sin(4(c+dx))-64\tan\left(\frac{1}{2}(c+dx)\right)\right)}{96d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(60*(c + d*x) - 432*Cos[c + d*x] - 16*Cos[3*(c +
d*x)] + 64*Cot[(c + d*x)/2] - 24*Csc[(c + d*x)/2]^2 + 480*Log[Cos[(c + d*x)
/2]] - 480*Log[Sin[(c + d*x)/2]] + 24*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x]^
3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 24*Sin[2*(c + d*
x)] - 3*Sin[4*(c + d*x)] - 64*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.20, size = 224, normalized size = 1.46

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)\right.}{}$
risch	$\frac{5a^2x}{8}+\frac{ia^2e^{4i(dx+c)}}{64d}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{9a^2e^{i(dx+c)}}{4d}-\frac{9a^2e^{-i(dx+c)}}{4d}-\frac{ia^2e^{-2i(dx+c)}}{8d}-\frac{ia^2e^{-4i(dx+c)}}{64d}+\frac{2a^2(}{}$
norman	$\frac{-\frac{a^2}{24d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{11a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{3a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{15a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{15a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{3a^2\left(\tan^{10}\left(\right.\right.}{8d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+2*a
^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*co
s(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/sin(d*x+c)*cos(d*x+c)^7-(co
s(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.50, size = 190, normalized size = 1.24

$$-\frac{4\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2+3\left(15dx+15c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^2-4\left(15dx+15c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/24*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 3*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 4*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^2)/d
```

Fricas [A]
time = 0.39, size = 219, normalized size = 1.43

$$\frac{6\,a^{2}\cos\left(dx+c\right)^{7}-3\,a^{2}\cos\left(dx+c\right)^{5}+20\,a^{2}\cos\left(dx+c\right)^{3}-15\,a^{2}\cos\left(dx+c\right)+60\,\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\,\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\left(16\,a^{2}\cos\left(dx+c\right)^{5}-15\,a^{2}dx\cos\left(dx+c\right)^{2}+80\,a^{2}\cos\left(dx+c\right)^{3}+15\,a^{2}dx-120\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*a^2*cos(d*x + c)^7 - 3*a^2*cos(d*x + c)^5 + 20*a^2*cos(d*x + c)^3 -
 15*a^2*cos(d*x + c) + 60*(a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 60*(a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2)*sin(d*x + c) - (16*a^2*cos(d*x + c)^5 - 15*a^2*d*x*cos(d*x + c)^2 + 80
*a^2*cos(d*x + c)^3 + 15*a^2*d*x - 120*a^2*cos(d*x + c))*sin(d*x + c))/((d*
cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.56, size = 274, normalized size = 1.79

$$\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,(dx+c)a^{2}-120\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{220\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-6\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}+\frac{2\,\left(15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-144\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-336\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-304\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-112\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 15*(d*x +
 c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2
*c) + (220*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a
^2*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 + 2*(15*a^2*tan(1/2*d
*x + 1/2*c)^7 - 144*a^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*tan(1/2*d*x + 1/2*c)
^5 - 336*a^2*tan(1/2*d*x + 1/2*c)^4 + 9*a^2*tan(1/2*d*x + 1/2*c)^3 - 304*a^
```

```
2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) - 112*a^2)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 8.93, size = 384, normalized size = 2.51

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}-\frac{5a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{5a^2\,\mathrm{atan}\left(\frac{25a^4}{16\left(\frac{25a^4}{2}+\frac{25a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)}-\frac{25a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{25a^4}{2}+\frac{25a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)}\right)}{4d}-\frac{5a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}-\frac{-15a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+98a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\frac{41a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}+232a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{104a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{644a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}-8a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{248a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-\frac{11a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a^2}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (5*a
^2*log(tan(c/2 + (d*x)/2)))/d - (5*a^2*atan((25*a^4)/(16*((25*a^4)/2 + (25*
a^4*tan(c/2 + (d*x)/2))/16)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a^4)/2 +
 (25*a^4*tan(c/2 + (d*x)/2))/16))))/(4*d) - (5*a^2*tan(c/2 + (d*x)/2))/(8*d
) - ((248*a^2*tan(c/2 + (d*x)/2)^3)/3 - (11*a^2*tan(c/2 + (d*x)/2)^2)/3 - 8
*a^2*tan(c/2 + (d*x)/2)^4 + (644*a^2*tan(c/2 + (d*x)/2)^5)/3 - (104*a^2*tan
(c/2 + (d*x)/2)^6)/3 + 232*a^2*tan(c/2 + (d*x)/2)^7 - (41*a^2*tan(c/2 + (d*
x)/2)^8)/3 + 98*a^2*tan(c/2 + (d*x)/2)^9 - 15*a^2*tan(c/2 + (d*x)/2)^10 + a
^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 +
(d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2
+ (d*x)/2)^11))
```

3.596 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=153

$$5a^2x+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{5a^2 c}{}$$

```
[Out] 5*a^2*x+5/8*a^2*arctanh(cos(d*x+c))/d-a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3
/d+4*a^2*cot(d*x+c)/d-2/3*a^2*cot(d*x+c)^3/d+5/8*a^2*cot(d*x+c)*csc(d*x+c)/
d-1/4*a^2*cot(d*x+c)*csc(d*x+c)^3/d+a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2951, 3852, 8, 3853, 3855, 2718, 2715, 2713}

$$-\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}+\frac{4a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot(c+dx)\csc(c+dx)}{8d}+5a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] 5*a^2*x + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cos[c + d*x])/d - (a^2
*Cos[c + d*x]^3)/(3*d) + (4*a^2*Cot[c + d*x])/d - (2*a^2*Cot[c + d*x]^3)/(3
*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d
*x]^3)/(4*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(6a^8 - 6a^8\csc^2(c+dx) - 2a^8\csc^3(c+dx) + 2a^8\csc^4(\right.}{}$$

$$= 6a^2x + a^2\int \csc^5(c+dx)\,dx - a^2\int \sin^3(c+dx)\,dx - ($$

$$= 6a^2x - \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cot(c+dx)\csc(c+dx)}{d} -$$

$$= 5a^2x + \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2\,\mathrm{co}}{}$$

$$= 5a^2x + \frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^2\cos(c+dx)}{d} - \frac{a^2}{}$$

Mathematica [A]

time = 0.90, size = 227, normalized size = 1.48

$$\frac{a^2(1+\sin(c+dx))^2\left(960(c+dx) - 240\cos(c+dx) - 16\cos(3(c+dx)) + 448\cot\left(\frac{1}{2}(c+dx)\right) + 30\csc^2\left(\frac{1}{2}(c+dx)\right) - 3\csc^4\left(\frac{1}{2}(c+dx)\right) + 120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 30\sec^2\left(\frac{1}{2}(c+dx)\right) + 3\sec^4\left(\frac{1}{2}(c+dx)\right) + 128\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) - 8\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) + 96\sin(2(c+dx)) - 448\tan\left(\frac{1}{2}(c+dx)\right)\right)}{192d\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(960*(c + d*x) - 240*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 448*Cot[(c + d*x)/2] + 30*Csc[(c + d*x)/2]^2 - 3*Csc[(c + d*x)/2]^
4 + 120*Log[Cos[(c + d*x)/2]] - 120*Log[Sin[(c + d*x)/2]] - 30*Sec[(c + d*x
)/2]^2 + 3*Sec[(c + d*x)/2]^4 + 128*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*C
sc[(c + d*x)/2]^4*Sin[c + d*x] + 96*Sin[2*(c + d*x)] - 448*Tan[(c + d*x)/2]
))/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.22, size = 246, normalized size = 1.61

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a^2\left(-\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2a^2\left(-\right.}{}$
risch	$5a^2x-\frac{a^2e^{3i(dx+c)}}{24d}-\frac{ia^2e^{2i(dx+c)}}{4d}-\frac{5a^2e^{i(dx+c)}}{8d}-\frac{5a^2e^{-i(dx+c)}}{8d}+\frac{ia^2e^{-2i(dx+c)}}{4d}-\frac{a^2e^{-3i(dx+c)}}{24d}-\frac{a^2(}{}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12d}+\frac{5a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{2a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{25a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{25a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{2a^2\left(\tan^{11}\left(\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*
cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)
))+2*a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+a^2
*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(
d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.50, size = 206, normalized size = 1.35

$$-\frac{4\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^2-16\left(15dx+15c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2+3a^2\left(\frac{2\left(9\cos(dx+c)^3-7\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-16\cos(dx+c)+15\log(\cos(dx+c)+1)-15\log(\cos(dx+c)-1)\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 - 16*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 +
 tan(d*x + c)^3))*a^2 + 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d
*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [**A**]
time = 0.40, size = 245, normalized size = 1.60

$$-\frac{16\,a^2\cos\left(dx+c\right)^7-240\,a^2dx\cos\left(dx+c\right)^4+16\,a^2\cos\left(dx+c\right)^5+480\,a^2dx\cos\left(dx+c\right)^2-50\,a^2\cos\left(dx+c\right)^3-240\,a^2dx+30\,a^2\cos\left(dx+c\right)-15\,\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\,\left(a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-16\,\left(3\,a^2\cos\left(dx+c\right)^5-20\,a^2\cos\left(dx+c\right)^3+15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*a^2*cos(d*x + c)^7 - 240*a^2*d*x*cos(d*x + c)^4 + 16*a^2*cos(d*x
+ c)^5 + 480*a^2*d*x*cos(d*x + c)^2 - 50*a^2*cos(d*x + c)^3 - 240*a^2*d*x +
 30*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2)
*log(1/2*cos(d*x + c) + 1/2) + 15*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^
2 + a^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a^2*cos(d*x + c)^5 - 20*a^2*c
os(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*
cos(d*x + c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 259, normalized size = 1.69

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)a^2-120\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-432\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{128\,\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}+\frac{250\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+432\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a^2 - 120*a^2*log(abs(tan(1/2*d*x
+ 1/2*c))) - 432*a^2*tan(1/2*d*x + 1/2*c) - 128*(3*a^2*tan(1/2*d*x + 1/2*c)
^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*ta
```

```
n(1/2*d*x + 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 + (250*a^2*tan(1
/2*d*x + 1/2*c)^4 + 432*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*a^2*tan(1/2*d*x + 1
/2*c)^2 - 16*a^2*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 8.94, size = 373, normalized size = 2.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}+\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-62\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\frac{320\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}-\frac{233\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{4}+136\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{449\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{12}+32\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4}-\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}-\frac{a^2}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}-\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,d}-\frac{10\,a^2\,\mathrm{atan}\left(\frac{100\,a^4}{\frac{25\,a^4}{2}+100\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{25\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,\left(\frac{25\,a^4}{2}+100\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d}-\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(12*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^2
*tan(c/2 + (d*x)/2)^4)/(64*d) + ((5*a^2*tan(c/2 + (d*x)/2)^2)/4 + 32*a^2*ta
n(c/2 + (d*x)/2)^3 - (449*a^2*tan(c/2 + (d*x)/2)^4)/12 + 136*a^2*tan(c/2 +
(d*x)/2)^5 - (233*a^2*tan(c/2 + (d*x)/2)^6)/4 + (320*a^2*tan(c/2 + (d*x)/2)
^7)/3 - 62*a^2*tan(c/2 + (d*x)/2)^8 + 4*a^2*tan(c/2 + (d*x)/2)^9 - a^2/4 -
(4*a^2*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d
*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*tan(c/2 + (d*x)/2)^10)) - (5*a^2*lo
g(tan(c/2 + (d*x)/2)))/(8*d) - (10*a^2*atan((100*a^4)/((25*a^4)/2 + 100*a^4
*tan(c/2 + (d*x)/2)) - (25*a^4*tan(c/2 + (d*x)/2))/(2*((25*a^4)/2 + 100*a^4
*tan(c/2 + (d*x)/2)))))/d - (9*a^2*tan(c/2 + (d*x)/2))/(4*d)
```

3.597 $\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=139

$$\frac{3a^2x}{2}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{9a^2\cot(c+dx)\csc(c+dx)}{4d}$$

```
[Out] 3/2*a^2*x-15/4*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d+a^2*cot(d*x+c)/
d-1/5*a^2*cot(d*x+c)^5/d+9/4*a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cot(d*x+c)
*csc(d*x+c)^3/d+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3853, 3852, 2718, 2715, 8}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{2d}+\frac{9a^2\cot(c+dx)\csc(c+dx)}{4d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (15*a^2*ArcTanh[Cos[c + d*x]])/(4*d) + (2*a^2*Cos[c + d*x])/d
 + (a^2*Cot[c + d*x])/d - (a^2*Cot[c + d*x]^5)/(5*d) + (9*a^2*Cot[c + d*x]*
Csc[c + d*x])/(4*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(2*d) + (a^2*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
```

```
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int \left(2a^8+6a^8\csc(c+dx)-6a^8\csc^3(c+dx)-2a^8\csc^4(c+dx)+2\right.}{} \\
&= 2a^2x+a^2\int \csc^6(c+dx)\,dx-a^2\int \sin^2(c+dx)\,dx-\left(2a^2\right)\int \csc \\
&= 2a^2x-\frac{6a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cot(c+}{} \\
&= \frac{3a^2x}{2}-\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\cot(c+d}{d} \\
&= \frac{3a^2x}{2}-\frac{15a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\cot(c+}{d}
\end{aligned}
$$

Mathematica [A]
time = 1.01, size = 264, normalized size = 1.90

$$\frac{a^2(1+\sin(c+dx))^2\left(240(c+dx)+320\cos(c+dx)+64\cot\left(\frac{1}{2}(c+dx)\right)+90\csc^2\left(\frac{1}{2}(c+dx)\right)-5\csc^4\left(\frac{1}{2}(c+dx)\right)-600\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+600\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-90\sec^2\left(\frac{1}{2}(c+dx)\right)+5\sec^4\left(\frac{1}{2}(c+dx)\right)-56\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+\frac{7}{2}\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-\frac{1}{2}\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+40\sin(2(c+dx))-64\tan\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{160d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(240*(c + d*x) + 320*Cos[c + d*x] + 64*Cot[(c + d
*x)/2] + 90*Csc[(c + d*x)/2]^2 - 5*Csc[(c + d*x)/2]^4 - 600*Log[Cos[(c + d*
x)/2]] + 600*Log[Sin[(c + d*x)/2]] - 90*Sec[(c + d*x)/2]^2 + 5*Sec[(c + d*x
)/2]^4 - 56*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (7*Csc[(c + d*x)/2]^4*Sin[c
 + d*x])/2 - (Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 40*Sin[2*(c + d*x)] - 64
*Tan[(c + d*x)/2] + Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(160*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.24, size = 217, normalized size = 1.56

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2a^2\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2a^2\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
risch	$\frac{3a^2x}{2}-\frac{ia^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{a^2\mathrm{e}^{i(dx+c)}}{d}+\frac{a^2\mathrm{e}^{-i(dx+c)}}{d}+\frac{ia^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{a^2\left(45\,\mathrm{e}^{9i(dx+c)}+80i\mathrm{e}^{6i(dx+c)}-50\,\mathrm{e}^{7i(dx+c)}\right.}{}$
norman	$\frac{-\frac{a^2}{160d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32d}+\frac{3a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{7a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{79a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{47a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{47a^2\left(\tan^8\left(\right.\right.}{32d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a^2*(-1/4/
sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/
8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5+5/4*c
os(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.51, size = 184, normalized size = 1.32

$$\frac{20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-15\,a^2\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos(dx+c)+15\,\log(\cos(dx+c)+1)-15\,\log(\cos(dx+c)-1)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan
(d*x + c)^5 + tan(d*x + c)^3))*a^2 - 8*(15*d*x + 15*c + (15*tan(d*x + c)^4
```

```
- 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^2 - 15*a^2*(2*(9*cos(d*x + c)^3 -
 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
+ 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(127) = 254$.
time = 0.42, size = 265, normalized size = 1.91

$$-\frac{20\,a^{2}\cos\left(dx+c\right)^{7}-92\,a^{2}\cos\left(dx+c\right)^{5}+140\,a^{2}\cos\left(dx+c\right)^{3}-60\,a^{2}\cos\left(dx+c\right)+75\,\left(a^{2}\cos\left(dx+c\right)^{4}-2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-75\,\left(a^{2}\cos\left(dx+c\right)^{4}-2\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-10\,\left(6\,a^{2}dx\cos\left(dx+c\right)^{4}+8\,a^{2}\cos\left(dx+c\right)^{5}-12\,a^{2}dx\cos\left(dx+c\right)^{2}-25\,a^{2}\cos\left(dx+c\right)^{3}+6\,a^{2}dx+15\,a^{2}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/40*(20*a^2*cos(d*x + c)^7 - 92*a^2*cos(d*x + c)^5 + 140*a^2*cos(d*x + c)
^3 - 60*a^2*cos(d*x + c) + 75*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 +
a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 75*(a^2*cos(d*x + c)^4 - 2*
a^2*cos(d*x + c)^2 + a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(6
*a^2*d*x*cos(d*x + c)^4 + 8*a^2*cos(d*x + c)^5 - 12*a^2*d*x*cos(d*x + c)^2
- 25*a^2*cos(d*x + c)^3 + 6*a^2*d*x + 15*a^2*cos(d*x + c))*sin(d*x + c))/((
d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(127) = 254$.
time = 0.54, size = 272, normalized size = 1.96

$$\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+240\,(dx+c)a^{2}+600\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-70\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{160\,\left(a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}-\frac{1370\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-70\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-80\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/160*(a^2*tan(1/2*d*x + 1/2*c)^5 + 5*a^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 80*a^2*tan(1/2*d*x + 1/2*c)^2 + 240*(d*x + c)*a^2 +
600*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 70*a^2*tan(1/2*d*x + 1/2*c) - 160*
(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*
x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (1370*a^2*tan(1/2*d*x
```

```
+ 1/2*c)^5 - 70*a^2*tan(1/2*d*x + 1/2*c)^4 - 80*a^2*tan(1/2*d*x + 1/2*c)^3
- 5*a^2*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 8.90, size = 363, normalized size = 2.61

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{15\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,d}+\frac{-18\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+144\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+61\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+159\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{79\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}+14\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{3\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{5}-a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{a^2}{5}}{d\,\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+64\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}+\frac{3\,a^2\,\mathrm{atan}\left(\frac{9\,a^4}{\frac{45\,a^4}{2}-9\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{45\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,\left(\frac{45\,a^4}{2}-9\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}\right)}{d}-\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^4)/(32*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(32*d) - (a^
2*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a^
2*log(tan(c/2 + (d*x)/2)))/(4*d) + ((3*a^2*tan(c/2 + (d*x)/2)^2)/5 + 14*a^2
*tan(c/2 + (d*x)/2)^3 + (79*a^2*tan(c/2 + (d*x)/2)^4)/5 + 159*a^2*tan(c/2 +
 (d*x)/2)^5 + 61*a^2*tan(c/2 + (d*x)/2)^6 + 144*a^2*tan(c/2 + (d*x)/2)^7 -
18*a^2*tan(c/2 + (d*x)/2)^8 - a^2/5 - a^2*tan(c/2 + (d*x)/2))/(d*(32*tan(c/
2 + (d*x)/2)^5 + 64*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9)) + (3*a
^2*atan((9*a^4)/((45*a^4)/2 - 9*a^4*tan(c/2 + (d*x)/2)) + (45*a^4*tan(c/2 +
 (d*x)/2))/(2*((45*a^4)/2 - 9*a^4*tan(c/2 + (d*x)/2)))))/d - (7*a^2*tan(c/2
 + (d*x)/2))/(16*d)
```

3.598 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$-2a^2x-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}+$$

```
[Out] -2*a^2*x-25/16*a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d-2*a^2*cot(d*x+c)/
d+2/3*a^2*cot(d*x+c)^3/d-2/5*a^2*cot(d*x+c)^5/d+7/16*a^2*cot(d*x+c)*csc(d*x
+c)/d+7/24*a^2*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.17, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2951, 3855, 3852, 8, 3853, 2718}

$$\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}+\frac{2a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot(c+dx)}{d}-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}+\frac{7a^2\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{7a^2\cot(c+dx)\csc(c+dx)}{16d}-2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -2*a^2*x - (25*a^2*ArcTanh[Cos[c + d*x]])/(16*d) + (a^2*Cos[c + d*x])/d - (
2*a^2*Cot[c + d*x])/d + (2*a^2*Cot[c + d*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^
5)/(5*d) + (7*a^2*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (7*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^2\,dx &= \frac{\int (-2a^8+2a^8\csc(c+dx)+6a^8\csc^2(c+dx)-6a^8\csc^4
\\
&= -2a^2x+a^2\int \csc^7(c+dx)\,dx-a^2\int \sin(c+dx)\,dx+
\\
&= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a
\\
&= -2a^2x-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}-\frac{2
\\
&= -2a^2x-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{4d}+\frac{a^2\cos(c+dx)}{d}-\frac{2
\\
&= -2a^2x-\frac{25a^2\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^2\cos(c+dx)}{d}-
\end{aligned}
$$

Mathematica [A]
time = 1.43, size = 270, normalized size = 1.72

$$
-\frac{a^2\left(-1920\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)(1472-210\csc(c+dx))+\csc^6\left(\frac{1}{2}(c+dx)\right)(12+5\csc(c+dx))-2\csc^4\left(\frac{1}{2}(c+dx)\right)(82+15\csc(c+dx))+120\csc(c+dx)\left(32(c+dx)+25\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-25\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-2(241+327\cos(c+dx)+92\cos(2(c+dx)))\sec^6\left(\frac{1}{2}(c+dx)\right)+840\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)+480\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-320\csc^7(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)(1+\sin(c+dx))^2}{1920d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1920*(a^2*(-1920*Cot[c + d*x] + Csc[(c + d*x)/2]^2*(1472 - 210*Csc[c + d
*x]) + Csc[(c + d*x)/2]^6*(12 + 5*Csc[c + d*x]) - 2*Csc[(c + d*x)/2]^4*(82
+ 15*Csc[c + d*x]) + 120*Csc[c + d*x]*(32*(c + d*x) + 25*Log[Cos[(c + d*x)/
2]] - 25*Log[Sin[(c + d*x)/2]]) - 2*(241 + 327*Cos[c + d*x] + 92*Cos[2*(c +
```

```
d*x)])*Sec[(c + d*x)/2]^6 + 840*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 + 480*Cs
c[c + d*x]^5*Sin[(c + d*x)/2]^4 - 320*Csc[c + d*x]^7*Sin[(c + d*x)/2]^6)*Si
n[c + d*x]*(1 + Sin[c + d*x])^2)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4
)
```

Maple [A]

time = 0.25, size = 239, normalized size = 1.52

method	result
risch	$-2a^2x+\frac{a^2 e^{i(dx+c)}}{2d}+\frac{a^2 e^{-i(dx+c)}}{2d}-\frac{a^2\left(105\, e^{11i(dx+c)}-595\, e^{9i(dx+c)}+1440 i e^{10i(dx+c)}-150\, e^{7i(dx+c)}-4320 i e\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right.}{}$
norman	$-\frac{a^2}{384d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80d}+\frac{a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{29a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}+\frac{7a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48d}-\frac{263a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{59a^2\left(\tan^7\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/1
6/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*
x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*
cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(
d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.51, size = 220, normalized size = 1.40

$$-\frac{64\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-5\,a^2\left(\frac{2\left(33\cos(dx+c)^5-40\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}+15\log(\cos(dx+c)+1)-15\log(\cos(dx+c)-1)\right)+30\,a^2\left(\frac{2\left(9\cos(dx+c)^3-7\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-16\cos(dx+c)+15\log(\cos(dx+c)+1)-15\log(\cos(dx+c)-1)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^2 - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*a^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(145) = 290$.
time = 0.40, size = 303, normalized size = 1.93

$$-\frac{960\,a^2dx\cos\left(dx+c\right)^6-480\,a^2\cos\left(dx+c\right)^7-2880\,a^2dx\cos\left(dx+c\right)^4+1650\,a^2\cos\left(dx+c\right)^5+2880\,a^2dx\cos\left(dx+c\right)^2-2000\,a^2\cos\left(dx+c\right)^3-960\,a^2dx+750\,a^2\cos\left(dx+c\right)+375\,\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-375\,\left(a^2\cos\left(dx+c\right)^6-3\,a^2\cos\left(dx+c\right)^4+3\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-64\,\left(23\,a^2\cos\left(dx+c\right)^5-35\,a^2\cos\left(dx+c\right)^3+15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a^2*d*x*cos(d*x + c)^6 - 480*a^2*cos(d*x + c)^7 - 2880*a^2*d*x*
cos(d*x + c)^4 + 1650*a^2*cos(d*x + c)^5 + 2880*a^2*d*x*cos(d*x + c)^2 - 20
00*a^2*cos(d*x + c)^3 - 960*a^2*d*x + 750*a^2*cos(d*x + c) + 375*(a^2*cos(d
*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(
d*x + c) + 1/2) - 375*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*co
s(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2) - 64*(23*a^2*cos(d*x + c)^
5 - 35*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 259, normalized size = 1.65

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-255\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3840\,(dx+c)a^2+3000\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+2640\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3840\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{7350\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+2640\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-255\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 255*a^2*tan(1/
2*d*x + 1/2*c)^2 - 3840*(d*x + c)*a^2 + 3000*a^2*log(abs(tan(1/2*d*x + 1/2*
c))) + 2640*a^2*tan(1/2*d*x + 1/2*c) + 3840*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1
) - (7350*a^2*tan(1/2*d*x + 1/2*c)^6 + 2640*a^2*tan(1/2*d*x + 1/2*c)^5 - 25
5*a^2*tan(1/2*d*x + 1/2*c)^4 - 280*a^2*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
1/2*d*x + 1/2*c)^2 + 24*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/tan(1/2*d*x + 1/2
*c)^6)/d
```

Mupad [B]
time = 11.01, size = 657, normalized size = 4.18

$$\frac{5\,a^2\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-5\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+24\,a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}-24\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-10\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-256\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}-270\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+2360\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9-255\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4095\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-2360\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+270\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+256\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+10\,a^2\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3000\,a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+3000\,a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+7680\,a^2\,\mathrm{atan}\left(\frac{32\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+7680\,a^2\,\mathrm{atan}\left(\frac{32\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+32\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{1920\,d\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left({\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (5*a^2*sin(c/2 + (d*x)/2)^14 - 5*a^2*cos(c/2 + (d*x)/2)^14 + 24*a^2*cos(c/2
 + (d*x)/2)*sin(c/2 + (d*x)/2)^13 - 24*a^2*cos(c/2 + (d*x)/2)^13*sin(c/2 +
(d*x)/2) - 10*a^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 256*a^2*cos(
c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^11 - 270*a^2*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2)^10 + 2360*a^2*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 255
*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 4095*a^2*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^6 - 2360*a^2*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
^5 + 270*a^2*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4 + 256*a^2*cos(c/2 +
 (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 10*a^2*cos(c/2 + (d*x)/2)^12*sin(c/2 +
(d*x)/2)^2 + 3000*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 +
(d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 3000*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 7680*a^2*atan((32*co
s(c/2 + (d*x)/2) - 25*sin(c/2 + (d*x)/2))/(25*cos(c/2 + (d*x)/2) + 32*sin(c
/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 7680*a^2*atan((
32*cos(c/2 + (d*x)/2) - 25*sin(c/2 + (d*x)/2))/(25*cos(c/2 + (d*x)/2) + 32*
sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/(1920*d*cos
(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d
*x)/2)^2))
```

3.599 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=162

$$-a^2x+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{5a^2\cot$$

```
[Out] -a^2*x+5/8*a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+1/3*a^2*cot(d*x+c)^3/
d-1/5*a^2*cot(d*x+c)^5/d-1/7*a^2*cot(d*x+c)^7/d-5/8*a^2*cot(d*x+c)*csc(d*x+
c)/d+5/12*a^2*cot(d*x+c)^3*csc(d*x+c)/d-1/3*a^2*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5a^2\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5a^2\cot(c+dx)\csc(c+dx)}{8d}-a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*a^2*ArcTanh[Cos[c + d*x]])/(8*d) - (a^2*Cot[c + d*x])/d + (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a^2*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^6(c+dx)+2a^2\cot^6(c+dx)\csc(c+dx)+a\right. \\
&= a^2\int \cot^6(c+dx)\,dx+a^2\int \cot^6(c+dx)\csc^2(c+dx \\
&= -\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^5(c+dx)\csc(c+dx)}{3d}-a^2 \\
&= \frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d} \\
&= -\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d} \\
&= -a^2x+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^2\cot(c+dx)}{d}+
\end{aligned}
$$

Mathematica [A]
time = 0.79, size = 262, normalized size = 1.62

$$\frac{a^2\left(-13440c-13440dx-9344\cot\left(\frac{1}{2}(c+dx)\right)-4620\csc^2\left(\frac{1}{2}(c+dx)\right)+8400\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8400\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4620\sec^2\left(\frac{1}{2}(c+dx)\right)-840\sec^4\left(\frac{1}{2}(c+dx)\right)+70\sec^6\left(\frac{1}{2}(c+dx)\right)-4624\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-\frac{15}{2}\csc^8\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+\csc^6\left(\frac{1}{2}(c+dx)\right)(-70+33\sin(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(840+289\sin(c+dx))+9344\tan\left(\frac{1}{2}(c+dx)\right)-66\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)+15\sec^6\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-13440*c - 13440*d*x - 9344*Cot[(c + d*x)/2] - 4620*Csc[(c + d*x)/2]^
2 + 8400*Log[Cos[(c + d*x)/2]] - 8400*Log[Sin[(c + d*x)/2]] + 4620*Sec[(c +
 d*x)/2]^2 - 840*Sec[(c + d*x)/2]^4 + 70*Sec[(c + d*x)/2]^6 - 4624*Csc[c +
d*x]^3*Sin[(c + d*x)/2]^4 - (15*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + Csc[(c
 + d*x)/2]^6*(-70 + 33*Sin[c + d*x]) + Csc[(c + d*x)/2]^4*(840 + 289*Sin[c
+ d*x]) + 9344*Tan[(c + d*x)/2] - 66*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] +
15*Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2]))/(13440*d)
```

Maple [A]

time = 0.24, size = 173, normalized size = 1.07

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc}{}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2a^2\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc}{}\right.}{d}$
risch	$-a^2x+\frac{a^2\left(-1680ie^{12i(dx+c)}+1155\,e^{13i(dx+c)}+10080ie^{10i(dx+c)}-980\,e^{11i(dx+c)}-16240ie^{8i(dx+c)}+2975\,e^{9i(dx+c)}+2\right.}{420d\left(e^{2i(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^2/sin(d*x+c)^7*cos(d*x+c)^7+2*a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)
^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d
*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+a
^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.54, size = 154, normalized size = 0.95

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-35\,a^2\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{240\,a^2}{\tan(dx+c)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^2 - 35*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(148) = 296$.
time = 0.39, size = 323, normalized size = 1.99

$$-\frac{2336 a^2 \cos(dx+c)^7 - 6496 a^2 \cos(dx+c)^5 + 5600 a^2 \cos(dx+c)^3 - 1680 a^2 \cos(dx+c) - 525 \left(a^2 \cos(dx+c)^6 - 3 a^2 \cos(dx+c)^4 + 3 a^2 \cos(dx+c)^2 - a^2\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 525 \left(a^2 \cos(dx+c)^6 - 3 a^2 \cos(dx+c)^4 + 3 a^2 \cos(dx+c)^2 - a^2\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 70 \left(24 a^2 dx \cos(dx+c)^6 - 72 a^2 dx \cos(dx+c)^4 - 33 a^2 \cos(dx+c)^5 + 72 a^2 dx \cos(dx+c)^2 + 40 a^2 \cos(dx+c)^3 - 24 a^2 dx - 15 a^2 \cos(dx+c)\right) \sin(dx+c)}{1680 \left(d \cos(dx+c)^6 - 3 d \cos(dx+c)^4 + 3 d \cos(dx+c)^2 - d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
[Out] -1/1680*(2336*a^2*cos(d*x + c)^7 - 6496*a^2*cos(d*x + c)^5 + 5600*a^2*cos(d
*x + c)^3 - 1680*a^2*cos(d*x + c) - 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x
 + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 525*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*x + c)^4 + 3*a^2*cos(d*x + c)^2
 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(24*a^2*d*x*cos(d*x
+ c)^6 - 72*a^2*d*x*cos(d*x + c)^4 - 33*a^2*cos(d*x + c)^5 + 72*a^2*d*x*cos
(d*x + c)^2 + 40*a^2*cos(d*x + c)^3 - 24*a^2*d*x - 15*a^2*cos(d*x + c))*sin
(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d
)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

Giac [A]
time = 0.51, size = 270, normalized size = 1.67

$$\frac{15 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 + 70 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 21 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 630 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 665 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 3150 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 13440 (dx+c) a^2 - 8400 a^2 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) + 8715 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \frac{21780 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 8715 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 - 3150 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 665 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 630 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 21 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 70 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 15 a^2}{\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7}}{13440 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a^2*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 - 630*a^2*tan(1/2*d*x + 1/2*c)^4 - 665*a^2*tan(
1/2*d*x + 1/2*c)^3 + 3150*a^2*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^2
- 8400*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8715*a^2*tan(1/2*d*x + 1/2*c) +
 (21780*a^2*tan(1/2*d*x + 1/2*c)^7 - 8715*a^2*tan(1/2*d*x + 1/2*c)^6 - 3150
*a^2*tan(1/2*d*x + 1/2*c)^5 + 665*a^2*tan(1/2*d*x + 1/2*c)^4 + 630*a^2*tan(
1/2*d*x + 1/2*c)^3 + 21*a^2*tan(1/2*d*x + 1/2*c)^2 - 70*a^2*tan(1/2*d*x + 1
/2*c) - 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 9.84, size = 351, normalized size = 2.17

$$\frac{19\,a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}-\frac{15\,a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}+\frac{3\,a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}-\frac{a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}-\frac{a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{15\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}-\frac{19\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}-\frac{3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}+\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{5\,a^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{83\,a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{83\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (19*a^2*cot(c/2 + (d*x)/2)^3)/(384*d) - (15*a^2*cot(c/2 + (d*x)/2)^2)/(64*d
) + (3*a^2*cot(c/2 + (d*x)/2)^4)/(64*d) + (a^2*cot(c/2 + (d*x)/2)^5)/(640*d
) - (a^2*cot(c/2 + (d*x)/2)^6)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(896*d)
 + (15*a^2*tan(c/2 + (d*x)/2)^2)/(64*d) - (19*a^2*tan(c/2 + (d*x)/2)^3)/(38
4*d) - (3*a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(64
0*d) + (a^2*tan(c/2 + (d*x)/2)^6)/(192*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(896
*d) - (2*a^2*atan((8*cos(c/2 + (d*x)/2) + 5*sin(c/2 + (d*x)/2))/(5*cos(c/2
+ (d*x)/2) - 8*sin(c/2 + (d*x)/2))))/d - (5*a^2*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/(8*d) - (83*a^2*cot(c/2 + (d*x)/2))/(128*d) + (83*a^2*tan(
c/2 + (d*x)/2))/(128*d)
```

3.600 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=182

$$\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{35a^2\cot(c+dx)\csc(c+dx)}{128d}+\frac{5a^2\cot^3(c+dx)\csc(c+dx)}{24d}-$$

```
[Out] 45/128*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-35/128*a^2*cot(d*x+
c)*csc(d*x+c)/d+5/24*a^2*cot(d*x+c)^3*csc(d*x+c)/d-1/6*a^2*cot(d*x+c)^5*csc
(d*x+c)/d-5/64*a^2*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a^2*cot(d*x+c)^3*csc(d*x+
c)^3/d-1/8*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3855, 2687, 30, 3853}

$$-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^2\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}+\frac{5a^2\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{5a^2\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{35a^2\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (45*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (35
*a^2*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x
])/(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x])/(6*d) - (5*a^2*Cot[c + d*x]*C
sc[c + d*x]^3)/(64*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^2
*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cot^6(c+dx)\csc(c+dx)+2a^2\cot^6(c+dx)\csc^2(\cdots\right.\\
&= a^2\int \cot^6(c+dx)\csc(c+dx)\,dx+a^2\int \cot^6(c+dx)\,\mathrm{c}\cdots\\
&= -\frac{a^2\cot^5(c+dx)\csc(c+dx)}{6d}-\frac{a^2\cot^5(c+dx)\csc^3(c\cdots}{8d}\cdots\\
&= -\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{a^2\cdots}{\cdots}\\
&= -\frac{2a^2\cot^7(c+dx)}{7d}-\frac{5a^2\cot(c+dx)\csc(c+dx)}{16d}+\frac{5a^2\cdots}{\cdots}\\
&= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{35a^2\cot\cdots}{\cdots}\\
&= \frac{45a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{35a^2\,\mathrm{c}\cdots}{\cdots}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(182) = 364.
time = 0.08, size = 401, normalized size = 2.20

$$a^2\left(\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{7d}-\frac{83\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{19\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{224d}+\frac{17\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{5\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{224d}+\frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^6\left(\frac{1}{2}(c+dx)\right)}{448d}-\frac{\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}+\frac{45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d}-\frac{45\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{83\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{17\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{7d}+\frac{19\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{224d}-\frac{5\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{224d}+\frac{\sec^6\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{448d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] a^2*(Cot[(c + d*x)/2]/(7*d) - (83*Csc[(c + d*x)/2]^2)/(512*d) - (19*Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(224*d) + (17*Csc[(c + d*x)/2]^4)/(1024*d) +
(5*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(224*d) + Csc[(c + d*x)/2]^6/(512*d
) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(448*d) - Csc[(c + d*x)/2]^8/(204
8*d) + (45*Log[Cos[(c + d*x)/2]])/(128*d) - (45*Log[Sin[(c + d*x)/2]])/(128
*d) + (83*Sec[(c + d*x)/2]^2)/(512*d) - (17*Sec[(c + d*x)/2]^4)/(1024*d) -
Sec[(c + d*x)/2]^6/(512*d) + Sec[(c + d*x)/2]^8/(2048*d) - Tan[(c + d*x)/2]
/(7*d) + (19*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(224*d) - (5*Sec[(c + d*x
)/2]^4*Tan[(c + d*x)/2])/(224*d) + (Sec[(c + d*x)/2]^6*Tan[(c + d*x)/2])/(4
48*d))
```

Maple [A]

time = 0.27, size = 255, normalized size = 1.40

method	result
risch	$\frac{a^2\left(581\,e^{15i(dx+c)}-2065\,e^{13i(dx+c)}-5376ie^{4i(dx+c)}+21\,e^{11i(dx+c)}+5376ie^{6i(dx+c)}-5705\,e^{9i(dx+c)}-1792ie^{12i(dx+c)}-\right.}{448}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(d\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/1
92/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+
c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-2
/7*a^2/sin(d*x+c)^7*cos(d*x+c)^7+a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/s
in(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5
/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.28, size = 221, normalized size = 1.21

$$-\frac{7\,a^2\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)-56\,a^2\left(\frac{2\left(33\cos(dx+c)^5-40\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}+15\log\left(\cos\left(dx+c\right)+1\right)-15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{1536\,a^2}{\tan(dx+c)^7}}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a^2/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.39, size = 255, normalized size = 1.40

$$-\frac{512\,a^2\cos\left(dx+c\right)^7\sin\left(dx+c\right)-1162\,a^2\cos\left(dx+c\right)^7+3066\,a^2\cos\left(dx+c\right)^5-2310\,a^2\cos\left(dx+c\right)^3+630\,a^2\cos\left(dx+c\right)-315\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+315\,\left(a^2\cos\left(dx+c\right)^8-4\,a^2\cos\left(dx+c\right)^6+6\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{1792\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1792*(512*a^2*cos(d*x + c)^7*sin(d*x + c) - 1162*a^2*cos(d*x + c)^7 + 30
66*a^2*cos(d*x + c)^5 - 2310*a^2*cos(d*x + c)^3 + 630*a^2*cos(d*x + c) - 31
5*(a^2*cos(d*x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2
*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6
*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 260, normalized size = 1.43

$$\frac{7\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+32\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-224\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1792\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-5040\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-1120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{13698\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+1120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1792\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-672\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+224\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-32\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{14336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/14336*(7*a^2*tan(1/2*d*x + 1/2*c)^8 + 32*a^2*tan(1/2*d*x + 1/2*c)^7 - 224
*a^2*tan(1/2*d*x + 1/2*c)^5 - 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 672*a^2*tan(
1/2*d*x + 1/2*c)^3 + 1792*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan
(1/2*d*x + 1/2*c))) - 1120*a^2*tan(1/2*d*x + 1/2*c) + (13698*a^2*tan(1/2*d*
```

```
x + 1/2*c)^8 + 1120*a^2*tan(1/2*d*x + 1/2*c)^7 - 1792*a^2*tan(1/2*d*x + 1/2
*c)^6 - 672*a^2*tan(1/2*d*x + 1/2*c)^5 + 280*a^2*tan(1/2*d*x + 1/2*c)^4 + 2
24*a^2*tan(1/2*d*x + 1/2*c)^3 - 32*a^2*tan(1/2*d*x + 1/2*c) - 7*a^2)/tan(1/
2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 11.16, size = 387, normalized size = 2.13

$$-\frac{a^2\left(7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+224\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-672\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-1792\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+1120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-1120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+1792\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+672\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-224\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+5040\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\right)}{14336\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] -(a^2*(7*cos(c/2 + (d*x)/2)^16 - 7*sin(c/2 + (d*x)/2)^16 - 32*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^15 + 32*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) +
 224*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 + 280*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^12 - 672*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 17
92*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 1120*cos(c/2 + (d*x)/2)^7*s
in(c/2 + (d*x)/2)^9 - 1120*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 1792
*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^6 + 672*cos(c/2 + (d*x)/2)^11*sin
(c/2 + (d*x)/2)^5 - 280*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 224*co
s(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^3 + 5040*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8))/(14336*d*cos(c/
2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

3.601 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5a^2\cot(c+dx)\csc(c+dx)}{64d}-\frac{5a^2\cot(c+dx)\,\mathrm{c}}{32d}$$

```
[Out] 5/64*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-1/9*a^2*cot(d*x+c)^9/
d+5/64*a^2*cot(d*x+c)*csc(d*x+c)/d-5/32*a^2*cot(d*x+c)*csc(d*x+c)^3/d+5/24*
a^2*cot(d*x+c)^3*csc(d*x+c)^3/d-1/4*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{24d}-\frac{5a^2\cot(c+dx)\csc^3(c+dx)}{32d}+\frac{5a^2\cot(c+dx)\csc(c+dx)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cos[c + d*x]])/(64*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2*
Cot[c + d*x]^9)/(9*d) + (5*a^2*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5*a^2*C
ot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d*x]^3)/
(24*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^2(c+dx)+2a^2\cot^6(c+dx)\csc\right.$$
$$= a^2\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^2\int \cot^6(c+dx$$
$$= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}(5a^2)\int \cot^4(c+d$$
$$= -\frac{a^2\cot^7(c+dx)}{7d} + \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{24d} -$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot(c+d}{3}$$
$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5a^2\cot(c+d}{64}$$
$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{64d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(152) = 304.

time = 0.97, size = 313, normalized size = 2.06

$$-\frac{a^2 \csc^9(c+dx) \left(72576 \cos(c+dx)+37632 \cos(3(c+dx))+6912 \cos(5(c+dx))-1728 \cos(7(c+dx))-704 \cos(9(c+dx))-39690 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx)+39690 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(c+dx)+36540 \sin(2(c+dx))+26460 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(3(c+dx))-26460 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(3(c+dx))+20916 \sin(4(c+dx))-11340 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(5(c+dx))+11340 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(5(c+dx))+16044 \sin(6(c+dx))+2835 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(7(c+dx))-2835 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(7(c+dx))+630 \sin(8(c+dx))-315 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) \sin(9(c+dx))+315 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \sin(9(c+dx))\right)}{1032192 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1032192*(a^2*Csc[c + d*x]^9*(72576*Cos[c + d*x] + 37632*Cos[3*(c + d*x)]
 + 6912*Cos[5*(c + d*x)] - 1728*Cos[7*(c + d*x)] - 704*Cos[9*(c + d*x)] - 3
9690*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 39690*Log[Sin[(c + d*x)/2]]*Sin[c
 + d*x] + 36540*Sin[2*(c + d*x)] + 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d
*x)] - 26460*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 20916*Sin[4*(c + d*x)
] - 11340*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 11340*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)] + 16044*Sin[6*(c + d*x)] + 2835*Log[Cos[(c + d*x)/2]]*
Sin[7*(c + d*x)] - 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] + 630*Sin[8*
(c + d*x)] - 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] + 315*Log[Sin[(c +
d*x)/2]]*Sin[9*(c + d*x)]))/d
```

Maple [A]

time = 0.30, size = 192, normalized size = 1.26

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\right.}{d}$
risch	$-\frac{a^2\left(-9216i e^{4i(dx+c)}+315 e^{17i(dx+c)}-8064i e^{8i(dx+c)}+8022 e^{15i(dx+c)}+48384i e^{6i(dx+c)}+10458 e^{13i(dx+c)}-10752i e^{12i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7)+2*
a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192/si
n(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)^5-
5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-1/7*a^
2/sin(d*x+c)^7*cos(d*x+c)^7)
```

Maxima [A]

time = 0.30, size = 155, normalized size = 1.02

$$-\frac{21\,a^2\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos(dx+c)+1\right)+15\log\left(\cos(dx+c)-1\right)\right)+\frac{1152\,a^2}{\tan(dx+c)^7}+\frac{128\left(9\tan(dx+c)^2+7\right)a^2}{\tan(dx+c)^9}}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/8064*(21*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)
^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
 1)) + 1152*a^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c
)^9)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(138) = 276$.
time = 0.41, size = 291, normalized size = 1.91

$$\frac{1408\,a^2\cos(dx+c)^9 - 2304\,a^2\cos(dx+c)^7 + 315\,\left(a^2\cos(dx+c)^8 - 4\,a^2\cos(dx+c)^6 + 6\,a^2\cos(dx+c)^4 - 4\,a^2\cos(dx+c)^2 + a^2\right)\log\left(\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 315\,\left(a^2\cos(dx+c)^8 - 4\,a^2\cos(dx+c)^6 + 6\,a^2\cos(dx+c)^4 - 4\,a^2\cos(dx+c)^2 + a^2\right)\log\left(-\frac{1}{2}\,\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 42\,\left(15\,a^2\cos(dx+c)^7 + 73\,a^2\cos(dx+c)^5 - 55\,a^2\cos(dx+c)^3 + 15\,a^2\cos(dx+c)\right)\sin(dx+c)}{8064\,\left(d\cos(dx+c)^8 - 4\,d\cos(dx+c)^6 + 6\,d\cos(dx+c)^4 - 4\,d\cos(dx+c)^2 + d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(1408*a^2*cos(d*x + c)^9 - 2304*a^2*cos(d*x + c)^7 + 315*(a^2*cos(d*
x + c)^8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)
^2 + a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315*(a^2*cos(d*x + c)^
8 - 4*a^2*cos(d*x + c)^6 + 6*a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + a^
2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a^2*cos(d*x + c)^7 +
73*a^2*cos(d*x + c)^5 - 55*a^2*cos(d*x + c)^3 + 15*a^2*cos(d*x + c))*sin(d*
x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*
cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(138) =$ 276.
time = 0.52, size = 324, normalized size = 2.13

$$\frac{14\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^9 + 63\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^8 + 18\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^7 - 336\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^6 - 504\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 504\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 + 1848\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 1008\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 5040\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 3276\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \frac{14258\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 + 3276\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 1008\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 1848\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 504\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 504\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 336\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 14\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{64512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a^2*tan(1/2*d*x + 1/2*c)^8 + 18
*a^2*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(
1/2*d*x + 1/2*c)^5 + 504*a^2*tan(1/2*d*x + 1/2*c)^4 + 1848*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 1008*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*log(abs(tan(1/2*d*x
 + 1/2*c))) - 3276*a^2*tan(1/2*d*x + 1/2*c) + (14258*a^2*tan(1/2*d*x + 1/2*
c)^9 + 3276*a^2*tan(1/2*d*x + 1/2*c)^8 - 1008*a^2*tan(1/2*d*x + 1/2*c)^7 -
1848*a^2*tan(1/2*d*x + 1/2*c)^6 - 504*a^2*tan(1/2*d*x + 1/2*c)^5 + 504*a^2*
tan(1/2*d*x + 1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*tan(1/2*d*
x + 1/2*c)^2 - 63*a^2*tan(1/2*d*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9
)/d
```

Mupad [B]

time = 9.41, size = 357, normalized size = 2.35

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{11\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4608\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4608\,d}-\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,d}+\frac{13\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}-\frac{13\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^5)/(128*d) - (11*a^2*cot(c/2 + (d*x)/2)^3)/(384*d)
- (a^2*cot(c/2 + (d*x)/2)^4)/(128*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(64*d) +
(a^2*cot(c/2 + (d*x)/2)^6)/(192*d) - (a^2*cot(c/2 + (d*x)/2)^7)/(3584*d) -
(a^2*cot(c/2 + (d*x)/2)^8)/(1024*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(4608*d) +
 (a^2*tan(c/2 + (d*x)/2)^2)/(64*d) + (11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d)
+ (a^2*tan(c/2 + (d*x)/2)^4)/(128*d) - (a^2*tan(c/2 + (d*x)/2)^5)/(128*d) -
 (a^2*tan(c/2 + (d*x)/2)^6)/(192*d) + (a^2*tan(c/2 + (d*x)/2)^7)/(3584*d) +
 (a^2*tan(c/2 + (d*x)/2)^8)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(4608*d)
- (5*a^2*log(tan(c/2 + (d*x)/2)))/(64*d) + (13*a^2*cot(c/2 + (d*x)/2))/(256
*d) - (13*a^2*tan(c/2 + (d*x)/2))/(256*d)
```

3.602 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=228

$$\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{2a^2\cot^9(c+dx)}{9d}+\frac{13a^2\cot(c+dx)\csc(c+dx)}{256d}-\frac{9a^2\cot(c+ \ldots$$

```
[Out] 13/256*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^
9/d+13/256*a^2*cot(d*x+c)*csc(d*x+c)/d-9/128*a^2*cot(d*x+c)*csc(d*x+c)^3/d+
5/48*a^2*cot(d*x+c)^3*csc(d*x+c)^3/d-1/8*a^2*cot(d*x+c)^5*csc(d*x+c)^3/d-1/
32*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^2*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*
a^2*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.28, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3853, 3855, 2687, 14}

$$-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{16d}+\frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{32d}-\frac{9a^2\cot(c+dx)\csc^3(c+dx)}{128d}+\frac{13a^2\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (13*a^2*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2*
a^2*Cot[c + d*x]^9)/(9*d) + (13*a^2*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (9
*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^2*Cot[c + d*x]^3*Csc[c + d
*x]^3)/(48*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^2*Cot[c + d*
x]*Csc[c + d*x]^5)/(32*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a
^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^3(c+dx)+2a^2\cot^6(c+dx)\csc^4\right.$$

$$= a^2\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a^2\int \cot^6(c+dx)$$

$$= -\frac{a^2\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{a^2\cot^5(c+dx)\csc^5(c}{10d}$$

$$= \frac{5a^2\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a^2\cot^5(c+dx)\csc^3(c}{8d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} - \frac{5a^2\cot(c+dx}{64}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9(c+dx)}{9d} + \frac{5a^2\cot(c+dx}{12}$$

$$= \frac{5a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot^9}{9}$$

$$= \frac{13a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{2a^2\cot}{}$$

Mathematica [A]

time = 0.85, size = 353, normalized size = 1.55

$$-\frac{a^2 \csc^{10}(c+dx) \left(2732940 \cos(c+dx)+1151640 \cos(3(c+dx))+388248 \cos(5(c+dx))-135870 \cos(7(c+dx))-8190 \cos(9(c+dx))-515970 \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+859950 \cos(2(c+dx)) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-491400 \cos(4(c+dx)) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+184275 \cos(6(c+dx)) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-40950 \cos(8(c+dx)) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4095 \cos(10(c+dx)) \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+515970 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-859950 \cos(2(c+dx)) \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+491400 \cos(4(c+dx)) \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-184275 \cos(6(c+dx)) \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+40950 \cos(8(c+dx)) \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4095 \cos(10(c+dx)) \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+1075200 \sin(2(c+dx))+1044480 \sin(4(c+dx))+414720 \sin(6(c+dx))+51200 \sin(8(c+dx))-5120 \sin(10(c+dx))\right)}{41287680 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/41287680*(a^2*Csc[c + d*x]^10*(2732940*Cos[c + d*x] + 1151640*Cos[3*(c +
 d*x)] + 388248*Cos[5*(c + d*x)] - 135870*Cos[7*(c + d*x)] - 8190*Cos[9*(c
+ d*x)] - 515970*Log[Cos[(c + d*x)/2]] + 859950*Cos[2*(c + d*x)]*Log[Cos[(c
 + d*x)/2]] - 491400*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 184275*Cos[6*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40950*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)
/2]] + 4095*Cos[10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 515970*Log[Sin[(c + d
*x)/2]] - 859950*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 491400*Cos[4*(c +
 d*x)]*Log[Sin[(c + d*x)/2]] - 184275*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]
] + 40950*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 4095*Cos[10*(c + d*x)]*L
og[Sin[(c + d*x)/2]] + 1075200*Sin[2*(c + d*x)] + 1044480*Sin[4*(c + d*x)]
+ 414720*Sin[6*(c + d*x)] + 51200*Sin[8*(c + d*x)] - 5120*Sin[10*(c + d*x)]
))/d
```

Maple [A]

time = 0.32, size = 312, normalized size = 1.37

method	result
risch	$-\frac{a^2\left(4095\,e^{19i(dx+c)}+67935\,e^{17i(dx+c)}-194124\,e^{15i(dx+c)}-92160i e^{4i(dx+c)}-575820\,e^{13i(dx+c)}-829440i e^{6i(dx+c)}-\ldots\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\ldots\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x+c)^8*cos(d*x+c)^7-1
/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/sin(d
*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+c)^3-3/256*cos(d*x+c
)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/
63/sin(d*x+c)^7*cos(d*x+c)^7)+a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(
d*x+c)^6*cos(d*x+c)^7+1/192/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*co
s(d*x+c)^7-1/128*cos(d*x+c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(
csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.33, size = 273, normalized size = 1.20

$$-\frac{63\,a^2\left(\frac{2\,(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c))}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)+210\,a^2\left(\frac{2\,(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c))}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log(\cos(dx+c)+1)+15\,\log(\cos(dx+c)-1)\right)+\frac{5120\,(9\,\tan(dx+c)^2+7)a^2}{\tan(dx+c)^9}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^2*(2*(15*cos(d*x
+ c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x
+ c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 +
 7)*a^2/tan(d*x + c)^9)/d
```

Fricas [A]
time = 0.43, size = 327, normalized size = 1.43

$$-\frac{8190\,a^2\cos(dx+c)^9+15540\,a^2\cos(dx+c)^7-69888\,a^2\cos(dx+c)^5+38220\,a^2\cos(dx+c)^3-8190\,a^2\cos(dx+c)-4095\,(a^2\cos(dx+c)^{10}-5\,a^2\cos(dx+c)^8+10\,a^2\cos(dx+c)^6-10\,a^2\cos(dx+c)^4+5\,a^2\cos(dx+c)^2-a^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+4095\,(a^2\cos(dx+c)^{10}-5\,a^2\cos(dx+c)^8+10\,a^2\cos(dx+c)^6-10\,a^2\cos(dx+c)^4+5\,a^2\cos(dx+c)^2-a^2)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5120\,(2\,a^2\cos(dx+c)^9-9\,a^2\cos(dx+c)^7)\sin(dx+c)}{161280\,(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(8190*a^2*cos(d*x + c)^9 + 15540*a^2*cos(d*x + c)^7 - 69888*a^2*c
os(d*x + c)^5 + 38220*a^2*cos(d*x + c)^3 - 8190*a^2*cos(d*x + c) - 4095*(a^
2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*c
os(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2) + 4
095*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 1
0*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) +
1/2) + 5120*(2*a^2*cos(d*x + c)^9 - 9*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 324, normalized size = 1.42

$$\frac{126 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{10} + 560 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{9} + 315 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{8} - 2160 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 3990 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 7560 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 13440 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 11340 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 65520 \, a^{2} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) - 30240 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + \frac{191906 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{10} + 30240 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{9} - 11340 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{8} - 13440 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 7560 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 3990 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 2160 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 315 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 560 \, a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 126 \, a^{2}}{\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{10}}}{1290240 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)

[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a^2*tan(1/2*d*x + 1/2*c)^9
 + 315*a^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*tan(1/2*d*x + 1/2*c)^7 - 3990*
a^2*tan(1/2*d*x + 1/2*c)^6 + 7560*a^2*tan(1/2*d*x + 1/2*c)^4 + 13440*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 11340*a^2*tan(1/2*d*x + 1/2*c)^2 - 65520*a^2*log(abs
(tan(1/2*d*x + 1/2*c))) - 30240*a^2*tan(1/2*d*x + 1/2*c) + (191906*a^2*tan(
1/2*d*x + 1/2*c)^10 + 30240*a^2*tan(1/2*d*x + 1/2*c)^9 - 11340*a^2*tan(1/2*
d*x + 1/2*c)^8 - 13440*a^2*tan(1/2*d*x + 1/2*c)^7 - 7560*a^2*tan(1/2*d*x +
1/2*c)^6 + 3990*a^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a^2*tan(1/2*d*x + 1/2*c)^
3 - 315*a^2*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*tan(1/2*d*x + 1/2*c) - 126*a^2
)/tan(1/2*d*x + 1/2*c)^10)/d
```

Mupad [B]
time = 9.43, size = 357, normalized size = 1.57

$$\frac{19\,a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{6144\,d}-\frac{a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,d}-\frac{3\,a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{512\,d}-\frac{9\,a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{1024\,d}+\frac{3\,a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{1792\,d}-\frac{a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{4096\,d}-\frac{a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{2304\,d}-\frac{a^2\,{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{10240\,d}+\frac{9\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{1024\,d}+\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,d}+\frac{3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{512\,d}-\frac{19\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{6144\,d}-\frac{3\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{1792\,d}+\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{4096\,d}+\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{2304\,d}+\frac{a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{10240\,d}-\frac{13\,a^2\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{256\,d}+\frac{3\,a^2\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{3\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^11,x)

[Out] (19*a^2*cot(c/2 + (d*x)/2)^6)/(6144*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(96*d)
- (3*a^2*cot(c/2 + (d*x)/2)^4)/(512*d) - (9*a^2*cot(c/2 + (d*x)/2)^2)/(1024
*d) + (3*a^2*cot(c/2 + (d*x)/2)^7)/(1792*d) - (a^2*cot(c/2 + (d*x)/2)^8)/(4
096*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(2304*d) - (a^2*cot(c/2 + (d*x)/2)^10)/
(10240*d) + (9*a^2*tan(c/2 + (d*x)/2)^2)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)
^3)/(96*d) + (3*a^2*tan(c/2 + (d*x)/2)^4)/(512*d) - (19*a^2*tan(c/2 + (d*x)
/2)^6)/(6144*d) - (3*a^2*tan(c/2 + (d*x)/2)^7)/(1792*d) + (a^2*tan(c/2 + (d
*x)/2)^8)/(4096*d) + (a^2*tan(c/2 + (d*x)/2)^9)/(2304*d) + (a^2*tan(c/2 + (
d*x)/2)^10)/(10240*d) - (13*a^2*log(tan(c/2 + (d*x)/2)))/(256*d) + (3*a^2*c
ot(c/2 + (d*x)/2))/(128*d) - (3*a^2*tan(c/2 + (d*x)/2))/(128*d)
```

3.603 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=194

$$\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{128d}-$$

```
[Out] 3/128*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-1/3*a^2*cot(d*x+c)^9
/d-1/11*a^2*cot(d*x+c)^11/d+3/128*a^2*cot(d*x+c)*csc(d*x+c)/d+1/64*a^2*cot(
d*x+c)*csc(d*x+c)^3/d-1/16*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/8*a^2*cot(d*x+c)
^3*csc(d*x+c)^5/d-1/5*a^2*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 14, 2691, 3853, 3855, 276}

$$-\frac{a^2\cot^{11}(c+dx)}{11d}-\frac{a^2\cot^9(c+dx)}{3d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{16d}+\frac{a^2\cot(c+dx)\csc^3(c+dx)}{64d}+\frac{3a^2\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (a^2
*Cot[c + d*x]^9)/(3*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*a^2*Cot[c + d*x]
*Csc[c + d*x])/(128*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^5)/(8*
d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^4(c+dx) + 2a^2\cot^6(c+dx)\csc^5\right.$$

$$= a^2\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^2\int \cot^6(c+dx)$$

$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\csc^5$$

$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{a^2\cot^5(c+dx)\csc^5(c+}{5d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx}{11d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx}{11d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{3d} - \frac{a^2\cot^{11}(c+dx}{11d}$$

$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(}{3}$$

Mathematica [A]
time = 2.52, size = 187, normalized size = 0.96

$$\frac{a^2(1+\sin(c+dx))^2\left(887040\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(1318400+1798400\cos(2(c+dx))+440320\cos(4(c+dx))-81280\cos(6(c+dx))-38400\cos(8(c+dx))+3200\cos(10(c+dx))+1073226\sin(c+dx)+869484\sin(3(c+dx))+727188\sin(5(c+dx))+40425\sin(7(c+dx))-3465\sin(9(c+dx)))\right)}{37847040d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(887040*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(1318400 + 1798400*Cos[2*(c + d*x)] +
 440320*Cos[4*(c + d*x)] - 81280*Cos[6*(c + d*x)] - 38400*Cos[8*(c + d*x)]
+ 3200*Cos[10*(c + d*x)] + 1073226*Sin[c + d*x] + 869484*Sin[3*(c + d*x)] +
 727188*Sin[5*(c + d*x)] + 40425*Sin[7*(c + d*x)] - 3465*Sin[9*(c + d*x)]))
)/(37847040*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.33, size = 248, normalized size = 1.28

method	result
derivativedivides	$a^2\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+}{640\sin(dx-}\right.$

default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+2a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(d}{640\sin(d}}{}$
risch	$-\frac{a^2(3465\,e^{21i(dx+c)}-56320ie^{4i(dx+c)}-36960\,e^{19i(dx+c)}-3294720ie^{8i(dx+c)}-767613\,e^{17i(dx+c)}+168960ie^{6i(dx+c)}-1}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8
/693/sin(d*x+c)^7*cos(d*x+c)^7)+2*a^2*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/8
0/sin(d*x+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c
)^4*cos(d*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256
*cos(d*x+c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/9/s
in(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7))
```

Maxima [A]
time = 0.30, size = 197, normalized size = 1.02

$$-\frac{693\,a^2\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{14080\left(9\,\tan(dx+c)^2+7\right)a^2}{\tan(dx+c)^9}+\frac{1280\left(99\,\tan(dx+c)^4+154\,\tan(dx+c)^2+63\right)a^2}{\tan(dx+c)^{11}}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*a^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(176) = 352$.
time = 0.41, size = 360, normalized size = 1.86

$$\frac{12800\,a^2\cos\left(dx+c\right)^{11}-70400\,a^2\cos\left(dx+c\right)^9+84480\,a^2\cos\left(dx+c\right)^7+3465\left(a^2\cos\left(dx+c\right)^{10}-5\,a^2\cos\left(dx+c\right)^8+10\,a^2\cos\left(dx+c\right)^6-10\,a^2\cos\left(dx+c\right)^4+5\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3465\left(a^2\cos\left(dx+c\right)^{10}-5\,a^2\cos\left(dx+c\right)^8+10\,a^2\cos\left(dx+c\right)^6-10\,a^2\cos\left(dx+c\right)^4+5\,a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-462\left(15\,a^2\cos\left(dx+c\right)^9-70\,a^2\cos\left(dx+c\right)^7-128\,a^2\cos\left(dx+c\right)^5+70\,a^2\cos\left(dx+c\right)^3-15\,a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{295680\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^8+10\,d\cos\left(dx+c\right)^6-10\,d\cos\left(dx+c\right)^4+5\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/295680*(12800*a^2*cos(d*x + c)^11 - 70400*a^2*cos(d*x + c)^9 + 84480*a^2*
cos(d*x + c)^7 + 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(d*x + c)^8 + 10*a^2*
cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x + c)^2 - a^2)*log(1/
```

```
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^2*cos(d*x + c)^10 - 5*a^2*cos(
d*x + c)^8 + 10*a^2*cos(d*x + c)^6 - 10*a^2*cos(d*x + c)^4 + 5*a^2*cos(d*x
 + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(15*a^2*cos(d
*x + c)^9 - 70*a^2*cos(d*x + c)^7 - 128*a^2*cos(d*x + c)^5 + 70*a^2*cos(d*x
 + c)^3 - 15*a^2*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^10 - 5*d*cos(
d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2
 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(176) = 352.
time = 0.56, size = 388, normalized size = 2.00

$$\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+462\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-2805\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2310\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+9240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+16170\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4620\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-55440\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-39270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{167422\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+39270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-4620\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-16170\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-9240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2310\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2805\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-462\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{2365440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/2365440*(105*a^2*tan(1/2*d*x + 1/2*c)^11 + 462*a^2*tan(1/2*d*x + 1/2*c)^1
0 + 385*a^2*tan(1/2*d*x + 1/2*c)^9 - 1155*a^2*tan(1/2*d*x + 1/2*c)^8 - 2805
*a^2*tan(1/2*d*x + 1/2*c)^7 - 2310*a^2*tan(1/2*d*x + 1/2*c)^6 + 1155*a^2*ta
n(1/2*d*x + 1/2*c)^5 + 9240*a^2*tan(1/2*d*x + 1/2*c)^4 + 16170*a^2*tan(1/2*
d*x + 1/2*c)^3 + 4620*a^2*tan(1/2*d*x + 1/2*c)^2 - 55440*a^2*log(abs(tan(1/
2*d*x + 1/2*c))) - 39270*a^2*tan(1/2*d*x + 1/2*c) + (167422*a^2*tan(1/2*d*x
 + 1/2*c)^11 + 39270*a^2*tan(1/2*d*x + 1/2*c)^10 - 4620*a^2*tan(1/2*d*x + 1
/2*c)^9 - 16170*a^2*tan(1/2*d*x + 1/2*c)^8 - 9240*a^2*tan(1/2*d*x + 1/2*c)^
7 - 1155*a^2*tan(1/2*d*x + 1/2*c)^6 + 2310*a^2*tan(1/2*d*x + 1/2*c)^5 + 280
5*a^2*tan(1/2*d*x + 1/2*c)^4 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 - 385*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 462*a^2*tan(1/2*d*x + 1/2*c) - 105*a^2)/tan(1/2*d*x
+ 1/2*c)^11)/d
```

Mupad [**B**]
time = 9.97, size = 433, normalized size = 2.23

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{1024\,d}-\frac{7\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{1024\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2048\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{512\,d}+\frac{17\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14336\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{6144\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{5120\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{22528\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{512\,d}+\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{1024\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{2048\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{1024\,d}-\frac{17\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{14336\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{6144\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{5120\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{22528\,d}-\frac{3\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,d}+\frac{17\,a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024\,d}-\frac{17\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^12,x)

[Out] (a^2*cot(c/2 + (d*x)/2)^6)/(1024*d) - (7*a^2*cot(c/2 + (d*x)/2)^3)/(1024*d)
 - (a^2*cot(c/2 + (d*x)/2)^4)/(256*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(2048*d)
 - (a^2*cot(c/2 + (d*x)/2)^2)/(512*d) + (17*a^2*cot(c/2 + (d*x)/2)^7)/(1433
6*d) + (a^2*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^2*cot(c/2 + (d*x)/2)^9)/(61
44*d) - (a^2*cot(c/2 + (d*x)/2)^10)/(5120*d) - (a^2*cot(c/2 + (d*x)/2)^11)/
(22528*d) + (a^2*tan(c/2 + (d*x)/2)^2)/(512*d) + (7*a^2*tan(c/2 + (d*x)/2)^
3)/(1024*d) + (a^2*tan(c/2 + (d*x)/2)^4)/(256*d) + (a^2*tan(c/2 + (d*x)/2)^
5)/(2048*d) - (a^2*tan(c/2 + (d*x)/2)^6)/(1024*d) - (17*a^2*tan(c/2 + (d*x)
/2)^7)/(14336*d) - (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^2*tan(c/2 + (d*
x)/2)^9)/(6144*d) + (a^2*tan(c/2 + (d*x)/2)^10)/(5120*d) + (a^2*tan(c/2 + (
d*x)/2)^11)/(22528*d) - (3*a^2*log(tan(c/2 + (d*x)/2)))/(128*d) + (17*a^2*c
ot(c/2 + (d*x)/2))/(1024*d) - (17*a^2*tan(c/2 + (d*x)/2))/(1024*d)
```

3.604 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=270

$$\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^{11}(c+dx)}{11d}+\frac{17a^2\cot(c+dx)\csc(c+}{1024d}$$

```
[Out] 17/1024*a^2*arctanh(cos(d*x+c))/d-2/7*a^2*cot(d*x+c)^7/d-4/9*a^2*cot(d*x+c)
^9/d-2/11*a^2*cot(d*x+c)^11/d+17/1024*a^2*cot(d*x+c)*csc(d*x+c)/d+17/1536*a
^2*cot(d*x+c)*csc(d*x+c)^3/d-11/384*a^2*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^2*
cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a^2*cot(d*x+c)^5*csc(d*x+c)^5/d-1/64*a^2*c
ot(d*x+c)*csc(d*x+c)^7/d+1/24*a^2*cot(d*x+c)^3*csc(d*x+c)^7/d-1/12*a^2*cot(
d*x+c)^5*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.31, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2952, 2691, 3853, 3855, 2687, 276}

$$-\frac{2a^2\cot^{11}(c+dx)}{11d}-\frac{4a^2\cot^9(c+dx)}{9d}-\frac{2a^2\cot^7(c+dx)}{7d}+\frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^2\cot^5(c+dx)\csc^7(c+dx)}{12d}-\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a^2\cot^3(c+dx)\csc^7(c+dx)}{24d}+\frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{16d}-\frac{a^2\cot(c+dx)\csc^7(c+dx)}{64d}-\frac{11a^2\cot(c+dx)\csc^5(c+dx)}{384d}+\frac{17a^2\cot(c+dx)\csc^3(c+dx)}{1536d}+\frac{17a^2\cot(c+dx)\csc(c+dx)}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (17*a^2*ArcTanh[Cos[c + d*x]])/(1024*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (4
*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Cot[c + d*x]^11)/(11*d) + (17*a^2*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (17*a^2*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (11*a^2*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (a^2*Cot[c + d*x]^3*Cs
c[c + d*x]^5)/(16*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^2*Co
t[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^2*Cot[c + d*x]^3*Csc[c + d*x]^7)/(24
*d) - (a^2*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\cot^6(c+dx)\csc^5(c+dx) + 2a^2\cot^6(c+dx)\csc^6\right.$$

$$= a^2\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^2\int \cot^6(c+dx)$$

$$= -\frac{a^2\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^2\cot^5(c+dx)\csc^7(c}{12d}$$

$$= \frac{a^2\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^2\cot^5(c+dx)\csc^5(c+}{10d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+}{11d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+}{11d}$$

$$= -\frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9(c+dx)}{9d} - \frac{2a^2\cot^{11}(c+}{11d}$$

$$= \frac{3a^2\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot^9}{9}$$

$$= \frac{17a^2\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{2a^2\cot^7(c+dx)}{7d} - \frac{4a^2\cot}{}$$

Mathematica [A]
time = 3.12, size = 197, normalized size = 0.73

$$\frac{a^2(1+\sin(c+dx))^2\left(30159360\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(65553642+67499586\cos(2(c+dx))+25966248\cos(4(c+dx))-6944091\cos(6(c+dx))-746130\cos(8(c+dx))+58905\cos(10(c+dx))+29655040\sin(c+dx)+51445760\sin(3(c+dx))+25600000\sin(5(c+dx))+3235840\sin(7(c+dx))-532480\sin(9(c+dx))+40960\sin(11(c+dx)))\right)}{1816657920d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(30159360*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(65553642 + 67499586*Cos[2*(c + d*x
)] + 25966248*Cos[4*(c + d*x)] - 6944091*Cos[6*(c + d*x)] - 746130*Cos[8*(c
 + d*x)] + 58905*Cos[10*(c + d*x)] + 29655040*Sin[c + d*x] + 51445760*Sin[3
*(c + d*x)] + 25600000*Sin[5*(c + d*x)] + 3235840*Sin[7*(c + d*x)] - 532480
*Sin[9*(c + d*x)] + 40960*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.34, size = 366, normalized size = 1.36

method	result

risch	$-\frac{a^2\left(58905\,\mathrm{e}^{23i(dx+c)}-687225\,\mathrm{e}^{21i(dx+c)}-7690221\,\mathrm{e}^{19i(dx+c)}+19022157\,\mathrm{e}^{17i(dx+c)}+5406720i\mathrm{e}^{4i(dx+c)}+93465834\,\mathrm{e}^{1}\right.}{}$
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{12\sin(dx+c)^{12}}-\frac{\cos^7(dx+c)}{24\sin(dx+c)^{10}}-\frac{\cos^7(dx+c)}{64\sin(dx+c)^{8}}-\frac{\cos^7(dx+c)}{384\sin(dx+c)^{6}}+\frac{\cos^7(dx+c)}{1536\sin(dx+c)^{4}}-\frac{\cos^7(dx+c)}{1024\sin(dx+c)^{2}}-\frac{\left(\cos^5(dx+c)\right)}{1024}-\frac{5}{}\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{12\sin(dx+c)^{12}}-\frac{\cos^7(dx+c)}{24\sin(dx+c)^{10}}-\frac{\cos^7(dx+c)}{64\sin(dx+c)^{8}}-\frac{\cos^7(dx+c)}{384\sin(dx+c)^{6}}+\frac{\cos^7(dx+c)}{1536\sin(dx+c)^{4}}-\frac{\cos^7(dx+c)}{1024\sin(dx+c)^{2}}-\frac{\left(\cos^5(dx+c)\right)}{1024}-\frac{5}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/12/sin(d*x+c)^12*cos(d*x+c)^7-1/24/sin(d*x+c)^10*cos(d*x+c)^7-
1/64/sin(d*x+c)^8*cos(d*x+c)^7-1/384/sin(d*x+c)^6*cos(d*x+c)^7+1/1536/sin(d
*x+c)^4*cos(d*x+c)^7-1/1024/sin(d*x+c)^2*cos(d*x+c)^7-1/1024*cos(d*x+c)^5-5
/3072*cos(d*x+c)^3-5/1024*cos(d*x+c)-5/1024*ln(csc(d*x+c)-cot(d*x+c)))+2*a^
2*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8/693/si
n(d*x+c)^7*cos(d*x+c)^7)+a^2*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x
+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d
*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+
c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.34, size = 323, normalized size = 1.20

$$-\frac{1155\,a^2\left(\frac{2\left(15\,\cos(dx+c)^{11}-85\,\cos(dx+c)^{9}+198\,\cos(dx+c)^{7}+198\,\cos(dx+c)^{5}-85\,\cos(dx+c)^{3}+15\,\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\,\cos(dx+c)^{10}+15\,\cos(dx+c)^{8}-20\,\cos(dx+c)^{6}+15\,\cos(dx+c)^{4}-6\,\cos(dx+c)^{2}+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+2772\,a^2\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}-128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{20480\left(99\,\tan(dx+c)^{4}+154\,\tan(dx+c)^{2}+63\right)a^2}{\tan(dx+c)^{11}}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^2*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 2772*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 20480*(99
*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^2/tan(d*x + c)^11)/d
```

Fricas [A]
time = 0.43, size = 384, normalized size = 1.42

$$-\frac{117810\,a^2\cos\left(dx+c\right)^{11}-667590\,a^2\cos\left(dx+c\right)^{9}+135828\,a^2\cos\left(dx+c\right)^{7}+1555092\,a^2\cos\left(dx+c\right)^{5}-667590\,a^2\cos\left(dx+c\right)^{3}+117810\,a^2\cos\left(dx+c\right)-58905\left(a^2\cos\left(dx+c\right)^{12}-6\,a^2\cos\left(dx+c\right)^{10}+15\,a^2\cos\left(dx+c\right)^{8}-20\,a^2\cos\left(dx+c\right)^{6}+15\,a^2\cos\left(dx+c\right)^{4}-6\,a^2\cos\left(dx+c\right)^{2}+a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+58905\left(a^2\cos\left(dx+c\right)^{12}-6\,a^2\cos\left(dx+c\right)^{10}+15\,a^2\cos\left(dx+c\right)^{8}-20\,a^2\cos\left(dx+c\right)^{6}+15\,a^2\cos\left(dx+c\right)^{4}-6\,a^2\cos\left(dx+c\right)^{2}+a^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+20480\left(8\,a^2\cos\left(dx+c\right)^{11}-44\,a^2\cos\left(dx+c\right)^{9}+99\,a^2\cos\left(dx+c\right)^{7}\right)\sin\left(dx+c\right)}{7096320\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^{8}-20\,d\cos\left(dx+c\right)^{6}+15\,d\cos\left(dx+c\right)^{4}-6\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="frica
s")

[Out] -1/7096320*(117810*a^2*cos(d*x + c)^11 - 667590*a^2*cos(d*x + c)^9 + 135828
*a^2*cos(d*x + c)^7 + 1555092*a^2*cos(d*x + c)^5 - 667590*a^2*cos(d*x + c)^
3 + 117810*a^2*cos(d*x + c) - 58905*(a^2*cos(d*x + c)^12 - 6*a^2*cos(d*x +
c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos(d*x + c)^6 + 15*a^2*cos(d*x + c)
^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(1/2*cos(d*x + c) + 1/2) + 58905*(a^2*c
os(d*x + c)^12 - 6*a^2*cos(d*x + c)^10 + 15*a^2*cos(d*x + c)^8 - 20*a^2*cos
(d*x + c)^6 + 15*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^2 + a^2)*log(-1/2*
cos(d*x + c) + 1/2) + 20480*(8*a^2*cos(d*x + c)^11 - 44*a^2*cos(d*x + c)^9
+ 99*a^2*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c
)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*
d*cos(d*x + c)^2 + d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A]
time = 0.50, size = 420, normalized size = 1.56

$$\frac{1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+5040\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+5544\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-6160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-24255\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-39600\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-27720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+55440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+162855\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+184800\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+55440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-942480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-554400\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2924714\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+554400\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-55440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-184800\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-162855\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-55440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+27720\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+39600\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24255\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+6160\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5544\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-5040\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1155\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}}}{56770560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^2,x, algorithm="giac"
)

[Out] 1/56770560*(1155*a^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*tan(1/2*d*x + 1/2*c
)^11 + 5544*a^2*tan(1/2*d*x + 1/2*c)^10 - 6160*a^2*tan(1/2*d*x + 1/2*c)^9 -
 24255*a^2*tan(1/2*d*x + 1/2*c)^8 - 39600*a^2*tan(1/2*d*x + 1/2*c)^7 - 2772
0*a^2*tan(1/2*d*x + 1/2*c)^6 + 55440*a^2*tan(1/2*d*x + 1/2*c)^5 + 162855*a^
2*tan(1/2*d*x + 1/2*c)^4 + 184800*a^2*tan(1/2*d*x + 1/2*c)^3 + 55440*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 942480*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 554400*a
^2*tan(1/2*d*x + 1/2*c) + (2924714*a^2*tan(1/2*d*x + 1/2*c)^12 + 554400*a^2
*tan(1/2*d*x + 1/2*c)^11 - 55440*a^2*tan(1/2*d*x + 1/2*c)^10 - 184800*a^2*t
an(1/2*d*x + 1/2*c)^9 - 162855*a^2*tan(1/2*d*x + 1/2*c)^8 - 55440*a^2*tan(1
/2*d*x + 1/2*c)^7 + 27720*a^2*tan(1/2*d*x + 1/2*c)^6 + 39600*a^2*tan(1/2*d*

```
x + 1/2*c)^5 + 24255*a^2*tan(1/2*d*x + 1/2*c)^4 + 6160*a^2*tan(1/2*d*x + 1/
2*c)^3 - 5544*a^2*tan(1/2*d*x + 1/2*c)^2 - 5040*a^2*tan(1/2*d*x + 1/2*c) -
1155*a^2)/tan(1/2*d*x + 1/2*c)^12)/d
```

Mupad [B]
time = 10.36, size = 471, normalized size = 1.74

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{5\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1536\,d}-\frac{47\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{5\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{7168\,d}+\frac{7\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{16384\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{9216\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{11264\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{49152\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{1536\,d}+\frac{47\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{1024\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{7168\,d}-\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{16384\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{9216\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{11264\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{49152\,d}-\frac{17\,a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d}+\frac{5\,a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,d}-\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^2)/sin(c + d*x)^13,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^6)/(2048*d) - (5*a^2*cot(c/2 + (d*x)/2)^3)/(1536*d)
 - (47*a^2*cot(c/2 + (d*x)/2)^4)/(16384*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(10
24*d) - (a^2*cot(c/2 + (d*x)/2)^2)/(1024*d) + (5*a^2*cot(c/2 + (d*x)/2)^7)/
(7168*d) + (7*a^2*cot(c/2 + (d*x)/2)^8)/(16384*d) + (a^2*cot(c/2 + (d*x)/2)
^9)/(9216*d) - (a^2*cot(c/2 + (d*x)/2)^10)/(10240*d) - (a^2*cot(c/2 + (d*x)
/2)^11)/(11264*d) - (a^2*cot(c/2 + (d*x)/2)^12)/(49152*d) + (a^2*tan(c/2 +
(d*x)/2)^2)/(1024*d) + (5*a^2*tan(c/2 + (d*x)/2)^3)/(1536*d) + (47*a^2*tan(
c/2 + (d*x)/2)^4)/(16384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(1024*d) - (a^2*ta
n(c/2 + (d*x)/2)^6)/(2048*d) - (5*a^2*tan(c/2 + (d*x)/2)^7)/(7168*d) - (7*a
^2*tan(c/2 + (d*x)/2)^8)/(16384*d) - (a^2*tan(c/2 + (d*x)/2)^9)/(9216*d) +
(a^2*tan(c/2 + (d*x)/2)^10)/(10240*d) + (a^2*tan(c/2 + (d*x)/2)^11)/(11264*
d) + (a^2*tan(c/2 + (d*x)/2)^12)/(49152*d) - (17*a^2*log(tan(c/2 + (d*x)/2)
))/(1024*d) + (5*a^2*cot(c/2 + (d*x)/2))/(512*d) - (5*a^2*tan(c/2 + (d*x)/2
))/(512*d)
```

3.605 $\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=224

$$\frac{27a^3x}{1024}-\frac{4a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^{13}(c+dx)}{13d}+\frac{27a^3\cos(c+dx)\sin(c+dx}{1024d}$$

```
[Out] 27/1024*a^3*x-4/7*a^3*cos(d*x+c)^7/d+a^3*cos(d*x+c)^9/d-6/11*a^3*cos(d*x+c)
^11/d+1/13*a^3*cos(d*x+c)^13/d+27/1024*a^3*cos(d*x+c)*sin(d*x+c)/d+9/512*a^
3*cos(d*x+c)^3*sin(d*x+c)/d+9/640*a^3*cos(d*x+c)^5*sin(d*x+c)/d-27/320*a^3*
cos(d*x+c)^7*sin(d*x+c)/d-9/40*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d-1/4*a^3*cos(
d*x+c)^7*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.31, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2648, 2715, 8, 2645, 276}

$$\frac{a^3\cos^{13}(c+dx)}{13d}-\frac{6a^3\cos^{11}(c+dx)}{11d}+\frac{a^3\cos^9(c+dx)}{d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{4d}-\frac{9a^3\sin^3(c+dx)\cos^7(c+dx)}{40d}-\frac{27a^3\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{9a^3\sin(c+dx)\cos^5(c+dx)}{640d}+\frac{9a^3\sin(c+dx)\cos^3(c+dx)}{512d}+\frac{27a^3\sin(c+dx)\cos(c+dx)}{1024d}+\frac{27a^3x}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (a^3*Cos[c + d*x]^9)/d - (
6*a^3*Cos[c + d*x]^11)/(11*d) + (a^3*Cos[c + d*x]^13)/(13*d) + (27*a^3*Cos[
c + d*x]*Sin[c + d*x])/(1024*d) + (9*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(512*
d) + (9*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(640*d) - (27*a^3*Cos[c + d*x]^7*S
in[c + d*x])/(320*d) - (9*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(40*d) - (a^3*
Cos[c + d*x]^7*Sin[c + d*x]^5)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^4(c+dx)+3a^3\cos^6(c+dx)\sin\right.$$

$$= a^3\int \cos^6(c+dx)\sin^4(c+dx)\,dx + a^3\int \cos^6(c+dx)$$

$$= -\frac{a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(}{4d}$$

$$= -\frac{3a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{9a^3\cos^7(c+dx)\sin^3}{40d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+}{11d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+}{11d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos^{11}(c+}{11d}$$

$$= \frac{3a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos}{1}$$

$$= \frac{27a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{a^3\cos^9(c+dx)}{d} - \frac{6a^3\cos}{}$$

Mathematica [A]
time = 1.57, size = 146, normalized size = 0.65

$$\frac{a^3(720720c + 1081080dx - 1401400\cos(c+dx) - 450450\cos(3(c+dx)) + 150150\cos(5(c+dx)) + 94380\cos(7(c+dx)) - 20020\cos(9(c+dx)) - 11830\cos(11(c+dx)) + 770\cos(13(c+dx)) + 80080\sin(2(c+dx)) - 385385\sin(4(c+dx)) - 40040\sin(6(c+dx)) + 65065\sin(8(c+dx)) + 8008\sin(10(c+dx)) - 5005\sin(12(c+dx)))}{41000960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(720720*c + 1081080*d*x - 1401400*Cos[c + d*x] - 450450*Cos[3*(c + d*x
)] + 150150*Cos[5*(c + d*x)] + 94380*Cos[7*(c + d*x)] - 20020*Cos[9*(c + d*
x)] - 11830*Cos[11*(c + d*x)] + 770*Cos[13*(c + d*x)] + 80080*Sin[2*(c + d*
x)] - 385385*Sin[4*(c + d*x)] - 40040*Sin[6*(c + d*x)] + 65065*Sin[8*(c + d
*x)] + 8008*Sin[10*(c + d*x)] - 5005*Sin[12*(c + d*x)]))/(41000960*d)
```

Maple [A]
time = 0.71, size = 308, normalized size = 1.38

method	result
risch	$-\frac{35a^3\cos(dx+c)}{1024d}+\frac{27a^3x}{1024}+\frac{a^3\cos(13dx+13c)}{53248d}-\frac{13a^3\cos(11dx+11c)}{45056d}-\frac{a^3\sin(12dx+12c)}{8192d}+\frac{a^3\sin(10dx+10c)}{5120d}$
derivativedivides	$a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.$
default	$a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+3*a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+3*a^3*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^
3*cos(d*x+c)^7-1/64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x
+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+a^3*(-1/13*sin(d*x+c
)^6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x
+c)^7-16/3003*cos(d*x+c)^7))
```

Maxima [A]
time = 0.29, size = 184, normalized size = 0.82

$$\frac{40960\left(231\cos(dx+c)^{13}-819\cos(dx+c)^{11}+1001\cos(dx+c)^{9}-429\cos(dx+c)^{7}\right)a^3-532480\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a^3+12012\left(32\sin(2dx+2c)^{5}+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)a^3+15015\left(4\sin(4dx+4c)^{3}+120dx+120c+9\sin(8dx+8c)-48\sin(4dx+4c)\right)a^3}{123002880d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/123002880*(40960*(231*cos(d*x + c)^13 - 819*cos(d*x + c)^11 + 1001*cos(d*
x + c)^9 - 429*cos(d*x + c)^7)*a^3 - 532480*(63*cos(d*x + c)^11 - 154*cos(d
*x + c)^9 + 99*cos(d*x + c)^7)*a^3 + 12012*(32*sin(2*d*x + 2*c)^5 + 120*d*x
 + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a^3 + 15015*(4*sin(4*d
*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*a
^3)/d
```

Fricas [**A**]
time = 0.42, size = 150, normalized size = 0.67

$$\frac{394240\,a^3\cos\left(dx+c\right)^{13}-2795520\,a^3\cos\left(dx+c\right)^{11}+5125120\,a^3\cos\left(dx+c\right)^{9}-2928640\,a^3\cos\left(dx+c\right)^{7}+135135\,a^3dx-1001\,\left(1280\,a^3\cos\left(dx+c\right)^{11}-3712\,a^3\cos\left(dx+c\right)^{9}+2864\,a^3\cos\left(dx+c\right)^{7}-72\,a^3\cos\left(dx+c\right)^{5}-90\,a^3\cos\left(dx+c\right)^{3}-135\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{5125120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/5125120*(394240*a^3*cos(d*x + c)^13 - 2795520*a^3*cos(d*x + c)^11 + 51251
20*a^3*cos(d*x + c)^9 - 2928640*a^3*cos(d*x + c)^7 + 135135*a^3*d*x - 1001*
(1280*a^3*cos(d*x + c)^11 - 3712*a^3*cos(d*x + c)^9 + 2864*a^3*cos(d*x + c)
^7 - 72*a^3*cos(d*x + c)^5 - 90*a^3*cos(d*x + c)^3 - 135*a^3*cos(d*x + c))*
sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(212) = 424$.
time = 4.73, size = 748, normalized size = 3.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((15*a**3*x*sin(c + d*x)**12/1024 + 45*a**3*x*sin(c + d*x)**10*cos
(c + d*x)**2/512 + 3*a**3*x*sin(c + d*x)**10/256 + 225*a**3*x*sin(c + d*x)*
*8*cos(c + d*x)**4/1024 + 15*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 7
5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 15*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 225*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 15*a*
*3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**10/512 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*
cos(c + d*x)**12/1024 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*sin(c + d*x
)**11*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(1024
*d) + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 99*a**3*sin(c + d*x)**7
```

```
*cos(c + d*x)**5/(512*d) + 7*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) -
 a**3*sin(c + d*x)**6*cos(c + d*x)**7/(7*d) - 99*a**3*sin(c + d*x)**5*cos(c
 + d*x)**7/(512*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 2*a**3*s
in(c + d*x)**4*cos(c + d*x)**9/(21*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)
**7/(7*d) - 85*a**3*sin(c + d*x)**3*cos(c + d*x)**9/(1024*d) - 7*a**3*sin(c
 + d*x)**3*cos(c + d*x)**7/(128*d) - 8*a**3*sin(c + d*x)**2*cos(c + d*x)**1
1/(231*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - 15*a**3*sin(c +
 d*x)*cos(c + d*x)**11/(1024*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*
d) - 16*a**3*cos(c + d*x)**13/(3003*d) - 8*a**3*cos(c + d*x)**11/(231*d), N
e(d, 0)), (x*(a*sin(c) + a)**3*sin(c)**4*cos(c)**6, True))
```

Giac [A]
time = 0.52, size = 225, normalized size = 1.00

$$\frac{27}{1024}a^3x+\frac{a^3\cos(13dx+13c)}{53248d}-\frac{13a^3\cos(11dx+11c)}{45056d}-\frac{a^3\cos(9dx+9c)}{2048d}+\frac{33a^3\cos(7dx+7c)}{14336d}+\frac{15a^3\cos(5dx+5c)}{4096d}-\frac{45a^3\cos(3dx+3c)}{4096d}-\frac{35a^3\cos(dx+c)}{1024d}-\frac{a^3\sin(12dx+12c)}{8192d}+\frac{a^3\sin(10dx+10c)}{5120d}+\frac{13a^3\sin(8dx+8c)}{8192d}-\frac{a^3\sin(6dx+6c)}{1024d}-\frac{77a^3\sin(4dx+4c)}{8192d}+\frac{a^3\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 27/1024*a^3*x + 1/53248*a^3*cos(13*d*x + 13*c)/d - 13/45056*a^3*cos(11*d*x
+ 11*c)/d - 1/2048*a^3*cos(9*d*x + 9*c)/d + 33/14336*a^3*cos(7*d*x + 7*c)/d
 + 15/4096*a^3*cos(5*d*x + 5*c)/d - 45/4096*a^3*cos(3*d*x + 3*c)/d - 35/102
4*a^3*cos(d*x + c)/d - 1/8192*a^3*sin(12*d*x + 12*c)/d + 1/5120*a^3*sin(10*
d*x + 10*c)/d + 13/8192*a^3*sin(8*d*x + 8*c)/d - 1/1024*a^3*sin(6*d*x + 6*c
)/d - 77/8192*a^3*sin(4*d*x + 4*c)/d + 1/512*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.19, size = 612, normalized size = 2.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (27*a^3*x)/1024 - ((27*a^3*(c + d*x))/1024 + (171*a^3*tan(c/2 + (d*x)/2)^3)
/256 - (1603*a^3*tan(c/2 + (d*x)/2)^5)/640 - (59523*a^3*tan(c/2 + (d*x)/2)^
7)/1280 + (305539*a^3*tan(c/2 + (d*x)/2)^9)/2560 - (93973*a^3*tan(c/2 + (d*
x)/2)^11)/640 + (93973*a^3*tan(c/2 + (d*x)/2)^15)/640 - (305539*a^3*tan(c/2
 + (d*x)/2)^17)/2560 + (59523*a^3*tan(c/2 + (d*x)/2)^19)/1280 + (1603*a^3*t
an(c/2 + (d*x)/2)^21)/640 - (171*a^3*tan(c/2 + (d*x)/2)^23)/256 - (27*a^3*t
an(c/2 + (d*x)/2)^25)/512 - a^3*((27*c)/1024 + (27*d*x)/1024 - 80/1001) + t
an(c/2 + (d*x)/2)^2*((351*a^3*(c + d*x))/1024 - a^3*((351*c)/1024 + (351*d*
x)/1024 - 80/77)) + tan(c/2 + (d*x)/2)^4*((1053*a^3*(c + d*x))/512 - a^3*((
1053*c)/512 + (1053*d*x)/512 - 480/77)) + tan(c/2 + (d*x)/2)^20*((3861*a^3*
(c + d*x))/512 - a^3*((3861*c)/512 + (3861*d*x)/512 - 32)) + tan(c/2 + (d*x
)/2)^6*((3861*a^3*(c + d*x))/512 - a^3*((3861*c)/512 + (3861*d*x)/512 + 64/
```

7)) + tan(c/2 + (d*x)/2)^14*((11583*a^3*(c + d*x))/256 - a^3*((11583*c)/256 + (11583*d*x)/256 - 320)) + tan(c/2 + (d*x)/2)^12*((11583*a^3*(c + d*x))/256 - a^3*((11583*c)/256 + (11583*d*x)/256 + 1280/7)) + tan(c/2 + (d*x)/2)^18*((19305*a^3*(c + d*x))/1024 - a^3*((19305*c)/1024 + (19305*d*x)/1024 - 16)) + tan(c/2 + (d*x)/2)^8*((19305*a^3*(c + d*x))/1024 - a^3*((19305*c)/1024 + (19305*d*x)/1024 - 288/7)) + tan(c/2 + (d*x)/2)^16*((34749*a^3*(c + d*x))/1024 - a^3*((34749*c)/1024 + (34749*d*x)/1024 + 48)) + tan(c/2 + (d*x)/2)^10*((34749*a^3*(c + d*x))/1024 - a^3*((34749*c)/1024 + (34749*d*x)/1024 - 1056/7)) + (27*a^3*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^13)

3.606 $\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=209

$$\frac{41a^3x}{1024}-\frac{4a^3\cos^7(c+dx)}{7d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{41a^3\cos(c+dx)\sin(c+dx)}{1024d}+\frac{41a^3\cos^3(c+}{15}$$

```
[Out] 41/1024*a^3*x-4/7*a^3*cos(d*x+c)^7/d+7/9*a^3*cos(d*x+c)^9/d-3/11*a^3*cos(d*
x+c)^11/d+41/1024*a^3*cos(d*x+c)*sin(d*x+c)/d+41/1536*a^3*cos(d*x+c)^3*sin(
d*x+c)/d+41/1920*a^3*cos(d*x+c)^5*sin(d*x+c)/d-41/320*a^3*cos(d*x+c)^7*sin(
d*x+c)/d-41/120*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d-1/12*a^3*cos(d*x+c)^7*sin(d
*x+c)^5/d
```

Rubi [A]
time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{3a^3\cos^{11}(c+dx)}{11d}+\frac{7a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\sin^5(c+dx)\cos^7(c+dx)}{12d}-\frac{41a^3\sin^3(c+dx)\cos^7(c+dx)}{120d}-\frac{41a^3\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{41a^3\sin(c+dx)\cos^5(c+dx)}{1920d}+\frac{41a^3\sin(c+dx)\cos^3(c+dx)}{1536d}+\frac{41a^3\sin(c+dx)\cos(c+dx)}{1024d}+\frac{41a^3x}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (41*a^3*x)/1024 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (7*a^3*Cos[c + d*x]^9)/(9*
d) - (3*a^3*Cos[c + d*x]^11)/(11*d) + (41*a^3*Cos[c + d*x]*Sin[c + d*x])/(1
024*d) + (41*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(1536*d) + (41*a^3*Cos[c + d*
x]^5*Sin[c + d*x])/(1920*d) - (41*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(320*d)
- (41*a^3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(120*d) - (a^3*Cos[c + d*x]^7*Sin[
c + d*x]^5)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^3(c+dx)+3a^3\cos^6(c+dx)\sin^4\right.$$

$$= a^3\int \cos^6(c+dx)\sin^3(c+dx)\,dx + a^3\int \cos^6(c+dx)\,\mathrm{s}$$

$$= -\frac{3a^3\cos^7(c+dx)\sin^3(c+dx)}{10d} - \frac{a^3\cos^7(c+dx)\sin^5(}{12d}$$

$$= -\frac{9a^3\cos^7(c+dx)\sin(c+dx)}{80d} - \frac{41a^3\cos^7(c+dx)\sin^3}{120d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+}{11d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+}{11d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^{11}(c+}{11d}$$

$$= \frac{9a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos^1}{1}$$

$$= \frac{41a^3x}{1024} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{7a^3\cos^9(c+dx)}{9d} - \frac{3a^3\cos}{}$$

Mathematica [A]
time = 1.09, size = 136, normalized size = 0.65

$$\frac{a^3(1247400c+1136520dx-1496880\cos(c+dx)-572880\cos(3(c+dx))+83160\cos(5(c+dx))+106920\cos(7(c+dx))+3080\cos(9(c+dx))-7560\cos(11(c+dx))+166320\sin(2(c+dx))-384615\sin(4(c+dx))-83160\sin(6(c+dx))+51975\sin(8(c+dx))+16632\sin(10(c+dx))-1155\sin(12(c+dx)))}{28385280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1247400*c + 1136520*d*x - 1496880*Cos[c + d*x] - 572880*Cos[3*(c + d*
x)] + 83160*Cos[5*(c + d*x)] + 106920*Cos[7*(c + d*x)] + 3080*Cos[9*(c + d*
x)] - 7560*Cos[11*(c + d*x)] + 166320*Sin[2*(c + d*x)] - 384615*Sin[4*(c +
d*x)] - 83160*Sin[6*(c + d*x)] + 51975*Sin[8*(c + d*x)] + 16632*Sin[10*(c +
 d*x)] - 1155*Sin[12*(c + d*x)]))/(28385280*d)
```

Maple [A]
time = 0.56, size = 272, normalized size = 1.30

method	result
risch	$-\frac{27a^3\cos(dx+c)}{512d}+\frac{41a^3x}{1024}-\frac{3a^3\cos(11dx+11c)}{11264d}+\frac{3a^3\sin(10dx+10c)}{5120d}-\frac{a^3\sin(12dx+12c)}{24576d}+\frac{a^3\cos(9dx+9c)}{9216d}-$

derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(d\right.}{}\right.}{}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+3a^3\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(d\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/10*si
n(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+3*a^3*(-1/11*
sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7
)+a^3*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*cos(d*x+c)^7-1/64*
cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c
))*sin(d*x+c)+5/1024*d*x+5/1024*c))
```

Maxima [A]
time = 0.28, size = 164, normalized size = 0.78

$$-\frac{122880\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a^{3}-450560\left(7\cos(dx+c)^{9}-9\cos(dx+c)^{7}\right)a^{3}-8316\left(32\sin(2dx+2c)^{5}+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)a^{3}-1155\left(4\sin(4dx+4c)^{3}+120dx+120c+9\sin(8dx+8c)-48\sin(4dx+4c)\right)a^{3}}{28385280d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/28385280*(122880*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
 c)^7)*a^3 - 450560*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 8316*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*a^3 - 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*c)
 - 48*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.43, size = 137, normalized size = 0.66

$$-\frac{967680a^{3}\cos(dx+c)^{11}-2759680a^{3}\cos(dx+c)^{9}+2027520a^{3}\cos(dx+c)^{7}-142065a^{3}dx+231\left(1280a^{3}\cos(dx+c)^{11}-7808a^{3}\cos(dx+c)^{9}+8496a^{3}\cos(dx+c)^{7}-328a^{3}\cos(dx+c)^{5}-410a^{3}\cos(dx+c)^{3}-615a^{3}\cos(dx+c)\right)\sin(dx+c)}{3548160d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(967680*a^3*cos(d*x + c)^11 - 2759680*a^3*cos(d*x + c)^9 + 20275
20*a^3*cos(d*x + c)^7 - 142065*a^3*d*x + 231*(1280*a^3*cos(d*x + c)^11 - 78
```

```
08*a^3*cos(d*x + c)^9 + 8496*a^3*cos(d*x + c)^7 - 328*a^3*cos(d*x + c)^5 -
410*a^3*cos(d*x + c)^3 - 615*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(201) = 402$.
time = 3.53, size = 699, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**12/1024 + 15*a**3*x*sin(c + d*x)**10*cos(
c + d*x)**2/512 + 9*a**3*x*sin(c + d*x)**10/256 + 75*a**3*x*sin(c + d*x)**8
*cos(c + d*x)**4/1024 + 45*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 25*
a**3*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 45*a**3*x*sin(c + d*x)**6*cos(
c + d*x)**4/128 + 75*a**3*x*sin(c + d*x)**4*cos(c + d*x)**8/1024 + 45*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**10/512 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*cos(
c + d*x)**12/1024 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*sin(c + d*x)**11
*cos(c + d*x)/(1024*d) + 85*a**3*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) +
 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 33*a**3*sin(c + d*x)**7*cos(
c + d*x)**5/(512*d) + 21*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) - 33*
a**3*sin(c + d*x)**5*cos(c + d*x)**7/(512*d) + 3*a**3*sin(c + d*x)**5*cos(c
 + d*x)**5/(10*d) - 3*a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 85*a**3*
sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 21*a**3*sin(c + d*x)**3*cos(c +
d*x)**7/(128*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(21*d) - a**3*sin(
c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**11/(1
024*d) - 9*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 8*a**3*cos(c + d*x)*
*11/(231*d) - 2*a**3*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a*sin(c) + a)**
3*sin(c)**3*cos(c)**6, True))
```

Giac [A]
time = 0.49, size = 208, normalized size = 1.00

$$\frac{41}{1024}a^3x-\frac{3\,a^3\cos\left(11\,dx+11\,c\right)}{11264\,d}+\frac{a^3\cos\left(9\,dx+9\,c\right)}{9216\,d}+\frac{27\,a^3\cos\left(7\,dx+7\,c\right)}{7168\,d}+\frac{3\,a^3\cos\left(5\,dx+5\,c\right)}{1024\,d}-\frac{31\,a^3\cos\left(3\,dx+3\,c\right)}{1536\,d}-\frac{27\,a^3\cos\left(dx+c\right)}{512\,d}-\frac{a^3\sin\left(12\,dx+12\,c\right)}{24576\,d}+\frac{3\,a^3\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{15\,a^3\sin\left(8\,dx+8\,c\right)}{8192\,d}-\frac{3\,a^3\sin\left(6\,dx+6\,c\right)}{1024\,d}-\frac{111\,a^3\sin\left(4\,dx+4\,c\right)}{8192\,d}+\frac{3\,a^3\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 41/1024*a^3*x - 3/11264*a^3*cos(11*d*x + 11*c)/d + 1/9216*a^3*cos(9*d*x + 9
*c)/d + 27/7168*a^3*cos(7*d*x + 7*c)/d + 3/1024*a^3*cos(5*d*x + 5*c)/d - 31
/1536*a^3*cos(3*d*x + 3*c)/d - 27/512*a^3*cos(d*x + c)/d - 1/24576*a^3*sin(
12*d*x + 12*c)/d + 3/5120*a^3*sin(10*d*x + 10*c)/d + 15/8192*a^3*sin(8*d*x
+ 8*c)/d - 3/1024*a^3*sin(6*d*x + 6*c)/d - 111/8192*a^3*sin(4*d*x + 4*c)/d
+ 3/512*a^3*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 11.02, size = 683, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (41*a^3*x)/1024 - ((1435*a^3*tan(c/2 + (d*x)/2)^3)/1536 - (36401*a^3*tan(c/
2 + (d*x)/2)^5)/2560 + (1263*a^3*tan(c/2 + (d*x)/2)^7)/2560 + (184331*a^3*t
an(c/2 + (d*x)/2)^9)/3840 - (35387*a^3*tan(c/2 + (d*x)/2)^11)/256 + (35387*
a^3*tan(c/2 + (d*x)/2)^13)/256 - (184331*a^3*tan(c/2 + (d*x)/2)^15)/3840 -
(1263*a^3*tan(c/2 + (d*x)/2)^17)/2560 + (36401*a^3*tan(c/2 + (d*x)/2)^19)/2
560 - (1435*a^3*tan(c/2 + (d*x)/2)^21)/1536 - (41*a^3*tan(c/2 + (d*x)/2)^23
)/512 + a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((41*c)/1024 + (41*d*x)/102
4 - 92/693) - tan(c/2 + (d*x)/2)^22*(a^3*((123*c)/256 + (123*d*x)/256) - 12
*a^3*((41*c)/1024 + (41*d*x)/1024)) - tan(c/2 + (d*x)/2)^2*(a^3*((123*c)/25
6 + (123*d*x)/256 - 368/231) - 12*a^3*((41*c)/1024 + (41*d*x)/1024)) + tan(
c/2 + (d*x)/2)^20*(66*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((1353*c)/512
 + (1353*d*x)/512 - 4)) + tan(c/2 + (d*x)/2)^4*(66*a^3*((41*c)/1024 + (41*d
*x)/1024) - a^3*((1353*c)/512 + (1353*d*x)/512 - 100/21)) + tan(c/2 + (d*x)
/2)^18*(220*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((2255*c)/256 + (2255*d
*x)/256 - 112/3)) + tan(c/2 + (d*x)/2)^6*(220*a^3*((41*c)/1024 + (41*d*x)/1
024) - a^3*((2255*c)/256 + (2255*d*x)/256 + 512/63)) + tan(c/2 + (d*x)/2)^1
4*(792*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((4059*c)/128 + (4059*d*x)/1
28 - 128)) + tan(c/2 + (d*x)/2)^10*(792*a^3*((41*c)/1024 + (41*d*x)/1024) -
 a^3*((4059*c)/128 + (4059*d*x)/128 + 160/7)) + tan(c/2 + (d*x)/2)^12*(924*
a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*((9471*c)/256 + (9471*d*x)/256 - 18
4/3)) + tan(c/2 + (d*x)/2)^16*(495*a^3*((41*c)/1024 + (41*d*x)/1024) - a^3*
((20295*c)/1024 + (20295*d*x)/1024 + 36)) + tan(c/2 + (d*x)/2)^8*(495*a^3*(
(41*c)/1024 + (41*d*x)/1024) - a^3*((20295*c)/1024 + (20295*d*x)/1024 - 712
/7)) + (41*a^3*tan(c/2 + (d*x)/2))/512)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

3.607 $\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$\frac{19a^3x}{256}-\frac{4a^3\cos^7(c+dx)}{7d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{19a^3\cos(c+dx)\sin(c+dx)}{256d}+\frac{19a^3\cos^3(c+dx)}{38\ldots}$$

```
[Out] 19/256*a^3*x-4/7*a^3*cos(d*x+c)^7/d+5/9*a^3*cos(d*x+c)^9/d-1/11*a^3*cos(d*x
+c)^11/d+19/256*a^3*cos(d*x+c)*sin(d*x+c)/d+19/384*a^3*cos(d*x+c)^3*sin(d*x
+c)/d+19/480*a^3*cos(d*x+c)^5*sin(d*x+c)/d-19/80*a^3*cos(d*x+c)^7*sin(d*x+c
)/d-3/10*a^3*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.25, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2648, 2715, 8, 2645, 14, 276}

$$-\frac{a^3\cos^{11}(c+dx)}{11d}+\frac{5a^3\cos^9(c+dx)}{9d}-\frac{4a^3\cos^7(c+dx)}{7d}-\frac{3a^3\sin^3(c+dx)\cos^7(c+dx)}{10d}-\frac{19a^3\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{19a^3\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{19a^3\sin(c+dx)\cos^3(c+dx)}{384d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{256d}+\frac{19a^3x}{256}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*x)/256 - (4*a^3*Cos[c + d*x]^7)/(7*d) + (5*a^3*Cos[c + d*x]^9)/(9*d
) - (a^3*Cos[c + d*x]^11)/(11*d) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(256*
d) + (19*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (19*a^3*Cos[c + d*x]^5*
Sin[c + d*x])/(480*d) - (19*a^3*Cos[c + d*x]^7*Sin[c + d*x])/(80*d) - (3*a^
3*Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)\sin^2(c+dx) + 3a^3\cos^6(c+dx)\sin^3\right.$$

$$= a^3\int \cos^6(c+dx)\sin^2(c+dx)\,dx + a^3\int \cos^6(c+dx)\,\mathrm{s}$$

$$= -\frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3a^3\cos^7(c+dx)\sin^3(c}{10d}$$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{48d} - \frac{19a^3\cos^7(c+dx)\sin(c+}{80d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+d}{11d}$$

$$= -\frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}(c+d}{11d}$$

$$= \frac{5a^3x}{128} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^{11}}{1}$$

$$= \frac{19a^3x}{256} - \frac{4a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^9(c+dx)}{9d} - \frac{a^3\cos^1}{}$$

Mathematica [A]

time = 0.87, size = 126, normalized size = 0.69

$$\frac{a^3(415800c+526680dx-568260\cos(c+dx)-244860\cos(3(c+dx))+6930\cos(5(c+dx))+40590\cos(7(c+dx))+8470\cos(9(c+dx))-630\cos(11(c+dx))+152460\sin(2(c+dx))-138600\sin(4(c+dx))-57750\sin(6(c+dx))+3465\sin(8(c+dx))+4158\sin(10(c+dx)))}{7096320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(415800*c + 526680*d*x - 568260*Cos[c + d*x] - 244860*Cos[3*(c + d*x)]
 + 6930*Cos[5*(c + d*x)] + 40590*Cos[7*(c + d*x)] + 8470*Cos[9*(c + d*x)] -
 630*Cos[11*(c + d*x)] + 152460*Sin[2*(c + d*x)] - 138600*Sin[4*(c + d*x)]
- 57750*Sin[6*(c + d*x)] + 3465*Sin[8*(c + d*x)] + 4158*Sin[10*(c + d*x)]))
/(7096320*d)
```

Maple [A]

time = 0.40, size = 236, normalized size = 1.29

method	result
risch	$-\frac{41a^3\cos(dx+c)}{512d} + \frac{19a^3x}{256} - \frac{a^3\cos(11dx+11c)}{11264d} + \frac{3a^3\sin(10dx+10c)}{5120d} + \frac{11a^3\cos(9dx+9c)}{9216d} + \frac{a^3\sin(8dx+8c)}{2048d} +$
derivativedivides	$a^3\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8} + \frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48} + \frac{5dx}{128} + \frac{5c}{128}\right) + 3a^3\left(-\frac{\left(\sin^2(dx+c)\right)(\mathrm{co}}{9}\right.$

default	$\frac{a^3\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+3a^3\left(-\frac{\left(\sin^2(dx+c)\right)(}{9}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+3*a^3*(-1/9*sin(d*x+c)^2*cos
(d*x+c)^7-2/63*cos(d*x+c)^7)+3*a^3*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*co
s(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+3/256*d*x+3/256*c)+a^3*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*si
n(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7))
```

Maxima [A]
time = 0.29, size = 164, normalized size = 0.90

$$-\frac{10240\left(63\cos(dx+c)^{11}-154\cos(dx+c)^{9}+99\cos(dx+c)^{7}\right)a^{3}-337920\left(7\cos(dx+c)^{9}-9\cos(dx+c)^{7}\right)a^{3}-2079\left(32\sin(2dx+2c)^{5}+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)a^{3}-2310\left(64\sin(2dx+2c)^{3}+120dx+120c-3\sin(8dx+8c)-24\sin(4dx+4c)\right)a^{3}}{7096320d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/7096320*(10240*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c
)^7)*a^3 - 337920*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^3 - 2079*(32*sin(
2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c)
)*a^3 - 2310*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
- 24*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.41, size = 124, normalized size = 0.68

$$-\frac{80640a^{3}\cos(dx+c)^{11}-492800a^{3}\cos(dx+c)^{9}+506880a^{3}\cos(dx+c)^{7}-65835a^{3}dx-231\left(1152a^{3}\cos(dx+c)^{9}-2064a^{3}\cos(dx+c)^{7}+152a^{3}\cos(dx+c)^{5}+190a^{3}\cos(dx+c)^{3}+285a^{3}\cos(dx+c)\right)\sin(dx+c)}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/887040*(80640*a^3*cos(d*x + c)^11 - 492800*a^3*cos(d*x + c)^9 + 506880*a
^3*cos(d*x + c)^7 - 65835*a^3*d*x - 231*(1152*a^3*cos(d*x + c)^9 - 2064*a^3
*cos(d*x + c)^7 + 152*a^3*cos(d*x + c)^5 + 190*a^3*cos(d*x + c)^3 + 285*a^3
*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(175) = 350$.
time = 2.65, size = 597, normalized size = 3.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((9*a**3*x*sin(c + d*x)**10/256 + 45*a**3*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 5*a**3*x*sin(c + d*x)**8/128 + 45*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**4/128 + 5*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 45*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**4/64 + 45*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 5*a**3*x*sin(c +
 d*x)**2*cos(c + d*x)**6/32 + 9*a**3*x*cos(c + d*x)**10/256 + 5*a**3*x*cos(
c + d*x)**8/128 + 9*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 21*a**3*sin
(c + d*x)**7*cos(c + d*x)**3/(128*d) + 5*a**3*sin(c + d*x)**7*cos(c + d*x)/
(128*d) + 3*a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c + d
*x)**5*cos(c + d*x)**3/(384*d) - a**3*sin(c + d*x)**4*cos(c + d*x)**7/(7*d)
 - 21*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*a**3*sin(c + d*x)**
3*cos(c + d*x)**5/(384*d) - 4*a**3*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) -
 3*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 9*a**3*sin(c + d*x)*cos(c +
 d*x)**9/(256*d) - 5*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 8*a**3*cos
(c + d*x)**11/(693*d) - 2*a**3*cos(c + d*x)**9/(21*d), Ne(d, 0)), (x*(a*sin
(c) + a)**3*sin(c)**2*cos(c)**6, True))
```

Giac [**A**]
time = 0.59, size = 191, normalized size = 1.04

$$\frac{19}{256}a^3x-\frac{a^3\cos(11\,dx+11\,c)}{11264\,d}+\frac{11\,a^3\cos(9\,dx+9\,c)}{9216\,d}+\frac{41\,a^3\cos(7\,dx+7\,c)}{7168\,d}+\frac{a^3\cos(5\,dx+5\,c)}{1024\,d}-\frac{53\,a^3\cos(3\,dx+3\,c)}{1536\,d}-\frac{41\,a^3\cos(dx+c)}{512\,d}+\frac{3\,a^3\sin(10\,dx+10\,c)}{5120\,d}+\frac{a^3\sin(8\,dx+8\,c)}{2048\,d}-\frac{25\,a^3\sin(6\,dx+6\,c)}{3072\,d}-\frac{5\,a^3\sin(4\,dx+4\,c)}{256\,d}+\frac{11\,a^3\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 19/256*a^3*x - 1/11264*a^3*cos(11*d*x + 11*c)/d + 11/9216*a^3*cos(9*d*x + 9
*c)/d + 41/7168*a^3*cos(7*d*x + 7*c)/d + 1/1024*a^3*cos(5*d*x + 5*c)/d - 53
/1536*a^3*cos(3*d*x + 3*c)/d - 41/512*a^3*cos(d*x + c)/d + 3/5120*a^3*sin(1
0*d*x + 10*c)/d + 1/2048*a^3*sin(8*d*x + 8*c)/d - 25/3072*a^3*sin(6*d*x + 6
*c)/d - 5/256*a^3*sin(4*d*x + 4*c)/d + 11/512*a^3*sin(2*d*x + 2*c)/d
```

Mupad [**B**]
time = 12.12, size = 543, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + a*sin(c + d*x))^3,x)

[Out] (19*a^3*x)/256 - ((19*a^3*(c + d*x))/256 - (13*a^3*tan(c/2 + (d*x)/2)^3)/12
 - (32417*a^3*tan(c/2 + (d*x)/2)^5)/1920 + (466*a^3*tan(c/2 + (d*x)/2)^7)/1
5 - (2937*a^3*tan(c/2 + (d*x)/2)^9)/64 + (2937*a^3*tan(c/2 + (d*x)/2)^13)/6
4 - (466*a^3*tan(c/2 + (d*x)/2)^15)/15 + (32417*a^3*tan(c/2 + (d*x)/2)^17)/
1920 + (13*a^3*tan(c/2 + (d*x)/2)^19)/12 - (19*a^3*tan(c/2 + (d*x)/2)^21)/1
28 - a^3*((19*c)/256 + (19*d*x)/256 - 148/693) + tan(c/2 + (d*x)/2)^2*((209
*a^3*(c + d*x))/256 - a^3*((209*c)/256 + (209*d*x)/256 - 148/63)) + tan(c/2
 + (d*x)/2)^18*((1045*a^3*(c + d*x))/256 - a^3*((1045*c)/256 + (1045*d*x)/2
56 - 12)) + tan(c/2 + (d*x)/2)^4*((1045*a^3*(c + d*x))/256 - a^3*((1045*c)/
256 + (1045*d*x)/256 + 16/63)) + tan(c/2 + (d*x)/2)^14*((3135*a^3*(c + d*x)
)/128 - a^3*((3135*c)/128 + (3135*d*x)/128 - 16/3)) + tan(c/2 + (d*x)/2)^16
*((3135*a^3*(c + d*x))/256 - a^3*((3135*c)/256 + (3135*d*x)/256 - 44/3)) +
tan(c/2 + (d*x)/2)^8*((3135*a^3*(c + d*x))/128 - a^3*((3135*c)/128 + (3135*
d*x)/128 - 456/7)) + tan(c/2 + (d*x)/2)^6*((3135*a^3*(c + d*x))/256 - a^3*(
(3135*c)/256 + (3135*d*x)/256 - 144/7)) + tan(c/2 + (d*x)/2)^10*((4389*a^3*
(c + d*x))/128 - a^3*((4389*c)/128 + (4389*d*x)/128 + 24)) + tan(c/2 + (d*x
)/2)^12*((4389*a^3*(c + d*x))/128 - a^3*((4389*c)/128 + (4389*d*x)/128 - 36
8/3)) + (19*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^11)
```

3.608 $\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$\frac{33a^3x}{256}-\frac{33a^3\cos^7(c+dx)}{560d}+\frac{33a^3\cos(c+dx)\sin(c+dx)}{256d}+\frac{11a^3\cos^3(c+dx)\sin(c+dx)}{128d}+\frac{11a^3\cos^5(c+dx)\,\mathrm{s}}{160d}$$

```
[Out] 33/256*a^3*x-33/560*a^3*cos(d*x+c)^7/d+33/256*a^3*cos(d*x+c)*sin(d*x+c)/d+1
1/128*a^3*cos(d*x+c)^3*sin(d*x+c)/d+11/160*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/
30*a*cos(d*x+c)^7*(a+a*sin(d*x+c))^2/d-1/10*cos(d*x+c)^7*(a+a*sin(d*x+c))^3
/d-11/240*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{33a^3\cos^7(c+dx)}{560d}-\frac{11\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{240d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{11a^3\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{33a^3\sin(c+dx)\cos(c+dx)}{256d}+\frac{33a^3x}{256}-\frac{\cos^7(c+dx)(a\sin(c+dx)+a)^3}{10d}-\frac{a\cos^7(c+dx)(a\sin(c+dx)+a)^2}{30d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*x)/256 - (33*a^3*Cos[c + d*x]^7)/(560*d) + (33*a^3*Cos[c + d*x]*Sin
[c + d*x])/(256*d) + (11*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (11*a^3
*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) - (a*Cos[c + d*x]^7*(a + a*Sin[c + d*
x])^2)/(30*d) - (Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*Cos[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(240*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)\sin(c+dx)(a+a\sin(c+dx))^3\,dx &= -\frac{\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d}+\frac{3}{10}\int \cos^6(c+dx \\
&= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d}-\frac{\cos^7(c+dx)(a}{1} \\
&= -\frac{a\cos^7(c+dx)(a+a\sin(c+dx))^2}{30d}-\frac{\cos^7(c+dx)(a}{1} \\
&= -\frac{33a^3\cos^7(c+dx)}{560d}-\frac{a\cos^7(c+dx)(a+a\sin(c+dx)}{30d} \\
&= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{11a^3\cos^5(c+dx)\sin(c+dx)}{160d}- \\
&= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{11a^3\cos^3(c+dx)\sin(c+dx)}{128d}+ \\
&= -\frac{33a^3\cos^7(c+dx)}{560d}+\frac{33a^3\cos(c+dx)\sin(c+dx)}{256d}+ \\
&= \frac{33a^3x}{256}-\frac{33a^3\cos^7(c+dx)}{560d}+\frac{33a^3\cos(c+dx)\sin(c+}{256d}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 116, normalized size = 0.64

$$\frac{a^3(31500c+27720dx-31920\cos(c+dx)-16800\cos(3(c+dx))-3360\cos(5(c+dx))+600\cos(7(c+dx))+280\cos(9(c+dx))+10500\sin(2(c+dx))-5880\sin(4(c+dx))-3570\sin(6(c+dx))-525\sin(8(c+dx))+42\sin(10(c+dx)))}{215040d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(31500*c + 27720*d*x - 31920*Cos[c + d*x] - 16800*Cos[3*(c + d*x)] - 3
360*Cos[5*(c + d*x)] + 600*Cos[7*(c + d*x)] + 280*Cos[9*(c + d*x)] + 10500*
Sin[2*(c + d*x)] - 5880*Sin[4*(c + d*x)] - 3570*Sin[6*(c + d*x)] - 525*Sin[
8*(c + d*x)] + 42*Sin[10*(c + d*x)]))/(215040*d)
```

Maple [A]
time = 0.31, size = 198, normalized size = 1.09

method	result
risch	$\frac{33a^3x}{256} - \frac{19a^3\cos(dx+c)}{128d} + \frac{a^3\sin(10dx+10c)}{5120d} + \frac{a^3\cos(9dx+9c)}{768d} - \frac{5a^3\sin(8dx+8c)}{2048d} + \frac{5a^3\cos(7dx+7c)}{1792d} - \frac{17a^3}{}$
derivativedivides	$\frac{-\frac{a^3\left(\cos^7(dx+c)\right)}{7}+3a^3\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+3a}{}$
default	$\frac{-\frac{a^3\left(\cos^7(dx+c)\right)}{7}+3a^3\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^3*cos(d*x+c)^7+3*a^3*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*
x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+3*a^
3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a^3*(-1/10*sin(d*x+c)^
3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x
+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.29, size = 141, normalized size = 0.78

$$-\frac{30720\,a^3\cos\left(dx+c\right)^7 - 10240\,\left(7\,\cos\left(dx+c\right)^9 - 9\,\cos\left(dx+c\right)^7\right)a^3 - 21\,\left(32\,\sin\left(2\,dx+2\,c\right)^5 + 120\,dx + 120\,c + 5\,\sin\left(8\,dx+8\,c\right) - 40\,\sin\left(4\,dx+4\,c\right)\right)a^3 - 210\,\left(64\,\sin\left(2\,dx+2\,c\right)^3 + 120\,dx + 120\,c - 3\,\sin\left(8\,dx+8\,c\right) - 24\,\sin\left(4\,dx+4\,c\right)\right)a^3}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/215040*(30720*a^3*cos(d*x + c)^7 - 10240*(7*cos(d*x + c)^9 - 9*cos(d*x +
 c)^7)*a^3 - 21*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*
c) - 40*sin(4*d*x + 4*c))*a^3 - 210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*
c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^3)/d
```

Fricas [A]
time = 0.40, size = 111, normalized size = 0.61

$$\frac{8960\,a^3\cos\left(dx+c\right)^9 - 15360\,a^3\cos\left(dx+c\right)^7 + 3465\,a^3dx + 21\,\left(128\,a^3\cos\left(dx+c\right)^9 - 656\,a^3\cos\left(dx+c\right)^7 + 88\,a^3\cos\left(dx+c\right)^5 + 110\,a^3\cos\left(dx+c\right)^3 + 165\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/26880*(8960*a^3*cos(d*x + c)^9 - 15360*a^3*cos(d*x + c)^7 + 3465*a^3*d*x + 21*(128*a^3*cos(d*x + c)^9 - 656*a^3*cos(d*x + c)^7 + 88*a^3*cos(d*x + c)^5 + 110*a^3*cos(d*x + c)^3 + 165*a^3*cos(d*x + c))*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(170) = 340$.
time = 1.86, size = 542, normalized size = 2.99

$$\begin{cases} \frac{3a^3x\sin^{10}(c+dx)}{256} + \frac{15a^3x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{15a^3x\sin^8(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{15a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15a^3x\sin^4(c+dx)\cos^6(c+dx)}{128} + \frac{45a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15a^3x\sin^2(c+dx)\cos^8(c+dx)}{256} + \frac{15a^3x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3a^3x\cos^{10}(c+dx)}{256} + \frac{15a^3x\cos^8(c+dx)}{128} + \frac{3a^3\sin^9(c+dx)\cos(c+dx)}{256d} + \frac{7a^3\sin^7(c+dx)\cos^3(c+dx)}{128d} + \frac{15a^3\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{a^3\sin^5(c+dx)\cos^5(c+dx)}{10d} + \frac{55a^3\sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{7a^3\sin^3(c+dx)\cos^7(c+dx)}{128d} + \frac{73a^3\sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{3a^3\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{3a^3\sin(c+dx)\cos^9(c+dx)}{256d} - \frac{15a^3\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{2a^3\cos^9(c+dx)}{21d} - \frac{a^3\cos^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)^3\sin(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((3*a**3*x*sin(c + d*x)**10/256 + 15*a**3*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 15*a**3*x*sin(c + d*x)**8/128 + 15*a**3*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 15*a**3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*a**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**8/256 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*a**3*x*cos(c + d*x)**10/256 + 15*a**3*x*cos(c + d*x)**8/128 + 3*a**3*sin(c + d*x)**9*cos(c + d*x)/(256*d) + 7*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 15*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + a**3*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 55*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 7*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 3*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 15*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 2*a**3*cos(c + d*x)**9/(21*d) - a**3*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a*sin(c) + a)**3*sin(c)*cos(c)**6, True))

Giac [A]
time = 0.57, size = 174, normalized size = 0.96

$$\frac{33}{256}a^3x + \frac{a^3\cos(9dx+9c)}{768d} + \frac{5a^3\cos(7dx+7c)}{1792d} - \frac{a^3\cos(5dx+5c)}{64d} - \frac{5a^3\cos(3dx+3c)}{64d} - \frac{19a^3\cos(dx+c)}{128d} + \frac{a^3\sin(10dx+10c)}{5120d} - \frac{5a^3\sin(8dx+8c)}{2048d} - \frac{17a^3\sin(6dx+6c)}{1024d} - \frac{7a^3\sin(4dx+4c)}{256d} + \frac{25a^3\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 33/256*a^3*x + 1/768*a^3*cos(9*d*x + 9*c)/d + 5/1792*a^3*cos(7*d*x + 7*c)/d - 1/64*a^3*cos(5*d*x + 5*c)/d - 5/64*a^3*cos(3*d*x + 3*c)/d - 19/128*a^3*cos(d*x + c)/d + 1/5120*a^3*sin(10*d*x + 10*c)/d - 5/2048*a^3*sin(8*d*x + 8*c)/d - 17/1024*a^3*sin(6*d*x + 6*c)/d - 7/256*a^3*sin(4*d*x + 4*c)/d + 25/512*a^3*sin(2*d*x + 2*c)/d

Mupad [B]

time = 10.96, size = 572, normalized size = 3.16

$$\frac{33\,a^3\,x}{256}-\frac{\frac{333\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{32}-\frac{577\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160}-\frac{705\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{128}-\frac{2749\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{64}+\frac{2749\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{64}-\frac{333\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}+\frac{577\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{160}+\frac{705\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{128}-\frac{33\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{128}+a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}-\frac{10}{21}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}\,\left(10\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{165\,c}{128}+\frac{165\,d\,x}{128}-2\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(10\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{165\,c}{128}+\frac{165\,d\,x}{128}-\frac{58}{21}\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(120\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{495\,c}{32}+\frac{495\,d\,x}{32}-8\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(120\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{495\,c}{32}+\frac{495\,d\,x}{32}-\frac{344}{7}\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}\,\left(45\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{1485\,c}{256}+\frac{1485\,d\,x}{256}-18\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(45\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{1485\,c}{256}+\frac{1485\,d\,x}{256}-\frac{24}{7}\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(252\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{2079\,c}{64}+\frac{2079\,d\,x}{64}-60\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(210\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{3465\,c}{128}+\frac{3465\,d\,x}{128}-28\right)\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(210\,a^3\,\left(\frac{33\,c}{256}+\frac{33\,d\,x}{256}\right)-a^3\,\left(\frac{3465\,c}{128}+\frac{3465\,d\,x}{128}-72\right)\right)+\frac{33\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (33*a^3*x)/256 - ((333*a^3*tan(c/2 + (d*x)/2)^7)/32 - (577*a^3*tan(c/2 + (d
*x)/2)^5)/160 - (705*a^3*tan(c/2 + (d*x)/2)^3)/128 - (2749*a^3*tan(c/2 + (d
*x)/2)^9)/64 + (2749*a^3*tan(c/2 + (d*x)/2)^11)/64 - (333*a^3*tan(c/2 + (d*
x)/2)^13)/32 + (577*a^3*tan(c/2 + (d*x)/2)^15)/160 + (705*a^3*tan(c/2 + (d*
x)/2)^17)/128 - (33*a^3*tan(c/2 + (d*x)/2)^19)/128 + a^3*((33*c)/256 + (33*
d*x)/256) - a^3*((33*c)/256 + (33*d*x)/256 - 10/21) + tan(c/2 + (d*x)/2)^18
*(10*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((165*c)/128 + (165*d*x)/128 - 2
)) + tan(c/2 + (d*x)/2)^2*(10*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((165*c
)/128 + (165*d*x)/128 - 58/21)) + tan(c/2 + (d*x)/2)^14*(120*a^3*((33*c)/25
6 + (33*d*x)/256) - a^3*((495*c)/32 + (495*d*x)/32 - 8)) + tan(c/2 + (d*x)/
2)^6*(120*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((495*c)/32 + (495*d*x)/32
 - 344/7)) + tan(c/2 + (d*x)/2)^16*(45*a^3*((33*c)/256 + (33*d*x)/256) - a^3
*((1485*c)/256 + (1485*d*x)/256 - 18)) + tan(c/2 + (d*x)/2)^4*(45*a^3*((33*
c)/256 + (33*d*x)/256) - a^3*((1485*c)/256 + (1485*d*x)/256 - 24/7)) + tan(
c/2 + (d*x)/2)^10*(252*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((2079*c)/64 +
 (2079*d*x)/64 - 60)) + tan(c/2 + (d*x)/2)^8*(210*a^3*((33*c)/256 + (33*d*x
)/256) - a^3*((3465*c)/128 + (3465*d*x)/128 - 28)) + tan(c/2 + (d*x)/2)^12*
(210*a^3*((33*c)/256 + (33*d*x)/256) - a^3*((3465*c)/128 + (3465*d*x)/128 -
 72)) + (33*a^3*tan(c/2 + (d*x)/2))/128)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.609 $\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=185

$$\frac{125a^3x}{128}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^7(c+dx)}{7d}+125$$

```
[Out] 125/128*a^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)
^3/d+1/5*a^3*cos(d*x+c)^5/d-3/7*a^3*cos(d*x+c)^7/d+125/128*a^3*cos(d*x+c)*s
in(d*x+c)/d+125/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+25/48*a^3*cos(d*x+c)^5*si
n(d*x+c)/d-1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2952, 2715, 8, 2672, 308, 212, 2645, 30, 2648}

$$-\frac{3a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{25a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{125a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{125a^3\sin(c+dx)\cos(c+dx)}{128d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{125a^3x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (125*a^3*x)/128 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Cos[c + d*x])/d + (a
^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cos[c + d*x]
^7)/(7*d) + (125*a^3*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (125*a^3*Cos[c +
d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d)
- (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(3a^3\cos^6(c+dx)+a^3\cos^5(c+dx)\cot(c+dx)+3a\right.$$
$$= a^3\int \cos^5(c+dx)\cot(c+dx)\,dx + a^3\int \cos^6(c+dx)\,\mathrm{s}$$
$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{2d} - \frac{a^3\cos^7(c+dx)\sin(c+a}{8d}$$
$$= -\frac{3a^3\cos^7(c+dx)}{7d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{8d} + \frac{2}{}$$
$$= \frac{a^3\cos(c+dx)}{d} + \frac{a^3\cos^3(c+dx)}{3d} + \frac{a^3\cos^5(c+dx)}{5d} -$$
$$= \frac{15a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} + \frac{a}{}$$
$$= \frac{125a^3x}{128} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} +$$

Mathematica [A]

time = 0.45, size = 122, normalized size = 0.66

$$\frac{a^3\left(105000c+105000dx+122640\cos(c+dx)+560\cos(3(c+dx))-3696\cos(5(c+dx))-720\cos(7(c+dx))-107520\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+107520\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+77280\sin(2(c+dx))+14280\sin(4(c+dx))+1120\sin(6(c+dx))-105\sin(8(c+dx))\right)}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(105000*c + 105000*d*x + 122640*Cos[c + d*x] + 560*Cos[3*(c + d*x)] -
3696*Cos[5*(c + d*x)] - 720*Cos[7*(c + d*x)] - 107520*Log[Cos[(c + d*x)/2]]
 + 107520*Log[Sin[(c + d*x)/2]] + 77280*Sin[2*(c + d*x)] + 14280*Sin[4*(c +
 d*x)] + 1120*Sin[6*(c + d*x)] - 105*Sin[8*(c + d*x)]))/(107520*d)
```

Maple [A]

time = 0.24, size = 177, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{}$
default	$\frac{a^3\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{}$

risch	$\frac{125a^3x}{128}+\frac{73a^3e^{i(dx+c)}}{128d}+\frac{73a^3e^{-i(dx+c)}}{128d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^3\sin(8dx+8c)}{1024d}-\frac{3a^3\cos}{}$
norman	$\frac{\frac{125a^3x}{128}+\frac{232a^3}{105d}-\frac{259a^3\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{1861a^3\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}-\frac{1817a^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{24a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{1817a^3(t}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)-3/7*a^3*cos(d*x+c)^7+a^3*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c))

Maxima [A]
time = 0.29, size = 171, normalized size = 0.92

$$-\frac{46080\,a^3\cos\left(dx+c\right)^7-3584\left(6\cos\left(dx+c\right)^5+10\cos\left(dx+c\right)^3+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-35\left(64\sin\left(2dx+2c\right)^3+120\,dx+120\,c-3\sin\left(8dx+8c\right)-24\sin\left(4dx+4c\right)\right)a^3+1680\left(4\sin\left(2dx+2c\right)^3-60\,dx-60\,c-9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a^3}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/107520*(46080*a^3*cos(d*x + c)^7 - 3584*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 35*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a^3 + 1680*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3)/d

Fricas [A]
time = 0.42, size = 154, normalized size = 0.83

$$-\frac{5760\,a^3\cos\left(dx+c\right)^7-2688\,a^3\cos\left(dx+c\right)^5-4480\,a^3\cos\left(dx+c\right)^3-13125\,a^3dx-13440\,a^3\cos\left(dx+c\right)+6720\,a^3\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-6720\,a^3\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+35\left(48\,a^3\cos\left(dx+c\right)^7-200\,a^3\cos\left(dx+c\right)^5-250\,a^3\cos\left(dx+c\right)^3-375\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/13440*(5760*a^3*cos(d*x + c)^7 - 2688*a^3*cos(d*x + c)^5 - 4480*a^3*cos(d*x + c)^3 - 13125*a^3*d*x - 13440*a^3*cos(d*x + c) + 6720*a^3*log(1/2*cos(d*x + c) + 1/2) - 6720*a^3*log(-1/2*cos(d*x + c) + 1/2) + 35*(48*a^3*cos(d*x + c)^7 - 200*a^3*cos(d*x + c)^5 - 250*a^3*cos(d*x + c)^3 - 375*a^3*cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.54, size = 277, normalized size = 1.50

$$\frac{13125\,(dx+c)a^3 + 13440\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - \frac{2\left(27195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+65135\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-161280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+63595\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-286720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+133175\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-519680\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-133175\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-544768\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-63595\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-254464\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-65135\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-118784\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-27195\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14848\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^8}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/13440*(13125*(d*x + c)*a^3 + 13440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2
*(27195*a^3*tan(1/2*d*x + 1/2*c)^15 + 65135*a^3*tan(1/2*d*x + 1/2*c)^13 - 1
61280*a^3*tan(1/2*d*x + 1/2*c)^12 + 63595*a^3*tan(1/2*d*x + 1/2*c)^11 - 286
720*a^3*tan(1/2*d*x + 1/2*c)^10 + 133175*a^3*tan(1/2*d*x + 1/2*c)^9 - 51968
0*a^3*tan(1/2*d*x + 1/2*c)^8 - 133175*a^3*tan(1/2*d*x + 1/2*c)^7 - 544768*a
^3*tan(1/2*d*x + 1/2*c)^6 - 63595*a^3*tan(1/2*d*x + 1/2*c)^5 - 254464*a^3*t
an(1/2*d*x + 1/2*c)^4 - 65135*a^3*tan(1/2*d*x + 1/2*c)^3 - 118784*a^3*tan(1
/2*d*x + 1/2*c)^2 - 27195*a^3*tan(1/2*d*x + 1/2*c) - 14848*a^3)/(tan(1/2*d*
x + 1/2*c)^2 + 1)^8)/d
```

Mupad [B]
time = 10.85, size = 429, normalized size = 2.32

$$\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{125\,a^3\,\mathrm{atan}\left(\frac{15625\,a^6}{4096\left(\frac{125\,a^6}{32}-\frac{15625\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4096}\right)}+\frac{125\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32\left(\frac{125\,a^6}{32}-\frac{15625\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4096}\right)}\right)}{64\,d}+\frac{-\frac{259\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}-\frac{1861\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}+24\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-\frac{1817\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{3}-\frac{3805\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{232\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{3}+\frac{3805\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{192}+\frac{1216\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{15}+\frac{1817\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}+\frac{568\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{15}+\frac{1861\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{192}+\frac{1856\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{105}+\frac{259\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}+\frac{232\,a^3}{105}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (125*a^3*atan((15625*a^6)/(4096*((125*a^6
)/32 - (15625*a^6*tan(c/2 + (d*x)/2))/4096)) + (125*a^6*tan(c/2 + (d*x)/2))
/(32*((125*a^6)/32 - (15625*a^6*tan(c/2 + (d*x)/2))/4096))))/(64*d) + ((185
6*a^3*tan(c/2 + (d*x)/2)^2)/105 + (1861*a^3*tan(c/2 + (d*x)/2)^3)/192 + (56
8*a^3*tan(c/2 + (d*x)/2)^4)/15 + (1817*a^3*tan(c/2 + (d*x)/2)^5)/192 + (121
6*a^3*tan(c/2 + (d*x)/2)^6)/15 + (3805*a^3*tan(c/2 + (d*x)/2)^7)/192 + (232
*a^3*tan(c/2 + (d*x)/2)^8)/3 - (3805*a^3*tan(c/2 + (d*x)/2)^9)/192 + (128*a
^3*tan(c/2 + (d*x)/2)^10)/3 - (1817*a^3*tan(c/2 + (d*x)/2)^11)/192 + 24*a^3
*tan(c/2 + (d*x)/2)^12 - (1861*a^3*tan(c/2 + (d*x)/2)^13)/192 - (259*a^3*ta
n(c/2 + (d*x)/2)^15)/64 + (232*a^3)/105 + (259*a^3*tan(c/2 + (d*x)/2))/64)/
(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2
)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*
x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.610 $\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=173

$$-\frac{15a^3x}{16}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^5(c+dx)}{5d}-\frac{a^3\cos^7(c+dx)}{7d}-a$$

```
[Out] -15/16*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d+a^3*cos(d*x+c)^
3/d+3/5*a^3*cos(d*x+c)^5/d-1/7*a^3*cos(d*x+c)^7/d-a^3*cot(d*x+c)/d+15/16*a^
3*cos(d*x+c)*sin(d*x+c)/d-11/8*a^3*cos(d*x+c)*sin(d*x+c)^3/d+1/2*a^3*cos(d*
x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.19, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 2718, 2715, 2713}

$$-\frac{a^3\cos^7(c+dx)}{7d}+\frac{3a^3\cos^5(c+dx)}{5d}+\frac{a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{2d}-\frac{11a^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{15a^3\sin(c+dx)\cos(c+dx)}{16d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{15a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*a^3*x)/16 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d +
 (a^3*Cos[c + d*x]^3)/d + (3*a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]^
7)/(7*d) - (a^3*Cot[c + d*x])/d + (15*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d)
 - (11*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(8*d) + (a^3*Cos[c + d*x]*Sin[c + d
*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(3a^9\csc(c+dx)+a^9\csc^2(c+dx)-8a^9\sin(c+dx)\right.}{}$$

$$= a^3\int \csc^2(c+dx)\,dx - a^3\int \sin^7(c+dx)\,dx + \left(3a^3\right)$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\,\mathrm{c}}{}$$

$$= -3a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d}$$

$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{a^3\cos}{}$$

$$= -\frac{15a^3x}{16} - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx}{d}$$

Mathematica [A]
time = 1.58, size = 168, normalized size = 0.97

$$\frac{(a+a\sin(c+dx))^3\left(-2100(c+dx)+9065\cos(c+dx)+875\cos(3(c+dx))+49\cos(5(c+dx))-5\cos(7(c+dx))-1120\cot\left(\frac{1}{2}(c+dx)\right)-6720\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6720\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+455\sin(2(c+dx))+245\sin(4(c+dx))+35\sin(6(c+dx))+1120\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2240d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-2100*(c + d*x) + 9065*Cos[c + d*x] + 875*Cos[3*(c
 + d*x)] + 49*Cos[5*(c + d*x)] - 5*Cos[7*(c + d*x)] - 1120*Cot[(c + d*x)/2]
 - 6720*Log[Cos[(c + d*x)/2]] + 6720*Log[Sin[(c + d*x)/2]] + 455*Sin[2*(c +
 d*x)] + 245*Sin[4*(c + d*x)] + 35*Sin[6*(c + d*x)] + 1120*Tan[(c + d*x)/2]
))/(2240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]

time = 0.21, size = 180, normalized size = 1.04

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+3a^3\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+\right.}{3}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+3a^3\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+\right.}{3}\right.}{}$
risch	$-\frac{15a^3x}{16}-\frac{13ia^3e^{2i(dx+c)}}{128d}+\frac{259a^3e^{i(dx+c)}}{128d}+\frac{259a^3e^{-i(dx+c)}}{128d}+\frac{13ia^3e^{-2i(dx+c)}}{128d}-\frac{2ia^3}{d\left(e^{2i(dx+c)}-1\right)}+\frac{3a^3\ln(e}{}$
norman	$\frac{\frac{16a^3\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{136a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{176a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^3}{2d}-\frac{9a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{21a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{29a^3\left(\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/sin(d*x+c)*cos(d*x+c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*co
s(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+3*a^3*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c
)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(
d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)-1/7*a^3*cos(d*x+c)^7)
```

Maxima [A]

time = 0.50, size = 186, normalized size = 1.08

$$-\frac{320\,a^3\cos\left(dx+c\right)^7-224\,\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3+35\,\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^3+280\,\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+25\,\tan(dx+c)^2+8}{\tan(dx+c)^5+2\,\tan(dx+c)^3+\tan(dx+c)}\right)a^3}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2240*(320*a^3*cos(d*x + c)^7 - 224*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
```

```
^3 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a^3 + 280*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
 c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A]
time = 0.42, size = 173, normalized size = 1.00

$$-\frac{280\,a^3\cos\left(dx+c\right)^7-70\,a^3\cos\left(dx+c\right)^5-175\,a^3\cos\left(dx+c\right)^3+840\,a^3\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-840\,a^3\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+525\,a^3\cos\left(dx+c\right)+\left(80\,a^3\cos\left(dx+c\right)^7-336\,a^3\cos\left(dx+c\right)^5-560\,a^3\cos\left(dx+c\right)^3+525\,a^3dx-1680\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{560\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/560*(280*a^3*cos(d*x + c)^7 - 70*a^3*cos(d*x + c)^5 - 175*a^3*cos(d*x +
c)^3 + 840*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 840*a^3*log(-1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) + 525*a^3*cos(d*x + c) + (80*a^3*cos(d*x +
 c)^7 - 336*a^3*cos(d*x + c)^5 - 560*a^3*cos(d*x + c)^3 + 525*a^3*d*x - 168
0*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.53, size = 290, normalized size = 1.68

$$-\frac{525\,\left(dx+c\right)a^3-1680\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{280\,\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-4480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-980\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-20160\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-38080\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-49280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-945\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-32256\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+980\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12992\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2496\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(525*(d*x + c)*a^3 - 1680*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 280*a
^3*tan(1/2*d*x + 1/2*c) + 280*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*
x + 1/2*c) + 2*(525*a^3*tan(1/2*d*x + 1/2*c)^13 - 4480*a^3*tan(1/2*d*x + 1/
2*c)^12 - 980*a^3*tan(1/2*d*x + 1/2*c)^11 - 20160*a^3*tan(1/2*d*x + 1/2*c)^
10 + 945*a^3*tan(1/2*d*x + 1/2*c)^9 - 38080*a^3*tan(1/2*d*x + 1/2*c)^8 - 49
280*a^3*tan(1/2*d*x + 1/2*c)^6 - 945*a^3*tan(1/2*d*x + 1/2*c)^5 - 32256*a^3
*tan(1/2*d*x + 1/2*c)^4 + 980*a^3*tan(1/2*d*x + 1/2*c)^3 - 12992*a^3*tan(1/
2*d*x + 1/2*c)^2 - 525*a^3*tan(1/2*d*x + 1/2*c) - 2496*a^3)/(tan(1/2*d*x +
1/2*c)^2 + 1)^7)/d
```

Mupad [B]

time = 9.38, size = 429, normalized size = 2.48

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{15\,a^3\,\mathrm{atan}\left(\frac{225\,a^6}{64\,\left(\frac{45\,a^6}{4}+\frac{225\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}-\frac{45\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{45\,a^6}{4}+\frac{225\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{\frac{19\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}}{4}-32\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}-144\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+\frac{111\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{4}-272\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+35\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-352\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\frac{113\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{4}-\frac{1152\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{5}+28\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-\frac{464\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{5}+\frac{13\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}-\frac{624\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+a^3}{d\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}+14\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}+42\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+70\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+70\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+42\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+14\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d + (15*a^3*atan((225*a^6)/(64*((45*a^6)/4
+ (225*a^6*tan(c/2 + (d*x)/2))/64)) - (45*a^6*tan(c/2 + (d*x)/2))/(4*((45*a
^6)/4 + (225*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + (a^3*tan(c/2 + (d*x)/2)
)/(2*d) - ((13*a^3*tan(c/2 + (d*x)/2)^2)/4 - (464*a^3*tan(c/2 + (d*x)/2)^3)
/5 + 28*a^3*tan(c/2 + (d*x)/2)^4 - (1152*a^3*tan(c/2 + (d*x)/2)^5)/5 + (113
*a^3*tan(c/2 + (d*x)/2)^6)/4 - 352*a^3*tan(c/2 + (d*x)/2)^7 + 35*a^3*tan(c/
2 + (d*x)/2)^8 - 272*a^3*tan(c/2 + (d*x)/2)^9 + (111*a^3*tan(c/2 + (d*x)/2)
^10)/4 - 144*a^3*tan(c/2 + (d*x)/2)^11 - 32*a^3*tan(c/2 + (d*x)/2)^13 + (19
*a^3*tan(c/2 + (d*x)/2)^14)/4 + a^3 - (624*a^3*tan(c/2 + (d*x)/2))/35)/(d*(
2*tan(c/2 + (d*x)/2) + 14*tan(c/2 + (d*x)/2)^3 + 42*tan(c/2 + (d*x)/2)^5 +
70*tan(c/2 + (d*x)/2)^7 + 70*tan(c/2 + (d*x)/2)^9 + 42*tan(c/2 + (d*x)/2)^1
1 + 14*tan(c/2 + (d*x)/2)^13 + 2*tan(c/2 + (d*x)/2)^15))
```

3.611 $\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=181

$$-\frac{85a^3x}{16}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3\cos(c+dx)}{d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cot(c+dx)}{d}-$$

```
[Out] -85/16*a^3*x-1/2*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d+2/3*a^3*cos(d*x
+c)^3/d+3/5*a^3*cos(d*x+c)^5/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*
x+c)/d-43/16*a^3*cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d
+1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.20, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2951, 3852, 8, 3853, 3855, 2718, 2715, 2713}

$$\frac{3a^3\cos^5(c+dx)}{5d}+\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{5a^3\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{43a^3\sin(c+dx)\cos(c+dx)}{16d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{85a^3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-85*a^3*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/(2*d) + (a^3*Cos[c + d*x])/d +
 (2*a^3*Cos[c + d*x]^3)/(3*d) + (3*a^3*Cos[c + d*x]^5)/(5*d) - (3*a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) - (43*a^3*Cos[c + d*x]*S
in[c + d*x])/(16*d) + (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^3*Cos
[c + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int (-8a^9 + 3a^9\csc^2(c+dx) + a^9\csc^3(c+dx) - 6a^9 \mathrm{si}}{}$$

$$= -8a^3x + a^3\int \csc^3(c+dx)\,dx - a^3\int \sin^6(c+dx)\,dx$$

$$= -8a^3x + \frac{6a^3\cos(c+dx)}{d} - \frac{a^3\cot(c+dx)\csc(c+dx}{2d}$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} +$$

$$= -5a^3x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d} +$$

$$= -\frac{85a^3x}{16} - \frac{a^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{a^3\cos(c+dx)}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 664 vs. 2(181) = 362.
time = 6.26, size = 664, normalized size = 3.67

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-85*(c + d*x)*(a + a*Sin[c + d*x])^3)/(16*d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^6) + (15*Cos[c + d*x]*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^6) + (17*Cos[3*(c + d*x)]*(a + a*Sin[c + d*x])^3)/(48
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*Cos[5*(c + d*x)]*(a + a*Si
n[c + d*x])^3)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) - (3*Cot[(c +
 d*x)/2]*(a + a*Sin[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^6) - (Csc[(c + d*x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6) - (Log[Cos[(c + d*x)/2]]*(a + a*Sin[c + d*x])^3)/(2*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Log[Sin[(c + d*x)/2]]*(a + a*Si
n[c + d*x])^3)/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (Sec[(c + d*
x)/2]^2*(a + a*Sin[c + d*x])^3)/(8*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
6) - (81*(a + a*Sin[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6) - (3*(a + a*Sin[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + ((a + a*Sin[c + d*x])^3*Sin[6*(c
+ d*x)])/(192*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6) + (3*(a + a*Sin[c
+ d*x])^3*Tan[(c + d*x)/2])/(2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.23, size = 236, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-
5/2*cos(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/sin(d*x+c)*cos(d*x+
c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15
/8*c)+3*a^3*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot
(d*x+c)))+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+
c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.50, size = 239, normalized size = 1.32

$$\frac{96\left(6\cos\left(dx+c\right)^{5}+10\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}-80\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}-5\left(4\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a^{3}-360\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}+25\tan(dx+c)^{2}+8}{\tan(dx+c)^{5}+2\tan(dx+c)^{3}+\tan(dx+c)}\right)a^{3}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*(96*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(
cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 80*(4*cos(d*x + c)^3 -
6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
 + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x -
60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 360*(15*d*x + 15*c +
 (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x +
c)^3 + tan(d*x + c)))*a^3)/d
```

Fricas [A]
time = 0.43, size = 212, normalized size = 1.17

$$\frac{144\,a^{3}\cos\left(dx+c\right)^{7}+16\,a^{3}\cos\left(dx+c\right)^{5}-1275\,a^{3}dx\cos\left(dx+c\right)^{2}+80\,a^{3}\cos\left(dx+c\right)^{3}+1275\,a^{3}dx-120\,a^{3}\cos\left(dx+c\right)-60\left(a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+60\left(a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\left(8\,a^{3}\cos\left(dx+c\right)^{7}-34\,a^{3}\cos\left(dx+c\right)^{5}-85\,a^{3}\cos\left(dx+c\right)^{3}+255\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(144*a^3*cos(d*x + c)^7 + 16*a^3*cos(d*x + c)^5 - 1275*a^3*d*x*cos(d*
x + c)^2 + 80*a^3*cos(d*x + c)^3 + 1275*a^3*d*x - 120*a^3*cos(d*x + c) - 60
*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 60*(a^3*cos(d*x +
 c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) + 5*(8*a^3*cos(d*x + c)^7 - 34*a^
3*cos(d*x + c)^5 - 85*a^3*cos(d*x + c)^3 + 255*a^3*cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.56, size = 306, normalized size = 1.69

$$\frac{30\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1275\,(dx+c)a^3+120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{30\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{2\left(645\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1440\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1735\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+3360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5440\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1735\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1824\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-645\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+544\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(30*a^3*tan(1/2*d*x + 1/2*c)^2 - 1275*(d*x + c)*a^3 + 120*a^3*log(abs
(tan(1/2*d*x + 1/2*c))) + 360*a^3*tan(1/2*d*x + 1/2*c) - 30*(6*a^3*tan(1/2*
d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2
+ 2*(645*a^3*tan(1/2*d*x + 1/2*c)^11 + 1440*a^3*tan(1/2*d*x + 1/2*c)^10 + 1
735*a^3*tan(1/2*d*x + 1/2*c)^9 + 3360*a^3*tan(1/2*d*x + 1/2*c)^8 + 450*a^3*
tan(1/2*d*x + 1/2*c)^7 + 5440*a^3*tan(1/2*d*x + 1/2*c)^6 - 450*a^3*tan(1/2*
d*x + 1/2*c)^5 + 4800*a^3*tan(1/2*d*x + 1/2*c)^4 - 1735*a^3*tan(1/2*d*x + 1
/2*c)^3 + 1824*a^3*tan(1/2*d*x + 1/2*c)^2 - 645*a^3*tan(1/2*d*x + 1/2*c) +
544*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 8.92, size = 438, normalized size = 2.42

$$\frac{\frac{31\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{2}+\frac{95\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{2}+\frac{131\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6}+109\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-75\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{1043\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{6}-135\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+150\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{887\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6}+\frac{533\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{10}-\frac{115\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{227\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a^3}{2}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+60\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+60\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{85\,a^3\,\mathrm{atan}\left(\frac{7225\,a^6}{64\left(\frac{85\,a^6}{8}+\frac{7225\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}-\frac{85\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(\frac{85\,a^6}{8}+\frac{7225\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}\right)}\right)}{8\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] ((227*a^3*tan(c/2 + (d*x)/2)^2)/15 - (115*a^3*tan(c/2 + (d*x)/2)^3)/2 + (53
3*a^3*tan(c/2 + (d*x)/2)^4)/10 - (887*a^3*tan(c/2 + (d*x)/2)^5)/6 + 150*a^3
*tan(c/2 + (d*x)/2)^6 - 135*a^3*tan(c/2 + (d*x)/2)^7 + (1043*a^3*tan(c/2 +
(d*x)/2)^8)/6 - 75*a^3*tan(c/2 + (d*x)/2)^9 + 109*a^3*tan(c/2 + (d*x)/2)^10
 + (131*a^3*tan(c/2 + (d*x)/2)^11)/6 + (95*a^3*tan(c/2 + (d*x)/2)^12)/2 + (
31*a^3*tan(c/2 + (d*x)/2)^13)/2 - a^3/2 - 6*a^3*tan(c/2 + (d*x)/2))/(d*(4*t
an(c/2 + (d*x)/2)^2 + 24*tan(c/2 + (d*x)/2)^4 + 60*tan(c/2 + (d*x)/2)^6 + 8
0*tan(c/2 + (d*x)/2)^8 + 60*tan(c/2 + (d*x)/2)^10 + 24*tan(c/2 + (d*x)/2)^1
2 + 4*tan(c/2 + (d*x)/2)^14)) + (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*log
(tan(c/2 + (d*x)/2)))/(2*d) + (85*a^3*atan((7225*a^6)/(64*((85*a^6)/8 + (72
25*a^6*tan(c/2 + (d*x)/2))/64)) - (85*a^6*tan(c/2 + (d*x)/2))/(8*((85*a^6)/
8 + (7225*a^6*tan(c/2 + (d*x)/2))/64))))/(8*d) + (3*a^3*tan(c/2 + (d*x)/2))
/(2*d)
```

3.612 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=176

$$-\frac{25a^3x}{8}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cos(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{a^3\cot(c+dx)}{d}-a$$

```
[Out] -25/8*a^3*x+13/2*a^3*arctanh(cos(d*x+c))/d-5*a^3*cos(d*x+c)/d-2/3*a^3*cos(d
*x+c)^3/d+1/5*a^3*cos(d*x+c)^5/d-a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3/
2*a^3*cot(d*x+c)*csc(d*x+c)/d-23/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/4*a^3*cos(
d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2951, 3855, 3853, 3852, 2718, 2715, 8, 2713}

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{2a^3\cos^3(c+dx)}{3d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{23a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}-\frac{25a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-25*a^3*x)/8 + (13*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cos[c + d*x])
/d - (2*a^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*Cot[c
 + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])
/(2*d) - (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (3*a^3*Cos[c + d*x]*Sin
[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(-6a^9 - 8a^9\csc(c+dx) + 3a^9\csc^3(c+dx) + a^9\csc\right.}{}$$

$$= -6a^3x + a^3\int \csc^4(c+dx)\,dx - a^3\int \sin^5(c+dx)\,dx$$

$$= -6a^3x + \frac{8a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{6a^3\cos(c+dx)}{d}$$

$$= -2a^3x + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a^3\cos(c+dx)}{d}$$

$$= -\frac{25a^3x}{8} + \frac{13a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a^3\cos(c+d}{d}$$

Mathematica [A]
time = 1.01, size = 219, normalized size = 1.24

$$\frac{a^3(1+\sin(c+dx))^3\left(-1500(c+dx)-2580\cos(c+dx)-50\cos(3(c+dx))+6\cos(5(c+dx))-160\cot\left(\frac{1}{2}(c+dx)\right)-180\csc^2\left(\frac{1}{2}(c+dx)\right)+3120\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3120\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+180\sec^2\left(\frac{1}{2}(c+dx)\right)+160\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-10\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-600\sin(2(c+dx))-45\sin(4(c+dx))+160\tan\left(\frac{1}{2}(c+dx)\right)\right)}{480d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-1500*(c + d*x) - 2580*Cos[c + d*x] - 50*Cos[3*(
c + d*x)] + 6*Cos[5*(c + d*x)] - 160*Cot[(c + d*x)/2] - 180*Csc[(c + d*x)/2
]^2 + 3120*Log[Cos[(c + d*x)/2]] - 3120*Log[Sin[(c + d*x)/2]] + 180*Sec[(c
+ d*x)/2]^2 + 160*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 10*Csc[(c + d*x)/2]^4
*Sin[c + d*x] - 600*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 160*Tan[(c + d
*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.25, size = 272, normalized size = 1.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+3*a
^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*co
s(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/sin(d*x+c)*cos(d*x+c)^7-(
cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+
a^3*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))
))
```

Maxima [A]
time = 0.50, size = 246, normalized size = 1.40

$$\frac{4\left(6\cos\left(dx+c\right)^5+10\cos\left(dx+c\right)^3+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-30\left(4\cos\left(dx+c\right)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-45\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^3+20\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^3}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*(4*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(c
os(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 30*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*a^3 - 45*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x +
c)))*a^3 + 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/
(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3)/d
```

Fricas [A]
time = 0.41, size = 231, normalized size = 1.31

$$\frac{90\,a^{3}\cos\left(dx+c\right)^{7}+75\,a^{3}\cos\left(dx+c\right)^{5}-500\,a^{3}\cos\left(dx+c\right)^{3}+375\,a^{3}\cos\left(dx+c\right)+390\,\left(a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-390\,\left(a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\left(24\,a^{3}\cos\left(dx+c\right)^{7}-104\,a^{3}\cos\left(dx+c\right)^{5}-375\,a^{3}dx\cos\left(dx+c\right)^{2}-520\,a^{3}\cos\left(dx+c\right)^{3}+375\,a^{3}dx+780\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(90*a^3*cos(d*x + c)^7 + 75*a^3*cos(d*x + c)^5 - 500*a^3*cos(d*x + c)
^3 + 375*a^3*cos(d*x + c) + 390*(a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 390*(a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + (24*a^3*cos(d*x + c)^7 - 104*a^3*cos(d*x + c)^5
- 375*a^3*d*x*cos(d*x + c)^2 - 520*a^3*cos(d*x + c)^3 + 375*a^3*d*x + 780*a
^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 292, normalized size = 1.66

$$\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+45\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-375\,\left(dx+c\right)a^{3}-780\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+45\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{5\,\left(286\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}+\frac{2\,\left(345\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-720\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+330\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2880\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-3680\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-330\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2560\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-345\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-656\,a^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(5*a^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 - 375*(
d*x + c)*a^3 - 780*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 45*a^3*tan(1/2*d*x
+ 1/2*c) + 5*(286*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c)^2
 - 9*a^3*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3 + 2*(345*a^3*ta
n(1/2*d*x + 1/2*c)^9 - 720*a^3*tan(1/2*d*x + 1/2*c)^8 + 330*a^3*tan(1/2*d*x
 + 1/2*c)^7 - 2880*a^3*tan(1/2*d*x + 1/2*c)^6 - 3680*a^3*tan(1/2*d*x + 1/2*
c)^4 - 330*a^3*tan(1/2*d*x + 1/2*c)^3 - 2560*a^3*tan(1/2*d*x + 1/2*c)^2 - 3
45*a^3*tan(1/2*d*x + 1/2*c) - 656*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 8.99, size = 429, normalized size = 2.44

$$\frac{3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}-\frac{13\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-43\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+99\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}-\frac{86\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{3}+399\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\frac{95\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{3}+\frac{1562\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{3}+\frac{232\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{3}+\frac{1114\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{3}+\frac{193\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{3}+\frac{1537\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{15}+\frac{14\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{3}+3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{3}}{d\,\left(8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}+40\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+80\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+80\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+40\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}-\frac{25\,a^3\,\mathrm{atan}\left(\frac{625\,a^6}{16\,\left(\frac{325\,a^6}{4}-\frac{625\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{325\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{325\,a^6}{4}-\frac{625\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)

[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (13*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) - ((14*a^3*tan(c/2 + (d*x)/2)^2)/3 + (1537*a^3*tan(c/2 + (d*x)/2)^3)/15 + (193*a^3*tan(c/2 + (d*x)/2)^4)/3 + (1114*a^3*tan(c/2 + (d*x)/2)^5)/3 + (232*a^3*tan(c/2 + (d*x)/2)^6)/3 + (1562*a^3*tan(c/2 + (d*x)/2)^7)/3 + (95*a^3*tan(c/2 + (d*x)/2)^8)/3 + 399*a^3*tan(c/2 + (d*x)/2)^9 - (86*a^3*tan(c/2 + (d*x)/2)^10)/3 + 99*a^3*tan(c/2 + (d*x)/2)^11 - 43*a^3*tan(c/2 + (d*x)/2)^12 + a^3/3 + 3*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^5 + 80*tan(c/2 + (d*x)/2)^7 + 80*tan(c/2 + (d*x)/2)^9 + 40*tan(c/2 + (d*x)/2)^11 + 8*tan(c/2 + (d*x)/2)^13)) - (25*a^3*atan((625*a^6)/(16*((325*a^6)/4 - (625*a^6*tan(c/2 + (d*x)/2))/16)) + (325*a^6*tan(c/2 + (d*x)/2))/(4*((325*a^6)/4 - (625*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) + (3*a^3*tan(c/2 + (d*x)/2))/(8*d)

3.613 $\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=178

$$\frac{45a^3x}{8}+\frac{45a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}-3a$$

```
[Out] 45/8*a^3*x+45/8*a^3*arctanh(cos(d*x+c))/d-5*a^3*cos(d*x+c)/d-a^3*cos(d*x+c)
^3/d+5*a^3*cot(d*x+c)/d-a^3*cot(d*x+c)^3/d-3/8*a^3*cot(d*x+c)*csc(d*x+c)/d-
1/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d+3/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*c
os(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2713, 2715}

$$-\frac{a^3\cos^3(c+dx)}{d}-\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{d}+\frac{5a^3\cot(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{45a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a^3\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{8d}+\frac{45a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (45*a^3*x)/8 + (45*a^3*ArcTanh[Cos[c + d*x]])/(8*d) - (5*a^3*Cos[c + d*x])/
d - (a^3*Cos[c + d*x]^3)/d + (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/
d - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d*x
]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]*Si
n[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(6a^9 - 6a^9\csc(c+dx) - 8a^9\csc^2(c+dx) + 3a^9\csc^4(c\right.}{}$$

$$= 6a^3x + a^3\int \csc^5(c+dx)\,dx - a^3\int \sin^4(c+dx)\,dx + ($$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{8a^3\cos(c+dx)}{d} - \frac{a^3}{}$$

$$= 6a^3x + \frac{6a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{5a^3\cos(c+dx)}{d} - \frac{a^3}{}$$

$$= \frac{45a^3x}{8} + \frac{45a^3\tanh^{-1}(\cos(c+dx))}{8d} - \frac{5a^3\cos(c+dx)}{d} -$$

Mathematica [A]
time = 0.60, size = 235, normalized size = 1.32

$$\frac{a^3(1+\sin(c+dx))^3\left(360(c+dx)-368\cos(c+dx)-16\cos(3(c+dx))+192\cot\left(\frac{1}{2}(c+dx)\right)-6\csc^2\left(\frac{1}{2}(c+dx)\right)-\csc^4\left(\frac{1}{2}(c+dx)\right)+360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+6\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+64\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-4\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+16\sin(2(c+dx))-2\sin(4(c+dx))-192\tan\left(\frac{1}{2}(c+dx)\right)\right)}{64d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(360*(c + d*x) - 368*Cos[c + d*x] - 16*Cos[3*(c +
 d*x)] + 192*Cot[(c + d*x)/2] - 6*Csc[(c + d*x)/2]^2 - Csc[(c + d*x)/2]^4 +
 360*Log[Cos[(c + d*x)/2]] - 360*Log[Sin[(c + d*x)/2]] + 6*Sec[(c + d*x)/2]
^2 + Sec[(c + d*x)/2]^4 + 64*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 4*Csc[(c +
 d*x)/2]^4*Sin[c + d*x] + 16*Sin[2*(c + d*x)] - 2*Sin[4*(c + d*x)] - 192*Ta
n[(c + d*x)/2]))/(64*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.24, size = 312, normalized size = 1.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*
cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)
))+3*a^3*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+3*a
^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*co
s(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-1/sin(d*x+c)*cos(d*x+c)^7-(co
s(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.52, size = 268, normalized size = 1.51

$$-\frac{4\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^3+2\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^3-8\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^3+a^3\left(\frac{2\left(9\cos(dx+c)^3-7\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-16\cos(dx+c)+15\log(\cos(dx+c)+1)-15\log(\cos(dx+c)-1)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 + 2*(15*
d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
2*tan(d*x + c)^3 + tan(d*x + c)))*a^3 - 8*(15*d*x + 15*c + (15*tan(d*x + c)
^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^3 + a^3*(2
*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1
) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))
/d
```

Fricas [A]
time = 0.43, size = 258, normalized size = 1.45

$$-\frac{16\,a^{3}\cos\left(dx+c\right)^{7}-90\,a^{3}dx\cos\left(dx+c\right)^{4}+48\,a^{3}\cos\left(dx+c\right)^{5}+180\,a^{3}dx\cos\left(dx+c\right)^{2}-150\,a^{3}\cos\left(dx+c\right)^{3}-90\,a^{3}dx+90\,a^{3}\cos\left(dx+c\right)-45\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+45\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(2\,a^{3}\cos\left(dx+c\right)^{7}-9\,a^{3}\cos\left(dx+c\right)^{5}+60\,a^{3}\cos\left(dx+c\right)^{3}-45\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*a^3*cos(d*x + c)^7 - 90*a^3*d*x*cos(d*x + c)^4 + 48*a^3*cos(d*x +
 c)^5 + 180*a^3*d*x*cos(d*x + c)^2 - 150*a^3*cos(d*x + c)^3 - 90*a^3*d*x +
90*a^3*cos(d*x + c) - 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*
log(1/2*cos(d*x + c) + 1/2) + 45*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^2
 + a^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*cos(d*x + c)^7 - 9*a^3*cos(
d*x + c)^5 + 60*a^3*cos(d*x + c)^3 - 45*a^3*cos(d*x + c))*sin(d*x + c))/(d*
cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 313, normalized size = 1.76

$$\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+360\left(dx+c\right)a^{3}-360\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-184\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{250\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+136\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-32\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+552\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-837\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1248\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1100\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+736\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-556\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+152\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{4}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^3*tan
(1/2*d*x + 1/2*c)^2 + 360*(d*x + c)*a^3 - 360*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) - 184*a^3*tan(1/2*d*x + 1/2*c) + (250*a^3*tan(1/2*d*x + 1/2*c)^12 + 1
36*a^3*tan(1/2*d*x + 1/2*c)^11 - 32*a^3*tan(1/2*d*x + 1/2*c)^10 + 552*a^3*t
an(1/2*d*x + 1/2*c)^9 - 837*a^3*tan(1/2*d*x + 1/2*c)^8 + 1248*a^3*tan(1/2*d
*x + 1/2*c)^7 - 1100*a^3*tan(1/2*d*x + 1/2*c)^6 + 736*a^3*tan(1/2*d*x + 1/2
*c)^5 - 556*a^3*tan(1/2*d*x + 1/2*c)^4 + 152*a^3*tan(1/2*d*x + 1/2*c)^3 - 1
2*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d
*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^4)/d
```

Mupad [B]
time = 8.96, size = 419, normalized size = 2.35

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{45\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{-34\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+258\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-138\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{2337\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}-312\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+525\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-184\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{403\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}-38\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+96\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{45\,a^3\,\mathrm{atan}\left(\frac{2025\,a^6}{16\,\left(\frac{2025\,a^6}{16}+\frac{2025\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}-\frac{2025\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,\left(\frac{2025\,a^6}{16}+\frac{2025\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}-\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(8*d) + (a^3*
tan(c/2 + (d*x)/2)^4)/(64*d) - (45*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) - (3*
a^3*tan(c/2 + (d*x)/2)^2 - 38*a^3*tan(c/2 + (d*x)/2)^3 + (403*a^3*tan(c/2 +
 (d*x)/2)^4)/2 - 184*a^3*tan(c/2 + (d*x)/2)^5 + 525*a^3*tan(c/2 + (d*x)/2)^
6 - 312*a^3*tan(c/2 + (d*x)/2)^7 + (2337*a^3*tan(c/2 + (d*x)/2)^8)/4 - 138*
a^3*tan(c/2 + (d*x)/2)^9 + 258*a^3*tan(c/2 + (d*x)/2)^10 - 34*a^3*tan(c/2 +
 (d*x)/2)^11 + a^3/4 + 2*a^3*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^
4 + 64*tan(c/2 + (d*x)/2)^6 + 96*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2 + (d*x)/
2)^10 + 16*tan(c/2 + (d*x)/2)^12)) - (45*a^3*atan((2025*a^6)/(16*((2025*a^6
)/16 + (2025*a^6*tan(c/2 + (d*x)/2))/16)) - (2025*a^6*tan(c/2 + (d*x)/2))/(
16*((2025*a^6)/16 + (2025*a^6*tan(c/2 + (d*x)/2))/16))))/(4*d) - (23*a^3*ta
n(c/2 + (d*x)/2))/(8*d)
```

3.614 $\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$\frac{13a^3x}{2}-\frac{25a^3\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3}{}$$

```
[Out] 13/2*a^3*x-25/8*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+
c)^3/d+5*a^3*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-1/5*a^3*cot(d*x+c)^5/d+23/
8*a^3*cot(d*x+c)*csc(d*x+c)/d-3/4*a^3*cot(d*x+c)*csc(d*x+c)^3/d+3/2*a^3*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 3855, 3852, 8, 3853, 2715, 2713}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}+\frac{5a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a^3\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{23a^3\cot(c+dx)\csc(c+dx)}{8d}+\frac{13a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*a^3*x)/2 - (25*a^3*ArcTanh[Cos[c + d*x]])/(8*d) + (a^3*Cos[c + d*x])/d
- (a^3*Cos[c + d*x]^3)/(3*d) + (5*a^3*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]
^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) + (23*a^3*Cot[c + d*x]*Csc[c + d*x])
/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d) + (3*a^3*Cos[c + d*x]*Si
n[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9+6a^9\csc(c+dx)-6a^9\csc^2(c+dx)-8a^9\csc^3(c+dx)+3\right.}{}$$

$$= 8a^3x+a^3\int \csc^6(c+dx)\,dx-a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \csc$$

$$= 8a^3x-\frac{6a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cot(c+dx)\csc(c+dx)}{d}-\frac{3}{}$$

$$= \frac{13a^3x}{2}-\frac{2a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+}{3d}$$

$$= \frac{13a^3x}{2}-\frac{25a^3\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+}{3d}$$

Mathematica [A]
time = 1.26, size = 271, normalized size = 1.55

$$\frac{a^3(1+\sin(c+dx))^3\left(6240(c+dx)+720\cos(c+dx)-80\cos(3(c+dx))+2624\cot\left(\frac{1}{2}(c+dx)\right)+690\csc^2\left(\frac{1}{2}(c+dx)\right)-45\csc^4\left(\frac{1}{2}(c+dx)\right)-3000\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3000\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-690\sec^2\left(\frac{1}{2}(c+dx)\right)+45\sec^4\left(\frac{1}{2}(c+dx)\right)+304\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-19\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-3\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+720\sin(2(c+dx))-2624\tan\left(\frac{1}{2}(c+dx)\right)+6\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{960d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(6240*(c + d*x) + 720*Cos[c + d*x] - 80*Cos[3*(c
+ d*x)] + 2624*Cot[(c + d*x)/2] + 690*Csc[(c + d*x)/2]^2 - 45*Csc[(c + d*x)
/2]^4 - 3000*Log[Cos[(c + d*x)/2]] + 3000*Log[Sin[(c + d*x)/2]] - 690*Sec[(
c + d*x)/2]^2 + 45*Sec[(c + d*x)/2]^4 + 304*Csc[c + d*x]^3*Sin[(c + d*x)/2]
^4 - 19*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 3*Csc[(c + d*x)/2]^6*Sin[c + d*x]
 + 720*Sin[2*(c + d*x)] - 2624*Tan[(c + d*x)/2] + 6*Sec[(c + d*x)/2]^4*Tan[
(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.28, size = 287, normalized size = 1.64 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/4/
sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/
8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/3/
sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5+5/4
*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+a^3*(-1/2/sin(d*x+
c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(d*x+c)-5/2*ln(c
sc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.58, size = 250, normalized size = 1.43

$$-\frac{20\left(4\cos\left(dx+c\right)^{3}-\frac{6\cos(dx+c)}{\cos(dx+c)^{2}-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^{3}-120\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}+10\tan(dx+c)^{2}-2}{\tan(dx+c)^{5}+\tan(dx+c)^{3}}\right)a^{3}+16\left(15\,dx+15\,c+\frac{15\tan(dx+c)^{4}-5\tan(dx+c)^{2}+3}{\tan(dx+c)^{5}}\right)a^{3}+45\,a^{3}\left(\frac{2\left(9\cos(dx+c)^{3}-7\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}-16\cos\left(dx+c\right)+15\log\left(\cos\left(dx+c\right)+1\right)-15\log\left(\cos\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(20*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^3 - 120*
(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^
5 + tan(d*x + c)^3))*a^3 + 16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d
*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 45*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*
x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(
cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.42, size = 278, normalized size = 1.59

$$-\frac{360\,a^{3}\cos\left(dx+c\right)^{7}-2392\,a^{3}\cos\left(dx+c\right)^{5}+3640\,a^{3}\cos\left(dx+c\right)^{3}-1560\,a^{3}\cos\left(dx+c\right)+375\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-375\left(a^{3}\cos\left(dx+c\right)^{4}-2\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10\left(8\,a^{3}\cos\left(dx+c\right)^{7}-156\,a^{3}dx\cos\left(dx+c\right)^{4}-40\,a^{3}\cos\left(dx+c\right)^{5}+312\,a^{3}dx\cos\left(dx+c\right)^{2}+125\,a^{3}\cos\left(dx+c\right)^{3}-156\,a^{3}dx-75\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(360*a^3*cos(d*x + c)^7 - 2392*a^3*cos(d*x + c)^5 + 3640*a^3*cos(d*x
 + c)^3 - 1560*a^3*cos(d*x + c) + 375*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x +
 c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 375*(a^3*cos(d*x +
c)^4 - 2*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 10*(8*a^3*cos(d*x + c)^7 - 156*a^3*d*x*cos(d*x + c)^4 - 40*a^3*cos(d*x
+ c)^5 + 312*a^3*d*x*cos(d*x + c)^2 + 125*a^3*cos(d*x + c)^3 - 156*a^3*d*x
- 75*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 276, normalized size = 1.58

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+50\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6240\,(dx+c)a^3+3000\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-2580\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{320\,\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}-\frac{6850\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2580\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+50\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^3*tan(1/2*d*x + 1/2*c)^4 + 50*a^
3*tan(1/2*d*x + 1/2*c)^3 - 600*a^3*tan(1/2*d*x + 1/2*c)^2 + 6240*(d*x + c)*
a^3 + 3000*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2580*a^3*tan(1/2*d*x + 1/2*
c) - 320*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 9*
a^3*tan(1/2*d*x + 1/2*c) - 4*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3 - (6850*a^
3*tan(1/2*d*x + 1/2*c)^5 - 2580*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a^3*tan(1/
2*d*x + 1/2*c)^3 + 50*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^3*tan(1/2*d*x + 1/2
*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 8.92, size = 408, normalized size = 2.33

$$\frac{5\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{96\,d}-\frac{5\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{3\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}+\frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}+\frac{25\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{13\,a^3\,\mathrm{atan}\left(\frac{169\,a^6}{\frac{325\,a^6}{4}-169\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{325\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{325\,a^6}{4}-169\,a^6\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}+\frac{-10\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+20\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+\frac{769\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{3}+\frac{373\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+\frac{1744\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{5}+\frac{589\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{6}+\frac{402\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{5}+\frac{31\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{34\,a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{15}-\frac{3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{a^3}{5}}{d\,\left(32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+96\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9+96\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}-\frac{43\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (5*a^3*tan(c/2 + (d*x)/2)^3)/(96*d) - (5*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) +
(3*a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) +
(25*a^3*log(tan(c/2 + (d*x)/2)))/(8*d) + (13*a^3*atan((169*a^6)/((325*a^6)/
4 - 169*a^6*tan(c/2 + (d*x)/2)) + (325*a^6*tan(c/2 + (d*x)/2))/(4*((325*a^6
)/4 - 169*a^6*tan(c/2 + (d*x)/2)))))/d + ((31*a^3*tan(c/2 + (d*x)/2)^3)/2 -
 (34*a^3*tan(c/2 + (d*x)/2)^2)/15 + (402*a^3*tan(c/2 + (d*x)/2)^4)/5 + (589
*a^3*tan(c/2 + (d*x)/2)^5)/6 + (1744*a^3*tan(c/2 + (d*x)/2)^6)/5 + (373*a^3
*tan(c/2 + (d*x)/2)^7)/2 + (769*a^3*tan(c/2 + (d*x)/2)^8)/3 + 20*a^3*tan(c/
2 + (d*x)/2)^9 - 10*a^3*tan(c/2 + (d*x)/2)^10 - a^3/5 - (3*a^3*tan(c/2 + (d
*x)/2))/2)/(d*(32*tan(c/2 + (d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c
/2 + (d*x)/2)^9 + 32*tan(c/2 + (d*x)/2)^11)) - (43*a^3*tan(c/2 + (d*x)/2))/
(16*d)
```

3.615 $\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=182

$$-\frac{a^3x}{2}-\frac{85a^3\tanh^{-1}(\cos(c+dx))}{16d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot^5(c+dx)}{5d}+4$$

```
[Out] -1/2*a^3*x-85/16*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-a^3*cot(d*x+c
)/d+2/3*a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/d+43/16*a^3*cot(d*x+c)*csc(
d*x+c)/d-5/24*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a^3*cot(d*x+c)*csc(d*x+c)^5
/d+1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2951, 3855, 3852, 8, 3853, 2718, 2715}

$$\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{85a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a^3\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{43a^3\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*x) - (85*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (3*a^3*Cos[c + d*x])
/d - (a^3*Cot[c + d*x])/d + (2*a^3*Cot[c + d*x]^3)/(3*d) - (3*a^3*Cot[c + d
*x]^5)/(5*d) + (43*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d) - (5*a^3*Cot[c + d
*x]*Csc[c + d*x]^3)/(24*d) - (a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(6*d) + (a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(8a^9\csc(c+dx)+6a^9\csc^2(c+dx)-6a^9\csc^3(c+dx\right.}{}$$

$$= a^3\int \csc^7(c+dx)\,dx - a^3\int \sin^2(c+dx)\,dx + \left(3a^3\right)\int$$

$$= -\frac{8a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} + \frac{3a^3\cot(}{}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a}{}$$

$$= -\frac{a^3x}{2} - \frac{5a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a}{}$$

$$= -\frac{a^3x}{2} - \frac{85a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{3a^3\cos(c+dx)}{d} -$$

Mathematica [A]
time = 1.66, size = 289, normalized size = 1.59

$$\frac{a^3(1+\sin(c+dx))^3\left(-960(c+dx)+5760\cos(c+dx)-2176\cot\left(\frac{1}{2}(c+dx)\right)+1290\csc^2\left(\frac{1}{2}(c+dx)\right)-30\csc^4\left(\frac{1}{2}(c+dx)\right)-5\csc^6\left(\frac{1}{2}(c+dx)\right)-10200\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+10200\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-1290\sec^2\left(\frac{1}{2}(c+dx)\right)+30\sec^4\left(\frac{1}{2}(c+dx)\right)+5\sec^6\left(\frac{1}{2}(c+dx)\right)-3296\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+206\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-18\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+480\sin(2(c+dx))+2176\tan\left(\frac{1}{2}(c+dx)\right)+36\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{1920d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-960*(c + d*x) + 5760*Cos[c + d*x] - 2176*Cot[(c
 + d*x)/2] + 1290*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*x)/2]^4 - 5*Csc[(c + d
*x)/2]^6 - 10200*Log[Cos[(c + d*x)/2]] + 10200*Log[Sin[(c + d*x)/2]] - 1290
*Sec[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 3296*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 206*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 18
*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 480*Sin[2*(c + d*x)] + 2176*Tan[(c + d*x
)/2] + 36*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*d*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.25, size = 323, normalized size = 1.77

method	result
risch	$-\frac{a^3 x}{2}-\frac{i a^3 e^{2i(dx+c)}}{8d}+\frac{3a^3 e^{i(dx+c)}}{2d}+\frac{3a^3 e^{-i(dx+c)}}{2d}+\frac{i a^3 e^{-2i(dx+c)}}{8d}-\frac{a^3(645\, e^{11i(dx+c)}-1735\, e^{9i(dx+c)}+14}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{16}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/1
6/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*
x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+3*a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^
2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(cs
c(d*x+c)-cot(d*x+c)))+a^3*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*co
s(d*x+c)^7+4/3*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5
/2*d*x+5/2*c))
```

Maxima [A]
time = 0.50, size = 275, normalized size = 1.51

$$\frac{80\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^3-96\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+5\,a^3\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-90\,a^3\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(80*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan
(d*x + c)^5 + tan(d*x + c)^3))*a^3 - 96*(15*d*x + 15*c + (15*tan(d*x + c)^4
 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5)*a^3 + 5*a^3*(2*(33*cos(d*x + c)^5
- 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 +
 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1
)) - 90*a^3*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d
*x + c) - 1)))/d
```

Fricas [A]
time = 0.42, size = 316, normalized size = 1.74

$$-\frac{240\,a^3dx\cos\left(dx+c\right)^6-1440\,a^3\cos\left(dx+c\right)^7-720\,a^3dx\cos\left(dx+c\right)^4+5610\,a^3\cos\left(dx+c\right)^5+720\,a^3dx\cos\left(dx+c\right)^2-6800\,a^3\cos\left(dx+c\right)^3-240\,a^3dx+2550\,a^3\cos\left(dx+c\right)+1275\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-1275\,\left(a^3\cos\left(dx+c\right)^6-3\,a^3\cos\left(dx+c\right)^4+3\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-16\,\left(15\,a^3\cos\left(dx+c\right)^7+23\,a^3\cos\left(dx+c\right)^5-35\,a^3\cos\left(dx+c\right)^3+15\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(240*a^3*d*x*cos(d*x + c)^6 - 1440*a^3*cos(d*x + c)^7 - 720*a^3*d*x*
cos(d*x + c)^4 + 5610*a^3*cos(d*x + c)^5 + 720*a^3*d*x*cos(d*x + c)^2 - 680
0*a^3*cos(d*x + c)^3 - 240*a^3*d*x + 2550*a^3*cos(d*x + c) + 1275*(a^3*cos(
d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos
(d*x + c) + 1/2) - 1275*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2) - 16*(15*a^3*cos(d*x + c
)^7 + 23*a^3*cos(d*x + c)^5 - 35*a^3*cos(d*x + c)^3 + 15*a^3*cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 307, normalized size = 1.69

$$\frac{5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1215\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-960\,(dx+c)a^3+10200\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+1800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{1920\,\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}-\frac{24990\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1215\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-340\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a
^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1/2*c)^3 - 1215*a^3*tan(1
/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a^3 + 10200*a^3*log(abs(tan(1/2*d*x + 1/2
*c))) + 1800*a^3*tan(1/2*d*x + 1/2*c) - 1920*(a^3*tan(1/2*d*x + 1/2*c)^3 -
6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^2 - (24990*a^3*tan(1/2*d*x + 1/2*c)^6 + 1800*a^3*tan(1/2
*d*x + 1/2*c)^5 - 1215*a^3*tan(1/2*d*x + 1/2*c)^4 - 340*a^3*tan(1/2*d*x + 1
/2*c)^3 + 45*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 5*a
^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 8.94, size = 396, normalized size = 2.18

$$\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}-\frac{17\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{81\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{85\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,d}+\frac{a^3\,\mathrm{atan}\left(\frac{a^6}{\frac{85\,a^6}{8}+a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{85\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,\left(\frac{85\,a^6}{8}+a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}-\frac{124\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{849\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+\frac{134\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{927\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{578\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{15}-\frac{112\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{134\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}+\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+\frac{a^3}{6}}{d\,\left(64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+128\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right)}+\frac{15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^4)/(128*d) - (17*a^3*tan(c/2 + (d*x)/2)^3)/(96*d)
 - (81*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) + (3*a^3*tan(c/2 + (d*x)/2)^5)/(16
0*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) + (85*a^3*log(tan(c/2 + (d*x)/2))
)/(16*d) + (a^3*atan(a^6/((85*a^6)/8 + a^6*tan(c/2 + (d*x)/2)) - (85*a^6*ta
n(c/2 + (d*x)/2))/(8*((85*a^6)/8 + a^6*tan(c/2 + (d*x)/2)))))/d - ((11*a^3*
tan(c/2 + (d*x)/2)^2)/6 - (134*a^3*tan(c/2 + (d*x)/2)^3)/15 - (112*a^3*tan(
c/2 + (d*x)/2)^4)/3 + (578*a^3*tan(c/2 + (d*x)/2)^5)/15 - (927*a^3*tan(c/2
+ (d*x)/2)^6)/2 + (134*a^3*tan(c/2 + (d*x)/2)^7)/3 - (849*a^3*tan(c/2 + (d*
x)/2)^8)/2 + 124*a^3*tan(c/2 + (d*x)/2)^9 + a^3/6 + (6*a^3*tan(c/2 + (d*x)/
2))/5)/(d*(64*tan(c/2 + (d*x)/2)^6 + 128*tan(c/2 + (d*x)/2)^8 + 64*tan(c/2
+ (d*x)/2)^10)) + (15*a^3*tan(c/2 + (d*x)/2))/(16*d)
```

3.616 $\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=172

$$-3a^3x-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot^5(c+dx)}{5d}-\frac{a^3}{}$$

```
[Out] -3*a^3*x-15/16*a^3*arctanh(cos(d*x+c))/d+a^3*cos(d*x+c)/d-3*a^3*cot(d*x+c)/
d+a^3*cot(d*x+c)^3/d-3/5*a^3*cot(d*x+c)^5/d-1/7*a^3*cot(d*x+c)^7/d-15/16*a^
3*cot(d*x+c)*csc(d*x+c)/d+11/8*a^3*cot(d*x+c)*csc(d*x+c)^3/d-1/2*a^3*cot(d*
x+c)*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.19, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3852, 8, 3853, 3855, 2718}

$$\frac{a^3\cos(c+dx)}{d}-\frac{a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{d}-\frac{3a^3\cot(c+dx)}{d}-\frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{2d}+\frac{11a^3\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{15a^3\cot(c+dx)\csc(c+dx)}{16d}-3a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -3*a^3*x - (15*a^3*ArcTanh[Cos[c + d*x]])/(16*d) + (a^3*Cos[c + d*x])/d - (
3*a^3*Cot[c + d*x])/d + (a^3*Cot[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/(5*
d) - (a^3*Cot[c + d*x]^7)/(7*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x])/(16*d)
 + (11*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d*x]*Csc[c + d
*x]^5)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int\left(-3a^9+8a^9\csc^2(c+dx)+6a^9\csc^3(c+dx)-6a^9\,\mathrm{c}\right.}{}$$

$$= -3a^3x + a^3\int \csc^8(c+dx)\,dx - a^3\int \sin(c+dx)\,dx$$

$$= -3a^3x + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+dx)\csc(c+dx)}{d}$$

$$= -3a^3x - \frac{3a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^3\cos(c+dx)}{d} -$$

$$= -3a^3x + \frac{a^3\cos(c+dx)}{d} - \frac{3a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(}{c}$$

$$= -3a^3x - \frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d} + \frac{a^3\cos(c+dx)}{d}$$

Mathematica [A]
time = 1.00, size = 292, normalized size = 1.70

$$\frac{a^3\left(-13440c-13440dx+4480\cos(c+dx)-9984\cot\left(\tfrac{1}{2}(c+dx)\right)-1050\csc^2\left(\tfrac{1}{2}(c+dx)\right)+350\csc^4\left(\tfrac{1}{2}(c+dx)\right)-35\csc^6\left(\tfrac{1}{2}(c+dx)\right)-4200\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+4200\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+1050\sec^2\left(\tfrac{1}{2}(c+dx)\right)-350\sec^4\left(\tfrac{1}{2}(c+dx)\right)+35\sec^6\left(\tfrac{1}{2}(c+dx)\right)-7664\csc^3(c+dx)\sin^4\left(\tfrac{1}{2}(c+dx)\right)+479\csc^4\left(\tfrac{1}{2}(c+dx)\right)\sin(c+dx)-17\csc^6\left(\tfrac{1}{2}(c+dx)\right)\sin(c+dx)-\tfrac{5}{2}\csc^8\left(\tfrac{1}{2}(c+dx)\right)\sin(c+dx)+9984\tan\left(\tfrac{1}{2}(c+dx)\right)+34\sec^4\left(\tfrac{1}{2}(c+dx)\right)\tan\left(\tfrac{1}{2}(c+dx)\right)+5\sec^6\left(\tfrac{1}{2}(c+dx)\right)\tan\left(\tfrac{1}{2}(c+dx)\right)\right)}{4480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-13440*c - 13440*d*x + 4480*Cos[c + d*x] - 9984*Cot[(c + d*x)/2] - 10
50*Csc[(c + d*x)/2]^2 + 350*Csc[(c + d*x)/2]^4 - 35*Csc[(c + d*x)/2]^6 - 42
```

```
00*Log[Cos[(c + d*x)/2]] + 4200*Log[Sin[(c + d*x)/2]] + 1050*Sec[(c + d*x)/
2]^2 - 350*Sec[(c + d*x)/2]^4 + 35*Sec[(c + d*x)/2]^6 - 7664*Csc[c + d*x]^3
*Sin[(c + d*x)/2]^4 + 479*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 17*Csc[(c + d*x
)/2]^6*Sin[c + d*x] - (5*Csc[(c + d*x)/2]^8*Sin[c + d*x])/2 + 9984*Tan[(c +
 d*x)/2] + 34*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 5*Sec[(c + d*x)/2]^6*Ta
n[(c + d*x)/2]))/(4480*d)
```

Maple [A]
time = 0.25, size = 261, normalized size = 1.52

method	result
risch	$-3a^3x+\frac{a^3 e^{i(dx+c)}}{2d}+\frac{a^3 e^{-i(dx+c)}}{2d}+\frac{a^3\left(-4480i e^{12i(dx+c)}+525\,e^{13i(dx+c)}+20160i e^{10i(dx+c)}+980\,e^{11i(dx+c)}-38\right.}{}$
derivativedivides	$\frac{-\frac{a^3\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+3a^3\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc\right.}{}}{}$
default	$\frac{-\frac{a^3\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+3a^3\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(\csc\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^3/sin(d*x+c)^7*cos(d*x+c)^7+3*a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)
^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d
*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+3
*a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+a^3*(-1/4/sin(d*
x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(
d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.51, size = 233, normalized size = 1.35

$$-\frac{224\left(15\,dx+15\,c+\frac{15\tan(dx+c)^4-5\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-35\,a^3\left(\frac{2\left(33\cos(dx+c)^5-40\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}+15\log\left(\cos(dx+c)+1\right)-15\log\left(\cos(dx+c)-1\right)\right)+70\,a^3\left(\frac{2\left(9\cos(dx+c)^3-7\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-16\cos(dx+c)+15\log\left(\cos(dx+c)+1\right)-15\log\left(\cos(dx+c)-1\right)\right)+\frac{160\,a^3}{\tan(dx+c)^7}}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/1120*(224*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*a^3 - 35*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 70*a^3*(2*(9*cos(d*x
+ c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d
*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 160*a^3/ta
n(d*x + c)^7)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(160) = 320$.
time = 0.40, size = 336, normalized size = 1.95

$$-\frac{4992\,a^{3}\cos\left(dx+c\right)^{7}-12992\,a^{3}\cos\left(dx+c\right)^{5}+11200\,a^{3}\cos\left(dx+c\right)^{3}-3360\,a^{3}\cos\left(dx+c\right)+525\,\left(a^{3}\cos\left(dx+c\right)^{6}-3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-525\,\left(a^{3}\cos\left(dx+c\right)^{6}-3\,a^{3}\cos\left(dx+c\right)^{4}+3\,a^{3}\cos\left(dx+c\right)^{2}-a^{3}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+70\,\left(48\,a^{3}dx\cos\left(dx+c\right)^{6}-16\,a^{3}\cos\left(dx+c\right)^{7}-144\,a^{3}dx\cos\left(dx+c\right)^{4}+33\,a^{3}\cos\left(dx+c\right)^{5}+144\,a^{3}dx\cos\left(dx+c\right)^{2}-40\,a^{3}\cos\left(dx+c\right)^{3}-48\,a^{3}dx+15\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1120\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(4992*a^3*cos(d*x + c)^7 - 12992*a^3*cos(d*x + c)^5 + 11200*a^3*cos
(d*x + c)^3 - 3360*a^3*cos(d*x + c) + 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d
*x + c)^4 + 3*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 525*(a^3*cos(d*x + c)^6 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)
^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(48*a^3*d*x*cos(d*
x + c)^6 - 16*a^3*cos(d*x + c)^7 - 144*a^3*d*x*cos(d*x + c)^4 + 33*a^3*cos(
d*x + c)^5 + 144*a^3*d*x*cos(d*x + c)^2 - 40*a^3*cos(d*x + c)^3 - 48*a^3*d*
x + 15*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c
)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.59, size = 291, normalized size = 1.69

$$\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+35\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+49\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-245\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-875\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+455\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-13440\,(dx+c)a^{3}+4200\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+9065\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{8960\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}-\frac{10890\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9065\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+455\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-875\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-245\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+49\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^3*tan(1/2*d*x + 1/2*c)^6 + 49*a
^3*tan(1/2*d*x + 1/2*c)^5 - 245*a^3*tan(1/2*d*x + 1/2*c)^4 - 875*a^3*tan(1/
2*d*x + 1/2*c)^3 + 455*a^3*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*a^3 + 4
200*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9065*a^3*tan(1/2*d*x + 1/2*c) + 89
60*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - (10890*a^3*tan(1/2*d*x + 1/2*c)^7 + 9
065*a^3*tan(1/2*d*x + 1/2*c)^6 + 455*a^3*tan(1/2*d*x + 1/2*c)^5 - 875*a^3*t
an(1/2*d*x + 1/2*c)^4 - 245*a^3*tan(1/2*d*x + 1/2*c)^3 + 49*a^3*tan(1/2*d*x
```

```
 + 1/2*c)^2 + 35*a^3*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^7)/
d
```

Mupad [B]
time = 9.04, size = 388, normalized size = 2.26

$$\frac{13\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{25\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128\,d}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{15\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,d}+\frac{6\,a^3\,\mathrm{atan}\left(\frac{36\,a^6}{\frac{45\,a^6}{4}+36\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{45\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{45\,a^6}{4}+36\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}-\frac{259\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-243\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+234\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{118\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{54\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a^3}{7}}{d\,\left(128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}+\frac{259\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (13*a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (25*a^3*tan(c/2 + (d*x)/2)^3)/(128*
d) - (7*a^3*tan(c/2 + (d*x)/2)^4)/(128*d) + (7*a^3*tan(c/2 + (d*x)/2)^5)/(6
40*d) + (a^3*tan(c/2 + (d*x)/2)^6)/(128*d) + (a^3*tan(c/2 + (d*x)/2)^7)/(89
6*d) + (15*a^3*log(tan(c/2 + (d*x)/2)))/(16*d) + (6*a^3*atan((36*a^6)/((45*
a^6)/4 + 36*a^6*tan(c/2 + (d*x)/2)) - (45*a^6*tan(c/2 + (d*x)/2))/(4*((45*a
^6)/4 + 36*a^6*tan(c/2 + (d*x)/2)))))/d - ((54*a^3*tan(c/2 + (d*x)/2)^2)/35
 - 6*a^3*tan(c/2 + (d*x)/2)^3 - (118*a^3*tan(c/2 + (d*x)/2)^4)/5 + 6*a^3*ta
n(c/2 + (d*x)/2)^5 + 234*a^3*tan(c/2 + (d*x)/2)^6 - 243*a^3*tan(c/2 + (d*x)
/2)^7 + 259*a^3*tan(c/2 + (d*x)/2)^8 + a^3/7 + a^3*tan(c/2 + (d*x)/2))/(d*(
128*tan(c/2 + (d*x)/2)^7 + 128*tan(c/2 + (d*x)/2)^9)) + (259*a^3*tan(c/2 +
(d*x)/2))/(128*d)
```

3.617 $\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=238

$$-a^3x+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot(c+dx)}{d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{3a^3\cot^7(c+dx)}{7d}-1$$

```
[Out] -a^3*x+125/128*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+1/3*a^3*cot(d*x+c
)^3/d-1/5*a^3*cot(d*x+c)^5/d-3/7*a^3*cot(d*x+c)^7/d-115/128*a^3*cot(d*x+c)*
csc(d*x+c)/d+5/8*a^3*cot(d*x+c)^3*csc(d*x+c)/d-1/2*a^3*cot(d*x+c)^5*csc(d*x
+c)/d-5/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d+5/48*a^3*cot(d*x+c)^3*csc(d*x+c)^3
/d-1/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2952, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$-\frac{3a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^3\cot^5(c+dx)\csc(c+dx)}{2d}+\frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{48d}+\frac{5a^3\cot^3(c+dx)\csc(c+dx)}{8d}-\frac{5a^3\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{115a^3\cot(c+dx)\csc(c+dx)}{128d}-a^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (125*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (a^3*Cot[c + d*x])/d +
 (a^3*Cot[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a^3*Cot[c + d
*x]^7)/(7*d) - (115*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a^3*Cot[c +
 d*x]^3*Csc[c + d*x])/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/(2*d) - (5*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*x
]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx) + 3a^3\cot^6(c+dx)\csc(c+dx) + 3\right.$$

$$= a^3\int \cot^6(c+dx)\,dx + a^3\int \cot^6(c+dx)\csc^3(c+dx$$

$$= -\frac{a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^5(c+dx)\csc(c+dx)}{2d} - \frac{a^3}{}$$

$$= \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d} - \frac{3a^3\cot^7(c+dx)}{7d}$$

$$= -\frac{a^3\cot(c+dx)}{d} + \frac{a^3\cot^3(c+dx)}{3d} - \frac{a^3\cot^5(c+dx)}{5d}$$

$$= -a^3x + \frac{15a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3\cot(c+dx)}{d} +$$

$$= -a^3x + \frac{125a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{a^3\cot(c+dx)}{d}$$

Mathematica [A]
time = 0.85, size = 279, normalized size = 1.17

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-215040*c - 215040*d*x - 118784*Cot[(c + d*x)/2] - 108780*Csc[(c + d*
x)/2]^2 + 210000*Log[Cos[(c + d*x)/2]] - 210000*Log[Sin[(c + d*x)/2]] + 108
780*Sec[(c + d*x)/2]^2 - 17010*Sec[(c + d*x)/2]^4 + 700*Sec[(c + d*x)/2]^6
+ 105*Sec[(c + d*x)/2]^8 + 71936*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c
 + d*x)/2]^4*(17010 - 4496*Sin[c + d*x]) - 15*Csc[(c + d*x)/2]^8*(7 + 24*Si
n[c + d*x]) + 4*Csc[(c + d*x)/2]^6*(-175 + 732*Sin[c + d*x]) + 118784*Tan[(
c + d*x)/2] - 5856*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2] + 720*Sec[(c + d*x)/
2]^6*Tan[(c + d*x)/2]))/(215040*d)
```

Maple [A]
time = 0.27, size = 296, normalized size = 1.24

method	result
risch	$-a^3x + \frac{a^3\left(27195\,e^{15i(dx+c)} - 65135\,e^{13i(dx+c)} + 63595\,e^{11i(dx+c)} + 161280ie^{12i(dx+c)} - 133175\,e^{9i(dx+c)} - 286720ie^{1}\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8} - \frac{\cos^7(dx+c)}{48\sin(dx+c)^6} + \frac{\cos^7(dx+c)}{192\sin(dx+c)^4} - \frac{\cos^7(dx+c)}{128\sin(dx+c)^2} - \frac{\left(\cos^5(dx+c)\right)}{128} - \frac{5\left(\cos^3(dx+c)\right)}{384} - \frac{5\cos(dx+c)}{128} - \frac{5\ln(\csc(}{}\right.}{}$

default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(dx-}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/1
92/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+
c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-3
/7*a^3/sin(d*x+c)^7*cos(d*x+c)^7+3*a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24
/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5
-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-1/
5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

<u>Maxima</u> [A]
time = 0.50, size = 265, normalized size = 1.11

$$-\frac{1792\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3+35\,a^3\left(\frac{2\left(15\,\cos(dx+c)^7+73\,\cos(dx+c)^5-55\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-840\,a^3\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{11520\,a^3}{\tan(dx+c)^7}}{26880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/26880*(1792*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/
tan(d*x + c)^5)*a^3 + 35*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55
*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(c
os(d*x + c) - 1)) - 840*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*
cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) +
15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 11520*a^3/tan(d*x +
c)^7)/d
```

<u>Fricas</u> [A]
time = 0.42, size = 362, normalized size = 1.52

$$-\frac{26880\,a^3dx\cos\left(dx+c\right)^8-107520\,a^3dx\cos\left(dx+c\right)^6-54390\,a^3\cos\left(dx+c\right)^7+161280\,a^3dx\cos\left(dx+c\right)^4+127750\,a^3\cos\left(dx+c\right)^5-107520\,a^3dx\cos\left(dx+c\right)^2-96250\,a^3\cos\left(dx+c\right)^3+26880\,a^3dx+26250\,a^3\cos\left(dx+c\right)-13125\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+13125\left(a^3\cos\left(dx+c\right)^8-4\,a^3\cos\left(dx+c\right)^6+6\,a^3\cos\left(dx+c\right)^4-4\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-256\left(116\,a^3\cos\left(dx+c\right)^7-406\,a^3\cos\left(dx+c\right)^5+350\,a^3\cos\left(dx+c\right)^3-105\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{26880\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(26880*a^3*d*x*cos(d*x + c)^8 - 107520*a^3*d*x*cos(d*x + c)^6 - 54
390*a^3*cos(d*x + c)^7 + 161280*a^3*d*x*cos(d*x + c)^4 + 127750*a^3*cos(d*x
 + c)^5 - 107520*a^3*d*x*cos(d*x + c)^2 - 96250*a^3*cos(d*x + c)^3 + 26880*
```

```
a^3*d*x + 26250*a^3*cos(d*x + c) - 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*
x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d
*x + c) + 1/2) + 13125*(a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*c
os(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2) -
256*(116*a^3*cos(d*x + c)^7 - 406*a^3*cos(d*x + c)^5 + 350*a^3*cos(d*x + c)
^3 - 105*a^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x +
c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 302, normalized size = 1.27

$$\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3696\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-14280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+77280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-215040\,(dx+c)a^3-210000\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+122640\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{570750\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-122640\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-77280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+560\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+14280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3696\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-720\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/215040*(105*a^3*tan(1/2*d*x + 1/2*c)^8 + 720*a^3*tan(1/2*d*x + 1/2*c)^7 +
 1120*a^3*tan(1/2*d*x + 1/2*c)^6 - 3696*a^3*tan(1/2*d*x + 1/2*c)^5 - 14280*
a^3*tan(1/2*d*x + 1/2*c)^4 - 560*a^3*tan(1/2*d*x + 1/2*c)^3 + 77280*a^3*tan
(1/2*d*x + 1/2*c)^2 - 215040*(d*x + c)*a^3 - 210000*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + 122640*a^3*tan(1/2*d*x + 1/2*c) + (570750*a^3*tan(1/2*d*x + 1
/2*c)^8 - 122640*a^3*tan(1/2*d*x + 1/2*c)^7 - 77280*a^3*tan(1/2*d*x + 1/2*c
)^6 + 560*a^3*tan(1/2*d*x + 1/2*c)^5 + 14280*a^3*tan(1/2*d*x + 1/2*c)^4 + 3
696*a^3*tan(1/2*d*x + 1/2*c)^3 - 1120*a^3*tan(1/2*d*x + 1/2*c)^2 - 720*a^3*
tan(1/2*d*x + 1/2*c) - 105*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [**B**]
time = 10.30, size = 389, normalized size = 1.63

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d}-\frac{23\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{17\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}+\frac{11\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{640\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}-\frac{3\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{896\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}+\frac{23\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{384\,d}-\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{256\,d}-\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{640\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{896\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048\,d}-\frac{2\,a^3\,\mathrm{atan}\left(\frac{128\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+125\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{125\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-128\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{125\,a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{128\,d}-\frac{73\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d}+\frac{73\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (a^3*cot(c/2 + (d*x)/2)^3)/(384*d) - (23*a^3*cot(c/2 + (d*x)/2)^2)/(64*d) +
 (17*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) + (11*a^3*cot(c/2 + (d*x)/2)^5)/(640
```

```
*d) - (a^3*cot(c/2 + (d*x)/2)^6)/(192*d) - (3*a^3*cot(c/2 + (d*x)/2)^7)/(89
6*d) - (a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) + (23*a^3*tan(c/2 + (d*x)/2)^2)/
(64*d) - (a^3*tan(c/2 + (d*x)/2)^3)/(384*d) - (17*a^3*tan(c/2 + (d*x)/2)^4)
/(256*d) - (11*a^3*tan(c/2 + (d*x)/2)^5)/(640*d) + (a^3*tan(c/2 + (d*x)/2)^
6)/(192*d) + (3*a^3*tan(c/2 + (d*x)/2)^7)/(896*d) + (a^3*tan(c/2 + (d*x)/2)
^8)/(2048*d) - (2*a^3*atan((128*cos(c/2 + (d*x)/2) + 125*sin(c/2 + (d*x)/2)
)/(125*cos(c/2 + (d*x)/2) - 128*sin(c/2 + (d*x)/2))))/d - (125*a^3*log(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(128*d) - (73*a^3*cot(c/2 + (d*x)/2))/(
128*d) + (73*a^3*tan(c/2 + (d*x)/2))/(128*d)
```

3.618 $\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=200

$$\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^9(c+dx)}{9d}-\frac{25a^3\cot(c+dx)\csc(c+dx)}{128d}+\frac{5a^3\cot^3(c+dx)}{2\cdots}$$

```
[Out] 55/128*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-1/9*a^3*cot(d*x+c)^
9/d-25/128*a^3*cot(d*x+c)*csc(d*x+c)/d+5/24*a^3*cot(d*x+c)^3*csc(d*x+c)/d-1
/6*a^3*cot(d*x+c)^5*csc(d*x+c)/d-15/64*a^3*cot(d*x+c)*csc(d*x+c)^3/d+5/16*a
^3*cot(d*x+c)^3*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2691, 3855, 2687, 30, 3853, 14}

$$-\frac{a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}-\frac{a^3\cot^5(c+dx)\csc(c+dx)}{6d}+\frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{16d}+\frac{5a^3\cot^3(c+dx)\csc(c+dx)}{24d}-\frac{15a^3\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{25a^3\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (55*a^3*ArcTanh[Cos[c + d*x]])/(128*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(9*d) - (25*a^3*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (5*a
^3*Cot[c + d*x]^3*Csc[c + d*x])/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x])/
(6*d) - (15*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(64*d) + (5*a^3*Cot[c + d*x]^3
*Csc[c + d*x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc(c+dx)+3a^3\cot^6(c+dx)\csc^2(\right.$$

$$= a^3\int \cot^6(c+dx)\csc(c+dx)\,dx + a^3\int \cot^6(c+dx)\,c$$

$$= -\frac{a^3\cot^5(c+dx)\csc(c+dx)}{6d} - \frac{3a^3\cot^5(c+dx)\csc^3(c}{8d}$$

$$= -\frac{3a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc(c+dx)}{24d} - \frac{a^3}{}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{9d} - \frac{5a^3\cot(c+dx}{16d}$$

$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{16d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(}{9}$$

$$= \frac{55a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 459 vs. 2(200) = 400.
time = 0.10, size = 459, normalized size = 2.30

$$a^3\left(\frac{29\cot\left(\frac{1}{2}(c+dx)\right)}{126d}-\frac{73\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{4163\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32256d}-\frac{13\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{319\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)}{10752d}+\frac{17\csc^6\left(\frac{1}{2}(c+dx)\right)}{1536d}-\frac{53\cot\left(\frac{1}{2}(c+dx)\right)\csc^6\left(\frac{1}{2}(c+dx)\right)}{32256d}-\frac{3\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^8\left(\frac{1}{2}(c+dx)\right)}{4608d}+\frac{55\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d}-\frac{55\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{73\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{13\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{17\sec^6\left(\frac{1}{2}(c+dx)\right)}{1536d}+\frac{3\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{29\tan\left(\frac{1}{2}(c+dx)\right)}{126d}+\frac{4163\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{32256d}-\frac{319\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{10752d}+\frac{53\sec^6\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{32256d}+\frac{\sec^8\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{4608d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((29*Cot[(c + d*x)/2])/(126*d) - (73*Csc[(c + d*x)/2]^2)/(512*d) - (416
3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(32256*d) - (13*Csc[(c + d*x)/2]^4)/
(1024*d) + (319*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(10752*d) + (17*Csc[(c
 + d*x)/2]^6)/(1536*d) - (53*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^6)/(32256*d)
 - (3*Csc[(c + d*x)/2]^8)/(2048*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^8)/
(4608*d) + (55*Log[Cos[(c + d*x)/2]])/(128*d) - (55*Log[Sin[(c + d*x)/2]])/
(128*d) + (73*Sec[(c + d*x)/2]^2)/(512*d) + (13*Sec[(c + d*x)/2]^4)/(1024*d
) - (17*Sec[(c + d*x)/2]^6)/(1536*d) + (3*Sec[(c + d*x)/2]^8)/(2048*d) - (2
9*Tan[(c + d*x)/2])/(126*d) + (4163*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3
2256*d) - (319*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(10752*d) + (53*Sec[(c
+ d*x)/2]^6*Tan[(c + d*x)/2])/(32256*d) + (Sec[(c + d*x)/2]^8*Tan[(c + d*x)
/2])/(4608*d))
```

Maple [**A**]
time = 0.29, size = 297, normalized size = 1.48

method	result
risch	$\frac{a^3\left(69120ie^{4i(dx+c)}+4599\,e^{17i(dx+c)}+145152ie^{8i(dx+c)}-39858\,e^{15i(dx+c)}-193536ie^{6i(dx+c)}-2142\,e^{13i(dx+c)}+118272\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+3a^3\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+3a^3\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7)+3*
a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192/si
n(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)^5-
5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-3/7*a^
3/sin(d*x+c)^7*cos(d*x+c)^7+a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*
x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*c
os(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 246, normalized size = 1.23

$$-\frac{63\,a^{3}\left(\frac{2\,\left(15\,\cos\left(dx+c\right)^{7}+73\,\cos\left(dx+c\right)^{5}-55\,\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)-168\,a^{3}\left(\frac{2\,\left(33\,\cos\left(dx+c\right)^{5}-40\,\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{6912\,a^{3}}{\tan\left(dx+c\right)^{7}}+\frac{256\,\left(9\,\tan\left(dx+c\right)^{2}+7\right)a^{3}}{\tan\left(dx+c\right)^{9}}}{16128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/16128*(63*a^3*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c
)^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^
4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c)
 - 1)) - 168*a^3*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c)
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d
*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 6912*a^3/tan(d*x + c)^7 + 256*(9
*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d
```

Fricas [A]
time = 0.40, size = 291, normalized size = 1.46

$$\frac{7424\,a^{3}\cos\left(dx+c\right)^{9}-9216\,a^{3}\cos\left(dx+c\right)^{7}+3465\,\left(a^{3}\cos\left(dx+c\right)^{8}-4\,a^{3}\cos\left(dx+c\right)^{6}+6\,a^{3}\cos\left(dx+c\right)^{4}-4\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3465\,\left(a^{3}\cos\left(dx+c\right)^{8}-4\,a^{3}\cos\left(dx+c\right)^{6}+6\,a^{3}\cos\left(dx+c\right)^{4}-4\,a^{3}\cos\left(dx+c\right)^{2}+a^{3}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+42\,\left(219\,a^{3}\cos\left(dx+c\right)^{7}-803\,a^{3}\cos\left(dx+c\right)^{5}+605\,a^{3}\cos\left(dx+c\right)^{3}-165\,a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16128\,\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/16128*(7424*a^3*cos(d*x + c)^9 - 9216*a^3*cos(d*x + c)^7 + 3465*(a^3*cos(
d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x +
c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3465*(a^3*cos(d*x +
c)^8 - 4*a^3*cos(d*x + c)^6 + 6*a^3*cos(d*x + c)^4 - 4*a^3*cos(d*x + c)^2 +
 a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(219*a^3*cos(d*x + c)^
7 - 803*a^3*cos(d*x + c)^5 + 605*a^3*cos(d*x + c)^3 - 165*a^3*cos(d*x + c))
*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4
 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 324, normalized size = 1.62

$$\frac{28 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^9 + 189 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^8 + 324 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^7 - 672 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 - 3024 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 - 1512 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 9744 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + 18144 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 55440 a^3 \log\left(\left|\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right|\right) - 16632 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) + \frac{156838 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^9 + 16632 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^8 - 18144 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^7 - 9744 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^6 + 1512 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^5 + 3024 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 + 672 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 - 324 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 - 189 a^3 \tan\left(\frac{1}{2} d x + \frac{1}{2} c\right) - 28 a^3}{\tan\left(\frac{1}{2} d x + \frac{1}{2} c\right)^9}}{129024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/129024*(28*a^3*tan(1/2*d*x + 1/2*c)^9 + 189*a^3*tan(1/2*d*x + 1/2*c)^8 +
324*a^3*tan(1/2*d*x + 1/2*c)^7 - 672*a^3*tan(1/2*d*x + 1/2*c)^6 - 3024*a^3*
tan(1/2*d*x + 1/2*c)^5 - 1512*a^3*tan(1/2*d*x + 1/2*c)^4 + 9744*a^3*tan(1/2
*d*x + 1/2*c)^3 + 18144*a^3*tan(1/2*d*x + 1/2*c)^2 - 55440*a^3*log(abs(tan(
1/2*d*x + 1/2*c))) - 16632*a^3*tan(1/2*d*x + 1/2*c) + (156838*a^3*tan(1/2*d
*x + 1/2*c)^9 + 16632*a^3*tan(1/2*d*x + 1/2*c)^8 - 18144*a^3*tan(1/2*d*x +
1/2*c)^7 - 9744*a^3*tan(1/2*d*x + 1/2*c)^6 + 1512*a^3*tan(1/2*d*x + 1/2*c)^
5 + 3024*a^3*tan(1/2*d*x + 1/2*c)^4 + 672*a^3*tan(1/2*d*x + 1/2*c)^3 - 324*
a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/tan(1/2
*d*x + 1/2*c)^9)/d
```

Mupad [B]
time = 9.40, size = 357, normalized size = 1.78

$$\frac{3 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^4}{256 d} - \frac{29 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{384 d} - \frac{9 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^2}{64 d} + \frac{3 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^5}{128 d} + \frac{a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^6}{192 d} - \frac{9 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^7}{3584 d} - \frac{3 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^8}{2048 d} - \frac{a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)^9}{4608 d} + \frac{9 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^2}{64 d} + \frac{29 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{384 d} - \frac{3 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^4}{256 d} - \frac{3 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^5}{128 d} - \frac{a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^6}{192 d} + \frac{9 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^7}{3584 d} + \frac{3 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^8}{2048 d} + \frac{a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^9}{4608 d} - \frac{55 a^3 \ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)\right)}{128 d} + \frac{33 a^3 \cot\left(\frac{c}{2}+\frac{d x}{2}\right)}{256 d} - \frac{33 a^3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{256 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^10,x)
```

```
[Out] (3*a^3*cot(c/2 + (d*x)/2)^4)/(256*d) - (29*a^3*cot(c/2 + (d*x)/2)^3)/(384*d
) - (9*a^3*cot(c/2 + (d*x)/2)^2)/(64*d) + (3*a^3*cot(c/2 + (d*x)/2)^5)/(128
*d) + (a^3*cot(c/2 + (d*x)/2)^6)/(192*d) - (9*a^3*cot(c/2 + (d*x)/2)^7)/(35
84*d) - (3*a^3*cot(c/2 + (d*x)/2)^8)/(2048*d) - (a^3*cot(c/2 + (d*x)/2)^9)/
(4608*d) + (9*a^3*tan(c/2 + (d*x)/2)^2)/(64*d) + (29*a^3*tan(c/2 + (d*x)/2)
^3)/(384*d) - (3*a^3*tan(c/2 + (d*x)/2)^4)/(256*d) - (3*a^3*tan(c/2 + (d*x)
/2)^5)/(128*d) - (a^3*tan(c/2 + (d*x)/2)^6)/(192*d) + (9*a^3*tan(c/2 + (d*x
)/2)^7)/(3584*d) + (3*a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) + (a^3*tan(c/2 + (
d*x)/2)^9)/(4608*d) - (55*a^3*log(tan(c/2 + (d*x)/2)))/(128*d) + (33*a^3*co
t(c/2 + (d*x)/2))/(256*d) - (33*a^3*tan(c/2 + (d*x)/2))/(256*d)
```

3.619 $\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=228

$$\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^9(c+dx)}{3d}+\frac{33a^3\cot(c+dx)\csc(c+dx)}{256d}-\frac{29a^3\cot(c+d}{12}$$

```
[Out] 33/256*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-1/3*a^3*cot(d*x+c)^
9/d+33/256*a^3*cot(d*x+c)*csc(d*x+c)/d-29/128*a^3*cot(d*x+c)*csc(d*x+c)^3/d
+5/16*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d-3/8*a^3*cot(d*x+c)^5*csc(d*x+c)^3/d-1
/32*a^3*cot(d*x+c)*csc(d*x+c)^5/d+1/16*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10
*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.30, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 30, 2691, 3853, 3855, 14}

$$-\frac{a^3\cot^9(c+dx)}{3d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{16d}+\frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{16d}-\frac{a^3\cot(c+dx)\csc^5(c+dx)}{32d}-\frac{29a^3\cot(c+dx)\csc^3(c+dx)}{128d}+\frac{33a^3\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (33*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a^
3*Cot[c + d*x]^9)/(3*d) + (33*a^3*Cot[c + d*x]*Csc[c + d*x])/(256*d) - (29*
a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) + (5*a^3*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^3)/(8*d) - (a^3*Cot[c + d
*x]*Csc[c + d*x]^5)/(32*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (
a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^6(c+dx)\csc^3\right.$$

$$= a^3\int \cot^6(c+dx)\csc^2(c+dx)\,dx + a^3\int \cot^6(c+dx)$$

$$= -\frac{3a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{a^3\cot^5(c+dx)\csc^5(}{10d}$$

$$= -\frac{a^3\cot^7(c+dx)}{7d} + \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{16d} - 3a$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} - \frac{15a^3\cot(c+dx}{64}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{3d} + \frac{15a^3\cot(c+dx}{128}$$

$$= \frac{15a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - a^3\cot^9$$

$$= \frac{33a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - a^3\cot^9$$

Mathematica [A]
time = 1.46, size = 365, normalized size = 1.60

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(51200*Cot[(c + d*x)/2] + 13860*Csc[(c + d*x)/2]^
2 + 55440*Log[Cos[(c + d*x)/2]] - 55440*Log[Sin[(c + d*x)/2]] - 13860*Sec[(
c + d*x)/2]^2 + 19320*Sec[(c + d*x)/2]^4 - 5250*Sec[(c + d*x)/2]^6 + 315*Se
c[(c + d*x)/2]^8 + 42*Sec[(c + d*x)/2]^10 + 164800*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 3840*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + Csc[(c + d*x)/2]^6*(52
50 - 60*Sin[c + d*x]) - 14*Csc[(c + d*x)/2]^10*(3 + 10*Sin[c + d*x]) + 5*Cs
c[(c + d*x)/2]^8*(-63 + 172*Sin[c + d*x]) - 20*Csc[(c + d*x)/2]^4*(966 + 51
5*Sin[c + d*x]) - 51200*Tan[(c + d*x)/2] - 1720*Sec[(c + d*x)/2]^6*Tan[(c +
 d*x)/2] + 280*Sec[(c + d*x)/2]^8*Tan[(c + d*x)/2]))/(430080*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.32, size = 334, normalized size = 1.46

method	result

risch	$-\frac{a^3\left(3465\,\mathrm{e}^{19i(dx+c)}+74025\,\mathrm{e}^{17i(dx+c)}+46080i\mathrm{e}^{4i(dx+c)}-48468\,\mathrm{e}^{15i(dx+c)}+376320i\mathrm{e}^{8i(dx+c)}-139860\,\mathrm{e}^{13i(dx+c)}-66\right.}{}$
derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-3\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x+c)^8*cos(d*x+c)^7-1
/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/sin(d
*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+c)^3-3/256*cos(d*x+c
)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/
63/sin(d*x+c)^7*cos(d*x+c)^7)+3*a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/si
n(d*x+c)^6*cos(d*x+c)^7+1/192/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*
cos(d*x+c)^7-1/128*cos(d*x+c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*l
n(csc(d*x+c)-cot(d*x+c)))-1/7*a^3/sin(d*x+c)^7*cos(d*x+c)^7)
```

Maxima [A]
time = 0.32, size = 286, normalized size = 1.25

$$-\frac{21\,a^3\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+210\,a^3\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{7680\,a^3}{\tan(dx+c)^7}+\frac{2560\left(9\tan(dx+c)^2+7\right)a^3}{\tan(dx+c)^9}}{53760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/53760*(21*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*a^3*(2*(15*cos(d*x +
 c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x +
 c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*lo
g(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 7680*a^3/tan(d*x + c)^7 +
 2560*(9*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9)/d
```

Fricas [A]
time = 0.43, size = 327, normalized size = 1.43

$$-\frac{6930\,a^3\cos\left(dx+c\right)^9+21420\,a^3\cos\left(dx+c\right)^7-59136\,a^3\cos\left(dx+c\right)^5+32340\,a^3\cos\left(dx+c\right)^3-6930\,a^3\cos\left(dx+c\right)-3465\left(a^3\cos\left(dx+c\right)^{10}-5\,a^3\cos\left(dx+c\right)^8+10\,a^3\cos\left(dx+c\right)^6-10\,a^3\cos\left(dx+c\right)^4+5\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3465\left(a^3\cos\left(dx+c\right)^{10}-5\,a^3\cos\left(dx+c\right)^8+10\,a^3\cos\left(dx+c\right)^6-10\,a^3\cos\left(dx+c\right)^4+5\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2560\left(5\,a^3\cos\left(dx+c\right)^9-12\,a^3\cos\left(dx+c\right)^7\right)\sin\left(dx+c\right)}{53760\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^8+10\,d\cos\left(dx+c\right)^6-10\,d\cos\left(dx+c\right)^4+5\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/53760*(6930*a^3*cos(d*x + c)^9 + 21420*a^3*cos(d*x + c)^7 - 59136*a^3*co
s(d*x + c)^5 + 32340*a^3*cos(d*x + c)^3 - 6930*a^3*cos(d*x + c) - 3465*(a^3
*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*co
s(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(1/2*cos(d*x + c) + 1/2) + 34
65*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10
*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1
/2) + 2560*(5*a^3*cos(d*x + c)^9 - 12*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*
cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x +
 c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 356, normalized size = 1.56

$$\frac{42\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3570\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-3360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5880\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+16800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+10500\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-55440\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-31920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{162382\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+31920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-10500\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-16800\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-5880\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3360\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3570\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-525\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-42\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{430080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/430080*(42*a^3*tan(1/2*d*x + 1/2*c)^10 + 280*a^3*tan(1/2*d*x + 1/2*c)^9 +
 525*a^3*tan(1/2*d*x + 1/2*c)^8 - 600*a^3*tan(1/2*d*x + 1/2*c)^7 - 3570*a^3
*tan(1/2*d*x + 1/2*c)^6 - 3360*a^3*tan(1/2*d*x + 1/2*c)^5 + 5880*a^3*tan(1/
2*d*x + 1/2*c)^4 + 16800*a^3*tan(1/2*d*x + 1/2*c)^3 + 10500*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 55440*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 31920*a^3*tan(1/2*
d*x + 1/2*c) + (162382*a^3*tan(1/2*d*x + 1/2*c)^10 + 31920*a^3*tan(1/2*d*x
+ 1/2*c)^9 - 10500*a^3*tan(1/2*d*x + 1/2*c)^8 - 16800*a^3*tan(1/2*d*x + 1/2
*c)^7 - 5880*a^3*tan(1/2*d*x + 1/2*c)^6 + 3360*a^3*tan(1/2*d*x + 1/2*c)^5 +
 3570*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a^3*tan(1/2*d*x + 1/2*c)^3 - 525*a^3
*tan(1/2*d*x + 1/2*c)^2 - 280*a^3*tan(1/2*d*x + 1/2*c) - 42*a^3)/tan(1/2*d*
x + 1/2*c)^10)/d
```

Mupad [B]
time = 9.65, size = 395, normalized size = 1.73

$$\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{128\,d}-\frac{7\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}-\frac{25\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}+\frac{17\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}+\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}-\frac{5\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{1536\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}+\frac{25\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{1024\,d}+\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{128\,d}+\frac{7\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{512\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{17\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2048\,d}-\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3584\,d}+\frac{5\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{4096\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{1536\,d}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{10240\,d}-\frac{33\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{256\,d}+\frac{19\,a^3\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}-\frac{19\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^11,x)

[Out] (a^3*cot(c/2 + (d*x)/2)^5)/(128*d) - (5*a^3*cot(c/2 + (d*x)/2)^3)/(128*d) - (7*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) - (25*a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (17*a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) + (5*a^3*cot(c/2 + (d*x)/2)^7)/(3584*d) - (5*a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)^9)/(1536*d) - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) + (25*a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (5*a^3*tan(c/2 + (d*x)/2)^3)/(128*d) + (7*a^3*tan(c/2 + (d*x)/2)^4)/(512*d) - (a^3*tan(c/2 + (d*x)/2)^5)/(128*d) - (17*a^3*tan(c/2 + (d*x)/2)^6)/(2048*d) - (5*a^3*tan(c/2 + (d*x)/2)^7)/(3584*d) + (5*a^3*tan(c/2 + (d*x)/2)^8)/(4096*d) + (a^3*tan(c/2 + (d*x)/2)^9)/(1536*d) + (a^3*tan(c/2 + (d*x)/2)^10)/(10240*d) - (33*a^3*log(tan(c/2 + (d*x)/2)))/(256*d) + (19*a^3*cot(c/2 + (d*x)/2))/(256*d) - (19*a^3*tan(c/2 + (d*x)/2))/(256*d)

3.620 $\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=246

$$\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{a^3\cot^{11}(c+dx)}{11d}+\frac{19a^3\cot(c+dx)\csc(c+d}{256d}$$

```
[Out] 19/256*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-5/9*a^3*cot(d*x+c)^
9/d-1/11*a^3*cot(d*x+c)^11/d+19/256*a^3*cot(d*x+c)*csc(d*x+c)/d-7/128*a^3*c
ot(d*x+c)*csc(d*x+c)^3/d+5/48*a^3*cot(d*x+c)^3*csc(d*x+c)^3/d-1/8*a^3*cot(d
*x+c)^5*csc(d*x+c)^3/d-3/32*a^3*cot(d*x+c)*csc(d*x+c)^5/d+3/16*a^3*cot(d*x+
c)^3*csc(d*x+c)^5/d-3/10*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.31, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2691, 3853, 3855, 2687, 14, 276}

$$-\frac{a^3\cot^{11}(c+dx)}{11d}-\frac{5a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}-\frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d}+\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{16d}+\frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{48d}-\frac{3a^3\cot(c+dx)\csc^5(c+dx)}{32d}-\frac{7a^3\cot(c+dx)\csc^3(c+dx)}{128d}+\frac{19a^3\cot(c+dx)\csc(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (19*a^3*ArcTanh[Cos[c + d*x]])/(256*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (5*
a^3*Cot[c + d*x]^9)/(9*d) - (a^3*Cot[c + d*x]^11)/(11*d) + (19*a^3*Cot[c +
d*x]*Csc[c + d*x]])/(256*d) - (7*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*d) +
(5*a^3*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*d) - (a^3*Cot[c + d*x]^5*Csc[c +
d*x]^3)/(8*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(32*d) + (3*a^3*Cot[c +
 d*x]^3*Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^3(c+dx)+3a^3\cot^6(c+dx)\csc^4\right.$$

$$= a^3\int \cot^6(c+dx)\csc^3(c+dx)\,dx + a^3\int \cot^6(c+dx)$$

$$= -\frac{a^3\cot^5(c+dx)\csc^3(c+dx)}{8d} - \frac{3a^3\cot^5(c+dx)\csc^5(}{10d}$$

$$= \frac{5a^3\cot^3(c+dx)\csc^3(c+dx)}{48d} - \frac{a^3\cot^5(c+dx)\csc^3(c}{8d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+d}{11d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9(c+dx)}{9d} - \frac{a^3\cot^{11}(c+d}{11d}$$

$$= \frac{5a^3\tanh^{-1}(\cos(c+dx))}{128d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot^9}{9}$$

$$= \frac{19a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{5a^3\cot}{}$$

Mathematica [A]
time = 2.82, size = 187, normalized size = 0.76

$$\frac{a^3(1+\sin(c+dx))^3\left(16853760\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(10050560+12423680\cos(2(c+dx))+839680\cos(4(c+dx))-2149120\cos(6(c+dx))-568320\cos(8(c+dx))+47360\cos(10(c+dx))+14477694\sin(c+dx)+5875716\sin(3(c+dx))+7902972\sin(5(c+dx))-414645\sin(7(c+dx))-65835\sin(9(c+dx)))\right)}{227082240d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(16853760*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(10050560 + 12423680*Cos[2*(c + d*x
)] + 839680*Cos[4*(c + d*x)] - 2149120*Cos[6*(c + d*x)] - 568320*Cos[8*(c +
 d*x)] + 47360*Cos[10*(c + d*x)] + 14477694*Sin[c + d*x] + 5875716*Sin[3*(c
 + d*x)] + 7902972*Sin[5*(c + d*x)] - 414645*Sin[7*(c + d*x)] - 65835*Sin[9
*(c + d*x)])))/(227082240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.36, size = 372, normalized size = 1.51

method	result
risch	$-\frac{a^3\left(65835\,e^{21i(dx+c)}+112640ie^{4i(dx+c)}+480480\,e^{19i(dx+c)}-28892160ie^{8i(dx+c)}-7488327\,e^{17i(dx+c)}+9123840ie^{6i(dx+}\right.}{}$

derivativedivides	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+3a^3\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(d}{640\sin(d}\right.}{}$
default	$\frac{a^3\left(-\frac{\cos^7(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^7(dx+c)\right)}{99\sin(dx+c)^9}-\frac{8\left(\cos^7(dx+c)\right)}{693\sin(dx+c)^7}\right)+3a^3\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(d}{640\sin(d}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8
/693/sin(d*x+c)^7*cos(d*x+c)^7)+3*a^3*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/8
0/sin(d*x+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c
)^4*cos(d*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256
*cos(d*x+c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/9
/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7)+a^3*(-1/8/sin(d*
x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192/sin(d*x+c)^4*cos(d
*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)^5-5/384*cos(d*x+c)
^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.30, size = 308, normalized size = 1.25

$$-\frac{2079\,a^3\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+2310\,a^3\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{84480\left(9\tan(dx+c)^2+7\right)a^3}{\tan(dx+c)^9}+\frac{2560\left(99\tan(dx+c)^4+154\tan(dx+c)^2+63\right)a^3}{\tan(dx+c)^{11}}}{1774080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/1774080*(2079*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*
x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*
x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2310*a^3*(2*(15*cos(
d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) -
15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 84480*(9*tan(d*x + c
)^2 + 7)*a^3/tan(d*x + c)^9 + 2560*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2
+ 63)*a^3/tan(d*x + c)^11)/d
```

Fricas [A]
time = 0.42, size = 360, normalized size = 1.46

$$\frac{189440\,a^3\cos\left(dx+c\right)^{11}-1041920\,a^3\cos\left(dx+c\right)^9+1013760\,a^3\cos\left(dx+c\right)^7+65835\left(a^3\cos\left(dx+c\right)^{10}-5\,a^3\cos\left(dx+c\right)^8+10\,a^3\cos\left(dx+c\right)^6-10\,a^3\cos\left(dx+c\right)^4+5\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-65835\left(a^3\cos\left(dx+c\right)^{10}-5\,a^3\cos\left(dx+c\right)^8+10\,a^3\cos\left(dx+c\right)^6-10\,a^3\cos\left(dx+c\right)^4+5\,a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-462\left(285\,a^3\cos\left(dx+c\right)^9-50\,a^3\cos\left(dx+c\right)^7-2432\,a^3\cos\left(dx+c\right)^5+1330\,a^3\cos\left(dx+c\right)^3-285\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1774080\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^8+10\,d\cos\left(dx+c\right)^6-10\,d\cos\left(dx+c\right)^4+5\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] 1/1774080*(189440*a^3*cos(d*x + c)^11 - 1041920*a^3*cos(d*x + c)^9 + 101376
0*a^3*cos(d*x + c)^7 + 65835*(a^3*cos(d*x + c)^10 - 5*a^3*cos(d*x + c)^8 +
10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*cos(d*x + c)^2 - a^3)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 65835*(a^3*cos(d*x + c)^10 - 5*
a^3*cos(d*x + c)^8 + 10*a^3*cos(d*x + c)^6 - 10*a^3*cos(d*x + c)^4 + 5*a^3*
cos(d*x + c)^2 - a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 462*(285*
a^3*cos(d*x + c)^9 - 50*a^3*cos(d*x + c)^7 - 2432*a^3*cos(d*x + c)^5 + 1330
*a^3*cos(d*x + c)^3 - 285*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 388, normalized size = 1.58

$$\frac{630\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4158\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+8470\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-40590\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-57750\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6930\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+138600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+244860\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+152460\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1053360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-568260\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3181018\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+568260\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-152460\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-244860\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-138600\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-6930\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+57750\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+40590\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3465\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-8470\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4158\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-630\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{14192640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/14192640*(630*a^3*tan(1/2*d*x + 1/2*c)^11 + 4158*a^3*tan(1/2*d*x + 1/2*c)
^10 + 8470*a^3*tan(1/2*d*x + 1/2*c)^9 - 3465*a^3*tan(1/2*d*x + 1/2*c)^8 - 4
0590*a^3*tan(1/2*d*x + 1/2*c)^7 - 57750*a^3*tan(1/2*d*x + 1/2*c)^6 + 6930*a
^3*tan(1/2*d*x + 1/2*c)^5 + 138600*a^3*tan(1/2*d*x + 1/2*c)^4 + 244860*a^3*
tan(1/2*d*x + 1/2*c)^3 + 152460*a^3*tan(1/2*d*x + 1/2*c)^2 - 1053360*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 568260*a^3*tan(1/2*d*x + 1/2*c) + (3181018*a
^3*tan(1/2*d*x + 1/2*c)^11 + 568260*a^3*tan(1/2*d*x + 1/2*c)^10 - 152460*a^
3*tan(1/2*d*x + 1/2*c)^9 - 244860*a^3*tan(1/2*d*x + 1/2*c)^8 - 138600*a^3*t
an(1/2*d*x + 1/2*c)^7 - 6930*a^3*tan(1/2*d*x + 1/2*c)^6 + 57750*a^3*tan(1/2
*d*x + 1/2*c)^5 + 40590*a^3*tan(1/2*d*x + 1/2*c)^4 + 3465*a^3*tan(1/2*d*x +
 1/2*c)^3 - 8470*a^3*tan(1/2*d*x + 1/2*c)^2 - 4158*a^3*tan(1/2*d*x + 1/2*c)
 - 630*a^3)/tan(1/2*d*x + 1/2*c)^11)/d
```

Mupad [B]
time = 9.98, size = 433, normalized size = 1.76

$$\frac{25\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6144\,d}-\frac{53\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}-\frac{5\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{11\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{41\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14336\,d}+\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4096\,d}-\frac{11\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{18432\,d}-\frac{3\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{22528\,d}+\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{53\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3072\,d}+\frac{5\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{512\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2048\,d}-\frac{25\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{6144\,d}-\frac{41\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{14336\,d}-\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4096\,d}+\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{18432\,d}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{22528\,d}-\frac{19\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{256\,d}+\frac{41\,a^3\,\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}-\frac{41\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^12,x)
```

```
[Out] (25*a^3*cot(c/2 + (d*x)/2)^6)/(6144*d) - (53*a^3*cot(c/2 + (d*x)/2)^3)/(307
2*d) - (5*a^3*cot(c/2 + (d*x)/2)^4)/(512*d) - (a^3*cot(c/2 + (d*x)/2)^5)/(2
048*d) - (11*a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (41*a^3*cot(c/2 + (d*x)/2
)^7)/(14336*d) + (a^3*cot(c/2 + (d*x)/2)^8)/(4096*d) - (11*a^3*cot(c/2 + (d
*x)/2)^9)/(18432*d) - (3*a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (a^3*cot(c/
2 + (d*x)/2)^11)/(22528*d) + (11*a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (53*a
^3*tan(c/2 + (d*x)/2)^3)/(3072*d) + (5*a^3*tan(c/2 + (d*x)/2)^4)/(512*d) +
(a^3*tan(c/2 + (d*x)/2)^5)/(2048*d) - (25*a^3*tan(c/2 + (d*x)/2)^6)/(6144*d
) - (41*a^3*tan(c/2 + (d*x)/2)^7)/(14336*d) - (a^3*tan(c/2 + (d*x)/2)^8)/(4
096*d) + (11*a^3*tan(c/2 + (d*x)/2)^9)/(18432*d) + (3*a^3*tan(c/2 + (d*x)/2
)^10)/(10240*d) + (a^3*tan(c/2 + (d*x)/2)^11)/(22528*d) - (19*a^3*log(tan(c
/2 + (d*x)/2)))/(256*d) + (41*a^3*cot(c/2 + (d*x)/2))/(1024*d) - (41*a^3*ta
n(c/2 + (d*x)/2))/(1024*d)
```

3.621 $\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=270

$$\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{3a^3\cot^{11}(c+dx)}{11d}+\frac{41a^3\cot(c+dx)\csc(c+}{1024d}$$

```
[Out] 41/1024*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-7/9*a^3*cot(d*x+c)
^9/d-3/11*a^3*cot(d*x+c)^11/d+41/1024*a^3*cot(d*x+c)*csc(d*x+c)/d+41/1536*a
^3*cot(d*x+c)*csc(d*x+c)^3/d-35/384*a^3*cot(d*x+c)*csc(d*x+c)^5/d+3/16*a^3*
cot(d*x+c)^3*csc(d*x+c)^5/d-3/10*a^3*cot(d*x+c)^5*csc(d*x+c)^5/d-1/64*a^3*c
ot(d*x+c)*csc(d*x+c)^7/d+1/24*a^3*cot(d*x+c)^3*csc(d*x+c)^7/d-1/12*a^3*cot(
d*x+c)^5*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.34, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2687, 14, 2691, 3853, 3855, 276}

$$-\frac{3a^3\cot^{11}(c+dx)}{11d}-\frac{7a^3\cot^9(c+dx)}{9d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{12d}-\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{24d}+\frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{16d}-\frac{a^3\cot(c+dx)\csc^7(c+dx)}{64d}-\frac{35a^3\cot(c+dx)\csc^5(c+dx)}{384d}+\frac{41a^3\cot(c+dx)\csc^3(c+dx)}{1536d}+\frac{41a^3\cot(c+dx)\csc(c+dx)}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (41*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (7
*a^3*Cot[c + d*x]^9)/(9*d) - (3*a^3*Cot[c + d*x]^11)/(11*d) + (41*a^3*Cot[c
 + d*x]*Csc[c + d*x])/(1024*d) + (41*a^3*Cot[c + d*x]*Csc[c + d*x]^3)/(1536
*d) - (35*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(384*d) + (3*a^3*Cot[c + d*x]^3*
Csc[c + d*x]^5)/(16*d) - (3*a^3*Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*d) - (a^
3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*x]^3*Csc[c + d*x]^7)
/(24*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(12*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))^3\,dx = \int \left(a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^6(c+dx)\csc^5\right.$$

$$= a^3\int \cot^6(c+dx)\csc^4(c+dx)\,dx + a^3\int \cot^6(c+dx)$$

$$= -\frac{3a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(}{12d}$$

$$= \frac{3a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{3a^3\cot^5(c+dx)\csc^5(c}{10d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+}{11d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+}{11d}$$

$$= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9(c+dx)}{9d} - \frac{3a^3\cot^{11}(c+}{11d}$$

$$= \frac{9a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot^9}{9}$$

$$= \frac{41a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{7a^3\cot}{}$$

Mathematica [A]
time = 3.24, size = 197, normalized size = 0.73

$$\frac{a^3(1+\sin(c+dx))^3\left(72737280\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{11}(c+dx)(91311066+62609778\cos(2(c+dx))+22551144\cos(4(c+dx))-23426403\cos(6(c+dx))-1799490\cos(8(c+dx))+142065\cos(10(c+dx))+49776640\sin(c+dx)+84039680\sin(3(c+dx))+38118400\sin(5(c+dx))+2206720\sin(7(c+dx))-1530880\sin(9(c+dx))+117760\sin(11(c+dx)))\right)}{1816657920d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(72737280*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d
*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^11*(91311066 + 62609778*Cos[2*(c + d*x
)] + 22551144*Cos[4*(c + d*x)] - 23426403*Cos[6*(c + d*x)] - 1799490*Cos[8*
(c + d*x)] + 142065*Cos[10*(c + d*x)] + 49776640*Sin[c + d*x] + 84039680*Si
n[3*(c + d*x)] + 38118400*Sin[5*(c + d*x)] + 2206720*Sin[7*(c + d*x)] - 153
0880*Sin[9*(c + d*x)] + 117760*Sin[11*(c + d*x)])))/(1816657920*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.36, size = 408, normalized size = 1.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/12/sin(d*x+c)^12*cos(d*x+c)^7-1/24/sin(d*x+c)^10*cos(d*x+c)^7-
1/64/sin(d*x+c)^8*cos(d*x+c)^7-1/384/sin(d*x+c)^6*cos(d*x+c)^7+1/1536/sin(d
*x+c)^4*cos(d*x+c)^7-1/1024/sin(d*x+c)^2*cos(d*x+c)^7-1/1024*cos(d*x+c)^5-5
/3072*cos(d*x+c)^3-5/1024*cos(d*x+c)-5/1024*ln(csc(d*x+c)-cot(d*x+c)))+3*a^
3*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8/693/si
n(d*x+c)^7*cos(d*x+c)^7)+3*a^3*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d
*x+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos
(d*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*
x+c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-1/9/sin(d*x+
c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7))
```

Maxima [**A**]
time = 0.30, size = 348, normalized size = 1.29

$$-\frac{1155\,a^3\left(\frac{2\left(15\,\cos(dx+c)^{11}-85\,\cos(dx+c)^9+198\,\cos(dx+c)^7+198\,\cos(dx+c)^5-85\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^{12}-6\,\cos(dx+c)^{10}+15\,\cos(dx+c)^8-20\,\cos(dx+c)^6+15\,\cos(dx+c)^4-6\,\cos(dx+c)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+8316\,a^3\left(\frac{2\left(15\,\cos(dx+c)^9-70\,\cos(dx+c)^7-128\,\cos(dx+c)^5+70\,\cos(dx+c)^3-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^8+10\,\cos(dx+c)^6-10\,\cos(dx+c)^4+5\,\cos(dx+c)^2-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{112640\left(9\,\tan(dx+c)^2+7\right)a^3}{\tan(dx+c)^9}+\frac{30720\left(99\,\tan(dx+c)^4+154\,\tan(dx+c)^2+63\right)a^3}{\tan(dx+c)^{11}}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

```
[Out] -1/7096320*(1155*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos(d
*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(
d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 + 1
5*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*lo
g(cos(d*x + c) - 1)) + 8316*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 -
 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10
 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c
)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 112640*(9
*tan(d*x + c)^2 + 7)*a^3/tan(d*x + c)^9 + 30720*(99*tan(d*x + c)^4 + 154*ta
n(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11)/d
```

Fricas [**A**]
time = 0.43, size = 384, normalized size = 1.42

$$-\frac{284130\,a^3\cos\left(dx+c\right)^{11}-1610070\,a^3\cos\left(dx+c\right)^9-507276\,a^3\cos\left(dx+c\right)^7+3750516\,a^3\cos\left(dx+c\right)^5-1610070\,a^3\cos\left(dx+c\right)^3+284130\,a^3\cos\left(dx+c\right)-142065\left(a^3\cos\left(dx+c\right)^{12}-6\,a^3\cos\left(dx+c\right)^{10}+15\,a^3\cos\left(dx+c\right)^8-20\,a^3\cos\left(dx+c\right)^6+15\,a^3\cos\left(dx+c\right)^4-6\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+142065\left(a^3\cos\left(dx+c\right)^{12}-6\,a^3\cos\left(dx+c\right)^{10}+15\,a^3\cos\left(dx+c\right)^8-20\,a^3\cos\left(dx+c\right)^6+15\,a^3\cos\left(dx+c\right)^4-6\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+10240\left(46\,a^3\cos\left(dx+c\right)^{11}-253\,a^3\cos\left(dx+c\right)^9+396\,a^3\cos\left(dx+c\right)^7\right)\sin\left(dx+c\right)}{7096320\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^8-20\,d\cos\left(dx+c\right)^6+15\,d\cos\left(dx+c\right)^4-6\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")
```

```
[Out] -1/7096320*(284130*a^3*cos(d*x + c)^11 - 1610070*a^3*cos(d*x + c)^9 - 50727
6*a^3*cos(d*x + c)^7 + 3750516*a^3*cos(d*x + c)^5 - 1610070*a^3*cos(d*x + c
)^3 + 284130*a^3*cos(d*x + c) - 142065*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x
 + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x +
 c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x + c) + 1/2) + 142065*(a
^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3
```

```
*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(-
1/2*cos(d*x + c) + 1/2) + 10240*(46*a^3*cos(d*x + c)^11 - 253*a^3*cos(d*x +
 c)^9 + 396*a^3*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^12 - 6*d*cos(
d*x + c)^10 + 15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)
^4 - 6*d*cos(d*x + c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**13*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 420, normalized size = 1.56

$$\frac{1155 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{12}+7560 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{11}+16632 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{10}+3080 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{9}-51975 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8}-106920 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}-83160 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}+83160 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+384615 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4}+572880 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+166320 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}-2273040 a^{3} \log \left(\left|\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)\right|\right)-1496880 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)+\frac{7053722 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{12}+1496880 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{11}-166320 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{10}-572880 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{9}-384615 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8}-83160 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}+83160 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}+106920 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+51975 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4}-3080 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}-16632 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}-7560 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)-1155 a^{3}}{\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{12}}}{56770560 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^13*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/56770560*(1155*a^3*tan(1/2*d*x + 1/2*c)^12 + 7560*a^3*tan(1/2*d*x + 1/2*c
)^11 + 16632*a^3*tan(1/2*d*x + 1/2*c)^10 + 3080*a^3*tan(1/2*d*x + 1/2*c)^9
- 51975*a^3*tan(1/2*d*x + 1/2*c)^8 - 106920*a^3*tan(1/2*d*x + 1/2*c)^7 - 83
160*a^3*tan(1/2*d*x + 1/2*c)^6 + 83160*a^3*tan(1/2*d*x + 1/2*c)^5 + 384615*
a^3*tan(1/2*d*x + 1/2*c)^4 + 572880*a^3*tan(1/2*d*x + 1/2*c)^3 + 166320*a^3
*tan(1/2*d*x + 1/2*c)^2 - 2273040*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 1496
880*a^3*tan(1/2*d*x + 1/2*c) + (7053722*a^3*tan(1/2*d*x + 1/2*c)^12 + 14968
80*a^3*tan(1/2*d*x + 1/2*c)^11 - 166320*a^3*tan(1/2*d*x + 1/2*c)^10 - 57288
0*a^3*tan(1/2*d*x + 1/2*c)^9 - 384615*a^3*tan(1/2*d*x + 1/2*c)^8 - 83160*a^
3*tan(1/2*d*x + 1/2*c)^7 + 83160*a^3*tan(1/2*d*x + 1/2*c)^6 + 106920*a^3*ta
n(1/2*d*x + 1/2*c)^5 + 51975*a^3*tan(1/2*d*x + 1/2*c)^4 - 3080*a^3*tan(1/2*
d*x + 1/2*c)^3 - 16632*a^3*tan(1/2*d*x + 1/2*c)^2 - 7560*a^3*tan(1/2*d*x +
1/2*c) - 1155*a^3)/tan(1/2*d*x + 1/2*c)^12)/d
```

Mupad [**B**]
time = 10.40, size = 471, normalized size = 1.74

$$\frac{3 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{6}}{2048 d}-\frac{31 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{3}}{3072 d}-\frac{111 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{4}}{16384 d}-\frac{3 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{5}}{2048 d}-\frac{3 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{2}}{1024 d}+\frac{27 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{7}}{14336 d}+\frac{15 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{8}}{16384 d}-\frac{a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{9}}{18432 d}-\frac{3 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{10}}{10240 d}-\frac{3 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{11}}{22528 d}-\frac{a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)^{12}}{49152 d}+\frac{3 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{2}}{1024 d}+\frac{31 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{3}}{3072 d}+\frac{111 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{4}}{16384 d}+\frac{3 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{5}}{2048 d}-\frac{3 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{6}}{2048 d}-\frac{27 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{7}}{14336 d}-\frac{15 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{8}}{16384 d}+\frac{a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{9}}{18432 d}+\frac{3 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{10}}{10240 d}+\frac{3 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{11}}{22528 d}+\frac{a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)^{12}}{49152 d}-\frac{41 a^{3} \ln \left(\tan \left(\frac{c}{2}+\frac{d x}{2}\right)\right)}{1024 d}+\frac{27 a^{3} \cot \left(\frac{c}{2}+\frac{d x}{2}\right)}{1024 d}-\frac{27 a^{3} \tan \left(\frac{c}{2}+\frac{d x}{2}\right)}{1024 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^13,x)
```

```
[Out] (3*a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (31*a^3*cot(c/2 + (d*x)/2)^3)/(3072
*d) - (111*a^3*cot(c/2 + (d*x)/2)^4)/(16384*d) - (3*a^3*cot(c/2 + (d*x)/2)^
5)/(2048*d) - (3*a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (27*a^3*cot(c/2 + (d*
x)/2)^7)/(14336*d) + (15*a^3*cot(c/2 + (d*x)/2)^8)/(16384*d) - (a^3*cot(c/2
 + (d*x)/2)^9)/(18432*d) - (3*a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (3*a^3
*cot(c/2 + (d*x)/2)^11)/(22528*d) - (a^3*cot(c/2 + (d*x)/2)^12)/(49152*d) +
 (3*a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (31*a^3*tan(c/2 + (d*x)/2)^3)/(307
2*d) + (111*a^3*tan(c/2 + (d*x)/2)^4)/(16384*d) + (3*a^3*tan(c/2 + (d*x)/2)
^5)/(2048*d) - (3*a^3*tan(c/2 + (d*x)/2)^6)/(2048*d) - (27*a^3*tan(c/2 + (d
*x)/2)^7)/(14336*d) - (15*a^3*tan(c/2 + (d*x)/2)^8)/(16384*d) + (a^3*tan(c/
2 + (d*x)/2)^9)/(18432*d) + (3*a^3*tan(c/2 + (d*x)/2)^10)/(10240*d) + (3*a^
3*tan(c/2 + (d*x)/2)^11)/(22528*d) + (a^3*tan(c/2 + (d*x)/2)^12)/(49152*d)
- (41*a^3*log(tan(c/2 + (d*x)/2)))/(1024*d) + (27*a^3*cot(c/2 + (d*x)/2))/(
1024*d) - (27*a^3*tan(c/2 + (d*x)/2))/(1024*d)
```

3.622 $\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=286

$$\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^{13}(c+dx)}{13d}+\frac{27a^3\text{co}}{}$$

```
[Out] 27/1024*a^3*arctanh(cos(d*x+c))/d-4/7*a^3*cot(d*x+c)^7/d-a^3*cot(d*x+c)^9/d
-6/11*a^3*cot(d*x+c)^11/d-1/13*a^3*cot(d*x+c)^13/d+27/1024*a^3*cot(d*x+c)*c
sc(d*x+c)/d+9/512*a^3*cot(d*x+c)*csc(d*x+c)^3/d-3/128*a^3*cot(d*x+c)*csc(d*
x+c)^5/d+1/16*a^3*cot(d*x+c)^3*csc(d*x+c)^5/d-1/10*a^3*cot(d*x+c)^5*csc(d*x
+c)^5/d-3/64*a^3*cot(d*x+c)*csc(d*x+c)^7/d+1/8*a^3*cot(d*x+c)^3*csc(d*x+c)^
7/d-1/4*a^3*cot(d*x+c)^5*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.34, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2952, 2691, 3853, 3855, 2687, 276}

$$-\frac{a^3\cot^{13}(c+dx)}{13d}-\frac{6a^3\cot^{11}(c+dx)}{11d}-\frac{a^3\cot^9(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}+\frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d}-\frac{a^3\cot^5(c+dx)\csc^7(c+dx)}{4d}-\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d}+\frac{a^3\cot^3(c+dx)\csc^7(c+dx)}{8d}+\frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{16d}-\frac{3a^3\cot(c+dx)\csc^7(c+dx)}{64d}-\frac{3a^3\cot(c+dx)\csc^5(c+dx)}{128d}+\frac{9a^3\cot(c+dx)\csc^3(c+dx)}{512d}+\frac{27a^3\cot(c+dx)\csc(c+dx)}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (27*a^3*ArcTanh[Cos[c + d*x]])/(1024*d) - (4*a^3*Cot[c + d*x]^7)/(7*d) - (a
^3*Cot[c + d*x]^9)/d - (6*a^3*Cot[c + d*x]^11)/(11*d) - (a^3*Cot[c + d*x]^1
3)/(13*d) + (27*a^3*Cot[c + d*x]*Csc[c + d*x])/(1024*d) + (9*a^3*Cot[c + d*
x]*Csc[c + d*x]^3)/(512*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^5)/(128*d) +
(a^3*Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*
x]^5)/(10*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x]^7)/(64*d) + (a^3*Cot[c + d*
x]^3*Csc[c + d*x]^7)/(8*d) - (a^3*Cot[c + d*x]^5*Csc[c + d*x]^7)/(4*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^6(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cot^6(c+dx)\csc^5(c+dx)+3a^3\cot^6(c+dx)\csc^6\right.\\
&= a^3\int \cot^6(c+dx)\csc^5(c+dx)\,dx + a^3\int \cot^6(c+dx)\\
&= -\frac{a^3\cot^5(c+dx)\csc^5(c+dx)}{10d} - \frac{a^3\cot^5(c+dx)\csc^7(c}{4d}\\
&= \frac{a^3\cot^3(c+dx)\csc^5(c+dx)}{16d} - \frac{a^3\cot^5(c+dx)\csc^5(c+}{10d}\\
&= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+d}{11d}\\
&= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+d}{11d}\\
&= -\frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(c+dx)}{d} - \frac{6a^3\cot^{11}(c+d}{11d}\\
&= \frac{3a^3\tanh^{-1}(\cos(c+dx))}{256d} - \frac{4a^3\cot^7(c+dx)}{7d} - \frac{a^3\cot^9(}{d}\\
&= \frac{27a^3\tanh^{-1}(\cos(c+dx))}{1024d} - \frac{4a^3\cot^7(c+dx)}{7d} - a^3\cot^9
\end{aligned}
$$

Mathematica [A]
time = 4.36, size = 206, normalized size = 0.72

$$
\frac{a^3(1+\sin(c+dx))^3\left(138378240\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cot(c+dx)\csc^{12}(c+dx)(-200294400-243712000\cos(2(c+dx))-11079680\cos(4(c+dx))+43294720\cos(6(c+dx))+9420800\cos(8(c+dx))-1433600\cos(10(c+dx))+102400\cos(12(c+dx))-194159966\sin(c+dx)-182107926\sin(3(c+dx))-123736613\sin(5(c+dx))+4571567\sin(7(c+dx))+1846845\sin(9(c+dx))-135135\sin(11(c+dx)))\right)}{5248122880d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^8*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(138378240*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c +
d*x)/2]]) + Cot[c + d*x]*Csc[c + d*x]^12*(-200294400 - 243712000*Cos[2*(c +
 d*x)] - 11079680*Cos[4*(c + d*x)] + 43294720*Cos[6*(c + d*x)] + 9420800*Co
s[8*(c + d*x)] - 1433600*Cos[10*(c + d*x)] + 102400*Cos[12*(c + d*x)] - 194
159966*Sin[c + d*x] - 182107926*Sin[3*(c + d*x)] - 123736613*Sin[5*(c + d*x
)] + 4571567*Sin[7*(c + d*x)] + 1846845*Sin[9*(c + d*x)] - 135135*Sin[11*(c
 + d*x)])))/(5248122880*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.40, size = 444, normalized size = 1.55 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a^3*(-1/13/sin(d*x+c)^13*cos(d*x+c)^7-6/143/sin(d*x+c)^11*cos(d*x+c)^7
-8/429/sin(d*x+c)^9*cos(d*x+c)^7-16/3003/sin(d*x+c)^7*cos(d*x+c)^7)+3*a^3*(
-1/12/sin(d*x+c)^12*cos(d*x+c)^7-1/24/sin(d*x+c)^10*cos(d*x+c)^7-1/64/sin(d
*x+c)^8*cos(d*x+c)^7-1/384/sin(d*x+c)^6*cos(d*x+c)^7+1/1536/sin(d*x+c)^4*co
s(d*x+c)^7-1/1024/sin(d*x+c)^2*cos(d*x+c)^7-1/1024*cos(d*x+c)^5-5/3072*cos(
d*x+c)^3-5/1024*cos(d*x+c)-5/1024*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(-1/11/s
in(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8/693/sin(d*x+c)^7
*cos(d*x+c)^7)+a^3*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x+c)^8*cos(
d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d*x+c)^7-3/
1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+c)^3-3/256
*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c))))

Maxima [A]
time = 0.30, size = 368, normalized size = 1.29

$$-\frac{15015\,a^3\left(\frac{2\left(15\cos\left(dx+c\right)^{11}-85\cos\left(dx+c\right)^9+198\cos\left(dx+c\right)^7+198\cos\left(dx+c\right)^5-85\cos\left(dx+c\right)^3+15\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{12}-6\cos\left(dx+c\right)^{10}+15\cos\left(dx+c\right)^8-20\cos\left(dx+c\right)^6+15\cos\left(dx+c\right)^4-6\cos\left(dx+c\right)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+12012\,a^3\left(\frac{2\left(15\cos\left(dx+c\right)^9-70\cos\left(dx+c\right)^7-128\cos\left(dx+c\right)^5+70\cos\left(dx+c\right)^3-15\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{10}-5\cos\left(dx+c\right)^8+10\cos\left(dx+c\right)^6-10\cos\left(dx+c\right)^4+5\cos\left(dx+c\right)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{133120\left(99\tan\left(dx+c\right)^4+154\tan\left(dx+c\right)^2+63\right)a^3}{\tan\left(dx+c\right)^{11}}+\frac{10240\left(429\tan\left(dx+c\right)^6+1001\tan\left(dx+c\right)^4+819\tan\left(dx+c\right)^2+231\right)a^3}{\tan\left(dx+c\right)^{13}}}{30750720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="maxim
a")

[Out] -1/30750720*(15015*a^3*(2*(15*cos(d*x + c)^11 - 85*cos(d*x + c)^9 + 198*cos
(d*x + c)^7 + 198*cos(d*x + c)^5 - 85*cos(d*x + c)^3 + 15*cos(d*x + c))/(co
s(d*x + c)^12 - 6*cos(d*x + c)^10 + 15*cos(d*x + c)^8 - 20*cos(d*x + c)^6 +
 15*cos(d*x + c)^4 - 6*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 12012*a^3*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^
7 - 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)
^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x
+ c)^2 - 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 133120
*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 + 63)*a^3/tan(d*x + c)^11 + 10240*
(429*tan(d*x + c)^6 + 1001*tan(d*x + c)^4 + 819*tan(d*x + c)^2 + 231)*a^3/t
an(d*x + c)^13)/d

Fricas [A]
time = 0.42, size = 417, normalized size = 1.46

$$\frac{409600\,a^3\cos\left(dx+c\right)^{13}-2662400\,a^3\cos\left(dx+c\right)^{11}+7321600\,a^3\cos\left(dx+c\right)^9-5857280\,a^3\cos\left(dx+c\right)^7+135135\left(a^3\cos\left(dx+c\right)^{12}-6\,a^3\cos\left(dx+c\right)^{10}+15\,a^3\cos\left(dx+c\right)^8-20\,a^3\cos\left(dx+c\right)^6+15\,a^3\cos\left(dx+c\right)^4-6\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-135135\left(a^3\cos\left(dx+c\right)^{12}-6\,a^3\cos\left(dx+c\right)^{10}+15\,a^3\cos\left(dx+c\right)^8-20\,a^3\cos\left(dx+c\right)^6+15\,a^3\cos\left(dx+c\right)^4-6\,a^3\cos\left(dx+c\right)^2+a^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2002\left(135\,a^3\cos\left(dx+c\right)^{11}-765\,a^3\cos\left(dx+c\right)^9+758\,a^3\cos\left(dx+c\right)^7+1782\,a^3\cos\left(dx+c\right)^5-765\,a^3\cos\left(dx+c\right)^3+135\,a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{10250240\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^8-20\,d\cos\left(dx+c\right)^6+15\,d\cos\left(dx+c\right)^4-6\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="frica
s")

[Out] 1/10250240*(409600*a^3*cos(d*x + c)^13 - 2662400*a^3*cos(d*x + c)^11 + 7321
600*a^3*cos(d*x + c)^9 - 5857280*a^3*cos(d*x + c)^7 + 135135*(a^3*cos(d*x +
 c)^12 - 6*a^3*cos(d*x + c)^10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c
)^6 + 15*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^2 + a^3)*log(1/2*cos(d*x +

```
c) + 1/2)*sin(d*x + c) - 135135*(a^3*cos(d*x + c)^12 - 6*a^3*cos(d*x + c)^
10 + 15*a^3*cos(d*x + c)^8 - 20*a^3*cos(d*x + c)^6 + 15*a^3*cos(d*x + c)^4
- 6*a^3*cos(d*x + c)^2 + a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
002*(135*a^3*cos(d*x + c)^11 - 765*a^3*cos(d*x + c)^9 + 758*a^3*cos(d*x + c
)^7 + 1782*a^3*cos(d*x + c)^5 - 765*a^3*cos(d*x + c)^3 + 135*a^3*cos(d*x +
c))*sin(d*x + c))/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d*cos(d*x
+ c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x + c)^2 + d
)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**14*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 452, normalized size = 1.58

$$\frac{770\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5005\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+11830\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+8008\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-20020\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-65065\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-94380\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-40040\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+150150\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+385385\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+450450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80080\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2162160\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-1401400\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6875958\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+1401400\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-80080\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-450450\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-385385\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-150150\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+40040\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+94380\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+65065\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20020\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8008\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-11830\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-5005\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-770\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}}}{82001920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^14*(a+a*sin(d*x+c))^3,x, algorithm="giac"
)
```

```
[Out] 1/82001920*(770*a^3*tan(1/2*d*x + 1/2*c)^13 + 5005*a^3*tan(1/2*d*x + 1/2*c)
^12 + 11830*a^3*tan(1/2*d*x + 1/2*c)^11 + 8008*a^3*tan(1/2*d*x + 1/2*c)^10
- 20020*a^3*tan(1/2*d*x + 1/2*c)^9 - 65065*a^3*tan(1/2*d*x + 1/2*c)^8 - 943
80*a^3*tan(1/2*d*x + 1/2*c)^7 - 40040*a^3*tan(1/2*d*x + 1/2*c)^6 + 150150*a
^3*tan(1/2*d*x + 1/2*c)^5 + 385385*a^3*tan(1/2*d*x + 1/2*c)^4 + 450450*a^3*
tan(1/2*d*x + 1/2*c)^3 + 80080*a^3*tan(1/2*d*x + 1/2*c)^2 - 2162160*a^3*log
(abs(tan(1/2*d*x + 1/2*c))) - 1401400*a^3*tan(1/2*d*x + 1/2*c) + (6875958*a
^3*tan(1/2*d*x + 1/2*c)^13 + 1401400*a^3*tan(1/2*d*x + 1/2*c)^12 - 80080*a^
3*tan(1/2*d*x + 1/2*c)^11 - 450450*a^3*tan(1/2*d*x + 1/2*c)^10 - 385385*a^3
*tan(1/2*d*x + 1/2*c)^9 - 150150*a^3*tan(1/2*d*x + 1/2*c)^8 + 40040*a^3*tan
(1/2*d*x + 1/2*c)^7 + 94380*a^3*tan(1/2*d*x + 1/2*c)^6 + 65065*a^3*tan(1/2*
d*x + 1/2*c)^5 + 20020*a^3*tan(1/2*d*x + 1/2*c)^4 - 8008*a^3*tan(1/2*d*x +
1/2*c)^3 - 11830*a^3*tan(1/2*d*x + 1/2*c)^2 - 5005*a^3*tan(1/2*d*x + 1/2*c)
 - 770*a^3)/tan(1/2*d*x + 1/2*c)^13)/d
```

Mupad [B]
time = 10.90, size = 509, normalized size = 1.78

$$\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{45\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8192\,d}-\frac{77\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}-\frac{15\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192\,d}-\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{33\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{28672\,d}+\frac{13\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{16384\,d}+\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4096\,d}-\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{13\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{90112\,d}-\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{16384\,d}-\frac{a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{106496\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{1024\,d}+\frac{45\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8192\,d}+\frac{77\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16384\,d}+\frac{15\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8192\,d}-\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2048\,d}-\frac{33\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{28672\,d}-\frac{13\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{16384\,d}-\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4096\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}+\frac{13\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{90112\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{16384\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{106496\,d}-\frac{27\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{1024\,d}+\frac{35\,a^3\,\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2048\,d}-\frac{35\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2048\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^3)/sin(c + d*x)^14,x)

[Out] (a^3*cot(c/2 + (d*x)/2)^6)/(2048*d) - (45*a^3*cot(c/2 + (d*x)/2)^3)/(8192*d
) - (77*a^3*cot(c/2 + (d*x)/2)^4)/(16384*d) - (15*a^3*cot(c/2 + (d*x)/2)^5)
/(8192*d) - (a^3*cot(c/2 + (d*x)/2)^2)/(1024*d) + (33*a^3*cot(c/2 + (d*x)/2
)^7)/(28672*d) + (13*a^3*cot(c/2 + (d*x)/2)^8)/(16384*d) + (a^3*cot(c/2 + (
d*x)/2)^9)/(4096*d) - (a^3*cot(c/2 + (d*x)/2)^10)/(10240*d) - (13*a^3*cot(c
/2 + (d*x)/2)^11)/(90112*d) - (a^3*cot(c/2 + (d*x)/2)^12)/(16384*d) - (a^3*
cot(c/2 + (d*x)/2)^13)/(106496*d) + (a^3*tan(c/2 + (d*x)/2)^2)/(1024*d) + (
45*a^3*tan(c/2 + (d*x)/2)^3)/(8192*d) + (77*a^3*tan(c/2 + (d*x)/2)^4)/(1638
4*d) + (15*a^3*tan(c/2 + (d*x)/2)^5)/(8192*d) - (a^3*tan(c/2 + (d*x)/2)^6)/
(2048*d) - (33*a^3*tan(c/2 + (d*x)/2)^7)/(28672*d) - (13*a^3*tan(c/2 + (d*x
)/2)^8)/(16384*d) - (a^3*tan(c/2 + (d*x)/2)^9)/(4096*d) + (a^3*tan(c/2 + (d
*x)/2)^10)/(10240*d) + (13*a^3*tan(c/2 + (d*x)/2)^11)/(90112*d) + (a^3*tan(
c/2 + (d*x)/2)^12)/(16384*d) + (a^3*tan(c/2 + (d*x)/2)^13)/(106496*d) - (27
*a^3*log(tan(c/2 + (d*x)/2)))/(1024*d) + (35*a^3*cot(c/2 + (d*x)/2))/(2048*
d) - (35*a^3*tan(c/2 + (d*x)/2))/(2048*d)

3.623 $\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=178

$$-\frac{135a^4x}{16}+\frac{6a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{4a^4\cos(c+dx)}{d}+\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cot(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-$$

```
[Out] -135/16*a^4*x+6*a^4*arctanh(cos(d*x+c))/d-4*a^4*cos(d*x+c)/d+4/5*a^4*cos(d*
x+c)^5/d-4*a^4*cot(d*x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x
+c)/d-89/16*a^4*cos(d*x+c)*sin(d*x+c)/d+23/24*a^4*cos(d*x+c)*sin(d*x+c)^3/d
+1/6*a^4*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 3853, 2715, 2713}

$$\frac{4a^4\cos^5(c+dx)}{5d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{4a^4\cot(c+dx)}{d}+\frac{a^4\sin^5(c+dx)\cos(c+dx)}{6d}+\frac{23a^4\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{89a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{6a^4\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^4\cot(c+dx)\csc(c+dx)}{d}-\frac{135a^4x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-135*a^4*x)/16 + (6*a^4*ArcTanh[Cos[c + d*x]])/d - (4*a^4*Cos[c + d*x])/d
+ (4*a^4*Cos[c + d*x]^5)/(5*d) - (4*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]
^3)/(3*d) - (2*a^4*Cot[c + d*x]*Csc[c + d*x])/d - (89*a^4*Cos[c + d*x]*Sin[
c + d*x])/(16*d) + (23*a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a^4*Cos[c
 + d*x]*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-14a^{10}-8a^{10}\csc(c+dx)+3a^{10}\csc^2(c+dx)+4\right.}{}$$

$$= -14a^4x + a^4\int \csc^4(c+dx)\,dx - a^4\int \sin^6(c+dx)\,d$$

$$= -14a^4x + \frac{8a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^4\cot(c+dx)}{d}$$

$$= -7a^4x + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d}$$

$$= -\frac{65a^4x}{8} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx}{d}$$

$$= -\frac{135a^4x}{16} + \frac{6a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+d}{d}$$

Mathematica [A]
time = 1.21, size = 229, normalized size = 1.29

$$\frac{a^4(1+\sin(c+dx))^4\left(-8100(c+dx)-3360\cos(c+dx)+240\cos(3(c+dx))+48\cos(5(c+dx))-1760\cot\left(\frac{1}{2}(c+dx)\right)-480\csc^2\left(\frac{1}{2}(c+dx)\right)+5760\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-5760\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+480\sec^2\left(\frac{1}{2}(c+dx)\right)+320\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-20\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-2415\sin(2(c+dx))-135\sin(4(c+dx))+5\sin(6(c+dx))+1760\tan\left(\frac{1}{2}(c+dx)\right)\right)}{960d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-8100*(c + d*x) - 3360*Cos[c + d*x] + 240*Cos[3*
(c + d*x)] + 48*Cos[5*(c + d*x)] - 1760*Cot[(c + d*x)/2] - 480*Csc[(c + d*x
)/2]^2 + 5760*Log[Cos[(c + d*x)/2]] - 5760*Log[Sin[(c + d*x)/2]] + 480*Sec[
(c + d*x)/2]^2 + 320*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 20*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - 2415*Sin[2*(c + d*x)] - 135*Sin[4*(c + d*x)] + 5*Sin[6*(
c + d*x)] + 1760*Tan[(c + d*x)/2]))/(960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^8)
```

Maple [A]
time = 0.21, size = 320, normalized size = 1.80

method	result
risch	$-\frac{135a^4x}{16}+\frac{9ia^4e^{4i(dx+c)}}{128d}+\frac{a^4e^{5i(dx+c)}}{40d}+\frac{161ia^4e^{2i(dx+c)}}{128d}-\frac{161ia^4e^{-2i(dx+c)}}{128d}-\frac{7a^4e^{i(dx+c)}}{4d}-\frac{7a^4e^{-i(dx+c}}{4d}$
derivativedivides	$\frac{a^4\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+4a^4\left(-\frac{\cos^7(dx+c}{2\sin(dx+c}\right.}{}$
default	$\frac{a^4\left(-\frac{\cos^7(dx+c)}{3\sin(dx+c)^3}+\frac{4\left(\cos^7(dx+c)\right)}{3\sin(dx+c)}+\frac{4\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+4a^4\left(-\frac{\cos^7(dx+c}{2\sin(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+4*a
^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*co
s(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+6*a^4*(-1/sin(d*x+c)*cos(d*x+c)^7-(
cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+
4*a^4*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c
)))+a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1
6*d*x+5/16*c))
```

Maxima [A]
time = 0.51, size = 294, normalized size = 1.65

$$\frac{128\left(6\cos(dx+c)^5+10\cos(dx+c)^3+30\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^4-320\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^4-720\left(15dx+15c+\frac{15\tan(dx+c)^4+25\tan(dx+c)^2+8}{\tan(dx+c)^5+2\tan(dx+c)^3+\tan(dx+c)}\right)a^4+160\left(15dx+15c+\frac{15\tan(dx+c)^4+10\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^4}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/960*(128*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log
(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 320*(4*cos(d*x + c)^3
- 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x +
c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 - 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x
- 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 720*(15*d*x + 15*c
 + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x
+ c)^3 + tan(d*x + c)))*a^4 + 160*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*
tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4)/d
```

Fricas [**A**]
time = 0.42, size = 245, normalized size = 1.38

$$-\frac{40\,a^4\cos\left(dx+c\right)^9-390\,a^4\cos\left(dx+c\right)^7-405\,a^4\cos\left(dx+c\right)^5+2700\,a^4\cos\left(dx+c\right)^3-2025\,a^4\cos\left(dx+c\right)-720\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+720\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,\left(64\,a^4\cos\left(dx+c\right)^7-64\,a^4\cos\left(dx+c\right)^5-675\,a^4dx\cos\left(dx+c\right)^2-320\,a^4\cos\left(dx+c\right)^3+675\,a^4dx+480\,a^4\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*a^4*cos(d*x + c)^9 - 390*a^4*cos(d*x + c)^7 - 405*a^4*cos(d*x +
c)^5 + 2700*a^4*cos(d*x + c)^3 - 2025*a^4*cos(d*x + c) - 720*(a^4*cos(d*x +
 c)^2 - a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 720*(a^4*cos(d*x +
c)^2 - a^4)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(64*a^4*cos(d*x +
 c)^7 - 64*a^4*cos(d*x + c)^5 - 675*a^4*d*x*cos(d*x + c)^2 - 320*a^4*cos(d*
x + c)^3 + 675*a^4*d*x + 480*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x +
c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 324, normalized size = 1.82

$$\frac{10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2025\,(dx+c)a^4-1440\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+450\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{10\left(264\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-45\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}+\frac{2\left(1335\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3085\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-3840\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+1110\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7680\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1110\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-7680\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3085\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4608\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1335\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-768\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(10*a^4*tan(1/2*d*x + 1/2*c)^3 + 120*a^4*tan(1/2*d*x + 1/2*c)^2 - 202
5*(d*x + c)*a^4 - 1440*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 450*a^4*tan(1/2
```

```
*d*x + 1/2*c) + 10*(264*a^4*tan(1/2*d*x + 1/2*c)^3 - 45*a^4*tan(1/2*d*x + 1
/2*c)^2 - 12*a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(13
35*a^4*tan(1/2*d*x + 1/2*c)^11 + 3085*a^4*tan(1/2*d*x + 1/2*c)^9 - 3840*a^4
*tan(1/2*d*x + 1/2*c)^8 + 1110*a^4*tan(1/2*d*x + 1/2*c)^7 - 7680*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1110*a^4*tan(1/2*d*x + 1/2*c)^5 - 7680*a^4*tan(1/2*d*x +
 1/2*c)^4 - 3085*a^4*tan(1/2*d*x + 1/2*c)^3 - 4608*a^4*tan(1/2*d*x + 1/2*c)
^2 - 1335*a^4*tan(1/2*d*x + 1/2*c) - 768*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
6)/d
```

Mupad [B]
time = 8.98, size = 474, normalized size = 2.66

$$\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2d}+\frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}-\frac{6a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{135a^4\mathrm{atan}\left(\frac{18225a^8}{64\left(\frac{405a^8}{2}-\frac{18225a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}\right)}+\frac{405a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(\frac{405a^8}{2}-\frac{18225a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}\right)}\right)}{8d}-\frac{-74a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-\frac{346a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}+280a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+153a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+572a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+379a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+592a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{1312a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{1836a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}+184a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{376a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{5}+17a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a^4}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+120\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+160\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+120\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}+\frac{15a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + a*sin(c + d*x))^4)/sin(c + d*x)^4,x)
```

```
[Out] (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (6*a
^4*log(tan(c/2 + (d*x)/2)))/d - (135*a^4*atan((18225*a^8)/(64*((405*a^8)/2
- (18225*a^8*tan(c/2 + (d*x)/2))/64)) + (405*a^8*tan(c/2 + (d*x)/2))/(2*((4
05*a^8)/2 - (18225*a^8*tan(c/2 + (d*x)/2))/64))))/(8*d) - (17*a^4*tan(c/2 +
 (d*x)/2)^2 + (376*a^4*tan(c/2 + (d*x)/2)^3)/5 + 184*a^4*tan(c/2 + (d*x)/2)
^4 + (1836*a^4*tan(c/2 + (d*x)/2)^5)/5 + (1312*a^4*tan(c/2 + (d*x)/2)^6)/3
+ 592*a^4*tan(c/2 + (d*x)/2)^7 + 379*a^4*tan(c/2 + (d*x)/2)^8 + 572*a^4*tan
(c/2 + (d*x)/2)^9 + 153*a^4*tan(c/2 + (d*x)/2)^10 + 280*a^4*tan(c/2 + (d*x)
/2)^11 - (346*a^4*tan(c/2 + (d*x)/2)^12)/3 + 4*a^4*tan(c/2 + (d*x)/2)^13 -
74*a^4*tan(c/2 + (d*x)/2)^14 + a^4/3 + 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(
c/2 + (d*x)/2)^3 + 48*tan(c/2 + (d*x)/2)^5 + 120*tan(c/2 + (d*x)/2)^7 + 160
*tan(c/2 + (d*x)/2)^9 + 120*tan(c/2 + (d*x)/2)^11 + 48*tan(c/2 + (d*x)/2)^1
3 + 8*tan(c/2 + (d*x)/2)^15)) + (15*a^4*tan(c/2 + (d*x)/2))/(8*d)
```

3.624 $\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{3x}{128a}+\frac{\cos^5(c+dx)}{5ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos(c+dx)\sin(c+dx)}{128ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{64ad}-c$$

```
[Out] 3/128*x/a+1/5*cos(d*x+c)^5/a/d-2/7*cos(d*x+c)^7/a/d+1/9*cos(d*x+c)^9/a/d+3/
128*cos(d*x+c)*sin(d*x+c)/a/d+1/64*cos(d*x+c)^3*sin(d*x+c)/a/d-1/16*cos(d*x
+c)^5*sin(d*x+c)/a/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2648, 2715, 8, 2645, 276}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{2\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}+\frac{3\sin(c+dx)\cos(c+dx)}{128ad}+\frac{3x}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(128*a) + Cos[c + d*x]^5/(5*a*d) - (2*Cos[c + d*x]^7)/(7*a*d) + Cos[c
 + d*x]^9/(9*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]
^3*Sin[c + d*x])/(64*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} + \frac{\text{Subst}\Big(\int$$

$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} + \frac{\int \cos^4(c+dx)\,d}{16a}$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{64ad}$$

$$= \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(c+dx)}{128ad}$$

$$= \frac{3x}{128a} + \frac{\cos^5(c+dx)}{5ad} - \frac{2\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} + \frac{3\cos(c+dx)\sin(}{128ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 429 vs. 2(159) = 318.
time = 6.09, size = 429, normalized size = 2.70

$$\frac{2520(5c+6dx)\cos\left(\frac{c}{2}\right)+7560\cos\left(\frac{c}{2}+dx\right)+7560\cos\left(\frac{3c}{2}+dx\right)+1680\cos\left(\frac{5c}{2}+3dx\right)+1680\cos\left(\frac{7c}{2}+3dx\right)-2520\cos\left(\frac{7c}{2}+4dx\right)+2520\cos\left(\frac{9c}{2}+4dx\right)-1008\cos\left(\frac{9c}{2}+5dx\right)-1008\cos\left(\frac{11c}{2}+5dx\right)-180\cos\left(\frac{13c}{2}+7dx\right)-180\cos\left(\frac{15c}{2}+7dx\right)+315\cos\left(\frac{15c}{2}+8dx\right)-315\cos\left(\frac{17c}{2}+8dx\right)+140\cos\left(\frac{17c}{2}+9dx\right)+140\cos\left(\frac{19c}{2}+9dx\right)+12600\sin\left(\frac{c}{2}\right)+12600c\sin\left(\frac{c}{2}\right)+15120dx\sin\left(\frac{c}{2}\right)-7560\sin\left(\frac{c}{2}+dx\right)+7560\sin\left(\frac{3c}{2}+dx\right)-1680\sin\left(\frac{5c}{2}+3dx\right)+1680\sin\left(\frac{7c}{2}+3dx\right)-2520\sin\left(\frac{7c}{2}+4dx\right)-2520\sin\left(\frac{9c}{2}+4dx\right)+1008\sin\left(\frac{9c}{2}+5dx\right)-1008\sin\left(\frac{11c}{2}+5dx\right)+180\sin\left(\frac{13c}{2}+7dx\right)-180\sin\left(\frac{15c}{2}+7dx\right)+315\sin\left(\frac{15c}{2}+8dx\right)+315\sin\left(\frac{17c}{2}+8dx\right)-140\sin\left(\frac{17c}{2}+9dx\right)+140\sin\left(\frac{19c}{2}+9dx\right)}{645120ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2520*(5*c + 6*d*x)*Cos[c/2] + 7560*Cos[c/2 + d*x] + 7560*Cos[(3*c)/2 + d*x
] + 1680*Cos[(5*c)/2 + 3*d*x] + 1680*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/
2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2 + 5*d*x] - 1008*C
os[(11*c)/2 + 5*d*x] - 180*Cos[(13*c)/2 + 7*d*x] - 180*Cos[(15*c)/2 + 7*d*x
] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] + 140*Cos[(17*c)/
2 + 9*d*x] + 140*Cos[(19*c)/2 + 9*d*x] + 12600*Sin[c/2] + 12600*c*Sin[c/2]
+ 15120*d*x*Sin[c/2] - 7560*Sin[c/2 + d*x] + 7560*Sin[(3*c)/2 + d*x] - 1680
*Sin[(5*c)/2 + 3*d*x] + 1680*Sin[(7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*
x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*x] - 1008*Sin[(11*c
)/2 + 5*d*x] + 180*Sin[(13*c)/2 + 7*d*x] - 180*Sin[(15*c)/2 + 7*d*x] + 315*
Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 140*Sin[(17*c)/2 + 9*d*
x] + 140*Sin[(19*c)/2 + 9*d*x])/(645120*a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.15, size = 220, normalized size = 1.38

method	result
risch	$\frac{3x}{128a}+\frac{3\cos(dx+c)}{128ad}+\frac{\cos(9dx+9c)}{2304ad}+\frac{\sin(8dx+8c)}{1024ad}-\frac{\cos(7dx+7c)}{1792ad}-\frac{\cos(5dx+5c)}{320ad}-\frac{\sin(4dx+4c)}{128ad}+\frac{\cos(3d\cdots}{192\cdots}$
derivativedivides	$\frac{32\left(\frac{1}{630}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{155\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-169\cdots\right.}{\cdots}$
default	$\frac{32\left(\frac{1}{630}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70}-\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{155\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-169\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*((1/630-3/2048*tan(1/2*d*x+1/2*c)+1/70*tan(1/2*d*x+1/2*c)^2-13/1024*
tan(1/2*d*x+1/2*c)^3+2/35*tan(1/2*d*x+1/2*c)^4+155/1024*tan(1/2*d*x+1/2*c)^
5-1/5*tan(1/2*d*x+1/2*c)^6-169/1024*tan(1/2*d*x+1/2*c)^7+7/10*tan(1/2*d*x+1
/2*c)^8-1/2*tan(1/2*d*x+1/2*c)^10+169/1024*tan(1/2*d*x+1/2*c)^11+1/3*tan(1/
2*d*x+1/2*c)^12-155/1024*tan(1/2*d*x+1/2*c)^13+13/1024*tan(1/2*d*x+1/2*c)^1
5+3/2048*tan(1/2*d*x+1/2*c)^17)/(1+tan(1/2*d*x+1/2*c)^2)^9+3/2048*arctan(ta
n(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(143) = 286.
time = 0.52, size = 502, normalized size = 3.16

$$-\frac{\frac{\frac{945\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{9216\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8190\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{36864\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{97650\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{129024\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{106470\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{451584\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{322560\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{106470\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{215040\,\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{97650\,\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{8190\,\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{945\,\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-1024}{a+\frac{9\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{36\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{126\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84\,a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{36\,a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{9\,a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}+\frac{a\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}}-\frac{945\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20160*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 9216*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 36864*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 12
9024*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 451584*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 322560*sin(d*x +
c)^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11
- 215040*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos
(d*x + c) + 1)^13 - 8190*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*
x + c)^17/(cos(d*x + c) + 1)^17 - 1024)/(a + 9*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 126*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 84*a*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.39, size = 90, normalized size = 0.57

$$\frac{4480\cos\left(dx+c\right)^{9}-11520\cos\left(dx+c\right)^{7}+8064\cos\left(dx+c\right)^{5}+945\,dx+315\left(16\cos\left(dx+c\right)^{7}-24\cos\left(dx+c\right)^{5}+2\cos\left(dx+c\right)^{3}+3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40320*(4480*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4318 vs. $2(131) = 262$.
time = 105.75, size = 4318, normalized size = 27.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((945*d*x*tan(c/2 + d*x/2)**18/(40320*a*d*tan(c/2 + d*x/2)**18 + 3
62880*a*d*tan(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880
*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*
tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2
 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 8505*d*x*tan(c
/2 + d*x/2)**16/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/
2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**1
```

2 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 33
86880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*
d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 34020*d*x*tan(c/2 + d*x/2)**14/(40320*
a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520*a*d*ta
n(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2
 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x
/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2)**2 +
 40320*a*d) + 79380*d*x*tan(c/2 + d*x/2)**12/(40320*a*d*tan(c/2 + d*x/2)**1
8 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 33
86880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320
*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*ta
n(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 119070*d*
x*tan(c/2 + d*x/2)**10/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2
 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*
x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)*
*8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 36
2880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 119070*d*x*tan(c/2 + d*x/2)**8/
(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520
*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*
tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/
2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/
2)**2 + 40320*a*d) + 79380*d*x*tan(c/2 + d*x/2)**6/(40320*a*d*tan(c/2 + d*x
/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**1
4 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5
080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*
a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 340
20*d*x*tan(c/2 + d*x/2)**4/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan
(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2
+ d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x
/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4
+ 362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 8505*d*x*tan(c/2 + d*x/2)**
2/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 14515
20*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*
d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(
c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*
x/2)**2 + 40320*a*d) + 945*d*x/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d
*tan(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(
c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 +
 d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)
4 + 362880*a*d*tan(c/2 + d*x/2)2 + 40320*a*d) + 1890*tan(c/2 + d*x/2)**
17/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451
520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a
*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan
(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d
*x/2)**2 + 40320*a*d) + 16380*tan(c/2 + d*x/2)**15/(40320*a*d*tan(c/2 + d*x

/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**1
4 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5
080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*
a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) - 195
300*tan(c/2 + d*x/2)**13/(40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c
/2 + d*x/2)**16 + 1451520*a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 +
d*x/2)**12 + 5080320*a*d*tan(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2
)**8 + 3386880*a*d*tan(c/2 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 +
362880*a*d*tan(c/2 + d*x/2)**2 + 40320*a*d) + 430080*tan(c/2 + d*x/2)**12/(
40320*a*d*tan(c/2 + d*x/2)**18 + 362880*a*d*tan(c/2 + d*x/2)**16 + 1451520*
a*d*tan(c/2 + d*x/2)**14 + 3386880*a*d*tan(c/2 + d*x/2)**12 + 5080320*a*d*t
an(c/2 + d*x/2)**10 + 5080320*a*d*tan(c/2 + d*x/2)**8 + 3386880*a*d*tan(c/2
 + d*x/2)**6 + 1451520*a*d*tan(c/2 + d*x/2)**4 + 362880*a*d*tan(c/2 + d*x/2
)**2 + 40320*a*d) + 212940*tan(c/2 + d*x/2)**11/(40320*a*d*tan(c/2 + d*x/2)
18 + 362880*a*d*tan(c/2 + d*x/2)16 + 145152...

Giac [A]
time = 0.45, size = 218, normalized size = 1.37

$$\frac{\frac{945\,(dx+c)}{a}+\frac{2\left(945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}+8190\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}-97650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+215040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+106470\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-322560\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+451584\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}-106470\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-129024\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+97650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+36864\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-8190\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+9216\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1024\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{9}a}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/40320*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2*d*
x + 1/2*c)^15 - 97650*tan(1/2*d*x + 1/2*c)^13 + 215040*tan(1/2*d*x + 1/2*c)
^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 322560*tan(1/2*d*x + 1/2*c)^10 + 451
584*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(1/2*d*x + 1/2*c)^7 - 129024*tan(1/2
*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*c)^5 + 36864*tan(1/2*d*x + 1/2*c)
^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 9216*tan(1/2*d*x + 1/2*c)^2 - 945*tan(1/
2*d*x + 1/2*c) + 1024)/((tan(1/2*d*x + 1/2*c)^2 + 1)^9*a))/d

Mupad [B]
time = 11.41, size = 211, normalized size = 1.33

$$\frac{3\,x}{128\,a}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32}-\frac{155\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{112\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{5}-\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32}-\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5}+\frac{155\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{32}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{35}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32}+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{16}{315}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)

[Out] (3*x)/(128*a) + ((16*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/64 -
 (13*tan(c/2 + (d*x)/2)^3)/32 + (64*tan(c/2 + (d*x)/2)^4)/35 + (155*tan(c/2
 + (d*x)/2)^5)/32 - (32*tan(c/2 + (d*x)/2)^6)/5 - (169*tan(c/2 + (d*x)/2)^7
)/32 + (112*tan(c/2 + (d*x)/2)^8)/5 - 16*tan(c/2 + (d*x)/2)^10 + (169*tan(c
/2 + (d*x)/2)^11)/32 + (32*tan(c/2 + (d*x)/2)^12)/3 - (155*tan(c/2 + (d*x)/
2)^13)/32 + (13*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^17)/64 +
16/315)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)

3.625 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=141

$$-\frac{3x}{128a}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{3\cos(c+dx)\sin(c+dx)}{128ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{16ad}$$

```
[Out] -3/128*x/a-1/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d-3/128*cos(d*x+c)*sin(d
*x+c)/a/d-1/64*cos(d*x+c)^3*sin(d*x+c)/a/d+1/16*cos(d*x+c)^5*sin(d*x+c)/a/d
+1/8*cos(d*x+c)^5*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2645, 14, 2648, 2715, 8}

$$\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{16ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{3\sin(c+dx)\cos(c+dx)}{128ad}-\frac{3x}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(128*a) - Cos[c + d*x]^5/(5*a*d) + Cos[c + d*x]^7/(7*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(128*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a*d) +
(Cos[c + d*x]^5*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(8
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a} \\
&= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a} - \frac{\mathrm{Subst}\left(\int x^4\right.}{} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^5(c+dx)\sin^3(c+dx)}{8ad} - \frac{\int \cos^4(c+dx)\,dx}{16a} \\
&= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{\cos^5(c+dx)}{16a} \\
&= -\frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^3(c+dx)}{64} \\
&= -\frac{3x}{128a} - \frac{\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} - \frac{3\cos(c+dx)\sin(c+dx)}{128ad} - \cos^3
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(141) = 282.
time = 6.17, size = 375, normalized size = 2.66

$$
\frac{1680(c-dx)\cos\left(\frac{c}{2}\right)-1680\cos\left(\frac{c}{2}+dx\right)-1680\cos\left(\frac{3c}{2}+dx\right)-560\cos\left(\frac{5c}{2}+3dx\right)-560\cos\left(\frac{7c}{2}+3dx\right)+280\cos\left(\frac{7c}{2}+4dx\right)-280\cos\left(\frac{9c}{2}+4dx\right)+112\cos\left(\frac{9c}{2}+5dx\right)+112\cos\left(\frac{11c}{2}+5dx\right)+80\cos\left(\frac{13c}{2}+7dx\right)+80\cos\left(\frac{15c}{2}+7dx\right)-35\cos\left(\frac{15c}{2}+8dx\right)+35\cos\left(\frac{17c}{2}+8dx\right)-3360\sin\left(\frac{c}{2}\right)+1680c\sin\left(\frac{c}{2}\right)-1680dx\sin\left(\frac{c}{2}\right)+1680\sin\left(\frac{c}{2}+dx\right)-1680\sin\left(\frac{3c}{2}+dx\right)+560\sin\left(\frac{5c}{2}+3dx\right)-560\sin\left(\frac{7c}{2}+3dx\right)+280\sin\left(\frac{7c}{2}+4dx\right)+280\sin\left(\frac{9c}{2}+4dx\right)-112\sin\left(\frac{9c}{2}+5dx\right)+112\sin\left(\frac{11c}{2}+5dx\right)-80\sin\left(\frac{13c}{2}+7dx\right)+80\sin\left(\frac{15c}{2}+7dx\right)-35\sin\left(\frac{15c}{2}+8dx\right)-35\sin\left(\frac{17c}{2}+8dx\right)}{71680ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1680*(c - d*x)*Cos[c/2] - 1680*Cos[c/2 + d*x] - 1680*Cos[(3*c)/2 + d*x] -
560*Cos[(5*c)/2 + 3*d*x] - 560*Cos[(7*c)/2 + 3*d*x] + 280*Cos[(7*c)/2 + 4*d
*x] - 280*Cos[(9*c)/2 + 4*d*x] + 112*Cos[(9*c)/2 + 5*d*x] + 112*Cos[(11*c)/
2 + 5*d*x] + 80*Cos[(13*c)/2 + 7*d*x] + 80*Cos[(15*c)/2 + 7*d*x] - 35*Cos[(
15*c)/2 + 8*d*x] + 35*Cos[(17*c)/2 + 8*d*x] - 3360*Sin[c/2] + 1680*c*Sin[c/
2] - 1680*d*x*Sin[c/2] + 1680*Sin[c/2 + d*x] - 1680*Sin[(3*c)/2 + d*x] + 56
0*Sin[(5*c)/2 + 3*d*x] - 560*Sin[(7*c)/2 + 3*d*x] + 280*Sin[(7*c)/2 + 4*d*x
] + 280*Sin[(9*c)/2 + 4*d*x] - 112*Sin[(9*c)/2 + 5*d*x] + 112*Sin[(11*c)/2
+ 5*d*x] - 80*Sin[(13*c)/2 + 7*d*x] + 80*Sin[(15*c)/2 + 7*d*x] - 35*Sin[(15
*c)/2 + 8*d*x] - 35*Sin[(17*c)/2 + 8*d*x])/(71680*a*d*(Cos[c/2] + Sin[c/2])
)
```

Maple [A]
time = 0.13, size = 207, normalized size = 1.47

method	result
risch	$-\frac{3x}{128a}-\frac{3\cos(dx+c)}{64ad}-\frac{\sin(8dx+8c)}{1024ad}+\frac{\cos(7dx+7c)}{448ad}+\frac{\cos(5dx+5c)}{320ad}+\frac{\sin(4dx+4c)}{128ad}-\frac{\cos(3dx+3c)}{64ad}$
derivativedivides	$\frac{16\left(-\frac{1}{140}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}-\frac{333\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\right.}{(}$
default	$\frac{16\left(-\frac{1}{140}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024}-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}-\frac{333\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1024}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\right.}{(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*((-1/140+3/1024*tan(1/2*d*x+1/2*c)-2/35*tan(1/2*d*x+1/2*c)^2+23/1024
*tan(1/2*d*x+1/2*c)^3+1/20*tan(1/2*d*x+1/2*c)^4-333/1024*tan(1/2*d*x+1/2*c)
^5-2/5*tan(1/2*d*x+1/2*c)^6+671/1024*tan(1/2*d*x+1/2*c)^7-1/4*tan(1/2*d*x+1
/2*c)^8-671/1024*tan(1/2*d*x+1/2*c)^9+333/1024*tan(1/2*d*x+1/2*c)^11-1/4*ta
n(1/2*d*x+1/2*c)^12-23/1024*tan(1/2*d*x+1/2*c)^13-3/1024*tan(1/2*d*x+1/2*c)
^15)/(1+tan(1/2*d*x+1/2*c)^2)^8-3/1024*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(127) = 254.
time = 0.53, size = 461, normalized size = 3.27

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{2048\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1792\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{11655\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{23485\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{8960\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{23485\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{11655\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{8960\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-256}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{2240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2240*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 2048*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1792*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 11655*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14336*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 23485*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 8960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 23485*sin(d*x + c)^9/(cos
(d*x + c) + 1)^9 + 11655*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8960*sin(d
*x + c)^12/(cos(d*x + c) + 1)^12 - 805*sin(d*x + c)^13/(cos(d*x + c) + 1)^1
3 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 256)/(a + 8*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(
d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x +
c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
a)/d
```

Fricas [A]
time = 0.38, size = 80, normalized size = 0.57

$$\frac{640\,\cos\left(dx+c\right)^{7}-896\,\cos\left(dx+c\right)^{5}-105\,dx-35\,\left(16\,\cos\left(dx+c\right)^{7}-24\,\cos\left(dx+c\right)^{5}+2\,\cos\left(dx+c\right)^{3}+3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*cos(d*x + c)^7 - 896*cos(d*x + c)^5 - 105*d*x - 35*(16*cos(d*x
+ c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c
))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(116) = 232$.
time = 71.02, size = 3580, normalized size = 25.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**16/(4480*a*d*tan(c/2 + d*x/2)**16 + 3
5840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*
d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/
2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)
**2 + 4480*a*d) - 840*d*x*tan(c/2 + d*x/2)**14/(4480*a*d*tan(c/2 + d*x/2)**
16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250
880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*
tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 +
d*x/2)**2 + 4480*a*d) - 2940*d*x*tan(c/2 + d*x/2)**12/(4480*a*d*tan(c/2 + d
```

*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**1
2 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8 + 2508
80*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan
(c/2 + d*x/2)**2 + 4480*a*d) - 5880*d*x*tan(c/2 + d*x/2)**10/(4480*a*d*tan(
c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/2 + d*
x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x/2)**8
 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 + 35840*
a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 7350*d*x*tan(c/2 + d*x/2)**8/(4480*a*
d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(c/
2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d*x
/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4 +
35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 5880*d*x*tan(c/2 + d*x/2)**6/(4
480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*
tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2
 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)
4 + 35840*a*d*tan(c/2 + d*x/2)2 + 4480*a*d) - 2940*d*x*tan(c/2 + d*x/2)
4/(4480*a*d*tan(c/2 + d*x/2)16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 12544
0*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*t
an(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 +
d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 840*d*x*tan(c/2 + d
*x/2)**2/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 +
125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*
a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c
/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 105*d*x/(4480*
a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*tan(
c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 + d
*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**4
+ 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 210*tan(c/2 + d*x/2)**15/(448
0*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d*ta
n(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/2 +
 d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2)**
4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 1610*tan(c/2 + d*x/2)**13/(
4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440*a*d
*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*tan(c/
2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d*x/2
)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 17920*tan(c/2 + d*x/2)**
12/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 125440
*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*d*ta
n(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2 + d
*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) + 23310*tan(c/2 + d*x/
2)**11/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 + 12
5440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600*a*
d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(c/2
 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 46970*tan(c/2 +
d*x/2)**9/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**14 +

```
 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313600
*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*tan(
c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 17920*tan(c/2
 + d*x/2)**8/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)**1
4 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 + 313
600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*d*t
an(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) + 46970*tan(
c/2 + d*x/2)**7/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x/2)
**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250880*a*d*tan(c/2 + d*x/2)**10 +
313600*a*d*tan(c/2 + d*x/2)**8 + 250880*a*d*tan(c/2 + d*x/2)**6 + 125440*a*
d*tan(c/2 + d*x/2)**4 + 35840*a*d*tan(c/2 + d*x/2)**2 + 4480*a*d) - 28672*t
an(c/2 + d*x/2)**6/(4480*a*d*tan(c/2 + d*x/2)**16 + 35840*a*d*tan(c/2 + d*x
/2)**14 + 125440*a*d*tan(c/2 + d*x/2)**12 + 250...
```

Giac [A]
time = 0.45, size = 205, normalized size = 1.45

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\,\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+8960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-11655\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+23485\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+8960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-23485\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+14336\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+11655\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1792\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2048\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+256\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4480*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 8960*tan(1/2*d*x + 1/2*c)^12 - 11655*tan(1/2*d*x + 1/2*c)^11
 + 23485*tan(1/2*d*x + 1/2*c)^9 + 8960*tan(1/2*d*x + 1/2*c)^8 - 23485*tan(1
/2*d*x + 1/2*c)^7 + 14336*tan(1/2*d*x + 1/2*c)^6 + 11655*tan(1/2*d*x + 1/2*
c)^5 - 1792*tan(1/2*d*x + 1/2*c)^4 - 805*tan(1/2*d*x + 1/2*c)^3 + 2048*tan(
1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a))/d
```

Mupad [B]
time = 11.62, size = 199, normalized size = 1.41

$$-\frac{3\,x}{128\,a}-\frac{\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{23\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{333\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+\frac{671\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{671\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{64}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5}+\frac{333\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{64}-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}-\frac{23\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{64}+\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{4}{35}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (3*x)/(128*a) - ((32*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/64
 - (23*tan(c/2 + (d*x)/2)^3)/64 - (4*tan(c/2 + (d*x)/2)^4)/5 + (333*tan(c/2
 + (d*x)/2)^5)/64 + (32*tan(c/2 + (d*x)/2)^6)/5 - (671*tan(c/2 + (d*x)/2)^7
)/64 + 4*tan(c/2 + (d*x)/2)^8 + (671*tan(c/2 + (d*x)/2)^9)/64 - (333*tan(c/
2 + (d*x)/2)^11)/64 + 4*tan(c/2 + (d*x)/2)^12 + (23*tan(c/2 + (d*x)/2)^13)/
64 + (3*tan(c/2 + (d*x)/2)^15)/64 + 4/35)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8
)
```

3.626 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{x}{16a}+\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos(c+dx)\sin(c+dx)}{16ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{24ad}-\frac{\cos^5(c+dx)\sin(c+$$

```
[Out] 1/16*x/a+1/5*cos(d*x+c)^5/a/d-1/7*cos(d*x+c)^7/a/d+1/16*cos(d*x+c)*sin(d*x+
c)/a/d+1/24*cos(d*x+c)^3*sin(d*x+c)/a/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2918, 2648, 2715, 8, 2645, 14}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{\sin(c+dx)\cos(c+dx)}{16ad}+\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/(16*a) + Cos[c + d*x]^5/(5*a*d) - Cos[c + d*x]^7/(7*a*d) + (Cos[c + d*x]*
Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c +
d*x]^5*Sin[c + d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a} \\
&= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^4(c+dx)\,dx}{6a} + \frac{\text{Subst}\left(\int x^4(1-x^2)\,dx, \right.}{ad} \\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} + \frac{\int \cos^2(c+dx)\,dx}{8a} + \\
&= \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin}{24ad} \\
&= \frac{x}{16a} + \frac{\cos^5(c+dx)}{5ad} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(115) = 230.
time = 8.28, size = 351, normalized size = 3.05

$$\frac{\frac{420x}{a}+\frac{315\cos(c)\cos(dx)}{ad}+\frac{105\cos(3c)\cos(3dx)}{ad}-\frac{21\cos(5c)\cos(5dx)}{ad}-\frac{15\cos(7c)\cos(7dx)}{ad}+\frac{105\cos(2dx)\sin(2c)}{ad}-\frac{105\cos(4dx)\sin(4c)}{ad}-\frac{35\cos(6dx)\sin(6c)}{ad}-\frac{315\sin(c)\sin(dx)}{ad}+\frac{105\cos(2c)\sin(2dx)}{ad}-\frac{105\sin(3c)\sin(3dx)}{ad}-\frac{105\cos(4c)\sin(4dx)}{ad}+\frac{21\sin(5c)\sin(5dx)}{ad}-\frac{35\cos(6c)\sin(6dx)}{ad}+\frac{15\sin(7c)\sin(7dx)}{ad}-\frac{525\sin\left(\frac{dx}{2}\right)}{ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{525\sin(c+dx)}{2ad(1+\sin(c+dx))}+\frac{525\sin^2\left(\frac{1}{2}(c+dx)\right)}{d(a+a\sin(c+dx))}}{6720}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((420*x)/a + (315*Cos[c]*Cos[d*x])/(a*d) + (105*Cos[3*c]*Cos[3*d*x])/(a*d)
- (21*Cos[5*c]*Cos[5*d*x])/(a*d) - (15*Cos[7*c]*Cos[7*d*x])/(a*d) + (105*Co
```

```
s[2*d*x]*Sin[2*c])/(a*d) - (105*Cos[4*d*x]*Sin[4*c])/(a*d) - (35*Cos[6*d*x]
*Sin[6*c])/(a*d) - (315*Sin[c]*Sin[d*x])/(a*d) + (105*Cos[2*c]*Sin[2*d*x])/
(a*d) - (105*Sin[3*c]*Sin[3*d*x])/(a*d) - (105*Cos[4*c]*Sin[4*d*x])/(a*d) +
 (21*Sin[5*c]*Sin[5*d*x])/(a*d) - (35*Cos[6*c]*Sin[6*d*x])/(a*d) + (15*Sin[
7*c]*Sin[7*d*x])/(a*d) - (525*Sin[(d*x)/2])/(a*d*(Cos[c/2] + Sin[c/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])) + (525*Sin[c + d*x])/(2*a*d*(1 + Sin[c +
 d*x])) + (525*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x])))/6720
```

Maple [A]
time = 0.25, size = 179, normalized size = 1.56 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*((1/64*tan(1/2*d*x+1/2*c)^13-11/48*tan(1/2*d*x+1/2*c)^11+1/2*tan(1/2*
d*x+1/2*c)^10+31/192*tan(1/2*d*x+1/2*c)^9-1/2*tan(1/2*d*x+1/2*c)^8+tan(1/2*
d*x+1/2*c)^6-31/192*tan(1/2*d*x+1/2*c)^5-1/5*tan(1/2*d*x+1/2*c)^4+11/48*tan
(1/2*d*x+1/2*c)^3+1/10*tan(1/2*d*x+1/2*c)^2-1/64*tan(1/2*d*x+1/2*c)+1/70)/(
1+tan(1/2*d*x+1/2*c)^2)^7+1/64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(103) = 206$.
time = 0.50, size = 400, normalized size = 3.48

$$-\frac{\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{672\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1540\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1344\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1085\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6720\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{3360\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1085\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3360\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1540\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{105\,\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-96}{a+\frac{7\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7\,a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{105\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 3360*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 - 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x +
 c)^13/(cos(d*x + c) + 1)^13 - 96)/(a + 7*a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 21*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + 35*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a*sin(d*x +
 c)^10/(cos(d*x + c) + 1)^10 + 7*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 +
a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a)/d
```

Fricas [A]
time = 0.39, size = 70, normalized size = 0.61

$$-\frac{240\,\cos\left(dx+c\right)^{7}-336\,\cos\left(dx+c\right)^{5}-105\,dx+35\,\left(8\,\cos\left(dx+c\right)^{5}-2\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1680*(240*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x
+ c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2773 vs. $2(90) = 180$.
time = 44.04, size = 2773, normalized size = 24.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((105*d*x*tan(c/2 + d*x/2)**14/(1680*a*d*tan(c/2 + d*x/2)**14 + 11
760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*t
an(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*
x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x
/2)**12/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 3
5280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*t
an(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*
x/2)**2 + 1680*a*d) + 2205*d*x*tan(c/2 + d*x/2)**10/(1680*a*d*tan(c/2 + d*x
/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 +
 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*
tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*
tan(c/2 + d*x/2)**8/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 3675*d*x*tan(c/2 + d*x/2)**6/(1680*a*d*ta
n(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d
*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 +
 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
+ 2205*d*x*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*t
an(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 +
 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 735*d*x*tan(c/2 + d*x/2)**2/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
1680*a*d) + 105*d*x/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*
x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 +
 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*
tan(c/2 + d*x/2)**2 + 1680*a*d) + 210*tan(c/2 + d*x/2)**13/(1680*a*d*tan(c/
2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2
```

```
)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 352
80*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 30
80*tan(c/2 + d*x/2)**11/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2
+ d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)*
*8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*
a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 6720*tan(c/2 + d*x/2)**10/(1680*a*d*t
an(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 +
d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6
+ 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d)
 + 2170*tan(c/2 + d*x/2)**9/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(
c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x
/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11
760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 6720*tan(c/2 + d*x/2)**8/(1680*a*
d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2
 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)*
*6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a
*d) + 13440*tan(c/2 + d*x/2)**6/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*
tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 +
 d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4
+ 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 2170*tan(c/2 + d*x/2)**5/(168
0*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan
(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x
/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 16
80*a*d) - 2688*tan(c/2 + d*x/2)**4/(1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a
*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/
2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)*
*4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) + 3080*tan(c/2 + d*x/2)**3/(
1680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*
tan(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 +
d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 +
 1680*a*d) + 1344*tan(c/2 + d*x/2)**2/(1680*a*d*tan(c/2 + d*x/2)**14 + 1176
0*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*tan(c/2 + d*x/2)**10 + 58800*a*d*tan
(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d*x/2)**6 + 35280*a*d*tan(c/2 + d*x/
2)**4 + 11760*a*d*tan(c/2 + d*x/2)**2 + 1680*a*d) - 210*tan(c/2 + d*x/2)/(1
680*a*d*tan(c/2 + d*x/2)**14 + 11760*a*d*tan(c/2 + d*x/2)**12 + 35280*a*d*t
an(c/2 + d*x/2)**10 + 58800*a*d*tan(c/2 + d*x/2)**8 + 58800*a*d*tan(c/2 + d
*x/2)**6 + 35280*a*d*tan(c/2 + d*x/2)**4 + 1176...
```

Giac [A]
time = 0.57, size = 179, normalized size = 1.56

$$\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-1540\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1085\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3360\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6720\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1085\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1344\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1540\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+672\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^13 - 1540*tan(1/2*d*x
 + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10 + 1085*tan(1/2*d*x + 1/2*c)^9 -
 3360*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*d*x + 1/2*c)^6 - 1085*tan(1/2*d
*x + 1/2*c)^5 - 1344*tan(1/2*d*x + 1/2*c)^4 + 1540*tan(1/2*d*x + 1/2*c)^3 +
 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 96)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a))/d
```

Mupad [B]
time = 12.75, size = 172, normalized size = 1.50

$$\frac{x}{16\,a}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{8}-\frac{11\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{6}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{31\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{24}-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}-\frac{31\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{24}-\frac{8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{5}+\frac{11\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{6}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{5}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{4}{35}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] x/(16*a) + ((4*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 + (11*tan(c/2
 + (d*x)/2)^3)/6 - (8*tan(c/2 + (d*x)/2)^4)/5 - (31*tan(c/2 + (d*x)/2)^5)/2
4 + 8*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^8 + (31*tan(c/2 + (d*x)/2
)^9)/24 + 4*tan(c/2 + (d*x)/2)^10 - (11*tan(c/2 + (d*x)/2)^11)/6 + tan(c/2
+ (d*x)/2)^13/8 + 4/35)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.627 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{16a}-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos(c+dx)\sin(c+dx)}{16ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}$$

```
[Out] -1/16*x/a-1/5*cos(d*x+c)^5/a/d-1/16*cos(d*x+c)*sin(d*x+c)/a/d-1/24*cos(d*x+
c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2645, 30, 2648, 2715, 8}

$$-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/16*x/a - Cos[c + d*x]^5/(5*a*d) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) -
 (Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(6*
a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \frac{\int \cos^4(c+dx)\,dx}{6a} - \frac{\mathrm{Subst}\left(\int x^4\,dx, x, \cos(c+d\right.}{ad} \\
&= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{6ad} - \int \\
&= -\frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad} + \cos \\
&= -\frac{x}{16a} - \frac{\cos^5(c+dx)}{5ad} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{24ad}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 377 vs. 2(97) = 194.
time = 3.45, size = 377, normalized size = 3.89

$$
-\frac{-30(5c-4dx)\cos\left(\frac{c}{2}\right)+120\cos\left(\frac{c}{2}+dx\right)+120\cos\left(\frac{3c}{2}+dx\right)+15\cos\left(\frac{3c}{2}+2dx\right)-15\cos\left(\frac{5c}{2}+2dx\right)+60\cos\left(\frac{5c}{2}+3dx\right)+60\cos\left(\frac{7c}{2}+3dx\right)-15\cos\left(\frac{7c}{2}+4dx\right)+15\cos\left(\frac{9c}{2}+4dx\right)+12\cos\left(\frac{9c}{2}+5dx\right)+12\cos\left(\frac{11c}{2}+5dx\right)-5\cos\left(\frac{11c}{2}+6dx\right)+5\cos\left(\frac{13c}{2}+6dx\right)+300\sin\left(\frac{c}{2}\right)-150c\sin\left(\frac{c}{2}\right)+120dx\sin\left(\frac{c}{2}\right)-120\sin\left(\frac{c}{2}+dx\right)+120\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)-60\sin\left(\frac{5c}{2}+3dx\right)+60\sin\left(\frac{7c}{2}+3dx\right)-15\sin\left(\frac{7c}{2}+4dx\right)-15\sin\left(\frac{9c}{2}+4dx\right)-12\sin\left(\frac{9c}{2}+5dx\right)+12\sin\left(\frac{11c}{2}+5dx\right)-5\sin\left(\frac{11c}{2}+6dx\right)-5\sin\left(\frac{13c}{2}+6dx\right)}{1920ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(-30*(5*c - 4*d*x)*Cos[c/2] + 120*Cos[c/2 + d*x] + 120*Cos[(3*c)/2
+ d*x] + 15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] + 60*Cos[(5*c)/2
 + 3*d*x] + 60*Cos[(7*c)/2 + 3*d*x] - 15*Cos[(7*c)/2 + 4*d*x] + 15*Cos[(9*c
```

```
)/2 + 4*d*x] + 12*Cos[(9*c)/2 + 5*d*x] + 12*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(
11*c)/2 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 300*Sin[c/2] - 150*c*Sin[c/2]
+ 120*d*x*Sin[c/2] - 120*Sin[c/2 + d*x] + 120*Sin[(3*c)/2 + d*x] + 15*Sin[(
3*c)/2 + 2*d*x] + 15*Sin[(5*c)/2 + 2*d*x] - 60*Sin[(5*c)/2 + 3*d*x] + 60*Si
n[(7*c)/2 + 3*d*x] - 15*Sin[(7*c)/2 + 4*d*x] - 15*Sin[(9*c)/2 + 4*d*x] - 12
*Sin[(9*c)/2 + 5*d*x] + 12*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x]
- 5*Sin[(13*c)/2 + 6*d*x])/(a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(87) = 174$.
time = 0.20, size = 181, normalized size = 1.87

method	result
risch	$-\frac{x}{16a}-\frac{\cos(dx+c)}{8ad}+\frac{\sin(6dx+6c)}{192ad}-\frac{\cos(5dx+5c)}{80ad}+\frac{\sin(4dx+4c)}{64ad}-\frac{\cos(3dx+3c)}{16ad}-\frac{\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{\frac{4\left(-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{47\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{13\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{13(\tan}{}\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{ad}$
default	$\frac{\frac{4\left(-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{47\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{13\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{13(\tan}{}\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{ad}$
norman	$\frac{-\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}-\frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{x}{16a}-\frac{11}{40ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{20ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{7x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a}-\frac{7x\left(\tan^{13}\left(\frac{dx}{2}+\right.\right.}{16a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*((-1/32*tan(1/2*d*x+1/2*c)^11-1/2*tan(1/2*d*x+1/2*c)^10+47/96*tan(1/2
*d*x+1/2*c)^9-1/2*tan(1/2*d*x+1/2*c)^8-13/16*tan(1/2*d*x+1/2*c)^7-tan(1/2*d
*x+1/2*c)^6+13/16*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4-47/96*tan(1/2*d
*x+1/2*c)^3-1/10*tan(1/2*d*x+1/2*c)^2+1/32*tan(1/2*d*x+1/2*c)-1/10)/(1+tan(
1/2*d*x+1/2*c)^2)^6-1/32*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(87) = 174$.
time = 0.50, size = 379, normalized size = 3.91

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{235\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{390\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{480\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{240\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{235\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-48}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 235*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(co
```

```
s(d*x + c) + 1)^4 + 390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 480*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 390*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 240*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 235*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*
x + c) + 1)^11 - 48)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.37, size = 60, normalized size = 0.62

$$-\frac{48 \cos{(dx+c)}^{5}+15\,dx-5\left(8 \cos{(dx+c)}^{5}-2 \cos{(dx+c)}^{3}-3 \cos{(dx+c)}\right)\sin{(dx+c)}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*cos(d*x + c)^5 + 15*d*x - 5*(8*cos(d*x + c)^5 - 2*cos(d*x + c)^3
 - 3*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2307 vs. 2(76) = 152.
time = 27.34, size = 2307, normalized size = 23.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**12/(240*a*d*tan(c/2 + d*x/2)**12 + 144
0*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/
2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2
 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1
440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(
c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)*
*2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 +
 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*ta
n(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2
)**2 + 240*a*d) - 300*d*x*tan(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12
 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*
tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x
/2)**2 + 240*a*d) - 225*d*x*tan(c/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**
12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*
d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d
```

```
*x/2)**2 + 240*a*d) - 90*d*x*tan(c/2 + d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)*
*12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a
*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 +
d*x/2)**2 + 240*a*d) - 15*d*x/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(
c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)
**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d
) - 30*tan(c/2 + d*x/2)**11/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/
2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**
6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d)
- 480*tan(c/2 + d*x/2)**10/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2
 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6
 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) +
 470*tan(c/2 + d*x/2)**9/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 +
 d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 +
 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 4
80*tan(c/2 + d*x/2)**8/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d
*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3
600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 780
*tan(c/2 + d*x/2)**7/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x
/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 360
0*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 960*t
an(c/2 + d*x/2)**6/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2
)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*
a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 780*tan
(c/2 + d*x/2)**5/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)*
*10 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*
d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 960*tan(c
/2 + d*x/2)**4/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**1
0 + 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*
tan(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 470*tan(c/2
 + d*x/2)**3/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10
+ 3600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*ta
n(c/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 96*tan(c/2 +
d*x/2)**2/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3
600*a*d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c
/2 + d*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) + 30*tan(c/2 + d*x
/2)/(240*a*d*tan(c/2 + d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*
d*tan(c/2 + d*x/2)**8 + 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d
*x/2)**4 + 1440*a*d*tan(c/2 + d*x/2)**2 + 240*a*d) - 96/(240*a*d*tan(c/2 +
d*x/2)**12 + 1440*a*d*tan(c/2 + d*x/2)**10 + 3600*a*d*tan(c/2 + d*x/2)**8 +
 4800*a*d*tan(c/2 + d*x/2)**6 + 3600*a*d*tan(c/2 + d*x/2)**4 + 1440*a*d*tan
(c/2 + d*x/2)**2 + 240*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a),
 True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(87) =

174.
time = 0.44, size = 179, normalized size = 1.85

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-235\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+235\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+48\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*x +
1/2*c)^10 - 235*tan(1/2*d*x + 1/2*c)^9 + 240*tan(1/2*d*x + 1/2*c)^8 + 390*t
an(1/2*d*x + 1/2*c)^7 + 480*tan(1/2*d*x + 1/2*c)^6 - 390*tan(1/2*d*x + 1/2*
c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 235*tan(1/2*d*x + 1/2*c)^3 + 48*tan(1/2
*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 48)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*a))/d
```

Mupad [B]
time = 12.49, size = 173, normalized size = 1.78

$$-\frac{x}{16\,a}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{2}{5}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - x/(16*a) - ((2*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/8 + (47*tan(c
/2 + (d*x)/2)^3)/24 + 4*tan(c/2 + (d*x)/2)^4 - (13*tan(c/2 + (d*x)/2)^5)/4
+ 4*tan(c/2 + (d*x)/2)^6 + (13*tan(c/2 + (d*x)/2)^7)/4 + 2*tan(c/2 + (d*x)/
2)^8 - (47*tan(c/2 + (d*x)/2)^9)/24 + 2*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (
d*x)/2)^11/8 + 2/5)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.628 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=101

$$-\frac{3x}{8a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

```
[Out] -3/8*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d-3/8*co
s(d*x+c)*sin(d*x+c)/a/d-1/4*cos(d*x+c)^3*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2672, 308, 212, 2715, 8}

$$\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{3\sin(c+dx)\cos(c+dx)}{8ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(8*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c + d
*x]^3/(3*a*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*a*d) - (Cos[c + d*x]^3*Sin
[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^4(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a}\\ &= -\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{3\int \cos^2(c+dx)\,dx}{4a}-\frac{\operatorname{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\cos\right.}{ad}\\ &= -\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}-\frac{3\int 1\,dx}{8a}-\frac{\operatorname{Subst}(}{}\\ &= -\frac{3x}{8a}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos(c+dx)\sin(c+dx)}{8ad}-\frac{\cos^3(c+}{}\\ &= -\frac{3x}{8a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}-\frac{3\cos(c+dx)}{8a}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 86, normalized size = 0.85

$$\frac{120\cos(c+dx)+8\cos(3(c+dx))-3\left(4\left(3c+3dx+8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+8\sin(2(c+dx))+\sin(4(c+dx))\right)}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (120*Cos[c + d*x] + 8*Cos[3*(c + d*x)] - 3*(4*(3*c + 3*d*x + 8*Log[Cos[(c +
 d*x)/2]] - 8*Log[Sin[(c + d*x)/2]]) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*a*d)
```

Maple [A]
time = 0.23, size = 139, normalized size = 1.38

method	result
risch	$-\frac{3x}{8a}+\frac{5\,\mathrm{e}^{i(dx+c)}}{8ad}+\frac{5\,\mathrm{e}^{-i(dx+c)}}{8ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{ad}-\frac{\sin(4dx+4c)}{32ad}+\frac{\cos(3dx+3c)}{12ad}-\sin($
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-10\left(\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{da}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-10\left(\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{da}$
norman	$-\frac{3x}{8a}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{15x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{15x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{15x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{15x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{15x\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(tan(1/2*d*x+1/2*c))-2*(-5/8*tan(1/2*d*x+1/2*c)^7-2*tan(1/2*d*x+1/
2*c)^6+3/8*tan(1/2*d*x+1/2*c)^5-4*tan(1/2*d*x+1/2*c)^4-3/8*tan(1/2*d*x+1/2*
c)^3-10/3*tan(1/2*d*x+1/2*c)^2+5/8*tan(1/2*d*x+1/2*c)-4/3)/(1+tan(1/2*d*x+1
/2*c)^2)^4-3/4*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(93) = 186$.
time = 0.51, size = 280, normalized size = 2.77

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{96\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{48\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-32}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 96*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 9*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 48*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 32)/(a + 4*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8) + 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 12*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.41, size = 84, normalized size = 0.83

$$\frac{8\cos(dx+c)^3-9\,dx-3\left(2\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)+24\cos(dx+c)-12\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*cos(d*x + c)^3 - 9*d*x - 3*(2*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(
d*x + c) + 24*cos(d*x + c) - 12*log(1/2*cos(d*x + c) + 1/2) + 12*log(-1/2*c
os(d*x + c) + 1/2))/(a*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^6(c+dx)\csc(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A]

time = 0.45, size = 143, normalized size = 1.42

$$-\frac{\frac{9\,(dx+c)}{a}-\frac{24\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\,\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+96\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+32\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(9*(d*x + c)/a - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(15*tan(1/2*
d*x + 1/2*c)^7 + 48*tan(1/2*d*x + 1/2*c)^6 - 9*tan(1/2*d*x + 1/2*c)^5 + 96*
tan(1/2*d*x + 1/2*c)^4 + 9*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)
^2 - 15*tan(1/2*d*x + 1/2*c) + 32)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B]

time = 10.43, size = 225, normalized size = 2.23

$$\frac{3\,\mathrm{atan}\left(\frac{9}{16\,\left(\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{3}{2}\right)}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{3}{2}\right)}\right)}{4\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{8}{3}}{d\,\left(a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atan(9/(16*((9*tan(c/2 + (d*x)/2))/16 + 3/2)) - (3*tan(c/2 + (d*x)/2))/(
2*((9*tan(c/2 + (d*x)/2))/16 + 3/2))))/(4*a*d) + log(tan(c/2 + (d*x)/2))/(a
*d) + ((20*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/4 + (3*tan(c/2
+ (d*x)/2)^3)/4 + 8*tan(c/2 + (d*x)/2)^4 - (3*tan(c/2 + (d*x)/2)^5)/4 + 4*t
an(c/2 + (d*x)/2)^6 + (5*tan(c/2 + (d*x)/2)^7)/4 + 8/3)/(d*(a + 4*a*tan(c/2
 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan
(c/2 + (d*x)/2)^8))
```

3.629 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{3x}{2a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}$$

```
[Out] -3/2*x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-3/2*co
t(d*x+c)/a/d+1/2*cos(d*x+c)^2*cot(d*x+c)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2918, 2671, 294, 327, 209, 2672, 308, 212}

$$-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{3\cot(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(2*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c + d
*x]^3/(3*a*d) - (3*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^2*Cot[c + d*x])/(2
*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a} + \frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^2(c+dx)\cot(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)\,dx, x, \cos(c+dx\right.}{ad}$$

$$= -\frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+dx)}{2ad} + \frac{\cos^2(c+dx)\cot(c+dx}{2ad}$$

$$= -\frac{3x}{2a} + \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\cos(c+dx)}{ad} - \frac{\cos^3(c+dx)}{3ad} - \frac{3\cot(c+}{2ad}$$

Mathematica [A]

time = 0.57, size = 122, normalized size = 1.28

$$-\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^2\left(27\cos(c+dx)+6\left(6c+6dx+5\cos(c+dx)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)+\cos(3(c+dx))(-3+2\sin(c+dx))\right)\tan\left(\frac{1}{2}(c+dx)\right)}{48ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/48*((1 + Cot[(c + d*x)/2])^2*(27*Cos[c + d*x] + 6*(6*c + 6*d*x + 5*Cos[c
 + d*x] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]])*Sin[c + d*x] +
 Cos[3*(c + d*x)]*(-3 + 2*Sin[c + d*x]))*Tan[(c + d*x)/2])/(a*d*(1 + Sin[c
+ d*x]))
```

Maple [A]

time = 0.24, size = 121, normalized size = 1.27

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{2}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-6}{2da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{2}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-6}{2da}$
risch	$-\frac{3x}{2a}+\frac{i\,\mathrm{e}^{2i(dx+c)}}{8ad}-\frac{5\,\mathrm{e}^{i(dx+c)}}{8ad}-\frac{5\,\mathrm{e}^{-i(dx+c)}}{8ad}-\frac{i\,\mathrm{e}^{-2i(dx+c)}}{8ad}-\frac{2i}{ad\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{ad}+\ln(\mathrm{e}^{i(}$
norman	$\frac{\frac{2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{13\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{3x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{6x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-6x(t}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-8
*(-1/4*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4+tan(1/2*d*x+1/2*c)^2+1/4*t
an(1/2*d*x+1/2*c)+2/3)/(1+tan(1/2*d*x+1/2*c)^2)^3-6*arctan(tan(1/2*d*x+1/2*
c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(87) = 174$.
time = 0.55, size = 277, normalized size = 2.92

$$-\frac{\frac{\frac{16\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{24\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{9\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}+\frac{18\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\,\sin(dx+c)}{a(\cos(dx+c)+1)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((16*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 24*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7) + 18*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3*sin(d*x + c)/(a*(cos(
d*x + c) + 1)))/d
```

Fricas [A]
time = 0.41, size = 104, normalized size = 1.09

$$\frac{3\,\cos\left(dx+c\right)^3-\left(2\,\cos\left(dx+c\right)^3+9\,dx+6\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,\cos\left(dx+c\right)}{6\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 9*d*x + 6*cos(d*x + c))*sin(d*x
 + c) + 3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - 9*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^6\left(c+dx\right)\csc^2\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 147, normalized size = 1.55

$$-\frac{\frac{9\,(dx+c)}{a}+\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\,\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)/a + 6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x +
1/2*c)/a - 3*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) - 2*(3*t
an(1/2*d*x + 1/2*c)^5 - 12*tan(1/2*d*x + 1/2*c)^4 - 12*tan(1/2*d*x + 1/2*c)
^2 - 3*tan(1/2*d*x + 1/2*c) - 8)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 9.03, size = 229, normalized size = 2.41

$$\frac{3\,\mathrm{atan}\left(\frac{6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-6}+\frac{9}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-6}\right)}{a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}-\frac{-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atan((6*tan(c/2 + (d*x)/2))/(9*tan(c/2 + (d*x)/2) - 6) + 9/(9*tan(c/2 +
(d*x)/2) - 6)))/(a*d) - log(tan(c/2 + (d*x)/2))/(a*d) - ((16*tan(c/2 + (d*x
)/2))/3 + 5*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^3 + 3*tan(c/2 + (d*
x)/2)^4 + 8*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 + 1)/(d*(2*a*tan(c/
2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^3 + 6*a*tan(c/2 + (d*x)/2)^5 + 2*a*ta
n(c/2 + (d*x)/2)^7)) + tan(c/2 + (d*x)/2)/(2*a*d)
```

3.630 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{3x}{2a}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+d$$

```
[Out] 3/2*x/a+3/2*arctanh(cos(d*x+c))/a/d-3/2*cos(d*x+c)/a/d+3/2*cot(d*x+c)/a/d-1
/2*cos(d*x+c)^2*cot(d*x+c)/a/d-1/2*cos(d*x+c)*cot(d*x+c)^2/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2918, 2672, 294, 327, 212, 2671, 209}

$$-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(2*a) + (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (3*Cos[c + d*x])/(2*a*d)
+ (3*Cot[c + d*x])/(2*a*d) - (Cos[c + d*x]^2*Cot[c + d*x])/(2*a*d) - (Cos[c
 + d*x]*Cot[c + d*x]^2)/(2*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^2(c+dx)\cot^2(c+dx)\,dx}{a}+\frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\cot(c+d\right.}{ad}\\ &= -\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}+\frac{3\text{Subst}\left(\int \frac{x^2}{1-x^2}\right.}{}\\ &= -\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot(c+dx)}{2ad}-\frac{\cos(c+d}{}\\ &= \frac{3x}{2a}+\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{3\cos(c+dx)}{2ad}+\frac{3\cot(c+dx)}{2ad}-\frac{\cos^2(c+}{}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 152, normalized size = 1.43

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-12c-12dx+12\cos(c+dx)-4\cos(3(c+dx))-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+12\cos(2(c+dx))\left(c+dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-10\sin(2(c+dx))+\sin(4(c+dx))\right)}{64ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/64*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-12*c - 12*d*x + 12*Cos[c +
 d*x] - 4*Cos[3*(c + d*x)] - 12*Log[Cos[(c + d*x)/2]] + 12*Cos[2*(c + d*x)]
*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 12*Log[Sin[(c
+ d*x)/2]] - 10*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x
]))
```

Maple [A]
time = 0.24, size = 140, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{4da}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{4da}$
risch	$\frac{3x}{2a}-\frac{i e^{2i(dx+c)}}{8ad}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}+\frac{i e^{-2i(dx+c)}}{8ad}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+2i e^{2i(dx+c)}-2i}{ad\left(e^{2i(dx+c)}-1\right)^2}-\frac{3\ln\left(e^{i(dx+c)}-\right.}{2ad}$
norman	$-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{3x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{9x(t}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/2*
c)^2+2/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2*c))+16*(-1/4*tan(1/2*d*x+1/2
*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+1/4*tan(1/2*d*x+1/2*c)-1/2)/(1+tan(1/2*d*x+1
/2*c)^2)^2+12*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(94) = 188.
time = 0.52, size = 261, normalized size = 2.46

$$-\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a}-\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{18\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{17\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)/a - (4*sin(d*x + c)/(cos(d*x + c) + 1) - 18*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 17*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6) - 24*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a + 12*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.41, size = 126, normalized size = 1.19

$$\frac{6\,dx\cos\left(dx+c\right)^{2}-4\,\cos\left(dx+c\right)^{3}-6\,dx+3\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+6\,\cos\left(dx+c\right)}{4\,\left(ad\cos\left(dx+c\right)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(6*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 6*d*x + 3*(cos(d*x + c)^2 -
1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
c) + 1/2) + 2*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) + 6*cos(d*x +
c))/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.47, size = 167, normalized size = 1.58

$$\frac{\frac{12\,(dx+c)}{a}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}+\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(12*(d*x + c)/a - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a*tan(1/2*d*x
+ 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + (6*tan(1/2*d*x + 1/2*c)^6 - 4*
tan(1/2*d*x + 1/2*c)^5 - 5*tan(1/2*d*x + 1/2*c)^4 + 16*tan(1/2*d*x + 1/2*c)
^3 - 12*tan(1/2*d*x + 1/2*c)^2 + 4*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x
+ 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a))/d
```

Mupad [B]
time = 8.98, size = 223, normalized size = 2.10

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d}-\frac{3\,\mathrm{atan}\left(\frac{9}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+9}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+9}\right)}{a\,d}-\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a\,d}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{1}{2}}{d\left(4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (3*atan(9/(9*tan(c/2 + (d*x)/2) + 9) - (9*ta
n(c/2 + (d*x)/2))/(9*tan(c/2 + (d*x)/2) + 9)))/(a*d) - (3*log(tan(c/2 + (d*
x)/2)))/(2*a*d) - (9*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 8*tan(c/
2 + (d*x)/2)^3 + (17*tan(c/2 + (d*x)/2)^4)/2 + 2*tan(c/2 + (d*x)/2)^5 + 1/2
)/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 8*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (
d*x)/2)^6)) - tan(c/2 + (d*x)/2)/(2*a*d)
```

3.631 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{x}{a}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\cos(c+dx)}{2ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] x/a-3/2*arctanh(cos(d*x+c))/a/d+3/2*cos(d*x+c)/a/d+cot(d*x+c)/a/d+1/2*cos(d
*x+c)*cot(d*x+c)^2/a/d-1/3*cot(d*x+c)^3/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2918, 3554, 8, 2672, 294, 327, 212}

$$\frac{3\cos(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Cos[c + d*x])/(2*a*d) + Cot[c
+ d*x]/(a*d) + (Cos[c + d*x]*Cot[c + d*x]^2)/(2*a*d) - Cot[c + d*x]^3/(3*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos(c+dx)\cot^3(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\,dx}{a}\\
&= -\frac{\cot^3(c+dx)}{3ad}-\frac{\int \cot^2(c+dx)\,dx}{a}+\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= \frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\int 1\,dx}{a}-\frac{3\text{Subs}}{}\\
&= \frac{x}{a}+\frac{3\cos(c+dx)}{2ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^2(c+dx)}{2ad}-\frac{\cot^3(c+d}{3ad}\\
&= \frac{x}{a}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\cos(c+dx)}{2ad}+\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\,\text{c}}{2a}
\end{aligned}
$$

Mathematica [A]
time = 0.67, size = 138, normalized size = 1.47

$$
\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(12\left(2c+2dx-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)-2\cos(3(c+dx))(4+3\sin(c+dx))+9\sin(2(c+dx))\right)}{192ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]

[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(12*(2*c + 2*d*x - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3 - 2*Cos[3*(c + d*x)]*(4 + 3*Sin[c + d*x]) + 9*Sin[2*(c + d*x)]))/(192*a*d*(1 + Sin[c + d*x]))

Maple [**A**]
time = 0.24, size = 125, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{8da}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+12\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{8da}$
risch	$\frac{x}{a}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}-\frac{-12ie^{4i(dx+c)}+3e^{5i(dx+c)}+12ie^{2i(dx+c)}-8i-3e^{i(dx+c)}}{3ad\left(e^{2i(dx+c)}-1\right)^3}-\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2ad}+\frac{3\ln}{}$
norman	$\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{2\left(\tan^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2-5*tan(1/2*d*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3+1/tan(1/2*d*x+1/2*c)^2+5/tan(1/2*d*x+1/2*c)+12*ln(tan(1/2*d*x+1/2*c))+16/(1+tan(1/2*d*x+1/2*c)^2)+16*arctan(tan(1/2*d*x+1/2*c)))

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(86) = 172.
time = 0.50, size = 240, normalized size = 2.55

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{51\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{36\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/24*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 51*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1)/(a*sin(d*x

```
+ c)^3/(cos(d*x + c) + 1)^3 + a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) - 48*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 36*log(sin(d*x + c)/(cos(d*x + c
) + 1))/a)/d
```

Fricas [A]
time = 0.40, size = 148, normalized size = 1.57

$$\frac{16 \cos\left(dx+c\right)^{3}-9 \left(\cos\left(dx+c\right)^{2}-1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right) \sin\left(dx+c\right)+9 \left(\cos\left(dx+c\right)^{2}-1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right) \sin\left(dx+c\right)+6 \left(2 dx \cos\left(dx+c\right)^{2}+2 \cos\left(dx+c\right)^{3}-2 dx-3 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)-12 \cos\left(dx+c\right)}{12 \left(ad \cos\left(dx+c\right)^{2}-ad\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 6*(2*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 2*d*x - 3*cos(d*x + c
))*sin(d*x + c) - 12*cos(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c)
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.48, size = 157, normalized size = 1.67

$$\frac{\frac{24 \left(dx+c\right)}{a}+\frac{36 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a}+\frac{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-3 a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-15 a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{3}}+\frac{48}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+1\right) a}-\frac{66 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}}}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)/a + 36*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d
*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c))
/a^3 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - (66*tan(1/2*d*x + 1/2*c)^3 - 1
5*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a*tan(1/2*d*x + 1/2
*c)^3))/d
```

Mupad [B]
time = 8.97, size = 212, normalized size = 2.26

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{2\,\mathrm{atan}\left(\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}+\frac{4}{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6}\right)}{a\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{14\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}}{d\,\left(8\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (2*atan((6*t
an(c/2 + (d*x)/2))/(4*tan(c/2 + (d*x)/2) - 6) + 4/(4*tan(c/2 + (d*x)/2) - 6
)))/(a*d) + (3*log(tan(c/2 + (d*x)/2)))/(2*a*d) + (tan(c/2 + (d*x)/2) + (14
*tan(c/2 + (d*x)/2)^2)/3 + 17*tan(c/2 + (d*x)/2)^3 + 5*tan(c/2 + (d*x)/2)^4
 - 1/3)/(d*(8*a*tan(c/2 + (d*x)/2)^3 + 8*a*tan(c/2 + (d*x)/2)^5)) - (5*tan(
c/2 + (d*x)/2))/(8*a*d)
```

3.632 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{x}{a}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}$$

```
[Out] -x/a-3/8*arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d+1/3*cot(d*x+c)^3/a/d+3/8*co
t(d*x+c)*csc(d*x+c)/a/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2918, 2691, 3855, 3554, 8}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]/(a*d) + Cot[c + d
*x]^3/(3*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]^3*Csc
[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^4(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}\\
&= \frac{\cot^3(c+dx)}{3ad}-\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a}\\
&= -\frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot^3(c+dx)}{4a}\\
&= -\frac{x}{a}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)}{ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{3\cot(c+d}{8}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(102) = 204.
time = 0.48, size = 232, normalized size = 2.27

$$-\frac{\csc^4(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(72c+72dx+18\cos(c+dx)+30\cos(3(c+dx))+24c\cos(4(c+dx))+24dx\cos(4(c+dx))+27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+9\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12\cos(2(c+dx))\left(8c+8dx+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-27\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-9\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+32\sin(2(c+dx))-32\sin(4(c+dx))\right)}{192ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(72*c + 72*d
*x + 18*Cos[c + d*x] + 30*Cos[3*(c + d*x)] + 24*c*Cos[4*(c + d*x)] + 24*d*x
*Cos[4*(c + d*x)] + 27*Log[Cos[(c + d*x)/2]] + 9*Cos[4*(c + d*x)]*Log[Cos[(
c + d*x)/2]] - 12*Cos[2*(c + d*x)]*(8*c + 8*d*x + 3*Log[Cos[(c + d*x)/2]] -
 3*Log[Sin[(c + d*x)/2]]) - 27*Log[Sin[(c + d*x)/2]] - 9*Cos[4*(c + d*x)]*L
og[Sin[(c + d*x)/2]] + 32*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(a*d*(1
+ Sin[c + d*x]))
```

Maple [A]
time = 0.24, size = 136, normalized size = 1.33

method	result

derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\cdots}{16da}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\cdots}{16da}$
risch	$-\frac{x}{a}-\frac{48i\,\mathrm{e}^{6i(dx+c)}+15\,\mathrm{e}^{7i(dx+c)}-96i\,\mathrm{e}^{4i(dx+c)}+9\,\mathrm{e}^{5i(dx+c)}+80i\,\mathrm{e}^{2i(dx+c)}+9\,\mathrm{e}^{3i(dx+c)}-32i+15\,\mathrm{e}^{i(dx+c)}}{12ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}-\frac{3\ln\left(\mathrm{e}^{i(dx\cdots}\right.}{8ad\cdots}$
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{29\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{91\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{91\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{29\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{5\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(1/4*tan(1/2*d*x+1/2*c)^4-2/3*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1
/2*c)^2+10*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4+2/3/tan(1/2*d*x+1/2*
c)^3+2/tan(1/2*d*x+1/2*c)^2-10/tan(1/2*d*x+1/2*c)+6*ln(tan(1/2*d*x+1/2*c))-
32*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(94) = 188$.
time = 0.50, size = 217, normalized size = 2.13

$$\frac{\frac{\frac{120\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}-\frac{384\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((120*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a - 384*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + 72*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 2
4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 - 3)*(cos(d*x + c) + 1)^4/(a*sin(d*x + c)^4))/d
```

Fricas [A]
time = 0.38, size = 171, normalized size = 1.68

$$-\frac{48\,dx\cos(dx+c)^4-96\,dx\cos(dx+c)^2+30\cos(dx+c)^3+48\,dx+9\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-9\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-16\left(4\cos(dx+c)^3-3\cos(dx+c)\right)\sin(dx+c)-18\cos(dx+c)}{48\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

[Out] -1/48*(48*d*x*cos(d*x + c)^4 - 96*d*x*cos(d*x + c)^2 + 30*cos(d*x + c)^3 + 48*d*x + 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 9*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c) - 18*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep

Giac [A]
time = 0.49, size = 167, normalized size = 1.64

$$-\frac{\frac{192\,(dx+c)}{a}-\frac{72\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/192*(192*(d*x + c)/a - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a^3*tan(1/2*d*x + 1/2*c))/a^4 + (150*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^4))/d

Mupad [B]
time = 9.42, size = 317, normalized size = 3.11

$$\frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8-8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+384\,\mathrm{atan}\left(\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+72\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{192\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)

[Out] (3*sin(c/2 + (d*x)/2)^8 - 3*cos(c/2 + (d*x)/2)^8 - 8*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^7 + 8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 24*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^5 - 120*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2 + 384*atan((8*cos(c/2 + (d*x)/2) - 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4 + 72*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)/(192*a*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)

3.633 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}$$

```
[Out] 3/8*arctanh(cos(d*x+c))/a/d-1/5*cot(d*x+c)^5/a/d-3/8*cot(d*x+c)*csc(d*x+c)/
a/d+1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a} + \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a}$$
$$= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} + \frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a} + \frac{\operatorname{Subst}\left(\int x^4\,dx, x, -\right.}{ad}$$
$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot^3(c+dx)\csc(c+dx)}{4ad} - \frac{3\int \csc(}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{\cot^5(c+dx)}{5ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot^3(c+dx}{4}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(82) = 164.
time = 0.57, size = 189, normalized size = 2.30

$$-\frac{\csc^5(c+dx)\left(80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))-150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+20\sin(2(c+dx))+75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-50\sin(4(c+dx))-15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{640ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/640*(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c
+ d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 1
5*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(
c + d*x)]))/(a*d)
```

Maple [A]
time = 0.28, size = 148, normalized size = 1.80

method	result

risch	$\frac{-40i\mathrm{e}^{8i(dx+c)}+25\,\mathrm{e}^{9i(dx+c)}-10\,\mathrm{e}^{7i(dx+c)}-80i\mathrm{e}^{4i(dx+c)}+10\,\mathrm{e}^{3i(dx+c)}-8i-25\,\mathrm{e}^{i(dx+c)}}{20ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^{5}}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{8ad}-\frac{3\ln\left(\mathrm{e}^{i(d}\right.}{8}$
derivativedivides	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-}{32da}$
default	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}-}{32da}$
norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}+\frac{3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{3\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{3\left(\mathrm{ta}\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a*(1/5*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4-tan(1/2*d*x+1/2
*c)^3+4*tan(1/2*d*x+1/2*c)^2+2*tan(1/2*d*x+1/2*c)+1/2/tan(1/2*d*x+1/2*c)^4-
4/tan(1/2*d*x+1/2*c)^2-1/5/tan(1/2*d*x+1/2*c)^5-12*ln(tan(1/2*d*x+1/2*c))-2
/tan(1/2*d*x+1/2*c)+1/tan(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(74) = 148$.
time = 0.31, size = 234, normalized size = 2.85

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{2\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{40\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{20\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-2\right)(\cos(dx+c)+1)^{5}}{a\sin(dx+c)^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.
time = 0.40, size = 155, normalized size = 1.89

$$-\frac{16\cos\left(dx+c\right)^{5}-15\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-10\left(5\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(74) = 148.
time = 0.47, size = 187, normalized size = 2.28

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^5}-\frac{274\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-20\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

Mupad [**B**]
time = 9.04, size = 183, normalized size = 2.23

$$\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{32\,a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,a\,d}-\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}-\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{5}\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(32*a*d) - tan(c/2 + (d
*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) - (3*log(tan(c/2 + (d*x)
/2)))/(8*a*d) + tan(c/2 + (d*x)/2)/(16*a*d) - (cot(c/2 + (d*x)/2)^5*(4*tan(
c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)/2 + 2*tan(c/2
+ (d*x)/2)^4 + 1/5))/(32*a*d)
```

3.634 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{x}{8a^2}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{\mathrm{co}}{-}$$

```
[Out] -1/8*x/a^2-2/3*cos(d*x+c)^3/a^2/d+3/5*cos(d*x+c)^5/a^2/d-1/7*cos(d*x+c)^7/a
^2/d-1/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d+1/3*
cos(d*x+c)^3*sin(d*x+c)^3/a^2/d
```

Rubi [A]

time = 0.24, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{\cos^7(c+dx)}{7a^2d}+\frac{3\cos^5(c+dx)}{5a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*x/a^2 - (2*Cos[c + d*x]^3)/(3*a^2*d) + (3*Cos[c + d*x]^5)/(5*a^2*d) -
Cos[c + d*x]^7/(7*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d) + (Cos[c +
 d*x]^3*Sin[c + d*x])/(4*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int (a^2\cos^2(c+dx)\sin^3(c+dx) - 2a^2\cos^2(c+dx)\sin^4(c+dx) + a^2\cos^2(c}{a^4}$$
$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^2} + \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^2} - \frac{2\int \cos^2}{}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int x^2(\right.}{}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{3a^2d} - \frac{\int \cos^2(c+dx)\,dx}{4a^2}$$
$$= -\frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin(c+dx}{8a^2d}$$
$$= -\frac{x}{8a^2} - \frac{2\cos^3(c+dx)}{3a^2d} + \frac{3\cos^5(c+dx)}{5a^2d} - \frac{\cos^7(c+dx)}{7a^2d} - \frac{\cos(c+dx)\sin}{8a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 418 vs. 2(135) = 270.
time = 2.12, size = 418, normalized size = 3.10

$$-\frac{210(1+8dx)\cos\left(\frac{c}{2}\right)+1365\cos\left(\frac{c}{2}+dx\right)+1365\cos\left(\frac{3c}{2}+dx\right)-210\cos\left(\frac{3c}{2}+2dx\right)+210\cos\left(\frac{5c}{2}+2dx\right)+175\cos\left(\frac{5c}{2}+3dx\right)+175\cos\left(\frac{7c}{2}+3dx\right)-210\cos\left(\frac{7c}{2}+4dx\right)+210\cos\left(\frac{9c}{2}+4dx\right)-147\cos\left(\frac{9c}{2}+5dx\right)-147\cos\left(\frac{11c}{2}+5dx\right)+70\cos\left(\frac{11c}{2}+6dx\right)-70\cos\left(\frac{13c}{2}+6dx\right)+15\cos\left(\frac{13c}{2}+7dx\right)+15\cos\left(\frac{15c}{2}+7dx\right)-210\sin\left(\frac{c}{2}\right)+1680dx\sin\left(\frac{c}{2}\right)-1365\sin\left(\frac{c}{2}+dx\right)+1365\sin\left(\frac{3c}{2}+dx\right)-210\sin\left(\frac{3c}{2}+2dx\right)-210\sin\left(\frac{5c}{2}+2dx\right)-175\sin\left(\frac{5c}{2}+3dx\right)+175\sin\left(\frac{7c}{2}+3dx\right)-210\sin\left(\frac{7c}{2}+4dx\right)-210\sin\left(\frac{9c}{2}+4dx\right)+147\sin\left(\frac{9c}{2}+5dx\right)-147\sin\left(\frac{11c}{2}+5dx\right)+70\sin\left(\frac{11c}{2}+6dx\right)+70\sin\left(\frac{13c}{2}+6dx\right)-15\sin\left(\frac{13c}{2}+7dx\right)+15\sin\left(\frac{15c}{2}+7dx\right)}{13440a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(210*(1 + 8*d*x)*Cos[c/2] + 1365*Cos[c/2 + d*x] + 1365*Cos[(3*c)/2
 + d*x] - 210*Cos[(3*c)/2 + 2*d*x] + 210*Cos[(5*c)/2 + 2*d*x] + 175*Cos[(5*
c)/2 + 3*d*x] + 175*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*C
os[(9*c)/2 + 4*d*x] - 147*Cos[(9*c)/2 + 5*d*x] - 147*Cos[(11*c)/2 + 5*d*x]
+ 70*Cos[(11*c)/2 + 6*d*x] - 70*Cos[(13*c)/2 + 6*d*x] + 15*Cos[(13*c)/2 + 7
*d*x] + 15*Cos[(15*c)/2 + 7*d*x] - 210*Sin[c/2] + 1680*d*x*Sin[c/2] - 1365*
Sin[c/2 + d*x] + 1365*Sin[(3*c)/2 + d*x] - 210*Sin[(3*c)/2 + 2*d*x] - 210*S
in[(5*c)/2 + 2*d*x] - 175*Sin[(5*c)/2 + 3*d*x] + 175*Sin[(7*c)/2 + 3*d*x] -
 210*Sin[(7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] + 147*Sin[(9*c)/2 + 5*
d*x] - 147*Sin[(11*c)/2 + 5*d*x] + 70*Sin[(11*c)/2 + 6*d*x] + 70*Sin[(13*c)
/2 + 6*d*x] - 15*Sin[(13*c)/2 + 7*d*x] + 15*Sin[(15*c)/2 + 7*d*x])/(a^2*d*(
Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.15, size = 181, normalized size = 1.34

method	result
risch	$-\frac{x}{8a^2}-\frac{13\cos(dx+c)}{64a^2d}-\frac{\cos(7dx+7c)}{448d\,a^2}-\frac{\sin(6dx+6c)}{96a^2d}+\frac{7\cos(5dx+5c)}{320d\,a^2}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{5\cos(3dx+3c)}{192d\,a^2}+$
derivativedivides	$\frac{\frac{16\left(-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{13\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}+\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{97}{}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}}{a^2d}$
default	$\frac{\frac{16\left(-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{5\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{97\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}-\frac{13\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}+\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{97}{}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^2*((-1/64*tan(1/2*d*x+1/2*c)^13-5/48*tan(1/2*d*x+1/2*c)^11-1/4*tan(1
/2*d*x+1/2*c)^10+97/192*tan(1/2*d*x+1/2*c)^9-13/12*tan(1/2*d*x+1/2*c)^8+1/6
*tan(1/2*d*x+1/2*c)^6-97/192*tan(1/2*d*x+1/2*c)^5-3/10*tan(1/2*d*x+1/2*c)^4
+5/48*tan(1/2*d*x+1/2*c)^3-11/60*tan(1/2*d*x+1/2*c)^2+1/64*tan(1/2*d*x+1/2*
c)-11/420)/(1+tan(1/2*d*x+1/2*c)^2)^7-1/64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(121) = 242$.
time = 0.52, size = 416, normalized size = 3.08

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{1232\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{700\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2016\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3395\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1120\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{7280\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3395\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1680\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{700\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{105\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-176}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1232*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 700*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2016*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3395*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1120*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 7280*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 3395*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1680*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 700*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 176)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 80, normalized size = 0.59

$$-\frac{120 \cos(dx+c)^7 - 504 \cos(dx+c)^5 + 560 \cos(dx+c)^3 + 105\,dx + 35\left(8 \cos(dx+c)^5 - 14 \cos(dx+c)^3 + 3 \cos(dx+c)\right)\sin(dx+c)}{840\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*cos(d*x + c)^7 - 504*cos(d*x + c)^5 + 560*cos(d*x + c)^3 + 105*
d*x + 35*(8*cos(d*x + c)^5 - 14*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x +
c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3046 vs. 2(121) = 242.
time = 122.93, size = 3046, normalized size = 22.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**14/(840*a**2*d*tan(c/2 + d*x/2)**14 +
 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 294
00*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a*
*2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) -
735*d*x*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d
*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*ta
n(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2
 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan
(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 +
d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x
/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**
4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/
2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 +
 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 294
00*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**
2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/2)**6/(840*a
**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*
d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*ta
n(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
+ d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/
2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 +
d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/
2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2
```

+ 840*a**2*d) - 735*d*x*tan(c/2 + d*x/2)**2/(840*a**2*d*tan(c/2 + d*x/2)**1
4 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 +
29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640
*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d)
 - 105*d*x/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)*
*12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8
+ 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 588
0*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 210*tan(c/2 + d*x/2)**13/(840*
a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2
*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*t
an(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
 + d*x/2)**2 + 840*a**2*d) - 1400*tan(c/2 + d*x/2)**11/(840*a**2*d*tan(c/2
+ d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*
x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)
6 + 17640*a2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 +
840*a**2*d) - 3360*tan(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 +
5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 2940
0*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**
2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) + 6
790*tan(c/2 + d*x/2)**9/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(
c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2
 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*
x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 14560*tan(c/2 + d
*x/2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**1
2 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 +
29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*
a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) + 2240*tan(c/2 + d*x/2)**6/(840*a*
*2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d
*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan
(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 +
 d*x/2)**2 + 840*a**2*d) - 6790*tan(c/2 + d*x/2)**5/(840*a**2*d*tan(c/2 + d
*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2
)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6
 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840
*a**2*d) - 4032*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880
*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a*
*2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*
tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) + 1400*
tan(c/2 + d*x/2)**3/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2
+ d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d
*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)
4 + 5880*a2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2464*tan(c/2 + d*x/2)
2/(840*a2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 1
7640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d...

Giac [A]
time = 0.51, size = 179, normalized size = 1.33

$$-\frac{\frac{105\,(dx+c)}{a^2}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+700\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+7280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+3395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-700\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1232\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+176\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 700*tan(1/2*d*
x + 1/2*c)^11 + 1680*tan(1/2*d*x + 1/2*c)^10 - 3395*tan(1/2*d*x + 1/2*c)^9
+ 7280*tan(1/2*d*x + 1/2*c)^8 - 1120*tan(1/2*d*x + 1/2*c)^6 + 3395*tan(1/2*
d*x + 1/2*c)^5 + 2016*tan(1/2*d*x + 1/2*c)^4 - 700*tan(1/2*d*x + 1/2*c)^3 +
 1232*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*
x + 1/2*c)^2 + 1)^7*a^2))/d
```

Mupad [B]
time = 12.66, size = 173, normalized size = 1.28

$$-\frac{x}{8\,a^2}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{4}+\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{3}+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-\frac{97\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{12}+\frac{52\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{3}-\frac{8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{3}+\frac{97\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{12}+\frac{24\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{5}-\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{3}+\frac{44\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{15}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{44}{105}}{a^2\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/(8*a^2) - ((44*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/4 - (5*tan
(c/2 + (d*x)/2)^3)/3 + (24*tan(c/2 + (d*x)/2)^4)/5 + (97*tan(c/2 + (d*x)/2)
^5)/12 - (8*tan(c/2 + (d*x)/2)^6)/3 + (52*tan(c/2 + (d*x)/2)^8)/3 - (97*tan
(c/2 + (d*x)/2)^9)/12 + 4*tan(c/2 + (d*x)/2)^10 + (5*tan(c/2 + (d*x)/2)^11)
/3 + tan(c/2 + (d*x)/2)^13/4 + 44/105)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.635 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{3x}{16a^2}+\frac{\cos^5(c+dx)}{10a^2d}+\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}$$

```
[Out] 3/16*x/a^2+1/10*cos(d*x+c)^5/a^2/d+3/16*cos(d*x+c)*sin(d*x+c)/a^2/d+1/8*cos
(d*x+c)^3*sin(d*x+c)/a^2/d+1/6*cos(d*x+c)^3*(a-a*sin(d*x+c))^3/a^5/d
```

Rubi [A]

time = 0.18, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2954, 2949, 2748, 2715, 8}

$$\frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d}+\frac{\cos^5(c+dx)}{10a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{8a^2d}+\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) + Cos[c + d*x]^5/(10*a^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(16*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(8*a^2*d) + (Cos[c + d*x]^3*(a -
 a*Sin[c + d*x])^3)/(6*a^5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
```

```
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)(a-a\sin(c+dx))\,dx}{2a^3} \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))^3}{6a^5d} + \frac{\int \cos^4(c+dx)\,dx}{2a^2} \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^3(c+dx)(a-a\sin(c+dx))}{6a^5d} \\
&= \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} + \cos \\
&= \frac{3x}{16a^2} + \frac{\cos^5(c+dx)}{10a^2d} + \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(104) = 208.
time = 1.39, size = 362, normalized size = 3.48

$$\frac{360dx\cos\left(\frac{c}{2}\right)+240\cos\left(\frac{c}{2}+dx\right)+240\cos\left(\frac{3c}{2}+dx\right)-15\cos\left(\frac{3c}{2}+2dx\right)+15\cos\left(\frac{5c}{2}+2dx\right)+40\cos\left(\frac{5c}{2}+3dx\right)+40\cos\left(\frac{7c}{2}+3dx\right)-45\cos\left(\frac{7c}{2}+4dx\right)+45\cos\left(\frac{9c}{2}+4dx\right)-24\cos\left(\frac{9c}{2}+5dx\right)-24\cos\left(\frac{11c}{2}+5dx\right)+5\cos\left(\frac{11c}{2}+6dx\right)-5\cos\left(\frac{13c}{2}+6dx\right)+50\sin\left(\frac{c}{2}\right)+360dx\sin\left(\frac{c}{2}\right)-240\sin\left(\frac{c}{2}+dx\right)+240\sin\left(\frac{3c}{2}+dx\right)-15\sin\left(\frac{3c}{2}+2dx\right)-15\sin\left(\frac{5c}{2}+2dx\right)-40\sin\left(\frac{5c}{2}+3dx\right)+40\sin\left(\frac{7c}{2}+3dx\right)-45\sin\left(\frac{7c}{2}+4dx\right)-45\sin\left(\frac{9c}{2}+4dx\right)+24\sin\left(\frac{9c}{2}+5dx\right)-24\sin\left(\frac{11c}{2}+5dx\right)+5\sin\left(\frac{11c}{2}+6dx\right)+5\sin\left(\frac{13c}{2}+6dx\right)}{1920a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (360*d*x*Cos[c/2] + 240*Cos[c/2 + d*x] + 240*Cos[(3*c)/2 + d*x] - 15*Cos[(3
*c)/2 + 2*d*x] + 15*Cos[(5*c)/2 + 2*d*x] + 40*Cos[(5*c)/2 + 3*d*x] + 40*Cos
[(7*c)/2 + 3*d*x] - 45*Cos[(7*c)/2 + 4*d*x] + 45*Cos[(9*c)/2 + 4*d*x] - 24*
Cos[(9*c)/2 + 5*d*x] - 24*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(11*c)/2 + 6*d*x] -
 5*Cos[(13*c)/2 + 6*d*x] + 50*Sin[c/2] + 360*d*x*Sin[c/2] - 240*Sin[c/2 + d
*x] + 240*Sin[(3*c)/2 + d*x] - 15*Sin[(3*c)/2 + 2*d*x] - 15*Sin[(5*c)/2 + 2
```

```
*d*x] - 40*Sin[(5*c)/2 + 3*d*x] + 40*Sin[(7*c)/2 + 3*d*x] - 45*Sin[(7*c)/2
+ 4*d*x] - 45*Sin[(9*c)/2 + 4*d*x] + 24*Sin[(9*c)/2 + 5*d*x] - 24*Sin[(11*c
)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13*c)/2 + 6*d*x])/(1920*a^2
*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.15, size = 153, normalized size = 1.47

method	result
risch	$\frac{3x}{16a^2}+\frac{\cos(dx+c)}{4a^2d}+\frac{\sin(6dx+6c)}{192a^2d}-\frac{\cos(5dx+5c)}{40d\,a^2}-\frac{3\sin(4dx+4c)}{64a^2d}+\frac{\cos(3dx+3c)}{24d\,a^2}-\frac{\sin(2dx+2c)}{64a^2d}$
derivativedivides	$\frac{8\left(\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{13\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}+\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{25\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{13\left(\tan^{3}\right.}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{a^2d}$
default	$\frac{8\left(\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{13\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192}+\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{25\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{13\left(\tan^{3}\right.}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*((3/64*tan(1/2*d*x+1/2*c)^11-13/192*tan(1/2*d*x+1/2*c)^9+tan(1/2*d*
x+1/2*c)^8-25/32*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*d*x+1/2*c)^6+25/32*tan(1/
2*d*x+1/2*c)^5+13/192*tan(1/2*d*x+1/2*c)^3+2/5*tan(1/2*d*x+1/2*c)^2-3/64*ta
n(1/2*d*x+1/2*c)+1/15)/(1+tan(1/2*d*x+1/2*c)^2)^6+3/64*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(95) = 190.
time = 0.52, size = 353, normalized size = 3.39

$$-\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{384\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{65\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{750\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{640\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{750\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{960\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{65\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-64}{a^2+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 65*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 750*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 750*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 960*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 65*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 - 64)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^2*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
```

15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d

Fricas [A]
time = 0.39, size = 70, normalized size = 0.67

$$-\frac{96\cos(dx+c)^5-160\cos(dx+c)^3-45\,dx-5\left(8\cos(dx+c)^5-26\cos(dx+c)^3+9\cos(dx+c)\right)\sin(dx+c)}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(96*cos(d*x + c)^5 - 160*cos(d*x + c)^3 - 45*d*x - 5*(8*cos(d*x + c)^5 - 26*cos(d*x + c)^3 + 9*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2271 vs. $2(94) = 188$.
time = 77.86, size = 2271, normalized size = 21.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((45*d*x*tan(c/2 + d*x/2)**12/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 270*d*x*tan(c/2 + d*x/2)**10/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 675*d*x*tan(c/2 + d*x/2)**8/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 900*d*x*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 675*d*x*tan(c/2 + d*x/2)**4/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 270*d*x*tan(c/2 + d*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) +
```

```
45*d*x/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10
+ 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*
a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d)
+ 90*tan(c/2 + d*x/2)**11/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*ta
n(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2
+ d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2
)**2 + 240*a**2*d) - 130*tan(c/2 + d*x/2)**9/(240*a**2*d*tan(c/2 + d*x/2)**
12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4
800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**
2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 1920*tan(c/2 + d*x/2)**8/(240*a**2*
d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan
(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 +
d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) - 1500*tan(c/2 +
d*x/2)**7/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**
10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 36
00*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*
d) + 1280*tan(c/2 + d*x/2)**6/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*
d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(
c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d
*x/2)**2 + 240*a**2*d) + 1500*tan(c/2 + d*x/2)**5/(240*a**2*d*tan(c/2 + d*x
/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**
8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 144
0*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 130*tan(c/2 + d*x/2)**3/(240*a
**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2)**10 + 3600*a**2*d
*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 + 3600*a**2*d*tan(c/
2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a**2*d) + 768*tan(c/2
 + d*x/2)**2/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*tan(c/2 + d*x/2
)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2 + d*x/2)**6 +
 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/2)**2 + 240*a*
*2*d) - 90*tan(c/2 + d*x/2)/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*
tan(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/
2 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x
/2)**2 + 240*a**2*d) + 128/(240*a**2*d*tan(c/2 + d*x/2)**12 + 1440*a**2*d*t
an(c/2 + d*x/2)**10 + 3600*a**2*d*tan(c/2 + d*x/2)**8 + 4800*a**2*d*tan(c/2
 + d*x/2)**6 + 3600*a**2*d*tan(c/2 + d*x/2)**4 + 1440*a**2*d*tan(c/2 + d*x/
2)**2 + 240*a**2*d), Ne(d, 0)), (x*sin(c)**2*cos(c)**6/(a*sin(c) + a)**2, T
rue))
```

Giac [A]

time = 0.44, size = 153, normalized size = 1.47

$$\frac{\frac{45\,(dx+c)}{a^2}+\frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-65\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+65\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 65*tan(1/2*d*x +
1/2*c)^9 + 960*tan(1/2*d*x + 1/2*c)^8 - 750*tan(1/2*d*x + 1/2*c)^7 + 640*ta
n(1/2*d*x + 1/2*c)^6 + 750*tan(1/2*d*x + 1/2*c)^5 + 65*tan(1/2*d*x + 1/2*c)
^3 + 384*tan(1/2*d*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*a^2))/d
```

Mupad [B]
time = 11.70, size = 146, normalized size = 1.40

$$\frac{3\,x}{16\,a^2}+\frac{\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\frac{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{8}{15}}{a^2\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (3*x)/(16*a^2) + ((16*tan(c/2 + (d*x)/2)^2)/5 - (3*tan(c/2 + (d*x)/2))/8 +
(13*tan(c/2 + (d*x)/2)^3)/24 + (25*tan(c/2 + (d*x)/2)^5)/4 + (16*tan(c/2 +
(d*x)/2)^6)/3 - (25*tan(c/2 + (d*x)/2)^7)/4 + 8*tan(c/2 + (d*x)/2)^8 - (13*
tan(c/2 + (d*x)/2)^9)/24 + (3*tan(c/2 + (d*x)/2)^11)/8 + 8/15)/(a^2*d*(tan(
c/2 + (d*x)/2)^2 + 1)^6)
```

3.636 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{x}{4a^2}-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{4a^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d}-\frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2}$$

```
[Out] -1/4*x/a^2-2/15*cos(d*x+c)^5/a^2/d-1/4*cos(d*x+c)*sin(d*x+c)/a^2/d-1/6*cos(
d*x+c)^3*sin(d*x+c)/a^2/d-1/3*cos(d*x+c)^7/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2938, 2761, 2715, 8}

$$-\frac{2\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{x}{4a^2}-\frac{\cos^7(c+dx)}{3d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*x/a^2 - (2*Cos[c + d*x]^5)/(15*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(4
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(6*a^2*d) - Cos[c + d*x]^7/(3*d*(a
+ a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{3a}\\ &= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \frac{2\int \cos^4(c+dx)\,dx}{3a^2}\\ &= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^7(c+dx)}{3d(a+a\sin(c+dx))^2} - \int c\\ &= -\frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d} - 3c\\ &= -\frac{x}{4a^2} - \frac{2\cos^5(c+dx)}{15a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{6a^2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 262 vs. 2(100) = 200.
time = 0.94, size = 262, normalized size = 2.62

$$\frac{-5(5+24dx)\cos\left(\frac{c}{2}\right) - 90\cos\left(\frac{c}{2}+dx\right) - 90\cos\left(\frac{3c}{2}+dx\right) - 25\cos\left(\frac{5c}{2}+3dx\right) - 25\cos\left(\frac{7c}{2}+3dx\right) + 15\cos\left(\frac{7c}{2}+4dx\right) - 15\cos\left(\frac{9c}{2}+4dx\right) + 3\cos\left(\frac{9c}{2}+5dx\right) + 3\cos\left(\frac{11c}{2}+5dx\right) + 25\sin\left(\frac{c}{2}\right) - 120dx\sin\left(\frac{c}{2}\right) + 90\sin\left(\frac{c}{2}+dx\right) - 90\sin\left(\frac{3c}{2}+dx\right) + 25\sin\left(\frac{5c}{2}+3dx\right) - 25\sin\left(\frac{7c}{2}+3dx\right) + 15\sin\left(\frac{7c}{2}+4dx\right) + 15\sin\left(\frac{9c}{2}+4dx\right) - 3\sin\left(\frac{9c}{2}+5dx\right) + 3\sin\left(\frac{11c}{2}+5dx\right)}{480a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(5 + 24*d*x)*Cos[c/2] - 90*Cos[c/2 + d*x] - 90*Cos[(3*c)/2 + d*x] - 25*
Cos[(5*c)/2 + 3*d*x] - 25*Cos[(7*c)/2 + 3*d*x] + 15*Cos[(7*c)/2 + 4*d*x] -
15*Cos[(9*c)/2 + 4*d*x] + 3*Cos[(9*c)/2 + 5*d*x] + 3*Cos[(11*c)/2 + 5*d*x]
+ 25*Sin[c/2] - 120*d*x*Sin[c/2] + 90*Sin[c/2 + d*x] - 90*Sin[(3*c)/2 + d*x
] + 25*Sin[(5*c)/2 + 3*d*x] - 25*Sin[(7*c)/2 + 3*d*x] + 15*Sin[(7*c)/2 + 4*
d*x] + 15*Sin[(9*c)/2 + 4*d*x] - 3*Sin[(9*c)/2 + 5*d*x] + 3*Sin[(11*c)/2 +
5*d*x])/(480*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.25, size = 142, normalized size = 1.42

method	result

risch	$-\frac{x}{4a^2}-\frac{3\cos(dx+c)}{8a^2d}+\frac{\cos(5dx+5c)}{80d\,a^2}+\frac{\sin(4dx+4c)}{16a^2d}-\frac{5\cos(3dx+3c)}{48d\,a^2}$
derivativedivides	$\frac{\frac{4\left(-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^2\left(\frac{d}{}\right.\right.}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^2}$
default	$\frac{\frac{4\left(-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^2\left(\frac{d}{}\right.\right.}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^2}$
norman	$\frac{-\frac{11x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{65\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{11\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6da}-\frac{x}{4a}-\frac{23}{30ad}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5ad}-\frac{11\left(\tan^{15}\left(\frac{dx}{2}\right.\right.}{6da}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*((-1/8*tan(1/2*d*x+1/2*c)^9-1/2*tan(1/2*d*x+1/2*c)^8+3/4*tan(1/2*d*
x+1/2*c)^7-2*tan(1/2*d*x+1/2*c)^6-1/3*tan(1/2*d*x+1/2*c)^4-3/4*tan(1/2*d*x+
1/2*c)^3-2/3*tan(1/2*d*x+1/2*c)^2+1/8*tan(1/2*d*x+1/2*c)-7/30)/(1+tan(1/2*d
*x+1/2*c)^2)^5-1/8*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(90) = 180$.
time = 0.51, size = 310, normalized size = 3.10

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{90\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{40\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{90\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-28}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 80*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 90*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 240*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 90*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9 - 28)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^2*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 60, normalized size = 0.60

$$\frac{12\cos(dx+c)^5-40\cos(dx+c)^3-15\,dx+15\left(2\cos(dx+c)^3-\cos(dx+c)\right)\sin(dx+c)}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*cos(d*x + c)^5 - 40*cos(d*x + c)^3 - 15*d*x + 15*(2*cos(d*x + c)^3
 - cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(90) = 180$.
time = 48.16, size = 1720, normalized size = 17.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(60*a**2*d*tan(c/2 + d*x/2)**10 + 3
00*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d
*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 75*d*x
*tan(c/2 + d*x/2)**8/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 +
 d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**
4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 150*d*x*tan(c/2 + d*x/2)*
*6/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a
**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan
(c/2 + d*x/2)**2 + 60*a**2*d) - 150*d*x*tan(c/2 + d*x/2)**4/(60*a**2*d*tan(
c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*
x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 +
 60*a**2*d) - 75*d*x*tan(c/2 + d*x/2)**2/(60*a**2*d*tan(c/2 + d*x/2)**10 +
300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*
d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 15*d*
x/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a*
*2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(
c/2 + d*x/2)**2 + 60*a**2*d) - 30*tan(c/2 + d*x/2)**9/(60*a**2*d*tan(c/2 +
d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**
6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a*
*2*d) - 120*tan(c/2 + d*x/2)**8/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*
d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2
 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) + 180*tan(c/2 +
d*x/2)**7/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8
+ 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**
2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 480*tan(c/2 + d*x/2)**6/(60*a**2*d*t
an(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 +
 d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**
2 + 60*a**2*d) - 80*tan(c/2 + d*x/2)**4/(60*a**2*d*tan(c/2 + d*x/2)**10 + 3
00*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d
*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 180*ta
```

```
n(c/2 + d*x/2)**3/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*
x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 +
 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 160*tan(c/2 + d*x/2)**2/(60*
a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*t
an(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 +
d*x/2)**2 + 60*a**2*d) + 30*tan(c/2 + d*x/2)/(60*a**2*d*tan(c/2 + d*x/2)**1
0 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a**2*d*tan(c/2 + d*x/2)**6 + 600*a
**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(c/2 + d*x/2)**2 + 60*a**2*d) - 5
6/(60*a**2*d*tan(c/2 + d*x/2)**10 + 300*a**2*d*tan(c/2 + d*x/2)**8 + 600*a*
*2*d*tan(c/2 + d*x/2)**6 + 600*a**2*d*tan(c/2 + d*x/2)**4 + 300*a**2*d*tan(
c/2 + d*x/2)**2 + 60*a**2*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a)
**2, True))
```

Giac [A]
time = 0.46, size = 140, normalized size = 1.40

$$-\frac{\frac{15\,(dx+c)}{a^2}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+28\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15*(d*x + c)/a^2 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1
/2*c)^8 - 90*tan(1/2*d*x + 1/2*c)^7 + 240*tan(1/2*d*x + 1/2*c)^6 + 40*tan(1
/2*d*x + 1/2*c)^4 + 90*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 -
 15*tan(1/2*d*x + 1/2*c) + 28)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

Mupad [B]
time = 9.00, size = 81, normalized size = 0.81

$$\frac{\cos\left(c+d\,x\right)^5}{5\,a^2\,d}-\frac{2\cos\left(c+d\,x\right)^3}{3\,a^2\,d}-\frac{x}{4\,a^2}+\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{2\,a^2\,d}-\frac{\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] cos(c + d*x)^5/(5*a^2*d) - (2*cos(c + d*x)^3)/(3*a^2*d) - x/(4*a^2) + (cos(
c + d*x)^3*sin(c + d*x))/(2*a^2*d) - (cos(c + d*x)*sin(c + d*x))/(4*a^2*d)
```

3.637 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{a^2d}$$

```
[Out] -x/a^2-arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d-1/3*cos(d*x+c)^3/a^2/d-co
s(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2954, 2952, 2715, 8, 2672, 327, 212, 2645, 30}

$$-\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) - Cos[c + d
*x]^3/(3*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-2a^2\cos^2(c+dx)+a^2\cos(c+dx)\cot(c+dx)+a^2\cos^2(c+dx)\sin(c+\right.}{a^4}$$
$$= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^2} + \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2} - \frac{2\int \cos^2(c-}{a^2}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} - \frac{\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{a^2d} - \frac{\text{Sub}}{}$$
$$= -\frac{x}{a^2} + \frac{\cos(c+dx)}{a^2d} - \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{a^2d} - \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= -\frac{x}{a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} - \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos(c+dx)\,\text{s}}{a^2d}$$

Mathematica [A]

time = 0.24, size = 69, normalized size = 0.95

$$-\frac{-9\cos(c+dx)+\cos(3(c+dx))+6\left(2\left(c+dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin(2(c+dx))\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/12*(-9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*(2*(c + d*x + Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]]) + Sin[2*(c + d*x)]))/(a^2*d)
```

Maple [A]

time = 0.28, size = 87, normalized size = 1.19

method	result
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{1}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$
risch	$-\frac{x}{a^2}+\frac{3\,\mathrm{e}^{i(dx+c)}}{8d\,a^2}+\frac{3\,\mathrm{e}^{-i(dx+c)}}{8d\,a^2}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a^2d}-\frac{\cos(3dx+3c)}{12d\,a^2}-\frac{\sin(2dx+2c)}{2a^2d}$
norman	$\frac{10\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{x}{a}+\frac{4}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{8x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{25x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(ln(tan(1/2*d*x+1/2*c))-4*(-1/2*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/
2*c)^2+1/2*tan(1/2*d*x+1/2*c)-1/3)/(1+tan(1/2*d*x+1/2*c)^2)^3-2*arctan(tan(
1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(71) = 142$.
time = 0.50, size = 188, normalized size = 2.58

$$-\frac{\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-2\right)}{a^2+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a^2 - 3*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 71, normalized size = 0.97

$$-\frac{2\cos(dx+c)^3+6dx+6\cos(dx+c)\sin(dx+c)-6\cos(dx+c)+3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{6a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*cos(d*x + c)^3 + 6*d*x + 6*cos(d*x + c)*sin(d*x + c) - 6*cos(d*x +
c) + 3*log(1/2*cos(d*x + c) + 1/2) - 3*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^6(c+dx)\csc(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.45, size = 91, normalized size = 1.25

$$-\frac{\frac{3\,(dx+c)}{a^2}-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)/a^2 - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(3*tan(1/2
*d*x + 1/2*c)^5 + 6*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

Mupad [B]
time = 9.47, size = 167, normalized size = 2.29

$$\frac{2\,\mathrm{atan}\left(\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4}\right)}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4}{3}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*atan(4/(4*tan(c/2 + (d*x)/2) + 4) - (4*tan(c/2 + (d*x)/2))/(4*tan(c/2 +
(d*x)/2) + 4)))/(a^2*d) + log(tan(c/2 + (d*x)/2))/(a^2*d) + (4*tan(c/2 + (d
*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 2*tan(c/2 + (d*x)/2)^5 + 4/3)/(d*(3*a^2*t
an(c/2 + (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6
 + a^2))
```

3.638 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=74

$$-\frac{x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}$$

```
[Out] -1/2*x/a^2+2*arctanh(cos(d*x+c))/a^2/d-2*cos(d*x+c)/a^2/d-cot(d*x+c)/a^2/d+
1/2*cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.241$, Rules used = {2954, 2788, 3855, 3852, 8, 2718, 2715}

$$-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*x/a^2 + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(-2a^4\csc(c+dx)+a^4\csc^2(c+dx)+2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)}{a^6} \\
&= \frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\,dx}{a^2}+\frac{2\int \sin(c+}{a^2} \\
&= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{\int 1\,dx}{2a^2} \\
&= -\frac{x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 116, normalized size = 1.57

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-2(c+dx)-8\cos(c+dx)-2\cot\left(\frac{1}{2}(c+dx)\right)+8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sin(2(c+dx))+2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4d(a+a\sin(c+dx))^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-2*(c + d*x) - 8*Cos[c + d*x] - 2*Cot[(c + d*x)/2] + 8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)] + 2*Tan[(c + d*x)/2]))/(4*d*(a + a*Sin[c + d*x])^2)

Maple [A]
time = 0.29, size = 112, normalized size = 1.51

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-2\arctan\left(\tan\left(\frac{d}{}\right.\right.}{2d\,a^2}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-2\arctan\left(\tan\left(\frac{d}{}\right.\right.}{2d\,a^2}$
risch	$-\frac{x}{2a^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8a^2d}-\frac{\mathrm{e}^{i(dx+c)}}{d\,a^2}-\frac{\mathrm{e}^{-i(dx+c)}}{d\,a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{2i}{a^2d\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{a^2d}+\frac{2\ln(\mathrm{e}}{}$
norman	$\frac{-\frac{25\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{1}{2ad}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{13x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{9x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c))+16*(-1/8*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+1/8*tan(1/2*d*x+1/2*c)-1/2)/(1+tan(1/2*d*x+1/2*c)^2)^2-2*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(70) = 140.
time = 0.50, size = 202, normalized size = 2.73

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/2*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d

Fricas [A]
time = 0.39, size = 88, normalized size = 1.19

$$-\frac{\cos\left(dx+c\right)^{3}+\left(dx+4\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-2\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\cos\left(dx+c\right)}{2\,a^{2}d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(cos(d*x + c)^3 + (d*x + 4*cos(d*x + c))*sin(d*x + c) - 2*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
+ cos(d*x + c))/(a^2*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 131, normalized size = 1.77

$$-\frac{\frac{dx+c}{a^{2}}+\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x +
1/2*c)/a^2 - (4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 2*(t
an(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4
)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 9.06, size = 175, normalized size = 2.36

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^2\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1}{d\,\left(2\,a^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\,a^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{1}{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4}+\frac{4\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-4}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (8*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)
^3 + 3*tan(c/2 + (d*x)/2)^4 + 1)/(d*(4*a^2*tan(c/2 + (d*x)/2)^3 + 2*a^2*tan
(c/2 + (d*x)/2)^5 + 2*a^2*tan(c/2 + (d*x)/2))) - (2*log(tan(c/2 + (d*x)/2))
)/(a^2*d) + atan(1/(tan(c/2 + (d*x)/2) - 4) + (4*tan(c/2 + (d*x)/2))/(tan(c
/2 + (d*x)/2) - 4))/(a^2*d)
```

3.639 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] 2*x/a^2-1/2*arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d+2*cot(d*x+c)/a^2/d-1
/2*cot(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A]
time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.241$, Rules used = {2954, 2951, 3852, 8, 3853, 3855, 2718}

$$\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 - ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]/(a^2*d) + (2*Cot
[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(2a^4-2a^4\csc^2(c+dx)+a^4\csc^3(c+dx)-a^4\sin(c+dx)\right)\,dx}{a^6}\\ &= \frac{2x}{a^2}+\frac{\int \csc^3(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\,dx}{a^2}-\frac{2\int \csc^2(c+dx)\,dx}{a^2}\\ &= \frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{2a^2}+\frac{2\mathrm{Subs}}{}\\ &= \frac{2x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)}{2a}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 134, normalized size = 1.84

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(16(c+dx)+8\cos(c+dx)+8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)-8\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(16*(c + d*x) + 8*Cos[c + d*x] + 8
*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Si
n[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 8*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin
[c + d*x])^2)
```

Maple [A]
time = 0.29, size = 101, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+16\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{4d\,a^2}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+16\arctan\left(\tan\left(\frac{dx}{2}\right.\right.}{4d\,a^2}$
risch	$\frac{2x}{a^2}+\frac{e^{i(dx+c)}}{2d\,a^2}+\frac{e^{-i(dx+c)}}{2d\,a^2}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+4ie^{2i(dx+c)}-4i}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2a^2d}$
norman	$\frac{-\frac{1}{8ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{5\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{6x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{12x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{20x}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/
2*c)^2+4/tan(1/2*d*x+1/2*c)+2*ln(tan(1/2*d*x+1/2*c))+8/(1+tan(1/2*d*x+1/2*c
)^2)+16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(69) = 138$.
time = 0.52, size = 204, normalized size = 2.79

$$\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^2}+\frac{32\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{4\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((8*sin(d*x + c)/(cos(d*x + c) + 1) + 15*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)/(a^2*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (8*sin(d*x +
c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 32*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 4*log(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2)/d
```

Fricas [A]
time = 0.40, size = 118, normalized size = 1.62

$$\frac{8\,dx\cos\left(dx+c\right)^{2}+4\,\cos\left(dx+c\right)^{3}-8\,dx-\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-8\,\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)}{4\,\left(a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(8*d*x*cos(d*x + c)^2 + 4*cos(d*x + c)^3 - 8*d*x - (cos(d*x + c)^2 - 1)
*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 8*cos(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^2*d*cos(d*x + c)^2
 - a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.48, size = 128, normalized size = 1.75

$$\frac{\frac{16\,(dx+c)}{a^{2}}+\frac{4\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}+\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{16}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}-\frac{6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*(d*x + c)/a^2 + 4*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 16/((tan(1/2*d*x + 1/2*
c)^2 + 1)*a^2) - (6*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a
^2*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 9.10, size = 186, normalized size = 2.55

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{4\,\mathrm{atan}\left(\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}+\frac{16}{16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4}\right)}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (4*atan((4*tan(c/2 + (d*x)/2))/(16*tan(c/2 + (d*x)/2) - 4) + 16/(16*tan(c/2 + (d*x)/2) - 4)))/(a^2*d) + log(tan(c/2 + (d*x)/2))/(2*a^2*d) + (4*tan(c/2 + (d*x)/2) + (15*tan(c/2 + (d*x)/2)^2)/2 + 4*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)/(a^2*d)

3.640 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{x}{a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

```
[Out] -x/a^2-arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+co
t(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cot[c + d*x]/(a^2*d) - Cot[c + d
*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^2(c+dx) - 2a^2\cot^2(c+dx)\csc(c+dx) + a^2\cot^2(c+dx)\csc^2(c\right.}{a^4}$$

$$= \frac{\int \cot^2(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^2} - \frac{2\int \cot^2(c+dx)\csc}{a^2}$$

$$= -\frac{\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - \frac{\int 1\,dx}{a^2} + \frac{\int \csc(c+dx)\,dx}{a^2} + \frac{S}{}$$

$$= -\frac{x}{a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3a^2d} + \frac{\cot(c+dx)}{a^2}$$

Mathematica [A]
time = 0.90, size = 124, normalized size = 1.70

$$-\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(6\cos(c+dx)-2\cos(3(c+dx))+12\left(c+dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)-6\sin(2(c+dx))\right)\tan\left(\frac{1}{2}(c+dx)\right)}{96a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/96*((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(6*Cos[c + d*x] - 2*Cos[
3*(c + d*x)] + 12*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])
*Sin[c + d*x]^3 - 6*Sin[2*(c + d*x)])*Tan[(c + d*x)/2])/(a^2*d*(1 + Sin[c +
 d*x])^2)
```

Maple [A]
time = 0.29, size = 110, normalized size = 1.51

method	result
risch	$-\frac{x}{a^2}-\frac{2\left(3\,e^{5i(dx+c)}-6ie^{2i(dx+c)}+2i-3\,e^{i(dx+c)}\right)}{3a^2d\left(e^{2i(dx+c)}-1\right)^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^2}$
norman	$-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2+3*tan(1/2*d*x+1/
2*c)-1/3/tan(1/2*d*x+1/2*c)^3+2/tan(1/2*d*x+1/2*c)^2-3/tan(1/2*d*x+1/2*c)+8
*ln(tan(1/2*d*x+1/2*c))-16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(71) = 142$.
time = 0.51, size = 176, normalized size = 2.41

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{48\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 48*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 + 24*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (6*s
```

```
in(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)
*(cos(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.40, size = 139, normalized size = 1.90

$$-\frac{4\cos(dx+c)^3+3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+6\left(dx\cos(dx+c)^2-dx+\cos(dx+c)\right)\sin(dx+c)-6\cos(dx+c)}{6\left(a^2d\cos(dx+c)^2-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/6*(4*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) + 6*(d*x*cos(d*x + c)^2 - d*x + cos(d*x + c))*sin(d*x + c) - 6*cos(d*
x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.44, size = 137, normalized size = 1.88

$$-\frac{\frac{24(dx+c)}{a^2}-\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{44\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/a^2 - 24*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (44*tan(1
/2*d*x + 1/2*c)^3 + 9*tan(1/2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/
(a^2*tan(1/2*d*x + 1/2*c)^3) - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*
d*x + 1/2*c)^2 + 9*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 9.30, size = 261, normalized size = 3.58

$$\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+9\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-9\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+48\,\mathrm{atan}\left(\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+24\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^2d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (sin(c/2 + (d*x)/2)^6 - cos(c/2 + (d*x)/2)^6 - 6*cos(c/2 + (d*x)/2)*sin(c/2
 + (d*x)/2)^5 + 6*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2) + 9*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^4 - 9*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2
+ 48*atan((cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2) + s
in(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3 + 24*log(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^
3)/(24*a^2*d*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)
```

3.641 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}$$

```
[Out] 5/8*arctanh(cos(d*x+c))/a^2/d+2/3*cot(d*x+c)^3/a^2/d-3/8*cot(d*x+c)*csc(d*x
+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2691, 3855, 2687, 30, 3853}

$$\frac{2\cot^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(8*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot
[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^2(c+dx)\csc(c+dx)-2a^2\cot^2(c+dx)\csc^2(c+dx)+a^2\cot^2(c\right.}{a^4}$$
$$= \frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2}-\frac{2\int \cot^2}{}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{\int \csc^3(c+dx)\,dx}{4a^2}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(}{}$$
$$= \frac{5\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot}{}$$

Mathematica [A]

time = 0.94, size = 116, normalized size = 1.41

$$\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-33\cos(c+dx)+60\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^4(c+dx)+\cos(3(c+dx))(9+16\sin(c+dx))+24\sin(2(c+dx))\right)}{1536a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(-33*Cos[c + d*x] + 60*(Log[Cos[(c
 + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 + Cos[3*(c + d*x)]*(9 +
 16*Sin[c + d*x]) + 24*Sin[2*(c + d*x)]))/(1536*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.30, size = 124, normalized size = 1.51

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^2}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^2}$
risch	$\frac{-48i\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{7i(dx+c)}+48i\mathrm{e}^{4i(dx+c)}-33\,\mathrm{e}^{5i(dx+c)}-16i\mathrm{e}^{2i(dx+c)}-33\,\mathrm{e}^{3i(dx+c)}+16i+9\,\mathrm{e}^{i(dx+c)}}{12a^2d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{8a^2d}$
norman	$\frac{-\frac{1}{64ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{17\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{7\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{\tan^{13}(}{64}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^2*(1/4*tan(1/2*d*x+1/2*c)^4-4/3*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x
+1/2*c)^2+4*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4-10*ln(tan(1/2*d*x+1
/2*c))-2/tan(1/2*d*x+1/2*c)^2-4/tan(1/2*d*x+1/2*c)+4/3/tan(1/2*d*x+1/2*c)^3
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(74) = 148.
time = 0.29, size = 194, normalized size = 2.37

$$\frac{\frac{\frac{48\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{48\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-3\right)(\cos(dx+c)+1)^4}{a^2\sin(dx+c)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/192*((48*sin(d*x + c)/(cos(d*x + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (16*si
n(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 48
```

```
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3)*(cos(d*x + c) + 1)^4/(a^2*sin(d*x
 + c)^4))/d
```

Fricas [A]
time = 0.39, size = 138, normalized size = 1.68

$$\frac{32 \cos\left(dx+c\right)^{3} \sin\left(dx+c\right)+18 \cos\left(dx+c\right)^{3}+15 \left(\cos\left(dx+c\right)^{4}-2 \cos\left(dx+c\right)^{2}+1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)-15 \left(\cos\left(dx+c\right)^{4}-2 \cos\left(dx+c\right)^{2}+1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)-30 \cos\left(dx+c\right)}{48 \left(a^{2} d \cos\left(dx+c\right)^{4}-2 a^{2} d \cos\left(dx+c\right)^{2}+a^{2} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(32*cos(d*x + c)^3*sin(d*x + c) + 18*cos(d*x + c)^3 + 15*(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) - 30*cos(d*x + c))/(a
^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(74) = 148.
time = 0.54, size = 158, normalized size = 1.93

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{2}}-\frac{250 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-48 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-24 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+16 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-3}{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}}-\frac{3 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-16 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+24 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+48 a^{6} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{8}}}{192 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (250*tan(1/2*d*x + 1/2*c)^
4 - 48*tan(1/2*d*x + 1/2*c)^3 - 24*tan(1/2*d*x + 1/2*c)^2 + 16*tan(1/2*d*x
+ 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4) - (3*a^6*tan(1/2*d*x + 1/2*c)^4
- 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 24*a^6*tan(1/2*d*x + 1/2*c)^2 + 48*a^6*ta
n(1/2*d*x + 1/2*c))/a^8)/d
```

Mupad [B]
time = 9.03, size = 151, normalized size = 1.84

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}-\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{4}\right)}{16\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + tan(c/2
+ (d*x)/2)^4/(64*a^2*d) - (5*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) + tan(c/2 +
 (d*x)/2)/(4*a^2*d) - (cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2)^2 - (4*ta
n(c/2 + (d*x)/2))/3 + 4*tan(c/2 + (d*x)/2)^3 + 1/4))/(16*a^2*d)
```

3.642 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}$$

```
[Out] -1/4*arctanh(cos(d*x+c))/a^2/d-2/3*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a^2/
d-1/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)*csc(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2788, 3852, 8, 3853, 3855}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*ArcTanh[Cos[c + d*x]]/(a^2*d) - (2*Cot[c + d*x]^3)/(3*a^2*d) - Cot[c +
 d*x]^5/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(4*a^2*d) + (Cot[c + d*x]*C
sc[c + d*x]^3)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(-a^4\csc^2(c+dx)+2a^4\csc^3(c+dx)-2a^4\csc^5(c+dx)+a^4\csc^6(c+dx)\right)\,dx}{a^6}\\ &= -\frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^6(c+dx)\,dx}{a^2}+\frac{2\int \csc^3(c+dx)\,dx}{a^2}-\frac{2\int \csc^5(c+d}{a^2}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{2a^2d}+\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{3\int}{}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+}{4a^2d}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}-\frac{2\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot(c+dx)\csc(c+}{4a^2d}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(200\cos(c+dx)+20\cos(3(c+dx))-28\cos(5(c+dx))+150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-150\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-180\sin(2(c+dx))-75\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+75\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-30\sin(4(c+dx))+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{960a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(Csc[c + d*x]^5*(200*Cos[c + d*x] + 20*Cos[3*(c + d*x)] - 28*Cos[5*(
c + d*x)] + 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*Log[Sin[(c + d*x)/
2]]*Sin[c + d*x] - 180*Sin[2*(c + d*x)] - 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c
 + d*x)] + 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*Sin[4*(c + d*x)]
+ 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[
5*(c + d*x)]))/(a^2*d)
```

Maple [A]
time = 0.34, size = 122, normalized size = 1.22

method	result

derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{32d\,a^2}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{32d\,a^2}$
risch	$\frac{60ie^{8i(dx+c)}+15\,e^{9i(dx+c)}-240ie^{6i(dx+c)}+90\,e^{7i(dx+c)}+40ie^{4i(dx+c)}-80ie^{2i(dx+c)}-90\,e^{3i(dx+c)}+28i-15\,e^{i(dx+c)}}{30a^2d\left(e^{2i(dx+c)}-1\right)^5}+$
norman	$\frac{-\frac{1}{160ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}+\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480ad}-\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{11\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160ad}-\frac{11(\tan}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a\left(\tan\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4+5/3*tan(1/2*d*x+1
/2*c)^3-6*tan(1/2*d*x+1/2*c)+1/tan(1/2*d*x+1/2*c)^4-1/5/tan(1/2*d*x+1/2*c)^
5+8*ln(tan(1/2*d*x+1/2*c))+6/tan(1/2*d*x+1/2*c)-5/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(90) = 180$.
time = 0.28, size = 195, normalized size = 1.95

$$-\frac{\frac{\frac{90\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{25\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{90\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((90*sin(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5)/a^2 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (15*s
in(d*x + c)/(cos(d*x + c) + 1) - 25*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*
x + c)^5))/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.67

$$\frac{56\cos(dx+c)^5-80\cos(dx+c)^3-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+30\left(\cos(dx+c)^3+\cos(dx+c)\right)\sin(dx+c)}{120\left(a^2d\cos(dx+c)^4-2a^2d\cos(dx+c)^2+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(56*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*(c
os(d*x + c)^3 + cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.51, size = 157, normalized size = 1.57

$$\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2} - \frac{274 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-90 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+25 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+3}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5} + \frac{3 a^8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5-15 a^8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+25 a^8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-90 a^8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{10}}}{480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (274*tan(1/2*d*x + 1/2*c)^5
 - 90*tan(1/2*d*x + 1/2*c)^4 + 25*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x +
 1/2*c) + 3)/(a^2*tan(1/2*d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 -
 15*a^8*tan(1/2*d*x + 1/2*c)^4 + 25*a^8*tan(1/2*d*x + 1/2*c)^3 - 90*a^8*tan
(1/2*d*x + 1/2*c))/a^10)/d
```

Mupad [B]
time = 9.02, size = 149, normalized size = 1.49

$$\frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^3}{96 a^2 d} - \frac{\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^4}{32 a^2 d} + \frac{\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^5}{160 a^2 d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)\right)}{4 a^2 d} - \frac{3 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{16 a^2 d} + \frac{\cot\left(\frac{c}{2}+\frac{d x}{2}\right)^5 \left(6 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^4 - \frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^2}{3} + \tan\left(\frac{c}{2}+\frac{d x}{2}\right) - \frac{1}{5}\right)}{32 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^3)/(96*a^2*d) - tan(c/2 + (d*x)/2)^4/(32*a^2*d) + tan
(c/2 + (d*x)/2)^5/(160*a^2*d) + log(tan(c/2 + (d*x)/2))/(4*a^2*d) - (3*tan(
c/2 + (d*x)/2))/(16*a^2*d) + (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)/2) - (5
*tan(c/2 + (d*x)/2)^2)/3 + 6*tan(c/2 + (d*x)/2)^4 - 1/5))/(32*a^2*d)
```

3.643 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^2d}-\frac{5\cot(c+dx)\csc^3(c+a}{24a^2d}$$

```
[Out] 3/16*arctanh(cos(d*x+c))/a^2/d+2/3*cot(d*x+c)^3/a^2/d+2/5*cot(d*x+c)^5/a^2/
d+3/16*cot(d*x+c)*csc(d*x+c)/a^2/d-5/24*cot(d*x+c)*csc(d*x+c)^3/a^2/d-1/6*c
ot(d*x+c)*csc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2691, 3853, 3855, 2687, 14}

$$\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (2*Co
t[c + d*x]^5)/(5*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (5*Cot
[c + d*x]*Csc[c + d*x]^3)/(24*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2
*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^2(c+dx)\csc^3(c+dx) - 2a^2\cot^2(c+dx)\csc^4(c+dx) + a^2\cot^2(c\right.}{a^4}$$

$$= \frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^2} - \frac{2\int \cot^2(}{}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^5(c+dx)}{6a^2d} - \frac{\int \csc^5(c+dx)\,dx}{6a^2}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{5\cot(c+dx)\csc^3(c+dx)}{24a^2d} - \frac{\cot(c+dx)\csc^5(c}{6a^2d}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc}{16a^2d}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^3(c+dx)}{3a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\,\mathrm{cs}}{16a^2d}$$

Mathematica [A]
time = 0.49, size = 229, normalized size = 1.85

$$-\frac{\csc^6(c+dx)\left(1500\cos(c+dx)-130\cos(3(c+dx))-90\cos(5(c+dx))-450\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+675\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-270\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+45\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+450\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-675\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+270\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-45\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-960\sin(2(c+dx))-384\sin(4(c+dx))+64\sin(6(c+dx))\right)}{7680a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(1500*Cos[c + d*x] - 130*Cos[3*(c + d*x)] - 90*Cos[
5*(c + d*x)] - 450*Log[Cos[(c + d*x)/2]] + 675*Cos[2*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] - 270*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 45*Cos[6*(c + d*x
)]*Log[Cos[(c + d*x)/2]] + 450*Log[Sin[(c + d*x)/2]] - 675*Cos[2*(c + d*x)]
*Log[Sin[(c + d*x)/2]] + 270*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 45*Co
s[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 960*Sin[2*(c + d*x)] - 384*Sin[4*(c
+ d*x)] + 64*Sin[6*(c + d*x)]))/(a^2*d)
```

Maple [A]
time = 0.36, size = 176, normalized size = 1.42

method	result
risch	$-\frac{45\,e^{11i(dx+c)}-960ie^{8i(dx+c)}+65\,e^{9i(dx+c)}+640ie^{6i(dx+c)}-750\,e^{7i(dx+c)}-750\,e^{5i(dx+c)}+384ie^{2i(dx+c)}+65\,e^{3i(dx+c)}\ldots}{120a^2d\left(e^{2i(dx+c)}-1\right)^6}$
derivativedivides	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots}}{64d\,a^2}$
default	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots}}{64d\,a^2}$
norman	$\frac{-\frac{1}{384ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{640ad}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480ad}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}-\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{16ad}-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{80ad}\ldots}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^2*(1/6*tan(1/2*d*x+1/2*c)^6-4/5*tan(1/2*d*x+1/2*c)^5+3/2*tan(1/2*d
*x+1/2*c)^4-4/3*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+8*tan(1/2*d*x
+1/2*c)+4/5/tan(1/2*d*x+1/2*c)^5-3/2/tan(1/2*d*x+1/2*c)^4-1/6/tan(1/2*d*x+1
/2*c)^6-12*ln(tan(1/2*d*x+1/2*c))+1/2/tan(1/2*d*x+1/2*c)^2-8/tan(1/2*d*x+1/
2*c)+4/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(112) = 224$.
time = 0.29, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5\right)(\cos(dx+c)+1)^6}{a^2\sin(dx+c)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6)/a^2 - 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2
 + (24*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) +
1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [**A**]
time = 0.39, size = 196, normalized size = 1.58

$$-\frac{90\,\cos\left(dx+c\right)^{5}-80\,\cos\left(dx+c\right)^{3}-45\,\left(\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+45\,\left(\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-64\,\left(2\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)-90\,\cos\left(dx+c\right)}{480\,\left(a^{2}d\cos\left(dx+c\right)^{6}-3\,a^{2}d\cos\left(dx+c\right)^{4}+3\,a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(90*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 45*(cos(d*x + c)^6 - 3*cos(
d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 45*(cos(d*
x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) +
 1/2) - 64*(2*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*sin(d*x + c) - 90*cos(d*x
+ c))/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)
^2 - a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 216, normalized size = 1.74

$$-\frac{\frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{882\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}-\frac{5\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-24\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+45\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-40\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+240\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{12}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (882*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*
x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) - 5)/(a^
2*tan(1/2*d*x + 1/2*c)^6) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(1/
2*d*x + 1/2*c)^5 + 45*a^10*tan(1/2*d*x + 1/2*c)^4 - 40*a^10*tan(1/2*d*x + 1
/2*c)^3 - 15*a^10*tan(1/2*d*x + 1/2*c)^2 + 240*a^10*tan(1/2*d*x + 1/2*c))/a
^12)/d
```

Mupad [B]
time = 10.11, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-40\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+360\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{1920\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 24*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 45*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 40*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 240*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 240*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x
)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 40*cos(c/2 + (d*x)/
2)^9*sin(c/2 + (d*x)/2)^3 + 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 +
 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/
2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.644 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$-\frac{23x}{16a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx)\sin(c+dx)}{16a^3d}+\frac{23\cos(c+dx)\sin^3(c+dx)}{24a^3d}$$

```
[Out] -23/16*x/a^3-4*cos(d*x+c)/a^3/d+7/3*cos(d*x+c)^3/a^3/d-3/5*cos(d*x+c)^5/a^3
/d+23/16*cos(d*x+c)*sin(d*x+c)/a^3/d+23/24*cos(d*x+c)*sin(d*x+c)^3/a^3/d+1/
6*cos(d*x+c)*sin(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2713, 2715, 8}

$$-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{\sin^5(c+dx)\cos(c+dx)}{6a^3d}+\frac{23\sin^3(c+dx)\cos(c+dx)}{24a^3d}+\frac{23\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{23x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) - (4*Cos[c + d*x])/(a^3*d) + (7*Cos[c + d*x]^3)/(3*a^3*d)
- (3*Cos[c + d*x]^5)/(5*a^3*d) + (23*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d)
+ (23*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^3*d) + (Cos[c + d*x]*Sin[c + d*x]^
5)/(6*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
```

```
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(a^3\sin^3(c+dx)-3a^3\sin^4(c+dx)+3a^3\sin^5(c+dx)-a^3\sin^6(c+dx\right.}{a^6} \\
&= \frac{\int \sin^3(c+dx)\,dx}{a^3}-\frac{\int \sin^6(c+dx)\,dx}{a^3}-\frac{3\int \sin^4(c+dx)\,dx}{a^3}+\frac{3\int \sin^5(c}{a} \\
&= \frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{\cos(c+dx)\sin^5(c+dx)}{6a^3d}-\frac{5\int \sin^4(c+dx)}{6a^3} \\
&= -\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{9\cos(c+dx)\sin(c-}{8a^3d} \\
&= -\frac{9x}{8a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+dx}{16} \\
&= -\frac{23x}{16a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{7\cos^3(c+dx)}{3a^3d}-\frac{3\cos^5(c+dx)}{5a^3d}+\frac{23\cos(c+}{1}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(129) = 258.
time = 1.50, size = 366, normalized size = 2.84

$$\frac{-3(3+920dx)\cos\left(\frac{c}{2}\right)-2520\cos\left(\frac{c}{2}+dx\right)-2520\cos\left(\frac{3c}{2}+dx\right)+945\cos\left(\frac{3c}{2}+2dx\right)-945\cos\left(\frac{5c}{2}+2dx\right)+380\cos\left(\frac{5c}{2}+3dx\right)+380\cos\left(\frac{7c}{2}+3dx\right)-135\cos\left(\frac{7c}{2}+4dx\right)+135\cos\left(\frac{9c}{2}+4dx\right)-36\cos\left(\frac{9c}{2}+5dx\right)-36\cos\left(\frac{11c}{2}+5dx\right)+5\cos\left(\frac{11c}{2}+6dx\right)-5\cos\left(\frac{13c}{2}+6dx\right)+9\sin\left(\frac{c}{2}\right)-2760dx\sin\left(\frac{c}{2}\right)+2520\sin\left(\frac{c}{2}+dx\right)-2520\sin\left(\frac{3c}{2}+dx\right)+945\sin\left(\frac{3c}{2}+2dx\right)+945\sin\left(\frac{5c}{2}+2dx\right)-380\sin\left(\frac{5c}{2}+3dx\right)+380\sin\left(\frac{7c}{2}+3dx\right)-135\sin\left(\frac{7c}{2}+4dx\right)-135\sin\left(\frac{9c}{2}+4dx\right)+36\sin\left(\frac{9c}{2}+5dx\right)-36\sin\left(\frac{11c}{2}+5dx\right)+5\sin\left(\frac{11c}{2}+6dx\right)+5\sin\left(\frac{13c}{2}+6dx\right)}{1920a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(3 + 920*d*x)*Cos[c/2] - 2520*Cos[c/2 + d*x] - 2520*Cos[(3*c)/2 + d*x]
+ 945*Cos[(3*c)/2 + 2*d*x] - 945*Cos[(5*c)/2 + 2*d*x] + 380*Cos[(5*c)/2 + 3
*d*x] + 380*Cos[(7*c)/2 + 3*d*x] - 135*Cos[(7*c)/2 + 4*d*x] + 135*Cos[(9*c)
/2 + 4*d*x] - 36*Cos[(9*c)/2 + 5*d*x] - 36*Cos[(11*c)/2 + 5*d*x] + 5*Cos[(1
1*c)/2 + 6*d*x] - 5*Cos[(13*c)/2 + 6*d*x] + 9*Sin[c/2] - 2760*d*x*Sin[c/2]
```

```
+ 2520*Sin[c/2 + d*x] - 2520*Sin[(3*c)/2 + d*x] + 945*Sin[(3*c)/2 + 2*d*x]
+ 945*Sin[(5*c)/2 + 2*d*x] - 380*Sin[(5*c)/2 + 3*d*x] + 380*Sin[(7*c)/2 + 3
*d*x] - 135*Sin[(7*c)/2 + 4*d*x] - 135*Sin[(9*c)/2 + 4*d*x] + 36*Sin[(9*c)/
2 + 5*d*x] - 36*Sin[(11*c)/2 + 5*d*x] + 5*Sin[(11*c)/2 + 6*d*x] + 5*Sin[(13
*c)/2 + 6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.18, size = 168, normalized size = 1.30

method	result
risch	$-\frac{23x}{16a^3}-\frac{21\cos(dx+c)}{8a^3d}+\frac{\sin(6dx+6c)}{192d\,a^3}-\frac{3\cos(5dx+5c)}{80d\,a^3}-\frac{9\sin(4dx+4c)}{64d\,a^3}+\frac{19\cos(3dx+3c)}{48d\,a^3}+\frac{63\sin(2dx+2c)}{64d\,a^3}$
derivativedivides	$\frac{16\left(\frac{-\frac{23\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{391\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{75\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{17\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{75\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-4}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}\right.}{d\,a^3}$
default	$\frac{16\left(\frac{-\frac{23\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{391\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{75\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-\frac{17\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{75\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}-4}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}\right.}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*((-23/128*tan(1/2*d*x+1/2*c)^11-391/384*tan(1/2*d*x+1/2*c)^9-1/4*t
an(1/2*d*x+1/2*c)^8-75/64*tan(1/2*d*x+1/2*c)^7-17/6*tan(1/2*d*x+1/2*c)^6+75
/64*tan(1/2*d*x+1/2*c)^5-4*tan(1/2*d*x+1/2*c)^4+391/384*tan(1/2*d*x+1/2*c)^
3-17/10*tan(1/2*d*x+1/2*c)^2+23/128*tan(1/2*d*x+1/2*c)-17/60)/(1+tan(1/2*d*
x+1/2*c)^2)^6-23/128*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(117) = 234$.
time = 0.52, size = 373, normalized size = 2.89

$$\frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}-\frac{3264\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{7680\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2250\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5440\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2250\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{480\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1955\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{345\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-544}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{345\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) - 3264*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1955*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7680*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 2250*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5440*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2250*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 480*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 1955*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 - 345*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 544)/(a^3 + 6*a^3
```

```
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 20*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 6*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*s
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^3)/d
```

Fricas [A]
time = 0.40, size = 78, normalized size = 0.60

$$-\frac{144\,\cos\left(dx+c\right)^{5}-560\,\cos\left(dx+c\right)^{3}+345\,dx-5\left(8\,\cos\left(dx+c\right)^{5}-62\,\cos\left(dx+c\right)^{3}+123\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+960\,\cos\left(dx+c\right)}{240\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(144*cos(d*x + c)^5 - 560*cos(d*x + c)^3 + 345*d*x - 5*(8*cos(d*x +
c)^5 - 62*cos(d*x + c)^3 + 123*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c
))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2404 vs. $2(122) = 244$.
time = 180.60, size = 2404, normalized size = 18.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-345*d*x*tan(c/2 + d*x/2)**12/(240*a**3*d*tan(c/2 + d*x/2)**12 +
 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*
a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*
tan(c/2 + d*x/2)**2 + 240*a**3*d) - 2070*d*x*tan(c/2 + d*x/2)**10/(240*a**3
*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*ta
n(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 +
 d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 5175*d*x*tan(c
/2 + d*x/2)**8/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x
/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6
 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*
a**3*d) - 6900*d*x*tan(c/2 + d*x/2)**6/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1
440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a*
*3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*ta
n(c/2 + d*x/2)**2 + 240*a**3*d) - 5175*d*x*tan(c/2 + d*x/2)**4/(240*a**3*d*
tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c
/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*
x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 2070*d*x*tan(c/2
```

```
+ d*x/2)**2/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)
**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 +
3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**
3*d) - 345*d*x/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x
/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6
 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*
a**3*d) - 690*tan(c/2 + d*x/2)**11/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*
a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d
*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/
2 + d*x/2)**2 + 240*a**3*d) - 3910*tan(c/2 + d*x/2)**9/(240*a**3*d*tan(c/2
+ d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x
/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4
+ 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 960*tan(c/2 + d*x/2)**8/(
240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a
**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*t
an(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 4500*t
an(c/2 + d*x/2)**7/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 +
 d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2
)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 +
240*a**3*d) - 10880*tan(c/2 + d*x/2)**6/(240*a**3*d*tan(c/2 + d*x/2)**12 +
1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a
**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*t
an(c/2 + d*x/2)**2 + 240*a**3*d) + 4500*tan(c/2 + d*x/2)**5/(240*a**3*d*tan
(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2
+ d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2
)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 15360*tan(c/2 + d*x/
2)**4/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 +
 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a
**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) +
 3910*tan(c/2 + d*x/2)**3/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*ta
n(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2
+ d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2
)**2 + 240*a**3*d) - 6528*tan(c/2 + d*x/2)**2/(240*a**3*d*tan(c/2 + d*x/2)*
*12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 +
4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a*
*3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) + 690*tan(c/2 + d*x/2)/(240*a**3*d*t
an(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/
2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x
/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 1088/(240*a**3*d*t
an(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/
2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x
/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d), Ne(d, 0)), (x*sin(c
)**3*cos(c)**6/(a*sin(c) + a)**3, True))
```

Giac [A]

time = 0.51, size = 166, normalized size = 1.29

$$-\frac{\frac{345\,(dx+c)}{a^3}+\frac{2\left(345\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+2250\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+5440\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-2250\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7680\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1955\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3264\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-345\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+544\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 + 2*(345*tan(1/2*d*x + 1/2*c)^11 + 1955*tan(1/2*d
*x + 1/2*c)^9 + 480*tan(1/2*d*x + 1/2*c)^8 + 2250*tan(1/2*d*x + 1/2*c)^7 +
5440*tan(1/2*d*x + 1/2*c)^6 - 2250*tan(1/2*d*x + 1/2*c)^5 + 7680*tan(1/2*d*
x + 1/2*c)^4 - 1955*tan(1/2*d*x + 1/2*c)^3 + 3264*tan(1/2*d*x + 1/2*c)^2 -
345*tan(1/2*d*x + 1/2*c) + 544)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^3))/d
```

Mupad [B]
time = 11.56, size = 160, normalized size = 1.24

$$-\frac{23\,x}{16\,a^3}-\frac{\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{136\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{136\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{68}{15}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (23*x)/(16*a^3) - ((136*tan(c/2 + (d*x)/2)^2)/5 - (23*tan(c/2 + (d*x)/2))
/8 - (391*tan(c/2 + (d*x)/2)^3)/24 + 64*tan(c/2 + (d*x)/2)^4 - (75*tan(c/2
+ (d*x)/2)^5)/4 + (136*tan(c/2 + (d*x)/2)^6)/3 + (75*tan(c/2 + (d*x)/2)^7)/
4 + 4*tan(c/2 + (d*x)/2)^8 + (391*tan(c/2 + (d*x)/2)^9)/24 + (23*tan(c/2 +
(d*x)/2)^11)/8 + 68/15)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.645 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{13x}{8a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{\cos^5(c+dx)}{5a^3d}-\frac{13\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d}$$

```
[Out] 13/8*x/a^3+4*cos(d*x+c)/a^3/d-5/3*cos(d*x+c)^3/a^3/d+1/5*cos(d*x+c)^5/a^3/d
-13/8*cos(d*x+c)*sin(d*x+c)/a^3/d-3/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2948, 2836, 2715, 8, 2713}

$$\frac{\cos^5(c+dx)}{5a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{4\cos(c+dx)}{a^3d}-\frac{3\sin^3(c+dx)\cos(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*x)/(8*a^3) + (4*Cos[c + d*x])/(a^3*d) - (5*Cos[c + d*x]^3)/(3*a^3*d) +
Cos[c + d*x]^5/(5*a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) - (3*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
```

```
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(a^3\sin^2(c+dx)-3a^3\sin^3(c+dx)+3a^3\sin^4(c+dx)-a^3\sin^5(c+dx\right.}{a^6}\\ &= \frac{\int \sin^2(c+dx)\,dx}{a^3}-\frac{\int \sin^5(c+dx)\,dx}{a^3}-\frac{3\int \sin^3(c+dx)\,dx}{a^3}+\frac{3\int \sin^4(c}{a}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{3\cos(c+dx)\sin^3(c+dx)}{4a^3d}+\frac{\int 1\,dx}{2a^3}+\frac{9\int \sin}{}\\ &= \frac{x}{2a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{\cos^5(c+dx)}{5a^3d}-\frac{13\cos(c+dx)\sin}{8a^3d}\\ &= \frac{13x}{8a^3}+\frac{4\cos(c+dx)}{a^3d}-\frac{5\cos^3(c+dx)}{3a^3d}+\frac{\cos^5(c+dx)}{5a^3d}-\frac{13\cos(c+dx)\sin}{8a^3d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 310 vs. 2(105) = 210.
time = 1.66, size = 310, normalized size = 2.95

$$\frac{1560dx\cos\left(\frac{c}{2}\right)+1380\cos\left(\frac{c}{2}+dx\right)+1380\cos\left(\frac{3c}{2}+dx\right)-480\cos\left(\frac{3c}{2}+2dx\right)+480\cos\left(\frac{5c}{2}+2dx\right)-170\cos\left(\frac{5c}{2}+3dx\right)-170\cos\left(\frac{7c}{2}+3dx\right)+45\cos\left(\frac{7c}{2}+4dx\right)-45\cos\left(\frac{9c}{2}+4dx\right)+6\cos\left(\frac{9c}{2}+5dx\right)+6\cos\left(\frac{11c}{2}+5dx\right)+10\sin\left(\frac{c}{2}\right)+1560dx\sin\left(\frac{c}{2}\right)-1380\sin\left(\frac{c}{2}+dx\right)+1380\sin\left(\frac{3c}{2}+dx\right)-480\sin\left(\frac{3c}{2}+2dx\right)-480\sin\left(\frac{5c}{2}+2dx\right)+170\sin\left(\frac{5c}{2}+3dx\right)-170\sin\left(\frac{7c}{2}+3dx\right)+45\sin\left(\frac{7c}{2}+4dx\right)+45\sin\left(\frac{9c}{2}+4dx\right)-6\sin\left(\frac{9c}{2}+5dx\right)+6\sin\left(\frac{11c}{2}+5dx\right)}{960a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1560*d*x*Cos[c/2] + 1380*Cos[c/2 + d*x] + 1380*Cos[(3*c)/2 + d*x] - 480*Co
s[(3*c)/2 + 2*d*x] + 480*Cos[(5*c)/2 + 2*d*x] - 170*Cos[(5*c)/2 + 3*d*x] -
170*Cos[(7*c)/2 + 3*d*x] + 45*Cos[(7*c)/2 + 4*d*x] - 45*Cos[(9*c)/2 + 4*d*x
] + 6*Cos[(9*c)/2 + 5*d*x] + 6*Cos[(11*c)/2 + 5*d*x] + 10*Sin[c/2] + 1560*d
*x*Sin[c/2] - 1380*Sin[c/2 + d*x] + 1380*Sin[(3*c)/2 + d*x] - 480*Sin[(3*c)
/2 + 2*d*x] - 480*Sin[(5*c)/2 + 2*d*x] + 170*Sin[(5*c)/2 + 3*d*x] - 170*Sin
[(7*c)/2 + 3*d*x] + 45*Sin[(7*c)/2 + 4*d*x] + 45*Sin[(9*c)/2 + 4*d*x] - 6*S
in[(9*c)/2 + 5*d*x] + 6*Sin[(11*c)/2 + 5*d*x])/(960*a^3*d*(Cos[c/2] + Sin[c
/2]))
```

Maple [A]
time = 0.17, size = 129, normalized size = 1.23

method	result
risch	$\frac{13x}{8a^3}+\frac{23\cos(dx+c)}{8a^3d}+\frac{\cos(5dx+5c)}{80d\,a^3}+\frac{3\sin(4dx+4c)}{32d\,a^3}-\frac{17\cos(3dx+3c)}{48d\,a^3}-\frac{\sin(2dx+2c)}{d\,a^3}$
derivativedivides	$\frac{\frac{8\left(\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{29\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{19\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-13\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^3}$
default	$\frac{\frac{8\left(\frac{13\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{29\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{25\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{19\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-13\tan\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^3*((13/32*tan(1/2*d*x+1/2*c)^9+25/16*tan(1/2*d*x+1/2*c)^7+3/2*tan(1/2
*d*x+1/2*c)^6+29/6*tan(1/2*d*x+1/2*c)^4-25/16*tan(1/2*d*x+1/2*c)^3+19/6*tan
(1/2*d*x+1/2*c)^2-13/32*tan(1/2*d*x+1/2*c)+19/30)/(1+tan(1/2*d*x+1/2*c)^2)^
5+13/32*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(95) = 190.
time = 0.56, size = 290, normalized size = 2.76

$$-\frac{\frac{\frac{195\sin(dx+c)}{\cos(dx+c)+1}-\frac{1520\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{750\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{750\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{195\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-304}{a^3+\frac{5a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{195\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/60*((195*sin(d*x + c)/(cos(d*x + c) + 1) - 1520*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 750*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2320*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 750*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 - 195*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
304)/(a^3 + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^
3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a^3*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10) - 195*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.38, size = 68, normalized size = 0.65

$$\frac{24\cos(dx+c)^5-200\cos(dx+c)^3+195\,dx+15\left(6\cos(dx+c)^3-19\cos(dx+c)\right)\sin(dx+c)+480\cos(dx+c)}{120\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(24*cos(d*x + c)^5 - 200*cos(d*x + c)^3 + 195*d*x + 15*(6*cos(d*x + c
)^3 - 19*cos(d*x + c))*sin(d*x + c) + 480*cos(d*x + c))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1608 vs. $2(99) = 198$.
time = 121.48, size = 1608, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((195*d*x*tan(c/2 + d*x/2)**10/(120*a**3*d*tan(c/2 + d*x/2)**10 +
600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**
3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 97
5*d*x*tan(c/2 + d*x/2)**8/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan
(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 +
d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 1950*d*x*tan(c/2
 + d*x/2)**6/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)
**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 6
00*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 1950*d*x*tan(c/2 + d*x/2)**4/
(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a*
*3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan
(c/2 + d*x/2)**2 + 120*a**3*d) + 975*d*x*tan(c/2 + d*x/2)**2/(120*a**3*d*ta
n(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 +
 d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)*
*2 + 120*a**3*d) + 195*d*x/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*ta
n(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 +
 d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 390*tan(c/2 + d
*x/2)**9/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8
+ 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a
**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 1500*tan(c/2 + d*x/2)**7/(120*a**
3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan
(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d
*x/2)**2 + 120*a**3*d) + 1440*tan(c/2 + d*x/2)**6/(120*a**3*d*tan(c/2 + d*x
/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6
+ 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a*
*3*d) + 4640*tan(c/2 + d*x/2)**4/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**
3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan
(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) - 1500*tan(
c/2 + d*x/2)**3/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x
```

```
/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4
+ 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d) + 3040*tan(c/2 + d*x/2)**2/(
120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**
3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(
c/2 + d*x/2)**2 + 120*a**3*d) - 390*tan(c/2 + d*x/2)/(120*a**3*d*tan(c/2 +
d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)*
*6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**2 + 120
*a**3*d) + 608/(120*a**3*d*tan(c/2 + d*x/2)**10 + 600*a**3*d*tan(c/2 + d*x/
2)**8 + 1200*a**3*d*tan(c/2 + d*x/2)**6 + 1200*a**3*d*tan(c/2 + d*x/2)**4 +
 600*a**3*d*tan(c/2 + d*x/2)**2 + 120*a**3*d), Ne(d, 0)), (x*sin(c)**2*cos(
c)**6/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.49, size = 127, normalized size = 1.21

$$\frac{\frac{195\,(dx+c)}{a^3}+\frac{2\left(195\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+750\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+720\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+2320\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-750\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1520\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-195\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+304\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(195*(d*x + c)/a^3 + 2*(195*tan(1/2*d*x + 1/2*c)^9 + 750*tan(1/2*d*x
+ 1/2*c)^7 + 720*tan(1/2*d*x + 1/2*c)^6 + 2320*tan(1/2*d*x + 1/2*c)^4 - 750
*tan(1/2*d*x + 1/2*c)^3 + 1520*tan(1/2*d*x + 1/2*c)^2 - 195*tan(1/2*d*x + 1
/2*c) + 304)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^3))/d
```

Mupad [B]
time = 9.03, size = 95, normalized size = 0.90

$$\frac{13\,x}{8\,a^3}+\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{5\cos\left(c+d\,x\right)^3}{3\,a^3\,d}+\frac{\cos\left(c+d\,x\right)^5}{5\,a^3\,d}+\frac{3\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}-\frac{19\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (13*x)/(8*a^3) + (4*cos(c + d*x))/(a^3*d) - (5*cos(c + d*x)^3)/(3*a^3*d) +
cos(c + d*x)^5/(5*a^3*d) + (3*cos(c + d*x)^3*sin(c + d*x))/(4*a^3*d) - (19*
cos(c + d*x)*sin(c + d*x))/(8*a^3*d)
```

3.646 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=84

$$-\frac{15x}{8a^3}-\frac{4\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{a^3d}+\frac{15\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos(c+dx)\sin^3(c+dx)}{4a^3d}$$

```
[Out] -15/8*x/a^3-4*cos(d*x+c)/a^3/d+cos(d*x+c)^3/a^3/d+15/8*cos(d*x+c)*sin(d*x+c
)/a^3/d+1/4*cos(d*x+c)*sin(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2938, 2758, 2761, 2715, 8}

$$-\frac{5\cos^3(c+dx)}{4a^3d}-\frac{3\cos^5(c+dx)}{4d\,(a^3\sin(c+dx)+a^3)}-\frac{15\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{15x}{8a^3}-\frac{\cos^7(c+dx)}{d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*x)/(8*a^3) - (5*Cos[c + d*x]^3)/(4*a^3*d) - (15*Cos[c + d*x]*Sin[c + d
*x])/(8*a^3*d) - Cos[c + d*x]^7/(d*(a + a*Sin[c + d*x])^3) - (3*Cos[c + d*x
]^5)/(4*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3}-\frac{3\int \frac{\cos^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a}\\
&= -\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3}-\frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)}-\frac{15\int \frac{\cos^4(c+dx)}{a+a\sin(c+dx)}\,dx}{4a^2}\\
&= -\frac{5\cos^3(c+dx)}{4a^3d}-\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3}-\frac{3\cos^5(c+dx)}{4d\left(a^3+a^3\sin(c+dx)\right)}-\frac{15\int
\end{aligned}$$

$$= -\frac{5\cos^3(c+dx)}{4a^3d}-\frac{15\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx))^3}-\frac{}{4a}$$

$$= -\frac{15x}{8a^3}-\frac{5\cos^3(c+dx)}{4a^3d}-\frac{15\cos(c+dx)\sin(c+dx)}{8a^3d}-\frac{\cos^7(c+dx)}{d(a+a\sin(c+dx)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 255 vs. 2(84) = 168.
time = 1.03, size = 255, normalized size = 3.04

$$-\frac{(1+120dx)\cos\left(\frac{c}{2}\right)+104\cos\left(\frac{c}{2}+dx\right)+104\cos\left(\frac{3c}{2}+dx\right)-32\cos\left(\frac{3c}{2}+2dx\right)+32\cos\left(\frac{5c}{2}+2dx\right)-8\cos\left(\frac{5c}{2}+3dx\right)-8\cos\left(\frac{7c}{2}+3dx\right)+\cos\left(\frac{7c}{2}+4dx\right)-\cos\left(\frac{9c}{2}+4dx\right)-\sin\left(\frac{c}{2}\right)+120dx\sin\left(\frac{c}{2}\right)-104\sin\left(\frac{c}{2}+dx\right)+104\sin\left(\frac{3c}{2}+dx\right)-32\sin\left(\frac{3c}{2}+2dx\right)-32\sin\left(\frac{5c}{2}+2dx\right)+8\sin\left(\frac{5c}{2}+3dx\right)-8\sin\left(\frac{7c}{2}+3dx\right)+\sin\left(\frac{7c}{2}+4dx\right)+\sin\left(\frac{9c}{2}+4dx\right)}{64a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/64*((1 + 120*d*x)*Cos[c/2] + 104*Cos[c/2 + d*x] + 104*Cos[(3*c)/2 + d*x]
 - 32*Cos[(3*c)/2 + 2*d*x] + 32*Cos[(5*c)/2 + 2*d*x] - 8*Cos[(5*c)/2 + 3*d*
```

```
x] - 8*Cos[(7*c)/2 + 3*d*x] + Cos[(7*c)/2 + 4*d*x] - Cos[(9*c)/2 + 4*d*x] -
 Sin[c/2] + 120*d*x*Sin[c/2] - 104*Sin[c/2 + d*x] + 104*Sin[(3*c)/2 + d*x]
- 32*Sin[(3*c)/2 + 2*d*x] - 32*Sin[(5*c)/2 + 2*d*x] + 8*Sin[(5*c)/2 + 3*d*x
] - 8*Sin[(7*c)/2 + 3*d*x] + Sin[(7*c)/2 + 4*d*x] + Sin[(9*c)/2 + 4*d*x])/(
a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.17, size = 129, normalized size = 1.54

method	result
risch	$-\frac{15x}{8a^3}-\frac{13\cos(dx+c)}{4a^3d}-\frac{\sin(4dx+4c)}{32d\,a^3}+\frac{\cos(3dx+3c)}{4d\,a^3}+\frac{\sin(2dx+2c)}{d\,a^3}$
derivativedivides	$\frac{\frac{4\left(-\frac{15\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{23\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+15}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^3}$
default	$\frac{\frac{4\left(-\frac{15\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{23\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{9\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{23\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{11\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+15}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*((-15/16*tan(1/2*d*x+1/2*c)^7-1/2*tan(1/2*d*x+1/2*c)^6-23/16*tan(1/
2*d*x+1/2*c)^5-9/2*tan(1/2*d*x+1/2*c)^4+23/16*tan(1/2*d*x+1/2*c)^3-11/2*tan
(1/2*d*x+1/2*c)^2+15/16*tan(1/2*d*x+1/2*c)-3/2)/(1+tan(1/2*d*x+1/2*c)^2)^4-
15/16*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(78) = 156.
time = 0.54, size = 267, normalized size = 3.18

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{88\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{23\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{72\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{23\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-24}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((15*sin(d*x + c)/(cos(d*x + c) + 1) - 88*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 23*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 72*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 23*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 24)/(a^3 + 4
*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) - 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.36, size = 58, normalized size = 0.69

$$\frac{8 \cos(dx+c)^3 - 15\,dx - \left(2 \cos(dx+c)^3 - 17 \cos(dx+c)\right)\sin(dx+c) - 32 \cos(dx+c)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*cos(d*x + c)^3 - 15*d*x - (2*cos(d*x + c)^3 - 17*cos(d*x + c))*sin(d
*x + c) - 32*cos(d*x + c))/(a^3*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. 2(78) = 156.
time = 78.33, size = 1246, normalized size = 14.83

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**8/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a
**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c
/2 + d*x/2)**2 + 8*a**3*d) - 60*d*x*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 +
 d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4
+ 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 90*d*x*tan(c/2 + d*x/2)**4/(8
*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan
(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 60*d*x*tan(c
/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**
6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*
d) - 15*d*x/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 +
 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d)
- 30*tan(c/2 + d*x/2)**7/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2
+ d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2
 + 8*a**3*d) - 16*tan(c/2 + d*x/2)**6/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a*
*3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/
2 + d*x/2)**2 + 8*a**3*d) - 46*tan(c/2 + d*x/2)**5/(8*a**3*d*tan(c/2 + d*x/
2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*
a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 144*tan(c/2 + d*x/2)**4/(8*a**3*d*
tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d
*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) + 46*tan(c/2 + d*x/2)*
*3/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)**6 + 48*a**3*
d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**3*d) - 176*tan
(c/2 + d*x/2)**2/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x/2)
**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*a**
```

```
3*d) + 30*tan(c/2 + d*x/2)/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/
2 + d*x/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)*
*2 + 8*a**3*d) - 48/(8*a**3*d*tan(c/2 + d*x/2)**8 + 32*a**3*d*tan(c/2 + d*x
/2)**6 + 48*a**3*d*tan(c/2 + d*x/2)**4 + 32*a**3*d*tan(c/2 + d*x/2)**2 + 8*
a**3*d), Ne(d, 0)), (x*sin(c)*cos(c)**6/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.48, size = 127, normalized size = 1.51

$$-\frac{\frac{15\,(dx+c)}{a^3}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-23\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+88\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^7 + 8*tan(1/2*d*x + 1/2
*c)^6 + 23*tan(1/2*d*x + 1/2*c)^5 + 72*tan(1/2*d*x + 1/2*c)^4 - 23*tan(1/2*
d*x + 1/2*c)^3 + 88*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 24)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

Mupad [B]
time = 9.00, size = 78, normalized size = 0.93

$$\frac{\cos\left(c+d\,x\right)^3}{a^3\,d}-\frac{4\,\cos\left(c+d\,x\right)}{a^3\,d}-\frac{15\,x}{8\,a^3}-\frac{\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{4\,a^3\,d}+\frac{17\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] cos(c + d*x)^3/(a^3*d) - (4*cos(c + d*x))/(a^3*d) - (15*x)/(8*a^3) - (cos(c
 + d*x)^3*sin(c + d*x))/(4*a^3*d) + (17*cos(c + d*x)*sin(c + d*x))/(8*a^3*d
)
```

3.647 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{7x}{2a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}$$

```
[Out] -7/2*x/a^3-arctanh(cos(d*x+c))/a^3/d-3*cos(d*x+c)/a^3/d+1/2*cos(d*x+c)*sin(
d*x+c)/a^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2948, 2836, 3855, 2718, 2715, 8}

$$-\frac{3\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{7x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*x)/(2*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
 (Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(-3a^3+a^3\csc(c+dx)+3a^3\sin(c+dx)-a^3\sin^2(c+dx)\right)\,dx}{a^6}\\ &= -\frac{3x}{a^3}+\frac{\int \csc(c+dx)\,dx}{a^3}-\frac{\int \sin^2(c+dx)\,dx}{a^3}+\frac{3\int \sin(c+dx)\,dx}{a^3}\\ &= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\\ &= -\frac{7x}{2a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 63, normalized size = 1.05

$$\frac{-12\cos(c+dx)-2\left(7c+7dx+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sin(2(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-12*Cos[c + d*x] - 2*(7*c + 7*d*x + 2*Log[Cos[(c + d*x)/2]] - 2*Log[Sin[(c
 + d*x)/2]]) + Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [A]
time = 0.36, size = 87, normalized size = 1.45

method	result

derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-7\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$
risch	$-\frac{7x}{2a^3}-\frac{3\,e^{i(dx+c)}}{2d\,a^3}-\frac{3\,e^{-i(dx+c)}}{2d\,a^3}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{d\,a^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}+\frac{\sin(2dx+2c)}{4d\,a^3}$
norman	$\frac{-\frac{35x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{205\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}-\frac{7x}{2a}-\frac{6}{ad}-\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{245x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{105x\left(\tan^{13}\left(\right.\right.}{2a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(ln(tan(1/2*d*x+1/2*c))-2*(1/2*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1
/2*c)^2-1/2*tan(1/2*d*x+1/2*c)+3)/(1+tan(1/2*d*x+1/2*c)^2)^2-7*arctan(tan(1
/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(56) = 112$.
time = 0.55, size = 161, normalized size = 2.68

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-6}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{7\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 7*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^3 + log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)
/d
```

Fricas [A]
time = 0.38, size = 59, normalized size = 0.98

$$-\frac{7\,dx-\cos(dx+c)\sin(dx+c)+6\,\cos(dx+c)+\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

[Out] -1/2*(7*d*x - cos(d*x + c)*sin(d*x + c) + 6*cos(d*x + c) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d*x + c) + 1/2))/(a^3*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\cos^6(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)

[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x)/a**3

Giac [A]
time = 0.46, size = 89, normalized size = 1.48

$$-\frac{\frac{7(dx+c)}{a^3}-\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}+\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/2*(7*(d*x + c)/a^3 - 2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(tan(1/2*d*x + 1/2*c)^3 + 6*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3))/d

Mupad [B]
time = 9.44, size = 150, normalized size = 2.50

$$\frac{7\,\mathrm{atan}\left(\frac{49}{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+14}-\frac{14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+14}\right)}{a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+6}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)

[Out] (7*atan(49/(49*tan(c/2 + (d*x)/2) + 14) - (14*tan(c/2 + (d*x)/2))/(49*tan(c/2 + (d*x)/2) + 14)))/(a^3*d) + log(tan(c/2 + (d*x)/2))/(a^3*d) - (6*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3 + 6)/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + a^3*tan(c/2 + (d*x)/2)^4 + a^3))

3.648 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=49

$$\frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}$$

```
[Out] 3*x/a^3+3*arctanh(cos(d*x+c))/a^3/d+cos(d*x+c)/a^3/d-cot(d*x+c)/a^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2948, 2836, 3855, 3852, 8, 2718}

$$\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) + Cos[c + d*x]/(a^3*d) - Cot[
c + d*x]/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(3a^3-3a^3\csc(c+dx)+a^3\csc^2(c+dx)-a^3\sin(c+dx)\right)\,dx}{a^6}\\ &= \frac{3x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}-\frac{\int \sin(c+dx)\,dx}{a^3}-\frac{3\int \csc(c+dx)\,dx}{a^3}\\ &= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{a^3d}\\ &= \frac{3x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(49) = 98.
time = 0.36, size = 106, normalized size = 2.16

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)+2\cos(c+dx)-\cot\left(\frac{1}{2}(c+dx)\right)+6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) + 2*Cos[c + d*x] - Co
t[(c + d*x)/2] + 6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + Tan[(c
 + d*x)/2]))/(2*d*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.37, size = 73, normalized size = 1.49

method	result

derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+12\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+12\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^3}$
risch	$\frac{3x}{a^3}+\frac{e^{i(dx+c)}}{2d\,a^3}+\frac{e^{-i(dx+c)}}{2d\,a^3}-\frac{2i}{a^3d\left(e^{2i(dx+c)}-1\right)}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{d\,a^3}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}$
norman	$\frac{\frac{3x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{81\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{2ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{42x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{15x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2*c))+4/(1+tan(1/2*d*x+1/2*c)^2)+12*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(49) = 98$.

time = 0.51, size = 158, normalized size = 3.22

$$\frac{\frac{\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1}{\frac{a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}+\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 1/2*((4*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d

Fricas [A]

time = 0.42, size = 82, normalized size = 1.67

$$\frac{2\left(3\,dx+\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)}{2\,a^3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/2*(2*(3*d*x + cos(d*x + c))*sin(d*x + c) + 3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*cos(d*x + c))/(a^3*d*sin(d*x + c))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(49) = 98.
time = 0.47, size = 111, normalized size = 2.27

$$\frac{\frac{6\,(dx+c)}{a^3}-\frac{6\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)/a^3 - 6*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + tan(1/2*d*x +
 1/2*c)/a^3 + (2*tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + 6*tan(1/
2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^3))/
d
```

Mupad [**B**]
time = 9.48, size = 151, normalized size = 3.08

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{6\,\mathrm{atan}\left(\frac{36}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+36}-\frac{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+36}\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (6*ata
n(36/(36*tan(c/2 + (d*x)/2) + 36) - (36*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (
d*x)/2) + 36)))/(a^3*d) - (tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) + 1)
/(d*(2*a^3*tan(c/2 + (d*x)/2)^3 + 2*a^3*tan(c/2 + (d*x)/2)))
```

3.649 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=60

$$-\frac{x}{a^3}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] -x/a^3-7/2*arctanh(cos(d*x+c))/a^3/d+3*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(
d*x+c)/a^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2948, 2836, 3855, 3852, 8, 3853}

$$\frac{3\cot(c+dx)}{a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (7*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cot[c + d*x])/(a^3*d) -
 (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{\int \left(-a^3+3a^3\csc(c+dx)-3a^3\csc^2(c+dx)+a^3\csc^3(c+dx)\right)\,dx}{a^6}\\
&= -\frac{x}{a^3}+\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{3\int \csc(c+dx)\,dx}{a^3}-\frac{3\int \csc^2(c+dx)\,dx}{a^3}\\
&= -\frac{x}{a^3}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{\int \csc(c+dx)}{2a^3}\\
&= -\frac{x}{a^3}-\frac{7\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(60) = 120.
time = 0.34, size = 126, normalized size = 2.10

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-8(c+dx)+12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-28\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+28\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)-12\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a+a\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-8*(c + d*x) + 12*Cot[(c + d*x)/2
] - Csc[(c + d*x)/2]^2 - 28*Log[Cos[(c + d*x)/2]] + 28*Log[Sin[(c + d*x)/2]
] + Sec[(c + d*x)/2]^2 - 12*Tan[(c + d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.38, size = 84, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+14\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3}$
risch	$-\frac{x}{a^3}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+6i e^{2i(dx+c)}-6i}{a^3 d\left(e^{2i(dx+c)}-1\right)^2}-\frac{7\ln\left(e^{i(dx+c)}+1\right)}{2d\,a^3}+\frac{7\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^3}$
norman	$\frac{\frac{229\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{7\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}-\frac{1}{8ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{5x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x(t}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-6*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/
2*c)^2+6/tan(1/2*d*x+1/2*c)+14*ln(tan(1/2*d*x+1/2*c))-8*arctan(tan(1/2*d*x+
1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(56) = 112$.

time = 0.54, size = 138, normalized size = 2.30

$$-\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{16\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{28\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)^2}{a^3\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/a^3 + 16*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 28*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^3 - (12*sin(d*x + c)/(cos(d*x + c) + 1) - 1)*(c
os(d*x + c) + 1)^2/(a^3*sin(d*x + c)^2))/d
```

Fricas [A]

time = 0.39, size = 109, normalized size = 1.82

$$-\frac{4\,dx\cos(dx+c)^2-4\,dx+7\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-7\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12\cos(dx+c)\sin(dx+c)-2\cos(dx+c)}{4\left(a^3d\cos(dx+c)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*d*x*cos(d*x + c)^2 - 4*d*x + 7*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x
 + c) + 1/2) - 7*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 12*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.51, size = 108, normalized size = 1.80

$$-\frac{\frac{8\,(dx+c)}{a^3}-\frac{28\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)/a^3 - 28*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (42*tan(1/2
*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^2)
 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 9.35, size = 161, normalized size = 2.68

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{2\,\mathrm{atan}\left(\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^3\,d}+\frac{7\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,a^3\,d}+\frac{3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - cot(c/2 + (d*x)/2)^2/(8*a^3*d) + (2*atan((
2*cos(c/2 + (d*x)/2) - 7*sin(c/2 + (d*x)/2))/(7*cos(c/2 + (d*x)/2) + 2*sin(
c/2 + (d*x)/2))))/(a^3*d) + (7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(2*a^3*d) + (3*cot(c/2 + (d*x)/2))/(2*a^3*d) - (3*tan(c/2 + (d*x)/2))/(2*a^
3*d)
```

3.650 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=72

$$\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] 5/2*arctanh(cos(d*x+c))/a^3/d-4*cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+3/2
*cot(d*x+c)*csc(d*x+c)/a^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2948, 2836, 3855, 3852, 8, 3853}

$$-\frac{\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - Cot[c + d*
x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(-a^3\csc(c+dx)+3a^3\csc^2(c+dx)-3a^3\csc^3(c+dx)+a^3\csc^4(c+d\right.}{a^6}\\ &= -\frac{\int \csc(c+dx)\,dx}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{3\int \csc^2(c+dx)\,dx}{a^3}-\frac{3\int \csc^3(}{}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3\int \csc(c+dx)\,dx}{2a^3}-\\ &= \frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\,\mathrm{cs}}{2a^3d}\end{aligned}$$

Mathematica [A]
time = 0.92, size = 115, normalized size = 1.60

$$-\frac{\csc^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(30\cos(c+dx)-22\cos(3(c+dx))-60\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)-18\sin(2(c+dx))\right)}{24a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/24*(Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(30*Cos[c + d
*x] - 22*Cos[3*(c + d*x)] - 60*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2
]])*Sin[c + d*x]^3 - 18*Sin[2*(c + d*x)]))/(a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.37, size = 98, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8d\,a^3}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{15}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{8d\,a^3}$
risch	$-\frac{18i\,e^{4i(dx+c)}+9\,e^{5i(dx+c)}-48i\,e^{2i(dx+c)}+22i-9\,e^{i(dx+c)}}{3a^3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{5\ln\left(e^{i(dx+c)}+1\right)}{2d\,a^3}-\frac{5\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^3}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{6ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{6da}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}-\frac{400\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{95\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)^2+15*tan(1/2*d*x+1
/2*c)-15/tan(1/2*d*x+1/2*c)-20*ln(tan(1/2*d*x+1/2*c))+3/tan(1/2*d*x+1/2*c)^
2-1/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(66) = 132$.
time = 0.30, size = 153, normalized size = 2.12

$$\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((45*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 60*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3 + (9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.38, size = 123, normalized size = 1.71

$$-\frac{44\cos(dx+c)^3-15\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+15\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+18\cos(dx+c)\sin(dx+c)-48\cos(dx+c)}{12\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(44*cos(d*x + c)^3 - 15*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 18*cos(d*x + c)*sin(d*x + c) - 48*cos(d*x + c))/((a^3*d*cos(d*x
 + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.50, size = 128, normalized size = 1.78

$$-\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (110*tan(1/2*d*x + 1/2*c)^3
- 45*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x
+ 1/2*c)^3) - (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 +
45*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

Mupad [B]
time = 9.25, size = 119, normalized size = 1.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}\right)}{8\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) - (5*l
og(tan(c/2 + (d*x)/2)))/(2*a^3*d) + (15*tan(c/2 + (d*x)/2))/(8*a^3*d) - (co
t(c/2 + (d*x)/2)^3*(15*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2) + 1/3))/
(8*a^3*d)
```

3.651 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=93

$$-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}$$

```
[Out] -15/8*arctanh(cos(d*x+c))/a^3/d+4*cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-15/8*
cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2948, 2836, 3852, 8, 3853, 3855}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{4\cot(c+dx)}{a^3d}-\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{15\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (4*Cot[c + d*x])/(a^3*d) + Cot[c +
d*x]^3/(a^3*d) - (15*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x]*C
sc[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc^2(c+dx)+3a^3\csc^3(c+dx)-3a^3\csc^4(c+dx)+a^3\csc^5(c+d$$
$$= -\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{\int \csc^5(c+dx)\,dx}{a^3}+\frac{3\int \csc^3(c+dx)\,dx}{a^3}-\frac{3\int \csc^4($$
$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{3\int \csc^3(c+dx)}{4a^3}$$
$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx)}{8a^3}$$
$$= -\frac{15\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{15\cot(c+dx}{8a}$$

Mathematica [A]
time = 2.06, size = 125, normalized size = 1.34

$$-\frac{\csc^4(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(46\cos(c+dx)+120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^4(c+dx)+6\cos(3(c+dx))(-5+8\sin(c+dx))-56\sin(2(c+dx))\right)}{64a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/64*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(46*Cos[c + d
*x] + 120*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^4 +
6*Cos[3*(c + d*x)]*(-5 + 8*Sin[c + d*x]) - 56*Sin[2*(c + d*x)]))/(a^3*d*(1
+ Sin[c + d*x])^3)
```

Maple [A]
time = 0.38, size = 124, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{26}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+30\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{16d\,a^3}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{26}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+30\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{16d\,a^3}$
risch	$\frac{15\,e^{7i(dx+c)}-23\,e^{5i(dx+c)}+8ie^{6i(dx+c)}-23\,e^{3i(dx+c)}-72ie^{4i(dx+c)}+15\,e^{i(dx+c)}+88ie^{2i(dx+c)}-24i}{4a^3d\left(e^{2i(dx+c)}-1\right)^4}+\frac{15\ln\left(e^{i(dx+c)}-1\right.}{8d\,a^3}$
norman	$\frac{-\frac{1}{64ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{17\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{3\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}-\frac{3\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+}{\left(1+\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^3*(1/4*tan(1/2*d*x+1/2*c)^4-2*tan(1/2*d*x+1/2*c)^3+8*tan(1/2*d*x+1
/2*c)^2-26*tan(1/2*d*x+1/2*c)+26/tan(1/2*d*x+1/2*c)+2/tan(1/2*d*x+1/2*c)^3+
30*ln(tan(1/2*d*x+1/2*c))-8/tan(1/2*d*x+1/2*c)^2-1/4/tan(1/2*d*x+1/2*c)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(87) = 174.
time = 0.31, size = 195, normalized size = 2.10

$$-\frac{\frac{\frac{104\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{32\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{104\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)(\cos(dx+c)+1)^4}{a^3\sin(dx+c)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((104*sin(d*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4)/a^3 - 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d
*x + c)/(cos(d*x + c) + 1) - 32*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 104*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x +
 c)^4))/d
```

Fricas [A]
time = 0.41, size = 149, normalized size = 1.60

$$\frac{30\cos(dx+c)^3-15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-16\left(3\cos(dx+c)^3-4\cos(dx+c)\right)\sin(dx+c)-34\cos(dx+c)}{16\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(30*cos(d*x + c)^3 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/
2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2
*cos(d*x + c) + 1/2) - 16*(3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c)
- 34*cos(d*x + c))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.53, size = 156, normalized size = 1.68

$$\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{250\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-104\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}+\frac{a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+32\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-104\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{12}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (250*tan(1/2*d*x + 1/2*c)^4
- 104*tan(1/2*d*x + 1/2*c)^3 + 32*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x +
1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*
a^9*tan(1/2*d*x + 1/2*c)^3 + 32*a^9*tan(1/2*d*x + 1/2*c)^2 - 104*a^9*tan(1/
2*d*x + 1/2*c))/a^12)/d
```

Mupad [B]
time = 9.29, size = 151, normalized size = 1.62

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d}+\frac{15\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^3\,d}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{4}\right)}{16\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(2*a^3*d) - tan(c/2 + (d*x)/2)^3/(8*a^3*d) + tan(c/2 +
 (d*x)/2)^4/(64*a^3*d) + (15*log(tan(c/2 + (d*x)/2)))/(8*a^3*d) - (13*tan(c
/2 + (d*x)/2))/(8*a^3*d) + (cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 8*
tan(c/2 + (d*x)/2)^2 + 26*tan(c/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

3.652 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=114

$$\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{3\cot(c+$$

```
[Out] 13/8*arctanh(cos(d*x+c))/a^3/d-4*cot(d*x+c)/a^3/d-5/3*cot(d*x+c)^3/a^3/d-1/
5*cot(d*x+c)^5/a^3/d+13/8*cot(d*x+c)*csc(d*x+c)/a^3/d+3/4*cot(d*x+c)*csc(d*
x+c)^3/a^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2787, 2836, 3853, 3855, 3852}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{4\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{13\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x])/(a^3*d) - (5*Cot[c
+ d*x]^3)/(3*a^3*d) - Cot[c + d*x]^5/(5*a^3*d) + (13*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^6(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\csc^3(c+dx)+3a^3\csc^4(c+dx)-3a^3\csc^5(c+dx)+a^3\csc^6(c+dx)\right)\,d}{a^6}$$
$$= -\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \csc^6(c+dx)\,dx}{a^3}+\frac{3\int \csc^4(c+dx)\,dx}{a^3}-\frac{3\int \csc^5(c+}{a^3}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\int \csc(c+dx)\,dx}{2a^3}-\frac{9\int}{}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13\,\mathrm{co}}{}$$
$$= \frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot(c+dx)}{a^3d}-\frac{5\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{13}{}$$

Mathematica [A]
time = 1.30, size = 189, normalized size = 1.66

$$\frac{\csc^5(c+dx)\left(-1600\cos(c+dx)+1520\cos(3(c+dx))-304\cos(5(c+dx))+1950\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-1950\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+1500\sin(2(c+dx))-975\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+975\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-390\sin(4(c+dx))+195\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-195\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{1920a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^5*(-1600*Cos[c + d*x] + 1520*Cos[3*(c + d*x)] - 304*Cos[5*(c
+ d*x)] + 1950*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1950*Log[Sin[(c + d*x)/
2]]*Sin[c + d*x] + 1500*Sin[2*(c + d*x)] - 975*Log[Cos[(c + d*x)/2]]*Sin[3*
(c + d*x)] + 975*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 390*Sin[4*(c + d*
x)] + 195*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 195*Log[Sin[(c + d*x)/2]
]*Sin[5*(c + d*x)]))/(1920*a^3*d)
```

Maple [A]
time = 0.39, size = 150, normalized size = 1.32

method	result
risch	$-\frac{195\,e^{9i(dx+c)}-720ie^{6i(dx+c)}-750\,e^{7i(dx+c)}+2320ie^{4i(dx+c)}-1520ie^{2i(dx+c)}+750\,e^{3i(dx+c)}+304i-195\,e^{i(dx+c)}}{60a^3d\left(e^{2i(dx+c)}-1\right)^5}-\ \frac{13}{}$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{46}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-52\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{32d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{17\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+46\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{46}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-52\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{32d\,a^3}$
norman	$\frac{-\frac{1}{160ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}-\frac{13\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{13\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{48ad}+\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right.}{192da}}{\tan\left(\frac{dx}{2}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-3/2*tan(1/2*d*x+1/2*c)^4+17/3*tan(1/2*
d*x+1/2*c)^3-16*tan(1/2*d*x+1/2*c)^2+46*tan(1/2*d*x+1/2*c)-46/tan(1/2*d*x+1
/2*c)-52*ln(tan(1/2*d*x+1/2*c))+3/2/tan(1/2*d*x+1/2*c)^4+16/tan(1/2*d*x+1/2
*c)^2-17/3/tan(1/2*d*x+1/2*c)^3-1/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(104) = 208$.
time = 0.34, size = 234, normalized size = 2.05

$$\frac{\frac{\frac{1380\sin(dx+c)}{\cos(dx+c)+1}-\frac{480\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{170\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1560\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{45\sin(dx+c)}{\cos(dx+c)+1}-\frac{170\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{480\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1380\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+1)^5}{a^3\sin(dx+c)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/960*((1380*sin(d*x + c)/(cos(d*x + c) + 1) - 480*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 170*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 1560*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + (45*sin(d*x + c)/(cos(d*x + c) + 1)
 - 170*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 480*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 1380*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1
)^5/(a^3*sin(d*x + c)^5))/d
```

Fricas [A]
time = 0.38, size = 179, normalized size = 1.57

$$-\frac{608\cos(dx+c)^5-1520\cos(dx+c)^3-195\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+195\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+30\left(13\cos(dx+c)^3-19\cos(dx+c)\right)\sin(dx+c)+960\cos(dx+c)}{240\left(a^3d\cos(dx+c)^4-2a^3d\cos(dx+c)^2+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/240*(608*cos(d*x + c)^5 - 1520*cos(d*x + c)^3 - 195*(cos(d*x + c)^4 - 2*
cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 195*(cos(d*x
 + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
 30*(13*cos(d*x + c)^3 - 19*cos(d*x + c))*sin(d*x + c) + 960*cos(d*x + c))/
((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]

time = 0.50, size = 187, normalized size = 1.64

$$-\frac{\frac{1560\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3562\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1380\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+480\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-170\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}-\frac{6\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-45\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+170\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-480\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1380\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(1560*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (3562*tan(1/2*d*x + 1/2*c
)^5 - 1380*tan(1/2*d*x + 1/2*c)^4 + 480*tan(1/2*d*x + 1/2*c)^3 - 170*tan(1/
2*d*x + 1/2*c)^2 + 45*tan(1/2*d*x + 1/2*c) - 6)/(a^3*tan(1/2*d*x + 1/2*c)^5
) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 170*a
^12*tan(1/2*d*x + 1/2*c)^3 - 480*a^12*tan(1/2*d*x + 1/2*c)^2 + 1380*a^12*ta
n(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]

time = 9.85, size = 291, normalized size = 2.55

$$-\frac{6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-170\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+480\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7-1380\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+1380\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-480\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+170\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1560\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{960\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(6*cos(c/2 + (d*x)/2)^10 - 6*sin(c/2 + (d*x)/2)^10 + 45*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 170*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 480*cos(c/2 + (d*x)/2)^3*sin(c/2
+ (d*x)/2)^7 - 1380*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 1380*cos(c/
2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 480*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^3 + 170*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 1560*log(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/
(960*a^3*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.653 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=267

$$\frac{a^3\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx\right.}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^3*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+3*a^3*cos(d*x+c)*hypergeom([-5/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+3*a^3*cos(d*x+c)*hypergeom([-5/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*s
in(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)+a^3*cos(d*x+c)*hypergeom([-5/2
, 1/2*n+2],[1/2*n+3],sin(d*x+c)^2)*sin(d*x+c)^(4+n)/d/(4+n)/(cos(d*x+c)^2)^
(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2952, 2657}

$$\frac{a^3\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\cos(c+dx)\sin^{n+3}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+3}{2};\frac{n+5}{2};\sin^2(c+dx)\right)}{d(n+3)\sqrt{\cos^2(c+dx)}}+\frac{a^3\cos(c+dx)\sin^{n+4}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+4}{2};\frac{n+6}{2};\sin^2(c+dx)\right)}{d(n+4)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 +
 n))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2]) + (a^3*Cos[c + d*x]*Hypergeometric2F1
[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(4 + n))/(d*(4 +
n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx &= \int \left(a^3\cos^6(c+dx)\sin^n(c+dx)+3a^3\cos^6(c+dx)\sin\right.\\ &= a^3\int \cos^6(c+dx)\sin^n(c+dx)\,dx+a^3\int \cos^6(c+dx\\ &= \frac{a^3\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(}{d(1+n)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 188, normalized size = 0.70

$$\frac{a^3\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{1+n}(c+dx)\left(\frac{{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)}{1+n}+\sin(c+dx)\left(\frac{3\,{}_2F_1\left(-\frac{5}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)}{2+n}+\sin(c+dx)\left(\frac{3\,{}_2F_1\left(-\frac{5}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(c+dx)\right)}{3+n}+\frac{{}_2F_1\left(-\frac{5}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(c+dx)\right)\sin(c+dx)}{4+n}\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(Hypergeometric
2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]/(1 + n) + Sin[c + d*x]*((3*
Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 + n) + Si
n[c + d*x]*((3*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2
])/(3 + n) + (Hypergeometric2F1[-5/2, (4 + n)/2, (6 + n)/2, Sin[c + d*x]^2]
*Sin[c + d*x])/(4 + n)))))/d
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \left(\cos^6(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a^3*cos(d*x + c)^8 - 4*a^3*cos(d*x + c)^6 + (a^3*cos(d*x + c)^
8 - 4*a^3*cos(d*x + c)^6)*sin(d*x + c))*sin(d*x + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)^n\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^3, x)
```

3.654 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=200

$$\frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+d\right.}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a^2*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*
x+c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+2*a^2*cos(d*x+c)*hypergeom([-5/2, 1
+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/
2)+a^2*cos(d*x+c)*hypergeom([-5/2, 3/2+1/2*n],[5/2+1/2*n],sin(d*x+c)^2)*sin
(d*x+c)^(3+n)/d/(3+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2952, 2657}

$$\frac{a^2\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{2a^2\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}+\frac{a^2\cos(c+dx)\sin^{n+3}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+3}{2};\frac{n+5}{2};\sin^2(c+dx)\right)}{d(n+3)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*Cos[c
+ d*x]*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c
+ d*x]^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2]) + (a^2*Cos[c + d*x]*Hyperg
eometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3 + n
))/(d*(3 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(a^2\cos^6(c+dx)\sin^n(c+dx)+2a^2\cos^6(c+dx)\sin^1\right.\\ &= a^2\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a^2\int \cos^6(c+dx)\\ &= \frac{a^2\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c}{d(1+n)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 164, normalized size = 0.82

$$\frac{a^2\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{1+n}(c+dx)\left((6+5n+n^2)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+(1+n)\sin(c+dx)\left(2(3+n)\,{}_2F_1\left(-\frac{5}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)+(2+n)\,{}_2F_1\left(-\frac{5}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(c+dx)\right)\sin(c+dx)\right)\right)}{d(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((6 + 5*n + n^2
)*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + (1 + n)*S
in[c + d*x]*(2*(3 + n)*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2] + (2 + n)*Hypergeometric2F1[-5/2, (3 + n)/2, (5 + n)/2, Sin[c + d
*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \left(\cos^6(dx+c)\right)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-(a^2*cos(d*x + c)^8 - 2*a^2*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*c
os(d*x + c)^6)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)^n\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x))^2, x)
```

3.655 $\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)\,s}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-5/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+a*cos(d*x+c)*hypergeom([-5/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2917, 2657}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{a\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-5/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (a*Cos[c + d*x]
*Hypergeometric2F1[-5/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^6(c+dx)\sin^n(c+dx)\,dx + a\int \cos^6(c+dx)\,\mathrm{si}$$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos^6(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
[Out] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^n*(a + a*Sin[c + d*x]), x]
```

Maple [F]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(\cos^6(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
[Out] int(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*cos(d*x + c)^6*sin(d*x + c) + a*cos(d*x + c)^6)*sin(d*x + c)^n,
 x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)^n\,\left(a+a\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^6*sin(c + d*x)^n*(a + a*sin(c + d*x)), x)
```

3.656 $\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^8(c+dx)}{8d}-\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^{10}(c+dx)}{10d}+\frac{3a\sin^{11}(c+dx)}{11d}+\frac{a\sin^{12}(c+dx)}{4d}-\frac{a\sin^1}{}$$

```
[Out] 1/7*a*sin(d*x+c)^7/d+1/8*a*sin(d*x+c)^8/d-1/3*a*sin(d*x+c)^9/d-3/10*a*sin(d
*x+c)^10/d+3/11*a*sin(d*x+c)^11/d+1/4*a*sin(d*x+c)^12/d-1/13*a*sin(d*x+c)^1
3/d-1/14*a*sin(d*x+c)^14/d
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^{14}(c+dx)}{14d}-\frac{a\sin^{13}(c+dx)}{13d}+\frac{a\sin^{12}(c+dx)}{4d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{3a\sin^{10}(c+dx)}{10d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^8(c+dx)}{8d}+\frac{a\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^8)/(8*d) - (a*Sin[c + d*x]^9)/(3
*d) - (3*a*Sin[c + d*x]^10)/(10*d) + (3*a*Sin[c + d*x]^11)/(11*d) + (a*Sin[
c + d*x]^12)/(4*d) - (a*Sin[c + d*x]^13)/(13*d) - (a*Sin[c + d*x]^14)/(14*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^4}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int (a^7x^6+a^6x^7-3a^5x^8-3a^4x^9+3a^3x^{10}+3a^2x}{a^{13}d}$$
$$= \frac{a\sin^7(c+dx)}{7d}+\frac{a\sin^8(c+dx)}{8d}-\frac{a\sin^9(c+dx)}{3d}-\frac{3a\,\mathrm{si}}{}$$

Mathematica [A]
time = 0.69, size = 117, normalized size = 0.91

$$-\frac{a(525525\cos(2(c+dx))-105105\cos(6(c+dx))+21021\cos(10(c+dx))-2145\cos(14(c+dx))-1201200\sin(c+dx)+300300\sin(3(c+dx))+180180\sin(5(c+dx))-51480\sin(7(c+dx))-40040\sin(9(c+dx))+5460\sin(11(c+dx))+4620\sin(13(c+dx)))}{246005760d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/246005760*(a*(525525*Cos[2*(c + d*x)] - 105105*Cos[6*(c + d*x)] + 21021*
Cos[10*(c + d*x)] - 2145*Cos[14*(c + d*x)] - 1201200*Sin[c + d*x] + 300300*
Sin[3*(c + d*x)] + 180180*Sin[5*(c + d*x)] - 51480*Sin[7*(c + d*x)] - 40040
*Sin[9*(c + d*x)] + 5460*Sin[11*(c + d*x)] + 4620*Sin[13*(c + d*x)]))/d
```

Maple [A]
time = 0.62, size = 166, normalized size = 1.29

method	result
risch	$\frac{5a\sin(dx+c)}{1024d}+\frac{a\cos(14dx+14c)}{114688d}-\frac{a\sin(13dx+13c)}{53248d}-\frac{a\sin(11dx+11c)}{45056d}-\frac{7a\cos(10dx+10c)}{81920d}+\frac{a\sin(9dx+9c)}{6144d}+$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^8(dx+c)\right)}{13}-\frac{5\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{143}-\frac{5\sin(dx+c)\left(\cos^8(dx+c)\right)}{429}+\frac{5\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\right.}{3003}\right.}{}$
default	$\frac{a\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^8(dx+c)\right)}{13}-\frac{5\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{143}-\frac{5\sin(dx+c)\left(\cos^8(dx+c)\right)}{429}+\frac{5\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\right.}{3003}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d*x+c)^8-5/4
29*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*c
```

```
os(d*x+c)^2)*sin(d*x+c))+a*(-1/14*sin(d*x+c)^6*cos(d*x+c)^8-1/28*sin(d*x+c)
^4*cos(d*x+c)^8-1/70*sin(d*x+c)^2*cos(d*x+c)^8-1/280*cos(d*x+c)^8))
```

Maxima [A]
time = 0.28, size = 94, normalized size = 0.73

$$-\frac{8580\,a\sin\left(dx+c\right)^{14}+9240\,a\sin\left(dx+c\right)^{13}-30030\,a\sin\left(dx+c\right)^{12}-32760\,a\sin\left(dx+c\right)^{11}+36036\,a\sin\left(dx+c\right)^{10}+40040\,a\sin\left(dx+c\right)^{9}-15015\,a\sin\left(dx+c\right)^{8}-17160\,a\sin\left(dx+c\right)^{7}}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/120120*(8580*a*sin(d*x + c)^14 + 9240*a*sin(d*x + c)^13 - 30030*a*sin(d*
x + c)^12 - 32760*a*sin(d*x + c)^11 + 36036*a*sin(d*x + c)^10 + 40040*a*sin
(d*x + c)^9 - 15015*a*sin(d*x + c)^8 - 17160*a*sin(d*x + c)^7)/d
```

Fricas [A]
time = 0.40, size = 128, normalized size = 0.99

$$\frac{8580\,a\cos\left(dx+c\right)^{14}-30030\,a\cos\left(dx+c\right)^{12}+36036\,a\cos\left(dx+c\right)^{10}-15015\,a\cos\left(dx+c\right)^{8}-40\,\left(231\,a\cos\left(dx+c\right)^{12}-567\,a\cos\left(dx+c\right)^{10}+371\,a\cos\left(dx+c\right)^{8}-5\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/120120*(8580*a*cos(d*x + c)^14 - 30030*a*cos(d*x + c)^12 + 36036*a*cos(d*
x + c)^10 - 15015*a*cos(d*x + c)^8 - 40*(231*a*cos(d*x + c)^12 - 567*a*cos(
d*x + c)^10 + 371*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^
4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

Sympy [A]
time = 6.58, size = 184, normalized size = 1.43

$$\begin{cases}\frac{16a\sin^{13}{\left(c+dx\right)}}{3003d}+\frac{8a\sin^{11}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{231d}+\frac{2a\sin^{9}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{21d}+\frac{a\sin^{7}{\left(c+dx\right)}\cos^{6}{\left(c+dx\right)}}{7d}-\frac{a\sin^{6}{\left(c+dx\right)}\cos^{8}{\left(c+dx\right)}}{8d}-\frac{3a\sin^{4}{\left(c+dx\right)}\cos^{10}{\left(c+dx\right)}}{40d}-\frac{a\sin^{2}{\left(c+dx\right)}\cos^{12}{\left(c+dx\right)}}{40d}-\frac{a\cos^{14}{\left(c+dx\right)}}{280d} & \text{for}\: d\neq 0\\x\left(a\sin{\left(c\right)}+a\right)\sin^{6}{\left(c\right)}\cos^{7}{\left(c\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + 8*a*sin(c + d*x)**11*cos(c + d*
x)**2/(231*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)
**7*cos(c + d*x)**6/(7*d) - a*sin(c + d*x)**6*cos(c + d*x)**8/(8*d) - 3*a*s
in(c + d*x)**4*cos(c + d*x)**10/(40*d) - a*sin(c + d*x)**2*cos(c + d*x)**12
/(40*d) - a*cos(c + d*x)**14/(280*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**
6*cos(c)**7, True))
```

Giac [A]
time = 0.58, size = 163, normalized size = 1.26

$$\frac{a\cos\left(14\,dx+14\,c\right)}{114688\,d}-\frac{7\,a\cos\left(10\,dx+10\,c\right)}{81920\,d}+\frac{7\,a\cos\left(6\,dx+6\,c\right)}{16384\,d}-\frac{35\,a\cos\left(2\,dx+2\,c\right)}{16384\,d}-\frac{a\sin\left(13\,dx+13\,c\right)}{53248\,d}-\frac{a\sin\left(11\,dx+11\,c\right)}{45056\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{6144\,d}+\frac{3\,a\sin\left(7\,dx+7\,c\right)}{14336\,d}-\frac{3\,a\sin\left(5\,dx+5\,c\right)}{4096\,d}-\frac{5\,a\sin\left(3\,dx+3\,c\right)}{4096\,d}+\frac{5\,a\sin\left(dx+c\right)}{1024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/114688*a*cos(14*d*x + 14*c)/d - 7/81920*a*cos(10*d*x + 10*c)/d + 7/16384*a*cos(6*d*x + 6*c)/d - 35/16384*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x + 13*c)/d - 1/45056*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d + 3/14336*a*sin(7*d*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(3*d*x + 3*c)/d + 5/1024*a*sin(d*x + c)/d

Mupad [B]
time = 0.10, size = 93, normalized size = 0.72

$$\frac{-\frac{a\sin(c+dx)^{14}}{14}-\frac{a\sin(c+dx)^{13}}{13}+\frac{a\sin(c+dx)^{12}}{4}+\frac{3a\sin(c+dx)^{11}}{11}-\frac{3a\sin(c+dx)^{10}}{10}-\frac{a\sin(c+dx)^{9}}{3}+\frac{a\sin(c+dx)^{8}}{8}+\frac{a\sin(c+dx)^{7}}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^7*sin(c + d*x)^6*(a + a*sin(c + d*x)),x)

[Out] ((a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^8)/8 - (a*sin(c + d*x)^9)/3 - (3*a*sin(c + d*x)^10)/10 + (3*a*sin(c + d*x)^11)/11 + (a*sin(c + d*x)^12)/4 - (a*sin(c + d*x)^13)/13 - (a*sin(c + d*x)^14)/14)/d

3.657 $\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^9(c+dx)}{3d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/5*a*cos(d*x+c)^10/d-1/12*a*cos(d*x+c)^12/d+1/7*a*si
n(d*x+c)^7/d-1/3*a*sin(d*x+c)^9/d+3/11*a*sin(d*x+c)^11/d-1/13*a*sin(d*x+c)^
13/d
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 2645, 272, 45, 2644, 276}

$$-\frac{a\sin^{13}(c+dx)}{13d}+\frac{3a\sin^{11}(c+dx)}{11d}-\frac{a\sin^9(c+dx)}{3d}+\frac{a\sin^7(c+dx)}{7d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(3*d) + (3*a*Sin[c
+ d*x]^11)/(11*d) - (a*Sin[c + d*x]^13)/(13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sin^5(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^7(c+dx)\sin^5(c+dx)\,dx + a\int \cos^7(c+dx)\sin^6\\ &= -\frac{a\text{Subst}\left(\int x^7(1-x^2)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\Big(}{}\\ &= -\frac{a\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\text{Subst}\Big(\int}{}\\ &= \frac{a\sin^7(c+dx)}{7d} - \frac{a\sin^9(c+dx)}{3d} + \frac{3a\sin^{11}(c+dx)}{11d} - \frac{a\,\text{s}}{}\\ &= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} +\end{aligned}$$

Mathematica [A]
time = 0.50, size = 137, normalized size = 1.21

$$-\frac{a(600600\cos(2(c+dx))+75075\cos(4(c+dx))-100100\cos(6(c+dx))-30030\cos(8(c+dx))+12012\cos(10(c+dx))+5005\cos(12(c+dx))-600600\sin(c+dx)+150150\sin(3(c+dx))+90090\sin(5(c+dx))-25740\sin(7(c+dx))-20020\sin(9(c+dx))+2730\sin(11(c+dx))+2310\sin(13(c+dx)))}{123002880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/123002880*(a*(600600*Cos[2*(c + d*x)] + 75075*Cos[4*(c + d*x)] - 100100*
Cos[6*(c + d*x)] - 30030*Cos[8*(c + d*x)] + 12012*Cos[10*(c + d*x)] + 5005*
```

```
Cos[12*(c + d*x)] - 600600*Sin[c + d*x] + 150150*Sin[3*(c + d*x)] + 90090*S
in[5*(c + d*x)] - 25740*Sin[7*(c + d*x)] - 20020*Sin[9*(c + d*x)] + 2730*Si
n[11*(c + d*x)] + 2310*Sin[13*(c + d*x)]))/d
```

Maple [A]
time = 0.47, size = 148, normalized size = 1.31

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{12}-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{30}-\frac{\left(\cos^{8}(dx+c)\right)}{120}\right)+a\left(-\frac{\left(\sin^{5}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{13}-\frac{5\left(\sin^{3}(dx+\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{4}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{12}-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{30}-\frac{\left(\cos^{8}(dx+c)\right)}{120}\right)+a\left(-\frac{\left(\sin^{5}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{13}-\frac{5\left(\sin^{3}(dx+\right.}{}\right.}{d}$
risch	$\frac{5a\sin(dx+c)}{1024d}-\frac{a\cos(12dx+12c)}{24576d}-\frac{a\sin(11dx+11c)}{45056d}-\frac{a\cos(10dx+10c)}{10240d}-\frac{a\sin(13dx+13c)}{53248d}+\frac{a\sin(9dx+9c)}{6144d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/12*sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/12
0*cos(d*x+c)^8)+a*(-1/13*sin(d*x+c)^5*cos(d*x+c)^8-5/143*sin(d*x+c)^3*cos(d
*x+c)^8-5/429*sin(d*x+c)*cos(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x
+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 0.83

$$-\frac{9240\,a\sin\left(dx+c\right)^{13}+10010\,a\sin\left(dx+c\right)^{12}-32760\,a\sin\left(dx+c\right)^{11}-36036\,a\sin\left(dx+c\right)^{10}+40040\,a\sin\left(dx+c\right)^{9}+45045\,a\sin\left(dx+c\right)^{8}-17160\,a\sin\left(dx+c\right)^{7}-20020\,a\sin\left(dx+c\right)^{6}}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120120*(9240*a*sin(d*x + c)^13 + 10010*a*sin(d*x + c)^12 - 32760*a*sin(d
*x + c)^11 - 36036*a*sin(d*x + c)^10 + 40040*a*sin(d*x + c)^9 + 45045*a*sin
(d*x + c)^8 - 17160*a*sin(d*x + c)^7 - 20020*a*sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.39, size = 117, normalized size = 1.04

$$-\frac{10010\,a\cos\left(dx+c\right)^{12}-24024\,a\cos\left(dx+c\right)^{10}+15015\,a\cos\left(dx+c\right)^{8}+40\,\left(231\,a\cos\left(dx+c\right)^{12}-567\,a\cos\left(dx+c\right)^{10}+371\,a\cos\left(dx+c\right)^{8}-5\,a\cos\left(dx+c\right)^{6}-6\,a\cos\left(dx+c\right)^{4}-8\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)}{120120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/120120*(10010*a*cos(d*x + c)^12 - 24024*a*cos(d*x + c)^10 + 15015*a*cos(
d*x + c)^8 + 40*(231*a*cos(d*x + c)^12 - 567*a*cos(d*x + c)^10 + 371*a*cos(
d*x + c)^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c))/d
```

Sympy [A]
time = 4.85, size = 160, normalized size = 1.42

$$\begin{cases} \frac{16a\sin^{13}(c+dx)}{3003d} + \frac{8a\sin^{11}(c+dx)\cos^2(c+dx)}{231d} + \frac{2a\sin^9(c+dx)\cos^4(c+dx)}{21d} + \frac{a\sin^7(c+dx)\cos^6(c+dx)}{7d} - \frac{a\sin^4(c+dx)\cos^8(c+dx)}{8d} - \frac{a\sin^2(c+dx)\cos^{10}(c+dx)}{20d} - \frac{a\cos^{12}(c+dx)}{120d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^5(c)\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**13/(3003*d) + 8*a*sin(c + d*x)**11*cos(c + d*
x)**2/(231*d) + 2*a*sin(c + d*x)**9*cos(c + d*x)**4/(21*d) + a*sin(c + d*x)
**7*cos(c + d*x)**6/(7*d) - a*sin(c + d*x)**4*cos(c + d*x)**8/(8*d) - a*sin
(c + d*x)**2*cos(c + d*x)**10/(20*d) - a*cos(c + d*x)**12/(120*d), Ne(d, 0)
), (x*(a*sin(c) + a)*sin(c)**5*cos(c)**7, True))
```

Giac [A]
time = 0.54, size = 193, normalized size = 1.71

$$-\frac{a\cos(12dx+12c)}{24576d} - \frac{a\cos(10dx+10c)}{10240d} + \frac{a\cos(8dx+8c)}{4096d} + \frac{5a\cos(6dx+6c)}{6144d} - \frac{5a\cos(4dx+4c)}{8192d} - \frac{5a\cos(2dx+2c)}{1024d} - \frac{a\sin(13dx+13c)}{53248d} - \frac{a\sin(11dx+11c)}{45056d} + \frac{a\sin(9dx+9c)}{6144d} + \frac{3a\sin(7dx+7c)}{14336d} - \frac{3a\sin(5dx+5c)}{4096d} - \frac{5a\sin(3dx+3c)}{4096d} + \frac{5a\sin(dx+c)}{1024d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d - 1/53248*a*sin(13*d*x + 13*c)/d - 1/450
56*a*sin(11*d*x + 11*c)/d + 1/6144*a*sin(9*d*x + 9*c)/d + 3/14336*a*sin(7*d
*x + 7*c)/d - 3/4096*a*sin(5*d*x + 5*c)/d - 5/4096*a*sin(3*d*x + 3*c)/d + 5
/1024*a*sin(d*x + c)/d
```

Mupad [B]
time = 8.82, size = 93, normalized size = 0.82

$$\frac{-\frac{a\sin(c+dx)^{13}}{13} - \frac{a\sin(c+dx)^{12}}{12} + \frac{3a\sin(c+dx)^{11}}{11} + \frac{3a\sin(c+dx)^{10}}{10} - \frac{a\sin(c+dx)^9}{3} - \frac{3a\sin(c+dx)^8}{8} + \frac{a\sin(c+dx)^7}{7} + \frac{a\sin(c+dx)^6}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^6)/6 + (a*sin(c + d*x)^7)/7 - (3*a*sin(c + d*x)^8)/8 - (a*
sin(c + d*x)^9)/3 + (3*a*sin(c + d*x)^10)/10 + (3*a*sin(c + d*x)^11)/11 - (
a*sin(c + d*x)^12)/12 - (a*sin(c + d*x)^13)/13)/d
```

3.658 $\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\sin^5(c+dx)}{5d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^9(c+dx)}{3d}-\frac{a\sin$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/5*a*cos(d*x+c)^10/d-1/12*a*cos(d*x+c)^12/d+1/5*a*si
n(d*x+c)^5/d-3/7*a*sin(d*x+c)^7/d+1/3*a*sin(d*x+c)^9/d-1/11*a*sin(d*x+c)^11
/d
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 2644, 276, 2645, 272, 45}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}-\frac{a\cos^{12}(c+dx)}{12d}+\frac{a\cos^{10}(c+dx)}{5d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Cos[c + d*x]^10)/(5*d) - (a*Cos[c + d*x]^12)
/(12*d) + (a*Sin[c + d*x]^5)/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c
+ d*x]^9)/(3*d) - (a*Sin[c + d*x]^11)/(11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^7(c+dx)\sin^4(c+dx)\,dx + a\int \cos^7(c+dx)\sin^5\\ &= -\frac{a\text{Subst}\left(\int x^7(1-x^2)^2\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\right.}{}\\ &= -\frac{a\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \cos^2(c+dx)\right)}{2d} + \frac{a\text{Subst}\left(\int\right.}{}\\ &= \frac{a\sin^5(c+dx)}{5d} - \frac{3a\sin^7(c+dx)}{7d} + \frac{a\sin^9(c+dx)}{3d} - \frac{a\,\text{si}}{}\\ &= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{5d} - \frac{a\cos^{12}(c+dx)}{12d} +\end{aligned}$$

Mathematica [A]
time = 0.41, size = 127, normalized size = 1.12

$$-\frac{a(46200\cos(2(c+dx))+5775\cos(4(c+dx))-7700\cos(6(c+dx))-2310\cos(8(c+dx))+924\cos(10(c+dx))+385\cos(12(c+dx))-129360\sin(c+dx)+18480\sin(3(c+dx))+20328\sin(5(c+dx))+1320\sin(7(c+dx))-3080\sin(9(c+dx))-840\sin(11(c+dx)))}{9461760d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/9461760*(a*(46200*Cos[2*(c + d*x)] + 5775*Cos[4*(c + d*x)] - 7700*Cos[6*
(c + d*x)] - 2310*Cos[8*(c + d*x)] + 924*Cos[10*(c + d*x)] + 385*Cos[12*(c
```

```
+ d*x)] - 129360*Sin[c + d*x] + 18480*Sin[3*(c + d*x)] + 20328*Sin[5*(c + d
*x)] + 1320*Sin[7*(c + d*x)] - 3080*Sin[9*(c + d*x)] - 840*Sin[11*(c + d*x)
]))/d
```

Maple [A]
time = 0.36, size = 130, normalized size = 1.15

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^{8}(dx+c)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{231}\right)+}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{8}(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^{8}(dx+c)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}(dx+c)\right)}{5}+\frac{8\left(\cos^{2}(dx+c)\right)}{5}\right)\sin(dx+c)}{231}\right)+}{d}$
risch	$-\frac{a\cos(12dx+12c)}{24576d}+\frac{7a\sin(dx+c)}{512d}+\frac{a\sin(11dx+11c)}{11264d}-\frac{a\cos(10dx+10c)}{10240d}+\frac{a\sin(9dx+9c)}{3072d}+\frac{a\cos(8dx+8c)}{4096d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+c)^8+1/231*
(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/12*
sin(d*x+c)^4*cos(d*x+c)^8-1/30*sin(d*x+c)^2*cos(d*x+c)^8-1/120*cos(d*x+c)^8
))
```

Maxima [A]
time = 0.28, size = 94, normalized size = 0.83

$$-\frac{770\,a\sin\left(dx+c\right)^{12}+840\,a\sin\left(dx+c\right)^{11}-2772\,a\sin\left(dx+c\right)^{10}-3080\,a\sin\left(dx+c\right)^{9}+3465\,a\sin\left(dx+c\right)^{8}+3960\,a\sin\left(dx+c\right)^{7}-1540\,a\sin\left(dx+c\right)^{6}-1848\,a\sin\left(dx+c\right)^{5}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(770*a*sin(d*x + c)^12 + 840*a*sin(d*x + c)^11 - 2772*a*sin(d*x + c
)^10 - 3080*a*sin(d*x + c)^9 + 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^
7 - 1540*a*sin(d*x + c)^6 - 1848*a*sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.40, size = 106, normalized size = 0.94

$$-\frac{770\,a\cos\left(dx+c\right)^{12}-1848\,a\cos\left(dx+c\right)^{10}+1155\,a\cos\left(dx+c\right)^{8}-8\left(105\,a\cos\left(dx+c\right)^{10}-140\,a\cos\left(dx+c\right)^{8}+5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}+8\,a\cos\left(dx+c\right)^{2}+16\,a\right)\sin\left(dx+c\right)}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/9240*(770*a*cos(d*x + c)^12 - 1848*a*cos(d*x + c)^10 + 1155*a*cos(d*x +
c)^8 - 8*(105*a*cos(d*x + c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6
 + 6*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A]
time = 3.62, size = 160, normalized size = 1.42

$$\begin{cases} \frac{16a\sin^{11}(c+dx)}{1155d} + \frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d} + \frac{6a\sin^{7}(c+dx)\cos^{4}(c+dx)}{35d} + \frac{a\sin^{5}(c+dx)\cos^{6}(c+dx)}{5d} - \frac{a\sin^{4}(c+dx)\cos^{8}(c+dx)}{8d} - \frac{a\sin^{2}(c+dx)\cos^{10}(c+dx)}{20d} - \frac{a\cos^{12}(c+dx)}{120d} & \text{for } d \neq 0 \\ x(a\sin(c)+a)\sin^{4}(c)\cos^{7}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**11/(1155*d) + 8*a*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 6*a*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + a*sin(c + d*x)*
*5*cos(c + d*x)**6/(5*d) - a*sin(c + d*x)**4*cos(c + d*x)**8/(8*d) - a*sin(
c + d*x)**2*cos(c + d*x)**10/(20*d) - a*cos(c + d*x)**12/(120*d), Ne(d, 0))
, (x*(a*sin(c) + a)*sin(c)**4*cos(c)**7, True))
```

Giac [A]
time = 0.54, size = 178, normalized size = 1.58

$$-\frac{a\cos(12\,dx+12\,c)}{24576\,d} - \frac{a\cos(10\,dx+10\,c)}{10240\,d} + \frac{a\cos(8\,dx+8\,c)}{4096\,d} + \frac{5\,a\cos(6\,dx+6\,c)}{6144\,d} - \frac{5\,a\cos(4\,dx+4\,c)}{8192\,d} - \frac{5\,a\cos(2\,dx+2\,c)}{1024\,d} + \frac{a\sin(11\,dx+11\,c)}{11264\,d} + \frac{a\sin(9\,dx+9\,c)}{3072\,d} - \frac{a\sin(7\,dx+7\,c)}{7168\,d} - \frac{11\,a\sin(5\,dx+5\,c)}{5120\,d} - \frac{a\sin(3\,dx+3\,c)}{512\,d} + \frac{7\,a\sin(dx+c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24576*a*cos(12*d*x + 12*c)/d - 1/10240*a*cos(10*d*x + 10*c)/d + 1/4096*a
*cos(8*d*x + 8*c)/d + 5/6144*a*cos(6*d*x + 6*c)/d - 5/8192*a*cos(4*d*x + 4*
c)/d - 5/1024*a*cos(2*d*x + 2*c)/d + 1/11264*a*sin(11*d*x + 11*c)/d + 1/307
2*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x
+ 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d + 7/512*a*sin(d*x + c)/d
```

Mupad [B]
time = 0.08, size = 93, normalized size = 0.82

$$\frac{-\frac{a\sin(c+d\,x)^{12}}{12} - \frac{a\sin(c+d\,x)^{11}}{11} + \frac{3\,a\sin(c+d\,x)^{10}}{10} + \frac{a\sin(c+d\,x)^{9}}{3} - \frac{3\,a\sin(c+d\,x)^{8}}{8} - \frac{3\,a\sin(c+d\,x)^{7}}{7} + \frac{a\sin(c+d\,x)^{6}}{6} + \frac{a\sin(c+d\,x)^{5}}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/6 - (3*a*sin(c + d*x)^7)/7 - (3*
a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/3 + (3*a*sin(c + d*x)^10)/10 - (a*
sin(c + d*x)^11)/11 - (a*sin(c + d*x)^12)/12)/d
```

3.659 $\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{a\cos^{10}(c+dx)}{10d}+\frac{a\sin^5(c+dx)}{5d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^9(c+dx)}{3d}-\frac{a\sin^{11}(c+dx)}{11d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/10*a*cos(d*x+c)^10/d+1/5*a*sin(d*x+c)^5/d-3/7*a*sin
(d*x+c)^7/d+1/3*a*sin(d*x+c)^9/d-1/11*a*sin(d*x+c)^11/d
```

Rubi [A]
time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2645, 14, 2644, 276}

$$-\frac{a\sin^{11}(c+dx)}{11d}+\frac{a\sin^9(c+dx)}{3d}-\frac{3a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^5)
/(5*d) - (3*a*Sin[c + d*x]^7)/(7*d) + (a*Sin[c + d*x]^9)/(3*d) - (a*Sin[c +
 d*x]^11)/(11*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^3(c+dx)\,dx + a\int \cos^7(c+dx)\sin^4$$

$$= -\frac{a\text{Subst}\left(\int x^7(1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^7-x^9)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int (x\right.}{}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^5(c+dx)}{5d} - \frac{3}{}$$

Mathematica [A]
time = 0.43, size = 117, normalized size = 1.21

$$\frac{a(-16170\cos(2(c+dx))-4620\cos(4(c+dx))+1155\cos(6(c+dx))+1155\cos(8(c+dx))+231\cos(10(c+dx))+16170\sin(c+dx)-2310\sin(3(c+dx))-2541\sin(5(c+dx))-165\sin(7(c+dx))+385\sin(9(c+dx))+105\sin(11(c+dx)))}{1182720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(-16170*Cos[2*(c + d*x)] - 4620*Cos[4*(c + d*x)] + 1155*Cos[6*(c + d*x)]
 + 1155*Cos[8*(c + d*x)] + 231*Cos[10*(c + d*x)] + 16170*Sin[c + d*x] - 231
0*Sin[3*(c + d*x)] - 2541*Sin[5*(c + d*x)] - 165*Sin[7*(c + d*x)] + 385*Sin
[9*(c + d*x)] + 105*Sin[11*(c + d*x)]))/(1182720*d)
```

Maple [A]
time = 0.28, size = 112, normalized size = 1.15

method	result

derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{10}-\frac{\left(\cos^8(dx+c)\right)}{40}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^6(dx\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+c)\right)}{10}-\frac{\left(\cos^8(dx+c)\right)}{40}\right)+a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^8(dx+c)\right)}{11}-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^6(dx\right.}{}\right.}{d}$
risch	$\frac{7a\sin(dx+c)}{512d}+\frac{a\sin(11dx+11c)}{11264d}+\frac{a\cos(10dx+10c)}{5120d}+\frac{a\sin(9dx+9c)}{3072d}+\frac{a\cos(8dx+8c)}{1024d}-\frac{a\sin(7dx+7c)}{7168d}+\frac{a\,c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d*x+c)^8)+a*(-1/11*sin(d*x
+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+6/
5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]

time = 0.28, size = 94, normalized size = 0.97

$$-\frac{840\,a\sin(dx+c)^{11}+924\,a\sin(dx+c)^{10}-3080\,a\sin(dx+c)^{9}-3465\,a\sin(dx+c)^{8}+3960\,a\sin(dx+c)^{7}+4620\,a\sin(dx+c)^{6}-1848\,a\sin(dx+c)^{5}-2310\,a\sin(dx+c)^{4}}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/9240*(840*a*sin(d*x + c)^11 + 924*a*sin(d*x + c)^10 - 3080*a*sin(d*x + c
)^9 - 3465*a*sin(d*x + c)^8 + 3960*a*sin(d*x + c)^7 + 4620*a*sin(d*x + c)^6
 - 1848*a*sin(d*x + c)^5 - 2310*a*sin(d*x + c)^4)/d
```

Fricas [A]

time = 0.39, size = 95, normalized size = 0.98

$$\frac{924\,a\cos(dx+c)^{10}-1155\,a\cos(dx+c)^{8}+8\,\left(105\,a\cos(dx+c)^{10}-140\,a\cos(dx+c)^{8}+5\,a\cos(dx+c)^{6}+6\,a\cos(dx+c)^{4}+8\,a\cos(dx+c)^{2}+16\,a\right)\sin(dx+c)}{9240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/9240*(924*a*cos(d*x + c)^10 - 1155*a*cos(d*x + c)^8 + 8*(105*a*cos(d*x +
c)^10 - 140*a*cos(d*x + c)^8 + 5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 + 8*
a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d
```

Sympy [A]

time = 2.63, size = 138, normalized size = 1.42

$$\begin{cases} \frac{16a\sin^{11}(c+dx)}{1155d}+\frac{8a\sin^{9}(c+dx)\cos^{2}(c+dx)}{105d}+\frac{6a\sin^{7}(c+dx)\cos^{4}(c+dx)}{35d}+\frac{a\sin^{5}(c+dx)\cos^{6}(c+dx)}{5d}-\frac{a\sin^{2}(c+dx)\cos^{8}(c+dx)}{8d}-\frac{a\cos^{10}(c+dx)}{40d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)\sin^{3}(c)\cos^{7}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**11/(1155*d) + 8*a*sin(c + d*x)**9*cos(c + d*x
)**2/(105*d) + 6*a*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + a*sin(c + d*x)*
*5*cos(c + d*x)**6/(5*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(
c + d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**3*cos(c)**7, True
))
```

Giac [A]
time = 0.56, size = 163, normalized size = 1.68

$$\frac{a\cos(10\,dx+10\,c)}{5120\,d}+\frac{a\cos(8\,dx+8\,c)}{1024\,d}+\frac{a\cos(6\,dx+6\,c)}{1024\,d}-\frac{a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{512\,d}+\frac{a\sin(11\,dx+11\,c)}{11264\,d}+\frac{a\sin(9\,dx+9\,c)}{3072\,d}-\frac{a\sin(7\,dx+7\,c)}{7168\,d}-\frac{11\,a\sin(5\,dx+5\,c)}{5120\,d}-\frac{a\sin(3\,dx+3\,c)}{512\,d}+\frac{7\,a\sin(dx+c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d +
1/11264*a*sin(11*d*x + 11*c)/d + 1/3072*a*sin(9*d*x + 9*c)/d - 1/7168*a*sin
(7*d*x + 7*c)/d - 11/5120*a*sin(5*d*x + 5*c)/d - 1/512*a*sin(3*d*x + 3*c)/d
 + 7/512*a*sin(d*x + c)/d
```

Mupad [B]
time = 0.08, size = 93, normalized size = 0.96

$$\frac{-\frac{a\sin(c+d\,x)^{11}}{11}-\frac{a\sin(c+d\,x)^{10}}{10}+\frac{a\sin(c+d\,x)^9}{3}+\frac{3\,a\sin(c+d\,x)^8}{8}-\frac{3\,a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{2}+\frac{a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 + (a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/2 - (3*a*
sin(c + d*x)^7)/7 + (3*a*sin(c + d*x)^8)/8 + (a*sin(c + d*x)^9)/3 - (a*sin(
c + d*x)^10)/10 - (a*sin(c + d*x)^11)/11)/d
```

3.660 $\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{a\cos^{10}(c+dx)}{10d}+\frac{a\sin^3(c+dx)}{3d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{a\sin^9(c+dx)}{9d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/10*a*cos(d*x+c)^10/d+1/3*a*sin(d*x+c)^3/d-3/5*a*sin
(d*x+c)^5/d+3/7*a*sin(d*x+c)^7/d-1/9*a*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2644, 276, 2645, 14}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{a\cos^{10}(c+dx)}{10d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Cos[c + d*x]^10)/(10*d) + (a*Sin[c + d*x]^3)
/(3*d) - (3*a*Sin[c + d*x]^5)/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c
 + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^7(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cos^7(c+dx)\sin^2(c+dx)\,dx + a\int \cos^7(c+dx)\sin^3$$

$$= -\frac{a\text{Subst}\left(\int x^7(1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int\right.}{}$$

$$= \frac{a\text{Subst}\left(\int (x^2-3x^4+3x^6-x^8)\,dx, x, \sin(c+dx)\right)}{d} - \frac{a}{}$$

$$= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\cos^{10}(c+dx)}{10d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3}{}$$

Mathematica [A]
time = 0.32, size = 97, normalized size = 1.00

$$-\frac{a(4410\cos(2(c+dx))+1260\cos(4(c+dx))-315\cos(6(c+dx))-315\cos(8(c+dx))-63\cos(10(c+dx))-17640\sin(c+dx)+2016\sin(5(c+dx))+900\sin(7(c+dx))+140\sin(9(c+dx)))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/322560*(a*(4410*Cos[2*(c + d*x)] + 1260*Cos[4*(c + d*x)] - 315*Cos[6*(c
+ d*x)] - 315*Cos[8*(c + d*x)] - 63*Cos[10*(c + d*x)] - 17640*Sin[c + d*x]
+ 2016*Sin[5*(c + d*x)] + 900*Sin[7*(c + d*x)] + 140*Sin[9*(c + d*x)]))/d
```

Maple [A]
time = 0.21, size = 94, normalized size = 0.97

method	result

derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+\right.}{10}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)+a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^8(dx+\right.}{10}\right.}{d}$
risch	$\frac{7a\sin(dx+c)}{128d}+\frac{a\cos(10dx+10c)}{5120d}-\frac{a\sin(9dx+9c)}{2304d}+\frac{a\cos(8dx+8c)}{1024d}-\frac{5a\sin(7dx+7c)}{1792d}+\frac{a\cos(6dx+6c)}{1024d}-\frac{a\sin}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)
^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*co
s(d*x+c)^8))
```

Maxima [A]

time = 0.27, size = 94, normalized size = 0.97

$$-\frac{252\,a\sin(dx+c)^{10}+280\,a\sin(dx+c)^{9}-945\,a\sin(dx+c)^{8}-1080\,a\sin(dx+c)^{7}+1260\,a\sin(dx+c)^{6}+1512\,a\sin(dx+c)^{5}-630\,a\sin(dx+c)^{4}-840\,a\sin(dx+c)^{3}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(252*a*sin(d*x + c)^10 + 280*a*sin(d*x + c)^9 - 945*a*sin(d*x + c)^
8 - 1080*a*sin(d*x + c)^7 + 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 -
 630*a*sin(d*x + c)^4 - 840*a*sin(d*x + c)^3)/d
```

Fricas [A]

time = 0.37, size = 84, normalized size = 0.87

$$\frac{252\,a\cos(dx+c)^{10}-315\,a\cos(dx+c)^{8}-8\left(35\,a\cos(dx+c)^{8}-5\,a\cos(dx+c)^{6}-6\,a\cos(dx+c)^{4}-8\,a\cos(dx+c)^{2}-16\,a\right)\sin(dx+c)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*a*cos(d*x + c)^10 - 315*a*cos(d*x + c)^8 - 8*(35*a*cos(d*x + c)
^8 - 5*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*s
in(d*x + c))/d
```

Sympy [A]

time = 1.80, size = 138, normalized size = 1.42

$$\begin{cases}\frac{16a\sin^9(c+dx)}{315d}+\frac{8a\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{2a\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{a\sin^3(c+dx)\cos^6(c+dx)}{3d}-\frac{a\sin^2(c+dx)\cos^8(c+dx)}{8d}-\frac{a\cos^{10}(c+dx)}{40d} & \text{for } d\neq 0\\ x(a\sin(c)+a)\sin^2(c)\cos^7(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - a*cos(c +
d*x)**10/(40*d), Ne(d, 0)), (x*(a*sin(c) + a)*sin(c)**2*cos(c)**7, True))
```

Giac [A]

time = 0.49, size = 133, normalized size = 1.37

$$\frac{a\cos(10dx+10c)}{5120d}+\frac{a\cos(8dx+8c)}{1024d}+\frac{a\cos(6dx+6c)}{1024d}-\frac{a\cos(4dx+4c)}{256d}-\frac{7a\cos(2dx+2c)}{512d}-\frac{a\sin(9dx+9c)}{2304d}-\frac{5a\sin(7dx+7c)}{1792d}-\frac{a\sin(5dx+5c)}{160d}+\frac{7a\sin(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/5120*a*cos(10*d*x + 10*c)/d + 1/1024*a*cos(8*d*x + 8*c)/d + 1/1024*a*cos(
6*d*x + 6*c)/d - 1/256*a*cos(4*d*x + 4*c)/d - 7/512*a*cos(2*d*x + 2*c)/d -
1/2304*a*sin(9*d*x + 9*c)/d - 5/1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d
*x + 5*c)/d + 7/128*a*sin(d*x + c)/d
```

Mupad [B]

time = 8.95, size = 93, normalized size = 0.96

$$\frac{-\frac{a\sin(c+dx)^{10}}{10}-\frac{a\sin(c+dx)^9}{9}+\frac{3a\sin(c+dx)^8}{8}+\frac{3a\sin(c+dx)^7}{7}-\frac{a\sin(c+dx)^6}{2}-\frac{3a\sin(c+dx)^5}{5}+\frac{a\sin(c+dx)^4}{4}+\frac{a\sin(c+dx)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 + (a*sin(c + d*x)^4)/4 - (3*a*sin(c + d*x)^5)/5 - (a*
sin(c + d*x)^6)/2 + (3*a*sin(c + d*x)^7)/7 + (3*a*sin(c + d*x)^8)/8 - (a*si
n(c + d*x)^9)/9 - (a*sin(c + d*x)^10)/10)/d
```

3.661 $\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cos^8(c+dx)}{8d}+\frac{a\sin^3(c+dx)}{3d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{a\sin^9(c+dx)}{9d}$$

```
[Out] -1/8*a*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-3/5*a*sin(d*x+c)^5/d+3/7*a*sin(d
*x+c)^7/d-1/9*a*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 2645, 30, 2644, 276}

$$-\frac{a\sin^9(c+dx)}{9d}+\frac{3a\sin^7(c+dx)}{7d}-\frac{3a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cos[c + d*x]^8)/d + (a*Sin[c + d*x]^3)/(3*d) - (3*a*Sin[c + d*x]^5)
/(5*d) + (3*a*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^9)/(9*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}\int \cos^7(c+dx)\sin(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cos^7(c+dx)\sin(c+dx)\,dx + a\int \cos^7(c+dx)\sin^2(c\\ &= -\frac{a\text{Subst}\left(\int x^7\,dx,x,\cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int x^2(1-x^2\right.}{}\\ &= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\text{Subst}\left(\int (x^2-3x^4+3x^6-x^8)\,dx,x\right.}{d}\\ &= -\frac{a\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{3a\sin^5(c+dx)}{5d} + \frac{3a}{}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 60, normalized size = 0.74

$$\frac{a(-1260\cos^8(c+dx)+(1606+1389\cos(2(c+dx))+330\cos(4(c+dx))+35\cos(6(c+dx)))\sin^3(c+dx))}{10080d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]*(a + a*Sin[c + d*x]),x]
[Out] (a*(-1260*Cos[c + d*x]^8 + (1606 + 1389*Cos[2*(c + d*x)] + 330*Cos[4*(c + d
*x)] + 35*Cos[6*(c + d*x)])*Sin[c + d*x]^3))/(10080*d)
```

Maple [A]
time = 0.20, size = 74, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8}+a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)}{d}$

default	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8}+a\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)}{d}$
risch	$\frac{7a\sin(dx+c)}{128d}-\frac{a\sin(9dx+9c)}{2304d}-\frac{a\cos(8dx+8c)}{1024d}-\frac{5a\sin(7dx+7c)}{1792d}-\frac{a\cos(6dx+6c)}{128d}-\frac{a\sin(5dx+5c)}{160d}-\frac{7a\cos}{}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{632a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}-\frac{2848a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*a*cos(d*x+c)^8+a*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]

time = 0.30, size = 94, normalized size = 1.16

$$-\frac{280\,a\sin(dx+c)^9+315\,a\sin(dx+c)^8-1080\,a\sin(dx+c)^7-1260\,a\sin(dx+c)^6+1512\,a\sin(dx+c)^5+1890\,a\sin(dx+c)^4-840\,a\sin(dx+c)^3-1260\,a\sin(dx+c)^2}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 1080*a*sin(d*x + c)^
7 - 1260*a*sin(d*x + c)^6 + 1512*a*sin(d*x + c)^5 + 1890*a*sin(d*x + c)^4 -
 840*a*sin(d*x + c)^3 - 1260*a*sin(d*x + c)^2)/d
```

Fricas [A]

time = 0.38, size = 73, normalized size = 0.90

$$-\frac{315\,a\cos(dx+c)^8+8\left(35\,a\cos(dx+c)^8-5\,a\cos(dx+c)^6-6\,a\cos(dx+c)^4-8\,a\cos(dx+c)^2-16\,a\right)\sin(dx+c)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(315*a*cos(d*x + c)^8 + 8*(35*a*cos(d*x + c)^8 - 5*a*cos(d*x + c)^6
 - 6*a*cos(d*x + c)^4 - 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c))/d
```

Sympy [A]

time = 1.36, size = 114, normalized size = 1.41

$$\begin{cases} \frac{16a\sin^9(c+dx)}{315d}+\frac{8a\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{2a\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{a\sin^3(c+dx)\cos^6(c+dx)}{3d}-\frac{a\cos^8(c+dx)}{8d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)\sin(c)\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*a*sin(c + d*x)**9/(315*d) + 8*a*sin(c + d*x)**7*cos(c + d*x)*
*2/(35*d) + 2*a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + a*sin(c + d*x)**3*c
os(c + d*x)**6/(3*d) - a*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(a*sin(c) + a
)*sin(c)*cos(c)**7, True))
```

Giac [A]
time = 0.47, size = 118, normalized size = 1.46

$$-\frac{a\cos(8\,dx+8\,c)}{1024\,d}-\frac{a\cos(6\,dx+6\,c)}{128\,d}-\frac{7\,a\cos(4\,dx+4\,c)}{256\,d}-\frac{7\,a\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(9\,dx+9\,c)}{2304\,d}-\frac{5\,a\sin(7\,dx+7\,c)}{1792\,d}-\frac{a\sin(5\,dx+5\,c)}{160\,d}+\frac{7\,a\sin(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1024*a*cos(8*d*x + 8*c)/d - 1/128*a*cos(6*d*x + 6*c)/d - 7/256*a*cos(4*d
*x + 4*c)/d - 7/128*a*cos(2*d*x + 2*c)/d - 1/2304*a*sin(9*d*x + 9*c)/d - 5/
1792*a*sin(7*d*x + 7*c)/d - 1/160*a*sin(5*d*x + 5*c)/d + 7/128*a*sin(d*x +
c)/d
```

Mupad [B]
time = 9.02, size = 93, normalized size = 1.15

$$\frac{-\frac{a\sin(c+d\,x)^9}{9}-\frac{a\sin(c+d\,x)^8}{8}+\frac{3\,a\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{2}-\frac{3\,a\sin(c+d\,x)^5}{5}-\frac{3\,a\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)*(a + a*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^3)/3 - (3*a*sin(c + d*x)^4)/4 - (3*
a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^6)/2 + (3*a*sin(c + d*x)^7)/7 - (a*si
n(c + d*x)^8)/8 - (a*sin(c + d*x)^9)/9)/d
```

3.662 $\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin$$

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d-3/2*a*sin(d*x+c)^2/d-a*sin(d*x+c)^3/d+3/4
*a*sin(d*x+c)^4/d+3/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6/d-1/7*a*sin(d*x+c
)^7/d
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^6(c+dx)}{6d}+\frac{3a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d - (3*a*Sin[c + d*x]^2)/(2*d) -
 (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d) + (3*a*Sin[c + d*x]^5)/(
5*d) - (a*Sin[c + d*x]^6)/(6*d) - (a*Sin[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^6(c+dx)\cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6+\frac{a^7}{x}-3a^5x-3a^4x^2+3a^3x^3+3a^2x^4-ax\right.\right.}{a^6 d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\ldots$$

Mathematica [A]

time = 0.10, size = 106, normalized size = 0.90

$$\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^5(c+dx)}{5d}-\frac{a\sin^7(c+dx)}{7d}+\frac{a(12\log(\sin(c+dx))-18\sin^2(c+dx)+9\sin^4(c+dx)-2\sin^6(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^5)/(5*d) - (a
*Sin[c + d*x]^7)/(7*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*S
in[c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

Maple [A]

time = 0.16, size = 85, normalized size = 0.72

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^6(dx+c)\right)}{6}+\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(d}{7}}{d}$
default	$\frac{a\left(\frac{\left(\cos^6(dx+c)\right)}{6}+\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{a\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(d}{7}}{d}$
risch	$-iax-\frac{2iac}{d}+\frac{29a\,e^{2i(dx+c)}}{128d}+\frac{29a\,e^{-2i(dx+c)}}{128d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{35a\sin(dx+c)}{64d}+\frac{a\sin(7dx+7c)}{448d}+\frac{a\,c}{}$
norman	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{86a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{424a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{86a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{4a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a*(1/6*cos(d*x+c)^6+1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/7*a*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))

Maxima [A]
time = 0.30, size = 91, normalized size = 0.77

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-252\,a\sin\left(dx+c\right)^{5}-315\,a\sin\left(dx+c\right)^{4}+420\,a\sin\left(dx+c\right)^{3}+630\,a\sin\left(dx+c\right)^{2}-420\,a\log\left(\sin\left(dx+c\right)\right)-420\,a\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 - 315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*log(sin(d*x + c)) - 420*a*sin(d*x + c))/d

Fricas [A]
time = 0.41, size = 96, normalized size = 0.81

$$\frac{70\,a\cos\left(dx+c\right)^{6}+105\,a\cos\left(dx+c\right)^{4}+210\,a\cos\left(dx+c\right)^{2}+420\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+12\,\left(5\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}+8\,a\cos\left(dx+c\right)^{2}+16\,a\right)\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/420*(70*a*cos(d*x + c)^6 + 105*a*cos(d*x + c)^4 + 210*a*cos(d*x + c)^2 + 420*a*log(1/2*sin(d*x + c)) + 12*(5*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c))/d

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**7*csc(d*x+c)*(a+a*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep

Giac [A]
time = 0.49, size = 92, normalized size = 0.78

$$-\frac{60\,a\sin\left(dx+c\right)^{7}+70\,a\sin\left(dx+c\right)^{6}-252\,a\sin\left(dx+c\right)^{5}-315\,a\sin\left(dx+c\right)^{4}+420\,a\sin\left(dx+c\right)^{3}+630\,a\sin\left(dx+c\right)^{2}-420\,a\log\left(|\sin\left(dx+c\right)|\right)-420\,a\sin\left(dx+c\right)}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/420*(60*a*sin(d*x + c)^7 + 70*a*sin(d*x + c)^6 - 252*a*sin(d*x + c)^5 - 315*a*sin(d*x + c)^4 + 420*a*sin(d*x + c)^3 + 630*a*sin(d*x + c)^2 - 420*a*log(abs(sin(d*x + c))) - 420*a*sin(d*x + c))/d

Mupad [B]
time = 9.16, size = 160, normalized size = 1.36

$$\frac{a\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a\,{\cos\left(c+d\,x\right)}^2}{2\,d}+\frac{a\,{\cos\left(c+d\,x\right)}^4}{4\,d}+\frac{a\,{\cos\left(c+d\,x\right)}^6}{6\,d}+\frac{16\,a\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,a\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a\,{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a\,{\cos\left(c+d\,x\right)}^6\,\sin\left(c+d\,x\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (a*cos(c
+ d*x)^6)/(6*d) + (16*a*sin(c + d*x))/(35*d) + (8*a*cos(c + d*x)^2*sin(c +
d*x))/(35*d) + (6*a*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a*cos(c + d*x)^6
*sin(c + d*x))/(7*d)
```

3.663 $\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{3a\sin(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^4(c+dx)}{4d}-\frac{a\,\mathrm{s}}{}$$

```
[Out] -a*csc(d*x+c)/d+a*ln(sin(d*x+c))/d-3*a*sin(d*x+c)/d-3/2*a*sin(d*x+c)^2/d+a*
sin(d*x+c)^3/d+3/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)^5/d-1/6*a*sin(d*x+c)^6
/d
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^6(c+dx)}{6d}-\frac{a\sin^5(c+dx)}{5d}+\frac{3a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}-\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (a*Log[Sin[c + d*x]])/d - (3*a*Sin[c + d*x])/d - (3
*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/d + (3*a*Sin[c + d*x]^4)/(4*d
) - (a*Sin[c + d*x]^5)/(5*d) - (a*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^5+\frac{a^7}{x^2}+\frac{a^6}{x}-3a^4x+3a^3x^2+3a^2x^3-ax\right.\right.}{a^5 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}-\frac{3a\sin(c+dx)}{d}-$$

Mathematica [A]

time = 0.10, size = 102, normalized size = 0.89

$$-\frac{a\csc(c+dx)}{d}-\frac{3a\sin(c+dx)}{d}+\frac{a\sin^3(c+dx)}{d}-\frac{a\sin^5(c+dx)}{5d}+\frac{a(12\log(\sin(c+dx))-18\sin^2(c+dx)+9\sin^4(c+dx)-2\sin^6(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) + (a*(12*Log[Sin[c + d*x]] - 18*Sin[c + d*x]^2 + 9*Sin[
c + d*x]^4 - 2*Sin[c + d*x]^6))/(12*d)
```

Maple [A]

time = 0.14, size = 105, normalized size = 0.92

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(\frac{\left(\cos^6(dx+c)\right)}{6}+\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(\frac{\left(\cos^6(dx+c)\right)}{6}+\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos\right.}{}\right.}{d}$
risch	$\frac{19ia\,e^{i(dx+c)}}{16d}-iax+\frac{29a\,e^{2i(dx+c)}}{128d}+\frac{29a\,e^{-2i(dx+c)}}{128d}-\frac{2ia\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2iac}{d}-\frac{19ia\,e^{-i(dx+c)}}{16d}+\frac{a\ln\left(e^{2i(\right.}{}$
norman	$\frac{-\frac{a}{2d}-\frac{19a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{65a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{599a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10d}-\frac{599a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10d}-\frac{65a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{19a\left(\tan^{12}\right.}{}}{\left(1+\tan^2\left(\frac{d}{}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*
cos(d*x+c)^2)*sin(d*x+c))+a*(1/6*cos(d*x+c)^6+1/4*cos(d*x+c)^4+1/2*cos(d*x+
c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.31, size = 91, normalized size = 0.80

$$-\frac{10\,a\sin\left(dx+c\right)^6+12\,a\sin\left(dx+c\right)^5-45\,a\sin\left(dx+c\right)^4-60\,a\sin\left(dx+c\right)^3+90\,a\sin\left(dx+c\right)^2-60\,a\log\left(\sin\left(dx+c\right)\right)+180\,a\sin\left(dx+c\right)+\frac{60\,a}{\sin(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 180*a*si
n(d*x + c) + 60*a/sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 113, normalized size = 0.99

$$\frac{48\,a\cos\left(dx+c\right)^6+96\,a\cos\left(dx+c\right)^4+384\,a\cos\left(dx+c\right)^2+240\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+5\left(8\,a\cos\left(dx+c\right)^6+12\,a\cos\left(dx+c\right)^4+24\,a\cos\left(dx+c\right)^2-19\,a\right)\sin\left(dx+c\right)-768\,a}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^6 + 96*a*cos(d*x + c)^4 + 384*a*cos(d*x + c)^2 + 2
40*a*log(1/2*sin(d*x + c))*sin(d*x + c) + 5*(8*a*cos(d*x + c)^6 + 12*a*cos(
d*x + c)^4 + 24*a*cos(d*x + c)^2 - 19*a)*sin(d*x + c) - 768*a)/(d*sin(d*x +
 c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.49, size = 101, normalized size = 0.89

$$-\frac{10\,a\sin\left(dx+c\right)^6+12\,a\sin\left(dx+c\right)^5-45\,a\sin\left(dx+c\right)^4-60\,a\sin\left(dx+c\right)^3+90\,a\sin\left(dx+c\right)^2-60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+180\,a\sin\left(dx+c\right)+\frac{60\,(a\sin(dx+c)+a)}{\sin(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(10*a*sin(d*x + c)^6 + 12*a*sin(d*x + c)^5 - 45*a*sin(d*x + c)^4 - 60
*a*sin(d*x + c)^3 + 90*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 180
*a*sin(d*x + c) + 60*(a*sin(d*x + c) + a)/sin(d*x + c))/d
```

Mupad [B]
time = 9.33, size = 340, normalized size = 2.98

$$\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}-\frac{6\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{d}+\frac{18\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{d}-\frac{104\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{3\,d}+\frac{44\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{d}-\frac{32\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{d}+\frac{32\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{3\,d}-\frac{13\,a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{14\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{112\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{136\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{96\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{32\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{5\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d - (6*a*cos(c/2 + (d*x)/2)^2)/d + (18*a*cos(c/2 + (d*x)/2)^4)/d -
(104*a*cos(c/2 + (d*x)/2)^6)/(3*d) + (44*a*cos(c/2 + (d*x)/2)^8)/d - (32*a*
cos(c/2 + (d*x)/2)^10)/d + (32*a*cos(c/2 + (d*x)/2)^12)/(3*d) - (13*a*cos(c
/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c
/2 + (d*x)/2)) + (14*a*cos(c/2 + (d*x)/2)^3)/(d*sin(c/2 + (d*x)/2)) - (112*
a*cos(c/2 + (d*x)/2)^5)/(5*d*sin(c/2 + (d*x)/2)) + (136*a*cos(c/2 + (d*x)/2
)^7)/(5*d*sin(c/2 + (d*x)/2)) - (96*a*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 +
(d*x)/2)) + (32*a*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2))
```

3.664 $\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\log(\sin(c+dx))}{d}-\frac{3a\sin(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{a\sin^3(c+dx)}{d}-a\,\mathrm{s}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-3*a*ln(sin(d*x+c))/d-3*a*sin(d*x+c)/d+
3/2*a*sin(d*x+c)^2/d+a*sin(d*x+c)^3/d-1/4*a*sin(d*x+c)^4/d-1/5*a*sin(d*x+c)
^5/d
```

Rubi [A]
time = 0.07, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^4(c+dx)}{4d}+\frac{a\sin^3(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-\frac{3a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{3a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (3*a*Log[Sin[c + d*x]])/
d - (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(2*d) + (a*Sin[c + d*x]^3)/
d - (a*Sin[c + d*x]^4)/(4*d) - (a*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(-3a^4+\frac{a^7}{x^3}+\frac{a^6}{x^2}-\frac{3a^5}{x}+3a^3x+3a^2x^2-ax^3\right.\right.}{a^4 d}$$

$$= -\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\log(\sin(c+dx))}{d}$$

Mathematica [A]

time = 0.09, size = 100, normalized size = 0.87

$$-\frac{a\csc(c+dx)}{d}-\frac{3a\sin(c+dx)}{d}+\frac{a\sin^3(c+dx)}{d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a(2\csc^2(c+dx)+12\log(\sin(c+dx))-6\sin^2(c+dx)+\sin^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (3*a*Sin[c + d*x])/d + (a*Sin[c + d*x]^3)/d - (a*Si
n[c + d*x]^5)/(5*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Sin[c
 + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

Maple [A]

time = 0.18, size = 125, normalized size = 1.09

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^6(dx+c)\right)}{2}-\frac{3\left(\cos^4(dx+c)\right)}{4}-\frac{3\left(\cos^2(dx+c)\right)}{2}-3\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)\right.\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^6(dx+c)\right)}{2}-\frac{3\left(\cos^4(dx+c)\right)}{4}-\frac{3\left(\cos^2(dx+c)\right)}{2}-3\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)\right.\right.}{d}$
risch	$3iax+\frac{3ia\,e^{3i(dx+c)}}{32d}-\frac{5a\,e^{2i(dx+c)}}{16d}+\frac{19ia\,e^{i(dx+c)}}{16d}-\frac{19ia\,e^{-i(dx+c)}}{16d}-\frac{5a\,e^{-2i(dx+c)}}{16d}-\frac{3ia\,e^{-3i(dx+c)}}{32d}+6a$
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{9a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{47a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{182a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{47a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{9a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^8-1/2*cos(d*x+c)^6-3/4*cos(d*x+c)^4-3/
2*cos(d*x+c)^2-3*ln(sin(d*x+c)))+a*(-1/sin(d*x+c)*cos(d*x+c)^8-(16/5+cos(d*
x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.78

$$-\frac{4\,a\sin\left(dx+c\right)^{5}+5\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{10\,(2\,a\sin(dx+c)+a)}{\sin(dx+c)^{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + 10*(2*a*sin(
d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.39, size = 124, normalized size = 1.08

$$-\frac{40\,a\cos\left(dx+c\right)^{6}+120\,a\cos\left(dx+c\right)^{4}-255\,a\cos\left(dx+c\right)^{2}+480\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+32\,\left(a\cos\left(dx+c\right)^{6}+2\,a\cos\left(dx+c\right)^{4}+8\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)+15\,a}{160\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/160*(40*a*cos(d*x + c)^6 + 120*a*cos(d*x + c)^4 - 255*a*cos(d*x + c)^2 +
 480*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 32*(a*cos(d*x + c)^6 +
2*a*cos(d*x + c)^4 + 8*a*cos(d*x + c)^2 - 16*a)*sin(d*x + c) + 15*a)/(d*cos
(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.48, size = 104, normalized size = 0.90

$$-\frac{4\,a\sin\left(dx+c\right)^{5}+5\,a\sin\left(dx+c\right)^{4}-20\,a\sin\left(dx+c\right)^{3}-30\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)-\frac{10\,\left(9\,a\sin(dx+c)^{2}-2\,a\sin(dx+c)-a\right)}{\sin(dx+c)^{2}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(4*a*sin(d*x + c)^5 + 5*a*sin(d*x + c)^4 - 20*a*sin(d*x + c)^3 - 30*a
*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - 10*(9*a
*sin(d*x + c)^2 - 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 9.17, size = 311, normalized size = 2.70

$$\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{26\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{47\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+74\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-\frac{107\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+\frac{628\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}-51\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+84\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+34\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a/2 + 2*a*tan(c/2 + (d*x)/2) + (5*
a*tan(c/2 + (d*x)/2)^2)/2 + 34*a*tan(c/2 + (d*x)/2)^3 - 19*a*tan(c/2 + (d*x
)/2)^4 + 84*a*tan(c/2 + (d*x)/2)^5 - 51*a*tan(c/2 + (d*x)/2)^6 + (628*a*tan
(c/2 + (d*x)/2)^7)/5 - (107*a*tan(c/2 + (d*x)/2)^8)/2 + 74*a*tan(c/2 + (d*x
)/2)^9 - (47*a*tan(c/2 + (d*x)/2)^10)/2 + 26*a*tan(c/2 + (d*x)/2)^11)/(d*(4
*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 +
 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^
12)) - (a*tan(c/2 + (d*x)/2))/(2*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a
*log(tan(c/2 + (d*x)/2)))/d
```

3.665 $\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{3a\log(\sin(c+dx))}{d}+\frac{3a\sin(c+dx)}{d}+\frac{3a\sin^2(c+dx)}{2d}-a\,\mathrm{si}$$

```
[Out] 3*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-3*a*ln(sin(d*x+c
))/d+3*a*sin(d*x+c)/d+3/2*a*sin(d*x+c)^2/d-1/3*a*sin(d*x+c)^3/d-1/4*a*sin(d
*x+c)^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^3(c+dx)}{3d}+\frac{3a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}-\frac{3a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x])/d + (3*a*Sin[c + d*x]^2)/(
2*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^4(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3 d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^3 + \frac{a^7}{x^4} + \frac{a^6}{x^3} - \frac{3a^5}{x^2} - \frac{3a^4}{x} + 3a^2x - ax^2 - x^3\right.\right.}{a^3 d}$$

$$= \frac{3a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\csc^3(c+dx)}{3d} - \frac{3a\,l}{}$$

Mathematica [A]

time = 0.15, size = 103, normalized size = 0.87

$$\frac{3a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} + \frac{3a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a(2\csc^2(c+dx) + 12\log(\sin(c+dx)) - 6\sin^2(c+dx) + \sin^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) - (a*(2*Csc[c + d*x]^2 + 12*Log[Sin[c + d*x]] - 6*Si
n[c + d*x]^2 + Sin[c + d*x]^4))/(4*d)
```

Maple [A]

time = 0.16, size = 143, normalized size = 1.21

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{3\sin(dx+c)^3}+\frac{5\left(\cos^8(dx+c)\right)}{3\sin(dx+c)}+\frac{5\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}-\frac{(\mathrm{cc}}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{3\sin(dx+c)^3}+\frac{5\left(\cos^8(dx+c)\right)}{3\sin(dx+c)}+\frac{5\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{3}\right)+a\left(-\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}-\frac{(\mathrm{cc}}{}\right.}{d}$
risch	$3iax-\frac{a\,e^{4i(dx+c)}}{64d}-\frac{ia\,e^{3i(dx+c)}}{24d}-\frac{5a\,e^{2i(dx+c)}}{16d}-\frac{11ia\,e^{i(dx+c)}}{8d}+\frac{11ia\,e^{-i(dx+c)}}{8d}-\frac{5a\,e^{-2i(dx+c)}}{16d}+\frac{ia\,e^{-3}}{2}$
norman	$\frac{-\frac{a}{24d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{29a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{101a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{231a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{231a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{101a\left(\tan^{10}\left(\frac{d}{}\right.\right.}{8d}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^8+5/3/sin(d*x+c)*cos(d*x+c)^8+5/3*(16/
5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/2/sin(d
*x+c)^2*cos(d*x+c)^8-1/2*cos(d*x+c)^6-3/4*cos(d*x+c)^4-3/2*cos(d*x+c)^2-3*l
n(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.78

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-18\,a\sin\left(dx+c\right)^{2}+36\,a\log\left(\sin\left(dx+c\right)\right)-36\,a\sin\left(dx+c\right)-\frac{2\,\left(18\,a\sin\left(dx+c\right)^{2}-3\,a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(sin(d*x + c)) - 36*a*sin(d*x + c) - 2*(18*a*sin(d*x + c)^2 - 3*a*sin(d
*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.41, size = 139, normalized size = 1.18

$$-\frac{32\,a\cos\left(dx+c\right)^{6}+192\,a\cos\left(dx+c\right)^{4}-768\,a\cos\left(dx+c\right)^{2}+288\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\left(8\,a\cos\left(dx+c\right)^{6}+24\,a\cos\left(dx+c\right)^{4}-51\,a\cos\left(dx+c\right)^{2}+3\,a\right)\sin\left(dx+c\right)+512\,a}{96\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^6 + 192*a*cos(d*x + c)^4 - 768*a*cos(d*x + c)^2 +
288*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(8*a*cos(
d*x + c)^6 + 24*a*cos(d*x + c)^4 - 51*a*cos(d*x + c)^2 + 3*a)*sin(d*x + c)
+ 512*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.48, size = 104, normalized size = 0.88

$$-\frac{3\,a\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-18\,a\sin\left(dx+c\right)^{2}+36\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-36\,a\sin\left(dx+c\right)-\frac{2\,\left(33\,a\sin\left(dx+c\right)^{3}+18\,a\sin\left(dx+c\right)^{2}-3\,a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*a*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 36*a
*log(abs(sin(d*x + c))) - 36*a*sin(d*x + c) - 2*(33*a*sin(d*x + c)^3 + 18*a
*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 9.07, size = 300, normalized size = 2.54

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+47\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{499\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+60\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{562\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+42\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+90\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(
tan(c/2 + (d*x)/2)))/d + ((29*a*tan(c/2 + (d*x)/2)^2)/3 - a*tan(c/2 + (d*x)
/2) - a/3 - 4*a*tan(c/2 + (d*x)/2)^3 + 90*a*tan(c/2 + (d*x)/2)^4 + 42*a*tan
(c/2 + (d*x)/2)^5 + (562*a*tan(c/2 + (d*x)/2)^6)/3 + 60*a*tan(c/2 + (d*x)/2
)^7 + (499*a*tan(c/2 + (d*x)/2)^8)/3 + 47*a*tan(c/2 + (d*x)/2)^9 + 59*a*tan
(c/2 + (d*x)/2)^10)/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2 + (d*x)/2)^5 +
48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(c/2 + (d*x)/2)^11
))
```

3.666 $\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{3a\log(\sin(c+dx))}{d}+\frac{3a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] 3*a*csc(d*x+c)/d+3/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)
^4/d+3*a*ln(sin(d*x+c))/d+3*a*sin(d*x+c)/d-1/2*a*sin(d*x+c)^2/d-1/3*a*sin(d
*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin^2(c+dx)}{2d}+\frac{3a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{3a\csc(c+dx)}{d}+\frac{3a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d
) - (a*Csc[c + d*x]^4)/(4*d) + (3*a*Log[Sin[c + d*x]])/d + (3*a*Sin[c + d*x
])/d - (a*Sin[c + d*x]^2)/(2*d) - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^7}{x^5}+\frac{a^6}{x^4}-\frac{3a^5}{x^3}-\frac{3a^4}{x^2}+\frac{3a^3}{x}-ax-x^2\right)\right.}{a^2 d}$$
$$= \frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a}{}$$

Mathematica [A]

time = 0.37, size = 105, normalized size = 0.89

$$\frac{3a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{3a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}+\frac{a(6\csc^2(c+dx)-\csc^4(c+dx)+12\log(\sin(c+dx))-2\sin^2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (3*a*Sin[c + d*x])/d - (a
*Sin[c + d*x]^3)/(3*d) + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin
[c + d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

Maple [A]

time = 0.16, size = 161, normalized size = 1.36

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{\left(\cos^6(dx+c)\right)}{2}+\frac{3\left(\cos^4(dx+c)\right)}{4}+\frac{3\left(\cos^2(dx+c)\right)}{2}+3\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{3\sin(dx+c)^3}+\frac{5(}{3}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{\left(\cos^6(dx+c)\right)}{2}+\frac{3\left(\cos^4(dx+c)\right)}{4}+\frac{3\left(\cos^2(dx+c)\right)}{2}+3\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{3\sin(dx+c)^3}+\frac{5(}{3}\right.}{d}$
risch	$-3iax-\frac{ia\,e^{3i(dx+c)}}{24d}+\frac{a\,e^{2i(dx+c)}}{8d}-\frac{11ia\,e^{i(dx+c)}}{8d}+\frac{11ia\,e^{-i(dx+c)}}{8d}+\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{ia\,e^{-3i(dx+c)}}{24d}-\frac{6ia}{d}$
norman	$\frac{-\frac{a}{64d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{17a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{91a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{35a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{91a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^8+1/2/sin(d*x+c)^2*cos(d*x+c)^8+1/2*cos(d*x+c)^6+3/4*cos(d*x+c)^4+3/2*cos(d*x+c)^2+3*ln(sin(d*x+c)))+a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^8+5/3/sin(d*x+c)*cos(d*x+c)^8+5/3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))

Maxima [A]
time = 0.28, size = 92, normalized size = 0.78

$$-\frac{4\,a\sin\left(dx+c\right)^{3}+6\,a\sin\left(dx+c\right)^{2}-36\,a\log\left(\sin\left(dx+c\right)\right)-36\,a\sin\left(dx+c\right)-\frac{36\,a\sin(dx+c)^{3}+18\,a\sin(dx+c)^{2}-4\,a\sin(dx+c)-3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(sin(d*x + c)) - 36*a*sin(d*x + c) - (36*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d

Fricas [A]
time = 0.38, size = 142, normalized size = 1.20

$$\frac{6\,a\cos\left(dx+c\right)^{6}-15\,a\cos\left(dx+c\right)^{4}-6\,a\cos\left(dx+c\right)^{2}+36\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+4\,\left(a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-24\,a\cos\left(dx+c\right)^{2}+16\,a\right)\sin\left(dx+c\right)+12\,a}{12\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/12*(6*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 - 6*a*cos(d*x + c)^2 + 36*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c)) + 4*(a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 24*a*cos(d*x + c)^2 + 16*a)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5*(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.52, size = 103, normalized size = 0.87

$$-\frac{4\,a\sin\left(dx+c\right)^{3}+6\,a\sin\left(dx+c\right)^{2}-36\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-36\,a\sin\left(dx+c\right)+\frac{75\,a\sin(dx+c)^{4}-36\,a\sin(dx+c)^{3}-18\,a\sin(dx+c)^{2}+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(4*a*sin(d*x + c)^3 + 6*a*sin(d*x + c)^2 - 36*a*log(abs(sin(d*x + c))
) - 36*a*sin(d*x + c) + (75*a*sin(d*x + c)^4 - 36*a*sin(d*x + c)^3 - 18*a*s
in(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 9.01, size = 290, normalized size = 2.46

$$\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{118\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-27\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{644\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{69\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{4}+160\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{57\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{4}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+48\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (11*a*tan(c/2 + (d*x)/2))/(8*d) + ((17*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan
(c/2 + (d*x)/2))/3 - a/4 + 20*a*tan(c/2 + (d*x)/2)^3 + (57*a*tan(c/2 + (d*x
)/2)^4)/4 + 160*a*tan(c/2 + (d*x)/2)^5 - (69*a*tan(c/2 + (d*x)/2)^6)/4 + (6
44*a*tan(c/2 + (d*x)/2)^7)/3 - 27*a*tan(c/2 + (d*x)/2)^8 + 118*a*tan(c/2 +
(d*x)/2)^9)/(d*(16*tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(
c/2 + (d*x)/2)^8 + 16*tan(c/2 + (d*x)/2)^10)) - (3*a*log(tan(c/2 + (d*x)/2)
^2 + 1))/d + (5*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(
24*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) + (3*a*log(tan(c/2 + (d*x)/2)))/d
```

3.667 $\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\log(\sin(c+dx))}{d}-$$

```
[Out] -3*a*csc(d*x+c)/d+3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d+3*a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*sin(d*x+c)
^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}+\frac{3a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d -
 (a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (3*a*Log[Sin[c + d*x
]])/d - (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^7}{x^6}+\frac{a^6}{x^5}-\frac{3a^5}{x^4}-\frac{3a^4}{x^3}+\frac{3a^3}{x^2}+\frac{3a^2}{x}-x\right)\,dx\right.}{ad}$$
$$= -\frac{3a\csc(c+dx)}{d}+\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}-$$

Mathematica [A]
time = 0.15, size = 102, normalized size = 0.89

$$-\frac{3a\csc(c+dx)}{d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\sin(c+dx)}{d}+\frac{a(6\csc^2(c+dx)-\csc^4(c+dx)+12\log(\sin(c+dx))-2\sin^2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*Sin[c + d*x])/d + (a*(6*Csc[c + d*x]^2 - Csc[c + d*x]^4 + 12*Log[Sin[c +
d*x]] - 2*Sin[c + d*x]^2))/(4*d)
```

Maple [A]
time = 0.18, size = 179, normalized size = 1.56

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{\cos^8}{4\sin}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{\cos^8}{4\sin}\right.}{d}$
risch	$-3iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{a\,e^{-2i(dx+c)}}{8d}-\frac{6iac}{d}-\frac{2ia\left(15\,e^{9i(dx+c)}-40\,e^{7i(dx+c)}-15\right.}{}$
norman	$\frac{-\frac{a}{160d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{13a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{9a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{161a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{175a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{175a\left(\tan^8\left(\frac{dx}{2}\right.\right.}{32d}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin(
d*x+c)*cos(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*s
```

```
in(d*x+c))+a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^8+1/2/sin(d*x+c)^2*cos(d*x+c)^8+
1/2*cos(d*x+c)^6+3/4*cos(d*x+c)^4+3/2*cos(d*x+c)^2+3*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.29, size = 91, normalized size = 0.79

$$-\frac{10\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\sin\left(dx+c\right)\right)+20\,a\sin\left(dx+c\right)+\frac{60\,a\sin(dx+c)^{4}-30\,a\sin(dx+c)^{3}-20\,a\sin(dx+c)^{2}+5\,a\sin(dx+c)+4\,a}{\sin(dx+c)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(sin(d*x + c)) + 20*a*sin(d*x + c) + (
60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*a*sin(d*x + c)^2 + 5*a*sin(d
*x + c) + 4*a)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.39, size = 157, normalized size = 1.37

$$\frac{20\,a\cos\left(dx+c\right)^{6}-120\,a\cos\left(dx+c\right)^{4}+160\,a\cos\left(dx+c\right)^{2}+60\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,\left(2\,a\cos\left(dx+c\right)^{6}-5\,a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+4\,a\right)\sin\left(dx+c\right)-64\,a}{20\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/20*(20*a*cos(d*x + c)^6 - 120*a*cos(d*x + c)^4 + 160*a*cos(d*x + c)^2 + 6
0*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*sin(d*x + c))*sin(d*x
 + c) + 5*(2*a*cos(d*x + c)^6 - 5*a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + 4
*a)*sin(d*x + c) - 64*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d
*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 103, normalized size = 0.90

$$-\frac{10\,a\sin\left(dx+c\right)^{2}-60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+20\,a\sin\left(dx+c\right)+\frac{137\,a\sin(dx+c)^{5}+60\,a\sin(dx+c)^{4}-30\,a\sin(dx+c)^{3}-20\,a\sin(dx+c)^{2}+5\,a\sin(dx+c)+4\,a}{\sin(dx+c)^{5}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/20*(10*a*sin(d*x + c)^2 - 60*a*log(abs(sin(d*x + c))) + 20*a*sin(d*x + c
) + (137*a*sin(d*x + c)^5 + 60*a*sin(d*x + c)^4 - 30*a*sin(d*x + c)^3 - 20*
a*sin(d*x + c)^2 + 5*a*sin(d*x + c) + 4*a)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 9.10, size = 281, normalized size = 2.44

$$\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{102\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+54\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+137\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{39\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\frac{161\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}-9\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{13\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}}{d\,\left(32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (5*a*tan(c/2 + (d*x)/2)^2)/(16*d) - (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d -
 (19*a*tan(c/2 + (d*x)/2))/(16*d) + (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (a*
tan(c/2 + (d*x)/2)^4)/(64*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*a*log(
tan(c/2 + (d*x)/2)))/d - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (13*a*tan(c/2 +
(d*x)/2)^2)/5 - 9*a*tan(c/2 + (d*x)/2)^3 + (161*a*tan(c/2 + (d*x)/2)^4)/5 -
 (39*a*tan(c/2 + (d*x)/2)^5)/2 + 137*a*tan(c/2 + (d*x)/2)^6 + 54*a*tan(c/2
+ (d*x)/2)^7 + 102*a*tan(c/2 + (d*x)/2)^8)/(d*(32*tan(c/2 + (d*x)/2)^5 + 64
*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9))
```

3.668 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\log\ldots$$

```
[Out] -3*a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/
d-1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)
/d
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 90}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c\right.}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+d}{4d}$$

Mathematica [A]
time = 0.28, size = 111, normalized size = 0.97

$$-\frac{3a\csc(c+dx)}{d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d}-\frac{a\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

Maple [A]
time = 0.20, size = 143, normalized size = 1.24

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^6(dx+c)\right)}{6}+\frac{\left(\cot^4(dx+c)\right)}{4}-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\text{co}\right.\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cot^6(dx+c)\right)}{6}+\frac{\left(\cot^4(dx+c)\right)}{4}-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\text{co}\right.\right.}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(45ie^{10i(dx+c)}+45\,e^{11i(dx+c)}-90ie^{8i(dx+c)}-165\,e^{9i(dx+c)}+170ie^{6i}\right.}{}$
norman	$\frac{-\frac{a}{384d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{7a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{25a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{35a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{35a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))
+a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin(d*x+
c)*cos(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d
*x+c)))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.79

$$-\frac{60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{180\,a\sin(dx+c)^5+90\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-45\,a\sin(dx+c)^2+12\,a\sin(dx+c)+10\,a}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.40, size = 158, normalized size = 1.37

$$\frac{90\,a\cos\left(dx+c\right)^4-135\,a\cos\left(dx+c\right)^2-60\,\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-12\,\left(5\,a\cos\left(dx+c\right)^6-30\,a\cos\left(dx+c\right)^4+40\,a\cos\left(dx+c\right)^2-16\,a\right)\sin\left(dx+c\right)+55\,a}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 104, normalized size = 0.90

$$-\frac{60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)-\frac{147\,a\sin(dx+c)^6-180\,a\sin(dx+c)^5-90\,a\sin(dx+c)^4+60\,a\sin(dx+c)^3+45\,a\sin(dx+c)^2-12\,a\sin(dx+c)-10\,a}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

Mupad [B]
time = 10.12, size = 267, normalized size = 2.32

$$\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,d}-\frac{29\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{19\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a\,\left(1920\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-1920\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{1920\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{51\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{16}+\frac{29\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{128}+\frac{35\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{32}+\frac{25\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{128}-\frac{7\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{80}-\frac{11\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{384}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{160}+\frac{a}{384}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (
19*a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) - (a*tan(
c/2 + (d*x)/2)^5)/(160*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*(1920*log
(tan(c/2 + (d*x)/2)) - 1920*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) - (cot
(c/2 + (d*x)/2)^6*(a/384 + (a*tan(c/2 + (d*x)/2))/160 - (11*a*tan(c/2 + (d*
x)/2)^2)/384 - (7*a*tan(c/2 + (d*x)/2)^3)/80 + (25*a*tan(c/2 + (d*x)/2)^4)/
128 + (35*a*tan(c/2 + (d*x)/2)^5)/32 + (29*a*tan(c/2 + (d*x)/2)^6)/128 + (5
1*a*tan(c/2 + (d*x)/2)^7)/16))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.669 $\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^7(\cdots}{7\cdots}$$

```
[Out] a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d-a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4/d+3
/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d-a*ln(sin(d*x+
c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^6(c+dx)}{6d}+\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/d + (3
*a*Csc[c + d*x]^4)/(4*d) + (3*a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)/
(6*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^8(a-x)^3(a+x)^4}{x^8}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^8}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a^7}{x^8}+\frac{a^6}{x^7}-\frac{3a^5}{x^6}-\frac{3a^4}{x^5}+\frac{3a^3}{x^4}+\frac{3a^2}{x^3}-\frac{a}{x^2}-\frac{1}{x}\right) d\right.}{d}$$
$$= \frac{a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\,\text{cs}}{}$$

Mathematica [A]

time = 0.28, size = 115, normalized size = 0.97

$$\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*a*Csc[c + d*x]^5)/(5*d) - (a
*Csc[c + d*x]^7)/(7*d) - (a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c
+ d*x]^6 + 12*Log[Cos[c + d*x]] + 12*Log[Tan[c + d*x]]))/(12*d)
```

Maple [A]

time = 0.22, size = 161, normalized size = 1.35

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}\right.}{d}$
risch	$iax+\frac{2iac}{d}+\frac{2ia\left(105\,\mathrm{e}^{13i(dx+c)}-210\,\mathrm{e}^{11i(dx+c)}-315i\mathrm{e}^{12i(dx+c)}+903\,\mathrm{e}^{9i(dx+c)}+945i\mathrm{e}^{10i(dx+c)}-636\,\mathrm{e}^{7i(dx+c)}-1820\right.}{105d\left(\mathrm{e}^{2i(\right.}}$
norman	$-\frac{a}{896d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{384d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1120d}+\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{7a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{25a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}+\frac{7a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{32d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/35/sin(d*x+c)^5*cos(d*x+c)^8-1/35/
sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*cos(d*x+c)^8+1/7*(16/5+cos(d*x+c)^
6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/6*cot(d*x+c)^6+1/4*c
ot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 94, normalized size = 0.79

$$-\frac{420\,a\log\left(\sin\left(dx+c\right)\right)-\frac{420\,a\sin\left(dx+c\right)^{6}-630\,a\sin\left(dx+c\right)^{5}-420\,a\sin\left(dx+c\right)^{4}+315\,a\sin\left(dx+c\right)^{3}+252\,a\sin\left(dx+c\right)^{2}-70\,a\sin\left(dx+c\right)-60\,a}{\sin\left(dx+c\right)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/420*(420*a*log(sin(d*x + c)) - (420*a*sin(d*x + c)^6 - 630*a*sin(d*x + c
)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c)^3 + 252*a*sin(d*x + c)^2 -
70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

Fricas [A]
time = 0.39, size = 172, normalized size = 1.45

$$\frac{420\,a\cos\left(dx+c\right)^{6}-840\,a\cos\left(dx+c\right)^{4}+672\,a\cos\left(dx+c\right)^{2}-420\,\left(a\cos\left(dx+c\right)^{6}-3\,a\cos\left(dx+c\right)^{4}+3\,a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+35\,\left(18\,a\cos\left(dx+c\right)^{4}-27\,a\cos\left(dx+c\right)^{2}+11\,a\right)\sin\left(dx+c\right)-192\,a}{420\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(420*a*cos(d*x + c)^6 - 840*a*cos(d*x + c)^4 + 672*a*cos(d*x + c)^2 -
 420*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1
/2*sin(d*x + c))*sin(d*x + c) + 35*(18*a*cos(d*x + c)^4 - 27*a*cos(d*x + c)
^2 + 11*a)*sin(d*x + c) - 192*a)/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 +
3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 106, normalized size = 0.89

$$-\frac{420\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{1089\,a\sin\left(dx+c\right)^{7}+420\,a\sin\left(dx+c\right)^{6}-630\,a\sin\left(dx+c\right)^{5}-420\,a\sin\left(dx+c\right)^{4}+315\,a\sin\left(dx+c\right)^{3}+252\,a\sin\left(dx+c\right)^{2}-70\,a\sin\left(dx+c\right)-60\,a}{\sin\left(dx+c\right)^{7}}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/420*(420*a*log(abs(sin(d*x + c))) - (1089*a*sin(d*x + c)^7 + 420*a*sin(d
*x + c)^6 - 630*a*sin(d*x + c)^5 - 420*a*sin(d*x + c)^4 + 315*a*sin(d*x + c
)^3 + 252*a*sin(d*x + c)^2 - 70*a*sin(d*x + c) - 60*a)/sin(d*x + c)^7)/d
```

Mupad [B]
time = 9.32, size = 270, normalized size = 2.27

$$\frac{35\,a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}+\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{29\,a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{128\,d}-\frac{7\,a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{128\,d}+\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32\,d}+\frac{7\,a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{640\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{384\,d}-\frac{a\,{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{896\,d}-\frac{29\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{128\,d}-\frac{7\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{128\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{32\,d}+\frac{7\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{640\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{384\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{896\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (35*a*cot(c/2 + (d*x)/2))/(128*d) + (35*a*tan(c/2 + (d*x)/2))/(128*d) + (a*
log(tan(c/2 + (d*x)/2)^2 + 1))/d - (29*a*cot(c/2 + (d*x)/2)^2)/(128*d) - (7
*a*cot(c/2 + (d*x)/2)^3)/(128*d) + (a*cot(c/2 + (d*x)/2)^4)/(32*d) + (7*a*c
ot(c/2 + (d*x)/2)^5)/(640*d) - (a*cot(c/2 + (d*x)/2)^6)/(384*d) - (a*cot(c/
2 + (d*x)/2)^7)/(896*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) - (7*a*tan(c/
2 + (d*x)/2)^3)/(128*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) + (7*a*tan(c/2 +
(d*x)/2)^5)/(640*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*tan(c/2 + (d*x)
/2)^7)/(896*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.670 $\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{a\cot^8(c+dx)}{8d}+\frac{a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d+a*csc(d*x+c)/d-a*csc(d*x+c)^3/d+3/5*a*csc(d*x+c)^5/d-
1/7*a*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2913, 2687, 30, 2686, 200}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^7(c+dx)}{7d}+\frac{3a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{d}+\frac{a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc(c+dx)\,dx + a\int \cot^7(c+dx)\csc^2$$

$$= -\frac{a\text{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (-1+\right.}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\text{Subst}\left(\int (-1+3x^2-3x^4+x^6)\,dx\right.}{d}$$

$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a}{}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$-\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{d} + \frac{3a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/d + (3*
a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(68) = 136.
time = 0.22, size = 138, normalized size = 1.86

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8\sin(dx+c)^8}+a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(\right.}{5}\right.}{7}\right.}{d}$

default	$\frac{-\frac{a\left(\cos^8(dx+c)\right)}{8\sin(dx+c)^8}+a\left(-\frac{\cos^8(dx+c)}{7\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{35\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{35\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{7\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2\right.}{}\right.}{7}\right.}{d}$
risch	$\frac{2ia\left(35i\,e^{14i(dx+c)}+35\,e^{15i(dx+c)}-105\,e^{13i(dx+c)}+245i\,e^{10i(dx+c)}+371\,e^{11i(dx+c)}-513\,e^{9i(dx+c)}+245i\,e^{6i(dx+c)}+513\,e^{7}\right.}{35d\left(e^{2i(dx+c)}-1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/8*a/sin(d*x+c)^8*cos(d*x+c)^8+a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^8+1/3
5/sin(d*x+c)^5*cos(d*x+c)^8-1/35/sin(d*x+c)^3*cos(d*x+c)^8+1/7/sin(d*x+c)*c
os(d*x+c)^8+1/7*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d
*x+c)))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.24

$$\frac{280\,a\sin(dx+c)^7+140\,a\sin(dx+c)^6-280\,a\sin(dx+c)^5-210\,a\sin(dx+c)^4+168\,a\sin(dx+c)^3+140\,a\sin(dx+c)^2-40\,a\sin(dx+c)-35\,a}{280\,d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.37, size = 131, normalized size = 1.77

$$-\frac{140\,a\cos(dx+c)^6-210\,a\cos(dx+c)^4+140\,a\cos(dx+c)^2+8\left(35\,a\cos(dx+c)^6-70\,a\cos(dx+c)^4+56\,a\cos(dx+c)^2-16\,a\right)\sin(dx+c)-35\,a}{280\left(d\cos(dx+c)^8-4\,d\cos(dx+c)^6+6\,d\cos(dx+c)^4-4\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*a*cos(d*x + c)^6 - 210*a*cos(d*x + c)^4 + 140*a*cos(d*x + c)^2
+ 8*(35*a*cos(d*x + c)^6 - 70*a*cos(d*x + c)^4 + 56*a*cos(d*x + c)^2 - 16*a
)*sin(d*x + c) - 35*a)/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 92, normalized size = 1.24

$$\frac{280\,a\sin\left(dx+c\right)^{7}+140\,a\sin\left(dx+c\right)^{6}-280\,a\sin\left(dx+c\right)^{5}-210\,a\sin\left(dx+c\right)^{4}+168\,a\sin\left(dx+c\right)^{3}+140\,a\sin\left(dx+c\right)^{2}-40\,a\sin\left(dx+c\right)-35\,a}{280\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/280*(280*a*sin(d*x + c)^7 + 140*a*sin(d*x + c)^6 - 280*a*sin(d*x + c)^5 -
 210*a*sin(d*x + c)^4 + 168*a*sin(d*x + c)^3 + 140*a*sin(d*x + c)^2 - 40*a*
sin(d*x + c) - 35*a)/(d*sin(d*x + c)^8)
```

Mupad [B]
time = 9.39, size = 91, normalized size = 1.23

$$-\frac{-a\sin\left(c+d\,x\right)^{7}-\frac{a\sin\left(c+d\,x\right)^{6}}{2}+a\sin\left(c+d\,x\right)^{5}+\frac{3\,a\sin\left(c+d\,x\right)^{4}}{4}-\frac{3\,a\sin\left(c+d\,x\right)^{3}}{5}-\frac{a\sin\left(c+d\,x\right)^{2}}{2}+\frac{a\,\sin\left(c+d\,x\right)}{7}+\frac{a}{8}}{d\sin\left(c+d\,x\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(a/8 + (a*sin(c + d*x))/7 - (a*sin(c + d*x)^2)/2 - (3*a*sin(c + d*x)^3)/5
+ (3*a*sin(c + d*x)^4)/4 + a*sin(c + d*x)^5 - (a*sin(c + d*x)^6)/2 - a*sin(
c + d*x)^7)/(d*sin(c + d*x)^8)
```

3.671 $\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^8(c+dx)}{8d}+\frac{a\csc^3(c+dx)}{3d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d+1/3*a*csc(d*x+c)^3/d-3/5*a*csc(d*x+c)^5/d+3/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2913, 2686, 276, 2687, 30}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^3(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^2(c+dx)\,dx + a\int \cot^7(c+dx)\operatorname{csc}$$

$$= -\frac{a\text{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int x^2(-1\right.}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\text{Subst}\left(\int\left(-x^2+3x^4-3x^6+x^8\right)d\right.}{d}$$

$$= -\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc^5(c+dx)}{5d} + $$

Mathematica [A]
time = 0.05, size = 81, normalized size = 1.00

$$-\frac{a\cot^8(c+dx)}{8d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3a\csc^5(c+dx)}{5d} + \frac{3a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d + (a*Csc[c + d*x]^3)/(3*d) - (3*a*Csc[c + d*x]^5)
/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(71) = 142$.
time = 0.21, size = 156, normalized size = 1.93

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{63\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{315\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{315\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2\right.}{}\right.}{63}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{63\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{315\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{315\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{63\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2\right.}{}\right.}{63}\right.}{d}$
risch	$-\frac{2a\left(420ie^{15i(dx+c)}+315\,e^{16i(dx+c)}+504ie^{13i(dx+c)}-315\,e^{14i(dx+c)}+2844ie^{11i(dx+c)}+2205\,e^{12i(dx+c)}+1424ie^{9i(dx+c)}\right.}{315}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x+c)^8+1/315
/sin(d*x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*
cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))-1/8*a/sin(d*x+c)^8*cos(d*x+c)^8)
```

Maxima [A]
time = 0.28, size = 92, normalized size = 1.14

$$\frac{1260\,a\sin\left(dx+c\right)^7+840\,a\sin\left(dx+c\right)^6-1890\,a\sin\left(dx+c\right)^5-1512\,a\sin\left(dx+c\right)^4+1260\,a\sin\left(dx+c\right)^3+1080\,a\sin\left(dx+c\right)^2-315\,a\sin\left(dx+c\right)-280\,a}{2520\,d\sin\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A]
time = 0.37, size = 139, normalized size = 1.72

$$-\frac{840\,a\cos\left(dx+c\right)^6-1008\,a\cos\left(dx+c\right)^4+576\,a\cos\left(dx+c\right)^2+315\left(4\,a\cos\left(dx+c\right)^6-6\,a\cos\left(dx+c\right)^4+4\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-128\,a}{2520\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(840*a*cos(d*x + c)^6 - 1008*a*cos(d*x + c)^4 + 576*a*cos(d*x + c)^
2 + 315*(4*a*cos(d*x + c)^6 - 6*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 - a)*
```

```
sin(d*x + c) - 128*a)/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
 + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.54, size = 92, normalized size = 1.14

$$\frac{1260\,a\sin\left(dx+c\right)^{7}+840\,a\sin\left(dx+c\right)^{6}-1890\,a\sin\left(dx+c\right)^{5}-1512\,a\sin\left(dx+c\right)^{4}+1260\,a\sin\left(dx+c\right)^{3}+1080\,a\sin\left(dx+c\right)^{2}-315\,a\sin\left(dx+c\right)-280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2520*(1260*a*sin(d*x + c)^7 + 840*a*sin(d*x + c)^6 - 1890*a*sin(d*x + c)^
5 - 1512*a*sin(d*x + c)^4 + 1260*a*sin(d*x + c)^3 + 1080*a*sin(d*x + c)^2 -
 315*a*sin(d*x + c) - 280*a)/(d*sin(d*x + c)^9)
```

Mupad [B]
time = 9.23, size = 92, normalized size = 1.14

$$-\frac{-\frac{a\sin(c+d\,x)^7}{2}-\frac{a\sin(c+d\,x)^6}{3}+\frac{3\,a\sin(c+d\,x)^5}{4}+\frac{3\,a\sin(c+d\,x)^4}{5}-\frac{a\sin(c+d\,x)^3}{2}-\frac{3\,a\sin(c+d\,x)^2}{7}+\frac{a\,\sin(c+d\,x)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^10,x)
[Out] -(a/9 + (a*sin(c + d*x))/8 - (3*a*sin(c + d*x)^2)/7 - (a*sin(c + d*x)^3)/2
+ (3*a*sin(c + d*x)^4)/5 + (3*a*sin(c + d*x)^5)/4 - (a*sin(c + d*x)^6)/3 -
(a*sin(c + d*x)^7)/2)/(d*sin(c + d*x)^9)
```

3.672 $\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^{10}(c+dx)}{10d}+\frac{a\csc^3(c+dx)}{3d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/10*a*cot(d*x+c)^10/d+1/3*a*csc(d*x+c)^3/d-3/5*a*csc
(d*x+c)^5/d+3/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 14, 2686, 276}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{9d}+\frac{3a\csc^7(c+dx)}{7d}-\frac{3a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^3)
/(3*d) - (3*a*Csc[c + d*x]^5)/(5*d) + (3*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c
 + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^7(c+dx)\csc^4(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^7(c+dx)\csc^3(c+dx)\,dx + a\int \cot^7(c+dx)\,\mathrm{csc} \\
&= -\frac{a\text{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}}{} \\
&= -\frac{a\text{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^3(c+dx)}{3d} - \frac{3}{}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 86, normalized size = 0.89

$$-\frac{a\csc^3(c+dx)\left(-840-630\csc(c+dx)+1512\csc^2(c+dx)+1260\csc^3(c+dx)-1080\csc^4(c+dx)-945\csc^5(c+dx)+280\csc^6(c+dx)+252\csc^7(c+dx)\right)}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2520*(a*Csc[c + d*x]^3*(-840 - 630*Csc[c + d*x] + 1512*Csc[c + d*x]^2 +
1260*Csc[c + d*x]^3 - 1080*Csc[c + d*x]^4 - 945*Csc[c + d*x]^5 + 280*Csc[c
+ d*x]^6 + 252*Csc[c + d*x]^7))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(85) = 170$.
time = 0.24, size = 176, normalized size = 1.81

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{40\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{63\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{315\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{315\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{63\sin(dx+c)}+\frac{\left(\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{40\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{63\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{315\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{315\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{63\sin(dx+c)}+\frac{\left(\right.}{}\right.}{d}$
risch	$-\frac{4ia\left(315i\mathrm{e}^{16i(dx+c)}+210\,\mathrm{e}^{17i(dx+c)}+630i\mathrm{e}^{14i(dx+c)}+42\,\mathrm{e}^{15i(dx+c)}+2205i\mathrm{e}^{12i(dx+c)}+1170\,\mathrm{e}^{13i(dx+c)}+1764i\mathrm{e}^{10i(dx+\right.}{315d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8)+a*
(-1/9/sin(d*x+c)^9*cos(d*x+c)^8-1/63/sin(d*x+c)^7*cos(d*x+c)^8+1/315/sin(d*
x+c)^5*cos(d*x+c)^8-1/315/sin(d*x+c)^3*cos(d*x+c)^8+1/63/sin(d*x+c)*cos(d*x
+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
))
```

Maxima [A]

time = 0.28, size = 92, normalized size = 0.95

$$\frac{840\,a\sin(dx+c)^7+630\,a\sin(dx+c)^6-1512\,a\sin(dx+c)^5-1260\,a\sin(dx+c)^4+1080\,a\sin(dx+c)^3+945\,a\sin(dx+c)^2-280\,a\sin(dx+c)-252\,a}{2520\,d\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

Fricas [A]

time = 0.36, size = 144, normalized size = 1.48

$$\frac{630\,a\cos(dx+c)^6-630\,a\cos(dx+c)^4+315\,a\cos(dx+c)^2+8\left(105\,a\cos(dx+c)^6-126\,a\cos(dx+c)^4+72\,a\cos(dx+c)^2-16\,a\right)\sin(dx+c)-63\,a}{2520\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2520*(630*a*cos(d*x + c)^6 - 630*a*cos(d*x + c)^4 + 315*a*cos(d*x + c)^2
+ 8*(105*a*cos(d*x + c)^6 - 126*a*cos(d*x + c)^4 + 72*a*cos(d*x + c)^2 - 16
```

```
*a)*sin(d*x + c) - 63*a)/(d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11*(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 92, normalized size = 0.95

$$\frac{840\,a\sin\left(dx+c\right)^{7}+630\,a\sin\left(dx+c\right)^{6}-1512\,a\sin\left(dx+c\right)^{5}-1260\,a\sin\left(dx+c\right)^{4}+1080\,a\sin\left(dx+c\right)^{3}+945\,a\sin\left(dx+c\right)^{2}-280\,a\sin\left(dx+c\right)-252\,a}{2520\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2520*(840*a*sin(d*x + c)^7 + 630*a*sin(d*x + c)^6 - 1512*a*sin(d*x + c)^5
 - 1260*a*sin(d*x + c)^4 + 1080*a*sin(d*x + c)^3 + 945*a*sin(d*x + c)^2 - 2
80*a*sin(d*x + c) - 252*a)/(d*sin(d*x + c)^10)
```

Mupad [**B**]
time = 9.21, size = 92, normalized size = 0.95

$$-\frac{-\frac{a\sin(c+d\,x)^7}{3}-\frac{a\sin(c+d\,x)^6}{4}+\frac{3\,a\sin(c+d\,x)^5}{5}+\frac{a\sin(c+d\,x)^4}{2}-\frac{3\,a\sin(c+d\,x)^3}{7}-\frac{3\,a\sin(c+d\,x)^2}{8}+\frac{a\,\sin(c+d\,x)}{9}+\frac{a}{10}}{d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^11,x)
[Out] -(a/10 + (a*sin(c + d*x))/9 - (3*a*sin(c + d*x)^2)/8 - (3*a*sin(c + d*x)^3)
/7 + (a*sin(c + d*x)^4)/2 + (3*a*sin(c + d*x)^5)/5 - (a*sin(c + d*x)^6)/4 -
 (a*sin(c + d*x)^7)/3)/(d*sin(c + d*x)^10)
```

3.673 $\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^{10}(c+dx)}{10d}+\frac{a\csc^5(c+dx)}{5d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^9(c+dx)}{3d}-\frac{a\csc^{11}(c+dx)}{11d}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/10*a*cot(d*x+c)^10/d+1/5*a*csc(d*x+c)^5/d-3/7*a*csc
(d*x+c)^7/d+1/3*a*csc(d*x+c)^9/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2686, 276, 2687, 14}

$$-\frac{a\cot^{10}(c+dx)}{10d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d - (a*Cot[c + d*x]^10)/(10*d) + (a*Csc[c + d*x]^5)
/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^9)/(3*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^7(c+dx)\csc^5(c+dx)(a+a\sin(c+dx))\,dx = a\int \cot^7(c+dx)\csc^4(c+dx)\,dx + a\int \cot^7(c+dx)\operatorname{csc}$$

$$= -\frac{a\text{Subst}\left(\int x^4(-1+x^2)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^7+x^9)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{a\text{Subst}(\int}{}$$

$$= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{10d} + \frac{a\csc^5(c+dx)}{5d} - \frac{3}{}$$

Mathematica [A]
time = 0.11, size = 86, normalized size = 0.89

$$-\frac{a\csc^4(c+dx)\left(-2310-1848\csc(c+dx)+4620\csc^2(c+dx)+3960\csc^3(c+dx)-3465\csc^4(c+dx)-3080\csc^5(c+dx)+924\csc^6(c+dx)+840\csc^7(c+dx)\right)}{9240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/9240*(a*Csc[c + d*x]^4*(-2310 - 1848*Csc[c + d*x] + 4620*Csc[c + d*x]^2
+ 3960*Csc[c + d*x]^3 - 3465*Csc[c + d*x]^4 - 3080*Csc[c + d*x]^5 + 924*Csc
[c + d*x]^6 + 840*Csc[c + d*x]^7))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(85) = 170$.
time = 0.24, size = 194, normalized size = 2.00

method	result

risch	$\frac{4a\left(1848ie^{17i(dx+c)}+1155\,e^{18i(dx+c)}+4752ie^{15i(dx+c)}+1155\,e^{16i(dx+c)}+13640ie^{13i(dx+c)}+5775\,e^{14i(dx+c)}+13280ie^{11}\right.}{}$
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^8(dx+c)}{33\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{231\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{1155\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{1155\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{231\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^8(dx+c)}{33\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{231\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{1155\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{1155\sin(dx+c)^3}+\frac{\cos^8(dx+c)}{231\sin(dx+c)}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin(d*x+c)^9*cos(d*x+c)^8-1/2
31/sin(d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*cos(d*x+c)^8-1/1155/sin(d*
x+c)^3*cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8+1/231*(16/5+cos(d*x+c)^6+
6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/10/sin(d*x+c)^10*cos(d
*x+c)^8-1/40/sin(d*x+c)^8*cos(d*x+c)^8))
```

Maxima [A]

time = 0.28, size = 92, normalized size = 0.95

$$\frac{2310\,a\sin\left(dx+c\right)^7+1848\,a\sin\left(dx+c\right)^6-4620\,a\sin\left(dx+c\right)^5-3960\,a\sin\left(dx+c\right)^4+3465\,a\sin\left(dx+c\right)^3+3080\,a\sin\left(dx+c\right)^2-924\,a\sin\left(dx+c\right)-840\,a}{9240\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

Fricas [A]

time = 0.38, size = 152, normalized size = 1.57

$$\frac{1848\,a\cos\left(dx+c\right)^6-1584\,a\cos\left(dx+c\right)^4+704\,a\cos\left(dx+c\right)^2+231\left(10\,a\cos\left(dx+c\right)^6-10\,a\cos\left(dx+c\right)^4+5\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-128\,a}{9240\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^8+10\,d\cos\left(dx+c\right)^6-10\,d\cos\left(dx+c\right)^4+5\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/9240*(1848*a*cos(d*x + c)^6 - 1584*a*cos(d*x + c)^4 + 704*a*cos(d*x + c)^
2 + 231*(10*a*cos(d*x + c)^6 - 10*a*cos(d*x + c)^4 + 5*a*cos(d*x + c)^2 - a
```

```
)*sin(d*x + c) - 128*a)/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos
(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 92, normalized size = 0.95

$$\frac{2310\,a\sin\left(dx+c\right)^{7}+1848\,a\sin\left(dx+c\right)^{6}-4620\,a\sin\left(dx+c\right)^{5}-3960\,a\sin\left(dx+c\right)^{4}+3465\,a\sin\left(dx+c\right)^{3}+3080\,a\sin\left(dx+c\right)^{2}-924\,a\sin\left(dx+c\right)-840\,a}{9240\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(2310*a*sin(d*x + c)^7 + 1848*a*sin(d*x + c)^6 - 4620*a*sin(d*x + c)
^5 - 3960*a*sin(d*x + c)^4 + 3465*a*sin(d*x + c)^3 + 3080*a*sin(d*x + c)^2
- 924*a*sin(d*x + c) - 840*a)/(d*sin(d*x + c)^11)
```

Mupad [B]
time = 9.28, size = 92, normalized size = 0.95

$$-\frac{-\frac{a\sin(c+d\,x)^7}{4}-\frac{a\sin(c+d\,x)^6}{5}+\frac{a\sin(c+d\,x)^5}{2}+\frac{3\,a\sin(c+d\,x)^4}{7}-\frac{3\,a\sin(c+d\,x)^3}{8}-\frac{a\sin(c+d\,x)^2}{3}+\frac{a\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (a*sin(c + d*x))/10 - (a*sin(c + d*x)^2)/3 - (3*a*sin(c + d*x)^3)/
8 + (3*a*sin(c + d*x)^4)/7 + (a*sin(c + d*x)^5)/2 - (a*sin(c + d*x)^6)/5 -
(a*sin(c + d*x)^7)/4)/(d*sin(c + d*x)^11)
```

3.674 $\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{12}(c+dx)}{12d}+\frac{a\csc^5(c+dx)}{5d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^9(c+dx)}{3d}-\frac{a\csc$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/5*a*cot(d*x+c)^10/d-1/12*a*cot(d*x+c)^12/d+1/5*a*cs
c(d*x+c)^5/d-3/7*a*csc(d*x+c)^7/d+1/3*a*csc(d*x+c)^9/d-1/11*a*csc(d*x+c)^11
/d
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 2687, 272, 45, 2686, 276}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{11}(c+dx)}{11d}+\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^7(c+dx)}{7d}+\frac{a\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^5)/(5*d) - (3*a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c
+ d*x]^9)/(3*d) - (a*Csc[c + d*x]^11)/(11*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$
\begin{aligned}
\int \cot^7(c+dx)\csc^6(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^7(c+dx)\csc^5(c+dx)\,dx + a\int \cot^7(c+dx)\csc^6 \\
&= -\frac{a\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\mathrm{Subst}}{} \\
&= -\frac{a\mathrm{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\mathrm{Subst}\left(\int\right.}{} \\
&= \frac{a\csc^5(c+dx)}{5d} - \frac{3a\csc^7(c+dx)}{7d} + \frac{a\csc^9(c+dx)}{3d} - \frac{a\,c}{} \\
&= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} +
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.76

$$
-\frac{a\csc^{12}(c+dx)(1617+3003\cos(2(c+dx))+1155\cos(4(c+dx))+385\cos(6(c+dx))-45\sin(c+dx)+1111\sin(3(c+dx))+363\sin(5(c+dx))+231\sin(7(c+dx)))}{73920d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/73920*(a*Csc[c + d*x]^12*(1617 + 3003*Cos[2*(c + d*x)] + 1155*Cos[4*(c +
 d*x)] + 385*Cos[6*(c + d*x)] - 45*Sin[c + d*x] + 1111*Sin[3*(c + d*x)] + 3
63*Sin[5*(c + d*x)] + 231*Sin[7*(c + d*x)]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(99) = 198$.

time = 0.28, size = 212, normalized size = 1.88

method	result
risch	$\frac{32ia\left(385i e^{18i(dx+c)}+231\,e^{19i(dx+c)}+1155i e^{16i(dx+c)}+363\,e^{17i(dx+c)}+3003i e^{14i(dx+c)}+1111\,e^{15i(dx+c)}+3234i e^{12i(dx}\right.}{1155d}$
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{12\sin(dx+c)^{12}}-\frac{\cos^8(dx+c)}{30\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{120\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^8(dx+c)}{33\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{231\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{1155\sin(dx+c}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{12\sin(dx+c)^{12}}-\frac{\cos^8(dx+c)}{30\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{120\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{11\sin(dx+c)^{11}}-\frac{\cos^8(dx+c)}{33\sin(dx+c)^9}-\frac{\cos^8(dx+c)}{231\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{1155\sin(dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/12/sin(d*x+c)^12*cos(d*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^8-1/
120/sin(d*x+c)^8*cos(d*x+c)^8)+a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^8-1/33/sin
(d*x+c)^9*cos(d*x+c)^8-1/231/sin(d*x+c)^7*cos(d*x+c)^8+1/1155/sin(d*x+c)^5*
cos(d*x+c)^8-1/1155/sin(d*x+c)^3*cos(d*x+c)^8+1/231/sin(d*x+c)*cos(d*x+c)^8
+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.81

$$\frac{1848\,a\sin\left(dx+c\right)^7+1540\,a\sin\left(dx+c\right)^6-3960\,a\sin\left(dx+c\right)^5-3465\,a\sin\left(dx+c\right)^4+3080\,a\sin\left(dx+c\right)^3+2772\,a\sin\left(dx+c\right)^2-840\,a\sin\left(dx+c\right)-770\,a}{9240\,d\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

Fricas [A]
time = 0.39, size = 153, normalized size = 1.35

$$-\frac{1540\,a\cos\left(dx+c\right)^6-1155\,a\cos\left(dx+c\right)^4+462\,a\cos\left(dx+c\right)^2+8\left(231\,a\cos\left(dx+c\right)^6-198\,a\cos\left(dx+c\right)^4+88\,a\cos\left(dx+c\right)^2-16\,a\right)\sin\left(dx+c\right)-77\,a}{9240\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^8-20\,d\cos\left(dx+c\right)^6+15\,d\cos\left(dx+c\right)^4-6\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/9240*(1540*a*cos(d*x + c)^6 - 1155*a*cos(d*x + c)^4 + 462*a*cos(d*x + c)
^2 + 8*(231*a*cos(d*x + c)^6 - 198*a*cos(d*x + c)^4 + 88*a*cos(d*x + c)^2 -
 16*a)*sin(d*x + c) - 77*a)/(d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 + 15*d
*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d*x +
 c)^2 + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 92, normalized size = 0.81

$$\frac{1848\,a\sin\left(dx+c\right)^{7}+1540\,a\sin\left(dx+c\right)^{6}-3960\,a\sin\left(dx+c\right)^{5}-3465\,a\sin\left(dx+c\right)^{4}+3080\,a\sin\left(dx+c\right)^{3}+2772\,a\sin\left(dx+c\right)^{2}-840\,a\sin\left(dx+c\right)-770\,a}{9240\,d\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/9240*(1848*a*sin(d*x + c)^7 + 1540*a*sin(d*x + c)^6 - 3960*a*sin(d*x + c)
^5 - 3465*a*sin(d*x + c)^4 + 3080*a*sin(d*x + c)^3 + 2772*a*sin(d*x + c)^2
- 840*a*sin(d*x + c) - 770*a)/(d*sin(d*x + c)^12)
```

Mupad [**B**]
time = 9.19, size = 92, normalized size = 0.81

$$-\frac{-\frac{a\sin(c+d\,x)^7}{5}-\frac{a\sin(c+d\,x)^6}{6}+\frac{3\,a\sin(c+d\,x)^5}{7}+\frac{3\,a\sin(c+d\,x)^4}{8}-\frac{a\sin(c+d\,x)^3}{3}-\frac{3\,a\sin(c+d\,x)^2}{10}+\frac{a\,\sin(c+d\,x)}{11}+\frac{a}{12}}{d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^13,x)
```

```
[Out] -(a/12 + (a*sin(c + d*x))/11 - (3*a*sin(c + d*x)^2)/10 - (a*sin(c + d*x)^3)
/3 + (3*a*sin(c + d*x)^4)/8 + (3*a*sin(c + d*x)^5)/7 - (a*sin(c + d*x)^6)/6
 - (a*sin(c + d*x)^7)/5)/(d*sin(c + d*x)^12)
```

3.675 $\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=113

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^{12}(c+dx)}{12d}+\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{3d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\cs}{}$$

```
[Out] -1/8*a*cot(d*x+c)^8/d-1/5*a*cot(d*x+c)^10/d-1/12*a*cot(d*x+c)^12/d+1/7*a*cs
c(d*x+c)^7/d-1/3*a*csc(d*x+c)^9/d+3/11*a*csc(d*x+c)^11/d-1/13*a*csc(d*x+c)^
13/d
```

Rubi [A]
time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 2686, 276, 2687, 272, 45}

$$-\frac{a\cot^{12}(c+dx)}{12d}-\frac{a\cot^{10}(c+dx)}{5d}-\frac{a\cot^8(c+dx)}{8d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{a\csc^9(c+dx)}{3d}+\frac{a\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Cot[c + d*x]^8)/d - (a*Cot[c + d*x]^10)/(5*d) - (a*Cot[c + d*x]^12)
/(12*d) + (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(3*d) + (3*a*Csc[c
+ d*x]^11)/(11*d) - (a*Csc[c + d*x]^13)/(13*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^7(c+dx)\csc^7(c+dx)(a+a\sin(c+dx))\,dx &= a\int \cot^7(c+dx)\csc^6(c+dx)\,dx + a\int \cot^7(c+dx)\,\text{csc} \\
&= -\frac{a\text{Subst}\left(\int x^6(-1+x^2)^3\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\text{Subst}}{} \\
&= -\frac{a\text{Subst}\left(\int x^3(1+x)^2\,dx, x, \cot^2(c+dx)\right)}{2d} - \frac{a\text{Subst}(\int}{} \\
&= \frac{a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{3d} + \frac{3a\csc^{11}(c+dx)}{11d} - \frac{a}{} \\
&= -\frac{a\cot^8(c+dx)}{8d} - \frac{a\cot^{10}(c+dx)}{5d} - \frac{a\cot^{12}(c+dx)}{12d} +
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 86, normalized size = 0.76

$$-\frac{a\csc^{13}(c+dx)(40200+70460\cos(2(c+dx))+28600\cos(4(c+dx))+8580\cos(6(c+dx))+3003\sin(c+dx)+24024\sin(3(c+dx))+10010\sin(5(c+dx))+5005\sin(7(c+dx)))}{1921920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1921920*(a*Csc[c + d*x]^13*(40200 + 70460*Cos[2*(c + d*x)] + 28600*Cos[4
*(c + d*x)] + 8580*Cos[6*(c + d*x)] + 3003*Sin[c + d*x] + 24024*Sin[3*(c +
d*x)] + 10010*Sin[5*(c + d*x)] + 5005*Sin[7*(c + d*x)]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(99) = 198$.
time = 0.28, size = 230, normalized size = 2.04

method	result
risch	$-\frac{32a(8580ie^{19i(dx+c)}+5005\,e^{20i(dx+c)}+28600ie^{17i(dx+c)}+10010\,e^{18i(dx+c)}+70460ie^{15i(dx+c)}+24024\,e^{16i(dx+c)}+80}{}$
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^8(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^8(dx+c)\right)}{429\sin(dx+c)^9}-\frac{5\left(\cos^8(dx+c)\right)}{3003\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{3003\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{3003\sin(dx+c)^3}+\frac{5\left(\cos^8(dx+c)\right)}{3003\sin(dx+c)}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^8(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^8(dx+c)\right)}{429\sin(dx+c)^9}-\frac{5\left(\cos^8(dx+c)\right)}{3003\sin(dx+c)^7}+\frac{\cos^8(dx+c)}{3003\sin(dx+c)^5}-\frac{\cos^8(dx+c)}{3003\sin(dx+c)^3}+\frac{5\left(\cos^8(dx+c)\right)}{3003\sin(dx+c)}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/13/sin(d*x+c)^13*cos(d*x+c)^8-5/143/sin(d*x+c)^11*cos(d*x+c)^8-5
/429/sin(d*x+c)^9*cos(d*x+c)^8-5/3003/sin(d*x+c)^7*cos(d*x+c)^8+1/3003/sin(
d*x+c)^5*cos(d*x+c)^8-1/3003/sin(d*x+c)^3*cos(d*x+c)^8+5/3003/sin(d*x+c)*co
s(d*x+c)^8+5/3003*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin
(d*x+c))+a*(-1/12/sin(d*x+c)^12*cos(d*x+c)^8-1/30/sin(d*x+c)^10*cos(d*x+c)^
8-1/120/sin(d*x+c)^8*cos(d*x+c)^8))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.81

$$\frac{20020\,a\sin\left(dx+c\right)^7+17160\,a\sin\left(dx+c\right)^6-45045\,a\sin\left(dx+c\right)^5-40040\,a\sin\left(dx+c\right)^4+36036\,a\sin\left(dx+c\right)^3+32760\,a\sin\left(dx+c\right)^2-10010\,a\sin\left(dx+c\right)-9240\,a}{120120\,d\sin\left(dx+c\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

Fricas [A]
time = 0.40, size = 161, normalized size = 1.42

$$-\frac{17160\,a\cos\left(dx+c\right)^6-11440\,a\cos\left(dx+c\right)^4+4160\,a\cos\left(dx+c\right)^2+1001\,\left(20\,a\cos\left(dx+c\right)^6-15\,a\cos\left(dx+c\right)^4+6\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-640\,a}{120120\,\left(d\cos\left(dx+c\right)^{12}-6\,d\cos\left(dx+c\right)^{10}+15\,d\cos\left(dx+c\right)^8-20\,d\cos\left(dx+c\right)^6+15\,d\cos\left(dx+c\right)^4-6\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/120120*(17160*a*cos(d*x + c)^6 - 11440*a*cos(d*x + c)^4 + 4160*a*cos(d*x
 + c)^2 + 1001*(20*a*cos(d*x + c)^6 - 15*a*cos(d*x + c)^4 + 6*a*cos(d*x + c
)^2 - a)*sin(d*x + c) - 640*a)/((d*cos(d*x + c)^12 - 6*d*cos(d*x + c)^10 +
15*d*cos(d*x + c)^8 - 20*d*cos(d*x + c)^6 + 15*d*cos(d*x + c)^4 - 6*d*cos(d
*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**14*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 92, normalized size = 0.81

$$\frac{20020\,a\sin\left(dx+c\right)^{7}+17160\,a\sin\left(dx+c\right)^{6}-45045\,a\sin\left(dx+c\right)^{5}-40040\,a\sin\left(dx+c\right)^{4}+36036\,a\sin\left(dx+c\right)^{3}+32760\,a\sin\left(dx+c\right)^{2}-10010\,a\sin\left(dx+c\right)-9240\,a}{120120\,d\sin\left(dx+c\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^14*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(20020*a*sin(d*x + c)^7 + 17160*a*sin(d*x + c)^6 - 45045*a*sin(d*x
 + c)^5 - 40040*a*sin(d*x + c)^4 + 36036*a*sin(d*x + c)^3 + 32760*a*sin(d*x
 + c)^2 - 10010*a*sin(d*x + c) - 9240*a)/(d*sin(d*x + c)^13)
```

Mupad [B]
time = 9.35, size = 92, normalized size = 0.81

$$-\frac{-\frac{a\sin(c+d\,x)^7}{6}-\frac{a\sin(c+d\,x)^6}{7}+\frac{3\,a\sin(c+d\,x)^5}{8}+\frac{a\sin(c+d\,x)^4}{3}-\frac{3\,a\sin(c+d\,x)^3}{10}-\frac{3\,a\sin(c+d\,x)^2}{11}+\frac{a\,\sin(c+d\,x)}{12}+\frac{a}{13}}{d\sin\left(c+d\,x\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^14,x)
```

```
[Out] -(a/13 + (a*sin(c + d*x))/12 - (3*a*sin(c + d*x)^2)/11 - (3*a*sin(c + d*x)^
3)/10 + (a*sin(c + d*x)^4)/3 + (3*a*sin(c + d*x)^5)/8 - (a*sin(c + d*x)^6)/
7 - (a*sin(c + d*x)^7)/6)/(d*sin(c + d*x)^13)
```

3.676 $\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{8d}-\frac{a\csc^9(c+dx)}{3d}-\frac{3a\csc^{10}(c+dx)}{10d}+\frac{3a\csc^{11}(c+dx)}{11d}+\frac{a\csc^{12}(c+dx)}{4d}-\frac{a\csc$$

```
[Out] 1/7*a*csc(d*x+c)^7/d+1/8*a*csc(d*x+c)^8/d-1/3*a*csc(d*x+c)^9/d-3/10*a*csc(d
*x+c)^10/d+3/11*a*csc(d*x+c)^11/d+1/4*a*csc(d*x+c)^12/d-1/13*a*csc(d*x+c)^1
3/d-1/14*a*csc(d*x+c)^14/d
```

Rubi [A]
time = 0.07, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a\csc^{14}(c+dx)}{14d}-\frac{a\csc^{13}(c+dx)}{13d}+\frac{a\csc^{12}(c+dx)}{4d}+\frac{3a\csc^{11}(c+dx)}{11d}-\frac{3a\csc^{10}(c+dx)}{10d}-\frac{a\csc^9(c+dx)}{3d}+\frac{a\csc^8(c+dx)}{8d}+\frac{a\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Csc[c + d*x]^7)/(7*d) + (a*Csc[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^9)/(3
*d) - (3*a*Csc[c + d*x]^10)/(10*d) + (3*a*Csc[c + d*x]^11)/(11*d) + (a*Csc[
c + d*x]^12)/(4*d) - (a*Csc[c + d*x]^13)/(13*d) - (a*Csc[c + d*x]^14)/(14*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^7(c+dx)\csc^8(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a^{15}(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^8\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^{15}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \left(\frac{a^7}{x^{15}} + \frac{a^6}{x^{14}} - \frac{3a^5}{x^{13}} - \frac{3a^4}{x^{12}} + \frac{3a^3}{x^{11}} + \frac{3a^2}{x^{10}} - \frac{a}{x^9} - \frac{1}{x^8}\right.\right.}{d}$$
$$= \frac{a\csc^7(c+dx)}{7d} + \frac{a\csc^8(c+dx)}{8d} - \frac{a\csc^9(c+dx)}{3d} - \frac{3a\,c}{}$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 0.67

$$-\frac{a\csc^{14}(c+dx)(76362+129129\cos(2(c+dx))+54054\cos(4(c+dx))+15015\cos(6(c+dx))+9940\sin(c+dx)+41860\sin(3(c+dx))+20020\sin(5(c+dx))+8580\sin(7(c+dx)))}{3843840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*Csc[c + d*x]^8*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3843840*(a*Csc[c + d*x]^14*(76362 + 129129*Cos[2*(c + d*x)] + 54054*Cos[
4*(c + d*x)] + 15015*Cos[6*(c + d*x)] + 9940*Sin[c + d*x] + 41860*Sin[3*(c
+ d*x)] + 20020*Sin[5*(c + d*x)] + 8580*Sin[7*(c + d*x)]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(113) = 226$.
time = 0.36, size = 248, normalized size = 1.92

method	result
risch	$-\frac{32ia\left(15015ie^{20i(dx+c)}+8580\,e^{21i(dx+c)}+54054ie^{18i(dx+c)}+20020\,e^{19i(dx+c)}+129129ie^{16i(dx+c)}+41860\,e^{17i(dx+c)}+1\right.}{}$
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{14\sin(dx+c)^{14}}-\frac{\cos^8(dx+c)}{28\sin(dx+c)^{12}}-\frac{\cos^8(dx+c)}{70\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{280\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^8(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^8(dx+c)\right)}{429\sin(dx+c)^9}\right.}{}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{14\sin(dx+c)^{14}}-\frac{\cos^8(dx+c)}{28\sin(dx+c)^{12}}-\frac{\cos^8(dx+c)}{70\sin(dx+c)^{10}}-\frac{\cos^8(dx+c)}{280\sin(dx+c)^8}\right)+a\left(-\frac{\cos^8(dx+c)}{13\sin(dx+c)^{13}}-\frac{5\left(\cos^8(dx+c)\right)}{143\sin(dx+c)^{11}}-\frac{5\left(\cos^8(dx+c)\right)}{429\sin(dx+c)^9}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/14/sin(d*x+c)^14*cos(d*x+c)^8-1/28/sin(d*x+c)^12*cos(d*x+c)^8-1/
70/sin(d*x+c)^10*cos(d*x+c)^8-1/280/sin(d*x+c)^8*cos(d*x+c)^8)+a*(-1/13/sin
(d*x+c)^13*cos(d*x+c)^8-5/143/sin(d*x+c)^11*cos(d*x+c)^8-5/429/sin(d*x+c)^9
*cos(d*x+c)^8-5/3003/sin(d*x+c)^7*cos(d*x+c)^8+1/3003/sin(d*x+c)^5*cos(d*x+
c)^8-1/3003/sin(d*x+c)^3*cos(d*x+c)^8+5/3003/sin(d*x+c)*cos(d*x+c)^8+5/3003
*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [**A**]
time = 0.28, size = 92, normalized size = 0.71

$$\frac{17160\,a\sin\left(dx+c\right)^{7}+15015\,a\sin\left(dx+c\right)^{6}-40040\,a\sin\left(dx+c\right)^{5}-36036\,a\sin\left(dx+c\right)^{4}+32760\,a\sin\left(dx+c\right)^{3}+30030\,a\sin\left(dx+c\right)^{2}-9240\,a\sin\left(dx+c\right)-8580\,a}{120120\,d\sin\left(dx+c\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

Fricas [**A**]
time = 0.39, size = 166, normalized size = 1.29

$$\frac{15015\,a\cos\left(dx+c\right)^{6}-9009\,a\cos\left(dx+c\right)^{4}+3003\,a\cos\left(dx+c\right)^{2}+40\,\left(429\,a\cos\left(dx+c\right)^{6}-286\,a\cos\left(dx+c\right)^{4}+104\,a\cos\left(dx+c\right)^{2}-16\,a\right)\sin\left(dx+c\right)-429\,a}{120120\,\left(d\cos\left(dx+c\right)^{14}-7\,d\cos\left(dx+c\right)^{12}+21\,d\cos\left(dx+c\right)^{10}-35\,d\cos\left(dx+c\right)^{8}+35\,d\cos\left(dx+c\right)^{6}-21\,d\cos\left(dx+c\right)^{4}+7\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/120120*(15015*a*cos(d*x + c)^6 - 9009*a*cos(d*x + c)^4 + 3003*a*cos(d*x +
 c)^2 + 40*(429*a*cos(d*x + c)^6 - 286*a*cos(d*x + c)^4 + 104*a*cos(d*x + c
)^2 - 16*a)*sin(d*x + c) - 429*a)/(d*cos(d*x + c)^14 - 7*d*cos(d*x + c)^12
+ 21*d*cos(d*x + c)^10 - 35*d*cos(d*x + c)^8 + 35*d*cos(d*x + c)^6 - 21*d*c
os(d*x + c)^4 + 7*d*cos(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**15*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 92, normalized size = 0.71

$$\frac{17160\,a\sin\left(dx+c\right)^{7}+15015\,a\sin\left(dx+c\right)^{6}-40040\,a\sin\left(dx+c\right)^{5}-36036\,a\sin\left(dx+c\right)^{4}+32760\,a\sin\left(dx+c\right)^{3}+30030\,a\sin\left(dx+c\right)^{2}-9240\,a\sin\left(dx+c\right)-8580\,a}{120120\,d\sin\left(dx+c\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^15*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120120*(17160*a*sin(d*x + c)^7 + 15015*a*sin(d*x + c)^6 - 40040*a*sin(d*x
 + c)^5 - 36036*a*sin(d*x + c)^4 + 32760*a*sin(d*x + c)^3 + 30030*a*sin(d*x
 + c)^2 - 9240*a*sin(d*x + c) - 8580*a)/(d*sin(d*x + c)^14)
```

Mupad [B]
time = 9.24, size = 92, normalized size = 0.71

$$-\frac{-\frac{a\sin(c+d\,x)^7}{7}-\frac{a\sin(c+d\,x)^6}{8}+\frac{a\sin(c+d\,x)^5}{3}+\frac{3\,a\sin(c+d\,x)^4}{10}-\frac{3\,a\sin(c+d\,x)^3}{11}-\frac{a\sin(c+d\,x)^2}{4}+\frac{a\,\sin(c+d\,x)}{13}+\frac{a}{14}}{d\sin\left(c+d\,x\right)^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(a + a*sin(c + d*x)))/sin(c + d*x)^15,x)
```

```
[Out] -(a/14 + (a*sin(c + d*x))/13 - (a*sin(c + d*x)^2)/4 - (3*a*sin(c + d*x)^3)/
11 + (3*a*sin(c + d*x)^4)/10 + (a*sin(c + d*x)^5)/3 - (a*sin(c + d*x)^6)/8
- (a*sin(c + d*x)^7)/7)/(d*sin(c + d*x)^14)
```

3.677 $\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{8ad}-\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{5ad}+\frac{\sin^{11}(c+dx)}{11ad}-\frac{\sin^{12}(c+dx)}{12ad}$$

```
[Out] 1/7*sin(d*x+c)^7/a/d-1/8*sin(d*x+c)^8/a/d-2/9*sin(d*x+c)^9/a/d+1/5*sin(d*x+
c)^10/a/d+1/11*sin(d*x+c)^11/a/d-1/12*sin(d*x+c)^12/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin^{12}(c+dx)}{12ad}+\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{5ad}-\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(8*a*d) - (2*Sin[c + d*x]^9)/(9*a*d
) + Sin[c + d*x]^10/(5*a*d) + Sin[c + d*x]^11/(11*a*d) - Sin[c + d*x]^12/(1
2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^6(a+x)^2}{a^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^6(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^6-a^4x^7-2a^3x^8+2a^2x^9+ax^{10}-x^{11}\right)dx, x, a\sin(c+dx)\right)}{a^{13}d}$$
$$= \frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{8ad}-\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{5ad}+\frac{\sin^{11}(c+}{11ad}$$

Mathematica [A]
time = 0.44, size = 68, normalized size = 0.62

$$\frac{\sin^7(c+dx)\left(3960-3465\sin(c+dx)-6160\sin^2(c+dx)+5544\sin^3(c+dx)+2520\sin^4(c+dx)-2310\sin^5(c+dx)\right)}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^7*(3960 - 3465*Sin[c + d*x] - 6160*Sin[c + d*x]^2 + 5544*Sin[
c + d*x]^3 + 2520*Sin[c + d*x]^4 - 2310*Sin[c + d*x]^5))/(27720*a*d)
```

Maple [A]
time = 0.18, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{\left(\sin^{12}(dx+c)\right)}{12}+\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{5}-\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{8}+\frac{\left(\sin^{7}(dx+c)\right)}{7}}{da}$
default	$\frac{-\frac{\left(\sin^{12}(dx+c)\right)}{12}+\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{5}-\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{8}+\frac{\left(\sin^{7}(dx+c)\right)}{7}}{da}$
risch	$\frac{5\sin(dx+c)}{512ad}-\frac{\cos(12dx+12c)}{24576ad}-\frac{\sin(11dx+11c)}{11264ad}+\frac{\cos(10dx+10c)}{10240ad}+\frac{\sin(9dx+9c)}{9216ad}+\frac{\cos(8dx+8c)}{4096ad}+\frac{5\sin(7dx+}{7168ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/12*sin(d*x+c)^12+1/11*sin(d*x+c)^11+1/5*sin(d*x+c)^10-2/9*sin(d*x
+c)^9-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.63

$$-\frac{2310\,\sin\left(dx+c\right)^{12}-2520\,\sin\left(dx+c\right)^{11}-5544\,\sin\left(dx+c\right)^{10}+6160\,\sin\left(dx+c\right)^{9}+3465\,\sin\left(dx+c\right)^{8}-3960\,\sin\left(dx+c\right)^{7}}{27720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

Fricas [**A**]
time = 0.40, size = 109, normalized size = 1.00

$$-\frac{2310 \cos(dx+c)^{12} - 8316 \cos(dx+c)^{10} + 10395 \cos(dx+c)^{8} - 4620 \cos(dx+c)^{6} + 40 \left(63 \cos(dx+c)^{10} - 161 \cos(dx+c)^{8} + 113 \cos(dx+c)^{6} - 3 \cos(dx+c)^{4} - 4 \cos(dx+c)^{2} - 8\right) \sin(dx+c)}{27720\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/27720*(2310*cos(d*x + c)^12 - 8316*cos(d*x + c)^10 + 10395*cos(d*x + c)^
8 - 4620*cos(d*x + c)^6 + 40*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113
*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a
*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 69, normalized size = 0.63

$$-\frac{2310 \sin(dx+c)^{12} - 2520 \sin(dx+c)^{11} - 5544 \sin(dx+c)^{10} + 6160 \sin(dx+c)^{9} + 3465 \sin(dx+c)^{8} - 3960 \sin(dx+c)^{7}}{27720\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/27720*(2310*sin(d*x + c)^12 - 2520*sin(d*x + c)^11 - 5544*sin(d*x + c)^1
0 + 6160*sin(d*x + c)^9 + 3465*sin(d*x + c)^8 - 3960*sin(d*x + c)^7)/(a*d)
```

Mupad [**B**]
time = 0.08, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^7}{7\,a} - \frac{\sin(c+d\,x)^8}{8\,a} - \frac{2\sin(c+d\,x)^9}{9\,a} + \frac{\sin(c+d\,x)^{10}}{5\,a} + \frac{\sin(c+d\,x)^{11}}{11\,a} - \frac{\sin(c+d\,x)^{12}}{12\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^6)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(8*a) - (2*sin(c + d*x)^9)/(9*a) + s
in(c + d*x)^10/(5*a) + sin(c + d*x)^11/(11*a) - sin(c + d*x)^12/(12*a))/d
```

3.678 $\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\sin^6(c+dx)}{6ad}-\frac{\sin^7(c+dx)}{7ad}-\frac{\sin^8(c+dx)}{4ad}+\frac{2\sin^9(c+dx)}{9ad}+\frac{\sin^{10}(c+dx)}{10ad}-\frac{\sin^{11}(c+dx)}{11ad}$$

```
[Out] 1/6*sin(d*x+c)^6/a/d-1/7*sin(d*x+c)^7/a/d-1/4*sin(d*x+c)^8/a/d+2/9*sin(d*x+
c)^9/a/d+1/10*sin(d*x+c)^10/a/d-1/11*sin(d*x+c)^11/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2915, 12, 90}

$$-\frac{\sin^{11}(c+dx)}{11ad}+\frac{\sin^{10}(c+dx)}{10ad}+\frac{2\sin^9(c+dx)}{9ad}-\frac{\sin^8(c+dx)}{4ad}-\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^6/(6*a*d) - Sin[c + d*x]^7/(7*a*d) - Sin[c + d*x]^8/(4*a*d) +
(2*Sin[c + d*x]^9)/(9*a*d) + Sin[c + d*x]^10/(10*a*d) - Sin[c + d*x]^11/(11
*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3x^5(a+x)^2}{a^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (a-x)^3x^5(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\text{Subst}\left(\int \left(a^5x^5-a^4x^6-2a^3x^7+2a^2x^8+ax^9-x^{10}\right)\,dx, x, a\sin(c+dx)\right)}{a^{12}d}$$
$$= \frac{\sin^6(c+dx)}{6ad} - \frac{\sin^7(c+dx)}{7ad} - \frac{\sin^8(c+dx)}{4ad} + \frac{2\sin^9(c+dx)}{9ad} + \frac{\sin^{10}(c+}{10ad}$$

Mathematica [A]
time = 0.65, size = 68, normalized size = 0.62

$$\frac{\sin^6(c+dx)\left(2310-1980\sin(c+dx)-3465\sin^2(c+dx)+3080\sin^3(c+dx)+1386\sin^4(c+dx)-1260\sin^5(c+dx)\right)}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^6*(2310 - 1980*Sin[c + d*x] - 3465*Sin[c + d*x]^2 + 3080*Sin[
c + d*x]^3 + 1386*Sin[c + d*x]^4 - 1260*Sin[c + d*x]^5))/(13860*a*d)
```

Maple [A]
time = 0.16, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{4}-\frac{\left(\sin^{7}(dx+c)\right)}{7}+\frac{\left(\sin^{6}(dx+c)\right)}{6}}{da}$
default	$\frac{-\frac{\left(\sin^{11}(dx+c)\right)}{11}+\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{2\left(\sin^{9}(dx+c)\right)}{9}-\frac{\left(\sin^{8}(dx+c)\right)}{4}-\frac{\left(\sin^{7}(dx+c)\right)}{7}+\frac{\left(\sin^{6}(dx+c)\right)}{6}}{da}$
risch	$-\frac{5\sin(dx+c)}{512ad}+\frac{\sin(11dx+11c)}{11264ad}-\frac{\cos(10dx+10c)}{5120ad}-\frac{\sin(9dx+9c)}{9216ad}-\frac{5\sin(7dx+7c)}{7168ad}+\frac{5\cos(6dx+6c)}{3072ad}+\frac{\sin(5c}{102}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/11*sin(d*x+c)^11+1/10*sin(d*x+c)^10+2/9*sin(d*x+c)^9-1/4*sin(d*x+
c)^8-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6)
```

Maxima [A]
time = 0.32, size = 69, normalized size = 0.63

$$-\frac{1260\sin(dx+c)^{11}-1386\sin(dx+c)^{10}-3080\sin(dx+c)^{9}+3465\sin(dx+c)^{8}+1980\sin(dx+c)^{7}-2310\sin(dx+c)^{6}}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

Fricas [A]
time = 0.39, size = 99, normalized size = 0.91

$$-\frac{1386 \cos(dx+c)^{10} - 3465 \cos(dx+c)^{8} + 2310 \cos(dx+c)^{6} - 20 \left(63 \cos(dx+c)^{10} - 161 \cos(dx+c)^{8} + 113 \cos(dx+c)^{6} - 3 \cos(dx+c)^{4} - 4 \cos(dx+c)^{2} - 8\right) \sin(dx+c)}{13860\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/13860*(1386*cos(d*x + c)^10 - 3465*cos(d*x + c)^8 + 2310*cos(d*x + c)^6
- 20*(63*cos(d*x + c)^10 - 161*cos(d*x + c)^8 + 113*cos(d*x + c)^6 - 3*cos(
d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2280 vs. 2(82) = 164.
time = 222.25, size = 2280, normalized size = 20.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((36960*tan(c/2 + d*x/2)**16/(3465*a*d*tan(c/2 + d*x/2)**22 + 3811
5*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*t
an(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/
2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d
*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4
+ 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d) - 63360*tan(c/2 + d*x/2)**15/(3
465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*
tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/
2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d
*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**
6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a
*d) - 36960*tan(c/2 + d*x/2)**14/(3465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d
*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/
2 + d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d
*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)
**8 + 571725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 381
15*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d) + 140800*tan(c/2 + d*x/2)**13/(3465*
a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(
```

```
c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/2 +
d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2
)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**6 +
190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d)
+ 59136*tan(c/2 + d*x/2)**12/(3465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan
(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 +
d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2
)**12 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8
+ 571725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a
*d*tan(c/2 + d*x/2)**2 + 3465*a*d) - 236800*tan(c/2 + d*x/2)**11/(3465*a*d*
tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2
+ d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/
2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2)**1
0 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**6 + 1905
75*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d) + 59
136*tan(c/2 + d*x/2)**10/(3465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2
 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/
2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**1
2 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 57
1725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*t
an(c/2 + d*x/2)**2 + 3465*a*d) + 140800*tan(c/2 + d*x/2)**9/(3465*a*d*tan(c
/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x
/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**1
4 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1
143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*
d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d) - 36960*t
an(c/2 + d*x/2)**8/(3465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x
/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16
 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 16
00830*a*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a
*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2
 + d*x/2)**2 + 3465*a*d) - 63360*tan(c/2 + d*x/2)**7/(3465*a*d*tan(c/2 + d*
x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20 + 190575*a*d*tan(c/2 + d*x/2)**18
 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143450*a*d*tan(c/2 + d*x/2)**14 + 160
0830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a*d*tan(c/2 + d*x/2)**10 + 1143450*
a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(c/2 + d*x/2)**6 + 190575*a*d*tan(c
/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/2)**2 + 3465*a*d) + 36960*tan(c/2
+ d*x/2)**6/(3465*a*d*tan(c/2 + d*x/2)**22 + 38115*a*d*tan(c/2 + d*x/2)**20
 + 190575*a*d*tan(c/2 + d*x/2)**18 + 571725*a*d*tan(c/2 + d*x/2)**16 + 1143
450*a*d*tan(c/2 + d*x/2)**14 + 1600830*a*d*tan(c/2 + d*x/2)**12 + 1600830*a
*d*tan(c/2 + d*x/2)**10 + 1143450*a*d*tan(c/2 + d*x/2)**8 + 571725*a*d*tan(
c/2 + d*x/2)**6 + 190575*a*d*tan(c/2 + d*x/2)**4 + 38115*a*d*tan(c/2 + d*x/
2)**2 + 3465*a*d), Ne(d, 0)), (x*sin(c)**5*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A]

time = 0.48, size = 69, normalized size = 0.63

$$-\frac{1260 \sin \left(dx+c\right)^{11}-1386 \sin \left(dx+c\right)^{10}-3080 \sin \left(dx+c\right)^{9}+3465 \sin \left(dx+c\right)^{8}+1980 \sin \left(dx+c\right)^{7}-2310 \sin \left(dx+c\right)^{6}}{13860\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(1260*sin(d*x + c)^11 - 1386*sin(d*x + c)^10 - 3080*sin(d*x + c)^9
 + 3465*sin(d*x + c)^8 + 1980*sin(d*x + c)^7 - 2310*sin(d*x + c)^6)/(a*d)
```

Mupad [B]
time = 8.99, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^6}{6\,a}-\frac{\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^8}{4\,a}+\frac{2\sin(c+d\,x)^9}{9\,a}+\frac{\sin(c+d\,x)^{10}}{10\,a}-\frac{\sin(c+d\,x)^{11}}{11\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^5)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^6/(6*a) - sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(4*a) + (2*si
n(c + d*x)^9)/(9*a) + sin(c + d*x)^10/(10*a) - sin(c + d*x)^11/(11*a))/d
```

3.679 $\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{2\sin^7(c+dx)}{7ad}+\frac{\sin^8(c+dx)}{4ad}+\frac{\sin^9(c+dx)}{9ad}-\frac{\sin^{10}(c+dx)}{10ad}$$

```
[Out] 1/5*sin(d*x+c)^5/a/d-1/6*sin(d*x+c)^6/a/d-2/7*sin(d*x+c)^7/a/d+1/4*sin(d*x+
c)^8/a/d+1/9*sin(d*x+c)^9/a/d-1/10*sin(d*x+c)^10/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin^{10}(c+dx)}{10ad}+\frac{\sin^9(c+dx)}{9ad}+\frac{\sin^8(c+dx)}{4ad}-\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^6(c+dx)}{6ad}+\frac{\sin^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^5/(5*a*d) - Sin[c + d*x]^6/(6*a*d) - (2*Sin[c + d*x]^7)/(7*a*d
) + Sin[c + d*x]^8/(4*a*d) + Sin[c + d*x]^9/(9*a*d) - Sin[c + d*x]^10/(10*a
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a-x)^3x^4(a+x)^2}{a^4}\,dx,x,a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\operatorname{Subst}\left(\int (a-x)^3x^4(a+x)^2\,dx,x,a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(a^5x^4-a^4x^5-2a^3x^6+2a^2x^7+ax^8-x^9\right)\,dx,x,a\sin(c+dx)\right)}{a^{11}d}$$
$$= \frac{\sin^5(c+dx)}{5ad}-\frac{\sin^6(c+dx)}{6ad}-\frac{2\sin^7(c+dx)}{7ad}+\frac{\sin^8(c+dx)}{4ad}+\frac{\sin^9(c+d}{9ad}$$

Mathematica [A]
time = 0.30, size = 68, normalized size = 0.62

$$\frac{\sin^5(c+dx)\left(252-210\sin(c+dx)-360\sin^2(c+dx)+315\sin^3(c+dx)+140\sin^4(c+dx)-126\sin^5(c+dx)\right)}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^5*(252 - 210*Sin[c + d*x] - 360*Sin[c + d*x]^2 + 315*Sin[c + d*x]^3 + 140*Sin[c + d*x]^4 - 126*Sin[c + d*x]^5))/(1260*a*d)
```

Maple [A]
time = 0.17, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{\left(\sin^9(dx+c)\right)}{9}+\frac{\left(\sin^8(dx+c)\right)}{4}-\frac{2\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{\left(\sin^5(dx+c)\right)}{5}}{da}$
default	$\frac{-\frac{\left(\sin^{10}(dx+c)\right)}{10}+\frac{\left(\sin^9(dx+c)\right)}{9}+\frac{\left(\sin^8(dx+c)\right)}{4}-\frac{2\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{\left(\sin^5(dx+c)\right)}{5}}{da}$
risch	$\frac{3\sin(dx+c)}{128ad}+\frac{\cos(10dx+10c)}{5120ad}+\frac{\sin(9dx+9c)}{2304ad}+\frac{\sin(7dx+7c)}{1792ad}-\frac{5\cos(6dx+6c)}{3072ad}-\frac{\sin(5dx+5c)}{320ad}-\frac{\sin(3dx+3c)}{192ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/10*sin(d*x+c)^10+1/9*sin(d*x+c)^9+1/4*sin(d*x+c)^8-2/7*sin(d*x+c)^7-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5)
```

Maxima [A]
time = 0.31, size = 69, normalized size = 0.63

$$-\frac{126\,\sin\left(dx+c\right)^{10}-140\,\sin\left(dx+c\right)^{9}-315\,\sin\left(dx+c\right)^{8}+360\,\sin\left(dx+c\right)^{7}+210\,\sin\left(dx+c\right)^{6}-252\,\sin\left(dx+c\right)^{5}}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

Fricas [A]
time = 0.37, size = 89, normalized size = 0.82

$$\frac{126\cos(dx+c)^{10}-315\cos(dx+c)^{8}+210\cos(dx+c)^{6}+4\left(35\cos(dx+c)^{8}-50\cos(dx+c)^{6}+3\cos(dx+c)^{4}+4\cos(dx+c)^{2}+8\right)\sin(dx+c)}{1260\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1260*(126*cos(d*x + c)^10 - 315*cos(d*x + c)^8 + 210*cos(d*x + c)^6 + 4*(
35*cos(d*x + c)^8 - 50*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2
 + 8)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2093 vs. $2(82) = 164$.
time = 157.08, size = 2093, normalized size = 19.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((2016*tan(c/2 + d*x/2)**15/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a
*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/
2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2
)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 141
75*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 3360
*tan(c/2 + d*x/2)**14/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*
x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14
 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a
*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2
 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 1440*tan(c/2 + d*x/
2)**13/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 1417
5*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan
(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*
x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 3
150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 6720*tan(c/2 + d*x/2)**12/(315*a*d
*tan(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 +
 d*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**
12 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800
```

```
*a*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2
 + d*x/2)**2 + 315*a*d) + 3520*tan(c/2 + d*x/2)**11/(315*a*d*tan(c/2 + d*x/
2)**20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 3
7800*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*
tan(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 +
d*x/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 +
315*a*d) - 12096*tan(c/2 + d*x/2)**10/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*
a*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c
/2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/
2)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14
175*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 352
0*tan(c/2 + d*x/2)**9/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*
x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14
+ 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a
*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2
+ d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 6720*tan(c/2 + d*x/
2)**8/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 14175
*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan(
c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*x
/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 31
50*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 1440*tan(c/2 + d*x/2)**7/(315*a*d*t
an(c/2 + d*x/2)**20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d
*x/2)**16 + 37800*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12
 + 79380*a*d*tan(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a
*d*tan(c/2 + d*x/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 +
 d*x/2)**2 + 315*a*d) - 3360*tan(c/2 + d*x/2)**6/(315*a*d*tan(c/2 + d*x/2)*
*20 + 3150*a*d*tan(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 3780
0*a*d*tan(c/2 + d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan
(c/2 + d*x/2)**10 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x
/2)**6 + 14175*a*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315
*a*d) + 2016*tan(c/2 + d*x/2)**5/(315*a*d*tan(c/2 + d*x/2)**20 + 3150*a*d*t
an(c/2 + d*x/2)**18 + 14175*a*d*tan(c/2 + d*x/2)**16 + 37800*a*d*tan(c/2 +
d*x/2)**14 + 66150*a*d*tan(c/2 + d*x/2)**12 + 79380*a*d*tan(c/2 + d*x/2)**1
0 + 66150*a*d*tan(c/2 + d*x/2)**8 + 37800*a*d*tan(c/2 + d*x/2)**6 + 14175*a
*d*tan(c/2 + d*x/2)**4 + 3150*a*d*tan(c/2 + d*x/2)**2 + 315*a*d), Ne(d, 0))
, (x*sin(c)**4*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A]

time = 0.44, size = 69, normalized size = 0.63

$$-\frac{126 \sin (dx+c)^{10}-140 \sin (dx+c)^{9}-315 \sin (dx+c)^{8}+360 \sin (dx+c)^{7}+210 \sin (dx+c)^{6}-252 \sin (dx+c)^{5}}{1260\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(126*sin(d*x + c)^10 - 140*sin(d*x + c)^9 - 315*sin(d*x + c)^8 + 36
0*sin(d*x + c)^7 + 210*sin(d*x + c)^6 - 252*sin(d*x + c)^5)/(a*d)
```

Mupad [B]
time = 8.94, size = 83, normalized size = 0.76

$$\frac{\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{6\,a}-\frac{2\sin(c+d\,x)^7}{7\,a}+\frac{\sin(c+d\,x)^8}{4\,a}+\frac{\sin(c+d\,x)^9}{9\,a}-\frac{\sin(c+d\,x)^{10}}{10\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a) - (2*sin(c + d*x)^7)/(7*a) + s
in(c + d*x)^8/(4*a) + sin(c + d*x)^9/(9*a) - sin(c + d*x)^10/(10*a))/d
```

3.680 $\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cos^6(c+dx)}{6ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^9(c+dx)}{9ad}$$

```
[Out] -1/6*cos(d*x+c)^6/a/d+1/8*cos(d*x+c)^8/a/d-1/5*sin(d*x+c)^5/a/d+2/7*sin(d*x
+c)^7/a/d-1/9*sin(d*x+c)^9/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2914, 2645, 14, 2644, 276}

$$-\frac{\sin^9(c+dx)}{9ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*Cos[c + d*x]^6/(a*d) + Cos[c + d*x]^8/(8*a*d) - Sin[c + d*x]^5/(5*a*d)
 + (2*Sin[c + d*x]^7)/(7*a*d) - Sin[c + d*x]^9/(9*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a}-\frac{\int \cos^5(c+dx)\sin^4(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^5(1-x^2)\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^4(1-x^2)^2\,dx,x,\text{s}\right.}{ad}\\ &= -\frac{\text{Subst}\left(\int (x^5-x^7)\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx,x\right.}{ad}\\ &= -\frac{\cos^6(c+dx)}{6ad}+\frac{\cos^8(c+dx)}{8ad}-\frac{\sin^5(c+dx)}{5ad}+\frac{2\sin^7(c+dx)}{7ad}-\frac{\sin^9(c}{9a}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 68, normalized size = 0.75

$$\frac{\sin^4(c+dx)\left(630-504\sin(c+dx)-840\sin^2(c+dx)+720\sin^3(c+dx)+315\sin^4(c+dx)-280\sin^5(c+dx)\right)}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^4*(630 - 504*Sin[c + d*x] - 840*Sin[c + d*x]^2 + 720*Sin[c +
d*x]^3 + 315*Sin[c + d*x]^4 - 280*Sin[c + d*x]^5))/(2520*a*d)
```

Maple [A]
time = 0.13, size = 69, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{\left(\sin^9(dx+c)\right)}{9}+\frac{\left(\sin^8(dx+c)\right)}{8}+\frac{2\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{3}-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{da}$

default	$\frac{-\frac{\left(\sin^9(dx+c)\right)}{9}+\frac{\left(\sin^8(dx+c)\right)}{8}+\frac{2\left(\sin^7(dx+c)\right)}{7}-\frac{\left(\sin^6(dx+c)\right)}{3}-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}}{da}$
risch	$-\frac{3\sin(dx+c)}{128ad}-\frac{\sin(9dx+9c)}{2304ad}+\frac{\cos(8dx+8c)}{1024ad}-\frac{\sin(7dx+7c)}{1792ad}+\frac{\cos(6dx+6c)}{384ad}+\frac{\sin(5dx+5c)}{320ad}-\frac{\cos(4dx+4c)}{256ad}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/9*sin(d*x+c)^9+1/8*sin(d*x+c)^8+2/7*sin(d*x+c)^7-1/3*sin(d*x+c)^6
-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.76

$$-\frac{280\sin\left(dx+c\right)^{9}-315\sin\left(dx+c\right)^{8}-720\sin\left(dx+c\right)^{7}+840\sin\left(dx+c\right)^{6}+504\sin\left(dx+c\right)^{5}-630\sin\left(dx+c\right)^{4}}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

Fricas [A]
time = 0.37, size = 79, normalized size = 0.87

$$\frac{315\cos\left(dx+c\right)^{8}-420\cos\left(dx+c\right)^{6}-8\left(35\cos\left(dx+c\right)^{8}-50\cos\left(dx+c\right)^{6}+3\cos\left(dx+c\right)^{4}+4\cos\left(dx+c\right)^{2}+8\right)\sin\left(dx+c\right)}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*cos(d*x + c)^8 - 420*cos(d*x + c)^6 - 8*(35*cos(d*x + c)^8 - 50
*cos(d*x + c)^6 + 3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*x + c))/(a
*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(68) = 136$.
time = 100.93, size = 1906, normalized size = 20.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((1260*tan(c/2 + d*x/2)**14/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a
*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/
```

2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2
)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835
*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 2016*tan(c/2 + d*x/2)**13/(315*a*d*ta
n(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*
x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10
 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d
*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 420*tan(c/
2 + d*x/2)**12/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**1
6 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690
*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c
/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)*
*2 + 315*a*d) + 3456*tan(c/2 + d*x/2)**11/(315*a*d*tan(c/2 + d*x/2)**18 + 2
835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*t
an(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 +
d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 +
 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 2520*tan(c/2 + d*x/2)**10/(315*a
*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2
 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)
10 + 39690*a*d*tan(c/2 + d*x/2)8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 1134
0*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 6976*
tan(c/2 + d*x/2)**9/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/
2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 +
39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*
tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*
x/2)**2 + 315*a*d) + 2520*tan(c/2 + d*x/2)**8/(315*a*d*tan(c/2 + d*x/2)**18
 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a
*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/
2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)*
*4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 3456*tan(c/2 + d*x/2)**7/(31
5*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(
c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x
/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 1
1340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) - 42
0*tan(c/2 + d*x/2)**6/(315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*
x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12
 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*
d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 +
d*x/2)**2 + 315*a*d) - 2016*tan(c/2 + d*x/2)**5/(315*a*d*tan(c/2 + d*x/2)**
18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*tan(c/2 + d*x/2)**14 + 26460
*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d*x/2)**10 + 39690*a*d*tan(
c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 + 11340*a*d*tan(c/2 + d*x/2
)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d) + 1260*tan(c/2 + d*x/2)**4/(
315*a*d*tan(c/2 + d*x/2)**18 + 2835*a*d*tan(c/2 + d*x/2)**16 + 11340*a*d*ta
n(c/2 + d*x/2)**14 + 26460*a*d*tan(c/2 + d*x/2)**12 + 39690*a*d*tan(c/2 + d
*x/2)**10 + 39690*a*d*tan(c/2 + d*x/2)**8 + 26460*a*d*tan(c/2 + d*x/2)**6 +

```
11340*a*d*tan(c/2 + d*x/2)**4 + 2835*a*d*tan(c/2 + d*x/2)**2 + 315*a*d), N
e(d, 0)), (x*sin(c)**3*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A]
time = 0.46, size = 69, normalized size = 0.76

$$-\frac{280 \sin\left(dx+c\right)^{9} - 315 \sin\left(dx+c\right)^{8} - 720 \sin\left(dx+c\right)^{7} + 840 \sin\left(dx+c\right)^{6} + 504 \sin\left(dx+c\right)^{5} - 630 \sin\left(dx+c\right)^{4}}{2520\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2520*(280*sin(d*x + c)^9 - 315*sin(d*x + c)^8 - 720*sin(d*x + c)^7 + 840
*sin(d*x + c)^6 + 504*sin(d*x + c)^5 - 630*sin(d*x + c)^4)/(a*d)
```

Mupad [B]
time = 0.06, size = 83, normalized size = 0.91

$$\frac{\frac{\sin(c+d\,x)^4}{4\,a} - \frac{\sin(c+d\,x)^5}{5\,a} - \frac{\sin(c+d\,x)^6}{3\,a} + \frac{2\sin(c+d\,x)^7}{7\,a} + \frac{\sin(c+d\,x)^8}{8\,a} - \frac{\sin(c+d\,x)^9}{9\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4/(4*a) - sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(3*a) + (2*si
n(c + d*x)^7)/(7*a) + sin(c + d*x)^8/(8*a) - sin(c + d*x)^9/(9*a))/d
```

3.681 $\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cos^6(c+dx)}{6ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] 1/6*cos(d*x+c)^6/a/d-1/8*cos(d*x+c)^8/a/d+1/3*sin(d*x+c)^3/a/d-2/5*sin(d*x+
c)^5/a/d+1/7*sin(d*x+c)^7/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2914, 2644, 276, 2645, 14}

$$\frac{\sin^7(c+dx)}{7ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^6/(6*a*d) - Cos[c + d*x]^8/(8*a*d) + Sin[c + d*x]^3/(3*a*d) -
(2*Sin[c + d*x]^5)/(5*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a}-\frac{\int \cos^5(c+dx)\sin^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^5(1-x^2)\,dx,x,\cos(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2(1-x^2)^2\,dx,x,\sin(c\right.}{ad}\\ &= \frac{\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int (x^5-x^7)\,dx,x,\text{co}\right.}{ad}\\ &= \frac{\cos^6(c+dx)}{6ad}-\frac{\cos^8(c+dx)}{8ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{2\sin^5(c+dx)}{5ad}+\frac{\sin^7(c+d}{7ad}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 68, normalized size = 0.75

$$\frac{\sin^3(c+dx)\left(280-210\sin(c+dx)-336\sin^2(c+dx)+280\sin^3(c+dx)+120\sin^4(c+dx)-105\sin^5(c+dx)\right)}{840ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^3*(280 - 210*Sin[c + d*x] - 336*Sin[c + d*x]^2 + 280*Sin[c +
d*x]^3 + 120*Sin[c + d*x]^4 - 105*Sin[c + d*x]^5))/(840*a*d)
```

Maple [A]
time = 0.14, size = 69, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{\left(\sin^8(dx+c)\right)}{8}+\frac{\left(\sin^7(dx+c)\right)}{7}+\frac{\left(\sin^6(dx+c)\right)}{3}-\frac{2\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}}{da}$

default	$\frac{-\frac{\left(\sin^8(dx+c)\right)}{8}+\frac{\left(\sin^7(dx+c)\right)}{7}+\frac{\left(\sin^6(dx+c)\right)}{3}-\frac{2\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}}{da}$
risch	$\frac{5\sin(dx+c)}{64ad}-\frac{\cos(8dx+8c)}{1024ad}-\frac{\sin(7dx+7c)}{448ad}-\frac{\cos(6dx+6c)}{384ad}-\frac{3\sin(5dx+5c)}{320ad}+\frac{\cos(4dx+4c)}{256ad}-\frac{\sin(3dx+3c)}{192ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/8*sin(d*x+c)^8+1/7*sin(d*x+c)^7+1/3*sin(d*x+c)^6-2/5*sin(d*x+c)^5
-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.76

$$-\frac{105\sin\left(dx+c\right)^{8}-120\sin\left(dx+c\right)^{7}-280\sin\left(dx+c\right)^{6}+336\sin\left(dx+c\right)^{5}+210\sin\left(dx+c\right)^{4}-280\sin\left(dx+c\right)^{3}}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

Fricas [A]
time = 0.37, size = 69, normalized size = 0.76

$$-\frac{105\cos\left(dx+c\right)^{8}-140\cos\left(dx+c\right)^{6}+8\left(15\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}-8\right)\sin\left(dx+c\right)}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(105*cos(d*x + c)^8 - 140*cos(d*x + c)^6 + 8*(15*cos(d*x + c)^6 - 3*
cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1719 vs. $2(68) = 136$.
time = 74.92, size = 1719, normalized size = 18.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((280*tan(c/2 + d*x/2)**13/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d
*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 +
d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 +
```

```
2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 420
*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x
/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7
350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c
/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 56*tan(c/2 + d*x/
2)**11/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*
a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2
 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4
+ 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 560*tan(c/2 + d*x/2)**10/(105*a*
d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 +
d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 +
 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(
c/2 + d*x/2)**2 + 105*a*d) + 688*tan(c/2 + d*x/2)**9/(105*a*d*tan(c/2 + d*x
/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 58
80*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c
/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2
 + 105*a*d) - 1400*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**16 + 840*
a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2
 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6
 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) +
688*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d
*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 +
 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan
(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 560*tan(c/2 + d
*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 294
0*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c
/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**
4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 56*tan(c/2 + d*x/2)**5/(105*a*
d*tan(c/2 + d*x/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 +
d*x/2)**12 + 5880*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 +
 5880*a*d*tan(c/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(
c/2 + d*x/2)**2 + 105*a*d) - 420*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x
/2)**16 + 840*a*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 58
80*a*d*tan(c/2 + d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c
/2 + d*x/2)**6 + 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2
 + 105*a*d) + 280*tan(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**16 + 840*a
*d*tan(c/2 + d*x/2)**14 + 2940*a*d*tan(c/2 + d*x/2)**12 + 5880*a*d*tan(c/2
+ d*x/2)**10 + 7350*a*d*tan(c/2 + d*x/2)**8 + 5880*a*d*tan(c/2 + d*x/2)**6
+ 2940*a*d*tan(c/2 + d*x/2)**4 + 840*a*d*tan(c/2 + d*x/2)**2 + 105*a*d), Ne
(d, 0)), (x*sin(c)**2*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A]
time = 0.48, size = 69, normalized size = 0.76

$$-\frac{105\,\sin\left(dx+c\right)^{8}-120\,\sin\left(dx+c\right)^{7}-280\,\sin\left(dx+c\right)^{6}+336\,\sin\left(dx+c\right)^{5}+210\,\sin\left(dx+c\right)^{4}-280\,\sin\left(dx+c\right)^{3}}{840\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(105*sin(d*x + c)^8 - 120*sin(d*x + c)^7 - 280*sin(d*x + c)^6 + 336*
sin(d*x + c)^5 + 210*sin(d*x + c)^4 - 280*sin(d*x + c)^3)/(a*d)
```

Mupad [B]
time = 8.96, size = 83, normalized size = 0.91

$$\frac{\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{4\,a}-\frac{2\,\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{3\,a}+\frac{\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^8}{8\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(4*a) - (2*sin(c + d*x)^5)/(5*a) + s
in(c + d*x)^6/(3*a) + sin(c + d*x)^7/(7*a) - sin(c + d*x)^8/(8*a))/d
```

3.682 $\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$-\frac{\cos^6(c+dx)}{6ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^7(c+dx)}{7ad}$$

```
[Out] -1/6*cos(d*x+c)^6/a/d-1/3*sin(d*x+c)^3/a/d+2/5*sin(d*x+c)^5/a/d-1/7*sin(d*x
+c)^7/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2914, 2645, 30, 2644, 276}

$$-\frac{\sin^7(c+dx)}{7ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^3(c+dx)}{3ad}-\frac{\cos^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*Cos[c + d*x]^6/(a*d) - Sin[c + d*x]^3/(3*a*d) + (2*Sin[c + d*x]^5)/(5*
a*d) - Sin[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^5(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^5(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^5\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^6(c+dx)}{6ad}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(c+dx)\right)}{ad}\\ &= -\frac{\cos^6(c+dx)}{6ad}-\frac{\sin^3(c+dx)}{3ad}+\frac{2\sin^5(c+dx)}{5ad}-\frac{\sin^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 68, normalized size = 0.93

$$\frac{\sin^2(c+dx)\left(105-70\sin(c+dx)-105\sin^2(c+dx)+84\sin^3(c+dx)+35\sin^4(c+dx)-30\sin^5(c+dx)\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^2*(105 - 70*Sin[c + d*x] - 105*Sin[c + d*x]^2 + 84*Sin[c + d*
x]^3 + 35*Sin[c + d*x]^4 - 30*Sin[c + d*x]^5))/(210*a*d)
```

Maple [A]
time = 0.23, size = 69, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{\left(\sin^7(dx+c)\right)}{7}+\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{2\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{2}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{da}$
default	$\frac{-\frac{\left(\sin^7(dx+c)\right)}{7}+\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{2\left(\sin^5(dx+c)\right)}{5}-\frac{\left(\sin^4(dx+c)\right)}{2}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}}{da}$

risch	$-\frac{5\sin(dx+c)}{64ad}+\frac{\sin(7dx+7c)}{448ad}-\frac{\cos(6dx+6c)}{192ad}+\frac{3\sin(5dx+5c)}{320ad}-\frac{\cos(4dx+4c)}{32ad}+\frac{\sin(3dx+3c)}{192ad}-\frac{5\cos(2dx+2c)}{64ad}$
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{4\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{52\left(\tan^5\right.}{1}}{(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/7*sin(d*x+c)^7+1/6*sin(d*x+c)^6+2/5*sin(d*x+c)^5-1/2*sin(d*x+c)^4
-1/3*sin(d*x+c)^3+1/2*sin(d*x+c)^2)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.95

$$-\frac{30\sin\left(dx+c\right)^{7}-35\sin\left(dx+c\right)^{6}-84\sin\left(dx+c\right)^{5}+105\sin\left(dx+c\right)^{4}+70\sin\left(dx+c\right)^{3}-105\sin\left(dx+c\right)^{2}}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin
(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.81

$$-\frac{35\cos\left(dx+c\right)^{6}-2\left(15\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}-8\right)\sin\left(dx+c\right)}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*cos(d*x + c)^6 - 2*(15*cos(d*x + c)^6 - 3*cos(d*x + c)^4 - 4*cos
(d*x + c)^2 - 8)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. $2(54) = 108$.
time = 45.57, size = 1530, normalized size = 20.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d
*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
```

```
d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 7
35*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*tan(c/2 + d*x/2)**11/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 210*ta
n(c/2 + d*x/2)**10/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)
**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*
a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 +
d*x/2)**2 + 105*a*d) + 224*tan(c/2 + d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**1
4 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d
*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*
x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 700*tan(c/2 + d*x/2)**8/
(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan
(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2
)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d
) - 912*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) + 700*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 224*tan(c/2 + d
*x/2)**5/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 220
5*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/
2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2
+ 105*a*d) + 210*tan(c/2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*
d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 +
 d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*tan(c/2 + d*x/2)**3/(105*a*d*t
an(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x
/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 220
5*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 210*ta
n(c/2 + d*x/2)**2/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)*
*12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a
*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d
*x/2)**2 + 105*a*d), Ne(d, 0)), (x*sin(c)*cos(c)**7/(a*sin(c) + a), True))
```

Giac [A]

time = 0.43, size = 69, normalized size = 0.95

$$-\frac{30\,\sin\left(dx+c\right)^{7}-35\,\sin\left(dx+c\right)^{6}-84\,\sin\left(dx+c\right)^{5}+105\,\sin\left(dx+c\right)^{4}+70\,\sin\left(dx+c\right)^{3}-105\,\sin\left(dx+c\right)^{2}}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/210*(30*sin(d*x + c)^7 - 35*sin(d*x + c)^6 - 84*sin(d*x + c)^5 + 105*sin
(d*x + c)^4 + 70*sin(d*x + c)^3 - 105*sin(d*x + c)^2)/(a*d)
```

Mupad [B]
time = 9.13, size = 83, normalized size = 1.14

$$\frac{\frac{\sin(c+d\,x)^2}{2\,a}-\frac{\sin(c+d\,x)^3}{3\,a}-\frac{\sin(c+d\,x)^4}{2\,a}+\frac{2\sin(c+d\,x)^5}{5\,a}+\frac{\sin(c+d\,x)^6}{6\,a}-\frac{\sin(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2/(2*a) - sin(c + d*x)^3/(3*a) - sin(c + d*x)^4/(2*a) + (2*si
n(c + d*x)^5)/(5*a) + sin(c + d*x)^6/(6*a) - sin(c + d*x)^7/(7*a))/d
```

3.683 $\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{(a-a\sin(c+dx))^4}{a^5d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^6}{6a^7d}$$

```
[Out] -(a-a*sin(d*x+c))^4/a^5/d+4/5*(a-a*sin(d*x+c))^5/a^6/d-1/6*(a-a*sin(d*x+c))
^6/a^7/d
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2746, 45}

$$-\frac{(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a - a*Sin[c + d*x])^4/(a^5*d)) + (4*(a - a*Sin[c + d*x])^5)/(5*a^6*d) -
(a - a*Sin[c + d*x])^6/(6*a^7*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= \frac{\text{Subst}\left(\int \left(4a^2(a-x)^3-4a(a-x)^4+(a-x)^5\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}\\
&= -\frac{(a-a\sin(c+dx))^4}{a^5d}+\frac{4(a-a\sin(c+dx))^5}{5a^6d}-\frac{(a-a\sin(c+dx))^6}{6a^7d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 66, normalized size = 0.97

$$-\frac{\sin(c+dx)\left(-30+15\sin(c+dx)+20\sin^2(c+dx)-15\sin^3(c+dx)-6\sin^4(c+dx)+5\sin^5(c+dx)\right)}{30ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/30*(Sin[c + d*x]*(-30 + 15*Sin[c + d*x] + 20*Sin[c + d*x]^2 - 15*Sin[c +
 d*x]^3 - 6*Sin[c + d*x]^4 + 5*Sin[c + d*x]^5))/(a*d)
```

Maple [A]
time = 0.23, size = 65, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{2}-\frac{2\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
default	$\frac{-\frac{\left(\sin^6(dx+c)\right)}{6}+\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{2}-\frac{2\left(\sin^3(dx+c)\right)}{3}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
risch	$\frac{5\sin(dx+c)}{8ad}+\frac{\cos(6dx+6c)}{192ad}+\frac{\sin(5dx+5c)}{80ad}+\frac{\cos(4dx+4c)}{32ad}+\frac{5\sin(3dx+3c)}{48ad}+\frac{5\cos(2dx+2c)}{64ad}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{14\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{14\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{14\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{42(\tan^6}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7(\text{ta}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/6*sin(d*x+c)^6+1/5*sin(d*x+c)^5+1/2*sin(d*x+c)^4-2/3*sin(d*x+c)^3
-1/2*sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 67, normalized size = 0.99

$$-\frac{5\sin(dx+c)^6-6\sin(dx+c)^5-15\sin(dx+c)^4+20\sin(dx+c)^3+15\sin(dx+c)^2-30\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

Fricas [A]
time = 0.38, size = 49, normalized size = 0.72

$$\frac{5\cos(dx+c)^6+2\left(3\cos(dx+c)^4+4\cos(dx+c)^2+8\right)\sin(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(5*cos(d*x + c)^6 + 2*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sin(d*
x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(54) = 108$.
time = 24.65, size = 1096, normalized size = 16.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*tan(c/2 + d*x/2)**11/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*ta
n(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)
**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) -
30*tan(c/2 + d*x/2)**10/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x
/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a
*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 70*tan(c/2
+ d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 22
5*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 +
 d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**7
/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c
/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4
+ 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 100*tan(c/2 + d*x/2)**6/(15*a*d*ta
n(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)
**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*ta
n(c/2 + d*x/2)**2 + 15*a*d) + 156*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x
/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a
*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x
/2)**2 + 15*a*d) + 70*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**12 + 90
*a*d*tan(c/2 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 +
 d*x/2)**6 + 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*
a*d) - 30*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2
 + d*x/2)**10 + 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 +
 225*a*d*tan(c/2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*ta
n(c/2 + d*x/2)/(15*a*d*tan(c/2 + d*x/2)**12 + 90*a*d*tan(c/2 + d*x/2)**10 +
 225*a*d*tan(c/2 + d*x/2)**8 + 300*a*d*tan(c/2 + d*x/2)**6 + 225*a*d*tan(c/
2 + d*x/2)**4 + 90*a*d*tan(c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*cos(c)*
*7/(a*sin(c) + a), True))
```

Giac [A]
time = 0.45, size = 67, normalized size = 0.99

$$-\frac{5\,\sin\left(dx+c\right)^{6}-6\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}+20\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)^{2}-30\,\sin\left(dx+c\right)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*sin(d*x + c)^6 - 6*sin(d*x + c)^5 - 15*sin(d*x + c)^4 + 20*sin(d*x
 + c)^3 + 15*sin(d*x + c)^2 - 30*sin(d*x + c))/(a*d)
```

Mupad [B]
time = 9.14, size = 80, normalized size = 1.18

$$\frac{\frac{\sin(c+d\,x)}{a}-\frac{\sin(c+d\,x)^2}{2\,a}-\frac{2\sin(c+d\,x)^3}{3\,a}+\frac{\sin(c+d\,x)^4}{2\,a}+\frac{\sin(c+d\,x)^5}{5\,a}-\frac{\sin(c+d\,x)^6}{6\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)/a - sin(c + d*x)^2/(2*a) - (2*sin(c + d*x)^3)/(3*a) + sin(c +
 d*x)^4/(2*a) + sin(c + d*x)^5/(5*a) - sin(c + d*x)^6/(6*a))/d
```

3.684 $\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{ad}+\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^4(c+dx)}{4ad}-\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d-sin(d*x+c)^2/a/d+2/3*sin(d*x+c)^3/a/d+1/4
*sin(d*x+c)^4/a/d-1/5*sin(d*x+c)^5/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}+\frac{2\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{ad}-\frac{\sin(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(a*d) + (2*Si
n[c + d*x]^3)/(3*a*d) + Sin[c + d*x]^4/(4*a*d) - Sin[c + d*x]^5/(5*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x}\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^5}{x}-2a^3x+2a^2x^2+ax^3-x^4\right)\,dx, x, a\sin(c+dx)\right)}{a^6 d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\sin(c+dx)}{ad} - \frac{\sin^2(c+dx)}{ad} + \frac{2\sin^3(c+dx)}{3ad} + \frac{\sin^4(c+}{4ad}$$

Mathematica [A]
time = 0.05, size = 68, normalized size = 0.69

$$\frac{60\log(\sin(c+dx)) - 60\sin(c+dx) - 60\sin^2(c+dx) + 40\sin^3(c+dx) + 15\sin^4(c+dx) - 12\sin^5(c+dx)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (60*Log[Sin[c + d*x]] - 60*Sin[c + d*x] - 60*Sin[c + d*x]^2 + 40*Sin[c + d*
x]^3 + 15*Sin[c + d*x]^4 - 12*Sin[c + d*x]^5)/(60*a*d)
```

Maple [A]
time = 0.23, size = 64, normalized size = 0.65

method	result
derivativedivides	$\frac{-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{2\left(\sin^3(dx+c)\right)}{3}-\left(\sin^2(dx+c)\right)-\sin(dx+c)+\ln(\sin(dx+c))}{da}$
default	$\frac{-\frac{\left(\sin^5(dx+c)\right)}{5}+\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{2\left(\sin^3(dx+c)\right)}{3}-\left(\sin^2(dx+c)\right)-\sin(dx+c)+\ln(\sin(dx+c))}{da}$
risch	$-\frac{ix}{a}+\frac{3\,e^{2i(dx+c)}}{16ad}+\frac{3\,e^{-2i(dx+c)}}{16ad}-\frac{2ic}{ad}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}-\frac{5\sin(dx+c)}{8ad}-\frac{\sin(5dx+5c)}{80ad}+\frac{\cos(4dx+4c)}{32ad}-$
norman	$\frac{\frac{2}{ad}+\frac{2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{10\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{10\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{40\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/5*sin(d*x+c)^5+1/4*sin(d*x+c)^4+2/3*sin(d*x+c)^3-sin(d*x+c)^2-sin
(d*x+c)+ln(sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.72

$$-\frac{\frac{12 \sin(dx+c)^5-15 \sin(dx+c)^4-40 \sin(dx+c)^3+60 \sin(dx+c)^2+60 \sin(dx+c)}{a}-\frac{60 \log(\sin(dx+c))}{a}}{60\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 40*sin(d*x + c)^3 + 60*sin(
d*x + c)^2 + 60*sin(d*x + c))/a - 60*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.39, size = 70, normalized size = 0.71

$$\frac{15 \cos(dx+c)^4+30 \cos(dx+c)^2-4 \left(3 \cos(dx+c)^4+4 \cos(dx+c)^2+8\right) \sin(dx+c)+60 \log\left(\frac{1}{2} \sin(dx+c)\right)}{60\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^4 + 4*cos(d
*x + c)^2 + 8)*sin(d*x + c) + 60*log(1/2*sin(d*x + c)))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 88, normalized size = 0.89

$$\frac{\frac{60 \log(|\sin(dx+c)|)}{a}-\frac{12 a^4 \sin(dx+c)^5-15 a^4 \sin(dx+c)^4-40 a^4 \sin(dx+c)^3+60 a^4 \sin(dx+c)^2+60 a^4 \sin(dx+c)}{a^5}}{60\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(sin(d*x + c)))/a - (12*a^4*sin(d*x + c)^5 - 15*a^4*sin(d*x
 + c)^4 - 40*a^4*sin(d*x + c)^3 + 60*a^4*sin(d*x + c)^2 + 60*a^4*sin(d*x +
c))/a^5)/d
```

Mupad [B]
time = 9.29, size = 140, normalized size = 1.41

$$\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a\,d}-\frac{\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{a\,d}-\frac{8\,\sin\left(c+d\,x\right)}{15\,a\,d}+\frac{{\cos\left(c+d\,x\right)}^2}{2\,a\,d}+\frac{{\cos\left(c+d\,x\right)}^4}{4\,a\,d}-\frac{4\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{15\,a\,d}-\frac{{\cos\left(c+d\,x\right)}^4\,\sin\left(c+d\,x\right)}{5\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - log(1/cos(c/2 + (d*x)/2)
^2)/(a*d) - (8*sin(c + d*x))/(15*a*d) + cos(c + d*x)^2/(2*a*d) + cos(c + d*
x)^4/(4*a*d) - (4*cos(c + d*x)^2*sin(c + d*x))/(15*a*d) - (cos(c + d*x)^4*s
in(c + d*x))/(5*a*d)
```

3.685 $\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^4(c+dx)}{4ad}$$

```
[Out] -csc(d*x+c)/a/d-ln(sin(d*x+c))/a/d-2*sin(d*x+c)/a/d+sin(d*x+c)^2/a/d+1/3*si
n(d*x+c)^3/a/d-1/4*sin(d*x+c)^4/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin^4(c+dx)}{4ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{ad}-\frac{2\sin(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[Sin[c + d*x]]/(a*d) - (2*Sin[c + d*x])/(a*d) +
Sin[c + d*x]^2/(a*d) + Sin[c + d*x]^3/(3*a*d) - Sin[c + d*x]^4/(4*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^2}\,dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^3+\frac{a^5}{x^2}-\frac{a^4}{x}+2a^2x+ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{a^5d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}-\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{ad}+\frac{\sin^3(c}{3a}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.69

$$-\frac{12\csc(c+dx)+12\log(\sin(c+dx))+24\sin(c+dx)-12\sin^2(c+dx)-4\sin^3(c+dx)+3\sin^4(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/12*(12*Csc[c + d*x] + 12*Log[Sin[c + d*x]] + 24*Sin[c + d*x] - 12*Sin[c
+ d*x]^2 - 4*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4)/(a*d)
```

Maple [A]
time = 0.25, size = 64, normalized size = 0.67

method	result
derivativedivides	$\frac{-\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}+\sin^2(dx+c)-2\sin(dx+c)}{da}$
default	$\frac{-\ln(\sin(dx+c))-\frac{1}{\sin(dx+c)}-\frac{\left(\sin^4(dx+c)\right)}{4}+\frac{\left(\sin^3(dx+c)\right)}{3}+\sin^2(dx+c)-2\sin(dx+c)}{da}$
risch	$\frac{ix}{a}-\frac{3\,\mathrm{e}^{2i(dx+c)}}{16ad}+\frac{7i\mathrm{e}^{i(dx+c)}}{8ad}-\frac{7i\mathrm{e}^{-i(dx+c)}}{8ad}-\frac{3\,\mathrm{e}^{-2i(dx+c)}}{16ad}+\frac{2ic}{ad}-\frac{2i\mathrm{e}^{i(dx+c)}}{da(\mathrm{e}^{2i(dx+c)}-1)}-\frac{\ln(\mathrm{e}^{2i(dx+c)}-1)}{ad}-$
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{13\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{13\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{43\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{43\left(\tan^9\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-ln(sin(d*x+c))-1/sin(d*x+c)-1/4*sin(d*x+c)^4+1/3*sin(d*x+c)^3+sin(d
*x+c)^2-2*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 74, normalized size = 0.78

$$-\frac{\frac{3 \sin(dx+c)^4-4 \sin(dx+c)^3-12 \sin(dx+c)^2+24 \sin(dx+c)}{a}+\frac{12 \log(\sin(dx+c))}{a}+\frac{12}{a \sin(dx+c)}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/12*((3*sin(d*x + c)^4 - 4*sin(d*x + c)^3 - 12*sin(d*x + c)^2 + 24*sin(d*
x + c))/a + 12*log(sin(d*x + c))/a + 12/(a*sin(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.89

$$\frac{32 \cos(dx+c)^4+128 \cos(dx+c)^2-3 \left(8 \cos(dx+c)^4+16 \cos(dx+c)^2-11\right) \sin(dx+c)-96 \log\left(\frac{1}{2} \sin(dx+c)\right) \sin(dx+c)-256}{96\, ad \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/96*(32*cos(d*x + c)^4 + 128*cos(d*x + c)^2 - 3*(8*cos(d*x + c)^4 + 16*cos
(d*x + c)^2 - 11)*sin(d*x + c) - 96*log(1/2*sin(d*x + c))*sin(d*x + c) - 25
6)/(a*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.47, size = 95, normalized size = 1.00

$$-\frac{\frac{12 \log(|\sin(dx+c)|)}{a}-\frac{12 (\sin(dx+c)-1)}{a \sin(dx+c)}+\frac{3 a^3 \sin(dx+c)^4-4 a^3 \sin(dx+c)^3-12 a^3 \sin(dx+c)^2+24 a^3 \sin(dx+c)}{a^4}}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/12*(12*log(abs(sin(d*x + c)))/a - 12*(sin(d*x + c) - 1)/(a*sin(d*x + c))
 + (3*a^3*sin(d*x + c)^4 - 4*a^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 2
4*a^3*sin(d*x + c))/a^4)/d
```

Mupad [B]
time = 9.36, size = 272, normalized size = 2.86

$$\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a\,d}-\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a\,d}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{a\,d}-\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{a\,d}+\frac{\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{a\,d}-\frac{\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a\,d}+\frac{20\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,a\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3\,a\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3\,a\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{9\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^2)/(a*d) - (8*cos(c/2 + (d*x)/2)^4)/(a*d) + (8*cos(c/
2 + (d*x)/2)^6)/(a*d) - (4*cos(c/2 + (d*x)/2)^8)/(a*d) + log(1/cos(c/2 + (d
*x)/2)^2)/(a*d) - log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) + (20*co
s(c/2 + (d*x)/2)^3)/(3*a*d*sin(c/2 + (d*x)/2)) - (16*cos(c/2 + (d*x)/2)^5)/
(3*a*d*sin(c/2 + (d*x)/2)) + (8*cos(c/2 + (d*x)/2)^7)/(3*a*d*sin(c/2 + (d*x
)/2)) - (9*cos(c/2 + (d*x)/2))/(2*a*d*sin(c/2 + (d*x)/2)) - sin(c/2 + (d*x)
/2)/(2*a*d*cos(c/2 + (d*x)/2))
```

3.686 $\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}-\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-2*ln(sin(d*x+c))/a/d+2*sin(d*x+c)/a/d+1
/2*sin(d*x+c)^2/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{2\sin(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (2*Log[Sin[c + d*x]])/(a*d) +
 (2*Sin[c + d*x])/(a*d) + Sin[c + d*x]^2/(2*a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(2a^2+\frac{a^5}{x^3}-\frac{a^4}{x^2}-\frac{2a^3}{x}+ax-x^2\right)\,dx, x, a\sin(c+dx)\right)}{a^4 d}$$

$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{2\log(\sin(c+dx))}{ad}+\frac{2\sin(c+dx)}{ad}+\frac{\sin^2(c}{2a}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 0.68

$$\frac{6\csc(c+dx)-3\csc^2(c+dx)-12\log(\sin(c+dx))+12\sin(c+dx)+3\sin^2(c+dx)-2\sin^3(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6*Csc[c + d*x] - 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x]
 + 3*Sin[c + d*x]^2 - 2*Sin[c + d*x]^3)/(6*a*d)
```

Maple [A]
time = 0.28, size = 64, normalized size = 0.66

method	result
derivativedivides	$\frac{-2\ln(\sin(dx+c))-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}+2\sin(dx+c)}{da}$
default	$\frac{-2\ln(\sin(dx+c))-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}-\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{\left(\sin^2(dx+c)\right)}{2}+2\sin(dx+c)}{da}$
risch	$\frac{2ix}{a}-\frac{ie^{3i(dx+c)}}{24ad}-\frac{e^{2i(dx+c)}}{8ad}-\frac{7ie^{i(dx+c)}}{8ad}+\frac{7ie^{-i(dx+c)}}{8ad}-\frac{e^{-2i(dx+c)}}{8ad}+\frac{ie^{-3i(dx+c)}}{24ad}+\frac{4ic}{ad}+\frac{2i(-ie^{2i(dx+}}{ad(}$
norman	$\frac{\frac{6\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{52\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{52(\tan^7}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan\left(\frac{d}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2*ln(sin(d*x+c))-1/2/sin(d*x+c)^2+1/sin(d*x+c)-1/3*sin(d*x+c)^3+1/2
*sin(d*x+c)^2+2*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.76

$$-\frac{\frac{2\sin(dx+c)^3-3\sin(dx+c)^2-12\sin(dx+c)}{a}+\frac{12\log(\sin(dx+c))}{a}-\frac{3(2\sin(dx+c)-1)}{a\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*sin(d*x + c)^3 - 3*sin(d*x + c)^2 - 12*sin(d*x + c))/a + 12*log(si
n(d*x + c))/a - 3*(2*sin(d*x + c) - 1)/(a*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.38, size = 91, normalized size = 0.94

$$-\frac{6\cos(dx+c)^4-9\cos(dx+c)^2+24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4\left(\cos(dx+c)^4+4\cos(dx+c)^2-8\right)\sin(dx+c)-3}{12\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 24*(cos(d*x + c)^2 - 1)*log(1/
2*sin(d*x + c)) - 4*(cos(d*x + c)^4 + 4*cos(d*x + c)^2 - 8)*sin(d*x + c) -
3)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.45, size = 94, normalized size = 0.97

$$-\frac{\frac{12\log(|\sin(dx+c)|)}{a}+\frac{2a^2\sin(dx+c)^3-3a^2\sin(dx+c)^2-12a^2\sin(dx+c)}{a^3}-\frac{3\left(6\sin(dx+c)^2+2\sin(dx+c)-1\right)}{a\sin(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*log(abs(sin(d*x + c)))/a + (2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x +
 c)^2 - 12*a^2*sin(d*x + c))/a^3 - 3*(6*sin(d*x + c)^2 + 2*sin(d*x + c) - 1
)/(a*sin(d*x + c)^2))/d
```

Mupad [B]
time = 9.23, size = 231, normalized size = 2.38

$$\frac{18\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2}+\frac{82\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{2}}{d\left(4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}+\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2) - (3*tan(c/2 + (d*x)/2)^2)/2 + 22*tan(c/2 + (d*x)/2)^
3 + (13*tan(c/2 + (d*x)/2)^4)/2 + (82*tan(c/2 + (d*x)/2)^5)/3 + (15*tan(c/2
 + (d*x)/2)^6)/2 + 18*tan(c/2 + (d*x)/2)^7 - 1/2)/(d*(4*a*tan(c/2 + (d*x)/2
)^2 + 12*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^6 + 4*a*tan(c/2 +
 (d*x)/2)^8)) - (2*log(tan(c/2 + (d*x)/2)))/(a*d) - tan(c/2 + (d*x)/2)^2/(8
*a*d) + tan(c/2 + (d*x)/2)/(2*a*d) + (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d
)
```

3.687 $\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\log(\sin(c+dx))}{ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}$$

```
[Out] 2*csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d+2*ln(sin(d*x+c))
/a/d+sin(d*x+c)/a/d-1/2*sin(d*x+c)^2/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}+\frac{2\csc(c+dx)}{ad}+\frac{2\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) +
(2*Log[Sin[c + d*x]])/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^4(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^4}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^5}{x^4}-\frac{a^4}{x^3}-\frac{2a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\log(\sin(c+dx))}{ad}+\frac{\sin(c}{a}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 0.68

$$\frac{12\csc(c+dx)+3\csc^2(c+dx)-2\csc^3(c+dx)+12\log(\sin(c+dx))+6\sin(c+dx)-3\sin^2(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
[Out] (12*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 2*Csc[c + d*x]^3 + 12*Log[Sin[c + d*x
]] + 6*Sin[c + d*x] - 3*Sin[c + d*x]^2)/(6*a*d)
```

Maple [A]
time = 0.28, size = 64, normalized size = 0.66

method	result
derivativedivides	$\frac{2\ln(\sin(dx+c))+\frac{1}{2\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}+\frac{2}{\sin(dx+c)}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
default	$\frac{2\ln(\sin(dx+c))+\frac{1}{2\sin(dx+c)^2}-\frac{1}{3\sin(dx+c)^3}+\frac{2}{\sin(dx+c)}-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)}{da}$
risch	$-\frac{2ix}{a}+\frac{e^{2i(dx+c)}}{8ad}-\frac{ie^{i(dx+c)}}{2ad}+\frac{ie^{-i(dx+c)}}{2ad}+\frac{e^{-2i(dx+c)}}{8ad}-\frac{4ic}{ad}+\frac{2i\left(6\,e^{5i(dx+c)}-8\,e^{3i(dx+c)}+3ie^{4i(dx+c)}+6\,e\right.}{3da\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{7\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(2*ln(sin(d*x+c))+1/2/sin(d*x+c)^2-1/3/sin(d*x+c)^3+2/sin(d*x+c)-1/2*
sin(d*x+c)^2+sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.75

$$-\frac{\frac{3\left(\sin(dx+c)^2-2\sin(dx+c)\right)}{a}-\frac{12\log(\sin(dx+c))}{a}-\frac{12\sin(dx+c)^2+3\sin(dx+c)-2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 - 2*sin(d*x + c))/a - 12*log(sin(d*x + c))/a - (12*
sin(d*x + c)^2 + 3*sin(d*x + c) - 2)/(a*sin(d*x + c)^3))/d
```

Fricas [A]
time = 0.38, size = 107, normalized size = 1.10

$$-\frac{12\cos(dx+c)^4-24\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-48\cos(dx+c)^2-3\left(2\cos(dx+c)^4-3\cos(dx+c)^2-1\right)\sin(dx+c)+32}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^4 - 24*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c))*si
n(d*x + c) - 48*cos(d*x + c)^2 - 3*(2*cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 1
)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.44, size = 87, normalized size = 0.90

$$\frac{\frac{12\log(|\sin(dx+c)|)}{a}-\frac{3\left(a\sin(dx+c)^2-2a\sin(dx+c)\right)}{a^2}-\frac{22\sin(dx+c)^3-12\sin(dx+c)^2-3\sin(dx+c)+2}{a\sin(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*log(abs(sin(d*x + c)))/a - 3*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
a^2 - (22*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 3*sin(d*x + c) + 2)/(a*sin(d
*x + c)^3))/d
```

Mupad [B]
time = 9.17, size = 221, normalized size = 2.28

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}+\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{89\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{3}}{d\left(8a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) + (2*log(tan(c
/2 + (d*x)/2)))/(a*d) + (7*tan(c/2 + (d*x)/2))/(8*a*d) + (tan(c/2 + (d*x)/2
) + (19*tan(c/2 + (d*x)/2)^2)/3 + 2*tan(c/2 + (d*x)/2)^3 + (89*tan(c/2 + (d
*x)/2)^4)/3 - 15*tan(c/2 + (d*x)/2)^5 + 23*tan(c/2 + (d*x)/2)^6 - 1/3)/(d*(
8*a*tan(c/2 + (d*x)/2)^3 + 16*a*tan(c/2 + (d*x)/2)^5 + 8*a*tan(c/2 + (d*x)/
2)^7)) - (2*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

3.688 $\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=94

$$-\frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{ad}$$

```
[Out] -2*csc(d*x+c)/a/d+csc(d*x+c)^2/a/d+1/3*csc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/
d+ln(sin(d*x+c))/a/d-sin(d*x+c)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\sin(c+dx)}{ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{ad}-\frac{2\csc(c+dx)}{ad}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(a*d) + Csc[c + d*x]^3/(3*a*d) - C
sc[c + d*x]^4/(4*a*d) + Log[Sin[c + d*x]]/(a*d) - Sin[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^5(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{x^5}-\frac{a^4}{x^4}-\frac{2a^3}{x^3}+\frac{2a^2}{x^2}+\frac{a}{x}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{2\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\log(\sin(c}{ad}$$

Mathematica [A]
time = 0.22, size = 66, normalized size = 0.70

$$-\frac{24\csc(c+dx)-12\csc^2(c+dx)-4\csc^3(c+dx)+3\csc^4(c+dx)-12\log(\sin(c+dx))+12\sin(c+dx)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/12*(24*Csc[c + d*x] - 12*Csc[c + d*x]^2 - 4*Csc[c + d*x]^3 + 3*Csc[c + d
*x]^4 - 12*Log[Sin[c + d*x]] + 12*Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.27, size = 62, normalized size = 0.66

method	result
derivativedivides	$\frac{-\sin(dx+c)+\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}+\frac{1}{3\sin(dx+c)^3}-\frac{2}{\sin(dx+c)}}{da}$
default	$\frac{-\sin(dx+c)+\ln(\sin(dx+c))+\frac{1}{\sin(dx+c)^2}-\frac{1}{4\sin(dx+c)^4}+\frac{1}{3\sin(dx+c)^3}-\frac{2}{\sin(dx+c)}}{da}$
risch	$-\frac{ix}{a}+\frac{ie^{i(dx+c)}}{2ad}-\frac{ie^{-i(dx+c)}}{2ad}-\frac{2ic}{ad}-\frac{4i(-3ie^{6i(dx+c)}+3\,e^{7i(dx+c)}+3ie^{4i(dx+c)}-7\,e^{5i(dx+c)}-3ie^{2i(dx+c)}+7\,e^{3i(}}{3ad(e^{2i(dx+c)}-1)^4}$
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan(\frac{dx}{2}+\frac{c}{2})}{192ad}+\frac{19(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{96ad}-\frac{61(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{96ad}-\frac{61(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{96ad}+\frac{19(\tan^{11}(\frac{dx}{2}+\frac{c}{2}))}{96ad}+\frac{5(\tan^{12}(\frac{dx}{2}+\frac{c}{2}))}{192ad}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^2\tan(\frac{dx}{2}+\frac{c}{2})^4(\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-sin(d*x+c)+ln(sin(d*x+c))+1/sin(d*x+c)^2-1/4/sin(d*x+c)^4+1/3/sin(d
*x+c)^3-2/sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.77

$$\frac{\frac{12 \log(\sin(dx+c))}{a} - \frac{12 \sin(dx+c)}{a} - \frac{24 \sin(dx+c)^3 - 12 \sin(dx+c)^2 - 4 \sin(dx+c) + 3}{a \sin(dx+c)^4}}{12\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*log(sin(d*x + c))/a - 12*sin(d*x + c)/a - (24*sin(d*x + c)^3 - 12*
sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x + c)^4))/d
```

Fricas [A]
time = 0.39, size = 104, normalized size = 1.11

$$-\frac{12 \cos(dx+c)^2 - 12 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \sin(dx+c)\right) + 4 \left(3 \cos(dx+c)^4 - 12 \cos(dx+c)^2 + 8\right) \sin(dx+c) - 9}{12 \left(ad \cos(dx+c)^4 - 2\, ad \cos(dx+c)^2 + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*cos(d*x + c)^2 - 12*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c)) + 4*(3*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 8)*sin(d*x + c
) - 9)/(a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.46, size = 83, normalized size = 0.88

$$\frac{\frac{12 \log(|\sin(dx+c)|)}{a} - \frac{12 \sin(dx+c)}{a} - \frac{25 \sin(dx+c)^4 + 24 \sin(dx+c)^3 - 12 \sin(dx+c)^2 - 4 \sin(dx+c) + 3}{a \sin(dx+c)^4}}{12\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*log(abs(sin(d*x + c)))/a - 12*sin(d*x + c)/a - (25*sin(d*x + c)^4
+ 24*sin(d*x + c)^3 - 12*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*sin(d*x +
c)^4))/d
```

Mupad [B]
time = 9.34, size = 214, normalized size = 2.28

$$\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{-46\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{1}{4}}{d\,\left(16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*tan(c/2 + (d*x)/2)^2)/(16*a*d) + tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2
 + (d*x)/2)^4/(64*a*d) + log(tan(c/2 + (d*x)/2))/(a*d) + ((2*tan(c/2 + (d*x
)/2))/3 + (11*tan(c/2 + (d*x)/2)^2)/4 - (40*tan(c/2 + (d*x)/2)^3)/3 + 3*tan
(c/2 + (d*x)/2)^4 - 46*tan(c/2 + (d*x)/2)^5 - 1/4)/(d*(16*a*tan(c/2 + (d*x)
/2)^4 + 16*a*tan(c/2 + (d*x)/2)^6)) - (7*tan(c/2 + (d*x)/2))/(8*a*d) - log(
tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.689 $\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{ad}+\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^5(c+dx)}{5ad}-\frac{\log(\sin(c+dx))}{ad}$$

```
[Out] -csc(d*x+c)/a/d-csc(d*x+c)^2/a/d+2/3*csc(d*x+c)^3/a/d+1/4*csc(d*x+c)^4/a/d-
1/5*csc(d*x+c)^5/a/d-ln(sin(d*x+c))/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{\csc^5(c+dx)}{5ad}+\frac{\csc^4(c+dx)}{4ad}+\frac{2\csc^3(c+dx)}{3ad}-\frac{\csc^2(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}-\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Csc[c + d*x]^2/(a*d) + (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^4/(4*a*d) - Csc[c + d*x]^5/(5*a*d) - Log[Sin[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^6(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^6}-\frac{a^4}{x^5}-\frac{2a^3}{x^4}+\frac{2a^2}{x^3}+\frac{a}{x^2}-\frac{1}{x}\right)dx, x, a\sin(c+dx)\right)}{ad}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{ad}+\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^4(c+dx)}{4ad}-\frac{\csc^5(c+d}{5ad}$$

Mathematica [A]

time = 0.08, size = 68, normalized size = 0.68

$$-\frac{60\csc(c+dx)+60\csc^2(c+dx)-40\csc^3(c+dx)-15\csc^4(c+dx)+12\csc^5(c+dx)+60\log(\sin(c+dx))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
[Out] -1/60*(60*Csc[c + d*x] + 60*Csc[c + d*x]^2 - 40*Csc[c + d*x]^3 - 15*Csc[c +
 d*x]^4 + 12*Csc[c + d*x]^5 + 60*Log[Sin[c + d*x]])/(a*d)
```

Maple [A]

time = 0.27, size = 68, normalized size = 0.68

method	result
derivativedivides	$\frac{-\ln(\sin(dx+c))-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}-\frac{1}{\sin(dx+c)^2}+\frac{2}{3\sin(dx+c)^3}}{da}$
default	$\frac{-\ln(\sin(dx+c))-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}-\frac{1}{\sin(dx+c)^2}+\frac{2}{3\sin(dx+c)^3}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{2i(15\,e^{9i(dx+c)}-20\,e^{7i(dx+c)}+30ie^{8i(dx+c)}+58\,e^{5i(dx+c)}-60ie^{6i(dx+c)}-20\,e^{3i(dx+c)}+60ie^{4i(dx+c)}+15\,e^{i(}}{15da\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}+\frac{59\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}-\frac{121\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}-\frac{83\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{83\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{121\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{960ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-ln(sin(d*x+c))-1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4-1/sin(d*x+c)-1/sin
(d*x+c)^2+2/3/sin(d*x+c)^3)
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.70

$$-\frac{\frac{60 \log(\sin(dx+c))}{a} + \frac{60 \sin(dx+c)^4+60 \sin(dx+c)^3-40 \sin(dx+c)^2-15 \sin(dx+c)+12}{a \sin(dx+c)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*log(sin(d*x + c))/a + (60*sin(d*x + c)^4 + 60*sin(d*x + c)^3 - 40
*sin(d*x + c)^2 - 15*sin(d*x + c) + 12)/(a*sin(d*x + c)^5))/d
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.18

$$-\frac{60 \cos(dx+c)^4 + 60 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \sin(dx+c)\right) \sin(dx+c) - 80 \cos(dx+c)^2 - 15 \left(4 \cos(dx+c)^2 - 3\right) \sin(dx+c) + 32}{60 \left(ad \cos(dx+c)^4 - 2 ad \cos(dx+c)^2 + ad\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*cos(d*x + c)^4 + 60*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*sin(d*x + c))*sin(d*x + c) - 80*cos(d*x + c)^2 - 15*(4*cos(d*x + c)^2 -
3)*sin(d*x + c) + 32)/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*si
n(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 82, normalized size = 0.82

$$-\frac{\frac{60 \log(|\sin(dx+c)|)}{a} - \frac{137 \sin(dx+c)^5-60 \sin(dx+c)^4-60 \sin(dx+c)^3+40 \sin(dx+c)^2+15 \sin(dx+c)-12}{a \sin(dx+c)^5}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*log(abs(sin(d*x + c)))/a - (137*sin(d*x + c)^5 - 60*sin(d*x + c)^
4 - 60*sin(d*x + c)^3 + 40*sin(d*x + c)^2 + 15*sin(d*x + c) - 12)/(a*sin(d*
x + c)^5))/d
```

Mupad [B]

time = 9.24, size = 204, normalized size = 2.04

$$\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{5}\right)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*tan(c/2 + (d*x)/2)^3)/(96*a*d) - (3*tan(c/2 + (d*x)/2)^2)/(16*a*d) + tan
(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^5/(160*a*d) - log(tan(c/2 +
 (d*x)/2))/(a*d) - (5*tan(c/2 + (d*x)/2))/(16*a*d) + log(tan(c/2 + (d*x)/2)
^2 + 1)/(a*d) - (cot(c/2 + (d*x)/2)^5*(6*tan(c/2 + (d*x)/2)^3 - (5*tan(c/2
+ (d*x)/2)^2)/3 - tan(c/2 + (d*x)/2)/2 + 10*tan(c/2 + (d*x)/2)^4 + 1/5))/(3
2*a*d)
```

3.690 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d+csc(d*x+c)/a/d-2/3*csc(d*x+c)^3/a/d+1/5*csc(d*x+c)^5/
a/d
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*Cot[c + d*x]^6/(a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d)
 + Csc[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^5\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^6(c+dx)}{6ad}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.90

$$\frac{\csc^6(c+dx)(-15\cos(4(c+dx))+78\sin(c+dx)-5(5+7\sin(3(c+dx))-3\sin(5(c+dx))))}{240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

Maple [A]
time = 0.27, size = 67, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{1}{5\sin(dx+c)^5}-\frac{2}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}-\frac{1}{6\sin(dx+c)^6}+\frac{1}{\sin(dx+c)}}{da}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{1}{5\sin(dx+c)^5}-\frac{2}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}-\frac{1}{6\sin(dx+c)^6}+\frac{1}{\sin(dx+c)}}{da}$
risch	$\frac{2i\left(-15i\mathrm{e}^{10i(dx+c)}+15\,\mathrm{e}^{11i(dx+c)}-35\,\mathrm{e}^{9i(dx+c)}-50i\mathrm{e}^{6i(dx+c)}+78\,\mathrm{e}^{7i(dx+c)}-78\,\mathrm{e}^{5i(dx+c)}-15i\mathrm{e}^{2i(dx+c)}+35\,\mathrm{e}^{3i(dx+c)}-15\right.}{15ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$

norman	$\frac{-\frac{1}{384ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1920ad}+\frac{7\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320ad}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{35\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}+\frac{35\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}+\frac{35\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128ad}-}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2/sin(d*x+c)^2+1/5/sin(d*x+c)^5-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^4
-1/6/sin(d*x+c)^6+1/sin(d*x+c))
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.97

$$\frac{30\sin\left(dx+c\right)^5-15\sin\left(dx+c\right)^4-20\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)^2+6\sin\left(dx+c\right)-5}{30\,ad\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.36, size = 96, normalized size = 1.41

$$\frac{15\cos\left(dx+c\right)^4-15\cos\left(dx+c\right)^2-2\left(15\cos\left(dx+c\right)^4-20\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)+5}{30\left(ad\cos\left(dx+c\right)^6-3\,ad\cos\left(dx+c\right)^4+3\,ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos
(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)
^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 66, normalized size = 0.97

$$\frac{30 \sin(dx+c)^5 - 15 \sin(dx+c)^4 - 20 \sin(dx+c)^3 + 15 \sin(dx+c)^2 + 6 \sin(dx+c) - 5}{30\, ad \sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Mupad [B]
time = 9.04, size = 63, normalized size = 0.93

$$\frac{\sin(c+d\,x)^5 - \frac{\sin(c+d\,x)^4}{2} - \frac{2\sin(c+d\,x)^3}{3} + \frac{\sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{5} - \frac{1}{6}}{a\,d\sin(c+d\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/5 + sin(c + d*x)^2/2 - (2*sin(c + d*x)^3)/3 - sin(c + d*x)^4/
2 + sin(c + d*x)^5 - 1/6)/(a*d*sin(c + d*x)^6)
```

3.691 $\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] 1/6*cot(d*x+c)^6/a/d-1/3*csc(d*x+c)^3/a/d+2/5*csc(d*x+c)^5/a/d-1/7*csc(d*x+
c)^7/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2914, 2686, 276, 2687, 30}

$$\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) - Csc[c + d*x]^3/(3*a*d) + (2*Csc[c + d*x]^5)/(5*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\ &= \frac{\cot^6(c+dx)}{6ad}-\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(c+dx)\right)}{ad}\\ &= \frac{\cot^6(c+dx)}{6ad}-\frac{\csc^3(c+dx)}{3ad}+\frac{2\csc^5(c+dx)}{5ad}-\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 68, normalized size = 0.93

$$\frac{\csc^2(c+dx)\left(105-70\csc(c+dx)-105\csc^2(c+dx)+84\csc^3(c+dx)+35\csc^4(c+dx)-30\csc^5(c+dx)\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^2*(105 - 70*Csc[c + d*x] - 105*Csc[c + d*x]^2 + 84*Csc[c + d*
x]^3 + 35*Csc[c + d*x]^4 - 30*Csc[c + d*x]^5))/(210*a*d)
```

Maple [A]
time = 0.29, size = 69, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{1}{7\sin(dx+c)^7}+\frac{1}{6\sin(dx+c)^6}-\frac{1}{2\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}+\frac{2}{5\sin(dx+c)^5}}{da}$
default	$\frac{-\frac{1}{7\sin(dx+c)^7}+\frac{1}{6\sin(dx+c)^6}-\frac{1}{2\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}+\frac{2}{5\sin(dx+c)^5}}{da}$

risch	$-\frac{2\left(-140i\mathrm{e}^{11i(dx+c)}+105\,\mathrm{e}^{12i(dx+c)}-112i\mathrm{e}^{9i(dx+c)}-105\,\mathrm{e}^{10i(dx+c)}-456i\mathrm{e}^{7i(dx+c)}+350\,\mathrm{e}^{8i(dx+c)}-112i\mathrm{e}^{5i(dx+c)}-35\right.}{105da\left(\mathrm{e}^{2i(dx+c)}-1\right)^{7}}$
norman	$\frac{-\frac{1}{896ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{672ad}+\frac{7\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}-\frac{7\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{640ad}-\frac{7\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384ad}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}+\frac{7\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{7(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/7/sin(d*x+c)^7+1/6/sin(d*x+c)^6-1/2/sin(d*x+c)^4-1/3/sin(d*x+c)^3
+1/2/sin(d*x+c)^2+2/5/sin(d*x+c)^5)
```

Maxima [A]
time = 0.34, size = 66, normalized size = 0.90

$$\frac{105\sin(dx+c)^{5}-70\sin(dx+c)^{4}-105\sin(dx+c)^{3}+84\sin(dx+c)^{2}+35\sin(dx+c)-30}{210\,ad\sin(dx+c)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

Fricas [A]
time = 0.36, size = 104, normalized size = 1.42

$$\frac{70\cos(dx+c)^{4}-56\cos(dx+c)^{2}-35\left(3\cos(dx+c)^{4}-3\cos(dx+c)^{2}+1\right)\sin(dx+c)+16}{210\left(ad\cos(dx+c)^{6}-3\,ad\cos(dx+c)^{4}+3\,ad\cos(dx+c)^{2}-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*cos(d*x + c)^4 - 56*cos(d*x + c)^2 - 35*(3*cos(d*x + c)^4 - 3*cos
(d*x + c)^2 + 1)*sin(d*x + c) + 16)/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x +
c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 66, normalized size = 0.90

$$\frac{105 \sin\left(dx+c\right)^{5} - 70 \sin\left(dx+c\right)^{4} - 105 \sin\left(dx+c\right)^{3} + 84 \sin\left(dx+c\right)^{2} + 35 \sin\left(dx+c\right) - 30}{210\, ad \sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/210*(105*sin(d*x + c)^5 - 70*sin(d*x + c)^4 - 105*sin(d*x + c)^3 + 84*sin
(d*x + c)^2 + 35*sin(d*x + c) - 30)/(a*d*sin(d*x + c)^7)
```

Mupad [B]
time = 8.97, size = 66, normalized size = 0.90

$$\frac{105 \sin\left(c+d\,x\right)^{5} - 70 \sin\left(c+d\,x\right)^{4} - 105 \sin\left(c+d\,x\right)^{3} + 84 \sin\left(c+d\,x\right)^{2} + 35 \sin\left(c+d\,x\right) - 30}{210\, a\, d \sin\left(c+d\,x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
```

```
[Out] (35*sin(c + d*x) + 84*sin(c + d*x)^2 - 105*sin(c + d*x)^3 - 70*sin(c + d*x)
^4 + 105*sin(c + d*x)^5 - 30)/(210*a*d*sin(c + d*x)^7)
```

3.692 $\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{\cot^6(c+dx)}{6ad}-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] -1/6*cot(d*x+c)^6/a/d-1/8*cot(d*x+c)^8/a/d+1/3*csc(d*x+c)^3/a/d-2/5*csc(d*x
+c)^5/a/d+1/7*csc(d*x+c)^7/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2914, 2687, 14, 2686, 276}

$$-\frac{\cot^8(c+dx)}{8ad}-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/6*Cot[c + d*x]^6/(a*d) - Cot[c + d*x]^8/(8*a*d) + Csc[c + d*x]^3/(3*a*d)
 - (2*Csc[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,-\right.}{ad}\\ &= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,-\right.}{ad}\\ &= -\frac{\cot^6(c+dx)}{6ad}-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{2\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+}{7ac}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 68, normalized size = 0.75

$$\frac{\csc^3(c+dx)\left(280-210\csc(c+dx)-336\csc^2(c+dx)+280\csc^3(c+dx)+120\csc^4(c+dx)-105\csc^5(c+dx)\right)}{840ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^3*(280 - 210*Csc[c + d*x] - 336*Csc[c + d*x]^2 + 280*Csc[c +
d*x]^3 + 120*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5))/(840*a*d)
```

Maple [A]
time = 0.28, size = 69, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{2}{5\sin(dx+c)^5}+\frac{1}{3\sin(dx+c)^6}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{3\sin(dx+c)^3}+\frac{1}{7\sin(dx+c)^7}-\frac{1}{4\sin(dx+c)^4}}{da}$

default	$\frac{-\frac{2}{5\sin(dx+c)^5}+\frac{1}{3\sin(dx+c)^6}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{3\sin(dx+c)^3}+\frac{1}{7\sin(dx+c)^7}-\frac{1}{4\sin(dx+c)^4}}{da}$
risch	$-\frac{4i(-105i e^{12i(dx+c)}+70\,e^{13i(dx+c)}-140i e^{10i(dx+c)}-14\,e^{11i(dx+c)}-350i e^{8i(dx+c)}+172\,e^{9i(dx+c)}-140i e^{6i(dx+c)}-172}{105ad(e^{2i(dx+c)}-1)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2/5/sin(d*x+c)^5+1/3/sin(d*x+c)^6-1/8/sin(d*x+c)^8+1/3/sin(d*x+c)^3
+1/7/sin(d*x+c)^7-1/4/sin(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.73

$$\frac{280\,\sin\left(dx+c\right)^5-210\,\sin\left(dx+c\right)^4-336\,\sin\left(dx+c\right)^3+280\,\sin\left(dx+c\right)^2+120\,\sin\left(dx+c\right)-105}{840\,ad\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.36, size = 107, normalized size = 1.18

$$-\frac{210\,\cos\left(dx+c\right)^4-140\,\cos\left(dx+c\right)^2-8\left(35\,\cos\left(dx+c\right)^4-28\,\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)+35}{840\left(ad\cos\left(dx+c\right)^8-4\,ad\cos\left(dx+c\right)^6+6\,ad\cos\left(dx+c\right)^4-4\,ad\cos\left(dx+c\right)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*cos(d*x + c)^4 - 140*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 28
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x
 + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 66, normalized size = 0.73

$$\frac{280 \sin\left(dx+c\right)^{5} - 210 \sin\left(dx+c\right)^{4} - 336 \sin\left(dx+c\right)^{3} + 280 \sin\left(dx+c\right)^{2} + 120 \sin\left(dx+c\right) - 105}{840\, ad \sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 336*sin(d*x + c)^3 + 280*s
in(d*x + c)^2 + 120*sin(d*x + c) - 105)/(a*d*sin(d*x + c)^8)
```

Mupad [B]
time = 9.04, size = 66, normalized size = 0.73

$$\frac{280 \sin\left(c+d\,x\right)^{5} - 210 \sin\left(c+d\,x\right)^{4} - 336 \sin\left(c+d\,x\right)^{3} + 280 \sin\left(c+d\,x\right)^{2} + 120\,\sin\left(c+d\,x\right) - 105}{840\,a\,d \sin\left(c+d\,x\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (120*sin(c + d*x) + 280*sin(c + d*x)^2 - 336*sin(c + d*x)^3 - 210*sin(c + d
*x)^4 + 280*sin(c + d*x)^5 - 105)/(840*a*d*sin(c + d*x)^8)
```

3.693 $\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^6(c+dx)}{6ad}+\frac{\cot^8(c+dx)}{8ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[Out] 1/6*cot(d*x+c)^6/a/d+1/8*cot(d*x+c)^8/a/d-1/5*csc(d*x+c)^5/a/d+2/7*csc(d*x+
c)^7/a/d-1/9*csc(d*x+c)^9/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2914, 2686, 276, 2687, 14}

$$\frac{\cot^8(c+dx)}{8ad}+\frac{\cot^6(c+dx)}{6ad}-\frac{\csc^9(c+dx)}{9ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^6/(6*a*d) + Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]^5/(5*a*d) +
(2*Csc[c + d*x]^7)/(7*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^5(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^5(c+dx)\,dx}{a}$$
$$= -\frac{\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\csc(c+dx)\right)}{ad}+\frac{\mathrm{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,\cdot\right.}{ad}$$
$$= \frac{\mathrm{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\right.}{ad}$$
$$= \frac{\cot^6(c+dx)}{6ad}+\frac{\cot^8(c+dx)}{8ad}-\frac{\csc^5(c+dx)}{5ad}+\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^9(c+d}{9ad}$$

Mathematica [A]
time = 0.12, size = 68, normalized size = 0.75

$$\frac{\csc^4(c+dx)\left(630-504\csc(c+dx)-840\csc^2(c+dx)+720\csc^3(c+dx)+315\csc^4(c+dx)-280\csc^5(c+dx)\right)}{2520ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^4*(630 - 504*Csc[c + d*x] - 840*Csc[c + d*x]^2 + 720*Csc[c +
d*x]^3 + 315*Csc[c + d*x]^4 - 280*Csc[c + d*x]^5))/(2520*a*d)
```

Maple [A]
time = 0.34, size = 69, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{2}{7\sin(dx+c)^7}+\frac{1}{8\sin(dx+c)^8}-\frac{1}{3\sin(dx+c)^6}-\frac{1}{9\sin(dx+c)^9}-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}}{da}$

default	$\frac{\frac{2}{7\sin(dx+c)^7}+\frac{1}{8\sin(dx+c)^8}-\frac{1}{3\sin(dx+c)^6}-\frac{1}{9\sin(dx+c)^9}-\frac{1}{5\sin(dx+c)^5}+\frac{1}{4\sin(dx+c)^4}}{da}$
risch	$\frac{-\frac{32i e^{13i(dx+c)}}{5}+4\,e^{14i(dx+c)}-\frac{384i e^{11i(dx+c)}}{35}+\frac{4\,e^{12i(dx+c)}}{3}-\frac{6976i e^{9i(dx+c)}}{315}+8\,e^{10i(dx+c)}-\frac{384i e^{7i(dx+c)}}{35}-8\,e^{8i(dx+c)}-}{da\left(e^{2i(dx+c)}-1\right)^9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(2/7/sin(d*x+c)^7+1/8/sin(d*x+c)^8-1/3/sin(d*x+c)^6-1/9/sin(d*x+c)^9-
1/5/sin(d*x+c)^5+1/4/sin(d*x+c)^4)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.73

$$\frac{630\sin(dx+c)^5-504\sin(dx+c)^4-840\sin(dx+c)^3+720\sin(dx+c)^2+315\sin(dx+c)-280}{2520\,ad\sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.26

$$-\frac{504\cos(dx+c)^4-288\cos(dx+c)^2-105\left(6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\sin(dx+c)+64}{2520\left(ad\cos(dx+c)^8-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^4-4\,ad\cos(dx+c)^2+ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*cos(d*x + c)^4 - 288*cos(d*x + c)^2 - 105*(6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1)*sin(d*x + c) + 64)/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d
*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 66, normalized size = 0.73

$$\frac{630 \sin(dx+c)^5 - 504 \sin(dx+c)^4 - 840 \sin(dx+c)^3 + 720 \sin(dx+c)^2 + 315 \sin(dx+c) - 280}{2520\, ad \sin(dx+c)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2520*(630*sin(d*x + c)^5 - 504*sin(d*x + c)^4 - 840*sin(d*x + c)^3 + 720*
sin(d*x + c)^2 + 315*sin(d*x + c) - 280)/(a*d*sin(d*x + c)^9)
```

Mupad [B]
time = 9.04, size = 65, normalized size = 0.71

$$\frac{\frac{\sin(c+dx)^5}{4} - \frac{\sin(c+dx)^4}{5} - \frac{\sin(c+dx)^3}{3} + \frac{2\sin(c+dx)^2}{7} + \frac{\sin(c+dx)}{8} - \frac{1}{9}}{a\,d\sin(c+dx)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^10*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/8 + (2*sin(c + d*x)^2)/7 - sin(c + d*x)^3/3 - sin(c + d*x)^4/
5 + sin(c + d*x)^5/4 - 1/9)/(a*d*sin(c + d*x)^9)
```

3.694 $\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}-\frac{2\csc^7(c+dx)}{7ad}+\frac{\csc^8(c+dx)}{4ad}+\frac{\csc^9(c+dx)}{9ad}-\frac{\csc^{10}(c+dx)}{10ad}$$

```
[Out] 1/5*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d-2/7*csc(d*x+c)^7/a/d+1/4*csc(d*x+
c)^8/a/d+1/9*csc(d*x+c)^9/a/d-1/10*csc(d*x+c)^10/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\csc^{10}(c+dx)}{10ad}+\frac{\csc^9(c+dx)}{9ad}+\frac{\csc^8(c+dx)}{4ad}-\frac{2\csc^7(c+dx)}{7ad}-\frac{\csc^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^5/(5*a*d) - Csc[c + d*x]^6/(6*a*d) - (2*Csc[c + d*x]^7)/(7*a*d
) + Csc[c + d*x]^8/(4*a*d) + Csc[c + d*x]^9/(9*a*d) - Csc[c + d*x]^10/(10*a
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{11}(a-x)^3(a+x)^2}{x^{11}}\,dx,x,a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{11}}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(\frac{a^5}{x^{11}}-\frac{a^4}{x^{10}}-\frac{2a^3}{x^9}+\frac{2a^2}{x^8}+\frac{a}{x^7}-\frac{1}{x^6}\right)\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^5(c+dx)}{5ad}-\frac{\csc^6(c+dx)}{6ad}-\frac{2\csc^7(c+dx)}{7ad}+\frac{\csc^8(c+dx)}{4ad}+\frac{\csc^9(c+a}{9ad}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.62

$$\frac{\csc^5(c+dx)\left(252-210\csc(c+dx)-360\csc^2(c+dx)+315\csc^3(c+dx)+140\csc^4(c+dx)-126\csc^5(c+dx)\right)}{1260ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^5*(252 - 210*Csc[c + d*x] - 360*Csc[c + d*x]^2 + 315*Csc[c + d*x]^3 + 140*Csc[c + d*x]^4 - 126*Csc[c + d*x]^5))/(1260*a*d)
```

Maple [A]
time = 0.38, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{10\sin(dx+c)^{10}}-\frac{2}{7\sin(dx+c)^7}+\frac{1}{9\sin(dx+c)^9}+\frac{1}{4\sin(dx+c)^8}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{da}$
default	$\frac{-\frac{1}{10\sin(dx+c)^{10}}-\frac{2}{7\sin(dx+c)^7}+\frac{1}{9\sin(dx+c)^9}+\frac{1}{4\sin(dx+c)^8}+\frac{1}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}}{da}$
risch	$\frac{32i\left(-105i e^{14i(dx+c)}+63\,e^{15i(dx+c)}-210i e^{12i(dx+c)}+45\,e^{13i(dx+c)}-378i e^{10i(dx+c)}+110\,e^{11i(dx+c)}-210i e^{8i(dx+c)}-110\,e\right.}{315ad\left(e^{2i(dx+c)}-1\right)^{10}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/10/sin(d*x+c)^10-2/7/sin(d*x+c)^7+1/9/sin(d*x+c)^9+1/4/sin(d*x+c)^8+1/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6)
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.61

$$\frac{252\sin(dx+c)^5-210\sin(dx+c)^4-360\sin(dx+c)^3+315\sin(dx+c)^2+140\sin(dx+c)-126}{1260\,ad\sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

Fricas [A]
time = 0.37, size = 120, normalized size = 1.10

$$\frac{210 \cos (dx+c)^4-105 \cos (dx+c)^2-4 \left(63 \cos (dx+c)^4-36 \cos (dx+c)^2+8\right) \sin (dx+c)+21}{1260 \left(ad\cos (dx+c)^{10}-5\,ad\cos (dx+c)^8+10\,ad\cos (dx+c)^6-10\,ad\cos (dx+c)^4+5\,ad\cos (dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*cos(d*x + c)^4 - 105*cos(d*x + c)^2 - 4*(63*cos(d*x + c)^4 - 36
*cos(d*x + c)^2 + 8)*sin(d*x + c) + 21)/(a*d*cos(d*x + c)^10 - 5*a*d*cos(d*
x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d*x +
c)^2 - a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 66, normalized size = 0.61

$$\frac{252 \sin (dx+c)^5-210 \sin (dx+c)^4-360 \sin (dx+c)^3+315 \sin (dx+c)^2+140 \sin (dx+c)-126}{1260\,ad\sin (dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1260*(252*sin(d*x + c)^5 - 210*sin(d*x + c)^4 - 360*sin(d*x + c)^3 + 315*
sin(d*x + c)^2 + 140*sin(d*x + c) - 126)/(a*d*sin(d*x + c)^10)
```

Mupad [B]
time = 9.08, size = 66, normalized size = 0.61

$$\frac{252\sin (c+d\,x)^5-210\sin (c+d\,x)^4-360\sin (c+d\,x)^3+315\sin (c+d\,x)^2+140\,\sin (c+d\,x)-126}{1260\,a\,d\sin (c+d\,x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^11*(a + a*sin(c + d*x))),x)
```

```
[Out] (140*sin(c + d*x) + 315*sin(c + d*x)^2 - 360*sin(c + d*x)^3 - 210*sin(c + d
*x)^4 + 252*sin(c + d*x)^5 - 126)/(1260*a*d*sin(c + d*x)^10)
```

3.695 $\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\csc^6(c+dx)}{6ad}-\frac{\csc^7(c+dx)}{7ad}-\frac{\csc^8(c+dx)}{4ad}+\frac{2\csc^9(c+dx)}{9ad}+\frac{\csc^{10}(c+dx)}{10ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

```
[Out] 1/6*csc(d*x+c)^6/a/d-1/7*csc(d*x+c)^7/a/d-1/4*csc(d*x+c)^8/a/d+2/9*csc(d*x+
c)^9/a/d+1/10*csc(d*x+c)^10/a/d-1/11*csc(d*x+c)^11/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{10ad}+\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{4ad}-\frac{\csc^7(c+dx)}{7ad}+\frac{\csc^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^6/(6*a*d) - Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(4*a*d) +
(2*Csc[c + d*x]^9)/(9*a*d) + Csc[c + d*x]^10/(10*a*d) - Csc[c + d*x]^11/(11
*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{12}(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^5\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{12}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{a^5}{x^{12}} - \frac{a^4}{x^{11}} - \frac{2a^3}{x^{10}} + \frac{2a^2}{x^9} + \frac{a}{x^8} - \frac{1}{x^7}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^6(c+dx)}{6ad} - \frac{\csc^7(c+dx)}{7ad} - \frac{\csc^8(c+dx)}{4ad} + \frac{2\csc^9(c+dx)}{9ad} + \frac{\csc^{10}(c+}{10ad}$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.62

$$\frac{\csc^6(c+dx)\left(2310 - 1980\csc(c+dx) - 3465\csc^2(c+dx) + 3080\csc^3(c+dx) + 1386\csc^4(c+dx) - 1260\csc^5(c+dx)\right)}{13860ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
[Out] (Csc[c + d*x]^6*(2310 - 1980*Csc[c + d*x] - 3465*Csc[c + d*x]^2 + 3080*Csc[
c + d*x]^3 + 1386*Csc[c + d*x]^4 - 1260*Csc[c + d*x]^5))/(13860*a*d)
```

Maple [A]
time = 0.40, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{2}{9\sin(dx+c)^9} - \frac{1}{7\sin(dx+c)^7} - \frac{1}{11\sin(dx+c)^{11}} + \frac{1}{6\sin(dx+c)^6} + \frac{1}{10\sin(dx+c)^{10}} - \frac{1}{4\sin(dx+c)^8}}{da}$
default	$\frac{\frac{2}{9\sin(dx+c)^9} - \frac{1}{7\sin(dx+c)^7} - \frac{1}{11\sin(dx+c)^{11}} + \frac{1}{6\sin(dx+c)^6} + \frac{1}{10\sin(dx+c)^{10}} - \frac{1}{4\sin(dx+c)^8}}{da}$
risch	$-\frac{32\left(-1980i e^{15i(dx+c)} + 1155 e^{16i(dx+c)} - 4400i e^{13i(dx+c)} + 1155 e^{14i(dx+c)} - 7400i e^{11i(dx+c)} + 1848 e^{12i(dx+c)} - 4400i e^{9i(}\right.}{3465da\left(e^{2i(dx+c)}-1\right)^{11}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(2/9/sin(d*x+c)^9-1/7/sin(d*x+c)^7-1/11/sin(d*x+c)^11+1/6/sin(d*x+c)^
6+1/10/sin(d*x+c)^10-1/4/sin(d*x+c)^8)
```

Maxima [A]
time = 0.28, size = 66, normalized size = 0.61

$$\frac{2310\sin(dx+c)^5 - 1980\sin(dx+c)^4 - 3465\sin(dx+c)^3 + 3080\sin(dx+c)^2 + 1386\sin(dx+c) - 1260}{13860\,ad\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

Fricas [A]
time = 0.37, size = 128, normalized size = 1.17

$$\frac{1980 \cos(dx+c)^4 - 880 \cos(dx+c)^2 - 231 \left(10 \cos(dx+c)^4 - 5 \cos(dx+c)^2 + 1\right)\sin(dx+c) + 160}{13860 \left(ad\cos(dx+c)^{10} - 5\,ad\cos(dx+c)^8 + 10\,ad\cos(dx+c)^6 - 10\,ad\cos(dx+c)^4 + 5\,ad\cos(dx+c)^2 - ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*cos(d*x + c)^4 - 880*cos(d*x + c)^2 - 231*(10*cos(d*x + c)^4
- 5*cos(d*x + c)^2 + 1)*sin(d*x + c) + 160)/((a*d*cos(d*x + c)^10 - 5*a*d*c
os(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d*cos(d
*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 66, normalized size = 0.61

$$\frac{2310 \sin(dx+c)^5 - 1980 \sin(dx+c)^4 - 3465 \sin(dx+c)^3 + 3080 \sin(dx+c)^2 + 1386 \sin(dx+c) - 1260}{13860\,ad\sin(dx+c)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13860*(2310*sin(d*x + c)^5 - 1980*sin(d*x + c)^4 - 3465*sin(d*x + c)^3 +
3080*sin(d*x + c)^2 + 1386*sin(d*x + c) - 1260)/(a*d*sin(d*x + c)^11)
```

Mupad [B]
time = 9.04, size = 65, normalized size = 0.60

$$\frac{\frac{\sin(c+d\,x)^5}{6} - \frac{\sin(c+d\,x)^4}{7} - \frac{\sin(c+d\,x)^3}{4} + \frac{2\sin(c+d\,x)^2}{9} + \frac{\sin(c+d\,x)}{10} - \frac{1}{11}}{a\,d\sin(c+d\,x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^12*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/10 + (2*sin(c + d*x)^2)/9 - sin(c + d*x)^3/4 - sin(c + d*x)^4
/7 + sin(c + d*x)^5/6 - 1/11)/(a*d*sin(c + d*x)^11)
```

3.696 $\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\csc^7(c+dx)}{7ad}-\frac{\csc^8(c+dx)}{8ad}-\frac{2\csc^9(c+dx)}{9ad}+\frac{\csc^{10}(c+dx)}{5ad}+\frac{\csc^{11}(c+dx)}{11ad}-\frac{\csc^{12}(c+dx)}{12ad}$$

```
[Out] 1/7*csc(d*x+c)^7/a/d-1/8*csc(d*x+c)^8/a/d-2/9*csc(d*x+c)^9/a/d+1/5*csc(d*x+
c)^10/a/d+1/11*csc(d*x+c)^11/a/d-1/12*csc(d*x+c)^12/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{\csc^{12}(c+dx)}{12ad}+\frac{\csc^{11}(c+dx)}{11ad}+\frac{\csc^{10}(c+dx)}{5ad}-\frac{2\csc^9(c+dx)}{9ad}-\frac{\csc^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]^7/(7*a*d) - Csc[c + d*x]^8/(8*a*d) - (2*Csc[c + d*x]^9)/(9*a*d
) + Csc[c + d*x]^10/(5*a*d) + Csc[c + d*x]^11/(11*a*d) - Csc[c + d*x]^12/(1
2*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^7(c+dx)\csc^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{a^{13}(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{(a-x)^3(a+x)^2}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{a^5}{x^{13}} - \frac{a^4}{x^{12}} - \frac{2a^3}{x^{11}} + \frac{2a^2}{x^{10}} + \frac{a}{x^9} - \frac{1}{x^8}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^7(c+dx)}{7ad} - \frac{\csc^8(c+dx)}{8ad} - \frac{2\csc^9(c+dx)}{9ad} + \frac{\csc^{10}(c+dx)}{5ad} + \frac{\csc^{11}(c+}{11ad}$$

Mathematica [A]
time = 0.08, size = 68, normalized size = 0.62

$$\frac{\csc^7(c+dx)\,(3960 - 3465\csc(c+dx) - 6160\csc^2(c+dx) + 5544\csc^3(c+dx) + 2520\csc^4(c+dx) - 2310\csc^5(c+dx))}{27720ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^7*Csc[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^7*(3960 - 3465*Csc[c + d*x] - 6160*Csc[c + d*x]^2 + 5544*Csc[
c + d*x]^3 + 2520*Csc[c + d*x]^4 - 2310*Csc[c + d*x]^5))/(27720*a*d)
```

Maple [A]
time = 0.46, size = 69, normalized size = 0.63

method	result
derivativedivides	$\frac{-\frac{2}{9\sin(dx+c)^9}+\frac{1}{7\sin(dx+c)^7}+\frac{1}{11\sin(dx+c)^{11}}-\frac{1}{12\sin(dx+c)^{12}}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{5\sin(dx+c)^{10}}}{da}$
default	$\frac{-\frac{2}{9\sin(dx+c)^9}+\frac{1}{7\sin(dx+c)^7}+\frac{1}{11\sin(dx+c)^{11}}-\frac{1}{12\sin(dx+c)^{12}}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{5\sin(dx+c)^{10}}}{da}$
risch	$-\frac{32i(-3465i e^{16i(dx+c)}+1980\,e^{17i(dx+c)}-8316i e^{14i(dx+c)}+2420\,e^{15i(dx+c)}-13398i e^{12i(dx+c)}+3000\,e^{13i(dx+c)}-8316i e}{3465da\left(e^{2i(dx+c)}-1\right)^{12}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-2/9/sin(d*x+c)^9+1/7/sin(d*x+c)^7+1/11/sin(d*x+c)^11-1/12/sin(d*x+c
)^12-1/8/sin(d*x+c)^8+1/5/sin(d*x+c)^10)
```

Maxima [A]
time = 0.28, size = 66, normalized size = 0.61

$$\frac{3960\,\sin\left(dx+c\right)^5 - 3465\,\sin\left(dx+c\right)^4 - 6160\,\sin\left(dx+c\right)^3 + 5544\,\sin\left(dx+c\right)^2 + 2520\,\sin\left(dx+c\right) - 2310}{27720\,ad\sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

Fricas [A]
time = 0.38, size = 131, normalized size = 1.20

$$-\frac{3465 \cos\left(dx+c\right)^{4}-1386 \cos\left(dx+c\right)^{2}-40 \left(99 \cos\left(dx+c\right)^{4}-44 \cos\left(dx+c\right)^{2}+8\right) \sin\left(dx+c\right)+231}{27720 \left(ad \cos\left(dx+c\right)^{12}-6 \, ad \cos\left(dx+c\right)^{10}+15 \, ad \cos\left(dx+c\right)^{8}-20 \, ad \cos\left(dx+c\right)^{6}+15 \, ad \cos\left(dx+c\right)^{4}-6 \, ad \cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/27720*(3465*cos(d*x + c)^4 - 1386*cos(d*x + c)^2 - 40*(99*cos(d*x + c)^4
 - 44*cos(d*x + c)^2 + 8)*sin(d*x + c) + 231)/(a*d*cos(d*x + c)^12 - 6*a*d*
cos(d*x + c)^10 + 15*a*d*cos(d*x + c)^8 - 20*a*d*cos(d*x + c)^6 + 15*a*d*co
s(d*x + c)^4 - 6*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*csc(d*x+c)**13/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 66, normalized size = 0.61

$$\frac{3960 \sin\left(dx+c\right)^{5}-3465 \sin\left(dx+c\right)^{4}-6160 \sin\left(dx+c\right)^{3}+5544 \sin\left(dx+c\right)^{2}+2520 \sin\left(dx+c\right)-2310}{27720 \, ad \sin\left(dx+c\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*csc(d*x+c)^13/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/27720*(3960*sin(d*x + c)^5 - 3465*sin(d*x + c)^4 - 6160*sin(d*x + c)^3 +
5544*sin(d*x + c)^2 + 2520*sin(d*x + c) - 2310)/(a*d*sin(d*x + c)^12)
```

Mupad [B]
time = 9.14, size = 65, normalized size = 0.60

$$\frac{\frac{\sin(c+d\,x)^5}{7}-\frac{\sin(c+d\,x)^4}{8}-\frac{2\sin(c+d\,x)^3}{9}+\frac{\sin(c+d\,x)^2}{5}+\frac{\sin(c+d\,x)}{11}-\frac{1}{12}}{a\,d\sin\left(c+d\,x\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(sin(c + d*x)^13*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/11 + sin(c + d*x)^2/5 - (2*sin(c + d*x)^3)/9 - sin(c + d*x)^4
/8 + sin(c + d*x)^5/7 - 1/12)/(a*d*sin(c + d*x)^12)
```

3.697 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=184

$$\frac{a^3\sin^{1+n}(c+dx)}{d(1+n)}+\frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)}-\frac{8a^3\sin^{4+n}(c+dx)}{d(4+n)}-\frac{6a^3\sin^{5+n}(c+dx)}{d(5+n)}+\frac{6a^3\sin^{6+n}(c+dx)}{d(6+n)}+\frac{8a^3\sin\ldots}{\ldots}$$

```
[Out] a^3*sin(d*x+c)^(1+n)/d/(1+n)+3*a^3*sin(d*x+c)^(2+n)/d/(2+n)-8*a^3*sin(d*x+c
)^(4+n)/d/(4+n)-6*a^3*sin(d*x+c)^(5+n)/d/(5+n)+6*a^3*sin(d*x+c)^(6+n)/d/(6+
n)+8*a^3*sin(d*x+c)^(7+n)/d/(7+n)-3*a^3*sin(d*x+c)^(9+n)/d/(9+n)-a^3*sin(d*
x+c)^(10+n)/d/(10+n)
```

Rubi [A]
time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 90}

$$\frac{a^3\sin^{n+1}(c+dx)}{d(n+1)}+\frac{3a^3\sin^{n+2}(c+dx)}{d(n+2)}-\frac{8a^3\sin^{n+4}(c+dx)}{d(n+4)}-\frac{6a^3\sin^{n+5}(c+dx)}{d(n+5)}+\frac{6a^3\sin^{n+6}(c+dx)}{d(n+6)}+\frac{8a^3\sin^{n+7}(c+dx)}{d(n+7)}-\frac{3a^3\sin^{n+9}(c+dx)}{d(n+9)}-\frac{a^3\sin^{n+10}(c+dx)}{d(n+10)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (3*a^3*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (8*a^3*Sin[c + d*x]^(4 + n))/(d*(4 + n)) - (6*a^3*Sin[c + d*x]^(5
+ n))/(d*(5 + n)) + (6*a^3*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (8*a^3*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (3*a^3*Sin[c + d*x]^(9 + n))/(d*(9 + n)) - (
a^3*Sin[c + d*x]^(10 + n))/(d*(10 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^6\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^9\left(\frac{x}{a}\right)^n + 3a^9\left(\frac{x}{a}\right)^{1+n} - 8a^9\left(\frac{x}{a}\right)^{3+n} - 6a^9\left(\frac{x}{a}\right)^{4}\right.\right.}{}$$
$$= \frac{a^3\sin^{1+n}(c+dx)}{d(1+n)} + \frac{3a^3\sin^{2+n}(c+dx)}{d(2+n)} - \frac{8a^3\sin^{4+n}(c}{d(4+n}$$

Mathematica [A]
time = 0.68, size = 126, normalized size = 0.68

$$\frac{a^3\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{3\sin(c+dx)}{2+n}-\frac{8\sin^3(c+dx)}{4+n}-\frac{6\sin^4(c+dx)}{5+n}+\frac{6\sin^5(c+dx)}{6+n}+\frac{8\sin^6(c+dx)}{7+n}-\frac{3\sin^8(c+dx)}{9+n}-\frac{\sin^9(c+dx)}{10+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (3*Sin[c + d*x])/(2 + n) - (8*Sin
[c + d*x]^3)/(4 + n) - (6*Sin[c + d*x]^4)/(5 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (8*Sin[c + d*x]^6)/(7 + n) - (3*Sin[c + d*x]^8)/(9 + n) - Sin[c + d*x
]^9/(10 + n)))/d
```

Maple [F]
time = 0.39, size = 0, normalized size = 0.00

$$\int \left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x)
```

Maxima [A]
time = 0.28, size = 165, normalized size = 0.90

$$-\frac{\frac{a^3\sin(dx+c)^{n+10}}{n+10}+\frac{3a^3\sin(dx+c)^{n+9}}{n+9}-\frac{8a^3\sin(dx+c)^{n+7}}{n+7}-\frac{6a^3\sin(dx+c)^{n+6}}{n+6}+\frac{6a^3\sin(dx+c)^{n+5}}{n+5}+\frac{8a^3\sin(dx+c)^{n+4}}{n+4}-\frac{3a^3\sin(dx+c)^{n+2}}{n+2}-\frac{a^3\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -(a^3*sin(d*x + c)^(n + 10)/(n + 10) + 3*a^3*sin(d*x + c)^(n + 9)/(n + 9) -
 8*a^3*sin(d*x + c)^(n + 7)/(n + 7) - 6*a^3*sin(d*x + c)^(n + 6)/(n + 6) +
6*a^3*sin(d*x + c)^(n + 5)/(n + 5) + 8*a^3*sin(d*x + c)^(n + 4)/(n + 4) - 3
*a^3*sin(d*x + c)^(n + 2)/(n + 2) - a^3*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(184) = 368$.
time = 0.44, size = 697, normalized size = 3.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] ((a^3*n^7 + 34*a^3*n^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804
*a^3*n^2 + 35844*a^3*n + 15120*a^3)*cos(d*x + c)^10 - 5*(a^3*n^7 + 34*a^3*n
^6 + 472*a^3*n^5 + 3442*a^3*n^4 + 14083*a^3*n^3 + 31804*a^3*n^2 + 35844*a^3
*n + 15120*a^3)*cos(d*x + c)^8 + 192*a^3*n^4 + 4*(a^3*n^7 + 28*a^3*n^6 + 30
4*a^3*n^5 + 1618*a^3*n^4 + 4375*a^3*n^3 + 5554*a^3*n^2 + 2520*a^3*n)*cos(d*
x + c)^6 + 4224*a^3*n^3 + 31488*a^3*n^2 + 24*(a^3*n^6 + 24*a^3*n^5 + 208*a^
3*n^4 + 786*a^3*n^3 + 1231*a^3*n^2 + 630*a^3*n)*cos(d*x + c)^4 + 87936*a^3*
n + 60480*a^3 + 96*(a^3*n^5 + 22*a^3*n^4 + 164*a^3*n^3 + 458*a^3*n^2 + 315*
a^3*n)*cos(d*x + c)^2 - (3*(a^3*n^7 + 35*a^3*n^6 + 497*a^3*n^5 + 3689*a^3*n
^4 + 15302*a^3*n^3 + 34916*a^3*n^2 + 39640*a^3*n + 16800*a^3)*cos(d*x + c)^
8 - 192*a^3*n^4 - 4*(a^3*n^7 + 31*a^3*n^6 + 385*a^3*n^5 + 2485*a^3*n^4 + 89
74*a^3*n^3 + 18004*a^3*n^2 + 18360*a^3*n + 7200*a^3)*cos(d*x + c)^6 - 4224*
a^3*n^3 - 31488*a^3*n^2 - 24*(a^3*n^6 + 26*a^3*n^5 + 255*a^3*n^4 + 1210*a^3
*n^3 + 2924*a^3*n^2 + 3384*a^3*n + 1440*a^3)*cos(d*x + c)^4 - 93696*a^3*n -
 92160*a^3 - 96*(a^3*n^5 + 23*a^3*n^4 + 186*a^3*n^3 + 652*a^3*n^2 + 968*a^3
*n + 480*a^3)*cos(d*x + c)^2)*sin(d*x + c))*sin(d*x + c)^n/(d*n^8 + 44*d*n^
7 + 812*d*n^6 + 8162*d*n^5 + 48503*d*n^4 + 172634*d*n^3 + 353884*d*n^2 + 37
3560*d*n + 151200*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 41196 vs. $2(158) = 316$.
time = 120.54, size = 41196, normalized size = 223.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**3*sin(c)**n*cos(c)**7, Eq(d, 0)), (-a**3*log(s
in(c + d*x))/d + 48*a**3/(35*d*sin(c + d*x)) - a**3*cos(c + d*x)**2/(2*d*si
n(c + d*x)**2) + 3*a**3/(8*d*sin(c + d*x)**2) - 24*a**3*cos(c + d*x)**2/(35
```

*d*sin(c + d*x)**3) + 16*a**3/(315*d*sin(c + d*x)**3) + a**3*cos(c + d*x)**
4/(4*d*sin(c + d*x)**4) - 3*a**3*cos(c + d*x)**2/(8*d*sin(c + d*x)**4) + 18
*a**3*cos(c + d*x)**4/(35*d*sin(c + d*x)**5) - 8*a**3*cos(c + d*x)**2/(105*
d*sin(c + d*x)**5) - a**3*cos(c + d*x)**6/(6*d*sin(c + d*x)**6) + 3*a**3*co
s(c + d*x)**4/(8*d*sin(c + d*x)**6) - 3*a**3*cos(c + d*x)**6/(7*d*sin(c + d
*x)**7) + 2*a**3*cos(c + d*x)**4/(21*d*sin(c + d*x)**7) - 3*a**3*cos(c + d*
x)**6/(8*d*sin(c + d*x)**8) - a**3*cos(c + d*x)**6/(9*d*sin(c + d*x)**9), E
q(n, -10)), (-3*a**3*log(sin(c + d*x))/d - 16*a**3*sin(c + d*x)/(5*d) - 8*a
3*cos(c + d*x)2/(5*d*sin(c + d*x)) + 48*a**3/(35*d*sin(c + d*x)) - 3*a*
*3*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) + a**3/(8*d*sin(c + d*x)**2) + 2*a
3*cos(c + d*x)4/(5*d*sin(c + d*x)**3) - 24*a**3*cos(c + d*x)**2/(35*d*s
in(c + d*x)**3) + 3*a**3*cos(c + d*x)**4/(4*d*sin(c + d*x)**4) - a**3*cos(c
 + d*x)**2/(8*d*sin(c + d*x)**4) - a**3*cos(c + d*x)**6/(5*d*sin(c + d*x)**
5) + 18*a**3*cos(c + d*x)**4/(35*d*sin(c + d*x)**5) - a**3*cos(c + d*x)**6/
(2*d*sin(c + d*x)**6) + a**3*cos(c + d*x)**4/(8*d*sin(c + d*x)**6) - 3*a**3
*cos(c + d*x)**6/(7*d*sin(c + d*x)**7) - a**3*cos(c + d*x)**6/(8*d*sin(c +
d*x)**8), Eq(n, -9)), (-a**3*tan(c/2 + d*x/2)**18/(896*d*tan(c/2 + d*x/2)**
11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 7*a**3*tan(c
/2 + d*x/2)**17/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 +
896*d*tan(c/2 + d*x/2)**7) - 9*a**3*tan(c/2 + d*x/2)**16/(896*d*tan(c/2 + d
*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) + 56*a*
*3*tan(c/2 + d*x/2)**15/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/
2)**9 + 896*d*tan(c/2 + d*x/2)**7) + 188*a**3*tan(c/2 + d*x/2)**14/(896*d*t
an(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**
7) - 196*a**3*tan(c/2 + d*x/2)**13/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan
(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 2548*a**3*tan(c/2 + d*x/2)*
*12/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/
2 + d*x/2)**7) + 196*a**3*tan(c/2 + d*x/2)**11/(896*d*tan(c/2 + d*x/2)**11
+ 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 14014*a**3*tan(
c/2 + d*x/2)**10/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 +
 896*d*tan(c/2 + d*x/2)**7) - 882*a**3*tan(c/2 + d*x/2)**9/(896*d*tan(c/2 +
 d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 140
14*a**3*tan(c/2 + d*x/2)**8/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 +
d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) + 196*a**3*tan(c/2 + d*x/2)**7/(896*
d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2
)**7) - 2548*a**3*tan(c/2 + d*x/2)**6/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*
tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 196*a**3*tan(c/2 + d*x/2
)**5/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c
/2 + d*x/2)**7) + 188*a**3*tan(c/2 + d*x/2)**4/(896*d*tan(c/2 + d*x/2)**11
+ 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) + 56*a**3*tan(c/2
 + d*x/2)**3/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896
*d*tan(c/2 + d*x/2)**7) - 9*a**3*tan(c/2 + d*x/2)**2/(896*d*tan(c/2 + d*x/2
)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7) - 7*a**3*ta
n(c/2 + d*x/2)/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c/2 + d*x/2)**9 + 8
96*d*tan(c/2 + d*x/2)**7) - a**3/(896*d*tan(c/2 + d*x/2)**11 + 1792*d*tan(c

```
/2 + d*x/2)**9 + 896*d*tan(c/2 + d*x/2)**7), Eq(n, -8)), (-15360*a**3*log(t
an(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**12/(1920*d*tan(c/2 + d*x/2)**12 +
 5760*d*tan(c/2 + d*x/2)**10 + 5760*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2
+ d*x/2)**6) - 46080*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**10
/(1920*d*tan(c/2 + d*x/2)**12 + 5760*d*tan(c/2 + d*x/2)**10 + 5760*d*tan(c/
2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 46080*a**3*log(tan(c/2 + d*x/
2)**2 + 1)*tan(c/2 + d*x/2)**8/(1920*d*tan(c/2 + d*x/2)**12 + 5760*d*tan(c/
2 + d*x/2)**10 + 5760*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) -
 15360*a**3*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(1920*d*tan(c/
2 + d*x/2)**12 + 5760*d*tan(c/2 + d*x/2)**10 + 5760*d*tan(c/2 + d*x/2)**8 +
 1920*d*tan(c/2 + d*x/2)**6) + 15360*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d
*x/2)**12/(1920*d*tan(c/2 + d*x/2)**12 + 5760*d*tan(c/2 + d*x/2)**10 + 5760
*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 46080*a**3*log(tan(c
/2 + d*x/2))*tan(c/2 + d*x/2)**10/(1920*d*tan(c/2 + d*x/2)**12 + 5760*d*tan
(c/2 + d*x/2)**10 + 5760*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6
) + 46080*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(1920*d*tan(c/2 +
d*x/2)**12 + 5760*d*tan(c/2 + d*x/2)**10 + 5760*d*tan(c/2 + d*x/2)**8 + 192
0*d*tan(c/2 + d*x/2)**6) + 15360*a**3*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2
)**6/(1920*d*tan(c/2 + d*x/2)**12 + 5760*d*tan(...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(184) = 368$.
time = 0.60, size = 1360, normalized size = 7.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^10 + 18*n^2*sin(d*x + c)^n*sin(d*x + c)^
10 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 104*n*sin(d*x + c)^n*sin(d*x + c
)^10 - 60*n^2*sin(d*x + c)^n*sin(d*x + c)^8 + 192*sin(d*x + c)^n*sin(d*x +
c)^10 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 - 372*n*sin(d*x + c)^n*sin(d*x
+ c)^8 + 66*n^2*sin(d*x + c)^n*sin(d*x + c)^6 - 720*sin(d*x + c)^n*sin(d*x
+ c)^8 - n^3*sin(d*x + c)^n*sin(d*x + c)^4 + 456*n*sin(d*x + c)^n*sin(d*x +
 c)^6 - 24*n^2*sin(d*x + c)^n*sin(d*x + c)^4 + 960*sin(d*x + c)^n*sin(d*x +
 c)^6 - 188*n*sin(d*x + c)^n*sin(d*x + c)^4 - 480*sin(d*x + c)^n*sin(d*x +
c)^4)*a^3/(n^4 + 28*n^3 + 284*n^2 + 1232*n + 1920) + 3*(n^3*sin(d*x + c)^n*
sin(d*x + c)^9 + 15*n^2*sin(d*x + c)^n*sin(d*x + c)^9 - 3*n^3*sin(d*x + c)^
n*sin(d*x + c)^7 + 71*n*sin(d*x + c)^n*sin(d*x + c)^9 - 51*n^2*sin(d*x + c)
^n*sin(d*x + c)^7 + 105*sin(d*x + c)^n*sin(d*x + c)^9 + 3*n^3*sin(d*x + c)^
n*sin(d*x + c)^5 - 261*n*sin(d*x + c)^n*sin(d*x + c)^7 + 57*n^2*sin(d*x + c
)^n*sin(d*x + c)^5 - 405*sin(d*x + c)^n*sin(d*x + c)^7 - n^3*sin(d*x + c)^n
*sin(d*x + c)^3 + 333*n*sin(d*x + c)^n*sin(d*x + c)^5 - 21*n^2*sin(d*x + c)
^n*sin(d*x + c)^3 + 567*sin(d*x + c)^n*sin(d*x + c)^5 - 143*n*sin(d*x + c)^
```

```
n*sin(d*x + c)^3 - 315*sin(d*x + c)^n*sin(d*x + c)^3)*a^3/(n^4 + 24*n^3 + 2
06*n^2 + 744*n + 945) + 3*(n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 12*n^2*sin(d
*x + c)^n*sin(d*x + c)^8 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 + 44*n*sin(d
*x + c)^n*sin(d*x + c)^8 - 42*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 48*sin(d*
x + c)^n*sin(d*x + c)^8 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^4 - 168*n*sin(d
*x + c)^n*sin(d*x + c)^6 + 48*n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 192*sin(d
*x + c)^n*sin(d*x + c)^6 - n^3*sin(d*x + c)^n*sin(d*x + c)^2 + 228*n*sin(d*
x + c)^n*sin(d*x + c)^4 - 18*n^2*sin(d*x + c)^n*sin(d*x + c)^2 + 288*sin(d*
x + c)^n*sin(d*x + c)^4 - 104*n*sin(d*x + c)^n*sin(d*x + c)^2 - 192*sin(d*x
 + c)^n*sin(d*x + c)^2)*a^3/(n^4 + 20*n^3 + 140*n^2 + 400*n + 384) + (n^3*s
in(d*x + c)^n*sin(d*x + c)^7 + 9*n^2*sin(d*x + c)^n*sin(d*x + c)^7 - 3*n^3*
sin(d*x + c)^n*sin(d*x + c)^5 + 23*n*sin(d*x + c)^n*sin(d*x + c)^7 - 33*n^2
*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 + 3*n^3*s
in(d*x + c)^n*sin(d*x + c)^3 - 93*n*sin(d*x + c)^n*sin(d*x + c)^5 + 39*n^2*
sin(d*x + c)^n*sin(d*x + c)^3 - 63*sin(d*x + c)^n*sin(d*x + c)^5 - n^3*sin(
d*x + c)^n*sin(d*x + c) + 141*n*sin(d*x + c)^n*sin(d*x + c)^3 - 15*n^2*sin(
d*x + c)^n*sin(d*x + c) + 105*sin(d*x + c)^n*sin(d*x + c)^3 - 71*n*sin(d*x
+ c)^n*sin(d*x + c) - 105*sin(d*x + c)^n*sin(d*x + c))*a^3/(n^4 + 16*n^3 +
86*n^2 + 176*n + 105))/d
```

Mupad [B]
time = 17.09, size = 1130, normalized size = 6.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (3*a^3*sin(c + d*x)^n*(6117676*n + 3058196*n^2 + 755233*n^3 + 109542*n^4 +
9800*n^5 + 502*n^6 + 11*n^7 + 3714480))/(256*d*(373560*n + 353884*n^2 + 172
634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (5*a^3
*sin(c + d*x)^n*cos(8*c + 8*d*x)*(35844*n + 31804*n^2 + 14083*n^3 + 3442*n^
4 + 472*n^5 + 34*n^6 + n^7 + 15120))/(256*d*(373560*n + 353884*n^2 + 172634
*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(
c + d*x)^n*cos(10*c + 10*d*x)*(35844*n + 31804*n^2 + 14083*n^3 + 3442*n^4 +
 472*n^5 + 34*n^6 + n^7 + 15120))/(512*d*(373560*n + 353884*n^2 + 172634*n^
3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*sin(c +
 d*x)*sin(c + d*x)^n*(n*16168200i + n^2*7143148i + n^3*1614322i + n^4*21508
3i + n^5*18019i + n^6*889i + n^7*19i + 13759200i)*1i)/(128*d*(373560*n + 35
3884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151
200)) - (3*a^3*sin(c + d*x)^n*cos(6*c + 6*d*x)*(1320260*n + 1100668*n^2 + 4
46515*n^3 + 97426*n^4 + 11608*n^5 + 706*n^6 + 17*n^7 + 579600))/(512*d*(373
560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 +
 n^8 + 151200)) - (a^3*sin(c + d*x)^n*cos(4*c + 4*d*x)*(1729500*n + 1246276
*n^2 + 413653*n^3 + 71710*n^4 + 6760*n^5 + 334*n^6 + 7*n^7 + 831600))/(64*d
```

*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*cos(2*c + 2*d*x)*(122059*n^3 - 2395364*n^2 - 9293340*n + 119842*n^4 + 17176*n^5 + 1042*n^6 + 25*n^7 - 6879600))/(256*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*sin(9*c + 9*d*x)*(n*39640i + n^2*34916i + n^3*15302i + n^4*3689i + n^5*497i + n^6*35i + n^7*1i + 16800i)*3i)/(256*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*97464i + n^2*117044i + n^3*66110i + n^4*18845i + n^5*2741i + n^6*191i + n^7*5i + 30240i)*1i)/(64*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) + (a^3*sin(c + d*x)^n*sin(7*c + 7*d*x)*(n*538680i + n^2*445172i + n^3*177758i + n^4*37709i + n^5*4277i + n^6*239i + n^7*5i + 237600i)*1i)/(256*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200)) - (a^3*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*763320i + n^2*586164i + n^3*211966i + n^4*40253i + n^5*4149i + n^6*223i + n^7*5i + 352800i)*3i)/(64*d*(373560*n + 353884*n^2 + 172634*n^3 + 48503*n^4 + 8162*n^5 + 812*n^6 + 44*n^7 + n^8 + 151200))

3.698 $\int \cos^7(c+dx) \sin^n(c+dx)(a+a \sin(c+dx))^2 \, dx$

Optimal. Leaf size=184

$$\frac{a^2 \sin^{1+n}(c+dx)}{d(1+n)}+\frac{2a^2 \sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a^2 \sin^{3+n}(c+dx)}{d(3+n)}-\frac{6a^2 \sin^{4+n}(c+dx)}{d(4+n)}+\frac{6a^2 \sin^{6+n}(c+dx)}{d(6+n)}+\frac{2a^2 \sin}{d(}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a^2*sin(d*x+c)^(2+n)/d/(2+n)-2*a^2*sin(d*x+c
)^(3+n)/d/(3+n)-6*a^2*sin(d*x+c)^(4+n)/d/(4+n)+6*a^2*sin(d*x+c)^(6+n)/d/(6+
n)+2*a^2*sin(d*x+c)^(7+n)/d/(7+n)-2*a^2*sin(d*x+c)^(8+n)/d/(8+n)-a^2*sin(d*
x+c)^(9+n)/d/(9+n)
```

Rubi [A]

time = 0.12, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2915, 90}

$$\frac{a^2 \sin^{n+1}(c+dx)}{d(n+1)}+\frac{2a^2 \sin^{n+2}(c+dx)}{d(n+2)}-\frac{2a^2 \sin^{n+3}(c+dx)}{d(n+3)}-\frac{6a^2 \sin^{n+4}(c+dx)}{d(n+4)}+\frac{6a^2 \sin^{n+6}(c+dx)}{d(n+6)}+\frac{2a^2 \sin^{n+7}(c+dx)}{d(n+7)}-\frac{2a^2 \sin^{n+8}(c+dx)}{d(n+8)}-\frac{a^2 \sin^{n+9}(c+dx)}{d(n+9)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a^2*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - (2*a^2*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (6*a^2*Sin[c + d*x]^(4
+ n))/(d*(4 + n)) + (6*a^2*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (2*a^2*Sin[c
 + d*x]^(7 + n))/(d*(7 + n)) - (2*a^2*Sin[c + d*x]^(8 + n))/(d*(8 + n)) - (
a^2*Sin[c + d*x]^(9 + n))/(d*(9 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^5\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^8\left(\frac{x}{a}\right)^n + 2a^8\left(\frac{x}{a}\right)^{1+n} - 2a^8\left(\frac{x}{a}\right)^{2+n} - 6a^8\left(\frac{x}{a}\right.\right.\right.}{}$$
$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2a^2\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a^2\sin^{3+n}(}{d(3+}$$

Mathematica [A]
time = 0.55, size = 126, normalized size = 0.68

$$\frac{a^2\sin^{1+n}(c+dx)\left(\frac{1}{1+n}+\frac{2\sin(c+dx)}{2+n}-\frac{2\sin^2(c+dx)}{3+n}-\frac{6\sin^3(c+dx)}{4+n}+\frac{6\sin^5(c+dx)}{6+n}+\frac{2\sin^6(c+dx)}{7+n}-\frac{2\sin^7(c+dx)}{8+n}-\frac{\sin^8(c+dx)}{9+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n)*((1 + n)^(-1) + (2*Sin[c + d*x])/(2 + n) - (2*Sin
[c + d*x]^2)/(3 + n) - (6*Sin[c + d*x]^3)/(4 + n) + (6*Sin[c + d*x]^5)/(6 +
 n) + (2*Sin[c + d*x]^6)/(7 + n) - (2*Sin[c + d*x]^7)/(8 + n) - Sin[c + d*x
]^8/(9 + n)))/d
```

Maple [F]
time = 0.47, size = 0, normalized size = 0.00

$$\int \left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x)
```

Maxima [A]
time = 0.30, size = 165, normalized size = 0.90

$$-\frac{\frac{a^2\sin(dx+c)^{n+9}}{n+9}+\frac{2a^2\sin(dx+c)^{n+8}}{n+8}-\frac{2a^2\sin(dx+c)^{n+7}}{n+7}-\frac{6a^2\sin(dx+c)^{n+6}}{n+6}+\frac{6a^2\sin(dx+c)^{n+4}}{n+4}+\frac{2a^2\sin(dx+c)^{n+3}}{n+3}-\frac{2a^2\sin(dx+c)^{n+2}}{n+2}-\frac{a^2\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -(a^2*sin(d*x + c)^(n + 9)/(n + 9) + 2*a^2*sin(d*x + c)^(n + 8)/(n + 8) - 2
*a^2*sin(d*x + c)^(n + 7)/(n + 7) - 6*a^2*sin(d*x + c)^(n + 6)/(n + 6) + 6*
a^2*sin(d*x + c)^(n + 4)/(n + 4) + 2*a^2*sin(d*x + c)^(n + 3)/(n + 3) - 2*a
^2*sin(d*x + c)^(n + 2)/(n + 2) - a^2*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(184) = 368$.
time = 0.42, size = 628, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a^2*n^7 + 32*a^2*n^6 + 414*a^2*n^5 + 2788*a^2*n^4 + 10469*a^2*n^3 + 21
708*a^2*n^2 + 22716*a^2*n + 9072*a^2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 26*a^2*
n^6 + 258*a^2*n^5 + 1240*a^2*n^4 + 3029*a^2*n^3 + 3534*a^2*n^2 + 1512*a^2*n
)*cos(d*x + c)^6 - 96*a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 22*a^2*n^5 + 1
70*a^2*n^4 + 560*a^2*n^3 + 789*a^2*n^2 + 378*a^2*n)*cos(d*x + c)^4 - 12480*
a^2*n^2 - 28800*a^2*n - 48*(a^2*n^5 + 20*a^2*n^4 + 130*a^2*n^3 + 300*a^2*n^
2 + 189*a^2*n)*cos(d*x + c)^2 - 18144*a^2 + ((a^2*n^7 + 31*a^2*n^6 + 391*a^
2*n^5 + 2581*a^2*n^4 + 9544*a^2*n^3 + 19564*a^2*n^2 + 20304*a^2*n + 8064*a^
2)*cos(d*x + c)^8 - 2*(a^2*n^7 + 29*a^2*n^6 + 343*a^2*n^5 + 2135*a^2*n^4 +
7504*a^2*n^3 + 14756*a^2*n^2 + 14832*a^2*n + 5760*a^2)*cos(d*x + c)^6 - 96*
a^2*n^4 - 1920*a^2*n^3 - 12*(a^2*n^6 + 24*a^2*n^5 + 223*a^2*n^4 + 1020*a^2*
n^3 + 2404*a^2*n^2 + 2736*a^2*n + 1152*a^2)*cos(d*x + c)^4 - 13440*a^2*n^2
- 38400*a^2*n - 48*(a^2*n^5 + 21*a^2*n^4 + 160*a^2*n^3 + 540*a^2*n^2 + 784*
a^2*n + 384*a^2)*cos(d*x + c)^2 - 36864*a^2)*sin(d*x + c))*sin(d*x + c)^n/(
d*n^8 + 40*d*n^7 + 670*d*n^6 + 6100*d*n^5 + 32773*d*n^4 + 105460*d*n^3 + 19
6380*d*n^2 + 190800*d*n + 72576*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29818 vs. $2(158) = 316$.
time = 69.07, size = 29818, normalized size = 162.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**2*sin(c)**n*cos(c)**7, Eq(d, 0)), (-a**2*log(s
in(c + d*x))/d + 32*a**2/(35*d*sin(c + d*x)) - a**2*cos(c + d*x)**2/(2*d*si
n(c + d*x)**2) + a**2/(8*d*sin(c + d*x)**2) - 16*a**2*cos(c + d*x)**2/(35*d
*sin(c + d*x)**3) + a**2*cos(c + d*x)**4/(4*d*sin(c + d*x)**4) - a**2*cos(c
 + d*x)**2/(8*d*sin(c + d*x)**4) + 12*a**2*cos(c + d*x)**4/(35*d*sin(c + d*
```

x)**5) - a**2*cos(c + d*x)**6/(6*d*sin(c + d*x)**6) + a**2*cos(c + d*x)**4/(8*d*sin(c + d*x)**6) - 2*a**2*cos(c + d*x)**6/(7*d*sin(c + d*x)**7) - a**2*cos(c + d*x)**6/(8*d*sin(c + d*x)**8), Eq(n, -9)), (-2*a**2*log(sin(c + d*x))/d - 16*a**2*sin(c + d*x)/(5*d) - 8*a**2*cos(c + d*x)**2/(5*d*sin(c + d*x)) + 16*a**2/(35*d*sin(c + d*x)) - a**2*cos(c + d*x)**2/(d*sin(c + d*x)**2) + 2*a**2*cos(c + d*x)**4/(5*d*sin(c + d*x)**3) - 8*a**2*cos(c + d*x)**2/(35*d*sin(c + d*x)**3) + a**2*cos(c + d*x)**4/(2*d*sin(c + d*x)**4) - a**2*cos(c + d*x)**6/(5*d*sin(c + d*x)**5) + 6*a**2*cos(c + d*x)**4/(35*d*sin(c + d*x)**5) - a**2*cos(c + d*x)**6/(3*d*sin(c + d*x)**6) - a**2*cos(c + d*x)**6/(7*d*sin(c + d*x)**7), Eq(n, -8)), (-3840*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**10/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 7680*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 3840*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 3840*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**10/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 7680*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 3840*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 5*a**2*tan(c/2 + d*x/2)**16/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 24*a**2*tan(c/2 + d*x/2)**15/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 20*a**2*tan(c/2 + d*x/2)**14/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 312*a**2*tan(c/2 + d*x/2)**13/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 220*a**2*tan(c/2 + d*x/2)**12/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 3864*a**2*tan(c/2 + d*x/2)**11/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 21000*a**2*tan(c/2 + d*x/2)**9/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 4230*a**2*tan(c/2 + d*x/2)**8/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 21000*a**2*tan(c/2 + d*x/2)**7/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 3864*a**2*tan(c/2 + d*x/2)**5/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 220*a**2*tan(c/2 + d*x/2)**4/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 312*a**2*tan(c/2 + d*x/2)**3/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) + 20*a**2*tan(c/2 + d*x/2)**2/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 24*a**2*tan(c/2 + d*x/2)/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6) - 5*a**2/(1920*d*tan(c/2 + d*x/2)**10 + 3840*d*tan(c/2 + d*x/2)**8 + 1920*d*tan(c/2 + d*x/2)**6), Eq(n, -7)), (-2880*

```
a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**11/(480*d*tan(c/2 + d*x
/2)**11 + 1440*d*tan(c/2 + d*x/2)**9 + 1440*d*tan(c/2 + d*x/2)**7 + 480*d*t
an(c/2 + d*x/2)**5) - 8640*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/
2)**9/(480*d*tan(c/2 + d*x/2)**11 + 1440*d*tan(c/2 + d*x/2)**9 + 1440*d*tan
(c/2 + d*x/2)**7 + 480*d*tan(c/2 + d*x/2)**5) - 8640*a**2*log(tan(c/2 + d*x
/2)**2 + 1)*tan(c/2 + d*x/2)**7/(480*d*tan(c/2 + d*x/2)**11 + 1440*d*tan(c/
2 + d*x/2)**9 + 1440*d*tan(c/2 + d*x/2)**7 + 480*d*tan(c/2 + d*x/2)**5) - 2
880*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(480*d*tan(c/2 +
d*x/2)**11 + 1440*d*tan(c/2 + d*x/2)**9 + 1440*d*tan(c/2 + d*x/2)**7 + 480*
d*tan(c/2 + d*x/2)**5) + 2880*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**
11/(480*d*tan(c/2 + d*x/2)**11 + 1440*d*tan(c/2 + d*x/2)**9 + 1440*d*tan(c/
2 + d*x/2)**7 + 480*d*tan(c/2 + d*x/2)**5) + 8640*a**2*log(tan(c/2 + d*x/2)
)*tan(c/2 + d*x/2)**9/(480*d*tan(c/2 + d*x/2)**11 + 1440*d*tan(c/2 + d*x/2)
**9 + 1440*d*tan(c/2 + d*x/2)**7 + 480*d*tan(c/2 + d*x/2)**5) + 8640*a**2*l
og(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**7/(480*d*tan(c/2 + d*x/2)**11 + 1440
*d*tan(c/2 + d*x/2)**9 + 1440*d*tan(c/2 + d*x/2)**7 + 480*d*tan(c/2 + d*x/2
)**5) + 2880*a**2*log(tan(c/2 + d*x/2))*tan(c/2...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(184) = 368$.
time = 0.54, size = 1019, normalized size = 5.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^9 + 15*n^2*sin(d*x + c)^n*sin(d*x + c)^9
 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^7 + 71*n*sin(d*x + c)^n*sin(d*x + c)^9
 - 51*n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 105*sin(d*x + c)^n*sin(d*x + c)^9
 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^5 - 261*n*sin(d*x + c)^n*sin(d*x + c)^
7 + 57*n^2*sin(d*x + c)^n*sin(d*x + c)^5 - 405*sin(d*x + c)^n*sin(d*x + c)^
7 - n^3*sin(d*x + c)^n*sin(d*x + c)^3 + 333*n*sin(d*x + c)^n*sin(d*x + c)^5
 - 21*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 567*sin(d*x + c)^n*sin(d*x + c)^5
 - 143*n*sin(d*x + c)^n*sin(d*x + c)^3 - 315*sin(d*x + c)^n*sin(d*x + c)^3)
*a^2/(n^4 + 24*n^3 + 206*n^2 + 744*n + 945) + 2*(n^3*sin(d*x + c)^n*sin(d*x
 + c)^8 + 12*n^2*sin(d*x + c)^n*sin(d*x + c)^8 - 3*n^3*sin(d*x + c)^n*sin(d
*x + c)^6 + 44*n*sin(d*x + c)^n*sin(d*x + c)^8 - 42*n^2*sin(d*x + c)^n*sin(
d*x + c)^6 + 48*sin(d*x + c)^n*sin(d*x + c)^8 + 3*n^3*sin(d*x + c)^n*sin(d*
x + c)^4 - 168*n*sin(d*x + c)^n*sin(d*x + c)^6 + 48*n^2*sin(d*x + c)^n*sin(
d*x + c)^4 - 192*sin(d*x + c)^n*sin(d*x + c)^6 - n^3*sin(d*x + c)^n*sin(d*x
 + c)^2 + 228*n*sin(d*x + c)^n*sin(d*x + c)^4 - 18*n^2*sin(d*x + c)^n*sin(d
*x + c)^2 + 288*sin(d*x + c)^n*sin(d*x + c)^4 - 104*n*sin(d*x + c)^n*sin(d*
x + c)^2 - 192*sin(d*x + c)^n*sin(d*x + c)^2)*a^2/(n^4 + 20*n^3 + 140*n^2 +
 400*n + 384) + (n^3*sin(d*x + c)^n*sin(d*x + c)^7 + 9*n^2*sin(d*x + c)^n*s
```

```
in(d*x + c)^7 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^5 + 23*n*sin(d*x + c)^n*s
in(d*x + c)^7 - 33*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*sin(d*x + c)^n*si
n(d*x + c)^7 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)^3 - 93*n*sin(d*x + c)^n*si
n(d*x + c)^5 + 39*n^2*sin(d*x + c)^n*sin(d*x + c)^3 - 63*sin(d*x + c)^n*sin
(d*x + c)^5 - n^3*sin(d*x + c)^n*sin(d*x + c) + 141*n*sin(d*x + c)^n*sin(d*
x + c)^3 - 15*n^2*sin(d*x + c)^n*sin(d*x + c) + 105*sin(d*x + c)^n*sin(d*x
+ c)^3 - 71*n*sin(d*x + c)^n*sin(d*x + c) - 105*sin(d*x + c)^n*sin(d*x + c)
)*a^2/(n^4 + 16*n^3 + 86*n^2 + 176*n + 105))/d
```

Mupad [B]
time = 16.45, size = 1142, normalized size = 6.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x)^n*(n*1507788i + n^2*868332i + n^3*238585i + n^4*37844i +
n^5*3702i + n^6*208i + n^7*5i + 843696i))/(64*d*(n*190800i + n^2*196380i +
n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i
)) - (a^2*sin(c + d*x)^n*sin(9*c + 9*d*x)*(20304*n + 19564*n^2 + 9544*n^3 +
 2581*n^4 + 391*n^5 + 31*n^6 + n^7 + 8064)*1i)/(256*d*(n*190800i + n^2*1963
80i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i +
72576i)) + (a^2*sin(c + d*x)*sin(c + d*x)^n*(6799248*n + 3169500*n^2 + 7706
32*n^3 + 111993*n^4 + 10267*n^5 + 555*n^6 + 13*n^7 + 5588352)*1i)/(128*d*(n
*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i +
n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^n*cos(8*c + 8*d*x)*(n*22716
i + n^2*21708i + n^3*10469i + n^4*2788i + n^5*414i + n^6*32i + n^7*1i + 907
2i))/(64*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i
+ n^6*670i + n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^n*cos(6*c + 6*
d*x)*(n*43920i + n^2*39882i + n^3*17909i + n^4*4336i + n^5*570i + n^6*38i +
 n^7*1i + 18144i))/(16*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773
i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i)) - (a^2*sin(c + d*x)^
n*cos(4*c + 4*d*x)*(n*140868i + n^2*111816i + n^3*41669i + n^4*7996i + n^5*
822i + n^6*44i + n^7*1i + 63504i))/(16*d*(n*190800i + n^2*196380i + n^3*105
460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i + 72576i)) + (a
^2*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n^3*2549i - n^2*59958i - n*186480i + n^
4*3568i + n^5*570i + n^6*38i + n^7*1i - 127008i))/(16*d*(n*190800i + n^2*19
6380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8*1i
+ 72576i)) - (a^2*sin(c + d*x)^n*sin(7*c + 7*d*x)*(23472*n + 18900*n^2 + 67
76*n^3 + 987*n^4 - 7*n^5 - 15*n^6 - n^7 + 10368)*1i)/(256*d*(n*190800i + n^
2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i + n^7*40i + n^8
*1i + 72576i)) + (a^2*sin(c + d*x)^n*sin(5*c + 5*d*x)*(178128*n + 165132*n^
2 + 76280*n^3 + 19149*n^4 + 2627*n^5 + 183*n^6 + 5*n^7 + 72576)*1i)/(64*d*(
n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*6100i + n^6*670i +
```

```
 n^7*40i + n^8*1i + 72576i)) + (a^2*sin(c + d*x)^n*sin(3*c + 3*d*x)*(112094
4*n + 889556*n^2 + 338024*n^3 + 68603*n^4 + 7661*n^5 + 449*n^6 + 11*n^7 + 5
08032)*1i)/(64*d*(n*190800i + n^2*196380i + n^3*105460i + n^4*32773i + n^5*
6100i + n^6*670i + n^7*40i + n^8*1i + 72576i))
```

3.699 $\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=167

$$\frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{a\sin^{2+n}(c+dx)}{d(2+n)}-\frac{3a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{3a\sin^{4+n}(c+dx)}{d(4+n)}+\frac{3a\sin^{5+n}(c+dx)}{d(5+n)}+\frac{3a\sin^{6+n}(c}{d(6+}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+a*sin(d*x+c)^(2+n)/d/(2+n)-3*a*sin(d*x+c)^(3+n)/
d/(3+n)-3*a*sin(d*x+c)^(4+n)/d/(4+n)+3*a*sin(d*x+c)^(5+n)/d/(5+n)+3*a*sin(d
*x+c)^(6+n)/d/(6+n)-a*sin(d*x+c)^(7+n)/d/(7+n)-a*sin(d*x+c)^(8+n)/d/(8+n)
```

Rubi [A]

time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2915, 90}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}+\frac{a\sin^{n+2}(c+dx)}{d(n+2)}-\frac{3a\sin^{n+3}(c+dx)}{d(n+3)}-\frac{3a\sin^{n+4}(c+dx)}{d(n+4)}+\frac{3a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{3a\sin^{n+6}(c+dx)}{d(n+6)}-\frac{a\sin^{n+7}(c+dx)}{d(n+7)}-\frac{a\sin^{n+8}(c+dx)}{d(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (a*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (3*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (3*a*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (3*a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (3*a*Sin[c + d*x]^(6 + n
))/(d*(6 + n)) - (a*Sin[c + d*x]^(7 + n))/(d*(7 + n)) - (a*Sin[c + d*x]^(8
+ n))/(d*(8 + n))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)^4\,dx, x, a\sin(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^7\left(\frac{x}{a}\right)^n + a^7\left(\frac{x}{a}\right)^{1+n} - 3a^7\left(\frac{x}{a}\right)^{2+n} - 3a^7\left(\frac{x}{a}\right)^{3+n}\right.\right.}{}$$
$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{a\sin^{2+n}(c+dx)}{d(2+n)} - \frac{3a\sin^{3+n}(c+dx}{d(3+n)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 659 vs. 2(167) = 334.
time = 2.25, size = 659, normalized size = 3.95

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*Sin[c + d*x]^n*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n)*(1755648 + 2521536*n + 1486096*n^2 + 481280*n^3 + 9
4012*n^4 + 11084*n^5 + 724*n^6 + 20*n^7 + 6*(114816 + 262064*n + 219828*n^2
 + 90640*n^3 + 20499*n^4 + 2611*n^5 + 177*n^6 + 5*n^7)*Cos[2*(c + d*x)] + 1
2*(10368 + 25776*n + 24372*n^2 + 11584*n^3 + 3027*n^4 + 439*n^5 + 33*n^6 +
n^7)*Cos[4*(c + d*x)] + 11520*Cos[6*(c + d*x)] + 29664*n*Cos[6*(c + d*x)] +
 29512*n^2*Cos[6*(c + d*x)] + 15008*n^3*Cos[6*(c + d*x)] + 4270*n^4*Cos[6*(
c + d*x)] + 686*n^5*Cos[6*(c + d*x)] + 58*n^6*Cos[6*(c + d*x)] + 2*n^7*Cos[
6*(c + d*x)] + 468720*Sin[c + d*x] + 879324*n*Sin[c + d*x] + 552236*n^2*Sin
[c + d*x] + 167669*n^3*Sin[c + d*x] + 28904*n^4*Sin[c + d*x] + 3050*n^5*Sin
[c + d*x] + 188*n^6*Sin[c + d*x] + 5*n^7*Sin[c + d*x] + 186480*Sin[3*(c + d
*x)] + 439836*n*Sin[3*(c + d*x)] + 384948*n^2*Sin[3*(c + d*x)] + 165273*n^3
*Sin[3*(c + d*x)] + 38232*n^4*Sin[3*(c + d*x)] + 4866*n^5*Sin[3*(c + d*x)]
+ 324*n^6*Sin[3*(c + d*x)] + 9*n^7*Sin[3*(c + d*x)] + 45360*Sin[5*(c + d*x)
] + 114252*n*Sin[5*(c + d*x)] + 110036*n^2*Sin[5*(c + d*x)] + 53525*n^3*Sin
[5*(c + d*x)] + 14360*n^4*Sin[5*(c + d*x)] + 2138*n^5*Sin[5*(c + d*x)] + 16
4*n^6*Sin[5*(c + d*x)] + 5*n^7*Sin[5*(c + d*x)] + 5040*Sin[7*(c + d*x)] + 1
3068*n*Sin[7*(c + d*x)] + 13132*n^2*Sin[7*(c + d*x)] + 6769*n^3*Sin[7*(c +
d*x)] + 1960*n^4*Sin[7*(c + d*x)] + 322*n^5*Sin[7*(c + d*x)] + 28*n^6*Sin[7
*(c + d*x)] + n^7*Sin[7*(c + d*x)]))/(64*d*(1 + n)*(2 + n)*(3 + n)*(4 + n)*
(5 + n)*(6 + n)*(7 + n)*(8 + n))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+a\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x)
```

Maxima [A]
time = 0.29, size = 148, normalized size = 0.89

$$-\frac{\frac{a\sin(dx+c)^{n+8}}{n+8}+\frac{a\sin(dx+c)^{n+7}}{n+7}-\frac{3\,a\sin(dx+c)^{n+6}}{n+6}-\frac{3\,a\sin(dx+c)^{n+5}}{n+5}+\frac{3\,a\sin(dx+c)^{n+4}}{n+4}+\frac{3\,a\sin(dx+c)^{n+3}}{n+3}-\frac{a\sin(dx+c)^{n+2}}{n+2}-\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*sin(d*x + c)^(n + 8)/(n + 8) + a*sin(d*x + c)^(n + 7)/(n + 7) - 3*a*sin
(d*x + c)^(n + 6)/(n + 6) - 3*a*sin(d*x + c)^(n + 5)/(n + 5) + 3*a*sin(d*x
+ c)^(n + 4)/(n + 4) + 3*a*sin(d*x + c)^(n + 3)/(n + 3) - a*sin(d*x + c)^(n
 + 2)/(n + 2) - a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(167) = 334.
time = 0.42, size = 445, normalized size = 2.66

$$-\frac{\left(\left(an^{7}+28\,an^{6}+322\,an^{5}+1960\,an^{4}+6769\,an^{3}+13132\,an^{2}+13068\,an+5040\,a\right)\cos\left(dx+c\right)^{8}-\left(an^{7}+22\,an^{6}+190\,an^{5}+820\,an^{4}+1849\,an^{3}+2038\,an^{2}+840\,an\right)\cos\left(dx+c\right)^{6}-48\,an^{4}-6\left(an^{6}+18\,an^{5}+118\,an^{4}+348\,an^{3}+457\,an^{2}+210\,an\right)\cos\left(dx+c\right)^{4}-768\,an^{3}-4128\,an^{2}-24\left(an^{5}+16\,an^{4}+86\,an^{3}+176\,an^{2}+105\,an\right)\cos\left(dx+c\right)^{2}-8448\,an-\left(\left(an^{7}+29\,an^{6}+343\,an^{5}+2135\,an^{4}+7504\,an^{3}+14756\,an^{2}+14832\,an+5760\,a\right)\cos\left(dx+c\right)^{6}+48\,an^{4}+6\left(an^{6}+24\,an^{5}+223\,an^{4}+1020\,an^{3}+2404\,an^{2}+2736\,an+1152\,a\right)\cos\left(dx+c\right)^{4}+960\,an^{3}+6720\,an^{2}+24\left(an^{5}+21\,an^{4}+160\,an^{3}+540\,an^{2}+784\,an+384\,a\right)\cos\left(dx+c\right)^{2}+19200\,an+18432\,a\right)\sin\left(dx+c\right)-5040\,a\right)\sin\left(dx+c\right)^{n}}{dn^{8}+36\,dn^{7}+546\,dn^{6}+4536\,dn^{5}+22449\,dn^{4}+67284\,dn^{3}+118124\,dn^{2}+109584\,dn+40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a*n^7 + 28*a*n^6 + 322*a*n^5 + 1960*a*n^4 + 6769*a*n^3 + 13132*a*n^2 + 1
3068*a*n + 5040*a)*cos(d*x + c)^8 - (a*n^7 + 22*a*n^6 + 190*a*n^5 + 820*a*n
^4 + 1849*a*n^3 + 2038*a*n^2 + 840*a*n)*cos(d*x + c)^6 - 48*a*n^4 - 6*(a*n^
6 + 18*a*n^5 + 118*a*n^4 + 348*a*n^3 + 457*a*n^2 + 210*a*n)*cos(d*x + c)^4
- 768*a*n^3 - 4128*a*n^2 - 24*(a*n^5 + 16*a*n^4 + 86*a*n^3 + 176*a*n^2 + 10
5*a*n)*cos(d*x + c)^2 - 8448*a*n - ((a*n^7 + 29*a*n^6 + 343*a*n^5 + 2135*a*
n^4 + 7504*a*n^3 + 14756*a*n^2 + 14832*a*n + 5760*a)*cos(d*x + c)^6 + 48*a*
n^4 + 6*(a*n^6 + 24*a*n^5 + 223*a*n^4 + 1020*a*n^3 + 2404*a*n^2 + 2736*a*n
+ 1152*a)*cos(d*x + c)^4 + 960*a*n^3 + 6720*a*n^2 + 24*(a*n^5 + 21*a*n^4 +
160*a*n^3 + 540*a*n^2 + 784*a*n + 384*a)*cos(d*x + c)^2 + 19200*a*n + 18432
*a)*sin(d*x + c) - 5040*a)*sin(d*x + c)^n/(d*n^8 + 36*d*n^7 + 546*d*n^6 + 4
536*d*n^5 + 22449*d*n^4 + 67284*d*n^3 + 118124*d*n^2 + 109584*d*n + 40320*d
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 19968 vs. 2(141) = 282.
time = 38.79, size = 19968, normalized size = 119.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n*(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)*sin(c)**n*cos(c)**7, Eq(d, 0)), (-a*log(sin(c +
 d*x))/d + 16*a/(35*d*sin(c + d*x)) - a*cos(c + d*x)**2/(2*d*sin(c + d*x)**
2) - 8*a*cos(c + d*x)**2/(35*d*sin(c + d*x)**3) + a*cos(c + d*x)**4/(4*d*si
n(c + d*x)**4) + 6*a*cos(c + d*x)**4/(35*d*sin(c + d*x)**5) - a*cos(c + d*x
)**6/(6*d*sin(c + d*x)**6) - a*cos(c + d*x)**6/(7*d*sin(c + d*x)**7), Eq(n,
 -8)), (-a*log(sin(c + d*x))/d - 16*a*sin(c + d*x)/(5*d) - 8*a*cos(c + d*x)
**2/(5*d*sin(c + d*x)) - a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) + 2*a*cos(
c + d*x)**4/(5*d*sin(c + d*x)**3) + a*cos(c + d*x)**4/(4*d*sin(c + d*x)**4)
 - a*cos(c + d*x)**6/(5*d*sin(c + d*x)**5) - a*cos(c + d*x)**6/(6*d*sin(c +
 d*x)**6), Eq(n, -7)), (-960*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2
)**9/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2
 + d*x/2)**5) - 1920*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**7/(32
0*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2
)**5) - 960*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(320*d*tan(c
/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) + 9
60*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**9/(320*d*tan(c/2 + d*x/2)**9 +
 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) + 1920*a*log(tan(c/
2 + d*x/2))*tan(c/2 + d*x/2)**7/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2
+ d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) + 960*a*log(tan(c/2 + d*x/2))*tan(
c/2 + d*x/2)**5/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 32
0*d*tan(c/2 + d*x/2)**5) - 2*a*tan(c/2 + d*x/2)**14/(320*d*tan(c/2 + d*x/2)
**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) - 5*a*tan(c/2
+ d*x/2)**13/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d
*tan(c/2 + d*x/2)**5) + 26*a*tan(c/2 + d*x/2)**12/(320*d*tan(c/2 + d*x/2)**
9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) + 90*a*tan(c/2 +
 d*x/2)**11/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*
tan(c/2 + d*x/2)**5) - 322*a*tan(c/2 + d*x/2)**10/(320*d*tan(c/2 + d*x/2)**
9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) - 1750*a*tan(c/2
 + d*x/2)**8/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d
*tan(c/2 + d*x/2)**5) - 830*a*tan(c/2 + d*x/2)**7/(320*d*tan(c/2 + d*x/2)**
9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) - 1750*a*tan(c/2
 + d*x/2)**6/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d
*tan(c/2 + d*x/2)**5) - 322*a*tan(c/2 + d*x/2)**4/(320*d*tan(c/2 + d*x/2)**
9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) + 90*a*tan(c/2 +
 d*x/2)**3/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*t
an(c/2 + d*x/2)**5) + 26*a*tan(c/2 + d*x/2)**2/(320*d*tan(c/2 + d*x/2)**9 +
 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2 + d*x/2)**5) - 5*a*tan(c/2 + d*x
/2)/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 + 320*d*tan(c/2
+ d*x/2)**5) - 2*a/(320*d*tan(c/2 + d*x/2)**9 + 640*d*tan(c/2 + d*x/2)**7 +
 320*d*tan(c/2 + d*x/2)**5), Eq(n, -6)), (-576*a*log(tan(c/2 + d*x/2)**2 +
1)*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2
```

```
)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 1728*a*log(
tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**10 +
576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x
/2)**4) - 1728*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(192*d*ta
n(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6
+ 192*d*tan(c/2 + d*x/2)**4) - 576*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
 d*x/2)**4/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*
tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 576*a*log(tan(c/2 + d*x/
2))*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/
2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 1728*a*log
(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**10 + 576*d*
tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4
) + 1728*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2
)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c
/2 + d*x/2)**4) + 576*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(192*d*ta
n(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6
+ 192*d*tan(c/2 + d*x/2)**4) - 3*a*tan(c/2 + d*x/2)**14/(192*d*tan(c/2 + d*
x/2)**10 + 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*ta
n(c/2 + d*x/2)**4) - 8*a*tan(c/2 + d*x/2)**13/(192*d*tan(c/2 + d*x/2)**10 +
 576*d*tan(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*
x/2)**4) + 51*a*tan(c/2 + d*x/2)**12/(192*d*tan(c/2 + d*x/2)**10 + 576*d*ta
n(c/2 + d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4)
+ 240*a*tan(c/2 + d*x/2)**11/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 +
d*x/2)**8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 2184*a
*tan(c/2 + d*x/2)**9/(192*d*tan(c/2 + d*x/2)**10 + 576*d*tan(c/2 + d*x/2)**
8 + 576*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 +...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 674 vs. 2(167) = 334.
time = 0.46, size = 674, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((n^3*sin(d*x + c)^n*sin(d*x + c)^8 + 12*n^2*sin(d*x + c)^n*sin(d*x + c)^8
 - 3*n^3*sin(d*x + c)^n*sin(d*x + c)^6 + 44*n*sin(d*x + c)^n*sin(d*x + c)^8
 - 42*n^2*sin(d*x + c)^n*sin(d*x + c)^6 + 48*sin(d*x + c)^n*sin(d*x + c)^8
+ 3*n^3*sin(d*x + c)^n*sin(d*x + c)^4 - 168*n*sin(d*x + c)^n*sin(d*x + c)^6
 + 48*n^2*sin(d*x + c)^n*sin(d*x + c)^4 - 192*sin(d*x + c)^n*sin(d*x + c)^6
 - n^3*sin(d*x + c)^n*sin(d*x + c)^2 + 228*n*sin(d*x + c)^n*sin(d*x + c)^4
- 18*n^2*sin(d*x + c)^n*sin(d*x + c)^2 + 288*sin(d*x + c)^n*sin(d*x + c)^4
- 104*n*sin(d*x + c)^n*sin(d*x + c)^2 - 192*sin(d*x + c)^n*sin(d*x + c)^2)*
a/(n^4 + 20*n^3 + 140*n^2 + 400*n + 384) + (n^3*sin(d*x + c)^n*sin(d*x + c)
```

```
^7 + 9*n^2*sin(d*x + c)^n*sin(d*x + c)^7 - 3*n^3*sin(d*x + c)^n*sin(d*x + c
)^5 + 23*n*sin(d*x + c)^n*sin(d*x + c)^7 - 33*n^2*sin(d*x + c)^n*sin(d*x +
c)^5 + 15*sin(d*x + c)^n*sin(d*x + c)^7 + 3*n^3*sin(d*x + c)^n*sin(d*x + c)
^3 - 93*n*sin(d*x + c)^n*sin(d*x + c)^5 + 39*n^2*sin(d*x + c)^n*sin(d*x + c
)^3 - 63*sin(d*x + c)^n*sin(d*x + c)^5 - n^3*sin(d*x + c)^n*sin(d*x + c) +
141*n*sin(d*x + c)^n*sin(d*x + c)^3 - 15*n^2*sin(d*x + c)^n*sin(d*x + c) +
105*sin(d*x + c)^n*sin(d*x + c)^3 - 71*n*sin(d*x + c)^n*sin(d*x + c) - 105*
sin(d*x + c)^n*sin(d*x + c))*a/(n^4 + 16*n^3 + 86*n^2 + 176*n + 105))/d
```

Mupad [B]
time = 15.90, size = 901, normalized size = 5.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*sin(c + d*x)^n*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*sin(c + d*x)^n*(879324*n + 552236*n^2 + 167669*n^3 + 28904*n^4 + 3050*n^
5 + 188*n^6 + 5*n^7 + 468720))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 +
22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n
*cos(2*c + 2*d*x)*(109872*n + 41822*n^2 + 599*n^3 - 2332*n^4 - 454*n^5 - 34
*n^6 - n^7 + 70560))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 +
 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*sin(7*c +
7*d*x)*(n*14832i + n^2*14756i + n^3*7504i + n^4*2135i + n^5*343i + n^6*29i
+ n^7*1i + 5760i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4
+ 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*sin(5*c +
 5*d*x)*(n*139824i + n^2*131476i + n^3*62000i + n^4*16027i + n^5*2291i + n^
6*169i + n^7*5i + 56448i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22
449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*s
in(3*c + 3*d*x)*(n*210512i + n^2*171084i + n^3*67472i + n^4*14445i + n^5*17
33i + n^6*111i + n^7*3i + 94080i)*3i)/(64*d*(109584*n + 118124*n^2 + 67284*
n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c +
d*x)^n*cos(8*c + 8*d*x)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))/(128*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449
*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*cos(
6*c + 6*d*x)*(25296*n + 24226*n^2 + 11689*n^3 + 3100*n^4 + 454*n^5 + 34*n^6
 + n^7 + 10080))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)^n*cos(4*c + 4*d*
x)*(81396*n + 68728*n^2 + 27937*n^3 + 5968*n^4 + 682*n^5 + 40*n^6 + n^7 + 3
5280))/(32*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 54
6*n^6 + 36*n^7 + n^8 + 40320)) - (a*sin(c + d*x)*sin(c + d*x)^n*(n*1735344i
 + n^2*826612i + n^3*209360i + n^4*32515i + n^5*3251i + n^6*193i + n^7*5i +
 1411200i)*1i)/(64*d*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

3.700 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$\frac{\sin^{1+n}(c+dx)}{ad(1+n)}-\frac{\sin^{2+n}(c+dx)}{ad(2+n)}-\frac{2\sin^{3+n}(c+dx)}{ad(3+n)}+\frac{2\sin^{4+n}(c+dx)}{ad(4+n)}+\frac{\sin^{5+n}(c+dx)}{ad(5+n)}-\frac{\sin^{6+n}(c+dx)}{ad(6+n)}$$

```
[Out] sin(d*x+c)^(1+n)/a/d/(1+n)-sin(d*x+c)^(2+n)/a/d/(2+n)-2*sin(d*x+c)^(3+n)/a/
d/(3+n)+2*sin(d*x+c)^(4+n)/a/d/(4+n)+sin(d*x+c)^(5+n)/a/d/(5+n)-sin(d*x+c)^
(6+n)/a/d/(6+n)
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 90}

$$\frac{\sin^{n+1}(c+dx)}{ad(n+1)}-\frac{\sin^{n+2}(c+dx)}{ad(n+2)}-\frac{2\sin^{n+3}(c+dx)}{ad(n+3)}+\frac{2\sin^{n+4}(c+dx)}{ad(n+4)}+\frac{\sin^{n+5}(c+dx)}{ad(n+5)}-\frac{\sin^{n+6}(c+dx)}{ad(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a*d*(1 + n)) - Sin[c + d*x]^(2 + n)/(a*d*(2 + n)) - (
2*Sin[c + d*x]^(3 + n))/(a*d*(3 + n)) + (2*Sin[c + d*x]^(4 + n))/(a*d*(4 +
n)) + Sin[c + d*x]^(5 + n)/(a*d*(5 + n)) - Sin[c + d*x]^(6 + n)/(a*d*(6 + n
))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3 \left(\frac{x}{a}\right)^n (a+x)^2\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^5\left(\frac{x}{a}\right)^n - a^5\left(\frac{x}{a}\right)^{1+n} - 2a^5\left(\frac{x}{a}\right)^{2+n} + 2a^5\left(\frac{x}{a}\right)^{3+n} + a^5\left(\frac{x}{a}\right)^{4+n} - a^5\right.\right.}{a^7 d}$$

$$= \frac{\sin^{1+n}(c+dx)}{ad(1+n)} - \frac{\sin^{2+n}(c+dx)}{ad(2+n)} - \frac{2\sin^{3+n}(c+dx)}{ad(3+n)} + \frac{2\sin^{4+n}(c+dx)}{ad(4+n)} +$$

Mathematica [A]
time = 0.22, size = 95, normalized size = 0.69

$$\frac{\sin^{1+n}(c+dx)\left(\frac{1}{1+n} - \frac{\sin(c+dx)}{2+n} - \frac{2\sin^2(c+dx)}{3+n} + \frac{2\sin^3(c+dx)}{4+n} + \frac{\sin^4(c+dx)}{5+n} - \frac{\sin^5(c+dx)}{6+n}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - Sin[c + d*x]/(2 + n) - (2*Sin[c + d*x
]^2)/(3 + n) + (2*Sin[c + d*x]^3)/(4 + n) + Sin[c + d*x]^4/(5 + n) - Sin[c
+ d*x]^5/(6 + n)))/(a*d)
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)}{a+a\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x)
```

Maxima [A]
time = 0.32, size = 241, normalized size = 1.76

$$-\frac{\left((n^5+15n^4+85n^3+225n^2+274n+120)\sin(dx+c)^6 - (n^5+16n^4+95n^3+260n^2+324n+144)\sin(dx+c)^5 - 2(n^5+17n^4+107n^3+307n^2+396n+180)\sin(dx+c)^4 + 2(n^5+18n^4+121n^3+372n^2+508n+240)\sin(dx+c)^3 + (n^5+19n^4+137n^3+461n^2+702n+360)\sin(dx+c)^2 - (n^5+20n^4+155n^3+580n^2+1044n+720)\sin(dx+c)\right)\sin(dx+c)^n}{(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*sin(d*x + c)^6 - (n^5 + 1
6*n^4 + 95*n^3 + 260*n^2 + 324*n + 144)*sin(d*x + c)^5 - 2*(n^5 + 17*n^4 +
```

```
107*n^3 + 307*n^2 + 396*n + 180)*sin(d*x + c)^4 + 2*(n^5 + 18*n^4 + 121*n^3
 + 372*n^2 + 508*n + 240)*sin(d*x + c)^3 + (n^5 + 19*n^4 + 137*n^3 + 461*n^
2 + 702*n + 360)*sin(d*x + c)^2 - (n^5 + 20*n^4 + 155*n^3 + 580*n^2 + 1044*
n + 720)*sin(d*x + c))*sin(d*x + c)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*a*d)
```

Fricas [A]
time = 0.39, size = 243, normalized size = 1.77

$$\frac{\left(\left(n^{5}+15\,n^{4}+85\,n^{3}+225\,n^{2}+274\,n+120\right)\cos\left(dx+c\right)^{6}-\left(n^{5}+11\,n^{4}+41\,n^{3}+61\,n^{2}+30\,n\right)\cos\left(dx+c\right)^{4}-8\,n^{3}-4\,\left(n^{4}+9\,n^{3}+23\,n^{2}+15\,n\right)\cos\left(dx+c\right)^{2}-72\,n^{2}+\left(\left(n^{5}+16\,n^{4}+95\,n^{3}+260\,n^{2}+324\,n+144\right)\cos\left(dx+c\right)^{4}+8\,n^{3}+4\,\left(n^{4}+13\,n^{3}+56\,n^{2}+92\,n+48\right)\cos\left(dx+c\right)^{2}+96\,n^{2}+352\,n+384\right)\sin\left(dx+c\right)-184\,n-120\right)\sin\left(dx+c\right)^{n}}{adn^{6}+21\,adn^{5}+175\,adn^{4}+735\,adn^{3}+1624\,adn^{2}+1764\,adn+720\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*cos(d*x + c)^6 - (n^5 + 11
*n^4 + 41*n^3 + 61*n^2 + 30*n)*cos(d*x + c)^4 - 8*n^3 - 4*(n^4 + 9*n^3 + 23
*n^2 + 15*n)*cos(d*x + c)^2 - 72*n^2 + ((n^5 + 16*n^4 + 95*n^3 + 260*n^2 +
324*n + 144)*cos(d*x + c)^4 + 8*n^3 + 4*(n^4 + 13*n^3 + 56*n^2 + 92*n + 48)
*cos(d*x + c)^2 + 96*n^2 + 352*n + 384)*sin(d*x + c) - 184*n - 120)*sin(d*x
 + c)^n/(a*d*n^6 + 21*a*d*n^5 + 175*a*d*n^4 + 735*a*d*n^3 + 1624*a*d*n^2 +
1764*a*d*n + 720*a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,5,0,0]%%%}+%%%{1,[0,1,4,0,0]%%%}+%%%{2,[0,1,3,0,0]%%%
}+%%%{
```

Mupad [B]
time = 14.29, size = 568, normalized size = 4.15

$$\frac{\sin\left(c+d\,x\right)^n\,\cos\left(6\,c+6\,d\,x\right)\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{\sin\left(c+d\,x\right)^n\,\left(4\,n^5+92\,n^4+948\,n^3+4516\,n^2+8936\,n+5280\right)}{64\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{\sin\left(c+d\,x\right)\,\sin\left(c+d\,x\right)^n\,\left(n^5\,4{}\mathrm{i}+n^4\,96{}\mathrm{i}+n^3\,1052{}\mathrm{i}+n^2\,5904{}\mathrm{i}+n\,15504{}\mathrm{i}+14400{}\mathrm{i}\right)\,1{}\mathrm{i}}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{\sin\left(c+d\,x\right)^n\,\cos\left(4\,c+4\,d\,x\right)\,\left(2\,n^5+46\,n^4+346\,n^3+1106\,n^2+1524\,n+720\right)}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{\sin\left(c+d\,x\right)^n\,\cos\left(2\,c+2\,d\,x\right)\,\left(-n^5-15\,n^4+43\,n^3+927\,n^2+2670\,n+1800\right)}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{\sin\left(c+d\,x\right)^n\,\sin\left(5\,c+5\,d\,x\right)\,\left(n^5\,2{}\mathrm{i}+n^4\,32{}\mathrm{i}+n^3\,190{}\mathrm{i}+n^2\,520{}\mathrm{i}+n\,648{}\mathrm{i}+288{}\mathrm{i}\right)\,1{}\mathrm{i}}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{\sin\left(c+d\,x\right)^n\,\sin\left(3\,c+3\,d\,x\right)\,\left(n^5\,6{}\mathrm{i}+n^4\,128{}\mathrm{i}+n^3\,986{}\mathrm{i}+n^2\,3352{}\mathrm{i}+n\,4888{}\mathrm{i}+2400{}\mathrm{i}\right)\,1{}\mathrm{i}}{32\,a\,d\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)
) - (sin(c + d*x)^n*(8936*n + 4516*n^2 + 948*n^3 + 92*n^4 + 4*n^5 + 5280))/
(64*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (si
n(c + d*x)*sin(c + d*x)^n*(n*15504i + n^2*5904i + n^3*1052i + n^4*96i + n^5
*4i + 14400i)*1i)/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
 n^6 + 720)) + (sin(c + d*x)^n*cos(4*c + 4*d*x)*(1524*n + 1106*n^2 + 346*n^
3 + 46*n^4 + 2*n^5 + 720))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
 21*n^5 + n^6 + 720)) + (sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2
+ 43*n^3 - 15*n^4 - n^5 + 1800))/(32*a*d*(1764*n + 1624*n^2 + 735*n^3 + 175
*n^4 + 21*n^5 + n^6 + 720)) - (sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*648i + n^
2*520i + n^3*190i + n^4*32i + n^5*2i + 288i)*1i)/(32*a*d*(1764*n + 1624*n^2
 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (sin(c + d*x)^n*sin(3*c + 3*d
*x)*(n*4888i + n^2*3352i + n^3*986i + n^4*128i + n^5*6i + 2400i)*1i)/(32*a*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

3.701 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{1+n}(c+dx)}{a^2d(1+n)}-\frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)}+\frac{2\sin^{4+n}(c+dx)}{a^2d(4+n)}-\frac{\sin^{5+n}(c+dx)}{a^2d(5+n)}$$

```
[Out] sin(d*x+c)^(1+n)/a^2/d/(1+n)-2*sin(d*x+c)^(2+n)/a^2/d/(2+n)+2*sin(d*x+c)^(4
+n)/a^2/d/(4+n)-sin(d*x+c)^(5+n)/a^2/d/(5+n)
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 76}

$$\frac{\sin^{n+1}(c+dx)}{a^2d(n+1)}-\frac{2\sin^{n+2}(c+dx)}{a^2d(n+2)}+\frac{2\sin^{n+4}(c+dx)}{a^2d(n+4)}-\frac{\sin^{n+5}(c+dx)}{a^2d(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^2*d*(1 + n)) - (2*Sin[c + d*x]^(2 + n))/(a^2*d*(2 +
 n)) + (2*Sin[c + d*x]^(4 + n))/(a^2*d*(4 + n)) - Sin[c + d*x]^(5 + n)/(a^2
*d*(5 + n))
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n(a+x)\,dx,x,a\sin(c+dx)\right)}{a^7d}\\ &= \frac{\text{Subst}\left(\int \left(a^4\left(\frac{x}{a}\right)^n-2a^4\left(\frac{x}{a}\right)^{1+n}+2a^4\left(\frac{x}{a}\right)^{3+n}-a^4\left(\frac{x}{a}\right)^{4+n}\right)\,dx,x,a\sin(c+\right.}{a^7d}\\ &= \frac{\sin^{1+n}(c+dx)}{a^2d(1+n)}-\frac{2\sin^{2+n}(c+dx)}{a^2d(2+n)}+\frac{2\sin^{4+n}(c+dx)}{a^2d(4+n)}-\frac{\sin^{5+n}(c+dx)}{a^2d(5+n)}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 117, normalized size = 1.27

$$\frac{\sin^{1+n}(c+dx)\left(40+38n+11n^2+n^3-2(20+29n+10n^2+n^3)\sin(c+dx)+2(10+17n+8n^2+n^3)\sin^3(c+dx)-(8+14n+7n^2+n^3)\sin^4(c+dx)\right)}{a^2d(1+n)(2+n)(4+n)(5+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(40 + 38*n + 11*n^2 + n^3 - 2*(20 + 29*n + 10*n^2 + n
^3)*Sin[c + d*x] + 2*(10 + 17*n + 8*n^2 + n^3)*Sin[c + d*x]^3 - (8 + 14*n +
 7*n^2 + n^3)*Sin[c + d*x]^4))/(a^2*d*(1 + n)*(2 + n)*(4 + n)*(5 + n))
```

Maple [F]
time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(\cos^7(dx+c))(\sin^n(dx+c))}{(a+a\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x)
```

Maxima [A]
time = 0.32, size = 126, normalized size = 1.37

$$-\frac{\left((n^3+7n^2+14n+8)\sin(dx+c)^5-2(n^3+8n^2+17n+10)\sin(dx+c)^4+2(n^3+10n^2+29n+20)\sin(dx+c)^2-(n^3+11n^2+38n+40)\sin(dx+c)\right)\sin(dx+c)^n}{(n^4+12n^3+49n^2+78n+40)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 7*n^2 + 14*n + 8)*sin(d*x + c)^5 - 2*(n^3 + 8*n^2 + 17*n + 10)*sin
(d*x + c)^4 + 2*(n^3 + 10*n^2 + 29*n + 20)*sin(d*x + c)^2 - (n^3 + 11*n^2 +
```

```
38*n + 40)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 12*n^3 + 49*n^2 + 78*n + 4
0)*a^2*d)
```

Fricas [A]
time = 0.39, size = 169, normalized size = 1.84

$$\frac{\left(2\left(n^{3}+8n^{2}+17n+10\right)\cos\left(dx+c\right)^{4}-2\left(n^{3}+6n^{2}+5n\right)\cos\left(dx+c\right)^{2}-4n^{2}-\left(\left(n^{3}+7n^{2}+14n+8\right)\cos\left(dx+c\right)^{4}-2\left(n^{3}+7n^{2}+14n+8\right)\cos\left(dx+c\right)^{2}-4n^{2}-24n-32\right)\sin\left(dx+c\right)-24n-20\right)\sin\left(dx+c\right)^{n}}{a^{2}dn^{4}+12a^{2}dn^{3}+49a^{2}dn^{2}+78a^{2}dn+40a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (2*(n^3 + 8*n^2 + 17*n + 10)*cos(d*x + c)^4 - 2*(n^3 + 6*n^2 + 5*n)*cos(d*x
 + c)^2 - 4*n^2 - ((n^3 + 7*n^2 + 14*n + 8)*cos(d*x + c)^4 - 2*(n^3 + 7*n^2
 + 14*n + 8)*cos(d*x + c)^2 - 4*n^2 - 24*n - 32)*sin(d*x + c) - 24*n - 20)*
sin(d*x + c)^n/(a^2*d*n^4 + 12*a^2*d*n^3 + 49*a^2*d*n^2 + 78*a^2*d*n + 40*a
^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,4,0,0]%%%}+%%%{2,[0,1,3,0,0]%%%}+%%%{-2,[0,1,1,0,0]%%
%}+%%%
```

Mupad [B]
time = 11.26, size = 280, normalized size = 3.04

$$\frac{\sin\left(c+d\,x\right)^n\,\left(560\,\sin\left(c+d\,x\right)-260\,n+160\,\cos\left(2\,c+2\,d\,x\right)+40\,\cos\left(4\,c+4\,d\,x\right)+40\,\sin\left(3\,c+3\,d\,x\right)-8\,\sin\left(5\,c+5\,d\,x\right)+468\,n\,\sin\left(c+d\,x\right)+192\,n\,\cos\left(2\,c+2\,d\,x\right)+68\,n\,\cos\left(4\,c+4\,d\,x\right)+70\,n\,\sin\left(3\,c+3\,d\,x\right)-14\,n\,\sin\left(5\,c+5\,d\,x\right)+106\,n^2\,\sin\left(c+d\,x\right)+6\,n^3\,\sin\left(c+d\,x\right)-64\,n^2-4\,n^3+32\,n^2\,\cos\left(2\,c+2\,d\,x\right)+32\,n^2\,\cos\left(4\,c+4\,d\,x\right)+4\,n^3\,\cos\left(4\,c+4\,d\,x\right)+35\,n^2\,\sin\left(3\,c+3\,d\,x\right)+5\,n^3\,\sin\left(3\,c+3\,d\,x\right)-7\,n^2\,\sin\left(5\,c+5\,d\,x\right)-n^3\,\sin\left(5\,c+5\,d\,x\right)-200\right)}{16\,a^2\,d\,\left(n^4+12\,n^3+49\,n^2+78\,n+40\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^n*(560*sin(c + d*x) - 260*n + 160*cos(2*c + 2*d*x) + 40*cos(4
*c + 4*d*x) + 40*sin(3*c + 3*d*x) - 8*sin(5*c + 5*d*x) + 468*n*sin(c + d*x)
 + 192*n*cos(2*c + 2*d*x) + 68*n*cos(4*c + 4*d*x) + 70*n*sin(3*c + 3*d*x) -
 14*n*sin(5*c + 5*d*x) + 106*n^2*sin(c + d*x) + 6*n^3*sin(c + d*x) - 64*n^2
 - 4*n^3 + 32*n^2*cos(2*c + 2*d*x) + 32*n^2*cos(4*c + 4*d*x) + 4*n^3*cos(4*
c + 4*d*x) + 35*n^2*sin(3*c + 3*d*x) + 5*n^3*sin(3*c + 3*d*x) - 7*n^2*sin(5
*c + 5*d*x) - n^3*sin(5*c + 5*d*x) - 200))/(16*a^2*d*(78*n + 49*n^2 + 12*n^
3 + n^4 + 40))
```

3.702 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{\sin^{1+n}(c+dx)}{a^3d(1+n)}-\frac{3\sin^{2+n}(c+dx)}{a^3d(2+n)}+\frac{3\sin^{3+n}(c+dx)}{a^3d(3+n)}-\frac{\sin^{4+n}(c+dx)}{a^3d(4+n)}$$

```
[Out] sin(d*x+c)^(1+n)/a^3/d/(1+n)-3*sin(d*x+c)^(2+n)/a^3/d/(2+n)+3*sin(d*x+c)^(3
+n)/a^3/d/(3+n)-sin(d*x+c)^(4+n)/a^3/d/(4+n)
```

Rubi [A]
time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 45}

$$\frac{\sin^{n+1}(c+dx)}{a^3d(n+1)}-\frac{3\sin^{n+2}(c+dx)}{a^3d(n+2)}+\frac{3\sin^{n+3}(c+dx)}{a^3d(n+3)}-\frac{\sin^{n+4}(c+dx)}{a^3d(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sin[c + d*x]^(1 + n)/(a^3*d*(1 + n)) - (3*Sin[c + d*x]^(2 + n))/(a^3*d*(2 +
 n)) + (3*Sin[c + d*x]^(3 + n))/(a^3*d*(3 + n)) - Sin[c + d*x]^(4 + n)/(a^3
*d*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\mathrm{Subst}\left(\int (a-x)^3\left(\frac{x}{a}\right)^n\,dx,x,a\sin(c+dx)\right)}{a^7d} \\ &= \frac{\mathrm{Subst}\left(\int \left(a^3\left(\frac{x}{a}\right)^n-3a^3\left(\frac{x}{a}\right)^{1+n}+3a^3\left(\frac{x}{a}\right)^{2+n}-a^3\left(\frac{x}{a}\right)^{3+n}\right)\,dx,x,a\sin(c+\right.}{a^7d} \\ &= \frac{\sin^{1+n}(c+dx)}{a^3d(1+n)}-\frac{3\sin^{2+n}(c+dx)}{a^3d(2+n)}+\frac{3\sin^{3+n}(c+dx)}{a^3d(3+n)}-\frac{\sin^{4+n}(c+dx)}{a^3d(4+n)}\end{aligned}$$

Mathematica [**A**]
time = 0.15, size = 66, normalized size = 0.72

$$\frac{\sin^{1+n}(c+dx)\left(\frac{1}{1+n}-\frac{3\sin(c+dx)}{2+n}+\frac{3\sin^2(c+dx)}{3+n}-\frac{\sin^3(c+dx)}{4+n}\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((1 + n)^(-1) - (3*Sin[c + d*x])/(2 + n) + (3*Sin[c +
 d*x]^2)/(3 + n) - Sin[c + d*x]^3/(4 + n)))/(a^3*d)
```

Maple [**F**]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)}{\left(a+a\sin(dx+c)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x)
```

Maxima [**A**]
time = 0.30, size = 126, normalized size = 1.37

$$-\frac{\left((n^3+6\,n^2+11\,n+6)\sin(dx+c)^4-3\,(n^3+7\,n^2+14\,n+8)\sin(dx+c)^3+3\,(n^3+8\,n^2+19\,n+12)\sin(dx+c)^2-(n^3+9\,n^2+26\,n+24)\sin(dx+c)\right)\sin(dx+c)^n}{(n^4+10\,n^3+35\,n^2+50\,n+24)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*sin(d*x + c)^4 - 3*(n^3 + 7*n^2 + 14*n + 8)*sin(
d*x + c)^3 + 3*(n^3 + 8*n^2 + 19*n + 12)*sin(d*x + c)^2 - (n^3 + 9*n^2 + 26
```

```
*n + 24)*sin(d*x + c))*sin(d*x + c)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*
a^3*d)
```

Fricas [A]
time = 0.40, size = 160, normalized size = 1.74

$$-\frac{\left((n^3+6n^2+11n+6)\cos(dx+c)^4+4n^3-(5n^3+36n^2+79n+48)\cos(dx+c)^2+30n^2-(4n^3-3(n^3+7n^2+14n+8)\cos(dx+c)^2+30n^2+68n+48)\sin(dx+c)+68n+42\right)\sin(dx+c)^n}{a^3dn^4+10a^3dn^3+35a^3dn^2+50a^3dn+24a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -((n^3 + 6*n^2 + 11*n + 6)*cos(d*x + c)^4 + 4*n^3 - (5*n^3 + 36*n^2 + 79*n
+ 48)*cos(d*x + c)^2 + 30*n^2 - (4*n^3 - 3*(n^3 + 7*n^2 + 14*n + 8)*cos(d*x
 + c)^2 + 30*n^2 + 68*n + 48)*sin(d*x + c) + 68*n + 42)*sin(d*x + c)^n/(a^3
*d*n^4 + 10*a^3*d*n^3 + 35*a^3*d*n^2 + 50*a^3*d*n + 24*a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{-1,[0,1,3,0,0]%%%}+%%%{3,[0,1,2,0,0]%%%}+%%%{-3,[0,1,1,0,0]%%
%}+%%%
```

Mupad [B]
time = 10.61, size = 242, normalized size = 2.63

$$-\frac{\sin(c+d\,x)^n\,\left(261\,n-336\,\sin(c+d\,x)-168\,\cos(2\,c+2\,d\,x)+6\,\cos(4\,c+4\,d\,x)+48\,\sin(3\,c+3\,d\,x)-460\,n\,\sin(c+d\,x)-272\,n\,\cos(2\,c+2\,d\,x)+11\,n\,\cos(4\,c+4\,d\,x)+84\,n\,\sin(3\,c+3\,d\,x)-198\,n^2\,\sin(c+d\,x)-26\,n^3\,\sin(c+d\,x)+114\,n^2+15\,n^3-120\,n^2\,\cos(2\,c+2\,d\,x)-16\,n^3\,\cos(2\,c+2\,d\,x)+6\,n^2\,\cos(4\,c+4\,d\,x)+n^3\,\cos(4\,c+4\,d\,x)+42\,n^2\,\sin(3\,c+3\,d\,x)+6\,n^3\,\sin(3\,c+3\,d\,x)+162\right)}{8\,a^3\,d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] -(sin(c + d*x)^n*(261*n - 336*sin(c + d*x) - 168*cos(2*c + 2*d*x) + 6*cos(4
*c + 4*d*x) + 48*sin(3*c + 3*d*x) - 460*n*sin(c + d*x) - 272*n*cos(2*c + 2*
d*x) + 11*n*cos(4*c + 4*d*x) + 84*n*sin(3*c + 3*d*x) - 198*n^2*sin(c + d*x)
 - 26*n^3*sin(c + d*x) + 114*n^2 + 15*n^3 - 120*n^2*cos(2*c + 2*d*x) - 16*n
^3*cos(2*c + 2*d*x) + 6*n^2*cos(4*c + 4*d*x) + n^3*cos(4*c + 4*d*x) + 42*n^
2*sin(3*c + 3*d*x) + 6*n^3*sin(3*c + 3*d*x) + 162))/(8*a^3*d*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24))
```

3.703 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=109

$$-\frac{7\sin^{1+n}(c+dx)}{a^4d(1+n)}+\frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4d(1+n)}+\frac{4\sin^{2+n}(c+dx)}{a^4d(2+n)}-\frac{\sin^{3+n}(c+dx)}{a^4d(3+n)}$$

```
[Out] -7*sin(d*x+c)^(1+n)/a^4/d/(1+n)+8*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin
(d*x+c)^(1+n)/a^4/d/(1+n)+4*sin(d*x+c)^(2+n)/a^4/d/(2+n)-sin(d*x+c)^(3+n)/a
^4/d/(3+n)
```

Rubi [A]
time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2915, 90, 45, 66}

$$\frac{8\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^4d(n+1)}-\frac{7\sin^{n+1}(c+dx)}{a^4d(n+1)}+\frac{4\sin^{n+2}(c+dx)}{a^4d(n+2)}-\frac{\sin^{n+3}(c+dx)}{a^4d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-7*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (8*Hypergeometric2F1[1, 1 + n,
2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(a^4*d*(1 + n)) + (4*Sin[c + d*
x]^(2 + n))/(a^4*d*(2 + n)) - Sin[c + d*x]^(3 + n)/(a^4*d*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(-4a^2\left(\frac{x}{a}\right)^n - 2a(a-x)\left(\frac{x}{a}\right)^n - (a-x)^2\left(\frac{x}{a}\right)^n + \frac{8a^3\left(\frac{x}{a}\right)^n}{a+x}\right) dx, x, a\right.}{a^7 d}$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4 d(1+n)} - \frac{\text{Subst}\left(\int (a-x)^2\left(\frac{x}{a}\right)^n dx, x, a\sin(c+dx)\right)}{a^7 d} - \frac{2\text{Subs}}{}$$

$$= -\frac{4\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d(1+n)} -$$

$$= -\frac{7\sin^{1+n}(c+dx)}{a^4 d(1+n)} + \frac{8\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^4 d(1+n)} +$$

Mathematica [A]
time = 0.19, size = 104, normalized size = 0.95

$$\frac{-\frac{7a^3\sin^{1+n}(c+dx)}{1+n} + \frac{8a^3\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{1+n} + \frac{4a^3\sin^{2+n}(c+dx)}{2+n} - \frac{a^3\sin^{3+n}(c+dx)}{3+n}}{a^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^4,x]
[Out] ((-7*a^3*Sin[c + d*x]^(1 + n))/(1 + n) + (8*a^3*Hypergeometric2F1[1, 1 + n,
 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(1 + n) + (4*a^3*Sin[c + d*x]^
(2 + n))/(2 + n) - (a^3*Sin[c + d*x]^(3 + n))/(3 + n))/(a^7*d)
```

Maple [F]
time = 1.66, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^7(dx+c)\right)\left(\sin^n(dx+c)\right)}{\left(a+a\sin(dx+c)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(a^4*cos(d*x + c)^4 - 8*a^4*cos(d*x
+ c)^2 + 8*a^4 - 4*(a^4*cos(d*x + c)^2 - 2*a^4)*sin(d*x + c)), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^4, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d x)^7 \sin (c+d x)^n}{(a+a \sin (c+d x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4,x)
[Out] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^4, x)
```

3.704 $\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx$

Optimal. Leaf size=160

$$-\frac{4(3+2n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^5d(1+n)}-\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\,(a^7+a^7\sin(c+dx))}+\frac{\sin^{1+n}(}{}$$

```
[Out] -4*(3+2*n)*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/a^5/d/(1+
n)-sin(d*x+c)^(1+n)*(a-a*sin(d*x+c))^2/d/(2+n)/(a^7+a^7*sin(d*x+c))+sin(d*x
+c)^(1+n)*(a*(8*n^2+30*n+27)+a*(7+2*n)*sin(d*x+c))/d/(n^2+3*n+2)/(a^6+a^6*s
in(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2915, 102, 151, 66}

$$-\frac{(a-a\sin(c+dx))^2\sin^{n+1}(c+dx)}{d(n+2)\,(a^7\sin(c+dx)+a^7)}+\frac{\sin^{n+1}(c+dx)\,(a(2n+7)\sin(c+dx)+a(8n^2+30n+27))}{d\,(n^2+3n+2)\,(a^6\sin(c+dx)+a^6)}-\frac{4(2n+3)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{a^5d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (-4*(3 + 2*n)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x
]^(1 + n))/(a^5*d*(1 + n)) - (Sin[c + d*x]^(1 + n)*(a - a*Sin[c + d*x])^2)/
(d*(2 + n)*(a^7 + a^7*Sin[c + d*x])) + (Sin[c + d*x]^(1 + n)*(a*(27 + 30*n
+ 8*n^2) + a*(7 + 2*n)*Sin[c + d*x]))/(d*(2 + 3*n + n^2)*(a^6 + a^6*Sin[c +
 d*x]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)
*(c + d*x)^(n + 1), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(c+dx)\sin^n(c+dx)}{(a+a\sin(c+dx))^5}\,dx &= \frac{\text{Subst}\left(\int \frac{(a-x)^3\left(\frac{x}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{a^7d}\\ &= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{(a-x)\left(\frac{x}{a}\right)^n(a(3+2n)+(-7-2n)x)}{(a+x)^2}\right.}{a^6d(2+n)}\\ &= -\frac{\sin^{1+n}(c+dx)(a-a\sin(c+dx))^2}{d(2+n)\left(a^7+a^7\sin(c+dx)\right)} + \frac{\sin^{1+n}(c+dx)\left(a\left(27+30n+8n^2\right)\right.}{d(1+n)(2+n)\left(a^6+a\right.}\\ &= -\frac{4(3+2n)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{a^5d(1+n)} - \frac{\sin^{1+n}(c-}{d(2+n}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 108, normalized size = 0.68

$$\frac{\sin^{1+n}(c+dx)\left(26+29n+8n^2+(9+4n)\sin(c+dx)-(1+n)\sin^2(c+dx)-4\left(6+7n+2n^2\right)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))(1+\sin(c+dx))\right)}{a^5d(1+n)(2+n)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Sin[c + d*x]^n)/(a + a*Sin[c + d*x])^5,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(26 + 29*n + 8*n^2 + (9 + 4*n)*Sin[c + d*x] - (1 + n)
*Sin[c + d*x]^2 - 4*(6 + 7*n + 2*n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -S
in[c + d*x]]*(1 + Sin[c + d*x])))/(a^5*d*(1 + n)*(2 + n)*(1 + Sin[c + d*x])
)
```

Maple [**F**]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{7}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{\left(a+a\sin\left(dx+c\right)\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

```
[Out] int(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="fricas
")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^7/(5*a^5*cos(d*x + c)^4 - 20*a^5*cos(d
*x + c)^2 + 16*a^5 + (a^5*cos(d*x + c)^4 - 12*a^5*cos(d*x + c)^2 + 16*a^5)*
sin(d*x + c)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*sin(d*x+c)**n/(a+a*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*sin(d*x+c)^n/(a+a*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^7/(a*sin(d*x + c) + a)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)^n}{{\left(a+a\,\sin\left(c+d\,x\right)\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^5,x)
```

```
[Out] int((cos(c + d*x)^7*sin(c + d*x)^n)/(a + a*sin(c + d*x))^5, x)
```

3.705 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=209

$$-\frac{5x}{1024a}-\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}-\frac{5\cos(c+dx)\sin(c+dx)}{1024ad}-\frac{5\cos^3(c+dx)\sin(c+d}{1536ad}$$

```
[Out] -5/1024*x/a-1/7*cos(d*x+c)^7/a/d+2/9*cos(d*x+c)^9/a/d-1/11*cos(d*x+c)^11/a/
d-5/1024*cos(d*x+c)*sin(d*x+c)/a/d-5/1536*cos(d*x+c)^3*sin(d*x+c)/a/d-1/384
*cos(d*x+c)^5*sin(d*x+c)/a/d+1/64*cos(d*x+c)^7*sin(d*x+c)/a/d+1/24*cos(d*x+
c)^7*sin(d*x+c)^3/a/d+1/12*cos(d*x+c)^7*sin(d*x+c)^5/a/d
```

Rubi [A]

time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2645, 276, 2648, 2715, 8}

$$-\frac{\cos^{11}(c+dx)}{11ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^7(c+dx)}{12ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{24ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{64ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{384ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{1536ad}-\frac{5\sin(c+dx)\cos(c+dx)}{1024ad}-\frac{5x}{1024a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(1024*a) - Cos[c + d*x]^7/(7*a*d) + (2*Cos[c + d*x]^9)/(9*a*d) - Cos
[c + d*x]^11/(11*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(1024*a*d) - (5*Cos[c
 + d*x]^3*Sin[c + d*x])/(1536*a*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(384*a*d
) + (Cos[c + d*x]^7*Sin[c + d*x])/(64*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3
)/(24*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^5)/(12*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a}-\frac{\int \cos^6(c+dx)\sin^6(c+dx)\,dx}{a}\\
&= \frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad}-\frac{5\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{12a}-\frac{\mathrm{Subst}\left(\int x\right.}{}\\
&= \frac{\cos^7(c+dx)\sin^3(c+dx)}{24ad}+\frac{\cos^7(c+dx)\sin^5(c+dx)}{12ad}-\frac{\int \cos^6(c+dx)\sin}{8a}\\
&= -\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}+\frac{\cos^7(c+dx)\sin(c+d}{64ad}\\
&= -\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}-\frac{\cos^5(c+dx)\sin(c+d}{384ad}\\
&= -\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}-\frac{5\cos^3(c+dx)\sin(c+}{1536ad}\\
&= -\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}-\frac{5\cos(c+dx)\sin(c+d}{1024ad}\\
&= -\frac{5x}{1024a}-\frac{\cos^7(c+dx)}{7ad}+\frac{2\cos^9(c+dx)}{9ad}-\frac{\cos^{11}(c+dx)}{11ad}-\frac{5\cos(c+dx)}{1024}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 518 vs. 2(209) = 418.

time = 10.23, size = 518, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/11354112*(55440*d*x*Cos[c/2] + 55440*Cos[c/2 + d*x] + 55440*Cos[(3*c)/2
+ d*x] + 18480*Cos[(5*c)/2 + 3*d*x] + 18480*Cos[(7*c)/2 + 3*d*x] - 10395*Co
s[(7*c)/2 + 4*d*x] + 10395*Cos[(9*c)/2 + 4*d*x] - 5544*Cos[(9*c)/2 + 5*d*x]
 - 5544*Cos[(11*c)/2 + 5*d*x] - 3960*Cos[(13*c)/2 + 7*d*x] - 3960*Cos[(15*c
)/2 + 7*d*x] + 2079*Cos[(15*c)/2 + 8*d*x] - 2079*Cos[(17*c)/2 + 8*d*x] + 61
6*Cos[(17*c)/2 + 9*d*x] + 616*Cos[(19*c)/2 + 9*d*x] + 504*Cos[(21*c)/2 + 11
*d*x] + 504*Cos[(23*c)/2 + 11*d*x] - 231*Cos[(23*c)/2 + 12*d*x] + 231*Cos[(
25*c)/2 + 12*d*x] + 99792*Sin[c/2] + 55440*d*x*Sin[c/2] - 55440*Sin[c/2 + d
*x] + 55440*Sin[(3*c)/2 + d*x] - 18480*Sin[(5*c)/2 + 3*d*x] + 18480*Sin[(7*
c)/2 + 3*d*x] - 10395*Sin[(7*c)/2 + 4*d*x] - 10395*Sin[(9*c)/2 + 4*d*x] + 5
544*Sin[(9*c)/2 + 5*d*x] - 5544*Sin[(11*c)/2 + 5*d*x] + 3960*Sin[(13*c)/2 +
 7*d*x] - 3960*Sin[(15*c)/2 + 7*d*x] + 2079*Sin[(15*c)/2 + 8*d*x] + 2079*Si
n[(17*c)/2 + 8*d*x] - 616*Sin[(17*c)/2 + 9*d*x] + 616*Sin[(19*c)/2 + 9*d*x]
 - 504*Sin[(21*c)/2 + 11*d*x] + 504*Sin[(23*c)/2 + 11*d*x] - 231*Sin[(23*c)
/2 + 12*d*x] - 231*Sin[(25*c)/2 + 12*d*x])/(a*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.19, size = 311, normalized size = 1.49

method	result
risch	$-\frac{5x}{1024a}-\frac{5\cos(dx+c)}{512ad}+\frac{\sin(12dx+12c)}{24576ad}-\frac{\cos(11dx+11c)}{11264ad}-\frac{\cos(9dx+9c)}{9216ad}-\frac{3\sin(8dx+8c)}{8192ad}+\frac{5\cos(7dx+7c}{7168ad}$
derivativedivides	$\frac{64\left(-\frac{1}{2772}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32768}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{231}+\frac{175\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{98304}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{42}+\frac{311\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32768}+\frac{11\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}\right.}{}$
default	$\frac{64\left(-\frac{1}{2772}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{32768}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{231}+\frac{175\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{98304}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{42}+\frac{311\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32768}+\frac{11\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a*((-1/2772+5/32768*tan(1/2*d*x+1/2*c)-1/231*tan(1/2*d*x+1/2*c)^2+175/
98304*tan(1/2*d*x+1/2*c)^3-1/42*tan(1/2*d*x+1/2*c)^4+311/32768*tan(1/2*d*x+
1/2*c)^5+11/126*tan(1/2*d*x+1/2*c)^6-8361/32768*tan(1/2*d*x+1/2*c)^7-3/7*ta
n(1/2*d*x+1/2*c)^8+42259/49152*tan(1/2*d*x+1/2*c)^9+3/14*tan(1/2*d*x+1/2*c)
^10-25295/16384*tan(1/2*d*x+1/2*c)^11-1/6*tan(1/2*d*x+1/2*c)^12+25295/16384
*tan(1/2*d*x+1/2*c)^13-1/2*tan(1/2*d*x+1/2*c)^14-42259/49152*tan(1/2*d*x+1/
```

```
2*c)^15+1/4*tan(1/2*d*x+1/2*c)^16+8361/32768*tan(1/2*d*x+1/2*c)^17-1/6*tan(
1/2*d*x+1/2*c)^18-311/32768*tan(1/2*d*x+1/2*c)^19-175/98304*tan(1/2*d*x+1/2
*c)^21-5/32768*tan(1/2*d*x+1/2*c)^23)/(1+tan(1/2*d*x+1/2*c)^2)^12-5/32768*a
rctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(189) = 378$.
time = 0.53, size = 705, normalized size = 3.37

$$\frac{\frac{\frac{3465 \sin(dx+c)}{\cos(dx+c)+1} - \frac{98304 \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{40425 \sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} - \frac{540672 \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{215523 \sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} + \frac{1982464 \sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} - \frac{5794173 \sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} - \frac{9732096 \sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} + \frac{19523658 \sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}} + \frac{4866048 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{35058870 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{3784704 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{35058870 \sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{11354112 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} - \frac{19523658 \sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}} + \frac{5677056 \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} + \frac{5794173 \sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}} - \frac{3784704 \sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} - \frac{215523 \sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}} - \frac{40425 \sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}} - \frac{3465 \sin(dx+c)^{23}}{(\cos(dx+c)+1)^{23}} - 8192}{a + \frac{12 a \sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{66 a \sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{220 a \sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} + \frac{495 a \sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} + \frac{792 a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{924 a \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{792 a \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{495 a \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} + \frac{220 a \sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} + \frac{66 a \sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}} + \frac{12 a \sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}} + \frac{a \sin(dx+c)^{24}}{(\cos(dx+c)+1)^{24}}} - \frac{3465 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{354816 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/354816*((3465*sin(d*x + c)/(cos(d*x + c) + 1) - 98304*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 40425*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 540672*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 215523*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 1982464*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5794173*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 - 9732096*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 19523658*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4866048*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - 35058870*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 3784704*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12 + 35058870*sin(d*x + c)^13/(cos(d*x + c) + 1
)^13 - 11354112*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 19523658*sin(d*x +
c)^15/(cos(d*x + c) + 1)^15 + 5677056*sin(d*x + c)^16/(cos(d*x + c) + 1)^16
 + 5794173*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 3784704*sin(d*x + c)^18/
(cos(d*x + c) + 1)^18 - 215523*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 4042
5*sin(d*x + c)^21/(cos(d*x + c) + 1)^21 - 3465*sin(d*x + c)^23/(cos(d*x + c
) + 1)^23 - 8192)/(a + 12*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 66*a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 220*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 + 495*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 792*a*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 + 924*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 792*a*sin(d
*x + c)^14/(cos(d*x + c) + 1)^14 + 495*a*sin(d*x + c)^16/(cos(d*x + c) + 1)
^16 + 220*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 66*a*sin(d*x + c)^20/(c
os(d*x + c) + 1)^20 + 12*a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22 + a*sin(d*
x + c)^24/(cos(d*x + c) + 1)^24) - 3465*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a)/d
```

Fricas [A]
time = 0.40, size = 110, normalized size = 0.53

$$-\frac{64512 \cos(dx+c)^{11} - 157696 \cos(dx+c)^{9} + 101376 \cos(dx+c)^{7} + 3465 \, dx - 231 \left(256 \cos(dx+c)^{11} - 640 \cos(dx+c)^{9} + 432 \cos(dx+c)^{7} - 8 \cos(dx+c)^{5} - 10 \cos(dx+c)^{3} - 15 \cos(dx+c)\right) \sin(dx+c)}{709632 \, a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/709632*(64512*cos(d*x + c)^11 - 157696*cos(d*x + c)^9 + 101376*cos(d*x +
 c)^7 + 3465*d*x - 231*(256*cos(d*x + c)^11 - 640*cos(d*x + c)^9 + 432*cos(
d*x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*
x + c))/(a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 309, normalized size = 1.48

$$-\frac{\frac{3465\,(dx+c)}{a}+\frac{2\left(3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{23}+40425\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{21}+215523\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{19}+3784704\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{18}-5794173\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{17}-5677056\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{16}+19523658\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+11354112\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{14}-35058870\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+3784704\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}+35058870\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-4866048\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-19523658\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+9732096\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+5794173\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-1982464\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-215523\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+540672\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-40425\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+98304\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+8192\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{12}a}}{709632\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/709632*(3465*(d*x + c)/a + 2*(3465*tan(1/2*d*x + 1/2*c)^23 + 40425*tan(1
/2*d*x + 1/2*c)^21 + 215523*tan(1/2*d*x + 1/2*c)^19 + 3784704*tan(1/2*d*x +
 1/2*c)^18 - 5794173*tan(1/2*d*x + 1/2*c)^17 - 5677056*tan(1/2*d*x + 1/2*c)
^16 + 19523658*tan(1/2*d*x + 1/2*c)^15 + 11354112*tan(1/2*d*x + 1/2*c)^14 -
 35058870*tan(1/2*d*x + 1/2*c)^13 + 3784704*tan(1/2*d*x + 1/2*c)^12 + 35058
870*tan(1/2*d*x + 1/2*c)^11 - 4866048*tan(1/2*d*x + 1/2*c)^10 - 19523658*ta
n(1/2*d*x + 1/2*c)^9 + 9732096*tan(1/2*d*x + 1/2*c)^8 + 5794173*tan(1/2*d*x
 + 1/2*c)^7 - 1982464*tan(1/2*d*x + 1/2*c)^6 - 215523*tan(1/2*d*x + 1/2*c)^
5 + 540672*tan(1/2*d*x + 1/2*c)^4 - 40425*tan(1/2*d*x + 1/2*c)^3 + 98304*ta
n(1/2*d*x + 1/2*c)^2 - 3465*tan(1/2*d*x + 1/2*c) + 8192)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^12*a))/d
```

Mupad [B]
time = 11.85, size = 303, normalized size = 1.45

$$-\frac{5\,x}{1024\,a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{23}}{512}+\frac{175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{21}}{1536}+\frac{311\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{512}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}}{3}-\frac{8361\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{512}-16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+\frac{42259\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{768}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-\frac{25295\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{256}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}+\frac{25295\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{256}-\frac{96\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{7}-\frac{42259\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{768}+\frac{192\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{7}+\frac{8361\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{512}-\frac{352\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{63}-\frac{311\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{512}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{21}-\frac{175\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{1536}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{231}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{512}+\frac{16}{693}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^5)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (5*x)/(1024*a) - ((64*tan(c/2 + (d*x)/2)^2)/231 - (5*tan(c/2 + (d*x)/2))/
512 - (175*tan(c/2 + (d*x)/2)^3)/1536 + (32*tan(c/2 + (d*x)/2)^4)/21 - (311
*tan(c/2 + (d*x)/2)^5)/512 - (352*tan(c/2 + (d*x)/2)^6)/63 + (8361*tan(c/2
```

```
+ (d*x)/2)^7)/512 + (192*tan(c/2 + (d*x)/2)^8)/7 - (42259*tan(c/2 + (d*x)/2
)^9)/768 - (96*tan(c/2 + (d*x)/2)^10)/7 + (25295*tan(c/2 + (d*x)/2)^11)/256
 + (32*tan(c/2 + (d*x)/2)^12)/3 - (25295*tan(c/2 + (d*x)/2)^13)/256 + 32*ta
n(c/2 + (d*x)/2)^14 + (42259*tan(c/2 + (d*x)/2)^15)/768 - 16*tan(c/2 + (d*x
)/2)^16 - (8361*tan(c/2 + (d*x)/2)^17)/512 + (32*tan(c/2 + (d*x)/2)^18)/3 +
 (311*tan(c/2 + (d*x)/2)^19)/512 + (175*tan(c/2 + (d*x)/2)^21)/1536 + (5*ta
n(c/2 + (d*x)/2)^23)/512 + 16/693)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^12)
```

3.706 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{3x}{256a}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^{11}(c+dx)}{11ad}+\frac{3\cos(c+dx)\sin(c+dx)}{256ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{128ad}+$$

```
[Out] 3/256*x/a+1/7*cos(d*x+c)^7/a/d-2/9*cos(d*x+c)^9/a/d+1/11*cos(d*x+c)^11/a/d+
3/256*cos(d*x+c)*sin(d*x+c)/a/d+1/128*cos(d*x+c)^3*sin(d*x+c)/a/d+1/160*cos
(d*x+c)^5*sin(d*x+c)/a/d-3/80*cos(d*x+c)^7*sin(d*x+c)/a/d-1/10*cos(d*x+c)^7
*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2648, 2715, 8, 2645, 276}

$$\frac{\cos^{11}(c+dx)}{11ad}-\frac{2\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}-\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{128ad}+\frac{3\sin(c+dx)\cos(c+dx)}{256ad}+\frac{3x}{256a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(256*a) + Cos[c + d*x]^7/(7*a*d) - (2*Cos[c + d*x]^9)/(9*a*d) + Cos[c
 + d*x]^11/(11*a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(256*a*d) + (Cos[c + d*
x]^3*Sin[c + d*x])/(128*a*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) - (3
*Cos[c + d*x]^7*Sin[c + d*x])/(80*a*d) - (Cos[c + d*x]^7*Sin[c + d*x]^3)/(1
0*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^5(c+dx)\,dx}{a}$$

$$= -\frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\Big(\int \ldots}{\ldots}$$

$$= -\frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} - \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} + \frac{3\int \cos^6(c+dx \ldots}{80a}$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} \ldots$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} \ldots$$

$$= \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin(c+dx \ldots}{256ad}$$

$$= \frac{3x}{256a} + \frac{\cos^7(c+dx)}{7ad} - \frac{2\cos^9(c+dx)}{9ad} + \frac{\cos^{11}(c+dx)}{11ad} + \frac{3\cos(c+dx)\sin \ldots}{256ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 573 vs. 2(183) = 366.
time = 8.31, size = 573, normalized size = 3.13

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((97020*c)/(a*d) + (83160*x)/a - (103950*Cos[c]*Cos[d*x])/(a*d) + (66990*Co
s[3*c]*Cos[3*d*x])/(a*d) - (24948*Cos[5*c]*Cos[5*d*x])/(a*d) + (1980*Cos[7*
c]*Cos[7*d*x])/(a*d) + (173250*Cos[c + d*x])/(a*d) - (43890*Cos[3*(c + d*x)
])/(a*d) + (18018*Cos[5*(c + d*x)])/(a*d) - (6930*Cos[7*(c + d*x)])/(a*d) +
 (770*Cos[9*(c + d*x)])/(a*d) + (630*Cos[11*(c + d*x)])/(a*d) + (90090*Cos[
2*d*x]*Sin[2*c])/(a*d) - (55440*Cos[4*d*x]*Sin[4*c])/(a*d) + (4620*Cos[6*d*
x]*Sin[6*c])/(a*d) + (103950*Sin[c]*Sin[d*x])/(a*d) + (90090*Cos[2*c]*Sin[2
*d*x])/(a*d) - (66990*Sin[3*c]*Sin[3*d*x])/(a*d) - (55440*Cos[4*c]*Sin[4*d*
x])/(a*d) + (24948*Sin[5*c]*Sin[5*d*x])/(a*d) + (4620*Cos[6*c]*Sin[6*d*x])/
(a*d) - (1980*Sin[7*c]*Sin[7*d*x])/(a*d) - (76230*Sin[(d*x)/2])/(a*d*(Cos[c
/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) - (20790*Sin[(c + d*
x)/2])/(a*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (48510*Sin[c + d*x])/(
a*d*(1 + Sin[c + d*x])) + (97020*Sin[(c + d*x)/2]^2)/(d*(a + a*Sin[c + d*x]
)) - (76230*Sin[2*(c + d*x)])/(a*d) + (27720*Sin[4*(c + d*x)])/(a*d) - (115
50*Sin[6*(c + d*x)])/(a*d) + (3465*Sin[8*(c + d*x)])/(a*d) + (1386*Sin[10*(
c + d*x)])/(a*d))/7096320
```

Maple [A]

time = 0.16, size = 272, normalized size = 1.49

method	result
risch	$\frac{5\cos(dx+c)}{512ad}+\frac{3x}{256a}+\frac{\cos(11dx+11c)}{11264ad}+\frac{\sin(10dx+10c)}{5120ad}+\frac{\cos(9dx+9c)}{9216ad}+\frac{\sin(8dx+8c)}{2048ad}-\frac{5\cos(7dx+7c)}{7168ad}-$
derivativedivides	$\frac{32\left(\frac{1}{1386}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}+\frac{3323\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-27\right.}{}$
default	$\frac{32\left(\frac{1}{1386}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}+\frac{3323\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-27\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*((1/1386-3/4096*tan(1/2*d*x+1/2*c)+1/126*tan(1/2*d*x+1/2*c)^2-1/128*
tan(1/2*d*x+1/2*c)^3+5/126*tan(1/2*d*x+1/2*c)^4+3323/20480*tan(1/2*d*x+1/2*
c)^5-3/14*tan(1/2*d*x+1/2*c)^6-27/80*tan(1/2*d*x+1/2*c)^7+15/14*tan(1/2*d*x
+1/2*c)^8+841/2048*tan(1/2*d*x+1/2*c)^9-3/2*tan(1/2*d*x+1/2*c)^10+11/6*tan(
1/2*d*x+1/2*c)^12-841/2048*tan(1/2*d*x+1/2*c)^13-5/6*tan(1/2*d*x+1/2*c)^14+
27/80*tan(1/2*d*x+1/2*c)^15+1/3*tan(1/2*d*x+1/2*c)^16-3323/20480*tan(1/2*d*
x+1/2*c)^17+1/128*tan(1/2*d*x+1/2*c)^19+3/4096*tan(1/2*d*x+1/2*c)^21)/(1+ta
n(1/2*d*x+1/2*c)^2)^11+3/4096*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(165) = 330$.
time = 0.52, size = 624, normalized size = 3.41

$$-\frac{\frac{\frac{10395 \sin(dx+c)}{\cos(dx+c)+1} - \frac{112640 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{110880 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{563200 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2302839 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{3041280 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{4790016 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{15206400 \sin(dx+c)^8}{(\cos(dx+c)+1)^8} - \frac{5828130 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} + \frac{21288960 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} - \frac{26019840 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{5828130 \sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} + \frac{11827200 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} - \frac{4790016 \sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}} - \frac{4730880 \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} + \frac{2302839 \sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}} - \frac{110880 \sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}} - \frac{10395 \sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}} - 10240}{a + \frac{11 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{55 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{165 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{330 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{462 a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{462 a \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{330 a \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{165 a \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} + \frac{55 a \sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} + \frac{11 a \sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}} + \frac{a \sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}}} - \frac{10395 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{443520\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/443520*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 112640*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 563200*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2302839*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 + 3041280*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4790016*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 15206400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 582
8130*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 21288960*sin(d*x + c)^10/(cos(d*
x + c) + 1)^10 - 26019840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 5828130*s
in(d*x + c)^13/(cos(d*x + c) + 1)^13 + 11827200*sin(d*x + c)^14/(cos(d*x +
c) + 1)^14 - 4790016*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 4730880*sin(d*
x + c)^16/(cos(d*x + c) + 1)^16 + 2302839*sin(d*x + c)^17/(cos(d*x + c) + 1
)^17 - 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21
/(cos(d*x + c) + 1)^21 - 10240)/(a + 11*a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 55*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 330*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 462*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12
 + 330*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a*sin(d*x + c)^16/(cos
(d*x + c) + 1)^16 + 55*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + 11*a*sin(d
*x + c)^20/(cos(d*x + c) + 1)^20 + a*sin(d*x + c)^22/(cos(d*x + c) + 1)^22)
 - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.40, size = 100, normalized size = 0.55

$$\frac{80640 \cos(dx+c)^{11} - 197120 \cos(dx+c)^9 + 126720 \cos(dx+c)^7 + 10395\, dx + 693\, (128 \cos(dx+c)^9 - 176 \cos(dx+c)^7 + 8 \cos(dx+c)^5 + 10 \cos(dx+c)^3 + 15 \cos(dx+c)) \sin(dx+c)}{887040\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/887040*(80640*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 126720*cos(d*x +
c)^7 + 10395*d*x + 693*(128*cos(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x
 + c)^5 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 6409 vs. $2(153) = 306$.
time = 237.21, size = 6409, normalized size = 35.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)

```
[Out] Piecewise((10395*d*x*tan(c/2 + d*x/2)**22/(887040*a*d*tan(c/2 + d*x/2)**22
+ 9757440*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 14
6361600*a*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409
812480*a*d*tan(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 2927
23200*a*d*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 4878720
0*a*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) +
 114345*d*x*tan(c/2 + d*x/2)**20/(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440
*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a
*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*
d*tan(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d
*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan
(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 571725*d
*x*tan(c/2 + d*x/2)**18/(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(
c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(c/
2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*d*tan(c/2
 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d*tan(c/2
+ d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2 + d*
x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 1715175*d*x*tan(c
/2 + d*x/2)**16/(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*
x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(c/2 + d*x/
2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*d*tan(c/2 + d*x/2
)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d*tan(c/2 + d*x/2)
**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2 + d*x/2)**4
+ 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 3430350*d*x*tan(c/2 + d*x
/2)**14/(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20
 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(c/2 + d*x/2)**16 +
 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*d*tan(c/2 + d*x/2)**12 +
409812480*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d*tan(c/2 + d*x/2)**8 + 14
6361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2 + d*x/2)**4 + 975744
0*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 4802490*d*x*tan(c/2 + d*x/2)**12/
(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20 + 48787
200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(c/2 + d*x/2)**16 + 2927232
00*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*d*tan(c/2 + d*x/2)**12 + 40981248
0*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d*tan(c/2 + d*x/2)**8 + 146361600*
a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*ta
n(c/2 + d*x/2)**2 + 887040*a*d) + 4802490*d*x*tan(c/2 + d*x/2)**10/(887040*
a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*
tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*t
an(c/2 + d*x/2)**14 + 409812480*a*d*tan(c/2 + d*x/2)**12 + 409812480*a*d*ta
n(c/2 + d*x/2)**10 + 292723200*a*d*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(
c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 +
```

```
d*x/2)**2 + 887040*a*d) + 3430350*d*x*tan(c/2 + d*x/2)**8/(887040*a*d*tan(c
/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 +
 d*x/2)**18 + 146361600*a*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 +
d*x/2)**14 + 409812480*a*d*tan(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d
*x/2)**10 + 292723200*a*d*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x
/2)**6 + 48787200*a*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2
 + 887040*a*d) + 1715175*d*x*tan(c/2 + d*x/2)**6/(887040*a*d*tan(c/2 + d*x/
2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**
18 + 146361600*a*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**1
4 + 409812480*a*d*tan(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10
 + 292723200*a*d*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 +
48787200*a*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040
*a*d) + 571725*d*x*tan(c/2 + d*x/2)**4/(887040*a*d*tan(c/2 + d*x/2)**22 + 9
757440*a*d*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 14636
1600*a*d*tan(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812
480*a*d*tan(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 2927232
00*a*d*tan(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a
*d*tan(c/2 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 11
4345*d*x*tan(c/2 + d*x/2)**2/(887040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d
*tan(c/2 + d*x/2)**20 + 48787200*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*t
an(c/2 + d*x/2)**16 + 292723200*a*d*tan(c/2 + d*x/2)**14 + 409812480*a*d*ta
n(c/2 + d*x/2)**12 + 409812480*a*d*tan(c/2 + d*x/2)**10 + 292723200*a*d*tan
(c/2 + d*x/2)**8 + 146361600*a*d*tan(c/2 + d*x/2)**6 + 48787200*a*d*tan(c/2
 + d*x/2)**4 + 9757440*a*d*tan(c/2 + d*x/2)**2 + 887040*a*d) + 10395*d*x/(8
87040*a*d*tan(c/2 + d*x/2)**22 + 9757440*a*d*tan(c/2 + d*x/2)**20 + 4878720
0*a*d*tan(c/2 + d*x/2)**18 + 146361600*a*d*tan(...
```

Giac [A]

time = 0.48, size = 270, normalized size = 1.48

$$\frac{\frac{10395\,(dx+c)}{a}+\frac{2\left(10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}+110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}-2302839\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+4730880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}+4790016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-11827200\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-5828130\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+26019840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-21288960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+5828130\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+15206400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4790016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3041280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2302839\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+563200\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+112640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10240\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{11}a}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/887040*(10395*(d*x + c)/a + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*tan
(1/2*d*x + 1/2*c)^19 - 2302839*tan(1/2*d*x + 1/2*c)^17 + 4730880*tan(1/2*d*
x + 1/2*c)^16 + 4790016*tan(1/2*d*x + 1/2*c)^15 - 11827200*tan(1/2*d*x + 1/
2*c)^14 - 5828130*tan(1/2*d*x + 1/2*c)^13 + 26019840*tan(1/2*d*x + 1/2*c)^1
2 - 21288960*tan(1/2*d*x + 1/2*c)^10 + 5828130*tan(1/2*d*x + 1/2*c)^9 + 152
06400*tan(1/2*d*x + 1/2*c)^8 - 4790016*tan(1/2*d*x + 1/2*c)^7 - 3041280*tan
(1/2*d*x + 1/2*c)^6 + 2302839*tan(1/2*d*x + 1/2*c)^5 + 563200*tan(1/2*d*x +
 1/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 112640*tan(1/2*d*x + 1/2*c)^2 -
 10395*tan(1/2*d*x + 1/2*c) + 10240)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a))/d
```

Mupad [B]

time = 11.60, size = 263, normalized size = 1.44

$$\frac{3x}{256a}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{128}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{4}-\frac{3323\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{640}+\frac{32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{3}+\frac{54\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{5}-\frac{80\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3}-\frac{841\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{176\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}-48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{841\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64}+\frac{240\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{7}-\frac{54\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{5}-\frac{48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{7}+\frac{3323\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{640}+\frac{80\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{63}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{4}+\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{63}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}+\frac{16}{693}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^4)/(a + a*sin(c + d*x)),x)
```

```
[Out] (3*x)/(256*a) + ((16*tan(c/2 + (d*x)/2)^2)/63 - (3*tan(c/2 + (d*x)/2))/128
- tan(c/2 + (d*x)/2)^3/4 + (80*tan(c/2 + (d*x)/2)^4)/63 + (3323*tan(c/2 + (
d*x)/2)^5)/640 - (48*tan(c/2 + (d*x)/2)^6)/7 - (54*tan(c/2 + (d*x)/2)^7)/5
+ (240*tan(c/2 + (d*x)/2)^8)/7 + (841*tan(c/2 + (d*x)/2)^9)/64 - 48*tan(c/2
 + (d*x)/2)^10 + (176*tan(c/2 + (d*x)/2)^12)/3 - (841*tan(c/2 + (d*x)/2)^13
)/64 - (80*tan(c/2 + (d*x)/2)^14)/3 + (54*tan(c/2 + (d*x)/2)^15)/5 + (32*ta
n(c/2 + (d*x)/2)^16)/3 - (3323*tan(c/2 + (d*x)/2)^17)/640 + tan(c/2 + (d*x)
/2)^19/4 + (3*tan(c/2 + (d*x)/2)^21)/128 + 16/693)/(a*d*(tan(c/2 + (d*x)/2)
^2 + 1)^11)
```

3.707 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$-\frac{3x}{256a}-\frac{\cos^7(c+dx)}{7ad}+\frac{\cos^9(c+dx)}{9ad}-\frac{3\cos(c+dx)\sin(c+dx)}{256ad}-\frac{\cos^3(c+dx)\sin(c+dx)}{128ad}-\frac{\cos^5(c+dx)\sin(}{160ad}$$

```
[Out] -3/256*x/a-1/7*cos(d*x+c)^7/a/d+1/9*cos(d*x+c)^9/a/d-3/256*cos(d*x+c)*sin(d
*x+c)/a/d-1/128*cos(d*x+c)^3*sin(d*x+c)/a/d-1/160*cos(d*x+c)^5*sin(d*x+c)/a
/d+3/80*cos(d*x+c)^7*sin(d*x+c)/a/d+1/10*cos(d*x+c)^7*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2645, 14, 2648, 2715, 8}

$$\frac{\cos^9(c+dx)}{9ad}-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^3(c+dx)\cos^7(c+dx)}{10ad}+\frac{3\sin(c+dx)\cos^7(c+dx)}{80ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{160ad}-\frac{\sin(c+dx)\cos^3(c+dx)}{128ad}-\frac{3\sin(c+dx)\cos(c+dx)}{256ad}-\frac{3x}{256a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*x)/(256*a) - Cos[c + d*x]^7/(7*a*d) + Cos[c + d*x]^9/(9*a*d) - (3*Cos[c
 + d*x]*Sin[c + d*x])/(256*a*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(128*a*d) -
 (Cos[c + d*x]^5*Sin[c + d*x])/(160*a*d) + (3*Cos[c + d*x]^7*Sin[c + d*x])/
(80*a*d) + (Cos[c + d*x]^7*Sin[c + d*x]^3)/(10*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^4(c+dx)\,dx}{a} \\
&= \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{10a} - \frac{\text{Subst}(\int \ldots}{} \\
&= \frac{3\cos^7(c+dx)\sin(c+dx)}{80ad} + \frac{\cos^7(c+dx)\sin^3(c+dx)}{10ad} - \frac{3\int \cos^6(c+dx \ldots}{80a} \\
&= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{160ad} + \frac{3\cos^7(c+d \ldots}{8 \ldots} \\
&= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{\cos^3(c+dx)\sin(c+dx)}{128ad} - \frac{\cos^5(c+dx \ldots}{16 \ldots} \\
&= -\frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos^3(c+dx \ldots}{12 \ldots} \\
&= -\frac{3x}{256a} - \frac{\cos^7(c+dx)}{7ad} + \frac{\cos^9(c+dx)}{9ad} - \frac{3\cos(c+dx)\sin(c+dx)}{256ad} - \frac{\cos \ldots}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 533 vs. 2(165) = 330.
time = 10.31, size = 533, normalized size = 3.23

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1290240*(-1260*(25*c - 12*d*x)*Cos[c/2] + 15120*Cos[c/2 + d*x] + 15120*C
os[(3*c)/2 + d*x] + 1260*Cos[(3*c)/2 + 2*d*x] - 1260*Cos[(5*c)/2 + 2*d*x] +
 6720*Cos[(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 +
 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 630*Cos[(11*c)/2 + 6*d*x] + 630*Cos[(
13*c)/2 + 6*d*x] - 1080*Cos[(13*c)/2 + 7*d*x] - 1080*Cos[(15*c)/2 + 7*d*x]
+ 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x] - 280*Cos[(17*c)/2
+ 9*d*x] - 280*Cos[(19*c)/2 + 9*d*x] + 126*Cos[(19*c)/2 + 10*d*x] - 126*Cos
[(21*c)/2 + 10*d*x] + 37800*Sin[c/2] - 31500*c*Sin[c/2] + 15120*d*x*Sin[c/2
] - 15120*Sin[c/2 + d*x] + 15120*Sin[(3*c)/2 + d*x] + 1260*Sin[(3*c)/2 + 2*
d*x] + 1260*Sin[(5*c)/2 + 2*d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*
c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] - 630
*Sin[(11*c)/2 + 6*d*x] - 630*Sin[(13*c)/2 + 6*d*x] + 1080*Sin[(13*c)/2 + 7*
d*x] - 1080*Sin[(15*c)/2 + 7*d*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17
*c)/2 + 8*d*x] + 280*Sin[(17*c)/2 + 9*d*x] - 280*Sin[(19*c)/2 + 9*d*x] + 12
6*Sin[(19*c)/2 + 10*d*x] + 126*Sin[(21*c)/2 + 10*d*x])/(a*d*(Cos[c/2] + Sin
[c/2]))
```

Maple [A]

time = 0.15, size = 259, normalized size = 1.57

method	result
risch	$-\frac{3x}{256a}-\frac{3\cos(dx+c)}{128ad}-\frac{\sin(10dx+10c)}{5120ad}+\frac{\cos(9dx+9c)}{2304ad}-\frac{\sin(8dx+8c)}{2048ad}+\frac{3\cos(7dx+7c)}{1792ad}+\frac{\sin(6dx+6c)}{1024ad}+\sin$
derivativedivides	$\frac{16\left(-\frac{1}{252}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}-\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}+\frac{29\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2048}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{867\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2560}-\frac{9\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}+\frac{51}{}\right.}{}$
default	$\frac{16\left(-\frac{1}{252}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2048}-\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{126}+\frac{29\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2048}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{867\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2560}-\frac{9\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}+\frac{51}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*((-1/252+3/2048*tan(1/2*d*x+1/2*c)-5/126*tan(1/2*d*x+1/2*c)^2+29/204
8*tan(1/2*d*x+1/2*c)^3+1/14*tan(1/2*d*x+1/2*c)^4-867/2560*tan(1/2*d*x+1/2*c
)^5-9/14*tan(1/2*d*x+1/2*c)^6+519/512*tan(1/2*d*x+1/2*c)^7-1879/1024*tan(1/
2*d*x+1/2*c)^9-1/2*tan(1/2*d*x+1/2*c)^10+1879/1024*tan(1/2*d*x+1/2*c)^11-5/
6*tan(1/2*d*x+1/2*c)^12-519/512*tan(1/2*d*x+1/2*c)^13+1/6*tan(1/2*d*x+1/2*c
)^14+867/2560*tan(1/2*d*x+1/2*c)^15-1/4*tan(1/2*d*x+1/2*c)^16-29/2048*tan(1
/2*d*x+1/2*c)^17-3/2048*tan(1/2*d*x+1/2*c)^19)/(1+tan(1/2*d*x+1/2*c)^2)^10-
3/2048*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(149) = 298$.

time = 0.55, size = 583, normalized size = 3.53

$$\frac{\frac{\frac{945 \sin(dx+c)}{\cos(dx+c)+1} - \frac{25600 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{9135 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{46080 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{218484 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{414720 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{653940 \sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{1183770 \sin(dx+c)^9}{(\cos(dx+c)+1)^9} - \frac{322560 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{1183770 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{537600 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} - \frac{653940 \sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} + \frac{107520 \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{218484 \sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}} - \frac{161280 \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} - \frac{9135 \sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}} - \frac{945 \sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}} - 2560}{a + \frac{10 a \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{45 a \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{120 a \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{210 a \sin(dx+c)^8}{(\cos(dx+c)+1)^8} + \frac{252 a \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{210 a \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{120 a \sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{45 a \sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}} + \frac{10 a \sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}} + \frac{a \sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}} - \frac{945 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{40320\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/40320*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 25600*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 9135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 46080*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 218484*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4
14720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 653940*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 1183770*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 322560*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10 + 1183770*sin(d*x + c)^11/(cos(d*x + c) + 1)^
11 - 537600*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 653940*sin(d*x + c)^13/
(cos(d*x + c) + 1)^13 + 107520*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 2184
84*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 161280*sin(d*x + c)^16/(cos(d*x
+ c) + 1)^16 - 9135*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 945*sin(d*x + c
)^19/(cos(d*x + c) + 1)^19 - 2560)/(a + 10*a*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 45*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 210*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 252*a*sin(d
*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a*sin(d*x + c)^12/(cos(d*x + c) + 1)
^12 + 120*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45*a*sin(d*x + c)^16/(c
os(d*x + c) + 1)^16 + 10*a*sin(d*x + c)^18/(cos(d*x + c) + 1)^18 + a*sin(d*
x + c)^20/(cos(d*x + c) + 1)^20) - 945*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A]

time = 0.40, size = 90, normalized size = 0.55

$$\frac{8960 \cos(dx+c)^9 - 11520 \cos(dx+c)^7 - 945\, dx - 63\left(128 \cos(dx+c)^9 - 176 \cos(dx+c)^7 + 8 \cos(dx+c)^5 + 10 \cos(dx+c)^3 + 15 \cos(dx+c)\right) \sin(dx+c)}{80640\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80640*(8960*cos(d*x + c)^9 - 11520*cos(d*x + c)^7 - 945*d*x - 63*(128*cos
(d*x + c)^9 - 176*cos(d*x + c)^7 + 8*cos(d*x + c)^5 + 10*cos(d*x + c)^3 + 1
5*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5501 vs. $2(138) = 276$.

time = 165.68, size = 5501, normalized size = 33.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-945*d*x*tan(c/2 + d*x/2)**20/(80640*a*d*tan(c/2 + d*x/2)**20 +
806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 967680
0*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a
*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*ta
n(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 +
d*x/2)**2 + 80640*a*d) - 9450*d*x*tan(c/2 + d*x/2)**18/(80640*a*d*tan(c/2 +
 d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2
)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**1
2 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 +
9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*
a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 42525*d*x*tan(c/2 + d*x/2)**16/(8064
0*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*
tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(
c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2
 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/
2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 113400*d*x*tan(c/2 +
d*x/2)**14/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**1
8 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 + 1
6934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 1693
4400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a*
d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 19845
0*d*x*tan(c/2 + d*x/2)**12/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan
(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2
+ d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d
*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)
**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80
640*a*d) - 238140*d*x*tan(c/2 + d*x/2)**10/(80640*a*d*tan(c/2 + d*x/2)**20
+ 806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676
800*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280
*a*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*
tan(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2
+ d*x/2)**2 + 80640*a*d) - 198450*d*x*tan(c/2 + d*x/2)**8/(80640*a*d*tan(c/
2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*
x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)
**12 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8
 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 8064
00*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 113400*d*x*tan(c/2 + d*x/2)**6/(8
0640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a
*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*t
an(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(
c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d
```

```
*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 42525*d*x*tan(c/2
+ d*x/2)**4/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x/2)**
18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**14 +
16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x/2)**10 + 169
34400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628800*a
*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) - 9450
*d*x*tan(c/2 + d*x/2)**2/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c
/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 +
d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x
/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**
6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 8064
0*a*d) - 945*d*x/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan(c/2 + d*x
/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2 + d*x/2)**
14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d*x/2)**10
+ 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)**6 + 3628
800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80640*a*d) -
 1890*tan(c/2 + d*x/2)**19/(80640*a*d*tan(c/2 + d*x/2)**20 + 806400*a*d*tan
(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a*d*tan(c/2
+ d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*tan(c/2 + d
*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c/2 + d*x/2)
**6 + 3628800*a*d*tan(c/2 + d*x/2)**4 + 806400*a*d*tan(c/2 + d*x/2)**2 + 80
640*a*d) - 18270*tan(c/2 + d*x/2)**17/(80640*a*d*tan(c/2 + d*x/2)**20 + 806
400*a*d*tan(c/2 + d*x/2)**18 + 3628800*a*d*tan(c/2 + d*x/2)**16 + 9676800*a
*d*tan(c/2 + d*x/2)**14 + 16934400*a*d*tan(c/2 + d*x/2)**12 + 20321280*a*d*
tan(c/2 + d*x/2)**10 + 16934400*a*d*tan(c/2 + d*x/2)**8 + 9676800*a*d*tan(c
/2 + d*x/2)**6 + 3628800*a*d*tan(c/2 + d*x/2)**...
```

Giac [A]
time = 0.45, size = 257, normalized size = 1.56

$$-\frac{\frac{945\,(dx+c)}{a}+\frac{2\left(945\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}+9135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+161280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-218484\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-107520\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+653940\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+537600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-1183770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+322560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1183770\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-653940\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+414720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+218484\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-46080\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-9135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+25600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-945\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2560\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{10}a}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(945*(d*x + c)/a + 2*(945*tan(1/2*d*x + 1/2*c)^19 + 9135*tan(1/2*d
*x + 1/2*c)^17 + 161280*tan(1/2*d*x + 1/2*c)^16 - 218484*tan(1/2*d*x + 1/2*
c)^15 - 107520*tan(1/2*d*x + 1/2*c)^14 + 653940*tan(1/2*d*x + 1/2*c)^13 + 5
37600*tan(1/2*d*x + 1/2*c)^12 - 1183770*tan(1/2*d*x + 1/2*c)^11 + 322560*ta
n(1/2*d*x + 1/2*c)^10 + 1183770*tan(1/2*d*x + 1/2*c)^9 - 653940*tan(1/2*d*x
 + 1/2*c)^7 + 414720*tan(1/2*d*x + 1/2*c)^6 + 218484*tan(1/2*d*x + 1/2*c)^5
 - 46080*tan(1/2*d*x + 1/2*c)^4 - 9135*tan(1/2*d*x + 1/2*c)^3 + 25600*tan(1
/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 2560)/((tan(1/2*d*x + 1/2*c)
^2 + 1)^10*a))/d
```

Mupad [B]

time = 11.52, size = 251, normalized size = 1.52

$$-\frac{3\,x}{256\,a}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{128}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-\frac{867\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{160}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{3}+\frac{519\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}+\frac{40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-\frac{1879\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1879\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64}-\frac{519\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32}+\frac{72\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{7}+\frac{867\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{7}-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{128}+\frac{40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{63}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}+\frac{4}{63}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x)),x)
```

```
[Out] - (3*x)/(256*a) - ((40*tan(c/2 + (d*x)/2)^2)/63 - (3*tan(c/2 + (d*x)/2))/12
8 - (29*tan(c/2 + (d*x)/2)^3)/128 - (8*tan(c/2 + (d*x)/2)^4)/7 + (867*tan(c
/2 + (d*x)/2)^5)/160 + (72*tan(c/2 + (d*x)/2)^6)/7 - (519*tan(c/2 + (d*x)/2
)^7)/32 + (1879*tan(c/2 + (d*x)/2)^9)/64 + 8*tan(c/2 + (d*x)/2)^10 - (1879*
tan(c/2 + (d*x)/2)^11)/64 + (40*tan(c/2 + (d*x)/2)^12)/3 + (519*tan(c/2 + (
d*x)/2)^13)/32 - (8*tan(c/2 + (d*x)/2)^14)/3 - (867*tan(c/2 + (d*x)/2)^15)/
160 + 4*tan(c/2 + (d*x)/2)^16 + (29*tan(c/2 + (d*x)/2)^17)/128 + (3*tan(c/2
 + (d*x)/2)^19)/128 + 4/63)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.708 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{5x}{128a}+\frac{\cos^7(c+dx)}{7ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{48ad}$$

```
[Out] 5/128*x/a+1/7*cos(d*x+c)^7/a/d-1/9*cos(d*x+c)^9/a/d+5/128*cos(d*x+c)*sin(d*
x+c)/a/d+5/192*cos(d*x+c)^3*sin(d*x+c)/a/d+1/48*cos(d*x+c)^5*sin(d*x+c)/a/d
-1/8*cos(d*x+c)^7*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2648, 2715, 8, 2645, 14}

$$-\frac{\cos^9(c+dx)}{9ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}+\frac{5\sin(c+dx)\cos(c+dx)}{128ad}+\frac{5x}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(128*a) + Cos[c + d*x]^7/(7*a*d) - Cos[c + d*x]^9/(9*a*d) + (5*Cos[c
+ d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d)
+ (Cos[c + d*x]^5*Sin[c + d*x])/(48*a*d) - (Cos[c + d*x]^7*Sin[c + d*x])/(8
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^3(c+dx)\,dx}{a} \\
&= -\frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{\int \cos^6(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^6(1-x^2)\,dx,\right.}{ad} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} - \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} + \frac{5\int \cos^4(c+dx)\,dx}{48a} \\
&= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} + \frac{\cos^5(c+dx)\,s}{48a} \\
&= \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)}{192a} \\
&= \frac{5x}{128a} + \frac{\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 479 vs. 2(139) = 278.
time = 5.88, size = 479, normalized size = 3.45

$$
-\frac{2520(c-2dx)\cos\left(\frac{c}{2}\right)-1512\cos\left(\frac{c}{2}+dx\right)-1512\cos\left(\frac{3c}{2}+dx\right)-1008\cos\left(\frac{3c}{2}+2dx\right)+1008\cos\left(\frac{5c}{2}+2dx\right)-672\cos\left(\frac{5c}{2}+3dx\right)-672\cos\left(\frac{7c}{2}+3dx\right)+504\cos\left(\frac{7c}{2}+4dx\right)-504\cos\left(\frac{9c}{2}+4dx\right)+336\cos\left(\frac{11c}{2}+6dx\right)-336\cos\left(\frac{13c}{2}+6dx\right)+108\cos\left(\frac{13c}{2}+7dx\right)+108\cos\left(\frac{15c}{2}+7dx\right)+63\cos\left(\frac{15c}{2}+8dx\right)-63\cos\left(\frac{17c}{2}+8dx\right)+28\cos\left(\frac{17c}{2}+9dx\right)+28\cos\left(\frac{19c}{2}+9dx\right)-7560\sin\left(\frac{c}{2}\right)+2520c\sin\left(\frac{c}{2}\right)-5040dx\sin\left(\frac{c}{2}\right)+1512\sin\left(\frac{c}{2}+dx\right)-1512\sin\left(\frac{3c}{2}+dx\right)-1008\sin\left(\frac{3c}{2}+2dx\right)-1008\sin\left(\frac{5c}{2}+2dx\right)+672\sin\left(\frac{5c}{2}+3dx\right)-672\sin\left(\frac{7c}{2}+3dx\right)+504\sin\left(\frac{7c}{2}+4dx\right)+504\sin\left(\frac{9c}{2}+4dx\right)+336\sin\left(\frac{11c}{2}+6dx\right)+336\sin\left(\frac{13c}{2}+6dx\right)-108\sin\left(\frac{13c}{2}+7dx\right)+108\sin\left(\frac{15c}{2}+7dx\right)+63\sin\left(\frac{15c}{2}+8dx\right)+63\sin\left(\frac{17c}{2}+8dx\right)-28\sin\left(\frac{17c}{2}+9dx\right)+28\sin\left(\frac{19c}{2}+9dx\right)}{129024ad\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

[Out] -1/129024*(2520*(c - 2*d*x)*Cos[c/2] - 1512*Cos[c/2 + d*x] - 1512*Cos[(3*c)/2 + d*x] - 1008*Cos[(3*c)/2 + 2*d*x] + 1008*Cos[(5*c)/2 + 2*d*x] - 672*Cos[(5*c)/2 + 3*d*x] - 672*Cos[(7*c)/2 + 3*d*x] + 504*Cos[(7*c)/2 + 4*d*x] - 504*Cos[(9*c)/2 + 4*d*x] + 336*Cos[(11*c)/2 + 6*d*x] - 336*Cos[(13*c)/2 + 6*d*x] + 108*Cos[(13*c)/2 + 7*d*x] + 108*Cos[(15*c)/2 + 7*d*x] + 63*Cos[(15*c)/2 + 8*d*x] - 63*Cos[(17*c)/2 + 8*d*x] + 28*Cos[(17*c)/2 + 9*d*x] + 28*Cos[(19*c)/2 + 9*d*x] - 7560*Sin[c/2] + 2520*c*Sin[c/2] - 5040*d*x*Sin[c/2] + 1512*Sin[c/2 + d*x] - 1512*Sin[(3*c)/2 + d*x] - 1008*Sin[(3*c)/2 + 2*d*x] - 1008*Sin[(5*c)/2 + 2*d*x] + 672*Sin[(5*c)/2 + 3*d*x] - 672*Sin[(7*c)/2 + 3*d*x] + 504*Sin[(7*c)/2 + 4*d*x] + 504*Sin[(9*c)/2 + 4*d*x] + 336*Sin[(11*c)/2 + 6*d*x] + 336*Sin[(13*c)/2 + 6*d*x] - 108*Sin[(13*c)/2 + 7*d*x] + 108*Sin[(15*c)/2 + 7*d*x] + 63*Sin[(15*c)/2 + 8*d*x] + 63*Sin[(17*c)/2 + 8*d*x] - 28*Sin[(17*c)/2 + 9*d*x] + 28*Sin[(19*c)/2 + 9*d*x])/(a*d*(Cos[c/2] + Sin[c/2]))

Maple [A]
time = 0.14, size = 233, normalized size = 1.68

method	result
risch	$\frac{5x}{128a}+\frac{3\cos(dx+c)}{128ad}-\frac{\cos(9dx+9c)}{2304ad}-\frac{\sin(8dx+8c)}{1024ad}-\frac{3\cos(7dx+7c)}{1792ad}-\frac{\sin(6dx+6c)}{192ad}-\frac{\sin(4dx+4c)}{128ad}+\frac{\cos(3\cdots}{9\cdots}$
derivativedivides	$\frac{8\left(\frac{1}{126}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}+\frac{191\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{83\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+145\cdots\right.}{\cdots}$
default	$\frac{8\left(\frac{1}{126}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}+\frac{191\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{83\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+145\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 8/d/a*((1/126-5/512*tan(1/2*d*x+1/2*c)+1/14*tan(1/2*d*x+1/2*c)^2+191/768*tan(1/2*d*x+1/2*c)^3-3/14*tan(1/2*d*x+1/2*c)^4-83/256*tan(1/2*d*x+1/2*c)^5+3/2*tan(1/2*d*x+1/2*c)^6+145/256*tan(1/2*d*x+1/2*c)^7-3/2*tan(1/2*d*x+1/2*c)^8+5/2*tan(1/2*d*x+1/2*c)^10-145/256*tan(1/2*d*x+1/2*c)^11-5/6*tan(1/2*d*x+1/2*c)^12+83/256*tan(1/2*d*x+1/2*c)^13+1/2*tan(1/2*d*x+1/2*c)^14-191/768*tan(1/2*d*x+1/2*c)^15+5/512*tan(1/2*d*x+1/2*c)^17)/(1+tan(1/2*d*x+1/2*c)^2)^9+5/512*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. 2(125) = 250.
time = 0.57, size = 522, normalized size = 3.76

$$-\frac{\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{2304\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8022\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6912\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10458\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{48384\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{18270\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{48384\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{80640\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{18270\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{26880\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{10458\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{16128\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{8022\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{315\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-256}{a+\frac{9a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{36a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{126a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{36a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{9a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}+\frac{a\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}}-\frac{315\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{4032d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4032*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 2304*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 8022*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6912*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 10458*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4838
4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 18270*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 48384*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 80640*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + 18270*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 26880
*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 10458*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 16128*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 8022*sin(d*x + c)
^15/(cos(d*x + c) + 1)^15 - 315*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 256
)/(a + 9*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 84*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 126*a*sin(d*x +
 c)^8/(cos(d*x + c) + 1)^8 + 126*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
84*a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a*sin(d*x + c)^14/(cos(d*x
+ c) + 1)^14 + 9*a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + a*sin(d*x + c)^1
8/(cos(d*x + c) + 1)^18) - 315*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.39, size = 80, normalized size = 0.58

$$-\frac{896 \cos\left(dx+c\right)^{9}-1152 \cos\left(dx+c\right)^{7}-315\, dx+21 \left(48 \cos\left(dx+c\right)^{7}-8 \cos\left(dx+c\right)^{5}-10 \cos\left(dx+c\right)^{3}-15 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{8064\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*cos(d*x + c)^9 - 1152*cos(d*x + c)^7 - 315*d*x + 21*(48*cos(d*
x + c)^7 - 8*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x
+ c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4490 vs. 2(116) = 232.
time = 108.29, size = 4490, normalized size = 32.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((315*d*x*tan(c/2 + d*x/2)**18/(8064*a*d*tan(c/2 + d*x/2)**18 + 72
576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d
*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(
c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x
/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 2835*d*x*tan(c/2 + d*x
/2)**16/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 2
90304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*
```

a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan
(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x
/2)**2 + 8064*a*d) + 11340*d*x*tan(c/2 + d*x/2)**14/(8064*a*d*tan(c/2 + d*x
/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14
+ 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016
064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*t
an(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 26460*d*x*
tan(c/2 + d*x/2)**12/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d
*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**
12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 6
77376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*
tan(c/2 + d*x/2)**2 + 8064*a*d) + 39690*d*x*tan(c/2 + d*x/2)**10/(8064*a*d*
tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2
+ d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/
2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 +
 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d)
 + 39690*d*x*tan(c/2 + d*x/2)**8/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d
*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/
2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d
*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4
+ 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 26460*d*x*tan(c/2 + d*x/2)**6
/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a
*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan
(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 +
d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2
+ 8064*a*d) + 11340*d*x*tan(c/2 + d*x/2)**4/(8064*a*d*tan(c/2 + d*x/2)**18
+ 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376
*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*
tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 +
 d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 2835*d*x*tan(c/2 +
 d*x/2)**2/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16
+ 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 10160
64*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*
tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 +
d*x/2)**2 + 8064*a*d) + 315*d*x/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*
tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2
 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*
x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 +
 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 630*tan(c/2 + d*x/2)**17/(8064
*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan
(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 +
 d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)
6 + 290304*a*d*tan(c/2 + d*x/2)4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064
*a*d) - 16044*tan(c/2 + d*x/2)**15/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a
*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(

```
c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 +
 d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**
4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) + 32256*tan(c/2 + d*x/2)**14/
(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*
d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(
c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d
*x/2)**6 + 290304*a*d*tan(c/2 + d*x/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 +
 8064*a*d) + 20916*tan(c/2 + d*x/2)**13/(8064*a*d*tan(c/2 + d*x/2)**18 + 72
576*a*d*tan(c/2 + d*x/2)**16 + 290304*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d
*tan(c/2 + d*x/2)**12 + 1016064*a*d*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(
c/2 + d*x/2)**8 + 677376*a*d*tan(c/2 + d*x/2)**6 + 290304*a*d*tan(c/2 + d*x
/2)**4 + 72576*a*d*tan(c/2 + d*x/2)**2 + 8064*a*d) - 53760*tan(c/2 + d*x/2)
**12/(8064*a*d*tan(c/2 + d*x/2)**18 + 72576*a*d*tan(c/2 + d*x/2)**16 + 2903
04*a*d*tan(c/2 + d*x/2)**14 + 677376*a*d*tan(c/2 + d*x/2)**12 + 1016064*a*d
*tan(c/2 + d*x/2)**10 + 1016064*a*d*tan(c/2 + d...
```

Giac [A]
time = 0.44, size = 231, normalized size = 1.66

$$\frac{\frac{315\,(dx+c)}{a}+\frac{2\left(315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}-8022\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+16128\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+10458\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-26880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-18270\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+80640\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-48384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+18270\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+48384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-10458\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6912\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8022\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2304\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+256\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{9}a}}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8064*(315*(d*x + c)/a + 2*(315*tan(1/2*d*x + 1/2*c)^17 - 8022*tan(1/2*d*x
 + 1/2*c)^15 + 16128*tan(1/2*d*x + 1/2*c)^14 + 10458*tan(1/2*d*x + 1/2*c)^1
3 - 26880*tan(1/2*d*x + 1/2*c)^12 - 18270*tan(1/2*d*x + 1/2*c)^11 + 80640*t
an(1/2*d*x + 1/2*c)^10 - 48384*tan(1/2*d*x + 1/2*c)^8 + 18270*tan(1/2*d*x +
 1/2*c)^7 + 48384*tan(1/2*d*x + 1/2*c)^6 - 10458*tan(1/2*d*x + 1/2*c)^5 - 6
912*tan(1/2*d*x + 1/2*c)^4 + 8022*tan(1/2*d*x + 1/2*c)^3 + 2304*tan(1/2*d*x
 + 1/2*c)^2 - 315*tan(1/2*d*x + 1/2*c) + 256)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^9*a))/d
```

Mupad [B]
time = 11.60, size = 224, normalized size = 1.61

$$\frac{5\,x}{128\,a}+\frac{\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}-\frac{191\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{96}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\frac{83\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32}-\frac{20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3}-\frac{145\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{145\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32}+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{83\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{32}-\frac{12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{7}+\frac{191\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{7}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{4}{63}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x)),x)
```

```
[Out] (5*x)/(128*a) + ((4*tan(c/2 + (d*x)/2)^2)/7 - (5*tan(c/2 + (d*x)/2))/64 + (
191*tan(c/2 + (d*x)/2)^3)/96 - (12*tan(c/2 + (d*x)/2)^4)/7 - (83*tan(c/2 +
(d*x)/2)^5)/32 + 12*tan(c/2 + (d*x)/2)^6 + (145*tan(c/2 + (d*x)/2)^7)/32 -
12*tan(c/2 + (d*x)/2)^8 + 20*tan(c/2 + (d*x)/2)^10 - (145*tan(c/2 + (d*x)/2
)^11)/32 - (20*tan(c/2 + (d*x)/2)^12)/3 + (83*tan(c/2 + (d*x)/2)^13)/32 + 4
*tan(c/2 + (d*x)/2)^14 - (191*tan(c/2 + (d*x)/2)^15)/96 + (5*tan(c/2 + (d*x
)/2)^17)/64 + 4/63)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.709 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{5x}{128a}-\frac{\cos^7(c+dx)}{7ad}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}-\frac{5\cos^3(c+dx)\sin(c+dx)}{192ad}-\frac{\cos^5(c+dx)\sin(c+dx)}{48ad}+\frac{\text{co}}{}$$

```
[Out] -5/128*x/a-1/7*cos(d*x+c)^7/a/d-5/128*cos(d*x+c)*sin(d*x+c)/a/d-5/192*cos(d
*x+c)^3*sin(d*x+c)/a/d-1/48*cos(d*x+c)^5*sin(d*x+c)/a/d+1/8*cos(d*x+c)^7*si
n(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2918, 2645, 30, 2648, 2715, 8}

$$-\frac{\cos^7(c+dx)}{7ad}+\frac{\sin(c+dx)\cos^7(c+dx)}{8ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{48ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{192ad}-\frac{5\sin(c+dx)\cos(c+dx)}{128ad}-\frac{5x}{128a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - Cos[c + d*x]^7/(7*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(12
8*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(192*a*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(48*a*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \cos^6(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a}$$

$$= \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} - \frac{\int \cos^6(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^6\,dx, x, \cos(c+d\right.}{ad}$$

$$= -\frac{\cos^7(c+dx)}{7ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} + \frac{\cos^7(c+dx)\sin(c+dx)}{8ad} - \frac{5\int}{}$$

$$= -\frac{\cos^7(c+dx)}{7ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} - \frac{\cos^5(c+dx)\sin(c+dx)}{48ad} + \cdots$$

$$= -\frac{\cos^7(c+dx)}{7ad} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} - \frac{5\cos^3(c+dx)\sin(c+dx)}{192ad} - \cdots$$

$$= -\frac{5x}{128a} - \frac{\cos^7(c+dx)}{7ad} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} - \frac{5\cos^3(c+dx)\sin(c+}{192ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 481 vs. 2(121) = 242.
time = 7.81, size = 481, normalized size = 3.98

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/43008*(-336*(7*c - 5*d*x)*Cos[c/2] + 1680*Cos[c/2 + d*x] + 1680*Cos[(3*c
)/2 + d*x] + 336*Cos[(3*c)/2 + 2*d*x] - 336*Cos[(5*c)/2 + 2*d*x] + 1008*Cos
[(5*c)/2 + 3*d*x] + 1008*Cos[(7*c)/2 + 3*d*x] - 168*Cos[(7*c)/2 + 4*d*x] +
168*Cos[(9*c)/2 + 4*d*x] + 336*Cos[(9*c)/2 + 5*d*x] + 336*Cos[(11*c)/2 + 5*
d*x] - 112*Cos[(11*c)/2 + 6*d*x] + 112*Cos[(13*c)/2 + 6*d*x] + 48*Cos[(13*c
)/2 + 7*d*x] + 48*Cos[(15*c)/2 + 7*d*x] - 21*Cos[(15*c)/2 + 8*d*x] + 21*Cos
[(17*c)/2 + 8*d*x] + 4704*Sin[c/2] - 2352*c*Sin[c/2] + 1680*d*x*Sin[c/2] -
1680*Sin[c/2 + d*x] + 1680*Sin[(3*c)/2 + d*x] + 336*Sin[(3*c)/2 + 2*d*x] +
336*Sin[(5*c)/2 + 2*d*x] - 1008*Sin[(5*c)/2 + 3*d*x] + 1008*Sin[(7*c)/2 + 3
*d*x] - 168*Sin[(7*c)/2 + 4*d*x] - 168*Sin[(9*c)/2 + 4*d*x] - 336*Sin[(9*c)
/2 + 5*d*x] + 336*Sin[(11*c)/2 + 5*d*x] - 112*Sin[(11*c)/2 + 6*d*x] - 112*S
in[(13*c)/2 + 6*d*x] - 48*Sin[(13*c)/2 + 7*d*x] + 48*Sin[(15*c)/2 + 7*d*x]
- 21*Sin[(15*c)/2 + 8*d*x] - 21*Sin[(17*c)/2 + 8*d*x])/(a*d*(Cos[c/2] + Sin
[c/2]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(109) = 218$.
time = 0.10, size = 233, normalized size = 1.93

method	result
risch	$-\frac{5x}{128a}-\frac{5\cos(dx+c)}{64ad}+\frac{\sin(8dx+8c)}{1024ad}-\frac{\cos(7dx+7c)}{448ad}+\frac{\sin(6dx+6c)}{192ad}-\frac{\cos(5dx+5c)}{64ad}+\frac{\sin(4dx+4c)}{128ad}-\frac{3\,\mathrm{co}}{}$
derivativedivides	$\frac{4\left(-\frac{1}{14}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{397\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{895\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\cdots\right.}{}$
default	$\frac{4\left(-\frac{1}{14}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{256}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{14}-\frac{397\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{895\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{768}-\frac{3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*((-1/14+5/256*tan(1/2*d*x+1/2*c)-1/14*tan(1/2*d*x+1/2*c)^2-397/768*ta
n(1/2*d*x+1/2*c)^3-3/2*tan(1/2*d*x+1/2*c)^4+895/768*tan(1/2*d*x+1/2*c)^5-3/
2*tan(1/2*d*x+1/2*c)^6-1765/768*tan(1/2*d*x+1/2*c)^7-5/2*tan(1/2*d*x+1/2*c)
^8+1765/768*tan(1/2*d*x+1/2*c)^9-5/2*tan(1/2*d*x+1/2*c)^10-895/768*tan(1/2*
d*x+1/2*c)^11-1/2*tan(1/2*d*x+1/2*c)^12+397/768*tan(1/2*d*x+1/2*c)^13-1/2*t
an(1/2*d*x+1/2*c)^14-5/256*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*x+1/2*c)^2)^
8-5/256*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(109) = 218$.
time = 0.50, size = 501, normalized size = 4.14

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{384\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2779\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{8064\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6265\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{8064\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{12355\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{13440\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{12355\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{13440\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6265\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{2688\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{2779\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{2688\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-384}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 384*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 2779*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8064*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 6265*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8064*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 12355*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 - 13440*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 12355*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 13440*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6265*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 - 2688*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2 + 2779*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 2688*sin(d*x + c)^14/(cos(
d*x + c) + 1)^14 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 384)/(a + 8*
a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 56*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 + 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*
x + c)^12/(cos(d*x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14
 + a*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.41, size = 70, normalized size = 0.58

$$-\frac{384\cos\left(dx+c\right)^{7}+105\,dx-7\left(48\cos\left(dx+c\right)^{7}-8\cos\left(dx+c\right)^{5}-10\cos\left(dx+c\right)^{3}-15\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(384*cos(d*x + c)^7 + 105*d*x - 7*(48*cos(d*x + c)^7 - 8*cos(d*x +
c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3888 vs. 2(102) = 204.
time = 71.46, size = 3888, normalized size = 32.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**16/(2688*a*d*tan(c/2 + d*x/2)**16 + 2
1504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 150528*a*d
*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*d*tan(c/2
 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 + d*x/2)**
2 + 2688*a*d) - 840*d*x*tan(c/2 + d*x/2)**14/(2688*a*d*tan(c/2 + d*x/2)**16
 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 150528
```

*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*d*tan
(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 + d*x/
2)**2 + 2688*a*d) - 2940*d*x*tan(c/2 + d*x/2)**12/(2688*a*d*tan(c/2 + d*x/2
)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12 + 1
50528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528*a*
d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/2 +
 d*x/2)**2 + 2688*a*d) - 5880*d*x*tan(c/2 + d*x/2)**10/(2688*a*d*tan(c/2 +
d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**1
2 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1505
28*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(
c/2 + d*x/2)**2 + 2688*a*d) - 7350*d*x*tan(c/2 + d*x/2)**8/(2688*a*d*tan(c/
2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2
)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 +
150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*
tan(c/2 + d*x/2)**2 + 2688*a*d) - 5880*d*x*tan(c/2 + d*x/2)**6/(2688*a*d*ta
n(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d
*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**
8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*
a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 2940*d*x*tan(c/2 + d*x/2)**4/(2688*a*
d*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2
 + d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/
2)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21
504*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 840*d*x*tan(c/2 + d*x/2)**2/(2688
*a*d*tan(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(
c/2 + d*x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d
*x/2)**8 + 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 +
 21504*a*d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 105*d*x/(2688*a*d*tan(c/2 + d*
x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
+ 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528
*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/
2 + d*x/2)**2 + 2688*a*d) - 210*tan(c/2 + d*x/2)**15/(2688*a*d*tan(c/2 + d*
x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
+ 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150528
*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c/
2 + d*x/2)**2 + 2688*a*d) - 5376*tan(c/2 + d*x/2)**14/(2688*a*d*tan(c/2 + d
*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**12
 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 15052
8*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(c
/2 + d*x/2)**2 + 2688*a*d) + 5558*tan(c/2 + d*x/2)**13/(2688*a*d*tan(c/2 +
d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**1
2 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1505
28*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan(
c/2 + d*x/2)**2 + 2688*a*d) - 5376*tan(c/2 + d*x/2)**12/(2688*a*d*tan(c/2 +
 d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)**
12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 150

```
528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*tan
(c/2 + d*x/2)**2 + 2688*a*d) - 12530*tan(c/2 + d*x/2)**11/(2688*a*d*tan(c/2
 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/2)
**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 + 1
50528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d*t
an(c/2 + d*x/2)**2 + 2688*a*d) - 26880*tan(c/2 + d*x/2)**10/(2688*a*d*tan(c
/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x/
2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8 +
 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*d
*tan(c/2 + d*x/2)**2 + 2688*a*d) + 24710*tan(c/2 + d*x/2)**9/(2688*a*d*tan(
c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*x
/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 188160*a*d*tan(c/2 + d*x/2)**8
+ 150528*a*d*tan(c/2 + d*x/2)**6 + 75264*a*d*tan(c/2 + d*x/2)**4 + 21504*a*
d*tan(c/2 + d*x/2)**2 + 2688*a*d) - 26880*tan(c/2 + d*x/2)**8/(2688*a*d*tan
(c/2 + d*x/2)**16 + 21504*a*d*tan(c/2 + d*x/2)**14 + 75264*a*d*tan(c/2 + d*
x/2)**12 + 150528*a*d*tan(c/2 + d*x/2)**10 + 18...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(109) = 218$.
time = 0.45, size = 231, normalized size = 1.91

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-2779\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2688\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+6265\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+13440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-12355\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+13440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+12355\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+8064\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-6265\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+8064\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2779\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+384\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+384\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 2688*tan(1/2*d*
x + 1/2*c)^14 - 2779*tan(1/2*d*x + 1/2*c)^13 + 2688*tan(1/2*d*x + 1/2*c)^12
 + 6265*tan(1/2*d*x + 1/2*c)^11 + 13440*tan(1/2*d*x + 1/2*c)^10 - 12355*tan
(1/2*d*x + 1/2*c)^9 + 13440*tan(1/2*d*x + 1/2*c)^8 + 12355*tan(1/2*d*x + 1/
2*c)^7 + 8064*tan(1/2*d*x + 1/2*c)^6 - 6265*tan(1/2*d*x + 1/2*c)^5 + 8064*t
an(1/2*d*x + 1/2*c)^4 + 2779*tan(1/2*d*x + 1/2*c)^3 + 384*tan(1/2*d*x + 1/2
*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 384)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a))
/d
```

Mupad [B]
time = 11.68, size = 225, normalized size = 1.86

$$-\frac{5\,x}{128\,a}-\frac{\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-\frac{397\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\frac{895\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{1765\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{1765\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{192}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{895\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{397\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{7}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{2}{7}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x)),x)
```

```
[Out] - (5*x)/(128*a) - ((2*tan(c/2 + (d*x)/2)^2)/7 - (5*tan(c/2 + (d*x)/2))/64 +
 (397*tan(c/2 + (d*x)/2)^3)/192 + 6*tan(c/2 + (d*x)/2)^4 - (895*tan(c/2 + (
```

```
d*x)/2)^5)/192 + 6*tan(c/2 + (d*x)/2)^6 + (1765*tan(c/2 + (d*x)/2)^7)/192 +
 10*tan(c/2 + (d*x)/2)^8 - (1765*tan(c/2 + (d*x)/2)^9)/192 + 10*tan(c/2 + (
d*x)/2)^10 + (895*tan(c/2 + (d*x)/2)^11)/192 + 2*tan(c/2 + (d*x)/2)^12 - (3
97*tan(c/2 + (d*x)/2)^13)/192 + 2*tan(c/2 + (d*x)/2)^14 + (5*tan(c/2 + (d*x
)/2)^15)/64 + 2/7)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.710 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{5x}{16a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{5\cos(c+dx)\sin(c+dx)}{16ad}-\frac{5\cos^3(c$$

```
[Out] -5/16*x/a-arctanh(cos(d*x+c))/a/d+cos(d*x+c)/a/d+1/3*cos(d*x+c)^3/a/d+1/5*c
os(d*x+c)^5/a/d-5/16*cos(d*x+c)*sin(d*x+c)/a/d-5/24*cos(d*x+c)^3*sin(d*x+c)
/a/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2672, 308, 212, 2715, 8}

$$\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos(c+dx)}{ad}-\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}-\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}-\frac{5\sin(c+dx)\cos(c+dx)}{16ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{5x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(16*a) - ArcTanh[Cos[c + d*x]]/(a*d) + Cos[c + d*x]/(a*d) + Cos[c +
d*x]^3/(3*a*d) + Cos[c + d*x]^5/(5*a*d) - (5*Cos[c + d*x]*Sin[c + d*x])/(16
*a*d) - (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) - (Cos[c + d*x]^5*Sin[c +
d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
```

```
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(c+dx)\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cos^6(c+dx)\,dx}{a}+\frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a}\\
&= -\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}-\frac{5\int \cos^4(c+dx)\,dx}{6a}-\frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx,x,\text{co}\right.}{ad}\\
&= -\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}-\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}-\frac{5\int \cos^2(c+dx}{8a}\\
&= \frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos^5(c+dx)}{5ad}-\frac{5\cos(c+dx)\sin(c+dx)}{16ad}-\\
&= -\frac{5x}{16a}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\cos(c+dx)}{ad}+\frac{\cos^3(c+dx)}{3ad}+\frac{\cos^5(c+d}{5ad}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 102, normalized size = 0.71

$$-\frac{300c+300dx-1320\cos(c+dx)-140\cos(3(c+dx))-12\cos(5(c+dx))+960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+225\sin(2(c+dx))+45\sin(4(c+dx))+5\sin(6(c+dx))}{960ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(300*c + 300*d*x - 1320*Cos[c + d*x] - 140*Cos[3*(c + d*x)] - 12*Cos
[5*(c + d*x)] + 960*Log[Cos[(c + d*x)/2]] - 960*Log[Sin[(c + d*x)/2]] + 225
*Sin[2*(c + d*x)] + 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])/(a*d)
```

Maple [A]
time = 0.29, size = 191, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(tan(1/2*d*x+1/2*c))-2*(-11/16*tan(1/2*d*x+1/2*c)^11-3*tan(1/2*d*x
+1/2*c)^10+5/48*tan(1/2*d*x+1/2*c)^9-9*tan(1/2*d*x+1/2*c)^8-15/8*tan(1/2*d*
x+1/2*c)^7-46/3*tan(1/2*d*x+1/2*c)^6+15/8*tan(1/2*d*x+1/2*c)^5-14*tan(1/2*d
*x+1/2*c)^4-5/48*tan(1/2*d*x+1/2*c)^3-31/5*tan(1/2*d*x+1/2*c)^2+11/16*tan(1
/2*d*x+1/2*c)-23/15)/(1+tan(1/2*d*x+1/2*c)^2)^6-5/8*arctan(tan(1/2*d*x+1/2*
c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(131) = 262.
time = 0.51, size = 402, normalized size = 2.81

$$-\frac{\frac{\frac{165\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{1488\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{25\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3360\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{450\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3680\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{450\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2160\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{25\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{720\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{165\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-368}{a+\frac{6\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{75\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{120\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/120*((165*sin(d*x + c)/(cos(d*x + c) + 1) - 1488*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 25*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 3360*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 450*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3680*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 - 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 25*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 - 720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 165*sin(d*x + c)^11/(
cos(d*x + c) + 1)^11 - 368)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/(
cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 75*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a - 120*log(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A]
time = 0.39, size = 104, normalized size = 0.73

$$\frac{48\,\cos(dx+c)^5+80\,\cos(dx+c)^3-75\,dx-5\,\left(8\,\cos(dx+c)^5+10\,\cos(dx+c)^3+15\,\cos(dx+c)\right)\sin(dx+c)+240\,\cos(dx+c)-120\,\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+120\,\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*cos(d*x + c)^5 + 80*cos(d*x + c)^3 - 75*d*x - 5*(8*cos(d*x + c)^5
 + 10*cos(d*x + c)^3 + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c) - 1
20*log(1/2*cos(d*x + c) + 1/2) + 120*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.43, size = 195, normalized size = 1.36

$$-\frac{\frac{75\,(dx+c)}{a}-\frac{240\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+3680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+25\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1488\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+368\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(75*(d*x + c)/a - 240*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(165*tan(
1/2*d*x + 1/2*c)^11 + 720*tan(1/2*d*x + 1/2*c)^10 - 25*tan(1/2*d*x + 1/2*c)
^9 + 2160*tan(1/2*d*x + 1/2*c)^8 + 450*tan(1/2*d*x + 1/2*c)^7 + 3680*tan(1/
2*d*x + 1/2*c)^6 - 450*tan(1/2*d*x + 1/2*c)^5 + 3360*tan(1/2*d*x + 1/2*c)^4
 + 25*tan(1/2*d*x + 1/2*c)^3 + 1488*tan(1/2*d*x + 1/2*c)^2 - 165*tan(1/2*d*
x + 1/2*c) + 368)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a))/d
```

Mupad [B]
time = 11.83, size = 305, normalized size = 2.13

$$\frac{5\,\mathrm{atan}\left(\frac{25}{64\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{5}{4}\right)}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{5}{4}\right)}\right)}{8\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{92\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}-\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{62\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{46}{15}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atan(25/(64*((25*tan(c/2 + (d*x)/2))/64 + 5/4)) - (5*tan(c/2 + (d*x)/2))
/(4*((25*tan(c/2 + (d*x)/2))/64 + 5/4))))/(8*a*d) + log(tan(c/2 + (d*x)/2))
/(a*d) + ((62*tan(c/2 + (d*x)/2)^2)/5 - (11*tan(c/2 + (d*x)/2))/8 + (5*tan(
c/2 + (d*x)/2)^3)/24 + 28*tan(c/2 + (d*x)/2)^4 - (15*tan(c/2 + (d*x)/2)^5)/
4 + (92*tan(c/2 + (d*x)/2)^6)/3 + (15*tan(c/2 + (d*x)/2)^7)/4 + 18*tan(c/2
+ (d*x)/2)^8 - (5*tan(c/2 + (d*x)/2)^9)/24 + 6*tan(c/2 + (d*x)/2)^10 + (11*
tan(c/2 + (d*x)/2)^11)/8 + 46/15)/(d*(a + 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*t
an(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8
 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12))
```

3.711 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=137

$$-\frac{15x}{8a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}-\frac{15\cot(c+dx)}{8ad}+\frac{5\cos^2(c+dx)\cot(}{8ad}$$

```
[Out] -15/8*x/a+arctanh(cos(d*x+c))/a/d-cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d-1/5*c
os(d*x+c)^5/a/d-15/8*cot(d*x+c)/a/d+5/8*cos(d*x+c)^2*cot(d*x+c)/a/d+1/4*cos
(d*x+c)^4*cot(d*x+c)/a/d
```

Rubi [A]

time = 0.11, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2918, 2671, 294, 327, 209, 2672, 308, 212}

$$-\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos(c+dx)}{ad}-\frac{15\cot(c+dx)}{8ad}+\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-15*x)/(8*a) + ArcTanh[Cos[c + d*x]]/(a*d) - Cos[c + d*x]/(a*d) - Cos[c +
d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) - (15*Cot[c + d*x])/(8*a*d) + (5*Co
s[c + d*x]^2*Cot[c + d*x])/(8*a*d) + (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^5(c+dx)\cot(c+dx)\,dx}{a}+\frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a}$$

$$=\frac{\operatorname{Subst}\left(\int \frac{x^6}{1-x^2}\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx,x,\cot(c+dx)\right)}{ad}$$

$$=\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}+\frac{\operatorname{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)\,dx,x,\cos(c+\right.}{ad}$$

$$=-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{5\cos^2(c+dx)\cot(c+dx}{8ad}$$

$$=\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}-\frac{15\,c}{}$$

$$=-\frac{15x}{8a}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+d}{5ad}$$

Mathematica [A]

time = 0.54, size = 146, normalized size = 1.07

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(1200\cos(c+dx)-225\cos(3(c+dx))-15\cos(5(c+dx))+1800c\sin(c+dx)+1800dx\sin(c+dx)-960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+590\sin(2(c+dx))+64\sin(4(c+dx))+6\sin(6(c+dx))\right)}{1920ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(1200*Cos[c + d*x] - 225*Cos[3*(
c + d*x)] - 15*Cos[5*(c + d*x)] + 1800*c*Sin[c + d*x] + 1800*d*x*Sin[c + d*
x] - 960*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 960*Log[Sin[(c + d*x)/2]]*Sin
[c + d*x] + 590*Sin[2*(c + d*x)] + 64*Sin[4*(c + d*x)] + 6*Sin[6*(c + d*x)]
))/(a*d)
```

Maple [A]

time = 0.29, size = 177, normalized size = 1.29

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{9\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan\right.}}{2da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{9\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+6\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\left(1+\tan\right.}}{2da}$
risch	$-\frac{15x}{8a}+\frac{i e^{2i(dx+c)}}{4ad}-\frac{11\,e^{i(dx+c)}}{16ad}-\frac{11\,e^{-i(dx+c)}}{16ad}-\frac{i e^{-2i(dx+c)}}{4ad}-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}+\frac{\ln(e}{}$

norman	$-\frac{\frac{15x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}-\frac{65\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{1}{2ad}-\frac{61\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{45x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a}-\frac{15x\left(\tan^{13}\left(\frac{dx}{2}+\right.\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-4
*(-9/8*tan(1/2*d*x+1/2*c)^9+3*tan(1/2*d*x+1/2*c)^8-5/4*tan(1/2*d*x+1/2*c)^7
+6*tan(1/2*d*x+1/2*c)^6+28/3*tan(1/2*d*x+1/2*c)^4+5/4*tan(1/2*d*x+1/2*c)^3+
14/3*tan(1/2*d*x+1/2*c)^2+9/8*tan(1/2*d*x+1/2*c)+23/15)/(1+tan(1/2*d*x+1/2*
c)^2)^5-15/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(125) = 250$.
time = 0.51, size = 379, normalized size = 2.77

$$-\frac{\frac{\frac{184\sin(dx+c)}{\cos(dx+c)+1}+\frac{285\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{560\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{450\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1120\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{300\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{360\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{105\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+30}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{10a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{a\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}+\frac{225\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{30\sin(dx+c)}{a(\cos(dx+c)+1)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((184*sin(d*x + c)/(cos(d*x + c) + 1) + 285*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 560*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 450*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 1120*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 300*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3
60*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 105*sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 + 30)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 10*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 5*a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11) + 225*arctan(sin(d*x + c)/(cos(d*x + c
) + 1))/a + 60*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 30*sin(d*x + c)/(a*
(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.38, size = 124, normalized size = 0.91

$$\frac{30\cos(dx+c)^5+75\cos(dx+c)^3-\left(24\cos(dx+c)^5+40\cos(dx+c)^3+225\,dx+120\cos(dx+c)\right)\sin(dx+c)+60\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-60\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-225\cos(dx+c)}{120\,ad\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(30*cos(d*x + c)^5 + 75*cos(d*x + c)^3 - (24*cos(d*x + c)^5 + 40*cos(
d*x + c)^3 + 225*d*x + 120*cos(d*x + c))*sin(d*x + c) + 60*log(1/2*cos(d*x
```

```
+ c) + 1/2)*sin(d*x + c) - 60*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2
25*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.45, size = 199, normalized size = 1.45

$$-\frac{\frac{225\,(dx+c)}{a}+\frac{120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{60\,\left(2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\,\left(135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-184\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(225*(d*x + c)/a + 120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2
*d*x + 1/2*c)/a - 60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c))
- 2*(135*tan(1/2*d*x + 1/2*c)^9 - 360*tan(1/2*d*x + 1/2*c)^8 + 150*tan(1/2*
d*x + 1/2*c)^7 - 720*tan(1/2*d*x + 1/2*c)^6 - 1120*tan(1/2*d*x + 1/2*c)^4 -
 150*tan(1/2*d*x + 1/2*c)^3 - 560*tan(1/2*d*x + 1/2*c)^2 - 135*tan(1/2*d*x
+ 1/2*c) - 184)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*a))/d
```

Mupad [B]
time = 9.04, size = 296, normalized size = 2.16

$$\frac{15\,\mathrm{atan}\left(\frac{15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{225\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{15}{2}\right)}+\frac{225}{16\,\left(\frac{225\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-\frac{15}{2}\right)}\right)}{4\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{112\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{3}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{56\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{2}+\frac{92\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+1}{d\,\left(2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+20\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+20\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+10\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (15*atan((15*tan(c/2 + (d*x)/2))/(2*((225*tan(c/2 + (d*x)/2))/16 - 15/2)) +
 225/(16*((225*tan(c/2 + (d*x)/2))/16 - 15/2))))/(4*a*d) - log(tan(c/2 + (d
*x)/2))/(a*d) - ((92*tan(c/2 + (d*x)/2))/15 + (19*tan(c/2 + (d*x)/2)^2)/2 +
 (56*tan(c/2 + (d*x)/2)^3)/3 + 15*tan(c/2 + (d*x)/2)^4 + (112*tan(c/2 + (d*
x)/2)^5)/3 + 10*tan(c/2 + (d*x)/2)^6 + 24*tan(c/2 + (d*x)/2)^7 + 12*tan(c/2
 + (d*x)/2)^9 - (7*tan(c/2 + (d*x)/2)^10)/2 + 1)/(d*(2*a*tan(c/2 + (d*x)/2)
 + 10*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^5 + 20*a*tan(c/2 + (
d*x)/2)^7 + 10*a*tan(c/2 + (d*x)/2)^9 + 2*a*tan(c/2 + (d*x)/2)^11)) + tan(c
/2 + (d*x)/2)/(2*a*d)
```

3.712 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{15x}{8a}+\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{5\cos(c+dx)}{2ad}-\frac{5\cos^3(c+dx)}{6ad}+\frac{15\cot(c+dx)}{8ad}-\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}-$$

```
[Out] 15/8*x/a+5/2*arctanh(cos(d*x+c))/a/d-5/2*cos(d*x+c)/a/d-5/6*cos(d*x+c)^3/a/
d+15/8*cot(d*x+c)/a/d-5/8*cos(d*x+c)^2*cot(d*x+c)/a/d-1/4*cos(d*x+c)^4*cot(
d*x+c)/a/d-1/2*cos(d*x+c)^3*cot(d*x+c)^2/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2918, 2672, 294, 308, 212, 2671, 327, 209}

$$-\frac{5\cos^3(c+dx)}{6ad}-\frac{5\cos(c+dx)}{2ad}+\frac{15\cot(c+dx)}{8ad}-\frac{\cos^4(c+dx)\cot(c+dx)}{4ad}-\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*x)/(8*a) + (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (5*Cos[c + d*x])/(2*a*d)
 - (5*Cos[c + d*x]^3)/(6*a*d) + (15*Cot[c + d*x])/(8*a*d) - (5*Cos[c + d*x]
^2*Cot[c + d*x])/(8*a*d) - (Cos[c + d*x]^4*Cot[c + d*x])/(4*a*d) - (Cos[c +
 d*x]^3*Cot[c + d*x]^2)/(2*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^4(c+dx)\cot^2(c+dx)\,dx}{a} + \frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \cot(c+d\ldots\right.}{ad}$$

$$= -\frac{\cos^4(c+dx)\cot(c+dx)}{4ad} - \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad} + \frac{5\text{Subst}\left(\int \frac{x^{4}}{(1+x\ldots}\right.}{\ldots}$$

$$= -\frac{5\cos^2(c+dx)\cot(c+dx)}{8ad} - \frac{\cos^4(c+dx)\cot(c+dx)}{4ad} - \frac{\cos^3(c+dx)\,c\ldots}{2a\ldots}$$

$$= -\frac{5\cos(c+dx)}{2ad} - \frac{5\cos^3(c+dx)}{6ad} + \frac{15\cot(c+dx)}{8ad} - \frac{5\cos^2(c+dx)\cot(\ldots}{8ad}$$

$$= \frac{15x}{8a} + \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{5\cos(c+dx)}{2ad} - \frac{5\cos^3(c+dx)}{6ad} + \frac{15\cot\ldots}{8\ldots}$$

Mathematica [A]
time = 0.38, size = 179, normalized size = 1.19

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-360c-360dx+400\cos(c+dx)-200\cos(3(c+dx))-8\cos(5(c+dx))-480\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120\cos(2(c+dx))\left(3c+3dx+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-285\sin(2(c+dx))+42\sin(4(c+dx))+3\sin(6(c+dx))\right)}{1536ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1536*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(-360*c - 360*d*x + 400*Co
s[c + d*x] - 200*Cos[3*(c + d*x)] - 8*Cos[5*(c + d*x)] - 480*Log[Cos[(c + d
*x)/2]] + 120*Cos[2*(c + d*x)]*(3*c + 3*d*x + 4*Log[Cos[(c + d*x)/2]] - 4*L
og[Sin[(c + d*x)/2]]) + 480*Log[Sin[(c + d*x)/2]] - 285*Sin[2*(c + d*x)] +
42*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.30, size = 192, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.}{\ldots}}{4d\ldots}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-9\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\ldots\right.\right.}{\ldots}}{4d\ldots}$
risch	$\frac{15x}{8a}-\frac{e^{3i(dx+c)}}{24ad}-\frac{ie^{2i(dx+c)}}{4ad}-\frac{9\,e^{i(dx+c)}}{8ad}-\frac{9\,e^{-i(dx+c)}}{8ad}+\frac{ie^{-2i(dx+c)}}{4ad}-\frac{e^{-3i(dx+c)}}{24ad}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}\ldots}{ad\left(e^{2i(dx+\ldots}\right.}$

norman	$-\frac{21\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{3\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8da}-\frac{1}{8ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}+\frac{15x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+\frac{15x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a*(1/2*tan(1/2*d*x+1/2*c)^2-2*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/2*
c)^2+2/tan(1/2*d*x+1/2*c)-10*ln(tan(1/2*d*x+1/2*c))+8*(-9/8*tan(1/2*d*x+1/2
*c)^7-3*tan(1/2*d*x+1/2*c)^6-1/8*tan(1/2*d*x+1/2*c)^5-7*tan(1/2*d*x+1/2*c)^
4+1/8*tan(1/2*d*x+1/2*c)^3-19/3*tan(1/2*d*x+1/2*c)^2+9/8*tan(1/2*d*x+1/2*c)
-7/3)/(1+tan(1/2*d*x+1/2*c)^2)^4+15*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(134) = 268$.
time = 0.51, size = 383, normalized size = 2.55

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{124\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{102\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{322\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{78\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{348\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{42\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{147\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{42\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-3}{\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{3\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a}+\frac{90\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/24*((12*sin(d*x + c)/(cos(d*x + c) + 1) - 124*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 102*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 322*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 78*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 348*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 42*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 147*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 - 42*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
- 3)/(a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 6*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 + a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 3*(4*sin(d
*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a + 90*ar
ctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*log(sin(d*x + c)/(cos(d*x + c)
 + 1))/a)/d
```

Fricas [A]
time = 0.38, size = 148, normalized size = 0.99

$$-\frac{8\cos(dx+c)^5-45dx\cos(dx+c)^2+40\cos(dx+c)^3+45dx-30\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+30\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(2\cos(dx+c)^5+5\cos(dx+c)^3-15\cos(dx+c)\right)\sin(dx+c)-60\cos(dx+c)}{24\left(ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*cos(d*x + c)^5 - 45*d*x*cos(d*x + c)^2 + 40*cos(d*x + c)^3 + 45*d*
x - 30*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 30*(cos(d*x + c)^
2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 3*(2*cos(d*x + c)^5 + 5*cos(d*x + c)^
```

```
3 - 15*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c))/(a*d*cos(d*x + c)^2 -
a*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.51, size = 216, normalized size = 1.44

$$\frac{\frac{45\,(dx+c)}{a}-\frac{60\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{3\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^2}+\frac{3\,\left(30\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{2\,\left(27\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+168\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+152\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+56\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(45*(d*x + c)/a - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a*tan(1/2*d
*x + 1/2*c)^2 - 4*a*tan(1/2*d*x + 1/2*c))/a^2 + 3*(30*tan(1/2*d*x + 1/2*c)^
2 + 4*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)^2) - 2*(27*tan(1/2*
d*x + 1/2*c)^7 + 72*tan(1/2*d*x + 1/2*c)^6 + 3*tan(1/2*d*x + 1/2*c)^5 + 168
*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)^3 + 152*tan(1/2*d*x + 1/2*
c)^2 - 27*tan(1/2*d*x + 1/2*c) + 56)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a))/d
```

Mupad [B]
time = 9.02, size = 303, normalized size = 2.02

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{15\,\mathrm{atan}\left(\frac{225}{16\,\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{75}{4}\right)}-\frac{75\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(\frac{225\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{75}{4}\right)}\right)}{4\,a\,d}-\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{49\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}-7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+58\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{161\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{62\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{2}}{d\,\left(4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (15*atan(225/(16*((225*tan(c/2 + (d*x)/2))/1
6 + 75/4)) - (75*tan(c/2 + (d*x)/2))/(4*((225*tan(c/2 + (d*x)/2))/16 + 75/4
))))/(4*a*d) - (5*log(tan(c/2 + (d*x)/2)))/(2*a*d) - ((62*tan(c/2 + (d*x)/2
)^2)/3 - 2*tan(c/2 + (d*x)/2) - 17*tan(c/2 + (d*x)/2)^3 + (161*tan(c/2 + (d
*x)/2)^4)/3 - 13*tan(c/2 + (d*x)/2)^5 + 58*tan(c/2 + (d*x)/2)^6 - 7*tan(c/2
 + (d*x)/2)^7 + (49*tan(c/2 + (d*x)/2)^8)/2 + 7*tan(c/2 + (d*x)/2)^9 + 1/2)
/(d*(4*a*tan(c/2 + (d*x)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4 + 24*a*tan(c/2 +
(d*x)/2)^6 + 16*a*tan(c/2 + (d*x)/2)^8 + 4*a*tan(c/2 + (d*x)/2)^10)) - tan(
c/2 + (d*x)/2)/(2*a*d)
```

3.713 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{5x}{2a}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{5\cos(c+dx)}{2ad}+\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}-\frac{5\cot}{}$$

```
[Out] 5/2*x/a-5/2*arctanh(cos(d*x+c))/a/d+5/2*cos(d*x+c)/a/d+5/6*cos(d*x+c)^3/a/d
+5/2*cot(d*x+c)/a/d+1/2*cos(d*x+c)^3*cot(d*x+c)^2/a/d-5/6*cot(d*x+c)^3/a/d+
1/2*cos(d*x+c)^2*cot(d*x+c)^3/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2918, 2671, 294, 308, 209, 2672, 212}

$$\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cos(c+dx)}{2ad}-\frac{5\cot^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{5x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(2*a) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) + (5*Cos[c + d*x])/(2*a*d)
+ (5*Cos[c + d*x]^3)/(6*a*d) + (5*Cot[c + d*x])/(2*a*d) + (Cos[c + d*x]^3*C
ot[c + d*x]^2)/(2*a*d) - (5*Cot[c + d*x]^3)/(6*a*d) + (Cos[c + d*x]^2*Cot[c
 + d*x]^3)/(2*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^3(c+dx)\cot^3(c+dx)\,dx}{a}+\frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{ad}$$

$$= \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{5\text{Subst}\left(\int \frac{x^4}{1-x^2}\right.}{}$$

$$= \frac{\cos^3(c+dx)\cot^2(c+dx)}{2ad}+\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{5\text{Subst}\left(\int (-1\right.}{}$$

$$= \frac{5\cos(c+dx)}{2ad}+\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cot(c+dx)}{2ad}+\frac{\cos^3(c+dx)\cot^2(c+}{2ad}$$

$$= \frac{5x}{2a}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{5\cos(c+dx)}{2ad}+\frac{5\cos^3(c+dx)}{6ad}+\frac{5\cot(c}{2a}$$

Mathematica [A]
time = 0.58, size = 197, normalized size = 1.35

$$-\frac{\csc^3(c+dx)\left(-30\cos(c+dx)+65\cos(3(c+dx))-3\cos(5(c+dx))-180c\sin(c+dx)-180dx\sin(c+dx)+180\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-180\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-75\sin(2(c+dx))+60c\sin(3(c+dx))+60dx\sin(3(c+dx))-60\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+60\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+24\sin(4(c+dx))+\sin(6(c+dx))\right)}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/96*(Csc[c + d*x]^3*(-30*Cos[c + d*x] + 65*Cos[3*(c + d*x)] - 3*Cos[5*(c
+ d*x)] - 180*c*Sin[c + d*x] - 180*d*x*Sin[c + d*x] + 180*Log[Cos[(c + d*x)
/2]]*Sin[c + d*x] - 180*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 75*Sin[2*(c +
d*x)] + 60*c*Sin[3*(c + d*x)] + 60*d*x*Sin[3*(c + d*x)] - 60*Log[Cos[(c + d
*x)/2]]*Sin[3*(c + d*x)] + 60*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 24*S
in[4*(c + d*x)] + Sin[6*(c + d*x)]))/(a*d)
```

Maple [A]
time = 0.30, size = 177, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{9}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ -}{8da}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{9}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+20\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ -}{8da}$
risch	$\frac{5x}{2a}-\frac{i e^{2i(dx+c)}}{8ad}+\frac{9\,e^{i(dx+c)}}{8ad}+\frac{9\,e^{-i(dx+c)}}{8ad}+\frac{i e^{-2i(dx+c)}}{8ad}-\frac{-18i e^{4i(dx+c)}+3\,e^{5i(dx+c)}+24i e^{2i(dx+c)}-14i-3\,e^{i(}}{3ad\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{85\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{12da}-\frac{1}{24ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{12ad}+\frac{\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{5x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{5x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{10x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2-9*tan(1/2*d*x+1/2*c)
-1/3/tan(1/2*d*x+1/2*c)^3+1/tan(1/2*d*x+1/2*c)^2+9/tan(1/2*d*x+1/2*c)+20*ln
(tan(1/2*d*x+1/2*c))+16*(-1/2*tan(1/2*d*x+1/2*c)^5+3*tan(1/2*d*x+1/2*c)^4+4
*tan(1/2*d*x+1/2*c)^2+1/2*tan(1/2*d*x+1/2*c)+7/3)/(1+tan(1/2*d*x+1/2*c)^2)^
3+40*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(130) = 260.
time = 0.50, size = 362, normalized size = 2.48

$$-\frac{\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{121\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{102\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{201\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{80\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{147\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-1}{\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((27*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - (3*sin(d*x + c)/(cos(d*x
 + c) + 1) + 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 121*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 + 102*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 201*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 + 80*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 147*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 1)/(a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 3*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9) - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.39, size = 168, normalized size = 1.15

$$-\frac{6\cos(dx+c)^5-40\cos(dx+c)^3+15\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-15\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\left(2\cos(dx+c)^5+15\,dx\cos(dx+c)^2+10\cos(dx+c)^3-15\,dx-15\cos(dx+c)\right)\sin(dx+c)+30\cos(dx+c)}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*(cos(d*x + c)^2 - 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 2*(2*cos(d*x + c)^5 + 15*d*x*cos(d*x + c)^2
+ 10*cos(d*x + c)^3 - 15*d*x - 15*cos(d*x + c))*sin(d*x + c) + 30*cos(d*x +
 c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.44, size = 228, normalized size = 1.56

$$\frac{\frac{180\,(dx+c)}{a}+\frac{180\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{3\left(a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-27a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^3}-\frac{110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-111\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-306\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-253\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-72\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^3a}}{72\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/72*(180*(d*x + c)/a + 180*log(abs(tan(1/2*d*x + 1/2*c)))/a + 3*(a^2*tan(1
/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2
*c))/a^3 - (110*tan(1/2*d*x + 1/2*c)^9 - 9*tan(1/2*d*x + 1/2*c)^8 - 111*tan
(1/2*d*x + 1/2*c)^7 - 240*tan(1/2*d*x + 1/2*c)^6 - 273*tan(1/2*d*x + 1/2*c)
^5 - 306*tan(1/2*d*x + 1/2*c)^4 - 253*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d
*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 3)/((tan(1/2*d*x + 1/2*c)^3 + tan(
1/2*d*x + 1/2*c))^3*a))/d
```

Mupad [B]
time = 9.06, size = 290, normalized size = 1.99

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}-\frac{5\,\mathrm{atan}\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}+\frac{25}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}\right)}{a\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+49\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+67\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+34\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}}{d\,\left(8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (5*atan((25*
tan(c/2 + (d*x)/2))/(25*tan(c/2 + (d*x)/2) - 25) + 25/(25*tan(c/2 + (d*x)/2
) - 25)))/(a*d) + (5*log(tan(c/2 + (d*x)/2)))/(2*a*d) + (tan(c/2 + (d*x)/2)
 + 8*tan(c/2 + (d*x)/2)^2 + (121*tan(c/2 + (d*x)/2)^3)/3 + 34*tan(c/2 + (d*
x)/2)^4 + 67*tan(c/2 + (d*x)/2)^5 + (80*tan(c/2 + (d*x)/2)^6)/3 + 49*tan(c/
2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 - 1/3)/(d*(8*a*tan(c/2 + (d*x)/2)^3 +
 24*a*tan(c/2 + (d*x)/2)^5 + 24*a*tan(c/2 + (d*x)/2)^7 + 8*a*tan(c/2 + (d*x
)/2)^9)) - (9*tan(c/2 + (d*x)/2))/(8*a*d)
```

3.714 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{5x}{2a}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{15\cos(c+dx)}{8ad}-\frac{5\cot(c+dx)}{2ad}+\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-$$

```
[Out] -5/2*x/a-15/8*arctanh(cos(d*x+c))/a/d+15/8*cos(d*x+c)/a/d-5/2*cot(d*x+c)/a/
d+5/8*cos(d*x+c)*cot(d*x+c)^2/a/d+5/6*cot(d*x+c)^3/a/d-1/2*cos(d*x+c)^2*cot
(d*x+c)^3/a/d-1/4*cos(d*x+c)*cot(d*x+c)^4/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2918, 2672, 294, 327, 212, 2671, 308, 209}

$$\frac{15\cos(c+dx)}{8ad}+\frac{5\cot^3(c+dx)}{6ad}-\frac{5\cot(c+dx)}{2ad}-\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad}-\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}+\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{5x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*x)/(2*a) - (15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Cos[c + d*x])/(8*a*
d) - (5*Cot[c + d*x])/(2*a*d) + (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + (
5*Cot[c + d*x]^3)/(6*a*d) - (Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*a*d) - (Cos[
c + d*x]*Cot[c + d*x]^4)/(4*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos^2(c+dx)\cot^4(c+dx)\,dx}{a} + \frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(c+dx\right.}{ad}$$

$$= -\frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{\cos(c+dx)\cot^4(c+dx)}{4ad} + \frac{5\text{Subst}\left(\int \frac{x^4}{(1-x\right.}{}$$

$$= \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} - \frac{\cos^2(c+dx)\cot^3(c+dx)}{2ad} - \frac{\cos(c+dx)\cot}{4ad}$$

$$= \frac{15\cos(c+dx)}{8ad} - \frac{5\cot(c+dx)}{2ad} + \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{5\cot^3(c+}{6ad}$$

$$= -\frac{5x}{2a} - \frac{15\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{15\cos(c+dx)}{8ad} - \frac{5\cot(c+dx)}{2ad} + \frac{5\cos}{}$$

Mathematica [A]
time = 0.54, size = 252, normalized size = 1.68

$$-\frac{\csc^4(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(180c+180dx-30\cos(c+dx)+90\cos(3(c+dx))+60c\cos(4(c+dx))+60dx\cos(4(c+dx))-12\cos(5(c+dx))+135\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+45\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-60\cos(2(c+dx))\left(4c+4dx+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-135\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-45\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+95\sin(2(c+dx))-68\sin(4(c+dx))+3\sin(6(c+dx))\right)}{192ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/192*(Csc[c + d*x]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(180*c + 180
*d*x - 30*Cos[c + d*x] + 90*Cos[3*(c + d*x)] + 60*c*Cos[4*(c + d*x)] + 60*d
*x*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 135*Log[Cos[(c + d*x)/2]] + 45*
Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 60*Cos[2*(c + d*x)]*(4*c + 4*d*x +
 3*Log[Cos[(c + d*x)/2]] - 3*Log[Sin[(c + d*x)/2]]) - 135*Log[Sin[(c + d*x)
/2]] - 45*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 95*Sin[2*(c + d*x)] - 68
*Sin[4*(c + d*x)] + 3*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.30, size = 192, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{16}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}}{16}$

risch	$-\frac{5x}{2a}+\frac{i\mathrm{e}^{2i(dx+c)}}{8ad}+\frac{\mathrm{e}^{i(dx+c)}}{2ad}+\frac{\mathrm{e}^{-i(dx+c)}}{2ad}-\frac{i\mathrm{e}^{-2i(dx+c)}}{8ad}-\frac{72i\mathrm{e}^{6i(dx+c)}+27\,\mathrm{e}^{7i(dx+c)}-168i\mathrm{e}^{4i(dx+c)}-3\,\mathrm{e}^{5i(dx+}}{12ad\left(\mathrm{e}^{2i(dx+}\right.}$
norman	$\frac{-\frac{1}{64ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{192ad}+\frac{47\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad}-\frac{51\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{51\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}-\frac{47\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192da}-\frac{5\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192da}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a*(1/4*tan(1/2*d*x+1/2*c)^4-2/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1
/2*c)^2+18*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4+2/3/tan(1/2*d*x+1/2*
c)^3+4/tan(1/2*d*x+1/2*c)^2-18/tan(1/2*d*x+1/2*c)+30*ln(tan(1/2*d*x+1/2*c))
-32*(-1/2*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2+1/2*tan(1/2*d*x+1/2*c)-
1)/(1+tan(1/2*d*x+1/2*c)^2)^2-80*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(134) = 268$.
time = 0.53, size = 340, normalized size = 2.27

$$\frac{\frac{\frac{216\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a}+\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}+\frac{42\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{200\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{477\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{616\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{432\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3}{\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{960\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/192*((216*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4)/a + (8*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 477*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 616*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4
32*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 3)/(a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 960*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a + 360*log(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

Fricas [A]
time = 0.41, size = 191, normalized size = 1.27

$$-\frac{120\,dx\cos(dx+c)^4-48\cos(dx+c)^5-240\,dx\cos(dx+c)^2+150\cos(dx+c)^3+120\,dx+45\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-45\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+8\left(3\cos(dx+c)^5-20\cos(dx+c)^3+15\cos(dx+c)\right)\sin(dx+c)-90\cos(dx+c)}{48\left(ad\cos(dx+c)^4-2\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(120*d*x*cos(d*x + c)^4 - 48*cos(d*x + c)^5 - 240*d*x*cos(d*x + c)^2
+ 150*cos(d*x + c)^3 + 120*d*x + 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
```

```
*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 8*(3*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 15*
cos(d*x + c))*sin(d*x + c) - 90*cos(d*x + c))/(a*d*cos(d*x + c)^4 - 2*a*d*c
os(d*x + c)^2 + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 224, normalized size = 1.49

$$-\frac{\frac{480\,(dx+c)}{a}-\frac{360\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{192\,\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}a}-\frac{3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+216\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{4}}+\frac{750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+216\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(480*(d*x + c)/a - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a - 192*(tan(1
/2*d*x + 1/2*c)^3 + 2*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 2)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a) - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 216*a^3*tan(1/2*d*x
+ 1/2*c))/a^4 + (750*tan(1/2*d*x + 1/2*c)^4 + 216*tan(1/2*d*x + 1/2*c)^3 -
48*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/
2*c)^4))/d
```

Mupad [**B**]
time = 9.03, size = 286, normalized size = 1.91

$$\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4\,a\,d}+\frac{5\,\mathrm{atan}\left(\frac{25}{25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{75}{4}}-\frac{75\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,\left(25\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{75}{4}\right)}\right)}{a\,d}+\frac{15\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}+\frac{-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-\frac{154\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{3}+\frac{159\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{4}-\frac{50\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\frac{7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{1}{4}}{d\,\left(16\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+32\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}+\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (
d*x)/2)^2/(4*a*d) + (5*atan(25/(25*tan(c/2 + (d*x)/2) + 75/4) - (75*tan(c/2
 + (d*x)/2))/(4*(25*tan(c/2 + (d*x)/2) + 75/4))))/(a*d) + (15*log(tan(c/2 +
 (d*x)/2)))/(8*a*d) + ((2*tan(c/2 + (d*x)/2))/3 + (7*tan(c/2 + (d*x)/2)^2)/
2 - (50*tan(c/2 + (d*x)/2)^3)/3 + (159*tan(c/2 + (d*x)/2)^4)/4 - (154*tan(c
/2 + (d*x)/2)^5)/3 + 36*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 - 1/4
)/(d*(16*a*tan(c/2 + (d*x)/2)^4 + 32*a*tan(c/2 + (d*x)/2)^6 + 16*a*tan(c/2
+ (d*x)/2)^8)) + (9*tan(c/2 + (d*x)/2))/(8*a*d)
```

3.715 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{x}{a}+\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{15\cos(c+dx)}{8ad}-\frac{\cot(c+dx)}{ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{\cot^3(c+dx)}{3ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -x/a+15/8*arctanh(cos(d*x+c))/a/d-15/8*cos(d*x+c)/a/d-cot(d*x+c)/a/d-5/8*co
s(d*x+c)*cot(d*x+c)^2/a/d+1/3*cot(d*x+c)^3/a/d+1/4*cos(d*x+c)*cot(d*x+c)^4/
a/d-1/5*cot(d*x+c)^5/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2918, 3554, 8, 2672, 294, 327, 212}

$$-\frac{15\cos(c+dx)}{8ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)\cot^4(c+dx)}{4ad}-\frac{5\cos(c+dx)\cot^2(c+dx)}{8ad}+\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) + (15*ArcTanh[Cos[c + d*x]])/(8*a*d) - (15*Cos[c + d*x])/(8*a*d) - C
ot[c + d*x]/(a*d) - (5*Cos[c + d*x]*Cot[c + d*x]^2)/(8*a*d) + Cot[c + d*x]^
3/(3*a*d) + (Cos[c + d*x]*Cot[c + d*x]^4)/(4*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cos(c+dx)\cot^5(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\,dx}{a}$$

$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{\int \cot^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right.}{ad}$$

$$= \frac{\cot^3(c+dx)}{3ad} + \frac{\cos(c+dx)\cot^4(c+dx)}{4ad} - \frac{\cot^5(c+dx)}{5ad} + \frac{\int \cot^2(c+dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{\cot^3(c+dx)}{3ad} + \frac{\cos(c+dx}{}$$

$$= -\frac{x}{a} - \frac{15\cos(c+dx)}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{5\cos(c+dx)\cot^2(c+dx)}{8ad} + \frac{\cot^3}{}$$

$$= -\frac{x}{a} + \frac{15\tanh^{-1}(\cos(c+dx))}{8ad} - \frac{15\cos(c+dx)}{8ad} - \frac{\cot(c+dx)}{ad} - \frac{5\cos(c}{}$$

Mathematica [A]
time = 0.72, size = 264, normalized size = 1.91

$$-\frac{\csc^5(c+dx)\left(400\cos(c+dx)-200\cos(3(c+dx))+184\cos(5(c+dx))+1200c\sin(c+dx)+1200dx\sin(c+dx)-2250\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+2250\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+600\sin(2(c+dx))-600c\sin(3(c+dx))-600dx\sin(3(c+dx))+1125\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-1125\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-510\sin(4(c+dx))+120c\sin(5(c+dx))+120dx\sin(5(c+dx))-225\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+225\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+60\sin(6(c+dx))\right)}{1920ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(400*Cos[c + d*x] - 200*Cos[3*(c + d*x)] + 184*Cos[
5*(c + d*x)] + 1200*c*Sin[c + d*x] + 1200*d*x*Sin[c + d*x] - 2250*Log[Cos[(
c + d*x)/2]]*Sin[c + d*x] + 2250*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 600*S
in[2*(c + d*x)] - 600*c*Sin[3*(c + d*x)] - 600*d*x*Sin[3*(c + d*x)] + 1125*
Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 1125*Log[Sin[(c + d*x)/2]]*Sin[3*(
c + d*x)] - 510*Sin[4*(c + d*x)] + 120*c*Sin[5*(c + d*x)] + 120*d*x*Sin[5*(
c + d*x)] - 225*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 225*Log[Sin[(c + d
*x)/2]]*Sin[5*(c + d*x)] + 60*Sin[6*(c + d*x)]))/(a*d)
```

Maple [A]
time = 0.31, size = 179, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}{32da}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+22\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}}{32da}$
risch	$-\frac{x}{a}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}+\frac{-360ie^{8i(dx+c)}+135\,e^{9i(dx+c)}+720ie^{6i(dx+c)}-150\,e^{7i(dx+c)}-1120ie^{4i(dx+c)}+560ie}{60ad\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{1}{160ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{320ad}+\frac{73\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960ad}-\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120ad}-\frac{23\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}+\frac{23\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}+\frac{19\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{120ad}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a*(1/5*tan(1/2*d*x+1/2*c)^5-1/2*tan(1/2*d*x+1/2*c)^4-7/3*tan(1/2*d*x
+1/2*c)^3+8*tan(1/2*d*x+1/2*c)^2+22*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*
c)^5+1/2/tan(1/2*d*x+1/2*c)^4+7/3/tan(1/2*d*x+1/2*c)^3-8/tan(1/2*d*x+1/2*c)
^2-22/tan(1/2*d*x+1/2*c)-60*ln(tan(1/2*d*x+1/2*c))-64/(1+tan(1/2*d*x+1/2*c)
^2)-64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(126) = 252.
time = 0.53, size = 319, normalized size = 2.31

$$\frac{\frac{\frac{660\sin(dx+c)}{\cos(dx+c)+1}+\frac{240\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{64\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{225\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{590\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2160\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{660\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-6}{\frac{a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}-\frac{1920\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{1800\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*((660*sin(d*x + c)/(cos(d*x + c) + 1) + 240*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a + (15*sin(d*x
+ c)/(cos(d*x + c) + 1) + 64*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 225*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 - 590*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 2160*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 660*sin(d*x + c)^6/(cos(d*x + c
) + 1)^6 - 6)/(a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7) - 1920*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 1800*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.38, size = 211, normalized size = 1.53

$$-\frac{368\cos\left(dx+c\right)^{5}-560\cos\left(dx+c\right)^{3}-225\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+225\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+30\left(8\,dx\cos\left(dx+c\right)^{4}+8\cos\left(dx+c\right)^{5}-16\,dx\cos\left(dx+c\right)^{2}-25\cos\left(dx+c\right)^{3}+8\,dx+15\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+240\cos\left(dx+c\right)}{240\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*cos(d*x + c)^5 - 560*cos(d*x + c)^3 - 225*(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 225*(cos(d*x
+ c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
30*(8*d*x*cos(d*x + c)^4 + 8*cos(d*x + c)^5 - 16*d*x*cos(d*x + c)^2 - 25*co
s(d*x + c)^3 + 8*d*x + 15*cos(d*x + c))*sin(d*x + c) + 240*cos(d*x + c))/((
a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 217, normalized size = 1.57

$$-\frac{\frac{960\,(dx+c)}{a}+\frac{1800\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{1920}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a}-\frac{6\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+660\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{5}}-\frac{4110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-660\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/960*(960*(d*x + c)/a + 1800*log(abs(tan(1/2*d*x + 1/2*c)))/a + 1920/((ta
n(1/2*d*x + 1/2*c)^2 + 1)*a) - (6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*tan(1
/2*d*x + 1/2*c)^4 - 70*a^4*tan(1/2*d*x + 1/2*c)^3 + 240*a^4*tan(1/2*d*x + 1
/2*c)^2 + 660*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (4110*tan(1/2*d*x + 1/2*c)^5
- 660*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/2*d*x + 1/2*c)^3 + 70*tan(1/2*d*x
+ 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 6)/(a*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 9.03, size = 279, normalized size = 2.02

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a\,d}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,a\,d}+\frac{2\,\mathrm{atan}\left(\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{15}{2}\right)}+\frac{4}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{15}{2}}\right)}{a\,d}-\frac{15\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d}-\frac{22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+72\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{1}{5}}{d\left(32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(4*a*d) - (7*tan(c/2 + (d*x)/2)^3)/(96*a*d) - tan(c/2
+ (d*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) + (2*atan((15*tan(c/
2 + (d*x)/2))/(2*(4*tan(c/2 + (d*x)/2) - 15/2)) + 4/(4*tan(c/2 + (d*x)/2) -
 15/2)))/(a*d) - (15*log(tan(c/2 + (d*x)/2)))/(8*a*d) - ((15*tan(c/2 + (d*x
)/2)^3)/2 - (32*tan(c/2 + (d*x)/2)^2)/15 - tan(c/2 + (d*x)/2)/2 + (59*tan(c
/2 + (d*x)/2)^4)/3 + 72*tan(c/2 + (d*x)/2)^5 + 22*tan(c/2 + (d*x)/2)^6 + 1/
5)/(d*(32*a*tan(c/2 + (d*x)/2)^5 + 32*a*tan(c/2 + (d*x)/2)^7)) + (11*tan(c/
2 + (d*x)/2))/(16*a*d)
```

3.716 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$\frac{x}{a}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{5\cot(c+dx)\csc(c+dx)}{16ad}+\frac{5\cot^3(c-}{}$$

```
[Out] x/a+5/16*arctanh(cos(d*x+c))/a/d+cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/5*co
t(d*x+c)^5/a/d-5/16*cot(d*x+c)*csc(d*x+c)/a/d+5/24*cot(d*x+c)^3*csc(d*x+c)/
a/d-1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2918, 2691, 3855, 3554, 8}

$$\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)}{ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}-\frac{5\cot(c+dx)\csc(c+dx)}{16ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + (5*ArcTanh[Cos[c + d*x]])/(16*a*d) + Cot[c + d*x]/(a*d) - Cot[c + d*x
]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(16*a*
d) + (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) - (Cot[c + d*x]^5*Csc[c + d*x
])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}\\
&= \frac{\cot^5(c+dx)}{5ad}-\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}\\
&= -\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}+\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}-\frac{\cot^5(c+dx)}{6a}\\
&= \frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{5\cot(c+dx)\csc(c+dx)}{16ad}+\\
&= \frac{x}{a}+\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 317 vs. 2(142) = 284.
time = 0.72, size = 317, normalized size = 2.23

$$
-\frac{\csc^6(c+dx)\left(\cos\left(\tfrac{1}{2}(c+dx)\right)+\sin\left(\tfrac{1}{2}(c+dx)\right)\right)^2\left(-2400c-2400dx+900\cos(c+dx)+50\cos(3(c+dx))-1440c\cos(4(c+dx))-1440dx\cos(4(c+dx))+330\cos(5(c+dx))+240c\cos(6(c+dx))+240dx\cos(6(c+dx))-750\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-450\cos(4(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+75\cos(6(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+225\cos(2(c+dx))\left(16(c+dx)+5\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-5\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+750\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+450\cos(4(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-75\cos(6(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-1200\sin(2(c+dx))+768\sin(4(c+dx))-368\sin(6(c+dx))\right)}{7680ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2400*c -
2400*d*x + 900*Cos[c + d*x] + 50*Cos[3*(c + d*x)] - 1440*c*Cos[4*(c + d*x)]
 - 1440*d*x*Cos[4*(c + d*x)] + 330*Cos[5*(c + d*x)] + 240*c*Cos[6*(c + d*x)
] + 240*d*x*Cos[6*(c + d*x)] - 750*Log[Cos[(c + d*x)/2]] - 450*Cos[4*(c + d
*x)]*Log[Cos[(c + d*x)/2]] + 75*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 22
5*Cos[2*(c + d*x)]*(16*(c + d*x) + 5*Log[Cos[(c + d*x)/2]] - 5*Log[Sin[(c +
 d*x)/2]]) + 750*Log[Sin[(c + d*x)/2]] + 450*Cos[4*(c + d*x)]*Log[Sin[(c +
d*x)/2]] - 75*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1200*Sin[2*(c + d*x)
] + 768*Sin[4*(c + d*x)] - 368*Sin[6*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.34, size = 188, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-44\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\tan\left(\frac{dx}{2}\right.}}{}$
default	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{15\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-44\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\tan\left(\frac{dx}{2}\right.}}{}$
risch	$\frac{x}{a}+\frac{720i\mathrm{e}^{10i(dx+c)}+165\,\mathrm{e}^{11i(dx+c)}-2160i\mathrm{e}^{8i(dx+c)}+25\,\mathrm{e}^{9i(dx+c)}+3680i\mathrm{e}^{6i(dx+c)}+450\,\mathrm{e}^{7i(dx+c)}-3360i\mathrm{e}^{4i(dx+c)}+45}{120ad\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$
norman	$\frac{x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{1}{384ad}+\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1920ad}+\frac{13\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480ad}-\frac{11}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a*(1/6*tan(1/2*d*x+1/2*c)^6-2/5*tan(1/2*d*x+1/2*c)^5-3/2*tan(1/2*d*x
+1/2*c)^4+14/3*tan(1/2*d*x+1/2*c)^3+15/2*tan(1/2*d*x+1/2*c)^2-44*tan(1/2*d*
x+1/2*c)-1/6/tan(1/2*d*x+1/2*c)^6+2/5/tan(1/2*d*x+1/2*c)^5+3/2/tan(1/2*d*x+
1/2*c)^4-14/3/tan(1/2*d*x+1/2*c)^3-15/2/tan(1/2*d*x+1/2*c)^2+44/tan(1/2*d*x
+1/2*c)-20*ln(tan(1/2*d*x+1/2*c))+128*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(130) = 260.
time = 0.50, size = 298, normalized size = 2.10

$$-\frac{\frac{\frac{1320\sin(dx+c)}{\cos(dx+c)+1}-\frac{225\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{140\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{12\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a}-\frac{3840\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{600\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{140\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{225\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1320\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5\right)(\cos(dx+c)+1)^6}{a\sin(dx+c)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/1920*((1320*sin(d*x + c)/(cos(d*x + c) + 1) - 225*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 12*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6)/a - 3840*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a + 600*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (12*sin(d*x + c)/(cos(d
*x + c) + 1) + 45*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 140*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3 - 225*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1320*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c) + 1)^6/(a*sin(d*x + c)^6)
)/d
```

Fricas [A]
time = 0.40, size = 236, normalized size = 1.66

$$\frac{480\,dx\cos(dx+c)^6-1440\,dx\cos(dx+c)^4+330\cos(dx+c)^5+1440\,dx\cos(dx+c)^2-400\cos(dx+c)^3-480\,dx+75\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-75\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-32\left(23\cos(dx+c)^5-35\cos(dx+c)^3+15\cos(dx+c)\right)\sin(dx+c)+150\cos(dx+c)}{480\left(ad\cos(dx+c)^6-3ad\cos(dx+c)^4+3ad\cos(dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(480*d*x*cos(d*x + c)^6 - 1440*d*x*cos(d*x + c)^4 + 330*cos(d*x + c)^
5 + 1440*d*x*cos(d*x + c)^2 - 400*cos(d*x + c)^3 - 480*d*x + 75*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
 - 75*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*c
os(d*x + c) + 1/2) - 32*(23*cos(d*x + c)^5 - 35*cos(d*x + c)^3 + 15*cos(d*x
 + c))*sin(d*x + c) + 150*cos(d*x + c))/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x
 + c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 224, normalized size = 1.58

$$\frac{\frac{1920\,(dx+c)}{a}-\frac{600\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{5\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-12\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-45\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+140\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+225\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1320\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}+\frac{1470\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-225\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-140\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(1920*(d*x + c)/a - 600*log(abs(tan(1/2*d*x + 1/2*c)))/a + (5*a^5*ta
n(1/2*d*x + 1/2*c)^6 - 12*a^5*tan(1/2*d*x + 1/2*c)^5 - 45*a^5*tan(1/2*d*x +
 1/2*c)^4 + 140*a^5*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(1/2*d*x + 1/2*c)^2
 - 1320*a^5*tan(1/2*d*x + 1/2*c))/a^6 + (1470*tan(1/2*d*x + 1/2*c)^6 + 1320
*tan(1/2*d*x + 1/2*c)^5 - 225*tan(1/2*d*x + 1/2*c)^4 - 140*tan(1/2*d*x + 1/
2*c)^3 + 45*tan(1/2*d*x + 1/2*c)^2 + 12*tan(1/2*d*x + 1/2*c) - 5)/(a*tan(1/
2*d*x + 1/2*c)^6))/d
```

Mupad [B]
time = 10.39, size = 413, normalized size = 2.91

$$-\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-12\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-225\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+1320\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7-1320\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+225\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3-45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3840\,\mathrm{atan}\left(\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+16\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+600\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{1920\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 12*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 12*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 45*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 140*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^9 - 225*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + 1320*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 1320*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2)^5 + 225*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2 +
(d*x)/2)^9*sin(c/2 + (d*x)/2)^3 - 45*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/
2)^2 + 3840*atan((16*cos(c/2 + (d*x)/2) - 5*sin(c/2 + (d*x)/2))/(5*cos(c/2
+ (d*x)/2) + 16*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2
)^6 + 600*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^6)/(1920*a*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.717 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)}{6a}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d+5/16*cot(d*x+c)*csc(d*x+
c)/a/d-5/24*cot(d*x+c)^3*csc(d*x+c)/a/d+1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}+\frac{\text{Subst}\left(\int x^6\,dx,x,-\right.}{ad}\\ &= -\frac{\cot^7(c+dx)}{7ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot}{}\\ &= -\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c}{}\\ &= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+}{}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 284 vs. 2(106) = 212.
time = 0.68, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\tfrac{1}{2}(c+dx)\right)+\sec\left(\tfrac{1}{2}(c+dx)\right)\right)^2\left(1680\cos(c+dx)+1008\cos(3(c+dx))+336\cos(5(c+dx))+48\cos(7(c+dx))+3675\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(c+dx)-3675\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(c+dx)-1190\sin(2(c+dx))-2205\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+2205\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+392\sin(4(c+dx))+735\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-735\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-462\sin(6(c+dx))-105\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))+105\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))\right)}{86016ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/86016*(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[
c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x
)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*S
in[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c +
 d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)]
 + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*S
in[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c
 + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(a*d*(1 + Sin[c + d
*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(96) = 192$.

time = 0.32, size = 200, normalized size = 1.89

method	result
risch	$-\frac{-336i\,e^{12i(dx+c)}+231\,e^{13i(dx+c)}-196\,e^{11i(dx+c)}-1680i\,e^{8i(dx+c)}+595\,e^{9i(dx+c)}-1008i\,e^{4i(dx+c)}-595\,e^{5i(dx+c)}+196\,e}{168ad\left(e^{2i(dx+c)}-1\right)^{7}}$
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan\left(\frac{d}{2}\right.}{}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan\left(\frac{d}{2}\right.}{}$
norman	$\frac{-\frac{1}{896ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{672ad}+\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{96ad}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64ad}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{5\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\right.\right.}{32ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a*(1/7*tan(1/2*d*x+1/2*c)^7-1/3*tan(1/2*d*x+1/2*c)^6-tan(1/2*d*x+1/
2*c)^5+3*tan(1/2*d*x+1/2*c)^4+3*tan(1/2*d*x+1/2*c)^3-15*tan(1/2*d*x+1/2*c)^
2-5*tan(1/2*d*x+1/2*c)-3/tan(1/2*d*x+1/2*c)^4+15/tan(1/2*d*x+1/2*c)^2-1/7/t
an(1/2*d*x+1/2*c)^7+1/3/tan(1/2*d*x+1/2*c)^6+40*ln(tan(1/2*d*x+1/2*c))+5/ta
n(1/2*d*x+1/2*c)-3/tan(1/2*d*x+1/2*c)^3+1/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(96) = 192$.

time = 0.30, size = 315, normalized size = 2.97

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{315\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{63\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{63\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{7\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{3\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{63\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{63\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{315\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{105\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-3\right)(\cos(dx+c)+1)^{7}}{a\sin(dx+c)^{7}}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(96) = 192$.
time = 0.39, size = 198, normalized size = 1.87

$$\frac{96 \cos\left(dx+c\right)^{7} - 105 \left(\cos\left(dx+c\right)^{6} - 3 \cos\left(dx+c\right)^{4} + 3 \cos\left(dx+c\right)^{2} - 1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) \sin\left(dx+c\right) + 105 \left(\cos\left(dx+c\right)^{6} - 3 \cos\left(dx+c\right)^{4} + 3 \cos\left(dx+c\right)^{2} - 1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right) + \frac{1}{2}\right) \sin\left(dx+c\right) - 14 \left(33 \cos\left(dx+c\right)^{5} - 40 \cos\left(dx+c\right)^{3} + 15 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{672 \left(ad \cos\left(dx+c\right)^{6} - 3 \, ad \cos\left(dx+c\right)^{4} + 3 \, ad \cos\left(dx+c\right)^{2} - ad\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(96) = 192$.
time = 0.49, size = 244, normalized size = 2.30

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)\right|\right)}{a} + \frac{3 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{7} - 7 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{6} - 21 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{5} + 63 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{4} + 63 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{3} - 315 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{2} - 105 \, a^{6} \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)}{a^{7}} - \frac{2178 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{7} - 105 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{6} - 315 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{5} + 63 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{4} + 63 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{3} - 21 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{2} - 7 \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right) + 3}{a \tan\left(\frac{1}{2} \, dx + \frac{1}{2} \, c\right)^{7}}}{2688 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

Mupad [**B**]
time = 10.62, size = 387, normalized size = 3.65

$$\frac{3 \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 3 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} - 7 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right) \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} + 7 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 21 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 63 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} + 63 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - 315 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9} - 105 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 105 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 315 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5} - 63 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - 63 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3} + 21 \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} + 840 \ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2688\,a\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7} \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/2)*s
in(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 21*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^11 + 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c/2 +
 (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 105*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)
/2)^8 + 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^5 - 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
- 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^12*
sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c
/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a*d*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)
```

3.718 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{128ad}-\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}+\frac{5\cot^3(c+dx)}{48}$$

```
[Out] 5/128*arctanh(cos(d*x+c))/a/d+1/7*cot(d*x+c)^7/a/d+5/128*cot(d*x+c)*csc(d*x
+c)/a/d-5/64*cot(d*x+c)*csc(d*x+c)^3/a/d+5/48*cot(d*x+c)^3*csc(d*x+c)^3/a/d
-1/8*cot(d*x+c)^5*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2918, 2691, 3853, 3855, 2687, 30}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}+\frac{5\cot(c+dx)\csc(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Cos[c + d*x]])/(128*a*d) + Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(128*a*d) - (5*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a*d) + (5
*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^3)/
(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^8(c+dx)\csc(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a}\\
&= -\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a}-\\
&= \frac{\cot^7(c+dx)}{7ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}+\\
&= \frac{\cot^7(c+dx)}{7ad}-\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}+\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}-\\
&= \frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{128ad}-\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}+\\
&= \frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{128ad}-
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 291 vs. 2(134) = 268.
time = 1.07, size = 291, normalized size = 2.17

$$\frac{\csc^8(c+dx)\left(-24710\cos(c+dx)-12530\cos(3(c+dx))-5558\cos(5(c+dx))-210\cos(7(c+dx))+3675\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-5880\cos(2(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2940\cos(4(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-840\cos(6(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+105\cos(8(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3675\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+5880\cos(2(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2940\cos(4(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+840\cos(6(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-105\cos(8(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+5376\sin(2(c+dx))+5376\sin(4(c+dx))+2304\sin(6(c+dx))+384\sin(8(c+dx))\right)}{344064ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-24710*Cos[c + d*x] - 12530*Cos[3*(c + d*x)] - 5558*Cos[5*
(c + d*x)] - 210*Cos[7*(c + d*x)] + 3675*Log[Cos[(c + d*x)/2]] - 5880*Cos[2
*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2940*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)
/2]] - 840*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 105*Cos[8*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] - 3675*Log[Sin[(c + d*x)/2]] + 5880*Cos[2*(c + d*x)]*Lo
g[Sin[(c + d*x)/2]] - 2940*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 840*Cos
[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[8*(c + d*x)]*Log[Sin[(c + d*x
)/2]] + 5376*Sin[2*(c + d*x)] + 5376*Sin[4*(c + d*x)] + 2304*Sin[6*(c + d*x
)] + 384*Sin[8*(c + d*x)]))/(344064*a*d)
```

Maple [A]
time = 0.30, size = 226, normalized size = 1.69

method	result
derivativedivides	$\frac{\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)-6\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tan^{2}\left(\frac{d}{2}\right.\right.}{}$
default	$\frac{\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)-6\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tan^{2}\left(\frac{d}{2}\right.\right.}{}$
risch	$-\frac{-8064ie^{4i(dx+c)}+105\,e^{15i(dx+c)}+8064ie^{6i(dx+c)}+2779\,e^{13i(dx+c)}-2688ie^{12i(dx+c)}+6265\,e^{11i(dx+c)}+384ie^{2i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d/a*(1/8*tan(1/2*d*x+1/2*c)^8-2/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*d*
x+1/2*c)^6+2*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4-6*tan(1/2*d*x+1/2*c)
^3+2*tan(1/2*d*x+1/2*c)^2+10*tan(1/2*d*x+1/2*c)+6/tan(1/2*d*x+1/2*c)^3-1/ta
n(1/2*d*x+1/2*c)^4-2/tan(1/2*d*x+1/2*c)^2+2/7/tan(1/2*d*x+1/2*c)^7+2/3/tan(
1/2*d*x+1/2*c)^6-10*ln(tan(1/2*d*x+1/2*c))-10/tan(1/2*d*x+1/2*c)-1/8/tan(1/
2*d*x+1/2*c)^8-2/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(122) = 244$.
time = 0.28, size = 354, normalized size = 2.64

$$\frac{\frac{\frac{1680\sin(dx+c)}{\cos(dx+c)+1}+\frac{336\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1008\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{168\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{336\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{112\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{48\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{21\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a}-\frac{1680\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{48\sin(dx+c)}{\cos(dx+c)+1}+\frac{112\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{336\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{168\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{1680\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-21\right)(\cos(dx+c)+1)^8}{a\sin(dx+c)^8}}{43008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/43008*((1680*sin(d*x + c)/(cos(d*x + c) + 1) + 336*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1008*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 168*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 336*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 112*sin
```

```
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 48*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 +
 21*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a - 1680*log(sin(d*x + c)/(cos(d*x
 + c) + 1))/a + (48*sin(d*x + c)/(cos(d*x + c) + 1) + 112*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 168*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 33
6*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1680*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 21)*(cos(d*x + c) + 1)^8/(a*sin(d*x + c)^8))/d
```

Fricas [**A**]
time = 0.40, size = 216, normalized size = 1.61

$$\frac{768 \cos(dx+c)^7 \sin(dx+c) - 210 \cos(dx+c)^7 - 1022 \cos(dx+c)^5 + 770 \cos(dx+c)^3 + 105 \left(\cos(dx+c)^8 - 4 \cos(dx+c)^6 + 6 \cos(dx+c)^4 - 4 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 105 \left(\cos(dx+c)^8 - 4 \cos(dx+c)^6 + 6 \cos(dx+c)^4 - 4 \cos(dx+c)^2 + 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 210 \cos(dx+c)}{5376 \left(ad \cos(dx+c)^8 - 4 ad \cos(dx+c)^6 + 6 ad \cos(dx+c)^4 - 4 ad \cos(dx+c)^2 + ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/5376*(768*cos(d*x + c)^7*sin(d*x + c) - 210*cos(d*x + c)^7 - 1022*cos(d*x
 + c)^5 + 770*cos(d*x + c)^3 + 105*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*c
os(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) - 105*(co
s(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*
log(-1/2*cos(d*x + c) + 1/2) - 210*cos(d*x + c))/(a*d*cos(d*x + c)^8 - 4*a*
d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(122) = 244.
time = 0.49, size = 274, normalized size = 2.04

$$-\frac{\frac{1680 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a} - \frac{21 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 - 48 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 112 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 336 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 168 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 1008 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 336 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1680 a^7 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^8} - \frac{4566 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8 - 1680 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^7 - 336 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 1008 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 - 168 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 - 336 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 112 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 48 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 21}{a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^8}}{43008 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/43008*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a - (21*a^7*tan(1/2*d*x + 1/2
*c)^8 - 48*a^7*tan(1/2*d*x + 1/2*c)^7 - 112*a^7*tan(1/2*d*x + 1/2*c)^6 + 33
6*a^7*tan(1/2*d*x + 1/2*c)^5 + 168*a^7*tan(1/2*d*x + 1/2*c)^4 - 1008*a^7*ta
n(1/2*d*x + 1/2*c)^3 + 336*a^7*tan(1/2*d*x + 1/2*c)^2 + 1680*a^7*tan(1/2*d*
```

```
x + 1/2*c))/a^8 - (4566*tan(1/2*d*x + 1/2*c)^8 - 1680*tan(1/2*d*x + 1/2*c)^
7 - 336*tan(1/2*d*x + 1/2*c)^6 + 1008*tan(1/2*d*x + 1/2*c)^5 - 168*tan(1/2*
d*x + 1/2*c)^4 - 336*tan(1/2*d*x + 1/2*c)^3 + 112*tan(1/2*d*x + 1/2*c)^2 +
48*tan(1/2*d*x + 1/2*c) - 21)/(a*tan(1/2*d*x + 1/2*c)^8))/d
```

Mupad [B]
time = 11.39, size = 435, normalized size = 3.25

$$-\frac{21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-21\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+48\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-48\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+112\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-336\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-168\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-336\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+336\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+168\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+336\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-112\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1680\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{43008\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^16 - 21*sin(c/2 + (d*x)/2)^16 + 48*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^15 - 48*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) + 11
2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 - 336*cos(c/2 + (d*x)/2)^3*sin
(c/2 + (d*x)/2)^13 - 168*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12 + 1008*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 336*cos(c/2 + (d*x)/2)^6*sin(c
/2 + (d*x)/2)^10 - 1680*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 + 1680*co
s(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 336*cos(c/2 + (d*x)/2)^10*sin(c/2
 + (d*x)/2)^6 - 1008*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 168*cos(c
/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 + 336*cos(c/2 + (d*x)/2)^13*sin(c/2 +
 (d*x)/2)^3 - 112*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 1680*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)
^8)/(43008*a*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

3.719 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{\cot^9(c+dx)}{9ad}-\frac{5\cot(c+dx)\csc(c+dx)}{128ad}+\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}$$

```
[Out] -5/128*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d-1/9*cot(d*x+c)^9/a/d-5/
128*cot(d*x+c)*csc(d*x+c)/a/d+5/64*cot(d*x+c)*csc(d*x+c)^3/a/d-5/48*cot(d*x
+c)^3*csc(d*x+c)^3/a/d+1/8*cot(d*x+c)^5*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2687, 14, 2691, 3853, 3855}

$$-\frac{\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{128ad}+\frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad}-\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad}+\frac{5\cot(c+dx)\csc^3(c+dx)}{64ad}-\frac{5\cot(c+dx)\csc(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(128*a*d) - Cot[c + d*x]^7/(7*a*d) - Cot[c + d*x
]^9/(9*a*d) - (5*Cot[c + d*x]*Csc[c + d*x])/(128*a*d) + (5*Cot[c + d*x]*Csc
[c + d*x]^3)/(64*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x]^3)/(48*a*d) + (Cot[c
 + d*x]^5*Csc[c + d*x]^3)/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^3(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a}$$
$$= \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} + \frac{5\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{8a} + \frac{\mathrm{Subst}(\int$$
$$= -\frac{5\cot^3(c+dx)\csc^3(c+dx)}{48ad} + \frac{\cot^5(c+dx)\csc^3(c+dx)}{8ad} - \frac{5\int \cot^2(c+$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} + \frac{5\cot(c+dx)\csc^3(c+dx)}{64ad} - \frac{5\cot^3(c+$$
$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} - \frac{5\cot(c+dx)\csc(c+dx)}{128ad} + \frac{5\cot(c+d$$
$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{128ad} - \frac{\cot^7(c+dx)}{7ad} - \frac{\cot^9(c+dx)}{9ad} - \frac{5\cot(c+dx)}{128a$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(152) = 304.
time = 0.94, size = 313, normalized size = 2.06

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2064384*(Csc[c + d*x]^9*(129024*Cos[c + d*x] + 75264*Cos[3*(c + d*x)] +
23040*Cos[5*(c + d*x)] + 2304*Cos[7*(c + d*x)] - 256*Cos[9*(c + d*x)] + 396
90*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 39690*Log[Sin[(c + d*x)/2]]*Sin[c +
 d*x] - 36540*Sin[2*(c + d*x)] - 26460*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 26460*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 20916*Sin[4*(c + d*x)]
+ 11340*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 11340*Log[Sin[(c + d*x)/2]
]*Sin[5*(c + d*x)] - 16044*Sin[6*(c + d*x)] - 2835*Log[Cos[(c + d*x)/2]]*Si
n[7*(c + d*x)] + 2835*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 630*Sin[8*(c
 + d*x)] + 315*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 315*Log[Sin[(c + d*
x)/2]]*Sin[9*(c + d*x)]))/(a*d)
```

Maple [A]
time = 0.35, size = 228, normalized size = 1.50

method	result
derivativedivides	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^{2}\left(\frac{dx}{2}\right.\right.}{}$
default	$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^{2}\left(\frac{dx}{2}\right.\right.}{}$
risch	$\frac{315\,e^{17i(dx+c)}-6912ie^{4i(dx+c)}+8022\,e^{15i(dx+c)}-48384ie^{6i(dx+c)}+10458\,e^{13i(dx+c)}-26880ie^{12i(dx+c)}+18270\,e^{11i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/512/d/a*(1/9*tan(1/2*d*x+1/2*c)^9-1/4*tan(1/2*d*x+1/2*c)^8-3/7*tan(1/2*d*
x+1/2*c)^7+4/3*tan(1/2*d*x+1/2*c)^6-2*tan(1/2*d*x+1/2*c)^4+8/3*tan(1/2*d*x+
1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2-6*tan(1/2*d*x+1/2*c)+2/tan(1/2*d*x+1/2*c)^4
+4/tan(1/2*d*x+1/2*c)^2+6/tan(1/2*d*x+1/2*c)-4/3/tan(1/2*d*x+1/2*c)^6+20*ln
(tan(1/2*d*x+1/2*c))+1/4/tan(1/2*d*x+1/2*c)^8+3/7/tan(1/2*d*x+1/2*c)^7-8/3/
tan(1/2*d*x+1/2*c)^3-1/9/tan(1/2*d*x+1/2*c)^9)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(138) = 276$.
time = 0.28, size = 355, normalized size = 2.34

$$-\frac{\frac{\frac{1512\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1008\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{672\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{504\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{336\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{108\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{63\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{28\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}-\frac{5040\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{63\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{108\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{336\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{504\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{672\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1008\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1512\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-28\right)(\cos(dx+c)+1)^9}{a\sin(dx+c)^9}}{129024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/129024*((1512*sin(d*x + c)/(cos(d*x + c) + 1) + 1008*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 108*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 + 63*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 - 28*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a - 5040*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a - (63*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 336*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 504*sin(d*
x + c)^5/(cos(d*x + c) + 1)^5 - 672*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1
008*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1512*sin(d*x + c)^8/(cos(d*x + c)
 + 1)^8 - 28)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c)^9))/d
```

Fricas [**A**]
time = 0.39, size = 249, normalized size = 1.64

$$\frac{512\,\cos\left(dx+c\right)^{9}-2304\,\cos\left(dx+c\right)^{7}-315\,\left(\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+315\,\left(\cos\left(dx+c\right)^{8}-4\,\cos\left(dx+c\right)^{6}+6\,\cos\left(dx+c\right)^{4}-4\,\cos\left(dx+c\right)^{2}+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+42\,\left(15\,\cos\left(dx+c\right)^{7}+73\,\cos\left(dx+c\right)^{5}-55\,\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16128\,\left(ad\cos\left(dx+c\right)^{8}-4\,ad\cos\left(dx+c\right)^{6}+6\,ad\cos\left(dx+c\right)^{4}-4\,ad\cos\left(dx+c\right)^{2}+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/16128*(512*cos(d*x + c)^9 - 2304*cos(d*x + c)^7 - 315*(cos(d*x + c)^8 - 4
*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 315*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d
*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 + 42*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d
*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*
cos(d*x + c)^4 - 4*a*d*cos(d*x + c)^2 + a*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 273, normalized size = 1.80

$$\frac{\frac{5040\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{28\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-63\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-108\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+336\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-504\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+672\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1008\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1512\,a^{8}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{9}}-\frac{14258\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1512\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1008\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+672\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-504\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+336\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-108\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+28}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}}}{129024\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/129024*(5040*log(abs(tan(1/2*d*x + 1/2*c)))/a + (28*a^8*tan(1/2*d*x + 1/2
*c)^9 - 63*a^8*tan(1/2*d*x + 1/2*c)^8 - 108*a^8*tan(1/2*d*x + 1/2*c)^7 + 33
6*a^8*tan(1/2*d*x + 1/2*c)^6 - 504*a^8*tan(1/2*d*x + 1/2*c)^4 + 672*a^8*tan
(1/2*d*x + 1/2*c)^3 - 1008*a^8*tan(1/2*d*x + 1/2*c)^2 - 1512*a^8*tan(1/2*d*
x + 1/2*c))/a^9 - (14258*tan(1/2*d*x + 1/2*c)^9 - 1512*tan(1/2*d*x + 1/2*c)
^8 - 1008*tan(1/2*d*x + 1/2*c)^7 + 672*tan(1/2*d*x + 1/2*c)^6 - 504*tan(1/2
*d*x + 1/2*c)^5 + 336*tan(1/2*d*x + 1/2*c)^3 - 108*tan(1/2*d*x + 1/2*c)^2 -
 63*tan(1/2*d*x + 1/2*c) + 28)/(a*tan(1/2*d*x + 1/2*c)^9))/d
```

Mupad [B]
time = 12.55, size = 435, normalized size = 2.86

$$\frac{28\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-28\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}+63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-108\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+336\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}-504\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+672\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-1008\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-1512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+1512\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+1008\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-672\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+504\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-336\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+108\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+5040\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{129024\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^10*(a + a*sin(c + d*x))),x)
```

```
[Out] (28*sin(c/2 + (d*x)/2)^18 - 28*cos(c/2 + (d*x)/2)^18 - 63*cos(c/2 + (d*x)/2
)*sin(c/2 + (d*x)/2)^17 + 63*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2) - 108
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 + 336*cos(c/2 + (d*x)/2)^3*sin(
c/2 + (d*x)/2)^15 - 504*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13 + 672*co
s(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 - 1008*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^11 - 1512*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 1512*co
s(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^8 + 1008*cos(c/2 + (d*x)/2)^11*sin(c
/2 + (d*x)/2)^7 - 672*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 504*cos(
c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^5 - 336*cos(c/2 + (d*x)/2)^15*sin(c/2
+ (d*x)/2)^3 + 108*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 5040*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2
)^9)/(129024*a*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

3.720 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot(c+dx)\csc(c+dx)}{256ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{128ad}-$$

```
[Out] 3/256*arctanh(cos(d*x+c))/a/d+1/7*cot(d*x+c)^7/a/d+1/9*cot(d*x+c)^9/a/d+3/2
56*cot(d*x+c)*csc(d*x+c)/a/d+1/128*cot(d*x+c)*csc(d*x+c)^3/a/d-1/32*cot(d*x
+c)*csc(d*x+c)^5/a/d+1/16*cot(d*x+c)^3*csc(d*x+c)^5/a/d-1/10*cot(d*x+c)^5*c
sc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2691, 3853, 3855, 2687, 14}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{\cot^7(c+dx)}{7ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}-\frac{\cot(c+dx)\csc^5(c+dx)}{32ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{128ad}+\frac{3\cot(c+dx)\csc(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(256*a*d) + Cot[c + d*x]^7/(7*a*d) + Cot[c + d*x]
^9/(9*a*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(256*a*d) + (Cot[c + d*x]*Csc[c
+ d*x]^3)/(128*a*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(32*a*d) + (Cot[c + d*x
]^3*Csc[c + d*x]^5)/(16*a*d) - (Cot[c + d*x]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^4(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a}$$

$$= -\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} - \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a} - \frac{\text{Subst}\big(\int x$$

$$= \frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad} - \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} + \frac{3\int \cot^2(c+dx)\,c}{16a}$$

$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} - \frac{\cot(c+dx)\csc^5(c+dx)}{32ad} + \frac{\cot^3(c+dx)\,cs}{16ad}$$

$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{\cot(c+dx)\csc^3(c+dx)}{128ad} - \frac{\cot(c+dx)\,csc}{32ad}$$

$$= \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{3\cot(c+dx)\csc(c+dx)}{256ad} + \frac{\cot(c+dx)\,csc}{128ac}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{256ad} + \frac{\cot^7(c+dx)}{7ad} + \frac{\cot^9(c+dx)}{9ad} + \frac{3\cot(c+dx)\csc(}{256ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 386 vs. 2(176) = 352.
time = 1.07, size = 386, normalized size = 2.19

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/165150720*(Csc[c + d*x]^9*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(23675
40*Cos[c + d*x] + 1307880*Cos[3*(c + d*x)] + 436968*Cos[5*(c + d*x)] + 1827
0*Cos[7*(c + d*x)] - 1890*Cos[9*(c + d*x)] - 119070*Log[Cos[(c + d*x)/2]] +
 198450*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 113400*Cos[4*(c + d*x)]*Lo
g[Cos[(c + d*x)/2]] + 42525*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 9450*C
os[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 945*Cos[10*(c + d*x)]*Log[Cos[(c +
d*x)/2]] + 119070*Log[Sin[(c + d*x)/2]] - 198450*Cos[2*(c + d*x)]*Log[Sin[(
c + d*x)/2]] + 113400*Cos[4*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 42525*Cos[6*
(c + d*x)]*Log[Sin[(c + d*x)/2]] + 9450*Cos[8*(c + d*x)]*Log[Sin[(c + d*x)/
2]] - 945*Cos[10*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 537600*Sin[2*(c + d*x)]
 - 522240*Sin[4*(c + d*x)] - 207360*Sin[6*(c + d*x)] - 25600*Sin[8*(c + d*x
)] + 2560*Sin[10*(c + d*x)]))/(a*d*(1 + Csc[c + d*x]))
```

Maple [A]
time = 0.40, size = 252, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10}-\frac{2\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16\left(\tan^{3}\left(\right.\right.}{3}}{}$
default	$\frac{\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10}-\frac{2\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16\left(\tan^{3}\left(\right.\right.}{3}}{}$
risch	$-\frac{945\,e^{19i(dx+c)}+46080ie^{4i(dx+c)}-9135\,e^{17i(dx+c)}+414720ie^{6i(dx+c)}-218484\,e^{15i(dx+c)}-537600ie^{12i(dx+c)}-653940}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024/d/a*(1/10*tan(1/2*d*x+1/2*c)^10-2/9*tan(1/2*d*x+1/2*c)^9-1/4*tan(1/2
*d*x+1/2*c)^8+6/7*tan(1/2*d*x+1/2*c)^7-1/2*tan(1/2*d*x+1/2*c)^6+2*tan(1/2*d
*x+1/2*c)^4-16/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d*x+1/2*c)^2+12*tan(1/2*d*x+1
/2*c)-12/tan(1/2*d*x+1/2*c)-2/tan(1/2*d*x+1/2*c)^4+16/3/tan(1/2*d*x+1/2*c)^
3-1/tan(1/2*d*x+1/2*c)^2+1/2/tan(1/2*d*x+1/2*c)^6-12*ln(tan(1/2*d*x+1/2*c))
-6/7/tan(1/2*d*x+1/2*c)^7+1/4/tan(1/2*d*x+1/2*c)^8-1/10/tan(1/2*d*x+1/2*c)^
10+2/9/tan(1/2*d*x+1/2*c)^9)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(160) = 320$.
time = 0.29, size = 394, normalized size = 2.24

$$\frac{\frac{\frac{15120 \sin(dx+c)}{\cos(dx+c)+1}+\frac{1260 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6720 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{630 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1080 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{280 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{126 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}{a}-\frac{15120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{280 \sin(dx+c)}{\cos(dx+c)+1}+\frac{315 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1080 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{630 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2520 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{6720 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{1260 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15120 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-126\right)(\cos(dx+c)+1)^{10}}{a \sin(dx+c)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/1290240*((15120*sin(d*x + c)/(cos(d*x + c) + 1) + 1260*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10
80*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) +
 1)^8 - 280*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 126*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10)/a - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (280*s
in(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
1080*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - 2520*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 6720*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 15120*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(a*sin(d*x + c)
^10))/d
```

Fricas [**A**]
time = 0.40, size = 272, normalized size = 1.55

$$-\frac{1890\,\cos\left(dx+c\right)^{9}-8820\,\cos\left(dx+c\right)^{7}-16128\,\cos\left(dx+c\right)^{5}+8820\,\cos\left(dx+c\right)^{3}-945\,\left(\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+945\,\left(\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2560\,\left(2\,\cos\left(dx+c\right)^{9}-9\,\cos\left(dx+c\right)^{7}\right)\sin\left(dx+c\right)-1890\,\cos\left(dx+c\right)}{161280\,\left(ad\cos\left(dx+c\right)^{10}-5\,ad\cos\left(dx+c\right)^{8}+10\,ad\cos\left(dx+c\right)^{6}-10\,ad\cos\left(dx+c\right)^{4}+5\,ad\cos\left(dx+c\right)^{2}-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/161280*(1890*cos(d*x + c)^9 - 8820*cos(d*x + c)^7 - 16128*cos(d*x + c)^5
 + 8820*cos(d*x + c)^3 - 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d
*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c)
+ 1/2) + 945*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 2560*(
2*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*sin(d*x + c) - 1890*cos(d*x + c))/(a*d
*cos(d*x + c)^10 - 5*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*co
s(d*x + c)^4 + 5*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 303, normalized size = 1.72

$$-\frac{\frac{15120\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{126\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-280\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-315\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1080\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-630\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+2520\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-6720\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1260\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15120\,a^{9}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}-\frac{44286\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-15120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1260\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2520\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+630\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1080\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-126}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1290240*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (126*a^9*tan(1/2*d*x +
 1/2*c)^10 - 280*a^9*tan(1/2*d*x + 1/2*c)^9 - 315*a^9*tan(1/2*d*x + 1/2*c)^
8 + 1080*a^9*tan(1/2*d*x + 1/2*c)^7 - 630*a^9*tan(1/2*d*x + 1/2*c)^6 + 2520
*a^9*tan(1/2*d*x + 1/2*c)^4 - 6720*a^9*tan(1/2*d*x + 1/2*c)^3 + 1260*a^9*ta
n(1/2*d*x + 1/2*c)^2 + 15120*a^9*tan(1/2*d*x + 1/2*c))/a^10 - (44286*tan(1/
2*d*x + 1/2*c)^10 - 15120*tan(1/2*d*x + 1/2*c)^9 - 1260*tan(1/2*d*x + 1/2*c
)^8 + 6720*tan(1/2*d*x + 1/2*c)^7 - 2520*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1
/2*d*x + 1/2*c)^4 - 1080*tan(1/2*d*x + 1/2*c)^3 + 315*tan(1/2*d*x + 1/2*c)^
2 + 280*tan(1/2*d*x + 1/2*c) - 126)/(a*tan(1/2*d*x + 1/2*c)^10))/d
```

Mupad [B]
time = 14.00, size = 483, normalized size = 2.74

$$-\frac{126\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}-126\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}-280\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-1080\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}+630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-1260\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-15120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+15120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+1260\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+1080\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+15120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{1290240\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^11*(a + a*sin(c + d*x))),x)
```

```
[Out] -(126*cos(c/2 + (d*x)/2)^20 - 126*sin(c/2 + (d*x)/2)^20 + 280*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^19 - 280*cos(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2)
+ 315*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^18 - 1080*cos(c/2 + (d*x)/2)^
3*sin(c/2 + (d*x)/2)^17 + 630*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^16 -
2520*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 + 6720*cos(c/2 + (d*x)/2)^7
*sin(c/2 + (d*x)/2)^13 - 1260*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 -
15120*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^11 + 15120*cos(c/2 + (d*x)/2)
^11*sin(c/2 + (d*x)/2)^9 + 1260*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^8
- 6720*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^7 + 2520*cos(c/2 + (d*x)/2)
^14*sin(c/2 + (d*x)/2)^6 - 630*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 +
 1080*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2)^3 - 315*cos(c/2 + (d*x)/2)^1
8*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)/(1290240*a*d*cos(c/2 + (d*x)/2)
^10*sin(c/2 + (d*x)/2)^10)
```

3.721 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^{11}(c+dx)}{11ad}-\frac{3\cot(c+dx)\csc(c+dx)}{256ad}-\cot(c+d$$

```
[Out] -3/256*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d-2/9*cot(d*x+c)^9/a/d-1/
11*cot(d*x+c)^11/a/d-3/256*cot(d*x+c)*csc(d*x+c)/a/d-1/128*cot(d*x+c)*csc(d
*x+c)^3/a/d+1/32*cot(d*x+c)*csc(d*x+c)^5/a/d-1/16*cot(d*x+c)^3*csc(d*x+c)^5
/a/d+1/10*cot(d*x+c)^5*csc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2918, 2687, 276, 2691, 3853, 3855}

$$-\frac{\cot^{11}(c+dx)}{11ad}-\frac{2\cot^9(c+dx)}{9ad}-\frac{\cot^7(c+dx)}{7ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{256ad}+\frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{32ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{128ad}-\frac{3\cot(c+dx)\csc(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(256*a*d) - Cot[c + d*x]^7/(7*a*d) - (2*Cot[c +
d*x]^9)/(9*a*d) - Cot[c + d*x]^11/(11*a*d) - (3*Cot[c + d*x]*Csc[c + d*x])/
(256*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(128*a*d) + (Cot[c + d*x]*Csc[c +
 d*x]^5)/(32*a*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5)/(16*a*d) + (Cot[c + d*x
]^5*Csc[c + d*x]^5)/(10*a*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\int \cot^6(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot^6(c+dx)\csc^6(c+dx)\,dx}{a}$$

$$= \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} + \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x\right.}{}$$

$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{16ad} + \frac{\cot^5(c+dx)\csc^5(c+dx)}{10ad} - \frac{3\int \cot^2(c+d}{}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} + \frac{\cot(c+dx)\csc^5(c+}{32ad}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{\cot(c+dx)\csc^3(c+}{128ad}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx)}{11ad} - \frac{3\cot(c+dx)\csc(c+}{256ad}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{256ad} - \frac{\cot^7(c+dx)}{7ad} - \frac{2\cot^9(c+dx)}{9ad} - \frac{\cot^{11}(c+dx}{11ad}$$

Mathematica [A]

time = 2.04, size = 187, normalized size = 0.96

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\cot(c+dx)\csc^{10}(c+dx)(6840320+9973760\cos(2(c+dx))+3543040\cos(4(c+dx))+343040\cos(6(c+dx))-61440\cos(8(c+dx))+5120\cos(10(c+dx))-3219678\sin(c+dx)-2608452\sin(3(c+dx))-2181564\sin(5(c+dx))-121275\sin(7(c+dx))+10395\sin(9(c+dx)))\right)}{227082240ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(-2661120*(Log[Cos[(c + d*x)/2]] -
 Log[Sin[(c + d*x)/2]]) - Cot[c + d*x]*Csc[c + d*x]^10*(6840320 + 9973760*C
os[2*(c + d*x)] + 3543040*Cos[4*(c + d*x)] + 343040*Cos[6*(c + d*x)] - 6144
0*Cos[8*(c + d*x)] + 5120*Cos[10*(c + d*x)] - 3219678*Sin[c + d*x] - 260845
2*Sin[3*(c + d*x)] - 2181564*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 1
0395*Sin[9*(c + d*x)])))/(227082240*a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.43, size = 302, normalized size = 1.56

method	result
risch	$\frac{10395\,e^{21i(dx+c)}-110880\,e^{19i(dx+c)}+563200ie^{4i(dx+c)}-2302839\,e^{17i(dx+c)}+15206400ie^{8i(dx+c)}-4790016\,e^{15i(dx+c)}+3}{}$
derivativedivides	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)-}{}$
default	$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2048/d/a*(1/11*tan(1/2*d*x+1/2*c)^11-1/5*tan(1/2*d*x+1/2*c)^10-1/9*tan(1/
2*d*x+1/2*c)^9+1/2*tan(1/2*d*x+1/2*c)^8-5/7*tan(1/2*d*x+1/2*c)^7+tan(1/2*d*
x+1/2*c)^6+tan(1/2*d*x+1/2*c)^5-4*tan(1/2*d*x+1/2*c)^4+10/3*tan(1/2*d*x+1/2
*c)^3-2*tan(1/2*d*x+1/2*c)^2-10*tan(1/2*d*x+1/2*c)+4/tan(1/2*d*x+1/2*c)^4+1
0/tan(1/2*d*x+1/2*c)+2/tan(1/2*d*x+1/2*c)^2-1/2/tan(1/2*d*x+1/2*c)^8-1/tan(
1/2*d*x+1/2*c)^6+24*ln(tan(1/2*d*x+1/2*c))-1/11/tan(1/2*d*x+1/2*c)^11-10/3/
tan(1/2*d*x+1/2*c)^3+1/9/tan(1/2*d*x+1/2*c)^9+1/5/tan(1/2*d*x+1/2*c)^10+5/7
/tan(1/2*d*x+1/2*c)^7-1/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(176) = 352$.
time = 0.28, size = 475, normalized size = 2.45

$$-\frac{\frac{\frac{69300\sin(dx+c)}{\cos(dx+c)+1}+\frac{13860\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{23100\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{27720\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6930\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6930\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4950\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3465\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{770\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1386\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{630\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}-\frac{166320\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{1386\sin(dx+c)}{\cos(dx+c)+1}+\frac{770\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3465\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4950\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6930\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6930\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{27720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{23100\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{13860\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{69300\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-630\right)(\cos(dx+c)+1)^{11}}{a\sin(dx+c)^{11}}}{14192640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/14192640*((69300*sin(d*x + c)/(cos(d*x + c) + 1) + 13860*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 23100*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 - 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4950*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 770*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11)/a - 166320*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a - (1386*sin(d*x + c)/(cos(d*x + c) + 1) + 770*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4950*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 27720*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 - 23100*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 + 69300*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 630
)*(cos(d*x + c) + 1)^11/(a*sin(d*x + c)^11))/d
```

Fricas [**A**]
time = 0.40, size = 302, normalized size = 1.56

$$\frac{20480\,\cos\left(dx+c\right)^{11}-112640\,\cos\left(dx+c\right)^{9}+253440\,\cos\left(dx+c\right)^{7}-10395\,\left(\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10395\,\left(\cos\left(dx+c\right)^{10}-5\,\cos\left(dx+c\right)^{8}+10\,\cos\left(dx+c\right)^{6}-10\,\cos\left(dx+c\right)^{4}+5\,\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+1386\,\left(15\,\cos\left(dx+c\right)^{9}-70\,\cos\left(dx+c\right)^{7}-128\,\cos\left(dx+c\right)^{5}+70\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1774080\,\left(ad\cos\left(dx+c\right)^{10}-5\,ad\cos\left(dx+c\right)^{8}+10\,ad\cos\left(dx+c\right)^{6}-10\,ad\cos\left(dx+c\right)^{4}+5\,ad\cos\left(dx+c\right)^{2}-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1774080*(20480*cos(d*x + c)^11 - 112640*cos(d*x + c)^9 + 253440*cos(d*x +
 c)^7 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*
cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 10*c
os(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x + c)^5 + 7
0*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^10 - 5
*a*d*cos(d*x + c)^8 + 10*a*d*cos(d*x + c)^6 - 10*a*d*cos(d*x + c)^4 + 5*a*d
*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(176) = 352.
time = 0.50, size = 360, normalized size = 1.86

$$\frac{\frac{166320 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}+\frac{630\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1386\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-770\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+3465\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-4950\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+6930\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6930\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-27720\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+23100\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-13860\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-69300\,a^{10}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{11}}-\frac{502266\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-69300\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-13860\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+23100\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-27720\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+6930\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+6930\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-4950\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3465\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-770\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1386\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+630}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{14192640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/14192640*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a + (630*a^10*tan(1/2*d*x
 + 1/2*c)^11 - 1386*a^10*tan(1/2*d*x + 1/2*c)^10 - 770*a^10*tan(1/2*d*x + 1
/2*c)^9 + 3465*a^10*tan(1/2*d*x + 1/2*c)^8 - 4950*a^10*tan(1/2*d*x + 1/2*c)
^7 + 6930*a^10*tan(1/2*d*x + 1/2*c)^6 + 6930*a^10*tan(1/2*d*x + 1/2*c)^5 -
27720*a^10*tan(1/2*d*x + 1/2*c)^4 + 23100*a^10*tan(1/2*d*x + 1/2*c)^3 - 138
60*a^10*tan(1/2*d*x + 1/2*c)^2 - 69300*a^10*tan(1/2*d*x + 1/2*c))/a^11 - (5
02266*tan(1/2*d*x + 1/2*c)^11 - 69300*tan(1/2*d*x + 1/2*c)^10 - 13860*tan(1
/2*d*x + 1/2*c)^9 + 23100*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*
c)^7 + 6930*tan(1/2*d*x + 1/2*c)^6 + 6930*tan(1/2*d*x + 1/2*c)^5 - 4950*tan
(1/2*d*x + 1/2*c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 770*tan(1/2*d*x + 1/2*c
)^2 - 1386*tan(1/2*d*x + 1/2*c) + 630)/(a*tan(1/2*d*x + 1/2*c)^11))/d
```

Mupad [**B**]
time = 16.27, size = 579, normalized size = 2.98

$$\frac{630\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{22}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{22}-1386\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}+1386\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+3465\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}-4950\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}+6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-27720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+23100\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-13860\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-69300\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+69300\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+13860\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-23100\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+27720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+4950\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-3465\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+770\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+166320\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{14192640\,a\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^12*(a + a*sin(c + d*x))),x)
```

```
[Out] (630*sin(c/2 + (d*x)/2)^22 - 630*cos(c/2 + (d*x)/2)^22 - 1386*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^21 + 1386*cos(c/2 + (d*x)/2)^21*sin(c/2 + (d*x)/2)
 - 770*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^20 + 3465*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^19 - 4950*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^18
+ 6930*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^17 + 6930*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^16 - 27720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^15
 + 23100*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^14 - 13860*cos(c/2 + (d*x)
/2)^9*sin(c/2 + (d*x)/2)^13 - 69300*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^12 + 69300*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^10 + 13860*cos(c/2 +
(d*x)/2)^13*sin(c/2 + (d*x)/2)^9 - 23100*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d
*x)/2)^8 + 27720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^7 - 6930*cos(c/2
+ (d*x)/2)^16*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^17*sin(c/2 + (
d*x)/2)^5 + 4950*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^4 - 3465*cos(c/2
+ (d*x)/2)^19*sin(c/2 + (d*x)/2)^3 + 770*cos(c/2 + (d*x)/2)^20*sin(c/2 + (d
*x)/2)^2 + 166320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x
)/2)^11*sin(c/2 + (d*x)/2)^11)/(14192640*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2
+ (d*x)/2)^11)
```

3.722 $\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$-\frac{3x}{128a^2}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{128a^2d}-\cos^3(c$$

```
[Out] -3/128*x/a^2-2/5*cos(d*x+c)^5/a^2/d+5/7*cos(d*x+c)^7/a^2/d-4/9*cos(d*x+c)^9
/a^2/d+1/11*cos(d*x+c)^11/a^2/d-3/128*cos(d*x+c)*sin(d*x+c)/a^2/d-1/64*cos(
d*x+c)^3*sin(d*x+c)/a^2/d+1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d+1/8*cos(d*x+c)
^5*sin(d*x+c)^3/a^2/d+1/5*cos(d*x+c)^5*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2952, 2645, 276, 2648, 2715, 8}

$$\frac{\cos^{11}(c+dx)}{11a^2d}-\frac{4\cos^9(c+dx)}{9a^2d}+\frac{5\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^5(c+dx)\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{64a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{128a^2d}-\frac{3x}{128a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(128*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (5*Cos[c + d*x]^7)/(7*a^2
*d) - (4*Cos[c + d*x]^9)/(9*a^2*d) + Cos[c + d*x]^11/(11*a^2*d) - (3*Cos[c
+ d*x]*Sin[c + d*x])/(128*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(64*a^2*d)
 + (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^5(c+dx) - 2a^2\cos^4(c+dx)\sin^6(c+dx) + a^2\cos^4(\right.}{a^4}$$

$$= \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^7(c+dx)\,dx}{a^2} - \frac{2\int \mathrm{co}}{}$$

$$= \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{\mathrm{Subst}\left(\int x\right.}{}$$

$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{3\int \cos^4(c+dx)}{8a}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} + \frac{\mathrm{co}}{}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{\mathrm{co}}{}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+dx)}{11a^2d} - \frac{3}{}$$

$$= -\frac{3x}{128a^2} - \frac{2\cos^5(c+dx)}{5a^2d} + \frac{5\cos^7(c+dx)}{7a^2d} - \frac{4\cos^9(c+dx)}{9a^2d} + \frac{\cos^{11}(c+}{11a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 638 vs. $2(203) = 406$.
time = 7.66, size = 638, normalized size = 3.14

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4620*(-5 + 222*c - 36*d*x)*Cos[c/2] - 131670*Cos[c/2 + d*x] - 131670*Cos[(
3*c)/2 + d*x] + 13860*Cos[(3*c)/2 + 2*d*x] - 13860*Cos[(5*c)/2 + 2*d*x] - 2
5410*Cos[(5*c)/2 + 3*d*x] - 25410*Cos[(7*c)/2 + 3*d*x] + 27720*Cos[(7*c)/2
+ 4*d*x] - 27720*Cos[(9*c)/2 + 4*d*x] + 18711*Cos[(9*c)/2 + 5*d*x] + 18711*
Cos[(11*c)/2 + 5*d*x] - 6930*Cos[(11*c)/2 + 6*d*x] + 6930*Cos[(13*c)/2 + 6*
d*x] + 1485*Cos[(13*c)/2 + 7*d*x] + 1485*Cos[(15*c)/2 + 7*d*x] - 3465*Cos[(
15*c)/2 + 8*d*x] + 3465*Cos[(17*c)/2 + 8*d*x] - 2695*Cos[(17*c)/2 + 9*d*x]
- 2695*Cos[(19*c)/2 + 9*d*x] + 1386*Cos[(19*c)/2 + 10*d*x] - 1386*Cos[(21*c
)/2 + 10*d*x] + 315*Cos[(21*c)/2 + 11*d*x] + 315*Cos[(23*c)/2 + 11*d*x] - 6
46800*Sin[c/2] + 1025640*c*Sin[c/2] - 166320*d*x*Sin[c/2] + 131670*Sin[c/2
+ d*x] - 131670*Sin[(3*c)/2 + d*x] + 13860*Sin[(3*c)/2 + 2*d*x] + 13860*Sin
[(5*c)/2 + 2*d*x] + 25410*Sin[(5*c)/2 + 3*d*x] - 25410*Sin[(7*c)/2 + 3*d*x]
```

```
 + 27720*Sin[(7*c)/2 + 4*d*x] + 27720*Sin[(9*c)/2 + 4*d*x] - 18711*Sin[(9*c
)/2 + 5*d*x] + 18711*Sin[(11*c)/2 + 5*d*x] - 6930*Sin[(11*c)/2 + 6*d*x] - 6
930*Sin[(13*c)/2 + 6*d*x] - 1485*Sin[(13*c)/2 + 7*d*x] + 1485*Sin[(15*c)/2
+ 7*d*x] - 3465*Sin[(15*c)/2 + 8*d*x] - 3465*Sin[(17*c)/2 + 8*d*x] + 2695*S
in[(17*c)/2 + 9*d*x] - 2695*Sin[(19*c)/2 + 9*d*x] + 1386*Sin[(19*c)/2 + 10*
d*x] + 1386*Sin[(21*c)/2 + 10*d*x] - 315*Sin[(21*c)/2 + 11*d*x] + 315*Sin[(
23*c)/2 + 11*d*x])/(7096320*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.19, size = 272, normalized size = 1.34

method	result
risch	$\frac{\cos(11dx+11c)}{11264d\,a^2}-\frac{19\cos(dx+c)}{512a^2d}-\frac{3x}{128a^2}+\frac{\sin(10dx+10c)}{2560a^2d}-\frac{7\cos(9dx+9c)}{9216d\,a^2}-\frac{\sin(8dx+8c)}{1024a^2d}+\frac{3\cos(7dx+7c)}{7168d\,a^2}-$
derivativedivides	$\frac{64\left(-\frac{17}{13860}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1260}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{17\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{252}+\frac{773\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{28}-\right.}{}$
default	$\frac{64\left(-\frac{17}{13860}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1260}+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{17\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{252}+\frac{773\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20480}-\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{28}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^2*((-17/13860+3/4096*tan(1/2*d*x+1/2*c)-17/1260*tan(1/2*d*x+1/2*c)^2
+1/128*tan(1/2*d*x+1/2*c)^3-17/252*tan(1/2*d*x+1/2*c)^4+773/20480*tan(1/2*d
*x+1/2*c)^5-1/28*tan(1/2*d*x+1/2*c)^6-37/80*tan(1/2*d*x+1/2*c)^7+5/28*tan(1
/2*d*x+1/2*c)^8+1207/2048*tan(1/2*d*x+1/2*c)^9-29/20*tan(1/2*d*x+1/2*c)^10+
53/60*tan(1/2*d*x+1/2*c)^12-1207/2048*tan(1/2*d*x+1/2*c)^13-7/12*tan(1/2*d*
x+1/2*c)^14+37/80*tan(1/2*d*x+1/2*c)^15-1/6*tan(1/2*d*x+1/2*c)^16-773/20480
*tan(1/2*d*x+1/2*c)^17-1/128*tan(1/2*d*x+1/2*c)^19-3/4096*tan(1/2*d*x+1/2*c
)^21)/(1+tan(1/2*d*x+1/2*c)^2)^11-3/4096*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(183) = 366.
time = 0.51, size = 648, normalized size = 3.19

$$\frac{\frac{\frac{10395\sin(dx+c)}{\cos(dx+c)+1}-\frac{191488\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{110880\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{957440\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{535689\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{506880\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6564096\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{2534400\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{8364510\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{20579328\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{12536832\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8364510\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{8279040\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{6564096\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{2365440\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}-\frac{535689\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{110880\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}-\frac{10395\sin(dx+c)^{21}}{(\cos(dx+c)+1)^{21}}-17408}{a^2+\frac{11a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{165a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{330a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{462a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{462a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{330a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{165a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}+\frac{55a^2\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}+\frac{11a^2\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}+\frac{a^2\sin(dx+c)^{22}}{(\cos(dx+c)+1)^{22}}}-\frac{10395\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{221760\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/221760*((10395*sin(d*x + c)/(cos(d*x + c) + 1) - 191488*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 110880*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 957440*si
```

```
n(d*x + c)^4/(cos(d*x + c) + 1)^4 + 535689*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - 506880*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6564096*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 2534400*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 8364510
*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 20579328*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 + 12536832*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8364510*sin(d
*x + c)^13/(cos(d*x + c) + 1)^13 - 8279040*sin(d*x + c)^14/(cos(d*x + c) +
1)^14 + 6564096*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 2365440*sin(d*x + c
)^16/(cos(d*x + c) + 1)^16 - 535689*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 -
 110880*sin(d*x + c)^19/(cos(d*x + c) + 1)^19 - 10395*sin(d*x + c)^21/(cos(
d*x + c) + 1)^21 - 17408)/(a^2 + 11*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 55*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 165*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 330*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 462*a^2*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 + 462*a^2*sin(d*x + c)^12/(cos(d*x + c)
 + 1)^12 + 330*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 165*a^2*sin(d*x
+ c)^16/(cos(d*x + c) + 1)^16 + 55*a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^1
8 + 11*a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20 + a^2*sin(d*x + c)^22/(cos
(d*x + c) + 1)^22) - 10395*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.40, size = 110, normalized size = 0.54

$$\frac{40320\,\cos\left(dx+c\right)^{11}-197120\,\cos\left(dx+c\right)^{9}+316800\,\cos\left(dx+c\right)^{7}-177408\,\cos\left(dx+c\right)^{5}-10395\,dx+693\,\left(128\,\cos\left(dx+c\right)^{9}-336\,\cos\left(dx+c\right)^{7}+248\,\cos\left(dx+c\right)^{5}-10\,\cos\left(dx+c\right)^{3}-15\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{443520\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/443520*(40320*cos(d*x + c)^11 - 197120*cos(d*x + c)^9 + 316800*cos(d*x +
c)^7 - 177408*cos(d*x + c)^5 - 10395*d*x + 693*(128*cos(d*x + c)^9 - 336*co
s(d*x + c)^7 + 248*cos(d*x + c)^5 - 10*cos(d*x + c)^3 - 15*cos(d*x + c))*si
n(d*x + c))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 270, normalized size = 1.33

$$-\frac{\frac{10395\,(dx+c)}{a^{2}}+\frac{2\left(10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{21}+110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{19}+535689\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+2365440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{16}-6564096\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+8279040\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}+8364510\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-12536832\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+20579328\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-8364510\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-2534400\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6564096\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+506880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-535689\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+957440\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-110880\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+191488\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10395\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17408\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{11}a^{2}}}{443520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/443520*(10395*(d*x + c)/a^2 + 2*(10395*tan(1/2*d*x + 1/2*c)^21 + 110880*tan(1/2*d*x + 1/2*c)^19 + 535689*tan(1/2*d*x + 1/2*c)^17 + 2365440*tan(1/2*d*x + 1/2*c)^16 - 6564096*tan(1/2*d*x + 1/2*c)^15 + 8279040*tan(1/2*d*x + 1/2*c)^14 + 8364510*tan(1/2*d*x + 1/2*c)^13 - 12536832*tan(1/2*d*x + 1/2*c)^12 + 20579328*tan(1/2*d*x + 1/2*c)^10 - 8364510*tan(1/2*d*x + 1/2*c)^9 - 2534400*tan(1/2*d*x + 1/2*c)^8 + 6564096*tan(1/2*d*x + 1/2*c)^7 + 506880*tan(1/2*d*x + 1/2*c)^6 - 535689*tan(1/2*d*x + 1/2*c)^5 + 957440*tan(1/2*d*x + 1/2*c)^4 - 110880*tan(1/2*d*x + 1/2*c)^3 + 191488*tan(1/2*d*x + 1/2*c)^2 - 10395*tan(1/2*d*x + 1/2*c) + 17408)/((tan(1/2*d*x + 1/2*c)^2 + 1)^11*a^2))/d

Mupad [B]
time = 11.51, size = 264, normalized size = 1.30

$$-\frac{3\,x}{128\,a^2}-\frac{\frac{3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{21}}{64}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{2}+\frac{773\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{320}+\frac{32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}}{3}-\frac{148\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{5}+\frac{112\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}}{3}+\frac{1207\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}-\frac{848\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{15}+\frac{464\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{5}-\frac{1207\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{32}-\frac{80\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{7}+\frac{148\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{5}+\frac{16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{7}-\frac{773\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{320}+\frac{272\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{63}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{2}+\frac{272\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{315}-\frac{3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{272}{3465}}{a^2\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^8*sin(c + d*x)^5)/(a + a*sin(c + d*x))^2,x)

[Out] - (3*x)/(128*a^2) - ((272*tan(c/2 + (d*x)/2)^2)/315 - (3*tan(c/2 + (d*x)/2))/64 - tan(c/2 + (d*x)/2)^3/2 + (272*tan(c/2 + (d*x)/2)^4)/63 - (773*tan(c/2 + (d*x)/2)^5)/320 + (16*tan(c/2 + (d*x)/2)^6)/7 + (148*tan(c/2 + (d*x)/2)^7)/5 - (80*tan(c/2 + (d*x)/2)^8)/7 - (1207*tan(c/2 + (d*x)/2)^9)/32 + (464*tan(c/2 + (d*x)/2)^10)/5 - (848*tan(c/2 + (d*x)/2)^12)/15 + (1207*tan(c/2 + (d*x)/2)^13)/32 + (112*tan(c/2 + (d*x)/2)^14)/3 - (148*tan(c/2 + (d*x)/2)^15)/5 + (32*tan(c/2 + (d*x)/2)^16)/3 + (773*tan(c/2 + (d*x)/2)^17)/320 + tan(c/2 + (d*x)/2)^19/2 + (3*tan(c/2 + (d*x)/2)^21)/64 + 272/3465)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^11)

3.723 $\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=185

$$\frac{9x}{256a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^9(c+dx)}{9a^2d}+\frac{9\cos(c+dx)\sin(c+dx)}{256a^2d}+\frac{3\cos^3(c+dx)\sin(c+d}{128a^2d}$$

```
[Out] 9/256*x/a^2+2/5*cos(d*x+c)^5/a^2/d-4/7*cos(d*x+c)^7/a^2/d+2/9*cos(d*x+c)^9/
a^2/d+9/256*cos(d*x+c)*sin(d*x+c)/a^2/d+3/128*cos(d*x+c)^3*sin(d*x+c)/a^2/d
-3/32*cos(d*x+c)^5*sin(d*x+c)/a^2/d-3/16*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-1/
10*cos(d*x+c)^5*sin(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.31, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2952, 2648, 2715, 8, 2645, 276}

$$\frac{2\cos^9(c+dx)}{9a^2d}-\frac{4\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^5(c+dx)\cos^5(c+dx)}{10a^2d}-\frac{3\sin^3(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{3\sin(c+dx)\cos^5(c+dx)}{32a^2d}+\frac{3\sin(c+dx)\cos^3(c+dx)}{128a^2d}+\frac{9\sin(c+dx)\cos(c+dx)}{256a^2d}+\frac{9x}{256a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9*x)/(256*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (4*Cos[c + d*x]^7)/(7*a^2*
d) + (2*Cos[c + d*x]^9)/(9*a^2*d) + (9*Cos[c + d*x]*Sin[c + d*x])/(256*a^2*
d) + (3*Cos[c + d*x]^3*Sin[c + d*x])/(128*a^2*d) - (3*Cos[c + d*x]^5*Sin[c
+ d*x])/(32*a^2*d) - (3*Cos[c + d*x]^5*Sin[c + d*x]^3)/(16*a^2*d) - (Cos[c
+ d*x]^5*Sin[c + d*x]^5)/(10*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^4(c+dx) - 2a^2\cos^4(c+dx)\sin^5(c+dx) + a^2\cos^4(\right.}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^6(c+dx)\,dx}{a^2} - \frac{2\int \mathrm{co}}{}$$
$$= -\frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{\cos^5(c+dx)\sin^5(c+dx)}{10a^2d} + \frac{3\int \cos^4(c+d}{}$$
$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{3\cos^5(c+dx)\sin^3(c+dx)}{16a^2d} - \frac{\cos^5(c+dx)\,\mathrm{s}}{10a}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{\cos^3(c+dx)\sin(c+}{64a^2d}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{3\cos(c+dx)\sin(c+}{128a^2d}$$
$$= \frac{3x}{128a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{9\cos(c+d}{25}$$
$$= \frac{9x}{256a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{4\cos^7(c+dx)}{7a^2d} + \frac{2\cos^9(c+dx)}{9a^2d} + \frac{9\cos(c+d}{25}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 585 vs. $2(185) = 370$.
time = 5.34, size = 585, normalized size = 3.16

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2520*(187*c - 18*d*x)*Cos[c/2] + 30240*Cos[c/2 + d*x] + 30240*Cos[(3*c)/2
 + d*x] - 1260*Cos[(3*c)/2 + 2*d*x] + 1260*Cos[(5*c)/2 + 2*d*x] + 6720*Cos[
(5*c)/2 + 3*d*x] + 6720*Cos[(7*c)/2 + 3*d*x] - 7560*Cos[(7*c)/2 + 4*d*x] +
7560*Cos[(9*c)/2 + 4*d*x] - 4032*Cos[(9*c)/2 + 5*d*x] - 4032*Cos[(11*c)/2 +
 5*d*x] + 630*Cos[(11*c)/2 + 6*d*x] - 630*Cos[(13*c)/2 + 6*d*x] - 720*Cos[(
13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 945*Cos[(15*c)/2 + 8*d*x] -
945*Cos[(17*c)/2 + 8*d*x] + 560*Cos[(17*c)/2 + 9*d*x] + 560*Cos[(19*c)/2 +
9*d*x] - 126*Cos[(19*c)/2 + 10*d*x] + 126*Cos[(21*c)/2 + 10*d*x] + 327180*S
in[c/2] - 471240*c*Sin[c/2] + 45360*d*x*Sin[c/2] - 30240*Sin[c/2 + d*x] + 3
0240*Sin[(3*c)/2 + d*x] - 1260*Sin[(3*c)/2 + 2*d*x] - 1260*Sin[(5*c)/2 + 2*
d*x] - 6720*Sin[(5*c)/2 + 3*d*x] + 6720*Sin[(7*c)/2 + 3*d*x] - 7560*Sin[(7*
c)/2 + 4*d*x] - 7560*Sin[(9*c)/2 + 4*d*x] + 4032*Sin[(9*c)/2 + 5*d*x] - 403
```

```
2*Sin[(11*c)/2 + 5*d*x] + 630*Sin[(11*c)/2 + 6*d*x] + 630*Sin[(13*c)/2 + 6*
d*x] + 720*Sin[(13*c)/2 + 7*d*x] - 720*Sin[(15*c)/2 + 7*d*x] + 945*Sin[(15*
c)/2 + 8*d*x] + 945*Sin[(17*c)/2 + 8*d*x] - 560*Sin[(17*c)/2 + 9*d*x] + 560
*Sin[(19*c)/2 + 9*d*x] - 126*Sin[(19*c)/2 + 10*d*x] - 126*Sin[(21*c)/2 + 10
*d*x])/(1290240*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.20, size = 257, normalized size = 1.39

method	result
risch	$\frac{9x}{256a^2}+\frac{3\cos(dx+c)}{64a^2d}-\frac{\sin(10dx+10c)}{5120a^2d}+\frac{\cos(9dx+9c)}{1152d\,a^2}+\frac{3\sin(8dx+8c)}{2048a^2d}-\frac{\cos(7dx+7c)}{896d\,a^2}+\frac{\sin(6dx+6c)}{1024a^2d}-\frac{\cos\cdots}{1\cdots}$
derivativedivides	$\frac{32\left(\frac{1}{315}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63}-\frac{87\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4096}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{553\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5120}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{491(\cdots}{\cdots}\right.}{\cdots}$
default	$\frac{32\left(\frac{1}{315}-\frac{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4096}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63}-\frac{87\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4096}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{553\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5120}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{491(\cdots}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*((1/315-9/4096*tan(1/2*d*x+1/2*c)+2/63*tan(1/2*d*x+1/2*c)^2-87/409
6*tan(1/2*d*x+1/2*c)^3+1/7*tan(1/2*d*x+1/2*c)^4+553/5120*tan(1/2*d*x+1/2*c)
^5-2/7*tan(1/2*d*x+1/2*c)^6+491/1024*tan(1/2*d*x+1/2*c)^7+tan(1/2*d*x+1/2*c
)^8-2555/2048*tan(1/2*d*x+1/2*c)^9+2/5*tan(1/2*d*x+1/2*c)^10+2555/2048*tan(
1/2*d*x+1/2*c)^11-1/3*tan(1/2*d*x+1/2*c)^12-491/1024*tan(1/2*d*x+1/2*c)^13+
2/3*tan(1/2*d*x+1/2*c)^14-553/5120*tan(1/2*d*x+1/2*c)^15+87/4096*tan(1/2*d*
x+1/2*c)^17+9/4096*tan(1/2*d*x+1/2*c)^19)/(1+tan(1/2*d*x+1/2*c)^2)^10+9/409
6*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(167) = 334$.
time = 0.53, size = 605, normalized size = 3.27

$$-\frac{\frac{\frac{2835\sin(dx+c)}{\cos(dx+c)+1}-\frac{40960\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27405\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{184320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{139356\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{368640\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{618660\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{1290240\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{1609650\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{516096\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{1609650\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{430080\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{618660\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{860160\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{139356\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{27405\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-\frac{2835\sin(dx+c)^{19}}{(\cos(dx+c)+1)^{19}}-4096}{a^2+\frac{10a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{45a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{120a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{210a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{252a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{210a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{120a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{45a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}+\frac{10a^2\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}+\frac{a^2\sin(dx+c)^{20}}{(\cos(dx+c)+1)^{20}}}-\frac{2835\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/40320*((2835*sin(d*x + c)/(cos(d*x + c) + 1) - 40960*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + 27405*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 184320*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 - 139356*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 + 368640*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 618660*sin(d*x + c)^7/(cos(
```

```
d*x + c) + 1)^7 - 1290240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 1609650*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 516096*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 - 1609650*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 430080*sin(d*x + c)^
12/(cos(d*x + c) + 1)^12 + 618660*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 8
60160*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 139356*sin(d*x + c)^15/(cos(d
*x + c) + 1)^15 - 27405*sin(d*x + c)^17/(cos(d*x + c) + 1)^17 - 2835*sin(d*
x + c)^19/(cos(d*x + c) + 1)^19 - 4096)/(a^2 + 10*a^2*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 45*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 120*a^2*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 210*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8 + 252*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 210*a^2*sin(d*x + c)^12
/(cos(d*x + c) + 1)^12 + 120*a^2*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + 45
*a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16 + 10*a^2*sin(d*x + c)^18/(cos(d*
x + c) + 1)^18 + a^2*sin(d*x + c)^20/(cos(d*x + c) + 1)^20) - 2835*arctan(s
in(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 100, normalized size = 0.54

$$\frac{17920 \cos\left(dx+c\right)^{9}-46080 \cos\left(dx+c\right)^{7}+32256 \cos\left(dx+c\right)^{5}+2835 \, dx-63 \left(128 \cos\left(dx+c\right)^{9}-496 \cos\left(dx+c\right)^{7}+488 \cos\left(dx+c\right)^{5}-30 \cos\left(dx+c\right)^{3}-45 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{80640 \, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*cos(d*x + c)^9 - 46080*cos(d*x + c)^7 + 32256*cos(d*x + c)^5
 + 2835*d*x - 63*(128*cos(d*x + c)^9 - 496*cos(d*x + c)^7 + 488*cos(d*x + c
)^5 - 30*cos(d*x + c)^3 - 45*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 257, normalized size = 1.39

$$\frac{\frac{2835 \, (dx+c)}{a^{2}}+\frac{2 \left(2835 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{19}+27405 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{17}-139356 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{15}+860160 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{14}-618660 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{13}-430080 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{12}+1609650 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{11}+516096 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10}-1609650 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+1290240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8}+618660 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}-368640 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+139356 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+184320 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-27405 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+40960 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-2835 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+4096\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+1\right)^{10} a^{2}}}{80640 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/80640*(2835*(d*x + c)/a^2 + 2*(2835*tan(1/2*d*x + 1/2*c)^19 + 27405*tan(1
/2*d*x + 1/2*c)^17 - 139356*tan(1/2*d*x + 1/2*c)^15 + 860160*tan(1/2*d*x +
1/2*c)^14 - 618660*tan(1/2*d*x + 1/2*c)^13 - 430080*tan(1/2*d*x + 1/2*c)^12
 + 1609650*tan(1/2*d*x + 1/2*c)^11 + 516096*tan(1/2*d*x + 1/2*c)^10 - 16096
50*tan(1/2*d*x + 1/2*c)^9 + 1290240*tan(1/2*d*x + 1/2*c)^8 + 618660*tan(1/2
*d*x + 1/2*c)^7 - 368640*tan(1/2*d*x + 1/2*c)^6 + 139356*tan(1/2*d*x + 1/2*
c)^5 + 184320*tan(1/2*d*x + 1/2*c)^4 - 27405*tan(1/2*d*x + 1/2*c)^3 + 40960
*tan(1/2*d*x + 1/2*c)^2 - 2835*tan(1/2*d*x + 1/2*c) + 4096)/((tan(1/2*d*x +
 1/2*c)^2 + 1)^10*a^2))/d
```

Mupad [B]
time = 11.63, size = 250, normalized size = 1.35

$$\frac{9\,x}{256\,a^2}+\frac{\frac{9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{128}+\frac{87\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{128}-\frac{553\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{160}+\frac{64\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}}{3}-\frac{491\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}-\frac{32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{3}+\frac{2555\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{64}+\frac{64\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{5}-\frac{2555\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{64}+32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\frac{491\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{32}-\frac{64\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{7}+\frac{553\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{160}+\frac{32\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{7}-\frac{87\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{128}+\frac{64\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{63}-\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}+\frac{32}{315}}{a^2\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^4)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (9*x)/(256*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/63 - (9*tan(c/2 + (d*x)/2))/12
8 - (87*tan(c/2 + (d*x)/2)^3)/128 + (32*tan(c/2 + (d*x)/2)^4)/7 + (553*tan(
c/2 + (d*x)/2)^5)/160 - (64*tan(c/2 + (d*x)/2)^6)/7 + (491*tan(c/2 + (d*x)/
2)^7)/32 + 32*tan(c/2 + (d*x)/2)^8 - (2555*tan(c/2 + (d*x)/2)^9)/64 + (64*t
an(c/2 + (d*x)/2)^10)/5 + (2555*tan(c/2 + (d*x)/2)^11)/64 - (32*tan(c/2 + (
d*x)/2)^12)/3 - (491*tan(c/2 + (d*x)/2)^13)/32 + (64*tan(c/2 + (d*x)/2)^14)
/3 - (553*tan(c/2 + (d*x)/2)^15)/160 + (87*tan(c/2 + (d*x)/2)^17)/128 + (9*
tan(c/2 + (d*x)/2)^19)/128 + 32/315)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^10)
```

3.724 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=159

$$-\frac{3x}{64a^2}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{\cos^9(c+dx)}{9a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{64a^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{32a^2d}$$

```
[Out] -3/64*x/a^2-2/5*cos(d*x+c)^5/a^2/d+3/7*cos(d*x+c)^7/a^2/d-1/9*cos(d*x+c)^9/
a^2/d-3/64*cos(d*x+c)*sin(d*x+c)/a^2/d-1/32*cos(d*x+c)^3*sin(d*x+c)/a^2/d+1
/8*cos(d*x+c)^5*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d
```

Rubi [A]

time = 0.26, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2954, 2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{\cos^9(c+dx)}{9a^2d}+\frac{3\cos^7(c+dx)}{7a^2d}-\frac{2\cos^5(c+dx)}{5a^2d}+\frac{\sin^3(c+dx)\cos^5(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{32a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{64a^2d}-\frac{3x}{64a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(64*a^2) - (2*Cos[c + d*x]^5)/(5*a^2*d) + (3*Cos[c + d*x]^7)/(7*a^2*
d) - Cos[c + d*x]^9/(9*a^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(64*a^2*d) -
(Cos[c + d*x]^3*Sin[c + d*x])/(32*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^5*Sin[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cos^4(c+dx)\sin^3(c+dx) - 2a^2\cos^4(c+dx)\sin^4(c+dx) + a^2\cos^4(\right.}{a^4}$$

$$= \frac{\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^5(c+dx)\,dx}{a^2} - \frac{2\int \mathrm{co}}{}$$

$$= \frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d} - \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{4a^2} - \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{\cos^5(c+dx)\sin(c+dx)}{8a^2d} + \frac{\cos^5(c+dx)\sin^3(c+dx)}{4a^2d} - \frac{\int \cos^4(c+dx)\,dx}{8a^2}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{\cos^3(c+dx)\sin(c+}{32a^2d}$$

$$= -\frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{3\cos(c+dx)\sin(c+}{64a^2d}$$

$$= -\frac{3x}{64a^2} - \frac{2\cos^5(c+dx)}{5a^2d} + \frac{3\cos^7(c+dx)}{7a^2d} - \frac{\cos^9(c+dx)}{9a^2d} - \frac{3\cos(c+dx}{64}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 430 vs. 2(159) = 318.
time = 3.74, size = 430, normalized size = 2.70

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/322560*(420*(7 + 330*c + 36*d*x)*Cos[c/2] + 11340*Cos[c/2 + d*x] + 11340
*Cos[(3*c)/2 + d*x] + 3360*Cos[(5*c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x]
 - 2520*Cos[(7*c)/2 + 4*d*x] + 2520*Cos[(9*c)/2 + 4*d*x] - 1008*Cos[(9*c)/2
 + 5*d*x] - 1008*Cos[(11*c)/2 + 5*d*x] - 450*Cos[(13*c)/2 + 7*d*x] - 450*Co
s[(15*c)/2 + 7*d*x] + 315*Cos[(15*c)/2 + 8*d*x] - 315*Cos[(17*c)/2 + 8*d*x]
 + 70*Cos[(17*c)/2 + 9*d*x] + 70*Cos[(19*c)/2 + 9*d*x] - 78960*Sin[c/2] + 1
38600*c*Sin[c/2] + 15120*d*x*Sin[c/2] - 11340*Sin[c/2 + d*x] + 11340*Sin[(3
*c)/2 + d*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(7*c)/2 + 3*d*x] - 2520
*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 1008*Sin[(9*c)/2 + 5*d*
x] - 1008*Sin[(11*c)/2 + 5*d*x] + 450*Sin[(13*c)/2 + 7*d*x] - 450*Sin[(15*c
)/2 + 7*d*x] + 315*Sin[(15*c)/2 + 8*d*x] + 315*Sin[(17*c)/2 + 8*d*x] - 70*S
in[(17*c)/2 + 9*d*x] + 70*Sin[(19*c)/2 + 9*d*x])/(a^2*d*(Cos[c/2] + Sin[c/2
]))
```

Maple [**A**]
time = 0.16, size = 233, normalized size = 1.47

method	result
risch	$-\frac{3x}{64a^2}-\frac{9\cos(dx+c)}{128a^2d}-\frac{\cos(9dx+9c)}{2304d\,a^2}-\frac{\sin(8dx+8c)}{512a^2d}+\frac{5\cos(7dx+7c)}{1792d\,a^2}+\frac{\cos(5dx+5c)}{160d\,a^2}+\frac{\sin(4dx+4c)}{64a^2d}-\frac{\cos(}{4}$
derivativedivides	$\frac{16\left(-\frac{13}{1260}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}-\frac{13\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{17\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140}-\frac{155\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}+\right.}{}$
default	$\frac{16\left(-\frac{13}{1260}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}-\frac{13\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}-\frac{17\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140}-\frac{155\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{256}+\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^2*((-13/1260+3/512*tan(1/2*d*x+1/2*c)-13/140*tan(1/2*d*x+1/2*c)^2+13
/256*tan(1/2*d*x+1/2*c)^3-17/140*tan(1/2*d*x+1/2*c)^4-155/256*tan(1/2*d*x+1
/2*c)^5+1/20*tan(1/2*d*x+1/2*c)^6+169/256*tan(1/2*d*x+1/2*c)^7-41/20*tan(1/
2*d*x+1/2*c)^8+3/4*tan(1/2*d*x+1/2*c)^10-169/256*tan(1/2*d*x+1/2*c)^11-11/1
2*tan(1/2*d*x+1/2*c)^12+155/256*tan(1/2*d*x+1/2*c)^13-1/4*tan(1/2*d*x+1/2*c
)^14-13/256*tan(1/2*d*x+1/2*c)^15-3/512*tan(1/2*d*x+1/2*c)^17)/(1+tan(1/2*d
*x+1/2*c)^2)^9-3/512*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(143) = 286$.

time = 0.52, size = 542, normalized size = 3.41

$$\frac{\frac{\frac{945\sin(dx+c)}{\cos(dx+c)+1}-\frac{14976\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8190\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19584\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{97650\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{8064\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{106470\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{330624\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120960\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{106470\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{147840\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{97650\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{40320\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}-\frac{8190\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-\frac{945\sin(dx+c)^{17}}{(\cos(dx+c)+1)^{17}}-1664}{a^2+\frac{9a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{36a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{84a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{126a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{84a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{36a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{9a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}+\frac{a^2\sin(dx+c)^{18}}{(\cos(dx+c)+1)^{18}}}-\frac{945\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/10080*((945*sin(d*x + c)/(cos(d*x + c) + 1) - 14976*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 8190*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19584*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 97650*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 80
64*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 106470*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - 330624*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120960*sin(d*x + c)
^10/(cos(d*x + c) + 1)^10 - 106470*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
147840*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 97650*sin(d*x + c)^13/(cos(d
*x + c) + 1)^13 - 40320*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 - 8190*sin(d*
x + c)^15/(cos(d*x + c) + 1)^15 - 945*sin(d*x + c)^17/(cos(d*x + c) + 1)^17
 - 1664)/(a^2 + 9*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 36*a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 + 84*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
126*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*a^2*sin(d*x + c)^10/(cos(
```

```
d*x + c) + 1)^10 + 84*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 36*a^2*si
n(d*x + c)^14/(cos(d*x + c) + 1)^14 + 9*a^2*sin(d*x + c)^16/(cos(d*x + c) +
 1)^16 + a^2*sin(d*x + c)^18/(cos(d*x + c) + 1)^18) - 945*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.38, size = 90, normalized size = 0.57

$$-\frac{2240 \cos\left(dx+c\right)^{9}-8640 \cos\left(dx+c\right)^{7}+8064 \cos\left(dx+c\right)^{5}+945\, dx+315 \left(16 \cos\left(dx+c\right)^{7}-24 \cos\left(dx+c\right)^{5}+2 \cos\left(dx+c\right)^{3}+3 \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{20160\, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*cos(d*x + c)^9 - 8640*cos(d*x + c)^7 + 8064*cos(d*x + c)^5 +
 945*d*x + 315*(16*cos(d*x + c)^7 - 24*cos(d*x + c)^5 + 2*cos(d*x + c)^3 +
3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 231, normalized size = 1.45

$$-\frac{\frac{945\,(dx+c)}{a^{2}}+\frac{2\left(945 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{17}+8190 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+40320 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{14}-97650 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+147840 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+106470 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-120960 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+330624 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-106470 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-8064 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+97650 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+19584 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8190 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+14976 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-945 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1664\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{9} a^{2}}}{20160\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20160*(945*(d*x + c)/a^2 + 2*(945*tan(1/2*d*x + 1/2*c)^17 + 8190*tan(1/2
*d*x + 1/2*c)^15 + 40320*tan(1/2*d*x + 1/2*c)^14 - 97650*tan(1/2*d*x + 1/2*
c)^13 + 147840*tan(1/2*d*x + 1/2*c)^12 + 106470*tan(1/2*d*x + 1/2*c)^11 - 1
20960*tan(1/2*d*x + 1/2*c)^10 + 330624*tan(1/2*d*x + 1/2*c)^8 - 106470*tan(
1/2*d*x + 1/2*c)^7 - 8064*tan(1/2*d*x + 1/2*c)^6 + 97650*tan(1/2*d*x + 1/2*
c)^5 + 19584*tan(1/2*d*x + 1/2*c)^4 - 8190*tan(1/2*d*x + 1/2*c)^3 + 14976*t
an(1/2*d*x + 1/2*c)^2 - 945*tan(1/2*d*x + 1/2*c) + 1664)/((tan(1/2*d*x + 1/
2*c)^2 + 1)^9*a^2))/d
```

Mupad [B]
time = 11.68, size = 225, normalized size = 1.42

$$-\frac{3x}{64a^2}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{32}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{16}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-\frac{155\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{16}+\frac{44\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{3}+\frac{169\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{16}-12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{164\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{5}-\frac{169\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{16}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{5}+\frac{155\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{16}+\frac{68\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{35}-\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{16}+\frac{52\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{35}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32}+\frac{52}{315}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - (3*x)/(64*a^2) - ((52*tan(c/2 + (d*x)/2)^2)/35 - (3*tan(c/2 + (d*x)/2))/3
2 - (13*tan(c/2 + (d*x)/2)^3)/16 + (68*tan(c/2 + (d*x)/2)^4)/35 + (155*tan(
c/2 + (d*x)/2)^5)/16 - (4*tan(c/2 + (d*x)/2)^6)/5 - (169*tan(c/2 + (d*x)/2)
^7)/16 + (164*tan(c/2 + (d*x)/2)^8)/5 - 12*tan(c/2 + (d*x)/2)^10 + (169*tan
(c/2 + (d*x)/2)^11)/16 + (44*tan(c/2 + (d*x)/2)^12)/3 - (155*tan(c/2 + (d*x
)/2)^13)/16 + 4*tan(c/2 + (d*x)/2)^14 + (13*tan(c/2 + (d*x)/2)^15)/16 + (3*
tan(c/2 + (d*x)/2)^17)/32 + 52/315)/(a^2*d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.725 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$\frac{11x}{128a^2}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{11\cos(c+dx)\sin(c+dx)}{128a^2d}+\frac{11\cos^3(c+dx)\sin(c+dx)}{192a^2d}-\frac{11\cos^5(c$$

```
[Out] 11/128*x/a^2+2/5*cos(d*x+c)^5/a^2/d-2/7*cos(d*x+c)^7/a^2/d+11/128*cos(d*x+c
)*sin(d*x+c)/a^2/d+11/192*cos(d*x+c)^3*sin(d*x+c)/a^2/d-11/48*cos(d*x+c)^5*
sin(d*x+c)/a^2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2952, 2648, 2715, 8, 2645, 14}

$$-\frac{2\cos^7(c+dx)}{7a^2d}+\frac{2\cos^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{11\sin(c+dx)\cos^5(c+dx)}{48a^2d}+\frac{11\sin(c+dx)\cos^3(c+dx)}{192a^2d}+\frac{11\sin(c+dx)\cos(c+dx)}{128a^2d}+\frac{11x}{128a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (2*Cos[c + d*x]^5)/(5*a^2*d) - (2*Cos[c + d*x]^7)/(7*a^2
*d) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) + (11*Cos[c + d*x]^3*Sin[c
 + d*x])/(192*a^2*d) - (11*Cos[c + d*x]^5*Sin[c + d*x])/(48*a^2*d) - (Cos[c
 + d*x]^5*Sin[c + d*x]^3)/(8*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^4(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int (a^2\cos^4(c+dx)\sin^2(c+dx) - 2a^2\cos^4(c+dx)\sin^3(c+dx) + a^2\cos^4(}{a^4}$$
$$= \frac{\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{2\int \text{co}}{}$$
$$= -\frac{\cos^5(c+dx)\sin(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{\int \cos^4(c+dx)}{6a^2}$$
$$= \frac{\cos^3(c+dx)\sin(c+dx)}{24a^2d} - \frac{11\cos^5(c+dx)\sin(c+dx)}{48a^2d} - \frac{\cos^5(c+dx)\sin}{8a^2d}$$
$$= \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{\cos(c+dx)\sin(c+dx)}{16a^2d} + \frac{11\cos^3(c+}{19}$$
$$= \frac{x}{16a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} + \frac{1}{}$$
$$= \frac{11x}{128a^2} + \frac{2\cos^5(c+dx)}{5a^2d} - \frac{2\cos^7(c+dx)}{7a^2d} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 481 vs. $2(141) = 282$.
time = 2.61, size = 481, normalized size = 3.41

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (9240*(15*c + 2*d*x)*Cos[c/2] + 10080*Cos[c/2 + d*x] + 10080*Cos[(3*c)/2 +
d*x] + 1680*Cos[(3*c)/2 + 2*d*x] - 1680*Cos[(5*c)/2 + 2*d*x] + 3360*Cos[(5*
c)/2 + 3*d*x] + 3360*Cos[(7*c)/2 + 3*d*x] - 2520*Cos[(7*c)/2 + 4*d*x] + 252
0*Cos[(9*c)/2 + 4*d*x] - 672*Cos[(9*c)/2 + 5*d*x] - 672*Cos[(11*c)/2 + 5*d*
x] - 560*Cos[(11*c)/2 + 6*d*x] + 560*Cos[(13*c)/2 + 6*d*x] - 480*Cos[(13*c)
/2 + 7*d*x] - 480*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*d*x] - 105*C
os[(17*c)/2 + 8*d*x] - 79800*Sin[c/2] + 138600*c*Sin[c/2] + 18480*d*x*Sin[c
/2] - 10080*Sin[c/2 + d*x] + 10080*Sin[(3*c)/2 + d*x] + 1680*Sin[(3*c)/2 +
2*d*x] + 1680*Sin[(5*c)/2 + 2*d*x] - 3360*Sin[(5*c)/2 + 3*d*x] + 3360*Sin[(
7*c)/2 + 3*d*x] - 2520*Sin[(7*c)/2 + 4*d*x] - 2520*Sin[(9*c)/2 + 4*d*x] + 6
72*Sin[(9*c)/2 + 5*d*x] - 672*Sin[(11*c)/2 + 5*d*x] - 560*Sin[(11*c)/2 + 6*
d*x] - 560*Sin[(13*c)/2 + 6*d*x] + 480*Sin[(13*c)/2 + 7*d*x] - 480*Sin[(15*
c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c)/2 + 8*d*x])/(215
040*a^2*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.17, size = 203, normalized size = 1.44

method	result
risch	$\frac{11x}{128a^2}+\frac{3\cos(dx+c)}{32a^2d}+\frac{\sin(8dx+8c)}{1024a^2d}-\frac{\cos(7dx+7c)}{224d\,a^2}-\frac{\sin(6dx+6c)}{192a^2d}-\frac{\cos(5dx+5c)}{160d\,a^2}-\frac{3\sin(4dx+4c)}{128a^2d}+\frac{\cos(3c}{32c}$
derivativedivides	$\frac{8\left(\frac{1}{35}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}+\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{259\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{1103\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-226\right.}{(1+}$
default	$\frac{8\left(\frac{1}{35}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{512}+\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35}+\frac{259\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{1103\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1536}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-226\right.}{(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*((1/35-11/512*tan(1/2*d*x+1/2*c)+8/35*tan(1/2*d*x+1/2*c)^2+259/1536
*tan(1/2*d*x+1/2*c)^3-1/5*tan(1/2*d*x+1/2*c)^4+1103/1536*tan(1/2*d*x+1/2*c)
^5+8/5*tan(1/2*d*x+1/2*c)^6-2261/1536*tan(1/2*d*x+1/2*c)^7+tan(1/2*d*x+1/2*
c)^8+2261/1536*tan(1/2*d*x+1/2*c)^9-1103/1536*tan(1/2*d*x+1/2*c)^11+tan(1/2
*d*x+1/2*c)^12-259/1536*tan(1/2*d*x+1/2*c)^13+11/512*tan(1/2*d*x+1/2*c)^15)
/(1+tan(1/2*d*x+1/2*c)^2)^8+11/512*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(127) = 254$.
time = 0.55, size = 479, normalized size = 3.40

$$-\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}-\frac{12288\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{9065\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10752\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{38605\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{86016\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{79135\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{53760\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{79135\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{38605\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{53760\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{9065\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1155\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-1536}{a^2+\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{1155\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) - 12288*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 - 9065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10752*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 38605*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 8
6016*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 79135*sin(d*x + c)^7/(cos(d*x +
c) + 1)^7 - 53760*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 79135*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 38605*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 5376
0*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 9065*sin(d*x + c)^13/(cos(d*x + c
) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15 - 1536)/(a^2 + 8*a^2
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
```

```
^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 80, normalized size = 0.57

$$-\frac{3840 \cos (dx+c)^7-5376 \cos (dx+c)^5-1155 dx-35 (48 \cos (dx+c)^7-136 \cos (dx+c)^5+22 \cos (dx+c)^3+33 \cos (dx+c)) \sin (dx+c)}{13440 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/13440*(3840*cos(d*x + c)^7 - 5376*cos(d*x + c)^5 - 1155*d*x - 35*(48*cos
(d*x + c)^7 - 136*cos(d*x + c)^5 + 22*cos(d*x + c)^3 + 33*cos(d*x + c))*sin
(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3934 vs. $2(134) = 268$.
time = 182.10, size = 3934, normalized size = 27.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((1155*d*x*tan(c/2 + d*x/2)**16/(13440*a**2*d*tan(c/2 + d*x/2)**16
 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**12
+ 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 +
752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107
520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) + 9240*d*x*tan(c/2 + d*x/2)*
*14/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14
 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)**10
+ 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 3
76320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 + 1344
0*a**2*d) + 32340*d*x*tan(c/2 + d*x/2)**12/(13440*a**2*d*tan(c/2 + d*x/2)**
16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**1
2 + 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8
+ 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 1
07520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) + 64680*d*x*tan(c/2 + d*x/
2)**10/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)*
*14 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)**
10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6
+ 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 + 1
3440*a**2*d) + 80850*d*x*tan(c/2 + d*x/2)**8/(13440*a**2*d*tan(c/2 + d*x/2)
**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)*
```

*12 + 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**
8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 +
 107520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) + 64680*d*x*tan(c/2 + d*
x/2)**6/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)
14 + 376320*a2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)*
*10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6
 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 +
13440*a**2*d) + 32340*d*x*tan(c/2 + d*x/2)**4/(13440*a**2*d*tan(c/2 + d*x/2
)**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)
12 + 752640*a2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)*
*8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4
+ 107520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) + 9240*d*x*tan(c/2 + d*
x/2)**2/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)
14 + 376320*a2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)*
*10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6
 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 +
13440*a**2*d) + 1155*d*x/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d
*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*
tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*ta
n(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c
/2 + d*x/2)**2 + 13440*a**2*d) + 2310*tan(c/2 + d*x/2)**15/(13440*a**2*d*ta
n(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan
(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(
c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2
 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) - 18130*ta
n(c/2 + d*x/2)**13/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c
/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/
2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2
+ d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d
*x/2)**2 + 13440*a**2*d) + 107520*tan(c/2 + d*x/2)**12/(13440*a**2*d*tan(c/
2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2
 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2
+ d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d
*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) - 77210*tan(c/
2 + d*x/2)**11/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 +
 d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 +
d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*
x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520*a**2*d*tan(c/2 + d*x/2
)**2 + 13440*a**2*d) + 158270*tan(c/2 + d*x/2)**9/(13440*a**2*d*tan(c/2 + d
*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/2)**14 + 376320*a**2*d*tan(c/2 + d*
x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2)**10 + 940800*a**2*d*tan(c/2 + d*x
/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)**6 + 376320*a**2*d*tan(c/2 + d*x/2)
4 + 107520*a2*d*tan(c/2 + d*x/2)**2 + 13440*a**2*d) + 107520*tan(c/2 +
d*x/2)**8/(13440*a**2*d*tan(c/2 + d*x/2)**16 + 107520*a**2*d*tan(c/2 + d*x/
2)**14 + 376320*a**2*d*tan(c/2 + d*x/2)**12 + 752640*a**2*d*tan(c/2 + d*x/2

```
)**10 + 940800*a**2*d*tan(c/2 + d*x/2)**8 + 752640*a**2*d*tan(c/2 + d*x/2)*
*6 + 376320*a**2*d*tan(c/2 + d*x/2)**4 + 107520...
```

Giac [A]
time = 0.46, size = 205, normalized size = 1.45

$$\frac{\frac{1155\,(dx+c)}{a^2}+\frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}-9065\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+53760\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-38605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+79135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+53760\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-79135\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+86016\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+38605\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10752\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9065\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12288\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1536\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^{2}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 - 9065*tan(1/
2*d*x + 1/2*c)^13 + 53760*tan(1/2*d*x + 1/2*c)^12 - 38605*tan(1/2*d*x + 1/2
*c)^11 + 79135*tan(1/2*d*x + 1/2*c)^9 + 53760*tan(1/2*d*x + 1/2*c)^8 - 7913
5*tan(1/2*d*x + 1/2*c)^7 + 86016*tan(1/2*d*x + 1/2*c)^6 + 38605*tan(1/2*d*x
 + 1/2*c)^5 - 10752*tan(1/2*d*x + 1/2*c)^4 + 9065*tan(1/2*d*x + 1/2*c)^3 +
12288*tan(1/2*d*x + 1/2*c)^2 - 1155*tan(1/2*d*x + 1/2*c) + 1536)/((tan(1/2*
d*x + 1/2*c)^2 + 1)^8*a^2))/d
```

Mupad [B]
time = 11.70, size = 198, normalized size = 1.40

$$\frac{11\,x}{128\,a^2}+\frac{\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{259\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{1103\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+\frac{2261\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{2261\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{192}+\frac{64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5}+\frac{1103\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192}-\frac{8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}+\frac{259\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35}-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{8}{35}}{a^2\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (11*x)/(128*a^2) + ((64*tan(c/2 + (d*x)/2)^2)/35 - (11*tan(c/2 + (d*x)/2))/
64 + (259*tan(c/2 + (d*x)/2)^3)/192 - (8*tan(c/2 + (d*x)/2)^4)/5 + (1103*ta
n(c/2 + (d*x)/2)^5)/192 + (64*tan(c/2 + (d*x)/2)^6)/5 - (2261*tan(c/2 + (d*
x)/2)^7)/192 + 8*tan(c/2 + (d*x)/2)^8 + (2261*tan(c/2 + (d*x)/2)^9)/192 - (
1103*tan(c/2 + (d*x)/2)^11)/192 + 8*tan(c/2 + (d*x)/2)^12 - (259*tan(c/2 +
(d*x)/2)^13)/192 + (11*tan(c/2 + (d*x)/2)^15)/64 + 8/35)/(a^2*d*(tan(c/2 +
(d*x)/2)^2 + 1)^8)
```

3.726 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$-\frac{x}{8a^2}-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{8a^2d}-\frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d}-\frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d}-\frac{\text{co}}{5d(a+}$$

```
[Out] -1/8*x/a^2-2/35*cos(d*x+c)^7/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)/a^2/d-1/12*cos
(d*x+c)^3*sin(d*x+c)/a^2/d-1/15*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/5*cos(d*x+c
)^9/d/(a+a*sin(d*x+c))^2
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2938, 2761, 2715, 8}

$$-\frac{2\cos^7(c+dx)}{35a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{15a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{12a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}-\frac{x}{8a^2}-\frac{\cos^9(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*x/a^2 - (2*Cos[c + d*x]^7)/(35*a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(12*a^2*d) - (Cos[c + d*x]^5*Sin[c
+ d*x])/(15*a^2*d) - Cos[c + d*x]^9/(5*d*(a + a*Sin[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
```

```
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= -\frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \frac{\cos^8(c+dx)}{a+a\sin(c+dx)}\,dx}{5a} \\
&= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\int \cos^6(c+dx)\,dx}{5a^2} \\
&= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \frac{\cos^9(c+dx)}{5d(a+a\sin(c+dx))^2} - \int \\
&= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{15a^2d} - \\
&= -\frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{12a^2d} - \\
&= -\frac{x}{8a^2} - \frac{2\cos^7(c+dx)}{35a^2d} - \frac{\cos(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^3(c+dx)\sin(c+
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 418 vs. $2(124) = 248$.
time = 3.79, size = 418, normalized size = 3.37

$$-\frac{70(7+24dx)\cos\left(\frac{c}{2}\right)+1155\cos\left(\frac{c}{2}+dx\right)+1155\cos\left(\frac{3c}{2}+dx\right)+210\cos\left(\frac{3c}{2}+2dx\right)-210\cos\left(\frac{5c}{2}+2dx\right)+525\cos\left(\frac{5c}{2}+3dx\right)+525\cos\left(\frac{7c}{2}+3dx\right)-210\cos\left(\frac{7c}{2}+4dx\right)+210\cos\left(\frac{9c}{2}+4dx\right)+63\cos\left(\frac{9c}{2}+5dx\right)+63\cos\left(\frac{11c}{2}+5dx\right)-70\cos\left(\frac{11c}{2}+6dx\right)+70\cos\left(\frac{13c}{2}+6dx\right)-15\cos\left(\frac{13c}{2}+7dx\right)-15\cos\left(\frac{15c}{2}+7dx\right)-490\sin\left(\frac{c}{2}\right)+1680dx\sin\left(\frac{c}{2}\right)-1155\sin\left(\frac{c}{2}+dx\right)+1155\sin\left(\frac{3c}{2}+dx\right)+210\sin\left(\frac{3c}{2}+2dx\right)+210\sin\left(\frac{5c}{2}+2dx\right)-525\sin\left(\frac{5c}{2}+3dx\right)+525\sin\left(\frac{7c}{2}+3dx\right)-210\sin\left(\frac{7c}{2}+4dx\right)-210\sin\left(\frac{9c}{2}+4dx\right)-63\sin\left(\frac{9c}{2}+5dx\right)+63\sin\left(\frac{11c}{2}+5dx\right)-70\sin\left(\frac{11c}{2}+6dx\right)-70\sin\left(\frac{13c}{2}+6dx\right)+15\sin\left(\frac{13c}{2}+7dx\right)-15\sin\left(\frac{15c}{2}+7dx\right)}{13440a^2d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(70*(7 + 24*d*x)*Cos[c/2] + 1155*Cos[c/2 + d*x] + 1155*Cos[(3*c)/2
 + d*x] + 210*Cos[(3*c)/2 + 2*d*x] - 210*Cos[(5*c)/2 + 2*d*x] + 525*Cos[(5*
c)/2 + 3*d*x] + 525*Cos[(7*c)/2 + 3*d*x] - 210*Cos[(7*c)/2 + 4*d*x] + 210*C
os[(9*c)/2 + 4*d*x] + 63*Cos[(9*c)/2 + 5*d*x] + 63*Cos[(11*c)/2 + 5*d*x] -
70*Cos[(11*c)/2 + 6*d*x] + 70*Cos[(13*c)/2 + 6*d*x] - 15*Cos[(13*c)/2 + 7*d
*x] - 15*Cos[(15*c)/2 + 7*d*x] - 490*Sin[c/2] + 1680*d*x*Sin[c/2] - 1155*Si
n[c/2 + d*x] + 1155*Sin[(3*c)/2 + d*x] + 210*Sin[(3*c)/2 + 2*d*x] + 210*Sin
[(5*c)/2 + 2*d*x] - 525*Sin[(5*c)/2 + 3*d*x] + 525*Sin[(7*c)/2 + 3*d*x] - 2
10*Sin[(7*c)/2 + 4*d*x] - 210*Sin[(9*c)/2 + 4*d*x] - 63*Sin[(9*c)/2 + 5*d*x
] + 63*Sin[(11*c)/2 + 5*d*x] - 70*Sin[(11*c)/2 + 6*d*x] - 70*Sin[(13*c)/2 +
```

```
6*d*x] + 15*Sin[(13*c)/2 + 7*d*x] - 15*Sin[(15*c)/2 + 7*d*x])/(a^2*d*(Cos[
c/2] + Sin[c/2]))
```

Maple [A]
time = 0.16, size = 194, normalized size = 1.56

method	result
risch	$-\frac{x}{8a^2}-\frac{11\cos(dx+c)}{64a^2d}+\frac{\cos(7dx+7c)}{448d\,a^2}+\frac{\sin(6dx+6c)}{96a^2d}-\frac{3\cos(5dx+5c)}{320d\,a^2}+\frac{\sin(4dx+4c)}{32a^2d}-\frac{5\cos(3dx+3c)}{64d\,a^2}-\sin$
derivativedivides	$\frac{4\left(-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{31\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\left(\tan\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
default	$\frac{4\left(-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{11\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12}-2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{31\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48}-\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\left(\tan\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*((-1/16*tan(1/2*d*x+1/2*c)^13-1/2*tan(1/2*d*x+1/2*c)^12+11/12*tan(1
/2*d*x+1/2*c)^11-2*tan(1/2*d*x+1/2*c)^10-31/48*tan(1/2*d*x+1/2*c)^9-1/2*tan
(1/2*d*x+1/2*c)^8-4*tan(1/2*d*x+1/2*c)^6+31/48*tan(1/2*d*x+1/2*c)^5-7/10*ta
n(1/2*d*x+1/2*c)^4-11/12*tan(1/2*d*x+1/2*c)^3-2/5*tan(1/2*d*x+1/2*c)^2+1/16
*tan(1/2*d*x+1/2*c)-9/70)/(1+tan(1/2*d*x+1/2*c)^2)^7-1/16*arctan(tan(1/2*d*
x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. 2(112) = 224.
time = 0.50, size = 436, normalized size = 3.52

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{672\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1540\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1176\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1085\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{840\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{1085\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3360\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{1540\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{840\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{105\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-216}{a^2+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/420*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 672*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1540*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1176*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 1085*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6720*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 840*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 1085*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3360*sin(d*x + c)^10/(cos(d*x
+ c) + 1)^10 + 1540*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 840*sin(d*x + c
)^12/(cos(d*x + c) + 1)^12 - 105*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 21
6)/(a^2 + 7*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 21*a^2*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 35*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 35*a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^2*sin(d*x + c)^10/(cos(d*x + c)
```

```
 + 1)^10 + 7*a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + a^2*sin(d*x + c)^1
4/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)
/d
```

Fricas [A]
time = 0.38, size = 70, normalized size = 0.56

$$\frac{120 \cos\left(dx+c\right)^{7}-336 \cos\left(dx+c\right)^{5}-105\,dx+35\,\left(8 \cos\left(dx+c\right)^{5}-2 \cos\left(dx+c\right)^{3}-3 \cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*cos(d*x + c)^7 - 336*cos(d*x + c)^5 - 105*d*x + 35*(8*cos(d*x +
c)^5 - 2*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3196 vs. $2(112) = 224$.
time = 126.64, size = 3196, normalized size = 25.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**14/(840*a**2*d*tan(c/2 + d*x/2)**14 +
 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 294
00*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a*
*2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) -
735*d*x*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d
*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*ta
n(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2
 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan
(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 +
d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x
/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**
4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/
2)**8/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 +
 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 294
00*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**
2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3675*d*x*tan(c/2 + d*x/2)**6/(840*a
**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*
d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*ta
n(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
+ d*x/2)**2 + 840*a**2*d) - 2205*d*x*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/
2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 +
```

d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/
2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2
+ 840*a**2*d) - 735*d*x*tan(c/2 + d*x/2)**2/(840*a**2*d*tan(c/2 + d*x/2)**1
4 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 +
29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640
*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d)
 - 105*d*x/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)*
*12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8
+ 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 588
0*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 210*tan(c/2 + d*x/2)**13/(840*
a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2
*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*t
an(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2
 + d*x/2)**2 + 840*a**2*d) - 1680*tan(c/2 + d*x/2)**12/(840*a**2*d*tan(c/2
+ d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*
x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)
6 + 17640*a2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 +
840*a**2*d) + 3080*tan(c/2 + d*x/2)**11/(840*a**2*d*tan(c/2 + d*x/2)**14 +
5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 2940
0*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**
2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 6
720*tan(c/2 + d*x/2)**10/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan
(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/
2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d
*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2170*tan(c/2 + d
*x/2)**9/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**1
2 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 +
29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*
a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 1680*tan(c/2 + d*x/2)**8/(840*a*
*2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d
*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan
(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 +
 d*x/2)**2 + 840*a**2*d) - 13440*tan(c/2 + d*x/2)**6/(840*a**2*d*tan(c/2 +
d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/
2)**10 + 29400*a**2*d*tan(c/2 + d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**
6 + 17640*a**2*d*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 84
0*a**2*d) + 2170*tan(c/2 + d*x/2)**5/(840*a**2*d*tan(c/2 + d*x/2)**14 + 588
0*a**2*d*tan(c/2 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a
2*d*tan(c/2 + d*x/2)8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d
*tan(c/2 + d*x/2)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 2352
*tan(c/2 + d*x/2)**4/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2
 + d*x/2)**12 + 17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*d*tan(c/2 +
d*x/2)**8 + 29400*a**2*d*tan(c/2 + d*x/2)**6 + 17640*a**2*d*tan(c/2 + d*x/2
)**4 + 5880*a**2*d*tan(c/2 + d*x/2)**2 + 840*a**2*d) - 3080*tan(c/2 + d*x/2
)**3/(840*a**2*d*tan(c/2 + d*x/2)**14 + 5880*a**2*d*tan(c/2 + d*x/2)**12 +

```
17640*a**2*d*tan(c/2 + d*x/2)**10 + 29400*a**2*...
```

Giac [A]
time = 0.46, size = 192, normalized size = 1.55

$$-\frac{\frac{105\,(dx+c)}{a^2}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}-1540\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+1085\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+840\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+6720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-1085\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1176\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1540\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+672\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+216\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{7}a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(105*(d*x + c)/a^2 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 840*tan(1/2*d*
x + 1/2*c)^12 - 1540*tan(1/2*d*x + 1/2*c)^11 + 3360*tan(1/2*d*x + 1/2*c)^10
 + 1085*tan(1/2*d*x + 1/2*c)^9 + 840*tan(1/2*d*x + 1/2*c)^8 + 6720*tan(1/2*
d*x + 1/2*c)^6 - 1085*tan(1/2*d*x + 1/2*c)^5 + 1176*tan(1/2*d*x + 1/2*c)^4
+ 1540*tan(1/2*d*x + 1/2*c)^3 + 672*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*
x + 1/2*c) + 216)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*a^2))/d
```

Mupad [B]
time = 12.65, size = 186, normalized size = 1.50

$$-\frac{x}{8\,a^2}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{12}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-\frac{31\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{12}+\frac{14\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{18}{35}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] - x/(8*a^2) - ((8*tan(c/2 + (d*x)/2)^2)/5 - tan(c/2 + (d*x)/2)/4 + (11*tan(
c/2 + (d*x)/2)^3)/3 + (14*tan(c/2 + (d*x)/2)^4)/5 - (31*tan(c/2 + (d*x)/2)^
5)/12 + 16*tan(c/2 + (d*x)/2)^6 + 2*tan(c/2 + (d*x)/2)^8 + (31*tan(c/2 + (d
*x)/2)^9)/12 + 8*tan(c/2 + (d*x)/2)^10 - (11*tan(c/2 + (d*x)/2)^11)/3 + 2*t
an(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^13/4 + 18/35)/(a^2*d*(tan(c/2 + (
d*x)/2)^2 + 1)^7)
```

3.727 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{3x}{4a^2}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^5(c+dx)}{5a^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{4a^2d}-\cos^3(c+$$

```
[Out] -3/4*x/a^2-arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a^2/d+1/3*cos(d*x+c)^3/a^2/
d-1/5*cos(d*x+c)^5/a^2/d-3/4*cos(d*x+c)*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)^3*s
in(d*x+c)/a^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2954, 2952, 2715, 8, 2672, 308, 212, 2645, 30}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos^3(c+dx)}{2a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{4a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{3x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-3*x)/(4*a^2) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a^2*d) + Cos
[c + d*x]^3/(3*a^2*d) - Cos[c + d*x]^5/(5*a^2*d) - (3*Cos[c + d*x]*Sin[c +
d*x])/(4*a^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^3(c+dx)\cot(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-2a^2\cos^4(c+dx)+a^2\cos^3(c+dx)\cot(c+dx)+a^2\cos^4(c+dx)\sin(c\right.}{a^4}$$

$$= \frac{\int \cos^3(c+dx)\cot(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2} - \frac{2\int \cos^4(c}{a}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d} - \frac{3\int \cos^2(c+dx)\,dx}{2a^2} - \frac{\text{Subst}\left(\int x^4\,dx, x, \cos(c\right.}{a^2d}$$

$$= -\frac{\cos^5(c+dx)}{5a^2d} - \frac{3\cos(c+dx)\sin(c+dx)}{4a^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2a^2d} - \frac{3}{}$$

$$= -\frac{3x}{4a^2} + \frac{\cos(c+dx)}{a^2d} + \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx)}{5a^2d} - \frac{3\cos(c+dx)\sin(c}{4a^2d}$$

$$= -\frac{3x}{4a^2} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} + \frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^5(c+dx}{5a^2d}$$

Mathematica [A]
time = 0.50, size = 93, normalized size = 0.78

$$\frac{270\cos(c+dx)+5\cos(3(c+dx))-3\left(60c+60dx+\cos(5(c+dx))+80\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-80\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+40\sin(2(c+dx))+5\sin(4(c+dx))\right)}{240a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (270*Cos[c + d*x] + 5*Cos[3*(c + d*x)] - 3*(60*c + 60*d*x + Cos[5*(c + d*x)
] + 80*Log[Cos[(c + d*x)/2]] - 80*Log[Sin[(c + d*x)/2]] + 40*Sin[2*(c + d*x
)] + 5*Sin[4*(c + d*x)]))/(240*a^2*d)
```

Maple [A]
time = 0.24, size = 152, normalized size = 1.28

method	result
risch	$-\frac{3x}{4a^2}+\frac{9\,e^{i(dx+c)}}{16d\,a^2}+\frac{9\,e^{-i(dx+c)}}{16d\,a^2}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\cos(5dx+5c)}{80d\,a^2}-\frac{\sin(4dx+4c)}{16a^2d}+\frac{\cos(3}{48}$
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}\right.\right.}{4}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^2}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(-\frac{5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{\left(\tan^3\left(\frac{dx}{2}\right.\right.}{4}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(ln(tan(1/2*d*x+1/2*c))-4*(-5/8*tan(1/2*d*x+1/2*c)^9-1/2*tan(1/2*d*
x+1/2*c)^8-1/4*tan(1/2*d*x+1/2*c)^7-3*tan(1/2*d*x+1/2*c)^6-8/3*tan(1/2*d*x+
1/2*c)^4+1/4*tan(1/2*d*x+1/2*c)^3-7/3*tan(1/2*d*x+1/2*c)^2+5/8*tan(1/2*d*x+
1/2*c)-17/30)/(1+tan(1/2*d*x+1/2*c)^2)^5-3/2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(109) = 218$.
time = 0.51, size = 333, normalized size = 2.80

$$-\frac{\frac{\frac{75\sin(dx+c)}{\cos(dx+c)+1}-\frac{280\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{320\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{360\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{30\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{60\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{75\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-68}{a^2+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*((75*sin(d*x + c)/(cos(d*x + c) + 1) - 280*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 320*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 30*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 - 60*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 75*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 - 68)/(a^2 + 5*a^2*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 10*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^2*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 + 5*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x
 + c) + 1))/a^2 - 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 94, normalized size = 0.79

$$-\frac{12\cos(dx+c)^5-20\cos(dx+c)^3+45\,dx+15\left(2\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)-60\cos(dx+c)+30\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-30\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{60\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 20*cos(d*x + c)^3 + 45*d*x + 15*(2*cos(d*x + c)^
3 + 3*cos(d*x + c))*sin(d*x + c) - 60*cos(d*x + c) + 30*log(1/2*cos(d*x + c
) + 1/2) - 30*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.45, size = 156, normalized size = 1.31

$$-\frac{\frac{45\,(dx+c)}{a^2}-\frac{60\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+360\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+320\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+68\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5a^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(45*(d*x + c)/a^2 - 60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(75*tan
(1/2*d*x + 1/2*c)^9 + 60*tan(1/2*d*x + 1/2*c)^8 + 30*tan(1/2*d*x + 1/2*c)^7
 + 360*tan(1/2*d*x + 1/2*c)^6 + 320*tan(1/2*d*x + 1/2*c)^4 - 30*tan(1/2*d*x
 + 1/2*c)^3 + 280*tan(1/2*d*x + 1/2*c)^2 - 75*tan(1/2*d*x + 1/2*c) + 68)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^5*a^2))/d
```

Mupad [B]
time = 10.76, size = 262, normalized size = 2.20

$$\frac{3\,\mathrm{atan}\left(\frac{9}{4\left(\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+3\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}+3}\right)}{2\,a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\frac{28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}+\frac{34}{15}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (3*atan(9/(4*((9*tan(c/2 + (d*x)/2))/4 + 3)) - (3*tan(c/2 + (d*x)/2))/((9*t
an(c/2 + (d*x)/2))/4 + 3)))/(2*a^2*d) + log(tan(c/2 + (d*x)/2))/(a^2*d) + (
(28*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/2 - tan(c/2 + (d*x)/2)
^3 + (32*tan(c/2 + (d*x)/2)^4)/3 + 12*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^7 + 2*tan(c/2 + (d*x)/2)^8 + (5*tan(c/2 + (d*x)/2)^9)/2 + 34/15)/(d*(5
*a^2*tan(c/2 + (d*x)/2)^2 + 10*a^2*tan(c/2 + (d*x)/2)^4 + 10*a^2*tan(c/2 +
(d*x)/2)^6 + 5*a^2*tan(c/2 + (d*x)/2)^8 + a^2*tan(c/2 + (d*x)/2)^10 + a^2))
```

3.728 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{9x}{8a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{8a^2d}-\cos($$

```
[Out] -9/8*x/a^2+2*arctanh(cos(d*x+c))/a^2/d-2*cos(d*x+c)/a^2/d-2/3*cos(d*x+c)^3/
a^2/d-cot(d*x+c)/a^2/d+1/8*cos(d*x+c)*sin(d*x+c)/a^2/d-1/4*cos(d*x+c)*sin(d
*x+c)^3/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2951, 3855, 3852, 8, 2718, 2715, 2713}

$$-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{9x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*x)/(8*a^2) + (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - (2*Cos[c + d*x])/(a^2*
d) - (2*Cos[c + d*x]^3)/(3*a^2*d) - Cot[c + d*x]/(a^2*d) + (Cos[c + d*x]*Si
n[c + d*x])/(8*a^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos^2(c+dx)\cot^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-a^6-2a^6\csc(c+dx)+a^6\csc^2(c+dx)+4a^6\sin(c+dx)-a^6\sin^2(c+\right.}{a^8}$$

$$= -\frac{x}{a^2}+\frac{\int \csc^2(c+dx)\,dx}{a^2}-\frac{\int \sin^2(c+dx)\,dx}{a^2}+\frac{\int \sin^4(c+dx)\,dx}{a^2}-\frac{2\int c}{}$$

$$= -\frac{x}{a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{4\cos(c+dx)}{a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}-$$

$$= -\frac{3x}{2a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c}{a^2}$$

$$= -\frac{9x}{8a^2}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2\cos(c+dx)}{a^2d}-\frac{2\cos^3(c+dx)}{3a^2d}-\frac{\cot(c}{a^2}$$

Mathematica [A]
time = 1.07, size = 128, normalized size = 1.10

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-108(c+dx)-240\cos(c+dx)-16\cos(3(c+dx))-48\cot\left(\frac{1}{2}(c+dx)\right)+192\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-192\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\sin(4(c+dx))+48\tan\left(\frac{1}{2}(c+dx)\right)\right)}{96d(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-108*(c + d*x) - 240*Cos[c + d*x]
 - 16*Cos[3*(c + d*x)] - 48*Cot[(c + d*x)/2] + 192*Log[Cos[(c + d*x)/2]] -
192*Log[Sin[(c + d*x)/2]] + 3*Sin[4*(c + d*x)] + 48*Tan[(c + d*x)/2]))/(96*
d*(a + a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.23, size = 164, normalized size = 1.41

method	result
risch	$-\frac{9x}{8a^2}-\frac{5\,e^{i(dx+c)}}{4d\,a^2}-\frac{5\,e^{-i(dx+c)}}{4d\,a^2}-\frac{2i}{a^2d\left(e^{2i(dx+c)}-1\right)}-\frac{2\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{2\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{\sin(4dx+4c\cdots}{32a^2d}$
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}}{2d\,a^2}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4\left(\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}}{2d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c))
-4*(1/8*tan(1/2*d*x+1/2*c)^7+4*tan(1/2*d*x+1/2*c)^6-7/8*tan(1/2*d*x+1/2*c)^
5+8*tan(1/2*d*x+1/2*c)^4+7/8*tan(1/2*d*x+1/2*c)^3+20/3*tan(1/2*d*x+1/2*c)^2
-1/8*tan(1/2*d*x+1/2*c)+8/3)/(1+tan(1/2*d*x+1/2*c)^2)^4-9/2*arctan(tan(1/2*
d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(108) = 216$.
time = 0.50, size = 348, normalized size = 3.00

$$-\frac{\frac{\frac{64\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{160\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{57\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{192\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{96\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{9\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+6}{\frac{a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}+\frac{27\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{24\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{6\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*((64*sin(d*x + c)/(cos(d*x + c) + 1) + 21*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 160*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 57*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 192*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 96*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 9*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 + 6)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^2*sin(d*x + c)^9/
(cos(d*x + c) + 1)^9) + 27*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 24
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 6*sin(d*x + c)/(a^2*(cos(d*x +
c) + 1)))/d
```

Fricas [A]
time = 0.39, size = 113, normalized size = 0.97

$$-\frac{6\cos(dx+c)^5-9\cos(dx+c)^3+\left(16\cos(dx+c)^3+27\,dx+48\cos(dx+c)\right)\sin(dx+c)-24\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+24\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+27\cos(dx+c)}{24\,a^2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*cos(d*x + c)^5 - 9*cos(d*x + c)^3 + (16*cos(d*x + c)^3 + 27*d*x +
48*cos(d*x + c))*sin(d*x + c) - 24*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 + 24*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 27*cos(d*x + c))/(a^2*d*s
in(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.48, size = 186, normalized size = 1.60

$$-\frac{\frac{27\,(dx+c)}{a^2}+\frac{48\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{12\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{12\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+192\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)/a^2 + 48*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 12*tan(1/
2*d*x + 1/2*c)/a^2 - 12*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2
```

```
*c)) + 2*(3*tan(1/2*d*x + 1/2*c)^7 + 96*tan(1/2*d*x + 1/2*c)^6 - 21*tan(1/2
*d*x + 1/2*c)^5 + 192*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^3 +
160*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 64)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a^2))/d
```

Mupad [B]
time = 9.21, size = 279, normalized size = 2.41

$$\frac{9\,\mathrm{atan}\left(\frac{9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{81\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-9}+\frac{81}{16\left(\frac{81\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}-9\right)}\right)}{4\,a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{80\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1}{d\,\left(2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (9*atan((9*tan(c/2 + (d*x)/2))/((81*tan(c/2 + (d*x)/2))/16 - 9) + 81/(16*((
81*tan(c/2 + (d*x)/2))/16 - 9))))/(4*a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(
a^2*d) - ((32*tan(c/2 + (d*x)/2))/3 + (7*tan(c/2 + (d*x)/2)^2)/2 + (80*tan(
c/2 + (d*x)/2)^3)/3 + (19*tan(c/2 + (d*x)/2)^4)/2 + 32*tan(c/2 + (d*x)/2)^5
 + tan(c/2 + (d*x)/2)^6/2 + 16*tan(c/2 + (d*x)/2)^7 + (3*tan(c/2 + (d*x)/2)
^8)/2 + 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 12*a^2*tan(c/2 + (d*x)/2)^5 + 8
*a^2*tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)^9 + 2*a^2*tan(c/2 + (d
*x)/2))) + tan(c/2 + (d*x)/2)/(2*a^2*d)
```

3.729 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=97

$$\frac{3x}{a^2}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{\cos(c+dx)\sin(c+dx)}{a^2d}$$

```
[Out] 3*x/a^2+1/2*arctanh(cos(d*x+c))/a^2/d+1/3*cos(d*x+c)^3/a^2/d+2*cot(d*x+c)/a
^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d+cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2718, 2715, 2713}

$$\frac{\cos^3(c+dx)}{3a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{3x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*x)/a^2 + ArcTanh[Cos[c + d*x]]/(2*a^2*d) + Cos[c + d*x]^3/(3*a^2*d) + (2
*Cot[c + d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cos[c + d
*x]*Sin[c + d*x])/(a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cos(c+dx)\cot^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int (4a^6 - a^6\csc(c+dx) - 2a^6\csc^2(c+dx) + a^6\csc^3(c+dx) - a^6\sin(c+$$
$$= \frac{4x}{a^2} - \frac{\int \csc(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \sin(c+dx)\,dx}{a^2} + \frac{\int \sin^3(c$$
$$= \frac{4x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cos$$
$$= \frac{3x}{a^2} + \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{\cos^3(c+dx)}{3a^2d} + \frac{2\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)}{2a^2}$$

Mathematica [A]

time = 1.34, size = 158, normalized size = 1.63

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(6\cos(c+dx)+2\cos(3(c+dx))+3\left(24c+24dx+8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+4\sin(2(c+dx))-8\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{24a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(6*Cos[c + d*x] + 2*Cos[3*(c + d*x
)] + 3*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 + 4*Log[Cos
[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + 4*Sin[2*(c
+ d*x)] - 8*Tan[(c + d*x)/2])))/(24*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]

time = 0.25, size = 140, normalized size = 1.44

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{4d\,a^2}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{4d\,a^2}$
risch	$\frac{3x}{a^2}+\frac{e^{3i(dx+c)}}{24d\,a^2}-\frac{i e^{2i(dx+c)}}{4a^2d}+\frac{e^{i(dx+c)}}{8d\,a^2}+\frac{e^{-i(dx+c)}}{8d\,a^2}+\frac{i e^{-2i(dx+c)}}{4a^2d}+\frac{e^{-3i(dx+c)}}{24d\,a^2}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+4ie^{2i}}{d\,a^2\left(e^{2i(dx+c)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/
2*c)^2+4/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))+16*(-1/2*tan(1/2*d*x+1
```

```
/2*c)^5+1/2*tan(1/2*d*x+1/2*c)^4+1/2*tan(1/2*d*x+1/2*c)+1/6)/(1+tan(1/2*d*x
+1/2*c)^2)^3+24*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(91) = 182$.

time = 0.50, size = 330, normalized size = 3.40

$$\frac{\frac{\frac{24\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{72\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{45\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{24\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-3}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{144\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((24*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 72*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 45*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 - 24*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3)/(a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 3*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d*x + c)^8/(cos(d*x
 + c) + 1)^8) - 3*(8*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/a^2 + 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 12
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [**A**]

time = 0.40, size = 140, normalized size = 1.44

$$\frac{4\cos(dx+c)^5+36\,dx\cos(dx+c)^2-4\cos(dx+c)^3-36\,dx+3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12\left(\cos(dx+c)^3-3\cos(dx+c)\right)\sin(dx+c)+6\cos(dx+c)}{12\left(a^2d\cos(dx+c)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*cos(d*x + c)^5 + 36*d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - 36*d*x
+ 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^2 -
1)*log(-1/2*cos(d*x + c) + 1/2) + 12*(cos(d*x + c)^3 - 3*cos(d*x + c))*sin(
d*x + c) + 6*cos(d*x + c))/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.46, size = 168, normalized size = 1.73

$$\frac{\frac{72\,(dx+c)}{a^2}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{3\,\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4}+\frac{3\,\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{16\,\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(72*(d*x + c)/a^2 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(a^2*tan
(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 + 3*(6*tan(1/2*d*x +
1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 16*(3
*tan(1/2*d*x + 1/2*c)^5 - 3*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x + 1/2*c)
 - 1)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^2))/d
```

Mupad [B]
time = 9.07, size = 270, normalized size = 2.78

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{6\,\mathrm{atan}\left(\frac{36}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}\right)}{a^2\,d}+\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{15\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (6*atan(36/(36*tan(c/2 + (d*x)/2) + 6) - (
6*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (d*x)/2) + 6)))/(a^2*d) + (4*tan(c/2 +
(d*x)/2) + (7*tan(c/2 + (d*x)/2)^2)/6 + 20*tan(c/2 + (d*x)/2)^3 - (3*tan(c/
2 + (d*x)/2)^4)/2 + 12*tan(c/2 + (d*x)/2)^5 + (15*tan(c/2 + (d*x)/2)^6)/2 -
 4*tan(c/2 + (d*x)/2)^7 - 1/2)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 12*a^2*tan(
c/2 + (d*x)/2)^4 + 12*a^2*tan(c/2 + (d*x)/2)^6 + 4*a^2*tan(c/2 + (d*x)/2)^8
)) - log(tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)/(a^2*d)
```

3.730 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{2a^2}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+$$

```
[Out] -1/2*x/a^2-3*arctanh(cos(d*x+c))/a^2/d+2*cos(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/
a^2/d+cot(d*x+c)*csc(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2788, 3855, 3852, 8, 3853, 2718, 2715}

$$\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}-\frac{x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*x/a^2 - (3*ArcTanh[Cos[c + d*x]])/(a^2*d) + (2*Cos[c + d*x])/(a^2*d) -
 Cot[c + d*x]^3/(3*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(a^2*d) - (Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(-a^6+4a^6\csc(c+dx)-a^6\csc^2(c+dx)-2a^6\csc^3(c+dx)+a^6\csc^4(c\right.}{a^8}$$

$$= -\frac{x}{a^2}-\frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \csc^4(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\,dx}{a^2}-\frac{2\int c}{}$$

$$= -\frac{x}{a^2}-\frac{4\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{a^2d}$$

$$= -\frac{x}{2a^2}-\frac{3\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+}{}$$

Mathematica [A]
time = 1.70, size = 184, normalized size = 1.90

$$-\frac{\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^4\sec^2\left(\frac{1}{2}(c+dx)\right)\left(30\cos(c+dx)-\cos(3(c+dx))+3\left(\cos(5(c+dx))+8\left(c+dx-6\cos(c+dx)+2\cos(3(c+dx))+6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\cos(2(c+dx))\left(c+dx+6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+dx)\right)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{768a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/768*((1 + Cot[(c + d*x)/2])^4*Sec[(c + d*x)/2]^2*(30*Cos[c + d*x] - Cos[
3*(c + d*x)] + 3*(Cos[5*(c + d*x)] + 8*(c + d*x - 6*Cos[c + d*x] + 2*Cos[3*
(c + d*x)] + 6*Log[Cos[(c + d*x)/2]] - Cos[2*(c + d*x)]*(c + d*x + 6*Log[Co
s[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]]) - 6*Log[Sin[(c + d*x)/2]])*Sin[c
 + d*x]))*Tan[(c + d*x)/2])/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 164, normalized size = 1.69

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^2}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+24\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^2}$
risch	$-\frac{x}{2a^2}+\frac{i\,\mathrm{e}^{2i(dx+c)}}{8a^2d}+\frac{\mathrm{e}^{i(dx+c)}}{d\,a^2}+\frac{\mathrm{e}^{-i(dx+c)}}{d\,a^2}-\frac{i\,\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{2\left(-3i\,\mathrm{e}^{4i(dx+c)}+3\,\mathrm{e}^{5i(dx+c)}-i-3\,\mathrm{e}^{i(dx+c)}\right)}{3a^2d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3}-3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^2*(1/3*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*
c)-1/3/tan(1/2*d*x+1/2*c)^3+2/tan(1/2*d*x+1/2*c)^2+1/tan(1/2*d*x+1/2*c)+24*
ln(tan(1/2*d*x+1/2*c))-16*(-1/2*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2
+1/2*tan(1/2*d*x+1/2*c)-2)/(1+tan(1/2*d*x+1/2*c)^2)^2-8*arctan(tan(1/2*d*x+
1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(91) = 182.
time = 0.50, size = 306, normalized size = 3.15

$$\frac{\frac{\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{108\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{102\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{27\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-1}{\frac{a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}-\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{72\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*((6*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^4/(cos(d*x
```

```
+ c) + 1)^4 + 102*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 27*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 1)/(a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*a^2*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7) - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^2 + 72*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.40, size = 161, normalized size = 1.66

$$\frac{3 \cos(dx+c)^5 - 4 \cos(dx+c)^3 - 9\left(\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 9\left(\cos(dx+c)^2 - 1\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 3\left(dx\cos(dx+c)^2 - 4\cos(dx+c)^3 - dx + 6\cos(dx+c)\right)\sin(dx+c) + 3\cos(dx+c)}{6\left(a^2 d\cos(dx+c)^2 - a^2 d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 - 9*(cos(d*x + c)^2 - 1)*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 9*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 3*(d*x*cos(d*x + c)^2 - 4*cos(d*x + c)^3 - d*x +
6*cos(d*x + c))*sin(d*x + c) + 3*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2
*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(91) = 182.
time = 0.49, size = 194, normalized size = 2.00

$$-\frac{\frac{12\,(dx+c)}{a^2} - \frac{72\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{24\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2} + \frac{132\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3} - \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(12*(d*x + c)/a^2 - 72*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 24*(tan(1
/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((
tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (132*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/
2*d*x + 1/2*c)^2 - 6*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3)
```

```
 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1
/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 9.16, size = 253, normalized size = 2.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^2\,d}+\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+34\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{19\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{3}}{d\,\left(8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^2\,d}+\frac{\mathrm{atan}\left(\frac{1}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}-\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - tan(c/2 + (d*x)/2)^2/(4*a^2*d) + (3*log(t
an(c/2 + (d*x)/2)))/(a^2*d) + (2*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2/
3 + 36*tan(c/2 + (d*x)/2)^3 - (19*tan(c/2 + (d*x)/2)^4)/3 + 34*tan(c/2 + (d
*x)/2)^5 + 9*tan(c/2 + (d*x)/2)^6 - 1/3)/(d*(8*a^2*tan(c/2 + (d*x)/2)^3 + 1
6*a^2*tan(c/2 + (d*x)/2)^5 + 8*a^2*tan(c/2 + (d*x)/2)^7)) - tan(c/2 + (d*x)
/2)/(8*a^2*d) + atan(1/(tan(c/2 + (d*x)/2) + 6) - (6*tan(c/2 + (d*x)/2))/(t
an(c/2 + (d*x)/2) + 6))/(a^2*d)
```

3.731 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=116

$$-\frac{2x}{a^2}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cos(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\cot(c+$$

```
[Out] -2*x/a^2+9/8*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/a^2/d-2*cot(d*x+c)/a^2/d+
2/3*cot(d*x+c)^3/a^2/d+1/8*cot(d*x+c)*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d
*x+c)^3/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2718}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{2\cot^3(c+dx)}{3a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 + (9*ArcTanh[Cos[c + d*x]])/(8*a^2*d) - Cos[c + d*x]/(a^2*d) - (
2*Cot[c + d*x])/(a^2*d) + (2*Cot[c + d*x]^3)/(3*a^2*d) + (Cot[c + d*x]*Csc[
c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\csc(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\
&= \frac{\int \left(-2a^6-a^6\csc(c+dx)+4a^6\csc^2(c+dx)-a^6\csc^3(c+dx)-2a^6\csc^4\right.}{a^8}\\
&= -\frac{2x}{a^2}-\frac{\int \csc(c+dx)\,dx}{a^2}-\frac{\int \csc^3(c+dx)\,dx}{a^2}+\frac{\int \csc^5(c+dx)\,dx}{a^2}+\frac{\int \mathrm{si}}{}\\
&= -\frac{2x}{a^2}+\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\\
&= -\frac{2x}{a^2}+\frac{3\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cos(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c}{3a^2}\\
&= -\frac{2x}{a^2}+\frac{9\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{\cos(c+dx)}{a^2d}-\frac{2\cot(c+dx)}{a^2d}+\frac{2\cot^3(c}{3a^2}
\end{aligned}$$

Mathematica [A]
time = 1.77, size = 219, normalized size = 1.89

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^4\left(192\cot(c+dx)+\csc^2\left(\frac{1}{2}(c+dx)\right)(128-6\csc(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(-8+3\csc(c+dx))+8\left(3\csc(c+dx)\left(16(c+dx)-9\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+9\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-(7+8\cos(c+dx))\sec^4\left(\frac{1}{2}(c+dx)\right)+3\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-6\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^5(c+dx)}{3072a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^2,x]

[Out] -1/3072*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^4*(192*Cot[c + d*x] + Csc[(c + d*x)/2]^2*(128 - 6*Csc[c + d*x]) + Csc[(c + d*x)/2]^4*(-8 + 3*Csc[c + d*x]) + 8*(3*Csc[c + d*x]*(16*(c + d*x) - 9*Log[Cos[(c + d*x)/2]] + 9*Log[Sin[(c + d*x)/2]]) - (7 + 8*Cos[c + d*x])*Sec[(c + d*x)/2]^4 + 3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^2 - 6*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4))*Sin[c + d*x]^5)/(a^2*d*(1 + Sin[c + d*x])^2)

Maple [A]
time = 0.28, size = 127, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{20}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-18\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{16d\,a^2}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{20}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-18\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{16d\,a^2}$
risch	$-\frac{2x}{a^2}-\frac{e^{i(dx+c)}}{2d\,a^2}-\frac{e^{-i(dx+c)}}{2d\,a^2}-\frac{96ie^{6i(dx+c)}+3\,e^{7i(dx+c)}-192ie^{4i(dx+c)}+21\,e^{5i(dx+c)}+160ie^{2i(dx+c)}+21\,e^{3i(dx+c)}}{12a^2d\left(e^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/16/d/a^2*(1/4*tan(1/2*d*x+1/2*c)^4-4/3*tan(1/2*d*x+1/2*c)^3+20*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4+4/3/tan(1/2*d*x+1/2*c)^3-20/tan(1/2*d*x+1/2*c)-18*ln(tan(1/2*d*x+1/2*c))-32/(1+tan(1/2*d*x+1/2*c)^2)-64*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(108) = 216.
time = 0.50, size = 263, normalized size = 2.27

$$\frac{\frac{\frac{16\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{224\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{384\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-3}{\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^2}-\frac{768\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{216\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/192*((16*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 224*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 384*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3)/(a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (240*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x + c) + 1)

```
^3 + 3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/a^2 - 768*arctan(sin(d*x + c)/(
cos(d*x + c) + 1))/a^2 - 216*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 187, normalized size = 1.61

$$-\frac{96\,dx\cos\left(dx+c\right)^{4}+48\,\cos\left(dx+c\right)^{5}-192\,dx\cos\left(dx+c\right)^{2}-90\,\cos\left(dx+c\right)^{3}+96\,dx-27\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+27\,\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-32\,\left(4\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+54\,\cos\left(dx+c\right)}{48\,\left(a^{2}d\cos\left(dx+c\right)^{4}-2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(96*d*x*cos(d*x + c)^4 + 48*cos(d*x + c)^5 - 192*d*x*cos(d*x + c)^2 -
 90*cos(d*x + c)^3 + 96*d*x - 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*lo
g(1/2*cos(d*x + c) + 1/2) + 27*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(
-1/2*cos(d*x + c) + 1/2) - 32*(4*cos(d*x + c)^3 - 3*cos(d*x + c))*sin(d*x +
 c) + 54*cos(d*x + c))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2
*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 159, normalized size = 1.37

$$-\frac{\frac{384\,\left(dx+c\right)}{a^{2}}+\frac{216\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}+\frac{384}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{2}}-\frac{450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}-\frac{3\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{8}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/192*(384*(d*x + c)/a^2 + 216*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 384/((
tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - (450*tan(1/2*d*x + 1/2*c)^4 - 240*tan(1/
2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*d*x + 1/2*c)^4
) - (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^6*tan(1/2*d*x + 1/2*c)^3 + 240*a^6
*tan(1/2*d*x + 1/2*c))/a^8)/d
```

Mupad [B]
time = 9.08, size = 232, normalized size = 2.00

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a^2\,d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{12\,a^2\,d}+\frac{4\,\mathrm{atan}\left(\frac{9\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9}+\frac{16}{16\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9}\right)}{a^2\,d}-\frac{9\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a^2\,d}-\frac{20\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{56\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{4}-\frac{4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{1}{4}}{d\,\left(16\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}+\frac{5\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) - tan(c/2 + (d*x)/2)^3/(12*a^2*d) + (4*atan((9*tan(c/2 + (d*x)/2))/(16*tan(c/2 + (d*x)/2) - 9) + 16/(16*tan(c/2 + (d*x)/2) - 9)))/(a^2*d) - (9*log(tan(c/2 + (d*x)/2)))/(8*a^2*d) - (tan(c/2 + (d*x)/2)^2/4 - (4*tan(c/2 + (d*x)/2))/3 + (56*tan(c/2 + (d*x)/2)^3)/3 + 32*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^5 + 1/4)/(d*(16*a^2*tan(c/2 + (d*x)/2)^4 + 16*a^2*tan(c/2 + (d*x)/2)^6)) + (5*tan(c/2 + (d*x)/2))/(4*a^2*d)

3.732 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=118

$$\frac{x}{a^2}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot^3(c+$$

```
[Out] x/a^2+3/4*arctanh(cos(d*x+c))/a^2/d+cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d
-1/5*cot(d*x+c)^5/a^2/d-3/4*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cot(d*x+c)^3*cs
c(d*x+c)/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 3554, 8, 2691, 3855, 2687, 30}

$$-\frac{\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (3*ArcTanh[Cos[c + d*x]])/(4*a^2*d) + Cot[c + d*x]/(a^2*d) - Cot[c
+ d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(5*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x
])/(4*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)-2a^2\cot^4(c+dx)\csc(c+dx)+a^2\cot^4(c+dx)\csc^2(c\right.}{a^4} \\
&= \frac{\int \cot^4(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(c+dx)\csc}{a^2} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}+\frac{\cot^3(c+dx)\csc(c+dx)}{2a^2d}-\frac{\int \cot^2(c+dx)\,dx}{a^2}+\frac{3\int \cot^2}{} \\
&= \frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{4a^2d}+ \\
&= \frac{x}{a^2}+\frac{3\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 254 vs. 2(118) = 236.
time = 0.81, size = 254, normalized size = 2.15

$$\frac{\csc^5(c+dx)\left(-40\cos(c+dx)-220\cos(3(c+dx))+68\cos(5(c+dx))+600c\sin(c+dx)+600dx\sin(c+dx)+450\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-450\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-60\sin(2(c+dx))-300c\sin(3(c+dx))-300dx\sin(3(c+dx))-225\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+225\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+150\sin(4(c+dx))+60c\sin(5(c+dx))+60dx\sin(5(c+dx))+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-45\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{960a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*Cos[c + d*x] - 220*Cos[3*(c + d*x)] + 68*Cos[5*(c + d*
x)] + 600*c*Sin[c + d*x] + 600*d*x*Sin[c + d*x] + 450*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x] - 450*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 60*Sin[2*(c + d*x)
] - 300*c*Sin[3*(c + d*x)] - 300*d*x*Sin[3*(c + d*x)] - 225*Log[Cos[(c + d*
x)/2]]*Sin[3*(c + d*x)] + 225*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 150*
Sin[4*(c + d*x)] + 60*c*Sin[5*(c + d*x)] + 60*d*x*Sin[5*(c + d*x)] + 45*Log
[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 45*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d
*x)]))/(960*a^2*d)
```

Maple [A]
time = 0.28, size = 160, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{32d\,a^2}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-18\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{32d\,a^2}$
risch	$\frac{x}{a^2}+\frac{60ie^{8i(dx+c)}+75\,e^{9i(dx+c)}-360ie^{6i(dx+c)}-30\,e^{7i(dx+c)}+320ie^{4i(dx+c)}-280ie^{2i(dx+c)}+30\,e^{3i(dx+c)}+68i-75\,e^{i}}{30a^2d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4+1/3*tan(1/2*d*x+1
/2*c)^3+8*tan(1/2*d*x+1/2*c)^2-18*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)
^5-1/3/tan(1/2*d*x+1/2*c)^3+1/tan(1/2*d*x+1/2*c)^4-8/tan(1/2*d*x+1/2*c)^2+1
8/tan(1/2*d*x+1/2*c)-24*ln(tan(1/2*d*x+1/2*c))+64*arctan(tan(1/2*d*x+1/2*c)
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(108) = 216.
time = 0.49, size = 258, normalized size = 2.19

$$-\frac{\frac{\frac{270\sin(dx+c)}{\cos(dx+c)+1}-\frac{120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}-\frac{960\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{270\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-3\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*((270*sin(d*x + c)/(cos(d*x + c) + 1) - 120*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 - 960*arctan
(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 360*log(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^2 - (15*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 270*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 3)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [**A**]
time = 0.40, size = 207, normalized size = 1.75

$$\frac{136 \cos(dx+c)^5 - 280 \cos(dx+c)^3 + 45 (\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) - 45 (\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 30 (4dx\cos(dx+c)^4 - 8dx\cos(dx+c)^2 + 5\cos(dx+c)^3 + 4dx - 3\cos(dx+c))\sin(dx+c) + 120\cos(dx+c)}{120 (a^2 d\cos(dx+c)^4 - 2a^2 d\cos(dx+c)^2 + a^2 d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(136*cos(d*x + c)^5 - 280*cos(d*x + c)^3 + 45*(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(cos(d*x + c
)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 30*
(4*d*x*cos(d*x + c)^4 - 8*d*x*cos(d*x + c)^2 + 5*cos(d*x + c)^3 + 4*d*x - 3
*cos(d*x + c))*sin(d*x + c) + 120*cos(d*x + c))/((a^2*d*cos(d*x + c)^4 - 2*
a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 195, normalized size = 1.65

$$\frac{\frac{480\,(dx+c)}{a^2} - \frac{360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2} + \frac{822\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 270\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 3}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5} + \frac{3a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 15a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 5a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 120a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 270a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{10}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(480*(d*x + c)/a^2 - 360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + (822*ta
n(1/2*d*x + 1/2*c)^5 + 270*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c
)^3 - 5*tan(1/2*d*x + 1/2*c)^2 + 15*tan(1/2*d*x + 1/2*c) - 3)/(a^2*tan(1/2*
d*x + 1/2*c)^5) + (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 15*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5*a^8*tan(1/2*d*x + 1/2*c)^3 + 120*a^8*tan(1/2*d*x + 1/2*c)^2 - 270*
a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

Mupad [B]
time = 9.85, size = 365, normalized size = 3.09

$$-\frac{3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+270\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6-270\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+960\,\mathrm{atan}\left(\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+360\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{480\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 + 15*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^9 - 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 5*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
(d*x)/2)^7 + 270*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 270*cos(c/2 +
(d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/
2)^3 + 5*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 960*atan((4*cos(c/2 +
(d*x)/2) - 3*sin(c/2 + (d*x)/2))/(3*cos(c/2 + (d*x)/2) + 4*sin(c/2 + (d*x)/
2)))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5 + 360*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(480*a^2*d*
cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.733 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=132

$$-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{5\cot(c+dx)\csc(c+dx)}{16a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\,c}{8a}$$

```
[Out] -7/16*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+5/16*cot(d*x+c)*csc(
d*x+c)/a^2/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a^2/d+1/8*cot(d*x+c)*csc(d*x+c)^3/
a^2/d-1/6*cot(d*x+c)^3*csc(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2691, 3855, 2687, 30, 3853}

$$\frac{2\cot^5(c+dx)}{5a^2d}-\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{8a^2d}+\frac{5\cot(c+dx)\csc(c+dx)}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(16*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (5*C
ot[c + d*x]*Csc[c + d*x])/(16*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x])/(4*a^2
*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x
]^3)/(6*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cot^4(c+dx)\csc(c+dx) - 2a^2\cot^4(c+dx)\csc^2(c+dx) + a^2\cot^4(c\right.}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2} - \frac{2\int \cot^4}{}$$

$$= -\frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} - \frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d} - \frac{\int \cot^2(c+dx)\,c}{2a}$$

$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{3\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc(c+dx)}{4a^2d} + $$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - $$

$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{16a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{5\cot(c+dx)\csc(c+dx)}{16a^2d} - $$

Mathematica [A]
time = 1.34, size = 145, normalized size = 1.10

$$\frac{\csc^6(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(3360\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^6(c+dx)+60\cos(c+dx)(-11+32\sin(c+dx))+6\cos(5(c+dx))(45+32\sin(c+dx))+10\cos(3(c+dx))(-89+96\sin(c+dx))\right)}{7680a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(3360*(-Log[Cos[(c
+ d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^6 + 60*Cos[c + d*x]*(-11 +
 32*Sin[c + d*x]) + 6*Cos[5*(c + d*x)]*(45 + 32*Sin[c + d*x]) + 10*Cos[3*(c
 + d*x)]*(-89 + 96*Sin[c + d*x])))/(7680*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.33, size = 176, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64d\,a^2}$
default	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{17\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64d\,a^2}$
risch	$-\frac{-480i\,e^{10i(dx+c)}+135\,e^{11i(dx+c)}+480i\,e^{8i(dx+c)}-445\,e^{9i(dx+c)}-960i\,e^{6i(dx+c)}-330\,e^{7i(dx+c)}+960i\,e^{4i(dx+c)}-330\,e^{5i(}}{120a^2d\left(e^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^2*(1/6*tan(1/2*d*x+1/2*c)^6-4/5*tan(1/2*d*x+1/2*c)^5+1/2*tan(1/2*d
*x+1/2*c)^4+4*tan(1/2*d*x+1/2*c)^3-17/2*tan(1/2*d*x+1/2*c)^2-8*tan(1/2*d*x+
1/2*c)-4/tan(1/2*d*x+1/2*c)^3-1/6/tan(1/2*d*x+1/2*c)^6+28*ln(tan(1/2*d*x+1/
2*c))+8/tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/2*c)^4+4/5/tan(1/2*d*x+1/2*c)^
5+17/2/tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(120) = 240$.
time = 0.31, size = 275, normalized size = 2.08

$$-\frac{\frac{\frac{240\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{24\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^2}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{24\sin(dx+c)}{\cos(dx+c)+1}-\frac{15\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{120\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{255\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{240\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-5\right)(\cos(dx+c)+1)^6}{a^2\sin(dx+c)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1920*((240*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6)/a^2 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/
a^2 - (24*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 120*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 255*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 240*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x +
c) + 1)^6/(a^2*sin(d*x + c)^6))/d
```

Fricas [**A**]
time = 0.37, size = 183, normalized size = 1.39

$$-\frac{192 \cos \left(dx+c\right)^{5} \sin \left(dx+c\right)+270 \cos \left(dx+c\right)^{5}-560 \cos \left(dx+c\right)^{3}+105 \left(\cos \left(dx+c\right)^{6}-3 \cos \left(dx+c\right)^{4}+3 \cos \left(dx+c\right)^{2}-1\right) \log \left(\frac{1}{2} \cos \left(dx+c\right)+\frac{1}{2}\right)-105 \left(\cos \left(dx+c\right)^{6}-3 \cos \left(dx+c\right)^{4}+3 \cos \left(dx+c\right)^{2}-1\right) \log \left(-\frac{1}{2} \cos \left(dx+c\right)+\frac{1}{2}\right)+210 \cos \left(dx+c\right)}{480 \left(a^{2} d \cos \left(dx+c\right)^{6}-3 a^{2} d \cos \left(dx+c\right)^{4}+3 a^{2} d \cos \left(dx+c\right)^{2}-a^{2} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(192*cos(d*x + c)^5*sin(d*x + c) + 270*cos(d*x + c)^5 - 560*cos(d*x
+ c)^3 + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log
(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) + 210*cos(d*x + c))/(a^2*d*cos(
d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 215, normalized size = 1.63

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{2}}-\frac{2058 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}-255 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+120 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-24 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5}{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}}+\frac{5 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}-24 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+15 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+120 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}-255 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-240 a^{10} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{12}}}{1920 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (2058*tan(1/2*d*x + 1/2*c)
^6 - 240*tan(1/2*d*x + 1/2*c)^5 - 255*tan(1/2*d*x + 1/2*c)^4 + 120*tan(1/2*
d*x + 1/2*c)^3 + 15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 5)/(
a^2*tan(1/2*d*x + 1/2*c)^6) + (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^10*tan(
```

```
1/2*d*x + 1/2*c)^5 + 15*a^10*tan(1/2*d*x + 1/2*c)^4 + 120*a^10*tan(1/2*d*x
+ 1/2*c)^3 - 255*a^10*tan(1/2*d*x + 1/2*c)^2 - 240*a^10*tan(1/2*d*x + 1/2*c
))/a^12)/d
```

Mupad [B]
time = 10.34, size = 339, normalized size = 2.57

$$\frac{5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-255\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+255\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{1920\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (5*sin(c/2 + (d*x)/2)^12 - 5*cos(c/2 + (d*x)/2)^12 - 24*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^11 + 24*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) + 15*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 120*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^9 - 255*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 240*cos(c/2
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 240*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*
x)/2)^5 + 255*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 120*cos(c/2 + (d*
x)/2)^9*sin(c/2 + (d*x)/2)^3 - 15*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^
2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin
(c/2 + (d*x)/2)^6)/(1920*a^2*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.734 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=124

$$\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+$$

```
[Out] 1/8*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-1/7*cot(d*x+c)^7/a^2/d
+1/8*cot(d*x+c)*csc(d*x+c)/a^2/d-7/12*cot(d*x+c)*csc(d*x+c)^3/a^2/d+1/3*cot
(d*x+c)*csc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2788, 3852, 8, 3853, 3855}

$$-\frac{\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}-\frac{7\cot(c+dx)\csc^3(c+dx)}{12a^2d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(8*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*
x]^7/(7*a^2*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (7*Cot[c + d*x]*Cs
c[c + d*x]^3)/(12*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \left(a^6\csc^2(c+dx) - 2a^6\csc^3(c+dx) - a^6\csc^4(c+dx) + 4a^6\csc^5(c+dx) - a^6 \right.}{a^8}$$
$$= \frac{\int \csc^2(c+dx)\,dx}{a^2} - \frac{\int \csc^4(c+dx)\,dx}{a^2} - \frac{\int \csc^6(c+dx)\,dx}{a^2} + \frac{\int \csc^8(c+dx)\,dx}{a^2}$$
$$= \frac{\cot(c+dx)\csc(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{a^2d} + \frac{\cot(c+dx)\csc^5(c+dx)}{3a^2d}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} - \frac{\cot(c+dx)\csc(c+dx}{2a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} + \frac{\cot(c+dx)\csc(c+}{8a^2d}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{8a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7a^2d} + \frac{\cot(c+dx)\csc(c+dx}{8a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 251 vs. 2(124) = 248.
time = 0.78, size = 251, normalized size = 2.02

$$-\frac{\csc^7(c+dx)\left(5880\cos(c+dx)+2184\cos(3(c+dx))-168\cos(5(c+dx))-216\cos(7(c+dx))-3675\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+3675\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-2170\sin(2(c+dx))+2205\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-2205\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-3080\sin(4(c+dx))-735\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+735\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-210\sin(6(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))-105\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))\right)}{53760a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/53760*(Csc[c + d*x]^7*(5880*Cos[c + d*x] + 2184*Cos[3*(c + d*x)] - 168*C
os[5*(c + d*x)] - 216*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c +
 d*x] + 3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] - 2170*Sin[2*(c + d*x)] + 2
205*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin
[3*(c + d*x)] - 3080*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c
+ d*x)] + 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)]
 + 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*S
in[7*(c + d*x)]))/(a^2*d)
```

Maple [A]
time = 0.32, size = 202, normalized size = 1.63

method	result
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan}{}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan}{}$
risch	$-\frac{840ie^{12i(dx+c)}+105\,e^{13i(dx+c)}-3360ie^{10i(dx+c)}+1540\,e^{11i(dx+c)}+840ie^{8i(dx+c)}+1085\,e^{9i(dx+c)}-6720ie^{6i(dx+c)}+1}{420a^{2}d\left(e^{2i(dx+c)}-1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7-2/3*tan(1/2*d*x+1/2*c)^6+3/5*tan(1/2*
d*x+1/2*c)^5+2*tan(1/2*d*x+1/2*c)^4-5*tan(1/2*d*x+1/2*c)^3+2*tan(1/2*d*x+1/
2*c)^2+11*tan(1/2*d*x+1/2*c)-1/7/tan(1/2*d*x+1/2*c)^7+5/tan(1/2*d*x+1/2*c)^
3+2/3/tan(1/2*d*x+1/2*c)^6-16*ln(tan(1/2*d*x+1/2*c))-11/tan(1/2*d*x+1/2*c)-
2/tan(1/2*d*x+1/2*c)^4-3/5/tan(1/2*d*x+1/2*c)^5-2/tan(1/2*d*x+1/2*c)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(112) = 224$.
time = 0.28, size = 314, normalized size = 2.53

$$\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}+\frac{210\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{525\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{210\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{63\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{70\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{2}}-\frac{1680\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}+\frac{\left(\frac{70\sin(dx+c)}{\cos(dx+c)+1}-\frac{63\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{210\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{525\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{210\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1155\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-15\right)(\cos(dx+c)+1)^{7}}{a^{2}\sin(dx+c)^{7}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/13440*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 210*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 525*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 210*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 70*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^2
 - 1680*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (70*sin(d*x + c)/(cos(d*
x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 210*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 525*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 210*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 - 1155*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 1
5)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

Fricas [A]
time = 0.40, size = 216, normalized size = 1.74

$$-\frac{432\cos(dx+c)^{7}-672\cos(dx+c)^{5}-105\left(\cos(dx+c)^{6}-3\cos(dx+c)^{4}+3\cos(dx+c)^{2}-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+105\left(\cos(dx+c)^{6}-3\cos(dx+c)^{4}+3\cos(dx+c)^{2}-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+70\left(3\cos(dx+c)^{5}+8\cos(dx+c)^{3}-3\cos(dx+c)\right)\sin(dx+c)}{1680\left(a^{2}d\cos(dx+c)^{6}-3a^{2}d\cos(dx+c)^{4}+3a^{2}d\cos(dx+c)^{2}-a^{2}d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(432*cos(d*x + c)^7 - 672*cos(d*x + c)^5 - 105*(cos(d*x + c)^6 - 3*
cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 70*(3*cos(d*x + c)^5 + 8*cos(d*x + c
)^3 - 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*
x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(112) = 224.
time = 0.53, size = 245, normalized size = 1.98

$$-\frac{\frac{1680\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{2}}-\frac{4356\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-210\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-63\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}-\frac{15\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-70\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+63\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+210\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-525\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+210\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1155\,a^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{14}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13440*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (4356*tan(1/2*d*x + 1/2
*c)^7 - 1155*tan(1/2*d*x + 1/2*c)^6 - 210*tan(1/2*d*x + 1/2*c)^5 + 525*tan(
1/2*d*x + 1/2*c)^4 - 210*tan(1/2*d*x + 1/2*c)^3 - 63*tan(1/2*d*x + 1/2*c)^2
 + 70*tan(1/2*d*x + 1/2*c) - 15)/(a^2*tan(1/2*d*x + 1/2*c)^7) - (15*a^12*ta
n(1/2*d*x + 1/2*c)^7 - 70*a^12*tan(1/2*d*x + 1/2*c)^6 + 63*a^12*tan(1/2*d*x
 + 1/2*c)^5 + 210*a^12*tan(1/2*d*x + 1/2*c)^4 - 525*a^12*tan(1/2*d*x + 1/2*
c)^3 + 210*a^12*tan(1/2*d*x + 1/2*c)^2 + 1155*a^12*tan(1/2*d*x + 1/2*c))/a^
14)/d
```

Mupad [**B**]
time = 10.84, size = 387, normalized size = 3.12

$$-\frac{15\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-15\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+70\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-70\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-210\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+525\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-210\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-1155\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+1155\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+210\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-525\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+210\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+63\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1680\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{13440\,a^{2}\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(15*cos(c/2 + (d*x)/2)^14 - 15*sin(c/2 + (d*x)/2)^14 + 70*cos(c/2 + (d*x)/
2)*sin(c/2 + (d*x)/2)^13 - 70*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 63
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 210*cos(c/2 + (d*x)/2)^3*sin(
c/2 + (d*x)/2)^11 + 525*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 210*co
s(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 1155*cos(c/2 + (d*x)/2)^6*sin(c/2
 + (d*x)/2)^8 + 1155*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 210*cos(c/
2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 - 525*cos(c/2 + (d*x)/2)^10*sin(c/2 + (
d*x)/2)^4 + 210*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 63*cos(c/2 + (
d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 1680*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(13440*a^2*d*cos(c/2 + (d
*x)/2)^7*sin(c/2 + (d*x)/2)^7)
```

3.735 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^7(c+dx)}{7a^2d}-\frac{11\cot(c+dx)\csc(c+dx)}{128a^2d}+\frac{7\cot(c+dx)\csc^3(c+}{64a^2d}$$

```
[Out] -11/128*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+2/7*cot(d*x+c)^7/a
^2/d-11/128*cot(d*x+c)*csc(d*x+c)/a^2/d+7/64*cot(d*x+c)*csc(d*x+c)^3/a^2/d-
1/6*cot(d*x+c)^3*csc(d*x+c)^3/a^2/d+1/16*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/8*
cot(d*x+c)^3*csc(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2691, 3853, 3855, 2687, 14}

$$\frac{2\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d}+\frac{7\cot(c+dx)\csc^3(c+dx)}{64a^2d}-\frac{11\cot(c+dx)\csc(c+dx)}{128a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(128*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (2
*Cot[c + d*x]^7)/(7*a^2*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (
7*Cot[c + d*x]*Csc[c + d*x]^3)/(64*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^3)
/(6*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(16*a^2*d) - (Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \cot^4(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^3(c+dx)-2a^2\cot^4(c+dx)\csc^4(c+dx)+a^2\cot^4(c\right.}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^3(c+dx)\,dx}{a^2}+\frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2}-\frac{2\int \cot^4(}{} \\
&= -\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}-\frac{3\int \cot^2(c+dx)}{8a} \\
&= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^2d}-\frac{\cot^3(c+dx)\csc^3(c+dx)}{6a^2d}+\frac{\cot(c+dx)\csc^5(c}{16a^2d} \\
&= \frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^7(c+dx)}{7a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{16a^2d}+\frac{7\cot(c+dx)}{64a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{16a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^7(c+dx)}{7a^2d}-\frac{11\cot(c+dx)}{128a} \\
&= -\frac{11\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{2\cot^7(c+dx)}{7a^2d}-\frac{11\cot(c+d}{12}
\end{aligned}
$$

Mathematica [A]
time = 0.65, size = 291, normalized size = 1.65

$$
-\frac{\csc^8(c+dx)\left(158270\cos(c+dx)+77210\cos(3(c+dx))-18130\cos(5(c+dx))-2310\cos(7(c+dx))+40425\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-64680\cos(2(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+32340\cos(4(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-9240\cos(6(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+1155\cos(8(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-40425\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+64680\cos(2(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-32340\cos(4(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+9240\cos(6(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-1155\cos(8(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-86016\sin(2(c+dx))-64512\sin(4(c+dx))-12288\sin(6(c+dx))+1536\sin(8(c+dx))\right)}{1720320a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/1720320*(Csc[c + d*x]^8*(158270*Cos[c + d*x] + 77210*Cos[3*(c + d*x)] -
18130*Cos[5*(c + d*x)] - 2310*Cos[7*(c + d*x)] + 40425*Log[Cos[(c + d*x)/2]
] - 64680*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 32340*Cos[4*(c + d*x)]*L
og[Cos[(c + d*x)/2]] - 9240*Cos[6*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1155*C
os[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 40425*Log[Sin[(c + d*x)/2]] + 64680
*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 32340*Cos[4*(c + d*x)]*Log[Sin[(c
 + d*x)/2]] + 9240*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 1155*Cos[8*(c +
 d*x)]*Log[Sin[(c + d*x)/2]] - 86016*Sin[2*(c + d*x)] - 64512*Sin[4*(c + d*
x)] - 12288*Sin[6*(c + d*x)] + 1536*Sin[8*(c + d*x)]))/(a^2*d)
```

Maple [A]
time = 0.36, size = 228, normalized size = 1.30

method	result
risch	$\frac{1155\,e^{15i(dx+c)}-53760ie^{12i(dx+c)}+9065\,e^{13i(dx+c)}-38605\,e^{11i(dx+c)}-53760ie^{8i(dx+c)}-79135\,e^{9i(dx+c)}+86016ie^{6i(dx+}}{6720a^2d\left(e^{2i(dx+\right.}}$

derivativedivides	$\frac{\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\tan^{2}\right.}{}$
default	$\frac{\frac{\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\tan^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d/a^2*(1/8*tan(1/2*d*x+1/2*c)^8-4/7*tan(1/2*d*x+1/2*c)^7+2/3*tan(1/2*
d*x+1/2*c)^6+4/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d*x+1/2*c)^4+4*tan(1/2*d*x+
1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2-12*tan(1/2*d*x+1/2*c)+4/7/tan(1/2*d*x+1/2*c
)^7-4/tan(1/2*d*x+1/2*c)^3-2/3/tan(1/2*d*x+1/2*c)^6+22*ln(tan(1/2*d*x+1/2*c
))+3/tan(1/2*d*x+1/2*c)^4-1/8/tan(1/2*d*x+1/2*c)^8-4/5/tan(1/2*d*x+1/2*c)^5
+2/tan(1/2*d*x+1/2*c)^2+12/tan(1/2*d*x+1/2*c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(160) = 320$.
time = 0.29, size = 355, normalized size = 2.02

$$-\frac{\frac{\frac{10080 \sin(dx+c)}{\cos(dx+c)+1}+\frac{1680 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3360 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{672 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{560 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{480 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{105 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a^2}-\frac{18480 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{480 \sin(dx+c)}{\cos(dx+c)+1}-\frac{560 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{672 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3360 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1680 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{10080 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}-105\right)(\cos(dx+c)+1)^8}{a^2 \sin(dx+c)^8}}{215040\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/215040*((10080*sin(d*x + c)/(cos(d*x + c) + 1) + 1680*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 - 3360*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 672*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 56
0*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 480*sin(d*x + c)^7/(cos(d*x + c) +
1)^7 - 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^2 - 18480*log(sin(d*x + c
)/(cos(d*x + c) + 1))/a^2 - (480*sin(d*x + c)/(cos(d*x + c) + 1) - 560*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 - 672*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 3360*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 1680*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 10080*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^2*sin(d*x + c)^8))/d
```

Fricas [A]
time = 0.39, size = 239, normalized size = 1.36

$$\frac{2310 \cos(dx+c)^7 + 490 \cos(dx+c)^5 - 8470 \cos(dx+c)^3 - 1155 \left(\cos(dx+c)^8 - 4 \cos(dx+c)^6 + 6 \cos(dx+c)^4 - 4 \cos(dx+c)^2 + 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 1155 \left(\cos(dx+c)^8 - 4 \cos(dx+c)^6 + 6 \cos(dx+c)^4 - 4 \cos(dx+c)^2 + 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 1536 \left(2 \cos(dx+c)^7 - 7 \cos(dx+c)^5\right) \sin(dx+c) + 2310 \cos(dx+c)}{26880 \left(a^2 d \cos(dx+c)^8 - 4 a^2 d \cos(dx+c)^6 + 6 a^2 d \cos(dx+c)^4 - 4 a^2 d \cos(dx+c)^2 + a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/26880*(2310*cos(d*x + c)^7 + 490*cos(d*x + c)^5 - 8470*cos(d*x + c)^3 - 1
155*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^
2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 1155*(cos(d*x + c)^8 - 4*cos(d*x + c)^
6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2) -
 1536*(2*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*sin(d*x + c) + 2310*cos(d*x + c
))/(a^2*d*cos(d*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4
- 4*a^2*d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 273, normalized size = 1.55

$$\frac{\frac{18480\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{50226\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-1680\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+3360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2520\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+672\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+560\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-480\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+105}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8}+\frac{105a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-480a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+560a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+672a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2520a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3360a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-1680a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10080a^{14}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{16}}}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/215040*(18480*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (50226*tan(1/2*d*x + 1
/2*c)^8 - 10080*tan(1/2*d*x + 1/2*c)^7 - 1680*tan(1/2*d*x + 1/2*c)^6 + 3360
*tan(1/2*d*x + 1/2*c)^5 - 2520*tan(1/2*d*x + 1/2*c)^4 + 672*tan(1/2*d*x + 1
/2*c)^3 + 560*tan(1/2*d*x + 1/2*c)^2 - 480*tan(1/2*d*x + 1/2*c) + 105)/(a^2
*tan(1/2*d*x + 1/2*c)^8) + (105*a^14*tan(1/2*d*x + 1/2*c)^8 - 480*a^14*tan(
1/2*d*x + 1/2*c)^7 + 560*a^14*tan(1/2*d*x + 1/2*c)^6 + 672*a^14*tan(1/2*d*x
 + 1/2*c)^5 - 2520*a^14*tan(1/2*d*x + 1/2*c)^4 + 3360*a^14*tan(1/2*d*x + 1/
2*c)^3 - 1680*a^14*tan(1/2*d*x + 1/2*c)^2 - 10080*a^14*tan(1/2*d*x + 1/2*c)
)/a^16)/d
```

Mupad [B]
time = 11.96, size = 435, normalized size = 2.47

$$\frac{105\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-480\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+480\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+672\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-10080\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+10080\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+1680\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-672\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3-560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+18480\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{215040\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (105*sin(c/2 + (d*x)/2)^16 - 105*cos(c/2 + (d*x)/2)^16 - 480*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^15 + 480*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2) +
```

560*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 672*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^13 - 2520*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12 + 3360*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 - 1680*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 - 10080*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9 + 10080*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 + 1680*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^6 - 3360*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5 + 2520*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 672*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^3 - 560*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2 + 18480*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)/(215040*a^2*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)

3.736 $\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{\cot^9(c+dx)}{9a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{64a^2d}+\frac{\cot(c+dx}{3}$$

```
[Out] 3/64*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-3/7*cot(d*x+c)^7/a^2/
d-1/9*cot(d*x+c)^9/a^2/d+3/64*cot(d*x+c)*csc(d*x+c)/a^2/d+1/32*cot(d*x+c)*c
sc(d*x+c)^3/a^2/d-1/8*cot(d*x+c)*csc(d*x+c)^5/a^2/d+1/4*cot(d*x+c)^3*csc(d*
x+c)^5/a^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2687, 14, 2691, 3853, 3855, 276}

$$-\frac{\cot^9(c+dx)}{9a^2d}-\frac{3\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d}+\frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{8a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{32a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{64a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(64*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (3*Co
t[c + d*x]^7)/(7*a^2*d) - Cot[c + d*x]^9/(9*a^2*d) + (3*Cot[c + d*x]*Csc[c
+ d*x])/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(32*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^5)/(8*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^5)/(4*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int (a^2\cot^4(c+dx)\csc^4(c+dx) - 2a^2\cot^4(c+dx)\csc^5(c+dx) + a^2\cot^4(c}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2} - \frac{2\int \cot}{}$$
$$= \frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d} + \frac{3\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{4a^2} + \frac{\mathrm{Subst}(\int x}{}$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{8a^2d} + \frac{\cot^3(c+dx)\csc^5(c+dx)}{4a^2d} - \frac{\int \csc^5(c+dx)\,d}{8a^2}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d} + \frac{\cot(c+dx)\csc^3(c+d}{32a^2d}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d} + \frac{3\cot(c+dx)\csc(c+}{64a^2d}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{64a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{3\cot^7(c+dx)}{7a^2d} - \frac{\cot^9(c+dx)}{9a^2d}$$

Mathematica [A]
time = 1.25, size = 313, normalized size = 1.86

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^9*(-451584*Cos[c + d*x] - 155904*Cos[3*(c + d*x)] + 20736*Cos
[5*(c + d*x)] + 14976*Cos[7*(c + d*x)] - 1664*Cos[9*(c + d*x)] + 119070*Log
[Cos[(c + d*x)/2]]*Sin[c + d*x] - 119070*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 + 212940*Sin[2*(c + d*x)] - 79380*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] +
 79380*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 195300*Sin[4*(c + d*x)] + 3
4020*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 34020*Log[Sin[(c + d*x)/2]]*S
in[5*(c + d*x)] + 16380*Sin[6*(c + d*x)] - 8505*Log[Cos[(c + d*x)/2]]*Sin[7
*(c + d*x)] + 8505*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)] - 1890*Sin[8*(c +
 d*x)] + 945*Log[Cos[(c + d*x)/2]]*Sin[9*(c + d*x)] - 945*Log[Sin[(c + d*x)
/2]]*Sin[9*(c + d*x)]))/(5160960*a^2*d)
```

Maple [A]
time = 0.43, size = 202, normalized size = 1.20

method	result

derivativedivides	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+18\tan\left(\right.}{}$
default	$\frac{\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+18\tan\left(\right.}{}$
risch	$-\frac{945\,e^{17i(dx+c)}+19584ie^{4i(dx+c)}-8190\,e^{15i(dx+c)}+8064ie^{6i(dx+c)}-97650\,e^{13i(dx+c)}+147840ie^{12i(dx+c)}-106470\,e^{11}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/512/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9-1/2*tan(1/2*d*x+1/2*c)^8+5/7*tan(1/2*d*x+1/2*c)^7-8/5*tan(1/2*d*x+1/2*c)^5+4*tan(1/2*d*x+1/2*c)^4-16/3*tan(1/2*d*x+1/2*c)^3+18*tan(1/2*d*x+1/2*c)-1/9/tan(1/2*d*x+1/2*c)^9-18/tan(1/2*d*x+1/2*c)+1/2/tan(1/2*d*x+1/2*c)^8+16/3/tan(1/2*d*x+1/2*c)^3-24*ln(tan(1/2*d*x+1/2*c))-5/7/tan(1/2*d*x+1/2*c)^7-4/tan(1/2*d*x+1/2*c)^4+8/5/tan(1/2*d*x+1/2*c)^5)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(152) = 304$.
time = 0.30, size = 314, normalized size = 1.87

$$\frac{\frac{\frac{11340\sin(dx+c)}{\cos(dx+c)+1}-\frac{3360\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2520\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1008\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{450\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{70\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}-\frac{15120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{315\sin(dx+c)}{\cos(dx+c)+1}-\frac{450\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1008\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2520\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3360\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{11340\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-70\right)(\cos(dx+c)+1)^9}{a^2\sin(dx+c)^9}}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/322560*((11340*sin(d*x + c)/(cos(d*x + c) + 1) - 3360*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2520*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1008*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 450*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 70*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 - 15120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + (315*sin(d*x + c)/(cos(d*x + c) + 1) - 450*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1008*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2520*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3360*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11340*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 70)*(cos(d*x + c) + 1)^9/(a^2*sin(d*x + c)^9))/d

Fricas [A]
time = 0.40, size = 269, normalized size = 1.60

$$-\frac{3328\cos(dx+c)^9-14976\cos(dx+c)^7+16128\cos(dx+c)^5-945\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+945\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+630\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)\sin(dx+c)}{40320\left(a^2d\cos(dx+c)^8-4a^2d\cos(dx+c)^6+6a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^2+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/40320*(3328*cos(d*x + c)^9 - 14976*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 - 945*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 945*(cos(d*x + c)^8 -
4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 630*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11
*cos(d*x + c)^3 + 3*cos(d*x + c))*sin(d*x + c))/((a^2*d*cos(d*x + c)^8 - 4*
a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^2 + a^
2*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**10/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 245, normalized size = 1.46

$$-\frac{\frac{15120 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2} - \frac{42774 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9-11340 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8+3360 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6-2520 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+1008 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-450 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+315 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-70}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9} - \frac{70 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9-315 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8+450 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7-1008 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+2520 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-3360 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+11340 a^{16} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{18}}}{322560 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^10/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/322560*(15120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (42774*tan(1/2*d*x +
1/2*c)^9 - 11340*tan(1/2*d*x + 1/2*c)^8 + 3360*tan(1/2*d*x + 1/2*c)^6 - 252
0*tan(1/2*d*x + 1/2*c)^5 + 1008*tan(1/2*d*x + 1/2*c)^4 - 450*tan(1/2*d*x +
1/2*c)^2 + 315*tan(1/2*d*x + 1/2*c) - 70)/(a^2*tan(1/2*d*x + 1/2*c)^9) - (7
0*a^16*tan(1/2*d*x + 1/2*c)^9 - 315*a^16*tan(1/2*d*x + 1/2*c)^8 + 450*a^16*
tan(1/2*d*x + 1/2*c)^7 - 1008*a^16*tan(1/2*d*x + 1/2*c)^5 + 2520*a^16*tan(1
/2*d*x + 1/2*c)^4 - 3360*a^16*tan(1/2*d*x + 1/2*c)^3 + 11340*a^16*tan(1/2*d
*x + 1/2*c))/a^18)/d
```

Mupad [B]
time = 12.56, size = 387, normalized size = 2.30

$$-\frac{70\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-70\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-315\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-11340\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+11340\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-3360\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+2520\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}-1008\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+450\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+15120\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{322560\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^10*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(70*cos(c/2 + (d*x)/2)^18 - 70*sin(c/2 + (d*x)/2)^18 + 315*cos(c/2 + (d*x)
/2)*sin(c/2 + (d*x)/2)^17 - 315*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2) -
450*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^16 + 1008*cos(c/2 + (d*x)/2)^4*
sin(c/2 + (d*x)/2)^14 - 2520*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^13 + 3
360*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^12 - 11340*cos(c/2 + (d*x)/2)^8
*sin(c/2 + (d*x)/2)^10 + 11340*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^8 -
 3360*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^6 + 2520*cos(c/2 + (d*x)/2)^
13*sin(c/2 + (d*x)/2)^5 - 1008*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^4 +
 450*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2 + 15120*log(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)/(322560*
a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^9)
```

3.737 $\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=218

$$-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^9(c+dx)}{9a^2d}-\frac{9\cot(c+dx)\csc(c+dx)}{256a^2d}-\frac{3\cot(c$$

```
[Out] -9/256*arctanh(cos(d*x+c))/a^2/d+2/5*cot(d*x+c)^5/a^2/d+4/7*cot(d*x+c)^7/a^
2/d+2/9*cot(d*x+c)^9/a^2/d-9/256*cot(d*x+c)*csc(d*x+c)/a^2/d-3/128*cot(d*x+
c)*csc(d*x+c)^3/a^2/d+9/160*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/8*cot(d*x+c)^3*
csc(d*x+c)^5/a^2/d+3/80*cot(d*x+c)*csc(d*x+c)^7/a^2/d-1/10*cot(d*x+c)^3*csc
(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.33, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2952, 2691, 3853, 3855, 2687, 276}

$$\frac{2\cot^9(c+dx)}{9a^2d}+\frac{4\cot^7(c+dx)}{7a^2d}+\frac{2\cot^5(c+dx)}{5a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d}-\frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d}-\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d}+\frac{3\cot(c+dx)\csc^7(c+dx)}{80a^2d}+\frac{9\cot(c+dx)\csc^5(c+dx)}{160a^2d}-\frac{3\cot(c+dx)\csc^3(c+dx)}{128a^2d}-\frac{9\cot(c+dx)\csc(c+dx)}{256a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(256*a^2*d) + (2*Cot[c + d*x]^5)/(5*a^2*d) + (4*
Cot[c + d*x]^7)/(7*a^2*d) + (2*Cot[c + d*x]^9)/(9*a^2*d) - (9*Cot[c + d*x]*
Csc[c + d*x])/(256*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x]^3)/(128*a^2*d) + (
9*Cot[c + d*x]*Csc[c + d*x]^5)/(160*a^2*d) - (Cot[c + d*x]^3*Csc[c + d*x]^5
)/(8*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x]^7)/(80*a^2*d) - (Cot[c + d*x]^3*
Csc[c + d*x]^7)/(10*a^2*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int (a^2\cot^4(c+dx)\csc^5(c+dx) - 2a^2\cot^4(c+dx)\csc^6(c+dx) + a^2\cot^4(c}{a^4}$$

$$= \frac{\int \cot^4(c+dx)\csc^5(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^7(c+dx)\,dx}{a^2} - \frac{2\int \cot}{}$$

$$= -\frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} - \frac{\cot^3(c+dx)\csc^7(c+dx)}{10a^2d} - \frac{3\int \cot^2(c+dx}{1}$$

$$= \frac{\cot(c+dx)\csc^5(c+dx)}{16a^2d} - \frac{\cot^3(c+dx)\csc^5(c+dx)}{8a^2d} + \frac{3\cot(c+dx)\csc^7(}{80a^2d}$$

$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{\cot(c+dx)\csc^3(c+d}{64a^2d}$$

$$= \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+dx)}{9a^2d} - \frac{3\cot(c+dx)\csc(c+}{128a^2d}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+d}{9a^2d}$$

$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{256a^2d} + \frac{2\cot^5(c+dx)}{5a^2d} + \frac{4\cot^7(c+dx)}{7a^2d} + \frac{2\cot^9(c+d}{9a^2d}$$

Mathematica [**A**]
time = 1.53, size = 353, normalized size = 1.62

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^10*(-3219300*Cos[c + d*x] - 1237320*Cos[3*(c + d*x)] + 278712
*Cos[5*(c + d*x)] + 54810*Cos[7*(c + d*x)] - 5670*Cos[9*(c + d*x)] - 357210
*Log[Cos[(c + d*x)/2]] + 595350*Cos[2*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 34
0200*Cos[4*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 127575*Cos[6*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 28350*Cos[8*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2835*Cos[
10*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 357210*Log[Sin[(c + d*x)/2]] - 595350
*Cos[2*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 340200*Cos[4*(c + d*x)]*Log[Sin[(
c + d*x)/2]] - 127575*Cos[6*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 28350*Cos[8*
(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2835*Cos[10*(c + d*x)]*Log[Sin[(c + d*x)
/2]] + 1720320*Sin[2*(c + d*x)] + 1228800*Sin[4*(c + d*x)] + 184320*Sin[6*(
c + d*x)] - 40960*Sin[8*(c + d*x)] + 4096*Sin[10*(c + d*x)]))/(41287680*a^2
*d)
```

Maple [**A**]
time = 0.48, size = 278, normalized size = 1.28

method	result
risch	$2835\,e^{19i(dx+c)}-27405\,e^{17i(dx+c)}-184320ie^{4i(dx+c)}-139356\,e^{15i(dx+c)}-1290240ie^{8i(dx+c)}+618660\,e^{13i(dx+c)}-36864$
derivativedivides	$\frac{\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10}-\frac{4\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-6\left(\tan^{4}\left(\right.\right.}{}$
default	$\frac{\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10}-\frac{4\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{4\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-6\left(\tan^{4}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1024/d/a^2*(1/10*tan(1/2*d*x+1/2*c)^10-4/9*tan(1/2*d*x+1/2*c)^9+3/4*tan(1
/2*d*x+1/2*c)^8-4/7*tan(1/2*d*x+1/2*c)^7-1/2*tan(1/2*d*x+1/2*c)^6+16/5*tan(
1/2*d*x+1/2*c)^5-6*tan(1/2*d*x+1/2*c)^4+16/3*tan(1/2*d*x+1/2*c)^3+tan(1/2*d
*x+1/2*c)^2-24*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)^2+4/7/tan(1/2*d*x+1/
2*c)^7+4/9/tan(1/2*d*x+1/2*c)^9-1/10/tan(1/2*d*x+1/2*c)^10+1/2/tan(1/2*d*x+
1/2*c)^6+36*ln(tan(1/2*d*x+1/2*c))-3/4/tan(1/2*d*x+1/2*c)^8+24/tan(1/2*d*x+
1/2*c)+6/tan(1/2*d*x+1/2*c)^4-16/3/tan(1/2*d*x+1/2*c)^3-16/5/tan(1/2*d*x+1/
2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(198) = 396$.

time = 0.29, size = 435, normalized size = 2.00

$$-\frac{\frac{\frac{30240\sin(dx+c)}{\cos(dx+c)+1}-\frac{1260\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6720\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{7560\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4032\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{630\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{945\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{560\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{126\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}{a^2}-\frac{45360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}-\frac{\left(\frac{560\sin(dx+c)}{\cos(dx+c)+1}-\frac{945\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{720\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{630\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4032\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7560\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{1260\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{30240\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-126\right)(\cos(dx+c)+1)^{10}}{a^2\sin(dx+c)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/1290240*((30240*sin(d*x + c)/(cos(d*x + c) + 1) - 1260*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 6720*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 7560*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
630*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 720*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 - 945*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 560*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 126*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)/a^2 - 45360*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - (560*sin(d*x + c)/(cos(d*x + c) +
1) - 945*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 720*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 630*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4032*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 + 7560*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6720*sin
(d*x + c)^7/(cos(d*x + c) + 1)^7 - 1260*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 30240*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 126)*(cos(d*x + c) + 1)^10/(
a^2*sin(d*x + c)^10))/d
```

Fricas [A]
time = 0.42, size = 294, normalized size = 1.35

$$\frac{5670 \cos\left(dx+c\right)^{9}-26460 \cos\left(dx+c\right)^{7}+16128 \cos\left(dx+c\right)^{5}+26460 \cos\left(dx+c\right)^{3}-2835 \left(\cos\left(dx+c\right)^{10}-5 \cos\left(dx+c\right)^{8}+10 \cos\left(dx+c\right)^{6}-10 \cos\left(dx+c\right)^{4}+5 \cos\left(dx+c\right)^{2}-1\right) \log\left(\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)+2835 \left(\cos\left(dx+c\right)^{10}-5 \cos\left(dx+c\right)^{8}+10 \cos\left(dx+c\right)^{6}-10 \cos\left(dx+c\right)^{4}+5 \cos\left(dx+c\right)^{2}-1\right) \log\left(-\frac{1}{2} \cos\left(dx+c\right)+\frac{1}{2}\right)-1024 \left(8 \cos\left(dx+c\right)^{9}-36 \cos\left(dx+c\right)^{7}+63 \cos\left(dx+c\right)^{5}\right) \sin\left(dx+c\right)-5670 \cos\left(dx+c\right)}{161280 \left(a^{2} d \cos\left(dx+c\right)^{10}-5 a^{2} d \cos\left(dx+c\right)^{8}+10 a^{2} d \cos\left(dx+c\right)^{6}-10 a^{2} d \cos\left(dx+c\right)^{4}+5 a^{2} d \cos\left(dx+c\right)^{2}-a^{2} d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/161280*(5670*cos(d*x + c)^9 - 26460*cos(d*x + c)^7 + 16128*cos(d*x + c)^5
 + 26460*cos(d*x + c)^3 - 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos
(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c
) + 1/2) + 2835*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 + 10*cos(d*x + c)^6 - 1
0*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2) - 102
4*(8*cos(d*x + c)^9 - 36*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*sin(d*x + c) -
 5670*cos(d*x + c))/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^
2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2
*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 331, normalized size = 1.52

$$\frac{\frac{45360 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^{2}}-\frac{132858 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10}-30240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+1260 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8}+6720 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}-7560 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+4032 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}-630 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}-720 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+945 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-560 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+126}{a^{2} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10}}+\frac{126 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10}-560 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+945 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8}-720 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}-630 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+4032 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}-7560 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+6720 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+1260 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-30240 a^{18} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{20}}}{1290240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(45360*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (132858*tan(1/2*d*x +
 1/2*c)^10 - 30240*tan(1/2*d*x + 1/2*c)^9 + 1260*tan(1/2*d*x + 1/2*c)^8 + 6
720*tan(1/2*d*x + 1/2*c)^7 - 7560*tan(1/2*d*x + 1/2*c)^6 + 4032*tan(1/2*d*x
 + 1/2*c)^5 - 630*tan(1/2*d*x + 1/2*c)^4 - 720*tan(1/2*d*x + 1/2*c)^3 + 945
*tan(1/2*d*x + 1/2*c)^2 - 560*tan(1/2*d*x + 1/2*c) + 126)/(a^2*tan(1/2*d*x
+ 1/2*c)^10) + (126*a^18*tan(1/2*d*x + 1/2*c)^10 - 560*a^18*tan(1/2*d*x + 1
/2*c)^9 + 945*a^18*tan(1/2*d*x + 1/2*c)^8 - 720*a^18*tan(1/2*d*x + 1/2*c)^7
```

```
 - 630*a^18*tan(1/2*d*x + 1/2*c)^6 + 4032*a^18*tan(1/2*d*x + 1/2*c)^5 - 756
0*a^18*tan(1/2*d*x + 1/2*c)^4 + 6720*a^18*tan(1/2*d*x + 1/2*c)^3 + 1260*a^1
8*tan(1/2*d*x + 1/2*c)^2 - 30240*a^18*tan(1/2*d*x + 1/2*c))/a^20)/d
```

Mupad [B]
time = 14.34, size = 531, normalized size = 2.44

$$\frac{126\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}-126\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}-560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}+560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+4032\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-7560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+1260\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-30240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+30240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-1260\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+7560\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}-4032\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+630\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-945\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+45360\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{1290240\,a^2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^11*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (126*sin(c/2 + (d*x)/2)^20 - 126*cos(c/2 + (d*x)/2)^20 - 560*cos(c/2 + (d*x
)/2)*sin(c/2 + (d*x)/2)^19 + 560*cos(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2) +
 945*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^18 - 720*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^17 - 630*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^16 + 40
32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^15 - 7560*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^14 + 6720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^13 + 12
60*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 - 30240*cos(c/2 + (d*x)/2)^9*
sin(c/2 + (d*x)/2)^11 + 30240*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9 -
1260*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^8 - 6720*cos(c/2 + (d*x)/2)^1
3*sin(c/2 + (d*x)/2)^7 + 7560*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 -
4032*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^5 + 630*cos(c/2 + (d*x)/2)^16
*sin(c/2 + (d*x)/2)^4 + 720*cos(c/2 + (d*x)/2)^17*sin(c/2 + (d*x)/2)^3 - 94
5*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^2 + 45360*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)/(1290240*
a^2*d*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^10)
```

3.738 $\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=210

$$\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{\cot^{11}(c+dx)}{11a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{128a^2d}$$

```
[Out] 3/128*arctanh(cos(d*x+c))/a^2/d-2/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2
/d-4/9*cot(d*x+c)^9/a^2/d-1/11*cot(d*x+c)^11/a^2/d+3/128*cot(d*x+c)*csc(d*x
+c)/a^2/d+1/64*cot(d*x+c)*csc(d*x+c)^3/a^2/d+1/80*cot(d*x+c)*csc(d*x+c)^5/a
^2/d-3/40*cot(d*x+c)*csc(d*x+c)^7/a^2/d+1/5*cot(d*x+c)^3*csc(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.28, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2952, 2687, 276, 2691, 3853, 3855}

$$-\frac{\cot^{11}(c+dx)}{11a^2d}-\frac{4\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^5(c+dx)}{5a^2d}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d}+\frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d}-\frac{3\cot(c+dx)\csc^7(c+dx)}{40a^2d}+\frac{\cot(c+dx)\csc^5(c+dx)}{80a^2d}+\frac{\cot(c+dx)\csc^3(c+dx)}{64a^2d}+\frac{3\cot(c+dx)\csc(c+dx)}{128a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (5*C
ot[c + d*x]^7)/(7*a^2*d) - (4*Cot[c + d*x]^9)/(9*a^2*d) - Cot[c + d*x]^11/(
11*a^2*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(128*a^2*d) + (Cot[c + d*x]*Csc[c
 + d*x]^3)/(64*a^2*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(80*a^2*d) - (3*Cot[c
 + d*x]*Csc[c + d*x]^7)/(40*a^2*d) + (Cot[c + d*x]^3*Csc[c + d*x]^7)/(5*a^2
*d)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \cot^4(c+dx)\csc^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int (a^2\cot^4(c+dx)\csc^6(c+dx) - 2a^2\cot^4(c+dx)\csc^7(c+dx) + a^2\cot^4(c}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2} + \frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2} - \frac{2\int \cot}{}$$
$$= \frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d} + \frac{3\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{5a^2} + \frac{\text{Subst}\Big(\int x}{}$$
$$= -\frac{3\cot(c+dx)\csc^7(c+dx)}{40a^2d} + \frac{\cot^3(c+dx)\csc^7(c+dx)}{5a^2d} - \frac{3\int \csc^7(c+dx}{40a^2}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{\cot}{}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{\cot}{}$$
$$= -\frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d} - \frac{\cot^{11}(c+dx)}{11a^2d} + \frac{3\,\mathrm{c}}{}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{5\cot^7(c+dx)}{7a^2d} - \frac{4\cot^9(c+dx)}{9a^2d}$$

Mathematica [A]
time = 3.80, size = 186, normalized size = 0.89

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(2661120\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cot(c+dx)\csc^{10}(c+dx)(-5402624-5752832\cos(2(c+dx))+346112\cos(4(c+dx))+583168\cos(6(c+dx))-104448\cos(8(c+dx))+8704\cos(10(c+dx))+2457378\sin(c+dx)+5907132\sin(3(c+dx))+656964\sin(5(c+dx))-121275\sin(7(c+dx))+10395\sin(9(c+dx)))\right)}{113541120a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(2661120*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) + Cot[c + d*x]*Csc[c + d*x]^10*(-5402624 - 5752832*C
os[2*(c + d*x)] + 346112*Cos[4*(c + d*x)] + 583168*Cos[6*(c + d*x)] - 10444
8*Cos[8*(c + d*x)] + 8704*Cos[10*(c + d*x)] + 2457378*Sin[c + d*x] + 590713
2*Sin[3*(c + d*x)] + 656964*Sin[5*(c + d*x)] - 121275*Sin[7*(c + d*x)] + 10
395*Sin[9*(c + d*x)])))/(113541120*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.50, size = 304, normalized size = 1.45

method	result
risch	$-\frac{10395\,\mathrm{e}^{21i(dx+c)}-110880\,\mathrm{e}^{19i(dx+c)}+957440i\mathrm{e}^{4i(dx+c)}+535689\,\mathrm{e}^{17i(dx+c)}-2534400i\mathrm{e}^{8i(dx+c)}+6564096\,\mathrm{e}^{15i(dx+c)}-5}{}$

derivativedivides	$\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{7\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{27(\tan}{}$
default	$\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{7\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{27(\tan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2048/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11-2/5*tan(1/2*d*x+1/2*c)^10+7/9*tan(
1/2*d*x+1/2*c)^9-tan(1/2*d*x+1/2*c)^8+3/7*tan(1/2*d*x+1/2*c)^7+2*tan(1/2*d*
x+1/2*c)^6-27/5*tan(1/2*d*x+1/2*c)^5+8*tan(1/2*d*x+1/2*c)^4-22/3*tan(1/2*d*
x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+38*tan(1/2*d*x+1/2*c)+22/3/tan(1/2*d*x+1/
2*c)^3+4/tan(1/2*d*x+1/2*c)^2-7/9/tan(1/2*d*x+1/2*c)^9-48*ln(tan(1/2*d*x+1/
2*c))-3/7/tan(1/2*d*x+1/2*c)^7-1/11/tan(1/2*d*x+1/2*c)^11-38/tan(1/2*d*x+1/
2*c)-2/tan(1/2*d*x+1/2*c)^6-8/tan(1/2*d*x+1/2*c)^4+1/tan(1/2*d*x+1/2*c)^8+2
/5/tan(1/2*d*x+1/2*c)^10+27/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(190) = 380$.

time = 0.28, size = 474, normalized size = 2.26

$$\frac{\frac{\frac{131670\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{13860\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{25410\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{27720\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{18711\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{6930\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1485\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3465\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{2695\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1386\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{315\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2}-\frac{166320\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{\left(\frac{1386\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{2695\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3465\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1485\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6930\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{18711\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{27720\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{25410\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{13860\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{131670\,\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-315\right)(\cos(dx+c)+1)^{11}}{a^2\sin(dx+c)^{11}}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] 1/7096320*((131670*sin(d*x + c)/(cos(d*x + c) + 1) - 13860*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - 25410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 27720*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 - 18711*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 6930*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 1485*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 3465*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 2695*sin(d*x + c)
^9/(cos(d*x + c) + 1)^9 - 1386*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 315*
sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 - 166320*log(sin(d*x + c)/(cos(d
*x + c) + 1))/a^2 + (1386*sin(d*x + c)/(cos(d*x + c) + 1) - 2695*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 3465*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1485
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6930*sin(d*x + c)^5/(cos(d*x + c) +
1)^5 + 18711*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 27720*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7 + 25410*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 13860*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - 131670*sin(d*x + c)^10/(cos(d*x + c) + 1)^
10 - 315)*(cos(d*x + c) + 1)^11/(a^2*sin(d*x + c)^11))/d
```

Fricas [A]

time = 0.41, size = 324, normalized size = 1.54

$$-\frac{34816\,\cos(dx+c)^{11}-191488\,\cos(dx+c)^{9}+430848\,\cos(dx+c)^{7}-354816\,\cos(dx+c)^{5}-10395\,\left(\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+10395\,\left(\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+1386\,\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}+128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)\sin(dx+c)}{887040\,\left(a^2d\cos(dx+c)^{10}-5\,a^2d\cos(dx+c)^{8}+10\,a^2d\cos(dx+c)^{6}-10\,a^2d\cos(dx+c)^{4}+5\,a^2d\cos(dx+c)^{2}-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/887040*(34816*cos(d*x + c)^11 - 191488*cos(d*x + c)^9 + 430848*cos(d*x +
 c)^7 - 354816*cos(d*x + c)^5 - 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 10395*(cos(d*x + c)^10 - 5*cos(d*x + c)^8 +
10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) + 1386*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7
+ 128*cos(d*x + c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))*sin(d*x + c))/(
(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x + c)^8 + 10*a^2*d*cos(d*x + c)^6 -
 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**12/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 361, normalized size = 1.72

$$-\frac{\frac{166320\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{502266\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-131670\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+13860\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+25410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-27720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+18711\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-6930\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1485\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3465\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2695\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1386\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-315}{a^2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}-\frac{315\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-1386\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+2695\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-3465\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1485\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+6930\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-18711\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+27720\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-25410\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-13860\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+131670\,a^{20}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{22}}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^12/(a+a*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] -1/7096320*(166320*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - (502266*tan(1/2*d*x
 + 1/2*c)^11 - 131670*tan(1/2*d*x + 1/2*c)^10 + 13860*tan(1/2*d*x + 1/2*c)^
9 + 25410*tan(1/2*d*x + 1/2*c)^8 - 27720*tan(1/2*d*x + 1/2*c)^7 + 18711*tan
(1/2*d*x + 1/2*c)^6 - 6930*tan(1/2*d*x + 1/2*c)^5 - 1485*tan(1/2*d*x + 1/2*
c)^4 + 3465*tan(1/2*d*x + 1/2*c)^3 - 2695*tan(1/2*d*x + 1/2*c)^2 + 1386*tan
(1/2*d*x + 1/2*c) - 315)/(a^2*tan(1/2*d*x + 1/2*c)^11) - (315*a^20*tan(1/2*
d*x + 1/2*c)^11 - 1386*a^20*tan(1/2*d*x + 1/2*c)^10 + 2695*a^20*tan(1/2*d*x
 + 1/2*c)^9 - 3465*a^20*tan(1/2*d*x + 1/2*c)^8 + 1485*a^20*tan(1/2*d*x + 1/
2*c)^7 + 6930*a^20*tan(1/2*d*x + 1/2*c)^6 - 18711*a^20*tan(1/2*d*x + 1/2*c)
^5 + 27720*a^20*tan(1/2*d*x + 1/2*c)^4 - 25410*a^20*tan(1/2*d*x + 1/2*c)^3
- 13860*a^20*tan(1/2*d*x + 1/2*c)^2 + 131670*a^20*tan(1/2*d*x + 1/2*c))/a^2
2)/d
```

Mupad [B]
time = 16.55, size = 579, normalized size = 2.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^12*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -(315*cos(c/2 + (d*x)/2)^22 - 315*sin(c/2 + (d*x)/2)^22 + 1386*cos(c/2 + (d
*x)/2)*sin(c/2 + (d*x)/2)^21 - 1386*cos(c/2 + (d*x)/2)^21*sin(c/2 + (d*x)/2
) - 2695*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^20 + 3465*cos(c/2 + (d*x)/
2)^3*sin(c/2 + (d*x)/2)^19 - 1485*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
8 - 6930*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^17 + 18711*cos(c/2 + (d*x)
/2)^6*sin(c/2 + (d*x)/2)^16 - 27720*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)
^15 + 25410*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^14 + 13860*cos(c/2 + (d
*x)/2)^9*sin(c/2 + (d*x)/2)^13 - 131670*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*
x)/2)^12 + 131670*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^10 - 13860*cos(c
/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^9 - 25410*cos(c/2 + (d*x)/2)^14*sin(c/2
 + (d*x)/2)^8 + 27720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)^7 - 18711*co
s(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^6 + 6930*cos(c/2 + (d*x)/2)^17*sin(c
/2 + (d*x)/2)^5 + 1485*cos(c/2 + (d*x)/2)^18*sin(c/2 + (d*x)/2)^4 - 3465*co
s(c/2 + (d*x)/2)^19*sin(c/2 + (d*x)/2)^3 + 2695*cos(c/2 + (d*x)/2)^20*sin(c
/2 + (d*x)/2)^2 + 166320*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^11)/(7096320*a^2*d*cos(c/2 + (d*x)/2)^11*
sin(c/2 + (d*x)/2)^11)
```

3.739 $\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{29x}{128a^3}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{3\cos^7(c+dx)}{7a^3d}-\frac{29\cos(c+dx)\sin(c+dx)}{128a^3d}+\frac{29\cos^3(c+dx)\sin(c+dx)}{64a^3d}$$

```
[Out] -29/128*x/a^3-4/3*cos(d*x+c)^3/a^3/d+7/5*cos(d*x+c)^5/a^3/d-3/7*cos(d*x+c)^
7/a^3/d-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d+29/64*cos(d*x+c)^3*sin(d*x+c)/a^
3/d+29/48*cos(d*x+c)^3*sin(d*x+c)^3/a^3/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a^3
/d
```

Rubi [A]
time = 0.33, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2954, 2952, 2645, 14, 2648, 2715, 8, 276}

$$-\frac{3\cos^7(c+dx)}{7a^3d}+\frac{7\cos^5(c+dx)}{5a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{29\sin^3(c+dx)\cos^3(c+dx)}{48a^3d}+\frac{29\sin(c+dx)\cos^3(c+dx)}{64a^3d}-\frac{29\sin(c+dx)\cos(c+dx)}{128a^3d}-\frac{29x}{128a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (4*Cos[c + d*x]^3)/(3*a^3*d) + (7*Cos[c + d*x]^5)/(5*a^
3*d) - (3*Cos[c + d*x]^7)/(7*a^3*d) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a
^3*d) + (29*Cos[c + d*x]^3*Sin[c + d*x])/(64*a^3*d) + (29*Cos[c + d*x]^3*Si
n[c + d*x]^3)/(48*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x]^5)/(8*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(a^3\cos^2(c+dx)\sin^3(c+dx) - 3a^3\cos^2(c+dx)\sin^4(c+dx) + 3a^3\cos^2(\right.}{a^6}$$
$$= \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^2}{}$$
$$= \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8a^3d} - \frac{5\int \cos^2(c+dx)\,s}{8a^3}$$
$$= \frac{3\cos^3(c+dx)\sin(c+dx)}{8a^3d} + \frac{29\cos^3(c+dx)\sin^3(c+dx)}{48a^3d} + \frac{\cos^3(c+dx)\,s}{8a^3}$$
$$= -\frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{3\cos(c+dx)\sin(c+}{16a^3d}$$
$$= -\frac{3x}{16a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+d}{12}$$
$$= -\frac{29x}{128a^3} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{7\cos^5(c+dx)}{5a^3d} - \frac{3\cos^7(c+dx)}{7a^3d} - \frac{29\cos(c+}{1}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 482 vs. $2(161) = 322$.
time = 3.35, size = 482, normalized size = 2.99

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (84*(-7 + 12870*c - 580*d*x)*Cos[c/2] - 38640*Cos[c/2 + d*x] - 38640*Cos[(3
*c)/2 + d*x] + 6720*Cos[(3*c)/2 + 2*d*x] - 6720*Cos[(5*c)/2 + 2*d*x] - 3920
*Cos[(5*c)/2 + 3*d*x] - 3920*Cos[(7*c)/2 + 3*d*x] + 5880*Cos[(7*c)/2 + 4*d*
x] - 5880*Cos[(9*c)/2 + 4*d*x] + 4368*Cos[(9*c)/2 + 5*d*x] + 4368*Cos[(11*c
)/2 + 5*d*x] - 2240*Cos[(11*c)/2 + 6*d*x] + 2240*Cos[(13*c)/2 + 6*d*x] - 72
0*Cos[(13*c)/2 + 7*d*x] - 720*Cos[(15*c)/2 + 7*d*x] + 105*Cos[(15*c)/2 + 8*
d*x] - 105*Cos[(17*c)/2 + 8*d*x] - 998928*Sin[c/2] + 1081080*c*Sin[c/2] - 4
8720*d*x*Sin[c/2] + 38640*Sin[c/2 + d*x] - 38640*Sin[(3*c)/2 + d*x] + 6720*
Sin[(3*c)/2 + 2*d*x] + 6720*Sin[(5*c)/2 + 2*d*x] + 3920*Sin[(5*c)/2 + 3*d*x
] - 3920*Sin[(7*c)/2 + 3*d*x] + 5880*Sin[(7*c)/2 + 4*d*x] + 5880*Sin[(9*c)/
2 + 4*d*x] - 4368*Sin[(9*c)/2 + 5*d*x] + 4368*Sin[(11*c)/2 + 5*d*x] - 2240*
Sin[(11*c)/2 + 6*d*x] - 2240*Sin[(13*c)/2 + 6*d*x] + 720*Sin[(13*c)/2 + 7*d
*x] - 720*Sin[(15*c)/2 + 7*d*x] + 105*Sin[(15*c)/2 + 8*d*x] + 105*Sin[(17*c
)/2 + 8*d*x])/(215040*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]

time = 0.17, size = 220, normalized size = 1.37

method	result
risch	$-\frac{29x}{128a^3}-\frac{23\cos(dx+c)}{64a^3d}+\frac{\sin(8dx+8c)}{1024d\,a^3}-\frac{3\cos(7dx+7c)}{448d\,a^3}-\frac{\sin(6dx+6c)}{48d\,a^3}+\frac{13\cos(5dx+5c)}{320d\,a^3}+\frac{7\sin(4dx+4c)}{128d\,a^3}$
derivativedivides	$\frac{16\left(-\frac{19}{420}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024}-\frac{38\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105}+\frac{667\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3072}-\frac{61\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60}-\frac{1465\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3072}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{15}\right.}{}$
default	$\frac{16\left(-\frac{19}{420}+\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1024}-\frac{38\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105}+\frac{667\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3072}-\frac{61\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60}-\frac{1465\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3072}+\frac{2\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{15}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*((-19/420+29/1024*tan(1/2*d*x+1/2*c)-38/105*tan(1/2*d*x+1/2*c)^2+6
67/3072*tan(1/2*d*x+1/2*c)^3-61/60*tan(1/2*d*x+1/2*c)^4-1465/3072*tan(1/2*d
*x+1/2*c)^5+2/15*tan(1/2*d*x+1/2*c)^6-5117/3072*tan(1/2*d*x+1/2*c)^7-19/12*
tan(1/2*d*x+1/2*c)^8+5117/3072*tan(1/2*d*x+1/2*c)^9-8/3*tan(1/2*d*x+1/2*c)^
10+1465/3072*tan(1/2*d*x+1/2*c)^11-1/4*tan(1/2*d*x+1/2*c)^12-667/3072*tan(1
/2*d*x+1/2*c)^13-29/1024*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*x+1/2*c)^2)^8-
29/1024*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(145) = 290.

time = 0.50, size = 499, normalized size = 3.10

$$\frac{\frac{\frac{3045\sin(dx+c)}{\cos(dx+c)+1}-\frac{38912\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{23345\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{109312\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{51275\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{14336\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{179095\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{170240\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{179095\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{286720\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{51275\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{26880\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{23345\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{3045\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}-4864}{a^3+\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{3045\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) - 38912*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 23345*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 109312*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 51275*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
14336*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 179095*sin(d*x + c)^7/(cos(d*x
+ c) + 1)^7 - 170240*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 179095*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 - 286720*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 51275*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 26880*sin(d*x + c)^12/(cos(d
*x + c) + 1)^12 - 23345*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - 3045*sin(d*
x + c)^15/(cos(d*x + c) + 1)^15 - 4864)/(a^3 + 8*a^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a^3*sin(d*x
```

```
 + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 +
 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^3*sin(d*x + c)^12/(cos
(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a^3*sin(d
*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(d*x + c)/(cos(d*x + c)
+ 1))/a^3)/d
```

Fricas [A]
time = 0.38, size = 90, normalized size = 0.56

$$-\frac{5760\cos\left(dx+c\right)^{7}-18816\cos\left(dx+c\right)^{5}+17920\cos\left(dx+c\right)^{3}+3045\,dx-35\left(48\cos\left(dx+c\right)^{7}-328\cos\left(dx+c\right)^{5}+454\cos\left(dx+c\right)^{3}-87\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{13440\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/13440*(5760*cos(d*x + c)^7 - 18816*cos(d*x + c)^5 + 17920*cos(d*x + c)^3
 + 3045*d*x - 35*(48*cos(d*x + c)^7 - 328*cos(d*x + c)^5 + 454*cos(d*x + c)
^3 - 87*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 218, normalized size = 1.35

$$-\frac{\frac{3045\,(dx+c)}{a^{3}}+\frac{2\left(3045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{15}+23345\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+26880\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{12}-51275\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+286720\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-179095\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+170240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+179095\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-14336\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+51275\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+109312\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-23345\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+38912\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3045\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4864\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{8}a^{3}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 + 23345*tan(
1/2*d*x + 1/2*c)^13 + 26880*tan(1/2*d*x + 1/2*c)^12 - 51275*tan(1/2*d*x + 1
/2*c)^11 + 286720*tan(1/2*d*x + 1/2*c)^10 - 179095*tan(1/2*d*x + 1/2*c)^9 +
 170240*tan(1/2*d*x + 1/2*c)^8 + 179095*tan(1/2*d*x + 1/2*c)^7 - 14336*tan(
1/2*d*x + 1/2*c)^6 + 51275*tan(1/2*d*x + 1/2*c)^5 + 109312*tan(1/2*d*x + 1/
2*c)^4 - 23345*tan(1/2*d*x + 1/2*c)^3 + 38912*tan(1/2*d*x + 1/2*c)^2 - 3045
*tan(1/2*d*x + 1/2*c) + 4864)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a^3))/d
```

Mupad [B]

time = 11.94, size = 212, normalized size = 1.32

$$-\frac{29\,x}{128\,a^3}-\frac{\frac{29\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{667\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{1465\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{192}+\frac{128\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}-\frac{5117\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+\frac{76\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\frac{5117\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{192}-\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{15}+\frac{1465\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192}+\frac{244\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{15}-\frac{667\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{192}+\frac{608\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{105}-\frac{29\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}+\frac{76}{105}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^3)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (29*x)/(128*a^3) - ((608*tan(c/2 + (d*x)/2)^2)/105 - (29*tan(c/2 + (d*x)/
2))/64 - (667*tan(c/2 + (d*x)/2)^3)/192 + (244*tan(c/2 + (d*x)/2)^4)/15 + (
1465*tan(c/2 + (d*x)/2)^5)/192 - (32*tan(c/2 + (d*x)/2)^6)/15 + (5117*tan(c
/2 + (d*x)/2)^7)/192 + (76*tan(c/2 + (d*x)/2)^8)/3 - (5117*tan(c/2 + (d*x)/
2)^9)/192 + (128*tan(c/2 + (d*x)/2)^10)/3 - (1465*tan(c/2 + (d*x)/2)^11)/19
2 + 4*tan(c/2 + (d*x)/2)^12 + (667*tan(c/2 + (d*x)/2)^13)/192 + (29*tan(c/2
 + (d*x)/2)^15)/64 + 76/105)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^8)
```

3.740 $\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=133

$$\frac{5x}{16a^3}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{\cos^7(c+dx)}{7a^3d}+\frac{5\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{5\cos^3(c+dx)\sin(c+dx)}{8a^3d}-\text{c}$$

```
[Out] 5/16*x/a^3+4/3*cos(d*x+c)^3/a^3/d-cos(d*x+c)^5/a^3/d+1/7*cos(d*x+c)^7/a^3/d
+5/16*cos(d*x+c)*sin(d*x+c)/a^3/d-5/8*cos(d*x+c)^3*sin(d*x+c)/a^3/d-1/2*cos
(d*x+c)^3*sin(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2648, 2715, 8, 2645, 14, 276}

$$\frac{\cos^7(c+dx)}{7a^3d}-\frac{\cos^5(c+dx)}{a^3d}+\frac{4\cos^3(c+dx)}{3a^3d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{2a^3d}-\frac{5\sin(c+dx)\cos^3(c+dx)}{8a^3d}+\frac{5\sin(c+dx)\cos(c+dx)}{16a^3d}+\frac{5x}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(16*a^3) + (4*Cos[c + d*x]^3)/(3*a^3*d) - Cos[c + d*x]^5/(a^3*d) + Co
s[c + d*x]^7/(7*a^3*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(16*a^3*d) - (5*Cos[
c + d*x]^3*Sin[c + d*x])/(8*a^3*d) - (Cos[c + d*x]^3*Sin[c + d*x]^3)/(2*a^3
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)\sin^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos^2(c+dx)\sin^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int (a^3\cos^2(c+dx)\sin^2(c+dx) - 3a^3\cos^2(c+dx)\sin^3(c+dx) + 3a^3\cos^2(}{a^6}$$

$$= \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a^3} - \frac{3\int \cos^2}{}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{2a^3d} + \frac{\int \cos^2(c+dx)\,d}{4a^3}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{5\cos^3(c+dx)\sin(c+dx)}{8a^3d} - \frac{\cos^3(c+dx)\sin^3(c}{2a^3d}$$

$$= \frac{x}{8a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(c}{16a^3d}$$

$$= \frac{5x}{16a^3} + \frac{4\cos^3(c+dx)}{3a^3d} - \frac{\cos^5(c+dx)}{a^3d} + \frac{\cos^7(c+dx)}{7a^3d} + \frac{5\cos(c+dx)\sin(}{16a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 429 vs. 2(133) = 266.
time = 8.09, size = 429, normalized size = 3.23

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-168*(99*c - 5*d*x)*Cos[c/2] + 609*Cos[c/2 + d*x] + 609*Cos[(3*c)/2 + d*x]
 - 63*Cos[(3*c)/2 + 2*d*x] + 63*Cos[(5*c)/2 + 2*d*x] + 91*Cos[(5*c)/2 + 3*d
*x] + 91*Cos[(7*c)/2 + 3*d*x] - 105*Cos[(7*c)/2 + 4*d*x] + 105*Cos[(9*c)/2
+ 4*d*x] - 63*Cos[(9*c)/2 + 5*d*x] - 63*Cos[(11*c)/2 + 5*d*x] + 21*Cos[(11*
c)/2 + 6*d*x] - 21*Cos[(13*c)/2 + 6*d*x] + 3*Cos[(13*c)/2 + 7*d*x] + 3*Cos[
(15*c)/2 + 7*d*x] + 16996*Sin[c/2] - 16632*c*Sin[c/2] + 840*d*x*Sin[c/2] -
609*Sin[c/2 + d*x] + 609*Sin[(3*c)/2 + d*x] - 63*Sin[(3*c)/2 + 2*d*x] - 63*
Sin[(5*c)/2 + 2*d*x] - 91*Sin[(5*c)/2 + 3*d*x] + 91*Sin[(7*c)/2 + 3*d*x] -
105*Sin[(7*c)/2 + 4*d*x] - 105*Sin[(9*c)/2 + 4*d*x] + 63*Sin[(9*c)/2 + 5*d*
x] - 63*Sin[(11*c)/2 + 5*d*x] + 21*Sin[(11*c)/2 + 6*d*x] + 21*Sin[(13*c)/2
+ 6*d*x] - 3*Sin[(13*c)/2 + 7*d*x] + 3*Sin[(15*c)/2 + 7*d*x])/(2688*a^3*d*(
Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.17, size = 179, normalized size = 1.35

method	result
risch	$\frac{5x}{16a^3}+\frac{29\cos(dx+c)}{64a^3d}+\frac{\cos(7dx+7c)}{448d\,a^3}+\frac{\sin(6dx+6c)}{64d\,a^3}-\frac{3\cos(5dx+5c)}{64d\,a^3}-\frac{5\sin(4dx+4c)}{64d\,a^3}+\frac{13\cos(3dx+3c)}{192d\,a^3}-$
derivativedivides	$\frac{\frac{8\left(\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{119\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{23\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+11\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}}{d\,a^3}$
default	$\frac{\frac{8\left(\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{119\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{23\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}+\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+11\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^3*((5/64*tan(1/2*d*x+1/2*c)^13+3/16*tan(1/2*d*x+1/2*c)^11+3/2*tan(1/2
*d*x+1/2*c)^10-119/64*tan(1/2*d*x+1/2*c)^9+23/6*tan(1/2*d*x+1/2*c)^8+1/3*ta
n(1/2*d*x+1/2*c)^6+119/64*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4-3/16*ta
n(1/2*d*x+1/2*c)^3+5/6*tan(1/2*d*x+1/2*c)^2-5/64*tan(1/2*d*x+1/2*c)+5/42)/(
1+tan(1/2*d*x+1/2*c)^2)^7+5/64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(121) = 242$.
time = 0.57, size = 416, normalized size = 3.13

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{1120\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{252\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1344\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2499\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{448\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{5152\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{2499\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2016\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{252\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{105\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-160}{a^3+\frac{7a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{21a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{35a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{35a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{21a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{7a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/168*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 1120*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 252*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1344*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 2499*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 448*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 5152*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
 + 2499*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2016*sin(d*x + c)^10/(cos(d*x
 + c) + 1)^10 - 252*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105*sin(d*x + c
)^13/(cos(d*x + c) + 1)^13 - 160)/(a^3 + 7*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 21*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 35*a^3*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 + 35*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 21*a^
3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 7*a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12 + a^3*sin(d*x + c)^14/(cos(d*x + c) + 1)^14) - 105*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.37, size = 80, normalized size = 0.60

$$\frac{48\cos(dx+c)^7-336\cos(dx+c)^5+448\cos(dx+c)^3+105\,dx+21\left(8\cos(dx+c)^5-18\cos(dx+c)^3+5\cos(dx+c)\right)\sin(dx+c)}{336\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/336*(48*cos(d*x + c)^7 - 336*cos(d*x + c)^5 + 448*cos(d*x + c)^3 + 105*d*
x + 21*(8*cos(d*x + c)^5 - 18*cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c)
)/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 179, normalized size = 1.35

$$\frac{\frac{105\,(dx+c)}{a^3}+\frac{2\left(105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+252\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+2016\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}-2499\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+5152\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+448\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+2499\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1344\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-252\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+1120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+160\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^7a^3}}{336\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/336*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^13 + 252*tan(1/2*d*x
 + 1/2*c)^11 + 2016*tan(1/2*d*x + 1/2*c)^10 - 2499*tan(1/2*d*x + 1/2*c)^9 +
 5152*tan(1/2*d*x + 1/2*c)^8 + 448*tan(1/2*d*x + 1/2*c)^6 + 2499*tan(1/2*d*
x + 1/2*c)^5 + 1344*tan(1/2*d*x + 1/2*c)^4 - 252*tan(1/2*d*x + 1/2*c)^3 + 1
120*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 160)/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^7*a^3))/d
```

Mupad [B]
time = 12.64, size = 172, normalized size = 1.29

$$\frac{5\,x}{16\,a^3}+\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{2}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{119\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{92\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{3}+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{119\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{8}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2}+\frac{20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{20}{21}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x)^2)/(a + a*sin(c + d*x))^3,x)
```

```
[Out] (5*x)/(16*a^3) + ((20*tan(c/2 + (d*x)/2)^2)/3 - (5*tan(c/2 + (d*x)/2))/8 -
(3*tan(c/2 + (d*x)/2)^3)/2 + 8*tan(c/2 + (d*x)/2)^4 + (119*tan(c/2 + (d*x)/
2)^5)/8 + (8*tan(c/2 + (d*x)/2)^6)/3 + (92*tan(c/2 + (d*x)/2)^8)/3 - (119*t
an(c/2 + (d*x)/2)^9)/8 + 12*tan(c/2 + (d*x)/2)^10 + (3*tan(c/2 + (d*x)/2)^1
1)/2 + (5*tan(c/2 + (d*x)/2)^13)/8 + 20/21)/(a^3*d*(tan(c/2 + (d*x)/2)^2 +
1)^7)
```

3.741 $\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$-\frac{7x}{16a^3}-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{7\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d}-\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3}-$$ 6d(

```
[Out] -7/16*x/a^3-7/30*cos(d*x+c)^5/a^3/d-7/16*cos(d*x+c)*sin(d*x+c)/a^3/d-7/24*c
os(d*x+c)^3*sin(d*x+c)/a^3/d-1/3*cos(d*x+c)^9/d/(a+a*sin(d*x+c))^3-1/6*cos(
d*x+c)^7/d/(a^3+a^3*sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2938, 2758, 2761, 2715, 8}

$$-\frac{7\cos^5(c+dx)}{30a^3d}-\frac{\cos^7(c+dx)}{6d\left(a^3\sin(c+dx)+a^3\right)}-\frac{7\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{7\sin(c+dx)\cos(c+dx)}{16a^3d}-\frac{7x}{16a^3}-\frac{\cos^9(c+dx)}{3d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*x)/(16*a^3) - (7*Cos[c + d*x]^5)/(30*a^3*d) - (7*Cos[c + d*x]*Sin[c + d
*x])/(16*a^3*d) - (7*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - Cos[c + d*x]
^9/(3*d*(a + a*Sin[c + d*x])^3) - Cos[c + d*x]^7/(6*d*(a^3 + a^3*Sin[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
```

```
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^8(c+dx)\sin(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\int \frac{\cos^8(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{a} \\
&= -\frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\cos^7(c+dx)}{6d\,(a^3+a^3\sin(c+dx))} - \frac{7\int \frac{\cos^6(c+dx)}{a+a\sin(c+dx)}\,dx}{6a^2} \\
&= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \frac{\cos^7(c+dx)}{6d\,(a^3+a^3\sin(c+dx))} - 7 \\
&= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^9(c+dx)}{3d(a+a\sin(c+dx))^3} - \\
&= -\frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{24a^3d} \\
&= -\frac{7x}{16a^3} - \frac{7\cos^5(c+dx)}{30a^3d} - \frac{7\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{7\cos^3(c+dx)\sin(}{24a^3d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(131) = 262.
time = 1.13, size = 366, normalized size = 2.79

$$
\frac{-21(1+40dx)\cos\left(\frac{c}{2}\right)-600\cos\left(\frac{c}{2}+dx\right)-600\cos\left(\frac{3c}{2}+dx\right)+15\cos\left(\frac{3c}{2}+2dx\right)-15\cos\left(\frac{5c}{2}+2dx\right)-140\cos\left(\frac{5c}{2}+3dx\right)-140\cos\left(\frac{7c}{2}+3dx\right)+105\cos\left(\frac{7c}{2}+4dx\right)-105\cos\left(\frac{9c}{2}+4dx\right)+36\cos\left(\frac{9c}{2}+5dx\right)+36\cos\left(\frac{11c}{2}+5dx\right)-5\cos\left(\frac{11c}{2}+6dx\right)+5\cos\left(\frac{13c}{2}+6dx\right)+21\sin\left(\frac{c}{2}\right)-840dx\sin\left(\frac{c}{2}\right)+600\sin\left(\frac{c}{2}+dx\right)-600\sin\left(\frac{3c}{2}+dx\right)+15\sin\left(\frac{3c}{2}+2dx\right)+15\sin\left(\frac{5c}{2}+2dx\right)+140\sin\left(\frac{5c}{2}+3dx\right)-140\sin\left(\frac{7c}{2}+3dx\right)+105\sin\left(\frac{7c}{2}+4dx\right)+105\sin\left(\frac{9c}{2}+4dx\right)-36\sin\left(\frac{9c}{2}+5dx\right)+36\sin\left(\frac{11c}{2}+5dx\right)-5\sin\left(\frac{11c}{2}+6dx\right)-5\sin\left(\frac{13c}{2}+6dx\right)}{1920a^3d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^8*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-21*(1 + 40*d*x)*Cos[c/2] - 600*Cos[c/2 + d*x] - 600*Cos[(3*c)/2 + d*x] +
15*Cos[(3*c)/2 + 2*d*x] - 15*Cos[(5*c)/2 + 2*d*x] - 140*Cos[(5*c)/2 + 3*d*x
] - 140*Cos[(7*c)/2 + 3*d*x] + 105*Cos[(7*c)/2 + 4*d*x] - 105*Cos[(9*c)/2 +
 4*d*x] + 36*Cos[(9*c)/2 + 5*d*x] + 36*Cos[(11*c)/2 + 5*d*x] - 5*Cos[(11*c)
/2 + 6*d*x] + 5*Cos[(13*c)/2 + 6*d*x] + 21*Sin[c/2] - 840*d*x*Sin[c/2] + 60
0*Sin[c/2 + d*x] - 600*Sin[(3*c)/2 + d*x] + 15*Sin[(3*c)/2 + 2*d*x] + 15*Si
n[(5*c)/2 + 2*d*x] + 140*Sin[(5*c)/2 + 3*d*x] - 140*Sin[(7*c)/2 + 3*d*x] +
105*Sin[(7*c)/2 + 4*d*x] + 105*Sin[(9*c)/2 + 4*d*x] - 36*Sin[(9*c)/2 + 5*d*
x] + 36*Sin[(11*c)/2 + 5*d*x] - 5*Sin[(11*c)/2 + 6*d*x] - 5*Sin[(13*c)/2 +
6*d*x])/(1920*a^3*d*(Cos[c/2] + Sin[c/2]))
```

Maple [A]
time = 0.15, size = 181, normalized size = 1.38

method	result
risch	$-\frac{7x}{16a^3}-\frac{5\cos(dx+c)}{8a^3d}-\frac{\sin(6dx+6c)}{192d\,a^3}+\frac{3\cos(5dx+5c)}{80d\,a^3}+\frac{7\sin(4dx+4c)}{64d\,a^3}-\frac{7\cos(3dx+3c)}{48d\,a^3}+\frac{\sin(2dx+2c)}{64d\,a^3}$
derivativedivides	$\frac{\frac{4\left(-\frac{7\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{73\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{9\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{37\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{11\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-37(\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{d\,a^3}$
default	$\frac{\frac{4\left(-\frac{7\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}-\frac{\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{73\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96}-\frac{9\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{37\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}-\frac{11\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-37(\cdots\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*((-7/32*tan(1/2*d*x+1/2*c)^11-1/2*tan(1/2*d*x+1/2*c)^10+73/96*tan(1
/2*d*x+1/2*c)^9-9/2*tan(1/2*d*x+1/2*c)^8+37/16*tan(1/2*d*x+1/2*c)^7-11/3*ta
n(1/2*d*x+1/2*c)^6-37/16*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4-73/96*ta
n(1/2*d*x+1/2*c)^3-17/10*tan(1/2*d*x+1/2*c)^2+7/32*tan(1/2*d*x+1/2*c)-11/30
)/(1+tan(1/2*d*x+1/2*c)^2)^6-7/32*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(119) = 238.
time = 0.52, size = 393, normalized size = 3.00

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{816\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{365\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{480\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1110\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{1760\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1110\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2160\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{365\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{240\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{105\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-176}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/120*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 816*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 365*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 480*sin(d*x + c)^4/(
```

```
cos(d*x + c) + 1)^4 - 1110*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1760*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + 1110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 2160*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 365*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 240*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 - 176)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^3*sin(
d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [**A**]
time = 0.39, size = 70, normalized size = 0.53

$$\frac{144\,\cos\left(dx+c\right)^{5}-320\,\cos\left(dx+c\right)^{3}-105\,dx-5\,\left(8\,\cos\left(dx+c\right)^{5}-50\,\cos\left(dx+c\right)^{3}+21\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*d*x - 5*(8*cos(d*x + c
)^5 - 50*cos(d*x + c)^3 + 21*cos(d*x + c))*sin(d*x + c))/(a^3*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2535 vs. $2(121) = 242$.
time = 216.16, size = 2535, normalized size = 19.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-105*d*x*tan(c/2 + d*x/2)**12/(240*a**3*d*tan(c/2 + d*x/2)**12 +
 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*
a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*
tan(c/2 + d*x/2)**2 + 240*a**3*d) - 630*d*x*tan(c/2 + d*x/2)**10/(240*a**3*
d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan
(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 +
d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 1575*d*x*tan(c/
2 + d*x/2)**8/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/
2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6
+ 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a
**3*d) - 2100*d*x*tan(c/2 + d*x/2)**6/(240*a**3*d*tan(c/2 + d*x/2)**12 + 14
40*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**
3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan
(c/2 + d*x/2)**2 + 240*a**3*d) - 1575*d*x*tan(c/2 + d*x/2)**4/(240*a**3*d*t
an(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/
```

2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x
/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 630*d*x*tan(c/2 +
d*x/2)**2/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**
10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 36
00*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*
d) - 105*d*x/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2
)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 +
 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a*
*3*d) - 210*tan(c/2 + d*x/2)**11/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a*
*3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*t
an(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2
+ d*x/2)**2 + 240*a**3*d) - 480*tan(c/2 + d*x/2)**10/(240*a**3*d*tan(c/2 +
d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2
)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 +
1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) + 730*tan(c/2 + d*x/2)**9/(24
0*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**
3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan
(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 4320*tan
(c/2 + d*x/2)**8/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d
*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)*
*6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 24
0*a**3*d) + 2220*tan(c/2 + d*x/2)**7/(240*a**3*d*tan(c/2 + d*x/2)**12 + 144
0*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3
*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(
c/2 + d*x/2)**2 + 240*a**3*d) - 3520*tan(c/2 + d*x/2)**6/(240*a**3*d*tan(c/
2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d
*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**
4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 2220*tan(c/2 + d*x/2)**
5/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 360
0*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*
d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 960
*tan(c/2 + d*x/2)**4/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2
 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x
/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2
+ 240*a**3*d) - 730*tan(c/2 + d*x/2)**3/(240*a**3*d*tan(c/2 + d*x/2)**12 +
1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a
3*d*tan(c/2 + d*x/2)6 + 3600*a**3*d*tan(c/2 + d*x/2)**4 + 1440*a**3*d*t
an(c/2 + d*x/2)**2 + 240*a**3*d) - 1632*tan(c/2 + d*x/2)**2/(240*a**3*d*tan
(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*a**3*d*tan(c/2
+ d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*tan(c/2 + d*x/2
)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) + 210*tan(c/2 + d*x/2)
/(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600
*a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d
*tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d) - 352/
(240*a**3*d*tan(c/2 + d*x/2)**12 + 1440*a**3*d*tan(c/2 + d*x/2)**10 + 3600*

```
a**3*d*tan(c/2 + d*x/2)**8 + 4800*a**3*d*tan(c/2 + d*x/2)**6 + 3600*a**3*d*
tan(c/2 + d*x/2)**4 + 1440*a**3*d*tan(c/2 + d*x/2)**2 + 240*a**3*d), Ne(d,
0)), (x*sin(c)*cos(c)**8/(a*sin(c) + a)**3, True))
```

Giac [A]
time = 0.48, size = 179, normalized size = 1.37

$$-\frac{\frac{105\,(dx+c)}{a^3}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+1760\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+816\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+176\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(105*(d*x + c)/a^3 + 2*(105*tan(1/2*d*x + 1/2*c)^11 + 240*tan(1/2*d*
x + 1/2*c)^10 - 365*tan(1/2*d*x + 1/2*c)^9 + 2160*tan(1/2*d*x + 1/2*c)^8 -
1110*tan(1/2*d*x + 1/2*c)^7 + 1760*tan(1/2*d*x + 1/2*c)^6 + 1110*tan(1/2*d*
x + 1/2*c)^5 + 480*tan(1/2*d*x + 1/2*c)^4 + 365*tan(1/2*d*x + 1/2*c)^3 + 81
6*tan(1/2*d*x + 1/2*c)^2 - 105*tan(1/2*d*x + 1/2*c) + 176)/((tan(1/2*d*x +
1/2*c)^2 + 1)^6*a^3))/d
```

Mupad [B]
time = 12.49, size = 173, normalized size = 1.32

$$-\frac{7\,x}{16\,a^3}-\frac{\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{73\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+18\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{37\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{44\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{37\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{73\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{34\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}-\frac{7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{22}{15}}{a^3\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^8*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)
```

```
[Out] - (7*x)/(16*a^3) - ((34*tan(c/2 + (d*x)/2)^2)/5 - (7*tan(c/2 + (d*x)/2))/8
+ (73*tan(c/2 + (d*x)/2)^3)/24 + 4*tan(c/2 + (d*x)/2)^4 + (37*tan(c/2 + (d*
x)/2)^5)/4 + (44*tan(c/2 + (d*x)/2)^6)/3 - (37*tan(c/2 + (d*x)/2)^7)/4 + 18
*tan(c/2 + (d*x)/2)^8 - (73*tan(c/2 + (d*x)/2)^9)/24 + 2*tan(c/2 + (d*x)/2)
^10 + (7*tan(c/2 + (d*x)/2)^11)/8 + 22/15)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1
)^6)
```

3.742 $\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$-\frac{13x}{8a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{a^3d}-\frac{\cos^3(c+dx)}{a^3d}-\frac{13\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+}{4a^3d}$$

```
[Out] -13/8*x/a^3-arctanh(cos(d*x+c))/a^3/d+cos(d*x+c)/a^3/d-cos(d*x+c)^3/a^3/d-1
3/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2954, 2952, 2715, 8, 2672, 327, 212, 2645, 30, 2648}

$$-\frac{\cos^3(c+dx)}{a^3d}+\frac{\cos(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}-\frac{13\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{13x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-13*x)/(8*a^3) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(a^3*d) - Co
s[c + d*x]^3/(a^3*d) - (13*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d) + (Cos[c +
d*x]^3*Sin[c + d*x])/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cos(c+dx)\cot(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\cos^2(c+dx)+a^3\cos(c+dx)\cot(c+dx)+3a^3\cos^2(c+dx)\sin(c\right.}{a^6}$$
$$= \frac{\int \cos(c+dx)\cot(c+dx)\,dx}{a^3} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{3\int \cos^2(c}{a}$$
$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\int \cos^2(c+dx)\,dx}{4a^3}$$
$$= -\frac{3x}{2a^3} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+dx)\sin(c+dx)}{8a^3d} + \frac{\cos^3(c}{}$$
$$= -\frac{13x}{8a^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{a^3d} - \frac{\cos^3(c+dx)}{a^3d} - \frac{13\cos(c+d}{8}$$

Mathematica [A]
time = 0.27, size = 80, normalized size = 0.81

$$\frac{-52c-52dx+8\cos(c+dx)-8\cos(3(c+dx))-32\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+32\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-24\sin(2(c+dx))+\sin(4(c+dx))}{32a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*Cot[c + d*x])/(a + a*Sin[c + d*x])^3,x]
[Out] (-52*c - 52*d*x + 8*Cos[c + d*x] - 8*Cos[3*(c + d*x)] - 32*Log[Cos[(c + d*x
)/2]] + 32*Log[Sin[(c + d*x)/2]] - 24*Sin[2*(c + d*x)] + Sin[4*(c + d*x)])/
(32*a^3*d)
```

Maple [A]
time = 0.22, size = 125, normalized size = 1.26

method	result
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\tan}{}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^3}$
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(-\frac{11\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{19\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{19\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{11\tan}{}\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}}{d\,a^3}$
risch	$-\frac{13x}{8a^3}+\frac{e^{i(dx+c)}}{8d\,a^3}+\frac{e^{-i(dx+c)}}{8d\,a^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{d\,a^3}+\frac{\sin(4dx+4c)}{32d\,a^3}-\frac{\cos(3dx+3c)}{4d\,a^3}-\frac{3\sin(2d}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(ln(tan(1/2*d*x+1/2*c))-2*(-11/8*tan(1/2*d*x+1/2*c)^7+2*tan(1/2*d*x
+1/2*c)^6-19/8*tan(1/2*d*x+1/2*c)^5+19/8*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x
+1/2*c)^2+11/8*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4-13/4*arctan(t
an(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(93) = 186$.
time = 0.52, size = 269, normalized size = 2.72

$$-\frac{\frac{\frac{11 \sin(dx+c)}{\cos(dx+c)+1}-\frac{16 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{19 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{19 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{16 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{11 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4 a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{13 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{4 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((11*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 19*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 19*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 + 16*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 11*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6
*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 13*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 - 4*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.41, size = 84, normalized size = 0.85

$$-\frac{8 \cos(dx+c)^3+13 dx-\left(2 \cos(dx+c)^3-13 \cos(dx+c)\right) \sin(dx+c)-8 \cos(dx+c)+4 \log\left(\frac{1}{2} \cos(dx+c)+\frac{1}{2}\right)-4 \log\left(-\frac{1}{2} \cos(dx+c)+\frac{1}{2}\right)}{8 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*cos(d*x + c)^3 + 13*d*x - (2*cos(d*x + c)^3 - 13*cos(d*x + c))*sin(
d*x + c) - 8*cos(d*x + c) + 4*log(1/2*cos(d*x + c) + 1/2) - 4*log(-1/2*cos(
d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.46, size = 129, normalized size = 1.30

$$-\frac{\frac{13\,(dx+c)}{a^3}-\frac{8\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{2\left(11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-19\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(13*(d*x + c)/a^3 - 8*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 2*(11*tan(1
/2*d*x + 1/2*c)^7 - 16*tan(1/2*d*x + 1/2*c)^6 + 19*tan(1/2*d*x + 1/2*c)^5 -
 19*tan(1/2*d*x + 1/2*c)^3 + 16*tan(1/2*d*x + 1/2*c)^2 - 11*tan(1/2*d*x + 1
/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

Mupad [B]
time = 11.00, size = 222, normalized size = 2.24

$$\frac{13\,\mathrm{atan}\left(\frac{169}{16\left(\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{13}{2}\right)}-\frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{169\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{13}{2}\right)}\right)}{4\,a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{19\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (13*atan(169/(16*((169*tan(c/2 + (d*x)/2))/16 + 13/2)) - (13*tan(c/2 + (d*x
)/2))/(2*((169*tan(c/2 + (d*x)/2))/16 + 13/2))))/(4*a^3*d) + log(tan(c/2 +
(d*x)/2))/(a^3*d) - ((11*tan(c/2 + (d*x)/2))/4 - 4*tan(c/2 + (d*x)/2)^2 + (
19*tan(c/2 + (d*x)/2)^3)/4 - (19*tan(c/2 + (d*x)/2)^5)/4 + 4*tan(c/2 + (d*x
)/2)^6 - (11*tan(c/2 + (d*x)/2)^7)/4)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2 + 6*a^
3*tan(c/2 + (d*x)/2)^4 + 4*a^3*tan(c/2 + (d*x)/2)^6 + a^3*tan(c/2 + (d*x)/2
)^8 + a^3))
```

3.743 $\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$\frac{x}{2a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}$$

```
[Out] 1/2*x/a^3+3*arctanh(cos(d*x+c))/a^3/d-3*cos(d*x+c)/a^3/d+1/3*cos(d*x+c)^3/a
^3/d-cot(d*x+c)/a^3/d+3/2*cos(d*x+c)*sin(d*x+c)/a^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2788, 3855, 3852, 8, 2718, 2715, 2713}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}+\frac{3\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/(2*a^3) + (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - (3*Cos[c + d*x])/(a^3*d) +
Cos[c + d*x]^3/(3*a^3*d) - Cot[c + d*x]/(a^3*d) + (3*Cos[c + d*x]*Sin[c + d
*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(c+dx)\cot^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{\int \left(2a^5-3a^5\csc(c+dx)+a^5\csc^2(c+dx)+2a^5\sin(c+dx)-3a^5\sin^2(c-\right.}{a^8}\\ &= \frac{2x}{a^3}+\frac{\int \csc^2(c+dx)\,dx}{a^3}+\frac{\int \sin^3(c+dx)\,dx}{a^3}+\frac{2\int \sin(c+dx)\,dx}{a^3}-\frac{3\int \mathrm{cs}}{}\\ &= \frac{2x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cos(c+dx)}{a^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2a^3d}-\\ &= \frac{x}{2a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{\cot(c+d}{a^3d}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 126, normalized size = 1.37

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(6(c+dx)-33\cos(c+dx)+\cos(3(c+dx))-6\cot\left(\frac{1}{2}(c+dx)\right)+36\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+9\sin(2(c+dx))+6\tan\left(\frac{1}{2}(c+dx)\right)\right)}{12d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Cot[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(6*(c + d*x) - 33*Cos[c + d*x] + C
os[3*(c + d*x)] - 6*Cot[(c + d*x)/2] + 36*Log[Cos[(c + d*x)/2]] - 36*Log[Si
n[(c + d*x)/2]] + 9*Sin[2*(c + d*x)] + 6*Tan[(c + d*x)/2]))/(12*d*(a + a*Si
n[c + d*x])^3)
```

Maple [A]
time = 0.24, size = 125, normalized size = 1.36

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{2d\,a^3}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-6\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{-6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-24\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{2d\,a^3}$
risch	$\frac{x}{2a^3}-\frac{3i e^{2i(dx+c)}}{8d\,a^3}-\frac{11\,e^{i(dx+c)}}{8d\,a^3}-\frac{11\,e^{-i(dx+c)}}{8d\,a^3}+\frac{3i e^{-2i(dx+c)}}{8d\,a^3}-\frac{2i}{a^3 d\left(e^{2i(dx+c)}-1\right)}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{d\,a^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2*c))
+8*(-3/4*tan(1/2*d*x+1/2*c)^5-tan(1/2*d*x+1/2*c)^4-3*tan(1/2*d*x+1/2*c)^2+3
/4*tan(1/2*d*x+1/2*c)-4/3)/(1+tan(1/2*d*x+1/2*c)^2)^3+2*arctan(tan(1/2*d*x+
1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(86) = 172$.
time = 0.51, size = 285, normalized size = 3.10

$$-\frac{\frac{\frac{32\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{9\,\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{72\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{9\,\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+\frac{24\,\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}+\frac{21\,\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}+3}{\frac{a^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{3\,a^{3}\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\,a^{3}\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}+\frac{a^{3}\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}}-\frac{6\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}+\frac{18\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^{3}}-\frac{3\,\sin\left(dx+c\right)}{a^{3}\left(\cos\left(dx+c\right)+1\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*((32*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 72*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 9*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 24*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 3)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^3*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
+ a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 6*arctan(sin(d*x + c)/(cos(d*x
```

```
+ c) + 1))/a^3 + 18*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 3*sin(d*x +
c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.40, size = 104, normalized size = 1.13

$$-\frac{9 \cos(dx+c)^3 - (2 \cos(dx+c)^3 + 3 dx - 18 \cos(dx+c)) \sin(dx+c) - 9 \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 9 \log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) - 3 \cos(dx+c)}{6 a^3 d \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*cos(d*x + c)^3 - (2*cos(d*x + c)^3 + 3*d*x - 18*cos(d*x + c))*sin(d
*x + c) - 9*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 9*log(-1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 3*cos(d*x + c))/(a^3*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.48, size = 147, normalized size = 1.60

$$\frac{\frac{3(dx+c)}{a^3} - \frac{18 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} + \frac{3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3} + \frac{3\left(6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)} - \frac{2\left(9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 12 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 36 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 9 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 16\right)}{\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 1\right)^3 a^3}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)/a^3 - 18*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*d*
x + 1/2*c)/a^3 + 3*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c))
- 2*(9*tan(1/2*d*x + 1/2*c)^5 + 12*tan(1/2*d*x + 1/2*c)^4 + 36*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 16)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a
^3))/d
```

Mupad [B]
time = 9.12, size = 231, normalized size = 2.51

$$\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2 a^3 d} - \frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 8 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 24 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + \frac{32 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{3} + 1}{d\left(2 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + 6 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 6 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 2 a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)} - \frac{3 \ln\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^3 d} - \frac{\mathrm{atan}\left(\frac{1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 6} - \frac{6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 6}\right)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((32*tan(c/2 + (d*x)/2))/3 - 3*tan(c/2 + (d*
x)/2)^2 + 24*tan(c/2 + (d*x)/2)^3 + 3*tan(c/2 + (d*x)/2)^4 + 8*tan(c/2 + (d
*x)/2)^5 + 7*tan(c/2 + (d*x)/2)^6 + 1)/(d*(6*a^3*tan(c/2 + (d*x)/2)^3 + 6*a
^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (d*x)/2)^7 + 2*a^3*tan(c/2 + (d*x
)/2))) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - atan(1/(tan(c/2 + (d*x)/2) +
 6) - (6*tan(c/2 + (d*x)/2))/(tan(c/2 + (d*x)/2) + 6))/(a^3*d)
```

3.744 $\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=98

$$\frac{5x}{2a^3}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cos(c+dx)\sin(c+dx}{2a^3d}$$

```
[Out] 5/2*x/a^3-5/2*arctanh(cos(d*x+c))/a^3/d+3*cos(d*x+c)/a^3/d+3*cot(d*x+c)/a^3
/d-1/2*cot(d*x+c)*csc(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2718, 2715}

$$\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{5x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(2*a^3) - (5*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (3*Cos[c + d*x])/(a^3
*d) + (3*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
```

```
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)\csc(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(2a^5+2a^5\csc(c+dx)-3a^5\csc^2(c+dx)+a^5\csc^3(c+dx)-3a^5\sin(c\right.}{a^8} \\
&= \frac{2x}{a^3}+\frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \sin^2(c+dx)\,dx}{a^3}+\frac{2\int \csc(c+dx)\,dx}{a^3}-\frac{3\int c}{} \\
&= \frac{2x}{a^3}-\frac{2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{3\cos(c+dx)}{a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d} \\
&= \frac{5x}{2a^3}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)}{a^3d}-\frac{\cot(c+}{}
\end{aligned}$$

Mathematica [A]
time = 0.62, size = 144, normalized size = 1.47

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(20(c+dx)+24\cos(c+dx)+12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-20\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)-2\sin(2(c+dx))-12\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(20*(c + d*x) + 24*Cos[c + d*x] +
12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] + 20*Lo
g[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 - 2*Sin[2*(c + d*x)] - 12*Tan[(c +
 d*x)/2]))/(8*d*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.26, size = 140, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+10\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{4d\,a^3}$
risch	$\frac{5x}{2a^3}+\frac{i\,e^{2i(dx+c)}}{8d\,a^3}+\frac{3\,e^{i(dx+c)}}{2d\,a^3}+\frac{3\,e^{-i(dx+c)}}{2d\,a^3}-\frac{i\,e^{-2i(dx+c)}}{8d\,a^3}+\frac{e^{3i(dx+c)}+e^{i(dx+c)}+6i\,e^{2i(dx+c)}-6i}{a^3d\left(e^{2i(dx+c)}-1\right)^2}+\frac{5\ln\left(e^{i(dx-}\right.}{2d\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/2*tan(1/2*d*x+1/2*c)^2-6*tan(1/2*d*x+1/2*c)-1/2/tan(1/2*d*x+1/
2*c)^2+6/tan(1/2*d*x+1/2*c)+10*ln(tan(1/2*d*x+1/2*c))+16*(1/4*tan(1/2*d*x+1
/2*c)^3+3/2*tan(1/2*d*x+1/2*c)^2-1/4*tan(1/2*d*x+1/2*c)+3/2)/(1+tan(1/2*d*x
+1/2*c)^2)^2+20*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(90) = 180.
time = 0.49, size = 267, normalized size = 2.72

$$\frac{\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{46\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{47\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{\frac{12\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}{a^3}+\frac{40\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{20\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*((12*sin(d*x + c)/(cos(d*x + c) + 1) + 46*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 16*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 47*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - (12*sin(d*x + c)/(cos(d*x + c) + 1)
 - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 40*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^3 + 20*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.40, size = 130, normalized size = 1.33

$$\frac{10\,dx\cos\left(dx+c\right)^{2}+12\,\cos\left(dx+c\right)^{3}-10\,dx-5\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+5\,\left(\cos\left(dx+c\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,\left(\cos\left(dx+c\right)^{3}+5\,\cos\left(dx+c\right)\right)\sin\left(dx+c\right)-10\,\cos\left(dx+c\right)}{4\,\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(10*d*x*cos(d*x + c)^2 + 12*cos(d*x + c)^3 - 10*d*x - 5*(cos(d*x + c)^2
 - 1)*log(1/2*cos(d*x + c) + 1/2) + 5*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(cos(d*x + c)^3 + 5*cos(d*x + c))*sin(d*x + c) - 10*cos(d*
x + c))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.50, size = 172, normalized size = 1.76

$$\frac{\frac{20\,(dx+c)}{a^{3}}+\frac{20\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}-\frac{10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-20\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}a^{3}}+\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(20*(d*x + c)/a^3 + 20*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (10*tan(1/2
*d*x + 1/2*c)^6 - 20*tan(1/2*d*x + 1/2*c)^5 - 27*tan(1/2*d*x + 1/2*c)^4 - 1
6*tan(1/2*d*x + 1/2*c)^3 - 36*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2
*c) + 1)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan
(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 9.24, size = 228, normalized size = 2.33

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{5\,\mathrm{atan}\left(\frac{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}+\frac{25}{25\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-25}\right)}{a^3\,d}+\frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{47\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (5*atan((25*tan(c/2 + (d*x)/2))/(25*tan(c/
2 + (d*x)/2) - 25) + 25/(25*tan(c/2 + (d*x)/2) - 25)))/(a^3*d) + (5*log(tan
(c/2 + (d*x)/2)))/(2*a^3*d) + (6*tan(c/2 + (d*x)/2) + 23*tan(c/2 + (d*x)/2)
^2 + 8*tan(c/2 + (d*x)/2)^3 + (47*tan(c/2 + (d*x)/2)^4)/2 + 10*tan(c/2 + (d
*x)/2)^5 - 1/2)/(d*(4*a^3*tan(c/2 + (d*x)/2)^2 + 8*a^3*tan(c/2 + (d*x)/2)^4
 + 4*a^3*tan(c/2 + (d*x)/2)^6)) - (3*tan(c/2 + (d*x)/2))/(2*a^3*d)
```

3.745 $\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=92

$$-\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cos(c+dx)}{a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

```
[Out] -3*x/a^3-1/2*arctanh(cos(d*x+c))/a^3/d-cos(d*x+c)/a^3/d-3*cot(d*x+c)/a^3/d-1/3*cot(d*x+c)^3/a^3/d+3/2*cot(d*x+c)*csc(d*x+c)/a^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2954, 2951, 3855, 3852, 8, 3853, 2718}

$$-\frac{\cos(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*x)/a^3 - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - Cos[c + d*x]/(a^3*d) - (3*Cot[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a^3*d) + (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[ExpandTrig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m + p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (GtQ[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^2(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-3a^5+2a^5\csc(c+dx)+2a^5\csc^2(c+dx)-3a^5\csc^3(c+dx)+a^5\csc^4\right.}{a^8}$$

$$= -\frac{3x}{a^3}+\frac{\int \csc^4(c+dx)\,dx}{a^3}+\frac{\int \sin(c+dx)\,dx}{a^3}+\frac{2\int \csc(c+dx)\,dx}{a^3}+\frac{2\int c}{}$$

$$= -\frac{3x}{a^3}-\frac{2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cos(c+dx)}{a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}$$

$$= -\frac{3x}{a^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cos(c+dx)}{a^3d}-\frac{3\cot(c+dx)}{a^3d}-\frac{\cot^3(c+dx}{3a^3d}$$

Mathematica [A]
time = 1.77, size = 132, normalized size = 1.43

$$\frac{\csc^3(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-12\left(6(c+dx)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^3(c+dx)+2\cos(3(c+dx))(8+3\sin(c+dx))+6\cos(c+dx)(-4+5\sin(c+dx))\right)}{24a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-12*(6*(c + d*x) +
 Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3 + 2*Cos[3*(c
 + d*x)]*(8 + 3*Sin[c + d*x]) + 6*Cos[c + d*x]*(-4 + 5*Sin[c + d*x])))/(24*
a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.27, size = 127, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{11}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^3}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}-\frac{11}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{8d\,a^3}$
risch	$-\frac{3x}{a^3}-\frac{e^{i(dx+c)}}{2d\,a^3}-\frac{e^{-i(dx+c)}}{2d\,a^3}-\frac{12i e^{4i(dx+c)}+9\,e^{5i(dx+c)}-36i e^{2i(dx+c)}+16i-9\,e^{i(dx+c)}}{3a^3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2d\,a^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^3*(1/3*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)^2+11*tan(1/2*d*x+1
/2*c)-1/3/tan(1/2*d*x+1/2*c)^3+3/tan(1/2*d*x+1/2*c)^2-11/tan(1/2*d*x+1/2*c)
+4*ln(tan(1/2*d*x+1/2*c))-16/(1+tan(1/2*d*x+1/2*c)^2)-48*arctan(tan(1/2*d*x
+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(86) = 172$.
time = 0.49, size = 242, normalized size = 2.63

$$\frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{34\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{39\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{33\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-1}{\frac{a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}+\frac{\frac{33\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{144\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 34*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 39*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 33*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 1)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^3*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5) + (33*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 -
 144*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 12*log(sin(d*x + c)/(cos
(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.38, size = 150, normalized size = 1.63

$$-\frac{32\cos(dx+c)^3+3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\left(\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+6\left(6\,dx\cos(dx+c)^2+2\cos(dx+c)^3-6\,dx+\cos(dx+c)\right)\sin(dx+c)-36\cos(dx+c)}{12\left(a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(32*cos(d*x + c)^3 + 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) + 6*(6*d*x*cos(d*x + c)^2 + 2*cos(d*x + c)^3 - 6*d*x + cos(d*x + c)
)*sin(d*x + c) - 36*cos(d*x + c))/((a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x +
 c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 157, normalized size = 1.71

$$-\frac{\frac{72\,(dx+c)}{a^3}-\frac{12\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}+\frac{22\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}{a^3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}-\frac{a^6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+33\,a^6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/24*(72*(d*x + c)/a^3 - 12*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 48/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3) + (22*tan(1/2*d*x + 1/2*c)^3 + 33*tan(1/2*d*x
+ 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 1)/(a^3*tan(1/2*d*x + 1/2*c)^3) - (a^
6*tan(1/2*d*x + 1/2*c)^3 - 9*a^6*tan(1/2*d*x + 1/2*c)^2 + 33*a^6*tan(1/2*d*
x + 1/2*c))/a^9)/d
```

Mupad [B]
time = 9.26, size = 219, normalized size = 2.38

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^3\,d}-\frac{3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}+\frac{6\,\mathrm{atan}\left(\frac{36}{36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}-\frac{6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6}\right)}{a^3\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^3\,d}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d}-\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{34\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1}{3}}{d\,\left(8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (3*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) + (6*a
tan(36/(36*tan(c/2 + (d*x)/2) + 6) - (6*tan(c/2 + (d*x)/2))/(36*tan(c/2 + (
d*x)/2) + 6)))/(a^3*d) + log(tan(c/2 + (d*x)/2))/(2*a^3*d) + (11*tan(c/2 +
(d*x)/2))/(8*a^3*d) - ((34*tan(c/2 + (d*x)/2)^2)/3 - 3*tan(c/2 + (d*x)/2) +
 13*tan(c/2 + (d*x)/2)^3 + 11*tan(c/2 + (d*x)/2)^4 + 1/3)/(d*(8*a^3*tan(c/2
 + (d*x)/2)^3 + 8*a^3*tan(c/2 + (d*x)/2)^5))
```

3.746 $\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{x}{a^3}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}$$

```
[Out] x/a^3+13/8*arctanh(cos(d*x+c))/a^3/d+cot(d*x+c)/a^3/d+cot(d*x+c)^3/a^3/d-11
/8*cot(d*x+c)*csc(d*x+c)/a^3/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2954, 2952, 3554, 8, 2691, 3855, 2687, 30, 3853}

$$\frac{\cot^3(c+dx)}{a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{11\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 + (13*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + Cot[c + d*x]/(a^3*d) + Cot[c
 + d*x]^3/(a^3*d) - (11*Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) - (Cot[c + d*x
]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \cot^2(c+dx)\csc^3(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^2(c+dx)+3a^3\cot^2(c+dx)\csc(c+dx)-3a^3\cot^2(c+dx)\,\mathrm{cs}\right.}{a^6} \\
&= -\frac{\int \cot^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a^3}+\frac{3\int \cot^2(c+dx)}{a^3} \\
&= \frac{\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\int \\
&= \frac{x}{a^3}+\frac{3\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+}{} \\
&= \frac{x}{a^3}+\frac{13\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{\cot(c+dx)}{a^3d}+\frac{\cot^3(c+dx)}{a^3d}-\frac{11\cot(c+}{}
\end{aligned}
$$

Mathematica [A]

time = 1.83, size = 165, normalized size = 1.70

$$
\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-22\csc^2\left(\frac{1}{2}(c+dx)\right)+22\sec^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)+8\left(8c+8dx+13\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-13\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)\right)+\csc^4\left(\frac{1}{2}(c+dx)\right)(-1+4\sin(c+dx))\right)}{64a^3d(1+\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^5)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-22*Csc[(c + d*x)/2]^2 + 22*Sec[(
c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 8*(8*c + 8*d*x + 13*Log[Cos[(c + d*x)/
2]] - 13*Log[Sin[(c + d*x)/2]] - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4) + Csc
[(c + d*x)/2]^4*(-1 + 4*Sin[c + d*x])))/(64*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]

time = 0.30, size = 136, normalized size = 1.40

method	result
derivativedivides	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+}{16d\,a^3}$
default	$\frac{\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{6}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+}{16d\,a^3}$
risch	$\frac{x}{a^3}+\frac{11\,\mathrm{e}^{7i(dx+c)}-19\,\mathrm{e}^{5i(dx+c)}-16i\mathrm{e}^{6i(dx+c)}-19\,\mathrm{e}^{3i(dx+c)}+11\,\mathrm{e}^{i(dx+c)}+16i\mathrm{e}^{2i(dx+c)}}{4d\,a^3\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}-\frac{13\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{8d\,a^3}+\frac{13\,\mathrm{l}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/d/a^3*(1/4*tan(1/2*d*x+1/2*c)^4-2*tan(1/2*d*x+1/2*c)^3+6*tan(1/2*d*x+1
/2*c)^2-2*tan(1/2*d*x+1/2*c)-1/4/tan(1/2*d*x+1/2*c)^4+2/tan(1/2*d*x+1/2*c)^
3-6/tan(1/2*d*x+1/2*c)^2+2/tan(1/2*d*x+1/2*c)-26*ln(tan(1/2*d*x+1/2*c))+32*
arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(91) = 182$.
time = 0.49, size = 218, normalized size = 2.25

$$-\frac{\frac{\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{a^3}-\frac{128\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{104\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{24\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-1\right)(\cos(dx+c)+1)^4}{a^3\sin(dx+c)^4}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/64*((8*sin(d*x + c)/(cos(d*x + c) + 1) - 24*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4)/a^3 - 128*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 + 104*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (8*sin(d*x + c)/(cos(d*x + c) + 1)
- 24*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 8*sin(d*x + c)^3/(cos(d*x + c) +
 1)^3 - 1)*(cos(d*x + c) + 1)^4/(a^3*sin(d*x + c)^4))/d
```

Fricas [**A**]
time = 0.43, size = 164, normalized size = 1.69

$$\frac{16\,dx\cos(dx+c)^4-32\,dx\cos(dx+c)^2+22\cos(dx+c)^3+16\,dx+13\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-13\left(\cos(dx+c)^4-2\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+16\cos(dx+c)\sin(dx+c)-26\cos(dx+c)}{16\left(a^3d\cos(dx+c)^4-2\,a^3d\cos(dx+c)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(16*d*x*cos(d*x + c)^4 - 32*d*x*cos(d*x + c)^2 + 22*cos(d*x + c)^3 + 1
6*d*x + 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1
/2) - 13*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) + 16*cos(d*x + c)*sin(d*x + c) - 26*cos(d*x + c))/(a^3*d*cos(d*x + c)^4
- 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A]
time = 0.51, size = 166, normalized size = 1.71

$$\frac{\frac{192\,(dx+c)}{a^3}-\frac{312\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{650\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}+\frac{3\left(a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^{12}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(192*(d*x + c)/a^3 - 312*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + (650*ta
n(1/2*d*x + 1/2*c)^4 + 24*tan(1/2*d*x + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^
2 + 24*tan(1/2*d*x + 1/2*c) - 3)/(a^3*tan(1/2*d*x + 1/2*c)^4) + 3*(a^9*tan(
1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*c)^3 + 24*a^9*tan(1/2*d*x + 1/
2*c)^2 - 8*a^9*tan(1/2*d*x + 1/2*c))/a^12)/d
```

Mupad [B]
time = 9.53, size = 315, normalized size = 3.25

$$-\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+128\,\mathrm{atan}\left(\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-13\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{13\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+104\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^5*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(cos(c/2 + (d*x)/2)^8 - sin(c/2 + (d*x)/2)^8 + 8*cos(c/2 + (d*x)/2)*sin(c/
2 + (d*x)/2)^7 - 8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - 24*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^6 + 8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^
5 - 8*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 24*cos(c/2 + (d*x)/2)^6*s
in(c/2 + (d*x)/2)^2 + 128*atan((8*cos(c/2 + (d*x)/2) - 13*sin(c/2 + (d*x)/2
))/(13*cos(c/2 + (d*x)/2) + 8*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^4 + 104*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
+ (d*x)/2)^4*sin(c/2 + (d*x)/2)^4)/(64*a^3*d*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^4)
```

3.747 $\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=100

$$-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{5a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}$$

```
[Out] -7/8*arctanh(cos(d*x+c))/a^3/d-4/3*cot(d*x+c)^3/a^3/d-1/5*cot(d*x+c)^5/a^3/
d+1/8*cot(d*x+c)*csc(d*x+c)/a^3/d+3/4*cot(d*x+c)*csc(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2691, 3855, 2687, 30, 3853, 14}

$$-\frac{\cot^5(c+dx)}{5a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}+\frac{\cot(c+dx)\csc(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-7*ArcTanh[Cos[c + d*x]])/(8*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[c
 + d*x]^5/(5*a^3*d) + (Cot[c + d*x]*Csc[c + d*x])/(8*a^3*d) + (3*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^6(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^4(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int (-a^3\cot^2(c+dx)\csc(c+dx)+3a^3\cot^2(c+dx)\csc^2(c+dx)-3a^3\cot^2}{a^6}$$

$$= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a^3} + \frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3} + \frac{3\int \cot}{}$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d} + \frac{\int \csc(c+dx)\,dx}{2a^3} +$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{\cot^3(c+dx)}{a^3d} + \frac{\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{3\cot(}{}$$

$$= -\frac{7\tanh^{-1}(\cos(c+dx))}{8a^3d} - \frac{4\cot^3(c+dx)}{3a^3d} - \frac{\cot^5(c+dx)}{5a^3d} + \frac{\cot(c+dx)\mathrm{cs}}{8a^3d}$$

Mathematica [A]

time = 1.23, size = 189, normalized size = 1.89

$$-\frac{\csc^5(c+dx)\left(560\cos(c+dx)-40\cos(3(c+dx))-136\cos(5(c+dx))+1050\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-1050\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-780\sin(2(c+dx))-525\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+525\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+30\sin(4(c+dx))+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-105\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{1920a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^6)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(Csc[c + d*x]^5*(560*Cos[c + d*x] - 40*Cos[3*(c + d*x)] - 136*Cos[5
*(c + d*x)] + 1050*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 1050*Log[Sin[(c + d
*x)/2]]*Sin[c + d*x] - 780*Sin[2*(c + d*x)] - 525*Log[Cos[(c + d*x)/2]]*Sin
[3*(c + d*x)] + 525*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 30*Sin[4*(c +
d*x)] + 105*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 105*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)]))/(a^3*d)
```

Maple [A]

time = 0.31, size = 150, normalized size = 1.50

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^3}$
risch	$-\frac{-360i\,e^{8i(dx+c)}+15\,e^{9i(dx+c)}+960i\,e^{6i(dx+c)}-390\,e^{7i(dx+c)}-400i\,e^{4i(dx+c)}+320i\,e^{2i(dx+c)}+390\,e^{3i(dx+c)}-136i-15\,e^{i}}{60a^3d\left(e^{2i(dx+c)}-1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5-3/2*tan(1/2*d*x+1/2*c)^4+13/3*tan(1/2*
d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2-14*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1
/2*c)^5+14/tan(1/2*d*x+1/2*c)+3/2/tan(1/2*d*x+1/2*c)^4+28*ln(tan(1/2*d*x+1/
2*c))-13/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(90) = 180$.
time = 0.28, size = 235, normalized size = 2.35

$$-\frac{\frac{\frac{420 \sin(dx+c)}{\cos(dx+c)+1}+\frac{120 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{130 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{45 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{45 \sin(dx+c)}{\cos(dx+c)+1}-\frac{130 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{120 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{420 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-6\right)(\cos(dx+c)+1)^5}{a^3 \sin(dx+c)^5}}{960\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/960*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 120*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 130*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 840*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (45*sin(d*x + c)/(cos(d*x + c) + 1)
- 130*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 120*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 420*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6)*(cos(d*x + c) + 1)^
5/(a^3*sin(d*x + c)^5))/d
```

Fricas [**A**]
time = 0.42, size = 169, normalized size = 1.69

$$\frac{272 \cos(dx+c)^5-320 \cos(dx+c)^3-105 \left(\cos(dx+c)^4-2 \cos(dx+c)^2+1\right) \log\left(\frac{1}{2} \cos(dx+c)+\frac{1}{2}\right) \sin(dx+c)+105 \left(\cos(dx+c)^4-2 \cos(dx+c)^2+1\right) \log\left(-\frac{1}{2} \cos(dx+c)+\frac{1}{2}\right) \sin(dx+c)-30 \left(\cos(dx+c)^3-7 \cos(dx+c)\right) \sin(dx+c)}{240 \left(a^3 d \cos(dx+c)^4-2 a^3 d \cos(dx+c)^2+a^3 d\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(272*cos(d*x + c)^5 - 320*cos(d*x + c)^3 - 105*(cos(d*x + c)^4 - 2*co
s(d*x + c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x +
 c)^4 - 2*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3
0*(cos(d*x + c)^3 - 7*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(90) = 180.
time = 0.54, size = 186, normalized size = 1.86

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}} - \frac{1918 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-420 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-120 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+130 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-45 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6}{a^{3} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}} + \frac{6\,a^{12} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-45\,a^{12} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+130\,a^{12} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,a^{12} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-420\,a^{12} \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (1918*tan(1/2*d*x + 1/2*c)^
5 - 420*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 130*tan(1/2*d
*x + 1/2*c)^2 - 45*tan(1/2*d*x + 1/2*c) + 6)/(a^3*tan(1/2*d*x + 1/2*c)^5) +
 (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^12*tan(1/2*d*x + 1/2*c)^4 + 130*a^12
*tan(1/2*d*x + 1/2*c)^3 - 120*a^12*tan(1/2*d*x + 1/2*c)^2 - 420*a^12*tan(1/
2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 9.60, size = 291, normalized size = 2.91

$$\frac{6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+45\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+130\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+420\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+120\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-130\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{960\,a^{3}\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^6*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (6*sin(c/2 + (d*x)/2)^10 - 6*cos(c/2 + (d*x)/2)^10 - 45*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^9 + 45*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) + 130*cos
(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 - 120*cos(c/2 + (d*x)/2)^3*sin(c/2 +
 (d*x)/2)^7 - 420*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 420*cos(c/2 +
 (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 120*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)
/2)^3 - 130*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/(960
*a^3*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)
```

3.748 $\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=124

$$\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)\csc(c+dx)}{16a^3d}-\frac{17\cot(c+dx)\csc^3(c+}{24a^3d}$$

[Out] 7/16*arctanh(cos(d*x+c))/a^3/d+4/3*cot(d*x+c)^3/a^3/d+3/5*cot(d*x+c)^5/a^3/d+7/16*cot(d*x+c)*csc(d*x+c)/a^3/d-17/24*cot(d*x+c)*csc(d*x+c)^3/a^3/d-1/6*cot(d*x+c)*csc(d*x+c)^5/a^3/d

Rubi [A]

time = 0.25, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.296$, Rules used = {2954, 2952, 2687, 30, 2691, 3853, 3855, 14}

$$\frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}+\frac{7\cot(c+dx)\csc(c+dx)}{16a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]

[Out] (7*ArcTanh[Cos[c + d*x]])/(16*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (3*Cot[c + d*x]^5)/(5*a^3*d) + (7*Cot[c + d*x]*Csc[c + d*x])/(16*a^3*d) - (17*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^3*d)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^5(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc^2(c+dx)+3a^3\cot^2(c+dx)\csc^3(c+dx)-3a^3\cot\right.}{a^6}$$
$$= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int c}{}$$
$$= -\frac{3\cot(c+dx)\csc^3(c+dx)}{4a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6a^3d}-\frac{\int \csc^5(c+dx)}{6a^3}$$
$$= \frac{\cot^3(c+dx)}{3a^3d}+\frac{3\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{17\cot(c+dx)\csc^3(c+dx)}{24a^3d}-$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)}{16a^3}$$
$$= \frac{7\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}+\frac{7\cot(c+dx)}{16a^3}$$

Mathematica [A]
time = 0.75, size = 242, normalized size = 1.95

$$\frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(-704\cot\left(\frac{1}{2}(c+dx)\right)+210\csc^2\left(\frac{1}{2}(c+dx)\right)+840\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-840\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-210\sec^2\left(\frac{1}{2}(c+dx)\right)+90\sec^4\left(\frac{1}{2}(c+dx)\right)+5\sec^6\left(\frac{1}{2}(c+dx)\right)-544\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)(-5+18\sin(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(-90+34\sin(c+dx))+704\tan\left(\frac{1}{2}(c+dx)\right)-36\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{1920a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^7)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(-704*Cot[(c + d*x)/2] + 210*Csc[(
c + d*x)/2]^2 + 840*Log[Cos[(c + d*x)/2]] - 840*Log[Sin[(c + d*x)/2]] - 210
*Sec[(c + d*x)/2]^2 + 90*Sec[(c + d*x)/2]^4 + 5*Sec[(c + d*x)/2]^6 - 544*Cs
c[c + d*x]^3*Sin[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6*(-5 + 18*Sin[c + d*x])
 + Csc[(c + d*x)/2]^4*(-90 + 34*Sin[c + d*x]) + 704*Tan[(c + d*x)/2] - 36*S
ec[(c + d*x)/2]^4*Tan[(c + d*x)/2]))/(1920*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]
time = 0.33, size = 176, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64d\,a^3}$
default	$\frac{\frac{\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64d\,a^3}$
risch	$-\frac{240i e^{10i(dx+c)}+105\,e^{11i(dx+c)}-2160i e^{8i(dx+c)}+365\,e^{9i(dx+c)}+1760i e^{6i(dx+c)}-1110\,e^{7i(dx+c)}-480i e^{4i(dx+c)}-1110}{120d\,a^3\left(e^{2i(dx+c)}-1\right)^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/d/a^3*(1/6*tan(1/2*d*x+1/2*c)^6-6/5*tan(1/2*d*x+1/2*c)^5+7/2*tan(1/2*d
*x+1/2*c)^4-14/3*tan(1/2*d*x+1/2*c)^3-1/2*tan(1/2*d*x+1/2*c)^2+20*tan(1/2*d
*x+1/2*c)-1/6/tan(1/2*d*x+1/2*c)^6-20/tan(1/2*d*x+1/2*c)-7/2/tan(1/2*d*x+1/
2*c)^4-28*ln(tan(1/2*d*x+1/2*c))+14/3/tan(1/2*d*x+1/2*c)^3+1/2/tan(1/2*d*x+
1/2*c)^2+6/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(112) = 224$.
time = 0.28, size = 274, normalized size = 2.21

$$\frac{\frac{\frac{600 \sin(dx+c)}{\cos(dx+c)+1} - \frac{15 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{140 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{105 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{36 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}{a^3} - \frac{840 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} + \frac{\left(\frac{36 \sin(dx+c)}{\cos(dx+c)+1} - \frac{105 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{140 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{600 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 5\right)(\cos(dx+c)+1)^6}{a^3 \sin(dx+c)^6}}{1920\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1920*((600*sin(d*x + c)/(cos(d*x + c) + 1) - 15*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6)/a^3 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a
^3 + (36*sin(d*x + c)/(cos(d*x + c) + 1) - 105*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 140*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 600*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5)*(cos(d*x + c
) + 1)^6/(a^3*sin(d*x + c)^6))/d
```

Fricas [A]
time = 0.45, size = 196, normalized size = 1.58

$$-\frac{210 \cos(dx+c)^5 - 80 \cos(dx+c)^3 - 105 \left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 105 \left(\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 32 \left(11 \cos(dx+c)^5 - 20 \cos(dx+c)^3\right) \sin(dx+c) - 210 \cos(dx+c)}{480 \left(a^3 d \cos(dx+c)^6 - 3 a^3 d \cos(dx+c)^4 + 3 a^3 d \cos(dx+c)^2 - a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/480*(210*cos(d*x + c)^5 - 80*cos(d*x + c)^3 - 105*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) + 105*(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c
) + 1/2) - 32*(11*cos(d*x + c)^5 - 20*cos(d*x + c)^3)*sin(d*x + c) - 210*co
s(d*x + c))/(a^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*
x + c)^2 - a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 216, normalized size = 1.74

$$-\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{2058\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-600\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}-\frac{5a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-36a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+105a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-140a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+600a^{15}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{18}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1920*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2058*tan(1/2*d*x + 1/2*c
)^6 - 600*tan(1/2*d*x + 1/2*c)^5 + 15*tan(1/2*d*x + 1/2*c)^4 + 140*tan(1/2*
d*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) - 5)/
(a^3*tan(1/2*d*x + 1/2*c)^6) - (5*a^15*tan(1/2*d*x + 1/2*c)^6 - 36*a^15*tan
(1/2*d*x + 1/2*c)^5 + 105*a^15*tan(1/2*d*x + 1/2*c)^4 - 140*a^15*tan(1/2*d*
x + 1/2*c)^3 - 15*a^15*tan(1/2*d*x + 1/2*c)^2 + 600*a^15*tan(1/2*d*x + 1/2*
c))/a^18)/d
```

Mupad [**B**]
time = 10.18, size = 339, normalized size = 2.73

$$-\frac{5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8-600\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+600\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5-15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{1920\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^7*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(5*cos(c/2 + (d*x)/2)^12 - 5*sin(c/2 + (d*x)/2)^12 + 36*cos(c/2 + (d*x)/2)
*sin(c/2 + (d*x)/2)^11 - 36*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2) - 105*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 140*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 600*cos(c/
2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 600*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d
*x)/2)^5 - 15*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 140*cos(c/2 + (d*
x)/2)^9*sin(c/2 + (d*x)/2)^3 + 105*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)
^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*si
n(c/2 + (d*x)/2)^6)/(1920*a^3*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

3.749 $\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=140

$$-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{\cot^7(c+dx)}{7a^3d}-\frac{5\cot(c+dx)\csc(c+dx)}{16a^3d}+\frac{\cot(c+d}{}$$

```
[Out] -5/16*arctanh(cos(d*x+c))/a^3/d-4/3*cot(d*x+c)^3/a^3/d-cot(d*x+c)^5/a^3/d-1
/7*cot(d*x+c)^7/a^3/d-5/16*cot(d*x+c)*csc(d*x+c)/a^3/d+1/8*cot(d*x+c)*csc(d
*x+c)^3/a^3/d+1/2*cot(d*x+c)*csc(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2788, 3853, 3855, 3852}

$$-\frac{\cot^7(c+dx)}{7a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^3(c+dx)}{8a^3d}-\frac{5\cot(c+dx)\csc(c+dx)}{16a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a^3*d) - (4*Cot[c + d*x]^3)/(3*a^3*d) - Cot[
c + d*x]^5/(a^3*d) - Cot[c + d*x]^7/(7*a^3*d) - (5*Cot[c + d*x]*Csc[c + d*x
])/(16*a^3*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^3*d) + (Cot[c + d*x]*Csc
[c + d*x]^5)/(2*a^3*d)
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^8(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \left(a^5\csc^3(c+dx)-3a^5\csc^4(c+dx)+2a^5\csc^5(c+dx)+2a^5\csc^6(c+dx)-\right.}{a^8} \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^3}+\frac{\int \csc^8(c+dx)\,dx}{a^3}+\frac{2\int \csc^5(c+dx)\,dx}{a^3}+\frac{2\int \csc^6(c+dx}{a^3} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)}{2a^3d}+\frac{\cot(c+dx)\csc^5(c+d}{2a^3d} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{\cot^7(c+dx)}{7a^3d}-\frac{5\,\mathrm{co}}{} \\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{4a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{\cot^7(c+dx)}{7a^3d}-\frac{5}{} \\
&= -\frac{5\tanh^{-1}(\cos(c+dx))}{16a^3d}-\frac{4\cot^3(c+dx)}{3a^3d}-\frac{\cot^5(c+dx)}{a^3d}-\frac{\cot^7(c+dx)}{7a^3d}-\frac{5}{}
\end{aligned}
$$

Mathematica [A]
time = 0.70, size = 251, normalized size = 1.79

$$\frac{\csc^7(c+dx)\left(-4704\cos(c+dx)+672\cos(3(c+dx))+1120\cos(5(c+dx))-160\cos(7(c+dx))-3675\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(c+dx)+3675\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(c+dx)+4998\sin(2(c+dx))+2205\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-2205\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+504\sin(4(c+dx))-735\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+735\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-210\sin(6(c+dx))+105\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))-105\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))\right)}{21504a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^7*(-4704*Cos[c + d*x] + 672*Cos[3*(c + d*x)] + 1120*Cos[5*(c
+ d*x)] - 160*Cos[7*(c + d*x)] - 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] +
3675*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 4998*Sin[2*(c + d*x)] + 2205*Log[
Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] - 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c +
d*x)] + 504*Sin[4*(c + d*x)] - 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] +
 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 210*Sin[6*(c + d*x)] + 105*Lo
g[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] - 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c +
 d*x)]))/(21504*a^3*d)
```

Maple [A]
time = 0.34, size = 200, normalized size = 1.43

method	result

risch	$\frac{105\,\mathrm{e}^{13i(dx+c)}-2016i\mathrm{e}^{10i(dx+c)}-252\,\mathrm{e}^{11i(dx+c)}+5152i\mathrm{e}^{8i(dx+c)}-2499\,\mathrm{e}^{9i(dx+c)}-448i\mathrm{e}^{6i(dx+c)}+1344i\mathrm{e}^{4i(dx+c)}+2499\,\mathrm{e}^{5}}{168a^3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^7}$
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-29\tan\left(\right.}{}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-29\tan\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/128/d/a^3*(1/7*tan(1/2*d*x+1/2*c)^7-tan(1/2*d*x+1/2*c)^6+3*tan(1/2*d*x+1/2*c)^5-5*tan(1/2*d*x+1/2*c)^4+13/3*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2-29*tan(1/2*d*x+1/2*c)+1/tan(1/2*d*x+1/2*c)^6+29/tan(1/2*d*x+1/2*c)-1/7/tan(1/2*d*x+1/2*c)^7+5/tan(1/2*d*x+1/2*c)^4+40*ln(tan(1/2*d*x+1/2*c))-3/tan(1/2*d*x+1/2*c)^5-13/3/tan(1/2*d*x+1/2*c)^3-3/tan(1/2*d*x+1/2*c)^2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(128) = 256$.
time = 0.29, size = 315, normalized size = 2.25

$$-\frac{\frac{\frac{609\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{63\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{91\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{105\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{21\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}-\frac{840\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}-\frac{\left(\frac{21\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{63\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{105\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{91\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{609\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-3\right)(\cos(dx+c)+1)^7}{a^3\sin(dx+c)^7}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -1/2688*((609*sin(d*x + c)/(cos(d*x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 91*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 105*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 21*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 - 840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - (21*sin(d*x + c)/(cos(d*x + c) + 1) - 63*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 105*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 91*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 609*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c) + 1)^7/(a^3*sin(d*x + c)^7))/d

Fricas [A]
time = 0.38, size = 226, normalized size = 1.61

$$\frac{320\,\cos(dx+c)^7-1120\,\cos(dx+c)^5+896\,\cos(dx+c)^3-105\left(\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+105\left(\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+42\left(5\,\cos(dx+c)^5-8\,\cos(dx+c)^3-5\,\cos(dx+c)\right)\sin(dx+c)}{672\left(a^3d\cos(dx+c)^6-3\,a^3d\cos(dx+c)^4+3\,a^3d\cos(dx+c)^2-a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] 1/672*(320*cos(d*x + c)^7 - 1120*cos(d*x + c)^5 + 896*cos(d*x + c)^3 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 42*(5*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 5*cos(d*x + c))*sin(d*x + c))/((a^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**8*csc(d*x+c)**8/(a+a*sin(d*x+c))**3,x)

[Out] Timed out

Giac [A]
time = 0.52, size = 244, normalized size = 1.74

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{2178\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-609\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+91\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}+\frac{3a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-21a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+63a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-105a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+91a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+63a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-609a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{21}}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^8*csc(d*x+c)^8/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2178*tan(1/2*d*x + 1/2*c)^7 - 609*tan(1/2*d*x + 1/2*c)^6 + 63*tan(1/2*d*x + 1/2*c)^5 + 91*tan(1/2*d*x + 1/2*c)^4 - 105*tan(1/2*d*x + 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2*d*x + 1/2*c)^7) + (3*a^18*tan(1/2*d*x + 1/2*c)^7 - 21*a^18*tan(1/2*d*x + 1/2*c)^6 + 63*a^18*tan(1/2*d*x + 1/2*c)^5 - 105*a^18*tan(1/2*d*x + 1/2*c)^4 + 91*a^18*tan(1/2*d*x + 1/2*c)^3 + 63*a^18*tan(1/2*d*x + 1/2*c)^2 - 609*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d

Mupad [B]
time = 10.86, size = 387, normalized size = 2.76

$$\frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+91\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9-609\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+609\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5-91\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3-63\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+840\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{2688\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^8/(sin(c + d*x)^8*(a + a*sin(c + d*x))^3),x)

[Out] (3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 21*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^13 + 21*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 63*co

```
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^11 + 91*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 + 63*cos(c/2
 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 609*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*
x)/2)^8 + 609*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 63*cos(c/2 + (d*x
)/2)^9*sin(c/2 + (d*x)/2)^5 - 91*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
 + 105*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 - 63*cos(c/2 + (d*x)/2)^1
2*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos
(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a^3*d*cos(c/2 + (d*x)/2)^7*si
n(c/2 + (d*x)/2)^7)
```

3.750 $\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=166

$$\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{29\cot(c+dx)\csc(c+dx)}{128a^3d}+\frac{29\,\mathrm{co}}{}$$

```
[Out] 29/128*arctanh(cos(d*x+c))/a^3/d+4/3*cot(d*x+c)^3/a^3/d+7/5*cot(d*x+c)^5/a^
3/d+3/7*cot(d*x+c)^7/a^3/d+29/128*cot(d*x+c)*csc(d*x+c)/a^3/d+29/192*cot(d*
x+c)*csc(d*x+c)^3/a^3/d-23/48*cot(d*x+c)*csc(d*x+c)^5/a^3/d-1/8*cot(d*x+c)*
csc(d*x+c)^7/a^3/d
```

Rubi [A]

time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2954, 2952, 2687, 14, 2691, 3853, 3855, 276}

$$\frac{3\cot^7(c+dx)}{7a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{23\cot(c+dx)\csc^5(c+dx)}{48a^3d}+\frac{29\cot(c+dx)\csc^3(c+dx)}{192a^3d}+\frac{29\cot(c+dx)\csc(c+dx)}{128a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (29*ArcTanh[Cos[c + d*x]])/(128*a^3*d) + (4*Cot[c + d*x]^3)/(3*a^3*d) + (7*
Cot[c + d*x]^5)/(5*a^3*d) + (3*Cot[c + d*x]^7)/(7*a^3*d) + (29*Cot[c + d*x]
*Csc[c + d*x])/(128*a^3*d) + (29*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^3*d) -
 (23*Cot[c + d*x]*Csc[c + d*x]^5)/(48*a^3*d) - (Cot[c + d*x]*Csc[c + d*x]^7
)/(8*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)\csc(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \cot^2(c+dx)\csc^7(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{\int \left(-a^3\cot^2(c+dx)\csc^4(c+dx)+3a^3\cot^2(c+dx)\csc^5(c+dx)-3a^3\,\mathrm{co}\right.}{a^6}$$

$$= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int \cot^2(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int \mathrm{c}}{}$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)}{2a^3d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8a^3d}-\frac{\int \csc^7(c+dx)\,d}{8a^3}$$

$$= \frac{\cot(c+dx)\csc^3(c+dx)}{8a^3d}-\frac{23\cot(c+dx)\csc^5(c+dx)}{48a^3d}-\frac{\cot(c+dx)\csc^7}{8a^3d}$$

$$= \frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}+\frac{3\cot(c+dx)\csc(c+}{16a^3d}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{16a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+dx)}{7a^3d}$$

$$= \frac{29\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{4\cot^3(c+dx)}{3a^3d}+\frac{7\cot^5(c+dx)}{5a^3d}+\frac{3\cot^7(c+d}{7a^3d}$$

Mathematica [**A**]
time = 4.84, size = 317, normalized size = 1.91

$$-\frac{\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^6\left(\csc^4\left(\frac{1}{2}(c+dx)\right)(1328-210\csc(c+dx))+15\csc^8\left(\frac{1}{2}(c+dx)\right)(-24+7\csc(c+dx))+4\csc^6\left(\frac{1}{2}(c+dx)\right)(-276+455\csc(c+dx))-4\csc^2\left(\frac{1}{2}(c+dx)\right)(-4864+3045\csc(c+dx))-8\left(6090\csc(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{1}{4}(2833+4616\cos(c+dx)+1907\cos(2(c+dx))+304\cos(3(c+dx)))\sec^8\left(\frac{1}{2}(c+dx)\right)-6090\csc^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-420\csc^5(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+14560\csc^7(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)+3360\csc^9(c+dx)\sin^8\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin^7(c+dx)}{13762560a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^8*Csc[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/13762560*((Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^6*(Csc[(c + d*x)/2]^4*(1
328 - 210*Csc[c + d*x]) + 15*Csc[(c + d*x)/2]^8*(-24 + 7*Csc[c + d*x]) + 4*
Csc[(c + d*x)/2]^6*(-276 + 455*Csc[c + d*x]) - 4*Csc[(c + d*x)/2]^2*(-4864
+ 3045*Csc[c + d*x]) - 8*(6090*Csc[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Si
n[(c + d*x)/2]]) + ((2833 + 4616*Cos[c + d*x] + 1907*Cos[2*(c + d*x)] + 304
*Cos[3*(c + d*x)])*Sec[(c + d*x)/2]^8)/4 - 6090*Csc[c + d*x]^3*Sin[(c + d*x
)/2]^2 - 420*Csc[c + d*x]^5*Sin[(c + d*x)/2]^4 + 14560*Csc[c + d*x]^7*Sin[(
c + d*x)/2]^6 + 3360*Csc[c + d*x]^9*Sin[(c + d*x)/2]^8))*Sin[c + d*x]^7)/(a
^3*d*(1 + Sin[c + d*x])^3)
```

Maple [**A**]
time = 0.38, size = 228, normalized size = 1.37

method	result
risch	$-\frac{3045\,\mathrm{e}^{15i(dx+c)}-26880i\mathrm{e}^{12i(dx+c)}-23345\,\mathrm{e}^{13i(dx+c)}+286720i\mathrm{e}^{10i(dx+c)}-51275\,\mathrm{e}^{11i(dx+c)}-170240i\mathrm{e}^{8i(dx+c)}+1790}{}$

derivativedivides	$\frac{\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{26\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-8\left(\tan^2\left(\right.\right.}{}$
default	$\frac{\frac{\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}-\frac{6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{26\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+7\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-8\left(\tan^2\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/256/d/a^3*(1/8*tan(1/2*d*x+1/2*c)^8-6/7*tan(1/2*d*x+1/2*c)^7+8/3*tan(1/2*
d*x+1/2*c)^6-26/5*tan(1/2*d*x+1/2*c)^5+7*tan(1/2*d*x+1/2*c)^4-14/3*tan(1/2*
d*x+1/2*c)^3-8*tan(1/2*d*x+1/2*c)^2+46*tan(1/2*d*x+1/2*c)-1/8/tan(1/2*d*x+1
/2*c)^8-46/tan(1/2*d*x+1/2*c)+6/7/tan(1/2*d*x+1/2*c)^7-7/tan(1/2*d*x+1/2*c)
^4+26/5/tan(1/2*d*x+1/2*c)^5-58*ln(tan(1/2*d*x+1/2*c))+14/3/tan(1/2*d*x+1/2
*c)^3+8/tan(1/2*d*x+1/2*c)^2-8/3/tan(1/2*d*x+1/2*c)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(150) = 300$.
time = 0.28, size = 354, normalized size = 2.13

$$\frac{\frac{\frac{38640\sin(dx+c)}{\cos(dx+c)+1}-\frac{6720\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3920\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5880\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4368\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2240\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{720\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}{a^3}-\frac{48720\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{\left(\frac{720\sin(dx+c)}{\cos(dx+c)+1}-\frac{2240\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4368\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5880\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3920\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{6720\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{38640\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-105\right)(\cos(dx+c)+1)^8}{a^3\sin(dx+c)^8}}{215040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/215040*((38640*sin(d*x + c)/(cos(d*x + c) + 1) - 6720*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - 3920*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5880*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4368*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 22
40*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 720*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 105*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)/a^3 - 48720*log(sin(d*x +
c)/(cos(d*x + c) + 1))/a^3 + (720*sin(d*x + c)/(cos(d*x + c) + 1) - 2240*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4368*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 5880*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3920*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 6720*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 38640*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 105)*(cos(d*x + c) + 1)^8/(a^3*sin(d*x + c)^8))/
d
```

Fricas [A]
time = 0.40, size = 249, normalized size = 1.50

$$-\frac{6090\cos(dx+c)^7-22330\cos(dx+c)^5+13510\cos(dx+c)^3-3045\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3045\left(\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-256\left(38\cos(dx+c)^7-133\cos(dx+c)^5+140\cos(dx+c)^3\right)\sin(dx+c)+6090\cos(dx+c)}{26880\left(a^3d\cos(dx+c)^8-4a^3d\cos(dx+c)^6+6a^3d\cos(dx+c)^4-4a^3d\cos(dx+c)^2+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/26880*(6090*cos(d*x + c)^7 - 22330*cos(d*x + c)^5 + 13510*cos(d*x + c)^3
 - 3045*(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x +
 c)^2 + 1)*log(1/2*cos(d*x + c) + 1/2) + 3045*(cos(d*x + c)^8 - 4*cos(d*x +
 c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1)*log(-1/2*cos(d*x + c) + 1/
2) - 256*(38*cos(d*x + c)^7 - 133*cos(d*x + c)^5 + 140*cos(d*x + c)^3)*sin(
d*x + c) + 6090*cos(d*x + c))/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*cos(d*x + c)^
6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8*csc(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 274, normalized size = 1.65

$$-\frac{\frac{48720 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3} - \frac{132414 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8 - 38640 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 + 6720 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 + 3920 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 - 5880 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 + 4368 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 2240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 720 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right) - 105}{a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8} - \frac{105 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8 - 720 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 + 2240 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6 - 4368 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5 + 5880 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 - 3920 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3 - 6720 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 + 38640 a^{21} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^{24}}}{215040 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8*csc(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/215040*(48720*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (132414*tan(1/2*d*x +
 1/2*c)^8 - 38640*tan(1/2*d*x + 1/2*c)^7 + 6720*tan(1/2*d*x + 1/2*c)^6 + 39
20*tan(1/2*d*x + 1/2*c)^5 - 5880*tan(1/2*d*x + 1/2*c)^4 + 4368*tan(1/2*d*x
+ 1/2*c)^3 - 2240*tan(1/2*d*x + 1/2*c)^2 + 720*tan(1/2*d*x + 1/2*c) - 105)/
(a^3*tan(1/2*d*x + 1/2*c)^8) - (105*a^21*tan(1/2*d*x + 1/2*c)^8 - 720*a^21*
tan(1/2*d*x + 1/2*c)^7 + 2240*a^21*tan(1/2*d*x + 1/2*c)^6 - 4368*a^21*tan(1
/2*d*x + 1/2*c)^5 + 5880*a^21*tan(1/2*d*x + 1/2*c)^4 - 3920*a^21*tan(1/2*d*
x + 1/2*c)^3 - 6720*a^21*tan(1/2*d*x + 1/2*c)^2 + 38640*a^21*tan(1/2*d*x +
1/2*c))/a^24)/d
```

Mupad [**B**]
time = 11.50, size = 435, normalized size = 2.62

$$-\frac{105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 105\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{15} - 720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\sin\left(\frac{c}{2}+\frac{dx}{2}\right) - 2240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 4368\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} - 5880\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 3920\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 38640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + 38640\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - 6720\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6 - 3920\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 5880\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - 4368\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 2240\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 48720\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{215040\,a^3\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(sin(c + d*x)^9*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -(105*cos(c/2 + (d*x)/2)^16 - 105*sin(c/2 + (d*x)/2)^16 + 720*cos(c/2 + (d*
x)/2)*sin(c/2 + (d*x)/2)^15 - 720*cos(c/2 + (d*x)/2)^15*sin(c/2 + (d*x)/2)
- 2240*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^14 + 4368*cos(c/2 + (d*x)/2)
^3*sin(c/2 + (d*x)/2)^13 - 5880*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^12
+ 3920*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^11 + 6720*cos(c/2 + (d*x)/2)
^6*sin(c/2 + (d*x)/2)^10 - 38640*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^9
+ 38640*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^7 - 6720*cos(c/2 + (d*x)/2)
^10*sin(c/2 + (d*x)/2)^6 - 3920*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5
+ 5880*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^4 - 4368*cos(c/2 + (d*x)/2)
^13*sin(c/2 + (d*x)/2)^3 + 2240*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2
+ 48720*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin
(c/2 + (d*x)/2)^8)/(215040*a^3*d*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^8)
```

3.751 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$-\frac{3ax}{2}+\frac{2a\cos(c+dx)}{d}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -3/2*a*x+2*a*cos(d*x+c)/d-1/3*a*cos(d*x+c)^3/d+a*sec(d*x+c)/d+3/2*a*tan(d*x
+c)/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2671, 294, 327, 209, 2670, 276}

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{2a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*a*Cos[c + d*x])/d - (a*Cos[c + d*x]^3)/(3*d) + (a*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin^2(c+dx)\tan^2(c+dx)\,dx + a\int \sin^3(c+dx)\tan$$

$$= -\frac{a\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{x}{(1+}\right.}{}$$

$$= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right.\right.}{d}$$

$$= \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+dx)}{d} + \frac{3a\,\mathrm{t}}{}$$

$$= -\frac{3ax}{2} + \frac{2a\cos(c+dx)}{d} - \frac{a\cos^3(c+dx)}{3d} + \frac{a\sec(c+d}{d}$$

Mathematica [A]
time = 0.24, size = 82, normalized size = 1.00

$$-\frac{3a(c+dx)}{2d} + \frac{7a\cos(c+dx)}{4d} - \frac{a\cos(3(c+dx))}{12d} + \frac{a\sec(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*a*Cos[c + d*x])/(4*d) - (a*Cos[3*(c + d*x)])/(1
2*d) + (a*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.17, size = 104, normalized size = 1.27

method	result
risch	$-\frac{3ax}{2}+\frac{7a\,\mathrm{e}^{i(dx+c)}}{8d}+\frac{7a\,\mathrm{e}^{-i(dx+c)}}{8d}+\frac{2a}{d\left(\mathrm{e}^{i(dx+c)}-i\right)}-\frac{a\cos(3dx+3c)}{12d}+\frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\mathrm{co}\right.}{d}$
default	$\frac{a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\mathrm{co}\right.}{d}$
norman	$\frac{\frac{3ax}{2}-\frac{16a}{3d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3ax\left(\tan^6\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c)+a*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)
*cos(d*x+c)))
```

Maxima [A]
time = 0.58, size = 75, normalized size = 0.91

$$-\frac{2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)a+3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan\left(dx+c\right)\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a)/d
```

Fricas [A]
time = 0.37, size = 130, normalized size = 1.59

$$-\frac{2\,a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^3+9\,adx-12\,a\cos\left(dx+c\right)^2+3\left(3\,adx-5\,a\right)\cos\left(dx+c\right)-\left(2\,a\cos\left(dx+c\right)^3+9\,adx+3\,a\cos\left(dx+c\right)^2-9\,a\cos\left(dx+c\right)+6\,a\right)\sin\left(dx+c\right)-6\,a}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^4 - a*cos(d*x + c)^3 + 9*a*d*x - 12*a*cos(d*x + c)^2
 + 3*(3*a*d*x - 5*a)*cos(d*x + c) - (2*a*cos(d*x + c)^3 + 9*a*d*x + 3*a*cos
(d*x + c)^2 - 9*a*cos(d*x + c) + 6*a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c) -
 d*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 105, normalized size = 1.28

$$-\frac{9\,(dx+c)a+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\,\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a*tan(1/2*d*x
+ 1/2*c)^5 - 6*a*tan(1/2*d*x + 1/2*c)^4 - 24*a*tan(1/2*d*x + 1/2*c)^2 - 3*a
*tan(1/2*d*x + 1/2*c) - 10*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 14.82, size = 257, normalized size = 3.13

$$\frac{\left(\frac{a\,(9\,c+9\,d\,x-18)}{6}-\frac{3\,a\,(c+d\,x)}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{9\,a\,(c+d\,x)}{2}-\frac{a\,(27\,c+27\,d\,x-18)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{a\,(27\,c+27\,d\,x-48)}{6}-\frac{9\,a\,(c+d\,x)}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{9\,a\,(c+d\,x)}{2}-\frac{a\,(27\,c+27\,d\,x-48)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{a\,(27\,c+27\,d\,x-78)}{6}-\frac{9\,a\,(c+d\,x)}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(\frac{3\,a\,(c+d\,x)}{2}-\frac{a\,(9\,c+9\,d\,x-14)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,(9\,c+9\,d\,x-32)}{6}-\frac{3\,a\,(c+d\,x)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}-\frac{3\,a\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] ((a*(9*c + 9*d*x - 32))/6 - tan(c/2 + (d*x)/2)*((a*(9*c + 9*d*x - 14))/6 -
(3*a*(c + d*x))/2) - (3*a*(c + d*x))/2 + tan(c/2 + (d*x)/2)^6*((a*(9*c + 9*
d*x - 18))/6 - (3*a*(c + d*x))/2) - tan(c/2 + (d*x)/2)^5*((a*(27*c + 27*d*x
 - 18))/6 - (9*a*(c + d*x))/2) - tan(c/2 + (d*x)/2)^3*((a*(27*c + 27*d*x -
48))/6 - (9*a*(c + d*x))/2) + tan(c/2 + (d*x)/2)^4*((a*(27*c + 27*d*x - 48)
)/6 - (9*a*(c + d*x))/2) + tan(c/2 + (d*x)/2)^2*((a*(27*c + 27*d*x - 78))/6
 - (9*a*(c + d*x))/2))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 +
1)^3) - (3*a*x)/2
```

3.752 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$-\frac{3ax}{2}+\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -3/2*a*x+a*cos(d*x+c)/d+a*sec(d*x+c)/d+3/2*a*tan(d*x+c)/d-1/2*a*sin(d*x+c)^
2*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2917, 2670, 14, 2671, 294, 327, 209}

$$\frac{a\cos(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}+\frac{a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*a*Tan[c + d*x])/(
2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a\int \sin(c+dx)\tan^2(c+dx)\,dx + a\int \sin^2(c+dx)\tan^2(\\
&= -\frac{a\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\right.}{}\\
&= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,\right.}{d}\\
&= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d} - \frac{a\sin^2}{}\\
&= -\frac{3ax}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3a\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.97

$$-\frac{3a(c+dx)}{2d} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sin(2(c+dx))}{4d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (a*Sin[2
*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.12, size = 94, normalized size = 1.45

method	result
risch	$-\frac{3ax}{2}+\frac{a\,{\mathrm e}^{i(dx+c)}}{2d}+\frac{a\,{\mathrm e}^{-i(dx+c)}}{2d}+\frac{2a}{d\left({\mathrm e}^{i(dx+c)}-i\right)}+\frac{a\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)\right)+a\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}$
default	$\frac{a\left(\frac{\sin^{4}(dx+c)}{\cos(dx+c)}+\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)\right)+a\left(\frac{\sin^{5}(dx+c)}{\cos(dx+c)}+\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}$
norman	$\frac{\frac{3ax}{2}-\frac{4a}{d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3ax\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3ax\left(\tan^{6}\left(\frac{d}{2}\cdots\right.\right.}{2}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(sin(d*x+c)^
5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.51, size = 62, normalized size = 0.95

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a
- 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.60

$$\frac{a\cos\left(dx+c\right)^{3}-3\,adx+2\,a\cos\left(dx+c\right)^{2}-3\,\left(adx-a\right)\cos\left(dx+c\right)+\left(3\,adx+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+2\,a\right)\sin\left(dx+c\right)+2\,a}{2\,\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)^3 - 3*a*d*x + 2*a*cos(d*x + c)^2 - 3*(a*d*x - a)*cos(d*
x + c) + (3*a*d*x + a*cos(d*x + c)^2 - a*cos(d*x + c) + 2*a)*sin(d*x + c) +
 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin^3(c+dx)\sec^2(c+dx)\,dx + \int \sin^4(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*
sec(c + d*x)**2, x))
```

Giac [A]
time = 0.45, size = 90, normalized size = 1.38

$$-\frac{3(dx+c)a + \frac{4a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1} + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)*a + 4*a/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 11.42, size = 160, normalized size = 2.46

$$\frac{\left(\frac{a(3dx-6)}{2}-\frac{3adx}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(3adx-\frac{a(6dx-6)}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{a(6dx-10)}{2}-3adx\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\left(\frac{3adx}{2}-\frac{a(3dx-2)}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a(3dx-8)}{2}-\frac{3adx}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}-\frac{3ax}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] ((a*(3*d*x - 8))/2 - tan(c/2 + (d*x)/2)*((a*(3*d*x - 2))/2 - (3*a*d*x)/2) -
 tan(c/2 + (d*x)/2)^3*((a*(6*d*x - 6))/2 - 3*a*d*x) + tan(c/2 + (d*x)/2)^4*
((a*(3*d*x - 6))/2 - (3*a*d*x)/2) + tan(c/2 + (d*x)/2)^2*((a*(6*d*x - 10))/
2 - 3*a*d*x) - (3*a*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)
^2 + 1)^2) - (3*a*x)/2
```

3.753 $\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=39

$$-ax+\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -a*x+a*cos(d*x+c)/d+a*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2787, 2825, 12, 2814, 2727}

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx \\
&= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx \\
&= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.21

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A]
time = 0.09, size = 59, normalized size = 1.51

method	result	size
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{2a}{d\left(e^{i(dx+c)}-i\right)}$	56
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
default	$\frac{a(\tan(dx+c)-dx-c)+a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59

norman	$\frac{ax-\frac{4a}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 1.00

$$-\frac{(dx+c-\tan(dx+c))a-a\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.
time = 0.40, size = 80, normalized size = 2.05

$$-\frac{adx-a\cos(dx+c)^2+(adx-2a)\cos(dx+c)-(adx-a\cos(dx+c)+a)\sin(dx+c)-a}{d\cos(dx+c)-d\sin(dx+c)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^2(c+dx)\sec^2(c+dx)\,dx+\int\sin^3(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*
sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.
time = 0.48, size = 81, normalized size = 2.08

$$-\frac{(dx+c)a+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) + 2*a)
/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) -
1))/d
```

Mupad [B]
time = 9.21, size = 99, normalized size = 2.54

$$\frac{(a\,(d\,x-2)-a\,d\,x)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+(a\,d\,x-a\,(d\,x-2))\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,(d\,x-4)-a\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (a*(d*x - 4) - tan(c/2 + (d*x)/2)*(a*(d*x - 2) - a*d*x) + tan(c/2 + (d*x)/2
)^2*(a*(d*x - 2) - a*d*x) - a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2)^2 + 1)) - a*x
```

3.754 $\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$-ax+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -a*x+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2917, 2686, 8, 3554}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(a*x) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = a\int \sec(c+dx)\tan(c+dx)\,dx + a\int \tan^2(c+dx)\,dx$$
$$= \frac{a\tan(c+dx)}{d} - a\int 1\,dx + \frac{a\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= -ax + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Mathematica [A]
time = 0.02, size = 36, normalized size = 1.33

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.06, size = 32, normalized size = 1.19

method	result	size
risch	$-ax + \frac{2a}{d\left(e^{i(dx+c)}-i\right)}$	26
derivativedivides	$\frac{\frac{a}{\cos(dx+c)}+a(\tan(dx+c)-dx-c)}{d}$	32
default	$\frac{\frac{a}{\cos(dx+c)}+a(\tan(dx+c)-dx-c)}{d}$	32
norman	$\frac{ax-\frac{2a}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a/cos(d*x+c)+a*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.56, size = 32, normalized size = 1.19

$$-\frac{(dx+c-\tan(dx+c))a - \frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a/cos(d*x + c))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.36, size = 60, normalized size = 2.22

$$-\frac{adx + (adx - a)\cos(dx + c) - (adx + a)\sin(dx + c) - a}{d\cos(dx + c) - d\sin(dx + c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x + (a*d*x - a)*cos(d*x + c) - (a*d*x + a)*sin(d*x + c) - a)/(d*cos(d
*x + c) - d*sin(d*x + c) + d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sin(c + dx)\sec^2(c + dx)\,dx + \int \sin^2(c + dx)\sec^2(c + dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**2, x))
```

Giac [**A**]
time = 0.43, size = 29, normalized size = 1.07

$$-\frac{(dx + c)a + \frac{2a}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

Mupad [**B**]
time = 8.98, size = 24, normalized size = 0.89

$$-a\,x - \frac{2\,a}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - a*x - (2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.755 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2702, 327, 213, 3852, 8}

$$\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \sec^2(c+dx)\,dx + a\int \csc(c+dx)\sec^2(c+dx)\,dx\\
&= -\frac{a\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{a\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,d\right.}{}\\
&= \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x,\right.}{d}\\
&= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+d}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.56

$$-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.14, size = 41, normalized size = 1.14

method	result	size
derivativedivides	$\frac{a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\tan(dx+c)}{d}$	41

default	$\frac{a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+a\tan(dx+c)}{d}$	41
risch	$\frac{2a}{d(e^{i(dx+c)}-i)}-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}$	57
norman	$\frac{-\frac{2a}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 48, normalized size = 1.33

$$\frac{a\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)+2\,a\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*a*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(36) = 72$.
time = 0.39, size = 108, normalized size = 3.00

$$\frac{2\,a\cos(dx+c)-(a\cos(dx+c)-a\sin(dx+c)+a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a\cos(dx+c)-a\sin(dx+c)+a)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\,a\sin(dx+c)+2\,a}{2\,(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c) - (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(1/2*cos(d
*x + c) + 1/2) + (a*cos(d*x + c) - a*sin(d*x + c) + a)*log(-1/2*cos(d*x + c
) + 1/2) + 2*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc(c+dx)\sec^2(c+dx)\,dx+\int\sin(c+dx)\csc(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x)**2, x) + Integral(sin(c + d*x)*csc(c
+ d*x)*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.44, size = 34, normalized size = 0.94

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*a/(tan(1/2*d*x + 1/2*c) - 1))/d
```

Mupad [B]
time = 8.95, size = 35, normalized size = 0.97

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)),x)
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (2*a)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.756 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+a*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2700, 14, 2702, 327, 213}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec$$
$$= \frac{a\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1+x^2}{x^2}\right.}{}$$
$$= \frac{a\sec(c+dx)}{d} + \frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+d}{d}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 1.42

$$-\frac{a\cot(c+dx)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.15, size = 61, normalized size = 1.27

method	result
derivativedivides	$\frac{a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
risch	$\frac{-4a-2ia\,\mathrm{e}^{i(dx+c)}+2a\,\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}-i\right)d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{a}{2d}-\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+a*(1/cos(d*x+c)+ln(csc(d*x+c)
-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 59, normalized size = 1.23

$$\frac{a\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(48) = 96$.
time = 0.38, size = 165, normalized size = 3.44

$$-\frac{4\,a\cos\left(dx+c\right)^2+2\,a\cos\left(dx+c\right)+\left(a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\left(2\,a\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)-2\,a}{2\left(d\cos\left(dx+c\right)^2+\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) + (a*cos(d*x + c)^2 + (a*cos(d*
x + c) + a)*sin(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) - (a*cos(d*x + c)
^2 + (a*cos(d*x + c) + a)*sin(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/2) -
2*(2*a*cos(d*x + c) + a)*sin(d*x + c) - 2*a)/(d*cos(d*x + c)^2 + (d*cos(d*x
 + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 87, normalized size = 1.81

$$\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*log(abs(tan(1/2*d*x + 1/2*c))) + a*tan(1/2*d*x + 1/2*c) - (a*tan(1
/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2*d*x + 1/2*c)^2 -
 tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.97, size = 77, normalized size = 1.60

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d - (a - 5*a*tan
(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2))
```

3.757 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+3/2*a*sec(d*x+c)/d-1/2*a*csc(d*
x+c)^2*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2702, 294, 327, 213, 2700, 14}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^2(c+dx)\,dx + a\int \csc^3(c+dx)\,\text{se}$$

$$= \frac{a\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1+}{x^2}\right.}{}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(}{2d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{3a\sec(c+}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(75) = 150.

time = 1.09, size = 172, normalized size = 2.29

$$-\frac{2a\cot(2(c+dx))}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.16, size = 83, normalized size = 1.11

method	result
derivativedivides	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
risch	$\frac{a\left(-3i e^{3i(dx+c)}+3\,e^{4i(dx+c)}+i e^{i(dx+c)}-5\,e^{2i(dx+c)}+4\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)d}-\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}$
norman	$\frac{\frac{a}{8d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{9a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{2a\left(\tan\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d
*x+c)))+a*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.12

$$\frac{a\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-4\,a\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(69) = 138.
time = 0.38, size = 261, normalized size = 3.48

$$\frac{8\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(4\,a\cos\left(dx+c\right)^2+a\cos\left(dx+c\right)-2\,a\right)\sin\left(dx+c\right)-4\,a}{4\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(8*a*cos(d*x + c)^3 + 6*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 3*(a*cos(
d*x + c)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin
(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 + a*cos(d*
x + c)^2 - a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*sin(d*x + c) - a)*log(-1
/2*cos(d*x + c) + 1/2) + 2*(4*a*cos(d*x + c)^2 + a*cos(d*x + c) - 2*a)*sin(
d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*
cos(d*x + c)^2 - d)*sin(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.44, size = 102, normalized size = 1.36

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*a*t
an(1/2*d*x + 1/2*c) - 16*a/(tan(1/2*d*x + 1/2*c) - 1) - (18*a*tan(1/2*d*x +
 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 8.90, size = 113, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + (3*a*tan(c/2 + (d*x)/2))/2 - 10*a*tan
(c/2 + (d*x)/2)^2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3)) +
(a*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.758 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}+\frac{3a\sec(c+dx)}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a\,\mathrm{ta}}{}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-2*a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d+3/2*a*se
c(d*x+c)/d-1/2*a*csc(d*x+c)^2*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2917, 2700, 276, 2702, 294, 327, 213}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot^3(c+dx)}{3d}-\frac{2a\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a*Cot[c + d*x])/d - (a*Cot[c + d*x]
^3)/(3*d) + (3*a*Sec[c + d*x])/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d
) + (a*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))\,dx &= a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + a\int \csc^4(c+dx)\,\mathrm{se}\\ &= \frac{a\mathrm{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\mathrm{Subst}\left(\int \frac{(1+}{}\right.}{}\\ &= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\right.}{d}\\ &= -\frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} + \frac{3a\sec(c+dx)}{2d} -\\ &= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a\cot(c+dx)}{d} - \frac{a\cot^3(c}{3d}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(91) = 182.

time = 3.82, size = 205, normalized size = 2.25

$$-\frac{5a\cot(c+dx)}{3d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*Cot[c + d*x])/(3*d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*d) + (3*a*Log[Sin[(c
 + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])) + (a*Tan[c + d*x])/d
```

Maple [A]

time = 0.16, size = 102, normalized size = 1.12

method	result
derivativedivides	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\mathrm{co}}{2}\right.}{d}$
default	$\frac{a\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\mathrm{co}}{2}\right.}{d}$
risch	$\frac{-9ia\,e^{5i(dx+c)}+9a\,e^{6i(dx+c)}+24ia\,e^{3i(dx+c)}-24a\,e^{4i(dx+c)}-7ia\,e^{i(dx+c)}+39a\,e^{2i(dx+c)}-16a}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}-i\right)d}+\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}-$
norman	$\frac{\frac{a}{24d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{35a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{9a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{35a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{7a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+
c))+a*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d*
x+c))))
```

Maxima [A]

time = 0.29, size = 98, normalized size = 1.08

$$\frac{3\,a\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-4\,a\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan\left(dx+c\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log
(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*a*((6*tan(d*x + c)^2 + 1)
/tan(d*x + c)^3 - 3*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(83) = 166$.
time = 0.39, size = 308, normalized size = 3.38

$$-\frac{32\,a\cos\left(dx+c\right)^{4}+14\,a\cos\left(dx+c\right)^{3}-48\,a\cos\left(dx+c\right)^{2}-18\,a\cos\left(dx+c\right)+9\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-9\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+\left(a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-a\right)\sin\left(dx+c\right)+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,\left(16\,a\cos\left(dx+c\right)^{3}+9\,a\cos\left(dx+c\right)^{2}-15\,a\cos\left(dx+c\right)-6\,a\right)\sin\left(dx+c\right)+12\,a}{12\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(32*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 48*a*cos(d*x + c)^2 - 18
*a*cos(d*x + c) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(1/2*cos(
d*x + c) + 1/2) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + (a*cos(d*x + c
)^3 + a*cos(d*x + c)^2 - a*cos(d*x + c) - a)*sin(d*x + c) + a)*log(-1/2*cos
(d*x + c) + 1/2) - 2*(16*a*cos(d*x + c)^3 + 9*a*cos(d*x + c)^2 - 15*a*cos(d
*x + c) - 6*a)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
+ (d*cos(d*x + c)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) +
 d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.45, size = 130, normalized size = 1.43

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{66\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 36*a*log(abs(
tan(1/2*d*x + 1/2*c))) + 21*a*tan(1/2*d*x + 1/2*c) - 48*a/(tan(1/2*d*x + 1/
2*c) - 1) - (66*a*tan(1/2*d*x + 1/2*c)^3 + 21*a*tan(1/2*d*x + 1/2*c)^2 + 3*
a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.99, size = 144, normalized size = 1.58

$$\frac{7\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{-23\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+6\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{2\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a}{3}}{d\,\left(8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,d}+\frac{3\,a\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (7*a*tan(c/2 + (d*x)/2))/(8*d) - (a/3 + (2*a*tan(c/2 + (d*x)/2))/3 + 6*a*ta
n(c/2 + (d*x)/2)^2 - 23*a*tan(c/2 + (d*x)/2)^3)/(d*(8*tan(c/2 + (d*x)/2)^3
- 8*tan(c/2 + (d*x)/2)^4)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 +
(d*x)/2)^3)/(24*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.759 $\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-3a^2x+\frac{3a^2\cos(c+dx)}{d}-\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\sec(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}$$

```
[Out] -3*a^2*x+3*a^2*cos(d*x+c)/d-1/3*a^2*cos(d*x+c)^3/d+2*a^2*sec(d*x+c)/d+3*a^2
*tan(d*x+c)/d-a^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2952, 2670, 14, 2671, 294, 327, 209, 276}

$$-\frac{a^2\cos^3(c+dx)}{3d}+\frac{3a^2\cos(c+dx)}{d}+\frac{3a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d}-3a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a^2*x + (3*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]^3)/(3*d) + (2*a^2*Sec
[c + d*x])/d + (3*a^2*Tan[c + d*x])/d - (a^2*Sin[c + d*x]^2*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= \int \left(a^2\sin(c+dx)\tan^2(c+dx)+2a^2\sin^2(c+dx)\tan^2\right. \\
&= a^2\int \sin(c+dx)\tan^2(c+dx)\,dx + a^2\int \sin^3(c+dx)\,\mathrm{t} \\
&= -\frac{a^2\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a^2\mathrm{Subst}\left(\int \frac{(1}{}\right.}{} \\
&= -\frac{a^2\sin^2(c+dx)\tan(c+dx)}{d} - \frac{a^2\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\right.}{d} \\
&= \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(c+dx)}{d} + \\
&= -3a^2x + \frac{3a^2\cos(c+dx)}{d} - \frac{a^2\cos^3(c+dx)}{3d} + \frac{2a^2\sec(}{d}
\end{aligned}
$$

Mathematica [A]

time = 0.37, size = 161, normalized size = 1.81

$$
-\frac{a^2(1+\sin(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(36c+36dx-33\cos(c+dx)+\cos(3(c+dx))-6\sin(2(c+dx)))-\sin\left(\frac{1}{2}(c+dx)\right)(48+36c+36dx-33\cos(c+dx)+\cos(3(c+dx))-6\sin(2(c+dx)))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/12*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(36*c + 36*d*x - 33*Cos[c
 + d*x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)]) - Sin[(c + d*x)/2]*(48 + 3
6*c + 36*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 6*Sin[2*(c + d*x)])))/(
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^4)
```

Maple [A]

time = 0.14, size = 148, normalized size = 1.66

method	result
risch	$-3a^2x+\frac{11a^2\mathrm{e}^{i(dx+c)}}{8d}+\frac{11a^2\mathrm{e}^{-i(dx+c)}}{8d}+\frac{4a^2}{d\left(\mathrm{e}^{i(dx+c)}-i\right)}-\frac{a^2\cos(3dx+3c)}{12d}+\frac{a^2\sin(2dx+2c)}{2d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+2a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a^2}{d}$
default	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+2a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+a^2}{d}$

norman	$\frac{3a^2x-\frac{28a^2}{3d}-\frac{6a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{10a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+6a^2x}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
a^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
))
```

Maxima [A]
time = 0.51, size = 98, normalized size = 1.10

$$-\frac{\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)a^2+3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan\left(dx+c\right)\right)a^2-3\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*((cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^2 + 3*(3*d*x + 3
*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 152, normalized size = 1.71

$$-\frac{a^2\cos\left(dx+c\right)^4-2\,a^2\cos\left(dx+c\right)^3+9\,a^2dx-9\,a^2\cos\left(dx+c\right)^2-6\,a^2+3\left(3\,a^2dx-4\,a^2\right)\cos\left(dx+c\right)-\left(a^2\cos\left(dx+c\right)^3+9\,a^2dx+3\,a^2\cos\left(dx+c\right)^2-6\,a^2\cos\left(dx+c\right)+6\,a^2\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^3 + 9*a^2*d*x - 9*a^2*cos(d*x
 + c)^2 - 6*a^2 + 3*(3*a^2*d*x - 4*a^2)*cos(d*x + c) - (a^2*cos(d*x + c)^3
+ 9*a^2*d*x + 3*a^2*cos(d*x + c)^2 - 6*a^2*cos(d*x + c) + 6*a^2)*sin(d*x +
c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 119, normalized size = 1.34

$$-\frac{9\,(dx+c)a^2+\frac{12\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a^2 + 12*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(3*a^2*tan(1/
2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^4 - 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) - 8*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/
d
```

Mupad [B]
time = 14.71, size = 288, normalized size = 3.24

$$-3\,a^2\,x-\frac{3\,a^2\,(c+d\,x)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(3\,a^2\,(c+d\,x)-\frac{a^2\,(9\,c+9\,d\,x-10)}{3}\right)-\frac{a^2\,(9\,c+9\,d\,x-28)}{3}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\left(3\,a^2\,(c+d\,x)-\frac{a^2\,(9\,c+9\,d\,x-18)}{3}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\left(9\,a^2\,(c+d\,x)-\frac{a^2\,(27\,c+27\,d\,x-18)}{3}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\left(9\,a^2\,(c+d\,x)-\frac{a^2\,(27\,c+27\,d\,x-36)}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\left(9\,a^2\,(c+d\,x)-\frac{a^2\,(27\,c+27\,d\,x-48)}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\left(9\,a^2\,(c+d\,x)-\frac{a^2\,(27\,c+27\,d\,x-66)}{3}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - 3*a^2*x - (3*a^2*(c + d*x) - tan(c/2 + (d*x)/2)*(3*a^2*(c + d*x) - (a^2*(
9*c + 9*d*x - 10))/3) - (a^2*(9*c + 9*d*x - 28))/3 + tan(c/2 + (d*x)/2)^6*(
3*a^2*(c + d*x) - (a^2*(9*c + 9*d*x - 18))/3) - tan(c/2 + (d*x)/2)^5*(9*a^2
*(c + d*x) - (a^2*(27*c + 27*d*x - 18))/3) - tan(c/2 + (d*x)/2)^3*(9*a^2*(c
 + d*x) - (a^2*(27*c + 27*d*x - 36))/3) + tan(c/2 + (d*x)/2)^4*(9*a^2*(c +
d*x) - (a^2*(27*c + 27*d*x - 48))/3) + tan(c/2 + (d*x)/2)^2*(9*a^2*(c + d*x
) - (a^2*(27*c + 27*d*x - 66))/3))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2)^2 + 1)^3)
```

3.760 $\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=71

$$-\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -5/2*a^2*x+2*a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2788, 2727, 2718, 2715, 8}

$$\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= a^2\int\left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx\\ &= -2a^2x-a^2\int\sin^2(c+dx)\,dx-\left(2a^2\right)\int\frac{1}{-1+\sin(c+dx)}\,dx-\left(\right.\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin}{2d}\\ &= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(71) = 142.
time = 0.28, size = 145, normalized size = 2.04

$$-\frac{a^2(1+\sin(c+dx))^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-8\cos(c+dx)-\sin(2(c+dx)))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(8+5c+5dx)+8\cos(c+dx)+\sin(2(c+dx)))\right)}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/4*(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c +
d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c
 + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.12, size = 117, normalized size = 1.65

method	result
risch	$-\frac{5a^2x}{2}+\frac{a^2e^{i(dx+c)}}{d}+\frac{a^2e^{-i(dx+c)}}{d}+\frac{4a^2}{d\left(e^{i(dx+c)}-i\right)}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$

norman	$\frac{\frac{5a^2x}{2}-\frac{8a^2}{d}-\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{6a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{5a^2x(}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.52, size = 84, normalized size = 1.18

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^2+2\left(dx+c-\tan\left(dx+c\right)\right)a^2-4\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A]
time = 0.36, size = 125, normalized size = 1.76

$$\frac{a^2\cos\left(dx+c\right)^3-5\,a^2dx+4\,a^2\cos\left(dx+c\right)^2+4\,a^2-\left(5\,a^2dx-7\,a^2\right)\cos\left(dx+c\right)+\left(5\,a^2dx+a^2\cos\left(dx+c\right)^2-3\,a^2\cos\left(dx+c\right)+4\,a^2\right)\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 2\sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\sin^4\left(c+dx\right)\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(2*sin(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.43, size = 102, normalized size = 1.44

$$-\frac{5\,(dx+c)a^2+\frac{8\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\,\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^2 + 8*a^2/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^2*tan(1/2*d
*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4
*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 11.39, size = 183, normalized size = 2.58

$$-\frac{5\,a^2\,x}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2\,(5\,d\,x-6)}{2}-\frac{5\,a^2\,d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2\,(5\,d\,x-10)}{2}-\frac{5\,a^2\,d\,x}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^2\,(10\,d\,x-10)}{2}-5\,a^2\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2\,(10\,d\,x-22)}{2}-5\,a^2\,d\,x\right)-\frac{a^2\,(5\,d\,x-16)}{2}+\frac{5\,a^2\,d\,x}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (5*a^2*x)/2 - (tan(c/2 + (d*x)/2)*((a^2*(5*d*x - 6))/2 - (5*a^2*d*x)/2) -
 tan(c/2 + (d*x)/2)^4*((a^2*(5*d*x - 10))/2 - (5*a^2*d*x)/2) + tan(c/2 + (d
*x)/2)^3*((a^2*(10*d*x - 10))/2 - 5*a^2*d*x) - tan(c/2 + (d*x)/2)^2*((a^2*(
10*d*x - 22))/2 - 5*a^2*d*x) - (a^2*(5*d*x - 16))/2 + (5*a^2*d*x)/2)/(d*(ta
n(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.761 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=43

$$-2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}$$

```
[Out] -2*a^2*x+2*a^2*cos(d*x+c)/d+sec(d*x+c)*(a+a*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {2934, 2718}

$$\frac{2a^2\cos(c+dx)}{d}-2a^2x+\frac{\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a^2*x + (2*a^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + a*Sin[c + d*x])^2)/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx &= \frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(2a)\int (a+a\sin(c+\\ &= -2a^2x+\frac{\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(2a^2)\int \sin(\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+a\sin(c+dx}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.

time = 0.26, size = 90, normalized size = 2.09

$$\frac{\left(-2(c+dx)+\cos(c+dx)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)(a+a\sin(c+dx))^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] ((-2*(c + d*x) + Cos[c + d*x] + (4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]))*(a + a*Sin[c + d*x])^2)/(d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^4)
```

Maple [A]

time = 0.13, size = 76, normalized size = 1.77

method	result
risch	$-2a^2x+\frac{a^2 e^{i(dx+c)}}{2d}+\frac{a^2 e^{-i(dx+c)}}{2d}+\frac{4a^2}{d\left(e^{i(dx+c)}-i\right)}$
derivativedivides	$\frac{\frac{a^2}{\cos(dx+c)}+2a^2(\tan(dx+c)-dx-c)+a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{\frac{a^2}{\cos(dx+c)}+2a^2(\tan(dx+c)-dx-c)+a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
norman	$\frac{2a^2x-\frac{6a^2}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/cos(d*x+c)+2*a^2*(tan(d*x+c)-d*x-c)+a^2*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]

time = 0.50, size = 57, normalized size = 1.33

$$-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)a^2-a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a^2 - a^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(43) = 86$.
time = 0.38, size = 101, normalized size = 2.35

$$-\frac{2\,a^2dx-a^2\cos\left(dx+c\right)^2-2\,a^2+\left(2\,a^2dx-3\,a^2\right)\cos\left(dx+c\right)-\left(2\,a^2dx-a^2\cos\left(dx+c\right)+2\,a^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*d*x - a^2*cos(d*x + c)^2 - 2*a^2 + (2*a^2*d*x - 3*a^2)*cos(d*x + c)
 - (2*a^2*d*x - a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d
*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sin\left(c+dx\right)\sec^2\left(c+dx\right)\,dx+\int 2\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)\,dx+\int\sin^3\left(c+dx\right)\sec^2\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(2*sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(43) = 86$.
time = 0.44, size = 89, normalized size = 2.07

$$-\frac{2\left((dx+c)a^2+\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a^2 + (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c
) + 3*a^2)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x +
 1/2*c) - 1))/d
```

Mupad [B]
time = 9.18, size = 117, normalized size = 2.72

$$-2\,a^2\,x-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2\,\left(d\,x-1\right)-2\,a^2\,d\,x\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2\,\left(d\,x-2\right)-2\,a^2\,d\,x\right)-2\,a^2\,\left(d\,x-3\right)+2\,a^2\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - 2*a^2*x - (tan(c/2 + (d*x)/2)*(2*a^2*(d*x - 1) - 2*a^2*d*x) - tan(c/2 + (
d*x)/2)^2*(2*a^2*(d*x - 2) - 2*a^2*d*x) - 2*a^2*(d*x - 3) + 2*a^2*d*x)/(d*(
tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.762 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=44

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}$$

[Out] -a^2*arctanh(cos(d*x+c))/d+2*a^2*sec(d*x+c)/d+2*a^2*tan(d*x+c)/d

Rubi [A]
time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2952, 3852, 8, 2702, 327, 213, 2686}

$$\frac{2a^2\tan(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]

[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (2*a^2*Sec[c + d*x])/d + (2*a^2*Tan[c + d*x])/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx &= \int \left(2a^2\sec^2(c+dx)+a^2\csc(c+dx)\sec^2(c+dx)+a^2 \text{s}\right.\\ &= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx + a^2\int \sec(c+dx)\tan\\ &= \frac{a^2\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{d}+\frac{a^2\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{d}\\ &= \frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}+\frac{a^2\text{Subst}\left(\int \frac{1}{-1+x^2}\,d\right.}{d}\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c}{d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 1.57

$$\frac{a^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

[Out] (a^2*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (4*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d

Maple [A]
time = 0.19, size = 58, normalized size = 1.32

method	result
derivativedivides	$\frac{a^{2}\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^{2}\tan(dx+c)+\frac{a^{2}}{\cos(dx+c)}}{d}$
default	$\frac{a^{2}\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2a^{2}\tan(dx+c)+\frac{a^{2}}{\cos(dx+c)}}{d}$
risch	$\frac{4a^{2}}{d\left(e^{i(dx+c)}-i\right)}+\frac{a^{2}\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^{2}\ln\left(e^{i(dx+c)}+1\right)}{d}$
norman	$\frac{-\frac{4a^{2}}{d}-\frac{4a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^{2}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^{2}\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{a^{2}\ln\left(\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*tan(d*x+c)+a^2/cos(d*x+c))

Maxima [A]
time = 0.29, size = 65, normalized size = 1.48

$$\frac{a^{2}\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+4\,a^{2}\tan\left(dx+c\right)+\frac{2\,a^{2}}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^2*tan(d*x + c) + 2*a^2/cos(d*x + c))/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(44) = 88.
time = 0.36, size = 126, normalized size = 2.86

$$\frac{4\,a^{2}\cos\left(dx+c\right)+4\,a^{2}\sin\left(dx+c\right)+4\,a^{2}-\left(a^{2}\cos\left(dx+c\right)-a^{2}\sin\left(dx+c\right)+a^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^{2}\cos\left(dx+c\right)-a^{2}\sin\left(dx+c\right)+a^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/2*(4*a^2*cos(d*x + c) + 4*a^2*sin(d*x + c) + 4*a^2 - (a^2*cos(d*x + c) - a^2*sin(d*x + c) + a^2)*log(1/2*cos(d*x + c) + 1/2) + (a^2*cos(d*x + c) - a

```
^2*sin(d*x + c) + a^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c) - d*si
n(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 38, normalized size = 0.86

$$\frac{a^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{4\,a^2}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 4*a^2/(tan(1/2*d*x + 1/2*c) - 1))/d
```

Mupad [**B**]
time = 8.89, size = 39, normalized size = 0.89

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)),x)
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (4*a^2)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.763 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=58

$$-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}+\frac{2a^2\tan(c+dx)}{d}$$

```
[Out] -2*a^2*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+2*a^2*sec(d*x+c)/d+2*a^2*tan(
d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2952, 3852, 8, 2702, 327, 213, 2700, 14}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2a^2\sec(c+dx)}{d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a^2*Sec[c + d*
x])/d + (2*a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\sec^2(c+dx)+2a^2\csc(c+dx)\sec^2(c+dx)+a^2\right.$$
$$= a^2\int \sec^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\sec^2(c+dx)\,d$$
$$= -\frac{a^2\text{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{d} + \frac{a^2\text{Subst}\left(\int \frac{1+x^2}{x^2}\right.}{}$$
$$= \frac{2a^2\sec(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{a^2\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right.\right.}{}$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2a^2\sec(}{d}$$

Mathematica [A]
time = 0.30, size = 96, normalized size = 1.66

$$\frac{a^2\left(-\cot\left(\frac{1}{2}(c+dx)\right)-4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-Cot[(c + d*x)/2] - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]]
 + (8*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d*
x)/2]))/(2*d)
```

Maple [A]
time = 0.19, size = 76, normalized size = 1.31

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+a^2\tan(dx+c)}{d}$
default	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2a^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+a^2\tan(dx+c)}{d}$
risch	$\frac{-6a^2-2ia^2e^{i(dx+c)}+4a^2e^{2i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)d}-\frac{2a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{a^2}{2d}-\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{4a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\tan\left(\frac{dx}{2}+\right.}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a^2*(1/cos(d*x+c)+ln(csc(
d*x+c)-cot(d*x+c)))+a^2*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.24

$$\frac{a^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-a^2\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+a^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + a^2*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(58) = 116$.
time = 0.36, size = 192, normalized size = 3.31

$$-\frac{3\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-2\,a^2+\left(a^2\cos\left(dx+c\right)^2-a^2+\left(a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2\cos\left(dx+c\right)^2-a^2+\left(a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(3\,a^2\cos\left(dx+c\right)+2\,a^2\right)\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^2+\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c)^2 - a
^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (
a^2*cos(d*x + c)^2 - a^2 + (a^2*cos(d*x + c) + a^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - (3*a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.46, size = 98, normalized size = 1.69

$$\frac{4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2*tan(1/2*d*x + 1/2*c)^2 + 7*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(tan(1/2*d*x
+ 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.93, size = 86, normalized size = 1.48

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d - (a^2 - 9*a^2*tan(c/2 + (d*x)/2))/(d*(2*
tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2)) + (a^2*tan(c/2 + (d*x)/2))/(2
*d)
```

3.764 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{2a^2\tan(c+dx)}{d}$$

```
[Out] -5/2*a^2*arctanh(cos(d*x+c))/d-2*a^2*cot(d*x+c)/d+5/2*a^2*sec(d*x+c)/d-1/2*
a^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2952, 2702, 327, 213, 2700, 14, 294}

$$\frac{2a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}+\frac{5a^2\sec(c+dx)}{2d}-\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-5*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (2*a^2*Cot[c + d*x])/d + (5*a^2*Sec[
c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a^2*Tan[c +
d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^2\,dx = \int \left(a^2\csc(c+dx)\sec^2(c+dx)+2a^2\csc^2(c+dx)\sec^2(\right.$$

$$= a^2\int \csc(c+dx)\sec^2(c+dx)\,dx + a^2\int \csc^3(c+dx)\,\mathrm{se}$$

$$= \frac{a^2\mathrm{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\mathrm{Subst}\left(\int \frac{}{}\right.}{}$$

$$= \frac{a^2\sec(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{a^2\mathrm{Subst}}{}$$

$$= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec(}{2d}$$

$$= -\frac{5a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^2\cot(c+dx)}{d} + \frac{5a^2\sec}{2}$$

Mathematica [A]
time = 0.77, size = 124, normalized size = 1.44

$$\frac{a^2\left(-8\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-20\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+20\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{32\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+8\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 20*Log[Cos[(c + d*x)/2]] +
 20*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (32*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]) + 8*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.22, size = 117, normalized size = 1.36

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+a^2\left(\frac{}{\cos(}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+a^2\left(\frac{}{\cos(}\right.}{d}$
risch	$\frac{a^2\left(-5i e^{3i(dx+c)}+5\,e^{4i(dx+c)}+3i e^{i(dx+c)}-11\,e^{2i(dx+c)}+8\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)d}+\frac{5a^2\ln\left(e^{i(dx+c)}-1\right)}{2d}-\frac{5a^2\ln\left(e^{i(dx+c)}+1\right)}{2d}$
norman	$\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2}{8d}-\frac{4a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\right.\right.}{8d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+2*a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+a^2*(1/cos(d*x+c)+ln
(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 124, normalized size = 1.44

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+2\,a^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-8\,a^2\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^2*(2/cos(d*x + c) - log(
```

```
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a^2*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(80) = 160$.
time = 0.35, size = 300, normalized size = 3.49

$$\frac{16\,a^{2}\cos\left(dx+c\right)^{3}+10\,a^{2}\cos\left(dx+c\right)^{2}-14\,a^{2}\cos\left(dx+c\right)-8\,a^{2}-5\left(a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)-a^{2}-\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+5\left(a^{2}\cos\left(dx+c\right)^{3}+a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)-a^{2}-\left(a^{2}\cos\left(dx+c\right)^{2}-a^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(8\,a^{2}\cos\left(dx+c\right)^{2}+3\,a^{2}\cos\left(dx+c\right)-4\,a^{2}\right)\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(16*a^2*cos(d*x + c)^3 + 10*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) -
8*a^2 - 5*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2
 - (a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 5
*(a^2*cos(d*x + c)^3 + a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - a^2 - (a^2*c
os(d*x + c)^2 - a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*a^2*
cos(d*x + c)^2 + 3*a^2*cos(d*x + c) - 4*a^2)*sin(d*x + c))/(d*cos(d*x + c)^
3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x + c)
 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.47, size = 116, normalized size = 1.35

$$\frac{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+20\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{32\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{30\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 20*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 8
*a^2*tan(1/2*d*x + 1/2*c) - 32*a^2/(tan(1/2*d*x + 1/2*c) - 1) - (30*a^2*tan
(1/2*d*x + 1/2*c)^2 + 8*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c
)^2)/d
```

Mupad [B]
time = 8.96, size = 124, normalized size = 1.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{5\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}-\frac{-20\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{7\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a^2}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (5*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) +
(a^2*tan(c/2 + (d*x)/2))/d - (a^2/2 - 20*a^2*tan(c/2 + (d*x)/2)^2 + (7*a^2*
tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3)
)
```

3.765 $\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=111

$$-\frac{51a^3x}{8}+\frac{7a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{19a^3\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos(c+dx)\ldots}{4\ldots}$$

```
[Out] -51/8*a^3*x+7*a^3*cos(d*x+c)/d-a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1-sin
(d*x+c))+19/8*a^3*cos(d*x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2951, 2727, 2718, 2715, 8, 2713}

$$-\frac{a^3\cos^3(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}+\frac{a^3\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{19a^3\sin(c+dx)\cos(c+dx)}{8d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{51a^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-51*a^3*x)/8 + (7*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/d + (4*a^3*Co
s[c + d*x])/(d*(1 - Sin[c + d*x])) + (19*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*
d) + (a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-4a\right.$$
$$=-4a^3x-a^3\int\sin^4(c+dx)\,dx-\left(3a^3\right)\int\sin^3(c+dx)$$
$$=-4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{2a^3}{}$$
$$=-6a^3x+\frac{7a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{4a^3\text{co}}{d(1-\text{si}}$$
$$=-\frac{51a^3x}{8}+\frac{7a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{d}+\frac{4a^3}{d(1-}$$

Mathematica [A]
time = 0.57, size = 125, normalized size = 1.13

$$\frac{(a+a\sin(c+dx))^3\left(-204(c+dx)+200\cos(c+dx)-8\cos(3(c+dx))+\frac{256\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+40\sin(2(c+dx))-\sin(4(c+dx))\right)}{32d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-204*(c + d*x) + 200*Cos[c + d*x] - 8*Cos[3*(c + d
*x)] + (256*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 40*Si
n[2*(c + d*x)] - Sin[4*(c + d*x)]))/(32*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(105) = 210$.
time = 0.17, size = 212, normalized size = 1.91

method	result
risch	$-\frac{51a^3x}{8}+\frac{25a^3e^{i(dx+c)}}{8d}+\frac{25a^3e^{-i(dx+c)}}{8d}+\frac{8a^3}{d\left(e^{i(dx+c)}-i\right)}-\frac{a^3\sin(4dx+4c)}{32d}-\frac{a^3\cos(3dx+3c)}{4d}+\frac{5a^3\sin(2dx}{4d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\right.}{}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3\left(\right.}{}$
norman	$\frac{\frac{51a^3x}{8}-\frac{20a^3}{d}-\frac{51a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{34a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{69a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{34a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{51a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{1}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
3*a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+a^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*
x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.51, size = 162, normalized size = 1.46

$$-\frac{8\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)a^3+\left(15\,dx+15\,c-\frac{9\tan(dx+c)^3+7\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}-8\tan\left(dx+c\right)\right)a^3+12\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan\left(dx+c\right)\right)a^3-8\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + (15*d*x +
15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)
^2 + 1) - 8*tan(d*x + c))*a^3 + 12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c
)^2 + 1) - 2*tan(d*x + c))*a^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 178, normalized size = 1.60

$$-\frac{2\,a^3\cos\left(dx+c\right)^5+8\,a^3\cos\left(dx+c\right)^4-15\,a^3\cos\left(dx+c\right)^3+51\,a^3dx-56\,a^3\cos\left(dx+c\right)^2-32\,a^3+\left(51\,a^3dx-67\,a^3\right)\cos\left(dx+c\right)+\left(2\,a^3\cos\left(dx+c\right)^4-6\,a^3\cos\left(dx+c\right)^3-51\,a^3dx-21\,a^3\cos\left(dx+c\right)^2+35\,a^3\cos\left(dx+c\right)-32\,a^3\right)\sin\left(dx+c\right)}{8\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(2*a^3*cos(d*x + c)^5 + 8*a^3*cos(d*x + c)^4 - 15*a^3*cos(d*x + c)^3 +
 51*a^3*d*x - 56*a^3*cos(d*x + c)^2 - 32*a^3 + (51*a^3*d*x - 67*a^3)*cos(d*
x + c) + (2*a^3*cos(d*x + c)^4 - 6*a^3*cos(d*x + c)^3 - 51*a^3*d*x - 21*a^3
*cos(d*x + c)^2 + 35*a^3*cos(d*x + c) - 32*a^3)*sin(d*x + c))/(d*cos(d*x +
c) - d*sin(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 167, normalized size = 1.50

$$-\frac{51\,(dx+c)a^3+\frac{64\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}+\frac{2\left(19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-32\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-144\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-160\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-19\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(51*(d*x + c)*a^3 + 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(19*a^3*tan(
1/2*d*x + 1/2*c)^7 - 32*a^3*tan(1/2*d*x + 1/2*c)^6 + 27*a^3*tan(1/2*d*x + 1
/2*c)^5 - 144*a^3*tan(1/2*d*x + 1/2*c)^4 - 27*a^3*tan(1/2*d*x + 1/2*c)^3 -
160*a^3*tan(1/2*d*x + 1/2*c)^2 - 19*a^3*tan(1/2*d*x + 1/2*c) - 48*a^3)/(tan
(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [**B**]
time = 14.70, size = 363, normalized size = 3.27

$$-\frac{51\,a^3\,x}{8}-\frac{\frac{51\,a^3\,\left(c+d\,x\right)}{8}-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{8}-\frac{a^3\,\left(51\,c+51\,d\,x-58\right)}{8}\right)-\frac{a^3\,\left(51\,c+51\,d\,x-160\right)}{8}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{8}-\frac{a^3\,\left(51\,c+51\,d\,x-102\right)}{8}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(204\,c+204\,d\,x-102\right)}{8}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(204\,c+204\,d\,x-266\right)}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(204\,c+204\,d\,x-374\right)}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{51\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(204\,c+204\,d\,x-538\right)}{8}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{153\,a^3\,\left(c+d\,x\right)}{4}-\frac{a^3\,\left(306\,c+306\,d\,x-342\right)}{8}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{153\,a^3\,\left(c+d\,x\right)}{4}-\frac{a^3\,\left(306\,c+306\,d\,x-618\right)}{8}\right)}{d\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (51*a^3*x)/8 - ((51*a^3*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((51*a^3*(c + d
*x))/8 - (a^3*(51*c + 51*d*x - 58))/8) - (a^3*(51*c + 51*d*x - 160))/8 + ta
n(c/2 + (d*x)/2)^8*((51*a^3*(c + d*x))/8 - (a^3*(51*c + 51*d*x - 102))/8) -
 tan(c/2 + (d*x)/2)^7*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d*x - 102))
/8) - tan(c/2 + (d*x)/2)^3*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d*x -
```

266))/8) + tan(c/2 + (d*x)/2)^6*((51*a^3*(c + d*x))/2 - (a^3*(204*c + 204*d
*x - 374))/8) + tan(c/2 + (d*x)/2)^2*((51*a^3*(c + d*x))/2 - (a^3*(204*c +
204*d*x - 538))/8) - tan(c/2 + (d*x)/2)^5*((153*a^3*(c + d*x))/4 - (a^3*(30
6*c + 306*d*x - 342))/8) + tan(c/2 + (d*x)/2)^4*((153*a^3*(c + d*x))/4 - (a
^3*(306*c + 306*d*x - 618))/8))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x
)/2)^2 + 1)^4)

3.766 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -11/2*a^3*x+5*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(1
-sin(d*x+c))+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {2788, 2727, 2718, 2715, 8, 2713}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx &= a^2\int \left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-3a\sin^2(c+dx)\right.\\ &= -4a^3x-a^3\int \sin^3(c+dx)\,dx-\left(3a^3\right)\int \sin^2(c+dx)\,dx-\left(4a^3\right)\int\\ &= -4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\,\mathrm{si}}{2d}\\ &= -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 115, normalized size = 1.29

$$\frac{(a+a\sin(c+dx))^3\left(-66(c+dx)+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+9\sin(2(c+dx))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.13, size = 167, normalized size = 1.88

method	result
risch	$-\frac{11a^3x}{2}+\frac{19a^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{19a^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{8a^3}{d(\mathrm{e}^{i(dx+c)}-i)}-\frac{a^3\cos(3dx+3c)}{12d}+\frac{3a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^3(\tan(dx+c)-dx-c)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\mathrm{co}\cdots}{d}$
default	$\frac{a^3(\tan(dx+c)-dx-c)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\mathrm{co}\cdots}{d}$
norman	$\frac{\frac{11a^3x}{2}-\frac{52a^3}{3d}-\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{21a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{21a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{11a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\cdots}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*c
os(d*x+c)-3/2*d*x-3/2*c)+a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3
*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.52, size = 117, normalized size = 1.31

$$-\frac{2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos\left(dx+c\right)}-6\cos\left(dx+c\right)\right)a^3+9\left(3\,dx+3\,c-\frac{\tan\left(dx+c\right)}{\tan\left(dx+c\right)^2+1}-2\tan\left(dx+c\right)\right)a^3+6\left(dx+c-\tan\left(dx+c\right)\right)a^3-18\,a^3\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 154, normalized size = 1.73

$$-\frac{2\,a^3\cos\left(dx+c\right)^4-7\,a^3\cos\left(dx+c\right)^3+33\,a^3dx-30\,a^3\cos\left(dx+c\right)^2-24\,a^3+3\left(11\,a^3dx-15\,a^3\right)\cos\left(dx+c\right)-\left(2\,a^3\cos\left(dx+c\right)^3+33\,a^3dx+9\,a^3\cos\left(dx+c\right)^2-21\,a^3\cos\left(dx+c\right)+24\,a^3\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)-d\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
```

```
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int \sin^2(c+dx)\sec^2(c+dx)\,dx + \int 3\sin^3(c+dx)\sec^2(c+dx)\,dx + \int 3\sin^4(c+dx)\sec^2(c+dx)\,dx + \int \sin^5(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x
)**3*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)**4*sec(c + d*x)**2, x) +
 Integral(sin(c + d*x)**5*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.45, size = 119, normalized size = 1.34

$$-\frac{33\,(dx+c)a^3 + \frac{48\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1} + \frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-28\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)*a^3 + 48*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(9*a^3*tan(1
/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 - 60*a^3*tan(1/2*d*x + 1/
2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 28*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
3)/d
```

Mupad [B]
time = 14.76, size = 288, normalized size = 3.24

$$-\frac{11\,a^3\,x}{2}-\frac{\frac{11\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{11\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(33\,c+33\,d\,x-38)}{6}\right)-\frac{a^3\,(33\,c+33\,d\,x-104)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{11\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(33\,c+33\,d\,x-66)}{6}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{33\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(99\,c+99\,d\,x-66)}{6}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{33\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(99\,c+99\,d\,x-120)}{6}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{33\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(99\,c+99\,d\,x-192)}{6}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{33\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(99\,c+99\,d\,x-246)}{6}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (11*a^3*x)/2 - ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((11*a^3*(c + d
*x))/2 - (a^3*(33*c + 33*d*x - 38))/6) - (a^3*(33*c + 33*d*x - 104))/6 + ta
n(c/2 + (d*x)/2)^6*((11*a^3*(c + d*x))/2 - (a^3*(33*c + 33*d*x - 66))/6) -
tan(c/2 + (d*x)/2)^5*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6)
- tan(c/2 + (d*x)/2)^3*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 120))/
6) + tan(c/2 + (d*x)/2)^4*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 192
))/6) + tan(c/2 + (d*x)/2)^2*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x -
246))/6))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.767 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=67

$$-\frac{9a^3x}{2}+\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{\sec(c+dx)(a+a\sin(c+dx))^3}{d}$$

```
[Out] -9/2*a^3*x+6*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+sec(d*x+c)*(a
+a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2934, 2723}

$$\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{9a^3x}{2}+\frac{\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-9*a^3*x)/2 + (6*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(
2*d) + (Sec[c + d*x]*(a + a*Sin[c + d*x])^3)/d
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx=\frac{\sec(c+dx)(a+a\sin(c+dx))^3}{d}-(3a)\int(a+a\sin(c+$$

$$=-\frac{9a^3x}{2}+\frac{6a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 145 vs. 2(67) = 134.
time = 0.35, size = 145, normalized size = 2.16

$$-\frac{a^3(1+\sin(c+dx))^3\left(\cos\left(\frac{1}{2}(c+dx)\right)(18(c+dx)-12\cos(c+dx)-\sin(2(c+dx)))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(16+9c+9dx)+12\cos(c+dx)+\sin(2(c+dx)))\right)}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/4*(a^3*(1 + Sin[c + d*x])^3*(Cos[(c + d*x)/2]*(18*(c + d*x) - 12*Cos[c +
 d*x] - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(16 + 9*c + 9*d*x) + 12*Co
s[c + d*x] + Sin[2*(c + d*x)])))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.
time = 0.12, size = 130, normalized size = 1.94

method	result
risch	$-\frac{9a^3x}{2}+\frac{3a^3e^{i(dx+c)}}{2d}+\frac{3a^3e^{-i(dx+c)}}{2d}+\frac{8a^3}{d\left(e^{i(dx+c)}-i\right)}+\frac{a^3\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{\frac{a^3}{\cos(dx+c)}+3a^3(\tan(dx+c)-dx-c)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(}{}\right.\right.}{d}$
default	$\frac{\frac{a^3}{\cos(dx+c)}+3a^3(\tan(dx+c)-dx-c)+3a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(}{}\right.\right.}{d}$
norman	$\frac{\frac{9a^3x}{2}-\frac{14a^3}{d}-\frac{9a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{23a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{18a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{23a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+9a^3x}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/cos(d*x+c)+3*a^3*(tan(d*x+c)-d*x-c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)
+(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/
2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.50, size = 97, normalized size = 1.45

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^3+6\left(dx+c-\tan\left(dx+c\right)\right)a^3-6\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-\frac{2\,a^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
3 + 6*(d*x + c - tan(d*x + c))*a^3 - 6*a^3*(1/cos(d*x + c) + cos(d*x + c))
- 2*a^3/cos(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 125, normalized size = 1.87

$$\frac{a^3\cos(dx+c)^3-9a^3dx+6a^3\cos(dx+c)^2+8a^3-(9a^3dx-13a^3)\cos(dx+c)+(9a^3dx+a^3\cos(dx+c)^2-5a^3\cos(dx+c)+8a^3)\sin(dx+c)}{2(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^3*cos(d*x + c)^3 - 9*a^3*d*x + 6*a^3*cos(d*x + c)^2 + 8*a^3 - (9*a^3
*d*x - 13*a^3)*cos(d*x + c) + (9*a^3*d*x + a^3*cos(d*x + c)^2 - 5*a^3*cos(d
*x + c) + 8*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\sin(c+dx)\sec^2(c+dx)\,dx+\int 3\sin^2(c+dx)\sec^2(c+dx)\,dx+\int 3\sin^3(c+dx)\sec^2(c+dx)\,dx+\int\sin^4(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(sin(c + d*x)*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(3*sin(c + d*x)**3*sec(c + d*x)**2, x) + In
tegral(sin(c + d*x)**4*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.48, size = 102, normalized size = 1.52

$$-\frac{9(dx+c)a^3+\frac{16a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(9*(d*x + c)*a^3 + 16*a^3/(tan(1/2*d*x + 1/2*c) - 1) + 2*(a^3*tan(1/2*
d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) -
6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 11.39, size = 183, normalized size = 2.73

$$-\frac{9a^3x}{2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^3(9dx-10)}{2}-\frac{9a^3dx}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^3(9dx-18)}{2}-\frac{9a^3dx}{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^3(18dx-14)}{2}-9a^3dx\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^3(18dx-42)}{2}-9a^3dx\right)-\frac{a^3(9dx-28)}{2}+\frac{9a^3dx}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (9*a^3*x)/2 - (tan(c/2 + (d*x)/2)*((a^3*(9*d*x - 10))/2 - (9*a^3*d*x)/2)
- tan(c/2 + (d*x)/2)^4*((a^3*(9*d*x - 18))/2 - (9*a^3*d*x)/2) + tan(c/2 + (
d*x)/2)^3*((a^3*(18*d*x - 14))/2 - 9*a^3*d*x) - tan(c/2 + (d*x)/2)^2*((a^3*
(18*d*x - 42))/2 - 9*a^3*d*x) - (a^3*(9*d*x - 28))/2 + (9*a^3*d*x)/2)/(d*(t
an(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.768 $\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=48

$$-a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -a^3*x-a^3*arctanh(cos(d*x+c))/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2951, 3855, 2727}

$$\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+a^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (4*a^3*Cos[c + d*x])/(d*(1 - Sin
[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int\left(-a+a\csc(c+dx)-\frac{4a}{-1+\sin(c+dx)}\right)dx$$
$$= -a^3x+a^3\int\csc(c+dx)\,dx-\left(4a^3\right)\int\frac{1}{-1+\sin(c+dx}$$
$$= -a^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

Mathematica [A]
time = 0.19, size = 74, normalized size = 1.54

$$-\frac{a^3\left(c+dx+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
[Out] -((a^3*(c + d*x + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]] - (8*Sin[(c
 + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])))/d)
```

Maple [A]
time = 0.21, size = 77, normalized size = 1.60

method	result
risch	$-a^3x+\frac{8a^3}{d\left(e^{i(dx+c)}-i\right)}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}$
derivativedivides	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\tan(dx+c)+\frac{3a^3}{\cos(dx+c)}+a^3(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\tan(dx+c)+\frac{3a^3}{\cos(dx+c)}+a^3(\tan(dx+c)-dx-c)}{d}$
norman	$\frac{a^3x-\frac{8a^3}{d}-\frac{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{24a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2a^3x\left(\right.}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*tan(d*x+c)+3*a^3/co
s(d*x+c)+a^3*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.51, size = 84, normalized size = 1.75

$$-\frac{2\,(dx+c-\tan\,(dx+c))a^3-a^3\left(\frac{2}{\cos(dx+c)}-\log\,(\cos\,(dx+c)+1)+\log\,(\cos\,(dx+c)-1)\right)-6\,a^3\tan\,(dx+c)-\frac{6\,a^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^3*tan(d*x + c) - 6*a^3/cos(d*x + c))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(46) = 92$.
time = 0.37, size = 151, normalized size = 3.15

$$-\frac{2\,a^3dx-8\,a^3+2\,(a^3dx-4\,a^3)\cos{(dx+c)}+(a^3\cos{(dx+c)}-a^3\sin{(dx+c)}+a^3)\log{\left(\frac{1}{2}\cos{(dx+c)}+\frac{1}{2}\right)}-(a^3\cos{(dx+c)}-a^3\sin{(dx+c)}+a^3)\log{\left(-\frac{1}{2}\cos{(dx+c)}+\frac{1}{2}\right)}-2\,(a^3dx+4\,a^3)\sin{(dx+c)}}{2\,(d\cos{(dx+c)}-d\sin{(dx+c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*d*x - 8*a^3 + 2*(a^3*d*x - 4*a^3)*cos(d*x + c) + (a^3*cos(d*x +
 c) - a^3*sin(d*x + c) + a^3)*log(1/2*cos(d*x + c) + 1/2) - (a^3*cos(d*x +
c) - a^3*sin(d*x + c) + a^3)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*d*x + 4*
a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.47, size = 49, normalized size = 1.02

$$-\frac{(dx+c)a^3-a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*a^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 8*a^3/(tan(1/2*d*x +
 1/2*c) - 1))/d
```

Mupad [B]
time = 8.93, size = 112, normalized size = 2.33

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{4\,a^6}{4\,a^6+4\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^6+4\,a^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{8\,a^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d + (2*a^3*atan((4*a^6)/(4*a^6 + 4*a^6*tan(c/
2 + (d*x)/2)) - (4*a^6*tan(c/2 + (d*x)/2))/(4*a^6 + 4*a^6*tan(c/2 + (d*x)/2
))))/d - (8*a^3)/(d*(tan(c/2 + (d*x)/2) - 1))
```

3.769 $\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=56

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -3*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x
+c))
```

Rubi [A]
time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2951, 3855, 3852, 8, 2727}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (4*a^3*Cos[c + d*
x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int \left(3a\csc(c+dx)+a\csc^2(c+dx)-\frac{4a}{-1+\sin(c+\right.$$
$$=a^3\int \csc^2(c+dx)\,dx+\left(3a^3\right)\int \csc(c+dx)\,dx-\left(4a^3\right)$$
$$=-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{a^3\mathrm{S}}{}$$
$$=-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{4a^3\cos}{d(1-\sin}$$

Mathematica [A]

time = 0.45, size = 96, normalized size = 1.71

$$\frac{a^3\left(-\cot\left(\frac{1}{2}(c+dx)\right)-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(-Cot[(c + d*x)/2] - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]]
 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + Tan[(c + d
*x)/2]))/(2*d)
```

Maple [A]

time = 0.20, size = 89, normalized size = 1.59

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\tan(dx+c)+\frac{a^3}{\cos(dx+c)}}{d}$
default	$\frac{a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^3\tan(dx+c)+\frac{a^3}{\cos(dx+c)}}{d}$
risch	$\frac{-10a^3-2ia^3\mathrm{e}^{i(dx+c)}+8a^3\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}-i\right)d}+\frac{3a^3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}-\frac{3a^3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}$

norman	$\frac{\frac{a^3}{2d}-\frac{15a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{25a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{24a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{25a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{15a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a^3(\tan}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*a^3*(1/cos(d*x+c)+ln(csc(
d*x+c)-cot(d*x+c)))+3*a^3*tan(d*x+c)+a^3/cos(d*x+c))
```

Maxima [A]
time = 0.29, size = 88, normalized size = 1.57

$$\frac{3\,a^3\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a^3\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+6\,a^3\tan\left(dx+c\right)+\frac{2\,a^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^3*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
 - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a^3*tan(d*x + c) + 2*a^3/cos(d
*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(54) = 108.
time = 0.38, size = 194, normalized size = 3.46

$$-\frac{10\,a^3\cos\left(dx+c\right)^2+2\,a^3\cos\left(dx+c\right)-8\,a^3+3\left(a^3\cos\left(dx+c\right)^2-a^3+\left(a^3\cos\left(dx+c\right)+a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a^3\cos\left(dx+c\right)^2-a^3+\left(a^3\cos\left(dx+c\right)+a^3\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\left(5\,a^3\cos\left(dx+c\right)+4\,a^3\right)\sin\left(dx+c\right)}{2\left(d\cos\left(dx+c\right)^2+\left(d\cos\left(dx+c\right)+d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(10*a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) - 8*a^3 + 3*(a^3*cos(d*x +
 c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) +
 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3 + (a^3*cos(d*x + c) + a^3)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2) - 2*(5*a^3*cos(d*x + c) + 4*a^3)*sin(d*x + c
))/(d*cos(d*x + c)^2 + (d*cos(d*x + c) + d)*sin(d*x + c) - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.47, size = 98, normalized size = 1.75

$$\frac{6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+14\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (3*a
^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 8.97, size = 86, normalized size = 1.54

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3-17\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3 - 17*a^3*tan(c/2 + (d*x)/2))/(d*(2
*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2)) + (a^3*tan(c/2 + (d*x)/2))/(
2*d)
```

3.770 $\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=80

$$-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -9/2*a^3*arctanh(cos(d*x+c))/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d*
x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3855, 3852, 8, 3853, 2727}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
+ d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx &= a^2\int \Big(4a\csc(c+dx)+3a\csc^2(c+dx)+a\csc^3(c+dx\\ &= a^3\int \csc^3(c+dx)\,dx+\left(3a^3\right)\int \csc^2(c+dx)\,dx+\big(4a^3\\ &= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}\\ &= -\frac{9a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(\end{aligned}$$

Mathematica [A]
time = 0.78, size = 124, normalized size = 1.55

$$\frac{a^3\left(-12\cot\left(\frac{1}{2}(c+dx)\right)-\csc^2\left(\frac{1}{2}(c+dx)\right)-36\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+36\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{64\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+12\tan\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-12*Cot[(c + d*x)/2] - Csc[(c + d*x)/2]^2 - 36*Log[Cos[(c + d*x)/2]]
+ 36*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^2 + (64*Sin[(c + d*x)/2])/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]) + 12*Tan[(c + d*x)/2]))/(8*d)
```

Maple [A]
time = 0.26, size = 128, normalized size = 1.60

method	result

risch	$\frac{a^3\left(-7i\,\mathrm{e}^{3i(dx+c)}+9\,\mathrm{e}^{4i(dx+c)}+5i\,\mathrm{e}^{i(dx+c)}-21\,\mathrm{e}^{2i(dx+c)}+14\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}-i\right)d}+\frac{9a^3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{2d}-\frac{9a^3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{2d}$
derivativedivides	$\frac{a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(\frac{}{\cos}\right.}{d}$
default	$\frac{a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^3\left(\frac{}{\cos}\right.}{d}$
norman	$\frac{\frac{a^3}{8d}+\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{13a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{27a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{49a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{27a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{13a^3\left(\tan^9\right.}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+3*a^3*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*a^3*(1/cos(d*x+c)+
ln(csc(d*x+c)-cot(d*x+c)))+a^3*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 135, normalized size = 1.69

$$\frac{a^3\left(\frac{2\left(3\cos\left(dx+c\right)^2-2\right)}{\cos\left(dx+c\right)^3-\cos\left(dx+c\right)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+6a^3\left(\frac{2}{\cos\left(dx+c\right)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12a^3\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)+4a^3\tan\left(dx+c\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^3*(1/tan(d*x + c) - tan(d
*x + c)) + 4*a^3*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(74) = 148$.
time = 0.40, size = 300, normalized size = 3.75

$$\frac{28\,a^3\cos\left(dx+c\right)^3+18\,a^3\cos\left(dx+c\right)^2-26\,a^3\cos\left(dx+c\right)-16\,a^3-9\left(a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-a^3-\left(a^3\cos\left(dx+c\right)^2-a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+9\left(a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-a^3-\left(a^3\cos\left(dx+c\right)^2-a^3\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(14\,a^3\cos\left(dx+c\right)^2+5\,a^3\cos\left(dx+c\right)-8\,a^3\right)\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(28*a^3*cos(d*x + c)^3 + 18*a^3*cos(d*x + c)^2 - 26*a^3*cos(d*x + c) -
16*a^3 - 9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^
3 - (a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
```

```
9*(a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - a^3 - (a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^
3*cos(d*x + c)^2 + 5*a^3*cos(d*x + c) - 8*a^3)*sin(d*x + c))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2 - d*cos(d*x + c) - (d*cos(d*x + c)^2 - d)*sin(d*x +
c) - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 116, normalized size = 1.45

$$\frac{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{64\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{54\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 1
2*a^3*tan(1/2*d*x + 1/2*c) - 64*a^3/(tan(1/2*d*x + 1/2*c) - 1) - (54*a^3*ta
n(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2
*c)^2)/d
```

Mupad [B]
time = 8.95, size = 125, normalized size = 1.56

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{9\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-38\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{11\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a^3}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (9*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) +
(3*a^3*tan(c/2 + (d*x)/2))/(2*d) - (a^3/2 - 38*a^3*tan(c/2 + (d*x)/2)^2 + (
11*a^3*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x
)/2)^3))
```

3.771 $\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{5a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}$$

```
[Out] -11/2*a^3*arctanh(cos(d*x+c))/d-5*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3
/2*a^3*cot(d*x+c)*csc(d*x+c)/d+4*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3855, 3852, 8, 3853, 2727}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{5a^3\cot(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (5*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (4*a^3*Cos[c +
 d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^2\int \Big(4a\csc(c+dx)+4a\csc^2(c+dx)+3a\csc^3(c+d$$
$$= a^3\int \csc^4(c+dx)\,dx + \left(3a^3\right)\int \csc^3(c+dx)\,dx + \left(4a^3\right.$$
$$= -\frac{4a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{3a^3\cot(c+dx)\csc(c+dx}{2d}$$
$$= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{5a^3\cot(c+dx)}{d} - \frac{a^3\cot}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 211 vs. 2(98) = 196.
time = 6.17, size = 211, normalized size = 2.15

$$a^3\left(-\frac{7\cot\left(\frac{1}{2}(c+dx)\right)}{3d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}-\frac{11\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{11\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{7\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-7*Cot[(c + d*x)/2])/(3*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c +
 d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (11*Log[Cos[(c + d*x)/2]])/(2*d) + (1
1*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + (8*Sin[(c +
 d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (7*Tan[(c + d*x)/2])/
(3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d))
```

Maple [A]
time = 0.25, size = 167, normalized size = 1.70

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+ \dots}{}\right.}{d}$
default	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+ \dots}{}\right.}{d}$
risch	$\frac{-33ia^3e^{5i(dx+c)}+33a^3e^{6i(dx+c)}+60ia^3e^{3i(dx+c)}-96a^3e^{4i(dx+c)}-19ia^3e^{i(dx+c)}+123a^3e^{2i(dx+c)}-52a^3}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}-i\right)d}-\frac{11a^3\ln(e^{i \dots}}{2 \dots}$
norman	$\frac{\frac{a^3}{24d}+\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{59a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{45a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{231a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{51a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{231a^3(\tan \dots}{}}{\tan \dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+3*a^3*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-
cot(d*x+c)))+3*a^3*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+a^3*(1/cos(d*x+c)
+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 160, normalized size = 1.63

$$\frac{9\,a^3\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+6\,a^3\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-36\,a^3\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)-4\,a^3\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan\left(dx+c\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*l
og(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a^3*(2/cos(d*x + c) - l
og(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a^3*(1/tan(d*x + c) - ta
n(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(90) = 180$.
time = 0.37, size = 354, normalized size = 3.61

$$-\frac{104\,a^3\cos\left(dx+c\right)^4+38\,a^3\cos\left(dx+c\right)^3-156\,a^3\cos\left(dx+c\right)^2-42\,a^3\cos\left(dx+c\right)+48\,a^3+33\,\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3+\left(a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-33\,\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^2+a^3+\left(a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-a^3\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,\left(52\,a^3\cos\left(dx+c\right)^3+33\,a^3\cos\left(dx+c\right)^2-45\,a^3\cos\left(dx+c\right)-24\,a^3\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 38*a^3*cos(d*x + c)^3 - 156*a^3*cos(d*x + c
)^2 - 42*a^3*cos(d*x + c) + 48*a^3 + 33*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x
 + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a^3*cos(d*x + c)
 - a^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4
- 2*a^3*cos(d*x + c)^2 + a^3 + (a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 - a
^3*cos(d*x + c) - a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(52*a
^3*cos(d*x + c)^3 + 33*a^3*cos(d*x + c)^2 - 45*a^3*cos(d*x + c) - 24*a^3)*s
in(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + (d*cos(d*x + c)^3 + d
*cos(d*x + c)^2 - d*cos(d*x + c) - d)*sin(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 148, normalized size = 1.51

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{192\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}-\frac{242\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+57\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 57*a^3*tan(1/2*d*x + 1/2*c) - 192*a^3/(tan(
1/2*d*x + 1/2*c) - 1) - (242*a^3*tan(1/2*d*x + 1/2*c)^3 + 57*a^3*tan(1/2*d*
x + 1/2*c)^2 + 9*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 8.97, size = 160, normalized size = 1.63

$$\frac{3\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{-83\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{8\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a^3}{3}}{d\,\left(8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-8\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}+\frac{11\,a^3\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}+\frac{19\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (3*a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (1
6*a^3*tan(c/2 + (d*x)/2)^2 - 83*a^3*tan(c/2 + (d*x)/2)^3 + a^3/3 + (8*a^3*t
an(c/2 + (d*x)/2))/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*x)/2)^4))
 + (11*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) + (19*a^3*tan(c/2 + (d*x)/2))/(8*
d)
```

3.772 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=83

$$\frac{x}{a}+\frac{\cos(c+dx)}{ad}+\frac{2\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] x/a+cos(d*x+c)/a/d+2*sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d-tan(d*x+c)/a/d+1/3
*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2918, 3554, 8, 2670, 276}

$$\frac{\cos(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Cos[c + d*x]/(a*d) + (2*Sec[c + d*x])/(a*d) - Sec[c + d*x]^3/(3*a*d)
- Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \tan^4(c+dx)\,dx}{a}-\frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a}\\
&= \frac{\tan^3(c+dx)}{3ad}-\frac{\int \tan^2(c+dx)\,dx}{a}+\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{ad}\\
&= -\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}+\frac{\int 1\,dx}{a}+\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx,x,\text{co}\right.}{ad}\\
&= \frac{x}{a}+\frac{\cos(c+dx)}{ad}+\frac{2\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c}{3a}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 148, normalized size = 1.78

$$
\frac{18+2(-11+6c+6dx)\cos(c+dx)+14\cos(2(c+dx))+11\sin(c+dx)-11\sin(2(c+dx))+6c\sin(2(c+dx))+6dx\sin(2(c+dx))+3\sin(3(c+dx))}{12ad\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (18 + 2*(-11 + 6*c + 6*d*x)*Cos[c + d*x] + 14*Cos[2*(c + d*x)] + 11*Sin[c +
 d*x] - 11*Sin[2*(c + d*x)] + 6*c*Sin[2*(c + d*x)] + 6*d*x*Sin[2*(c + d*x)]
 + 3*Sin[3*(c + d*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.18, size = 99, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{32}{16+16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{2\left(\tan\left(\frac{dx}{2}\right.\right.}}{ad}$
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{32}{16+16\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{5}{2\left(\tan\left(\frac{dx}{2}\right.\right.}}{ad}$

risch	$\frac{x}{a}+\frac{\mathrm{e}^{i(dx+c)}}{2ad}+\frac{\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{\frac{4\,\mathrm{e}^{i(dx+c)}}{3}+4i\mathrm{e}^{2i(dx+c)}+4\,\mathrm{e}^{3i(dx+c)}+\frac{8i}{3}}{\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}da}$
norman	$\frac{\frac{x\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x}{a}+\frac{8\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{2x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x(}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*(-1/64/(tan(1/2*d*x+1/2*c)-1)+1/16/(1+tan(1/2*d*x+1/2*c)^2)+1/16*arc
tan(tan(1/2*d*x+1/2*c))-1/48/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2
*c)+1)^2+5/64/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(79) = 158$.
time = 0.51, size = 236, normalized size = 2.84

$$\frac{2\left(\frac{\frac{13\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{6\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{3\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+8}{a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{2\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{3\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*((13*sin(d*x + c)/(cos(d*x + c) + 1) + 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 6*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8)/(a + 2*a*sin(d*x +
c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 2*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6) + 3*arctan(sin(d*x + c)/(cos(d*x + c) +
1))/a)/d
```

Fricas [A]
time = 0.36, size = 80, normalized size = 0.96

$$\frac{3\,dx\cos\left(dx+c\right)+7\,\cos\left(dx+c\right)^{2}+\left(3\,dx\cos\left(dx+c\right)+3\,\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)+1}{3\,\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cos(d*x + c) + 7*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) + 3*cos(d*
x + c)^2 + 2)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 125, normalized size = 1.51

$$\frac{\frac{6(dx+c)}{a} - \frac{3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)a} + \frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+17}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)/a - 3*(tan(1/2*d*x + 1/2*c)^2 - 4*tan(1/2*d*x + 1/2*c) + 5
)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c)
- 1)*a) + (15*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x + 1/2*c) + 17)/(a*(ta
n(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 13.52, size = 129, normalized size = 1.55

$$\frac{x}{a} - \frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 - 4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3} + \frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3} + \frac{16}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{dx}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] x/a - ((26*tan(c/2 + (d*x)/2))/3 + (4*tan(c/2 + (d*x)/2)^2)/3 - (4*tan(c/2
+ (d*x)/2)^3)/3 - 4*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + 16/3)/(
a*d*(tan(c/2 + (d*x)/2) + 1)^3*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 +
 tan(c/2 + (d*x)/2)^3 - 1))
```

3.773 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{x}{a}-\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] -x/a-sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+tan(d*x+c)/a/d-1/3*tan(d*x+c)^3/a/
d
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2918, 2686, 3554, 8}

$$-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d) -
 Tan[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \tan^4(c+dx)\,dx}{a}\\ &= -\frac{\tan^3(c+dx)}{3ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx\right.}{ad}\\ &= -\frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad} - \frac{\int 1\,dx}{a}\\ &= -\frac{x}{a} - \frac{\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{3ad} + \frac{\tan(c+dx)}{ad} - \frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 111, normalized size = 1.59

$$\frac{4\cos(2(c+dx)) - 2\sin(c+dx) + (-5+6c+6dx)\cos(c+dx)(1+\sin(c+dx))}{6ad\left(-\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (4*Cos[2*(c + d*x)] - 2*Sin[c + d*x] + (-5 + 6*c + 6*d*x)*Cos[c + d*x]*(1 +
 Sin[c + d*x]))/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.17, size = 82, normalized size = 1.17

method	result
risch	$-\frac{x}{a} - \frac{2\left(3\,e^{3i(dx+c)}+4i+5\,e^{i(dx+c)}\right)}{3\left(e^{i(dx+c)}+i\right)^3\left(e^{i(dx+c)}-i\right)ad}$
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - 2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{ad}$
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} - 2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3} - \frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} - \frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{ad}$
norman	$\frac{\frac{x}{a} + \frac{x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} - \frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} - \frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} + \frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a} - \frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} - \frac{2x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} - \frac{x\left(\tan^6\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 16/d/a*(-1/32/(tan(1/2*d*x+1/2*c)-1)-1/8*arctan(tan(1/2*d*x+1/2*c))+1/24/(tan(1/2*d*x+1/2*c)+1)^3-1/16/(tan(1/2*d*x+1/2*c)+1)^2-3/32/(tan(1/2*d*x+1/2*c)+1))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(66) = 132$.
time = 0.57, size = 154, normalized size = 2.20

$$-\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+2}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] -2/3*((sin(d*x + c)/(cos(d*x + c) + 1) - 6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d

Fricas [A]
time = 0.35, size = 70, normalized size = 1.00

$$-\frac{3\,dx\cos\left(dx+c\right)+4\,\cos\left(dx+c\right)^2+\left(3\,dx\cos\left(dx+c\right)-1\right)\sin\left(dx+c\right)-2}{3\left(ad\cos\left(dx+c\right)\sin\left(dx+c\right)+ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/3*(3*d*x*cos(d*x + c) + 4*cos(d*x + c)^2 + (3*d*x*cos(d*x + c) - 1)*sin(d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.46, size = 77, normalized size = 1.10

$$-\frac{\frac{6\,(dx+c)}{a}+\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)/a + 3/(a*(tan(1/2*d*x + 1/2*c) - 1)) + (9*tan(1/2*d*x + 1
/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 11.18, size = 79, normalized size = 1.13

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{4}{3}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2))/3 - 4*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^3
 + 4/3)/(a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3) - x/a
```

3.774 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {2785, 2687, 30, 2686}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
```

```
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(50) = 100.
time = 0.10, size = 106, normalized size = 2.12

$$\frac{6-10\cos(c+dx)+2\cos(2(c+dx))+8\sin(c+dx)-5\sin(2(c+dx))}{12ad\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.15, size = 70, normalized size = 1.40

method	result	size
norman	$\frac{-\frac{4}{3ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	54
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{ad}$	70
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}}{ad}$	70
risch	$\frac{2i\,e^{2i(dx+c)}+2\,e^{3i(dx+c)}+\frac{2i}{3}-\frac{2\,e^{i(dx+c)}}{3}}{\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^3 da}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.30, size = 90, normalized size = 1.80

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

Fricas [A]
time = 0.34, size = 47, normalized size = 0.94

$$\frac{\cos(dx+c)^2+2\sin(dx+c)+1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.45, size = 68, normalized size = 1.36

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 8.90, size = 47, normalized size = 0.94

$$-\frac{4\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) + 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 +
(d*x)/2) + 1)^3)
```

3.775 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] 1/3*sec(d*x+c)^3/a/d-1/3*tan(d*x+c)^3/a/d
```

Rubi [A]

time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2918, 2686, 30, 2687}

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^3/(3*a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{\sec^3(c+dx)}{3ad}-\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 104 vs. $2(37) = 74$.

time = 0.10, size = 104, normalized size = 2.81

$$\frac{-3+\cos(c+dx)+\cos(2(c+dx))-2\sin(c+dx)+\frac{1}{2}\sin(2(c+dx))}{6ad\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3 + Cos[c + d*x] + Cos[2*(c + d*x)] - 2*Sin[c + d*x] + Sin[2*(c + d*x)]/2
)/(6*a*d*(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.

time = 0.14, size = 70, normalized size = 1.89

method	result	size
risch	$\frac{2i\left(2i\,e^{i(dx+c)}+3\,e^{2i(dx+c)}-1\right)}{3\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^3 da}$	63
derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}}{ad}$	70
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8}}{ad}$	70
norman	$\frac{-\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2}{3ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(-1/8/(tan(1/2*d*x+1/2*c)-1)+1/6/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/
2*d*x+1/2*c)+1)^2+1/8/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(33) = 66$.
time = 0.28, size = 110, normalized size = 2.97

$$\frac{2\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(co
s(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

Fricas [A]
time = 0.33, size = 47, normalized size = 1.27

$$-\frac{\cos(dx+c)^2-\sin(dx+c)-2}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 - sin(d*x + c) - 2)/(a*d*cos(d*x + c)*sin(d*x + c) + a
*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.44, size = 57, normalized size = 1.54

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*
(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 8.91, size = 60, normalized size = 1.62

$$-\frac{2\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(2*tan(c/2 + (d*x)/2) + 3*tan(c/2 + (d*x)/2)^2 + 1))/(3*a*d*(tan(c/2 +
(d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^3)
```

3.776 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d-tan(d*x+c)/a/d
-1/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2918, 2702, 308, 213, 3852}

$$-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) - Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \sec^4(c+dx)\,dx}{a}+\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx\right.}{ad}\\ &= -\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\sec(c+d\right.}{ad}\\ &= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\text{Subst}\left(\int \frac{1}{-1+x}\right.}{}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}-\frac{\tan^3}{}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 149, normalized size = 1.89

$$\frac{-6\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{11}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]] + (Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^(-2) + Sin[(c + d*x)/2]*(3/(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]) - 2/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 - 11/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])))/(6*a*d)
```

Maple [A]
time = 0.19, size = 79, normalized size = 1.00

method	result	size

derivativedivides	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	79
default	$\frac{-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{5}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	79
norman	$\frac{-\frac{8}{3ad}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	91
risch	$\frac{4i e^{2i(dx+c)}+2\,e^{3i(dx+c)}+\frac{4i}{3}+\frac{2\,e^{i(dx+c)}}{3}}{\left(e^{i(dx+c)}+i\right)^{3}\left(e^{i(dx+c)}-i\right)ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{ad}$	112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2/(tan(1/2*d*x+1/2*c)-1)+2/3/(tan(1/2*d*x+1/2*c)+1)^3-1/(tan(1/2*
d*x+1/2*c)+1)^2+5/2/(tan(1/2*d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.29, size = 136, normalized size = 1.72

$$\frac{\frac{2\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+4\right)}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 4)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.46

$$\frac{4\cos(dx+c)^{2}-3\left(\cos(dx+c)\sin(dx+c)+\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)\sin(dx+c)+\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\sin(dx+c)+4}{6\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(d*x + c)^2 - 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)*sin(d*x + c) + cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*sin(d*x + c) + 4)/(a*d*cos(d*x + c)*sin(d*x + c)
 + a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)\sec^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.43, size = 83, normalized size = 1.05

$$\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3/(a*(tan(1/2*d*x + 1/2*c) - 1))
+ (15*tan(1/2*d*x + 1/2*c)^2 + 24*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(1/2*d*
x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 9.90, size = 92, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{8}{3}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((10*tan(c/2 + (d*x)/2))/3 - 2*tan(c/2 + (d
*x)/2)^3 + 8/3)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 2*a*tan(c/2 + (d*x)/2)^3 -
 a*tan(c/2 + (d*x)/2)^4))
```

3.777 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+
2*tan(d*x+c)/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2918, 2700, 276, 2702, 308, 213}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) + (2*Tan[c + d*x])/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx,x,\tan(c+dx)\right.}{ad}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,d\right.}{ad}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}+\frac{\tan^3(c+d}{3ad}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{2\tan}{}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(93) = 186.
time = 0.44, size = 245, normalized size = 2.63

$$-\frac{\csc^3(c+dx)\left(2+10\cos(2(c+dx))+8\cos(3(c+dx))+3\cos(3(c+dx))\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-3\cos(3(c+dx))\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(-8-3\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)+4\sin(c+dx)-16\sin(2(c+dx))-6\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+6\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+8\sin(3(c+dx))\right)}{3ad\left(\csc\left(\tfrac{1}{2}(c+dx)\right)-\sec\left(\tfrac{1}{2}(c+dx)\right)\right)\left(\csc\left(\tfrac{1}{2}(c+dx)\right)+\sec\left(\tfrac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(Csc[c + d*x]^3*(2 + 10*Cos[2*(c + d*x)] + 8*Cos[3*(c + d*x)] + 3*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2
```

```
]] + Cos[c + d*x]*(-8 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])
+ 4*Sin[c + d*x] - 16*Sin[2*(c + d*x)] - 6*Log[Cos[(c + d*x)/2]]*Sin[2*(c +
 d*x)] + 6*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(a
*d*(Csc[(c + d*x)/2] - Sec[(c + d*x)/2])*(Csc[(c + d*x)/2] + Sec[(c + d*x)/
2])*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.23, size = 104, normalized size = 1.12

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{7}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}$
risch	$-\frac{2\left(-2\,{\rm e}^{3i(dx+c)}+6i{\rm e}^{4i(dx+c)}-13\,{\rm e}^{i(dx+c)}-8i+2i{\rm e}^{2i(dx+c)}+3\,{\rm e}^{5i(dx+c)}\right)}{3\left({\rm e}^{2i(dx+c)}-1\right)\left({\rm e}^{i(dx+c)}+i\right)^{3}\left({\rm e}^{i(dx+c)}-i\right)ad}-\frac{\ln\left({\rm e}^{i(dx+c)}-1\right)}{ad}+\frac{\ln\left({\rm e}^{i(dx+c)}+1\right)}{ad}$
norman	$\frac{\frac{1}{2ad}-\frac{6\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{3ad}-\frac{13\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{17\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-1/(tan(1/2*d*x+1/2*c)-1)-4/3/(tan(1/2*d*x+1/2*c
)+1)^3+2/(tan(1/2*d*x+1/2*c)+1)^2-7/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/
2*c)-2*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(89) = 178$.
time = 0.29, size = 215, normalized size = 2.31

$$-\frac{\frac{\frac{22\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{30\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+3}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((22*sin(d*x + c)/(cos(d*x + c) + 1) + 8*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/(a*sin(d*x + c)/(cos(d*x + c) + 1) + 2*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5) + 6*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 3
*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.35, size = 162, normalized size = 1.74

$$\frac{10\cos\left(dx+c\right)^{2}+3\left(\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)\sin\left(dx+c\right)-\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(8\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-4}{6\left(ad\cos\left(dx+c\right)^{3}-ad\cos\left(dx+c\right)\sin\left(dx+c\right)-ad\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 - cos(d*x + c)*sin(d*x + c) - co
s(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 - cos(d*x + c)*
sin(d*x + c) - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(8*cos(d*x +
c)^2 - 1)*sin(d*x + c) - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c)*sin(d*x
+ c) - a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.47, size = 133, normalized size = 1.43

$$-\frac{\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a}+\frac{21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+36\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+19}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{3}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*tan(1/2*d*x + 1/2*c)/a - 3*(ta
n(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/((tan(1/2*d*x + 1/2*c)^2
 - tan(1/2*d*x + 1/2*c))*a) + (21*tan(1/2*d*x + 1/2*c)^2 + 36*tan(1/2*d*x +
 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

Mupad [B]
time = 9.11, size = 150, normalized size = 1.61

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,d}-\frac{-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+\frac{22\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+1}{d\,\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a*d) - ((22*tan(c/2 + (d*x)/2))/3 + (8*tan(c/2 + (d*x
)/2)^2)/3 - 10*tan(c/2 + (d*x)/2)^3 - 9*tan(c/2 + (d*x)/2)^4 + 1)/(d*(2*a*t
an(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 + (d*x)/2)^4 - 2
*a*tan(c/2 + (d*x)/2)^5)) - log(tan(c/2 + (d*x)/2))/(a*d)
```

3.778 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{9x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan^5}{1\cdots}$$

```
[Out] -9/2*x/a^2-2*cos(d*x+c)/a^2/d-6*sec(d*x+c)/a^2/d+2*sec(d*x+c)^3/a^2/d-2/5*s
ec(d*x+c)^5/a^2/d+9/2*tan(d*x+c)/a^2/d-3/2*tan(d*x+c)^3/a^2/d+9/10*tan(d*x+
c)^5/a^2/d-1/2*sin(d*x+c)^2*tan(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2954, 2789, 3554, 8, 2670, 276, 2671, 294, 308, 209}

$$-\frac{2\cos(c+dx)}{a^2d}+\frac{9\tan^5(c+dx)}{10a^2d}-\frac{3\tan^3(c+dx)}{2a^2d}+\frac{9\tan(c+dx)}{2a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}-\frac{\sin^2(c+dx)\tan^5(c+dx)}{2a^2d}-\frac{9x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*x)/(2*a^2) - (2*Cos[c + d*x])/(a^2*d) - (6*Sec[c + d*x])/(a^2*d) + (2*S
ec[c + d*x]^3)/(a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (9*Tan[c + d*x])/(2
*a^2*d) - (3*Tan[c + d*x]^3)/(2*a^2*d) + (9*Tan[c + d*x]^5)/(10*a^2*d) - (S
in[c + d*x]^2*Tan[c + d*x]^5)/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int (a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}\\ &= \frac{\int \left(a^2\tan^6(c+dx)-2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\tan^6(c\right.}{a^4}\\ &= \frac{\int \tan^6(c+dx)\,dx}{a^2}+\frac{\int \sin^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sin(c+dx)\tan^6}{a^2}\\ &= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)^2}\,dx,x,\tan(c+dx)\right)}{a^2d}\\ &= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{\sin^2(c+dx)\tan^5(c+dx)}{2a^2d}+\frac{\int \tan^2(c+}{a^2}\\ &= -\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(}{}\\ &= -\frac{x}{a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\\ &= -\frac{9x}{2a^2}-\frac{2\cos(c+dx)}{a^2d}-\frac{6\sec(c+dx)}{a^2d}+\frac{2\sec^3(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}\end{aligned}$$

Mathematica [A]
time = 0.42, size = 191, normalized size = 1.28

$$-\frac{500+10(-103+90c+90dx)\cos(c+dx)+544\cos(2(c+dx))+206\cos(3(c+dx))-180c\cos(3(c+dx))-180dx\cos(3(c+dx))-20\cos(4(c+dx))+250\sin(c+dx)-824\sin(2(c+dx))+720c\sin(2(c+dx))+720dx\sin(2(c+dx))+351\sin(3(c+dx))+5\sin(5(c+dx))}{160a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/160*(500 + 10*(-103 + 90*c + 90*d*x)*Cos[c + d*x] + 544*Cos[2*(c + d*x)]
 + 206*Cos[3*(c + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)]
 - 20*Cos[4*(c + d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] + 720*c*Si
n[2*(c + d*x)] + 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*
(c + d*x)])/(a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.34, size = 168, normalized size = 1.13

method	result
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-9\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^2}$

default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8\left(\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8}+\frac{1}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-9\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^2}$
risch	$-\frac{9x}{2a^2}-\frac{ie^{2i(dx+c)}}{8a^2d}-\frac{e^{i(dx+c)}}{d\,a^2}-\frac{e^{-i(dx+c)}}{d\,a^2}+\frac{ie^{-2i(dx+c)}}{8a^2d}-\frac{2\left(-40\,e^{3i(dx+c)}+75ie^{4i(dx+c)}+30\,e^{5i(dx+c)}-78\,e\right.}{5\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)}$
norman	$\frac{-\frac{9x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{132\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{9x}{2a}+\frac{64}{5ad}+\frac{211\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5ad}-\frac{81x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{18x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{81x\left(\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^2*(-1/512/(tan(1/2*d*x+1/2*c)-1)-1/16*(1/8*tan(1/2*d*x+1/2*c)^3+1/2
*tan(1/2*d*x+1/2*c)^2-1/8*tan(1/2*d*x+1/2*c)+1/2)/(1+tan(1/2*d*x+1/2*c)^2)^
2-9/128*arctan(tan(1/2*d*x+1/2*c))-1/160/(tan(1/2*d*x+1/2*c)+1)^5+1/64/(tan
(1/2*d*x+1/2*c)+1)^4+1/128/(tan(1/2*d*x+1/2*c)+1)^3-7/256/(tan(1/2*d*x+1/2*
c)+1)^2-31/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(137) = 274$.

time = 0.58, size = 421, normalized size = 2.83

$$-\frac{\frac{\frac{211\sin(dx+c)}{\cos(dx+c)+1}+\frac{268\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{212\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{174\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{300\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{300\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{180\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{45\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+64}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{7a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{7a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5*((211*sin(d*x + c)/(cos(d*x + c) + 1) + 268*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 212*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 - 174*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 300*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 300*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 180*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 45*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + 64)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 7*a^2*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 - 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 7*a^2*sin(d*x + c)^8/(cos
(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x
+ c)^10/(cos(d*x + c) + 1)^10) + 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^2)/d
```

Fricas [A]

time = 0.36, size = 132, normalized size = 0.89

$$-\frac{45\,dx\cos(dx+c)^3+10\,\cos(dx+c)^4-90\,dx\cos(dx+c)-78\,\cos(dx+c)^2-\left(5\,\cos(dx+c)^4+90\,dx\cos(dx+c)+84\,\cos(dx+c)^2-6\right)\sin(dx+c)+4}{10\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/10*(45*d*x*cos(d*x + c)^3 + 10*cos(d*x + c)^4 - 90*d*x*cos(d*x + c) - 78*cos(d*x + c)^2 - (5*cos(d*x + c)^4 + 90*d*x*cos(d*x + c) + 84*cos(d*x + c)^2 - 6)*sin(d*x + c) + 4)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A]
time = 0.51, size = 160, normalized size = 1.07

$$-\frac{\frac{90\,(dx+c)}{a^2}+\frac{20\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+690\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+181}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/20*(90*(d*x + c)/a^2 + 20*(tan(1/2*d*x + 1/2*c)^3 + 4*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + 5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (155*tan(1/2*d*x + 1/2*c)^4 + 690*tan(1/2*d*x + 1/2*c)^3 + 1120*tan(1/2*d*x + 1/2*c)^2 + 750*tan(1/2*d*x + 1/2*c) + 181)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

Mupad [B]
time = 17.67, size = 172, normalized size = 1.15

$$\frac{-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-60\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-60\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{174\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{84\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+\frac{212\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{268\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{211\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+\frac{64}{5}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}-\frac{9\,x}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^6/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)

[Out] ((211*tan(c/2 + (d*x)/2))/5 + (268*tan(c/2 + (d*x)/2)^2)/5 + (212*tan(c/2 + (d*x)/2)^3)/5 + (84*tan(c/2 + (d*x)/2)^4)/5 - (174*tan(c/2 + (d*x)/2)^5)/5 - 60*tan(c/2 + (d*x)/2)^6 - 60*tan(c/2 + (d*x)/2)^7 - 36*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^9 + 64/5)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (9*x)/(2*a^2)

3.779 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{2x}{a^2}+\frac{\cos(c+dx)}{a^2d}+\frac{4\sec(c+dx)}{a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan^5(c}{5a^2}$$

```
[Out] 2*x/a^2+cos(d*x+c)/a^2/d+4*sec(d*x+c)/a^2/d-5/3*sec(d*x+c)^3/a^2/d+2/5*sec(
d*x+c)^5/a^2/d-2*tan(d*x+c)/a^2/d+2/3*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a
^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2686, 200, 3554, 8, 2670, 276}

$$\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{5\sec^3(c+dx)}{3a^2d}+\frac{4\sec(c+dx)}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + Cos[c + d*x]/(a^2*d) + (4*Sec[c + d*x])/(a^2*d) - (5*Sec[c + d*
x]^3)/(3*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) +
 (2*Tan[c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4}$$

$$= \frac{\int (a^2\sec(c+dx)\tan^5(c+dx) - 2a^2\tan^6(c+dx) + a^2\sin(c+dx)\tan^6(c}{a^4}$$

$$= \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2} + \frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a^2} - \frac{2\int \tan}{}$$

$$= -\frac{2\tan^5(c+dx)}{5a^2d} + \frac{2\int \tan^4(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+\right.}{a^2d}$$

$$= \frac{2\tan^3(c+dx)}{3a^2d} - \frac{2\tan^5(c+dx)}{5a^2d} - \frac{2\int \tan^2(c+dx)\,dx}{a^2} - \frac{\text{Subst}\left(\int (-1\right.}{}$$

$$= \frac{\cos(c+dx)}{a^2d} + \frac{4\sec(c+dx)}{a^2d} - \frac{5\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2\tan(c}{a^2}$$

$$= \frac{2x}{a^2} + \frac{\cos(c+dx)}{a^2d} + \frac{4\sec(c+dx)}{a^2d} - \frac{5\sec^3(c+dx)}{3a^2d} + \frac{2\sec^5(c+dx)}{5a^2d} - \frac{2}{}$$

Mathematica [A]
time = 0.40, size = 148, normalized size = 1.23

$$\frac{\sec(c+dx)(550+(-995+600c+600dx)\cos(c+dx)+376\cos(2(c+dx))+199\cos(3(c+dx))-120c\cos(3(c+dx))-120dx\cos(3(c+dx))-30\cos(4(c+dx))+400\sin(c+dx)-796\sin(2(c+dx))+480c\sin(2(c+dx))+480dx\sin(2(c+dx))+304\sin(3(c+dx)))}{240a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(550 + (-995 + 600*c + 600*d*x)*Cos[c + d*x] + 376*Cos[2*(c +
 d*x)] + 199*Cos[3*(c + d*x)] - 120*c*Cos[3*(c + d*x)] - 120*d*x*Cos[3*(c +
 d*x)] - 30*Cos[4*(c + d*x)] + 400*Sin[c + d*x] - 796*Sin[2*(c + d*x)] + 48
0*c*Sin[2*(c + d*x)] + 480*d*x*Sin[2*(c + d*x)] + 304*Sin[3*(c + d*x)]))/(2
40*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.27, size = 129, normalized size = 1.08

method	result
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{64}{32+32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{}{3\left(\tan\left(\frac{d}{}\right.\right.}}{d\,a^2}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{64}{32+32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{}{3\left(\tan\left(\frac{d}{}\right.\right.}}{d\,a^2}$
risch	$\frac{2x}{a^2}+\frac{e^{i(dx+c)}}{2d\,a^2}+\frac{e^{-i(dx+c)}}{2d\,a^2}+\frac{20i\,e^{4i(dx+c)}+8\,e^{5i(dx+c)}+\frac{32i\,e^{2i(dx+c)}}{3}-\frac{40\,e^{3i(dx+c)}}{3}-\frac{92i}{15}-\frac{248\,e^{i(dx+c)}}{15}}{\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^5 d\,a^2}$

norman	$\frac{-\frac{2x}{a}+\frac{128\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{8x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{16x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{24x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{26x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{16x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^2*(-1/256/(tan(1/2*d*x+1/2*c)-1)+1/32/(1+tan(1/2*d*x+1/2*c)^2)+1/16*
arctan(tan(1/2*d*x+1/2*c))+1/80/(tan(1/2*d*x+1/2*c)+1)^5-1/32/(tan(1/2*d*x+
1/2*c)+1)^4-1/192/(tan(1/2*d*x+1/2*c)+1)^3+5/128/(tan(1/2*d*x+1/2*c)+1)^2+1
7/256/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(112) = 224$.
time = 0.51, size = 335, normalized size = 2.79

$$\frac{4\left(\frac{\frac{97\sin(dx+c)}{\cos(dx+c)+1}+\frac{108\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{27\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{40\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{85\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{60\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+28}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{4a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/15*((97*sin(d*x + c)/(cos(d*x + c) + 1) + 108*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 27*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 40*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 85*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 60*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 28)/(a^2
+ 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^2*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 4*a^2*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 6*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4*a^2*s
in(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 122, normalized size = 1.02

$$\frac{30\,dx\cos(dx+c)^3+15\,\cos(dx+c)^4-60\,dx\cos(dx+c)-62\,\cos(dx+c)^2-2\left(30\,dx\cos(dx+c)+38\,\cos(dx+c)^2+3\right)\sin(dx+c)-9}{15\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(30*d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 - 60*d*x*cos(d*x + c) - 62*
cos(d*x + c)^2 - 2*(30*d*x*cos(d*x + c) + 38*cos(d*x + c)^2 + 3)*sin(d*x +
```

```
c) - 9)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d
*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 151, normalized size = 1.26

$$\frac{\frac{120\,(dx+c)}{a^2}-\frac{15\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^2}+\frac{255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1170\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1960\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1310\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+313}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(120*(d*x + c)/a^2 - 15*(tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*
c) + 9)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1
/2*c) - 1)*a^2) + (255*tan(1/2*d*x + 1/2*c)^4 + 1170*tan(1/2*d*x + 1/2*c)^3
 + 1960*tan(1/2*d*x + 1/2*c)^2 + 1310*tan(1/2*d*x + 1/2*c) + 313)/(a^2*(tan
(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [**B**]
time = 15.38, size = 156, normalized size = 1.30

$$\frac{2\,x}{a^2}-\frac{-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{68\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{36\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{144\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{388\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{112}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*x)/a^2 - ((388*tan(c/2 + (d*x)/2))/15 + (144*tan(c/2 + (d*x)/2)^2)/5 + (
36*tan(c/2 + (d*x)/2)^3)/5 - (32*tan(c/2 + (d*x)/2)^4)/3 - (68*tan(c/2 + (d
*x)/2)^5)/3 - 16*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + 112/15)/(a
^2*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2
+ tan(c/2 + (d*x)/2)^3 - 1))
```

3.780 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{a^2}-\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}$$

```
[Out] -x/a^2-2*sec(d*x+c)/a^2/d+4/3*sec(d*x+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+tan
(d*x+c)/a^2/d-1/3*tan(d*x+c)^3/a^2/d+2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2687, 30, 2686, 200, 3554, 8}

$$\frac{2\tan^5(c+dx)}{5a^2d}-\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*Sec[c + d*x])/(a^2*d) + (4*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec
[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]/(a^2*d) - Tan[c + d*x]^3/(3*a^2*d) +
(2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^4(c+dx)\,dx}{a^4}$$
$$= \frac{\int (a^2\sec^2(c+dx)\tan^4(c+dx)-2a^2\sec(c+dx)\tan^5(c+dx)+a^2\tan^6(}{a^4}$$
$$= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec(c+dx)\mathrm{ta}}{a^2}$$
$$= \frac{\tan^5(c+dx)}{5a^2d}-\frac{\int \tan^4(c+dx)\,dx}{a^2}+\frac{\mathrm{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2}{}$$
$$= -\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\int \tan^2(c+dx)\,dx}{a^2}-\frac{2\mathrm{Subst}\left(\int (1-2\right.}{}$$
$$= -\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-\frac{\tan^3}{3}$$
$$= -\frac{x}{a^2}-\frac{2\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan(c+dx)}{a^2d}-$$

Mathematica [A]
time = 0.39, size = 143, normalized size = 1.35

$$-\frac{\sec(c+dx)\left(20+\frac{5}{4}(-89+60c+60dx)\cos(c+dx)+44\cos(2(c+dx))+\frac{89}{4}\cos(3(c+dx))-15c\cos(3(c+dx))-15dx\cos(3(c+dx))-10\sin(c+dx)-89\sin(2(c+dx))+60c\sin(2(c+dx))+60dx\sin(2(c+dx))+26\sin(3(c+dx))\right)}{60a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/60*(Sec[c + d*x]*(20 + (5*(-89 + 60*c + 60*d*x)*Cos[c + d*x])/4 + 44*Cos
[2*(c + d*x)] + (89*Cos[3*(c + d*x)])/4 - 15*c*Cos[3*(c + d*x)] - 15*d*x*Co
s[3*(c + d*x)] - 10*Sin[c + d*x] - 89*Sin[2*(c + d*x)] + 60*c*Sin[2*(c + d*
x)] + 60*d*x*Sin[2*(c + d*x)] + 26*Sin[3*(c + d*x)]))/(a^2*d*(1 + Sin[c + d
*x])^2)
```

Maple [A]
time = 0.25, size = 112, normalized size = 1.06

method	result
risch	$-\frac{x}{a^2}-\frac{4\left(-10\,{\rm e}^{3i(dx+c)}+30i{\rm e}^{4i(dx+c)}+15\,{\rm e}^{5i(dx+c)}-37\,{\rm e}^{i(dx+c)}+35i{\rm e}^{2i(dx+c)}-13i\right)}{15\left({\rm e}^{i(dx+c)}-i\right)\left({\rm e}^{i(dx+c)}+i\right)^5 d\,a^2}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d\,a^2}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d\,a^2}$
norman	$\frac{\frac{x}{a}-\frac{8\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{212\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{7x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{8x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{6x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{6x(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*(-1/128/(tan(1/2*d*x+1/2*c)-1)-1/16*arctan(tan(1/2*d*x+1/2*c))-1/4
0/(tan(1/2*d*x+1/2*c)+1)^5+1/16/(tan(1/2*d*x+1/2*c)+1)^4-1/96/(tan(1/2*d*x+
1/2*c)+1)^3-3/64/(tan(1/2*d*x+1/2*c)+1)^2-7/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(98) = 196$.
time = 0.50, size = 249, normalized size = 2.35

$$-\frac{2\left(\frac{\frac{49\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{70\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{60\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+16}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -2/15*((49*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 70*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 60*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 16)/(a^2 + 4*a^2
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.35, size = 112, normalized size = 1.06

$$-\frac{15\,dx\cos\left(dx+c\right)^{3}-30\,dx\cos\left(dx+c\right)-22\,\cos\left(dx+c\right)^{2}-\left(30\,dx\cos\left(dx+c\right)+26\,\cos\left(dx+c\right)^{2}-9\right)\sin\left(dx+c\right)+6}{15\,\left(a^{2}d\cos\left(dx+c\right)^{3}-2\,a^{2}d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 - 30*d*x*cos(d*x + c) - 22*cos(d*x + c)^2 - (3
0*d*x*cos(d*x + c) + 26*cos(d*x + c)^2 - 9)*sin(d*x + c) + 6)/(a^2*d*cos(d*
x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.47, size = 103, normalized size = 0.97

$$-\frac{\frac{60\,(dx+c)}{a^{2}}+\frac{15}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+510\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+610\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+143}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(60*(d*x + c)/a^2 + 15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) + (105*tan(1/
2*d*x + 1/2*c)^4 + 510*tan(1/2*d*x + 1/2*c)^3 + 920*tan(1/2*d*x + 1/2*c)^2
+ 610*tan(1/2*d*x + 1/2*c) + 143)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 14.09, size = 105, normalized size = 0.99

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{98\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{32}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}-\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((98*tan(c/2 + (d*x)/2))/15 + (8*tan(c/2 + (d*x)/2)^2)/3 - (28*tan(c/2 + (d
*x)/2)^3)/3 - 8*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + 32/15)/(a^2
*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^5) - x/a^2
```

3.781 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\sec(c+dx)}{a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}$$

```
[Out] sec(d*x+c)/a^2/d-sec(d*x+c)^3/a^2/d+2/5*sec(d*x+c)^5/a^2/d-2/5*tan(d*x+c)^5
/a^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.259$, Rules used = {2954, 2952, 2686, 14, 2687, 30, 200}

$$-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(a^2*d) - Sec[c + d*x]^3/(a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d
) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\sec^3(c+dx)\tan^3(c+dx)-2a^2\sec^2(c+dx)\tan^4(c+dx)+a^2\sec(c-\right.}{a^4} \\
&= \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a^2}-\frac{2\int \sec^2(}{} \\
&= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\sec(\right.}{a^2d} \\
&= -\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int\right.}{} \\
&= \frac{\sec(c+dx)}{a^2d}-\frac{\sec^3(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 84, normalized size = 1.27

$$\frac{\sec(c+dx)(40-65\cos(c+dx)-8\cos(2(c+dx))+13\cos(3(c+dx))+40\sin(c+dx)-52\sin(2(c+dx))+8\sin(3(c+dx)))}{80a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(40 - 65*Cos[c + d*x] - 8*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*
x)] + 40*Sin[c + d*x] - 52*Sin[2*(c + d*x)] + 8*Sin[3*(c + d*x)]))/(80*a^2*
d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.22, size = 100, normalized size = 1.52

method	result
risch	$\frac{4 i \mathrm{e}^{4 i (d x+c)}+2 \mathrm{e}^{5 i (d x+c)}-4 \mathrm{e}^{3 i (d x+c)}-\frac{4 i}{5}-\frac{6 \mathrm{e}^{i (d x+c)}}{5}}{\left(\mathrm{e}^{i (d x+c)}+i\right)^{5} \left(\mathrm{e}^{i (d x+c)}-i\right) d\, a^{2}}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{2}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{16}{64 \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+}}{d\, a^{2}}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{2}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{2\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{16}{64 \tan\left(\frac{d x}{2}+\frac{c}{2}\right)+}}{d\, a^{2}}$
norman	$\frac{-\frac{4\left(\tan^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}-\frac{4}{5 a d}-\frac{16 \tan\left(\frac{d x}{2}+\frac{c}{2}\right)}{5 a d}-\frac{24\left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{5 a d}-\frac{16\left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{5 a d}}{\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{d x}{2}+\frac{c}{2}\right)+1\right)^{5} a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^2*(-1/64/(tan(1/2*d*x+1/2*c)-1)+1/20/(tan(1/2*d*x+1/2*c)+1)^5-1/8/(t
an(1/2*d*x+1/2*c)+1)^4+1/16/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*
c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(62) = 124$.
time = 0.29, size = 164, normalized size = 2.48

$$\frac{4 \left(\frac{4 \sin\left(d x+c\right)}{\cos\left(d x+c\right)+1}+\frac{5 \sin\left(d x+c\right)^{2}}{\left(\cos\left(d x+c\right)+1\right)^{2}}+1\right)}{5 \left(a^{2}+\frac{4 a^{2} \sin\left(d x+c\right)}{\cos\left(d x+c\right)+1}+\frac{5 a^{2} \sin\left(d x+c\right)^{2}}{\left(\cos\left(d x+c\right)+1\right)^{2}}-\frac{5 a^{2} \sin\left(d x+c\right)^{4}}{\left(\cos\left(d x+c\right)+1\right)^{4}}-\frac{4 a^{2} \sin\left(d x+c\right)^{5}}{\left(\cos\left(d x+c\right)+1\right)^{5}}-\frac{a^{2} \sin\left(d x+c\right)^{6}}{\left(\cos\left(d x+c\right)+1\right)^{6}}\right) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/5*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6)*d)
```

Fricas [A]
time = 0.34, size = 76, normalized size = 1.15

$$\frac{\cos(dx+c)^2-2\left(\cos(dx+c)^2+1\right)\sin(dx+c)-3}{5\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(cos(d*x + c)^2 - 2*(cos(d*x + c)^2 + 1)*sin(d*x + c) - 3)/(a^2*d*cos(d
*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 94, normalized size = 1.42

$$-\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (5*tan(1/2*d*x + 1/2*c)^4 + 30*
tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 50*tan(1/2*d*x + 1/2*c
) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 9.27, size = 111, normalized size = 1.68

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^6)/5 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/
5 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/(a^2*d*(cos(c/2 + (d*x)/2)
 - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.782 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=73

$$\frac{2\sec^3(c+dx)}{3a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{\tan^3(c+dx)}{3a^2d} + \frac{2\tan^5(c+dx)}{5a^2d}$$

```
[Out] 2/3*sec(d*x+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+1/3*tan(d*x+c)^3/a^2/d+2/5*ta
n(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2790, 2687, 14, 2686, 30}

$$\frac{2\tan^5(c+dx)}{5a^2d} + \frac{\tan^3(c+dx)}{3a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(a^2\sec^4(c+dx)\tan^2(c+dx)-2a^2\sec^3(c+dx)\tan^3(c+dx)+a^2\sec^2(c+dx\right.}{a^4}\\ &= \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a^2}+\frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int \sec^3(c+}{}\\ &= \frac{\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int x^2(1+x^2)\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\text{Su}}{}\\ &= \frac{\tan^5(c+dx)}{5a^2d}+\frac{\text{Subst}\left(\int (x^2+x^4)\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\text{Subst}\left(\int (-x^2+x^4)\right.}{a^2d}\\ &= \frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 86, normalized size = 1.18

$$-\frac{\sec(c+dx)\left(-20+\frac{55}{4}\cos(c+dx)+4\cos(2(c+dx))-\frac{11}{4}\cos(3(c+dx))-35\sin(c+dx)+11\sin(2(c+dx))+\sin(3(c+dx))\right)}{60a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/60*(Sec[c + d*x]*(-20 + (55*Cos[c + d*x])/4 + 4*Cos[2*(c + d*x)] - (11*C
os[3*(c + d*x)])/4 - 35*Sin[c + d*x] + 11*Sin[2*(c + d*x)] + Sin[3*(c + d*x
)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 100, normalized size = 1.37

method	result
risch	$\frac{2i\left(20i\,\mathrm{e}^{3i(dx+c)}+15\,\mathrm{e}^{4i(dx+c)}-4i\,\mathrm{e}^{i(dx+c)}-20\,\mathrm{e}^{2i(dx+c)}+1\right)}{15\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^5 d\,a^2}$

norman	$\frac{-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8}{15ad}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}}{d\,a^2}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{5}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{8}{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*(-1/32/(tan(1/2*d*x+1/2*c)-1)-1/10/(tan(1/2*d*x+1/2*c)+1)^5+1/4/(ta
n(1/2*d*x+1/2*c)+1)^4-5/24/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c
)+1)^2+1/32/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(65) = 130$.

time = 0.29, size = 184, normalized size = 2.52

$$\frac{8\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{15\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^2 + 4*a^2*sin(d*x + c
)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)
^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)*d)
```

Fricas [A]

time = 0.33, size = 77, normalized size = 1.05

$$\frac{2\cos(dx+c)^2+\left(\cos(dx+c)^2-9\right)\sin(dx+c)-6}{15\left(a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)\sin(dx+c)-2\,a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/15*(2*cos(d*x + c)^2 + (cos(d*x + c)^2 - 9)*sin(d*x + c) - 6)/(a^2*d*cos(
d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.49, size = 94, normalized size = 1.29

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 + 9
0*tan(1/2*d*x + 1/2*c)^3 + 80*tan(1/2*d*x + 1/2*c)^2 + 70*tan(1/2*d*x + 1/2
*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 9.31, size = 132, normalized size = 1.81

$$\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}{15\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (8*cos(c/2 + (d*x)/2)^3*(cos(c/2 + (d*x)/2)^3 + 5*sin(c/2 + (d*x)/2)^3 + 5*
cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^2*sin(c/2 +
(d*x)/2)))/(15*a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (
d*x)/2) + sin(c/2 + (d*x)/2))^5)
```

3.783 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2}-\frac{2\sec(c+dx)}{15d\,(a^2+a^2\sin(c+dx))}+\frac{4\tan(c+dx)}{15a^2d}$$

[Out] 1/5*sec(d*x+c)/d/(a+a*sin(d*x+c))^2-2/15*sec(d*x+c)/d/(a^2+a^2*sin(d*x+c))+4/15*tan(d*x+c)/a^2/d

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2938, 2751, 3852, 8}

$$\frac{4\tan(c+dx)}{15a^2d}-\frac{2\sec(c+dx)}{15d\,(a^2\sin(c+dx)+a^2)}+\frac{\sec(c+dx)}{5d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]/(5*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x])/(15*d*(a^2 + a
^2*Sin[c + d*x])) + (4*Tan[c + d*x])/(15*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} + \frac{2\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{5a}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\int \sec^2(c+dx)\,dx}{15a^2}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} - \frac{4\mathrm{Subst}(\int 1\,dx, x, -\mathrm{ta}}{15a^2d}$$
$$= \frac{\sec(c+dx)}{5d(a+a\sin(c+dx))^2} - \frac{2\sec(c+dx)}{15d\left(a^2+a^2\sin(c+dx)\right)} + \frac{4\tan(c+dx)}{15a^2d}$$

Mathematica [A]
time = 0.17, size = 82, normalized size = 1.15

$$-\frac{\sec(c+dx)(-80-5\cos(c+dx)+64\cos(2(c+dx))+\cos(3(c+dx))-80\sin(c+dx)-4\sin(2(c+dx))+16\sin(3(c+dx)))}{240a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/240*(Sec[c + d*x]*(-80 - 5*Cos[c + d*x] + 64*Cos[2*(c + d*x)] + Cos[3*(c
 + d*x)] - 80*Sin[c + d*x] - 4*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)]))/(a^
2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 100, normalized size = 1.41

method	result
risch	$-\frac{8\left(5i\,e^{2i(dx+c)}+5\,e^{3i(dx+c)}-i-4\,e^{i(dx+c)}\right)}{15\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^5 d\,a^2}$
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{7}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^2}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{7}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}}{d\,a^2}$

norman	$\frac{-\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2}{15ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*(-1/16/(tan(1/2*d*x+1/2*c)-1)+1/5/(tan(1/2*d*x+1/2*c)+1)^5-1/2/(tan
(1/2*d*x+1/2*c)+1)^4+7/12/(tan(1/2*d*x+1/2*c)+1)^3-3/8/(tan(1/2*d*x+1/2*c)+
1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(65) = 130$.
time = 0.30, size = 204, normalized size = 2.87

$$\frac{2\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)}{15\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/15*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 - 4*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x
 + c) + 1)^6)*d)
```

Fricas [A]
time = 0.34, size = 80, normalized size = 1.13

$$\frac{8\cos\left(dx+c\right)^2+2\left(2\cos\left(dx+c\right)^2-3\right)\sin\left(dx+c\right)-9}{15\left(a^2d\cos\left(dx+c\right)^3-2a^2d\cos\left(dx+c\right)\sin\left(dx+c\right)-2a^2d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(d*x + c)^2 + 2*(2*cos(d*x + c)^2 - 3)*sin(d*x + c) - 9)/(a^2*d*
cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin\left(c+dx\right)\sec^2\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.45, size = 94, normalized size = 1.32

$$-\frac{\frac{15}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-50\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(15/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) - (15*tan(1/2*d*x + 1/2*c)^4 - 3
0*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*d*x + 1/2*c)^2 - 50*tan(1/2*d*x + 1/2
*c) - 7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 9.39, size = 159, normalized size = 2.24

$$\frac{\frac{2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((2*cos(c/2 + (d*x)/2)^6)/15 + (8*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/
15 + 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + (8*cos(c/2 + (d*x)/2)^3*
sin(c/2 + (d*x)/2)^3)/3 + (8*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/3)/
(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(
c/2 + (d*x)/2))^5)
```

3.784 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan^5}{5}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d+2/5*sec(
d*x+c)^5/a^2/d-2*tan(d*x+c)/a^2/d-4/3*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a
^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2686, 30}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x])/(a^2*d) - (4*Tan[
c + d*x]^3)/(3*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}\\ &= \frac{\int \left(-2a^2\sec^6(c+dx)+a^2\csc(c+dx)\sec^6(c+dx)+a^2\sec^5(c+dx)\tan(c\right.}{a^4}\\ &= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2}+\frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a^2}-\frac{2\int \sec^6(}{}\\ &= \frac{\mathrm{Subst}\left(\int x^4\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{2\mathrm{S}}{}\\ &= \frac{\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{4\tan^3(c+dx)}{3a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{\mathrm{Subst}}{}\\ &= \frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{4\tan^3(c-}{3a^2d}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}-\end{aligned}$$

Mathematica [A]

time = 0.35, size = 196, normalized size = 1.70

$$\frac{\sec(c+dx)\left(280+136\cos(2(c+dx))+79\cos(3(c+dx))+60\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-5\cos(c+dx)\left(79+60\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-60\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-60\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+160\sin(c+dx)-316\sin(2(c+dx))-240\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+240\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+64\sin(3(c+dx))\right)}{240a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]*(280 + 136*Cos[2*(c + d*x)] + 79*Cos[3*(c + d*x)] + 60*Cos[3*
(c + d*x)]*Log[Cos[(c + d*x)/2]] - 5*Cos[c + d*x]*(79 + 60*Log[Cos[(c + d*x
)/2]] - 60*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2
]] + 160*Sin[c + d*x] - 316*Sin[2*(c + d*x)] - 240*Log[Cos[(c + d*x)/2]]*Si
n[2*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 64*Sin[3*(c +
 d*x)]))/(240*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]

time = 0.28, size = 109, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{11}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{17}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^2}$
default	$\frac{-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{11}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{7}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{17}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^2}$
risch	$\frac{8i e^{4i(dx+c)}+2\,e^{5i(dx+c)}+\frac{8i e^{2i(dx+c)}}{3}-\frac{28\,e^{3i(dx+c)}}{3}-\frac{32i}{15}-\frac{98\,e^{i(dx+c)}}{15}}{\left(e^{i(dx+c)}+i\right)^5\left(e^{i(dx+c)}-i\right)a^2d}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{a^2d}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{a^2d}$

norman	$\frac{\frac{8\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{52}{15ad}+\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{28\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{148\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/4/(tan(1/2*d*x+1/2*c)-1)+4/5/(tan(1/2*d*x+1/2*c)+1)^5-2/(tan(1/
2*d*x+1/2*c)+1)^4+11/3/(tan(1/2*d*x+1/2*c)+1)^3-7/2/(tan(1/2*d*x+1/2*c)+1)^
2+17/4/(tan(1/2*d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(107) = 214$.
time = 0.30, size = 250, normalized size = 2.17

$$\frac{\frac{4\left(\frac{37\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{30\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+13\right)}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/15*(4*(37*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 30*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 13)/(a^2 + 4*a^
2*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 15*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.36, size = 168, normalized size = 1.46

$$-\frac{34\cos(dx+c)^2+15\left(\cos(dx+c)^3-2\cos(dx+c)\sin(dx+c)-2\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-15\left(\cos(dx+c)^3-2\cos(dx+c)\sin(dx+c)-2\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+4\left(8\cos(dx+c)^2+3\right)\sin(dx+c)+18}{30\left(a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)\sin(dx+c)-2a^2d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(34*cos(d*x + c)^2 + 15*(cos(d*x + c)^3 - 2*cos(d*x + c)*sin(d*x + c)
 - 2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(cos(d*x + c)^3 - 2*cos
(d*x + c)*sin(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(
8*cos(d*x + c)^2 + 3)*sin(d*x + c) + 18)/(a^2*d*cos(d*x + c)^3 - 2*a^2*d*co
s(d*x + c)*sin(d*x + c) - 2*a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.48, size = 109, normalized size = 0.95

$$\frac{\frac{60\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{15}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{255\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+810\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+710\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+193}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 15/(a^2*(tan(1/2*d*x + 1/2*c)
 - 1)) + (255*tan(1/2*d*x + 1/2*c)^4 + 810*tan(1/2*d*x + 1/2*c)^3 + 1120*ta
n(1/2*d*x + 1/2*c)^2 + 710*tan(1/2*d*x + 1/2*c) + 193)/(a^2*(tan(1/2*d*x +
1/2*c) + 1)^5))/d
```

Mupad [B]
time = 10.78, size = 117, normalized size = 1.02

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{148\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{52}{15}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((148*tan(c/2 + (d*x)/2))/15 + (28*tan(c/
2 + (d*x)/2)^2)/3 - (8*tan(c/2 + (d*x)/2)^3)/3 - 8*tan(c/2 + (d*x)/2)^4 - 4
*tan(c/2 + (d*x)/2)^5 + 52/15)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (
d*x)/2) + 1)^5)
```

3.785 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}+\frac{4\tan(c+dx)}{a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d-2/3*sec(d*x
+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d+4*tan(d*x+c)/a^2/d+5/3*tan(d*x+c)^3/a^2/
d+2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2700, 276}

$$\frac{2\tan^5(c+dx)}{5a^2d}+\frac{5\tan^3(c+dx)}{3a^2d}+\frac{4\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) + (4
*Tan[c + d*x])/(a^2*d) + (5*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*x]^5)/
(5*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^2(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\sec^6(c+dx) - 2a^2\csc(c+dx)\sec^6(c+dx) + a^2\csc^2(c+dx)\sec^6(c+\right.}{a^4}$$
$$= \frac{\int \sec^6(c+dx)\,dx}{a^2} + \frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a^2} - \frac{2\int \csc(c+dx)\sec^6(}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2d} - \frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan\right.}{a^2d}$$
$$= \frac{\tan(c+dx)}{a^2d} + \frac{2\tan^3(c+dx)}{3a^2d} + \frac{\tan^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+\right.\right.}{a^2d}$$
$$= -\frac{\cot(c+dx)}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} - \frac{2\sec^5(c+dx)}{5a^2d} + \frac{4\tan(}{a^2}$$
$$= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{a^2d} - \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{3a^2d} - $$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 289 vs. 2(130) = 260.
time = 0.55, size = 289, normalized size = 2.22

$$-\frac{\csc^3(c+dx)\left(40+48\cos(2(c+dx))+112\cos(3(c+dx))-28\cos(4(c+dx))+60\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4\cos(c+dx)\left(28+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-60\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+58\sin(c+dx)-168\sin(2(c+dx))-90\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+90\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+82\sin(3(c+dx))+28\sin(4(c+dx))+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))\right)}{15a^2d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/15*(Csc[c + d*x]^3*(40 + 48*Cos[2*(c + d*x)] + 112*Cos[3*(c + d*x)] - 28
*Cos[4*(c + d*x)] + 60*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*Cos[c + d
*x]*(28 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]]) - 60*Cos[3*(
c + d*x)]*Log[Sin[(c + d*x)/2]] + 58*Sin[c + d*x] - 168*Sin[2*(c + d*x)] -
90*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 90*Log[Sin[(c + d*x)/2]]*Sin[2*
(c + d*x)] + 82*Sin[3*(c + d*x)] + 28*Sin[4*(c + d*x)] + 15*Log[Cos[(c + d*
x)/2]]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(a^2*
d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.32, size = 134, normalized size = 1.03

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{26}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{9}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{}{2(\tan}}{2d\,a^2}$

default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{26}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{9}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{\cdots}{2(\mathrm{t}\cdots}}{2d\,a^{2}}$
risch	$-\frac{4\left(-85\,\mathrm{e}^{5i(dx+c)}+60i\mathrm{e}^{6i(dx+c)}-40i\mathrm{e}^{4i(dx+c)}+97\,\mathrm{e}^{i(dx+c)}-108i\mathrm{e}^{2i(dx+c)}+28i-27\,\mathrm{e}^{3i(dx+c)}+15\,\mathrm{e}^{7i(dx+c)}\right)}{15\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}\left(\mathrm{e}^{i(dx+c)}-i\right)a^{2}d}+\frac{2\ln(\mathrm{e}\cdots}{\cdots}$
norman	$\frac{\frac{1}{2ad}-\frac{55\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{7\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{29\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{199\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{30ad}+\frac{188\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{7(\mathrm{t}\cdots}{\cdots}}{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-1/2/(tan(1/2*d*x+1/2*c)-1)-8/5/(tan(1/2*d*x+1
/2*c)+1)^5+4/(tan(1/2*d*x+1/2*c)+1)^4-26/3/(tan(1/2*d*x+1/2*c)+1)^3+9/(tan(
1/2*d*x+1/2*c)+1)^2-31/2/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-4*ln(t
an(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(122) = 244$.

time = 0.28, size = 309, normalized size = 2.38

$$-\frac{\frac{\frac{244\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{571\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{320\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{475\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{660\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{255\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+15}{\frac{a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{5\,a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{5\,a^{2}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{4\,a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}+\frac{60\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}-\frac{15\,\sin(dx+c)}{a^{2}(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*((244*sin(d*x + c)/(cos(d*x + c) + 1) + 571*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 320*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 475*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 660*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 255*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 15)/(a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 4
*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^2*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 - 5*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*a^2*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 60*log(
sin(d*x + c)/(cos(d*x + c) + 1))/a^2 - 15*sin(d*x + c)/(a^2*(cos(d*x + c) +
 1)))/d
```

Fricas [A]

time = 0.37, size = 218, normalized size = 1.68

$$-\frac{56\,\cos(dx+c)^{4}-80\,\cos(dx+c)^{2}-15\left(2\,\cos(dx+c)^{3}+\left(\cos(dx+c)^{3}-2\,\cos(dx+c)\right)\sin(dx+c)-2\,\cos(dx+c)\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+15\left(2\,\cos(dx+c)^{3}+\left(\cos(dx+c)^{3}-2\,\cos(dx+c)\right)\sin(dx+c)-2\,\cos(dx+c)\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-2\left(41\,\cos(dx+c)^{2}-3\right)\sin(dx+c)+9}{15\left(2\,a^{2}d\cos(dx+c)^{3}-2\,a^{2}d\cos(dx+c)+\left(a^{2}d\cos(dx+c)^{3}-2\,a^{2}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/15*(56*cos(d*x + c)^4 - 80*cos(d*x + c)^2 - 15*(2*cos(d*x + c)^3 + (cos(
d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c) - 2*cos(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) + 15*(2*cos(d*x + c)^3 + (cos(d*x + c)^3 - 2*cos(d*x + c))*sin
(d*x + c) - 2*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(41*cos(d*x +
c)^2 - 3)*sin(d*x + c) + 9)/(2*a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c)
+ (a^2*d*cos(d*x + c)^3 - 2*a^2*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]

time = 0.48, size = 161, normalized size = 1.24

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{30\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{15\left(4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^2}+\frac{465\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1590\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2240\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1450\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+383}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/60*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 30*tan(1/2*d*x + 1/2*c)/a^2
 - 15*(4*tan(1/2*d*x + 1/2*c)^2 - 7*tan(1/2*d*x + 1/2*c) + 2)/((tan(1/2*d*x
 + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^2) + (465*tan(1/2*d*x + 1/2*c)^4 + 15
90*tan(1/2*d*x + 1/2*c)^3 + 2240*tan(1/2*d*x + 1/2*c)^2 + 1450*tan(1/2*d*x
+ 1/2*c) + 383)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]

time = 10.78, size = 216, normalized size = 1.66

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-44\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{95\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{571\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{244\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+1}{d\left(-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((244*
tan(c/2 + (d*x)/2))/15 + (571*tan(c/2 + (d*x)/2)^2)/15 + (64*tan(c/2 + (d*x
)/2)^3)/3 - (95*tan(c/2 + (d*x)/2)^4)/3 - 44*tan(c/2 + (d*x)/2)^5 - 17*tan(
c/2 + (d*x)/2)^6 + 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^2 + 10*a^2*tan(c/2 + (d*
x)/2)^3 - 10*a^2*tan(c/2 + (d*x)/2)^5 - 8*a^2*tan(c/2 + (d*x)/2)^6 - 2*a^2*
tan(c/2 + (d*x)/2)^7 + 2*a^2*tan(c/2 + (d*x)/2)))
```

3.786 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec(c+dx)}{2a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}-\frac{\csc^2(c+dx)\sec^5(}{2a^2d}$$

```
[Out] -9/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+9/2*sec(d*x+c)/a^2/d+3/2*
sec(d*x+c)^3/a^2/d+9/10*sec(d*x+c)^5/a^2/d-1/2*csc(d*x+c)^2*sec(d*x+c)^5/a^
2/d-6*tan(d*x+c)/a^2/d-2*tan(d*x+c)^3/a^2/d-2/5*tan(d*x+c)^5/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2702, 308, 213, 2700, 276, 294}

$$-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{9\sec^5(c+dx)}{10a^2d}+\frac{3\sec^3(c+dx)}{2a^2d}+\frac{9\sec(c+dx)}{2a^2d}-\frac{9\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^5(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-9*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (9*Sec[c
+ d*x])/(2*a^2*d) + (3*Sec[c + d*x]^3)/(2*a^2*d) + (9*Sec[c + d*x]^5)/(10*a
^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^5)/(2*a^2*d) - (6*Tan[c + d*x])/(a^2*d
) - (2*Tan[c + d*x]^3)/(a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^6(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int \left(a^2\csc(c+dx)\sec^6(c+dx) - 2a^2\csc^2(c+dx)\sec^6(c+dx) + a^2\csc^3(c\right.}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^6(c+dx)\,dx}{a^2} - \frac{2\int \csc}{}$$

$$= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{a^2 d} + \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(c+dx)\right.}{a^2 d}$$

$$= -\frac{\csc^2(c+dx)\sec^5(c+dx)}{2a^2 d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \sec(\right.}{a^2 d}$$

$$= \frac{2\cot(c+dx)}{a^2 d} + \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{\sec^5(c+dx)}{5a^2 d} - \frac{\csc^2(c+d}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{2\cot(c+dx)}{a^2 d} + \frac{9\sec(c+dx)}{2a^2 d} + \frac{3\sec^3(c+dx)}{2a^2 d}$$

$$= -\frac{9\tanh^{-1}(\cos(c+dx))}{2a^2 d} + \frac{2\cot(c+dx)}{a^2 d} + \frac{9\sec(c+dx)}{2a^2 d} + \frac{3\sec^3(c+dx}{2a^2 d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 328 vs. 2(158) = 316.
time = 0.51, size = 328, normalized size = 2.08

$$-\frac{\csc^2(c+dx)\sec(c+dx)\left(-348+176\cos(2(c+dx))-651\cos(3(c+dx))+332\cos(4(c+dx))+93\cos(5(c+dx))-630\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+90\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+18\cos(c+dx)\left(31+30\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-30\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+630\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-90\cos(5(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-432\sin(c+dx)+744\sin(2(c+dx))+720\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-720\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-176\sin(3(c+dx))-372\sin(4(c+dx))-360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+128\sin(5(c+dx))\right)}{320a^2 d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/320*(Csc[c + d*x]^2*Sec[c + d*x]*(-348 + 176*Cos[2*(c + d*x)] - 651*Cos[
3*(c + d*x)] + 332*Cos[4*(c + d*x)] + 93*Cos[5*(c + d*x)] - 630*Cos[3*(c +
d*x)]*Log[Cos[(c + d*x)/2]] + 90*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 1
8*Cos[c + d*x]*(31 + 30*Log[Cos[(c + d*x)/2]] - 30*Log[Sin[(c + d*x)/2]]) +
 630*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 90*Cos[5*(c + d*x)]*Log[Sin[(
c + d*x)/2]] - 432*Sin[c + d*x] + 744*Sin[2*(c + d*x)] + 720*Log[Cos[(c + d
*x)/2]]*Sin[2*(c + d*x)] - 720*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 176
*Sin[3*(c + d*x)] - 372*Sin[4*(c + d*x)] - 360*Log[Cos[(c + d*x)/2]]*Sin[4*
(c + d*x)] + 360*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 128*Sin[5*(c + d*
x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [**A**]
time = 0.34, size = 162, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{20}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{dx}{\right.\right.}}}{4d\,a^{2}}$
default	$\frac{-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{20}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{2}{\left(\tan\left(\frac{dx}{2}\right.\right.}}{4d\,a^{2}}$
risch	$\frac{180i\,e^{8i(dx+c)}+45\,e^{9i(dx+c)}-300i\,e^{6i(dx+c)}-300\,e^{7i(dx+c)}-84i\,e^{4i(dx+c)}+174\,e^{5i(dx+c)}+268i\,e^{2i(dx+c)}+212\,e^{3i(dx+c)}-64}{5\left(e^{2i(dx+c)}-1\right)^{2}\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^{5}a^{2}d}$
norman	$\frac{\frac{1}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{25\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{43\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}-\frac{43\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{93\left(\tan^{6}\left(\frac{dx}{2}\right.\right.}{4ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(-1/(tan(1/2*d*x+1/2*c)-1)+1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x
+1/2*c)+16/5/(tan(1/2*d*x+1/2*c)+1)^5-8/(tan(1/2*d*x+1/2*c)+1)^4+20/(tan(1/
2*d*x+1/2*c)+1)^3-22/(tan(1/2*d*x+1/2*c)+1)^2+49/(tan(1/2*d*x+1/2*c)+1)-1/2
/tan(1/2*d*x+1/2*c)^2+4/tan(1/2*d*x+1/2*c)+18*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(146) = 292$.
time = 0.29, size = 354, normalized size = 2.24

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{567\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1448\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{985\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{820\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1355\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{520\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-5}{\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{4a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{5a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{4a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{5\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{2}}+\frac{180\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/40*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 567*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 1448*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 985*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 820*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1355*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 520*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)
/(a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 5*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 5*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 4*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^2*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 5*(8*sin(d*x + c)/(cos(d*x + c) + 1)
- sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 180*log(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 260, normalized size = 1.65

$$-\frac{166\cos(dx+c)^{4}-144\cos(dx+c)^{2}+45\left(\cos(dx+c)^{5}-3\cos(dx+c)^{3}-2\left(\cos(dx+c)^{3}-\cos(dx+c)\right)\sin(dx+c)+2\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-45\left(\cos(dx+c)^{5}-3\cos(dx+c)^{3}-2\left(\cos(dx+c)^{3}-\cos(dx+c)\right)\sin(dx+c)+2\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+4\left(32\cos(dx+c)^{4}-35\cos(dx+c)^{2}-2\right)\sin(dx+c)-12}{20\left(a^{2}d\cos(dx+c)^{5}-3a^{2}d\cos(dx+c)^{3}+2a^{2}d\cos(dx+c)-2\left(a^{2}d\cos(dx+c)^{3}-a^{2}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20*(166*cos(d*x + c)^4 - 144*cos(d*x + c)^2 + 45*(cos(d*x + c)^5 - 3*cos
(d*x + c)^3 - 2*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x +
c))*log(1/2*cos(d*x + c) + 1/2) - 45*(cos(d*x + c)^5 - 3*cos(d*x + c)^3 - 2
*(cos(d*x + c)^3 - cos(d*x + c))*sin(d*x + c) + 2*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 4*(32*cos(d*x + c)^4 - 35*cos(d*x + c)^2 - 2)*sin(d*x +
 c) - 12)/(a^2*d*cos(d*x + c)^5 - 3*a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
+ c) - 2*(a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 187, normalized size = 1.18

$$\frac{\frac{180 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2}+\frac{5 \left(a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-8 a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right)}{a^4}-\frac{10}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}-\frac{5 \left(54 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-8 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2}+\frac{2 \left(245 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+870 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+1240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+810 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+211\right)}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^5}}{40 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40*(180*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 5*(a^2*tan(1/2*d*x + 1/2*c)^
2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 10/(a^2*(tan(1/2*d*x + 1/2*c) - 1)) -
 5*(54*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*
x + 1/2*c)^2) + 2*(245*tan(1/2*d*x + 1/2*c)^4 + 870*tan(1/2*d*x + 1/2*c)^3
+ 1240*tan(1/2*d*x + 1/2*c)^2 + 810*tan(1/2*d*x + 1/2*c) + 211)/(a^2*(tan(1
/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [**B**]
time = 10.09, size = 191, normalized size = 1.21

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}+\frac{9\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(-13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{271\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{8}-\frac{41\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\frac{197\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{8}+\frac{181\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{567\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{40}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}-\frac{1}{8}\right)}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (9*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - ta
n(c/2 + (d*x)/2)/(a^2*d) - (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2)/2 + (5
67*tan(c/2 + (d*x)/2)^2)/40 + (181*tan(c/2 + (d*x)/2)^3)/5 + (197*tan(c/2 +
 (d*x)/2)^4)/8 - (41*tan(c/2 + (d*x)/2)^5)/2 - (271*tan(c/2 + (d*x)/2)^6)/8
 - 13*tan(c/2 + (d*x)/2)^7 - 1/8))/(a^2*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2
 + (d*x)/2) + 1)^5)
```

3.787 $\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$\frac{3x}{a^3}+\frac{\cos(c+dx)}{a^3d}+\frac{7\sec(c+dx)}{a^3d}-\frac{5\sec^3(c+dx)}{a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{a^3d}$$

```
[Out] 3*x/a^3+cos(d*x+c)/a^3/d+7*sec(d*x+c)/a^3/d-5*sec(d*x+c)^3/a^3/d+13/5*sec(d
*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d-3*tan(d*x+c)/a^3/d+tan(d*x+c)^3/a^3/d-
3/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]

time = 0.25, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2954, 2952, 2687, 30, 2686, 200, 3554, 8, 2670, 276}

$$\frac{\cos(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{13\sec^5(c+dx)}{5a^3d}-\frac{5\sec^3(c+dx)}{a^3d}+\frac{7\sec(c+dx)}{a^3d}+\frac{3x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*x)/a^3 + Cos[c + d*x]/(a^3*d) + (7*Sec[c + d*x])/(a^3*d) - (5*Sec[c + d*
x]^3)/(a^3*d) + (13*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d
) - (3*Tan[c + d*x])/(a^3*d) + Tan[c + d*x]^3/(a^3*d) - (3*Tan[c + d*x]^5)/
(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(a^3\sec^2(c+dx)\tan^6(c+dx) - 3a^3\sec(c+dx)\tan^7(c+dx) + 3a^3\tan^8\right.}{a^6}$$
$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^3} - \frac{3\int \mathrm{sec}}{}$$
$$= \frac{3\tan^7(c+dx)}{7a^3d} - \frac{3\int \tan^6(c+dx)\,dx}{a^3} + \frac{\mathrm{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^3d} -$$
$$= -\frac{3\tan^5(c+dx)}{5a^3d} + \frac{4\tan^7(c+dx)}{7a^3d} + \frac{3\int \tan^4(c+dx)\,dx}{a^3} + \frac{\mathrm{Subst}\left(\int (1+\right.}{}$$
$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7}{7}$$
$$= \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7}{7}$$
$$= \frac{3x}{a^3} + \frac{\cos(c+dx)}{a^3d} + \frac{7\sec(c+dx)}{a^3d} - \frac{5\sec^3(c+dx)}{a^3d} + \frac{13\sec^5(c+dx)}{5a^3d} -$$

Mathematica [A]
time = 0.47, size = 224, normalized size = 1.48

$$\frac{8400+14(-1483+840c+840dx)\cos(c+dx)+5152\cos(2(c+dx))+8898\cos(3(c+dx))-5040c\cos(3(c+dx))-5040dx\cos(3(c+dx))-2288\cos(4(c+dx))+8008\sin(c+dx)-20762\sin(2(c+dx))+11760c\sin(2(c+dx))+11760dx\sin(2(c+dx))+6588\sin(3(c+dx))+1483\sin(4(c+dx))-840c\sin(4(c+dx))-840dx\sin(4(c+dx))-140\sin(5(c+dx))}{2240a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
[Out] (8400 + 14*(-1483 + 840*c + 840*d*x)*Cos[c + d*x] + 5152*Cos[2*(c + d*x)] +
 8898*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(c + d*x)
] - 2288*Cos[4*(c + d*x)] + 8008*Sin[c + d*x] - 20762*Sin[2*(c + d*x)] + 11
760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 6588*Sin[3*(c + d*x)]
 + 1483*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(c + d*x)
] - 140*Sin[5*(c + d*x)])/(2240*a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.37, size = 159, normalized size = 1.05

method	result
derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{128}{64+64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{}{5\left(\tan\left(\frac{d}{}\right.\right.}}{d\,a^3}$

default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{128}{64+64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+6\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{14}{5\left(\tan\left(\frac{dx}{2}\right.\right.}}{d\,a^{3}}$
risch	$\frac{3x}{a^{3}}+\frac{\mathrm{e}^{i(dx+c)}}{2d\,a^{3}}+\frac{\mathrm{e}^{-i(dx+c)}}{2d\,a^{3}}+\frac{-46i\mathrm{e}^{4i(dx+c)}-94\,\mathrm{e}^{5i(dx+c)}-\frac{254\,\mathrm{e}^{3i(dx+c)}}{5}+58i\mathrm{e}^{6i(dx+c)}+14\,\mathrm{e}^{7i(dx+c)}-\frac{434i\mathrm{e}^{2i(dx+c)}}{5}}{\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}d\,a^{3}}$
norman	$\frac{54x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{1996\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{36\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{3x}{a}-\frac{352}{35ad}-\frac{1902\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}+\frac{6\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{186x\left(\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^3*(-1/1024/(tan(1/2*d*x+1/2*c)-1)+1/64/(1+tan(1/2*d*x+1/2*c)^2)+3/6
4*arctan(tan(1/2*d*x+1/2*c))-1/112/(tan(1/2*d*x+1/2*c)+1)^7+1/32/(tan(1/2*d
*x+1/2*c)+1)^6-7/320/(tan(1/2*d*x+1/2*c)+1)^5-3/128/(tan(1/2*d*x+1/2*c)+1)^
4+1/256/(tan(1/2*d*x+1/2*c)+1)^3+17/512/(tan(1/2*d*x+1/2*c)+1)^2+49/1024/(t
an(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(143) = 286$.
time = 0.51, size = 421, normalized size = 2.79

$$\frac{2\left(\frac{\frac{951\sin(dx+c)}{\cos(dx+c)+1}+\frac{2010\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1980\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{574\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{966\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1890\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{1540\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{630\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{105\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+176}{a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{15a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{20a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{14a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{14a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{20a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{15a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/35*((951*sin(d*x + c)/(cos(d*x + c) + 1) + 2010*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1980*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 574*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 - 966*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1890*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 1540*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
 630*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9 + 176)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 15*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 20*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
+ 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 14*a^3*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 - 20*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 15*a^3*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.36, size = 159, normalized size = 1.05

$$\frac{315\,dx\cos\left(dx+c\right)^{3}+286\,\cos\left(dx+c\right)^{4}-420\,dx\cos\left(dx+c\right)-447\,\cos\left(dx+c\right)^{2}+\left(105\,dx\cos\left(dx+c\right)^{3}+35\,\cos\left(dx+c\right)^{4}-420\,dx\cos\left(dx+c\right)-438\,\cos\left(dx+c\right)^{2}-20\right)\sin\left(dx+c\right)-15}{35\left(3\,a^{3}d\cos\left(dx+c\right)^{3}-4\,a^{3}d\cos\left(dx+c\right)+\left(a^{3}d\cos\left(dx+c\right)^{3}-4\,a^{3}d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(315*d*x*cos(d*x + c)^3 + 286*cos(d*x + c)^4 - 420*d*x*cos(d*x + c) -
447*cos(d*x + c)^2 + (105*d*x*cos(d*x + c)^3 + 35*cos(d*x + c)^4 - 420*d*x*
cos(d*x + c) - 438*cos(d*x + c)^2 - 20)*sin(d*x + c) - 15)/(3*a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x +
c))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 177, normalized size = 1.17

$$\frac{\frac{840\,(dx+c)}{a^3} - \frac{35\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)a^3} + \frac{1715\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+11480\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31815\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+45920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35161\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+13832\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2221}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(840*(d*x + c)/a^3 - 35*(tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/
2*c) + 17)/((tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) - 1)*a^3) + (1715*tan(1/2*d*x + 1/2*c)^6 + 11480*tan(1/2*d*x + 1/2
*c)^5 + 31815*tan(1/2*d*x + 1/2*c)^4 + 45920*tan(1/2*d*x + 1/2*c)^3 + 35161
*tan(1/2*d*x + 1/2*c)^2 + 13832*tan(1/2*d*x + 1/2*c) + 2221)/(a^3*(tan(1/2*
d*x + 1/2*c) + 1)^7))/d
```

Mupad [**B**]
time = 15.50, size = 182, normalized size = 1.21

$$\frac{3\,x}{a^3} - \frac{-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 - 36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 88\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 - 108\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - \frac{276\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} + \frac{164\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5} + \frac{792\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{7} + \frac{804\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{7} + \frac{1902\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35} + \frac{352}{35}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (3*x)/a^3 - ((1902*tan(c/2 + (d*x)/2))/35 + (804*tan(c/2 + (d*x)/2)^2)/7 +
(792*tan(c/2 + (d*x)/2)^3)/7 + (164*tan(c/2 + (d*x)/2)^4)/5 - (276*tan(c/2
```

```
+ (d*x)/2)^5)/5 - 108*tan(c/2 + (d*x)/2)^6 - 88*tan(c/2 + (d*x)/2)^7 - 36*t
an(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^9 + 352/35)/(a^3*d*(tan(c/2 + (d
*x)/2) + 1)^7*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/
2)^3 - 1))
```

3.788 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=142

$$-\frac{x}{a^3}-\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan(c+dx)}{a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan^5(}{5}$$

```
[Out] -x/a^3-3*sec(d*x+c)/a^3/d+10/3*sec(d*x+c)^3/a^3/d-11/5*sec(d*x+c)^5/a^3/d+4
/7*sec(d*x+c)^7/a^3/d+tan(d*x+c)/a^3/d-1/3*tan(d*x+c)^3/a^3/d+1/5*tan(d*x+c
)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2954, 2952, 2686, 276, 2687, 30, 200, 3554, 8}

$$-\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) - (3*Sec[c + d*x])/(a^3*d) + (10*Sec[c + d*x]^3)/(3*a^3*d) - (11*S
ec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]/(a^3
*d) - Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) - (4*Tan[c + d*x]
^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(a^3\sec^3(c+dx)\tan^5(c+dx)-3a^3\sec^2(c+dx)\tan^6(c+dx)+3a^3\sec\right.}{a^6} \\
&= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{\int \tan^8(c+dx)\,dx}{a^3}-\frac{3\int \sec^2(c+dx)\tan}{a^3} \\
&= -\frac{\tan^7(c+dx)}{7a^3d}+\frac{\int \tan^6(c+dx)\,dx}{a^3}+\frac{\operatorname{Subst}\left(\int x^2(-1+x^2)^2\,dx,x,\sec(\right.}{a^3d} \\
&= \frac{\tan^5(c+dx)}{5a^3d}-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{\int \tan^4(c+dx)\,dx}{a^3}+\frac{\operatorname{Subst}\left(\int (x^2-2x\right.}{} \\
&= -\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}- \\
&= -\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+ \\
&= -\frac{x}{a^3}-\frac{3\sec(c+dx)}{a^3d}+\frac{10\sec^3(c+dx)}{3a^3d}-\frac{11\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+d}{7a^3d}
\end{aligned}
$$

Mathematica [A]
time = 0.56, size = 214, normalized size = 1.51

$$
-\frac{4200+14(-1663+840c+840dx)\cos(c+dx)+6272\cos(2(c+dx))+9978\cos(3(c+dx))-5040c\cos(3(c+dx))-5040dx\cos(3(c+dx))-1768\cos(4(c+dx))+2688\sin(c+dx)-23282\sin(2(c+dx))+11760c\sin(2(c+dx))+11760dx\sin(2(c+dx))+5568\sin(3(c+dx))+1663\sin(4(c+dx))-840c\sin(4(c+dx))-840dx\sin(4(c+dx))}{6720a^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/6720*(4200 + 14*(-1663 + 840*c + 840*d*x)*Cos[c + d*x] + 6272*Cos[2*(c +
 d*x)] + 9978*Cos[3*(c + d*x)] - 5040*c*Cos[3*(c + d*x)] - 5040*d*x*Cos[3*(
c + d*x)] - 1768*Cos[4*(c + d*x)] + 2688*Sin[c + d*x] - 23282*Sin[2*(c + d*
x)] + 11760*c*Sin[2*(c + d*x)] + 11760*d*x*Sin[2*(c + d*x)] + 5568*Sin[3*(c
 + d*x)] + 1663*Sin[4*(c + d*x)] - 840*c*Sin[4*(c + d*x)] - 840*d*x*Sin[4*(
c + d*x)])/(a^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.34, size = 142, normalized size = 1.00

method	result
risch	$-\frac{x}{a^3}-\frac{2\left(1155i\,e^{6i(dx+c)}+315\,e^{7i(dx+c)}-525i\,e^{4i(dx+c)}-1715\,e^{5i(dx+c)}-1939i\,e^{2i(dx+c)}-1379\,e^{3i(dx+c)}+221i+1011\right.}{105\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^7 d\,a^3}$

derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{18}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d\,a^3}$
default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{18}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d\,a^3}$
norman	$-\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{60\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{x}{a}+\frac{272}{105ad}+\frac{474\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{17x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{6x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{17x\left(\tan^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^3*(-1/512/(tan(1/2*d*x+1/2*c)-1)-1/32*arctan(tan(1/2*d*x+1/2*c))+1/5
6/(tan(1/2*d*x+1/2*c)+1)^7-1/16/(tan(1/2*d*x+1/2*c)+1)^6+9/160/(tan(1/2*d*x
+1/2*c)+1)^5+1/64/(tan(1/2*d*x+1/2*c)+1)^4-5/384/(tan(1/2*d*x+1/2*c)+1)^3-7
/256/(tan(1/2*d*x+1/2*c)+1)^2-15/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(130) = 260$.
time = 0.51, size = 335, normalized size = 2.36

$$-\frac{2\left(\frac{\frac{711\sin(dx+c)}{\cos(dx+c)+1}+\frac{1274\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{469\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1260\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{1435\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{630\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{105\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+136}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{14a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -2/105*((711*sin(d*x + c)/(cos(d*x + c) + 1) + 1274*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 469*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1260*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 1435*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 630*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 - 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 136)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) + 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.35, size = 150, normalized size = 1.06

$$-\frac{315\,dx\cos\left(dx+c\right)^3+221\cos\left(dx+c\right)^4-420\,dx\cos\left(dx+c\right)-417\cos\left(dx+c\right)^2+3\left(35\,dx\cos\left(dx+c\right)^3-140\,dx\cos\left(dx+c\right)-116\cos\left(dx+c\right)^2+15\right)\sin\left(dx+c\right)+60}{105\left(3\,a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx+c\right)+\left(a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/105*(315*d*x*cos(d*x + c)^3 + 221*cos(d*x + c)^4 - 420*d*x*cos(d*x + c)
- 417*cos(d*x + c)^2 + 3*(35*d*x*cos(d*x + c)^3 - 140*d*x*cos(d*x + c) - 11
6*cos(d*x + c)^2 + 15)*sin(d*x + c) + 60)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d
*cos(d*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 129, normalized size = 0.91

$$-\frac{\frac{840\,(dx+c)}{a^3}+\frac{105}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}+\frac{1575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+10920\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+31675\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48160\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36981\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+14392\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2281}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(840*(d*x + c)/a^3 + 105/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) + (1575*ta
n(1/2*d*x + 1/2*c)^6 + 10920*tan(1/2*d*x + 1/2*c)^5 + 31675*tan(1/2*d*x + 1
/2*c)^4 + 48160*tan(1/2*d*x + 1/2*c)^3 + 36981*tan(1/2*d*x + 1/2*c)^2 + 143
92*tan(1/2*d*x + 1/2*c) + 2281)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [**B**]
time = 15.65, size = 131, normalized size = 0.92

$$\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{82\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{134\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}+\frac{364\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{474\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+\frac{272}{105}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}-\frac{x}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((474*tan(c/2 + (d*x)/2))/35 + (364*tan(c/2 + (d*x)/2)^2)/15 + (134*tan(c/2
 + (d*x)/2)^3)/15 - 24*tan(c/2 + (d*x)/2)^4 - (82*tan(c/2 + (d*x)/2)^5)/3 -
 12*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + 272/105)/(a^3*d*(tan(c/
2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1)^7) - x/a^3
```

3.789 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{\sec(c+dx)}{a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}$$

```
[Out] sec(d*x+c)/a^3/d-2*sec(d*x+c)^3/a^3/d+9/5*sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)
^7/a^3/d+1/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2687, 14, 2686, 276, 30, 200}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{9\sec^5(c+dx)}{5a^3d}-\frac{2\sec^3(c+dx)}{a^3d}+\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(a^3*d) - (2*Sec[c + d*x]^3)/(a^3*d) + (9*Sec[c + d*x]^5)/(5*a
^3*d) - (4*Sec[c + d*x]^7)/(7*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (4*Tan[c
+ d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^4(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^4(c+dx) - 3a^3\sec^3(c+dx)\tan^5(c+dx) + 3a^3\sec\right.}{a^6}$$

$$= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^3} - \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{3\int \sec}{}$$

$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx, x, \text{t}\right.}{a^3d}$$

$$= \frac{3\tan^7(c+dx)}{7a^3d} - \frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \sec(c+dx)\right)}{a^3d} +$$

$$= \frac{\sec(c+dx)}{a^3d} - \frac{2\sec^3(c+dx)}{a^3d} + \frac{9\sec^5(c+dx)}{5a^3d} - \frac{4\sec^7(c+dx)}{7a^3d} + \frac{\tan^5(c}{5a}$$

Mathematica [A]
time = 0.35, size = 104, normalized size = 1.02

$$\frac{\sec(c+dx)(840-1946\cos(c+dx)-224\cos(2(c+dx))+834\cos(3(c+dx))-104\cos(4(c+dx))+1344\sin(c+dx)-1946\sin(2(c+dx))+64\sin(3(c+dx))+139\sin(4(c+dx)))}{2240a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 1946*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 834*Cos[3*(
c + d*x)] - 104*Cos[4*(c + d*x)] + 1344*Sin[c + d*x] - 1946*Sin[2*(c + d*x)
] + 64*Sin[3*(c + d*x)] + 139*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d
*x])^3)
```

Maple [A]
time = 0.29, size = 130, normalized size = 1.27

method	result
risch	$\frac{-\frac{2\,e^{3i(dx+c)}}{5}-\frac{22ie^{2i(dx+c)}}{5}-6ie^{4i(dx+c)}-10\,e^{5i(dx+c)}+\frac{86\,e^{i(dx+c)}}{35}+\frac{26i}{35}+6ie^{6i(dx+c)}+2\,e^{7i(dx+c)}}{\left(e^{i(dx+c)}+i\right)^{7}\left(e^{i(dx+c)}-i\right)d\,a^{3}}$
derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{22}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^{3}}$
default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{22}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^{3}}$
norman	$\frac{-\frac{32\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{16}{35ad}-\frac{32\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{96\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{256\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{416\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{464\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}}{a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^3*(-1/256/(tan(1/2*d*x+1/2*c)-1)-1/28/(tan(1/2*d*x+1/2*c)+1)^7+1/8/(
tan(1/2*d*x+1/2*c)+1)^6-11/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4+1/64/(tan(1/2*d*x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/25
6/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(94) = 188.
time = 0.29, size = 230, normalized size = 2.25

$$\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{35\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 16/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 14*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a^3 + 6*a^3*sin(d*x
+ c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a
^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.02

$$\frac{13 \cos\left(dx+c\right)^{4} - 6 \cos\left(dx+c\right)^{2} - 4 \left(\cos\left(dx+c\right)^{2} + 5\right) \sin\left(dx+c\right) - 15}{35 \left(3 a^{3} d \cos\left(dx+c\right)^{3} - 4 a^{3} d \cos\left(dx+c\right) + \left(a^{3} d \cos\left(dx+c\right)^{3} - 4 a^{3} d \cos\left(dx+c\right)\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(13*cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 + 5)*sin(d*x
 + c) - 15)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
 + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{4}\left(c+dx\right) \sec^{2}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3 \sin^{2}\left(c+dx\right)+3 \sin\left(c+dx\right)+1}\, dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.56, size = 120, normalized size = 1.18

$$-\frac{\frac{35}{a^{3} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)} - \frac{35 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+280 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+1015 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+2240 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+1673 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+616 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+93}{a^{3} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{7}}}{280 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 2240*tan(1/2*d*x
 + 1/2*c)^3 + 1673*tan(1/2*d*x + 1/2*c)^2 + 616*tan(1/2*d*x + 1/2*c) + 93)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 9.69, size = 135, normalized size = 1.32

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{35}+\frac{96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^8)/35 + (96*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)
)/35 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3)/5 + (32*cos(c/2 + (d*
x)/2)^6*sin(c/2 + (d*x)/2)^2)/5)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*
x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

3.790 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=88

$$\frac{\sec^3(c+dx)}{a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{4\tan^7(c+dx)}{7a^3d}$$

```
[Out] sec(d*x+c)^3/a^3/d-7/5*sec(d*x+c)^5/a^3/d+4/7*sec(d*x+c)^7/a^3/d-3/5*tan(d*
x+c)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2686, 14, 2687, 276, 30}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{7\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(a^3*d) - (7*Sec[c + d*x]^5)/(5*a^3*d) + (4*Sec[c + d*x]^7)/
(7*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^5(c+dx)\tan^3(c+dx) - 3a^3\sec^4(c+dx)\tan^4(c+dx) + 3a^3\sec^3(\cdots\right.}{a^6}$$

$$= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{3\int \sec^4\cdots}{\cdots}$$

$$= -\frac{\mathrm{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^3d} + \frac{\mathrm{Subst}\left(\int x^4(-1+x^2)\,dx, x, \sec(c+dx\cdots\right.}{a^3d}$$

$$= -\frac{\tan^7(c+dx)}{7a^3d} + \frac{\mathrm{Subst}\left(\int (-x^4+x^6)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{3\mathrm{Subst}\left(\int (x^2\cdots\right.}{\cdots}$$

$$= \frac{\sec^3(c+dx)}{a^3d} - \frac{7\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} - \frac{3\tan^5(c+dx)}{5a^3d} - \frac{4\tan^7(\cdots}{7a\cdots}$$

Mathematica [A]

time = 0.28, size = 104, normalized size = 1.18

$$\frac{\sec(c+dx)(840-602\cos(c+dx)-448\cos(2(c+dx))+258\cos(3(c+dx))-8\cos(4(c+dx))+1008\sin(c+dx)-602\sin(2(c+dx))+48\sin(3(c+dx))+43\sin(4(c+dx)))}{2240a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(840 - 602*Cos[c + d*x] - 448*Cos[2*(c + d*x)] + 258*Cos[3*(c
 + d*x)] - 8*Cos[4*(c + d*x)] + 1008*Sin[c + d*x] - 602*Sin[2*(c + d*x)] +
48*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

Maple [A]
time = 0.27, size = 130, normalized size = 1.48

method	result
risch	$\frac{2i\left(-105\,\mathrm{e}^{4i(dx+c)}-56i\mathrm{e}^{3i(dx+c)}+21\,\mathrm{e}^{2i(dx+c)}-6i\mathrm{e}^{i(dx+c)}+1+70i\mathrm{e}^{5i(dx+c)}+35\,\mathrm{e}^{6i(dx+c)}\right)}{35\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}\left(\mathrm{e}^{i(dx+c)}-i\right)d\,a^{3}}$
derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{26}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{3}}$
default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{26}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{3}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{3}}$
norman	$\frac{-\frac{24\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{12}{35ad}-\frac{4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{72\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{36\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}-\frac{48\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}-\frac{44\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*(-1/128/(tan(1/2*d*x+1/2*c)-1)+1/14/(tan(1/2*d*x+1/2*c)+1)^7-1/4/(
tan(1/2*d*x+1/2*c)+1)^6+13/40/(tan(1/2*d*x+1/2*c)+1)^5-3/16/(tan(1/2*d*x+1/
2*c)+1)^4+1/32/(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(80) = 160$.
time = 0.31, size = 250, normalized size = 2.84

$$\frac{4\left(\frac{18\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{42\,\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{42\,\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{35\,\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}+3\right)}{35\left(a^{3}+\frac{6\,a^{3}\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{14\,a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{14\,a^{3}\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}-\frac{14\,a^{3}\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}-\frac{14\,a^{3}\sin\left(dx+c\right)^{6}}{\left(\cos\left(dx+c\right)+1\right)^{6}}-\frac{6\,a^{3}\sin\left(dx+c\right)^{7}}{\left(\cos\left(dx+c\right)+1\right)^{7}}-\frac{a^{3}\sin\left(dx+c\right)^{8}}{\left(\cos\left(dx+c\right)+1\right)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/35*(18*sin(d*x + c)/(cos(d*x + c) + 1) + 42*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 42*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 35*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 3)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) +
```

```
1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.36, size = 102, normalized size = 1.16

$$\frac{\cos\left(dx+c\right)^4+13\,\cos\left(dx+c\right)^2-3\,\left(\cos\left(dx+c\right)^2+5\right)\sin\left(dx+c\right)-20}{35\,\left(3\,a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx+c\right)+\left(a^3d\cos\left(dx+c\right)^3-4\,a^3d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 13*cos(d*x + c)^2 - 3*(cos(d*x + c)^2 + 5)*sin(d*x +
 c) - 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x +
 c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 120, normalized size = 1.36

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1001\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+392\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+61}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 +
280*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1120*tan(1/2*d*x
 + 1/2*c)^3 + 1001*tan(1/2*d*x + 1/2*c)^2 + 392*tan(1/2*d*x + 1/2*c) + 61)/
(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 9.73, size = 158, normalized size = 1.80

$$\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+18\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+42\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+42\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+35\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{35\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (4*cos(c/2 + (d*x)/2)^4*(3*cos(c/2 + (d*x)/2)^4 + 35*sin(c/2 + (d*x)/2)^4 +
 42*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^3 + 18*cos(c/2 + (d*x)/2)^3*sin(c
/2 + (d*x)/2) + 42*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^2))/(35*a^3*d*(c
os(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*
x)/2))^7)
```

3.791 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{4\tan^7(c+dx)}{7a^3d}$$

```
[Out] -1/3*sec(d*x+c)^3/a^3/d+sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d+1/3*tan(d
*x+c)^3/a^3/d+tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2790, 2687, 276, 2686, 14}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*Sec[c + d*x]^3/(a^3*d) + Sec[c + d*x]^5/(a^3*d) - (4*Sec[c + d*x]^7)/(
7*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + Tan[c + d*x]^5/(a^3*d) + (4*Tan[c + d
*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int (a^3\sec^6(c+dx)\tan^2(c+dx)-3a^3\sec^5(c+dx)\tan^3(c+dx)+3a^3\sec^4(c+}{a^6}\\ &= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{3\int \sec^5(c}{}\\ &= -\frac{\operatorname{Subst}\left(\int x^2(-1+x^2)^2\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\operatorname{Subst}\left(\int x^2(1+x^2)^2\,dx,x,\tan\right.}{a^3d}\\ &= -\frac{\operatorname{Subst}\left(\int (x^2-2x^4+x^6)\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\operatorname{Subst}\left(\int (x^2+2x^4+x^6)\,dx,\right.}{a^3d}\\ &= -\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{\tan^5(c+dx}{a^3d}\end{aligned}$$

Mathematica [A]

time = 0.27, size = 104, normalized size = 1.01

$$\frac{\sec(c+dx)(336-70\cos(c+dx)-224\cos(2(c+dx))+30\cos(3(c+dx))+16\cos(4(c+dx))+672\sin(c+dx)-70\sin(2(c+dx))-96\sin(3(c+dx))+5\sin(4(c+dx)))}{1344a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(336 - 70*Cos[c + d*x] - 224*Cos[2*(c + d*x)] + 30*Cos[3*(c +
 d*x)] + 16*Cos[4*(c + d*x)] + 672*Sin[c + d*x] - 70*Sin[2*(c + d*x)] - 96*
Sin[3*(c + d*x)] + 5*Sin[4*(c + d*x)]))/(1344*a^3*d*(1 + Sin[c + d*x])^3)
```

Maple [A]

time = 0.25, size = 130, normalized size = 1.26

method	result

risch	$-\frac{4\left(-14i\,\mathrm{e}^{2i(dx+c)}-28\,\mathrm{e}^{3i(dx+c)}+6\,\mathrm{e}^{i(dx+c)}+i+21i\,\mathrm{e}^{4i(dx+c)}+14\,\mathrm{e}^{5i(dx+c)}\right)}{21\left(\mathrm{e}^{i(dx+c)}-i\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}d\,a^{3}}$
derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{6}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{13}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d\,a^{3}}$
default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{6}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{5}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{13}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{d\,a^{3}}$
norman	$\frac{-\frac{4\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4}{21ad}-\frac{8\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{7ad}-\frac{8\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^3*(-1/64/(tan(1/2*d*x+1/2*c)-1)-1/7/(tan(1/2*d*x+1/2*c)+1)^7+1/2/(tan
(1/2*d*x+1/2*c)+1)^6-3/4/(tan(1/2*d*x+1/2*c)+1)^5+5/8/(tan(1/2*d*x+1/2*c)+1
)^4-13/48/(tan(1/2*d*x+1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(
1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(95) = 190$.

time = 0.31, size = 270, normalized size = 2.62

$$\frac{4\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{28\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{21\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+1\right)}{21\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/21*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 14*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 28*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 14*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*si
n(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d
*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]

time = 0.35, size = 104, normalized size = 1.01

$$-\frac{2\cos(dx+c)^{4}-9\cos(dx+c)^{2}-6\left(\cos(dx+c)^{2}-2\right)\sin(dx+c)+9}{21\left(3a^{3}d\cos(dx+c)^{3}-4a^{3}d\cos(dx+c)+\left(a^{3}d\cos(dx+c)^{3}-4a^{3}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/21*(2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 - 2)*sin(d*x
 + c) + 9)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(d*x
+ c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.49, size = 120, normalized size = 1.17

$$-\frac{\frac{21}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}-\frac{21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+168\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+161\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+224\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+63\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+56\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/168*(21/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (21*tan(1/2*d*x + 1/2*c)^6 +
168*tan(1/2*d*x + 1/2*c)^5 + 161*tan(1/2*d*x + 1/2*c)^4 + 224*tan(1/2*d*x +
 1/2*c)^3 + 63*tan(1/2*d*x + 1/2*c)^2 + 56*tan(1/2*d*x + 1/2*c) + 11)/(a^3*
(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 10.15, size = 183, normalized size = 1.78

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{21}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{7}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^8)/21 + (8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2))/
7 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^5)/3 + 4*cos(c/2 + (d*x)/2)^
4*sin(c/2 + (d*x)/2)^4 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3)/3 +
 (8*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^2)/3)/(a^3*d*(cos(c/2 + (d*x)/2
) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

3.792 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=99

$$\frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+dx)\right)}+\frac{6\tan(c+dx)}{35a^3d}$$

```
[Out] 1/7*sec(d*x+c)/d/(a+a*sin(d*x+c))^3-3/35*sec(d*x+c)/a/d/(a+a*sin(d*x+c))^2-
3/35*sec(d*x+c)/d/(a^3+a^3*sin(d*x+c))+6/35*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2938, 2751, 3852, 8}

$$\frac{6\tan(c+dx)}{35a^3d}-\frac{3\sec(c+dx)}{35d\left(a^3\sin(c+dx)+a^3\right)}-\frac{3\sec(c+dx)}{35ad(a\sin(c+dx)+a)^2}+\frac{\sec(c+dx)}{7d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]/(7*d*(a + a*Sin[c + d*x])^3) - (3*Sec[c + d*x])/(35*a*d*(a + a
*Sin[c + d*x])^2) - (3*Sec[c + d*x])/(35*d*(a^3 + a^3*Sin[c + d*x])) + (6*T
an[c + d*x])/(35*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
```

```
) && NeQ[2*m + p + 1, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}+\frac{3\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{7a}\\ &= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}+\frac{9\int \frac{\sec^2(c+dx)}{a+a\sin(c+dx)}\,dx}{35a^2}\\ &= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+\right.}\\ &= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+\right.}\\ &= \frac{\sec(c+dx)}{7d(a+a\sin(c+dx))^3}-\frac{3\sec(c+dx)}{35ad(a+a\sin(c+dx))^2}-\frac{3\sec(c+dx)}{35d\left(a^3+a^3\sin(c+\right.}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 104, normalized size = 1.05

$$\frac{\sec(c+dx)(560+182\cos(c+dx)-672\cos(2(c+dx))-78\cos(3(c+dx))+48\cos(4(c+dx))+672\sin(c+dx)+182\sin(2(c+dx))-288\sin(3(c+dx))-13\sin(4(c+dx)))}{2240a^3d(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(560 + 182*Cos[c + d*x] - 672*Cos[2*(c + d*x)] - 78*Cos[3*(c
+ d*x)] + 48*Cos[4*(c + d*x)] + 672*Sin[c + d*x] + 182*Sin[2*(c + d*x)] - 2
88*Sin[3*(c + d*x)] - 13*Sin[4*(c + d*x)]))/(2240*a^3*d*(1 + Sin[c + d*x])^
3)
```

Maple [A]
time = 0.23, size = 130, normalized size = 1.31

method	result
risch	$-\frac{4i\left(-42\,e^{2i(dx+c)}-18ie^{i(dx+c)}+3+35\,e^{4i(dx+c)}+42ie^{3i(dx+c)}\right)}{35\left(e^{i(dx+c)}-i\right)\left(e^{i(dx+c)}+i\right)^{7}d\,a^{3}}$

derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{34}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{7}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{9}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^{3}}$
default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{34}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{7}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{9}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^{3}}$
norman	$\frac{-\frac{6\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2}{35ad}-\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{6\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{16\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*(-1/32/(tan(1/2*d*x+1/2*c)-1)+2/7/(tan(1/2*d*x+1/2*c)+1)^7-1/(tan(1
/2*d*x+1/2*c)+1)^6+17/10/(tan(1/2*d*x+1/2*c)+1)^5-7/4/(tan(1/2*d*x+1/2*c)+1
)^4+9/8/(tan(1/2*d*x+1/2*c)+1)^3-7/16/(tan(1/2*d*x+1/2*c)+1)^2+1/32/(tan(1/
2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(91) = 182$.
time = 0.29, size = 290, normalized size = 2.93

$$-\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}-\frac{21\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{56\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{105\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{70\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{35\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}{35\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2/35*(6*sin(d*x + c)/(cos(d*x + c) + 1) - 21*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 56*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 105*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 70*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 35*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) +
14*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 - 14*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a
^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 106, normalized size = 1.07

$$-\frac{6\cos(dx+c)^{4}-27\cos(dx+c)^{2}-3\left(6\cos(dx+c)^{2}-5\right)\sin(dx+c)+20}{35\left(3a^{3}d\cos(dx+c)^{3}-4a^{3}d\cos(dx+c)+\left(a^{3}d\cos(dx+c)^{3}-4a^{3}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(6*cos(d*x + c)^4 - 27*cos(d*x + c)^2 - 3*(6*cos(d*x + c)^2 - 5)*sin(
d*x + c) + 20)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c) + (a^3*d*cos(
d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.53, size = 120, normalized size = 1.21

$$-\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-665\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-791\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-392\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-51}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) - 1)) - (35*tan(1/2*d*x + 1/2*c)^6 -
280*tan(1/2*d*x + 1/2*c)^5 - 665*tan(1/2*d*x + 1/2*c)^4 - 1120*tan(1/2*d*x
+ 1/2*c)^3 - 791*tan(1/2*d*x + 1/2*c)^2 - 392*tan(1/2*d*x + 1/2*c) - 51)/(a
^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 10.01, size = 206, normalized size = 2.08

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6-6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+21\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+56\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+105\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4+70\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+35\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6\right)}{35\,a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)^2*(35*sin(c/2 + (d*x)/2)^6 - cos(c/2 + (d*x)/2)^6 + 7
0*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^5 - 6*cos(c/2 + (d*x)/2)^5*sin(c/2
+ (d*x)/2) + 105*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 56*cos(c/2 + (
d*x)/2)^3*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^2))/(35*a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))*(cos(c/2 + (d*x)/2
) + sin(c/2 + (d*x)/2))^7)
```

3.793 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{10\tan^3(c}{3a^3c}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d+1/3*sec(d*x+c)^3/a^3/d+1/5*sec(
d*x+c)^5/a^3/d+4/7*sec(d*x+c)^7/a^3/d-3*tan(d*x+c)/a^3/d-10/3*tan(d*x+c)^3/
a^3/d-11/5*tan(d*x+c)^5/a^3/d-4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2686, 30, 2687, 276}

$$-\frac{4\tan^7(c+dx)}{7a^3d}-\frac{11\tan^5(c+dx)}{5a^3d}-\frac{10\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + (4*Sec[c + d*x]^7)/(7*a^3*d) - (3*Tan[
c + d*x])/(a^3*d) - (10*Tan[c + d*x]^3)/(3*a^3*d) - (11*Tan[c + d*x]^5)/(5*
a^3*d) - (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int \left(-3a^3\sec^8(c+dx)+a^3\csc(c+dx)\sec^8(c+dx)+3a^3\sec^7(c+dx)\tan(c\right.}{a^6}$$
$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^3} - \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{3\int \sec^8(c}{}$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int x^2(1+x^2)^2\,dx, x, \tan(c+d\right.}{a^3 d}$$
$$= \frac{3\sec^7(c+dx)}{7a^3d} - \frac{3\tan(c+dx)}{a^3d} - \frac{3\tan^3(c+dx)}{a^3d} - \frac{9\tan^5(c+dx)}{5a^3d} - \frac{3\tan^7}{7}$$
$$= \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{4\sec^7(c+dx)}{7a^3d} - \frac{3\tan(c+dx}{a^3d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\sec(c+dx)}{a^3d} + \frac{\sec^3(c+dx)}{3a^3d} + \frac{\sec^5(c+dx)}{5a^3d} + \frac{4\,\text{se}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(151) = 302.
time = 0.30, size = 341, normalized size = 2.26

$$\frac{60-\frac{120\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}-324\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+162\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-706\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3+353\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4-2281\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5-840\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6+840\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6+\frac{105\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}}{840d(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (60 - (120*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 324*Si
n[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 162*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - 706*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^3 + 353*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 2281*Sin[(c + d
*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 840*Log[Cos[(c + d*x)/2]]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6 + 840*Log[Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^6 + (105*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^6)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(840*d*(a + a
*Sin[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 139, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{42}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}-\frac{11}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{67}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{d\,a^3}$

default	$\frac{-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{42}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{11}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{67}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{d\,a^{3}}$
risch	$\frac{12i\,e^{6i(dx+c)}+2\,e^{7i(dx+c)}-24i\,e^{4i(dx+c)}-\frac{82\,e^{5i(dx+c)}}{3}-\frac{364i\,e^{2i(dx+c)}}{15}-\frac{134\,e^{3i(dx+c)}}{15}+\frac{272i}{105}+\frac{474\,e^{i(dx+c)}}{35}}{\left(e^{i(dx+c)}+i\right)^{7}\left(e^{i(dx+c)}-i\right)d\,a^{3}}+\frac{\ln\left(e^{i(dx+c}\right.}{d\,a^{3}}$
norman	$\frac{\frac{10\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{442}{105ad}+\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{98\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{22\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{394\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{554\left(\tan^{2}\left(\frac{dx}{2}+\right.\right.}{15ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/8/(tan(1/2*d*x+1/2*c)-1)+8/7/(tan(1/2*d*x+1/2*c)+1)^7-4/(tan(1/
2*d*x+1/2*c)+1)^6+42/5/(tan(1/2*d*x+1/2*c)+1)^5-11/(tan(1/2*d*x+1/2*c)+1)^4
+67/6/(tan(1/2*d*x+1/2*c)+1)^3-31/4/(tan(1/2*d*x+1/2*c)+1)^2+49/8/(tan(1/2*
d*x+1/2*c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(139) = 278$.
time = 0.30, size = 336, normalized size = 2.23

$$\frac{\frac{2\left(\frac{1011\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{1939\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1379\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{525\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{1715\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{1155\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{315\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+221\right)}{a^{3}+\frac{6\,a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{14\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\,a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{14\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{6\,a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{105\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(1011*sin(d*x + c)/(cos(d*x + c) + 1) + 1939*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 1379*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 525*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 1715*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1155*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 315*sin(d*x + c)^7/(cos(d*x + c) + 1)
^7 + 221)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 14*a^3*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*
a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^3*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 - 6*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a^3*sin(d*x + c)^8/
(cos(d*x + c) + 1)^8) + 105*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.37, size = 218, normalized size = 1.44

$$\frac{272\,\cos(dx+c)^{4}-594\,\cos(dx+c)^{2}-105\left(3\,\cos(dx+c)^{3}+\left(\cos(dx+c)^{3}-4\,\cos(dx+c)\right)\sin(dx+c)-4\,\cos(dx+c)\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+105\left(3\,\cos(dx+c)^{3}+\left(\cos(dx+c)^{3}-4\,\cos(dx+c)\right)\sin(dx+c)-4\,\cos(dx+c)\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-6\left(101\,\cos(dx+c)^{2}+15\right)\sin(dx+c)-120}{210\left(3\,a^{3}d\cos(dx+c)^{3}-4\,a^{3}d\cos(dx+c)+\left(a^{3}d\cos(dx+c)^{3}-4\,a^{3}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/210*(272*cos(d*x + c)^4 - 594*cos(d*x + c)^2 - 105*(3*cos(d*x + c)^3 + (c
os(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) - 4*cos(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + 105*(3*cos(d*x + c)^3 + (cos(d*x + c)^3 - 4*cos(d*x + c))
*sin(d*x + c) - 4*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 6*(101*cos(d
*x + c)^2 + 15)*sin(d*x + c) - 120)/(3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d
*x + c) + (a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.47, size = 135, normalized size = 0.89

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{105}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}+\frac{5145\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+24360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+54005\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+66080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+47691\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+18872\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3431}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105/(a^3*(tan(1/2*d*x + 1/2
*c) - 1)) + (5145*tan(1/2*d*x + 1/2*c)^6 + 24360*tan(1/2*d*x + 1/2*c)^5 + 5
4005*tan(1/2*d*x + 1/2*c)^4 + 66080*tan(1/2*d*x + 1/2*c)^3 + 47691*tan(1/2*
d*x + 1/2*c)^2 + 18872*tan(1/2*d*x + 1/2*c) + 3431)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^7))/d
```

Mupad [B]
time = 11.33, size = 143, normalized size = 0.95

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{98\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{394\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}+\frac{554\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{674\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+\frac{442}{105}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((674*tan(c/2 + (d*x)/2))/35 + (554*tan(c
/2 + (d*x)/2)^2)/15 + (394*tan(c/2 + (d*x)/2)^3)/15 - 10*tan(c/2 + (d*x)/2)
^4 - (98*tan(c/2 + (d*x)/2)^5)/3 - 22*tan(c/2 + (d*x)/2)^6 - 6*tan(c/2 + (d
*x)/2)^7 + 442/105)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2) + 1
)^7)
```

3.794 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=162

$$\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}+\frac{7\tan(c}{a^3}$$

```
[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-3*sec(d*x+c)/a^3/d-sec(d*x+c)^
3/a^3/d-3/5*sec(d*x+c)^5/a^3/d-4/7*sec(d*x+c)^7/a^3/d+7*tan(d*x+c)/a^3/d+5*
tan(d*x+c)^3/a^3/d+13/5*tan(d*x+c)^5/a^3/d+4/7*tan(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2700, 276, 2686, 30}

$$\frac{4\tan^7(c+dx)}{7a^3d}+\frac{13\tan^5(c+dx)}{5a^3d}+\frac{5\tan^3(c+dx)}{a^3d}+\frac{7\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])
/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (4*Sec[c
 + d*x]^7)/(7*a^3*d) + (7*Tan[c + d*x])/(a^3*d) + (5*Tan[c + d*x]^3)/(a^3*d
) + (13*Tan[c + d*x]^5)/(5*a^3*d) + (4*Tan[c + d*x]^7)/(7*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(3a^3\sec^8(c+dx)-3a^3\csc(c+dx)\sec^8(c+dx)+a^3\csc^2(c+dx)\sec^8(\right.}{a^6} \\
&= \frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^3}-\frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^3}+\frac{3\int \sec}{} \\
&= -\frac{\mathrm{Subst}\left(\int x^6\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\mathrm{Subst}\left(\int \frac{(1+x^2)^4}{x^2}\,dx,x,\tan(c+dx)\right)}{a^3d} \\
&= -\frac{\sec^7(c+dx)}{7a^3d}+\frac{3\tan(c+dx)}{a^3d}+\frac{3\tan^3(c+dx)}{a^3d}+\frac{9\tan^5(c+dx)}{5a^3d}+\frac{3\,\mathrm{ta}}{} \\
&= -\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{4\sec^7(}{7a} \\
&= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(162) = 324.
time = 0.68, size = 351, normalized size = 2.17

$$\frac{\csc^3(c+dx)\left(-966-440\cos(2(c+dx))-2640\cos(3(c+dx))+846\cos(4(c+dx))+176\cos(5(c+dx))-1575\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+105\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+14\cos(c+dx)\left(176+105\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-105\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+1575\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-105\cos(5(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-1316\sin(c+dx)+3520\sin(2(c+dx))+2100\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-2100\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-1380\sin(3(c+dx))-1056\sin(4(c+dx))-630\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+630\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+176\sin(5(c+dx))\right)}{140a^3d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(-966 - 440*Cos[2*(c + d*x)] - 2640*Cos[3*(c + d*x)] + 846*
Cos[4*(c + d*x)] + 176*Cos[5*(c + d*x)] - 1575*Cos[3*(c + d*x)]*Log[Cos[(c
+ d*x)/2]] + 105*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 14*Cos[c + d*x]*(
176 + 105*Log[Cos[(c + d*x)/2]] - 105*Log[Sin[(c + d*x)/2]]) + 1575*Cos[3*(
c + d*x)]*Log[Sin[(c + d*x)/2]] - 105*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]
] - 1316*Sin[c + d*x] + 3520*Sin[2*(c + d*x)] + 2100*Log[Cos[(c + d*x)/2]]*
Sin[2*(c + d*x)] - 2100*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 1380*Sin[3
*(c + d*x)] - 1056*Sin[4*(c + d*x)] - 630*Log[Cos[(c + d*x)/2]]*Sin[4*(c +
d*x)] + 630*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 176*Sin[5*(c + d*x)]))
/(140*a^3*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)*(1 + Sin[c + d*x])^3)
```

Maple [**A**]
time = 0.42, size = 164, normalized size = 1.01

method	result

derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{92}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{26}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{}{(\tan}}{2d\,a^{3}}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{92}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{26}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{}{(\tan}}{2d\,a^{3}}$
risch	$-\frac{2(-1540\,e^{7i(dx+c)}+630ie^{8i(dx+c)}+966\,e^{5i(dx+c)}-1890ie^{6i(dx+c)}+1980\,e^{3i(dx+c)}-951\,e^{i(dx+c)}+2010ie^{2i(dx+c)}-176i}{35\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^{7}\left(e^{i(dx+c)}-i\right)d\,a^{3}}$
norman	$\frac{-\frac{143\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{1}{2ad}-\frac{60\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{51\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{96\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{572\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}+\frac{6}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-1/4/(tan(1/2*d*x+1/2*c)-1)-16/7/(tan(1/2*d*x+
1/2*c)+1)^7+8/(tan(1/2*d*x+1/2*c)+1)^6-92/5/(tan(1/2*d*x+1/2*c)+1)^5+26/(ta
n(1/2*d*x+1/2*c)+1)^4-31/(tan(1/2*d*x+1/2*c)+1)^3+49/2/(tan(1/2*d*x+1/2*c)+
1)^2-111/4/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2
*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(154) = 308$.

time = 0.28, size = 395, normalized size = 2.44

$$-\frac{\frac{\frac{934\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3854\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6566\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3556\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{3710\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{7070\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{4270\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{1015\,\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+35}{\frac{a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{14\,a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{14\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{14\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{14\,a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{6\,a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}+\frac{210\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{35\,\sin(dx+c)}{a^{3}(\cos(dx+c)+1)}}{70\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/70*((934*sin(d*x + c)/(cos(d*x + c) + 1) + 3854*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 6566*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3556*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 3710*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 7070*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4270*sin(d*x + c)^7/(cos(d*x + c) + 1)^
7 - 1015*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)/(a^3*sin(d*x + c)/(cos(d
*x + c) + 1) + 6*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 14*a^3*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 14*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 1
4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 14*a^3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 6*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9) + 210*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 35
*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A]

time = 0.38, size = 265, normalized size = 1.64

$$\frac{846\cos(dx+c)^{4}-956\cos(dx+c)^{2}+105\left(\cos(dx+c)^{5}-5\cos(dx+c)^{3}-\left(3\cos(dx+c)^{3}-4\cos(dx+c)\right)\sin(dx+c)+4\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-105\left(\cos(dx+c)^{5}-5\cos(dx+c)^{3}-\left(3\cos(dx+c)^{3}-4\cos(dx+c)\right)\sin(dx+c)+4\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(176\cos(dx+c)^{4}-477\cos(dx+c)^{2}+15\right)\sin(dx+c)+40}{70\left(a^{3}d\cos(dx+c)^{5}-5\,a^{3}d\cos(dx+c)^{3}+4\,a^{3}d\cos(dx+c)-\left(3\,a^{3}d\cos(dx+c)^{3}-4\,a^{3}d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/70*(846*cos(d*x + c)^4 - 956*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 5*cos
(d*x + c)^3 - (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) - 105*(cos(d*x + c)^5 - 5*cos(d*x + c)^3
- (3*cos(d*x + c)^3 - 4*cos(d*x + c))*sin(d*x + c) + 4*cos(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2) + 2*(176*cos(d*x + c)^4 - 477*cos(d*x + c)^2 + 15)*s
in(d*x + c) + 40)/(a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^3 + 4*a^3*d*
cos(d*x + c) - (3*a^3*d*cos(d*x + c)^3 - 4*a^3*d*cos(d*x + c))*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.53, size = 187, normalized size = 1.15

$$-\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{35\left(12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-17\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^3}+\frac{3885\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+19880\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+45465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+57120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+41671\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+16632\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2931}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/280*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 140*tan(1/2*d*x + 1/2*c)/a
^3 - 35*(12*tan(1/2*d*x + 1/2*c)^2 - 17*tan(1/2*d*x + 1/2*c) + 4)/((tan(1/2
*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c))*a^3) + (3885*tan(1/2*d*x + 1/2*c)^6
 + 19880*tan(1/2*d*x + 1/2*c)^5 + 45465*tan(1/2*d*x + 1/2*c)^4 + 57120*tan(
1/2*d*x + 1/2*c)^3 + 41671*tan(1/2*d*x + 1/2*c)^2 + 16632*tan(1/2*d*x + 1/2
*c) + 2931)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 10.84, size = 274, normalized size = 1.69

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{-29\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-122\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-202\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-106\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{508\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+\frac{938\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{3854\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+\frac{934\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+1}{d\,\left(-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-28\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-28\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+28\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+28\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((934*tan(c/2 + (d*x)/2))/35 + (3854*tan(c/2
 + (d*x)/2)^2)/35 + (938*tan(c/2 + (d*x)/2)^3)/5 + (508*tan(c/2 + (d*x)/2)^
4)/5 - 106*tan(c/2 + (d*x)/2)^5 - 202*tan(c/2 + (d*x)/2)^6 - 122*tan(c/2 +
(d*x)/2)^7 - 29*tan(c/2 + (d*x)/2)^8 + 1)/(d*(12*a^3*tan(c/2 + (d*x)/2)^2 +
 28*a^3*tan(c/2 + (d*x)/2)^3 + 28*a^3*tan(c/2 + (d*x)/2)^4 - 28*a^3*tan(c/2
 + (d*x)/2)^6 - 28*a^3*tan(c/2 + (d*x)/2)^7 - 12*a^3*tan(c/2 + (d*x)/2)^8 -
 2*a^3*tan(c/2 + (d*x)/2)^9 + 2*a^3*tan(c/2 + (d*x)/2))) - (3*log(tan(c/2 +
 (d*x)/2)))/(a^3*d)
```

3.795 $\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=117

$$\frac{5ax}{2}-\frac{3a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{5a\tan(c+dx)}{2d}+\frac{5a\tan^3(c+dx)}{6d}-$$

```
[Out] 5/2*a*x-3*a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3*a*sec(d*x+c)/d+1/3*a*sec(d*
x+c)^3/d-5/2*a*tan(d*x+c)/d+5/6*a*tan(d*x+c)^3/d-1/2*a*sin(d*x+c)^2*tan(d*x
+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2671, 294, 308, 209, 2670, 276}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{3a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{3a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (5*a*x)/2 - (3*a*Cos[c + d*x])/d + (a*Cos[c + d*x]^3)/(3*d) - (3*a*Sec[c +
d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c
+ d*x]^3)/(6*d) - (a*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule Rubi steps

$$\int \sin^2(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sin^2(c+dx)\tan^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan$$

$$= -\frac{a\text{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{x}{(1+}\right.}{}$$

$$= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}\right.\right.}{}$$

$$= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a}{}$$

$$= -\frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx)}{d} + \frac{a}{}$$

$$= \frac{5ax}{2} - \frac{3a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} - \frac{3a\sec(c+dx}{d}$$

Mathematica [A]

time = 0.29, size = 84, normalized size = 0.72

$$\frac{a(30c+30dx-33\cos(c+dx)+\cos(3(c+dx))-36\sec(c+dx)+4\sec^3(c+dx)-3\sin(2(c+dx))-28\tan(c+dx)+4\sec^2(c+dx)\tan(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 33*Cos[c + d*x] + Cos[3*(c + d*x)] - 36*Sec[c + d*x] +
4*Sec[c + d*x]^3 - 3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*
Tan[c + d*x]))/(12*d)
```

Maple [A]

time = 0.19, size = 164, normalized size = 1.40

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+a\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\right.}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+a\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\right.}{d}$
risch	$\frac{5ax}{2}+\frac{a\,e^{3i(dx+c)}}{24d}+\frac{ia\,e^{2i(dx+c)}}{8d}-\frac{11a\,e^{i(dx+c)}}{8d}-\frac{11a\,e^{-i(dx+c)}}{8d}-\frac{ia\,e^{-2i(dx+c)}}{8d}+\frac{a\,e^{-3i(dx+c)}}{24d}-\frac{2(5a\,e^{i(}}{}$
norman	$\frac{-\frac{5ax}{2}+\frac{32a}{3d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{14a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{14a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{5a\left(\tan^{11}\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(
d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+a*(1/3
*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(d*x+c)
^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]

time = 0.52, size = 96, normalized size = 0.82

$$\frac{2\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a+\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a)/d
```

Fricas [A]
time = 0.37, size = 108, normalized size = 0.92

$$\frac{a\cos(dx+c)^4 - 15\,adx\cos(dx+c) + 29\,a\cos(dx+c)^2 + \left(2\,a\cos(dx+c)^4 + 15\,adx\cos(dx+c) - 15\,a\cos(dx+c)^2 - 4\,a\right)\sin(dx+c) + 2\,a}{6\,(d\cos(dx+c)\sin(dx+c) - d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 29*a*cos(d*x + c)^2 + (2*a*
cos(d*x + c)^4 + 15*a*d*x*cos(d*x + c) - 15*a*cos(d*x + c)^2 - 4*a)*sin(d*x
 + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.50, size = 182, normalized size = 1.56

$$\frac{15\,(dx+c)a - \frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1} + \frac{33\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8 - 102\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 200\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 330\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 402\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 410\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 264\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 61\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a - 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (33*a*tan(1/2*d*x +
1/2*c)^8 - 102*a*tan(1/2*d*x + 1/2*c)^7 + 200*a*tan(1/2*d*x + 1/2*c)^6 - 33
0*a*tan(1/2*d*x + 1/2*c)^5 + 402*a*tan(1/2*d*x + 1/2*c)^4 - 410*a*tan(1/2*d
*x + 1/2*c)^3 + 264*a*tan(1/2*d*x + 1/2*c)^2 - 150*a*tan(1/2*d*x + 1/2*c) +
 61*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

Mupad [B]
time = 14.96, size = 306, normalized size = 2.62

$$\frac{5\,a\,x}{2} + \frac{\left(5\,a\,d\,x - \frac{a\,(30\,d\,x-30)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \left(\frac{a\,(45\,d\,x-60)}{6} - \frac{15\,a\,d\,x}{2}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + \left(10\,a\,d\,x - \frac{a\,(60\,d\,x-80)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \left(\frac{a\,(30\,d\,x-100)}{6} - 5\,a\,d\,x\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \left(5\,a\,d\,x - \frac{a\,(30\,d\,x-28)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \left(\frac{a\,(60\,d\,x-176)}{6} - 10\,a\,d\,x\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \left(\frac{15\,a\,d\,x}{2} - \frac{a\,(45\,d\,x-132)}{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \left(\frac{a\,(30\,d\,x-98)}{6} - 5\,a\,d\,x\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - \frac{a\,(15\,d\,x-64)}{6} + \frac{5\,a\,d\,x}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (5*a*x)/2 + (tan(c/2 + (d*x)/2)*((a*(30*d*x - 98))/6 - 5*a*d*x) - (a*(15*d*
x - 64))/6 - tan(c/2 + (d*x)/2)^4*((a*(30*d*x - 28))/6 - 5*a*d*x) - tan(c/2
 + (d*x)/2)^9*((a*(30*d*x - 30))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^8*((a*(4
5*d*x - 60))/6 - (15*a*d*x)/2) + tan(c/2 + (d*x)/2)^6*((a*(30*d*x - 100))/6
 - 5*a*d*x) - tan(c/2 + (d*x)/2)^7*((a*(60*d*x - 80))/6 - 10*a*d*x) - tan(c
/2 + (d*x)/2)^2*((a*(45*d*x - 132))/6 - (15*a*d*x)/2) + tan(c/2 + (d*x)/2)^
3*((a*(60*d*x - 176))/6 - 10*a*d*x) + 6*a*tan(c/2 + (d*x)/2)^5 + (5*a*d*x)/
2)/(d*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 +
 tan(c/2 + (d*x)/2)^3 - 1)^3)
```

3.796 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{5ax}{2}-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{5a\tan(c+dx)}{2d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}$$

```
[Out] 5/2*a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-5/2*a*tan(d*x+
c)/d+5/6*a*tan(d*x+c)^3/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2917, 2670, 276, 2671, 294, 308, 209}

$$-\frac{a\cos(c+dx)}{d}+\frac{5a\tan^3(c+dx)}{6d}-\frac{5a\tan(c+dx)}{2d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}-\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (5*a*x)/2 - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/
(3*d) - (5*a*Tan[c + d*x])/(2*d) + (5*a*Tan[c + d*x]^3)/(6*d) - (a*Sin[c +
d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx &= a\int \sin(c+dx)\tan^4(c+dx)\,dx + a\int \sin^2(c+dx)\tan\\ &= -\frac{a\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{}{(1+}\right.}{}\\ &= -\frac{a\sin^2(c+dx)\tan^3(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right.\right.}{d}\\ &= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a}{}\\ &= -\frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{5a}{}\\ &= \frac{5ax}{2} - \frac{a\cos(c+dx)}{d} - \frac{2a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.75

$$\frac{a(30c+30dx-12\cos(c+dx)-24\sec(c+dx)+4\sec^3(c+dx)-3\sin(2(c+dx))-28\tan(c+dx)+4\sec^2(c+dx)\tan(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*(30*c + 30*d*x - 12*Cos[c + d*x] - 24*Sec[c + d*x] + 4*Sec[c + d*x]^3 -
3*Sin[2*(c + d*x)] - 28*Tan[c + d*x] + 4*Sec[c + d*x]^2*Tan[c + d*x]))/(12*
d)
```

Maple [A]
time = 0.16, size = 154, normalized size = 1.52

method	result
risch	$\frac{5ax}{2}+\frac{ia\,e^{2i(dx+c)}}{8d}-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}-\frac{ia\,e^{-2i(dx+c)}}{8d}-\frac{2a\left(-3ie^{2i(dx+c)}+6\,e^{3i(dx+c)}-7i+8\,e^{i(dx+c)}\right)}{3\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^{3}d}$
derivativedivides	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx\right.}{}\right.}{d}$
default	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx\right.}{}\right.}{d}$
norman	$\frac{-\frac{5ax}{2}+\frac{16a}{3d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{20a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{22a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{20a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5ax\left(\tan^2\left(\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin
(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*co
s(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.56, size = 87, normalized size = 0.86

$$\frac{\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)a-2\,a\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a - 2*a*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos
(d*x + c)))/d
```

Fricas [A]
time = 0.35, size = 98, normalized size = 0.97

$$\frac{3\,a\cos\left(dx+c\right)^{4}-15\,adx\cos\left(dx+c\right)+17\,a\cos\left(dx+c\right)^{2}+\left(15\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)-4\,a}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^4 - 15*a*d*x*cos(d*x + c) + 17*a*cos(d*x + c)^2 + (15
*a*d*x*cos(d*x + c) - 3*a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 4*a)/(d*cos(
d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.45, size = 134, normalized size = 1.33

$$\frac{15\left(dx+c\right)a+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{6\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}+\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + 6*(a*tan(1/2*d*x + 1
/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(1
/2*d*x + 1/2*c)^2 + 1)^2 + (21*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x
+ 1/2*c) + 23*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 14.62, size = 243, normalized size = 2.41

$$\frac{5\,a\,x}{2}-\frac{\left(\frac{a\,\left(30\,d\,x-30\right)}{6}-5\,a\,d\,x\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(5\,a\,d\,x-\frac{a\,\left(30\,d\,x-60\right)}{6}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{a\,\left(30\,d\,x-50\right)}{6}-5\,a\,d\,x\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{20\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\left(5\,a\,d\,x-\frac{a\,\left(30\,d\,x-14\right)}{6}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{a\,\left(30\,d\,x-4\right)}{6}-5\,a\,d\,x\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(5\,a\,d\,x-\frac{a\,\left(30\,d\,x-34\right)}{6}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,\left(15\,d\,x-32\right)}{6}-\frac{5\,a\,d\,x}{2}}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (5*a*x)/2 - ((a*(15*d*x - 32))/6 - tan(c/2 + (d*x)/2)*((a*(30*d*x - 34))/6
- 5*a*d*x) + tan(c/2 + (d*x)/2)^2*((a*(30*d*x - 4))/6 - 5*a*d*x) - tan(c/2
+ (d*x)/2)^3*((a*(30*d*x - 14))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^7*((a*(30
*d*x - 30))/6 - 5*a*d*x) + tan(c/2 + (d*x)/2)^5*((a*(30*d*x - 50))/6 - 5*a*
d*x) - tan(c/2 + (d*x)/2)^6*((a*(30*d*x - 60))/6 - 5*a*d*x) + (20*a*tan(c/2
 + (d*x)/2)^4)/3 - (5*a*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d
*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.797 $\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$ax-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3
*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2789, 3554, 8, 2670, 276}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2789

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)\,dx\\ &= a\int \tan^4(c+dx)\,dx+a\int \sin(c+dx)\tan^4(c+dx)\,dx\\ &= \frac{a\tan^3(c+dx)}{3d}-a\int \tan^2(c+dx)\,dx-\frac{a\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(\right.}{d}\\ &= -\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+a\int 1\,dx-\frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\right.\right.}{}\\ &= ax-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+d}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 1.12

$$\frac{a\tan^{-1}(\tan(c+dx))}{d}-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.15, size = 98, normalized size = 1.36

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
default	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
risch	$ax-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}-\frac{4\left(-2ia+a\,e^{i(dx+c)}-3ia\,e^{2i(dx+c)}+3a\,e^{3i(dx+c)}\right)}{3\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^3 d}$

norman	$\frac{ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-ax+\frac{16a}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{14a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{14a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2ax\left(\tan^2\left(\right.\right.}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3
-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.58, size = 65, normalized size = 0.90

$$\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-a\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 88, normalized size = 1.22

$$-\frac{3\,adx\cos\left(dx+c\right)-7\,a\cos\left(dx+c\right)^2-\left(3\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^2-2\,a\right)\sin\left(dx+c\right)-a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.48, size = 124, normalized size = 1.72

$$\frac{6\,(dx+c)a-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a - 3*(a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)
 + 5*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/
2*c) + 1) + (15*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*x + 1/2*c) + 17*a
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 12.72, size = 185, normalized size = 2.57

$$a\,x+\frac{\left(2\,a\,d\,x-\frac{a\,(6\,d\,x-6)}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{a\,(3\,d\,x-12)}{3}-a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\left(a\,d\,x-\frac{a\,(3\,d\,x-4)}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(\frac{a\,(6\,d\,x-26)}{3}-2\,a\,d\,x\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a\,(3\,d\,x-16)}{3}+a\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] a*x + (tan(c/2 + (d*x)/2)*((a*(6*d*x - 26))/3 - 2*a*d*x) - (a*(3*d*x - 16))
/3 - tan(c/2 + (d*x)/2)^2*((a*(3*d*x - 4))/3 - a*d*x) - tan(c/2 + (d*x)/2)^
5*((a*(6*d*x - 6))/3 - 2*a*d*x) + tan(c/2 + (d*x)/2)^4*((a*(3*d*x - 12))/3
- a*d*x) + (4*a*tan(c/2 + (d*x)/2)^3)/3 + a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1
)^3*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.798 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=60

$$ax-\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] a*x-a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2917, 2686, 3554, 8}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] a*x - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d +
(a*Tan[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx &= a\int \sec(c+dx)\tan^3(c+dx)\,dx + a\int \tan^4(c+dx)\,dx \\
&= \frac{a\tan^3(c+dx)}{3d} - a\int \tan^2(c+dx)\,dx + \frac{a\text{Subst}\left(\int (-1+ \right.}{} \\
&= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\,\text{ta}}{} \\
&= ax - \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} +
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 69, normalized size = 1.15

$$
\frac{a\tan^{-1}(\tan(c+dx))}{d} - \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} - \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.12, size = 88, normalized size = 1.47

method	result
risch	$ax - \frac{2a\left(3\,\mathrm{e}^{3i(dx+c)}-4i+5\,\mathrm{e}^{i(dx+c)}\right)}{3\left(\mathrm{e}^{i(dx+c)}+i\right)\left(\mathrm{e}^{i(dx+c)}-i\right)^3 d}$
derivativedivides	$\frac{a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$
default	$\frac{a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$
norman	$\frac{ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-ax+\frac{4a}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{14a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{14a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+si
n(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.58, size = 55, normalized size = 0.92

$$\frac{\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-\frac{\left(3\,\cos\left(dx+c\right)^2-1\right)a}{\cos\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - (3*cos(d*x + c)^2
- 1)*a/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 75, normalized size = 1.25

$$-\frac{3\,adx\cos\left(dx+c\right)-4\,a\cos\left(dx+c\right)^2-\left(3\,adx\cos\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a}{3\,\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 4*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) + a
)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 74, normalized size = 1.23

$$\frac{6\,\left(dx+c\right)a+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*a + 3*a/(tan(1/2*d*x + 1/2*c) + 1) + (9*a*tan(1/2*d*x + 1/
2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 11*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 10.14, size = 117, normalized size = 1.95

$$a\,x-\frac{\left(\frac{a\,\left(6\,d\,x-6\right)}{3}-2\,a\,d\,x\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\left(2\,a\,d\,x-\frac{a\,\left(6\,d\,x-2\right)}{3}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\,\left(3\,d\,x-4\right)}{3}-a\,d\,x}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] a*x - ((a*(3*d*x - 4))/3 - tan(c/2 + (d*x)/2)*((a*(6*d*x - 2))/3 - 2*a*d*x)
 + tan(c/2 + (d*x)/2)^3*((a*(6*d*x - 6))/3 - 2*a*d*x) + 4*a*tan(c/2 + (d*x)
/2)^2 - a*d*x)/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1))
```

3.799 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=45

$$-\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] -a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2917, 2687, 30, 2686}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sec^2(c+dx)\tan^2(c+dx)\,dx + a\int \sec(c+dx)\tan^{3}$$
$$= \frac{a\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int (-1+x^2)\right.}{d}$$
$$= -\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 1.00

$$-\frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*Sec[c + d*x])/d) + (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.13, size = 82, normalized size = 1.82

method	result	size
risch	$-\frac{2\left(-3ia\,e^{2i(dx+c)}-a\,e^{i(dx+c)}+3a\,e^{3i(dx+c)}-ia\right)}{3\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)d}$	76
derivativedivides	$\frac{\frac{a\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)}{d}$	82
default	$\frac{\frac{a\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)}{d}$	82
norman	$\frac{\frac{4a}{3d}-\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a*sin(d*x+c)^3/cos(d*x+c)^3+a*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*s
in(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.87

$$\frac{a\tan\left(dx+c\right)^3-\frac{\left(3\,\cos\left(dx+c\right)^2-1\right)a}{\cos\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/3*(a*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.09

$$\frac{a\cos\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)+a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/3*(a*cos(d*x + c)^2 - 2*a*sin(d*x + c) + a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^2\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**3*
sec(c + d*x)**4, x))
```

Giac [A]
time = 0.46, size = 67, normalized size = 1.49

$$-\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 - 12*a*t
an(1/2*d*x + 1/2*c) + 5*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.03, size = 74, normalized size = 1.64

$$-\frac{4\,a\,\left({\sin\left(c+d\,x\right)}^2+2\,\sin\left(c+d\,x\right)+4\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\sin\left(2\,c+2\,d\,x\right)-4\right)}{3\,d\,\left(8\,{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,\sin\left(2\,c+2\,d\,x\right)-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] -(4*a*(2*sin(c + d*x) + sin(2*c + 2*d*x) + 4*sin(c/2 + (d*x)/2)^2 + sin(c +
 d*x)^2 - 4))/(3*d*(2*sin(2*c + 2*d*x) + 8*sin(c/2 + (d*x)/2)^2 - 4))
```

3.800 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=33

$$\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/3*a*sec(d*x+c)^3/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$, Rules used = {2917, 2686, 30, 2687}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = a\int \sec^3(c+dx)\tan(c+dx)\,dx + a\int \sec^2(c+dx)\tan^2($$
$$= \frac{a\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int x^2\,dx, x, \tan\right.}{d}$$
$$= \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$\frac{a\sec^3(c+dx)}{3d} + \frac{a\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] (a*Sec[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.10, size = 36, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\frac{a}{3\cos(dx+c)^3}+\frac{a\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$	36
default	$\frac{\frac{a}{3\cos(dx+c)^3}+\frac{a\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$	36
risch	$-\frac{2i\left(-2ia\,e^{i(dx+c)}-a+3a\,e^{2i(dx+c)}\right)}{3\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)d}$	64
norman	$\frac{-\frac{2a}{3d}-\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a/cos(d*x+c)^3+1/3*a*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 26, normalized size = 0.79

$$\frac{a\tan\left(dx+c\right)^3+\frac{a}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(a*tan(d*x + c)^3 + a/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.34, size = 50, normalized size = 1.52

$$\frac{a\cos\left(dx+c\right)^{2}+a\sin\left(dx+c\right)-2\,a}{3\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(d*x + c)^2 + a*sin(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c)
- d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{4}\left(c+dx\right)dx+\int\sin^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**4, x))
```

Giac [A]
time = 0.42, size = 53, normalized size = 1.61

$$\frac{\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a/(tan(1/2*d*x + 1/2*c) + 1) - (3*a*tan(1/2*d*x + 1/2*c)^2 + a)/(tan
(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.03, size = 50, normalized size = 1.52

$$-\frac{2\,a\,\left(\cos\left(c+d\,x\right)+1\right)\,\left(\cos\left(c+d\,x\right)+\sin\left(c+d\,x\right)-2\right)}{3\,d\,\left(2\,\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] -(2*a*(cos(c + d*x) + 1)*(cos(c + d*x) + sin(c + d*x) - 2))/(3*d*(2*cos(c +
 d*x) - sin(2*c + 2*d*x)))
```

3.801 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=68

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d+a*tan(d*x+c)/d
+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2917, 2702, 308, 213, 3852}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3
*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \sec^4(c+dx)\,dx + a\int \csc(c+dx)\sec^4(c+dx)\,dx$$
$$= \frac{a\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} - \frac{a\text{Subst}\left(\int (1+x^2)\right.}{}$$
$$= \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d} + \frac{a\text{Subst}\left(\int \left(1+x^2+\right.\right.}{}$$
$$= \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx}{3d}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 1.25

$$-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A]
time = 0.20, size = 64, normalized size = 0.94

method	result
derivativedivides	$\frac{a\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$

default	$\frac{a\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)}{d}$
risch	$\frac{2a\left(-6i\,\mathrm{e}^{2i(dx+c)}+3\,\mathrm{e}^{3i(dx+c)}-2i+\mathrm{e}^{i(dx+c)}\right)}{3\left(\mathrm{e}^{i(dx+c)}+i\right)\left(\mathrm{e}^{i(dx+c)}-i\right)^3 d}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}$
norman	$\frac{-\frac{8a}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))-a*(-2/3-1/
3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 73, normalized size = 1.07

$$\frac{2\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a+a\left(\frac{2\left(3\cos\left(dx+c\right)^2+1\right)}{\cos\left(dx+c\right)^3}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + a*(2*(3*cos(d*x + c)^2 + 1)/co
s(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 126, normalized size = 1.85

$$-\frac{4\,a\cos\left(dx+c\right)^2+3\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)-a\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,a\sin\left(dx+c\right)+4\,a}{6\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)*sin(d*x + c) - a*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*a*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.45, size = 81, normalized size = 1.19

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a/(tan(1/2*d*x + 1/2*c) + 1) -
(15*a*tan(1/2*d*x + 1/2*c)^2 - 24*a*tan(1/2*d*x + 1/2*c) + 13*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.94, size = 90, normalized size = 1.32

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-\frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{8\,a}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - ((8*a)/3 - (10*a*tan(c/2 + (d*x)/2))/3 + 2*
a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^3 +
 tan(c/2 + (d*x)/2)^4 - 1))
```

3.802 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d
+2*a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2700, 276, 2702, 308, 213}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Sec[c + d*x]^3)/(3*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^4(c+dx)\,dx + a\int \csc^2(c+dx)\sec^4($$
$$= \frac{a\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\right.}{}$$
$$= \frac{a\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, \tan(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{2a\,\text{t}}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} - \frac{a\cot(c+dx)}{d} + \frac{a\sec(c+dx}{d}$$

Mathematica [A]
time = 0.05, size = 109, normalized size = 1.35

$$-\frac{a\cot(c+dx)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\sec(c+dx)}{d} + \frac{a\sec^3(c+dx)}{3d} + \frac{5a\tan(c+dx)}{3d} + \frac{a\sec^2(c+dx)\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/
2]])/d + (a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d) + (5*a*Tan[c + d*x])
/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*d)
```

Maple [A]
time = 0.19, size = 90, normalized size = 1.11

method	result
derivativedivides	$\frac{a\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+a\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+a\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
risch	$\frac{-\frac{4a\,e^{3i(dx+c)}}{3}-4ia\,e^{4i(dx+c)}-\frac{26a\,e^{i(dx+c)}}{3}+\frac{16ia}{3}-\frac{4ia\,e^{2i(dx+c)}}{3}+2a\,e^{5i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^3 d}-\frac{a\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a\ln\left(e^{i(dx+c)}-\right.}{d}$
norman	$\frac{\frac{a}{2d}-\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{7a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{11a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{4a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c
))+a*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 83, normalized size = 1.02

$$\frac{2\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a+a\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(3*cos(d
*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c
) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(77) = 154.
time = 0.37, size = 170, normalized size = 2.10

$$-\frac{10\,a\cos(dx+c)^2+3\left(a\cos(dx+c)^3+a\cos(dx+c)\sin(dx+c)-a\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(a\cos(dx+c)^3+a\cos(dx+c)\sin(dx+c)-a\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(8\,a\cos(dx+c)^2-a\right)\sin(dx+c)-4\,a}{6\left(d\cos(dx+c)^3+d\cos(dx+c)\sin(dx+c)-d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^3 + a*cos(d*x + c)*sin(d*x +
c) - a*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c)^3 + a*
cos(d*x + c)*sin(d*x + c) - a*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(8*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.46, size = 129, normalized size = 1.59

$$\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\,a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*a*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - 3*(a*t
an(1/2*d*x + 1/2*c)^2 + 3*a*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 + tan(1/2*d*x + 1/2*c)) - (21*a*tan(1/2*d*x + 1/2*c)^2 - 36*a*tan(1/2*d*
x + 1/2*c) + 19*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.10, size = 145, normalized size = 1.79

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a}{d\left(-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (a*log(tan(c/2 + (d*x)/2)))/d - (a - (22*a*t
an(c/2 + (d*x)/2))/3 + (8*a*tan(c/2 + (d*x)/2)^2)/3 + 10*a*tan(c/2 + (d*x)/
2)^3 - 9*a*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*
x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5))
```

3.803 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=110

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec(c+dx)}{2d}+\frac{5a\sec^3(c+dx)}{6d}-\frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d}+\frac{2a}{}$$

```
[Out] -5/2*a*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+5/2*a*sec(d*x+c)/d+5/6*a*sec(d*
x+c)^3/d-1/2*a*csc(d*x+c)^2*sec(d*x+c)^3/d+2*a*tan(d*x+c)/d+1/3*a*tan(d*x+c
)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2702, 294, 308, 213, 2700, 276}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{2a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{5a\sec^3(c+dx)}{6d}+\frac{5a\sec(c+dx)}{2d}-\frac{5a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x])/d + (5*a*Sec[c + d*x]
)/(2*d) + (5*a*Sec[c + d*x]^3)/(6*d) - (a*Csc[c + d*x]^2*Sec[c + d*x]^3)/(2
*d) + (2*a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^4(c+dx)\,dx + a\int \csc^3(c+dx)\sec^4$$

$$= \frac{a\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{a\text{Subst}\left(\int \frac{(1+x}{x^2}\right.}{}$$

$$= -\frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{a\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\right.}{d}$$

$$= -\frac{a\cot(c+dx)}{d} - \frac{a\csc^2(c+dx)\sec^3(c+dx)}{2d} + \frac{2a\tan(}{}$$

$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+dx)}{2d} + \frac{5a\sec^3(c+dx)}{6d} - $$

$$= -\frac{5a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)}{d} + \frac{5a\sec(c+}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(110) = 220.

time = 6.08, size = 359, normalized size = 3.26

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{5a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{5a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{13a\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{a}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{13a\sin\left(\frac{1}{2}(c+dx)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{5a\tan(c+dx)}{3d}+\frac{a\sec^2(c+dx)\tan(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (5*a*Log[Cos[(c + d*
x)/2]])/(2*d) + (5*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/
(8*d) + a/(12*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (a*Sin[(c + d*x)
/2])/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (13*a*Sin[(c + d*x)/2]
)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (a*Sin[(c + d*x)/2])/(6*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + a/(12*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2) - (13*a*Sin[(c + d*x)/2])/(6*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (5*a*Tan[c + d*x])/(3*d) + (a*Sec[c + d*x]^2*Tan[c + d*x])/(3*
d)
```

Maple [A]
time = 0.24, size = 120, normalized size = 1.09

method	result
derivativedivides	$\frac{a\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+3\right.}{d}$
default	$\frac{a\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+3\right.}{d}$
risch	$\frac{a\left(-30i e^{6i(dx+c)}+15\,e^{7i(dx+c)}+20i e^{4i(dx+c)}-25\,e^{5i(dx+c)}+2i e^{2i(dx+c)}-7\,e^{3i(dx+c)}-16i+17\,e^{i(dx+c)}\right)}{3\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)d}-\frac{5a\ln(e^{i(dx}}{2d}$
norman	$\frac{\frac{a}{8d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{5a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{11a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos(d
*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+a*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin
(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c)))
```

Maxima [A]
time = 0.28, size = 106, normalized size = 0.96

$$\frac{4\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a+a\left(\frac{2\left(15\cos(dx+c)^4-10\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a + a*(2*(15*cos
(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(100) = 200$.
time = 0.37, size = 222, normalized size = 2.02

$$\frac{32\,a\cos\left(dx+c\right)^{4}-18\,a\cos\left(dx+c\right)^{2}-15\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)-8\,a}{12\,\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(32*a*cos(d*x + c)^4 - 18*a*cos(d*x + c)^2 - 15*(a*cos(d*x + c)^3 - a*
cos(d*x + c) - (a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c) - (a*cos(d*x + c)
^3 - a*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(a*cos(
d*x + c)^2 + 2*a)*sin(d*x + c) - 8*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c) -
(d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 148, normalized size = 1.35

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+60\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{12\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}-\frac{3\,\left(30\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-\frac{4\,\left(27\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*tan(1/2*d*x + 1/2*c)^2 + 60*a*log(abs(tan(1/2*d*x + 1/2*c))) + 12
*a*tan(1/2*d*x + 1/2*c) + 12*a/(tan(1/2*d*x + 1/2*c) + 1) - 3*(30*a*tan(1/2
*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c)^2 - 4*
(27*a*tan(1/2*d*x + 1/2*c)^2 - 48*a*tan(1/2*d*x + 1/2*c) + 25*a)/(tan(1/2*d
*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 8.98, size = 180, normalized size = 1.64

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{-18\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{23\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+\frac{67\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-\frac{68\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a}{2}}{d\left(-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + a*tan(c/2 + (d*x)/2) - (68*a*tan(c/2
+ (d*x)/2)^2)/3 + (67*a*tan(c/2 + (d*x)/2)^3)/3 + (23*a*tan(c/2 + (d*x)/2)^
4)/2 - 18*a*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 +
(d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5 - 4*tan(c/2 + (d*x)/2)^6)) + (a*tan(c/2
 + (d*x)/2)^2)/(8*d) + (5*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.804 $\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{7a^2x}{2}-\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 7/2*a^2*x-2*a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))^2-11/3*a^2
*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 120, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2787, 2844, 3056, 2813}

$$\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
```

```
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c}{a-a\sin(c+dx)}$$
$$= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}$$
$$= \frac{7a^2x}{2} - \frac{16a^2\cos(c+dx)}{3d} - \frac{7a^2\cos(c+dx)\sin(c+dx)}{2d} - \frac{8a^2\cos(}{3d(}$$

Mathematica [A]
time = 0.89, size = 159, normalized size = 1.57

$$-\frac{a^2\left(-21(7+12c+12dx)\cos\left(\frac{1}{2}(c+dx)\right)+(239+84c+84dx)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx)\right)+2(50+56c+56dx+(-27+28c+28dx)\cos(c+dx)-6\cos(2(c+dx))-\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/48*(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x
)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2]
+ 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c +
 d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^3)
```

Maple [A]
time = 0.17, size = 186, normalized size = 1.84

method	result
risch	$\frac{7a^2x}{2}+\frac{ia^2 e^{2i(dx+c)}}{8d}-\frac{a^2 e^{i(dx+c)}}{d}-\frac{a^2 e^{-i(dx+c)}}{d}-\frac{ia^2 e^{-2i(dx+c)}}{8d}-\frac{2(-21ia^2 e^{i(dx+c)}+12a^2 e^{2i(dx+c)}-11a^2)}{3(e^{i(dx+c)}-i)^3 d}$
derivativedivides	$\frac{a^2\left(\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+dx+c\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4(\sin^2(dx+c))}{3}\right)\cos(dx+c)\right)+}{d}$
default	$\frac{a^2\left(\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+dx+c\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4(\sin^2(dx+c))}{3}\right)\cos(dx+c)\right)+}{d}$
norman	$\frac{-\frac{7a^2x}{2}+\frac{32a^2}{3d}+\frac{7a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{28a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{50a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{28a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{7a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7a}{}}{(\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a^2*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin
(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.51, size = 120, normalized size = 1.19

$$\frac{\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^2+2\left(\tan(dx+c)^3+3\,dx+3\,c-3\tan(dx+c)\right)a^2-4\,a^2\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos(dx+c)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(89) = 178.
time = 0.36, size = 196, normalized size = 1.94

$$\frac{3\,a^2\cos(dx+c)^4-6\,a^2\cos(dx+c)^3-42\,a^2dx+(21\,a^2dx+31\,a^2)\cos(dx+c)^2-2\,a^2-(21\,a^2dx-38\,a^2)\cos(dx+c)-\left(3\,a^2\cos(dx+c)^3-42\,a^2dx+9\,a^2\cos(dx+c)^2+2\,a^2-(21\,a^2dx-40\,a^2)\cos(dx+c)\right)\sin(dx+c)}{6\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.45, size = 135, normalized size = 1.34

$$\frac{21\,(dx+c)a^2+\frac{6\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{4\left(9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(21*(d*x + c)*a^2 + 6*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x +
 1/2*c)^2 - a^2*tan(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(9*a^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 10*a^2)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 14.69, size = 287, normalized size = 2.84

$$\frac{7\,a^2\,x}{2}+\frac{\frac{7\,a^2\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{21\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(63\,c+63\,d\,x-150)}{6}\right)-\frac{a^2\,(21\,c+21\,d\,x-64)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\left(\frac{21\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(63\,c+63\,d\,x-42)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\left(\frac{35\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(105\,c+105\,d\,x-126)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\left(\frac{35\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(105\,c+105\,d\,x-194)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\left(\frac{49\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(147\,c+147\,d\,x-196)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\left(\frac{49\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(147\,c+147\,d\,x-252)}{6}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] (7*a^2*x)/2 + ((7*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((21*a^2*(c + d*x))
/2 - (a^2*(63*c + 63*d*x - 150))/6) - (a^2*(21*c + 21*d*x - 64))/6 + tan(c/
2 + (d*x)/2)^6*((21*a^2*(c + d*x))/2 - (a^2*(63*c + 63*d*x - 42))/6) - tan(
c/2 + (d*x)/2)^5*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 126))/6) +
 tan(c/2 + (d*x)/2)^2*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 194))
/6) + tan(c/2 + (d*x)/2)^4*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d*x -
196))/6) - tan(c/2 + (d*x)/2)^3*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*d
*x - 252))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.805 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=86

$$2a^2x-\frac{4a^2\cos(c+dx)}{3d}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] 2*a^2*x-4/3*a^2*cos(d*x+c)/d-2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(
d*x+c)*sin(d*x+c)^2/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.17, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.259$, Rules used = {2948, 2844, 3047, 3102, 12, 2814, 2727}

$$\frac{a^4\sin^2(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{4a^2\cos(c+dx)}{3d}-\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+2a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] 2*a^2*x - (4*a^2*Cos[c + d*x])/(3*d) - (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c +
 d*x])) + (a^4*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = a^4\int \frac{\sin^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\sin(c+dx)(-2a-}{a-a\sin(}$$
$$= \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{-2a\sin(c+dx)-}{a-a\sin(}$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}a\int$$
$$= -\frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} - (2a^3$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} -$$
$$= 2a^2x - \frac{4a^2\cos(c+dx)}{3d} + \frac{a^4\cos(c+dx)\sin^2(c+dx)}{3d(a-a\sin(c+dx))^2} -$$

Mathematica [A]
time = 0.75, size = 131, normalized size = 1.52

$$\frac{a^2(1+\sin(c+dx))^2\left(6c+6dx-3\cos(c+dx)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-7+8\sin(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(6*c + 6*d*x - 3*Cos[c + d*x] + (Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-7 + 8*Sin[c + d*x]))/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^4)
```

Maple [A]
time = 0.17, size = 162, normalized size = 1.88

method	result
risch	$2a^2x-\frac{a^2e^{i(dx+c)}}{2d}-\frac{a^2e^{-i(dx+c)}}{2d}-\frac{2a^2\left(-15ie^{i(dx+c)}+9e^{2i(dx+c)}-8\right)}{3d\left(e^{i(dx+c)}-i\right)^3}$
derivativedivides	$\frac{a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+2a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\right.}{d}$

default	$\frac{a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+2a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
norman	$-2a^2x+\frac{20a^2}{3d}+\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{56a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+2*a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+a^2*(1/
3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*s
in(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.49, size = 95, normalized size = 1.10

$$\frac{2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2-a^2\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-\frac{\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 - a^2*((6*cos(d*
x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - (3*cos(d*x + c)^2 - 1)*a^2
/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(81) = 162.
time = 0.34, size = 168, normalized size = 1.95

$$-\frac{3\,a^2\cos\left(dx+c\right)^3+12\,a^2dx-\left(6\,a^2dx+11\,a^2\right)\cos\left(dx+c\right)^2+a^2+\left(6\,a^2dx-13\,a^2\right)\cos\left(dx+c\right)-\left(12\,a^2dx-3\,a^2\cos\left(dx+c\right)^2-a^2+2\left(3\,a^2dx-7\,a^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^2*cos(d*x + c)^3 + 12*a^2*d*x - (6*a^2*d*x + 11*a^2)*cos(d*x + c)
^2 + a^2 + (6*a^2*d*x - 13*a^2)*cos(d*x + c) - (12*a^2*d*x - 3*a^2*cos(d*x
+ c)^2 - a^2 + 2*(3*a^2*d*x - 7*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.48, size = 86, normalized size = 1.00

$$\frac{2\left(3\,(dx+c)a^2-\frac{3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(d*x + c)*a^2 - 3*a^2/(tan(1/2*d*x + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*
d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 7*a^2)/(tan(1/2*d*x + 1/2*c)
 - 1)^3)/d
```

Mupad [B]
time = 12.16, size = 182, normalized size = 2.12

$$2\,a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,a^2\,\left(9\,d\,x-24\right)}{3}-6\,a^2\,d\,x\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{2\,a^2\,\left(9\,d\,x-6\right)}{3}-6\,a^2\,d\,x\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{2\,a^2\,\left(12\,d\,x-18\right)}{3}-8\,a^2\,d\,x\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{2\,a^2\,\left(12\,d\,x-22\right)}{3}-8\,a^2\,d\,x\right)-\frac{2\,a^2\,\left(3\,d\,x-10\right)}{3}+2\,a^2\,d\,x}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] 2*a^2*x + (tan(c/2 + (d*x)/2)*((2*a^2*(9*d*x - 24))/3 - 6*a^2*d*x) - tan(c/
2 + (d*x)/2)^4*((2*a^2*(9*d*x - 6))/3 - 6*a^2*d*x) + tan(c/2 + (d*x)/2)^3*(
(2*a^2*(12*d*x - 18))/3 - 8*a^2*d*x) - tan(c/2 + (d*x)/2)^2*((2*a^2*(12*d*x
 - 22))/3 - 8*a^2*d*x) - (2*a^2*(3*d*x - 10))/3 + 2*a^2*d*x)/(d*(tan(c/2 +
(d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.806 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=63

$$a^2x-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] a^2*x-5/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(d*x+c)/d/(a-a*sin(d*x
+c))^2
```

Rubi [A]
time = 0.15, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2948, 2837, 2814, 2727}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{5a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+a^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] a^2*x - (5*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) + (a^4*Cos[c + d*x])/
(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = a^4\int \frac{\sin^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{-2a-3a\sin(c+dx)}{a-a\sin(c+dx)}$$
$$= a^2x + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{1}{3}(5a^3)\int \frac{1}{a-a\sin(c+}$$
$$= a^2x + \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2} - \frac{5a^3\cos(c+dx)}{3d(a-a\sin(c+dx))}$$

Mathematica [A]
time = 0.04, size = 79, normalized size = 1.25

$$\frac{a^2\tan^{-1}(\tan(c+dx))}{d} - \frac{2a^2\sec(c+dx)}{d} + \frac{2a^2\sec^3(c+dx)}{3d} - \frac{a^2\tan(c+dx)}{d} + \frac{2a^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
[Out] (a^2*ArcTan[Tan[c + d*x]])/d - (2*a^2*Sec[c + d*x])/d + (2*a^2*Sec[c + d*x]
^3)/(3*d) - (a^2*Tan[c + d*x])/d + (2*a^2*Tan[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.16, size = 114, normalized size = 1.81

method	result
risch	$a^2x - \frac{2\left(-9ia^2e^{i(dx+c)}+6a^2e^{2i(dx+c)}-5a^2\right)}{3\left(e^{i(dx+c)}-i\right)^3 d}$
derivativedivides	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$
default	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+2a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$

norman	$\frac{a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^2x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2x+\frac{8a^2}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{44a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{1}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+2*a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(
d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.51, size = 71, normalized size = 1.13

$$\frac{a^2\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^2-\frac{2\left(3\,\cos(dx+c)^2-1\right)a^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a
^2 - 2*(3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(58) = 116$.
time = 0.36, size = 141, normalized size = 2.24

$$-\frac{6\,a^2dx-\left(3\,a^2dx+5\,a^2\right)\cos\left(dx+c\right)^2+a^2+\left(3\,a^2dx-4\,a^2\right)\cos\left(dx+c\right)-\left(6\,a^2dx-a^2+\left(3\,a^2dx-5\,a^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(6*a^2*d*x - (3*a^2*d*x + 5*a^2)*cos(d*x + c)^2 + a^2 + (3*a^2*d*x - 4
*a^2)*cos(d*x + c) - (6*a^2*d*x - a^2 + (3*a^2*d*x - 5*a^2)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*si
n(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 67, normalized size = 1.06

$$\frac{3\,(dx+c)a^2+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^2 + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*tan(1/2*d*x
+ 1/2*c) + 4*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.29, size = 102, normalized size = 1.62

$$a^2\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2\,(9\,d\,x-18)}{3}-3\,a^2\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2\,(9\,d\,x-6)}{3}-3\,a^2\,d\,x\right)-\frac{a^2\,(3\,d\,x-8)}{3}+a^2\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] a^2*x + (tan(c/2 + (d*x)/2)*((a^2*(9*d*x - 18))/3 - 3*a^2*d*x) - tan(c/2 +
(d*x)/2)^2*((a^2*(9*d*x - 6))/3 - 3*a^2*d*x) - (a^2*(3*d*x - 8))/3 + a^2*d*
x)/(d*(tan(c/2 + (d*x)/2) - 1)^3)
```

3.807 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=60

$$-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d}-\frac{2a^2\tan(c+dx)}{3d}$$

```
[Out] -2/3*a^2*sec(d*x+c)/d+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^2/d-2/3*a^2*tan(d*x
+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2934, 2748, 3852, 8}

$$-\frac{2a^2\tan(c+dx)}{3d}-\frac{2a^2\sec(c+dx)}{3d}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Sec[c + d*x])/(3*d) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2)/(3*d)
 - (2*a^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} - \frac{1}{3}(2a)\int \sec^2(c+dx$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} -$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} +$$
$$= -\frac{2a^2\sec(c+dx)}{3d} + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} -$$

Mathematica [A]
time = 0.21, size = 72, normalized size = 1.20

$$\frac{a^2\left(3\cos\left(\frac{1}{2}(c+dx)\right)-2\cos\left(\frac{3}{2}(c+dx)\right)-3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
[Out] (a^2*(3*Cos[(c + d*x)/2] - 2*Cos[(3*(c + d*x))/2] - 3*Sin[(c + d*x)/2]))/(3
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [A]
time = 0.14, size = 99, normalized size = 1.65

method	result
risch	$-\frac{2a^2\left(-3i\,e^{i(dx+c)}+3\,e^{2i(dx+c)}-2\right)}{3d\left(e^{i(dx+c)}-i\right)^3}$
derivativedivides	$\frac{\frac{a^2}{3\cos(dx+c)^3}+\frac{2a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)}{d}$
default	$\frac{\frac{a^2}{3\cos(dx+c)^3}+\frac{2a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+a^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)}{d}$
norman	$\frac{\frac{2a^2}{3d}-\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{32a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^2\left(\tan^2\right.}{3}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^2/cos(d*x+c)^3+2/3*a^2*sin(d*x+c)^3/cos(d*x+c)^3+a^2*(1/3*sin(d*
x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*
x+c)))
```

Maxima [A]
time = 0.28, size = 56, normalized size = 0.93

$$\frac{2\,a^2\tan\left(dx+c\right)^3-\frac{\left(3\,\cos\left(dx+c\right)^2-1\right)a^2}{\cos\left(dx+c\right)^3}+\frac{a^2}{\cos\left(dx+c\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*tan(d*x + c)^3 - (3*cos(d*x + c)^2 - 1)*a^2/cos(d*x + c)^3 + a^2
/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.35, size = 98, normalized size = 1.63

$$\frac{2\,a^2\cos\left(dx+c\right)^2+a^2\cos\left(dx+c\right)-a^2-\left(2\,a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) - a^2 - (2*a^2*cos(d*x + c) +
a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2
*d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sin\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int 2\sin^2\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(sin(c + d*x)*sec(c + d*x)**4, x) + Integral(2*sin(c + d*x)**
2*sec(c + d*x)**4, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.42, size = 38, normalized size = 0.63

$$-\frac{2\left(3\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
[Out] -2/3*(3*a^2*tan(1/2*d*x + 1/2*c) - a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

Mupad [B]
time = 9.14, size = 34, normalized size = 0.57

$$-\frac{2\,a^{2}\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
[Out] -(2*a^2*(3*tan(c/2 + (d*x)/2) - 1))/(3*d*(tan(c/2 + (d*x)/2) - 1)^3)
```

3.808 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=73

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+4/3*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cos(
d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2948, 2845, 3057, 12, 3855}

$$\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*ArcTanh[Cos[c + d*x]])/d) + (4*a^2*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])) + (a^4*Cos[c + d*x])/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx &= a^4\int \frac{\csc(c+dx)}{(a-a\sin(c+dx))^2}\,dx\\ &= \frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{\csc(c+dx)(3a+a\sin(}{a-a\sin(c+dx}\\ &= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}\int 3a\\ &= \frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)}{3d(a-a\sin(c+dx))^2}+a^2\int \mathrm{cs}\\ &= -\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^2\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{a}{3d(a}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 142, normalized size = 1.95

$$\frac{a^2(1+\sin(c+dx))^2\left(-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{1}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-5+4\sin(c+dx))}{\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2) + (2*Sin[(c + d*x)/2]*(-5 +
```

```
4*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A]
time = 0.24, size = 81, normalized size = 1.11

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^2}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-2a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^2}{3\cos(dx+c)^3}}{d}$
risch	$\frac{2a^2\left(-9i\mathrm{e}^{i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}-4\right)}{3d\left(\mathrm{e}^{i(dx+c)}-i\right)^3}+\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}$
norman	$\frac{-\frac{10a^2}{3d}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{16a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{16a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2\left(\tan^4\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))-2*a^2*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*a^2/cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 90, normalized size = 1.23

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^2+a^2\left(\frac{2\left(3\cos\left(dx+c\right)^2+1\right)}{\cos\left(dx+c\right)^3}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2a^2}{\cos\left(dx+c\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1
)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a
^2/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(68) = 136$.
time = 0.37, size = 231, normalized size = 3.16

$$-\frac{8a^2\cos\left(dx+c\right)^2+10a^2\cos\left(dx+c\right)+2a^2+3\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2a^2+\left(a^2\cos\left(dx+c\right)+2a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2a^2+\left(a^2\cos\left(dx+c\right)+2a^2\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\left(4a^2\cos\left(dx+c\right)-a^2\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2d\right)\sin\left(dx+c\right)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*a^2*cos(d*x + c)^2 + 10*a^2*cos(d*x + c) + 2*a^2 + 3*(a^2*cos(d*x +
 c)^2 - a^2*cos(d*x + c) - 2*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))
*log(1/2*cos(d*x + c) + 1/2) - 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2
*a^2 + (a^2*cos(d*x + c) + 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(4*a^2*cos(d*x + c) - a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.47, size = 73, normalized size = 1.00

$$\frac{3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(6*a^2*tan(1/2*d*x + 1/2*c)^2
 - 9*a^2*tan(1/2*d*x + 1/2*c) + 5*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.50, size = 98, normalized size = 1.34

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-6\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{10\,a^2}{3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (4*a^2*tan(c/2 + (d*x)/2)^2 + (10*a^2)/3
- 6*a^2*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)
^2 + tan(c/2 + (d*x)/2)^3 - 1))
```

3.809 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{10a^2\cot(c+dx)}{3d}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}+\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}$$

```
[Out] -2*a^2*arctanh(cos(d*x+c))/d-10/3*a^2*cot(d*x+c)/d+2*a^2*cot(d*x+c)/d/(1-si
n(d*x+c))+1/3*a^4*cot(d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2948, 2845, 3057, 2827, 3852, 8, 3855}

$$\frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{10a^2\cot(c+dx)}{3d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*ArcTanh[Cos[c + d*x]])/d - (10*a^2*Cot[c + d*x])/(3*d) + (2*a^2*Cot
[c + d*x])/(d*(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x])/(3*d*(a - a*Sin[c +
d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^2(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^2(c+dx)(4a+2a}{a-a\sin(c+}$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}\int \mathrm{cs}$$
$$= \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)}{3d(a-a\sin(c+dx))^2} + (2a^2)$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cot(c+dx)}{d(1-\sin(c+dx))} + \frac{}{3d}$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{10a^2\cot(c+dx)}{3d} + \frac{2a}{d(1}$$

Mathematica [A]

time = 0.65, size = 135, normalized size = 1.55

$$\frac{a^2\left(-3\cot\left(\frac{1}{2}(c+dx)\right)-12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)(-8+7\sin(c+dx))}{\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-3*Cot[(c + d*x)/2] - 12*Log[Cos[(c + d*x)/2]] + 12*Log[Sin[(c + d*x)
/2]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-8
+ 7*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c + d*
x)/2]))/(6*d)
```

Maple [A]

time = 0.25, size = 118, normalized size = 1.36

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c)\right.}{d}$
default	$\frac{a^2\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2a^2\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c)\right.}{d}$
risch	$\frac{-\frac{44a^2e^{2i(dx+c)}}{3}-12ie^{3i(dx+c)}a^2+\frac{20a^2}{3}+16ia^2e^{i(dx+c)}+4a^2e^{4i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)^3d}-\frac{2a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{2a^2\ln\left(e^{i(dx+c)}-1\right)}{d}$

norman	$\frac{\frac{a^2}{2d}-\frac{7a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{35a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{10a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{35a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{7a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\left(\tan^{12}\right.}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x
+c))+2*a^2*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))-a^2*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.31, size = 107, normalized size = 1.23

$$\frac{\left(\tan\left(dx+c\right)^3-\frac{3}{\tan\left(dx+c\right)}+6\,\tan\left(dx+c\right)\right)a^2+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2+a^2\left(\frac{2\left(3\,\cos\left(dx+c\right)^2+1\right)}{\cos\left(dx+c\right)^3}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + (tan(d*x + c)
^3 + 3*tan(d*x + c))*a^2 + a^2*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(82) = 164$.
time = 0.37, size = 329, normalized size = 3.78

$$\frac{10\,a^2\cos\left(dx+c\right)^3-4\,a^2\cos\left(dx+c\right)^2-13\,a^2\cos\left(dx+c\right)+a^2-3\left(a^2\cos\left(dx+c\right)^3+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2\,a^2-\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2\,a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a^2\cos\left(dx+c\right)^3+2\,a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2\,a^2-\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)-2\,a^2\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(10\,a^2\cos\left(dx+c\right)^2+14\,a^2\cos\left(dx+c\right)+a^2\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)-2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 13*a^2*cos(d*x + c) + a
^2 - 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^
2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) + 3*(a^2*cos(d*x + c)^3 + 2*a^2*cos(d*x + c)^2 - a^2*cos
(d*x + c) - 2*a^2 - (a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2) + (10*a^2*cos(d*x + c)^2 + 14*a^2*cos(d
*x + c) + a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos
(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 118, normalized size = 1.36

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,(4\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+a^2)}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)}-\frac{4\left(9\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-15\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+8\,a^2\right)}{(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - 3
*(4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) - 4*(9*a^2*tan(1/2
*d*x + 1/2*c)^2 - 15*a^2*tan(1/2*d*x + 1/2*c) + 8*a^2)/(tan(1/2*d*x + 1/2*c
) - 1)^3)/d
```

Mupad [B]
time = 9.45, size = 144, normalized size = 1.66

$$\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-13\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+23\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{41\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a^2}{d\,\left(-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (2*a^2*log(tan(c/2 + (d*x)/2)))/d - (23*a^2*tan(c/2 + (d*x)/2)^2 - 13*a^2*t
an(c/2 + (d*x)/2)^3 + a^2 - (41*a^2*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 +
(d*x)/2) - 6*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d
*x)/2)^4)) + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

3.810 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cdots}{3\cdots}$$

```
[Out] -7/2*a^2*arctanh(cos(d*x+c))/d-16/3*a^2*cot(d*x+c)/d-7/2*a^2*cot(d*x+c)*csc
(d*x+c)/d+8/3*a^2*cot(d*x+c)*csc(d*x+c)/d/(1-sin(d*x+c))+1/3*a^4*cot(d*x+c)
*csc(d*x+c)/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2948, 2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{16a^2\cot(c+dx)}{3d}-\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d}-\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d}+\frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (16*a^2*Cot[c + d*x])/(3*d) - (7*a^2
*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (8*a^2*Cot[c + d*x]*Csc[c + d*x])/(3*d*
(1 - Sin[c + d*x])) + (a^4*Cot[c + d*x]*Csc[c + d*x])/(3*d*(a - a*Sin[c + d
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2948

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^4\int \frac{\csc^3(c+dx)}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^4\cot(c+dx)\csc(c+dx)}{3d(a-a\sin(c+dx))^2} + \frac{1}{3}a^2\int \frac{\csc^3(c+dx)(5a+}{a-a\sin(c}$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx}{3d(a-a\sin(c+dx))^2}$$
$$= \frac{8a^2\cot(c+dx)\csc(c+dx)}{3d(1-\sin(c+dx))} + \frac{a^4\cot(c+dx)\csc(c+dx}{3d(a-a\sin(c+dx))^2}$$
$$= -\frac{7a^2\cot(c+dx)\csc(c+dx)}{2d} + \frac{8a^2\cot(c+dx)\csc(c+}{3d(1-\sin(c+dx)}$$
$$= -\frac{7a^2\tanh^{-1}(\cos(c+dx))}{2d} - \frac{16a^2\cot(c+dx)}{3d} - \frac{7a^2\,\mathrm{co}}{}$$

Mathematica [A]

time = 1.37, size = 190, normalized size = 1.52

$$\frac{a^2\left(-24\cot\left(\frac{1}{2}(c+dx)\right)-3\csc^2\left(\frac{1}{2}(c+dx)\right)-84\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+84\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{8}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{160\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+24\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-24*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 84*Log[Cos[(c + d*x)/2]
] + 84*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 8/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^3 + (160*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) +
24*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A]

time = 0.30, size = 164, normalized size = 1.31

method	result
risch	$\frac{a^2\left(-63i\mathrm{e}^{5i(dx+c)}+21\,\mathrm{e}^{6i(dx+c)}+126i\mathrm{e}^{3i(dx+c)}-98\,\mathrm{e}^{4i(dx+c)}-75i\mathrm{e}^{i(dx+c)}+97\,\mathrm{e}^{2i(dx+c)}-32\right)}{3\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}-i\right)^3 d}-\frac{7a^2\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{2d}+$
derivativedivides	$\frac{a^2\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\right.}{d}$
default	$\frac{a^2\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2a^2\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\right.}{d}$
norman	$\frac{\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2}{8d}-\frac{10a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{19a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{28a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{19a^2\left(\tan^9\left(\frac{dx}{2}\right.\right.}{3d}}{\tan\left(\frac{dx}{2}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos
(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+2*a^2*(1/3/sin(d*x+c)/cos(d*x+c)^3+4
/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+a^2*(1/3/cos(d*x+c)^3+1/cos(d*x+c)
+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 160, normalized size = 1.28

$$\frac{8\left(\tan\left(dx+c\right)^{3}-\frac{3}{\tan\left(dx+c\right)}+6\,\tan\left(dx+c\right)\right)a^{2}+a^{2}\left(\frac{2\left(15\,\cos\left(dx+c\right)^{4}-10\,\cos\left(dx+c\right)^{2}-2\right)}{\cos\left(dx+c\right)^{5}-\cos\left(dx+c\right)^{3}}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+2\,a^{2}\left(\frac{2\left(3\,\cos\left(dx+c\right)^{2}+1\right)}{\cos\left(dx+c\right)^{3}}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(8*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^2 + a^2*(2*(15
*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3)
- 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^2*(2*(3*cos(d*
x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c)
 - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(114) = 228$.
time = 0.37, size = 428, normalized size = 3.42

$$-\frac{64\,a^{2}\cos\left(dx+c\right)^{4}+86\,a^{2}\cos\left(dx+c\right)^{3}-54\,a^{2}\cos\left(dx+c\right)^{2}-80\,a^{2}\cos\left(dx+c\right)-4\,a^{2}+21\left(a^{2}\cos\left(dx+c\right)^{4}-a^{2}\cos\left(dx+c\right)^{3}-3\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)+2\,a^{2}+\left(a^{2}\cos\left(dx+c\right)^{3}+2\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)-2\,a^{2}\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-21\left(a^{2}\cos\left(dx+c\right)^{4}-a^{2}\cos\left(dx+c\right)^{3}-3\,a^{2}\cos\left(dx+c\right)^{2}+a^{2}\cos\left(dx+c\right)+2\,a^{2}+\left(a^{2}\cos\left(dx+c\right)^{3}+2\,a^{2}\cos\left(dx+c\right)^{2}-a^{2}\cos\left(dx+c\right)-2\,a^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-2\left(32\,a^{2}\cos\left(dx+c\right)^{3}-11\,a^{2}\cos\left(dx+c\right)^{2}-38\,a^{2}\cos\left(dx+c\right)+2\,a^{2}\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^{4}-d\cos\left(dx+c\right)^{3}-3\,d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)^{3}+2\,d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)-2\,d\right)\sin\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(64*a^2*cos(d*x + c)^4 + 86*a^2*cos(d*x + c)^3 - 54*a^2*cos(d*x + c)^
2 - 80*a^2*cos(d*x + c) - 4*a^2 + 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)
^3 - 3*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3
+ 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) - 21*(a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 3*a^2*cos
(d*x + c)^2 + a^2*cos(d*x + c) + 2*a^2 + (a^2*cos(d*x + c)^3 + 2*a^2*cos(d*
x + c)^2 - a^2*cos(d*x + c) - 2*a^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
1/2) - 2*(32*a^2*cos(d*x + c)^3 - 11*a^2*cos(d*x + c)^2 - 38*a^2*cos(d*x +
c) + 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos(d*
x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*cos(
d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 150, normalized size = 1.20

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+84\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(42\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{16\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^2*log(abs(tan(1/2*d*x + 1/2*c)))
+ 24*a^2*tan(1/2*d*x + 1/2*c) - 3*(42*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*ta
n(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^2 - 16*(12*a^2*tan(1/2*d*x +
 1/2*c)^2 - 21*a^2*tan(1/2*d*x + 1/2*c) + 11*a^2)/(tan(1/2*d*x + 1/2*c) - 1
)^3)/d
```

Mupad [B]
time = 9.07, size = 182, normalized size = 1.46

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{7\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-36\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{135\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{239\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a^2}{2}}{d\,\left(-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (7*a^2*log(tan(c/2 + (d*x)/2)))/(2*d) -
((135*a^2*tan(c/2 + (d*x)/2)^3)/2 - (239*a^2*tan(c/2 + (d*x)/2)^2)/6 - 36*a
^2*tan(c/2 + (d*x)/2)^4 + a^2/2 + (5*a^2*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(c
/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^4 - 4*tan
(c/2 + (d*x)/2)^5)) + (a^2*tan(c/2 + (d*x)/2))/d
```

3.811 $\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 17/2*a^3*x-6*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d+2/3*a^3*cos(d*x+c)/d/(
1-sin(d*x+c))^2-25/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))-3/2*a^3*cos(d*x+c)*sin
(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 2729, 2727, 2718, 2715, 8, 2713}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{6a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx &= a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c\right.\\&=7a^3x+a^3\int\sin^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+(\\&=7a^3x-\frac{5a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx}\\&=\frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 1.47, size = 177, normalized size = 1.49

$$\frac{(a+a\sin(c+dx))^3\left(102(c+dx)-69\cos(c+dx)+\cos(3(c+dx))+\frac{8}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{16\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-9\sin(2(c+dx))\right)}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
```

```
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(109) = 218$.
time = 0.18, size = 266, normalized size = 2.24

method	result
risch	$\frac{17a^3x}{2} + \frac{3ia^3 e^{2i(dx+c)}}{8d} - \frac{23a^3 e^{i(dx+c)}}{8d} - \frac{23a^3 e^{-i(dx+c)}}{8d} - \frac{3ia^3 e^{-2i(dx+c)}}{8d} - \frac{2\left(-48ia^3 e^{i(dx+c)}+27a^3 e^{2i(dx+c}\right.}{3\left(e^{i(dx+c)}-i\right)^3 d}$
norman	$\frac{-\frac{17a^3x}{2}+\frac{80a^3}{3d}+\frac{17a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{17a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{54a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{32a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{54a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\right.\right.}$
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right.}{}$
default	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^3*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+3*a^3*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(s
in(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+a^3
*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin(d
*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [**A**]
time = 0.51, size = 165, normalized size = 1.39

$$\frac{2\left(\cos\left(dx+c\right)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos\left(dx+c\right)\right)a^3+3\left(2\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan\left(dx+c\right)\right)a^3+2\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\tan\left(dx+c\right)\right)a^3-6\,a^3\left(\frac{6\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
```

```
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(105) = 210$.
time = 0.36, size = 220, normalized size = 1.85

$$\frac{2 a^{3} \cos \left(d x+c\right)^{5}+7 a^{3} \cos \left(d x+c\right)^{4}-22 a^{3} \cos \left(d x+c\right)^{3}-102 a^{3} d x-4 a^{3}+\left(51 a^{3} d x+77 a^{3}\right) \cos \left(d x+c\right)^{2}-\left(51 a^{3} d x-100 a^{3}\right) \cos \left(d x+c\right)+\left(2 a^{3} \cos \left(d x+c\right)^{4}-5 a^{3} \cos \left(d x+c\right)^{3}+102 a^{3} d x-27 a^{3} \cos \left(d x+c\right)^{2}-4 a^{3}+\left(51 a^{3} d x-104 a^{3}\right) \cos \left(d x+c\right)\right) \sin \left(d x+c\right)}{6 \left(d \cos \left(d x+c\right)^{2}-d \cos \left(d x+c\right)+\left(d \cos \left(d x+c\right)+2 d\right) \sin \left(d x+c\right)-2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.50, size = 187, normalized size = 1.57

$$\frac{51 \left(d x+c\right) a^{3}+\frac{2 \left(51 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8}-153 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7}+289 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6}-459 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5}+501 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4}-511 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}+327 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}-189 a^{3} \tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)+80 a^{3}\right)}{\left(\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3}-\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2}+\tan \left(\frac{1}{2} d x+\frac{1}{2} c\right)-1\right)^{3}}}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^3 + 2*(51*a^3*tan(1/2*d*x + 1/2*c)^8 - 153*a^3*tan(1/2*
d*x + 1/2*c)^7 + 289*a^3*tan(1/2*d*x + 1/2*c)^6 - 459*a^3*tan(1/2*d*x + 1/2
*c)^5 + 501*a^3*tan(1/2*d*x + 1/2*c)^4 - 511*a^3*tan(1/2*d*x + 1/2*c)^3 + 3
27*a^3*tan(1/2*d*x + 1/2*c)^2 - 189*a^3*tan(1/2*d*x + 1/2*c) + 80*a^3)/(tan
(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)^3)
/d
```

Mupad [B]
time = 14.92, size = 317, normalized size = 2.66

$$\frac{17\,a^3\,x}{2}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(153\,d\,x-378\right)}{6}-\frac{51\,a^3\,d\,x}{2}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{a^3\,\left(153\,d\,x-102\right)}{6}-\frac{51\,a^3\,d\,x}{2}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{a^3\,\left(306\,d\,x-306\right)}{6}-51\,a^3\,d\,x\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(306\,d\,x-654\right)}{6}-51\,a^3\,d\,x\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{a^3\,\left(510\,d\,x-578\right)}{6}-85\,a^3\,d\,x\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{a^3\,\left(510\,d\,x-1022\right)}{6}-85\,a^3\,d\,x\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{a^3\,\left(612\,d\,x-918\right)}{6}-102\,a^3\,d\,x\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{a^3\,\left(612\,d\,x-1002\right)}{6}-102\,a^3\,d\,x\right)-\frac{a^3\,\left(51\,d\,x-160\right)}{6}+\frac{17\,a^3\,d\,x}{2}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (17*a^3*x)/2 + (tan(c/2 + (d*x)/2)*((a^3*(153*d*x - 378))/6 - (51*a^3*d*x)/
2) - tan(c/2 + (d*x)/2)^8*((a^3*(153*d*x - 102))/6 - (51*a^3*d*x)/2) + tan(
c/2 + (d*x)/2)^7*((a^3*(306*d*x - 306))/6 - 51*a^3*d*x) - tan(c/2 + (d*x)/2
)^2*((a^3*(306*d*x - 654))/6 - 51*a^3*d*x) - tan(c/2 + (d*x)/2)^6*((a^3*(51
0*d*x - 578))/6 - 85*a^3*d*x) + tan(c/2 + (d*x)/2)^3*((a^3*(510*d*x - 1022)
)/6 - 85*a^3*d*x) + tan(c/2 + (d*x)/2)^5*((a^3*(612*d*x - 918))/6 - 102*a^3
*d*x) - tan(c/2 + (d*x)/2)^4*((a^3*(612*d*x - 1002))/6 - 102*a^3*d*x) - (a^
3*(51*d*x - 160))/6 + (17*a^3*d*x)/2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d
*x)/2)^2 + tan(c/2 + (d*x)/2)^3 - 1)^3)
```

3.812 $\int \sec(c + dx)(a + a\sin(c + dx))^3 \tan^3(c + dx)\,dx$

Optimal. Leaf size=101

$$\frac{11a^3x}{2}-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 11/2*a^3*x-3*a^3*cos(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2-19/3*a^
3*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2951, 2729, 2727, 2718, 2715, 8}

$$-\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{19a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (11*a^3*x)/2 - (3*a^3*Cos[c + d*x])/d + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (19*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^3*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx &= a^4\int\left(\frac{5}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{7}{a(-1+\sin(c+dx\right.\\ &= 5a^3x+a^3\int\sin^2(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+\\ &= 5a^3x-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{7a}{d(1\\ &= \frac{11a^3x}{2}-\frac{3a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{1}{3a}\end{aligned}$$

Mathematica [A]
time = 1.07, size = 159, normalized size = 1.57

$$-\frac{a^3\left(-3(89+132c+132dx)\cos\left(\frac{1}{2}(c+dx)\right)+(403+132c+132dx)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-9\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx)\right)+2(86+88c+88dx+(-43+44c+44dx)\cos(c+dx)-10\cos(2(c+dx))-\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/48*(a^3*(-3*(89 + 132*c + 132*d*x)*Cos[(c + d*x)/2] + (403 + 132*c + 132
*d*x)*Cos[(3*(c + d*x))/2] + 3*(-9*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))
/2] + 2*(86 + 88*c + 88*d*x + (-43 + 44*c + 44*d*x)*Cos[c + d*x] - 10*Cos[2
*(c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(93) = 186$.
time = 0.18, size = 246, normalized size = 2.44

method	result
risch	$\frac{11a^3x}{2}+\frac{ia^3 e^{2i(dx+c)}}{8d}-\frac{3a^3 e^{i(dx+c)}}{2d}-\frac{3a^3 e^{-i(dx+c)}}{2d}-\frac{ia^3 e^{-2i(dx+c)}}{8d}-\frac{2a^3\left(-36i e^{i(dx+c)}+21 e^{2i(dx+c)}-19\right)}{3d\left(e^{i(dx+c)}-i\right)^3}$
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\right.}{}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^3\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\right.}{}$
norman	$\frac{-\frac{11a^3x}{2}+\frac{52a^3}{3d}+\frac{11a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{11a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{50a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{128a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{50a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\mathrm{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+
sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^3*(
1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3
*sin(d*x+c)^2)*cos(d*x+c))+a^3*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c
)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+
c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.51, size = 145, normalized size = 1.44

$$\frac{\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)a^3+6\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3-6\,a^3\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-\frac{2\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^3 + 6*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 2*
(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(89) = 178$.

time = 0.36, size = 196, normalized size = 1.94

$$\frac{3\,a^3\cos\left(dx+c\right)^4-12\,a^3\cos\left(dx+c\right)^3-66\,a^3dx-4\,a^3+\left(33\,a^3dx+53\,a^3\right)\cos\left(dx+c\right)^2-\left(33\,a^3dx-64\,a^3\right)\cos\left(dx+c\right)-\left(3\,a^3\cos\left(dx+c\right)^3-66\,a^3dx+15\,a^3\cos\left(dx+c\right)^2+4\,a^3-\left(33\,a^3dx-68\,a^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^3*cos(d*x + c)^4 - 12*a^3*cos(d*x + c)^3 - 66*a^3*d*x - 4*a^3 + (3
3*a^3*d*x + 53*a^3)*cos(d*x + c)^2 - (33*a^3*d*x - 64*a^3)*cos(d*x + c) - (
3*a^3*cos(d*x + c)^3 - 66*a^3*d*x + 15*a^3*cos(d*x + c)^2 + 4*a^3 - (33*a^3
*d*x - 68*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x +
c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.48, size = 135, normalized size = 1.34

$$\frac{33\left(dx+c\right)a^3+\frac{6\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{4\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*(d*x + c)*a^3 + 6*(a^3*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*tan(1/2*d*x +
 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 6*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 4*(15*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 17*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 14.96, size = 287, normalized size = 2.84

$$\frac{11\,a^3\,x}{2}+\frac{\frac{11\,a^3\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-246\right)}{6}\right)-\frac{a^3\,\left(33\,c+33\,d\,x-104\right)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{33\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(99\,c+99\,d\,x-66\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{55\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(165\,c+165\,d\,x-198\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{55\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(165\,c+165\,d\,x-322\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{77\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(231\,c+231\,d\,x-308\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{77\,a^3\,\left(c+d\,x\right)}{2}-\frac{a^3\,\left(231\,c+231\,d\,x-420\right)}{6}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] (11*a^3*x)/2 + ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((33*a^3*(c + d*x
))/2 - (a^3*(99*c + 99*d*x - 246))/6) - (a^3*(33*c + 33*d*x - 104))/6 + tan
(c/2 + (d*x)/2)^6*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))/6) - t
an(c/2 + (d*x)/2)^5*((55*a^3*(c + d*x))/2 - (a^3*(165*c + 165*d*x - 198))/6
) + tan(c/2 + (d*x)/2)^2*((55*a^3*(c + d*x))/2 - (a^3*(165*c + 165*d*x - 32
2))/6) + tan(c/2 + (d*x)/2)^4*((77*a^3*(c + d*x))/2 - (a^3*(231*c + 231*d*x
 - 308))/6) - tan(c/2 + (d*x)/2)^3*((77*a^3*(c + d*x))/2 - (a^3*(231*c + 23
1*d*x - 420))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^
2)
```

3.813 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=77

$$3a^3x-\frac{3a^3\cos(c+dx)}{d}-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}+\frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d}$$

```
[Out] 3*a^3*x-3*a^3*cos(d*x+c)/d-2*a^5*cos(d*x+c)^3/d/(a-a*sin(d*x+c))^2+1/3*sec(
d*x+c)^3*(a+a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.172$, Rules used = {2950, 2749, 2759, 2761, 8}

$$-\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2}-\frac{3a^3\cos(c+dx)}{d}+3a^3x+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] 3*a^3*x - (3*a^3*Cos[c + d*x])/d - (2*a^5*Cos[c + d*x]^3)/(d*(a - a*Sin[c +
 d*x])^2) + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2950

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(
p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*m)), x] - Dist[1/g^2, Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && EqQ[a^2 - b^2, 0] && EqQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \int \sec^2(c+dx)(a+$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^6\int \frac{\cos^4(c+d}{(a-a\sin(c+}$$
$$= -\frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+dx)(a+a\sin(c+d}{3d}$$
$$= -\frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3(c+d}{}$$
$$= 3a^3x - \frac{3a^3\cos(c+dx)}{d} - \frac{2a^5\cos^3(c+dx)}{d(a-a\sin(c+dx))^2} + \frac{\sec^3}{}$$

Mathematica [A]
time = 0.90, size = 133, normalized size = 1.73

$$\frac{a^3(1+\sin(c+dx))^3\left(9c+9dx-3\cos(c+dx)+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-11+13\sin(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(9*c + 9*d*x - 3*Cos[c + d*x] + 2/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-11 + 13*Sin[c + d*x]))/(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^6)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(75) = 150$.
time = 0.17, size = 184, normalized size = 2.39

method	result
risch	$3a^3x - \frac{a^3 e^{i(dx+c)}}{2d} - \frac{a^3 e^{-i(dx+c)}}{2d} - \frac{2\left(-24ia^3 e^{i(dx+c)} - 13a^3 + 15a^3 e^{2i(dx+c)}\right)}{3\left(e^{i(dx+c)} - i\right)^3 d}$
derivativedivides	$\frac{\frac{a^3\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3} + 3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3} - \frac{\sin^4(dx+c)}{3\cos(dx+c)} - \frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right) + 3a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3} - \tan(dx+c) + dx + c\right)}{d}$
default	$\frac{\frac{a^3\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3} + 3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3} - \frac{\sin^4(dx+c)}{3\cos(dx+c)} - \frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right) + 3a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3} - \tan(dx+c) + dx + c\right)}{d}$
norman	$\frac{-3a^3x + \frac{28a^3}{3d} + \frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} - \frac{14a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} - \frac{44a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{128a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d} - \frac{44a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^3*sin(d*x+c)^3/cos(d*x+c)^3+3*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*ta
n(d*x+c)^3-tan(d*x+c)+d*x+c)+a^3*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^
6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.55, size = 107, normalized size = 1.39

$$\frac{a^3\tan(dx+c)^3 + 3\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)a^3 - a^3\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3} + 3\,\cos(dx+c)\right) - \frac{3\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/3*(a^3*tan(d*x + c)^3 + 3*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)) - 3*(3*
cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(76) = 152$.
time = 0.36, size = 169, normalized size = 2.19

$$-\frac{3\,a^3\cos(dx+c)^3 + 18\,a^3dx + 2\,a^3 - (9\,a^3dx + 16\,a^3)\cos(dx+c)^2 + (9\,a^3dx - 17\,a^3)\cos(dx+c) - (18\,a^3dx - 3\,a^3\cos(dx+c)^2 - 2\,a^3 + (9\,a^3dx - 19\,a^3)\cos(dx+c))\sin(dx+c)}{3\left(d\cos(dx+c)^2 - d\cos(dx+c) + (d\cos(dx+c) + 2\,d)\sin(dx+c) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/3*(3*a^3*cos(d*x + c)^3 + 18*a^3*d*x + 2*a^3 - (9*a^3*d*x + 16*a^3)*cos(
d*x + c)^2 + (9*a^3*d*x - 17*a^3)*cos(d*x + c) - (18*a^3*d*x - 3*a^3*cos(d*
x + c)^2 - 2*a^3 + (9*a^3*d*x - 19*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.50, size = 87, normalized size = 1.13

$$\frac{9\,(dx+c)a^3-\frac{6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{2\,\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(9*(d*x + c)*a^3 - 6*a^3/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(9*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 11*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

Mupad [B]
time = 12.30, size = 182, normalized size = 2.36

$$3\,a^3\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,(27\,d\,x-66)}{3}-9\,a^3\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^3\,(27\,d\,x-18)}{3}-9\,a^3\,d\,x\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^3\,(36\,d\,x-54)}{3}-12\,a^3\,d\,x\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^3\,(36\,d\,x-58)}{3}-12\,a^3\,d\,x\right)-\frac{a^3\,(9\,d\,x-28)}{3}+3\,a^3\,d\,x}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] 3*a^3*x + (tan(c/2 + (d*x)/2)*((a^3*(27*d*x - 66))/3 - 9*a^3*d*x) - tan(c/2
 + (d*x)/2)^4*((a^3*(27*d*x - 18))/3 - 9*a^3*d*x) + tan(c/2 + (d*x)/2)^3*((
a^3*(36*d*x - 54))/3 - 12*a^3*d*x) - tan(c/2 + (d*x)/2)^2*((a^3*(36*d*x - 5
8))/3 - 12*a^3*d*x) - (a^3*(9*d*x - 28))/3 + 3*a^3*d*x)/(d*(tan(c/2 + (d*x)
/2) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.814 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$a^3x+\frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d}-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] a^3*x+1/3*sec(d*x+c)^3*(a+a*sin(d*x+c))^3/d-2*a^5*cos(d*x+c)/d/(a^2-a^2*sin
(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {2934, 2749, 2759, 8}

$$a^3x-\frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+\frac{\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] a^3*x + (Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*Cos[c + d*x]
)/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a\int \sec^2(c+dx)(a-$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - a^5\int \frac{\cos^2(c+dx}{(a-a\sin(c+}$$
$$= \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx\right.}$$
$$= a^3x + \frac{\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{2a^5\cos(c-}{d\left(a^2-a^2\sin\right.}$$

Mathematica [A]
time = 0.90, size = 107, normalized size = 1.67

$$-\frac{a^3\left(-9(2+c+dx)\cos\left(\frac{1}{2}(c+dx)\right)+(14+3c+3dx)\cos\left(\frac{3}{2}(c+dx)\right)+6(2(2+c+dx)+(c+dx)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -1/6*(a^3*(-9*(2 + c + d*x)*Cos[(c + d*x)/2] + (14 + 3*c + 3*d*x)*Cos[(3*(c
 + d*x))/2] + 6*(2*(2 + c + d*x) + (c + d*x)*Cos[c + d*x])*Sin[(c + d*x)/2]
))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.
time = 0.16, size = 126, normalized size = 1.97

method	result
risch	$a^3x-\frac{2a^3\left(-12i\,e^{i(dx+c)}+9\,e^{2i(dx+c)}-7\right)}{3d\left(e^{i(dx+c)}-i\right)^3}$
derivativedivides	$\frac{\frac{a^3}{3\cos(dx+c)^3}+\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c\right.}{d}$

default	$\frac{\frac{a^3}{3\cos(dx+c)^3}+\frac{a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+3a^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+\right.}{d}$
norman	$a^3x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^3x+\frac{10a^3}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{26a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{36a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{11}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^3/cos(d*x+c)^3+a^3*sin(d*x+c)^3/cos(d*x+c)^3+3*a^3*(1/3*sin(d*x+
c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 84, normalized size = 1.31

$$\frac{3\,a^3\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^3-\frac{3\left(3\,\cos(dx+c)^2-1\right)a^3}{\cos(dx+c)^3}+\frac{a^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))
*a^3 - 3*(3*cos(d*x + c)^2 - 1)*a^3/cos(d*x + c)^3 + a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.
time = 0.34, size = 143, normalized size = 2.23

$$-\frac{6\,a^3dx+2\,a^3-\left(3\,a^3dx+7\,a^3\right)\cos\left(dx+c\right)^2+\left(3\,a^3dx-5\,a^3\right)\cos\left(dx+c\right)-\left(6\,a^3dx-2\,a^3+\left(3\,a^3dx-7\,a^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a^3*d*x + 2*a^3 - (3*a^3*d*x + 7*a^3)*cos(d*x + c)^2 + (3*a^3*d*x -
 5*a^3)*cos(d*x + c) - (6*a^3*d*x - 2*a^3 + (3*a^3*d*x - 7*a^3)*cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d
)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 67, normalized size = 1.05

$$\frac{3\,(dx+c)a^3+\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^3 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*tan(1/2*d*x
 + 1/2*c) + 5*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.26, size = 102, normalized size = 1.59

$$a^3\,x+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(9\,d\,x-24\right)}{3}-3\,a^3\,d\,x\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(9\,d\,x-6\right)}{3}-3\,a^3\,d\,x\right)-\frac{a^3\,\left(3\,d\,x-10\right)}{3}+a^3\,d\,x}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] a^3*x + (tan(c/2 + (d*x)/2)*((a^3*(9*d*x - 24))/3 - 3*a^3*d*x) - tan(c/2 +
(d*x)/2)^2*((a^3*(9*d*x - 6))/3 - 3*a^3*d*x) - (a^3*(3*d*x - 10))/3 + a^3*d
*x)/(d*(tan(c/2 + (d*x)/2) - 1)^3)
```

3.815 $\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}$$

```
[Out] -a^3*arctanh(cos(d*x+c))/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+5/3*a^3*co
s(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]

time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2951, 3855, 2729, 2727}

$$\frac{5a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*ArcTanh[Cos[c + d*x]])/d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d
*x])^2) + (5*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{\csc(c+dx)}{a}+\frac{2}{a(-1+\sin(c+dx))^2}-\frac{}{a(-1+\text{s}}\right.$$
$$= a^3\int\csc(c+dx)\,dx - a^3\int\frac{1}{-1+\sin(c+dx)}\,dx + \left(2a^3\right.$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{a^3}{d(1}$$
$$= -\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{5a}{3d(}$$

Mathematica [A]
time = 0.47, size = 144, normalized size = 2.00

$$\frac{a^3(1+\sin(c+dx))^3\left(-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)(-7+5\sin(c+dx))}{\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^3*(-3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2
]] + 2/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-7 +
5*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3))/(3*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.27, size = 101, normalized size = 1.40

method	result
risch	$\frac{2a^3\left(-12i e^{i(dx+c)}+3\,e^{2i(dx+c)}-5\right)}{3d\left(e^{i(dx+c)}-i\right)^3}+\frac{a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}$
derivativedivides	$\frac{a^3\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^3}{\cos(dx+c)^3}+\frac{a^3\left(\sin^3(dx\right.}{3\cos(dx+}}{d}$
default	$\frac{a^3\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)-3a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^3}{\cos(dx+c)^3}+\frac{a^3\left(\sin^3(dx\right.}{3\cos(dx+}}{d}$

norman	$\frac{-\frac{14a^3}{3d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{10a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{50a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{20a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{68a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{20a^3(t}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))-3*a^3*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3/cos(d*x+c)^3+1/3*a^3*sin(d*x+c)^3/cos(
d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 103, normalized size = 1.43

$$\frac{2\,a^3\tan\left(dx+c\right)^3+6\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^3+a^3\left(\frac{2\left(3\,\cos\left(dx+c\right)^2+1\right)}{\cos\left(dx+c\right)^3}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{6\,a^3}{\cos\left(dx+c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*tan(d*x + c)^3 + 6*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(
2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(c
os(d*x + c) - 1)) + 6*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(64) = 128.
time = 0.36, size = 231, normalized size = 3.21

$$-\frac{10\,a^3\cos\left(dx+c\right)^2+14\,a^3\cos\left(dx+c\right)+4\,a^3+3\left(a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-2\,a^3+\left(a^3\cos\left(dx+c\right)+2\,a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a^3\cos\left(dx+c\right)^2-a^3\cos\left(dx+c\right)-2\,a^3+\left(a^3\cos\left(dx+c\right)+2\,a^3\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\left(5\,a^3\cos\left(dx+c\right)-2\,a^3\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(10*a^3*cos(d*x + c)^2 + 14*a^3*cos(d*x + c) + 4*a^3 + 3*(a^3*cos(d*x
+ c)^2 - a^3*cos(d*x + c) - 2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c)
)*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) -
2*a^3 + (a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/
2) - 2*(5*a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos
(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 73, normalized size = 1.01

$$\frac{3\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^2
 - 12*a^3*tan(1/2*d*x + 1/2*c) + 7*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.23, size = 98, normalized size = 1.36

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{6\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{14\,a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (6*a^3*tan(c/2 + (d*x)/2)^2 + (14*a^3)/3
- 8*a^3*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2) - 3*tan(c/2 + (d*x)/2)
^2 + tan(c/2 + (d*x)/2)^3 - 1))
```

3.816 $\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}$$

```
[Out] -3*a^3*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d
*x+c))^2+11/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 3855, 3852, 8, 2729, 2727}

$$-\frac{a^3\cot(c+dx)}{d}+\frac{11a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (2*a^3*Cos[c + d*
x])/(3*d*(1 - Sin[c + d*x])^2) + (11*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
```

```
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx = a^4\int\left(\frac{3\csc(c+dx)}{a}+\frac{\csc^2(c+dx)}{a}+\frac{2}{a(-1+\sin(c+\right.$$
$$= a^3\int \csc^2(c+dx)\,dx + \left(2a^3\right)\int \frac{1}{(-1+\sin(c+dx))^2}\,dx$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{}{d(}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{2a^3\cos}{3d(1-\sin}$$

Mathematica [A]
time = 0.72, size = 135, normalized size = 1.57

$$\frac{a^3\left(-3\cot\left(\frac{1}{2}(c+dx)\right)-18\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+18\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)(-13+11\sin(c+dx))}{\left(-\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+3\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-3*Cot[(c + d*x)/2] - 18*Log[Cos[(c + d*x)/2]] + 18*Log[Sin[(c + d*x)
/2]] + 4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*Sin[(c + d*x)/2]*(-13
 + 11*Sin[c + d*x]))/(-Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 3*Tan[(c +
d*x)/2]))/(6*d)
```

Maple [A]
time = 0.27, size = 131, normalized size = 1.52

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^3\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c)\right.}{d}$
default	$\frac{a^3\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^3\left(\frac{1}{3\cos(dx+c)^3}+\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c)\right.}{d}$
risch	$\frac{-\frac{58a^3e^{2i(dx+c)}}{3}-18ia^3e^{3i(dx+c)}+\frac{28a^3}{3}+22ia^3e^{i(dx+c)}+6a^3e^{4i(dx+c)}}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)^3d}+\frac{3a^3\ln\left(e^{i(dx+c)}-1\right)}{d}-\frac{3a^3\ln\left(e^{i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{a^3}{2d}-\frac{21a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{14a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{133a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{21a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{44a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-21a^3}{\tan(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x
+c))+3*a^3*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))-3*a^3*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*a^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.36, size = 123, normalized size = 1.43

$$\frac{2\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a^3+6\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+3a^3\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log\left(\cos(dx+c)+1\right)+3\log\left(\cos(dx+c)-1\right)\right)+\frac{2a^3}{\cos(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 6*(tan(d*x
+ c)^3 + 3*tan(d*x + c))*a^3 + 3*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)
^3 - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*a^3/cos(d*x + c
)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(78) = 156.
time = 0.37, size = 334, normalized size = 3.88

$$\frac{28a^3\cos(dx+c)^3-10a^3\cos(dx+c)^2-34a^3\cos(dx+c)+4a^3-9\left(a^3\cos(dx+c)^3+2a^3\cos(dx+c)^2-a^3\cos(dx+c)-2a^3-\left(a^3\cos(dx+c)^2-a^3\cos(dx+c)-2a^3\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+9\left(a^3\cos(dx+c)^3+2a^3\cos(dx+c)^2-a^3\cos(dx+c)-2a^3-\left(a^3\cos(dx+c)^2-a^3\cos(dx+c)-2a^3\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(14a^3\cos(dx+c)^2+19a^3\cos(dx+c)+2a^3\right)\sin(dx+c)}{6\left(d\cos(dx+c)^3+2d\cos(dx+c)^2-d\cos(dx+c)-\left(d\cos(dx+c)^2-d\cos(dx+c)-2d\right)\sin(dx+c)-2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(28*a^3*cos(d*x + c)^3 - 10*a^3*cos(d*x + c)^2 - 34*a^3*cos(d*x + c) +
4*a^3 - 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2
*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/
2*cos(d*x + c) + 1/2) + 9*(a^3*cos(d*x + c)^3 + 2*a^3*cos(d*x + c)^2 - a^3*
cos(d*x + c) - 2*a^3 - (a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(14*a^3*cos(d*x + c)^2 + 19*a^3*
cos(d*x + c) + 2*a^3)*sin(d*x + c))/(d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2
- d*cos(d*x + c) - (d*cos(d*x + c)^2 - d*cos(d*x + c) - 2*d)*sin(d*x + c) -
 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 118, normalized size = 1.37

$$\frac{18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\,\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{4\,\left(15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) - 3
*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c) - 4*(15*a^3*tan(1/
2*d*x + 1/2*c)^2 - 24*a^3*tan(1/2*d*x + 1/2*c) + 13*a^3)/(tan(1/2*d*x + 1/2
*c) - 1)^3)/d
```

Mupad [B]
time = 9.21, size = 144, normalized size = 1.67

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-21\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+35\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{61\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+a^3}{d\,\left(-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^2),x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)))/d - (35*a^3*tan(c/2 + (d*x)/2)^2 - 21*a^3*t
an(c/2 + (d*x)/2)^3 + a^3 - (61*a^3*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 +
(d*x)/2) - 6*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d
*x)/2)^4)) + (a^3*tan(c/2 + (d*x)/2))/(2*d)
```

3.817 $\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=110

$$-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^3\cos}{3d(1-\text{si}}$$

```
[Out] -11/2*a^3*arctanh(cos(d*x+c))/d-3*a^3*cot(d*x+c)/d-1/2*a^3*cot(d*x+c)*csc(d
*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+17/3*a^3*cos(d*x+c)/d/(1-sin(
d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 3853, 2729, 2727}

$$-\frac{3a^3\cot(c+dx)}{d}+\frac{17a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-11*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^
2) + (17*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^4\int\left(\frac{5\csc(c+dx)}{a}+\frac{3\csc^2(c+dx)}{a}+\frac{\csc^3(c+dx)}{a}+\right.\\ &= a^3\int\csc^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx\\ &= -\frac{5a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)\csc(c+dx)}{2d}\\ &= -\frac{11a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)}{d}-\frac{a^3\cot}{}\end{aligned}$$

Mathematica [A]
time = 1.39, size = 190, normalized size = 1.73

$$\frac{a^3\left(-36\cot\left(\frac{1}{2}(c+dx)\right)-3\csc^2\left(\frac{1}{2}(c+dx)\right)-132\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+132\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{16}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{32\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{272\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+36\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-36*Cot[(c + d*x)/2] - 3*Csc[(c + d*x)/2]^2 - 132*Log[Cos[(c + d*x)/2
]] + 132*Log[Sin[(c + d*x)/2]] + 3*Sec[(c + d*x)/2]^2 + 16/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2 + (32*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3 + (272*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
 + 36*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A]
time = 0.27, size = 188, normalized size = 1.71

method	result
risch	$\frac{a^3\left(-99i e^{5i(dx+c)}+33\,e^{6i(dx+c)}+210i e^{3i(dx+c)}-154\,e^{4i(dx+c)}-123i e^{i(dx+c)}+161\,e^{2i(dx+c)}-52\right)}{3\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^3 d}+\frac{11a^3\ln\left(e^{i(dx+c)}-\right.}{2d}$
derivativedivides	$a^3\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}\right.$
default	$a^3\left(\frac{1}{3\sin(dx+c)^2\cos(dx+c)^3}-\frac{5}{6\sin(dx+c)^2\cos(dx+c)}+\frac{5}{2\cos(dx+c)}+\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^3\left(\frac{1}{3\sin(dx+c)\cos(dx+c)^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos
(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^3*(1/3/sin(d*x+c)/cos(d*x+c)^3+4
/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+3*a^3*(1/3/cos(d*x+c)^3+1/cos(d*x+
c)+ln(csc(d*x+c)-cot(d*x+c)))-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.35, size = 182, normalized size = 1.65

$$\frac{12\left(\tan(dx+c)^3-\frac{3}{\tan(dx+c)}+6\tan(dx+c)\right)a^3+4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+a^3\left(\frac{2\left(15\cos(dx+c)^4-10\cos(dx+c)^2-2\right)}{\cos(dx+c)^5-\cos(dx+c)^3}-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)+6a^3\left(\frac{2\left(3\cos(dx+c)^2+1\right)}{\cos(dx+c)^3}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 + 3*tan(d*x + c))*a^3 + a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c
)^2 - 2)/(cos(d*x + c)^5 - cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*
log(cos(d*x + c) - 1)) + 6*a^3*(2*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(98) = 196$.
time = 0.36, size = 428, normalized size = 3.89

$$-\frac{104\,a^3\cos(dx+c)^4+142\,a^3\cos(dx+c)^3-90\,a^3\cos(dx+c)^2-136\,a^3\cos(dx+c)-8\,a^3+33\left(a^3\cos(dx+c)^4-a^3\cos(dx+c)^3-3\,a^3\cos(dx+c)^2+a^3\cos(dx+c)+2\,a^3+\left(a^3\cos(dx+c)^3+2\,a^3\cos(dx+c)^2-a^3\cos(dx+c)-2\,a^3\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-33\left(a^3\cos(dx+c)^4-a^3\cos(dx+c)^3-3\,a^3\cos(dx+c)^2+a^3\cos(dx+c)+2\,a^3+\left(a^3\cos(dx+c)^3+2\,a^3\cos(dx+c)^2-a^3\cos(dx+c)-2\,a^3\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(52\,a^3\cos(dx+c)^3-19\,a^3\cos(dx+c)^2-64\,a^3\cos(dx+c)+4\,a^3\right)\sin(dx+c)}{12\left(d\cos(dx+c)^4-d\cos(dx+c)^3-3\,d\cos(dx+c)^2+d\cos(dx+c)+\left(d\cos(dx+c)^3+2\,d\cos(dx+c)^2-d\cos(dx+c)-2\,d\right)\sin(dx+c)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 + 142*a^3*cos(d*x + c)^3 - 90*a^3*cos(d*x + c
)^2 - 136*a^3*cos(d*x + c) - 8*a^3 + 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x +
 c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)
^3 + 2*a^3*cos(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) - 33*(a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*
cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 + (a^3*cos(d*x + c)^3 + 2*a^3*cos
(d*x + c)^2 - a^3*cos(d*x + c) - 2*a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(52*a^3*cos(d*x + c)^3 - 19*a^3*cos(d*x + c)^2 - 64*a^3*cos(d*x
 + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 3*d*cos
(d*x + c)^2 + d*cos(d*x + c) + (d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 - d*c
os(d*x + c) - 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.64, size = 150, normalized size = 1.36

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+132\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{3\left(66\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{16\left(21\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\,a^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 132*a^3*log(abs(tan(1/2*d*x + 1/2*c)))
 + 36*a^3*tan(1/2*d*x + 1/2*c) - 3*(66*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*
tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2 - 16*(21*a^3*tan(1/2*d*x
 + 1/2*c)^2 - 36*a^3*tan(1/2*d*x + 1/2*c) + 19*a^3)/(tan(1/2*d*x + 1/2*c) -
 1)^3)/d
```

Mupad [B]
time = 9.14, size = 183, normalized size = 1.66

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{11\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{-62\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{227\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{403\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{6}+\frac{9\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a^3}{2}}{d\,\left(-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (11*a^3*log(tan(c/2 + (d*x)/2)))/(2*d) -
 ((227*a^3*tan(c/2 + (d*x)/2)^3)/2 - (403*a^3*tan(c/2 + (d*x)/2)^2)/6 - 62*
a^3*tan(c/2 + (d*x)/2)^4 + a^3/2 + (9*a^3*tan(c/2 + (d*x)/2))/2)/(d*(4*tan(
c/2 + (d*x)/2)^2 - 12*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^4 - 4*ta
n(c/2 + (d*x)/2)^5)) + (3*a^3*tan(c/2 + (d*x)/2))/(2*d)
```

3.818 $\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=128

$$-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}+\frac{2a^3\cos(c+}{3d(1-\sin(c+}$$

```
[Out] -17/2*a^3*arctanh(cos(d*x+c))/d-6*a^3*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-3
/2*a^3*cot(d*x+c)*csc(d*x+c)/d+2/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))^2+23/3*a
^3*cos(d*x+c)/d/(1-sin(d*x+c))
```

Rubi [A]

time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2951, 3855, 3852, 8, 3853, 2729, 2727}

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot(c+dx)}{d}+\frac{23a^3\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^3\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-17*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (6*a^3*Cot[c + d*x])/d - (a^3*Cot[c
 + d*x]^3)/(3*d) - (3*a^3*Cot[c + d*x]*Csc[c + d*x])/(2*d) + (2*a^3*Cos[c +
 d*x])/(3*d*(1 - Sin[c + d*x])^2) + (23*a^3*Cos[c + d*x])/(3*d*(1 - Sin[c +
 d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
```

```
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)\sec^4(c+dx)(a+a\sin(c+dx))^3\,dx &= a^4\int\left(\frac{7\csc(c+dx)}{a}+\frac{5\csc^2(c+dx)}{a}+\frac{3\csc^3(c+dx}{a}\right.\\ &= a^3\int\csc^4(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\\ &= -\frac{7a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^3\cot(c+dx)\csc(c+}{2d}\\ &= -\frac{17a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{6a^3\cot(c+dx)}{d}-\frac{a^3\mathrm{co}}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 287 vs. 2(128) = 256.
time = 6.17, size = 287, normalized size = 2.24

$$a^3\left(-\frac{17\cot\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}-\frac{17\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{17\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{2}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{46\sin\left(\frac{1}{2}(c+dx)\right)}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{17\tan\left(\frac{1}{2}(c+dx)\right)}{6d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] a^3*((-17*Cot[(c + d*x)/2])/(6*d) - (3*Csc[(c + d*x)/2]^2)/(8*d) - (Cot[(c
+ d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) - (17*Log[Cos[(c + d*x)/2]])/(2*d) + (
17*Log[Sin[(c + d*x)/2]])/(2*d) + (3*Sec[(c + d*x)/2]^2)/(8*d) + 2/(3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (4*Sin[(c + d*x)/2])/(3*d*(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3) + (46*Sin[(c + d*x)/2])/(3*d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) + (17*Tan[(c + d*x)/2])/(6*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*d))
```

Maple [A]

time = 0.31, size = 232, normalized size = 1.81

method	result
risch	$\frac{-153 i a^{3} e^{7 i (d x +c)}+51 a^{3} e^{8 i (d x +c)}+459 i a^{3} e^{5 i (d x +c)}-289 a^{3} e^{6 i (d x +c)}-511 i a^{3} e^{3 i (d x +c)}+501 a^{3} e^{4 i (d x +c)}+189 i a^{3} e^{i (d x +c)}-}{3 \left(e^{2 i (d x +c)}-1\right)^{3} \left(e^{i (d x +c)}-i \right)^{3} d}$
derivativedivides	$\frac{a^{3} \left(\frac{1}{3 \sin \left(d x +c \right)^{3} \cos \left(d x +c \right)^{3}}-\frac{2}{3 \sin \left(d x +c \right)^{3} \cos \left(d x +c \right)}+\frac{8}{3 \sin \left(d x +c \right) \cos \left(d x +c \right)}-\frac{16 \cot \left(d x +c \right)}{3}\right)+3 a^{3} \left(\frac{1}{3 \sin \left(d x +c \right)^{2} \cos \left(d x +c \right)^{3}}-\frac{}{6 \mathrm{s}}\right.}{}$
default	$\frac{a^{3} \left(\frac{1}{3 \sin \left(d x +c \right)^{3} \cos \left(d x +c \right)^{3}}-\frac{2}{3 \sin \left(d x +c \right)^{3} \cos \left(d x +c \right)}+\frac{8}{3 \sin \left(d x +c \right) \cos \left(d x +c \right)}-\frac{16 \cot \left(d x +c \right)}{3}\right)+3 a^{3} \left(\frac{1}{3 \sin \left(d x +c \right)^{2} \cos \left(d x +c \right)^{3}}-\frac{}{6 \mathrm{s}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3/sin(d*x+c)^3/cos(d*x+c)^3-2/3/sin(d*x+c)^3/cos(d*x+c)+8/3/sin
(d*x+c)/cos(d*x+c)-16/3*cot(d*x+c))+3*a^3*(1/3/sin(d*x+c)^2/cos(d*x+c)^3-5/
6/sin(d*x+c)^2/cos(d*x+c)+5/2/cos(d*x+c)+5/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a
^3*(1/3/sin(d*x+c)/cos(d*x+c)^3+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+a
^3*(1/3/cos(d*x+c)^3+1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.34, size = 205, normalized size = 1.60

$$\frac{12 \left(\tan \left(dx+c\right)^{3}-\frac{3}{\tan \left(dx+c\right)}+6 \tan \left(dx+c\right)\right)a^{3}+4 \left(\tan \left(dx+c\right)^{3}-\frac{9 \tan \left(dx+c\right)^{2}+1}{\tan \left(dx+c\right)^{3}}+9 \tan \left(dx+c\right)\right)a^{3}+3 a^{3} \left(\frac{2 \left(15 \cos \left(dx+c\right)^{4}-10 \cos \left(dx+c\right)^{2}-2\right)}{\cos \left(dx+c\right)^{5}-\cos \left(dx+c\right)^{3}}-15 \log \left(\cos \left(dx+c\right)+1\right)+15 \log \left(\cos \left(dx+c\right)-1\right)\right)+2 a^{3} \left(\frac{2 \left(3 \cos \left(dx+c\right)^{2}+1\right)}{\cos \left(dx+c\right)^{3}}-3 \log \left(\cos \left(dx+c\right)+1\right)+3 \log \left(\cos \left(dx+c\right)-1\right)\right)}{12 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(12*(tan(d*x + c)^3 - 3/tan(d*x + c) + 6*tan(d*x + c))*a^3 + 4*(tan(d*
x + c)^3 - (9*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 + 9*tan(d*x + c))*a^3 + 3*
a^3*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(cos(d*x + c)^5 - cos(d*
x + c)^3) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 2*a^3*(2
*(3*cos(d*x + c)^2 + 1)/cos(d*x + c)^3 - 3*log(cos(d*x + c) + 1) + 3*log(co
s(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(114) = 228$.
time = 0.36, size = 528, normalized size = 4.12

$$\frac{160\,a^3\cos(dx+c)^5 - 58\,a^3\cos(dx+c)^4 - 356\,a^3\cos(dx+c)^3 + 70\,a^3\cos(dx+c)^2 + 200\,a^3\cos(dx+c) - 8\,a^3 - 51\,\left(a^3\cos(dx+c)^5 + 2\,a^3\cos(dx+c)^4 - 2\,a^3\cos(dx+c)^3 - 4\,a^3\cos(dx+c)^2 + a^3\cos(dx+c) + 2\,a^3 - \left(a^3\cos(dx+c)^4 - a^3\cos(dx+c)^3 - 3\,a^3\cos(dx+c)^2 + a^3\cos(dx+c) + 2\,a^3\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 51\,\left(a^3\cos(dx+c)^5 + 2\,a^3\cos(dx+c)^4 - 2\,a^3\cos(dx+c)^3 - 4\,a^3\cos(dx+c)^2 + a^3\cos(dx+c) + 2\,a^3 - \left(a^3\cos(dx+c)^4 - a^3\cos(dx+c)^3 - 3\,a^3\cos(dx+c)^2 + a^3\cos(dx+c) + 2\,a^3\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right) + 2\,\left(80\,a^3\cos(dx+c)^4 + 109\,a^3\cos(dx+c)^3 - 69\,a^3\cos(dx+c)^2 - 104\,a^3\cos(dx+c) - 4\,a^3\right)\sin(dx+c)}{12\,\left(d\cos(dx+c)^5 + 2\,d\cos(dx+c)^4 - 2\,d\cos(dx+c)^3 - 4\,d\cos(dx+c)^2 + d\cos(dx+c) - \left(d\cos(dx+c)^4 - d\cos(dx+c)^3 - 3\,d\cos(dx+c)^2 + d\cos(dx+c) + 2\,d\right)\sin(dx+c) + 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(160*a^3*cos(d*x + c)^5 - 58*a^3*cos(d*x + c)^4 - 356*a^3*cos(d*x + c)
^3 + 70*a^3*cos(d*x + c)^2 + 200*a^3*cos(d*x + c) - 8*a^3 - 51*(a^3*cos(d*x
 + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 - 4*a^3*cos(d*x + c)^
2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x + c)^4 - a^3*cos(d*x + c)^3 - 3
*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + 51*(a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 2*a^3*cos(
d*x + c)^3 - 4*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*a^3 - (a^3*cos(d*x
 + c)^4 - a^3*cos(d*x + c)^3 - 3*a^3*cos(d*x + c)^2 + a^3*cos(d*x + c) + 2*
a^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(80*a^3*cos(d*x + c)^4
+ 109*a^3*cos(d*x + c)^3 - 69*a^3*cos(d*x + c)^2 - 104*a^3*cos(d*x + c) - 4
*a^3)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 2*d*cos(d*x +
c)^3 - 4*d*cos(d*x + c)^2 + d*cos(d*x + c) - (d*cos(d*x + c)^4 - d*cos(d*x
+ c)^3 - 3*d*cos(d*x + c)^2 + d*cos(d*x + c) + 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 194, normalized size = 1.52

$$\frac{a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 9\,a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 204\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 69\,a^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{187\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6 - 60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 405\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 394\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x + 1/2*c)^2 + 204*a^3*l
og(abs(tan(1/2*d*x + 1/2*c))) + 69*a^3*tan(1/2*d*x + 1/2*c) - (187*a^3*tan(
1/2*d*x + 1/2*c)^6 - 60*a^3*tan(1/2*d*x + 1/2*c)^5 - 405*a^3*tan(1/2*d*x +
```

```
1/2*c)^4 + 394*a^3*tan(1/2*d*x + 1/2*c)^3 - 45*a^3*tan(1/2*d*x + 1/2*c)^2 -
 6*a^3*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x +
1/2*c))^3)/d
```

Mupad [B]
time = 11.41, size = 239, normalized size = 1.87

$$\frac{a^3\left(6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+45\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-581\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+897\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-303\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-181\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+45\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-204\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+612\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-612\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+204\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+1\right)}{24\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^4*sin(c + d*x)^4),x)
```

```
[Out] (a^3*(6*tan(c/2 + (d*x)/2) + 45*tan(c/2 + (d*x)/2)^2 - 581*tan(c/2 + (d*x)/
2)^3 + 897*tan(c/2 + (d*x)/2)^4 - 303*tan(c/2 + (d*x)/2)^5 - 181*tan(c/2 +
(d*x)/2)^6 + 45*tan(c/2 + (d*x)/2)^7 + 6*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (
d*x)/2)^9 - 204*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 612*log(tan(
c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 - 612*log(tan(c/2 + (d*x)/2))*tan(c/2
+ (d*x)/2)^5 + 204*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1))/(24*d
*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)/2) - 1)^3)
```

3.819 $\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{35a^4\cos(c+dx)}{8d}$$

```
[Out] 163/8*a^4*x-16*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d+4/3*a^4*cos(d*x+c)/d
/(1-sin(d*x+c))^2-56/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))-35/8*a^4*cos(d*x+c)*
sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2788, 2729, 2727, 2718, 2715, 8, 2713}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{163a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx &= a^4\int \left(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+
\right.\\
&= 16a^4x+a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \frac{1}{(-1+\sin(c+dx))^2}\,dx+\\
&= 16a^4x-\frac{12a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{20a^4\cos(c+}{d(1-\sin(c+}\\
&= 20a^4x-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx)}\\
&= \frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx}{3d(1-\sin(c+dx}
\end{aligned}
$$

Mathematica [A]
time = 1.09, size = 252, normalized size = 1.76

$$
\frac{a^4\left(24(209+489c+489dx)\cos\left(\frac{1}{2}(c+dx)\right)-24(453+163c+163dx)\cos\left(\frac{3}{2}(c+dx)\right)+885\cos\left(\frac{5}{2}(c+dx)\right)-129\cos\left(\frac{7}{2}(c+dx)\right)-23\cos\left(\frac{9}{2}(c+dx)\right)+3\cos\left(\frac{11}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)+3704\sin\left(\frac{3}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912dx\sin\left(\frac{3}{2}(c+dx)\right)+885\sin\left(\frac{5}{2}(c+dx)\right)+129\sin\left(\frac{7}{2}(c+dx)\right)-23\sin\left(\frac{9}{2}(c+dx)\right)-3\sin\left(\frac{11}{2}(c+dx)\right)\right)}{384d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(131) = 262$.
time = 0.25, size = 360, normalized size = 2.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+4*a^4*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+6*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(s
in(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a
^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+sin
(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*sin(d*x+c
)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*x+c)^7+7/6*sin(d*x+c)^5
+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/8*d*x+35/8*c))
```

Maxima [A]
time = 0.60, size = 238, normalized size = 1.66

$$\frac{32\left(\cos\left(dx+c\right)^{3}-\frac{9\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}-9\cos\left(dx+c\right)\right)a^{4}+\left(8\tan\left(dx+c\right)^{3}+105\,dx+105\,c-\frac{3\left(13\tan(dx+c)^{3}+11\tan(dx+c)\right)}{\tan(dx+c)^{4}+2\tan(dx+c)^{2}+1}-72\tan\left(dx+c\right)\right)a^{4}+24\left(2\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\tan\left(dx+c\right)\right)a^{4}+8\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\tan\left(dx+c\right)\right)a^{4}-32\,a^{4}\left(\frac{6\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\cos\left(dx+c\right)\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

Fricas [A]
time = 0.36, size = 247, normalized size = 1.73

$$-\frac{6\,a^{4}\cos\left(dx+c\right)^{6}-20\,a^{4}\cos\left(dx+c\right)^{5}-85\,a^{4}\cos\left(dx+c\right)^{4}+214\,a^{4}\cos\left(dx+c\right)^{3}+978\,a^{4}dx+32\,a^{4}-\left(489\,a^{4}dx+721\,a^{4}\right)\cos\left(dx+c\right)^{2}+\left(489\,a^{4}dx-962\,a^{4}\right)\cos\left(dx+c\right)-\left(6\,a^{4}\cos\left(dx+c\right)^{5}+26\,a^{4}\cos\left(dx+c\right)^{4}-59\,a^{4}\cos\left(dx+c\right)^{3}+978\,a^{4}dx-273\,a^{4}\cos\left(dx+c\right)^{2}-32\,a^{4}+\left(489\,a^{4}dx-994\,a^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)^{2}-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.70, size = 200, normalized size = 1.40

$$\frac{489\,(dx+c)a^4+\frac{64\left(12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{2\left(105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-288\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+129\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1056\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-129\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-105\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-352\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(489*(d*x + c)*a^4 + 64*(12*a^4*tan(1/2*d*x + 1/2*c)^2 - 27*a^4*tan(1/
2*d*x + 1/2*c) + 13*a^4)/(tan(1/2*d*x + 1/2*c) - 1)^3 + 2*(105*a^4*tan(1/2*
d*x + 1/2*c)^7 - 288*a^4*tan(1/2*d*x + 1/2*c)^6 + 129*a^4*tan(1/2*d*x + 1/2
*c)^5 - 1056*a^4*tan(1/2*d*x + 1/2*c)^4 - 129*a^4*tan(1/2*d*x + 1/2*c)^3 -
1120*a^4*tan(1/2*d*x + 1/2*c)^2 - 105*a^4*tan(1/2*d*x + 1/2*c) - 352*a^4)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 16.83, size = 437, normalized size = 3.06

$$\frac{163\,a^4\,x}{8}+\frac{\frac{163\,a^4\,\left(c+d\,x\right)}{8}-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{489\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(1467\,c+1467\,d\,x-3630\right)}{24}\right)-\frac{a^4\,\left(489\,c+489\,d\,x-1536\right)}{24}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(\frac{489\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(1467\,c+1467\,d\,x-978\right)}{24}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{1141\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(3423\,c+3423\,d\,x-2934\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{1141\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(3423\,c+3423\,d\,x-7818\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{2119\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(6357\,c+6357\,d\,x-6520\right)}{24}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{2119\,a^4\,\left(c+d\,x\right)}{8}-\frac{a^4\,\left(6357\,c+6357\,d\,x-13448\right)}{24}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{1467\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(8802\,c+8802\,d\,x-11736\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{1467\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(8802\,c+8802\,d\,x-15912\right)}{24}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{1793\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(10758\,c+10758\,d\,x-15364\right)}{24}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{1793\,a^4\,\left(c+d\,x\right)}{4}-\frac{a^4\,\left(10758\,c+10758\,d\,x-18428\right)}{24}\right)}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (163*a^4*x)/8 + ((163*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((489*a^4*(c +
d*x))/8 - (a^4*(1467*c + 1467*d*x - 3630))/24) - (a^4*(489*c + 489*d*x - 15
36))/24 + tan(c/2 + (d*x)/2)^10*((489*a^4*(c + d*x))/8 - (a^4*(1467*c + 146
7*d*x - 978))/24) - tan(c/2 + (d*x)/2)^9*((1141*a^4*(c + d*x))/8 - (a^4*(34
23*c + 3423*d*x - 2934))/24) + tan(c/2 + (d*x)/2)^2*((1141*a^4*(c + d*x))/8
 - (a^4*(3423*c + 3423*d*x - 7818))/24) + tan(c/2 + (d*x)/2)^8*((2119*a^4*(
c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 6520))/24) - tan(c/2 + (d*x)/2)^3*(
(2119*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 13448))/24) - tan(c/2 +
(d*x)/2)^7*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x - 11736))/24)
+ tan(c/2 + (d*x)/2)^4*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x -
15912))/24) + tan(c/2 + (d*x)/2)^6*((1793*a^4*(c + d*x))/4 - (a^4*(10758*c
+ 10758*d*x - 15364))/24) - tan(c/2 + (d*x)/2)^5*((1793*a^4*(c + d*x))/4 -
(a^4*(10758*c + 10758*d*x - 18428))/24))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan
(c/2 + (d*x)/2)^2 + 1)^4)
```

3.820 $\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{17a^4x}{2}-\frac{4a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{a^4\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 17/2*a^4*x-4*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))^2-32/3*a^
4*cos(d*x+c)/d/(1-sin(d*x+c))-1/2*a^4*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2951, 2729, 2727, 2718, 2715, 8}

$$-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{32a^4\cos(c+dx)}{3d(1-\sin(c+dx))}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}+\frac{17a^4x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] (17*a^4*x)/2 - (4*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x])/(3*d*(1 - Sin[
c + d*x])^2) - (32*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])) - (a^4*Cos[c
+ d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2951

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^4\tan^2(c+dx)\,dx = a^4\int\left(8+\frac{4}{(-1+\sin(c+dx))^2}+\frac{12}{-1+\sin(c+dx)}\right.$$
$$= 8a^4x + a^4\int \sin^2(c+dx)\,dx + \left(4a^4\right)\int\frac{1}{(-1+\sin(c$$
$$= 8a^4x - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{12}{d(1}$$
$$= \frac{17a^4x}{2} - \frac{4a^4\cos(c+dx)}{d} + \frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2} - \frac{}{3}$$

Mathematica [A]
time = 1.40, size = 158, normalized size = 1.56

$$-\frac{a^4\left(-3(161+204c+204dx)\cos\left(\frac{1}{2}(c+dx)\right)+(647+204c+204dx)\cos\left(\frac{3}{2}(c+dx)\right)-39\cos\left(\frac{5}{2}(c+dx)\right)+3\cos\left(\frac{7}{2}(c+dx)\right)+6(146+136c+136dx+(-59+68c+68dx)\cos(c+dx)-14\cos(2(c+dx))-\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] -1/48*(a^4*(-3*(161 + 204*c + 204*d*x)*Cos[(c + d*x)/2] + (647 + 204*c + 20
4*d*x)*Cos[(3*(c + d*x))/2] - 39*Cos[(5*(c + d*x))/2] + 3*Cos[(7*(c + d*x))
/2] + 6*(146 + 136*c + 136*d*x + (-59 + 68*c + 68*d*x)*Cos[c + d*x] - 14*Co
s[2*(c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(93) = 186$.
time = 0.20, size = 268, normalized size = 2.65

method	result
risch	$\frac{17a^4x}{2} + \frac{ia^4 e^{2i(dx+c)}}{8d} - \frac{2a^4 e^{i(dx+c)}}{d} - \frac{2a^4 e^{-i(dx+c)}}{d} - \frac{ia^4 e^{-2i(dx+c)}}{8d} - \frac{8(-15ia^4 e^{i(dx+c)}+9a^4 e^{2i(dx+c)}-8a^4)}{3(e^{i(dx+c)}-i)^3 d}$
derivativedivides	$\frac{\frac{a^4(\sin^3(dx+c))}{3\cos(dx+c)^3}+4a^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{(2+\sin^2(dx+c))\cos(dx+c)}{3}\right)+6a^4\left(\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+dx+c\right)+4}{}$
default	$\frac{\frac{a^4(\sin^3(dx+c))}{3\cos(dx+c)^3}+4a^4\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{(2+\sin^2(dx+c))\cos(dx+c)}{3}\right)+6a^4\left(\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+dx+c\right)+4}{}$
norman	$\frac{-\frac{17a^4x}{2}+\frac{80a^4}{3d}+\frac{17a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{26a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{307a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{188a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{307a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^4*sin(d*x+c)^3/cos(d*x+c)^3+4*a^4*(1/3*sin(d*x+c)^4/cos(d*x+c)^3
-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(1/3*ta
n(d*x+c)^3-tan(d*x+c)+d*x+c)+4*a^4*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c
)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*sin
(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*si
n(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.49, size = 158, normalized size = 1.56

$$\frac{2\,a^4\tan\left(dx+c\right)^3+\left(2\,\tan\left(dx+c\right)^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\,\tan\left(dx+c\right)\right)a^4+12\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^4-8\,a^4\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-\frac{8\left(3\,\cos(dx+c)^2-1\right)a^4}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 1/6*(2*a^4*tan(d*x + c)^3 + (2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x +
 c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c))*a^4 + 12*(tan(d*x + c)^3 + 3*d*
x + 3*c - 3*tan(d*x + c))*a^4 - 8*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^
3 + 3*cos(d*x + c)) - 8*(3*cos(d*x + c)^2 - 1)*a^4/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(89) = 178$.

time = 0.37, size = 197, normalized size = 1.95

$$\frac{3\,a^4\cos\left(dx+c\right)^4-18\,a^4\cos\left(dx+c\right)^3-102\,a^4dx-8\,a^4+17\,\left(3\,a^4dx+5\,a^4\right)\cos\left(dx+c\right)^2-\left(51\,a^4dx-98\,a^4\right)\cos\left(dx+c\right)-\left(3\,a^4\cos\left(dx+c\right)^3-102\,a^4dx+21\,a^4\cos\left(dx+c\right)^2+8\,a^4-\left(51\,a^4dx-106\,a^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/6*(3*a^4*cos(d*x + c)^4 - 18*a^4*cos(d*x + c)^3 - 102*a^4*d*x - 8*a^4 + 1
7*(3*a^4*d*x + 5*a^4)*cos(d*x + c)^2 - (51*a^4*d*x - 98*a^4)*cos(d*x + c) -
 (3*a^4*cos(d*x + c)^3 - 102*a^4*d*x + 21*a^4*cos(d*x + c)^2 + 8*a^4 - (51*
a^4*d*x - 106*a^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*
x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.62, size = 135, normalized size = 1.34

$$\frac{51\,(dx+c)a^4+\frac{6\,\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{16\,\left(6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(51*(d*x + c)*a^4 + 6*(a^4*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2*d*x +
 1/2*c)^2 - a^4*tan(1/2*d*x + 1/2*c) - 8*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^
2 + 16*(6*a^4*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/2*c) + 7*a^4)
/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 14.85, size = 287, normalized size = 2.84

$$\frac{17\,a^4\,x}{2}+\frac{\frac{17\,a^4\,\left(c+d\,x\right)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{51\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(153\,c+153\,d\,x-378\right)}{6}\right)-\frac{a^4\,\left(51\,c+51\,d\,x-160\right)}{6}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{51\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(153\,c+153\,d\,x-102\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{85\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(255\,c+255\,d\,x-306\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{85\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(255\,c+255\,d\,x-494\right)}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{119\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(357\,c+357\,d\,x-460\right)}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{119\,a^4\,\left(c+d\,x\right)}{2}-\frac{a^4\,\left(357\,c+357\,d\,x-660\right)}{6}\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^4)/cos(c + d*x)^4,x)
```

```
[Out] (17*a^4*x)/2 + ((17*a^4*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((51*a^4*(c + d*x
))/2 - (a^4*(153*c + 153*d*x - 378))/6) - (a^4*(51*c + 51*d*x - 160))/6 + t
an(c/2 + (d*x)/2)^6*((51*a^4*(c + d*x))/2 - (a^4*(153*c + 153*d*x - 102))/6
) - tan(c/2 + (d*x)/2)^5*((85*a^4*(c + d*x))/2 - (a^4*(255*c + 255*d*x - 30
6))/6) + tan(c/2 + (d*x)/2)^2*((85*a^4*(c + d*x))/2 - (a^4*(255*c + 255*d*x
 - 494))/6) + tan(c/2 + (d*x)/2)^4*((119*a^4*(c + d*x))/2 - (a^4*(357*c + 3
57*d*x - 460))/6) - tan(c/2 + (d*x)/2)^3*((119*a^4*(c + d*x))/2 - (a^4*(357
*c + 357*d*x - 660))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^
2 + 1)^2)
```

3.821 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{x}{a}-\frac{\cos(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan(c+dx)}{ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -x/a-cos(d*x+c)/a/d-3*sec(d*x+c)/a/d+sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+
tan(d*x+c)/a/d-1/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2918, 3554, 8, 2670, 276}

$$-\frac{\cos(c+dx)}{ad}+\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{ad}-\frac{3\sec(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(x/a) - Cos[c + d*x]/(a*d) - (3*Sec[c + d*x])/(a*d) + Sec[c + d*x]^3/(a*d)
 - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]/(a*d) - Tan[c + d*x]^3/(3*a*d) + T
an[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \tan^6(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\tan^6(c+dx)\,dx}{a} \\
&= \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{ad} \\
&= -\frac{\tan^3(c+dx)}{3ad} + \frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}\right.\right.}{} \\
&= -\frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(c+d}{ad} \\
&= -\frac{x}{a} - \frac{\cos(c+dx)}{ad} - \frac{3\sec(c+dx)}{ad} + \frac{\sec^3(c+dx)}{ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan(}{}
\end{aligned}
$$

Mathematica [A]
time = 0.51, size = 224, normalized size = 1.91

$$
-\frac{1200+18(-103+40c+40dx)\cos(c+dx)+1568\cos(2(c+dx))-618\cos(3(c+dx))+240c\cos(3(c+dx))+240dx\cos(3(c+dx))+304\cos(4(c+dx))+216\sin(c+dx)-618\sin(2(c+dx))+240c\sin(2(c+dx))+240dx\sin(2(c+dx))+532\sin(3(c+dx))-309\sin(4(c+dx))+120c\sin(4(c+dx))+120dx\sin(4(c+dx))+60\sin(5(c+dx))}{960ad\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(1200 + 18*(-103 + 40*c + 40*d*x)*Cos[c + d*x] + 1568*Cos[2*(c + d*x
)] - 618*Cos[3*(c + d*x)] + 240*c*Cos[3*(c + d*x)] + 240*d*x*Cos[3*(c + d*x
)] + 304*Cos[4*(c + d*x)] + 216*Sin[c + d*x] - 618*Sin[2*(c + d*x)] + 240*c
*Sin[2*(c + d*x)] + 240*d*x*Sin[2*(c + d*x)] + 532*Sin[3*(c + d*x)] - 309*S
in[4*(c + d*x)] + 120*c*Sin[4*(c + d*x)] + 120*d*x*Sin[4*(c + d*x)] + 60*Si
n[5*(c + d*x)])/(a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^5)
```

Maple [A]
time = 0.29, size = 159, normalized size = 1.36

method	result

derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{ad}$
risch	$-\frac{x}{a}-\frac{e^{i(dx+c)}}{2ad}-\frac{e^{-i(dx+c)}}{2ad}-\frac{2\left(63\,e^{3i(dx+c)}+105ie^{4i(dx+c)}+91ie^{2i(dx+c)}+75\,e^{5i(dx+c)}+e^{i(dx+c)}+23i+45ie^{6i(dx}\right.}{15\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^5da}$
norman	$\frac{x}{a}-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{88\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-\frac{2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{4x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+7/1
024/(tan(1/2*d*x+1/2*c)-1)-1/320/(tan(1/2*d*x+1/2*c)+1)^5+1/128/(tan(1/2*d*
x+1/2*c)+1)^4+1/384/(tan(1/2*d*x+1/2*c)+1)^3-3/256/(tan(1/2*d*x+1/2*c)+1)^2
-23/1024/(tan(1/2*d*x+1/2*c)+1)-1/64/(1+tan(1/2*d*x+1/2*c)^2)-1/64*arctan(t
an(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(111) = 222$.

time = 0.50, size = 400, normalized size = 3.42

$$-\frac{2\left(\frac{\frac{81\sin(dx+c)}{\cos(dx+c)+1}-\frac{78\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{172\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{26\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{22\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{70\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{20\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{30\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{15\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+48}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{4a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/15*((81*sin(d*x + c)/(cos(d*x + c) + 1) - 78*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 172*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 26*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 22*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 70*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 20*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 30*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8 - 15*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 4
8)/(a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - a*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 4*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 4*a*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 - a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.37, size = 108, normalized size = 0.92

$$-\frac{15\,dx\cos(dx+c)^3+38\,\cos(dx+c)^4+11\,\cos(dx+c)^2+\left(15\,dx\cos(dx+c)^3+15\,\cos(dx+c)^4+22\,\cos(dx+c)^2-4\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*d*x*cos(d*x + c)^3 + 38*cos(d*x + c)^4 + 11*cos(d*x + c)^2 + (15*
d*x*cos(d*x + c)^3 + 15*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 4)*sin(d*x + c
) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.74, size = 149, normalized size = 1.27

$$-\frac{\frac{120\,(dx+c)}{a}+\frac{240}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{5\left(21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1560\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2570\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1720\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+413}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*(d*x + c)/a + 240/((tan(1/2*d*x + 1/2*c)^2 + 1)*a) - 5*(21*tan(
1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 23)/(a*(tan(1/2*d*x + 1/2*c)
 - 1)^3) + (345*tan(1/2*d*x + 1/2*c)^4 + 1560*tan(1/2*d*x + 1/2*c)^3 + 2570
*tan(1/2*d*x + 1/2*c)^2 + 1720*tan(1/2*d*x + 1/2*c) + 413)/(a*(tan(1/2*d*x
+ 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 18.67, size = 172, normalized size = 1.47

$$\frac{-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+\frac{28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{44\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}-\frac{52\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}-\frac{344\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{15}-\frac{52\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{5}+\frac{54\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}+\frac{32}{5}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] ((54*tan(c/2 + (d*x)/2))/5 - (52*tan(c/2 + (d*x)/2)^2)/5 - (344*tan(c/2 + (
d*x)/2)^3)/15 - (52*tan(c/2 + (d*x)/2)^4)/15 + (44*tan(c/2 + (d*x)/2)^5)/15
 + (28*tan(c/2 + (d*x)/2)^6)/3 + (8*tan(c/2 + (d*x)/2)^7)/3 - 4*tan(c/2 + (
d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + 32/5)/(a*d*(tan(c/2 + (d*x)/2) - 1)^3*
(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)) - x/a
```

3.822 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{x}{a}+\frac{\sec(c+dx)}{ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] x/a+sec(d*x+c)/a/d-2/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)^5/a/d-tan(d*x+c)/a/d
+1/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2918, 2686, 200, 3554, 8}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{2\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] x/a + Sec[c + d*x]/(a*d) - (2*Sec[c + d*x]^3)/(3*a*d) + Sec[c + d*x]^5/(5*a
*d) - Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \tan^6(c+dx)\,dx}{a}$$
$$= -\frac{\tan^5(c+dx)}{5ad} + \frac{\int \tan^4(c+dx)\,dx}{a} + \frac{\mathrm{Subst}\left(\int (-1+x^2)^2\,dx, x, \sec(c+a\right.}{ad}$$
$$= \frac{\tan^3(c+dx)}{3ad} - \frac{\tan^5(c+dx)}{5ad} - \frac{\int \tan^2(c+dx)\,dx}{a} + \frac{\mathrm{Subst}(\int (1-2x^2+x}{a}$$
$$= \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c+dx}{3ad}$$
$$= \frac{x}{a} + \frac{\sec(c+dx)}{ad} - \frac{2\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan(c+dx)}{ad} + \frac{\tan^3(c}{3a}$$

Mathematica [A]
time = 0.46, size = 191, normalized size = 1.82

$$-\frac{\sec^3(c+dx)\left(-25+\left(\frac{267}{4}-90c-90dx\right)\cos(c+dx)-16\cos(2(c+dx))+\frac{89}{4}\cos(3(c+dx))-30c\cos(3(c+dx))-30dx\cos(3(c+dx))-23\cos(4(c+dx))+8\sin(c+dx)+\frac{89}{4}\sin(2(c+dx))-30c\sin(2(c+dx))-30dx\sin(2(c+dx))+16\sin(3(c+dx))+\frac{89}{8}\sin(4(c+dx))-15c\sin(4(c+dx))-15dx\sin(4(c+dx))\right)}{120ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]^3*(-25 + (267/4 - 90*c - 90*d*x)*Cos[c + d*x] - 16*Cos
[2*(c + d*x)] + (89*Cos[3*(c + d*x)])/4 - 30*c*Cos[3*(c + d*x)] - 30*d*x*Co
s[3*(c + d*x)] - 23*Cos[4*(c + d*x)] + 8*Sin[c + d*x] + (89*Sin[2*(c + d*x)
])/4 - 30*c*Sin[2*(c + d*x)] - 30*d*x*Sin[2*(c + d*x)] + 16*Sin[3*(c + d*x)
] + (89*Sin[4*(c + d*x)])/8 - 15*c*Sin[4*(c + d*x)] - 15*d*x*Sin[4*(c + d*x
)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.22, size = 127, normalized size = 1.21

method	result

risch	$\frac{x}{a}+\frac{-2i\mathrm{e}^{6i(dx+c)}+2\,\mathrm{e}^{7i(dx+c)}+\frac{10i\mathrm{e}^{4i(dx+c)}}{3}+\frac{26\,\mathrm{e}^{5i(dx+c)}}{3}+\frac{62i\mathrm{e}^{2i(dx+c)}}{15}+\frac{146\,\mathrm{e}^{3i(dx+c)}}{15}+\frac{46i}{15}+\frac{62\,\mathrm{e}^{i(dx+c)}}{15}}{\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}da}$
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{ad}$
norman	$\frac{x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x}{a}-\frac{44\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+\frac{4x\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2+5/51
2/(tan(1/2*d*x+1/2*c)-1)+1/160/(tan(1/2*d*x+1/2*c)+1)^5-1/64/(tan(1/2*d*x+1
/2*c)+1)^4+1/64/(tan(1/2*d*x+1/2*c)+1)^2+11/512/(tan(1/2*d*x+1/2*c)+1)+1/32
*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(97) = 194$.
time = 0.51, size = 318, normalized size = 3.03

$$\frac{2\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{46\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{13\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{100\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{35\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{30\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+8}{a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*((sin(d*x + c)/(cos(d*x + c) + 1) - 46*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 13*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 100*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 35*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 30*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 8)/(a + 2*a*
sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*arctan(sin(d
*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]
time = 0.36, size = 98, normalized size = 0.93

$$\frac{15\,dx\cos\left(dx+c\right)^{3}+23\,\cos\left(dx+c\right)^{4}-19\,\cos\left(dx+c\right)^{2}+\left(15\,dx\cos\left(dx+c\right)^{3}-8\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+4}{15\,\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cos(d*x + c)^3 + 23*cos(d*x + c)^4 - 19*cos(d*x + c)^2 + (15*d
*x*cos(d*x + c)^3 - 8*cos(d*x + c)^2 + 1)*sin(d*x + c) + 4)/(a*d*cos(d*x +
c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.68, size = 130, normalized size = 1.24

$$\frac{\frac{120\,(dx+c)}{a}+\frac{5\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+17\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{3\left(55\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+260\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+300\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+71\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(120*(d*x + c)/a + 5*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/
2*c) + 17)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(55*tan(1/2*d*x + 1/2*c)^4
+ 260*tan(1/2*d*x + 1/2*c)^3 + 450*tan(1/2*d*x + 1/2*c)^2 + 300*tan(1/2*d*x
 + 1/2*c) + 71)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 16.14, size = 131, normalized size = 1.25

$$\frac{x}{a}-\frac{-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{40\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{26\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{15}-\frac{92\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{16}{15}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] x/a - ((2*tan(c/2 + (d*x)/2))/15 - (92*tan(c/2 + (d*x)/2)^2)/15 - (26*tan(c
/2 + (d*x)/2)^3)/15 + (40*tan(c/2 + (d*x)/2)^4)/3 + (14*tan(c/2 + (d*x)/2)^
5)/3 - 4*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + 16/15)/(a*d*(tan(c
/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.823 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sec(c+dx)}{ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -sec(d*x+c)/a/d+2/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/5*tan(d*x+c)^5/
a/d
```

Rubi [A]
time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2686, 200}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\tan^5(c+dx)}{5ad} - \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= -\frac{\sec(c+dx)}{ad} + \frac{2\sec^3(c+dx)}{3ad} - \frac{\sec^5(c+dx)}{5ad} + \frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.54

$$-\frac{\sec^3(c+dx)(200-534\cos(c+dx)+288\cos(2(c+dx))-178\cos(3(c+dx))+24\cos(4(c+dx))-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx)))}{960ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178
*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c +
d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(63) = 126$.
time = 0.20, size = 130, normalized size = 1.88

method	result
norman	$\frac{\frac{16}{15ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}-\frac{32\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{32\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

risch	$-\frac{2\left(25i\,e^{4i(dx+c)}+5\,e^{5i(dx+c)}+21i\,e^{2i(dx+c)}+13\,e^{3i(dx+c)}+15i\,e^{6i(dx+c)}+15\,e^{7i(dx+c)}-9\,e^{i(dx+c)}+3i\right)}{15\left(e^{i(dx+c)}+i\right)^{5}\left(e^{i(dx+c)}-i\right)^{3}ad}$
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(63) = 126$.
time = 0.29, size = 214, normalized size = 3.10

$$-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.09

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2+4\left(3\cos(dx+c)^2-1\right)\sin(dx+c)-1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.78, size = 120, normalized size = 1.74

$$\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+73}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 9.55, size = 73, normalized size = 1.06

$$\frac{16\left(-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] (16*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2)^3
 + 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.824 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -1/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)^5/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2918, 2686, 14, 2687, 30}

$$-\frac{\tan^5(c+dx)}{5ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*Sec[c + d*x]^3/(a*d) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^4\,dx, x, \tan(c+dx)\right)}{ad} + \frac{\operatorname{Subst}\left(\int x^2(-1+x^2)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= -\frac{\tan^5(c+dx)}{5ad} + \frac{\operatorname{Subst}\left(\int (-x^2+x^4)\,dx, x, \sec(c+dx)\right)}{ad}\\ &= -\frac{\sec^3(c+dx)}{3ad} + \frac{\sec^5(c+dx)}{5ad} - \frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 106, normalized size = 1.93

$$\frac{\sec^3(c+dx)(40+66\cos(c+dx)-192\cos(2(c+dx))+22\cos(3(c+dx))+24\cos(4(c+dx))+16\sin(c+dx)+22\sin(2(c+dx))-48\sin(3(c+dx))+11\sin(4(c+dx)))}{960ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^3*(40 + 66*Cos[c + d*x] - 192*Cos[2*(c + d*x)] + 22*Cos[3*(c
+ d*x)] + 24*Cos[4*(c + d*x)] + 16*Sin[c + d*x] + 22*Sin[2*(c + d*x)] - 48*
Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.
time = 0.20, size = 115, normalized size = 2.09

method	result
risch	$-\frac{2i\left(8i\,e^{3i(dx+c)}+9\,e^{2i(dx+c)}+6i\,e^{i(dx+c)}-5\,e^{4i(dx+c)}-3+10i\,e^{5i(dx+c)}+15\,e^{6i(dx+c)}\right)}{15\left(e^{i(dx+c)}+i\right)^5\left(e^{i(dx+c)}-i\right)^3 da}$
norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4}{15ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}-\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{16}{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-128}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{16}{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-128}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2+1/128/
(tan(1/2*d*x+1/2*c)-1)+1/40/(tan(1/2*d*x+1/2*c)+1)^5-1/16/(tan(1/2*d*x+1/2*
c)+1)^4+1/24/(tan(1/2*d*x+1/2*c)+1)^3-1/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(49) = 98$.
time = 0.29, size = 234, normalized size = 4.25

$$-\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{15\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*s
in(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.36

$$\frac{3\cos(dx+c)^{4}-9\cos(dx+c)^{2}-\left(3\cos(dx+c)^{2}-1\right)\sin(dx+c)+4}{15\left(ad\cos(dx+c)^{3}\sin(dx+c)+ad\cos(dx+c)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - (3*cos(d*x + c)^2 - 1)*sin(d*x
+ c) + 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(49) = 98.
time = 0.57, size = 120, normalized size = 2.18

$$\frac{\frac{5\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a*(tan(1
/2*d*x + 1/2*c) - 1)^3) - (15*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/2
*c)^3 + 10*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(1/
2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 9.86, size = 86, normalized size = 1.56

$$-\frac{4\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
 + 15*tan(c/2 + (d*x)/2)^4 - 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/
2 + (d*x)/2) + 1)^5)
```

3.825 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] 1/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+
c)^5/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2918, 2687, 14, 2686}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + Tan[c + d*x]^3/(3*a*d) +
Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a}-\frac{\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^2(-1+x^2)\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2(1+x^2)\,dx,x,\text{ta}\right.}{ad}\\ &= -\frac{\text{Subst}\left(\int(-x^2+x^4)\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int(x^2+x^4)\,dx,x,\tan(\right.}{ad}\\ &= \frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]

time = 0.25, size = 106, normalized size = 1.45

$$-\frac{\sec^3(c+dx)(-80+66\cos(c+dx)-32\cos(2(c+dx))+22\cos(3(c+dx))-16\cos(4(c+dx))-224\sin(c+dx)+22\sin(2(c+dx))+32\sin(3(c+dx))+11\sin(4(c+dx)))}{960ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(-80 + 66*Cos[c + d*x] - 32*Cos[2*(c + d*x)] + 22*Co
s[3*(c + d*x)] - 16*Cos[4*(c + d*x)] - 224*Sin[c + d*x] + 22*Sin[2*(c + d*x
)] + 32*Sin[3*(c + d*x)] + 11*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]

time = 0.20, size = 130, normalized size = 1.78

method	result
risch	$\frac{-\frac{16\,\mathrm{e}^{3i(dx+c)}}{15}+\frac{4i\mathrm{e}^{4i(dx+c)}}{3}+\frac{8i\mathrm{e}^{2i(dx+c)}}{15}+\frac{8\,\mathrm{e}^{5i(dx+c)}}{3}+\frac{8\,\mathrm{e}^{i(dx+c)}}{15}+\frac{4i}{15}}{\left(\mathrm{e}^{i(dx+c)}+i\right)^5\left(\mathrm{e}^{i(dx+c)}-i\right)^3 da}$
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}}{ad}$

norman	$\frac{-\frac{4\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{4}{15ad}-\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}+\frac{8\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/64/(t
an(1/2*d*x+1/2*c)-1)-1/20/(tan(1/2*d*x+1/2*c)+1)^5+1/8/(tan(1/2*d*x+1/2*c)+
1)^4-1/8/(tan(1/2*d*x+1/2*c)+1)^3+1/16/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1
/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(65) = 130$.
time = 0.28, size = 254, normalized size = 3.48

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x +
 c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.35, size = 73, normalized size = 1.00

$$\frac{2\cos(dx+c)^4-\cos(dx+c)^2-2\left(\cos(dx+c)^2-2\right)\sin(dx+c)+1}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 2)*sin(d*x +
c) + 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^2(c+dx)\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.59, size = 109, normalized size = 1.49

$$-\frac{\frac{5\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{3\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+50\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) -
 3*(5*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 50*tan(1/2*d*x +
 1/2*c)^2 + 40*tan(1/2*d*x + 1/2*c) + 9)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/
d
```

Mupad [B]
time = 10.16, size = 99, normalized size = 1.36

$$-\frac{4\left(10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{15\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(4*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^3
 + 5*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^5 + 1))/(15*a*d*(tan(c/2
+ (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.826 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sec^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] 1/5*sec(d*x+c)^5/a/d-1/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2918, 2686, 30, 2687, 14}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]^3/(3*a*d) - Tan[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^5(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec^4(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int x^4\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2(1+x^2)\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{\sec^5(c+dx)}{5ad}-\frac{\text{Subst}\left(\int (x^2+x^4)\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{\sec^5(c+dx)}{5ad}-\frac{\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 106, normalized size = 1.93

$$-\frac{\sec^3(c+dx)(-240+54\cos(c+dx)+32\cos(2(c+dx))+18\cos(3(c+dx))+16\cos(4(c+dx))-96\sin(c+dx)+18\sin(2(c+dx))-32\sin(3(c+dx))+9\sin(4(c+dx)))}{960ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/960*(Sec[c + d*x]^3*(-240 + 54*Cos[c + d*x] + 32*Cos[2*(c + d*x)] + 18*C
os[3*(c + d*x)] + 16*Cos[4*(c + d*x)] - 96*Sin[c + d*x] + 18*Sin[2*(c + d*x
)] - 32*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(49) = 98$.
time = 0.20, size = 130, normalized size = 2.36

method	result
risch	$\frac{4i\left(-2\,e^{2i(dx+c)}+2i e^{i(dx+c)}-1+6i e^{3i(dx+c)}+15\,e^{4i(dx+c)}\right)}{15\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^5 da}$
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{ad}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{ad}$

norman	$$-\frac{\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2}{5ad}-\frac{2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5ad}-\frac{4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-3/32/(t
an(1/2*d*x+1/2*c)-1)-1/4/(tan(1/2*d*x+1/2*c)+1)^4+1/10/(tan(1/2*d*x+1/2*c)+
1)^5+1/3/(tan(1/2*d*x+1/2*c)+1)^3-1/4/(tan(1/2*d*x+1/2*c)+1)^2+3/32/(tan(1/
2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(49) = 98$.
time = 0.29, size = 274, normalized size = 4.98

$$\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+3\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 10*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 3)/((a + 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c)
+ 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8)*d)
```

Fricas [A]
time = 0.34, size = 75, normalized size = 1.36

$$-\frac{2\cos(dx+c)^4-\cos(dx+c)^2-\left(2\cos(dx+c)^2+1\right)\sin(dx+c)-4}{15\left(ad\cos(dx+c)^3\sin(dx+c)+ad\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 - cos(d*x + c)^2 - (2*cos(d*x + c)^2 + 1)*sin(d*x +
 c) - 4)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin(c+dx)\sec^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(49) = 98.

time = 0.51, size = 120, normalized size = 2.18

$$-\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+60\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+70\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 60*tan(1/2*d*x + 1/
2*c)^3 + 70*tan(1/2*d*x + 1/2*c)^2 + 20*tan(1/2*d*x + 1/2*c) + 13)/(a*(tan(
1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]

time = 10.85, size = 112, normalized size = 2.04

$$-\frac{2\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] -(2*(6*tan(c/2 + (d*x)/2) + 9*tan(c/2 + (d*x)/2)^2 - 8*tan(c/2 + (d*x)/2)^3
 + 5*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^5 + 15*tan(c/2 + (d*x)/2)
^6 + 3))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.827 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=115

$$-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan(c+dx)}{ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+1/5*sec(d*x+c)
^5/a/d-tan(d*x+c)/a/d-2/3*tan(d*x+c)^3/a/d-1/5*tan(d*x+c)^5/a/d
```

Rubi [A]

time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2918, 2702, 308, 213, 3852}

$$-\frac{\tan^5(c+dx)}{5ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan(c+dx)}{ad}+\frac{\sec^5(c+dx)}{5ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/(3*a*d
) + Sec[c + d*x]^5/(5*a*d) - Tan[c + d*x]/(a*d) - (2*Tan[c + d*x]^3)/(3*a*d
) - Tan[c + d*x]^5/(5*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \sec^6(c+dx)\,dx}{a}+\frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c\right.}{ad}\\ &= -\frac{\tan(c+dx)}{ad}-\frac{2\tan^3(c+dx)}{3ad}-\frac{\tan^5(c+dx)}{5ad}+\frac{\text{Subst}\left(\int \left(1+x^2+x^4+\right.\right.}{}\\ &= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan(c+dx)}{ad}-\frac{2\tan^3(c+dx)}{3ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec^5(c+dx)}{5ad}-\frac{\tan}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 267 vs. 2(115) = 230.
time = 0.48, size = 267, normalized size = 2.32

$$-\frac{\sec^3(c+dx)\left(-100-76\cos(2(c+dx))+\frac{149}{4}\cos(3(c+dx))-8\cos(4(c+dx))+30\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(\frac{447}{4}+90\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-90\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-30\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-22\sin(c+dx)+\frac{149}{4}\sin(2(c+dx))+30\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-30\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))-14\sin(3(c+dx))+\frac{149}{8}\sin(4(c+dx))+15\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))\right)}{120ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/120*(Sec[c + d*x]^3*(-100 - 76*Cos[2*(c + d*x)] + (149*Cos[3*(c + d*x)])
/4 - 8*Cos[4*(c + d*x)] + 30*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + Cos[c
 + d*x]*(447/4 + 90*Log[Cos[(c + d*x)/2]] - 90*Log[Sin[(c + d*x)/2]]) - 30*
Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 22*Sin[c + d*x] + (149*Sin[2*(c +
d*x)])/4 + 30*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] - 30*Log[Sin[(c + d*x)
/2]]*Sin[2*(c + d*x)] - 14*Sin[3*(c + d*x)] + (149*Sin[4*(c + d*x)])/8 + 15
*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] - 15*Log[Sin[(c + d*x)/2]]*Sin[4*(c
 + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]

time = 0.26, size = 139, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{7}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{da}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{7}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{da}$
risch	$\frac{4i e^{6i(dx+c)}+2\,e^{7i(dx+c)}+\frac{40i e^{4i(dx+c)}}{3}+\frac{14\,e^{5i(dx+c)}}{3}+\frac{92i e^{2i(dx+c)}}{15}+\frac{26\,e^{3i(dx+c)}}{15}+\frac{16i}{15}+\frac{2\,e^{i(dx+c)}}{15}}{\left(e^{i(dx+c)}+i\right)^{5}\left(e^{i(dx+c)}-i\right)^{3}da}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{ad}$
norman	$\frac{-\frac{10\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{46}{15ad}-\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{26\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{62\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}+\frac{62\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/6/(tan(1/2*d*x+1/2*c)-1)^3-1/4/(tan(1/2*d*x+1/2*c)-1)^2-7/8/(tan(
1/2*d*x+1/2*c)-1)+2/5/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*d*x+1/2*c)+1)^4+2
/(tan(1/2*d*x+1/2*c)+1)^3-2/(tan(1/2*d*x+1/2*c)+1)^2+23/8/(tan(1/2*d*x+1/2*
c)+1)+ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(107) = 214.

time = 0.30, size = 320, normalized size = 2.78

$$\frac{\frac{2\left(\frac{31\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{31\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{73\,\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{25\,\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{65\,\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\,\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{15\,\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+23\right)}{a+\frac{2\,a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\,a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6\,a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2\,a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{15\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/15*(2*(31*sin(d*x + c)/(cos(d*x + c) + 1) - 31*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 73*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 25*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 65*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 23)/(a +
 2*a*sin(d*x + c)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - 6*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 + 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin
(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A]

time = 0.38, size = 149, normalized size = 1.30

$$\frac{16\,\cos\left(dx+c\right)^{4}+22\,\cos\left(dx+c\right)^{2}-15\left(\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{3}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{3}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(7\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)+8}{30\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(16*cos(d*x + c)^4 + 22*cos(d*x + c)^2 - 15*(cos(d*x + c)^3*sin(d*x +
c) + cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^3*sin(d
*x + c) + cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(7*cos(d*x + c)^
2 + 1)*sin(d*x + c) + 8)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c
)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc (c+dx) \sec^4 (c+dx)}{\sin (c+dx)+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A]
time = 0.50, size = 136, normalized size = 1.18

$$\frac{\frac{120 \log \left(\left|\tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)\right|\right)}{a}-\frac{5 \left(21 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}-36 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)+19\right)}{a \left(\tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)-1\right)^{3}}+\frac{3 \left(115 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{4}+380 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{3}+530 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^{2}+340 \tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)+91\right)}{a \left(\tan \left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)+1\right)^{5}}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 5*(21*tan(1/2*d*x + 1/2*c)^2
- 36*tan(1/2*d*x + 1/2*c) + 19)/(a*(tan(1/2*d*x + 1/2*c) - 1)^3) + 3*(115*t
an(1/2*d*x + 1/2*c)^4 + 380*tan(1/2*d*x + 1/2*c)^3 + 530*tan(1/2*d*x + 1/2*
c)^2 + 340*tan(1/2*d*x + 1/2*c) + 91)/(a*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

Mupad [B]
time = 11.53, size = 143, normalized size = 1.24

$$\frac{\ln \left(\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{-2\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+2\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\frac{26\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{3}+\frac{10\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{3}-\frac{146\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{15}-\frac{62\,{\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{15}+\frac{62\,\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{46}{15}}{a\,d\,{\left(\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3\,{\left(\tan \left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - ((62*tan(c/2 + (d*x)/2))/15 - (62*tan(c/2 +
 (d*x)/2)^2)/15 - (146*tan(c/2 + (d*x)/2)^3)/15 + (10*tan(c/2 + (d*x)/2)^4)
/3 + (26*tan(c/2 + (d*x)/2)^5)/3 + 2*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*
x)/2)^7 + 46/15)/(a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5
)
```

3.828 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{3\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{ad}$$

```
[Out] arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d-sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d-
1/5*sec(d*x+c)^5/a/d+3*tan(d*x+c)/a/d+tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2918, 2700, 276, 2702, 308, 213}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{\tan^3(c+dx)}{ad}+\frac{3\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}+\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d) - Sec[c + d*x]/(a*d) - Sec
[c + d*x]^3/(3*a*d) - Sec[c + d*x]^5/(5*a*d) + (3*Tan[c + d*x])/(a*d) + Tan
[c + d*x]^3/(a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\sec^6(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\sec^6(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2}\,dx,x,\tan(c+dx)\right.}{ad}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+3x^2+x^4\right)\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+x^2+x^4\right.\right.}{}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{3\tan(c+d}{ad}\\
&= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}-\frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}-\frac{\sec^5(}{5}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 341 vs. 2(126) = 252.
time = 0.46, size = 341, normalized size = 2.71

$$
-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(176+1216\cos(2(c+dx))+149\cos(3(c+dx))+528\cos(4(c+dx))+149\cos(5(c+dx))+120\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-120\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-120\cos(5(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(c+dx)\left(-298-240\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+240\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+352\sin(c+dx)-596\sin(2(c+dx))-480\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+480\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+864\sin(3(c+dx))-298\sin(4(c+dx))-240\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+240\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+384\sin(5(c+dx))\right)}{3840ad(1+\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3840*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]^3*(176 + 1216*Cos[2
*(c + d*x)] + 149*Cos[3*(c + d*x)] + 528*Cos[4*(c + d*x)] + 149*Cos[5*(c +
```

```
d*x)] + 120*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 120*Cos[5*(c + d*x)]*L
og[Cos[(c + d*x)/2]] - 120*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 120*Cos
[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]*(-298 - 240*Log[Cos[(c +
 d*x)/2]] + 240*Log[Sin[(c + d*x)/2]]) + 352*Sin[c + d*x] - 596*Sin[2*(c +
d*x)] - 480*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)] + 480*Log[Sin[(c + d*x)/
2]]*Sin[2*(c + d*x)] + 864*Sin[3*(c + d*x)] - 298*Sin[4*(c + d*x)] - 240*Lo
g[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 240*Log[Sin[(c + d*x)/2]]*Sin[4*(c +
 d*x)] + 384*Sin[5*(c + d*x)]))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.30, size = 164, normalized size = 1.30

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{9}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{}{3(}}{2da}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{9}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{}{3(}}{2da}$
risch	$-\frac{2\left(20\,\mathrm{e}^{7i(dx+c)}+30i\mathrm{e}^{8i(dx+c)}-22\,\mathrm{e}^{5i(dx+c)}+70i\mathrm{e}^{6i(dx+c)}-172\,\mathrm{e}^{3i(dx+c)}-81\,\mathrm{e}^{i(dx+c)}-78i\mathrm{e}^{2i(dx+c)}-48i+26i\mathrm{e}^{4i(dx+c}\right.}{15\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}ad}$
norman	$\frac{\frac{1}{2ad}+\frac{20\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{19\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{35\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{23\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{133\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a*(tan(1/2*d*x+1/2*c)-1/3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(tan(1/2*d*x+1
/2*c)-1)^2-9/4/(tan(1/2*d*x+1/2*c)-1)-4/5/(tan(1/2*d*x+1/2*c)+1)^5+2/(tan(1
/2*d*x+1/2*c)+1)^4-14/3/(tan(1/2*d*x+1/2*c)+1)^3+5/(tan(1/2*d*x+1/2*c)+1)^2
-39/4/(tan(1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(120) = 240.
time = 0.30, size = 379, normalized size = 3.01

$$-\frac{\frac{\frac{122\sin(dx+c)}{\cos(dx+c)+1}-\frac{26\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{454\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{252\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{510\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{330\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{210\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{195\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+15}{\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{2a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{6a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{6a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{2a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{a\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{15\sin(dx+c)}{a(\cos(dx+c)+1)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*((122*sin(d*x + c)/(cos(d*x + c) + 1) - 26*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 454*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 252*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 510*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 330*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6 - 210*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 195
```

```
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15)/(a*sin(d*x + c)/(cos(d*x + c) +
1) + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*a*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 6*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*a*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - a*sin(d*x + c)^9/(cos(d*x + c) + 1)^9) + 30
*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - 15*sin(d*x + c)/(a*(cos(d*x + c)
+ 1)))/d
```

Fricas [A]
time = 0.38, size = 194, normalized size = 1.54

$$\frac{66 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + 15 \left(\cos(dx+c)^5 - \cos(dx+c)^3 \sin(dx+c) - \cos(dx+c)^3\right) \log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 15 \left(\cos(dx+c)^5 - \cos(dx+c)^3 \sin(dx+c) - \cos(dx+c)^3\right) \log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 2 \left(48 \cos(dx+c)^4 - 9 \cos(dx+c)^2 - 1\right) \sin(dx+c) - 8}{30 \left(ad \cos(dx+c)^5 - ad \cos(dx+c)^3 \sin(dx+c) - ad \cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(66*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 15*(cos
(d*x + c)^5 - cos(d*x + c)^3*sin(d*x + c) - cos(d*x + c)^3)*log(-1/2*cos(d*
x + c) + 1/2) + 2*(48*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 1)*sin(d*x + c) -
 8)/(a*d*cos(d*x + c)^5 - a*d*cos(d*x + c)^3*sin(d*x + c) - a*d*cos(d*x + c
)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.76, size = 178, normalized size = 1.41

$$-\frac{\frac{120 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{60 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a} - \frac{60 \left(2 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} + \frac{5 \left(27 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 48 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+25\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3} + \frac{585 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 2040 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 2890 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 1880 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+493}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - 60*tan(1/2*d*x + 1/2*c)/a -
60*(2*tan(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)) + 5*(27*tan(1/2*d*
x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a*(tan(1/2*d*x + 1/2*c) - 1)^
3) + (585*tan(1/2*d*x + 1/2*c)^4 + 2040*tan(1/2*d*x + 1/2*c)^3 + 2890*tan(1
```

```
/2*d*x + 1/2*c)^2 + 1880*tan(1/2*d*x + 1/2*c) + 493)/(a*(tan(1/2*d*x + 1/2*
c) + 1)^5))/d
```

Mupad [B]
time = 10.82, size = 257, normalized size = 2.04

$$\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+14\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-22\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-34\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{5}+\frac{454\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{15}+\frac{26\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}-\frac{122\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15}-1}{d\left(-2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] ((26*tan(c/2 + (d*x)/2)^2)/15 - (122*tan(c/2 + (d*x)/2))/15 + (454*tan(c/2
+ (d*x)/2)^3)/15 + (84*tan(c/2 + (d*x)/2)^4)/5 - 34*tan(c/2 + (d*x)/2)^5 -
22*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^7 + 13*tan(c/2 + (d*x)/2)^8
 - 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 +
 (d*x)/2)^3 - 12*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^6 + 4*a*t
an(c/2 + (d*x)/2)^7 - 4*a*tan(c/2 + (d*x)/2)^8 - 2*a*tan(c/2 + (d*x)/2)^9))
 - log(tan(c/2 + (d*x)/2))/(a*d) + tan(c/2 + (d*x)/2)/(2*a*d)
```

3.829 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=155

$$-\frac{2x}{a^2}-\frac{\cos(c+dx)}{a^2d}-\frac{5\sec(c+dx)}{a^2d}+\frac{3\sec^3(c+dx)}{a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(c}{3a^2}$$

```
[Out] -2*x/a^2-cos(d*x+c)/a^2/d-5*sec(d*x+c)/a^2/d+3*sec(d*x+c)^3/a^2/d-7/5*sec(d
*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d+2*tan(d*x+c)/a^2/d-2/3*tan(d*x+c)^3/a^
2/d+2/5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2686, 200, 3554, 8, 2670, 276}

$$-\frac{\cos(c+dx)}{a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+\frac{3\sec^3(c+dx)}{a^2d}-\frac{5\sec(c+dx)}{a^2d}-\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*x)/a^2 - Cos[c + d*x]/(a^2*d) - (5*Sec[c + d*x])/(a^2*d) + (3*Sec[c + d
*x]^3)/(a^2*d) - (7*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d
) + (2*Tan[c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(3*a^2*d) + (2*Tan[c + d*
x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec(c+dx)(a-a\sin(c+dx))^2\tan^7(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\sec(c+dx)\tan^7(c+dx)-2a^2\tan^8(c+dx)+a^2\sin(c+dx)\tan^8(c\right.}{a^4} \\
&= \frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2}+\frac{\int \sin(c+dx)\tan^8(c+dx)\,dx}{a^2}-\frac{2\int \tan^8(}{} \\
&= -\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\int \tan^6(c+dx)\,dx}{a^2}-\frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^4}{x^8}\,dx,x,\cos(c+d\right.}{a^2d} \\
&= \frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\int \tan^4(c+dx)\,dx}{a^2}-\frac{\operatorname{Subst}\left(\int \left(1+\frac{1}{x^8}\right.\right.}{} \\
&= -\frac{\cos(c+dx)}{a^2d}-\frac{5\sec(c+dx)}{a^2d}+\frac{3\sec^3(c+dx)}{a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(}{7} \\
&= -\frac{\cos(c+dx)}{a^2d}-\frac{5\sec(c+dx)}{a^2d}+\frac{3\sec^3(c+dx)}{a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(}{7} \\
&= -\frac{2x}{a^2}-\frac{\cos(c+dx)}{a^2d}-\frac{5\sec(c+dx)}{a^2d}+\frac{3\sec^3(c+dx)}{a^2d}-\frac{7\sec^5(c+dx)}{5a^2d}+
\end{aligned}
$$

Mathematica [A]
time = 0.58, size = 267, normalized size = 1.72

$$
-\frac{11172+42(-551+280c+280dx)\cos(c+dx)+14834\cos(2(c+dx))-4959\cos(3(c+dx))+2520c\cos(3(c+dx))+2520dx\cos(3(c+dx))+1852\cos(4(c+dx))+1653\cos(5(c+dx))-840c\cos(5(c+dx))-840dx\cos(5(c+dx))-210\cos(6(c+dx))+5488\sin(c+dx)-13224\sin(2(c+dx))+6720c\sin(2(c+dx))+6720dx\sin(2(c+dx))+8376\sin(3(c+dx))-6612\sin(4(c+dx))+3360c\sin(4(c+dx))+3360dx\sin(4(c+dx))+2248\sin(5(c+dx))}{6720a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/6720*(11172 + 42*(-551 + 280*c + 280*d*x)*Cos[c + d*x] + 14834*Cos[2*(c
+ d*x)] - 4959*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*
(c + d*x)] + 1852*Cos[4*(c + d*x)] + 1653*Cos[5*(c + d*x)] - 840*c*Cos[5*(c
 + d*x)] - 840*d*x*Cos[5*(c + d*x)] - 210*Cos[6*(c + d*x)] + 5488*Sin[c + d
*x] - 13224*Sin[2*(c + d*x)] + 6720*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c
+ d*x)] + 8376*Sin[3*(c + d*x)] - 6612*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c +
 d*x)] + 3360*d*x*Sin[4*(c + d*x)] + 2248*Sin[5*(c + d*x)])/(a^2*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.44, size = 189, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 256/d/a^2*(-1/3072/(tan(1/2*d*x+1/2*c)-1)^3-1/2048/(tan(1/2*d*x+1/2*c)-1)^2
+1/512/(tan(1/2*d*x+1/2*c)-1)+1/448/(tan(1/2*d*x+1/2*c)+1)^7-1/128/(tan(1/2
```

```
*d*x+1/2*c)+1)^6+3/640/(tan(1/2*d*x+1/2*c)+1)^5+1/128/(tan(1/2*d*x+1/2*c)+1
)^4-1/3072/(tan(1/2*d*x+1/2*c)+1)^3-23/2048/(tan(1/2*d*x+1/2*c)+1)^2-9/512/
(tan(1/2*d*x+1/2*c)+1)-1/128/(1+tan(1/2*d*x+1/2*c)^2)-1/64*arctan(tan(1/2*d
*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(145) = 290$.

time = 0.54, size = 507, normalized size = 3.27

$$-\frac{4\left(\frac{\frac{759\sin(dx+c)}{\cos(dx+c)+1}+\frac{444\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1249\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1816\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{454\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{616\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1274\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{560\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{385\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{420\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{105\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+216}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{8a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{11a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{4a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{4a^2\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -4/105*((759*sin(d*x + c)/(cos(d*x + c) + 1) + 444*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 1249*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1816*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 454*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 616*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 1274*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
+ 560*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 385*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 420*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 105*sin(d*x + c)^11/
(cos(d*x + c) + 1)^11 + 216)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) +
 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*a^2*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 11*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 11*
a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 4*a^2*sin(d*x + c)^9/(cos(d*x + c
) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)
^11/(cos(d*x + c) + 1)^11 - a^2*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 10
5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A]

time = 0.36, size = 150, normalized size = 0.97

$$-\frac{210\,dx\cos(dx+c)^5+105\cos(dx+c)^6-420\,dx\cos(dx+c)^3-389\cos(dx+c)^4-173\cos(dx+c)^2-2\left(210\,dx\cos(dx+c)^3+281\cos(dx+c)^4+51\cos(dx+c)^2-5\right)\sin(dx+c)+25}{105\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(210*d*x*cos(d*x + c)^5 + 105*cos(d*x + c)^6 - 420*d*x*cos(d*x + c)^
3 - 389*cos(d*x + c)^4 - 173*cos(d*x + c)^2 - 2*(210*d*x*cos(d*x + c)^3 + 2
81*cos(d*x + c)^4 + 51*cos(d*x + c)^2 - 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*
x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.61, size = 175, normalized size = 1.13

$$-\frac{\frac{1680\,(dx+c)}{a^2}+\frac{1680}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}-\frac{35\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{3780\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+25095\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+68845\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+98350\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75222\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+29659\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4777}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*(d*x + c)/a^2 + 1680/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^2) - 35*(
12*tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 13)/(a^2*(tan(1/2*d*x
 + 1/2*c) - 1)^3) + (3780*tan(1/2*d*x + 1/2*c)^6 + 25095*tan(1/2*d*x + 1/2*
c)^5 + 68845*tan(1/2*d*x + 1/2*c)^4 + 98350*tan(1/2*d*x + 1/2*c)^3 + 75222*
tan(1/2*d*x + 1/2*c)^2 + 29659*tan(1/2*d*x + 1/2*c) + 4777)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 17.73, size = 198, normalized size = 1.28

$$\frac{-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{44\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}+\frac{64\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{728\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15}+\frac{352\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}-\frac{1816\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{105}-\frac{7264\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{105}-\frac{4996\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{105}+\frac{592\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+\frac{1012\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}+\frac{288}{35}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{2\,x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((1012*tan(c/2 + (d*x)/2))/35 + (592*tan(c/2 + (d*x)/2)^2)/35 - (4996*tan(c
/2 + (d*x)/2)^3)/105 - (7264*tan(c/2 + (d*x)/2)^4)/105 - (1816*tan(c/2 + (d
*x)/2)^5)/105 + (352*tan(c/2 + (d*x)/2)^6)/15 + (728*tan(c/2 + (d*x)/2)^7)/
15 + (64*tan(c/2 + (d*x)/2)^8)/3 - (44*tan(c/2 + (d*x)/2)^9)/3 - 16*tan(c/2
 + (d*x)/2)^10 - 4*tan(c/2 + (d*x)/2)^11 + 288/35)/(a^2*d*(tan(c/2 + (d*x)/
2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7*(tan(c/2 + (d*x)/2)^2 + 1)) - (2*x)/a^
2
```

3.830 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=140

$$\frac{x}{a^2}+\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan^5(c+dx)}{5a^2d}$$

```
[Out] x/a^2+2*sec(d*x+c)/a^2/d-2*sec(d*x+c)^3/a^2/d+6/5*sec(d*x+c)^5/a^2/d-2/7*se
c(d*x+c)^7/a^2/d-tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d-1/5*tan(d*x+c)^5/a
^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]

time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2687, 30, 2686, 200, 3554, 8}

$$\frac{2\tan^7(c+dx)}{7a^2d}-\frac{\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{6\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{a^2d}+\frac{2\sec(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] x/a^2 + (2*Sec[c + d*x])/(a^2*d) - (2*Sec[c + d*x]^3)/(a^2*d) + (6*Sec[c +
d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) - Tan[c + d*x]/(a^2*d) + T
an[c + d*x]^3/(3*a^2*d) - Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*
a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \sec^2(c+dx)(a-a\sin(c+dx))^2\tan^6(c+dx)\,dx}{a^4}$$

$$= \frac{\int \left(a^2\sec^2(c+dx)\tan^6(c+dx) - 2a^2\sec(c+dx)\tan^7(c+dx) + a^2\tan^8(\right.}{a^4}$$

$$= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2} + \frac{\int \tan^8(c+dx)\,dx}{a^2} - \frac{2\int \sec(c+dx)\,\mathrm{ta}}{a^2}$$

$$= \frac{\tan^7(c+dx)}{7a^2d} - \frac{\int \tan^6(c+dx)\,dx}{a^2} + \frac{\mathrm{Subst}\left(\int x^6\,dx, x, \tan(c+dx)\right)}{a^2d} - \frac{2}{}$$

$$= -\frac{\tan^5(c+dx)}{5a^2d} + \frac{2\tan^7(c+dx)}{7a^2d} + \frac{\int \tan^4(c+dx)\,dx}{a^2} - \frac{2\mathrm{Subst}\left(\int (-1+\right.}{}$$

$$= \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} + \frac{\tan^3}{}$$

$$= \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} - \frac{\tan(}{}$$

$$= \frac{x}{a^2} + \frac{2\sec(c+dx)}{a^2d} - \frac{2\sec^3(c+dx)}{a^2d} + \frac{6\sec^5(c+dx)}{5a^2d} - \frac{2\sec^7(c+dx)}{7a^2d} -$$

Mathematica [A]
time = 0.45, size = 257, normalized size = 1.84

$$\frac{4032 + 42(-381 + 280c + 280dx)\cos(c+dx) + 5504\cos(2(c+dx)) - 3429\cos(3(c+dx)) + 2520c\cos(3(c+dx)) + 2520dx\cos(3(c+dx)) + 2752\cos(4(c+dx)) + 1143\cos(5(c+dx)) - 840c\cos(5(c+dx)) - 840dx\cos(5(c+dx)) + 2128\sin(c+dx) - 9144\sin(2(c+dx)) + 6720c\sin(2(c+dx)) + 6720dx\sin(2(c+dx)) + 456\sin(3(c+dx)) - 4572\sin(4(c+dx)) + 3360c\sin(4(c+dx)) + 3360dx\sin(4(c+dx)) + 1528\sin(5(c+dx))}{13440a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4032 + 42*(-381 + 280*c + 280*d*x)*Cos[c + d*x] + 5504*Cos[2*(c + d*x)] -
3429*Cos[3*(c + d*x)] + 2520*c*Cos[3*(c + d*x)] + 2520*d*x*Cos[3*(c + d*x)]
 + 2752*Cos[4*(c + d*x)] + 1143*Cos[5*(c + d*x)] - 840*c*Cos[5*(c + d*x)] -
 840*d*x*Cos[5*(c + d*x)] + 2128*Sin[c + d*x] - 9144*Sin[2*(c + d*x)] + 672
0*c*Sin[2*(c + d*x)] + 6720*d*x*Sin[2*(c + d*x)] + 456*Sin[3*(c + d*x)] - 4
572*Sin[4*(c + d*x)] + 3360*c*Sin[4*(c + d*x)] + 3360*d*x*Sin[4*(c + d*x)]
+ 1528*Sin[5*(c + d*x)])/(13440*a^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.39, size = 172, normalized size = 1.23

method	result
risch	$\frac{x}{a^2} + \frac{-\frac{48\,e^{5i(dx+c)}}{5} + 28ie^{6i(dx+c)} + \frac{344ie^{4i(dx+c)}}{15} + 8\,e^{7i(dx+c)} - \frac{2216\,e^{3i(dx+c)}}{105} + 6ie^{8i(dx+c)} + 4\,e^{9i(dx+c)} + \frac{172ie^{2i(dx+c}}{35}}{\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^7 d\,a^2}$

derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{a^{2}d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{a^{2}d}$
norman	$\frac{x\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{8\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{x}{a}-\frac{64}{35ad}-\frac{186\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}+\frac{5x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4x\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{5x\left(\tan^{2}\left(\frac{dx}{2}+\right.\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^2*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2
+3/1024/(tan(1/2*d*x+1/2*c)-1)-1/224/(tan(1/2*d*x+1/2*c)+1)^7+1/64/(tan(1/2
*d*x+1/2*c)+1)^6-1/80/(tan(1/2*d*x+1/2*c)+1)^5-1/128/(tan(1/2*d*x+1/2*c)+1)
^4+5/1536/(tan(1/2*d*x+1/2*c)+1)^3+11/1024/(tan(1/2*d*x+1/2*c)+1)^2+13/1024
/(tan(1/2*d*x+1/2*c)+1)+1/64*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(130) = 260$.
time = 0.53, size = 421, normalized size = 3.01

$$\frac{2\left(\frac{\frac{279\sin(dx+c)}{\cos(dx+c)+1}-\frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1048\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{364\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{980\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{280\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{420\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{105\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+96}{a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 2/105*((279*sin(d*x + c)/(cos(d*x + c) + 1) - 132*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 1048*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 364*sin(d*x + c)^4/
(cos(d*x + c) + 1)^4 + 1554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 980*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - 280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
420*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 105*sin(d*x + c)^9/(cos(d*x + c)
+ 1)^9 + 96)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14
*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*arctan(sin(d*x + c)/(cos(d*x +
c) + 1))/a^2)/d
```

Fricas [A]
time = 0.37, size = 140, normalized size = 1.00

$$\frac{105\,dx\cos(dx+c)^5-210\,dx\cos(dx+c)^3-172\cos(dx+c)^4+86\cos(dx+c)^2-\left(210\,dx\cos(dx+c)^3+191\cos(dx+c)^4-129\cos(dx+c)^2+25\right)\sin(dx+c)-10}{105\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/105*(105*d*x*cos(d*x + c)^5 - 210*d*x*cos(d*x + c)^3 - 172*cos(d*x + c)^4
 + 86*cos(d*x + c)^2 - (210*d*x*cos(d*x + c)^3 + 191*cos(d*x + c)^4 - 129*c
os(d*x + c)^2 + 25)*sin(d*x + c) - 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(
d*x + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**A**]
time = 0.60, size = 155, normalized size = 1.11

$$\frac{\frac{840\,(dx+c)}{a^2}+\frac{35\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{1365\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+9345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+26600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+39410\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30261\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+11837\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1886}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)/a^2 + 35*(9*tan(1/2*d*x + 1/2*c)^2 - 21*tan(1/2*d*x +
1/2*c) + 10)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (1365*tan(1/2*d*x + 1/2*c
)^6 + 9345*tan(1/2*d*x + 1/2*c)^5 + 26600*tan(1/2*d*x + 1/2*c)^4 + 39410*ta
n(1/2*d*x + 1/2*c)^3 + 30261*tan(1/2*d*x + 1/2*c)^2 + 11837*tan(1/2*d*x + 1
/2*c) + 1886)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [**B**]
time = 16.89, size = 156, normalized size = 1.11

$$\frac{x}{a^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}-\frac{148\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{104\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{2096\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{105}+\frac{88\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}-\frac{186\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}-\frac{64}{35}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] x/a^2 + ((88*tan(c/2 + (d*x)/2)^2)/35 - (186*tan(c/2 + (d*x)/2))/35 + (2096
*tan(c/2 + (d*x)/2)^3)/105 + (104*tan(c/2 + (d*x)/2)^4)/15 - (148*tan(c/2 +
 (d*x)/2)^5)/5 - (56*tan(c/2 + (d*x)/2)^6)/3 + (16*tan(c/2 + (d*x)/2)^7)/3
+ 8*tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^9 - 64/35)/(a^2*d*(tan(c/2
+ (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7)
```

3.831 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}$$

```
[Out] -sec(d*x+c)/a^2/d+4/3*sec(d*x+c)^3/a^2/d-sec(d*x+c)^5/a^2/d+2/7*sec(d*x+c)^
7/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2686, 276, 2687, 30, 200}

$$-\frac{2\tan^7(c+dx)}{7a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]/(a^2*d)) + (4*Sec[c + d*x]^3)/(3*a^2*d) - Sec[c + d*x]^5/(a^
2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^2\tan^5(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\sec^3(c+dx)\tan^5(c+dx)-2a^2\sec^2(c+dx)\tan^6(c+dx)+a^2\sec(c\right.}{a^4} \\
&= \frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a^2}-\frac{2\int \sec}{} \\
&= \frac{\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\mathrm{s}\right.}{a^2d} \\
&= -\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\operatorname{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+ \\
&= -\frac{\sec(c+dx)}{a^2d}+\frac{4\sec^3(c+dx)}{3a^2d}-\frac{\sec^5(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^7(}{7a}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 126, normalized size = 1.48

$$-\frac{\sec^3(c+dx)(42-182\cos(c+dx)+104\cos(2(c+dx))-39\cos(3(c+dx))-18\cos(4(c+dx))+13\cos(5(c+dx))+28\sin(c+dx)-104\sin(2(c+dx))+66\sin(3(c+dx))-52\sin(4(c+dx))+6\sin(5(c+dx)))}{336a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/336*(Sec[c + d*x]^3*(42 - 182*Cos[c + d*x] + 104*Cos[2*(c + d*x)] - 39*C
os[3*(c + d*x)] - 18*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 28*Sin[c + d*
x] - 104*Sin[2*(c + d*x)] + 66*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 6*S
in[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.35, size = 145, normalized size = 1.71

method	result
risch	$-\frac{2\left(42i e^{8i(dx+c)}+21\,e^{9i(dx+c)}+56i e^{6i(dx+c)}-28\,e^{7i(dx+c)}+28i e^{4i(dx+c)}-42\,e^{5i(dx+c)}-24i e^{2i(dx+c)}-76\,e^{3i(dx+c)}-6i-\right.}{21\left(e^{i(dx+c)}-i\right)^{3}\left(e^{i(dx+c)}+i\right)^{7} d\,a^{2}}$
derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{64}{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{a^{2}d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{64}{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{a^{2}d}$
norman	$\frac{-\frac{32\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{16}{21ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{21ad}+\frac{64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{64\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{176\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{128\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}}{a\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^2*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)+1/112/(tan(1/2*d*x+1/2*c)+1)^7-1/32/(tan(1/2*d*x
+1/2*c)+1)^6+1/32/(tan(1/2*d*x+1/2*c)+1)^5-5/768/(tan(1/2*d*x+1/2*c)+1)^3-3
/512/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(79) = 158.
time = 0.31, size = 296, normalized size = 3.48

$$-\frac{16\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)}{21\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{4a^{2}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -16/21*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*
```

```
x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 -
 a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A]
time = 0.36, size = 104, normalized size = 1.22

$$-\frac{9\,\cos\left(dx+c\right)^{4}-22\,\cos\left(dx+c\right)^{2}-2\,\left(3\,\cos\left(dx+c\right)^{4}+6\,\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)+5}{21\,\left(a^{2}d\cos\left(dx+c\right)^{5}-2\,a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/21*(9*cos(d*x + c)^4 - 22*cos(d*x + c)^2 - 2*(3*cos(d*x + c)^4 + 6*cos(d
*x + c)^2 - 1)*sin(d*x + c) + 5)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.77, size = 146, normalized size = 1.72

$$\frac{\frac{7\,\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{42\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1750\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1344\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+511\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+79}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(6*tan(1/2*d*x + 1/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 7)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (42*tan(1/2*d*x + 1/2*c)^6 + 315*tan(1/2*d*x +
1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1750*tan(1/2*d*x + 1/2*c)^3 + 1344
*tan(1/2*d*x + 1/2*c)^2 + 511*tan(1/2*d*x + 1/2*c) + 79)/(a^2*(tan(1/2*d*x
+ 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 12.37, size = 160, normalized size = 1.88

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{7}-\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{21}-\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{3}}{a^{2}\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{3}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)

[Out] -((16*cos(c/2 + (d*x)/2)^10)/21 + (64*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2))/21 - (32*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/3 - (128*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/21 + (16*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2)/7)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)

3.832 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2\sec^3(c+dx)}{3a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+dx)}{7a^2d}$$

```
[Out] -2/3*sec(d*x+c)^3/a^2/d+4/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+1/5*t
an(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2790, 2687, 14, 2686, 276, 30}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*Sec[c + d*x]^3)/(3*a^2*d) + (4*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d
*x]^7)/(7*a^2*d) + Tan[c + d*x]^5/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \left(a^2\sec^4(c+dx)\tan^4(c+dx)-2a^2\sec^3(c+dx)\tan^5(c+dx)+a^2\sec^2(c+dx\right.}{a^4}\\
&= \frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2}+\frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a^2}-\frac{2\int \sec^3(c+}{}\\
&= \frac{\operatorname{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\mathrm{Su}}{}\\
&= \frac{\tan^7(c+dx)}{7a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,\tan(c+dx)\right)}{a^2d}-\frac{2\operatorname{Subst}\left(\int \left(x^2-2x^4+\right.\right.}{a}\\
&= -\frac{2\sec^3(c+dx)}{3a^2d}+\frac{4\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+}{7a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 126, normalized size = 1.38

$$-\frac{\sec^3(c+dx)(672-1442\cos(c+dx)+1664\cos(2(c+dx))-309\cos(3(c+dx))-288\cos(4(c+dx))+103\cos(5(c+dx))-1232\sin(c+dx)-824\sin(2(c+dx))+1896\sin(3(c+dx))-412\sin(4(c+dx))-72\sin(5(c+dx)))}{13440a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/13440*(Sec[c + d*x]^3*(672 - 1442*Cos[c + d*x] + 1664*Cos[2*(c + d*x)] -
 309*Cos[3*(c + d*x)] - 288*Cos[4*(c + d*x)] + 103*Cos[5*(c + d*x)] - 1232*
Sin[c + d*x] - 824*Sin[2*(c + d*x)] + 1896*Sin[3*(c + d*x)] - 412*Sin[4*(c
+ d*x)] - 72*Sin[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.33, size = 160, normalized size = 1.76

method	result
risch	$-\frac{2i\left(140i\,e^{7i(dx+c)}+105\,e^{8i(dx+c)}+84i\,e^{5i(dx+c)}-140\,e^{6i(dx+c)}+68i\,e^{3i(dx+c)}+14\,e^{4i(dx+c)}-36i\,e^{i(dx+c)}-132\,e^{2i(dx+c}}{105\left(e^{i(dx+c)}+i\right)^{7}\left(e^{i(dx+c)}-i\right)^{3}a^{2}d}$
norman	$\frac{-\frac{32\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{32}{105ad}+\frac{128\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{105ad}+\frac{32\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{256\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}-\frac{64\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}$
derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{32}{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{12}{5\left(\tan\left(\frac{dx}{2}+\right.}}{a^{2}d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{32}{256\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-256}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{12}{5\left(\tan\left(\frac{dx}{2}+\right.}}{a^{2}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^2*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2+1/
256/(tan(1/2*d*x+1/2*c)-1)-1/56/(tan(1/2*d*x+1/2*c)+1)^7+1/16/(tan(1/2*d*x+
1/2*c)+1)^6-3/40/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/2*c)+1)^4+1/3
84/(tan(1/2*d*x+1/2*c)+1)^3-1/256/(tan(1/2*d*x+1/2*c)+1)^2-1/256/(tan(1/2*d
*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(81) = 162.
time = 0.34, size = 316, normalized size = 3.47

$$-\frac{32\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+1\right)}{105\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{4a^{2}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -32/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^2 + 4*a^2*s
in(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4
*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c
) + 1)^10)*d)
```

Fricas [A]
time = 0.35, size = 103, normalized size = 1.13

$$-\frac{18\cos(dx+c)^4-44\cos(dx+c)^2+\left(9\cos(dx+c)^4-66\cos(dx+c)^2+25\right)\sin(dx+c)+10}{105\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(18*cos(d*x + c)^4 - 44*cos(d*x + c)^2 + (9*cos(d*x + c)^4 - 66*cos(
d*x + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 146, normalized size = 1.60

$$\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+735\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2030\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2030\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+1701\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+707\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+116}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 4)/(a^2*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 735*tan(1/2*d*x +
 1/2*c)^5 + 2030*tan(1/2*d*x + 1/2*c)^4 + 2030*tan(1/2*d*x + 1/2*c)^3 + 170
1*tan(1/2*d*x + 1/2*c)^2 + 707*tan(1/2*d*x + 1/2*c) + 116)/(a^2*(tan(1/2*d*
x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 12.38, size = 184, normalized size = 2.02

$$-\frac{\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{105}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{35}-\frac{256\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}-\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{15}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{5}}{a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] -((32*cos(c/2 + (d*x)/2)^10)/105 + (128*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x
)/2))/105 - (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^5)/5 - (64*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 - (256*cos(c/2 + (d*x)/2)^7*sin(c/2 +
 (d*x)/2)^3)/105 + (32*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2)/35)/(a^2*
d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2
 + (d*x)/2))^7)
```

3.833 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(c+dx)}{7a^2d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^2/d-3/5*sec(d*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d-2/5*ta
n(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2954, 2952, 2686, 14, 2687, 276}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x]^5)/(5*a^2*d) + (2*Sec[c + d*x]^7
)/(7*a^2*d) - (2*Tan[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^2\tan^3(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\sec^5(c+dx)\tan^3(c+dx)-2a^2\sec^4(c+dx)\tan^4(c+dx)+a^2\sec^3(\ldots\right.}{a^4}\\
&= \frac{\int \sec^5(c+dx)\tan^3(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\tan^5(c+dx)\,dx}{a^2}-\frac{2\int \mathrm{se}\ldots}{\ldots}\\
&= \frac{\mathrm{Subst}\left(\int x^4\left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\ldots\right.}{a^2d}\\
&= \frac{\mathrm{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int \left(-x^4+x^6\right)\,dx,x\ldots\right.}{a^2d}\\
&= \frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan^5(c+dx)}{5a^2d}-\frac{2\tan\ldots}{\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(672-182\cos(c+dx)-736\cos(2(c+dx))-39\cos(3(c+dx))+192\cos(4(c+dx))+13\cos(5(c+dx))+448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-52\sin(4(c+dx))+48\sin(5(c+dx)))}{6720a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(672 - 182*Cos[c + d*x] - 736*Cos[2*(c + d*x)] - 39*Cos[3*(
c + d*x)] + 192*Cos[4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] -
 104*Sin[2*(c + d*x)] - 144*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin
[5*(c + d*x)]))/(6720*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.32, size = 130, normalized size = 1.43

method	result
risch	$\frac{\frac{8i e^{6i(dx+c)}}{3}+\frac{8\,e^{7i(dx+c)}}{3}+\frac{8i e^{4i(dx+c)}}{15}-\frac{16\,e^{5i(dx+c)}}{5}+\frac{24i e^{2i(dx+c)}}{35}+\frac{88\,e^{3i(dx+c)}}{105}-\frac{8i}{35}-\frac{32\,e^{i(dx+c)}}{35}}{\left(e^{i(dx+c)}+i\right)^{7}\left(e^{i(dx+c)}-i\right)^{3} d\, a^{2}}$
derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{14}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{a^{2} d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}+\frac{14}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}}{a^{2} d}$
norman	$\frac{-\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{4}{105ad}-\frac{4\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{52\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{105ad}-\frac{4\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{32\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3} a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^2*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+1/
28/(tan(1/2*d*x+1/2*c)+1)^7-1/8/(tan(1/2*d*x+1/2*c)+1)^6+7/40/(tan(1/2*d*x+
1/2*c)+1)^5-1/8/(tan(1/2*d*x+1/2*c)+1)^4+7/192/(tan(1/2*d*x+1/2*c)+1)^3+1/1
28/(tan(1/2*d*x+1/2*c)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(81) = 162.
time = 0.37, size = 336, normalized size = 3.69

$$\frac{4\left(\frac{4\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{91\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{84\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{105\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}{105\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{4a^{2}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 4/105*(4*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 91*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 1)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*
a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2
```

```
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) +
 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.14

$$-\frac{24\cos\left(dx+c\right)^{4}-47\cos\left(dx+c\right)^{2}+2\left(6\cos\left(dx+c\right)^{4}-9\cos\left(dx+c\right)^{2}+5\right)\sin\left(dx+c\right)+25}{105\left(a^{2}d\cos\left(dx+c\right)^{5}-2\,a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*cos(d*x + c)^4 - 47*cos(d*x + c)^2 + 2*(6*cos(d*x + c)^4 - 9*cos
(d*x + c)^2 + 5)*sin(d*x + c) + 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.69, size = 120, normalized size = 1.32

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{105\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1015\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1330\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1302\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+469\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+67}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3)
- (105*tan(1/2*d*x + 1/2*c)^5 + 1015*tan(1/2*d*x + 1/2*c)^4 + 1330*tan(1/2*
d*x + 1/2*c)^3 + 1302*tan(1/2*d*x + 1/2*c)^2 + 469*tan(1/2*d*x + 1/2*c) + 6
7)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 14.07, size = 207, normalized size = 2.27

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{105}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{35}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{105}+\frac{52\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{15}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{5}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{a^{2}\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{3}\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^10)/105 + (16*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2
))/105 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + (16*cos(c/2 + (d*x)/
2)^5*sin(c/2 + (d*x)/2)^5)/5 + (52*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^
4)/15 - (32*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (4*cos(c/2 + (
d*x)/2)^8*sin(c/2 + (d*x)/2)^2)/35)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 +
(d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

3.834 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7(c+dx)}{7a^2d}$$

```
[Out] 2/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+1/3*tan(d*x+c)^3/a^2/d+3/5*ta
n(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2954, 2952, 2687, 276, 2686, 14}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^3(c+dx)}{3a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{2\sec^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Sec[c + d*x]^5)/(5*a^2*d) - (2*Sec[c + d*x]^7)/(7*a^2*d) + Tan[c + d*x]^
3/(3*a^2*d) + (3*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \sec^6(c+dx)(a-a\sin(c+dx))^2\tan^2(c+dx)\,dx}{a^4}\\ &= \frac{\int \left(a^2\sec^6(c+dx)\tan^2(c+dx)-2a^2\sec^5(c+dx)\tan^3(c+dx)+a^2\sec^4(\right.}{a^4}\\ &= \frac{\int \sec^6(c+dx)\tan^2(c+dx)\,dx}{a^2}+\frac{\int \sec^4(c+dx)\tan^4(c+dx)\,dx}{a^2}-\frac{2\int \sec}{}\\ &= \frac{\mathrm{Subst}\left(\int x^4(1+x^2)\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int x^2(1+x^2)^2\,dx,x,\tan(\right.}{a^2d}\\ &= \frac{\mathrm{Subst}\left(\int (x^4+x^6)\,dx,x,\tan(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int (x^2+2x^4+x^6)\,dx,x,t\right.}{a^2d}\\ &= \frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{2\tan^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 126, normalized size = 1.38

$$\frac{\sec^3(c+dx)(1344-714\cos(c+dx)+128\cos(2(c+dx))-153\cos(3(c+dx))+64\cos(4(c+dx))+51\cos(5(c+dx))+3136\sin(c+dx)-408\sin(2(c+dx))-48\sin(3(c+dx))-204\sin(4(c+dx))+16\sin(5(c+dx)))}{13440a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(1344 - 714*Cos[c + d*x] + 128*Cos[2*(c + d*x)] - 153*Cos[3
*(c + d*x)] + 64*Cos[4*(c + d*x)] + 51*Cos[5*(c + d*x)] + 3136*Sin[c + d*x]
 - 408*Sin[2*(c + d*x)] - 48*Sin[3*(c + d*x)] - 204*Sin[4*(c + d*x)] + 16*S
in[5*(c + d*x)]))/(13440*a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]
time = 0.31, size = 160, normalized size = 1.76

method	result
risch	$\frac{4i\left(84i\,\mathrm{e}^{5i(dx+c)}+105\,\mathrm{e}^{6i(dx+c)}+8i\,\mathrm{e}^{3i(dx+c)}-91\,\mathrm{e}^{4i(dx+c)}+4i\,\mathrm{e}^{i(dx+c)}+3\,\mathrm{e}^{2i(dx+c)}-1\right)}{105\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}a^{2}d}$
derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^{2}d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{16}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^{2}d}$
norman	$\frac{-\frac{16\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{8}{35ad}-\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{24\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{88\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}+}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^2*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-1/64/
(tan(1/2*d*x+1/2*c)-1)-1/14/(tan(1/2*d*x+1/2*c)+1)^7+1/4/(tan(1/2*d*x+1/2*c
)+1)^6-2/5/(tan(1/2*d*x+1/2*c)+1)^5+3/8/(tan(1/2*d*x+1/2*c)+1)^4-19/96/(tan
(1/2*d*x+1/2*c)+1)^3+3/64/(tan(1/2*d*x+1/2*c)+1)^2+1/64/(tan(1/2*d*x+1/2*c)
+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(81) = 162.
time = 0.32, size = 356, normalized size = 3.91

$$\frac{8\left(\frac{12\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{11\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{7\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{42\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{35\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{35\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+3\right)}{105\left(a^{2}+\frac{4a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{8a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{14a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{8a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{3a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{4a^{2}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 8/105*(12*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 + 11*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 7*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 42*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 35*sin(d*x + c)^6/(
cos(d*x + c) + 1)^6 + 35*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3)/((a^2 + 4
*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(
```

```
cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10)*d)
```

Fricas [A]
time = 0.36, size = 103, normalized size = 1.13

$$-\frac{4\,\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+\left(2\,\cos\left(dx+c\right)^{4}-3\,\cos\left(dx+c\right)^{2}+25\right)\sin\left(dx+c\right)+10}{105\,\left(a^{2}d\cos\left(dx+c\right)^{5}-2\,a^{2}d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,a^{2}d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(4*cos(d*x + c)^4 - 2*cos(d*x + c)^2 + (2*cos(d*x + c)^4 - 3*cos(d*x
 + c)^2 + 25)*sin(d*x + c) + 10)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x +
c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x)
+ 1), x)/a**2
```

Giac [A]
time = 0.58, size = 146, normalized size = 1.60

$$-\frac{\frac{35\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right)}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+945\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1820\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2450\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1617\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+749\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+122}{a^{2}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (105*tan(1/2*d*x + 1/2*c)^6 + 945*tan(1/2*d*x
+ 1/2*c)^5 + 1820*tan(1/2*d*x + 1/2*c)^4 + 2450*tan(1/2*d*x + 1/2*c)^3 + 16
17*tan(1/2*d*x + 1/2*c)^2 + 749*tan(1/2*d*x + 1/2*c) + 122)/(a^2*(tan(1/2*d
*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 14.34, size = 231, normalized size = 2.54

$$\frac{\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{35}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{35}+\frac{24\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{35}+\frac{88\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{105}-\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{15}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{5}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{3}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{3}}{a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((8*cos(c/2 + (d*x)/2)^10)/35 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)
)/35 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)/3 + (8*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^6)/3 + (16*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)
^5)/5 - (8*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 + (88*cos(c/2 + (d
*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (24*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*
x)/2)^2)/35)/(a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 +
(d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

3.835 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=93

$$\frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)}+\frac{8\tan(c+dx)}{35a^2d}+\frac{8\tan^3(c+dx)}{105a^2d}$$

```
[Out] 1/7*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^2-2/35*sec(d*x+c)^3/d/(a^2+a^2*sin(d*x+
c))+8/35*tan(d*x+c)/a^2/d+8/105*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2938, 2751, 3852}

$$\frac{8\tan^3(c+dx)}{105a^2d}+\frac{8\tan(c+dx)}{35a^2d}-\frac{2\sec^3(c+dx)}{35d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\sec^3(c+dx)}{7d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Sec[c + d*x]^3/(7*d*(a + a*Sin[c + d*x])^2) - (2*Sec[c + d*x]^3)/(35*d*(a^2
 + a^2*Sin[c + d*x])) + (8*Tan[c + d*x])/(35*a^2*d) + (8*Tan[c + d*x]^3)/(1
05*a^2*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}+\frac{2\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{7a}\\ &= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)}+\frac{8\int \sec^4(c+dx)\,dx}{35a^2}\\ &= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)}-\frac{8\mathrm{Subst}\left(\int \left(1+x^2\right) \ldots\right.}{35\ldots}\\ &= \frac{\sec^3(c+dx)}{7d(a+a\sin(c+dx))^2}-\frac{2\sec^3(c+dx)}{35d\left(a^2+a^2\sin(c+dx)\right)}+\frac{8\tan(c+dx)}{35a^2d}+\frac{8\tan\ldots}{\ldots}\end{aligned}$$

Mathematica [A]

time = 0.22, size = 134, normalized size = 1.44

$$-\frac{\sec^3(c+dx)\left(-84+\frac{21}{4}\cos(c+dx)+32\cos(2(c+dx))+\frac{9}{8}\cos(3(c+dx))+16\cos(4(c+dx))-\frac{3}{8}\cos(5(c+dx))-56\sin(c+dx)+3\sin(2(c+dx))-12\sin(3(c+dx))+\frac{3}{2}\sin(4(c+dx))+4\sin(5(c+dx))\right)}{420a^2d(1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/420*(Sec[c + d*x]^3*(-84 + (21*Cos[c + d*x])/4 + 32*Cos[2*(c + d*x)] + (
9*Cos[3*(c + d*x)])/8 + 16*Cos[4*(c + d*x)] - (3*Cos[5*(c + d*x)])/8 - 56*S
in[c + d*x] + 3*Sin[2*(c + d*x)] - 12*Sin[3*(c + d*x)] + (3*Sin[4*(c + d*x)
])/2 + 4*Sin[5*(c + d*x)]))/(a^2*d*(1 + Sin[c + d*x])^2)
```

Maple [A]

time = 0.30, size = 160, normalized size = 1.72

method	result
risch	$-\frac{32\left(14i\,e^{4i(dx+c)}+21\,e^{5i(dx+c)}+3i\,e^{2i(dx+c)}-8\,e^{3i(dx+c)}-i-4\,e^{i(dx+c)}\right)}{105\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^7 d\,a^2}$
derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{18}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.}\ldots}{a^2d}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{18}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.}\ldots}{a^2d}$

norman	$\frac{\frac{8\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{6}{35ad}-\frac{2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{8\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{35ad}-\frac{136\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}-\frac{4(\text{ta}}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^2*(-1/48/(tan(1/2*d*x+1/2*c)-1)^3-1/32/(tan(1/2*d*x+1/2*c)-1)^2-1/16/
(tan(1/2*d*x+1/2*c)-1)+1/7/(tan(1/2*d*x+1/2*c)+1)^7-1/2/(tan(1/2*d*x+1/2*c)
+1)^6+9/10/(tan(1/2*d*x+1/2*c)+1)^5-1/(tan(1/2*d*x+1/2*c)+1)^4+35/48/(tan(1
/2*d*x+1/2*c)+1)^3-11/32/(tan(1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+
1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(85) = 170$.
time = 0.30, size = 376, normalized size = 4.04

$$\frac{2\left(\frac{36\sin(dx+c)}{\cos(dx+c)+1}+\frac{132\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{68\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{14\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{140\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{140\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{105\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+9\right)}{105\left(a^2+\frac{4a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2/105*(36*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 68*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 14*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4 - 84*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 140*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 + 140*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 105*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8 + 9)/((a^2 + 4*a^2*sin(d*x + c)/(cos(d*x +
 c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 3*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A]
time = 0.35, size = 104, normalized size = 1.12

$$\frac{32\cos(dx+c)^4-16\cos(dx+c)^2+2\left(8\cos(dx+c)^4-12\cos(dx+c)^2-5\right)\sin(dx+c)-25}{105\left(a^2d\cos(dx+c)^5-2a^2d\cos(dx+c)^3\sin(dx+c)-2a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(32*cos(d*x + c)^4 - 16*cos(d*x + c)^2 + 2*(8*cos(d*x + c)^4 - 12*cos
(d*x + c)^2 - 5)*sin(d*x + c) - 25)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x
 + c)^3*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^4(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x)/a**2
```

Giac [A]
time = 0.58, size = 146, normalized size = 1.57

$$-\frac{\frac{35\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+105\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-910\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-756\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-427\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-31}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^2*(ta
n(1/2*d*x + 1/2*c) - 1)^3) - (210*tan(1/2*d*x + 1/2*c)^6 + 105*tan(1/2*d*x
+ 1/2*c)^5 - 175*tan(1/2*d*x + 1/2*c)^4 - 910*tan(1/2*d*x + 1/2*c)^3 - 756*
tan(1/2*d*x + 1/2*c)^2 - 427*tan(1/2*d*x + 1/2*c) - 31)/(a^2*(tan(1/2*d*x +
 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 14.35, size = 254, normalized size = 2.73

$$\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(9\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8+36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+132\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2+68\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3+14\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-84\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6+140\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7+105\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8\right)}{105\,a^2\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (2*cos(c/2 + (d*x)/2)^2*(9*cos(c/2 + (d*x)/2)^8 + 105*sin(c/2 + (d*x)/2)^8
+ 140*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^7 + 36*cos(c/2 + (d*x)/2)^7*sin
(c/2 + (d*x)/2) + 140*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^6 - 84*cos(c/
2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^5 + 14*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*
x)/2)^4 + 68*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^3 + 132*cos(c/2 + (d*x
)/2)^6*sin(c/2 + (d*x)/2)^2))/(105*a^2*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d
*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^7)
```

3.836 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(c-}{a^2d}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d+1/5*sec(
d*x+c)^5/a^2/d+2/7*sec(d*x+c)^7/a^2/d-2*tan(d*x+c)/a^2/d-2*tan(d*x+c)^3/a^2
/d-6/5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2686, 30}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{2\tan(c+dx)}{a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d) + Sec[c + d*x]^5/(5*a^2*d) + (2*Sec[c + d*x]^7)/(7*a^2*d) - (2*Tan[
c + d*x])/(a^2*d) - (2*Tan[c + d*x]^3)/(a^2*d) - (6*Tan[c + d*x]^5)/(5*a^2*
d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(-2a^2\sec^8(c+dx)+a^2\csc(c+dx)\sec^8(c+dx)+a^2\sec^7(c+dx)\tan(c\right.}{a^4}$$
$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2}+\frac{\int \sec^7(c+dx)\tan(c+dx)\,dx}{a^2}-\frac{2\int \sec^8(c}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^6\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^2d}+\frac{2\text{Su}}{}$$
$$= \frac{\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx)}{a^2d}-\frac{2\tan^3(c+dx)}{a^2d}-\frac{6\tan^5(c+dx)}{5a^2d}-\frac{2\tan^7(}{7a}$$
$$= \frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\tan(c+dx}{a^2d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec^5(c+dx)}{5a^2d}+\frac{2\text{se}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 352 vs. $2(149) = 298$.
time = 0.46, size = 352, normalized size = 2.36

$$\frac{6216+5312\cos(2(c+dx))-1677\cos(3(c+dx))+696\cos(4(c+dx))+559\cos(5(c+dx))-1260\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+420\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-14\cos(c+dx)\left(559+420\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-420\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+1260\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-420\cos(5(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2464\sin(c+dx)-4472\sin(2(c+dx))-3360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+3360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(2(c+dx))+2208\sin(3(c+dx))-2236\sin(4(c+dx))-1680\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+1680\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+384\sin(5(c+dx))}{6720a^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (6216 + 5312*Cos[2*(c + d*x)] - 1677*Cos[3*(c + d*x)] + 696*Cos[4*(c + d*x)
] + 559*Cos[5*(c + d*x)] - 1260*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 42
0*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 14*Cos[c + d*x]*(559 + 420*Log[C
os[(c + d*x)/2]] - 420*Log[Sin[(c + d*x)/2]]) + 1260*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 420*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 2464*Sin[c
+ d*x] - 4472*Sin[2*(c + d*x)] - 3360*Log[Cos[(c + d*x)/2]]*Sin[2*(c + d*x)
] + 3360*Log[Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] + 2208*Sin[3*(c + d*x)] - 2
236*Sin[4*(c + d*x)] - 1680*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 1680*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)])/(6720*a^2*d*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^7)
```

Maple [A]
time = 0.39, size = 169, normalized size = 1.13

method	result

derivativedivides	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{22}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}\cdots}{d\,a^2}$
default	$\frac{-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{2}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}+\frac{22}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}\cdots}{d\,a^2}$
risch	$\frac{8i\mathrm{e}^{8i(dx+c)}+2\,\mathrm{e}^{9i(dx+c)}+\frac{56i\mathrm{e}^{6i(dx+c)}}{3}-\frac{16\,\mathrm{e}^{7i(dx+c)}}{3}+\frac{104i\mathrm{e}^{4i(dx+c)}}{15}-\frac{148\,\mathrm{e}^{5i(dx+c)}}{5}-\frac{88i\mathrm{e}^{2i(dx+c)}}{35}-\frac{2096\,\mathrm{e}^{3i(dx+c)}}{105}-\frac{64i}{35}\cdots}{\left(\mathrm{e}^{i(dx+c)}+i\right)^7\left(\mathrm{e}^{i(dx+c)}-i\right)^3 d\,a^2}$
norman	$\frac{-\frac{48\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{277}{105ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{14\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{67\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}+\frac{134\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}{15ad}\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/12/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^2-1/2/(t
an(1/2*d*x+1/2*c)-1)+4/7/(tan(1/2*d*x+1/2*c)+1)^7-2/(tan(1/2*d*x+1/2*c)+1)^
6+22/5/(tan(1/2*d*x+1/2*c)+1)^5-6/(tan(1/2*d*x+1/2*c)+1)^4+79/12/(tan(1/2*d
*x+1/2*c)+1)^3-39/8/(tan(1/2*d*x+1/2*c)+1)^2+9/2/(tan(1/2*d*x+1/2*c)+1)+ln(
tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(139) = 278$.

time = 0.32, size = 422, normalized size = 2.83

$$\frac{\frac{2\left(\frac{554\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{258\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1108\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1204\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{504\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1470\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{420\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{315\,\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{210\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+191\right)}{a^2+\frac{4\,a^2\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\,a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{14\,a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{14\,a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{8\,a^2\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3\,a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4\,a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{105\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(2*(554*sin(d*x + c)/(cos(d*x + c) + 1) + 258*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 1108*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1204*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 + 504*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1470*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + 420*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 210*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 + 191)/(a^2 + 4*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 8*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- 14*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 14*a^2*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + 8*a^2*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 3*a^2*sin(d*x
+ c)^8/(cos(d*x + c) + 1)^8 - 4*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a
^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 105*log(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^2)/d
```

Fricas [A]

time = 0.38, size = 200, normalized size = 1.34

$$-\frac{174\,\cos(dx+c)^4+158\,\cos(dx+c)^2+105\left(\cos(dx+c)^5-2\,\cos(dx+c)^3\sin(dx+c)-2\,\cos(dx+c)^3\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-105\left(\cos(dx+c)^5-2\,\cos(dx+c)^3\sin(dx+c)-2\,\cos(dx+c)^3\right)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+4\left(48\,\cos(dx+c)^4+33\,\cos(dx+c)^2+5\right)\sin(dx+c)+50}{210\left(a^2d\cos(dx+c)^5-2\,a^2d\cos(dx+c)^3\sin(dx+c)-2\,a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/210*(174*cos(d*x + c)^4 + 158*cos(d*x + c)^2 + 105*(cos(d*x + c)^5 - 2*c
os(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2)
- 105*(cos(d*x + c)^5 - 2*cos(d*x + c)^3*sin(d*x + c) - 2*cos(d*x + c)^3)*l
og(-1/2*cos(d*x + c) + 1/2) + 4*(48*cos(d*x + c)^4 + 33*cos(d*x + c)^2 + 5)
*sin(d*x + c) + 50)/(a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3*sin(d*x
+ c) - 2*a^2*d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 161, normalized size = 1.08

$$\frac{\frac{840 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{35\left(12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{3780\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+18585\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+41755\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+51730\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+37506\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+14917\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2671}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 35*(12*tan(1/2*d*x + 1/2*c)
^2 - 21*tan(1/2*d*x + 1/2*c) + 11)/(a^2*(tan(1/2*d*x + 1/2*c) - 1)^3) + (37
80*tan(1/2*d*x + 1/2*c)^6 + 18585*tan(1/2*d*x + 1/2*c)^5 + 41755*tan(1/2*d*
x + 1/2*c)^4 + 51730*tan(1/2*d*x + 1/2*c)^3 + 37506*tan(1/2*d*x + 1/2*c)^2
+ 14917*tan(1/2*d*x + 1/2*c) + 2671)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 12.34, size = 169, normalized size = 1.13

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+28\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{48\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}-\frac{344\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}-\frac{2216\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{105}+\frac{172\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{35}+\frac{1108\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}+\frac{382}{105}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((1108*tan(c/2 + (d*x)/2))/105 + (172*tan
(c/2 + (d*x)/2)^2)/35 - (2216*tan(c/2 + (d*x)/2)^3)/105 - (344*tan(c/2 + (d
*x)/2)^4)/15 + (48*tan(c/2 + (d*x)/2)^5)/5 + 28*tan(c/2 + (d*x)/2)^6 + 8*ta
n(c/2 + (d*x)/2)^7 - 6*tan(c/2 + (d*x)/2)^8 - 4*tan(c/2 + (d*x)/2)^9 + 382/
105)/(a^2*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^7)
```

3.837 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}+5\tan$$

```
[Out] 2*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d-2/3*sec(d*x
+c)^3/a^2/d-2/5*sec(d*x+c)^5/a^2/d-2/7*sec(d*x+c)^7/a^2/d+5*tan(d*x+c)/a^2/
d+3*tan(d*x+c)^3/a^2/d+7/5*tan(d*x+c)^5/a^2/d+2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2700, 276}

$$\frac{2\tan^7(c+dx)}{7a^2d}+\frac{7\tan^5(c+dx)}{5a^2d}+\frac{3\tan^3(c+dx)}{a^2d}+\frac{5\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec^7(c+dx)}{7a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Sec[c + d*x])
/(a^2*d) - (2*Sec[c + d*x]^3)/(3*a^2*d) - (2*Sec[c + d*x]^5)/(5*a^2*d) - (2
*Sec[c + d*x]^7)/(7*a^2*d) + (5*Tan[c + d*x])/(a^2*d) + (3*Tan[c + d*x]^3)/
(a^2*d) + (7*Tan[c + d*x]^5)/(5*a^2*d) + (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\int \csc^2(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4} \\
&= \frac{\int (a^2\sec^8(c+dx)-2a^2\csc(c+dx)\sec^8(c+dx)+a^2\csc^2(c+dx)\sec^8(c}{a^4} \\
&= \frac{\int \sec^8(c+dx)\,dx}{a^2}+\frac{\int \csc^2(c+dx)\sec^8(c+dx)\,dx}{a^2}-\frac{2\int \csc(c+dx)\sec}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}-\frac{\text{Subst}\left(\int (1+3x^2+3x^4+x^6)\,dx,\right.}{a^2d} \\
&= \frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{a^2d}+\frac{3\tan^5(c+dx)}{5a^2d}+\frac{\tan^7(c+dx)}{7a^2d}+\frac{\text{Subst}(\int}{} \\
&= -\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-\frac{2\sec^5(c+dx)}{5a^2d}-\frac{2\sec}{} \\
&= \frac{2\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}-\frac{2\sec^3(c+dx)}{3a^2d}-
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(164) = 328.

time = 5.60, size = 329, normalized size = 2.01

$$
\frac{-840\cot\left(\frac{1}{2}(c+dx)\right)+3360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{35}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{60}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}-\frac{288}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{997}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{70}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{910}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{120}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}+\frac{576}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}+\frac{1994}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{9554}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)+840\tan\left(\frac{1}{2}(c+dx)\right)}{1680a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-840*Cot[(c + d*x)/2] + 3360*Log[Cos[(c + d*x)/2]] - 3360*Log[Sin[(c + d*x
)/2]] + 35/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - 60/(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^6 - 288/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 997/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + Sin[(c + d*x)/2]*(70/(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^3 + 910/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 120
/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + 576/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^5 + 1994/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 9554/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])) + 840*Tan[(c + d*x)/2])/(1680*a^2*d)
```

Maple [A]

time = 0.44, size = 194, normalized size = 1.18

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6}-\frac{}{5(}}{}$

default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{5}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{}{5(\mathrm{ta}}}{}$
risch	$-\frac{4\left(-385\,\mathrm{e}^{9i(dx+c)}+420i\mathrm{e}^{10i(dx+c)}-1274\,\mathrm{e}^{7i(dx+c)}+560i\mathrm{e}^{8i(dx+c)}-616i\mathrm{e}^{6i(dx+c)}+1249\,\mathrm{e}^{3i(dx+c)}-1816i\mathrm{e}^{4i(dx+c)}+759\right.}{105\left(\mathrm{e}^{2i(dx+c)}-1\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}a^{2}d}$
norman	$\frac{\frac{1}{2ad}+\frac{266\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{93\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{17\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{53\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{57\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(tan(1/2*d*x+1/2*c)-1/6/(tan(1/2*d*x+1/2*c)-1)^3-1/4/(tan(1/2*d*x
+1/2*c)-1)^2-5/4/(tan(1/2*d*x+1/2*c)-1)-8/7/(tan(1/2*d*x+1/2*c)+1)^7+4/(tan
(1/2*d*x+1/2*c)+1)^6-48/5/(tan(1/2*d*x+1/2*c)+1)^5+14/(tan(1/2*d*x+1/2*c)+1
)^4-107/6/(tan(1/2*d*x+1/2*c)+1)^3+59/4/(tan(1/2*d*x+1/2*c)+1)^2-75/4/(tan(
1/2*d*x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-4*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(154) = 308$.
time = 0.36, size = 481, normalized size = 2.93

$$-\frac{\frac{\frac{1828\sin(dx+c)}{\cos(dx+c)+1}+\frac{3847\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1656\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{12734\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{7952\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{9702\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{12600\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{315\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{5460\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{2205\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+105}{\frac{a^{2}\sin(dx+c)}{\cos(dx+c)+1}+\frac{4a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{2}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{8a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{14a^{2}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{14a^{2}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{8a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{3a^{2}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{4a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{a^{2}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}+\frac{420\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}-\frac{105\sin(dx+c)}{a^{2}(\cos(dx+c)+1)}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/210*((1828*sin(d*x + c)/(cos(d*x + c) + 1) + 3847*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 1656*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 12734*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 7952*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 9702
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 12600*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 - 315*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 5460*sin(d*x + c)^9/(cos(
d*x + c) + 1)^9 - 2205*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 105)/(a^2*si
n(d*x + c)/(cos(d*x + c) + 1) + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 8*a^2*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 - 14*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 14*a^2*sin(d*x +
 c)^7/(cos(d*x + c) + 1)^7 + 8*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 3*
a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^2*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10 - a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11) + 420*log(sin(d*x +
 c)/(cos(d*x + c) + 1))/a^2 - 105*sin(d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.37, size = 250, normalized size = 1.52

$$-\frac{432\cos(dx+c)^{6}-660\cos(dx+c)^{4}+98\cos(dx+c)^{2}-105\left(2\cos(dx+c)^{5}-2\cos(dx+c)^{3}+\left(\cos(dx+c)^{5}-2\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+105\left(2\cos(dx+c)^{5}-2\cos(dx+c)^{3}+\left(\cos(dx+c)^{5}-2\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(327\cos(dx+c)^{4}-41\cos(dx+c)^{2}-5\right)\sin(dx+c)+25}{105\left(2a^{2}d\cos(dx+c)^{5}-2a^{2}d\cos(dx+c)^{3}+\left(a^{2}d\cos(dx+c)^{5}-2a^{2}d\cos(dx+c)^{3}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(432*cos(d*x + c)^6 - 660*cos(d*x + c)^4 + 98*cos(d*x + c)^2 - 105*(
2*cos(d*x + c)^5 - 2*cos(d*x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*s
in(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(2*cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + (cos(d*x + c)^5 - 2*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*(327*cos(d*x + c)^4 - 41*cos(d*x + c)^2 - 5)*sin(d*x + c
) + 25)/(2*a^2*d*cos(d*x + c)^5 - 2*a^2*d*cos(d*x + c)^3 + (a^2*d*cos(d*x +
 c)^5 - 2*a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.62, size = 204, normalized size = 1.24

$$-\frac{\frac{1680 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^2}-\frac{420 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^2}-\frac{420 \left(4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}{a^2 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}+\frac{35 \left(15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-27 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+14\right)}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^3}+\frac{7875 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6+41055 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+94640 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+119630 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+87507 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+34979 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+6122}{a^2 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^7}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(1680*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 420*tan(1/2*d*x + 1/2*c)/
a^2 - 420*(4*tan(1/2*d*x + 1/2*c) - 1)/(a^2*tan(1/2*d*x + 1/2*c)) + 35*(15*
tan(1/2*d*x + 1/2*c)^2 - 27*tan(1/2*d*x + 1/2*c) + 14)/(a^2*(tan(1/2*d*x +
1/2*c) - 1)^3) + (7875*tan(1/2*d*x + 1/2*c)^6 + 41055*tan(1/2*d*x + 1/2*c)^
5 + 94640*tan(1/2*d*x + 1/2*c)^4 + 119630*tan(1/2*d*x + 1/2*c)^3 + 87507*ta
n(1/2*d*x + 1/2*c)^2 + 34979*tan(1/2*d*x + 1/2*c) + 6122)/(a^2*(tan(1/2*d*x
 + 1/2*c) + 1)^7))/d
```

Mupad [B]
time = 11.24, size = 331, normalized size = 2.02

$$\frac{21 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+52 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-120 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{462 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{5}+\frac{1136 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15}+\frac{12734 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{105}+\frac{552 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{35}-\frac{3847 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{105}-\frac{1828 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}-1}{d \left(-2 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-8 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-6 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+16 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+28 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-28 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-16 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+6 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2 a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{2 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2 d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((552*tan(c/2 + (d*x)/2)^3)/35 - (3847*tan(c/2 + (d*x)/2)^2)/105 - (1828*ta
n(c/2 + (d*x)/2))/105 + (12734*tan(c/2 + (d*x)/2)^4)/105 + (1136*tan(c/2 +
(d*x)/2)^5)/15 - (462*tan(c/2 + (d*x)/2)^6)/5 - 120*tan(c/2 + (d*x)/2)^7 +
3*tan(c/2 + (d*x)/2)^8 + 52*tan(c/2 + (d*x)/2)^9 + 21*tan(c/2 + (d*x)/2)^10
 - 1)/(d*(8*a^2*tan(c/2 + (d*x)/2)^2 + 6*a^2*tan(c/2 + (d*x)/2)^3 - 16*a^2*
tan(c/2 + (d*x)/2)^4 - 28*a^2*tan(c/2 + (d*x)/2)^5 + 28*a^2*tan(c/2 + (d*x)
/2)^7 + 16*a^2*tan(c/2 + (d*x)/2)^8 - 6*a^2*tan(c/2 + (d*x)/2)^9 - 8*a^2*ta
n(c/2 + (d*x)/2)^10 - 2*a^2*tan(c/2 + (d*x)/2)^11 + 2*a^2*tan(c/2 + (d*x)/2
))) - (2*log(tan(c/2 + (d*x)/2)))/(a^2*d) + tan(c/2 + (d*x)/2)/(2*a^2*d)
```

3.838 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec(c+dx)}{2a^2d}+\frac{11\sec^3(c+dx)}{6a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}+\frac{11\sec^7(c+d}{14a^2d}$$

```
[Out] -11/2*arctanh(cos(d*x+c))/a^2/d+2*cot(d*x+c)/a^2/d+11/2*sec(d*x+c)/a^2/d+11
/6*sec(d*x+c)^3/a^2/d+11/10*sec(d*x+c)^5/a^2/d+11/14*sec(d*x+c)^7/a^2/d-1/2
*csc(d*x+c)^2*sec(d*x+c)^7/a^2/d-8*tan(d*x+c)/a^2/d-4*tan(d*x+c)^3/a^2/d-8/
5*tan(d*x+c)^5/a^2/d-2/7*tan(d*x+c)^7/a^2/d
```

Rubi [A]

time = 0.25, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2702, 308, 213, 2700, 276, 294}

$$-\frac{2\tan^7(c+dx)}{7a^2d}-\frac{8\tan^5(c+dx)}{5a^2d}-\frac{4\tan^3(c+dx)}{a^2d}-\frac{8\tan(c+dx)}{a^2d}+\frac{2\cot(c+dx)}{a^2d}+\frac{11\sec^7(c+dx)}{14a^2d}+\frac{11\sec^5(c+dx)}{10a^2d}+\frac{11\sec^3(c+dx)}{6a^2d}+\frac{11\sec(c+dx)}{2a^2d}-\frac{11\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^7(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-11*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (2*Cot[c + d*x])/(a^2*d) + (11*Sec[
c + d*x])/(2*a^2*d) + (11*Sec[c + d*x]^3)/(6*a^2*d) + (11*Sec[c + d*x]^5)/(
10*a^2*d) + (11*Sec[c + d*x]^7)/(14*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^7
)/(2*a^2*d) - (8*Tan[c + d*x])/(a^2*d) - (4*Tan[c + d*x]^3)/(a^2*d) - (8*Ta
n[c + d*x]^5)/(5*a^2*d) - (2*Tan[c + d*x]^7)/(7*a^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \csc^3(c+dx)\sec^8(c+dx)(a-a\sin(c+dx))^2\,dx}{a^4}$$

$$= \frac{\int (a^2\csc(c+dx)\sec^8(c+dx) - 2a^2\csc^2(c+dx)\sec^8(c+dx) + a^2\csc^3(c}{a^4}$$

$$= \frac{\int \csc(c+dx)\sec^8(c+dx)\,dx}{a^2} + \frac{\int \csc^3(c+dx)\sec^8(c+dx)\,dx}{a^2} - \frac{2\int \csc}{}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \sec(c+dx)\right.}{a^2d}$$

$$= -\frac{\csc^2(c+dx)\sec^7(c+dx)}{2a^2d} + \frac{\text{Subst}\left(\int \left(1+x^2+x^4+x^6+\frac{1}{-1+x^2}\right)\,dx, x\right.}{a^2d}$$

$$= \frac{2\cot(c+dx)}{a^2d} + \frac{\sec(c+dx)}{a^2d} + \frac{\sec^3(c+dx)}{3a^2d} + \frac{\sec^5(c+dx)}{5a^2d} + \frac{\sec^7(c+d}{7a^2d}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{11\sec(c+dx)}{2a^2d} + \frac{11\sec^3(c+d}{6a^2d}$$

$$= -\frac{11\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{2\cot(c+dx)}{a^2d} + \frac{11\sec(c+dx)}{2a^2d} + \frac{11\sec^3(c+}{6a^2d}$$

Mathematica [A]

time = 0.42, size = 277, normalized size = 1.43

$$\frac{-36960\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7 + 36960\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7 + \frac{\csc^2(c+dx)(4510-6908\cos(c+dx)-563\cos(2(c+dx))+4396\cos(3(c+dx))-5390\cos(4(c+dx))+3140\cos(5(c+dx))-1917\cos(6(c+dx))-628\cos(7(c+dx))+4488\sin(c+dx)-7536\sin(2(c+dx))+3836\sin(3(c+dx))-780\sin(5(c+dx))+2512\sin(6(c+dx))-768\sin(7(c+dx)))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}}{6720d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-36960*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + 369
60*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7 + (Csc[c +
 d*x]^2*(4510 - 6908*Cos[c + d*x] - 563*Cos[2*(c + d*x)] + 4396*Cos[3*(c +
d*x)] - 5390*Cos[4*(c + d*x)] + 3140*Cos[5*(c + d*x)] - 1917*Cos[6*(c + d*x
)] - 628*Cos[7*(c + d*x)] + 4488*Sin[c + d*x] - 7536*Sin[2*(c + d*x)] + 383
6*Sin[3*(c + d*x)] - 780*Sin[5*(c + d*x)] + 2512*Sin[6*(c + d*x)] - 768*Sin
[7*(c + d*x)]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)/(6720*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin[c + d*x])^2)
```

Maple [A]

time = 0.59, size = 222, normalized size = 1.14

method	result

derivativedivides	$\frac{-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{}{\left(\tan\left(\right.\right.}}{}$
default	$\frac{-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{3}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{16}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^7}-\frac{}{\left(\tan\left(\right.\right.}}{}$
risch	$\frac{4620ie^{12i(dx+c)}+1155\,e^{13i(dx+c)}+1540ie^{10i(dx+c)}-5390\,e^{11i(dx+c)}-12936ie^{8i(dx+c)}-9779\,e^{9i(dx+c)}-3960ie^{6i(dx+c)}+9}{105\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^3\left(e\right.}$
norman	$\frac{\frac{1}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{434\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}+\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8da}-\frac{55\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{291\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{53\left(\tan^9\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^2*(-1/3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(tan(1/2*d*x+1/2*c)-1)^2-3/(ta
n(1/2*d*x+1/2*c)-1)+1/2*tan(1/2*d*x+1/2*c)^2-4*tan(1/2*d*x+1/2*c)+16/7/(tan
(1/2*d*x+1/2*c)+1)^7-8/(tan(1/2*d*x+1/2*c)+1)^6+104/5/(tan(1/2*d*x+1/2*c)+1
)^5-32/(tan(1/2*d*x+1/2*c)+1)^4+139/3/(tan(1/2*d*x+1/2*c)+1)^3-83/2/(tan(1/
2*d*x+1/2*c)+1)^2+67/(tan(1/2*d*x+1/2*c)+1)-1/2/tan(1/2*d*x+1/2*c)^2+4/tan(
1/2*d*x+1/2*c)+22*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(178) = 356$.
time = 0.36, size = 526, normalized size = 2.71

$$\frac{\frac{\frac{420\sin(dx+c)}{\cos(dx+c)+1}+\frac{15173\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{38432\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{894\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{95344\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{77182\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{61992\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{101115\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{11340\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{33495\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{14280\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-105}{\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{4a^2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{8a^2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{14a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{14a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{8a^2\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{3a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{4a^2\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{105\left(\frac{8\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2}+\frac{4620\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/840*((420*sin(d*x + c)/(cos(d*x + c) + 1) + 15173*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 38432*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 894*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 95344*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 77182
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 61992*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + 101115*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 11340*sin(d*x + c)^9/(
cos(d*x + c) + 1)^9 - 33495*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 14280*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 105)/(a^2*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 4*a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 - 8*a^2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14*a^2
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 14*a^2*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 8*a^2*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 3*a^2*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 4*a^2*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^2*s
```

```
in(d*x + c)^12/(cos(d*x + c) + 1)^12) - 105*(8*sin(d*x + c)/(cos(d*x + c) +
 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^2 + 4620*log(sin(d*x + c)/(cos
(d*x + c) + 1))/a^2)/d
```

Fricas [A]
time = 0.39, size = 292, normalized size = 1.51

$$-\frac{3834 \cos(dx+c)^6 - 3056 \cos(dx+c)^4 - 468 \cos(dx+c)^2 + 1155 \left(\cos(dx+c)^7 - 3 \cos(dx+c)^5 + 2 \cos(dx+c)^3 - 2 \left(\cos(dx+c)^5 - \cos(dx+c)^3\right) \sin(dx+c)\right) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) - 1155 \left(\cos(dx+c)^7 - 3 \cos(dx+c)^5 + 2 \cos(dx+c)^3 - 2 \left(\cos(dx+c)^5 - \cos(dx+c)^3\right) \sin(dx+c)\right) \log\left(-\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) + 4 \left(768 \cos(dx+c)^6 - 765 \cos(dx+c)^4 - 98 \cos(dx+c)^2 - 10\right) \sin(dx+c) - 100}{420 \left(a^2 d \cos(dx+c)^7 - 3 a^2 d \cos(dx+c)^5 + 2 a^2 d \cos(dx+c)^3 - 2 \left(a^2 d \cos(dx+c)^5 - a^2 d \cos(dx+c)^3\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/420*(3834*cos(d*x + c)^6 - 3056*cos(d*x + c)^4 - 468*cos(d*x + c)^2 + 11
55*(cos(d*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^
5 - cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 1155*(cos(d
*x + c)^7 - 3*cos(d*x + c)^5 + 2*cos(d*x + c)^3 - 2*(cos(d*x + c)^5 - cos(d
*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(768*cos(d*x + c)
^6 - 765*cos(d*x + c)^4 - 98*cos(d*x + c)^2 - 10)*sin(d*x + c) - 100)/(a^2*
d*cos(d*x + c)^7 - 3*a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^3 - 2*(a^2
*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [A]
time = 0.53, size = 238, normalized size = 1.23

$$\frac{\frac{4620 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^2} + \frac{105 \left(a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a^4} - \frac{105 \left(66 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 8 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)}{a^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2} - \frac{35 \left(18 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 33 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 17\right)}{a^2 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)^3} + \frac{14070 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^6 + 75705 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 177205 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 226450 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 166488 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 66661 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 11533}{a^2 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^7}}{840 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(4620*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 105*(a^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^4 - 105*(66*tan(1/2*d*x + 1/2*c)^2 -
8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d*x + 1/2*c)^2) - 35*(18*tan(1/2*d
*x + 1/2*c)^2 - 33*tan(1/2*d*x + 1/2*c) + 17)/(a^2*(tan(1/2*d*x + 1/2*c) -
1)^3) + (14070*tan(1/2*d*x + 1/2*c)^6 + 75705*tan(1/2*d*x + 1/2*c)^5 + 1772
05*tan(1/2*d*x + 1/2*c)^4 + 226450*tan(1/2*d*x + 1/2*c)^3 + 166488*tan(1/2*
```

```
d*x + 1/2*c)^2 + 66661*tan(1/2*d*x + 1/2*c) + 11533)/(a^2*(tan(1/2*d*x + 1/
2*c) + 1)^7))/d
```

Mupad [B]
time = 10.77, size = 243, normalized size = 1.25

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d}+\frac{11\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-\frac{319\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{8}+\frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2}+\frac{963\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{8}+\frac{369\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{5}-\frac{5513\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{60}-\frac{11918\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{105}+\frac{149\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{140}+\frac{4804\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{105}+\frac{15173\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{840}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}-\frac{1}{8}\right)}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + (11*log(tan(c/2 + (d*x)/2)))/(2*a^2*d) - t
an(c/2 + (d*x)/2)/(a^2*d) - (cot(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2)/2 + (
15173*tan(c/2 + (d*x)/2)^2)/840 + (4804*tan(c/2 + (d*x)/2)^3)/105 + (149*ta
n(c/2 + (d*x)/2)^4)/140 - (11918*tan(c/2 + (d*x)/2)^5)/105 - (5513*tan(c/2
+ (d*x)/2)^6)/60 + (369*tan(c/2 + (d*x)/2)^7)/5 + (963*tan(c/2 + (d*x)/2)^8
)/8 + (27*tan(c/2 + (d*x)/2)^9)/2 - (319*tan(c/2 + (d*x)/2)^10)/8 - 17*tan(
c/2 + (d*x)/2)^11 - 1/8))/(a^2*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x
)/2) + 1)^7)
```

3.839 $\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$\frac{x}{a^3}+\frac{3\sec(c+dx)}{a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{\tan^3(}{3}$$

```
[Out] x/a^3+3*sec(d*x+c)/a^3/d-13/3*sec(d*x+c)^3/a^3/d+21/5*sec(d*x+c)^5/a^3/d-15
/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-tan(d*x+c)/a^3/d+1/3*tan(d*x+c
)^3/a^3/d-1/5*tan(d*x+c)^5/a^3/d+1/7*tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^
3/d
```

Rubi [A]
time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2954, 2952, 2686, 276, 2687, 30, 200, 3554, 8}

$$-\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{15\sec^7(c+dx)}{7a^3d}+\frac{21\sec^5(c+dx)}{5a^3d}-\frac{13\sec^3(c+dx)}{3a^3d}+\frac{3\sec(c+dx)}{a^3d}+\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] x/a^3 + (3*Sec[c + d*x])/(a^3*d) - (13*Sec[c + d*x]^3)/(3*a^3*d) + (21*Sec[
c + d*x]^5)/(5*a^3*d) - (15*Sec[c + d*x]^7)/(7*a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - Tan[c + d*x]/(a^3*d) + Tan[c + d*x]^3/(3*a^3*d) - Tan[c + d*x]^
5/(5*a^3*d) + Tan[c + d*x]^7/(7*a^3*d) - (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^3(c+dx)(a-a\sin(c+dx))^3\tan^7(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^3(c+dx)\tan^7(c+dx) - 3a^3\sec^2(c+dx)\tan^8(c+dx) + 3a^3\sec\right.}{a^6}$$

$$= \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{\int \tan^{10}(c+dx)\,dx}{a^3} - \frac{3\int \sec^2(c+dx)\,t}{a^3}$$

$$= -\frac{\tan^9(c+dx)}{9a^3d} + \frac{\int \tan^8(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, \sec(\right.}{a^3d}$$

$$= \frac{\tan^7(c+dx)}{7a^3d} - \frac{4\tan^9(c+dx)}{9a^3d} - \frac{\int \tan^6(c+dx)\,dx}{a^3} + \frac{\text{Subst}\left(\int (-x^2+3\right.}{}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4}{}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4}{}$$

$$= \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx)}{7a^3d} + \frac{4}{}$$

$$= \frac{x}{a^3} + \frac{3\sec(c+dx)}{a^3d} - \frac{13\sec^3(c+dx)}{3a^3d} + \frac{21\sec^5(c+dx)}{5a^3d} - \frac{15\sec^7(c+dx}{7a^3d}$$

Mathematica [A]
time = 0.41, size = 273, normalized size = 1.53

$$\frac{169344 - 675036\cos(c+dx) + 362880(c+dx)\cos(c+dx) + 173952\cos(2(c+dx)) - 37502\cos(3(c+dx)) + 20160(c+dx)\cos(3(c+dx)) + 54912\cos(4(c+dx)) + 112506\cos(5(c+dx)) - 60480(c+dx)\cos(5(c+dx)) - 21376\cos(6(c+dx)) + 93312\sin(c+dx) - 506277\sin(2(c+dx)) + 272160(c+dx)\sin(2(c+dx)) + 125248\sin(3(c+dx)) - 225012\sin(4(c+dx)) + 120960(c+dx)\sin(4(c+dx)) + 67776\sin(5(c+dx)) + 18751\sin(6(c+dx)) - 10080(c+dx)\sin(6(c+dx))}{322560d\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (169344 - 675036*Cos[c + d*x] + 362880*(c + d*x)*Cos[c + d*x] + 173952*Cos[
2*(c + d*x)] - 37502*Cos[3*(c + d*x)] + 20160*(c + d*x)*Cos[3*(c + d*x)] +
54912*Cos[4*(c + d*x)] + 112506*Cos[5*(c + d*x)] - 60480*(c + d*x)*Cos[5*(c
 + d*x)] - 21376*Cos[6*(c + d*x)] + 93312*Sin[c + d*x] - 506277*Sin[2*(c +
d*x)] + 272160*(c + d*x)*Sin[2*(c + d*x)] + 125248*Sin[3*(c + d*x)] - 22501
2*Sin[4*(c + d*x)] + 120960*(c + d*x)*Sin[4*(c + d*x)] + 67776*Sin[5*(c + d
*x)] + 18751*Sin[6*(c + d*x)] - 10080*(c + d*x)*Sin[6*(c + d*x)])/(322560*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.29, size = 202, normalized size = 1.13

method	result
risch	$\frac{x}{a^3}+\frac{20i\mathrm{e}^{10i(dx+c)}+6\,\mathrm{e}^{11i(dx+c)}+40i\mathrm{e}^{8i(dx+c)}-\frac{50\,\mathrm{e}^{9i(dx+c)}}{3}-\frac{168i\mathrm{e}^{6i(dx+c)}}{5}-\frac{428\,\mathrm{e}^{7i(dx+c)}}{5}-\frac{2608i\mathrm{e}^{4i(dx+c)}}{35}-\frac{2348\,\mathrm{e}^{5i(dx}}{35}}{\left(\mathrm{e}^{i(dx+c)}+i\right)^9\left(\mathrm{e}^{i(dx+c)}-i\right)^3 d\,a^3}$
derivativedivides	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{40}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
default	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{7}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{40}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 256/d/a^3*(-1/6144/(tan(1/2*d*x+1/2*c)-1)^3-1/4096/(tan(1/2*d*x+1/2*c)-1)^2
+7/8192/(tan(1/2*d*x+1/2*c)-1)+1/288/(tan(1/2*d*x+1/2*c)+1)^9-1/64/(tan(1/2
*d*x+1/2*c)+1)^8+5/224/(tan(1/2*d*x+1/2*c)+1)^7-1/192/(tan(1/2*d*x+1/2*c)+1
)^6-21/2560/(tan(1/2*d*x+1/2*c)+1)^5-3/1024/(tan(1/2*d*x+1/2*c)+1)^4+3/1024
/(tan(1/2*d*x+1/2*c)+1)^3+13/2048/(tan(1/2*d*x+1/2*c)+1)^2+57/8192/(tan(1/2
*d*x+1/2*c)+1)+1/128*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(162) = 324$.
time = 0.56, size = 487, normalized size = 2.74

$$\frac{2\left(\frac{\frac{1893\sin(dx+c)}{\cos(dx+c)+1}+\frac{2526\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2939\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{9936\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3546\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{11172\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9702\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{3675\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{1890\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+368}{a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^3\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{315\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2/315*((1893*sin(d*x + c)/(cos(d*x + c) + 1) + 2526*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 2939*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 9936*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4 - 3546*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 11172*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 9702*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - 3675*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 1890*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10 - 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 368)/(a^3 +
6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10
/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12) + 315*arctan(sin(d*x + c)/(cos(d*x +
 c) + 1))/a^3)/d
```

Fricas [A]
time = 0.39, size = 177, normalized size = 0.99

$$\frac{945\,dx\cos(dx+c)^5+668\cos(dx+c)^6-1260\,dx\cos(dx+c)^3-1431\cos(dx+c)^4+465\cos(dx+c)^2+\left(315\,dx\cos(dx+c)^5-1260\,dx\cos(dx+c)^3-1059\cos(dx+c)^4+305\cos(dx+c)^2-35\right)\sin(dx+c)-70}{315\left(3\,a^3d\cos(dx+c)^5-4\,a^3d\cos(dx+c)^3+\left(a^3d\cos(dx+c)^5-4\,a^3d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(945*d*x*cos(d*x + c)^5 + 668*cos(d*x + c)^6 - 1260*d*x*cos(d*x + c)^
3 - 1431*cos(d*x + c)^4 + 465*cos(d*x + c)^2 + (315*d*x*cos(d*x + c)^5 - 12
60*d*x*cos(d*x + c)^3 - 1059*cos(d*x + c)^4 + 305*cos(d*x + c)^2 - 35)*sin(
d*x + c) - 70)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*co
s(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.67, size = 181, normalized size = 1.02

$$\frac{\frac{10080\,(dx+c)}{a^3}+\frac{105\left(21\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+23\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{17955\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+160020\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+624960\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1387260\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1884582\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1556268\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+774792\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+215748\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25967}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^9}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*(d*x + c)/a^3 + 105*(21*tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*
d*x + 1/2*c) + 23)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3) + (17955*tan(1/2*d*x
+ 1/2*c)^8 + 160020*tan(1/2*d*x + 1/2*c)^7 + 624960*tan(1/2*d*x + 1/2*c)^6
+ 1387260*tan(1/2*d*x + 1/2*c)^5 + 1884582*tan(1/2*d*x + 1/2*c)^4 + 1556268
*tan(1/2*d*x + 1/2*c)^3 + 774792*tan(1/2*d*x + 1/2*c)^2 + 215748*tan(1/2*d*
x + 1/2*c) + 25967)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 17.81, size = 169, normalized size = 0.95

$$\frac{x}{a^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}-\frac{308\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}-\frac{1064\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{788\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{35}+\frac{2208\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{35}+\frac{5878\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{315}-\frac{1684\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{105}-\frac{1262\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}-\frac{736}{315}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] x/a^3 + ((5878*tan(c/2 + (d*x)/2)^3)/315 - (1684*tan(c/2 + (d*x)/2)^2)/105
- (1262*tan(c/2 + (d*x)/2))/105 + (2208*tan(c/2 + (d*x)/2)^4)/35 + (788*tan
(c/2 + (d*x)/2)^5)/35 - (1064*tan(c/2 + (d*x)/2)^6)/15 - (308*tan(c/2 + (d*
x)/2)^7)/5 + (70*tan(c/2 + (d*x)/2)^9)/3 + 12*tan(c/2 + (d*x)/2)^10 + 2*tan
(c/2 + (d*x)/2)^11 - 736/315)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 +
(d*x)/2) + 1)^9)
```

3.840 $\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{\sec(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}+\frac{4\tan^9(c+dx)}{9a^3d}$$

```
[Out] -sec(d*x+c)/a^3/d+7/3*sec(d*x+c)^3/a^3/d-3*sec(d*x+c)^5/a^3/d+13/7*sec(d*x+
c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/d+1/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x+c)^9/a
^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2954, 2952, 2687, 14, 2686, 276, 30, 200}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{\tan^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{13\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{a^3d}+\frac{7\sec^3(c+dx)}{3a^3d}-\frac{\sec(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]/(a^3*d)) + (7*Sec[c + d*x]^3)/(3*a^3*d) - (3*Sec[c + d*x]^5)
/(a^3*d) + (13*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + T
an[c + d*x]^7/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \sec^4(c+dx)(a-a\sin(c+dx))^3\tan^6(c+dx)\,dx}{a^6}$$

$$= \frac{\int \left(a^3\sec^4(c+dx)\tan^6(c+dx) - 3a^3\sec^3(c+dx)\tan^7(c+dx) + 3a^3\sec^2\right.}{a^6}$$

$$= \frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3} - \frac{\int \sec(c+dx)\tan^9(c+dx)\,dx}{a^3} - \frac{3\int \sec^3}{}$$

$$= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)^4\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int x^6\left(1+x^2\right)\,dx, x, \tan\right.}{a^3d}$$

$$= \frac{\tan^9(c+dx)}{3a^3d} - \frac{\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx, x, \sec(c+dx)\right)}{a^3d} +$$

$$= -\frac{\sec(c+dx)}{a^3d} + \frac{7\sec^3(c+dx)}{3a^3d} - \frac{3\sec^5(c+dx)}{a^3d} + \frac{13\sec^7(c+dx)}{7a^3d} - \frac{4\sec}{}$$

Mathematica [A]
time = 0.52, size = 185, normalized size = 1.53

$$\frac{-9408+36252\cos(c+dx)-12384\cos(2(c+dx))+2014\cos(3(c+dx))+4800\cos(4(c+dx))-6042\cos(5(c+dx))+608\cos(6(c+dx))-2304\sin(c+dx)+27189\sin(2(c+dx))-16256\sin(3(c+dx))+12084\sin(4(c+dx))+384\sin(5(c+dx))-1007\sin(6(c+dx))}{64512d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-9408 + 36252*Cos[c + d*x] - 12384*Cos[2*(c + d*x)] + 2014*Cos[3*(c + d*x)
] + 4800*Cos[4*(c + d*x)] - 6042*Cos[5*(c + d*x)] + 608*Cos[6*(c + d*x)] -
2304*Sin[c + d*x] + 27189*Sin[2*(c + d*x)] - 16256*Sin[3*(c + d*x)] + 12084
*Sin[4*(c + d*x)] + 384*Sin[5*(c + d*x)] - 1007*Sin[6*(c + d*x)])/(64512*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^3*(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.46, size = 190, normalized size = 1.57

method	result
risch	$-\frac{2(51\,e^{i(dx+c)}-294ie^{6i(dx+c)}+189ie^{10i(dx+c)}-39ie^{2i(dx+c)}+63ie^{8i(dx+c)}-306\,e^{5i(dx+c)}+235\,e^{3i(dx+c)}-378\,e^{7i(dx+}}{63\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^9 d\,a^3}$
derivativedivides	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{44}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{44}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}$
norman	$\frac{-\frac{96\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}+\frac{32}{63ad}+\frac{64\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{21ad}+\frac{64\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9ad}+\frac{64\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9ad}-\frac{64\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63ad}-\frac{832\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{63ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3 a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 128/d/a^3*(-1/3072/(tan(1/2*d*x+1/2*c)-1)^3-1/2048/(tan(1/2*d*x+1/2*c)-1)^2
+5/4096/(tan(1/2*d*x+1/2*c)-1)-1/144/(tan(1/2*d*x+1/2*c)+1)^9+1/32/(tan(1/2
*d*x+1/2*c)+1)^8-11/224/(tan(1/2*d*x+1/2*c)+1)^7+5/192/(tan(1/2*d*x+1/2*c)+
1)^6+1/256/(tan(1/2*d*x+1/2*c)+1)^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4-1/384/(t
an(1/2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-5/4096/(tan(1/2*d*x+1
/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(111) = 222.
time = 0.29, size = 362, normalized size = 2.99

$$-\frac{32\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+1\right)}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^3\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -32/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a^3 + 6*a^3*s
in(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*
x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
- 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d
*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x
 + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A]
time = 0.36, size = 130, normalized size = 1.07

$$-\frac{19 \cos \left(dx+c\right)^{6}+9 \cos \left(dx+c\right)^{4}-51 \cos \left(dx+c\right)^{2}+2 \left(3 \cos \left(dx+c\right)^{4}-34 \cos \left(dx+c\right)^{2}+7\right) \sin \left(dx+c\right)+7}{63 \left(3 a^{3} d \cos \left(dx+c\right)^{5}-4 a^{3} d \cos \left(dx+c\right)^{3}+\left(a^{3} d \cos \left(dx+c\right)^{5}-4 a^{3} d \cos \left(dx+c\right)^{3}\right) \sin \left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/63*(19*cos(d*x + c)^6 + 9*cos(d*x + c)^4 - 51*cos(d*x + c)^2 + 2*(3*cos(
d*x + c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) + 7)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**6/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.64, size = 172, normalized size = 1.42

$$\frac{\frac{21 \left(15 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-36 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+17\right)}{a^{3} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^{3}}-\frac{315 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8}+3024 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}+13020 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+32760 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+51282 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+43008 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+20988 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+5688 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+667}{a^{3} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{9}}}{2016 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^6/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 36*tan(1/2*d*x + 1/2*c) + 17)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 3024*tan(1/2*
d*x + 1/2*c)^7 + 13020*tan(1/2*d*x + 1/2*c)^6 + 32760*tan(1/2*d*x + 1/2*c)^
5 + 51282*tan(1/2*d*x + 1/2*c)^4 + 43008*tan(1/2*d*x + 1/2*c)^3 + 20988*tan
(1/2*d*x + 1/2*c)^2 + 5688*tan(1/2*d*x + 1/2*c) + 667)/(a^3*(tan(1/2*d*x +
1/2*c) + 1)^9))/d
```

Mupad [B]
time = 13.17, size = 184, normalized size = 1.52

$$-\frac{\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{63}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{63}-\frac{96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{7}-\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{7}}{a^3\,d\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^3\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((32*cos(c/2 + (d*x)/2)^12)/63 + (64*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/21 - (128*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/7 - (96*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/7 + (64*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d
*x)/2)^3)/63 + (128*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*
(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 +
 (d*x)/2))^9)
```

3.841 $\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$-\frac{\sec^3(c+dx)}{a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}-\frac{4\tan^9(c+dx)}{9a^3d}$$

```
[Out] -sec(d*x+c)^3/a^3/d+2*sec(d*x+c)^5/a^3/d-11/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*
x+c)^9/a^3/d-3/7*tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2954, 2952, 2686, 276, 2687, 14, 30}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{3\tan^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Sec[c + d*x]^3/(a^3*d)) + (2*Sec[c + d*x]^5)/(a^3*d) - (11*Sec[c + d*x]^7
)/(7*a^3*d) + (4*Sec[c + d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x]^7)/(7*a^3*d) -
 (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sec^5(c+dx)(a-a\sin(c+dx))^3\tan^5(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(a^3\sec^5(c+dx)\tan^5(c+dx)-3a^3\sec^4(c+dx)\tan^6(c+dx)+3a^3\sec^3\right.}{a^6} \\
&= \frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{\int \sec^2(c+dx)\tan^8(c+dx)\,dx}{a^3}-\frac{3\int \sec}{} \\
&= -\frac{\operatorname{Subst}\left(\int x^8\,dx,x,\tan(c+dx)\right)}{a^3d}+\frac{\operatorname{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(c+\right.}{a^3d} \\
&= -\frac{\tan^9(c+dx)}{9a^3d}+\frac{\operatorname{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{3\operatorname{Subst}(}{} \\
&= -\frac{\sec^3(c+dx)}{a^3d}+\frac{2\sec^5(c+dx)}{a^3d}-\frac{11\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\,\mathrm{t}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 185, normalized size = 1.76

$$
\frac{-1344+8676\cos(c+dx)-11232\cos(2(c+dx))+482\cos(3(c+dx))+4416\cos(4(c+dx))-1446\cos(5(c+dx))-32\cos(6(c+dx))-1152\sin(c+dx)+6507\sin(2(c+dx))-8128\sin(3(c+dx))+2892\sin(4(c+dx))+192\sin(5(c+dx))-241\sin(6(c+dx))}{64512d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-1344 + 8676*Cos[c + d*x] - 11232*Cos[2*(c + d*x)] + 482*Cos[3*(c + d*x)]
+ 4416*Cos[4*(c + d*x)] - 1446*Cos[5*(c + d*x)] - 32*Cos[6*(c + d*x)] - 115
2*Sin[c + d*x] + 6507*Sin[2*(c + d*x)] - 8128*Sin[3*(c + d*x)] + 2892*Sin[4
*(c + d*x)] + 192*Sin[5*(c + d*x)] - 241*Sin[6*(c + d*x)])/(64512*d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a
 + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.39, size = 190, normalized size = 1.81

method	result
risch	$-\frac{2i\left(-128i\mathrm{e}^{3i(dx+c)}+75\,\mathrm{e}^{2i(dx+c)}+6i\mathrm{e}^{i(dx+c)}-162\,\mathrm{e}^{4i(dx+c)}-1-36i\mathrm{e}^{5i(dx+c)}-42\,\mathrm{e}^{6i(dx+c)}-189\,\mathrm{e}^{8i(dx+c)}+126i\mathrm{e}^{9i(dx+}\right.}{63\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{9}d\,a^{3}}$
derivativedivides	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{48}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
default	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{48}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{}$
norman	$\frac{-\frac{64\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}+\frac{16}{63ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{21ad}+\frac{208\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63ad}+\frac{128\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{63ad}-\frac{80\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{544\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{63ad}}{a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64/d/a^3*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2+
3/2048/(tan(1/2*d*x+1/2*c)-1)+1/72/(tan(1/2*d*x+1/2*c)+1)^9-1/16/(tan(1/2*d
*x+1/2*c)+1)^8+3/28/(tan(1/2*d*x+1/2*c)+1)^7-1/12/(tan(1/2*d*x+1/2*c)+1)^6+
3/128/(tan(1/2*d*x+1/2*c)+1)^5+1/256/(tan(1/2*d*x+1/2*c)+1)^4-1/768/(tan(1/
2*d*x+1/2*c)+1)^3-1/512/(tan(1/2*d*x+1/2*c)+1)^2-3/2048/(tan(1/2*d*x+1/2*c)
+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(97) = 194$.
time = 0.32, size = 382, normalized size = 3.64

$$-\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{42\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+1\right)}{63\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{2a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{12a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^{3}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 1
2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x +
c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x +
c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A]
time = 0.36, size = 128, normalized size = 1.22

$$\frac{\cos\left(dx+c\right)^{6}-36\,\cos\left(dx+c\right)^{4}+57\,\cos\left(dx+c\right)^{2}-\left(3\,\cos\left(dx+c\right)^{4}-34\,\cos\left(dx+c\right)^{2}+7\right)\sin\left(dx+c\right)-14}{63\,\left(3\,a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}+\left(a^{3}d\cos\left(dx+c\right)^{5}-4\,a^{3}d\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/63*(cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 57*cos(d*x + c)^2 - (3*cos(d*x +
 c)^4 - 34*cos(d*x + c)^2 + 7)*sin(d*x + c) - 14)/(3*a^3*d*cos(d*x + c)^5 -
 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*s
in(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.63, size = 172, normalized size = 1.64

$$\frac{\frac{21\,\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-24\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}-\frac{189\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1764\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7224\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+16380\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+19026\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+16380\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8352\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2340\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+281}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{9}}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2016*(21*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a^3*(
tan(1/2*d*x + 1/2*c) - 1)^3) - (189*tan(1/2*d*x + 1/2*c)^8 + 1764*tan(1/2*d
```

```
*x + 1/2*c)^7 + 7224*tan(1/2*d*x + 1/2*c)^6 + 16380*tan(1/2*d*x + 1/2*c)^5
+ 19026*tan(1/2*d*x + 1/2*c)^4 + 16380*tan(1/2*d*x + 1/2*c)^3 + 8352*tan(1/
2*d*x + 1/2*c)^2 + 2340*tan(1/2*d*x + 1/2*c) + 281)/(a^3*(tan(1/2*d*x + 1/2
*c) + 1)^9))/d
```

Mupad [B]
time = 13.19, size = 208, normalized size = 1.98

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{63}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}+\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{21}+\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{63}-\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{7}-\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{7}-\frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}}{a^3\,d\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^3\,{\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((16*cos(c/2 + (d*x)/2)^12)/63 + (32*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/21 - (32*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/3 - (64*cos(c/2 +
(d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/7 - (48*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*
x)/2)^4)/7 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/63 + (64*cos(c/
2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(
c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.842 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=127

$$\frac{\sec^3(c+dx)}{3a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^5(c+dx)}{5a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{4\tan^9(c+dx)}{9a^3d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^3/d-6/5*sec(d*x+c)^5/a^3/d+9/7*sec(d*x+c)^7/a^3/d-4/9*se
c(d*x+c)^9/a^3/d+1/5*tan(d*x+c)^5/a^3/d+5/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x+
c)^9/a^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2790, 2687, 276, 2686, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{5\tan^7(c+dx)}{7a^3d}+\frac{\tan^5(c+dx)}{5a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{9\sec^7(c+dx)}{7a^3d}-\frac{6\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^3*d) - (6*Sec[c + d*x]^5)/(5*a^3*d) + (9*Sec[c + d*x]^7
)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + Tan[c + d*x]^5/(5*a^3*d) + (5*
Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \left(a^3\sec^6(c+dx)\tan^4(c+dx) - 3a^3\sec^5(c+dx)\tan^5(c+dx) + 3a^3\sec^4(c+a\right.}{a^6}$$

$$= \frac{\int \sec^6(c+dx)\tan^4(c+dx)\,dx}{a^3} - \frac{\int \sec^3(c+dx)\tan^7(c+dx)\,dx}{a^3} - \frac{3\int \sec^5(c+}{}$$

$$= -\frac{\operatorname{Subst}\left(\int x^2(-1+x^2)^3\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\operatorname{Subst}\left(\int x^4(1+x^2)^2\,dx, x, \tan(\right.}{a^3d}$$

$$= -\frac{\operatorname{Subst}\left(\int (-x^2+3x^4-3x^6+x^8)\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\operatorname{Subst}\left(\int (x^4+2x^6+\right.}{a}$$

$$= \frac{\sec^3(c+dx)}{3a^3d} - \frac{6\sec^5(c+dx)}{5a^3d} + \frac{9\sec^7(c+dx)}{7a^3d} - \frac{4\sec^9(c+dx)}{9a^3d} + \frac{\tan^5(c+dx}{5a^3d}$$

Mathematica [A]

time = 0.23, size = 185, normalized size = 1.46

$$\frac{5376+1116\cos(c+dx)-21312\cos(2(c+dx))+62\cos(3(c+dx))+8448\cos(4(c+dx))-186\cos(5(c+dx))-704\cos(6(c+dx))+39168\sin(c+dx)+837\sin(2(c+dx))-28288\sin(3(c+dx))+372\sin(4(c+dx))+4224\sin(5(c+dx))-31\sin(6(c+dx))}{322560d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 + 1116*Cos[c + d*x] - 21312*Cos[2*(c + d*x)] + 62*Cos[3*(c + d*x)] +
8448*Cos[4*(c + d*x)] - 186*Cos[5*(c + d*x)] - 704*Cos[6*(c + d*x)] + 39168
*Sin[c + d*x] + 837*Sin[2*(c + d*x)] - 28288*Sin[3*(c + d*x)] + 372*Sin[4*(
c + d*x)] + 4224*Sin[5*(c + d*x)] - 31*Sin[6*(c + d*x)])/(322560*d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a
+ a*Sin[c + d*x])^3)
```

Maple [A]

time = 0.35, size = 175, normalized size = 1.38

method	result
risch	$\frac{\frac{88\,\mathrm{e}^{i(dx+c)}}{105}-\frac{176i\mathrm{e}^{2i(dx+c)}}{105}-\frac{32\,\mathrm{e}^{7i(dx+c)}}{5}+4i\mathrm{e}^{8i(dx+c)}+\frac{48\,\mathrm{e}^{5i(dx+c)}}{35}-\frac{16i\mathrm{e}^{6i(dx+c)}}{15}-\frac{928\,\mathrm{e}^{3i(dx+c)}}{315}+\frac{8i\mathrm{e}^{4i(dx+c)}}{35}+\frac{8\,\mathrm{e}^{9i(dx}}{3}}{\left(\mathrm{e}^{i(dx+c)}+i\right)^{9}\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}a^{3}d}$
norman	$\frac{-\frac{32\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{16}{315ad}-\frac{288\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{32\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{105ad}+\frac{64\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}+\frac{32\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{315ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a^{2}}$
derivativedivides	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{32}{1024\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1024}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{}{7\left(\tan\left(\frac{d}{2}\right.\right.}}{a^{3}d}$
default	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{32}{1024\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1024}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{}{7\left(\tan\left(\frac{d}{2}\right.\right.}}{a^{3}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^3*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2+1/
1024/(tan(1/2*d*x+1/2*c)-1)-1/36/(tan(1/2*d*x+1/2*c)+1)^9+1/8/(tan(1/2*d*x+
1/2*c)+1)^8-13/56/(tan(1/2*d*x+1/2*c)+1)^7+11/48/(tan(1/2*d*x+1/2*c)+1)^6-3
9/320/(tan(1/2*d*x+1/2*c)+1)^5+3/128/(tan(1/2*d*x+1/2*c)+1)^4+1/192/(tan(1/
2*d*x+1/2*c)+1)^3-1/1024/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(113) = 226$.
time = 0.30, size = 402, normalized size = 3.17

$$-\frac{16\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{162\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{126\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{126\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+1\right)}{315\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{2a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{12a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^{3}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -16/315*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 162*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 126*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6 - 126*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 1)/((a^
3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36
*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x +
 c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 -
a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A]
time = 0.35, size = 130, normalized size = 1.02

$$\frac{22 \cos(dx+c)^6 - 99 \cos(dx+c)^4 + 120 \cos(dx+c)^2 - 2\left(33 \cos(dx+c)^4 - 80 \cos(dx+c)^2 + 35\right)\sin(dx+c) - 35}{315\left(3 a^3 d \cos(dx+c)^5 - 4 a^3 d \cos(dx+c)^3 + \left(a^3 d \cos(dx+c)^5 - 4 a^3 d \cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/315*(22*cos(d*x + c)^6 - 99*cos(d*x + c)^4 + 120*cos(d*x + c)^2 - 2*(33*c
os(d*x + c)^4 - 80*cos(d*x + c)^2 + 35)*sin(d*x + c) - 35)/(3*a^3*d*cos(d*x
 + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.71, size = 159, normalized size = 1.25

$$\frac{\frac{105\left(3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-12 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5\right)}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^3}-\frac{315 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8+2520 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7+7140 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6-1638 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4-8232 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-2988 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-432 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-13}{a^3\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^9}}{10080\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(105*(3*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 5)/(a^3*
(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 + 2520*tan(1/2*
d*x + 1/2*c)^7 + 7140*tan(1/2*d*x + 1/2*c)^6 - 1638*tan(1/2*d*x + 1/2*c)^4
- 8232*tan(1/2*d*x + 1/2*c)^3 - 2988*tan(1/2*d*x + 1/2*c)^2 - 432*tan(1/2*d
*x + 1/2*c) - 13)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 14.33, size = 232, normalized size = 1.83

$$-\frac{\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{315}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{105}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{315}-\frac{48\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{35}-\frac{288\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{35}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{5}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{5}}{a^3\, d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] -((16*cos(c/2 + (d*x)/2)^12)/315 + (32*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x
)/2))/105 - (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7)/5 - (32*cos(c/2
+ (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/5 - (288*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2)^5)/35 - (48*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/35 + (32*co
s(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/315 + (64*cos(c/2 + (d*x)/2)^10*si
n(c/2 + (d*x)/2)^2)/105)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.843 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=105

$$\frac{3\sec^5(c+dx)}{5a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{4\tan^9(c+dx)}{9a^3d}$$

```
[Out] 3/5*sec(d*x+c)^5/a^3/d-sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-3/5*tan(d*
x+c)^5/a^3/d-tan(d*x+c)^7/a^3/d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2954, 2952, 2686, 14, 2687, 276}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{\tan^7(c+dx)}{a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{3\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*Sec[c + d*x]^5)/(5*a^3*d) - Sec[c + d*x]^7/(a^3*d) + (4*Sec[c + d*x]^9)/
(9*a^3*d) - (3*Tan[c + d*x]^5)/(5*a^3*d) - Tan[c + d*x]^7/(a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sec^7(c+dx)(a-a\sin(c+dx))^3\tan^3(c+dx)\,dx}{a^6}\\
&= \frac{\int \left(a^3\sec^7(c+dx)\tan^3(c+dx)-3a^3\sec^6(c+dx)\tan^4(c+dx)+3a^3\sec^5\right.}{a^6}\\
&= \frac{\int \sec^7(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^3}-\frac{3\int \text{se}}{}\\
&= \frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)\,dx,x,\tan\right.}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(-x^6+x^8\right)\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int \left(x^6+x^8\right)\,dx,x,\tan(c\right.}{a^3d}\\
&= \frac{3\sec^5(c+dx)}{5a^3d}-\frac{\sec^7(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\tan^5(c+dx)}{5a^3d}-\frac{\tan^7(}{a}
\end{aligned}$$

Mathematica [A]
time = 0.25, size = 185, normalized size = 1.76

$$\frac{5376-1764\cos(c+dx)-4032\cos(2(c+dx))-98\cos(3(c+dx))+768\cos(4(c+dx))+294\cos(5(c+dx))-64\cos(6(c+dx))+4608\sin(c+dx)-1323\sin(2(c+dx))-128\sin(3(c+dx))-588\sin(4(c+dx))+384\sin(5(c+dx))+49\sin(6(c+dx))}{46080d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5376 - 1764*Cos[c + d*x] - 4032*Cos[2*(c + d*x)] - 98*Cos[3*(c + d*x)] + 7
68*Cos[4*(c + d*x)] + 294*Cos[5*(c + d*x)] - 64*Cos[6*(c + d*x)] + 4608*Sin
[c + d*x] - 1323*Sin[2*(c + d*x)] - 128*Sin[3*(c + d*x)] - 588*Sin[4*(c + d
*x)] + 384*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)])/(46080*d*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*Sin
[c + d*x])^3)
```

Maple [A]
time = 0.37, size = 190, normalized size = 1.81

method	result
risch	$\frac{4i\left(2i e^{3i(dx+c)}-12\,e^{2i(dx+c)}-6i e^{i(dx+c)}+18\,e^{4i(dx+c)}+1-84\,e^{6i(dx+c)}-18i e^{5i(dx+c)}+45\,e^{8i(dx+c)}+54i e^{7i(dx+c)}\right)}{45\left(e^{i(dx+c)}+i\right)^{9}\left(e^{i(dx+c)}-i\right)^{3} d\,a^{3}}$
derivativedivides	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$
default	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$
norman	$\frac{-\frac{112\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{4}{45ad}-\frac{24\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{4\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{8\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15ad}-\frac{16\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-8}{a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^3*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/18/(tan(1/2*d*x+1/2*c)+1)^9-1/4/(tan(1/2*d*x+1
/2*c)+1)^8+1/2/(tan(1/2*d*x+1/2*c)+1)^7-7/12/(tan(1/2*d*x+1/2*c)+1)^6+67/16
0/(tan(1/2*d*x+1/2*c)+1)^5-11/64/(tan(1/2*d*x+1/2*c)+1)^4+5/192/(tan(1/2*d*
x+1/2*c)+1)^3+1/128/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(97) = 194.
time = 0.29, size = 422, normalized size = 4.02

$$\frac{4\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{18\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{84\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{54\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{45\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+1\right)}{45\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^3\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/45*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 12*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 18*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 + 54*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 45*sin(d*x +
c)^8/(cos(d*x + c) + 1)^8 + 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1
```

```
) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d
*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^1
2)*d)
```

Fricas [**A**]
time = 0.36, size = 130, normalized size = 1.24

$$\frac{2\cos(dx+c)^6-9\cos(dx+c)^4+15\cos(dx+c)^2-(6\cos(dx+c)^4-5\cos(dx+c)^2+5)\sin(dx+c)-10}{45\left(3a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3+(a^3d\cos(dx+c)^5-4a^3d\cos(dx+c)^3)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 - 9*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (6*cos(d*x
+ c)^4 - 5*cos(d*x + c)^2 + 5)*sin(d*x + c) - 10)/(3*a^3*d*cos(d*x + c)^5 -
 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*s
in(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.63, size = 161, normalized size = 1.53

$$-\frac{\frac{15\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+540\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+5940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+8298\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+6372\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3528\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+972\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{1440d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/1440*(15*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^
3) - (45*tan(1/2*d*x + 1/2*c)^8 + 540*tan(1/2*d*x + 1/2*c)^7 + 3120*tan(1/2
*d*x + 1/2*c)^6 + 5940*tan(1/2*d*x + 1/2*c)^5 + 8298*tan(1/2*d*x + 1/2*c)^4
 + 6372*tan(1/2*d*x + 1/2*c)^3 + 3528*tan(1/2*d*x + 1/2*c)^2 + 972*tan(1/2*
d*x + 1/2*c) + 113)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 16.12, size = 255, normalized size = 2.43

$$\frac{\frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{45}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{15}+\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{45}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{5}+\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5}+\frac{112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{15}+\frac{24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{a^3\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((4*cos(c/2 + (d*x)/2)^12)/45 + (8*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
)/15 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (24*cos(c/2 + (d*x)/2)
^5*sin(c/2 + (d*x)/2)^7)/5 + (112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6
)/15 + (8*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/5 + (8*cos(c/2 + (d*x)
/2)^8*sin(c/2 + (d*x)/2)^4)/5 + (8*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^
3)/45 + (16*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/15)/(a^3*d*(cos(c/2
 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2
))^9)
```

3.844 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$-\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{4\tan^9(c+dx)}{9a^3d}$$

```
[Out] -1/5*sec(d*x+c)^5/a^3/d+5/7*sec(d*x+c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/d+1/3*t
an(d*x+c)^3/a^3/d+6/5*tan(d*x+c)^5/a^3/d+9/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x
+c)^9/a^3/d
```

Rubi [A]
time = 0.25, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2954, 2952, 2687, 276, 2686, 14}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{9\tan^7(c+dx)}{7a^3d}+\frac{6\tan^5(c+dx)}{5a^3d}+\frac{\tan^3(c+dx)}{3a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{\sec^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*Sec[c + d*x]^5/(a^3*d) + (5*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x
]^9)/(9*a^3*d) + Tan[c + d*x]^3/(3*a^3*d) + (6*Tan[c + d*x]^5)/(5*a^3*d) +
(9*Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \sec^8(c+dx)(a-a\sin(c+dx))^3\tan^2(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(a^3\sec^8(c+dx)\tan^2(c+dx)-3a^3\sec^7(c+dx)\tan^3(c+dx)+3a^3\sec^6\right.}{a^6} \\
&= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{\int \sec^5(c+dx)\tan^5(c+dx)\,dx}{a^3}-\frac{3\int \sec}{} \\
&= -\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x\right.}{a^3d} \\
&= -\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\sec(c+dx)\right)}{a^3d}+\frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6\right.\right.}{a} \\
&= -\frac{\sec^5(c+dx)}{5a^3d}+\frac{5\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\tan^3(c+dx)}{3a^3d}+\frac{6\tan^5}{5}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 185, normalized size = 1.46

$$
\frac{32256-9684\cos(c+dx)-6912\cos(2(c+dx))-538\cos(3(c+dx))-3072\cos(4(c+dx))+1614\cos(5(c+dx))+256\cos(6(c+dx))+73728\sin(c+dx)-7263\sin(2(c+dx))+512\sin(3(c+dx))-3228\sin(4(c+dx))-1536\sin(5(c+dx))+269\sin(6(c+dx))}{322560d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (32256 - 9684*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] - 538*Cos[3*(c + d*x)] -
 3072*Cos[4*(c + d*x)] + 1614*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 737
28*Sin[c + d*x] - 7263*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] - 3228*Sin[4
*(c + d*x)] - 1536*Sin[5*(c + d*x)] + 269*Sin[6*(c + d*x)])/(322560*d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*
(a + a*Sin[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 190, normalized size = 1.50

method	result
risch	$-\frac{16\left(6\,\mathrm{e}^{i(dx+c)}+i-2\,\mathrm{e}^{3i(dx+c)}+126i\mathrm{e}^{6i(dx+c)}-12i\mathrm{e}^{2i(dx+c)}-162\,\mathrm{e}^{5i(dx+c)}+126\,\mathrm{e}^{7i(dx+c)}-27i\mathrm{e}^{4i(dx+c)}\right)}{315\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{9}d\,a^{3}}$
derivativedivides	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{60}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{60}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}$
norman	$\frac{-\frac{16\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{44}{315ad}-\frac{8\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{4\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{32\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35ad}-\frac{88\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{105ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^3*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2-3/2
56/(tan(1/2*d*x+1/2*c)-1)-1/9/(tan(1/2*d*x+1/2*c)+1)^9+1/2/(tan(1/2*d*x+1/2
*c)+1)^8-15/14/(tan(1/2*d*x+1/2*c)+1)^7+17/12/(tan(1/2*d*x+1/2*c)+1)^6-99/8
0/(tan(1/2*d*x+1/2*c)+1)^5+23/32/(tan(1/2*d*x+1/2*c)+1)^4-1/4/(tan(1/2*d*x+
1/2*c)+1)^3+1/32/(tan(1/2*d*x+1/2*c)+1)^2+3/256/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(113) = 226.
time = 0.30, size = 442, normalized size = 3.48

$$\frac{4\left(\frac{66\sin(dx+c)}{\cos(dx+c)+1}+\frac{132\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{232\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{18\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{108\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{84\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{504\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{315\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{210\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+11\right)}{315\left(a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{2a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{12a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^{3}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 4/315*(66*sin(d*x + c)/(cos(d*x + c) + 1) + 132*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 232*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 18*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 108*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 84*sin(d*x + c
```

```
)^6/(cos(d*x + c) + 1)^6 + 504*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 315*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 210*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 11)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c)
+ 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*
sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A]
time = 0.34, size = 130, normalized size = 1.02

$$-\frac{8\cos\left(dx+c\right)^{6}-36\cos\left(dx+c\right)^{4}+15\cos\left(dx+c\right)^{2}-2\left(12\cos\left(dx+c\right)^{4}-10\cos\left(dx+c\right)^{2}-35\right)\sin\left(dx+c\right)+35}{315\left(3a^{3}d\cos\left(dx+c\right)^{5}-4a^{3}d\cos\left(dx+c\right)^{3}+\left(a^{3}d\cos\left(dx+c\right)^{5}-4a^{3}d\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/315*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - 2*(12*co
s(d*x + c)^4 - 10*cos(d*x + c)^2 - 35)*sin(d*x + c) + 35)/(3*a^3*d*cos(d*x
+ c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x +
 c)^3)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)*
*2 + 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]
time = 0.58, size = 172, normalized size = 1.35

$$-\frac{\frac{105\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{945\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+23940\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+42840\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+41958\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+32592\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+14148\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+5112\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+673}{a^{3}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{9}}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(105*(9*tan(1/2*d*x + 1/2*c)^2 - 12*tan(1/2*d*x + 1/2*c) + 7)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (945*tan(1/2*d*x + 1/2*c)^8 + 10080*tan(1/
```

```
2*d*x + 1/2*c)^7 + 23940*tan(1/2*d*x + 1/2*c)^6 + 42840*tan(1/2*d*x + 1/2*c
)^5 + 41958*tan(1/2*d*x + 1/2*c)^4 + 32592*tan(1/2*d*x + 1/2*c)^3 + 14148*t
an(1/2*d*x + 1/2*c)^2 + 5112*tan(1/2*d*x + 1/2*c) + 673)/(a^3*(tan(1/2*d*x
+ 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 14.87, size = 279, normalized size = 2.20

$$\frac{\frac{44\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{315}+\frac{88\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{105}+\frac{176\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{105}+\frac{928\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{315}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{35}+\frac{48\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{35}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{15}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{5}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{3}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((44*cos(c/2 + (d*x)/2)^12)/315 + (88*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)
/2))/105 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9)/3 + 4*cos(c/2 + (d
*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)
^7)/5 + (16*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/15 + (48*cos(c/2 + (
d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/35 + (8*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x
)/2)^4)/35 + (928*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/315 + (176*cos
(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2)/105)/(a^3*d*(cos(c/2 + (d*x)/2) -
sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.845 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=123

$$\frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+dx)\right)}+\frac{4\tan(c+dx)}{21a^3d}+\frac{4\tan^3(c+dx)}{63a^3d}$$

```
[Out] 1/9*sec(d*x+c)^3/d/(a+a*sin(d*x+c))^3-1/21*sec(d*x+c)^3/a/d/(a+a*sin(d*x+c)
)^2-1/21*sec(d*x+c)^3/d/(a^3+a^3*sin(d*x+c))+4/21*tan(d*x+c)/a^3/d+4/63*tan
(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2938, 2751, 3852}

$$\frac{4\tan^3(c+dx)}{63a^3d}+\frac{4\tan(c+dx)}{21a^3d}-\frac{\sec^3(c+dx)}{21d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\sec^3(c+dx)}{21ad(a\sin(c+dx)+a)^2}+\frac{\sec^3(c+dx)}{9d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Sec[c + d*x]^3/(9*d*(a + a*Sin[c + d*x])^3) - Sec[c + d*x]^3/(21*a*d*(a + a
*Sin[c + d*x])^2) - Sec[c + d*x]^3/(21*d*(a^3 + a^3*Sin[c + d*x])) + (4*Tan
[c + d*x])/(21*a^3*d) + (4*Tan[c + d*x]^3)/(63*a^3*d)
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}+\frac{\int \frac{\sec^4(c+dx)}{(a+a\sin(c+dx))^2}\,dx}{3a}\\ &= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}+\frac{5\int \frac{\sec^4(c+dx)}{a+a\sin(c+dx)}\,dx}{21a^2}\\ &= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+\right.}\\ &= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+\right.}\\ &= \frac{\sec^3(c+dx)}{9d(a+a\sin(c+dx))^3}-\frac{\sec^3(c+dx)}{21ad(a+a\sin(c+dx))^2}-\frac{\sec^3(c+dx)}{21d\left(a^3+a^3\sin(c+\right.}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 185, normalized size = 1.50

$$\frac{10752+900\cos(c+dx)-6912\cos(2(c+dx))+50\cos(3(c+dx))-3072\cos(4(c+dx))-150\cos(5(c+dx))+256\cos(6(c+dx))+9216\sin(c+dx)+675\sin(2(c+dx))+512\sin(3(c+dx))+300\sin(4(c+dx))-1536\sin(5(c+dx))-25\sin(6(c+dx))}{64512d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a+a\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (10752 + 900*Cos[c + d*x] - 6912*Cos[2*(c + d*x)] + 50*Cos[3*(c + d*x)] - 3
072*Cos[4*(c + d*x)] - 150*Cos[5*(c + d*x)] + 256*Cos[6*(c + d*x)] + 9216*S
in[c + d*x] + 675*Sin[2*(c + d*x)] + 512*Sin[3*(c + d*x)] + 300*Sin[4*(c +
d*x)] - 1536*Sin[5*(c + d*x)] - 25*Sin[6*(c + d*x)])/(64512*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a + a*S
in[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 190, normalized size = 1.54

method	result
risch	$-\frac{16i\left(2i e^{3i(dx+c)}-12\,e^{2i(dx+c)}-6i e^{i(dx+c)}-27\,e^{4i(dx+c)}+1+36i e^{5i(dx+c)}+42\,e^{6i(dx+c)}\right)}{63\left(e^{i(dx+c)}-i\right)^3\left(e^{i(dx+c)}+i\right)^9 d\,a^3}$
derivativedivides	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{64}{7\left(\tan\left(\frac{dx}{2}+\right.\right.}$

default	$\frac{-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{5}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{64}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
norman	$\frac{\frac{4\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2}{63ad}-\frac{4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{36\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{21ad}+\frac{8(\tan}{}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/a^3*(-1/96/(tan(1/2*d*x+1/2*c)-1)^3-1/64/(tan(1/2*d*x+1/2*c)-1)^2-5/128
/(tan(1/2*d*x+1/2*c)-1)+2/9/(tan(1/2*d*x+1/2*c)+1)^9-1/(tan(1/2*d*x+1/2*c)+
1)^8+16/7/(tan(1/2*d*x+1/2*c)+1)^7-10/3/(tan(1/2*d*x+1/2*c)+1)^6+27/8/(tan(
1/2*d*x+1/2*c)+1)^5-39/16/(tan(1/2*d*x+1/2*c)+1)^4+59/48/(tan(1/2*d*x+1/2*c
)+1)^3-13/32/(tan(1/2*d*x+1/2*c)+1)^2+5/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(113) = 226$.
time = 0.31, size = 442, normalized size = 3.59

$$\frac{2\left(\frac{6\sin(dx+c)}{\cos(dx+c)+1}+\frac{75\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{128\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{162\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{42\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{189\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{126\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{63\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+1\right)}{63\left(a^3+\frac{6a^3\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2a^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{27a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{36a^3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{36a^3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{27a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2a^3\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{12a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^3\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2/63*(6*sin(d*x + c)/(cos(d*x + c) + 1) + 75*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 128*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 162*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 - 36*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 42*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + 189*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 126*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 + 63*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 1)/((a^3 + 6*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 12*a^3*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 27*a^3*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 27*a^3*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 12*a^3*sin
(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a^3*sin(d*x + c)^11/(cos(d*x + c) +
1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A]
time = 0.35, size = 130, normalized size = 1.06

$$-\frac{8\cos\left(dx+c\right)^6-36\cos\left(dx+c\right)^4+15\cos\left(dx+c\right)^2-\left(24\cos\left(dx+c\right)^4-20\cos\left(dx+c\right)^2-7\right)\sin\left(dx+c\right)+14}{63\left(3a^3d\cos\left(dx+c\right)^5-4a^3d\cos\left(dx+c\right)^3+\left(a^3d\cos\left(dx+c\right)^5-4a^3d\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/63*(8*cos(d*x + c)^6 - 36*cos(d*x + c)^4 + 15*cos(d*x + c)^2 - (24*cos(d
*x + c)^4 - 20*cos(d*x + c)^2 - 7)*sin(d*x + c) + 14)/(3*a^3*d*cos(d*x + c)
^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^
3)*sin(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)\sec^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2
+ 3*sin(c + d*x) + 1), x)/a**3
```

Giac [A]

time = 0.60, size = 172, normalized size = 1.40

$$-\frac{\frac{21\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+13\right)}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{315\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-756\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-4200\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-11340\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-14994\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-13356\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6768\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2196\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-209}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^9}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2016*(21*(15*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 13)/(a^3
*(tan(1/2*d*x + 1/2*c) - 1)^3) - (315*tan(1/2*d*x + 1/2*c)^8 - 756*tan(1/2*
d*x + 1/2*c)^7 - 4200*tan(1/2*d*x + 1/2*c)^6 - 11340*tan(1/2*d*x + 1/2*c)^5
 - 14994*tan(1/2*d*x + 1/2*c)^4 - 13356*tan(1/2*d*x + 1/2*c)^3 - 6768*tan(1
/2*d*x + 1/2*c)^2 - 2196*tan(1/2*d*x + 1/2*c) - 209)/(a^3*(tan(1/2*d*x + 1/
2*c) + 1)^9))/d
```

Mupad [B]

time = 15.00, size = 279, normalized size = 2.27

$$\frac{\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{63}+\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{21}+\frac{50\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{21}+\frac{256\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{63}+\frac{36\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{7}-\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{7}-\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+6\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9+2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{a^3\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + a*sin(c + d*x))^3),x)
```

```
[Out] ((2*cos(c/2 + (d*x)/2)^12)/63 + (4*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)
)/21 + 2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 4*cos(c/2 + (d*x)/2)^
3*sin(c/2 + (d*x)/2)^9 + 6*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - (4*c
os(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/3 - (8*cos(c/2 + (d*x)/2)^7*sin(c
/2 + (d*x)/2)^5)/7 + (36*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/7 + (25
6*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)/63 + (50*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^2)/21)/(a^3*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))
^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.846 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\tan(c+dx)}{a^3d}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d+sec(d*x+c)/a^3/d+1/3*sec(d*x+c)^3/a^3/d+1/5*sec(
d*x+c)^5/a^3/d+1/7*sec(d*x+c)^7/a^3/d+4/9*sec(d*x+c)^9/a^3/d-3*tan(d*x+c)/a
^3/d-13/3*tan(d*x+c)^3/a^3/d-21/5*tan(d*x+c)^5/a^3/d-15/7*tan(d*x+c)^7/a^3/
d-4/9*tan(d*x+c)^9/a^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2686, 30, 2687, 276}

$$-\frac{4\tan^9(c+dx)}{9a^3d}-\frac{15\tan^7(c+dx)}{7a^3d}-\frac{21\tan^5(c+dx)}{5a^3d}-\frac{13\tan^3(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4\sec^9(c+dx)}{9a^3d}+\frac{\sec^7(c+dx)}{7a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec(c+dx)}{a^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^3*d)) + Sec[c + d*x]/(a^3*d) + Sec[c + d*x]^3/(3
*a^3*d) + Sec[c + d*x]^5/(5*a^3*d) + Sec[c + d*x]^7/(7*a^3*d) + (4*Sec[c +
d*x]^9)/(9*a^3*d) - (3*Tan[c + d*x])/(a^3*d) - (13*Tan[c + d*x]^3)/(3*a^3*d
) - (21*Tan[c + d*x]^5)/(5*a^3*d) - (15*Tan[c + d*x]^7)/(7*a^3*d) - (4*Tan[
c + d*x]^9)/(9*a^3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\int \csc(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{\int \left(-3a^3\sec^{10}(c+dx)+a^3\csc(c+dx)\sec^{10}(c+dx)+3a^3\sec^9(c+dx)\tan\right.}{a^6} \\
&= \frac{\int \csc(c+dx)\sec^{10}(c+dx)\,dx}{a^3}-\frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^3}-\frac{3\int \sec^{10}}{} \\
&= \frac{\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,\tan(c+d\right.}{a^3d} \\
&= \frac{\sec^9(c+dx)}{3a^3d}-\frac{3\tan(c+dx)}{a^3d}-\frac{4\tan^3(c+dx)}{a^3d}-\frac{18\tan^5(c+dx)}{5a^3d}-\frac{12\tan}{7} \\
&= \frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^7(c+dx)}{7a^3d}+\frac{4\sec^9(c+dx}{9a^3d} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\sec(c+dx)}{a^3d}+\frac{\sec^3(c+dx)}{3a^3d}+\frac{\sec^5(c+dx)}{5a^3d}+\frac{\sec^7}{}
\end{aligned}
$$

Mathematica [A]

time = 0.96, size = 204, normalized size = 1.09

$$
\frac{-322560\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+322560\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{357504-510876\cos(c+dx)+317952\cos(2(c+dx))-28382\cos(3(c+dx))+20352\cos(4(c+dx))+85146\cos(5(c+dx))-11776\cos(6(c+dx))+196992\sin(c+dx)-383157\sin(2(c+dx))+211648\sin(3(c+dx))-170292\sin(4(c+dx))+50496\sin(5(c+dx))+14191\sin(6(c+dx))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^9}}{322560a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-322560*Log[Cos[(c + d*x)/2]] + 322560*Log[Sin[(c + d*x)/2]] + (357504 - 5
10876*Cos[c + d*x] + 317952*Cos[2*(c + d*x)] - 28382*Cos[3*(c + d*x)] + 203
52*Cos[4*(c + d*x)] + 85146*Cos[5*(c + d*x)] - 11776*Cos[6*(c + d*x)] + 196
992*Sin[c + d*x] - 383157*Sin[2*(c + d*x)] + 211648*Sin[3*(c + d*x)] - 1702
92*Sin[4*(c + d*x)] + 50496*Sin[5*(c + d*x)] + 14191*Sin[6*(c + d*x)])/((Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
9))/(322560*a^3*d)
```

Maple [A]

time = 0.48, size = 199, normalized size = 1.06

method	result
risch	$\frac{12i\,e^{10i(dx+c)}+2\,e^{11i(dx+c)}-\frac{70\,e^{9i(dx+c)}}{3}-\frac{1064i\,e^{6i(dx+c)}}{15}-\frac{308\,e^{7i(dx+c)}}{5}-\frac{2208i\,e^{4i(dx+c)}}{35}-\frac{788\,e^{5i(dx+c)}}{35}-\frac{1684i\,e^{2i(dx+c)}}{105}}{\left(e^{i(dx+c)}+i\right)^9\left(e^{i(dx+c)}-i\right)^3 d\,a^3}$
derivativedivides	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{9}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}+\frac{72}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

default	$-\frac{1}{24\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{9}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{8}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}-\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}+\frac{72}{7\left(\tan\left(\frac{dx}{2}+\ldots\right.\right.}$
norman	$\frac{-\frac{168\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{1336}{315ad}+\frac{6\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{20\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{50\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{40\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{428\left(\tan^{7}\left(\ldots\right.\right.}{5ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a^{2}\left(\mathrm{t}\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/24/(tan(1/2*d*x+1/2*c)-1)^3-1/16/(tan(1/2*d*x+1/2*c)-1)^2-9/32/
(tan(1/2*d*x+1/2*c)-1)+8/9/(tan(1/2*d*x+1/2*c)+1)^9-4/(tan(1/2*d*x+1/2*c)+1
)^8+72/7/(tan(1/2*d*x+1/2*c)+1)^7-52/3/(tan(1/2*d*x+1/2*c)+1)^6+219/10/(tan
(1/2*d*x+1/2*c)+1)^5-83/4/(tan(1/2*d*x+1/2*c)+1)^4+193/12/(tan(1/2*d*x+1/2*
c)+1)^3-75/8/(tan(1/2*d*x+1/2*c)+1)^2+201/32/(tan(1/2*d*x+1/2*c)+1)+ln(tan(
1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(171) = 342$.
time = 0.31, size = 508, normalized size = 2.72

$$\frac{\frac{2\left(\frac{3063\sin(dx+c)}{\cos(dx+c)+1}+\frac{4866\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1289\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{11736\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{10566\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{5292\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{13482\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{6300\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{2625\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{3150\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{945\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+668\right)}{a^{3}+\frac{6a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{12a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{2a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{27a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{36a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{36a^{3}\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{27a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{2a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{12a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{6a^{3}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}+\frac{315\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/315*(2*(3063*sin(d*x + c)/(cos(d*x + c) + 1) + 4866*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 1289*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11736*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 - 10566*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 52
92*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13482*sin(d*x + c)^7/(cos(d*x + c)
 + 1)^7 + 6300*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2625*sin(d*x + c)^9/(c
os(d*x + c) + 1)^9 - 3150*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 945*sin(d
*x + c)^11/(cos(d*x + c) + 1)^11 + 668)/(a^3 + 6*a^3*sin(d*x + c)/(cos(d*x
+ c) + 1) + 12*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*a^3*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 - 27*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 36*a
^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 36*a^3*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 + 27*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 2*a^3*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 - 12*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 6*a
^3*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a^3*sin(d*x + c)^12/(cos(d*x + c
) + 1)^12) + 315*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A]
time = 0.37, size = 250, normalized size = 1.34

$$\frac{736\cos(dx+c)^{6}-1422\cos(dx+c)^{4}-510\cos(dx+c)^{2}-315\left(3\cos(dx+c)^{5}-4\cos(dx+c)^{3}+\left(\cos(dx+c)^{5}-4\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+315\left(3\cos(dx+c)^{5}-4\cos(dx+c)^{3}+\left(\cos(dx+c)^{5}-4\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2\left(789\cos(dx+c)^{4}+235\cos(dx+c)^{2}+35\right)\sin(dx+c)-140}{630\left(3a^{3}d\cos(dx+c)^{5}-4a^{3}d\cos(dx+c)^{3}+\left(a^{3}d\cos(dx+c)^{5}-4a^{3}d\cos(dx+c)^{3}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/630*(736*cos(d*x + c)^6 - 1422*cos(d*x + c)^4 - 510*cos(d*x + c)^2 - 315*
(3*cos(d*x + c)^5 - 4*cos(d*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 315*(3*cos(d*x + c)^5 - 4*cos(d
*x + c)^3 + (cos(d*x + c)^5 - 4*cos(d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(
d*x + c) + 1/2) - 2*(789*cos(d*x + c)^4 + 235*cos(d*x + c)^2 + 35)*sin(d*x
+ c) - 140)/(3*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3 + (a^3*d*cos(d
*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 187, normalized size = 1.00

$$\frac{\frac{10080 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^3}-\frac{105 \left(27 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-48 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+25\right)}{a^3 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^3}+\frac{63315 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8+412020 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7+1273440 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^6+2324700 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^5+2731302 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+2097228 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3+1032552 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+297828 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+40127}{a^3 \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^9}}{10080 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/10080*(10080*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 105*(27*tan(1/2*d*x + 1
/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 25)/(a^3*(tan(1/2*d*x + 1/2*c) - 1)^3)
+ (63315*tan(1/2*d*x + 1/2*c)^8 + 412020*tan(1/2*d*x + 1/2*c)^7 + 1273440*t
an(1/2*d*x + 1/2*c)^6 + 2324700*tan(1/2*d*x + 1/2*c)^5 + 2731302*tan(1/2*d*
x + 1/2*c)^4 + 2097228*tan(1/2*d*x + 1/2*c)^3 + 1032552*tan(1/2*d*x + 1/2*c
)^2 + 297828*tan(1/2*d*x + 1/2*c) + 40127)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^
9))/d
```

Mupad [**B**]
time = 12.56, size = 195, normalized size = 1.04

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\frac{50\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}+40\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{428\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}+\frac{168\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5}-\frac{2348\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{35}-\frac{2608\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{35}-\frac{2578\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{315}+\frac{3244\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{105}+\frac{2042\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}+\frac{1336}{315}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((2042*tan(c/2 + (d*x)/2))/105 + (3244*tan(c/2 + (d*x)/2)^2)/105 - (2578*tan(c/2 + (d*x)/2)^3)/315 - (2608*tan(c/2 + (d*x)/2)^4)/35 - (2348*tan(c/2 + (d*x)/2)^5)/35 + (168*tan(c/2 + (d*x)/2)^6)/5 + (428*tan(c/2 + (d*x)/2)^7)/5 + 40*tan(c/2 + (d*x)/2)^8 - (50*tan(c/2 + (d*x)/2)^9)/3 - 20*tan(c/2 + (d*x)/2)^10 - 6*tan(c/2 + (d*x)/2)^11 + 1336/315)/(a^3*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^9)

3.847 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=200

$$\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{3\sec^7(c+dx)}{7a^3d}-\frac{4\sec^9(c}{9a^3}$$

[Out] 3*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^3/d-3*sec(d*x+c)/a^3/d-sec(d*x+c)^3/a^3/d-3/5*sec(d*x+c)^5/a^3/d-3/7*sec(d*x+c)^7/a^3/d-4/9*sec(d*x+c)^9/a^3/d+8*tan(d*x+c)/a^3/d+22/3*tan(d*x+c)^3/a^3/d+28/5*tan(d*x+c)^5/a^3/d+17/7*tan(d*x+c)^7/a^3/d+4/9*tan(d*x+c)^9/a^3/d

Rubi [A]
time = 0.26, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2954, 2952, 3852, 2702, 308, 213, 2700, 276, 2686, 30}

$$\frac{4\tan^9(c+dx)}{9a^3d}+\frac{17\tan^7(c+dx)}{7a^3d}+\frac{28\tan^5(c+dx)}{5a^3d}+\frac{22\tan^3(c+dx)}{3a^3d}+\frac{8\tan(c+dx)}{a^3d}-\frac{\cot(c+dx)}{a^3d}-\frac{4\sec^9(c+dx)}{9a^3d}-\frac{3\sec^7(c+dx)}{7a^3d}-\frac{3\sec^5(c+dx)}{5a^3d}-\frac{\sec^3(c+dx)}{a^3d}-\frac{3\sec(c+dx)}{a^3d}+\frac{3\tanh^{-1}(\cos(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]

[Out] (3*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[c + d*x]/(a^3*d) - (3*Sec[c + d*x])/(a^3*d) - Sec[c + d*x]^3/(a^3*d) - (3*Sec[c + d*x]^5)/(5*a^3*d) - (3*Sec[c + d*x]^7)/(7*a^3*d) - (4*Sec[c + d*x]^9)/(9*a^3*d) + (8*Tan[c + d*x])/(a^3*d) + (22*Tan[c + d*x]^3)/(3*a^3*d) + (28*Tan[c + d*x]^5)/(5*a^3*d) + (17*Tan[c + d*x]^7)/(7*a^3*d) + (4*Tan[c + d*x]^9)/(9*a^3*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)(a-a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{\int (3a^3\sec^{10}(c+dx) - 3a^3\csc(c+dx)\sec^{10}(c+dx) + a^3\csc^2(c+dx)\sec^{10}}{a^6}$$
$$= \frac{\int \csc^2(c+dx)\sec^{10}(c+dx)\,dx}{a^3} - \frac{\int \sec^9(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{3\int \sec^1}{}$$
$$= -\frac{\text{Subst}\left(\int x^8\,dx, x, \sec(c+dx)\right)}{a^3d} + \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^2}\,dx, x, \tan(c+dx)\right)}{a^3d} -$$
$$= -\frac{\sec^9(c+dx)}{9a^3d} + \frac{3\tan(c+dx)}{a^3d} + \frac{4\tan^3(c+dx)}{a^3d} + \frac{18\tan^5(c+dx)}{5a^3d} + \frac{12\,\mathrm{t}}{}$$
$$= -\frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\sec^5(c+dx)}{5a^3d} - \frac{3\sec^7(c}{7a^3}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{3\sec(c+dx)}{a^3d} - \frac{\sec^3(c+dx)}{a^3d} - \frac{3\,\mathrm{s}}{}$$

Mathematica [A]
time = 0.47, size = 230, normalized size = 1.15

$$\frac{1935360\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 1935360\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{\csc(c+dx)(-590976+1083321\cos(c+dx)-653248\cos(2(c+dx))-601845\cos(3(c+dx))+340096\cos(4(c+dx))-521599\cos(5(c+dx))+259008\cos(6(c+dx))+40123\cos(7(c+dx))-707328\sin(c+dx)+1364182\sin(2(c+dx))-1161600\sin(3(c+dx))+320984\sin(4(c+dx))-329344\sin(5(c+dx))-240738\sin(6(c+dx))+53248\sin(7(c+dx)))}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^9}}{645120a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (1935360*Log[Cos[(c + d*x)/2]] - 1935360*Log[Sin[(c + d*x)/2]] + (Csc[c + d
*x]*(-590976 + 1083321*Cos[c + d*x] - 653248*Cos[2*(c + d*x)] - 601845*Cos[
3*(c + d*x)] + 340096*Cos[4*(c + d*x)] - 521599*Cos[5*(c + d*x)] + 259008*C
os[6*(c + d*x)] + 40123*Cos[7*(c + d*x)] - 707328*Sin[c + d*x] + 1364182*Si
n[2*(c + d*x)] - 1161600*Sin[3*(c + d*x)] + 320984*Sin[4*(c + d*x)] - 32934
4*Sin[5*(c + d*x)] - 240738*Sin[6*(c + d*x)] + 53248*Sin[7*(c + d*x)]))/((C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^9))/(645120*a^3*d)
```

Maple [A]
time = 0.58, size = 224, normalized size = 1.12

method	result
derivativedivides	$\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{}{7(}$
default	$\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{11}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^8}-\frac{}{7(}$

risch	$-\frac{2\left(15962 i e^{2i(dx+c)}-9039\,e^{i(dx+c)}+945\,e^{13i(dx+c)}-11412 i e^{6i(dx+c)}+30630 i e^{4i(dx+c)}-1664 i-5670 i e^{10i(dx+c)}-1808\cdots\right.}{315\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+\cdots}\right.}$
norman	$\frac{\frac{1}{2ad}+\frac{2492\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}+\frac{\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{102\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{57\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}-\frac{601\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{134\left(\tan^{9}\left(\frac{dx}{2}\cdots\right.\right.}{ad}}{\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^3*(tan(1/2*d*x+1/2*c)-1/12/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*
x+1/2*c)-1)^2-11/16/(tan(1/2*d*x+1/2*c)-1)-16/9/(tan(1/2*d*x+1/2*c)+1)^9+8/
(tan(1/2*d*x+1/2*c)+1)^8-152/7/(tan(1/2*d*x+1/2*c)+1)^7+116/3/(tan(1/2*d*x+
1/2*c)+1)^6-267/5/(tan(1/2*d*x+1/2*c)+1)^5+111/2/(tan(1/2*d*x+1/2*c)+1)^4-5
0/(tan(1/2*d*x+1/2*c)+1)^3+67/2/(tan(1/2*d*x+1/2*c)+1)^2-501/16/(tan(1/2*d*
x+1/2*c)+1)-1/tan(1/2*d*x+1/2*c)-6*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(186) = 372$.
time = 0.31, size = 567, normalized size = 2.84

$$-\frac{\frac{\frac{8786\sin(dx+c)}{\cos(dx+c)+1}+\frac{35076\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{43062\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{41753\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{152172\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{99072\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{93324\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{157689\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{44730\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{50820\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{42210\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{10395\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+315}{\frac{a^{3}\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{12a^{3}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{27a^{3}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{36a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{36a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{27a^{3}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{2a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{12a^{3}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{6a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{a^{3}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}+\frac{1890\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{3}}-\frac{315\sin(dx+c)}{a^{3}(\cos(dx+c)+1)}}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/630*((8786*sin(d*x + c)/(cos(d*x + c) + 1) + 35076*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 43062*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 41753*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 152172*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
99072*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 93324*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7 + 157689*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 44730*sin(d*x + c
)^9/(cos(d*x + c) + 1)^9 - 50820*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 42
210*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 10395*sin(d*x + c)^12/(cos(d*x
+ c) + 1)^12 + 315)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^3*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 12*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*
a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 27*a^3*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 - 36*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 36*a^3*sin(d*x + c
)^8/(cos(d*x + c) + 1)^8 + 27*a^3*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2*a
^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 12*a^3*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 6*a^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - a^3*sin(d*x +
c)^13/(cos(d*x + c) + 1)^13) + 1890*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^
3 - 315*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 0.38, size = 297, normalized size = 1.48

$$\frac{8094\cos(dx+c)^{6}-9484\cos(dx+c)^{4}+620\cos(dx+c)^{2}+945\left(\cos(dx+c)^{7}-5\cos(dx+c)^{5}+4\cos(dx+c)^{3}-\left(3\cos(dx+c)^{5}-4\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-945\left(\cos(dx+c)^{7}-5\cos(dx+c)^{5}+4\cos(dx+c)^{3}-\left(3\cos(dx+c)^{5}-4\cos(dx+c)^{3}\right)\sin(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(1664\cos(dx+c)^{6}-4653\cos(dx+c)^{4}+285\cos(dx+c)^{2}+35\right)\sin(dx+c)+140}{630\left(a^{3}d\cos(dx+c)^{7}-5a^{3}d\cos(dx+c)^{5}+4a^{3}d\cos(dx+c)^{3}-\left(3a^{3}d\cos(dx+c)^{5}-4a^{3}d\cos(dx+c)^{3}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/630*(8094*cos(d*x + c)^6 - 9484*cos(d*x + c)^4 + 620*cos(d*x + c)^2 + 945
*(cos(d*x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5
- 4*cos(d*x + c)^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 945*(cos(d*
x + c)^7 - 5*cos(d*x + c)^5 + 4*cos(d*x + c)^3 - (3*cos(d*x + c)^5 - 4*cos(
d*x + c)^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(1664*cos(d*x +
c)^6 - 4653*cos(d*x + c)^4 + 285*cos(d*x + c)^2 + 35)*sin(d*x + c) + 140)/(
a^3*d*cos(d*x + c)^7 - 5*a^3*d*cos(d*x + c)^5 + 4*a^3*d*cos(d*x + c)^3 - (3
*a^3*d*cos(d*x + c)^5 - 4*a^3*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.62, size = 230, normalized size = 1.15

$$-\frac{\frac{30240\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5040\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{5040\,\left(6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{105\,\left(33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+31\right)}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}+\frac{157815\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+1093680\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3488940\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+6524280\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7788186\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6052704\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2995596\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+864504\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+113591}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^9}}{10080\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/10080*(30240*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5040*tan(1/2*d*x + 1/2
*c)/a^3 - 5040*(6*tan(1/2*d*x + 1/2*c) - 1)/(a^3*tan(1/2*d*x + 1/2*c)) + 10
5*(33*tan(1/2*d*x + 1/2*c)^2 - 60*tan(1/2*d*x + 1/2*c) + 31)/(a^3*(tan(1/2*
d*x + 1/2*c) - 1)^3) + (157815*tan(1/2*d*x + 1/2*c)^8 + 1093680*tan(1/2*d*x
 + 1/2*c)^7 + 3488940*tan(1/2*d*x + 1/2*c)^6 + 6524280*tan(1/2*d*x + 1/2*c)
^5 + 7788186*tan(1/2*d*x + 1/2*c)^4 + 6052704*tan(1/2*d*x + 1/2*c)^3 + 2995
596*tan(1/2*d*x + 1/2*c)^2 + 864504*tan(1/2*d*x + 1/2*c) + 113591)/(a^3*(ta
n(1/2*d*x + 1/2*c) + 1)^9))/d
```

Mupad [B]
time = 11.32, size = 390, normalized size = 1.95

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}-\frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{-33\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-134\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-\frac{484\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}+142\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{2503\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{5}+\frac{4444\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{15}-\frac{11008\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{35}-\frac{16908\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{35}-\frac{41753\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{315}+\frac{14354\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{105}+\frac{11692\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{105}+\frac{8786\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{315}+1}{d\,\left(-2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-24\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+54\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+72\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-72\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-54\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+12\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (3*log(tan(c/2 + (d*x)/2)))/(a^3*d) - ((8786
*tan(c/2 + (d*x)/2))/315 + (11692*tan(c/2 + (d*x)/2)^2)/105 + (14354*tan(c/
2 + (d*x)/2)^3)/105 - (41753*tan(c/2 + (d*x)/2)^4)/315 - (16908*tan(c/2 + (
d*x)/2)^5)/35 - (11008*tan(c/2 + (d*x)/2)^6)/35 + (4444*tan(c/2 + (d*x)/2)^
7)/15 + (2503*tan(c/2 + (d*x)/2)^8)/5 + 142*tan(c/2 + (d*x)/2)^9 - (484*tan
(c/2 + (d*x)/2)^10)/3 - 134*tan(c/2 + (d*x)/2)^11 - 33*tan(c/2 + (d*x)/2)^1
2 + 1)/(d*(12*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*tan(c/2 + (d*x)/2)^3 + 4*a^
3*tan(c/2 + (d*x)/2)^4 - 54*a^3*tan(c/2 + (d*x)/2)^5 - 72*a^3*tan(c/2 + (d*
x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^8 + 54*a^3*tan(c/2 + (d*x)/2)^9 - 4*a^3
*tan(c/2 + (d*x)/2)^10 - 24*a^3*tan(c/2 + (d*x)/2)^11 - 12*a^3*tan(c/2 + (d
*x)/2)^12 - 2*a^3*tan(c/2 + (d*x)/2)^13 + 2*a^3*tan(c/2 + (d*x)/2)))
```

3.848 $\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$\frac{4\sec^5(c+dx)}{5a^4d}-\frac{16\sec^7(c+dx)}{7a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{\tan^5(c+dx)}{5a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{16\tan^9(c}{9a^4}$$

```
[Out] 4/5*sec(d*x+c)^5/a^4/d-16/7*sec(d*x+c)^7/a^4/d+20/9*sec(d*x+c)^9/a^4/d-8/11
*sec(d*x+c)^11/a^4/d+1/5*tan(d*x+c)^5/a^4/d+9/7*tan(d*x+c)^7/a^4/d+16/9*tan
(d*x+c)^9/a^4/d+8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2790, 2687, 276, 2686, 14}

$$\frac{8\tan^{11}(c+dx)}{11a^4d}+\frac{16\tan^9(c+dx)}{9a^4d}+\frac{9\tan^7(c+dx)}{7a^4d}+\frac{\tan^5(c+dx)}{5a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{16\sec^7(c+dx)}{7a^4d}+\frac{4\sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Sec[c + d*x]^5)/(5*a^4*d) - (16*Sec[c + d*x]^7)/(7*a^4*d) + (20*Sec[c +
d*x]^9)/(9*a^4*d) - (8*Sec[c + d*x]^11)/(11*a^4*d) + Tan[c + d*x]^5/(5*a^4*
d) + (9*Tan[c + d*x]^7)/(7*a^4*d) + (16*Tan[c + d*x]^9)/(9*a^4*d) + (8*Tan[
c + d*x]^11)/(11*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(a^4\sec^8(c+dx)\tan^4(c+dx)-4a^4\sec^7(c+dx)\tan^5(c+dx)+6a^4\sec^6(c+\right.}{}$$

$$= \frac{\int \sec^8(c+dx)\tan^4(c+dx)\,dx}{a^4}+\frac{\int \sec^4(c+dx)\tan^8(c+dx)\,dx}{a^4}-\frac{4\int \sec^7(c+}{}$$

$$= \frac{\text{Subst}\left(\int x^8(1+x^2)\,dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int x^4(1+x^2)^3\,dx,x,\tan(c+\right.}{a^4d}$$

$$= \frac{\text{Subst}\left(\int (x^8+x^{10})\,dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int (x^4+3x^6+3x^8+x^{10})\,dx,\right.}{a^4d}$$

$$= \frac{4\sec^5(c+dx)}{5a^4d}-\frac{16\sec^7(c+dx)}{7a^4d}+\frac{20\sec^9(c+dx)}{9a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{\tan^5(}{5}$$

Mathematica [A]
time = 0.37, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(168960-78903\cos(c+dx)-183040\cos(2(c+dx))+8767\cos(3(c+dx))+62464\cos(4(c+dx))+19925\cos(5(c+dx))-15616\cos(6(c+dx))-797\cos(7(c+dx))+501600\sin(c+dx)-70136\sin(2(c+dx))-200288\sin(3(c+dx))-25504\sin(4(c+dx))+48800\sin(5(c+dx))+6376\sin(6(c+dx))-1952\sin(7(c+dx)))}{3548160a^4d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(168960 - 78903*Cos[c + d*x] - 183040*Cos[2*(c + d*x)] + 87
67*Cos[3*(c + d*x)] + 62464*Cos[4*(c + d*x)] + 19925*Cos[5*(c + d*x)] - 156
16*Cos[6*(c + d*x)] - 797*Cos[7*(c + d*x)] + 501600*Sin[c + d*x] - 70136*Si
n[2*(c + d*x)] - 200288*Sin[3*(c + d*x)] - 25504*Sin[4*(c + d*x)] + 48800*S
in[5*(c + d*x)] + 6376*Sin[6*(c + d*x)] - 1952*Sin[7*(c + d*x)]))/(3548160*
a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.44, size = 190, normalized size = 1.31

method	result
risch	$\frac{4i(5544i\mathrm{e}^{9i(dx+c)}+3465\,\mathrm{e}^{10i(dx+c)}-5280i\mathrm{e}^{7i(dx+c)}-10857\,\mathrm{e}^{8i(dx+c)}+176i\mathrm{e}^{5i(dx+c)}+4818\,\mathrm{e}^{6i(dx+c)}-1952i\mathrm{e}^{3i(dx+c)}-27}{3465(\mathrm{e}^{i(dx+c)}+i)^{11}(\mathrm{e}^{i(dx+c)}-i)^{3}a^4d}$
derivativedivides	$-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{176}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{28}{\left(\tan\left(\frac{dx}{2}+\right.\right.}$
default	$-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{176}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^9}+\frac{28}{\left(\tan\left(\frac{dx}{2}+\right.\right.}$
norman	$\frac{-\frac{320\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{64}{3465ad}-\frac{32\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{704\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105ad}-\frac{128\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{1504\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315ad}-\frac{512\tan(}{3465}}{a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 32/d/a^4*(-1/1536/(tan(1/2*d*x+1/2*c)-1)^3-1/1024/(tan(1/2*d*x+1/2*c)-1)^2-
1/22/(tan(1/2*d*x+1/2*c)+1)^11+1/4/(tan(1/2*d*x+1/2*c)+1)^10-11/18/(tan(1/2
*d*x+1/2*c)+1)^9+7/8/(tan(1/2*d*x+1/2*c)+1)^8-179/224/(tan(1/2*d*x+1/2*c)+1
)^7+89/192/(tan(1/2*d*x+1/2*c)+1)^6-49/320/(tan(1/2*d*x+1/2*c)+1)^5+1/64/(t
an(1/2*d*x+1/2*c)+1)^4+7/1536/(tan(1/2*d*x+1/2*c)+1)^3+1/1024/(tan(1/2*d*x+
1/2*c)+1)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(129) = 258$.
time = 0.31, size = 488, normalized size = 3.37

$$\frac{32\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{50\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{64\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{22\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{517\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{726\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{1650\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{924\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{693\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+2\right)}{3465\left(a^4+\frac{8a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{25a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{32a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{11a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{88a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{99a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{99a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{88a^4\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{11a^4\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{32a^4\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{25a^4\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^4\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{a^4\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 32/3465*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + 64*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 22*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 517*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 726*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 1650*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 924*
sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 693*sin(d*x + c)^9/(cos(d*x + c) + 1)
^9 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)
^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c
) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)
^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a
^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x
```

```
+ c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)
*d)
```

Fricas [A]
time = 0.34, size = 153, normalized size = 1.06

$$-\frac{488\cos(dx+c)^6-1220\cos(dx+c)^4+1120\cos(dx+c)^2+\left(122\cos(dx+c)^6-915\cos(dx+c)^4+1400\cos(dx+c)^2-735\right)\sin(dx+c)-420}{3465\left(a^4d\cos(dx+c)^7-8a^4d\cos(dx+c)^5+8a^4d\cos(dx+c)^3-4\left(a^4d\cos(dx+c)^5-2a^4d\cos(dx+c)^3\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
[Out] -1/3465*(488*cos(d*x + c)^6 - 1220*cos(d*x + c)^4 + 1120*cos(d*x + c)^2 + (
122*cos(d*x + c)^6 - 915*cos(d*x + c)^4 + 1400*cos(d*x + c)^2 - 735)*sin(d*
x + c) - 420)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(
d*x + c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c)
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
[Out] Timed out
```

Giac [A]
time = 0.62, size = 172, normalized size = 1.19

$$-\frac{\frac{1155\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{3465\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+47355\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+309540\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+588588\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+891198\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+747450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+481140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+172700\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+35233\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3203}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{11}}}{110880\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c) - 1)/(a^4*(tan(1/2*d*x + 1/2*c) - 1
)^3) - (3465*tan(1/2*d*x + 1/2*c)^9 + 47355*tan(1/2*d*x + 1/2*c)^8 + 309540
*tan(1/2*d*x + 1/2*c)^7 + 588588*tan(1/2*d*x + 1/2*c)^6 + 891198*tan(1/2*d*
x + 1/2*c)^5 + 747450*tan(1/2*d*x + 1/2*c)^4 + 481140*tan(1/2*d*x + 1/2*c)^
3 + 172700*tan(1/2*d*x + 1/2*c)^2 + 35233*tan(1/2*d*x + 1/2*c) + 3203)/(a^4
*(tan(1/2*d*x + 1/2*c) + 1)^11))/d
```

Mupad [B]
time = 16.85, size = 279, normalized size = 1.92

$$\frac{\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3465}+\frac{512\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3465}+\frac{320\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{693}+\frac{2048\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3465}-\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{315}+\frac{1504\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{315}+\frac{704\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{105}+\frac{320\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{21}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{15}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{5}}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((64*cos(c/2 + (d*x)/2)^14)/3465 + (512*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*
x)/2))/3465 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9)/5 + (128*cos(c
/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8)/15 + (320*cos(c/2 + (d*x)/2)^7*sin(c/
2 + (d*x)/2)^7)/21 + (704*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/105 +
(1504*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)/315 - (64*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^4)/315 + (2048*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*
x)/2)^3)/3465 + (320*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2)/693)/(a^4*
d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2
 + (d*x)/2))^11)
```

3.849 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=145

$$-\frac{\sec^5(c+dx)}{5a^4d}+\frac{9\sec^7(c+dx)}{7a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{20\tan$$

```
[Out] -1/5*sec(d*x+c)^5/a^4/d+9/7*sec(d*x+c)^7/a^4/d-16/9*sec(d*x+c)^9/a^4/d+8/11
*sec(d*x+c)^11/a^4/d-4/5*tan(d*x+c)^5/a^4/d-16/7*tan(d*x+c)^7/a^4/d-20/9*ta
n(d*x+c)^9/a^4/d-8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A]
time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2954, 2952, 2686, 14, 2687, 276}

$$-\frac{8\tan^{11}(c+dx)}{11a^4d}-\frac{20\tan^9(c+dx)}{9a^4d}-\frac{16\tan^7(c+dx)}{7a^4d}-\frac{4\tan^5(c+dx)}{5a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{9\sec^7(c+dx)}{7a^4d}-\frac{\sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/5*Sec[c + d*x]^5/(a^4*d) + (9*Sec[c + d*x]^7)/(7*a^4*d) - (16*Sec[c + d*
x]^9)/(9*a^4*d) + (8*Sec[c + d*x]^11)/(11*a^4*d) - (4*Tan[c + d*x]^5)/(5*a^
4*d) - (16*Tan[c + d*x]^7)/(7*a^4*d) - (20*Tan[c + d*x]^9)/(9*a^4*d) - (8*T
an[c + d*x]^11)/(11*a^4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2954

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[(g*Cos[e + f*x])^(2*m + p)*((d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])
^m), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\int \sec^9(c+dx)(a-a\sin(c+dx))^4\tan^3(c+dx)\,dx}{a^8}\\
&= \frac{\int \left(a^4\sec^9(c+dx)\tan^3(c+dx)-4a^4\sec^8(c+dx)\tan^4(c+dx)+6a^4\sec^7(\right.}{}\\
&= \frac{\int \sec^9(c+dx)\tan^3(c+dx)\,dx}{a^4}+\frac{\int \sec^5(c+dx)\tan^7(c+dx)\,dx}{a^4}-\frac{4\int \sec^8}{}\\
&= \frac{\operatorname{Subst}\left(\int x^8(-1+x^2)\,dx,x,\sec(c+dx)\right)}{a^4d}+\frac{\operatorname{Subst}\left(\int x^4(-1+x^2)^3\,dx,x,\mathrm{se}\right.}{a^4d}\\
&= \frac{\operatorname{Subst}\left(\int (-x^4+3x^6-3x^8+x^{10})\,dx,x,\sec(c+dx)\right)}{a^4d}+\frac{\operatorname{Subst}\left(\int (-x^8+x\right.}{}\\
&= -\frac{\sec^5(c+dx)}{5a^4d}+\frac{9\sec^7(c+dx)}{7a^4d}-\frac{16\sec^9(c+dx)}{9a^4d}+\frac{8\sec^{11}(c+dx)}{11a^4d}-\frac{4\,\mathrm{ta}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.36, size = 166, normalized size = 1.14

$$\frac{\sec^3(c+dx)(844800-215721\cos(c+dx)-619520\cos(2(c+dx))+23969\cos(3(c+dx))+32768\cos(4(c+dx))+54475\cos(5(c+dx))-8192\cos(6(c+dx))-2179\cos(7(c+dx))+844800\sin(c+dx)-191752\sin(2(c+dx))+11264\sin(3(c+dx))-69728\sin(4(c+dx))+25600\sin(5(c+dx))+17432\sin(6(c+dx))-1024\sin(7(c+dx)))}{7096320a^4d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(844800 - 215721*Cos[c + d*x] - 619520*Cos[2*(c + d*x)] + 2
3969*Cos[3*(c + d*x)] + 32768*Cos[4*(c + d*x)] + 54475*Cos[5*(c + d*x)] - 8
192*Cos[6*(c + d*x)] - 2179*Cos[7*(c + d*x)] + 844800*Sin[c + d*x] - 191752
*Sin[2*(c + d*x)] + 11264*Sin[3*(c + d*x)] - 69728*Sin[4*(c + d*x)] + 25600
*Sin[5*(c + d*x)] + 17432*Sin[6*(c + d*x)] - 1024*Sin[7*(c + d*x)]))/(70963
20*a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A]
time = 0.40, size = 220, normalized size = 1.52

method	result
risch	$-\frac{32\left(924 i \mathrm{e}^{8 i(dx+c)}+693\, \mathrm{e}^{9 i(dx+c)}-726 i \mathrm{e}^{6 i(dx+c)}-1650\, \mathrm{e}^{7 i(dx+c)}-22 i \mathrm{e}^{4 i(dx+c)}+517\, \mathrm{e}^{5 i(dx+c)}-50 i \mathrm{e}^{2 i(dx+c)}-64\, \mathrm{e}^{3 i(d}\right.}{3465\left(\mathrm{e}^{i(dx+c)}+i\right)^{11}\left(\mathrm{e}^{i(dx+c)}-i\right)^{3} d\, a^{4}}$
derivativedivides	$\frac{-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}+\frac{1}{9\left(\tan\left(\frac{d}{2}\right.\right.}}{}$
default	$\frac{-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}-\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}+\frac{1}{9\left(\tan\left(\frac{d}{2}\right.\right.}}{}$
norman	$\frac{-\frac{128\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{244}{3465ad}-\frac{4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{32\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{188\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{1016\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315ad}-\frac{64\left(\tan^{5}\right.}{3}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3} a^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 16/d/a^4*(-1/768/(tan(1/2*d*x+1/2*c)-1)^3-1/512/(tan(1/2*d*x+1/2*c)-1)^2-1/
512/(tan(1/2*d*x+1/2*c)-1)+1/11/(tan(1/2*d*x+1/2*c)+1)^11-1/2/(tan(1/2*d*x+
1/2*c)+1)^10+23/18/(tan(1/2*d*x+1/2*c)+1)^9-2/(tan(1/2*d*x+1/2*c)+1)^8+235/
112/(tan(1/2*d*x+1/2*c)+1)^7-145/96/(tan(1/2*d*x+1/2*c)+1)^6+29/40/(tan(1/2
*d*x+1/2*c)+1)^5-13/64/(tan(1/2*d*x+1/2*c)+1)^4+13/768/(tan(1/2*d*x+1/2*c)+
1)^3+3/512/(tan(1/2*d*x+1/2*c)+1)^2+1/512/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(129) = 258.
time = 0.33, size = 508, normalized size = 3.50

$$\frac{4\left(\frac{488\sin(dx+c)}{\cos(dx+c)+1}+\frac{1525\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1952\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{2794\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{176\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{4818\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{5280\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{10857\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{5544\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{3465\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+61\right)}{3465\left(a^{4}+\frac{8a^{4}\sin(dx+c)}{\cos(dx+c)+1}+\frac{25a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{32a^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{11a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{88a^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{99a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{99a^{4}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{88a^{4}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{11a^{4}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{32a^{4}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{25a^{4}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^{4}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{a^{4}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 4/3465*(488*sin(d*x + c)/(cos(d*x + c) + 1) + 1525*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 1952*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2794*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 176*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 4818*sin
```

```
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5280*sin(d*x + c)^7/(cos(d*x + c) + 1)^7
 + 10857*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 5544*sin(d*x + c)^9/(cos(d*x
 + c) + 1)^9 + 3465*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 61)/((a^4 + 8*a
^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 11*a^4*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 99*a^4*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 11*a^4*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 25*a
^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a^4*sin(d*x + c)^13/(cos(d*x +
 c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c) + 1)^14)*d)
```

Fricas [A]
time = 0.35, size = 154, normalized size = 1.06

$$-\frac{128 \cos\left(dx+c\right)^{6}-320 \cos\left(dx+c\right)^{4}+805 \cos\left(dx+c\right)^{2}+4 \left(8 \cos\left(dx+c\right)^{6}-60 \cos\left(dx+c\right)^{4}+35 \cos\left(dx+c\right)^{2}-105\right) \sin\left(dx+c\right)-735}{3465 \left(a^{4} d \cos\left(dx+c\right)^{7}-8 a^{4} d \cos\left(dx+c\right)^{5}+8 a^{4} d \cos\left(dx+c\right)^{3}-4 \left(a^{4} d \cos\left(dx+c\right)^{5}-2 a^{4} d \cos\left(dx+c\right)^{3}\right) \sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] -1/3465*(128*cos(d*x + c)^6 - 320*cos(d*x + c)^4 + 805*cos(d*x + c)^2 + 4*(
8*cos(d*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 - 105)*sin(d*x + c
) - 735)/(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x +
 c)^3 - 4*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.63, size = 198, normalized size = 1.37

$$-\frac{\frac{1155 \left(3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+2\right)}{a^{4} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)^{3}}-\frac{3465 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{10}+45045 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{9}+279510 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{8}+669900 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{7}+1205358 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{6}+1334718 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{5}+1144440 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}+627660 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{3}+257345 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+57013 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+5498}{a^{4} \left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)^{11}}}{110880 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/110880*(1155*(3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 2)/(a^
4*(tan(1/2*d*x + 1/2*c) - 1)^3) - (3465*tan(1/2*d*x + 1/2*c)^10 + 45045*tan
```

```
(1/2*d*x + 1/2*c)^9 + 279510*tan(1/2*d*x + 1/2*c)^8 + 669900*tan(1/2*d*x +
1/2*c)^7 + 1205358*tan(1/2*d*x + 1/2*c)^6 + 1334718*tan(1/2*d*x + 1/2*c)^5
+ 1144440*tan(1/2*d*x + 1/2*c)^4 + 627660*tan(1/2*d*x + 1/2*c)^3 + 257345*t
an(1/2*d*x + 1/2*c)^2 + 57013*tan(1/2*d*x + 1/2*c) + 5498)/(a^4*(tan(1/2*d*
x + 1/2*c) + 1)^11))/d
```

Mupad [B]
time = 15.89, size = 303, normalized size = 2.09

$$\frac{\frac{244\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{3465}+\frac{1952\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3465}+\frac{1220\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{693}+\frac{7808\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{3465}+\frac{1016\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{315}+\frac{64\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{315}+\frac{584\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{105}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{21}+\frac{188\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{15}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{5}+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((244*cos(c/2 + (d*x)/2)^14)/3465 + (1952*cos(c/2 + (d*x)/2)^13*sin(c/2 + (
d*x)/2))/3465 + 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 + (32*cos(c/2
+ (d*x)/2)^5*sin(c/2 + (d*x)/2)^9)/5 + (188*cos(c/2 + (d*x)/2)^6*sin(c/2 +
(d*x)/2)^8)/15 + (128*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/21 + (584*
cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/105 + (64*cos(c/2 + (d*x)/2)^9*s
in(c/2 + (d*x)/2)^5)/315 + (1016*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4
)/315 + (7808*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3)/3465 + (1220*cos(
c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2)/693)/(a^4*d*(cos(c/2 + (d*x)/2) - s
in(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^11)
```

3.850 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=143

$$-\frac{4\sec^7(c+dx)}{7a^4d}+\frac{4\sec^9(c+dx)}{3a^4d}-\frac{8\sec^{11}(c+dx)}{11a^4d}+\frac{\tan^3(c+dx)}{3a^4d}+\frac{2\tan^5(c+dx)}{a^4d}+\frac{25\tan^7(c+dx)}{7a^4d}+\frac{8\tan^9(c}{3a^4}$$

```
[Out] -4/7*sec(d*x+c)^7/a^4/d+4/3*sec(d*x+c)^9/a^4/d-8/11*sec(d*x+c)^11/a^4/d+1/3
*tan(d*x+c)^3/a^4/d+2*tan(d*x+c)^5/a^4/d+25/7*tan(d*x+c)^7/a^4/d+8/3*tan(d*
x+c)^9/a^4/d+8/11*tan(d*x+c)^11/a^4/d
```

Rubi [A]
time = 0.23, antiderivative size = 184, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2949, 2751, 3852, 8}

$$\frac{8\tan(c+dx)}{231a^4d}-\frac{4\sec(c+dx)}{231d\,(a^4\sin(c+dx)+a^4)}-\frac{4\sec(c+dx)}{231d\,(a^2\sin(c+dx)+a^2)^2}+\frac{\sec^3(c+dx)}{6ad(a\sin(c+dx)+a)^3}-\frac{5\sec(c+dx)}{231ad(a\sin(c+dx)+a)^3}-\frac{\sec(c+dx)}{33d(a\sin(c+dx)+a)^4}-\frac{a\sec(c+dx)}{22d(a\sin(c+dx)+a)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -1/22*(a*Sec[c + d*x])/(d*(a + a*Sin[c + d*x])^5) - Sec[c + d*x]/(33*d*(a +
 a*Sin[c + d*x])^4) - (5*Sec[c + d*x])/(231*a*d*(a + a*Sin[c + d*x])^3) + S
ec[c + d*x]^3/(6*a*d*(a + a*Sin[c + d*x])^3) - (4*Sec[c + d*x])/(231*d*(a^2
 + a^2*Sin[c + d*x])^2) - (4*Sec[c + d*x])/(231*d*(a^4 + a^4*Sin[c + d*x]))
 + (8*Tan[c + d*x])/(231*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2949

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(
p + 1))*((a + b*Sin[e + f*x])^(m + 1)/(2*b*f*g*(m + 1))), x] + Dist[a/(2*g^
2), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{1}{2}a\int \frac{\sec^2(c+dx)}{(a+a\sin(c+dx))^5}\,dx \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} + \frac{\sec^3(c+dx)}{6ad(a+a\sin(c+dx))^3} + \frac{3}{11}\int \frac{\sec^2(c-}{(a+a\sin(} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} + \frac{\sec^3(c+dx}{6ad(a+a\sin(c+} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+d}{231ad(a+a\sin(} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+d}{231ad(a+a\sin(} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+d}{231ad(a+a\sin(} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+d}{231ad(a+a\sin(} \\
&= -\frac{a\sec(c+dx)}{22d(a+a\sin(c+dx))^5} - \frac{\sec(c+dx)}{33d(a+a\sin(c+dx))^4} - \frac{5\sec(c+d}{231ad(a+a\sin(}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 166, normalized size = 1.16

$$\frac{\sec^3(c+dx)(11264-1287\cos(c+dx)-5632\cos(2(c+dx))+143\cos(3(c+dx))-2048\cos(4(c+dx))+325\cos(5(c+dx))+512\cos(6(c+dx))-13\cos(7(c+dx))+26048\sin(c+dx)-1144\sin(2(c+dx))-704\sin(3(c+dx))-416\sin(4(c+dx))-1600\sin(5(c+dx))+104\sin(6(c+dx))+64\sin(7(c+dx)))}{118272a^4d(1+\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]^3*(11264 - 1287*Cos[c + d*x] - 5632*Cos[2*(c + d*x)] + 143*Co
s[3*(c + d*x)] - 2048*Cos[4*(c + d*x)] + 325*Cos[5*(c + d*x)] + 512*Cos[6*(
c + d*x)] - 13*Cos[7*(c + d*x)] + 26048*Sin[c + d*x] - 1144*Sin[2*(c + d*x)
] - 704*Sin[3*(c + d*x)] - 416*Sin[4*(c + d*x)] - 1600*Sin[5*(c + d*x)] + 1
```

```
04*Sin[6*(c + d*x)] + 64*Sin[7*(c + d*x)]))/(118272*a^4*d*(1 + Sin[c + d*x]
)^4)
```

Maple [A]
time = 0.39, size = 218, normalized size = 1.52

method	result
risch	$-\frac{16i\left(176i\mathrm{e}^{7i(dx+c)}+154\,\mathrm{e}^{8i(dx+c)}-88i\mathrm{e}^{5i(dx+c)}-253\,\mathrm{e}^{6i(dx+c)}-32i\mathrm{e}^{3i(dx+c)}+11\,\mathrm{e}^{4i(dx+c)}+8i\mathrm{e}^{i(dx+c)}+25\,\mathrm{e}^{2i(dx+c)}-\cdots\right.}{231\left(\mathrm{e}^{i(dx+c)}-i\right)^{3}\left(\mathrm{e}^{i(dx+c)}+i\right)^{11}d\,a^{4}}$
derivativedivides	$\frac{-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{64}{3\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}\cdots}{\cdots}$
default	$\frac{-\frac{1}{48\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{32\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{11\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{10}}-\frac{64}{3\left(\tan\left(\frac{dx}{2}+\cdots\right.\right.}\cdots}{\cdots}$
norman	$\frac{-\frac{80\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{21ad}-\frac{16}{231ad}-\frac{8\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{16\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{32\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7ad}-\frac{32\left(\tan^{5}\left(\frac{dx}{2}+\cdots\right.\right.}{7ad}\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{11}a^{3}\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 8/d/a^4*(-1/384/(tan(1/2*d*x+1/2*c)-1)^3-1/256/(tan(1/2*d*x+1/2*c)-1)^2-1/1
28/(tan(1/2*d*x+1/2*c)-1)-2/11/(tan(1/2*d*x+1/2*c)+1)^11+1/(tan(1/2*d*x+1/2
*c)+1)^10-8/3/(tan(1/2*d*x+1/2*c)+1)^9+9/2/(tan(1/2*d*x+1/2*c)+1)^8-295/56/
(tan(1/2*d*x+1/2*c)+1)^7+71/16/(tan(1/2*d*x+1/2*c)+1)^6-43/16/(tan(1/2*d*x+
1/2*c)+1)^5+9/8/(tan(1/2*d*x+1/2*c)+1)^4-109/384/(tan(1/2*d*x+1/2*c)+1)^3+5
/256/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(129) = 258$.
time = 0.36, size = 528, normalized size = 3.69

$$\frac{8\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1}+\frac{50\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{141\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{132\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{132\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{44\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{110\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{154\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{308\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{154\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{77\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+2\right)}{231\left(a^{4}+\frac{8a^{4}\sin(dx+c)}{\cos(dx+c)+1}+\frac{25a^{4}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{32a^{4}\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{11a^{4}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{88a^{4}\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{99a^{4}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{99a^{4}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{88a^{4}\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{11a^{4}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{32a^{4}\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{25a^{4}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{8a^{4}\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{a^{4}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] 8/231*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 50*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 141*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 132*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 132*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 44*sin(d*x + c
)^6/(cos(d*x + c) + 1)^6 + 110*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 154*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 308*sin(d*x + c)^9/(cos(d*x + c) + 1)^9
 + 154*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 77*sin(d*x + c)^11/(cos(d*x
+ c) + 1)^11 + 2)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 25*a^4*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 32*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1
```

```
)^3 - 11*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 88*a^4*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 - 99*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 99*a^4*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 88*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9 + 11*a^4*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 32*a^4*sin(d*x + c)^11
/(cos(d*x + c) + 1)^11 - 25*a^4*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 8*a
^4*sin(d*x + c)^13/(cos(d*x + c) + 1)^13 - a^4*sin(d*x + c)^14/(cos(d*x + c
) + 1)^14)*d)
```

Fricas [A]
time = 0.37, size = 153, normalized size = 1.07

$$\frac{32\cos\left(dx+c\right)^{6}-80\cos\left(dx+c\right)^{4}+28\cos\left(dx+c\right)^{2}+\left(8\cos\left(dx+c\right)^{6}-60\cos\left(dx+c\right)^{4}+35\cos\left(dx+c\right)^{2}+49\right)\sin\left(dx+c\right)+28}{231\left(a^{4}d\cos\left(dx+c\right)^{7}-8\,a^{4}d\cos\left(dx+c\right)^{5}+8\,a^{4}d\cos\left(dx+c\right)^{3}-4\left(a^{4}d\cos\left(dx+c\right)^{5}-2\,a^{4}d\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] 1/231*(32*cos(d*x + c)^6 - 80*cos(d*x + c)^4 + 28*cos(d*x + c)^2 + (8*cos(d
*x + c)^6 - 60*cos(d*x + c)^4 + 35*cos(d*x + c)^2 + 49)*sin(d*x + c) + 28)/
(a^4*d*cos(d*x + c)^7 - 8*a^4*d*cos(d*x + c)^5 + 8*a^4*d*cos(d*x + c)^3 - 4
*(a^4*d*cos(d*x + c)^5 - 2*a^4*d*cos(d*x + c)^3)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^{2}\left(c+dx\right)\sec^{4}\left(c+dx\right)}{\sin^{4}\left(c+dx\right)+4\sin^{3}\left(c+dx\right)+6\sin^{2}\left(c+dx\right)+4\sin\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)*
*3 + 6*sin(c + d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A]
time = 0.59, size = 198, normalized size = 1.38

$$-\frac{\frac{77\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5\right)}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^{3}}-\frac{462\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{10}+5775\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+14399\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+29260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+30800\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+27874\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+12650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+6556\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+1210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+935\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+127}{a^{4}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^{11}}}{7392\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/7392*(77*(6*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) + 5)/(a^4*(t
an(1/2*d*x + 1/2*c) - 1)^3) - (462*tan(1/2*d*x + 1/2*c)^10 + 5775*tan(1/2*d
*x + 1/2*c)^9 + 14399*tan(1/2*d*x + 1/2*c)^8 + 29260*tan(1/2*d*x + 1/2*c)^7
```

```
 + 30800*tan(1/2*d*x + 1/2*c)^6 + 27874*tan(1/2*d*x + 1/2*c)^5 + 12650*tan(
1/2*d*x + 1/2*c)^4 + 6556*tan(1/2*d*x + 1/2*c)^3 + 1210*tan(1/2*d*x + 1/2*c
)^2 + 935*tan(1/2*d*x + 1/2*c) + 127)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^11))/
d
```

Mupad [B]
time = 16.00, size = 327, normalized size = 2.29

$$\frac{\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{231}+\frac{128\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{231}+\frac{400\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{231}+\frac{376\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{77}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{7}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{7}-\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{21}+\frac{80\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{21}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{3}+\frac{32\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{3}+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{3}+\frac{8\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{3}}{a^4\,d\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^3\,\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + a*sin(c + d*x))^4),x)
```

```
[Out] ((16*cos(c/2 + (d*x)/2)^14)/231 + (128*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x
)/2))/231 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^11)/3 + (16*cos(c/2
+ (d*x)/2)^4*sin(c/2 + (d*x)/2)^10)/3 + (32*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^9)/3 + (16*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8)/3 + (80*cos(
c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/21 - (32*cos(c/2 + (d*x)/2)^8*sin(c/
2 + (d*x)/2)^6)/21 + (32*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)/7 + (32
*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4)/7 + (376*cos(c/2 + (d*x)/2)^11
*sin(c/2 + (d*x)/2)^3)/77 + (400*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2
)/231)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^3*(cos(c/2 + (d*x)/
2) + sin(c/2 + (d*x)/2))^11)
```

3.851 $\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=133

$$-\frac{39a\log(1-\sin(c+dx))}{16d}-\frac{9a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{2d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}$$

```
[Out] -39/16*a*ln(1-sin(d*x+c))/d-9/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*
sin(d*x+c)^2/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-5/4*a^2/d/(a-a*sin(d*x+c))-1/8*
a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin^2(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{39a\log(1-\sin(c+dx))}{16d}-\frac{9a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-39*a*Log[1 - Sin[c + d*x]])/(16*d) - (9*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/(2*d) + a^3/(8*d*(a - a*Sin[c + d*
x])^2) - (5*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sin(c+dx)(a+a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^6}{a^6(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(-a+\frac{a^4}{4(a-x)^3}-\frac{5a^3}{4(a-x)^2}+\frac{39a^2}{16(a-x)}-x+\frac{a^3}{8(a+x)^2}\right.\right.}{ad}$$

$$= -\frac{39a\log(1-\sin(c+dx))}{16d}-\frac{9a\log(1+\sin(c+dx))}{16d}-$$

Mathematica [A]

time = 0.36, size = 133, normalized size = 1.00

$$-\frac{a(12\log(\cos(c+dx))+6\sec^2(c+dx)-\sec^4(c+dx)+2\sin^2(c+dx))}{4d}-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{5a(6\sec^3(c+dx)\tan(c+dx)-8\sec(c+dx)\tan^3(c+dx)-3(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A]

time = 0.21, size = 167, normalized size = 1.26

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sin^8(dx+c}{4\cos(dx+c}\right.}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\sin^8(dx+c}{4\cos(dx+c}\right.}{d}$
risch	$3iax+\frac{a\,e^{2i(dx+c)}}{8d}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{a\,e^{-2i(dx+c)}}{8d}+\frac{6iac}{d}+\frac{ia\left(6ie^{4i(dx+c)}+9\,e^{5i(dx+c)}-6ie^{2i(dx}\right.}{4\left(e^{i(dx+c)}-i\right)^4\left(e^{i(}\right.}$
norman	$\frac{\frac{12a}{d}+\frac{12a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{15a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{25a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{11a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{11a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{25a\left(\tan^9\left(\frac{dx}{2}+\right.\right.}{4d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin
(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+
a*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+
c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 106, normalized size = 0.80

$$-\frac{8\,a\sin\left(dx+c\right)^{2}+9\,a\log\left(\sin\left(dx+c\right)+1\right)+39\,a\log\left(\sin\left(dx+c\right)-1\right)+16\,a\sin\left(dx+c\right)-\frac{2\left(9\,a\sin\left(dx+c\right)^{2}+3\,a\sin\left(dx+c\right)-10\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*a*sin(d*x + c)^2 + 9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x +
c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) - 10
*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.40, size = 172, normalized size = 1.29

$$\frac{8\,a\cos\left(dx+c\right)^{4}+6\,a\cos\left(dx+c\right)^{2}-9\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-39\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(4\,a\cos\left(dx+c\right)^{4}+6\,a\cos\left(dx+c\right)^{2}-3\,a\right)\sin\left(dx+c\right)+2\,a}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*a*cos(d*x + c)^4 + 6*a*cos(d*x + c)^2 - 9*(a*cos(d*x + c)^2*sin(d*x
 + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 39*(a*cos(d*x + c)^2*sin(
d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(4*a*cos(d*x + c)^4
 + 6*a*cos(d*x + c)^2 - 3*a)*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d*x
+ c) - d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.57, size = 113, normalized size = 0.85

$$-\frac{16\,a\sin\left(dx+c\right)^{2}+18\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+78\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a\sin\left(dx+c\right)-\frac{2\,(9\,a\sin\left(dx+c\right)+7\,a)}{\sin\left(dx+c\right)+1}-\frac{117\,a\sin\left(dx+c\right)^{2}-194\,a\sin\left(dx+c\right)+81\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(16*a*sin(d*x + c)^2 + 18*a*log(abs(sin(d*x + c) + 1)) + 78*a*log(abs
(sin(d*x + c) - 1)) + 32*a*sin(d*x + c) - 2*(9*a*sin(d*x + c) + 7*a)/(sin(d
*x + c) + 1) - (117*a*sin(d*x + c)^2 - 194*a*sin(d*x + c) + 81*a)/(sin(d*x
+ c) - 1)^2)/d
```

Mupad [B]
time = 9.82, size = 286, normalized size = 2.15

$$-\frac{\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2}+7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-\frac{7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+7\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{9\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}-\frac{39\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((3*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + 7*a*tan(c/2 +
 (d*x)/2)^3 - (7*a*tan(c/2 + (d*x)/2)^4)/2 + (11*a*tan(c/2 + (d*x)/2)^5)/2
- (7*a*tan(c/2 + (d*x)/2)^6)/2 + 7*a*tan(c/2 + (d*x)/2)^7 + (3*a*tan(c/2 +
(d*x)/2)^8)/2 - (15*a*tan(c/2 + (d*x)/2)^9)/4)/(d*(tan(c/2 + (d*x)/2)^2 - 2
*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^5 - 2*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + tan(c
/2 + (d*x)/2)^10 + 1)) - (9*a*log(tan(c/2 + (d*x)/2) + 1))/(8*d) - (39*a*lo
g(tan(c/2 + (d*x)/2) - 1))/(8*d) + (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.852 $\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=115

$$-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}$$

```
[Out] -23/16*a*ln(1-sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d+1/8*a^
3/d/(a-a*sin(d*x+c))^2-a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2786, 90\}$

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\sin(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{4(a-x)^3}-\frac{a^2}{(a-x)^2}+\frac{23a}{16(a-x)}-\frac{a^2}{8(a+x)^2}+\frac{7a}{16(a+x)}\right)dx, x\right.}{d}$$
$$= -\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}$$

Mathematica [A]

time = 0.21, size = 123, normalized size = 1.07

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}-\frac{5a(6\sec^3(c+dx)\tan(c+dx)-8\sec(c+dx)\tan^3(c+dx)-3(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A]

time = 0.17, size = 121, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin}{}\right.}{d}$
default	$\frac{a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin}{}\right.}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}+\frac{i(2ia\,e^{2i(dx+c)}+9a\,e^{i(dx+c)}-2ia\,e^{4i(dx+c)}+6a\,e^{3i(dx+c)}+9a\,e^{5i(dx+c)})}{4\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^2 d}$
norman	$\frac{-\frac{15a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{10a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{10a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{15a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{6a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+a*(1/4*sin(d*x+c)
^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+
c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.83

$$\frac{7\,a\log\left(\sin\left(dx+c\right)+1\right)-23\,a\log\left(\sin\left(dx+c\right)-1\right)-16\,a\sin\left(dx+c\right)+\frac{2\,\left(9\,a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-6\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 159, normalized size = 1.38

$$\frac{16\,a\cos\left(dx+c\right)^{4}+2\,a\cos\left(dx+c\right)^{2}+7\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-23\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(8\,a\cos\left(dx+c\right)^{2}+a\right)\sin\left(dx+c\right)-6\,a}{16\,\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.54, size = 101, normalized size = 0.88

$$\frac{14\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-46\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-32\,a\sin\left(dx+c\right)-\frac{2\,\left(7\,a\sin\left(dx+c\right)+5\,a\right)}{\sin\left(dx+c\right)+1}+\frac{69\,a\sin\left(dx+c\right)^{2}-106\,a\sin\left(dx+c\right)+41\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) - 3
2*a*sin(d*x + c) - 2*(7*a*sin(d*x + c) + 5*a)/(sin(d*x + c) + 1) + (69*a*si
n(d*x + c)^2 - 106*a*sin(d*x + c) + 41*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 9.29, size = 235, normalized size = 2.04

$$-\frac{\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{23\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}+\frac{7\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((11*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/
2 + (d*x)/2)^3)/4 - 5*a*tan(c/2 + (d*x)/2)^4 + (11*a*tan(c/2 + (d*x)/2)^5)/
4 + (11*a*tan(c/2 + (d*x)/2)^6)/2 - (15*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(2*ta
n(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 2*tan(
c/2 + (d*x)/2)^5 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (2
3*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + (7*a*log(tan(c/2 + (d*x)/2) + 1))/
(8*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.853 $\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(1+\sin(c+dx))}{16d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{}{8d(a}$$

```
[Out] -11/16*a*ln(1-sin(d*x+c))/d-5/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*
x+c))^2-3/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}-\frac{5a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) - (5*a*Log[1 + Sin[c + d*x]])/(16*d) +
 a^3/(8*d*(a - a*Sin[c + d*x])^2) - (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^
2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))\tan^4(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{a^2}{4(a-x)^3} - \frac{3a}{4(a-x)^2} + \frac{11}{16(a-x)} + \frac{a}{8(a+x)^2} - \frac{5}{16(a+x}\right.\right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} - \frac{5a\log(1+\sin(c+dx))}{16d} + \frac{}{8}$$

Mathematica [A]
time = 0.18, size = 106, normalized size = 1.01

$$\frac{a\sec(c+dx)\tan^3(c+dx)}{d} - \frac{a(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d} - \frac{a(6\sec^3(c+dx)\tan(c+dx) - 3(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A]
time = 0.16, size = 111, normalized size = 1.06

method	result
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4} - \frac{\sin^5(dx+c)}{8\cos(dx+c)^2} - \frac{\left(\sin^3(dx+c)\right)}{8} - \frac{3\sin(dx+c)}{8} + \frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + a\left(\frac{\left(\tan^4(dx+c)\right)}{4} - \frac{\left(\tan^2(dx+c)\right)}{2} - \right.}{d}$
default	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4} - \frac{\sin^5(dx+c)}{8\cos(dx+c)^2} - \frac{\left(\sin^3(dx+c)\right)}{8} - \frac{3\sin(dx+c)}{8} + \frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + a\left(\frac{\left(\tan^4(dx+c)\right)}{4} - \frac{\left(\tan^2(dx+c)\right)}{2} - \right.}{d}$
risch	$iax + \frac{2iac}{d} + \frac{ia\left(6i e^{4i(dx+c)} + 5e^{5i(dx+c)} - 6i e^{2i(dx+c)} + 14e^{3i(dx+c)} + 5e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx+c)}-i\right)^4 d} - \frac{11a\ln\left(e^{i(dx+c)}-i\right)}{8d} - \frac{5a\ln(e^{i(}}{8}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d} + \frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{11a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} + \frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{3a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d} + \frac{6a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{6a(t}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin
(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/4*tan(d*x+c)^4
-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.82

$$-\frac{5\,a\log\left(\sin\left(dx+c\right)+1\right)+11\,a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(5\,a\sin\left(dx+c\right)^{2}+3\,a\sin\left(dx+c\right)-6\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 2*(5*a*sin(
d*x + c)^2 + 3*a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin
(d*x + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 136, normalized size = 1.30

$$\frac{10\,a\cos\left(dx+c\right)^{2}-5\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-11\,\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-6\,a\sin\left(dx+c\right)+2\,a}{16\,\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 - 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2
*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.55, size = 93, normalized size = 0.89

$$-\frac{10\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+22\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(5\,a\sin\left(dx+c\right)+3\,a\right)}{\sin\left(dx+c\right)+1}-\frac{33\,a\sin\left(dx+c\right)^{2}-42\,a\sin\left(dx+c\right)+13\,a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
2*(5*a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) - (33*a*sin(d*x + c)^2 - 42*a
*sin(d*x + c) + 13*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 9.26, size = 205, normalized size = 1.95

$$\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}-\frac{5\,a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{8\,d}-\frac{11\,a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{8\,d}+\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}-\frac{9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (5*a*log(tan(c/2 + (d*x)/2) + 1))/(8*
d) - (11*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + ((3*a*tan(c/2 + (d*x)/2))/4
 + (a*tan(c/2 + (d*x)/2)^2)/2 - (9*a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 +
 (d*x)/2)^4)/2 + (3*a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + t
an(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan
(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.854 $\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=84

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a+a\sin(c+dx))}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-1/2*a^2/d/(a-a*sin
(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2915, 12, 90, 212}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}+\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/
(2*d*(a - a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))\tan^3(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a}{4(a-x)^3} - \frac{1}{2(a-x)^2} - \frac{1}{8(a+x)^2} + \frac{3}{8(a^2-x^2)}\right)\,dx, x$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{2d(a-a\sin(c+dx))} + \frac{}{8d(a}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{}{2d(}$$

Mathematica [A]

time = 0.16, size = 84, normalized size = 1.00

$$\frac{a\sec(c+dx)\tan^3(c+dx)}{d} + \frac{a\tan^4(c+dx)}{4d} - \frac{a\left(6\sec^3(c+dx)\tan(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (a*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
])))/(8*d)
```

Maple [A]

time = 0.15, size = 98, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$

risch	$\frac{i\left(-2ia\,\mathrm{e}^{4i(dx+c)}-2a\,\mathrm{e}^{3i(dx+c)}+2ia\,\mathrm{e}^{2i(dx+c)}+5a\,\mathrm{e}^{5i(dx+c)}+5a\,\mathrm{e}^{i(dx+c)}\right)}{4\left(\mathrm{e}^{i(dx+c)}-i\right)^{4}\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}d}-\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{4a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a}{}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a*sin(d*x+c)^4/cos(d*x+c)^4+a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*s
in(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.02

$$\frac{3\,a\log\left(\sin\left(dx+c\right)+1\right)-3\,a\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-2\,a\right)}{\sin\left(dx+c\right)^{3}-\sin\left(dx+c\right)^{2}-\sin\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*a*log(sin(d*x + c) + 1) - 3*a*log(sin(d*x + c) - 1) + 2*(5*a*sin(d*
x + c)^2 - a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x
 + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 136, normalized size = 1.62

$$\frac{10\,a\cos\left(dx+c\right)^{2}+3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)-6\,a}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(10*a*cos(d*x + c)^2 + 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) - 3*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
+ c)^2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*
sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.56, size = 90, normalized size = 1.07

$$\frac{6\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(3\,a\sin(dx+c)+a)}{\sin(dx+c)+1}+\frac{9\,a\sin(dx+c)^2-2\,a\sin(dx+c)-3\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(6*a*log(abs(sin(d*x + c) + 1)) - 6*a*log(abs(sin(d*x + c) - 1)) - 2*(
3*a*sin(d*x + c) + a)/(sin(d*x + c) + 1) + (9*a*sin(d*x + c)^2 - 2*a*sin(d*
x + c) - 3*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 14.46, size = 167, normalized size = 1.99

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((3*a*tan(c/2 + (d*x)/2)^2)/2 - (3*
a*tan(c/2 + (d*x)/2))/4 + (a*tan(c/2 + (d*x)/2)^3)/2 + (3*a*tan(c/2 + (d*x)
/2)^4)/2 - (3*a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2
 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 +
 (d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.855 $\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=84

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a+a\sin(c+dx))}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2-1/4*a^2/d/(a-a*si
n(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2915, 12, 90, 212}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(
4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))\tan^2(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{1}{4(a-x)^3} - \frac{1}{4a(a-x)^2} + \frac{1}{8a(a+x)^2} - \frac{1}{8a(a^2-x^2)}\right) d\right.}{d}$$
$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{a^2}{4d(a-a\sin(c+dx))} - \frac{}{8d(a}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} - \frac{}{4d(}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.88

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.14, size = 88, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
default	$\frac{a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
risch	$\frac{ia\left(6i e^{4i(dx+c)}+e^{5i(dx+c)}-6i e^{2i(dx+c)}+6\,e^{3i(dx+c)}+e^{i(dx+c)}\right)}{4\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^2 d}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{8d}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{8d}$

norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^6\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin
(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*a*sin(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.00

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a\sin(dx+c)^2+3\,a\sin(dx+c)-2\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 + 3*a*sin(d*x + c) - 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x +
c) + 1))/d
```

Fricas [A]
time = 0.37, size = 135, normalized size = 1.61

$$\frac{2\,a\cos\left(dx+c\right)^2-\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-6\,a\sin\left(dx+c\right)+2\,a}{16\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 6*a*sin(d*x + c) + 2*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 91, normalized size = 1.08

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,(a\sin(dx+c)-a)}{\sin(dx+c)+1}+\frac{3\,a\sin(dx+c)^2-14\,a\sin(dx+c)+7\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) - a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 14*a*sin(d*
x + c) + 7*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 14.46, size = 167, normalized size = 1.99

$$-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*tan(c/2 + (d*x)/2))/4 - (a*tan(
c/2 + (d*x)/2)^2)/2 + (5*a*tan(c/2 + (d*x)/2)^3)/2 - (a*tan(c/2 + (d*x)/2)^
4)/2 + (a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2 + (d*
x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 + (d*x)
/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.856 $\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=61

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a+a\sin(c+dx))}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+1/8*a^2/d/(a+a*si
n(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2915, 12, 78, 212}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))\tan(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\text{Subst}\left(\int \frac{x}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^4\text{Subst}\left(\int \left(\frac{1}{4a(a-x)^3} - \frac{1}{8a^2(a+x)^2} - \frac{1}{8a^2(a^2-x^2)}\right)\,dx, x, a\sin$$

$$= \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{a^2}{8d(a+a\sin(c+dx))} - \frac{a^2\text{Sub}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3}{8d(a-a\sin(c+dx))^2} + \frac{}{8d(}$$

Mathematica [A]

time = 0.03, size = 74, normalized size = 1.21

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\sec^4(c+dx)}{4d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^4)/(4*d) - (a*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A]

time = 0.13, size = 80, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{a}{4\cos(dx+c)^4}+a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{a}{4\cos(dx+c)^4}+a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$

risch	$\frac{i\left(2ia\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{i(dx+c)}-10a\,\mathrm{e}^{3i(dx+c)}-2ia\,\mathrm{e}^{4i(dx+c)}+a\,\mathrm{e}^{5i(dx+c)}\right)}{4\left(\mathrm{e}^{i(dx+c)}-i\right)^{4}\left(\mathrm{e}^{i(dx+c)}+i\right)^{2}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8d}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}$
norman	$\frac{\frac{2a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+a(\tan^{9}\cdots}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a/cos(d*x+c)^4+a*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/c
os(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.38

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a\sin(dx+c)^{2}-a\sin(dx+c)+2a\right)}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^2 - a*sin(d*x + c) + 2*a)/(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c)
 + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(56) = 112$.
time = 0.39, size = 135, normalized size = 2.21

$$\frac{2\,a\cos\left(dx+c\right)^{2}-\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,a\sin\left(dx+c\right)-6\,a}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*a*cos(d*x + c)^2 - (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(sin(d*x + c) + 1) + (a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) + 2*a*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d
*x + c) - d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec^{5}\left(c+dx\right)\,dx+\int\sin^{2}\left(c+dx\right)\sec^{5}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x)**5, x) + Integral(sin(c + d*x)**2*sec
(c + d*x)**5, x))
```

Giac [A]
time = 0.52, size = 91, normalized size = 1.49

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\,\left(a\sin\left(dx+c\right)+3\,a\right)}{\sin\left(dx+c\right)+1}+\frac{3\,a\sin\left(dx+c\right)^{2}-6\,a\sin\left(dx+c\right)-a}{\left(\sin\left(dx+c\right)-1\right)^{2}}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*a*log(abs(sin(d*x + c) + 1)) - 2*a*log(abs(sin(d*x + c) - 1)) - 2*
(a*sin(d*x + c) + 3*a)/(sin(d*x + c) + 1) + (3*a*sin(d*x + c)^2 - 6*a*sin(d
*x + c) - a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 14.59, size = 167, normalized size = 2.74

$$-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{4}+\frac{3\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{2}-\frac{3\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{3\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((a*tan(c/2 + (d*x)/2))/4 + (3*a*ta
n(c/2 + (d*x)/2)^2)/2 - (3*a*tan(c/2 + (d*x)/2)^3)/2 + (3*a*tan(c/2 + (d*x)
/2)^4)/2 + (a*tan(c/2 + (d*x)/2)^5)/4)/(d*(2*tan(c/2 + (d*x)/2) + tan(c/2 +
 (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 2*tan(c/2 + (
d*x)/2)^5 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.857 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=117

$$-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}-\frac{5a\log(1+\sin(c+dx))}{16d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{}{2d(a-a}$$

```
[Out] -11/16*a*ln(1-sin(d*x+c))/d+a*ln(sin(d*x+c))/d-5/16*a*ln(1+sin(d*x+c))/d+1/
8*a^3/d/(a-a*sin(d*x+c))^2+1/2*a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*
x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{11a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}-\frac{5a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-11*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c + d*x]])/d - (5*a*Log[1
 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(2*d*(a -
 a*Sin[c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{1}{4a^3(a-x)^3} + \frac{1}{2a^4(a-x)^2} + \frac{11}{16a^5(a-x)} + \frac{1}{a^5x} - \frac{}{8a^4(}\right.\right.}{d}$$
$$= -\frac{11a\log(1-\sin(c+dx))}{16d} + \frac{a\log(\sin(c+dx))}{d} - \frac{5a\log}{}$$

Mathematica [A]

time = 0.17, size = 99, normalized size = 0.85

$$-\frac{a(4\log(\cos(c+dx)) - 4\log(\sin(c+dx)) - 2\sec^2(c+dx) - \sec^4(c+dx))}{4d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3a(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/4*(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec
[c + d*x]^4))/d + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]

time = 0.21, size = 82, normalized size = 0.70

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c}{8}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c}{8}\right.}{d}$
risch	$-\frac{i\left(2ia\,e^{4i(dx+c)}+18a\,e^{3i(dx+c)}-2ia\,e^{2i(dx+c)}+3a\,e^{i(dx+c)}+3a\,e^{5i(dx+c)}\right)}{4\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^2d}-\frac{11a\ln\left(e^{i(dx+c)}-i\right)}{8d}-\frac{5a\ln\left(e^{i(dx+c)}+i\right)}{8d}$
norman	$\frac{\frac{4a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a\left(\tan^9\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+a*(-(-1/4*sec(d*x
+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 95, normalized size = 0.81

$$-\frac{5\,a\log\left(\sin\left(dx+c\right)+1\right)+11\,a\log\left(\sin\left(dx+c\right)-1\right)-16\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(3\,a\sin(dx+c)^2+a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(5*a*log(sin(d*x + c) + 1) + 11*a*log(sin(d*x + c) - 1) - 16*a*log(si
n(d*x + c)) + 2*(3*a*sin(d*x + c)^2 + a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3
 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.39, size = 175, normalized size = 1.50

$$-\frac{6\,a\cos\left(dx+c\right)^2-16\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+5\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+11\left(a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,a\sin\left(dx+c\right)+6\,a}{16\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x +
 c)^2)*log(1/2*sin(d*x + c)) + 5*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x
 + c)^2)*log(sin(d*x + c) + 1) + 11*(a*cos(d*x + c)^2*sin(d*x + c) - a*cos(
d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*a*sin(d*x + c) + 6*a)/(d*cos(d*x + c
)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.53, size = 104, normalized size = 0.89

$$-\frac{10\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+22\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-32\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{2\,(5\,a\sin(dx+c)+7\,a)}{\sin(dx+c)+1}-\frac{33\,a\sin(dx+c)^2-82\,a\sin(dx+c)+53\,a}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(10*a*log(abs(sin(d*x + c) + 1)) + 22*a*log(abs(sin(d*x + c) - 1)) -
32*a*log(abs(sin(d*x + c))) - 2*(5*a*sin(d*x + c) + 7*a)/(sin(d*x + c) + 1)
 - (33*a*sin(d*x + c)^2 - 82*a*sin(d*x + c) + 53*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 0.10, size = 99, normalized size = 0.85

$$\frac{a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{3\,a\,{\sin\left(c+d\,x\right)}^2}{8}+\frac{a\,\sin\left(c+d\,x\right)}{8}-\frac{3\,a}{4}}{d\,\left({\cos\left(c+d\,x\right)}^2+{\sin\left(c+d\,x\right)}^3-\sin\left(c+d\,x\right)\right)}-\frac{11\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}-\frac{5\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a*log(sin(c + d*x)))/d - ((a*sin(c + d*x))/8 - (3*a)/4 + (3*a*sin(c + d*x)
^2)/8)/(d*(cos(c + d*x)^2 - sin(c + d*x) + sin(c + d*x)^3)) - (11*a*log(sin
(c + d*x) - 1))/(16*d) - (5*a*log(sin(c + d*x) + 1))/(16*d)
```

3.858 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}+\frac{7a\log(1+\sin(c+dx))}{16d}+\frac{a^3}{8d(a-a\sin(c+d$$

[Out] -a*csc(d*x+c)/d-23/16*a*ln(1-sin(d*x+c))/d+a*ln(sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+3/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{3a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc(c+dx)}{d}-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{a\log(\sin(c+dx))}{d}+\frac{7a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]

[Out] -((a*Csc[c + d*x])/d) - (23*a*Log[1 - Sin[c + d*x]])/(16*d) + (a*Log[Sin[c + d*x]])/d + (7*a*Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + (3*a^2)/(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{4a^4(a-x)^3} + \frac{3}{4a^5(a-x)^2} + \frac{23}{16a^6(a-x)} + \frac{1}{a^5x^2} + \frac{1}{a^6}\right.\right.}{d}$$

$$= -\frac{a\csc(c+dx)}{d} - \frac{23a\log(1-\sin(c+dx))}{16d} + \frac{a\log(\sin(c}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.14, size = 76, normalized size = 0.59

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a(4\log(\cos(c+dx)) - 4\log(\sin(c+dx)) - 2\sec^2(c+dx) - \sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

Maple [A]
time = 0.27, size = 101, normalized size = 0.78

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}\right.}{d}$
risch	$-\frac{ia\left(-22ie^{6i(dx+c)}+15\,e^{7i(dx+c)}-20ie^{4i(dx+c)}+11\,e^{5i(dx+c)}-22ie^{2i(dx+c)}-11\,e^{3i(dx+c)}-15\,e^{i(dx+c)}\right)}{4\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^2d} - \frac{23a\ln(e^{i(dx-}}{8d}$
norman	$\frac{\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{2d}+\frac{13a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(d*
x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+l
n(tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 114, normalized size = 0.88

$$\frac{7\,a\log\left(\sin\left(dx+c\right)+1\right)-23\,a\log\left(\sin\left(dx+c\right)-1\right)+16\,a\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(15\,a\sin\left(dx+c\right)^3-11\,a\sin\left(dx+c\right)^2-14\,a\sin\left(dx+c\right)+8\,a\right)}{\sin\left(dx+c\right)^4-\sin\left(dx+c\right)^3-\sin\left(dx+c\right)^2+\sin\left(dx+c\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) + 16*a*log(sin
(d*x + c)) - 2*(15*a*sin(d*x + c)^3 - 11*a*sin(d*x + c)^2 - 14*a*sin(d*x +
c) + 8*a)/(sin(d*x + c)^4 - sin(d*x + c)^3 - sin(d*x + c)^2 + sin(d*x + c))
)/d
```

Fricas [A]
time = 0.39, size = 229, normalized size = 1.78

$$-\frac{22\,a\cos\left(dx+c\right)^2-16\left(a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-7\left(a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+23\left(a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-a\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(15\,a\cos\left(dx+c\right)^2-a\right)\sin\left(dx+c\right)-6\,a}{16\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(22*a*cos(d*x + c)^2 - 16*(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) - 7*(a*cos(d*x + c)^4 + a*
cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 23*
(a*cos(d*x + c)^4 + a*cos(d*x + c)^2*sin(d*x + c) - a*cos(d*x + c)^2)*log(-
sin(d*x + c) + 1) - 2*(15*a*cos(d*x + c)^2 - a)*sin(d*x + c) - 6*a)/(d*cos(
d*x + c)^4 + d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.70, size = 121, normalized size = 0.94

$$\frac{14\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-46\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{23\,a\sin\left(dx+c\right)^2+59\,a\sin\left(dx+c\right)+32\,a}{\sin\left(dx+c\right)^2+\sin\left(dx+c\right)}+\frac{69\,a\sin\left(dx+c\right)^2-162\,a\sin\left(dx+c\right)+97\,a}{\left(\sin\left(dx+c\right)-1\right)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(14*a*log(abs(sin(d*x + c) + 1)) - 46*a*log(abs(sin(d*x + c) - 1)) + 3
2*a*log(abs(sin(d*x + c))) - (23*a*sin(d*x + c)^2 + 59*a*sin(d*x + c) + 32*
a)/(sin(d*x + c)^2 + sin(d*x + c)) + (69*a*sin(d*x + c)^2 - 162*a*sin(d*x +
 c) + 97*a)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 0.10, size = 118, normalized size = 0.91

$$\frac{a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{15\,a\,\sin\left(c+d\,x\right)^3}{8}-\frac{11\,a\,\sin\left(c+d\,x\right)^2}{8}-\frac{7\,a\,\sin\left(c+d\,x\right)}{4}+a}{d\,\left({\sin\left(c+d\,x\right)}^4-{\sin\left(c+d\,x\right)}^3-{\sin\left(c+d\,x\right)}^2+\sin\left(c+d\,x\right)\right)}-\frac{23\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}+\frac{7\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (a*log(sin(c + d*x)))/d - (a - (7*a*sin(c + d*x))/4 - (11*a*sin(c + d*x)^2)
/8 + (15*a*sin(c + d*x)^3)/8)/(d*(sin(c + d*x) - sin(c + d*x)^2 - sin(c + d
*x)^3 + sin(c + d*x)^4)) - (23*a*log(sin(c + d*x) - 1))/(16*d) + (7*a*log(s
in(c + d*x) + 1))/(16*d)
```

3.859 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{39a\log(1-\sin(c+dx))}{16d}+\frac{3a\log(\sin(c+dx))}{d}-\frac{9a\log(1+\sin(c+dx))}{16d}+\frac{}{8}$$

```
[Out] -a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-39/16*a*ln(1-sin(d*x+c))/d+3*a*ln(sin(
d*x+c))/d-9/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*sin(d*x+c))^2+a^2/d/(a-a
*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{39a\log(1-\sin(c+dx))}{16d}+\frac{3a\log(\sin(c+dx))}{d}-\frac{9a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (39*a*Log[1 - Sin[c + d*
x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d - (9*a*Log[1 + Sin[c + d*x]])/(16*d
) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + a^2/(d*(a - a*Sin[c + d*x])) + a^2/(
8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \left(\frac{1}{4a^5(a-x)^3} + \frac{1}{a^6(a-x)^2} + \frac{39}{16a^7(a-x)} + \frac{1}{a^5x^3} + \frac{1}{a^6x}\right.\right.}{d}$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{39a\log(1-\sin(c+$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.52, size = 86, normalized size = 0.60

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a(2\csc^2(c+dx)+12\log(\cos(c+dx))-12\log(\sin(c+dx))-4\sec^2(c+dx)-\sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
 d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.28, size = 129, normalized size = 0.90

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)}\right.}{d}$
risch	$-\frac{i\left(-6ia\,e^{8i(dx+c)}+15a\,e^{9i(dx+c)}-14ia\,e^{6i(dx+c)}+28a\,e^{7i(dx+c)}+14ia\,e^{4i(dx+c)}-22a\,e^{5i(dx+c)}+6ia\,e^{2i(dx+c)}+28a\,e^{3i(d}\right.}{4\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^4\left(e^{i(dx+c)}+i\right)^2d}$
norman	$\frac{-\frac{a}{8d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{13a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{13a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/sin
(d*x+c)^2+3*ln(tan(d*x+c)))+a*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/c
os(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 127, normalized size = 0.89

$$-\frac{9\,a\log\left(\sin\left(dx+c\right)+1\right)+39\,a\log\left(\sin\left(dx+c\right)-1\right)-48\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,a\sin\left(dx+c\right)^4-3\,a\sin\left(dx+c\right)^3-22\,a\sin\left(dx+c\right)^2+4\,a\sin\left(dx+c\right)+4\,a\right)}{\sin\left(dx+c\right)^5-\sin\left(dx+c\right)^4-\sin\left(dx+c\right)^3+\sin\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(9*a*log(sin(d*x + c) + 1) + 39*a*log(sin(d*x + c) - 1) - 48*a*log(si
n(d*x + c)) + 2*(15*a*sin(d*x + c)^4 - 3*a*sin(d*x + c)^3 - 22*a*sin(d*x +
c)^2 + 4*a*sin(d*x + c) + 4*a)/(sin(d*x + c)^5 - sin(d*x + c)^4 - sin(d*x +
 c)^3 + sin(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(136) = 272.
time = 0.40, size = 294, normalized size = 2.06

$$\frac{30\,a\cos\left(dx+c\right)^4-16\,a\cos\left(dx+c\right)^2+48\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2-\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-9\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2-\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-39\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2-\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,a\cos\left(dx+c\right)^2+a\right)\sin\left(dx+c\right)-6\,a}{16\left(d\cos\left(dx+c\right)^4-d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^4-d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(30*a*cos(d*x + c)^4 - 16*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^4 - a*
cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(1/
2*sin(d*x + c)) - 9*(a*cos(d*x + c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^
4 - a*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 39*(a*cos(d*x +
 c)^4 - a*cos(d*x + c)^2 - (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x +
c))*log(-sin(d*x + c) + 1) + 2*(3*a*cos(d*x + c)^2 + a)*sin(d*x + c) - 6*a)
/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - d*cos(d*x + c)^
2)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 125, normalized size = 0.87

$$-\frac{36\,a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+156\,a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-192\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{4\,(9\,a\sin(dx+c)+11\,a)}{\sin(dx+c)+1}+\frac{27\,a\sin(dx+c)^4+74\,a\sin(dx+c)^3-141\,a\sin(dx+c)^2+32\,a}{\left(\sin(dx+c)^2-\sin(dx+c)\right)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/64*(36*a*log(abs(sin(d*x + c) + 1)) + 156*a*log(abs(sin(d*x + c) - 1)) -
 192*a*log(abs(sin(d*x + c))) - 4*(9*a*sin(d*x + c) + 11*a)/(sin(d*x + c) +
 1) + (27*a*sin(d*x + c)^4 + 74*a*sin(d*x + c)^3 - 141*a*sin(d*x + c)^2 + 3
2*a)/(sin(d*x + c)^2 - sin(d*x + c))^2)/d
```

Mupad [B]
time = 9.31, size = 134, normalized size = 0.94

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{39\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}-\frac{9\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{16\,d}-\frac{\frac{15\,a\sin(c+d\,x)^4}{8}-\frac{3\,a\sin(c+d\,x)^3}{8}-\frac{11\,a\sin(c+d\,x)^2}{4}+\frac{a\,\sin(c+d\,x)}{2}+\frac{a}{2}}{d\,\left({\sin\left(c+d\,x\right)}^5-{\sin\left(c+d\,x\right)}^4-{\sin\left(c+d\,x\right)}^3+{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (39*a*log(sin(c + d*x) - 1))/(16*d) - (9*a*log(
sin(c + d*x) + 1))/(16*d) - (a/2 + (a*sin(c + d*x))/2 - (11*a*sin(c + d*x)^
2)/4 - (3*a*sin(c + d*x)^3)/8 + (15*a*sin(c + d*x)^4)/8)/(d*(sin(c + d*x)^2
 - sin(c + d*x)^3 - sin(c + d*x)^4 + sin(c + d*x)^5))
```

3.860 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$-\frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{59a\log(1-\sin(c+dx))}{16d}+\frac{3a\log(\sin(c+dx))}{d}+\frac{11a\log(1}{}$$

```
[Out] -3*a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-59/16*a*ln(1-si
n(d*x+c))/d+3*a*ln(sin(d*x+c))/d+11/16*a*ln(1+sin(d*x+c))/d+1/8*a^3/d/(a-a*
sin(d*x+c))^2+5/4*a^2/d/(a-a*sin(d*x+c))-1/8*a^2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3}{8d(a-a\sin(c+dx))^2}+\frac{5a^2}{4d(a-a\sin(c+dx))}-\frac{a^2}{8d(a\sin(c+dx)+a)}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}-\frac{59a\log(1-\sin(c+dx))}{16d}+\frac{3a\log(\sin(c+dx))}{d}+\frac{11a\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
 - (59*a*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*Log[Sin[c + d*x]])/d + (11*a*
Log[1 + Sin[c + d*x]])/(16*d) + a^3/(8*d*(a - a*Sin[c + d*x])^2) + (5*a^2)/
(4*d*(a - a*Sin[c + d*x])) - a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\text{Subst}\left(\int \left(\frac{1}{4a^6(a-x)^3}+\frac{5}{4a^7(a-x)^2}+\frac{59}{16a^8(a-x)}+\frac{1}{a^5x^4}+\frac{1}{a^6}\right.\right.}{}$$

$$= -\frac{3a\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{59}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.82, size = 90, normalized size = 0.56

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}-\frac{a(2\csc^2(c+dx)+12\log(\cos(c+dx))-12\log(\sin(c+dx))-4\sec^2(c+dx)-\sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/d
- (a*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.23, size = 147, normalized size = 0.91

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\right.}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\right.}{d}$
risch	$-\frac{ia\left(-138i\,e^{10i(dx+c)}+105\,e^{11i(dx+c)}+136i\,e^{8i(dx+c)}-101\,e^{9i(dx+c)}+260i\,e^{6i(dx+c)}-158\,e^{7i(dx+c)}+136i\,e^{4i(dx+c)}+158\,e\right.}{12\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx+c)}-i\right)^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)^3/cos(d*x+c)^4-7/12/sin(d*x+c)^3/cos(d*x+c)^2+35/24/
sin(d*x+c)/cos(d*x+c)^2-35/8/sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c)))+a*(
```

```
1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/sin(d*x+c)^
2+3*ln(tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 138, normalized size = 0.85

$$\frac{33\,a\log\left(\sin\left(dx+c\right)+1\right)-177\,a\log\left(\sin\left(dx+c\right)-1\right)+144\,a\log\left(\sin\left(dx+c\right)\right)-\frac{2\,\left(105\,a\sin\left(dx+c\right)^5-69\,a\sin\left(dx+c\right)^4-106\,a\sin\left(dx+c\right)^3+52\,a\sin\left(dx+c\right)^2+4\,a\sin\left(dx+c\right)+8\,a\right)}{\sin\left(dx+c\right)^6-\sin\left(dx+c\right)^5-\sin\left(dx+c\right)^4+\sin\left(dx+c\right)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(33*a*log(sin(d*x + c) + 1) - 177*a*log(sin(d*x + c) - 1) + 144*a*log(
sin(d*x + c)) - 2*(105*a*sin(d*x + c)^5 - 69*a*sin(d*x + c)^4 - 106*a*sin(d
*x + c)^3 + 52*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 8*a)/(sin(d*x + c)^6 -
 sin(d*x + c)^5 - sin(d*x + c)^4 + sin(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(150) = 300.
time = 0.40, size = 343, normalized size = 2.12

$$-\frac{138\,a\cos\left(dx+c\right)^4-172\,a\cos\left(dx+c\right)^2-144\,\left(a\cos\left(dx+c\right)^6-2\,a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-33\,\left(a\cos\left(dx+c\right)^6-2\,a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+177\,\left(a\cos\left(dx+c\right)^6-2\,a\cos\left(dx+c\right)^4+a\cos\left(dx+c\right)^2+\left(a\cos\left(dx+c\right)^4-a\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(105\,a\cos\left(dx+c\right)^4-104\,a\cos\left(dx+c\right)^2+3\,a\right)\sin\left(dx+c\right)+18\,a}{48\,\left(d\cos\left(dx+c\right)^6-2\,d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^2+\left(d\cos\left(dx+c\right)^4-d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(138*a*cos(d*x + c)^4 - 172*a*cos(d*x + c)^2 - 144*(a*cos(d*x + c)^6
- 2*a*cos(d*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c
)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 33*(a*cos(d*x + c)^6 - 2*a*cos(d
*x + c)^4 + a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*
x + c))*log(sin(d*x + c) + 1) + 177*(a*cos(d*x + c)^6 - 2*a*cos(d*x + c)^4
+ a*cos(d*x + c)^2 + (a*cos(d*x + c)^4 - a*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-sin(d*x + c) + 1) - 2*(105*a*cos(d*x + c)^4 - 104*a*cos(d*x + c)^2 + 3*a
)*sin(d*x + c) + 18*a)/(d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4 + d*cos(d*x +
 c)^2 + (d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 149, normalized size = 0.92

$$\frac{66\,a\log\left(|\sin\left(dx+c\right)+1|\right)-354\,a\log\left(|\sin\left(dx+c\right)-1|\right)+288\,a\log\left(|\sin\left(dx+c\right)|\right)-\frac{6\,(11\,a\sin(dx+c)+13\,a)}{\sin(dx+c)+1}+\frac{3\left(177\,a\sin(dx+c)^2-394\,a\sin(dx+c)+221\,a\right)}{(\sin(dx+c)-1)^2}-\frac{16\left(33\,a\sin(dx+c)^3+18\,a\sin(dx+c)^2+3\,a\sin(dx+c)+2\,a\right)}{\sin(dx+c)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/96*(66*a*log(abs(sin(d*x + c) + 1)) - 354*a*log(abs(sin(d*x + c) - 1)) +
288*a*log(abs(sin(d*x + c))) - 6*(11*a*sin(d*x + c) + 13*a)/(sin(d*x + c) +
 1) + 3*(177*a*sin(d*x + c)^2 - 394*a*sin(d*x + c) + 221*a)/(sin(d*x + c) -
 1)^2 - 16*(33*a*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*a*sin(d*x + c) +
2*a)/sin(d*x + c)^3)/d
```

Mupad [B]
time = 0.11, size = 145, normalized size = 0.90

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{35\,a\sin(c+d\,x)^5}{8}-\frac{23\,a\sin(c+d\,x)^4}{8}-\frac{53\,a\sin(c+d\,x)^3}{12}+\frac{13\,a\sin(c+d\,x)^2}{6}+\frac{a\,\sin(c+d\,x)}{6}+\frac{a}{3}}{d\,\left(\sin\left(c+d\,x\right)^6-\sin\left(c+d\,x\right)^5-\sin\left(c+d\,x\right)^4+\sin\left(c+d\,x\right)^3\right)}-\frac{59\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{16\,d}+\frac{11\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^4),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (a/3 + (a*sin(c + d*x))/6 + (13*a*sin(c + d*x)^
2)/6 - (53*a*sin(c + d*x)^3)/12 - (23*a*sin(c + d*x)^4)/8 + (35*a*sin(c + d
*x)^5)/8)/(d*(sin(c + d*x)^3 - sin(c + d*x)^4 - sin(c + d*x)^5 + sin(c + d*
x)^6)) - (59*a*log(sin(c + d*x) - 1))/(16*d) + (11*a*log(sin(c + d*x) + 1))
/(16*d)
```

3.861 $\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+d}$$

[Out] -31/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-9/4*a^3/d/(a-a*sin(d*x+c))

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2786, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]

[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx, x, a\,\text{si}\right.}{d}$$
$$= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+}{d}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.63

$$-\frac{a^2\left(31\log(1-\sin(c+dx))+\log(1+\sin(c+dx))-\frac{2}{(-1+\sin(c+dx))^2}-\frac{18}{-1+\sin(c+dx)}+16\sin(c+dx)+4\sin^2(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
[Out] -1/8*(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/
d
```

Maple [A]
time = 0.24, size = 206, normalized size = 1.73

method	result
risch	$4ia^2x+\frac{a^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{a^2e^{-2i(dx+c)}}{8d}+\frac{8ia^2c}{d}+\frac{i(-9a^2e^{i(dx+c)}-16ia^2e^{2i(dx+c}}{2(e^{i(dx+c)}-i)^4}$
derivativedivides	$\frac{a^2\left(\frac{\tan^4(dx+c)}{4}-\frac{\tan^2(dx+c)}{2}-\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\sin^7(dx+c)}{8\cos(dx+c)^2}-\frac{3\sin^5(dx+c)}{8}-\frac{5\sin^3(dx+c)}{8}-\frac{1}{}\right.}{}$
default	$\frac{a^2\left(\frac{\tan^4(dx+c)}{4}-\frac{\tan^2(dx+c)}{2}-\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\sin^7(dx+c)}{8\cos(dx+c)^2}-\frac{3\sin^5(dx+c)}{8}-\frac{5\sin^3(dx+c)}{8}-\frac{1}{}\right.}{}$
norman	$\frac{\frac{16a^2\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{16a^2\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{15a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{25a^2\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{11a^2\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{11a^2\tan^7\left(\frac{dx}{2}+\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+2*a^2*(1/4*sin(
d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*si
n(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/4*sin(d*x
```

```
+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d
*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 96, normalized size = 0.81

$$-\frac{4\,a^2\sin\left(dx+c\right)^2+a^2\log\left(\sin\left(dx+c\right)+1\right)+31\,a^2\log\left(\sin\left(dx+c\right)-1\right)+16\,a^2\sin\left(dx+c\right)-\frac{2\,\left(9\,a^2\sin(dx+c)-8\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.40, size = 168, normalized size = 1.41

$$\frac{4\,a^2\cos\left(dx+c\right)^4+22\,a^2\cos\left(dx+c\right)^2-12\,a^2-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-31\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(4\,a^2\cos\left(dx+c\right)^2-5\,a^2\right)\sin\left(dx+c\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.51, size = 102, normalized size = 0.86

$$-\frac{8\,a^2\sin\left(dx+c\right)^2+2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+62\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a^2\sin\left(dx+c\right)-\frac{93\,a^2\sin(dx+c)^2-150\,a^2\sin(dx+c)+61\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(8*a^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c) + 1)) + 62*a^2*log
(abs(sin(d*x + c) - 1)) + 32*a^2*sin(d*x + c) - (93*a^2*sin(d*x + c)^2 - 15
0*a^2*sin(d*x + c) + 61*a^2)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 9.23, size = 283, normalized size = 2.38

$$\frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{\frac{15\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-22\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{61\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-36\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{61\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-22\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{15\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+14\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-12\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{31\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (4*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2) + 1))
/(4*d) - ((61*a^2*tan(c/2 + (d*x)/2)^3)/2 - 22*a^2*tan(c/2 + (d*x)/2)^2 - 3
6*a^2*tan(c/2 + (d*x)/2)^4 + (61*a^2*tan(c/2 + (d*x)/2)^5)/2 - 22*a^2*tan(c
/2 + (d*x)/2)^6 + (15*a^2*tan(c/2 + (d*x)/2)^7)/2 + (15*a^2*tan(c/2 + (d*x)
/2))/2)/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 12*tan(c/2 + (d
*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 +
(d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (31*a^2*
log(tan(c/2 + (d*x)/2) - 1))/(4*d)
```

3.862 $\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{a^2\sin(c+dx)}{d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}$$

[Out] -17/8*a^2*ln(1-sin(d*x+c))/d+1/8*a^2*ln(1+sin(d*x+c))/d-a^2*sin(d*x+c)/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-7/4*a^3/d/(a-a*sin(d*x+c))

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\sin(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]

[Out] (-17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[1 + Sin[c + d*x]])/(8*d) - (a^2*Sin[c + d*x])/d + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (7*a^3)/(4*d*(a - a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \frac{x^4}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(-1+\frac{a^3}{2(a-x)^3}-\frac{7a^2}{4(a-x)^2}+\frac{17a}{8(a-x)}+\frac{a}{8(a+x)}\right)\,dx$$
$$= -\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(1+\sin(c+dx))}{8d}-$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 0.66

$$-\frac{a^2\left(17\log(1-\sin(c+dx))-\log(1+\sin(c+dx))-\frac{2}{(-1+\sin(c+dx))^2}-\frac{14}{-1+\sin(c+dx)}+8\sin(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/8*(a^2*(17*Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c
 + d*x])^2 - 14/(-1 + Sin[c + d*x]) + 8*Sin[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(93) = 186$.
time = 0.21, size = 201, normalized size = 1.99

method	result
risch	$2ia^2x+\frac{ia^2e^{i(dx+c)}}{2d}-\frac{ia^2e^{-i(dx+c)}}{2d}+\frac{4ia^2c}{d}+\frac{ia^2\left(-12ie^{2i(dx+c)}+7e^{3i(dx+c)}-7e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-i\right)^4}-\frac{17a^2\ln(e^{i(dx+c)}}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2a^2\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c\right)}{2}\right.}{}$
default	$\frac{a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2a^2\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c\right)}{2}\right.}{}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{15a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{13a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{13a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\text{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a^2*(1/4*tan(d*
x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+a^2*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-
3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*
x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 83, normalized size = 0.82

$$\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-17\,a^2\log\left(\sin\left(dx+c\right)-1\right)-8\,a^2\sin\left(dx+c\right)+\frac{2\,\left(7\,a^2\sin\left(dx+c\right)-6\,a^2\right)}{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) - 8*a^2*sin(d
*x + c) + 2*(7*a^2*sin(d*x + c) - 6*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/d
```

Fricas [A]
time = 0.40, size = 154, normalized size = 1.52

$$\frac{16\,a^2\cos\left(dx+c\right)^2-4\,a^2+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-17\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(4\,a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(16*a^2*cos(d*x + c)^2 - 4*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(sin(d*x + c) + 1) - 17*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x
+ c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos(d*x + c)^2 - a^2)*sin(d
*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.60, size = 88, normalized size = 0.87

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-34\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-16\,a^2\sin\left(dx+c\right)+\frac{51\,a^2\sin(dx+c)^2-74\,a^2\sin(dx+c)+27\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
- 16*a^2*sin(d*x + c) + (51*a^2*sin(d*x + c)^2 - 74*a^2*sin(d*x + c) + 27*a
^2)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 9.27, size = 225, normalized size = 2.23

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{17\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d}-\frac{\frac{9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-14\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-14\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{9\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}+\frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2) + 1))/(4*d) - (17*a^2*log(tan(c/2 + (d*x)/2) -
1))/(4*d) - (17*a^2*tan(c/2 + (d*x)/2)^3 - 14*a^2*tan(c/2 + (d*x)/2)^2 - 14
*a^2*tan(c/2 + (d*x)/2)^4 + (9*a^2*tan(c/2 + (d*x)/2)^5)/2 + (9*a^2*tan(c/2
 + (d*x)/2))/2)/(d*(7*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 8*tan(c
/2 + (d*x)/2)^3 + 7*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^5 + tan(c/2
 + (d*x)/2)^6 + 1)) + (2*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.863 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}$$

```
[Out] -7/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d
*x+c))^2-5/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) +
a^4/(4*d*(a - a*Sin[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(\frac{a^2}{2(a-x)^3}-\frac{5a}{4(a-x)^2}+\frac{7}{8(a-x)}-\frac{1}{8(a+x)}\right)\,dx,x,\right.}{d}$$
$$= -\frac{7a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}+$$

Mathematica [A]
time = 0.32, size = 91, normalized size = 1.05

$$\frac{a^2(3\tanh^{-1}(\sin(c+dx))-6\sec^3(c+dx)\tan(c+dx)+\sec(c+dx)\tan(c+dx)(3+8\tan^2(c+dx))-2(2\log(\cos(c+dx))+\tan^2(c+dx)-\tan^4(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(3*ArcTanh[Sin[c + d*x]] - 6*Sec[c + d*x]^3*Tan[c + d*x] + Sec[c + d*x
]*Tan[c + d*x]*(3 + 8*Tan[c + d*x]^2) - 2*(2*Log[Cos[c + d*x]] + Tan[c + d*
x]^2 - Tan[c + d*x]^4)))/(4*d)
```

Maple [A]
time = 0.19, size = 137, normalized size = 1.57

method	result
risch	$ia^2x+\frac{2ia^2c}{d}+\frac{i\left(-8ia^2e^{2i(dx+c)}-5a^2e^{i(dx+c)}+5e^{3i(dx+c)}a^2\right)}{2\left(e^{i(dx+c)}-i\right)^4d}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{4d}-\frac{7a^2\ln\left(e^{i(dx+c)}-i\right)}{4d}$
derivativedivides	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^4(dx+\right.}{4}\right.}{d}$
default	$\frac{\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a^2\left(\frac{\left(\tan^4(dx+\right.}{4}\right.}{d}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{5a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{15a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{15a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^2*sin(d*x+c)^4/cos(d*x+c)^4+2*a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+a^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.83

$$-\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)+7\,a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(5\,a^2\sin\left(dx+c\right)-4\,a^2\right)}{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 2*(5*a^2*si
n(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.39, size = 125, normalized size = 1.44

$$-\frac{10\,a^2\sin\left(dx+c\right)-8\,a^2+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)+7\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(10*a^2*sin(d*x + c) - 8*a^2 + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(sin(d*x + c) + 1) + 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x +
c) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.60, size = 78, normalized size = 0.90

$$-\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+14\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{21\,a^2\sin\left(dx+c\right)^2-22\,a^2\sin\left(dx+c\right)+5\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - (21*a^2*sin(d*x + c)^2 - 22*a^2*sin(d*x + c) + 5*a^2)/(sin(d*x + c) - 1)
^2)/d
```

Mupad [B]
time = 9.30, size = 166, normalized size = 1.91

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{7\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d}-\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}-\frac{\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-4\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{3\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (7*a^2*log(tan(c/2 + (d*x)/2) - 1))
/(4*d) - (a^2*log(tan(c/2 + (d*x)/2) + 1))/(4*d) - ((3*a^2*tan(c/2 + (d*x)/
2)^3)/2 - 4*a^2*tan(c/2 + (d*x)/2)^2 + (3*a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*
tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(
c/2 + (d*x)/2)^4 + 1))
```

3.864 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}$$

```
[Out] 1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-3/4*a^3/d/(a-a*s
in(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.138$, Rules used = {2915, 12, 90, 212}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{3a^3}{4d(a-a\sin(c+dx))}+\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(4*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) - (3*a
^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{x^2}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3} - \frac{3}{4(a-x)^2} + \frac{1}{4(a^2-x^2)}\right)\,dx, x, a\sin(c$$
$$= \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{3a^3}{4d(a-a\sin(c+dx))} + \frac{a^3\text{Su}}{}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{a^4}{4d(a-a\sin(c+dx))^2} - \frac{}{4d}$$

Mathematica [A]
time = 0.08, size = 39, normalized size = 0.61

$$\frac{a^2\left(\tanh^{-1}(\sin(c+dx)) + \frac{-2+3\sin(c+dx)}{(-1+\sin(c+dx))^2}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (a^2*(ArcTanh[Sin[c + d*x]] + (-2 + 3*Sin[c + d*x])/(-1 + Sin[c + d*x])^2))
/(4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(58) = 116$.
time = 0.18, size = 167, normalized size = 2.61

method	result
risch	$\frac{ia^2\left(-4i e^{2i(dx+c)}+3\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-i\right)^4} + \frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{4d} - \frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\right.}{d}$

default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}\right.}{d}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{7a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{13a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{13a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a^2*sin(d*x+c)^4/cos(d*x+c)^4+
a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*
x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.12

$$\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(3\,a^2\sin(dx+c)-2\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) + 2*(3*a^2*sin(d
*x + c) - 2*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(60) = 120.
time = 0.37, size = 125, normalized size = 1.95

$$-\frac{6\,a^2\sin\left(dx+c\right)-4\,a^2-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(6*a^2*sin(d*x + c) - 4*a^2 - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
- 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.56, size = 77, normalized size = 1.20

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{3\,a^2\sin(dx+c)^2+6\,a^2\sin(dx+c)-5\,a^2}{(\sin(dx+c)-1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 2*a^2*log(abs(sin(d*x + c) - 1)) +
 (3*a^2*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 5*a^2)/(sin(d*x + c) - 1)^2)/
d
```

Mupad [B]
time = 11.55, size = 123, normalized size = 1.92

$$\frac{a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d) - ((a^2*tan(c/2 + (d*x)/2)^3)/2 - 2*a
^2*tan(c/2 + (d*x)/2)^2 + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/2 + (d*x)
/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^
4 + 1))
```

3.865 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}$$

```
[Out] -1/4*a^2*arctanh(sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-1/4*a^3/d/(a-a*
sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.148$, Rules used = {2915, 12, 78, 212}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/4*(a^2*ArcTanh[Sin[c + d*x]])/d + a^4/(4*d*(a - a*Sin[c + d*x])^2) - a^3
/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x}{a(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{x}{(a-x)^3(a+x)}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \left(\frac{1}{2(a-x)^3}-\frac{1}{4a(a-x)^2}-\frac{1}{4a(a^2-x^2)}\right)\,dx,x,a\sin(\right.}{d}$$
$$= \frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{a^3}{4d(a-a\sin(c+dx))}-\frac{a^3\text{Su}}{}$$
$$= -\frac{a^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{}{4a}$$

Mathematica [A]
time = 0.06, size = 36, normalized size = 0.56

$$-\frac{a^2\left(\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)}{(-1+\sin(c+dx))^2}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
[Out] -1/4*(a^2*(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(-1 + Sin[c + d*x])^2))/d
```

Maple [A]
time = 0.17, size = 106, normalized size = 1.66

method	result
risch	$\frac{i\left(e^{3i(dx+c)}a^2-a^2e^{i(dx+c)}\right)}{2\left(e^{i(dx+c)}-i\right)^4 d}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{4d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)}{4d}$
derivativedivides	$\frac{\frac{a^2}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$

default	$\frac{\frac{a^2}{4\cos(dx+c)^4}+2a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{9a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{11a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\mathrm{ta}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^2/cos(d*x+c)^4+2*a^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+
c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*a^2*sin
(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 64, normalized size = 1.00

$$-\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)-a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,a^2\sin(dx+c)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) - a^2*log(sin(d*x + c) - 1) - 2*a^2*sin(d*x
 + c)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 120, normalized size = 1.88

$$-\frac{2\,a^2\sin\left(dx+c\right)+\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*sin(d*x + c) + (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2
)*log(sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)
*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A]
time = 0.47, size = 95, normalized size = 1.48

$$-\frac{a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)-a^2\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)+\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)\right)-6\,a^2}{\frac{1}{\sin(dx+c)}+\sin(dx+c)-2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(a^2*log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - a^2*log(abs(1/sin(
d*x + c) + sin(d*x + c) - 2)) + (a^2*(1/sin(d*x + c) + sin(d*x + c)) - 6*a^
2)/(1/sin(d*x + c) + sin(d*x + c) - 2))/d
```

Mupad [B]
time = 11.13, size = 106, normalized size = 1.66

$$\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] ((a^2*tan(c/2 + (d*x)/2)^3)/2 + (a^2*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/2 +
 (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*
x)/2)^4 + 1)) - (a^2*atanh(tan(c/2 + (d*x)/2)))/(2*d)
```

3.866 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=101

$$-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(1+\sin(c+dx))}{8d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{}{4d(a-c}$$

[Out] -7/8*a^2*ln(1-sin(d*x+c))/d+a^2*ln(sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d +1/4*a^4/d/(a-a*sin(d*x+c))^2+3/4*a^3/d/(a-a*sin(d*x+c))

Rubi [A]
time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 84}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{3a^3}{4d(a-a\sin(c+dx))}-\frac{7a^2\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]

[Out] (-7*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (a^2*Log[Sin[c + d*x]])/d - (a^2*Log [1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (3*a^3)/(4*d *(a - a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\text{Subst}\left(\int \frac{a}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{1}{(a-x)^3x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{1}{2a^2(a-x)^3} + \frac{3}{4a^3(a-x)^2} + \frac{7}{8a^4(a-x)} + \frac{1}{a^4x} - \frac{}{8a^4(}\right.\right.}{d}$$
$$= -\frac{7a^2\log(1-\sin(c+dx))}{8d} + \frac{a^2\log(\sin(c+dx))}{d} - \frac{a^2\text{lo}}{}$$

Mathematica [A]
time = 0.22, size = 66, normalized size = 0.65

$$-\frac{a^2\left(7\log(1-\sin(c+dx)) - 8\log(\sin(c+dx)) + \log(1+\sin(c+dx)) - \frac{2}{(-1+\sin(c+dx))^2} + \frac{6}{-1+\sin(c+dx)}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(7*Log[1 - Sin[c + d*x]] - 8*Log[Sin[c + d*x]] + Log[1 + Sin[c +
d*x]] - 2/(-1 + Sin[c + d*x])^2 + 6/(-1 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.27, size = 100, normalized size = 0.99

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+2a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(d}{8}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+2a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(d}{8}\right.}{d}$
risch	$-\frac{i\left(-3a^2e^{i(dx+c)}-8ia^2e^{2i(dx+c)}+3\,e^{3i(dx+c)}a^2\right)}{2\left(e^{i(dx+c)}-i\right)^4d}-\frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{4d}-\frac{7a^2\ln\left(e^{i(dx+c)}-i\right)}{4d}+\frac{a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{8a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{13a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+2*a^2*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*a
^2/cos(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.83

$$-\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)+7\,a^2\log\left(\sin\left(dx+c\right)-1\right)-8\,a^2\log\left(\sin\left(dx+c\right)\right)+\frac{2\,\left(3\,a^2\sin\left(dx+c\right)-4\,a^2\right)}{\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 7*a^2*log(sin(d*x + c) - 1) - 8*a^2*log(s
in(d*x + c)) + 2*(3*a^2*sin(d*x + c) - 4*a^2)/(sin(d*x + c)^2 - 2*sin(d*x +
 c) + 1))/d
```

Fricas [A]
time = 0.39, size = 166, normalized size = 1.64

$$\frac{6\,a^2\sin\left(dx+c\right)-8\,a^2+8\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-7\,\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*sin(d*x + c) - 8*a^2 + 8*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(1/2*sin(d*x + c)) - (a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c)
 - 2*a^2)*log(sin(d*x + c) + 1) - 7*(a^2*cos(d*x + c)^2 + 2*a^2*sin(d*x + c
) - 2*a^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 91, normalized size = 0.90

$$-\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+14\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-16\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{21\,a^2\sin\left(dx+c\right)^2-54\,a^2\sin\left(dx+c\right)+37\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) + 14*a^2*log(abs(sin(d*x + c) - 1))
 - 16*a^2*log(abs(sin(d*x + c))) - (21*a^2*sin(d*x + c)^2 - 54*a^2*sin(d*x
+ c) + 37*a^2)/(sin(d*x + c) - 1)^2)/d
```

Mupad [B]
time = 0.08, size = 91, normalized size = 0.90

$$\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{\frac{3\,a^2\,\sin\left(c+d\,x\right)}{4}-a^2}{d\,\left({\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1\right)}-\frac{7\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^2*log(sin(c + d*x)))/d - (a^2*log(sin(c + d*x) + 1))/(8*d) - ((3*a^2*sin
(c + d*x))/4 - a^2)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x) + 1)) - (7*a^2*log(
sin(c + d*x) - 1))/(8*d)
```

3.867 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\log(1+\sin(c+dx))}{8d}+\frac{a^4}{4d(a-a\sin(c$$

[Out] -a^2*csc(d*x+c)/d-17/8*a^2*ln(1-sin(d*x+c))/d+2*a^2*ln(sin(d*x+c))/d+1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+5/4*a^3/d/(a-a*sin(d*x+c)
)

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2915, 12, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{5a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc(c+dx)}{d}-\frac{17a^2\log(1-\sin(c+dx))}{8d}+\frac{2a^2\log(\sin(c+dx))}{d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]

[Out] -((a^2*Csc[c + d*x])/d) - (17*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (2*a^2*Log[Sin[c + d*x]])/d + (a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (5*a^3)/(4*d*(a - a*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^3x^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{2a^3(a-x)^3} + \frac{5}{4a^4(a-x)^2} + \frac{17}{8a^5(a-x)} + \frac{1}{a^4x^2} + \frac{2}{a^5}\right.\right.}{d}$$

$$= -\frac{a^2\csc(c+dx)}{d} - \frac{17a^2\log(1-\sin(c+dx))}{8d} + \frac{2a^2\log}{}$$

Mathematica [A]

time = 0.20, size = 74, normalized size = 0.64

$$\frac{a^2\left(-8\csc(c+dx) - 17\log(1-\sin(c+dx)) + 16\log(\sin(c+dx)) + \log(1+\sin(c+dx)) + \frac{2}{(-1+\sin(c+dx))^2} - \frac{10}{-1+\sin(c+dx)}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(-8*Csc[c + d*x] - 17*Log[1 - Sin[c + d*x]] + 16*Log[Sin[c + d*x]] + L
og[1 + Sin[c + d*x]] + 2/(-1 + Sin[c + d*x])^2 - 10/(-1 + Sin[c + d*x])))/(
8*d)
```

Maple [A]

time = 0.29, size = 154, normalized size = 1.33

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4} + \frac{5}{8\sin(dx+c)\cos(dx+c)^2} - \frac{15}{8\sin(dx+c)} + \frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + 2a^2\left(\frac{1}{4\cos(dx+c)^4} + \frac{1}{2\cos(dx+c}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4} + \frac{5}{8\sin(dx+c)\cos(dx+c)^2} - \frac{15}{8\sin(dx+c)} + \frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + 2a^2\left(\frac{1}{4\cos(dx+c)^4} + \frac{1}{2\cos(dx+c}\right.}{d}$
risch	$-\frac{ia^2\left(-28i e^{4i(dx+c)} + 9e^{5i(dx+c)} + 28i e^{2i(dx+c)} - 34e^{3i(dx+c)} + 9e^{i(dx+c)}\right)}{2\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)^4 d} + \frac{a^2\ln\left(e^{i(dx+c)}+i\right)}{4d} - \frac{17a^2\ln\left(e^{i(dx+c)}-i\right.}{4d}$
norman	$\frac{\frac{8a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{8a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{a^2}{2d} + \frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{9a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{7a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d} + \frac{7a^2\left(\tan^8\left(\frac{dx}{2}\right.\right.}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(
d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a^2*(1/4/cos(d*x+c)^4+1/2/cos(d*x+
c)^2+ln(tan(d*x+c)))+a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/
8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.90

$$\frac{a^2 \log (\sin (dx+c)+1) - 17 a^2 \log (\sin (dx+c)-1) + 16 a^2 \log (\sin (dx+c)) - \frac{2 \left(9 a^2 \sin(dx+c)^2 - 14 a^2 \sin(dx+c) + 4 a^2\right)}{\sin(dx+c)^3 - 2 \sin(dx+c)^2 + \sin(dx+c)}}{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*(a^2*log(sin(d*x + c) + 1) - 17*a^2*log(sin(d*x + c) - 1) + 16*a^2*log(
sin(d*x + c)) - 2*(9*a^2*sin(d*x + c)^2 - 14*a^2*sin(d*x + c) + 4*a^2)/(sin
(d*x + c)^3 - 2*sin(d*x + c)^2 + sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(110) = 220.
time = 0.40, size = 240, normalized size = 2.07

$$\frac{18 a^2 \cos (dx+c)^2 + 28 a^2 \sin (dx+c) - 26 a^2 + 16 \left(2 a^2 \cos (dx+c)^2 - 2 a^2 - \left(a^2 \cos (dx+c)^2 - 2 a^2\right) \sin (dx+c)\right) \log \left(\frac{1}{2} \sin (dx+c)\right) + \left(2 a^2 \cos (dx+c)^2 - 2 a^2 - \left(a^2 \cos (dx+c)^2 - 2 a^2\right) \sin (dx+c)\right) \log (\sin (dx+c)+1) - 17 \left(2 a^2 \cos (dx+c)^2 - 2 a^2 - \left(a^2 \cos (dx+c)^2 - 2 a^2\right) \sin (dx+c)\right) \log (-\sin (dx+c)+1)}{8 \left(2 d \cos (dx+c)^2 - \left(d \cos (dx+c)^2 - 2 d\right) \sin (dx+c) - 2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 + 28*a^2*sin(d*x + c) - 26*a^2 + 16*(2*a^2*cos(d
*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + (2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2*cos(d*x + c)^2 - 2*a^2)*si
n(d*x + c))*log(sin(d*x + c) + 1) - 17*(2*a^2*cos(d*x + c)^2 - 2*a^2 - (a^2
*cos(d*x + c)^2 - 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x
 + c)^2 - (d*cos(d*x + c)^2 - 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.69, size = 115, normalized size = 0.99

$$\frac{2\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-34\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+32\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{16\,\left(2\,a^2\sin\left(dx+c\right)+a^2\right)}{\sin\left(dx+c\right)}+\frac{51\,a^2\sin\left(dx+c\right)^2-122\,a^2\sin\left(dx+c\right)+75\,a^2}{\left(\sin\left(dx+c\right)-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2*log(abs(sin(d*x + c) + 1)) - 34*a^2*log(abs(sin(d*x + c) - 1))
+ 32*a^2*log(abs(sin(d*x + c))) - 16*(2*a^2*sin(d*x + c) + a^2)/sin(d*x + c
) + (51*a^2*sin(d*x + c)^2 - 122*a^2*sin(d*x + c) + 75*a^2)/(sin(d*x + c) -
 1)^2)/d
```

Mupad [B]
time = 9.05, size = 110, normalized size = 0.95

$$\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{17\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}+\frac{2\,a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{9\,a^2\,\sin\left(c+d\,x\right)^2}{4}-\frac{7\,a^2\,\sin\left(c+d\,x\right)}{2}+a^2}{d\,\left(\sin\left(c+d\,x\right)^3-2\,\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (a^2*log(sin(c + d*x) + 1))/(8*d) - (17*a^2*log(sin(c + d*x) - 1))/(8*d) +
(2*a^2*log(sin(c + d*x)))/d - (a^2 - (7*a^2*sin(c + d*x))/2 + (9*a^2*sin(c
+ d*x)^2)/4)/(d*(sin(c + d*x) - 2*sin(c + d*x)^2 + sin(c + d*x)^3))
```

3.868 $\int \csc^3(c+dx) \sec^5(c+dx)(a+a\sin(c+dx))^2 \, dx$

Optimal. Leaf size=134

$$-\frac{2a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(1+\sin(c+dx$$

```
[Out] -2*a^2*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-31/8*a^2*ln(1-sin(d*x+c))/d+4*a^
2*ln(sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/(a-a*sin(d*x+c))^2+
7/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A]

time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{7a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2a^2\csc(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}+\frac{4a^2\log(\sin(c+dx))}{d}-\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (31*a^2*Log[1 - Sin[
c + d*x]])/(8*d) + (4*a^2*Log[Sin[c + d*x]])/d - (a^2*Log[1 + Sin[c + d*x]]
)/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (7*a^3)/(4*d*(a - a*Sin[c + d*
x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \frac{1}{(a-x)^3x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \left(\frac{1}{2a^4(a-x)^3} + \frac{7}{4a^5(a-x)^2} + \frac{31}{8a^6(a-x)} + \frac{1}{a^4x^3} + \frac{}{a^5}\right.\right.}{d}$$
$$= -\frac{2a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{31a^2\log(1-\sin}{8d}$$

Mathematica [A]
time = 0.81, size = 84, normalized size = 0.63

$$-\frac{a^2\left(16\csc(c+dx)+4\csc^2(c+dx)+31\log(1-\sin(c+dx))-32\log(\sin(c+dx))+\log(1+\sin(c+dx))-\frac{2}{(-1+\sin(c+dx))^2}+\frac{14}{-1+\sin(c+dx)}\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(16*Csc[c + d*x] + 4*Csc[c + d*x]^2 + 31*Log[1 - Sin[c + d*x]] -
32*Log[Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d*x])^2 + 14
/(-1 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.28, size = 166, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+2a^2\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(dx}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+2a^2\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(dx}\right.}{d}$
risch	$-\frac{i\left(-44ia^2e^{6i(dx+c)}+15a^2e^{7i(dx+c)}+72ia^2e^{4i(dx+c)}-61a^2e^{5i(dx+c)}-44ia^2e^{2i(dx+c)}+61e^{3i(dx+c)}a^2-15a^2e^{i(dx+c)}\right)}{2\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^4d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/s
in(d*x+c)^2+3*ln(tan(d*x+c)))+2*a^2*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*
```

```
x+c)/cos(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*(1/4/
cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 119, normalized size = 0.89

$$-\frac{a^2\log\left(\sin\left(dx+c\right)+1\right)+31\,a^2\log\left(\sin\left(dx+c\right)-1\right)-32\,a^2\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,a^2\sin\left(dx+c\right)^3-22\,a^2\sin\left(dx+c\right)^2+4\,a^2\sin\left(dx+c\right)+2\,a^2\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^3+\sin\left(dx+c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x + c) - 1) - 32*a^2*log
(sin(d*x + c)) + 2*(15*a^2*sin(d*x + c)^3 - 22*a^2*sin(d*x + c)^2 + 4*a^2*s
in(d*x + c) + 2*a^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(126) = 252$.
time = 0.41, size = 302, normalized size = 2.25

$$-\frac{44\,a^2\cos\left(dx+c\right)^2-40\,a^2-32\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2+2\left(a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2+2\left(a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+31\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2+2\left(a^2\cos\left(dx+c\right)^2-a^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(15\,a^2\cos\left(dx+c\right)^2-19\,a^2\right)\sin\left(dx+c\right)}{8\left(d\cos\left(dx+c\right)^4-3\,d\cos\left(dx+c\right)^2+2\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/8*(44*a^2*cos(d*x + c)^2 - 40*a^2 - 32*(a^2*cos(d*x + c)^4 - 3*a^2*cos(d
*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + (a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos
(d*x + c)^2 - a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) + 31*(a^2*cos(d*x +
c)^4 - 3*a^2*cos(d*x + c)^2 + 2*a^2 + 2*(a^2*cos(d*x + c)^2 - a^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(15*a^2*cos(d*x + c)^2 - 19*a^2)*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*(d*cos(d*x + c)^2 - d)*sin(
d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 125, normalized size = 0.93

$$-\frac{4\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+124\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-128\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{3\,a^2\sin(dx+c)^4+114\,a^2\sin(dx+c)^3-173\,a^2\sin(dx+c)^2+32\,a^2\sin(dx+c)+16\,a^2}{\left(\sin(dx+c)^2-\sin(dx+c)\right)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/32*(4*a^2*log(abs(sin(d*x + c) + 1)) + 124*a^2*log(abs(sin(d*x + c) - 1)
) - 128*a^2*log(abs(sin(d*x + c))) + (3*a^2*sin(d*x + c)^4 + 114*a^2*sin(d*
x + c)^3 - 173*a^2*sin(d*x + c)^2 + 32*a^2*sin(d*x + c) + 16*a^2)/(sin(d*x
+ c)^2 - sin(d*x + c))^2)/d
```

Mupad [B]
time = 0.10, size = 126, normalized size = 0.94

$$\frac{4\,a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{\frac{15\,a^2\,\sin(c+d\,x)^3}{4}-\frac{11\,a^2\,\sin(c+d\,x)^2}{2}+a^2\,\sin\left(c+d\,x\right)+\frac{a^2}{2}}{d\,\left(\sin\left(c+d\,x\right)^4-2\,\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2\right)}-\frac{31\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (4*a^2*log(sin(c + d*x)))/d - (a^2*log(sin(c + d*x) + 1))/(8*d) - (a^2*sin(
c + d*x) + a^2/2 - (11*a^2*sin(c + d*x)^2)/2 + (15*a^2*sin(c + d*x)^3)/4)/(
d*(sin(c + d*x)^2 - 2*sin(c + d*x)^3 + sin(c + d*x)^4)) - (31*a^2*log(sin(c
 + d*x) - 1))/(8*d)
```

3.869 $\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=150

$$-\frac{4a^2\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{49a^2\log(1-\sin(c+dx))}{8d}+\frac{6a^2\log(\sin(c+dx))}{d}+\frac{a^2\,\mathrm{l}}{}$$

```
[Out] -4*a^2*csc(d*x+c)/d-a^2*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-49/8*a^2*ln(1
-sin(d*x+c))/d+6*a^2*ln(sin(d*x+c))/d+1/8*a^2*ln(1+sin(d*x+c))/d+1/4*a^4/d/
(a-a*sin(d*x+c))^2+9/4*a^3/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^4}{4d(a-a\sin(c+dx))^2}+\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{a^2\csc^2(c+dx)}{d}-\frac{4a^2\csc(c+dx)}{d}-\frac{49a^2\log(1-\sin(c+dx))}{8d}+\frac{6a^2\log(\sin(c+dx))}{d}+\frac{a^2\log(\sin(c+dx)+1)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*
d) - (49*a^2*Log[1 - Sin[c + d*x]])/(8*d) + (6*a^2*Log[Sin[c + d*x]])/d + (
a^2*Log[1 + Sin[c + d*x]])/(8*d) + a^4/(4*d*(a - a*Sin[c + d*x])^2) + (9*a^
3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^4}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\text{Subst}\left(\int \frac{1}{(a-x)^3x^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\text{Subst}\left(\int \left(\frac{1}{2a^5(a-x)^3} + \frac{9}{4a^6(a-x)^2} + \frac{49}{8a^7(a-x)} + \frac{1}{a^4x^4} + \frac{2}{a^5}\right.\right.}{d}$$
$$= -\frac{4a^2\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{d} - \frac{a^2\csc^3(c+dx)}{3d}$$

Mathematica [A]

time = 6.06, size = 133, normalized size = 0.89

$$\frac{a^9\left(-\frac{4\csc(c+dx)}{a^7} - \frac{\csc^2(c+dx)}{a^7} - \frac{\csc^3(c+dx)}{3a^7} - \frac{49\log(1-\sin(c+dx))}{8a^7} + \frac{6\log(\sin(c+dx))}{a^7} + \frac{\log(1+\sin(c+dx))}{8a^7} + \frac{1}{4a^5(a-a\sin(c+dx))^2} + \frac{9}{4a^6(a-a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^9*((-4*Csc[c + d*x])/a^7 - Csc[c + d*x]^2/a^7 - Csc[c + d*x]^3/(3*a^7) -
 (49*Log[1 - Sin[c + d*x]])/(8*a^7) + (6*Log[Sin[c + d*x]])/a^7 + Log[1 + S
in[c + d*x]]/(8*a^7) + 1/(4*a^5*(a - a*Sin[c + d*x])^2) + 9/(4*a^6*(a - a*S
in[c + d*x]))))/d
```

Maple [A]

time = 0.29, size = 219, normalized size = 1.46

method	result
risch	$-\frac{ia^2\left(-228ie^{8i(dx+c)}+75\,e^{9i(dx+c)}+652ie^{6i(dx+c)}-412\,e^{7i(dx+c)}-652ie^{4i(dx+c)}+738\,e^{5i(dx+c)}+228ie^{2i(dx+c)}-412\,e^{3i}\right.}{6\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}-i\right)^4d}$
derivativedivides	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2}{}$
default	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/sin(d*x+c)^3/cos(d*x+c)^4-7/12/sin(d*x+c)^3/cos(d*x+c)^2+35/2
4/sin(d*x+c)/cos(d*x+c)^2-35/8/sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c)))+2
```

```
*a^2*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/sin(d
*x+c)^2+3*ln(tan(d*x+c)))+a^2*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/c
os(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 133, normalized size = 0.89

$$\frac{3\,a^2\log\left(\sin\left(dx+c\right)+1\right)-147\,a^2\log\left(\sin\left(dx+c\right)-1\right)+144\,a^2\log\left(\sin\left(dx+c\right)\right)-\frac{2\,\left(75\,a^2\sin\left(dx+c\right)^4-114\,a^2\sin\left(dx+c\right)^3+28\,a^2\sin\left(dx+c\right)^2+4\,a^2\sin\left(dx+c\right)+4\,a^2\right)}{\sin\left(dx+c\right)^5-2\,\sin\left(dx+c\right)^4+\sin\left(dx+c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(3*a^2*log(sin(d*x + c) + 1) - 147*a^2*log(sin(d*x + c) - 1) + 144*a^2
*log(sin(d*x + c)) - 2*(75*a^2*sin(d*x + c)^4 - 114*a^2*sin(d*x + c)^3 + 28
*a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + 4*a^2)/(sin(d*x + c)^5 - 2*sin(d
*x + c)^4 + sin(d*x + c)^3))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(142) = 284.
time = 0.39, size = 370, normalized size = 2.47

$$\frac{150\,a^2\cos\left(dx+c\right)^4-356\,a^2\cos\left(dx+c\right)^2+214\,a^2+144\,\left(2\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+2\,a^2-\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+3\,\left(2\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+2\,a^2-\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-147\,\left(2\,a^2\cos\left(dx+c\right)^4-4\,a^2\cos\left(dx+c\right)^2+2\,a^2-\left(a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+2\,a^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+4\,\left(57\,a^2\cos\left(dx+c\right)^2-55\,a^2\right)\sin\left(dx+c\right)}{24\,\left(2\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^4-3\,d\cos\left(dx+c\right)^2+2\,d\right)\sin\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(150*a^2*cos(d*x + c)^4 - 356*a^2*cos(d*x + c)^2 + 214*a^2 + 144*(2*a^
2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a
^2*cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) + 3*(2*a^2*c
os(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*
cos(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 147*(2*a^2*co
s(d*x + c)^4 - 4*a^2*cos(d*x + c)^2 + 2*a^2 - (a^2*cos(d*x + c)^4 - 3*a^2*c
os(d*x + c)^2 + 2*a^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 4*(57*a^2*cos
(d*x + c)^2 - 55*a^2)*sin(d*x + c))/(2*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^
2 - (d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2*d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+a*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A]
time = 0.52, size = 142, normalized size = 0.95

$$\frac{6\,a^2\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-294\,a^2\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+288\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{3\left(147\,a^2\sin(dx+c)^2-330\,a^2\sin(dx+c)+187\,a^2\right)}{(\sin(dx+c)-1)^2}-\frac{16\left(33\,a^2\sin(dx+c)^3+12\,a^2\sin(dx+c)^2+3\,a^2\sin(dx+c)+a^2\right)}{\sin(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+a*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/48*(6*a^2*log(abs(sin(d*x + c) + 1)) - 294*a^2*log(abs(sin(d*x + c) - 1)) + 288*a^2*log(abs(sin(d*x + c))) + 3*(147*a^2*sin(d*x + c)^2 - 330*a^2*sin (d*x + c) + 187*a^2)/(sin(d*x + c) - 1)^2 - 16*(33*a^2*sin(d*x + c)^3 + 12* a^2*sin(d*x + c)^2 + 3*a^2*sin(d*x + c) + a^2)/sin(d*x + c)^3)/d

Mupad [B]
time = 9.05, size = 140, normalized size = 0.93

$$\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{8\,d}-\frac{49\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{8\,d}-\frac{\frac{25\,a^2\sin(c+d\,x)^4}{4}-\frac{19\,a^2\sin(c+d\,x)^3}{2}+\frac{7\,a^2\sin(c+d\,x)^2}{3}+\frac{a^2\,\sin(c+d\,x)}{3}+\frac{a^2}{3}}{d\,\left(\sin\left(c+d\,x\right)^5-2\sin\left(c+d\,x\right)^4+\sin\left(c+d\,x\right)^3\right)}+\frac{6\,a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^4),x)

[Out] (a^2*log(sin(c + d*x) + 1))/(8*d) - (49*a^2*log(sin(c + d*x) - 1))/(8*d) - ((a^2*sin(c + d*x))/3 + a^2/3 + (7*a^2*sin(c + d*x)^2)/3 - (19*a^2*sin(c + d*x)^3)/2 + (25*a^2*sin(c + d*x)^4)/4)/(d*(sin(c + d*x)^3 - 2*sin(c + d*x)^4 + sin(c + d*x)^5)) + (6*a^2*log(sin(c + d*x)))/d

3.870 $\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=114

$$-\frac{10a^3\log(1-\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-$$

```
[Out] -10*a^3*ln(1-sin(d*x+c))/d-6*a^3*sin(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^
3*sin(d*x+c)^3/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-5*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{2786, 45\}$

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{5a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{6a^3\sin(c+dx)}{d}-\frac{10a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]

[Out] (-10*a^3*Log[1 - Sin[c + d*x]])/d - (6*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
 d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d) + a^5/(2*d*(a - a*Sin[c + d*x])
^2) - (5*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-6a^2+\frac{a^5}{(a-x)^3}-\frac{5a^4}{(a-x)^2}+\frac{10a^3}{a-x}-3ax-x^2\right)dx, x, a\sin(c$$
$$= -\frac{10a^3\log(1-\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-$$

Mathematica [A]
time = 0.24, size = 73, normalized size = 0.64

$$-\frac{a^3\left(60\log(1-\sin(c+dx))+\frac{27-30\sin(c+dx)}{(-1+\sin(c+dx))^2}+36\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/6*(a^3*(60*Log[1 - Sin[c + d*x]] + (27 - 30*Sin[c + d*x])/(-1 + Sin[c +
d*x])^2 + 36*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(108) = 216.
time = 0.23, size = 302, normalized size = 2.65

method	result
risch	$10ia^3x-\frac{ia^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{25ia^3e^{i(dx+c)}}{8d}-\frac{25ia^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{ia^3e^{-3i(dx+c)}}{24d}$
derivativedivides	$a^3\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-1$
default	$a^3\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^3\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-1$
norman	$\frac{20a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{20a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{64a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{64a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{20a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{40a^3\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+3*a^3*(1/4*sin(
d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*si
```

```
n(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/4*sin(d
*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin
(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+a^3*(1/4*sin(d*x+c)^9/cos(d*x+
c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*sin(d*x+c)^7-7/8*sin(d*x+c)^5-35/24*
sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [**A**]
time = 0.28, size = 96, normalized size = 0.84

$$-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+60\,a^3\log\left(\sin\left(dx+c\right)-1\right)+36\,a^3\sin\left(dx+c\right)-\frac{3\,\left(10\,a^3\sin(dx+c)-9\,a^3\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 60*a^3*log(sin(d*x + c)
 - 1) + 36*a^3*sin(d*x + c) - 3*(10*a^3*sin(d*x + c) - 9*a^3)/(sin(d*x + c)
^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [**A**]
time = 0.38, size = 141, normalized size = 1.24

$$\frac{10\,a^3\cos\left(dx+c\right)^4+115\,a^3\cos\left(dx+c\right)^2-80\,a^3-120\,\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,a^3\cos\left(dx+c\right)^4-24\,a^3\cos\left(dx+c\right)^2+37\,a^3\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(10*a^3*cos(d*x + c)^4 + 115*a^3*cos(d*x + c)^2 - 80*a^3 - 120*(a^3*co
s(d*x + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) + 2*(2*a^
3*cos(d*x + c)^4 - 24*a^3*cos(d*x + c)^2 + 37*a^3)*sin(d*x + c))/(d*cos(d*x
 + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(110) = 220.
time = 0.56, size = 242, normalized size = 2.12

$$\frac{30\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-60\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{55\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6+36\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+183\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+80\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+183\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+36\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+55\,a^3}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1\right)^3}+\frac{125\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4-524\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+804\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-524\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+125\,a^3}{(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(30*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 60*a^3*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - (55*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^3*tan(1/2*d*x + 1/2*c)
^5 + 183*a^3*tan(1/2*d*x + 1/2*c)^4 + 80*a^3*tan(1/2*d*x + 1/2*c)^3 + 183*a
^3*tan(1/2*d*x + 1/2*c)^2 + 36*a^3*tan(1/2*d*x + 1/2*c) + 55*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3 + (125*a^3*tan(1/2*d*x + 1/2*c)^4 - 524*a^3*tan(1/2*d
*x + 1/2*c)^3 + 804*a^3*tan(1/2*d*x + 1/2*c)^2 - 524*a^3*tan(1/2*d*x + 1/2*
c) + 125*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 11.56, size = 321, normalized size = 2.82

$$\frac{10\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-60\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{500\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+184\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{500\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{320\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-60\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+22\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+9\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{20\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (10*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((320*a^3*tan(c/2 + (d*x)/2)^3)/
3 - 60*a^3*tan(c/2 + (d*x)/2)^2 - (500*a^3*tan(c/2 + (d*x)/2)^4)/3 + 184*a^
3*tan(c/2 + (d*x)/2)^5 - (500*a^3*tan(c/2 + (d*x)/2)^6)/3 + (320*a^3*tan(c/
2 + (d*x)/2)^7)/3 - 60*a^3*tan(c/2 + (d*x)/2)^8 + 20*a^3*tan(c/2 + (d*x)/2)
^9 + 20*a^3*tan(c/2 + (d*x)/2))/(d*(9*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d
*x)/2) - 16*tan(c/2 + (d*x)/2)^3 + 22*tan(c/2 + (d*x)/2)^4 - 24*tan(c/2 + (
d*x)/2)^5 + 22*tan(c/2 + (d*x)/2)^6 - 16*tan(c/2 + (d*x)/2)^7 + 9*tan(c/2 +
 (d*x)/2)^8 - 4*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10 + 1)) - (20*a^
3*log(tan(c/2 + (d*x)/2) - 1))/d
```

3.871 $\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=96

$$-\frac{6a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(c+dx)}{2d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}$$

```
[Out] -6*a^3*ln(1-sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*sin(d*x+c)^2/d+1/2*a^5
/d/(a-a*sin(d*x+c))^2-4*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 45}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{4a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin^2(c+dx)}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-6*a^3*Log[1 - Sin[c + d*x]])/d - (3*a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*
x]^2)/(2*d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (4*a^4)/(d*(a - a*Sin[c +
d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^4}{a^4(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \frac{x^4}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(-3a+\frac{a^4}{(a-x)^3}-\frac{4a^3}{(a-x)^2}+\frac{6a^2}{a-x}-x\right)\,dx,x,a\,\text{s}\right.}{d}$$
$$= -\frac{6a^3\log(1-\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\sin^2(}{2}$$

Mathematica [A]
time = 0.29, size = 61, normalized size = 0.64

$$-\frac{a^3\left(12\log(1-\sin(c+dx))+\frac{7-8\sin(c+dx)}{(-1+\sin(c+dx))^2}+6\sin(c+dx)+\sin^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] -1/2*(a^3*(12*Log[1 - Sin[c + d*x]] + (7 - 8*Sin[c + d*x])/(-1 + Sin[c + d*
x])^2 + 6*Sin[c + d*x] + Sin[c + d*x]^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(92) = 184.
time = 0.21, size = 282, normalized size = 2.94

method	result
risch	$6ia^3x+\frac{a^3e^{2i(dx+c)}}{8d}+\frac{3ia^3e^{i(dx+c)}}{2d}-\frac{3ia^3e^{-i(dx+c)}}{2d}+\frac{a^3e^{-2i(dx+c)}}{8d}+\frac{12ia^3c}{d}+\frac{2ia^3\left(-7ie^{2i(dx+c)}+4e^{3i(dx}\right.}{d(e^{i(dx+c)}-i}$
derivativedivides	$\frac{a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c\right)}{2}\right.}{}$
default	$\frac{a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c\right)}{2}\right.}{}$
norman	$\frac{\frac{64a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{64a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{44a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{48a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(\text{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/4*tan(d*
x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+3*a^3*(1/4*sin(d*x+c)^7/cos(d*x+c)^
4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(
d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+a^3*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1
/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+
c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 82, normalized size = 0.85

$$-\frac{a^3\sin\left(dx+c\right)^2+12\,a^3\log\left(\sin\left(dx+c\right)-1\right)+6\,a^3\sin\left(dx+c\right)-\frac{8\,a^3\sin(dx+c)-7\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(a^3*sin(d*x + c)^2 + 12*a^3*log(sin(d*x + c) - 1) + 6*a^3*sin(d*x + c
) - (8*a^3*sin(d*x + c) - 7*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 128, normalized size = 1.33

$$\frac{2\,a^3\cos\left(dx+c\right)^4+19\,a^3\cos\left(dx+c\right)^2-8\,a^3-24\,\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(4\,a^3\cos\left(dx+c\right)^2-3\,a^3\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*a^3*cos(d*x + c)^4 + 19*a^3*cos(d*x + c)^2 - 8*a^3 - 24*(a^3*cos(d*x
 + c)^2 + 2*a^3*sin(d*x + c) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(4*a^3*cos
(d*x + c)^2 - 3*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(94) = 188.
time = 0.68, size = 209, normalized size = 2.18

$$\frac{6\,a^{3}\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-12\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{9\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}+\frac{25\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-106\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+164\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-106\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (9*a^3*tan(1/2*d*x + 1/2*c)^4 + 6*a^3*tan(1/2*d*x + 1/2*c)^3 + 20
*a^3*tan(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) + 9*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^2 + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 106*a^3*tan(1/2*d*
x + 1/2*c)^3 + 164*a^3*tan(1/2*d*x + 1/2*c)^2 - 106*a^3*tan(1/2*d*x + 1/2*c
) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [**B**]
time = 10.93, size = 263, normalized size = 2.74

$$\frac{6\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{12\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-36\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+52\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}-64\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+52\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-36\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+12\,a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}-\frac{12\,a^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (6*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (52*a^3*tan(c/2 + (d*x)/2)^3 - 36
*a^3*tan(c/2 + (d*x)/2)^2 - 64*a^3*tan(c/2 + (d*x)/2)^4 + 52*a^3*tan(c/2 +
(d*x)/2)^5 - 36*a^3*tan(c/2 + (d*x)/2)^6 + 12*a^3*tan(c/2 + (d*x)/2)^7 + 12
*a^3*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)
- 12*tan(c/2 + (d*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)
^5 + 8*tan(c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8
 + 1)) - (12*a^3*log(tan(c/2 + (d*x)/2) - 1))/d
```

3.872 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=78

$$-\frac{3a^3\log(1-\sin(c+dx))}{d}-\frac{a^3\sin(c+dx)}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}$$

```
[Out] -3*a^3*ln(1-sin(d*x+c))/d-a^3*sin(d*x+c)/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-3*a
^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 45}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\sin(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*a^3*Log[1 - Sin[c + d*x]])/d - (a^3*Sin[c + d*x])/d + a^5/(2*d*(a - a*S
in[c + d*x])^2) - (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^3}{a^3(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{x^3}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\text{Subst}\left(\int \left(-1+\frac{a^3}{(a-x)^3}-\frac{3a^2}{(a-x)^2}+\frac{3a}{a-x}\right)\,dx, x, a\sin(c$$
$$= -\frac{3a^3\log(1-\sin(c+dx))}{d}-\frac{a^3\sin(c+dx)}{d}+\frac{}{2d(a-c}$$

Mathematica [A]
time = 0.31, size = 53, normalized size = 0.68

$$-\frac{a^3\left(6\log(1-\sin(c+dx))+\frac{5-6\sin(c+dx)}{(-1+\sin(c+dx))^2}+2\sin(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] -1/2*(a^3*(6*Log[1 - Sin[c + d*x]] + (5 - 6*Sin[c + d*x])/(-1 + Sin[c + d*x
])^2 + 2*Sin[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(76) = 152$.
time = 0.22, size = 223, normalized size = 2.86

method	result
risch	$3ia^3x+\frac{ia^3e^{i(dx+c)}}{2d}-\frac{ia^3e^{-i(dx+c)}}{2d}+\frac{6ia^3c}{d}+\frac{2i\left(-3a^3e^{i(dx+c)}-5ia^3e^{2i(dx+c)}+3e^{3i(dx+c)}a^3\right)}{\left(e^{i(dx+c)}-i\right)^4 d}-\frac{6a^3\ln(e^{i(dx}}{d}$
derivativedivides	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{\left(\tan^4(dx\right.}{4}\right.}{}$
default	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{\left(\tan^4(dx\right.}{4}\right.}{}$
norman	$\frac{\frac{10a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{10a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{38a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{56a^3\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3*a^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+3*a^3*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))
+a^3*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d
*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.90

$$-\frac{6\,a^3\log\left(\sin\left(dx+c\right)-1\right)+2\,a^3\sin\left(dx+c\right)-\frac{6\,a^3\sin(dx+c)-5\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) + 2*a^3*sin(d*x + c) - (6*a^3*sin(d*x + c
) - 5*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.39, size = 110, normalized size = 1.41

$$\frac{4\,a^3\cos\left(dx+c\right)^2+a^3-6\,\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(a^3\cos\left(dx+c\right)^2+a^3\right)\sin\left(dx+c\right)}{2\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(4*a^3*cos(d*x + c)^2 + a^3 - 6*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(-sin(d*x + c) + 1) - 2*(a^3*cos(d*x + c)^2 + a^3)*sin(d*x +
c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(78) = 156.
time = 0.53, size = 178, normalized size = 2.28

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-108\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+170\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-108\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^2 + 2*a^3*tan(1/2*d*x + 1/2*c)
+ 3*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 10
8*a^3*tan(1/2*d*x + 1/2*c)^3 + 170*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a^3*tan
(1/2*d*x + 1/2*c) + 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 10.13, size = 205, normalized size = 2.63

$$\frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{6\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}-\frac{6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-18\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+20\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-18\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (3*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (6*a^3*log(tan(c/2 + (d*x)/2) - 1
))/d - (20*a^3*tan(c/2 + (d*x)/2)^3 - 18*a^3*tan(c/2 + (d*x)/2)^2 - 18*a^3*
tan(c/2 + (d*x)/2)^4 + 6*a^3*tan(c/2 + (d*x)/2)^5 + 6*a^3*tan(c/2 + (d*x)/2
))/(d*(7*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 8*tan(c/2 + (d*x)/2)
^3 + 7*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6
 + 1))
```

3.873 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}$$

```
[Out] -a^3*ln(1-sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2-2*a^4/d/(a-a*sin(d*x+c
))
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2915, 12, 45}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}-\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + a^5/(2*d*(a - a*Sin[c + d*x])^2) - (2*a^
4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x^2}{a^2(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \frac{x^2}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{a^2}{(a-x)^3} - \frac{2a}{(a-x)^2} + \frac{1}{a-x}\right)\,dx, x, a\sin(c+dx\right.}{d}$$
$$= -\frac{a^3\log(1-\sin(c+dx))}{d} + \frac{a^5}{2d(a-a\sin(c+dx))^2} - $$

Mathematica [A]
time = 0.19, size = 45, normalized size = 0.70

$$-\frac{a^3\left(2\log(1-\sin(c+dx)) + \frac{3-4\sin(c+dx)}{(-1+\sin(c+dx))^2}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
[Out] -1/2*(a^3*(2*Log[1 - Sin[c + d*x]] + (3 - 4*Sin[c + d*x])/(-1 + Sin[c + d*x
])^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(62) = 124.
time = 0.19, size = 202, normalized size = 3.16

method	result
risch	$i a^3 x + \frac{2 i a^3 c}{d} + \frac{2 i a^3 \left(-3 i e^{2 i (dx+c)} + 2 e^{3 i (dx+c)} - 2 e^{i (dx+c)}\right)}{d \left(e^{i (dx+c)} - i\right)^4} - \frac{2 a^3 \ln\left(e^{i (dx+c)} - i\right)}{d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4} + \frac{\sin^3(dx+c)}{8\cos(dx+c)^2} + \frac{\sin(dx+c)}{8} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + \frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4} + 3a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4} - \frac{\sin^5(dx+c)}{8\cos(dx+c)^2}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4} + \frac{\sin^3(dx+c)}{8\cos(dx+c)^2} + \frac{\sin(dx+c)}{8} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + \frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4} + 3a^3\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4} - \frac{\sin^5(dx+c)}{8\cos(dx+c)^2}\right.}{d}$
norman	$\frac{\frac{14a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{14a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{52a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{52a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} - \frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d} + \frac{4a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{(\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*a^3*sin(d*x+c)^4/cos(d*x+c)^4+
3*a^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(
d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a^3*(1/4*tan(d*x+c)^
4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.29, size = 59, normalized size = 0.92

$$-\frac{2\,a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{4\,a^3\sin(dx+c)-3\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - (4*a^3*sin(d*x + c) - 3*a^3)/(sin(d*x +
 c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.38, size = 86, normalized size = 1.34

$$-\frac{4\,a^3\sin\left(dx+c\right)-3\,a^3+2\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(4*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.63, size = 125, normalized size = 1.95

$$\frac{6\,a^3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+186\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-112\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^3*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*a^3*log(abs(tan(1/2*d*x + 1
/2*c) - 1)) + (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 112*a^3*tan(1/2*d*x + 1/2*c)
^3 + 186*a^3*tan(1/2*d*x + 1/2*c)^2 - 112*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3
)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 9.71, size = 313, normalized size = 4.89

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(4\,a^3\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)\right)+a^3\,\left(4\,\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)-8\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)+2\right)\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^3\,\left(4\,a^3\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)\right)+a^3\,\left(4\,\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)-8\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)+2\right)\right)-{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\,\left(6\,a^3\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)\right)+a^3\,\left(6\,\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)-12\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)+6\right)\right)}{d\,{\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}^4}-\frac{a^3\,\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)*(4*a^3*(2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 +
 (d*x)/2)^2 + 1)) + a^3*(4*log(tan(c/2 + (d*x)/2)^2 + 1) - 8*log(tan(c/2 +
(d*x)/2) - 1) + 2)) + tan(c/2 + (d*x)/2)^3*(4*a^3*(2*log(tan(c/2 + (d*x)/2)
 - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) + a^3*(4*log(tan(c/2 + (d*x)/2)^2 +
1) - 8*log(tan(c/2 + (d*x)/2) - 1) + 2)) - tan(c/2 + (d*x)/2)^2*(6*a^3*(2*l
og(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) + a^3*(6*log(ta
n(c/2 + (d*x)/2)^2 + 1) - 12*log(tan(c/2 + (d*x)/2) - 1) + 6)))/(d*(tan(c/2
 + (d*x)/2) - 1)^4) - (a^3*(2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (
d*x)/2)^2 + 1)))/d
```

3.874 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=31

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

```
[Out] 1/2*a^5*sin(d*x+c)^2/d/(a-a*sin(d*x+c))^2
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 37}

$$\frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (a^5*Sin[c + d*x]^2)/(2*d*(a - a*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3\tan(c+dx)\,dx = \frac{a^5\text{Subst}\left(\int \frac{x}{a(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^4\text{Subst}\left(\int \frac{x}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\sin^2(c+dx)}{2d(a-a\sin(c+dx))^2}$$

Mathematica [A]
time = 0.04, size = 30, normalized size = 0.97

$$\frac{a^3\sin^2(c+dx)}{2d(1-\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (a^3*Sin[c + d*x]^2)/(2*d*(1 - Sin[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(29) = 58$.
time = 0.17, size = 181, normalized size = 5.84

method	result
risch	$\frac{2i\left(-a^3 e^{i(dx+c)}+e^{3i(dx+c)}a^3-ia^3 e^{2i(dx+c)}\right)}{\left(e^{i(dx+c)}-i\right)^4 d}$
derivativedivides	$\frac{\frac{a^3}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+a^3 (\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}}{d}$
default	$\frac{\frac{a^3}{4\cos(dx+c)^4}+3a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+a^3 (\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}}{d}$
norman	$\frac{\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{44a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{44a^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8a^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{32a^3 (\tan^5 (\frac{dx}{2}+}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4 (1-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3/cos(d*x+c)^4+3*a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+
c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*a^3*sin
(d*x+c)^4/cos(d*x+c)^4+a^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/
```

```
cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))
)
```

Maxima [A]
time = 0.27, size = 42, normalized size = 1.35

$$\frac{2\,a^3 \sin\left(dx+c\right) - a^3}{2\left(\sin\left(dx+c\right)^2 - 2\,\sin\left(dx+c\right) + 1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] 1/2*(2*a^3*sin(d*x + c) - a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c) + 1)*d)
```

Fricas [A]
time = 0.36, size = 44, normalized size = 1.42

$$-\frac{2\,a^3 \sin\left(dx+c\right) - a^3}{2\left(d\cos\left(dx+c\right)^2 + 2\,d\sin\left(dx+c\right) - 2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] -1/2*(2*a^3*sin(d*x + c) - a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+a*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.49, size = 32, normalized size = 1.03

$$\frac{2\,a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{d\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] 2*a^3*tan(1/2*d*x + 1/2*c)^2/(d*(tan(1/2*d*x + 1/2*c) - 1)^4)
```

Mupad [B]
time = 9.29, size = 30, normalized size = 0.97

$$\frac{a^3 \sin(c+d\,x)^2}{8\,d\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
[Out] (a^3*sin(c + d*x)^2)/(8*d*cos(c/2 + pi/4 + (d*x)/2)^4)
```

3.875 $\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}$$

```
[Out] -a^3*ln(1-sin(d*x+c))/d+a^3*ln(sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2+a
^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2915, 12, 46}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{a^4}{d(a-a\sin(c+dx))}-\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Log[1 - Sin[c + d*x]])/d) + (a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a
- a*Sin[c + d*x])^2) + a^4/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\text{Subst}\left(\int \frac{a}{(a-x)^3x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \frac{1}{(a-x)^3x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^6\text{Subst}\left(\int \left(\frac{1}{a(a-x)^3}+\frac{1}{a^2(a-x)^2}+\frac{1}{a^3(a-x)}+\frac{1}{a^3x}\right)\,dx, x, a$$
$$= -\frac{a^3\log(1-\sin(c+dx))}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{}{2d(a-}$$

Mathematica [A]
time = 0.32, size = 54, normalized size = 0.70

$$\frac{a^3\left(-2\log(1-\sin(c+dx))+2\log(\sin(c+dx))+\frac{3-2\sin(c+dx)}{(-1+\sin(c+dx))^2}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(-2*Log[1 - Sin[c + d*x]] + 2*Log[Sin[c + d*x]] + (3 - 2*Sin[c + d*x])
/(-1 + Sin[c + d*x])^2))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(75) = 150$.
time = 0.27, size = 165, normalized size = 2.14

method	result
risch	$-\frac{2i\left(-a^3e^{i(dx+c)}-3ia^3e^{2i(dx+c)}+e^{3i(dx+c)}a^3\right)}{\left(e^{i(dx+c)}-i\right)^4d}-\frac{2a^3\ln\left(e^{i(dx+c)}-i\right)}{d}+\frac{a^3\ln\left(e^{2i(dx+c)}-1\right)}{d}$
derivativedivides	$\frac{a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(d}{8}}{d}$
default	$\frac{a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(d}{8}}{d}$
norman	$\frac{\frac{10a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{10a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{26a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{26a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{28a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{28a^3\left(\tan^8\right.}{d}}{\left(t\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a^3*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*a
^3/cos(d*x+c)^4+a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x
+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.91

$$-\frac{2\,a^3\log\left(\sin\left(dx+c\right)-1\right)-2\,a^3\log\left(\sin\left(dx+c\right)\right)+\frac{2\,a^3\sin(dx+c)-3\,a^3}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*log(sin(d*x + c) - 1) - 2*a^3*log(sin(d*x + c)) + (2*a^3*sin(d*
x + c) - 3*a^3)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.37, size = 126, normalized size = 1.64

$$\frac{2\,a^3\sin\left(dx+c\right)-3\,a^3+2\,\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-2\,\left(a^3\cos\left(dx+c\right)^2+2\,a^3\sin\left(dx+c\right)-2\,a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*sin(d*x + c) - 3*a^3 + 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x + c
) - 2*a^3)*log(1/2*sin(d*x + c)) - 2*(a^3*cos(d*x + c)^2 + 2*a^3*sin(d*x +
c) - 2*a^3)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) -
2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 123, normalized size = 1.60

$$-\frac{12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{25\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-76\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+114\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-76\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 76*a^3*tan(1/2*d*x + 1/2*c)^
3 + 114*a^3*tan(1/2*d*x + 1/2*c)^2 - 76*a^3*tan(1/2*d*x + 1/2*c) + 25*a^3)/
(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 9.12, size = 61, normalized size = 0.79

$$\frac{2\,a^3\,\mathrm{atanh}(2\,\sin\left(c+d\,x\right)-1)}{d}-\frac{a^3\,\sin\left(c+d\,x\right)-\frac{3\,a^3}{2}}{d\,\left({\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (2*a^3*atanh(2*sin(c + d*x) - 1))/d - (a^3*sin(c + d*x) - (3*a^3)/2)/(d*(si
n(c + d*x)^2 - 2*sin(c + d*x) + 1))
```

3.876 $\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=93

$$-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}$$

```
[Out] -a^3*csc(d*x+c)/d-3*a^3*ln(1-sin(d*x+c))/d+3*a^3*ln(sin(d*x+c))/d+1/2*a^5/d
/(a-a*sin(d*x+c))^2+2*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 46}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{2a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc(c+dx)}{d}-\frac{3a^3\log(1-\sin(c+dx))}{d}+\frac{3a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a^3*Log[1 - Sin[c + d*x]])/d + (3*a^3*Log[Sin[
c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2) + (2*a^4)/(d*(a - a*Sin[c +
 d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^2}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^3x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{a^2(a-x)^3} + \frac{2}{a^3(a-x)^2} + \frac{3}{a^4(a-x)} + \frac{1}{a^3x^2} + \frac{3}{a^4x}\right)\right.}{d}$$
$$= -\frac{a^3\csc(c+dx)}{d} - \frac{3a^3\log(1-\sin(c+dx))}{d} + \frac{3a^3\log(s}{}$$

Mathematica [A]

time = 0.17, size = 63, normalized size = 0.68

$$\frac{a^3\left(-2\csc(c+dx) - 6\log(1-\sin(c+dx)) + 6\log(\sin(c+dx)) + \frac{1}{(-1+\sin(c+dx))^2} - \frac{4}{-1+\sin(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
[Out] (a^3*(-2*Csc[c + d*x] - 6*Log[1 - Sin[c + d*x]] + 6*Log[Sin[c + d*x]] + (-1
 + Sin[c + d*x])^(-2) - 4/(-1 + Sin[c + d*x])))/(2*d)
```

Maple [A]

time = 0.23, size = 168, normalized size = 1.81

method	result
risch	$-\frac{2ia^3\left(-9i e^{4i(dx+c)}+3\,e^{5i(dx+c)}+9i e^{2i(dx+c)}-10\,e^{3i(dx+c)}+3\,e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}-i\right)^4 d} - \frac{6a^3\ln\left(e^{i(dx+c)}-i\right)}{d} + \frac{3a^3\ln\left(e^{2i(dx+c)}-1\right.}{d}$
derivativedivides	$\frac{a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(
d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/4/cos(d*x+c)^4+1/2/cos(d*x+
c)^2+ln(tan(d*x+c)))+3*a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+
3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*a^3/cos(d*x+c)^4)
```

Maxima [A]
time = 0.30, size = 90, normalized size = 0.97

$$-\frac{6\,a^{3}\log\left(\sin\left(dx+c\right)-1\right)-6\,a^{3}\log\left(\sin\left(dx+c\right)\right)+\frac{6\,a^{3}\sin\left(dx+c\right)^{2}-9\,a^{3}\sin\left(dx+c\right)+2\,a^{3}}{\sin\left(dx+c\right)^{3}-2\,\sin\left(dx+c\right)^{2}+\sin\left(dx+c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(6*a^3*log(sin(d*x + c) - 1) - 6*a^3*log(sin(d*x + c)) + (6*a^3*sin(d*
x + c)^2 - 9*a^3*sin(d*x + c) + 2*a^3)/(sin(d*x + c)^3 - 2*sin(d*x + c)^2 +
 sin(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 185, normalized size = 1.99

$$\frac{6\,a^{3}\cos\left(dx+c\right)^{2}+9\,a^{3}\sin\left(dx+c\right)-8\,a^{3}+6\,\left(2\,a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}-\left(a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-6\,\left(2\,a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}-\left(a^{3}\cos\left(dx+c\right)^{2}-2\,a^{3}\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,\left(2\,d\cos\left(dx+c\right)^{2}-\left(d\cos\left(dx+c\right)^{2}-2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/2*(6*a^3*cos(d*x + c)^2 + 9*a^3*sin(d*x + c) - 8*a^3 + 6*(2*a^3*cos(d*x +
 c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) - 6*(2*a^3*cos(d*x + c)^2 - 2*a^3 - (a^3*cos(d*x + c)^2 - 2*a^3)*sin(
d*x + c))*log(-sin(d*x + c) + 1))/(2*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 -
 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 166, normalized size = 1.78

$$-\frac{12\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-6\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{25\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-88\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+130\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-88\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,a^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{4}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(12*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 6*a^3*log(abs(tan(1/2*d*x
 + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) + (6*a^3*tan(1/2*d*x + 1/2*c) + a^3)
/tan(1/2*d*x + 1/2*c) - (25*a^3*tan(1/2*d*x + 1/2*c)^4 - 88*a^3*tan(1/2*d*x
 + 1/2*c)^3 + 130*a^3*tan(1/2*d*x + 1/2*c)^2 - 88*a^3*tan(1/2*d*x + 1/2*c)
+ 25*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 0.08, size = 80, normalized size = 0.86

$$\frac{6\,a^3\,\mathrm{atanh}(2\,\sin\left(c+d\,x\right)-1)}{d}-\frac{3\,a^3\,\sin\left(c+d\,x\right)^2-\frac{9\,a^3\,\sin(c+d\,x)}{2}+a^3}{d\,\left(\sin\left(c+d\,x\right)^3-2\,\sin\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (6*a^3*atanh(2*sin(c + d*x) - 1))/d - (a^3 - (9*a^3*sin(c + d*x))/2 + 3*a^3
*sin(c + d*x)^2)/(d*(sin(c + d*x) - 2*sin(c + d*x)^2 + sin(c + d*x)^3))
```

3.877 $\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=111

$$-\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+\frac{6a^3\log(\sin(c+dx))}{d}+\frac{a^5}{2d(a-a\sin(c+dx))^2}$$

```
[Out] -3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-6*a^3*ln(1-sin(d*x+c))/d+6*a^3*l
n(sin(d*x+c))/d+1/2*a^5/d/(a-a*sin(d*x+c))^2+3*a^4/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 46}

$$\frac{a^5}{2d(a-a\sin(c+dx))^2}+\frac{3a^4}{d(a-a\sin(c+dx))}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}-\frac{6a^3\log(1-\sin(c+dx))}{d}+\frac{6a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (6*a^3*Log[1 - Sin[c
 + d*x]])/d + (6*a^3*Log[Sin[c + d*x]])/d + a^5/(2*d*(a - a*Sin[c + d*x])^2
) + (3*a^4)/(d*(a - a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^5\text{Subst}\left(\int \frac{a^3}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \frac{1}{(a-x)^3x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^8\text{Subst}\left(\int \left(\frac{1}{a^3(a-x)^3} + \frac{3}{a^4(a-x)^2} + \frac{6}{a^5(a-x)} + \frac{1}{a^3x^3} + \frac{3}{a^4x^2}\right.\right.}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{6a^3\log(1-\sin(}{d}$$

Mathematica [A]
time = 0.54, size = 73, normalized size = 0.66

$$-\frac{a^3\left(6\csc(c+dx) + \csc^2(c+dx) + 12\log(1-\sin(c+dx)) - 12\log(\sin(c+dx)) - \frac{1}{(-1+\sin(c+dx))^2} + \frac{6}{-1+\sin(c+dx)}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^3*(6*Csc[c + d*x] + Csc[c + d*x]^2 + 12*Log[1 - Sin[c + d*x]] - 12*
Log[Sin[c + d*x]] - (-1 + Sin[c + d*x])^(-2) + 6/(-1 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.27, size = 215, normalized size = 1.94

method	result
risch	$-\frac{4i\left(-9ia^3e^{6i(dx+c)}+3a^3e^{7i(dx+c)}+16ia^3e^{4i(dx+c)}-13a^3e^{5i(dx+c)}-9ia^3e^{2i(dx+c)}+13e^{3i(dx+c)}a^3-3a^3e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)^2\left(e^{i(dx+c)}-i\right)^4d} - \frac{12}{}$
derivativedivides	$a^3\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(dx}\right.$
default	$a^3\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(dx}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/s
in(d*x+c)^2+3*ln(tan(d*x+c)))+3*a^3*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*
x+c)/cos(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*(1/
```

```
4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+a^3*(-(-1/4*sec(d*x+c)^3-3/
8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 103, normalized size = 0.93

$$-\frac{12\,a^3\log\left(\sin\left(dx+c\right)-1\right)-12\,a^3\log\left(\sin\left(dx+c\right)\right)+\frac{12\,a^3\sin(dx+c)^3-18\,a^3\sin(dx+c)^2+4\,a^3\sin(dx+c)+a^3}{\sin(dx+c)^4-2\,\sin(dx+c)^3+\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/2*(12*a^3*log(sin(d*x + c) - 1) - 12*a^3*log(sin(d*x + c)) + (12*a^3*sin
(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 + 4*a^3*sin(d*x + c) + a^3)/(sin(d*x +
c)^4 - 2*sin(d*x + c)^3 + sin(d*x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(109) = 218$.
time = 0.38, size = 235, normalized size = 2.12

$$-\frac{18\,a^3\cos\left(dx+c\right)^2-17\,a^3-12\,\left(a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^2+2\,a^3+2\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\sin\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+12\,\left(a^3\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^2+2\,a^3+2\,\left(a^3\cos\left(dx+c\right)^2-a^3\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-4\,\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3\right)\sin\left(dx+c\right)}{2\,\left(d\cos\left(dx+c\right)^4-3\,d\cos\left(dx+c\right)^2+2\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)+2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(18*a^3*cos(d*x + c)^2 - 17*a^3 - 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d
*x + c)^2 + 2*a^3 + 2*(a^3*cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(1/2*sin(
d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^2 + 2*a^3 + 2*(a^3*
cos(d*x + c)^2 - a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 4*(3*a^3*cos(d
*x + c)^2 - 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4 - 3*d*cos(d*x + c)^2 + 2
*(d*cos(d*x + c)^2 - d)*sin(d*x + c) + 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 198, normalized size = 1.78

$$-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+96\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-48\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{72\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+12\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+a^3}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2}-\frac{8\,\left(25\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4-92\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+136\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-92\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+25\,a^3\right)}{(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 96*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 48*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^3*tan(1/2*d*x + 1/2*c) + (
72*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*
d*x + 1/2*c)^2 - 8*(25*a^3*tan(1/2*d*x + 1/2*c)^4 - 92*a^3*tan(1/2*d*x + 1/
2*c)^3 + 136*a^3*tan(1/2*d*x + 1/2*c)^2 - 92*a^3*tan(1/2*d*x + 1/2*c) + 25*
a^3)/(tan(1/2*d*x + 1/2*c) - 1)^4)/d
```

Mupad [B]
time = 0.09, size = 97, normalized size = 0.87

$$\frac{12\,a^3\,\mathrm{atanh}(2\,\sin\left(c+d\,x\right)-1)}{d}-\frac{6\,a^3\,\sin\left(c+d\,x\right)^3-9\,a^3\,\sin\left(c+d\,x\right)^2+2\,a^3\,\sin\left(c+d\,x\right)+\frac{a^3}{2}}{d\,\left(\sin\left(c+d\,x\right)^4-2\,\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (12*a^3*atanh(2*sin(c + d*x) - 1))/d - (2*a^3*sin(c + d*x) + a^3/2 - 9*a^3*
sin(c + d*x)^2 + 6*a^3*sin(c + d*x)^3)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^
3 + sin(c + d*x)^4))
```

3.878 $\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{515\log(1-\sin(c+dx))}{256ad}-\frac{1795\log(1+\sin(c+dx))}{256ad}+\frac{5\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin^3(c+dx)}{3ad}+\frac{}{96d(a-}$$

```
[Out] 515/256*ln(1-sin(d*x+c))/a/d-1795/256*ln(1+sin(d*x+c))/a/d+5*sin(d*x+c)/a/d
-1/2*sin(d*x+c)^2/a/d+1/3*sin(d*x+c)^3/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3-17
/128*a/d/(a-a*sin(d*x+c))^2+125/128/d/(a-a*sin(d*x+c))+1/64*a^3/d/(a+a*sin(
d*x+c))^4-3/16*a^2/d/(a+a*sin(d*x+c))^3+71/64*a/d/(a+a*sin(d*x+c))^2-5/d/(a
+a*sin(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{3a^2}{16d(a\sin(c+dx)+a)^3}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}-\frac{17a}{128d(a-a\sin(c+dx))^2}+\frac{71a}{64d(a\sin(c+dx)+a)^2}+\frac{125}{128d(a-a\sin(c+dx))}-\frac{5}{d(a\sin(c+dx)+a)}+\frac{5\sin(c+dx)}{ad}+\frac{515\log(1-\sin(c+dx))}{256ad}-\frac{1795\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (1795*Log[1 + Sin[c + d*x]])/(256*a
*d) + (5*Sin[c + d*x])/(a*d) - Sin[c + d*x]^2/(2*a*d) + Sin[c + d*x]^3/(3*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (17*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 125/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
 - (3*a^2)/(16*d*(a + a*Sin[c + d*x])^3) + (71*a)/(64*d*(a + a*Sin[c + d*x]
)^2) - 5/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^7\text{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^{11}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^4 d} \\
&= \frac{\text{Subst}\left(\int \left(5a^2 + \frac{a^6}{32(a-x)^4} - \frac{17a^5}{64(a-x)^3} + \frac{125a^4}{128(a-x)^2} - \frac{515a^3}{256(a-x)} - ax + x^2 - \frac{a}{16(a}\right.\right.}{a^4 d} \\
&= \frac{515\log(1-\sin(c+dx))}{256ad} - \frac{1795\log(1+\sin(c+dx))}{256ad} + \frac{5\sin(c+dx)}{ad} - \frac{\text{si}}{}
\end{aligned}
$$

Mathematica [A]

time = 6.10, size = 153, normalized size = 0.65

$$
\frac{1545\log(1-\sin(c+dx)) - 5385\log(1+\sin(c+dx)) + \frac{8}{(1-\sin(c+dx))^3} - \frac{102}{(1-\sin(c+dx))^2} + \frac{750}{1-\sin(c+dx)} + 3840\sin(c+dx) - 384\sin^2(c+dx) + 256\sin^3(c+dx) + \frac{12}{(1+\sin(c+dx))^4} - \frac{144}{(1+\sin(c+dx))^3} + \frac{852}{(1+\sin(c+dx))^2} - \frac{3840}{1+\sin(c+dx)}}{768ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^4*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (1545*Log[1 - Sin[c + d*x]] - 5385*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d
*x])^3 - 102/(1 - Sin[c + d*x])^2 + 750/(1 - Sin[c + d*x]) + 3840*Sin[c + d
*x] - 384*Sin[c + d*x]^2 + 256*Sin[c + d*x]^3 + 12/(1 + Sin[c + d*x])^4 - 1
44/(1 + Sin[c + d*x])^3 + 852/(1 + Sin[c + d*x])^2 - 3840/(1 + Sin[c + d*x]
))/(768*a*d)
```

Maple [A]

time = 0.28, size = 143, normalized size = 0.61

method	result
derivativedivides	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \frac{\left(\sin^2(dx+c)\right)}{2} + 5\sin(dx+c) + \frac{1}{64(1+\sin(dx+c))^4} - \frac{3}{16(1+\sin(dx+c))^3} + \frac{71}{64(1+\sin(dx+c))^2} - \frac{5}{1+\sin(dx+c)} - \frac{1795\ln(1}{}}{da}$
default	$\frac{\frac{\left(\sin^3(dx+c)\right)}{3} - \frac{\left(\sin^2(dx+c)\right)}{2} + 5\sin(dx+c) + \frac{1}{64(1+\sin(dx+c))^4} - \frac{3}{16(1+\sin(dx+c))^3} + \frac{71}{64(1+\sin(dx+c))^2} - \frac{5}{1+\sin(dx+c)} - \frac{1795\ln(1}{}}{da}$
risch	$\frac{5ix}{a} + \frac{ie^{3i(dx+c)}}{24ad} + \frac{e^{2i(dx+c)}}{8ad} - \frac{21ie^{i(dx+c)}}{8ad} + \frac{21ie^{-i(dx+c)}}{8ad} + \frac{e^{-2i(dx+c)}}{8ad} - \frac{ie^{-3i(dx+c)}}{24ad} + \frac{10ic}{ad} - \frac{i(-750ie^{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/3*sin(d*x+c)^3-1/2*sin(d*x+c)^2+5*sin(d*x+c)+1/64/(1+sin(d*x+c))^4
-3/16/(1+sin(d*x+c))^3+71/64/(1+sin(d*x+c))^2-5/(1+sin(d*x+c))-1795/256*ln(
1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-17/128/(sin(d*x+c)-1)^2-125/128/(sin(d*
x+c)-1)+515/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 209, normalized size = 0.89

$$-\frac{\frac{2\left(2295\sin(dx+c)^6+375\sin(dx+c)^5-5480\sin(dx+c)^4-680\sin(dx+c)^3+4473\sin(dx+c)^2+313\sin(dx+c)-1232\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{128\left(2\sin(dx+c)^3-3\sin(dx+c)^2+30\sin(dx+c)\right)}{a}+\frac{5385\log(\sin(dx+c)+1)}{a}-\frac{1545\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/768*(2*(2295*sin(d*x + c)^6 + 375*sin(d*x + c)^5 - 5480*sin(d*x + c)^4 -
 680*sin(d*x + c)^3 + 4473*sin(d*x + c)^2 + 313*sin(d*x + c) - 1232)/(a*sin
(d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 +
3*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 128*(2*sin(
d*x + c)^3 - 3*sin(d*x + c)^2 + 30*sin(d*x + c))/a + 5385*log(sin(d*x + c)
+ 1)/a - 1545*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.43, size = 207, normalized size = 0.88

$$\frac{256\cos(dx+c)^{10}-3968\cos(dx+c)^8-686\cos(dx+c)^6+2810\cos(dx+c)^4-796\cos(dx+c)^2-5385\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+1545\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)+2\left(64\cos(dx+c)^8+1952\cos(dx+c)^6+375\cos(dx+c)^4-70\cos(dx+c)^2+8\right)\sin(dx+c)+112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(256*cos(d*x + c)^10 - 3968*cos(d*x + c)^8 - 686*cos(d*x + c)^6 + 281
0*cos(d*x + c)^4 - 796*cos(d*x + c)^2 - 5385*(cos(d*x + c)^6*sin(d*x + c) +
 cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 1545*(cos(d*x + c)^6*sin(d*x + c)
+ cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(64*cos(d*x + c)^8 + 1952*cos(
d*x + c)^6 + 375*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112
)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 179, normalized size = 0.76

$$-\frac{\frac{21540 \log\left(|\sin\left(dx+c\right)+1|\right)}{a} - \frac{6180 \log\left(|\sin\left(dx+c\right)-1|\right)}{a} - \frac{512\left(2 a^{2} \sin\left(dx+c\right)^{3} - 3 a^{2} \sin\left(dx+c\right)^{2} + 30 a^{2} \sin\left(dx+c\right)\right)}{a^{3}} + \frac{2\left(5665 \sin\left(dx+c\right)^{3} - 15495 \sin\left(dx+c\right)^{2} + 14199 \sin\left(dx+c\right) - 4353\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}} - \frac{44875 \sin\left(dx+c\right)^{4} + 164140 \sin\left(dx+c\right)^{3} + 226578 \sin\left(dx+c\right)^{2} + 139660 \sin\left(dx+c\right) + 32395}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1
))/a - 512*(2*a^2*sin(d*x + c)^3 - 3*a^2*sin(d*x + c)^2 + 30*a^2*sin(d*x +
c))/a^3 + 2*(5665*sin(d*x + c)^3 - 15495*sin(d*x + c)^2 + 14199*sin(d*x + c
) - 4353)/(a*(sin(d*x + c) - 1)^3) - (44875*sin(d*x + c)^4 + 164140*sin(d*x
 + c)^3 + 226578*sin(d*x + c)^2 + 139660*sin(d*x + c) + 32395)/(a*(sin(d*x
+ c) + 1)^4))/d
```

Mupad [B]
time = 10.64, size = 567, normalized size = 2.40

$$\frac{515 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{128\, a\, d} - \frac{1795 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{128\, a\, d} - \frac{-\frac{1155 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{64} - \frac{835 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}}{32} + \frac{3205 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{64} + \frac{305 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{4} + \frac{41 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{16} + \frac{53 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{24} - \frac{5521 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{48} - \frac{2387 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{12} + \frac{6697 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96} + \frac{6901 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{48} + \frac{6697 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{96} - \frac{2387 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{12} - \frac{5521 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{48} + \frac{53 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{24} + \frac{41 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{16} + \frac{305 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{4} + \frac{3205 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{64} - \frac{835 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32} - \frac{1155 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19} - 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} - 6 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17} - 3 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 8 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 12 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} - 12 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 12 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7} + 8 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} - 3 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - 6 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3} - 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right) + a\right)} + \frac{5 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{a\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (515*log(tan(c/2 + (d*x)/2) - 1))/(128*a*d) - (1795*log(tan(c/2 + (d*x)/2)
+ 1))/(128*a*d) - ((3205*tan(c/2 + (d*x)/2)^3)/64 - (835*tan(c/2 + (d*x)/2)
^2)/32 - (1155*tan(c/2 + (d*x)/2))/64 + (305*tan(c/2 + (d*x)/2)^4)/4 + (41*
tan(c/2 + (d*x)/2)^5)/16 + (53*tan(c/2 + (d*x)/2)^6)/24 - (5521*tan(c/2 + (
d*x)/2)^7)/48 - (2387*tan(c/2 + (d*x)/2)^8)/12 + (6697*tan(c/2 + (d*x)/2)^9
)/96 + (6901*tan(c/2 + (d*x)/2)^10)/48 + (6697*tan(c/2 + (d*x)/2)^11)/96 -
(2387*tan(c/2 + (d*x)/2)^12)/12 - (5521*tan(c/2 + (d*x)/2)^13)/48 + (53*tan
(c/2 + (d*x)/2)^14)/24 + (41*tan(c/2 + (d*x)/2)^15)/16 + (305*tan(c/2 + (d*
x)/2)^16)/4 + (3205*tan(c/2 + (d*x)/2)^17)/64 - (835*tan(c/2 + (d*x)/2)^18)
/32 - (1155*tan(c/2 + (d*x)/2)^19)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 2*a
*tan(c/2 + (d*x)/2)^2 - 6*a*tan(c/2 + (d*x)/2)^3 - 3*a*tan(c/2 + (d*x)/2)^4
 + 8*a*tan(c/2 + (d*x)/2)^6 + 16*a*tan(c/2 + (d*x)/2)^7 + 2*a*tan(c/2 + (d*
x)/2)^8 - 12*a*tan(c/2 + (d*x)/2)^9 - 12*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan
(c/2 + (d*x)/2)^11 + 2*a*tan(c/2 + (d*x)/2)^12 + 16*a*tan(c/2 + (d*x)/2)^13
 + 8*a*tan(c/2 + (d*x)/2)^14 - 3*a*tan(c/2 + (d*x)/2)^16 - 6*a*tan(c/2 + (d
*x)/2)^17 - 2*a*tan(c/2 + (d*x)/2)^18 + 2*a*tan(c/2 + (d*x)/2)^19 + a*tan(c
/2 + (d*x)/2)^20)) + (5*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

3.879 $\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{325\log(1-\sin(c+dx))}{256ad}+\frac{955\log(1+\sin(c+dx))}{256ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-$$

```
[Out] 325/256*ln(1-sin(d*x+c))/a/d+955/256*ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1/
2*sin(d*x+c)^2/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3-15/128*a/d/(a-a*sin(d*x+c)
)^2+95/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4+1/6*a^2/d/(a+a*
sin(d*x+c))^3-55/64*a/d/(a+a*sin(d*x+c))^2+105/32/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{6d(a\sin(c+dx)+a)^3}+\frac{\sin^2(c+dx)}{2ad}-\frac{15a}{128d(a-a\sin(c+dx))^2}-\frac{55a}{64d(a\sin(c+dx)+a)^2}+\frac{95}{128d(a-a\sin(c+dx))}+\frac{105}{32d(a\sin(c+dx)+a)}-\frac{\sin(c+dx)}{ad}+\frac{325\log(1-\sin(c+dx))}{256ad}+\frac{955\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (325*Log[1 - Sin[c + d*x]])/(256*a*d) + (955*Log[1 + Sin[c + d*x]])/(256*a*
d) - Sin[c + d*x]/(a*d) + Sin[c + d*x]^2/(2*a*d) + a^2/(96*d*(a - a*Sin[c +
 d*x])^3) - (15*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c
+ d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) + a^2/(6*d*(a + a*Sin[c + d*x]
)^3) - (55*a)/(64*d*(a + a*Sin[c + d*x])^2) + 105/(32*d*(a + a*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^7\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^3d}\\
&= \frac{\text{Subst}\left(\int \left(-a+\frac{a^5}{32(a-x)^4}-\frac{15a^4}{64(a-x)^3}+\frac{95a^3}{128(a-x)^2}-\frac{325a^2}{256(a-x)}+x+\frac{a^6}{16(a+x)^5}-\right.\right.}{a^3d}\\
&= \frac{325\log(1-\sin(c+dx))}{256ad}+\frac{955\log(1+\sin(c+dx))}{256ad}-\frac{\sin(c+dx)}{ad}+\frac{\sin^2(}{}
\end{aligned}$$

Mathematica [A]

time = 6.12, size = 143, normalized size = 0.65

$$\frac{975\log(1-\sin(c+dx))+2865\log(1+\sin(c+dx))+\frac{8}{(1-\sin(c+dx))^3}-\frac{90}{(1-\sin(c+dx))^2}+\frac{570}{1-\sin(c+dx)}-768\sin(c+dx)+384\sin^2(c+dx)-\frac{12}{(1+\sin(c+dx))^4}+\frac{128}{(1+\sin(c+dx))^3}-\frac{660}{(1+\sin(c+dx))^2}+\frac{2520}{1+\sin(c+dx)}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (975*Log[1 - Sin[c + d*x]] + 2865*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 90/(1 - Sin[c + d*x])^2 + 570/(1 - Sin[c + d*x]) - 768*Sin[c + d*x]
 + 384*Sin[c + d*x]^2 - 12/(1 + Sin[c + d*x])^4 + 128/(1 + Sin[c + d*x])^3
- 660/(1 + Sin[c + d*x])^2 + 2520/(1 + Sin[c + d*x]))/(768*a*d)
```

Maple [A]

time = 0.28, size = 133, normalized size = 0.60

method	result
derivativedivides	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{55}{64(1+\sin(dx+c))^2}+\frac{105}{32(1+\sin(dx+c))}+\frac{955\ln(1+\sin(dx+c))}{256}-}{da}$
default	$\frac{\frac{\left(\sin^2(dx+c)\right)}{2}-\sin(dx+c)-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{55}{64(1+\sin(dx+c))^2}+\frac{105}{32(1+\sin(dx+c))}+\frac{955\ln(1+\sin(dx+c))}{256}-}{da}$
risch	$-\frac{5ix}{a}-\frac{e^{2i(dx+c)}}{8ad}+\frac{ie^{i(dx+c)}}{2ad}-\frac{ie^{-i(dx+c)}}{2ad}-\frac{e^{-2i(dx+c)}}{8ad}-\frac{10ic}{ad}+\frac{i\left(-1890ie^{12i(dx+c)}+975\,e^{13i(dx+c)}-3030i\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/2*sin(d*x+c)^2-sin(d*x+c)-1/64/(1+sin(d*x+c))^4+1/6/(1+sin(d*x+c))
^3-55/64/(1+sin(d*x+c))^2+105/32/(1+sin(d*x+c))+955/256*ln(1+sin(d*x+c))-1/
96/(sin(d*x+c)-1)^3-15/128/(sin(d*x+c)-1)^2-95/128/(sin(d*x+c)-1)+325/256*l
n(sin(d*x+c)-1))
```

Maxima [A]
time = 0.27, size = 197, normalized size = 0.90

$$\frac{\frac{2\left(975\,\sin\left(dx+c\right)^{6}-945\,\sin\left(dx+c\right)^{5}-3240\,\sin\left(dx+c\right)^{4}+1560\,\sin\left(dx+c\right)^{3}+3489\,\sin\left(dx+c\right)^{2}-671\,\sin\left(dx+c\right)-1232\right)}{a\sin\left(dx+c\right)^{7}+a\sin\left(dx+c\right)^{6}-3\,a\sin\left(dx+c\right)^{5}-3\,a\sin\left(dx+c\right)^{4}+3\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-a}+\frac{384\left(\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)\right)}{a}+\frac{2865\,\log\left(\sin\left(dx+c\right)+1\right)}{a}+\frac{975\,\log\left(\sin\left(dx+c\right)-1\right)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/768*(2*(975*sin(d*x + c)^6 - 945*sin(d*x + c)^5 - 3240*sin(d*x + c)^4 + 1
560*sin(d*x + c)^3 + 3489*sin(d*x + c)^2 - 671*sin(d*x + c) - 1232)/(a*sin(
d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3
*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 384*(sin(d*x
 + c)^2 - 2*sin(d*x + c))/a + 2865*log(sin(d*x + c) + 1)/a + 975*log(sin(d*
x + c) - 1)/a)/d
```

Fricas [A]
time = 0.43, size = 197, normalized size = 0.90

$$\frac{384\,\cos\left(dx+c\right)^{8}+1374\,\cos\left(dx+c\right)^{6}+630\,\cos\left(dx+c\right)^{4}-132\,\cos\left(dx+c\right)^{2}+2865\,\left(\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{6}\right)\log\left(\sin\left(dx+c\right)+1\right)+975\,\left(\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{6}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(192\,\cos\left(dx+c\right)^{8}+288\,\cos\left(dx+c\right)^{6}-945\,\cos\left(dx+c\right)^{4}+330\,\cos\left(dx+c\right)^{2}-56\right)\sin\left(dx+c\right)+16}{768\,\left(ad\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/768*(384*cos(d*x + c)^8 + 1374*cos(d*x + c)^6 + 630*cos(d*x + c)^4 - 132*
cos(d*x + c)^2 + 2865*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(si
n(d*x + c) + 1) + 975*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-s
in(d*x + c) + 1) - 2*(192*cos(d*x + c)^8 + 288*cos(d*x + c)^6 - 945*cos(d*x
 + c)^4 + 330*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

[Out] Timed out

Giac [A]
time = 0.58, size = 161, normalized size = 0.73

$$\frac{\frac{11460 \log(|\sin(dx+c)+1|)}{a} + \frac{3900 \log(|\sin(dx+c)-1|)}{a} + \frac{1536 \left(a \sin(dx+c)^2 - 2 a \sin(dx+c)\right)}{a^2} - \frac{2 \left(3575 \sin(dx+c)^3 - 9585 \sin(dx+c)^2 + 8625 \sin(dx+c) - 2599\right)}{a(\sin(dx+c)-1)^3} - \frac{23875 \sin(dx+c)^4 + 85420 \sin(dx+c)^3 + 115650 \sin(dx+c)^2 + 70028 \sin(dx+c) + 15971}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1)
)/a + 1536*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 2*(3575*sin(d*x + c)
^3 - 9585*sin(d*x + c)^2 + 8625*sin(d*x + c) - 2599)/(a*(sin(d*x + c) - 1)^
3) - (23875*sin(d*x + c)^4 + 85420*sin(d*x + c)^3 + 115650*sin(d*x + c)^2 +
 70028*sin(d*x + c) + 15971)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 10.34, size = 512, normalized size = 2.33

$$\frac{325 \ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)-1\right)}{128 a d} + \frac{955 \ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)+1\right)}{128 a d} + \frac{-\frac{315 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{17}}{64} + \frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{16}}{32} + \frac{265 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{15}}{8} + \frac{195 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{14}}{32} - \frac{1217 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{13}}{16} - \frac{2389 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{12}}{96} + \frac{1189 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{11}}{24} + \frac{767 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{10}}{32} + \frac{6845 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{9}}{96} + \frac{767 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{8}}{32} + \frac{1189 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{7}}{24} - \frac{2389 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{6}}{96} - \frac{1217 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{5}}{16} + \frac{195 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{4}}{32} + \frac{265 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{3}}{8} + \frac{5 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{2}}{32} - \frac{315 \tan\left(\frac{c}{2}+\frac{d x}{2}\right)}{64}}{d \left(a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{18} + 2 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{17} - 3 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{16} - 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{15} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{13} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{12} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{11} - 6 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{10} - 20 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{9} - 6 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{8} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{7} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{6} + 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{5} - 8 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{3} - 3 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right)^{2} + 2 a \tan\left(\frac{c}{2}+\frac{d x}{2}\right) + a\right)} - \frac{5 \ln\left(\tan\left(\frac{c}{2}+\frac{d x}{2}\right)^2+1\right)}{a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^10/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (325*log(tan(c/2 + (d*x)/2) - 1))/(128*a*d) + (955*log(tan(c/2 + (d*x)/2) +
 1))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^2)/32 - (315*tan(c/2 + (d*x)/2))/64
 + (265*tan(c/2 + (d*x)/2)^3)/8 + (195*tan(c/2 + (d*x)/2)^4)/32 - (1217*tan
(c/2 + (d*x)/2)^5)/16 - (2389*tan(c/2 + (d*x)/2)^6)/96 + (1189*tan(c/2 + (d
*x)/2)^7)/24 + (767*tan(c/2 + (d*x)/2)^8)/32 + (6845*tan(c/2 + (d*x)/2)^9)/
96 + (767*tan(c/2 + (d*x)/2)^10)/32 + (1189*tan(c/2 + (d*x)/2)^11)/24 - (23
89*tan(c/2 + (d*x)/2)^12)/96 - (1217*tan(c/2 + (d*x)/2)^13)/16 + (195*tan(c
/2 + (d*x)/2)^14)/32 + (265*tan(c/2 + (d*x)/2)^15)/8 + (5*tan(c/2 + (d*x)/2
)^16)/32 - (315*tan(c/2 + (d*x)/2)^17)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) -
 3*a*tan(c/2 + (d*x)/2)^2 - 8*a*tan(c/2 + (d*x)/2)^3 + 8*a*tan(c/2 + (d*x)/
2)^5 + 8*a*tan(c/2 + (d*x)/2)^6 + 8*a*tan(c/2 + (d*x)/2)^7 - 6*a*tan(c/2 +
(d*x)/2)^8 - 20*a*tan(c/2 + (d*x)/2)^9 - 6*a*tan(c/2 + (d*x)/2)^10 + 8*a*ta
n(c/2 + (d*x)/2)^11 + 8*a*tan(c/2 + (d*x)/2)^12 + 8*a*tan(c/2 + (d*x)/2)^13
 - 8*a*tan(c/2 + (d*x)/2)^15 - 3*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d
*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18)) - (5*log(tan(c/2 + (d*x)/2)^2 + 1))/(
a*d)
```

3.880 $\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{443\log(1+\sin(c+dx))}{256ad}+\frac{\sin(c+dx)}{ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{13a}{128d(a-a\sin(}$$

```
[Out] 187/256*ln(1-sin(d*x+c))/a/d-443/256*ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d+1/
96*a^2/d/(a-a*sin(d*x+c))^3-13/128*a/d/(a-a*sin(d*x+c))^2+69/128/d/(a-a*sin
(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+c))^4-7/48*a^2/d/(a+a*sin(d*x+c))^3+41/64*
a/d/(a+a*sin(d*x+c))^2-2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{7a^2}{48d(a\sin(c+dx)+a)^3}-\frac{13a}{128d(a-a\sin(c+dx))^2}+\frac{41a}{64d(a\sin(c+dx)+a)^2}+\frac{69}{128d(a-a\sin(c+dx))}-\frac{2}{d(a\sin(c+dx)+a)}+\frac{\sin(c+dx)}{ad}+\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{443\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (187*Log[1 - Sin[c + d*x]])/(256*a*d) - (443*Log[1 + Sin[c + d*x]])/(256*a*
d) + Sin[c + d*x]/(a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (13*a)/(128*d
*(a - a*Sin[c + d*x])^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a +
 a*Sin[c + d*x])^4) - (7*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (41*a)/(64*d*
(a + a*Sin[c + d*x])^2) - 2/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{x^9}{a^9(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^9}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^4}{32(a-x)^4}-\frac{13a^3}{64(a-x)^3}+\frac{69a^2}{128(a-x)^2}-\frac{187a}{256(a-x)}-\frac{a^5}{16(a+x)^5}+\frac{7a^4}{16(a+x}\right.\right.}{a^2d}$$
$$= \frac{187\log(1-\sin(c+dx))}{256ad}-\frac{443\log(1+\sin(c+dx))}{256ad}+\frac{\sin(c+dx)}{ad}+\frac{}{96d(}$$

Mathematica [A]

time = 6.11, size = 133, normalized size = 0.67

$$\frac{561\log(1-\sin(c+dx))-1329\log(1+\sin(c+dx))+\frac{8}{(1-\sin(c+dx))^3}-\frac{78}{(1-\sin(c+dx))^2}+\frac{414}{1-\sin(c+dx)}+768\sin(c+dx)+\frac{12}{(1+\sin(c+dx))^4}-\frac{112}{(1+\sin(c+dx))^3}+\frac{492}{(1+\sin(c+dx))^2}-\frac{1536}{1+\sin(c+dx)}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
[Out] (561*Log[1 - Sin[c + d*x]] - 1329*Log[1 + Sin[c + d*x]] + 8/(1 - Sin[c + d*
x])^3 - 78/(1 - Sin[c + d*x])^2 + 414/(1 - Sin[c + d*x]) + 768*Sin[c + d*x]
 + 12/(1 + Sin[c + d*x])^4 - 112/(1 + Sin[c + d*x])^3 + 492/(1 + Sin[c + d*
x])^2 - 1536/(1 + Sin[c + d*x]))/(768*a*d)
```

Maple [A]

time = 0.27, size = 121, normalized size = 0.61

method	result
derivativedivides	$\frac{\sin(dx+c)+\frac{1}{64(1+\sin(dx+c))^4}-\frac{7}{48(1+\sin(dx+c))^3}+\frac{41}{64(1+\sin(dx+c))^2}-\frac{2}{1+\sin(dx+c)}-\frac{443\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-}{da}$
default	$\frac{\sin(dx+c)+\frac{1}{64(1+\sin(dx+c))^4}-\frac{7}{48(1+\sin(dx+c))^3}+\frac{41}{64(1+\sin(dx+c))^2}-\frac{2}{1+\sin(dx+c)}-\frac{443\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-}{da}$
risch	$\frac{ix}{a}-\frac{ie^{i(dx+c)}}{2ad}+\frac{ie^{-i(dx+c)}}{2ad}+\frac{2ic}{ad}-\frac{i(975\,e^{i(dx+c)}-810ie^{4i(dx+c)}+810ie^{10i(dx+c)}+2502\,e^{11i(dx+c)}+2502\,e^{3i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(sin(d*x+c)+1/64/(1+sin(d*x+c))^4-7/48/(1+sin(d*x+c))^3+41/64/(1+sin(
d*x+c))^2-2/(1+sin(d*x+c))-443/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-1
3/128/(sin(d*x+c)-1)^2-69/128/(sin(d*x+c)-1)+187/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 186, normalized size = 0.93

$$-\frac{\frac{2\left(975\sin(dx+c)^6+207\sin(dx+c)^5-2088\sin(dx+c)^4-360\sin(dx+c)^3+1569\sin(dx+c)^2+161\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{1329\log(\sin(dx+c)+1)}{a}-\frac{561\log(\sin(dx+c)-1)}{a}-\frac{768\sin(dx+c)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(975*sin(d*x + c)^6 + 207*sin(d*x + c)^5 - 2088*sin(d*x + c)^4 -
360*sin(d*x + c)^3 + 1569*sin(d*x + c)^2 + 161*sin(d*x + c) - 400)/(a*sin(d
*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*
a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 1329*log(sin(
d*x + c) + 1)/a - 561*log(sin(d*x + c) - 1)/a - 768*sin(d*x + c)/a)/d
```

Fricas [A]
time = 0.43, size = 187, normalized size = 0.94

$$-\frac{768\cos(dx+c)^8+1182\cos(dx+c)^6-1674\cos(dx+c)^4+636\cos(dx+c)^2+1329\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)-561\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(384\cos(dx+c)^6+207\cos(dx+c)^4-54\cos(dx+c)^2+8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(768*cos(d*x + c)^8 + 1182*cos(d*x + c)^6 - 1674*cos(d*x + c)^4 + 63
6*cos(d*x + c)^2 + 1329*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
sin(d*x + c) + 1) - 561*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(
-sin(d*x + c) + 1) - 2*(384*cos(d*x + c)^6 + 207*cos(d*x + c)^4 - 54*cos(d*
x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*co
s(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 147, normalized size = 0.74

$$-\frac{\frac{5316\log(|\sin(dx+c)+1|)}{a}-\frac{2244\log(|\sin(dx+c)-1|)}{a}-\frac{3072\sin(dx+c)}{a}+\frac{2\left(2057\sin(dx+c)^3-5343\sin(dx+c)^2+4671\sin(dx+c)-1369\right)}{a(\sin(dx+c)-1)^3}-\frac{11075\sin(dx+c)^4+38156\sin(dx+c)^3+49986\sin(dx+c)^2+29356\sin(dx+c)+6499}{a(\sin(dx+c)+1)^4}}{3072d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1)
)/a - 3072*sin(d*x + c)/a + 2*(2057*sin(d*x + c)^3 - 5343*sin(d*x + c)^2 +
4671*sin(d*x + c) - 1369)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4
+ 38156*sin(d*x + c)^3 + 49986*sin(d*x + c)^2 + 29356*sin(d*x + c) + 6499)/
(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 10.14, size = 485, normalized size = 2.44

$$\frac{\frac{315\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{251\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{32}-\frac{1411\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}-\frac{607\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{16}+\frac{2183\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{64}+\frac{6287\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}-\frac{2749\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}-\frac{803\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{24}-\frac{2749\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{192}+\frac{6287\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{96}+\frac{2183\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{64}-\frac{607\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{16}-\frac{1411\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{64}+\frac{251\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{315\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+18a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+18a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-10a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\frac{187\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{128\,a\,d}-\frac{443\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{128\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((315*tan(c/2 + (d*x)/2))/64 + (251*tan(c/2 + (d*x)/2)^2)/32 - (1411*tan(c/
2 + (d*x)/2)^3)/64 - (607*tan(c/2 + (d*x)/2)^4)/16 + (2183*tan(c/2 + (d*x)/
2)^5)/64 + (6287*tan(c/2 + (d*x)/2)^6)/96 - (2749*tan(c/2 + (d*x)/2)^7)/192
 - (803*tan(c/2 + (d*x)/2)^8)/24 - (2749*tan(c/2 + (d*x)/2)^9)/192 + (6287*
tan(c/2 + (d*x)/2)^10)/96 + (2183*tan(c/2 + (d*x)/2)^11)/64 - (607*tan(c/2
+ (d*x)/2)^12)/16 - (1411*tan(c/2 + (d*x)/2)^13)/64 + (251*tan(c/2 + (d*x)/
2)^14)/32 + (315*tan(c/2 + (d*x)/2)^15)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2)
- 4*a*tan(c/2 + (d*x)/2)^2 - 10*a*tan(c/2 + (d*x)/2)^3 + 4*a*tan(c/2 + (d*x
)/2)^4 + 18*a*tan(c/2 + (d*x)/2)^5 + 4*a*tan(c/2 + (d*x)/2)^6 - 10*a*tan(c/
2 + (d*x)/2)^7 - 10*a*tan(c/2 + (d*x)/2)^8 - 10*a*tan(c/2 + (d*x)/2)^9 + 4*
a*tan(c/2 + (d*x)/2)^10 + 18*a*tan(c/2 + (d*x)/2)^11 + 4*a*tan(c/2 + (d*x)/
2)^12 - 10*a*tan(c/2 + (d*x)/2)^13 - 4*a*tan(c/2 + (d*x)/2)^14 + 2*a*tan(c/
2 + (d*x)/2)^15 + a*tan(c/2 + (d*x)/2)^16)) + (187*log(tan(c/2 + (d*x)/2) -
 1))/(128*a*d) - (443*log(tan(c/2 + (d*x)/2) + 1))/(128*a*d) + log(tan(c/2
+ (d*x)/2)^2 + 1)/(a*d)
```

3.881 $\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=188

$$\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{163\log(1+\sin(c+dx))}{256ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{}{128}$$

```
[Out] 93/256*ln(1-sin(d*x+c))/a/d+163/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*si
n(d*x+c))^3-11/128*a/d/(a-a*sin(d*x+c))^2+47/128/d/(a-a*sin(d*x+c))-1/64*a^
3/d/(a+a*sin(d*x+c))^4+1/8*a^2/d/(a+a*sin(d*x+c))^3-29/64*a/d/(a+a*sin(d*x+
c))^2+35/32/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{8d(a\sin(c+dx)+a)^3}-\frac{11a}{128d(a-a\sin(c+dx))^2}-\frac{29a}{64d(a\sin(c+dx)+a)^2}+\frac{47}{128d(a-a\sin(c+dx))}+\frac{35}{32d(a\sin(c+dx)+a)}+\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{163\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (93*Log[1 - Sin[c + d*x]])/(256*a*d) + (163*Log[1 + Sin[c + d*x]])/(256*a*d
) + a^2/(96*d*(a - a*Sin[c + d*x])^3) - (11*a)/(128*d*(a - a*Sin[c + d*x])^
2) + 47/(128*d*(a - a*Sin[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) +
a^2/(8*d*(a + a*Sin[c + d*x])^3) - (29*a)/(64*d*(a + a*Sin[c + d*x])^2) + 3
5/(32*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{x^8}{a^8(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{32(a-x)^4} - \frac{11a^2}{64(a-x)^3} + \frac{47a}{128(a-x)^2} - \frac{93}{256(a-x)} + \frac{a^4}{16(a+x)^5} - \frac{3a^3}{8(a+x)^4} + \right.\right.}{ad}$$
$$= \frac{93\log(1-\sin(c+dx))}{256ad} + \frac{163\log(1+\sin(c+dx))}{256ad} + \frac{a^2}{96d(a-a\sin(c+dx)}$$

Mathematica [A]
time = 2.60, size = 117, normalized size = 0.62

$$\frac{279\log(1-\sin(c+dx)) + 489\log(1+\sin(c+dx)) + \frac{2(-400-295\sin(c+dx)+1113\sin^2(c+dx)+728\sin^3(c+dx)-1000\sin^4(c+dx)-489\sin^5(c+dx)+279\sin^6(c+dx))}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{768ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
[Out] (279*Log[1 - Sin[c + d*x]] + 489*Log[1 + Sin[c + d*x]] + (2*(-400 - 295*Sin
[c + d*x] + 1113*Sin[c + d*x]^2 + 728*Sin[c + d*x]^3 - 1000*Sin[c + d*x]^4
- 489*Sin[c + d*x]^5 + 279*Sin[c + d*x]^6))/((-1 + Sin[c + d*x])^3*(1 + Sin
[c + d*x])^4))/(768*a*d)
```

Maple [A]
time = 0.42, size = 115, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{8(1+\sin(dx+c))^3}-\frac{29}{64(1+\sin(dx+c))^2}+\frac{35}{32(1+\sin(dx+c))}+\frac{163\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
default	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{8(1+\sin(dx+c))^3}-\frac{29}{64(1+\sin(dx+c))^2}+\frac{35}{32(1+\sin(dx+c))}+\frac{163\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
risch	$-\frac{ix}{a}-\frac{2ic}{ad}+\frac{i(-978i e^{12i(dx+c)}+279\,e^{13i(dx+c)}-934i e^{10i(dx+c)}+2326\,e^{11i(dx+c)}-1748i e^{8i(dx+c)}+5993\,e^{9i(dx+c)}+1}{192(e^{i(dx+c)}+}$
norman	$\frac{-\frac{35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{35\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{29\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{29\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32da}-\frac{139\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\frac{139\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/64/(1+sin(d*x+c))^4+1/8/(1+sin(d*x+c))^3-29/64/(1+sin(d*x+c))^2+3
5/32/(1+sin(d*x+c))+163/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-11/128/(
sin(d*x+c)-1)^2-47/128/(sin(d*x+c)-1)+93/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 175, normalized size = 0.93

$$\frac{\frac{2\left(279\,\sin(dx+c)^6-489\,\sin(dx+c)^5-1000\,\sin(dx+c)^4+728\,\sin(dx+c)^3+1113\,\sin(dx+c)^2-295\,\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3\,a\sin(dx+c)^5-3\,a\sin(dx+c)^4+3\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{489\,\log(\sin(dx+c)+1)}{a}+\frac{279\,\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(279*sin(d*x + c)^6 - 489*sin(d*x + c)^5 - 1000*sin(d*x + c)^4 + 7
28*sin(d*x + c)^3 + 1113*sin(d*x + c)^2 - 295*sin(d*x + c) - 400)/(a*sin(d*
x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a
*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 489*log(sin(d*
x + c) + 1)/a + 279*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.41, size = 167, normalized size = 0.89

$$\frac{558\,\cos(dx+c)^6+326\,\cos(dx+c)^4-100\,\cos(dx+c)^2+489\,\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+279\,\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)+2\,\left(489\,\cos(dx+c)^4-250\,\cos(dx+c)^2+56\right)\sin(dx+c)+16}{768\,\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(558*cos(d*x + c)^6 + 326*cos(d*x + c)^4 - 100*cos(d*x + c)^2 + 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(4
89*cos(d*x + c)^4 - 250*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*
x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

[Out] Timed out

Giac [A]
time = 0.64, size = 136, normalized size = 0.72

$$\frac{\frac{1956\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}+\frac{1116\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a}-\frac{2\,\left(1023\,\sin\left(dx+c\right)^{3}-2505\,\sin\left(dx+c\right)^{2}+2073\,\sin\left(dx+c\right)-575\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{4075\,\sin\left(dx+c\right)^{4}+12940\,\sin\left(dx+c\right)^{3}+15762\,\sin\left(dx+c\right)^{2}+8620\,\sin\left(dx+c\right)+1771}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1))
/a - 2*(1023*sin(d*x + c)^3 - 2505*sin(d*x + c)^2 + 2073*sin(d*x + c) - 575
)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*x + c)^4 + 12940*sin(d*x + c)^3 +
15762*sin(d*x + c)^2 + 8620*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/
d
```

Mupad [B]
time = 9.36, size = 432, normalized size = 2.30

$$\frac{-\frac{35\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{64}+\frac{29\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{32}+\frac{629\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{96}-\frac{365\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{96}-\frac{5399\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{192}+\frac{203\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}}{48}+\frac{3019\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}}{48}+\frac{203\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}}{48}-\frac{5399\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{192}-\frac{365\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}}{96}+\frac{629\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{96}+\frac{29\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}}{32}-\frac{35\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left(a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+2\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}-5\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-12\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+9\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+30\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}-5\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-40\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}-5\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+30\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}+9\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-12\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}-5\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+2\,a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{93\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{128\,a\,d}+\frac{163\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{128\,a\,d}-\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((29*tan(c/2 + (d*x)/2)^2)/32 - (35*tan(c/2 + (d*x)/2))/64 + (629*tan(c/2 +
 (d*x)/2)^3)/96 - (365*tan(c/2 + (d*x)/2)^4)/96 - (5399*tan(c/2 + (d*x)/2)^
5)/192 + (203*tan(c/2 + (d*x)/2)^6)/48 + (3019*tan(c/2 + (d*x)/2)^7)/48 + (
203*tan(c/2 + (d*x)/2)^8)/48 - (5399*tan(c/2 + (d*x)/2)^9)/192 - (365*tan(c
/2 + (d*x)/2)^10)/96 + (629*tan(c/2 + (d*x)/2)^11)/96 + (29*tan(c/2 + (d*x)
/2)^12)/32 - (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2)
- 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x
)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/
2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a
*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2
)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) + (93*log(tan(
c/2 + (d*x)/2) - 1))/(128*a*d) + (163*log(tan(c/2 + (d*x)/2) + 1))/(128*a*d
) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.882 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+$$

```
[Out] -35/128*arctanh(sin(d*x+c))/a/d+35/128*sec(d*x+c)*tan(d*x+c)/a/d-35/192*sec
(d*x+c)*tan(d*x+c)^3/a/d+7/48*sec(d*x+c)*tan(d*x+c)^5/a/d-1/8*sec(d*x+c)*ta
n(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A]

time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec(c+dx)}{192ad}+\frac{35\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}$$
$$= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^7\,dx, x, \text{t}\right.}{ad}$$
$$= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+dx)}{8ad} + \frac{\tan^8(c+dx)}{8ad} - \frac{35\int \sec(}{}$$
$$= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad} - \frac{\sec(c+dx)\tan^7(c+}{8ad}$$
$$= \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+dx)}{192ad} + \frac{7\sec(c+dx)\tan^5(c+}{48ad}$$
$$= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{35\sec(c+dx)\tan(c+dx)}{128ad} - \frac{35\sec(c+dx)\tan^3(c+d}{192ad}$$

Mathematica [A]
time = 0.66, size = 101, normalized size = 0.78

$$-\frac{105\tanh^{-1}(\sin(c+dx)) + \frac{-48+57\sin(c+dx)+249\sin^2(c+dx)-136\sin^3(c+dx)-424\sin^4(c+dx)+87\sin^5(c+dx)+279\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*
x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Si
n[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

Maple [A]
time = 0.36, size = 115, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{19}{64(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
default	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{19}{64(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
risch	$-\frac{i(-174ie^{2i(dx+c)}+279\,e^{i(dx+c)}+218ie^{10i(dx+c)}+300ie^{8i(dx+c)}-300ie^{6i(dx+c)}-218ie^{4i(dx+c)}+174ie^{12i(dx+c)}+279}{192(e^{i(dx+c)}+i)^8(e^{i(dx+c)}-i)^6}$
norman	$\frac{\frac{35(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{32ad}+\frac{35(\tan^{12}(\frac{dx}{2}+\frac{c}{2}))}{32ad}+\frac{231(\tan^6(\frac{dx}{2}+\frac{c}{2}))}{16ad}+\frac{231(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{16ad}-\frac{25(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{16ad}+\frac{35\tan(\frac{dx}{2}+\frac{c}{2})}{64ad}+\frac{35(}{}}{(\tan(\frac{dx}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/64/(1+sin(d*x+c))^4-5/48/(1+sin(d*x+c))^3+19/64/(1+sin(d*x+c))^2-1
/2/(1+sin(d*x+c))-35/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-9/128/(sin(
d*x+c)-1)^2-29/128/(sin(d*x+c)-1)+35/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(279\sin(dx+c)^6+87\sin(dx+c)^5-424\sin(dx+c)^4-136\sin(dx+c)^3+249\sin(dx+c)^2+57\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3\,a\sin(dx+c)^5-3\,a\sin(dx+c)^4+3\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{105\log(\sin(dx+c)+1)}{a}-\frac{105\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.28

$$-\frac{558\cos(dx+c)^6-826\cos(dx+c)^4+476\cos(dx+c)^2+105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)-105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(87\cos(dx+c)^4-38\cos(dx+c)^2+8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
```

```
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.57, size = 136, normalized size = 1.05

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a}-\frac{420 \log(|\sin(dx+c)-1|)}{a}+\frac{2\left(385 \sin(dx+c)^3-807 \sin(dx+c)^2+567 \sin(dx+c)-129\right)}{a(\sin(dx+c)-1)^3}-\frac{875 \sin(dx+c)^4+1964 \sin(dx+c)^3+1554 \sin(dx+c)^2+396 \sin(dx+c)-21}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 17.05, size = 388, normalized size = 2.98

$$\frac{\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}-\frac{595\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}+\frac{791\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{16}-\frac{25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{16}+\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{16}+\frac{791\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}-\frac{595\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{96}-\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
[Out] ((35*tan(c/2 + (d*x)/2))/64 + (35*tan(c/2 + (d*x)/2)^2)/32 - (245*tan(c/2 +
 (d*x)/2)^3)/96 - (595*tan(c/2 + (d*x)/2)^4)/96 + (791*tan(c/2 + (d*x)/2)^5
)/192 + (231*tan(c/2 + (d*x)/2)^6)/16 - (25*tan(c/2 + (d*x)/2)^7)/16 + (231
*tan(c/2 + (d*x)/2)^8)/16 + (791*tan(c/2 + (d*x)/2)^9)/192 - (595*tan(c/2 +
 (d*x)/2)^10)/96 - (245*tan(c/2 + (d*x)/2)^11)/96 + (35*tan(c/2 + (d*x)/2)^
12)/32 + (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*
a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)
^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 +
(d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan
(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12
 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (35*atanh(tan(c/
2 + (d*x)/2)))/(64*a*d)
```

3.883 $\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=134

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{5\sec(c+dx)\tan(c+dx)}{128ad}+\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad}-\frac{5\sec^3(c+dx)\tan^3(c+dx}{48ad}$$

```
[Out] -5/128*arctanh(sin(d*x+c))/a/d-5/128*sec(d*x+c)*tan(d*x+c)/a/d+5/64*sec(d*x
+c)^3*tan(d*x+c)/a/d-5/48*sec(d*x+c)^3*tan(d*x+c)^3/a/d+1/8*sec(d*x+c)^3*ta
n(d*x+c)^5/a/d-1/8*tan(d*x+c)^8/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2914, 2691, 3853, 3855, 2687, 30}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}-\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c+dx)}{64ad}-\frac{5\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a
*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (5*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(48*a*d) + (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) - Tan[c + d*x]^8
/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a} \\
&= \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a} - \frac{\text{Subst}\left(\int x^{\cdots}\right.}{\cdots} \\
&= -\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad} - \frac{\tan^8(c+dx)}{8ad} \cdots \\
&= \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad} + \frac{\sec^3(c+dx)\tan^{\cdots}}{8ad} \cdots \\
&= -\frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{64ad} - \frac{5\sec^3(c+dx)\cdots}{48a\cdots} \\
&= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad} - \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(\cdots}{64ad}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 101, normalized size = 0.75

$$-\frac{15\tanh^{-1}(\sin(c+dx)) + \frac{48+63\sin(c+dx)-129\sin^2(c+dx)-184\sin^3(c+dx)+104\sin^4(c+dx)+177\sin^5(c+dx)-15\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(15*ArcTanh[Sin[c + d*x]] + (48 + 63*Sin[c + d*x] - 129*Sin[c + d*x]
^2 - 184*Sin[c + d*x]^3 + 104*Sin[c + d*x]^4 + 177*Sin[c + d*x]^5 - 15*Sin[
c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

Maple [A]
time = 0.34, size = 115, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{12(1+\sin(dx+c))^3}-\frac{11}{64(1+\sin(dx+c))^2}+\frac{5}{32(1+\sin(dx+c))}-\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin}}{da}$
default	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{12(1+\sin(dx+c))^3}-\frac{11}{64(1+\sin(dx+c))^2}+\frac{5}{32(1+\sin(dx+c))}-\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin}}{da}$
risch	$\frac{i\left(-354i e^{12i(dx+c)}+15 e^{13i(dx+c)}+298i e^{10i(dx+c)}+326 e^{11i(dx+c)}-1140i e^{8i(dx+c)}+625 e^{9i(dx+c)}+1140i e^{6i(dx+c)}+114\right.}{192\left(e^{i(dx+c)}-i\right)^6\left(e^{i(dx+c)}+i\right)^8 dc}$
norman	$\frac{\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{5\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{33\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{33\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{289\left(\tan\right.}{1}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/64/(1+sin(d*x+c))^4+1/12/(1+sin(d*x+c))^3-11/64/(1+sin(d*x+c))^2+
5/32/(1+sin(d*x+c))-5/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-7/128/(sin
(d*x+c)-1)^2-15/128/(sin(d*x+c)-1)+5/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.31

$$\frac{\frac{2\left(15\sin(dx+c)^6-177\sin(dx+c)^5-104\sin(dx+c)^4+184\sin(dx+c)^3+129\sin(dx+c)^2-63\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(15*sin(d*x + c)^6 - 177*sin(d*x + c)^5 - 104*sin(d*x + c)^4 + 184
*sin(d*x + c)^3 + 129*sin(d*x + c)^2 - 63*sin(d*x + c) - 48)/(a*sin(d*x + c
)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(
d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c)
 + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.25

$$\frac{30\cos(dx+c)^6+118\cos(dx+c)^4-68\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)+2\left(177\cos(dx+c)^4-170\cos(dx+c)^2+56\right)\sin(dx+c)+16}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(30*cos(d*x + c)^6 + 118*cos(d*x + c)^4 - 68*cos(d*x + c)^2 - 15*(cos
(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(177*c
os(d*x + c)^4 - 170*cos(d*x + c)^2 + 56)*sin(d*x + c) + 16)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 136, normalized size = 1.01

$$-\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a} - \frac{60 \log(|\sin(dx+c)-1|)}{a} + \frac{2\left(55 \sin(dx+c)^3+15 \sin(dx+c)^2-111 \sin(dx+c)+57\right)}{a(\sin(dx+c)-1)^3} - \frac{125 \sin(dx+c)^4+980 \sin(dx+c)^3+1662 \sin(dx+c)^2+1140 \sin(dx+c)+285}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
+ 2*(55*sin(d*x + c)^3 + 15*sin(d*x + c)^2 - 111*sin(d*x + c) + 57)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 980*sin(d*x + c)^3 + 1662*sin(d*x
 + c)^2 + 1140*sin(d*x + c) + 285)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 17.12, size = 388, normalized size = 2.90

$$\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}-\frac{85\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}+\frac{113\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{33\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{16}+\frac{289\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{16}+\frac{33\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{16}+\frac{113\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}-\frac{85\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{96}-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+30\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (5*tan(c/2 + (d*x)/2)^2)/32 - (35*tan(c/2 + (d
*x)/2)^3)/96 - (85*tan(c/2 + (d*x)/2)^4)/96 + (113*tan(c/2 + (d*x)/2)^5)/19
2 + (33*tan(c/2 + (d*x)/2)^6)/16 + (289*tan(c/2 + (d*x)/2)^7)/16 + (33*tan(
c/2 + (d*x)/2)^8)/16 + (113*tan(c/2 + (d*x)/2)^9)/192 - (85*tan(c/2 + (d*x)
/2)^10)/96 - (35*tan(c/2 + (d*x)/2)^11)/96 + (5*tan(c/2 + (d*x)/2)^12)/32 +
```

```
 (5*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2
 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a
*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^
7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d
*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*ta
n(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (5*atanh(tan(c/2 + (d*x)/
2)))/(64*a*d)
```

3.884 $\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad}+\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad}-$$

```
[Out] 5/128*arctanh(sin(d*x+c))/a/d+5/128*sec(d*x+c)*tan(d*x+c)/a/d-5/64*sec(d*x+
c)^3*tan(d*x+c)/a/d+5/48*sec(d*x+c)^3*tan(d*x+c)^3/a/d-1/8*sec(d*x+c)^3*tan
(d*x+c)^5/a/d+1/6*tan(d*x+c)^6/a/d+1/8*tan(d*x+c)^8/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2687, 14, 2691, 3853, 3855}

$$\frac{\tan^8(c+dx)}{8ad}+\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^5(c+dx)\sec^3(c+dx)}{8ad}+\frac{5\tan^3(c+dx)\sec^3(c+dx)}{48ad}-\frac{5\tan(c+dx)\sec^3(c+dx)}{64ad}+\frac{5\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) - (5*Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) + (5*Sec[c + d*x]^3*Tan[c + d
*x]^3)/(48*a*d) - (Sec[c + d*x]^3*Tan[c + d*x]^5)/(8*a*d) + Tan[c + d*x]^6/
(6*a*d) + Tan[c + d*x]^8/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a}-\frac{\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{a}\\ &= -\frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad}+\frac{5\int \sec^3(c+dx)\tan^4(c+dx)\,dx}{8a}+\frac{\text{Subst}(\\ &= \frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad}-\frac{\sec^3(c+dx)\tan^5(c+dx)}{8ad}-\frac{5\int \sec^3(c+\\ &= -\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad}+\frac{5\sec^3(c+dx)\tan^3(c+dx)}{48ad}-\frac{\sec^3(c+dx\\ &= \frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\frac{5\sec^3(c+dx)\tan(c+dx)}{64ad}+\frac{5\sec^3(c+dx)}{48a}\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\frac{5\sec^3(c+dx)\tan(}{64ad}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 92, normalized size = 0.61

$$\frac{15\tanh^{-1}(\sin(c+dx))-\frac{4}{(-1+\sin(c+dx))^3}-\frac{15}{(-1+\sin(c+dx))^2}-\frac{15}{-1+\sin(c+dx)}+\frac{6}{(1+\sin(c+dx))^4}-\frac{24}{(1+\sin(c+dx))^3}+\frac{30}{(1+\sin(c+dx))^2}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 - 15/(-1 + Sin[c + d*x]
)^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 24/(1 + Sin[c + d*x
])^3 + 30/(1 + Sin[c + d*x])^2)/(384*a*d)
```

Maple [A]
time = 0.34, size = 103, normalized size = 0.68

method	result
derivativedivides	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{16(1+\sin(dx+c))^3}+\frac{5}{64(1+\sin(dx+c))^2}+\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{5}{128(\sin(dx+c)-1)^2}-\frac{}{128(\sin(}}{da}$
default	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{16(1+\sin(dx+c))^3}+\frac{5}{64(1+\sin(dx+c))^2}+\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{5}{128(\sin(dx+c)-1)^2}-\frac{}{128(\sin(}}{da}$
risch	$-\frac{i\left(-30i e^{2i(dx+c)}+15\,e^{i(dx+c)}-86i e^{10i(dx+c)}+140i e^{8i(dx+c)}-140i e^{6i(dx+c)}+86i e^{4i(dx+c)}+30i e^{12i(dx+c)}+15\,e^{13i(dx+c}\right.}{192\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^6 da}$
norman	$\frac{-\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{5\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{413\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{413\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{85(\tan}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/64/(1+sin(d*x+c))^4-1/16/(1+sin(d*x+c))^3+5/64/(1+sin(d*x+c))^2+5/
256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-5/128/(sin(d*x+c)-1)^2-5/128/(si
n(d*x+c)-1)-5/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 173, normalized size = 1.14

$$-\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5+88\sin(dx+c)^4-8\sin(dx+c)^3-63\sin(dx+c)^2+\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 + 88*sin(d*x + c)^4 - 8*si
n(d*x + c)^3 - 63*sin(d*x + c)^2 + sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1)/a
 + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.10

$$-\frac{30\cos(dx+c)^6-266\cos(dx+c)^4+316\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(15\cos(dx+c)^4-22\cos(dx+c)^2+8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 266*cos(d*x + c)^4 + 316*cos(d*x + c)^2 - 15*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(co
s(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*
cos(d*x + c)^4 - 22*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x +
c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [A]
time = 0.56, size = 136, normalized size = 0.89

$$\frac{\frac{60 \log(|\sin(dx+c)+1|)}{a} - \frac{60 \log(|\sin(dx+c)-1|)}{a} + \frac{2\left(55 \sin(dx+c)^3 - 225 \sin(dx+c)^2 + 225 \sin(dx+c) - 71\right)}{a(\sin(dx+c)-1)^3} - \frac{125 \sin(dx+c)^4 + 500 \sin(dx+c)^3 + 510 \sin(dx+c)^2 + 212 \sin(dx+c) + 29}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 225*sin(d*x + c) - 71)/(a*(sin
(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 510*sin(d*x
+ c)^2 + 212*sin(d*x + c) + 29)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 17.21, size = 388, normalized size = 2.55

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d} + \frac{-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32} + \frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96} + \frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96} - \frac{113\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192} + \frac{413\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{48} + \frac{157\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{48} + \frac{413\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{48} - \frac{113\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192} + \frac{85\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{96} + \frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{32} - \frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} - 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} - 12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} + 9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 30\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9} - 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} - 40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7} - 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + 30\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5} + 9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4} - 12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3} - 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((35*tan(c/2 + (d*x)/2)^3)/96 - (5
*tan(c/2 + (d*x)/2)^2)/32 - (5*tan(c/2 + (d*x)/2))/64 + (85*tan(c/2 + (d*x)
/2)^4)/96 - (113*tan(c/2 + (d*x)/2)^5)/192 + (413*tan(c/2 + (d*x)/2)^6)/48
+ (157*tan(c/2 + (d*x)/2)^7)/48 + (413*tan(c/2 + (d*x)/2)^8)/48 - (113*tan(
c/2 + (d*x)/2)^9)/192 + (85*tan(c/2 + (d*x)/2)^10)/96 + (35*tan(c/2 + (d*x)
/2)^11)/96 - (5*tan(c/2 + (d*x)/2)^12)/32 - (5*tan(c/2 + (d*x)/2)^13)/64)/(
d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (
```

```
d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(
c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 3
0*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)
/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2
+ (d*x)/2)^14))
```

3.885 $\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{3\sec(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{64ad}-\frac{\sec^5(c+dx)\tan(c+dx)}{16ad}+\sec$$

```
[Out] 3/128*arctanh(sin(d*x+c))/a/d+3/128*sec(d*x+c)*tan(d*x+c)/a/d+1/64*sec(d*x+
c)^3*tan(d*x+c)/a/d-1/16*sec(d*x+c)^5*tan(d*x+c)/a/d+1/8*sec(d*x+c)^5*tan(d
*x+c)^3/a/d-1/6*tan(d*x+c)^6/a/d-1/8*tan(d*x+c)^8/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2691, 3853, 3855, 2687, 14}

$$-\frac{\tan^8(c+dx)}{8ad}-\frac{\tan^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{64ad}+\frac{3\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(128*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*
d) + (Sec[c + d*x]^3*Tan[c + d*x])/(64*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])
/(16*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*a*d) - Tan[c + d*x]^6/(6*a*d
) - Tan[c + d*x]^8/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a}-\frac{\int \sec^4(c+dx)\tan^5(c+dx)\,dx}{a}\\ &= \frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad}-\frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a}-\frac{\mathrm{Subst}\left(\int x\right.}{}\\ &= -\frac{\sec^5(c+dx)\tan(c+dx)}{16ad}+\frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad}+\frac{\int \sec^5(c+dx)\,d}{16a}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{64ad}-\frac{\sec^5(c+dx)\tan(c+dx)}{16ad}+\frac{\sec^5(c+dx)\tan^3(}{8ad}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{64ad}-\frac{\sec^5(c+dx)\tan(}{16ad}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{3\sec(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^3(c+dx)\tan(c+}{64ad}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 101, normalized size = 0.67

$$\frac{9\tanh^{-1}(\sin(c+dx))+\frac{16+25\sin(c+dx)-39\sin^2(c+dx)-72\sin^3(c+dx)+24\sin^4(c+dx)-9\sin^5(c+dx)-9\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (9*ArcTanh[Sin[c + d*x]] + (16 + 25*Sin[c + d*x] - 39*Sin[c + d*x]^2 - 72*S
in[c + d*x]^3 + 24*Sin[c + d*x]^4 - 9*Sin[c + d*x]^5 - 9*Sin[c + d*x]^6)/((
-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A]
time = 0.33, size = 115, normalized size = 0.77

method	result
derivativedivides	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{24(1+\sin(dx+c))^3}-\frac{1}{64(1+\sin(dx+c))^2}-\frac{1}{32(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin}$ ...
default	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{24(1+\sin(dx+c))^3}-\frac{1}{64(1+\sin(dx+c))^2}-\frac{1}{32(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin}$ ...
risch	$-\frac{i(18i e^{12i(dx+c)}+9 e^{13i(dx+c)}-666i e^{10i(dx+c)}+42 e^{11i(dx+c)}+1108i e^{8i(dx+c)}+375 e^{9i(dx+c)}-1108i e^{6i(dx+c)}+172 e^{7}}{192(e^{i(dx+c)}+i)^8(e^{i(dx+c)}-i)^6 da}$ ...
norman	$\frac{-\frac{3\tan(\frac{dx}{2}+\frac{c}{2})}{64ad}-\frac{3(\tan^{13}(\frac{dx}{2}+\frac{c}{2}))}{64da}-\frac{3(\tan^{2}(\frac{dx}{2}+\frac{c}{2}))}{32ad}-\frac{3(\tan^{12}(\frac{dx}{2}+\frac{c}{2}))}{32ad}+\frac{43(\tan^{6}(\frac{dx}{2}+\frac{c}{2}))}{48ad}+\frac{43(\tan^{8}(\frac{dx}{2}+\frac{c}{2}))}{48ad}+\frac{17(\tan}{}}{(\tan(\frac{dx}{2}+\frac{c}{2})}$ ...

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/64/(1+sin(d*x+c))^4+1/24/(1+sin(d*x+c))^3-1/64/(1+sin(d*x+c))^2-1
/32/(1+sin(d*x+c))+3/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-3/128/(sin(
d*x+c)-1)^2+1/128/(sin(d*x+c)-1)-3/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.17

$$-\frac{\frac{2\left(9\sin(dx+c)^6+9\sin(dx+c)^5-24\sin(dx+c)^4+72\sin(dx+c)^3+39\sin(dx+c)^2-25\sin(dx+c)-16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{9\log(\sin(dx+c)+1)}{a}+\frac{9\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 + 72*sin
(d*x + c)^3 + 39*sin(d*x + c)^2 - 25*sin(d*x + c) - 16)/(a*sin(d*x + c)^7 +
 a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x +
 c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/
a + 9*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.11

$$-\frac{18\cos(dx+c)^6-6\cos(dx+c)^4+36\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(9\cos(dx+c)^4-90\cos(dx+c)^2+56\right)\sin(dx+c)-16}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 + 36*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 - 90*cos(d*x + c)^2 + 56)*sin(d*x + c) - 16)/(a*d*cos(d*x + c)^6*s
in(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.54, size = 136, normalized size = 0.91

$$\frac{\frac{36\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{36\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{2\left(33\sin\left(dx+c\right)^{3}-87\sin\left(dx+c\right)^{2}+39\sin\left(dx+c\right)-1\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{75\sin\left(dx+c\right)^{4}+396\sin\left(dx+c\right)^{3}+786\sin\left(dx+c\right)^{2}+556\sin\left(dx+c\right)+139}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a +
 2*(33*sin(d*x + c)^3 - 87*sin(d*x + c)^2 + 39*sin(d*x + c) - 1)/(a*(sin(d*
x + c) - 1)^3) - (75*sin(d*x + c)^4 + 396*sin(d*x + c)^3 + 786*sin(d*x + c)
^2 + 556*sin(d*x + c) + 139)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 17.15, size = 388, normalized size = 2.59

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{32}+\frac{387\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64}+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{48}+\frac{299\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{48}+\frac{43\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{48}+\frac{387\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{64}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{32}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{32}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+30\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+30\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((7*tan(c/2 + (d*x)/2)^3)/32 - (3*
tan(c/2 + (d*x)/2)^2)/32 - (3*tan(c/2 + (d*x)/2))/64 + (17*tan(c/2 + (d*x)/
2)^4)/32 + (387*tan(c/2 + (d*x)/2)^5)/64 + (43*tan(c/2 + (d*x)/2)^6)/48 + (
299*tan(c/2 + (d*x)/2)^7)/48 + (43*tan(c/2 + (d*x)/2)^8)/48 + (387*tan(c/2
+ (d*x)/2)^9)/64 + (17*tan(c/2 + (d*x)/2)^10)/32 + (7*tan(c/2 + (d*x)/2)^11
```

```
)/32 - (3*tan(c/2 + (d*x)/2)^12)/32 - (3*tan(c/2 + (d*x)/2)^13)/64)/(d*(a +
 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2
)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 +
(d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*ta
n(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11
 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x
)/2)^14))
```

3.886 $\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=150

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{3\sec(c+dx)\tan(c+dx)}{128ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{64ad}+$$

```
[Out] -3/128*arctanh(sin(d*x+c))/a/d-1/6*sec(d*x+c)^6/a/d+1/8*sec(d*x+c)^8/a/d-3/
128*sec(d*x+c)*tan(d*x+c)/a/d-1/64*sec(d*x+c)^3*tan(d*x+c)/a/d+1/16*sec(d*x
+c)^5*tan(d*x+c)/a/d-1/8*sec(d*x+c)^5*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2686, 14, 2691, 3853, 3855}

$$\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^6(c+dx)}{6ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{16ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{64ad}-\frac{3\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(128*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x
]^8/(8*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^3*Tan
[c + d*x])/(64*a*d) + (Sec[c + d*x]^5*Tan[c + d*x])/(16*a*d) - (Sec[c + d*x
]^5*Tan[c + d*x]^3)/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a}-\frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{a}\\ &= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad}+\frac{3\int \sec^5(c+dx)\tan^2(c+dx)\,dx}{8a}+\frac{\text{Subst}(}{}\\ &= \frac{\sec^5(c+dx)\tan(c+dx)}{16ad}-\frac{\sec^5(c+dx)\tan^3(c+dx)}{8ad}-\frac{\int \sec^5(c+dx)\,d}{16a}\\ &= -\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{64ad}+\frac{\sec^5(c+dx}{1}\\ &= -\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{3\sec(c+dx)\tan(c+dx)}{128ad}-\frac{\sec^3(c+d}{6}\\ &= -\frac{3\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{3\sec(c+dx)\,t}{128a}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 92, normalized size = 0.61

$$-\frac{9\tanh^{-1}(\sin(c+dx))+\frac{4}{(-1+\sin(c+dx))^3}+\frac{3}{(-1+\sin(c+dx))^2}-\frac{9}{-1+\sin(c+dx)}-\frac{6}{(1+\sin(c+dx))^4}+\frac{8}{(1+\sin(c+dx))^3}+\frac{6}{(1+\sin(c+dx))^2}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(9*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 + 3/(-1 + Sin[c +
 d*x])^2 - 9/(-1 + Sin[c + d*x]) - 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c +
d*x])^3 + 6/(1 + Sin[c + d*x])^2)/(a*d)
```

Maple [A]
time = 0.33, size = 103, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{48(1+\sin(dx+c))^3}-\frac{1}{64(1+\sin(dx+c))^2}-\frac{3\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{1}{128(\sin(dx+c)-1)^2}+\frac{}{128(\sin(}}{da}$
default	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{48(1+\sin(dx+c))^3}-\frac{1}{64(1+\sin(dx+c))^2}-\frac{3\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{1}{128(\sin(dx+c)-1)^2}+\frac{}{128(\sin(}}{da}$
risch	$\frac{i\left(-18i e^{2i(dx+c)}+9\,e^{i(dx+c)}+102i e^{10i(dx+c)}-172i e^{8i(dx+c)}+172i e^{6i(dx+c)}-102i e^{4i(dx+c)}+18i e^{12i(dx+c)}+9\,e^{13i(dx+c)}+\right.}{192\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^6 da}$
norman	$\frac{\frac{111\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{111\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{3\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{277(\tan}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/64/(1+sin(d*x+c))^4-1/48/(1+sin(d*x+c))^3-1/64/(1+sin(d*x+c))^2-3/
256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-1/128/(sin(d*x+c)-1)^2+3/128/(si
n(d*x+c)-1)+3/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 175, normalized size = 1.17

$$\frac{\frac{2\left(9\sin(dx+c)^6+9\sin(dx+c)^5-24\sin(dx+c)^4-24\sin(dx+c)^3-57\sin(dx+c)^2+7\sin(dx+c)+16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{9\log(\sin(dx+c)+1)}{a}+\frac{9\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(9*sin(d*x + c)^6 + 9*sin(d*x + c)^5 - 24*sin(d*x + c)^4 - 24*sin(
d*x + c)^3 - 57*sin(d*x + c)^2 + 7*sin(d*x + c) + 16)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 9*log(sin(d*x + c) + 1)/a
+ 9*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 167, normalized size = 1.11

$$\frac{18\cos(dx+c)^6-6\cos(dx+c)^4-156\cos(dx+c)^2-9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+9\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(9\cos(dx+c)^4+6\cos(dx+c)^2-8\right)\sin(dx+c)+112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(18*cos(d*x + c)^6 - 6*cos(d*x + c)^4 - 156*cos(d*x + c)^2 - 9*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 9*(cos(d*x
 + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*
x + c)^4 + 6*cos(d*x + c)^2 - 8)*sin(d*x + c) + 112)/(a*d*cos(d*x + c)^6*si
n(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.54, size = 136, normalized size = 0.91

$$-\frac{\frac{36\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{36\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{2\,\left(33\,\sin\left(dx+c\right)^3-135\,\sin\left(dx+c\right)^2+183\,\sin\left(dx+c\right)-65\right)}{a\left(\sin\left(dx+c\right)-1\right)^3}-\frac{75\,\sin\left(dx+c\right)^4+300\,\sin\left(dx+c\right)^3+402\,\sin\left(dx+c\right)^2+140\,\sin\left(dx+c\right)+11}{a\left(\sin\left(dx+c\right)+1\right)^4}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(36*log(abs(sin(d*x + c) + 1))/a - 36*log(abs(sin(d*x + c) - 1))/a
+ 2*(33*sin(d*x + c)^3 - 135*sin(d*x + c)^2 + 183*sin(d*x + c) - 65)/(a*(si
n(d*x + c) - 1)^3) - (75*sin(d*x + c)^4 + 300*sin(d*x + c)^3 + 402*sin(d*x
+ c)^2 + 140*sin(d*x + c) + 11)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 17.02, size = 388, normalized size = 2.59

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{32}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{32}+\frac{125\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64}+\frac{277\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{48}-\frac{43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{48}+\frac{277\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{48}+\frac{125\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{64}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{32}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/64 + (3*tan(c/2 + (d*x)/2)^2)/32 - (7*tan(c/2 + (d*
x)/2)^3)/32 + (111*tan(c/2 + (d*x)/2)^4)/32 + (125*tan(c/2 + (d*x)/2)^5)/64
 + (277*tan(c/2 + (d*x)/2)^6)/48 - (43*tan(c/2 + (d*x)/2)^7)/48 + (277*tan(
c/2 + (d*x)/2)^8)/48 + (125*tan(c/2 + (d*x)/2)^9)/64 + (111*tan(c/2 + (d*x)
/2)^10)/32 - (7*tan(c/2 + (d*x)/2)^11)/32 + (3*tan(c/2 + (d*x)/2)^12)/32 +
```

```
(3*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2
+ (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*
tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7
 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*
x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan
(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (3*atanh(tan(c/2 + (d*x)/2
)))/(64*a*d)
```

3.887 $\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{5\sec(c+dx)\tan(c+dx)}{128ad}-\frac{5\sec^3(c+dx)\tan(c+dx}{192ad}$$

```
[Out] -5/128*arctanh(sin(d*x+c))/a/d+1/6*sec(d*x+c)^6/a/d-1/8*sec(d*x+c)^8/a/d-5/
128*sec(d*x+c)*tan(d*x+c)/a/d-5/192*sec(d*x+c)^3*tan(d*x+c)/a/d-1/48*sec(d*
x+c)^5*tan(d*x+c)/a/d+1/8*sec(d*x+c)^7*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2691, 3853, 3855, 2686, 14}

$$-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}-\frac{5\tan(c+dx)\sec^3(c+dx)}{192ad}-\frac{5\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x
]^8/(8*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(128*a*d) - (5*Sec[c + d*x]^3*T
an[c + d*x])/(192*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) + (Sec[c +
d*x]^7*Tan[c + d*x])/(8*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^6(c+dx)\tan^3(c+dx)\,dx}{a} \\
&= \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} - \frac{\int \sec^7(c+dx)\,dx}{8a} - \frac{\operatorname{Subst}\left(\int x^5(-1+x^2)\,dx,\right.}{ad} \\
&= -\frac{\sec^5(c+dx)\tan(c+dx)}{48ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} - \frac{5\int \sec^5(c+dx)}{48a} \\
&= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} - \frac{\sec^5(c+dx)}{48} \\
&= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec(c+dx)\tan(c+dx)}{128ad} - \frac{5\sec^3(c+dx)}{192} \\
&= -\frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{8ad} - \frac{5\sec(c+dx)\,\mathrm{ta}}{128ad}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 92, normalized size = 0.62

$$-\frac{15\tanh^{-1}(\sin(c+dx)) + \frac{4}{(-1+\sin(c+dx))^3} - \frac{3}{(-1+\sin(c+dx))^2} - \frac{3}{-1+\sin(c+dx)} + \frac{6}{(1+\sin(c+dx))^4} - \frac{6}{(1+\sin(c+dx))^2} - \frac{12}{1+\sin(c+dx)}}{384ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]

[Out] -1/384*(15*ArcTanh[Sin[c + d*x]] + 4/(-1 + Sin[c + d*x])^3 - 3/(-1 + Sin[c + d*x])^2 - 3/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 - 6/(1 + Sin[c + d*x])^2 - 12/(1 + Sin[c + d*x]))/(a*d)

Maple [A]
time = 0.34, size = 103, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{64(1+\sin(dx+c))^2}+\frac{1}{32+32\sin(dx+c)}-\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{1}{128(\sin(dx+c)-1)^2}+\frac{1}{128\sin}}{da}$
default	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{64(1+\sin(dx+c))^2}+\frac{1}{32+32\sin(dx+c)}-\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{1}{128(\sin(dx+c)-1)^2}+\frac{1}{128\sin}}{da}$
risch	$\frac{i(30i e^{12i(dx+c)}+15\,e^{13i(dx+c)}+170i e^{10i(dx+c)}+70\,e^{11i(dx+c)}-1652i e^{8i(dx+c)}+113\,e^{9i(dx+c)}+1652i e^{6i(dx+c)}+628\,e^{7i(}}{192(e^{i(dx+c)}-i)^6(e^{i(dx+c)}+i)^8 da}$
norman	$\frac{\frac{5\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{5\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{35\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{35\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{355(\tan^7}{4}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-1/64/(1+sin(d*x+c))^4+1/64/(1+sin(d*x+c))^2+1/32/(1+sin(d*x+c))-5/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3+1/128/(sin(d*x+c)-1)^2+1/128/(sin(d*x+c)-1)+5/256*ln(sin(d*x+c)-1))

Maxima [A]
time = 0.28, size = 175, normalized size = 1.18

$$\frac{\frac{2\left(15\sin(dx+c)^6+15\sin(dx+c)^5-40\sin(dx+c)^4-40\sin(dx+c)^3+33\sin(dx+c)^2-31\sin(dx+c)-16\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*sin(d*x + c)^3 + 33*sin(d*x + c)^2 - 31*sin(d*x + c) - 16)/(a*sin(d*x + c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d

Fricas [A]
time = 0.37, size = 167, normalized size = 1.13

$$\frac{30\cos(dx+c)^6-10\cos(dx+c)^4-4\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(15\cos(dx+c)^4+10\cos(dx+c)^2-56\right)\sin(dx+c)+16}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos(d*x + c)^4 + 10*cos(d*x + c)^2 - 56)*sin(d*x + c) + 16)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**7*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep

Giac [A]
time = 0.60, size = 136, normalized size = 0.92

$$-\frac{\frac{60\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{60\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{2\,\left(55\,\sin\left(dx+c\right)^{3}-177\,\sin\left(dx+c\right)^{2}+177\,\sin\left(dx+c\right)-39\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{125\,\sin\left(dx+c\right)^{4}+596\,\sin\left(dx+c\right)^{3}+1086\,\sin\left(dx+c\right)^{2}+884\,\sin\left(dx+c\right)+221}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] -1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a + 2*(55*sin(d*x + c)^3 - 177*sin(d*x + c)^2 + 177*sin(d*x + c) - 39)/(a*(sin(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 596*sin(d*x + c)^3 + 1086*sin(d*x + c)^2 + 884*sin(d*x + c) + 221)/(a*(sin(d*x + c) + 1)^4))/d

Mupad [B]
time = 17.12, size = 388, normalized size = 2.62

$$\frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}+\frac{221\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}+\frac{43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{96}+\frac{625\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{48}+\frac{355\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{48}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{48}+\frac{625\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}+\frac{43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{96}+\frac{221\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{32}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+30a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^2/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)

[Out] ((5*tan(c/2 + (d*x)/2))/64 + (5*tan(c/2 + (d*x)/2)^2)/32 + (221*tan(c/2 + (d*x)/2)^3)/96 + (43*tan(c/2 + (d*x)/2)^4)/96 + (625*tan(c/2 + (d*x)/2)^5)/192 + (35*tan(c/2 + (d*x)/2)^6)/48 + (355*tan(c/2 + (d*x)/2)^7)/48 + (35*tan(c/2 + (d*x)/2)^8)/48 + (625*tan(c/2 + (d*x)/2)^9)/192 + (43*tan(c/2 + (d*x)/2)^10)/96 + (221*tan(c/2 + (d*x)/2)^11)/96 + (5*tan(c/2 + (d*x)/2)^12)/32

+ (5*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (5*atanh(tan(c/2 + (d*x)/2)))/(64*a*d)

3.888 $\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{\sec^8(c+dx)}{8ad}+\frac{5\sec(c+dx)\tan(c+dx)}{128ad}+\frac{5\sec^3(c+dx)\tan(c+dx)}{192ad}+\frac{\sec^5(c+dx)\tan(c+dx)}{48ad}$$

```
[Out] 5/128*arctanh(sin(d*x+c))/a/d+1/8*sec(d*x+c)^8/a/d+5/128*sec(d*x+c)*tan(d*x
+c)/a/d+5/192*sec(d*x+c)^3*tan(d*x+c)/a/d+1/48*sec(d*x+c)^5*tan(d*x+c)/a/d-
1/8*sec(d*x+c)^7*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2914, 2686, 30, 2691, 3853, 3855}

$$\frac{\sec^8(c+dx)}{8ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{8ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{48ad}+\frac{5\tan(c+dx)\sec^3(c+dx)}{192ad}+\frac{5\tan(c+dx)\sec(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(128*a*d) + Sec[c + d*x]^8/(8*a*d) + (5*Sec[c + d
*x]*Tan[c + d*x])/(128*a*d) + (5*Sec[c + d*x]^3*Tan[c + d*x])/(192*a*d) + (
Sec[c + d*x]^5*Tan[c + d*x])/(48*a*d) - (Sec[c + d*x]^7*Tan[c + d*x])/(8*a*
d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^8(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= -\frac{\sec^7(c+dx)\tan(c+dx)}{8ad} + \frac{\int \sec^7(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^7\,dx, x, \sec(c\right.}{ad}$$
$$= \frac{\sec^8(c+dx)}{8ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{48ad} - \frac{\sec^7(c+dx)\tan(c+dx)}{8ad} + 5$$
$$= \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} + \frac{\sec^5(c+dx)\tan(c+dx)}{48ad} -$$
$$= \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec^3(c+dx)\tan(c+dx)}{192ad} +$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{\sec^8(c+dx)}{8ad} + \frac{5\sec(c+dx)\tan(c+dx)}{128ad} + \frac{5\sec}{}$$

Mathematica [A]
time = 0.66, size = 92, normalized size = 0.71

$$\frac{15\tanh^{-1}(\sin(c+dx)) - \frac{4}{(-1+\sin(c+dx))^3} + \frac{9}{(-1+\sin(c+dx))^2} - \frac{15}{-1+\sin(c+dx)} + \frac{6}{(1+\sin(c+dx))^4} + \frac{8}{(1+\sin(c+dx))^3} + \frac{6}{(1+\sin(c+dx))^2}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - 4/(-1 + Sin[c + d*x])^3 + 9/(-1 + Sin[c + d*x])
^2 - 15/(-1 + Sin[c + d*x]) + 6/(1 + Sin[c + d*x])^4 + 8/(1 + Sin[c + d*x])
^3 + 6/(1 + Sin[c + d*x])^2)/(384*a*d)
```

Maple [A]
time = 0.32, size = 103, normalized size = 0.79

method	result
derivativedivides	$\frac{\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{48(1+\sin(dx+c))^3}+\frac{1}{64(1+\sin(dx+c))^2}+\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{3}{128(\sin(dx+c)-1)^2}-\frac{\cdots}{128(\sin(\cdots}}{da}$
default	$\frac{\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{48(1+\sin(dx+c))^3}+\frac{1}{64(1+\sin(dx+c))^2}+\frac{5\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{3}{128(\sin(dx+c)-1)^2}-\frac{\cdots}{128(\sin(\cdots}}{da}$
risch	$-\frac{i\left(15\,e^{i(dx+c)}-170ie^{4i(dx+c)}+396ie^{8i(dx+c)}+70\,e^{11i(dx+c)}-30ie^{2i(dx+c)}+70\,e^{3i(dx+c)}+30ie^{12i(dx+c)}+113\,e^{5i(dx+c)}-\cdots\right.}{192\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^6 da}$
norman	$\frac{\frac{95\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{95\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{95\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{5\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{59\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{59\left(\tan^{12}\cdots\right.}{32\cdots}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/64/(1+sin(d*x+c))^4+1/48/(1+sin(d*x+c))^3+1/64/(1+sin(d*x+c))^2+5/
256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3+3/128/(sin(d*x+c)-1)^2-5/128/(si
n(d*x+c)-1)-5/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.35

$$-\frac{\frac{2\left(15\,\sin(dx+c)^6+15\,\sin(dx+c)^5-40\,\sin(dx+c)^4-40\,\sin(dx+c)^3+33\,\sin(dx+c)^2+33\,\sin(dx+c)+48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3\,a\sin(dx+c)^5-3\,a\sin(dx+c)^4+3\,a\sin(dx+c)^3+3\,a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{15\,\log(\sin(dx+c)+1)}{a}+\frac{15\,\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(15*sin(d*x + c)^6 + 15*sin(d*x + c)^5 - 40*sin(d*x + c)^4 - 40*s
in(d*x + c)^3 + 33*sin(d*x + c)^2 + 33*sin(d*x + c) + 48)/(a*sin(d*x + c)^7
 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x
 + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 15*log(sin(d*x + c) +
1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.28

$$-\frac{30\,\cos(dx+c)^6-10\,\cos(dx+c)^4-4\,\cos(dx+c)^2-15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-2\left(15\,\cos(dx+c)^4+10\,\cos(dx+c)^2+8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(30*cos(d*x + c)^6 - 10*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 15*(cos(
d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*(cos(d
*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*cos
(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^
6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 136, normalized size = 1.05

$$\frac{\frac{60\,\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{60\,\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}+\frac{2\left(55\,\sin\left(dx+c\right)^3-225\,\sin\left(dx+c\right)^2+321\,\sin\left(dx+c\right)-167\right)}{a\left(\sin\left(dx+c\right)-1\right)^3}-\frac{125\,\sin\left(dx+c\right)^4+500\,\sin\left(dx+c\right)^3+702\,\sin\left(dx+c\right)^2+340\,\sin\left(dx+c\right)-35}{a\left(\sin\left(dx+c\right)+1\right)^4}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a +
 2*(55*sin(d*x + c)^3 - 225*sin(d*x + c)^2 + 321*sin(d*x + c) - 167)/(a*(si
n(d*x + c) - 1)^3) - (125*sin(d*x + c)^4 + 500*sin(d*x + c)^3 + 702*sin(d*x
 + c)^2 + 340*sin(d*x + c) - 35)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [**B**]
time = 17.04, size = 388, normalized size = 2.98

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{64\,a\,d}+\frac{-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{59\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{32}+\frac{163\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{96}+\frac{149\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{96}-\frac{625\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+\frac{95\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{16}+\frac{95\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{16}+\frac{95\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{16}-\frac{625\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{192}+\frac{149\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{96}+\frac{163\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96}+\frac{59\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{32}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}-5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-12\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}+9\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+30\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}-5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-40\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}-5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+30\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}+9\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-12\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}-5\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(64*a*d) + ((59*tan(c/2 + (d*x)/2)^2)/32 - (5
*tan(c/2 + (d*x)/2))/64 + (163*tan(c/2 + (d*x)/2)^3)/96 + (149*tan(c/2 + (d
*x)/2)^4)/96 - (625*tan(c/2 + (d*x)/2)^5)/192 + (95*tan(c/2 + (d*x)/2)^6)/1
6 + (95*tan(c/2 + (d*x)/2)^7)/16 + (95*tan(c/2 + (d*x)/2)^8)/16 - (625*tan(
c/2 + (d*x)/2)^9)/192 + (149*tan(c/2 + (d*x)/2)^10)/96 + (163*tan(c/2 + (d*
x)/2)^11)/96 + (59*tan(c/2 + (d*x)/2)^12)/32 - (5*tan(c/2 + (d*x)/2)^13)/64
)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2
```

```
+ (d*x)/2)^3 + 9*a*tan(c/2 + (d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*t
an(c/2 + (d*x)/2)^6 - 40*a*tan(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8
+ 30*a*tan(c/2 + (d*x)/2)^9 + 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d
*x)/2)^11 - 5*a*tan(c/2 + (d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c
/2 + (d*x)/2)^14))
```

3.889 $\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}+\frac{15}{128d(a-a\sin(c+dx))}-\frac{}{64d}$$

```
[Out] 35/128*arctanh(sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+5/128*a/d/(a-a
*sin(d*x+c))^2+15/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/24
*a^2/d/(a+a*sin(d*x+c))^3-5/64*a/d/(a+a*sin(d*x+c))^2-5/32/d/(a+a*sin(d*x+c
))
```

Rubi [A]
time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{24d(a\sin(c+dx)+a)^3}+\frac{5a}{128d(a-a\sin(c+dx))^2}-\frac{5a}{64d(a\sin(c+dx)+a)^2}+\frac{15}{128d(a-a\sin(c+dx))}-\frac{5}{32d(a\sin(c+dx)+a)}+\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (35*ArcTanh[Sin[c + d*x]])/(128*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) +
(5*a)/(128*d*(a - a*Sin[c + d*x])^2) + 15/(128*d*(a - a*Sin[c + d*x])) - a^
3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(24*d*(a + a*Sin[c + d*x])^3) - (5*a)
/(64*d*(a + a*Sin[c + d*x])^2) - 5/(32*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

])

Rubi steps

$$\int \frac{\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{1}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\text{Subst}\left(\int \left(\frac{1}{32a^5(a-x)^4} + \frac{5}{64a^6(a-x)^3} + \frac{15}{128a^7(a-x)^2} + \frac{1}{16a^4(a+x)^5} + \frac{1}{8a^5(a+x)^4} + \frac{5}{32a^6(a+x)}\right.\right.}{d}$$
$$= \frac{a^2}{96d(a-a\sin(c+dx))^3} + \frac{5a}{128d(a-a\sin(c+dx))^2} + \frac{15}{128d(a-a\sin(c+dx))} - $$
$$= \frac{35\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{a^2}{96d(a-a\sin(c+dx))^3} + \frac{5a}{128d(a-a\sin(c+dx))^2} + \frac{}{1}$$

Mathematica [A]

time = 0.36, size = 145, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left(48-105\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8-231\sin(c+dx)-231\sin^2(c+dx)+280\sin^3(c+dx)+280\sin^4(c+dx)-105\sin^5(c+dx)-105\sin^6(c+dx)\right)}{384ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/384*(Sec[c + d*x]^6*(48 - 105*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] -
Sin[(c + d*x)/2])^6*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8 - 231*Sin[c + d
*x] - 231*Sin[c + d*x]^2 + 280*Sin[c + d*x]^3 + 280*Sin[c + d*x]^4 - 105*Si
n[c + d*x]^5 - 105*Sin[c + d*x]^6))/(a*d*(1 + Sin[c + d*x]))
```

Maple [A]

time = 0.35, size = 115, normalized size = 0.70

method	result
derivativedivides	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{24(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(dx+c))^2}-\frac{5}{32(1+\sin(dx+c))}+\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{}{128(\sin}}{da}$
default	$\frac{-\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{24(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(dx+c))^2}-\frac{5}{32(1+\sin(dx+c))}+\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}+\frac{}{128(\sin}}{da}$
risch	$-\frac{i\left(210i\,e^{12i(dx+c)}+105\,e^{13i(dx+c)}+1190i\,e^{10i(dx+c)}+490\,e^{11i(dx+c)}+2772i\,e^{8i(dx+c)}+791\,e^{9i(dx+c)}-2772i\,e^{6i(dx+c)}+30\right.}{192\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^6}$
norman	$\frac{\frac{25\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{25\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{25\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16ad}+\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{93\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{29\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{29\left(\tan\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/64/(1+sin(d*x+c))^4-1/24/(1+sin(d*x+c))^3-5/64/(1+sin(d*x+c))^2-5
/32/(1+sin(d*x+c))+35/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3+5/128/(sin
(d*x+c)-1)^2-15/128/(sin(d*x+c)-1)-35/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 175, normalized size = 1.06

$$-\frac{\frac{2\left(105\sin(dx+c)^6+105\sin(dx+c)^5-280\sin(dx+c)^4-280\sin(dx+c)^3+231\sin(dx+c)^2+231\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{105\log(\sin(dx+c)+1)}{a}+\frac{105\log(\sin(dx+c)-1)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(105*sin(d*x + c)^6 + 105*sin(d*x + c)^5 - 280*sin(d*x + c)^4 - 2
80*sin(d*x + c)^3 + 231*sin(d*x + c)^2 + 231*sin(d*x + c) - 48)/(a*sin(d*x
+ c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*s
in(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 105*log(sin(d*x
+ c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.39, size = 167, normalized size = 1.01

$$-\frac{210\cos(dx+c)^6-70\cos(dx+c)^4-28\cos(dx+c)^2-105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)+105\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)-14\left(15\cos(dx+c)^4+10\cos(dx+c)^2+8\right)\sin(dx+c)-16}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(210*cos(d*x + c)^6 - 70*cos(d*x + c)^4 - 28*cos(d*x + c)^2 - 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 105*(c
os(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 14*(1
5*cos(d*x + c)^4 + 10*cos(d*x + c)^2 + 8)*sin(d*x + c) - 16)/(a*d*cos(d*x +
 c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^7(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

[Out] Integral(sec(c + d*x)**7/(sin(c + d*x) + 1), x)/a

Giac [A]
time = 0.48, size = 136, normalized size = 0.82

$$\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a} - \frac{420 \log(|\sin(dx+c)-1|)}{a} + \frac{2\left(385 \sin(dx+c)^3-1335 \sin(dx+c)^2+1575 \sin(dx+c)-641\right)}{a(\sin(dx+c)-1)^3} - \frac{875 \sin(dx+c)^4+3980 \sin(dx+c)^3+6930 \sin(dx+c)^2+5548 \sin(dx+c)+1771}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")

[Out] 1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/a + 2*(385*sin(d*x + c)^3 - 1335*sin(d*x + c)^2 + 1575*sin(d*x + c) - 641)/(a*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 3980*sin(d*x + c)^3 + 6930*sin(d*x + c)^2 + 5548*sin(d*x + c) + 1771)/(a*(sin(d*x + c) + 1)^4))/d

Mupad [B]
time = 0.24, size = 158, normalized size = 0.96

$$\frac{35\,\mathrm{atanh}(\sin\left(c+d\,x\right))}{128\,a\,d}+\frac{\frac{35\sin(c+d\,x)^6}{128}+\frac{35\sin(c+d\,x)^5}{128}-\frac{35\sin(c+d\,x)^4}{48}-\frac{35\sin(c+d\,x)^3}{48}+\frac{77\sin(c+d\,x)^2}{128}+\frac{77\sin(c+d\,x)}{128}-\frac{1}{8}}{d\,\left(-a\sin\left(c+d\,x\right)^7-a\sin\left(c+d\,x\right)^6+3\,a\sin\left(c+d\,x\right)^5+3\,a\sin\left(c+d\,x\right)^4-3\,a\sin\left(c+d\,x\right)^3-3\,a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)

[Out] (35*atanh(sin(c + d*x)))/(128*a*d) + ((77*sin(c + d*x))/128 + (77*sin(c + d*x)^2)/128 - (35*sin(c + d*x)^3)/48 - (35*sin(c + d*x)^4)/48 + (35*sin(c + d*x)^5)/128 + (35*sin(c + d*x)^6)/128 - 1/8)/(d*(a + a*sin(c + d*x) - 3*a*sin(c + d*x)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 - a*sin(c + d*x)^6 - a*sin(c + d*x)^7))

3.890 $\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$-\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{163\log(1+\sin(c+dx))}{256ad}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{}{128d(a-}$$

```
[Out] -93/256*ln(1-sin(d*x+c))/a/d+ln(sin(d*x+c))/a/d-163/256*ln(1+sin(d*x+c))/a/
d+1/96*a^2/d/(a-a*sin(d*x+c))^3+7/128*a/d/(a-a*sin(d*x+c))^2+29/128/d/(a-a*
sin(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+c))^4+1/16*a^2/d/(a+a*sin(d*x+c))^3+11/
64*a/d/(a+a*sin(d*x+c))^2+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{a^2}{16d(a\sin(c+dx)+a)^3}+\frac{7a}{128d(a-a\sin(c+dx))^2}+\frac{11a}{64d(a\sin(c+dx)+a)^2}+\frac{29}{128d(a-a\sin(c+dx))}+\frac{1}{2d(a\sin(c+dx)+a)}-\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{163\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-93*Log[1 - Sin[c + d*x]])/(256*a*d) + Log[Sin[c + d*x]]/(a*d) - (163*Log[
1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (7*a)/(1
28*d*(a - a*Sin[c + d*x])^2) + 29/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*
(a + a*Sin[c + d*x])^4) + a^2/(16*d*(a + a*Sin[c + d*x])^3) + (11*a)/(64*d*
(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^7\text{Subst}\left(\int \frac{a}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \frac{1}{(a-x)^4x(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^8\text{Subst}\left(\int \left(\frac{1}{32a^6(a-x)^4}+\frac{7}{64a^7(a-x)^3}+\frac{29}{128a^8(a-x)^2}+\frac{93}{256a^9(a-x)}+\frac{1}{a^9x}-\frac{1}{16a^5(a+\ldots}\right.\right.}{d}\\ &= -\frac{93\log(1-\sin(c+dx))}{256ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{163\log(1+\sin(c+dx))}{256ad}+\ldots\end{aligned}$$

Mathematica [A]

time = 6.12, size = 189, normalized size = 0.94

$$\frac{a^8\left(-\frac{93\log(1-\sin(c+dx))}{256a^9}+\frac{\log(\sin(c+dx))}{a^9}-\frac{163\log(1+\sin(c+dx))}{256a^9}+\frac{1}{96a^6(a-a\sin(c+dx))^3}+\frac{7}{128a^7(a-a\sin(c+dx))^2}+\frac{29}{128a^8(a-a\sin(c+dx))}+\frac{1}{64a^5(a+a\sin(c+dx))^4}+\frac{1}{16a^6(a+a\sin(c+dx))^3}+\frac{11}{64a^7(a+a\sin(c+dx))^2}+\frac{1}{2a^8(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^8*((-93*Log[1 - Sin[c + d*x]])/(256*a^9) + Log[Sin[c + d*x]]/a^9 - (163*
Log[1 + Sin[c + d*x]])/(256*a^9) + 1/(96*a^6*(a - a*Sin[c + d*x])^3) + 7/(1
28*a^7*(a - a*Sin[c + d*x])^2) + 29/(128*a^8*(a - a*Sin[c + d*x])) + 1/(64*
a^5*(a + a*Sin[c + d*x])^4) + 1/(16*a^6*(a + a*Sin[c + d*x])^3) + 11/(64*a^
7*(a + a*Sin[c + d*x])^2) + 1/(2*a^8*(a + a*Sin[c + d*x]))))/d
```

Maple [A]

time = 0.42, size = 122, normalized size = 0.60

method	result
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{11}{64(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}-\frac{163\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-\ldots}}{da}$
default	$\frac{\ln(\sin(dx+c))+\frac{1}{64(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{11}{64(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}-\frac{163\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-\ldots}}{da}$
risch	$\frac{i(174ie^{2i(dx+c)}+105\,e^{i(dx+c)}+105\,e^{13i(dx+c)}-730ie^{10i(dx+c)}-812ie^{8i(dx+c)}+812ie^{6i(dx+c)}+730ie^{4i(dx+c)}-174ie^{12i(dx\ldots}}{192\left(e^{i(dx+c)}+i\right)^8\left(e^{i(dx+c)}-i\right)^6d\ldots}$
norman	$\frac{-\frac{93\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}-\frac{93\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64da}+\frac{163\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{163\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{437\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{437\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}-\ldots}{\left(\tan\left(\ldots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(sin(d*x+c))+1/64/(1+sin(d*x+c))^4+1/16/(1+sin(d*x+c))^3+11/64/(1+
sin(d*x+c))^2+1/2/(1+sin(d*x+c))-163/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-
1)^3+7/128/(sin(d*x+c)-1)^2-29/128/(sin(d*x+c)-1)-93/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 187, normalized size = 0.93

$$\frac{\frac{2\left(105\sin(dx+c)^6-87\sin(dx+c)^5-472\sin(dx+c)^4+200\sin(dx+c)^3+711\sin(dx+c)^2-121\sin(dx+c)-400\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}-\frac{489\log(\sin(dx+c)+1)}{a}-\frac{279\log(\sin(dx+c)-1)}{a}+\frac{768\log(\sin(dx+c))}{a}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/768*(2*(105*sin(d*x + c)^6 - 87*sin(d*x + c)^5 - 472*sin(d*x + c)^4 + 200
*sin(d*x + c)^3 + 711*sin(d*x + c)^2 - 121*sin(d*x + c) - 400)/(a*sin(d*x +
 c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*si
n(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) - 489*log(sin(d*x +
 c) + 1)/a - 279*log(sin(d*x + c) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.43, size = 202, normalized size = 1.00

$$\frac{210\cos(dx+c)^6+314\cos(dx+c)^4+164\cos(dx+c)^2+768\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-489\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(\sin(dx+c)+1)-279\left(\cos(dx+c)^6\sin(dx+c)+\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)+2\left(87\cos(dx+c)^4+26\cos(dx+c)^2+8\right)\sin(dx+c)+112}{768\left(ad\cos(dx+c)^6\sin(dx+c)+ad\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(d*x + c)^6 + 314*cos(d*x + c)^4 + 164*cos(d*x + c)^2 + 768*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(1/2*sin(d*x + c)) - 489*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 279*(
cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8
7*cos(d*x + c)^4 + 26*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)/(a*d*cos(d*x
+ c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.49, size = 149, normalized size = 0.74

$$-\frac{\frac{1956 \log(|\sin(dx+c)+1|)}{a} + \frac{1116 \log(|\sin(dx+c)-1|)}{a} - \frac{3072 \log(|\sin(dx+c)|)}{a} - \frac{2\left(1023 \sin(dx+c)^3 - 3417 \sin(dx+c)^2 + 3849 \sin(dx+c) - 1471\right)}{a(\sin(dx+c)-1)^3} - \frac{4075 \sin(dx+c)^4 + 17836 \sin(dx+c)^3 + 29586 \sin(dx+c)^2 + 22156 \sin(dx+c) + 6379}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(1956*log(abs(sin(d*x + c) + 1))/a + 1116*log(abs(sin(d*x + c) - 1)
)/a - 3072*log(abs(sin(d*x + c)))/a - 2*(1023*sin(d*x + c)^3 - 3417*sin(d*x
 + c)^2 + 3849*sin(d*x + c) - 1471)/(a*(sin(d*x + c) - 1)^3) - (4075*sin(d*
x + c)^4 + 17836*sin(d*x + c)^3 + 29586*sin(d*x + c)^2 + 22156*sin(d*x + c)
 + 6379)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]

time = 0.16, size = 191, normalized size = 0.95

$$\frac{\ln(\sin(c+d\,x))}{a\,d} - \frac{163\,\ln(\sin(c+d\,x)+1)}{256\,a\,d} - \frac{93\,\ln(\sin(c+d\,x)-1)}{256\,a\,d} + \frac{-\frac{35\sin(c+d\,x)^6}{128} + \frac{29\sin(c+d\,x)^5}{128} + \frac{59\sin(c+d\,x)^4}{48} - \frac{25\sin(c+d\,x)^3}{48} - \frac{237\sin(c+d\,x)^2}{128} + \frac{121\,\sin(c+d\,x)}{384} + \frac{25}{24}}{d\,\left(-a\sin(c+d\,x)^7 - a\sin(c+d\,x)^6 + 3\,a\sin(c+d\,x)^5 + 3\,a\sin(c+d\,x)^4 - 3\,a\sin(c+d\,x)^3 - 3\,a\sin(c+d\,x)^2 + a\,\sin(c+d\,x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - (163*log(sin(c + d*x) + 1))/(256*a*d) - (93*log(s
in(c + d*x) - 1))/(256*a*d) + ((121*sin(c + d*x))/384 - (237*sin(c + d*x)^2
)/128 - (25*sin(c + d*x)^3)/48 + (59*sin(c + d*x)^4)/48 + (29*sin(c + d*x)^
5)/128 - (35*sin(c + d*x)^6)/128 + 25/24)/(d*(a + a*sin(c + d*x) - 3*a*sin(
c + d*x)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 -
 a*sin(c + d*x)^6 - a*sin(c + d*x)^7))
```

3.891 $\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=217

$$-\frac{\csc(c+dx)}{ad}-\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{443\log(1+\sin(c+dx))}{256ad}+\frac{a^2}{96d(a-a\sin(c+dx}$$

```
[Out] -csc(d*x+c)/a/d-187/256*ln(1-sin(d*x+c))/a/d-ln(sin(d*x+c))/a/d+443/256*ln(
1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+9/128*a/d/(a-a*sin(d*x+c))^
2+47/128/d/(a-a*sin(d*x+c))-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/12*a^2/d/(a+a*s
in(d*x+c))^3-19/64*a/d/(a+a*sin(d*x+c))^2-35/32/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.16, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{12d(a\sin(c+dx)+a)^3}+\frac{9a}{128d(a-a\sin(c+dx))^2}-\frac{19a}{64d(a\sin(c+dx)+a)^2}+\frac{47}{128d(a-a\sin(c+dx))}-\frac{35}{32d(a\sin(c+dx)+a)}-\frac{\csc(c+dx)}{ad}-\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{443\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (187*Log[1 - Sin[c + d*x]])/(256*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (443*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin
[c + d*x])^3) + (9*a)/(128*d*(a - a*Sin[c + d*x])^2) + 47/(128*d*(a - a*Sin
[c + d*x])) - a^3/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(12*d*(a + a*Sin[c +
d*x])^3) - (19*a)/(64*d*(a + a*Sin[c + d*x])^2) - 35/(32*d*(a + a*Sin[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^7\text{Subst}\left(\int \frac{a^2}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a^9\text{Subst}\left(\int \frac{1}{(a-x)^4x^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{a^9\text{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^4}+\frac{9}{64a^8(a-x)^3}+\frac{47}{128a^9(a-x)^2}+\frac{187}{256a^{10}(a-x)}+\frac{1}{a^9x^2}-\frac{1}{a^{10}x}\right.\right.}{} \\
&= -\frac{\csc(c+dx)}{ad}-\frac{187\log(1-\sin(c+dx))}{256ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{443\log(1-}{2}
\end{aligned}$$

Mathematica [A]
time = 6.10, size = 201, normalized size = 0.93

$$\frac{a^9\left(-\frac{\csc(c+dx)}{a^{10}}-\frac{187\log(1-\sin(c+dx))}{256a^{10}}-\frac{\log(\sin(c+dx))}{a^{10}}+\frac{443\log(1+\sin(c+dx))}{256a^{10}}+\frac{1}{96a^7(a-a\sin(c+dx))^3}+\frac{9}{128a^8(a-a\sin(c+dx))^2}+\frac{47}{128a^9(a-a\sin(c+dx))}-\frac{1}{64a^6(a+a\sin(c+dx))^4}-\frac{1}{12a^7(a+a\sin(c+dx))^3}-\frac{19}{64a^8(a+a\sin(c+dx))^2}-\frac{35}{32a^9(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^9*(-(Csc[c + d*x]/a^10) - (187*Log[1 - Sin[c + d*x]])/(256*a^10) - Log[S
in[c + d*x]]/a^10 + (443*Log[1 + Sin[c + d*x]])/(256*a^10) + 1/(96*a^7*(a -
 a*Sin[c + d*x])^3) + 9/(128*a^8*(a - a*Sin[c + d*x])^2) + 47/(128*a^9*(a -
 a*Sin[c + d*x])) - 1/(64*a^6*(a + a*Sin[c + d*x])^4) - 1/(12*a^7*(a + a*Si
n[c + d*x])^3) - 19/(64*a^8*(a + a*Sin[c + d*x])^2) - 35/(32*a^9*(a + a*Sin
[c + d*x]))))/d
```

Maple [A]
time = 0.31, size = 134, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))-\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{19}{64(1+\sin(dx+c))^2}-\frac{35}{32(1+\sin(dx+c))}+\frac{443\ln(1+\sin(dx+c))}{256}}{da}$
default	$\frac{-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))-\frac{1}{64(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{19}{64(1+\sin(dx+c))^2}-\frac{35}{32(1+\sin(dx+c))}+\frac{443\ln(1+\sin(dx+c))}{256}}{da}$
risch	$-\frac{i\left(1506i\,\mathrm{e}^{14i(dx+c)}+945\,\mathrm{e}^{15i(dx+c)}+7284i\,\mathrm{e}^{12i(dx+c)}+4233\,\mathrm{e}^{13i(dx+c)}+12574i\,\mathrm{e}^{10i(dx+c)}+6549\,\mathrm{e}^{11i(dx+c)}+6424i\,\mathrm{e}^{8i(dx-}\right.}{192\left(\mathrm{e}^{2i(dx+c)}-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)-ln(sin(d*x+c))-1/64/(1+sin(d*x+c))^4-1/12/(1+sin(d*x+c
))^3-19/64/(1+sin(d*x+c))^2-35/32/(1+sin(d*x+c))+443/256*ln(1+sin(d*x+c))-1
/96/(sin(d*x+c)-1)^3+9/128/(sin(d*x+c)-1)^2-47/128/(sin(d*x+c)-1)-187/256*l
n(sin(d*x+c)-1))
```

Maxima [**A**]
time = 0.29, size = 205, normalized size = 0.94

$$-\frac{\frac{2\left(945\,\sin\left(dx+c\right)^{7}+753\,\sin\left(dx+c\right)^{6}-2712\,\sin\left(dx+c\right)^{5}-2040\,\sin\left(dx+c\right)^{4}+2559\,\sin\left(dx+c\right)^{3}+1727\,\sin\left(dx+c\right)^{2}-784\,\sin\left(dx+c\right)-384\right)}{a\sin\left(dx+c\right)^{8}+a\sin\left(dx+c\right)^{7}-3\,a\sin\left(dx+c\right)^{6}-3\,a\sin\left(dx+c\right)^{5}+3\,a\sin\left(dx+c\right)^{4}+3\,a\sin\left(dx+c\right)^{3}-a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)}-\frac{1329\,\log\left(\sin\left(dx+c\right)+1\right)}{a}+\frac{561\,\log\left(\sin\left(dx+c\right)-1\right)}{a}+\frac{768\,\log\left(\sin\left(dx+c\right)\right)}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(945*sin(d*x + c)^7 + 753*sin(d*x + c)^6 - 2712*sin(d*x + c)^5 -
2040*sin(d*x + c)^4 + 2559*sin(d*x + c)^3 + 1727*sin(d*x + c)^2 - 784*sin(d
*x + c) - 384)/(a*sin(d*x + c)^8 + a*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 -
3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c)^3 - a*sin(d*x +
c)^2 - a*sin(d*x + c)) - 1329*log(sin(d*x + c) + 1)/a + 561*log(sin(d*x + c
) - 1)/a + 768*log(sin(d*x + c))/a)/d
```

Fricas [**A**]
time = 0.42, size = 258, normalized size = 1.19

$$\frac{1506\,\cos\left(dx+c\right)^{6}-438\,\cos\left(dx+c\right)^{4}-188\,\cos\left(dx+c\right)^{2}-768\left(\cos\left(dx+c\right)^{8}-\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-\cos\left(dx+c\right)^{6}\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+1329\left(\cos\left(dx+c\right)^{8}-\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-\cos\left(dx+c\right)^{6}\right)\log\left(\sin\left(dx+c\right)+1\right)-561\left(\cos\left(dx+c\right)^{8}-\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-\cos\left(dx+c\right)^{6}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(945\,\cos\left(dx+c\right)^{6}-123\,\cos\left(dx+c\right)^{4}-30\,\cos\left(dx+c\right)^{2}-8\right)\sin\left(dx+c\right)-112}{768\left(ad\cos\left(dx+c\right)^{8}-ad\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)-ad\cos\left(dx+c\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(1506*cos(d*x + c)^6 - 438*cos(d*x + c)^4 - 188*cos(d*x + c)^2 - 768*
(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x + c)^6)*log(1/2*sin
(d*x + c)) + 1329*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x + c) - cos(d*x +
 c)^6)*log(sin(d*x + c) + 1) - 561*(cos(d*x + c)^8 - cos(d*x + c)^6*sin(d*x
 + c) - cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(945*cos(d*x + c)^6 - 12
3*cos(d*x + c)^4 - 30*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
+ c)^8 - a*d*cos(d*x + c)^6*sin(d*x + c) - a*d*cos(d*x + c)^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 170, normalized size = 0.78

$$\frac{\frac{5316\,\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{2244\,\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}-\frac{3072\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{3072\,\left(\sin\left(dx+c\right)-1\right)}{a\sin\left(dx+c\right)}+\frac{2\,\left(2057\,\sin\left(dx+c\right)^{3}-6735\,\sin\left(dx+c\right)^{2}+7407\,\sin\left(dx+c\right)-2745\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{11075\,\sin\left(dx+c\right)^{4}+47660\,\sin\left(dx+c\right)^{3}+77442\,\sin\left(dx+c\right)^{2}+56460\,\sin\left(dx+c\right)+15651}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(5316*log(abs(sin(d*x + c) + 1))/a - 2244*log(abs(sin(d*x + c) - 1))
/a - 3072*log(abs(sin(d*x + c)))/a + 3072*(sin(d*x + c) - 1)/(a*sin(d*x + c
)) + 2*(2057*sin(d*x + c)^3 - 6735*sin(d*x + c)^2 + 7407*sin(d*x + c) - 274
5)/(a*(sin(d*x + c) - 1)^3) - (11075*sin(d*x + c)^4 + 47660*sin(d*x + c)^3
+ 77442*sin(d*x + c)^2 + 56460*sin(d*x + c) + 15651)/(a*(sin(d*x + c) + 1)^
4))/d
```

Mupad [B]
time = 9.33, size = 212, normalized size = 0.98

$$\frac{443\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{256\,a\,d}-\frac{187\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{256\,a\,d}-\frac{-\frac{315\,\sin\left(c+d\,x\right)^7}{128}-\frac{251\,\sin\left(c+d\,x\right)^6}{128}+\frac{113\,\sin\left(c+d\,x\right)^5}{16}+\frac{85\,\sin\left(c+d\,x\right)^4}{16}-\frac{853\,\sin\left(c+d\,x\right)^3}{128}-\frac{1727\,\sin\left(c+d\,x\right)^2}{384}+\frac{49\,\sin\left(c+d\,x\right)}{24}+1}{d\,\left(-a\,\sin\left(c+d\,x\right)^8-a\,\sin\left(c+d\,x\right)^7+3\,a\,\sin\left(c+d\,x\right)^6+3\,a\,\sin\left(c+d\,x\right)^5-3\,a\,\sin\left(c+d\,x\right)^4-3\,a\,\sin\left(c+d\,x\right)^3+a\,\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (443*log(sin(c + d*x) + 1))/(256*a*d) - (187*log(sin(c + d*x) - 1))/(256*a*
d) - ((49*sin(c + d*x))/24 - (1727*sin(c + d*x)^2)/384 - (853*sin(c + d*x)^
3)/128 + (85*sin(c + d*x)^4)/16 + (113*sin(c + d*x)^5)/16 - (251*sin(c + d*
x)^6)/128 - (315*sin(c + d*x)^7)/128 + 1)/(d*(a*sin(c + d*x) + a*sin(c + d*
x)^2 - 3*a*sin(c + d*x)^3 - 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 + 3*a*s
in(c + d*x)^6 - a*sin(c + d*x)^7 - a*sin(c + d*x)^8)) - log(sin(c + d*x))/(
a*d)
```

3.892 $\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{325\log(1-\sin(c+dx))}{256ad}+\frac{5\log(\sin(c+dx))}{ad}-\frac{955\log(1+\sin(c+dx))}{256ad}+\frac{}{96d(a}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-325/256*ln(1-sin(d*x+c))/a/d+5*ln(sin(d
*x+c))/a/d-955/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d/(a-a*sin(d*x+c))^3+11/12
8*a/d/(a-a*sin(d*x+c))^2+69/128/d/(a-a*sin(d*x+c))+1/64*a^3/d/(a+a*sin(d*x+
c))^4+5/48*a^2/d/(a+a*sin(d*x+c))^3+29/64*a/d/(a+a*sin(d*x+c))^2+2/d/(a+a*s
in(d*x+c))
```

Rubi [A]

time = 0.16, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}+\frac{5a^2}{48d(a\sin(c+dx)+a)^3}+\frac{11a}{128d(a-a\sin(c+dx))^2}+\frac{29a}{64d(a\sin(c+dx)+a)^2}+\frac{69}{128d(a-a\sin(c+dx))}+\frac{2}{d(a\sin(c+dx)+a)}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{325\log(1-\sin(c+dx))}{256ad}+\frac{5\log(\sin(c+dx))}{ad}-\frac{955\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (325*Log[1 - Sin[c + d*x]])/(
256*a*d) + (5*Log[Sin[c + d*x]])/(a*d) - (955*Log[1 + Sin[c + d*x]])/(256*a
*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (11*a)/(128*d*(a - a*Sin[c + d*x]
)^2) + 69/(128*d*(a - a*Sin[c + d*x])) + a^3/(64*d*(a + a*Sin[c + d*x])^4)
+ (5*a^2)/(48*d*(a + a*Sin[c + d*x])^3) + (29*a)/(64*d*(a + a*Sin[c + d*x])
^2) + 2/(d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{a^3}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\text{Subst}\left(\int \frac{1}{(a-x)^4x^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{10}\text{Subst}\left(\int \left(\frac{1}{32a^8(a-x)^4} + \frac{11}{64a^9(a-x)^3} + \frac{69}{128a^{10}(a-x)^2} + \frac{325}{256a^{11}(a-x)} + \frac{1}{a^9x^3} - \frac{1}{a^{10}\ldots}\right.\right.}{}$$

$$= \frac{\csc(c+dx)}{ad} - \frac{\csc^2(c+dx)}{2ad} - \frac{325\log(1-\sin(c+dx))}{256ad} + \frac{5\log(\sin(c+dx)\ldots}{ad}$$

Mathematica [A]

time = 6.14, size = 213, normalized size = 0.92

$$\frac{a^{10}\left(\frac{\csc(c+dx)}{a^{11}} - \frac{\csc^2(c+dx)}{2a^{11}} - \frac{325\log(1-\sin(c+dx))}{256a^{11}} + \frac{5\log(\sin(c+dx))}{a^{11}} - \frac{955\log(1+\sin(c+dx))}{256a^{11}} + \frac{1}{96a^8(a-a\sin(c+dx))^3} + \frac{11}{128a^9(a-a\sin(c+dx))^2} + \frac{69}{128a^{10}(a-a\sin(c+dx))} + \frac{1}{64a^7(a+a\sin(c+dx))^4} + \frac{5}{48a^8(a+a\sin(c+dx))^3} + \frac{29}{64a^9(a+a\sin(c+dx))^2} + \frac{2}{a^{10}(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*(Csc[c + d*x]/a^11 - Csc[c + d*x]^2/(2*a^11) - (325*Log[1 - Sin[c + d
*x]])/(256*a^11) + (5*Log[Sin[c + d*x]])/a^11 - (955*Log[1 + Sin[c + d*x]])
/(256*a^11) + 1/(96*a^8*(a - a*Sin[c + d*x])^3) + 11/(128*a^9*(a - a*Sin[c
+ d*x])^2) + 69/(128*a^10*(a - a*Sin[c + d*x])) + 1/(64*a^7*(a + a*Sin[c +
d*x])^4) + 5/(48*a^8*(a + a*Sin[c + d*x])^3) + 29/(64*a^9*(a + a*Sin[c + d*
x])^2) + 2/(a^10*(a + a*Sin[c + d*x]))))/d
```

Maple [A]

time = 0.50, size = 142, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}+5\ln(\sin(dx+c))+\frac{1}{64(1+\sin(dx+c))^4}+\frac{5}{48(1+\sin(dx+c))^3}+\frac{29}{64(1+\sin(dx+c))^2}+\frac{2}{1+\sin(dx+c)}-\frac{955\ln(\ldots}{}}{da}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}+5\ln(\sin(dx+c))+\frac{1}{64(1+\sin(dx+c))^4}+\frac{5}{48(1+\sin(dx+c))^3}+\frac{29}{64(1+\sin(dx+c))^2}+\frac{2}{1+\sin(dx+c)}-\frac{955\ln(\ldots}{}}{da}$
risch	$\frac{i\left(-1170i\,e^{4i(dx+c)}+945\,e^{i(dx+c)}+9512\,e^{11i(dx+c)}+6360\,e^{15i(dx+c)}+14604\,e^{13i(dx+c)}-4602i\,e^{8i(dx+c)}-4778i\,e^{6i(dx+c)}-\ldots\right.}{192\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-1/2/sin(d*x+c)^2+1/sin(d*x+c)+5*ln(sin(d*x+c))+1/64/(1+sin(d*x+c))^4+5/48/(1+sin(d*x+c))^3+29/64/(1+sin(d*x+c))^2+2/(1+sin(d*x+c))-955/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3+11/128/(sin(d*x+c)-1)^2-69/128/(sin(d*x+c)-1)-325/256*ln(sin(d*x+c)-1))

Maxima [A]
time = 0.31, size = 217, normalized size = 0.94

$$\frac{\frac{2\left(945\sin(dx+c)^8-15\sin(dx+c)^7-3480\sin(dx+c)^6-120\sin(dx+c)^5+4479\sin(dx+c)^4+319\sin(dx+c)^3-2192\sin(dx+c)^2-192\sin(dx+c)+192\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-3a\sin(dx+c)^7-3a\sin(dx+c)^6+3a\sin(dx+c)^5+3a\sin(dx+c)^4-a\sin(dx+c)^3-a\sin(dx+c)^2}-\frac{2865\log(\sin(dx+c)+1)}{a}-\frac{975\log(\sin(dx+c)-1)}{a}+\frac{3840\log(\sin(dx+c))}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/768*(2*(945*sin(d*x + c)^8 - 15*sin(d*x + c)^7 - 3480*sin(d*x + c)^6 - 120*sin(d*x + c)^5 + 4479*sin(d*x + c)^4 + 319*sin(d*x + c)^3 - 2192*sin(d*x + c)^2 - 192*sin(d*x + c) + 192)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 3*a*sin(d*x + c)^7 - 3*a*sin(d*x + c)^6 + 3*a*sin(d*x + c)^5 + 3*a*sin(d*x + c)^4 - a*sin(d*x + c)^3 - a*sin(d*x + c)^2) - 2865*log(sin(d*x + c) + 1)/a - 975*log(sin(d*x + c) - 1)/a + 3840*log(sin(d*x + c))/a)/d

Fricas [A]
time = 0.40, size = 311, normalized size = 1.34

$$\frac{1890\cos(dx+c)^8-600\cos(dx+c)^6-582\cos(dx+c)^4-212\cos(dx+c)^2+3840\left(\cos(dx+c)^8-\cos(dx+c)^6+\left(\cos(dx+c)^8-\cos(dx+c)^6\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-2865\left(\cos(dx+c)^8-\cos(dx+c)^6+\left(\cos(dx+c)^8-\cos(dx+c)^6\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-975\left(\cos(dx+c)^8-\cos(dx+c)^6+\left(\cos(dx+c)^8-\cos(dx+c)^6\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(15\cos(dx+c)^6-165\cos(dx+c)^4-34\cos(dx+c)^2-8\right)\sin(dx+c)-112}{768\left(ad\cos(dx+c)^8-ad\cos(dx+c)^6+\left(ad\cos(dx+c)^8-ad\cos(dx+c)^6\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/768*(1890*cos(d*x + c)^8 - 600*cos(d*x + c)^6 - 582*cos(d*x + c)^4 - 212*cos(d*x + c)^2 + 3840*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) - 2865*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(d*x + c) + 1) - 975*(cos(d*x + c)^8 - cos(d*x + c)^6 + (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 165*cos(d*x + c)^4 - 34*cos(d*x + c)^2 - 8)*sin(d*x + c) - 112)/(a*d*cos(d*x + c)^8 - a*d*cos(d*x + c)^6 + (a*d*cos(d*x + c)^8 - a*d*cos(d*x + c)^6)*sin(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 182, normalized size = 0.78

$$-\frac{\frac{11460\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a}+\frac{3900\,\log\left(|\sin\left(dx+c\right)-1|\right)}{a}-\frac{15360\,\log\left(|\sin\left(dx+c\right)|\right)}{a}+\frac{1536\,\left(15\,\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)}{a\sin\left(dx+c\right)^{2}}-\frac{2\,\left(3575\,\sin\left(dx+c\right)^{3}-11553\,\sin\left(dx+c\right)^{2}+12513\,\sin\left(dx+c\right)-4551\right)}{a\left(\sin\left(dx+c\right)-1\right)^{3}}-\frac{23875\,\sin\left(dx+c\right)^{4}+101644\,\sin\left(dx+c\right)^{3}+163074\,\sin\left(dx+c\right)^{2}+117036\,\sin\left(dx+c\right)+31779}{a\left(\sin\left(dx+c\right)+1\right)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(11460*log(abs(sin(d*x + c) + 1))/a + 3900*log(abs(sin(d*x + c) - 1
))/a - 15360*log(abs(sin(d*x + c)))/a + 1536*(15*sin(d*x + c)^2 - 2*sin(d*x
 + c) + 1)/(a*sin(d*x + c)^2) - 2*(3575*sin(d*x + c)^3 - 11553*sin(d*x + c)
^2 + 12513*sin(d*x + c) - 4551)/(a*(sin(d*x + c) - 1)^3) - (23875*sin(d*x +
 c)^4 + 101644*sin(d*x + c)^3 + 163074*sin(d*x + c)^2 + 117036*sin(d*x + c)
 + 31779)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 9.24, size = 223, normalized size = 0.96

$$\frac{5\,\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{955\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{256\,a\,d}-\frac{325\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{256\,a\,d}+\frac{-\frac{315\sin(c+d\,x)^8}{128}+\frac{5\sin(c+d\,x)^7}{128}+\frac{145\sin(c+d\,x)^6}{16}+\frac{5\sin(c+d\,x)^5}{16}-\frac{1493\sin(c+d\,x)^4}{128}-\frac{319\sin(c+d\,x)^3}{384}+\frac{137\sin(c+d\,x)^2}{24}+\frac{\sin(c+d\,x)}{2}-\frac{1}{2}}{d\,\left(-a\sin\left(c+d\,x\right)^9-a\sin\left(c+d\,x\right)^8+3\,a\sin\left(c+d\,x\right)^7+3\,a\sin\left(c+d\,x\right)^6-3\,a\sin\left(c+d\,x\right)^5-3\,a\sin\left(c+d\,x\right)^4+a\sin\left(c+d\,x\right)^3+a\sin\left(c+d\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (5*log(sin(c + d*x)))/(a*d) - (955*log(sin(c + d*x) + 1))/(256*a*d) - (325*
log(sin(c + d*x) - 1))/(256*a*d) + (sin(c + d*x)/2 + (137*sin(c + d*x)^2)/2
4 - (319*sin(c + d*x)^3)/384 - (1493*sin(c + d*x)^4)/128 + (5*sin(c + d*x)^
5)/16 + (145*sin(c + d*x)^6)/16 + (5*sin(c + d*x)^7)/128 - (315*sin(c + d*x
)^8)/128 - 1/2)/(d*(a*sin(c + d*x)^2 + a*sin(c + d*x)^3 - 3*a*sin(c + d*x)^
4 - 3*a*sin(c + d*x)^5 + 3*a*sin(c + d*x)^6 + 3*a*sin(c + d*x)^7 - a*sin(c
+ d*x)^8 - a*sin(c + d*x)^9))
```

3.893 $\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=253

$$-\frac{5\csc(c+dx)}{ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{\csc^3(c+dx)}{3ad}-\frac{515\log(1-\sin(c+dx))}{256ad}-\frac{5\log(\sin(c+dx))}{ad}+\frac{1795\log(1+s$$

```
[Out] -5*csc(d*x+c)/a/d+1/2*csc(d*x+c)^2/a/d-1/3*csc(d*x+c)^3/a/d-515/256*ln(1-si
n(d*x+c))/a/d-5*ln(sin(d*x+c))/a/d+1795/256*ln(1+sin(d*x+c))/a/d+1/96*a^2/d
/(a-a*sin(d*x+c))^3+13/128*a/d/(a-a*sin(d*x+c))^2+95/128/d/(a-a*sin(d*x+c))
-1/64*a^3/d/(a+a*sin(d*x+c))^4-1/8*a^2/d/(a+a*sin(d*x+c))^3-41/64*a/d/(a+a*
sin(d*x+c))^2-105/32/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^3}{64d(a\sin(c+dx)+a)^4}+\frac{a^2}{96d(a-a\sin(c+dx))^3}-\frac{a^2}{8d(a\sin(c+dx)+a)^3}+\frac{13a}{128d(a-a\sin(c+dx))^2}-\frac{41a}{64d(a\sin(c+dx)+a)^2}+\frac{95}{128d(a-a\sin(c+dx))}-\frac{105}{32d(a\sin(c+dx)+a)}-\frac{\csc^3(c+dx)}{3ad}+\frac{\csc^2(c+dx)}{2ad}-\frac{5\csc(c+dx)}{ad}-\frac{515\log(1-\sin(c+dx))}{256ad}-\frac{5\log(\sin(c+dx))}{ad}+\frac{1795\log(\sin(c+dx)+1)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*Csc[c + d*x])/(a*d) + Csc[c + d*x]^2/(2*a*d) - Csc[c + d*x]^3/(3*a*d) -
 (515*Log[1 - Sin[c + d*x]])/(256*a*d) - (5*Log[Sin[c + d*x]])/(a*d) + (179
5*Log[1 + Sin[c + d*x]])/(256*a*d) + a^2/(96*d*(a - a*Sin[c + d*x])^3) + (1
3*a)/(128*d*(a - a*Sin[c + d*x])^2) + 95/(128*d*(a - a*Sin[c + d*x])) - a^3
/(64*d*(a + a*Sin[c + d*x])^4) - a^2/(8*d*(a + a*Sin[c + d*x])^3) - (41*a)/
(64*d*(a + a*Sin[c + d*x])^2) - 105/(32*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)\sec^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{a^4}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\text{Subst}\left(\int \frac{1}{(a-x)^4x^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^{11}\text{Subst}\left(\int \left(\frac{1}{32a^9(a-x)^4} + \frac{13}{64a^{10}(a-x)^3} + \frac{95}{128a^{11}(a-x)^2} + \frac{515}{256a^{12}(a-x)} + \frac{1}{a^9x^4} - \frac{}{a^1}\right.\right.}{}$$

$$= -\frac{5\csc(c+dx)}{ad} + \frac{\csc^2(c+dx)}{2ad} - \frac{\csc^3(c+dx)}{3ad} - \frac{515\log(1-\sin(c+dx))}{256ad}$$

Mathematica [A]

time = 6.11, size = 231, normalized size = 0.91

$$\frac{a^{11}\left(-\frac{5\csc(c+dx)}{a^{12}} + \frac{\csc^2(c+dx)}{2a^{12}} - \frac{\csc^3(c+dx)}{3a^{12}} - \frac{515\log(1-\sin(c+dx))}{256a^{12}} - \frac{5\log(\sin(c+dx))}{a^{12}} + \frac{1795\log(1+\sin(c+dx))}{256a^{12}} + \frac{1}{96a^9(a-a\sin(c+dx))^3} + \frac{13}{128a^{10}(a-a\sin(c+dx))^2} + \frac{95}{128a^{11}(a-a\sin(c+dx))} - \frac{1}{64a^8(a+a\sin(c+dx))^4} - \frac{1}{8a^9(a+a\sin(c+dx))^3} - \frac{41}{64a^{10}(a+a\sin(c+dx))^2} - \frac{105}{32a^{11}(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^4*Sec[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*((-5*Csc[c + d*x])/a^12 + Csc[c + d*x]^2/(2*a^12) - Csc[c + d*x]^3/(3
*a^12) - (515*Log[1 - Sin[c + d*x]])/(256*a^12) - (5*Log[Sin[c + d*x]])/a^1
2 + (1795*Log[1 + Sin[c + d*x]])/(256*a^12) + 1/(96*a^9*(a - a*Sin[c + d*x]
)^3) + 13/(128*a^10*(a - a*Sin[c + d*x])^2) + 95/(128*a^11*(a - a*Sin[c + d
*x])) - 1/(64*a^8*(a + a*Sin[c + d*x])^4) - 1/(8*a^9*(a + a*Sin[c + d*x])^3
) - 41/(64*a^10*(a + a*Sin[c + d*x])^2) - 105/(32*a^11*(a + a*Sin[c + d*x])
)))/d
```

Maple [A]

time = 0.76, size = 154, normalized size = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{3\sin(dx+c)^3} + \frac{1}{2\sin(dx+c)^2} - \frac{5}{\sin(dx+c)} - 5\ln(\sin(dx+c)) - \frac{1}{64(1+\sin(dx+c))^4} - \frac{1}{8(1+\sin(dx+c))^3} - \frac{41}{64(1+\sin(dx+c))^2} - \frac{10}{32(1+\sin}}{da}$
default	$\frac{-\frac{1}{3\sin(dx+c)^3} + \frac{1}{2\sin(dx+c)^2} - \frac{5}{\sin(dx+c)} - 5\ln(\sin(dx+c)) - \frac{1}{64(1+\sin(dx+c))^4} - \frac{1}{8(1+\sin(dx+c))^3} - \frac{41}{64(1+\sin(dx+c))^2} - \frac{10}{32(1+\sin}}{da}$

risch	$-\frac{i\left(14640 i e^{4 i(dx+c)}+3465\, e^{19 i(dx+c)}-38192 i e^{8 i(dx+c)}+9615\, e^{17 i(dx+c)}+5010 i e^{18 i(dx+c)}-492\, e^{15 i(dx+c)}-424 i e^{6 i(dx-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2-5/sin(d*x+c)-5*ln(sin(d*x+c))-1/6
4/(1+sin(d*x+c))^4-1/8/(1+sin(d*x+c))^3-41/64/(1+sin(d*x+c))^2-105/32/(1+si
n(d*x+c))+1795/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3+13/128/(sin(d*x+c
)-1)^2-95/128/(sin(d*x+c)-1)-515/256*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 227, normalized size = 0.90

$$-\frac{\frac{2\left(3465 \sin(dx+c)^{9}+2505 \sin(dx+c)^{8}-10200 \sin(dx+c)^{7}-6840 \sin(dx+c)^{6}+10023 \sin(dx+c)^{5}+5863 \sin(dx+c)^{4}-3344 \sin(dx+c)^{3}-1344 \sin(dx+c)^{2}+64 \sin(dx+c)-128\right)}{a \sin(dx+c)^{10}+a \sin(dx+c)^{9}-3 a \sin(dx+c)^{8}-3 a \sin(dx+c)^{7}+3 a \sin(dx+c)^{6}+3 a \sin(dx+c)^{5}-a \sin(dx+c)^{4}-a \sin(dx+c)^{3}}-\frac{5385 \log(\sin(dx+c)+1)}{a}+\frac{1545 \log(\sin(dx+c)-1)}{a}+\frac{3840 \log(\sin(dx+c))}{a}}{768\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(3465*sin(d*x + c)^9 + 2505*sin(d*x + c)^8 - 10200*sin(d*x + c)^7
 - 6840*sin(d*x + c)^6 + 10023*sin(d*x + c)^5 + 5863*sin(d*x + c)^4 - 3344*
sin(d*x + c)^3 - 1344*sin(d*x + c)^2 + 64*sin(d*x + c) - 128)/(a*sin(d*x +
c)^10 + a*sin(d*x + c)^9 - 3*a*sin(d*x + c)^8 - 3*a*sin(d*x + c)^7 + 3*a*si
n(d*x + c)^6 + 3*a*sin(d*x + c)^5 - a*sin(d*x + c)^4 - a*sin(d*x + c)^3) -
5385*log(sin(d*x + c) + 1)/a + 1545*log(sin(d*x + c) - 1)/a + 3840*log(sin(
d*x + c))/a)/d
```

Fricas [A]
time = 0.43, size = 360, normalized size = 1.42

$$\frac{5010 \cos(dx+c)^{8}-6360 \cos(dx+c)^{6}+746 \cos(dx+c)^{4}+236 \cos(dx+c)^{2}-3840\left(\cos(dx+c)^{10}-2 \cos(dx+c)^{8}+\cos(dx+c)^{6}-\left(\cos(dx+c)^{8}-\cos(dx+c)^{6}\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+5385\left(\cos(dx+c)^{10}-2 \cos(dx+c)^{8}+\cos(dx+c)^{6}-\left(\cos(dx+c)^{8}-\cos(dx+c)^{6}\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-1545\left(\cos(dx+c)^{10}-2 \cos(dx+c)^{8}+\cos(dx+c)^{6}-\left(\cos(dx+c)^{8}-\cos(dx+c)^{6}\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(3465 \cos(dx+c)^{8}-3660 \cos(dx+c)^{6}+213 \cos(dx+c)^{4}+38 \cos(dx+c)^{2}+8\right)\sin(dx+c)+112}{768\left(ad\cos(dx+c)^{10}-2 ad\cos(dx+c)^{8}+ad\cos(dx+c)^{6}-\left(ad\cos(dx+c)^{8}-ad\cos(dx+c)^{6}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/768*(5010*cos(d*x + c)^8 - 6360*cos(d*x + c)^6 + 746*cos(d*x + c)^4 + 236
*cos(d*x + c)^2 - 3840*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6
 - (cos(d*x + c)^8 - cos(d*x + c)^6)*sin(d*x + c))*log(1/2*sin(d*x + c)) +
5385*(cos(d*x + c)^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8
 - cos(d*x + c)^6)*sin(d*x + c))*log(sin(d*x + c) + 1) - 1545*(cos(d*x + c)
^10 - 2*cos(d*x + c)^8 + cos(d*x + c)^6 - (cos(d*x + c)^8 - cos(d*x + c)^6)
*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(3465*cos(d*x + c)^8 - 3660*cos(d
*x + c)^6 + 213*cos(d*x + c)^4 + 38*cos(d*x + c)^2 + 8)*sin(d*x + c) + 112)
/(a*d*cos(d*x + c)^10 - 2*a*d*cos(d*x + c)^8 + a*d*cos(d*x + c)^6 - (a*d*co
s(d*x + c)^8 - a*d*cos(d*x + c)^6)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 187, normalized size = 0.74

$$\frac{\frac{21540 \log\left(|\sin(dx+c)+1|\right)}{a} - \frac{6180 \log\left(|\sin(dx+c)-1|\right)}{a} - \frac{15360 \log\left(|\sin(dx+c)|\right)}{a} + \frac{19745 \sin(dx+c)^6 - 76875 \sin(dx+c)^5 + 111723 \sin(dx+c)^4 - 74081 \sin(dx+c)^3 + 23040 \sin(dx+c)^2 - 4608 \sin(dx+c) + 1024}{\left(\sin(dx+c)^2 - \sin(dx+c)\right)^3 a} - \frac{44875 \sin(dx+c)^4 + 189580 \sin(dx+c)^3 + 301458 \sin(dx+c)^2 + 214060 \sin(dx+c) + 57355}{a(\sin(dx+c)+1)^4}}{3072\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/3072*(21540*log(abs(sin(d*x + c) + 1))/a - 6180*log(abs(sin(d*x + c) - 1)
)/a - 15360*log(abs(sin(d*x + c)))/a + (19745*sin(d*x + c)^6 - 76875*sin(d*
x + c)^5 + 111723*sin(d*x + c)^4 - 74081*sin(d*x + c)^3 + 23040*sin(d*x + c
)^2 - 4608*sin(d*x + c) + 1024)/((sin(d*x + c)^2 - sin(d*x + c))^3*a) - (44
875*sin(d*x + c)^4 + 189580*sin(d*x + c)^3 + 301458*sin(d*x + c)^2 + 214060
*sin(d*x + c) + 57355)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]
time = 9.26, size = 233, normalized size = 0.92

$$\frac{\frac{1155 \sin(c+d\,x)^9}{128} + \frac{835 \sin(c+d\,x)^8}{128} - \frac{425 \sin(c+d\,x)^7}{16} - \frac{285 \sin(c+d\,x)^6}{16} + \frac{3341 \sin(c+d\,x)^5}{128} + \frac{5863 \sin(c+d\,x)^4}{384} - \frac{209 \sin(c+d\,x)^3}{24} - \frac{7 \sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{6} - \frac{1}{3}}{d\,\left(-a \sin\left(c+d\,x\right)^{10} - a \sin\left(c+d\,x\right)^9 + 3\,a \sin\left(c+d\,x\right)^8 + 3\,a \sin\left(c+d\,x\right)^7 - 3\,a \sin\left(c+d\,x\right)^6 - 3\,a \sin\left(c+d\,x\right)^5 + a \sin\left(c+d\,x\right)^4 + a \sin\left(c+d\,x\right)^3\right)} - \frac{515 \ln\left(\sin\left(c+d\,x\right)-1\right)}{256\,a\,d} + \frac{1795 \ln\left(\sin\left(c+d\,x\right)+1\right)}{256\,a\,d} - \frac{5 \ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*sin(c + d*x)^4*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/6 - (7*sin(c + d*x)^2)/2 - (209*sin(c + d*x)^3)/24 + (5863*si
n(c + d*x)^4)/384 + (3341*sin(c + d*x)^5)/128 - (285*sin(c + d*x)^6)/16 - (
425*sin(c + d*x)^7)/16 + (835*sin(c + d*x)^8)/128 + (1155*sin(c + d*x)^9)/1
28 - 1/3)/(d*(a*sin(c + d*x)^3 + a*sin(c + d*x)^4 - 3*a*sin(c + d*x)^5 - 3*
a*sin(c + d*x)^6 + 3*a*sin(c + d*x)^7 + 3*a*sin(c + d*x)^8 - a*sin(c + d*x)
^9 - a*sin(c + d*x)^10)) - (515*log(sin(c + d*x) - 1))/(256*a*d) + (1795*lo
g(sin(c + d*x) + 1))/(256*a*d) - (5*log(sin(c + d*x)))/(a*d)
```

3.894 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{a^2\sec^3(c+dx)}{3d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\tan^7(c+dx)}{7d}$$

```
[Out] 1/3*a^2*sec(d*x+c)^3/d-3/5*a^2*sec(d*x+c)^5/d+2/7*a^2*sec(d*x+c)^7/d+2/5*a^
2*tan(d*x+c)^5/d+2/7*a^2*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2952, 2686, 14, 2687, 276}

$$\frac{2a^2\tan^7(c+dx)}{7d}+\frac{2a^2\tan^5(c+dx)}{5d}+\frac{2a^2\sec^7(c+dx)}{7d}-\frac{3a^2\sec^5(c+dx)}{5d}+\frac{a^2\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*Sec[c + d*x]^3)/(3*d) - (3*a^2*Sec[c + d*x]^5)/(5*d) + (2*a^2*Sec[c +
d*x]^7)/(7*d) + (2*a^2*Tan[c + d*x]^5)/(5*d) + (2*a^2*Tan[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2952

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \int \left(a^2\sec^5(c+dx)\tan^3(c+dx)+2a^2\sec^4(c+dx)\tan\right.$$
$$= a^2\int \sec^5(c+dx)\tan^3(c+dx)\,dx + a^2\int \sec^3(c+dx)$$
$$= \frac{a^2\text{Subst}\left(\int x^4(-1+x^2)\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Subst}}{}$$
$$= \frac{a^2\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sec(c+dx)\right)}{d} + \frac{a^2\text{Su}}{}$$
$$= \frac{a^2\sec^3(c+dx)}{3d} - \frac{3a^2\sec^5(c+dx)}{5d} + \frac{2a^2\sec^7(c+dx)}{7d}$$

Mathematica [A]
time = 0.73, size = 139, normalized size = 1.53

$$-\frac{a^2\sec^7(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(-672+182\cos(c+dx)+736\cos(2(c+dx))+39\cos(3(c+dx))-192\cos(4(c+dx))-13\cos(5(c+dx))+448\sin(c+dx)-104\sin(2(c+dx))-144\sin(3(c+dx))-52\sin(4(c+dx))+48\sin(5(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -1/6720*(a^2*Sec[c + d*x]^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-672 +
 182*Cos[c + d*x] + 736*Cos[2*(c + d*x)] + 39*Cos[3*(c + d*x)] - 192*Cos[4*
(c + d*x)] - 13*Cos[5*(c + d*x)] + 448*Sin[c + d*x] - 104*Sin[2*(c + d*x)]
- 144*Sin[3*(c + d*x)] - 52*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(81) = 162$.
time = 0.18, size = 248, normalized size = 2.73

method	result
risch	$\frac{8a^2\left(-35i\,e^{6i(dx+c)}+35\,e^{7i(dx+c)}-7i\,e^{4i(dx+c)}-42\,e^{5i(dx+c)}-9i\,e^{2i(dx+c)}+11\,e^{3i(dx+c)}+3i-12\,e^{i(dx+c)}\right)}{105\left(e^{i(dx+c)}+i\right)^3\left(e^{i(dx+c)}-i\right)^7 d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+2a^2\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(c\right.}{35\cos(d}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^4(dx+c)}{7\cos(dx+c)^7}+\frac{3\left(\sin^4(dx+c)\right)}{35\cos(dx+c)^5}+\frac{\sin^4(dx+c)}{35\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{35\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{35}\right)+2a^2\left(\frac{\sin^5(dx+c)}{7\cos(dx+c)^7}+\frac{2\left(\sin^5(c\right.}{35\cos(d}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35
*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^
2)*cos(d*x+c))+2*a^2*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d
*x+c)^5)+a^2*(1/7*sin(d*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5-
1/105*sin(d*x+c)^6/cos(d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.31, size = 91, normalized size = 1.00

$$\frac{6\left(5\tan\left(dx+c\right)^7+7\tan\left(dx+c\right)^5\right)a^2+\frac{\left(35\cos(dx+c)^4-42\cos(dx+c)^2+15\right)a^2}{\cos(dx+c)^7}-\frac{3\left(7\cos(dx+c)^2-5\right)a^2}{\cos(dx+c)^7}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/105*(6*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a^2 + (35*cos(d*x + c)^4 - 4
2*cos(d*x + c)^2 + 15)*a^2/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*a^2/co
s(d*x + c)^7)/d
```

Fricas [A]
time = 0.37, size = 115, normalized size = 1.26

$$-\frac{24\,a^2\cos\left(dx+c\right)^4-47\,a^2\cos\left(dx+c\right)^2+25\,a^2-2\left(6\,a^2\cos\left(dx+c\right)^4-9\,a^2\cos\left(dx+c\right)^2+5\,a^2\right)\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^5+2\,d\cos\left(dx+c\right)^3\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/105*(24*a^2*cos(d*x + c)^4 - 47*a^2*cos(d*x + c)^2 + 25*a^2 - 2*(6*a^2*c
os(d*x + c)^4 - 9*a^2*cos(d*x + c)^2 + 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)
^5 + 2*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*sin(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 138, normalized size = 1.52

$$-\frac{\frac{35\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}-\frac{105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1015\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1302\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+469\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-67\,a^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*sin(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(35*(3*a^2*tan(1/2*d*x + 1/2*c) + a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3
- (105*a^2*tan(1/2*d*x + 1/2*c)^5 - 1015*a^2*tan(1/2*d*x + 1/2*c)^4 + 1330*
a^2*tan(1/2*d*x + 1/2*c)^3 - 1302*a^2*tan(1/2*d*x + 1/2*c)^2 + 469*a^2*tan(
1/2*d*x + 1/2*c) - 67*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

Mupad [**B**]
time = 13.52, size = 215, normalized size = 2.36

$$\frac{4\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+91\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-84\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+105\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\right)}{105\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^7\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + a*sin(c + d*x))^2)/cos(c + d*x)^8,x)
```

```
[Out] (4*a^2*cos(c/2 + (d*x)/2)^3*(cos(c/2 + (d*x)/2)^7 + 105*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^6 - 4*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2) - 84*cos(c
/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^5 + 91*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d
*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^3 + 3*cos(c/2 + (d*x)/
2)^5*sin(c/2 + (d*x)/2)^2))/(105*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)
)^7*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^3)
```

3.895 $\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{843\log(1-\sin(c+dx))}{512ad}-\frac{2229\log(1+\sin(c+dx))}{512ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{a^3}{256d(a-a\sin(c+dx)}$$

```
[Out] -843/512*ln(1-sin(d*x+c))/a/d-2229/512*ln(1+sin(d*x+c))/a/d+sin(d*x+c)/a/d-
1/2*sin(d*x+c)^2/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4-3/64*a^2/d/(a-a*sin(d*x
+c))^3+141/512*a/d/(a-a*sin(d*x+c))^2-39/32/d/(a-a*sin(d*x+c))-1/160*a^4/d/
(a+a*sin(d*x+c))^5+19/256*a^3/d/(a+a*sin(d*x+c))^4-53/128*a^2/d/(a+a*sin(d*
x+c))^3+765/512*a/d/(a+a*sin(d*x+c))^2-1155/256/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{19a^3}{256d(a\sin(c+dx)+a)^4}-\frac{3a^2}{64d(a-a\sin(c+dx))^3}-\frac{53a^2}{128d(a\sin(c+dx)+a)^3}-\frac{\sin^2(c+dx)}{2ad}+\frac{141a}{512d(a-a\sin(c+dx))^2}+\frac{765a}{512d(a\sin(c+dx)+a)^2}-\frac{39}{32d(a-a\sin(c+dx))}-\frac{1155}{256d(a\sin(c+dx)+a)}+\frac{\sin(c+dx)}{ad}-\frac{843\log(1-\sin(c+dx))}{512ad}-\frac{2229\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-843*Log[1 - Sin[c + d*x]])/(512*a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^2/(2*a*d) + a^3/(256*d*(a - a*Sin[
c + d*x])^4) - (3*a^2)/(64*d*(a - a*Sin[c + d*x])^3) + (141*a)/(512*d*(a -
a*Sin[c + d*x])^2) - 39/(32*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin
[c + d*x])^5) + (19*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (53*a^2)/(128*d*(
a + a*Sin[c + d*x])^3) + (765*a)/(512*d*(a + a*Sin[c + d*x])^2) - 1155/(256
*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\text{Subst}\left(\int \frac{x^{12}}{a^{12}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{12}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$

$$= \frac{\text{Subst}\left(\int \left(a+\frac{a^6}{64(a-x)^5}-\frac{9a^5}{64(a-x)^4}+\frac{141a^4}{256(a-x)^3}-\frac{39a^3}{32(a-x)^2}+\frac{843a^2}{512(a-x)}-x+ \ldots\right.\right.}{\ldots}$$

$$= -\frac{843\log(1-\sin(c+dx))}{512ad}-\frac{2229\log(1+\sin(c+dx))}{512ad}+\frac{\sin(c+dx)}{ad}- \ldots$$

Mathematica [A]
time = 6.12, size = 169, normalized size = 0.64

$$-\frac{4215\log(1-\sin(c+dx))+11145\log(1+\sin(c+dx))-\frac{10}{(1-\sin(c+dx))^4}+\frac{120}{(1-\sin(c+dx))^3}-\frac{705}{(1-\sin(c+dx))^2}+\frac{3120}{1-\sin(c+dx)}-2560\sin(c+dx)+1280\sin^2(c+dx)+\frac{16}{(1+\sin(c+dx))^5}-\frac{190}{(1+\sin(c+dx))^4}+\frac{1060}{(1+\sin(c+dx))^3}-\frac{3825}{(1+\sin(c+dx))^2}+\frac{11550}{1+\sin(c+dx)}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(4215*Log[1 - Sin[c + d*x]] + 11145*Log[1 + Sin[c + d*x]] - 10/(1 -
 Sin[c + d*x])^4 + 120/(1 - Sin[c + d*x])^3 - 705/(1 - Sin[c + d*x])^2 + 31
20/(1 - Sin[c + d*x]) - 2560*Sin[c + d*x] + 1280*Sin[c + d*x]^2 + 16/(1 + S
in[c + d*x])^5 - 190/(1 + Sin[c + d*x])^4 + 1060/(1 + Sin[c + d*x])^3 - 382
5/(1 + Sin[c + d*x])^2 + 11550/(1 + Sin[c + d*x]))/(a*d)
```

Maple [A]
time = 0.35, size = 155, normalized size = 0.59

method	result
derivativedivides	$\frac{-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)-\frac{1}{160(1+\sin(dx+c))^5}+\frac{19}{256(1+\sin(dx+c))^4}-\frac{53}{128(1+\sin(dx+c))^3}+\frac{765}{512(1+\sin(dx+c))^2}-\frac{1155}{256(1+\sin(dx+\ldots}}{d\ldots}$
default	$\frac{-\frac{\left(\sin^2(dx+c)\right)}{2}+\sin(dx+c)-\frac{1}{160(1+\sin(dx+c))^5}+\frac{19}{256(1+\sin(dx+c))^4}-\frac{53}{128(1+\sin(dx+c))^3}+\frac{765}{512(1+\sin(dx+c))^2}-\frac{1155}{256(1+\sin(dx+\ldots}}{d\ldots}$
risch	$\frac{6ix}{a}+\frac{e^{2i(dx+c)}}{8ad}-\frac{ie^{i(dx+c)}}{2ad}+\frac{ie^{-i(dx+c)}}{2ad}+\frac{e^{-2i(dx+c)}}{8ad}+\frac{12ic}{ad}-\frac{i\left(31370ie^{4i(dx+c)}+4215\,e^{17i(dx+c)}+20922ie^{8\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2*sin(d*x+c)^2+sin(d*x+c)-1/160/(1+sin(d*x+c))^5+19/256/(1+sin(d*
x+c))^4-53/128/(1+sin(d*x+c))^3+765/512/(1+sin(d*x+c))^2-1155/256/(1+sin(d*
x+c))-2229/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4+3/64/(sin(d*x+c)-1)^
3+141/512/(sin(d*x+c)-1)^2+39/32/(sin(d*x+c)-1)-843/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 236, normalized size = 0.89

$$-\frac{\frac{2\left(4215\sin(dx+c)^8-5385\sin(dx+c)^7-18655\sin(dx+c)^6+13345\sin(dx+c)^5+30113\sin(dx+c)^4-11487\sin(dx+c)^3-21257\sin(dx+c)^2+3383\sin(dx+c)+5568\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{1280\left(\sin(dx+c)^2-2\sin(dx+c)\right)}{a}+\frac{11145\log(\sin(dx+c)+1)}{a}+\frac{4215\log(\sin(dx+c)-1)}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2560*(2*(4215*sin(d*x + c)^8 - 5385*sin(d*x + c)^7 - 18655*sin(d*x + c)^
6 + 13345*sin(d*x + c)^5 + 30113*sin(d*x + c)^4 - 11487*sin(d*x + c)^3 - 21
257*sin(d*x + c)^2 + 3383*sin(d*x + c) + 5568)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 1280*(sin(d*x + c)^2 - 2*sin(d*x + c))/a + 11145*log(sin(d*x + c)
+ 1)/a + 4215*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.43, size = 217, normalized size = 0.82

$$-\frac{1280\cos(dx+c)^{10}+6510\cos(dx+c)^8+3590\cos(dx+c)^6-1124\cos(dx+c)^4+272\cos(dx+c)^2+11145\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+4215\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(640\cos(dx+c)^{10}+960\cos(dx+c)^8-5385\cos(dx+c)^6+2810\cos(dx+c)^4-952\cos(dx+c)^2+144\right)\sin(dx+c)-32}{2560\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2560*(1280*cos(d*x + c)^10 + 6510*cos(d*x + c)^8 + 3590*cos(d*x + c)^6 -
 1124*cos(d*x + c)^4 + 272*cos(d*x + c)^2 + 11145*(cos(d*x + c)^8*sin(d*x +
 c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) + 4215*(cos(d*x + c)^8*sin(d*x
+ c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(640*cos(d*x + c)^10 + 96
0*cos(d*x + c)^8 - 5385*cos(d*x + c)^6 + 2810*cos(d*x + c)^4 - 952*cos(d*x
+ c)^2 + 144)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos
(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**12/(a+a*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.61, size = 181, normalized size = 0.69

$$-\frac{\frac{44580\log\left(|\sin\left(dx+c\right)+1|\right)}{a}+\frac{16860\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{5120\left(a\sin\left(dx+c\right)^{2}-2a\sin\left(dx+c\right)\right)}{a^{2}}-\frac{5\left(7025\sin\left(dx+c\right)^{4}-25604\sin\left(dx+c\right)^{3}+35226\sin\left(dx+c\right)^{2}-21644\sin\left(dx+c\right)+5005\right)}{a\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{101791\sin\left(dx+c\right)^{5}+462755\sin\left(dx+c\right)^{4}+848410\sin\left(dx+c\right)^{3}+782370\sin\left(dx+c\right)^{2}+362335\sin\left(dx+c\right)+67347}{a\left(\sin\left(dx+c\right)+1\right)^{5}}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^12/(a+a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a + 5120*(a*sin(d*x + c)^2 - 2*a*sin(d*x + c))/a^2 - 5*(7025*sin(d*x +
 c)^4 - 25604*sin(d*x + c)^3 + 35226*sin(d*x + c)^2 - 21644*sin(d*x + c) +
5005)/(a*(sin(d*x + c) - 1)^4) - (101791*sin(d*x + c)^5 + 462755*sin(d*x +
c)^4 + 848410*sin(d*x + c)^3 + 782370*sin(d*x + c)^2 + 362335*sin(d*x + c)
+ 67347)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 11.20, size = 648, normalized size = 2.45

$$\frac{\frac{693\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{128}-\frac{75\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}}{64}-\frac{3153\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{64}-\frac{87\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}}{64}+\frac{111333\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{640}+\frac{1331\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{40}-\frac{4559\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{16}-\frac{1823\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{20}+\frac{42953\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{320}+\frac{11713\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{160}+\frac{43457\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{160}+\frac{11713\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{160}+\frac{42953\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{320}-\frac{1823\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{20}-\frac{4559\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{16}+\frac{1331\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{40}+\frac{111333\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{640}-\frac{87\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}-\frac{3153\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{64}-\frac{75\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{693\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{22}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}+7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+26a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}+5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-22a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-22a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+26a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{843\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{256\,a\,d}-\frac{2229\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{256\,a\,d}+\frac{6\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^12/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
[Out] ((693*tan(c/2 + (d*x)/2))/128 - (75*tan(c/2 + (d*x)/2)^2)/64 - (3153*tan(c/
2 + (d*x)/2)^3)/64 - (87*tan(c/2 + (d*x)/2)^4)/64 + (111333*tan(c/2 + (d*x)
/2)^5)/640 + (1331*tan(c/2 + (d*x)/2)^6)/40 - (4559*tan(c/2 + (d*x)/2)^7)/1
6 - (1823*tan(c/2 + (d*x)/2)^8)/20 + (42953*tan(c/2 + (d*x)/2)^9)/320 + (11
713*tan(c/2 + (d*x)/2)^10)/160 + (43457*tan(c/2 + (d*x)/2)^11)/160 + (11713
*tan(c/2 + (d*x)/2)^12)/160 + (42953*tan(c/2 + (d*x)/2)^13)/320 - (1823*tan
(c/2 + (d*x)/2)^14)/20 - (4559*tan(c/2 + (d*x)/2)^15)/16 + (1331*tan(c/2 +
(d*x)/2)^16)/40 + (111333*tan(c/2 + (d*x)/2)^17)/640 - (87*tan(c/2 + (d*x)/
2)^18)/64 - (3153*tan(c/2 + (d*x)/2)^19)/64 - (75*tan(c/2 + (d*x)/2)^20)/64
 + (693*tan(c/2 + (d*x)/2)^21)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 5*a*ta
n(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 7*a*tan(c/2 + (d*x)/2)^4 +
 26*a*tan(c/2 + (d*x)/2)^5 + 5*a*tan(c/2 + (d*x)/2)^6 - 16*a*tan(c/2 + (d*x
)/2)^7 - 22*a*tan(c/2 + (d*x)/2)^8 - 28*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c
/2 + (d*x)/2)^10 + 56*a*tan(c/2 + (d*x)/2)^11 + 14*a*tan(c/2 + (d*x)/2)^12
- 28*a*tan(c/2 + (d*x)/2)^13 - 22*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 +
(d*x)/2)^15 + 5*a*tan(c/2 + (d*x)/2)^16 + 26*a*tan(c/2 + (d*x)/2)^17 + 7*a*
tan(c/2 + (d*x)/2)^18 - 12*a*tan(c/2 + (d*x)/2)^19 - 5*a*tan(c/2 + (d*x)/2)
^20 + 2*a*tan(c/2 + (d*x)/2)^21 + a*tan(c/2 + (d*x)/2)^22)) - (843*log(tan(
c/2 + (d*x)/2) - 1))/(256*a*d) - (2229*log(tan(c/2 + (d*x)/2) + 1))/(256*a*
d) + (6*log(tan(c/2 + (d*x)/2)^2 + 1))/(a*d)
```

3.896 $\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$-\frac{437\log(1-\sin(c+dx))}{512ad}+\frac{949\log(1+\sin(c+dx))}{512ad}-\frac{\sin(c+dx)}{ad}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{a}{24d(a-a\sin}$$

```
[Out] -437/512*ln(1-sin(d*x+c))/a/d+949/512*ln(1+sin(d*x+c))/a/d-sin(d*x+c)/a/d+1
/256*a^3/d/(a-a*sin(d*x+c))^4-1/24*a^2/d/(a-a*sin(d*x+c))^3+109/512*a/d/(a-
a*sin(d*x+c))^2-203/256/d/(a-a*sin(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5-1
7/256*a^3/d/(a+a*sin(d*x+c))^4+125/384*a^2/d/(a+a*sin(d*x+c))^3-515/512*a/d
/(a+a*sin(d*x+c))^2+5/2/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.18, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{17a^3}{256d(a\sin(c+dx)+a)^4}-\frac{a^2}{24d(a-a\sin(c+dx))^3}+\frac{125a^2}{384d(a\sin(c+dx)+a)^3}+\frac{109a}{512d(a-a\sin(c+dx))^2}-\frac{515a}{512d(a\sin(c+dx)+a)^2}-\frac{203}{256d(a-a\sin(c+dx))}+\frac{5}{2d(a\sin(c+dx)+a)}-\frac{\sin(c+dx)}{ad}-\frac{437\log(1-\sin(c+dx))}{512ad}+\frac{949\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-437*Log[1 - Sin[c + d*x]])/(512*a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a
*d) - Sin[c + d*x]/(a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - a^2/(24*d*(
a - a*Sin[c + d*x])^3) + (109*a)/(512*d*(a - a*Sin[c + d*x])^2) - 203/(256*
d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) - (17*a^3)/(25
6*d*(a + a*Sin[c + d*x])^4) + (125*a^2)/(384*d*(a + a*Sin[c + d*x])^3) - (5
15*a)/(512*d*(a + a*Sin[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\text{Subst}\left(\int \frac{x^{11}}{a^{11}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{11}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^5}{64(a-x)^5}-\frac{a^4}{8(a-x)^4}+\frac{109a^3}{256(a-x)^3}-\frac{203a^2}{256(a-x)^2}+\frac{437a}{512(a-x)}-\frac{a^5}{32(a-\ldots}\right.\right.}{\ldots}$$

$$= -\frac{437\log(1-\sin(c+dx))}{512ad}+\frac{949\log(1+\sin(c+dx))}{512ad}-\frac{\sin(c+dx)}{ad}+\frac{\ldots}{25\ldots}$$

Mathematica [A]

time = 6.15, size = 159, normalized size = 0.64

$$-\frac{6555\log(1-\sin(c+dx))-14235\log(1+\sin(c+dx))-\frac{30}{(1-\sin(c+dx))^4}+\frac{320}{(1-\sin(c+dx))^3}-\frac{1635}{(1-\sin(c+dx))^2}+\frac{6090}{1-\sin(c+dx)}+7680\sin(c+dx)-\frac{48}{(1+\sin(c+dx))^5}+\frac{510}{(1+\sin(c+dx))^4}-\frac{2500}{(1+\sin(c+dx))^3}+\frac{7725}{(1+\sin(c+dx))^2}-\frac{19200}{1+\sin(c+dx)}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(6555*Log[1 - Sin[c + d*x]] - 14235*Log[1 + Sin[c + d*x]] - 30/(1 -
 Sin[c + d*x])^4 + 320/(1 - Sin[c + d*x])^3 - 1635/(1 - Sin[c + d*x])^2 + 6
090/(1 - Sin[c + d*x]) + 7680*Sin[c + d*x] - 48/(1 + Sin[c + d*x])^5 + 510/
(1 + Sin[c + d*x])^4 - 2500/(1 + Sin[c + d*x])^3 + 7725/(1 + Sin[c + d*x])^
2 - 19200/(1 + Sin[c + d*x]))/(a*d)
```

Maple [A]

time = 0.29, size = 147, normalized size = 0.60

method	result
derivativedivides	$\frac{-\sin(dx+c)+\frac{1}{160(1+\sin(dx+c))^5}-\frac{17}{256(1+\sin(dx+c))^4}+\frac{125}{384(1+\sin(dx+c))^3}-\frac{515}{512(1+\sin(dx+c))^2}+\frac{5}{2(1+\sin(dx+c))}+\frac{949\ln(1+\sin\ldots}{512}}{da}$
default	$\frac{-\sin(dx+c)+\frac{1}{160(1+\sin(dx+c))^5}-\frac{17}{256(1+\sin(dx+c))^4}+\frac{125}{384(1+\sin(dx+c))^3}-\frac{515}{512(1+\sin(dx+c))^2}+\frac{5}{2(1+\sin(dx+c))}+\frac{949\ln(1+\sin\ldots}{512}}{da}$
risch	$-\frac{ix}{a}+\frac{ie^{i(dx+c)}}{2ad}-\frac{ie^{-i(dx+c)}}{2ad}-\frac{2ic}{ad}+\frac{i\left(-21090ie^{4i(dx+c)}+12645\,e^{i(dx+c)}-16594ie^{8i(dx+c)}-37738ie^{6i(dx+c)}+2\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-sin(d*x+c)+1/160/(1+sin(d*x+c))^5-17/256/(1+sin(d*x+c))^4+125/384/(
1+sin(d*x+c))^3-515/512/(1+sin(d*x+c))^2+5/2/(1+sin(d*x+c))+949/512*ln(1+si
n(d*x+c))+1/256/(sin(d*x+c)-1)^4+1/24/(sin(d*x+c)-1)^3+109/512/(sin(d*x+c)-
1)^2+203/256/(sin(d*x+c)-1)-437/512*ln(sin(d*x+c)-1))
```

Maxima [**A**]
time = 0.29, size = 225, normalized size = 0.91

$$\frac{\frac{2\left(12645\sin(dx+c)^8+3045\sin(dx+c)^7-36765\sin(dx+c)^6-7965\sin(dx+c)^5+42339\sin(dx+c)^4+7139\sin(dx+c)^3-22171\sin(dx+c)^2-2171\sin(dx+c)+4384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{14235\log(\sin(dx+c)+1)}{a}-\frac{6555\log(\sin(dx+c)-1)}{a}-\frac{7680\sin(dx+c)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] 1/7680*(2*(12645*sin(d*x + c)^8 + 3045*sin(d*x + c)^7 - 36765*sin(d*x + c)^
6 - 7965*sin(d*x + c)^5 + 42339*sin(d*x + c)^4 + 7139*sin(d*x + c)^3 - 2217
1*sin(d*x + c)^2 - 2171*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x
+ c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a
*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c)
+ a) + 14235*log(sin(d*x + c) + 1)/a - 6555*log(sin(d*x + c) - 1)/a - 7680*
sin(d*x + c)/a)/d
```

Fricas [**A**]
time = 0.42, size = 207, normalized size = 0.84

$$\frac{7680\cos(dx+c)^{10}+17610\cos(dx+c)^8-27630\cos(dx+c)^6+15828\cos(dx+c)^4-5584\cos(dx+c)^2+14235\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)-6555\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(3840\cos(dx+c)^8+3045\cos(dx+c)^6-1170\cos(dx+c)^4+344\cos(dx+c)^2-48\right)\sin(dx+c)+864}{7680\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/7680*(7680*cos(d*x + c)^10 + 17610*cos(d*x + c)^8 - 27630*cos(d*x + c)^6
+ 15828*cos(d*x + c)^4 - 5584*cos(d*x + c)^2 + 14235*(cos(d*x + c)^8*sin(d*
x + c) + cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x + c)^8*sin(d
*x + c) + cos(d*x + c)^8)*log(-sin(d*x + c) + 1) - 2*(3840*cos(d*x + c)^8 +
 3045*cos(d*x + c)^6 - 1170*cos(d*x + c)^4 + 344*cos(d*x + c)^2 - 48)*sin(d
*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**11/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 167, normalized size = 0.68

$$\frac{\frac{56940\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{26220\log\left(|\sin\left(dx+c\right)-1|\right)}{a}-\frac{30720\sin\left(dx+c\right)}{a}+\frac{5\left(10925\sin\left(dx+c\right)^{4}-38828\sin\left(dx+c\right)^{3}+52242\sin\left(dx+c\right)^{2}-31444\sin\left(dx+c\right)+7129\right)}{a\left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{130013\sin\left(dx+c\right)^{5}+573265\sin\left(dx+c\right)^{4}+1023830\sin\left(dx+c\right)^{3}+922030\sin\left(dx+c\right)^{2}+417605\sin\left(dx+c\right)+75961}{a\left(\sin\left(dx+c\right)+1\right)^{5}}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^11/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*sin(d*x + c)/a + 5*(10925*sin(d*x + c)^4 - 38828*sin(d*x + c)
^3 + 52242*sin(d*x + c)^2 - 31444*sin(d*x + c) + 7129)/(a*(sin(d*x + c) - 1
)^4) - (130013*sin(d*x + c)^5 + 573265*sin(d*x + c)^4 + 1023830*sin(d*x + c
)^3 + 922030*sin(d*x + c)^2 + 417605*sin(d*x + c) + 75961)/(a*(sin(d*x + c)
 + 1)^5))/d
```

Mupad [B]
time = 10.85, size = 595, normalized size = 2.41

$$\frac{949\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{256\,a\,d}-\frac{437\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{256\,a\,d}-\frac{\frac{693\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{128}+\frac{565\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}}{64}-\frac{4439\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{963\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{16}+\frac{7091\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{80}+\frac{40031\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{240}-\frac{12829\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{120}-\frac{17969\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{80}+\frac{39491\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{960}+\frac{49513\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{480}+\frac{39491\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{960}-\frac{17969\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{80}-\frac{12829\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{120}+\frac{40031\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{240}+\frac{7091\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{80}-\frac{963\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{16}-\frac{4439\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{128}+\frac{565\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{693\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}-6a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}-14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}+13a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-8a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-8a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+40a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+13a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-6a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (949*log(tan(c/2 + (d*x)/2) + 1))/(256*a*d) - (437*log(tan(c/2 + (d*x)/2) -
 1))/(256*a*d) - ((693*tan(c/2 + (d*x)/2))/128 + (565*tan(c/2 + (d*x)/2)^2)
/64 - (4439*tan(c/2 + (d*x)/2)^3)/128 - (963*tan(c/2 + (d*x)/2)^4)/16 + (70
91*tan(c/2 + (d*x)/2)^5)/80 + (40031*tan(c/2 + (d*x)/2)^6)/240 - (12829*tan
(c/2 + (d*x)/2)^7)/120 - (17969*tan(c/2 + (d*x)/2)^8)/80 + (39491*tan(c/2 +
 (d*x)/2)^9)/960 + (49513*tan(c/2 + (d*x)/2)^10)/480 + (39491*tan(c/2 + (d*
x)/2)^11)/960 - (17969*tan(c/2 + (d*x)/2)^12)/80 - (12829*tan(c/2 + (d*x)/2
)^13)/120 + (40031*tan(c/2 + (d*x)/2)^14)/240 + (7091*tan(c/2 + (d*x)/2)^15
)/80 - (963*tan(c/2 + (d*x)/2)^16)/16 - (4439*tan(c/2 + (d*x)/2)^17)/128 +
(565*tan(c/2 + (d*x)/2)^18)/64 + (693*tan(c/2 + (d*x)/2)^19)/128)/(d*(a + 2
*a*tan(c/2 + (d*x)/2) - 6*a*tan(c/2 + (d*x)/2)^2 - 14*a*tan(c/2 + (d*x)/2)^
3 + 13*a*tan(c/2 + (d*x)/2)^4 + 40*a*tan(c/2 + (d*x)/2)^5 - 8*a*tan(c/2 + (
d*x)/2)^6 - 56*a*tan(c/2 + (d*x)/2)^7 - 14*a*tan(c/2 + (d*x)/2)^8 + 28*a*ta
n(c/2 + (d*x)/2)^9 + 28*a*tan(c/2 + (d*x)/2)^10 + 28*a*tan(c/2 + (d*x)/2)^1
1 - 14*a*tan(c/2 + (d*x)/2)^12 - 56*a*tan(c/2 + (d*x)/2)^13 - 8*a*tan(c/2 +
 (d*x)/2)^14 + 40*a*tan(c/2 + (d*x)/2)^15 + 13*a*tan(c/2 + (d*x)/2)^16 - 14
*a*tan(c/2 + (d*x)/2)^17 - 6*a*tan(c/2 + (d*x)/2)^18 + 2*a*tan(c/2 + (d*x)/
2)^19 + a*tan(c/2 + (d*x)/2)^20)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
```

3.897 $\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{193\log(1-\sin(c+dx))}{512ad}-\frac{319\log(1+\sin(c+dx))}{512ad}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}$$

```
[Out] -193/512*ln(1-sin(d*x+c))/a/d-319/512*ln(1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a
*sin(d*x+c))^4-7/192*a^2/d/(a-a*sin(d*x+c))^3+81/512*a/d/(a-a*sin(d*x+c))^2
-61/128/d/(a-a*sin(d*x+c))-1/160*a^4/d/(a+a*sin(d*x+c))^5+15/256*a^3/d/(a+a
*sin(d*x+c))^4-95/384*a^2/d/(a+a*sin(d*x+c))^3+325/512*a/d/(a+a*sin(d*x+c))
^2-315/256/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.16, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{15a^3}{256d(a\sin(c+dx)+a)^4}-\frac{7a^2}{192d(a-a\sin(c+dx))^3}-\frac{95a^2}{384d(a\sin(c+dx)+a)^3}+\frac{81a}{512d(a-a\sin(c+dx))^2}+\frac{325a}{512d(a\sin(c+dx)+a)^2}-\frac{61}{128d(a-a\sin(c+dx))}-\frac{315}{256d(a\sin(c+dx)+a)}-\frac{193\log(1-\sin(c+dx))}{512ad}-\frac{319\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) - (319*Log[1 + Sin[c + d*x]])/(512*a
*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) - (7*a^2)/(192*d*(a - a*Sin[c + d*
x])^3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) - 61/(128*d*(a - a*Sin[c + d
*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) + (15*a^3)/(256*d*(a + a*Sin[c +
 d*x])^4) - (95*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a
*Sin[c + d*x])^2) - 315/(256*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\text{Subst}\left(\int \frac{x^{10}}{a^{10}(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^{10}}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{64(a-x)^5} - \frac{7a^3}{64(a-x)^4} + \frac{81a^2}{256(a-x)^3} - \frac{61a}{128(a-x)^2} + \frac{193}{512(a-x)} + \frac{a^5}{32(a+x)^6} - \right.\right.}{a}$$

$$= -\frac{193\log(1-\sin(c+dx))}{512ad} - \frac{319\log(1+\sin(c+dx))}{512ad} + \frac{a^3}{256d(a-a\sin(c+}$$

Mathematica [A]

time = 4.66, size = 137, normalized size = 0.59

$$-\frac{2895\log(1-\sin(c+dx)) + 4785\log(1+\sin(c+dx)) + \frac{2(4384+3439\sin(c+dx)-16561\sin^2(c+dx)-12151\sin^3(c+dx)+23049\sin^4(c+dx)+14985\sin^5(c+dx)-13815\sin^6(c+dx)-6705\sin^7(c+dx)+2895\sin^8(c+dx))}{(-1+\sin(c+dx))^4(1+\sin(c+dx))^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(2895*Log[1 - Sin[c + d*x]] + 4785*Log[1 + Sin[c + d*x]] + (2*(4384
 + 3439*Sin[c + d*x] - 16561*Sin[c + d*x]^2 - 12151*Sin[c + d*x]^3 + 23049*
Sin[c + d*x]^4 + 14985*Sin[c + d*x]^5 - 13815*Sin[c + d*x]^6 - 6705*Sin[c +
 d*x]^7 + 2895*Sin[c + d*x]^8))/((-1 + Sin[c + d*x])^4*(1 + Sin[c + d*x])^5
))/(a*d)
```

Maple [A]

time = 0.27, size = 139, normalized size = 0.60

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{15}{256(1+\sin(dx+c))^4}-\frac{95}{384(1+\sin(dx+c))^3}+\frac{325}{512(1+\sin(dx+c))^2}-\frac{315}{256(1+\sin(dx+c))}-\frac{319\ln(1+\sin(dx+c))}{512}+\frac{}{2}}{da}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{15}{256(1+\sin(dx+c))^4}-\frac{95}{384(1+\sin(dx+c))^3}+\frac{325}{512(1+\sin(dx+c))^2}-\frac{315}{256(1+\sin(dx+c))}-\frac{319\ln(1+\sin(dx+c))}{512}+\frac{}{2}}{da}$
risch	$\frac{ix}{a}+\frac{2ic}{ad}-\frac{i\left(26010i\,e^{4i(dx+c)}+2895\,e^{17i(dx+c)}+3146i\,e^{8i(dx+c)}+32100\,e^{15i(dx+c)}+71042i\,e^{6i(dx+c)}+118284\,e^{13i(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5+15/256/(1+sin(d*x+c))^4-95/384/(1+sin(d*x+c)
)^3+325/512/(1+sin(d*x+c))^2-315/256/(1+sin(d*x+c))-319/512*ln(1+sin(d*x+c)
)+1/256/(sin(d*x+c)-1)^4+7/192/(sin(d*x+c)-1)^3+81/512/(sin(d*x+c)-1)^2+61/
128/(sin(d*x+c)-1)-193/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 0.92

$$-\frac{\frac{2\left(2895 \sin(dx+c)^8-6705 \sin(dx+c)^7-13815 \sin(dx+c)^6+14985 \sin(dx+c)^5+23049 \sin(dx+c)^4-12151 \sin(dx+c)^3-16561 \sin(dx+c)^2+3439 \sin(dx+c)+4384\right)}{a \sin(dx+c)^9+a \sin(dx+c)^8-4 a \sin(dx+c)^7-4 a \sin(dx+c)^6+6 a \sin(dx+c)^5+6 a \sin(dx+c)^4-4 a \sin(dx+c)^3-4 a \sin(dx+c)^2+a \sin(dx+c)+a}+\frac{4785 \log(\sin(dx+c)+1)}{a}+\frac{2895 \log(\sin(dx+c)-1)}{a}}{7680\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/7680*(2*(2895*sin(d*x + c)^8 - 6705*sin(d*x + c)^7 - 13815*sin(d*x + c)^
6 + 14985*sin(d*x + c)^5 + 23049*sin(d*x + c)^4 - 12151*sin(d*x + c)^3 - 16
561*sin(d*x + c)^2 + 3439*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*
x + c)^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6
*a*sin(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c
) + a) + 4785*log(sin(d*x + c) + 1)/a + 2895*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.42, size = 187, normalized size = 0.80

$$-\frac{5790 \cos(dx+c)^8+4470 \cos(dx+c)^6-2052 \cos(dx+c)^4+656 \cos(dx+c)^2+4785\left(\cos(dx+c)^8 \sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+2895\left(\cos(dx+c)^8 \sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)+2\left(6705 \cos(dx+c)^6-5130 \cos(dx+c)^4+2296 \cos(dx+c)^2-432\right)\sin(dx+c)-96}{7680\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/7680*(5790*cos(d*x + c)^8 + 4470*cos(d*x + c)^6 - 2052*cos(d*x + c)^4 +
656*cos(d*x + c)^2 + 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(sin(d*x + c) + 1) + 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(-sin(d*x + c) + 1) + 2*(6705*cos(d*x + c)^6 - 5130*cos(d*x + c)^4 + 2296
*cos(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c)
+ a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**10/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 156, normalized size = 0.67

$$-\frac{\frac{19140 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}+\frac{11580 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}-\frac{5 \left(4825 \sin\left(dx+c\right)^{4}-16372 \sin\left(dx+c\right)^{3}+21138 \sin\left(dx+c\right)^{2}-12236 \sin\left(dx+c\right)+2669\right)}{a \left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{43703 \sin\left(dx+c\right)^{5}+180715 \sin\left(dx+c\right)^{4}+305330 \sin\left(dx+c\right)^{3}+261130 \sin\left(dx+c\right)^{2}+112415 \sin\left(dx+c\right)+19411}{a \left(\sin\left(dx+c\right)+1\right)^{5}}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^10/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 5*(4825*sin(d*x + c)^4 - 16372*sin(d*x + c)^3 + 21138*sin(d*x + c)
^2 - 12236*sin(d*x + c) + 2669)/(a*(sin(d*x + c) - 1)^4) - (43703*sin(d*x +
 c)^5 + 180715*sin(d*x + c)^4 + 305330*sin(d*x + c)^3 + 261130*sin(d*x + c)
^2 + 112415*sin(d*x + c) + 19411)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 9.40, size = 539, normalized size = 2.31

$$\frac{\frac{63 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{65 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{233 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{413 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64}+\frac{6527 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160}-\frac{14911 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960}-\frac{59737 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480}+\frac{12763 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960}+\frac{45791 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{12763 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960}-\frac{59737 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480}-\frac{14911 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960}+\frac{6527 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160}+\frac{413 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}-\frac{233 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}-\frac{65 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{63 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d \left(a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{193 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{256\, a\, d}-\frac{319 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{256\, a\, d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{a\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^10/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((63*tan(c/2 + (d*x)/2))/128 - (65*tan(c/2 + (d*x)/2)^2)/64 - (233*tan(c/2
+ (d*x)/2)^3)/32 + (413*tan(c/2 + (d*x)/2)^4)/64 + (6527*tan(c/2 + (d*x)/2)
^5)/160 - (14911*tan(c/2 + (d*x)/2)^6)/960 - (59737*tan(c/2 + (d*x)/2)^7)/4
80 + (12763*tan(c/2 + (d*x)/2)^8)/960 + (45791*tan(c/2 + (d*x)/2)^9)/192 +
(12763*tan(c/2 + (d*x)/2)^10)/960 - (59737*tan(c/2 + (d*x)/2)^11)/480 - (14
911*tan(c/2 + (d*x)/2)^12)/960 + (6527*tan(c/2 + (d*x)/2)^13)/160 + (413*ta
n(c/2 + (d*x)/2)^14)/64 - (233*tan(c/2 + (d*x)/2)^15)/32 - (65*tan(c/2 + (d
*x)/2)^16)/64 + (63*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)
/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 +
 (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a
*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2
)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c
/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14
- 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d
*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18)) - (193*log(tan(c/2 + (d*x)/2) - 1))/(
256*a*d) - (319*log(tan(c/2 + (d*x)/2) + 1))/(256*a*d) + log(tan(c/2 + (d*x
)/2)^2 + 1)/(a*d)
```

3.898 $\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{63\sec(c+dx)\tan(c+dx)}{256ad}+\frac{21\sec(c+dx)\tan^3(c+dx)}{128ad}-\frac{21\sec(c+dx)\tan^5(c+dx)}{160ad}$$

```
[Out] 63/256*arctanh(sin(d*x+c))/a/d-63/256*sec(d*x+c)*tan(d*x+c)/a/d+21/128*sec(
d*x+c)*tan(d*x+c)^3/a/d-21/160*sec(d*x+c)*tan(d*x+c)^5/a/d+9/80*sec(d*x+c)*
tan(d*x+c)^7/a/d-1/10*sec(d*x+c)*tan(d*x+c)^9/a/d+1/10*tan(d*x+c)^10/a/d
```

Rubi [A]

time = 0.13, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^9(c+dx)\sec(c+dx)}{10ad}+\frac{9\tan^7(c+dx)\sec(c+dx)}{80ad}-\frac{21\tan^5(c+dx)\sec(c+dx)}{160ad}+\frac{21\tan^3(c+dx)\sec(c+dx)}{128ad}-\frac{63\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) - (63*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (21*Sec[c + d*x]*Tan[c + d*x]^3)/(128*a*d) - (21*Sec[c + d*x]*Tan[c
+ d*x]^5)/(160*a*d) + (9*Sec[c + d*x]*Tan[c + d*x]^7)/(80*a*d) - (Sec[c + d
*x]*Tan[c + d*x]^9)/(10*a*d) + Tan[c + d*x]^10/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^9(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^{10}(c+dx)\,dx}{a}\\
&= -\frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{9\int \sec(c+dx)\tan^8(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \text{t}\right.}{ad}\\
&= \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+dx)}{10ad} + \frac{\tan^{10}(c+dx)}{10ad} - \frac{63\int \sec}{}\\
&= -\frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c+dx)}{80ad} - \frac{\sec(c+dx)\tan^9(c+}{10ad}\\
&= \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(c+dx)}{160ad} + \frac{9\sec(c+dx)\tan^7(c-}{80ad}\\
&= -\frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx)}{128ad} - \frac{21\sec(c+dx)\tan^5(}{160ad}\\
&= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{63\sec(c+dx)\tan(c+dx)}{256ad} + \frac{21\sec(c+dx)\tan^3(c+dx}{128ad}\end{aligned}$$

Mathematica [A]
time = 1.63, size = 122, normalized size = 0.79

$$\frac{630\tanh^{-1}(\sin(c+dx)) + \frac{2(128-187\sin(c+dx)-827\sin^2(c+dx)+643\sin^3(c+dx)+1923\sin^4(c+dx)-765\sin^5(c+dx)-2045\sin^6(c+dx)+325\sin^7(c+dx)+965\sin^8(c+dx))}{(-1+\sin(c+dx))^4(1+\sin(c+dx))^5}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (630*ArcTanh[Sin[c + d*x]] + (2*(128 - 187*Sin[c + d*x] - 827*Sin[c + d*x]^
2 + 643*Sin[c + d*x]^3 + 1923*Sin[c + d*x]^4 - 765*Sin[c + d*x]^5 - 2045*Si
n[c + d*x]^6 + 325*Sin[c + d*x]^7 + 965*Sin[c + d*x]^8))/((-1 + Sin[c + d*x
])^4*(1 + Sin[c + d*x])^5))/(2560*a*d)
```

Maple [A]
time = 0.25, size = 139, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{13}{256(1+\sin(dx+c))^4}+\frac{23}{128(1+\sin(dx+c))^3}-\frac{187}{512(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{63\ln(1+\sin(dx+c))}{512}+\frac{}{256(\mathrm{si}}}{da}$
default	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{13}{256(1+\sin(dx+c))^4}+\frac{23}{128(1+\sin(dx+c))^3}-\frac{187}{512(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{63\ln(1+\sin(dx+c))}{512}+\frac{}{256(\mathrm{si}}}{da}$
risch	$i\left(-1570i e^{4i(dx+c)}-658i e^{8i(dx+c)}+965\,e^{i(dx+c)}+460\,e^{15i(dx+c)}-1484\,e^{11i(dx+c)}+15470\,e^{9i(dx+c)}-3626i e^{6i(dx+c)}+36\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/160/(1+sin(d*x+c))^5-13/256/(1+sin(d*x+c))^4+23/128/(1+sin(d*x+c))
^3-187/512/(1+sin(d*x+c))^2+1/2/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))+1/25
6/(sin(d*x+c)-1)^4+1/32/(sin(d*x+c)-1)^3+57/512/(sin(d*x+c)-1)^2+65/256/(si
n(d*x+c)-1)-63/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 214, normalized size = 1.39

$$\frac{\frac{2\left(965\sin(dx+c)^8+325\sin(dx+c)^7-2045\sin(dx+c)^6-765\sin(dx+c)^5+1923\sin(dx+c)^4+643\sin(dx+c)^3-827\sin(dx+c)^2-187\sin(dx+c)+128\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{315\log(\sin(dx+c)+1)}{a}-\frac{315\log(\sin(dx+c)-1)}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(965*sin(d*x + c)^8 + 325*sin(d*x + c)^7 - 2045*sin(d*x + c)^6 -
765*sin(d*x + c)^5 + 1923*sin(d*x + c)^4 + 643*sin(d*x + c)^3 - 827*sin(d*x
 + c)^2 - 187*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*
a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x +
c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 315*
log(sin(d*x + c) + 1)/a - 315*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 187, normalized size = 1.21

$$\frac{1930\cos(dx+c)^8-3630\cos(dx+c)^6+3156\cos(dx+c)^4-1488\cos(dx+c)^2+315\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)-315\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(325\cos(dx+c)^6-210\cos(dx+c)^4+88\cos(dx+c)^2-16\right)\sin(dx+c)+288}{2560\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(1930*cos(d*x + c)^8 - 3630*cos(d*x + c)^6 + 3156*cos(d*x + c)^4 - 1
488*cos(d*x + c)^2 + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
(sin(d*x + c) + 1) - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log
```

```
(-sin(d*x + c) + 1) - 2*(325*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 88*cos(d
*x + c)^2 - 16)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 156, normalized size = 1.01

$$\frac{\frac{1260\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{1260\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{5\left(525\sin(dx+c)^4-1580\sin(dx+c)^3+1818\sin(dx+c)^2-932\sin(dx+c)+177\right)}{a(\sin(dx+c)-1)^4}-\frac{2877\sin(dx+c)^5+9265\sin(dx+c)^4+12030\sin(dx+c)^3+7430\sin(dx+c)^2+1965\sin(dx+c)+113}{a(\sin(dx+c)+1)^5}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 1580*sin(d*x + c)^3 + 1818*sin(d*x + c)^2 - 9
32*sin(d*x + c) + 177)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 92
65*sin(d*x + c)^4 + 12030*sin(d*x + c)^3 + 7430*sin(d*x + c)^2 + 1965*sin(d
*x + c) + 113)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 17.05, size = 497, normalized size = 3.23

$$\frac{63\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a\,d}-\frac{\frac{63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}+\frac{63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}-\frac{483\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64}+\frac{1407\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160}+\frac{8043\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{320}-\frac{1779\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{160}-\frac{15159\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{320}+\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64}-\frac{15159\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{320}-\frac{1779\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{160}+\frac{8043\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{320}+\frac{1407\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160}-\frac{483\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}-\frac{105\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}+\frac{63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{63\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (63*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) - ((63*tan(c/2 + (d*x)/2))/128 + (
63*tan(c/2 + (d*x)/2)^2)/64 - (105*tan(c/2 + (d*x)/2)^3)/32 - (483*tan(c/2
+ (d*x)/2)^4)/64 + (1407*tan(c/2 + (d*x)/2)^5)/160 + (8043*tan(c/2 + (d*x)/
2)^6)/320 - (1779*tan(c/2 + (d*x)/2)^7)/160 - (15159*tan(c/2 + (d*x)/2)^8)/
320 + (245*tan(c/2 + (d*x)/2)^9)/64 - (15159*tan(c/2 + (d*x)/2)^10)/320 - (
1779*tan(c/2 + (d*x)/2)^11)/160 + (8043*tan(c/2 + (d*x)/2)^12)/320 + (1407*
tan(c/2 + (d*x)/2)^13)/160 - (483*tan(c/2 + (d*x)/2)^14)/64 - (105*tan(c/2
+ (d*x)/2)^15)/32 + (63*tan(c/2 + (d*x)/2)^16)/64 + (63*tan(c/2 + (d*x)/2)^
17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*t
an(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5
```

```
 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 +
(d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*
a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)
/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(
c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

3.899 $\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=160

$$\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{7\sec(c+dx)\tan(c+dx)}{256ad}-\frac{7\sec^3(c+dx)\tan(c+dx)}{128ad}+\frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad}-$$

```
[Out] 7/256*arctanh(sin(d*x+c))/a/d+7/256*sec(d*x+c)*tan(d*x+c)/a/d-7/128*sec(d*x
+c)^3*tan(d*x+c)/a/d+7/96*sec(d*x+c)^3*tan(d*x+c)^3/a/d-7/80*sec(d*x+c)^3*t
an(d*x+c)^5/a/d+1/10*sec(d*x+c)^3*tan(d*x+c)^7/a/d-1/10*tan(d*x+c)^10/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2914, 2691, 3853, 3855, 2687, 30}

$$-\frac{\tan^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}-\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}+\frac{7\tan^3(c+dx)\sec^3(c+dx)}{96ad}-\frac{7\tan(c+dx)\sec^3(c+dx)}{128ad}+\frac{7\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*
d) - (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (7*Sec[c + d*x]^3*Tan[c +
d*x]^3)/(96*a*d) - (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) + (Sec[c + d*
x]^3*Tan[c + d*x]^7)/(10*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^8(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a} - \frac{\int \sec^2(c+dx)\tan^9(c+dx)\,dx}{a}$$

$$= \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{\tan^{10}(c+dx}{10ad}$$

$$= \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} + \frac{\sec^3(c+dx)}{10}$$

$$= -\frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} - \frac{7\sec^3(c+d}{8}$$

$$= \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} + \frac{7\sec^3(c+dx)\,\text{t}}{96a}$$

$$= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+dx)\tan(c}{128ad}$$

Mathematica [A]
time = 2.12, size = 121, normalized size = 0.76

$$\frac{210\tanh^{-1}(\sin(c+dx)) + \frac{-768-978\sin(c+dx)+2862\sin^2(c+dx)+3842\sin^3(c+dx)-3838\sin^4(c+dx)-5630\sin^5(c+dx)+2050\sin^6(c+dx)+3630\sin^7(c+dx)-210\sin^8(c+dx)}{(-1+\sin(c+dx))^4(1+\sin(c+dx))^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^8)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + (-768 - 978*Sin[c + d*x] + 2862*Sin[c + d*x]^2
 + 3842*Sin[c + d*x]^3 - 3838*Sin[c + d*x]^4 - 5630*Sin[c + d*x]^5 + 2050*S
in[c + d*x]^6 + 3630*Sin[c + d*x]^7 - 210*Sin[c + d*x]^8)/((-1 + Sin[c + d*
x])^4*(1 + Sin[c + d*x])^5))/(7680*a*d)
```

Maple [A]
time = 0.27, size = 139, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{11}{256(1+\sin(dx+c))^4}-\frac{47}{384(1+\sin(dx+c))^3}+\frac{93}{512(1+\sin(dx+c))^2}-\frac{35}{256(1+\sin(dx+c))}+\frac{7\ln(1+\sin(dx+c))}{512}+\frac{}{256}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{11}{256(1+\sin(dx+c))^4}-\frac{47}{384(1+\sin(dx+c))^3}+\frac{93}{512(1+\sin(dx+c))^2}-\frac{35}{256(1+\sin(dx+c))}+\frac{7\ln(1+\sin(dx+c))}{512}+\frac{}{256}$
risch	$-\frac{i\left(-2890i e^{4i(dx+c)}+105\,e^{17i(dx+c)}-23674i e^{8i(dx+c)}+3260\,e^{15i(dx+c)}+25102i e^{6i(dx+c)}+9044\,e^{13i(dx+c)}-25102i e^{12i(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5+11/256/(1+sin(d*x+c))^4-47/384/(1+sin(d*x+c)
)^3+93/512/(1+sin(d*x+c))^2-35/256/(1+sin(d*x+c))+7/512*ln(1+sin(d*x+c))+1/
256/(sin(d*x+c)-1)^4+5/192/(sin(d*x+c)-1)^3+37/512/(sin(d*x+c)-1)^2+7/64/(s
in(d*x+c)-1)-7/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 214, normalized size = 1.34

$$-\frac{\frac{2\left(105\,\sin(dx+c)^8-1815\,\sin(dx+c)^7-1025\,\sin(dx+c)^6+2815\,\sin(dx+c)^5+1919\,\sin(dx+c)^4-1921\,\sin(dx+c)^3-1431\,\sin(dx+c)^2+489\,\sin(dx+c)+384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4\,a\sin(dx+c)^7-4\,a\sin(dx+c)^6+6\,a\sin(dx+c)^5+6\,a\sin(dx+c)^4-4\,a\sin(dx+c)^3-4\,a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\,\log(\sin(dx+c)+1)}{a}+\frac{105\,\log(\sin(dx+c)-1)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 - 1815*sin(d*x + c)^7 - 1025*sin(d*x + c)^6
+ 2815*sin(d*x + c)^5 + 1919*sin(d*x + c)^4 - 1921*sin(d*x + c)^3 - 1431*si
n(d*x + c)^2 + 489*sin(d*x + c) + 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8
 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d
*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) -
 105*log(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.42, size = 187, normalized size = 1.17

$$-\frac{210\,\cos\left(dx+c\right)^8+1210\,\cos\left(dx+c\right)^6-1052\,\cos\left(dx+c\right)^4+496\,\cos\left(dx+c\right)^2-105\left(\cos\left(dx+c\right)^8\sin\left(dx+c\right)+\cos\left(dx+c\right)^8\right)\log\left(\sin\left(dx+c\right)+1\right)+105\left(\cos\left(dx+c\right)^8\sin\left(dx+c\right)+\cos\left(dx+c\right)^8\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(1815\,\cos\left(dx+c\right)^6-2630\,\cos\left(dx+c\right)^4+1736\,\cos\left(dx+c\right)^2-432\right)\sin\left(dx+c\right)-96}{7680\left(ad\cos\left(dx+c\right)^8\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 + 1210*cos(d*x + c)^6 - 1052*cos(d*x + c)^4 + 4
96*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) + 2*(1815*cos(d*x + c)^6 - 2630*cos(d*x + c)^4 + 1736*co
s(d*x + c)^2 - 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a
*d*cos(d*x + c)^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 156, normalized size = 0.98

$$\frac{\frac{420\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{420\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{5\left(175\sin(dx+c)^4-28\sin(dx+c)^3-522\sin(dx+c)^2+588\sin(dx+c)-189\right)}{a(\sin(dx+c)-1)^4}-\frac{959\sin(dx+c)^5+8995\sin(dx+c)^4+20810\sin(dx+c)^3+21810\sin(dx+c)^2+11055\sin(dx+c)+2211}{a(\sin(dx+c)+1)^5}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 28*sin(d*x + c)^3 - 522*sin(d*x + c)^2 + 588*si
n(d*x + c) - 189)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 8995*sin
(d*x + c)^4 + 20810*sin(d*x + c)^3 + 21810*sin(d*x + c)^2 + 11055*sin(d*x +
 c) + 2211)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [**B**]
time = 16.77, size = 496, normalized size = 3.10

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96}+\frac{161\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{192}-\frac{469\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{480}-\frac{2681\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960}+\frac{593\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480}+\frac{5053\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960}+\frac{10841\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{5053\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960}+\frac{593\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480}-\frac{2681\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960}-\frac{469\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{480}+\frac{161\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{192}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}-\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((35*tan(c/2 + (d*x)/2)^3)/96 - (
7*tan(c/2 + (d*x)/2)^2)/64 - (7*tan(c/2 + (d*x)/2))/128 + (161*tan(c/2 + (d
*x)/2)^4)/192 - (469*tan(c/2 + (d*x)/2)^5)/480 - (2681*tan(c/2 + (d*x)/2)^6
```

)/960 + (593*tan(c/2 + (d*x)/2)^7)/480 + (5053*tan(c/2 + (d*x)/2)^8)/960 + (10841*tan(c/2 + (d*x)/2)^9)/192 + (5053*tan(c/2 + (d*x)/2)^10)/960 + (593*tan(c/2 + (d*x)/2)^11)/480 - (2681*tan(c/2 + (d*x)/2)^12)/960 - (469*tan(c/2 + (d*x)/2)^13)/480 + (161*tan(c/2 + (d*x)/2)^14)/192 + (35*tan(c/2 + (d*x)/2)^15)/96 - (7*tan(c/2 + (d*x)/2)^16)/64 - (7*tan(c/2 + (d*x)/2)^17)/128) /(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))

3.900 $\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{7\sec(c+dx)\tan(c+dx)}{256ad}+\frac{7\sec^3(c+dx)\tan(c+dx)}{128ad}-\frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad}$$

```
[Out] -7/256*arctanh(sin(d*x+c))/a/d-7/256*sec(d*x+c)*tan(d*x+c)/a/d+7/128*sec(d*
x+c)^3*tan(d*x+c)/a/d-7/96*sec(d*x+c)^3*tan(d*x+c)^3/a/d+7/80*sec(d*x+c)^3*
tan(d*x+c)^5/a/d-1/10*sec(d*x+c)^3*tan(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d+1/
10*tan(d*x+c)^10/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2687, 14, 2691, 3853, 3855}

$$\frac{\tan^{10}(c+dx)}{10ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^7(c+dx)\sec^3(c+dx)}{10ad}+\frac{7\tan^5(c+dx)\sec^3(c+dx)}{80ad}-\frac{7\tan^3(c+dx)\sec^3(c+dx)}{96ad}+\frac{7\tan(c+dx)\sec^3(c+dx)}{128ad}-\frac{7\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (7*Sec[c + d*x]^3*Tan[c +
 d*x]^3)/(96*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x]^5)/(80*a*d) - (Sec[c + d
*x]^3*Tan[c + d*x]^7)/(10*a*d) + Tan[c + d*x]^8/(8*a*d) + Tan[c + d*x]^10/(
10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a} - \frac{\int \sec^3(c+dx)\tan^8(c+dx)\,dx}{a}\\
&= -\frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} + \frac{7\int \sec^3(c+dx)\tan^6(c+dx)\,dx}{10a} + \frac{\text{Subst}\big(\int\\
&= \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx)\tan^7(c+dx)}{10ad} - \frac{7\int \sec^3(c+dx\\
&= -\frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)\tan^5(c+dx)}{80ad} - \frac{\sec^3(c+dx\\
&= \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)\tan^3(c+dx)}{96ad} + \frac{7\sec^3(c+dx)}{80a\\
&= -\frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{7\sec^3(c+dx)}{96a\\
&= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(}{128ad}\end{aligned}$$

Mathematica [A]

time = 1.23, size = 124, normalized size = 0.70

$$-\frac{210\tanh^{-1}(\sin(c+dx))-\frac{30}{(1-\sin(c+dx))^4}+\frac{160}{(1-\sin(c+dx))^3}-\frac{315}{(1-\sin(c+dx))^2}+\frac{210}{1-\sin(c+dx)}-\frac{48}{(1+\sin(c+dx))^5}+\frac{270}{(1+\sin(c+dx))^4}-\frac{580}{(1+\sin(c+dx))^3}+\frac{525}{(1+\sin(c+dx))^2}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^7)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(210*ArcTanh[Sin[c + d*x]] - 30/(1 - Sin[c + d*x])^4 + 160/(1 - Sin
[c + d*x])^3 - 315/(1 - Sin[c + d*x])^2 + 210/(1 - Sin[c + d*x]) - 48/(1 +
Sin[c + d*x])^5 + 270/(1 + Sin[c + d*x])^4 - 580/(1 + Sin[c + d*x])^3 + 525
/(1 + Sin[c + d*x])^2)/(a*d)
```

Maple [A]
time = 0.25, size = 127, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{9}{256(1+\sin(dx+c))^4}+\frac{29}{384(1+\sin(dx+c))^3}-\frac{35}{512(1+\sin(dx+c))^2}-\frac{7\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}+\frac{}{48}}{da}$
default	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{9}{256(1+\sin(dx+c))^4}+\frac{29}{384(1+\sin(dx+c))^3}-\frac{35}{512(1+\sin(dx+c))^2}-\frac{7\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}+\frac{}{48}}{da}$
risch	$i(950ie^{4i(dx+c)}+3206ie^{8i(dx+c)}+105\,e^{i(dx+c)}-4420\,e^{15i(dx+c)}-29372\,e^{11i(dx+c)}+24710\,e^{9i(dx+c)}-1778ie^{6i(dx+c)}+1$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/160/(1+sin(d*x+c))^5-9/256/(1+sin(d*x+c))^4+29/384/(1+sin(d*x+c))^
3-35/512/(1+sin(d*x+c))^2-7/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4+1/4
8/(sin(d*x+c)-1)^3+21/512/(sin(d*x+c)-1)^2+7/256/(sin(d*x+c)-1)+7/512*ln(si
n(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 214, normalized size = 1.20

$$\frac{\frac{2\left(105\sin(dx+c)^8+105\sin(dx+c)^7+895\sin(dx+c)^6-65\sin(dx+c)^5-961\sin(dx+c)^4-\sin(dx+c)^3+489\sin(dx+c)^2+9\sin(dx+c)-96\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\log(\sin(dx+c)+1)}{a}+\frac{105\log(\sin(dx+c)-1)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 + 895*sin(d*x + c)^6 - 6
5*sin(d*x + c)^5 - 961*sin(d*x + c)^4 - sin(d*x + c)^3 + 489*sin(d*x + c)^2
 + 9*sin(d*x + c) - 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
+ c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log(sin(d*
x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.41, size = 187, normalized size = 1.05

$$\frac{210 \cos\left(dx+c\right)^{8} - 2630 \cos\left(dx+c\right)^{6} + 4708 \cos\left(dx+c\right)^{4} - 3344 \cos\left(dx+c\right)^{2} - 105 \left(\cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + \cos\left(dx+c\right)^{8}\right) \log\left(\sin\left(dx+c\right) + 1\right) + 105 \left(\cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + \cos\left(dx+c\right)^{8}\right) \log\left(-\sin\left(dx+c\right) + 1\right) - 2 \left(105 \cos\left(dx+c\right)^{6} - 250 \cos\left(dx+c\right)^{4} + 184 \cos\left(dx+c\right)^{2} - 48\right) \sin\left(dx+c\right) + 864}{7680 \left(ad \cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + ad \cos\left(dx+c\right)^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 2630*cos(d*x + c)^6 + 4708*cos(d*x + c)^4 - 33
44*cos(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(
-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 - 250*cos(d*x + c)^4 + 184*cos(d
*x + c)^2 - 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*
cos(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.66, size = 156, normalized size = 0.88

$$-\frac{\frac{420 \log\left(|\sin\left(dx+c\right)+1|\right)}{a} - \frac{420 \log\left(|\sin\left(dx+c\right)-1|\right)}{a} + \frac{5 \left(175 \sin\left(dx+c\right)^{4} - 868 \sin\left(dx+c\right)^{3} + 1302 \sin\left(dx+c\right)^{2} - 828 \sin\left(dx+c\right) + 195\right)}{a \left(\sin\left(dx+c\right) - 1\right)^{4}} - \frac{959 \sin\left(dx+c\right)^{5} + 4795 \sin\left(dx+c\right)^{4} + 7490 \sin\left(dx+c\right)^{3} + 5610 \sin\left(dx+c\right)^{2} + 2055 \sin\left(dx+c\right) + 291}{a \left(\sin\left(dx+c\right) + 1\right)^{5}}}{30720 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1302*sin(d*x + c)^2 - 828
*sin(d*x + c) + 195)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 7490*sin(d*x + c)^3 + 5610*sin(d*x + c)^2 + 2055*sin(d*x +
 c) + 291)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 16.70, size = 496, normalized size = 2.79

$$\frac{\frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128} + \frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64} - \frac{35 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96} - \frac{161 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{192} + \frac{469 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{480} + \frac{2681 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960} - \frac{593 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480} + \frac{25667 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960} + \frac{1447 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192} + \frac{25667 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960} - \frac{593 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480} + \frac{2681 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960} + \frac{469 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{480} - \frac{161 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{192} - \frac{35 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96} + \frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64} + \frac{7 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d \left(a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17} - 7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15} + 20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} - 28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 140 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} + 14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7} - 28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5} + 20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3} - 7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right) + a\right)} - \frac{7 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128 \, a \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((7*tan(c/2 + (d*x)/2))/128 + (7*tan(c/2 + (d*x)/2)^2)/64 - (35*tan(c/2 + (
d*x)/2)^3)/96 - (161*tan(c/2 + (d*x)/2)^4)/192 + (469*tan(c/2 + (d*x)/2)^5)
/480 + (2681*tan(c/2 + (d*x)/2)^6)/960 - (593*tan(c/2 + (d*x)/2)^7)/480 + (
25667*tan(c/2 + (d*x)/2)^8)/960 + (1447*tan(c/2 + (d*x)/2)^9)/192 + (25667*
tan(c/2 + (d*x)/2)^10)/960 - (593*tan(c/2 + (d*x)/2)^11)/480 + (2681*tan(c/
2 + (d*x)/2)^12)/960 + (469*tan(c/2 + (d*x)/2)^13)/480 - (161*tan(c/2 + (d*
x)/2)^14)/192 - (35*tan(c/2 + (d*x)/2)^15)/96 + (7*tan(c/2 + (d*x)/2)^16)/6
4 + (7*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan
(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 +
 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d
*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*ta
n(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)
^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/
2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a
*tan(c/2 + (d*x)/2)^18)) - (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

3.901 $\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=176

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{3\sec(c+dx)\tan(c+dx)}{256ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^5(c+dx)\tan(c+dx)}{32ad}-\text{se}$$

```
[Out] -3/256*arctanh(sin(d*x+c))/a/d-3/256*sec(d*x+c)*tan(d*x+c)/a/d-1/128*sec(d*
x+c)^3*tan(d*x+c)/a/d+1/32*sec(d*x+c)^5*tan(d*x+c)/a/d-1/16*sec(d*x+c)^5*ta
n(d*x+c)^3/a/d+1/10*sec(d*x+c)^5*tan(d*x+c)^5/a/d-1/8*tan(d*x+c)^8/a/d-1/10
*tan(d*x+c)^10/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2691, 3853, 3855, 2687, 14}

$$-\frac{\tan^{10}(c+dx)}{10ad}-\frac{\tan^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}-\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{32ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{128ad}-\frac{3\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a
*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*x
])/(32*a*d) - (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) + (Sec[c + d*x]^5*Ta
n[c + d*x]^5)/(10*a*d) - Tan[c + d*x]^8/(8*a*d) - Tan[c + d*x]^10/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a}-\frac{\int \sec^4(c+dx)\tan^7(c+dx)\,dx}{a}\\
&= \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad}-\frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a}-\frac{\operatorname{Subst}\left(\int x\right.}{}\\
&= -\frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad}+\frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad}+\frac{3\int \sec^5(c+}{}\\
&= \frac{\sec^5(c+dx)\tan(c+dx)}{32ad}-\frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad}+\frac{\sec^5(c+dx)\tan}{10ad}\\
&= -\frac{\sec^3(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^5(c+dx)\tan(c+dx)}{32ad}-\frac{\sec^5(c+dx)\tan}{16ad}\\
&= -\frac{3\sec(c+dx)\tan(c+dx)}{256ad}-\frac{\sec^3(c+dx)\tan(c+dx)}{128ad}+\frac{\sec^5(c+dx)\tan}{32ad}\\
&= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{3\sec(c+dx)\tan(c+dx)}{256ad}-\frac{\sec^3(c+dx)\tan(}{128ad}
\end{aligned}
$$

Mathematica [A]

time = 1.91, size = 122, normalized size = 0.69

$$-\frac{30\tanh^{-1}(\sin(c+dx))-\frac{2(32+47\sin(c+dx)-113\sin^2(c+dx)-183\sin^3(c+dx)+137\sin^4(c+dx)+265\sin^5(c+dx)-55\sin^6(c+dx)+15\sin^7(c+dx)+15\sin^8(c+dx))}{(-1+\sin(c+dx))^4(1+\sin(c+dx))^5}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^6)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(30*ArcTanh[Sin[c + d*x]] - (2*(32 + 47*Sin[c + d*x] - 113*Sin[c +
d*x]^2 - 183*Sin[c + d*x]^3 + 137*Sin[c + d*x]^4 + 265*Sin[c + d*x]^5 - 55*
Sin[c + d*x]^6 + 15*Sin[c + d*x]^7 + 15*Sin[c + d*x]^8))/((-1 + Sin[c + d*x
])^4*(1 + Sin[c + d*x])^5))/(a*d)
```

Maple [A]

time = 0.26, size = 139, normalized size = 0.79

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{7}{256(1+\sin(dx+c))^4}-\frac{5}{128(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{5}{256(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{7}{256(1+\sin(dx+c))^4}-\frac{5}{128(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{5}{256(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
risch	$\frac{i(2330i e^{4i(dx+c)}+15\,e^{17i(dx+c)}+10698i e^{8i(dx+c)}+100\,e^{15i(dx+c)}-5374i e^{6i(dx+c)}+1292\,e^{13i(dx+c)}+5374i e^{12i(dx+c)}+92}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5+7/256/(1+sin(d*x+c))^4-5/128/(1+sin(d*x+c))^
3+5/512/(1+sin(d*x+c))^2+5/256/(1+sin(d*x+c))-3/512*ln(1+sin(d*x+c))+1/256/
(sin(d*x+c)-1)^4+1/64/(sin(d*x+c)-1)^3+9/512/(sin(d*x+c)-1)^2-1/128/(sin(d*
x+c)-1)+3/512*ln(sin(d*x+c)-1))
```

Maxima [A]

time = 0.30, size = 214, normalized size = 1.22

$$\frac{\frac{2\left(15\sin(dx+c)^8+15\sin(dx+c)^7-55\sin(dx+c)^6+265\sin(dx+c)^5+137\sin(dx+c)^4-183\sin(dx+c)^3-113\sin(dx+c)^2+47\sin(dx+c)+32\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 + 265*
sin(d*x + c)^5 + 137*sin(d*x + c)^4 - 183*sin(d*x + c)^3 - 113*sin(d*x + c)
^2 + 47*sin(d*x + c) + 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d
*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(
d*x + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 187, normalized size = 1.06

$$\frac{30 \cos\left(dx+c\right)^{8} - 10 \cos\left(dx+c\right)^{6} + 124 \cos\left(dx+c\right)^{4} - 112 \cos\left(dx+c\right)^{2} - 15 \left(\cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + \cos\left(dx+c\right)^{8}\right) \log\left(\sin\left(dx+c\right)+1\right) + 15 \left(\cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + \cos\left(dx+c\right)^{8}\right) \log\left(-\sin\left(dx+c\right)+1\right) - 2 \left(15 \cos\left(dx+c\right)^{6} - 310 \cos\left(dx+c\right)^{4} + 392 \cos\left(dx+c\right)^{2} - 144\right) \sin\left(dx+c\right) + 32}{2560 \left(ad \cos\left(dx+c\right)^{8} \sin\left(dx+c\right) + ad \cos\left(dx+c\right)^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 + 124*cos(d*x + c)^4 - 112*co
s(d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(15*cos(d*x + c)^6 - 310*cos(d*x + c)^4 + 392*cos(d*x + c)^
2 - 144)*sin(d*x + c) + 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 156, normalized size = 0.89

$$-\frac{\frac{60 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a} - \frac{60 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a} + \frac{5 \left(25 \sin\left(dx+c\right)^{4} - 84 \sin\left(dx+c\right)^{3} + 66 \sin\left(dx+c\right)^{2} - 12 \sin\left(dx+c\right) - 3\right)}{a \left(\sin\left(dx+c\right)-1\right)^{4}} - \frac{137 \sin\left(dx+c\right)^{5} + 885 \sin\left(dx+c\right)^{4} + 2270 \sin\left(dx+c\right)^{3} + 2470 \sin\left(dx+c\right)^{2} + 1265 \sin\left(dx+c\right) + 253}{a \left(\sin\left(dx+c\right)+1\right)^{5}}}{10240 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 84*sin(d*x + c)^3 + 66*sin(d*x + c)^2 - 12*sin(d*
x + c) - 3)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 885*sin(d*x +
c)^4 + 2270*sin(d*x + c)^3 + 2470*sin(d*x + c)^2 + 1265*sin(d*x + c) + 253)
/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 16.64, size = 496, normalized size = 2.82

$$\frac{\frac{3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128} + \frac{3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64} - \frac{5 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32} - \frac{23 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64} + \frac{67 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160} + \frac{383 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{320} + \frac{2841 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{160} + \frac{741 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{320} + \frac{1377 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64} + \frac{741 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{320} + \frac{2841 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{160} + \frac{383 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{320} + \frac{67 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160} - \frac{23 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64} - \frac{5 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32} + \frac{3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64} + \frac{3 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d \left(a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17} - 7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} - 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15} + 20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} - 28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} - 112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 140 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} + 14 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} - 112 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7} - 28 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 56 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5} + 20 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} - 16 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3} - 7 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 2 a \tan\left(\frac{c}{2}+\frac{dx}{2}\right) + a\right)} - \frac{3 \operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128 \, a \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/128 + (3*tan(c/2 + (d*x)/2)^2)/64 - (5*tan(c/2 + (d
*x)/2)^3)/32 - (23*tan(c/2 + (d*x)/2)^4)/64 + (67*tan(c/2 + (d*x)/2)^5)/160
 + (383*tan(c/2 + (d*x)/2)^6)/320 + (2841*tan(c/2 + (d*x)/2)^7)/160 + (741*
tan(c/2 + (d*x)/2)^8)/320 + (1377*tan(c/2 + (d*x)/2)^9)/64 + (741*tan(c/2 +
 (d*x)/2)^10)/320 + (2841*tan(c/2 + (d*x)/2)^11)/160 + (383*tan(c/2 + (d*x)
/2)^12)/320 + (67*tan(c/2 + (d*x)/2)^13)/160 - (23*tan(c/2 + (d*x)/2)^14)/6
4 - (5*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^16)/64 + (3*tan(c/
2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/
2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2
 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14
*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)
/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*ta
n(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^
15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d
*x)/2)^18)) - (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

3.902 $\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\sec^6(c+dx)}{6ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^{10}(c+dx)}{10ad}+\frac{3\sec(c+dx)\tan(c+dx)}{256ad}+\frac{\sec^3(c+dx}{12}$$

```
[Out] 3/256*arctanh(sin(d*x+c))/a/d+1/6*sec(d*x+c)^6/a/d-1/4*sec(d*x+c)^8/a/d+1/1
0*sec(d*x+c)^10/a/d+3/256*sec(d*x+c)*tan(d*x+c)/a/d+1/128*sec(d*x+c)^3*tan(
d*x+c)/a/d-1/32*sec(d*x+c)^5*tan(d*x+c)/a/d+1/16*sec(d*x+c)^5*tan(d*x+c)^3/
a/d-1/10*sec(d*x+c)^5*tan(d*x+c)^5/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2914, 2686, 272, 45, 2691, 3853, 3855}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{4ad}+\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^5(c+dx)\sec^5(c+dx)}{10ad}+\frac{\tan^3(c+dx)\sec^5(c+dx)}{16ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{32ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{128ad}+\frac{3\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^6/(6*a*d) - Sec[c + d*x]
^8/(4*a*d) + Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*
x])/(32*a*d) + (Sec[c + d*x]^5*Tan[c + d*x]^3)/(16*a*d) - (Sec[c + d*x]^5*T
an[c + d*x]^5)/(10*a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec^5(c+dx)\tan^6(c+dx)\,dx}{a}$$
$$= -\frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} + \frac{\int \sec^5(c+dx)\tan^4(c+dx)\,dx}{2a} + \frac{\text{Subst}\Big(\int$$
$$= \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\tan^5(c+dx)}{10ad} - \frac{3\int \sec^5(c+dx}{}$$
$$= -\frac{\sec^5(c+dx)\tan(c+dx)}{32ad} + \frac{\sec^5(c+dx)\tan^3(c+dx)}{16ad} - \frac{\sec^5(c+dx)\,\mathrm{t}}{10a}$$
$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad}$$
$$= \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx)}{256ad}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^6(c+dx)}{6ad} - \frac{\sec^8(c+dx)}{4ad} + \frac{\sec^{10}(c+dx)}{10ad} +$$

Mathematica [A]
time = 3.73, size = 116, normalized size = 0.60

$$\frac{90\tanh^{-1}(\sin(c+dx)) + \frac{30}{(-1+\sin(c+dx))^4} + \frac{80}{(-1+\sin(c+dx))^3} + \frac{15}{(-1+\sin(c+dx))^2} - \frac{90}{-1+\sin(c+dx)} + \frac{48}{(1+\sin(c+dx))^5} - \frac{150}{(1+\sin(c+dx))^4} + \frac{100}{(1+\sin(c+dx))^3} + \frac{75}{(1+\sin(c+dx))^2}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x]^5)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 80/(-1 + Sin[c + d*x
])^3 + 15/(-1 + Sin[c + d*x])^2 - 90/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c +
d*x])^5 - 150/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + Sin
[c + d*x])^2)/(7680*a*d)
```

Maple [A]
time = 0.23, size = 127, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{1}{160(1+\sin(dx+c))^5} - \frac{5}{256(1+\sin(dx+c))^4} + \frac{5}{384(1+\sin(dx+c))^3} + \frac{5}{512(1+\sin(dx+c))^2} + \frac{3\ln(1+\sin(dx+c))}{512} + \frac{1}{256(\sin(dx+c)-1)^4} + \frac{}{96}}{da}$
default	$\frac{\frac{1}{160(1+\sin(dx+c))^5} - \frac{5}{256(1+\sin(dx+c))^4} + \frac{5}{384(1+\sin(dx+c))^3} + \frac{5}{512(1+\sin(dx+c))^2} + \frac{3\ln(1+\sin(dx+c))}{512} + \frac{1}{256(\sin(dx+c)-1)^4} + \frac{}{96}}{da}$
risch	$-\frac{i\left(-690ie^{4i(dx+c)}+45\,e^{i(dx+c)}-3746ie^{8i(dx+c)}+23252\,e^{11i(dx+c)}-40610\,e^{9i(dx+c)}+300\,e^{15i(dx+c)}+1798ie^{6i(dx+c)}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/160/(1+sin(d*x+c))^5-5/256/(1+sin(d*x+c))^4+5/384/(1+sin(d*x+c))^3
+5/512/(1+sin(d*x+c))^2+3/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4+1/96/
(sin(d*x+c)-1)^3+1/512/(sin(d*x+c)-1)^2-3/256/(sin(d*x+c)-1)-3/512*ln(sin(d
*x+c)-1))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 1.10

$$-\frac{\frac{2\left(45\,\sin\left(dx+c\right)^{8}+45\,\sin\left(dx+c\right)^{7}-165\,\sin\left(dx+c\right)^{6}-165\,\sin\left(dx+c\right)^{5}-549\,\sin\left(dx+c\right)^{4}+91\,\sin\left(dx+c\right)^{3}+301\,\sin\left(dx+c\right)^{2}-19\,\sin\left(dx+c\right)-64\right)}{a\sin\left(dx+c\right)^{9}+a\sin\left(dx+c\right)^{8}-4\,a\sin\left(dx+c\right)^{7}-4\,a\sin\left(dx+c\right)^{6}+6\,a\sin\left(dx+c\right)^{5}+6\,a\sin\left(dx+c\right)^{4}-4\,a\sin\left(dx+c\right)^{3}-4\,a\sin\left(dx+c\right)^{2}+a\sin\left(dx+c\right)+a}-\frac{45\,\log\left(\sin\left(dx+c\right)+1\right)}{a}+\frac{45\,\log\left(\sin\left(dx+c\right)-1\right)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 - 549*sin(d*x + c)^4 + 91*sin(d*x + c)^3 + 301*sin(d*x + c
)^2 - 19*sin(d*x + c) - 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(
d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 -
 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(sin
(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.41, size = 187, normalized size = 0.96

$$-\frac{90\,\cos\left(dx+c\right)^{8}-30\,\cos\left(dx+c\right)^{6}-1548\,\cos\left(dx+c\right)^{4}+2224\,\cos\left(dx+c\right)^{2}-45\,\left(\cos\left(dx+c\right)^{8}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{8}\right)\log\left(\sin\left(dx+c\right)+1\right)+45\,\left(\cos\left(dx+c\right)^{8}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{8}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(45\,\cos\left(dx+c\right)^{6}+30\,\cos\left(dx+c\right)^{4}-104\,\cos\left(dx+c\right)^{2}+48\right)\sin\left(dx+c\right)-864}{7680\,\left(ad\cos\left(dx+c\right)^{8}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 1548*cos(d*x + c)^4 + 2224
*cos(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin
(d*x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 104*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*
x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.65, size = 156, normalized size = 0.80

$$\frac{\frac{180 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a} - \frac{180 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a} + \frac{5 \left(75 \sin\left(dx+c\right)^{4} - 372 \sin\left(dx+c\right)^{3} + 678 \sin\left(dx+c\right)^{2} - 476 \sin\left(dx+c\right) + 119\right)}{a \left(\sin\left(dx+c\right)-1\right)^{4}} - \frac{411 \sin\left(dx+c\right)^{5} + 2055 \sin\left(dx+c\right)^{4} + 3810 \sin\left(dx+c\right)^{3} + 2810 \sin\left(dx+c\right)^{2} + 955 \sin\left(dx+c\right) + 119}{a \left(\sin\left(dx+c\right)+1\right)^{5}}}{30720 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 372*sin(d*x + c)^3 + 678*sin(d*x + c)^2 - 476*si
n(d*x + c) + 119)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2055*sin
(d*x + c)^4 + 3810*sin(d*x + c)^3 + 2810*sin(d*x + c)^2 + 955*sin(d*x + c)
+ 119)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 16.62, size = 496, normalized size = 2.56

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64}-\frac{67\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160}+\frac{9091\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960}+\frac{1717\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480}+\frac{18257\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960}-\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{18257\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960}+\frac{1717\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480}+\frac{9091\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960}-\frac{67\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^3)/32 - (3
*tan(c/2 + (d*x)/2)^2)/64 - (3*tan(c/2 + (d*x)/2))/128 + (23*tan(c/2 + (d*x
)/2)^4)/64 - (67*tan(c/2 + (d*x)/2)^5)/160 + (9091*tan(c/2 + (d*x)/2)^6)/96
0 + (1717*tan(c/2 + (d*x)/2)^7)/480 + (18257*tan(c/2 + (d*x)/2)^8)/960 - (3
5*tan(c/2 + (d*x)/2)^9)/192 + (18257*tan(c/2 + (d*x)/2)^10)/960 + (1717*tan
(c/2 + (d*x)/2)^11)/480 + (9091*tan(c/2 + (d*x)/2)^12)/960 - (67*tan(c/2 +
(d*x)/2)^13)/160 + (23*tan(c/2 + (d*x)/2)^14)/64 + (5*tan(c/2 + (d*x)/2)^15
)/32 - (3*tan(c/2 + (d*x)/2)^16)/64 - (3*tan(c/2 + (d*x)/2)^17)/128)/(d*(a
+ 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/
2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2
 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 14
0*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*
x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*t
an(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^
16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

3.903 $\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=192

$$\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^{10}(c+dx)}{10ad}+\frac{3\sec(c+dx)\tan(c+dx)}{256ad}+\frac{\sec^3(c+dx)}{128}$$

```
[Out] 3/256*arctanh(sin(d*x+c))/a/d-1/6*sec(d*x+c)^6/a/d+1/4*sec(d*x+c)^8/a/d-1/1
0*sec(d*x+c)^10/a/d+3/256*sec(d*x+c)*tan(d*x+c)/a/d+1/128*sec(d*x+c)^3*tan(
d*x+c)/a/d+1/160*sec(d*x+c)^5*tan(d*x+c)/a/d-3/80*sec(d*x+c)^7*tan(d*x+c)/a
/d+1/10*sec(d*x+c)^7*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2914, 2691, 3853, 3855, 2686, 272, 45}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{4ad}-\frac{\sec^6(c+dx)}{6ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}-\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{\tan(c+dx)\sec^5(c+dx)}{160ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{128ad}+\frac{3\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^6/(6*a*d) + Sec[c + d*x]
^8/(4*a*d) - Sec[c + d*x]^10/(10*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(256*
a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(128*a*d) + (Sec[c + d*x]^5*Tan[c + d*
x])/(160*a*d) - (3*Sec[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^7*
Tan[c + d*x]^3)/(10*a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a} - \frac{\int \sec^6(c+dx)\tan^5(c+dx)\,dx}{a}$$

$$= \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} - \frac{\text{Subst}\Big(\int$$

$$= -\frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+d}{80a}$$

$$= \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} - \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^7(c+dx)\tan^3}{10ad}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{128ad}$$

$$= -\frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3\sec(c+dx)\tan(c+dx}{256ad}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^6(c+dx)}{6ad} + \frac{\sec^8(c+dx)}{4ad} - \frac{\sec^{10}(c+dx)}{10ad} + \frac{3}{}$$

Mathematica [A]

time = 4.24, size = 116, normalized size = 0.60

$$\frac{90\tanh^{-1}(\sin(c+dx)) + \frac{30}{(-1+\sin(c+dx))^4} + \frac{40}{(-1+\sin(c+dx))^3} - \frac{45}{(-1+\sin(c+dx))^2} - \frac{48}{(1+\sin(c+dx))^5} + \frac{90}{(1+\sin(c+dx))^4} + \frac{20}{(1+\sin(c+dx))^3} - \frac{45}{(1+\sin(c+dx))^2} - \frac{90}{1+\sin(c+dx)}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Tan[c + d*x]^4)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (90*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 + 40/(-1 + Sin[c + d*x
])^3 - 45/(-1 + Sin[c + d*x])^2 - 48/(1 + Sin[c + d*x])^5 + 90/(1 + Sin[c +
 d*x])^4 + 20/(1 + Sin[c + d*x])^3 - 45/(1 + Sin[c + d*x])^2 - 90/(1 + Sin[
c + d*x]))/(7680*a*d)
```

Maple [A]

time = 0.23, size = 127, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{3}{256(1+\sin(dx+c))^4}+\frac{1}{384(1+\sin(dx+c))^3}-\frac{3}{512(1+\sin(dx+c))^2}-\frac{3}{256(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}+\frac{3}{256(1+\sin(dx+c))^4}+\frac{1}{384(1+\sin(dx+c))^3}-\frac{3}{512(1+\sin(dx+c))^2}-\frac{3}{256(1+\sin(dx+c))}+\frac{3\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
risch	$-\frac{i(-690i e^{4i(dx+c)}+45\,e^{17i(dx+c)}-36514i e^{8i(dx+c)}+300\,e^{15i(dx+c)}+18182i e^{6i(dx+c)}+804\,e^{13i(dx+c)}-18182i e^{12i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5+3/256/(1+sin(d*x+c))^4+1/384/(1+sin(d*x+c))^
3-3/512/(1+sin(d*x+c))^2-3/256/(1+sin(d*x+c))+3/512*ln(1+sin(d*x+c))+1/256/
(sin(d*x+c)-1)^4+1/192/(sin(d*x+c)-1)^3-3/512/(sin(d*x+c)-1)^2-3/512*ln(sin
(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 1.11

$$-\frac{\frac{2\left(45\sin(dx+c)^8+45\sin(dx+c)^7-165\sin(dx+c)^6-165\sin(dx+c)^5+219\sin(dx+c)^4-421\sin(dx+c)^3-211\sin(dx+c)^2+109\sin(dx+c)+64\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{45\log(\sin(dx+c)+1)}{a}+\frac{45\log(\sin(dx+c)-1)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(45*sin(d*x + c)^8 + 45*sin(d*x + c)^7 - 165*sin(d*x + c)^6 - 16
5*sin(d*x + c)^5 + 219*sin(d*x + c)^4 - 421*sin(d*x + c)^3 - 211*sin(d*x +
c)^2 + 109*sin(d*x + c) + 64)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*si
n(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4
 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 45*log(s
in(d*x + c) + 1)/a + 45*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.41, size = 187, normalized size = 0.97

$$-\frac{90\cos(dx+c)^8-30\cos(dx+c)^6-12\cos(dx+c)^4+176\cos(dx+c)^2-45\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+45\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(45\cos(dx+c)^6+30\cos(dx+c)^4-616\cos(dx+c)^2+432\right)\sin(dx+c)-96}{7680\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(90*cos(d*x + c)^8 - 30*cos(d*x + c)^6 - 12*cos(d*x + c)^4 + 176*co
s(d*x + c)^2 - 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 45*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*
x + c) + 1) - 2*(45*cos(d*x + c)^6 + 30*cos(d*x + c)^4 - 616*cos(d*x + c)^2
 + 432)*sin(d*x + c) - 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x +
 c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 156, normalized size = 0.81

$$\frac{\frac{180 \log(|\sin(dx+c)+1|)}{a}-\frac{180 \log(|\sin(dx+c)-1|)}{a}+\frac{5\left(75 \sin(dx+c)^4-300 \sin(dx+c)^3+414 \sin(dx+c)^2-196 \sin(dx+c)+31\right)}{a(\sin(dx+c)-1)^4}-\frac{411 \sin(dx+c)^5+2415 \sin(dx+c)^4+5730 \sin(dx+c)^3+6730 \sin(dx+c)^2+3515 \sin(dx+c)+703}{a(\sin(dx+c)+1)^5}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(180*log(abs(sin(d*x + c) + 1))/a - 180*log(abs(sin(d*x + c) - 1))/
a + 5*(75*sin(d*x + c)^4 - 300*sin(d*x + c)^3 + 414*sin(d*x + c)^2 - 196*si
n(d*x + c) + 31)/(a*(sin(d*x + c) - 1)^4) - (411*sin(d*x + c)^5 + 2415*sin(
d*x + c)^4 + 5730*sin(d*x + c)^3 + 6730*sin(d*x + c)^2 + 3515*sin(d*x + c)
+ 703)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 16.71, size = 496, normalized size = 2.58

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64}+\frac{957\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160}+\frac{899\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960}+\frac{5813\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480}+\frac{1873\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960}+\frac{4061\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{1873\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960}+\frac{5813\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480}+\frac{899\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960}+\frac{957\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160}+\frac{23\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((5*tan(c/2 + (d*x)/2)^3)/32 - (3
*tan(c/2 + (d*x)/2)^2)/64 - (3*tan(c/2 + (d*x)/2))/128 + (23*tan(c/2 + (d*x
)/2)^4)/64 + (957*tan(c/2 + (d*x)/2)^5)/160 + (899*tan(c/2 + (d*x)/2)^6)/96
0 + (5813*tan(c/2 + (d*x)/2)^7)/480 + (1873*tan(c/2 + (d*x)/2)^8)/960 + (40
61*tan(c/2 + (d*x)/2)^9)/192 + (1873*tan(c/2 + (d*x)/2)^10)/960 + (5813*tan
(c/2 + (d*x)/2)^11)/480 + (899*tan(c/2 + (d*x)/2)^12)/960 + (957*tan(c/2 +
(d*x)/2)^13)/160 + (23*tan(c/2 + (d*x)/2)^14)/64 + (5*tan(c/2 + (d*x)/2)^15
)/32 - (3*tan(c/2 + (d*x)/2)^16)/64 - (3*tan(c/2 + (d*x)/2)^17)/128)/(d*(a
+ 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/
2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2
 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 14
0*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*
x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*t
an(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^
16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))
```

3.904 $\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\sec^8(c+dx)}{8ad}+\frac{\sec^{10}(c+dx)}{10ad}-\frac{3\sec(c+dx)\tan(c+dx)}{256ad}-\frac{\sec^3(c+dx)\tan(c+dx}{128ad}$$

```
[Out] -3/256*arctanh(sin(d*x+c))/a/d-1/8*sec(d*x+c)^8/a/d+1/10*sec(d*x+c)^10/a/d-
3/256*sec(d*x+c)*tan(d*x+c)/a/d-1/128*sec(d*x+c)^3*tan(d*x+c)/a/d-1/160*sec
(d*x+c)^5*tan(d*x+c)/a/d+3/80*sec(d*x+c)^7*tan(d*x+c)/a/d-1/10*sec(d*x+c)^7
*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2686, 14, 2691, 3853, 3855}

$$\frac{\sec^{10}(c+dx)}{10ad}-\frac{\sec^8(c+dx)}{8ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan^3(c+dx)\sec^7(c+dx)}{10ad}+\frac{3\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{\tan(c+dx)\sec^5(c+dx)}{160ad}-\frac{\tan(c+dx)\sec^3(c+dx)}{128ad}-\frac{3\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(256*a*d) - Sec[c + d*x]^8/(8*a*d) + Sec[c + d*x
]^10/(10*a*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (Sec[c + d*x]^3*T
an[c + d*x])/(128*a*d) - (Sec[c + d*x]^5*Tan[c + d*x])/(160*a*d) + (3*Sec[c
 + d*x]^7*Tan[c + d*x])/(80*a*d) - (Sec[c + d*x]^7*Tan[c + d*x]^3)/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a} - \frac{\int \sec^7(c+dx)\tan^4(c+dx)\,dx}{a}\\
&= -\frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} + \frac{3\int \sec^7(c+dx)\tan^2(c+dx)\,dx}{10a} + \frac{\text{Subst}\big(\int\\
&= \frac{3\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^7(c+dx)\tan^3(c+dx)}{10ad} - \frac{3\int \sec^7(c+dx)}{80a}\\
&= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^5(c+dx)\tan(c+dx)}{160ad} + \frac{3\sec^7(c+d}{8}\\
&= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{\sec^3(c+dx)\tan(c+dx)}{128ad} - \frac{\sec^5(c+dx)}{16}\\
&= -\frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\tan(c+dx)}{256ad} - \frac{\sec^3(c+dx}{12}\\
&= -\frac{3\tanh^{-1}(\sin(c+dx))}{256ad} - \frac{\sec^8(c+dx)}{8ad} + \frac{\sec^{10}(c+dx)}{10ad} - \frac{3\sec(c+dx)\,\text{t}}{256a}\end{aligned}$$

Mathematica [A]
time = 1.89, size = 104, normalized size = 0.60

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{10}{(-1+\sin(c+dx))^4} + \frac{15}{(-1+\sin(c+dx))^2} - \frac{30}{-1+\sin(c+dx)} - \frac{16}{(1+\sin(c+dx))^5} + \frac{10}{(1+\sin(c+dx))^4} + \frac{20}{(1+\sin(c+dx))^3} + \frac{15}{(1+\sin(c+dx))^2}}{2560ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*Tan[c + d*x]^3)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/2560*(30*ArcTanh[Sin[c + d*x]] - 10/(-1 + Sin[c + d*x])^4 + 15/(-1 + Sin
[c + d*x])^2 - 30/(-1 + Sin[c + d*x]) - 16/(1 + Sin[c + d*x])^5 + 10/(1 + S
in[c + d*x])^4 + 20/(1 + Sin[c + d*x])^3 + 15/(1 + Sin[c + d*x])^2)/(a*d)
```

Maple [A]
time = 0.23, size = 115, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{1}{256(1+\sin(dx+c))^4}-\frac{1}{128(1+\sin(dx+c))^3}-\frac{3}{512(1+\sin(dx+c))^2}-\frac{3\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}-\frac{5}{}}{da}$
default	$\frac{\frac{1}{160(1+\sin(dx+c))^5}-\frac{1}{256(1+\sin(dx+c))^4}-\frac{1}{128(1+\sin(dx+c))^3}-\frac{3}{512(1+\sin(dx+c))^2}-\frac{3\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}-\frac{5}{}}{da}$
risch	$\frac{i(-230i e^{4i(dx+c)}+15\,e^{i(dx+c)}-11364\,e^{11i(dx+c)}+13770\,e^{9i(dx+c)}+100\,e^{15i(dx+c)}+1482i e^{8i(dx+c)}-766i e^{6i(dx+c)}+766}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/160/(1+sin(d*x+c))^5-1/256/(1+sin(d*x+c))^4-1/128/(1+sin(d*x+c))^3
-3/512/(1+sin(d*x+c))^2-3/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4-3/512
/(sin(d*x+c)-1)^2+3/256/(sin(d*x+c)-1)+3/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 1.23

$$\frac{\frac{2\left(15\sin(dx+c)^8+15\sin(dx+c)^7-55\sin(dx+c)^6-55\sin(dx+c)^5+73\sin(dx+c)^4+73\sin(dx+c)^3+143\sin(dx+c)^2-17\sin(dx+c)-32\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{15\log(\sin(dx+c)+1)}{a}+\frac{15\log(\sin(dx+c)-1)}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(15*sin(d*x + c)^8 + 15*sin(d*x + c)^7 - 55*sin(d*x + c)^6 - 55*s
in(d*x + c)^5 + 73*sin(d*x + c)^4 + 73*sin(d*x + c)^3 + 143*sin(d*x + c)^2
 - 17*sin(d*x + c) - 32)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*sin(d*x
 + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^4 - 4*a
*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 15*log(sin(d*x
 + c) + 1)/a + 15*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.42, size = 187, normalized size = 1.07

$$\frac{30\cos(dx+c)^8-10\cos(dx+c)^6-4\cos(dx+c)^4-368\cos(dx+c)^2-15\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+15\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(15\cos(dx+c)^6+10\cos(dx+c)^4+8\cos(dx+c)^2-16\right)\sin(dx+c)+288}{2560\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(30*cos(d*x + c)^8 - 10*cos(d*x + c)^6 - 4*cos(d*x + c)^4 - 368*cos(
d*x + c)^2 - 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*x
+ c) + 1) + 15*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d*x
+ c) + 1) - 2*(15*cos(d*x + c)^6 + 10*cos(d*x + c)^4 + 8*cos(d*x + c)^2 - 1
6)*sin(d*x + c) + 288)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x + c)^
8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [A]
time = 0.51, size = 156, normalized size = 0.90

$$-\frac{\frac{60\log\left(|\sin\left(dx+c\right)+1|\right)}{a}-\frac{60\log\left(|\sin\left(dx+c\right)-1|\right)}{a}+\frac{5\left(25\sin\left(dx+c\right)^4-124\sin\left(dx+c\right)^3+234\sin\left(dx+c\right)^2-196\sin\left(dx+c\right)+53\right)}{a\left(\sin\left(dx+c\right)-1\right)^4}-\frac{137\sin\left(dx+c\right)^5+685\sin\left(dx+c\right)^4+1310\sin\left(dx+c\right)^3+1110\sin\left(dx+c\right)^2+305\sin\left(dx+c\right)+21}{a\left(\sin\left(dx+c\right)+1\right)^5}}{10240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(60*log(abs(sin(d*x + c) + 1))/a - 60*log(abs(sin(d*x + c) - 1))/a
 + 5*(25*sin(d*x + c)^4 - 124*sin(d*x + c)^3 + 234*sin(d*x + c)^2 - 196*sin
(d*x + c) + 53)/(a*(sin(d*x + c) - 1)^4) - (137*sin(d*x + c)^5 + 685*sin(d*
x + c)^4 + 1310*sin(d*x + c)^3 + 1110*sin(d*x + c)^2 + 305*sin(d*x + c) + 2
1)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 17.34, size = 496, normalized size = 2.85

$$\frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{32}+\frac{233\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{64}+\frac{323\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{160}+\frac{2687\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{320}-\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{160}+\frac{5349\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{320}+\frac{353\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{64}+\frac{5349\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{320}-\frac{231\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{160}+\frac{2687\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{320}+\frac{323\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{160}+\frac{233\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{64}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{32}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2))/128 + (3*tan(c/2 + (d*x)/2)^2)/64 - (5*tan(c/2 + (d
*x)/2)^3)/32 + (233*tan(c/2 + (d*x)/2)^4)/64 + (323*tan(c/2 + (d*x)/2)^5)/1
60 + (2687*tan(c/2 + (d*x)/2)^6)/320 - (231*tan(c/2 + (d*x)/2)^7)/160 + (53
49*tan(c/2 + (d*x)/2)^8)/320 + (353*tan(c/2 + (d*x)/2)^9)/64 + (5349*tan(c/
2 + (d*x)/2)^10)/320 - (231*tan(c/2 + (d*x)/2)^11)/160 + (2687*tan(c/2 + (d
```

*x)/2)^12)/320 + (323*tan(c/2 + (d*x)/2)^13)/160 + (233*tan(c/2 + (d*x)/2)^14)/64 - (5*tan(c/2 + (d*x)/2)^15)/32 + (3*tan(c/2 + (d*x)/2)^16)/64 + (3*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18)) - (3*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)

3.905 $\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=172

$$-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{\sec^{10}(c+dx)}{10ad}-\frac{7\sec(c+dx)\tan(c+dx)}{256ad}-\frac{7\sec^3(c+dx)\tan(c+dx}{384ad}$$

```
[Out] -7/256*arctanh(sin(d*x+c))/a/d+1/8*sec(d*x+c)^8/a/d-1/10*sec(d*x+c)^10/a/d-
7/256*sec(d*x+c)*tan(d*x+c)/a/d-7/384*sec(d*x+c)^3*tan(d*x+c)/a/d-7/480*sec
(d*x+c)^5*tan(d*x+c)/a/d-1/80*sec(d*x+c)^7*tan(d*x+c)/a/d+1/10*sec(d*x+c)^9
*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2914, 2691, 3853, 3855, 2686, 14}

$$-\frac{\sec^{10}(c+dx)}{10ad}+\frac{\sec^8(c+dx)}{8ad}-\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}-\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}-\frac{7\tan(c+dx)\sec^5(c+dx)}{480ad}-\frac{7\tan(c+dx)\sec^3(c+dx)}{384ad}-\frac{7\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^8/(8*a*d) - Sec[c + d*x
]^10/(10*a*d) - (7*Sec[c + d*x]*Tan[c + d*x])/(256*a*d) - (7*Sec[c + d*x]^3
*Tan[c + d*x])/(384*a*d) - (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) - (Sec
[c + d*x]^7*Tan[c + d*x])/(80*a*d) + (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a} - \frac{\int \sec^8(c+dx)\tan^3(c+dx)\,dx}{a}$$

$$= \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{\int \sec^9(c+dx)\,dx}{10a} - \frac{\mathrm{Subst}\left(\int x^7(-1+x^2)\,d\right.}{ad}$$

$$= -\frac{\sec^7(c+dx)\tan(c+dx)}{80ad} + \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} - \frac{7\int \sec^7(c+dx}{80a}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} - \frac{\sec^7(c+dx}{8}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} - \frac{7\sec^5(c+}{4}$$

$$= \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)\tan(c+dx)}{256ad} - \frac{7\sec^3(c+d}{38}$$

$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^8(c+dx)}{8ad} - \frac{\sec^{10}(c+dx)}{10ad} - \frac{7\sec(c+dx)}{256}$$

Mathematica [A]

time = 1.80, size = 122, normalized size = 0.71

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2(96+201\sin(c+dx)-279\sin^2(c+dx)+511\sin^3(c+dx)+511\sin^4(c+dx)-385\sin^5(c+dx)-385\sin^6(c+dx)+105\sin^7(c+dx)+105\sin^8(c+dx))}{(-1+\sin(c+dx))^4(1+\sin(c+dx))^5}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*Tan[c + d*x]^2)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/7680*(210*ArcTanh[Sin[c + d*x]] - (2*(96 + 201*Sin[c + d*x] - 279*Sin[c
+ d*x]^2 + 511*Sin[c + d*x]^3 + 511*Sin[c + d*x]^4 - 385*Sin[c + d*x]^5 - 3
85*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7 + 105*Sin[c + d*x]^8))/((-1 + Sin[c
+ d*x])^4*(1 + Sin[c + d*x])^5))/(a*d)
```

Maple [A]

time = 0.23, size = 139, normalized size = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}-\frac{1}{256(1+\sin(dx+c))^4}+\frac{1}{384(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{5}{256(1+\sin(dx+c))}-\frac{7\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}-\frac{1}{256(1+\sin(dx+c))^4}+\frac{1}{384(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{5}{256(1+\sin(dx+c))}-\frac{7\ln(1+\sin(dx+c))}{512}+\frac{}{256}}{da}$
risch	$\frac{i(-1610ie^{4i(dx+c)}+105\,e^{17i(dx+c)}+51334ie^{8i(dx+c)}+700\,e^{15i(dx+c)}-5362ie^{6i(dx+c)}+1876\,e^{13i(dx+c)}+5362ie^{12i(dx+c)}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5-1/256/(1+sin(d*x+c))^4+1/384/(1+sin(d*x+c))^
3+5/512/(1+sin(d*x+c))^2+5/256/(1+sin(d*x+c))-7/512*ln(1+sin(d*x+c))+1/256/
(sin(d*x+c)-1)^4-1/192/(sin(d*x+c)-1)^3+1/512/(sin(d*x+c)-1)^2+1/128/(sin(d
*x+c)-1)+7/512*ln(sin(d*x+c)-1))
```

Maxima [A]

time = 0.32, size = 214, normalized size = 1.24

$$\frac{\frac{2\left(105\sin(dx+c)^8+105\sin(dx+c)^7-385\sin(dx+c)^6-385\sin(dx+c)^5+511\sin(dx+c)^4+511\sin(dx+c)^3-279\sin(dx+c)^2+201\sin(dx+c)+96\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\log(\sin(dx+c)+1)}{a}+\frac{105\log(\sin(dx+c)-1)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 - 3
85*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x +
 c)^2 + 201*sin(d*x + c) + 96)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a*s
in(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c)^
4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*log
(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 187, normalized size = 1.09

$$\frac{210 \cos (dx+c)^8-70 \cos (dx+c)^6-28 \cos (dx+c)^4-16 \cos (dx+c)^2-105 \left(\cos (dx+c)^8 \sin (dx+c)+\cos (dx+c)^8\right) \log (\sin (dx+c)+1)+105 \left(\cos (dx+c)^8 \sin (dx+c)+\cos (dx+c)^8\right) \log (-\sin (dx+c)+1)-2 \left(105 \cos (dx+c)^6+70 \cos (dx+c)^4+56 \cos (dx+c)^2-432\right) \sin (dx+c)+96}{7680 \left(ad \cos (dx+c)^8 \sin (dx+c)+ad \cos (dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*cos
(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d*
x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(d
*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)^
2 - 432)*sin(d*x + c) + 96)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
+ c)^8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.60, size = 156, normalized size = 0.91

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a}-\frac{420 \log(|\sin(dx+c)-1|)}{a}+\frac{5\left(175 \sin(dx+c)^4-748 \sin(dx+c)^3+1182 \sin(dx+c)^2-788 \sin(dx+c)+155\right)}{a(\sin(dx+c)-1)^4}-\frac{959 \sin(dx+c)^5+5395 \sin(dx+c)^4+12290 \sin(dx+c)^3+14170 \sin(dx+c)^2+8135 \sin(dx+c)+1627}{a(\sin(dx+c)+1)^5}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 5*(175*sin(d*x + c)^4 - 748*sin(d*x + c)^3 + 1182*sin(d*x + c)^2 - 788
*sin(d*x + c) + 155)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 5395*
sin(d*x + c)^4 + 12290*sin(d*x + c)^3 + 14170*sin(d*x + c)^2 + 8135*sin(d*x
 + c) + 1627)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 16.88, size = 496, normalized size = 2.88

$$\frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{128}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}}{64}+\frac{221\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{96}+\frac{95\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{192}+\frac{2261\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{480}+\frac{889\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{960}+\frac{7343\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{480}+\frac{1603\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{960}+\frac{2471\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{192}+\frac{1603\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{960}+\frac{7343\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{480}+\frac{889\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}}{960}+\frac{2261\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{480}+\frac{95\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{192}+\frac{221\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{96}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{64}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+140a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+14a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-112a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}-28a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+56a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+20a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-7a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{128\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)

[Out] ((7*tan(c/2 + (d*x)/2))/128 + (7*tan(c/2 + (d*x)/2)^2)/64 + (221*tan(c/2 +
(d*x)/2)^3)/96 + (95*tan(c/2 + (d*x)/2)^4)/192 + (2261*tan(c/2 + (d*x)/2)^5
)/480 + (889*tan(c/2 + (d*x)/2)^6)/960 + (7343*tan(c/2 + (d*x)/2)^7)/480 +
(1603*tan(c/2 + (d*x)/2)^8)/960 + (2471*tan(c/2 + (d*x)/2)^9)/192 + (1603*t
an(c/2 + (d*x)/2)^10)/960 + (7343*tan(c/2 + (d*x)/2)^11)/480 + (889*tan(c/2
 + (d*x)/2)^12)/960 + (2261*tan(c/2 + (d*x)/2)^13)/480 + (95*tan(c/2 + (d*x
)/2)^14)/192 + (221*tan(c/2 + (d*x)/2)^15)/96 + (7*tan(c/2 + (d*x)/2)^16)/6
4 + (7*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan
(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 +
 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d
*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*ta
n(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)
^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/
2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a
*tan(c/2 + (d*x)/2)^18)) - (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d)
```

3.906 $\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\sec(c+dx)\tan(c+dx)}{256ad}+\frac{7\sec^3(c+dx)\tan(c+dx)}{384ad}+\frac{7\sec^5(c+d}{48}$$

```
[Out] 7/256*arctanh(sin(d*x+c))/a/d+1/10*sec(d*x+c)^10/a/d+7/256*sec(d*x+c)*tan(d
*x+c)/a/d+7/384*sec(d*x+c)^3*tan(d*x+c)/a/d+7/480*sec(d*x+c)^5*tan(d*x+c)/a
/d+1/80*sec(d*x+c)^7*tan(d*x+c)/a/d-1/10*sec(d*x+c)^9*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2914, 2686, 30, 2691, 3853, 3855}

$$\frac{\sec^{10}(c+dx)}{10ad}+\frac{7\tanh^{-1}(\sin(c+dx))}{256ad}-\frac{\tan(c+dx)\sec^9(c+dx)}{10ad}+\frac{\tan(c+dx)\sec^7(c+dx)}{80ad}+\frac{7\tan(c+dx)\sec^5(c+dx)}{480ad}+\frac{7\tan(c+dx)\sec^3(c+dx)}{384ad}+\frac{7\tan(c+dx)\sec(c+dx)}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(256*a*d) + Sec[c + d*x]^10/(10*a*d) + (7*Sec[c +
 d*x]*Tan[c + d*x])/(256*a*d) + (7*Sec[c + d*x]^3*Tan[c + d*x])/(384*a*d) +
 (7*Sec[c + d*x]^5*Tan[c + d*x])/(480*a*d) + (Sec[c + d*x]^7*Tan[c + d*x])/
(80*a*d) - (Sec[c + d*x]^9*Tan[c + d*x])/(10*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^8(c+dx)\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^{10}(c+dx)\tan(c+dx)\,dx}{a} - \frac{\int \sec^9(c+dx)\tan^2(c+dx)\,dx}{a} \\
&= -\frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{\int \sec^9(c+dx)\,dx}{10a} + \frac{\text{Subst}\left(\int x^9\,dx, x, \sec(c+\right.}{ad} \\
&= \frac{\sec^{10}(c+dx)}{10ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \frac{\sec^9(c+dx)\tan(c+dx)}{10ad} + \frac{7\int}{} \\
&= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \frac{\sec^7(c+dx)\tan(c+dx)}{80ad} - \\
&= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \frac{7\sec^5(c+dx)\tan(c+dx)}{480ad} + \\
&= \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec^3(c+dx)\tan(c+dx)}{384ad} + \\
&= \frac{7\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{\sec^{10}(c+dx)}{10ad} + \frac{7\sec(c+dx)\tan(c+dx)}{256ad} + \frac{7\sec}{}
\end{aligned}$$

Mathematica [A]
time = 3.67, size = 116, normalized size = 0.75

$$\frac{210\tanh^{-1}(\sin(c+dx)) + \frac{30}{(-1+\sin(c+dx))^4} - \frac{80}{(-1+\sin(c+dx))^3} + \frac{135}{(-1+\sin(c+dx))^2} - \frac{210}{-1+\sin(c+dx)} + \frac{48}{(1+\sin(c+dx))^5} + \frac{90}{(1+\sin(c+dx))^4} + \frac{100}{(1+\sin(c+dx))^3} + \frac{75}{(1+\sin(c+dx))^2}}{7680ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^8*Tan[c + d*x])/(a + a*Sin[c + d*x]),x]
```

```
[Out] (210*ArcTanh[Sin[c + d*x]] + 30/(-1 + Sin[c + d*x])^4 - 80/(-1 + Sin[c + d*
x])^3 + 135/(-1 + Sin[c + d*x])^2 - 210/(-1 + Sin[c + d*x]) + 48/(1 + Sin[c
 + d*x])^5 + 90/(1 + Sin[c + d*x])^4 + 100/(1 + Sin[c + d*x])^3 + 75/(1 + S
in[c + d*x])^2)/(7680*a*d)
```

Maple [A]
time = 0.21, size = 127, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{1}{160(1+\sin(dx+c))^5}+\frac{3}{256(1+\sin(dx+c))^4}+\frac{5}{384(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{7\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}-\ldots}{da}$
default	$\frac{\frac{1}{160(1+\sin(dx+c))^5}+\frac{3}{256(1+\sin(dx+c))^4}+\frac{5}{384(1+\sin(dx+c))^3}+\frac{5}{512(1+\sin(dx+c))^2}+\frac{7\ln(1+\sin(dx+c))}{512}+\frac{1}{256(\sin(dx+c)-1)^4}-\ldots}{da}$
risch	$-\frac{i\left(-1610ie^{4i(dx+c)}+105\,e^{i(dx+c)}+2372\,e^{11i(dx+c)}-108410\,e^{9i(dx+c)}+700\,e^{15i(dx+c)}-10106ie^{8i(dx+c)}-5362ie^{6i(dx+\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(1/160/(1+sin(d*x+c))^5+3/256/(1+sin(d*x+c))^4+5/384/(1+sin(d*x+c))^3
+5/512/(1+sin(d*x+c))^2+7/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4-1/96/
(sin(d*x+c)-1)^3+9/512/(sin(d*x+c)-1)^2-7/256/(sin(d*x+c)-1)-7/512*ln(sin(d
*x+c)-1))
```

Maxima [A]
time = 0.28, size = 214, normalized size = 1.39

$$-\frac{\frac{2\left(105\sin(dx+c)^8+105\sin(dx+c)^7-385\sin(dx+c)^6-385\sin(dx+c)^5+511\sin(dx+c)^4+511\sin(dx+c)^3-279\sin(dx+c)^2-279\sin(dx+c)-384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4\,a\sin(dx+c)^7-4\,a\sin(dx+c)^6+6\,a\sin(dx+c)^5+6\,a\sin(dx+c)^4-4\,a\sin(dx+c)^3-4\,a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{105\log(\sin(dx+c)+1)}{a}+\frac{105\log(\sin(dx+c)-1)}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(105*sin(d*x + c)^8 + 105*sin(d*x + c)^7 - 385*sin(d*x + c)^6 -
385*sin(d*x + c)^5 + 511*sin(d*x + c)^4 + 511*sin(d*x + c)^3 - 279*sin(d*x
+ c)^2 - 279*sin(d*x + c) - 384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 - 4*a
*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x + c
)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 105*l
og(sin(d*x + c) + 1)/a + 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.41, size = 187, normalized size = 1.21

$$-\frac{210\cos(dx+c)^8-70\cos(dx+c)^6-28\cos(dx+c)^4-16\cos(dx+c)^2-105\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+105\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-2\left(105\cos(dx+c)^6+70\cos(dx+c)^4+56\cos(dx+c)^2+48\right)\sin(dx+c)-864}{7680\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/7680*(210*cos(d*x + c)^8 - 70*cos(d*x + c)^6 - 28*cos(d*x + c)^4 - 16*co
s(d*x + c)^2 - 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(d
*x + c) + 1) + 105*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin(
d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c)
^2 + 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*sin(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**A**]
time = 0.55, size = 156, normalized size = 1.01

$$\frac{\frac{420\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{420\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{5\left(175\sin(dx+c)^4-868\sin(dx+c)^3+1662\sin(dx+c)^2-1484\sin(dx+c)+539\right)}{a(\sin(dx+c)-1)^4}-\frac{959\sin(dx+c)^5+4795\sin(dx+c)^4+9290\sin(dx+c)^3+8290\sin(dx+c)^2+2735\sin(dx+c)-293}{a(\sin(dx+c)+1)^5}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*sin(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 5*(175*sin(d*x + c)^4 - 868*sin(d*x + c)^3 + 1662*sin(d*x + c)^2 - 1484
*sin(d*x + c) + 539)/(a*(sin(d*x + c) - 1)^4) - (959*sin(d*x + c)^5 + 4795*
sin(d*x + c)^4 + 9290*sin(d*x + c)^3 + 8290*sin(d*x + c)^2 + 2735*sin(d*x +
 c) - 293)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [**B**]
time = 16.75, size = 496, normalized size = 3.22

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,a\,d}+\frac{-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{128}+\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{64}+\frac{163\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{96}+\frac{289\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{192}-\frac{2261\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{480}+\frac{12551\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{960}+\frac{6097\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{480}+\frac{11837\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{960}-\frac{2471\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{192}+\frac{11837\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{960}+\frac{6097\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{480}+\frac{12551\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{960}-\frac{2261\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{480}+\frac{289\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{192}+\frac{163\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{96}+\frac{121\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{64}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+56\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-28\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-112\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+140\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-112\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-28\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+56\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(128*a*d) + ((121*tan(c/2 + (d*x)/2)^2)/64 -
(7*tan(c/2 + (d*x)/2))/128 + (163*tan(c/2 + (d*x)/2)^3)/96 + (289*tan(c/2 +
 (d*x)/2)^4)/192 - (2261*tan(c/2 + (d*x)/2)^5)/480 + (12551*tan(c/2 + (d*x)
```

/2)^6)/960 + (6097*tan(c/2 + (d*x)/2)^7)/480 + (11837*tan(c/2 + (d*x)/2)^8)/960 - (2471*tan(c/2 + (d*x)/2)^9)/192 + (11837*tan(c/2 + (d*x)/2)^10)/960 + (6097*tan(c/2 + (d*x)/2)^11)/480 + (12551*tan(c/2 + (d*x)/2)^12)/960 - (2261*tan(c/2 + (d*x)/2)^13)/480 + (289*tan(c/2 + (d*x)/2)^14)/192 + (163*tan(c/2 + (d*x)/2)^15)/96 + (121*tan(c/2 + (d*x)/2)^16)/64 - (7*tan(c/2 + (d*x)/2)^17)/128)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 7*a*tan(c/2 + (d*x)/2)^2 - 16*a*tan(c/2 + (d*x)/2)^3 + 20*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^5 - 28*a*tan(c/2 + (d*x)/2)^6 - 112*a*tan(c/2 + (d*x)/2)^7 + 14*a*tan(c/2 + (d*x)/2)^8 + 140*a*tan(c/2 + (d*x)/2)^9 + 14*a*tan(c/2 + (d*x)/2)^10 - 112*a*tan(c/2 + (d*x)/2)^11 - 28*a*tan(c/2 + (d*x)/2)^12 + 56*a*tan(c/2 + (d*x)/2)^13 + 20*a*tan(c/2 + (d*x)/2)^14 - 16*a*tan(c/2 + (d*x)/2)^15 - 7*a*tan(c/2 + (d*x)/2)^16 + 2*a*tan(c/2 + (d*x)/2)^17 + a*tan(c/2 + (d*x)/2)^18))

3.907 $\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=210

$$\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}+\frac{21a}{512d(a-a\sin(c+dx))^2}+\frac{}{64d\ldots}$$

```
[Out] 63/256*arctanh(sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/64*a^2/d/(a
-a*sin(d*x+c))^3+21/512*a/d/(a-a*sin(d*x+c))^2+7/64/d/(a-a*sin(d*x+c))-1/16
0*a^4/d/(a+a*sin(d*x+c))^5-5/256*a^3/d/(a+a*sin(d*x+c))^4-5/128*a^2/d/(a+a*
sin(d*x+c))^3-35/512*a/d/(a+a*sin(d*x+c))^2-35/256/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2746, 46, 212}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{5a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{64d(a-a\sin(c+dx))^3}-\frac{5a^2}{128d(a\sin(c+dx)+a)^3}+\frac{21a}{512d(a-a\sin(c+dx))^2}-\frac{35a}{512d(a\sin(c+dx)+a)^2}+\frac{7}{64d(a-a\sin(c+dx))}-\frac{35}{256d(a\sin(c+dx)+a)}+\frac{63\tanh^{-1}(\sin(c+dx))}{256ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (63*ArcTanh[Sin[c + d*x]])/(256*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) +
 a^2/(64*d*(a - a*Sin[c + d*x])^3) + (21*a)/(512*d*(a - a*Sin[c + d*x])^2)
+ 7/(64*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*Sin[c + d*x])^5) - (5*a
^3)/(256*d*(a + a*Sin[c + d*x])^4) - (5*a^2)/(128*d*(a + a*Sin[c + d*x])^3)
 - (35*a)/(512*d*(a + a*Sin[c + d*x])^2) - 35/(256*d*(a + a*Sin[c + d*x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
```

```
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\int \frac{\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\text{Subst}\left(\int \frac{1}{(a-x)^5(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^9\text{Subst}\left(\int \left(\frac{1}{64a^6(a-x)^5} + \frac{3}{64a^7(a-x)^4} + \frac{21}{256a^8(a-x)^3} + \frac{7}{64a^9(a-x)^2} + \frac{1}{32a^5(a+x)^6} + \frac{5}{64a^6(a+\right.}{d}$$

$$= \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} + \frac{21a}{512d(a-a\sin(c+dx))^2}$$

$$= \frac{63\tanh^{-1}(\sin(c+dx))}{256ad} + \frac{a^3}{256d(a-a\sin(c+dx))^4} + \frac{a^2}{64d(a-a\sin(c+dx))^3} +$$

Mathematica [A]
time = 0.87, size = 165, normalized size = 0.79

$$\frac{\sec^8(c+dx)\left(-128+315\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^8\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^{10}+837\sin(c+dx)+837\sin^2(c+dx)-1533\sin^3(c+dx)-1533\sin^4(c+dx)+1155\sin^5(c+dx)+1155\sin^6(c+dx)-315\sin^7(c+dx)-315\sin^8(c+dx)\right)}{1280ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^8*(-128 + 315*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])^8*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^10 + 837*Sin[c + d*x]
+ 837*Sin[c + d*x]^2 - 1533*Sin[c + d*x]^3 - 1533*Sin[c + d*x]^4 + 1155*Sin
[c + d*x]^5 + 1155*Sin[c + d*x]^6 - 315*Sin[c + d*x]^7 - 315*Sin[c + d*x]^8
))/(1280*a*d*(1 + Sin[c + d*x]))
```

Maple [A]
time = 0.22, size = 139, normalized size = 0.66

method	result
derivativedivides	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}-\frac{5}{256(1+\sin(dx+c))^4}-\frac{5}{128(1+\sin(dx+c))^3}-\frac{35}{512(1+\sin(dx+c))^2}-\frac{35}{256(1+\sin(dx+c))}+\frac{63\ln(1+\sin(dx+c))}{512}+}{da}$
default	$\frac{-\frac{1}{160(1+\sin(dx+c))^5}-\frac{5}{256(1+\sin(dx+c))^4}-\frac{5}{128(1+\sin(dx+c))^3}-\frac{35}{512(1+\sin(dx+c))^2}-\frac{35}{256(1+\sin(dx+c))}+\frac{63\ln(1+\sin(dx+c))}{512}+}{da}$
risch	$-\frac{i\left(-4830ie^{4i(dx+c)}+315\,e^{17i(dx+c)}-30318ie^{8i(dx+c)}+2100\,e^{15i(dx+c)}-16086ie^{6i(dx+c)}+5628\,e^{13i(dx+c)}+16086ie^{12i}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/160/(1+sin(d*x+c))^5-5/256/(1+sin(d*x+c))^4-5/128/(1+sin(d*x+c))^
3-35/512/(1+sin(d*x+c))^2-35/256/(1+sin(d*x+c))+63/512*ln(1+sin(d*x+c))+1/2
56/(sin(d*x+c)-1)^4-1/64/(sin(d*x+c)-1)^3+21/512/(sin(d*x+c)-1)^2-7/64/(sin
(d*x+c)-1)-63/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 214, normalized size = 1.02

$$-\frac{\frac{2\left(315\sin(dx+c)^8+315\sin(dx+c)^7-1155\sin(dx+c)^6-1155\sin(dx+c)^5+1533\sin(dx+c)^4+1533\sin(dx+c)^3-837\sin(dx+c)^2-837\sin(dx+c)+128\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{315\log(\sin(dx+c)+1)}{a}+\frac{315\log(\sin(dx+c)-1)}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2560*(2*(315*sin(d*x + c)^8 + 315*sin(d*x + c)^7 - 1155*sin(d*x + c)^6 -
 1155*sin(d*x + c)^5 + 1533*sin(d*x + c)^4 + 1533*sin(d*x + c)^3 - 837*sin(
d*x + c)^2 - 837*sin(d*x + c) + 128)/(a*sin(d*x + c)^9 + a*sin(d*x + c)^8 -
 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin(d*x
 + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) - 3
15*log(sin(d*x + c) + 1)/a + 315*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A]
time = 0.40, size = 187, normalized size = 0.89

$$-\frac{630\cos(dx+c)^8-210\cos(dx+c)^6-84\cos(dx+c)^4-48\cos(dx+c)^2-315\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)+315\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)-6\left(105\cos(dx+c)^6+70\cos(dx+c)^4+56\cos(dx+c)^2+48\right)\sin(dx+c)-32}{2560\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2560*(630*cos(d*x + c)^8 - 210*cos(d*x + c)^6 - 84*cos(d*x + c)^4 - 48*c
os(d*x + c)^2 - 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(sin(
d*x + c) + 1) + 315*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*log(-sin
(d*x + c) + 1) - 6*(105*cos(d*x + c)^6 + 70*cos(d*x + c)^4 + 56*cos(d*x + c
)^2 + 48)*sin(d*x + c) - 32)/(a*d*cos(d*x + c)^8*sin(d*x + c) + a*d*cos(d*x
 + c)^8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]

time = 0.47, size = 156, normalized size = 0.74

$$\frac{\frac{1260 \log(|\sin(dx+c)+1|)}{a} - \frac{1260 \log(|\sin(dx+c)-1|)}{a} + \frac{5\left(525 \sin(dx+c)^4-2324 \sin(dx+c)^3+3906 \sin(dx+c)^2-2972 \sin(dx+c)+873\right)}{a(\sin(dx+c)-1)^4} - \frac{2877 \sin(dx+c)^5+15785 \sin(dx+c)^4+35070 \sin(dx+c)^3+39670 \sin(dx+c)^2+23085 \sin(dx+c)+5641}{a(\sin(dx+c)+1)^5}}{10240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/10240*(1260*log(abs(sin(d*x + c) + 1))/a - 1260*log(abs(sin(d*x + c) - 1)
)/a + 5*(525*sin(d*x + c)^4 - 2324*sin(d*x + c)^3 + 3906*sin(d*x + c)^2 - 2
972*sin(d*x + c) + 873)/(a*(sin(d*x + c) - 1)^4) - (2877*sin(d*x + c)^5 + 1
5785*sin(d*x + c)^4 + 35070*sin(d*x + c)^3 + 39670*sin(d*x + c)^2 + 23085*s
in(d*x + c) + 5641)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]

time = 9.41, size = 199, normalized size = 0.95

$$\frac{63\,\mathrm{atanh}(\sin(c+d\,x))}{256\,a\,d} - \frac{\frac{63\sin(c+d\,x)^8}{256} + \frac{63\sin(c+d\,x)^7}{256} - \frac{231\sin(c+d\,x)^6}{256} - \frac{231\sin(c+d\,x)^5}{256} + \frac{1533\sin(c+d\,x)^4}{1280} + \frac{1533\sin(c+d\,x)^3}{1280} - \frac{837\sin(c+d\,x)^2}{1280} - \frac{837\sin(c+d\,x)}{1280} + \frac{1}{10}}{d\,\left(a\sin(c+d\,x)^9 + a\sin(c+d\,x)^8 - 4\,a\sin(c+d\,x)^7 - 4\,a\sin(c+d\,x)^6 + 6\,a\sin(c+d\,x)^5 + 6\,a\sin(c+d\,x)^4 - 4\,a\sin(c+d\,x)^3 - 4\,a\sin(c+d\,x)^2 + a\,\sin(c+d\,x) + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*(a + a*sin(c + d*x))),x)
```

```
[Out] (63*atanh(sin(c + d*x)))/(256*a*d) - ((1533*sin(c + d*x)^3)/1280 - (837*sin
(c + d*x)^2)/1280 - (837*sin(c + d*x))/1280 + (1533*sin(c + d*x)^4)/1280 -
(231*sin(c + d*x)^5)/256 - (231*sin(c + d*x)^6)/256 + (63*sin(c + d*x)^7)/2
56 + (63*sin(c + d*x)^8)/256 + 1/10)/(d*(a + a*sin(c + d*x) - 4*a*sin(c + d
*x)^2 - 4*a*sin(c + d*x)^3 + 6*a*sin(c + d*x)^4 + 6*a*sin(c + d*x)^5 - 4*a*
sin(c + d*x)^6 - 4*a*sin(c + d*x)^7 + a*sin(c + d*x)^8 + a*sin(c + d*x)^9))
```

3.908 $\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=247

$$-\frac{193\log(1-\sin(c+dx))}{512ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{319\log(1+\sin(c+dx))}{512ad}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{}{48d(a-}$$

```
[Out] -193/512*ln(1-sin(d*x+c))/a/d+ln(sin(d*x+c))/a/d-319/512*ln(1+sin(d*x+c))/a
/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/48*a^2/d/(a-a*sin(d*x+c))^3+37/512*a/d/
(a-a*sin(d*x+c))^2+65/256/d/(a-a*sin(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5
+7/256*a^3/d/(a+a*sin(d*x+c))^4+29/384*a^2/d/(a+a*sin(d*x+c))^3+93/512*a/d/
(a+a*sin(d*x+c))^2+1/2/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2915, 12, 90}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{7a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{48d(a-a\sin(c+dx))^3}+\frac{29a^2}{384d(a\sin(c+dx)+a)^3}+\frac{37a}{512d(a-a\sin(c+dx))^2}+\frac{93a}{512d(a\sin(c+dx)+a)^2}+\frac{65}{256d(a-a\sin(c+dx))}+\frac{1}{2d(a\sin(c+dx)+a)}-\frac{193\log(1-\sin(c+dx))}{512ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{319\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-193*Log[1 - Sin[c + d*x]])/(512*a*d) + Log[Sin[c + d*x]]/(a*d) - (319*Log
[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(4
8*d*(a - a*Sin[c + d*x])^3) + (37*a)/(512*d*(a - a*Sin[c + d*x])^2) + 65/(2
56*d*(a - a*Sin[c + d*x])) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (7*a^3)/(
256*d*(a + a*Sin[c + d*x])^4) + (29*a^2)/(384*d*(a + a*Sin[c + d*x])^3) + (
93*a)/(512*d*(a + a*Sin[c + d*x])^2) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^9\text{Subst}\left(\int \frac{a}{(a-x)^5x(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^{10}\text{Subst}\left(\int \frac{1}{(a-x)^5x(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}\\
&= \frac{a^{10}\text{Subst}\left(\int \left(\frac{1}{64a^7(a-x)^5}+\frac{1}{16a^8(a-x)^4}+\frac{37}{256a^9(a-x)^3}+\frac{65}{256a^{10}(a-x)^2}+\frac{193}{512a^{11}(a-x)}\right.\right.}{}\\
&= -\frac{193\log(1-\sin(c+dx))}{512ad}+\frac{\log(\sin(c+dx))}{ad}-\frac{319\log(1+\sin(c+dx))}{512ad}
\end{aligned}
$$

Mathematica [A]

time = 6.15, size = 228, normalized size = 0.92

$$
\frac{a^{10}\left(-\frac{193\log(1-\sin(c+dx))}{512a^{11}}+\frac{\log(\sin(c+dx))}{a^{11}}-\frac{319\log(1+\sin(c+dx))}{512a^{11}}+\frac{1}{256a^7(a-a\sin(c+dx))^4}+\frac{1}{48a^8(a-a\sin(c+dx))^3}+\frac{37}{512a^9(a-a\sin(c+dx))^2}+\frac{65}{256a^{10}(a-a\sin(c+dx))}+\frac{1}{160a^6(a+a\sin(c+dx))^5}+\frac{7}{256a^7(a+a\sin(c+dx))^4}+\frac{29}{384a^8(a+a\sin(c+dx))^3}+\frac{93}{512a^9(a+a\sin(c+dx))^2}+\frac{1}{2a^{10}(a+a\sin(c+dx))}\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^10*((-193*Log[1 - Sin[c + d*x]])/(512*a^11) + Log[Sin[c + d*x]]/a^11 - (
319*Log[1 + Sin[c + d*x]])/(512*a^11) + 1/(256*a^7*(a - a*Sin[c + d*x])^4)
+ 1/(48*a^8*(a - a*Sin[c + d*x])^3) + 37/(512*a^9*(a - a*Sin[c + d*x])^2) +
 65/(256*a^10*(a - a*Sin[c + d*x])) + 1/(160*a^6*(a + a*Sin[c + d*x])^5) +
7/(256*a^7*(a + a*Sin[c + d*x])^4) + 29/(384*a^8*(a + a*Sin[c + d*x])^3) +
93/(512*a^9*(a + a*Sin[c + d*x])^2) + 1/(2*a^10*(a + a*Sin[c + d*x]))))/d
```

Maple [A]

time = 0.26, size = 146, normalized size = 0.59

method	result
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{160(1+\sin(dx+c))^5}+\frac{7}{256(1+\sin(dx+c))^4}+\frac{29}{384(1+\sin(dx+c))^3}+\frac{93}{512(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}-\frac{319\ln(1+s}{5}}{da}$
default	$\frac{\ln(\sin(dx+c))+\frac{1}{160(1+\sin(dx+c))^5}+\frac{7}{256(1+\sin(dx+c))^4}+\frac{29}{384(1+\sin(dx+c))^3}+\frac{93}{512(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}-\frac{319\ln(1+s}{5}}{da}$
risch	$\frac{i(12390ie^{4i(dx+c)}+25526ie^{8i(dx+c)}+29822ie^{6i(dx+c)}+945\,e^{i(dx+c)}+238948\,e^{11i(dx+c)}+457910\,e^{9i(dx+c)}-29822ie^{12i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(ln(sin(d*x+c))+1/160/(1+sin(d*x+c))^5+7/256/(1+sin(d*x+c))^4+29/384/
(1+sin(d*x+c))^3+93/512/(1+sin(d*x+c))^2+1/2/(1+sin(d*x+c))-319/512*ln(1+si
n(d*x+c))+1/256/(sin(d*x+c)-1)^4-1/48/(sin(d*x+c)-1)^3+37/512/(sin(d*x+c)-1
)^2-65/256/(sin(d*x+c)-1)-193/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 226, normalized size = 0.91

$$\frac{\frac{2\left(945\sin(dx+c)^8-975\sin(dx+c)^7-5385\sin(dx+c)^6+3255\sin(dx+c)^5+11319\sin(dx+c)^4-3721\sin(dx+c)^3-10831\sin(dx+c)^2+1489\sin(dx+c)+4384\right)}{a\sin(dx+c)^9+a\sin(dx+c)^8-4a\sin(dx+c)^7-4a\sin(dx+c)^6+6a\sin(dx+c)^5+6a\sin(dx+c)^4-4a\sin(dx+c)^3-4a\sin(dx+c)^2+a\sin(dx+c)+a}-\frac{4785\log(\sin(dx+c)+1)}{a}-\frac{2895\log(\sin(dx+c)-1)}{a}+\frac{7680\log(\sin(dx+c))}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/7680*(2*(945*sin(d*x + c)^8 - 975*sin(d*x + c)^7 - 5385*sin(d*x + c)^6 +
3255*sin(d*x + c)^5 + 11319*sin(d*x + c)^4 - 3721*sin(d*x + c)^3 - 10831*si
n(d*x + c)^2 + 1489*sin(d*x + c) + 4384)/(a*sin(d*x + c)^9 + a*sin(d*x + c)
^8 - 4*a*sin(d*x + c)^7 - 4*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 + 6*a*sin
(d*x + c)^4 - 4*a*sin(d*x + c)^3 - 4*a*sin(d*x + c)^2 + a*sin(d*x + c) + a)
 - 4785*log(sin(d*x + c) + 1)/a - 2895*log(sin(d*x + c) - 1)/a + 7680*log(s
in(d*x + c))/a)/d
```

Fricas [A]
time = 0.41, size = 222, normalized size = 0.90

$$\frac{1890\cos(dx+c)^8+3210\cos(dx+c)^6+1668\cos(dx+c)^4+1136\cos(dx+c)^2+7680\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-4785\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(\sin(dx+c)+1)-2895\left(\cos(dx+c)^8\sin(dx+c)+\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)+2\left(975\cos(dx+c)^6+330\cos(dx+c)^4+136\cos(dx+c)^2+48\right)\sin(dx+c)+864}{7680\left(ad\cos(dx+c)^8\sin(dx+c)+ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(1890*cos(d*x + c)^8 + 3210*cos(d*x + c)^6 + 1668*cos(d*x + c)^4 + 1
136*cos(d*x + c)^2 + 7680*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*lo
g(1/2*sin(d*x + c)) - 4785*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*l
og(sin(d*x + c) + 1) - 2895*(cos(d*x + c)^8*sin(d*x + c) + cos(d*x + c)^8)*
log(-sin(d*x + c) + 1) + 2*(975*cos(d*x + c)^6 + 330*cos(d*x + c)^4 + 136*c
os(d*x + c)^2 + 48)*sin(d*x + c) + 864)/(a*d*cos(d*x + c)^8*sin(d*x + c) +
a*d*cos(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 169, normalized size = 0.68

$$-\frac{\frac{19140 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a} + \frac{11580 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a} - \frac{30720 \log\left(\left|\sin\left(dx+c\right)\right|\right)}{a} - \frac{5 \left(4825 \sin\left(dx+c\right)^{4} - 20860 \sin\left(dx+c\right)^{3} + 34074 \sin\left(dx+c\right)^{2} - 24996 \sin\left(dx+c\right) + 6981\right)}{a\left(\sin\left(dx+c\right)-1\right)^{4}} - \frac{43703 \sin\left(dx+c\right)^{5} + 233875 \sin\left(dx+c\right)^{4} + 504050 \sin\left(dx+c\right)^{3} + 548250 \sin\left(dx+c\right)^{2} + 302175 \sin\left(dx+c\right) + 67995}{a\left(\sin\left(dx+c\right)+1\right)^{5}}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30720*(19140*log(abs(sin(d*x + c) + 1))/a + 11580*log(abs(sin(d*x + c) -
 1))/a - 30720*log(abs(sin(d*x + c)))/a - 5*(4825*sin(d*x + c)^4 - 20860*si
n(d*x + c)^3 + 34074*sin(d*x + c)^2 - 24996*sin(d*x + c) + 6981)/(a*(sin(d*
x + c) - 1)^4) - (43703*sin(d*x + c)^5 + 233875*sin(d*x + c)^4 + 504050*sin
(d*x + c)^3 + 548250*sin(d*x + c)^2 + 302175*sin(d*x + c) + 67995)/(a*(sin(
d*x + c) + 1)^5))/d
```

Mupad [B]
time = 0.23, size = 231, normalized size = 0.94

$$\frac{\frac{63\sin\left(c+d\,x\right)^8}{256}-\frac{65\sin\left(c+d\,x\right)^7}{256}-\frac{359\sin\left(c+d\,x\right)^6}{256}+\frac{217\sin\left(c+d\,x\right)^5}{256}+\frac{3773\sin\left(c+d\,x\right)^4}{1280}-\frac{3721\sin\left(c+d\,x\right)^3}{3840}-\frac{10831\sin\left(c+d\,x\right)^2}{3840}+\frac{1489\sin\left(c+d\,x\right)}{3840}+\frac{137}{120}}{d\,\left(a\sin\left(c+d\,x\right)^9+a\sin\left(c+d\,x\right)^8-4\,a\sin\left(c+d\,x\right)^7-4\,a\sin\left(c+d\,x\right)^6+6\,a\sin\left(c+d\,x\right)^5+6\,a\sin\left(c+d\,x\right)^4-4\,a\sin\left(c+d\,x\right)^3-4\,a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}-\frac{319\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{512\,a\,d}-\frac{193\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{512\,a\,d}+\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] ((1489*sin(c + d*x))/3840 - (10831*sin(c + d*x)^2)/3840 - (3721*sin(c + d*x
)^3)/3840 + (3773*sin(c + d*x)^4)/1280 + (217*sin(c + d*x)^5)/256 - (359*si
n(c + d*x)^6)/256 - (65*sin(c + d*x)^7)/256 + (63*sin(c + d*x)^8)/256 + 137
/120)/(d*(a + a*sin(c + d*x) - 4*a*sin(c + d*x)^2 - 4*a*sin(c + d*x)^3 + 6*
a*sin(c + d*x)^4 + 6*a*sin(c + d*x)^5 - 4*a*sin(c + d*x)^6 - 4*a*sin(c + d*
x)^7 + a*sin(c + d*x)^8 + a*sin(c + d*x)^9)) - (319*log(sin(c + d*x) + 1))/
(512*a*d) - (193*log(sin(c + d*x) - 1))/(512*a*d) + log(sin(c + d*x))/(a*d)
```

3.909 $\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{\csc(c+dx)}{ad}-\frac{437\log(1-\sin(c+dx))}{512ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{949\log(1+\sin(c+dx))}{512ad}+\frac{a^3}{256d(a-a\sin(c+dx$$

```
[Out] -csc(d*x+c)/a/d-437/512*ln(1-sin(d*x+c))/a/d-ln(sin(d*x+c))/a/d+949/512*ln(
1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+5/192*a^2/d/(a-a*sin(d*x+c
))^3+57/512*a/d/(a-a*sin(d*x+c))^2+61/128/d/(a-a*sin(d*x+c))-1/160*a^4/d/(a
+a*sin(d*x+c))^5-9/256*a^3/d/(a+a*sin(d*x+c))^4-47/384*a^2/d/(a+a*sin(d*x+c
))^3-187/512*a/d/(a+a*sin(d*x+c))^2-315/256/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$-\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}-\frac{9a^3}{256d(a\sin(c+dx)+a)^4}+\frac{5a^2}{192d(a-a\sin(c+dx))^3}-\frac{47a^2}{384d(a\sin(c+dx)+a)^3}+\frac{57a}{512d(a-a\sin(c+dx))^2}-\frac{187a}{512d(a\sin(c+dx)+a)^2}+\frac{61}{128d(a-a\sin(c+dx))}-\frac{315}{256d(a\sin(c+dx)+a)}-\frac{\csc(c+dx)}{ad}-\frac{437\log(1-\sin(c+dx))}{512ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{949\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (437*Log[1 - Sin[c + d*x]])/(512*a*d) - Log[Sin[c +
 d*x]]/(a*d) + (949*Log[1 + Sin[c + d*x]])/(512*a*d) + a^3/(256*d*(a - a*Si
n[c + d*x])^4) + (5*a^2)/(192*d*(a - a*Sin[c + d*x])^3) + (57*a)/(512*d*(a
- a*Sin[c + d*x])^2) + 61/(128*d*(a - a*Sin[c + d*x])) - a^4/(160*d*(a + a*
Sin[c + d*x])^5) - (9*a^3)/(256*d*(a + a*Sin[c + d*x])^4) - (47*a^2)/(384*d
*(a + a*Sin[c + d*x])^3) - (187*a)/(512*d*(a + a*Sin[c + d*x])^2) - 315/(25
6*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{a^9\text{Subst}\left(\int \frac{a^2}{(a-x)^5x^2(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^{11}\text{Subst}\left(\int \frac{1}{(a-x)^5x^2(a+x)^6}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^{11}\text{Subst}\left(\int \left(\frac{1}{64a^8(a-x)^5}+\frac{5}{64a^9(a-x)^4}+\frac{57}{256a^{10}(a-x)^3}+\frac{61}{128a^{11}(a-x)^2}+\frac{437}{512a^{12}(a\right.\right.}{}\\ &= -\frac{\csc(c+dx)}{ad}-\frac{437\log(1-\sin(c+dx))}{512ad}-\frac{\log(\sin(c+dx))}{ad}+\frac{949\log(1}{}\end{aligned}$$

Mathematica [A]

time = 6.17, size = 240, normalized size = 0.92

$$\frac{a^{11}\left(-\frac{\csc(c+dx)}{a^{12}}-\frac{437\log(1-\sin(c+dx))}{512a^{12}}-\frac{\log(\sin(c+dx))}{a^{12}}+\frac{949\log(1+\sin(c+dx))}{512a^{12}}+\frac{1}{256a^8(a-a\sin(c+dx))^4}+\frac{5}{192a^9(a-a\sin(c+dx))^3}+\frac{57}{512a^{10}(a-a\sin(c+dx))^2}+\frac{61}{128a^{11}(a-a\sin(c+dx))}-\frac{1}{160a^7(a+a\sin(c+dx))^5}-\frac{9}{256a^8(a+a\sin(c+dx))^4}-\frac{47}{384a^9(a+a\sin(c+dx))^3}-\frac{187}{512a^{10}(a+a\sin(c+dx))^2}-\frac{315}{256a^{11}(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^11*(-(Csc[c + d*x]/a^12) - (437*Log[1 - Sin[c + d*x]])/(512*a^12) - Log[
Sin[c + d*x]]/a^12 + (949*Log[1 + Sin[c + d*x]])/(512*a^12) + 1/(256*a^8*(a
 - a*Sin[c + d*x])^4) + 5/(192*a^9*(a - a*Sin[c + d*x])^3) + 57/(512*a^10*(
a - a*Sin[c + d*x])^2) + 61/(128*a^11*(a - a*Sin[c + d*x])) - 1/(160*a^7*(a
 + a*Sin[c + d*x])^5) - 9/(256*a^8*(a + a*Sin[c + d*x])^4) - 47/(384*a^9*(a
 + a*Sin[c + d*x])^3) - 187/(512*a^10*(a + a*Sin[c + d*x])^2) - 315/(256*a^
11*(a + a*Sin[c + d*x]))))/d
```

Maple [A]

time = 0.40, size = 158, normalized size = 0.60

method	result
derivativedivides	$-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))-\frac{1}{160(1+\sin(dx+c))^5}-\frac{9}{256(1+\sin(dx+c))^4}-\frac{47}{384(1+\sin(dx+c))^3}-\frac{187}{512(1+\sin(dx+c))^2}-\frac{315}{256(1+\sin(d}$
default	$-\frac{1}{\sin(dx+c)}-\ln(\sin(dx+c))-\frac{1}{160(1+\sin(dx+c))^5}-\frac{9}{256(1+\sin(dx+c))^4}-\frac{47}{384(1+\sin(dx+c))^3}-\frac{187}{512(1+\sin(dx+c))^2}-\frac{315}{256(1+\sin(d}$

risch	$-\frac{i\left(115560ie^{4i(dx+c)}+10395\,e^{19i(dx+c)}+431256ie^{8i(dx+c)}+66585\,e^{17i(dx+c)}+16950ie^{18i(dx+c)}+170184\,e^{15i(dx+c)}+320\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)-ln(sin(d*x+c))-1/160/(1+sin(d*x+c))^5-9/256/(1+sin(d*x
+c))^4-47/384/(1+sin(d*x+c))^3-187/512/(1+sin(d*x+c))^2-315/256/(1+sin(d*x+
c))+949/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4-5/192/(sin(d*x+c)-1)^3+
57/512/(sin(d*x+c)-1)^2-61/128/(sin(d*x+c)-1)-437/512*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 245, normalized size = 0.94

$$-\frac{\frac{2\left(10395\sin(dx+c)^9+8475\sin(dx+c)^8-40035\sin(dx+c)^7-31395\sin(dx+c)^6+57309\sin(dx+c)^5+42269\sin(dx+c)^4-35941\sin(dx+c)^3-23621\sin(dx+c)^2+8224\sin(dx+c)+3840\right)}{a\sin(dx+c)^{10}+a\sin(dx+c)^9-4a\sin(dx+c)^8-4a\sin(dx+c)^7+6a\sin(dx+c)^6+6a\sin(dx+c)^5-4a\sin(dx+c)^4-4a\sin(dx+c)^3+a\sin(dx+c)^2+a\sin(dx+c)}-\frac{14235\log(\sin(dx+c)+1)}{a}+\frac{6555\log(\sin(dx+c)-1)}{a}+\frac{7680\log(\sin(dx+c))}{a}}{7680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/7680*(2*(10395*sin(d*x + c)^9 + 8475*sin(d*x + c)^8 - 40035*sin(d*x + c)
^7 - 31395*sin(d*x + c)^6 + 57309*sin(d*x + c)^5 + 42269*sin(d*x + c)^4 - 3
5941*sin(d*x + c)^3 - 23621*sin(d*x + c)^2 + 8224*sin(d*x + c) + 3840)/(a*s
in(d*x + c)^10 + a*sin(d*x + c)^9 - 4*a*sin(d*x + c)^8 - 4*a*sin(d*x + c)^7
 + 6*a*sin(d*x + c)^6 + 6*a*sin(d*x + c)^5 - 4*a*sin(d*x + c)^4 - 4*a*sin(d
*x + c)^3 + a*sin(d*x + c)^2 + a*sin(d*x + c)) - 14235*log(sin(d*x + c) + 1
)/a + 6555*log(sin(d*x + c) - 1)/a + 7680*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.42, size = 278, normalized size = 1.06

$$\frac{16950\cos(dx+c)^8-5010\cos(dx+c)^6-2132\cos(dx+c)^4-1264\cos(dx+c)^2-7680\left(\cos(dx+c)^{10}-\cos(dx+c)^8\sin(dx+c)-\cos(dx+c)^8\right)\log\left(\frac{1}{2}\sin(dx+c)\right)+14235\left(\cos(dx+c)^{10}-\cos(dx+c)^8\sin(dx+c)-\cos(dx+c)^8\right)\log(\sin(dx+c)+1)-6555\left(\cos(dx+c)^{10}-\cos(dx+c)^8\sin(dx+c)-\cos(dx+c)^8\right)\log(-\sin(dx+c)+1)+2\left(10395\cos(dx+c)^8-1545\cos(dx+c)^6-426\cos(dx+c)^4-152\cos(dx+c)^2-48\right)\sin(dx+c)-864}{7680\left(ad\cos(dx+c)^{10}-ad\cos(dx+c)^8\sin(dx+c)-ad\cos(dx+c)^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/7680*(16950*cos(d*x + c)^8 - 5010*cos(d*x + c)^6 - 2132*cos(d*x + c)^4 -
1264*cos(d*x + c)^2 - 7680*(cos(d*x + c)^10 - cos(d*x + c)^8*sin(d*x + c) -
 cos(d*x + c)^8)*log(1/2*sin(d*x + c)) + 14235*(cos(d*x + c)^10 - cos(d*x +
 c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(sin(d*x + c) + 1) - 6555*(cos(d*x
+ c)^10 - cos(d*x + c)^8*sin(d*x + c) - cos(d*x + c)^8)*log(-sin(d*x + c) +
 1) + 2*(10395*cos(d*x + c)^8 - 1545*cos(d*x + c)^6 - 426*cos(d*x + c)^4 -
152*cos(d*x + c)^2 - 48)*sin(d*x + c) - 864)/(a*d*cos(d*x + c)^10 - a*d*cos
(d*x + c)^8*sin(d*x + c) - a*d*cos(d*x + c)^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.64, size = 190, normalized size = 0.73

$$\frac{\frac{56940 \log(|\sin(dx+c)+1|)}{a} - \frac{26220 \log(|\sin(dx+c)-1|)}{a} - \frac{30720 \log(|\sin(dx+c)|)}{a} + \frac{30720 (\sin(dx+c)-1)}{a \sin(dx+c)} + \frac{5 \left(10925 \sin(dx+c)^4 - 46628 \sin(dx+c)^3 + 75018 \sin(dx+c)^2 - 54012 \sin(dx+c) + 14721\right)}{a(\sin(dx+c)-1)^4} - \frac{130013 \sin(dx+c)^5 + 687865 \sin(dx+c)^4 + 1462550 \sin(dx+c)^3 + 1564350 \sin(dx+c)^2 + 843525 \sin(dx+c) + 184065}{a(\sin(dx+c)+1)^5}}{30720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30720*(56940*log(abs(sin(d*x + c) + 1))/a - 26220*log(abs(sin(d*x + c) -
1))/a - 30720*log(abs(sin(d*x + c)))/a + 30720*(sin(d*x + c) - 1)/(a*sin(d*
x + c)) + 5*(10925*sin(d*x + c)^4 - 46628*sin(d*x + c)^3 + 75018*sin(d*x +
c)^2 - 54012*sin(d*x + c) + 14721)/(a*(sin(d*x + c) - 1)^4) - (130013*sin(d
*x + c)^5 + 687865*sin(d*x + c)^4 + 1462550*sin(d*x + c)^3 + 1564350*sin(d*
x + c)^2 + 843525*sin(d*x + c) + 184065)/(a*(sin(d*x + c) + 1)^5))/d
```

Mupad [**B**]
time = 9.46, size = 252, normalized size = 0.96

$$\frac{949 \ln(\sin(c+dx)+1)}{512\, a\, d} - \frac{437 \ln(\sin(c+dx)-1)}{512\, a\, d} - \frac{\ln(\sin(c+dx))}{a\, d} - \frac{\frac{693 \sin(c+dx)^9}{256} + \frac{565 \sin(c+dx)^8}{256} - \frac{2669 \sin(c+dx)^7}{256} - \frac{2093 \sin(c+dx)^6}{256} + \frac{19103 \sin(c+dx)^5}{1280} + \frac{42269 \sin(c+dx)^4}{3840} - \frac{35941 \sin(c+dx)^3}{3840} - \frac{23621 \sin(c+dx)^2}{3840} + \frac{257 \sin(c+dx)}{120} + 1}{d\left(a \sin(c+dx)^{10} + a \sin(c+dx)^9 - 4 a \sin(c+dx)^8 - 4 a \sin(c+dx)^7 + 6 a \sin(c+dx)^6 + 6 a \sin(c+dx)^5 - 4 a \sin(c+dx)^4 - 4 a \sin(c+dx)^3 + a \sin(c+dx)^2 + a \sin(c+dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)^2*(a + a*sin(c + d*x))),x)
```

```
[Out] (949*log(sin(c + d*x) + 1))/(512*a*d) - (437*log(sin(c + d*x) - 1))/(512*a*
d) - log(sin(c + d*x))/(a*d) - ((257*sin(c + d*x))/120 - (23621*sin(c + d*x
)^2)/3840 - (35941*sin(c + d*x)^3)/3840 + (42269*sin(c + d*x)^4)/3840 + (19
103*sin(c + d*x)^5)/1280 - (2093*sin(c + d*x)^6)/256 - (2669*sin(c + d*x)^7
)/256 + (565*sin(c + d*x)^8)/256 + (693*sin(c + d*x)^9)/256 + 1)/(d*(a*sin(
c + d*x) + a*sin(c + d*x)^2 - 4*a*sin(c + d*x)^3 - 4*a*sin(c + d*x)^4 + 6*a
*sin(c + d*x)^5 + 6*a*sin(c + d*x)^6 - 4*a*sin(c + d*x)^7 - 4*a*sin(c + d*x
)^8 + a*sin(c + d*x)^9 + a*sin(c + d*x)^10))
```

3.910 $\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=279

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{843\log(1-\sin(c+dx))}{512ad}+\frac{6\log(\sin(c+dx))}{ad}-\frac{2229\log(1+\sin(c+dx))}{512ad}+\frac{}{256d(}$$

```
[Out] csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d-843/512*ln(1-sin(d*x+c))/a/d+6*ln(sin(d
*x+c))/a/d-2229/512*ln(1+sin(d*x+c))/a/d+1/256*a^3/d/(a-a*sin(d*x+c))^4+1/3
2*a^2/d/(a-a*sin(d*x+c))^3+81/512*a/d/(a-a*sin(d*x+c))^2+203/256/d/(a-a*sin
(d*x+c))+1/160*a^4/d/(a+a*sin(d*x+c))^5+11/256*a^3/d/(a+a*sin(d*x+c))^4+23/
128*a^2/d/(a+a*sin(d*x+c))^3+325/512*a/d/(a+a*sin(d*x+c))^2+5/2/d/(a+a*sin(
d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 12, 90}

$$\frac{a^4}{160d(a\sin(c+dx)+a)^5}+\frac{a^3}{256d(a-a\sin(c+dx))^4}+\frac{11a^3}{256d(a\sin(c+dx)+a)^4}+\frac{a^2}{32d(a-a\sin(c+dx))^3}+\frac{23a^2}{128d(a\sin(c+dx)+a)^3}+\frac{81a}{512d(a-a\sin(c+dx))^2}+\frac{325a}{512d(a\sin(c+dx)+a)^2}+\frac{203}{256d(a-a\sin(c+dx))}+\frac{5}{2d(a\sin(c+dx)+a)}-\frac{\csc^2(c+dx)}{2ad}+\frac{\csc(c+dx)}{ad}-\frac{843\log(1-\sin(c+dx))}{512ad}+\frac{6\log(\sin(c+dx))}{ad}-\frac{2229\log(\sin(c+dx)+1)}{512ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d) - (843*Log[1 - Sin[c + d*x]])/(
512*a*d) + (6*Log[Sin[c + d*x]])/(a*d) - (2229*Log[1 + Sin[c + d*x]])/(512*
a*d) + a^3/(256*d*(a - a*Sin[c + d*x])^4) + a^2/(32*d*(a - a*Sin[c + d*x])^
3) + (81*a)/(512*d*(a - a*Sin[c + d*x])^2) + 203/(256*d*(a - a*Sin[c + d*x]
)) + a^4/(160*d*(a + a*Sin[c + d*x])^5) + (11*a^3)/(256*d*(a + a*Sin[c + d*
x])^4) + (23*a^2)/(128*d*(a + a*Sin[c + d*x])^3) + (325*a)/(512*d*(a + a*Si
n[c + d*x])^2) + 5/(2*d*(a + a*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{a^9\text{Subst}\left(\int \frac{a^3}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{12}\text{Subst}\left(\int \frac{1}{(a-x)^5x^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^{12}\text{Subst}\left(\int \left(\frac{1}{64a^9(a-x)^5}+\frac{3}{32a^{10}(a-x)^4}+\frac{81}{256a^{11}(a-x)^3}+\frac{203}{256a^{12}(a-x)^2}+\frac{843}{512a^{13}(\cdots}\right.\right.}{}$$
$$= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}-\frac{843\log(1-\sin(c+dx))}{512ad}+\frac{6\log(\sin(c+d\cdots}{ad}$$

Mathematica [A]
time = 6.19, size = 254, normalized size = 0.91

$$\frac{a^{12}\left(\frac{\csc(c+dx)}{a^{13}}-\frac{\csc^2(c+dx)}{2a^{13}}-\frac{843\log(1-\sin(c+dx))}{512a^{13}}+\frac{6\log(\sin(c+dx))}{a^{13}}-\frac{2229\log(1+\sin(c+dx))}{512a^{13}}+\frac{1}{256a^9(a-a\sin(c+dx))^4}+\frac{1}{32a^{10}(a-a\sin(c+dx))^3}+\frac{81}{512a^{11}(a-a\sin(c+dx))^2}+\frac{203}{256a^{12}(a-a\sin(c+dx))}+\frac{1}{160a^8(a+a\sin(c+dx))^5}+\frac{11}{256a^9(a+a\sin(c+dx))^4}+\frac{23}{128a^{10}(a+a\sin(c+dx))^3}+\frac{325}{512a^{11}(a+a\sin(c+dx))^2}+\frac{5}{2a^{12}(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^9)/(a + a*Sin[c + d*x]),x]
```

```
[Out] (a^12*(Csc[c + d*x]/a^13 - Csc[c + d*x]^2/(2*a^13) - (843*Log[1 - Sin[c + d
*x]])/(512*a^13) + (6*Log[Sin[c + d*x]])/a^13 - (2229*Log[1 + Sin[c + d*x]]
)/(512*a^13) + 1/(256*a^9*(a - a*Sin[c + d*x])^4) + 1/(32*a^10*(a - a*Sin[c
 + d*x])^3) + 81/(512*a^11*(a - a*Sin[c + d*x])^2) + 203/(256*a^12*(a - a*S
in[c + d*x])) + 1/(160*a^8*(a + a*Sin[c + d*x])^5) + 11/(256*a^9*(a + a*Sin
[c + d*x])^4) + 23/(128*a^10*(a + a*Sin[c + d*x])^3) + 325/(512*a^11*(a + a
*Sin[c + d*x])^2) + 5/(2*a^12*(a + a*Sin[c + d*x]))))/d
```

Maple [A]
time = 0.63, size = 166, normalized size = 0.59

method	result
derivativedivides	$-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}+6\ln(\sin(dx+c))+\frac{1}{160(1+\sin(dx+c))^5}+\frac{11}{256(1+\sin(dx+c))^4}+\frac{23}{128(1+\sin(dx+c))^3}+\frac{325}{512(1+\sin(dx+c\cdots}$

default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{1}{\sin(dx+c)}+6\ln(\sin(dx+c))+\frac{1}{160(1+\sin(dx+c))^5}+\frac{11}{256(1+\sin(dx+c))^4}+\frac{23}{128(1+\sin(dx+c))^3}+\frac{325}{512(1+\sin(dx+c))^2}}{}$
risch	$\frac{i(-870ie^{4i(dx+c)}-58336ie^{8i(dx+c)}+3465\,e^{i(dx+c)}+85906\,e^{9i(dx+c)}-750ie^{20i(dx+c)}+182360\,e^{15i(dx+c)}+870ie^{18i(dx+c)}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/2/sin(d*x+c)^2+1/sin(d*x+c)+6*ln(sin(d*x+c))+1/160/(1+sin(d*x+c))
^5+11/256/(1+sin(d*x+c))^4+23/128/(1+sin(d*x+c))^3+325/512/(1+sin(d*x+c))^2
+5/2/(1+sin(d*x+c))-2229/512*ln(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^4-1/32/(
sin(d*x+c)-1)^3+81/512/(sin(d*x+c)-1)^2-203/256/(sin(d*x+c)-1)-843/512*ln(s
in(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 257, normalized size = 0.92

$$\frac{\frac{2\left(3465\sin(dx+c)^{10}-375\sin(dx+c)^{9}-16545\sin(dx+c)^{8}+735\sin(dx+c)^{7}+30303\sin(dx+c)^{6}+223\sin(dx+c)^{5}-25847\sin(dx+c)^{4}-1207\sin(dx+c)^{3}+9408\sin(dx+c)^{2}+640\sin(dx+c)-640\right)}{a\sin(dx+c)^{11}+a\sin(dx+c)^{10}-4a\sin(dx+c)^{9}-4a\sin(dx+c)^{8}+6a\sin(dx+c)^{7}+6a\sin(dx+c)^{6}-4a\sin(dx+c)^{5}-4a\sin(dx+c)^{4}+a\sin(dx+c)^{3}+a\sin(dx+c)^{2}}-\frac{11145\log(\sin(dx+c)+1)}{a}-\frac{4215\log(\sin(dx+c)-1)}{a}+\frac{15360\log(\sin(dx+c))}{a}}{2560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2560*(2*(3465*sin(d*x + c)^10 - 375*sin(d*x + c)^9 - 16545*sin(d*x + c)^8
 + 735*sin(d*x + c)^7 + 30303*sin(d*x + c)^6 + 223*sin(d*x + c)^5 - 25847*s
in(d*x + c)^4 - 1207*sin(d*x + c)^3 + 9408*sin(d*x + c)^2 + 640*sin(d*x + c
) - 640)/(a*sin(d*x + c)^11 + a*sin(d*x + c)^10 - 4*a*sin(d*x + c)^9 - 4*a*
sin(d*x + c)^8 + 6*a*sin(d*x + c)^7 + 6*a*sin(d*x + c)^6 - 4*a*sin(d*x + c)
^5 - 4*a*sin(d*x + c)^4 + a*sin(d*x + c)^3 + a*sin(d*x + c)^2) - 11145*log(
sin(d*x + c) + 1)/a - 4215*log(sin(d*x + c) - 1)/a + 15360*log(sin(d*x + c)
)/a)/d
```

Fricas [A]
time = 0.41, size = 331, normalized size = 1.19

$$\frac{6930\cos(dx+c)^{10}-1560\cos(dx+c)^{8}-2454\cos(dx+c)^{6}-884\cos(dx+c)^{4}-464\cos(dx+c)^{2}+15360\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}+\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}\right)\sin(dx+c)\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-11145\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}+\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-4215\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}+\left(\cos(dx+c)^{10}-\cos(dx+c)^{8}\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)+2\left(375\cos(dx+c)^{8}-765\cos(dx+c)^{6}-178\cos(dx+c)^{4}-56\cos(dx+c)^{2}-16\right)\sin(dx+c)-288}{2560\left(ad\cos(dx+c)^{10}-ad\cos(dx+c)^{8}+\left(ad\cos(dx+c)^{10}-ad\cos(dx+c)^{8}\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2560*(6930*cos(d*x + c)^10 - 1560*cos(d*x + c)^8 - 2454*cos(d*x + c)^6 -
884*cos(d*x + c)^4 - 464*cos(d*x + c)^2 + 15360*(cos(d*x + c)^10 - cos(d*x
+ c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(1/2*sin(d*x +
 c)) - 11145*(cos(d*x + c)^10 - cos(d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x
 + c)^8)*sin(d*x + c))*log(sin(d*x + c) + 1) - 4215*(cos(d*x + c)^10 - cos(
d*x + c)^8 + (cos(d*x + c)^10 - cos(d*x + c)^8)*sin(d*x + c))*log(-sin(d*x
```

```
+ c) + 1) + 2*(375*cos(d*x + c)^8 - 765*cos(d*x + c)^6 - 178*cos(d*x + c)^4
 - 56*cos(d*x + c)^2 - 16)*sin(d*x + c) - 288)/(a*d*cos(d*x + c)^10 - a*d*c
os(d*x + c)^8 + (a*d*cos(d*x + c)^10 - a*d*cos(d*x + c)^8)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.60, size = 202, normalized size = 0.72

$$-\frac{\frac{44580 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}+\frac{16860 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}-\frac{61440 \log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{5120 \left(18 \sin\left(dx+c\right)^{2}-2 \sin\left(dx+c\right)+1\right)}{a \sin\left(dx+c\right)^{2}}-\frac{5 \left(7025 \sin\left(dx+c\right)^{4}-29724 \sin\left(dx+c\right)^{3}+47346 \sin\left(dx+c\right)^{2}-33684 \sin\left(dx+c\right)+9045\right)}{a \left(\sin\left(dx+c\right)-1\right)^{4}}-\frac{101791 \sin\left(dx+c\right)^{5}+534555 \sin\left(dx+c\right)^{4}+1126810 \sin\left(dx+c\right)^{3}+1192850 \sin\left(dx+c\right)^{2}+634975 \sin\left(dx+c\right)+136235}{a \left(\sin\left(dx+c\right)+1\right)^{5}}}{10240 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/10240*(44580*log(abs(sin(d*x + c) + 1))/a + 16860*log(abs(sin(d*x + c) -
 1))/a - 61440*log(abs(sin(d*x + c)))/a + 5120*(18*sin(d*x + c)^2 - 2*sin(d
*x + c) + 1)/(a*sin(d*x + c)^2) - 5*(7025*sin(d*x + c)^4 - 29724*sin(d*x +
c)^3 + 47346*sin(d*x + c)^2 - 33684*sin(d*x + c) + 9045)/(a*(sin(d*x + c) -
 1)^4) - (101791*sin(d*x + c)^5 + 534555*sin(d*x + c)^4 + 1126810*sin(d*x +
 c)^3 + 1192850*sin(d*x + c)^2 + 634975*sin(d*x + c) + 136235)/(a*(sin(d*x
+ c) + 1)^5))/d
```

Mupad [**B**]
time = 9.64, size = 263, normalized size = 0.94

$$\frac{\frac{693 \sin(c+d\,x)^{10}}{256}-\frac{75 \sin(c+d\,x)^{9}}{256}-\frac{3309 \sin(c+d\,x)^{8}}{256}+\frac{147 \sin(c+d\,x)^{7}}{256}+\frac{30303 \sin(c+d\,x)^{6}}{1280}+\frac{223 \sin(c+d\,x)^{5}}{1280}-\frac{25847 \sin(c+d\,x)^{4}}{1280}-\frac{1207 \sin(c+d\,x)^{3}}{1280}+\frac{147 \sin(c+d\,x)^{2}}{20}+\frac{\sin(c+d\,x)}{2}-\frac{1}{2}}{d\,\left(a\,{\sin\left(c+d\,x\right)}^{11}+a\,{\sin\left(c+d\,x\right)}^{10}-4\,a\,{\sin\left(c+d\,x\right)}^{9}-4\,a\,{\sin\left(c+d\,x\right)}^{8}+6\,a\,{\sin\left(c+d\,x\right)}^{7}+6\,a\,{\sin\left(c+d\,x\right)}^{6}-4\,a\,{\sin\left(c+d\,x\right)}^{5}-4\,a\,{\sin\left(c+d\,x\right)}^{4}+a\,{\sin\left(c+d\,x\right)}^{3}+a\,{\sin\left(c+d\,x\right)}^{2}\right)}-\frac{2229\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{512\,a\,d}-\frac{843\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{512\,a\,d}+\frac{6\,\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^9*sin(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (sin(c + d*x)/2 + (147*sin(c + d*x)^2)/20 - (1207*sin(c + d*x)^3)/1280 - (2
5847*sin(c + d*x)^4)/1280 + (223*sin(c + d*x)^5)/1280 + (30303*sin(c + d*x)
^6)/1280 + (147*sin(c + d*x)^7)/256 - (3309*sin(c + d*x)^8)/256 - (75*sin(c
 + d*x)^9)/256 + (693*sin(c + d*x)^10)/256 - 1/2)/(d*(a*sin(c + d*x)^2 + a*
sin(c + d*x)^3 - 4*a*sin(c + d*x)^4 - 4*a*sin(c + d*x)^5 + 6*a*sin(c + d*x)
^6 + 6*a*sin(c + d*x)^7 - 4*a*sin(c + d*x)^8 - 4*a*sin(c + d*x)^9 + a*sin(c
 + d*x)^10 + a*sin(c + d*x)^11)) - (2229*log(sin(c + d*x) + 1))/(512*a*d) -
 (843*log(sin(c + d*x) - 1))/(512*a*d) + (6*log(sin(c + d*x)))/(a*d)
```

3.911 $\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=127

$$\frac{F_1\left(1+n;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+n;\sin(e+fx),-\sin(e+fx)\right)\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx)}{df(1+n)}$$

```
[Out] AppellF1(1+n,1/2-m+1/2*p,1/2+1/2*p,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(
g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/2+1/2*p)*(d*sin(f*x+e))^(1+n)*(1+sin(f*x+
e))^(1/2-m+1/2*p)*(a+a*sin(f*x+e))^m/d/f/(1+n)
```

Rubi [A]
time = 0.22, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3005, 2965, 140, 138}

$$\frac{\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(d\sin(e+fx))^{n+1}(g\sec(e+fx))^p(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(n+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+1);n+2;\sin(e+fx),-\sin(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (AppellF1[1 + n, (1 + p)/2, (1 - 2*m + p)/2, 2 + n, Sin[e + f*x], -Sin[e +
f*x]]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(d*Sin
[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(a + a*Sin[e + f*x]
)^m)/(d*f*(1 + n))
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2965

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[g*((g*Cos
[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])
```

```
^((p - 1)/2))), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3005

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(g*Cos[e + f*x])^p),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(d\sin(e+fx))^n(a+a\sin(e+fx))^m\,dx = ((g\cos(e+fx))^p(g\sec(e+fx))^p)\int (g\cos(e+$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+fx\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx)\right.}{}$$

$$= \frac{\left(\sec(e+fx)(g\sec(e+fx))^p(1-\sin(e+fx)\right.}{}$$

$$= \frac{F_1\left(1+n;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+n;\sin(e+f\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 347 vs. 2(127) = 254.
time = 3.57, size = 347, normalized size = 2.73

$$\frac{g(-3+p)F_1\left(\frac{1-p}{2};-n,1+m+n-p;\frac{3-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)(g\sec(e+fx))^{-1+p}(d\sin(e+fx))^n(a(1+\sin(e+fx)))^m}{f(-1+p)\left((-3+p)F_1\left(\frac{1-p}{2};-n,1+m+n-p;\frac{3-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+2\left(nF_1\left(\frac{3-p}{2};1-n,1+m+n-p;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)+(1+m+n-p)F_1\left(\frac{3-p}{2};-n,2+m+n-p;\frac{5-p}{2};\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right),-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)\right)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (g*(-3 + p)*AppellF1[(1 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi
 + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*(g*Sec[e + f*x])^(-1 + p)*(d*
Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^m)/(f*(-1 + p)*((-3 + p)*AppellF1[(1
 - p)/2, -n, 1 + m + n - p, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2
*e - Pi + 2*f*x)/4]^2] + 2*(n*AppellF1[(3 - p)/2, 1 - n, 1 + m + n - p, (5
```

```
- p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2] + (1 + m
 + n - p)*AppellF1[(3 - p)/2, -n, 2 + m + n - p, (5 - p)/2, Cot[(2*e + Pi +
 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Tan[(2*e - Pi + 2*f*x)/4]^2))
```

Maple [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int (g\sec (fx+e))^p (d\sin (fx+e))^n (a+a\sin (fx+e))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
[Out] int((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="maxima")
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="fricas")
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(d*sin(f*x+e))**n*(a+a*sin(f*x+e))**m,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m,x, algorithm
="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\sin(e+f\,x))^n\,\left(\frac{g}{\cos(e+f\,x)}\right)^p\,(a+a\,\sin(e+f\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^n*(g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((d*sin(e + f*x))^n*(g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m, x)
```

3.912 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{{}_2F_1\left(1+m,-n;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}}{af(1+m)}$$

```
[Out] hypergeom([-n, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)*(
c+d*sin(f*x+e))^n/a/f/(1+m)/(((c+d*sin(f*x+e))/(c-d))^n)
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2912, 72, 71}

$$\frac{(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}{}_2F_1\left(m+1,-n;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
```

```
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\operatorname{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^n\,dx, x, a\sin(e+f
\\
= \frac{\left((c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}\right)\operatorname{Subs}
\\
= \frac{{}_2F_1\left(1+m,-n;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+
$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 1.00

$$\frac{{}_2F_1\left(1+m,-n;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}}{af(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a
+ a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(1 + m)*((c + d*Sin[
e + f*x])/(c - d))^n)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \cos(fx+e)(a+a\sin(fx+e))^m(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (c+d\sin(e+fx))^n \cos(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(c + d*sin(e + f*x))**n*cos(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.913 $\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{1+n}}{d^5f(1+n)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{2+n}}{d^5f(2+n)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{3+n}}{d^5f(3+n)}-$$

```
[Out] a^4*(c-d)^4*(c+d*sin(f*x+e))^(1+n)/d^5/f/(1+n)-4*a^4*(c-d)^3*(c+d*sin(f*x+e
))^(2+n)/d^5/f/(2+n)+6*a^4*(c-d)^2*(c+d*sin(f*x+e))^(3+n)/d^5/f/(3+n)-4*a^4
*(c-d)*(c+d*sin(f*x+e))^(4+n)/d^5/f/(4+n)+a^4*(c+d*sin(f*x+e))^(5+n)/d^5/f/
(5+n)
```

Rubi [A]
time = 0.14, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2912, 45}

$$\frac{a^4(c-d)^4(c+d\sin(e+fx))^{n+1}}{d^5f(n+1)}-\frac{4a^4(c-d)^3(c+d\sin(e+fx))^{n+2}}{d^5f(n+2)}+\frac{6a^4(c-d)^2(c+d\sin(e+fx))^{n+3}}{d^5f(n+3)}-\frac{4a^4(c-d)(c+d\sin(e+fx))^{n+4}}{d^5f(n+4)}+\frac{a^4(c+d\sin(e+fx))^{n+5}}{d^5f(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c - d)^4*(c + d*Sin[e + f*x])^(1 + n))/(d^5*f*(1 + n)) - (4*a^4*(c -
d)^3*(c + d*Sin[e + f*x])^(2 + n))/(d^5*f*(2 + n)) + (6*a^4*(c - d)^2*(c +
d*Sin[e + f*x])^(3 + n))/(d^5*f*(3 + n)) - (4*a^4*(c - d)*(c + d*Sin[e + f*
x])^(4 + n))/(d^5*f*(4 + n)) + (a^4*(c + d*Sin[e + f*x])^(5 + n))/(d^5*f*(5
 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^4(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^4\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{a^4(c-d)^4\left(c+\frac{dx}{a}\right)^n}{d^4} - \frac{4a^4(c-d)^3\left(c+\frac{dx}{a}\right)^{1+n}}{d^4} + \cdots\right.\right.}{\cdots}$$

$$= \frac{a^4(c-d)^4(c+d\sin(e+fx))^{1+n}}{d^5f(1+n)} - \frac{4a^4(c-d)^3(\cdots}{\cdots}$$

Mathematica [A]
time = 0.45, size = 130, normalized size = 0.74

$$\frac{a^4(c+d\sin(e+fx))^{1+n}\left(\frac{(c-d)^4}{1+n} - \frac{4(c-d)^3(c+d\sin(e+fx))}{2+n} + \frac{6(c-d)^2(c+d\sin(e+fx))^2}{3+n} - \frac{4(c-d)(c+d\sin(e+fx))^3}{4+n} + \frac{(c+d\sin(e+fx))^4}{5+n}\right)}{d^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^4*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^4*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^4/(1 + n) - (4*(c - d)^3*(c + d*
Sin[e + f*x]))/(2 + n) + (6*(c - d)^2*(c + d*Sin[e + f*x])^2)/(3 + n) - (4*
(c - d)*(c + d*Sin[e + f*x])^3)/(4 + n) + (c + d*Sin[e + f*x])^4/(5 + n)))/
(d^5*f)
```

Maple [F]
time = 0.45, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\,(a+a\sin(fx+e))^4\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(175) = 350.
time = 0.31, size = 486, normalized size = 2.78

$$\frac{\frac{4\left(d^2(n+1)\sin(fx+e)^2+cdn\sin(fx+e)-c^2\right)(d\sin(fx+e)+c)^na^4}{(n^2+3n+2)d^2} + \frac{(d\sin(fx+e)+c)^{n+1}a^4}{d(n+1)} + \frac{6\left((n^2+3n+2)d^3\sin(fx+e)^3+(n^2+n)cd^2\sin(fx+e)^2-2c^2dn\sin(fx+e)+2c^3\right)(d\sin(fx+e)+c)^na^4}{(n^3+6n^2+11n+6)d^3} + \frac{4\left((n^3+6n^2+11n+6)d^4\sin(fx+e)^4+(n^3+3n^2+2n)cd^3\sin(fx+e)^3-3(n^2+n)c^2d^2\sin(fx+e)^2+6c^3dn\sin(fx+e)-6c^4\right)(d\sin(fx+e)+c)^na^4}{(n^4+10n^3+35n^2+50n+24)d^4} + \frac{\left((n^4+10n^3+35n^2+50n+24)d^5\sin(fx+e)^5+(n^4+6n^3+11n^2+6n)cd^4\sin(fx+e)^4-4(n^3+3n^2+2n)c^2d^3\sin(fx+e)^3+12(n^2+n)c^3d^2\sin(fx+e)^2-24c^4dn\sin(fx+e)+24c^5\right)(d\sin(fx+e)+c)^na^4}{(n^5+15n^4+85n^3+225n^2+274n+120)d^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (4*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^4/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^4/(d*(n +
 1)) + 6*((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)
^2 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^4/((n^3 + 6*n
^2 + 11*n + 6)*d^3) + 4*((n^3 + 6*n^2 + 11*n + 6)*d^4*sin(f*x + e)^4 + (n^3
 + 3*n^2 + 2*n)*c*d^3*sin(f*x + e)^3 - 3*(n^2 + n)*c^2*d^2*sin(f*x + e)^2 +
 6*c^3*d*n*sin(f*x + e) - 6*c^4)*(d*sin(f*x + e) + c)^n*a^4/((n^4 + 10*n^3
+ 35*n^2 + 50*n + 24)*d^4) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*sin(f
*x + e)^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*c*d^4*sin(f*x + e)^4 - 4*(n^3 + 3*
n^2 + 2*n)*c^2*d^3*sin(f*x + e)^3 + 12*(n^2 + n)*c^3*d^2*sin(f*x + e)^2 - 2
4*c^4*d*n*sin(f*x + e) + 24*c^5)*(d*sin(f*x + e) + c)^n*a^4/((n^5 + 15*n^4
+ 85*n^3 + 225*n^2 + 274*n + 120)*d^5))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(180) = 360$.
time = 0.46, size = 908, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (24*a^4*c^5 - 120*a^4*c^4*d + 240*a^4*c^3*d^2 - 240*a^4*c^2*d^3 + 120*a^4*c
*d^4 + 360*a^4*d^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (120*a^4*d^5 + (a^4*c*d^
4 + 4*a^4*d^5)*n^4 + 2*(3*a^4*c*d^4 + 22*a^4*d^5)*n^3 + (11*a^4*c*d^4 + 164
*a^4*d^5)*n^2 + 2*(3*a^4*c*d^4 + 122*a^4*d^5)*n)*cos(f*x + e)^4 - 16*(a^4*c
^2*d^3 - 5*a^4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 +
 32*a^4*c*d^4 + 50*a^4*d^5)*n^2 - 4*(120*a^4*d^5 + (2*a^4*c*d^4 + 3*a^4*d^5
)*n^4 - (3*a^4*c^2*d^3 - 18*a^4*c*d^4 - 35*a^4*d^5)*n^3 + (3*a^4*c^3*d^2 -
18*a^4*c^2*d^3 + 49*a^4*c*d^4 + 141*a^4*d^5)*n^2 + (3*a^4*c^3*d^2 - 15*a^4*
c^2*d^3 + 33*a^4*c*d^4 + 229*a^4*d^5)*n)*cos(f*x + e)^2 - 8*(3*a^4*c^4*d -
15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*d^5)*n + (384*a^4*d
^5 + 8*(a^4*c*d^4 + a^4*d^5)*n^4 + (a^4*d^5*n^4 + 10*a^4*d^5*n^3 + 35*a^4*d
^5*n^2 + 50*a^4*d^5*n + 24*a^4*d^5)*cos(f*x + e)^4 - 16*(a^4*c^2*d^3 - 5*a^
4*c*d^4 - 6*a^4*d^5)*n^3 + 8*(3*a^4*c^3*d^2 - 15*a^4*c^2*d^3 + 32*a^4*c*d^4
 + 50*a^4*d^5)*n^2 - 4*(72*a^4*d^5 + (a^4*c*d^4 + 2*a^4*d^5)*n^4 - (a^4*c^2
*d^3 - 8*a^4*c*d^4 - 23*a^4*d^5)*n^3 - (3*a^4*c^2*d^3 - 17*a^4*c*d^4 - 91*a
^4*d^5)*n^2 - 2*(a^4*c^2*d^3 - 5*a^4*c*d^4 - 71*a^4*d^5)*n)*cos(f*x + e)^2
- 8*(3*a^4*c^4*d - 15*a^4*c^3*d^2 + 31*a^4*c^2*d^3 - 35*a^4*c*d^4 - 84*a^4*
d^5)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/(d^5*f*n^5 + 15*d^5*f*n^4 + 85
*d^5*f*n^3 + 225*d^5*f*n^2 + 274*d^5*f*n + 120*d^5*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11900 vs. $2(150) = 300$.

time = 39.00, size = 11900, normalized size = 68.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**4*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**4*sin(e + f*x)**5/(5*f) + a**4*sin(e + f*x)**4/f + 2*a*
*4*sin(e + f*x)**3/f + 2*a**4*sin(e + f*x)**2/f + a**4*sin(e + f*x)/f), Eq(
d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**4*cos(e), Eq(f, 0)), (12*a**4*
c**4*log(c/d + sin(e + f*x))/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x)
+ 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*
sin(e + f*x)**4) + 25*a**4*c**4/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*
x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9
*f*sin(e + f*x)**4) + 48*a**4*c**3*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(
12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)*
*2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 88*a**4*c**
3*d*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d*
*7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)
**4) - 12*a**4*c**3*d/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c*
*2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e +
 f*x)**4) + 72*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(12*c
**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 +
 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 108*a**4*c**2*d
**2*sin(e + f*x)**2/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2
*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f
*x)**4) - 48*a**4*c**2*d**2*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*s
in(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3
+ 12*d**9*f*sin(e + f*x)**4) - 6*a**4*c**2*d**2/(12*c**4*d**5*f + 48*c**3*d
**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f
*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 48*a**4*c*d**3*log(c/d + sin(e + f*x)
)*sin(e + f*x)**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d
**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x
)**4) + 48*a**4*c*d**3*sin(e + f*x)**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin
(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 +
12*d**9*f*sin(e + f*x)**4) - 72*a**4*c*d**3*sin(e + f*x)**2/(12*c**4*d**5*f
 + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8
*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 24*a**4*c*d**3*sin(e + f*
x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f
*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 4*a**4*
c*d**3/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e
 + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) + 12*
a**4*d**4*log(c/d + sin(e + f*x))*sin(e + f*x)**4/(12*c**4*d**5*f + 48*c**3
*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e +
 f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 48*a**4*d**4*sin(e + f*x)**3/(12*c*
```

```
*4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f*sin(e + f*x)**2 +
48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4) - 36*a**4*d**4*sin
(e + f*x)**2/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x) + 72*c**2*d**7*f
*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*sin(e + f*x)**4)
 - 16*a**4*d**4*sin(e + f*x)/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x)
+ 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*f*
sin(e + f*x)**4) - 3*a**4*d**4/(12*c**4*d**5*f + 48*c**3*d**6*f*sin(e + f*x
) + 72*c**2*d**7*f*sin(e + f*x)**2 + 48*c*d**8*f*sin(e + f*x)**3 + 12*d**9*
f*sin(e + f*x)**4), Eq(n, -5)), (-12*a**4*c**4*log(c/d + sin(e + f*x))/(3*c
**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d*
*8*f*sin(e + f*x)**3) - 22*a**4*c**4/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e +
 f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 36*a**4*c*
*3*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**6*f*si
n(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 12*a*
*4*c**3*d*log(c/d + sin(e + f*x))/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*
x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 54*a**4*c**3*
d*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin
(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) + 22*a**4*c**3*d/(3*c**3*d**5*f +
9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e +
f*x)**3) - 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(3*c**
3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8
*f*sin(e + f*x)**3) + 36*a**4*c**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x
)/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2
+ 3*d**8*f*sin(e + f*x)**3) - 36*a**4*c**2*d**2*sin(e + f*x)**2/(3*c**3*d**
5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*si
n(e + f*x)**3) + 54*a**4*c**2*d**2*sin(e + f*x)/(3*c**3*d**5*f + 9*c**2*d**
6*f*sin(e + f*x) + 9*c*d**7*f*sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) -
 6*a**4*c**2*d**2/(3*c**3*d**5*f + 9*c**2*d**6*f*sin(e + f*x) + 9*c*d**7*f*
sin(e + f*x)**2 + 3*d**8*f*sin(e + f*x)**3) - 1...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. $2(180) = 360$.
time = 0.53, size = 1840, normalized size = 10.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^4*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n*n^4 - 4*(d*sin(f*x + e) + c
)^4*(d*sin(f*x + e) + c)^n*c*n^4 + 6*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e)
 + c)^n*c^2*n^4 - 4*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^4 +
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^4 + 10*(d*sin(f*x + e) +
 c)^5*(d*sin(f*x + e) + c)^n*n^3 - 44*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e
```

```
) + c)^n*c*n^3 + 72*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^3 -
 52*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^3 + 14*(d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^4*n^3 + 35*(d*sin(f*x + e) + c)^5*(d*sin(
f*x + e) + c)^n*n^2 - 164*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n
^2 + 294*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n^2 - 236*(d*sin
(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^3*n^2 + 71*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^4*n^2 + 50*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) +
 c)^n*n - 244*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c*n + 468*(d*si
n(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*c^2*n - 428*(d*sin(f*x + e) + c)^2
*(d*sin(f*x + e) + c)^n*c^3*n + 154*(d*sin(f*x + e) + c)*(d*sin(f*x + e) +
c)^n*c^4*n + 24*(d*sin(f*x + e) + c)^5*(d*sin(f*x + e) + c)^n - 120*(d*sin(
f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*c + 240*(d*sin(f*x + e) + c)^3*(d*si
n(f*x + e) + c)^n*c^2 - 240*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c
^3 + 120*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^4)*a^4/(d^4*n^5 + 15
*d^4*n^4 + 85*d^4*n^3 + 225*d^4*n^2 + 274*d^4*n + 120*d^4) + 4*((d*sin(f*x
+ e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n
^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(d*sin(f*x + e
) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n^
2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11*(d*sin(f*x +
 e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^3*(d*sin(f*x
+ e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2*n -
26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*sin(f*x + e) +
c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c
)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 - 24*(d*sin(f*
x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^4/(d^3*n^4 + 10*d^3*n^3 + 35*d^3*
n^2 + 50*d^3*n + 24*d^3) + 6*((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n
*n^2 - 2*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x +
 e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f
*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*
(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*
c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^4/(d^2*n^3 + 6*d^2
*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^4/(n + 1) + 4*((d
*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin
(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d
*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^4/((n^2 + 3*n + 2)*d))/(d*f)
```

Mupad [B]

time = 18.14, size = 863, normalized size = 4.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^4*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a^4*sin(5*e + 5*f*x)*(c + d*sin(e + f*x))^n*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)*1i)/(16*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) +
(a^4*(c + d*sin(e + f*x))^n*(c*d^4*960i - c^4*d*960i + d^5*n*2444i + c^5*19
2i + d^5*1320i - c^2*d^3*1920i + c^3*d^2*1920i + d^5*n^2*1436i + d^5*n^3*34
0i + d^5*n^4*28i - c^2*d^3*n*1744i + c^3*d^2*n*912i + c*d^4*n^2*1297i + c*d
^4*n^3*370i + c*d^4*n^4*35i - c^2*d^3*n^2*672i + c^3*d^2*n^2*144i - c^2*d^3
*n^3*80i + c*d^4*n*1730i - c^4*d*n*192i))/(8*d^5*f*(n*274i + n^2*225i + n^3
*85i + n^4*15i + n^5*1i + 120i)) + (a^4*sin(e + f*x)*(c + d*sin(e + f*x))^n
*(4290*d^4*n - 192*c^4*n + 2520*d^4 + 2507*d^4*n^2 + 594*d^4*n^3 + 49*d^4*n
^4 - 1968*c^2*d^2*n + 1912*c*d^3*n^2 + 192*c^3*d*n^2 + 576*c*d^3*n^3 + 56*c
*d^3*n^4 - 936*c^2*d^2*n^2 - 120*c^2*d^2*n^3 + 2160*c*d^3*n + 960*c^3*d*n)*
1i)/(8*d^4*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) + (a^
4*cos(4*e + 4*f*x)*(c + d*sin(e + f*x))^n*(d*20i + c*n*1i + d*n*4i)*(11*n +
 6*n^2 + n^3 + 6))/(8*d*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i +
 120i)) - (a^4*sin(3*e + 3*f*x)*(c + d*sin(e + f*x))^n*(3*n + n^2 + 2)*(251
*d^2*n - 16*c^2*n + 540*d^2 + 29*d^2*n^2 + 80*c*d*n + 16*c*d*n^2)*1i)/(16*d
^2*f*(n*274i + n^2*225i + n^3*85i + n^4*15i + n^5*1i + 120i)) - (a^4*cos(2*
e + 2*f*x)*(n + 1)*(c + d*sin(e + f*x))^n*(c^3*n*12i + d^3*n*312i + d^3*360
i + d^3*n^2*88i + d^3*n^3*8i + c*d^2*n^2*59i - c^2*d*n^2*12i + c*d^2*n^3*7i
 + c*d^2*n*126i - c^2*d*n*60i))/(2*d^3*f*(n*274i + n^2*225i + n^3*85i + n^4
*15i + n^5*1i + 120i))
```

3.914 $\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=139

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{1+n}}{d^4f(1+n)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{2+n}}{d^4f(2+n)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{3+n}}{d^4f(3+n)}+\frac{a}{}$$

```
[Out] -a^3*(c-d)^3*(c+d*sin(f*x+e))^(1+n)/d^4/f/(1+n)+3*a^3*(c-d)^2*(c+d*sin(f*x+
e))^(2+n)/d^4/f/(2+n)-3*a^3*(c-d)*(c+d*sin(f*x+e))^(3+n)/d^4/f/(3+n)+a^3*(c
+d*sin(f*x+e))^(4+n)/d^4/f/(4+n)
```

Rubi [A]
time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 45}

$$-\frac{a^3(c-d)^3(c+d\sin(e+fx))^{n+1}}{d^4f(n+1)}+\frac{3a^3(c-d)^2(c+d\sin(e+fx))^{n+2}}{d^4f(n+2)}-\frac{3a^3(c-d)(c+d\sin(e+fx))^{n+3}}{d^4f(n+3)}+\frac{a^3(c+d\sin(e+fx))^{n+4}}{d^4f(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a^3*(c - d)^3*(c + d*Sin[e + f*x])^(1 + n))/(d^4*f*(1 + n))) + (3*a^3*(c
 - d)^2*(c + d*Sin[e + f*x])^(2 + n))/(d^4*f*(2 + n)) - (3*a^3*(c - d)*(c +
 d*Sin[e + f*x])^(3 + n))/(d^4*f*(3 + n)) + (a^3*(c + d*Sin[e + f*x])^(4 +
n))/(d^4*f*(4 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^3\left(c+\frac{dx}{a}\right)^n dx, x, a\sin(e+fx\right)}{af}$$

$$= \frac{\text{Subst}\left(\int\left(-\frac{a^3(c-d)^3\left(c+\frac{dx}{a}\right)^n}{d^3}+\frac{3a^3(c-d)^2\left(c+\frac{dx}{a}\right)^{1-}}{d^3}\right.\right.}{}$$

$$= -\frac{a^3(c-d)^3(c+d\sin(e+fx))^{1+n}}{d^4f(1+n)}+\frac{3a^3(c-}{}$$

Mathematica [A]
time = 0.25, size = 105, normalized size = 0.76

$$\frac{a^3(c+d\sin(e+fx))^{1+n}\left(-\frac{(c-d)^3}{1+n}+\frac{3(c-d)^2(c+d\sin(e+fx))}{2+n}-\frac{3(c-d)(c+d\sin(e+fx))^2}{3+n}+\frac{(c+d\sin(e+fx))^3}{4+n}\right)}{d^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^3*(c + d*Sin[e + f*x])^(1 + n)*(-((c - d)^3/(1 + n)) + (3*(c - d)^2*(c +
 d*Sin[e + f*x]))/(2 + n) - (3*(c - d)*(c + d*Sin[e + f*x])^2)/(3 + n) + (c
 + d*Sin[e + f*x])^3/(4 + n)))/(d^4*f)
```

Maple [F]
time = 0.40, size = 0, normalized size = 0.00

$$\int \cos(fx+e)(a+a\sin(fx+e))^3(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(139) = 278.
time = 0.30, size = 313, normalized size = 2.25

$$\frac{\frac{3\left(d^2(n+1)\sin(fx+e)^2+cdn\sin(fx+e)-c^2\right)(d\sin(fx+e)+c)^na^3}{(n^2+3n+2)d^2}+\frac{(d\sin(fx+e)+c)^{n+1}a^3}{d(n+1)}+\frac{3\left((n^2+3n+2)d^3\sin(fx+e)^3+(n^2+n)cd^2\sin(fx+e)^2-2c^2dn\sin(fx+e)+2c^3\right)(d\sin(fx+e)+c)^na^3}{(n^3+6n^2+11n+6)d^3}+\frac{\left((n^3+6n^2+11n+6)d^4\sin(fx+e)^4+(n^3+3n^2+2n)cd^3\sin(fx+e)^3-3(n^2+n)c^2d^2\sin(fx+e)^2+6c^3dn\sin(fx+e)-6c^4\right)(d\sin(fx+e)+c)^na^3}{(n^4+10n^3+35n^2+50n+24)d^4}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (3*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^3/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^3/(d*(n +
 1)) + 3*((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)
^2 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^3/((n^3 + 6*n
^2 + 11*n + 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*sin(f*x + e)^4 + (n^3 +
 3*n^2 + 2*n)*c*d^3*sin(f*x + e)^3 - 3*(n^2 + n)*c^2*d^2*sin(f*x + e)^2 + 6
*c^3*d*n*sin(f*x + e) - 6*c^4)*(d*sin(f*x + e) + c)^n*a^3/((n^4 + 10*n^3 +
35*n^2 + 50*n + 24)*d^4))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(143) = 286$.
time = 0.44, size = 550, normalized size = 3.96

$$-\frac{\left(6\,a^3c^4-24\,a^3c^3d+36\,a^3c^2d^2-24\,a^3cd^3-42\,a^3d^4-\left(a^3d^4n^3+6\,a^3d^4n^2+11\,a^3d^4n+6\,a^3d^4\right)\cos\left(fx+e\right)^4-4\left(a^3cd^3+a^3d^4\right)n^3+6\left(a^3c^2d^2-4\,a^3cd^3-5\,a^3d^4\right)n^2+\left(48\,a^3d^4+\left(3\,a^3cd^3+5\,a^3d^4\right)n^3-3\left(a^3c^2d^2-5\,a^3cd^3-12\,a^3d^4\right)n^2-\left(3\,a^3c^2d^2-12\,a^3cd^3-79\,a^3d^4\right)n\right)\cos\left(fx+e\right)^2-2\left(3\,a^3c^3d-12\,a^3c^2d^2+19\,a^3cd^3+34\,a^3d^4\right)n-\left(48\,a^3d^4+4\left(a^3cd^3+a^3d^4\right)n^3-6\left(a^3c^2d^2-4\,a^3cd^3-5\,a^3d^4\right)n^2-\left(24\,a^3d^4+\left(a^3cd^3+3\,a^3d^4\right)n^3+3\left(a^3cd^3+7\,a^3d^4\right)n^2+2\left(a^3cd^3+21\,a^3d^4\right)n\right)\cos\left(fx+e\right)^2+2\left(3\,a^3c^3d-12\,a^3c^2d^2+19\,a^3cd^3+34\,a^3d^4\right)n\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right)+c\right)^n}{d^4fn^4+10\,d^4fn^3+35\,d^4fn^2+50\,d^4fn+24\,d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] -(6*a^3*c^4 - 24*a^3*c^3*d + 36*a^3*c^2*d^2 - 24*a^3*c*d^3 - 42*a^3*d^4 - (
a^3*d^4*n^3 + 6*a^3*d^4*n^2 + 11*a^3*d^4*n + 6*a^3*d^4)*cos(f*x + e)^4 - 4*
(a^3*c*d^3 + a^3*d^4)*n^3 + 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 - 5*a^3*d^4)*n^2 +
 (48*a^3*d^4 + (3*a^3*c*d^3 + 5*a^3*d^4)*n^3 - 3*(a^3*c^2*d^2 - 5*a^3*c*d^3
 - 12*a^3*d^4)*n^2 - (3*a^3*c^2*d^2 - 12*a^3*c*d^3 - 79*a^3*d^4)*n)*cos(f*x
 + e)^2 - 2*(3*a^3*c^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n -
(48*a^3*d^4 + 4*(a^3*c*d^3 + a^3*d^4)*n^3 - 6*(a^3*c^2*d^2 - 4*a^3*c*d^3 -
5*a^3*d^4)*n^2 - (24*a^3*d^4 + (a^3*c*d^3 + 3*a^3*d^4)*n^3 + 3*(a^3*c*d^3 +
 7*a^3*d^4)*n^2 + 2*(a^3*c*d^3 + 21*a^3*d^4)*n)*cos(f*x + e)^2 + 2*(3*a^3*c
^3*d - 12*a^3*c^2*d^2 + 19*a^3*c*d^3 + 34*a^3*d^4)*n)*sin(f*x + e))*(d*sin(
f*x + e) + c)^n/(d^4*f*n^4 + 10*d^4*f*n^3 + 35*d^4*f*n^2 + 50*d^4*f*n + 24*
d^4*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5596 vs. $2(117) = 234$.
time = 14.78, size = 5596, normalized size = 40.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**3*sin(e + f*x)**4/(4*f) + a**3*sin(e + f*x)**3/f + 3*a*
*3*sin(e + f*x)**2/(2*f) + a**3*sin(e + f*x)/f), Eq(d, 0)), (x*(c + d*sin(e
))**n*(a*sin(e) + a)**3*cos(e), Eq(f, 0)), (6*a**3*c**3*log(c/d + sin(e + f
*x))/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x
)**2 + 6*d**7*f*sin(e + f*x)**3) + 11*a**3*c**3/(6*c**3*d**4*f + 18*c**2*d*
```

*5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3)
 + 18*a**3*c**2*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(6*c**3*d**4*f + 18*
c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f
*x)**3) + 27*a**3*c**2*d*sin(e + f*x)/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e
 + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 6*a**3*
c**2*d/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f
*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 18*a**3*c*d**2*log(c/d + sin(e + f*x))
*sin(e + f*x)**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f
*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 18*a**3*c*d**2*sin(e + f*x)*
*2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)*
*2 + 6*d**7*f*sin(e + f*x)**3) - 18*a**3*c*d**2*sin(e + f*x)/(6*c**3*d**4*f
 + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin
(e + f*x)**3) - 3*a**3*c*d**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x)
+ 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) + 6*a**3*d**3*log
(c/d + sin(e + f*x))*sin(e + f*x)**3/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e
+ f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 18*a**3*
d**3*sin(e + f*x)**2/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d*
*6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3) - 9*a**3*d**3*sin(e + f*x)
/(6*c**3*d**4*f + 18*c**2*d**5*f*sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2
 + 6*d**7*f*sin(e + f*x)**3) - 2*a**3*d**3/(6*c**3*d**4*f + 18*c**2*d**5*f*
sin(e + f*x) + 18*c*d**6*f*sin(e + f*x)**2 + 6*d**7*f*sin(e + f*x)**3), Eq(
n, -4)), (-6*a**3*c**3*log(c/d + sin(e + f*x))/(2*c**2*d**4*f + 4*c*d**5*f*
sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 9*a**3*c**3/(2*c**2*d**4*f + 4*c
*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**2*d*log(c/d +
 sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d*
*6*f*sin(e + f*x)**2) + 6*a**3*c**2*d*log(c/d + sin(e + f*x))/(2*c**2*d**4*
f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 12*a**3*c**2*d*si
n(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)
2) + 9*a3*c**2*d/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*si
n(e + f*x)**2) - 6*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c
2*d4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12*a**3*
c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin
(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 12*a**3*c*d**2*sin(e + f*x)/(2*c**2
*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 3*a**3*c*d*
*2/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 6
*a**3*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)**2/(2*c**2*d**4*f + 4*c*d**
5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) + 2*a**3*d**3*sin(e + f*x)**3/
(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*sin(e + f*x)**2) - 6*a*
*3*d**3*sin(e + f*x)/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**6*f*si
n(e + f*x)**2) - a**3*d**3/(2*c**2*d**4*f + 4*c*d**5*f*sin(e + f*x) + 2*d**
6*f*sin(e + f*x)**2), Eq(n, -3)), (6*a**3*c**3*log(c/d + sin(e + f*x))/(2*c
*d**4*f + 2*d**5*f*sin(e + f*x)) + 6*a**3*c**3/(2*c*d**4*f + 2*d**5*f*sin(e
 + f*x)) + 6*a**3*c**2*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f +
 2*d**5*f*sin(e + f*x)) - 12*a**3*c**2*d*log(c/d + sin(e + f*x))/(2*c*d**4*
f + 2*d**5*f*sin(e + f*x)) - 12*a**3*c**2*d/(2*c*d**4*f + 2*d**5*f*sin(e +

```
f*x)) - 12*a**3*c*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f + 2
*d**5*f*sin(e + f*x)) + 6*a**3*c*d**2*log(c/d + sin(e + f*x))/(2*c*d**4*f +
 2*d**5*f*sin(e + f*x)) - 3*a**3*c*d**2*sin(e + f*x)**2/(2*c*d**4*f + 2*d**
5*f*sin(e + f*x)) + 6*a**3*c*d**2/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) + 6*
a**3*d**3*log(c/d + sin(e + f*x))*sin(e + f*x)/(2*c*d**4*f + 2*d**5*f*sin(e
 + f*x)) + a**3*d**3*sin(e + f*x)**3/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) +
 6*a**3*d**3*sin(e + f*x)**2/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)) - 2*a**3*
d**3/(2*c*d**4*f + 2*d**5*f*sin(e + f*x)), Eq(n, -2)), (-a**3*c**3*log(c/d
+ sin(e + f*x))/(d**4*f) + 3*a**3*c**2*log(c/d + sin(e + f*x))/(d**3*f) + a
**3*c**2*sin(e + f*x)/(d**3*f) - 3*a**3*c*log(c/d + sin(e + f*x))/(d**2*f)
- a**3*c*sin(e + f*x)**2/(2*d**2*f) - 3*a**3*c*sin(e + f*x)/(d**2*f) + a**3
*log(c/d + sin(e + f*x))/(d*f) + a**3*sin(e + f*x)**3/(3*d*f) + 3*a**3*sin(
e + f*x)**2/(2*d*f) + 3*a**3*sin(e + f*x)/(d*f), Eq(n, -1)), (-6*a**3*c**4*
(c + d*sin(e + f*x))**n/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50
*d**4*f*n + 24*d**4*f) + 6*a**3*c**3*d*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)/(d**4*f*n**4 + 10*d**4*f*n**3 + 35*d**4*f*n**2 + 50*d**4*f*n + 24*d**4*f
) + 6*a**3*c**3*d*n*(c + d*sin(e + f*x))**n/(d*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(143) = 286$.
time = 0.59, size = 1001, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^3 - 3*(d*sin(f*x + e) + c
)^3*(d*sin(f*x + e) + c)^n*c*n^3 + 3*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
 + c)^n*c^2*n^3 - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^3 + 6*(
d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n^2 - 21*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n^2 + 24*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n*c^2*n^2 - 9*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n^2 + 11
*(d*sin(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n*n - 42*(d*sin(f*x + e) + c)^
3*(d*sin(f*x + e) + c)^n*c*n + 57*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c^2*n - 26*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3*n + 6*(d*si
n(f*x + e) + c)^4*(d*sin(f*x + e) + c)^n - 24*(d*sin(f*x + e) + c)^3*(d*sin
(f*x + e) + c)^n*c + 36*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c^2 -
 24*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^3)*a^3/(d^3*n^4 + 10*d^3*
n^3 + 35*d^3*n^2 + 50*d^3*n + 24*d^3) + 3*((d*sin(f*x + e) + c)^3*(d*sin(f*
x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n^2 +
 (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n^2 + 3*(d*sin(f*x + e) +
c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) +
c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2*n + 2*(d*sin(f
```

```
*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6*(d*sin(f*x + e) + c)^2*(d*sin(f*x
 + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c^2)*a^3/(d^
2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^3/(n
 + 1) + 3*((d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*n - (d*sin(f*x + e
) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(f*x + e) + c)^2*(d*sin(f*x + e)
+ c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c)*a^3/((n^2 + 3*n +
 2)*d))/(d*f)
```

Mupad [B]
time = 13.58, size = 617, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a^3*(c + d*sin(e + f*x))^n*(192*c*d^3 + 192*c^3*d + 261*d^4*n + 336*d^4*si
n(e + f*x) - 48*c^4 + 162*d^4 - 288*c^2*d^2 - 168*d^4*cos(2*e + 2*f*x) + 6*
d^4*cos(4*e + 4*f*x) + 114*d^4*n^2 + 15*d^4*n^3 - 48*d^4*sin(3*e + 3*f*x) +
 460*d^4*n*sin(e + f*x) - 180*c^2*d^2*n + 132*c*d^3*n^2 + 20*c*d^3*n^3 - 27
2*d^4*n*cos(2*e + 2*f*x) + 11*d^4*n*cos(4*e + 4*f*x) - 84*d^4*n*sin(3*e + 3
*f*x) + 198*d^4*n^2*sin(e + f*x) + 26*d^4*n^3*sin(e + f*x) - 36*c^2*d^2*n^2
 - 120*d^4*n^2*cos(2*e + 2*f*x) - 16*d^4*n^3*cos(2*e + 2*f*x) + 6*d^4*n^2*c
os(4*e + 4*f*x) + d^4*n^3*cos(4*e + 4*f*x) - 42*d^4*n^2*sin(3*e + 3*f*x) -
6*d^4*n^3*sin(3*e + 3*f*x) + 256*c*d^3*n + 48*c^3*d*n + 12*c^2*d^2*n*cos(2*
e + 2*f*x) - 60*c*d^3*n^2*cos(2*e + 2*f*x) - 12*c*d^3*n^3*cos(2*e + 2*f*x)
- 6*c*d^3*n^2*sin(3*e + 3*f*x) - 2*c*d^3*n^3*sin(3*e + 3*f*x) - 48*c^2*d^2*
n^2*sin(e + f*x) + 300*c*d^3*n*sin(e + f*x) + 48*c^3*d*n*sin(e + f*x) + 12*
c^2*d^2*n^2*cos(2*e + 2*f*x) - 48*c*d^3*n*cos(2*e + 2*f*x) - 4*c*d^3*n*sin(
3*e + 3*f*x) - 192*c^2*d^2*n*sin(e + f*x) + 186*c*d^3*n^2*sin(e + f*x) + 30
*c*d^3*n^3*sin(e + f*x)))/(8*d^4*f*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

3.915 $\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=101

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{1+n}}{d^3f(1+n)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{2+n}}{d^3f(2+n)}+\frac{a^2(c+d\sin(e+fx))^{3+n}}{d^3f(3+n)}$$

```
[Out] a^2*(c-d)^2*(c+d*sin(f*x+e))^(1+n)/d^3/f/(1+n)-2*a^2*(c-d)*(c+d*sin(f*x+e))
^(2+n)/d^3/f/(2+n)+a^2*(c+d*sin(f*x+e))^(3+n)/d^3/f/(3+n)
```

Rubi [A]
time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 45}

$$\frac{a^2(c-d)^2(c+d\sin(e+fx))^{n+1}}{d^3f(n+1)}-\frac{2a^2(c-d)(c+d\sin(e+fx))^{n+2}}{d^3f(n+2)}+\frac{a^2(c+d\sin(e+fx))^{n+3}}{d^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c - d)^2*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(1 + n)) - (2*a^2*(c -
d)*(c + d*Sin[e + f*x])^(2 + n))/(d^3*f*(2 + n)) + (a^2*(c + d*Sin[e + f*x]
)^(3 + n))/(d^3*f*(3 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(c+\frac{dx}{a}\right)^n\,dx, x, a\sin(e+fx\right.}{af}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{a^2(c-d)^2\left(c+\frac{dx}{a}\right)^n}{d^2} - \frac{2a^2(c-d)\left(c+\frac{dx}{a}\right)^{1+n}}{d^2}\right.\right.}{af}$$

$$= \frac{a^2(c-d)^2(c+d\sin(e+fx))^{1+n}}{d^3f(1+n)} - \frac{2a^2(c-d)}{}$$

Mathematica [A]
time = 0.28, size = 78, normalized size = 0.77

$$\frac{a^2(c+d\sin(e+fx))^{1+n}\left(\frac{(c-d)^2}{1+n} - \frac{2(c-d)(c+d\sin(e+fx))}{2+n} + \frac{(c+d\sin(e+fx))^2}{3+n}\right)}{d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (a^2*(c + d*Sin[e + f*x])^(1 + n)*((c - d)^2/(1 + n) - (2*(c - d)*(c + d*Si
n[e + f*x]))/(2 + n) + (c + d*Sin[e + f*x])^2/(3 + n)))/(d^3*f)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\,(a+a\sin(fx+e))^2\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [A]
time = 0.32, size = 183, normalized size = 1.81

$$\frac{\frac{2\left(d^2(n+1)\sin(fx+e)^2+cdn\sin(fx+e)-c^2\right)(d\sin(fx+e)+c)^n a^2}{(n^2+3n+2)d^2} + \frac{(d\sin(fx+e)+c)^{n+1}a^2}{d(n+1)} + \frac{\left((n^2+3n+2)d^3\sin(fx+e)^3+(n^2+n)cd^2\sin(fx+e)^2-2c^2dn\sin(fx+e)+2c^3\right)(d\sin(fx+e)+c)^n a^2}{(n^3+6n^2+11n+6)d^3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] (2*(d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e)
+ c)^n*a^2/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a^2/(d*(n +
 1)) + ((n^2 + 3*n + 2)*d^3*sin(f*x + e)^3 + (n^2 + n)*c*d^2*sin(f*x + e)^2
 - 2*c^2*d*n*sin(f*x + e) + 2*c^3)*(d*sin(f*x + e) + c)^n*a^2/((n^3 + 6*n^2
 + 11*n + 6)*d^3))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(104) = 208$.
time = 0.42, size = 298, normalized size = 2.95

$$\frac{\left(2\,a^2c^3 - 6\,a^2c^2d + 6\,a^2cd^2 + 6\,a^2d^3 + 2\,(a^2cd^2 + a^2d^3)n^2 - (6\,a^2d^3 + (a^2cd^2 + 2\,a^2d^3)n^2 + (a^2cd^2 + 8\,a^2d^3)n)\cos\left(fx + e\right)^2 - 2\,(a^2c^2d - 3\,a^2cd^2 - 4\,a^2d^3)n + \left(8\,a^2d^3 + 2\,(a^2cd^2 + a^2d^3)n^2 - (a^2d^3n^2 + 3\,a^2d^3n + 2\,a^2d^3)\cos\left(fx + e\right)^2 - 2\,(a^2c^2d - 3\,a^2cd^2 - 4\,a^2d^3)n\right)\sin\left(fx + e\right)\right)\left(d\sin\left(fx + e\right) + c\right)^n}{d^3fn^3 + 6\,d^3fn^2 + 11\,d^3fn + 6\,d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] (2*a^2*c^3 - 6*a^2*c^2*d + 6*a^2*c*d^2 + 6*a^2*d^3 + 2*(a^2*c*d^2 + a^2*d^3
)*n^2 - (6*a^2*d^3 + (a^2*c*d^2 + 2*a^2*d^3)*n^2 + (a^2*c*d^2 + 8*a^2*d^3)*
n)*cos(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n + (8*a^2*d^3
+ 2*(a^2*c*d^2 + a^2*d^3)*n^2 - (a^2*d^3*n^2 + 3*a^2*d^3*n + 2*a^2*d^3)*cos
(f*x + e)^2 - 2*(a^2*c^2*d - 3*a^2*c*d^2 - 4*a^2*d^3)*n)*sin(f*x + e))*(d*s
in(f*x + e) + c)^n/(d^3*f*n^3 + 6*d^3*f*n^2 + 11*d^3*f*n + 6*d^3*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2159 vs. $2(85) = 170$.
time = 6.16, size = 2159, normalized size = 21.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a**2*sin(e + f*x)**3/(3*f) + a**2*sin(e + f*x)**2/f + a**2
*sin(e + f*x)/f), Eq(d, 0)), (x*(c + d*sin(e))**n*(a*sin(e) + a)**2*cos(e),
 Eq(f, 0)), (2*a**2*c**2*log(c/d + sin(e + f*x))/(2*c**2*d**3*f + 4*c*d**4*
f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 3*a**2*c**2/(2*c**2*d**3*f + 4
*c*d**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 4*a**2*c*d*log(c/d + s
in(e + f*x))*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5
*f*sin(e + f*x)**2) + 4*a**2*c*d*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*s
in(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - 2*a**2*c*d/(2*c**2*d**3*f + 4*c*d
**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) + 2*a**2*d**2*log(c/d + sin(
e + f*x))*sin(e + f*x)**2/(2*c**2*d**3*f + 4*c*d**4*f*sin(e + f*x) + 2*d**5
*f*sin(e + f*x)**2) - 4*a**2*d**2*sin(e + f*x)/(2*c**2*d**3*f + 4*c*d**4*f*
sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2) - a**2*d**2/(2*c**2*d**3*f + 4*c*d
**4*f*sin(e + f*x) + 2*d**5*f*sin(e + f*x)**2), Eq(n, -3)), (-2*a**2*c**2*l
og(c/d + sin(e + f*x))/(c*d**3*f + d**4*f*sin(e + f*x)) - 2*a**2*c**2/(c*d*
```

```
*3*f + d**4*f*sin(e + f*x)) - 2*a**2*c*d*log(c/d + sin(e + f*x))*sin(e + f*
x)/(c*d**3*f + d**4*f*sin(e + f*x)) + 2*a**2*c*d*log(c/d + sin(e + f*x))/(c
*d**3*f + d**4*f*sin(e + f*x)) + 2*a**2*c*d/(c*d**3*f + d**4*f*sin(e + f*x)
) + 2*a**2*d**2*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**3*f + d**4*f*sin
(e + f*x)) + a**2*d**2*sin(e + f*x)**2/(c*d**3*f + d**4*f*sin(e + f*x)) - a
**2*d**2/(c*d**3*f + d**4*f*sin(e + f*x)), Eq(n, -2)), (a**2*c**2*log(c/d +
 sin(e + f*x))/(d**3*f) - 2*a**2*c*log(c/d + sin(e + f*x))/(d**2*f) - a**2*
c*sin(e + f*x)/(d**2*f) + a**2*log(c/d + sin(e + f*x))/(d*f) + a**2*sin(e +
 f*x)**2/(2*d*f) + 2*a**2*sin(e + f*x)/(d*f), Eq(n, -1)), (2*a**2*c**3*(c +
 d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f)
- 2*a**2*c**2*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**
3*f*n**2 + 11*d**3*f*n + 6*d**3*f) - 2*a**2*c**2*d*n*(c + d*sin(e + f*x))**
n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) - 6*a**2*c**2*d*(c
 + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f
) + a**2*c*d**2*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 +
 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*c*d**2*n**2*(c + d*sin(e
+ f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3
*f) + a**2*c*d**2*n**2*(c + d*sin(e + f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2
 + 11*d**3*f*n + 6*d**3*f) + a**2*c*d**2*n*(c + d*sin(e + f*x))**n*sin(e +
f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*c*d
**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 1
1*d**3*f*n + 6*d**3*f) + 5*a**2*c*d**2*n*(c + d*sin(e + f*x))**n/(d**3*f*n*
*3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*c*d**2*(c + d*sin(e +
 f*x))**n/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d**
3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2
 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*d**3*n**2*(c + d*sin(e + f*x))**n*sin(e
 + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + a**2*d*
*3*n**2*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 +
 11*d**3*f*n + 6*d**3*f) + 3*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 8*a**2*d**3*
n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f*n**3 + 6*d**3*f*n**2 + 11
*d**3*f*n + 6*d**3*f) + 5*a**2*d**3*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/
(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 2*a**2*d**3*(c + d
*sin(e + f*x))**n*sin(e + f*x)**3/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*
n + 6*d**3*f) + 6*a**2*d**3*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**3*f
*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3*f) + 6*a**2*d**3*(c + d*sin(e
+ f*x))**n*sin(e + f*x)/(d**3*f*n**3 + 6*d**3*f*n**2 + 11*d**3*f*n + 6*d**3
*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(104) = 208$.
time = 0.43, size = 463, normalized size = 4.58

$$\frac{\frac{\left((d\sin(fx+e)+c)^3(d\sin(fx+e)+c)^n n^2-2\,(d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n cn^2+(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n c^2n^2+3\,(d\sin(fx+e)+c)^3(d\sin(fx+e)+c)^n n-8\,(d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n cn+5\,(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n c^2n+2\,(d\sin(fx+e)+c)^3(d\sin(fx+e)+c)^n-6\,(d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n c+6\,(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n c^2\right)a^2}{d^2n^3+6\,d^2n^2+11\,d^2n+6\,d^2}+\frac{(d\sin(fx+e)+c)^{n+1}a^2}{n+1}+\frac{2\left((d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n n-(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n cn+(d\sin(fx+e)+c)^2(d\sin(fx+e)+c)^n-2\,(d\sin(fx+e)+c)(d\sin(fx+e)+c)^n c\right)a^2}{(n^2+3\,n+2)d}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] (((d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n^2 - 2*(d*sin(f*x + e) + c
)^2*(d*sin(f*x + e) + c)^n*c*n^2 + (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c
)^n*c^2*n^2 + 3*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n*n - 8*(d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c*n + 5*(d*sin(f*x + e) + c)*(d*sin(
f*x + e) + c)^n*c^2*n + 2*(d*sin(f*x + e) + c)^3*(d*sin(f*x + e) + c)^n - 6
*(d*sin(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n*c + 6*(d*sin(f*x + e) + c)*(
d*sin(f*x + e) + c)^n*c^2)*a^2/(d^2*n^3 + 6*d^2*n^2 + 11*d^2*n + 6*d^2) + (
d*sin(f*x + e) + c)^(n + 1)*a^2/(n + 1) + 2*((d*sin(f*x + e) + c)^2*(d*sin(
f*x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*si
n(f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*
x + e) + c)^n*c)*a^2/((n^2 + 3*n + 2)*d))/(d*f)
```

Mupad [B]
time = 11.26, size = 302, normalized size = 2.99

$$\frac{a^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,\left(24\,c\,d^2-24\,c^2\,d+16\,d^3\,n+30\,d^3\,\sin\left(e+f\,x\right)+8\,c^3+12\,d^3-12\,d^3\,\cos\left(2\,e+2\,f\,x\right)+4\,d^3\,n^2-2\,d^3\,\sin\left(3\,e+3\,f\,x\right)+29\,d^3\,n\,\sin\left(e+f\,x\right)+6\,c\,d^2\,n^2-16\,d^3\,n\,\cos\left(2\,e+2\,f\,x\right)-3\,d^3\,n\,\sin\left(3\,e+3\,f\,x\right)+7\,d^3\,n^2\,\sin\left(e+f\,x\right)-4\,d^3\,n^2\,\cos\left(2\,e+2\,f\,x\right)-d^3\,n^2\,\sin\left(3\,e+3\,f\,x\right)+22\,c\,d^2\,n-8\,c^2\,d\,n-2\,c\,d^2\,n^2\,\cos\left(2\,e+2\,f\,x\right)+24\,c\,d^2\,n\,\sin\left(e+f\,x\right)-8\,c^2\,d\,n\,\sin\left(e+f\,x\right)-2\,c\,d^2\,n\,\cos\left(2\,e+2\,f\,x\right)+8\,c\,d^2\,n^2\,\sin\left(e+f\,x\right)\right)}{4\,d^3\,f\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a^2*(c + d*sin(e + f*x))^n*(24*c*d^2 - 24*c^2*d + 16*d^3*n + 30*d^3*sin(e
 + f*x) + 8*c^3 + 12*d^3 - 12*d^3*cos(2*e + 2*f*x) + 4*d^3*n^2 - 2*d^3*sin(3
*e + 3*f*x) + 29*d^3*n*sin(e + f*x) + 6*c*d^2*n^2 - 16*d^3*n*cos(2*e + 2*f*
x) - 3*d^3*n*sin(3*e + 3*f*x) + 7*d^3*n^2*sin(e + f*x) - 4*d^3*n^2*cos(2*e
 + 2*f*x) - d^3*n^2*sin(3*e + 3*f*x) + 22*c*d^2*n - 8*c^2*d*n - 2*c*d^2*n^2*
cos(2*e + 2*f*x) + 24*c*d^2*n*sin(e + f*x) - 8*c^2*d*n*sin(e + f*x) - 2*c*d
^2*n*cos(2*e + 2*f*x) + 8*c*d^2*n^2*sin(e + f*x)))/(4*d^3*f*(11*n + 6*n^2 +
 n^3 + 6))
```

3.916 $\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=61

$$-\frac{a(c-d)(c+d\sin(e+fx))^{1+n}}{d^2f(1+n)}+\frac{a(c+d\sin(e+fx))^{2+n}}{d^2f(2+n)}$$

```
[Out] -a*(c-d)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(1+n)+a*(c+d*sin(f*x+e))^(2+n)/d^2/f/
(2+n)
```

Rubi [A]
time = 0.06, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{a(c+d\sin(e+fx))^{n+2}}{d^2f(n+2)}-\frac{a(c-d)(c+d\sin(e+fx))^{n+1}}{d^2f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((a*(c - d)*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*(1 + n))) + (a*(c + d*Sin
[e + f*x])^(2 + n))/(d^2*f*(2 + n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\text{Subst}\left(\int (a+x)\left(c+\frac{dx}{a}\right)^n\,dx,x,a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a(c-d)\left(c+\frac{dx}{a}\right)^n}{d}+\frac{a\left(c+\frac{dx}{a}\right)^{1+n}}{d}\right)\,dx,x,\right.}{af}$$

$$= -\frac{a(c-d)(c+d\sin(e+fx))^{1+n}}{d^2f(1+n)}+\frac{a(c+d\sin(e}{d^2f(2-}$$

Mathematica [A]
time = 0.37, size = 52, normalized size = 0.85

$$\frac{a(c+d\sin(e+fx))^{1+n}(-c+d(2+n)+d(1+n)\sin(e+fx))}{d^2f(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] (a*(c + d*Sin[e + f*x])^(1 + n)*(-c + d*(2 + n) + d*(1 + n)*Sin[e + f*x]))/
(d^2*f*(1 + n)*(2 + n))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \cos(fx+e)(a+a\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 1.43

$$\frac{\frac{\left(d^2(n+1)\sin(fx+e)^2+cdn\sin(fx+e)-c^2\right)(d\sin(fx+e)+c)^na}{(n^2+3n+2)d^2}+\frac{(d\sin(fx+e)+c)^{n+1}a}{d(n+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxi
ma")
```

```
[Out] ((d^2*(n + 1)*sin(f*x + e)^2 + c*d*n*sin(f*x + e) - c^2)*(d*sin(f*x + e) +
c)^n*a/((n^2 + 3*n + 2)*d^2) + (d*sin(f*x + e) + c)^(n + 1)*a/(d*(n + 1)))/
f
```

Fricas [A]
time = 0.37, size = 119, normalized size = 1.95

$$-\frac{\left(ac^2 - 2\,acd - ad^2 + (ad^2n + ad^2)\cos\left(fx+e\right)^2 - (acd + ad^2)n - \left(2\,ad^2 + (acd + ad^2)n\right)\sin\left(fx+e\right)\right)\left(d\sin\left(fx+e\right) + c\right)^n}{d^2fn^2 + 3\,d^2fn + 2\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fric
as")
```

```
[Out] -(a*c^2 - 2*a*c*d - a*d^2 + (a*d^2*n + a*d^2)*cos(f*x + e)^2 - (a*c*d + a*d
^2)*n - (2*a*d^2 + (a*c*d + a*d^2)*n)*sin(f*x + e))*(d*sin(f*x + e) + c)^n/
(d^2*f*n^2 + 3*d^2*f*n + 2*d^2*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(49) = 98$.
time = 1.64, size = 586, normalized size = 9.61

$$\begin{cases} c^n\left(\frac{a\sin^2\left(e+fx\right)}{2f} + \frac{a\sin\left(e+fx\right)}{f}\right) & \text{for}\: d = 0 \\ x\left(c + d\sin\left(e\right)\right)^n\left(a\sin\left(e\right) + a\right)\cos\left(e\right) & \text{for}\: f = 0 \\ \frac{ac\log\left(\frac{c}{d}+\sin\left(e+fx\right)\right)}{cd^2f+d^3f\sin\left(e+fx\right)} + \frac{ac}{cd^2f+d^3f\sin\left(e+fx\right)} + \frac{ad\log\left(\frac{c}{d}+\sin\left(e+fx\right)\right)\sin\left(e+fx\right)}{cd^2f+d^3f\sin\left(e+fx\right)} - \frac{ad}{cd^2f+d^3f\sin\left(e+fx\right)} & \text{for}\: n = -2 \\ -\frac{ac\log\left(\frac{c}{d}+\sin\left(e+fx\right)\right)}{d^2f} + \frac{a\log\left(\frac{c}{d}+\sin\left(e+fx\right)\right)}{df} + \frac{a\sin\left(e+fx\right)}{df} & \text{for}\: n = -1 \\ -\frac{ac^2\left(c+d\sin\left(e+fx\right)\right)^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{acdn\left(c+d\sin\left(e+fx\right)\right)^n\sin\left(e+fx\right)}{d^2fn^2+3d^2fn+2d^2f} + \frac{acdn\left(c+d\sin\left(e+fx\right)\right)^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{2acd\left(c+d\sin\left(e+fx\right)\right)^n}{d^2fn^2+3d^2fn+2d^2f} + \frac{ad^2n\left(c+d\sin\left(e+fx\right)\right)^n\sin^2\left(e+fx\right)}{d^2fn^2+3d^2fn+2d^2f} + \frac{ad^2n\left(c+d\sin\left(e+fx\right)\right)^n\sin\left(e+fx\right)}{d^2fn^2+3d^2fn+2d^2f} + \frac{ad^2\left(c+d\sin\left(e+fx\right)\right)^n\sin^2\left(e+fx\right)}{d^2fn^2+3d^2fn+2d^2f} + \frac{2ad^2\left(c+d\sin\left(e+fx\right)\right)^n\sin\left(e+fx\right)}{d^2fn^2+3d^2fn+2d^2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Piecewise((c**n*(a*sin(e + f*x)**2/(2*f) + a*sin(e + f*x)/f), Eq(d, 0)), (x
*(c + d*sin(e))**n*(a*sin(e) + a)*cos(e), Eq(f, 0)), (a*c*log(c/d + sin(e +
 f*x))/(c*d**2*f + d**3*f*sin(e + f*x)) + a*c/(c*d**2*f + d**3*f*sin(e + f*
x)) + a*d*log(c/d + sin(e + f*x))*sin(e + f*x)/(c*d**2*f + d**3*f*sin(e + f
*x)) - a*d/(c*d**2*f + d**3*f*sin(e + f*x)), Eq(n, -2)), (-a*c*log(c/d + si
n(e + f*x))/(d**2*f) + a*log(c/d + sin(e + f*x))/(d*f) + a*sin(e + f*x)/(d*
f), Eq(n, -1)), (-a*c**2*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*f*n
+ 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3
*d**2*f*n + 2*d**2*f) + a*c*d*n*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d*
*2*f*n + 2*d**2*f) + 2*a*c*d*(c + d*sin(e + f*x))**n/(d**2*f*n**2 + 3*d**2*
f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*x)**2/(d**2*f*
n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*n*(c + d*sin(e + f*x))**n*sin(e + f*
x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + a*d**2*(c + d*sin(e + f*x))**n*s
in(e + f*x)**2/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f) + 2*a*d**2*(c + d*sin(
e + f*x))**n*sin(e + f*x)/(d**2*f*n**2 + 3*d**2*f*n + 2*d**2*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(63) = 126.

time = 0.44, size = 156, normalized size = 2.56

$$\frac{\frac{(d\sin(fx+e)+c)^{n+1}a}{n+1}+\frac{\left((d\sin(fx+e)+c)^{2}(d\sin(fx+e)+c)^{n}n-(d\sin(fx+e)+c)(d\sin(fx+e)+c)^{n}cn+(d\sin(fx+e)+c)^{2}(d\sin(fx+e)+c)^{n}-2\,(d\sin(fx+e)+c)(d\sin(fx+e)+c)^{n}c\right)a}{(n^{2}+3\,n+2)d}}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac
")
```

```
[Out] ((d*sin(f*x + e) + c)^(n + 1)*a/(n + 1) + ((d*sin(f*x + e) + c)^2*(d*sin(f*
x + e) + c)^n*n - (d*sin(f*x + e) + c)*(d*sin(f*x + e) + c)^n*c*n + (d*sin(
f*x + e) + c)^2*(d*sin(f*x + e) + c)^n - 2*(d*sin(f*x + e) + c)*(d*sin(f*x
+ e) + c)^n*c)*a/((n^2 + 3*n + 2)*d))/(d*f)
```

Mupad [B]

time = 10.08, size = 121, normalized size = 1.98

$$\frac{a\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n\,\left(4\,c\,d+d^2\,n+4\,d^2\,\sin\left(e+f\,x\right)+d^2\,\left(2\,{\sin\left(e+f\,x\right)}^2-1\right)-2\,c^2+d^2+2\,d^2\,n\,\sin\left(e+f\,x\right)+d^2\,n\,\left(2\,{\sin\left(e+f\,x\right)}^2-1\right)+2\,c\,d\,n+2\,c\,d\,n\,\sin\left(e+f\,x\right)\right)}{2\,d^2\,f\,\left(n^2+3\,n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
```

```
[Out] (a*(c + d*sin(e + f*x))^n*(4*c*d + d^2*n + 4*d^2*sin(e + f*x) + d^2*(2*sin(
e + f*x)^2 - 1) - 2*c^2 + d^2 + 2*d^2*n*sin(e + f*x) + d^2*n*(2*sin(e + f*x
)^2 - 1) + 2*c*d*n + 2*c*d*n*sin(e + f*x)))/(2*d^2*f*(3*n + n^2 + 2))
```

3.917 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)}$$

```
[Out] -hypergeom([1, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n)/a/
(c-d)/f/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$-\frac{(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{af(n+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{a+x}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= -\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)}$$

Mathematica [A]
time = 0.06, size = 60, normalized size = 1.00

$$-\frac{{}_2F_1\left(1,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a(c-d)f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
[Out] -((Hypergeometric2F1[1, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\cos(fx+e)(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxi
ma")
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{a+a\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.918 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=60

$$\frac{d\,{}_2F_1\left(2,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^2(c-d)^2f(1+n)}$$

```
[Out] d*hypergeom([2, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n)/a
^2/(c-d)^2/f/(1+n)
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$\frac{d(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^2f(n+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c + d*
Sin[e + f*x])^(1 + n))/(a^2*(c - d)^2*f*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^2}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{d\,{}_2F_1\left(2,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^2(c-d)^2f(1+n)}$$

Mathematica [A]
time = 0.05, size = 61, normalized size = 1.02

$$\frac{d\,{}_2F_1\left(2,1+n;2+n;-\frac{c+d\sin(e+fx)}{-c+d}\right)(c+d\sin(e+fx))^{1+n}}{a^2(-c+d)^2f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*Hypergeometric2F1[2, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^2*(-c + d)^2*f*(1 + n))
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{\cos(fx+e)(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(a^2*cos(f*x + e)^2 - 2*a^2*s
in(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

3.919 $\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=63

$$-\frac{d^2\,{}_2F_1\left(3,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^3(c-d)^3f(1+n)}$$

```
[Out] -d^2*hypergeom([3, 1+n],[2+n],(c+d*sin(f*x+e))/(c-d))*(c+d*sin(f*x+e))^(1+n
)/a^3/(c-d)^3/f/(1+n)
```

Rubi [A]
time = 0.10, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$-\frac{d^2(c+d\sin(e+fx))^{n+1}\,{}_2F_1\left(3,n+1;n+2;\frac{c+d\sin(e+fx)}{c-d}\right)}{a^3f(n+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((d^2*Hypergeometric2F1[3, 1 + n, 2 + n, (c + d*Sin[e + f*x])/(c - d)]*(c
+ d*Sin[e + f*x])^(1 + n))/(a^3*(c - d)^3*f*(1 + n)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(c+\frac{dx}{a}\right)^n}{(a+x)^3}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= -\frac{d^2\,{}_2F_1\left(3,1+n;2+n;\frac{c+d\sin(e+fx)}{c-d}\right)(c+d\sin(e+fx))^{1+n}}{a^3(c-d)^3f(1+n)}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 1.00

$$\frac{d^2\,{}_2F_1\left(3,1+n;2+n;-\frac{c+d\sin(e+fx)}{-c+d}\right)(c+d\sin(e+fx))^{1+n}}{a^3(-c+d)^3f(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (d^2*Hypergeometric2F1[3, 1 + n, 2 + n, -((c + d*Sin[e + f*x])/(-c + d))]*(
c + d*Sin[e + f*x])^(1 + n))/(a^3*(-c + d)^3*f*(1 + n))
```

Maple [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\cos(fx+e)\,(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)/(3*a^3*cos(f*x + e)^2 - 4*a^3
 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)/(a*sin(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int((cos(e + f*x)*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

3.920 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx$

Optimal. Leaf size=170

$$\frac{(c-d)^4(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{4(c-d)^3d(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}+\frac{6(c-d)^2d^2(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)}+4(c$$

```
[Out] (c-d)^4*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+4*(c-d)^3*d*(a+a*sin(f*x+e))^(2+m)
/a^2/f/(2+m)+6*(c-d)^2*d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)+4*(c-d)*d^3*(
a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)+d^4*(a+a*sin(f*x+e))^(5+m)/a^5/f/(5+m)
```

Rubi [A]

time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 45}

$$\frac{d^4(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{4d^3(c-d)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{6d^2(c-d)^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{4d(c-d)^3(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)^4(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((c - d)^4*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (4*(c - d)^3*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (6*(c - d)^2*d^2*(a + a*Sin[e
+ f*x])^(3 + m))/(a^3*f*(3 + m)) + (4*(c - d)*d^3*(a + a*Sin[e + f*x])^(4 +
 m))/(a^4*f*(4 + m)) + (d^4*(a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^4\,dx = \frac{\text{Subst}\left(\int (a+x)^m \left(c+\frac{dx}{a}\right)^4 dx, x, a\sin(e+f\right.}{af}$$

$$= \frac{\text{Subst}\left(\int \left((c-d)^4(a+x)^m + \frac{4(c-d)^3 d(a+x)^{1+m}}{a}\right.\right.}{}$$

$$= \frac{(c-d)^4(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{4(c-d)^3 d(}{a}$$

Mathematica [A]
time = 0.53, size = 143, normalized size = 0.84

$$\frac{(a(1+\sin(e+fx)))^{1+m}\left(\frac{a^4(c-d)^4}{1+m}+\frac{4a^4(c-d)^3d(1+\sin(e+fx))}{2+m}+\frac{6a^4(c-d)^2d^2(1+\sin(e+fx))^2}{3+m}+\frac{4a^4(c-d)d^3(1+\sin(e+fx))^3}{4+m}+\frac{d^4(a+a\sin(e+fx))^4}{5+m}\right)}{a^5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^4*(c - d)^4)/(1 + m) + (4*a^4*(c - d)^3
*d*(1 + Sin[e + f*x]))/(2 + m) + (6*a^4*(c - d)^2*d^2*(1 + Sin[e + f*x])^2)
/(3 + m) + (4*a^4*(c - d)*d^3*(1 + Sin[e + f*x])^3)/(4 + m) + (d^4*(a + a*S
in[e + f*x])^4)/(5 + m)))/(a^5*f)
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(175) = 350.
time = 0.30, size = 476, normalized size = 2.80

$$\frac{\frac{4\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)c^3d(\sin(fx+e)+1)^m}{m^2+3m+2}+\frac{6\left((m^2+3m+2)a^m\sin(fx+e)^3+(m^2+m)a^m\sin(fx+e)^2-2a^m m\sin(fx+e)+2a^m\right)c^2d^2(\sin(fx+e)+1)^m}{m^3+6m^2+11m+6}+\frac{4\left((m^3+6m^2+11m+6)a^m\sin(fx+e)^4+(m^3+3m^2+2m)a^m\sin(fx+e)^3-3(m^2+m)a^m\sin(fx+e)^2+6a^m m\sin(fx+e)-6a^m\right)cd^3(\sin(fx+e)+1)^m}{m^4+10m^3+35m^2+50m+24}+\frac{\left((m^4+10m^3+35m^2+50m+24)a^m\sin(fx+e)^5+(m^4+6m^3+11m^2+6m)a^m\sin(fx+e)^4-4(m^3+3m^2+2m)a^m\sin(fx+e)^3+12(m^2+m)a^m\sin(fx+e)^2-24a^m m\sin(fx+e)+24a^m\right)d^4(\sin(fx+e)+1)^m}{m^5+15m^4+85m^3+225m^2+274m+120}+\frac{(a\sin(fx+e)+a)^{m+1}c^4}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="ma
xima")
```

```
[Out] (4*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c^3*d*(sin(f*x +
 e) + 1)^m/(m^2 + 3*m + 2) + 6*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 +
 m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*c^2*d^2*(sin(f*x + e
) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + 4*((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x
 + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x
+ e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*c*d^3*(sin(f*x + e) + 1)^m/(m^4 + 10
*m^3 + 35*m^2 + 50*m + 24) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(f
*x + e)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4*(m^3 + 3*m^
2 + 2*m)*a^m*sin(f*x + e)^3 + 12*(m^2 + m)*a^m*sin(f*x + e)^2 - 24*a^m*m*si
n(f*x + e) + 24*a^m)*d^4*(sin(f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120) + (a*sin(f*x + e) + a)^(m + 1)*c^4/(a*(m + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(175) = 350$.
time = 0.43, size = 750, normalized size = 4.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="fr
icas")
```

```
[Out] ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 + d^4)*m^4 + ((4*c*d^3 + d^4)*m^4 + 1
20*c*d^3 + 2*(22*c*d^3 + 3*d^4)*m^3 + (164*c*d^3 + 11*d^4)*m^2 + 2*(122*c*d
^3 + 3*d^4)*m)*cos(f*x + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 +
 24*c^3*d + 30*c^2*d^2 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*
c^2*d^2 + 92*c*d^3 + 23*d^4)*m^2 - 2*((2*c^3*d + 3*c^2*d^2 + 4*c*d^3 + d^4)
*m^4 + 120*c^3*d + 120*c*d^3 + 2*(13*c^3*d + 15*c^2*d^2 + 19*c*d^3 + 3*d^4)
*m^3 + (118*c^3*d + 87*c^2*d^2 + 128*c*d^3 + 17*d^4)*m^2 + 2*(107*c^3*d + 3
0*c^2*d^2 + 107*c*d^3 + 6*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^3*d +
114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m + ((c^4 + 4*c^3*d + 6*c^2*d^2 + 4*c*d^3 +
 d^4)*m^4 + (d^4*m^4 + 10*d^4*m^3 + 35*d^4*m^2 + 50*d^4*m + 24*d^4)*cos(f*x
 + e)^4 + 120*c^4 + 240*c^2*d^2 + 24*d^4 + 2*(7*c^4 + 24*c^3*d + 30*c^2*d^2
 + 16*c*d^3 + 3*d^4)*m^3 + (71*c^4 + 188*c^3*d + 186*c^2*d^2 + 92*c*d^3 + 2
3*d^4)*m^2 - 2*((3*c^2*d^2 + 2*c*d^3 + d^4)*m^4 + 120*c^2*d^2 + 24*d^4 + 4*
(9*c^2*d^2 + 4*c*d^3 + 2*d^4)*m^3 + (147*c^2*d^2 + 34*c*d^3 + 29*d^4)*m^2 +
 2*(117*c^2*d^2 + 10*c*d^3 + 23*d^4)*m)*cos(f*x + e)^2 + 2*(77*c^4 + 120*c^
3*d + 114*c^2*d^2 + 80*c*d^3 + 9*d^4)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)
^m/(f*m^5 + 15*f*m^4 + 85*f*m^3 + 225*f*m^2 + 274*f*m + 120*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 9238 vs. $2(144) = 288$.
time = 23.84, size = 9238, normalized size = 54.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**4,x)

[Out] Piecewise((x*(c + d*sin(e))**4*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-3*c**
4/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e
+ f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) - 16*c**3*d*sin(e + f*x)/(1
2*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*
x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) - 4*c**3*d/(12*a**5*f*sin(e + f
*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*
sin(e + f*x) + 12*a**5*f) - 36*c**2*d**2*sin(e + f*x)**2/(12*a**5*f*sin(e +
 f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*
f*sin(e + f*x) + 12*a**5*f) - 24*c**2*d**2*sin(e + f*x)/(12*a**5*f*sin(e +
f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f
*sin(e + f*x) + 12*a**5*f) - 6*c**2*d**2/(12*a**5*f*sin(e + f*x)**4 + 48*a*
*5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) +
 12*a**5*f) - 48*c*d**3*sin(e + f*x)**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**
5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) +
12*a**5*f) - 72*c*d**3*sin(e + f*x)**2/(12*a**5*f*sin(e + f*x)**4 + 48*a**5
*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 1
2*a**5*f) - 48*c*d**3*sin(e + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*s
in(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a*
*5*f) - 12*c*d**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 +
72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 12*d**4*l
og(sin(e + f*x) + 1)*sin(e + f*x)**4/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f
*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*
a**5*f) + 48*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(12*a**5*f*sin(e +
f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f
*sin(e + f*x) + 12*a**5*f) + 72*d**4*log(sin(e + f*x) + 1)*sin(e + f*x)**2/
(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e +
f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 48*d**4*log(sin(e + f*x) +
1)*sin(e + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72
*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 12*d**4*log
(sin(e + f*x) + 1)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 +
 72*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 48*d**4*
sin(e + f*x)**3/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72
*a**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 108*d**4*si
n(e + f*x)**2/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a
**5*f*sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 88*d**4*sin(e
 + f*x)/(12*a**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*
sin(e + f*x)**2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 25*d**4/(12*a**5*f*
sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)**2 + 4
8*a**5*f*sin(e + f*x) + 12*a**5*f), Eq(m, -5)), (-c**4/(3*a**4*f*sin(e + f*
x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 6*c*
*3*d*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*
a**4*f*sin(e + f*x) + 3*a**4*f) - 2*c**3*d/(3*a**4*f*sin(e + f*x)**3 + 9*a*
*4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 18*c**2*d**2*sin
```

```
(e + f*x)**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*
f*sin(e + f*x) + 3*a**4*f) - 18*c**2*d**2*sin(e + f*x)/(3*a**4*f*sin(e + f*
x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 6*c*
*2*d**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin
(e + f*x) + 3*a**4*f) + 12*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(3*
a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) +
 3*a**4*f) + 36*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(3*a**4*f*sin(
e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f)
+ 36*c*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 +
9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) + 12*c*d**3*lo
g(sin(e + f*x) + 1)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 +
9*a**4*f*sin(e + f*x) + 3*a**4*f) + 36*c*d**3*sin(e + f*x)**2/(3*a**4*f*sin
(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f)
 + 54*c*d**3*sin(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)
**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) + 22*c*d**3/(3*a**4*f*sin(e + f*x)*
*3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 12*d**4
*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f
*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 36*d**4*log(sin(e +
f*x) + 1)*sin(e + f*x)**2/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)
**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) - 36*d**4*log(sin(e + f*x) + 1)*sin
(e + f*x)/(3*a**4*f*sin(e + f*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*s
in(e + f*x) + 3*a**4*f) - 12*d**4*log(sin(e + f*x) + 1)/(3*a**4*f*sin(e + f
*x)**3 + 9*a**4*f*sin(e + f*x)**2 + 9*a**4*f*sin(e + f*x) + 3*a**4*f) + 3*d
**4*sin(e + f*x)**4/(3*a**4*f*sin(e + f*x)**3 +...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1845 vs. $2(175) = 350$.
time = 0.50, size = 1845, normalized size = 10.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^4,x, algorithm="gi
ac")
```

```
[Out] (6*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c^2*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^
2) + 4*((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x +
e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3
```

```
 + 6*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x
 + e) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^
2 + 11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e)
 + a)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
 e) + a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*
(a*sin(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
*a^2 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*c*d^3/(a^3*m^4 +
 10*a^3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + ((a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*m^4 - 4*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a
*m^4 + 6*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^4 - 4*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*m^4 + (a*sin(f*x + e) + a)*(a*sin
(f*x + e) + a)^m*a^4*m^4 + 10*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m
*m^3 - 44*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^3 + 72*(a*sin(f
*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2*m^3 - 52*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^3*m^3 + 14*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
)^m*a^4*m^3 + 35*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 164*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 294*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a^2*m^2 - 236*(a*sin(f*x + e) + a)^2*(a*sin(f*x
+ e) + a)^m*a^3*m^2 + 71*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m^
2 + 50*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 244*(a*sin(f*x + e
) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 468*(a*sin(f*x + e) + a)^3*(a*sin(f*x
 + e) + a)^m*a^2*m - 428*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3*
m + 154*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^4*m + 24*(a*sin(f*x +
 e) + a)^5*(a*sin(f*x + e) + a)^m - 120*(a*sin(f*x + e) + a)^4*(a*sin(f*x +
 e) + a)^m*a + 240*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a^2 - 240*
(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^3 + 120*(a*sin(f*x + e) + a
)*(a*sin(f*x + e) + a)^m*a^4)*d^4/(a^4*m^5 + 15*a^4*m^4 + 85*a^4*m^3 + 225*
a^4*m^2 + 274*a^4*m + 120*a^4) + (a*sin(f*x + e) + a)^(m + 1)*c^4/(m + 1) +
 4*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*
(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m
- 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^3*d/((m^2 + 3*m + 2)*a
))/(a*f)
```

Mupad [B]
time = 16.87, size = 1656, normalized size = 9.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^4,x)
```

```
[Out] exp(- e*5i - f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*
1i)*1i)/2))^m*((exp(e*6i + f*x*6i)*(2464*c^4*m + 20*d^4*m + 1920*c^4 + 240*
```

d^4 + 2880*c^2*d^2 + 1136*c^4*m^2 + 224*c^4*m^3 + 16*c^4*m^4 + 206*d^4*m^2
+ 52*d^4*m^3 + 10*d^4*m^4 + 1776*c^2*d^2*m + 1200*c*d^3*m^2 + 3008*c^3*d*m^
2 + 384*c*d^3*m^3 + 768*c^3*d*m^3 + 48*c*d^3*m^4 + 64*c^3*d*m^4 + 1800*c^2*
d^2*m^2 + 672*c^2*d^2*m^3 + 72*c^2*d^2*m^4 + 2400*c*d^3*m + 3840*c^3*d*m))/
(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) - (exp(e*4i
+ f*x*4i)*(2464*c^4*m + 20*d^4*m + 1920*c^4 + 240*d^4 + 2880*c^2*d^2 + 1136
*c^4*m^2 + 224*c^4*m^3 + 16*c^4*m^4 + 206*d^4*m^2 + 52*d^4*m^3 + 10*d^4*m^4
 + 1776*c^2*d^2*m + 1200*c*d^3*m^2 + 3008*c^3*d*m^2 + 384*c*d^3*m^3 + 768*c
^3*d*m^3 + 48*c*d^3*m^4 + 64*c^3*d*m^4 + 1800*c^2*d^2*m^2 + 672*c^2*d^2*m^3
 + 72*c^2*d^2*m^4 + 2400*c*d^3*m + 3840*c^3*d*m))/(32*f*(m*274i + m^2*225i
+ m^3*85i + m^4*15i + m^5*1i + 120i)) - (d^4*(50*m + 35*m^2 + 10*m^3 + m^4
+ 24))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (ex
p(e*5i + f*x*5i)*(c^4*m*4928i - c^3*d*3840i - c*d^3*2400i + d^4*m*264i + c^
4*3840i + d^4*768i + c^2*d^2*7680i + c^4*m^2*2272i + c^4*m^3*448i + c^4*m^4
*32i + d^4*m^2*324i + d^4*m^3*72i + d^4*m^4*12i + c^2*d^2*m*5376i + c*d^3*m
^2*816i + c^3*d*m^2*2240i + c*d^3*m^3*336i + c^3*d*m^3*704i + c*d^3*m^4*48i
 + c^3*d*m^4*64i + c^2*d^2*m^2*3168i + c^2*d^2*m^3*960i + c^2*d^2*m^4*96i +
 c*d^3*m*1200i + c^3*d*m*832i))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15
i + m^5*1i + 120i)) + (d^4*exp(e*10i + f*x*10i)*(50*m + 35*m^2 + 10*m^3 + m
^4 + 24))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) +
(d^2*exp(e*2i + f*x*2i)*(3*m + m^2 + 2)*(216*c^2*m + 19*d^2*m + 480*c^2 + 6
0*d^2 + 24*c^2*m^2 + 5*d^2*m^2 + 80*c*d*m + 16*c*d*m^2))/(32*f*(m*274i + m^
2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) - (d^2*exp(e*8i + f*x*8i)*(3*m
 + m^2 + 2)*(216*c^2*m + 19*d^2*m + 480*c^2 + 60*d^2 + 24*c^2*m^2 + 5*d^2*m
^2 + 80*c*d*m + 16*c*d*m^2))/(32*f*(m*274i + m^2*225i + m^3*85i + m^4*15i +
 m^5*1i + 120i)) - (d*exp(e*3i + f*x*3i)*(m + 1)*(c*d^2*120i + c^3*m*188i +
 d^3*m*18i + c^3*240i + c^3*m^2*48i + c^3*m^3*4i + d^3*m^2*5i + d^3*m^3*1i
+ c*d^2*m^2*28i + c^2*d*m^2*54i + c*d^2*m^3*4i + c^2*d*m^3*6i + c*d^2*m*64i
 + c^2*d*m*120i))/(4*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 12
0i)) - (d*exp(e*7i + f*x*7i)*(m + 1)*(c*d^2*120i + c^3*m*188i + d^3*m*18i +
 c^3*240i + c^3*m^2*48i + c^3*m^3*4i + d^3*m^2*5i + d^3*m^3*1i + c*d^2*m^2*
28i + c^2*d*m^2*54i + c*d^2*m^3*4i + c^2*d*m^3*6i + c*d^2*m*64i + c^2*d*m*1
20i))/(4*f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (d^3*
exp(e*1i + f*x*1i)*(c*20i + c*m*4i + d*m*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*
f*(m*274i + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)) + (d^3*exp(e*9i
+ f*x*9i)*(c*20i + c*m*4i + d*m*1i)*(11*m + 6*m^2 + m^3 + 6))/(16*f*(m*274i
 + m^2*225i + m^3*85i + m^4*15i + m^5*1i + 120i)))

3.921 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx$

Optimal. Leaf size=133

$$\frac{(c-d)^3(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{3(c-d)^2d(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}+\frac{3(c-d)d^2(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)}+\frac{d^3(\ldots$$

```
[Out] (c-d)^3*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+3*(c-d)^2*d*(a+a*sin(f*x+e))^(2+m)
/a^2/f/(2+m)+3*(c-d)*d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)+d^3*(a+a*sin(f*
x+e))^(4+m)/a^4/f/(4+m)
```

Rubi [A]

time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 45}

$$\frac{d^3(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{3d^2(c-d)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{3d(c-d)^2(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)^3(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((c - d)^3*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (3*(c - d)^2*d*(a
+ a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (3*(c - d)*d^2*(a + a*Sin[e +
f*x])^(3 + m))/(a^3*f*(3 + m)) + (d^3*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*
(4 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^3\,dx = \frac{\operatorname{Subst}\left(\int (a+x)^m \left(c+\frac{dx}{a}\right)^3\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\operatorname{Subst}\left(\int \left((c-d)^3(a+x)^m + \frac{3(c-d)^2d(a+x)^{1+m}}{a} +\right.\right.}{}$$

$$= \frac{(c-d)^3(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{3(c-d)^2d(a}{a^2}$$

Mathematica [A]
time = 0.30, size = 113, normalized size = 0.85

$$\frac{(a(1+\sin(e+fx)))^{1+m}\left(\frac{a^3(c-d)^3}{1+m}+\frac{3a^3(c-d)^2d(1+\sin(e+fx))}{2+m}+\frac{3a^3(c-d)d^2(1+\sin(e+fx))^2}{3+m}+\frac{d^3(a+a\sin(e+fx))^3}{4+m}\right)}{a^4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^3*(c - d)^3)/(1 + m) + (3*a^3*(c - d)^2
*d*(1 + Sin[e + f*x]))/(2 + m) + (3*a^3*(c - d)*d^2*(1 + Sin[e + f*x])^2)/(
3 + m) + (d^3*(a + a*Sin[e + f*x])^3)/(4 + m)))/(a^4*f)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\,(a+a\sin(fx+e))^m\,(c+d\sin(fx+e))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(137) = 274.
time = 0.28, size = 307, normalized size = 2.31

$$\frac{\frac{3\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)c^2d(\sin(fx+e)+1)^m}{m^2+3m+2}+\frac{3\left((m^2+3m+2)a^m\sin(fx+e)^3+(m^2+m)a^m\sin(fx+e)^2-2a^m m\sin(fx+e)+2a^m\right)cd^2(\sin(fx+e)+1)^m}{m^3+6m^2+11m+6}+\frac{\left((m^3+6m^2+11m+6)a^m\sin(fx+e)^4+(m^3+3m^2+2m)a^m\sin(fx+e)^3-3(m^2+m)a^m\sin(fx+e)^2+6a^m m\sin(fx+e)-6a^m\right)d^3(\sin(fx+e)+1)^m}{m^4+10m^3+35m^2+50m+24}+\frac{(a\sin(fx+e)+a)^{m+1}c^3}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
```

[Out] (3*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c^2*d*(sin(f*x + e) + 1)^m/(m^2 + 3*m + 2) + 3*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*c*d^2*(sin(f*x + e) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*d^3*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + (a*sin(f*x + e) + a)^(m + 1)*c^3/(a*(m + 1)))/f

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(137) = 274$.
time = 0.39, size = 408, normalized size = 3.07

$$\frac{\left((d^3m^3 + 6\,d^3m^2 + 11\,d^3m + 6\,d^3)\cos\left(fx + e\right)^4 + (c^3 + 3\,c^2d + 3\,cd^2 + d^3)m^3 + 24\,c^3 + 24\,cd^2 + 3\,(3\,c^3 + 7\,c^2d + 5\,cd^2 + d^3)m^2 - ((3\,c^2d + 3\,cd^2 + 2\,d^3)m^3 + 36\,c^2d + 12\,d^3 + 3\,(8\,c^2d + 5\,cd^2 + 3\,d^3)m^2 + (57\,c^2d + 12\,cd^2 + 19\,d^3)m)\cos\left(fx + e\right)^2 + 2\,(13\,c^3 + 18\,c^2d + 9\,cd^2 + 4\,d^3)m + ((c^3 + 3\,c^2d + 3\,cd^2 + d^3)m^3 + 24\,c^3 + 24\,cd^2 + 3\,(3\,c^3 + 7\,c^2d + 5\,cd^2 + d^3)m^2 - ((3\,cd^2 + d^3)m^3 + 24\,cd^2 + 3\,(7\,cd^2 + d^3)m^2 + 2\,(21\,cd^2 + d^3)m)\cos\left(fx + e\right)^2 + 2\,(13\,c^3 + 18\,c^2d + 9\,cd^2 + 4\,d^3)m)\sin\left(fx + e\right)\right)\left(a\sin\left(fx + e\right) + a\right)^m}{fm^4 + 10\,fm^3 + 35\,fm^2 + 50\,fm + 24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] ((d^3*m^3 + 6*d^3*m^2 + 11*d^3*m + 6*d^3)*cos(f*x + e)^4 + (c^3 + 3*c^2*d + 3*c*d^2 + d^3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^3)*m^2 - ((3*c^2*d + 3*c*d^2 + 2*d^3)*m^3 + 36*c^2*d + 12*d^3 + 3*(8*c^2*d + 5*c*d^2 + 3*d^3)*m^2 + (57*c^2*d + 12*c*d^2 + 19*d^3)*m)*cos(f*x + e)^2 + 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m + ((c^3 + 3*c^2*d + 3*c*d^2 + d^3)*m^3 + 24*c^3 + 24*c*d^2 + 3*(3*c^3 + 7*c^2*d + 5*c*d^2 + d^3)*m^2 - ((3*c*d^2 + d^3)*m^3 + 24*c*d^2 + 3*(7*c*d^2 + d^3)*m^2 + 2*(21*c*d^2 + d^3)*m)*cos(f*x + e)^2 + 2*(13*c^3 + 18*c^2*d + 9*c*d^2 + 4*d^3)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)^m/(f*m^4 + 10*f*m^3 + 35*f*m^2 + 50*f*m + 24*f)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4310 vs. $2(112) = 224$.
time = 9.56, size = 4310, normalized size = 32.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**3,x)

[Out] Piecewise((x*(c + d*sin(e))**3*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-2*c**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 9*c**2*d*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 3*c**2*d/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*c*d**2*sin(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*c*d**2*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 6*c*d**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e

+ f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 6*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 6*d**3*log(sin(e + f*x) + 1)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 18*d**3*sin(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 27*d**3*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) + 11*d**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f), Eq(m, -4)), (-c**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*c**2*d*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 3*c**2*d/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 6*c*d**2*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*c*d**2*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 9*c*d**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 12*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 6*d**3*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**3*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 12*d**3*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 9*d**3/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f), Eq(m, -3)), (-2*c**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*d*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*d*log(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c**2*d/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2*log(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*c*d**2*sin(e + f*x)**2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 12*c*d**2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3*log(sin(e + f*x) + 1)/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + d**3*sin(e + f*x)**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f) - 3*d**3*sin(e + f*x)**2/(2*a**2*f*sin(e + f*x) + 2*a**2*f) + 6*d**3/(2*a**2*f*sin(e + f*x) + 2*a**2*f), Eq(m, -2)), (c**3*log(sin(e + f*x) + 1)/(a*f) - 3*c**2*d*log(sin(e + f*x) + 1)/(a*f) + 3*c**2*d*sin(e + f*x)/(a*f) + 3*c*d**2*log(sin(e + f*x) + 1)/(a*f) + 3*c*d**2*sin(e + f*x)**2/(2*a*f) - 3*c*d**2*sin(e + f*x)/(a*f) - d**3*log(sin(e + f*x) + 1)/(a*f) + d**3*sin(e + f*x)**3/(3*a*f) - d**3*sin(e + f*x)**2/(2*a*f) + d**3*sin(e + f*x)/(a*f), Eq(m, -1)), (c**3*m**3*(a*sin(

```
e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24
*f) + c**3*m**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 5
0*f*m + 24*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 +
10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 9*c**3*m**2*(a*sin(e + f*x) + a)**
m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 26*c**3*m*(a*sin(e + f
*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) +
 26*c**3*m*(a*sin(e + f*x) + a)**m/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m
 + 24*f) + 24*c**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3
 + 35*f*m**2 + 50*f*m + 24*f) + 24*c**3*(a*sin(e + f*x) + a)**m/(f*m**4 + 1
0*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c**2*d*m**3*(a*sin(e + f*x) + a)*
*m*sin(e + f*x)**2/(f*m**4 + 10*f*m**3 + 35*f*m**2 + 50*f*m + 24*f) + 3*c**
2*d*m**3*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**4 + 10*f*m**3 + 35*f*m*
*2 + 50*f*m + 24*f) + 24*c**2*d*m**2*(a*sin(e +...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(137) = 274$.
time = 0.46, size = 1003, normalized size = 7.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] (3*((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) +
 a)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) +
 a)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*si
n(f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m -
 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)
*(a*sin(f*x + e) + a)^m*a^2)*c*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2)
 + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^3 - 3*(a*sin(f*x + e) +
 a)^3*(a*sin(f*x + e) + a)^m*a*m^3 + 3*(a*sin(f*x + e) + a)^2*(a*sin(f*x +
e) + a)^m*a^2*m^3 - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^3 + 6
*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 21*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 24*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m*a^2*m^2 - 9*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m^2 +
11*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m - 42*(a*sin(f*x + e) + a
)^3*(a*sin(f*x + e) + a)^m*a*m + 57*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e)
+ a)^m*a^2*m - 26*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3*m + 6*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*s
in(f*x + e) + a)^m*a + 36*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2
 - 24*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a^3)*d^3/(a^3*m^4 + 10*a^
3*m^3 + 35*a^3*m^2 + 50*a^3*m + 24*a^3) + (a*sin(f*x + e) + a)^(m + 1)*c^3/
(m + 1) + 3*((a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*m - (a*sin(f*x +
```

```
 e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(f*x + e) + a)^2*(a*sin(f*x + e
) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a)*c^2*d/((m^2 + 3
*m + 2)*a))/(a*f)
```

Mupad [B]
time = 13.44, size = 703, normalized size = 5.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^3,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(192*c*d^2 - 144*c^2*d + 208*c^3*m + 21*d^3*m + 1
92*c^3*sin(e + f*x) + 192*c^3 - 30*d^3 - 24*d^3*cos(2*e + 2*f*x) + 6*d^3*co
s(4*e + 4*f*x) + 72*c^3*m^2 + 8*c^3*m^3 + 6*d^3*m^2 + 3*d^3*m^3 + 208*c^3*m
*sin(e + f*x) + 60*d^3*m*sin(e + f*x) - 144*c^2*d*cos(2*e + 2*f*x) + 60*c*d
^2*m^2 + 72*c^2*d*m^2 + 12*c*d^2*m^3 + 12*c^2*d*m^3 - 32*d^3*m*cos(2*e + 2*
f*x) + 11*d^3*m*cos(4*e + 4*f*x) - 48*c*d^2*sin(3*e + 3*f*x) + 72*c^3*m^2*s
in(e + f*x) + 8*c^3*m^3*sin(e + f*x) - 4*d^3*m*sin(3*e + 3*f*x) + 18*d^3*m^
2*sin(e + f*x) + 6*d^3*m^3*sin(e + f*x) - 12*d^3*m^2*cos(2*e + 2*f*x) - 4*d
^3*m^3*cos(2*e + 2*f*x) + 6*d^3*m^2*cos(4*e + 4*f*x) + d^3*m^3*cos(4*e + 4*
f*x) - 6*d^3*m^2*sin(3*e + 3*f*x) - 2*d^3*m^3*sin(3*e + 3*f*x) + 96*c*d^2*m
 + 60*c^2*d*m + 144*c*d^2*sin(e + f*x) - 60*c*d^2*m^2*cos(2*e + 2*f*x) - 96
*c^2*d*m^2*cos(2*e + 2*f*x) - 12*c*d^2*m^3*cos(2*e + 2*f*x) - 12*c^2*d*m^3*
cos(2*e + 2*f*x) - 42*c*d^2*m^2*sin(3*e + 3*f*x) - 6*c*d^2*m^3*sin(3*e + 3*
f*x) + 60*c*d^2*m*sin(e + f*x) + 288*c^2*d*m*sin(e + f*x) - 48*c*d^2*m*cos(
2*e + 2*f*x) - 228*c^2*d*m*cos(2*e + 2*f*x) - 84*c*d^2*m*sin(3*e + 3*f*x) +
 78*c*d^2*m^2*sin(e + f*x) + 168*c^2*d*m^2*sin(e + f*x) + 18*c*d^2*m^3*sin(
e + f*x) + 24*c^2*d*m^3*sin(e + f*x)))/(8*f*(50*m + 35*m^2 + 10*m^3 + m^4 +
 24))
```

3.922 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx$

Optimal. Leaf size=96

$$\frac{(c-d)^2(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{2(c-d)d(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}+\frac{d^2(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)}$$

```
[Out] (c-d)^2*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+2*(c-d)*d*(a+a*sin(f*x+e))^(2+m)/a
^2/f/(2+m)+d^2*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 45}

$$\frac{d^2(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{2d(c-d)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)^2(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((c - d)^2*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (2*(c - d)*d*(a +
a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) + (d^2*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(3 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^2\,dx = \frac{\text{Subst}\left(\int (a+x)^m\left(c+\frac{dx}{a}\right)^2\,dx,x,a\sin(e+fx)\right)}{af}$$
$$= \frac{\text{Subst}\left(\int \left((c-d)^2(a+x)^m+\frac{2(c-d)d(a+x)^{1+m}}{a}+\right.\right.}{af}$$
$$= \frac{(c-d)^2(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{2(c-d)d(a+}{a^2}$$

Mathematica [A]
time = 0.31, size = 83, normalized size = 0.86

$$\frac{(a(1+\sin(e+fx)))^{1+m}\left(\frac{a^2(c-d)^2}{1+m}+\frac{2a^2(c-d)d(1+\sin(e+fx))}{2+m}+\frac{d^2(a+a\sin(e+fx))^2}{3+m}\right)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*((a^2*(c - d)^2)/(1 + m) + (2*a^2*(c - d)*d
*(1 + Sin[e + f*x]))/(2 + m) + (d^2*(a + a*Sin[e + f*x])^2)/(3 + m)))/(a^3*
f)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \cos(fx+e)(a+a\sin(fx+e))^m(c+d\sin(fx+e))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x)
```

Maxima [A]
time = 0.28, size = 179, normalized size = 1.86

$$\frac{\frac{2\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)cd(\sin(fx+e)+1)^m}{m^2+3m+2}+\frac{\left((m^2+3m+2)a^m\sin(fx+e)^3+(m^2+m)a^m\sin(fx+e)^2-2a^m m\sin(fx+e)+2a^m\right)d^2(\sin(fx+e)+1)^m}{m^3+6m^2+11m+6}+\frac{(a\sin(fx+e)+a)^{m+1}c^2}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
```

```
[Out] (2*(a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*c*d*(sin(f*x + e
) + 1)^m/(m^2 + 3*m + 2) + ((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*
a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*d^2*(sin(f*x + e) + 1)^m
/(m^3 + 6*m^2 + 11*m + 6) + (a*sin(f*x + e) + a)^(m + 1)*c^2/(a*(m + 1)))/f
```

Fricas [A]
time = 0.38, size = 193, normalized size = 2.01

$$\frac{\left((c^2+2\,cd+d^2)m^2-((2\,cd+d^2)m^2+6\,cd+(8\,cd+d^2)m)\cos\left(fx+e\right)^2+6\,c^2+2\,d^2+(5\,c^2+6\,cd+d^2)m+((c^2+2\,cd+d^2)m^2-(d^2m^2+3\,d^2m+2\,d^2)\cos\left(fx+e\right)^2+6\,c^2+2\,d^2+(5\,c^2+6\,cd+d^2)m)\sin\left(fx+e\right)\right)(a\sin\left(fx+e\right)+a)^m}{fm^3+6\,fm^2+11\,fm+6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] ((c^2 + 2*c*d + d^2)*m^2 - ((2*c*d + d^2)*m^2 + 6*c*d + (8*c*d + d^2)*m)*co
s(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^2 + 6*c*d + d^2)*m + ((c^2 + 2*c*d + d^
2)*m^2 - (d^2*m^2 + 3*d^2*m + 2*d^2)*cos(f*x + e)^2 + 6*c^2 + 2*d^2 + (5*c^
2 + 6*c*d + d^2)*m)*sin(f*x + e))*(a*sin(f*x + e) + a)^m/(f*m^3 + 6*f*m^2 +
 11*f*m + 6*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(80) = 160$.
time = 3.37, size = 1622, normalized size = 16.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Piecewise((x*(c + d*sin(e))**2*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c**2/
(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*c*d*sin(e
 + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*c
*d/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*l
og(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*s
in(e + f*x) + 2*a**3*f) + 4*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(2*a**3
*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 2*d**2*log(sin(e +
 f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) +
4*d**2*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a
**3*f) + 3*d**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*
f), Eq(m, -3)), (-c**2/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f
*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d*log(sin(e + f*
x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) + 2*c*d/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin(e + f*x) + a**2*
f) - 2*d**2*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f) + d**2*sin
(e + f*x)**2/(a**2*f*sin(e + f*x) + a**2*f) - 2*d**2/(a**2*f*sin(e + f*x) +
 a**2*f), Eq(m, -2)), (c**2*log(sin(e + f*x) + 1)/(a*f) - 2*c*d*log(sin(e +
```

```
f*x) + 1)/(a*f) + 2*c*d*sin(e + f*x)/(a*f) + d**2*log(sin(e + f*x) + 1)/(a
*f) + d**2*sin(e + f*x)**2/(2*a*f) - d**2*sin(e + f*x)/(a*f), Eq(m, -1)), (
c**2*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m
+ 6*f) + c**2*m**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*
f) + 5*c**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11*
f*m + 6*f) + 5*c**2*m*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m +
 6*f) + 6*c**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2 + 11
*f*m + 6*f) + 6*c**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**2 + 11*f*m +
6*f) + 2*c*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**
2 + 11*f*m + 6*f) + 2*c*d*m**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3
 + 6*f*m**2 + 11*f*m + 6*f) + 8*c*d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)*
*2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c*d*m*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) - 2*c*d*m*(a*sin(e + f*x) + a)
**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 6*c*d*(a*sin(e + f*x) + a)**m*sin(
e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) - 6*c*d*(a*sin(e + f*x) + a)
**m/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + d**2*m**2*(a*sin(e + f*x) + a)**m*
sin(e + f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + d**2*m**2*(a*sin(e + f
*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) + 3*d**2*m*(
a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 + 6*f*m**2 + 11*f*m + 6*f) +
 d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**3 + 6*f*m**2 + 11*f*m
 + 6*f) - 2*d**2*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**3 + 6*f*m**2
+ 11*f*m + 6*f) + 2*d**2*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3/(f*m**3 +
6*f*m**2 + 11*f*m + 6*f) + 2*d**2*(a*sin(e + f*x) + a)**m/(f*m**3 + 6*f*m**
2 + 11*f*m + 6*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(99) = 198.

time = 0.42, size = 462, normalized size = 4.81

$$\frac{\frac{\left((a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m m^2-2\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a m^2+(a\sin(fx+e)+a)(a\sin(fx+e)+a)^m a^2 m^2+3\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m m-8\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a m+5\,(a\sin(fx+e)+a)(a\sin(fx+e)+a)^m a^2 m+2\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m-6\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a+6\,(a\sin(fx+e)+a)(a\sin(fx+e)+a)^m a^2\right)d^2}{a^2m^3+6\,a^2m^2+11\,a^2m+6\,a^2}+\frac{(a\sin(fx+e)+a)^{m+1}c^2}{m+1}+\frac{2\left((a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m m-(a\sin(fx+e)+a)(a\sin(fx+e)+a)^m a m+(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m-2\,(a\sin(fx+e)+a)(a\sin(fx+e)+a)^m a\right)cd}{(m^2+3\,m+2)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m^2 - 2*(a*sin(f*x + e) + a
)^2*(a*sin(f*x + e) + a)^m*a*m^2 + (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a
)^m*a^2*m^2 + 3*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 8*(a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a*m + 5*(a*sin(f*x + e) + a)*(a*sin(
f*x + e) + a)^m*a^2*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m - 6
*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a + 6*(a*sin(f*x + e) + a)*(
a*sin(f*x + e) + a)^m*a^2)*d^2/(a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2) + (
a*sin(f*x + e) + a)^(m + 1)*c^2/(m + 1) + 2*((a*sin(f*x + e) + a)^2*(a*sin(
f*x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*si
n(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*
x + e) + a)^m*a)*c*d/((m^2 + 3*m + 2)*a))/(a*f)
```

Mupad [B]
time = 11.48, size = 305, normalized size = 3.18

$$\frac{(a\,(\sin(e+f\,x)+1))^m\,\left(20\,c^2\,m-12\,c\,d+2\,d^2\,m+24\,c^2\,\sin(e+f\,x)+6\,d^2\,\sin(e+f\,x)+24\,c^2+8\,d^2+4\,c^2\,m^2+2\,d^2\,m^2-2\,d^2\,\sin(3\,e+3\,f\,x)+20\,c^2\,m\,\sin(e+f\,x)+d^2\,m\,\sin(e+f\,x)-2\,d^2\,m\,\cos(2\,e+2\,f\,x)+4\,c^2\,m^2\,\sin(e+f\,x)-3\,d^2\,m\,\sin(3\,e+3\,f\,x)+3\,d^2\,m^2\,\sin(e+f\,x)+8\,c\,d\,m-2\,d^2\,m^2\,\cos(2\,e+2\,f\,x)-d^2\,m^2\,\sin(3\,e+3\,f\,x)-12\,c\,d\,\cos(2\,e+2\,f\,x)+4\,c\,d\,m^2+24\,c\,d\,m\,\sin(e+f\,x)-16\,c\,d\,m\,\cos(2\,e+2\,f\,x)+8\,c\,d\,m^2\,\sin(e+f\,x)-4\,c\,d\,m^2\,\cos(2\,e+2\,f\,x)\right)}{4\,f\,\left(m^3+6\,m^2+11\,m+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(20*c^2*m - 12*c*d + 2*d^2*m + 24*c^2*sin(e + f*x
) + 6*d^2*sin(e + f*x) + 24*c^2 + 8*d^2 + 4*c^2*m^2 + 2*d^2*m^2 - 2*d^2*sin
(3*e + 3*f*x) + 20*c^2*m*sin(e + f*x) + d^2*m*sin(e + f*x) - 2*d^2*m*cos(2*
e + 2*f*x) + 4*c^2*m^2*sin(e + f*x) - 3*d^2*m*sin(3*e + 3*f*x) + 3*d^2*m^2*
sin(e + f*x) + 8*c*d*m - 2*d^2*m^2*cos(2*e + 2*f*x) - d^2*m^2*sin(3*e + 3*f
*x) - 12*c*d*cos(2*e + 2*f*x) + 4*c*d*m^2 + 24*c*d*m*sin(e + f*x) - 16*c*d*
m*cos(2*e + 2*f*x) + 8*c*d*m^2*sin(e + f*x) - 4*c*d*m^2*cos(2*e + 2*f*x)))/
(4*f*(11*m + 6*m^2 + m^3 + 6))
```

3.923 $\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{(c-d)(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{d(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}$$

```
[Out] (c-d)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+d*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{d(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(c-d)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((c - d)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (d*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a+x)^m \left(c+\frac{dx}{a}\right)\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\text{Subst}\left(\int \left((c-d)(a+x)^m + \frac{d(a+x)^{1+m}}{a}\right)\,dx, x,\right.}{af}$$

$$= \frac{(c-d)(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{d(a+a\sin(e}{a^2f(2+}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.86

$$\frac{(a(1+\sin(e+fx)))^{1+m}(-d+c(2+m)+d(1+m)\sin(e+fx))}{af(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-d + c*(2 + m) + d*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \cos(fx+e)(a+a\sin(fx+e))^m (c+d\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x)
```

Maxima [A]
time = 0.29, size = 87, normalized size = 1.47

$$\frac{\frac{\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)d(\sin(fx+e)+1)^m}{m^2+3m+2} + \frac{(a\sin(fx+e)+a)^{m+1}c}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] ((a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*d*(sin(f*x + e) +
1)^m/(m^2 + 3*m + 2) + (a*sin(f*x + e) + a)^(m + 1)*c/(a*(m + 1)))/f
```

Fricas [A]
time = 0.36, size = 73, normalized size = 1.24

$$-\frac{\left((dm+d)\cos\left(fx+e\right)^{2}-(c+d)m-\left((c+d)m+2\,c\right)\sin\left(fx+e\right)-2\,c\right)\left(a\sin\left(fx+e\right)+a\right)^{m}}{fm^{2}+3\,fm+2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((d*m + d)*cos(f*x + e)^2 - (c + d)*m - ((c + d)*m + 2*c)*sin(f*x + e) - 2
*c)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(46) = 92$.
time = 1.28, size = 428, normalized size = 7.25

$$\begin{cases} x(c+d\sin(e))\left(a\sin(e)+a\right)^{m}\cos(e) & \text{for}\; f=0 \\ -\frac{c}{a^{2}f\sin(e+fx)+a^{2}f}+\frac{d\log(\sin(e+fx)+1)\sin(e+fx)}{a^{2}f\sin(e+fx)+a^{2}f}+\frac{d\log(\sin(e+fx)+1)}{a^{2}f\sin(e+fx)+a^{2}f}+\frac{d}{a^{2}f\sin(e+fx)+a^{2}f} & \text{for}\; m=-2 \\ \frac{c\log(\sin(e+fx)+1)}{af}-\frac{d\log(\sin(e+fx)+1)}{af}+\frac{d\sin(e+fx)}{af} & \text{for}\; m=-1 \\ \frac{cm(a\sin(e+fx)+a)^{m}\sin(e+fx)}{fm^{2}+3fm+2f}+\frac{cm(a\sin(e+fx)+a)^{m}}{fm^{2}+3fm+2f}+\frac{2c(a\sin(e+fx)+a)^{m}\sin(e+fx)}{fm^{2}+3fm+2f}+\frac{2c(a\sin(e+fx)+a)^{m}}{fm^{2}+3fm+2f}+\frac{dm(a\sin(e+fx)+a)^{m}\sin^{2}(e+fx)}{fm^{2}+3fm+2f}+\frac{dm(a\sin(e+fx)+a)^{m}\sin(e+fx)}{fm^{2}+3fm+2f}+\frac{d(a\sin(e+fx)+a)^{m}\sin^{2}(e+fx)}{fm^{2}+3fm+2f}-\frac{d(a\sin(e+fx)+a)^{m}}{fm^{2}+3fm+2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(c + d*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-c/(a**2*
f*sin(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + d*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + d/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (c*log(sin(e + f*x) + 1)/
(a*f) - d*log(sin(e + f*x) + 1)/(a*f) + d*sin(e + f*x)/(a*f), Eq(m, -1)), (
c*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + c*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*c*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + d*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + d*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - d*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(61) = 122$.
time = 0.44, size = 156, normalized size = 2.64

$$\frac{\frac{(a\sin(fx+e)+a)^{m+1}c}{m+1}+\frac{\left((a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}m-(a\sin(fx+e)+a)(a\sin(fx+e)+a)^{m}am+(a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}-2\,(a\sin(fx+e)+a)(a\sin(fx+e)+a)^{m}a\right)d}{(m^{2}+3\,m+2)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*c/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*d/((m^2 + 3*m + 2)*a))/(a*f)
```

Mupad [B]
time = 9.76, size = 99, normalized size = 1.68

$$\frac{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)^m\,\left(4\,c-d+2\,c\,m+d\,m+4\,c\,\sin\left(e+f\,x\right)+d\,\left(2\,{\sin\left(e+f\,x\right)}^2-1\right)+2\,c\,m\,\sin\left(e+f\,x\right)+2\,d\,m\,\sin\left(e+f\,x\right)+d\,m\,\left(2\,{\sin\left(e+f\,x\right)}^2-1\right)\right)}{2\,f\,\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(4*c - d + 2*c*m + d*m + 4*c*sin(e + f*x) + d*(2*
sin(e + f*x)^2 - 1) + 2*c*m*sin(e + f*x) + 2*d*m*sin(e + f*x) + d*m*(2*sin(
e + f*x)^2 - 1)))/(2*f*(3*m + m^2 + 2))
```

3.924 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(1,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(1+m)}$$

```
[Out] hypergeom([1, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)/f/(1+m)
```

Rubi [A]

time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
```

```
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{c+d\sin(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{c+\frac{dx}{a}}\,dx,x,a\sin(e+fx)\right)}{af}\\ &= \frac{{}_2F_1\left(1,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 1.00

$$\frac{{}_2F_1\left(1,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x]),x]
[Out] (Hypergeometric2F1[1, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)*f*(1 + m))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^{m}}{c+d\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="fric
as")
[Out] integral((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{c+d\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
```

3.925 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(2,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^2f(1+m)}$$

```
[Out] hypergeom([2, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)^2/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(2,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^2}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{{}_2F_1\left(2,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^2f(1+m)}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 1.00

$$\frac{{}_2F_1\left(2,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^2f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^2,x]
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^2*f*(1 + m))
```

Maple [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\cos(fx+e)(a+a\sin(fx+e))^m}{(c+d\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="ma
xima")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(d^2*cos(f*x + e)^2 - 2*c*d*s
in(f*x + e) - c^2 - d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x)
```

3.926 $\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx$

Optimal. Leaf size=59

$$\frac{{}_2F_1\left(3,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^3f(1+m)}$$

```
[Out] hypergeom([3, 1+m],[2+m],-d*(1+sin(f*x+e))/(c-d))*(a+a*sin(f*x+e))^(1+m)/a/
(c-d)^3/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2912, 70}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(3,m+1;m+2;-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(m+1)(c-d)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(e+fx)(a+a\sin(e+fx))^m}{(c+d\sin(e+fx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{\left(c+\frac{dx}{a}\right)^3}\,dx, x, a\sin(e+fx)\right)}{af}$$
$$= \frac{{}_2F_1\left(3,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^3f(1+m)}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 1.00

$$\frac{{}_2F_1\left(3,1+m;2+m;-\frac{d(1+\sin(e+fx))}{c-d}\right)(a+a\sin(e+fx))^{1+m}}{a(c-d)^3f(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(c + d*Sin[e + f*x])^3,x]
[Out] (Hypergeometric2F1[3, 1 + m, 2 + m, -((d*(1 + Sin[e + f*x]))/(c - d))]*(a +
 a*Sin[e + f*x])^(1 + m))/(a*(c - d)^3*f*(1 + m))
```

Maple [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\cos(fx+e)\,(a+a\sin(fx+e))^m}{(c+d\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="ma
xima")
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="fr
icas")
```

```
[Out] integral(-(a*sin(f*x + e) + a)^m*cos(f*x + e)/(3*c*d^2*cos(f*x + e)^2 - c^3
 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m/(c+d*sin(f*x+e))^3,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cos(f*x + e)/(d*sin(f*x + e) + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\left(c+d\,\sin\left(e+f\,x\right)\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)
```

```
[Out] int((cos(e + f*x)*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x)
```

3.927 $\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$-\frac{{}_2F_1(1,2+m+n;2+m;1+\sin(c+dx))\sin^{1+n}(c+dx)(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

```
[Out] -hypergeom([1, 2+m+n],[2+m],1+sin(d*x+c))*sin(d*x+c)^(1+n)*(a+a*sin(d*x+c))
^(1+m)/a/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 68, 66}

$$\frac{(\sin(c+dx)+1)^{-m}\sin^{n+1}(c+dx)(a\sin(c+dx)+a)^m\,{}_2F_1(-m,n+1;n+2;-\sin(c+dx))}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 68

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n]), Int[(b*x)^m*(1 + d*(
x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ
[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2^
(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^n(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^n (a+x)^m\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\left((1+\sin(c+dx))^{-m}(a+a\sin(c+dx))^m\right)\text{Subst}\left(\int \left(\frac{x}{a}\right)\right.}{ad}$$
$$= \frac{{}_2F_1(-m,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)(1}{d(1+n)}$$

Mathematica [A]
time = 0.06, size = 61, normalized size = 1.13

$$\frac{{}_2F_1(-m,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)(1+\sin(c+dx))^{-m}(a+a\sin(c+dx))^m}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^n*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (Hypergeometric2F1[-m, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a
 + a*Sin[c + d*x])^m)/(d*(1 + n)*(1 + Sin[c + d*x])^m)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^n(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin (c+dx)+1))^m \sin^n (c+dx) \cos (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**n*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*sin(c + d*x)**n*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^n*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*sin(d*x + c)^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos (c+d\,x)\,\sin (c+d\,x)^n\,(a+a\,\sin (c+d\,x))^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^m,x)
[Out] int(cos(c + d*x)*sin(c + d*x)^n*(a + a*sin(c + d*x))^m, x)
```

3.928 $\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=134

$$\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}-\frac{4(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}+\frac{6(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}-\frac{4(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}+\frac{(a+$$

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)-4*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)+6*(a+
a*sin(d*x+c))^(3+m)/a^3/d/(3+m)-4*(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)+(a+a*s
in(d*x+c))^(5+m)/a^5/d/(5+m)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^{m+5}}{a^5d(m+5)}-\frac{4(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}+\frac{6(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{4(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (4*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (6*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) - (4*
(a + a*Sin[c + d*x])^(4 + m))/(a^4*d*(4 + m)) + (a + a*Sin[c + d*x])^(5 + m
)/(a^5*d*(5 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^4(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)^m}{a^4}\,dx, x, a\sin(c+dx)\right)}{ad}$$

$$= \frac{\text{Subst}\left(\int x^4(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^4(a+x)^m - 4a^3(a+x)^{1+m} + 6a^2(a+x)^{2+}\right.\right.}{a}$$

$$= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} - \frac{4(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} + $$

Mathematica [A]
time = 1.04, size = 150, normalized size = 1.12

$$\frac{(a(1+\sin(c+dx)))^{1+m}\left(\frac{7}{1+m} - \frac{40(1+\sin(c+dx))}{2+m} + \frac{84(1+\sin(c+dx))^2}{3+m} - \frac{64(1+\sin(c+dx))^3}{4+m} + \frac{16(1+\sin(c+dx))^4}{5+m} + \frac{3(6+m+m^2-2(2+3m+m^2)\cos(2(c+dx))-8(1+m)\sin(c+dx))}{(1+m)(2+m)(3+m)}\right)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^4*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(7/(1 + m) - (40*(1 + Sin[c + d*x]))/(2 + m
) + (84*(1 + Sin[c + d*x])^2)/(3 + m) - (64*(1 + Sin[c + d*x])^3)/(4 + m) +
 (16*(1 + Sin[c + d*x])^4)/(5 + m) + (3*(6 + m + m^2 - 2*(2 + 3*m + m^2)*Co
s[2*(c + d*x)] - 8*(1 + m)*Sin[c + d*x]))/((1 + m)*(2 + m)*(3 + m))))/(16*a
*d)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^4(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.32, size = 159, normalized size = 1.19

$$\frac{\left((m^4+10m^3+35m^2+50m+24)a^m\sin(dx+c)^5 + (m^4+6m^3+11m^2+6m)a^m\sin(dx+c)^4 - 4(m^3+3m^2+2m)a^m\sin(dx+c)^3 + 12(m^2+m)a^m\sin(dx+c)^2 - 24a^m m\sin(dx+c) + 24a^m\right)(\sin(dx+c)+1)^m}{(m^5+15m^4+85m^3+225m^2+274m+120)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(d*x + c)^5 + (m^4 + 6*m^3 + 11
*m^2 + 6*m)*a^m*sin(d*x + c)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(d*x + c)^3 +
 12*(m^2 + m)*a^m*sin(d*x + c)^2 - 24*a^m*m*sin(d*x + c) + 24*a^m)*(sin(d*x
 + c) + 1)^m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*d)
```

Fricas [A]
time = 0.39, size = 197, normalized size = 1.47

$$\frac{\left((m^4+6\,m^3+11\,m^2+6\,m)\cos\left(dx+c\right)^4+m^4+6\,m^3-2\,(m^4+6\,m^3+17\,m^2+12\,m)\cos\left(dx+c\right)^2+23\,m^2+\left((m^4+10\,m^3+35\,m^2+50\,m+24)\cos\left(dx+c\right)^4+m^4+6\,m^3-2\,(m^4+8\,m^3+29\,m^2+46\,m+24)\cos\left(dx+c\right)^2+23\,m^2+18\,m+24\right)\sin\left(dx+c\right)+18\,m+24\right)\left(a\sin\left(dx+c\right)+a\right)^m}{dm^5+15\,dm^4+85\,dm^3+225\,dm^2+274\,dm+120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^4 + 6*m^3 + 11*m^2 + 6*m)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 6*m^3
 + 17*m^2 + 12*m)*cos(d*x + c)^2 + 23*m^2 + ((m^4 + 10*m^3 + 35*m^2 + 50*m
+ 24)*cos(d*x + c)^4 + m^4 + 6*m^3 - 2*(m^4 + 8*m^3 + 29*m^2 + 46*m + 24)*c
os(d*x + c)^2 + 23*m^2 + 18*m + 24)*sin(d*x + c) + 18*m + 24)*(a*sin(d*x +
c) + a)^m/(d*m^5 + 15*d*m^4 + 85*d*m^3 + 225*d*m^2 + 274*d*m + 120*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2747 vs. $2(112) = 224$.
time = 17.53, size = 2747, normalized size = 20.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**4*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**4*cos(c), Eq(d, 0)), (12*log(sin(c +
 d*x) + 1)*sin(c + d*x)**4/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d
*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d) +
48*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(12*a**5*d*sin(c + d*x)**4 + 48*a*
*5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) +
 12*a**5*d) + 72*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(12*a**5*d*sin(c + d
*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*
sin(c + d*x) + 12*a**5*d) + 48*log(sin(c + d*x) + 1)*sin(c + d*x)/(12*a**5*
d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 +
 48*a**5*d*sin(c + d*x) + 12*a**5*d) + 12*log(sin(c + d*x) + 1)/(12*a**5*d*
sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 4
8*a**5*d*sin(c + d*x) + 12*a**5*d) + 48*sin(c + d*x)**3/(12*a**5*d*sin(c +
d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d
*sin(c + d*x) + 12*a**5*d) + 108*sin(c + d*x)**2/(12*a**5*d*sin(c + d*x)**4
 + 48*a**5*d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c
+ d*x) + 12*a**5*d) + 88*sin(c + d*x)/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*
d*sin(c + d*x)**3 + 72*a**5*d*sin(c + d*x)**2 + 48*a**5*d*sin(c + d*x) + 12
*a**5*d) + 25/(12*a**5*d*sin(c + d*x)**4 + 48*a**5*d*sin(c + d*x)**3 + 72*a
```

5*d*sin(c + d*x)2 + 48*a**5*d*sin(c + d*x) + 12*a**5*d), Eq(m, -5)), (-12*log(sin(c + d*x) + 1)*sin(c + d*x)**3/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 12*log(sin(c + d*x) + 1)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) + 3*sin(c + d*x)**4/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 36*sin(c + d*x)**2/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 54*sin(c + d*x)/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d) - 22/(3*a**4*d*sin(c + d*x)**3 + 9*a**4*d*sin(c + d*x)**2 + 9*a**4*d*sin(c + d*x) + 3*a**4*d), Eq(m, -4)), (12*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 12*log(sin(c + d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + sin(c + d*x)**4/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 4*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 24*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 18/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (-12*log(sin(c + d*x) + 1)*sin(c + d*x)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 12*log(sin(c + d*x) + 1)/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + sin(c + d*x)**4/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 2*sin(c + d*x)**3/(3*a**2*d*sin(c + d*x) + 3*a**2*d) + 6*sin(c + d*x)**2/(3*a**2*d*sin(c + d*x) + 3*a**2*d) - 12/(3*a**2*d*sin(c + d*x) + 3*a**2*d), Eq(m, -2)), (log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**4/(4*a*d) - sin(c + d*x)**3/(3*a*d) + sin(c + d*x)**2/(2*a*d) - sin(c + d*x)/(a*d), Eq(m, -1)), (m**4*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + m**4*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 10*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 6*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 4*m**3*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 35*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 11*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 12*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 12*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 50*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**5/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) + 6*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 8*m*(a*si

```
n(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**5 + 15*d*m**4 + 85*d*m**3 + 225*d*
m**2 + 274*d*m + 120*d) + 12*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m
**5 + 15*d*m**4 + 85*d*m**3 + 225*d*m**2 + 274*d*m + 120*d) - 24*m*(a*sin(c
 + d*x) + a)**m*sin(c + d*x)/(d*m**5 + 15*d*m**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(134) = 268.
time = 0.43, size = 791, normalized size = 5.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^4*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m*m^4 - 4*(a*sin(d*x + c) + a)
^4*(a*sin(d*x + c) + a)^m*a*m^4 + 6*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c)
+ a)^m*a^2*m^4 - 4*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^4 +
(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^4*m^4 + 10*(a*sin(d*x + c) +
a)^5*(a*sin(d*x + c) + a)^m*m^3 - 44*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c)
 + a)^m*a*m^3 + 72*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m^3 -
52*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^3 + 14*(a*sin(d*x +
c) + a)*(a*sin(d*x + c) + a)^m*a^4*m^3 + 35*(a*sin(d*x + c) + a)^5*(a*sin(d
*x + c) + a)^m*m^2 - 164*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m^
2 + 294*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m^2 - 236*(a*sin(
d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^3*m^2 + 71*(a*sin(d*x + c) + a)*(a
*sin(d*x + c) + a)^m*a^4*m^2 + 50*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) +
a)^m*m - 244*(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a*m + 468*(a*sin
(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*a^2*m - 428*(a*sin(d*x + c) + a)^2*
(a*sin(d*x + c) + a)^m*a^3*m + 154*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a
)^m*a^4*m + 24*(a*sin(d*x + c) + a)^5*(a*sin(d*x + c) + a)^m - 120*(a*sin(d
*x + c) + a)^4*(a*sin(d*x + c) + a)^m*a + 240*(a*sin(d*x + c) + a)^3*(a*sin
(d*x + c) + a)^m*a^2 - 240*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^
3 + 120*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^4)/((a^4*m^5 + 15*a^4
*m^4 + 85*a^4*m^3 + 225*a^4*m^2 + 274*a^4*m + 120*a^4)*a*d)
```

Mupad [B]
time = 12.19, size = 349, normalized size = 2.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^4*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(132*m + 240*sin(c + d*x) - 120*sin(3*c + 3*d*x)
+ 24*sin(5*c + 5*d*x) + 20*m*sin(c + d*x) - 144*m*cos(2*c + 2*d*x) + 12*m*c
os(4*c + 4*d*x) - 218*m*sin(3*c + 3*d*x) + 50*m*sin(5*c + 5*d*x) + 206*m^2*
```

```
sin(c + d*x) + 52*m^3*sin(c + d*x) + 10*m^4*sin(c + d*x) + 162*m^2 + 36*m^3
 + 6*m^4 - 184*m^2*cos(2*c + 2*d*x) - 48*m^3*cos(2*c + 2*d*x) - 8*m^4*cos(2
*c + 2*d*x) + 22*m^2*cos(4*c + 4*d*x) + 12*m^3*cos(4*c + 4*d*x) + 2*m^4*cos
(4*c + 4*d*x) - 127*m^2*sin(3*c + 3*d*x) - 34*m^3*sin(3*c + 3*d*x) - 5*m^4*
sin(3*c + 3*d*x) + 35*m^2*sin(5*c + 5*d*x) + 10*m^3*sin(5*c + 5*d*x) + m^4*
sin(5*c + 5*d*x) + 384))/(16*d*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 1
20))
```

3.929 $\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=108

$$-\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}+\frac{3(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}-\frac{3(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}+\frac{(a+a\sin(c+dx))^{4+m}}{a^4d(4+m)}$$

```
[Out] -(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)+3*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)-3*(a
+a*sin(d*x+c))^(3+m)/a^3/d/(3+m)+(a+a*sin(d*x+c))^(4+m)/a^4/d/(4+m)
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^{m+4}}{a^4d(m+4)}-\frac{3(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}+\frac{3(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (3*(a + a*Sin[c + d*x])^(2
+ m))/(a^2*d*(2 + m)) - (3*(a + a*Sin[c + d*x])^(3 + m))/(a^3*d*(3 + m)) +
(a + a*Sin[c + d*x])^(4 + m)/(a^4*d*(4 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^3(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\mathrm{Subst}\left(\int \frac{x^3(a+x)^m}{a^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\mathrm{Subst}\left(\int x^3(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^4d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(-a^3(a+x)^m + 3a^2(a+x)^{1+m} - 3a(a+x)^2\right.\right.}{a^4d}$$
$$= -\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} + \frac{3(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} -$$

Mathematica [A]
time = 0.42, size = 94, normalized size = 0.87

$$\frac{(a(1+\sin(c+dx)))^{1+m}\left(-6+6(1+m)\sin(c+dx)-3(2+3m+m^2)\sin^2(c+dx)+(6+11m+6m^2+m^3)\sin^3(c+dx)\right)}{ad(1+m)(2+m)(3+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^3*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 + 6*(1 + m)*Sin[c + d*x] - 3*(2 + 3*m +
 m^2)*Sin[c + d*x]^2 + (6 + 11*m + 6*m^2 + m^3)*Sin[c + d*x]^3))/(a*d*(1 +
m)*(2 + m)*(3 + m)*(4 + m))
```

Maple [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^3(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.28, size = 119, normalized size = 1.10

$$\frac{\left((m^3+6m^2+11m+6)a^m\sin(dx+c)^4+(m^3+3m^2+2m)a^m\sin(dx+c)^3-3(m^2+m)a^m\sin(dx+c)^2+6a^m m\sin(dx+c)-6a^m\right)(\sin(dx+c)+1)^m}{(m^4+10m^3+35m^2+50m+24)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(d*x + c)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(
d*x + c)^3 - 3*(m^2 + m)*a^m*sin(d*x + c)^2 + 6*a^m*m*sin(d*x + c) - 6*a^m)
*(sin(d*x + c) + 1)^m/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*d)
```

Fricas [A]
time = 0.42, size = 140, normalized size = 1.30

$$\frac{\left((m^3+6m^2+11m+6)\cos(dx+c)^4+m^3-(2m^3+9m^2+19m+12)\cos(dx+c)^2+3m^2+\left(m^3-(m^3+3m^2+2m)\cos(dx+c)^2+3m^2+8m\right)\sin(dx+c)+8m\right)(a\sin(dx+c)+a)^m}{dm^4+10dm^3+35dm^2+50dm+24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] ((m^3 + 6*m^2 + 11*m + 6)*cos(d*x + c)^4 + m^3 - (2*m^3 + 9*m^2 + 19*m + 12
)*cos(d*x + c)^2 + 3*m^2 + (m^3 - (m^3 + 3*m^2 + 2*m)*cos(d*x + c)^2 + 3*m^
2 + 8*m)*sin(d*x + c) + 8*m)*(a*sin(d*x + c) + a)^m/(d*m^4 + 10*d*m^3 + 35*
d*m^2 + 50*d*m + 24*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1508 vs. $2(88) = 176$.
time = 7.41, size = 1508, normalized size = 13.96

$$\begin{cases} x(a\sin(c)+a)^m\sin^3(c)\cos(c) & \text{for } d=0 \\ \frac{6\log(\sin(c+dx)+1)\sin^3(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{18\log(\sin(c+dx)+1)\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{18\log(\sin(c+dx)+1)\sin(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{6\log(\sin(c+dx)+1)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{18\sin^2(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{27\sin(c+dx)}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d}+\frac{11}{6a^4d\sin^3(c+dx)+18a^4d\sin^2(c+dx)+18a^4d\sin(c+dx)+6a^4d} & \text{for } m=-4 \\ -\frac{6\log(\sin(c+dx)+1)\sin^2(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}-\frac{12\log(\sin(c+dx)+1)\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}-\frac{6\log(\sin(c+dx)+1)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}+\frac{2\sin^3(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}-\frac{12\sin(c+dx)}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d}-\frac{9}{2a^3d\sin^2(c+dx)+4a^3d\sin(c+dx)+2a^3d} & \text{for } m=-3 \\ \frac{6\log(\sin(c+dx)+1)\sin(c+dx)}{2a^2d\sin(c+dx)+2a^2d}+\frac{6\log(\sin(c+dx)+1)}{2a^2d\sin(c+dx)+2a^2d}+\frac{\sin^3(c+dx)}{2a^2d\sin(c+dx)+2a^2d}-\frac{3\sin^2(c+dx)}{2a^2d\sin(c+dx)+2a^2d}+\frac{6}{2a^2d\sin(c+dx)+2a^2d} & \text{for } m=-2 \\ -\frac{\log(\sin(c+dx)+1)}{ad}+\frac{\sin^3(c+dx)}{3ad}-\frac{\sin^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{ad} & \text{for } m=-1 \\ \frac{m^3(a\sin(c+dx)+a)^m\sin^4(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{m^3(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{6m^2(a\sin(c+dx)+a)^m\sin^4(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{3m^2(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}-\frac{3m^2(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{11m(a\sin(c+dx)+a)^m\sin^4(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{2m(a\sin(c+dx)+a)^m\sin^3(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}-\frac{3m(a\sin(c+dx)+a)^m\sin^2(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{6m(a\sin(c+dx)+a)^m\sin(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}+\frac{6(a\sin(c+dx)+a)^m\sin^4(c+dx)}{dm^4+10dm^3+35dm^2+50dm+24d}-\frac{6(a\sin(c+dx)+a)^m}{dm^4+10dm^3+35dm^2+50dm+24d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**3*cos(c), Eq(d, 0)), (6*log(sin(c +
d*x) + 1)*sin(c + d*x)**3/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x
)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c
+ d*x)**2/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d
*sin(c + d*x) + 6*a**4*d) + 18*log(sin(c + d*x) + 1)*sin(c + d*x)/(6*a**4*d
*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a
**4*d) + 6*log(sin(c + d*x) + 1)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(
c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 18*sin(c + d*x)**2/(6*a*
*4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) +
 6*a**4*d) + 27*sin(c + d*x)/(6*a**4*d*sin(c + d*x)**3 + 18*a**4*d*sin(c +
d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d) + 11/(6*a**4*d*sin(c + d*x)**3
 + 18*a**4*d*sin(c + d*x)**2 + 18*a**4*d*sin(c + d*x) + 6*a**4*d), Eq(m, -4
)), (-6*log(sin(c + d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4
*a**3*d*sin(c + d*x) + 2*a**3*d) - 12*log(sin(c + d*x) + 1)*sin(c + d*x)/(2
*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) - 6*log(sin(c +
 d*x) + 1)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) +
2*sin(c + d*x)**3/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**
3*d) - 12*sin(c + d*x)/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) +
```

```
2*a**3*d) - 9/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d)
, Eq(m, -3)), (6*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**2*d*sin(c + d*x)
+ 2*a**2*d) + 6*log(sin(c + d*x) + 1)/(2*a**2*d*sin(c + d*x) + 2*a**2*d) +
sin(c + d*x)**3/(2*a**2*d*sin(c + d*x) + 2*a**2*d) - 3*sin(c + d*x)**2/(2*a
**2*d*sin(c + d*x) + 2*a**2*d) + 6/(2*a**2*d*sin(c + d*x) + 2*a**2*d), Eq(m
, -2)), (-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)**3/(3*a*d) - sin(c + d
*x)**2/(2*a*d) + sin(c + d*x)/(a*d), Eq(m, -1)), (m**3*(a*sin(c + d*x) + a)
**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + m**3
*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 +
50*d*m + 24*d) + 6*m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 1
0*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + 3*m**2*(a*sin(c + d*x) + a)**m*sin(
c + d*x)**3/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m**2*(a*si
n(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m
 + 24*d) + 11*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3
 + 35*d*m**2 + 50*d*m + 24*d) + 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3
/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) - 3*m*(a*sin(c + d*x) + a
)**m*sin(c + d*x)**2/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*d*m + 24*d) + 6*m
*(a*sin(c + d*x) + a)**m*sin(c + d*x)/(d*m**4 + 10*d*m**3 + 35*d*m**2 + 50*
d*m + 24*d) + 6*(a*sin(c + d*x) + a)**m*sin(c + d*x)**4/(d*m**4 + 10*d*m**3
 + 35*d*m**2 + 50*d*m + 24*d) - 6*(a*sin(c + d*x) + a)**m/(d*m**4 + 10*d*m*
*3 + 35*d*m**2 + 50*d*m + 24*d), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(108) = 216$.
time = 0.50, size = 508, normalized size = 4.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m^3 - 3*(a*sin(d*x + c) + a)
^3*(a*sin(d*x + c) + a)^m*a*m^3 + 3*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c)
+ a)^m*a^2*m^3 - (a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m^3 + 6*(a
*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m^2 - 21*(a*sin(d*x + c) + a)^3
*(a*sin(d*x + c) + a)^m*a*m^2 + 24*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) +
 a)^m*a^2*m^2 - 9*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m^2 + 11*
(a*sin(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m*m - 42*(a*sin(d*x + c) + a)^3
*(a*sin(d*x + c) + a)^m*a*m + 57*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a
)^m*a^2*m - 26*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3*m + 6*(a*sin
(d*x + c) + a)^4*(a*sin(d*x + c) + a)^m - 24*(a*sin(d*x + c) + a)^3*(a*sin(
d*x + c) + a)^m*a + 36*(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a^2 -
24*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a^3)/((a^3*m^4 + 10*a^3*m^3
+ 35*a^3*m^2 + 50*a^3*m + 24*a^3)*a*d)
```

Mupad [**B**]

time = 10.74, size = 224, normalized size = 2.07

$$\frac{(a(\sin(c+dx)+1))^m (21m - 24\cos(2c+2dx) + 6\cos(4c+4dx) + 60m\sin(c+dx) - 32m\cos(2c+2dx) + 11m\cos(4c+4dx) - 4m\sin(3c+3dx) + 18m^2\sin(c+dx) + 6m^3\sin(c+dx) + 6m^2 + 3m^3 - 12m^2\cos(2c+2dx) - 4m^3\cos(2c+2dx) + 6m^2\cos(4c+4dx) + m^3\cos(4c+4dx) - 6m^2\sin(3c+3dx) - 2m^3\sin(3c+3dx) - 30)}{8d\,(m^4 + 10m^3 + 35m^2 + 50m + 24)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^3*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(21*m - 24*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x)
+ 60*m*sin(c + d*x) - 32*m*cos(2*c + 2*d*x) + 11*m*cos(4*c + 4*d*x) - 4*m*s
in(3*c + 3*d*x) + 18*m^2*sin(c + d*x) + 6*m^3*sin(c + d*x) + 6*m^2 + 3*m^3
- 12*m^2*cos(2*c + 2*d*x) - 4*m^3*cos(2*c + 2*d*x) + 6*m^2*cos(4*c + 4*d*x)
 + m^3*cos(4*c + 4*d*x) - 6*m^2*sin(3*c + 3*d*x) - 2*m^3*sin(3*c + 3*d*x) -
 30))/(8*d*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

3.930 $\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=80

$$\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}-\frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}+\frac{(a+a\sin(c+dx))^{3+m}}{a^3d(3+m)}$$

```
[Out] (a+a*sin(d*x+c))^(1+m)/a/d/(1+m)-2*(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)+(a+a*
sin(d*x+c))^(3+m)/a^3/d/(3+m)
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^{m+3}}{a^3d(m+3)}-\frac{2(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}+\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] (a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m)) - (2*(a + a*Sin[c + d*x])^(2 + m
))/(a^2*d*(2 + m)) + (a + a*Sin[c + d*x])^(3 + m)/(a^3*d*(3 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+x)^m}{a^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x^2(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2(a+x)^m - 2a(a+x)^{1+m} + (a+x)^{2+m}\right)\,dx}{a^3d}$$
$$= \frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} - \frac{2(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)} + \frac{(a}{}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.96

$$-\frac{(a(1+\sin(c+dx)))^{1+m}\left(-6-3m-m^2+(2+3m+m^2)\cos(2(c+dx))+4(1+m)\sin(c+dx)\right)}{2ad(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
[Out] -1/2*((a*(1 + Sin[c + d*x]))^(1 + m)*(-6 - 3*m - m^2 + (2 + 3*m + m^2)*Cos[
2*(c + d*x)] + 4*(1 + m)*Sin[c + d*x]))/(a*d*(1 + m)*(2 + m)*(3 + m))
```

Maple [F]
time = 0.24, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\sin^2(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [A]
time = 0.28, size = 84, normalized size = 1.05

$$\frac{\left((m^2+3m+2)a^m\sin(dx+c)^3+(m^2+m)a^m\sin(dx+c)^2-2a^m m\sin(dx+c)+2a^m\right)(\sin(dx+c)+1)^m}{(m^3+6m^2+11m+6)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] ((m^2 + 3*m + 2)*a^m*sin(d*x + c)^3 + (m^2 + m)*a^m*sin(d*x + c)^2 - 2*a^m*
m*sin(d*x + c) + 2*a^m)*(sin(d*x + c) + 1)^m/((m^3 + 6*m^2 + 11*m + 6)*d)
```

Fricas [A]
time = 0.36, size = 93, normalized size = 1.16

$$-\frac{\left((m^2+m)\cos\left(dx+c\right)^2-m^2+\left((m^2+3\,m+2)\cos\left(dx+c\right)^2-m^2-m-2\right)\sin\left(dx+c\right)-m-2\right)\left(a\sin\left(dx+c\right)+a\right)^m}{dm^3+6\,dm^2+11\,dm+6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m^2 + m)*cos(d*x + c)^2 - m^2 + ((m^2 + 3*m + 2)*cos(d*x + c)^2 - m^2 -
m - 2)*sin(d*x + c) - m - 2)*(a*sin(d*x + c) + a)^m/(d*m^3 + 6*d*m^2 + 11*d
*m + 6*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(65) = 130.
time = 2.84, size = 697, normalized size = 8.71

$$\begin{cases} x(a\sin\left(c\right)+a)^m\sin^2\left(c\right)\cos\left(c\right) & \text{for } d=0 \\ \frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin^2\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{4\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{2\log\left(\sin\left(c+dx\right)+1\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{4\sin\left(c+dx\right)}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d}+\frac{3}{2a^3d\sin^2\left(c+dx\right)+4a^3d\sin\left(c+dx\right)+2a^3d} & \text{for } m=-3 \\ -\frac{2\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d}-\frac{2\log\left(\sin\left(c+dx\right)+1\right)}{a^2d\sin\left(c+dx\right)+a^2d}+\frac{\sin^2\left(c+dx\right)}{a^2d\sin\left(c+dx\right)+a^2d}-\frac{2}{a^2d\sin\left(c+dx\right)+a^2d} & \text{for } m=-2 \\ \frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin^2\left(c+dx\right)}{2ad}-\frac{\sin\left(c+dx\right)}{ad} & \text{for } m=-1 \\ \frac{m^2(a\sin\left(c+dx\right)+a)^m\sin^3\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}+\frac{m^2(a\sin\left(c+dx\right)+a)^m\sin^2\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}+\frac{3m(a\sin\left(c+dx\right)+a)^m\sin^3\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}+\frac{m(a\sin\left(c+dx\right)+a)^m\sin^2\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}-\frac{2m(a\sin\left(c+dx\right)+a)^m\sin\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}+\frac{2(a\sin\left(c+dx\right)+a)^m\sin^3\left(c+dx\right)}{dm^3+6dm^2+11dm+6d}+\frac{2(a\sin\left(c+dx\right)+a)^m}{dm^3+6dm^2+11dm+6d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)**2*cos(c), Eq(d, 0)), (2*log(sin(c +
d*x) + 1)*sin(c + d*x)**2/(2*a**3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x)
 + 2*a**3*d) + 4*log(sin(c + d*x) + 1)*sin(c + d*x)/(2*a**3*d*sin(c + d*x)*
*2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 2*log(sin(c + d*x) + 1)/(2*a**3*d*
sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 4*sin(c + d*x)/(2*a**
3*d*sin(c + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d) + 3/(2*a**3*d*sin(c
 + d*x)**2 + 4*a**3*d*sin(c + d*x) + 2*a**3*d), Eq(m, -3)), (-2*log(sin(c +
 d*x) + 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) - 2*log(sin(c + d*x)
 + 1)/(a**2*d*sin(c + d*x) + a**2*d) + sin(c + d*x)**2/(a**2*d*sin(c + d*x)
 + a**2*d) - 2/(a**2*d*sin(c + d*x) + a**2*d), Eq(m, -2)), (log(sin(c + d*x
) + 1)/(a*d) + sin(c + d*x)**2/(2*a*d) - sin(c + d*x)/(a*d), Eq(m, -1)), (m
**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2 + 11*d*m + 6
*d) + m**2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*m**2 + 11*
d*m + 6*d) + 3*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3 + 6*d*m**2
 + 11*d*m + 6*d) + m*(a*sin(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**3 + 6*d*
m**2 + 11*d*m + 6*d) - 2*m*(a*sin(c + d*x) + a)**m*sin(c + d*x)/(d*m**3 + 6
*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(c + d*x) + a)**m*sin(c + d*x)**3/(d*m**3
 + 6*d*m**2 + 11*d*m + 6*d) + 2*(a*sin(c + d*x) + a)**m/(d*m**3 + 6*d*m**2
+ 11*d*m + 6*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(80) = 160.

time = 0.44, size = 287, normalized size = 3.59

$$\frac{(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m m^2 - 2(a\sin(dx+c)+a)^2(a\sin(dx+c)+a)^m a m^2 + (a\sin(dx+c)+a)(a\sin(dx+c)+a)^m a^2 m^2 + 3(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m m - 8(a\sin(dx+c)+a)^2(a\sin(dx+c)+a)^m a m + 5(a\sin(dx+c)+a)(a\sin(dx+c)+a)^m a^2 m + 2(a\sin(dx+c)+a)^3(a\sin(dx+c)+a)^m - 6(a\sin(dx+c)+a)^2(a\sin(dx+c)+a)^m a + 6(a\sin(dx+c)+a)(a\sin(dx+c)+a)^m a^2}{(a^2m^3+6a^2m^2+11a^2m+6a^2)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m^2 - 2*(a*sin(d*x + c) + a)
^2*(a*sin(d*x + c) + a)^m*a*m^2 + (a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)
^m*a^2*m^2 + 3*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m*m - 8*(a*sin(d
*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a*m + 5*(a*sin(d*x + c) + a)*(a*sin(d
*x + c) + a)^m*a^2*m + 2*(a*sin(d*x + c) + a)^3*(a*sin(d*x + c) + a)^m - 6*
(a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*a + 6*(a*sin(d*x + c) + a)*(a
*sin(d*x + c) + a)^m*a^2)/((a^2*m^3 + 6*a^2*m^2 + 11*a^2*m + 6*a^2)*a*d)
```

Mupad [B]
time = 9.92, size = 138, normalized size = 1.72

$$\frac{(a\,(\sin(c+d\,x)+1))^m\,(2\,m+6\,\sin(c+d\,x)-2\,\sin(3\,c+3\,d\,x)+m\,\sin(c+d\,x)+2\,m\,(2\sin(c+d\,x)^2-1)-3\,m\,\sin(3\,c+3\,d\,x)+3\,m^2\,\sin(c+d\,x)+2\,m^2\,(2\sin(c+d\,x)^2-1)+2\,m^2-m^2\,\sin(3\,c+3\,d\,x)+8)}{4\,d\,(m^3+6\,m^2+11\,m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)^2*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(2*m + 6*sin(c + d*x) - 2*sin(3*c + 3*d*x) + m*si
n(c + d*x) + 2*m*(2*sin(c + d*x)^2 - 1) - 3*m*sin(3*c + 3*d*x) + 3*m^2*sin(
c + d*x) + 2*m^2*(2*sin(c + d*x)^2 - 1) + 2*m^2 - m^2*sin(3*c + 3*d*x) + 8)
)/(4*d*(11*m + 6*m^2 + m^3 + 6))
```

3.931 $\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=54

$$-\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)}+\frac{(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}$$

```
[Out] -(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)+(a+a*sin(d*x+c))^(2+m)/a^2/d/(2+m)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2912, 12, 45}

$$\frac{(a\sin(c+dx)+a)^{m+2}}{a^2d(m+2)}-\frac{(a\sin(c+dx)+a)^{m+1}}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((a + a*Sin[c + d*x])^(1 + m)/(a*d*(1 + m))) + (a + a*Sin[c + d*x])^(2 + m
)/(a^2*d*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)\sin(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^m}{a}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int x(a+x)^m\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-a(a+x)^m+(a+x)^{1+m}\right)\,dx, x, a\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{(a+a\sin(c+dx))^{1+m}}{ad(1+m)} + \frac{(a+a\sin(c+dx))^{2+m}}{a^2d(2+m)}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 0.80

$$\frac{(a(1+\sin(c+dx)))^{1+m}(-1+(1+m)\sin(c+dx))}{ad(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] ((a*(1 + Sin[c + d*x]))^(1 + m)*(-1 + (1 + m)*Sin[c + d*x]))/(a*d*(1 + m)*(
2 + m))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\sin(dx+c)\left(a+a\sin(dx+c)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [**A**]
time = 0.27, size = 56, normalized size = 1.04

$$\frac{\left(a^m(m+1)\sin(dx+c)^2+a^m m\sin(dx+c)-a^m\right)\left(\sin(dx+c)+1\right)^m}{(m^2+3m+2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] (a^m*(m + 1)*sin(d*x + c)^2 + a^m*m*sin(d*x + c) - a^m)*(sin(d*x + c) + 1)^
m/((m^2 + 3*m + 2)*d)
```

Fricas [A]
time = 0.36, size = 54, normalized size = 1.00

$$-\frac{\left((m+1)\cos\left(dx+c\right)^{2}-m\sin\left(dx+c\right)-m\right)\left(a\sin\left(dx+c\right)+a\right)^{m}}{dm^{2}+3\,dm+2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] -((m + 1)*cos(d*x + c)^2 - m*sin(d*x + c) - m)*(a*sin(d*x + c) + a)^m/(d*m^
2 + 3*d*m + 2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(41) = 82.
time = 1.16, size = 248, normalized size = 4.59

$$\begin{cases} x\left(a\sin\left(c\right)+a\right)^{m}\sin\left(c\right)\cos\left(c\right) & \text{for}\ d=0 \\ \frac{\log\left(\sin\left(c+dx\right)+1\right)\sin\left(c+dx\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{\log\left(\sin\left(c+dx\right)+1\right)}{a^{2}d\sin\left(c+dx\right)+a^{2}d}+\frac{1}{a^{2}d\sin\left(c+dx\right)+a^{2}d} & \text{for}\ m=-2 \\ -\frac{\log\left(\sin\left(c+dx\right)+1\right)}{ad}+\frac{\sin\left(c+dx\right)}{ad} & \text{for}\ m=-1 \\ \frac{m\left(a\sin\left(c+dx\right)+a\right)^{m}\sin^{2}\left(c+dx\right)}{dm^{2}+3dm+2d}+\frac{m\left(a\sin\left(c+dx\right)+a\right)^{m}\sin\left(c+dx\right)}{dm^{2}+3dm+2d}+\frac{\left(a\sin\left(c+dx\right)+a\right)^{m}\sin^{2}\left(c+dx\right)}{dm^{2}+3dm+2d}-\frac{\left(a\sin\left(c+dx\right)+a\right)^{m}}{dm^{2}+3dm+2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Piecewise((x*(a*sin(c) + a)**m*sin(c)*cos(c), Eq(d, 0)), (log(sin(c + d*x)
+ 1)*sin(c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + log(sin(c + d*x) + 1)/(a
**2*d*sin(c + d*x) + a**2*d) + 1/(a**2*d*sin(c + d*x) + a**2*d), Eq(m, -2))
, (-log(sin(c + d*x) + 1)/(a*d) + sin(c + d*x)/(a*d), Eq(m, -1)), (m*(a*sin
(c + d*x) + a)**m*sin(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) + m*(a*sin(c + d*x
) + a)**m*sin(c + d*x)/(d*m**2 + 3*d*m + 2*d) + (a*sin(c + d*x) + a)**m*sin
(c + d*x)**2/(d*m**2 + 3*d*m + 2*d) - (a*sin(c + d*x) + a)**m/(d*m**2 + 3*d
*m + 2*d), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(54) = 108.
time = 0.45, size = 120, normalized size = 2.22

$$\frac{\left(a\sin\left(dx+c\right)+a\right)^{2}\left(a\sin\left(dx+c\right)+a\right)^{m}m-\left(a\sin\left(dx+c\right)+a\right)\left(a\sin\left(dx+c\right)+a\right)^{m}am+\left(a\sin\left(dx+c\right)+a\right)^{2}\left(a\sin\left(dx+c\right)+a\right)^{m}-2\left(a\sin\left(dx+c\right)+a\right)\left(a\sin\left(dx+c\right)+a\right)^{m}a}{\left(m^{2}+3\,m+2\right)a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] ((a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m*m - (a*sin(d*x + c) + a)*(a*
sin(d*x + c) + a)^m*a*m + (a*sin(d*x + c) + a)^2*(a*sin(d*x + c) + a)^m - 2
*(a*sin(d*x + c) + a)*(a*sin(d*x + c) + a)^m*a)/((m^2 + 3*m + 2)*a^2*d)
```

Mupad [B]
time = 9.42, size = 62, normalized size = 1.15

$$\frac{(a\,(\sin{(c+d\,x)}+1))^m\,\left(\frac{m}{2}+m\,\sin{(c+d\,x)}+\frac{m\,\left(2\,\sin{(c+d\,x)}^2-1\right)}{2}+\sin{(c+d\,x)}^2-1\right)}{d\,\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(c + d*x)*(a + a*sin(c + d*x))^m,x)
```

```
[Out] ((a*(sin(c + d*x) + 1))^m*(m/2 + m*sin(c + d*x) + (m*(2*sin(c + d*x)^2 - 1)
)/2 + sin(c + d*x)^2 - 1))/(d*(3*m + m^2 + 2))
```

3.932 $\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{{}_2F_1(1,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

```
[Out] -hypergeom([1, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Rubi [A]

time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2786, 67}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)(a+a\sin(c+dx))^m\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 1.00

$$-\frac{{}_2F_1(1,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\csc\left(dx+c\right)\left(a+a\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

```
[Out] int(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(c+dx\right)+1\right)\right)^{m}\cos\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)*csc(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m}{\sin\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x),x)
```

```
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x), x)
```

3.933 $\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=42

$$\frac{{}_2F_1(2,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

[Out] hypergeom([2, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 67}

$$\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(2,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]

[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^(1 + m))/(a*d*(1 + m))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{a^2(a+x)^m}{x^2}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a\text{Subst}\left(\int \frac{(a+x)^m}{x^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1(2,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx$$
$$\overline{ad(1+m)}$$

Mathematica [A]
time = 0.04, size = 42, normalized size = 1.00

$$\frac{{}_2F_1(2,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]*(a + a*Sin[c + d*x])^m,x]
[Out] (Hypergeometric2F1[2, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x])^
(1 + m))/(a*d*(1 + m))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\csc^2(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(c+dx)+1))^m \cos(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2*(a+a*sin(d*x+c))**m,x)
[Out] Integral((a*(sin(c + d*x) + 1))**m*cos(c + d*x)*csc(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2*(a+a*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)\,\left(a+a\,\sin(c+d\,x)\right)^m}{\sin(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^2,x)
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^2, x)
```

3.934 $\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{{}_2F_1(3,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

```
[Out] -hypergeom([3, 1+m],[2+m],1+sin(d*x+c))*(a+a*sin(d*x+c))^(1+m)/a/d/(1+m)
```

Rubi [A]

time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 67}

$$-\frac{(a\sin(c+dx)+a)^{m+1}\,{}_2F_1(3,m+1;m+2;\sin(c+dx)+1)}{ad(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cot(c+dx)\csc^2(c+dx)(a+a\sin(c+dx))^m\,dx = \frac{\text{Subst}\left(\int \frac{a^3(a+x)^m}{x^3}\,dx, x, a\sin(c+dx)\right)}{ad}$$
$$= \frac{a^2\text{Subst}\left(\int \frac{(a+x)^m}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{{}_2F_1(3,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+d}{ad(1+m)}$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 1.00

$$-\frac{{}_2F_1(3,1+m;2+m;1+\sin(c+dx))(a+a\sin(c+dx))^{1+m}}{ad(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Csc[c + d*x]^2*(a + a*Sin[c + d*x])^m,x]
[Out] -((Hypergeometric2F1[3, 1 + m, 2 + m, 1 + Sin[c + d*x]]*(a + a*Sin[c + d*x]
)^(1 + m))/(a*d*(1 + m)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\left(\csc^3(dx+c)\right)(a+a\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
[Out] int(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3*(a+a*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3*(a+a*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + a)^m*cos(d*x + c)*csc(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(a+a\,\sin\left(c+d\,x\right)\right)}^m}{{\sin\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^3,x)
```

```
[Out] int((cos(c + d*x)*(a + a*sin(c + d*x))^m)/sin(c + d*x)^3, x)
```

3.935 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx$

Optimal. Leaf size=79

$$\frac{1}{8}a(4c+d)x-\frac{a(c+d)\cos^3(e+fx)}{3f}+\frac{a(4c+d)\cos(e+fx)\sin(e+fx)}{8f}-\frac{ad\cos^3(e+fx)\sin(e+fx)}{4f}$$

```
[Out] 1/8*a*(4*c+d)*x-1/3*a*(c+d)*cos(f*x+e)^3/f+1/8*a*(4*c+d)*cos(f*x+e)*sin(f*x
+e)/f-1/4*a*d*cos(f*x+e)^3*sin(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = $\{2939, 2748, 2715, 8\}$

$$-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}ax(4c+d)-\frac{d\cos^3(e+fx)(a\sin(e+fx)+a)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
```

```
[Out] (a*(4*c + d)*x)/8 - (a*(4*c + d)*Cos[e + f*x]^3)/(12*f) + (a*(4*c + d)*Cos[
e + f*x]*Sin[e + f*x])/(8*f) - (d*Cos[e + f*x]^3*(a + a*Sin[e + f*x]))/(4*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))\,dx = -\frac{d\cos^3(e+fx)(a+a\sin(e+fx))}{4f}+\frac{1}{4}(4c+d$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}-\frac{d\cos^3(e+fx)(a+$$
$$= -\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+d)\cos(e+f$$
$$= \frac{1}{8}a(4c+d)x-\frac{a(4c+d)\cos^3(e+fx)}{12f}+\frac{a(4c+$$

Mathematica [A]
time = 0.42, size = 64, normalized size = 0.81

$$-\frac{a(-12(4c+d)fx+24(c+d)\cos(e+fx)+8(c+d)\cos(3(e+fx))-24c\sin(2(e+fx))+3d\sin(4(e+fx)))}{96f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]
[Out] -1/96*(a*(-12*(4*c + d)*f*x + 24*(c + d)*Cos[e + f*x] + 8*(c + d)*Cos[3*(e
+ f*x)] - 24*c*Sin[2*(e + f*x)] + 3*d*Sin[4*(e + f*x)]))/f
```

Maple [A]
time = 0.20, size = 96, normalized size = 1.22

method	result
derivativedivides	$\frac{ad\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)-\frac{ac\left(\cos^3(fx+e)\right)}{3}-\frac{\left(\cos^3(fx+e)\right)ad}{3}+ac\left(\frac{\cos(fx+e)\sin(fx+e)}{2}\right.}{f}$
default	$\frac{ad\left(-\frac{\left(\cos^3(fx+e)\right)\sin(fx+e)}{4}+\frac{\cos(fx+e)\sin(fx+e)}{8}+\frac{fx}{8}+\frac{e}{8}\right)-\frac{ac\left(\cos^3(fx+e)\right)}{3}-\frac{\left(\cos^3(fx+e)\right)ad}{3}+ac\left(\frac{\cos(fx+e)\sin(fx+e)}{2}\right.}{f}$
risch	$\frac{axc}{2}+\frac{axd}{8}-\frac{a\cos(fx+e)c}{4f}-\frac{a\cos(fx+e)d}{4f}-\frac{ad\sin(4fx+4e)}{32f}-\frac{a\cos(3fx+3e)c}{12f}-\frac{a\cos(3fx+3e)d}{12f}+\frac{ac\sin}{}$

norman	$\left(\frac{1}{2}ac+\frac{1}{8}ad\right)x+\left(2ac+\frac{1}{2}ad\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(2ac+\frac{1}{2}ad\right)x\left(\tan^6\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(3ac+\frac{3}{4}ad\right)x\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\left(\frac{1}{2}ac+\frac{1}{8}ad\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(a*d*(-1/4*cos(f*x+e)^3*sin(f*x+e)+1/8*cos(f*x+e)*sin(f*x+e)+1/8*f*x+1/
8*e)-1/3*a*c*cos(f*x+e)^3-1/3*cos(f*x+e)^3*a*d+a*c*(1/2*cos(f*x+e)*sin(f*x+
e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.01

$$-\frac{32\,ac\cos\left(fx+e\right)^{3}+32\,ad\cos\left(fx+e\right)^{3}-24\left(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right)\right)ac-3\left(4\,fx+4\,e-\sin\left(4\,fx+4\,e\right)\right)ad}{96\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*a*c*cos(f*x + e)^3 + 32*a*d*cos(f*x + e)^3 - 24*(2*f*x + 2*e + si
n(2*f*x + 2*e))*a*c - 3*(4*f*x + 4*e - sin(4*f*x + 4*e))*a*d)/f
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.96

$$-\frac{8\left(ac+ad\right)\cos\left(fx+e\right)^{3}-3\left(4\,ac+ad\right)fx+3\left(2\,ad\cos\left(fx+e\right)^{3}-\left(4\,ac+ad\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(a*c + a*d)*cos(f*x + e)^3 - 3*(4*a*c + a*d)*f*x + 3*(2*a*d*cos(f*
x + e)^3 - (4*a*c + a*d)*cos(f*x + e))*sin(f*x + e))/f
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(71) = 142$.
time = 0.23, size = 199, normalized size = 2.52

$$\begin{cases} \frac{acx\sin^{2}\left(e+fx\right)}{2}+\frac{acx\cos^{2}\left(e+fx\right)}{2}+\frac{ac\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}-\frac{ac\cos^{3}\left(e+fx\right)}{3f}+\frac{adx\sin^{4}\left(e+fx\right)}{8}+\frac{adx\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+\frac{adx\cos^{4}\left(e+fx\right)}{8}+\frac{ad\sin^{3}\left(e+fx\right)\cos\left(e+fx\right)}{8f}-\frac{ad\sin\left(e+fx\right)\cos^{3}\left(e+fx\right)}{8f}-\frac{ad\cos^{3}\left(e+fx\right)}{3f} & \text{for}\: f\neq 0 \\ x\left(c+d\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)\cos^{2}\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x*sin(e + f*x)**2/2 + a*c*x*cos(e + f*x)**2/2 + a*c*sin(e +
f*x)*cos(e + f*x)/(2*f) - a*c*cos(e + f*x)**3/(3*f) + a*d*x*sin(e + f*x)**4
/8 + a*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + a*d*x*cos(e + f*x)**4/8 + a*
d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - a*d*sin(e + f*x)*cos(e + f*x)**3/(8*
f) - a*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(c + d*sin(e))*(a*sin(e) + a)
*cos(e)**2, True))
```

Giac [A]
time = 0.48, size = 87, normalized size = 1.10

$$\frac{1}{8}(4ac+ad)x-\frac{ad\sin(4fx+4e)}{32f}+\frac{ac\sin(2fx+2e)}{4f}-\frac{(ac+ad)\cos(3fx+3e)}{12f}-\frac{(ac+ad)\cos(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*a*c + a*d)*x - 1/32*a*d*sin(4*f*x + 4*e)/f + 1/4*a*c*sin(2*f*x + 2*e
)/f - 1/12*(a*c + a*d)*cos(3*f*x + 3*e)/f - 1/4*(a*c + a*d)*cos(f*x + e)/f
```

Mupad [B]
time = 10.24, size = 276, normalized size = 3.49

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{e}{2}+\frac{fx}{2}\right)(4c+d)}{4\left(ac+\frac{ad}{4}\right)}\right)(4c+d)}{4f}-\frac{\left(ac-\frac{ad}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^7+(2ac+2ad)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+\left(ac+\frac{7ad}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^5+(2ac+2ad)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+\left(-ac-\frac{7ad}{4}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^3+\left(\frac{2ac}{3}+\frac{2ad}{3}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+\left(\frac{ad}{4}-ac\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)+\frac{2ac}{3}+\frac{2ad}{3}}{f\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^8+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^6+6\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^4+4\tan\left(\frac{e}{2}+\frac{fx}{2}\right)^2+1\right)}-\frac{a(4c+d)\left(\mathrm{atan}\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)-\frac{fx}{2}\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(4*c + d))/(4*(a*c + (a*d)/4)))*(4*c + d))/(4
*f) - ((2*a*c)/3 + (2*a*d)/3 + tan(e/2 + (f*x)/2)^4*(2*a*c + 2*a*d) + tan(e
/2 + (f*x)/2)^6*(2*a*c + 2*a*d) + tan(e/2 + (f*x)/2)^2*((2*a*c)/3 + (2*a*d)
/3) + tan(e/2 + (f*x)/2)^7*(a*c - (a*d)/4) - tan(e/2 + (f*x)/2)^3*(a*c + (7
*a*d)/4) + tan(e/2 + (f*x)/2)^5*(a*c + (7*a*d)/4) - tan(e/2 + (f*x)/2)*(a*c
 - (a*d)/4))/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/
2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) - (a*(4*c + d)*(atan(tan(e/2 +
(f*x)/2)) - (f*x)/2))/(4*f)
```

3.936 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=123

$$-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)f}+\frac{2\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)\sqrt{d}\,f}$$

```
[Out] -2*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*2^(1/2)/a
^(3/2)/(c-d)/f+2*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*
x+e))^(1/2))*(c+d)^(1/2)/a^(3/2)/(c-d)/f/d^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2995, 3064, 2728, 212, 2852, 214}

$$\frac{2\sqrt{c+d}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{d}\,\cos(e+fx)}{\sqrt{c+d}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}\sqrt{d}\,f(c-d)}-\frac{2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\cos(e+fx)}{\sqrt{2}\,\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
]])])/(a^(3/2)*(c - d)*f) + (2*Sqrt[c + d]*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e +
f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(a^(3/2)*(c - d)*Sqrt[d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2852

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b*c + a*d - d*x^2), x
], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2995

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 3064

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}(c+d\sin(e+fx))}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\,(c+d\sin(e+fx))}\,dx}{a^2}$$

$$= \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{a(c-d)} - \frac{(c+d)\int \frac{\sqrt{a+a\sin(e+}}{c+d\sin(e+fx)}}{a^2(c-d)}$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a(c-d)f} + \frac{(2(c+}{}$$

$$= -\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}(c-d)f} + \frac{2\sqrt{c+d}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.04, size = 220, normalized size = 1.79

$$\frac{(-1)^{3/4}\left((-4-4i)\sqrt{d}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)+\sqrt[4]{-1}\sqrt{c+d}\left(\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}+\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)-\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)-\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\left(\sqrt{c+d}-\sqrt{d}\cos\left(\frac{1}{2}(e+fx)\right)+\sqrt{d}\sin\left(\frac{1}{2}(e+fx)\right)\right)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3}{\sqrt{d}\,(-c+d)f(a(1+\sin(e+fx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])),
x]
```

```
[Out] ((-1)^(3/4)*((-4 - 4*I)*Sqrt[d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e
 + f*x)/4])] + (-1)^(1/4)*Sqrt[c + d]*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d]
+ Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)
/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])
)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(Sqrt[d]*(-c + d)*f*(a*(1 + Sin[
e + f*x]))^(3/2))
```

Maple [A]
time = 8.42, size = 160, normalized size = 1.30

method	result
default	$-\frac{2(1+\sin(fx+e))\sqrt{-a(\sin(fx+e)-1)}\left(\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a(c+d)d}\;-}{a^{\frac{3}{2}}(c-d)\sqrt{a(c+d)d}\,\cos(fx+e)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/a^(3/2)*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(2^(1/2)*arctanh(1/2*(-
a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*(a*(c+d)*d)^(1/2)-arctanh((-a*(sin
(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*c-arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(1/2)*d)/(c-d)/(a*(c+d)*d)^(1/2)/cos(f*x+e)/
(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)),
 x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(104) = 208.
time = 0.45, size = 709, normalized size = 5.76

$$\left[-\frac{\sqrt{\frac{c+d}{ad}}\log\left(\frac{d^{2}\cos(fx+e)^{3}-(6cd+7d^{2})\cos(fx+e)^{2}-c^{2}-2cd-d^{2}-4\left(d^{2}\cos(fx+e)^{2}-cd-3d^{2}-(cd+2d^{2})\cos(fx+e)+(d^{2}\cos(fx+e)+cd+3d^{2})\sin(fx+e)\right)\sqrt{a\sin(fx+e)+a}\sqrt{\frac{c+d}{ad}}-(c^{2}+8cd+9d^{2})\cos(fx+e)+\left(d^{2}\cos(fx+e)^{2}-c^{2}-2cd-d^{2}+2(3cd+4d^{2})\cos(fx+e)\right)\sin(fx+e)}{d^{2}\cos(fx+e)^{3}+(2cd+d^{2})\cos(fx+e)^{2}-c^{2}-2cd-d^{2}-(c^{2}+d^{2})\cos(fx+e)+\left(d^{2}\cos(fx+e)^{2}-2cd\cos(fx+e)-c^{2}-2cd-d^{2}\right)\sin(fx+e)}\right)+\frac{2\sqrt{2}\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{2(ac-ad)f},\ \frac{\sqrt{-\frac{c+d}{ad}}\arctan\left(\frac{\sqrt{a\sin(fx+e)+a}(d\sin(fx+e)-c-2d)\sqrt{-\frac{c+d}{ad}}}{2(c+d)\cos(fx+e)}\right)-\frac{\sqrt{2}\log\left(-\frac{\cos(fx+e)^{2}-(\cos(fx+e)-2)\sin(fx+e)+\frac{2\sqrt{2}\sqrt{a\sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}}+3\cos(fx+e)+2}{\cos(fx+e)^{2}-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)}{\sqrt{a}}}{(ac-ad)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] [-1/2*(sqrt((c + d)/(a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*
x + e)^2 - c^2 - 2*c*d - d^2 - 4*(d^2*cos(f*x + e)^2 - c*d - 3*d^2 - (c*d +
 2*d^2)*cos(f*x + e) + (d^2*cos(f*x + e) + c*d + 3*d^2)*sin(f*x + e))*sqrt(
a*sin(f*x + e) + a)*sqrt((c + d)/(a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e
) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e
))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 2*sqrt(2)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c -
a*d)*f), (sqrt(-(c + d)/(a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f
*x + e) - c - 2*d)*sqrt(-(c + d)/(a*d))/((c + d)*cos(f*x + e))) - sqrt(2)*l
og(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*si
n(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e)
+ 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
/sqrt(a))/((a*c - a*d)*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(104) = 208.
time = 0.51, size = 224, normalized size = 1.82

$$\frac{\sqrt{2}\,\sqrt{a}\left(\frac{\sqrt{2}\,(c+d)\arctan\left(\frac{\sqrt{2}\,d\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)}{\sqrt{-cd-d^2}}\right)}{\left(a^2c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\right)\sqrt{-cd-d^2}}+\frac{\log\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^2c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}-\frac{\log\left(-\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{a^2c\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)-a^2d\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] sqrt(2)*sqrt(a)*(sqrt(2)*(c + d)*arctan(sqrt(2)*d*sin(-1/4*pi + 1/2*f*x + 1
/2*e)/sqrt(-c*d - d^2))/((a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d
```

```
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(-c*d - d^2)) + log(sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 1)/(a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - log(-sin(-1/4*pi + 1/2*f*x + 1/2*e) +
 1)/(a^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) - a^2*d*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))))/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(c+d\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),x)
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))), x)
```

3.937 $\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx$

Optimal. Leaf size=141

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}\,f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{c-d}\,f}$$

```
[Out] -2*arctanh(1/2*cos(f*x+e)*a^(1/2)*(c-d)^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2
)/(c+d*sin(f*x+e))^(1/2))*2^(1/2)/a^(3/2)/f/(c-d)^(1/2)+2*arctan(cos(f*x+e)
*a^(1/2)*d^(1/2)/(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2))/a^(3/2)/f/d
^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2995, 3061, 2861, 214, 2854, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}\,f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c-d}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}f\sqrt{c-d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c +
 d*Sin[e + f*x]])])/(a^(3/2)*Sqrt[d]*f) - (2*Sqrt[2]*ArcTanh[(Sqrt[a]*Sqrt[
c - d]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f
*x]])])/(a^(3/2)*Sqrt[c - d]*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2854

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]], x_Symbol] :> Dist[-2*(b/f), Subst[Int[1/(b + d*x^2), x], x
, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]))], x]
```

```
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rule 2861

```
Int[1/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e
_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[-2*(a/f), Subst[Int[1/(2*b^2 - (a*c
- b*d)*x^2), x], x, b*(Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Si
n[e + f*x]]))], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2995

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*(
(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[1/b^2, Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ
[2*m, 2*n]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dis
t[(A*b - a*B)/b, Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]),
 x], x] + Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(a+a\sin(e+fx))^{3/2}\sqrt{c+d\sin(e+fx)}}\,dx = \frac{\int \frac{a-a\sin(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\,dx}{a^2}$$

$$= -\frac{\int \frac{\sqrt{a+a\sin(e+fx)}}{\sqrt{c+d\sin(e+fx)}}\,dx}{a^2} + \frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}\sqrt{}}}{a}$$

$$= -\frac{4\text{Subst}\left(\int \frac{1}{2a^2-(ac-ad)x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c-}}\right.}{f}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{d}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}\sqrt{c+d\sin(e+fx)}}\right)}{a^{3/2}\sqrt{d}\,f} - \frac{2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 42.27, size = 208404, normalized size = 1478.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x
]]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4460 vs. $2(114) = 228$.
time = 0.33, size = 4461, normalized size = 31.64

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/
(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1
/2)*c*sin(f*x+e)+d)*d)^(1/2)*d^4*cos(f*x+e)+2*arctan(((c+d*sin(f*x+e))/((d^
2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)
^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c*d^3-ar
ctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2
)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(
f*x+e)+d)*d)^(1/2)*d^4+(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^
(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*
((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c^3*d+(d^2/c^2)
^(1/2)*arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e
))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-
d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)
^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c
)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)
*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2
)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*c+4*2^(1/2)*((c+d*s
in(f*x+e))/(1+cos(f*x+e)))^(1/2)*ln(2*((2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*
x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+c*sin(f*x+e)-d*sin(f*x+e)+cos(f*x+e)
*c-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*
c^2*d^2*sin(f*x+e)-8*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*ln(2*(
(2*c-2*d)^(1/2)*2^(1/2)*((c+d*sin(f*x+e))/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+
c*sin(f*x+e)-d*sin(f*x+e)+cos(f*x+e)*c-d*cos(f*x+e)-c+d)/(1-cos(f*x+e)+sin(
f*x+e)))*(-(d^2/c^2)^(1/2)*c)^(1/2)*c*d^3*sin(f*x+e)-(d^2/c^2)^(1/2)*arctan
```

```
(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1
/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+
e)+d)*d)^(1/2)*c^3*d*sin(f*x+e)+2*(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/
((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-
2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c^2*
d^2*sin(f*x+e)-(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*
sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*si
n(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c*d^3*sin(f*x+e)-(d^2/c
^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2
)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^
(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c^3*d*cos(f*x+e)+2*(d^2/c^2)^(1/2)*arctan(((
c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)
*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+
d)*d)^(1/2)*c^2*d^2*cos(f*x+e)-(d^2/c^2)^(1/2)*arctan(((c+d*sin(f*x+e))/((d
^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d
)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*c*d^3*c
os(f*x+e)+arctan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*
x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+
e)-d*cos(f*x+e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c
^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4
)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)
+d)*d)^(1/2)*(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(
1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*d*sin(f*x+e)-arc
tan(((d^2/c^2)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c
^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+
e)+d)*((d^2/c^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4
+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(
2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*
(-(d^2/c^2)^(1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+
d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*d*cos(f*x+e)+arctan(((d^2/c^2
)^(1/2)*c*sin(f*x+e)+d*cos(f*x+e)-d)/((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*s
in(f*x+e)+d)*d)^(1/2)/((d^2/c^2)^(1/2)*c*sin(f*x+e)-d*cos(f*x+e)+d)*((d^2/c
^2)^(1/2)*c^2-d^2)*c*((d^2/c^2)^(1/2)-1)/(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^
(1/2)*d^2*c^2+d^4*(d^2/c^2)^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2))*(2*c-2*d)^(1/2
)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*(-(d^2/c^2)^(
1/2)*c)^(1/2)*(((d^2/c^2)^(1/2)*c^4+6*(d^2/c^2)^(1/2)*d^2*c^2+d^4*(d^2/c^2)
^(1/2)-4*c^2*d^2-4*d^4)*c)^(1/2)*d+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2
)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+
d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)*d^4*sin(f*x+e)-arct
an(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*x+e)+d)*d)^(1/2)/(-(d^2/c^2)^
(1/2)*c)^(1/2))*(2*c-2*d)^(1/2)*((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*
x+e)+d)*d)^(1/2)*c^2*d^2+arctan(((c+d*sin(f*x+e))/((d^2/c^2)^(1/2)*c*sin(f*
x+e)+d)*d)^(1/2)/(-(d^2/c^2)^(1/2)*c)^(1/2))*(2...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((a*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) +
c)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(120) = 240$.
time = 0.80, size = 2161, normalized size = 15.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*(4*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos
(f*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*s
in(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(
f*x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x
+ e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(-a*d)*lo
g((128*a*d^4*cos(f*x + e)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 +
 a*d^4 + 128*(2*a*c*d^3 - a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*
d^3 + 13*a*d^4)*cos(f*x + e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*
a*d^4)*cos(f*x + e)^2 - 8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x
 + e)^3 - c^3 + 17*c^2*d - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d
^3)*cos(f*x + e)^2 - (c^3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16
*d^3*cos(f*x + e)^3 + c^3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d
^3)*cos(f*x + e)^2 - 2*(5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4
 - 28*a*c^3*d + 230*a*c^2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (12
8*a*d^4*cos(f*x + e)^4 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^
4 - 256*(a*c*d^3 - a*d^4)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*
a*d^4)*cos(f*x + e)^2 + 32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*c
os(f*x + e))*sin(f*x + e))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1
/2*(2*sqrt(2)*a*d*log(-((c - 3*d)*cos(f*x + e)^2 - 2*sqrt(2)*((c - d)*cos(f
*x + e) - (c - d)*sin(f*x + e) + c - d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin
(f*x + e) + c)/sqrt(a*c - a*d) + (3*c - d)*cos(f*x + e) - ((c - 3*d)*cos(f*
x + e) - 2*c - 2*d)*sin(f*x + e) + 2*c + 2*d)/(cos(f*x + e)^2 - (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a*c - a*d) - sqrt(a*d)*arcta
```

```
n(1/4*(8*d^2*cos(f*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x +
 e))*sqrt(a*d)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*c
os(f*x + e)^3 - (3*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d -
a*c*d^2 + 2*a*d^3)*cos(f*x + e))))/(a^2*d*f), 1/4*(8*sqrt(2)*a*d*sqrt(-1/(a
*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)
*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(-a*d)*log((128*a*d^4*cos(f*x + e
)^5 + a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 + 128*(2*a*c*d^3
- a*d^4)*cos(f*x + e)^4 - 32*(5*a*c^2*d^2 - 14*a*c*d^3 + 13*a*d^4)*cos(f*x
+ e)^3 - 32*(a*c^3*d - 2*a*c^2*d^2 + 9*a*c*d^3 - 4*a*d^4)*cos(f*x + e)^2 -
8*(16*d^3*cos(f*x + e)^4 + 24*(c*d^2 - d^3)*cos(f*x + e)^3 - c^3 + 17*c^2*d
 - 59*c*d^2 + 51*d^3 - 2*(5*c^2*d - 26*c*d^2 + 33*d^3)*cos(f*x + e)^2 - (c^
3 - 7*c^2*d + 31*c*d^2 - 25*d^3)*cos(f*x + e) + (16*d^3*cos(f*x + e)^3 + c^
3 - 17*c^2*d + 59*c*d^2 - 51*d^3 - 8*(3*c*d^2 - 5*d^3)*cos(f*x + e)^2 - 2*(
5*c^2*d - 14*c*d^2 + 13*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*d)*sqrt(a*
sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c) + (a*c^4 - 28*a*c^3*d + 230*a*c^
2*d^2 - 476*a*c*d^3 + 289*a*d^4)*cos(f*x + e) + (128*a*d^4*cos(f*x + e)^4 +
 a*c^4 + 4*a*c^3*d + 6*a*c^2*d^2 + 4*a*c*d^3 + a*d^4 - 256*(a*c*d^3 - a*d^4
)*cos(f*x + e)^3 - 32*(5*a*c^2*d^2 - 6*a*c*d^3 + 5*a*d^4)*cos(f*x + e)^2 +
32*(a*c^3*d - 7*a*c^2*d^2 + 15*a*c*d^3 - 9*a*d^4)*cos(f*x + e))*sin(f*x + e
))/(cos(f*x + e) + sin(f*x + e) + 1)))/(a^2*d*f), 1/2*(4*sqrt(2)*a*d*sqrt(-
1/(a*c - a*d))*arctan(sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)*sqrt(-1/(a*c - a*d))/cos(f*x + e)) - sqrt(a*d)*arctan(1/4*(8*d^2*cos(f
*x + e)^2 - c^2 + 6*c*d - 9*d^2 - 8*(c*d - d^2)*sin(f*x + e))*sqrt(a*d)*sqr
t(a*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*d^3*cos(f*x + e)^3 - (3
*a*c*d^2 - a*d^3)*cos(f*x + e)*sin(f*x + e) - (a*c^2*d - a*c*d^2 + 2*a*d^3)
*cos(f*x + e))))/(a^2*d*f)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{c+d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(c + d*sin(e +
f*x))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{3/2}\,\sqrt{c+d\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int(cos(e + f*x)^2/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),
 x)
```

3.938 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{2\sqrt{2}\,F_1\left(\frac{3}{2}+m;-\frac{1}{2},-n;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{1+m}(c+}{af(3+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 2*AppellF1(3/2+m,-n,-1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/((
(c+d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2997, 145, 144, 143}

$$\frac{2\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{3}{2};-\frac{1}{2},-n;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+3)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e +
f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2997

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] && !IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \sqrt{a-ax}\,(a+ax)^{\frac{1}{2}+m}(\ldots\right.}{f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+a\ldots}}$$

$$= \frac{\left(\sqrt{2}\,\cos(e+fx)\right)\text{Subst}\left(\int \sqrt{\frac{1}{2}-\frac{x}{2}}\,(a+\ldots\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{\ldots}}$$

$$= \frac{\left(\sqrt{2}\,\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+d\ldots}{\ldots}\right.\right.}{f\ldots}$$

$$= \frac{2\sqrt{2}\,F_1\left(\frac{3}{2}+m;-\frac{1}{2},-n;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e\ldots\right.}{\ldots}$$

Mathematica [A]

time = 0.57, size = 158, normalized size = 1.17

$$-\frac{4F_1\left(\frac{3}{2};-\frac{1}{2}-m,-n;\frac{5}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{-\frac{1}{2}-m}}{3f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[3/2, -1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c
+ d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4]^
2)^(-1/2 - m))/(3*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.939 $\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{16\sqrt{2}\,a^3F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx)}{3f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -16/3*a^3*AppellF1(3/2,-n,-7/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{16\sqrt{2}\,a^3(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^3*AppellF1[3/2, -7/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x
])^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx = \frac{(a^3\cos(e+fx))\,\text{Subst}\left(\int \sqrt{1-x}\,(1+x)^{7/2}(c\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin}}$$

$$= \frac{\left(a^3\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin}{-c}\right.\right.}{f\sqrt{}}$$

$$= -\frac{16\sqrt{2}\,a^3F_1\left(\frac{3}{2};-\frac{7}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)\right.}{}$$

Mathematica [F]
time = 107.13, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^3(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^3\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^4 - 4*a^3*cos(f*x + e)^2 + (a^3*cos(f*x + e)^
4 - 4*a^3*cos(f*x + e)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**3*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^3*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx))^3 (c+d\sin(e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^n, x)
```

3.940 $\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}\,a^2F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))}{3f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -8/3*a^2*AppellF1(3/2,-n,-5/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e)
)*cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))
/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{8\sqrt{2}\,a^2(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a^2*AppellF1[3/2, -5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 -
Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{(a^2\cos(e+fx))\,\mathrm{Subst}\left(\int \sqrt{1-x}\,(1+x)^{5/2}(c\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin}{-c}\right.\right.}{f\sqrt{}}$$

$$= -\frac{8\sqrt{2}\,a^2F_1\left(\frac{3}{2};-\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right.}{}$$

Mathematica [F]
time = 70.72, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))^2\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2*sin(f*x + e) - 2*a^2*c
os(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx))^2 (c+d\sin(e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.941 $\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{2}\,aF_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))^n}{3f\sqrt{1+\sin(e+fx)}}$$

[Out] -4/3*a*AppellF1(3/2,-n,-3/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)

Rubi [A]

time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2947, 144, 143}

$$-\frac{4\sqrt{2}\,a(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*Sqrt[2]*a*AppellF1[3/2, -3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^
n)/(3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2947

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c*g*((g
*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(1 + (d/c)*x)^((p + 1)/2)*(1 - (d/c)*x)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos(e+fx))\text{Subst}\left(\int \sqrt{1-x}\,(1+x)^{3/2}(c+\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(}$$

$$= \frac{\left(a\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e-}{-c-c}\right.\right.}{f\sqrt{1}}$$

$$= -\frac{4\sqrt{2}\,aF_1\left(\frac{3}{2};-\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\right.}{}$$

Mathematica [F]
time = 11.69, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+a\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^2 (a+a\sin(e+fx)) (c+d\sin(e+fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.942 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{2}\,F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))^n\left(\frac{c+}{}\right.}{3af\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/3*AppellF1(3/2,-n,1/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))/(c
+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2993, 2834, 144, 143}

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -1/3*(Sqrt[2]*AppellF1[3/2, 1/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin
[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n
)/(a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2834

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[c*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sq
rt[1 - Sin[e + f*x]])), Subst[Int[(a + b*x)^m*(Sqrt[1 + (d/c)*x]/Sqrt[1 - (
d/c)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m] && EqQ[c^2 - d^2, 0
]
```

Rule 2993

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\int (a-a\sin(e+fx))(c+d\sin(e+fx))^n\,dx}{a^2}$$

$$= \frac{\cos(e+fx)\text{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{\sqrt{1+x}}\,dx,x,\sin(e+fx)\right)}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \cdots\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(\cdots}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{3}{2};\frac{1}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+f\cdots}{3af\sqrt{1+\sin(\cdots}$$

Mathematica [A]
time = 0.72, size = 229, normalized size = 1.92

$$-\frac{\sec(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\,(c+d\sin(e+fx))^{1+n}\left(-\left((c+d)(2+n)F_1\left(1+n;\frac{1}{2},\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\right)+(1+n)F_1\left(2+n;\frac{1}{2},\frac{1}{2};3+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)(c+d\sin(e+fx))\right)}{ad(-c+d)f(1+n)(2+n)\sqrt{\frac{d(1+\sin(e+fx))}{-c+d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((Sec[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[-((d*(-1 + Sin
[e + f*x]))/(c + d))]*(c + d*Sin[e + f*x])^(1 + n)*(-((c + d)*(2 + n)*Appel
lF1[1 + n, 1/2, 1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*
x])/(c + d)]) + (1 + n)*AppellF1[2 + n, 1/2, 1/2, 3 + n, (c + d*Sin[e + f*x
])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])))/(a*d*(-c +
 d)*f*(1 + n)*(2 + n)*Sqrt[(d*(1 + Sin[e + f*x]))/(-c + d)]))
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{a+a\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
```

```
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.943 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(}{c+}\right.}{3\sqrt{2}\,a^2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/6*AppellF1(3/2,-n,3/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n/a^2/f/(((c+d*sin(f*x+e))/(c+d))^n
)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{3\sqrt{2}\,a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/3*(AppellF1[3/2, 3/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(Sqrt[
2]*a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \cdots\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin \cdots}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{3}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1\cdots}{3\sqrt{2}\,a^2 f\sqrt{1+\sin(\cdots}}$$

Mathematica [F]
time = 10.95, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]
```

Maple [F]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(fx+e)\right)\left(c+d\sin(fx+e)\right)^n}{\left(a+a\sin(fx+e)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

3.944 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=119

$$-\frac{F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(\right.}{c+}}{6\sqrt{2}\,a^3f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/12*AppellF1(3/2,-n,5/2,5/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d))^
n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{(1-\sin(e+fx))\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{6\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/6*(AppellF1[3/2, 5/2, -n, 5/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(Sqrt[
2]*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{\sqrt{1-x}\,(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \cdots\right.}{a^3 f\sqrt{1-\sin(e+fx)}\sqrt{1+\sin\cdots}}$$

$$= -\frac{F_1\left(\frac{3}{2};\frac{5}{2},-n;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1\cdots}{6\sqrt{2}\,a^3 f\sqrt{1+\sin(\cdots}}$$

Mathematica [F]
time = 22.48, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

Maple [F]
time = 1.77, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(a*sin(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

3.945 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt{2}\,F_1\left(\frac{5}{2}+m;-\frac{3}{2},-n;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\cos(e+fx)(a+a\sin(e+fx))^{2+m}(c+}{a^2f(5+2m)\sqrt{1-\sin(e+fx)}}$$

```
[Out] 4*AppellF1(5/2+m,-n,-3/2,7/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*
cos(f*x+e)*(a+a*sin(f*x+e))^(2+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(5+2*m)/
(((c+d*sin(f*x+e))/(c-d))^n)/(1-sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2997, 145, 144, 143}

$$\frac{4\sqrt{2}\cos(e+fx)(a\sin(e+fx)+a)^{m+2}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{5}{2};-\frac{3}{2},-n;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{a^2f(2m+5)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, -n, 7/2 + m, (1 + Sin[e + f*x])/2, -((d*
(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(2 + m)*(c
+ d*Sin[e + f*x])^n)/(a^2*f*(5 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e
+ f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2997

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[e + f*
x]/(a^(p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[I
nt[(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&
 IntegerQ[p/2] && !IntegerQ[m]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int (a-ax)^{3/2}(a+ax)^{\frac{3}{2}+m}\right.}{a^2 f\sqrt{a-a\sin(e+fx)}\,\sqrt{a+}}$$

$$= \frac{\left(2\sqrt{2}\,\cos(e+fx)\right)\text{Subst}\left(\int \left(\frac{1}{2}-\frac{x}{2}\right)^{3/2}(a+\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}\,\sqrt{}}$$

$$= \frac{\left(2\sqrt{2}\,\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{a(c+}{}\right.\right.}{a}$$

$$= \frac{4\sqrt{2}\,F_1\left(\frac{5}{2}+m;-\frac{3}{2},-n;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e\right.}{}$$

Mathematica [A]
time = 1.03, size = 160, normalized size = 1.19

$$-\frac{4F_1\left(\frac{5}{2};-\frac{3}{2}-m,-n;\frac{7}{2};\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right),\frac{2d\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^3(e+fx)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)^{-\frac{3}{2}-m}}{5f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-4*AppellF1[5/2, -3/2 - m, -n, 7/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[
(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[e + f*x]^3*(a*(1 + Sin[e + f*x]))^m*(
c + d*Sin[e + f*x])^n*Sin[(2*e - Pi + 2*f*x)/4]^2*(Sin[(2*e + Pi + 2*f*x)/4
]^2)^(-3/2 - m))/(5*f*((c + d*Sin[e + f*x])/(c + d))^n)
```

Maple [**F**]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.946 $\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=121

$$-\frac{16\sqrt{2}\,a^2F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx$$
$$5f\sqrt{1+\sin(e+fx)}$$

```
[Out] -16/5*a^2*AppellF1(5/2,-n,-7/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e
))*cos(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/f/(((c+d*sin(f*x+
e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{16\sqrt{2}\,a^2(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-16*Sqrt[2]*a^2*AppellF1[5/2, -7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 -
 Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^n)/(5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{(a^2\cos(e+fx))\,\text{Subst}\left(\int(1-x)^{3/2}(1+x)^{7/2}(\ldots\right.}{f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin\ldots}}$$

$$= \frac{\left(a^2\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin\ldots}{-c\ldots}\right.\right.}{f\sqrt{\ldots}}$$

$$= -\frac{16\sqrt{2}\,a^2F_1\left(\frac{5}{2};-\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)\ldots\right.}{\ldots}$$

Mathematica [F]
time = 1.19, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.51, size = 0, normalized size = 0.00

$$\int \left(\cos^4(fx+e)\right)(a+a\sin(fx+e))^2(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^6 - 2*a^2*cos(f*x + e)^4*sin(f*x + e) - 2*a^2*c
os(f*x + e)^4)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^4 (a+a\sin(e+fx))^2 (c+d\sin(e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.947 $\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=119

$$-\frac{8\sqrt{2}\,aF_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos^3(e+fx)(1-\sin(e+fx))(c+d\sin(e+fx))}{5f(1+\sin(e+fx))^{3/2}}$$

```
[Out] -8/5*a*AppellF1(5/2,-n,-5/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*
cos(f*x+e)^3*(1-sin(f*x+e))*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+sin(f*x+e))^(3/
2)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2947, 144, 143}

$$-\frac{8\sqrt{2}\,a(1-\sin(e+fx))\cos^3(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5f(\sin(e+fx)+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (-8*Sqrt[2]*a*AppellF1[5/2, -5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Si
n[e + f*x]))/(c + d)]*Cos[e + f*x]^3*(1 - Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n)/(5*f*(1 + Sin[e + f*x])^(3/2)*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2947

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c*g*((g
*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(1 + (d/c)*x)^((p + 1)/2)*(1 - (d/c)*x)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{(a\cos^3(e+fx))\,\mathrm{Subst}\left(\int(1-x)^{3/2}(1+x)^{5/2}(c\right.}{f(1-\sin(e+fx))^{3/2}(1+\sin(}$$

$$= \frac{\left(a\cos^3(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e}{-c-}\right.\right.}{f(1-}$$

$$= -\frac{8\sqrt{2}\,aF_1\left(\frac{5}{2};-\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\right.}{}$$

Mathematica [F]
time = 0.53, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((a*cos(f*x + e)^4*sin(f*x + e) + a*cos(f*x + e)^4)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^4*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.948 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{2\sqrt{2}\,F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx))^n}{5af\sqrt{1+\sin(e+fx)}}$$

```
[Out] -2/5*AppellF1(5/2,-n,-1/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(((c+d*sin(f*x+e))
/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{2\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5af\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] (-2*Sqrt[2]*AppellF1[5/2, -1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[
e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^
n)/(5*a*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int(1-x)^{3/2}\sqrt{1+x}\,(c+dx)^n\,dx,x,\sin(e+fx\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int(1\right.}{af\sqrt{1-\sin(e+fx)}\,\sqrt{1}}$$

$$= -\frac{2\sqrt{2}\,F_1\left(\frac{5}{2};-\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e-}{5af\sqrt{1+\sin}}$$

Mathematica [F]
time = 26.76, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]), x]
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(fx+e)\right)\left(c+d\sin(fx+e)\right)^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(e+f\,x\right)^4\,{\left(c+d\,\sin \left(e+f\,x\right)\right)}^n}{a+a\,\sin \left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.949 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{2}\,F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx))^n\left(\frac{c}{}\right.}{5a^2f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/5*AppellF1(5/2,-n,1/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos
(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n*2^(1/2)/a^2/f/(((c+d*sin(f*x+e)
)/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2993, 2863, 144, 143}

$$-\frac{\sqrt{2}\,(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5a^2f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -1/5*(Sqrt[2]*AppellF1[5/2, 1/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin
[e + f*x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])
^n)/(a^2*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2863

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^m*(Cos[e + f*x]/(f*Sqrt[1 + Sin[e +
 f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 + (b/a)*x)^(m - 1/2)*((c + d*
x)^n/Sqrt[1 - (b/a)*x]), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
 IntegerQ[m]
```

Rule 2993

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^(2*m), I
nt[(c + d*Sin[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\int (a-a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx}{a^4}$$

$$= \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{\sqrt{1+x}}\,dx, x, \sin(e+fx)\right)}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(}{}\right.}{a^2 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(}$$

$$= -\frac{\sqrt{2}\,F_1\left(\frac{5}{2};\frac{1}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+f}{5a^2 f\sqrt{1+\sin(}$$

Mathematica [F]
time = 15.90, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2, x
]

Maple [F]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^n}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)

[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
maxima")

[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
fricas")

[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^2*cos(f*x + e)^2 - 2*a^2
*sin(f*x + e) - 2*a^2), x)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

3.950 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=121

$$-\frac{F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{5\sqrt{2}\,a^3f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/10*AppellF1(5/2,-n,3/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^3/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{5\sqrt{2}\,a^3f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -1/5*(AppellF1[5/2, 3/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(Sqr
t[2]*a^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{3/2}}\,dx, x, \sin(e+fx)\right)}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \right.}{a^3 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{3}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1}{5\sqrt{2}\,a^3 f\sqrt{1+\sin}}$$

Mathematica [F]
time = 20.20, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3, x
]
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(fx+e)\right)\left(c+d\sin(fx+e)\right)^n}{\left(a+a\sin(fx+e)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(3*a^3*cos(f*x + e)^2 - 4*a
^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^4\,(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

3.951 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=121

$$-\frac{F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx))^n\left(\frac{c+d\sin}{c}\right.}{10\sqrt{2}\,a^4f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/20*AppellF1(5/2,-n,5/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^4/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{10\sqrt{2}\,a^4f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -1/10*(AppellF1[5/2, 5/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*
x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(Sq
rt[2]*a^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{5/2}}\,dx, x, \sin(e+fx)\right)}{a^4 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \frac{(}{}\right.}{a^4 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{5}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1}{10\sqrt{2}\,a^4 f\sqrt{1+\sin}}$$

Mathematica [F]
time = 28.52, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4, x
]
```

Maple [F]
time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(fx+e)\right)(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a^4*cos(f*x + e)^4 - 8*a^4*
cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (e+f x)^4 (c+d \sin (e+f x))^n}{(a+a \sin (e+f x))^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4, x)
```

3.952 $\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$

Optimal. Leaf size=121

$$-\frac{F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1-\sin(e+fx))^2(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}}{20\sqrt{2}\,a^5f\sqrt{1+\sin(e+fx)}}$$

```
[Out] -1/40*AppellF1(5/2,-n,7/2,7/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*co
s(f*x+e)*(1-sin(f*x+e))^2*(c+d*sin(f*x+e))^n/a^5/f/(((c+d*sin(f*x+e))/(c+d)
)^n)*2^(1/2)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2996, 144, 143}

$$-\frac{(1-\sin(e+fx))^2\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{20\sqrt{2}\,a^5f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
```

```
[Out] -1/20*(AppellF1[5/2, 7/2, -n, 7/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*
x]))/(c + d)]*Cos[e + f*x]*(1 - Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n)/(Sq
rt[2]*a^5*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2996

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^m*(Cos[e
 + f*x]/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]])), Subst[Int[(1 +
(b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[
e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[p/2] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int \frac{(1-x)^{3/2}(c+dx)^n}{(1+x)^{7/2}}\,dx,x,\sin(e+fx)\right)}{a^5 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(e+fx)}}$$

$$= \frac{\left(\cos(e+fx)(c+d\sin(e+fx))^n\left(-\frac{c+d\sin(e+fx)}{-c-d}\right)^{-n}\right)\text{Subst}\left(\int \right.}{a^5 f\sqrt{1-\sin(e+fx)}\,\sqrt{1+\sin(}$$

$$= -\frac{F_1\left(\frac{5}{2};\frac{7}{2},-n;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)\cos(e+fx)(1}{20\sqrt{2}\,a^5 f\sqrt{1+\sin}}$$

Mathematica [F]
time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(e+fx)(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^5}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5,x]
[Out] Integrate[(Cos[e + f*x]^4*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^5, x
]
```

Maple [F]
time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(fx+e)\right)\left(c+d\sin(fx+e)\right)^n}{\left(a+a\sin(fx+e)\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

```
[Out] int(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(5*a^5*cos(f*x + e)^4 - 20*a
^5*cos(f*x + e)^2 + 16*a^5 + (a^5*cos(f*x + e)^4 - 12*a^5*cos(f*x + e)^2 +
16*a^5)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^5,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^4/(a*sin(f*x + e) + a)^5, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(e+f\,x)^4\,(c+d\,\sin(e+f\,x))^n}{(a+a\,\sin(e+f\,x))^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^5,x)
[Out] int((cos(e + f*x)^4*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^5, x)
```

3.953 $\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{8(A-B)(a+a\sin(c+dx))^5}{5a^4d}-\frac{2(3A-5B)(a+a\sin(c+dx))^6}{3a^5d}+\frac{6(A-3B)(a+a\sin(c+dx))^7}{7a^6d}-\frac{(A-7B)}{}$$

```
[Out] 8/5*(A-B)*(a+a*sin(d*x+c))^5/a^4/d-2/3*(3*A-5*B)*(a+a*sin(d*x+c))^6/a^5/d+6
/7*(A-3*B)*(a+a*sin(d*x+c))^7/a^6/d-1/8*(A-7*B)*(a+a*sin(d*x+c))^8/a^7/d-1/
9*B*(a+a*sin(d*x+c))^9/a^8/d
```

Rubi [A]

time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^9}{9a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^8}{8a^7d}+\frac{6(A-3B)(a\sin(c+dx)+a)^7}{7a^6d}-\frac{2(3A-5B)(a\sin(c+dx)+a)^6}{3a^5d}+\frac{8(A-B)(a\sin(c+dx)+a)^5}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^4*d) - (2*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^6)/(3*a^5*d) + (6*(A - 3*B)*(a + a*Sin[c + d*x])^7)/(7*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^8)/(8*a^7*d) - (B*(a + a*Sin[c + d*x])^9)/(9*a
^8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^4\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(\right.}{a^7d}$$

$$= \frac{\text{Subst}\left(\int\left(8a^3(A-B)(a+x)^4-4a^2(3A-5B\right.\right.}{}$$

$$= \frac{8(A-B)(a+a\sin(c+dx))^5}{5a^4d} - \frac{2(3A-5B)(a}{3}$$

Mathematica [A]

time = 0.63, size = 194, normalized size = 1.45

$$\frac{a(1+\sin(c+dx))(-17640(A+B)\cos(2(c+dx))-8820(A+B)\cos(4(c+dx))-2520A\cos(6(c+dx))-2520B\cos(6(c+dx))-315A\cos(8(c+dx))-315B\cos(8(c+dx))+176400A\sin(c+dx)+17640B\sin(c+dx)+35280A\sin(3(c+dx))+7056A\sin(5(c+dx))-2016B\sin(5(c+dx))+720A\sin(7(c+dx))-900B\sin(7(c+dx))-140B\sin(9(c+dx)))}{322560d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(1 + Sin[c + d*x])*(-17640*(A + B)*Cos[2*(c + d*x)] - 8820*(A + B)*Cos[4
*(c + d*x)] - 2520*A*Cos[6*(c + d*x)] - 2520*B*Cos[6*(c + d*x)] - 315*A*Cos
[8*(c + d*x)] - 315*B*Cos[8*(c + d*x)] + 176400*A*Sin[c + d*x] + 17640*B*Si
n[c + d*x] + 35280*A*Sin[3*(c + d*x)] + 7056*A*Sin[5*(c + d*x)] - 2016*B*Si
n[5*(c + d*x)] + 720*A*Sin[7*(c + d*x)] - 900*B*Sin[7*(c + d*x)] - 140*B*Si
n[9*(c + d*x)]))/(322560*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)
```

Maple [A]

time = 0.48, size = 128, normalized size = 0.96

method	result
derivativedivides	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{aA\left(\cos^8(dx+c)\right)}{8}-\frac{B\left(\cos^8(\right.}{}}{d}$
default	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{aA\left(\cos^8(dx+c)\right)}{8}-\frac{B\left(\cos^8(\right.}{}}{d}$
risch	$\frac{35aA\sin(dx+c)}{64d}+\frac{7aB\sin(dx+c)}{128d}-\frac{aB\sin(9dx+9c)}{2304d}-\frac{a\cos(8dx+8c)A}{1024d}-\frac{a\cos(8dx+8c)B}{1024d}+\frac{\sin(7dx+7c)aA}{448d}$
norman	$\frac{(2aA+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14(aA+aB)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14(aA+aB)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{14(aA}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*a*A*cos(d*x+c)^8-1/8*B*cos(d*x+c)^8*
a+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.34, size = 134, normalized size = 1.00

$$-\frac{280\,Ba\sin\left(dx+c\right)^{9}+315\,(A+B)a\sin\left(dx+c\right)^{8}+360\,(A-3\,B)a\sin\left(dx+c\right)^{7}-1260\,(A+B)a\sin\left(dx+c\right)^{6}-1512\,(A-B)a\sin\left(dx+c\right)^{5}+1890\,(A+B)a\sin\left(dx+c\right)^{4}+840\,(3\,A-B)a\sin\left(dx+c\right)^{3}-1260\,(A+B)a\sin\left(dx+c\right)^{2}-2520\,Aa\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*a*sin(d*x + c)^9 + 315*(A + B)*a*sin(d*x + c)^8 + 360*(A - 3
*B)*a*sin(d*x + c)^7 - 1260*(A + B)*a*sin(d*x + c)^6 - 1512*(A - B)*a*sin(d
*x + c)^5 + 1890*(A + B)*a*sin(d*x + c)^4 + 840*(3*A - B)*a*sin(d*x + c)^3
- 1260*(A + B)*a*sin(d*x + c)^2 - 2520*A*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.72

$$-\frac{315\,(A+B)a\cos\left(dx+c\right)^{8}+8\,\left(35\,Ba\cos\left(dx+c\right)^{8}-5\,(9\,A+B)a\cos\left(dx+c\right)^{6}-6\,(9\,A+B)a\cos\left(dx+c\right)^{4}-8\,(9\,A+B)a\cos\left(dx+c\right)^{2}-16\,(9\,A+B)a\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(A + B)*a*cos(d*x + c)^8 + 8*(35*B*a*cos(d*x + c)^8 - 5*(9*A +
 B)*a*cos(d*x + c)^6 - 6*(9*A + B)*a*cos(d*x + c)^4 - 8*(9*A + B)*a*cos(d*x
 + c)^2 - 16*(9*A + B)*a)*sin(d*x + c))/d
```

Sympy [A]
time = 1.41, size = 228, normalized size = 1.70

$$\begin{cases} \frac{16Aa\sin^{7}{\left(c+dx \right)}}{35d} + \frac{8Aa\sin^{5}{\left(c+dx \right)}\cos^{2}{\left(c+dx \right)}}{5d} + \frac{2Aa\sin^{3}{\left(c+dx \right)}\cos^{4}{\left(c+dx \right)}}{d} + \frac{Aa\sin{\left(c+dx \right)}\cos^{6}{\left(c+dx \right)}}{d} - \frac{Aa\cos^{8}{\left(c+dx \right)}}{8d} + \frac{16Ba\sin^{9}{\left(c+dx \right)}}{315d} + \frac{8Ba\sin^{7}{\left(c+dx \right)}\cos^{2}{\left(c+dx \right)}}{35d} + \frac{2Ba\sin^{5}{\left(c+dx \right)}\cos^{4}{\left(c+dx \right)}}{5d} + \frac{Ba\sin^{3}{\left(c+dx \right)}\cos^{6}{\left(c+dx \right)}}{3d} - \frac{Ba\cos^{8}{\left(c+dx \right)}}{8d} & \text{for}\: d \neq 0 \\ x\left(A + B\sin{\left(c \right)}\right)\left(a\sin{\left(c \right)} + a\right)\cos^{7}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*a*cos(c + d*x)**8/(8*d) + 16*B*a*sin(c + d*x)**9/(315*d
) + 8*B*a*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*B*a*sin(c + d*x)**5*co
s(c + d*x)**4/(5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) - B*a*cos(c
 + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**7, Tr
ue))
```

Giac [A]
time = 0.51, size = 182, normalized size = 1.36

$$-\frac{Ba\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{7\,Aa\sin\left(3\,dx+3\,c\right)}{64\,d}-\frac{\left(Aa+Ba\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(Aa+Ba\right)\cos\left(6\,dx+6\,c\right)}{128\,d}-\frac{7\,\left(Aa+Ba\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,\left(Aa+Ba\right)\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{\left(4\,Aa-5\,Ba\right)\sin\left(7\,dx+7\,c\right)}{1792\,d}+\frac{\left(7\,Aa-2\,Ba\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{7\,\left(10\,Aa+Ba\right)\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*a*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(A*a
+ B*a)*cos(8*d*x + 8*c)/d - 1/128*(A*a + B*a)*cos(6*d*x + 6*c)/d - 7/256*(A
*a + B*a)*cos(4*d*x + 4*c)/d - 7/128*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*a)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*a)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*a)*sin(d*x + c)/d
```

Mupad [B]
time = 0.12, size = 134, normalized size = 1.00

$$-\frac{\frac{B\,a\,{\sin\left(c+d\,x\right)}^9}{9}+\frac{a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^8}{8}+\frac{a\,\left(A-3\,B\right)\,{\sin\left(c+d\,x\right)}^7}{7}-\frac{a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^6}{2}-\frac{3\,a\,\left(A-B\right)\,{\sin\left(c+d\,x\right)}^5}{5}+\frac{3\,a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^4}{4}+\frac{a\,\left(3\,A-B\right)\,{\sin\left(c+d\,x\right)}^3}{3}-\frac{a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^2}{2}-A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] -((a*sin(c + d*x)^3*(3*A - B))/3 - A*a*sin(c + d*x) - (a*sin(c + d*x)^2*(A
+ B))/2 + (3*a*sin(c + d*x)^4*(A + B))/4 - (a*sin(c + d*x)^6*(A + B))/2 + (
a*sin(c + d*x)^8*(A + B))/8 - (3*a*sin(c + d*x)^5*(A - B))/5 + (a*sin(c + d
*x)^7*(A - 3*B))/7 + (B*a*sin(c + d*x)^9)/9)/d
```

3.954 $\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{(A-B)(a+a\sin(c+dx))^4}{a^3d}-\frac{4(A-2B)(a+a\sin(c+dx))^5}{5a^4d}+\frac{(A-5B)(a+a\sin(c+dx))^6}{6a^5d}+\frac{B(a+a\sin(c+dx))^7}{7a^6d}$$

```
[Out] (A-B)*(a+a*sin(d*x+c))^4/a^3/d-4/5*(A-2*B)*(a+a*sin(d*x+c))^5/a^4/d+1/6*(A-
5*B)*(a+a*sin(d*x+c))^6/a^5/d+1/7*B*(a+a*sin(d*x+c))^7/a^6/d
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 78}

$$\frac{B(a\sin(c+dx)+a)^7}{7a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^6}{6a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^5}{5a^4d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(a^3*d) - (4*(A - 2*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^6)/(6*a^5*d) + (B*(a + a*S
in[c + d*x])^7)/(7*a^6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^3\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(\right.}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^3 - 4a(A-2B)(\right.\right.}{}$$

$$= \frac{(A-B)(a+a\sin(c+dx))^4}{a^3 d} - \frac{4(A-2B)(a+}{5a}$$

Mathematica [A]
time = 0.47, size = 130, normalized size = 1.27

$$-\frac{a(525(A+B)\cos(2(c+dx))+210(A+B)\cos(4(c+dx))+35A\cos(6(c+dx))+35B\cos(6(c+dx))-4200A\sin(c+dx)-525B\sin(c+dx)-700A\sin(3(c+dx))+35B\sin(3(c+dx))-84A\sin(5(c+dx))+63B\sin(5(c+dx))+15B\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(a*(525*(A + B)*Cos[2*(c + d*x)] + 210*(A + B)*Cos[4*(c + d*x)] + 3
5*A*Cos[6*(c + d*x)] + 35*B*Cos[6*(c + d*x)] - 4200*A*Sin[c + d*x] - 525*B*
Sin[c + d*x] - 700*A*Sin[3*(c + d*x)] + 35*B*Sin[3*(c + d*x)] - 84*A*Sin[5*
(c + d*x)] + 63*B*Sin[5*(c + d*x)] + 15*B*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.32, size = 108, normalized size = 1.06

method	result
derivativedivides	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{aA\left(\cos^6(dx+c)\right)}{6}-\frac{B\left(\cos^6(dx+c)\right)a}{6}+\frac{aA\left(\frac{8}{3}\right.}{}}{d}$
default	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{aA\left(\cos^6(dx+c)\right)}{6}-\frac{B\left(\cos^6(dx+c)\right)a}{6}+\frac{aA\left(\frac{8}{3}\right.}{}}{d}$
risch	$\frac{5aA\sin(dx+c)}{8d}+\frac{5aB\sin(dx+c)}{64d}-\frac{\sin(7dx+7c)aB}{448d}-\frac{a\cos(6dx+6c)A}{192d}-\frac{a\cos(6dx+6c)B}{192d}+\frac{\sin(5dx+5c)aA}{80d}-$
norman	$\frac{(2aA+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5(4aA}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*a*A*cos(d*x+c)^6-1/6*B*cos(d*x+c)^6*a+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.39, size = 104, normalized size = 1.02

$$\frac{30 Ba\sin(dx+c)^7+35(A+B)a\sin(dx+c)^6+42(A-2B)a\sin(dx+c)^5-105(A+B)a\sin(dx+c)^4-70(2A-B)a\sin(dx+c)^3+105(A+B)a\sin(dx+c)^2+210 Aa\sin(dx+c)}{210 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*a*sin(d*x + c)^7 + 35*(A + B)*a*sin(d*x + c)^6 + 42*(A - 2*B)*a
*sin(d*x + c)^5 - 105*(A + B)*a*sin(d*x + c)^4 - 70*(2*A - B)*a*sin(d*x + c
)^3 + 105*(A + B)*a*sin(d*x + c)^2 + 210*A*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.79

$$-\frac{35(A+B)a\cos(dx+c)^6+2\left(15 Ba\cos(dx+c)^6-3(7A+B)a\cos(dx+c)^4-4(7A+B)a\cos(dx+c)^2-8(7A+B)a\right)\sin(dx+c)}{210 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(A + B)*a*cos(d*x + c)^6 + 2*(15*B*a*cos(d*x + c)^6 - 3*(7*A + B
)*a*cos(d*x + c)^4 - 4*(7*A + B)*a*cos(d*x + c)^2 - 8*(7*A + B)*a)*sin(d*x
+ c))/d
```

Sympy [A]
time = 0.68, size = 178, normalized size = 1.75

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d}+\frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^4(c+dx)}{d}-\frac{Aa\cos^6(c+dx)}{6d}+\frac{8Ba\sin^7(c+dx)}{105d}+\frac{4Ba\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{Ba\sin^3(c+dx)\cos^4(c+dx)}{3d}-\frac{Ba\cos^6(c+dx)}{6d} & \text{for } d\neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*a*cos(c + d*x)**6/(6*d)
 + 8*B*a*sin(c + d*x)**7/(105*d) + 4*B*a*sin(c + d*x)**5*cos(c + d*x)**2/(1
5*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*cos(c + d*x)**6/(6*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**5, True))
```

Giac [A]
time = 0.50, size = 145, normalized size = 1.42

$$-\frac{Ba\sin(7dx+7c)}{448d}-\frac{(Aa+Ba)\cos(6dx+6c)}{192d}-\frac{(Aa+Ba)\cos(4dx+4c)}{32d}-\frac{5(Aa+Ba)\cos(2dx+2c)}{64d}+\frac{(4Aa-3Ba)\sin(5dx+5c)}{320d}+\frac{(20Aa-Ba)\sin(3dx+3c)}{192d}+\frac{5(8Aa+Ba)\sin(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*a*sin(7*d*x + 7*c)/d - 1/192*(A*a + B*a)*cos(6*d*x + 6*c)/d - 1/32
*(A*a + B*a)*cos(4*d*x + 4*c)/d - 5/64*(A*a + B*a)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*a)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*a)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*a)*sin(d*x + c)/d
```

Mupad [B]
time = 0.08, size = 102, normalized size = 1.00

$$\frac{\frac{B\,a\,\sin(c+d\,x)^7}{7}+\frac{a\,(A+B)\,\sin(c+d\,x)^6}{6}+\frac{a\,(A-2\,B)\,\sin(c+d\,x)^5}{5}-\frac{a\,(A+B)\,\sin(c+d\,x)^4}{2}-\frac{a\,(2\,A-B)\,\sin(c+d\,x)^3}{3}+\frac{a\,(A+B)\,\sin(c+d\,x)^2}{2}+A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x) - (a*sin(c + d*x)^3*(2*A - B))/3 + (a*sin(c + d*x)^2*(A +
 B))/2 - (a*sin(c + d*x)^4*(A + B))/2 + (a*sin(c + d*x)^6*(A + B))/6 + (a*s
in(c + d*x)^5*(A - 2*B))/5 + (B*a*sin(c + d*x)^7)/7)/d
```

3.955 $\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{2(A-B)(a+a\sin(c+dx))^3}{3a^2d}-\frac{(A-3B)(a+a\sin(c+dx))^4}{4a^3d}-\frac{B(a+a\sin(c+dx))^5}{5a^4d}$$

```
[Out] 2/3*(A-B)*(a+a*sin(d*x+c))^3/a^2/d-1/4*(A-3*B)*(a+a*sin(d*x+c))^4/a^3/d-1/5
*B*(a+a*sin(d*x+c))^5/a^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^5}{5a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^4}{4a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^3)/(3*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^4)/(4*a^3*d) - (B*(a + a*Sin[c + d*x])^5)/(5*a^4*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^2\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(\right.}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^2+(-A+3B)(a\right.\right.}{a^3d}$$
$$= \frac{2(A-B)(a+a\sin(c+dx))^3}{3a^2d} - \frac{(A-3B)(a+}{4}$$

Mathematica [A]
time = 0.59, size = 78, normalized size = 1.00

$$-\frac{a(-4(100A+11B)\sin(c+dx)+3\cos(4(c+dx))(5(A+B)+4B\sin(c+dx))+\cos(2(c+dx))(60(A+B)+(-80A+32B)\sin(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a*(-4*(100*A + 11*B)*Sin[c + d*x] + 3*Cos[4*(c + d*x)]*(5*(A + B) +
 4*B*Sin[c + d*x]) + Cos[2*(c + d*x)]*(60*(A + B) + (-80*A + 32*B)*Sin[c +
d*x])))/d
```

Maple [A]
time = 0.22, size = 88, normalized size = 1.13

method	result
derivativedivides	$\frac{aB\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{aA\left(\cos^4(dx+c)\right)}{4}-\frac{B\left(\cos^4(dx+c)\right)a}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$
default	$\frac{aB\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{aA\left(\cos^4(dx+c)\right)}{4}-\frac{B\left(\cos^4(dx+c)\right)a}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$
risch	$\frac{3aA\sin(dx+c)}{4d}+\frac{aB\sin(dx+c)}{8d}-\frac{\sin(5dx+5c)aB}{80d}-\frac{a\cos(4dx+4c)A}{32d}-\frac{a\cos(4dx+4c)B}{32d}+\frac{aA\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{(2aA+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(aA+aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(aA+aB)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2aA\tan(}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*a*A*cos(d*x+c)^4-1/4*B*cos(d*x+c)^4*a+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A]
time = 0.37, size = 72, normalized size = 0.92

$$-\frac{12\,Ba\sin\left(dx+c\right)^{5}+15\,(A+B)a\sin\left(dx+c\right)^{4}+20\,(A-B)a\sin\left(dx+c\right)^{3}-30\,(A+B)a\sin\left(dx+c\right)^{2}-60\,Aa\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*(A + B)*a*sin(d*x + c)^4 + 20*(A - B)*a*s
in(d*x + c)^3 - 30*(A + B)*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 65, normalized size = 0.83

$$-\frac{15\,(A+B)a\cos\left(dx+c\right)^{4}+4\,\left(3\,Ba\cos\left(dx+c\right)^{4}-(5\,A+B)a\cos\left(dx+c\right)^{2}-2\,(5\,A+B)a\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(A + B)*a*cos(d*x + c)^4 + 4*(3*B*a*cos(d*x + c)^4 - (5*A + B)*a*
cos(d*x + c)^2 - 2*(5*A + B)*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.30, size = 128, normalized size = 1.64

$$\begin{cases} \frac{2Aa\sin^{3}(c+dx)}{3d}+\frac{Aa\sin(c+dx)\cos^{2}(c+dx)}{d}-\frac{Aa\cos^{4}(c+dx)}{4d}+\frac{2Ba\sin^{5}(c+dx)}{15d}+\frac{Ba\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}-\frac{Ba\cos^{4}(c+dx)}{4d} & \text{for } d\neq 0 \\ x(A+B\sin(c))\left(a\sin(c)+a\right)\cos^{3}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*a*cos(c + d*x)**4/(4*d) + 2*B*a*sin(c + d*x)**5/(15*d) + B*a*sin(c + d
*x)**3*cos(c + d*x)**2/(3*d) - B*a*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a*sin(c) + a)*cos(c)**3, True))
```

Giac [A]
time = 0.45, size = 100, normalized size = 1.28

$$-\frac{12\,Ba\sin\left(dx+c\right)^{5}+15\,Aa\sin\left(dx+c\right)^{4}+15\,Ba\sin\left(dx+c\right)^{4}+20\,Aa\sin\left(dx+c\right)^{3}-20\,Ba\sin\left(dx+c\right)^{3}-30\,Aa\sin\left(dx+c\right)^{2}-30\,Ba\sin\left(dx+c\right)^{2}-60\,Aa\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*a*sin(d*x + c)^5 + 15*A*a*sin(d*x + c)^4 + 15*B*a*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*a*sin(d*x + c)^3 - 30*A*a*sin(d*x + c)^2 -
 30*B*a*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

Mupad [B]
time = 9.01, size = 72, normalized size = 0.92

$$-\frac{\frac{B\,a\,{\sin\left(c+d\,x\right)}^5}{5}+\frac{a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^4}{4}+\frac{a\,\left(A-B\right)\,{\sin\left(c+d\,x\right)}^3}{3}-\frac{a\,\left(A+B\right)\,{\sin\left(c+d\,x\right)}^2}{2}-A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] -((a*sin(c + d*x)^4*(A + B))/4 - (a*sin(c + d*x)^2*(A + B))/2 - A*a*sin(c +
 d*x) + (a*sin(c + d*x)^3*(A - B))/3 + (B*a*sin(c + d*x)^5)/5)/d
```

3.956 $\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=49

$$\frac{aA\sin(c+dx)}{d}+\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aB\sin^3(c+dx)}{3d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*a*(A+B)*sin(d*x+c)^2/d+1/3*a*B*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2912, 45}

$$\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{aB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + (a*(A + B)*Sin[c + d*x]^2)/(2*d) + (a*B*Sin[c + d*x]
^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(aA+(A+B)x+\frac{Bx^2}{a}\right)\,dx,x,a\sin(c+\right.}{ad}\\ &= \frac{aA\sin(c+dx)}{d}+\frac{a(A+B)\sin^2(c+dx)}{2d}+\frac{aB\sin}{}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 46, normalized size = 0.94

$$-\frac{a(-2(6A+B)\sin(c+dx)+\cos(2(c+dx))(3(A+B)+2B\sin(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/12*(a*(-2*(6*A + B)*Sin[c + d*x] + Cos[2*(c + d*x)]*(3*(A + B) + 2*B*Sin
[c + d*x])))/d
```

Maple [A]
time = 0.11, size = 44, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{aB\left(\sin^3(dx+c)\right)}{3}+\frac{(aA+aB)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)a}{d}$
default	$\frac{\frac{aB\left(\sin^3(dx+c)\right)}{3}+\frac{(aA+aB)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)a}{d}$
risch	$\frac{aA\sin(dx+c)}{d}+\frac{aB\sin(dx+c)}{4d}-\frac{\sin(3dx+3c)aB}{12d}-\frac{a\cos(2dx+2c)A}{4d}-\frac{a\cos(2dx+2c)B}{4d}$
norman	$\frac{\frac{(2aA+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2aA+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2aA\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2aA\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a*B*sin(d*x+c)^3+1/2*(A*a+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

Maxima [A]
time = 0.30, size = 42, normalized size = 0.86

$$\frac{2\,Ba\sin{(dx+c)}^3+3\,(A+B)a\sin{(dx+c)}^2+6\,Aa\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*(A + B)*a*sin(d*x + c)^2 + 6*A*a*sin(d*x + c)
)/d
```

Fricas [A]
time = 0.37, size = 48, normalized size = 0.98

$$-\frac{3\,(A+B)a\cos{(dx+c)}^2+2\,\left(Ba\cos{(dx+c)}^2-(3\,A+B)a\right)\sin{(dx+c)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/6*(3*(A + B)*a*cos(d*x + c)^2 + 2*(B*a*cos(d*x + c)^2 - (3*A + B)*a)*sin
(d*x + c))/d
```

Sympy [A]
time = 0.14, size = 75, normalized size = 1.53

$$\begin{cases} \frac{Aa\sin^2(c+dx)}{2d} + \frac{Aa\sin(c+dx)}{d} + \frac{Ba\sin^3(c+dx)}{3d} + \frac{Ba\sin^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)**2/(2*d) + A*a*sin(c + d*x)/d + B*a*sin(c + d*x
)**3/(3*d) + B*a*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin
(c) + a)*cos(c), True))
```

Giac [A]
time = 0.48, size = 52, normalized size = 1.06

$$\frac{2\,Ba\sin(dx+c)^3 + 3\,Aa\sin(dx+c)^2 + 3\,Ba\sin(dx+c)^2 + 6\,Aa\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*a*sin(d*x + c)^3 + 3*A*a*sin(d*x + c)^2 + 3*B*a*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

Mupad [B]
time = 0.07, size = 40, normalized size = 0.82

$$\frac{\frac{B\,a\sin(c+d\,x)^3}{3} + \frac{a\,(A+B)\sin(c+d\,x)^2}{2} + A\,a\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x) + (a*sin(c + d*x)^2*(A + B))/2 + (B*a*sin(c + d*x)^3)/3)/
d
```

3.957 $\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=34

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

[Out] -a*(A+B)*ln(1-sin(d*x+c))/d-a*B*sin(d*x+c)/d

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {2915, 45}

$$-\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]

[Out] -((a*(A + B)*Log[1 - Sin[c + d*x]])/d) - (a*B*Sin[c + d*x])/d

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{a\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a-x}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a\text{Subst}\left(\int \left(-\frac{B}{a}+\frac{A+B}{a-x}\right)\,dx,x,a\sin(c+dx)\right)}{d}\\ &= -\frac{a(A+B)\log(1-\sin(c+dx))}{d}-\frac{aB\sin(c+d}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 2.00

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}-\frac{aA\log(\cos(c+dx))}{d}-\frac{aB\log(\cos(c+dx))}{d}-\frac{aB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d - (a*A*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (a*B*Sin[c + d*x])/d
```

Maple [A]
time = 0.16, size = 29, normalized size = 0.85

method	result
derivativedivides	$-\frac{a(B\sin(dx+c)+(A+B)\ln(\sin(dx+c)-1))}{d}$
default	$-\frac{a(B\sin(dx+c)+(A+B)\ln(\sin(dx+c)-1))}{d}$
norman	$\frac{-\frac{2aB\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2aB\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(A+B)a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(A+B)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
risch	$iaxA+iaxB+\frac{iaB\,e^{i(dx+c)}}{2d}-\frac{iaB\,e^{-i(dx+c)}}{2d}+\frac{2iaAc}{d}+\frac{2iaBc}{d}-\frac{2a\ln\left(e^{i(dx+c)}-i\right)A}{d}-\frac{2a\ln\left(e^{i(dx+c)}-i\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(B*sin(d*x+c)+(A+B)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 29, normalized size = 0.85

$$-\frac{(A+B)a\log\left(\sin\left(dx+c\right)-1\right)+Ba\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -((A + B)*a*log(sin(d*x + c) - 1) + B*a*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.91

$$-\frac{(A+B)a\log\left(-\sin\left(dx+c\right)+1\right)+Ba\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -((A + B)*a*log(-sin(d*x + c) + 1) + B*a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sin(c + d*x)*sec(c + d*x), x) +
 Integral(B*sin(c + d*x)*sec(c + d*x), x) + Integral(B*sin(c + d*x)**2*sec(
c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(34) = 68.
time = 0.44, size = 114, normalized size = 3.35

$$\frac{(Aa+Ba)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)-2\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa+Ba}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((A*a + B*a)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a + B*a)*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - (A*a*tan(1/2*d*x + 1/2*c)^2 + B*a*tan(1/2*d*x + 1/2*
c)^2 + 2*B*a*tan(1/2*d*x + 1/2*c) + A*a + B*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)
)/d
```

Mupad [B]
time = 0.07, size = 35, normalized size = 1.03

$$-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(A\,a+B\,a\right)}{d}-\frac{B\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x),x)
```

```
[Out] - (log(sin(c + d*x) - 1)*(A*a + B*a))/d - (B*a*sin(c + d*x))/d
```

3.958 $\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}$$

```
[Out] 1/2*a*(A-B)*arctanh(sin(d*x+c))/d+1/2*a^2*(A+B)/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 78, 212}

$$\frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(A - B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(A + B))/(2*d*(a - a*Sin[c +
 d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{a^3\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{a^3\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^2}+\frac{A-B}{2a(a^2-x^2)}\right)\,dx, x, a\sin(c\right.}{d}\\ &= \frac{a^2(A+B)}{2d(a-a\sin(c+dx))}+\frac{(a^2(A-B))\,\text{Subst}\left(\int \frac{}{a}\right.}{}\\ &= \frac{a(A-B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(A+}{2d(a-a\sin(c}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.47, size = 260, normalized size = 5.53

$$\frac{a\left(2A+2B+iAdx-iBdx-2A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+A\log\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)-B\log\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)+2i(A-B)\tan^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)(-1+\sin(c+dx))+(A-B)\left(-idx+2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)\right)\sin(c+dx)\right)}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(2*A + 2*B + I*A*d*x - I*B*d*x - 2*A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^2] - B*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2]
 + (2*I)*(A - B)*ArcTan[Tan[(c + d*x)/2]]*(-1 + Sin[c + d*x]) + (A - B)*((-
I)*d*x + 2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2])*Sin[c + d*x]))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.
time = 0.25, size = 110, normalized size = 2.34

method	result
derivativedivides	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{aB}{2\cos(dx+c)^2}+\frac{aA}{2\cos(dx+c)^2}+aB\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx\right.}{d}$
default	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{aB}{2\cos(dx+c)^2}+\frac{aA}{2\cos(dx+c)^2}+aB\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx\right.}{d}$
risch	$-\frac{ia\,e^{i(dx+c)}(A+B)}{d\left(e^{i(dx+c)}-i\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)A}{2d}-\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)A}{2d}+\frac{a\ln\left(e^{i(dx+c)}-i\right)B}{2d}$

norman	$\frac{\frac{(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(A+B)a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(2aA+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(A+B)a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3(A+B)a\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a*B/
cos(d*x+c)^2+1/2*a*A/cos(d*x+c)^2+a*B*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*si
n(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 55, normalized size = 1.17

$$\frac{(A-B)a\log\left(\sin\left(dx+c\right)+1\right)-(A-B)a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,(A+B)a}{\sin(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A - B)*a*log(sin(d*x + c) + 1) - (A - B)*a*log(sin(d*x + c) - 1) - 2*
(A + B)*a/(sin(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.
time = 0.36, size = 90, normalized size = 1.91

$$-\frac{2\,(A+B)a-((A-B)a\sin\left(dx+c\right)-(A-B)a)\log\left(\sin\left(dx+c\right)+1\right)+((A-B)a\sin\left(dx+c\right)-(A-B)a)\log\left(-\sin\left(dx+c\right)+1\right)}{4\,(d\sin\left(dx+c\right)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(2*(A + B)*a - ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(sin(d*x + c) +
 1) + ((A - B)*a*sin(d*x + c) - (A - B)*a)*log(-sin(d*x + c) + 1))/(d*sin(d
*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^3\left(c+dx\right)dx+\int A\sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int B\sin^2\left(c+dx\right)\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**3
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**3, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**3, x))
```

Giac [A]
time = 0.46, size = 84, normalized size = 1.79

$$\frac{(Aa - Ba)\log(|\sin(dx + c) + 1|) - (Aa - Ba)\log(|\sin(dx + c) - 1|) + \frac{Aa\sin(dx+c) - Ba\sin(dx+c) - 3\,Aa - Ba}{\sin(dx+c) - 1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*a)*log(abs(sin(d*x + c) + 1)) - (A*a - B*a)*log(abs(sin(d*x +
 c) - 1)) + (A*a*sin(d*x + c) - B*a*sin(d*x + c) - 3*A*a - B*a)/(sin(d*x +
c) - 1))/d
```

Mupad [B]
time = 9.12, size = 43, normalized size = 0.91

$$\frac{a\,\mathrm{atanh}(\sin(c + d\,x))\,(A - B)}{2\,d} - \frac{\frac{A\,a}{2} + \frac{B\,a}{2}}{d\,(\sin(c + d\,x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(A - B))/(2*d) - ((A*a)/2 + (B*a)/2)/(d*(sin(c + d*x
) - 1))
```

3.959 $\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=100

$$\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}+\frac{a^2A}{4d(a-a\sin(c+dx))}-\frac{a^2(A-B)}{8d(a+a\sin(c+dx))}$$

```
[Out] 1/8*a*(3*A-B)*arctanh(sin(d*x+c))/d+1/8*a^3*(A+B)/d/(a-a*sin(d*x+c))^2+1/4*
a^2*A/d/(a-a*sin(d*x+c))-1/8*a^2*(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 78, 212}

$$\frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{8d(a\sin(c+dx)+a)}+\frac{a^2A}{4d(a-a\sin(c+dx))}+\frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(3*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(A + B))/(8*d*(a - a*Sin[c
 + d*x])^2) + (a^2*A)/(4*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(8*d*(a +
a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= \frac{a^5\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^5\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^3}+\frac{A}{4a^3(a-x)^2}+\frac{A-B}{8a^3(a+x)^2}+\right.\right.}{d}\\ &= \frac{a^3(A+B)}{8d(a-a\sin(c+dx))^2}+\frac{a^2A}{4d(a-a\sin(c+dx))}\\ &= \frac{a(3A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(A+}{8d(a-a\sin}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.13, size = 357, normalized size = 3.57

$$\frac{a\left(\frac{2(-A+B)}{d}+i(3A-B)x\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2-\frac{2i(3A-B)\tan^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d}-\frac{2(3A-B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d}+\frac{(3A-B)\log\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d}+\frac{2(A+B)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{4A\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}\right)(1+\sin(c+dx))}{16\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((2*(-A + B))/d + I*(3*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2
- ((2*I)*(3*A - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2)/d - (2*(3*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2)/d + ((3*A - B)*Log[(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/d + (2*(A + B)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*
x)/2])^4) + (4*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(d*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(16*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^4)
```

Maple [A]
time = 0.28, size = 141, normalized size = 1.41

method	result
derivativedivides	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{aB}{4\cos(dx+c)^4}+\frac{aA}{4\cos(dx+c)^4}+aB\left(\frac{\sin^3(}{4\cos(}\right.}{d}$

default	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{aB}{4\cos(dx+c)^4}+\frac{aA}{4\cos(dx+c)^4}+aB\left(\frac{\sin^3(dx-}{4\cos(dx-}\right.}{d}$
risch	$-\frac{ia\left(-6iA\,e^{4i(dx+c)}+3A\,e^{5i(dx+c)}+2iB\,e^{4i(dx+c)}-B\,e^{5i(dx+c)}+6iA\,e^{2i(dx+c)}+2A\,e^{3i(dx+c)}-2iB\,e^{2i(dx+c)}+10B\,e^{3i(dx}\right.}{4\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx+c)}-i\right)^4d}$
norman	$\frac{\frac{(4aA+4aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(4aA+4aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(aA+aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(aA+aB)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(aA+aB)(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+1/4*a*B/cos(d*x+c)^4+1/4*a*A/cos(d*x+c)^4+a*B*(1/4*sin(d*x+c)^
3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+
c)+tan(d*x+c))))
```

Maxima [A]

time = 0.39, size = 115, normalized size = 1.15

$$\frac{(3A-B)a\log\left(\sin\left(dx+c\right)+1\right)-(3A-B)a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((3A-B)a\sin(dx+c)^2-(3A-B)a\sin(dx+c)-2\,(A+B)a\right)}{\sin(dx+c)^3-\sin(dx+c)^2-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A - B)*a*log(sin(d*x + c) + 1) - (3*A - B)*a*log(sin(d*x + c) - 1)
 - 2*((3*A - B)*a*sin(d*x + c)^2 - (3*A - B)*a*sin(d*x + c) - 2*(A + B)*a)/
(sin(d*x + c)^3 - sin(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A]

time = 0.36, size = 182, normalized size = 1.82

$$-\frac{2\,(3\,A-B)a\cos\left(dx+c\right)^2+2\,(3\,A-B)a\sin\left(dx+c\right)-2\,(A-3\,B)a-\left((3\,A-B)a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-(3\,A-B)a\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((3\,A-B)a\cos\left(dx+c\right)^2\sin\left(dx+c\right)-(3\,A-B)a\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{16\,\left(d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A - B)*a*cos(d*x + c)^2 + 2*(3*A - B)*a*sin(d*x + c) - 2*(A - 3
*B)*a - ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A - B)*a*cos(d*x + c)
^2)*log(sin(d*x + c) + 1) + ((3*A - B)*a*cos(d*x + c)^2*sin(d*x + c) - (3*A
 - B)*a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1))/(d*cos(d*x + c)^2*sin(d*x +
 c) - d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^5(c+dx)\,dx+\int A\sin(c+dx)\sec^5(c+dx)\,dx+\int B\sin(c+dx)\sec^5(c+dx)\,dx+\int B\sin^2(c+dx)\sec^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**5, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**5
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**5, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**5, x))
```

Giac [A]
time = 0.50, size = 152, normalized size = 1.52

$$\frac{2\,(3\,Aa-Ba)\log\left(|\sin(dx+c)+1|\right)-2\,(3\,Aa-Ba)\log\left(|\sin(dx+c)-1|\right)-\frac{2\,(3\,Aa\sin(dx+c)-Ba\sin(dx+c)+5\,Aa-3\,Ba)}{\sin(dx+c)+1}+\frac{9\,Aa\sin(dx+c)^2-3\,Ba\sin(dx+c)^2-26\,Aa\sin(dx+c)+6\,Ba\sin(dx+c)+21\,Aa+Ba}{(\sin(dx+c)-1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 2*(3*A*a - B*a)*log(abs(
sin(d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c) - B*a*sin(d*x + c) + 5*A*a - 3*B
*a)/(sin(d*x + c) + 1) + (9*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 - 26*
A*a*sin(d*x + c) + 6*B*a*sin(d*x + c) + 21*A*a + B*a)/(sin(d*x + c) - 1)^2)
/d
```

Mupad [B]
time = 0.14, size = 98, normalized size = 0.98

$$\frac{a\,\mathrm{atanh}(\sin(c+d\,x))\,(3\,A-B)}{8\,d}-\frac{\left(\frac{B\,a}{8}-\frac{3\,A\,a}{8}\right)\,\sin(c+d\,x)^2+\left(\frac{3\,A\,a}{8}-\frac{B\,a}{8}\right)\,\sin(c+d\,x)+\frac{A\,a}{4}+\frac{B\,a}{4}}{d\,\left(-\sin(c+d\,x)^3+\sin(c+d\,x)^2+\sin(c+d\,x)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(3*A - B))/(8*d) - ((A*a)/4 + (B*a)/4 + sin(c + d*x)
*((3*A*a)/8 - (B*a)/8) - sin(c + d*x)^2*((3*A*a)/8 - (B*a)/8))/(d*(sin(c +
d*x) + sin(c + d*x)^2 - sin(c + d*x)^3 - 1))
```

3.960 $\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=157

$$\frac{a(5A-B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}+\frac{3a^2A}{16d(a-a\sin(c+dx))}$$

```
[Out] 1/16*a*(5*A-B)*arctanh(sin(d*x+c))/d+1/24*a^4*(A+B)/d/(a-a*sin(d*x+c))^3+1/
32*a^3*(3*A+B)/d/(a-a*sin(d*x+c))^2+3/16*a^2*A/d/(a-a*sin(d*x+c))-1/32*a^3*
(A-B)/d/(a+a*sin(d*x+c))^2-1/16*a^2*(2*A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 78, 212}

$$\frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3}+\frac{a^3(3A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{32d(a\sin(c+dx)+a)^2}-\frac{a^2(2A-B)}{16d(a\sin(c+dx)+a)}+\frac{3a^2A}{16d(a-a\sin(c+dx))}+\frac{a(5A-B)\tanh^{-1}(\sin(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(5*A - B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^4*(A + B))/(24*d*(a - a*Sin
[c + d*x])^3) + (a^3*(3*A + B))/(32*d*(a - a*Sin[c + d*x])^2) + (3*a^2*A)/(
16*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(32*d*(a + a*Sin[c + d*x])^2) -
(a^2*(2*A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{a^7\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{A+B}{8a^3(a-x)^4} + \frac{3A+B}{16a^4(a-x)^3} + \frac{3A}{16a^5(a-x)^2} \right.\right.}{}$$

$$= \frac{a^4(A+B)}{24d(a-a\sin(c+dx))^3} + \frac{a^3(3A+B)}{32d(a-a\sin(c+dx}$$

$$= \frac{a(5A-B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^4(A}{24d(a-a\sin}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.66, size = 451, normalized size = 2.87

$$\frac{a\left(\frac{3(-A+B)}{d} - \frac{6(2A-B)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}{d} + 3i(5A-B)x\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4 - \frac{6i(5A-B)\tan^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d} - \frac{6(5A-B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d} + \frac{3(5A-B)\log\left(\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d} + \frac{4(A+B)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6} + \frac{3(3A+B)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{18A\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}\right)(1+\sin(c+dx))}{96\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*((3*(-A + B))/d - (6*(2*A - B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/
d + (3*I)*(5*A - B)*x*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - ((6*I)*(5*A
 - B)*ArcTan[Tan[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d -
 (6*(5*A - B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^4)/d + (3*(5*A - B)*Log[(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/d + (4*(A + B)*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6) +
 (3*(3*A + B)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^4) + (18*A*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)/(d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2))*(1 + Sin[c + d*x]))/(96*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A]
time = 0.39, size = 169, normalized size = 1.08

method	result

derivativedivides	$\frac{aA\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{aB}{6\cos(dx+c)^6}+\frac{aA}{6\cos(dx+c)^6}}{d}$
default	$\frac{aA\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{aB}{6\cos(dx+c)^6}+\frac{aA}{6\cos(dx+c)^6}}{d}$
risch	$-\frac{ia\left(110iA\,e^{4i(dx+c)}+15A\,e^{9i(dx+c)}+22iB\,e^{6i(dx+c)}-3B\,e^{9i(dx+c)}-110iA\,e^{6i(dx+c)}+40A\,e^{7i(dx+c)}-30iA\,e^{8i(dx+c)}-8\right.}{}$
norman	$\frac{2(aA+aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(aA+aB)\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(aA+aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(aA+aB)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{10(4aA+4aB}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+1/6*a*B/cos(d*x+c)^6+1/6*a*A/cos(d*x+c)^6+
a*B*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d
*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.34, size = 171, normalized size = 1.09

$$\frac{3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-B)a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-B)a\sin(dx+c)^4-3\,(5\,A-B)a\sin(dx+c)^3-5\,(5\,A-B)a\sin(dx+c)^2+5\,(5\,A-B)a\sin(dx+c)+8\,(A+B)a\right)}{\sin(dx+c)^5-\sin(dx+c)^4-2\,\sin(dx+c)^3+2\,\sin(dx+c)^2+\sin(dx+c)-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A - B)*a*log(sin(d*x + c) + 1) - 3*(5*A - B)*a*log(sin(d*x + c)
- 1) - 2*(3*(5*A - B)*a*sin(d*x + c)^4 - 3*(5*A - B)*a*sin(d*x + c)^3 - 5*(
5*A - B)*a*sin(d*x + c)^2 + 5*(5*A - B)*a*sin(d*x + c) + 8*(A + B)*a)/(sin(
d*x + c)^5 - sin(d*x + c)^4 - 2*sin(d*x + c)^3 + 2*sin(d*x + c)^2 + sin(d*x
 + c) - 1))/d
```

Fricas [A]

time = 0.38, size = 222, normalized size = 1.41

$$-\frac{6\,(5\,A-B)a\cos\left(dx+c\right)^4-2\,(5\,A-B)a\cos\left(dx+c\right)^2-4\,(A-5\,B)a-3\left((5\,A-B)a\cos\left(dx+c\right)^4\sin\left(dx+c\right)-(5\,A-B)a\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left((5\,A-B)a\cos\left(dx+c\right)^4\sin\left(dx+c\right)-(5\,A-B)a\cos\left(dx+c\right)^4\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\,(5\,A-B)a\cos\left(dx+c\right)^2+2\,(5\,A-B)a\right)\sin\left(dx+c\right)}{96\left(d\cos\left(dx+c\right)^4\sin\left(dx+c\right)-d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A - B)*a*cos(d*x + c)^4 - 2*(5*A - B)*a*cos(d*x + c)^2 - 4*(A -
 5*B)*a - 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c) - (5*A - B)*a*cos(d*x
```

```
+ c)^4)*log(sin(d*x + c) + 1) + 3*((5*A - B)*a*cos(d*x + c)^4*sin(d*x + c)
- (5*A - B)*a*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(5*A - B)*a*cos
(d*x + c)^2 + 2*(5*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x + c) -
 d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.48, size = 201, normalized size = 1.28

$$\frac{6\,(5\,Aa-Ba)\log\left(|\sin\left(dx+c\right)+1|\right)-6\,(5\,Aa-Ba)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{3\left(15\,Aa\sin(dx+c)^2-3\,Ba\sin(dx+c)^2+38\,Aa\sin(dx+c)-10\,Ba\sin(dx+c)+25\,Aa-9\,Ba\right)}{(\sin(dx+c)+1)^2}+\frac{55\,Aa\sin(dx+c)^3-11\,Ba\sin(dx+c)^3-201\,Aa\sin(dx+c)^2+33\,Ba\sin(dx+c)^2+255\,Aa\sin(dx+c)-27\,Ba\sin(dx+c)-117\,Aa-3\,Ba}{(\sin(dx+c)-1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A*a - B*a)*log(abs(sin(d*x + c) + 1)) - 6*(5*A*a - B*a)*log(abs
(sin(d*x + c) - 1)) - 3*(15*A*a*sin(d*x + c)^2 - 3*B*a*sin(d*x + c)^2 + 38*
A*a*sin(d*x + c) - 10*B*a*sin(d*x + c) + 25*A*a - 9*B*a)/(sin(d*x + c) + 1)
^2 + (55*A*a*sin(d*x + c)^3 - 11*B*a*sin(d*x + c)^3 - 201*A*a*sin(d*x + c)^
2 + 33*B*a*sin(d*x + c)^2 + 255*A*a*sin(d*x + c) - 27*B*a*sin(d*x + c) - 11
7*A*a - 3*B*a)/(sin(d*x + c) - 1)^3)/d
```

Mupad [B]
time = 9.26, size = 155, normalized size = 0.99

$$\frac{a\,\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(5\,A-B\right)}{16\,d}-\frac{\left(\frac{5\,A\,a}{16}-\frac{B\,a}{16}\right)\,{\sin\left(c+d\,x\right)}^4+\left(\frac{B\,a}{16}-\frac{5\,A\,a}{16}\right)\,{\sin\left(c+d\,x\right)}^3+\left(\frac{5\,B\,a}{48}-\frac{25\,A\,a}{48}\right)\,{\sin\left(c+d\,x\right)}^2+\left(\frac{25\,A\,a}{48}-\frac{5\,B\,a}{48}\right)\,\sin\left(c+d\,x\right)+\frac{A\,a}{6}+\frac{B\,a}{6}}{d\,\left({\sin\left(c+d\,x\right)}^5-{\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^3+2\,{\sin\left(c+d\,x\right)}^2+\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^7,x)
```

```
[Out] (a*atanh(sin(c + d*x))*(5*A - B))/(16*d) - ((A*a)/6 + (B*a)/6 + sin(c + d*x
)*((25*A*a)/48 - (5*B*a)/48) - sin(c + d*x)^3*((5*A*a)/16 - (B*a)/16) + sin
(c + d*x)^4*((5*A*a)/16 - (B*a)/16) - sin(c + d*x)^2*((25*A*a)/48 - (5*B*a)
/48))/(d*(sin(c + d*x) + 2*sin(c + d*x)^2 - 2*sin(c + d*x)^3 - sin(c + d*x)
^4 + sin(c + d*x)^5 - 1))
```

3.961 $\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{5}{128}a(8A+B)x-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A+B)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5a(8A+B)\cos^3(c+dx)\sin}{192d}$$

```
[Out] 5/128*a*(8*A+B)*x-1/56*a*(8*A+B)*cos(d*x+c)^7/d+5/128*a*(8*A+B)*cos(d*x+c)*
sin(d*x+c)/d+5/192*a*(8*A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/48*a*(8*A+B)*cos(d
*x+c)^5*sin(d*x+c)/d-1/8*B*cos(d*x+c)^7*(a+a*sin(d*x+c))/d
```

Rubi [A]

time = 0.09, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2939, 2748, 2715, 8}

$$-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a(8A+B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a(8A+B)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}ax(8A+B)-\frac{B\cos^7(c+dx)(a\sin(c+dx)+a)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a*(8*A + B)*x)/128 - (a*(8*A + B)*Cos[c + d*x]^7)/(56*d) + (5*a*(8*A + B
)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a*(8*A + B)*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (a*(8*A + B)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) - (B*Cos
[c + d*x]^7*(a + a*Sin[c + d*x]))/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx &= -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))}{8d}+\frac{1}{8}(8A+\\ &= -\frac{a(8A+B)\cos^7(c+dx)}{56d}-\frac{B\cos^7(c+dx)(a\\ &= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{a(8A+B)\cos^5(c}{48}\\ &= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A+B)\cos^3(}{19}\\ &= -\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5a(8A+B)\cos(c}{12}\\ &= \frac{5}{128}a(8A+B)x-\frac{a(8A+B)\cos^7(c+dx)}{56d}+\frac{5}{}\end{aligned}$$

Mathematica [A]
time = 0.79, size = 164, normalized size = 1.19

$$-\frac{a(-6720Adx-840Bdx+1680(A+B)\cos(c+dx)+1008(A+B)\cos(3(c+dx))+336A\cos(5(c+dx))+336B\cos(5(c+dx))+48A\cos(7(c+dx))+48B\cos(7(c+dx))-5040A\sin(2(c+dx))-336B\sin(2(c+dx))-1008A\sin(4(c+dx))+168B\sin(4(c+dx))-112A\sin(6(c+dx))+112B\sin(6(c+dx))+21B\sin(8(c+dx)))}{21504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/21504*(a*(-6720*A*d*x - 840*B*d*x + 1680*(A + B)*Cos[c + d*x] + 1008*(A
+ B)*Cos[3*(c + d*x)] + 336*A*Cos[5*(c + d*x)] + 336*B*Cos[5*(c + d*x)] + 4
8*A*Cos[7*(c + d*x)] + 48*B*Cos[7*(c + d*x)] - 5040*A*Sin[2*(c + d*x)] - 33
6*B*Sin[2*(c + d*x)] - 1008*A*Sin[4*(c + d*x)] + 168*B*Sin[4*(c + d*x)] - 1
12*A*Sin[6*(c + d*x)] + 112*B*Sin[6*(c + d*x)] + 21*B*Sin[8*(c + d*x)]))/d
```

Maple [A]
time = 0.37, size = 138, normalized size = 1.00

method	result

derivativedivides	$\frac{aB\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{aA\left(\cos^7(dx+c)\right)}{7}-\frac{B(c}{}}{d}$
default	$\frac{aB\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)-\frac{aA\left(\cos^7(dx+c)\right)}{7}-\frac{B(c}{}}{d}$
risch	$\frac{5axA}{16}+\frac{5aBx}{128}-\frac{5a\cos(dx+c)A}{64d}-\frac{5a\cos(dx+c)B}{64d}-\frac{aB\sin(8dx+8c)}{1024d}-\frac{a\cos(7dx+7c)A}{448d}-\frac{a\cos(7dx+7c)B}{448d}+$
norman	$\frac{\left(\frac{5}{16}aA+\frac{5}{128}aB\right)x-\frac{2aA+2aB}{7d}+\left(\frac{35}{2}aA+\frac{35}{16}aB\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{35}{4}aA+\frac{35}{32}aB\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5}{2}aA+\frac{5}{16}aB\right)x\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-1/7*a*A*cos(d*x+c)^7-1/7*B*c
os(d*x+c)^7*a+a*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(
d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]

time = 0.30, size = 124, normalized size = 0.90

$$-\frac{3072\,Aa\cos\left(dx+c\right)^{7}+3072\,Ba\cos\left(dx+c\right)^{7}+112\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)Aa-7\,\left(64\,\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)Ba}{21504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/21504*(3072*A*a*cos(d*x + c)^7 + 3072*B*a*cos(d*x + c)^7 + 112*(4*sin(2*
d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*A*
a - 7*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*B*a)/d
```

Fricas [A]

time = 0.37, size = 97, normalized size = 0.70

$$-\frac{384\,(A+B)a\cos\left(dx+c\right)^{7}-105\,(8\,A+B)adx+7\,\left(48\,Ba\cos\left(dx+c\right)^{7}-8\,(8\,A+B)a\cos\left(dx+c\right)^{5}-10\,(8\,A+B)a\cos\left(dx+c\right)^{3}-15\,(8\,A+B)a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2688*(384*(A + B)*a*cos(d*x + c)^7 - 105*(8*A + B)*a*d*x + 7*(48*B*a*cos
(d*x + c)^7 - 8*(8*A + B)*a*cos(d*x + c)^5 - 10*(8*A + B)*a*cos(d*x + c)^3
- 15*(8*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(131) = 262.
time = 1.10, size = 416, normalized size = 3.01

$$\begin{cases} \frac{5Aax\sin^6(c+dx)}{16} + \frac{15Aax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15Aax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Aax\cos^6(c+dx)}{16} + \frac{5Aa\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5Aa\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{11Aa\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{Aa\cos^7(c+dx)}{7d} + \frac{5Bax\sin^8(c+dx)}{128} + \frac{5Bax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15Bax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{5Bax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{5Bax\cos^8(c+dx)}{128} + \frac{5Ba\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55Ba\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{73Ba\sin^3(c+dx)\cos^5(c+dx)}{384d} - \frac{5Ba\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{Ba\cos^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a*x*sin(c + d*x)**6/16 + 15*A*a*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 15*A*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*a*x*cos(c + d*
x)**6/16 + 5*A*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*a*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 11*A*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - A*a*
cos(c + d*x)**7/(7*d) + 5*B*a*x*sin(c + d*x)**8/128 + 5*B*a*x*sin(c + d*x)*
*6*cos(c + d*x)**2/32 + 15*B*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*B*a
*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 5*B*a*x*cos(c + d*x)**8/128 + 5*B*a
*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*B*a*sin(c + d*x)**5*cos(c + d*x)
**3/(384*d) + 73*B*a*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*B*a*sin(c
+ d*x)*cos(c + d*x)**7/(128*d) - B*a*cos(c + d*x)**7/(7*d), Ne(d, 0)), (x*(
A + B*sin(c))*(a*sin(c) + a)*cos(c)**6, True))
```

Giac [A]
time = 0.52, size = 176, normalized size = 1.28

$$\frac{5}{128}(8Aa+Ba)x - \frac{Ba\sin(8dx+8c)}{1024d} - \frac{(Aa+Ba)\cos(7dx+7c)}{448d} - \frac{(Aa+Ba)\cos(5dx+5c)}{64d} - \frac{3(Aa+Ba)\cos(3dx+3c)}{64d} - \frac{5(Aa+Ba)\cos(dx+c)}{64d} + \frac{(Aa-Ba)\sin(6dx+6c)}{192d} + \frac{(6Aa-Ba)\sin(4dx+4c)}{128d} + \frac{(15Aa+Ba)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 5/128*(8*A*a + B*a)*x - 1/1024*B*a*sin(8*d*x + 8*c)/d - 1/448*(A*a + B*a)*c
os(7*d*x + 7*c)/d - 1/64*(A*a + B*a)*cos(5*d*x + 5*c)/d - 3/64*(A*a + B*a)*
cos(3*d*x + 3*c)/d - 5/64*(A*a + B*a)*cos(d*x + c)/d + 1/192*(A*a - B*a)*si
n(6*d*x + 6*c)/d + 1/128*(6*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(15*A*a +
B*a)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.72, size = 504, normalized size = 3.65

$$\frac{5a\,\mathrm{atan}\left(\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(8A+B)}{64\left(\frac{5Aa}{8}+\frac{5Ba}{64}\right)}\right)(8A+B)}{64d} - \frac{5a(8A+B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{64d} - \frac{\left(\frac{11Aa}{8}-\frac{5Ba}{64}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15} + (2Aa+2Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + \left(\frac{61Aa}{24}+\frac{397Ba}{192}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} + (2Aa+2Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + \left(\frac{113Aa}{24}-\frac{895Ba}{192}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + (10Aa+10Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + \left(\frac{85Aa}{24}+\frac{1765Ba}{192}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9} + (10Aa+10Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + \left(-\frac{85Aa}{24}-\frac{1765Ba}{192}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7} + (6Aa+6Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + \left(\frac{895Ba}{192}-\frac{113Aa}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5} + (6Aa+6Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} + \left(-\frac{61Aa}{24}-\frac{397Ba}{192}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3} + \left(\frac{2Aa}{7}+\frac{2Ba}{7}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + \left(\frac{5Ba}{64}-\frac{11Aa}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{2Aa}{7} + \frac{2Ba}{7}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16} + 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14} + 28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8} + 56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6} + 28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4} + 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)

[Out] (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(8*A + B))/(64*((5*A*a)/8 + (5*B*a)/64)))
*(8*A + B))/(64*d) - (5*a*(8*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(
64*d) - ((2*A*a)/7 + (2*B*a)/7 - tan(c/2 + (d*x)/2)*((11*A*a)/8 - (5*B*a)/6
4) + tan(c/2 + (d*x)/2)^4*(6*A*a + 6*B*a) + tan(c/2 + (d*x)/2)^12*(2*A*a +
2*B*a) + tan(c/2 + (d*x)/2)^6*(6*A*a + 6*B*a) + tan(c/2 + (d*x)/2)^14*(2*A*
a + 2*B*a) + tan(c/2 + (d*x)/2)^2*((2*A*a)/7 + (2*B*a)/7) + tan(c/2 + (d*x)
/2)^8*(10*A*a + 10*B*a) + tan(c/2 + (d*x)/2)^10*(10*A*a + 10*B*a) + tan(c/2
 + (d*x)/2)^15*((11*A*a)/8 - (5*B*a)/64) - tan(c/2 + (d*x)/2)^3*((61*A*a)/2
4 + (397*B*a)/192) + tan(c/2 + (d*x)/2)^13*((61*A*a)/24 + (397*B*a)/192) -
tan(c/2 + (d*x)/2)^5*((113*A*a)/24 - (895*B*a)/192) + tan(c/2 + (d*x)/2)^11
*((113*A*a)/24 - (895*B*a)/192) - tan(c/2 + (d*x)/2)^7*((85*A*a)/24 + (1765
*B*a)/192) + tan(c/2 + (d*x)/2)^9*((85*A*a)/24 + (1765*B*a)/192))/(d*(8*tan
(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*
tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12
+ 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))
```

3.962 $\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=111

$$\frac{1}{16}a(6A+B)x-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a(6A+B)\cos^3(c+dx)\sin(c+dx)}{24d}$$

```
[Out] 1/16*a*(6*A+B)*x-1/30*a*(6*A+B)*cos(d*x+c)^5/d+1/16*a*(6*A+B)*cos(d*x+c)*si
n(d*x+c)/d+1/24*a*(6*A+B)*cos(d*x+c)^3*sin(d*x+c)/d-1/6*B*cos(d*x+c)^5*(a+a
*sin(d*x+c))/d
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2939, 2748, 2715, 8}

$$-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a(6A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax(6A+B)-\frac{B\cos^5(c+dx)(a\sin(c+dx)+a)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(6*A + B)*x)/16 - (a*(6*A + B)*Cos[c + d*x]^5)/(30*d) + (a*(6*A + B)*Cos
[c + d*x]*Sin[c + d*x])/(16*d) + (a*(6*A + B)*Cos[c + d*x]^3*Sin[c + d*x])/
(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x]))/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))}{6d}+\frac{1}{6}(6A+B$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}-\frac{B\cos^5(c+dx)(a+$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\cos^3(c+}{24d}$$
$$= -\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6A+B)\cos(c+}{16d}$$
$$= \frac{1}{16}a(6A+B)x-\frac{a(6A+B)\cos^5(c+dx)}{30d}+\frac{a(6}{}$$

Mathematica [A]
time = 0.54, size = 120, normalized size = 1.08

$$-\frac{a(-360Adx-60Bdx+120(A+B)\cos(c+dx)+60(A+B)\cos(3(c+dx))+12A\cos(5(c+dx))+12B\cos(5(c+dx))-240A\sin(2(c+dx))-15B\sin(2(c+dx))-30A\sin(4(c+dx))+15B\sin(4(c+dx))+5B\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] -1/960*(a*(-360*A*d*x - 60*B*d*x + 120*(A + B)*Cos[c + d*x] + 60*(A + B)*Co
s[3*(c + d*x)] + 12*A*Cos[5*(c + d*x)] + 12*B*Cos[5*(c + d*x)] - 240*A*Sin[
2*(c + d*x)] - 15*B*Sin[2*(c + d*x)] - 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(
c + d*x)] + 5*B*Sin[6*(c + d*x)]))/d
```

Maple [A]
time = 0.29, size = 118, normalized size = 1.06

method	result
derivativedivides	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{aA\left(\cos^5(dx+c)\right)}{5}-\frac{B\left(\cos^5(dx+c)\right)a}{5}+aA\left(\right.}{d}$

default	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{aA\left(\cos^5(dx+c)\right)}{5}-\frac{B\left(\cos^5(dx+c)\right)a}{5}+aA\left(\right.}{d}$
risch	$\frac{3axA}{8}+\frac{aBx}{16}-\frac{a\cos(dx+c)A}{8d}-\frac{a\cos(dx+c)B}{8d}-\frac{\sin(6dx+6c)aB}{192d}-\frac{a\cos(5dx+5c)A}{80d}-\frac{a\cos(5dx+5c)B}{80d}+$
norman	$\frac{(\frac{3}{8}aA+\frac{1}{16}aB)x+(\frac{3}{8}aA+\frac{1}{16}aB)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{9}{4}aA+\frac{3}{8}aB)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{9}{4}aA+\frac{3}{8}aB)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-1/5*a*A*cos(d*x+c)^5-1/5*B*cos(d*x+c)^5*a+a*A*(1
/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.30, size = 98, normalized size = 0.88

$$-\frac{192\,Aa\cos\left(dx+c\right)^5+192\,Ba\cos\left(dx+c\right)^5-30\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa-5\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Ba}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/960*(192*A*a*cos(d*x + c)^5 + 192*B*a*cos(d*x + c)^5 - 30*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 5*(4*sin(2*d*x + 2*c)^3 + 1
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*B*a)/d
```

Fricas [A]
time = 0.36, size = 81, normalized size = 0.73

$$-\frac{48\left(A+B\right)a\cos\left(dx+c\right)^5-15\left(6\,A+B\right)adx+5\left(8\,Ba\cos\left(dx+c\right)^5-2\left(6\,A+B\right)a\cos\left(dx+c\right)^3-3\left(6\,A+B\right)a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/240*(48*(A + B)*a*cos(d*x + c)^5 - 15*(6*A + B)*a*d*x + 5*(8*B*a*cos(d*x
 + c)^5 - 2*(6*A + B)*a*cos(d*x + c)^3 - 3*(6*A + B)*a*cos(d*x + c))*sin(d*
x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(100) = 200$.
time = 0.48, size = 306, normalized size = 2.76

$$\begin{cases} \frac{3Aax\sin^4(c+dx)}{8}+\frac{3Aax\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3Aax\cos^4(c+dx)}{8}+\frac{3Aa\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5Aa\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{Aa\cos^5(c+dx)}{5d}+\frac{Bax\sin^6(c+dx)}{16}+\frac{3Bax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3Bax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{Bax\cos^6(c+dx)}{16}+\frac{Ba\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{Ba\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{Ba\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{Ba\cos^5(c+dx)}{5d} & \text{for}\ d\neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a*x*sin(c + d*x)**4/8 + 3*A*a*x*sin(c + d*x)**2*cos(c + d*x)
**2/4 + 3*A*a*x*cos(c + d*x)**4/8 + 3*A*a*sin(c + d*x)**3*cos(c + d*x)/(8*d
) + 5*A*a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - A*a*cos(c + d*x)**5/(5*d) +
B*a*x*sin(c + d*x)**6/16 + 3*B*a*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B
*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + B*a*x*cos(c + d*x)**6/16 + B*a*si
n(c + d*x)**5*cos(c + d*x)/(16*d) + B*a*sin(c + d*x)**3*cos(c + d*x)**3/(6*
d) - B*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - B*a*cos(c + d*x)**5/(5*d), N
e(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)*cos(c)**4, True))
```

Giac [A]
time = 0.47, size = 133, normalized size = 1.20

$$\frac{1}{16}(6\,Aa+Ba)x-\frac{Ba\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{(Aa+Ba)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{(Aa+Ba)\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{(Aa+Ba)\cos\left(dx+c\right)}{8\,d}+\frac{(2\,Aa-Ba)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{(16\,Aa+Ba)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(6*A*a + B*a)*x - 1/192*B*a*sin(6*d*x + 6*c)/d - 1/80*(A*a + B*a)*cos(
5*d*x + 5*c)/d - 1/16*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/8*(A*a + B*a)*cos(
d*x + c)/d + 1/64*(2*A*a - B*a)*sin(4*d*x + 4*c)/d + 1/64*(16*A*a + B*a)*si
n(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.51, size = 391, normalized size = 3.52

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6\,A+B)}{8\left(\frac{3\,Aa}{4}+\frac{Ba}{8}\right)}\right)(6\,A+B)}{8\,d}-\frac{a\,(6\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{8\,d}-\frac{\left(\frac{5\,Aa}{4}-\frac{Ba}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+(2\,Aa+2\,Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\left(\frac{7\,Aa}{4}+\frac{47\,Ba}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+(2\,Aa+2\,Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+\left(\frac{Aa}{2}-\frac{13\,Ba}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+(4\,Aa+4\,Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+\left(\frac{13\,Ba}{4}-\frac{Aa}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+(4\,Aa+4\,Ba)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+\left(-\frac{7\,Aa}{4}-\frac{47\,Ba}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+\left(\frac{2\,Aa}{5}+\frac{2\,Ba}{5}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+\left(\frac{Ba}{8}-\frac{5\,Aa}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{2\,Aa}{5}+\frac{2\,Ba}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(6*A + B))/(8*((3*A*a)/4 + (B*a)/8)))*(6*A +
B))/(8*d) - (a*(6*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d) - ((2*
A*a)/5 + (2*B*a)/5 - tan(c/2 + (d*x)/2)*((5*A*a)/4 - (B*a)/8) + tan(c/2 + (
d*x)/2)^4*(4*A*a + 4*B*a) + tan(c/2 + (d*x)/2)^8*(2*A*a + 2*B*a) + tan(c/2
+ (d*x)/2)^6*(4*A*a + 4*B*a) + tan(c/2 + (d*x)/2)^10*(2*A*a + 2*B*a) + tan(
c/2 + (d*x)/2)^2*((2*A*a)/5 + (2*B*a)/5) - tan(c/2 + (d*x)/2)^5*((A*a)/2 -
(13*B*a)/4) + tan(c/2 + (d*x)/2)^7*((A*a)/2 - (13*B*a)/4) + tan(c/2 + (d*x)
/2)^11*((5*A*a)/4 - (B*a)/8) - tan(c/2 + (d*x)/2)^3*((7*A*a)/4 + (47*B*a)/2
4) + tan(c/2 + (d*x)/2)^9*((7*A*a)/4 + (47*B*a)/24))/(d*(6*tan(c/2 + (d*x)/
2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*
x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.963 $\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{1}{8}a(4A+B)x-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\cos(c+dx)\sin(c+dx)}{8d}-\frac{B\cos^3(c+dx)(a+a\sin(c+dx))}{4d}$$

```
[Out] 1/8*a*(4*A+B)*x-1/12*a*(4*A+B)*cos(d*x+c)^3/d+1/8*a*(4*A+B)*cos(d*x+c)*sin(
d*x+c)/d-1/4*B*cos(d*x+c)^3*(a+a*sin(d*x+c))/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2939, 2748, 2715, 8}

$$-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(4A+B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(4*A + B)*x)/8 - (a*(4*A + B)*Cos[c + d*x]^3)/(12*d) + (a*(4*A + B)*Cos[
c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x]))/(4*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))}{4d}+\frac{1}{4}(4A+B$$

$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}-\frac{B\cos^3(c+dx)(a+}{4d}$$

$$= -\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A+B)\cos(c+}{8d}$$

$$= \frac{1}{8}a(4A+B)x-\frac{a(4A+B)\cos^3(c+dx)}{12d}+\frac{a(4A}{}$$

Mathematica [A]
time = 0.65, size = 64, normalized size = 0.76

$$-\frac{a(-12(4A+B)dx+24(A+B)\cos(c+dx)+8(A+B)\cos(3(c+dx))-24A\sin(2(c+dx))+3B\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/96*(a*(-12*(4*A + B)*d*x + 24*(A + B)*Cos[c + d*x] + 8*(A + B)*Cos[3*(c
 + d*x)] - 24*A*Sin[2*(c + d*x)] + 3*B*Sin[4*(c + d*x)]))/d
```

Maple [A]
time = 0.19, size = 96, normalized size = 1.14

method	result
derivativedivides	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{aA\left(\cos^3(dx+c)\right)}{3}-\frac{B\left(\cos^3(dx+c)\right)a}{3}+aA\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
default	$\frac{aB\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{aA\left(\cos^3(dx+c)\right)}{3}-\frac{B\left(\cos^3(dx+c)\right)a}{3}+aA\left(\frac{\sin(dx+c)\cos(dx+c)}{2}\right.}{d}$
risch	$\frac{axA}{2}+\frac{aBx}{8}-\frac{a\cos(dx+c)A}{4d}-\frac{a\cos(dx+c)B}{4d}-\frac{\sin(4dx+4c)aB}{32d}-\frac{a\cos(3dx+3c)A}{12d}-\frac{a\cos(3dx+3c)B}{12d}+\sin($
norman	$\left(\frac{1}{2}aA+\frac{1}{8}aB\right)x+\left(2aA+\frac{1}{2}aB\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2aA+\frac{1}{2}aB\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3aA+\frac{3}{4}aB\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}aA\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*B*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)-1/3*a*A*cos(d*x+c)^3-1/3*B*cos(d*x+c)^3*a+a*A*(1/2*sin(d*x+c)*cos(d*x+
c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.88

$$-\frac{32\,Aa\cos\left(dx+c\right)^{3}+32\,Ba\cos\left(dx+c\right)^{3}-24\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa-3\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)Ba}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*A*a*cos(d*x + c)^3 + 32*B*a*cos(d*x + c)^3 - 24*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*a - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a)/d
```

Fricas [A]
time = 0.36, size = 65, normalized size = 0.77

$$-\frac{8\,(A+B)a\cos\left(dx+c\right)^{3}-3\,(4\,A+B)adx+3\,\left(2\,Ba\cos\left(dx+c\right)^{3}-(4\,A+B)a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/24*(8*(A + B)*a*cos(d*x + c)^3 - 3*(4*A + B)*a*d*x + 3*(2*B*a*cos(d*x +
c)^3 - (4*A + B)*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(75) = 150.
time = 0.22, size = 199, normalized size = 2.37

$$\begin{cases} \frac{Aax\sin^{2}\left(c+dx\right)}{2}+\frac{Aax\cos^{2}\left(c+dx\right)}{2}+\frac{Aa\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{Aa\cos^{3}\left(c+dx\right)}{3d}+\frac{Bax\sin^{4}\left(c+dx\right)}{8}+\frac{Bax\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{Bax\cos^{4}\left(c+dx\right)}{8}+\frac{Ba\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{Ba\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{Ba\cos^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0 \\ x(A+B\sin\left(c\right))\left(a\sin\left(c\right)+a\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*x*sin(c + d*x)**2/2 + A*a*x*cos(c + d*x)**2/2 + A*a*sin(c +
d*x)*cos(c + d*x)/(2*d) - A*a*cos(c + d*x)**3/(3*d) + B*a*x*sin(c + d*x)**4
/8 + B*a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + B*a*x*cos(c + d*x)**4/8 + B*
```

```
a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - B*a*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)
*cos(c)**2, True))
```

Giac [A]
time = 0.42, size = 83, normalized size = 0.99

$$\frac{1}{8}(4\,Aa+Ba)x-\frac{Ba\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{Aa\sin\left(2\,dx+2\,c\right)}{4\,d}-\frac{(Aa+Ba)\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{(Aa+Ba)\cos\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/8*(4*A*a + B*a)*x - 1/32*B*a*sin(4*d*x + 4*c)/d + 1/4*A*a*sin(2*d*x + 2*c
)/d - 1/12*(A*a + B*a)*cos(3*d*x + 3*c)/d - 1/4*(A*a + B*a)*cos(d*x + c)/d
```

Mupad [B]
time = 10.55, size = 276, normalized size = 3.29

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(4\,A+B)}{4\left(A\,a+\frac{B\,a}{4}\right)}\right)(4\,A+B)}{4\,d}-\frac{a\,(4\,A+B)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{4\,d}-\frac{\left(A\,a-\frac{B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+(2\,A\,a+2\,B\,a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(A\,a+\frac{7\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+(2\,A\,a+2\,B\,a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(-A\,a-\frac{7\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{2\,A\,a}{3}+\frac{2\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(\frac{B\,a}{4}-A\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,A\,a}{3}+\frac{2\,B\,a}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x)),x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(4*A + B))/(4*(A*a + (B*a)/4)))*(4*A + B))/(4
*d) - (a*(4*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(4*d) - ((2*A*a)/3
 + (2*B*a)/3 - tan(c/2 + (d*x)/2)*(A*a - (B*a)/4) + tan(c/2 + (d*x)/2)^4*(2
*A*a + 2*B*a) + tan(c/2 + (d*x)/2)^6*(2*A*a + 2*B*a) + tan(c/2 + (d*x)/2)^2
*((2*A*a)/3 + (2*B*a)/3) + tan(c/2 + (d*x)/2)^7*(A*a - (B*a)/4) - tan(c/2 +
 (d*x)/2)^3*(A*a + (7*B*a)/4) + tan(c/2 + (d*x)/2)^5*(A*a + (7*B*a)/4))/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
 tan(c/2 + (d*x)/2)^8 + 1))
```

3.964 $\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=29

$$-aBx+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}$$

```
[Out] -a*B*x+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2934, 8}

$$\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)}{d}-aBx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] -(a*B*x) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c + d*x]))/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx=\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))}{d}-(aB)$$
$$=-aBx+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx)}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(29) = 58.

time = 0.34, size = 85, normalized size = 2.93

$$\frac{a\left(-Bdx\cos\left(\frac{dx}{2}\right)+2(A+B)\sin\left(\frac{dx}{2}\right)+Bdx\sin\left(c+\frac{dx}{2}\right)\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(-(B*d*x*Cos[(d*x)/2]) + 2*(A + B)*Sin[(d*x)/2] + B*d*x*Sin[c + (d*x)/2]
))/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
```

Maple [A]

time = 0.14, size = 54, normalized size = 1.86

method	result
risch	$-aBx+\frac{2aA}{d\left(e^{i(dx+c)}-i\right)}+\frac{2aB}{d\left(e^{i(dx+c)}-i\right)}$
derivativedivides	$\frac{aA\tan(dx+c)+\frac{aB}{\cos(dx+c)}+\frac{aA}{\cos(dx+c)}+aB(\tan(dx+c)-dx-c)}{d}$
default	$\frac{aA\tan(dx+c)+\frac{aB}{\cos(dx+c)}+\frac{aA}{\cos(dx+c)}+aB(\tan(dx+c)-dx-c)}{d}$
norman	$\frac{aBx+aBx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2aA+2aB}{d}-\frac{(2aA+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4aA+4aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-aBx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*tan(d*x+c)+a*B/cos(d*x+c)+a*A/cos(d*x+c)+a*B*(tan(d*x+c)-d*x-c))
```

Maxima [A]

time = 0.52, size = 56, normalized size = 1.93

$$-\frac{(dx+c-\tan(dx+c))Ba-Aa\tan(dx+c)-\frac{Aa}{\cos(dx+c)}-\frac{Ba}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -((d*x + c - tan(d*x + c))*B*a - A*a*tan(d*x + c) - A*a/cos(d*x + c) - B*a/
cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.
time = 0.36, size = 73, normalized size = 2.52

$$-\frac{Badx - (A+B)a + (Badx - (A+B)a)\cos(dx+c) - (Badx + (A+B)a)\sin(dx+c)}{d\cos(dx+c) - d\sin(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -(B*a*d*x - (A + B)*a + (B*a*d*x - (A + B)*a)*cos(d*x + c) - (B*a*d*x + (A
+ B)*a)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^2(c+dx)\,dx + \int A\sin(c+dx)\sec^2(c+dx)\,dx + \int B\sin(c+dx)\sec^2(c+dx)\,dx + \int B\sin^2(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**2
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.44, size = 36, normalized size = 1.24

$$-\frac{(dx+c)Ba + \frac{2(Aa+Ba)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((d*x + c)*B*a + 2*(A*a + B*a)/(tan(1/2*d*x + 1/2*c) - 1))/d
```

Mupad [B]
time = 9.17, size = 33, normalized size = 1.14

$$-\frac{2\,A\,a+2\,B\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-B\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (2*A*a + 2*B*a)/(d*(tan(c/2 + (d*x)/2) - 1)) - B*a*x
```

3.965 $\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=50

$$\frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d}+\frac{a(2A-B)\tan(c+dx)}{3d}$$

```
[Out] 1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))/d+1/3*a*(2*A-B)*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2934, 3852, 8}

$$\frac{a(2A-B)\tan(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x]))/(3*d) + (a*(2*A - B)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} + \frac{1}{3}(a$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} - \frac{(a(2}{}$$
$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))}{3d} + \frac{a(2.}{}$$

Mathematica [A]
time = 0.51, size = 97, normalized size = 1.94

$$\frac{a\sec(c)\sec^3(c+dx)(1+\sin(c+dx))(6B\cos(c)-2(A+B)\cos(c+dx)+4A\cos(c+2dx)-2B\cos(c+2dx)+8A\sin(dx)-4B\sin(dx)+A\sin(2(c+dx))+B\sin(2(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] (a*Sec[c]*Sec[c + d*x]^3*(1 + Sin[c + d*x])*(6*B*Cos[c] - 2*(A + B)*Cos[c +
 d*x] + 4*A*Cos[c + 2*d*x] - 2*B*Cos[c + 2*d*x] + 8*A*Sin[d*x] - 4*B*Sin[d*
x] + A*Sin[2*(c + d*x)] + B*Sin[2*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.22, size = 72, normalized size = 1.44

method	result
derivativedivides	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{aB}{3\cos(dx+c)^3}+\frac{aA}{3\cos(dx+c)^3}+\frac{aB\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
default	$\frac{-aA\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{aB}{3\cos(dx+c)^3}+\frac{aA}{3\cos(dx+c)^3}+\frac{aB\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
risch	$-\frac{2ia\left(4iA\,e^{i(dx+c)}-2iB\,e^{i(dx+c)}+3B\,e^{2i(dx+c)}+2A-B\right)}{3\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^3 d}$
norman	$\frac{-\frac{2aA+2aB}{3d}-\frac{2(2aA+2aB)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(2aA+2aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4aA+4aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2(4aA+4aB)\left(\tan^4\left(\frac{dx}{2}\right.\right.}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*B/cos(d*x+c)^3+1/3*a*A/c
os(d*x+c)^3+1/3*a*B*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 59, normalized size = 1.18

$$\frac{Ba\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa+\frac{Aa}{\cos(dx+c)^3}+\frac{Ba}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(B*a*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + A*a/cos(d
*x + c)^3 + B*a/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.34, size = 69, normalized size = 1.38

$$-\frac{(2A-B)a\cos(dx+c)^2+(2A-B)a\sin(dx+c)-(A-2B)a}{3(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/3*((2*A - B)*a*cos(d*x + c)^2 + (2*A - B)*a*sin(d*x + c) - (A - 2*B)*a)/
(d*cos(d*x + c)*sin(d*x + c) - d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^4(c+dx)\,dx+\int A\sin(c+dx)\sec^4(c+dx)\,dx+\int B\sin(c+dx)\sec^4(c+dx)\,dx+\int B\sin^2(c+dx)\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*sin(c + d*x)*sec(c + d*x)**4
, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**4, x) + Integral(B*sin(c + d*x
)**2*sec(c + d*x)**4, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.
time = 0.48, size = 94, normalized size = 1.88

$$-\frac{\frac{3(Aa-Ba)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}+\frac{9Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7Aa+Ba}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(A*a - B*a)/(tan(1/2*d*x + 1/2*c) + 1) + (9*A*a*tan(1/2*d*x + 1/2*c
)^2 + 3*B*a*tan(1/2*d*x + 1/2*c)^2 - 12*A*a*tan(1/2*d*x + 1/2*c) + 7*A*a +
B*a)/(tan(1/2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [B]
time = 9.25, size = 107, normalized size = 2.14

$$\frac{\frac{2\,a\,\left(\frac{3\,B}{2}+A\,\cos(c+d\,x)+B\,\cos(c+d\,x)+2\,A\,\sin(c+d\,x)-B\,\sin(c+d\,x)+A\,\cos(2\,c+2\,d\,x)-\frac{B\,\cos(2\,c+2\,d\,x)}{2}\right)}{3}-\frac{4\,a\,\cos(c+d\,x)\,\left(\frac{A}{2}+\frac{B}{2}\right)}{3}}{d\,\left(2\,\cos\left(c+d\,x\right)-\sin\left(2\,c+2\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] ((2*a*((3*B)/2 + A*cos(c + d*x) + B*cos(c + d*x) + 2*A*sin(c + d*x) - B*sin
(c + d*x) + A*cos(2*c + 2*d*x) - (B*cos(2*c + 2*d*x))/2))/3 - (4*a*cos(c +
d*x)*(A/2 + B/2))/3)/(d*(2*cos(c + d*x) - sin(2*c + 2*d*x)))
```

3.966 $\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{a(4A-B)\tan^3(c+dx)}{15d}$$

```
[Out] 1/5*(A+B)*sec(d*x+c)^5*(a+a*sin(d*x+c))/d+1/5*a*(4*A-B)*tan(d*x+c)/d+1/15*a
*(4*A-B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2934, 3852}

$$\frac{a(4A-B)\tan^3(c+dx)}{15d}+\frac{a(4A-B)\tan(c+dx)}{5d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^5*(a + a*Sin[c + d*x]))/(5*d) + (a*(4*A - B)*Tan[c +
d*x])/(5*d) + (a*(4*A - B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} + \frac{1}{5}(a$$
$$= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} - \frac{(a(4$$
$$= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))}{5d} + \frac{a(4.$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 223 vs. 2(73) = 146.
time = 1.04, size = 223, normalized size = 3.05

$$\frac{a\sec(c)(240B\cos(c)-54(A+B)\cos(c+dx)-18A\cos(3(c+dx))-18B\cos(3(c+dx))+128A\cos(c+2dx)-32B\cos(c+2dx)+64A\cos(3c+4dx)-16B\cos(3c+4dx)+384A\sin(dx)-96B\sin(dx)+18A\sin(2(c+dx))+18B\sin(2(c+dx))+9A\sin(4(c+dx))+9B\sin(4(c+dx))+128A\sin(2c+3dx)-32B\sin(2c+3dx))}{960d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(240*B*Cos[c] - 54*(A + B)*Cos[c + d*x] - 18*A*Cos[3*(c + d*x)] -
 18*B*Cos[3*(c + d*x)] + 128*A*Cos[c + 2*d*x] - 32*B*Cos[c + 2*d*x] + 64*A*
Cos[3*c + 4*d*x] - 16*B*Cos[3*c + 4*d*x] + 384*A*Sin[d*x] - 96*B*Sin[d*x] +
 18*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + 9*A*Sin[4*(c + d*x)] + 9*B
*Sin[4*(c + d*x)] + 128*A*Sin[2*c + 3*d*x] - 32*B*Sin[2*c + 3*d*x]))/(960*d
*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^3)
```

Maple [**A**]
time = 0.32, size = 102, normalized size = 1.40

method	result
derivativedivides	$\frac{-aA\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{aB}{5\cos(dx+c)^5}+\frac{aA}{5\cos(dx+c)^5}+aB\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right.}{d}$
default	$\frac{-aA\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{aB}{5\cos(dx+c)^5}+\frac{aA}{5\cos(dx+c)^5}+aB\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right.}{d}$
risch	$-\frac{4ia\left(24iA\,e^{3i(dx+c)}-6iB\,e^{3i(dx+c)}+15B\,e^{4i(dx+c)}+8iA\,e^{i(dx+c)}+8A\,e^{2i(dx+c)}-2iB\,e^{i(dx+c)}-2B\,e^{2i(dx+c)}+4A-B\right)}{15\left(e^{i(dx+c)}+i\right)^3\left(e^{i(dx+c)}-i\right)^5 d}$
norman	$\frac{-\frac{2aA+2aB}{5d}-\frac{(2aA+2aB)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4aA+4aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{(4aA+4aB)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(6aA+6aB)\left(\tan^8\left(\frac{dx}{2}\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*a*B/cos
(d*x+c)^5+1/5*a*A/cos(d*x+c)^5+a*B*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(
d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.29, size = 86, normalized size = 1.18

$$\frac{\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Aa+\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)Ba+\frac{3\,Aa}{\cos\left(dx+c\right)^{5}}+\frac{3\,Ba}{\cos\left(dx+c\right)^{5}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a + (3*tan
(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a + 3*A*a/cos(d*x + c)^5 + 3*B*a/cos(d*x
+ c)^5)/d
```

Fricas [A]
time = 0.36, size = 112, normalized size = 1.53

$$-\frac{2\,(4\,A-B)a\cos\left(dx+c\right)^{4}-(4\,A-B)a\cos\left(dx+c\right)^{2}-(A-4\,B)a+\left(2\,(4\,A-B)a\cos\left(dx+c\right)^{2}+(4\,A-B)a\right)\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/15*(2*(4*A - B)*a*cos(d*x + c)^4 - (4*A - B)*a*cos(d*x + c)^2 - (A - 4*B
)*a + (2*(4*A - B)*a*cos(d*x + c)^2 + (4*A - B)*a)*sin(d*x + c))/(d*cos(d*x
 + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(67) = 134.
time = 0.51, size = 225, normalized size = 3.08

$$-\frac{\frac{5\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+13\,Aa-7\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}+\frac{165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+45\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-480\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-60\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+650\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+70\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-400\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-20\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+113\,Aa+13\,Ba}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/120*(5*(15*A*a*tan(1/2*d*x + 1/2*c)^2 - 9*B*a*tan(1/2*d*x + 1/2*c)^2 + 2
4*A*a*tan(1/2*d*x + 1/2*c) - 12*B*a*tan(1/2*d*x + 1/2*c) + 13*A*a - 7*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^3 + (165*A*a*tan(1/2*d*x + 1/2*c)^4 + 45*B*a*tan
(1/2*d*x + 1/2*c)^4 - 480*A*a*tan(1/2*d*x + 1/2*c)^3 - 60*B*a*tan(1/2*d*x +
 1/2*c)^3 + 650*A*a*tan(1/2*d*x + 1/2*c)^2 + 70*B*a*tan(1/2*d*x + 1/2*c)^2
- 400*A*a*tan(1/2*d*x + 1/2*c) - 20*B*a*tan(1/2*d*x + 1/2*c) + 113*A*a + 13
*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^5)/d
```

Mupad [B]
time = 11.14, size = 224, normalized size = 3.07

$$-\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{9\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)-\frac{15\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{3\,B\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}-B\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)+\frac{B\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{4}-\frac{73\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{25\,A\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{19\,A\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{3\,A\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}+\frac{7\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{5\,B\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{B\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{3\,B\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}\right)}{120\,d\,{\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}^3\,{\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^6,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((A*cos((5*c)/2 + (5*d*x)/2))/4 - (9*A*cos((3*c)/2 +
 (3*d*x)/2))/4 - A*cos((7*c)/2 + (7*d*x)/2) - (15*B*cos(c/2 + (d*x)/2))/4 +
 (3*B*cos((3*c)/2 + (3*d*x)/2))/2 - B*cos((5*c)/2 + (5*d*x)/2) + (B*cos((7*
c)/2 + (7*d*x)/2))/4 - (73*A*sin(c/2 + (d*x)/2))/8 + (25*A*sin((3*c)/2 + (3
*d*x)/2))/8 - (19*A*sin((5*c)/2 + (5*d*x)/2))/8 + (3*A*sin((7*c)/2 + (7*d*x
)/2))/8 + (7*B*sin(c/2 + (d*x)/2))/8 + (5*B*sin((3*c)/2 + (3*d*x)/2))/8 + (
B*sin((5*c)/2 + (5*d*x)/2))/8 + (3*B*sin((7*c)/2 + (7*d*x)/2))/8))/(120*d*c
os(c/2 - pi/4 + (d*x)/2)^3*cos(c/2 + pi/4 + (d*x)/2)^5)
```

3.967 $\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan^5(c+dx)}{35d}$$

```
[Out] 1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))/d+1/7*a*(6*A-B)*tan(d*x+c)/d+2/21*a
*(6*A-B)*tan(d*x+c)^3/d+1/35*a*(6*A-B)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2934, 3852}

$$\frac{a(6A-B)\tan^5(c+dx)}{35d}+\frac{2a(6A-B)\tan^3(c+dx)}{21d}+\frac{a(6A-B)\tan(c+dx)}{7d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+a)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x]))/(7*d) + (a*(6*A - B)*Tan[c +
d*x])/(7*d) + (2*a*(6*A - B)*Tan[c + d*x]^3)/(21*d) + (a*(6*A - B)*Tan[c +
d*x]^5)/(35*d)
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} + \frac{1}{7}(a$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} - \frac{(a(6$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))}{7d} + \frac{a(6.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(96) = 192.
time = 1.65, size = 315, normalized size = 3.28

$$\frac{a\sec(c)(8960B\cos(c)-1500(A+B)\cos(c+dx)-750A\cos(3(c+dx))-750B\cos(3(c+dx))-150A\cos(5(c+dx))-150B\cos(5(c+dx))+3840A\cos(c+2dx)-640B\cos(c+2dx)+3072A\cos(3c+4dx)-512B\cos(3c+4dx)+768A\cos(5c+6dx)-128B\cos(5c+6dx)+15360A\sin(dx)-2560B\sin(dx)+375A\sin(2(c+dx))+375B\sin(2(c+dx))+300A\sin(4(c+dx))+300B\sin(4(c+dx))+75A\sin(6(c+dx))+75B\sin(6(c+dx))+7680A\sin(2c+3dx)-1280B\sin(2c+3dx)+1536A\sin(4c+5dx)-256B\sin(4c+5dx))}{53760d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(8960*B*Cos[c] - 1500*(A + B)*Cos[c + d*x] - 750*A*Cos[3*(c + d*x
)] - 750*B*Cos[3*(c + d*x)] - 150*A*Cos[5*(c + d*x)] - 150*B*Cos[5*(c + d*x
)] + 3840*A*Cos[c + 2*d*x] - 640*B*Cos[c + 2*d*x] + 3072*A*Cos[3*c + 4*d*x]
 - 512*B*Cos[3*c + 4*d*x] + 768*A*Cos[5*c + 6*d*x] - 128*B*Cos[5*c + 6*d*x]
 + 15360*A*Sin[d*x] - 2560*B*Sin[d*x] + 375*A*Sin[2*(c + d*x)] + 375*B*Sin[
2*(c + d*x)] + 300*A*Sin[4*(c + d*x)] + 300*B*Sin[4*(c + d*x)] + 75*A*Sin[6
*(c + d*x)] + 75*B*Sin[6*(c + d*x)] + 7680*A*Sin[2*c + 3*d*x] - 1280*B*Sin[
2*c + 3*d*x] + 1536*A*Sin[4*c + 5*d*x] - 256*B*Sin[4*c + 5*d*x]))/(53760*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
])^5)
```

Maple [A]
time = 0.33, size = 130, normalized size = 1.35

method	result
derivativedivides	$\frac{-aA\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{aB}{7\cos(dx+c)^7}+\frac{aA}{7\cos(dx+c)^7}+aB\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\right.}{d}$
default	$\frac{-aA\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{aB}{7\cos(dx+c)^7}+\frac{aA}{7\cos(dx+c)^7}+aB\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\right.}{d}$
risch	$-\frac{16ia(120iA\,e^{5i(dx+c)}-20iB\,e^{5i(dx+c)}+70B\,e^{6i(dx+c)}+60iA\,e^{3i(dx+c)}+30A\,e^{4i(dx+c)}-10iB\,e^{3i(dx+c)}-5B\,e^{4i(dx+c)}}{105\left(e^{i(dx+c)}+i\right)^5\left(e^{i(dx+c)}-i\right)^7d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+1/7*a*B/cos(d*x+c)^7+1/7*a*A/cos(d*x+c)^7+a*B*(1/7*sin(d*x+c)^3/cos
(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.30, size = 107, normalized size = 1.11

$$\frac{3\left(5\tan\left(dx+c\right)^{7}+21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+35\tan\left(dx+c\right)\right)Aa+\left(15\tan\left(dx+c\right)^{7}+42\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}\right)Ba+\frac{15\,Aa}{\cos\left(dx+c\right)^{7}}+\frac{15\,Ba}{\cos\left(dx+c\right)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/105*(3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan
(d*x + c))*A*a + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3
)*B*a + 15*A*a/cos(d*x + c)^7 + 15*B*a/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.35, size = 149, normalized size = 1.55

$$-\frac{8\,(6\,A-B)a\cos\left(dx+c\right)^{6}-4\,(6\,A-B)a\cos\left(dx+c\right)^{4}-(6\,A-B)a\cos\left(dx+c\right)^{2}-3\,(A-6\,B)a+\left(8\,(6\,A-B)a\cos\left(dx+c\right)^{4}+4\,(6\,A-B)a\cos\left(dx+c\right)^{2}+3\,(6\,A-B)a\right)\sin\left(dx+c\right)}{105\,\left(d\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/105*(8*(6*A - B)*a*cos(d*x + c)^6 - 4*(6*A - B)*a*cos(d*x + c)^4 - (6*A
- B)*a*cos(d*x + c)^2 - 3*(A - 6*B)*a + (8*(6*A - B)*a*cos(d*x + c)^4 + 4*(
6*A - B)*a*cos(d*x + c)^2 + 3*(6*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^5*
sin(d*x + c) - d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(88) = 176.
time = 0.52, size = 345, normalized size = 3.59

$$-\frac{\frac{7\left(165\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-75\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+540\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-210\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+750\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-280\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+480\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-170\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+129\,Aa-49\,Ba\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{5}}+\frac{2205\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+525\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-10080\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1470\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+21945\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2555\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-26460\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2240\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+18963\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1407\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7476\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-434\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1383\,Aa+137\,Ba}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{7}}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/1680*(7*(165*A*a*tan(1/2*d*x + 1/2*c)^4 - 75*B*a*tan(1/2*d*x + 1/2*c)^4
+ 540*A*a*tan(1/2*d*x + 1/2*c)^3 - 210*B*a*tan(1/2*d*x + 1/2*c)^3 + 750*A*a
*tan(1/2*d*x + 1/2*c)^2 - 280*B*a*tan(1/2*d*x + 1/2*c)^2 + 480*A*a*tan(1/2*
d*x + 1/2*c) - 170*B*a*tan(1/2*d*x + 1/2*c) + 129*A*a - 49*B*a)/(tan(1/2*d*
x + 1/2*c) + 1)^5 + (2205*A*a*tan(1/2*d*x + 1/2*c)^6 + 525*B*a*tan(1/2*d*x
+ 1/2*c)^6 - 10080*A*a*tan(1/2*d*x + 1/2*c)^5 - 1470*B*a*tan(1/2*d*x + 1/2*
c)^5 + 21945*A*a*tan(1/2*d*x + 1/2*c)^4 + 2555*B*a*tan(1/2*d*x + 1/2*c)^4 -
 26460*A*a*tan(1/2*d*x + 1/2*c)^3 - 2240*B*a*tan(1/2*d*x + 1/2*c)^3 + 18963
*A*a*tan(1/2*d*x + 1/2*c)^2 + 1407*B*a*tan(1/2*d*x + 1/2*c)^2 - 7476*A*a*ta
n(1/2*d*x + 1/2*c) - 434*B*a*tan(1/2*d*x + 1/2*c) + 1383*A*a + 137*B*a)/(ta
n(1/2*d*x + 1/2*c) - 1)^7)/d
```

Mupad [B]
time = 12.64, size = 320, normalized size = 3.33

$$-\frac{a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}-\frac{75\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{105\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{9\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{3\,A\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{2}-\frac{35\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{65\,B\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{55\,B\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{35\,B\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}-\frac{19\,B\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}+\frac{B\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{4}-\frac{843\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{363\,A\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}-\frac{651\,A\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}+\frac{171\,A\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}-\frac{111\,A\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}+\frac{15\,A\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{32}+\frac{53\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{27\,B\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}+\frac{21\,B\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}+\frac{59\,B\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}+\frac{B\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}+\frac{15\,B\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{32}\right)}{3360\,d\,{\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}^5\,{\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^8,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((15*A*cos((5*c)/2 + (5*d*x)/2))/8 - (75*A*cos((3*c)
/2 + (3*d*x)/2))/8 - (105*A*cos((7*c)/2 + (7*d*x)/2))/16 + (9*A*cos((9*c)/2
 + (9*d*x)/2))/16 - (3*A*cos((11*c)/2 + (11*d*x)/2))/2 - (35*B*cos(c/2 + (d
*x)/2))/2 + (65*B*cos((3*c)/2 + (3*d*x)/2))/8 - (55*B*cos((5*c)/2 + (5*d*x)
/2))/8 + (35*B*cos((7*c)/2 + (7*d*x)/2))/16 - (19*B*cos((9*c)/2 + (9*d*x)/2
))/16 + (B*cos((11*c)/2 + (11*d*x)/2))/4 - (843*A*sin(c/2 + (d*x)/2))/16 +
(363*A*sin((3*c)/2 + (3*d*x)/2))/16 - (651*A*sin((5*c)/2 + (5*d*x)/2))/32 +
 (171*A*sin((7*c)/2 + (7*d*x)/2))/32 - (111*A*sin((9*c)/2 + (9*d*x)/2))/32
+ (15*A*sin((11*c)/2 + (11*d*x)/2))/32 + (53*B*sin(c/2 + (d*x)/2))/16 + (27
*B*sin((3*c)/2 + (3*d*x)/2))/16 + (21*B*sin((5*c)/2 + (5*d*x)/2))/32 + (59*
B*sin((7*c)/2 + (7*d*x)/2))/32 + (B*sin((9*c)/2 + (9*d*x)/2))/32 + (15*B*si
n((11*c)/2 + (11*d*x)/2))/32))/(3360*d*cos(c/2 - pi/4 + (d*x)/2)^5*cos(c/2
+ pi/4 + (d*x)/2)^7)
```

3.968 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan}{15d}$$

```
[Out] 1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))/d+1/9*a*(8*A-B)*tan(d*x+c)/d+1/9*a*
(8*A-B)*tan(d*x+c)^3/d+1/15*a*(8*A-B)*tan(d*x+c)^5/d+1/63*a*(8*A-B)*tan(d*x
+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2934, 3852}

$$\frac{a(8A-B)\tan^7(c+dx)}{63d}+\frac{a(8A-B)\tan^5(c+dx)}{15d}+\frac{a(8A-B)\tan^3(c+dx)}{9d}+\frac{a(8A-B)\tan(c+dx)}{9d}+\frac{(A+B)\sec^9(c+dx)(a\sin(c+dx)+a)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x]))/(9*d) + (a*(8*A - B)*Tan[c +
d*x])/(9*d) + (a*(8*A - B)*Tan[c + d*x]^3)/(9*d) + (a*(8*A - B)*Tan[c + d*x
]^5)/(15*d) + (a*(8*A - B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} + \frac{1}{9}(c$$

$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} - \frac{(a($$

$$= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))}{9d} + \frac{a(8$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(119) = 238.
time = 3.18, size = 407, normalized size = 3.42

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*Sec[c]*(645120*B*Cos[c] - 85750*(A + B)*Cos[c + d*x] - 51450*A*Cos[3*(c
+ d*x)] - 51450*B*Cos[3*(c + d*x)] - 17150*A*Cos[5*(c + d*x)] - 17150*B*Cos
[5*(c + d*x)] - 2450*A*Cos[7*(c + d*x)] - 2450*B*Cos[7*(c + d*x)] + 229376*
A*Cos[c + 2*d*x] - 28672*B*Cos[c + 2*d*x] + 229376*A*Cos[3*c + 4*d*x] - 286
72*B*Cos[3*c + 4*d*x] + 98304*A*Cos[5*c + 6*d*x] - 12288*B*Cos[5*c + 6*d*x]
 + 16384*A*Cos[7*c + 8*d*x] - 2048*B*Cos[7*c + 8*d*x] + 1146880*A*Sin[d*x]
- 143360*B*Sin[d*x] + 17150*A*Sin[2*(c + d*x)] + 17150*B*Sin[2*(c + d*x)] +
 17150*A*Sin[4*(c + d*x)] + 17150*B*Sin[4*(c + d*x)] + 7350*A*Sin[6*(c + d*
x)] + 7350*B*Sin[6*(c + d*x)] + 1225*A*Sin[8*(c + d*x)] + 1225*B*Sin[8*(c +
 d*x)] + 688128*A*Sin[2*c + 3*d*x] - 86016*B*Sin[2*c + 3*d*x] + 229376*A*Si
n[4*c + 5*d*x] - 28672*B*Sin[4*c + 5*d*x] + 32768*A*Sin[6*c + 7*d*x] - 4096
*B*Sin[6*c + 7*d*x]))/(5160960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^9*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^7)
```

Maple [A]
time = 0.44, size = 158, normalized size = 1.33

method	result
derivativedivides	$\frac{-aA\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)+\frac{aB}{9\cos(dx+c)^9}+\frac{aA}{9\cos(dx+c)^9}+}{d}$
default	$\frac{-aA\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)+\frac{aB}{9\cos(dx+c)^9}+\frac{aA}{9\cos(dx+c)^9}+}{d}$
risch	$-\frac{32ia\left(560iA\,e^{7i(dx+c)}-70iB\,e^{7i(dx+c)}+315B\,e^{8i(dx+c)}+16iA\,e^{i(dx+c)}+112A\,e^{6i(dx+c)}-2iB\,e^{i(dx+c)}-14B\,e^{6i(dx+c)}\right.}{315(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/d*(-a*A*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^4-
64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*a*B/cos(d*x+c)^9+1/9*a*A/cos(d*x+c)^9+a
*B*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(
d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.30, size = 126, normalized size = 1.06

$$\frac{\left(35\,\tan\left(dx+c\right)^{9}+180\,\tan\left(dx+c\right)^{7}+378\,\tan\left(dx+c\right)^{5}+420\,\tan\left(dx+c\right)^{3}+315\,\tan\left(dx+c\right)\right)Aa+\left(35\,\tan\left(dx+c\right)^{9}+135\,\tan\left(dx+c\right)^{7}+189\,\tan\left(dx+c\right)^{5}+105\,\tan\left(dx+c\right)^{3}\right)Ba+\frac{35\,Aa}{\cos(dx+c)^{9}}+\frac{35\,Ba}{\cos(dx+c)^{9}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="max
ima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a + (35*tan(d*x + c)^9 + 135*tan(d*x +
c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a + 35*A*a/cos(d*x + c)^9
 + 35*B*a/cos(d*x + c)^9)/d
```

Fricas [A]
time = 0.38, size = 185, normalized size = 1.55

$$-\frac{16\,(8\,A-B)a\cos\left(dx+c\right)^{8}-8\,(8\,A-B)a\cos\left(dx+c\right)^{6}-2\,(8\,A-B)a\cos\left(dx+c\right)^{4}-(8\,A-B)a\cos\left(dx+c\right)^{2}-5\,(A-8\,B)a+\left(16\,(8\,A-B)a\cos\left(dx+c\right)^{6}+8\,(8\,A-B)a\cos\left(dx+c\right)^{4}+6\,(8\,A-B)a\cos\left(dx+c\right)^{2}+5\,(8\,A-B)a\right)\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)^{7}\sin\left(dx+c\right)-d\cos\left(dx+c\right)^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fri
cas")
```

```
[Out] -1/315*(16*(8*A - B)*a*cos(d*x + c)^8 - 8*(8*A - B)*a*cos(d*x + c)^6 - 2*(8
*A - B)*a*cos(d*x + c)^4 - (8*A - B)*a*cos(d*x + c)^2 - 5*(A - 8*B)*a + (16
*(8*A - B)*a*cos(d*x + c)^6 + 8*(8*A - B)*a*cos(d*x + c)^4 + 6*(8*A - B)*a*
cos(d*x + c)^2 + 5*(8*A - B)*a)*sin(d*x + c))/(d*cos(d*x + c)^7*sin(d*x + c
) - d*cos(d*x + c)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(109) = 218.
time = 0.51, size = 465, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="gia
c")
```

```
[Out] -1/40320*(3*(9765*A*a*tan(1/2*d*x + 1/2*c)^6 - 3675*B*a*tan(1/2*d*x + 1/2*c
)^6 + 48720*A*a*tan(1/2*d*x + 1/2*c)^5 - 15960*B*a*tan(1/2*d*x + 1/2*c)^5 +
 109865*A*a*tan(1/2*d*x + 1/2*c)^4 - 33775*B*a*tan(1/2*d*x + 1/2*c)^4 + 136
640*A*a*tan(1/2*d*x + 1/2*c)^3 - 39760*B*a*tan(1/2*d*x + 1/2*c)^3 + 99183*A
*a*tan(1/2*d*x + 1/2*c)^2 - 28161*B*a*tan(1/2*d*x + 1/2*c)^2 + 39536*A*a*ta
n(1/2*d*x + 1/2*c) - 11032*B*a*tan(1/2*d*x + 1/2*c) + 7043*A*a - 2101*B*a)/
(tan(1/2*d*x + 1/2*c) + 1)^7 + (51345*A*a*tan(1/2*d*x + 1/2*c)^8 + 11025*B*
a*tan(1/2*d*x + 1/2*c)^8 - 322560*A*a*tan(1/2*d*x + 1/2*c)^7 - 47880*B*a*ta
n(1/2*d*x + 1/2*c)^7 + 976500*A*a*tan(1/2*d*x + 1/2*c)^6 + 117180*B*a*tan(1
/2*d*x + 1/2*c)^6 - 1753920*A*a*tan(1/2*d*x + 1/2*c)^5 - 168840*B*a*tan(1/2
*d*x + 1/2*c)^5 + 2037294*A*a*tan(1/2*d*x + 1/2*c)^4 + 165942*B*a*tan(1/2*d
*x + 1/2*c)^4 - 1550976*A*a*tan(1/2*d*x + 1/2*c)^3 - 106008*B*a*tan(1/2*d*x
 + 1/2*c)^3 + 760644*A*a*tan(1/2*d*x + 1/2*c)^2 + 47772*B*a*tan(1/2*d*x + 1
/2*c)^2 - 219456*A*a*tan(1/2*d*x + 1/2*c) - 12888*B*a*tan(1/2*d*x + 1/2*c)
+ 30089*A*a + 2657*B*a)/(tan(1/2*d*x + 1/2*c) - 1)^9)/d
```

Mupad [B]
time = 13.30, size = 416, normalized size = 3.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x)))/cos(c + d*x)^10,x)
```

```
[Out] -(a*cos(c/2 + (d*x)/2)*((329*A*cos((5*c)/2 + (5*d*x)/2))/64 - (1225*A*cos((
3*c)/2 + (3*d*x)/2))/64 - (133*A*cos((7*c)/2 + (7*d*x)/2))/8 + (21*A*cos((9
*c)/2 + (9*d*x)/2))/8 - (413*A*cos((11*c)/2 + (11*d*x)/2))/64 + (29*A*cos((
13*c)/2 + (13*d*x)/2))/64 - A*cos((15*c)/2 + (15*d*x)/2) - (315*B*cos(c/2 +
 (d*x)/2))/8 + (1295*B*cos((3*c)/2 + (3*d*x)/2))/64 - (1183*B*cos((5*c)/2 +
 (5*d*x)/2))/64 + 7*B*cos((7*c)/2 + (7*d*x)/2) - (21*B*cos((9*c)/2 + (9*d*x
)/2))/4 + (91*B*cos((11*c)/2 + (11*d*x)/2))/64 - (43*B*cos((13*c)/2 + (13*d
*x)/2))/64 + (B*cos((15*c)/2 + (15*d*x)/2))/8 - (17609*A*sin(c/2 + (d*x)/2)
```

```
)/128 + (8649*A*sin((3*c)/2 + (3*d*x)/2))/128 - (8159*A*sin((5*c)/2 + (5*d*
x)/2))/128 + (2783*A*sin((7*c)/2 + (7*d*x)/2))/128 - (2293*A*sin((9*c)/2 +
(9*d*x)/2))/128 + (501*A*sin((11*c)/2 + (11*d*x)/2))/128 - (291*A*sin((13*c
)/2 + (13*d*x)/2))/128 + (35*A*sin((15*c)/2 + (15*d*x)/2))/128 + (823*B*sin
(c/2 + (d*x)/2))/128 + (297*B*sin((3*c)/2 + (3*d*x)/2))/128 + (193*B*sin((5
*c)/2 + (5*d*x)/2))/128 + (479*B*sin((7*c)/2 + (7*d*x)/2))/128 + (11*B*sin(
(9*c)/2 + (9*d*x)/2))/128 + (213*B*sin((11*c)/2 + (11*d*x)/2))/128 - (3*B*s
in((13*c)/2 + (13*d*x)/2))/128 + (35*B*sin((15*c)/2 + (15*d*x)/2))/128))/(4
0320*d*cos(c/2 - pi/4 + (d*x)/2)^7*cos(c/2 + pi/4 + (d*x)/2)^9)
```

3.969 $\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{4(A-B)(a+a\sin(c+dx))^6}{3a^4d}-\frac{4(3A-5B)(a+a\sin(c+dx))^7}{7a^5d}+\frac{3(A-3B)(a+a\sin(c+dx))^8}{4a^6d}-\frac{(A-7B)}{}$$

```
[Out] 4/3*(A-B)*(a+a*sin(d*x+c))^6/a^4/d-4/7*(3*A-5*B)*(a+a*sin(d*x+c))^7/a^5/d+3
/4*(A-3*B)*(a+a*sin(d*x+c))^8/a^6/d-1/9*(A-7*B)*(a+a*sin(d*x+c))^9/a^7/d-1/
10*B*(a+a*sin(d*x+c))^10/a^8/d
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^{10}}{10a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^9}{9a^7d}+\frac{3(A-3B)(a\sin(c+dx)+a)^8}{4a^6d}-\frac{4(3A-5B)(a\sin(c+dx)+a)^7}{7a^5d}+\frac{4(A-B)(a\sin(c+dx)+a)^6}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^4*d) - (4*(3*A - 5*B)*(a + a*Sin[c
+ d*x])^7)/(7*a^5*d) + (3*(A - 3*B)*(a + a*Sin[c + d*x])^8)/(4*a^6*d) - ((A
 - 7*B)*(a + a*Sin[c + d*x])^9)/(9*a^7*d) - (B*(a + a*Sin[c + d*x])^10)/(10
*a^8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^5\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(\right.}{a^7d}$$
$$= \frac{\text{Subst}\Big(\int \Big(8a^3(A-B)(a+x)^5-4a^2(3A-5B)}{}$$
$$= \frac{4(A-B)(a+a\sin(c+dx))^6}{3a^4d} - \frac{4(3A-5B)(a}{7}$$

Mathematica [A]
time = 0.81, size = 86, normalized size = 0.64

$$-\frac{a^2(1+\sin(c+dx))^6(-325A+61B+6(115A-61B)\sin(c+dx)+(-525A+651B)\sin^2(c+dx)+28(5A-17B)\sin^3(c+dx)+126B\sin^4(c+dx))}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/1260*(a^2*(1 + Sin[c + d*x])^6*(-325*A + 61*B + 6*(115*A - 61*B)*Sin[c +
 d*x] + (-525*A + 651*B)*Sin[c + d*x]^2 + 28*(5*A - 17*B)*Sin[c + d*x]^3 +
126*B*Sin[c + d*x]^4))/d
```

Maple [A]
time = 0.68, size = 231, normalized size = 1.72

method	result
derivativedivides	$\frac{\frac{a^2A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{B\left(\cos^8(dx+c)\right)a^2}{8}-\frac{A\left(\cos^8(dx+c)\right)a^2}{4}+2Ba^2\left(-\frac{\sin(dx+c)}{}\right.}{}$
default	$\frac{\frac{a^2A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{B\left(\cos^8(dx+c)\right)a^2}{8}-\frac{A\left(\cos^8(dx+c)\right)a^2}{4}+2Ba^2\left(-\frac{\sin(dx+c)}{}\right.}{}$
risch	$\frac{77\sin(dx+c)a^2A}{128d}+\frac{7\sin(dx+c)Ba^2}{64d}+\frac{Ba^2\cos(10dx+10c)}{5120d}-\frac{\sin(9dx+9c)a^2A}{2304d}-\frac{\sin(9dx+9c)Ba^2}{1152d}-\frac{Aa^2\cos(8}{512}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)-1/8*B*cos(d*x+c)^8*a^2-1/4*A*cos(d*x+c)^8*a^2+2*B*a^2*(-1/9*sin(d*x+c)
*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
```

```
n(d*x+c))+a^2*A*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*a^2*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8))
```

Maxima [A]
time = 0.38, size = 168, normalized size = 1.25

$$-\frac{126\,Ba^2\sin(dx+c)^{10}+140\,(A+2\,B)a^2\sin(dx+c)^9+315\,(A-B)a^2\sin(dx+c)^8-360\,(A+3\,B)a^2\sin(dx+c)^7-1260\,Aa^2\sin(dx+c)^6+1512\,Ba^2\sin(dx+c)^5+630\,(3\,A+B)a^2\sin(dx+c)^4+840\,(A-B)a^2\sin(dx+c)^3-630\,(2\,A+B)a^2\sin(dx+c)^2-1260\,Aa^2\sin(dx+c)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/1260*(126*B*a^2*sin(d*x + c)^10 + 140*(A + 2*B)*a^2*sin(d*x + c)^9 + 315
*(A - B)*a^2*sin(d*x + c)^8 - 360*(A + 3*B)*a^2*sin(d*x + c)^7 - 1260*A*a^2
*sin(d*x + c)^6 + 1512*B*a^2*sin(d*x + c)^5 + 630*(3*A + B)*a^2*sin(d*x + c
)^4 + 840*(A - B)*a^2*sin(d*x + c)^3 - 630*(2*A + B)*a^2*sin(d*x + c)^2 - 1
260*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 127, normalized size = 0.95

$$\frac{126\,Ba^2\cos(dx+c)^{10}-315\,(A+B)a^2\cos(dx+c)^8-4\,\left(35\,(A+2\,B)a^2\cos(dx+c)^8-10\,(5\,A+B)a^2\cos(dx+c)^6-12\,(5\,A+B)a^2\cos(dx+c)^4-16\,(5\,A+B)a^2\cos(dx+c)^2-32\,(5\,A+B)a^2\right)\sin(dx+c)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1260*(126*B*a^2*cos(d*x + c)^10 - 315*(A + B)*a^2*cos(d*x + c)^8 - 4*(35*
(A + 2*B)*a^2*cos(d*x + c)^8 - 10*(5*A + B)*a^2*cos(d*x + c)^6 - 12*(5*A +
B)*a^2*cos(d*x + c)^4 - 16*(5*A + B)*a^2*cos(d*x + c)^2 - 32*(5*A + B)*a^2)
*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(126) = 252$.
time = 1.92, size = 389, normalized size = 2.90

$$\begin{cases} \frac{16Aa^2\sin^9(c+dx)}{315d}+\frac{8Aa^2\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16Aa^2\sin^7(c+dx)}{35d}+\frac{2Aa^2\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{Aa^2\sin^3(c+dx)\cos^6(c+dx)}{3d}+\frac{2Aa^2\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{Aa^2\sin(c+dx)\cos^6(c+dx)}{d}-\frac{Aa^2\cos^8(c+dx)}{4d}+\frac{32Ba^2\sin^9(c+dx)}{315d}+\frac{16Ba^2\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{4Ba^2\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{2Ba^2\sin^3(c+dx)\cos^6(c+dx)}{3d}-\frac{Ba^2\sin^2(c+dx)\cos^8(c+dx)}{8d}-\frac{Ba^2\cos^{10}(c+dx)}{40d}-\frac{Ba^2\cos^8(c+dx)}{8d} & \text{for } d\neq 0 \\ x(A+B\sin(c))\,(a\sin(c)+a)^2\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**9/(315*d) + 8*A*a**2*sin(c + d*x)**7*cos
(c + d*x)**2/(35*d) + 16*A*a**2*sin(c + d*x)**7/(35*d) + 2*A*a**2*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 8*A*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*
d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + 2*A*a**2*sin(c + d*x)**
```

```
3*cos(c + d*x)**4/d + A*a**2*sin(c + d*x)*cos(c + d*x)**6/d - A*a**2*cos(c
+ d*x)**8/(4*d) + 32*B*a**2*sin(c + d*x)**9/(315*d) + 16*B*a**2*sin(c + d*x
)**7*cos(c + d*x)**2/(35*d) + 4*B*a**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d
) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) - B*a**2*sin(c + d*x)**2
*cos(c + d*x)**8/(8*d) - B*a**2*cos(c + d*x)**10/(40*d) - B*a**2*cos(c + d*
x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**7, True
))
```

Giac [**A**]
time = 0.60, size = 239, normalized size = 1.78

$$\frac{Ba^2\cos(10\,dx+10\,c)}{5120\,d}-\frac{Aa^2\cos(8\,dx+8\,c)}{512\,d}+\frac{7\,Aa^2\sin(3\,dx+3\,c)}{64\,d}-\frac{(16\,Aa^2+7\,Ba^2)\cos(6\,dx+6\,c)}{1024\,d}-\frac{(7\,Aa^2+4\,Ba^2)\cos(4\,dx+4\,c)}{128\,d}-\frac{7\,(8\,Aa^2+5\,Ba^2)\cos(2\,dx+2\,c)}{512\,d}-\frac{(Aa^2+2\,Ba^2)\sin(9\,dx+9\,c)}{2304\,d}-\frac{(Aa^2+10\,Ba^2)\sin(7\,dx+7\,c)}{1792\,d}+\frac{(5\,Aa^2-4\,Ba^2)\sin(5\,dx+5\,c)}{320\,d}+\frac{7\,(11\,Aa^2+2\,Ba^2)\sin(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^2*cos(10*d*x + 10*c)/d - 1/512*A*a^2*cos(8*d*x + 8*c)/d + 7/64*A
*a^2*sin(3*d*x + 3*c)/d - 1/1024*(16*A*a^2 + 7*B*a^2)*cos(6*d*x + 6*c)/d -
1/128*(7*A*a^2 + 4*B*a^2)*cos(4*d*x + 4*c)/d - 7/512*(8*A*a^2 + 5*B*a^2)*co
s(2*d*x + 2*c)/d - 1/2304*(A*a^2 + 2*B*a^2)*sin(9*d*x + 9*c)/d - 1/1792*(A*
a^2 + 10*B*a^2)*sin(7*d*x + 7*c)/d + 1/320*(5*A*a^2 - 4*B*a^2)*sin(5*d*x +
5*c)/d + 7/128*(11*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

Mupad [**B**]
time = 9.20, size = 168, normalized size = 1.25

$$-\frac{\frac{2\,a^2\,\sin(c+d\,x)^3\,(A-B)}{3}-\frac{a^2\,\sin(c+d\,x)^2\,(2\,A+B)}{2}-A\,a^2\,\sin\left(c+d\,x\right)^6+\frac{a^2\,\sin(c+d\,x)^4\,(3\,A+B)}{2}+\frac{a^2\,\sin(c+d\,x)^8\,(A-B)}{4}-\frac{2\,a^2\,\sin(c+d\,x)^7\,(A+3\,B)}{7}+\frac{a^2\,\sin(c+d\,x)^9\,(A+2\,B)}{9}+\frac{6\,B\,a^2\,\sin(c+d\,x)^5}{5}+\frac{B\,a^2\,\sin(c+d\,x)^{10}}{10}-A\,a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((2*a^2*sin(c + d*x)^3*(A - B))/3 - (a^2*sin(c + d*x)^2*(2*A + B))/2 - A*a
^2*sin(c + d*x)^6 + (a^2*sin(c + d*x)^4*(3*A + B))/2 + (a^2*sin(c + d*x)^8*
(A - B))/4 - (2*a^2*sin(c + d*x)^7*(A + 3*B))/7 + (a^2*sin(c + d*x)^9*(A +
2*B))/9 + (6*B*a^2*sin(c + d*x)^5)/5 + (B*a^2*sin(c + d*x)^10)/10 - A*a^2*s
in(c + d*x))/d
```

3.970 $\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{4(A-B)(a+a\sin(c+dx))^5}{5a^3d}-\frac{2(A-2B)(a+a\sin(c+dx))^6}{3a^4d}+\frac{(A-5B)(a+a\sin(c+dx))^7}{7a^5d}+\frac{B(a+a\sin(c+dx))^8}{8a^6d}$$

```
[Out] 4/5*(A-B)*(a+a*sin(d*x+c))^5/a^3/d-2/3*(A-2*B)*(a+a*sin(d*x+c))^6/a^4/d+1/7
*(A-5*B)*(a+a*sin(d*x+c))^7/a^5/d+1/8*B*(a+a*sin(d*x+c))^8/a^6/d
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$\frac{B(a\sin(c+dx)+a)^8}{8a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^7}{7a^5d}-\frac{2(A-2B)(a\sin(c+dx)+a)^6}{3a^4d}+\frac{4(A-B)(a\sin(c+dx)+a)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^3*d) - (2*(A - 2*B)*(a + a*Sin[c +
d*x])^6)/(3*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^7)/(7*a^5*d) + (B*(a +
 a*Sin[c + d*x])^8)/(8*a^6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)^4\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(\right.}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(4a^2(A-B)(a+x)^4 - 4a(A-2B)(\right.\right.}{}$$

$$= \frac{4(A-B)(a+a\sin(c+dx))^5}{5a^3 d} - \frac{2(A-2B)(a+}{3}$$

Mathematica [A]
time = 0.24, size = 70, normalized size = 0.67

$$\frac{a^2(1+\sin(c+dx))^5\left(232A-47B-5(64A-47B)\sin(c+dx)+15(8A-19B)\sin^2(c+dx)+105B\sin^3(c+dx)\right)}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^5*(232*A - 47*B - 5*(64*A - 47*B)*Sin[c + d*x] + 15
*(8*A - 19*B)*Sin[c + d*x]^2 + 105*B*Sin[c + d*x]^3))/(840*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(97) = 194.
time = 0.50, size = 201, normalized size = 1.91

method	result
derivativedivides	$\frac{\frac{a^2 A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} - \frac{B\left(\cos^6(dx+c)\right)a^2}{6} - \frac{A\left(\cos^6(dx+c)\right)a^2}{3} + 2B\,a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7} + \right.}{}$
default	$\frac{\frac{a^2 A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5} - \frac{B\left(\cos^6(dx+c)\right)a^2}{6} - \frac{A\left(\cos^6(dx+c)\right)a^2}{3} + 2B\,a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7} + \right.}{}$
risch	$\frac{45\sin(dx+c)a^2 A}{64d} + \frac{5\sin(dx+c)B\,a^2}{32d} + \frac{B\,a^2\cos(8dx+8c)}{1024d} - \frac{\sin(7dx+7c)a^2 A}{448d} - \frac{\sin(7dx+7c)B\,a^2}{224d} - \frac{a^2\cos(6dx+}{96d}$
norman	$\frac{2\left(2a^2A+B\,a^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{2\left(2a^2A+B\,a^2\right)\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{4\left(2a^2A+2B\,a^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{4\left(2a^2A+2B\,a^2\right)\left(\tan^{12}\left(\frac{dx}{2}\right.\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*cos(d*x
+c)^6*a^2-1/3*A*cos(d*x+c)^6*a^2+2*B*a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a^2*A*(-1/7*sin(d*x+c)*cos
(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+B*a^2*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A]
time = 0.38, size = 142, normalized size = 1.35

$$\frac{105\,Ba^2\sin(dx+c)^8+120\,(A+2\,B)a^2\sin(dx+c)^7+140\,(2\,A-B)a^2\sin(dx+c)^6-168\,(A+4\,B)a^2\sin(dx+c)^5-210\,(4\,A+B)a^2\sin(dx+c)^4-280\,(A-2\,B)a^2\sin(dx+c)^3+420\,(2\,A+B)a^2\sin(dx+c)^2+840\,Aa^2\sin(dx+c)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*a^2*sin(d*x + c)^8 + 120*(A + 2*B)*a^2*sin(d*x + c)^7 + 140*(2
*A - B)*a^2*sin(d*x + c)^6 - 168*(A + 4*B)*a^2*sin(d*x + c)^5 - 210*(4*A +
B)*a^2*sin(d*x + c)^4 - 280*(A - 2*B)*a^2*sin(d*x + c)^3 + 420*(2*A + B)*a^
2*sin(d*x + c)^2 + 840*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 109, normalized size = 1.04

$$\frac{105\,Ba^2\cos(dx+c)^8-280\,(A+B)a^2\cos(dx+c)^6-8\,(15\,(A+2\,B)a^2\cos(dx+c)^6-6\,(4\,A+B)a^2\cos(dx+c)^4-8\,(4\,A+B)a^2\cos(dx+c)^2-16\,(4\,A+B)a^2)\sin(dx+c)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*a^2*cos(d*x + c)^8 - 280*(A + B)*a^2*cos(d*x + c)^6 - 8*(15*(A
 + 2*B)*a^2*cos(d*x + c)^6 - 6*(4*A + B)*a^2*cos(d*x + c)^4 - 8*(4*A + B)*a
^2*cos(d*x + c)^2 - 16*(4*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(99) = 198$.
time = 1.00, size = 309, normalized size = 2.94

$$\begin{cases} \frac{8Aa^2\sin^7(c+dx)}{105d}+\frac{4Aa^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{8Aa^2\sin^5(c+dx)}{15d}+\frac{Aa^2\sin^3(c+dx)\cos^4(c+dx)}{3d}+\frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{Aa^2\cos^6(c+dx)}{3d}+\frac{16Ba^2\sin^7(c+dx)}{105d}+\frac{8Ba^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{2Ba^2\sin^3(c+dx)\cos^4(c+dx)}{3d}-\frac{Ba^2\sin^2(c+dx)\cos^6(c+dx)}{6d}-\frac{Ba^2\cos^8(c+dx)}{24d}-\frac{Ba^2\cos^6(c+dx)}{6d} & \text{for } d\neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**7/(105*d) + 4*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(15*d) + 8*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)*
*3*cos(c + d*x)**4/(3*d) + 4*A*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) +
 A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a**2*cos(c + d*x)**6/(3*d) + 16*
```

```
B*a**2*sin(c + d*x)**7/(105*d) + 8*B*a**2*sin(c + d*x)**5*cos(c + d*x)**2/(
15*d) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a**2*sin(c + d*x
)**2*cos(c + d*x)**6/(6*d) - B*a**2*cos(c + d*x)**8/(24*d) - B*a**2*cos(c +
 d*x)**6/(6*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**5, T
rue))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(97) = 194.
time = 0.55, size = 202, normalized size = 1.92

$$\frac{Ba^2\cos(8\,dx+8\,c)}{1024\,d}-\frac{(4\,Aa^2+Ba^2)\cos(6\,dx+6\,c)}{384\,d}-\frac{(16\,Aa^2+9\,Ba^2)\cos(4\,dx+4\,c)}{256\,d}-\frac{(20\,Aa^2+13\,Ba^2)\cos(2\,dx+2\,c)}{128\,d}-\frac{(Aa^2+2\,Ba^2)\sin(7\,dx+7\,c)}{448\,d}+\frac{(Aa^2-6\,Ba^2)\sin(5\,dx+5\,c)}{320\,d}+\frac{(19\,Aa^2-2\,Ba^2)\sin(3\,dx+3\,c)}{192\,d}+\frac{5\,(9\,Aa^2+2\,Ba^2)\sin(dx+c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*a^2*cos(8*d*x + 8*c)/d - 1/384*(4*A*a^2 + B*a^2)*cos(6*d*x + 6*c)/
d - 1/256*(16*A*a^2 + 9*B*a^2)*cos(4*d*x + 4*c)/d - 1/128*(20*A*a^2 + 13*B*
a^2)*cos(2*d*x + 2*c)/d - 1/448*(A*a^2 + 2*B*a^2)*sin(7*d*x + 7*c)/d + 1/32
0*(A*a^2 - 6*B*a^2)*sin(5*d*x + 5*c)/d + 1/192*(19*A*a^2 - 2*B*a^2)*sin(3*d
*x + 3*c)/d + 5/64*(9*A*a^2 + 2*B*a^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.12, size = 140, normalized size = 1.33

$$\frac{\frac{a^2\sin(c+d\,x)^2\,(2\,A+B)}{2}-\frac{a^2\sin(c+d\,x)^3\,(A-2\,B)}{3}-\frac{a^2\sin(c+d\,x)^4\,(4\,A+B)}{4}-\frac{a^2\sin(c+d\,x)^5\,(A+4\,B)}{5}+\frac{a^2\sin(c+d\,x)^7\,(A+2\,B)}{7}+\frac{B\,a^2\sin(c+d\,x)^8}{8}+\frac{a^2\sin(c+d\,x)^6\,(2\,A-B)}{6}+A\,a^2\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2*(2*A + B))/2 - (a^2*sin(c + d*x)^3*(A - 2*B))/3 - (a^2
*sin(c + d*x)^4*(4*A + B))/4 - (a^2*sin(c + d*x)^5*(A + 4*B))/5 + (a^2*sin(
c + d*x)^7*(A + 2*B))/7 + (B*a^2*sin(c + d*x)^8)/8 + (a^2*sin(c + d*x)^6*(2
*A - B))/6 + A*a^2*sin(c + d*x))/d
```

3.971 $\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{(A-B)(a+a\sin(c+dx))^4}{2a^2d}-\frac{(A-3B)(a+a\sin(c+dx))^5}{5a^3d}-\frac{B(a+a\sin(c+dx))^6}{6a^4d}$$

```
[Out] 1/2*(A-B)*(a+a*sin(d*x+c))^4/a^2/d-1/5*(A-3*B)*(a+a*sin(d*x+c))^5/a^3/d-1/6
*B*(a+a*sin(d*x+c))^6/a^4/d
```

Rubi [A]
time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^6}{6a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^5}{5a^3d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(2*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*x]
)^5)/(5*a^3*d) - (B*(a + a*Sin[c + d*x])^6)/(6*a^4*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^3\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(c\right.}{a^3d}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A-B)(a+x)^3+(-A+3B)(a\right.\right.}{a^3d}$$

$$= \frac{(A-B)(a+a\sin(c+dx))^4}{2a^2d} - \frac{(A-3B)(a+a}{5a^3}$$

Mathematica [A]
time = 0.29, size = 66, normalized size = 0.85

$$\frac{a^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8(18A-9B+5B\cos(2(c+dx))-4(3A-4B)\sin(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*(18*A - 9*B + 5*B*Cos[2*(c + d
*x)] - 4*(3*A - 4*B)*Sin[c + d*x]))/(60*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(72) = 144.
time = 0.32, size = 171, normalized size = 2.19

method	result
derivativedivides	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{B\left(\cos^4(dx+c)\right)a^2}{4}-\frac{A\left(\cos^4(dx+c)\right)a^2}{2}+2Ba^2\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(\right.}{15}}{d}$
default	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{B\left(\cos^4(dx+c)\right)a^2}{4}-\frac{A\left(\cos^4(dx+c)\right)a^2}{2}+2Ba^2\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(\right.}{15}}{d}$
risch	$\frac{7\sin(dx+c)a^2A}{8d}+\frac{\sin(dx+c)Ba^2}{4d}+\frac{a^2\cos(6dx+6c)B}{192d}-\frac{\sin(5dx+5c)a^2A}{80d}-\frac{\sin(5dx+5c)Ba^2}{40d}-\frac{a^2\cos(4dx+4c)}{16d}$
norman	$\frac{\frac{(8a^2A+8Ba^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(8a^2A+8Ba^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(2a^2A+Ba^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(2a^2A+Ba^2)\left(\tan^{10}\left(\frac{dx}{2}+\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*cos(d*x+c)^4*a^2-1/2*A*cos
(d*x+c)^4*a^2+2*B*a^2*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*s
```

```
in(d*x+c))+a^2*A*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*
x+c))+B*a^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 1.23

$$-\frac{5\,Ba^2\sin\left(dx+c\right)^6+6\,(A+2\,B)a^2\sin\left(dx+c\right)^5+15\,Aa^2\sin\left(dx+c\right)^4-20\,Ba^2\sin\left(dx+c\right)^3-15\,(2\,A+B)a^2\sin\left(dx+c\right)^2-30\,Aa^2\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*(A + 2*B)*a^2*sin(d*x + c)^5 + 15*A*a^2*s
in(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 15*(2*A + B)*a^2*sin(d*x + c)^2 -
 30*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.17

$$\frac{5\,Ba^2\cos\left(dx+c\right)^6-15\,(A+B)a^2\cos\left(dx+c\right)^4-2\,\left(3\,(A+2\,B)a^2\cos\left(dx+c\right)^4-2\,(3\,A+B)a^2\cos\left(dx+c\right)^2-4\,(3\,A+B)a^2\right)\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*a^2*cos(d*x + c)^6 - 15*(A + B)*a^2*cos(d*x + c)^4 - 2*(3*(A + 2*
B)*a^2*cos(d*x + c)^4 - 2*(3*A + B)*a^2*cos(d*x + c)^2 - 4*(3*A + B)*a^2)*s
in(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(68) = 136$.
time = 0.47, size = 228, normalized size = 2.92

$$\begin{cases} \frac{2Aa^2\sin^5(c+dx)}{15d}+\frac{Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{2Aa^2\sin^3(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d}-\frac{Aa^2\cos^4(c+dx)}{2d}+\frac{4Ba^2\sin^5(c+dx)}{15d}+\frac{2Ba^2\sin^3(c+dx)\cos^2(c+dx)}{3d}-\frac{Ba^2\sin^2(c+dx)\cos^4(c+dx)}{4d}-\frac{Ba^2\cos^6(c+dx)}{12d}-\frac{Ba^2\cos^4(c+dx)}{4d} & \text{for}\ d\neq 0 \\ x(A+B\sin(c))\left(a\sin(c)+a\right)^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**5/(15*d) + A*a**2*sin(c + d*x)**3*cos(c +
 d*x)**2/(3*d) + 2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c
 + d*x)**2/d - A*a**2*cos(c + d*x)**4/(2*d) + 4*B*a**2*sin(c + d*x)**5/(15*
d) + 2*B*a**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*sin(c + d*x)**
2*cos(c + d*x)**4/(4*d) - B*a**2*cos(c + d*x)**6/(12*d) - B*a**2*cos(c + d*
x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**3, True
))
```

Giac [A]
time = 0.50, size = 116, normalized size = 1.49

$$-\frac{5\,Ba^2\sin\left(dx+c\right)^6+6\,Aa^2\sin\left(dx+c\right)^5+12\,Ba^2\sin\left(dx+c\right)^5+15\,Aa^2\sin\left(dx+c\right)^4-20\,Ba^2\sin\left(dx+c\right)^3-30\,Aa^2\sin\left(dx+c\right)^2-15\,Ba^2\sin\left(dx+c\right)^2-30\,Aa^2\sin\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*a^2*sin(d*x + c)^6 + 6*A*a^2*sin(d*x + c)^5 + 12*B*a^2*sin(d*x +
 c)^5 + 15*A*a^2*sin(d*x + c)^4 - 20*B*a^2*sin(d*x + c)^3 - 30*A*a^2*sin(d*
x + c)^2 - 15*B*a^2*sin(d*x + c)^2 - 30*A*a^2*sin(d*x + c))/d
```

Mupad [B]
time = 9.08, size = 96, normalized size = 1.23

$$-\frac{\frac{A\,a^2\,\sin\left(c+d\,x\right)^4}{2}-\frac{a^2\,\sin\left(c+d\,x\right)^2\,\left(2\,A+B\right)}{2}+\frac{a^2\,\sin\left(c+d\,x\right)^5\,\left(A+2\,B\right)}{5}-\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)^3}{3}+\frac{B\,a^2\,\sin\left(c+d\,x\right)^6}{6}-A\,a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((A*a^2*sin(c + d*x)^4)/2 - (a^2*sin(c + d*x)^2*(2*A + B))/2 + (a^2*sin(c
+ d*x)^5*(A + 2*B))/5 - (2*B*a^2*sin(c + d*x)^3)/3 + (B*a^2*sin(c + d*x)^6)
/6 - A*a^2*sin(c + d*x))/d
```

3.972 $\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{(A-B)(a+a\sin(c+dx))^3}{3ad}+\frac{B(a+a\sin(c+dx))^4}{4a^2d}$$

[Out] 1/3*(A-B)*(a+a*sin(d*x+c))^3/a/d+1/4*B*(a+a*sin(d*x+c))^4/a^2/d

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{B(a\sin(c+dx)+a)^4}{4a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^3}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]

[Out] ((A - B)*(a + a*Sin[c + d*x])^3)/(3*a*d) + (B*(a + a*Sin[c + d*x])^4)/(4*a^2*d)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{ad}$$

$$=\frac{\text{Subst}\left(\int \left((A-B)(a+x)^2+\frac{B(a+x)^3}{a}\right)\,dx,x,a\right.}{ad}$$

$$=\frac{(A-B)(a+a\sin(c+dx))^3}{3ad}+\frac{B(a+a\sin(c+\right.}{4a^2d}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.96

$$\frac{\frac{1}{3}(A-B)(a+a\sin(c+dx))^3+\frac{B(a+a\sin(c+dx))^4}{4a}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (((A - B)*(a + a*Sin[c + d*x])^3)/3 + (B*(a + a*Sin[c + d*x])^4)/(4*a))/(a*
d)
```

Maple [A]
time = 0.15, size = 75, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{B\,a^2\left(\sin^4(dx+c)\right)}{4}+\frac{\left(a^2A+2B\,a^2\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(2a^2A+B\,a^2\right)\left(\sin^2(dx+c)\right)}{2}+a^2A\sin(dx+c)}{d}$
default	$\frac{\frac{B\,a^2\left(\sin^4(dx+c)\right)}{4}+\frac{\left(a^2A+2B\,a^2\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(2a^2A+B\,a^2\right)\left(\sin^2(dx+c)\right)}{2}+a^2A\sin(dx+c)}{d}$
risch	$\frac{5\sin(dx+c)a^2A}{4d}+\frac{\sin(dx+c)B\,a^2}{2d}+\frac{a^2\cos(4dx+4c)B}{32d}-\frac{a^2A\sin(3dx+3c)}{12d}-\frac{\sin(3dx+3c)B\,a^2}{6d}-\frac{a^2\cos(2dx+2c)}{2d}\cdots$
norman	$\frac{\frac{2\left(2a^2A+B\,a^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(2a^2A+B\,a^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(4a^2A+4B\,a^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*B*a^2*sin(d*x+c)^4+1/3*(A*a^2+2*B*a^2)*sin(d*x+c)^3+1/2*(2*A*a^2+B
*a^2)*sin(d*x+c)^2+a^2*A*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 1.33

$$\frac{3\,Ba^2\sin\left(dx+c\right)^4+4\left(A+2\,B\right)a^2\sin\left(dx+c\right)^3+6\left(2\,A+B\right)a^2\sin\left(dx+c\right)^2+12\,Aa^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*(A + 2*B)*a^2*sin(d*x + c)^3 + 6*(2*A + B)
*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.35, size = 72, normalized size = 1.41

$$\frac{3\,Ba^2\cos\left(dx+c\right)^4-12\,(A+B)a^2\cos\left(dx+c\right)^2-4\,\left((A+2\,B)a^2\cos\left(dx+c\right)^2-2\,(2\,A+B)a^2\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*a^2*cos(d*x + c)^4 - 12*(A + B)*a^2*cos(d*x + c)^2 - 4*((A + 2*B)
*a^2*cos(d*x + c)^2 - 2*(2*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(41) = 82$.
time = 0.20, size = 117, normalized size = 2.29

$$\begin{cases} \frac{Aa^2\sin^3\left(c+dx\right)}{3d}+\frac{Aa^2\sin^2\left(c+dx\right)}{d}+\frac{Aa^2\sin\left(c+dx\right)}{d}+\frac{Ba^2\sin^4\left(c+dx\right)}{4d}+\frac{2Ba^2\sin^3\left(c+dx\right)}{3d}+\frac{Ba^2\sin^2\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x(A+B\sin\left(c\right))\left(a\sin\left(c\right)+a\right)^2\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)**2/d + A*a**2
*sin(c + d*x)/d + B*a**2*sin(c + d*x)**4/(4*d) + 2*B*a**2*sin(c + d*x)**3/(
3*d) + B*a**2*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
 + a)**2*cos(c), True))
```

Giac [A]
time = 0.49, size = 88, normalized size = 1.73

$$\frac{3\,Ba^2\sin\left(dx+c\right)^4+4\,Aa^2\sin\left(dx+c\right)^3+8\,Ba^2\sin\left(dx+c\right)^3+12\,Aa^2\sin\left(dx+c\right)^2+6\,Ba^2\sin\left(dx+c\right)^2+12\,Aa^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*a^2*sin(d*x + c)^4 + 4*A*a^2*sin(d*x + c)^3 + 8*B*a^2*sin(d*x + c
)^3 + 12*A*a^2*sin(d*x + c)^2 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

Mupad [B]
time = 9.10, size = 66, normalized size = 1.29

$$\frac{\frac{a^2\sin(c+d\,x)^2\,(2\,A+B)}{2}+\frac{a^2\sin(c+d\,x)^3\,(A+2\,B)}{3}+\frac{B\,a^2\sin(c+d\,x)^4}{4}+A\,a^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((a^2*sin(c + d*x)^2*(2*A + B))/2 + (a^2*sin(c + d*x)^3*(A + 2*B))/3 + (B*a
^2*sin(c + d*x)^4)/4 + A*a^2*sin(c + d*x))/d
```

3.973 $\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{B(a+a\sin(c+dx))^2}{2d}$$

```
[Out] -2*a^2*(A+B)*ln(1-sin(d*x+c))/d-a^2*(A+B)*sin(d*x+c)/d-1/2*B*(a+a*sin(d*x+c
))^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 78}

$$-\frac{a^2(A+B)\sin(c+dx)}{d}-\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*(A + B)*Log[1 - Sin[c + d*x]])/d - (a^2*(A + B)*Sin[c + d*x])/d - (
B*(a + a*Sin[c + d*x])^2)/(2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(-A-B+\frac{2a(A+B)}{a-x}-\frac{B(a+x)}{a}\right)\,dx, x,\right.}{d}$$
$$= -\frac{2a^2(A+B)\log(1-\sin(c+dx))}{d} - \frac{a^2(A+B)}{}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.85

$$\frac{a\left(-2a(A+B)\log(1-\sin(c+dx))-a(A+2B)\sin(c+dx)-\frac{1}{2}aB\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*(-2*a*(A + B)*Log[1 - Sin[c + d*x]] - a*(A + 2*B)*Sin[c + d*x] - (a*B*Si
n[c + d*x]^2)/2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(58) = 116.
time = 0.19, size = 133, normalized size = 2.22

method	result
derivativedivides	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^2\ln(\cos(dx+c))-2a^2A\ln(\cos(dx+c))+2B\,a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c}{d}$
default	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^2\ln(\cos(dx+c))-2a^2A\ln(\cos(dx+c))+2B\,a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c}{d}$
norman	$\frac{-\frac{2B\,a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2B\,a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(A+2B)a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4(A+2B)a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(A+2B)a^2\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
risch	$2ia^2xA+2ia^2xB+\frac{ie^{i(dx+c)}a^2A}{2d}+\frac{ie^{i(dx+c)}Ba^2}{d}-\frac{ia^2e^{-i(dx+c)}A}{2d}-\frac{ia^2e^{-i(dx+c)}B}{d}+\frac{4ia^2Ac}{d}+\frac{4ia^2}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))-B*a^2*ln(cos(d*x+c))-2*a^2*A*ln(cos(d*
x+c))+2*B*a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+a^2*A*(-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+B*a^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.30, size = 52, normalized size = 0.87

$$-\frac{Ba^2\sin\left(dx+c\right)^2+4\,(A+B)a^2\log\left(\sin\left(dx+c\right)-1\right)+2\,(A+2\,B)a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*a^2*sin(d*x + c)^2 + 4*(A + B)*a^2*log(sin(d*x + c) - 1) + 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 54, normalized size = 0.90

$$\frac{Ba^2\cos\left(dx+c\right)^2-4\,(A+B)a^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,(A+2\,B)a^2\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(B*a^2*cos(d*x + c)^2 - 4*(A + B)*a^2*log(-sin(d*x + c) + 1) - 2*(A + 2
*B)*a^2*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\sec\left(c+dx\right)dx+\int 2A\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 2B\sin^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin^{3}\left(c+dx\right)\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sin(c + d*x)*sec(c + d*x),
 x) + Integral(A*sin(c + d*x)**2*sec(c + d*x), x) + Integral(B*sin(c + d*x)
*sec(c + d*x), x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x), x) + Integra
l(B*sin(c + d*x)**3*sec(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(58) = 116.
time = 0.42, size = 220, normalized size = 3.67

$$\frac{2\,(Aa^2+Ba^2)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-4\,(Aa^2+Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{3\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Aa^2+3\,Ba^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] (2*(A*a^2 + B*a^2)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 4*(A*a^2 + B*a^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) - (3*A*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*B*a^2*
tan(1/2*d*x + 1/2*c)^4 + 2*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^2*tan(1/2*d
*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 8*B*a^2*tan(1/2*d*x + 1/2*
c)^2 + 2*A*a^2*tan(1/2*d*x + 1/2*c) + 4*B*a^2*tan(1/2*d*x + 1/2*c) + 3*A*a^
2 + 3*B*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 0.08, size = 63, normalized size = 1.05

$$-\frac{\sin\left(c+d\,x\right)\,\left(a^2\,\left(A+B\right)+B\,a^2\right)+\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(2\,A\,a^2+2\,B\,a^2\right)+\frac{B\,a^2\,{\sin\left(c+d\,x\right)}^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)*(a^2*(A + B) + B*a^2) + log(sin(c + d*x) - 1)*(2*A*a^2 + 2*B
*a^2) + (B*a^2*sin(c + d*x)^2)/2)/d
```

3.974 $\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^2B\log(1-\sin(c+dx))}{d}+\frac{a^3(A+B)}{d(a-a\sin(c+dx))}$$

```
[Out] a^2*B*ln(1-sin(d*x+c))/d+a^3*(A+B)/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2915, 45}

$$\frac{a^3(A+B)}{d(a-a\sin(c+dx))}+\frac{a^2B\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*B*Log[1 - Sin[c + d*x]])/d + (a^3*(A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^3\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{A+B}{(a-x)^2}-\frac{B}{a(a-x)}\right)\,dx, x, a\sin(c+dx\right.}{d}$$
$$= \frac{a^2B\log(1-\sin(c+dx))}{d}+\frac{a^3(A+B)}{d(a-a\sin(c+dx)}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.95

$$\frac{a^3\left(\frac{B\log(1-\sin(c+dx))}{a}+\frac{A+B}{a-a\sin(c+dx)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (a^3*((B*Log[1 - Sin[c + d*x]])/a + (A + B)/(a - a*Sin[c + d*x])))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(43) = 86.
time = 0.25, size = 189, normalized size = 4.40

method	result
risch	$-ia^2xB-\frac{2ia^2Bc}{d}-\frac{2ia^2\mathrm{e}^{i(dx+c)}(A+B)}{d\left(\mathrm{e}^{i(dx+c)}-i\right)^2}+\frac{2a^2\ln\left(\mathrm{e}^{i(dx+c)}-i\right)B}{d}$
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{Ba^2}{2\cos(dx+c)^2}+\frac{a^2A}{\cos(dx+c)^2}+2Ba^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(d}{}\right.}{d}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{Ba^2}{2\cos(dx+c)^2}+\frac{a^2A}{\cos(dx+c)^2}+2Ba^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(d}{}\right.}{d}$
norman	$\frac{-\frac{4a^2A+4Ba^2}{d}-\frac{\left(4a^2A+4Ba^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(10a^2A+10Ba^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(10a^2A+10Ba^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2(}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*B*
a^2/cos(d*x+c)^2+a^2*A/cos(d*x+c)^2+2*B*a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+
```

```
1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*A*(1/2*sin(d*x+c)^3/cos(d
*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*a^2*(1/2*tan(d*x+c)
^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.30, size = 37, normalized size = 0.86

$$\frac{Ba^2 \log(\sin(dx+c)-1) - \frac{(A+B)a^2}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] (B*a^2*log(sin(d*x + c) - 1) - (A + B)*a^2/(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.28

$$-\frac{(A+B)a^2 - (Ba^2 \sin(dx+c) - Ba^2) \log(-\sin(dx+c)+1)}{d \sin(dx+c) - d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -((A + B)*a^2 - (B*a^2*sin(d*x + c) - B*a^2)*log(-sin(d*x + c) + 1))/(d*sin
(d*x + c) - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sec^3(c+dx)\,dx + \int 2A \sin(c+dx) \sec^3(c+dx)\,dx + \int A \sin^2(c+dx) \sec^3(c+dx)\,dx + \int B \sin(c+dx) \sec^3(c+dx)\,dx + \int 2B \sin^2(c+dx) \sec^3(c+dx)\,dx + \int B \sin^3(c+dx) \sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sin(c + d*x)*sec(c + d*
x)**3, x) + Integral(A*sin(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*sin
(c + d*x)*sec(c + d*x)**3, x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x)**
3, x) + Integral(B*sin(c + d*x)**3*sec(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(45) = 90.
time = 0.54, size = 112, normalized size = 2.60

$$-\frac{Ba^2 \log\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right) - 2\,Ba^2 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{3\,Ba^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 2\,Aa^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 8\,Ba^2 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 3\,Ba^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -(B*a^2*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*B*a^2*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + (3*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 2*A*a^2*tan(1/2*d*x + 1/2*c)
- 8*B*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

Mupad [B]
time = 0.06, size = 44, normalized size = 1.02

$$\frac{B\,a^2\,\ln\left(\sin\left(c+d\,x\right)-1\right)}{d}-\frac{A\,a^2+B\,a^2}{d\,\left(\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (B*a^2*log(sin(c + d*x) - 1))/d - (A*a^2 + B*a^2)/(d*(sin(c + d*x) - 1))
```

3.975 $\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}$$

```
[Out] 1/4*a^2*(A-B)*arctanh(sin(d*x+c))/d+1/4*a^4*(A+B)/d/(a-a*sin(d*x+c))^2+1/4*
a^3*(A-B)/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}+\frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - B)*ArcTanh[Sin[c + d*x]])/(4*d) + (a^4*(A + B))/(4*d*(a - a*Sin[c
 + d*x])^2) + (a^3*(A - B))/(4*d*(a - a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^5\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^3}+\frac{A-B}{4a^2(a-x)^2}+\frac{A-B}{4a^2(a^2-x^2)}\right)\,d}{d}$$
$$= \frac{a^4(A+B)}{4d(a-a\sin(c+dx))^2}+\frac{a^3(A-B)}{4d(a-a\sin(c+dx))}$$
$$= \frac{a^2(A-B)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{a^4(A+}{4d(a-a\sin(}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.97

$$\frac{a^5\left(\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{4a^3}+\frac{A+B}{4a(a-a\sin(c+dx))^2}+\frac{A-B}{4a^2(a-a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (a^5*(((A - B)*ArcTanh[Sin[c + d*x]])/(4*a^3) + (A + B)/(4*a*(a - a*Sin[c +
 d*x])^2) + (A - B)/(4*a^2*(a - a*Sin[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(71) = 142$.
time = 0.30, size = 238, normalized size = 3.09

method	result
risch	$\frac{ia^2\left(4iA\,e^{2i(dx+c)}-A\,e^{3i(dx+c)}+B\,e^{3i(dx+c)}+A\,e^{i(dx+c)}-B\,e^{i(dx+c)}\right)}{2d\left(e^{i(dx+c)}-i\right)^4}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)A}{4d}+\frac{a^2\ln\left(e^{i(dx+c)}-i\right)B}{4d}+$
derivativedivides	$\frac{a^2A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{B\,a^2}{4\cos(dx+c)^4}+\frac{a^2A}{2\cos(dx+c)^4}+2B\,a^2\left(\frac{\sin^3}{4\cos}\right.}{}$
default	$\frac{a^2A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{B\,a^2}{4\cos(dx+c)^4}+\frac{a^2A}{2\cos(dx+c)^4}+2B\,a^2\left(\frac{\sin^3}{4\cos}\right.}{}$
norman	$\frac{\frac{(4a^2A+2B\,a^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(4a^2A+2B\,a^2)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(12a^2A+10B\,a^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(12a^2A+10B\,a^2)\left(\tan^{10}\left(\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))+1/4*B*a^2/cos(d*x+c)^4+1/2*a^2*A/cos(d*x+c)^4+2*B*a^2*(1/4*s
in(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*l
n(sec(d*x+c)+tan(d*x+c)))+a^2*A*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+
c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*B*a^2*s
in(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.31, size = 87, normalized size = 1.13

$$\frac{(A-B)a^2\log\left(\sin\left(dx+c\right)+1\right)-(A-B)a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((A-B)a^2\sin(dx+c)-2\,Aa^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/8*((A - B)*a^2*log(sin(d*x + c) + 1) - (A - B)*a^2*log(sin(d*x + c) - 1)
- 2*((A - B)*a^2*sin(d*x + c) - 2*A*a^2)/(sin(d*x + c)^2 - 2*sin(d*x + c) +
 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(73) = 146$.
time = 0.38, size = 161, normalized size = 2.09

$$\frac{2\,(A-B)a^2\sin\left(dx+c\right)-4\,Aa^2+\left((A-B)a^2\cos\left(dx+c\right)^2+2\,(A-B)a^2\sin\left(dx+c\right)-2\,(A-B)a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left((A-B)a^2\cos\left(dx+c\right)^2+2\,(A-B)a^2\sin\left(dx+c\right)-2\,(A-B)a^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{8\,\left(d\cos\left(dx+c\right)^2+2\,d\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8*(2*(A - B)*a^2*sin(d*x + c) - 4*A*a^2 + ((A - B)*a^2*cos(d*x + c)^2 + 2
*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(sin(d*x + c) + 1) - ((A - B)
*a^2*cos(d*x + c)^2 + 2*(A - B)*a^2*sin(d*x + c) - 2*(A - B)*a^2)*log(-sin(
d*x + c) + 1))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 130, normalized size = 1.69

$$\frac{2\left(Aa^{2}-Ba^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-2\left(Aa^{2}-Ba^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{3\,Aa^{2}\sin(dx+c)^{2}-3\,Ba^{2}\sin(dx+c)^{2}-10\,Aa^{2}\sin(dx+c)+10\,Ba^{2}\sin(dx+c)+11\,Aa^{2}-3\,Ba^{2}}{(\sin(dx+c)-1)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*(2*(A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 2*(A*a^2 - B*a^2)*log(
abs(sin(d*x + c) - 1)) + (3*A*a^2*sin(d*x + c)^2 - 3*B*a^2*sin(d*x + c)^2 -
 10*A*a^2*sin(d*x + c) + 10*B*a^2*sin(d*x + c) + 11*A*a^2 - 3*B*a^2)/(sin(d
*x + c) - 1)^2)/d
```

Mupad [B]
time = 0.11, size = 73, normalized size = 0.95

$$\frac{\frac{A\,a^2}{2}-\sin\left(c+d\,x\right)\,\left(\frac{A\,a^2}{4}-\frac{B\,a^2}{4}\right)}{d\,\left({\sin\left(c+d\,x\right)}^2-2\,\sin\left(c+d\,x\right)+1\right)}+\frac{a^2\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(A-B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] ((A*a^2)/2 - sin(c + d*x)*((A*a^2)/4 - (B*a^2)/4))/(d*(sin(c + d*x)^2 - 2*s
in(c + d*x) + 1)) + (a^2*atanh(sin(c + d*x))*(A - B))/(4*d)
```

3.976 $\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^4A}{8d(a-a\sin(c+dx))^2}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}$$

```
[Out] 1/8*a^2*(2*A-B)*arctanh(sin(d*x+c))/d+1/12*a^5*(A+B)/d/(a-a*sin(d*x+c))^3+1
/8*a^4*A/d/(a-a*sin(d*x+c))^2+1/16*a^3*(3*A-B)/d/(a-a*sin(d*x+c))-1/16*a^3*
(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A]

time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^4A}{8d(a-a\sin(c+dx))^2}+\frac{a^3(3A-B)}{16d(a-a\sin(c+dx))}-\frac{a^3(A-B)}{16d(a\sin(c+dx)+a)}+\frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(2*A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^5*(A + B))/(12*d*(a - a*Si
n[c + d*x])^3) + (a^4*A)/(8*d*(a - a*Sin[c + d*x])^2) + (a^3*(3*A - B))/(16
*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(16*d*(a + a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{a^7\mathrm{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^7\mathrm{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^4}+\frac{A}{4a^3(a-x)^3}+\frac{3A-B}{16a^4(a-x)^2}+\right.\right.}{d}$$
$$= \frac{a^5(A+B)}{12d(a-a\sin(c+dx))^3}+\frac{a^4A}{8d(a-a\sin(c+dx))}$$
$$= \frac{a^2(2A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^5(A}{12d(a-a\,\mathrm{si}}$$

Mathematica [A]
time = 0.49, size = 90, normalized size = 0.68

$$\frac{a^2\left(6(2A-B)\tanh^{-1}(\sin(c+dx))-\frac{4(A+B)}{(-1+\sin(c+dx))^3}+\frac{6A}{(-1+\sin(c+dx))^2}+\frac{-9A+3B}{-1+\sin(c+dx)}-\frac{3(A-B)}{1+\sin(c+dx)}\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (a^2*(6*(2*A - B)*ArcTanh[Sin[c + d*x]] - (4*(A + B))/(-1 + Sin[c + d*x])^3
 + (6*A)/(-1 + Sin[c + d*x])^2 + (-9*A + 3*B)/(-1 + Sin[c + d*x]) - (3*(A -
 B))/(1 + Sin[c + d*x])))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(122) = 244.
time = 0.33, size = 304, normalized size = 2.30

method	result
risch	$-\frac{ia^2\left(-24iA\,e^{6i(dx+c)}+6A\,e^{7i(dx+c)}+12iB\,e^{6i(dx+c)}-3B\,e^{7i(dx+c)}-16iA\,e^{4i(dx+c)}-26A\,e^{5i(dx+c)}-40iB\,e^{4i(dx+c)}+13\right.}{12\left(e^{i(dx+c)}+i\right)^2\left(e^{i(dx-\right.}}$
derivativedivides	$\frac{a^2A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^2}{6\cos(dx+c)^6}+\frac{a^2A}{3\cos(dx+c}}{}$

default	$\frac{a^2 A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^2}{6\cos(dx+c)^6}+\frac{a^2 A}{3\cos(dx-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+
c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))+1/6*B*a^2/cos(d*x+c)^6+1/3*a^2*A/cos(d*x
+c)^6+2*B*a^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+
1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c
)))+a^2*A*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16
*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+
B*a^2*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.31, size = 148, normalized size = 1.12

$$\frac{3\,(2\,A-B)a^2\log\left(\sin\left(dx+c\right)+1\right)-3\,(2\,A-B)a^2\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(2\,A-B)a^2\sin(dx+c)^3-6\,(2\,A-B)a^2\sin(dx+c)^2+(2\,A-B)a^2\sin(dx+c)+2\,(4\,A+B)a^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^3+2\,\sin(dx+c)-1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(2*A - B)*a^2*log(sin(d*x + c) + 1) - 3*(2*A - B)*a^2*log(sin(d*x +
 c) - 1) - 2*(3*(2*A - B)*a^2*sin(d*x + c)^3 - 6*(2*A - B)*a^2*sin(d*x + c)
^2 + (2*A - B)*a^2*sin(d*x + c) + 2*(4*A + B)*a^2)/(sin(d*x + c)^4 - 2*sin(
d*x + c)^3 + 2*sin(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(125) = 250$.
time = 0.36, size = 271, normalized size = 2.05

$$-\frac{12\,(2\,A-B)a^2\cos\left(dx+c\right)^2-8\,(A-2\,B)a^2-3\left((2\,A-B)a^2\cos\left(dx+c\right)^4+2\,(2\,A-B)a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,(2\,A-B)a^2\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left((2\,A-B)a^2\cos\left(dx+c\right)^4+2\,(2\,A-B)a^2\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,(2\,A-B)a^2\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(3\,(2\,A-B)a^2\cos\left(dx+c\right)^2-4\,(2\,A-B)a^2\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4+2\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/48*(12*(2*A - B)*a^2*cos(d*x + c)^2 - 8*(A - 2*B)*a^2 - 3*((2*A - B)*a^2
*cos(d*x + c)^4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)
*a^2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*((2*A - B)*a^2*cos(d*x + c)^
4 + 2*(2*A - B)*a^2*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A - B)*a^2*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) - 2*(3*(2*A - B)*a^2*cos(d*x + c)^2 - 4*(2*A
```

```
- B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2*sin(d*x + c)
 - 2*d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.56, size = 209, normalized size = 1.58

$$\frac{6\,(2\,Aa^2-Ba^2)\log\left(|\sin\left(dx+c\right)+1|\right)-6\,(2\,Aa^2-Ba^2)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{6\,(2\,Aa^2\sin(dx+c)-Ba^2\sin(dx+c)+3\,Aa^2-2\,Ba^2)}{\sin(dx+c)+1}+\frac{22\,Aa^2\sin(dx+c)^3-11\,Ba^2\sin(dx+c)^3-84\,Aa^2\sin(dx+c)^2+39\,Ba^2\sin(dx+c)^2+114\,Aa^2\sin(dx+c)-45\,Ba^2\sin(dx+c)-60\,Aa^2+9\,Ba^2}{(\sin(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A*a^2 - B*a^2)*log(abs(sin(d*x + c) + 1)) - 6*(2*A*a^2 - B*a^2)*
log(abs(sin(d*x + c) - 1)) - 6*(2*A*a^2*sin(d*x + c) - B*a^2*sin(d*x + c) +
 3*A*a^2 - 2*B*a^2)/(sin(d*x + c) + 1) + (22*A*a^2*sin(d*x + c)^3 - 11*B*a^
2*sin(d*x + c)^3 - 84*A*a^2*sin(d*x + c)^2 + 39*B*a^2*sin(d*x + c)^2 + 114*
A*a^2*sin(d*x + c) - 45*B*a^2*sin(d*x + c) - 60*A*a^2 + 9*B*a^2)/(sin(d*x +
 c) - 1)^3)/d
```

Mupad [B]
time = 9.22, size = 136, normalized size = 1.03

$$\frac{a^2\,\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(2\,A-B\right)}{8\,d}-\frac{{\sin\left(c+d\,x\right)}^3\,\left(\frac{A\,a^2}{4}-\frac{B\,a^2}{8}\right)-{\sin\left(c+d\,x\right)}^2\,\left(\frac{A\,a^2}{2}-\frac{B\,a^2}{4}\right)+\frac{A\,a^2}{3}+\frac{B\,a^2}{12}+\sin\left(c+d\,x\right)\,\left(\frac{A\,a^2}{12}-\frac{B\,a^2}{24}\right)}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^3+2\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^7,x)
```

```
[Out] (a^2*atanh(sin(c + d*x))*(2*A - B))/(8*d) - (sin(c + d*x)^3*((A*a^2)/4 - (B
*a^2)/8) - sin(c + d*x)^2*((A*a^2)/2 - (B*a^2)/4) + (A*a^2)/3 + (B*a^2)/12
+ sin(c + d*x)*((A*a^2)/12 - (B*a^2)/24))/(d*(2*sin(c + d*x) - 2*sin(c + d*
x)^3 + sin(c + d*x)^4 - 1))
```

3.977 $\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=196

$$\frac{5}{128}a^2(9A+2B)x-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2(9A+2B)\cos(c+dx)\sin(c+dx)}{128d}+\frac{5a^2(9A+2B)\cos^3(c}{192}$$

```
[Out] 5/128*a^2*(9*A+2*B)*x-1/56*a^2*(9*A+2*B)*cos(d*x+c)^7/d+5/128*a^2*(9*A+2*B)
*cos(d*x+c)*sin(d*x+c)/d+5/192*a^2*(9*A+2*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/48
*a^2*(9*A+2*B)*cos(d*x+c)^5*sin(d*x+c)/d-1/9*B*cos(d*x+c)^7*(a+a*sin(d*x+c)
)^2/d-1/72*(9*A+2*B)*cos(d*x+c)^7*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.14, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{(9A+2B)\cos^7(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{72d}+\frac{a^2(9A+2B)\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{5a^2(9A+2B)\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{5a^2(9A+2B)\sin(c+dx)\cos(c+dx)}{128d}+\frac{5}{128}a^2x(9A+2B)-\frac{B\cos^7(c+dx)(a\sin(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (5*a^2*(9*A + 2*B)*x)/128 - (a^2*(9*A + 2*B)*Cos[c + d*x]^7)/(56*d) + (5*a^
2*(9*A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*a^2*(9*A + 2*B)*Cos[c
 + d*x]^3*Sin[c + d*x])/(192*d) + (a^2*(9*A + 2*B)*Cos[c + d*x]^5*Sin[c + d
*x])/(48*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^2)/(9*d) - ((9*A + 2*B
)*Cos[c + d*x]^7*(a^2 + a^2*Sin[c + d*x]))/(72*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{1}{9}(9A+ \\
&= -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^2}{9d}-\frac{(9A+2 \\
&= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}-\frac{B\cos^7(c+dx)(a \\
&= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{a^2(9A+2B)\cos \\
&= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2(9A+2B)\cos \\
&= -\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}+\frac{5a^2(9A+2B)\cos \\
&= \frac{5}{128}a^2(9A+2B)x-\frac{a^2(9A+2B)\cos^7(c+dx)}{56d}
\end{aligned}$$

Mathematica [A]
time = 3.69, size = 216, normalized size = 1.10

$$-\frac{a^2\cos(c+dx)\left(2880A+1900B+\frac{2520(9A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+32(135A+86B)\cos(2(c+dx))+16(108A+59B)\cos(4(c+dx))+288A\cos(6(c+dx))+64B\cos(6(c+dx))-28B\cos(8(c+dx))-13671A\sin(c+dx)-2478B\sin(c+dx)-2457A\sin(3(c+dx))+462B\sin(3(c+dx))-63A\sin(5(c+dx))+546B\sin(5(c+dx))+63A\sin(7(c+dx))+126B\sin(7(c+dx))\right)}{32256d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/32256*(a^2*Cos[c + d*x]*(2880*A + 1900*B + (2520*(9*A + 2*B)*ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 32*(135*A + 86*B)*Cos[2
*(c + d*x)] + 16*(108*A + 59*B)*Cos[4*(c + d*x)] + 288*A*Cos[6*(c + d*x)] +
 64*B*Cos[6*(c + d*x)] - 28*B*Cos[8*(c + d*x)] - 13671*A*Sin[c + d*x] - 247
8*B*Sin[c + d*x] - 2457*A*Sin[3*(c + d*x)] + 462*B*Sin[3*(c + d*x)] - 63*A*
Sin[5*(c + d*x)] + 546*B*Sin[5*(c + d*x)] + 63*A*Sin[7*(c + d*x)] + 126*B*S
in[7*(c + d*x)]))/d
```

Maple [A]
time = 0.58, size = 245, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)-1/7*B*cos(d*x+c)^7*a^2-2/7*A*cos(d*x+c)^7*a^2+2*B*a^2*(-1/
8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/128*d*x+5/128*c)+a^2*A*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*
(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*
c)+B*a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A]
time = 0.39, size = 208, normalized size = 1.06

$$-\frac{18432 \, A a^{2} \cos\left(d x + c\right)^{7} + 9216 \, B a^{2} \cos\left(d x + c\right)^{7} - 21 \, {\left(64 \, \sin\left(2 \, d x + 2 \, c\right)^{3} + 120 \, d x + 120 \, c - 3 \, \sin\left(8 \, d x + 8 \, c\right) - 24 \, \sin\left(4 \, d x + 4 \, c\right)\right)} A a^{2} + 336 \, {\left(4 \, \sin\left(2 \, d x + 2 \, c\right)^{3} - 60 \, d x - 60 \, c - 9 \, \sin\left(4 \, d x + 4 \, c\right) - 48 \, \sin\left(2 \, d x + 2 \, c\right)\right)} A a^{2} - 1024 \, {\left(7 \, \cos\left(d x + c\right)^{9} - 9 \, \cos\left(d x + c\right)^{7}\right)} B a^{2} - 42 \, {\left(64 \, \sin\left(2 \, d x + 2 \, c\right)^{3} + 120 \, d x + 120 \, c - 3 \, \sin\left(8 \, d x + 8 \, c\right) - 24 \, \sin\left(4 \, d x + 4 \, c\right)\right)} B a^{2}}{64512 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/64512*(18432*A*a^2*cos(d*x + c)^7 + 9216*B*a^2*cos(d*x + c)^7 - 21*(64*s
in(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4
*c))*A*a^2 + 336*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c)
 - 48*sin(2*d*x + 2*c))*A*a^2 - 1024*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*
B*a^2 - 42*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) -
24*sin(4*d*x + 4*c))*B*a^2)/d
```

Fricas [A]
time = 0.38, size = 135, normalized size = 0.69

$$\frac{896 \, B a^{2} \cos\left(d x + c\right)^{9} - 2304 \, {\left(A + B\right)} a^{2} \cos\left(d x + c\right)^{7} + 315 \, {\left(9 \, A + 2 \, B\right)} a^{2} d x - 21 \, {\left(48 \, {\left(A + 2 \, B\right)} a^{2} \cos\left(d x + c\right)^{7} - 8 \, {\left(9 \, A + 2 \, B\right)} a^{2} \cos\left(d x + c\right)^{5} - 10 \, {\left(9 \, A + 2 \, B\right)} a^{2} \cos\left(d x + c\right)^{3} - 15 \, {\left(9 \, A + 2 \, B\right)} a^{2} \cos\left(d x + c\right)\right)} \sin\left(d x + c\right)}{8064 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/8064*(896*B*a^2*cos(d*x + c)^9 - 2304*(A + B)*a^2*cos(d*x + c)^7 + 315*(9
*A + 2*B)*a^2*d*x - 21*(48*(A + 2*B)*a^2*cos(d*x + c)^7 - 8*(9*A + 2*B)*a^2
*cos(d*x + c)^5 - 10*(9*A + 2*B)*a^2*cos(d*x + c)^3 - 15*(9*A + 2*B)*a^2*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(180) = 360$.
time = 1.42, size = 719, normalized size = 3.67

$$\begin{cases} \frac{5Aa^2x\sin^8(c+dx)}{128} + \frac{5Aa^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{5Aa^2x\sin^6(c+dx)}{16} + \frac{15Aa^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15Aa^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{5Aa^2x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{15Aa^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5Aa^2x\cos^8(c+dx)}{128} + \frac{5Aa^2x\cos^6(c+dx)}{16} + \frac{5Aa^2\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55Aa^2\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{5Aa^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{73Aa^2\sin^3(c+dx)\cos^5(c+dx)}{384d} + \frac{5Aa^2\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{5Aa^2\sin(c+dx)\cos^7(c+dx)}{128d} + \frac{11Aa^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{2Aa^2\cos^7(c+dx)}{7d} + \frac{5Ba^2x\sin^8(c+dx)}{64} + \frac{5Ba^2x\sin^6(c+dx)\cos^2(c+dx)}{16} + \frac{15Ba^2x\sin^4(c+dx)\cos^4(c+dx)}{32} + \frac{5Ba^2x\sin^2(c+dx)\cos^6(c+dx)}{16} + \frac{5Ba^2x\cos^8(c+dx)}{64} + \frac{5Ba^2\sin^7(c+dx)\cos(c+dx)}{64d} + \frac{55Ba^2\sin^5(c+dx)\cos^3(c+dx)}{192d} + \frac{73Ba^2\sin^3(c+dx)\cos^5(c+dx)}{192d} - \frac{Ba^2\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{5Ba^2\sin(c+dx)\cos^7(c+dx)}{64d} - \frac{2Ba^2\cos^9(c+dx)}{63d} - \frac{Ba^2\cos^7(c+dx)}{7d} & \text{for}\: d \neq 0 \\ x(A + B\sin(c))(a\sin(c) + a)^2\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*A*a**2*x*sin(c + d*x)**8/128 + 5*A*a**2*x*sin(c + d*x)**6*cos(
c + d*x)**2/32 + 5*A*a**2*x*sin(c + d*x)**6/16 + 15*A*a**2*x*sin(c + d*x)**
4*cos(c + d*x)**4/64 + 15*A*a**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 5*A
*a**2*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**2*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 5*A*a**2*x*cos(c + d*x)**8/128 + 5*A*a**2*x*cos(c + d*x)
**6/16 + 5*A*a**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**2*sin(c +
d*x)**5*cos(c + d*x)**3/(384*d) + 5*A*a**2*sin(c + d*x)**5*cos(c + d*x)/(16
*d) + 73*A*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 5*A*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) - 5*A*a**2*sin(c + d*x)*cos(c + d*x)**7/(128*
d) + 11*A*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**2*cos(c + d*x)*
*7/(7*d) + 5*B*a**2*x*sin(c + d*x)**8/64 + 5*B*a**2*x*sin(c + d*x)**6*cos(c
 + d*x)**2/16 + 15*B*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 5*B*a**2*x
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*B*a**2*x*cos(c + d*x)**8/64 + 5*B*a
**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*B*a**2*sin(c + d*x)**5*cos(c +
 d*x)**3/(192*d) + 73*B*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(192*d) - B*a*
*2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 5*B*a**2*sin(c + d*x)*cos(c + d*
x)**7/(64*d) - 2*B*a**2*cos(c + d*x)**9/(63*d) - B*a**2*cos(c + d*x)**7/(7*
d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**2*cos(c)**6, True))
```

Giac [A]
time = 0.55, size = 235, normalized size = 1.20

$$\frac{Ba^2\cos(9\,dx + 9\,c)}{2304\,d} - \frac{Ba^2\sin(6\,dx + 6\,c)}{96\,d} + \frac{5}{128}\,(9\,Aa^2 + 2\,Ba^2)x - \frac{(8\,Aa^2 + Ba^2)\cos(7\,dx + 7\,c)}{1792\,d} - \frac{(2\,Aa^2 + Ba^2)\cos(5\,dx + 5\,c)}{64\,d} - \frac{(18\,Aa^2 + 11\,Ba^2)\cos(3\,dx + 3\,c)}{192\,d} - \frac{(20\,Aa^2 + 13\,Ba^2)\cos(dx + c)}{128\,d} - \frac{(Aa^2 + 2\,Ba^2)\sin(8\,dx + 8\,c)}{1024\,d} + \frac{(5\,Aa^2 - 2\,Ba^2)\sin(4\,dx + 4\,c)}{128\,d} + \frac{(8\,Aa^2 + Ba^2)\sin(2\,dx + 2\,c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^2*cos(9*d*x + 9*c)/d - 1/96*B*a^2*sin(6*d*x + 6*c)/d + 5/128*(9*
A*a^2 + 2*B*a^2)*x - 1/1792*(8*A*a^2 + B*a^2)*cos(7*d*x + 7*c)/d - 1/64*(2*
A*a^2 + B*a^2)*cos(5*d*x + 5*c)/d - 1/192*(18*A*a^2 + 11*B*a^2)*cos(3*d*x +
 3*c)/d - 1/128*(20*A*a^2 + 13*B*a^2)*cos(d*x + c)/d - 1/1024*(A*a^2 + 2*B*
```

```
a^2)*sin(8*d*x + 8*c)/d + 1/128*(5*A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/
32*(8*A*a^2 + B*a^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.81, size = 622, normalized size = 3.17

$$\frac{5\,a^2\,\mathrm{atan}\left(\frac{5\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,A+2\,B\right)}{64\,\left(\frac{45\,A\,a^2}{64}+\frac{5\,B\,a^2}{32}\right)}\right)\,\left(9\,A+2\,B\right)}{64\,d}-\frac{5\,a^2\,\left(9\,A+2\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}-\frac{\frac{4\,A\,a^2}{7}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{83\,A\,a^2}{64}-\frac{5\,B\,a^2}{32}\right)+\frac{22\,B\,a^2}{63}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}\,\left(4\,A\,a^2+2\,B\,a^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(8\,A\,a^2+8\,B\,a^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}\,\left(\frac{8\,A\,a^2}{7}+\frac{8\,B\,a^2}{7}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}\,\left(32\,A\,a^2+4\,B\,a^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}\,\left(24\,A\,a^2+24\,B\,a^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(24\,A\,a^2+\frac{16\,B\,a^2}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(40\,A\,a^2+40\,B\,a^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}\,\left(\frac{88\,A\,a^2}{7}+\frac{32\,B\,a^2}{7}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}\,\left(\frac{83\,A\,a^2}{64}-\frac{5\,B\,a^2}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}\,\left(\frac{149\,A\,a^2}{32}-\frac{83\,B\,a^2}{16}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{149\,A\,a^2}{32}-\frac{83\,B\,a^2}{16}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}\,\left(\frac{189\,A\,a^2}{32}+\frac{191\,B\,a^2}{48}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}\,\left(\frac{189\,A\,a^2}{32}+\frac{191\,B\,a^2}{48}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}\,\left(\frac{409\,A\,a^2}{32}+\frac{145\,B\,a^2}{16}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{409\,A\,a^2}{32}+\frac{145\,B\,a^2}{16}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}+9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}+36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*atan((5*a^2*tan(c/2 + (d*x)/2)*(9*A + 2*B))/(64*((45*A*a^2)/64 + (5*
B*a^2)/32)))*(9*A + 2*B))/(64*d) - (5*a^2*(9*A + 2*B)*(atan(tan(c/2 + (d*x)
/2)) - (d*x)/2))/(64*d) - ((4*A*a^2)/7 - tan(c/2 + (d*x)/2)*((83*A*a^2)/64
- (5*B*a^2)/32) + (22*B*a^2)/63 + tan(c/2 + (d*x)/2)^16*(4*A*a^2 + 2*B*a^2)
 + tan(c/2 + (d*x)/2)^14*(8*A*a^2 + 8*B*a^2) + tan(c/2 + (d*x)/2)^2*((8*A*a
^2)/7 + (8*B*a^2)/7) + tan(c/2 + (d*x)/2)^8*(32*A*a^2 + 4*B*a^2) + tan(c/2
+ (d*x)/2)^6*(24*A*a^2 + 24*B*a^2) + tan(c/2 + (d*x)/2)^12*(24*A*a^2 + (16*
B*a^2)/3) + tan(c/2 + (d*x)/2)^10*(40*A*a^2 + 40*B*a^2) + tan(c/2 + (d*x)/2
)^4*((88*A*a^2)/7 + (32*B*a^2)/7) + tan(c/2 + (d*x)/2)^17*((83*A*a^2)/64 -
(5*B*a^2)/32) - tan(c/2 + (d*x)/2)^5*((149*A*a^2)/32 - (83*B*a^2)/16) + tan
(c/2 + (d*x)/2)^13*((149*A*a^2)/32 - (83*B*a^2)/16) - tan(c/2 + (d*x)/2)^3*
((189*A*a^2)/32 + (191*B*a^2)/48) + tan(c/2 + (d*x)/2)^15*((189*A*a^2)/32 +
 (191*B*a^2)/48) - tan(c/2 + (d*x)/2)^7*((409*A*a^2)/32 + (145*B*a^2)/16) +
 tan(c/2 + (d*x)/2)^11*((409*A*a^2)/32 + (145*B*a^2)/16))/(d*(9*tan(c/2 + (
d*x)/2)^2 + 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 + 126*tan(c/2
 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 + 84*tan(c/2 + (d*x)/2)^12 + 36*t
an(c/2 + (d*x)/2)^14 + 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 + 1)
)
```

3.978 $\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{1}{16}a^2(7A+2B)x-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7A+2B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^2(7A+2B)\cos^3(c+d$$

```
[Out] 1/16*a^2*(7*A+2*B)*x-1/30*a^2*(7*A+2*B)*cos(d*x+c)^5/d+1/16*a^2*(7*A+2*B)*c
os(d*x+c)*sin(d*x+c)/d+1/24*a^2*(7*A+2*B)*cos(d*x+c)^3*sin(d*x+c)/d-1/7*B*c
os(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/42*(7*A+2*B)*cos(d*x+c)^5*(a^2+a^2*sin(d
*x+c))/d
```

Rubi [A]
time = 0.12, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{(7A+2B)\cos^5(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{42d}+\frac{a^2(7A+2B)\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a^2(7A+2B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}a^2x(7A+2B)-\frac{B\cos^5(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 2*B)*x)/16 - (a^2*(7*A + 2*B)*Cos[c + d*x]^5)/(30*d) + (a^2*(7*
A + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^2*(7*A + 2*B)*Cos[c + d*x]^
3*Sin[c + d*x])/(24*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^2)/(7*d) -
((7*A + 2*B)*Cos[c + d*x]^5*(a^2 + a^2*Sin[c + d*x]))/(42*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{1}{7}(7A\cdots \\
&= -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^2}{7d}-\frac{(7A+\cdots}{} \\
&= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}-\frac{B\cos^5(c+dx)\cdots}{} \\
&= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7A+2B)\,\mathrm{co}\cdots}{} \\
&= -\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}+\frac{a^2(7A+2B)\,\mathrm{co}\cdots}{} \\
&= \frac{1}{16}a^2(7A+2B)x-\frac{a^2(7A+2B)\cos^5(c+dx)}{30d}\cdots
\end{aligned}
$$

Mathematica [A]
time = 1.40, size = 171, normalized size = 1.04

$$
-\frac{a^2\cos(c+dx)\left(504A+354B+\frac{420(7A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+(672A+447B)\cos(2(c+dx))+6(28A+13B)\cos(4(c+dx))-15B\cos(6(c+dx))-1645A\sin(c+dx)-350B\sin(c+dx)-140A\sin(3(c+dx))+140B\sin(3(c+dx))+35A\sin(5(c+dx))+70B\sin(5(c+dx))\right)}{3360d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/3360*(a^2*Cos[c + d*x]*(504*A + 354*B + (420*(7*A + 2*B)*ArcSin[Sqrt[1 -
 Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + (672*A + 447*B)*Cos[2*(c +
d*x)] + 6*(28*A + 13*B)*Cos[4*(c + d*x)] - 15*B*Cos[6*(c + d*x)] - 1645*A*S
in[c + d*x] - 350*B*Sin[c + d*x] - 140*A*Sin[3*(c + d*x)] + 140*B*Sin[3*(c
+ d*x)] + 35*A*Sin[5*(c + d*x)] + 70*B*Sin[5*(c + d*x)]))/d
```

Maple [A]
time = 0.39, size = 215, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5
*B*cos(d*x+c)^5*a^2-2/5*A*cos(d*x+c)^5*a^2+2*B*a^2*(-1/6*sin(d*x+c)*cos(d*x
+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+a^2*A*
(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+1/16*d*x+1/16*c)+B*a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A]
time = 0.31, size = 171, normalized size = 1.04

$$-\frac{2688\,Aa^2\cos\left(dx+c\right)^5+1344\,Ba^2\cos\left(dx+c\right)^5-35\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Aa^2-210\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^2-192\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)Ba^2-70\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Ba^2}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6720*(2688*A*a^2*cos(d*x + c)^5 + 1344*B*a^2*cos(d*x + c)^5 - 35*(4*sin(
2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^2 - 210*(12*d*x +
12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 192*(5*cos(d*x + c)^7
 - 7*cos(d*x + c)^5)*B*a^2 - 70*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*s
in(4*d*x + 4*c))*B*a^2)/d
```

Fricas [A]
time = 0.37, size = 115, normalized size = 0.70

$$\frac{240\,Ba^2\cos\left(dx+c\right)^7-672\,(A+B)a^2\cos\left(dx+c\right)^5+105\,(7\,A+2\,B)a^2dx-35\left(8\,(A+2\,B)a^2\cos\left(dx+c\right)^5-2\,(7\,A+2\,B)a^2\cos\left(dx+c\right)^3-3\,(7\,A+2\,B)a^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(240*B*a^2*cos(d*x + c)^7 - 672*(A + B)*a^2*cos(d*x + c)^5 + 105*(7*
A + 2*B)*a^2*d*x - 35*(8*(A + 2*B)*a^2*cos(d*x + c)^5 - 2*(7*A + 2*B)*a^2*c
os(d*x + c)^3 - 3*(7*A + 2*B)*a^2*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(146) = 292$.
time = 0.70, size = 539, normalized size = 3.27

$$\begin{cases} \frac{A a^2 x \sin^6(c+dx)}{16} + \frac{3 A a^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3 A a^2 x \sin^4(c+dx)}{8} + \frac{3 A a^2 x \sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3 A a^2 x \sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{A a^2 x \cos^6(c+dx)}{16} + \frac{3 A a^2 x \cos^4(c+dx)}{8} + \frac{A a^2 \sin^5(c+dx)\cos(c+dx)}{16d} + \frac{A a^2 \sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{3 A a^2 \sin^3(c+dx)\cos(c+dx)}{8d} - \frac{A a^2 \sin(c+dx)\cos^5(c+dx)}{16d} + \frac{5 A a^2 \sin(c+dx)\cos^3(c+dx)}{8d} - \frac{2 A a^2 \cos^5(c+dx)}{5d} + \frac{B a^2 x \sin^6(c+dx)}{8} + \frac{3 B a^2 x \sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3 B a^2 x \sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{B a^2 x \cos^6(c+dx)}{8} + \frac{B a^2 \sin^5(c+dx)\cos(c+dx)}{8d} + \frac{B a^2 \sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{B a^2 \sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{B a^2 \sin(c+dx)\cos^5(c+dx)}{8d} - \frac{2 B a^2 \cos^7(c+dx)}{35d} - \frac{B a^2 \cos^5(c+dx)}{5d} & \text{for}\ d \neq 0 \\ x(A + B\sin(c))(a\sin(c) + a)^2\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**6/16 + 3*A*a**2*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 3*A*a**2*x*sin(c + d*x)**4/8 + 3*A*a**2*x*sin(c + d*x)**2*cos
(c + d*x)**4/16 + 3*A*a**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + A*a**2*x*c
os(c + d*x)**6/16 + 3*A*a**2*x*cos(c + d*x)**4/8 + A*a**2*sin(c + d*x)**5*c
os(c + d*x)/(16*d) + A*a**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 3*A*a**
2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*sin(c + d*x)*cos(c + d*x)**5/
(16*d) + 5*A*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**2*cos(c + d*x
)**5/(5*d) + B*a**2*x*sin(c + d*x)**6/8 + 3*B*a**2*x*sin(c + d*x)**4*cos(c
+ d*x)**2/8 + 3*B*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + B*a**2*x*cos(c
 + d*x)**6/8 + B*a**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + B*a**2*sin(c + d
*x)**3*cos(c + d*x)**3/(3*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 2*B*a**2*cos(c + d*x)**7/(35
*d) - B*a**2*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**4, True))
```

Giac [A]
time = 0.58, size = 192, normalized size = 1.16

$$\frac{Ba^2\cos(7\,dx+7\,c)}{448\,d} + \frac{1}{16}\left(7\,Aa^2+2\,Ba^2\right)x - \frac{\left(8\,Aa^2+3\,Ba^2\right)\cos(5\,dx+5\,c)}{320\,d} - \frac{\left(8\,Aa^2+5\,Ba^2\right)\cos(3\,dx+3\,c)}{64\,d} - \frac{\left(16\,Aa^2+11\,Ba^2\right)\cos(dx+c)}{64\,d} - \frac{\left(Aa^2+2\,Ba^2\right)\sin(6\,dx+6\,c)}{192\,d} + \frac{\left(Aa^2-2\,Ba^2\right)\sin(4\,dx+4\,c)}{64\,d} + \frac{\left(17\,Aa^2+2\,Ba^2\right)\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/448*B*a^2*cos(7*d*x + 7*c)/d + 1/16*(7*A*a^2 + 2*B*a^2)*x - 1/320*(8*A*a^
2 + 3*B*a^2)*cos(5*d*x + 5*c)/d - 1/64*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)
/d - 1/64*(16*A*a^2 + 11*B*a^2)*cos(d*x + c)/d - 1/192*(A*a^2 + 2*B*a^2)*si
n(6*d*x + 6*c)/d + 1/64*(A*a^2 - 2*B*a^2)*sin(4*d*x + 4*c)/d + 1/64*(17*A*a
^2 + 2*B*a^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 11.00, size = 494, normalized size = 2.99

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(7\,A+2\,B)}{8\left(\frac{7\,Aa^2}{8}+\frac{Ba^2}{4}\right)}\right)(7\,A+2\,B)}{8\,d} - \frac{a^2\,(7\,A+2\,B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{8\,d} - \frac{\frac{4\,Aa^2}{5} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{9\,Aa^2}{8}-\frac{Ba^2}{4}\right) + \frac{18\,Ba^2}{35} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(4\,Aa^2+2\,Ba^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(12\,Aa^2+2\,Ba^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(8\,Aa^2+8\,Ba^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{8\,Aa^2}{5}+\frac{8\,Ba^2}{5}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{9\,Aa^2}{8}-\frac{Ba^2}{4}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(16\,Aa^2+16\,Ba^2\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(\frac{29\,Aa^2}{6}+\frac{11\,Ba^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{29\,Aa^2}{6}+\frac{11\,Ba^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{44\,Aa^2}{5}+\frac{14\,Ba^2}{5}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{23\,Aa^2}{24}-\frac{31\,Ba^2}{12}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{23\,Aa^2}{24}-\frac{31\,Ba^2}{12}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(7*A + 2*B))/(8*((7*A*a^2)/8 + (B*a^2)/4)
))*(7*A + 2*B))/(8*d) - (a^2*(7*A + 2*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/
2))/(8*d) - ((4*A*a^2)/5 - tan(c/2 + (d*x)/2)*((9*A*a^2)/8 - (B*a^2)/4) + (
18*B*a^2)/35 + tan(c/2 + (d*x)/2)^12*(4*A*a^2 + 2*B*a^2) + tan(c/2 + (d*x)/
2)^8*(12*A*a^2 + 2*B*a^2) + tan(c/2 + (d*x)/2)^10*(8*A*a^2 + 8*B*a^2) + tan
(c/2 + (d*x)/2)^2*((8*A*a^2)/5 + (8*B*a^2)/5) + tan(c/2 + (d*x)/2)^13*((9*A
*a^2)/8 - (B*a^2)/4) + tan(c/2 + (d*x)/2)^6*(16*A*a^2 + 16*B*a^2) - tan(c/2
 + (d*x)/2)^3*((29*A*a^2)/6 + (11*B*a^2)/3) + tan(c/2 + (d*x)/2)^11*((29*A*
a^2)/6 + (11*B*a^2)/3) + tan(c/2 + (d*x)/2)^4*((44*A*a^2)/5 + (14*B*a^2)/5)
 - tan(c/2 + (d*x)/2)^5*((23*A*a^2)/24 - (31*B*a^2)/12) + tan(c/2 + (d*x)/2
)^9*((23*A*a^2)/24 - (31*B*a^2)/12))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/
2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan
(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1))
```

3.979 $\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{1}{8}a^2(5A+2B)x-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}+\frac{a^2(5A+2B)\cos(c+dx)\sin(c+dx)}{8d}-\frac{B\cos^3(c+dx)(a+a\sin}{5d}$$

```
[Out] 1/8*a^2*(5*A+2*B)*x-1/12*a^2*(5*A+2*B)*cos(d*x+c)^3/d+1/8*a^2*(5*A+2*B)*cos
(d*x+c)*sin(d*x+c)/d-1/5*B*cos(d*x+c)^3*(a+a*sin(d*x+c))^2/d-1/20*(5*A+2*B)
*cos(d*x+c)^3*(a^2+a^2*sin(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{(5A+2B)\cos^3(c+dx)\left(a^2\sin(c+dx)+a^2\right)}{20d}+\frac{a^2(5A+2B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(5A+2B)-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A + 2*B)*x)/8 - (a^2*(5*A + 2*B)*Cos[c + d*x]^3)/(12*d) + (a^2*(5*A
 + 2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(5*d) - ((5*A + 2*B)*Cos[c + d*x]^3*(a^2 + a^2*Sin[c + d*x]))/(20
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{1}{5}(5A+\\ &= -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^2}{5d}-\frac{(5A+2\\ &= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}-\frac{B\cos^3(c+dx)(a\\ &= -\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}+\frac{a^2(5A+2B)\cos\\ &= \frac{1}{8}a^2(5A+2B)x-\frac{a^2(5A+2B)\cos^3(c+dx)}{12d}+\end{aligned}$$

Mathematica [A]
time = 0.86, size = 133, normalized size = 0.99

$$-\frac{a^2\cos(c+dx)\left(80A+62B+\frac{60(5A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+8(10A+7B)\cos(2(c+dx))-6B\cos(4(c+dx))-135A\sin(c+dx)-30B\sin(c+dx)+15A\sin(3(c+dx))+30B\sin(3(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/240*(a^2*Cos[c + d*x]*(80*A + 62*B + (60*(5*A + 2*B)*ArcSin[Sqrt[1 - Sin
[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 8*(10*A + 7*B)*Cos[2*(c + d*x)]
 - 6*B*Cos[4*(c + d*x)] - 135*A*Sin[c + d*x] - 30*B*Sin[c + d*x] + 15*A*Sin
[3*(c + d*x)] + 30*B*Sin[3*(c + d*x)]))/d
```

Maple [A]

time = 0.26, size = 182, normalized size = 1.36

method	result
risch	$\frac{5a^2xA}{8}+\frac{a^2xB}{4}-\frac{a^2\cos(dx+c)A}{2d}-\frac{3a^2\cos(dx+c)B}{8d}+\frac{a^2\cos(5dx+5c)B}{80d}-\frac{\sin(4dx+4c)a^2A}{32d}-\frac{\sin(4dx+4c)B}{16d}$
derivativedivides	$\frac{a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{B\left(\cos^3(dx+c)\right)a^2}{3}-\frac{2A\left(\cos^3(dx+c)\right)a^2}{3}+2Ba^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos}{8}\right.}{}$
default	$\frac{a^2A\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{B\left(\cos^3(dx+c)\right)a^2}{3}-\frac{2A\left(\cos^3(dx+c)\right)a^2}{3}+2Ba^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos}{8}\right.}{}$
norman	$\frac{(\frac{5}{8}a^2A+\frac{1}{4}Ba^2)x+(\frac{5}{8}a^2A+\frac{1}{4}Ba^2)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{25}{4}a^2A+\frac{5}{2}Ba^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{25}{4}a^2A+\frac{5}{2}Ba^2)x\left(\tan^6\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-1/3*B*cos(d*x+c)^3*a^2
-2/3*A*cos(d*x+c)^3*a^2+2*B*a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c
)*cos(d*x+c)+1/8*d*x+1/8*c)+a^2*A*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x
+c)*cos(d*x+c)+1/8*d*x+1/8*c)+B*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*co
s(d*x+c)^3))
```

Maxima [A]

time = 0.31, size = 134, normalized size = 1.00

$$-\frac{320\,Aa^2\cos(dx+c)^3+160\,Ba^2\cos(dx+c)^3-15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))Aa^2-120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2-32\,(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3)Ba^2-30\,(4\,dx+4\,c-\sin(4\,dx+4\,c))Ba^2}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/480*(320*A*a^2*cos(d*x + c)^3 + 160*B*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4
*c - sin(4*d*x + 4*c))*A*a^2 - 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 -
 32*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*B*a^2 - 30*(4*d*x + 4*c - sin(4*d
*x + 4*c))*B*a^2)/d
```

Fricas [A]

time = 0.37, size = 95, normalized size = 0.71

$$\frac{24\,Ba^2\cos(dx+c)^5-80\,(A+B)a^2\cos(dx+c)^3+15\,(5\,A+2\,B)a^2dx-15\,(2\,(A+2\,B)a^2\cos(dx+c)^3-(5\,A+2\,B)a^2\cos(dx+c))\sin(dx+c)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(24*B*a^2*cos(d*x + c)^5 - 80*(A + B)*a^2*cos(d*x + c)^3 + 15*(5*A +
2*B)*a^2*d*x - 15*(2*(A + 2*B)*a^2*cos(d*x + c)^3 - (5*A + 2*B)*a^2*cos(d*x
 + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(117) = 234$.
time = 0.33, size = 371, normalized size = 2.77

$$\begin{cases} \frac{Aa^2x\sin^4(c+dx)}{8} + \frac{Aa^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^2x\sin^2(c+dx)}{2} + \frac{Aa^2x\cos^4(c+dx)}{8} + \frac{Aa^2x\cos^2(c+dx)}{2} + \frac{Aa^2\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{Aa^2\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{Aa^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{2Aa^2\cos^3(c+dx)}{3d} + \frac{Ba^2x\sin^4(c+dx)}{4} + \frac{Ba^2x\sin^2(c+dx)\cos^2(c+dx)}{2} + \frac{Ba^2x\cos^4(c+dx)}{4} + \frac{Ba^2\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{Ba^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{Ba^2\sin(c+dx)\cos^3(c+dx)}{4d} - \frac{2Ba^2\cos^5(c+dx)}{15d} - \frac{Ba^2\cos^3(c+dx)}{3d} & \text{for}\ d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^2\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(c + d*x)**4/8 + A*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + A*a**2*x*sin(c + d*x)**2/2 + A*a**2*x*cos(c + d*x)**4/8 + A*a**2*
x*cos(c + d*x)**2/2 + A*a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**2*si
n(c + d*x)*cos(c + d*x)**3/(8*d) + A*a**2*sin(c + d*x)*cos(c + d*x)/(2*d) -
 2*A*a**2*cos(c + d*x)**3/(3*d) + B*a**2*x*sin(c + d*x)**4/4 + B*a**2*x*sin
(c + d*x)**2*cos(c + d*x)**2/2 + B*a**2*x*cos(c + d*x)**4/4 + B*a**2*sin(c
+ d*x)**3*cos(c + d*x)/(4*d) - B*a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d)
 - B*a**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*B*a**2*cos(c + d*x)**5/(15
*d) - B*a**2*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c)
+ a)**2*cos(c)**2, True))
```

Giac [A]
time = 0.52, size = 130, normalized size = 0.97

$$\frac{Ba^2\cos(5dx+5c)}{80d} + \frac{Aa^2\sin(2dx+2c)}{4d} + \frac{1}{8}(5Aa^2+2Ba^2)x - \frac{(8Aa^2+5Ba^2)\cos(3dx+3c)}{48d} - \frac{(4Aa^2+3Ba^2)\cos(dx+c)}{8d} - \frac{(Aa^2+2Ba^2)\sin(4dx+4c)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/80*B*a^2*cos(5*d*x + 5*c)/d + 1/4*A*a^2*sin(2*d*x + 2*c)/d + 1/8*(5*A*a^2
 + 2*B*a^2)*x - 1/48*(8*A*a^2 + 5*B*a^2)*cos(3*d*x + 3*c)/d - 1/8*(4*A*a^2
+ 3*B*a^2)*cos(d*x + c)/d - 1/32*(A*a^2 + 2*B*a^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 10.47, size = 367, normalized size = 2.74

$$\frac{a^2\,\mathrm{atan}\left(\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5A+2B)}{4\left(\frac{5Aa^2}{4}+\frac{Ba^2}{2}\right)}\right)(5A+2B)}{4d} - \frac{a^2(5A+2B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{4d} - \frac{\frac{4Aa^2}{3} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3Aa^2}{4}-\frac{Ba^2}{2}\right) + \frac{14Ba^2}{15} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(4Aa^2+2Ba^2) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{7Aa^2}{2}+3Ba^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{7Aa^2}{2}+3Ba^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{3Aa^2}{4}-\frac{Ba^2}{2}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(8Aa^2+8Ba^2) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{8Aa^2}{3}+\frac{8Ba^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{16Aa^2}{3}+\frac{4Ba^2}{3}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2,x)
```

```
[Out] (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(5*A + 2*B))/(4*((5*A*a^2)/4 + (B*a^2)/2)
))*(5*A + 2*B))/(4*d) - (a^2*(5*A + 2*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/
2))/(4*d) - ((4*A*a^2)/3 - tan(c/2 + (d*x)/2)*((3*A*a^2)/4 - (B*a^2)/2) + (
14*B*a^2)/15 + tan(c/2 + (d*x)/2)^8*(4*A*a^2 + 2*B*a^2) - tan(c/2 + (d*x)/2
)^3*((7*A*a^2)/2 + 3*B*a^2) + tan(c/2 + (d*x)/2)^7*((7*A*a^2)/2 + 3*B*a^2)
+ tan(c/2 + (d*x)/2)^9*((3*A*a^2)/4 - (B*a^2)/2) + tan(c/2 + (d*x)/2)^6*(8*
A*a^2 + 8*B*a^2) + tan(c/2 + (d*x)/2)^2*((8*A*a^2)/3 + (8*B*a^2)/3) + tan(c
/2 + (d*x)/2)^4*((16*A*a^2)/3 + (4*B*a^2)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 +
10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8
+ tan(c/2 + (d*x)/2)^10 + 1))
```

3.980 $\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=55

$$-a^2(A+2B)x+\frac{a^2(A+2B)\cos(c+dx)}{d}+\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^2}{d}$$

```
[Out] -a^2*(A+2*B)*x+a^2*(A+2*B)*cos(d*x+c)/d+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))^2
/d
```

Rubi [A]
time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2934, 2718}

$$\frac{a^2(A+2B)\cos(c+dx)}{d}-\left(a^2x(A+2B)\right)+\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -(a^2*(A + 2*B)*x) + (a^2*(A + 2*B)*Cos[c + d*x])/d + ((A + B)*Sec[c + d*x]
*(a + a*Sin[c + d*x])^2)/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx=\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^2}{d}-(a(A$$

$$=-a^2(A+2B)x+\frac{(A+B)\sec(c+dx)(a+a\sin}{d}$$

$$=-a^2(A+2B)x+\frac{a^2(A+2B)\cos(c+dx)}{d}+\frac{(A}{}$$

Mathematica [A]
time = 0.24, size = 91, normalized size = 1.65

$$\frac{a^2\sec(c+dx)\left(4A+5B+4(A+2B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{\cos^2(c+dx)}+B\cos(2(c+dx))+4A\sin(c+dx)+4B\sin(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*Sec[c + d*x]*(4*A + 5*B + 4*(A + 2*B)*ArcSin[Sqrt[1 - Sin[c + d*x]]/Sq
rt[2]]*Sqrt[Cos[c + d*x]^2] + B*Cos[2*(c + d*x)] + 4*A*Sin[c + d*x] + 4*B*S
in[c + d*x]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(55) = 110.
time = 0.17, size = 123, normalized size = 2.24

method	result
risch	$-a^2xA-2a^2xB+\frac{B\,a^2e^{i(dx+c)}}{2d}+\frac{B\,a^2e^{-i(dx+c)}}{2d}+\frac{4a^2A}{d(e^{i(dx+c)}-i)}+\frac{4a^2B}{d(e^{i(dx+c)}-i)}$
derivativedivides	$\frac{a^2A\tan(dx+c)+\frac{B\,a^2}{\cos(dx+c)}+\frac{2a^2A}{\cos(dx+c)}+2B\,a^2(\tan(dx+c)-dx-c)+a^2A(\tan(dx+c)-dx-c)+B\,a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin\right.}{d}$
default	$\frac{a^2A\tan(dx+c)+\frac{B\,a^2}{\cos(dx+c)}+\frac{2a^2A}{\cos(dx+c)}+2B\,a^2(\tan(dx+c)-dx-c)+a^2A(\tan(dx+c)-dx-c)+B\,a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin\right.}{d}$
norman	$\frac{(a^2A+2B\,a^2)x+(-2a^2A-4B\,a^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a^2A-2B\,a^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a^2A+4B\,a^2)x\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*tan(d*x+c)+B*a^2/cos(d*x+c)+2*a^2*A/cos(d*x+c)+2*B*a^2*(tan(d*x+
c)-d*x-c)+a^2*A*(tan(d*x+c)-d*x-c)+B*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*
x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.54, size = 104, normalized size = 1.89

$$-\frac{(dx+c-\tan(dx+c))Aa^2+2(dx+c-\tan(dx+c))Ba^2-Ba^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-Aa^2\tan(dx+c)-\frac{2Aa^2}{\cos(dx+c)}-\frac{Ba^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -((d*x + c - tan(d*x + c))*A*a^2 + 2*(d*x + c - tan(d*x + c))*B*a^2 - B*a^2
*(1/cos(d*x + c) + cos(d*x + c)) - A*a^2*tan(d*x + c) - 2*A*a^2/cos(d*x + c
) - B*a^2/cos(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(55) = 110.
time = 0.41, size = 128, normalized size = 2.33

$$-\frac{(A+2B)a^2dx - Ba^2\cos(dx+c)^2 - 2(A+B)a^2 + ((A+2B)a^2dx - (2A+3B)a^2)\cos(dx+c) - ((A+2B)a^2dx - Ba^2\cos(dx+c) + 2(A+B)a^2)\sin(dx+c)}{d\cos(dx+c) - d\sin(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -((A + 2*B)*a^2*d*x - B*a^2*cos(d*x + c)^2 - 2*(A + B)*a^2 + ((A + 2*B)*a^2
*d*x - (2*A + 3*B)*a^2)*cos(d*x + c) - ((A + 2*B)*a^2*d*x - B*a^2*cos(d*x +
 c) + 2*(A + B)*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^2(c+dx)\,dx + \int 2A\sin(c+dx)\sec^2(c+dx)\,dx + \int A\sin^2(c+dx)\sec^2(c+dx)\,dx + \int B\sin(c+dx)\sec^2(c+dx)\,dx + \int 2B\sin^2(c+dx)\sec^2(c+dx)\,dx + \int B\sin^3(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sin(c + d*x)*sec(c + d*
x)**2, x) + Integral(A*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*sin
(c + d*x)*sec(c + d*x)**2, x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*sin(c + d*x)**3*sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(55) = 110.
time = 0.44, size = 125, normalized size = 2.27

$$-\frac{(Aa^2 + 2Ba^2)(dx+c) + \frac{2\left(2Aa^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 2Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 2Aa^2 + 3Ba^2\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^2 + 2*B*a^2)*(d*x + c) + 2*(2*A*a^2*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^2
*tan(1/2*d*x + 1/2*c)^2 - B*a^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 + 3*B*a^2)/(
tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c) - 1)
)/d
```

Mupad [B]
time = 9.30, size = 110, normalized size = 2.00

$$-\frac{4\,A\,a^2+6\,B\,a^2+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(4\,A\,a^2+4\,B\,a^2\right)-2\,B\,a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-A\,a^2\,x-2\,B\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (4*A*a^2 + 6*B*a^2 + tan(c/2 + (d*x)/2)^2*(4*A*a^2 + 4*B*a^2) - 2*B*a^2*t
an(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(c/2
+ (d*x)/2)^3 - 1)) - A*a^2*x - 2*B*a^2*x
```

3.981 $\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=73

$$\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d}+\frac{a^2(A-2B)\tan(c+dx)}{3d}$$

```
[Out] 1/3*a^2*(A-2*B)*sec(d*x+c)/d+1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))^2/d+1/
3*a^2*(A-2*B)*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2934, 2748, 3852, 8}

$$\frac{a^2(A-2B)\tan(c+dx)}{3d}+\frac{a^2(A-2B)\sec(c+dx)}{3d}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(A - 2*B)*Sec[c + d*x])/(3*d) + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c +
 d*x])^2)/(3*d) + (a^2*(A - 2*B)*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^2}{3d} + \frac{1}{3}$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+d}{}$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+d}{}$$
$$= \frac{a^2(A-2B)\sec(c+dx)}{3d} + \frac{(A+B)\sec^3(c+d}{}$$

Mathematica [A]
time = 0.02, size = 121, normalized size = 1.66

$$\frac{2a^2A\sec^3(c+dx)}{3d} - \frac{a^2B\sec^3(c+dx)}{3d} + \frac{a^2A\sec^2(c+dx)\tan(c+dx)}{d} + \frac{a^2B\sec(c+dx)\tan^2(c+dx)}{d} - \frac{a^2A\tan^3(c+dx)}{3d} + \frac{2a^2B\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] (2*a^2*A*Sec[c + d*x]^3)/(3*d) - (a^2*B*Sec[c + d*x]^3)/(3*d) + (a^2*A*Sec[
c + d*x]^2*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]*Tan[c + d*x]^2)/d - (a^2*A
*Tan[c + d*x]^3)/(3*d) + (2*a^2*B*Tan[c + d*x]^3)/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(67) = 134$.
time = 0.24, size = 162, normalized size = 2.22

method	result
risch	$-\frac{2\left(-3iB\,a^2 e^{i(dx+c)}+3B\,a^2 e^{2i(dx+c)}+a^2A-2B\,a^2+3iA\,a^2 e^{i(dx+c)}\right)}{3\left(e^{i(dx+c)}-i\right)^3 d}$
derivativedivides	$\frac{-a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{B\,a^2}{3\cos(dx+c)^3}+\frac{2a^2A}{3\cos(dx+c)^3}+\frac{2B\,a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{a^2A\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+B\,a^2\Big(\frac{\sin^4}{3\cos}}{d}$
default	$\frac{-a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{B\,a^2}{3\cos(dx+c)^3}+\frac{2a^2A}{3\cos(dx+c)^3}+\frac{2B\,a^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{a^2A\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+B\,a^2\Big(\frac{\sin^4}{3\cos}}{d}$

norman	$\frac{-\frac{4a^2A-2B\,a^2}{3d}-\frac{(8a^2A+12B\,a^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(12a^2A+10B\,a^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2A\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^2*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*B*a^2/cos(d*x+c)^3+2/3*a
^2*A/cos(d*x+c)^3+2/3*B*a^2*sin(d*x+c)^3/cos(d*x+c)^3+1/3*a^2*A*sin(d*x+c)^
3/cos(d*x+c)^3+B*a^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*
x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.30, size = 108, normalized size = 1.48

$$\frac{Aa^2\tan(dx+c)^3+2\,Ba^2\tan(dx+c)^3+\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Aa^2-\frac{\left(3\,\cos(dx+c)^2-1\right)Ba^2}{\cos(dx+c)^3}+\frac{2\,Aa^2}{\cos(dx+c)^3}+\frac{Ba^2}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(A*a^2*tan(d*x + c)^3 + 2*B*a^2*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*ta
n(d*x + c))*A*a^2 - (3*cos(d*x + c)^2 - 1)*B*a^2/cos(d*x + c)^3 + 2*A*a^2/c
os(d*x + c)^3 + B*a^2/cos(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 120, normalized size = 1.64

$$-\frac{(A-2\,B)a^2\cos(dx+c)^2+(2\,A-B)a^2\cos(dx+c)+(A+B)a^2-\left((A-2\,B)a^2\cos(dx+c)-(A+B)a^2\right)\sin(dx+c)}{3\left(d\cos(dx+c)^2-d\cos(dx+c)+(d\cos(dx+c)+2\,d)\sin(dx+c)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*((A - 2*B)*a^2*cos(d*x + c)^2 + (2*A - B)*a^2*cos(d*x + c) + (A + B)*a
^2 - ((A - 2*B)*a^2*cos(d*x + c) - (A + B)*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^4(c+dx)\,dx+\int 2A\sin(c+dx)\sec^4(c+dx)\,dx+\int A\sin^2(c+dx)\sec^4(c+dx)\,dx+\int B\sin(c+dx)\sec^4(c+dx)\,dx+\int 2B\sin^2(c+dx)\sec^4(c+dx)\,dx+\int B\sin^3(c+dx)\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**4, x) + Integral(2*A*sin(c + d*x)*sec(c + d*
x)**4, x) + Integral(A*sin(c + d*x)**2*sec(c + d*x)**4, x) + Integral(B*sin
(c + d*x)*sec(c + d*x)**4, x) + Integral(2*B*sin(c + d*x)**2*sec(c + d*x)**
4, x) + Integral(B*sin(c + d*x)**3*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.47, size = 78, normalized size = 1.07

$$-\frac{2\left(3\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Aa^{2}-Ba^{2}\right)}{3\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/3*(3*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a
^2*tan(1/2*d*x + 1/2*c) + 2*A*a^2 - B*a^2)/(d*(tan(1/2*d*x + 1/2*c) - 1)^3)
```

Mupad [B]
time = 9.15, size = 77, normalized size = 1.05

$$-\frac{\sqrt{2}\,a^{2}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{B}{2}-\frac{5\,A}{2}+\frac{A\,\cos(c+d\,x)}{2}+\frac{B\,\cos(c+d\,x)}{2}+\frac{3\,A\,\sin(c+d\,x)}{2}-\frac{3\,B\,\sin(c+d\,x)}{2}\right)}{6\,d\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^4,x)
```

```
[Out] -(2^(1/2)*a^2*cos(c/2 + (d*x)/2)*(B/2 - (5*A)/2 + (A*cos(c + d*x))/2 + (B*c
os(c + d*x))/2 + (3*A*sin(c + d*x))/2 - (3*B*sin(c + d*x))/2))/(6*d*cos(c/2
 + pi/4 + (d*x)/2)^3)
```

3.982 $\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=104

$$\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^2}{5d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2}{}$$

```
[Out] 1/15*a^2*(3*A-2*B)*sec(d*x+c)^3/d+1/5*(A+B)*sec(d*x+c)^5*(a+a*sin(d*x+c))^2
/d+1/5*a^2*(3*A-2*B)*tan(d*x+c)/d+1/15*a^2*(3*A-2*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2748, 3852}

$$\frac{a^2(3A-2B)\tan^3(c+dx)}{15d}+\frac{a^2(3A-2B)\tan(c+dx)}{5d}+\frac{a^2(3A-2B)\sec^3(c+dx)}{15d}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(3*A - 2*B)*Sec[c + d*x]^3)/(15*d) + ((A + B)*Sec[c + d*x]^5*(a + a*Si
n[c + d*x])^2)/(5*d) + (a^2*(3*A - 2*B)*Tan[c + d*x])/(5*d) + (a^2*(3*A - 2
*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^6(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^2}{5d} + \frac{1}{5}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+}{}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+}{}$$
$$= \frac{a^2(3A-2B)\sec^3(c+dx)}{15d} + \frac{(A+B)\sec^5(c+}{}$$

Mathematica [A]
time = 0.02, size = 178, normalized size = 1.71

$$\frac{2a^2A\sec^5(c+dx)}{5d}+\frac{a^2B\sec^5(c+dx)}{15d}+\frac{a^2A\sec^4(c+dx)\tan(c+dx)}{d}+\frac{a^2B\sec^3(c+dx)\tan^2(c+dx)}{3d}-\frac{a^2A\sec^2(c+dx)\tan^3(c+dx)}{d}+\frac{2a^2B\sec^2(c+dx)\tan^3(c+dx)}{3d}+\frac{2a^2A\tan^5(c+dx)}{5d}-\frac{4a^2B\tan^5(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*A*Sec[c + d*x]^5)/(5*d) + (a^2*B*Sec[c + d*x]^5)/(15*d) + (a^2*A*Sec
[c + d*x]^4*Tan[c + d*x])/d + (a^2*B*Sec[c + d*x]^3*Tan[c + d*x]^2)/(3*d) -
 (a^2*A*Sec[c + d*x]^2*Tan[c + d*x]^3)/d + (2*a^2*B*Sec[c + d*x]^2*Tan[c +
d*x]^3)/(3*d) + (2*a^2*A*Tan[c + d*x]^5)/(5*d) - (4*a^2*B*Tan[c + d*x]^5)/(
15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(96) = 192.
time = 0.30, size = 231, normalized size = 2.22

method	result
risch	$-\frac{4\left(15iA\,a^2 e^{2i(dx+c)}+12A\,a^2 e^{i(dx+c)}+10B\,a^2 e^{3i(dx+c)}+2iB\,a^2-10iB\,a^2 e^{2i(dx+c)}-8B\,a^2 e^{i(dx+c)}-3iA\,a^2\right)}{15\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^5 d}$
derivativedivides	$\frac{-a^2A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{B\,a^2}{5\cos(dx+c)^5}+\frac{2a^2A}{5\cos(dx+c)^5}+2B\,a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c\right.}{15\cos(dx+c}\right.}{}$
default	$\frac{-a^2A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{B\,a^2}{5\cos(dx+c)^5}+\frac{2a^2A}{5\cos(dx+c)^5}+2B\,a^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c\right.}{15\cos(dx+c}\right.}{}$

norman	$\frac{-\frac{12a^2A+2B\,a^2}{15d}-\frac{2\left(2a^2A+B\,a^2\right)\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(6a^2A+5B\,a^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(18a^2A+13B\,a^2\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}-\frac{2(30}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^2*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*B*a^2
/cos(d*x+c)^5+2/5*a^2*A/cos(d*x+c)^5+2*B*a^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5
+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+a^2*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*
sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*
x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(
d*x+c)))
```

Maxima [A]
time = 0.33, size = 147, normalized size = 1.41

$$\frac{\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)Aa^2+\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)Aa^2+2\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)Ba^2-\frac{\left(5\,\cos(dx+c)^2-3\right)Ba^2}{\cos(dx+c)^5}+\frac{6\,Aa^2}{\cos(dx+c)^5}+\frac{3\,Ba^2}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^2 + (3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^2 + 2*(3*tan(d*x + c)^5 + 5*tan(d*x +
 c)^3)*B*a^2 - (5*cos(d*x + c)^2 - 3)*B*a^2/cos(d*x + c)^5 + 6*A*a^2/cos(d*
x + c)^5 + 3*B*a^2/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.35, size = 113, normalized size = 1.09

$$-\frac{4\,(3\,A-2\,B)a^2\cos\left(dx+c\right)^2-3\,(2\,A-3\,B)a^2-\left(2\,(3\,A-2\,B)a^2\cos\left(dx+c\right)^2-3\,(3\,A-2\,B)a^2\right)\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/15*(4*(3*A - 2*B)*a^2*cos(d*x + c)^2 - 3*(2*A - 3*B)*a^2 - (2*(3*A - 2*B
)*a^2*cos(d*x + c)^2 - 3*(3*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^3 +
 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.49, size = 192, normalized size = 1.85

$$-\frac{\frac{15\,(Aa^2-Ba^2)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{105\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-270\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+360\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-40\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-210\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+50\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+63\,Aa^2-7\,Ba^2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(15*(A*a^2 - B*a^2)/(tan(1/2*d*x + 1/2*c) + 1) + (105*A*a^2*tan(1/2*d
*x + 1/2*c)^4 + 15*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 270*A*a^2*tan(1/2*d*x + 1
/2*c)^3 + 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 360*A*a^2*tan(1/2*d*x + 1/2*c)^
2 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 210*A*a^2*tan(1/2*d*x + 1/2*c) + 50*B
*a^2*tan(1/2*d*x + 1/2*c) + 63*A*a^2 - 7*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^
5)/d
```

Mupad [B]
time = 9.36, size = 175, normalized size = 1.68

$$\frac{2\,a^2\,\left(\frac{5\,B\,\sin(c+d\,x)}{2}-\frac{15\,A\,\cos(c+d\,x)}{4}-\frac{5\,B\,\cos(c+d\,x)}{8}-\frac{15\,A\,\sin(c+d\,x)}{4}-\frac{5\,B}{2}-3\,A\,\cos\left(2\,c+2\,d\,x\right)+\frac{3\,A\,\cos(3\,c+3\,d\,x)}{4}+2\,B\,\cos\left(2\,c+2\,d\,x\right)+\frac{B\,\cos(3\,c+3\,d\,x)}{8}+3\,A\,\sin\left(2\,c+2\,d\,x\right)+\frac{3\,A\,\sin(3\,c+3\,d\,x)}{4}+\frac{B\,\sin(2\,c+2\,d\,x)}{2}-\frac{B\,\sin(3\,c+3\,d\,x)}{2}\right)}{15\,d\,\left(\frac{\cos(3\,c+3\,d\,x)}{4}-\frac{5\,\cos(c+d\,x)}{4}+\sin\left(2\,c+2\,d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^6,x)
```

```
[Out] (2*a^2*((5*B*sin(c + d*x))/2 - (15*A*cos(c + d*x))/4 - (5*B*cos(c + d*x))/8
 - (15*A*sin(c + d*x))/4 - (5*B)/2 - 3*A*cos(2*c + 2*d*x) + (3*A*cos(3*c +
3*d*x))/4 + 2*B*cos(2*c + 2*d*x) + (B*cos(3*c + 3*d*x))/8 + 3*A*sin(2*c + 2
*d*x) + (3*A*sin(3*c + 3*d*x))/4 + (B*sin(2*c + 2*d*x))/2 - (B*sin(3*c + 3*
d*x))/2))/(15*d*(cos(3*c + 3*d*x)/4 - (5*cos(c + d*x))/4 + sin(2*c + 2*d*x)
))
```

3.983 $\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{2a^2(5A-$$

```
[Out] 1/35*a^2*(5*A-2*B)*sec(d*x+c)^5/d+1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))^2
/d+1/7*a^2*(5*A-2*B)*tan(d*x+c)/d+2/21*a^2*(5*A-2*B)*tan(d*x+c)^3/d+1/35*a^
2*(5*A-2*B)*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2748, 3852}

$$\frac{a^2(5A-2B)\tan^5(c+dx)}{35d}+\frac{2a^2(5A-2B)\tan^3(c+dx)}{21d}+\frac{a^2(5A-2B)\tan(c+dx)}{7d}+\frac{a^2(5A-2B)\sec^5(c+dx)}{35d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*A - 2*B)*Sec[c + d*x]^5)/(35*d) + ((A + B)*Sec[c + d*x]^7*(a + a*Si
n[c + d*x])^2)/(7*d) + (a^2*(5*A - 2*B)*Tan[c + d*x])/(7*d) + (2*a^2*(5*A -
 2*B)*Tan[c + d*x]^3)/(21*d) + (a^2*(5*A - 2*B)*Tan[c + d*x]^5)/(35*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^2}{7d} + \frac{1}{7}$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+}{}$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+}{}$$
$$= \frac{a^2(5A-2B)\sec^5(c+dx)}{35d} + \frac{(A+B)\sec^7(c+}{}$$

Mathematica [A]
time = 0.31, size = 130, normalized size = 1.01

$$\frac{a^2((30A+9B)\sec^7(c+dx)+105A\sec^6(c+dx)\tan(c+dx)+21B\sec^5(c+dx)\tan^2(c+dx)-35(5A-2B)\sec^4(c+dx)\tan^3(c+dx)+28(5A-2B)\sec^2(c+dx)\tan^5(c+dx)+8(-5A+2B)\tan^7(c+dx))}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*((30*A + 9*B)*Sec[c + d*x]^7 + 105*A*Sec[c + d*x]^6*Tan[c + d*x] + 21*
B*Sec[c + d*x]^5*Tan[c + d*x]^2 - 35*(5*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x
]^3 + 28*(5*A - 2*B)*Sec[c + d*x]^2*Tan[c + d*x]^5 + 8*(-5*A + 2*B)*Tan[c +
 d*x]^7))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(119) = 238$.
time = 0.31, size = 295, normalized size = 2.29

method	result
risch	$-\frac{16(70iA\,a^2e^{4i(dx+c)}-28iB\,a^2e^{4i(dx+c)}+15iA\,a^2e^{2i(dx+c)}+42B\,a^2e^{5i(dx+c)}+40A\,a^2e^{3i(dx+c)}+20A\,a^2e^{i(dx+c)}-8B}{105\left(e^{i(dx+c)}+i\right)^3\left(e^{i(dx+c)}-i\right)^7 d}$
derivativedivides	$-a^2A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{B\,a^2}{7\cos(dx+c)^7}+\frac{2a^2A}{7\cos(dx+c)^7}+2B\,a^2\left(\frac{\sin^3(dx+}{7\cos(dx+}\right.$
default	$-a^2A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{B\,a^2}{7\cos(dx+c)^7}+\frac{2a^2A}{7\cos(dx+c)^7}+2B\,a^2\left(\frac{\sin^3(dx+}{7\cos(dx+}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^2*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*t
an(d*x+c)+1/7*B*a^2/cos(d*x+c)^7+2/7*a^2*A/cos(d*x+c)^7+2*B*a^2*(1/7*sin(d*
x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d
*x+c)^3)+a^2*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^
5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/3
5*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^
4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.31, size = 178, normalized size = 1.38

$$\frac{\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)Aa^{2}+3\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)Aa^{2}+2\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)Ba^{2}-\frac{3\left(7\cos(dx+c)^{2}-5\right)Ba^{2}}{\cos(dx+c)^{7}}+\frac{30\,Aa^{2}}{\cos(dx+c)^{7}}+\frac{15\,Ba^{2}}{\cos(dx+c)^{7}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/105*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^2 +
3*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x +
c))*A*a^2 + 2*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B
*a^2 - 3*(7*cos(d*x + c)^2 - 5)*B*a^2/cos(d*x + c)^7 + 30*A*a^2/cos(d*x + c
)^7 + 15*B*a^2/cos(d*x + c)^7)/d
```

Fricas [A]
time = 0.36, size = 157, normalized size = 1.22

$$-\frac{16\,(5\,A-2\,B)a^{2}\cos\left(dx+c\right)^{4}-8\,(5\,A-2\,B)a^{2}\cos\left(dx+c\right)^{2}-5\,(2\,A-5\,B)a^{2}-\left(8\,(5\,A-2\,B)a^{2}\cos\left(dx+c\right)^{4}-12\,(5\,A-2\,B)a^{2}\cos\left(dx+c\right)^{2}-5\,(5\,A-2\,B)a^{2}\right)\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{5}+2\,d\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/105*(16*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 8*(5*A - 2*B)*a^2*cos(d*x + c)^
2 - 5*(2*A - 5*B)*a^2 - (8*(5*A - 2*B)*a^2*cos(d*x + c)^4 - 12*(5*A - 2*B)*
a^2*cos(d*x + c)^2 - 5*(5*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^5 + 2
*d*cos(d*x + c)^3*sin(d*x + c) - 2*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(119) = 238.

time = 0.48, size = 325, normalized size = 2.52

$$-\frac{\frac{35\left(9\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-6\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,Aa^{2}-5\,Ba^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}+\frac{1365\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+210\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-5775\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12250\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-175\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-14350\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+910\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+10185\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-756\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3955\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+427\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+760\,Aa^{2}-31\,Ba^{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{7}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/840*(35*(9*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*B*a^2*tan(1/2*d*x + 1/2*c)^2
 + 15*A*a^2*tan(1/2*d*x + 1/2*c) - 9*B*a^2*tan(1/2*d*x + 1/2*c) + 8*A*a^2 -
 5*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (1365*A*a^2*tan(1/2*d*x + 1/2*c)^6
 + 210*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 5775*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 1
05*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 12250*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 175*
B*a^2*tan(1/2*d*x + 1/2*c)^4 - 14350*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 910*B*a
^2*tan(1/2*d*x + 1/2*c)^3 + 10185*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 756*B*a^2*
tan(1/2*d*x + 1/2*c)^2 - 3955*A*a^2*tan(1/2*d*x + 1/2*c) + 427*B*a^2*tan(1/
2*d*x + 1/2*c) + 760*A*a^2 - 31*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^7)/d
```

Mupad [B]

time = 12.23, size = 274, normalized size = 2.12

$$-\frac{a^{2}\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{25\,A\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{105\,A\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-\frac{95\,A\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}+\frac{15\,A\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{8}-21\,B\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{105\,B\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{41\,B\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{55\,B\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{9\,B\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{125\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{55\,A\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}-\frac{25\,A\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{2}+5\,A\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+\frac{5\,A\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{2}+\frac{37\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{19\,B\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}-\frac{B\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}+\frac{13\,B\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{4}-B\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\right)}{1680\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)^{3}\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^8,x)
```

```
[Out] -(a^2*cos(c/2 + (d*x)/2)*((25*A*cos((5*c)/2 + (5*d*x)/2))/4 - (105*A*cos((3
*c)/2 + (3*d*x)/2))/4 - (95*A*cos((7*c)/2 + (7*d*x)/2))/8 + (15*A*cos((9*c)
/2 + (9*d*x)/2))/8 - 21*B*cos(c/2 + (d*x)/2) + (105*B*cos((3*c)/2 + (3*d*x)
/2))/8 - (41*B*cos((5*c)/2 + (5*d*x)/2))/8 + (55*B*cos((7*c)/2 + (7*d*x)/2)
)/16 + (9*B*cos((9*c)/2 + (9*d*x)/2))/16 - (125*A*sin(c/2 + (d*x)/2))/2 + (
55*A*sin((3*c)/2 + (3*d*x)/2))/2 - (25*A*sin((5*c)/2 + (5*d*x)/2))/2 + 5*A*
sin((7*c)/2 + (7*d*x)/2) + (5*A*sin((9*c)/2 + (9*d*x)/2))/2 + (37*B*sin(c/2
 + (d*x)/2))/4 + (19*B*sin((3*c)/2 + (3*d*x)/2))/4 - (B*sin((5*c)/2 + (5*d*
x)/2))/4 + (13*B*sin((7*c)/2 + (7*d*x)/2))/4 - B*sin((9*c)/2 + (9*d*x)/2)))
/(1680*d*cos(c/2 - pi/4 + (d*x)/2)^3*cos(c/2 + pi/4 + (d*x)/2)^7)
```

3.984 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{a^2(7A-2B)\sec^7(c+dx)}{63d}+\frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^2}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+\frac{a^2(7A-2\ldots}{}$$

```
[Out] 1/63*a^2*(7*A-2*B)*sec(d*x+c)^7/d+1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))^2
/d+1/9*a^2*(7*A-2*B)*tan(d*x+c)/d+1/9*a^2*(7*A-2*B)*tan(d*x+c)^3/d+1/15*a^2
*(7*A-2*B)*tan(d*x+c)^5/d+1/63*a^2*(7*A-2*B)*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2748, 3852}

$$\frac{a^2(7A-2B)\tan^7(c+dx)}{63d}+\frac{a^2(7A-2B)\tan^5(c+dx)}{15d}+\frac{a^2(7A-2B)\tan^3(c+dx)}{9d}+\frac{a^2(7A-2B)\tan(c+dx)}{9d}+\frac{a^2(7A-2B)\sec^7(c+dx)}{63d}+\frac{(A+B)\sec^9(c+dx)(a\sin(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(7*A - 2*B)*Sec[c + d*x]^7)/(63*d) + ((A + B)*Sec[c + d*x]^9*(a + a*Si
n[c + d*x])^2)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x])/(9*d) + (a^2*(7*A - 2
*B)*Tan[c + d*x]^3)/(9*d) + (a^2*(7*A - 2*B)*Tan[c + d*x]^5)/(15*d) + (a^2*
(7*A - 2*B)*Tan[c + d*x]^7)/(63*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^2}{9d} + \frac{1}{9}$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c-}{}$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c-}{}$$
$$= \frac{a^2(7A-2B)\sec^7(c+dx)}{63d} + \frac{(A+B)\sec^9(c-}{}$$

Mathematica [A]
time = 0.41, size = 156, normalized size = 1.01

$$\frac{a^2(5(14A+5B)\sec^9(c+dx)+315A\sec^8(c+dx)\tan(c+dx)+45B\sec^7(c+dx)\tan^2(c+dx)-105(7A-2B)\sec^6(c+dx)\tan^3(c+dx)+126(7A-2B)\sec^4(c+dx)\tan^5(c+dx)-72(7A-2B)\sec^2(c+dx)\tan^7(c+dx)+16(7A-2B)\tan^9(c+dx))}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(5*(14*A + 5*B)*Sec[c + d*x]^9 + 315*A*Sec[c + d*x]^8*Tan[c + d*x] + 4
5*B*Sec[c + d*x]^7*Tan[c + d*x]^2 - 105*(7*A - 2*B)*Sec[c + d*x]^6*Tan[c +
d*x]^3 + 126*(7*A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^5 - 72*(7*A - 2*B)*Sec
[c + d*x]^2*Tan[c + d*x]^7 + 16*(7*A - 2*B)*Tan[c + d*x]^9))/(315*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(142) = 284$.
time = 0.39, size = 359, normalized size = 2.33

method	result
risch	$-\frac{32(315iA\,a^2e^{6i(dx+c)}-90iB\,a^2e^{6i(dx+c)}+133iA\,a^2e^{4i(dx+c)}-38iB\,a^2e^{4i(dx+c)}+7iA\,a^2e^{2i(dx+c)}+140A\,a^2e^{5i(dx+c)}}{315}$
derivativedivides	$\frac{-a^2A\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)+\frac{B\,a^2}{9\cos(dx+c)^9}+\frac{2a^2A}{9\cos(dx+c)^9}}{}$
default	$\frac{-a^2A\left(-\frac{128}{315}-\frac{\left(\sec^8(dx+c)\right)}{9}-\frac{8\left(\sec^6(dx+c)\right)}{63}-\frac{16\left(\sec^4(dx+c)\right)}{105}-\frac{64\left(\sec^2(dx+c)\right)}{315}\right)\tan(dx+c)+\frac{B\,a^2}{9\cos(dx+c)^9}+\frac{2a^2A}{9\cos(dx+c)^9}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/d*(-a^2*A*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^
4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*a^2/cos(d*x+c)^9+2/9*a^2*A/cos(d*x+
c)^9+2*B*a^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+
8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+a^2*A*(1/
9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)
^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+B*a^2*(1/9*sin(d*x+c)^4/c
os(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1
/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+
c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.32, size = 207, normalized size = 1.34

$$\frac{\left(35\,\tan\left(dx+c\right)^9+180\,\tan\left(dx+c\right)^7+378\,\tan\left(dx+c\right)^5+420\,\tan\left(dx+c\right)^3+315\,\tan\left(dx+c\right)\right)Aa^2+\left(35\,\tan\left(dx+c\right)^9+135\,\tan\left(dx+c\right)^7+189\,\tan\left(dx+c\right)^5+105\,\tan\left(dx+c\right)^3\right)Aa^2+2\left(35\,\tan\left(dx+c\right)^9+135\,\tan\left(dx+c\right)^7+189\,\tan\left(dx+c\right)^5+105\,\tan\left(dx+c\right)^3\right)Ba^2-\frac{5\left(9\,\cos(dx+c)^2-7\right)Ba^2}{\cos(dx+c)^9}+\frac{70\,Aa^2}{\cos(dx+c)^9}+\frac{35\,Ba^2}{\cos(dx+c)^9}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^2 + (35*tan(d*x + c)^9 + 135*tan(d*x
+ c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^2 + 2*(35*tan(d*x + c
)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^2 -
 5*(9*cos(d*x + c)^2 - 7)*B*a^2/cos(d*x + c)^9 + 70*A*a^2/cos(d*x + c)^9 +
35*B*a^2/cos(d*x + c)^9)/d
```

Fricas [A]
time = 0.38, size = 197, normalized size = 1.28

$$-\frac{32\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^6-16\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^4-4\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^2-7\,(2\,A-7\,B)a^2-\left(16\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^6-24\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^4-10\,(7\,A-2\,B)a^2\cos\left(dx+c\right)^2-7\,(7\,A-2\,B)a^2\right)\sin\left(dx+c\right)}{315\left(d\cos\left(dx+c\right)^7+2\,d\cos\left(dx+c\right)^5\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/315*(32*(7*A - 2*B)*a^2*cos(d*x + c)^6 - 16*(7*A - 2*B)*a^2*cos(d*x + c)
^4 - 4*(7*A - 2*B)*a^2*cos(d*x + c)^2 - 7*(2*A - 7*B)*a^2 - (16*(7*A - 2*B)
*a^2*cos(d*x + c)^6 - 24*(7*A - 2*B)*a^2*cos(d*x + c)^4 - 10*(7*A - 2*B)*a^
2*cos(d*x + c)^2 - 7*(7*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^7 + 2*d
*cos(d*x + c)^5*sin(d*x + c) - 2*d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(142) = 284.
time = 0.47, size = 461, normalized size = 2.99

$$-\frac{\frac{21 \left(435 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 225 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 1470 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 690 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 2060 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 940 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1330 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 590 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 353 \, A a^{2} - 163 \, B a^{2}\right)}{{\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 1\right)}^{5}} + \frac{31185 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{8} + 4725 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{8} - 185220 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 11340 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} + 546840 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 15120 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} - 961380 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} + 3780 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} + 1101618 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 24318 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 828492 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 33852 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 404208 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 19368 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 116172 \, A a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 6732 \, B a^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 16373 \, A a^{2} - 223 \, B a^{2}}{{\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 1\right)}^{9}}}{20160 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/20160*(21*(435*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 225*B*a^2*tan(1/2*d*x + 1/
2*c)^4 + 1470*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 690*B*a^2*tan(1/2*d*x + 1/2*c)
^3 + 2060*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 940*B*a^2*tan(1/2*d*x + 1/2*c)^2 +
 1330*A*a^2*tan(1/2*d*x + 1/2*c) - 590*B*a^2*tan(1/2*d*x + 1/2*c) + 353*A*a
^2 - 163*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^5 + (31185*A*a^2*tan(1/2*d*x + 1
/2*c)^8 + 4725*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 185220*A*a^2*tan(1/2*d*x + 1/
2*c)^7 - 11340*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 546840*A*a^2*tan(1/2*d*x + 1/
2*c)^6 + 15120*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 961380*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 3780*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 1101618*A*a^2*tan(1/2*d*x + 1/
2*c)^4 - 24318*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 828492*A*a^2*tan(1/2*d*x + 1/
2*c)^3 + 33852*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 404208*A*a^2*tan(1/2*d*x + 1/
2*c)^2 - 19368*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 116172*A*a^2*tan(1/2*d*x + 1/
2*c) + 6732*B*a^2*tan(1/2*d*x + 1/2*c) + 16373*A*a^2 - 223*B*a^2)/(tan(1/2*
d*x + 1/2*c) - 1)^9)/d
```

Mupad [B]
time = 12.95, size = 370, normalized size = 2.40

$$-\frac{a^2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{455\,A\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}-\frac{1575\,A\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{32}-35\,A\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)+7\,A\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)-\frac{259\,A\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{32}+\frac{35\,A\,\cos\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{32}-45\,B\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{1755\,B\,\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{64}-\frac{1115\,B\,\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{64}+10\,B\,\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)-2\,B\,\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)+\frac{103\,B\,\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{64}+\frac{25\,B\,\cos\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{64}-\frac{623\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+77\,A\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)-\frac{441\,A\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{175\,A\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}-\frac{35\,A\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{8}+\frac{21\,A\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{8}+\frac{7\,A\,\sin\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{4}+\frac{131\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{49\,B\,\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{27\,B\,\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{16}+\frac{125\,B\,\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}-\frac{25\,B\,\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}+\frac{33\,B\,\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{16}-\frac{B\,\sin\left(\frac{13\,c}{2}+\frac{13\,d\,x}{2}\right)}{2}\right)}{20160\,d\,{\cos\left(\frac{c}{2}-\frac{\pi }{4}+\frac{d\,x}{2}\right)}^5\,{\cos\left(\frac{c}{2}+\frac{\pi }{4}+\frac{d\,x}{2}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^10,x)
```

```
[Out] -(a^2*cos(c/2 + (d*x)/2)*((455*A*cos((5*c)/2 + (5*d*x)/2))/32 - (1575*A*cos
((3*c)/2 + (3*d*x)/2))/32 - 35*A*cos((7*c)/2 + (7*d*x)/2) + 7*A*cos((9*c)/2
 + (9*d*x)/2) - (259*A*cos((11*c)/2 + (11*d*x)/2))/32 + (35*A*cos((13*c)/2
+ (13*d*x)/2))/32 - 45*B*cos(c/2 + (d*x)/2) + (1755*B*cos((3*c)/2 + (3*d*x)
/2))/64 - (1115*B*cos((5*c)/2 + (5*d*x)/2))/64 + 10*B*cos((7*c)/2 + (7*d*x)
/2) - 2*B*cos((9*c)/2 + (9*d*x)/2) + (103*B*cos((11*c)/2 + (11*d*x)/2))/64
+ (25*B*cos((13*c)/2 + (13*d*x)/2))/64 - (623*A*sin(c/2 + (d*x)/2))/4 + 77*
A*sin((3*c)/2 + (3*d*x)/2) - (441*A*sin((5*c)/2 + (5*d*x)/2))/8 + (175*A*si
n((7*c)/2 + (7*d*x)/2))/8 - (35*A*sin((9*c)/2 + (9*d*x)/2))/8 + (21*A*sin((
```

```
11*c)/2 + (11*d*x)/2))/8 + (7*A*sin((13*c)/2 + (13*d*x)/2))/4 + (131*B*sin(
c/2 + (d*x)/2))/8 + (49*B*sin((3*c)/2 + (3*d*x)/2))/8 + (27*B*sin((5*c)/2 +
 (5*d*x)/2))/16 + (125*B*sin((7*c)/2 + (7*d*x)/2))/16 - (25*B*sin((9*c)/2 +
 (9*d*x)/2))/16 + (33*B*sin((11*c)/2 + (11*d*x)/2))/16 - (B*sin((13*c)/2 +
(13*d*x)/2))/2))/(20160*d*cos(c/2 - pi/4 + (d*x)/2)^5*cos(c/2 + pi/4 + (d*x
)/2)^9)
```

3.985 $\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{a^2(9A-2B)\sec^9(c+dx)}{99d}+\frac{(A+B)\sec^{11}(c+dx)(a+a\sin(c+dx))^2}{11d}+\frac{a^2(9A-2B)\tan(c+dx)}{11d}+\frac{4a^2(9A$$

```
[Out] 1/99*a^2*(9*A-2*B)*sec(d*x+c)^9/d+1/11*(A+B)*sec(d*x+c)^11*(a+a*sin(d*x+c))
^2/d+1/11*a^2*(9*A-2*B)*tan(d*x+c)/d+4/33*a^2*(9*A-2*B)*tan(d*x+c)^3/d+6/55
*a^2*(9*A-2*B)*tan(d*x+c)^5/d+4/77*a^2*(9*A-2*B)*tan(d*x+c)^7/d+1/99*a^2*(9
*A-2*B)*tan(d*x+c)^9/d
```

Rubi [A]
time = 0.11, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2748, 3852}

$$\frac{a^2(9A-2B)\tan^9(c+dx)}{99d}+\frac{4a^2(9A-2B)\tan^7(c+dx)}{77d}+\frac{6a^2(9A-2B)\tan^5(c+dx)}{55d}+\frac{4a^2(9A-2B)\tan^3(c+dx)}{33d}+\frac{a^2(9A-2B)\tan(c+dx)}{11d}+\frac{a^2(9A-2B)\sec^9(c+dx)}{99d}+\frac{(A+B)\sec^{11}(c+dx)(a\sin(c+dx)+a)^2}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(9*A - 2*B)*Sec[c + d*x]^9)/(99*d) + ((A + B)*Sec[c + d*x]^11*(a + a*S
in[c + d*x])^2)/(11*d) + (a^2*(9*A - 2*B)*Tan[c + d*x])/(11*d) + (4*a^2*(9*
A - 2*B)*Tan[c + d*x]^3)/(33*d) + (6*a^2*(9*A - 2*B)*Tan[c + d*x]^5)/(55*d)
 + (4*a^2*(9*A - 2*B)*Tan[c + d*x]^7)/(77*d) + (a^2*(9*A - 2*B)*Tan[c + d*x
]^9)/(99*d)
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{12}(c+dx)(a+a\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^{11}(c+dx)(a+a\sin(c+dx))^2}{11d}+\frac{1}{1} \\ &= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d}+\frac{(A+B)\sec^{11}(c+}{} \\ &= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d}+\frac{(A+B)\sec^{11}(c+}{} \\ &= \frac{a^2(9A-2B)\sec^9(c+dx)}{99d}+\frac{(A+B)\sec^{11}(c+}{}\end{aligned}$$

Mathematica [A]
time = 0.80, size = 181, normalized size = 1.01

$$\frac{a^2(35(18A+7B)\sec^{11}(c+dx)+3465A\sec^{10}(c+dx)\tan(c+dx)+385B\sec^9(c+dx)\tan^2(c+dx)-1155(9A-2B)\sec^8(c+dx)\tan^3(c+dx)+1848(9A-2B)\sec^6(c+dx)\tan^5(c+dx)-1584(9A-2B)\sec^4(c+dx)\tan^7(c+dx)+704(9A-2B)\sec^2(c+dx)\tan^9(c+dx)+128(-9A+2B)\tan^{11}(c+dx))}{3465d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a + a*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^2*(35*(18*A + 7*B)*Sec[c + d*x]^11 + 3465*A*Sec[c + d*x]^10*Tan[c + d*x]
 + 385*B*Sec[c + d*x]^9*Tan[c + d*x]^2 - 1155*(9*A - 2*B)*Sec[c + d*x]^8*Ta
n[c + d*x]^3 + 1848*(9*A - 2*B)*Sec[c + d*x]^6*Tan[c + d*x]^5 - 1584*(9*A -
 2*B)*Sec[c + d*x]^4*Tan[c + d*x]^7 + 704*(9*A - 2*B)*Sec[c + d*x]^2*Tan[c
+ d*x]^9 + 128*(-9*A + 2*B)*Tan[c + d*x]^11))/(3465*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(165) = 330$.
time = 0.57, size = 423, normalized size = 2.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/d*(-a^2*A*(-256/693-1/11*sec(d*x+c)^10-10/99*sec(d*x+c)^8-80/693*sec(d*x+
c)^6-32/231*sec(d*x+c)^4-128/693*sec(d*x+c)^2)*tan(d*x+c)+1/11*B*a^2/cos(d*
x+c)^11+2/11*a^2*A/cos(d*x+c)^11+2*B*a^2*(1/11*sin(d*x+c)^3/cos(d*x+c)^11+8
/99*sin(d*x+c)^3/cos(d*x+c)^9+16/231*sin(d*x+c)^3/cos(d*x+c)^7+64/1155*sin(
d*x+c)^3/cos(d*x+c)^5+128/3465*sin(d*x+c)^3/cos(d*x+c)^3)+a^2*A*(1/11*sin(d
```

```
*x+c)^3/cos(d*x+c)^11+8/99*sin(d*x+c)^3/cos(d*x+c)^9+16/231*sin(d*x+c)^3/co
s(d*x+c)^7+64/1155*sin(d*x+c)^3/cos(d*x+c)^5+128/3465*sin(d*x+c)^3/cos(d*x+
c)^3)+B*a^2*(1/11*sin(d*x+c)^4/cos(d*x+c)^11+7/99*sin(d*x+c)^4/cos(d*x+c)^9
+5/99*sin(d*x+c)^4/cos(d*x+c)^7+1/33*sin(d*x+c)^4/cos(d*x+c)^5+1/99*sin(d*x
+c)^4/cos(d*x+c)^3-1/99*sin(d*x+c)^4/cos(d*x+c)-1/99*(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A]
time = 0.32, size = 238, normalized size = 1.33

$$\frac{\left(315\,\tan\left(dx+c\right)^{11}+1540\,\tan\left(dx+c\right)^{9}+2970\,\tan\left(dx+c\right)^{7}+2772\,\tan\left(dx+c\right)^{5}+1155\,\tan\left(dx+c\right)^{3}\right)Aa^{2}+5\left(63\,\tan\left(dx+c\right)^{11}+385\,\tan\left(dx+c\right)^{9}+990\,\tan\left(dx+c\right)^{7}+1386\,\tan\left(dx+c\right)^{5}+1155\,\tan\left(dx+c\right)^{3}+693\,\tan\left(dx+c\right)\right)Aa^{2}+2\left(315\,\tan\left(dx+c\right)^{11}+1540\,\tan\left(dx+c\right)^{9}+2970\,\tan\left(dx+c\right)^{7}+2772\,\tan\left(dx+c\right)^{5}+1155\,\tan\left(dx+c\right)^{3}\right)Ba^{2}-\frac{35\left(11\,\cos\left(dx+c\right)^{2}-9\right)Ba^{2}}{\cos\left(dx+c\right)^{11}}+\frac{630\,Aa^{2}}{\cos\left(dx+c\right)^{11}}+\frac{315\,Ba^{2}}{\cos\left(dx+c\right)^{11}}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/3465*((315*tan(d*x + c)^11 + 1540*tan(d*x + c)^9 + 2970*tan(d*x + c)^7 +
2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*A*a^2 + 5*(63*tan(d*x + c)^11 +
385*tan(d*x + c)^9 + 990*tan(d*x + c)^7 + 1386*tan(d*x + c)^5 + 1155*tan(d*
x + c)^3 + 693*tan(d*x + c))*A*a^2 + 2*(315*tan(d*x + c)^11 + 1540*tan(d*x
+ c)^9 + 2970*tan(d*x + c)^7 + 2772*tan(d*x + c)^5 + 1155*tan(d*x + c)^3)*B
*a^2 - 35*(11*cos(d*x + c)^2 - 9)*B*a^2/cos(d*x + c)^11 + 630*A*a^2/cos(d*x
 + c)^11 + 315*B*a^2/cos(d*x + c)^11)/d
```

Fricas [A]
time = 0.40, size = 237, normalized size = 1.32

$$-\frac{256\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{8}-128\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{6}-32\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{4}-16\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{2}-45\,(2\,A-9\,B)a^{2}-\left(128\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{8}-192\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{6}-80\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{4}-56\,(9\,A-2\,B)a^{2}\cos\left(dx+c\right)^{2}-45\,(9\,A-2\,B)a^{2}\right)\sin\left(dx+c\right)}{3465\left(d\cos\left(dx+c\right)^{9}+2\,d\cos\left(dx+c\right)^{7}\sin\left(dx+c\right)-2\,d\cos\left(dx+c\right)^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] -1/3465*(256*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 128*(9*A - 2*B)*a^2*cos(d*x +
 c)^6 - 32*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 16*(9*A - 2*B)*a^2*cos(d*x + c)
^2 - 45*(2*A - 9*B)*a^2 - (128*(9*A - 2*B)*a^2*cos(d*x + c)^8 - 192*(9*A -
2*B)*a^2*cos(d*x + c)^6 - 80*(9*A - 2*B)*a^2*cos(d*x + c)^4 - 56*(9*A - 2*B
)*a^2*cos(d*x + c)^2 - 45*(9*A - 2*B)*a^2)*sin(d*x + c))/(d*cos(d*x + c)^9
+ 2*d*cos(d*x + c)^7*sin(d*x + c) - 2*d*cos(d*x + c)^7)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**12*(a+a*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(165) = 330.
time = 0.48, size = 597, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^12*(a+a*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac")

[Out] -1/443520*(33*(6825*A*a^2*tan(1/2*d*x + 1/2*c)^6 - 2940*B*a^2*tan(1/2*d*x + 1/2*c)^6 + 34965*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 13755*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 79800*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 30065*B*a^2*tan(1/2*d*x + 1/2*c)^4 + 100170*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 36470*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 73017*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 26166*B*a^2*tan(1/2*d*x + 1/2*c)^2 + 29169*A*a^2*tan(1/2*d*x + 1/2*c) - 10367*B*a^2*tan(1/2*d*x + 1/2*c) + 5142*A*a^2 - 1901*B*a^2)/(tan(1/2*d*x + 1/2*c) + 1)^7 + (661815*A*a^2*tan(1/2*d*x + 1/2*c)^10 + 97020*B*a^2*tan(1/2*d*x + 1/2*c)^10 - 5083155*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 405405*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 19355490*A*a^2*tan(1/2*d*x + 1/2*c)^8 + 952875*B*a^2*tan(1/2*d*x + 1/2*c)^8 - 45446940*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 1122660*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 72295146*A*a^2*tan(1/2*d*x + 1/2*c)^6 + 557172*B*a^2*tan(1/2*d*x + 1/2*c)^6 - 80611146*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 563178*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 63771840*A*a^2*tan(1/2*d*x + 1/2*c)^4 - 1126950*B*a^2*tan(1/2*d*x + 1/2*c)^4 - 35253900*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 955020*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 13119975*A*a^2*tan(1/2*d*x + 1/2*c)^2 - 406120*B*a^2*tan(1/2*d*x + 1/2*c)^2 - 2978811*A*a^2*tan(1/2*d*x + 1/2*c) + 97163*B*a^2*tan(1/2*d*x + 1/2*c) + 330966*A*a^2 - 13*B*a^2)/(tan(1/2*d*x + 1/2*c) - 1)^11)/d

Mupad [**B**]
time = 13.96, size = 466, normalized size = 2.60

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^2)/cos(c + d*x)^12,x)

[Out] -(a^2*cos(c/2 + (d*x)/2)*((8127*A*cos((5*c)/2 + (5*d*x)/2))/64 - (24255*A*cos((3*c)/2 + (3*d*x)/2))/64 - (21357*A*cos((7*c)/2 + (7*d*x)/2))/64 + (5229*A*cos((9*c)/2 + (9*d*x)/2))/64 - (8379*A*cos((11*c)/2 + (11*d*x)/2))/64 + (1467*A*cos((13*c)/2 + (13*d*x)/2))/64 - (2619*A*cos((15*c)/2 + (15*d*x)/2))/128 + (315*A*cos((17*c)/2 + (17*d*x)/2))/128 - 385*B*cos(c/2 + (d*x)/2) +

(30415*B*cos((3*c)/2 + (3*d*x)/2))/128 - (23247*B*cos((5*c)/2 + (5*d*x)/2))/128 + (12957*B*cos((7*c)/2 + (7*d*x)/2))/128 - (5789*B*cos((9*c)/2 + (9*d*x)/2))/128 + (3339*B*cos((11*c)/2 + (11*d*x)/2))/128 - (267*B*cos((13*c)/2 + (13*d*x)/2))/128 + (779*B*cos((15*c)/2 + (15*d*x)/2))/256 + (245*B*cos((17*c)/2 + (17*d*x)/2))/256 - (47889*A*sin(c/2 + (d*x)/2))/32 + (25713*A*sin((3*c)/2 + (3*d*x)/2))/32 - (21303*A*sin((5*c)/2 + (5*d*x)/2))/32 + (9207*A*sin((7*c)/2 + (7*d*x)/2))/32 - (4797*A*sin((9*c)/2 + (9*d*x)/2))/32 + (1917*A*sin((11*c)/2 + (11*d*x)/2))/32 - (27*A*sin((13*c)/2 + (13*d*x)/2))/32 + (171*A*sin((15*c)/2 + (15*d*x)/2))/32 + (9*A*sin((17*c)/2 + (17*d*x)/2))/2 + (7809*B*sin(c/2 + (d*x)/2))/64 + (2047*B*sin((3*c)/2 + (3*d*x)/2))/64 + (1383*B*sin((5*c)/2 + (5*d*x)/2))/64 + (3993*B*sin((7*c)/2 + (7*d*x)/2))/64 - (563*B*sin((9*c)/2 + (9*d*x)/2))/64 + (1843*B*sin((11*c)/2 + (11*d*x)/2))/64 - (373*B*sin((13*c)/2 + (13*d*x)/2))/64 + (309*B*sin((15*c)/2 + (15*d*x)/2))/64 - B*sin((17*c)/2 + (17*d*x)/2)))/(887040*d*cos(c/2 - pi/4 + (d*x)/2)^7*cos(c/2 + pi/4 + (d*x)/2)^11)

3.986 $\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{8(A-B)(a+a\sin(c+dx))^7}{7a^4d}-\frac{(3A-5B)(a+a\sin(c+dx))^8}{2a^5d}+\frac{2(A-3B)(a+a\sin(c+dx))^9}{3a^6d}-\frac{(A-7B)(\ldots}{}$$

```
[Out] 8/7*(A-B)*(a+a*sin(d*x+c))^7/a^4/d-1/2*(3*A-5*B)*(a+a*sin(d*x+c))^8/a^5/d+2
/3*(A-3*B)*(a+a*sin(d*x+c))^9/a^6/d-1/10*(A-7*B)*(a+a*sin(d*x+c))^10/a^7/d-
1/11*B*(a+a*sin(d*x+c))^11/a^8/d
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^{11}}{11a^8d}-\frac{(A-7B)(a\sin(c+dx)+a)^{10}}{10a^7d}+\frac{2(A-3B)(a\sin(c+dx)+a)^9}{3a^6d}-\frac{(3A-5B)(a\sin(c+dx)+a)^8}{2a^5d}+\frac{8(A-B)(a\sin(c+dx)+a)^7}{7a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[c + d*x])^7)/(7*a^4*d) - ((3*A - 5*B)*(a + a*Sin[c +
d*x])^8)/(2*a^5*d) + (2*(A - 3*B)*(a + a*Sin[c + d*x])^9)/(3*a^6*d) - ((A -
 7*B)*(a + a*Sin[c + d*x])^10)/(10*a^7*d) - (B*(a + a*Sin[c + d*x])^11)/(11
*a^8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^6\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin\right.}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^6-4a^2(3A-5B\right.\right.}{}$$
$$= \frac{8(A-B)(a+a\sin(c+dx))^7}{7a^4d} - \frac{(3A-5B)(a}{}$$

Mathematica [A]
time = 1.39, size = 86, normalized size = 0.64

$$-\frac{a^3(1+\sin(c+dx))^7\left(-484A+78B+14(77A-39B)\sin(c+dx)+(-847A+1029B)\sin^2(c+dx)+21(11A-37B)\sin^3(c+dx)+210B\sin^4(c+dx)\right)}{2310d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/2310*(a^3*(1 + Sin[c + d*x])^7*(-484*A + 78*B + 14*(77*A - 39*B)*Sin[c +
 d*x] + (-847*A + 1029*B)*Sin[c + d*x]^2 + 21*(11*A - 37*B)*Sin[c + d*x]^3
+ 210*B*Sin[c + d*x]^4))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(124) = 248.
time = 1.00, size = 345, normalized size = 2.57

method	result
derivativedivides	$\frac{\frac{a^3A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{B\left(\cos^8(dx+c)\right)a^3}{8}-\frac{3A\left(\cos^8(dx+c)\right)a^3}{8}+3Ba^3\left(-\frac{\sin(dx}{}\right.}{}$
default	$\frac{\frac{a^3A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{B\left(\cos^8(dx+c)\right)a^3}{8}-\frac{3A\left(\cos^8(dx+c)\right)a^3}{8}+3Ba^3\left(-\frac{\sin(dx}{}\right.}{}$
risch	$\frac{91\sin(dx+c)a^3A}{128d}+\frac{91a^3B\sin(dx+c)}{512d}+\frac{Ba^3\sin(11dx+11c)}{11264d}+\frac{a^3\cos(10dx+10c)A}{5120d}+\frac{3a^3\cos(10dx+10c)B}{5120d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/7*a^3*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)-1/8*B*cos(d*x+c)^8*a^3-3/8*A*cos(d*x+c)^8*a^3+3*B*a^3*(-1/9*sin(d*x+c)
```

```
*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))+3*a^3*A*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5
*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+3*B*a^3*(-1/10*sin(d*x+c)^2*cos
(d*x+c)^8-1/40*cos(d*x+c)^8)+a^3*A*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*co
s(d*x+c)^8)+B*a^3*(-1/11*sin(d*x+c)^3*cos(d*x+c)^8-1/33*sin(d*x+c)*cos(d*x+
c)^8+1/231*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
))
```

Maxima [**A**]
time = 0.30, size = 182, normalized size = 1.36

$$-\frac{210\,Ba^3\sin\left(dx+c\right)^{11}+231\,(A+3\,B)a^3\sin\left(dx+c\right)^{10}+770\,Aa^3\sin\left(dx+c\right)^{9}-2310\,Ba^3\sin\left(dx+c\right)^{8}-660\,(4\,A+3\,B)a^3\sin\left(dx+c\right)^{7}-2310\,(A-B)a^3\sin\left(dx+c\right)^{6}+924\,(3\,A+4\,B)a^3\sin\left(dx+c\right)^{5}+4620\,Aa^3\sin\left(dx+c\right)^{4}-2310\,Ba^3\sin\left(dx+c\right)^{3}-1155\,(3\,A+B)a^3\sin\left(dx+c\right)^{2}-2310\,Aa^3\sin\left(dx+c\right)}{2310\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2310*(210*B*a^3*sin(d*x + c)^11 + 231*(A + 3*B)*a^3*sin(d*x + c)^10 + 77
0*A*a^3*sin(d*x + c)^9 - 2310*B*a^3*sin(d*x + c)^8 - 660*(4*A + 3*B)*a^3*si
n(d*x + c)^7 - 2310*(A - B)*a^3*sin(d*x + c)^6 + 924*(3*A + 4*B)*a^3*sin(d*
x + c)^5 + 4620*A*a^3*sin(d*x + c)^4 - 2310*B*a^3*sin(d*x + c)^3 - 1155*(3*
A + B)*a^3*sin(d*x + c)^2 - 2310*A*a^3*sin(d*x + c))/d
```

Fricas [**A**]
time = 0.41, size = 155, normalized size = 1.16

$$\frac{231\,(A+3\,B)a^3\cos\left(dx+c\right)^{10}-1155\,(A+B)a^3\cos\left(dx+c\right)^{8}+2\,\left(105\,Ba^3\cos\left(dx+c\right)^{10}-35\,(11\,A+15\,B)a^3\cos\left(dx+c\right)^{8}+20\,(11\,A+3\,B)a^3\cos\left(dx+c\right)^{6}+24\,(11\,A+3\,B)a^3\cos\left(dx+c\right)^{4}+32\,(11\,A+3\,B)a^3\cos\left(dx+c\right)^{2}+64\,(11\,A+3\,B)a^3\right)\sin\left(dx+c\right)}{2310\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2310*(231*(A + 3*B)*a^3*cos(d*x + c)^10 - 1155*(A + B)*a^3*cos(d*x + c)^8
 + 2*(105*B*a^3*cos(d*x + c)^10 - 35*(11*A + 15*B)*a^3*cos(d*x + c)^8 + 20*
(11*A + 3*B)*a^3*cos(d*x + c)^6 + 24*(11*A + 3*B)*a^3*cos(d*x + c)^4 + 32*(
11*A + 3*B)*a^3*cos(d*x + c)^2 + 64*(11*A + 3*B)*a^3)*sin(d*x + c))/d
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(124) = 248$.
time = 2.84, size = 530, normalized size = 3.96

$$\begin{cases} \frac{16Aa^3\sin^9(c+dx)}{105d}+\frac{24Aa^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16Aa^3\sin^7(c+dx)}{35d}+\frac{6Aa^3\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8Aa^3\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{Aa^3\sin^3(c+dx)\cos^6(c+dx)}{d}+\frac{2Aa^3\sin^3(c+dx)\cos^4(c+dx)}{d}-\frac{Aa^3\sin^2(c+dx)\cos^8(c+dx)}{8d}+\frac{Aa^3\sin(c+dx)\cos^6(c+dx)}{d}-\frac{Aa^3\cos^{10}(c+dx)}{40d}-\frac{3Aa^3\cos^8(c+dx)}{8d}+\frac{16Ba^3\sin^{11}(c+dx)}{1155d}+\frac{8Ba^3\sin^9(c+dx)\cos^2(c+dx)}{105d}+\frac{16Ba^3\sin^9(c+dx)}{105d}+\frac{6Ba^3\sin^7(c+dx)\cos^4(c+dx)}{35d}+\frac{24Ba^3\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{Ba^3\sin^5(c+dx)\cos^6(c+dx)}{5d}+\frac{6Ba^3\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{Ba^3\sin^3(c+dx)\cos^6(c+dx)}{d}-\frac{3Ba^3\sin^2(c+dx)\cos^8(c+dx)}{8d}-\frac{3Ba^3\cos^{10}(c+dx)}{40d}-\frac{Ba^3\cos^8(c+dx)}{8d} & \text{for } d \neq 0 \\ x(A+B\sin(c))\left(a\sin(c)+a\right)^3\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**3*sin(c + d*x)**9/(105*d) + 24*A*a**3*sin(c + d*x)**7*co
s(c + d*x)**2/(35*d) + 16*A*a**3*sin(c + d*x)**7/(35*d) + 6*A*a**3*sin(c +
d*x)**5*cos(c + d*x)**4/(5*d) + 8*A*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5
*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**6/d + 2*A*a**3*sin(c + d*x)**3*c
os(c + d*x)**4/d - A*a**3*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) + A*a**3*si
n(c + d*x)*cos(c + d*x)**6/d - A*a**3*cos(c + d*x)**10/(40*d) - 3*A*a**3*co
s(c + d*x)**8/(8*d) + 16*B*a**3*sin(c + d*x)**11/(1155*d) + 8*B*a**3*sin(c
+ d*x)**9*cos(c + d*x)**2/(105*d) + 16*B*a**3*sin(c + d*x)**9/(105*d) + 6*B
*a**3*sin(c + d*x)**7*cos(c + d*x)**4/(35*d) + 24*B*a**3*sin(c + d*x)**7*co
s(c + d*x)**2/(35*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**6/(5*d) + 6*B*a
**3*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*a**3*sin(c + d*x)**3*cos(c +
d*x)**6/d - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - 3*B*a**3*cos(c
 + d*x)**10/(40*d) - B*a**3*cos(c + d*x)**8/(8*d), Ne(d, 0)), (x*(A + B*sin
(c))*(a*sin(c) + a)**3*cos(c)**7, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(124) = 248.
time = 0.51, size = 283, normalized size = 2.11

$$\frac{Ba^3\sin(11\,dx+11\,c)}{11264\,d}+\frac{(Aa^3+3\,Ba^3)\cos(10\,dx+10\,c)}{5120\,d}-\frac{(Aa^3-Ba^3)\cos(8\,dx+8\,c)}{512\,d}-\frac{(23\,Aa^3+5\,Ba^3)\cos(6\,dx+6\,c)}{1024\,d}-\frac{(11\,Aa^3+5\,Ba^3)\cos(4\,dx+4\,c)}{128\,d}-\frac{7\,(13\,Aa^3+7\,Ba^3)\cos(2\,dx+2\,c)}{512\,d}-\frac{(4\,Aa^3+3\,Ba^3)\sin(9\,dx+9\,c)}{3072\,d}-\frac{(44\,Aa^3+61\,Ba^3)\sin(7\,dx+7\,c)}{7168\,d}+\frac{(16\,Aa^3-107\,Ba^3)\sin(5\,dx+5\,c)}{5120\,d}+\frac{(56\,Aa^3-Ba^3)\sin(3\,dx+3\,c)}{512\,d}+\frac{91\,(4\,Aa^3+Ba^3)\sin(dx+c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/11264*B*a^3*sin(11*d*x + 11*c)/d + 1/5120*(A*a^3 + 3*B*a^3)*cos(10*d*x +
10*c)/d - 1/512*(A*a^3 - B*a^3)*cos(8*d*x + 8*c)/d - 1/1024*(23*A*a^3 + 5*B
*a^3)*cos(6*d*x + 6*c)/d - 1/128*(11*A*a^3 + 5*B*a^3)*cos(4*d*x + 4*c)/d -
7/512*(13*A*a^3 + 7*B*a^3)*cos(2*d*x + 2*c)/d - 1/3072*(4*A*a^3 + 3*B*a^3)*
sin(9*d*x + 9*c)/d - 1/7168*(44*A*a^3 + 61*B*a^3)*sin(7*d*x + 7*c)/d + 1/51
20*(16*A*a^3 - 107*B*a^3)*sin(5*d*x + 5*c)/d + 1/512*(56*A*a^3 - B*a^3)*sin
(3*d*x + 3*c)/d + 91/512*(4*A*a^3 + B*a^3)*sin(d*x + c)/d
```

Mupad [B]
time = 0.20, size = 177, normalized size = 1.32

$$\frac{\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}-\frac{A\,a^3\sin(c+d\,x)^9}{3}-2\,A\,a^3\sin\left(c+d\,x\right)^4+a^3\sin\left(c+d\,x\right)^6\,(A-B)-\frac{a^3\sin(c+d\,x)^{10}\,(A+3\,B)}{10}+B\,a^3\sin\left(c+d\,x\right)^3+B\,a^3\sin\left(c+d\,x\right)^8-\frac{B\,a^3\sin(c+d\,x)^{11}}{11}-\frac{2\,a^3\sin(c+d\,x)^5\,(3\,A+4\,B)}{5}+\frac{2\,a^3\sin(c+d\,x)^7\,(4\,A+3\,B)}{7}+A\,a^3\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 - (A*a^3*sin(c + d*x)^9)/3 - 2*A*a^3*sin(
c + d*x)^4 + a^3*sin(c + d*x)^6*(A - B) - (a^3*sin(c + d*x)^10*(A + 3*B))/1
0 + B*a^3*sin(c + d*x)^3 + B*a^3*sin(c + d*x)^8 - (B*a^3*sin(c + d*x)^11)/1
1 - (2*a^3*sin(c + d*x)^5*(3*A + 4*B))/5 + (2*a^3*sin(c + d*x)^7*(4*A + 3*B
))/7 + A*a^3*sin(c + d*x))/d
```

3.987 $\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{2(A-B)(a+a\sin(c+dx))^6}{3a^3d}-\frac{4(A-2B)(a+a\sin(c+dx))^7}{7a^4d}+\frac{(A-5B)(a+a\sin(c+dx))^8}{8a^5d}+\frac{B(a+a\sin(c+dx))^9}{9a^6d}$$

```
[Out] 2/3*(A-B)*(a+a*sin(d*x+c))^6/a^3/d-4/7*(A-2*B)*(a+a*sin(d*x+c))^7/a^4/d+1/8
*(A-5*B)*(a+a*sin(d*x+c))^8/a^5/d+1/9*B*(a+a*sin(d*x+c))^9/a^6/d
```

Rubi [A]
time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$\frac{B(a\sin(c+dx)+a)^9}{9a^6d}+\frac{(A-5B)(a\sin(c+dx)+a)^8}{8a^5d}-\frac{4(A-2B)(a\sin(c+dx)+a)^7}{7a^4d}+\frac{2(A-B)(a\sin(c+dx)+a)^6}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^6)/(3*a^3*d) - (4*(A - 2*B)*(a + a*Sin[c +
d*x])^7)/(7*a^4*d) + ((A - 5*B)*(a + a*Sin[c + d*x])^8)/(8*a^5*d) + (B*(a +
 a*Sin[c + d*x])^9)/(9*a^6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\mathrm{Subst}\left(\int (a-x)^2(a+x)^5\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin\right.}{a^5d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(4a^2(A-B)(a+x)^5-4a(A-2B)\right.\right.}{}$$

$$= \frac{2(A-B)(a+a\sin(c+dx))^6}{3a^3d} - \frac{4(A-2B)(a}{}$$

Mathematica [A]
time = 0.38, size = 70, normalized size = 0.67

$$\frac{a^3(1+\sin(c+dx))^6\,(111A-19B-6(27A-19B)\sin(c+dx)+21(3A-7B)\sin^2(c+dx)+56B\sin^3(c+dx))}{504d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^6*(111*A - 19*B - 6*(27*A - 19*B)*Sin[c + d*x] + 21
*(3*A - 7*B)*Sin[c + d*x]^2 + 56*B*Sin[c + d*x]^3))/(504*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(97) = 194$.
time = 0.68, size = 305, normalized size = 2.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*a^3*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*cos(d*x
+c)^6*a^3-1/2*A*cos(d*x+c)^6*a^3+3*B*a^3*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*a^3*A*(-1/7*sin(d*x+c)*c
os(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+3*B*a^3*(-
1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+a^3*A*(-1/8*sin(d*x+c)^2*c
os(d*x+c)^6-1/24*cos(d*x+c)^6)+B*a^3*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*s
in(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
))
```

Maxima [A]
time = 0.30, size = 158, normalized size = 1.50

$$\frac{56\,Ba^3\sin(dx+c)^9+63\,(A+3\,B)a^3\sin(dx+c)^8+72\,(3\,A+B)a^3\sin(dx+c)^7+84\,(A-5\,B)a^3\sin(dx+c)^6-504\,(A+B)a^3\sin(dx+c)^5-126\,(5\,A-B)a^3\sin(dx+c)^4+168\,(A+3\,B)a^3\sin(dx+c)^3+252\,(3\,A+B)a^3\sin(dx+c)^2+504\,Aa^3\sin(dx+c)}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/504*(56*B*a^3*sin(d*x + c)^9 + 63*(A + 3*B)*a^3*sin(d*x + c)^8 + 72*(3*A
+ B)*a^3*sin(d*x + c)^7 + 84*(A - 5*B)*a^3*sin(d*x + c)^6 - 504*(A + B)*a^3
*sin(d*x + c)^5 - 126*(5*A - B)*a^3*sin(d*x + c)^4 + 168*(A + 3*B)*a^3*sin(
d*x + c)^3 + 252*(3*A + B)*a^3*sin(d*x + c)^2 + 504*A*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.43, size = 129, normalized size = 1.23

$$\frac{63\,(A+3\,B)a^3\cos\left(dx+c\right)^8-336\,(A+B)a^3\cos\left(dx+c\right)^6+8\,\left(7\,Ba^3\cos\left(dx+c\right)^8-(27\,A+37\,B)a^3\cos\left(dx+c\right)^6+6\,(3\,A+B)a^3\cos\left(dx+c\right)^4+8\,(3\,A+B)a^3\cos\left(dx+c\right)^2+16\,(3\,A+B)a^3\right)\sin\left(dx+c\right)}{504\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/504*(63*(A + 3*B)*a^3*cos(d*x + c)^8 - 336*(A + B)*a^3*cos(d*x + c)^6 + 8
*(7*B*a^3*cos(d*x + c)^8 - (27*A + 37*B)*a^3*cos(d*x + c)^6 + 6*(3*A + B)*a
^3*cos(d*x + c)^4 + 8*(3*A + B)*a^3*cos(d*x + c)^2 + 16*(3*A + B)*a^3)*sin(
d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(99) = 198$.
time = 1.45, size = 418, normalized size = 3.98

$$\begin{cases} \frac{8Aa^3\sin^7(c+dx)}{35d} + \frac{4Aa^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{8Aa^3\sin^5(c+dx)}{15d} + \frac{Aa^3\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{4Aa^3\sin^3(c+dx)\cos^2(c+dx)}{3d} - \frac{Aa^3\sin^2(c+dx)\cos^6(c+dx)}{6d} + \frac{Aa^3\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Aa^3\cos^8(c+dx)}{24d} - \frac{Aa^3\cos^6(c+dx)}{2d} + \frac{8Ba^3\sin^9(c+dx)}{315d} + \frac{4Ba^3\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{8Ba^3\sin^7(c+dx)}{35d} + \frac{Ba^3\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{4Ba^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{Ba^3\sin^3(c+dx)\cos^4(c+dx)}{d} - \frac{Ba^3\sin^2(c+dx)\cos^6(c+dx)}{2d} - \frac{Ba^3\cos^8(c+dx)}{8d} - \frac{Ba^3\cos^6(c+dx)}{6d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**3*sin(c + d*x)**7/(35*d) + 4*A*a**3*sin(c + d*x)**5*cos(c
 + d*x)**2/(5*d) + 8*A*a**3*sin(c + d*x)**5/(15*d) + A*a**3*sin(c + d*x)**3
*cos(c + d*x)**4/d + 4*A*a**3*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - A*a**
3*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) + A*a**3*sin(c + d*x)*cos(c + d*x)*
*4/d - A*a**3*cos(c + d*x)**8/(24*d) - A*a**3*cos(c + d*x)**6/(2*d) + 8*B*a
**3*sin(c + d*x)**9/(315*d) + 4*B*a**3*sin(c + d*x)**7*cos(c + d*x)**2/(35*
d) + 8*B*a**3*sin(c + d*x)**7/(35*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)*
*4/(5*d) + 4*B*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + B*a**3*sin(c +
d*x)**3*cos(c + d*x)**4/d - B*a**3*sin(c + d*x)**2*cos(c + d*x)**6/(2*d) -
B*a**3*cos(c + d*x)**8/(8*d) - B*a**3*cos(c + d*x)**6/(6*d), Ne(d, 0)), (x*
(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(97) = 194$.
time = 0.55, size = 230, normalized size = 2.19

$$\frac{Ba^3\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{\left(Aa^3+3\,Ba^3\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(5\,Aa^3-Ba^3\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(25\,Aa^3+11\,Ba^3\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{\left(33\,Aa^3+19\,Ba^3\right)\cos\left(2\,dx+2\,c\right)}{128\,d}-\frac{\left(12\,Aa^3+11\,Ba^3\right)\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{\left(Aa^3+2\,Ba^3\right)\sin\left(5\,dx+5\,c\right)}{64\,d}+\frac{\left(17\,Aa^3-4\,Ba^3\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{11\,\left(10\,Aa^3+3\,Ba^3\right)\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2304*B*a^3*sin(9*d*x + 9*c)/d + 1/1024*(A*a^3 + 3*B*a^3)*cos(8*d*x + 8*c)
/d - 1/384*(5*A*a^3 - B*a^3)*cos(6*d*x + 6*c)/d - 1/256*(25*A*a^3 + 11*B*a^
3)*cos(4*d*x + 4*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(2*d*x + 2*c)/d - 1/
1792*(12*A*a^3 + 11*B*a^3)*sin(7*d*x + 7*c)/d - 1/64*(A*a^3 + 2*B*a^3)*sin(
5*d*x + 5*c)/d + 1/192*(17*A*a^3 - 4*B*a^3)*sin(3*d*x + 3*c)/d + 11/128*(10
*A*a^3 + 3*B*a^3)*sin(d*x + c)/d
```

Mupad [B]
time = 0.14, size = 156, normalized size = 1.49

$$\frac{\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}+\frac{a^3\sin(c+d\,x)^3\,(A+3\,B)}{3}+\frac{a^3\sin(c+d\,x)^7\,(3\,A+B)}{7}+\frac{a^3\sin(c+d\,x)^6\,(A-5\,B)}{6}+\frac{a^3\sin(c+d\,x)^8\,(A+3\,B)}{8}+\frac{B\,a^3\sin(c+d\,x)^9}{9}-\frac{a^3\sin(c+d\,x)^4\,(5\,A-B)}{4}+A\,a^3\,\sin\left(c+d\,x\right)-a^3\sin\left(c+d\,x\right)^5\,(A+B)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*sin(c + d*x)^3*(A + 3*B))/3 + (a^3
*sin(c + d*x)^7*(3*A + B))/7 + (a^3*sin(c + d*x)^6*(A - 5*B))/6 + (a^3*sin(
c + d*x)^8*(A + 3*B))/8 + (B*a^3*sin(c + d*x)^9)/9 - (a^3*sin(c + d*x)^4*(5
*A - B))/4 + A*a^3*sin(c + d*x) - a^3*sin(c + d*x)^5*(A + B))/d
```

3.988 $\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{2(A-B)(a+a\sin(c+dx))^5}{5a^2d}-\frac{(A-3B)(a+a\sin(c+dx))^6}{6a^3d}-\frac{B(a+a\sin(c+dx))^7}{7a^4d}$$

```
[Out] 2/5*(A-B)*(a+a*sin(d*x+c))^5/a^2/d-1/6*(A-3*B)*(a+a*sin(d*x+c))^6/a^3/d-1/7
*B*(a+a*sin(d*x+c))^7/a^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(c+dx)+a)^7}{7a^4d}-\frac{(A-3B)(a\sin(c+dx)+a)^6}{6a^3d}+\frac{2(A-B)(a\sin(c+dx)+a)^5}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[c + d*x])^5)/(5*a^2*d) - ((A - 3*B)*(a + a*Sin[c + d*
x])^6)/(6*a^3*d) - (B*(a + a*Sin[c + d*x])^7)/(7*a^4*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a-x)(a+x)^4\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin\right.}{a^3d}$$
$$= \frac{\text{Subst}\Big(\int \Big(2a(A-B)(a+x)^4+(-A+3B)(a}{a^3d}$$
$$= \frac{2(A-B)(a+a\sin(c+dx))^5}{5a^2d} - \frac{(A-3B)(a-}{6}$$

Mathematica [A]
time = 0.21, size = 53, normalized size = 0.68

$$-\frac{a^3(1+\sin(c+dx))^5\left(-49A+9B+5(7A-9B)\sin(c+dx)+30B\sin^2(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/210*(a^3*(1 + Sin[c + d*x])^5*(-49*A + 9*B + 5*(7*A - 9*B)*Sin[c + d*x]
+ 30*B*Sin[c + d*x]^2))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(72) = 144$.
time = 0.42, size = 265, normalized size = 3.40

method	result
risch	$\frac{9\sin(dx+c)a^3A}{8d}+\frac{27a^3B\sin(dx+c)}{64d}+\frac{\sin(7dx+7c)B\,a^3}{448d}+\frac{a^3\cos(6dx+6c)A}{192d}+\frac{a^3\cos(6dx+6c)B}{64d}-\frac{3\sin(5dx+}{80d}$
derivativedivides	$\frac{\frac{a^3A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{B\left(\cos^4(dx+c)\right)a^3}{4}-\frac{3A\left(\cos^4(dx+c)\right)a^3}{4}+3B\,a^3\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)}{15}\right.}{}$
default	$\frac{\frac{a^3A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{B\left(\cos^4(dx+c)\right)a^3}{4}-\frac{3A\left(\cos^4(dx+c)\right)a^3}{4}+3B\,a^3\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)}{15}\right.}{}$
norman	$\frac{\left(6a^3A+2B\,a^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(6a^3A+2B\,a^3\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(22a^3A+18B\,a^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(22a^3A+18B\,a^3\right)\left(\tan^{10}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^3*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*cos(d*x+c)^4*a^3-3/4*A*cos
(d*x+c)^4*a^3+3*B*a^3*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*s
in(d*x+c))+3*a^3*A*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(
```

```
d*x+c))+3*B*a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+a^3*A*(-
1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+B*a^3*(-1/7*sin(d*x+c)^3*c
os(d*x+c)^4-3/35*cos(d*x+c)^4*sin(d*x+c)+1/35*(2+cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 126, normalized size = 1.62

$$-\frac{30\,Ba^3\sin\left(dx+c\right)^7+35\,(A+3\,B)a^3\sin\left(dx+c\right)^6+42\,(3\,A+2\,B)a^3\sin\left(dx+c\right)^5+105\,(A-B)a^3\sin\left(dx+c\right)^4-70\,(2\,A+3\,B)a^3\sin\left(dx+c\right)^3-105\,(3\,A+B)a^3\sin\left(dx+c\right)^2-210\,Aa^3\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*(A + 3*B)*a^3*sin(d*x + c)^6 + 42*(3*A
 + 2*B)*a^3*sin(d*x + c)^5 + 105*(A - B)*a^3*sin(d*x + c)^4 - 70*(2*A + 3*B
)*a^3*sin(d*x + c)^3 - 105*(3*A + B)*a^3*sin(d*x + c)^2 - 210*A*a^3*sin(d*x
 + c))/d
```

Fricas [A]
time = 0.37, size = 115, normalized size = 1.47

$$\frac{35\,(A+3\,B)a^3\cos\left(dx+c\right)^6-210\,(A+B)a^3\cos\left(dx+c\right)^4+2\,\left(15\,Ba^3\cos\left(dx+c\right)^6-3\,(21\,A+29\,B)a^3\cos\left(dx+c\right)^4+8\,(7\,A+3\,B)a^3\cos\left(dx+c\right)^2+16\,(7\,A+3\,B)a^3\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/210*(35*(A + 3*B)*a^3*cos(d*x + c)^6 - 210*(A + B)*a^3*cos(d*x + c)^4 + 2
*(15*B*a^3*cos(d*x + c)^6 - 3*(21*A + 29*B)*a^3*cos(d*x + c)^4 + 8*(7*A + 3
*B)*a^3*cos(d*x + c)^2 + 16*(7*A + 3*B)*a^3)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(70) = 140$.
time = 0.69, size = 313, normalized size = 4.01

$$\begin{cases} \frac{2Aa^3\sin^5(c+dx)}{5d} + \frac{Aa^3\sin^3(c+dx)\cos^2(c+dx)}{d} + \frac{2Aa^3\sin^3(c+dx)}{3d} - \frac{Aa^3\sin^2(c+dx)\cos^4(c+dx)}{4d} + \frac{Aa^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{Aa^3\cos^6(c+dx)}{12d} - \frac{3Aa^3\cos^4(c+dx)}{4d} + \frac{2Ba^3\sin^7(c+dx)}{35d} + \frac{Ba^3\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Ba^3\sin^5(c+dx)}{5d} + \frac{Ba^3\sin^3(c+dx)\cos^2(c+dx)}{d} - \frac{3Ba^3\sin^2(c+dx)\cos^4(c+dx)}{4d} - \frac{Ba^3\cos^6(c+dx)}{4d} - \frac{Ba^3\cos^4(c+dx)}{4d} & \text{for}\ d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**3*sin(c + d*x)**5/(5*d) + A*a**3*sin(c + d*x)**3*cos(c +
d*x)**2/d + 2*A*a**3*sin(c + d*x)**3/(3*d) - A*a**3*sin(c + d*x)**2*cos(c +
 d*x)**4/(4*d) + A*a**3*sin(c + d*x)*cos(c + d*x)**2/d - A*a**3*cos(c + d*x
)**6/(12*d) - 3*A*a**3*cos(c + d*x)**4/(4*d) + 2*B*a**3*sin(c + d*x)**7/(35
*d) + B*a**3*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*B*a**3*sin(c + d*x)*
*5/(5*d) + B*a**3*sin(c + d*x)**3*cos(c + d*x)**2/d - 3*B*a**3*sin(c + d*x)
```

```
**2*cos(c + d*x)**4/(4*d) - B*a**3*cos(c + d*x)**6/(4*d) - B*a**3*cos(c + d
*x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**3, Tru
e))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(72) = 144$.
time = 0.51, size = 172, normalized size = 2.21

$$-\frac{30\,Ba^3\sin(dx+c)^7 + 35\,Aa^3\sin(dx+c)^6 + 105\,Ba^3\sin(dx+c)^6 + 126\,Aa^3\sin(dx+c)^5 + 84\,Ba^3\sin(dx+c)^5 + 105\,Aa^3\sin(dx+c)^4 - 105\,Ba^3\sin(dx+c)^4 - 140\,Aa^3\sin(dx+c)^3 - 210\,Ba^3\sin(dx+c)^3 - 315\,Aa^3\sin(dx+c)^2 - 105\,Ba^3\sin(dx+c)^2 - 210\,Aa^3\sin(dx+c)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/210*(30*B*a^3*sin(d*x + c)^7 + 35*A*a^3*sin(d*x + c)^6 + 105*B*a^3*sin(d
*x + c)^6 + 126*A*a^3*sin(d*x + c)^5 + 84*B*a^3*sin(d*x + c)^5 + 105*A*a^3*
sin(d*x + c)^4 - 105*B*a^3*sin(d*x + c)^4 - 140*A*a^3*sin(d*x + c)^3 - 210*
B*a^3*sin(d*x + c)^3 - 315*A*a^3*sin(d*x + c)^2 - 105*B*a^3*sin(d*x + c)^2
- 210*A*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 9.14, size = 126, normalized size = 1.62

$$-\frac{\frac{a^3\sin(c+d\,x)^4\,(A-B)}{2} - \frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2} + \frac{a^3\sin(c+d\,x)^6\,(A+3\,B)}{6} + \frac{B\,a^3\sin(c+d\,x)^7}{7} - \frac{a^3\sin(c+d\,x)^3\,(2\,A+3\,B)}{3} + \frac{a^3\sin(c+d\,x)^5\,(3\,A+2\,B)}{5} - A\,a^3\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] -((a^3*sin(c + d*x)^4*(A - B))/2 - (a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*
sin(c + d*x)^6*(A + 3*B))/6 + (B*a^3*sin(c + d*x)^7)/7 - (a^3*sin(c + d*x)^
3*(2*A + 3*B))/3 + (a^3*sin(c + d*x)^5*(3*A + 2*B))/5 - A*a^3*sin(c + d*x))
/d
```

3.989 $\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{(A-B)(a+a\sin(c+dx))^4}{4ad}+\frac{B(a+a\sin(c+dx))^5}{5a^2d}$$

```
[Out] 1/4*(A-B)*(a+a*sin(d*x+c))^4/a/d+1/5*B*(a+a*sin(d*x+c))^5/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {2912, 45}

$$\frac{B(a\sin(c+dx)+a)^5}{5a^2d}+\frac{(A-B)(a\sin(c+dx)+a)^4}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[c + d*x])^4)/(4*a*d) + (B*(a + a*Sin[c + d*x])^5)/(5*a^
2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^3\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^3+\frac{B(a+x)^4}{a}\right)\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{(A-B)(a+a\sin(c+dx))^4}{4ad}+\frac{B(a+a\sin(c+dx))^5}{5a^2d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 36, normalized size = 0.71

$$\frac{a^3(1+\sin(c+dx))^4(5A-B+4B\sin(c+dx))}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(1 + Sin[c + d*x])^4*(5*A - B + 4*B*Sin[c + d*x]))/(20*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.
time = 0.20, size = 98, normalized size = 1.92

method	result
derivativedivides	$\frac{\frac{B\,a^3\left(\sin^5(dx+c)\right)}{5}+\frac{\left(a^3A+3B\,a^3\right)\left(\sin^4(dx+c)\right)}{4}+\frac{\left(3a^3A+3B\,a^3\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(3a^3A+B\,a^3\right)\left(\sin^2(dx+c)\right)}{2}+a^3A\sin(dx+c)}{d}$
default	$\frac{\frac{B\,a^3\left(\sin^5(dx+c)\right)}{5}+\frac{\left(a^3A+3B\,a^3\right)\left(\sin^4(dx+c)\right)}{4}+\frac{\left(3a^3A+3B\,a^3\right)\left(\sin^3(dx+c)\right)}{3}+\frac{\left(3a^3A+B\,a^3\right)\left(\sin^2(dx+c)\right)}{2}+a^3A\sin(dx+c)}{d}$
risch	$\frac{7\sin(dx+c)a^3A}{4d}+\frac{7a^3B\sin(dx+c)}{8d}+\frac{\sin(5dx+5c)B\,a^3}{80d}+\frac{a^3\cos(4dx+4c)A}{32d}+\frac{3a^3\cos(4dx+4c)B}{32d}-\frac{\sin(3dx+3}{4d}$
norman	$\frac{\frac{\left(6a^3A+2B\,a^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(6a^3A+2B\,a^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(11a^3A+9B\,a^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(11a^3A+9B\,a^3\right)\left(\tan^6\left(\right.\right.}{d}}{\left(1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*B*a^3*sin(d*x+c)^5+1/4*(A*a^3+3*B*a^3)*sin(d*x+c)^4+1/3*(3*A*a^3+3
*B*a^3)*sin(d*x+c)^3+1/2*(3*A*a^3+B*a^3)*sin(d*x+c)^2+a^3*A*sin(d*x+c))
```

Maxima [A]
time = 0.32, size = 84, normalized size = 1.65

$$\frac{4\,Ba^3\sin\left(dx+c\right)^5+5\,(A+3\,B)a^3\sin\left(dx+c\right)^4+20\,(A+B)a^3\sin\left(dx+c\right)^3+10\,(3\,A+B)a^3\sin\left(dx+c\right)^2+20\,Aa^3\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*(A + 3*B)*a^3*sin(d*x + c)^4 + 20*(A + B)*
a^3*sin(d*x + c)^3 + 10*(3*A + B)*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c
))/d
```

Fricas [A]
time = 0.37, size = 94, normalized size = 1.84

$$\frac{5\,(A+3\,B)a^3\cos\left(dx+c\right)^4-40\,(A+B)a^3\cos\left(dx+c\right)^2+4\left(Ba^3\cos\left(dx+c\right)^4-(5\,A+7\,B)a^3\cos\left(dx+c\right)^2+2\,(5\,A+3\,B)a^3\right)\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/20*(5*(A + 3*B)*a^3*cos(d*x + c)^4 - 40*(A + B)*a^3*cos(d*x + c)^2 + 4*(B
*a^3*cos(d*x + c)^4 - (5*A + 7*B)*a^3*cos(d*x + c)^2 + 2*(5*A + 3*B)*a^3)*s
in(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(41) = 82.
time = 0.28, size = 151, normalized size = 2.96

$$\begin{cases} \frac{Aa^3\sin^4\left(c+dx\right)}{4d}+\frac{Aa^3\sin^3\left(c+dx\right)}{d}+\frac{3Aa^3\sin^2\left(c+dx\right)}{2d}+\frac{Aa^3\sin\left(c+dx\right)}{d}+\frac{Ba^3\sin^5\left(c+dx\right)}{5d}+\frac{3Ba^3\sin^4\left(c+dx\right)}{4d}+\frac{Ba^3\sin^3\left(c+dx\right)}{d}+\frac{Ba^3\sin^2\left(c+dx\right)}{2d} & \text{for } d\neq 0 \\ x(A+B\sin\left(c\right))\left(a\sin\left(c\right)+a\right)^3\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**3*sin(c + d*x)**4/(4*d) + A*a**3*sin(c + d*x)**3/d + 3*A*a*
*3*sin(c + d*x)**2/(2*d) + A*a**3*sin(c + d*x)/d + B*a**3*sin(c + d*x)**5/(
5*d) + 3*B*a**3*sin(c + d*x)**4/(4*d) + B*a**3*sin(c + d*x)**3/d + B*a**3*s
in(c + d*x)**2/(2*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(47) = 94.
time = 0.49, size = 116, normalized size = 2.27

$$\frac{4\,Ba^3\sin\left(dx+c\right)^5+5\,Aa^3\sin\left(dx+c\right)^4+15\,Ba^3\sin\left(dx+c\right)^4+20\,Aa^3\sin\left(dx+c\right)^3+20\,Ba^3\sin\left(dx+c\right)^3+30\,Aa^3\sin\left(dx+c\right)^2+10\,Ba^3\sin\left(dx+c\right)^2+20\,Aa^3\sin\left(dx+c\right)}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/20*(4*B*a^3*sin(d*x + c)^5 + 5*A*a^3*sin(d*x + c)^4 + 15*B*a^3*sin(d*x +
c)^4 + 20*A*a^3*sin(d*x + c)^3 + 20*B*a^3*sin(d*x + c)^3 + 30*A*a^3*sin(d*x
 + c)^2 + 10*B*a^3*sin(d*x + c)^2 + 20*A*a^3*sin(d*x + c))/d
```

Mupad [B]
time = 9.07, size = 81, normalized size = 1.59

$$\frac{\frac{a^3\sin(c+d\,x)^2\,(3\,A+B)}{2}+\frac{a^3\sin(c+d\,x)^4\,(A+3\,B)}{4}+\frac{B\,a^3\sin(c+d\,x)^5}{5}+A\,a^3\,\sin\left(c+d\,x\right)+a^3\sin\left(c+d\,x\right)^3\,(A+B)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] ((a^3*sin(c + d*x)^2*(3*A + B))/2 + (a^3*sin(c + d*x)^4*(A + 3*B))/4 + (B*a
^3*sin(c + d*x)^5)/5 + A*a^3*sin(c + d*x) + a^3*sin(c + d*x)^3*(A + B))/d
```

3.990 $\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{B(a+a\sin(c+dx))}{3d}$$

```
[Out] -4*a^3*(A+B)*ln(1-sin(d*x+c))/d-3*a^3*(A+B)*sin(d*x+c)/d-1/2*a^3*(A+B)*sin(
d*x+c)^2/d-1/3*B*(a+a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2915, 78}

$$-\frac{a^3(A+B)\sin^2(c+dx)}{2d}-\frac{3a^3(A+B)\sin(c+dx)}{d}-\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d}-\frac{B(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-4*a^3*(A + B)*Log[1 - Sin[c + d*x]])/d - (3*a^3*(A + B)*Sin[c + d*x])/d -
 (a^3*(A + B)*Sin[c + d*x]^2)/(2*d) - (B*(a + a*Sin[c + d*x])^3)/(3*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(-3a(A+B)+\frac{4a^2(A+B)}{a-x}-(A+B)\right.\right.}{d}$$
$$= -\frac{4a^3(A+B)\log(1-\sin(c+dx))}{d} - \frac{3a^3(A+}{}$$

Mathematica [A]
time = 0.09, size = 68, normalized size = 0.84

$$-\frac{a^3(24(A+B)\log(1-\sin(c+dx))+6(3A+4B)\sin(c+dx)+3(A+3B)\sin^2(c+dx)+2B\sin^3(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a^3*(24*(A + B)*Log[1 - Sin[c + d*x]] + 6*(3*A + 4*B)*Sin[c + d*x] +
3*(A + 3*B)*Sin[c + d*x]^2 + 2*B*Sin[c + d*x]^3))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(77) = 154$.
time = 0.22, size = 198, normalized size = 2.44

method	result
derivativedivides	$\frac{a^3A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^3\ln(\cos(dx+c))-3a^3A\ln(\cos(dx+c))+3B\,a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx}{}$
default	$\frac{a^3A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^3\ln(\cos(dx+c))-3a^3A\ln(\cos(dx+c))+3B\,a^3(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx}{}$
risch	$4ia^3xA+4ia^3xB+\frac{3ie^{i(dx+c)}a^3A}{2d}+\frac{17ie^{i(dx+c)}B\,a^3}{8d}-\frac{3ia^3e^{-i(dx+c)}A}{2d}-\frac{17ia^3e^{-i(dx+c)}B}{8d}+\frac{8ia^3Ac}{d}$
norman	$\frac{-\frac{2\left(a^3A+3B\,a^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^3A+3B\,a^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(2a^3A+6B\,a^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^3(27A+40B)\left(\tan^3\left(\frac{d}{}\right.\right.}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*A*ln(sec(d*x+c)+tan(d*x+c))-B*a^3*ln(cos(d*x+c))-3*a^3*A*ln(cos(d*
x+c))+3*B*a^3*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*A*(-sin(d*x+c)+
ln(sec(d*x+c)+tan(d*x+c)))+3*B*a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+a^3*A
```

```
*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+B*a^3*(-1/3*sin(d*x+c)^3-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 73, normalized size = 0.90

$$-\frac{2\,Ba^3\sin\left(dx+c\right)^3+3\,(A+3\,B)a^3\sin\left(dx+c\right)^2+24\,(A+B)a^3\log\left(\sin\left(dx+c\right)-1\right)+6\,(3\,A+4\,B)a^3\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*(2*B*a^3*sin(d*x + c)^3 + 3*(A + 3*B)*a^3*sin(d*x + c)^2 + 24*(A + B)*
a^3*log(sin(d*x + c) - 1) + 6*(3*A + 4*B)*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 77, normalized size = 0.95

$$\frac{3\,(A+3\,B)a^3\cos\left(dx+c\right)^2-24\,(A+B)a^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(Ba^3\cos\left(dx+c\right)^2-(9\,A+13\,B)a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A + 3*B)*a^3*cos(d*x + c)^2 - 24*(A + B)*a^3*log(-sin(d*x + c) + 1)
 + 2*(B*a^3*cos(d*x + c)^2 - (9*A + 13*B)*a^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec\left(c+dx\right)dx+\int 3A\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3A\sin^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\sin^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3B\sin^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3B\sin^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\sin^4\left(c+dx\right)\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sin(c + d*x)*sec(c + d*x),
 x) + Integral(3*A*sin(c + d*x)**2*sec(c + d*x), x) + Integral(A*sin(c + d*
x)**3*sec(c + d*x), x) + Integral(B*sin(c + d*x)*sec(c + d*x), x) + Integra
l(3*B*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*B*sin(c + d*x)**3*sec(c
 + d*x), x) + Integral(B*sin(c + d*x)**4*sec(c + d*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(77) = 154.
time = 0.47, size = 289, normalized size = 3.57

$$\frac{2\left(6\,(Aa^3+Ba^3)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-12\,(Aa^3+Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{11\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6+11\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6+9\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+12\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+36\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+42\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+18\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+28\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+36\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+42\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+9\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+12\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+11\,Aa^3+11\,Ba^3}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1\right)^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 2/3*(6*(A*a^3 + B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(A*a^3 + B*a^3)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - (11*A*a^3*tan(1/2*d*x + 1/2*c)^6 + 11
*B*a^3*tan(1/2*d*x + 1/2*c)^6 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 12*B*a^3*t
an(1/2*d*x + 1/2*c)^5 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 42*B*a^3*tan(1/2*
d*x + 1/2*c)^4 + 18*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 28*B*a^3*tan(1/2*d*x + 1
/2*c)^3 + 36*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 42*B*a^3*tan(1/2*d*x + 1/2*c)^2
 + 9*A*a^3*tan(1/2*d*x + 1/2*c) + 12*B*a^3*tan(1/2*d*x + 1/2*c) + 11*A*a^3
+ 11*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 0.09, size = 100, normalized size = 1.23

$$-\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{a^3\,\left(A+2\,B\right)}{2}+\frac{B\,a^3}{2}\right)+\sin\left(c+d\,x\right)\,\left(a^3\,\left(A+2\,B\right)+a^3\,\left(2\,A+B\right)+B\,a^3\right)+\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(4\,A\,a^3+4\,B\,a^3\right)+\frac{B\,a^3\,\sin\left(c+d\,x\right)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)^2*((a^3*(A + 2*B))/2 + (B*a^3)/2) + sin(c + d*x)*(a^3*(A + 2
*B) + a^3*(2*A + B) + B*a^3) + log(sin(c + d*x) - 1)*(4*A*a^3 + 4*B*a^3) +
(B*a^3*sin(c + d*x)^3)/3)/d
```

3.991 $\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}+\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}$$

```
[Out] a^3*(A+3*B)*ln(1-sin(d*x+c))/d+a^3*B*sin(d*x+c)/d+2*a^4*(A+B)/d/(a-a*sin(d*
x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$\frac{2a^4(A+B)}{d(a-a\sin(c+dx))}+\frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + 3*B)*Log[1 - Sin[c + d*x]])/d + (a^3*B*Sin[c + d*x])/d + (2*a^4*(
A + B))/(d*(a - a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^3\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{a}\right)}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\text{Subst}\left(\int \left(\frac{B}{a}+\frac{2a(A+B)}{(a-x)^2}+\frac{-A-3B}{a-x}\right)dx, x, a\,\text{si}\right.}{d}$$
$$= \frac{a^3(A+3B)\log(1-\sin(c+dx))}{d}+\frac{a^3B\sin(c}{d}$$

Mathematica [A]
time = 0.12, size = 48, normalized size = 0.77

$$\frac{a^3\left((A+3B)\log(1-\sin(c+dx))-\frac{2(A+B)}{-1+\sin(c+dx)}+B\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
[Out] (a^3*((A + 3*B)*Log[1 - Sin[c + d*x]] - (2*(A + B))/(-1 + Sin[c + d*x]) + B
*Sin[c + d*x]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(62) = 124.
time = 0.28, size = 273, normalized size = 4.40

method	result
risch	$-ia^3xA-3ia^3xB-\frac{ie^{i(dx+c)}B\,a^3}{2d}+\frac{ia^3e^{-i(dx+c)}B}{2d}-\frac{2ia^3Ac}{d}-\frac{6ia^3Bc}{d}-\frac{4ia^3e^{i(dx+c)}(A+B)}{d\left(e^{i(dx+c)}-i\right)^2}+\frac{2a^3}{}$
derivativedivides	$\frac{a^3A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{B\,a^3}{2\cos(dx+c)^2}+\frac{3a^3A}{2\cos(dx+c)^2}+3B\,a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\text{se}}{}\right.}{}$
default	$\frac{a^3A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{B\,a^3}{2\cos(dx+c)^2}+\frac{3a^3A}{2\cos(dx+c)^2}+3B\,a^3\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\text{se}}{}\right.}{}$
norman	$\frac{\frac{48a^3A+48B\,a^3}{4d}+\frac{\left(48a^3A+48B\,a^3\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(64a^3A+64B\,a^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{\left(64a^3A+64B\,a^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*B*
a^3/cos(d*x+c)^2+3/2*a^3*A/cos(d*x+c)^2+3*B*a^3*(1/2*sin(d*x+c)^3/cos(d*x+c
)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*A*(1/2*sin(d*x+c)^3
/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*B*a^3*(1/2*ta
n(d*x+c)^2+ln(cos(d*x+c)))+a^3*A*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+B*a^3*(1
/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x
+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.84

$$\frac{(A+3\,B)a^3\log\left(\sin\left(dx+c\right)-1\right)+Ba^3\sin\left(dx+c\right)-\frac{2\,(A+B)a^3}{\sin(dx+c)-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] ((A + 3*B)*a^3*log(sin(d*x + c) - 1) + B*a^3*sin(d*x + c) - 2*(A + B)*a^3/(
sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.37, size = 89, normalized size = 1.44

$$-\frac{Ba^3\cos\left(dx+c\right)^2+Ba^3\sin\left(dx+c\right)+(2\,A+B)a^3-\left((A+3\,B)a^3\sin\left(dx+c\right)-(A+3\,B)a^3\right)\log\left(-\sin\left(dx+c\right)+1\right)}{d\sin\left(dx+c\right)-d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -(B*a^3*cos(d*x + c)^2 + B*a^3*sin(d*x + c) + (2*A + B)*a^3 - ((A + 3*B)*a^
3*sin(d*x + c) - (A + 3*B)*a^3)*log(-sin(d*x + c) + 1))/(d*sin(d*x + c) - d
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(63) = 126.
time = 0.48, size = 228, normalized size = 3.68

$$-\frac{(Aa^3+3\,Ba^3)\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)-2\,(Aa^3+3\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+3\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+2\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+Aa^3+3\,Ba^3}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1}+\frac{3\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+9\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-10\,Aa^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-22\,Ba^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+3\,Aa^3+9\,Ba^3}{(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-1)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -((A*a^3 + 3*B*a^3)*log(tan(1/2*d*x + 1/2*c)^2 + 1) - 2*(A*a^3 + 3*B*a^3)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - (A*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*B*a^3
*tan(1/2*d*x + 1/2*c)^2 + 2*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 + 3*B*a^3)/(
tan(1/2*d*x + 1/2*c)^2 + 1) + (3*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 9*B*a^3*tan
(1/2*d*x + 1/2*c)^2 - 10*A*a^3*tan(1/2*d*x + 1/2*c) - 22*B*a^3*tan(1/2*d*x
+ 1/2*c) + 3*A*a^3 + 9*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1)^2)/d
```

Mupad [B]
time = 0.08, size = 63, normalized size = 1.02

$$\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(A\,a^3+3\,B\,a^3\right)-\frac{2\,A\,a^3+2\,B\,a^3}{\sin\left(c+d\,x\right)-1}+B\,a^3\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) - 1)*(A*a^3 + 3*B*a^3) - (2*A*a^3 + 2*B*a^3)/(sin(c + d*x
) - 1) + B*a^3*sin(c + d*x))/d
```

3.992 $\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2(A+B)d(a-a\sin(c+dx))^2}$$

```
[Out] 1/2*a^3*(a*A+a*B*sin(d*x+c))^2/(A+B)/d/(a-a*sin(d*x+c))^2
```

Rubi [A]

time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 37}

$$\frac{a^3(aA+aB\sin(c+dx))^2}{2d(A+B)(a-a\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(a*A + a*B*Sin[c + d*x])^2)/(2*(A + B)*d*(a - a*Sin[c + d*x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{a^5\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3}\,dx,x,a\sin(c+dx)\right)}{d}\\ &= \frac{a^3(aA+aB\sin(c+dx))^2}{2(A+B)d(a-a\sin(c+dx))^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 37, normalized size = 0.86

$$\frac{a^3(A+B\sin(c+dx))^2}{2(A+B)d(-1+\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A + B*Sin[c + d*x])^2)/(2*(A + B)*d*(-1 + Sin[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(41) = 82$.
time = 0.29, size = 337, normalized size = 7.84

method	result
risch	$-\frac{2\left(A\,a^3 e^{2i(dx+c)}+iB\,a^3 e^{i(dx+c)}-B\,a^3 e^{2i(dx+c)}-iB\,a^3 e^{3i(dx+c)}\right)}{\left(e^{i(dx+c)}-i\right)^4 d}$
derivativedivides	$a^3A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{B\,a^3}{4\cos(dx+c)^4}+\frac{3a^3A}{4\cos(dx+c)^4}+3B\,a^3\left(\frac{\text{si}}{4\,\text{c}}\right.$
default	$a^3A\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{B\,a^3}{4\cos(dx+c)^4}+\frac{3a^3A}{4\cos(dx+c)^4}+3B\,a^3\left(\frac{\text{si}}{4\,\text{c}}\right.$
norman	$\frac{4\left(7a^3A+5B\,a^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(7a^3A+5B\,a^3\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(36a^3A+44B\,a^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))+1/4*B*a^3/cos(d*x+c)^4+3/4*a^3*A/cos(d*x+c)^4+3*B*a^3*(1/4*s
in(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*l
n(sec(d*x+c)+tan(d*x+c)))+3*a^3*A*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*
x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*B*a^3
*sin(d*x+c)^4/cos(d*x+c)^4+1/4*a^3*A*sin(d*x+c)^4/cos(d*x+c)^4+B*a^3*(1/4*s
in(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8
*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.09

$$\frac{2\,Ba^3\sin\left(dx+c\right)+(A-B)a^3}{2\left(\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/((sin(d*x + c)^2 - 2*sin(d*x + c)
+ 1)*d)
```

Fricas [A]
time = 0.35, size = 49, normalized size = 1.14

$$-\frac{2 Ba^3 \sin(dx+c) + (A-B)a^3}{2 \left(d \cos(dx+c)^2 + 2 d \sin(dx+c) - 2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*B*a^3*sin(d*x + c) + (A - B)*a^3)/(d*cos(d*x + c)^2 + 2*d*sin(d*x +
 c) - 2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.48, size = 82, normalized size = 1.91

$$\frac{2 \left(Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + Ba^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + Aa^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c)^2 + B*a^3*tan(
1/2*d*x + 1/2*c)^2 + A*a^3*tan(1/2*d*x + 1/2*c))/(d*(tan(1/2*d*x + 1/2*c) -
 1)^4)
```

Mupad [B]
time = 9.15, size = 36, normalized size = 0.84

$$\frac{\frac{a^3\,(A-B)}{2}+B\,a^3\,\sin\left(c+d\,x\right)}{d\,{\left(\sin\left(c+d\,x\right)-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^5,x)
[Out] ((a^3*(A - B))/2 + B*a^3*sin(c + d*x))/(d*(sin(c + d*x) - 1)^2)
```

3.993 $\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}$$

```
[Out] 1/8*a^3*(A-B)*arctanh(sin(d*x+c))/d+1/6*a^6*(A+B)/d/(a-a*sin(d*x+c))^3+1/8*
a^5*(A-B)/d/(a-a*sin(d*x+c))^2+1/8*a^4*(A-B)/d/(a-a*sin(d*x+c))
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx))^2}+\frac{a^4(A-B)}{8d(a-a\sin(c+dx))}+\frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(A - B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^6*(A + B))/(6*d*(a - a*Sin[c
 + d*x])^3) + (a^5*(A - B))/(8*d*(a - a*Sin[c + d*x])^2) + (a^4*(A - B))/(8
*d*(a - a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^7\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{A+B}{2a(a-x)^4}+\frac{A-B}{4a^2(a-x)^3}+\frac{A-B}{8a^3(a-x)^2}+\right.\right.}{d}$$

$$= \frac{a^6(A+B)}{6d(a-a\sin(c+dx))^3}+\frac{a^5(A-B)}{8d(a-a\sin(c+dx)}$$

$$= \frac{a^3(A-B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^6(A}{6d(a-a\sin}$$

Mathematica [A]
time = 0.24, size = 95, normalized size = 0.90

$$\frac{a^3\left(2(-5A+B)-3(A-B)\tanh^{-1}(\sin(c+dx))\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^6+9(A-B)\sin(c+dx)-3(A-B)\sin^2(c+dx)\right)}{24d(-1+\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(2*(-5*A + B) - 3*(A - B)*ArcTanh[Sin[c + d*x]]*(Cos[(c + d*x)/2] - Si
n[(c + d*x)/2])^6 + 9*(A - B)*Sin[c + d*x] - 3*(A - B)*Sin[c + d*x]^2))/(24
*d*(-1 + Sin[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(97) = 194.
time = 0.30, size = 442, normalized size = 4.21

method	result
risch	$-\frac{ia^3\left(-18iA\,e^{4i(dx+c)}+3A\,e^{5i(dx+c)}+18iB\,e^{4i(dx+c)}-3B\,e^{5i(dx+c)}+18iA\,e^{2i(dx+c)}-46A\,e^{3i(dx+c)}-18iB\,e^{2i(dx+c)}+\right.}{12d\left(e^{i(dx+c)}-i\right)^6}$
derivativedivides	$\frac{a^3A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^3}{6\cos(dx+c)^6}+\frac{a^3A}{2\cos(dx-}}{}$
default	$\frac{a^3A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^3}{6\cos(dx+c)^6}+\frac{a^3A}{2\cos(dx-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+
c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))+1/6*B*a^3/cos(d*x+c)^6+1/2*a^3*A/cos(d*x
+c)^6+3*B*a^3*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+
1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c
)))+3*a^3*A*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/
16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c))
)+3*B*a^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+a^
3*A*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+B*a^3*(1
/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48*sin(d*x+c)
^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*x+c)+tan(d*
x+c))))
```

Maxima [A]
time = 0.29, size = 123, normalized size = 1.17

$$\frac{3(A-B)a^3\log(\sin(dx+c)+1)-3(A-B)a^3\log(\sin(dx+c)-1)-\frac{2\left(3(A-B)a^3\sin(dx+c)^2-9(A-B)a^3\sin(dx+c)+2(5A-B)a^3\right)}{\sin(dx+c)^3-3\sin(dx+c)^2+3\sin(dx+c)-1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(3*(A - B)*a^3*log(sin(d*x + c) + 1) - 3*(A - B)*a^3*log(sin(d*x + c)
- 1) - 2*(3*(A - B)*a^3*sin(d*x + c)^2 - 9*(A - B)*a^3*sin(d*x + c) + 2*(5*
A - B)*a^3)/(sin(d*x + c)^3 - 3*sin(d*x + c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(100) = 200$.
time = 0.38, size = 242, normalized size = 2.30

$$\frac{6(A-B)a^3\cos(dx+c)^2+18(A-B)a^3\sin(dx+c)-2(13A-5B)a^3+3\left(3(A-B)a^3\cos(dx+c)^2-4(A-B)a^3-\left((A-B)a^3\cos(dx+c)^2-4(A-B)a^3\right)\sin(dx+c)\right)\log(\sin(dx+c)+1)-3\left(3(A-B)a^3\cos(dx+c)^2-4(A-B)a^3-\left((A-B)a^3\cos(dx+c)^2-4(A-B)a^3\right)\sin(dx+c)\right)\log(-\sin(dx+c)+1)}{48\left(3d\cos(dx+c)^2-\left(d\cos(dx+c)^2-4d\right)\sin(dx+c)-4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(6*(A - B)*a^3*cos(d*x + c)^2 + 18*(A - B)*a^3*sin(d*x + c) - 2*(13*A
- 5*B)*a^3 + 3*(3*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3
*cos(d*x + c)^2 - 4*(A - B)*a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3
```

```
*(A - B)*a^3*cos(d*x + c)^2 - 4*(A - B)*a^3 - ((A - B)*a^3*cos(d*x + c)^2 -
 4*(A - B)*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(3*d*cos(d*x + c)^2 -
 (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.50, size = 158, normalized size = 1.50

$$\frac{6\left(Aa^{3}-Ba^{3}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-6\left(Aa^{3}-Ba^{3}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{11\,Aa^{3}\sin(dx+c)^{3}-11\,Ba^{3}\sin(dx+c)^{3}-45\,Aa^{3}\sin(dx+c)^{2}+45\,Ba^{3}\sin(dx+c)^{2}+69\,Aa^{3}\sin(dx+c)-69\,Ba^{3}\sin(dx+c)-51\,Aa^{3}+19\,Ba^{3}}{(\sin(dx+c)-1)^{3}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(A*a^3 - B*a^3)*log(abs(sin(d*x + c) + 1)) - 6*(A*a^3 - B*a^3)*log(
abs(sin(d*x + c) - 1)) + (11*A*a^3*sin(d*x + c)^3 - 11*B*a^3*sin(d*x + c)^3
 - 45*A*a^3*sin(d*x + c)^2 + 45*B*a^3*sin(d*x + c)^2 + 69*A*a^3*sin(d*x + c
) - 69*B*a^3*sin(d*x + c) - 51*A*a^3 + 19*B*a^3)/(sin(d*x + c) - 1)^3)/d
```

Mupad [B]
time = 9.14, size = 112, normalized size = 1.07

$$\frac{a^{3}\,\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(A-B\right)}{8\,d}-\frac{\sin\left(c+d\,x\right)^{2}\,\left(\frac{A\,a^{3}}{8}-\frac{B\,a^{3}}{8}\right)+\frac{5\,A\,a^{3}}{12}-\frac{B\,a^{3}}{12}-\sin\left(c+d\,x\right)\,\left(\frac{3\,A\,a^{3}}{8}-\frac{3\,B\,a^{3}}{8}\right)}{d\,\left(\sin\left(c+d\,x\right)^{3}-3\sin\left(c+d\,x\right)^{2}+3\,\sin\left(c+d\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^7,x)
```

```
[Out] (a^3*atanh(sin(c + d*x))*(A - B))/(8*d) - (sin(c + d*x)^2*((A*a^3)/8 - (B*a
^3)/8) + (5*A*a^3)/12 - (B*a^3)/12 - sin(c + d*x)*((3*A*a^3)/8 - (3*B*a^3)/
8))/(d*(3*sin(c + d*x) - 3*sin(c + d*x)^2 + sin(c + d*x)^3 - 1))
```

3.994 $\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=162

$$\frac{a^3(5A-3B)\tanh^{-1}(\sin(c+dx))}{32d}+\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^6A}{12d(a-a\sin(c+dx))^3}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx)}$$

```
[Out] 1/32*a^3*(5*A-3*B)*arctanh(sin(d*x+c))/d+1/16*a^7*(A+B)/d/(a-a*sin(d*x+c))^
4+1/12*a^6*A/d/(a-a*sin(d*x+c))^3+1/32*a^5*(3*A-B)/d/(a-a*sin(d*x+c))^2+1/1
6*a^4*(2*A-B)/d/(a-a*sin(d*x+c))-1/32*a^4*(A-B)/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^6A}{12d(a-a\sin(c+dx))^3}+\frac{a^5(3A-B)}{32d(a-a\sin(c+dx))^2}+\frac{a^4(2A-B)}{16d(a-a\sin(c+dx))}-\frac{a^4(A-B)}{32d(a\sin(c+dx)+a)}+\frac{a^3(5A-3B)\tanh^{-1}(\sin(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*d) + (a^7*(A + B))/(16*d*(a - a
*Sin[c + d*x])^4) + (a^6*A)/(12*d*(a - a*Sin[c + d*x])^3) + (a^5*(3*A - B))
/(32*d*(a - a*Sin[c + d*x])^2) + (a^4*(2*A - B))/(16*d*(a - a*Sin[c + d*x])
) - (a^4*(A - B))/(32*d*(a + a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^9(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{a^9\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^5(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a^9\text{Subst}\left(\int \left(\frac{A+B}{4a^2(a-x)^5}+\frac{A}{4a^3(a-x)^4}+\frac{3A-B}{16a^4(a-x)^3}\right.\right.}{}$$
$$= \frac{a^7(A+B)}{16d(a-a\sin(c+dx))^4}+\frac{a^6A}{12d(a-a\sin(c+d}$$
$$= \frac{a^3(5A-3B)\tanh^{-1}(\sin(c+dx))}{32d}+\frac{a^7(}{16d(a-}$$

Mathematica [A]
time = 0.56, size = 151, normalized size = 0.93

$$\frac{a^9\left(\frac{(5A-3B)\tanh^{-1}(\sin(c+dx))}{32a^6}+\frac{A+B}{16a^2(a-a\sin(c+dx))^4}+\frac{A}{12a^3(a-a\sin(c+dx))^3}+\frac{3A-B}{32a^4(a-a\sin(c+dx))^2}+\frac{2A-B}{16a^5(a-a\sin(c+dx))}-\frac{A-B}{32a^5(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^9*(((5*A - 3*B)*ArcTanh[Sin[c + d*x]])/(32*a^6) + (A + B)/(16*a^2*(a - a
*Sin[c + d*x])^4) + A/(12*a^3*(a - a*Sin[c + d*x])^3) + (3*A - B)/(32*a^4*(
a - a*Sin[c + d*x])^2) + (2*A - B)/(16*a^5*(a - a*Sin[c + d*x])) - (A - B)/
(32*a^5*(a + a*Sin[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(150) = 300$.
time = 0.37, size = 542, normalized size = 3.35 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(-(-1/8*sec(d*x+c)^7-7/48*sec(d*x+c)^5-35/192*sec(d*x+c)^3-35/12
8*sec(d*x+c))*tan(d*x+c)+35/128*ln(sec(d*x+c)+tan(d*x+c)))+1/8*B*a^3/cos(d*
x+c)^8+3/8*a^3*A/cos(d*x+c)^8+3*B*a^3*(1/8*sin(d*x+c)^3/cos(d*x+c)^8+5/48*s
in(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4+5/128*sin(d*x+c)^3/
```

```
cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d*x+c)))+3*a^3*A*(1/8
*sin(d*x+c)^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3
/cos(d*x+c)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec
(d*x+c)+tan(d*x+c)))+3*B*a^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+1/12*sin(d*x+c)
^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+a^3*A*(1/8*sin(d*x+c)^4/cos
(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+B*
a^3*(1/8*sin(d*x+c)^5/cos(d*x+c)^8+1/16*sin(d*x+c)^5/cos(d*x+c)^6+1/64*sin(
d*x+c)^5/cos(d*x+c)^4-1/128*sin(d*x+c)^5/cos(d*x+c)^2-1/128*sin(d*x+c)^3-3/
128*sin(d*x+c)+3/128*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 185, normalized size = 1.14

$$\frac{3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,A-3\,B)a^3\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,A-3\,B)a^3\sin(dx+c)^4-9\,(5\,A-3\,B)a^3\sin(dx+c)^3+7\,(5\,A-3\,B)a^3\sin(dx+c)^2+3\,(5\,A-3\,B)a^3\sin(dx+c)-32\,Aa^3\right)}{\sin(dx+c)^5-3\,\sin(dx+c)^4+2\,\sin(dx+c)^3+2\,\sin(dx+c)^2-3\,\sin(dx+c)+1}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/192*(3*(5*A - 3*B)*a^3*log(sin(d*x + c) + 1) - 3*(5*A - 3*B)*a^3*log(sin(
d*x + c) - 1) - 2*(3*(5*A - 3*B)*a^3*sin(d*x + c)^4 - 9*(5*A - 3*B)*a^3*sin
(d*x + c)^3 + 7*(5*A - 3*B)*a^3*sin(d*x + c)^2 + 3*(5*A - 3*B)*a^3*sin(d*x
+ c) - 32*A*a^3)/(sin(d*x + c)^5 - 3*sin(d*x + c)^4 + 2*sin(d*x + c)^3 + 2*
sin(d*x + c)^2 - 3*sin(d*x + c) + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(154) = 308$.
time = 0.38, size = 353, normalized size = 2.18

$$\frac{6\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^4-26\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2+12\,(3\,A-5\,B)a^3+3\,\left(3\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^4-4\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2-\left((5\,A-3\,B)a^3\cos\left(dx+c\right)^4-4\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(3\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^4-4\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2-\left((5\,A-3\,B)a^3\cos\left(dx+c\right)^4-4\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)+6\,\left(3\,(5\,A-3\,B)a^3\cos\left(dx+c\right)^2-2\,(5\,A-3\,B)a^3\right)\sin\left(dx+c\right)}{192\,\left(3\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2-\left(d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/192*(6*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 26*(5*A - 3*B)*a^3*cos(d*x + c)^2
 + 12*(3*A - 5*B)*a^3 + 3*(3*(5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)
*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x + c)^4 - 4*(5*A - 3*B)*a^3*c
os(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 3*(3*(5*A - 3*B)*a^3*c
os(d*x + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2 - ((5*A - 3*B)*a^3*cos(d*x
 + c)^4 - 4*(5*A - 3*B)*a^3*cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c)
 + 1) + 6*(3*(5*A - 3*B)*a^3*cos(d*x + c)^2 - 2*(5*A - 3*B)*a^3)*sin(d*x +
c))/(3*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 - (d*cos(d*x + c)^4 - 4*d*cos(
d*x + c)^2)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 237, normalized size = 1.46

$$\frac{12\,(5\,Aa^3-3\,Ba^3)\log\left(|\sin\left(dx+c\right)+1|\right)-12\,(5\,Aa^3-3\,Ba^3)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{12\,(5\,Aa^3\sin(dx+c)-3\,Ba^3\sin(dx+c)+7\,Aa^3-5\,Ba^3)}{\sin(dx+c)+1}+\frac{125\,Aa^3\sin(dx+c)^4-75\,Ba^3\sin(dx+c)^4-596\,Aa^3\sin(dx+c)^3+348\,Ba^3\sin(dx+c)^3+1110\,Aa^3\sin(dx+c)^2-618\,Ba^3\sin(dx+c)^2-996\,Aa^3\sin(dx+c)+492\,Ba^3\sin(dx+c)+405\,Aa^3-99\,Ba^3}{(\sin(dx+c)-1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/768*(12*(5*A*a^3 - 3*B*a^3)*log(abs(sin(d*x + c) + 1)) - 12*(5*A*a^3 - 3*
B*a^3)*log(abs(sin(d*x + c) - 1)) - 12*(5*A*a^3*sin(d*x + c) - 3*B*a^3*sin(
d*x + c) + 7*A*a^3 - 5*B*a^3)/(sin(d*x + c) + 1) + (125*A*a^3*sin(d*x + c)^
4 - 75*B*a^3*sin(d*x + c)^4 - 596*A*a^3*sin(d*x + c)^3 + 348*B*a^3*sin(d*x
+ c)^3 + 1110*A*a^3*sin(d*x + c)^2 - 618*B*a^3*sin(d*x + c)^2 - 996*A*a^3*s
in(d*x + c) + 492*B*a^3*sin(d*x + c) + 405*A*a^3 - 99*B*a^3)/(sin(d*x + c)
- 1)^4)/d
```

Mupad [**B**]
time = 9.18, size = 172, normalized size = 1.06

$$\frac{a^3\,\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(5\,A-3\,B\right)}{32\,d}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(\frac{5\,A\,a^3}{32}-\frac{3\,B\,a^3}{32}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{15\,A\,a^3}{32}-\frac{9\,B\,a^3}{32}\right)+{\sin\left(c+d\,x\right)}^2\,\left(\frac{35\,A\,a^3}{96}-\frac{7\,B\,a^3}{32}\right)-\frac{A\,a^3}{3}+\sin\left(c+d\,x\right)\,\left(\frac{5\,A\,a^3}{32}-\frac{3\,B\,a^3}{32}\right)}{d\,\left({\sin\left(c+d\,x\right)}^5-3\,{\sin\left(c+d\,x\right)}^4+2\,{\sin\left(c+d\,x\right)}^3+2\,{\sin\left(c+d\,x\right)}^2-3\,\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^9,x)
```

```
[Out] (a^3*atanh(sin(c + d*x))*(5*A - 3*B))/(32*d) - (sin(c + d*x)^4*((5*A*a^3)/3
2 - (3*B*a^3)/32) - sin(c + d*x)^3*((15*A*a^3)/32 - (9*B*a^3)/32) + sin(c +
 d*x)^2*((35*A*a^3)/96 - (7*B*a^3)/32) - (A*a^3)/3 + sin(c + d*x)*((5*A*a^3
)/32 - (3*B*a^3)/32))/(d*(2*sin(c + d*x)^2 - 3*sin(c + d*x) + 2*sin(c + d*x
)^3 - 3*sin(c + d*x)^4 + sin(c + d*x)^5 + 1))
```

3.995 $\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$\frac{11}{256}a^3(10A+3B)x-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+3B)\cos(c+dx)\sin(c+dx)}{256d}+\frac{11a^3(10A+3I}{}$$

```
[Out] 11/256*a^3*(10*A+3*B)*x-11/560*a^3*(10*A+3*B)*cos(d*x+c)^7/d+11/256*a^3*(10
*A+3*B)*cos(d*x+c)*sin(d*x+c)/d+11/384*a^3*(10*A+3*B)*cos(d*x+c)^3*sin(d*x+
c)/d+11/480*a^3*(10*A+3*B)*cos(d*x+c)^5*sin(d*x+c)/d-1/90*a*(10*A+3*B)*cos(
d*x+c)^7*(a+a*sin(d*x+c))^2/d-1/10*B*cos(d*x+c)^7*(a+a*sin(d*x+c))^3/d-11/7
20*(10*A+3*B)*cos(d*x+c)^7*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.18, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{11(10A+3B)\cos^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{720d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos^3(c+dx)}{384d}+\frac{11a^3(10A+3B)\sin(c+dx)\cos(c+dx)}{256d}+\frac{11}{256}a^3x(10A+3B)-\frac{a(10A+3B)\cos^7(c+dx)(a\sin(c+dx)+a)^2}{90d}-\frac{B\cos^7(c+dx)(a\sin(c+dx)+a)^3}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (11*a^3*(10*A + 3*B)*x)/256 - (11*a^3*(10*A + 3*B)*Cos[c + d*x]^7)/(560*d)
+ (11*a^3*(10*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (11*a^3*(10*A +
 3*B)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + (11*a^3*(10*A + 3*B)*Cos[c + d
*x]^5*Sin[c + d*x])/(480*d) - (a*(10*A + 3*B)*Cos[c + d*x]^7*(a + a*Sin[c +
 d*x])^2)/(90*d) - (B*Cos[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(10*d) - (11*(
10*A + 3*B)*Cos[c + d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(720*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
```

```
Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^7(c+dx)(a+a\sin(c+dx))^3}{10d}+\frac{1}{10}(10$$

$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx)}{90d}$$

$$= -\frac{a(10A+3B)\cos^7(c+dx)(a+a\sin(c+dx)}{90d}$$

$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}-\frac{a(10A+3I}{}$$

$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+}{}$$

$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+}{}$$

$$= -\frac{11a^3(10A+3B)\cos^7(c+dx)}{560d}+\frac{11a^3(10A+}{}$$

$$= \frac{11}{256}a^3(10A+3B)x-\frac{11a^3(10A+3B)\cos^7(c}{560d}$$

Mathematica [A]

time = 4.84, size = 238, normalized size = 1.03

$$-\frac{a^3\cos(c+dx)\left(47800A+28200B+\frac{27720(10A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+320(221A+123B)\cos(2(c+dx))+160(167A+69B)\cos(4(c+dx))+3520A\cos(6(c+dx))-960B\cos(6(c+dx))-280A\cos(8(c+dx))-840B\cos(8(c+dx))-161490A\sin(c+dx)-40131B\sin(c+dx)-19950A\sin(3(c+dx))+8631B\sin(3(c+dx))+4830A\sin(5(c+dx))+9009B\sin(5(c+dx))+1890A\sin(7(c+dx))+1701B\sin(7(c+dx))-126B\sin(9(c+dx))\right)}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/322560*(a^3*Cos[c + d*x]*(47800*A + 28200*B + (27720*(10*A + 3*B)*ArcSin
[Sqrt[1 - Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 320*(221*A + 123*B
)*Cos[2*(c + d*x)] + 160*(167*A + 69*B)*Cos[4*(c + d*x)] + 3520*A*Cos[6*(c
+ d*x)] - 960*B*Cos[6*(c + d*x)] - 280*A*Cos[8*(c + d*x)] - 840*B*Cos[8*(c
+ d*x)] - 161490*A*Sin[c + d*x] - 40131*B*Sin[c + d*x] - 19950*A*Sin[3*(c +
 d*x)] + 8631*B*Sin[3*(c + d*x)] + 4830*A*Sin[5*(c + d*x)] + 9009*B*Sin[5*(
c + d*x)] + 1890*A*Sin[7*(c + d*x)] + 1701*B*Sin[7*(c + d*x)] - 126*B*Sin[9
*(c + d*x)]))/d
```

Maple [A]

time = 0.79, size = 363, normalized size = 1.57

method	result
risch	$\frac{55a^3xA}{128}+\frac{33a^3Bx}{256}-\frac{33a^3\cos(dx+c)A}{128d}-\frac{19a^3\cos(dx+c)B}{128d}+\frac{B\,a^3\sin(10dx+10c)}{5120d}+\frac{a^3\cos(9dx+9c)A}{2304d}+\frac{a^3\,\mathrm{co}}{}$
derivativedivides	$\frac{a^3A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{B\left(\cos^7(dx+c)\right)a^3}{7}-\frac{3A\left(\cos^7(dx+c)\right)a^3}{7}+3B\,a^3\left(-\right.}{}$
default	$\frac{a^3A\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{B\left(\cos^7(dx+c)\right)a^3}{7}-\frac{3A\left(\cos^7(dx+c)\right)a^3}{7}+3B\,a^3\left(-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)-1/7*B*cos(d*x+c)^7*a^3-3/7*A*cos(d*x+c)^7*a^3+3*B*a^3*(-1/
8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/128*d*x+5/128*c)+3*a^3*A*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/4
8*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/12
8*c)+3*B*a^3*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+a^3*A*(-1/9
*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+B*a^3*(-1/10*sin(d*x+c)^3*cos
(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3
+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.31, size = 284, normalized size = 1.23

$$-\frac{276480\,Aa^3\cos(dx+c)^7 + 92160\,Ba^3\cos(dx+c)^7 - 10240\,(7\cos(dx+c)^9 - 9\cos(dx+c)^7)Aa^3 - 630\,(64\sin(2dx+2c)^3 + 120\,dx + 120\,c - 3\sin(8dx+8c) - 24\sin(4dx+4c))Aa^3 + 3360\,(4\sin(2dx+2c)^3 - 60\,dx - 60\,c - 9\sin(4dx+4c) - 48\sin(2dx+2c))Aa^3 - 30720\,(7\cos(dx+c)^9 - 9\cos(dx+c)^7)Ba^3 - 63\,(32\sin(2dx+2c)^5 + 120\,dx + 120\,c + 5\sin(8dx+8c) - 40\sin(4dx+4c))Ba^3 - 630\,(64\sin(2dx+2c)^3 + 120\,dx + 120\,c - 3\sin(8dx+8c) - 24\sin(4dx+4c))Ba^3}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/645120*(276480*A*a^3*cos(d*x + c)^7 + 92160*B*a^3*cos(d*x + c)^7 - 10240
*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*A*a^3 - 630*(64*sin(2*d*x + 2*c)^3 +
 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*A*a^3 + 3360*(
4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
2*c))*A*a^3 - 30720*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*B*a^3 - 63*(32*si
n(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*
c))*B*a^3 - 630*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*
c) - 24*sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A]
time = 0.39, size = 155, normalized size = 0.67

$$\frac{8960\,(A+3\,B)a^3\cos(dx+c)^9 - 46080\,(A+B)a^3\cos(dx+c)^7 + 3465\,(10\,A+3\,B)a^3dx + 21\,(384\,Ba^3\cos(dx+c)^9 - 48\,(30\,A+41\,B)a^3\cos(dx+c)^7 + 88\,(10\,A+3\,B)a^3\cos(dx+c)^5 + 110\,(10\,A+3\,B)a^3\cos(dx+c)^3 + 165\,(10\,A+3\,B)a^3\cos(dx+c))\sin(dx+c)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/80640*(8960*(A + 3*B)*a^3*cos(d*x + c)^9 - 46080*(A + B)*a^3*cos(d*x + c)
^7 + 3465*(10*A + 3*B)*a^3*d*x + 21*(384*B*a^3*cos(d*x + c)^9 - 48*(30*A +
41*B)*a^3*cos(d*x + c)^7 + 88*(10*A + 3*B)*a^3*cos(d*x + c)^5 + 110*(10*A +
 3*B)*a^3*cos(d*x + c)^3 + 165*(10*A + 3*B)*a^3*cos(d*x + c))*sin(d*x + c))
/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(219) = 438$.
time = 1.97, size = 1042, normalized size = 4.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((15*A*a**3*x*sin(c + d*x)**8/128 + 15*A*a**3*x*sin(c + d*x)**6*co
s(c + d*x)**2/32 + 5*A*a**3*x*sin(c + d*x)**6/16 + 45*A*a**3*x*sin(c + d*x)
**4*cos(c + d*x)**4/64 + 15*A*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 1
```

```
5*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 15*A*a**3*x*sin(c + d*x)**2
*cos(c + d*x)**4/16 + 15*A*a**3*x*cos(c + d*x)**8/128 + 5*A*a**3*x*cos(c +
d*x)**6/16 + 15*A*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*A*a**3*sin
(c + d*x)**5*cos(c + d*x)**3/(128*d) + 5*A*a**3*sin(c + d*x)**5*cos(c + d*x
)/(16*d) + 73*A*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + 5*A*a**3*sin
(c + d*x)**3*cos(c + d*x)**3/(6*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)**7
/(7*d) - 15*A*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) + 11*A*a**3*sin(c +
 d*x)*cos(c + d*x)**5/(16*d) - 2*A*a**3*cos(c + d*x)**9/(63*d) - 3*A*a**3*c
os(c + d*x)**7/(7*d) + 3*B*a**3*x*sin(c + d*x)**10/256 + 15*B*a**3*x*sin(c
+ d*x)**8*cos(c + d*x)**2/256 + 15*B*a**3*x*sin(c + d*x)**8/128 + 15*B*a**3
*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*B*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 15*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 45*B*a**3*
x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*B*a**3*x*sin(c + d*x)**2*cos(c +
d*x)**8/256 + 15*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*B*a**3*x*c
os(c + d*x)**10/256 + 15*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*sin(c + d*
x)**9*cos(c + d*x)/(256*d) + 7*B*a**3*sin(c + d*x)**7*cos(c + d*x)**3/(128*
d) + 15*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + B*a**3*sin(c + d*x)**
5*cos(c + d*x)**5/(10*d) + 55*B*a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d
) - 7*B*a**3*sin(c + d*x)**3*cos(c + d*x)**7/(128*d) + 73*B*a**3*sin(c + d*
x)**3*cos(c + d*x)**5/(128*d) - 3*B*a**3*sin(c + d*x)**2*cos(c + d*x)**7/(7
*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 15*B*a**3*sin(c + d*x
)*cos(c + d*x)**7/(128*d) - 2*B*a**3*cos(c + d*x)**9/(21*d) - B*a**3*cos(c
+ d*x)**7/(7*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*cos(c)**6,
True))
```

Giac [A]
time = 0.51, size = 273, normalized size = 1.18

$$\frac{Ba^3\sin(10\,dx+10\,c)}{5120\,d}+\frac{11}{256}\,(10\,Aa^3+3\,Ba^3)x+\frac{(Aa^3+3\,Ba^3)\cos(9\,dx+9\,c)}{2304\,d}-\frac{(9\,Aa^3-5\,Ba^3)\cos(7\,dx+7\,c)}{1792\,d}-\frac{(3\,Aa^3+Ba^3)\cos(5\,dx+5\,c)}{64\,d}-\frac{(29\,Aa^3+15\,Ba^3)\cos(3\,dx+3\,c)}{192\,d}-\frac{(33\,Aa^3+19\,Ba^3)\cos(dx+c)}{128\,d}-\frac{(6\,Aa^3+5\,Ba^3)\sin(8\,dx+8\,c)}{2048\,d}-\frac{(32\,Aa^3+51\,Ba^3)\sin(6\,dx+6\,c)}{3072\,d}+\frac{(6\,Aa^3-7\,Ba^3)\sin(4\,dx+4\,c)}{256\,d}+\frac{(144\,Aa^3+25\,Ba^3)\sin(2\,dx+2\,c)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*a^3*sin(10*d*x + 10*c)/d + 11/256*(10*A*a^3 + 3*B*a^3)*x + 1/2304*
(A*a^3 + 3*B*a^3)*cos(9*d*x + 9*c)/d - 1/1792*(9*A*a^3 - 5*B*a^3)*cos(7*d*x
 + 7*c)/d - 1/64*(3*A*a^3 + B*a^3)*cos(5*d*x + 5*c)/d - 1/192*(29*A*a^3 + 1
5*B*a^3)*cos(3*d*x + 3*c)/d - 1/128*(33*A*a^3 + 19*B*a^3)*cos(d*x + c)/d -
1/2048*(6*A*a^3 + 5*B*a^3)*sin(8*d*x + 8*c)/d - 1/3072*(32*A*a^3 + 51*B*a^3
)*sin(6*d*x + 6*c)/d + 1/256*(6*A*a^3 - 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/512
*(144*A*a^3 + 25*B*a^3)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.83, size = 711, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)
```

```
[Out] (11*a^3*atan((11*a^3*tan(c/2 + (d*x)/2)*(10*A + 3*B))/(128*((55*A*a^3)/64 +
 (33*B*a^3)/128)))*(10*A + 3*B))/(128*d) - (11*a^3*(10*A + 3*B)*(atan(tan(c
/2 + (d*x)/2)) - (d*x)/2))/(128*d) - ((58*A*a^3)/63 - tan(c/2 + (d*x)/2)*((
73*A*a^3)/64 - (33*B*a^3)/128) + (10*B*a^3)/21 + tan(c/2 + (d*x)/2)^18*(6*A
*a^3 + 2*B*a^3) + tan(c/2 + (d*x)/2)^16*(22*A*a^3 + 18*B*a^3) + tan(c/2 + (
d*x)/2)^8*(84*A*a^3 + 28*B*a^3) + tan(c/2 + (d*x)/2)^14*((136*A*a^3)/3 + 8*
B*a^3) + tan(c/2 + (d*x)/2)^4*((136*A*a^3)/7 + (24*B*a^3)/7) + tan(c/2 + (d
*x)/2)^10*(116*A*a^3 + 60*B*a^3) + tan(c/2 + (d*x)/2)^19*((73*A*a^3)/64 - (
33*B*a^3)/128) + tan(c/2 + (d*x)/2)^2*((202*A*a^3)/63 + (58*B*a^3)/21) + ta
n(c/2 + (d*x)/2)^12*((328*A*a^3)/3 + 72*B*a^3) - tan(c/2 + (d*x)/2)^7*((341
*A*a^3)/16 - (333*B*a^3)/32) + tan(c/2 + (d*x)/2)^13*((341*A*a^3)/16 - (333
*B*a^3)/32) + tan(c/2 + (d*x)/2)^6*((456*A*a^3)/7 + (344*B*a^3)/7) - tan(c/
2 + (d*x)/2)^5*((449*A*a^3)/48 + (577*B*a^3)/160) + tan(c/2 + (d*x)/2)^15*(
(449*A*a^3)/48 + (577*B*a^3)/160) - tan(c/2 + (d*x)/2)^3*((2117*A*a^3)/192
+ (705*B*a^3)/128) + tan(c/2 + (d*x)/2)^17*((2117*A*a^3)/192 + (705*B*a^3)/
128) - tan(c/2 + (d*x)/2)^9*((699*A*a^3)/32 + (2749*B*a^3)/64) + tan(c/2 +
(d*x)/2)^11*((699*A*a^3)/32 + (2749*B*a^3)/64))/(d*(10*tan(c/2 + (d*x)/2)^2
 + 45*tan(c/2 + (d*x)/2)^4 + 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)
/2)^8 + 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 + 120*tan(c/2
 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 + 10*tan(c/2 + (d*x)/2)^18 + tan(
c/2 + (d*x)/2)^20 + 1))
```

3.996 $\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=200

$$\frac{9}{128}a^3(8A+3B)x-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{9a^3(8A+3B)\cos(c+dx)\sin(c+dx)}{128d}+\frac{3a^3(8A+3B)\cos^3(c$$

```
[Out] 9/128*a^3*(8*A+3*B)*x-3/80*a^3*(8*A+3*B)*cos(d*x+c)^5/d+9/128*a^3*(8*A+3*B)
*cos(d*x+c)*sin(d*x+c)/d+3/64*a^3*(8*A+3*B)*cos(d*x+c)^3*sin(d*x+c)/d-1/56*
a*(8*A+3*B)*cos(d*x+c)^5*(a+a*sin(d*x+c))^2/d-1/8*B*cos(d*x+c)^5*(a+a*sin(d
*x+c))^3/d-3/112*(8*A+3*B)*cos(d*x+c)^5*(a^3+a^3*sin(d*x+c))/d
```

Rubi [A]
time = 0.16, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{3(8A+3B)\cos^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{112d}+\frac{3a^3(8A+3B)\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{9a^3(8A+3B)\sin(c+dx)\cos(c+dx)}{128d}+\frac{9}{128}a^3x(8A+3B)-\frac{a(8A+3B)\cos^5(c+dx)(a\sin(c+dx)+a)^2}{56d}-\frac{B\cos^5(c+dx)(a\sin(c+dx)+a)^3}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (9*a^3*(8*A + 3*B)*x)/128 - (3*a^3*(8*A + 3*B)*Cos[c + d*x]^5)/(80*d) + (9*
a^3*(8*A + 3*B)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (3*a^3*(8*A + 3*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - (a*(8*A + 3*B)*Cos[c + d*x]^5*(a + a*Sin
[c + d*x])^2)/(56*d) - (B*Cos[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(8*d) - (3
*(8*A + 3*B)*Cos[c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(112*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = -\frac{B\cos^5(c+dx)(a+a\sin(c+dx))^3}{8d}+\frac{1}{8}(8A$$
$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}$$
$$= -\frac{a(8A+3B)\cos^5(c+dx)(a+a\sin(c+dx))^2}{56d}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}-\frac{a(8A+3B)\cos}{}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{3a^3(8A+3B)}{}$$
$$= -\frac{3a^3(8A+3B)\cos^5(c+dx)}{80d}+\frac{9a^3(8A+3B)}{}$$
$$= \frac{9}{128}a^3(8A+3B)x-\frac{3a^3(8A+3B)\cos^5(c+d}{80d}$$

Mathematica [A]
time = 1.60, size = 183, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(4576A+2976B+\frac{2520(8A+3B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+16(373A+223B)\cos(2(c+dx))+32(41A+11B)\cos(4(c+dx))-80A\cos(6(c+dx))-240B\cos(6(c+dx))-10640A\sin(c+dx)-3045B\sin(c+dx)+1365B\sin(3(c+dx))+560A\sin(5(c+dx))+595B\sin(5(c+dx))-35B\sin(7(c+dx))\right)}{17920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/17920*(a^3*Cos[c + d*x]*(4576*A + 2976*B + (2520*(8*A + 3*B)*ArcSin[Sqrt
[1 - Sin[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(373*A + 223*B)*Cos[
2*(c + d*x)] + 32*(41*A + 11*B)*Cos[4*(c + d*x)] - 80*A*Cos[6*(c + d*x)] -
240*B*Cos[6*(c + d*x)] - 10640*A*Sin[c + d*x] - 3045*B*Sin[c + d*x] + 1365*
B*Sin[3*(c + d*x)] + 560*A*Sin[5*(c + d*x)] + 595*B*Sin[5*(c + d*x)] - 35*B
*Sin[7*(c + d*x)]))/d
```

Maple [A]
time = 0.53, size = 323, normalized size = 1.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5
*B*cos(d*x+c)^5*a^3-3/5*A*cos(d*x+c)^5*a^3+3*B*a^3*(-1/6*sin(d*x+c)*cos(d*x
+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*a^3*
A*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)+1/16*d*x+1/16*c)+3*B*a^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)
^5)+a^3*A*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+B*a^3*(-1/8*si
n(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2
*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A]
time = 0.31, size = 232, normalized size = 1.16

$$-\frac{21504\,Aa^{3}\cos\left(dx+c\right)^{5}+7168\,Ba^{3}\cos\left(dx+c\right)^{5}-1024\,\left(5\,\cos\left(dx+c\right)^{7}-7\,\cos\left(dx+c\right)^{5}\right)Aa^{3}-560\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Aa^{3}-1120\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^{3}-3072\,\left(5\,\cos\left(dx+c\right)^{7}-7\,\cos\left(dx+c\right)^{5}\right)Ba^{3}-560\,\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)Ba^{3}-35\,\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)Ba^{3}}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/35840*(21504*A*a^3*cos(d*x + c)^5 + 7168*B*a^3*cos(d*x + c)^5 - 1024*(5*
cos(d*x + c)^7 - 7*cos(d*x + c)^5)*A*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d
*x + 12*c - 3*sin(4*d*x + 4*c))*A*a^3 - 1120*(12*d*x + 12*c + sin(4*d*x + 4
*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5
)*B*a^3 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*B
*a^3 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A]
time = 0.38, size = 135, normalized size = 0.68

$$\frac{640\,(A+3\,B)a^{3}\cos\left(dx+c\right)^{7}-3584\,(A+B)a^{3}\cos\left(dx+c\right)^{5}+315\,(8\,A+3\,B)a^{3}dx+35\,\left(16\,Ba^{3}\cos\left(dx+c\right)^{7}-8\,(8\,A+11\,B)a^{3}\cos\left(dx+c\right)^{5}+6\,(8\,A+3\,B)a^{3}\cos\left(dx+c\right)^{3}+9\,(8\,A+3\,B)a^{3}\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4480*(640*(A + 3*B)*a^3*cos(d*x + c)^7 - 3584*(A + B)*a^3*cos(d*x + c)^5
+ 315*(8*A + 3*B)*a^3*d*x + 35*(16*B*a^3*cos(d*x + c)^7 - 8*(8*A + 11*B)*a^
3*cos(d*x + c)^5 + 6*(8*A + 3*B)*a^3*cos(d*x + c)^3 + 9*(8*A + 3*B)*a^3*cos
(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(189) = 378$.
time = 1.08, size = 823, normalized size = 4.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**6/16 + 9*A*a**3*x*sin(c + d*x)**4*cos(c
 + d*x)**2/16 + 3*A*a**3*x*sin(c + d*x)**4/8 + 9*A*a**3*x*sin(c + d*x)**2*c
os(c + d*x)**4/16 + 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*a**3
*x*cos(c + d*x)**6/16 + 3*A*a**3*x*cos(c + d*x)**4/8 + 3*A*a**3*sin(c + d*x
)**5*cos(c + d*x)/(16*d) + A*a**3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) + 3
*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*A*a**3*si
n(c + d*x)*cos(c + d*x)**3/(8*d) - 2*A*a**3*cos(c + d*x)**7/(35*d) - 3*A*a*
*3*cos(c + d*x)**5/(5*d) + 3*B*a**3*x*sin(c + d*x)**8/128 + 3*B*a**3*x*sin(
c + d*x)**6*cos(c + d*x)**2/32 + 3*B*a**3*x*sin(c + d*x)**6/16 + 9*B*a**3*x
*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*B*a**3*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 9*B*a**3*x*sin(c
 + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*cos(c + d*x)**8/128 + 3*B*a**3*x
*cos(c + d*x)**6/16 + 3*B*a**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*B*
a**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) + 3*B*a**3*sin(c + d*x)**5*cos
(c + d*x)/(16*d) - 11*B*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) + B*a*
*3*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*B*a**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 3*B*a**3*s
in(c + d*x)*cos(c + d*x)**5/(16*d) - 6*B*a**3*cos(c + d*x)**7/(35*d) - B*a*
*3*cos(c + d*x)**5/(5*d), Ne(d, 0)), (x*(A + B*sin(c))*(a*sin(c) + a)**3*co
s(c)**4, True))
```

Giac [A]
time = 0.62, size = 217, normalized size = 1.08

$$\frac{Ba^3\sin(8\,dx+8\,c)}{1024\,d}+\frac{9}{128}\left(8\,Aa^3+3\,Ba^3\right)x+\frac{\left(Aa^3+3\,Ba^3\right)\cos(7\,dx+7\,c)}{448\,d}-\frac{\left(11\,Aa^3+Ba^3\right)\cos(5\,dx+5\,c)}{320\,d}-\frac{\left(13\,Aa^3+7\,Ba^3\right)\cos(3\,dx+3\,c)}{64\,d}-\frac{\left(27\,Aa^3+17\,Ba^3\right)\cos(dx+c)}{64\,d}-\frac{\left(Aa^3+Ba^3\right)\sin(6\,dx+6\,c)}{64\,d}-\frac{\left(2\,Aa^3+7\,Ba^3\right)\sin(4\,dx+4\,c)}{128\,d}+\frac{\left(19\,Aa^3+3\,Ba^3\right)\sin(2\,dx+2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

[Out] 1/1024*B*a^3*sin(8*d*x + 8*c)/d + 9/128*(8*A*a^3 + 3*B*a^3)*x + 1/448*(A*a^3 + 3*B*a^3)*cos(7*d*x + 7*c)/d - 1/320*(11*A*a^3 + B*a^3)*cos(5*d*x + 5*c)/d - 1/64*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d - 1/64*(27*A*a^3 + 17*B*a^3)*cos(d*x + c)/d - 1/64*(A*a^3 + B*a^3)*sin(6*d*x + 6*c)/d - 1/128*(2*A*a^3 + 7*B*a^3)*sin(4*d*x + 4*c)/d + 1/64*(19*A*a^3 + 3*B*a^3)*sin(2*d*x + 2*c)/d

Mupad [B]
time = 10.69, size = 584, normalized size = 2.92

$$\frac{9\,a^3\,\mathrm{atan}\left(\frac{9\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,A+3\,B\right)}{64\,\left(\frac{9\,A\,a^3}{8}+\frac{27\,B\,a^3}{64}\right)}\right)\,\left(8\,A+3\,B\right)}{64\,d}-\frac{9\,a^3\,\left(8\,A+3\,B\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}-\frac{\frac{46\,A\,a^3}{35}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7\,A\,a^3}{8}-\frac{27\,B\,a^3}{64}\right)+\frac{26\,B\,a^3}{35}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(6\,A\,a^3+2\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(30\,A\,a^3+10\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(22\,A\,a^3+18\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}\,\left(46\,A\,a^3+26\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}\,\left(\frac{74\,A\,a^3}{5}+\frac{14\,B\,a^3}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}\,\left(\frac{7\,A\,a^3}{8}-\frac{27\,B\,a^3}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}\,\left(\frac{158\,A\,a^3}{35}+\frac{138\,B\,a^3}{35}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}\,\left(\frac{218\,A\,a^3}{5}+\frac{158\,B\,a^3}{5}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}\,\left(\frac{75\,A\,a^3}{8}+\frac{305\,B\,a^3}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{75\,A\,a^3}{8}+\frac{305\,B\,a^3}{64}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}\,\left(\frac{55\,A\,a^3}{8}+\frac{437\,B\,a^3}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{55\,A\,a^3}{8}+\frac{437\,B\,a^3}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}\,\left(\frac{13\,A\,a^3}{8}+\frac{919\,B\,a^3}{64}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}\,\left(\frac{13\,A\,a^3}{8}+\frac{919\,B\,a^3}{64}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+70\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+56\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+28\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+8\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)

[Out] (9*a^3*atan((9*a^3*tan(c/2 + (d*x)/2)*(8*A + 3*B))/(64*((9*A*a^3)/8 + (27*B*a^3)/64)))*(8*A + 3*B))/(64*d) - (9*a^3*(8*A + 3*B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d) - ((46*A*a^3)/35 - tan(c/2 + (d*x)/2)*((7*A*a^3)/8 - (27*B*a^3)/64) + (26*B*a^3)/35 + tan(c/2 + (d*x)/2)^14*(6*A*a^3 + 2*B*a^3) + tan(c/2 + (d*x)/2)^10*(30*A*a^3 + 10*B*a^3) + tan(c/2 + (d*x)/2)^12*(22*A*a^3 + 18*B*a^3) + tan(c/2 + (d*x)/2)^8*(46*A*a^3 + 26*B*a^3) + tan(c/2 + (d*x)/2)^4*((74*A*a^3)/5 + (14*B*a^3)/5) + tan(c/2 + (d*x)/2)^15*((7*A*a^3)/8 - (27*B*a^3)/64) + tan(c/2 + (d*x)/2)^2*((158*A*a^3)/35 + (138*B*a^3)/35) + tan(c/2 + (d*x)/2)^6*((218*A*a^3)/5 + (158*B*a^3)/5) - tan(c/2 + (d*x)/2)^3*((75*A*a^3)/8 + (305*B*a^3)/64) + tan(c/2 + (d*x)/2)^13*((75*A*a^3)/8 + (305*B*a^3)/64) - tan(c/2 + (d*x)/2)^5*((55*A*a^3)/8 + (437*B*a^3)/64) + tan(c/2 + (d*x)/2)^11*((55*A*a^3)/8 + (437*B*a^3)/64) + tan(c/2 + (d*x)/2)^7*((13*A*a^3)/8 + (919*B*a^3)/64) - tan(c/2 + (d*x)/2)^9*((13*A*a^3)/8 + (919*B*a^3)/64))/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1))

3.997 $\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{7}{16}a^3(2A+B)x-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(2A+B)\cos(c+dx)\sin(c+dx)}{16d}-\frac{a(2A+B)\cos^3(c+dx)}{10d}$$

```
[Out] 7/16*a^3*(2*A+B)*x-7/24*a^3*(2*A+B)*cos(d*x+c)^3/d+7/16*a^3*(2*A+B)*cos(d*x
+c)*sin(d*x+c)/d-1/10*a*(2*A+B)*cos(d*x+c)^3*(a+a*sin(d*x+c))^2/d-1/6*B*cos
(d*x+c)^3*(a+a*sin(d*x+c))^3/d-7/40*(2*A+B)*cos(d*x+c)^3*(a^3+a^3*sin(d*x+c
))/d
```

Rubi [A]
time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2939, 2757, 2748, 2715, 8}

$$-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{7(2A+B)\cos^3(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{40d}+\frac{7a^3(2A+B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{7}{16}a^3x(2A+B)-\frac{a(2A+B)\cos^3(c+dx)(a\sin(c+dx)+a)^2}{10d}-\frac{B\cos^3(c+dx)(a\sin(c+dx)+a)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (7*a^3*(2*A + B)*x)/16 - (7*a^3*(2*A + B)*Cos[c + d*x]^3)/(24*d) + (7*a^3*(
2*A + B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a*(2*A + B)*Cos[c + d*x]^3*(a
 + a*Sin[c + d*x])^2)/(10*d) - (B*Cos[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(6
*d) - (7*(2*A + B)*Cos[c + d*x]^3*(a^3 + a^3*Sin[c + d*x]))/(40*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2757

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[a*((2*m + p - 1)/(m + p)), Int[(g*C
os[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegersQ
[2*m, 2*p]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= -\frac{B\cos^3(c+dx)(a+a\sin(c+dx))^3}{6d}+\frac{1}{2}(2A+\\ &= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\\ &= -\frac{a(2A+B)\cos^3(c+dx)(a+a\sin(c+dx))^2}{10d}-\\ &= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}-\frac{a(2A+B)\cos^3(\\ &= -\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\frac{7a^3(2A+B)\cos\\ &= \frac{7}{16}a^3(2A+B)x-\frac{7a^3(2A+B)\cos^3(c+dx)}{24d}+\end{aligned}$$

Mathematica [A]
time = 1.04, size = 146, normalized size = 0.92

$$-\frac{a^3\cos(c+dx)\left(284A+212B+\frac{420(2A+B)\sin^{-1}\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)}{\sqrt{\cos^2(c+dx)}}+16(17A+11B)\cos(2(c+dx))-12(A+3B)\cos(4(c+dx))-330A\sin(c+dx)-95B\sin(c+dx)+90A\sin(3(c+dx))+110B\sin(3(c+dx))-5B\sin(5(c+dx))\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/480*(a^3*Cos[c + d*x]*(284*A + 212*B + (420*(2*A + B)*ArcSin[Sqrt[1 - Si
n[c + d*x]]/Sqrt[2]])/Sqrt[Cos[c + d*x]^2] + 16*(17*A + 11*B)*Cos[2*(c + d*
x)] - 12*(A + 3*B)*Cos[4*(c + d*x)] - 330*A*Sin[c + d*x] - 95*B*Sin[c + d*x
] + 90*A*Sin[3*(c + d*x)] + 110*B*Sin[3*(c + d*x)] - 5*B*Sin[5*(c + d*x)]))
/d
```

Maple [A]
time = 0.33, size = 279, normalized size = 1.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-1/3*B*cos(d*x+c)^3*a^3
-A*cos(d*x+c)^3*a^3+3*B*a^3*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*co
s(d*x+c)+1/8*d*x+1/8*c)+3*a^3*A*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c
)*cos(d*x+c)+1/8*d*x+1/8*c)+3*B*a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*co
s(d*x+c)^3)+a^3*A*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+B*a^3*
(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)
*cos(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.31, size = 199, normalized size = 1.25

$$-\frac{960\,Aa^3\cos\left(dx+c\right)^3+320\,Ba^3\cos\left(dx+c\right)^3-64\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)Aa^3-90\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)Aa^3-240\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3-192\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)Ba^3+5\,\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)Ba^3-90\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)Ba^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/960*(960*A*a^3*cos(d*x + c)^3 + 320*B*a^3*cos(d*x + c)^3 - 64*(3*cos(d*x
 + c)^5 - 5*cos(d*x + c)^3)*A*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*A*a
^3 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 - 192*(3*cos(d*x + c)^5 - 5
*cos(d*x + c)^3)*B*a^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*
d*x + 4*c))*B*a^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*B*a^3)/d
```

Fricas [A]
time = 0.38, size = 111, normalized size = 0.70

$$\frac{48\,(A+3\,B)a^3\cos\left(dx+c\right)^5-320\,(A+B)a^3\cos\left(dx+c\right)^3+105\,(2\,A+B)a^3dx+5\,\left(8\,Ba^3\cos\left(dx+c\right)^5-2\,(18\,A+25\,B)a^3\cos\left(dx+c\right)^3+21\,(2\,A+B)a^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(48*(A + 3*B)*a^3*cos(d*x + c)^5 - 320*(A + B)*a^3*cos(d*x + c)^3 + 1
05*(2*A + B)*a^3*d*x + 5*(8*B*a^3*cos(d*x + c)^5 - 2*(18*A + 25*B)*a^3*cos(
d*x + c)^3 + 21*(2*A + B)*a^3*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(151) = 302$.
time = 0.54, size = 588, normalized size = 3.70

$$\begin{cases} \frac{3Aa^3x\sin^4(c+dx)}{8} + \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Aa^3x\sin^2(c+dx)}{2} + \frac{3Aa^3x\cos^4(c+dx)}{8} + \frac{Aa^3x\cos^2(c+dx)}{2} + \frac{3Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{Aa^3\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{3Aa^3\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{Aa^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{2Aa^3\cos^5(c+dx)}{15d} - \frac{Aa^3\cos^3(c+dx)}{d} + \frac{Ba^3x\sin^6(c+dx)}{16} + \frac{3Ba^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3Ba^3x\sin^4(c+dx)}{8} + \frac{3Ba^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3Ba^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{Ba^3x\cos^6(c+dx)}{16} + \frac{3Ba^3x\cos^4(c+dx)}{8} + \frac{Ba^3\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{Ba^3\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{3Ba^3\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{Ba^3\sin^2(c+dx)\cos^3(c+dx)}{d} - \frac{Ba^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{3Ba^3\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{2Ba^3\cos^5(c+dx)}{5d} - \frac{Ba^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a\sin(c)+a)^3\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(c + d*x)**4/8 + 3*A*a**3*x*sin(c + d*x)**2*cos(c
+ d*x)**2/4 + A*a**3*x*sin(c + d*x)**2/2 + 3*A*a**3*x*cos(c + d*x)**4/8 + A
*a**3*x*cos(c + d*x)**2/2 + 3*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - A
*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 3*A*a**3*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) + A*a**3*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*A*a**3*cos(c +
d*x)**5/(15*d) - A*a**3*cos(c + d*x)**3/d + B*a**3*x*sin(c + d*x)**6/16 + 3
*B*a**3*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*B*a**3*x*sin(c + d*x)**4/8
 + 3*B*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 3*B*a**3*x*sin(c + d*x)*
*2*cos(c + d*x)**2/4 + B*a**3*x*cos(c + d*x)**6/16 + 3*B*a**3*x*cos(c + d*x
)**4/8 + B*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - B*a**3*sin(c + d*x)**
3*cos(c + d*x)**3/(6*d) + 3*B*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) - B*a
**3*sin(c + d*x)**2*cos(c + d*x)**3/d - B*a**3*sin(c + d*x)*cos(c + d*x)**5
/(16*d) - 3*B*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*B*a**3*cos(c + d*
x)**5/(5*d) - B*a**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a
*sin(c) + a)**3*cos(c)**2, True))
```

Giac [A]
time = 0.50, size = 165, normalized size = 1.04

$$\frac{Ba^3\sin(6dx+6c)}{192d} + \frac{7}{16}(2Aa^3+Ba^3)x + \frac{(Aa^3+3Ba^3)\cos(5dx+5c)}{80d} - \frac{(13Aa^3+7Ba^3)\cos(3dx+3c)}{48d} - \frac{(7Aa^3+5Ba^3)\cos(dx+c)}{8d} - \frac{(6Aa^3+7Ba^3)\sin(4dx+4c)}{64d} + \frac{(16Aa^3-Ba^3)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/192*B*a^3*sin(6*d*x + 6*c)/d + 7/16*(2*A*a^3 + B*a^3)*x + 1/80*(A*a^3 + 3
*B*a^3)*cos(5*d*x + 5*c)/d - 1/48*(13*A*a^3 + 7*B*a^3)*cos(3*d*x + 3*c)/d -
 1/8*(7*A*a^3 + 5*B*a^3)*cos(d*x + c)/d - 1/64*(6*A*a^3 + 7*B*a^3)*sin(4*d*
x + 4*c)/d + 1/64*(16*A*a^3 - B*a^3)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 10.73, size = 451, normalized size = 2.84

$$\frac{7a^3\,\mathrm{atan}\left(\frac{7a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2A+B)}{8\left(\frac{7Aa^3}{4}+\frac{7Ba^3}{8}\right)}\right)(2A+B)}{8d} - \frac{\frac{34Aa^3}{15} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Aa^3}{4}-\frac{7Ba^3}{8}\right) + \frac{22Ba^3}{15} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(6Aa^3+2Ba^3) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(12Aa^3+4Ba^3) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{Aa^3}{4}-\frac{7Ba^3}{8}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(22Aa^3+18Ba^3) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{13Aa^3}{2}+\frac{37Ba^3}{4}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{13Aa^3}{2}+\frac{37Ba^3}{4}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{38Aa^3}{5}+\frac{34Ba^3}{5}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{68Aa^3}{3}+\frac{44Ba^3}{3}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{27Aa^3}{4}+\frac{73Ba^3}{24}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{27Aa^3}{4}+\frac{73Ba^3}{24}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12} + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 1\right)} - \frac{7a^3(2A+B)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right) - \frac{dx}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)

[Out] (7*a^3*atan((7*a^3*tan(c/2 + (d*x)/2)*(2*A + B))/(8*((7*A*a^3)/4 + (7*B*a^3
)/8)))*(2*A + B))/(8*d) - ((34*A*a^3)/15 - tan(c/2 + (d*x)/2)*((A*a^3)/4 -
(7*B*a^3)/8) + (22*B*a^3)/15 + tan(c/2 + (d*x)/2)^10*(6*A*a^3 + 2*B*a^3) +
tan(c/2 + (d*x)/2)^4*(12*A*a^3 + 4*B*a^3) + tan(c/2 + (d*x)/2)^11*((A*a^3)/
4 - (7*B*a^3)/8) + tan(c/2 + (d*x)/2)^8*(22*A*a^3 + 18*B*a^3) - tan(c/2 + (
d*x)/2)^5*((13*A*a^3)/2 + (37*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((13*A*a^3)/
2 + (37*B*a^3)/4) + tan(c/2 + (d*x)/2)^2*((38*A*a^3)/5 + (34*B*a^3)/5) + ta
n(c/2 + (d*x)/2)^6*((68*A*a^3)/3 + (44*B*a^3)/3) - tan(c/2 + (d*x)/2)^3*((2
7*A*a^3)/4 + (73*B*a^3)/24) + tan(c/2 + (d*x)/2)^9*((27*A*a^3)/4 + (73*B*a^
3)/24))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 +
 (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
 (d*x)/2)^12 + 1)) - (7*a^3*(2*A + B)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))
/(8*d)
```

3.998 $\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=91

$$-\frac{3}{2}a^3(2A+3B)x+\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\cos(c+dx)\sin(c+dx)}{2d}+\frac{(A+B)\sec(c+dx)(a}{d}$$

```
[Out] -3/2*a^3*(2*A+3*B)*x+2*a^3*(2*A+3*B)*cos(d*x+c)/d+1/2*a^3*(2*A+3*B)*cos(d*x
+c)*sin(d*x+c)/d+(A+B)*sec(d*x+c)*(a+a*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2934, 2723}

$$\frac{2a^3(2A+3B)\cos(c+dx)}{d}+\frac{a^3(2A+3B)\sin(c+dx)\cos(c+dx)}{2d}-\frac{3}{2}a^3x(2A+3B)+\frac{(A+B)\sec(c+dx)(a\sin(c+dx)+a)^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^3*(2*A + 3*B)*x)/2 + (2*a^3*(2*A + 3*B)*Cos[c + d*x])/d + (a^3*(2*A +
 3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + ((A + B)*Sec[c + d*x]*(a + a*Sin[c
 + d*x])^3)/d
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx=\frac{(A+B)\sec(c+dx)(a+a\sin(c+dx))^3}{d}-(a(2$$

$$=-\frac{3}{2}a^3(2A+3B)x+\frac{2a^3(2A+3B)\cos(c+dx)}{d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.19, size = 82, normalized size = 0.90

$$\frac{\sec(c+dx)\left(4\sqrt{2}\,a^3(2A+3B)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1}{2}(1-\sin(c+dx))\right)\sqrt{1+\sin(c+dx)}-B(a+a\sin(c+dx))^3\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(4*Sqrt[2]*a^3*(2*A + 3*B)*Hypergeometric2F1[-3/2, -1/2, 1/2,
 (1 - Sin[c + d*x])/2]*Sqrt[1 + Sin[c + d*x]] - B*(a + a*Sin[c + d*x])^3))/
(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(87) = 174.
time = 0.20, size = 219, normalized size = 2.41

method	result
risch	$-3a^3xA-\frac{9a^3Bx}{2}+\frac{a^3e^{i(dx+c)}A}{2d}+\frac{3a^3e^{i(dx+c)}B}{2d}+\frac{a^3e^{-i(dx+c)}A}{2d}+\frac{3a^3e^{-i(dx+c)}B}{2d}+\frac{8a^3A}{d\left(e^{i(dx+c)}-i\right)}+$
derivativedivides	$\frac{a^3A\tan(dx+c)+\frac{Ba^3}{\cos(dx+c)}+\frac{3a^3A}{\cos(dx+c)}+3Ba^3(\tan(dx+c)-dx-c)+3a^3A(\tan(dx+c)-dx-c)+3Ba^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+s\right.}{}$
default	$\frac{a^3A\tan(dx+c)+\frac{Ba^3}{\cos(dx+c)}+\frac{3a^3A}{\cos(dx+c)}+3Ba^3(\tan(dx+c)-dx-c)+3a^3A(\tan(dx+c)-dx-c)+3Ba^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+s\right.}{}$
norman	$\frac{(3a^3A+\frac{9}{2}Ba^3)x+(-9a^3A-\frac{27}{2}Ba^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-3a^3A-\frac{9}{2}Ba^3)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(9a^3A+\frac{27}{2}Ba^3)x\left(\tan^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^3*A*tan(d*x+c)+B*a^3/cos(d*x+c)+3*a^3*A/cos(d*x+c)+3*B*a^3*(tan(d*x+
c)-d*x-c)+3*a^3*A*(tan(d*x+c)-d*x-c)+3*B*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+si
n(d*x+c)^2)*cos(d*x+c))+a^3*A*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos
(d*x+c))+B*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d
*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.54, size = 167, normalized size = 1.84

$$-\frac{6\,(dx+c-\tan(dx+c))Aa^3+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)Ba^3+6\,(dx+c-\tan(dx+c))Ba^3-2\,Aa^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-6\,Ba^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-2\,Aa^3\tan(dx+c)-\frac{6\,Aa^3}{\cos(dx+c)}-\frac{2\,Ba^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*A*a^3 + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*B*a^3 + 6*(d*x + c - tan(d*x + c))*B*a^3 -
 2*A*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 6*B*a^3*(1/cos(d*x + c) + cos(d*
x + c)) - 2*A*a^3*tan(d*x + c) - 6*A*a^3/cos(d*x + c) - 2*B*a^3/cos(d*x + c
))/d
```

Fricas [A]
time = 0.37, size = 173, normalized size = 1.90

$$\frac{Ba^3\cos(dx+c)^3 - 3(2A+3B)a^3dx + 2(A+3B)a^3\cos(dx+c)^2 + 8(A+B)a^3 - (3(2A+3B)a^3dx - (10A+13B)a^3)\cos(dx+c) + (3(2A+3B)a^3dx + Ba^3\cos(dx+c)^2 - (2A+5B)a^3\cos(dx+c) + 8(A+B)a^3)\sin(dx+c)}{2(d\cos(dx+c) - d\sin(dx+c) + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*a^3*cos(d*x + c)^3 - 3*(2*A + 3*B)*a^3*d*x + 2*(A + 3*B)*a^3*cos(d*x
 + c)^2 + 8*(A + B)*a^3 - (3*(2*A + 3*B)*a^3*d*x - (10*A + 13*B)*a^3)*cos(d
*x + c) + (3*(2*A + 3*B)*a^3*d*x + B*a^3*cos(d*x + c)^2 - (2*A + 5*B)*a^3*c
os(d*x + c) + 8*(A + B)*a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c)
 + d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec^2(c+dx)\,dx + \int 3A\sin(c+dx)\sec^2(c+dx)\,dx + \int 3A\sin^2(c+dx)\sec^2(c+dx)\,dx + \int A\sin^3(c+dx)\sec^2(c+dx)\,dx + \int B\sin(c+dx)\sec^2(c+dx)\,dx + \int 3B\sin^2(c+dx)\sec^2(c+dx)\,dx + \int 3B\sin^3(c+dx)\sec^2(c+dx)\,dx + \int B\sin^4(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sin(c + d*x)*sec(c + d*
x)**2, x) + Integral(3*A*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A*s
in(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x)*sec(c + d*x)**
2, x) + Integral(3*B*sin(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B*sin
(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*sin(c + d*x)**4*sec(c + d*x)*
*2, x))
```

Giac [A]
time = 0.48, size = 147, normalized size = 1.62

$$-\frac{3(2Aa^3 + 3Ba^3)(dx+c) + \frac{16(Aa^3+Ba^3)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1} + \frac{2\left(Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 6Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2Aa^3 - 6Ba^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(3*(2*A*a^3 + 3*B*a^3)*(d*x + c) + 16*(A*a^3 + B*a^3)/(tan(1/2*d*x + 1
/2*c) - 1) + 2*(B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1/2*d*x + 1/2*c)
^2 - 6*B*a^3*tan(1/2*d*x + 1/2*c)^2 - B*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3
- 6*B*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 11.51, size = 234, normalized size = 2.57

$$-\frac{10\,A\,a^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^3+5\,B\,a^3\right)+14\,B\,a^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,A\,a^3+7\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(8\,A\,a^3+9\,B\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(18\,A\,a^3+21\,B\,a^3\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}-\frac{3\,a^3\,\mathrm{atan}\left(\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A+3\,B\right)}{6\,A\,a^3+9\,B\,a^3}\right)\,\left(2\,A+3\,B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] - (10*A*a^3 - tan(c/2 + (d*x)/2)*(2*A*a^3 + 5*B*a^3) + 14*B*a^3 - tan(c/2 +
 (d*x)/2)^3*(2*A*a^3 + 7*B*a^3) + tan(c/2 + (d*x)/2)^4*(8*A*a^3 + 9*B*a^3)
+ tan(c/2 + (d*x)/2)^2*(18*A*a^3 + 21*B*a^3))/(d*(tan(c/2 + (d*x)/2) - 2*ta
n(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^4 + tan(c/
2 + (d*x)/2)^5 - 1)) - (3*a^3*atan((3*a^3*tan(c/2 + (d*x)/2)*(2*A + 3*B))/(
6*A*a^3 + 9*B*a^3))*(2*A + 3*B))/d
```

3.999 $\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=69

$$a^3Bx+\frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d}-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}$$

```
[Out] a^3*B*x+1/3*(A+B)*sec(d*x+c)^3*(a+a*sin(d*x+c))^3/d-2*a^5*B*cos(d*x+c)/d/(a
^2-a^2*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.129$, Rules used = {2934, 2749, 2759, 8}

$$a^3Bx-\frac{2a^5B\cos(c+dx)}{d\left(a^2-a^2\sin(c+dx)\right)}+\frac{(A+B)\sec^3(c+dx)(a\sin(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] a^3*B*x + ((A + B)*Sec[c + d*x]^3*(a + a*Sin[c + d*x])^3)/(3*d) - (2*a^5*B*
Cos[c + d*x])/(d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sec^4(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - (a$$

$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - (a$$

$$= \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx))^3}{3d} - \frac{}{d}$$

$$= a^3Bx + \frac{(A+B)\sec^3(c+dx)(a+a\sin(c+dx$$

Mathematica [A]
time = 0.85, size = 121, normalized size = 1.75

$$-\frac{a^3\left(-3(2A+3B(2+c+dx))\cos\left(\frac{1}{2}(c+dx)\right)+(2A+B(14+3c+3dx))\cos\left(\frac{3}{2}(c+dx)\right)+6B(2(2+c+dx)+(c+dx)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
[Out] -1/6*(a^3*(-3*(2*A + 3*B*(2 + c + d*x))*Cos[(c + d*x)/2] + (2*A + B*(14 + 3
*c + 3*d*x))*Cos[(3*(c + d*x))/2] + 6*B*(2*(2 + c + d*x) + (c + d*x)*Cos[c
+ d*x])*Sin[(c + d*x)/2]))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(67) = 134$.
time = 0.26, size = 248, normalized size = 3.59

method	result
risch	$a^3Bx - \frac{2\left(3A\,a^3 e^{2i(dx+c)} - 12iB\,a^3 e^{i(dx+c)} + 9B\,a^3 e^{2i(dx+c)} - a^3A - 7B\,a^3\right)}{3\left(e^{i(dx+c)} - i\right)^3 d}$
derivativedivides	$\frac{-a^3A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{B\,a^3}{3\cos(dx+c)^3}+\frac{a^3A}{\cos(dx+c)^3}+\frac{B\,a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a^3A\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+3B\,a^3\left(\frac{\sin^4(}{3\cos(}\right.}{}$

default	$\frac{-a^3 A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{B\,a^3}{3\cos(dx+c)^3}+\frac{a^3 A}{\cos(dx+c)^3}+\frac{B\,a^3\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a^3 A\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+3B\,a^3\left(\frac{\sin^4(dx}{3\cos(dx}\right.}{}$
norman	$\frac{\frac{\left(6a^3 A+2B\,a^3\right)\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+a^3 Bx\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a^3 Bx\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{20a^3 A-4B\,a^3}{3d}-a^3 Bx-\frac{\left(8a^3 A+24B\,a^3\right)(}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^3*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/3*B*a^3/cos(d*x+c)^3+a^3*A
/cos(d*x+c)^3+B*a^3*sin(d*x+c)^3/cos(d*x+c)^3+a^3*A*sin(d*x+c)^3/cos(d*x+c)
^3+3*B*a^3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(
2+sin(d*x+c)^2)*cos(d*x+c))+a^3*A*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*
x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/3*tan(d*x+c)^3-
tan(d*x+c)+d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(68) = 136$.
time = 0.51, size = 164, normalized size = 2.38

$$\frac{3\,Aa^3\tan\left(dx+c\right)^3+3\,Ba^3\tan\left(dx+c\right)^3+\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Aa^3+\left(\tan\left(dx+c\right)^3+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)Ba^3-\frac{\left(3\,\cos(dx+c)^2-1\right)Aa^3}{\cos(dx+c)^3}-\frac{3\left(3\,\cos(dx+c)^2-1\right)Ba^3}{\cos(dx+c)^3}+\frac{3\,Aa^3}{\cos(dx+c)^3}+\frac{Ba^3}{\cos(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/3*(3*A*a^3*tan(d*x + c)^3 + 3*B*a^3*tan(d*x + c)^3 + (tan(d*x + c)^3 + 3*
tan(d*x + c))*A*a^3 + (tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*B*a^3
 - (3*cos(d*x + c)^2 - 1)*A*a^3/cos(d*x + c)^3 - 3*(3*cos(d*x + c)^2 - 1)*B
*a^3/cos(d*x + c)^3 + 3*A*a^3/cos(d*x + c)^3 + B*a^3/cos(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(68) = 136$.
time = 0.37, size = 167, normalized size = 2.42

$$-\frac{6\,Ba^3dx+2\,(A+B)a^3-\left(3\,Ba^3dx+(A+7\,B)a^3\right)\cos\left(dx+c\right)^2+\left(3\,Ba^3dx+(A-5\,B)a^3\right)\cos\left(dx+c\right)-\left(6\,Ba^3dx-2\,(A+B)a^3+\left(3\,Ba^3dx-(A+7\,B)a^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+c\right)+2\,d\right)\sin\left(dx+c\right)-2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*B*a^3*d*x + 2*(A + B)*a^3 - (3*B*a^3*d*x + (A + 7*B)*a^3)*cos(d*x +
 c)^2 + (3*B*a^3*d*x + (A - 5*B)*a^3)*cos(d*x + c) - (6*B*a^3*d*x - 2*(A +
```

```
B)*a^3 + (3*B*a^3*d*x - (A + 7*B)*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 93, normalized size = 1.35

$$\frac{3\,(dx+c)Ba^3 - \frac{2\left(3\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Aa^3-5\,Ba^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(d*x + c)*B*a^3 - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*B*a^3*tan(1/
2*d*x + 1/2*c)^2 + 12*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^3 - 5*B*a^3)/(tan(1/
2*d*x + 1/2*c) - 1)^3)/d
```

Mupad [**B**]
time = 9.81, size = 140, normalized size = 2.03

$$B\,a^3\,x-\frac{\frac{a^3\,\left(2\,A-10\,B+3\,B\,\left(c+d\,x\right)\right)}{3}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^3\,\left(6\,A-6\,B+9\,B\,\left(c+d\,x\right)\right)}{3}-3\,B\,a^3\,\left(c+d\,x\right)\right)+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3\,\left(24\,B-9\,B\,\left(c+d\,x\right)\right)}{3}+3\,B\,a^3\,\left(c+d\,x\right)\right)-B\,a^3\,\left(c+d\,x\right)}{d\,{\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^4,x)
```

```
[Out] B*a^3*x - ((a^3*(2*A - 10*B + 3*B*(c + d*x)))/3 + tan(c/2 + (d*x)/2)^2*((a^
3*(6*A - 6*B + 9*B*(c + d*x)))/3 - 3*B*a^3*(c + d*x)) + tan(c/2 + (d*x)/2)*
((a^3*(24*B - 9*B*(c + d*x)))/3 + 3*B*a^3*(c + d*x)) - B*a^3*(c + d*x))/(d*
(tan(c/2 + (d*x)/2) - 1)^3)
```

3.1000 $\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d}+\frac{a^5(2A-3B)\cos(c+dx)}{15d\,(a^2-a^2\sin(c+dx))}$$

```
[Out] 1/15*a^5*(2*A-3*B)*cos(d*x+c)/d/(a-a*sin(d*x+c))^2+1/5*(A+B)*sec(d*x+c)^5*(
a+a*sin(d*x+c))^3/d+1/15*a^5*(2*A-3*B)*cos(d*x+c)/d/(a^2-a^2*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2934, 2749, 2729, 2727}

$$\frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{a^5(2A-3B)\cos(c+dx)}{15d\,(a^2-a^2\sin(c+dx))}+\frac{(A+B)\sec^5(c+dx)(a\sin(c+dx)+a)^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^5*(2*A - 3*B)*Cos[c + d*x])/(15*d*(a - a*Sin[c + d*x])^2) + ((A + B)*Sec
[c + d*x]^5*(a + a*Sin[c + d*x])^3)/(5*d) + (a^5*(2*A - 3*B)*Cos[c + d*x])/
(15*d*(a^2 - a^2*Sin[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2749

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d}+\frac{1}{5}\\ &= \frac{(A+B)\sec^5(c+dx)(a+a\sin(c+dx))^3}{5d}+\frac{1}{5}\\ &= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{(A+B)\sec^5(c+}{}\\ &= \frac{a^5(2A-3B)\cos(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{a^4(2A-3B)\cos(}{15d(a-a\sin(c-}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 94, normalized size = 0.88

$$-\frac{a^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(-16A+9B+(2A-3B)\cos(2(c+dx))+6(2A-3B)\sin(c+dx)\right)}{30d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/30*(a^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(-16*A + 9*B + (2*A - 3*B)
*Cos[2*(c + d*x)] + 6*(2*A - 3*B)*Sin[c + d*x]))/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(101) = 202$.
time = 0.23, size = 333, normalized size = 3.11

method	result
risch	$\frac{2ia^3\left(20iA\,e^{2i(dx+c)}-15iB\,e^{2i(dx+c)}+15B\,e^{3i(dx+c)}-2iA+10A\,e^{i(dx+c)}+3iB-15B\,e^{i(dx+c)}\right)}{15d\left(e^{i(dx+c)}-i\right)^5}$

derivativedivides	$\frac{-a^3A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{B\,a^3}{5\cos(dx+c)^5}+\frac{3a^3A}{5\cos(dx+c)^5}+3B\,a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right.}{}$
default	$\frac{-a^3A\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{B\,a^3}{5\cos(dx+c)^5}+\frac{3a^3A}{5\cos(dx+c)^5}+3B\,a^3\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^3*A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/5*B*a^3
/cos(d*x+c)^5+3/5*a^3*A/cos(d*x+c)^5+3*B*a^3*(1/5*sin(d*x+c)^3/cos(d*x+c)^5
+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+3*a^3*A*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/1
5*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*si
n(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*
cos(d*x+c))+a^3*A*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+
c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+1/5*B*a
^3*sin(d*x+c)^5/cos(d*x+c)^5)
```

Maxima [A]
time = 0.30, size = 188, normalized size = 1.76

$$\frac{3\,Ba^3\tan\left(dx+c\right)^5+\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)Aa^3+3\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)Aa^3+3\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)Ba^3-\frac{\left(5\,\cos(dx+c)^2-3\right)Aa^3}{\cos(dx+c)^5}-\frac{3\left(5\,\cos(dx+c)^2-3\right)Ba^3}{\cos(dx+c)^5}+\frac{9\,Aa^3}{\cos(dx+c)^5}+\frac{3\,Ba^3}{\cos(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/15*(3*B*a^3*tan(d*x + c)^5 + (3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*A*a^3 + 3*(3*tan(d*x + c)^5 + 5*tan(d*x + c)^3)*A*a^3 + 3*(3*t
an(d*x + c)^5 + 5*tan(d*x + c)^3)*B*a^3 - (5*cos(d*x + c)^2 - 3)*A*a^3/cos(
d*x + c)^5 - 3*(5*cos(d*x + c)^2 - 3)*B*a^3/cos(d*x + c)^5 + 9*A*a^3/cos(d*
x + c)^5 + 3*B*a^3/cos(d*x + c)^5)/d
```

Fricas [A]
time = 0.37, size = 188, normalized size = 1.76

$$\frac{(2\,A-3\,B)a^3\cos\left(dx+c\right)^3-2\,(2\,A-3\,B)a^3\cos\left(dx+c\right)^2-3\,(3\,A-2\,B)a^3\cos\left(dx+c\right)-3\,(A+B)a^3+\left((2\,A-3\,B)a^3\cos\left(dx+c\right)^2+3\,(2\,A-3\,B)a^3\cos\left(dx+c\right)-3\,(A+B)a^3\right)\sin\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^3+3\,d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^2-2\,d\cos\left(dx+c\right)-4\,d\right)\sin\left(dx+c\right)-4\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/15*((2*A - 3*B)*a^3*cos(d*x + c)^3 - 2*(2*A - 3*B)*a^3*cos(d*x + c)^2 - 3
*(3*A - 2*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3 + ((2*A - 3*B)*a^3*cos(d*x +
```

```
c)^2 + 3*(2*A - 3*B)*a^3*cos(d*x + c) - 3*(A + B)*a^3)*sin(d*x + c))/(d*cos
(d*x + c)^3 + 3*d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2
*d*cos(d*x + c) - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.47, size = 146, normalized size = 1.36

$$-\frac{2\left(15\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-30\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-20\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,Aa^{3}-3\,Ba^{3}\right)}{15\,d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -2/15*(15*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 30*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
15*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^
3*tan(1/2*d*x + 1/2*c)^2 - 20*A*a^3*tan(1/2*d*x + 1/2*c) + 15*B*a^3*tan(1/2
*d*x + 1/2*c) + 7*A*a^3 - 3*B*a^3)/(d*(tan(1/2*d*x + 1/2*c) - 1)^5)
```

Mupad [B]
time = 11.31, size = 113, normalized size = 1.06

$$-\frac{\sqrt{2}\,a^{3}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,B-\frac{53\,A}{4}+4\,A\,\cos\left(c+d\,x\right)+\frac{3\,B\,\cos(c+d\,x)}{2}+\frac{25\,A\,\sin(c+d\,x)}{2}-\frac{15\,B\,\sin(c+d\,x)}{2}+\frac{9\,A\,\cos(2\,c+2\,d\,x)}{4}-\frac{3\,B\,\cos(2\,c+2\,d\,x)}{2}-\frac{5\,A\,\sin(2\,c+2\,d\,x)}{4}\right)}{60\,d\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^6,x)
```

```
[Out] -(2^(1/2)*a^3*cos(c/2 + (d*x)/2)*(3*B - (53*A)/4 + 4*A*cos(c + d*x) + (3*B*
cos(c + d*x))/2 + (25*A*sin(c + d*x))/2 - (15*B*sin(c + d*x))/2 + (9*A*cos(
2*c + 2*d*x))/4 - (3*B*cos(2*c + 2*d*x))/2 - (5*A*sin(2*c + 2*d*x))/4))/(60
*d*cos(c/2 + pi/4 + (d*x)/2)^5)
```

3.1001 $\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$\frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{35d}+\frac{3a^3(4A-3B)\tan}{35d}$$

```
[Out] 1/7*(A+B)*sec(d*x+c)^7*(a+a*sin(d*x+c))^3/d+2/35*(4*A-3*B)*sec(d*x+c)^5*(a^
3+a^3*sin(d*x+c))/d+3/35*a^3*(4*A-3*B)*tan(d*x+c)/d+1/35*a^3*(4*A-3*B)*tan(
d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2755, 3852}

$$\frac{a^3(4A-3B)\tan^3(c+dx)}{35d}+\frac{3a^3(4A-3B)\tan(c+dx)}{35d}+\frac{2(4A-3B)\sec^5(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{35d}+\frac{(A+B)\sec^7(c+dx)(a\sin(c+dx)+a)^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^7*(a + a*Sin[c + d*x])^3)/(7*d) + (2*(4*A - 3*B)*Sec[
c + d*x]^5*(a^3 + a^3*Sin[c + d*x]))/(35*d) + (3*a^3*(4*A - 3*B)*Tan[c + d*
x])/(35*d) + (a^3*(4*A - 3*B)*Tan[c + d*x]^3)/(35*d)
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx = \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d}+\frac{1}{7}($$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d}+\frac{2(}{}$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d}+\frac{2(}{}$$
$$= \frac{(A+B)\sec^7(c+dx)(a+a\sin(c+dx))^3}{7d}+\frac{2(}{}$$

Mathematica [A]
time = 0.35, size = 135, normalized size = 1.17

$$\frac{a^3(35B+14(4A-3B)\cos(2(c+dx))+(-4A+3B)\cos(4(c+dx))+56A\sin(c+dx)-42B\sin(c+dx)-24A\sin(3(c+dx))+18B\sin(3(c+dx)))}{140d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^7\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^3*(35*B + 14*(4*A - 3*B)*Cos[2*(c + d*x)] + (-4*A + 3*B)*Cos[4*(c + d*x)
] + 56*A*Sin[c + d*x] - 42*B*Sin[c + d*x] - 24*A*Sin[3*(c + d*x)] + 18*B*Si
n[3*(c + d*x)]))/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(107) = 214.
time = 0.29, size = 435, normalized size = 3.78

method	result
risch	$\frac{4ia^3\left(56iA\,e^{3i(dx+c)}-42iB\,e^{3i(dx+c)}+35B\,e^{4i(dx+c)}-24iA\,e^{i(dx+c)}+56A\,e^{2i(dx+c)}+18iB\,e^{i(dx+c)}-42B\,e^{2i(dx+c)}-4A\right.}{35\left(e^{i(dx+c)}+i\right)\left(e^{i(dx+c)}-i\right)^7d}$
derivativedivides	$\frac{-a^3A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{B\,a^3}{7\cos(dx+c)^7}+\frac{3a^3A}{7\cos(dx+c)^7}+3B\,a^3\left(\frac{\sin^3(dx+}{7\cos(dx+}\right.}{}$
default	$\frac{-a^3A\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{B\,a^3}{7\cos(dx+c)^7}+\frac{3a^3A}{7\cos(dx+c)^7}+3B\,a^3\left(\frac{\sin^3(dx+}{7\cos(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^3*A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)+1/7*B*a^3/cos(d*x+c)^7+3/7*a^3*A/cos(d*x+c)^7+3*B*a^3*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3*a^3*A*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+a^3*A*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(107) = 214$.
time = 0.32, size = 228, normalized size = 1.98

$$\frac{\left(15 \tan\left(dx+c\right)^{7}+42 \tan\left(dx+c\right)^{5}+35 \tan\left(dx+c\right)^{3}\right)Aa^{3}+\left(5 \tan\left(dx+c\right)^{7}+21 \tan\left(dx+c\right)^{5}+35 \tan\left(dx+c\right)^{3}+35 \tan\left(dx+c\right)\right)Aa^{3}+\left(15 \tan\left(dx+c\right)^{7}+42 \tan\left(dx+c\right)^{5}+35 \tan\left(dx+c\right)^{3}\right)Ba^{3}+\left(5 \tan\left(dx+c\right)^{7}+7 \tan\left(dx+c\right)^{5}\right)Ba^{3}-\frac{\left(7 \cos(dx+c)^{2}-5\right)Aa^{3}}{\cos(dx+c)^{7}}-\frac{3\left(7 \cos(dx+c)^{2}-5\right)Ba^{3}}{\cos(dx+c)^{7}}+\frac{15\,Aa^{3}}{\cos(dx+c)^{7}}+\frac{5\,Ba^{3}}{\cos(dx+c)^{7}}}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/35*((15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*A*a^3 + (5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*tan(d*x + c))*A*a^3 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*B*a^3 + (5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*B*a^3 - (7*cos(d*x + c)^2 - 5)*A*a^3/cos(d*x + c)^7 - 3*(7*cos(d*x + c)^2 - 5)*B*a^3/cos(d*x + c)^7 + 15*A*a^3/cos(d*x + c)^7 + 5*B*a^3/cos(d*x + c)^7)/d

Fricas [A]
time = 0.36, size = 146, normalized size = 1.27

$$\frac{2\,(4\,A-3\,B)a^{3}\cos\left(dx+c\right)^{4}-9\,(4\,A-3\,B)a^{3}\cos\left(dx+c\right)^{2}+5\,(3\,A-4\,B)a^{3}+\left(6\,(4\,A-3\,B)a^{3}\cos\left(dx+c\right)^{2}-5\,(4\,A-3\,B)a^{3}\right)\sin\left(dx+c\right)}{35\,\left(3\,d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)-\left(d\cos\left(dx+c\right)^{3}-4\,d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/35*(2*(4*A - 3*B)*a^3*cos(d*x + c)^4 - 9*(4*A - 3*B)*a^3*cos(d*x + c)^2 + 5*(3*A - 4*B)*a^3 + (6*(4*A - 3*B)*a^3*cos(d*x + c)^2 - 5*(4*A - 3*B)*a^3)

```
*sin(d*x + c))/(3*d*cos(d*x + c)^3 - 4*d*cos(d*x + c) - (d*cos(d*x + c)^3 -
 4*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(107) = 214.
time = 0.49, size = 260, normalized size = 2.26

$$-\frac{\frac{35\,(Aa^3-Ba^3)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1}+\frac{525\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+35\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4025\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-665\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-4480\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1120\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-791\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+392\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+243\,Aa^3-51\,Ba^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/280*(35*(A*a^3 - B*a^3)/(tan(1/2*d*x + 1/2*c) + 1) + (525*A*a^3*tan(1/2*
d*x + 1/2*c)^6 + 35*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 1960*A*a^3*tan(1/2*d*x +
 1/2*c)^5 + 280*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 4025*A*a^3*tan(1/2*d*x + 1/2
*c)^4 - 665*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 4480*A*a^3*tan(1/2*d*x + 1/2*c)^
3 + 1120*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 3143*A*a^3*tan(1/2*d*x + 1/2*c)^2 -
 791*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 1176*A*a^3*tan(1/2*d*x + 1/2*c) + 392*B
*a^3*tan(1/2*d*x + 1/2*c) + 243*A*a^3 - 51*B*a^3)/(tan(1/2*d*x + 1/2*c) - 1
)^7)/d
```

Mupad [B]
time = 11.18, size = 213, normalized size = 1.85

$$-\frac{a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{35\,A\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{4}-\frac{91\,A\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{4}+A\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)-\frac{35\,B\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{21\,B\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{2}-\frac{3\,B\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{4}-\frac{233\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{121\,A\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}+\frac{61\,A\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}-\frac{13\,A\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}+\frac{61\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{23\,B\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{37\,B\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{B\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{8}\right)}{280\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^8,x)
```

```
[Out] -(a^3*cos(c/2 + (d*x)/2)*((35*A*cos((5*c)/2 + (5*d*x)/2))/4 - (91*A*cos((3*
c)/2 + (3*d*x)/2))/4 + A*cos((7*c)/2 + (7*d*x)/2) - (35*B*cos(c/2 + (d*x)/2
))/4 + (21*B*cos((3*c)/2 + (3*d*x)/2))/2 - (3*B*cos((7*c)/2 + (7*d*x)/2))/4
 - (233*A*sin(c/2 + (d*x)/2))/8 + (121*A*sin((3*c)/2 + (3*d*x)/2))/8 + (61*
A*sin((5*c)/2 + (5*d*x)/2))/8 - (13*A*sin((7*c)/2 + (7*d*x)/2))/8 + (61*B*s
in(c/2 + (d*x)/2))/8 + (23*B*sin((3*c)/2 + (3*d*x)/2))/8 - (37*B*sin((5*c)/
2 + (5*d*x)/2))/8 + (B*sin((7*c)/2 + (7*d*x)/2))/8))/(280*d*cos(c/2 - pi/4
+ (d*x)/2)*cos(c/2 + pi/4 + (d*x)/2)^7)
```

3.1002 $\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3+a^3\sin(c+dx)\right)}{21d}+\frac{5a^3(2A-B)\tan(c}{21d}$$

```
[Out] 1/9*(A+B)*sec(d*x+c)^9*(a+a*sin(d*x+c))^3/d+2/21*(2*A-B)*sec(d*x+c)^7*(a^3+
a^3*sin(d*x+c))/d+5/21*a^3*(2*A-B)*tan(d*x+c)/d+10/63*a^3*(2*A-B)*tan(d*x+c
)^3/d+1/21*a^3*(2*A-B)*tan(d*x+c)^5/d
```

Rubi [A]

time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2934, 2755, 3852}

$$\frac{a^3(2A-B)\tan^5(c+dx)}{21d}+\frac{10a^3(2A-B)\tan^3(c+dx)}{63d}+\frac{5a^3(2A-B)\tan(c+dx)}{21d}+\frac{2(2A-B)\sec^7(c+dx)\left(a^3\sin(c+dx)+a^3\right)}{21d}+\frac{(A+B)\sec^9(c+dx)(a\sin(c+dx)+a)^3}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*Sec[c + d*x]^9*(a + a*Sin[c + d*x])^3)/(9*d) + (2*(2*A - B)*Sec[c
+ d*x]^7*(a^3 + a^3*Sin[c + d*x]))/(21*d) + (5*a^3*(2*A - B)*Tan[c + d*x])/
(21*d) + (10*a^3*(2*A - B)*Tan[c + d*x]^3)/(63*d) + (a^3*(2*A - B)*Tan[c +
d*x]^5)/(21*d)
```

Rule 2755

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[-2*b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(p + 1))), x] + Dist[b^2*((2*m + p - 1)/(g^2*(p + 1))),
Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[
{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && Inte
gersQ[2*m, 2*p]
```

Rule 2934

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Dist[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1))), Int[(g*Cos[e + f
*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(c+dx)(a+a\sin(c+dx))^3(A+B\sin(c+dx))\,dx &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d}+\frac{1}{3}\\ &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d}+\frac{2}{}\\ &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d}+\frac{2}{}\\ &= \frac{(A+B)\sec^9(c+dx)(a+a\sin(c+dx))^3}{9d}+\frac{2}{}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 176, normalized size = 1.26

$$-\frac{a^3(-42B+27(-2A+B)\cos(2(c+dx))+12(-2A+B)\cos(4(c+dx))+2A\cos(6(c+dx))-B\cos(6(c+dx))-72A\sin(c+dx)+36B\sin(c+dx)-4A\sin(3(c+dx))+2B\sin(3(c+dx))+12A\sin(5(c+dx))-6B\sin(5(c+dx)))}{252d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^9\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a + a*Sin[c + d*x])^3*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/252*(a^3*(-42*B + 27*(-2*A + B)*Cos[2*(c + d*x)] + 12*(-2*A + B)*Cos[4*(
c + d*x)] + 2*A*Cos[6*(c + d*x)] - B*Cos[6*(c + d*x)] - 72*A*Sin[c + d*x] +
 36*B*Sin[c + d*x] - 4*A*Sin[3*(c + d*x)] + 2*B*Sin[3*(c + d*x)] + 12*A*Sin
[5*(c + d*x)] - 6*B*Sin[5*(c + d*x)]))/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^9*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(130) = 260$.
time = 0.36, size = 535, normalized size = 3.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/d*(-a^3*A*(-128/315-1/9*sec(d*x+c)^8-8/63*sec(d*x+c)^6-16/105*sec(d*x+c)^
4-64/315*sec(d*x+c)^2)*tan(d*x+c)+1/9*B*a^3/cos(d*x+c)^9+1/3*a^3*A/cos(d*x+
c)^9+3*B*a^3*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+
8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3*a^3*A*(
1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+
```

```
c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3*B*a^3*(1/9*sin(d*x+c)
^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)
^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(
d*x+c)^2)*cos(d*x+c))+a^3*A*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^
4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^
3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c))+B*a^3*(1/9
*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^
5/cos(d*x+c)^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(130) = 260$.
time = 0.32, size = 270, normalized size = 1.93

$$\frac{\left(35 \tan \left(dx+c\right)^{9}+180 \tan \left(dx+c\right)^{7}+378 \tan \left(dx+c\right)^{5}+420 \tan \left(dx+c\right)^{3}+315 \tan \left(dx+c\right)\right) A a^{3}+3 \left(35 \tan \left(dx+c\right)^{9}+135 \tan \left(dx+c\right)^{7}+189 \tan \left(dx+c\right)^{5}+105 \tan \left(dx+c\right)^{3}\right) A a^{3}+3 \left(35 \tan \left(dx+c\right)^{9}+135 \tan \left(dx+c\right)^{7}+189 \tan \left(dx+c\right)^{5}+105 \tan \left(dx+c\right)^{3}\right) B a^{3}+\left(35 \tan \left(dx+c\right)^{9}+90 \tan \left(dx+c\right)^{7}+63 \tan \left(dx+c\right)^{5}\right) B a^{3}-\frac{5 \left(9 \cos \left(dx+c\right)^{2}-7\right) A a^{3}}{\cos \left(dx+c\right)^{9}}-\frac{15 \left(9 \cos \left(dx+c\right)^{2}-7\right) B a^{3}}{\cos \left(dx+c\right)^{9}}+\frac{105 A a^{3}}{\cos \left(dx+c\right)^{9}}+\frac{35 B a^{3}}{\cos \left(dx+c\right)^{9}}}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="m
axima")
```

```
[Out] 1/315*((35*tan(d*x + c)^9 + 180*tan(d*x + c)^7 + 378*tan(d*x + c)^5 + 420*t
an(d*x + c)^3 + 315*tan(d*x + c))*A*a^3 + 3*(35*tan(d*x + c)^9 + 135*tan(d*
x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*A*a^3 + 3*(35*tan(d*x +
 c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x + c)^5 + 105*tan(d*x + c)^3)*B*a^3
 + (35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*B*a^3 - 5*(9
*cos(d*x + c)^2 - 7)*A*a^3/cos(d*x + c)^9 - 15*(9*cos(d*x + c)^2 - 7)*B*a^3
/cos(d*x + c)^9 + 105*A*a^3/cos(d*x + c)^9 + 35*B*a^3/cos(d*x + c)^9)/d
```

Fricas [A]
time = 0.38, size = 188, normalized size = 1.34

$$\frac{8 \left(2 A-B\right) a^{3} \cos \left(dx+c\right)^{6}-36 \left(2 A-B\right) a^{3} \cos \left(dx+c\right)^{4}+15 \left(2 A-B\right) a^{3} \cos \left(dx+c\right)^{2}+7 \left(A-2 B\right) a^{3}+\left(24 \left(2 A-B\right) a^{3} \cos \left(dx+c\right)^{4}-20 \left(2 A-B\right) a^{3} \cos \left(dx+c\right)^{2}-7 \left(2 A-B\right) a^{3}\right) \sin \left(dx+c\right)}{63 \left(3 d \cos \left(dx+c\right)^{5}-4 d \cos \left(dx+c\right)^{3}-\left(d \cos \left(dx+c\right)^{5}-4 d \cos \left(dx+c\right)^{3}\right) \sin \left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] 1/63*(8*(2*A - B)*a^3*cos(d*x + c)^6 - 36*(2*A - B)*a^3*cos(d*x + c)^4 + 15
*(2*A - B)*a^3*cos(d*x + c)^2 + 7*(A - 2*B)*a^3 + (24*(2*A - B)*a^3*cos(d*x
 + c)^4 - 20*(2*A - B)*a^3*cos(d*x + c)^2 - 7*(2*A - B)*a^3)*sin(d*x + c))/
(3*d*cos(d*x + c)^5 - 4*d*cos(d*x + c)^3 - (d*cos(d*x + c)^5 - 4*d*cos(d*x
+ c)^3)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a+a*sin(d*x+c))**3*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. 2(130) = 260.
time = 0.51, size = 393, normalized size = 2.81

$$-\frac{\frac{21\left(21\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+19\,Aa^{3}-13\,Ba^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}+\frac{3591\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+315\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-19656\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+756\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+56196\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-4200\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-95760\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+11340\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+107730\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-14994\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-79464\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+13356\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+38484\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-6768\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10944\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2196\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1615\,Aa^{3}-209\,Ba^{3}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^{9}}}{2016\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a+a*sin(d*x+c))^3*(A+B*sin(d*x+c)),x, algorithm="g
iac")
```

```
[Out] -1/2016*(21*(21*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*B*a^3*tan(1/2*d*x + 1/2*c
)^2 + 36*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) + 19*A*
a^3 - 13*B*a^3)/(tan(1/2*d*x + 1/2*c) + 1)^3 + (3591*A*a^3*tan(1/2*d*x + 1/
2*c)^8 + 315*B*a^3*tan(1/2*d*x + 1/2*c)^8 - 19656*A*a^3*tan(1/2*d*x + 1/2*c
)^7 + 756*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 56196*A*a^3*tan(1/2*d*x + 1/2*c)^6
 - 4200*B*a^3*tan(1/2*d*x + 1/2*c)^6 - 95760*A*a^3*tan(1/2*d*x + 1/2*c)^5 +
 11340*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 107730*A*a^3*tan(1/2*d*x + 1/2*c)^4 -
 14994*B*a^3*tan(1/2*d*x + 1/2*c)^4 - 79464*A*a^3*tan(1/2*d*x + 1/2*c)^3 +
13356*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 38484*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 6
768*B*a^3*tan(1/2*d*x + 1/2*c)^2 - 10944*A*a^3*tan(1/2*d*x + 1/2*c) + 2196*
B*a^3*tan(1/2*d*x + 1/2*c) + 1615*A*a^3 - 209*B*a^3)/(tan(1/2*d*x + 1/2*c)
- 1)^9)/d
```

Mupad [B]
time = 12.21, size = 322, normalized size = 2.30

$$-\frac{a^{3}\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{63\,A\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}-\frac{171\,A\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{145\,A\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{49\,A\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}+\frac{A\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{2}-\frac{21\,B\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{75\,B\cos\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{8}-\frac{21\,B\cos\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{8}+\frac{41\,B\cos\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{16}+\frac{7\,B\cos\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{16}-\frac{B\cos\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{4}-\frac{617\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{329\,A\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}-\frac{145\,A\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}+\frac{113\,A\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}+\frac{115\,A\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}-\frac{19\,A\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{32}+\frac{109\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}+\frac{35\,B\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)}{16}-\frac{43\,B\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)}{32}+\frac{59\,B\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)}{32}-\frac{47\,B\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)}{32}-\frac{B\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)}{32}\right)}{2016\,d\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)^{3}\cos\left(\frac{c}{2}+\frac{\pi}{4}+\frac{d\,x}{2}\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + a*sin(c + d*x))^3)/cos(c + d*x)^10,x)
```

```
[Out] -(a^3*cos(c/2 + (d*x)/2)*((63*A*cos((5*c)/2 + (5*d*x)/2))/8 - (171*A*cos((3
*c)/2 + (3*d*x)/2))/8 - (145*A*cos((7*c)/2 + (7*d*x)/2))/16 + (49*A*cos((9*
c)/2 + (9*d*x)/2))/16 + (A*cos((11*c)/2 + (11*d*x)/2))/2 - (21*B*cos(c/2 +
(d*x)/2))/2 + (75*B*cos((3*c)/2 + (3*d*x)/2))/8 - (21*B*cos((5*c)/2 + (5*d*
x)/2))/8 + (41*B*cos((7*c)/2 + (7*d*x)/2))/16 + (7*B*cos((9*c)/2 + (9*d*x)/
2))/16 - (B*cos((11*c)/2 + (11*d*x)/2))/4 - (617*A*sin(c/2 + (d*x)/2))/16 +
 (329*A*sin((3*c)/2 + (3*d*x)/2))/16 - (145*A*sin((5*c)/2 + (5*d*x)/2))/32
+ (113*A*sin((7*c)/2 + (7*d*x)/2))/32 + (115*A*sin((9*c)/2 + (9*d*x)/2))/32
 - (19*A*sin((11*c)/2 + (11*d*x)/2))/32 + (109*B*sin(c/2 + (d*x)/2))/16 + (
```

```
35*B*sin((3*c)/2 + (3*d*x)/2))/16 - (43*B*sin((5*c)/2 + (5*d*x)/2))/32 + (5
9*B*sin((7*c)/2 + (7*d*x)/2))/32 - (47*B*sin((9*c)/2 + (9*d*x)/2))/32 - (B*
sin((11*c)/2 + (11*d*x)/2))/32))/(2016*d*cos(c/2 - pi/4 + (d*x)/2)^3*cos(c/
2 + pi/4 + (d*x)/2)^9)
```

3.1003 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{B(a-a\,\mathrm{si}}{7}$$

```
[Out] -(A+B)*(a-a*sin(d*x+c))^4/a^5/d+4/5*(A+2*B)*(a-a*sin(d*x+c))^5/a^6/d-1/6*(A
+5*B)*(a-a*sin(d*x+c))^6/a^7/d+1/7*B*(a-a*sin(d*x+c))^7/a^8/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$\frac{B(a-a\sin(c+dx))^7}{7a^8d}-\frac{(A+5B)(a-a\sin(c+dx))^6}{6a^7d}+\frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d}-\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(((A + B)*(a - a*Sin[c + d*x])^4)/(a^5*d)) + (4*(A + 2*B)*(a - a*Sin[c + d
*x])^5)/(5*a^6*d) - ((A + 5*B)*(a - a*Sin[c + d*x])^6)/(6*a^7*d) + (B*(a -
a*Sin[c + d*x])^7)/(7*a^8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^2\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$

$$= \frac{\text{Subst}\Big(\int \Big(4a^2(A+B)(a-x)^3 - 4a(A+2B)(a-x)^4 + (A+5B)}{a^7d}$$

$$= -\frac{(A+B)(a-a\sin(c+dx))^4}{a^5d} + \frac{4(A+2B)(a-a\sin(c+dx))^5}{5a^6d} -$$

Mathematica [A]

time = 0.16, size = 69, normalized size = 0.66

$$-\frac{(-1+\sin(c+dx))^4\left(77A+19B+(98A+76B)\sin(c+dx)+5(7A+17B)\sin^2(c+dx)+30B\sin^3(c+dx)\right)}{210ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] -1/210*((-1 + Sin[c + d*x])^4*(77*A + 19*B + (98*A + 76*B)*Sin[c + d*x] + 5
*(7*A + 17*B)*Sin[c + d*x]^2 + 30*B*Sin[c + d*x]^3))/(a*d)
```

Maple [A]

time = 0.34, size = 107, normalized size = 1.02

method	result
derivativedivides	$\frac{-\frac{B\left(\sin^7(dx+c)\right)}{7}+\frac{(-A+B)\left(\sin^6(dx+c)\right)}{6}+\frac{(A+2B)\left(\sin^5(dx+c)\right)}{5}+\frac{(2A-2B)\left(\sin^4(dx+c)\right)}{4}+\frac{(-2A-B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\text{si}\right.}{2}}{da}$
default	$\frac{-\frac{B\left(\sin^7(dx+c)\right)}{7}+\frac{(-A+B)\left(\sin^6(dx+c)\right)}{6}+\frac{(A+2B)\left(\sin^5(dx+c)\right)}{5}+\frac{(2A-2B)\left(\sin^4(dx+c)\right)}{4}+\frac{(-2A-B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\text{si}\right.}{2}}{da}$
risch	$\frac{5A\sin(dx+c)}{8ad}-\frac{5B\sin(dx+c)}{64ad}+\frac{B\sin(7dx+7c)}{448ad}+\frac{\cos(6dx+6c)A}{192ad}-\frac{\cos(6dx+6c)B}{192ad}+\frac{\sin(5dx+5c)A}{80ad}+\frac{3\sin(5d}{32}$
norman	$\frac{\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2A\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2(10A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{2(10A-B)\left(\tan^{14}\right.}{3ad}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/7*B*sin(d*x+c)^7+1/6*(-A+B)*sin(d*x+c)^6+1/5*(A+2*B)*sin(d*x+c)^5
+1/4*(2*A-2*B)*sin(d*x+c)^4+1/3*(-2*A-B)*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)
^2+A*sin(d*x+c))
```

Maxima [A]

time = 0.29, size = 104, normalized size = 0.99

$$-\frac{30\,B\sin(dx+c)^7+35\,(A-B)\sin(dx+c)^6-42\,(A+2\,B)\sin(dx+c)^5-105\,(A-B)\sin(dx+c)^4+70\,(2\,A+B)\sin(dx+c)^3+105\,(A-B)\sin(dx+c)^2-210\,A\sin(dx+c)}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*(A - B)*sin(d*x + c)^6 - 42*(A + 2*B)*sin(
d*x + c)^5 - 105*(A - B)*sin(d*x + c)^4 + 70*(2*A + B)*sin(d*x + c)^3 + 105
*(A - B)*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

Fricas [**A**]
time = 0.38, size = 84, normalized size = 0.80

$$\frac{35\,(A-B)\cos\left(dx+c\right)^{6}+2\,\left(15\,B\cos\left(dx+c\right)^{6}+3\,(7\,A-B)\cos\left(dx+c\right)^{4}+4\,(7\,A-B)\cos\left(dx+c\right)^{2}+56\,A-8\,B\right)\sin\left(dx+c\right)}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/210*(35*(A - B)*cos(d*x + c)^6 + 2*(15*B*cos(d*x + c)^6 + 3*(7*A - B)*cos
(d*x + c)^4 + 4*(7*A - B)*cos(d*x + c)^2 + 56*A - 8*B)*sin(d*x + c))/(a*d)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3363 vs. $2(94) = 188$.
time = 42.50, size = 3363, normalized size = 32.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((210*A*tan(c/2 + d*x/2)**13/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a
*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2
+ d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 +
 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**12/(105*a
*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 +
 d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 +
 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 70
0*A*tan(c/2 + d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 +
 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(
c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**10/(105*a*d*tan(c/2 +
d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 +
 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan
(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 1582*A*tan(c/2
+ d*x/2)**9/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 +
2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan
```

(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)*
*2 + 105*a*d) - 700*A*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 7
35*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(
c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)*
*4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 2184*A*tan(c/2 + d*x/2)**7/(1
05*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c
/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)*
*6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
- 700*A*tan(c/2 + d*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2
 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**
8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*t
an(c/2 + d*x/2)**2 + 105*a*d) + 1582*A*tan(c/2 + d*x/2)**5/(105*a*d*tan(c/2
 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**1
0 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*
tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/
2 + d*x/2)**4/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12
+ 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*t
an(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2
)**2 + 105*a*d) + 700*A*tan(c/2 + d*x/2)**3/(105*a*d*tan(c/2 + d*x/2)**14 +
 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*ta
n(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2
)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 210*A*tan(c/2 + d*x/2)**2/(
105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(
c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)
6 + 2205*a*d*tan(c/2 + d*x/2)4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
 + 210*A*tan(c/2 + d*x/2)/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 +
 d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8
+ 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan
(c/2 + d*x/2)**2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)**12/(105*a*d*tan(c/2 +
 d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10
+ 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*ta
n(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) - 280*B*tan(c/2
+ d*x/2)**11/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 +
 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*ta
n(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)
2 + 105*a*d) + 210*B*tan(c/2 + d*x/2)10/(105*a*d*tan(c/2 + d*x/2)**14 +
 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*ta
n(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2
)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 224*B*tan(c/2 + d*x/2)**9/(
105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(
c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)
6 + 2205*a*d*tan(c/2 + d*x/2)4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d)
 + 700*B*tan(c/2 + d*x/2)**8/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/
2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(c/2 + d*x/2)*
*8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*tan(c/2 + d*x/2)**4 + 735*a*d*

```
tan(c/2 + d*x/2)**2 + 105*a*d) - 912*B*tan(c/2 + d*x/2)**7/(105*a*d*tan(c/2
 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12 + 2205*a*d*tan(c/2 + d*x/2)**1
0 + 3675*a*d*tan(c/2 + d*x/2)**8 + 3675*a*d*tan(c/2 + d*x/2)**6 + 2205*a*d*
tan(c/2 + d*x/2)**4 + 735*a*d*tan(c/2 + d*x/2)**2 + 105*a*d) + 700*B*tan(c/
2 + d*x/2)**6/(105*a*d*tan(c/2 + d*x/2)**14 + 735*a*d*tan(c/2 + d*x/2)**12
+ 2205*a*d*tan(c/2 + d*x/2)**10 + 3675*a*d*tan(...
```

Giac [A]
time = 0.46, size = 139, normalized size = 1.32

$$-\frac{30 B \sin(dx+c)^7 + 35 A \sin(dx+c)^6 - 35 B \sin(dx+c)^6 - 42 A \sin(dx+c)^5 - 84 B \sin(dx+c)^5 - 105 A \sin(dx+c)^4 + 105 B \sin(dx+c)^4 + 140 A \sin(dx+c)^3 + 70 B \sin(dx+c)^3 + 105 A \sin(dx+c)^2 - 105 B \sin(dx+c)^2 - 210 A \sin(dx+c)}{210 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/210*(30*B*sin(d*x + c)^7 + 35*A*sin(d*x + c)^6 - 35*B*sin(d*x + c)^6 - 4
2*A*sin(d*x + c)^5 - 84*B*sin(d*x + c)^5 - 105*A*sin(d*x + c)^4 + 105*B*sin
(d*x + c)^4 + 140*A*sin(d*x + c)^3 + 70*B*sin(d*x + c)^3 + 105*A*sin(d*x +
c)^2 - 105*B*sin(d*x + c)^2 - 210*A*sin(d*x + c))/(a*d)
```

Mupad [B]
time = 0.11, size = 124, normalized size = 1.18

$$-\frac{\frac{\sin(c+d\,x)^2\,(A-B)}{2\,a}+\frac{\sin(c+d\,x)^3\,(2\,A+B)}{3\,a}-\frac{\sin(c+d\,x)^5\,(A+2\,B)}{5\,a}+\frac{\sin(c+d\,x)^6\,(A-B)}{6\,a}+\frac{B\sin(c+d\,x)^7}{7\,a}-\frac{\sin(c+d\,x)^4\,(2\,A-2\,B)}{4\,a}-\frac{A\,\sin(c+d\,x)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] -((sin(c + d*x)^2*(A - B))/(2*a) + (sin(c + d*x)^3*(2*A + B))/(3*a) - (sin(
c + d*x)^5*(A + 2*B))/(5*a) + (sin(c + d*x)^6*(A - B))/(6*a) + (B*sin(c + d
*x)^7)/(7*a) - (sin(c + d*x)^4*(2*A - 2*B))/(4*a) - (A*sin(c + d*x))/a)/d
```

3.1004 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}+\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{B(a-a\sin(c+dx))^5}{5a^6d}$$

```
[Out] -2/3*(A+B)*(a-a*sin(d*x+c))^3/a^4/d+1/4*(A+3*B)*(a-a*sin(d*x+c))^4/a^5/d-1/
5*B*(a-a*sin(d*x+c))^5/a^6/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a-a\sin(c+dx))^5}{5a^6d}+\frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}-\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-2*(A + B)*(a - a*Sin[c + d*x])^3)/(3*a^4*d) + ((A + 3*B)*(a - a*Sin[c + d
*x])^4)/(4*a^5*d) - (B*(a - a*Sin[c + d*x])^5)/(5*a^6*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int (a-x)^2(a+x)\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(c+dx)\right)}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^2+(-A-3B)(a-x)^3+\frac{B(a-x)^4}{a}\right)\,d\right.}{a^5d}$$

$$= -\frac{2(A+B)(a-a\sin(c+dx))^3}{3a^4d} + \frac{(A+3B)(a-a\sin(c+dx))^4}{4a^5d}$$

Mathematica [A]
time = 0.11, size = 72, normalized size = 0.91

$$\frac{\sin(c+dx)\left(60A-30(A-B)\sin(c+dx)-20(A+B)\sin^2(c+dx)+15(A-B)\sin^3(c+dx)+12B\sin^4(c+dx)\right)}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]*(60*A - 30*(A - B)*Sin[c + d*x] - 20*(A + B)*Sin[c + d*x]^2 +
 15*(A - B)*Sin[c + d*x]^3 + 12*B*Sin[c + d*x]^4))/(60*a*d)
```

Maple [A]
time = 0.25, size = 75, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{B\left(\sin^5(dx+c)\right)}{5}+\frac{(A-B)\left(\sin^4(dx+c)\right)}{4}+\frac{(-A-B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{da}$
default	$\frac{\frac{B\left(\sin^5(dx+c)\right)}{5}+\frac{(A-B)\left(\sin^4(dx+c)\right)}{4}+\frac{(-A-B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{da}$
risch	$\frac{3A\sin(dx+c)}{4ad}-\frac{B\sin(dx+c)}{8ad}+\frac{\sin(5dx+5c)B}{80ad}+\frac{\cos(4dx+4c)A}{32ad}-\frac{\cos(4dx+4c)B}{32ad}+\frac{\sin(3dx+3c)A}{12ad}+\frac{\sin(3dx}{48}$
norman	$\frac{\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2A\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2(8A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{2(8A-B)\left(\tan^{10}\right.}{3ad}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(1/5*B*sin(d*x+c)^5+1/4*(A-B)*sin(d*x+c)^4+1/3*(-A-B)*sin(d*x+c)^3+1/
2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.91

$$\frac{12\,B\sin\left(dx+c\right)^5+15\,(A-B)\sin\left(dx+c\right)^4-20\,(A+B)\sin\left(dx+c\right)^3-30\,(A-B)\sin\left(dx+c\right)^2+60\,A\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*(A - B)*sin(d*x + c)^4 - 20*(A + B)*sin(d*x
+ c)^3 - 30*(A - B)*sin(d*x + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

Fricas [A]
time = 0.38, size = 66, normalized size = 0.84

$$\frac{15\,(A-B)\cos\left(dx+c\right)^{4}+4\,\left(3\,B\cos\left(dx+c\right)^{4}+(5\,A-B)\cos\left(dx+c\right)^{2}+10\,A-2\,B\right)\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/60*(15*(A - B)*cos(d*x + c)^4 + 4*(3*B*cos(d*x + c)^4 + (5*A - B)*cos(d*x
 + c)^2 + 10*A - 2*B)*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. 2(73) = 146.
time = 14.99, size = 1703, normalized size = 21.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((30*A*tan(c/2 + d*x/2)**9/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*t
an(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)
**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**8/(15*a
*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*
x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d
) + 80*A*tan(c/2 + d*x/2)**7/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2
+ d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 7
5*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**6/(15*a*d*tan(
c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6
 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 100
*A*tan(c/2 + d*x/2)**5/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/
2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*
tan(c/2 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 +
d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150
*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 80*A*tan(
c/2 + d*x/2)**3/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 +
 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2
```

```
 + d*x/2)**2 + 15*a*d) - 30*A*tan(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)*
*10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*ta
n(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*A*tan(c/2 + d
*x/2)/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*t
an(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)*
*2 + 15*a*d) + 30*B*tan(c/2 + d*x/2)**8/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a
*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*
x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**7/(
15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2
+ d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15
*a*d) + 30*B*tan(c/2 + d*x/2)**6/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(
c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4
 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 16*B*tan(c/2 + d*x/2)**5/(15*a*d*
tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2
)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) +
 30*B*tan(c/2 + d*x/2)**4/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d
*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a
*d*tan(c/2 + d*x/2)**2 + 15*a*d) - 40*B*tan(c/2 + d*x/2)**3/(15*a*d*tan(c/2
 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**8 + 150*a*d*tan(c/2 + d*x/2)**6 +
150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(c/2 + d*x/2)**2 + 15*a*d) + 30*B*t
an(c/2 + d*x/2)**2/(15*a*d*tan(c/2 + d*x/2)**10 + 75*a*d*tan(c/2 + d*x/2)**
8 + 150*a*d*tan(c/2 + d*x/2)**6 + 150*a*d*tan(c/2 + d*x/2)**4 + 75*a*d*tan(
c/2 + d*x/2)**2 + 15*a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**5/(a*sin(c)
 + a), True))
```

Giac [A]
time = 0.45, size = 95, normalized size = 1.20

$$\frac{12\,B\sin\left(dx+c\right)^{5}+15\,A\sin\left(dx+c\right)^{4}-15\,B\sin\left(dx+c\right)^{4}-20\,A\sin\left(dx+c\right)^{3}-20\,B\sin\left(dx+c\right)^{3}-30\,A\sin\left(dx+c\right)^{2}+30\,B\sin\left(dx+c\right)^{2}+60\,A\sin\left(dx+c\right)}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*(12*B*sin(d*x + c)^5 + 15*A*sin(d*x + c)^4 - 15*B*sin(d*x + c)^4 - 20*
A*sin(d*x + c)^3 - 20*B*sin(d*x + c)^3 - 30*A*sin(d*x + c)^2 + 30*B*sin(d*x
 + c)^2 + 60*A*sin(d*x + c))/(a*d)
```

Mupad [B]
time = 9.20, size = 82, normalized size = 1.04

$$\frac{\frac{\sin(c+d\,x)^4\,(A-B)}{4\,a}-\frac{\sin(c+d\,x)^2\,(A-B)}{2\,a}+\frac{B\sin(c+d\,x)^5}{5\,a}+\frac{A\,\sin(c+d\,x)}{a}-\frac{\sin(c+d\,x)^3\,(A+B)}{3\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] ((sin(c + d*x)^4*(A - B))/(4*a) - (sin(c + d*x)^2*(A - B))/(2*a) + (B*sin(c
 + d*x)^5)/(5*a) + (A*sin(c + d*x))/a - (sin(c + d*x)^3*(A + B))/(3*a))/d
```

3.1005 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=57

$$\frac{A\sin(c+dx)}{ad}-\frac{(A-B)\sin^2(c+dx)}{2ad}-\frac{B\sin^3(c+dx)}{3ad}$$

```
[Out] A*sin(d*x+c)/a/d-1/2*(A-B)*sin(d*x+c)^2/a/d-1/3*B*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 45}

$$-\frac{(A-B)\sin^2(c+dx)}{2ad}+\frac{A\sin(c+dx)}{ad}-\frac{B\sin^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (A*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x]^2)/(2*a*d) - (B*Sin[c + d*x]
^3)/(3*a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int (a-x)\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{\text{Subst}\left(\int \left(aA-(A-B)x-\frac{Bx^2}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^3d}\\ &= \frac{A\sin(c+dx)}{ad}-\frac{(A-B)\sin^2(c+dx)}{2ad}-\frac{B\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 44, normalized size = 0.77

$$\frac{\sin(c+dx)\left(6A-3(A-B)\sin(c+dx)-2B\sin^2(c+dx)\right)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]*(6*A - 3*(A - B)*Sin[c + d*x] - 2*B*Sin[c + d*x]^2))/(6*a*d)
```

Maple [A]
time = 0.19, size = 43, normalized size = 0.75

method	result
derivativedivides	$\frac{-\frac{B\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{da}$
default	$\frac{-\frac{B\left(\sin^3(dx+c)\right)}{3}+\frac{(-A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{da}$
risch	$\frac{A\sin(dx+c)}{ad}-\frac{B\sin(dx+c)}{4ad}+\frac{\sin(3dx+3c)B}{12ad}+\frac{\cos(2dx+2c)A}{4ad}-\frac{\cos(2dx+2c)B}{4ad}$
norman	$\frac{\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2A\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2B\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2(6A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{2(6A-B)\left(\tan^6\left(\frac{dx}{2}\right.\right.}{3ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(-1/3*B*sin(d*x+c)^3+1/2*(-A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 44, normalized size = 0.77

$$-\frac{2\,B\sin\left(dx+c\right)^3+3\,(A-B)\sin\left(dx+c\right)^2-6\,A\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*(A - B)*sin(d*x + c)^2 - 6*A*sin(d*x + c))/(a*
d)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 0.86

$$\frac{3\,(A-B)\cos\left(dx+c\right)^2+2\left(B\cos\left(dx+c\right)^2+3\,A-B\right)\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")

[Out] 1/6*(3*(A - B)*cos(d*x + c)^2 + 2*(B*cos(d*x + c)^2 + 3*A - B)*sin(d*x + c)
)/(a*d)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(44) = 88$.
time = 4.51, size = 588, normalized size = 10.32

$$\begin{cases} \frac{6A\tan^{5}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{6A\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{12A\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{6A\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{6A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{6B\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} - \frac{8B\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} + \frac{6B\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)+3ad} & \text{for}\: d \neq 0 \\ \frac{x(A+B\sin(c))\cos^{3}(c)}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)

[Out] Piecewise((6*A*tan(c/2 + d*x/2)**5/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c
/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c/2 + d*x/2)*
*4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 +
 d*x/2)**2 + 3*a*d) + 12*A*tan(c/2 + d*x/2)**3/(3*a*d*tan(c/2 + d*x/2)**6 +
 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 6*A*tan(c
/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a
*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6*A*tan(c/2 + d*x/2)/(3*a*d*tan(c/2 + d*x
/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) + 6
*B*tan(c/2 + d*x/2)**4/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)*
*4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d) - 8*B*tan(c/2 + d*x/2)**3/(3*a*d*ta
n(c/2 + d*x/2)**6 + 9*a*d*tan(c/2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 +
 3*a*d) + 6*B*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**6 + 9*a*d*tan(c/
2 + d*x/2)**4 + 9*a*d*tan(c/2 + d*x/2)**2 + 3*a*d), Ne(d, 0)), (x*(A + B*si
n(c))*cos(c)**3/(a*sin(c) + a), True))

Giac [A]
time = 0.46, size = 51, normalized size = 0.89

$$-\frac{2\,B\sin\left(dx+c\right)^{3}+3\,A\sin\left(dx+c\right)^{2}-3\,B\sin\left(dx+c\right)^{2}-6\,A\sin\left(dx+c\right)}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")

[Out] -1/6*(2*B*sin(d*x + c)^3 + 3*A*sin(d*x + c)^2 - 3*B*sin(d*x + c)^2 - 6*A*si
n(d*x + c))/(a*d)

Mupad [B]
time = 0.08, size = 47, normalized size = 0.82

$$\frac{\sin\left(c+d\,x\right)\,\left(6\,A-3\,A\,\sin\left(c+d\,x\right)+3\,B\,\sin\left(c+d\,x\right)-2\,B\,{\sin\left(c+d\,x\right)}^2\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(6*A - 3*A*sin(c + d*x) + 3*B*sin(c + d*x) - 2*B*sin(c + d*x)
^2))/(6*a*d)
```

3.1006 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{(A-B)\log(1+\sin(c+dx))}{ad}+\frac{B\sin(c+dx)}{ad}$$

```
[Out] (A-B)*ln(1+sin(d*x+c))/a/d+B*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2912, 45}

$$\frac{(A-B)\log(\sin(c+dx)+1)}{ad}+\frac{B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A - B)*Log[1 + Sin[c + d*x]])/(a*d) + (B*Sin[c + d*x])/(a*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{a+x}\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(\frac{B}{a}+\frac{A-B}{a+x}\right)\,dx,x,a\sin(c+dx)\right)}{ad}\\
&= \frac{(A-B)\log(1+\sin(c+dx))}{ad}+\frac{B\sin(c+dx)}{ad}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.86

$$\frac{(A-B)\log(1+\sin(c+dx))+B\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] ((A - B)*Log[1 + Sin[c + d*x]] + B*Sin[c + d*x])/(a*d)
```

Maple [A]
time = 0.11, size = 32, normalized size = 0.89

method	result
derivativedivides	$\frac{B\sin(dx+c)+(A-B)\ln(1+\sin(dx+c))}{da}$
default	$\frac{B\sin(dx+c)+(A-B)\ln(1+\sin(dx+c))}{da}$
risch	$-\frac{ixA}{a}+\frac{ixB}{a}-\frac{iB\,e^{i(dx+c)}}{2ad}+\frac{iB\,e^{-i(dx+c)}}{2ad}-\frac{2iAc}{ad}+\frac{2iBc}{ad}+\frac{2\ln(e^{i(dx+c)}+i)A}{ad}-\frac{2\ln(e^{i(dx+c)}+i)B}{ad}$
norman	$\frac{-\frac{2B}{ad}-\frac{2B\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2B\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{(A-B)\ln\left(1+\tan\right.}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(B*sin(d*x+c)+(A-B)*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 34, normalized size = 0.94

$$\frac{\frac{(A-B)\log(\sin(dx+c)+1)}{a}+\frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
[Out] ((A - B)*log(sin(d*x + c) + 1)/a + B*sin(d*x + c)/a)/d
```

Fricas [A]
time = 0.41, size = 31, normalized size = 0.86

$$\frac{(A-B)\log\left(\sin\left(dx+c\right)+1\right)+B\sin\left(dx+c\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] ((A - B)*log(sin(d*x + c) + 1) + B*sin(d*x + c))/(a*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(27) = 54$.
time = 0.27, size = 60, normalized size = 1.67

$$\begin{cases} \frac{A\log\left(\sin\left(c+dx\right)+1\right)}{ad} - \frac{B\log\left(\sin\left(c+dx\right)+1\right)}{ad} + \frac{B\sin\left(c+dx\right)}{ad} & \text{for } d \neq 0 \\ \frac{x(A+B\sin\left(c\right))\cos\left(c\right)}{a\sin\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*log(sin(c + d*x) + 1)/(a*d) - B*log(sin(c + d*x) + 1)/(a*d) +
B*sin(c + d*x)/(a*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)/(a*sin(c) + a), T
rue))
```

Giac [A]
time = 0.42, size = 35, normalized size = 0.97

$$\frac{\frac{(A-B)\log(|\sin(dx+c)+1|)}{a} + \frac{B\sin(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((A - B)*log(abs(sin(d*x + c) + 1))/a + B*sin(d*x + c)/a)/d
```

Mupad [B]
time = 9.23, size = 36, normalized size = 1.00

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(A-B\right)}{a\,d}+\frac{B\,\sin\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x)),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(A - B))/(a*d) + (B*sin(c + d*x))/(a*d)
```

3.1007 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a+a\sin(c+dx))}$$

```
[Out] 1/2*(A+B)*arctanh(sin(d*x+c))/a/d+1/2*(-A+B)/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2915, 78, 212}

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - (A - B)/(2*d*(a + a*Sin[c + d*x])
)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^2}+\frac{A+B}{2a(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{A-B}{2d(a+a\sin(c+dx))}+\frac{(A+B)\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+d$$
$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{A-B}{2d(a+a\sin(c+dx))}$$

Mathematica [A]
time = 0.05, size = 44, normalized size = 0.98

$$\frac{-A+B+(A+B)\tanh^{-1}(\sin(c+dx))(1+\sin(c+dx))}{2ad(1+\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (-A + B + (A + B)*ArcTanh[Sin[c + d*x]]*(1 + Sin[c + d*x]))/(2*a*d*(1 + Sin
[c + d*x]))
```

Maple [A]
time = 0.19, size = 62, normalized size = 1.38

method	result	size
derivativedivides	$\frac{-\frac{\frac{A}{2}-\frac{B}{2}}{1+\sin(dx+c)}+\left(\frac{B}{4}+\frac{A}{4}\right)\ln(1+\sin(dx+c))+\left(-\frac{A}{4}-\frac{B}{4}\right)\ln(\sin(dx+c)-1)}{da}$	62
default	$\frac{-\frac{\frac{A}{2}-\frac{B}{2}}{1+\sin(dx+c)}+\left(\frac{B}{4}+\frac{A}{4}\right)\ln(1+\sin(dx+c))+\left(-\frac{A}{4}-\frac{B}{4}\right)\ln(\sin(dx+c)-1)}{da}$	62
norman	$\frac{\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{(A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{(A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$	122
risch	$-\frac{ie^{i(dx+c)}(A-B)}{da\left(e^{i(dx+c)}+i\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)A}{2ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{2ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)A}{2ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)B}{2ad}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-(1/2*A-1/2*B)/(1+sin(d*x+c))+(1/4*B+1/4*A)*ln(1+sin(d*x+c))+(-1/4*A
-1/4*B)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 58, normalized size = 1.29

$$\frac{\frac{(A+B)\log(\sin(dx+c)+1)}{a} - \frac{(A+B)\log(\sin(dx+c)-1)}{a} - \frac{2\,(A-B)}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/4*((A + B)*log(sin(d*x + c) + 1)/a - (A + B)*log(sin(d*x + c) - 1)/a - 2*
(A - B)/(a*sin(d*x + c) + a))/d
```

Fricas [A]
time = 0.38, size = 73, normalized size = 1.62

$$\frac{((A+B)\sin(dx+c)+A+B)\log(\sin(dx+c)+1)-((A+B)\sin(dx+c)+A+B)\log(-\sin(dx+c)+1)-2\,A+2\,B}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*(((A + B)*sin(d*x + c) + A + B)*log(sin(d*x + c) + 1) - ((A + B)*sin(d*
x + c) + A + B)*log(-sin(d*x + c) + 1) - 2*A + 2*B)/(a*d*sin(d*x + c) + a*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\sin(c+dx)+1}\,dx + \int \frac{B\sin(c+dx)\sec(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x)*s
ec(c + d*x)/(sin(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.48, size = 79, normalized size = 1.76

$$\frac{\frac{(A+B)\log(|\sin(dx+c)+1|)}{a} - \frac{(A+B)\log(|\sin(dx+c)-1|)}{a} - \frac{A\sin(dx+c)+B\sin(dx+c)+3\,A-B}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*((A + B)*log(abs(sin(d*x + c) + 1))/a - (A + B)*log(abs(sin(d*x + c) -
1))/a - (A*sin(d*x + c) + B*sin(d*x + c) + 3*A - B)/(a*(sin(d*x + c) + 1)))
/d
```

Mupad [B]
time = 0.10, size = 43, normalized size = 0.96

$$\frac{\operatorname{atanh}(\sin(c+d\,x))\ (A+B)}{2\,a\,d}-\frac{\frac{A}{2}-\frac{B}{2}}{d\ (a+a\ \sin(c+d\,x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + a*sin(c + d*x))),x)
```

```
[Out] (atanh(sin(c + d*x))*(A + B))/(2*a*d) - (A/2 - B/2)/(d*(a + a*sin(c + d*x))
)
```

3.1008 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a+a\sin(c+dx))^2}-\frac{A}{4d(a+a\sin(c+dx))}$$

```
[Out] 1/8*(3*A+B)*arctanh(sin(d*x+c))/a/d+1/8*(A+B)/d/(a-a*sin(d*x+c))-1/8*a*(A-B
)/d/(a+a*sin(d*x+c))^2-1/4*A/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a\sin(c+dx)+a)^2}+\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{A}{4d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((3*A + B)*ArcTanh[Sin[c + d*x]])/(8*a*d) + (A + B)/(8*d*(a - a*Sin[c + d*x
])) - (a*(A - B))/(8*d*(a + a*Sin[c + d*x])^2) - A/(4*d*(a + a*Sin[c + d*x]
))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^3\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\text{Subst}\left(\int \left(\frac{A+B}{8a^3(a-x)^2}+\frac{A-B}{4a^2(a+x)^3}+\frac{A}{4a^3(a+x)^2}+\frac{3A+B}{8a^3(a^2-x^2)}\right)\,dx, x, a\right.}{d}$$

$$= \frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a(A-B)}{8d(a+a\sin(c+dx))^2}-\frac{A}{4d(a+a\sin(c+}$$

$$= \frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{A+B}{8d(a-a\sin(c+dx))}-\frac{a}{8d(a+}$$

Mathematica [A]
time = 0.21, size = 75, normalized size = 0.82

$$\frac{\frac{(3A+B)\tanh^{-1}(\sin(c+dx))}{a}+\frac{-A+B}{a(1+\sin(c+dx))^2}+\frac{A+B}{a-a\sin(c+dx)}-\frac{2A}{a+a\sin(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (((3*A + B)*ArcTanh[Sin[c + d*x]])/a + (-A + B)/(a*(1 + Sin[c + d*x])^2) +
(A + B)/(a - a*Sin[c + d*x]) - (2*A)/(a + a*Sin[c + d*x]))/(8*d)
```

Maple [A]
time = 0.26, size = 94, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{A}{4(1+\sin(dx+c))}-\frac{\frac{A}{4}-\frac{B}{4}}{2(1+\sin(dx+c))^2}+\left(\frac{3A}{16}+\frac{B}{16}\right)\ln(1+\sin(dx+c))+\left(-\frac{3A}{16}-\frac{B}{16}\right)\ln(\sin(dx+c)-1)-\frac{\frac{A}{8}+\frac{B}{8}}{\sin(dx+c)-1}}{da}$
default	$\frac{-\frac{A}{4(1+\sin(dx+c))}-\frac{\frac{A}{4}-\frac{B}{4}}{2(1+\sin(dx+c))^2}+\left(\frac{3A}{16}+\frac{B}{16}\right)\ln(1+\sin(dx+c))+\left(-\frac{3A}{16}-\frac{B}{16}\right)\ln(\sin(dx+c)-1)-\frac{\frac{A}{8}+\frac{B}{8}}{\sin(dx+c)-1}}{da}$
risch	$-\frac{i\left(6iA\,e^{4i(dx+c)}+3A\,e^{5i(dx+c)}+2iB\,e^{4i(dx+c)}+B\,e^{5i(dx+c)}-6iA\,e^{2i(dx+c)}+2A\,e^{3i(dx+c)}-2iB\,e^{2i(dx+c)}-10B\,e^{3i(dx-}\right.}{4\left(e^{i(dx+c)}+i\right)^4\left(e^{i(dx+c)}-i\right)^2da}$
norman	$\frac{\frac{(A+3B)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(A+3B)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{(A+3B)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{(7A+5B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{(7A+5B)\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{4ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/4*A/(1+sin(d*x+c))-1/2*(1/4*A-1/4*B)/(1+sin(d*x+c))^2+(3/16*A+1/1
6*B)*ln(1+sin(d*x+c))+(-3/16*A-1/16*B)*ln(sin(d*x+c)-1)-(1/8*A+1/8*B)/(sin(
d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 113, normalized size = 1.24

$$\frac{\frac{(3\,A+B)\log(\sin(dx+c)+1)}{a}-\frac{(3\,A+B)\log(\sin(dx+c)-1)}{a}-\frac{2\left((3\,A+B)\sin(dx+c)^2+(3\,A+B)\sin(dx+c)-2\,A+2\,B\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A + B)*log(sin(d*x + c) + 1)/a - (3*A + B)*log(sin(d*x + c) - 1)/a
 - 2*((3*A + B)*sin(d*x + c)^2 + (3*A + B)*sin(d*x + c) - 2*A + 2*B)/(a*sin
(d*x + c)^3 + a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

Fricas [A]
time = 0.38, size = 161, normalized size = 1.77

$$-\frac{2\,(3\,A+B)\cos\left(dx+c\right)^2-\left((3\,A+B)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+(3\,A+B)\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((3\,A+B)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+(3\,A+B)\cos\left(dx+c\right)^2\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,(3\,A+B)\sin\left(dx+c\right)-2\,A-6\,B}{16\left(ad\cos\left(dx+c\right)^2\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/16*(2*(3*A + B)*cos(d*x + c)^2 - ((3*A + B)*cos(d*x + c)^2*sin(d*x + c)
+ (3*A + B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((3*A + B)*cos(d*x + c)
^2*sin(d*x + c) + (3*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*A
 + B)*sin(d*x + c) - 2*A - 6*B)/(a*d*cos(d*x + c)^2*sin(d*x + c) + a*d*cos(
d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx+\int \frac{B\sin\left(c+dx\right)\sec^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x
)*sec(c + d*x)**3/(sin(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.49, size = 147, normalized size = 1.62

$$\frac{\frac{2\,(3\,A+B)\log\left(|\sin(dx+c)+1|\right)}{a}-\frac{2\,(3\,A+B)\log\left(|\sin(dx+c)-1|\right)}{a}+\frac{2\,(3\,A\sin(dx+c)+B\sin(dx+c)-5\,A-3\,B)}{a(\sin(dx+c)-1)}-\frac{9\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2+26\,A\sin(dx+c)+6\,B\sin(dx+c)+21\,A-B}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/32*(2*(3*A + B)*log(abs(sin(d*x + c) + 1))/a - 2*(3*A + B)*log(abs(sin(d*
x + c) - 1))/a + 2*(3*A*sin(d*x + c) + B*sin(d*x + c) - 5*A - 3*B)/(a*(sin(
d*x + c) - 1)) - (9*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 26*A*sin(d*x +
c) + 6*B*sin(d*x + c) + 21*A - B)/(a*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 0.13, size = 96, normalized size = 1.05

$$\frac{\left(\frac{3\,A}{8}+\frac{B}{8}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{3\,A}{8}+\frac{B}{8}\right)\,\sin\left(c+d\,x\right)-\frac{A}{4}+\frac{B}{4}}{d\,\left(-a\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}+\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(3\,A+B\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + a*sin(c + d*x))),x)
```

```
[Out] (B/4 - A/4 + sin(c + d*x)*((3*A)/8 + B/8) + sin(c + d*x)^2*((3*A)/8 + B/8))
/(d*(a + a*sin(c + d*x) - a*sin(c + d*x)^2 - a*sin(c + d*x)^3)) + (atanh(si
n(c + d*x))*(3*A + B))/(8*a*d)
```

3.1009 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=146

$$\frac{(5A+B)\tanh^{-1}(\sin(c+dx))}{16ad}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}-\frac{a^2(A-B)}{24d(a+a\sin(c+dx))^3}-$$

```
[Out] 1/16*(5*A+B)*arctanh(sin(d*x+c))/a/d+1/32*a*(A+B)/d/(a-a*sin(d*x+c))^2+1/16
*(2*A+B)/d/(a-a*sin(d*x+c))-1/24*a^2*(A-B)/d/(a+a*sin(d*x+c))^3-1/32*a*(3*A
-B)/d/(a+a*sin(d*x+c))^2-3/16*A/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$-\frac{a^2(A-B)}{24d(a\sin(c+dx)+a)^3}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}-\frac{a(3A-B)}{32d(a\sin(c+dx)+a)^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}+\frac{(5A+B)\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{3A}{16d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] ((5*A + B)*ArcTanh[Sin[c + d*x]])/(16*a*d) + (a*(A + B))/(32*d*(a - a*Sin[c
 + d*x])^2) + (2*A + B)/(16*d*(a - a*Sin[c + d*x])) - (a^2*(A - B))/(24*d*(
a + a*Sin[c + d*x])^3) - (a*(3*A - B))/(32*d*(a + a*Sin[c + d*x])^2) - (3*A
)/(16*d*(a + a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
```

```
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^5\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\text{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^3}+\frac{2A+B}{16a^5(a-x)^2}+\frac{A-B}{8a^3(a+x)^4}+\frac{3A-B}{16a^4(a+x)^3}+\frac{3}{16a^5(\ldots}\right.\right.}{d}$$

$$= \frac{a(A+B)}{32d(a-a\sin(c+dx))^2}+\frac{2A+B}{16d(a-a\sin(c+dx))}-\frac{a^2(A-}{24d(a+a\sin}$$

$$= \frac{(5A+B)\tanh^{-1}(\sin(c+dx))}{16ad}+\frac{a(A+B)}{32d(a-a\sin(c+dx))^2}+\frac{}{16d(a}$$

Mathematica [**A**]
time = 0.42, size = 105, normalized size = 0.72

$$\frac{6(5A+B)\tanh^{-1}(\sin(c+dx))+\frac{3(A+B)}{(-1+\sin(c+dx))^2}-\frac{6(2A+B)}{-1+\sin(c+dx)}-\frac{4(A-B)}{(1+\sin(c+dx))^3}+\frac{-9A+3B}{(1+\sin(c+dx))^2}-\frac{18A}{1+\sin(c+dx)}}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
[Out] (6*(5*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (6
*(2*A + B))/(-1 + Sin[c + d*x]) - (4*(A - B))/(1 + Sin[c + d*x])^3 + (-9*A
+ 3*B)/(1 + Sin[c + d*x])^2 - (18*A)/(1 + Sin[c + d*x]))/(96*a*d)
```

Maple [**A**]
time = 0.39, size = 132, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{3A}{16(1+\sin(dx+c))}-\frac{\frac{A}{8}-\frac{B}{8}}{3(1+\sin(dx+c))^3}-\frac{\frac{3A}{16}-\frac{B}{16}}{2(1+\sin(dx+c))^2}+\left(\frac{5A}{32}+\frac{B}{32}\right)\ln(1+\sin(dx+c))+\left(-\frac{5A}{32}-\frac{B}{32}\right)\ln(\sin(dx+c)-1)-\frac{}{2(\mathrm{s}}}{da}$
default	$\frac{-\frac{3A}{16(1+\sin(dx+c))}-\frac{\frac{A}{8}-\frac{B}{8}}{3(1+\sin(dx+c))^3}-\frac{\frac{3A}{16}-\frac{B}{16}}{2(1+\sin(dx+c))^2}+\left(\frac{5A}{32}+\frac{B}{32}\right)\ln(1+\sin(dx+c))+\left(-\frac{5A}{32}-\frac{B}{32}\right)\ln(\sin(dx+c)-1)-\frac{}{2(\mathrm{s}}}{da}$
risch	$-\frac{i\left(30iA\,\mathrm{e}^{8i(dx+c)}+15A\,\mathrm{e}^{9i(dx+c)}+22iB\,\mathrm{e}^{6i(dx+c)}+3B\,\mathrm{e}^{9i(dx+c)}-22iB\,\mathrm{e}^{4i(dx+c)}+40A\,\mathrm{e}^{7i(dx+c)}+110iA\,\mathrm{e}^{6i(dx+c)}+8\ldots\right.}{}$
norman	$\frac{\frac{(41A+37B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{(41A+37B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{(A-19B)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(3A+7B)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{(3A+7B)\left(\tan\ldots\right.}{4\ldots}}{\left(\tan\ldots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-3/16*A/(1+sin(d*x+c))-1/3*(1/8*A-1/8*B)/(1+sin(d*x+c))^3-1/2*(3/16*
A-1/16*B)/(1+sin(d*x+c))^2+(5/32*A+1/32*B)*ln(1+sin(d*x+c))+(-5/32*A-1/32*B
)*ln(sin(d*x+c)-1)-1/2*(-1/16*A-1/16*B)/(sin(d*x+c)-1)^2-(1/8*A+1/16*B)/(si
n(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 165, normalized size = 1.13

$$\frac{\frac{3\,(5\,A+B)\log\left(\sin\left(dx+c\right)+1\right)}{a}-\frac{3\,(5\,A+B)\log\left(\sin\left(dx+c\right)-1\right)}{a}-\frac{2\left(3\,(5\,A+B)\sin\left(dx+c\right)^4+3\,(5\,A+B)\sin\left(dx+c\right)^3-5\,(5\,A+B)\sin\left(dx+c\right)^2-5\,(5\,A+B)\sin\left(dx+c\right)+8\,A-8\,B\right)}{a\sin\left(dx+c\right)^5+a\sin\left(dx+c\right)^4-2\,a\sin\left(dx+c\right)^3-2\,a\sin\left(dx+c\right)^2+a\sin\left(dx+c\right)+a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A + B)*log(sin(d*x + c) + 1)/a - 3*(5*A + B)*log(sin(d*x + c) -
1)/a - 2*(3*(5*A + B)*sin(d*x + c)^4 + 3*(5*A + B)*sin(d*x + c)^3 - 5*(5*A
+ B)*sin(d*x + c)^2 - 5*(5*A + B)*sin(d*x + c) + 8*A - 8*B)/(a*sin(d*x + c)
^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*a*sin(d*x + c)^2 + a*sin(d*x
 + c) + a))/d
```

Fricas [A]
time = 0.37, size = 194, normalized size = 1.33

$$-\frac{6\,(5\,A+B)\cos\left(dx+c\right)^4-2\,(5\,A+B)\cos\left(dx+c\right)^2-3\left((5\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)+(5\,A+B)\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left((5\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)+(5\,A+B)\cos\left(dx+c\right)^4\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(3\,(5\,A+B)\cos\left(dx+c\right)^2+10\,A+2\,B\right)\sin\left(dx+c\right)-4\,A-20\,B}{96\left(ad\cos\left(dx+c\right)^4\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/96*(6*(5*A + B)*cos(d*x + c)^4 - 2*(5*A + B)*cos(d*x + c)^2 - 3*((5*A +
B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4)*log(sin(d*x + c)
 + 1) + 3*((5*A + B)*cos(d*x + c)^4*sin(d*x + c) + (5*A + B)*cos(d*x + c)^4
)*log(-sin(d*x + c) + 1) - 2*(3*(5*A + B)*cos(d*x + c)^2 + 10*A + 2*B)*sin(
d*x + c) - 4*A - 20*B)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^
4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^5\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx+\int \frac{B\sin\left(c+dx\right)\sec^5\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sin(c + d*x) + 1), x) + Integral(B*sin(c + d*x
)*sec(c + d*x)**5/(sin(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.49, size = 192, normalized size = 1.32

$$\frac{\frac{6\,(5\,A+B)\log(|\sin(dx+c)+1|)}{a}-\frac{6\,(5\,A+B)\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2-38\,A\sin(dx+c)-10\,B\sin(dx+c)+25\,A+9\,B\right)}{a(\sin(dx+c)-1)^2}-\frac{55\,A\sin(dx+c)^3+11\,B\sin(dx+c)^3+201\,A\sin(dx+c)^2+33\,B\sin(dx+c)^2+255\,A\sin(dx+c)+27\,B\sin(dx+c)+117\,A-3\,B}{a(\sin(dx+c)+1)^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/192*(6*(5*A + B)*log(abs(sin(d*x + c) + 1))/a - 6*(5*A + B)*log(abs(sin(d
*x + c) - 1))/a + 3*(15*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 - 38*A*sin(d*
x + c) - 10*B*sin(d*x + c) + 25*A + 9*B)/(a*(sin(d*x + c) - 1)^2) - (55*A*s
in(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 201*A*sin(d*x + c)^2 + 33*B*sin(d*x +
 c)^2 + 255*A*sin(d*x + c) + 27*B*sin(d*x + c) + 117*A - 3*B)/(a*(sin(d*x +
 c) + 1)^3))/d
```

Mupad [B]
time = 9.18, size = 151, normalized size = 1.03

$$\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(5\,A+B\right)}{16\,a\,d}-\frac{\left(\frac{5\,A}{16}+\frac{B}{16}\right)\,\sin\left(c+d\,x\right)^4+\left(\frac{5\,A}{16}+\frac{B}{16}\right)\,\sin\left(c+d\,x\right)^3+\left(-\frac{25\,A}{48}-\frac{5\,B}{48}\right)\,\sin\left(c+d\,x\right)^2+\left(-\frac{25\,A}{48}-\frac{5\,B}{48}\right)\,\sin\left(c+d\,x\right)+\frac{A}{6}-\frac{B}{6}}{d\,\left(a\sin\left(c+d\,x\right)^5+a\sin\left(c+d\,x\right)^4-2\,a\sin\left(c+d\,x\right)^3-2\,a\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + a*sin(c + d*x))),x)
```

```
[Out] (atanh(sin(c + d*x))*(5*A + B))/(16*a*d) - (A/6 - B/6 - sin(c + d*x)*((25*A
)/48 + (5*B)/48) + sin(c + d*x)^3*((5*A)/16 + B/16) + sin(c + d*x)^4*((5*A)
/16 + B/16) - sin(c + d*x)^2*((25*A)/48 + (5*B)/48))/(d*(a + a*sin(c + d*x)
 - 2*a*sin(c + d*x)^2 - 2*a*sin(c + d*x)^3 + a*sin(c + d*x)^4 + a*sin(c + d
*x)^5))
```

3.1010 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=205

$$\frac{5(7A+B)\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}+\frac{5(3A+B)}{128d(a-a\sin(c+dx)}$$

```
[Out] 5/128*(7*A+B)*arctanh(sin(d*x+c))/a/d+1/96*a^2*(A+B)/d/(a-a*sin(d*x+c))^3+1
/128*a*(5*A+3*B)/d/(a-a*sin(d*x+c))^2+5/128*(3*A+B)/d/(a-a*sin(d*x+c))-1/64
*a^3*(A-B)/d/(a+a*sin(d*x+c))^4-1/48*a^2*(2*A-B)/d/(a+a*sin(d*x+c))^3-1/64*
a*(5*A-B)/d/(a+a*sin(d*x+c))^2-5/32*A/d/(a+a*sin(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$-\frac{a^3(A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3}-\frac{a^2(2A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2}-\frac{a(5A-B)}{64d(a\sin(c+dx)+a)^2}+\frac{5(3A+B)}{128d(a-a\sin(c+dx))}+\frac{5(7A+B)\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{5A}{32d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*(7*A + B)*ArcTanh[Sin[c + d*x]])/(128*a*d) + (a^2*(A + B))/(96*d*(a - a*
Sin[c + d*x])^3) + (a*(5*A + 3*B))/(128*d*(a - a*Sin[c + d*x])^2) + (5*(3*A
 + B))/(128*d*(a - a*Sin[c + d*x])) - (a^3*(A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a^2*(2*A - B))/(48*d*(a + a*Sin[c + d*x])^3) - (a*(5*A - B))/(64*
d*(a + a*Sin[c + d*x])^2) - (5*A)/(32*d*(a + a*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+a\sin(c+dx)}\,dx = \frac{a^7\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{A+B}{32a^5(a-x)^4} + \frac{5A+3B}{64a^6(a-x)^3} + \frac{5(3A+B)}{128a^7(a-x)^2} + \frac{A-B}{16a^4(a+x)^5} + \frac{2}{16a}\right.\right.}{}$$

$$= \frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3} + \frac{a(5A+3B)}{128d(a-a\sin(c+dx))^2} + \frac{5(3A}{128d(a-a}$$

$$= \frac{5(7A+B)\tanh^{-1}(\sin(c+dx))}{128ad} + \frac{a^2(A+B)}{96d(a-a\sin(c+dx))^3} + \frac{}{128d}$$

Mathematica [A]
time = 0.69, size = 142, normalized size = 0.69

$$\frac{15(7A+B)\tanh^{-1}(\sin(c+dx)) + \frac{48(A-B)-33(7A+B)\sin(c+dx)-33(7A+B)\sin^2(c+dx)+40(7A+B)\sin^3(c+dx)+40(7A+B)\sin^4(c+dx)-15(7A+B)\sin^5(c+dx)-15(7A+B)\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x]),x]
```

```
[Out] (15*(7*A + B)*ArcTanh[Sin[c + d*x]] + (48*(A - B) - 33*(7*A + B)*Sin[c + d*
x] - 33*(7*A + B)*Sin[c + d*x]^2 + 40*(7*A + B)*Sin[c + d*x]^3 + 40*(7*A +
B)*Sin[c + d*x]^4 - 15*(7*A + B)*Sin[c + d*x]^5 - 15*(7*A + B)*Sin[c + d*x]
^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A]
time = 0.56, size = 170, normalized size = 0.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-5/32*A/(1+sin(d*x+c))-1/4*(1/16*A-1/16*B)/(1+sin(d*x+c))^4-1/3*(1/8
*A-1/16*B)/(1+sin(d*x+c))^3-1/2*(5/32*A-1/32*B)/(1+sin(d*x+c))^2+(35/256*A+
5/256*B)*ln(1+sin(d*x+c))+(-35/256*A-5/256*B)*ln(sin(d*x+c)-1)-1/2*(-5/64*A
-3/64*B)/(sin(d*x+c)-1)^2-1/3*(1/32*A+1/32*B)/(sin(d*x+c)-1)^3-(15/128*A+5/
128*B)/(sin(d*x+c)-1))
```

Maxima [A]
time = 0.33, size = 220, normalized size = 1.07

$$\frac{\frac{15\,(7\,A+B)\log\left(\sin\left(dx+c\right)+1\right)}{a}-\frac{15\,(7\,A+B)\log\left(\sin\left(dx+c\right)-1\right)}{a}-\frac{2\,\left(15\,(7\,A+B)\sin\left(dx+c\right)^{6}+15\,(7\,A+B)\sin\left(dx+c\right)^{5}-40\,(7\,A+B)\sin\left(dx+c\right)^{4}-40\,(7\,A+B)\sin\left(dx+c\right)^{3}+33\,(7\,A+B)\sin\left(dx+c\right)^{2}+33\,(7\,A+B)\sin\left(dx+c\right)-48\,A+48\,B\right)}{a\sin\left(dx+c\right)^{7}+a\sin\left(dx+c\right)^{6}-3\,a\sin\left(dx+c\right)^{5}-3\,a\sin\left(dx+c\right)^{4}+3\,a\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-a\sin\left(dx+c\right)-a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/768*(15*(7*A + B)*log(sin(d*x + c) + 1)/a - 15*(7*A + B)*log(sin(d*x + c)
 - 1)/a - 2*(15*(7*A + B)*sin(d*x + c)^6 + 15*(7*A + B)*sin(d*x + c)^5 - 40
*(7*A + B)*sin(d*x + c)^4 - 40*(7*A + B)*sin(d*x + c)^3 + 33*(7*A + B)*sin(
d*x + c)^2 + 33*(7*A + B)*sin(d*x + c) - 48*A + 48*B)/(a*sin(d*x + c)^7 + a
*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin(d*x + c
)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a))/d
```

Fricas [A]
time = 0.39, size = 224, normalized size = 1.09

$$-\frac{30\,(7\,A+B)\cos\left(dx+c\right)^{6}-10\,(7\,A+B)\cos\left(dx+c\right)^{4}-4\,(7\,A+B)\cos\left(dx+c\right)^{2}-15\,\left((7\,A+B)\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+(7\,A+B)\cos\left(dx+c\right)^{6}\right)\log\left(\sin\left(dx+c\right)+1\right)+15\,\left((7\,A+B)\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+(7\,A+B)\cos\left(dx+c\right)^{6}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(15\,(7\,A+B)\cos\left(dx+c\right)^{4}+10\,(7\,A+B)\cos\left(dx+c\right)^{2}+56\,A+8\,B\right)\sin\left(dx+c\right)-16\,A-112\,B}{768\,\left(ad\cos\left(dx+c\right)^{6}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/768*(30*(7*A + B)*cos(d*x + c)^6 - 10*(7*A + B)*cos(d*x + c)^4 - 4*(7*A
+ B)*cos(d*x + c)^2 - 15*((7*A + B)*cos(d*x + c)^6*sin(d*x + c) + (7*A + B)
*cos(d*x + c)^6)*log(sin(d*x + c) + 1) + 15*((7*A + B)*cos(d*x + c)^6*sin(d
*x + c) + (7*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(15*(7*A + B
)*cos(d*x + c)^4 + 10*(7*A + B)*cos(d*x + c)^2 + 56*A + 8*B)*sin(d*x + c) -
 16*A - 112*B)/(a*d*cos(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 236, normalized size = 1.15

$$\frac{\frac{60\,(7\,A+B)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a}-\frac{60\,(7\,A+B)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a}+\frac{2\,\left(385\,A\sin\left(dx+c\right)^{3}+55\,B\sin\left(dx+c\right)^{3}-1335\,A\sin\left(dx+c\right)^{2}-225\,B\sin\left(dx+c\right)^{2}+1575\,A\sin\left(dx+c\right)+321\,B\sin\left(dx+c\right)-641\,A-167\,B\right)}{a(\sin\left(dx+c\right)-1)^{3}}-\frac{875\,A\sin\left(dx+c\right)^{4}+125\,B\sin\left(dx+c\right)^{4}+3980\,A\sin\left(dx+c\right)^{3}+500\,B\sin\left(dx+c\right)^{3}+6930\,A\sin\left(dx+c\right)^{2}+702\,B\sin\left(dx+c\right)^{2}+5548\,A\sin\left(dx+c\right)+340\,B\sin\left(dx+c\right)+1771\,A-35\,B}{a(\sin\left(dx+c\right)+1)^{4}}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/3072*(60*(7*A + B)*log(abs(sin(d*x + c) + 1))/a - 60*(7*A + B)*log(abs(si
n(d*x + c) - 1))/a + 2*(385*A*sin(d*x + c)^3 + 55*B*sin(d*x + c)^3 - 1335*A
*sin(d*x + c)^2 - 225*B*sin(d*x + c)^2 + 1575*A*sin(d*x + c) + 321*B*sin(d*
x + c) - 641*A - 167*B)/(a*(sin(d*x + c) - 1)^3) - (875*A*sin(d*x + c)^4 +
125*B*sin(d*x + c)^4 + 3980*A*sin(d*x + c)^3 + 500*B*sin(d*x + c)^3 + 6930*
A*sin(d*x + c)^2 + 702*B*sin(d*x + c)^2 + 5548*A*sin(d*x + c) + 340*B*sin(d
*x + c) + 1771*A - 35*B)/(a*(sin(d*x + c) + 1)^4))/d
```

Mupad [B]

time = 9.35, size = 206, normalized size = 1.00

$$\frac{\left(\frac{35\,A}{128}+\frac{5\,B}{128}\right)\,\sin\left(c+d\,x\right)^6+\left(\frac{35\,A}{128}+\frac{5\,B}{128}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{35\,A}{48}-\frac{5\,B}{48}\right)\,\sin\left(c+d\,x\right)^4+\left(-\frac{35\,A}{48}-\frac{5\,B}{48}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{77\,A}{128}+\frac{11\,B}{128}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{77\,A}{128}+\frac{11\,B}{128}\right)\,\sin\left(c+d\,x\right)-\frac{A}{8}+\frac{B}{8}}{d\,\left(-a\,\sin\left(c+d\,x\right)^7-a\,\sin\left(c+d\,x\right)^6+3\,a\,\sin\left(c+d\,x\right)^5+3\,a\,\sin\left(c+d\,x\right)^4-3\,a\,\sin\left(c+d\,x\right)^3-3\,a\,\sin\left(c+d\,x\right)^2+a\,\sin\left(c+d\,x\right)+a\right)}+\frac{5\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(7\,A+B\right)}{128\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + a*sin(c + d*x))),x)
```

```
[Out] (B/8 - A/8 + sin(c + d*x)*((77*A)/128 + (11*B)/128) - sin(c + d*x)^3*((35*A
)/48 + (5*B)/48) - sin(c + d*x)^4*((35*A)/48 + (5*B)/48) + sin(c + d*x)^5*(
(35*A)/128 + (5*B)/128) + sin(c + d*x)^6*((35*A)/128 + (5*B)/128) + sin(c +
 d*x)^2*((77*A)/128 + (11*B)/128))/(d*(a + a*sin(c + d*x) - 3*a*sin(c + d*x
)^2 - 3*a*sin(c + d*x)^3 + 3*a*sin(c + d*x)^4 + 3*a*sin(c + d*x)^5 - a*sin(
c + d*x)^6 - a*sin(c + d*x)^7)) + (5*atanh(sin(c + d*x))*(7*A + B))/(128*a*
d)
```

3.1011 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{B(a-a\sin(c+dx))^6}{6a^8d}$$

```
[Out] -1/2*(A+B)*(a-a*sin(d*x+c))^4/a^6/d+1/5*(A+3*B)*(a-a*sin(d*x+c))^5/a^7/d-1/
6*B*(a-a*sin(d*x+c))^6/a^8/d
```

Rubi [A]
time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a-a\sin(c+dx))^6}{6a^8d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((A + B)*(a - a*Sin[c + d*x])^4)/(a^6*d) + ((A + 3*B)*(a - a*Sin[c + d
*x])^5)/(5*a^7*d) - (B*(a - a*Sin[c + d*x])^6)/(6*a^8*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)\left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(c+dx)\right)}{a^7d}$$

$$= \frac{\text{Subst}\left(\int \left(2a(A+B)(a-x)^3+(-A-3B)(a-x)^4+\frac{B(a-x)^5}{a}\right)\,\right.}{a^7d}$$

$$= -\frac{(A+B)(a-a\sin(c+dx))^4}{2a^6d}+\frac{(A+3B)(a-a\sin(c+dx))^5}{5a^7d}-$$

Mathematica [A]
time = 0.12, size = 52, normalized size = 0.66

$$-\frac{(-1+\sin(c+dx))^4\,(9A+2B+(6A+8B)\sin(c+dx)+5B\sin^2(c+dx))}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/30*((-1 + Sin[c + d*x])^4*(9*A + 2*B + (6*A + 8*B)*Sin[c + d*x] + 5*B*Si
n[c + d*x]^2))/(a^2*d)
```

Maple [A]
time = 0.27, size = 82, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{B\left(\sin^6(dx+c)\right)}{6}+\frac{(-A+2B)\left(\sin^5(dx+c)\right)}{5}+\frac{A\left(\sin^4(dx+c)\right)}{2}-\frac{2B\left(\sin^3(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{d\,a^2}$
default	$\frac{-\frac{B\left(\sin^6(dx+c)\right)}{6}+\frac{(-A+2B)\left(\sin^5(dx+c)\right)}{5}+\frac{A\left(\sin^4(dx+c)\right)}{2}-\frac{2B\left(\sin^3(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{d\,a^2}$
risch	$\frac{7\sin(dx+c)A}{8a^2d}-\frac{\sin(dx+c)B}{4a^2d}+\frac{B\cos(6dx+6c)}{192a^2d}-\frac{\sin(5dx+5c)A}{80a^2d}+\frac{\sin(5dx+5c)B}{40a^2d}+\frac{\cos(4dx+4c)A}{16a^2d}-\frac{\cos(4dx}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d/a^2*(-1/6*B*sin(d*x+c)^6+1/5*(-A+2*B)*sin(d*x+c)^5+1/2*A*sin(d*x+c)^4-2
/3*B*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c)^2+A*sin(d*x+c))
```

Maxima [A]
time = 0.31, size = 83, normalized size = 1.05

$$-\frac{5\,B\sin(dx+c)^6+6\,(A-2\,B)\sin(dx+c)^5-15\,A\sin(dx+c)^4+20\,B\sin(dx+c)^3+15\,(2\,A-B)\sin(dx+c)^2-30\,A\sin(dx+c)}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*(A - 2*B)*sin(d*x + c)^5 - 15*A*sin(d*x + c)^
4 + 20*B*sin(d*x + c)^3 + 15*(2*A - B)*sin(d*x + c)^2 - 30*A*sin(d*x + c))/
(a^2*d)
```

Fricas [A]
time = 0.37, size = 82, normalized size = 1.04

$$\frac{5 B \cos(dx+c)^6 + 15 (A-B) \cos(dx+c)^4 - 2 \left(3 (A-2 B) \cos(dx+c)^4 - 2 (3 A-B) \cos(dx+c)^2 - 12 A + 4 B\right) \sin(dx+c)}{30 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/30*(5*B*cos(d*x + c)^6 + 15*(A - B)*cos(d*x + c)^4 - 2*(3*(A - 2*B)*cos(d
*x + c)^4 - 2*(3*A - B)*cos(d*x + c)^2 - 12*A + 4*B)*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2705 vs. $2(70) = 140$.
time = 71.65, size = 2705, normalized size = 34.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((30*A*tan(c/2 + d*x/2)**11/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a
**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*ta
n(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*
x/2)**2 + 15*a**2*d) - 60*A*tan(c/2 + d*x/2)**10/(15*a**2*d*tan(c/2 + d*x/2
)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 3
00*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*
tan(c/2 + d*x/2)**2 + 15*a**2*d) + 150*A*tan(c/2 + d*x/2)**9/(15*a**2*d*tan
(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d
*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4
+ 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*A*tan(c/2 + d*x/2)**8/(1
5*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d
*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2
+ d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 204*A*tan(c/2 +
d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10
+ 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**
2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 120*
A*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 +
 d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)*
```

*6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a*
*2*d) + 204*A*tan(c/2 + d*x/2)**5/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2
*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c
/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2
)**2 + 15*a**2*d) - 120*A*tan(c/2 + d*x/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**
12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*
a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan
(c/2 + d*x/2)**2 + 15*a**2*d) + 150*A*tan(c/2 + d*x/2)**3/(15*a**2*d*tan(c/
2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/
2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 9
0*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 60*A*tan(c/2 + d*x/2)**2/(15*a*
*2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan
(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*
x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 30*A*tan(c/2 + d*x/2
)/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a*
*2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(
c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 30*B*tan(c/2
 + d*x/2)**10/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)*
*10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225
*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) -
80*B*tan(c/2 + d*x/2)**9/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/
2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/
2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15
*a**2*d) + 120*B*tan(c/2 + d*x/2)**8/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a
2*d*tan(c/2 + d*x/2)10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*ta
n(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*
x/2)**2 + 15*a**2*d) - 48*B*tan(c/2 + d*x/2)**7/(15*a**2*d*tan(c/2 + d*x/2)
12 + 90*a2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 30
0*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*t
an(c/2 + d*x/2)**2 + 15*a**2*d) + 20*B*tan(c/2 + d*x/2)**6/(15*a**2*d*tan(c
/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x
/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 +
90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 48*B*tan(c/2 + d*x/2)**5/(15*a
2*d*tan(c/2 + d*x/2)12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 225*a**2*d*ta
n(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d*tan(c/2 + d
*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) + 120*B*tan(c/2 + d*x
/2)**4/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x/2)**10 + 2
25*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 + 225*a**2*d
*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d) - 80*B*ta
n(c/2 + d*x/2)**3/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*tan(c/2 + d*x
/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 + d*x/2)**6 +
 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2 + 15*a**2*d
) + 30*B*tan(c/2 + d*x/2)**2/(15*a**2*d*tan(c/2 + d*x/2)**12 + 90*a**2*d*ta
n(c/2 + d*x/2)**10 + 225*a**2*d*tan(c/2 + d*x/2)**8 + 300*a**2*d*tan(c/2 +
d*x/2)**6 + 225*a**2*d*tan(c/2 + d*x/2)**4 + 90*a**2*d*tan(c/2 + d*x/2)**2

```
+ 15*a**2*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**7/(a*sin(c) + a)**2, Tru
e))
```

Giac [A]
time = 0.46, size = 95, normalized size = 1.20

$$-\frac{5 B \sin\left(dx+c\right)^{6}+6 A \sin\left(dx+c\right)^{5}-12 B \sin\left(dx+c\right)^{5}-15 A \sin\left(dx+c\right)^{4}+20 B \sin\left(dx+c\right)^{3}+30 A \sin\left(dx+c\right)^{2}-15 B \sin\left(dx+c\right)^{2}-30 A \sin\left(dx+c\right)}{30 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/30*(5*B*sin(d*x + c)^6 + 6*A*sin(d*x + c)^5 - 12*B*sin(d*x + c)^5 - 15*A
*sin(d*x + c)^4 + 20*B*sin(d*x + c)^3 + 30*A*sin(d*x + c)^2 - 15*B*sin(d*x
+ c)^2 - 30*A*sin(d*x + c))/(a^2*d)
```

Mupad [B]
time = 0.08, size = 98, normalized size = 1.24

$$-\frac{\frac{\sin(c+d\,x)^5\,(A-2\,B)}{5\,a^2}-\frac{A\sin(c+d\,x)^4}{2\,a^2}+\frac{2\,B\sin(c+d\,x)^3}{3\,a^2}+\frac{B\sin(c+d\,x)^6}{6\,a^2}+\frac{\sin(c+d\,x)^2\,(2\,A-B)}{2\,a^2}-\frac{A\,\sin(c+d\,x)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -((sin(c + d*x)^5*(A - 2*B))/(5*a^2) - (A*sin(c + d*x)^4)/(2*a^2) + (2*B*si
n(c + d*x)^3)/(3*a^2) + (B*sin(c + d*x)^6)/(6*a^2) + (sin(c + d*x)^2*(2*A -
 B))/(2*a^2) - (A*sin(c + d*x))/a^2)/d
```

3.1012 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=51

$$-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}+\frac{B(a-a\sin(c+dx))^4}{4a^6d}$$

```
[Out] -1/3*(A+B)*(a-a*sin(d*x+c))^3/a^5/d+1/4*B*(a-a*sin(d*x+c))^4/a^6/d
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 45}

$$\frac{B(a-a\sin(c+dx))^4}{4a^6d}-\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/3*((A + B)*(a - a*Sin[c + d*x])^3)/(a^5*d) + (B*(a - a*Sin[c + d*x])^4)/
(4*a^6*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int (a-x)^2\left(A+\frac{Bx}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= \frac{\text{Subst}\left(\int \left((A+B)(a-x)^2-\frac{B(a-x)^3}{a}\right)\,dx,x,a\sin(c+dx)\right)}{a^5d}\\
&= -\frac{(A+B)(a-a\sin(c+dx))^3}{3a^5d}+\frac{B(a-a\sin(c+dx))^4}{4a^6d}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 34, normalized size = 0.67

$$\frac{(-1+\sin(c+dx))^3(4A+B+3B\sin(c+dx))}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
[Out] ((-1 + Sin[c + d*x])^3*(4*A + B + 3*B*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A]
time = 0.33, size = 58, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{B\left(\sin^4(dx+c)\right)}{4}+\frac{(A-2B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{d\,a^2}$
default	$\frac{\frac{B\left(\sin^4(dx+c)\right)}{4}+\frac{(A-2B)\left(\sin^3(dx+c)\right)}{3}+\frac{(-2A+B)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)}{d\,a^2}$
risch	$\frac{5\sin(dx+c)A}{4a^2d}-\frac{\sin(dx+c)B}{2a^2d}+\frac{\cos(4dx+4c)B}{32a^2d}-\frac{\sin(3dx+3c)A}{12a^2d}+\frac{\sin(3dx+3c)B}{6a^2d}+\frac{\cos(2dx+2c)A}{2a^2d}-\frac{3\cos(2dx\cdots}{8a^2\cdots}$
norman	$\frac{\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2A\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(2A+2B)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(2A+2B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(12A+2B)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(12A\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
[Out] 1/d/a^2*(1/4*B*sin(d*x+c)^4+1/3*(A-2*B)*sin(d*x+c)^3+1/2*(-2*A+B)*sin(d*x+c
)^2+A*sin(d*x+c))
```

Maxima [A]
time = 0.31, size = 61, normalized size = 1.20

$$\frac{3\,B\sin{(dx+c)}^4+4\,(A-2\,B)\sin{(dx+c)}^3-6\,(2\,A-B)\sin{(dx+c)}^2+12\,A\sin{(dx+c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*(A - 2*B)*sin(d*x + c)^3 - 6*(2*A - B)*sin(d*x
 + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

Fricas [A]
time = 0.36, size = 64, normalized size = 1.25

$$\frac{3\,B\cos{(dx+c)}^4+12\,(A-B)\cos{(dx+c)}^2-4\,\left((A-2\,B)\cos{(dx+c)}^2-4\,A+2\,B\right)\sin{(dx+c)}}{12\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*B*cos(d*x + c)^4 + 12*(A - B)*cos(d*x + c)^2 - 4*((A - 2*B)*cos(d*x
 + c)^2 - 4*A + 2*B)*sin(d*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(44) = 88$.
time = 27.18, size = 1182, normalized size = 23.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((6*A*tan(c/2 + d*x/2)**7/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*
d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 3*a**2*d) - 12*A*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2
)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a
**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 26*A*tan(c/2 + d*x/2)**5/(3*a**2*d*
tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d
*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 24*A*tan(c/2 + d*x/2
)**4/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**
2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 26*A*
tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x
/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*
a**2*d) - 12*A*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*
d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 +
 d*x/2)**2 + 3*a**2*d) + 6*A*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**8
 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*
d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 6*B*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c
/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)
**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) - 16*B*tan(c/2 + d*x/2)**5/
(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*t
an(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*d) + 24*B*tan(c
/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan(c/2 + d*x/2)**
6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/2)**2 + 3*a**2*
d) - 16*B*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**8 + 12*a**2*d*tan
(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*tan(c/2 + d*x/
2)**2 + 3*a**2*d) + 6*B*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 +
 12*a**2*d*tan(c/2 + d*x/2)**6 + 18*a**2*d*tan(c/2 + d*x/2)**4 + 12*a**2*d*
tan(c/2 + d*x/2)**2 + 3*a**2*d), Ne(d, 0)), (x*(A + B*sin(c))*cos(c)**5/(a*
sin(c) + a)**2, True))
```

Giac [A]
time = 0.52, size = 73, normalized size = 1.43

$$\frac{3\,B\sin\left(dx+c\right)^{4}+4\,A\sin\left(dx+c\right)^{3}-8\,B\sin\left(dx+c\right)^{3}-12\,A\sin\left(dx+c\right)^{2}+6\,B\sin\left(dx+c\right)^{2}+12\,A\sin\left(dx+c\right)}{12\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(3*B*sin(d*x + c)^4 + 4*A*sin(d*x + c)^3 - 8*B*sin(d*x + c)^3 - 12*A*s
in(d*x + c)^2 + 6*B*sin(d*x + c)^2 + 12*A*sin(d*x + c))/(a^2*d)
```

Mupad [B]
time = 9.12, size = 68, normalized size = 1.33

$$\frac{\frac{\sin(c+d\,x)^3\,(A-2\,B)}{3\,a^2}+\frac{B\,\sin(c+d\,x)^4}{4\,a^2}-\frac{\sin(c+d\,x)^2\,(2\,A-B)}{2\,a^2}+\frac{A\,\sin(c+d\,x)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] ((sin(c + d*x)^3*(A - 2*B))/(3*a^2) + (B*sin(c + d*x)^4)/(4*a^2) - (sin(c +
 d*x)^2*(2*A - B))/(2*a^2) + (A*sin(c + d*x))/a^2)/d
```

3.1013 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{2(A-B)\log(1+\sin(c+dx))}{a^2d}-\frac{(A-B)\sin(c+dx)}{a^2d}-\frac{B(a-a\sin(c+dx))^2}{2a^4d}$$

```
[Out] 2*(A-B)*ln(1+sin(d*x+c))/a^2/d-(A-B)*sin(d*x+c)/a^2/d-1/2*B*(a-a*sin(d*x+c)
)^2/a^4/d
```

Rubi [A]
time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a-a\sin(c+dx))^2}{2a^4d}-\frac{(A-B)\sin(c+dx)}{a^2d}+\frac{2(A-B)\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*(A - B)*Log[1 + Sin[c + d*x]])/(a^2*d) - ((A - B)*Sin[c + d*x])/(a^2*d)
- (B*(a - a*Sin[c + d*x])^2)/(2*a^4*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)\left(A+\frac{Bx}{a}\right)}{a+x}\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-A+B+\frac{B(a-x)}{a}+\frac{2a(A-B)}{a+x}\right)\,dx, x, a\sin(c+dx)\right)}{a^3d}$$
$$= \frac{2(A-B)\log(1+\sin(c+dx))}{a^2d} - \frac{(A-B)\sin(c+dx)}{a^2d} - \frac{B(a-a\,\text{s}}{2}$$

Mathematica [A]
time = 0.07, size = 51, normalized size = 0.77

$$-\frac{B-4(A-B)\log(1+\sin(c+dx))+2(A-2B)\sin(c+dx)+B\sin^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(B - 4*(A - B)*Log[1 + Sin[c + d*x]] + 2*(A - 2*B)*Sin[c + d*x] + B*Si
n[c + d*x]^2)/(a^2*d)
```

Maple [A]
time = 0.29, size = 55, normalized size = 0.83

method	result
derivativedivides	$\frac{-\frac{B\left(\sin^2(dx+c)\right)}{2}-A\sin(dx+c)+2B\sin(dx+c)+(2A-2B)\ln(1+\sin(dx+c))}{d\,a^2}$
default	$\frac{-\frac{B\left(\sin^2(dx+c)\right)}{2}-A\sin(dx+c)+2B\sin(dx+c)+(2A-2B)\ln(1+\sin(dx+c))}{d\,a^2}$
risch	$-\frac{2ixA}{a^2}+\frac{2ixB}{a^2}+\frac{ie^{i(dx+c)}A}{2d\,a^2}-\frac{ie^{i(dx+c)}B}{d\,a^2}-\frac{ie^{-i(dx+c)}A}{2d\,a^2}+\frac{ie^{-i(dx+c)}B}{d\,a^2}-\frac{4iAc}{d\,a^2}+\frac{4iBc}{d\,a^2}+\frac{4\ln\left(e^{i(dx+c)}+\right.}{d\,a^2}$
norman	$\frac{-\frac{(12A-18B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(12A-18B)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2(12A-17B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2(12A-17B)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2(10A}{}}{\left(1+\text{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d/a^2*(-1/2*B*sin(d*x+c)^2-A*sin(d*x+c)+2*B*sin(d*x+c)+(2*A-2*B)*ln(1+sin
(d*x+c)))
```

Maxima [A]
time = 0.30, size = 54, normalized size = 0.82

$$\frac{\frac{4\,(A-B)\log(\sin(dx+c)+1)}{a^2}-\frac{B\sin(dx+c)^2+2\,(A-2\,B)\sin(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/2*(4*(A - B)*log(sin(d*x + c) + 1)/a^2 - (B*sin(d*x + c)^2 + 2*(A - 2*B)*
sin(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.36, size = 48, normalized size = 0.73

$$\frac{B\cos\left(dx+c\right)^2+4\,(A-B)\log\left(\sin\left(dx+c\right)+1\right)-2\,(A-2\,B)\sin\left(dx+c\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*cos(d*x + c)^2 + 4*(A - B)*log(sin(d*x + c) + 1) - 2*(A - 2*B)*sin(d
*x + c))/(a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(56) = 112$.
time = 8.95, size = 1096, normalized size = 16.61

$$\begin{cases} \frac{4A\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{8A\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{4A\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{2A\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{4A\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{2A\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{2A\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{2A\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{4B\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{8B\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{4B\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2B\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{4B\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{2B\log\left(\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{4B\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} - \frac{2B\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} + \frac{4B\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+2a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2d} & \text{for}\: d \neq 0 \\ \frac{x\left(A+B\sin\left(c\right)\right)\cos^3\left(c\right)}{\left(a\sin\left(c\right)+a\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((4*A*log(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/
2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 8*A*log(tan(c/2 +
d*x/2) + 1)*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(
c/2 + d*x/2)**2 + a**2*d) + 4*A*log(tan(c/2 + d*x/2) + 1)/(a**2*d*tan(c/2 +
 d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 2*A*log(tan(c/2 + d*x
/2)**2 + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(
c/2 + d*x/2)**2 + a**2*d) - 4*A*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/
2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d)
- 2*A*log(tan(c/2 + d*x/2)**2 + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*t
an(c/2 + d*x/2)**2 + a**2*d) - 2*A*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*
x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 2*A*tan(c/2 + d*x/2)/(a*
```

```
*2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 4*B*log
(tan(c/2 + d*x/2) + 1)*tan(c/2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*
a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) - 8*B*log(tan(c/2 + d*x/2) + 1)*tan(c/
2 + d*x/2)**2/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 +
a**2*d) - 4*B*log(tan(c/2 + d*x/2) + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**
2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 2*B*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/
2 + d*x/2)**4/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 +
a**2*d) + 4*B*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(a**2*d*tan(
c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 2*B*log(tan(c/2
+ d*x/2)**2 + 1)/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2
 + a**2*d) + 4*B*tan(c/2 + d*x/2)**3/(a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d
*tan(c/2 + d*x/2)**2 + a**2*d) - 2*B*tan(c/2 + d*x/2)**2/(a**2*d*tan(c/2 +
d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d) + 4*B*tan(c/2 + d*x/2)/(
a**2*d*tan(c/2 + d*x/2)**4 + 2*a**2*d*tan(c/2 + d*x/2)**2 + a**2*d), Ne(d,
0)), (x*(A + B*sin(c))*cos(c)**3/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.52, size = 92, normalized size = 1.39

$$-\frac{\frac{4\,(A-B)\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a^2}+\frac{(a\sin(dx+c)+a)^2\left(B+\frac{2\,\left(Aa^2-3\,Ba^2\right)}{(a\sin(dx+c)+a)a}\right)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(A - B)*log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a))
)/a^2 + (a*sin(d*x + c) + a)^2*(B + 2*(A*a^2 - 3*B*a^2)/((a*sin(d*x + c) +
a)*a))/a^4)/d
```

Mupad [B]
time = 0.07, size = 61, normalized size = 0.92

$$-\frac{2\,A\,\sin\left(c+d\,x\right)-4\,B\,\sin\left(c+d\,x\right)+B\,{\sin\left(c+d\,x\right)}^2-4\,A\,\ln\left(\sin\left(c+d\,x\right)+1\right)+4\,B\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] -(2*A*sin(c + d*x) - 4*B*sin(c + d*x) + B*sin(c + d*x)^2 - 4*A*log(sin(c +
d*x) + 1) + 4*B*log(sin(c + d*x) + 1))/(2*a^2*d)
```

3.1014 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=44

$$\frac{B\log(1+\sin(c+dx))}{a^2d}-\frac{A-B}{d\,(a^2+a^2\sin(c+dx))}$$

[Out] B*ln(1+sin(d*x+c))/a^2/d+(-A+B)/d/(a^2+a^2*sin(d*x+c))

Rubi [A]

time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2912, 45}

$$\frac{B\log(\sin(c+dx)+1)}{a^2d}-\frac{A-B}{d\,(a^2\sin(c+dx)+a^2)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]

[Out] (B*Log[1 + Sin[c + d*x]])/(a^2*d) - (A - B)/(d*(a^2 + a^2*Sin[c + d*x]))

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a+x)^2}\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{\text{Subst}\left(\int \left(\frac{A-B}{(a+x)^2}+\frac{B}{a(a+x)}\right)\,dx,x,a\sin(c+dx)\right)}{ad}\\ &= \frac{B\log(1+\sin(c+dx))}{a^2d}-\frac{A-B}{d\,(a^2+a^2\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 41, normalized size = 0.93

$$\frac{\frac{B\log(1+\sin(c+dx))}{a}-\frac{A-B}{a+a\sin(c+dx)}}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]

[Out] ((B*Log[1 + Sin[c + d*x]])/a - (A - B)/(a + a*Sin[c + d*x]))/(a*d)

Maple [A]
time = 0.20, size = 37, normalized size = 0.84

method	result
derivativedivides	$\frac{B\ln(1+\sin(dx+c))-\frac{A-B}{1+\sin(dx+c)}}{d\,a^2}$
default	$\frac{B\ln(1+\sin(dx+c))-\frac{A-B}{1+\sin(dx+c)}}{d\,a^2}$
risch	$-\frac{ixB}{a^2}-\frac{2iBc}{d\,a^2}-\frac{2i\mathrm{e}^{i(dx+c)}(A-B)}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)B}{d\,a^2}$
norman	$\frac{\frac{(2A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{(2A-2B)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(2A-2B)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(2A-2B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(4A-4B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d/a^2*(B*ln(1+sin(d*x+c))-(A-B)/(1+sin(d*x+c)))

Maxima [A]
time = 0.28, size = 43, normalized size = 0.98

$$-\frac{\frac{A-B}{a^2\sin(dx+c)+a^2}-\frac{B\log(\sin(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -((A - B)/(a^2*sin(d*x + c) + a^2) - B*log(sin(d*x + c) + 1)/a^2)/d

Fricas [A]
time = 0.37, size = 45, normalized size = 1.02

$$\frac{(B\sin(dx+c)+B)\log(\sin(dx+c)+1)-A+B}{a^2d\sin(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] ((B*sin(d*x + c) + B)*log(sin(d*x + c) + 1) - A + B)/(a^2*d*sin(d*x + c) +
a^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(34) = 68$.
time = 0.44, size = 121, normalized size = 2.75

$$\begin{cases} -\frac{A}{a^2d\sin(c+dx)+a^2d} + \frac{B\log(\sin(c+dx)+1)\sin(c+dx)}{a^2d\sin(c+dx)+a^2d} + \frac{B\log(\sin(c+dx)+1)}{a^2d\sin(c+dx)+a^2d} + \frac{B}{a^2d\sin(c+dx)+a^2d} & \text{for } d \neq 0 \\ \frac{x(A+B\sin(c))\cos(c)}{(a\sin(c)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-A/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)*sin(
c + d*x)/(a**2*d*sin(c + d*x) + a**2*d) + B*log(sin(c + d*x) + 1)/(a**2*d*s
in(c + d*x) + a**2*d) + B/(a**2*d*sin(c + d*x) + a**2*d), Ne(d, 0)), (x*(A
+ B*sin(c))*cos(c)/(a*sin(c) + a)**2, True))
```

Giac [A]
time = 0.46, size = 76, normalized size = 1.73

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|a\sin(dx+c)+a|}{(a\sin(dx+c)+a)^2|a|}\right)}{a} - \frac{1}{a\sin(dx+c)+a}\right)}{a} + \frac{A}{(a\sin(dx+c)+a)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(a*sin(d*x + c) + a)/((a*sin(d*x + c) + a)^2*abs(a)))/a - 1/(a*
sin(d*x + c) + a))/a + A/((a*sin(d*x + c) + a)*a))/d
```

Mupad [B]
time = 0.06, size = 41, normalized size = 0.93

$$\frac{B\,\ln\left(\sin\left(c+d\,x\right)+1\right)}{a^2\,d} - \frac{A-B}{a^2\,d\,\left(\sin\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + a*sin(c + d*x))^2,x)
```

```
[Out] (B*log(sin(c + d*x) + 1))/(a^2*d) - (A - B)/(a^2*d*(sin(c + d*x) + 1))
```

3.1015 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=71

$$\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a+a\sin(c+dx))^2}-\frac{A+B}{4d\,(a^2+a^2\sin(c+dx))}$$

```
[Out] 1/4*(A+B)*arctanh(sin(d*x+c))/a^2/d+1/4*(-A+B)/d/(a+a*sin(d*x+c))^2+1/4*(-A
-B)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 78, 212}

$$-\frac{A+B}{4d\,(a^2\sin(c+dx)+a^2)}+\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d}-\frac{A-B}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^2*d) - (A - B)/(4*d*(a + a*Sin[c + d*x
])^2) - (A + B)/(4*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a\text{Subst}\left(\int \left(\frac{A-B}{2a(a+x)^3}+\frac{A+B}{4a^2(a+x)^2}+\frac{A+B}{4a^2(a^2-x^2)}\right)\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{A-B}{4d(a+a\sin(c+dx))^2} - \frac{A+B}{4d\,(a^2+a^2\sin(c+dx))} + \frac{(A+B)\text{Sub}}{}$$

$$= \frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^2d} - \frac{A-B}{4d(a+a\sin(c+dx))^2} - \frac{}{4d\,(a^2+}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 0.97

$$\frac{a\left(\frac{(A+B)\tanh^{-1}(\sin(c+dx))}{4a^3}-\frac{A-B}{4a(a+a\sin(c+dx))^2}-\frac{A+B}{4a^2(a+a\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a*(((A + B)*ArcTanh[Sin[c + d*x]])/(4*a^3) - (A - B)/(4*a*(a + a*Sin[c + d
*x])^2) - (A + B)/(4*a^2*(a + a*Sin[c + d*x]))))/d
```

Maple [A]
time = 0.30, size = 81, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{\frac{A}{2}-\frac{B}{2}}{2(1+\sin(dx+c))^2}-\frac{\frac{B}{4}+\frac{A}{4}}{1+\sin(dx+c)}+\left(\frac{A}{8}+\frac{B}{8}\right)\ln(1+\sin(dx+c))+\left(-\frac{A}{8}-\frac{B}{8}\right)\ln(\sin(dx+c)-1)}{d\,a^2}$
default	$\frac{-\frac{\frac{A}{2}-\frac{B}{2}}{2(1+\sin(dx+c))^2}-\frac{\frac{B}{4}+\frac{A}{4}}{1+\sin(dx+c)}+\left(\frac{A}{8}+\frac{B}{8}\right)\ln(1+\sin(dx+c))+\left(-\frac{A}{8}-\frac{B}{8}\right)\ln(\sin(dx+c)-1)}{d\,a^2}$
risch	$-\frac{i\left(4iA\,e^{2i(dx+c)}+A\,e^{3i(dx+c)}+B\,e^{3i(dx+c)}-A\,e^{i(dx+c)}-B\,e^{i(dx+c)}\right)}{2d\,a^2\left(e^{i(dx+c)}+i\right)^4}+\frac{\ln\left(e^{i(dx+c)}+i\right)A}{4d\,a^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{4d\,a^2}-\frac{\ln}{}$
norman	$\frac{\frac{2A\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2A\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(3A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(3A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4 a}-\frac{(A+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/2*(1/2*A-1/2*B)/(1+sin(d*x+c))^2-(1/4*B+1/4*A)/(1+sin(d*x+c))+(
1/8*A+1/8*B)*ln(1+sin(d*x+c))+(-1/8*A-1/8*B)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 84, normalized size = 1.18

$$-\frac{\frac{2\,((A+B)\sin(dx+c)+2\,A)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{(A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{(A+B)\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] -1/8*(2*((A + B)*sin(d*x + c) + 2*A)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x +
c) + a^2) - (A + B)*log(sin(d*x + c) + 1)/a^2 + (A + B)*log(sin(d*x + c) -
1)/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(65) = 130.
time = 0.38, size = 134, normalized size = 1.89

$$\frac{((A+B)\cos(dx+c)^2-2\,(A+B)\sin(dx+c)-2\,A-2\,B)\log(\sin(dx+c)+1)-((A+B)\cos(dx+c)^2-2\,(A+B)\sin(dx+c)-2\,A-2\,B)\log(-\sin(dx+c)+1)+2\,(A+B)\sin(dx+c)+4\,A}{8\,(a^2d\cos(dx+c)^2-2\,a^2d\sin(dx+c)-2\,a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/8*(((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*B)*log(sin(
d*x + c) + 1) - ((A + B)*cos(d*x + c)^2 - 2*(A + B)*sin(d*x + c) - 2*A - 2*
B)*log(-sin(d*x + c) + 1) + 2*(A + B)*sin(d*x + c) + 4*A)/(a^2*d*cos(d*x +
c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx+\int \frac{B\sin(c+dx)\sec(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x) + Integ
ral(B*sin(c + d*x)*sec(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x))
/a**2
```

Giac [A]
time = 0.44, size = 104, normalized size = 1.46

$$\frac{\frac{2\,(A+B)\log(|\sin(dx+c)+1|)}{a^2} - \frac{2\,(A+B)\log(|\sin(dx+c)-1|)}{a^2} - \frac{3\,A\sin(dx+c)^2+3\,B\sin(dx+c)^2+10\,A\sin(dx+c)+10\,B\sin(dx+c)+11\,A+3\,B}{a^2(\sin(dx+c)+1)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/16*(2*(A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 2*(A + B)*log(abs(sin(d*x
+ c) - 1))/a^2 - (3*A*sin(d*x + c)^2 + 3*B*sin(d*x + c)^2 + 10*A*sin(d*x +
c) + 10*B*sin(d*x + c) + 11*A + 3*B)/(a^2*(sin(d*x + c) + 1)^2))/d
```

Mupad [B]
time = 9.25, size = 71, normalized size = 1.00

$$\frac{\mathrm{atanh}(\sin\left(c+d\,x\right))\,\left(A+B\right)}{4\,a^2\,d}-\frac{\frac{A}{2}+\sin\left(c+d\,x\right)\,\left(\frac{A}{4}+\frac{B}{4}\right)}{d\,\left(a^2\sin\left(c+d\,x\right)^2+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (atanh(sin(c + d*x))*(A + B))/(4*a^2*d) - (A/2 + sin(c + d*x)*(A/4 + B/4))/
(d*(2*a^2*sin(c + d*x) + a^2 + a^2*sin(c + d*x)^2))
```

3.1016 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=123

$$\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a+a\sin(c+dx))^3}-\frac{A}{8d(a+a\sin(c+dx))^2}+\frac{A+B}{16d\,(a^2-a^2\sin(c+dx))}-$$

```
[Out] 1/8*(2*A+B)*arctanh(sin(d*x+c))/a^2/d-1/12*a*(A-B)/d/(a+a*sin(d*x+c))^3-1/8
*A/d/(a+a*sin(d*x+c))^2+1/16*(A+B)/d/(a^2-a^2*sin(d*x+c))+1/16*(-3*A-B)/d/(
a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$\frac{A+B}{16d\,(a^2-a^2\sin(c+dx))}-\frac{3A+B}{16d\,(a^2\sin(c+dx)+a^2)}+\frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{a(A-B)}{12d(a\sin(c+dx)+a)^3}-\frac{A}{8d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ((2*A + B)*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (a*(A - B))/(12*d*(a + a*Sin[
c + d*x])^3) - A/(8*d*(a + a*Sin[c + d*x])^2) + (A + B)/(16*d*(a^2 - a^2*Si
n[c + d*x])) - (3*A + B)/(16*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^3\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^3\text{Subst}\left(\int \left(\frac{A+B}{16a^4(a-x)^2} + \frac{A-B}{4a^2(a+x)^4} + \frac{A}{4a^3(a+x)^3} + \frac{3A+B}{16a^4(a+x)^2} + \frac{2A+}{8a^4(a^2}\right.\right.}{d}$$

$$= -\frac{a(A-B)}{12d(a+a\sin(c+dx))^3} - \frac{A}{8d(a+a\sin(c+dx))^2} + \frac{A}{16d\,(a^2-a}$$

$$= \frac{(2A+B)\tanh^{-1}(\sin(c+dx))}{8a^2d} - \frac{a(A-B)}{12d(a+a\sin(c+dx))^3} - \frac{}{8d(a}$$

Mathematica [A]
time = 0.53, size = 87, normalized size = 0.71

$$-\frac{-6(2A+B)\tanh^{-1}(\sin(c+dx)) + \frac{3(A+B)}{-1+\sin(c+dx)} + \frac{4(A-B)}{(1+\sin(c+dx))^3} + \frac{6A}{(1+\sin(c+dx))^2} + \frac{3(3A+B)}{1+\sin(c+dx)}}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -1/48*(-6*(2*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])
 + (4*(A - B))/(1 + Sin[c + d*x])^3 + (6*A)/(1 + Sin[c + d*x])^2 + (3*(3*A
+ B))/(1 + Sin[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.41, size = 113, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{A}{8(1+\sin(dx+c))^2} - \frac{\frac{A}{4}-\frac{B}{4}}{3(1+\sin(dx+c))^3} + \left(\frac{A}{8}+\frac{B}{16}\right)\ln(1+\sin(dx+c)) - \frac{\frac{3A}{16}+\frac{B}{16}}{1+\sin(dx+c)} + \left(-\frac{A}{8}-\frac{B}{16}\right)\ln(\sin(dx+c)-1) - \frac{\frac{A}{16}+\frac{B}{16}}{\sin(dx+c}}{d\,a^2}$
default	$\frac{-\frac{A}{8(1+\sin(dx+c))^2} - \frac{\frac{A}{4}-\frac{B}{4}}{3(1+\sin(dx+c))^3} + \left(\frac{A}{8}+\frac{B}{16}\right)\ln(1+\sin(dx+c)) - \frac{\frac{3A}{16}+\frac{B}{16}}{1+\sin(dx+c)} + \left(-\frac{A}{8}-\frac{B}{16}\right)\ln(\sin(dx+c)-1) - \frac{\frac{A}{16}+\frac{B}{16}}{\sin(dx+c}}{d\,a^2}$
risch	$-\frac{i\left(24iA\,e^{6i(dx+c)}+6A\,e^{7i(dx+c)}+12iB\,e^{6i(dx+c)}+3B\,e^{7i(dx+c)}+16iA\,e^{4i(dx+c)}-26A\,e^{5i(dx+c)}-40iB\,e^{4i(dx+c)}-13B\right.}{12\left(e^{i(dx+c)}+i\right)^6\left(e^{i(dx+c}\right.}$
norman	$\frac{\frac{(2A+B)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{(2A+B)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{4(2A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} + \frac{4(2A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad} + \frac{(14A+19B)\left(\tan^5\left(\frac{d}{}\right.\right.}{6ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^6 a\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d/a^2*(-1/8*A/(1+sin(d*x+c))^2-1/3*(1/4*A-1/4*B)/(1+sin(d*x+c))^3+(1/8*A+
1/16*B)*ln(1+sin(d*x+c))-(3/16*A+1/16*B)/(1+sin(d*x+c))+(-1/8*A-1/16*B)*ln(
sin(d*x+c)-1)-(1/16*A+1/16*B)/(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 139, normalized size = 1.13

$$-\frac{\frac{2\left(3\,(2\,A+B)\sin(dx+c)^3+6\,(2\,A+B)\sin(dx+c)^2+(2\,A+B)\sin(dx+c)-8\,A+2\,B\right)}{a^2\sin(dx+c)^4+2\,a^2\sin(dx+c)^3-2\,a^2\sin(dx+c)-a^2}-\frac{3\,(2\,A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{3\,(2\,A+B)\log(\sin(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/48*(2*(3*(2*A + B)*sin(d*x + c)^3 + 6*(2*A + B)*sin(d*x + c)^2 + (2*A +
B)*sin(d*x + c) - 8*A + 2*B)/(a^2*sin(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2
*a^2*sin(d*x + c) - a^2) - 3*(2*A + B)*log(sin(d*x + c) + 1)/a^2 + 3*(2*A +
 B)*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(114) = 228$.
time = 0.36, size = 230, normalized size = 1.87

$$\frac{12\,(2\,A+B)\cos(dx+c)^2+3\left((2\,A+B)\cos(dx+c)^4-2\,(2\,A+B)\cos(dx+c)^2\sin(dx+c)-2\,(2\,A+B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((2\,A+B)\cos(dx+c)^4-2\,(2\,A+B)\cos(dx+c)^2\sin(dx+c)-2\,(2\,A+B)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)+2\left(3\,(2\,A+B)\cos(dx+c)^2-8\,A-4\,B\right)\sin(dx+c)-8\,A-16\,B}{48\left(a^2d\cos(dx+c)^4-2\,a^2d\cos(dx+c)^2\sin(dx+c)-2\,a^2d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/48*(12*(2*A + B)*cos(d*x + c)^2 + 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A +
B)*cos(d*x + c)^2*sin(d*x + c) - 2*(2*A + B)*cos(d*x + c)^2)*log(sin(d*x +
c) + 1) - 3*((2*A + B)*cos(d*x + c)^4 - 2*(2*A + B)*cos(d*x + c)^2*sin(d*x
+ c) - 2*(2*A + B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(3*(2*A + B)*
cos(d*x + c)^2 - 8*A - 4*B)*sin(d*x + c) - 8*A - 16*B)/(a^2*d*cos(d*x + c)^
4 - 2*a^2*d*cos(d*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx+\int \frac{B\sin(c+dx)\sec^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x) + In
tegral(B*sin(c + d*x)*sec(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1
), x))/a**2
```

Giac [A]
time = 0.53, size = 169, normalized size = 1.37

$$\frac{\frac{6\,(2\,A+B)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^2}-\frac{6\,(2\,A+B)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^2}+\frac{6\,(2\,A\sin\left(dx+c\right)+B\sin\left(dx+c\right)-3\,A-2\,B)}{a^2(\sin\left(dx+c\right)-1)}-\frac{22\,A\sin\left(dx+c\right)^3+11\,B\sin\left(dx+c\right)^3+84\,A\sin\left(dx+c\right)^2+39\,B\sin\left(dx+c\right)^2+114\,A\sin\left(dx+c\right)+45\,B\sin\left(dx+c\right)+60\,A+9\,B}{a^2(\sin\left(dx+c\right)+1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/96*(6*(2*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 6*(2*A + B)*log(abs(sin(
d*x + c) - 1))/a^2 + 6*(2*A*sin(d*x + c) + B*sin(d*x + c) - 3*A - 2*B)/(a^2
*(sin(d*x + c) - 1)) - (22*A*sin(d*x + c)^3 + 11*B*sin(d*x + c)^3 + 84*A*si
n(d*x + c)^2 + 39*B*sin(d*x + c)^2 + 114*A*sin(d*x + c) + 45*B*sin(d*x + c)
 + 60*A + 9*B)/(a^2*(sin(d*x + c) + 1)^3))/d
```

Mupad [B]
time = 0.15, size = 121, normalized size = 0.98

$$\frac{\left(\frac{A}{4}+\frac{B}{8}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{A}{2}+\frac{B}{4}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{A}{12}+\frac{B}{24}\right)\,\sin\left(c+d\,x\right)-\frac{A}{3}+\frac{B}{12}}{d\,\left(-a^2\sin\left(c+d\,x\right)^4-2\,a^2\sin\left(c+d\,x\right)^3+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}+\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(2\,A+B\right)}{8\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (B/12 - A/3 + sin(c + d*x)*(A/12 + B/24) + sin(c + d*x)^2*(A/2 + B/4) + sin
(c + d*x)^3*(A/4 + B/8))/(d*(2*a^2*sin(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^
3 - a^2*sin(c + d*x)^4)) + (atanh(sin(c + d*x))*(2*A + B))/(8*a^2*d)
```

3.1017 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{A+B}{64d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{32d(a+a\sin(c+dx))^4}-\frac{a(3A-B)}{48d(a+a\sin(c+dx))^3}$$

```
[Out] 5/64*(3*A+B)*arctanh(sin(d*x+c))/a^2/d+1/64*(A+B)/d/(a-a*sin(d*x+c))^2-1/32
*a^2*(A-B)/d/(a+a*sin(d*x+c))^4-1/48*a*(3*A-B)/d/(a+a*sin(d*x+c))^3-3/32*A/
d/(a+a*sin(d*x+c))^2+1/64*(5*A+3*B)/d/(a^2-a^2*sin(d*x+c))+1/32*(-5*A-B)/d/
(a^2+a^2*sin(d*x+c))
```

Rubi [A]

time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$-\frac{a^2(A-B)}{32d(a\sin(c+dx)+a)^4}+\frac{5A+3B}{64d\,(a^2-a^2\sin(c+dx))}-\frac{5A+B}{32d\,(a^2\sin(c+dx)+a^2)}+\frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{a(3A-B)}{48d(a\sin(c+dx)+a)^3}+\frac{A+B}{64d(a-a\sin(c+dx))^2}-\frac{3A}{32d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*(3*A + B)*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + (A + B)/(64*d*(a - a*Sin[c
 + d*x])^2) - (a^2*(A - B))/(32*d*(a + a*Sin[c + d*x])^4) - (a*(3*A - B))/(
48*d*(a + a*Sin[c + d*x])^3) - (3*A)/(32*d*(a + a*Sin[c + d*x])^2) + (5*A +
 3*B)/(64*d*(a^2 - a^2*Sin[c + d*x])) - (5*A + B)/(32*d*(a^2 + a^2*Sin[c +
d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^5\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^5\text{Subst}\left(\int \left(\frac{A+B}{32a^5(a-x)^3}+\frac{5A+3B}{64a^6(a-x)^2}+\frac{A-B}{8a^3(a+x)^5}+\frac{3A-B}{16a^4(a+x)^4}+\frac{3}{16a^5(}\right.\right.}{d}$$

$$= \frac{A+B}{64d(a-a\sin(c+dx))^2}-\frac{a^2(A-B)}{32d(a+a\sin(c+dx))^4}-\frac{a(3A}{48d(a+a\sin}$$

$$= \frac{5(3A+B)\tanh^{-1}(\sin(c+dx))}{64a^2d}+\frac{A+B}{64d(a-a\sin(c+dx))^2}-\frac{}{32d}$$

Mathematica [A]

time = 0.50, size = 123, normalized size = 0.69

$$\frac{15(3A+B)\tanh^{-1}(\sin(c+dx))+\frac{3(A+B)}{(-1+\sin(c+dx))^2}-\frac{3(5A+3B)}{-1+\sin(c+dx)}-\frac{6(A-B)}{(1+\sin(c+dx))^4}+\frac{4(-3A+B)}{(1+\sin(c+dx))^3}-\frac{18A}{(1+\sin(c+dx))^2}-\frac{6(5A+B)}{1+\sin(c+dx)}}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*(3*A + B)*ArcTanh[Sin[c + d*x]] + (3*(A + B))/(-1 + Sin[c + d*x])^2 - (
3*(5*A + 3*B))/(-1 + Sin[c + d*x]) - (6*(A - B))/(1 + Sin[c + d*x])^4 + (4*
(-3*A + B))/(1 + Sin[c + d*x])^3 - (18*A)/(1 + Sin[c + d*x])^2 - (6*(5*A +
B))/(1 + Sin[c + d*x]))/(192*a^2*d)
```

Maple [A]

time = 0.53, size = 151, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{3A}{32(1+\sin(dx+c))^2}-\frac{\frac{A}{8}-\frac{B}{8}}{4(1+\sin(dx+c))^4}-\frac{\frac{3A}{16}-\frac{B}{16}}{3(1+\sin(dx+c))^3}-\frac{\frac{5A}{32}+\frac{B}{32}}{1+\sin(dx+c)}+\left(\frac{15A}{128}+\frac{5B}{128}\right)\ln(1+\sin(dx+c))+\left(-\frac{15A}{128}-\frac{5B}{128}\right)\ln(}{d\,a^2}$
default	$\frac{-\frac{3A}{32(1+\sin(dx+c))^2}-\frac{\frac{A}{8}-\frac{B}{8}}{4(1+\sin(dx+c))^4}-\frac{\frac{3A}{16}-\frac{B}{16}}{3(1+\sin(dx+c))^3}-\frac{\frac{5A}{32}+\frac{B}{32}}{1+\sin(dx+c)}+\left(\frac{15A}{128}+\frac{5B}{128}\right)\ln(1+\sin(dx+c))+\left(-\frac{15A}{128}-\frac{5B}{128}\right)\ln(}{d\,a^2}$
risch	$-\frac{i\left(480iA\,e^{8i(dx+c)}+45A\,e^{11i(dx+c)}-952iB\,e^{6i(dx+c)}+15B\,e^{11i(dx+c)}+180iA\,e^{10i(dx+c)}-105A\,e^{9i(dx+c)}+160iB\,e^{4i(}\right.}{}$

norman	$\frac{(117A+103B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{(117A+103B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{48ad}+\frac{(143A+69B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{(143A+69B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32ad}+\frac{(17$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d/a^2*(-3/32*A/(1+sin(d*x+c))^2-1/4*(1/8*A-1/8*B)/(1+sin(d*x+c))^4-1/3*(3
/16*A-1/16*B)/(1+sin(d*x+c))^3-(5/32*A+1/32*B)/(1+sin(d*x+c))+(15/128*A+5/1
28*B)*ln(1+sin(d*x+c))+(-15/128*A-5/128*B)*ln(sin(d*x+c)-1)-1/2*(-1/32*A-1/
32*B)/(sin(d*x+c)-1)^2-(5/64*A+3/64*B)/(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 207, normalized size = 1.16

$$-\frac{\frac{2\left(15\,(3\,A+B)\sin(dx+c)^5+30\,(3\,A+B)\sin(dx+c)^4-10\,(3\,A+B)\sin(dx+c)^3-50\,(3\,A+B)\sin(dx+c)^2-17\,(3\,A+B)\sin(dx+c)+48\,A-16\,B\right)}{a^2\sin(dx+c)^6+2\,a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4\,a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{15\,(3\,A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{15\,(3\,A+B)\log(\sin(dx+c)-1)}{a^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/384*(2*(15*(3*A + B)*sin(d*x + c)^5 + 30*(3*A + B)*sin(d*x + c)^4 - 10*(
3*A + B)*sin(d*x + c)^3 - 50*(3*A + B)*sin(d*x + c)^2 - 17*(3*A + B)*sin(d*
x + c) + 48*A - 16*B)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x + c)^5 - a^2*sin(
d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c)
 + a^2) - 15*(3*A + B)*log(sin(d*x + c) + 1)/a^2 + 15*(3*A + B)*log(sin(d*x
 + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.39, size = 260, normalized size = 1.45

$$\frac{60\,(3\,A+B)\cos\left(dx+c\right)^4-20\,(3\,A+B)\cos\left(dx+c\right)^2+15\,\left((3\,A+B)\cos\left(dx+c\right)^6-2\,(3\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)-2\,(3\,A+B)\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)-15\,\left((3\,A+B)\cos\left(dx+c\right)^6-2\,(3\,A+B)\cos\left(dx+c\right)^4\sin\left(dx+c\right)-2\,(3\,A+B)\cos\left(dx+c\right)^4\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(15\,(3\,A+B)\cos\left(dx+c\right)^4-20\,(3\,A+B)\cos\left(dx+c\right)^2-36\,A-12\,B\right)\sin\left(dx+c\right)-24\,A-72\,B}{384\,\left(a^2d\cos\left(dx+c\right)^6-2\,a^2d\cos\left(dx+c\right)^4\sin\left(dx+c\right)-2\,a^2d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/384*(60*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c)^2 + 15*((3*A
 + B)*cos(d*x + c)^6 - 2*(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*((3*A + B)*cos(d*x + c)^6 - 2*
(3*A + B)*cos(d*x + c)^4*sin(d*x + c) - 2*(3*A + B)*cos(d*x + c)^4)*log(-si
n(d*x + c) + 1) + 2*(15*(3*A + B)*cos(d*x + c)^4 - 20*(3*A + B)*cos(d*x + c
)^2 - 36*A - 12*B)*sin(d*x + c) - 24*A - 72*B)/(a^2*d*cos(d*x + c)^6 - 2*a^
2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 214, normalized size = 1.20

$$\frac{\frac{60\,(3\,A+B)\log\left(|\sin(dx+c)+1|\right)}{a^2} - \frac{60\,(3\,A+B)\log\left(|\sin(dx+c)-1|\right)}{a^2} + \frac{6\,\left(45\,A\sin(dx+c)^2+15\,B\sin(dx+c)^2-110\,A\sin(dx+c)-42\,B\sin(dx+c)+69\,A+31\,B\right)}{a^2(\sin(dx+c)-1)^2} - \frac{375\,A\sin(dx+c)^4+125\,B\sin(dx+c)^4+1740\,A\sin(dx+c)^3+548\,B\sin(dx+c)^3+3114\,A\sin(dx+c)^2+894\,B\sin(dx+c)^2+2604\,A\sin(dx+c)+612\,B\sin(dx+c)+903\,A+93\,B}{a^2(\sin(dx+c)+1)^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/1536*(60*(3*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 60*(3*A + B)*log(abs(
sin(d*x + c) - 1))/a^2 + 6*(45*A*sin(d*x + c)^2 + 15*B*sin(d*x + c)^2 - 110
*A*sin(d*x + c) - 42*B*sin(d*x + c) + 69*A + 31*B)/(a^2*(sin(d*x + c) - 1)^
2) - (375*A*sin(d*x + c)^4 + 125*B*sin(d*x + c)^4 + 1740*A*sin(d*x + c)^3 +
 548*B*sin(d*x + c)^3 + 3114*A*sin(d*x + c)^2 + 894*B*sin(d*x + c)^2 + 2604
*A*sin(d*x + c) + 612*B*sin(d*x + c) + 903*A + 93*B)/(a^2*(sin(d*x + c) + 1
)^4))/d
```

Mupad [**B**]
time = 9.50, size = 193, normalized size = 1.08

$$\frac{\left(-\frac{15\,A}{64}-\frac{5\,B}{64}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{15\,A}{32}-\frac{5\,B}{32}\right)\,\sin\left(c+d\,x\right)^4+\left(\frac{5\,A}{32}+\frac{5\,B}{96}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{25\,A}{32}+\frac{25\,B}{96}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{17\,A}{64}+\frac{17\,B}{192}\right)\,\sin\left(c+d\,x\right)-\frac{A}{4}+\frac{B}{12}}{d\,\left(a^2\,\sin\left(c+d\,x\right)^6+2\,a^2\,\sin\left(c+d\,x\right)^5-a^2\,\sin\left(c+d\,x\right)^4-4\,a^2\,\sin\left(c+d\,x\right)^3-a^2\,\sin\left(c+d\,x\right)^2+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}+\frac{5\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(3\,A+B\right)}{64\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + a*sin(c + d*x))^2),x)
```

```
[Out] (B/12 - A/4 + sin(c + d*x)*((17*A)/64 + (17*B)/192) - sin(c + d*x)^4*((15*A
)/32 + (5*B)/32) + sin(c + d*x)^3*((5*A)/32 + (5*B)/96) - sin(c + d*x)^5*((
15*A)/64 + (5*B)/64) + sin(c + d*x)^2*((25*A)/32 + (25*B)/96))/(d*(2*a^2*si
n(c + d*x) + a^2 - a^2*sin(c + d*x)^2 - 4*a^2*sin(c + d*x)^3 - a^2*sin(c +
d*x)^4 + 2*a^2*sin(c + d*x)^5 + a^2*sin(c + d*x)^6)) + (5*atanh(sin(c + d*x
))*(3*A + B))/(64*a^2*d)
```

3.1018 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=236

$$\frac{7(4A+B)\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A+2B}{128d(a-a\sin(c+dx))^2}-\frac{a^3(A-B)}{80d(a+a\sin(c+dx)}$$

```
[Out] 7/128*(4*A+B)*arctanh(sin(d*x+c))/a^2/d+1/192*a*(A+B)/d/(a-a*sin(d*x+c))^3+
1/128*(3*A+2*B)/d/(a-a*sin(d*x+c))^2-1/80*a^3*(A-B)/d/(a+a*sin(d*x+c))^5-1/
64*a^2*(2*A-B)/d/(a+a*sin(d*x+c))^4-1/96*a*(5*A-B)/d/(a+a*sin(d*x+c))^3-5/6
4*A/d/(a+a*sin(d*x+c))^2+3/256*(7*A+3*B)/d/(a^2-a^2*sin(d*x+c))-5/256*(7*A+
B)/d/(a^2+a^2*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 78, 212}

$$-\frac{a^3(A-B)}{80d(a\sin(c+dx)+a)^5}-\frac{a^2(2A-B)}{64d(a\sin(c+dx)+a)^4}+\frac{3(7A+3B)}{256d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5(7A+B)}{256d\left(a^2\sin(c+dx)+a^2\right)}+\frac{7(4A+B)\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a(A+B)}{192d(a-a\sin(c+dx))^3}-\frac{a(5A-B)}{96d(a\sin(c+dx)+a)^3}+\frac{3A+2B}{128d(a-a\sin(c+dx))^2}-\frac{5A}{64d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (7*(4*A + B)*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + (a*(A + B))/(192*d*(a - a
*Sin[c + d*x])^3) + (3*A + 2*B)/(128*d*(a - a*Sin[c + d*x])^2) - (a^3*(A -
B))/(80*d*(a + a*Sin[c + d*x])^5) - (a^2*(2*A - B))/(64*d*(a + a*Sin[c + d*
x])^4) - (a*(5*A - B))/(96*d*(a + a*Sin[c + d*x])^3) - (5*A)/(64*d*(a + a*S
in[c + d*x])^2) + (3*(7*A + 3*B))/(256*d*(a^2 - a^2*Sin[c + d*x])) - (5*(7*
A + B))/(256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+a\sin(c+dx))^2}\,dx = \frac{a^7\text{Subst}\left(\int \frac{A+\frac{Bx}{a}}{(a-x)^4(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a^7\text{Subst}\left(\int \left(\frac{A+B}{64a^6(a-x)^4}+\frac{3A+2B}{64a^7(a-x)^3}+\frac{3(7A+3B)}{256a^8(a-x)^2}+\frac{A-B}{16a^4(a+x)^6}+\frac{2}{16a}\right.\right.}{}$$

$$= \frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{3A+2B}{128d(a-a\sin(c+dx))^2}-\frac{a^3(}{80d(a+a}$$

$$= \frac{7(4A+B)\tanh^{-1}(\sin(c+dx))}{128a^2d}+\frac{a(A+B)}{192d(a-a\sin(c+dx))^3}+\frac{}{128}$$

Mathematica [A]
time = 1.08, size = 160, normalized size = 0.68

$$\frac{210(4A+B)\tanh^{-1}(\sin(c+dx))-\frac{2(48(-8A+3B)+183(4A+B)\sin(c+dx)+462(4A+B)\sin^2(c+dx)-49(4A+B)\sin^3(c+dx)-560(4A+B)\sin^4(c+dx)-175(4A+B)\sin^5(c+dx)+210(4A+B)\sin^6(c+dx)+105(4A+B)\sin^7(c+dx))}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^5}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (210*(4*A + B)*ArcTanh[Sin[c + d*x]] - (2*(48*(-8*A + 3*B) + 183*(4*A + B)*
Sin[c + d*x] + 462*(4*A + B)*Sin[c + d*x]^2 - 49*(4*A + B)*Sin[c + d*x]^3 -
 560*(4*A + B)*Sin[c + d*x]^4 - 175*(4*A + B)*Sin[c + d*x]^5 + 210*(4*A + B
)*Sin[c + d*x]^6 + 105*(4*A + B)*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1
 + Sin[c + d*x])^5))/(3840*a^2*d)
```

Maple [A]
time = 0.52, size = 189, normalized size = 0.80

method	result
derivativedivides	$\frac{-\frac{5A}{64(1+\sin(dx+c))^2}-\frac{\frac{A}{16}-\frac{B}{16}}{5(1+\sin(dx+c))^5}-\frac{\frac{A}{8}-\frac{B}{16}}{4(1+\sin(dx+c))^4}-\frac{\frac{5A}{32}-\frac{B}{32}}{3(1+\sin(dx+c))^3}+\left(\frac{7A}{64}+\frac{7B}{256}\right)\ln(1+\sin(dx+c))-\frac{\frac{35A}{256}+\frac{5B}{256}}{1+\sin(dx+c)}+(}{d\,a^2}$

default	$\frac{-\frac{5A}{64(1+\sin(dx+c))^2}-\frac{\frac{A}{16}-\frac{B}{16}}{5(1+\sin(dx+c))^5}-\frac{\frac{A}{8}-\frac{B}{16}}{4(1+\sin(dx+c))^4}-\frac{\frac{5A}{32}-\frac{B}{32}}{3(1+\sin(dx+c))^3}+\left(\frac{7A}{64}+\frac{7B}{256}\right)\ln(1+\sin(dx+c))-\frac{\frac{35A}{256}+\frac{5B}{256}}{1+\sin(dx+c)}+\left(-\right.}{d\,a^2}$
risch	$-\frac{i\left(4800iA\,e^{8i(dx+c)}-29520iB\,e^{8i(dx+c)}+7840iA\,e^{4i(dx+c)}+1960iB\,e^{4i(dx+c)}+1680iA\,e^{2i(dx+c)}+420iB\,e^{2i(dx+c)}+316\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d/a^2*(-5/64*A/(1+sin(d*x+c))^2-1/5*(1/16*A-1/16*B)/(1+sin(d*x+c))^5-1/4*
(1/8*A-1/16*B)/(1+sin(d*x+c))^4-1/3*(5/32*A-1/32*B)/(1+sin(d*x+c))^3+(7/64*
A+7/256*B)*ln(1+sin(d*x+c))-(35/256*A+5/256*B)/(1+sin(d*x+c))+(-7/64*A-7/25
6*B)*ln(sin(d*x+c)-1)-1/2*(-3/64*A-1/32*B)/(sin(d*x+c)-1)^2-1/3*(1/64*A+1/6
4*B)/(sin(d*x+c)-1)^3-(21/256*A+9/256*B)/(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 252, normalized size = 1.07

$$-\frac{\frac{2\left(105\,(4\,A+B)\sin(dx+c)^7+210\,(4\,A+B)\sin(dx+c)^6-175\,(4\,A+B)\sin(dx+c)^5-560\,(4\,A+B)\sin(dx+c)^4-49\,(4\,A+B)\sin(dx+c)^3+462\,(4\,A+B)\sin(dx+c)^2+183\,(4\,A+B)\sin(dx+c)-384\,A+144\,B\right)}{a^2\sin(dx+c)^8+2\,a^2\sin(dx+c)^7-2\,a^2\sin(dx+c)^6-6\,a^2\sin(dx+c)^5+6\,a^2\sin(dx+c)^3+2\,a^2\sin(dx+c)^2-2\,a^2\sin(dx+c)-a^2}-\frac{105\,(4\,A+B)\log(\sin(dx+c)+1)}{a^2}+\frac{105\,(4\,A+B)\log(\sin(dx+c)-1)}{a^2}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/3840*(2*(105*(4*A + B)*sin(d*x + c)^7 + 210*(4*A + B)*sin(d*x + c)^6 - 1
75*(4*A + B)*sin(d*x + c)^5 - 560*(4*A + B)*sin(d*x + c)^4 - 49*(4*A + B)*s
in(d*x + c)^3 + 462*(4*A + B)*sin(d*x + c)^2 + 183*(4*A + B)*sin(d*x + c) -
 384*A + 144*B)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x
+ c)^6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2
 - 2*a^2*sin(d*x + c) - a^2) - 105*(4*A + B)*log(sin(d*x + c) + 1)/a^2 + 10
5*(4*A + B)*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A]
time = 0.40, size = 290, normalized size = 1.23

$$\frac{420\,(4\,A+B)\cos(dx+c)^6-140\,(4\,A+B)\cos(dx+c)^4-56\,(4\,A+B)\cos(dx+c)^2+105\,\left((4\,A+B)\cos(dx+c)^8-2\,(4\,A+B)\cos(dx+c)^6\sin(dx+c)-2\,(4\,A+B)\cos(dx+c)^6\right)\log(\sin(dx+c)+1)-105\,\left((4\,A+B)\cos(dx+c)^8-2\,(4\,A+B)\cos(dx+c)^6\sin(dx+c)-2\,(4\,A+B)\cos(dx+c)^6\right)\log(-\sin(dx+c)+1)+2\,\left(105\,(4\,A+B)\cos(dx+c)^6-140\,(4\,A+B)\cos(dx+c)^4-84\,(4\,A+B)\cos(dx+c)^2-256\,A-64\,B\right)\sin(dx+c)-128\,A-512\,B}{3840\,\left(a^2d\cos(dx+c)^8-2\,a^2d\cos(dx+c)^6\sin(dx+c)-2\,a^2d\cos(dx+c)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/3840*(420*(4*A + B)*cos(d*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 56*(4
*A + B)*cos(d*x + c)^2 + 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*
x + c)^6*sin(d*x + c) - 2*(4*A + B)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) -
 105*((4*A + B)*cos(d*x + c)^8 - 2*(4*A + B)*cos(d*x + c)^6*sin(d*x + c) -
```

```
2*(4*A + B)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(105*(4*A + B)*cos(d
*x + c)^6 - 140*(4*A + B)*cos(d*x + c)^4 - 84*(4*A + B)*cos(d*x + c)^2 - 25
6*A - 64*B)*sin(d*x + c) - 128*A - 512*B)/(a^2*d*cos(d*x + c)^8 - 2*a^2*d*c
os(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 258, normalized size = 1.09

$$\frac{\frac{420\,(4\,A+B)\log\left(|\sin(dx+c)+1|\right)}{a^2}-\frac{420\,(4\,A+B)\log\left(|\sin(dx+c)-1|\right)}{a^2}+\frac{10\left(308\,A\sin(dx+c)^3+77\,B\sin(dx+c)^3-1050\,A\sin(dx+c)^2-285\,B\sin(dx+c)^2+1212\,A\sin(dx+c)+363\,B\sin(dx+c)-478\,A-163\,B\right)}{a^2(\sin(dx+c)-1)^3}-\frac{3836\,A\sin(dx+c)^5+959\,B\sin(dx+c)^5+21280\,A\sin(dx+c)^4+5095\,B\sin(dx+c)^4+47960\,A\sin(dx+c)^3+10790\,B\sin(dx+c)^3+55360\,A\sin(dx+c)^2+11230\,B\sin(dx+c)^2+33260\,A\sin(dx+c)+5435\,B\sin(dx+c)+8608\,A+667\,B}{a^2(\sin(dx+c)+1)^5}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+a*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/15360*(420*(4*A + B)*log(abs(sin(d*x + c) + 1))/a^2 - 420*(4*A + B)*log(a
bs(sin(d*x + c) - 1))/a^2 + 10*(308*A*sin(d*x + c)^3 + 77*B*sin(d*x + c)^3
- 1050*A*sin(d*x + c)^2 - 285*B*sin(d*x + c)^2 + 1212*A*sin(d*x + c) + 363*
B*sin(d*x + c) - 478*A - 163*B)/(a^2*(sin(d*x + c) - 1)^3) - (3836*A*sin(d*
x + c)^5 + 959*B*sin(d*x + c)^5 + 21280*A*sin(d*x + c)^4 + 5095*B*sin(d*x +
 c)^4 + 47960*A*sin(d*x + c)^3 + 10790*B*sin(d*x + c)^3 + 55360*A*sin(d*x +
 c)^2 + 11230*B*sin(d*x + c)^2 + 33260*A*sin(d*x + c) + 5435*B*sin(d*x + c)
 + 8608*A + 667*B)/(a^2*(sin(d*x + c) + 1)^5))/d
```

Mupad [B]
time = 9.60, size = 240, normalized size = 1.02

$$\frac{\left(\frac{7\,A}{32}+\frac{7\,B}{128}\right)\,\sin\left(c+d\,x\right)^7+\left(\frac{7\,A}{16}+\frac{7\,B}{64}\right)\,\sin\left(c+d\,x\right)^6+\left(-\frac{35\,A}{96}-\frac{35\,B}{384}\right)\,\sin\left(c+d\,x\right)^5+\left(-\frac{7\,A}{6}-\frac{7\,B}{24}\right)\,\sin\left(c+d\,x\right)^4+\left(-\frac{49\,A}{480}-\frac{49\,B}{1920}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{77\,A}{80}+\frac{77\,B}{320}\right)\,\sin\left(c+d\,x\right)^2+\left(\frac{61\,A}{160}+\frac{61\,B}{640}\right)\,\sin\left(c+d\,x\right)-\frac{A}{5}+\frac{3\,B}{40}}{d\,\left(-a^2\sin\left(c+d\,x\right)^8-2\,a^2\sin\left(c+d\,x\right)^7+2\,a^2\sin\left(c+d\,x\right)^6+6\,a^2\sin\left(c+d\,x\right)^5-6\,a^2\sin\left(c+d\,x\right)^3-2\,a^2\sin\left(c+d\,x\right)^2+2\,a^2\,\sin\left(c+d\,x\right)+a^2\right)}+\frac{7\,\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(4\,A+B\right)}{128\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + a*sin(c + d*x))^2),x)
```

```
[Out] ((3*B)/40 - A/5 + sin(c + d*x)*((61*A)/160 + (61*B)/640) - sin(c + d*x)^4*(
(7*A)/6 + (7*B)/24) + sin(c + d*x)^6*((7*A)/16 + (7*B)/64) + sin(c + d*x)^7
*((7*A)/32 + (7*B)/128) - sin(c + d*x)^5*((35*A)/96 + (35*B)/384) + sin(c +
 d*x)^2*((77*A)/80 + (77*B)/320) - sin(c + d*x)^3*((49*A)/480 + (49*B)/1920
))/(d*(2*a^2*sin(c + d*x) + a^2 - 2*a^2*sin(c + d*x)^2 - 6*a^2*sin(c + d*x)
^3 + 6*a^2*sin(c + d*x)^5 + 2*a^2*sin(c + d*x)^6 - 2*a^2*sin(c + d*x)^7 - a
^2*sin(c + d*x)^8)) + (7*atanh(sin(c + d*x))*(4*A + B))/(128*a^2*d)
```

3.1019 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=170

$$-\frac{2^{\frac{1}{2}(1+2m+p)}a(Bm+A(1+m+p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+}{fg(1+p)(1+m+p)}$$

```
[Out] -2^(1/2+m+1/2*p)*a*(B*m+A*(1+m+p))*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*
p, 1/2-m-1/2*p],[3/2+1/2*p],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m-1/2*p
)*(a+a*sin(f*x+e))^(-1+m)/f/g/(1+p)/(1+m+p)-B*(g*cos(f*x+e))^(1+p)*(a+a*sin
(f*x+e))^m/f/g/(1+m+p)
```

Rubi [A]
time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2939, 2768, 72, 71}

$$-\frac{a2^{\frac{1}{2}(2m+p+1)}(A(m+p+1)+Bm)(a\sin(e+fx)+a)^{m-1}(g\cos(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m-p+1)}\,{}_2F_1\left(\frac{1}{2}(-2m-p+1),\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{fg(p+1)(m+p+1)}-\frac{B(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg(m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((2^((1 + 2*m + p)/2)*a*(B*m + A*(1 + m + p))*(g*Cos[e + f*x])^(1 + p)*Hyp
ergeometric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2]
*(1 + Sin[e + f*x])^((1 - 2*m - p)/2)*(a + a*Sin[e + f*x])^(-1 + m))/(f*g*(
1 + p)*(1 + m + p))) - (B*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/
(f*g*(1 + m + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))}{fg(1+m+p)}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))}{fg(1+m+p)}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))}{fg(1+m+p)}$$

$$= -\frac{2^{\frac{1}{2}(1+2m+p)}a\left(A+\frac{Bm}{1+m+p}\right)(g\cos(e+fx}{}$$

Mathematica [A]
time = 0.33, size = 154, normalized size = 0.91

$$-\frac{\cos(e+fx)(g\cos(e+fx))^p(1+\sin(e+fx))^{\frac{1}{2}(-1-2m-p)}(a(1+\sin(e+fx)))^m\left(2^{\frac{1}{2}(1+2m+p)}(Bm+A(1+m+p))\,{}_2F_1\left(\frac{1}{2}(1-2m-p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx))\right)+B(1+p)(1+\sin(e+fx))^{\frac{1}{2}(1+2m+p)}\right)}{f(1+p)(1+m+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(1 + Sin[e + f*x])^((-1 - 2*m - p)/2)*(a
*(1 + Sin[e + f*x]))^m*(2^((1 + 2*m + p)/2)*(B*m + A*(1 + m + p))*Hypergeom
etric2F1[(1 - 2*m - p)/2, (1 + p)/2, (3 + p)/2, (1 - Sin[e + f*x])/2] + B*(
1 + p)*(1 + Sin[e + f*x])^((1 + 2*m + p)/2)))/(f*(1 + p)*(1 + m + p)))
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p (a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="fricas")
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (g\cos(e+fx))^p (A+B\sin(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(g*cos(e + f*x))**p*(A + B*sin(e + f*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^p\,(A+B\sin(e+fx))\,(a+a\sin(e+fx))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.1020 $\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=159

$$\frac{8(A-B)(a+a\sin(e+fx))^{4+m}}{a^4f(4+m)}-\frac{4(3A-5B)(a+a\sin(e+fx))^{5+m}}{a^5f(5+m)}+\frac{6(A-3B)(a+a\sin(e+fx))^{6+m}}{a^6f(6+m)}-($$

```
[Out] 8*(A-B)*(a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)-4*(3*A-5*B)*(a+a*sin(f*x+e))^(5+
m)/a^5/f/(5+m)+6*(A-3*B)*(a+a*sin(f*x+e))^(6+m)/a^6/f/(6+m)-(A-7*B)*(a+a*si
n(f*x+e))^(7+m)/a^7/f/(7+m)-B*(a+a*sin(f*x+e))^(8+m)/a^8/f/(8+m)
```

Rubi [A]
time = 0.11, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(e+fx)+a)^{m+8}}{a^8f(m+8)}-\frac{(A-7B)(a\sin(e+fx)+a)^{m+7}}{a^7f(m+7)}+\frac{6(A-3B)(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}-\frac{4(3A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}+\frac{8(A-B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (8*(A - B)*(a + a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) - (4*(3*A - 5*B)*(
a + a*Sin[e + f*x])^(5 + m))/(a^5*f*(5 + m)) + (6*(A - 3*B)*(a + a*Sin[e +
f*x])^(6 + m))/(a^6*f*(6 + m)) - ((A - 7*B)*(a + a*Sin[e + f*x])^(7 + m))/(
a^7*f*(7 + m)) - (B*(a + a*Sin[e + f*x])^(8 + m))/(a^8*f*(8 + m))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a-x)^3(a+x)^{3+m}\left(A+\frac{Bx}{a}\right)\,dx, x,\right.}{a^7 f}$$

$$= \frac{\text{Subst}\left(\int \left(8a^3(A-B)(a+x)^{3+m} - 4a^2(3A\right.\right.}{}$$

$$= \frac{8(A-B)(a+a\sin(e+fx))^{4+m}}{a^4 f(4+m)} - \frac{4(3A-}{}$$

Mathematica [A]
time = 0.55, size = 132, normalized size = 0.83

$$\frac{(a(1+\sin(e+fx)))^{4+m}\left(\frac{8a^4(A-B)}{4+m} - \frac{4a^4(3A-5B)(1+\sin(e+fx))}{5+m} + \frac{6a^4(A-3B)(1+\sin(e+fx))^2}{6+m} - \frac{a^4(A-7B)(1+\sin(e+fx))^3}{7+m} - \frac{B(a+a\sin(e+fx))^4}{8+m}\right)}{a^8 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^7*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(4 + m)*((8*a^4*(A - B))/(4 + m) - (4*a^4*(3*A - 5*
B)*(1 + Sin[e + f*x]))/(5 + m) + (6*a^4*(A - 3*B)*(1 + Sin[e + f*x])^2)/(6
+ m) - (a^4*(A - 7*B)*(1 + Sin[e + f*x])^3)/(7 + m) - (B*(a + a*Sin[e + f*x
])^4)/(8 + m)))/(a^8*f)
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\cos^7(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1250 vs. 2(164) = 328.
time = 0.34, size = 1250, normalized size = 7.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -(((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*a^m*sin(f*x
 + e)^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(f*x +
 e)^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 + 30*(
m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 120*(m^3 + 3*m^2 + 2*m)*a^
m*sin(f*x + e)^3 + 360*(m^2 + m)*a^m*sin(f*x + e)^2 - 720*a^m*m*sin(f*x + e
) + 720*a^m)*A*(sin(f*x + e) + 1)^m/(m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 67
69*m^3 + 13132*m^2 + 13068*m + 5040) - 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 2
4)*a^m*sin(f*x + e)^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4
*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 + 12*(m^2 + m)*a^m*sin(f*x + e)^2 -
 24*a^m*m*sin(f*x + e) + 24*a^m)*A*(sin(f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*
m^3 + 225*m^2 + 274*m + 120) + 3*((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2
 + m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f*x + e) + 2*a^m)*A*(sin(f*x + e) +
1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*
m^3 + 13132*m^2 + 13068*m + 5040)*a^m*sin(f*x + e)^8 + (m^7 + 21*m^6 + 175*
m^5 + 735*m^4 + 1624*m^3 + 1764*m^2 + 720*m)*a^m*sin(f*x + e)^7 - 7*(m^6 +
15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*a^m*sin(f*x + e)^6 + 42*(m^5 +
 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 - 210*(m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(f*x + e)^4 + 840*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^
3 - 2520*(m^2 + m)*a^m*sin(f*x + e)^2 + 5040*a^m*m*sin(f*x + e) - 5040*a^m)
*B*(sin(f*x + e) + 1)^m/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67
284*m^3 + 118124*m^2 + 109584*m + 40320) - 3*((m^5 + 15*m^4 + 85*m^3 + 225*
m^2 + 274*m + 120)*a^m*sin(f*x + e)^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 2
4*m)*a^m*sin(f*x + e)^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*a^m*sin(f*x + e)^4
 + 20*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 60*(m^2 + m)*a^m*sin(f*x + e
)^2 + 120*a^m*m*sin(f*x + e) - 120*a^m)*B*(sin(f*x + e) + 1)^m/(m^6 + 21*m^
5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720) + 3*((m^3 + 6*m^2 + 11*m +
 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 3*(m^2 +
m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)*B*(sin(f*x + e) + 1)^
m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) - (a^m*(m + 1)*sin(f*x + e)^2 + a^m*m
*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3*m + 2) - (a*sin(f*x +
e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(164) = 328$.
time = 0.44, size = 342, normalized size = 2.15

$$-\frac{((Bm^4 + 22\,Bm^3 + 179\,Bm^2 + 638\,Bm + 840\,B)\cos(fx+e)^8 - ((A+B)m^4 + (17\,A + 9\,B)m^3 + 4\,(23\,A + 5\,B)m^2 + 160\,Am)\cos(fx+e)^6 - 12\,((A+B)m^3 + (11\,A + 3\,B)m^2 + 24\,Am)\cos(fx+e)^4 - 96\,((A+B)m^2 + 8\,Am)\cos(fx+e)^2 - 384\,(A+B)m - (((A+B)m^4 + (23\,A + 15\,B)m^3 + 2\,(97\,A + 37\,B)m^2 + 8\,(89\,A + 15\,B)m + 960\,A)\cos(fx+e)^6 + 12\,((A+B)m^3 + (15\,A + 7\,B)m^2 + 4\,(17\,A + 3\,B)m + 96\,A)\cos(fx+e)^4 + 96\,((A+B)m^2 + 2\,(5\,A + B)m + 16\,A)\cos(fx+e)^2 + 384\,(A+B)m + 3072\,A)\sin(fx+e) - 3072\,A)(a\sin(fx+e) + a)^m}{fm^5 + 30\,fm^4 + 355\,fm^3 + 2070\,fm^2 + 5944\,fm + 6720\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^4 + 22*B*m^3 + 179*B*m^2 + 638*B*m + 840*B)*cos(f*x + e)^8 - ((A + B
)*m^4 + (17*A + 9*B)*m^3 + 4*(23*A + 5*B)*m^2 + 160*A*m)*cos(f*x + e)^6 - 1
2*((A + B)*m^3 + (11*A + 3*B)*m^2 + 24*A*m)*cos(f*x + e)^4 - 96*((A + B)*m^
```

```
2 + 8*A*m)*cos(f*x + e)^2 - 384*(A + B)*m - (((A + B)*m^4 + (23*A + 15*B)*m
^3 + 2*(97*A + 37*B)*m^2 + 8*(89*A + 15*B)*m + 960*A)*cos(f*x + e)^6 + 12*(
(A + B)*m^3 + (15*A + 7*B)*m^2 + 4*(17*A + 3*B)*m + 96*A)*cos(f*x + e)^4 +
96*((A + B)*m^2 + 2*(5*A + B)*m + 16*A)*cos(f*x + e)^2 + 384*(A + B)*m + 30
72*A)*sin(f*x + e) - 3072*A)*(a*sin(f*x + e) + a)^m/(f*m^5 + 30*f*m^4 + 355
*f*m^3 + 2070*f*m^2 + 5944*f*m + 6720*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**7*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. 2(164) = 328.
time = 0.49, size = 1402, normalized size = 8.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^7*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^3 - 6*(a*sin(f*x + e) +
a)^6*(a*sin(f*x + e) + a)^m*a*m^3 + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x +
e) + a)^m*a^2*m^3 - 8*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^3*m^3
 + 15*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m^2 - 96*(a*sin(f*x + e
) + a)^6*(a*sin(f*x + e) + a)^m*a*m^2 + 204*(a*sin(f*x + e) + a)^5*(a*sin(f
*x + e) + a)^m*a^2*m^2 - 144*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3*m^2 + 74*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*m - 498*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a*m + 1128*(a*sin(f*x + e) + a)^5*(a*
sin(f*x + e) + a)^m*a^2*m - 856*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)
^m*a^3*m + 120*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m - 840*(a*sin(f
*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a + 2016*(a*sin(f*x + e) + a)^5*(a*si
n(f*x + e) + a)^m*a^2 - 1680*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*
a^3)*A/(a^6*m^4 + 22*a^6*m^3 + 179*a^6*m^2 + 638*a^6*m + 840*a^6) + ((a*sin
(f*x + e) + a)^8*(a*sin(f*x + e) + a)^m*m^4 - 7*(a*sin(f*x + e) + a)^7*(a*s
in(f*x + e) + a)^m*a*m^4 + 18*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m
*a^2*m^4 - 20*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^4 + 8*(a*
sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^4 + 22*(a*sin(f*x + e) + a
)^8*(a*sin(f*x + e) + a)^m*m^3 - 161*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e)
 + a)^m*a*m^3 + 432*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^3 -
```

```
500*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^3 + 208*(a*sin(f*x
 + e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^3 + 179*(a*sin(f*x + e) + a)^8*(a
*sin(f*x + e) + a)^m*m^2 - 1358*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)
^m*a*m^2 + 3798*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m^2 - 460
0*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m^2 + 2008*(a*sin(f*x +
 e) + a)^4*(a*sin(f*x + e) + a)^m*a^4*m^2 + 638*(a*sin(f*x + e) + a)^8*(a*s
in(f*x + e) + a)^m*m - 4984*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a
*m + 14472*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2*m - 18400*(a*s
in(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a^3*m + 8528*(a*sin(f*x + e) + a)
^4*(a*sin(f*x + e) + a)^m*a^4*m + 840*(a*sin(f*x + e) + a)^8*(a*sin(f*x + e
) + a)^m - 6720*(a*sin(f*x + e) + a)^7*(a*sin(f*x + e) + a)^m*a + 20160*(a*
sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*a^2 - 26880*(a*sin(f*x + e) + a)
^5*(a*sin(f*x + e) + a)^m*a^3 + 13440*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e
) + a)^m*a^4)*B/((a^6*m^5 + 30*a^6*m^4 + 355*a^6*m^3 + 2070*a^6*m^2 + 5944*
a^6*m + 6720*a^6)*a))/(a*f)
```

Mupad [B]
time = 17.90, size = 783, normalized size = 4.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^7*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] -exp(- e*8i - f*x*8i)*(a + a*sin(e + f*x))^m*((exp(e*8i + f*x*8i)*cos(4*e +
 4*f*x)*(23520*B - 8448*A*m + 17864*B*m - 4320*A*m^2 - 600*A*m^3 - 24*A*m^4
 + 3956*B*m^2 + 208*B*m^3 + 4*B*m^4))/(128*f*(5944*m + 2070*m^2 + 355*m^3 +
 30*m^4 + m^5 + 6720)) - (exp(e*8i + f*x*8i)*(786432*A - 58800*B + 237056*A
*m + 53644*B*m + 32320*A*m^2 + 2512*A*m^3 + 80*A*m^4 + 4814*B*m^2 + 332*B*m
^3 + 10*B*m^4))/(256*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720))
 - (exp(e*8i + f*x*8i)*cos(2*e + 2*f*x)*(77184*A*m - 47040*B - 35728*B*m +
20112*A*m^2 + 1788*A*m^3 + 60*A*m^4 - 376*B*m^2 + 76*B*m^3 + 4*B*m^4))/(128
*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (B*exp(e*8i + f*x
*8i)*cos(8*e + 8*f*x)*(638*m + 179*m^2 + 22*m^3 + m^4 + 840))/(128*f*(5944*
m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e*8i + f*x*8i)*sin(5*
e + 5*f*x)*(A*8i + A*m*1i + B*m*1i)*(706*m + 123*m^2 + 5*m^3 + 1176)*1i)/(6
4*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e*8i + f*x*
8i)*sin(3*e + 3*f*x)*(A*8i + A*m*1i + B*m*1i)*(1070*m + 93*m^2 + 3*m^3 + 19
60)*3i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) + (exp(e
*8i + f*x*8i)*cos(6*e + 6*f*x)*(9*m + m^2 + 20)*(84*B - 8*A*m + 26*B*m - A*
m^2 + B*m^2))/(32*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720)) +
(exp(e*8i + f*x*8i)*sin(7*e + 7*f*x)*(A*8i + A*m*1i + B*m*1i)*(74*m + 15*m^
2 + m^3 + 120)*1i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720
)) + (exp(e*8i + f*x*8i)*sin(e + f*x)*(A*8i + A*m*1i + B*m*1i)*(2578*m + 17
1*m^2 + 5*m^3 + 29400)*1i)/(64*f*(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^
5 + 6720)))
```

3.1021 $\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{4(A-B)(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)}-\frac{4(A-2B)(a+a\sin(e+fx))^{4+m}}{a^4f(4+m)}+\frac{(A-5B)(a+a\sin(e+fx))^{5+m}}{a^5f(5+m)}+\frac{B}{}$$

```
[Out] 4*(A-B)*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)-4*(A-2*B)*(a+a*sin(f*x+e))^(4+m)
/a^4/f/(4+m)+(A-5*B)*(a+a*sin(f*x+e))^(5+m)/a^5/f/(5+m)+B*(a+a*sin(f*x+e))^
(6+m)/a^6/f/(6+m)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$\frac{B(a\sin(e+fx)+a)^{m+6}}{a^6f(m+6)}+\frac{(A-5B)(a\sin(e+fx)+a)^{m+5}}{a^5f(m+5)}-\frac{4(A-2B)(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}+\frac{4(A-B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (4*(A - B)*(a + a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (4*(A - 2*B)*(a
+ a*Sin[e + f*x])^(4 + m))/(a^4*f*(4 + m)) + ((A - 5*B)*(a + a*Sin[e + f*x]
)^(5 + m))/(a^5*f*(5 + m)) + (B*(a + a*Sin[e + f*x])^(6 + m))/(a^6*f*(6 + m
))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\operatorname{Subst}\left(\int (a-x)^2(a+x)^{2+m}\left(A+\frac{Bx}{a}\right)\,dx,x,a}{a^5f}$$

$$= \frac{\operatorname{Subst}\left(\int \left(4a^2(A-B)(a+x)^{2+m}-4a(A-2\right.\right.}{}$$

$$= \frac{4(A-B)(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)} - \frac{4(A-2B}{}$$

Mathematica [A]
time = 0.29, size = 103, normalized size = 0.84

$$\frac{(a(1+\sin(e+fx)))^{3+m}\left(\frac{4a^3(A-B)}{3+m}-\frac{4a^3(A-2B)(1+\sin(e+fx))}{4+m}+\frac{a^3(A-5B)(1+\sin(e+fx))^2}{5+m}+\frac{B(a+a\sin(e+fx))^3}{6+m}\right)}{a^6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3 + m)*((4*a^3*(A - B))/(3 + m) - (4*a^3*(A - 2*B)
*(1 + Sin[e + f*x]))/(4 + m) + (a^3*(A - 5*B)*(1 + Sin[e + f*x])^2)/(5 + m)
 + (B*(a + a*Sin[e + f*x])^3)/(6 + m)))/(a^6*f)
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cos^5(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(127) = 254.
time = 0.32, size = 669, normalized size = 5.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] (((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*a^m*sin(f*x + e)^5 + (m^4 + 6*m^3 + 1
1*m^2 + 6*m)*a^m*sin(f*x + e)^4 - 4*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3
+ 12*(m^2 + m)*a^m*sin(f*x + e)^2 - 24*a^m*m*sin(f*x + e) + 24*a^m)*A*(sin(
f*x + e) + 1)^m/(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120) - 2*((m^2 +
 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*a^m*sin(f*x + e)^2 - 2*a^m*m*sin(f
*x + e) + 2*a^m)*A*(sin(f*x + e) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) + ((m^5 +
15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*a^m*sin(f*x + e)^6 + (m^5 + 10*m^4
 + 35*m^3 + 50*m^2 + 24*m)*a^m*sin(f*x + e)^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6
*m)*a^m*sin(f*x + e)^4 + 20*(m^3 + 3*m^2 + 2*m)*a^m*sin(f*x + e)^3 - 60*(m^
2 + m)*a^m*sin(f*x + e)^2 + 120*a^m*m*sin(f*x + e) - 120*a^m)*B*(sin(f*x +
e) + 1)^m/(m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720) - 2*
((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin(
f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m)
*B*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) + (a^m*(m + 1)*
sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3*
m + 2) + (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

Fricas [A]
time = 0.41, size = 228, normalized size = 1.85

$$-\frac{\left((B m^3 + 12\,B m^2 + 47\,B m + 60\,B)\cos\left(f x + e\right)^6 - ((A + B)m^3 + 3\,(3\,A + B)m^2 + 18\,A m)\cos\left(f x + e\right)^4 - 8\,((A + B)m^2 + 6\,A m)\cos\left(f x + e\right)^2 - 32\,(A + B)m - (((A + B)m^3 + (13\,A + 7\,B)m^2 + 6\,(9\,A + 2\,B)m + 72\,A)\cos\left(f x + e\right)^4 + 8\,((A + B)m^2 + 2\,(4\,A + B)m + 12\,A)\cos\left(f x + e\right)^2 + 32\,(A + B)m + 192\,A)\sin\left(f x + e\right) - 192\,A\right)\left(a\sin\left(f x + e\right) + a\right)^m}{f m^4 + 18\,f m^3 + 119\,f m^2 + 342\,f m + 360\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^3 + 12*B*m^2 + 47*B*m + 60*B)*cos(f*x + e)^6 - ((A + B)*m^3 + 3*(3*A
 + B)*m^2 + 18*A*m)*cos(f*x + e)^4 - 8*((A + B)*m^2 + 6*A*m)*cos(f*x + e)^2
 - 32*(A + B)*m - (((A + B)*m^3 + (13*A + 7*B)*m^2 + 6*(9*A + 2*B)*m + 72*A
)*cos(f*x + e)^4 + 8*((A + B)*m^2 + 2*(4*A + B)*m + 12*A)*cos(f*x + e)^2 +
32*(A + B)*m + 192*A)*sin(f*x + e) - 192*A)*(a*sin(f*x + e) + a)^m/(f*m^4 +
 18*f*m^3 + 119*f*m^2 + 342*f*m + 360*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22522 vs. $2(107) = 214$.
time = 150.02, size = 22522, normalized size = 183.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e)**5, Eq(f, 0)), (-32*A*
sin(e + f*x)**4/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 6
00*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e +
 f*x) + 60*a**6*f) - 100*A*sin(e + f*x)**3/(60*a**6*f*sin(e + f*x)**5 + 300
```

```
*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f
*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 16*A*sin(e + f*x)**2*cos(e
+ f*x)**2/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**
6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x)
+ 60*a**6*f) - 116*A*sin(e + f*x)**2/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*
f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2
 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 20*A*sin(e + f*x)*cos(e + f*x)**2
/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e
 + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6
*f) - 60*A*sin(e + f*x)/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x
)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f
*sin(e + f*x) + 60*a**6*f) - 12*A*cos(e + f*x)**4/(60*a**6*f*sin(e + f*x)**
5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*si
n(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 4*A*cos(e + f*x)**2/
(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e
+ f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*
f) - 12*A/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**
6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x)
+ 60*a**6*f) + 60*B*log(sin(e + f*x) + 1)*sin(e + f*x)**5/(60*a**6*f*sin(e
+ f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a
**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 300*B*log(si
n(e + f*x) + 1)*sin(e + f*x)**4/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin
(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 30
0*a**6*f*sin(e + f*x) + 60*a**6*f) + 600*B*log(sin(e + f*x) + 1)*sin(e + f*
x)**3/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*
sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60
*a**6*f) + 600*B*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(60*a**6*f*sin(e + f
*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6
*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 300*B*log(sin(e
 + f*x) + 1)*sin(e + f*x)/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f
*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6
*f*sin(e + f*x) + 60*a**6*f) + 60*B*log(sin(e + f*x) + 1)/(60*a**6*f*sin(e
+ f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a
**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 132*B*sin(e
+ f*x)**4/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**
6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x)
+ 60*a**6*f) + 30*B*sin(e + f*x)**3*cos(e + f*x)**2/(60*a**6*f*sin(e + f*x)
**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*
sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 480*B*sin(e + f*x)
**3/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*si
n(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a
**6*f) + 54*B*sin(e + f*x)**2*cos(e + f*x)**2/(60*a**6*f*sin(e + f*x)**5 +
300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e
+ f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 656*B*sin(e + f*x)**2/(6
0*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e +
```

```
f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f)
 - 15*B*sin(e + f*x)*cos(e + f*x)**4/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*
f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2
 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 30*B*sin(e + f*x)*cos(e + f*x)**2
/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e
 + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6
*f) + 400*B*sin(e + f*x)/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*
x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*
f*sin(e + f*x) + 60*a**6*f) - 3*B*cos(e + f*x)**4/(60*a**6*f*sin(e + f*x)**
5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e + f*x)**3 + 600*a**6*f*si
n(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*f) + 6*B*cos(e + f*x)**2/
(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**6*f*sin(e
+ f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x) + 60*a**6*
f) + 92*B/(60*a**6*f*sin(e + f*x)**5 + 300*a**6*f*sin(e + f*x)**4 + 600*a**
6*f*sin(e + f*x)**3 + 600*a**6*f*sin(e + f*x)**2 + 300*a**6*f*sin(e + f*x)
+ 60*a**6*f), Eq(m, -6)), (12*A*log(sin(e + f*x) + 1)*sin(e + f*x)**4/(12*a
**5*f*sin(e + f*x)**4 + 48*a**5*f*sin(e + f*x)**3 + 72*a**5*f*sin(e + f*x)*
*2 + 48*a**5*f*sin(e + f*x) + 12*a**5*f) + 48*A...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(127) = 254.
time = 0.46, size = 861, normalized size = 7.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] (((a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m^2 - 4*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a*m^2 + 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^2*m^2 + 7*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*m - 32*(a
*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a*m + 36*(a*sin(f*x + e) + a)^3
*(a*sin(f*x + e) + a)^m*a^2*m + 12*(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) +
 a)^m - 60*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*a + 80*(a*sin(f*x
+ e) + a)^3*(a*sin(f*x + e) + a)^m*a^2)*A/(a^4*m^3 + 12*a^4*m^2 + 47*a^4*m
+ 60*a^4) + ((a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^3 - 5*(a*sin(f
*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^3 + 8*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m*a^2*m^3 - 4*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^
m*a^3*m^3 + 12*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m^2 - 65*(a*si
n(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m^2 + 112*(a*sin(f*x + e) + a)^4
*(a*sin(f*x + e) + a)^m*a^2*m^2 - 60*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e)
 + a)^m*a^3*m^2 + 47*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m*m - 270*
(a*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a*m + 504*(a*sin(f*x + e) + a
)^4*(a*sin(f*x + e) + a)^m*a^2*m - 296*(a*sin(f*x + e) + a)^3*(a*sin(f*x +
```

```
e) + a)^m*a^3*m + 60*(a*sin(f*x + e) + a)^6*(a*sin(f*x + e) + a)^m - 360*(a
*sin(f*x + e) + a)^5*(a*sin(f*x + e) + a)^m*a + 720*(a*sin(f*x + e) + a)^4*
(a*sin(f*x + e) + a)^m*a^2 - 480*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a
)^m*a^3)*B/((a^4*m^4 + 18*a^4*m^3 + 119*a^4*m^2 + 342*a^4*m + 360*a^4)*a))/
(a*f)
```

Mupad [B]
time = 15.82, size = 517, normalized size = 4.20

$$-\mathrm{e}^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,\left(-\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\left(12288\,A-1200\,B+4016\,A\,m+1108\,B\,m+472\,A\,m^2+24\,A\,m^3+88\,B\,m^2+4\,B\,m^3\right)}{64\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}-\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,\left(1056\,A\,m-900\,B-705\,B\,m+272\,A\,m^2+16\,A\,m^3-4\,B\,m^2+B\,m^3\right)}{32\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}+\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,\left(m+3\right)\,\left(60\,B-12\,A\,m+27\,B\,m-2\,A\,m^2+B\,m^2\right)}{16\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}+\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(e+f\,x\right)\,\left(A\,6\mathrm{i}+A\,m\,1\mathrm{i}+B\,m\,1\mathrm{i}\right)\,\left(m^2+23\,m+300\right)\,1\mathrm{i}}{8\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}+\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(5\,e+5\,f\,x\right)\,\left(A\,6\mathrm{i}+A\,m\,1\mathrm{i}+B\,m\,1\mathrm{i}\right)\,\left(m^2+7\,m+12\right)\,1\mathrm{i}}{16\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}+\frac{B\,\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(6\,e+6\,f\,x\right)\,\left(m^3+12\,m^2+47\,m+60\right)}{32\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}+\frac{\mathrm{e}^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\sin\left(3\,e+3\,f\,x\right)\,\left(A\,6\mathrm{i}+A\,m\,1\mathrm{i}+B\,m\,1\mathrm{i}\right)\,\left(3\,m^2+53\,m+100\right)\,1\mathrm{i}}{16\,f\,\left(m^4+18\,m^3+119\,m^2+342\,m+360\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] -exp(- e*6i - f*x*6i)*(a + a*sin(e + f*x))^m*((exp(e*6i + f*x*6i)*cos(4*e +
 4*f*x)*(m + 3)*(60*B - 12*A*m + 27*B*m - 2*A*m^2 + B*m^2))/(16*f*(342*m +
119*m^2 + 18*m^3 + m^4 + 360)) - (exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(1056
*A*m - 900*B - 705*B*m + 272*A*m^2 + 16*A*m^3 - 4*B*m^2 + B*m^3))/(32*f*(34
2*m + 119*m^2 + 18*m^3 + m^4 + 360)) - (exp(e*6i + f*x*6i)*(12288*A - 1200*
B + 4016*A*m + 1108*B*m + 472*A*m^2 + 24*A*m^3 + 88*B*m^2 + 4*B*m^3))/(64*f
*(342*m + 119*m^2 + 18*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(e + f*x)
*(A*6i + A*m*1i + B*m*1i)*(23*m + m^2 + 300)*1i)/(8*f*(342*m + 119*m^2 + 18
*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*6i + A*m*1i +
B*m*1i)*(7*m + m^2 + 12)*1i)/(16*f*(342*m + 119*m^2 + 18*m^3 + m^4 + 360))
+ (B*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(47*m + 12*m^2 + m^3 + 60))/(32*f*
(342*m + 119*m^2 + 18*m^3 + m^4 + 360)) + (exp(e*6i + f*x*6i)*sin(3*e + 3*f
*x)*(A*6i + A*m*1i + B*m*1i)*(53*m + 3*m^2 + 100)*1i)/(16*f*(342*m + 119*m^
2 + 18*m^3 + m^4 + 360)))
```

3.1022 $\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=93

$$\frac{2(A-B)(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}-\frac{(A-3B)(a+a\sin(e+fx))^{3+m}}{a^3f(3+m)}-\frac{B(a+a\sin(e+fx))^{4+m}}{a^4f(4+m)}$$

```
[Out] 2*(A-B)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)-(A-3*B)*(a+a*sin(f*x+e))^(3+m)/a
^3/f/(3+m)-B*(a+a*sin(f*x+e))^(4+m)/a^4/f/(4+m)
```

Rubi [A]
time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2915, 78}

$$-\frac{B(a\sin(e+fx)+a)^{m+4}}{a^4f(m+4)}-\frac{(A-3B)(a\sin(e+fx)+a)^{m+3}}{a^3f(m+3)}+\frac{2(A-B)(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\operatorname{Subst}\left(\int (a-x)(a+x)^{1+m}\left(A+\frac{Bx}{a}\right)\,dx, x, a\,\mathrm{s}\right.}{a^3 f}$$

$$= \frac{\operatorname{Subst}\left(\int \left(2a(A-B)(a+x)^{1+m}+(-A+3B\right.\right.}{a}$$

$$= \frac{2(A-B)(a+a\sin(e+fx))^{2+m}}{a^2 f(2+m)} - \frac{(A-3B)}{}$$

Mathematica [A]
time = 0.22, size = 93, normalized size = 1.00

$$\frac{2(A-B)(a+a\sin(e+fx))^{2+m}}{a^2 f(2+m)} - \frac{(A-3B)(a+a\sin(e+fx))^{3+m}}{a^3 f(3+m)} - \frac{B(a+a\sin(e+fx))^{4+m}}{a^4 f(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (2*(A - B)*(a + a*Sin[e + f*x])^(2 + m))/(a^2*f*(2 + m)) - ((A - 3*B)*(a +
a*Sin[e + f*x])^(3 + m))/(a^3*f*(3 + m)) - (B*(a + a*Sin[e + f*x])^(4 + m))
/(a^4*f*(4 + m))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\cos^3(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(96) = 192.
time = 0.31, size = 298, normalized size = 3.20

$$-\frac{\frac{\left((m^2+3m+2)a^m\sin(fx+e)^3+(m^2+m)a^m\sin(fx+e)^2-2a^m m\sin(fx+e)+2a^m\right)A(\sin(fx+e)+1)^m}{m^3+6m^2+11m+6}+\frac{\left((m^3+6m^2+11m+6)a^m\sin(fx+e)^4+(m^3+3m^2+2m)a^m\sin(fx+e)^3-3(m^2+m)a^m\sin(fx+e)^2+6a^m m\sin(fx+e)-6a^m\right)B(\sin(fx+e)+1)^m}{m^4+10m^3+35m^2+50m+24}-\frac{\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)B(\sin(fx+e)+1)^m}{m^2+3m+2}-\frac{(a\sin(fx+e)+a)^{m+1}A}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] -((((m^2 + 3*m + 2)*a^m*sin(f*x + e)^3 + (m^2 + m)*a^m*sin(f*x + e)^2 - 2*a^
m*m*sin(f*x + e) + 2*a^m)*A*(sin(f*x + e) + 1)^m/(m^3 + 6*m^2 + 11*m + 6) +
 ((m^3 + 6*m^2 + 11*m + 6)*a^m*sin(f*x + e)^4 + (m^3 + 3*m^2 + 2*m)*a^m*sin
(f*x + e)^3 - 3*(m^2 + m)*a^m*sin(f*x + e)^2 + 6*a^m*m*sin(f*x + e) - 6*a^m
)*B*(sin(f*x + e) + 1)^m/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24) - (a^m*(m + 1)
*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) + 1)^m/(m^2 + 3
*m + 2) - (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

Fricas [A]
time = 0.40, size = 140, normalized size = 1.51

$$-\frac{((Bm^2+5\,Bm+6\,B)\cos{(fx+e)}^4-((A+B)m^2+4\,Am)\cos{(fx+e)}^2-4\,(A+B)m-(((A+B)m^2+2\,(3\,A+B)m+8\,A)\cos{(fx+e)}^2+4\,(A+B)m+16\,A)\sin{(fx+e)}-16\,A)(a\sin{(fx+e)}+a)^m}{fm^3+9\,fm^2+26\,fm+24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] -((B*m^2 + 5*B*m + 6*B)*cos(f*x + e)^4 - ((A + B)*m^2 + 4*A*m)*cos(f*x + e)
^2 - 4*(A + B)*m - (((A + B)*m^2 + 2*(3*A + B)*m + 8*A)*cos(f*x + e)^2 + 4*
(A + B)*m + 16*A)*sin(f*x + e) - 16*A)*(a*sin(f*x + e) + a)^m/(f*m^3 + 9*f*
m^2 + 26*f*m + 24*f)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5243 vs. $2(78) = 156$.
time = 17.09, size = 5243, normalized size = 56.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e)**3, Eq(f, 0)), (4*A*si
n(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a*
*4*f*sin(e + f*x) + 6*a**4*f) + 6*A*sin(e + f*x)/(6*a**4*f*sin(e + f*x)**3
+ 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 2*A*cos(
e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4
*f*sin(e + f*x) + 6*a**4*f) + 2*A/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin
(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 6*B*log(sin(e + f*x) +
1)*sin(e + f*x)**3/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 +
18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*B*log(sin(e + f*x) + 1)*sin(e + f*x
)**2/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(
e + f*x) + 6*a**4*f) - 18*B*log(sin(e + f*x) + 1)*sin(e + f*x)/(6*a**4*f*si
n(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4
*f) - 6*B*log(sin(e + f*x) + 1)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e
 + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 10*B*sin(e + f*x)**2/(6*a
**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x)
```

+ 6*a**4*f) - 3*B*sin(e + f*x)*cos(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 +
18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 18*B*sin(e
 + f*x)/(6*a**4*f*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*s
in(e + f*x) + 6*a**4*f) - B*cos(e + f*x)**2/(6*a**4*f*sin(e + f*x)**3 + 18*
a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a**4*f) - 8*B/(6*a**4*f
*sin(e + f*x)**3 + 18*a**4*f*sin(e + f*x)**2 + 18*a**4*f*sin(e + f*x) + 6*a
4*f), Eq(m, -4)), (-2*A*log(sin(e + f*x) + 1)*sin(e + f*x)2/(2*a**3*f*s
in(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - 4*A*log(sin(e + f*x) +
 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3
*f) - 2*A*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e
+ f*x) + 2*a**3*f) - 2*A*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*
sin(e + f*x) + 2*a**3*f) - A*cos(e + f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*
a**3*f*sin(e + f*x) + 2*a**3*f) - 2*A/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*
sin(e + f*x) + 2*a**3*f) + 6*B*log(sin(e + f*x) + 1)*sin(e + f*x)**2/(2*a**
3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 12*B*log(sin(e +
f*x) + 1)*sin(e + f*x)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) +
2*a**3*f) + 6*B*log(sin(e + f*x) + 1)/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*
sin(e + f*x) + 2*a**3*f) - 4*B*sin(e + f*x)**3/(2*a**3*f*sin(e + f*x)**2 +
4*a**3*f*sin(e + f*x) + 2*a**3*f) - 2*B*sin(e + f*x)*cos(e + f*x)**2/(2*a**
3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 14*B*sin(e + f*x)
/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) - B*cos(e +
f*x)**2/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f) + 10*
B/(2*a**3*f*sin(e + f*x)**2 + 4*a**3*f*sin(e + f*x) + 2*a**3*f), Eq(m, -3))
, (4*A*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/
2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 8*A*log(tan(e/2 + f*x/2) +
 1)*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*
x/2)**2 + a**2*f) + 4*A*log(tan(e/2 + f*x/2) + 1)/(a**2*f*tan(e/2 + f*x/2)*
*4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*log(tan(e/2 + f*x/2)**2 +
 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*
x/2)**2 + a**2*f) - 4*A*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(a
2*f*tan(e/2 + f*x/2)4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*lo
g(tan(e/2 + f*x/2)**2 + 1)/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 +
 f*x/2)**2 + a**2*f) - 2*A*tan(e/2 + f*x/2)**3/(a**2*f*tan(e/2 + f*x/2)**4
+ 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 2*A*tan(e/2 + f*x/2)/(a**2*f*tan
(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) - 4*B*log(tan(e/2
 + f*x/2) + 1)*tan(e/2 + f*x/2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*t
an(e/2 + f*x/2)**2 + a**2*f) - 8*B*log(tan(e/2 + f*x/2) + 1)*tan(e/2 + f*x/
2)**2/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f)
- 4*B*log(tan(e/2 + f*x/2) + 1)/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(
e/2 + f*x/2)**2 + a**2*f) + 2*B*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/
2)**4/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f)
+ 4*B*log(tan(e/2 + f*x/2)**2 + 1)*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f*
x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 2*B*log(tan(e/2 + f*x/2)
2 + 1)/(a2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*
f) + 4*B*tan(e/2 + f*x/2)**3/(a**2*f*tan(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2

```
 + f*x/2)**2 + a**2*f) - 2*B*tan(e/2 + f*x/2)**2/(a**2*f*tan(e/2 + f*x/2)**
4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f) + 4*B*tan(e/2 + f*x/2)/(a**2*f*t
an(e/2 + f*x/2)**4 + 2*a**2*f*tan(e/2 + f*x/2)**2 + a**2*f), Eq(m, -2)), (6
*A*tan(e/2 + f*x/2)**5/(3*a*f*tan(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)*
*4 + 9*a*f*tan(e/2 + f*x/2)**2 + 3*a*f) - 6*A*tan(e/2 + f*x/2)**4/(3*a*f*ta
n(e/2 + f*x/2)**6 + 9*a*f*tan(e/2 + f*x/2)**4 + 9*a*f*tan(e/2 + f*x/2)**2 +
 3*a*f) + 12*A*tan(e/2 + f*x/2)**3/(3*a*f*tan(e...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(96) = 192.
time = 0.46, size = 458, normalized size = 4.92

$$-\frac{\frac{\left((a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m m-2\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m am+2\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m-6\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a\right)A}{a^2m^2+5\,a^2m+6\,a^2}+\frac{\left((a\sin(fx+e)+a)^4(a\sin(fx+e)+a)^m m^2-3\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m am^2+2\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a^2m^2+5\,(a\sin(fx+e)+a)^4(a\sin(fx+e)+a)^m m-18\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m am+14\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a^2m+6\,(a\sin(fx+e)+a)^4(a\sin(fx+e)+a)^m-24\,(a\sin(fx+e)+a)^3(a\sin(fx+e)+a)^m a+24\,(a\sin(fx+e)+a)^2(a\sin(fx+e)+a)^m a^2\right)B}{(a^2m^3+9\,a^2m^2+26\,a^2m+24\,a^2)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] -(((a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*m - 2*(a*sin(f*x + e) + a)
^2*(a*sin(f*x + e) + a)^m*a*m + 2*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) +
a)^m - 6*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a)*A/(a^2*m^2 + 5*a^
2*m + 6*a^2) + ((a*sin(f*x + e) + a)^4*(a*sin(f*x + e) + a)^m*m^2 - 3*(a*si
n(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m^2 + 2*(a*sin(f*x + e) + a)^2*(
a*sin(f*x + e) + a)^m*a^2*m^2 + 5*(a*sin(f*x + e) + a)^4*(a*sin(f*x + e) +
a)^m*m - 18*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a*m + 14*(a*sin(f
*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2*m + 6*(a*sin(f*x + e) + a)^4*(a*s
in(f*x + e) + a)^m - 24*(a*sin(f*x + e) + a)^3*(a*sin(f*x + e) + a)^m*a + 2
4*(a*sin(f*x + e) + a)^2*(a*sin(f*x + e) + a)^m*a^2)*B/((a^2*m^3 + 9*a^2*m^
2 + 26*a^2*m + 24*a^2)*a))/(a*f)
```

Mupad [B]
time = 11.74, size = 272, normalized size = 2.92

$$\frac{(a\,(\sin(e+f\,x)+1))^m\,(128\,A-18\,B+48\,A\,m+17\,B\,m+144\,A\,\sin(e+f\,x)-24\,B\,\cos(2\,e+2\,f\,x)-6\,B\,\cos(4\,e+4\,f\,x)+4\,A\,m^2+B\,m^2+16\,A\,\sin(3\,e+3\,f\,x)+4\,A\,m^2\,\cos(2\,e+2\,f\,x)-B\,m^2\,\cos(4\,e+4\,f\,x)+2\,A\,m^2\,\sin(3\,e+3\,f\,x)+2\,B\,m^2\,\sin(3\,e+3\,f\,x)+44\,A\,m\,\sin(e+f\,x)+36\,B\,m\,\sin(e+f\,x)+16\,A\,m\,\cos(2\,e+2\,f\,x)-20\,B\,m\,\cos(2\,e+2\,f\,x)-5\,B\,m\,\cos(4\,e+4\,f\,x)+12\,A\,m\,\sin(3\,e+3\,f\,x)+2\,A\,m^2\,\sin(e+f\,x)+4\,B\,m\,\sin(3\,e+3\,f\,x)+2\,B\,m^2\,\sin(e+f\,x))}{8\,f\,(m^3+9\,m^2+26\,m+24)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(128*A - 18*B + 48*A*m + 17*B*m + 144*A*sin(e + f
*x) - 24*B*cos(2*e + 2*f*x) - 6*B*cos(4*e + 4*f*x) + 4*A*m^2 + B*m^2 + 16*A
*sin(3*e + 3*f*x) + 4*A*m^2*cos(2*e + 2*f*x) - B*m^2*cos(4*e + 4*f*x) + 2*A
*m^2*sin(3*e + 3*f*x) + 2*B*m^2*sin(3*e + 3*f*x) + 44*A*m*sin(e + f*x) + 36
*B*m*sin(e + f*x) + 16*A*m*cos(2*e + 2*f*x) - 20*B*m*cos(2*e + 2*f*x) - 5*B
*m*cos(4*e + 4*f*x) + 12*A*m*sin(3*e + 3*f*x) + 2*A*m^2*sin(e + f*x) + 4*B*
m*sin(3*e + 3*f*x) + 2*B*m^2*sin(e + f*x)))/(8*f*(26*m + 9*m^2 + m^3 + 24))
```

3.1023 $\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=59

$$\frac{(A-B)(a+a\sin(e+fx))^{1+m}}{af(1+m)}+\frac{B(a+a\sin(e+fx))^{2+m}}{a^2f(2+m)}$$

```
[Out] (A-B)*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)+B*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{B(a\sin(e+fx)+a)^{m+2}}{a^2f(m+2)}+\frac{(A-B)(a\sin(e+fx)+a)^{m+1}}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m)) + (B*(a + a*Sin[e + f*
x])^(2 + m))/(a^2*f*(2 + m))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{\text{Subst}\left(\int (a+x)^m \left(A+\frac{Bx}{a}\right)\,dx, x, a\sin(e+f\right.}{af}$$
$$= \frac{\text{Subst}\left(\int \left((A-B)(a+x)^m + \frac{B(a+x)^{1+m}}{a}\right)\,dx\right.}{af}$$
$$= \frac{(A-B)(a+a\sin(e+fx))^{1+m}}{af(1+m)} + \frac{B(a+a\sin}{a^2f(}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.86

$$\frac{(a(1+\sin(e+fx)))^{1+m}(-B+A(2+m)+B(1+m)\sin(e+fx))}{af(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1 + m)*(-B + A*(2 + m) + B*(1 + m)*Sin[e + f*x]))/
(a*f*(1 + m)*(2 + m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos(fx+e)\left(a+a\sin(fx+e)\right)^m \left(A+B\sin(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 1.47

$$\frac{\frac{\left(a^m(m+1)\sin(fx+e)^2+a^m m\sin(fx+e)-a^m\right)B(\sin(fx+e)+1)^m}{m^2+3m+2}+\frac{(a\sin(fx+e)+a)^{m+1}A}{a(m+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] ((a^m*(m + 1)*sin(f*x + e)^2 + a^m*m*sin(f*x + e) - a^m)*B*(sin(f*x + e) +
1)^m/(m^2 + 3*m + 2) + (a*sin(f*x + e) + a)^(m + 1)*A/(a*(m + 1)))/f
```

Fricas [**A**]
time = 0.39, size = 73, normalized size = 1.24

$$-\frac{\left((Bm+B)\cos\left(fx+e\right)^{2}-(A+B)m-((A+B)m+2\,A)\sin\left(fx+e\right)-2\,A\right)(a\sin\left(fx+e\right)+a)^{m}}{fm^{2}+3\,fm+2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] -((B*m + B)*cos(f*x + e)^2 - (A + B)*m - ((A + B)*m + 2*A)*sin(f*x + e) - 2
*A)*(a*sin(f*x + e) + a)^m/(f*m^2 + 3*f*m + 2*f)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(46) = 92$.
time = 1.43, size = 428, normalized size = 7.25

$$\begin{cases} x\left(A+B\sin\left(e\right)\right)\left(a\sin\left(e\right)+a\right)^{m}\cos\left(e\right) & \text{for}\: f=0 \\ -\frac{A}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B\log\left(\sin\left(e+fx\right)+1\right)\sin\left(e+fx\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B\log\left(\sin\left(e+fx\right)+1\right)}{a^{2}f\sin\left(e+fx\right)+a^{2}f}+\frac{B}{a^{2}f\sin\left(e+fx\right)+a^{2}f} & \text{for}\: m=-2 \\ \frac{A\log\left(\sin\left(e+fx\right)+1\right)}{af}-\frac{B\log\left(\sin\left(e+fx\right)+1\right)}{af}+\frac{B\sin\left(e+fx\right)}{af} & \text{for}\: m=-1 \\ \frac{Am\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{Am\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{2A\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{2A\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f}+\frac{Bm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin^{2}\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{Bm\left(a\sin\left(e+fx\right)+a\right)^{m}\sin\left(e+fx\right)}{fm^{2}+3fm+2f}+\frac{B\left(a\sin\left(e+fx\right)+a\right)^{m}\sin^{2}\left(e+fx\right)}{fm^{2}+3fm+2f}-\frac{B\left(a\sin\left(e+fx\right)+a\right)^{m}}{fm^{2}+3fm+2f} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((x*(A + B*sin(e))*(a*sin(e) + a)**m*cos(e), Eq(f, 0)), (-A/(a**2*
f*sin(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)*sin(e + f*x)/(a**2*f*sin
(e + f*x) + a**2*f) + B*log(sin(e + f*x) + 1)/(a**2*f*sin(e + f*x) + a**2*f
) + B/(a**2*f*sin(e + f*x) + a**2*f), Eq(m, -2)), (A*log(sin(e + f*x) + 1)/
(a*f) - B*log(sin(e + f*x) + 1)/(a*f) + B*sin(e + f*x)/(a*f), Eq(m, -1)), (
A*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + A*m*(a*si
n(e + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m*sin
(e + f*x)/(f*m**2 + 3*f*m + 2*f) + 2*A*(a*sin(e + f*x) + a)**m/(f*m**2 + 3*
f*m + 2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m +
2*f) + B*m*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(f*m**2 + 3*f*m + 2*f) + B*
(a*sin(e + f*x) + a)**m*sin(e + f*x)**2/(f*m**2 + 3*f*m + 2*f) - B*(a*sin(e
 + f*x) + a)**m/(f*m**2 + 3*f*m + 2*f), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(61) = 122$.
time = 0.42, size = 156, normalized size = 2.64

$$\frac{\frac{(a\sin(fx+e)+a)^{m+1}A}{m+1}+\frac{\left((a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}m-(a\sin(fx+e)+a)(a\sin(fx+e)+a)^{m}am+(a\sin(fx+e)+a)^{2}(a\sin(fx+e)+a)^{m}-2\,(a\sin(fx+e)+a)(a\sin(fx+e)+a)^{m}a\right)B}{(m^{2}+3\,m+2)a}}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] ((a*sin(f*x + e) + a)^(m + 1)*A/(m + 1) + ((a*sin(f*x + e) + a)^2*(a*sin(f*
x + e) + a)^m*m - (a*sin(f*x + e) + a)*(a*sin(f*x + e) + a)^m*a*m + (a*sin(
f*x + e) + a)^2*(a*sin(f*x + e) + a)^m - 2*(a*sin(f*x + e) + a)*(a*sin(f*x
+ e) + a)^m*a)*B/((m^2 + 3*m + 2)*a))/(a*f)
```

Mupad [B]
time = 10.17, size = 99, normalized size = 1.68

$$\frac{(a\,(\sin(e+f\,x)+1))^m\,\left(4\,A-B+2\,A\,m+B\,m+4\,A\,\sin(e+f\,x)+B\,\left(2\sin(e+f\,x)^2-1\right)+2\,A\,m\,\sin(e+f\,x)+2\,B\,m\,\sin(e+f\,x)+B\,m\,\left(2\sin(e+f\,x)^2-1\right)\right)}{2\,f\,\left(m^2+3\,m+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] ((a*(sin(e + f*x) + 1))^m*(4*A - B + 2*A*m + B*m + 4*A*sin(e + f*x) + B*(2*
sin(e + f*x)^2 - 1) + 2*A*m*sin(e + f*x) + 2*B*m*sin(e + f*x) + B*m*(2*sin(
e + f*x)^2 - 1)))/(2*f*(3*m + m^2 + 2))
```

3.1024 $\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=80

$$\frac{(A-B)(a+a\sin(e+fx))^m}{2fm}+\frac{(A+B)\,{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{4af(1+m)}$$

```
[Out] 1/2*(A-B)*(a+a*sin(f*x+e))^m/f/m+1/4*(A+B)*hypergeom([1, 1+m],[2+m],1/2+1/2
*sin(f*x+e))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2915, 80, 70}

$$\frac{(A+B)(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}+\frac{(A-B)(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] ((A - B)*(a + a*Sin[e + f*x])^m)/(2*f*m) + ((A + B)*Hypergeometric2F1[1, 1
+ m, 2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 +
 m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p
+ 1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && Sum
SimplerQ[p, 1]
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a\text{Subst}\left(\int \frac{(a+x)^{-1+m}\left(A+\frac{Bx}{a}\right)}{a-x}\,dx, x, a\sin(e+fx\right.}{f}$$
$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\text{Subs}}{}$$
$$= \frac{(A-B)(a+a\sin(e+fx))^m}{2fm} + \frac{(A+B)\,{}_2F_1(}{}$$

Mathematica [A]
time = 0.10, size = 71, normalized size = 0.89

$$\frac{(a(1+\sin(e+fx)))^m\left(2(A-B)(1+m)+(A+B)m\,{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(1+\sin(e+fx))\right)}{4fm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] ((a*(1 + Sin[e + f*x]))^m*(2*(A - B)*(1 + m) + (A + B)*m*Hypergeometric2F1[
1, 1 + m, 2 + m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \sec(fx+e)\,(a+a\sin(fx+e))^m\,(A+B\sin(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(f*x + e)*sin(f*x + e) + A*sec(f*x + e))*(a*sin(f*x + e) + a
)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e+fx)+1))^m (A+B\sin(e+fx))\sec(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sec(e + f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac
")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\cos(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x), x)
```

3.1025 $\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=100

$$-\frac{a(A(2-m)-Bm)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-1+m}}{4f(1-m)}+\frac{a^2(A+B)(a+a\sin}{2f(a-a\sin(}$$

```
[Out] -1/4*a*(A*(2-m)-B*m)*hypergeom([1, -1+m],[m],1/2+1/2*sin(f*x+e))*(a+a*sin(f
*x+e))^(-1+m)/f/(1-m)+1/2*a^2*(A+B)*(a+a*sin(f*x+e))^(-1+m)/f/(a-a*sin(f*x+
e))
```

Rubi [A]

time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 79, 70}

$$\frac{a^2(A+B)(a\sin(e+fx)+a)^{m-1}}{2f(a-a\sin(e+fx))}-\frac{a(A(2-m)-Bm)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/4*(a*(A*(2 - m) - B*m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x]
)/2]*(a + a*Sin[e + f*x])^(-1 + m))/(f*(1 - m)) + (a^2*(A + B)*(a + a*Sin[e
 + f*x])^(-1 + m))/(2*f*(a - a*Sin[e + f*x]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^3\mathrm{Subst}\left(\int \frac{(a+x)^{-2+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^2}\,dx, x, a\sin(e+f$$

$$= \frac{a^2(A+B)(a+a\sin(e+fx))^{-1+m}}{2f(a-a\sin(e+fx))} + \frac{(a^2(A(2$$

$$= -\frac{a(A(2-m)-Bm)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\right.}{4f(1-}$$

Mathematica [A]
time = 0.13, size = 82, normalized size = 0.82

$$-\frac{a\left(2(A+B)(-1+m)+(A(-2+m)+Bm)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(-1+\sin(e+fx))\right)(a(1+\sin(e+fx)))^{-1+m}}{4f(-1+m)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/4*(a*(2*(A + B)*(-1 + m) + (A*(-2 + m) + B*m)*Hypergeometric2F1[1, -1 +
m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]))*(a*(1 + Sin[e + f*x]))^(-1
 + m))/(f*(-1 + m)*(-1 + Sin[e + f*x]))
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^3(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^3*sin(f*x + e) + A*sec(f*x + e)^3)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin(e+f\,x))\,(a+a\,\sin(e+f\,x))^m}{\cos(e+f\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^3, x)
```

3.1026 $\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=104

$$-\frac{a^2(A(4-m)-Bm)\,{}_2F_1\left(2,-2+m;-1+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-2+m}}{16f(2-m)}+\frac{a^4(A+B)(a\cdots}{4f(a-\cdots}$$

```
[Out] -1/16*a^2*(A*(4-m)-B*m)*hypergeom([2, -2+m],[-1+m],1/2+1/2*sin(f*x+e))*(a+a
*sin(f*x+e))^(-2+m)/f/(2-m)+1/4*a^4*(A+B)*(a+a*sin(f*x+e))^(-2+m)/f/(a-a*si
n(f*x+e))^2
```

Rubi [A]

time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2915, 79, 70}

$$\frac{a^4(A+B)(a\sin(e+fx)+a)^{m-2}}{4f(a-a\sin(e+fx))^2}-\frac{a^2(A(4-m)-Bm)(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(2,m-2;m-1;\frac{1}{2}(\sin(e+fx)+1)\right)}{16f(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/16*(a^2*(A*(4 - m) - B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[
e + f*x])/2]*(a + a*Sin[e + f*x])^(-2 + m))/(f*(2 - m)) + (a^4*(A + B)*(a +
 a*Sin[e + f*x])^(-2 + m))/(4*f*(a - a*Sin[e + f*x])^2)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 2915

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d/b)*x)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{a^5\text{Subst}\left(\int \frac{(a+x)^{-3+m}\left(A+\frac{Bx}{a}\right)}{(a-x)^3}\,dx, x, a\sin(e+\ldots\right.}{f}$$

$$= \frac{a^4(A+B)(a+a\sin(e+fx))^{-2+m}}{4f(a-a\sin(e+fx))^2} + \frac{(a^4(A(\ldots}{\ldots}$$

$$= -\frac{a^2(A(4-m)-Bm)\,{}_2F_1(2,-2+m;-1+\ldots}{16f\ldots}$$

Mathematica [A]
time = 0.13, size = 76, normalized size = 0.73

$$\frac{a^2\left(-\frac{(A(-4+m)+Bm)\,{}_2F_1\left(2,-2+m;-1+m;\frac{1}{2}(1+\sin(e+fx))\right)}{-2+m}+\frac{4(A+B)}{(-1+\sin(e+fx))^2}\right)(a(1+\sin(e+fx)))^{-2+m}}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-(((A*(-4 + m) + B*m)*Hypergeometric2F1[2, -2 + m, -1 + m, (1 + Sin[e
 + f*x])/2])/(-2 + m)) + (4*(A + B))/(-1 + Sin[e + f*x])^2)*(a*(1 + Sin[e +
 f*x]))^(-2 + m))/(16*f)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\sec^5(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^5*sin(f*x + e) + A*sec(f*x + e)^5)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B\,\sin(e + f\,x))\,\left(a + a\,\sin(e + f\,x)\right)^m}{\cos(e + f\,x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^5,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^5, x)
```

3.1027 $\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{2^{\frac{7}{2}+m}a^3(Bm+A(7+m))\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{5}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a$$
$$7f(7+m)$$

```
[Out] -1/7*2^(7/2+m)*a^3*(B*m+A*(7+m))*cos(f*x+e)^7*hypergeom([7/2, -5/2-m],[9/2]
,1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-3+m)/f/(7+m
)-B*cos(f*x+e)^7*(a+a*sin(f*x+e))^m/f/(7+m)
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$-\frac{a^3 2^{m+\frac{7}{2}}(A(m+7)+Bm)\cos^7(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-3}\,{}_2F_1\left(\frac{7}{2},-m-\frac{5}{2};\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{7f(m+7)}-\frac{B\cos^7(e+fx)(a\sin(e+fx)+a)^m}{f(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/7*(2^(7/2 + m)*a^3*(B*m + A*(7 + m))*Cos[e + f*x]^7*Hypergeometric2F1[7/
2, -5/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a +
a*Sin[e + f*x])^(-3 + m))/(f*(7 + m)) - (B*Cos[e + f*x]^7*(a + a*Sin[e + f*
x])^m)/(f*(7 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \Big(A$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{(a^2}{}$$

$$= -\frac{B\cos^7(e+fx)(a+a\sin(e+fx))^m}{f(7+m)} + \frac{\Big(2^{\frac{}{}}}{}$$

$$= -\frac{2^{\frac{7}{2}+m}a^3\left(A+\frac{Bm}{7+m}\right)\cos^7(e+fx)\,{}_2F_1\left(\frac{7}{2},-\frac{5}{2}\right.}{}$$

Mathematica [A]
time = 0.67, size = 111, normalized size = 0.86

$$-\frac{\cos^7(e+fx)(1+\sin(e+fx))^{-\frac{7}{2}-m}(a(1+\sin(e+fx)))^m\left(2^{\frac{7}{2}+m}(Bm+A(7+m))\,{}_2F_1\left(\frac{7}{2},-\frac{5}{2}-m;\frac{9}{2};\frac{1}{2}(1-\sin(e+fx))\right)+7B(1+\sin(e+fx))^{\frac{7}{2}+m}\right)}{7f(7+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/7*(Cos[e + f*x]^7*(1 + Sin[e + f*x])^(-7/2 - m)*(a*(1 + Sin[e + f*x]))^m
*(2^(7/2 + m)*(B*m + A*(7 + m))*Hypergeometric2F1[7/2, -5/2 - m, 9/2, (1 -
Sin[e + f*x])/2] + 7*B*(1 + Sin[e + f*x])^(7/2 + m)))/(f*(7 + m))
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\cos^{6}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
[Out] integral((B*cos(f*x + e)^6*sin(f*x + e) + A*cos(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+fx)^6 \left(A+B\sin(e+fx)\right) \left(a+a\sin(e+fx)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cos(e + f*x)^6*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.1028 $\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=129

$$-\frac{2^{\frac{5}{2}+m}a^2(Bm+A(5+m))\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a}{5f(5+m)}$$

```
[Out] -1/5*2^(5/2+m)*a^2*(B*m+A*(5+m))*cos(f*x+e)^5*hypergeom([5/2, -3/2-m],[7/2]
,1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(5+m
)-B*cos(f*x+e)^5*(a+a*sin(f*x+e))^m/f/(5+m)
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$-\frac{a^2 2^{m+\frac{5}{2}}(A(m+5)+Bm)\cos^5(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-2}\,{}_2F_1\left(\frac{5}{2},-m-\frac{3}{2};\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5f(m+5)}-\frac{B\cos^5(e+fx)(a\sin(e+fx)+a)^m}{f(m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/5*(2^(5/2 + m)*a^2*(B*m + A*(5 + m))*Cos[e + f*x]^5*Hypergeometric2F1[5/
2, -3/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a +
a*Sin[e + f*x])^(-2 + m))/(f*(5 + m)) - (B*Cos[e + f*x]^5*(a + a*Sin[e + f*
x])^m)/(f*(5 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \Big(A$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{(a^2}{}$$

$$= -\frac{B\cos^5(e+fx)(a+a\sin(e+fx))^m}{f(5+m)} + \frac{\Big(2^{\frac{5}{2}}}{}$$

$$= -\frac{2^{\frac{5}{2}+m}a^2\left(A+\frac{Bm}{5+m}\right)\cos^5(e+fx)\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}\right.}{}$$

Mathematica [A]
time = 0.40, size = 111, normalized size = 0.86

$$-\frac{\cos^5(e+fx)(1+\sin(e+fx))^{-\frac{5}{2}-m}(a(1+\sin(e+fx)))^m\left(2^{\frac{5}{2}+m}(Bm+A(5+m))\,{}_2F_1\left(\frac{5}{2},-\frac{3}{2}-m;\frac{7}{2};\frac{1}{2}(1-\sin(e+fx))\right)+5B(1+\sin(e+fx))^{\frac{5}{2}+m}\right)}{5f(5+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/5*(Cos[e + f*x]^5*(1 + Sin[e + f*x])^(-5/2 - m)*(a*(1 + Sin[e + f*x]))^m
*(2^(5/2 + m)*(B*m + A*(5 + m))*Hypergeometric2F1[5/2, -3/2 - m, 7/2, (1 -
Sin[e + f*x])/2] + 5*B*(1 + Sin[e + f*x])^(5/2 + m)))/(f*(5 + m))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cos^4(fx+e)\right)(a+a\sin(fx+e))^m (A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
[Out] integral((B*cos(f*x + e)^4*sin(f*x + e) + A*cos(f*x + e)^4)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^4\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cos(e + f*x)^4*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.1029 $\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=127

$$-\frac{2^{\frac{3}{2}+m}a(Bm+A(3+m))\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a}{3f(3+m)}$$

```
[Out] -1/3*2^(3/2+m)*a*(B*m+A*(3+m))*cos(f*x+e)^3*hypergeom([3/2, -1/2-m],[5/2],1
/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^(-1+m)/f/(3+m)-
B*cos(f*x+e)^3*(a+a*sin(f*x+e))^m/f/(3+m)
```

Rubi [A]

time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$-\frac{a2^{m+\frac{3}{2}}(A(m+3)+Bm)\cos^3(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(\frac{3}{2},-m-\frac{1}{2};\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3f(m+3)}-\frac{B\cos^3(e+fx)(a\sin(e+fx)+a)^m}{f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/3*(2^(3/2 + m)*a*(B*m + A*(3 + m))*Cos[e + f*x]^3*Hypergeometric2F1[3/2,
 -1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*
Sin[e + f*x])^(-1 + m))/(f*(3 + m)) - (B*Cos[e + f*x]^3*(a + a*Sin[e + f*x]
)^m)/(f*(3 + m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \Big(A$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \frac{(a^2}{}$$

$$= -\frac{B\cos^3(e+fx)(a+a\sin(e+fx))^m}{f(3+m)} + \frac{\Big(2^{}}{}$$

$$= -\frac{2^{\frac{3}{2}+m}a\left(A+\frac{Bm}{3+m}\right)\cos^3(e+fx)\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}\right.}{}$$

Mathematica [A]
time = 0.24, size = 111, normalized size = 0.87

$$-\frac{\cos^3(e+fx)(1+\sin(e+fx))^{-\frac{3}{2}-m}(a(1+\sin(e+fx)))^m\left(2^{\frac{3}{2}+m}(Bm+A(3+m))\,{}_2F_1\left(\frac{3}{2},-\frac{1}{2}-m;\frac{5}{2};\frac{1}{2}(1-\sin(e+fx))\right)+3B(1+\sin(e+fx))^{\frac{3}{2}+m}\right)}{3f(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/3*(Cos[e + f*x]^3*(1 + Sin[e + f*x])^(-3/2 - m)*(a*(1 + Sin[e + f*x]))^m
*(2^(3/2 + m)*(B*m + A*(3 + m))*Hypergeometric2F1[3/2, -1/2 - m, 5/2, (1 -
Sin[e + f*x])/2] + 3*B*(1 + Sin[e + f*x])^(3/2 + m)))/(f*(3 + m))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*cos(f*x + e)^2*sin(f*x + e) + A*cos(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^m\left(A+B\sin\left(e+fx\right)\right)\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^2\,(A+B\,\sin(e+f\,x))\,\left(a+a\,\sin(e+f\,x)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)
```

```
[Out] int(cos(e + f*x)^2*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)
```

3.1030 $\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=123

$$\frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)}+\frac{2^{-\frac{1}{2}+m}(A(1-m)-Bm)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+}{f(1-m)}$$

[Out] B*sec(f*x+e)*(a+a*sin(f*x+e))^m/f/(1-m)+2^(-1/2+m)*(A*(1-m)-B*m)*hypergeom([-1/2, 3/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/f/(1-m)

Rubi [A]
time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$\frac{2^{m-\frac{1}{2}}(A(1-m)-Bm)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}+\frac{B\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)) + (2^(-1/2 + m)*(A*(1 -
 m) - B*m)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[
e + f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
```

```
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^2(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \Big(A-$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\big(a^2(A}{}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{\big(2^{-\frac{3}{2}+}}{}$$

$$= \frac{B\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)} + \frac{2^{-\frac{1}{2}+m}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 6.42, size = 3925, normalized size = 31.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^2*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] -1/4*((A + B)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(
a + a*Sin[e + f*x])^m*(-(AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) +
 (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi
/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)
^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(
```

```
-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi
/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m,
 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + P
i/2 - f*x)/4]^2)))/(f*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/
2 + f*x)/2])^2*(-1/2*(m*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(AppellF1[-1/2
, -2*m, 2*m, 1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*
(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[
(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/
4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/
2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1
[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)
/4]^2] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan
[(-e + Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2))) - ((Cos[(-e + Pi/2
- f*x)/4]^2)^(2*m)*Csc[(-e + Pi/2 - f*x)/4]^2*(-(AppellF1[-1/2, -2*m, 2*m,
1/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi
/2 - f*x)/4]^2)^(2*m)) + (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan
[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e +
Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m
, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + Appe
llF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 -
 f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)))/8 + ((Cos[(-e + Pi/2 - f*x)/4]^2
)^(2*m)*Cot[(-e + Pi/2 - f*x)/4]*(-(m*AppellF1[-1/2, -2*m, 2*m, 1/2, Tan[(-
e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4
]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]) - (Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(
m*AppellF1[1/2, 1 - 2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + m*Ap
pellF1[1/2, -2*m, 1 + 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2
 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) + (3*App
ellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]*(1 - Tan[(-e +
 Pi/2 - f*x)/4]^2)^(2*m))/(2*(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/
2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2
*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF
1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*
x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2)) + (3*Tan[(-e + Pi/2 - f*x)/4]^2*(-1/
3*(m*AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) -
(m*AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
 Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/3)*
(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Ta
n[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2,
 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2
] + AppellF1[3/2, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2])*Tan[(-e + Pi/2 - f*x)/4]^2) - (3*m*AppellF1[1/2, -2*m,
```

```
2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e
 + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^3*(1 - Tan[(-e + Pi/2 - f*x)/4
]^2)^(-1 + 2*m))/(3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2 - f*x)/4]^
2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 4*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, Tan
[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*m,
 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*Ta
n[(-e + Pi/2 - f*x)/4]^2) - (3*AppellF1[1/2, -2*m, 2*m, 3/2, Tan[(-e + Pi/2
 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2*(1 -
Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-2*m*(AppellF1[3/2, 1 - 2*m, 2*m, 5/2, T
an[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[3/2, -2*
m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*
Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4] + 3*(-1/3*(m*AppellF1[3
/2, 1 - 2*m, 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4
]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]) - (m*AppellF1[3/2
, -2*m, 1 + 2*m, 5/2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]
^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 -...
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^m\left(A+B\sin\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^2*sin(f*x + e) + A*sec(f*x + e)^2)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m}{{\cos\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^2,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^2, x)
```

3.1031 $\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)}+\frac{2^{-\frac{3}{2}+m}(A(3-m)-Bm)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec}{3af(3-m)}$$

```
[Out] B*sec(f*x+e)^3*(a+a*sin(f*x+e))^m/f/(3-m)+1/3*2^(-3/2+m)*(A*(3-m)-B*m)*hype
rgeom([-3/2, 5/2-m],[-1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^
(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/f/(3-m)
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$\frac{2^{m-\frac{3}{2}}(A(3-m)-Bm)\sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{5}{2}-m;-\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3af(3-m)}+\frac{B\sec^3(e+fx)(a\sin(e+fx)+a)^m}{f(3-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^m)/(f*(3 - m)) + (2^(-3/2 + m)*(A*(3
 - m) - B*m)*Hypergeometric2F1[-3/2, 5/2 - m, -1/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3
*a*f*(3 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \Big(A-$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{(a^2(A}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{\Big(2^{-\frac{5}{2}+}}{}$$

$$= \frac{B\sec^3(e+fx)(a+a\sin(e+fx))^m}{f(3-m)} + \frac{2^{-\frac{3}{2}+m}}{}$$

Mathematica [F]
time = 1.46, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] Integrate[Sec[e + f*x]^4*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^4(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^4*sin(f*x + e) + A*sec(f*x + e)^4)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\sin(e+fx))(a+a\sin(e+fx))^m}{\cos(e+fx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^4,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^4, x)
```

3.1032 $\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=135

$$\frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)}+\frac{2^{-\frac{5}{2}+m}(A(5-m)-Bm)\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec}{5a^2f(5-m)}$$

```
[Out] B*sec(f*x+e)^5*(a+a*sin(f*x+e))^m/f/(5-m)+1/5*2^(-5/2+m)*(A*(5-m)-B*m)*hype
rgeom([-5/2, 7/2-m],[-3/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^
(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/f/(5-m)
```

Rubi [A]
time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2939, 2768, 72, 71}

$$\frac{2^{m-\frac{5}{2}}(A(5-m)-Bm)\sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^{m+2}\,{}_2F_1\left(-\frac{5}{2},\frac{7}{2}-m;-\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{5a^2f(5-m)}+\frac{B\sec^5(e+fx)(a\sin(e+fx)+a)^m}{f(5-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
```

```
[Out] (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x])^m)/(f*(5 - m)) + (2^(-5/2 + m)*(A*(5
 - m) - B*m)*Hypergeometric2F1[-5/2, 7/2 - m, -3/2, (1 - Sin[e + f*x])/2]*S
ec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5
*a^2*f*(5 - m))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \Big(A-$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{(a^2(A}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{\Big(2^{-\frac{7}{2}+}}{}$$

$$= \frac{B\sec^5(e+fx)(a+a\sin(e+fx))^m}{f(5-m)} + \frac{2^{-\frac{5}{2}+m}}{}$$

Mathematica [**F**]
time = 3.14, size = 0, normalized size = 0.00

$$\int \sec^6(e+fx)(a+a\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]
[Out] Integrate[Sec[e + f*x]^6*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]
```

Maple [**F**]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\sec^6(fx+e)\right)(a+a\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

```
[Out] int(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(f*x + e)^6*sin(f*x + e) + A*sec(f*x + e)^6)*(a*sin(f*x + e)
 + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*sec(f*x + e)^6, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A+B\,\sin\left(e+f\,x\right))\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m}{{\cos\left(e+f\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^6,x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/cos(e + f*x)^6, x)
```

3.1033 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx$

Optimal. Leaf size=239

$$\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-4-p}}{fg(7+p)}+\frac{(3A-B(4+p))(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{cfg(5+p)(7+p)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-4-p)/f/g/(7+p)+(3*A-B*(4+p))*
(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-3-p)/c/f/g/(p^2+12*p+35)+2*(3*A-B*(
4+p))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/c^2/f/g/(5+p)/(p^2+10*p+
21)+2*(3*A-B*(4+p))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c^3/f/g/(p
^2+6*p+5)/(p^2+10*p+21)
```

Rubi [A]
time = 0.29, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2938, 2751, 2750}

$$\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^3fg(p+1)(p+3)(p+5)(p+7)}+\frac{2(3A-B(p+4))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{c^2fg(p+3)(p+5)(p+7)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-4}(g\cos(e+fx))^{p+1}}{fg(p+7)}+\frac{(3A-B(p+4))(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{cfg(p+5)(p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-4 - p))/(f*g*(7 +
p)) + ((3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3
- p))/(c*f*g*(5 + p)*(7 + p)) + (2*(3*A - B*(4 + p))*(g*Cos[e + f*x])^(1 +
p)*(c - c*Sin[e + f*x])^(-2 - p))/(c^2*f*g*(3 + p)*(5 + p)*(7 + p)) + (2*(3
*A - B*(4 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(c^
3*f*g*(1 + p)*(3 + p)*(5 + p)*(7 + p))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
```

```
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-4-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(7+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(7+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(7+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(7+p)}$$

Mathematica [A]
time = 0.39, size = 160, normalized size = 0.67

$$\frac{\cos(e+fx)(g\cos(e+fx))^p(c-c\sin(e+fx))^{-p}\left(-B(13+8p+p^2)+A(36+41p+12p^2+p^3)+(13+8p+p^2)\left(-3A+B(4+p)\right)\sin(e+fx)-2(4+p)(-3A+B(4+p))\sin^2(e+fx)+(-6A+2B(4+p))\sin^3(e+fx)\right)}{c^4f(1+p)(3+p)(5+p)(7+p)(-1+\sin(e+fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4
- p),x]
```

```
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(13 + 8*p + p^2)) + A*(36 + 41*p + 12
*p^2 + p^3) + (13 + 8*p + p^2)*(-3*A + B*(4 + p))*Sin[e + f*x] - 2*(4 + p)*
(-3*A + B*(4 + p))*Sin[e + f*x]^2 + (-6*A + 2*B*(4 + p))*Sin[e + f*x]^3))/(
c^4*f*(1 + p)*(3 + p)*(5 + p)*(7 + p)*(-1 + Sin[e + f*x])^4*(c - c*Sin[e +
f*x])^p)
```

Maple [F]
time = 1.19, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-4-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 4), x)
```

Fricas [**A**]
time = 0.40, size = 204, normalized size = 0.85

$$\frac{\left(2\,\left(Bp^2-(3\,A-8\,B)p-12\,A+16\,B\right)\cos\left(fx+e\right)^3+\left(Ap^3+3\,(4\,A-B)p^2+(47\,A-24\,B)p+60\,A-45\,B\right)\cos\left(fx+e\right)-\left(2\,\left(Bp-3\,A+4\,B\right)\cos\left(fx+e\right)^3-\left(Bp^3-3\,(A-4\,B)p^2-(24\,A-47\,B)p-45\,A+60\,B\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^p\left(-c\sin\left(fx+e\right)+c\right)^{-p-4}}{fp^4+16\,fp^3+86\,fp^2+176\,fp+105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="fricas")
```

```
[Out] (2*(B*p^2 - (3*A - 8*B)*p - 12*A + 16*B)*cos(f*x + e)^3 + (A*p^3 + 3*(4*A -
 B)*p^2 + (47*A - 24*B)*p + 60*A - 45*B)*cos(f*x + e) - (2*(B*p - 3*A + 4*B
)*cos(f*x + e)^3 - (B*p^3 - 3*(A - 4*B)*p^2 - (24*A - 47*B)*p - 45*A + 60*B
)*cos(f*x + e))*sin(f*x + e))*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 4)/(f*p^4 + 16*f*p^3 + 86*f*p^2 + 176*f*p + 105*f)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-4-p),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-4-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 4), x)
```

Mupad [B]
time = 18.42, size = 441, normalized size = 1.85

$$\frac{\cos\left(e+f\,x\right)\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^p\,\left(A\,168{}\mathrm{i}-B\,84{}\mathrm{i}+A\,p\,170{}\mathrm{i}-B\,p\,48{}\mathrm{i}+A\,p^2\,48{}\mathrm{i}+A\,p^3\,4{}\mathrm{i}-B\,p^2\,6{}\mathrm{i}\right)}{4\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+4}\,\left(p^4\,1{}\mathrm{i}+p^3\,16{}\mathrm{i}+p^2\,86{}\mathrm{i}+p\,176{}\mathrm{i}+105{}\mathrm{i}\right)}-\frac{\sin\left(4\,e+4\,f\,x\right)\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^p\,\left(4\,B-3\,A+B\,p\right)\,1{}\mathrm{i}}{4\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+4}\,\left(p^4\,1{}\mathrm{i}+p^3\,16{}\mathrm{i}+p^2\,86{}\mathrm{i}+p\,176{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\cos\left(3\,e+3\,f\,x\right)\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^p\,\left(p+4\right)\,\left(-A\,3{}\mathrm{i}+B\,4{}\mathrm{i}+B\,p\,1{}\mathrm{i}\right)}{2\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+4}\,\left(p^4\,1{}\mathrm{i}+p^3\,16{}\mathrm{i}+p^2\,86{}\mathrm{i}+p\,176{}\mathrm{i}+105{}\mathrm{i}\right)}+\frac{\sin\left(2\,e+2\,f\,x\right)\,{\left(g\,\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}}{2}\right)\right)}^p\,\left(4\,B-3\,A+B\,p\right)\,\left(p^2+8\,p+14\right)\,1{}\mathrm{i}}{2\,f\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+4}\,\left(p^4\,1{}\mathrm{i}+p^3\,16{}\mathrm{i}+p^2\,86{}\mathrm{i}+p\,176{}\mathrm{i}+105{}\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 4),
x)
```

```
[Out] (cos(e + f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(A*168i
 - B*84i + A*p*170i - B*p*48i + A*p^2*48i + A*p^3*4i - B*p^2*6i))/(4*f*(c -
 c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i + p^3*16i + p^4*1i + 105i)) - (s
in(4*e + 4*f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(4*B
- 3*A + B*p)*1i)/(4*f*(c - c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i + p^3*
16i + p^4*1i + 105i)) + (cos(3*e + 3*f*x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(
e*1i + f*x*1i)/2))^p*(p + 4)*(B*4i - A*3i + B*p*1i))/(2*f*(c - c*sin(e + f*
x))^(p + 4)*(p*176i + p^2*86i + p^3*16i + p^4*1i + 105i)) + (sin(2*e + 2*f*
x)*(g*(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^p*(4*B - 3*A + B*p)*
(8*p + p^2 + 14)*1i)/(2*f*(c - c*sin(e + f*x))^(p + 4)*(p*176i + p^2*86i +
p^3*16i + p^4*1i + 105i))
```

3.1034 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx$

Optimal. Leaf size=168

$$\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-3-p}}{fg(5+p)}+\frac{(2A-B(3+p))(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-}}{cfg(3+p)(5+p)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-3-p)/f/g/(5+p)+(2*A-B*(3+p))*
(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/c/f/g/(p^2+8*p+15)+(2*A-B*(3+p
))*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c^2/f/g/(3+p)/(p^2+6*p+5)
```

Rubi [A]

time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2938, 2751, 2750}

$$\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{c^2fg(p+1)(p+3)(p+5)}+\frac{(A+B)(c-c\sin(e+fx))^{-p-3}(g\cos(e+fx))^{p+1}}{fg(p+5)}+\frac{(2A-B(p+3))(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{cfg(p+3)(p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-3 - p))/(f*g*(5 +
p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2
- p))/(c*f*g*(3 + p)*(5 + p)) + ((2*A - B*(3 + p))*(g*Cos[e + f*x])^(1 + p)
*(c - c*Sin[e + f*x])^(-1 - p))/(c^2*f*g*(1 + p)*(3 + p)*(5 + p))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-3-p}\,dx &= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(5+p)}\\ &= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(5+p)}\\ &= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(5+p)}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 119, normalized size = 0.71

$$-\frac{\cos(e+fx)(g\cos(e+fx))^p(c-c\sin(e+fx))^{-p}\left(-B(3+p)+A(7+6p+p^2)+(3+p)(-2A+B(3+p))\sin(e+fx)+(2A-B(3+p))\sin^2(e+fx)\right)}{c^3f(1+p)(3+p)(5+p)(-1+\sin(e+fx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3
- p),x]
```

```
[Out] -((Cos[e + f*x]*(g*Cos[e + f*x])^p*(-(B*(3 + p)) + A*(7 + 6*p + p^2) + (3 +
 p)*(-2*A + B*(3 + p))*Sin[e + f*x] + (2*A - B*(3 + p))*Sin[e + f*x]^2))/(c
^3*f*(1 + p)*(3 + p)*(5 + p)*(-1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^p))
```

Maple [F]
time = 1.10, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(A+B\sin\left(fx+e\right)\right)\left(c-c\sin\left(fx+e\right)\right)^{-3-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 3), x)
```

Fricas [A]
time = 0.42, size = 137, normalized size = 0.82

$$\frac{\left((Bp-2\,A+3\,B)\cos\left(fx+e\right)^{3}+\left(Bp^{2}-2\,(A-3\,B)p-6\,A+9\,B\right)\cos\left(fx+e\right)\sin\left(fx+e\right)+\left(Ap^{2}+2\,(3\,A-B)p+9\,A-6\,B\right)\cos\left(fx+e\right)\right)\left(g\cos\left(fx+e\right)\right)^{p}\left(-c\sin\left(fx+e\right)+c\right)^{-p-3}}{fp^{3}+9\,fp^{2}+23\,fp+15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - 2*A + 3*B)*cos(f*x + e)^3 + (B*p^2 - 2*(A - 3*B)*p - 6*A + 9*B)*cos
(f*x + e)*sin(f*x + e) + (A*p^2 + 2*(3*A - B)*p + 9*A - 6*B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 3)/(f*p^3 + 9*f*p^2 + 23*f*p
 + 15*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-3-p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-3-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 3), x)
```

Mupad [B]
time = 11.94, size = 234, normalized size = 1.39

$$-\frac{(g\cos(e+fx))^p\,(30\,A\cos(e+fx)-15\,B\cos(e+fx)-2\,A\cos(3\,e+3\,f\,x)+3\,B\cos(3\,e+3\,f\,x)-12\,A\sin(2\,e+2\,f\,x)+18\,B\sin(2\,e+2\,f\,x)+2\,B\,p^2\sin(2\,e+2\,f\,x)+24\,A\,p\cos(e+fx)-5\,B\,p\cos(e+fx)+4\,A\,p^2\cos(e+fx)+B\,p\cos(3\,e+3\,f\,x)-4\,A\,p\sin(2\,e+2\,f\,x)+12\,B\,p\sin(2\,e+2\,f\,x))}{c^3\,f\,(-c\,(\sin(e+fx)-1))^p\,(p^3+9\,p^2+23\,p+15)\,(15\sin(e+fx)+6\cos(2\,e+2\,f\,x)-\sin(3\,e+3\,f\,x)-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 3),
x)
```

```
[Out] -((g*cos(e + f*x))^p*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos(3*e +
 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e + 2*f
*x) + 2*B*p^2*sin(2*e + 2*f*x) + 24*A*p*cos(e + f*x) - 5*B*p*cos(e + f*x) +
 4*A*p^2*cos(e + f*x) + B*p*cos(3*e + 3*f*x) - 4*A*p*sin(2*e + 2*f*x) + 12*
B*p*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^p*(23*p + 9*p^2 + p^3
 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x) - 10))
```

3.1035 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx$

Optimal. Leaf size=102

$$\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-2-p}}{fg(3+p)}+\frac{(A-B(2+p))(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1}}{cfg(1+p)(3+p)}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-2-p)/f/g/(3+p)+(A-B*(2+p))*(g
*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/c/f/g/(p^2+4*p+3)
```

Rubi [A]
time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2938, 2750}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-2}(g\cos(e+fx))^{p+1}}{fg(p+3)}+\frac{(A-B(p+2))(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{cfg(p+1)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-2 - p))/(f*g*(3 +
p)) + ((A - B*(2 + p))*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 -
p))/(c*f*g*(1 + p)*(3 + p))
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-2-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(3+p)}$$
$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+}{fg(3+p)}$$

Mathematica [A]
time = 0.13, size = 83, normalized size = 0.81

$$\frac{\cos(e+fx)(g\cos(e+fx))^p(c-c\sin(e+fx))^{-p}(-B+A(2+p)+(-A+B(2+p))\sin(e+fx))}{c^2f(1+p)(3+p)(-1+\sin(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2
- p),x]
[Out] (Cos[e + f*x]*(g*Cos[e + f*x])^p*(-B + A*(2 + p) + (-A + B*(2 + p))*Sin[e +
 f*x]))/(c^2*f*(1 + p)*(3 + p)*(-1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^p
)
```

Maple [F]
time = 1.06, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p(A+B\sin(fx+e))(c-c\sin(fx+e))^{-2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 2), x)
```

Fricas [A]
time = 0.41, size = 89, normalized size = 0.87

$$\frac{((Bp - A + 2\,B)\cos{(fx + e)}\sin{(fx + e)} + (Ap + 2\,A - B)\cos{(fx + e)})(g\cos{(fx + e)})^{p}\,(-c\sin{(fx + e)} + c)^{-p-2}}{fp^{2} + 4\,fp + 3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="fricas")
```

```
[Out] ((B*p - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (A*p + 2*A - B)*cos(f*x + e))*
(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p - 2)/(f*p^2 + 4*f*p + 3*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-2-p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-2-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 2), x)
```

Mupad [B]
time = 1.28, size = 129, normalized size = 1.26

$$-\frac{(g\,\cos{(e+f\,x)})^{p}\,(4\,A\,\cos{(e+f\,x)} - 2\,B\,\cos{(e+f\,x)} - A\,\sin{(2\,e+2\,f\,x)} + 2\,B\,\sin{(2\,e+2\,f\,x)} + 2\,A\,p\,\cos{(e+f\,x)} + B\,p\,\sin{(2\,e+2\,f\,x)})}{c^{2}\,f\,(-c\,(\sin{(e+f\,x)}-1))^{p}\,(4\,\sin{(e+f\,x)} + \cos{(2\,e+2\,f\,x)} - 3)\,(p^{2}+4\,p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 2),
x)
```

```
[Out] -((g*cos(e + f*x))^p*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e + 2*f
*x) + 2*B*sin(2*e + 2*f*x) + 2*A*p*cos(e + f*x) + B*p*sin(2*e + 2*f*x)))/(c
^2*f*(-c*(sin(e + f*x) - 1))^p*(4*sin(e + f*x) + cos(2*e + 2*f*x) - 3)*(4*p
 + p^2 + 3))
```

3.1036 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx$

Optimal. Leaf size=151

$$\frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))^{-1-p}}{fg(1+p)}-\frac{2^{\frac{1}{2}-\frac{p}{2}}B(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1+p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e\cdots\right.}{fg(1+\cdots}$$

```
[Out] (A+B)*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)-2^(1/2-1/2*p)*
B*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*p, 1/2+1/2*p],[3/2+1/2*p],1/2+1/2
*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)
```

Rubi [A]

time = 0.17, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2938, 2768, 72, 71}

$$\frac{(A+B)(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}}{fg(p+1)}-\frac{B2^{\frac{1}{2}-\frac{p}{2}}(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - p),x
]
```

```
[Out] ((A + B)*(g*Cos[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 +
p)) - (2^(1/2 - p/2)*B*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 + p)/2
, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 + p)/2
)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2938

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-1-p}\,dx = \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e}{fg(1+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e}{fg(1+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e}{fg(1+p)}$$

$$= \frac{(A+B)(g\cos(e+fx))^{1+p}(c-c\sin(e}{fg(1+p)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.45, size = 300, normalized size = 1.99

$$-\frac{2^{-p}(g\cos(e+fx))^p\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^{-2(-1-p)-2p}(c-c\sin(e+fx))^{-1-p}\left(\frac{1-\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\frac{1}{1+\cos(e+fx)}}}\right)^{2p}\left(\frac{1-\tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}}\right)^{-2p}\left(-iB(1+p)\,{}_2F_1\left(1,-p;1-p;-\frac{i\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)}{1+\tan\left(\frac{1}{2}(e+fx)\right)}\right)\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)+iB(1+p)\,{}_2F_1\left(1,-p;1-p;\frac{i\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)}{1+\tan\left(\frac{1}{2}(e+fx)\right)}\right)\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)+(A+B)p\left(1+\tan\left(\frac{1}{2}(e+fx)\right)\right)\right)}{fp(1+p)\left(-1+\tan\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1
- p),x]
```

```
[Out] -(((g*Cos[e + f*x])^p*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(-2*(-1 - p) -
2*p)*(c - c*Sin[e + f*x])^(-1 - p)*((1 - Tan[(e + f*x)/2])/Sqrt[(1 + Cos[e
+ f*x])^(-1)])^(2*p)*((-I)*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, ((-I)*
```

```
(-1 + Tan[(e + f*x)/2]))/(1 + Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) +
I*B*(1 + p)*Hypergeometric2F1[1, -p, 1 - p, (I*(-1 + Tan[(e + f*x)/2]))/(1
+ Tan[(e + f*x)/2])]*(-1 + Tan[(e + f*x)/2]) + (A + B)*p*(1 + Tan[(e + f*x)
/2])))/(2^p*f*p*(1 + p)*((1 - Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2*p)*(-1 + Tan[(e + f*x)/2])))
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{p} (A+B\sin(fx+e)) (c-c\sin(fx+e))^{-1-p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
- 1), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(-1-p),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(-1-p),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{p+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 1),
x)
```

```
[Out] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^(p + 1),
 x)
```

3.1037 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx$

Optimal. Leaf size=147

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}c(A+Bp)(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1+p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}(c-c\sin(e+f}{fg(1+p)}$$

```
[Out] 2^(1/2-1/2*p)*c*(B*p+A)*(g*cos(f*x+e))^(1+p)*hypergeom([1/2+1/2*p, 1/2+1/2*
p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c*sin(f*x+
e))^(-1-p)/f/g/(1+p)-B*(g*cos(f*x+e))^(1+p)/f/g/((c-c*sin(f*x+e))^p)
```

Rubi [A]

time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2939, 2768, 72, 71}

$$\frac{c2^{\frac{1}{2}-\frac{p}{2}}(A+Bp)(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}-\frac{B(c-c\sin(e+fx))^{-p}(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,x]
```

```
[Out] (2^(1/2 - p/2)*c*(A + B*p)*(g*Cos[e + f*x])^(1 + p)*Hypergeometric2F1[(1 +
p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^((1 +
p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e + f*x])^(1
 + p))/(f*g*(c - c*Sin[e + f*x])^p)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))}{fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))}{fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx))}{fg}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c(A+Bp)(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1+}{2}\right.}{}$$

Mathematica [A]
time = 0.40, size = 144, normalized size = 0.98

$$-\frac{2^{\frac{1}{2}(-1-p)}\cos(e+fx)(g\cos(e+fx))^p\left(2(A+Bp)\,{}_2F_1\left(\frac{1+p}{2},\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}+2^{\frac{1+p}{2}}B(1+p)(-1+\sin(e+fx))\right)(c-c\sin(e+fx))^{-p}}{f(1+p)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((g*Cos[e + f*x])^p*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^p,
x]
```

```
[Out] -((2^((-1 - p)/2)*Cos[e + f*x]*(g*Cos[e + f*x])^p*(2*(A + B*p)*Hypergeometr
ic2F1[(1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])))/(f*(1 + p)
*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{p} (c-c\sin(fx+e))^{-p} (A+B\sin(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p,
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p, x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^p),x, algorit
hm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p/(-c*sin(f*x + e) + c)^p,
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)}{(c-c\,\sin\left(e+f\,x\right))^p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^p,x)
```

```
[Out] int(((g*cos(e + f*x))^p*(A + B*sin(e + f*x)))/(c - c*sin(e + f*x))^p, x)
```

3.1038 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx$

Optimal. Leaf size=160

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}c^2(2A-B(1-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-1+p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}}{fg(1+p)}$$

```
[Out] 2^(1/2-1/2*p)*c^2*(2*A-B*(1-p))*(g*cos(f*x+e))^(1+p)*hypergeom([-1/2+1/2*p,
 1/2+1/2*p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*(c-c
*sin(f*x+e))^(-1-p)/f/g/(1+p)-1/2*B*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e))^(
1-p)/f/g
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2939, 2768, 72, 71}

$$\frac{c^2 2^{\frac{1}{2}-\frac{p}{2}}(2A-B(1-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-1}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{fg(p+1)}-\frac{B(c-c\sin(e+fx))^{1-p}(g\cos(e+fx))^{p+1}}{2fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 - p),x]
```

```
[Out] (2^(1/2 - p/2)*c^2*(2*A - B*(1 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(f*g*(1 + p)) - (B*(g*Cos[e
 + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(1 - p))/(2*f*g)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{1-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{2fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{2fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{2fg}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}c^2(2A-B(1-p))(g\cos(e+fx))^{1}}{}$$

Mathematica [A]
time = 0.45, size = 150, normalized size = 0.94

$$\frac{2^{\frac{1}{2}(-3-p)}c\cos(e+fx)(g\cos(e+fx))^p\left(-4(2A+B(-1+p))\,{}_2F_1\left(\frac{1}{2}(-1+p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}+2^{\frac{1+p}{2}}B(1+p)(-1+\sin(e+fx))^2\right)(c-c\sin(e+fx))^{-p}}{f(1+p)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(1 -
 p),x]
```

```
[Out] (2^((-3 - p)/2)*c*Cos[e + f*x]*(g*Cos[e + f*x])^p*(-4*(2*A + B*(-1 + p))*Hy
pergeometric2F1[(-1 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1
- Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f*x])^2
))/(f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p (A+B\sin(fx+e)) (c-c\sin(fx+e))^{1-p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

```
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-p),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-p),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos\left(e+f\,x\right))^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{1-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1 - p),x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1 - p), x
)
```

3.1039 $\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx$

Optimal. Leaf size=163

$$\frac{2^{\frac{5}{2}-\frac{p}{2}}c^3(3A-B(2-p))(g\cos(e+fx))^{1+p}\,{}_2F_1\left(\frac{1}{2}(-3+p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}}{3fg(1+p)}$$

```
[Out] 1/3*2^(5/2-1/2*p)*c^3*(3*A-B*(2-p))*(g*cos(f*x+e))^(1+p)*hypergeom([-3/2+1/
2*p, 1/2+1/2*p],[3/2+1/2*p],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+1/2*p)*
(c-c*sin(f*x+e))^(-1-p)/f/g/(1+p)-1/3*B*(g*cos(f*x+e))^(1+p)*(c-c*sin(f*x+e
))^(2-p)/f/g
```

Rubi [A]
time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2939, 2768, 72, 71}

$$\frac{c^3 2^{\frac{5}{2}-\frac{p}{2}}(3A-B(2-p))(1-\sin(e+fx))^{\frac{p+1}{2}}(c-c\sin(e+fx))^{-p-1}(g\cos(e+fx))^{p+1}\,{}_2F_1\left(\frac{p-3}{2},\frac{p+1}{2};\frac{p+3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{3fg(p+1)}-\frac{B(c-c\sin(e+fx))^{2-p}(g\cos(e+fx))^{p+1}}{3fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - p),x]
```

```
[Out] (2^(5/2 - p/2)*c^3*(3*A - B*(2 - p))*(g*Cos[e + f*x])^(1 + p)*Hypergeometri
c2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f
*x])^((1 + p)/2)*(c - c*Sin[e + f*x])^(-1 - p))/(3*f*g*(1 + p)) - (B*(g*Cos
[e + f*x])^(1 + p)*(c - c*Sin[e + f*x])^(2 - p))/(3*f*g)
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 72

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n]), Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 2768

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2))), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 2939

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a +
b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a
^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(A+B\sin(e+fx))(c-c\sin(e+fx))^{2-p}\,dx = -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{3fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{3fg}$$

$$= -\frac{B(g\cos(e+fx))^{1+p}(c-c\sin(e+fx)}{3fg}$$

$$= \frac{2^{\frac{5}{2}-\frac{p}{2}}c^3(3A-B(2-p))(g\cos(e+fx))^{1}}{}$$

Mathematica [A]
time = 0.61, size = 155, normalized size = 0.95

$$-\frac{2^{\frac{1}{2}(-1-p)}c^2\cos(e+fx)(g\cos(e+fx))^p\left(8(3A+B(-2+p))\,{}_2F_1\left(\frac{1}{2}(-3+p),\frac{1+p}{2};\frac{3+p}{2};\frac{1}{2}(1+\sin(e+fx))\right)(1-\sin(e+fx))^{\frac{1+p}{2}}+2^{\frac{1+p}{2}}B(1+p)(-1+\sin(e+fx))^3\right)(c-c\sin(e+fx))^{-p}}{3f(1+p)(-1+\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 -
 p),x]
```

```
[Out] -1/3*(2^((-1 - p)/2)*c^2*Cos[e + f*x]*(g*Cos[e + f*x])^p*(8*(3*A + B*(-2 +
p))*Hypergeometric2F1[(-3 + p)/2, (1 + p)/2, (3 + p)/2, (1 + Sin[e + f*x])/
2]*(1 - Sin[e + f*x])^((1 + p)/2) + 2^((1 + p)/2)*B*(1 + p)*(-1 + Sin[e + f
*x])^3))/(f*(1 + p)*(-1 + Sin[e + f*x])*(c - c*Sin[e + f*x])^p)
```

Maple [F]
time = 0.50, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{p} (A+B\sin(fx+e)) (c-c\sin(fx+e))^{2-p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
[Out] int((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
+ 2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-p),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-p),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^p*(-c*sin(f*x + e) + c)^(-p
 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos\left(e+f\,x\right))^p\,\left(A+B\,\sin\left(e+f\,x\right)\right)\,{\left(c-c\,\sin\left(e+f\,x\right)\right)}^{2-p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(2 - p),x)
```

```
[Out] int((g*cos(e + f*x))^p*(A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(2 - p), x
)
```

3.1040 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=32

$$\frac{A(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg}$$

[Out] A*(g*cos(f*x+e))^(1+p)*(a+a*sin(f*x+e))^m/f/g

Rubi [A]
time = 0.07, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2933}

$$\frac{A(a\sin(e+fx)+a)^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Sin[e + f*x]),x]

[Out] (A*(g*Cos[e + f*x])^(1 + p)*(a + a*Sin[e + f*x])^m)/(f*g)

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(Am-A(1+m+p)\sin(e+fx))\,dx = \frac{A(g\cos(e+fx))^{1+p}(a+a\sin(e+fx))^m}{fg}$$

Mathematica [A]
time = 0.13, size = 33, normalized size = 1.03

$$\frac{A\cos(e+fx)(g\cos(e+fx))^p(a(1+\sin(e+fx)))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(A*m - A*(1 + m + p)*Si
n[e + f*x]),x]
```

```
[Out] (A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a*(1 + Sin[e + f*x]))^m)/f
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \left(g \cos \left(f x +e \right)\right)^{p} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(A m -A \left(1+m +p \right) \sin \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] -integrate((A*(m + p + 1)*sin(f*x + e) - A*m)*(g*cos(f*x + e))^p*(a*sin(f*x
 + e) + a)^m, x)
```

Fricas [A]
time = 0.38, size = 36, normalized size = 1.12

$$\frac{\left(g \cos \left(f x + e\right)\right)^{p} {\left(a \sin \left(f x + e\right) + a\right)}^{m} A \cos \left(f x + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] (g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$- A \left(\int \left(- m \left(g \cos{\left(e + f x \right)}\right)^{p} \left(a \sin{\left(e + f x \right)} + a\right)^{m}\right)\, dx + \int \left(g \cos{\left(e + f x \right)}\right)^{p} \left(a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx + \int m \left(g \cos{\left(e + f x \right)}\right)^{p} \left(a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx + \int p \left(g \cos{\left(e + f x \right)}\right)^{p} \left(a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(A*m-A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] -A*(Integral(-m*(g*cos(e + f*x))**p*(a*sin(e + f*x) + a)**m, x) + Integral(
(g*cos(e + f*x))**p*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(m*(
g*cos(e + f*x))**p*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(p*(g
*cos(e + f*x))**p*(a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1878 vs. $2(34) = 68$.
time = 5.60, size = 1878, normalized size = 58.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(A*m-A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] (A*e^(-m*log(2) - p*log(2) + p*log(4*abs(g)*abs(tan(1/8*pi - 1/4*f*x - 1/4*
e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*m*log(2*abs(tan(1/8*pi - 1/4
*f*x - 1/4*e)^2 - 1)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + p*log(2*abs(t
an(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1))
+ m*log(abs(a)))*tan(1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2*f*
x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + pi*m*floor(1/2*f*x
/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor
(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + 1/2*pi
*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4)
+ pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/p
i + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2
*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) + g) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2
 - 1) - 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) - 3/4*pi
*m - 1/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 - A*e^(-m*log(2) - p*log(2) + p*log
(4*abs(g)*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)
^2 + 1)) + 2*m*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)/(tan(1/8*pi -
 1/4*f*x - 1/4*e)^2 + 1)) + p*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1
)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)))*tan(1/4*pi*p*sgn(
g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x +
1/2*e)^2 - 1)*sgn(g) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi
+ 1/2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2
*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - f
loor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi
 + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a)
+ 1/2) + 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) +
 g) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*p*sgn(tan(1/2*f*x +
 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) - 3/4*pi*m - 1/4*pi*p)^2 - A*e^(-m*log(2)
- p*log(2) + p*log(4*abs(g)*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi
- 1/4*f*x - 1/4*e)^2 + 1)) + 2*m*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2
- 1)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + p*log(2*abs(tan(1/8*pi - 1/4*
```

```
f*x - 1/4*e)^2 - 1)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)))
*tan(1/2*f*x + 1/2*e)^2 + A*e^(-m*log(2) - p*log(2) + p*log(4*abs(g)*abs(ta
n(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*m*l
og(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)/(tan(1/8*pi - 1/4*f*x - 1/4*e
)^2 + 1)) + p*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)/(tan(1/8*pi -
1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a))))/(f*tan(1/4*pi*p*sgn(g*tan(1/2*f*
x + 1/2*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
)*sgn(g) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi +
 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/
2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x
/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi
*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1/2) + 1/4
*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) + g) - 1/2*pi
*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) + 1/4*pi*m*sgn(a) - 3/4*pi*m - 1/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 + f*ta
n(1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn
(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - fl
oor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e
/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + 1/2*pi*p*floor(1/2*f*x/pi
 + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*
x/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*flo
or(-1/4*sgn(a) + 1/2) + 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 - 2*g*tan(1/2
*f*x + 1/2*e) + g) - 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) - 1/4*pi*p*sg
n(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) - 3/4*pi*m - 1/4*pi*p)^2 +
f*tan(1/2*f*x + 1/2*e)^2 + f)
```

Mupad [B]
time = 9.70, size = 33, normalized size = 1.03

$$\frac{A\,\cos\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(a\,\left(\sin\left(e+f\,x\right)+1\right)\right)}^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A*m - A*sin(e + f*x)*(m + p + 1))*(a + a*sin(e + f*
x))^m,x)
```

```
[Out] (A*cos(e + f*x)*(g*cos(e + f*x))^p*(a*(sin(e + f*x) + 1))^m)/f
```

3.1041 $\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx$

Optimal. Leaf size=34

$$-\frac{A(g\cos(e+fx))^{1+p}(a-a\sin(e+fx))^m}{fg}$$

```
[Out] -A*(g*cos(f*x+e))^(1+p)*(a-a*sin(f*x+e))^m/f/g
```

Rubi [A]
time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.025$, Rules used = {2933}

$$-\frac{A(a-a\sin(e+fx))^m(g\cos(e+fx))^{p+1}}{fg}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Sin[e + f*x]),x]
```

```
[Out] -((A*(g*Cos[e + f*x])^(1 + p)*(a - a*Sin[e + f*x])^m)/(f*g))
```

Rule 2933

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] /;
FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b
*c*(m + p + 1), 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a-a\sin(e+fx))^m(Am+A(1+m+p)\sin(e+fx))\,dx = -\frac{A(g\cos(e+fx))^{1+p}(a-a}{fg}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 1.03

$$-\frac{A\cos(e+fx)(g\cos(e+fx))^p(a-a\sin(e+fx))^m}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m*(A*m + A*(1 + m + p)*Si
n[e + f*x]),x]
```

```
[Out] -((A*Cos[e + f*x]*(g*Cos[e + f*x])^p*(a - a*Sin[e + f*x])^m)/f)
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(g \cos \left(f x +e \right)\right)^{p} \left(a -a \sin \left(f x +e \right)\right)^{m} \left(A m +A (1+m +p) \sin \left(f x +e \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="maxima")
```

```
[Out] integrate((A*(m + p + 1)*sin(f*x + e) + A*m)*(g*cos(f*x + e))^p*(-a*sin(f*x
 + e) + a)^m, x)
```

Fricas [A]
time = 0.42, size = 38, normalized size = 1.12

$$-\frac{\left(g \cos \left(f x + e\right)\right)^{p} {\left(-a \sin \left(f x + e\right) + a\right)}^{m} A \cos \left(f x + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="fricas")
```

```
[Out] -(g*cos(f*x + e))^p*(-a*sin(f*x + e) + a)^m*A*cos(f*x + e)/f
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A \left(\int m \left(g \cos{\left(e + f x \right)}\right)^{p} \left(- a \sin{\left(e + f x \right)} + a\right)^{m}\, dx + \int \left(g \cos{\left(e + f x \right)}\right)^{p} \left(- a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx + \int m \left(g \cos{\left(e + f x \right)}\right)^{p} \left(- a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx + \int p \left(g \cos{\left(e + f x \right)}\right)^{p} \left(- a \sin{\left(e + f x \right)} + a\right)^{m} \sin{\left(e + f x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a-a*sin(f*x+e))**m*(A*m+A*(1+m+p)*sin(f*x+e)),
x)
```

```
[Out] A*(Integral(m*(g*cos(e + f*x))**p*(-a*sin(e + f*x) + a)**m, x) + Integral((
g*cos(e + f*x))**p*(-a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(m*(
g*cos(e + f*x))**p*(-a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(p*(
g*cos(e + f*x))**p*(-a*sin(e + f*x) + a)**m*sin(e + f*x), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1863 vs. $2(36) = 72$.
time = 5.74, size = 1863, normalized size = 54.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a-a*sin(f*x+e))^m*(A*m+A*(1+m+p)*sin(f*x+e)),x,
 algorithm="giac")
```

```
[Out] -(A*e^(-m*log(2) - p*log(2) + p*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 -
 1)*abs(g)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(1/8*p
i - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + p*log(4*abs(
tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*l
og(abs(a)))*tan(-1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x +
1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + 1/2*pi*p*floor(1/2*f*x
/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + pi*m*floor(1/2
*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1/2*pi*p*f
loor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi
*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/2*e/pi +
1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2
+ 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)
+ 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1
/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 - A*e^(-m*log(2) - p*log(2) + p*log(2*abs
(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)*abs(g)/(tan(1/8*pi - 1/4*f*x - 1/4*e)
^2 + 1)) + 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f
*x - 1/4*e)^2 + 1)) + p*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*p
i - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)))*tan(-1/4*pi*p*sgn(g*tan(1/2*f
*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 -
1)*sgn(g) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e
/pi + 1/2) + 3/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1
/2*e/pi + 1/2) + 1/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*
x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + p
i*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4*
pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi*
m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1
) + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1/4*pi*p)^2 - A*e^(-m*log(2) - p*log(2) +
p*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 - 1)*abs(g)/(tan(1/8*pi - 1/4*f
*x - 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8
*pi - 1/4*f*x - 1/4*e)^2 + 1)) + p*log(4*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))
```

```
/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + m*log(abs(a)))*tan(1/2*f*x + 1/2*
e)^2 + A*e^(-m*log(2) - p*log(2) + p*log(2*abs(tan(1/8*pi - 1/4*f*x - 1/4*e
)^2 - 1)*abs(g)/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + 2*m*log(4*abs(tan(
1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1)) + p*log(4
*abs(tan(1/8*pi - 1/4*f*x - 1/4*e))/(tan(1/8*pi - 1/4*f*x - 1/4*e)^2 + 1))
+ m*log(abs(a))))/(f*tan(-1/4*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1
/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 - 1)*sgn(g) + 1/2*pi*p*floo
r(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 3/4) + pi*m*
floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + 1/4) + 1
/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*p*floor(1/2*f*x/pi + 1/
2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4*pi*p*sgn(g*tan(1/2*f*x +
1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi*m*sgn(tan(1/2*f*x + 1/2*e
)^2 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*m*sgn(a) + 1/4
*pi*m + 1/4*pi*p)^2*tan(1/2*f*x + 1/2*e)^2 + f*tan(-1/4*pi*p*sgn(g*tan(1/2*
f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g)*sgn(tan(1/2*f*x + 1/2*e)^2 -
 1)*sgn(g) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi + 1/2*
e/pi + 1/2) + 3/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f*x/pi +
1/2*e/pi + 1/2) + 1/4) + 1/2*pi*p*floor(1/2*f*x/pi + 1/2*e/pi - floor(1/2*f
*x/pi + 1/2*e/pi + 1/2) + 1/4) + pi*m*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) +
pi*p*floor(1/2*f*x/pi + 1/2*e/pi + 1/2) + pi*m*floor(-1/4*sgn(a) + 1) - 1/4
*pi*p*sgn(g*tan(1/2*f*x + 1/2*e)^2 + 2*g*tan(1/2*f*x + 1/2*e) + g) + 1/2*pi
*m*sgn(tan(1/2*f*x + 1/2*e)^2 - 1) + 1/4*pi*p*sgn(tan(1/2*f*x + 1/2*e)^2 -
1) + 1/4*pi*m*sgn(a) + 1/4*pi*m + 1/4*pi*p)^2 + f*tan(1/2*f*x + 1/2*e)^2 +
f)
```

Mupad [B]
time = 0.52, size = 35, normalized size = 1.03

$$-\frac{A\,\cos\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^p\,{\left(-a\,\left(\sin\left(e+f\,x\right)-1\right)\right)}^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(A*m + A*sin(e + f*x)*(m + p + 1))*(a - a*sin(e + f*
x))^m,x)
```

```
[Out] -(A*cos(e + f*x)*(g*cos(e + f*x))^p*(-a*(sin(e + f*x) - 1))^m)/f
```

3.1042 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=168

$$\frac{2^{\frac{1+p}{2}}gF_1\left(\frac{1}{2}(1+2m+p);\frac{1-p}{2},-n;\frac{1}{2}(3+2m+p);\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)(g\cos(e+fx))^{-1+p}(1-}{af(1+2m+p)}$$

```
[Out] 2^(1/2+1/2*p)*g*AppellF1(1/2+m+1/2*p,-n,1/2-1/2*p,3/2+m+1/2*p,-d*(1+sin(f*x
+e))/(c-d),1/2+1/2*sin(f*x+e))*(g*cos(f*x+e))^(-1+p)*(1-sin(f*x+e))^(1/2-1/
2*p)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n/a/f/(1+2*m+p)/(((c+d*sin(f*x
+e))/(c-d))^n)
```

Rubi [A]

time = 0.19, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3000, 145, 144, 143}

$$\frac{g2^{\frac{p+1}{2}}(1-\sin(e+fx))^{\frac{1-p}{2}}(a\sin(e+fx)+a)^{m+1}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m+p+1);\frac{1-p}{2},-n;\frac{1}{2}(2m+p+3);\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m+p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2^((1 + p)/2)*g*AppellF1[(1 + 2*m + p)/2, (1 - p)/2, -n, (3 + 2*m + p)/2,
(1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*(g*Cos[e + f*x])^(
-1 + p)*(1 - Sin[e + f*x])^((1 - p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c + d*
Sin[e + f*x])^n)/(a*f*(1 + 2*m + p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] &&  !GtQ[b/
(b*c - a*d), 0] &&  !SimplerQ[c + d*x, a + b*x] &&  !SimplerQ[e + f*x, a +
b*x]
```

Rule 3000

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[g*
((g*Cos[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(a-a\sin(e+fx))\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e\right.}{}$$

$$= \frac{\left(2^{-\frac{1}{2}+\frac{p}{2}}g(g\cos(e+fx))^{-1+p}(a-a\sin(e\right.}{}$$

$$= \frac{2^{\frac{1+p}{2}}gF_1\left(\frac{1}{2}(1+2m+p);\frac{1-p}{2},-n;\frac{1}{2}(3+2\right.}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 798 vs. 2(168) = 336.
time = 7.91, size = 798, normalized size = 4.75

$$-\frac{2F_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)(g\cos(e+fx))^p\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)(a(1+\sin(e+fx)))^m(c+d\sin(e+fx))^n\sin\left(\frac{1}{4}(2e+\pi+2fx)\right)}{f\left(F_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)+\frac{2(1+p)\left((c-d)nF_1\left(\frac{3+p}{2};1+m+n+p,1-n;\frac{5+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)-(c+d)(1+m+n+p)F_1\left(\frac{3+p}{2};2+m+n+p,-n;\frac{5+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\right)\cot^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}{(c+d)(3+p)}+pF_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sin(e+fx)-\frac{dnF_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\cos^2(e+fx)}{c+d\sin(e+fx)}+2(n+p)F_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)-\frac{2(c-d)nF_1\left(\frac{1+p}{2};1+m+n+p,-n;\frac{3+p}{2};-\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right),-\frac{(c-d)\tan^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d}\right)\sin^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{c+d\sin(e+fx)}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*
x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*(g*Cos[e + f*x])
^p*Cos[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^
n*Sin[(2*e + Pi + 2*f*x)/4])/(f*(AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3
+ p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^
2)/(c + d))] + (2*(1 + p)*((c - d)*n*AppellF1[(3 + p)/2, 1 + m + n + p, 1 -
 n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f
*x)/4]^2)/(c + d))] - (c + d)*(1 + m + n + p)*AppellF1[(3 + p)/2, 2 + m + n
 + p, -n, (5 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi
 + 2*f*x)/4]^2)/(c + d))])*Cot[(2*e + Pi + 2*f*x)/4]^2)/((c + d)*(3 + p)) +
 p*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[e + f*x] - (d
*n*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[e + f*x]^2)/(
c + d*Sin[e + f*x]) + 2*(n + p)*AppellF1[(1 + p)/2, 1 + m + n + p, -n, (3 +
 p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2
)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2 - (2*(c - d)*n*AppellF1[(1 + p)/2,
1 + m + n + p, -n, (3 + p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[
(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[
e + f*x])))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(g \cos \left(f x +e \right)\right)^{p} \left(a +a \sin \left(f x +e \right)\right)^{m} \left(c +d \sin \left(f x +e \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\,\cos(e+f\,x))^p\,(a+a\,\sin(e+f\,x))^m\,(c+d\,\sin(e+f\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.1043 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=149

$$-\frac{2^{\frac{5}{2}+\frac{p}{2}}a^2F_1\left(\frac{1+p}{2};\frac{1}{2}(-3-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{2}}{fg(1+p)}$$

```
[Out] -2^(5/2+1/2*p)*a^2*AppellF1(1/2+1/2*p,-n,-3/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+
e))/(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2
*p)*(c+d*sin(f*x+e))^n/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]
time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2999, 144, 143}

$$-\frac{a^2g2^{\frac{p+5}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-3),-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((5 + p)/2)*a^2*g*AppellF1[(1 + p)/2, (-3 - p)/2, -n, (3 + p)/2, (1 -
Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*
(1 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))\right.}{}$$

$$= \frac{\left(a^2g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))\right.}{}$$

$$= -\frac{2^{\frac{5+p}{2}}a^2gF_1\left(\frac{1+p}{2};\frac{1}{2}(-3-p),-n;\frac{3+p}{2};\frac{1}{2}(1\right.}{}$$

Mathematica [F]
time = 20.24, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,
 x]
```

Maple [F]
time = 0.70, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^p\,(a+a\sin(fx+e))^2\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*cos(f*x + e)
)^p*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^p (a+a\sin(e+fx))^2 (c+d\sin(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.1044 $\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=145

$$-\frac{2^{\frac{3}{2}+\frac{p}{2}}aF_1\left(\frac{1+p}{2};\frac{1}{2}(-1-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{\frac{1}{2}(}}{fg(1+p)}$$

```
[Out] -2^(3/2+1/2*p)*a*AppellF1(1/2+1/2*p,-n,-1/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e)
)/(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p
)*(c+d*sin(f*x+e))^n/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]

time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2947, 144, 143}

$$-\frac{ag2^{\frac{p+3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{1}{2}(-p-1),-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -((2^((3 + p)/2)*a*g*AppellF1[(1 + p)/2, (-1 - p)/2, -n, (3 + p)/2, (1 - Si
n[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1
 - Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2947

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c*g*((g
*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e + f*x])
^((p - 1)/2))), Subst[Int[(1 + (d/c)*x)^((p + 1)/2)*(1 - (d/c)*x)^((p - 1)/
2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

Rubi steps

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-}{2}}\right.}{}$$

$$= \frac{\left(ag(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-}{2}}\right.}{}$$

$$= -\frac{2^{\frac{3+p}{2}}agF_1\left(\frac{1+p}{2};\frac{1}{2}(-1-p),-n;\frac{3+p}{2};\frac{1}{2}(1-\right.}{}$$

Mathematica [F]
time = 5.02, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^p(a+a\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[(g*Cos[e + f*x])^p*(a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x
]
```

Maple [F]
time = 0.35, size = 0, normalized size = 0.00

$$\int \left(g\cos\left(fx+e\right)\right)^{p}\left(a+a\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (g\cos(e+fx))^p\,(c+d\sin(e+fx))^n\,dx+\int (g\cos(e+fx))^p\,(c+d\sin(e+fx))^n\sin(e+fx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] a*(Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n, x) + Integral((g*c
os(e + f*x))**p*(c + d*sin(e + f*x))**n*sin(e + f*x), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(a+a*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm
="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (g\cos(e+fx))^p\,(a+a\sin(e+fx))\,\left(c+d\sin(e+fx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int((g*cos(e + f*x))^p*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.1045 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{-\frac{1}{2}+\frac{p}{2}}F_1\left(\frac{1+p}{2};\frac{3-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{-1+\frac{1-p}{2}}(c}{afg(1+p)}$$

```
[Out] -2^(-1/2+1/2*p)*AppellF1(1/2+1/2*p,-n,3/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]

time = 0.15, antiderivative size = 155, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2999, 144, 143}

$$-\frac{g2^{\frac{p}{2}-\frac{1}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{3-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{af(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),x]
```

```
[Out] -((2^(-1/2 + p/2)*g*AppellF1[(1 + p)/2, (3 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a*f*
(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= -\frac{2^{-\frac{1}{2}+\frac{p}{2}}gF_1\left(\frac{1+p}{2};\frac{3-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+f}{c+d}\right.}{}$$

Mathematica [F]
time = 8.05, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]),
 x]
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p\,(c+d\sin(fx+e))^n}{a+a\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\cos{(e+fx)})^{p}(c+d\sin{(e+fx)})^{n}}{\sin{(e+fx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n/(sin(e + f*x) + 1), x)
/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\cos(e+fx))^p (c+d\sin(e+fx))^n}{a+a\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)
```

3.1046 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{-\frac{3}{2}+\frac{p}{2}}F_1\left(\frac{1+p}{2};\frac{5-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{-2+\frac{3-p}{2}}(c}{a^2fg(1+p)}$$

```
[Out] -2^(-3/2+1/2*p)*AppellF1(1/2+1/2*p,-n,5/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^2/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]

time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2999, 144, 143}

$$-\frac{g2^{\frac{p-3}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{5-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{a^2f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] -((2^((-3 + p)/2)*g*AppellF1[(1 + p)/2, (5 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^2*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-3+p)}gF_1\left(\frac{1+p}{2};\frac{5-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{}$$

Mathematica [F]
time = 11.70, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
2, x]
```

Maple [F]
time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p\,(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2
*a^2*sin(f*x + e) - 2*a^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(g\cos{(e+fx)})^p(c+d\sin{(e+fx)})^n}{\sin^2{(e+fx)}+2\sin{(e+fx)}+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*cos(e + f*x))**p*(c + d*sin(e + f*x))**n/(sin(e + f*x)**2 + 2*s
in(e + f*x) + 1), x)/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)
```

```
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)
```

3.1047 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{-\frac{5}{2}+\frac{p}{2}}F_1\left(\frac{1+p}{2};\frac{7-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{-3+\frac{5-p}{2}}(c}{a^3fg(1+p)}$$

```
[Out] -2^(-5/2+1/2*p)*AppellF1(1/2+1/2*p,-n,7/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^3/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]

time = 0.17, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2999, 144, 143}

$$-\frac{g2^{\frac{p-5}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{7-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{a^3f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] -((2^((-5 + p)/2)*g*AppellF1[(1 + p)/2, (7 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^3*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-5+p)}gF_1\left(\frac{1+p}{2};\frac{7-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{}$$

Mathematica [F]
time = 16.23, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
3, x]
```

Maple [F]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p\,(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3,
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(3*a^3*cos(f*x + e)^2 -
 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^3,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^3,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3,x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^3, x)
```

3.1048 $\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$

Optimal. Leaf size=149

$$-\frac{2^{-\frac{7}{2}+\frac{p}{2}}F_1\left(\frac{1+p}{2};\frac{9-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)(g\cos(e+fx))^{1+p}(1+\sin(e+fx))^{-4+\frac{7-p}{2}}(c}{a^4fg(1+p)}$$

```
[Out] -2^(-7/2+1/2*p)*AppellF1(1/2+1/2*p,-n,9/2-1/2*p,3/2+1/2*p,d*(1-sin(f*x+e))/
(c+d),1/2-1/2*sin(f*x+e))*(g*cos(f*x+e))^(1+p)*(1+sin(f*x+e))^(-1/2-1/2*p)*
(c+d*sin(f*x+e))^n/a^4/f/g/(1+p)/(((c+d*sin(f*x+e))/(c+d))^n)
```

Rubi [A]

time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2999, 144, 143}

$$-\frac{g2^{\frac{p-7}{2}}(1-\sin(e+fx))(\sin(e+fx)+1)^{\frac{1-p}{2}}(g\cos(e+fx))^{p-1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{p+1}{2};\frac{9-p}{2},-n;\frac{p+3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{a^4f(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^4,x]
```

```
[Out] -((2^((-7 + p)/2)*g*AppellF1[(1 + p)/2, (9 - p)/2, -n, (3 + p)/2, (1 - Sin[
e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(g*Cos[e + f*x])^(-1 + p)*(1 -
 Sin[e + f*x])*(1 + Sin[e + f*x])^((1 - p)/2)*(c + d*Sin[e + f*x])^n)/(a^4*
f*(1 + p)*((c + d*Sin[e + f*x])/(c + d))^n))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
```

```
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2999

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[a^
m*g*((g*Cos[e + f*x])^(p - 1)/(f*(1 + Sin[e + f*x])^((p - 1)/2)*(1 - Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(1 + (b/a)*x)^(m + (p - 1)/2)*(1 - (b/a)*x
)^((p - 1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e
, f, n, p}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx = \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= \frac{\left(g(g\cos(e+fx))^{-1+p}(1-\sin(e+fx))^{\frac{1-p}{2}}(1+\sin(e+fx))^{\frac{1}{}}\right.}{}$$

$$= -\frac{2^{\frac{1}{2}(-7+p)}gF_1\left(\frac{1+p}{2};\frac{9-p}{2},-n;\frac{3+p}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+}{c+d}\right.}{}$$

Mathematica [F]
time = 21.27, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^p(c+d\sin(e+fx))^n}{(a+a\sin(e+fx))^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4,x]
```

```
[Out] Integrate[((g*Cos[e + f*x])^p*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^
4, x]
```

Maple [F]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p\,(c+d\sin(fx+e))^n}{(a+a\sin(fx+e))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

```
[Out] int((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a^4*cos(f*x + e)^4 - 8*
a^4*cos(f*x + e)^2 + 8*a^4 - 4*(a^4*cos(f*x + e)^2 - 2*a^4)*sin(f*x + e)),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**4,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^4,x, algorit
hm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^4,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p\,\left(c+d\,\sin\left(e+f\,x\right)\right)^n}{\left(a+a\,\sin\left(e+f\,x\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4,x)
[Out] int(((g*cos(e + f*x))^p*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^4, x)
```

3.1049 $\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=175

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}F_1\left(\frac{1}{2}(1+2m-p);\frac{1+p}{2},-n;\frac{1}{2}(3+2m-p);\frac{1}{2}(1+\sin(e+fx)),-\frac{d(1+\sin(e+fx))}{c-d}\right)\sec(e+fx)(g\sec(e+f}{af(1+2m-p)}$$

```
[Out] 2^(1/2-1/2*p)*AppellF1(1/2+m-1/2*p,-n,1/2+1/2*p,3/2+m-1/2*p,-d*(1+sin(f*x+e
))/(c-d),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(g*sec(f*x+e))^p*(1-sin(f*x+e))^(1/
2+1/2*p)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n/a/f/(1+2*m-p)/(((c+d*sin
(f*x+e))/(c-d))^n)
```

Rubi [A]

time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3005, 3000, 145, 144, 143}

$$\frac{2^{\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{m+1}(g\sec(e+fx))^p(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(\frac{1}{2}(2m-p+1);\frac{p+1}{2},-n;\frac{1}{2}(2m-p+3);\frac{1}{2}(\sin(e+fx)+1),-\frac{d(\sin(e+fx)+1)}{c-d}\right)}{af(2m-p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (2^(1/2 - p/2)*AppellF1[(1 + 2*m - p)/2, (1 + p)/2, -n, (3 + 2*m - p)/2, (1
 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*(g*Sec[
e + f*x])^p*(1 - Sin[e + f*x])^((1 + p)/2)*(a + a*Sin[e + f*x])^(1 + m)*(c
+ d*Sin[e + f*x])^n)/(a*f*(1 + 2*m - p)*((c + d*Sin[e + f*x])/(c - d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 145

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
(b*((c + d*x)/(b*c - a*d)))^FracPart[n]), Int[(a + b*x)^m*(b*(c/(b*c - a*d)
) + b*d*(x/(b*c - a*d)))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 3000

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[g*
((g*Cos[e + f*x])^(p - 1)/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e
+ f*x])^((p - 1)/2))), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
1)/2)*(c + d*x)^n, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 3005

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(g*Cos[e + f*x])^p),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int (g\sec(e+fx))^p(a+a\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = ((g\cos(e+fx))^p(g\sec(e+fx))^p)\int (g\cos($$

$$= \frac{\Big(\sec(e+fx)(g\sec(e+fx))^p(a-a\sin(e+}{}$$

$$= \frac{\Big(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a}{}$$

$$= \frac{\Big(2^{-\frac{1}{2}-\frac{p}{2}}\sec(e+fx)(g\sec(e+fx))^p(a-a}{}$$

$$= \frac{2^{\frac{1}{2}-\frac{p}{2}}F_1\Big(\frac{1}{2}(1+2m-p);\frac{1+p}{2},-n;\frac{1}{2}(3+2m}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 852 vs. $2(175) = 350$.

time = 8.82, size = 852, normalized size = 4.87

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (2*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x
)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[(2*e + Pi + 2
*f*x)/4]*(g*Sec[e + f*x])^p*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n
*Sin[(2*e + Pi + 2*f*x)/4])/(f*(-AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3
- p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^
2)/(c + d))] + (2*(1 - p)*(-(((c - d)*n*AppellF1[(3 - p)/2, 1 + m + n - p,
1 - n, (5 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi +
2*f*x)/4]^2)/(c + d))])/(c + d)) + (1 + m + n - p)*AppellF1[(3 - p)/2, 2 +
m + n - p, -n, (5 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e
 - Pi + 2*f*x)/4]^2)/(c + d))])*Cot[(2*e + Pi + 2*f*x)/4]^2)/(3 - p) + p*Ap
pellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^
2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Sin[e + f*x] + (d*n*Ap
pellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^
2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c + d))]*Cos[e + f*x]^2)/(c + d
*Sin[e + f*x]) - 2*(n - p)*AppellF1[(1 - p)/2, 1 + m + n - p, -n, (3 - p)/2
, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e - Pi + 2*f*x)/4]^2)/(c
```

```
+ d))]*Sin[(2*e - Pi + 2*f*x)/4]^2 + (2*(c - d)*n*AppellF1[(1 - p)/2, 1 + m
 + n - p, -n, (3 - p)/2, -Tan[(2*e - Pi + 2*f*x)/4]^2, -(((c - d)*Tan[(2*e
- Pi + 2*f*x)/4]^2)/(c + d))]*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f
*x])))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int (g\sec(fx+e))^{p} (a+a\sin(fx+e))^{m} (c+d\sin(fx+e))^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p*(a+a*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="giac")
```

```
[Out] integrate((g*sec(f*x + e))^p*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{g}{\cos\left(e+f\,x\right)}\right)}^p\,{\left(a+a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int((g/cos(e + f*x))^p*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.1050 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{bx}{16}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}-\frac{b\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d}$$

```
[Out] 1/16*b*x-1/3*a*cos(d*x+c)^3/d+1/5*a*cos(d*x+c)^5/d+1/16*b*cos(d*x+c)*sin(d*
x+c)/d-1/8*b*cos(d*x+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^3*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2645, 14, 2648, 2715, 8}

$$\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^3)/(3*d) + (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) - (b*Cos
[c + d*x]^3*Sin[c + d*x]^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin^3(c+dx)\,dx+b\int \cos^2(c+dx)\sin^4\\ &= -\frac{b\cos^3(c+dx)\sin^3(c+dx)}{6d}+\frac{1}{2}b\int \cos^2(c+dx)\sin^2(c\\ &= -\frac{b\cos^3(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+dx)\sin^3(c+dx}{6d}\\ &= -\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c}{16d}\\ &= \frac{bx}{16}-\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin}{16d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.73

$$\frac{60bdx-120a\cos(c+dx)-20a\cos(3(c+dx))+12a\cos(5(c+dx))-15b\sin(2(c+dx))-15b\sin(4(c+dx))+5b\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
[Out] (60*b*d*x - 120*a*Cos[c + d*x] - 20*a*Cos[3*(c + d*x)] + 12*a*Cos[5*(c + d*
x)] - 15*b*Sin[2*(c + d*x)] - 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c + d*x)])
/(960*d)
```

Maple [A]
time = 0.19, size = 95, normalized size = 0.90

method	result
risch	$\frac{bx}{16}-\frac{a\cos(dx+c)}{8d}+\frac{b\sin(6dx+6c)}{192d}+\frac{a\cos(5dx+5c)}{80d}-\frac{b\sin(4dx+4c)}{64d}-\frac{a\cos(3dx+3c)}{48d}-\frac{b\sin(2dx+2c)}{64d}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+b\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)\cos(}{16}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+b\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)\cos(}{16}\right.}{d}$
norman	$\frac{\frac{bx}{16}-\frac{4a}{15d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{17b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{19b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{19b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{17b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{b\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{8d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b*(-1/6*sin(d*x+c
)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/1
6*d*x+1/16*c))
```

Maxima [A]
time = 0.27, size = 65, normalized size = 0.62

$$\frac{64\left(3\cos\left(dx+c\right)^{5}-5\cos\left(dx+c\right)^{3}\right)a-5\left(4\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\sin\left(4\,dx+4\,c\right)\right)b}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/960*(64*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a - 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b)/d
```

Fricas [A]
time = 0.36, size = 73, normalized size = 0.70

$$\frac{48\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\,bdx+5\left(8\,b\cos\left(dx+c\right)^{5}-14\,b\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(48*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*b*d*x + 5*(8*b*cos(d*
x + c)^5 - 14*b*cos(d*x + c)^3 + 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(92) = 184$.
time = 0.45, size = 192, normalized size = 1.83

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{2a\cos^5(c+dx)}{15d} + \frac{bx\sin^6(c+dx)}{16} + \frac{3bx\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3bx\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{bx\cos^6(c+dx)}{16} + \frac{b\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{b\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin^3(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a*cos(c + d*x)**5/(
15*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b*x*cos(c + d*x)**6/16 + b*sin(
c + d*x)**5*cos(c + d*x)/(16*d) - b*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) -
 b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)
**3*cos(c)**2, True))
```

Giac [A]
time = 0.43, size = 92, normalized size = 0.88

$$\frac{1}{16}\,bx + \frac{a\cos(5\,dx + 5\,c)}{80\,d} - \frac{a\cos(3\,dx + 3\,c)}{48\,d} - \frac{a\cos(dx + c)}{8\,d} + \frac{b\sin(6\,dx + 6\,c)}{192\,d} - \frac{b\sin(4\,dx + 4\,c)}{64\,d} - \frac{b\sin(2\,dx + 2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x + 1/80*a*cos(5*d*x + 5*c)/d - 1/48*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d + 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d - 1/6
4*b*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.98, size = 153, normalized size = 1.46

$$\frac{b\,x}{16} - \frac{-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} - \frac{17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24} + 4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + \frac{19\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3} - \frac{19\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} + \frac{17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24} + \frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8} + \frac{4\,a}{15}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/16 - ((4*a)/15 + (b*tan(c/2 + (d*x)/2))/8 + (8*a*tan(c/2 + (d*x)/2)^2
)/5 + (8*a*tan(c/2 + (d*x)/2)^6)/3 + 4*a*tan(c/2 + (d*x)/2)^8 + (17*b*tan(c
/2 + (d*x)/2)^3)/24 - (19*b*tan(c/2 + (d*x)/2)^5)/4 + (19*b*tan(c/2 + (d*x)
/2)^7)/4 - (17*b*tan(c/2 + (d*x)/2)^9)/24 - (b*tan(c/2 + (d*x)/2)^11)/8)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^6)
```

3.1051 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{ax}{8}-\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos^5(c+dx)}{5d}+\frac{a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*a*x-1/3*b*cos(d*x+c)^3/d+1/5*b*cos(d*x+c)^5/d+1/8*a*cos(d*x+c)*sin(d*x+
c)/d-1/4*a*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2917, 2648, 2715, 8, 2645, 14}

$$-\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{a\sin(c+dx)\cos(c+dx)}{8d}+\frac{ax}{8}+\frac{b\cos^5(c+dx)}{5d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/8 - (b*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]^5)/(5*d) + (a*Cos[c +
d*x]*Sin[c + d*x])/(8*d) - (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
```

```
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin^2(c+dx)\,dx+b\int \cos^2(c+dx)\sin^3\\ &= -\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}a\int \cos^2(c+dx)\,dx-\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\\ &= \frac{ax}{8}-\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos^5(c+dx)}{5d}+\frac{a\cos(c+dx)\sin}{8d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.73

$$\frac{60ac+60adx-60b\cos(c+dx)-10b\cos(3(c+dx))+6b\cos(5(c+dx))-15a\sin(4(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*c + 60*a*d*x - 60*b*Cos[c + d*x] - 10*b*Cos[3*(c + d*x)] + 6*b*Cos[5*
(c + d*x)] - 15*a*Sin[4*(c + d*x)])/(480*d)
```

Maple [A]
time = 0.14, size = 77, normalized size = 0.95

method	result
risch	$\frac{ax}{8}-\frac{b\cos(dx+c)}{8d}+\frac{b\cos(5dx+5c)}{80d}-\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(3dx+3c)}{48d}$
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^{3}(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b\left(-\frac{\left(\sin^{2}(dx+c)\right)\left(\cos^{3}(dx+c)\right)}{5}-\frac{2\left(\cos^{3}(dx+c)\right)}{15}\right)}{d}$
norman	$\frac{\frac{ax}{8}-\frac{4b}{15d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{3a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5ax\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{5ax\left(\tan^{4}\left(\frac{dx}{2}+\right.\right.}{4}}{\left(1+\tan^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*
c)+b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.64

$$\frac{15\,(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right))a+32\,\left(3\,\cos\left(dx+c\right)^{5}-5\,\cos\left(dx+c\right)^{3}\right)b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a + 32*(3*cos(d*x + c)^5 - 5*cos
(d*x + c)^3)*b)/d
```

Fricas [A]
time = 0.38, size = 62, normalized size = 0.77

$$\frac{24\,b\cos\left(dx+c\right)^{5}-40\,b\cos\left(dx+c\right)^{3}+15\,adx-15\,\left(2\,a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*b*cos(d*x + c)^5 - 40*b*cos(d*x + c)^3 + 15*a*d*x - 15*(2*a*cos(d
*x + c)^3 - a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(70) = 140$.
time = 0.31, size = 144, normalized size = 1.78

$$\begin{cases} \frac{ax\sin^{4}(c+dx)}{8}+\frac{ax\sin^{2}(c+dx)\cos^{2}(c+dx)}{4}+\frac{ax\cos^{4}(c+dx)}{8}+\frac{a\sin^{3}(c+dx)\cos(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^{3}(c+dx)}{8d}-\frac{b\sin^{2}(c+dx)\cos^{3}(c+dx)}{3d}-\frac{2b\cos^{5}(c+dx)}{15d} & \text{for } d\neq 0 \\ x(a+b\sin(c))\sin^{2}(c)\cos^{2}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**4/8 + a*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
a*x*cos(c + d*x)**4/8 + a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - a*sin(c + d*
x)*cos(c + d*x)**3/(8*d) - b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b*co
s(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**2*cos(c)**2, Tru
e))
```

Giac [A]
time = 0.47, size = 62, normalized size = 0.77

$$\frac{1}{8}\,ax+\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}-\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*a*x + 1/80*b*cos(5*d*x + 5*c)/d - 1/48*b*cos(3*d*x + 3*c)/d - 1/8*b*cos
(d*x + c)/d - 1/32*a*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 12.89, size = 125, normalized size = 1.54

$$\frac{a\,x}{8}-\frac{-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{4\,b}{15}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/8 - ((4*b)/15 + (a*tan(c/2 + (d*x)/2))/4 - (3*a*tan(c/2 + (d*x)/2)^3)
/2 + (3*a*tan(c/2 + (d*x)/2)^7)/2 - (a*tan(c/2 + (d*x)/2)^9)/4 + (4*b*tan(c
/2 + (d*x)/2)^2)/3 - (4*b*tan(c/2 + (d*x)/2)^4)/3 + 4*b*tan(c/2 + (d*x)/2)^
6)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.1052 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{bx}{8}-\frac{a\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*b*x-1/3*a*cos(d*x+c)^3/d+1/8*b*cos(d*x+c)*sin(d*x+c)/d-1/4*b*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {2917, 2645, 30, 2648, 2715, 8}

$$-\frac{a\cos^3(c+dx)}{3d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b\sin(c+dx)\cos(c+dx)}{8d}+\frac{bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(8*d) -
(b*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^2(c+dx)\sin(c+dx)\,dx+b\int \cos^2(c+dx)\sin^2(c\\
&= -\frac{b\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}b\int \cos^2(c+dx)\,dx-\frac{aS}{}\\
&= -\frac{a\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(c+}{}\\
&= \frac{bx}{8}-\frac{a\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos^3(}{}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 0.94

$$\frac{bx}{8}-\frac{a\cos^3(c+dx)}{3d}+\frac{1}{8}b\left(-\frac{\cos(4dx)\sin(4c)}{4d}-\frac{\cos(4c)\sin(4dx)}{4d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/8 - (a*Cos[c + d*x]^3)/(3*d) + (b*(-1/4*(Cos[4*d*x]*Sin[4*c])/d - (Co
s[4*c]*Sin[4*d*x])/(4*d)))/8
```

Maple [A]
time = 0.10, size = 57, normalized size = 0.88

method	result

risch	$\frac{bx}{8}-\frac{a\cos(dx+c)}{4d}-\frac{b\sin(4dx+4c)}{32d}-\frac{a\cos(3dx+3c)}{12d}$
derivativedivides	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+b\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3}+b\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
norman	$\frac{\frac{bx}{8}-\frac{2a}{3d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{7b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{7b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{3bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a*cos(d*x+c)^3+b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos
(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.60

$$-\frac{32\,a\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(32*a*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*b)/d
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.78

$$-\frac{8\,a\cos\left(dx+c\right)^3-3\,bdx+3\left(2\,b\cos\left(dx+c\right)^3-b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(8*a*cos(d*x + c)^3 - 3*b*d*x + 3*(2*b*cos(d*x + c)^3 - b*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(56) = 112$.
time = 0.18, size = 119, normalized size = 1.83

$$\begin{cases} -\frac{a\cos^3(c+dx)}{3d}+\frac{bx\sin^4(c+dx)}{8}+\frac{bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{bx\cos^4(c+dx)}{8}+\frac{b\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for } d\neq 0 \\ x(a+b\sin(c))\sin(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**3/(3*d) + b*x*sin(c + d*x)**4/8 + b*x*sin(c + d
*x)**2*cos(c + d*x)**2/4 + b*x*cos(c + d*x)**4/8 + b*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) - b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*s
in(c))*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.45, size = 47, normalized size = 0.72

$$\frac{1}{8}\,bx - \frac{a\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{a\cos\left(dx + c\right)}{4\,d} - \frac{b\sin\left(4\,dx + 4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*b*x - 1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/32*b*sin(4*d
*x + 4*c)/d
```

Mupad [B]
time = 12.66, size = 125, normalized size = 1.92

$$\frac{b\,x}{8} - \frac{-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - \frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4} + \frac{2\,a}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/8 - ((2*a)/3 + (b*tan(c/2 + (d*x)/2))/4 + (2*a*tan(c/2 + (d*x)/2)^2)/
3 + 2*a*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/2)^6 - (7*b*tan(c/2 + (d
*x)/2)^3)/4 + (7*b*tan(c/2 + (d*x)/2)^5)/4 - (b*tan(c/2 + (d*x)/2)^7)/4)/(d
*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.1053 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=51

$$\frac{bx}{2}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{b\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*b*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/2*b*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A]
time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2917, 2672, 327, 212, 2715, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/2 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (b*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\cot(c+dx)\,dx + b\int \cos^2(c+dx)\,dx \\
&= \frac{b\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}b\int 1\,dx - \frac{a\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx \right.}{} \\
&= \frac{bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+dx)\sin(c+dx)}{2d} - \frac{a\text{Subst}}{} \\
&= \frac{bx}{2} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{b\cos(c+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 1.45

$$
\frac{b(c+dx)}{2d} + \frac{a\cos(c+dx)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\sin(2(c+dx))}{4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a
*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.12, size = 55, normalized size = 1.08

method	result
derivativedivides	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$\frac{bx}{2}+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{bx}{2}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b*(1/2*sin(d*x+c)*cos(d*x+c)+
1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 57, normalized size = 1.12

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))b+2\,a(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*b + 2*a*(2*cos(d*x + c) - log(cos(d*x
 + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 60, normalized size = 1.18

$$\frac{bdx+b\cos(dx+c)\sin(dx+c)+2\,a\cos(dx+c)-a\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+a\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + b*cos(d*x + c)*sin(d*x + c) + 2*a*cos(d*x + c) - a*log(1/2*cos
(d*x + c) + 1/2) + a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))\cos^2(c+dx)\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [A]
time = 0.44, size = 87, normalized size = 1.71

$$\frac{(dx+c)b+2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*b + 2*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x +
1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) - 2*a)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 9.87, size = 157, normalized size = 3.08

$$\frac{b\,\mathrm{atan}\left(\frac{b^2}{2\,a\,b-b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{2\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a\,b-b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{-b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (b*atan(b^2/(2*a*b - b^2*tan(c/2 + (d*x)/2)) + (2*a*b*tan(c/2 + (d*x)/2))/(
2*a*b - b^2*tan(c/2 + (d*x)/2))))/d + (2*a + b*tan(c/2 + (d*x)/2) + 2*a*tan
(c/2 + (d*x)/2)^2 - b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.1054 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}$$

```
[Out] -a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d-a*cot(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2801, 2672, 327, 212, 3554, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx\\ &= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+\right.}{d}\\ &= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 75, normalized size = 1.83

$$\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

Maple [A]
time = 0.11, size = 49, normalized size = 1.20

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
default	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}-\frac{2ia}{d(e^{2i(dx+c)}-1)}-\frac{b\ln(e^{i(dx+c)}+1)}{d}+\frac{b\ln(e^{i(dx+c)}-1)}{d}$	91
norman	$\frac{-\frac{a}{2d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-cot(d*x+c)-d*x-c)+b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.32

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-b(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.
time = 0.37, size = 84, normalized size = 2.05

$$-\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2\,a\cos(dx+c)+2\,(adx-b\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\cos^2(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.
time = 0.45, size = 108, normalized size = 2.63

$$-\frac{6\,(dx+c)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

Mupad [B]
time = 9.54, size = 158, normalized size = 3.85

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^{2}}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^{2}+4\,b\,a}-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^{2}+4\,b\,a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(tan(c/2 + (d*x)/2)))/d - (a - 4*b*tan
(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*x)/2) + 2*tan(
c/2 + (d*x)/2)^3)) + (2*a*atan((4*a^2)/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2)) -
 (4*a*b*tan(c/2 + (d*x)/2))/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.1055 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$-bx+\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)}{d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -b*x+1/2*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)/d-1/2*a*cot(d*x+c)*csc(d*x+c)
/d
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {2917, 2691, 3855, 3554, 8}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(b*x) + (a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d - (a*Cot[c +
d*x]*Csc[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc(c+dx)\,dx + b\int \cot^2(c+dx)\,dx$$
$$= -\frac{b\cot(c+dx)}{d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}a\int \csc(c$$
$$= -bx + \frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} - \frac{a\cot(c-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 109, normalized size = 2.10

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{b\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1,
1/2, -Tan[c + d*x]^2])/d + (a*Log[Cos[(c + d*x)/2]])/(2*d) - (a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A]
time = 0.13, size = 71, normalized size = 1.37

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{a\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b(-\cot(dx+c)-dx-c)}{d}$
risch	$-bx-\frac{i\left(ia\,e^{3i(dx+c)}+ia\,e^{i(dx+c)}+2b\,e^{2i(dx+c)}-2b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-1\right)}{2d}$

norman	$\frac{-\frac{a}{8d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$ −

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot
(d*x+c)))+b*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 66, normalized size = 1.27

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)b-a\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 -
1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(48) = 96.
time = 0.39, size = 114, normalized size = 2.19

$$-\frac{4\,bdx\cos\left(dx+c\right)^2-4\,bdx-4\,b\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*d*x*cos(d*x + c)^2 - 4*b*d*x - 4*b*cos(d*x + c)*sin(d*x + c) - 2*
a*cos(d*x + c) - (a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + (a*co
s(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**3, x)
```

Giac [A]
time = 0.45, size = 95, normalized size = 1.83

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,(dx+c)b-4\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 8*(d*x + c)*b - 4*a*log(abs(tan(1/2*d*x + 1
/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(
1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 9.36, size = 151, normalized size = 2.90

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}-\frac{2\,b\,\mathrm{atan}\left(\frac{2\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (b*cot(c/2 + (d*x)/2))/(2*d) - (a*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) - (2*b*atan((2*b*cos(c/2 + (d*x)/2)
 + a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - 2*b*sin(c/2 + (d*x)/2))))/
d - (a*cot(c/2 + (d*x)/2)^2)/(8*d) + (a*tan(c/2 + (d*x)/2)^2)/(8*d)
```

3.1056 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot^3(c+dx)}{3d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] 1/2*b*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/2*b*cot(d*x+c)*csc(d*x+c
)/d
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2917, 2687, 30, 2691, 3855}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]^3)/(3*d) - (b*Cot[c + d*x
]*Csc[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^2(c+dx)\,dx + b\int \cot^2(c+dx)\csc($$

$$= -\frac{b\cot(c+dx)\csc(c+dx)}{2d} - \frac{1}{2}b\int \csc(c+dx)\,dx + \frac{a\mathrm{Su}}{}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\cot(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 95, normalized size = 1.83

$$-\frac{a\cot^3(c+dx)}{3d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(a*Cot[c + d*x]^3)/d - (b*Csc[(c + d*x)/2]^2)/(8*d) + (b*Log[Cos[(c +
d*x)/2]])/(2*d) - (b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2)/
(8*d)
```

Maple [A]
time = 0.14, size = 72, normalized size = 1.38

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$\frac{6ia\,e^{4i(dx+c)}+3b\,e^{5i(dx+c)}+2ia-3b\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{2d}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{2d}$

norman	$\frac{-\frac{a}{24d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} - b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a/sin(d*x+c)^3*cos(d*x+c)^3+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2
*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.17

$$\frac{3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{4\,a}{\tan(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - lo
g(cos(d*x + c) - 1)) - 4*a/tan(d*x + c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(46) = 92$.
time = 0.39, size = 119, normalized size = 2.29

$$\frac{4\,a\cos\left(dx+c\right)^3+6\,b\cos\left(dx+c\right)\sin\left(dx+c\right)+3\,\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)*sin(d*x + c) + 3*(b*cos(d*x + c
)^2 - b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(b*cos(d*x + c)^2 - b
)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*
x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\cos^2\left(c+dx\right)\csc^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**2*csc(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(46) = 92$.

time = 0.54, size = 115, normalized size = 2.21

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{22\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 - 12*b*log(abs(
tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x + 1/2*c) + (22*b*tan(1/2*d*x + 1/2
*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*
d*x + 1/2*c)^3)/d
```

Mupad [B]

time = 9.31, size = 111, normalized size = 2.13

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2))/(8*d) + (b*tan(c/2
 + (d*x)/2)^2)/(8*d) - (b*log(tan(c/2 + (d*x)/2)))/(2*d) - (cot(c/2 + (d*x)
/2)^3*(a/3 + b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(8*d)
```

3.1057 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 1/8*a*arctanh(cos(d*x+c))/d-1/3*b*cot(d*x+c)^3/d+1/8*a*cot(d*x+c)*csc(d*x+c
)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 30}

$$\frac{a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^3)/(3*d) + (a*Cot[c + d*x
]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c + d*x]^3)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^3(c+dx)\,dx + b\int \cot^2(c+dx)\csc^2$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}a\int \csc^3(c+dx)\,dx + $$
$$= -\frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+$$
$$= \frac{a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\cot^3(c+dx)}{3d} + \frac{a\cot(c+dx)}{8d}$$

Mathematica [A]
time = 0.03, size = 135, normalized size = 1.82

$$-\frac{b\cot^3(c+dx)}{3d}+\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(b*Cot[c + d*x]^3)/d + (a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Log[Cos[(c + d*x)/2]])/(8*d) - (a*Log[Sin[(c + d*x)/2]]
)/(8*d) - (a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.16, size = 90, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{b\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$
default	$\frac{a\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{b\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}}{d}$
risch	$-\frac{-24ib\,e^{6i(dx+c)}+3a\,e^{7i(dx+c)}+24ib\,e^{4i(dx+c)}+21a\,e^{5i(dx+c)}-8ib\,e^{2i(dx+c)}+21a\,e^{3i(dx+c)}+8ib+3a\,e^{i(dx+c)}}{12d\left(e^{2i(dx+c)}-1\right)^4}-\frac{a\ln(}{}$
norman	$\frac{-\frac{a}{64d}-\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{b(\text{ta}}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*co
s(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*b/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]

time = 0.29, size = 80, normalized size = 1.08

$$-\frac{3\,a\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,b}{\tan(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*a*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x +
 c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*b/tan(d*x
+ c)^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(66) = 132$.

time = 0.37, size = 137, normalized size = 1.85

$$-\frac{16\,b\cos\left(dx+c\right)^3\sin\left(dx+c\right)+6\,a\cos\left(dx+c\right)^3+6\,a\cos\left(dx+c\right)-3\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(16*b*cos(d*x + c)^3*sin(d*x + c) + 6*a*cos(d*x + c)^3 + 6*a*cos(d*x
+ c) - 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) +
 1/2) + 3*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c)
 + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 116, normalized size = 1.57

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{50\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*log(a
bs(tan(1/2*d*x + 1/2*c))) - 24*b*tan(1/2*d*x + 1/2*c) + (50*a*tan(1/2*d*x +
 1/2*c)^4 + 24*b*tan(1/2*d*x + 1/2*c)^3 - 8*b*tan(1/2*d*x + 1/2*c) - 3*a)/t
an(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 9.33, size = 112, normalized size = 1.51

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{a}{4}\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2))/(8*d) + (b*tan(c/2
 + (d*x)/2)^3)/(24*d) - (a*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x
)/2)^4*(a/4 + (2*b*tan(c/2 + (d*x)/2))/3 - 2*b*tan(c/2 + (d*x)/2)^3))/(16*d
)
```

3.1058 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=90

$$\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot^5(c+dx)}{5d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}$$

```
[Out] 1/8*b*arctanh(cos(d*x+c))/d-1/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)^5/d+1/8*b
*cot(d*x+c)*csc(d*x+c)/d-1/4*b*cot(d*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2687, 14, 2691, 3853, 3855}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^3)/(3*d) - (a*Cot[c + d*x
]^5)/(5*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]*Csc[c +
d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^2(c+dx)\csc^4(c+dx)\,dx + b\int \cot^2(c+dx)\csc^3$$

$$= -\frac{b\cot(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}b\int \csc^3(c+dx)\,dx + \frac{a}{}$$

$$= \frac{b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\csc^3(c+dx)}{4d} -$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx}{5d}$$

Mathematica [A]
time = 0.06, size = 177, normalized size = 1.97

$$\frac{2a\cot(c+dx)}{15d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Cot[c + d*x])/(15*d) + (b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x
)/2]^4)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*Cot[c + d*x]*C
sc[c + d*x]^4)/(5*d) + (b*Log[Cos[(c + d*x)/2]])/(8*d) - (b*Log[Sin[(c + d*
x)/2]])/(8*d) - (b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*
d)
```

Maple [A]
time = 0.17, size = 110, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+b\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
default	$\frac{a\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+b\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
risch	$-\frac{15b\,e^{9i(dx+c)}+240ia\,e^{6i(dx+c)}+90b\,e^{7i(dx+c)}+80ia\,e^{4i(dx+c)}+80ia\,e^{2i(dx+c)}-90b\,e^{3i(dx+c)}-16ia-15b\,e^{i(dx+c)}}{60d\left(e^{2i(dx+c)}-1\right)^{5}}-$
norman	$\frac{-\frac{a}{160d}-\frac{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60d}+\frac{5a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5a\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60d}+\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}-}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+b*(-
1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d*x+c)-
1/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 92, normalized size = 1.02

$$-\frac{15\,b\left(\frac{2\left(\cos(dx+c)^{3}+\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\left(5\,\tan(dx+c)^{2}+3\right)a}{\tan(dx+c)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(15*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 16*(5*tan(d
*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(80) = 160$.
time = 0.38, size = 169, normalized size = 1.88

$$\frac{32\,a\cos\left(dx+c\right)^{5}-80\,a\cos\left(dx+c\right)^{3}+15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-30\left(b\cos\left(dx+c\right)^{3}+b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(32*a*cos(d*x + c)^5 - 80*a*cos(d*x + c)^3 + 15*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos
(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 30*(b*cos(d*x + c)^3 + b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.47, size = 144, normalized size = 1.60

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-120\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{274\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+60\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 120*b*log(abs(tan(1/2*d*x + 1/2*c))) - 60*a*tan(1/2*d*
x + 1/2*c) + (274*b*tan(1/2*d*x + 1/2*c)^5 + 60*a*tan(1/2*d*x + 1/2*c)^4 -
10*a*tan(1/2*d*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 6*a)/tan(1/2*d*x
+ 1/2*c)^5)/d
```

Mupad [B]
time = 9.33, size = 143, normalized size = 1.59

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(96*d) - (a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/
2 + (d*x)/2)^5)/(160*d) + (b*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*log(tan(c/2
+ (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2
+ (a*tan(c/2 + (d*x)/2)^2)/3 - 2*a*tan(c/2 + (d*x)/2)^4))/(32*d)
```

3.1059 $\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=190

$$\frac{abx}{8}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{ab\cos(c+dx)\sin(c+dx)}{8d}-\frac{ab\cos(c+dx)\sin^3(c+dx)}{12d}$$

```
[Out] 1/8*a*b*x-1/35*(7*a^2+4*b^2)*cos(d*x+c)/d+1/105*(7*a^2+4*b^2)*cos(d*x+c)^3/
d-1/8*a*b*cos(d*x+c)*sin(d*x+c)/d-1/12*a*b*cos(d*x+c)*sin(d*x+c)^3/d+1/35*(
2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/d+1/21*a*b*cos(d*x+c)*sin(d*x+c)^5/d+1/7
*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3112, 3102, 2827, 2713, 2715, 8}

$$\frac{(7a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{(7a^2+4b^2)\cos(c+dx)}{35d}+\frac{(2a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{ab\sin^5(c+dx)\cos(c+dx)}{21d}+\frac{\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{ab\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{ab\sin(c+dx)\cos(c+dx)}{8d}+\frac{abx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((7*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + ((7*a^2 + 4*b^2)*Cos[c
+ d*x]^3)/(105*d) - (a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a*b*Cos[c + d*
x]*Sin[c + d*x]^3)/(12*d) + ((2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35
*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^5)/(21*d) + (Cos[c + d*x]*Sin[c + d*x]
^4*(a + b*Sin[c + d*x])^2)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \sin^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right) \\
&= \frac{\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2}{7d} + \frac{1}{7}\int \text{si} \\
&= \frac{ab\cos(c+dx)\sin^5(c+dx)}{21d} + \frac{\cos(c+dx)\sin^4(c+dx}{7d} \\
&= \frac{(2a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\,\text{s}}{21d} \\
&= \frac{(2a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{ab\cos(c+dx)\,\text{s}}{21d} \\
&= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{12d} + \frac{(2a^2-b^2)\cos(c+dx}{35d} \\
&= -\frac{(7a^2+4b^2)\cos(c+dx)}{35d} + \frac{(7a^2+4b^2)\cos^3(c+dx)}{105d} \\
&= \frac{abx}{8} - \frac{(7a^2+4b^2)\cos(c+dx)}{35d} + \frac{(7a^2+4b^2)\cos^3(c+}{105d}
\end{aligned}
$$

Mathematica [A]

time = 0.41, size = 132, normalized size = 0.69

$$
\frac{840abc+840abdx-105(8a^2+5b^2)\cos(c+dx)-35(4a^2+b^2)\cos(3(c+dx))+84a^2\cos(5(c+dx))+63b^2\cos(5(c+dx))-15b^2\cos(7(c+dx))-210ab\sin(2(c+dx))-210ab\sin(4(c+dx))+70ab\sin(6(c+dx))}{6720d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 5*b^2)*Cos[c + d*x] - 35*(4*a^2 + b
^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] + 63*b^2*Cos[5*(c + d*x)] -
15*b^2*Cos[7*(c + d*x)] - 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*x
)] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A]

time = 0.26, size = 150, normalized size = 0.79

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)}{1}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^3(dx+c)\right)}{6}-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{8}+\frac{\sin(dx+c)}{1}\right.}{d}$

risch	$\frac{abx}{8} - \frac{a^2\cos(dx+c)}{8d} - \frac{5b^2\cos(dx+c)}{64d} - \frac{b^2\cos(7dx+7c)}{448d} + \frac{ab\sin(6dx+6c)}{96d} + \frac{a^2\cos(5dx+5c)}{80d} + \frac{3\cos(5dx+5c)b^2}{320d}$
norman	$\frac{-\frac{28a^2+16b^2}{105d}+\frac{abx}{8}-\frac{4a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(8a^2-16b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(8a^2+16b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{\left(20a^2+32b^2\right)\left(\tan^8\left(\frac{dx}{2}\right.\right.}{3d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+2*a*b*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+
c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*c
os(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 104, normalized size = 0.55

$$\frac{224\left(3\cos\left(dx+c\right)^5-5\cos\left(dx+c\right)^3\right)a^2-35\left(4\sin\left(2dx+2c\right)^3-12dx-12c+3\sin\left(4dx+4c\right)\right)ab-32\left(15\cos\left(dx+c\right)^7-42\cos\left(dx+c\right)^5+35\cos\left(dx+c\right)^3\right)b^2}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3360*(224*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2 - 35*(4*sin(2*d*x + 2
*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b - 32*(15*cos(d*x + c)^7 - 4
2*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 0.36, size = 104, normalized size = 0.55

$$-\frac{120b^2\cos\left(dx+c\right)^7-168\left(a^2+2b^2\right)\cos\left(dx+c\right)^5-105abdx+280\left(a^2+b^2\right)\cos\left(dx+c\right)^3-35\left(8ab\cos\left(dx+c\right)^5-14ab\cos\left(dx+c\right)^3+3ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 2*b^2)*cos(d*x + c)^5 - 105*a*b
*d*x + 280*(a^2 + b^2)*cos(d*x + c)^3 - 35*(8*a*b*cos(d*x + c)^5 - 14*a*b*c
os(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.66, size = 275, normalized size = 1.45

$$\begin{cases} -\frac{a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{2a^2\cos^5(c+dx)}{15d} + \frac{abx\sin^6(c+dx)}{8} + \frac{3abx\sin^4(c+dx)\cos^2(c+dx)}{8} + \frac{3abx\sin^2(c+dx)\cos^4(c+dx)}{8} + \frac{abx\cos^6(c+dx)}{8} + \frac{ab\sin^5(c+dx)\cos(c+dx)}{8d} - \frac{ab\sin^3(c+dx)\cos^3(c+dx)}{3d} - \frac{ab\sin(c+dx)\cos^5(c+dx)}{8d} - \frac{b^2\sin^4(c+dx)\cos^3(c+dx)}{3d} - \frac{4b^2\sin^2(c+dx)\cos^5(c+dx)}{15d} - \frac{8b^2\cos^7(c+dx)}{105d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2\sin^3(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*a**2*cos(c + d*x
)**5/(15*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*sin(c + d*x)**4*cos(c + d*x
)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + a*b*x*cos(c + d*x)**6/
8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) - a*b*sin(c + d*x)**3*cos(c + d*
x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**4
*cos(c + d*x)**3/(3*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*
b**2*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos
(c)**2, True))
```

Giac [A]
time = 0.55, size = 141, normalized size = 0.74

$$\frac{1}{8}\,abx-\frac{b^2\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{ab\sin\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{ab\sin\left(2\,dx+2\,c\right)}{32\,d}+\frac{\left(4\,a^2+3\,b^2\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(4\,a^2+b^2\right)\cos\left(3\,dx+3\,c\right)}{192\,d}-\frac{\left(8\,a^2+5\,b^2\right)\cos\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x - 1/448*b^2*cos(7*d*x + 7*c)/d + 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d - 1/32*a*b*sin(2*d*x + 2*c)/d + 1/320*(4*a^2 + 3*
b^2)*cos(5*d*x + 5*c)/d - 1/192*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*
a^2 + 5*b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 12.97, size = 233, normalized size = 1.23

$$\frac{a\,b\,x}{8}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{8\,a^2}{3}-\frac{16\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{8\,a^2}{5}+\frac{16\,b^2}{5}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{20\,a^2}{3}+\frac{32\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{28\,a^2}{15}+\frac{16\,b^2}{15}\right)+4\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{4\,a^2}{15}+\frac{16\,b^2}{105}+\frac{5\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}-\frac{97\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{12}+\frac{97\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{12}-\frac{5\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{3}-\frac{a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{4}+\frac{a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((8*a^2)/3 - (16*b^2)/3) + tan(c/2 + (d*x
)/2)^4*((8*a^2)/5 + (16*b^2)/5) + tan(c/2 + (d*x)/2)^8*((20*a^2)/3 + (32*b^
2)/3) + tan(c/2 + (d*x)/2)^2*((28*a^2)/15 + (16*b^2)/15) + 4*a^2*tan(c/2 +
(d*x)/2)^10 + (4*a^2)/15 + (16*b^2)/105 + (5*a*b*tan(c/2 + (d*x)/2)^3)/3 -
(97*a*b*tan(c/2 + (d*x)/2)^5)/12 + (97*a*b*tan(c/2 + (d*x)/2)^9)/12 - (5*a*
b*tan(c/2 + (d*x)/2)^11)/3 - (a*b*tan(c/2 + (d*x)/2)^13)/4 + (a*b*tan(c/2 +
 (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.1060 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{1}{16}\left(2a^2+b^2\right)x-\frac{2ab\cos(c+dx)}{5d}+\frac{2ab\cos^3(c+dx)}{15d}-\frac{\left(2a^2+b^2\right)\cos(c+dx)\sin(c+dx)}{16d}+\frac{\left(2a^2-b^2\right)\cos(c+d}{24d}$$

[Out] 1/16*(2*a^2+b^2)*x-2/5*a*b*cos(d*x+c)/d+2/15*a*b*cos(d*x+c)^3/d-1/16*(2*a^2+b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/d+1/15*a*b*cos(d*x+c)*sin(d*x+c)^4/d+1/6*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/d

Rubi [A]
time = 0.28, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3112, 3102, 2827, 2715, 8, 2713}

$$\frac{\left(2a^2-b^2\right)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{\left(2a^2+b^2\right)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(2a^2+b^2\right)+\frac{2ab\cos^3(c+dx)}{15d}-\frac{2ab\cos(c+dx)}{5d}+\frac{ab\sin^4(c+dx)\cos(c+dx)}{15d}+\frac{\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{6d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]

[Out] ((2*a^2 + b^2)*x)/16 - (2*a*b*Cos[c + d*x])/(5*d) + (2*a*b*Cos[c + d*x]^3)/(15*d) - ((2*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) + (a*b*Cos[c + d*x]*Sin[c + d*x]^4)/(15*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(6*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,d$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int \sin^2$$
$$= \frac{ab\cos(c+dx)\sin^4(c+dx)}{15d} + \frac{\cos(c+dx)\sin^3(c+dx)(}{6d}$$
$$= \frac{(2a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin}{15d}$$
$$= \frac{(2a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{24d} + \frac{ab\cos(c+dx)\sin}{15d}$$
$$= -\frac{(2a^2+b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{(2a^2-b^2)\cos(c}{2}$$
$$= \frac{1}{16}\left(2a^2+b^2\right)x - \frac{2ab\cos(c+dx)}{5d} + \frac{2ab\cos^3(c+dx)}{15d} -$$

Mathematica [A]
time = 0.17, size = 120, normalized size = 0.74

$$\frac{120a^2c+60b^2c+120a^2dx+60b^2dx-240ab\cos(c+dx)-40ab\cos(3(c+dx))+24ab\cos(5(c+dx))-15b^2\sin(2(c+dx))-30a^2\sin(4(c+dx))-15b^2\sin(4(c+dx))+5b^2\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (120*a^2*c + 60*b^2*c + 120*a^2*d*x + 60*b^2*d*x - 240*a*b*Cos[c + d*x] - 4
0*a*b*Cos[3*(c + d*x)] + 24*a*b*Cos[5*(c + d*x)] - 15*b^2*Sin[2*(c + d*x)]
- 30*a^2*Sin[4*(c + d*x)] - 15*b^2*Sin[4*(c + d*x)] + 5*b^2*Sin[6*(c + d*x)
])/(960*d)
```

Maple [A]
time = 0.22, size = 141, normalized size = 0.87

method	result
risch	$\frac{a^2x}{8}+\frac{b^2x}{16}-\frac{ab\cos(dx+c)}{4d}+\frac{b^2\sin(6dx+6c)}{192d}+\frac{ab\cos(5dx+5c)}{40d}-\frac{a^2\sin(4dx+4c)}{32d}-\frac{\sin(4dx+4c)b^2}{64d}-\frac{ab\cos(3}{2}$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+b^2\left(-\frac{(}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+b^2\left(-\frac{(}{}\right.}{d}$

norman	$\frac{\left(\frac{a^2}{8}+\frac{b^2}{16}\right)x+\left(\frac{a^2}{8}+\frac{b^2}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{4}+\frac{3b^2}{8}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{4}+\frac{3b^2}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5a^2}{2}+\frac{5b}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)+2*a*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+b^2*(-1/6*sin
(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*cos(d*x+
c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.28, size = 92, normalized size = 0.56

$$\frac{30\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2+128\,(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3)ab-5\,(4\,\sin(2\,dx+2\,c)^3-12\,dx-12\,c+3\,\sin(4\,dx+4\,c))b^2}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/960*(30*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 128*(3*cos(d*x + c)^5 - 5*
cos(d*x + c)^3)*a*b - 5*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x
 + 4*c))*b^2)/d
```

Fricas [A]
time = 0.40, size = 103, normalized size = 0.63

$$\frac{96\,ab\cos(dx+c)^5-160\,ab\cos(dx+c)^3+15\,(2\,a^2+b^2)dx+5\,\left(8\,b^2\cos(dx+c)^5-2\,(6\,a^2+7\,b^2)\cos(dx+c)^3+3\,(2\,a^2+b^2)\cos(dx+c)\right)\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/240*(96*a*b*cos(d*x + c)^5 - 160*a*b*cos(d*x + c)^3 + 15*(2*a^2 + b^2)*d*
x + 5*(8*b^2*cos(d*x + c)^5 - 2*(6*a^2 + 7*b^2)*cos(d*x + c)^3 + 3*(2*a^2 +
 b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(148) = 296$.
time = 0.49, size = 309, normalized size = 1.90

$$\begin{cases} \frac{a^2x\sin^4(c+dx)}{8}+\frac{a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{a^2x\cos^4(c+dx)}{8}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{a^2\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{2ab\sin^2(c+dx)\cos^3(c+dx)}{3d}-\frac{4ab\cos^5(c+dx)}{15d}+\frac{b^2x\sin^6(c+dx)}{16}+\frac{3b^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3b^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{b^2x\cos^6(c+dx)}{16}+\frac{b^2\sin^5(c+dx)\cos(c+dx)}{16d}-\frac{b^2\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b^2\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))^2\sin^2(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**4/8 + a**2*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**2*x*cos(c + d*x)**4/8 + a**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x
)**3/(3*d) - 4*a*b*cos(c + d*x)**5/(15*d) + b**2*x*sin(c + d*x)**6/16 + 3*b
**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**4/16 + b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)
/(16*d) - b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*co
s(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**2,
True))
```

Giac [A]

time = 0.53, size = 115, normalized size = 0.71

$$\frac{1}{16}\left(2a^2+b^2\right)x+\frac{ab\cos\left(5dx+5c\right)}{40d}-\frac{ab\cos\left(3dx+3c\right)}{24d}-\frac{ab\cos\left(dx+c\right)}{4d}+\frac{b^2\sin\left(6dx+6c\right)}{192d}-\frac{b^2\sin\left(2dx+2c\right)}{64d}-\frac{\left(2a^2+b^2\right)\sin\left(4dx+4c\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*a^2 + b^2)*x + 1/40*a*b*cos(5*d*x + 5*c)/d - 1/24*a*b*cos(3*d*x + 3
*c)/d - 1/4*a*b*cos(d*x + c)/d + 1/192*b^2*sin(6*d*x + 6*c)/d - 1/64*b^2*si
n(2*d*x + 2*c)/d - 1/64*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]

time = 9.62, size = 112, normalized size = 0.69

$$-\frac{\frac{15a^2\sin(4c+4dx)}{2}+\frac{15b^2\sin(2c+2dx)}{4}+\frac{15b^2\sin(4c+4dx)}{4}-\frac{5b^2\sin(6c+6dx)}{4}+60ab\cos\left(c+dx\right)+10ab\cos\left(3c+3dx\right)-6ab\cos\left(5c+5dx\right)-30a^2dx-15b^2dx}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] -((15*a^2*sin(4*c + 4*d*x))/2 + (15*b^2*sin(2*c + 2*d*x))/4 + (15*b^2*sin(4
*c + 4*d*x))/4 - (5*b^2*sin(6*c + 6*d*x))/4 + 60*a*b*cos(c + d*x) + 10*a*b*
cos(3*c + 3*d*x) - 6*a*b*cos(5*c + 5*d*x) - 30*a^2*d*x - 15*b^2*d*x)/(240*d
)
```

3.1061 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{abx}{4}-\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}+\frac{ab\cos(c+dx)\sin(c+dx)}{4d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}-\frac{\cos^3(c+dx)(}{}$$

```
[Out] 1/4*a*b*x-1/30*(a^2+4*b^2)*cos(d*x+c)^3/d+1/4*a*b*cos(d*x+c)*sin(d*x+c)/d-1
/10*a*cos(d*x+c)^3*(a+b*sin(d*x+c))/d-1/5*cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d
```

Rubi [A]

time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2941, 2748, 2715, 8}

$$-\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}+\frac{ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - ((a^2 + 4*b^2)*Cos[c + d*x]^3)/(30*d) + (a*b*Cos[c + d*x]*Sin[c
 + d*x])/(4*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x]))/(10*d) - (Cos[c +
d*x]^3*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
```

```
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{5}\int \cos^2(c+dx)(2$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))}{10d}-\frac{\cos^3(c+dx)(a+b$$
$$= -\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}-\frac{a\cos^3(c+dx)(a+b\sin(c+$$
$$= -\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}+\frac{ab\cos(c+dx)\sin(c+dx)}{4d}-$$
$$= \frac{abx}{4}-\frac{(a^2+4b^2)\cos^3(c+dx)}{30d}+\frac{ab\cos(c+dx)\sin(c+d}{4d}$$

Mathematica [A]
time = 0.27, size = 77, normalized size = 0.73

$$\frac{-30(2a^2+b^2)\cos(c+dx)-5(4a^2+b^2)\cos(3(c+dx))+3b(20a(c+dx)+b\cos(5(c+dx))-5a\sin(4(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
[Out] (-30*(2*a^2 + b^2)*Cos[c + d*x] - 5*(4*a^2 + b^2)*Cos[3*(c + d*x)] + 3*b*(2
0*a*(c + d*x) + b*Cos[5*(c + d*x)] - 5*a*Sin[4*(c + d*x)]))/(240*d)
```

Maple [A]
time = 0.19, size = 94, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3}+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3\right.}{1}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3}+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3\right.}{1}\right.}{d}$
risch	$\frac{abx}{4}-\frac{a^2\cos(dx+c)}{4d}-\frac{b^2\cos(dx+c)}{8d}+\frac{\cos(5dx+5c)b^2}{80d}-\frac{ab\sin(4dx+4c)}{16d}-\frac{a^2\cos(3dx+3c)}{12d}-\frac{\cos(3dx+3c)b^2}{48d}$

norman	$\frac{-\frac{10a^2+4b^2}{15d}+\frac{abx}{4}-\frac{2a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(2a^2+2b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(4a^2-2b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(4a^2+4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{3d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2*cos(d*x+c)^3+2*a*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+
c)*cos(d*x+c)+1/8*d*x+1/8*c)+b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d
*x+c)^3))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.64

$$-\frac{80\,a^2\cos\left(dx+c\right)^3-15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab-16\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)b^2}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/240*(80*a^2*cos(d*x + c)^3 - 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b - 1
6*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 0.36, size = 73, normalized size = 0.69

$$\frac{12\,b^2\cos\left(dx+c\right)^5+15\,abdx-20\left(a^2+b^2\right)\cos\left(dx+c\right)^3-15\left(2\,ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^5 + 15*a*b*d*x - 20*(a^2 + b^2)*cos(d*x + c)^3 -
15*(2*a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.28, size = 172, normalized size = 1.62

$$\begin{cases} -\frac{a^2\cos^3\left(c+dx\right)}{3d}+\frac{abx\sin^4\left(c+dx\right)}{4}+\frac{abx\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{2}+\frac{abx\cos^4\left(c+dx\right)}{4}+\frac{ab\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{ab\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{4d}-\frac{b^2\sin^2\left(c+dx\right)\cos^3\left(c+dx\right)}{3d}-\frac{2b^2\cos^5\left(c+dx\right)}{15d} & \text{for}\ d\neq 0\\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**3/(3*d) + a*b*x*sin(c + d*x)**4/4 + a*b*x*si
n(c + d*x)**2*cos(c + d*x)**2/2 + a*b*x*cos(c + d*x)**4/4 + a*b*sin(c + d*x
)**3*cos(c + d*x)/(4*d) - a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - b**2*sin
```

```
(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**2*cos(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.47, size = 82, normalized size = 0.77

$$\frac{1}{4}\,abx + \frac{b^2\cos\left(5\,dx + 5\,c\right)}{80\,d} - \frac{ab\sin\left(4\,dx + 4\,c\right)}{16\,d} - \frac{\left(4\,a^2 + b^2\right)\cos\left(3\,dx + 3\,c\right)}{48\,d} - \frac{\left(2\,a^2 + b^2\right)\cos\left(dx + c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*a*b*x + 1/80*b^2*cos(5*d*x + 5*c)/d - 1/16*a*b*sin(4*d*x + 4*c)/d - 1/4
8*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/8*(2*a^2 + b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 12.77, size = 180, normalized size = 1.70

$$\frac{a\,b\,x}{4} - \frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6\left(4\,a^2 + 4\,b^2\right) + \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2\left(\frac{4\,a^2}{3} + \frac{4\,b^2}{3}\right) + \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4\left(\frac{8\,a^2}{3} - \frac{4\,b^2}{3}\right) + 2\,a^2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^8 + \frac{2\,a^2}{3} + \frac{4\,b^2}{15} - 3\,a\,b\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + 3\,a\,b\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^7 - \frac{a\,b\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^9}{2} + \frac{a\,b\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)}{2}}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/4 - (tan(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^2*((
4*a^2)/3 + (4*b^2)/3) + tan(c/2 + (d*x)/2)^4*((8*a^2)/3 - (4*b^2)/3) + 2*a^
2*tan(c/2 + (d*x)/2)^8 + (2*a^2)/3 + (4*b^2)/15 - 3*a*b*tan(c/2 + (d*x)/2)^
3 + 3*a*b*tan(c/2 + (d*x)/2)^7 - (a*b*tan(c/2 + (d*x)/2)^9)/2 + (a*b*tan(c/
2 + (d*x)/2))/2)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
```

3.1062 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=90

$$abx-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{(2a^2-b^2)\cos(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}$$

```
[Out] a*b*x-a^2*arctanh(cos(d*x+c))/d+1/3*(2*a^2-b^2)*cos(d*x+c)/d+1/3*a*b*cos(d*
x+c)*sin(d*x+c)/d+1/3*cos(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2968, 3129, 3112, 3102, 2814, 3855}

$$\frac{(2a^2-b^2)\cos(c+dx)}{3d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a*b*x - (a^2*ArcTanh[Cos[c + d*x]])/d + ((2*a^2 - b^2)*Cos[c + d*x])/(3*d)
+ (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) + (Cos[c + d*x]*(a + b*Sin[c + d*x]
)^2)/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \csc(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\,dx \\
&= \frac{\cos(c+dx)(a+b\sin(c+dx))^2}{3d}+\frac{1}{3}\int \csc(c+dx)(a+b \\
&= \frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{\cos(c+dx)(a+b\sin(c+dx \\
&= \frac{(2a^2-b^2)\cos(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx)}{3d}+\frac{co \\
&= abx+\frac{(2a^2-b^2)\cos(c+dx)}{3d}+\frac{ab\cos(c+dx)\sin(c+dx \\
&= abx-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{(2a^2-b^2)\cos(c+dx)}{3d}+
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 91, normalized size = 1.01

$$\frac{3(4a^2-b^2)\cos(c+dx)-b^2\cos(3(c+dx))+6a\left(2bc+2bdx-2a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\sin(2(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*(4*a^2 - b^2)*Cos[c + d*x] - b^2*Cos[3*(c + d*x)] + 6*a*(2*b*c + 2*b*d*x
 - 2*a*Log[Cos[(c + d*x)/2]] + 2*a*Log[Sin[(c + d*x)/2]] + b*Sin[2*(c + d*x
)]))/(12*d)
```

Maple [A]
time = 0.21, size = 72, normalized size = 0.80

method	result
derivativedivides	$\frac{a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+2ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^3(dx+c)\right)b^2}{3}}{d}$
default	$\frac{a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+2ab\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^3(dx+c)\right)b^2}{3}}{d}$
risch	$abx+\frac{a^2 e^{i(dx+c)}}{2d}-\frac{e^{i(dx+c)}b^2}{8d}+\frac{a^2 e^{-i(dx+c)}}{2d}-\frac{e^{-i(dx+c)}b^2}{8d}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{d}-\cdots$
norman	$\frac{abx+\frac{4a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(2a^2-2b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+abx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{6a^2-2b^2}{3d}+\frac{2ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(1/2*sin(d*x+c)*cos(d
*x+c)+1/2*d*x+1/2*c)-1/3*cos(d*x+c)^3*b^2)
```

Maxima [A]
time = 0.29, size = 74, normalized size = 0.82

$$-\frac{2\,b^2\cos\left(dx+c\right)^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab-3\,a^2\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*a*b - 3*a^2
*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 84, normalized size = 0.93

$$-\frac{2\,b^2\cos\left(dx+c\right)^3-6\,abdx-6\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-6\,a^2\cos\left(dx+c\right)+3\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^2*cos(d*x + c)^3 - 6*a*b*d*x - 6*a*b*cos(d*x + c)*sin(d*x + c) -
6*a^2*cos(d*x + c) + 3*a^2*log(1/2*cos(d*x + c) + 1/2) - 3*a^2*log(-1/2*cos
(d*x + c) + 1/2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))^2 \cos^2(c + dx)\csc(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [A]
time = 0.46, size = 133, normalized size = 1.48

$$\frac{3\,(dx + c)ab + 3\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - \frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 + 3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 - 6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 3\,a^2 + b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a*b + 3*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(3*a*b*tan(
1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)^4 + 3*b^2*tan(1/2*d*x + 1/2
*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 9.87, size = 225, normalized size = 2.50

$$\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^2-2\,b^2\right) + 4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 2\,a^2 - \frac{2\,b^2}{3} - 2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)} + \frac{2\,a\,b\,\mathrm{atan}\left(\frac{4\,a^2\,b^2}{4\,a^3\,b-4\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} + \frac{4\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b-4\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^4*(2*a^2 - 2*b^2) + 4
*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2 - (2*b^2)/3 - 2*a*b*tan(c/2 + (d*x)/2)^5
+ 2*a*b*tan(c/2 + (d*x)/2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/
2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (2*a*b*atan((4*a^2*b^2)/(4*a^3*b - 4*a^
2*b^2*tan(c/2 + (d*x)/2)) + (4*a^3*b*tan(c/2 + (d*x)/2))/(4*a^3*b - 4*a^2*b
^2*tan(c/2 + (d*x)/2))))/d
```

3.1063 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] -a^2*x+1/2*b^2*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d-a^2*cot(d*x
+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2801, 2715, 8, 2672, 327, 212, 3554}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right) \\ &= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int \text{co} \\ &= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int 1 \\ &= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin}{2d} \\ &= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(}{}\end{aligned}$$

Mathematica [A]
time = 0.29, size = 116, normalized size = 1.49

$$\frac{-4a^2c+2b^2c-4a^2dx+2b^2dx+8ab\cos(c+dx)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^2\sin(2(c+dx))+2a^2\tan\left(\frac{1}{2}(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

Maple [A]
time = 0.18, size = 79, normalized size = 1.01

method	result
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$-a^2x+\frac{b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d(e^{2i(dx+c)}-1)}-\frac{2ab\ln(e^{i(d}}{d}$
norman	$\frac{\left(-a^2+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(-a^2+\frac{b^2}{2}\right)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2a^2+b^2)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2}{2d}+\frac{a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{(a^2}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-cot(d*x+c)-d*x-c)+2*a*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b
^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.50, size = 79, normalized size = 1.01

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

Fricas [A]
time = 0.39, size = 118, normalized size = 1.51

$$-\frac{b^2\cos(dx+c)^3+2\,ab\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,ab\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+(2\,a^2-b^2)\cos(dx+c)+((2\,a^2-b^2)dx-4\,ab\cos(dx+c))\sin(dx+c)}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2\cos^2(c+dx)\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 148, normalized size = 1.90

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-(2\,a^2-b^2)(dx+c)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 9.71, size = 277, normalized size = 3.55

$$\frac{b^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)-2\,a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)+2\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\cos\left(c+d\,x\right)-\frac{b^2\,\cos(c+d\,x)}{8}+\frac{b^2\,\cos(3\,c+3\,d\,x)}{8}-a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\sin\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (b^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/
2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2 + (d*x)/2) + 4*a*b*co
s(c/2 + (d*x)/2))) - 2*a^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (
d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2
 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))) + 2*a*b*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/d - (a^2*cos(c + d*x) - (b^2*cos(c + d*x))/8 + (b^2*cos(3
*c + 3*d*x))/8 - a*b*sin(2*c + 2*d*x))/(d*sin(c + d*x))
```

3.1064 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=89

$$-2abx+\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}+\frac{3b^2\cos(c+dx)}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin}{2d}$$

```
[Out] -2*a*b*x+1/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/d+3/2*b^2*cos(d*x+c)/d-a*b*cot
(d*x+c)/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2968, 3127, 3110, 3102, 2814, 3855}

$$\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{ab\cot(c+dx)}{d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}-2abx+\frac{3b^2\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (3*b^2*Cos[c + d*x
])/(2*d) - (a*b*Cot[c + d*x])/d - (Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c +
 d*x])^2)/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^3(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)$$

$$= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{1}{2}\int \text{cs}$$

$$= -\frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c}{2d}$$

$$= \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+dx)\csc(c+}{}$$

$$= -2abx + \frac{3b^2\cos(c+dx)}{2d} - \frac{ab\cot(c+dx)}{d} - \frac{\cot(c+d}{}$$

$$= -2abx + \frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{3b^2\cos(c+}{2d}$$

Mathematica [A]
time = 0.62, size = 155, normalized size = 1.74

$$\frac{-16abc-16abdx+8b^2\cos(c+dx)-8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+8ab\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-16*a*b*c - 16*a*b*d*x + 8*b^2*Cos[c + d*x] - 8*a*b*Cot[(c + d*x)/2] - a^2
*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)
/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[
(c + d*x)/2]^2 + 8*a*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A]
time = 0.22, size = 102, normalized size = 1.15

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab(-\cot(dx+c)-dx-c)+b^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab(-\cot(dx+c)-dx-c)+b^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot}{d}$
risch	$-2abx+\frac{e^{i(dx+c)}b^2}{2d}+\frac{e^{-i(dx+c)}b^2}{2d}-\frac{ia\left(ia\,e^{3i(dx+c)}+ia\,e^{i(dx+c)}+4b\,e^{2i(dx+c)}-4b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a^2\ln\left(e^{i(dx+c)}-1\right)}{2d}+$
norman	$\frac{\frac{ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ab\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{8d}+\frac{a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(a^2-4b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{\left(a^2-4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-ab}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-c
ot(d*x+c)))+2*a*b*(-cot(d*x+c)-d*x-c)+b^2*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x
+c))))
```

Maxima [A]
time = 0.49, size = 103, normalized size = 1.16

$$-\frac{8\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab-a^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-2b^2(2\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/4*(8*(d*x + c + 1/tan(d*x + c))*a*b - a^2*(2*cos(d*x + c)/(cos(d*x + c)^
2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 2*b^2*(2*cos(d*x
+ c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(83) = 166.
time = 0.38, size = 168, normalized size = 1.89

$$-\frac{8\,abdx\cos\left(dx+c\right)^2-4\,b^2\cos\left(dx+c\right)^3-8\,abdx-8\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\left(a^2-2\,b^2\right)\cos\left(dx+c\right)-\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - 4*b^2*cos(d*x + c)^3 - 8*a*b*d*x - 8*a*b*c
os(d*x + c)*sin(d*x + c) - 2*(a^2 - 2*b^2)*cos(d*x + c) - ((a^2 - 2*b^2)*co
s(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*co
s(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2
 - d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)^2\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**2*csc(c + d*x)**3, x)
```

Giac [A]
time = 0.47, size = 148, normalized size = 1.66

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-16(dx+c)ab+8ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4\left(a^2-2b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+\frac{16b^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1}+\frac{6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*a*b + 8*a*b*tan(1/2*d*x + 1/
2*c) - 4*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 16*b^2/(tan(1/2*d*x
 + 1/2*c)^2 + 1) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2
*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 10.20, size = 397, normalized size = 4.46

$$\frac{\cos(c+dx)\left(\frac{a^2}{2}-\frac{b^2}{4}\right)-\frac{b^2}{2}+\frac{a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4}-\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{b^2\cos(2c+2dx)}{2}+\frac{b^2\cos(3c+3dx)}{4}-2ab\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)-\frac{a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{4}+\frac{b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(2c+2dx)}{2}+ab\sin(2c+2dx)+2ab\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab-2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{-\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)ab+2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}\right)\cos(2c+2dx)}{d\left(\cos(c+dx)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (cos(c + d*x)*(a^2/2 - b^2/4) - b^2/2 + (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/4 - (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (b^2
*cos(2*c + 2*d*x))/2 + (b^2*cos(3*c + 3*d*x))/4 - 2*a*b*atan((a^2*sin(c/2 +
 (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos
(c/2 + (d*x)/2) - a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))) - (a^
2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/4 + (b^2*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 + a*b*sin(2*c +
 2*d*x) + 2*a*b*atan((a^2*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 4
*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos(c/2 + (d*x)/2) - a^2*cos(c/2 + (d*x)/2)
 + 4*a*b*sin(c/2 + (d*x)/2)))*cos(2*c + 2*d*x))/(d*(cos(c + d*x)^2 - 1))
```

3.1065 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=96

$$-b^2x+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{(a^2-2b^2)\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d}$$

```
[Out] -b^2*x+a*b*arctanh(cos(d*x+c))/d+1/3*(a^2-2*b^2)*cot(d*x+c)/d-1/3*a*b*cot(d
*x+c)*csc(d*x+c)/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2968, 3127, 3110, 3100, 2814, 3855}

$$\frac{(a^2-2b^2)\cot(c+dx)}{3d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\cot(c+dx)\csc(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d}+b^2(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (a*b*ArcTanh[Cos[c + d*x]])/d + ((a^2 - 2*b^2)*Cot[c + d*x])/(3*
d) - (a*b*Cot[c + d*x]*Csc[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(
a + b*Sin[c + d*x])^2)/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\, d$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{3d} + \frac{1}{3}\int \text{cs}$$

$$= -\frac{ab\cot(c+dx)\csc(c+dx)}{3d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3d}$$

$$= \frac{(a^2-2b^2)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+dx)}{3d} -$$

$$= -b^2x + \frac{(a^2-2b^2)\cot(c+dx)}{3d} - \frac{ab\cot(c+dx)\csc(c+}{3d}$$

$$= -b^2x + \frac{ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{(a^2-2b^2)\cot(c+dx}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 538 vs. 2(96) = 192.
time = 6.14, size = 538, normalized size = 5.60

$$-\frac{b^2(c+dx)(b+a\csc(c+dx))^2\sin^2(c+dx)}{d(a+b\sin(c+dx))^2}+\frac{\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{6d(a+b\sin(c+dx))^2}-\frac{ab\csc^2\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{4d(a+b\sin(c+dx))^2}-\frac{a^2\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{24d(a+b\sin(c+dx))^2}+\frac{ab(b+a\csc(c+dx))^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)}{d(a+b\sin(c+dx))^2}-\frac{ab(b+a\csc(c+dx))^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)}{d(a+b\sin(c+dx))^2}+\frac{ab(b+a\csc(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)}{4d(a+b\sin(c+dx))^2}+\frac{(b+a\csc(c+dx))^2\sec\left(\frac{1}{2}(c+dx)\right)\left(-a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)}{6d(a+b\sin(c+dx))^2}+\frac{a^2(b+a\csc(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{24d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d
*x])^2)) + ((a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(6*d*(a + b*Sin[c + d*x])^2) - (a*
b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(4*d*(a + b*Sin
[c + d*x])^2) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x
])^2*Sin[c + d*x]^2)/(24*d*(a + b*Sin[c + d*x])^2) + (a*b*(b + a*Csc[c + d*
x])^2*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*Sin[c + d*x])^2) - (a
*b*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(d*(a + b*S
in[c + d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*
x]^2)/(4*d*(a + b*Sin[c + d*x])^2) + ((b + a*Csc[c + d*x])^2*Sec[(c + d*x)/
2]*(-(a^2*Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^2)/(6*d*
(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Si
n[c + d*x]^2*Tan[(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.21, size = 96, normalized size = 1.00

method	result

derivativedivides	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2ab\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b^2(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{-\frac{a^2\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+2ab\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b^2(-\cot(dx+c)-dx-c)}{d}$
risch	$-b^2x+\frac{2ia^2e^{4i(dx+c)}-2ib^2e^{4i(dx+c)}+2b\,e^{5i(dx+c)}a+4ib^2e^{2i(dx+c)}+\frac{2ia^2}{3}-2ib^2-2b\,e^{i(dx+c)}a}{d\left(e^{2i(dx+c)}-1\right)^3}+\frac{ab\ln\left(e^{i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{ab\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{24d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-b^2x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b^2x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^2x\left(\tan^7\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^2/sin(d*x+c)^3*cos(d*x+c)^3+2*a*b*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.85

$$-\frac{6\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^2-3\,ab\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{2\,a^2}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c + 1/tan(d*x + c))*b^2 - 3*a*b*(2*cos(d*x + c)/(cos(d*x + c
)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 2*a^2/tan(d*x +
 c)^3)/d
```

Fricas [A]
time = 0.38, size = 167, normalized size = 1.74

$$\frac{2\left(a^2-3\,b^2\right)\cos\left(dx+c\right)^3+6\,b^2\cos\left(dx+c\right)+3\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-6\left(b^2dx\cos\left(dx+c\right)^2-b^2dx-ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*(a^2 - 3*b^2)*cos(d*x + c)^3 + 6*b^2*cos(d*x + c) + 3*(a*b*cos(d*x +
 c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a*b*cos(d*x + c)
^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(b^2*d*x*cos(d*x +
c)^2 - b^2*d*x - a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*si
n(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 167, normalized size = 1.74

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,(dx+c)b^2-24\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{44\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x +
 c)*b^2 - 24*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 12*b^2*tan(1/2*d*x + 1/2*c) + (44*a*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 9.48, size = 231, normalized size = 2.41

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{a^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2*cot(c/2 + (d*x)/2)^3)/(24*d) + (a^
2*cot(c/2 + (d*x)/2))/(8*d) - (b^2*cot(c/2 + (d*x)/2))/(2*d) - (a^2*tan(c/2
 + (d*x)/2))/(8*d) + (b^2*tan(c/2 + (d*x)/2))/(2*d) - (2*b^2*atan((b*cos(c/
2 + (d*x)/2) + a*sin(c/2 + (d*x)/2))/(a*cos(c/2 + (d*x)/2) - b*sin(c/2 + (d
*x)/2))))/d - (a*b*cot(c/2 + (d*x)/2)^2)/(4*d) + (a*b*tan(c/2 + (d*x)/2)^2)
/(4*d) - (a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.1066 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=123

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{2ab\cot(c+dx)}{3d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{ab\cot(c+dx)\csc^2(c$$

```
[Out] 1/8*(a^2+4*b^2)*arctanh(cos(d*x+c))/d+2/3*a*b*cot(d*x+c)/d+1/8*(a^2-2*b^2)*
cot(d*x+c)*csc(d*x+c)/d-1/6*a*b*cot(d*x+c)*csc(d*x+c)^2/d-1/4*cot(d*x+c)*cs
c(d*x+c)^3*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3127, 3110, 3100, 2827, 3852, 8, 3855}

$$\frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}+\frac{2ab\cot(c+dx)}{3d}-\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (2*a*b*Cot[c + d*x])/(3*d) +
((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*b*Cot[c + d*x]*Csc[c +
 d*x]^2)/(6*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \csc^5(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right) \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{4d}+\frac{1}{4}\int \\
&= -\frac{ab\cot(c+dx)\csc^2(c+dx)}{6d}-\frac{\cot(c+dx)\csc^3(c+}{} \\
&= \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{ab\cot(c+dx)\operatorname{cs}}{6d} \\
&= \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{ab\cot(c+dx)\operatorname{cs}}{6d} \\
&= \frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cot(c+}{8d} \\
&= \frac{(a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{2ab\cot(c+dx)}{3d}+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 579 vs. $2(123) = 246$.
time = 6.14, size = 579, normalized size = 4.71

$$\frac{ab\cot\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{3d(a+b\sin(c+dx))^2}+\frac{(a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{32d(a+b\sin(c+dx))^2}-\frac{ab\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{12d(a+b\sin(c+dx))^2}-\frac{a^2\csc^4\left(\frac{1}{2}(c+dx)\right)(b+a\csc(c+dx))^2\sin^2(c+dx)}{64d(a+b\sin(c+dx))^2}+\frac{(a^2+4b^2)(b+a\csc(c+dx))^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)}{8d(a+b\sin(c+dx))^2}+\frac{(-a^2-4b^2)(b+a\csc(c+dx))^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin^2(c+dx)}{8d(a+b\sin(c+dx))^2}+\frac{(-a^2+4b^2)(b+a\csc(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)}{32d(a+b\sin(c+dx))^2}+\frac{a^2(b+a\csc(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)}{64d(a+b\sin(c+dx))^2}-\frac{ab(b+a\csc(c+dx))^2\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{3d(a+b\sin(c+dx))^2}+\frac{ab(b+a\csc(c+dx))^2\sec^2\left(\frac{1}{2}(c+dx)\right)\sin^2(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)}{12d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*Cot[(c + d*x)/2]*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(3*d*(a + b*Si
n[c + d*x])^2) + ((a^2 - 4*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^2*S
in[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) - (a*b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(12*d*(a + b*Sin[c + d*
x])^2) - (a^2*Csc[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2)/(64
*d*(a + b*Sin[c + d*x])^2) + ((a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Log[Cos[
(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*Sin[c + d*x])^2) + ((-a^2 - 4*b^2
)*(b + a*Csc[c + d*x])^2*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^2)/(8*d*(a + b*
Sin[c + d*x])^2) + ((-a^2 + 4*b^2)*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^
2*Sin[c + d*x]^2)/(32*d*(a + b*Sin[c + d*x])^2) + (a^2*(b + a*Csc[c + d*x])
^2*Sec[(c + d*x)/2]^4*Sin[c + d*x]^2)/(64*d*(a + b*Sin[c + d*x])^2) - (a*b*
(b + a*Csc[c + d*x])^2*Sin[c + d*x]^2*Tan[(c + d*x)/2])/(3*d*(a + b*Sin[c +
 d*x])^2) + (a*b*(b + a*Csc[c + d*x])^2*Sec[(c + d*x)/2]^2*Sin[c + d*x]^2*T
an[(c + d*x)/2])/(12*d*(a + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.26, size = 142, normalized size = 1.15

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{2ab\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+b^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{2ab\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+b^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\right.}{d}$
risch	$-\frac{-48iab\,e^{6i(dx+c)}+3a^2e^{7i(dx+c)}-12b^2e^{7i(dx+c)}+48iab\,e^{4i(dx+c)}+21a^2e^{5i(dx+c)}+12b^2e^{5i(dx+c)}-16iab\,e^{2i(dx+c)}+21\,e}{12d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a^2}{64d}+\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{(a^2+4b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{(a^2+4b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{(a^2+16b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{(a^2+16b^2)(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*
cos(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-2/3*a*b/sin(d*x+c)^3*cos(d*x+c)^3
+b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]

time = 0.28, size = 129, normalized size = 1.05

$$-\frac{3a^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-12b^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)+\frac{32ab}{\tan(dx+c)^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(3*a^2*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x
 + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*b^2*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c)
 - 1)) + 32*a*b/tan(d*x + c)^3)/d
```

Fricas [A]

time = 0.38, size = 200, normalized size = 1.63

$$-\frac{32ab\cos(dx+c)^3\sin(dx+c)+6(a^2-4b^2)\cos(dx+c)^3+6(a^2+4b^2)\cos(dx+c)-3\left((a^2+4b^2)\cos(dx+c)^4-2(a^2+4b^2)\cos(dx+c)^2+a^2+4b^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left((a^2+4b^2)\cos(dx+c)^4-2(a^2+4b^2)\cos(dx+c)^2+a^2+4b^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{48\left(d\cos(dx+c)^4-2d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(32*a*b*cos(d*x + c)^3*sin(d*x + c) + 6*(a^2 - 4*b^2)*cos(d*x + c)^3
+ 6*(a^2 + 4*b^2)*cos(d*x + c) - 3*((a^2 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 +
```

```
 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^2
 + 4*b^2)*cos(d*x + c)^4 - 2*(a^2 + 4*b^2)*cos(d*x + c)^2 + a^2 + 4*b^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.48, size = 182, normalized size = 1.48

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,(a^2+4\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{50\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+200\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 + 24*b^
2*tan(1/2*d*x + 1/2*c)^2 - 48*a*b*tan(1/2*d*x + 1/2*c) - 24*(a^2 + 4*b^2)*l
og(abs(tan(1/2*d*x + 1/2*c))) + (50*a^2*tan(1/2*d*x + 1/2*c)^4 + 200*b^2*ta
n(1/2*d*x + 1/2*c)^4 + 48*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 16*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 9.33, size = 165, normalized size = 1.34

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{8}+\frac{b^2}{2}\right)}{d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{4}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{16\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) - (log(tan(c/2 + (d*x)/2))*(a^2/8 + b^2/2
))/d + (b^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (cot(c/2 + (d*x)/2)^4*(2*b^2*tan(
c/2 + (d*x)/2)^2 + a^2/4 - 4*a*b*tan(c/2 + (d*x)/2)^3 + (4*a*b*tan(c/2 + (d
*x)/2))/3))/(16*d) + (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) - (a*b*tan(c/2 + (d*
x)/2))/(4*d)
```

3.1067 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=148

$$\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{ab\cot(c+dx)\csc(c+dx)}{4d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}$$

```
[Out] 1/4*a*b*arctanh(cos(d*x+c))/d+1/15*(2*a^2+5*b^2)*cot(d*x+c)/d+1/4*a*b*cot(d
*x+c)*csc(d*x+c)/d+1/15*(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^2/d-1/10*a*b*cot(
d*x+c)*csc(d*x+c)^3/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2968, 3127, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d}+\frac{ab\cot(c+dx)\csc(c+dx)}{4d}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*ArcTanh[Cos[c + d*x]])/(4*d) + ((2*a^2 + 5*b^2)*Cot[c + d*x])/(15*d) +
 (a*b*Cot[c + d*x]*Csc[c + d*x])/(4*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c
+ d*x]^2)/(15*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(10*d) - (Cot[c + d*x]
*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx &= \int \csc^6(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)\ d\\ &= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{5d}+\frac{1}{5}\int \mathrm{cs}\\ &= -\frac{ab\cot(c+dx)\csc^3(c+dx)}{10d}-\frac{\cot(c+dx)\csc^4(c+dx}{5d}\\ &= \frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{ab\cot(c+dx)\mathrm{cs}}{10d}\\ &= \frac{(a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{ab\cot(c+dx)\mathrm{cs}}{10d}\\ &= \frac{ab\cot(c+dx)\csc(c+dx)}{4d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^2}{15d}\\ &= \frac{ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{(2a^2+5b^2)\cot(c+dx)}{15d}+\frac{ab}{}\end{aligned}$$

Mathematica [A]
time = 0.59, size = 236, normalized size = 1.59

$$\frac{\csc^5(c+dx)\left(-40(4a^2+b^2)\cos(c+dx)+20(-2a^2+b^2)\cos(3(c+dx))+8a^2\cos(5(c+dx))+20b^2\cos(5(c+dx))+150ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-150ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-180ab\sin(2(c+dx))-75ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+75ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))-30ab\sin(4(c+dx))+15ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-15ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^5*(-40*(4*a^2 + b^2)*Cos[c + d*x] + 20*(-2*a^2 + b^2)*Cos[3*(
c + d*x)] + 8*a^2*Cos[5*(c + d*x)] + 20*b^2*Cos[5*(c + d*x)] + 150*a*b*Log[
Cos[(c + d*x)/2]]*Sin[c + d*x] - 150*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]
 - 180*a*b*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)]
 + 75*a*b*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 30*a*b*Sin[4*(c + d*x)]
+ 15*a*b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*a*b*Log[Sin[(c + d*x)/
2]]*Sin[5*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.25, size = 135, normalized size = 0.91

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+2ab\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{b^{2}\left(\cos^{3}\right.}{3\sin(d}}{d}$
default	$\frac{a^{2}\left(-\frac{\cos^{3}(dx+c)}{5\sin(dx+c)^{5}}-\frac{2\left(\cos^{3}(dx+c)\right)}{15\sin(dx+c)^{3}}\right)+2ab\left(-\frac{\cos^{3}(dx+c)}{4\sin(dx+c)^{4}}-\frac{\cos^{3}(dx+c)}{8\sin(dx+c)^{2}}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{b^{2}\left(\cos^{3}\right.}{3\sin(d}}{d}$
risch	$-\frac{-60ib^{2}e^{8i(dx+c)}+15ab\,e^{9i(dx+c)}+120ia^{2}e^{6i(dx+c)}+120ib^{2}e^{6i(dx+c)}+90ab\,e^{7i(dx+c)}+40ia^{2}e^{4i(dx+c)}-80ib^{2}e^{4i(dx+c)}}{30d\left(e^{2i(dx+c)}-1\right)^{5}}$
norman	$-\frac{a^{2}}{160d}+\frac{a^{2}\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{\left(5a^{2}+8b^{2}\right)\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{\left(5a^{2}+8b^{2}\right)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{\left(11a^{2}+20b^{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+\frac{\left(11a^{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+2*
a*b*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d
*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*b^2/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]

time = 0.29, size = 108, normalized size = 0.73

$$-\frac{15\,ab\left(\frac{2\left(\cos(dx+c)^{3}+\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\cos(dx+c)^{2}+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{40\,b^{2}}{\tan(dx+c)^{3}}+\frac{8\left(5\tan(dx+c)^{2}+3\right)a^{2}}{\tan(dx+c)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(15*a*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 40*b^2/ta
n(d*x + c)^3 + 8*(5*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A]

time = 0.39, size = 195, normalized size = 1.32

$$\frac{8\left(2\,a^{2}+5\,b^{2}\right)\cos\left(dx+c\right)^{5}-40\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}+15\left(ab\cos\left(dx+c\right)^{4}-2\,ab\cos\left(dx+c\right)^{2}+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(ab\cos\left(dx+c\right)^{4}-2\,ab\cos\left(dx+c\right)^{2}+ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-30\left(ab\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/120*(8*(2*a^2 + 5*b^2)*cos(d*x + c)^5 - 40*(a^2 + b^2)*cos(d*x + c)^3 + 1
5*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) +
```

```
1/2)*sin(d*x + c) - 15*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*lo
g(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(a*b*cos(d*x + c)^3 + a*b*cos(
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.47, size = 222, normalized size = 1.50

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{274\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 + 5*a^2
*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*log(abs(t
an(1/2*d*x + 1/2*c))) - 30*a^2*tan(1/2*d*x + 1/2*c) - 60*b^2*tan(1/2*d*x +
1/2*c) + (274*a*b*tan(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 +
60*b^2*tan(1/2*d*x + 1/2*c)^4 - 5*a^2*tan(1/2*d*x + 1/2*c)^2 - 20*b^2*tan(1
/2*d*x + 1/2*c)^2 - 15*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/tan(1/2*d*x + 1/2*
c)^5)/d
```

Mupad [B]
time = 9.36, size = 187, normalized size = 1.26

$$\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{3}+\frac{4\,b^2}{3}\right)-\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^2+4\,b^2\right)+\frac{a^2}{5}+a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{32\,d}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^2}{16}+\frac{b^2}{8}\right)}{d}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^2}{96}+\frac{b^2}{24}\right)}{d}+\frac{a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a\,b\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) - (cot(c/2 + (d*x)/2)^5*(tan(c/2 + (d*x)
/2)^2*(a^2/3 + (4*b^2)/3) - tan(c/2 + (d*x)/2)^4*(2*a^2 + 4*b^2) + a^2/5 +
a*b*tan(c/2 + (d*x)/2)))/(32*d) - (tan(c/2 + (d*x)/2)*(a^2/16 + b^2/8))/d +
 (tan(c/2 + (d*x)/2)^3*(a^2/96 + b^2/24))/d + (a*b*tan(c/2 + (d*x)/2)^4)/(3
2*d) - (a*b*log(tan(c/2 + (d*x)/2)))/(4*d)
```

3.1068 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{2ab\cot(c+dx)}{5d}+\frac{2ab\cot^3(c+dx)}{15d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{(a$$

```
[Out] 1/16*(a^2+2*b^2)*arctanh(cos(d*x+c))/d+2/5*a*b*cot(d*x+c)/d+2/15*a*b*cot(d*
x+c)^3/d+1/16*(a^2+2*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/24*(a^2-2*b^2)*cot(d*x+
c)*csc(d*x+c)^3/d-1/15*a*b*cot(d*x+c)*csc(d*x+c)^4/d-1/6*cot(d*x+c)*csc(d*x
+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3127, 3110, 3100, 2827, 3852, 3853, 3855}

$$\frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d}+\frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{2ab\cot^3(c+dx)}{15d}+\frac{2ab\cot(c+dx)}{5d}-\frac{ab\cot(c+dx)\csc^4(c+dx)}{15d}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (2*a*b*Cot[c + d*x])/(5*d) +
 (2*a*b*Cot[c + d*x]^3)/(15*d) + ((a^2 + 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(16*d) + ((a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*b*Cot[c +
d*x]*Csc[c + d*x]^4)/(15*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d
*x])^2)/(6*d)
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^2\left(1-\sin^2(c+dx)\right)$$

$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2}{6d} + \frac{1}{6}\int$$

$$= -\frac{ab\cot(c+dx)\csc^4(c+dx)}{15d} - \frac{\cot(c+dx)\csc^5(c+d}{6}$$

$$= \frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\,c}{15d}$$

$$= \frac{(a^2-2b^2)\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot(c+dx)\,c}{15d}$$

$$= \frac{(a^2+2b^2)\cot(c+dx)\csc(c+dx)}{16d} + \frac{(a^2-2b^2)\cot(c}{2}$$

$$= \frac{(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{2ab\cot(c+dx)}{5d} + \frac{2}{}$$

Mathematica [A]
time = 0.59, size = 296, normalized size = 1.74

$$\frac{256ab\cot\left(\frac{1}{2}(c+dx)\right)+30(a^2+2b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)+120a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+240b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-120a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-240b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-60b^2\sec^2\left(\frac{1}{2}(c+dx)\right)+30b^2\sec^4\left(\frac{1}{2}(c+dx)\right)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-64ab\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+768ab\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)-a\csc^6\left(\frac{1}{2}(c+dx)\right)(5a+12b\sin(c+dx))+\csc^4\left(\frac{1}{2}(c+dx)\right)(-30b^2+4ab\sin(c+dx))-256ab\tan\left(\frac{1}{2}(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (256*a*b*Cot[(c + d*x)/2] + 30*(a^2 + 2*b^2)*Csc[(c + d*x)/2]^2 + 120*a^2*L
og[Cos[(c + d*x)/2]] + 240*b^2*Log[Cos[(c + d*x)/2]] - 120*a^2*Log[Sin[(c +
 d*x)/2]] - 240*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c + d*x)/2]^2 - 60*
b^2*Sec[(c + d*x)/2]^2 + 30*b^2*Sec[(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]
^6 - 64*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[
(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a + 12*b*Sin[c + d*x]) + Csc[(c +
d*x)/2]^4*(-30*b^2 + 4*a*b*Sin[c + d*x]) - 256*a*b*Tan[(c + d*x)/2])/(1920*
d)
```

Maple [A]
time = 0.30, size = 199, normalized size = 1.17

method	result

derivativedivides	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+2ab\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right.}{15\sin(dx+c)^3}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+2ab\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right.}{15\sin(dx+c)^3}\right.}{d}$
risch	$-\frac{15a^2e^{11i(dx+c)}+30b^2e^{11i(dx+c)}+64iab-85a^2e^{9i(dx+c)}+150b^2e^{9i(dx+c)}-384iab\,e^{2i(dx+c)}-570a^2e^{7i(dx+c)}-180b^2e^{7i(d}}{1}$
norman	$-\frac{a^2}{384d}+\frac{a^2\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{\left(a^2+3b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{\left(a^2+3b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{\left(5a^2+6b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{\left(5a^2+6b^2\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^3-1/8/sin(d*x+c)^4*cos(d*x+c)^3-1/16
/sin(d*x+c)^2*cos(d*x+c)^3-1/16*cos(d*x+c)-1/16*ln(csc(d*x+c)-cot(d*x+c)))+
2*a*b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+b^2*(
-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(d*x+c)
-1/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.29, size = 186, normalized size = 1.09

$$-\frac{5\,a^2\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+30\,b^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{64\left(5\,\tan(dx+c)^2+3\right)ab}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^2*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 30*b^2*(2*(cos(d*x + c)^3 + cos(d*x + c
))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(co
s(d*x + c) - 1)) + 64*(5*tan(d*x + c)^2 + 3)*a*b/tan(d*x + c)^5)/d
```

Fricas [A]

time = 0.37, size = 283, normalized size = 1.66

$$-\frac{30\,(a^2+2\,b^2)\cos\left(dx+c\right)^5-80\,a^2\cos\left(dx+c\right)^3-30\,(a^2+2\,b^2)\cos\left(dx+c\right)-15\left(\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^6-3\,(a^2+2\,b^2)\cos\left(dx+c\right)^4+3\,(a^2+2\,b^2)\cos\left(dx+c\right)^2-a^2-2\,b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^6-3\,(a^2+2\,b^2)\cos\left(dx+c\right)^4+3\,(a^2+2\,b^2)\cos\left(dx+c\right)^2-a^2-2\,b^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+64\left(2\,ab\cos\left(dx+c\right)^5-5\,ab\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(30*(a^2 + 2*b^2)*cos(d*x + c)^5 - 80*a^2*cos(d*x + c)^3 - 30*(a^2 +
 2*b^2)*cos(d*x + c) - 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*c
os(d*x + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(1/2*cos(d
*x + c) + 1/2) + 15*((a^2 + 2*b^2)*cos(d*x + c)^6 - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^4 + 3*(a^2 + 2*b^2)*cos(d*x + c)^2 - a^2 - 2*b^2)*log(-1/2*cos(d*x +
c) + 1/2) + 64*(2*a*b*cos(d*x + c)^5 - 5*a*b*cos(d*x + c)^3)*sin(d*x + c))/
(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]

time = 0.48, size = 276, normalized size = 1.62

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{294\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+588\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-40\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 + 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*a*b*tan(1/2*
d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*a*b*tan(1/2*d*x + 1/2*
c) - 120*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^2*tan(1/2*d*
x + 1/2*c)^6 + 588*b^2*tan(1/2*d*x + 1/2*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c
)^5 + 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 40*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^
2*tan(1/2*d*x + 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d
*x + 1/2*c) - 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]

time = 9.44, size = 245, normalized size = 1.44

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{16}+\frac{b^2}{8}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{a^2}{6}-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2}+b^2\right)+\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}\right)}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{128}+\frac{b^2}{64}\right)}{d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{48\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{80\,d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) - (a^2*tan(c/2 + (d*x)/2)^2)/(128*d) - (
log(tan(c/2 + (d*x)/2))*(a^2/16 + b^2/8))/d - (cot(c/2 + (d*x)/2)^6*(a^2/6
- (a^2*tan(c/2 + (d*x)/2)^4)/2 + tan(c/2 + (d*x)/2)^2*(a^2/2 + b^2) + (4*a*
b*tan(c/2 + (d*x)/2)^3)/3 - 8*a*b*tan(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (
d*x)/2))/5))/(64*d) + (tan(c/2 + (d*x)/2)^4*(a^2/128 + b^2/64))/d + (a*b*ta
n(c/2 + (d*x)/2)^3)/(48*d) + (a*b*tan(c/2 + (d*x)/2)^5)/(80*d) - (a*b*tan(c
/2 + (d*x)/2))/(8*d)
```

3.1069 $\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{1}{16}a\left(2a^2+3b^2\right)x-\frac{b(21a^2+4b^2)\cos(c+dx)}{35d}+\frac{b(21a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{a(2a^2+3b^2)\cos(c+dx)\sin(c+dx)}{16d}$$

```
[Out] 1/16*a*(2*a^2+3*b^2)*x-1/35*b*(21*a^2+4*b^2)*cos(d*x+c)/d+1/105*b*(21*a^2+4
*b^2)*cos(d*x+c)^3/d-1/16*a*(2*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/56*a*(2
*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)^3/d+1/35*b*(a^2-b^2)*cos(d*x+c)*sin(d*x+c
)^4/d+1/14*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/d+1/7*cos(d*x+c)*si
n(d*x+c)^3*(a+b*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2968, 3129, 3128, 3112, 3102, 2827, 2715, 8, 2713}

$$\frac{b(21a^2+4b^2)\cos^3(c+dx)}{105d}-\frac{b(21a^2+4b^2)\cos(c+dx)}{35d}+\frac{b(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{35d}+\frac{a(2a^2-7b^2)\sin^3(c+dx)\cos(c+dx)}{56d}-\frac{a(2a^2+3b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}ax\left(2a^2+3b^2\right)+\frac{\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{7d}+\frac{a\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{14d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + 3*b^2)*x)/16 - (b*(21*a^2 + 4*b^2)*Cos[c + d*x])/(35*d) + (b*(2
1*a^2 + 4*b^2)*Cos[c + d*x]^3)/(105*d) - (a*(2*a^2 + 3*b^2)*Cos[c + d*x]*Si
n[c + d*x])/(16*d) + (a*(2*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(56*d)
 + (b*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(35*d) + (a*Cos[c + d*x]*Sin
[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(14*d) + (Cos[c + d*x]*Sin[c + d*x]^3*(
a + b*Sin[c + d*x])^3)/(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
```

```
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \cos^2(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \sin^2(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,d$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{7d} + \frac{1}{7}\int \sin^2$$

$$= \frac{a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2}{14d} + \frac{\cos(c-}{}$$

$$= \frac{b(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{35d} + \frac{a\cos(c+dx)\sin^3}{}$$

$$= \frac{a(2a^2-7b^2)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b(a^2-b^2)\cos(}{}$$

$$= \frac{a(2a^2-7b^2)\cos(c+dx)\sin^3(c+dx)}{56d} + \frac{b(a^2-b^2)\cos(}{}$$

$$= -\frac{a(2a^2+3b^2)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a(2a^2-7b^2)\,c}{}$$

$$= \frac{1}{16}a\left(2a^2+3b^2\right)x - \frac{b(21a^2+4b^2)\cos(c+dx)}{35d} + \frac{b(21a^2}{}$$

Mathematica [A]
time = 0.61, size = 157, normalized size = 0.68

$$\frac{-105b(24a^2+5b^2)\cos(c+dx)-35(12a^2b+b^3)\cos(3(c+dx))+63(4a^2b+b^3)\cos(5(c+dx))-15b^3\cos(7(c+dx))+105a(8a^2c+12b^2c+8a^2dx+12b^2dx-3b^2\sin(2(c+dx))-(2a^2+3b^2)\sin(4(c+dx))+b^2\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(24*a^2 + 5*b^2)*Cos[c + d*x] - 35*(12*a^2*b + b^3)*Cos[3*(c + d*x)
] + 63*(4*a^2*b + b^3)*Cos[5*(c + d*x)] - 15*b^3*Cos[7*(c + d*x)] + 105*a*(
8*a^2*c + 12*b^2*c + 8*a^2*d*x + 12*b^2*d*x - 3*b^2*Sin[2*(c + d*x)] - (2*a
^2 + 3*b^2)*Sin[4*(c + d*x)] + b^2*Sin[6*(c + d*x)]))/(6720*d)
```

Maple [A]
time = 0.31, size = 196, normalized size = 0.84

method	result
derivativedivides	$\frac{a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2}{}$
default	$\frac{a^3\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2}{}$
risch	$\frac{a^3x}{8}+\frac{3a\,b^2x}{16}-\frac{3a^2b\cos(dx+c)}{8d}-\frac{5b^3\cos(dx+c)}{64d}-\frac{b^3\cos(7dx+7c)}{448d}+\frac{a\,b^2\sin(6dx+6c)}{64d}+\frac{3b\cos(5dx+5c)a^2}{80d}$
norman	$\frac{-\frac{84a^2b+16b^3}{105d}+\frac{a\left(2a^2+3b^2\right)x}{16}-\frac{12a^2b\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(24a^2b-16b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{\left(24a^2b+16b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-(60a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/
8*c)+3*a^2*b*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1
/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*sin(d*x+c)*co
s(d*x+c)+1/16*d*x+1/16*c)+b^3*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+
c)^2*cos(d*x+c)^3-8/105*cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.56

$$\frac{210\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^3+1344\,\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^2b-105\,\left(4\,\sin(2\,dx+2\,c)^3-12\,dx-12\,c+3\,\sin(4\,dx+4\,c)\right)ab^2-64\,\left(15\,\cos(dx+c)^7-42\,\cos(dx+c)^5+35\,\cos(dx+c)^3\right)b^3}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/6720*(210*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^3 + 1344*(3*cos(d*x + c)^5 -
 5*cos(d*x + c)^3)*a^2*b - 105*(4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*si
n(4*d*x + 4*c))*a*b^2 - 64*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(
d*x + c)^3)*b^3)/d
```

Fricas [A]
time = 0.37, size = 141, normalized size = 0.61

$$-\frac{240\,b^3\cos(dx+c)^7-336\,(3\,a^2b+2\,b^3)\cos(dx+c)^5+560\,(3\,a^2b+b^3)\cos(dx+c)^3-105\,(2\,a^3+3\,ab^2)dx-105\,\left(8\,ab^2\cos(dx+c)^5-2\,(2\,a^3+7\,ab^2)\cos(dx+c)^3+(2\,a^3+3\,ab^2)\cos(dx+c)\right)\sin(dx+c)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1680*(240*b^3*cos(d*x + c)^7 - 336*(3*a^2*b + 2*b^3)*cos(d*x + c)^5 + 56
0*(3*a^2*b + b^3)*cos(d*x + c)^3 - 105*(2*a^3 + 3*a*b^2)*d*x - 105*(8*a*b^2
*cos(d*x + c)^5 - 2*(2*a^3 + 7*a*b^2)*cos(d*x + c)^3 + (2*a^3 + 3*a*b^2)*co
s(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.74, size = 394, normalized size = 1.70

$$\begin{cases} \frac{a^3 x \sin^4(c+dx)}{8} + \frac{a^3 x \sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{a^3 x \cos^4(c+dx)}{8} + \frac{a^3 \sin^3(c+dx)\cos(c+dx)}{8d} - \frac{a^3 \sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a^2 b \sin^2(c+dx)\cos^3(c+dx)}{d} - \frac{2a^2 b\cos^5(c+dx)}{5d} + \frac{3ab^2 x \sin^6(c+dx)}{16} + \frac{9ab^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{9ab^2 x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{3ab^2 x\cos^6(c+dx)}{16} + \frac{3ab^2\sin^5(c+dx)\cos(c+dx)}{16d} - \frac{ab^2\sin^3(c+dx)\cos^3(c+dx)}{2d} - \frac{3ab^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{b^3\sin^4(c+dx)\cos^3(c+dx)}{3d} - \frac{4b^3\sin^2(c+dx)\cos^5(c+dx)}{15d} - \frac{8b^3\cos^7(c+dx)}{105d} & \text{for}\ d \neq 0 \\ x(a+b\sin(c))^3\sin^2(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**4/8 + a**3*x*sin(c + d*x)**2*cos(c + d*x)**
2/4 + a**3*x*cos(c + d*x)**4/8 + a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) -
a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**2*b*sin(c + d*x)**2*cos(c + d*
x)**3/d - 2*a**2*b*cos(c + d*x)**5/(5*d) + 3*a*b**2*x*sin(c + d*x)**6/16 +
9*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*
cos(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)*
*5*cos(c + d*x)/(16*d) - a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a
*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b**3*sin(c + d*x)**4*cos(c + d*
x)**3/(3*d) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) - 8*b**3*cos(c
+ d*x)**7/(105*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**2, Tru
e))
```

Giac [A]
time = 0.59, size = 166, normalized size = 0.72

$$-\frac{b^3\cos(7\,dx+7\,c)}{448\,d} + \frac{ab^2\sin(6\,dx+6\,c)}{64\,d} - \frac{3\,ab^2\sin(2\,dx+2\,c)}{64\,d} + \frac{1}{16}\left(2\,a^3+3\,ab^2\right)x + \frac{3\left(4\,a^2b+b^3\right)\cos(5\,dx+5\,c)}{320\,d} - \frac{\left(12\,a^2b+b^3\right)\cos(3\,dx+3\,c)}{192\,d} - \frac{\left(24\,a^2b+5\,b^3\right)\cos(dx+c)}{64\,d} - \frac{\left(2\,a^3+3\,ab^2\right)\sin(4\,dx+4\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*b^3*cos(7*d*x + 7*c)/d + 1/64*a*b^2*sin(6*d*x + 6*c)/d - 3/64*a*b^2*
sin(2*d*x + 2*c)/d + 1/16*(2*a^3 + 3*a*b^2)*x + 3/320*(4*a^2*b + b^3)*cos(5
*d*x + 5*c)/d - 1/192*(12*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 1/64*(24*a^2*b
+ 5*b^3)*cos(d*x + c)/d - 1/64*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 10.66, size = 455, normalized size = 1.96

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)}{8\,\left(\frac{a^3}{4}+\frac{3\,a\,b^2}{8}\right)}\right)\,\left(2\,a^2+3\,b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{4\,a^2\,b}{5}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{5\,a\,b^2}{2}-a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{5\,a\,b^2}{2}-a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{a^3}{4}+\frac{3\,a\,b^2}{8}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{11\,a^3}{4}+\frac{97\,a\,b^2}{8}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{11\,a^3}{4}+\frac{97\,a\,b^2}{8}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(8\,a^2\,b-\frac{16\,b^3}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{24\,a^2\,b}{5}+\frac{16\,b^3}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(20\,a^2\,b+\frac{32\,b^3}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{28\,a^2\,b}{5}+\frac{16\,b^3}{15}\right)+\frac{16\,b^3}{105}+12\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a\,\left(2\,a^2+3\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2))/(8*((3*a*b^2)/8 + a^3/4)))*(
2*a^2 + 3*b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a*b^2)/8 + a^3/4) + (4*a^2*
b)/5 + tan(c/2 + (d*x)/2)^3*((5*a*b^2)/2 - a^3) - tan(c/2 + (d*x)/2)^11*((5
*a*b^2)/2 - a^3) - tan(c/2 + (d*x)/2)^13*((3*a*b^2)/8 + a^3/4) - tan(c/2 +
(d*x)/2)^5*((97*a*b^2)/8 + (11*a^3)/4) + tan(c/2 + (d*x)/2)^9*((97*a*b^2)/8
 + (11*a^3)/4) + tan(c/2 + (d*x)/2)^6*(8*a^2*b - (16*b^3)/3) + tan(c/2 + (d
*x)/2)^4*((24*a^2*b)/5 + (16*b^3)/5) + tan(c/2 + (d*x)/2)^8*(20*a^2*b + (32
*b^3)/3) + tan(c/2 + (d*x)/2)^2*((28*a^2*b)/5 + (16*b^3)/15) + (16*b^3)/105
 + 12*a^2*b*tan(c/2 + (d*x)/2)^10)/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2
+ (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c
/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) -
(a*(2*a^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

3.1070 $\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=163

$$\frac{1}{16}b\left(6a^2+b^2\right)x-\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}+\frac{b(6a^2+b^2)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(2a^2+5b^2)\cos^3(c+dx)(}{40d}$$

```
[Out] 1/16*b*(6*a^2+b^2)*x-1/120*a*(2*a^2+33*b^2)*cos(d*x+c)^3/d+1/16*b*(6*a^2+b^
2)*cos(d*x+c)*sin(d*x+c)/d-1/40*(2*a^2+5*b^2)*cos(d*x+c)^3*(a+b*sin(d*x+c))
/d-1/10*a*cos(d*x+c)^3*(a+b*sin(d*x+c))^2/d-1/6*cos(d*x+c)^3*(a+b*sin(d*x+c
))^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2941, 2748, 2715, 8}

$$-\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}-\frac{(2a^2+5b^2)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}+\frac{b(6a^2+b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}bx\left(6a^2+b^2\right)-\frac{\cos^3(c+dx)(a+b\sin(c+dx))^3}{6d}-\frac{a\cos^3(c+dx)(a+b\sin(c+dx))^2}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 + b^2)*x)/16 - (a*(2*a^2 + 33*b^2)*Cos[c + d*x]^3)/(120*d) + (b*(
6*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((2*a^2 + 5*b^2)*Cos[c + d
*x]^3*(a + b*Sin[c + d*x]))/(40*d) - (a*Cos[c + d*x]^3*(a + b*Sin[c + d*x])
^2)/(10*d) - (Cos[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^2(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos^3(c+dx)(a+b\sin(c+dx))^3}{6d}+\frac{1}{6}\int \cos^2(c+dx)$$
$$= -\frac{a\cos^3(c+dx)(a+b\sin(c+dx))^2}{10d}-\frac{\cos^3(c+dx)(a}{6}$$
$$= -\frac{(2a^2+5b^2)\cos^3(c+dx)(a+b\sin(c+dx))}{40d}-\frac{a\cos^3(}{}$$
$$= -\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}-\frac{(2a^2+5b^2)\cos^3(c+dx}{40d}$$
$$= -\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}+\frac{b(6a^2+b^2)\cos(c+dx)}{16d}$$
$$= \frac{1}{16}b(6a^2+b^2)\,x-\frac{a(2a^2+33b^2)\cos^3(c+dx)}{120d}+\frac{b(6a^2}{}$$

Mathematica [A]
time = 0.58, size = 138, normalized size = 0.85

$$\frac{-120a(2a^2+3b^2)\cos(c+dx)-20(4a^3+3ab^2)\cos(3(c+dx))+b(36ab\cos(5(c+dx))+5(72a^2c+18b^2c+72a^2dx+12b^2dx-3b^2\sin(2(c+dx))-3(6a^2+b^2)\sin(4(c+dx))+b^2\sin(6(c+dx))))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-120*a*(2*a^2 + 3*b^2)*Cos[c + d*x] - 20*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)
] + b*(36*a*b*Cos[5*(c + d*x)] + 5*(72*a^2*c + 18*b^2*c + 72*a^2*d*x + 12*b
^2*d*x - 3*b^2*Sin[2*(c + d*x)] - 3*(6*a^2 + b^2)*Sin[4*(c + d*x)] + b^2*Si
n[6*(c + d*x)])))/(960*d)
```

Maple [A]
time = 0.24, size = 158, normalized size = 0.97

method	result

derivativedivides	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^2b(-\frac{\sin(dx+c)(\cos^3(dx+c))}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8})+3a\,b^2(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(c}{}}{d}$
default	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^2b(-\frac{\sin(dx+c)(\cos^3(dx+c))}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8})+3a\,b^2(-\frac{(\sin^2(dx+c))(\cos^3(dx+c))}{5}-\frac{2(c}{}}{d}$
risch	$\frac{3a^2bx}{8}+\frac{b^3x}{16}-\frac{a^3\cos(dx+c)}{4d}-\frac{3a\,b^2\cos(dx+c)}{8d}+\frac{b^3\sin(6dx+6c)}{192d}+\frac{3a\,b^2\cos(5dx+5c)}{80d}-\frac{3b\sin(4dx+4c)a^2}{32d}-$
norman	$\frac{-\frac{10a^3+12a\,b^2}{15d}+\frac{b(6a^2+b^2)x}{16}-\frac{2a^3(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{d}-\frac{4a^3(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{d}-\frac{3(2a^3+4a\,b^2)(\tan^8(\frac{dx}{2}+\frac{c}{2}))}{d}-\frac{4(5a^3+6a\,b^2)(\tan^6(}{3d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*cos(d*x+c)^3+3*a^2*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*
x+c)*cos(d*x+c)+1/8*d*x+1/8*c)+3*a*b^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15
*cos(d*x+c)^3)+b^3*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c
)^3+1/16*sin(d*x+c)*cos(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.66

$$-\frac{320\,a^3\cos(dx+c)^3-90\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b-192\,(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3)ab^2+5\,(4\,\sin(2\,dx+2\,c)^3-12\,dx-12\,c+3\,\sin(4\,dx+4\,c))b^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(320*a^3*cos(d*x + c)^3 - 90*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2*b
- 192*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^2 + 5*(4*sin(2*d*x + 2*c)^3
 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A]
time = 0.37, size = 116, normalized size = 0.71

$$\frac{144\,ab^2\cos(dx+c)^5-80\,(a^3+3\,ab^2)\cos(dx+c)^3+15\,(6\,a^2b+b^3)dx+5\,(8\,b^3\cos(dx+c)^5-2\,(18\,a^2b+7\,b^3)\cos(dx+c)^3+3\,(6\,a^2b+b^3)\cos(dx+c))\sin(dx+c)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(144*a*b^2*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(6
*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b + 7*b^3)*cos(d*x
+ c)^3 + 3*(6*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.
time = 0.51, size = 340, normalized size = 2.09

$$\begin{cases} -\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^2bx\sin^4(c+dx)}{8}+\frac{3a^2bx\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2bx\cos^4(c+dx)}{8}+\frac{3a^2b\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3a^2b\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{ab^2\sin^2(c+dx)\cos^3(c+dx)}{d}-\frac{2ab^2\cos^5(c+dx)}{5d}+\frac{b^3x\sin^6(c+dx)}{16}+\frac{3b^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3b^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{b^3x\cos^6(c+dx)}{16}+\frac{b^3\sin^5(c+dx)\cos(c+dx)}{16d}-\frac{b^3\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b^3\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))^3\sin(c)\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**3/(3*d) + 3*a**2*b*x*sin(c + d*x)**4/8 + 3*a
**2*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a**2*b*x*cos(c + d*x)**4/8 +
3*a**2*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a**2*b*sin(c + d*x)*cos(c +
 d*x)**3/(8*d) - a*b**2*sin(c + d*x)**2*cos(c + d*x)**3/d - 2*a*b**2*cos(c
+ d*x)**5/(5*d) + b**3*x*sin(c + d*x)**6/16 + 3*b**3*x*sin(c + d*x)**4*cos(
c + d*x)**2/16 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b**3*x*cos(c
 + d*x)**6/16 + b**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) - b**3*sin(c + d*x
)**3*cos(c + d*x)**3/(6*d) - b**3*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d
, 0)), (x*(a + b*sin(c))**3*sin(c)*cos(c)**2, True))
```

Giac [A]
time = 0.56, size = 139, normalized size = 0.85

$$\frac{3\,ab^{2}\cos\left(5\,dx+5\,c\right)}{80\,d}+\frac{b^{3}\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b^{3}\sin\left(2\,dx+2\,c\right)}{64\,d}+\frac{1}{16}\left(6\,a^{2}b+b^{3}\right)x-\frac{\left(4\,a^{3}+3\,ab^{2}\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(2\,a^{3}+3\,ab^{2}\right)\cos\left(dx+c\right)}{8\,d}-\frac{\left(6\,a^{2}b+b^{3}\right)\sin\left(4\,dx+4\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/80*a*b^2*cos(5*d*x + 5*c)/d + 1/192*b^3*sin(6*d*x + 6*c)/d - 1/64*b^3*sin
(2*d*x + 2*c)/d + 1/16*(6*a^2*b + b^3)*x - 1/48*(4*a^3 + 3*a*b^2)*cos(3*d*x
 + 3*c)/d - 1/8*(2*a^3 + 3*a*b^2)*cos(d*x + c)/d - 1/64*(6*a^2*b + b^3)*sin
(4*d*x + 4*c)/d
```

Mupad [B]
time = 10.75, size = 425, normalized size = 2.61

$$\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2+b^2\right)}{8\left(\frac{3a^2b}{4}+\frac{b^3}{8}\right)}\right)\left(6a^2+b^2\right)}{8d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3a^2b}{4}+\frac{b^3}{8}\right)+4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\frac{4ab^2}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(6a^3+12ab^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(2a^3+\frac{24ab^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{20a^3}{3}+8ab^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{3a^2b}{4}+\frac{b^3}{8}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{9a^2b}{2}+\frac{19b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{9a^2b}{2}+\frac{19b^3}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{15a^2b}{4}-\frac{17b^3}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{15a^2b}{4}-\frac{17b^3}{24}\right)+\frac{2a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{b\left(6a^2+b^2\right)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (b*atan((b*tan(c/2 + (d*x)/2)*(6*a^2 + b^2))/(8*((3*a^2*b)/4 + b^3/8)))*(6*
a^2 + b^2))/(8*d) - (tan(c/2 + (d*x)/2)*((3*a^2*b)/4 + b^3/8) + 4*a^3*tan(c
/2 + (d*x)/2)^4 + 2*a^3*tan(c/2 + (d*x)/2)^10 + (4*a*b^2)/5 + tan(c/2 + (d*
x)/2)^8*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*((24*a*b^2)/5 + 2*a^3) +
tan(c/2 + (d*x)/2)^6*(8*a*b^2 + (20*a^3)/3) - tan(c/2 + (d*x)/2)^11*((3*a^2
*b)/4 + b^3/8) - tan(c/2 + (d*x)/2)^5*((9*a^2*b)/2 + (19*b^3)/4) + tan(c/2
+ (d*x)/2)^7*((9*a^2*b)/2 + (19*b^3)/4) - tan(c/2 + (d*x)/2)^3*((15*a^2*b)/
4 - (17*b^3)/24) + tan(c/2 + (d*x)/2)^9*((15*a^2*b)/4 - (17*b^3)/24) + (2*a
^3)/3)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 +
(d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
(d*x)/2)^12 + 1)) - (b*(6*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/
(8*d)
```

3.1071 $\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=136

$$\frac{1}{8}b\left(12a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a(a^2-2b^2)\cos(c+dx)}{2d}+\frac{b(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\,\mathrm{c}}{}$$

```
[Out] 1/8*b*(12*a^2+b^2)*x-a^3*arctanh(cos(d*x+c))/d+1/2*a*(a^2-2*b^2)*cos(d*x+c)
/d+1/8*b*(2*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)*(a+b*sin(d*x+
c))^2/d+1/4*cos(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [A]

time = 0.27, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2968, 3129, 3128, 3112, 3102, 2814, 3855}

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a(a^2-2b^2)\cos(c+dx)}{2d}+\frac{b(2a^2-b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}bx\left(12a^2+b^2\right)+\frac{a\cos(c+dx)(a+b\sin(c+dx))^2}{4d}+\frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(12*a^2 + b^2)*x)/8 - (a^3*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 - 2*b^2)*C
os[c + d*x])/(2*d) + (b*(2*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a
*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(4*d) + (Cos[c + d*x]*(a + b*Sin[c +
d*x])^3)/(4*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&& !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx$$

$$= \frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{1}{4}\int \csc(c+dx)(a+b$$

$$= \frac{a\cos(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\cos(c+dx)(a+b\sin}{4d}$$

$$= \frac{b(2a^2-b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a\cos(c+dx)(a+b}{4d}$$

$$= \frac{a(a^2-2b^2)\cos(c+dx)}{2d} + \frac{b(2a^2-b^2)\cos(c+dx)\sin(c+}{8d}$$

$$= \frac{1}{8}b\left(12a^2+b^2\right)x + \frac{a(a^2-2b^2)\cos(c+dx)}{2d} + \frac{b(2a^2-b^2)}{}$$

$$= \frac{1}{8}b\left(12a^2+b^2\right)x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} + \frac{a(a^2-2b^2)}{2}$$

Mathematica [A]

time = 0.22, size = 129, normalized size = 0.95

$$\frac{48a^2bc+4b^3c+48a^2bdx+4b^3dx+8a(4a^2-3b^2)\cos(c+dx)-8ab^2\cos(3(c+dx))-32a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+32a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+24a^2b\sin(2(c+dx))-b^3\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (48*a^2*b*c + 4*b^3*c + 48*a^2*b*d*x + 4*b^3*d*x + 8*a*(4*a^2 - 3*b^2)*Cos[
c + d*x] - 8*a*b^2*Cos[3*(c + d*x)] - 32*a^3*Log[Cos[(c + d*x)/2]] + 32*a^3
*Log[Sin[(c + d*x)/2]] + 24*a^2*b*Sin[2*(c + d*x)] - b^3*Sin[4*(c + d*x)])/
(32*d)
```

Maple [A]

time = 0.23, size = 117, normalized size = 0.86

method	result
derivativedivides	$\frac{a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-(\cos^3(dx+c))a\,b^2+b^3\left(-\frac{\sin(dx+c)\left(\cos^3(\right.}{4}\right.}{d}$
default	$\frac{a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a^2b\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-(\cos^3(dx+c))a\,b^2+b^3\left(-\frac{\sin(dx+c)\left(\cos^3(\right.}{4}\right.}{d}$
risch	$\frac{3a^2bx}{2}+\frac{b^3x}{8}+\frac{a^3e^{i(dx+c)}}{2d}-\frac{3\,e^{i(dx+c)}a\,b^2}{8d}+\frac{a^3e^{-i(dx+c)}}{2d}-\frac{3\,e^{-i(dx+c)}a\,b^2}{8d}-\frac{a^3\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a^3\ln\left(e^{i(}\right.}{}$

norman	$\frac{(\frac{3}{2}a^2b+\frac{1}{8}b^3)x+(6a^2b+\frac{1}{2}b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6a^2b+\frac{1}{2}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(9a^2b+\frac{3}{4}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^2b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(1/2*sin(d*x+c)*cos
(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^3*a*b^2+b^3*(-1/4*sin(d*x+c)*cos(d*x+c)^3
+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c))
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.74

$$-\frac{32\,ab^2\cos\left(dx+c\right)^3-24\,\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2b-\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^3-16\,a^3\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(32*a*b^2*cos(d*x + c)^3 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2*b
- (4*d*x + 4*c - sin(4*d*x + 4*c))*b^3 - 16*a^3*(2*cos(d*x + c) - log(cos(d
*x + c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.44, size = 116, normalized size = 0.85

$$-\frac{8\,ab^2\cos\left(dx+c\right)^3-8\,a^3\cos\left(dx+c\right)+4\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-4\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(12\,a^2b+b^3\right)dx+\left(2\,b^3\cos\left(dx+c\right)^3-\left(12\,a^2b+b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^3 - 8*a^3*cos(d*x + c) + 4*a^3*log(1/2*cos(d*x +
 c) + 1/2) - 4*a^3*log(-1/2*cos(d*x + c) + 1/2) - (12*a^2*b + b^3)*d*x + (2
*b^3*cos(d*x + c)^3 - (12*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\cos^2\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cos(c + d*x)**2*csc(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(126) = 252.
time = 0.53, size = 293, normalized size = 2.15

$$\frac{8\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\left(12\,a^{2}b+b^{3}\right)\left(dx+c\right)-\frac{2\,\left(12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-8\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+7\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+24\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^{3}+8\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + (12*a^2*b + b^3)*(d*x + c) - 2*
(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1
/2*d*x + 1/2*c)^6 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 + 7*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 +
 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 7*b^3*
tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*a*b^2*tan(1/2*d*
x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - 8
*a^3 + 8*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 11.12, size = 567, normalized size = 4.17

$$\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2\,a\,b^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(3\,a^2\,b-\frac{b^3}{4}\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\,\left(2\,a\,b^2-6\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^6\,\left(6\,a\,b^2-2\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4\,\left(6\,a\,b^2-6\,a^3\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^7\,\left(3\,a^2\,b-\frac{b^3}{4}\right)-{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^3\,\left(3\,a^2\,b+\frac{7\,b^3}{4}\right)+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^5\,\left(3\,a^2\,b+\frac{7\,b^3}{4}\right)-2\,a^3}{d\,\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\,\left(12\,a^2+b^2\right)\,\left(3\,a^2\,b+\frac{b^3}{4}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(12\,a^2+b^2\right)\,3{}\mathrm{i}}{4}\right)}{8}+\frac{b\,\left(12\,a^2+b^2\right)\,\left(3\,a^2\,b+\frac{b^3}{4}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(12\,a^2+b^2\right)\,3{}\mathrm{i}}{4}\right)}{8}}{2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{b^6}{16}+\frac{3\,a^2\,b^4}{2}+9\,a^4\,b^2\right)+6\,a^5\,b+\frac{a^3\,b^3}{2}-\frac{b\,\left(12\,a^2+b^2\right)\,\left(3\,a^2\,b+\frac{b^3}{4}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(12\,a^2+b^2\right)\,3{}\mathrm{i}}{4}\right)\,1{}\mathrm{i}}{8}+\frac{b\,\left(12\,a^2+b^2\right)\,\left(3\,a^2\,b+\frac{b^3}{4}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(12\,a^2+b^2\right)\,3{}\mathrm{i}}{4}\right)\,1{}\mathrm{i}}{8}}\right)\,\left(12\,a^2+b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (2*a*b^2 - tan(c/2 + (d*x)/2)*(3*a^2*b -
b^3/4) + tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^6*(6*a
*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)
^7*(3*a^2*b - b^3/4) - tan(c/2 + (d*x)/2)^3*(3*a^2*b + (7*b^3)/4) + tan(c/2
 + (d*x)/2)^5*(3*a^2*b + (7*b^3)/4) - 2*a^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6
*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
 + (b*atan(((b*(12*a^2 + b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) -
 (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)*3i)/4))/8 + (b*(12*a^2 + b^2)*(3*a^2*
b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)
*3i)/4))/8)/(2*tan(c/2 + (d*x)/2)*(b^6/16 + (3*a^2*b^4)/2 + 9*a^4*b^2) + 6*
a^5*b + (a^3*b^3)/2 - (b*(12*a^2 + b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 +
(d*x)/2) - (b*tan(c/2 + (d*x)/2)*(12*a^2 + b^2)*3i)/4)*1i)/8 + (b*(12*a^2 +
 b^2)*(3*a^2*b + b^3/4 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(
12*a^2 + b^2)*3i)/4)*1i)/8))*(12*a^2 + b^2))/(4*d)
```

3.1072 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+$$

```
[Out] -a^3*x+3/2*a*b^2*x-3*a^2*b*arctanh(cos(d*x+c))/d+3*a^2*b*cos(d*x+c)/d-1/3*b
^3*cos(d*x+c)^3/d-a^3*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 2715, 8, 2672, 327, 212, 3554, 2645, 30}

$$-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(3ab^2\cos^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+a^3\cot^2(c+\right. \\
&= a^3\int \cot^2(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx+\left(3a\right. \\
&= -\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d}-a^3\int 1\,dx+\frac{1}{2}(3 \\
&= -a^3x+\frac{3}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+}{d} \\
&= -a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-
\end{aligned}
$$

Mathematica [A]
time = 0.93, size = 143, normalized size = 1.40

$$
\frac{(36a^2b-3b^3)\cos(c+dx)-b^3\cos(3(c+dx))-6a^3\cot\left(\frac{1}{2}(c+dx)\right)+9ab^2\sin(2(c+dx))+6a\left(-2a^2c+3b^2c-2a^2dx+3b^2dx-6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\tan\left(\frac{1}{2}(c+dx)\right)\right)}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

Maple [A]
time = 0.21, size = 96, normalized size = 0.94

method	result
derivativedivides	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^3(dx+\right.}{3}}{d}$
default	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^3(dx+\right.}{3}}{d}$
risch	$-a^3x+\frac{3a\,b^2x}{2}-\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{b^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}-\frac{b^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{3ia\,b^2\mathrm{e}^{-}}{8}$
norman	$\frac{(-3a^3+\frac{9}{2}a\,b^2)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-3a^3+\frac{9}{2}a\,b^2)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a^3+\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-a^3+\frac{3}{2}a\,b^2)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-cot(d*x+c)-d*x-c)+3*a^2*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))
+3*a*b^2*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-1/3*cos(d*x+c)^3*b^3)
```

Maxima [A]
time = 0.50, size = 95, normalized size = 0.93

$$-\frac{4\,b^{3}\cos\left(dx+c\right)^{3}+12\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^{3}-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab^{2}-18\,a^{2}b\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 143, normalized size = 1.40

$$-\frac{9\,ab^{2}\cos\left(dx+c\right)^{3}+9\,a^{2}b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^{2}b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(2\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)+\left(2\,b^{3}\cos\left(dx+c\right)^{3}-18\,a^{2}b\cos\left(dx+c\right)+3\left(2\,a^{3}-3\,ab^{2}\right)dx\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^{3}\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cos(c + d*x)**2*csc(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(96) = 192.
time = 0.50, size = 199, normalized size = 1.95

$$\frac{18\,a^{2}b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^{3}-3\,ab^{2}\right)\left(dx+c\right)-\frac{3\left(6\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-18\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+6\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,a^{2}b+2\,b^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

Mupad [B]
time = 9.47, size = 289, normalized size = 2.83

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,\left(\frac{a\,b^2\,3{}\mathrm{i}}{2}-a^3\,1{}\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(a^3+6\,a\,b^2\right)-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a\,b^2-3\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(12\,a^2\,b-4\,b^3\right)-a^3+24\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,\left(2\,a^2-3\,b^2\right)\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*3i)/
2 - a^3*1i))/d + (tan(c/2 + (d*x)/2)*(12*a^2*b - (4*b^3)/3) - tan(c/2 + (d*
x)/2)^6*(6*a*b^2 + a^3) - 3*a^3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^2
*(6*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^5*(12*a^2*b - 4*b^3) - a^3 + 24*a^2
*b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3
+ 6*tan(c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (3*a^2*b*log(tan(c/2
+ (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 3*b^2)*1i)/(2*d)
```

3.1073 $\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$-\frac{1}{2}b(6a^2-b^2)\,x+\frac{a(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2d}+\frac{15ab^2\cos(c+dx)}{2d}+\frac{5b^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{3b\cot(\cdots}{}$$

```
[Out] -1/2*b*(6*a^2-b^2)*x+1/2*a*(a^2-6*b^2)*arctanh(cos(d*x+c))/d+15/2*a*b^2*cos
(d*x+c)/d+5/2*b^3*cos(d*x+c)*sin(d*x+c)/d-3/2*b*cot(d*x+c)*(a+b*sin(d*x+c))
^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [A]

time = 0.30, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2968, 3127, 3126, 3112, 3102, 2814, 3855}

$$\frac{a(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{1}{2}bx(6a^2-b^2)+\frac{15ab^2\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{2d}+\frac{5b^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(b*(6*a^2 - b^2)*x) + (a*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) +
(15*a*b^2*Cos[c + d*x])/(2*d) + (5*b^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) - (
3*b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]
*(a + b*Sin[c + d*x])^3)/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&& !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \csc^3(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\,dx \\
&= -\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{2d}+\frac{1}{2}\int \csc \\
&= -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^2}{2d}-\frac{\cot(c+dx)\csc(c \\
&= \frac{5b^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{3b\cot(c+dx)(a+b\sin(c}{2d} \\
&= \frac{15ab^2\cos(c+dx)}{2d}+\frac{5b^3\cos(c+dx)\sin(c+dx)}{2d}-\frac{3b\cot}{} \\
&= -\frac{1}{2}b\left(6a^2-b^2\right)x+\frac{15ab^2\cos(c+dx)}{2d}+\frac{5b^3\cos(c+dx)}{2d} \\
&= -\frac{1}{2}b\left(6a^2-b^2\right)x+\frac{a(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2d}+
\end{aligned}
$$

Mathematica [A]
time = 0.91, size = 192, normalized size = 1.39

$$
\frac{-24a^2bc+4b^3c-24a^2bdx+4b^3dx+24ab^2\cos(c+dx)-12a^2b\cot\left(\frac{1}{2}(c+dx)\right)-a^3\csc^2\left(\frac{1}{2}(c+dx)\right)+4a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-24ab^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+24ab^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^3\sec^2\left(\frac{1}{2}(c+dx)\right)+2b^3\sin(2(c+dx))+12a^2b\tan\left(\frac{1}{2}(c+dx)\right)}{8d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-24*a^2*b*c + 4*b^3*c - 24*a^2*b*d*x + 4*b^3*d*x + 24*a*b^2*Cos[c + d*x] -
 12*a^2*b*Cot[(c + d*x)/2] - a^3*Csc[(c + d*x)/2]^2 + 4*a^3*Log[Cos[(c + d*
x)/2]] - 24*a*b^2*Log[Cos[(c + d*x)/2]] - 4*a^3*Log[Sin[(c + d*x)/2]] + 24*
a*b^2*Log[Sin[(c + d*x)/2]] + a^3*Sec[(c + d*x)/2]^2 + 2*b^3*Sin[2*(c + d*x
)] + 12*a^2*b*Tan[(c + d*x)/2])/(8*d)
```

Maple [A]
time = 0.26, size = 132, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b(-\cot(dx+c)-dx-c)+3a\,b^2(\cos(dx+c)+\ln(\csc(dx+c)-\mathrm{c}}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b(-\cot(dx+c)-dx-c)+3a\,b^2(\cos(dx+c)+\ln(\csc(dx+c)-\mathrm{c}}{d}$
risch	$-3a^2bx+\frac{b^3x}{2}-\frac{ib^3\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3\,\mathrm{e}^{i(dx+c)}a\,b^2}{2d}+\frac{3\,\mathrm{e}^{-i(dx+c)}a\,b^2}{2d}+\frac{ib^3\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{ia^2\left(ia\,\mathrm{e}^{3i(dx+c)}+ia\,\mathrm{e}^{i(dx}}{d\left(\mathrm{e}^{2i(dx+}\right.}$

norman	$\frac{(-9a^2b+\frac{3}{2}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-9a^2b+\frac{3}{2}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-3a^2b+\frac{1}{2}b^3)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-3a^2b+\frac{1}{2}b^3)x\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-c
ot(d*x+c)))+3*a^2*b*(-cot(d*x+c)-d*x-c)+3*a*b^2*(cos(d*x+c)+ln(csc(d*x+c)-c
ot(d*x+c)))+b^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.51, size = 128, normalized size = 0.93

$$-\frac{12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2b-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^3-a^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log(\cos(dx+c)+1)-\log(\cos(dx+c)-1)\right)-6\,ab^2(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/4*(12*(d*x + c + 1/tan(d*x + c))*a^2*b - (2*d*x + 2*c + sin(2*d*x + 2*c)
)*b^3 - a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) -
log(cos(d*x + c) - 1)) - 6*a*b^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) +
log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 216, normalized size = 1.57

$$\frac{12\,ab^2\cos(dx+c)^3-2\,(6\,a^2b-b^3)dx\cos(dx+c)^2+2\,(6\,a^2b-b^3)dx+2\,(a^3-6\,ab^2)\cos(dx+c)-\left(a^3-6\,ab^2-(a^3-6\,ab^2)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(a^3-6\,ab^2-(a^3-6\,ab^2)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2\left(b^3\cos(dx+c)^3+(6\,a^2b-b^3)\cos(dx+c)\right)\sin(dx+c)}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/4*(12*a*b^2*cos(d*x + c)^3 - 2*(6*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 2*(6*
a^2*b - b^3)*d*x + 2*(a^3 - 6*a*b^2)*cos(d*x + c) - (a^3 - 6*a*b^2 - (a^3 -
 6*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 6*a*b^2 - (a
^3 - 6*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b^3*cos(d*x
 + c)^3 + (6*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(126) = 252.
time = 0.57, size = 272, normalized size = 1.97

$$\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4\left(6a^2b-b^3\right)(dx+c)-4\left(a^3-6ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+\frac{2a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-8b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+24ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^3}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 4*(6*a^2*
b - b^3)*(d*x + c) - 4*(a^3 - 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (2*
a^3*tan(1/2*d*x + 1/2*c)^6 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*b*tan
(1/2*d*x + 1/2*c)^5 - 8*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/
2*c)^4 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^3
+ 8*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b
*tan(1/2*d*x + 1/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c)
)^2)/d
```

Mupad [B]
time = 9.46, size = 585, normalized size = 4.24

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(3ab^2-\frac{a^3}{2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(6a^2b+4b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(24ab^2-\frac{a^3}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(24ab^2-a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(12a^2b-4b^3\right)+\frac{a^3}{2}+6a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{3a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\left(6a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6ab^2-a^3\right)-6a^2b+b^3-b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2-b^2\right)3\mathrm{i}\right)}{2}+\frac{b\left(6a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6ab^2-a^3\right)-6a^2b+b^3+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2-b^2\right)3\mathrm{i}\right)}{2}}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(36a^4b^2-12a^2b^4+b^6\right)+6ab^5+6a^5b-37a^3b^3-\frac{b\left(6a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6ab^2-a^3\right)-6a^2b+b^3-b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2-b^2\right)3\mathrm{i}\right)1\mathrm{i}}{2}+\frac{b\left(6a^2-b^2\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6ab^2-a^3\right)-6a^2b+b^3+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(6a^2-b^2\right)3\mathrm{i}\right)1\mathrm{i}}{2}}\right)\left(6a^2-b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - a^3/
2))/d - (tan(c/2 + (d*x)/2)^5*(6*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^4*(24*
a*b^2 - a^3/2) - tan(c/2 + (d*x)/2)^2*(24*a*b^2 - a^3) + tan(c/2 + (d*x)/2)
^3*(12*a^2*b - 4*b^3) + a^3/2 + 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 +
 (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6)) + (3*a^2*b*
tan(c/2 + (d*x)/2))/(2*d) + (b*atan(((b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(
6*a*b^2 - a^3) - 6*a^2*b + b^3 - b*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i))/2
+ (b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2 - a^3) - 6*a^2*b + b^3 + b*
tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i))/2)/(2*tan(c/2 + (d*x)/2)*(b^6 - 12*a^
2*b^4 + 36*a^4*b^2) + 6*a*b^5 + 6*a^5*b - 37*a^3*b^3 - (b*(6*a^2 - b^2)*(ta
n(c/2 + (d*x)/2)*(6*a*b^2 - a^3) - 6*a^2*b + b^3 - b*tan(c/2 + (d*x)/2)*(6*
a^2 - b^2)*3i)*1i)/2 + (b*(6*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2 - a^3)
 - 6*a^2*b + b^3 + b*tan(c/2 + (d*x)/2)*(6*a^2 - b^2)*3i)*1i)/2))*(6*a^2 -
b^2))/d
```

3.1074 $\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=138

$$-3ab^2x+\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}+\frac{11b^3\cos(c+dx)}{6d}+\frac{a(a^2-3b^2)\cot(c+dx)}{3d}-\frac{b\cot(c+dx)\csc(c$$

```
[Out] -3*a*b^2*x+1/2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/d+11/6*b^3*cos(d*x+c)/d+
1/3*a*(a^2-3*b^2)*cot(d*x+c)/d-1/2*b*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))
^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.31, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2968, 3127, 3126, 3110, 3102, 2814, 3855}

$$\frac{a(a^2-3b^2)\cot(c+dx)}{3d}+\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-3ab^2x-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d}-\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d}+\frac{11b^3\cos(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -3*a*b^2*x + (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) + (11*b^3*Cos[
c + d*x])/(6*d) + (a*(a^2 - 3*b^2)*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x]*Cs
c[c + d*x]*(a + b*Sin[c + d*x])^2)/(2*d) - (Cot[c + d*x]*Csc[c + d*x]^2*(a
+ b*Sin[c + d*x])^3)/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^4(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)$$
$$= -\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{3d} + \frac{1}{3}\int$$
$$= -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{2d} - \cot($$
$$= \frac{a(a^2-3b^2)\cot(c+dx)}{3d} - \frac{b\cot(c+dx)\csc(c+dx)(a}{2d}$$
$$= \frac{11b^3\cos(c+dx)}{6d} + \frac{a(a^2-3b^2)\cot(c+dx)}{3d} - \frac{b\cot(c}{}$$
$$= -3ab^2x + \frac{11b^3\cos(c+dx)}{6d} + \frac{a(a^2-3b^2)\cot(c+dx)}{3d}$$
$$= -3ab^2x + \frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} + \frac{11b^3\text{co}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 615 vs. 2(138) = 276.
time = 6.16, size = 615, normalized size = 4.46

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c
+ d*x])^3) + (b^3*Cos[c + d*x]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a
 + b*Sin[c + d*x])^3) + ((a^3*Cos[(c + d*x)/2] - 9*a*b^2*Cos[(c + d*x)/2])*
Csc[(c + d*x)/2]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c +
 d*x])^3) - (3*a^2*b*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]
^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2
*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(24*d*(a + b*Sin[c + d*x])^3) + ((3
*a^2*b - 2*b^3)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]*Sin[c + d*x]^3
)/(2*d*(a + b*Sin[c + d*x])^3) + ((-3*a^2*b + 2*b^3)*(b + a*Csc[c + d*x])^3
*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (3*a^
2*b*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3)/(8*d*(a + b*S
in[c + d*x])^3) + ((b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]*(-(a^3*Sin[(c +
d*x)/2]) + 9*a*b^2*Sin[(c + d*x)/2])*Sin[c + d*x]^3)/(6*d*(a + b*Sin[c + d*
```

```
x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[
(c + d*x)/2])/(24*d*(a + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.25, size = 127, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{a^3\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+3a^2b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a\,b^2(-\cot(dx+c)-dx-c)+b^3(\cos(dx+c}{d}$
default	$\frac{-\frac{a^3\left(\cos^3(dx+c)\right)}{3\sin(dx+c)^3}+3a^2b\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a\,b^2(-\cot(dx+c)-dx-c)+b^3(\cos(dx+c}{d}$
risch	$-3a\,b^2x+\frac{b^3e^{i(dx+c)}}{2d}+\frac{b^3e^{-i(dx+c)}}{2d}+\frac{a\left(6ia^2e^{4i(dx+c)}-18ib^2e^{4i(dx+c)}+9b\,e^{5i(dx+c)}a+36ib^2e^{2i(dx+c)}+2ia^2-18\right.}{3d\left(e^{2i(dx+c)}-1\right)^3}$
norman	$\frac{-\frac{a^3}{24d}+\frac{a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{\left(9a^2b-8b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{\left(21a^2b-16b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(33a^2b-32b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{3a\,b}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3/sin(d*x+c)^3*cos(d*x+c)^3+3*a^2*b*(-1/2/sin(d*x+c)^2*cos(d*x+
c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)))+3*a*b^2*(-cot(d*x+c)-d*x
-c)+b^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.51, size = 119, normalized size = 0.86

$$-\frac{36\left(dx+c+\frac{1}{\tan(dx+c)}\right)ab^2-9\,a^2b\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,b^3(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right))+\frac{4\,a^3}{\tan(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/12*(36*(d*x + c + 1/tan(d*x + c))*a*b^2 - 9*a^2*b*(2*cos(d*x + c)/(cos(d
*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) - 6*b^3*(2*
cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 4*a^3/tan(d
*x + c)^3)/d
```

Fricas [A]
time = 0.38, size = 231, normalized size = 1.67

$$\frac{36\,ab^2\cos\left(dx+c\right)+4\left(a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(3\,a^2b-2\,b^3-\left(3\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(3\,a^2b-2\,b^3-\left(3\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-6\left(6\,ab^2dx\cos\left(dx+c\right)^2-2\,b^3\cos\left(dx+c\right)^3-6\,ab^2dx-\left(3\,a^2b-2\,b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(36*a*b^2*cos(d*x + c) + 4*(a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(3*a^2*b
 - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/
2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(6*a*b^2*d*x*cos(d*x + c)^2 - 2*b^3*
cos(d*x + c)^3 - 6*a*b^2*d*x - (3*a^2*b - 2*b^3)*cos(d*x + c))*sin(d*x + c)
)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.55, size = 222, normalized size = 1.61

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab^2-3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{48\,b^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-12\,(3\,a^2b-2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{66\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-44\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x
 + c)*a*b^2 - 3*a^3*tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) +
48*b^3/(tan(1/2*d*x + 1/2*c)^2 + 1) - 12*(3*a^2*b - 2*b^3)*log(abs(tan(1/2*
d*x + 1/2*c))) + (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/2*d*x + 1/
2*c)^3 + 3*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9
*a^2*b*tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 10.71, size = 477, normalized size = 3.46

$$-\frac{\frac{a^3\,\cos(c+d\,x)}{4}-\frac{3\,b^3\,\sin(c+d\,x)}{4}+\frac{a^3\,\cos(3\,c+3\,d\,x)}{12}-\frac{b^3\,\sin(2\,c+2\,d\,x)}{4}+\frac{b^3\,\sin(3\,c+3\,d\,x)}{4}+\frac{b^3\,\sin(4\,c+4\,d\,x)}{8}-\frac{3\,a\,b^2\,\cos(3\,c+3\,d\,x)}{4}-\frac{3\,b^3\,\sin(c+d\,x)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}+\frac{3\,a^2\,b\,\sin(2\,c+2\,d\,x)}{4}+\frac{b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(3\,c+3\,d\,x)}{4}+\frac{3\,a\,b^2\,\cos(c+d\,x)}{4}-\frac{3\,a^2\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(3\,c+3\,d\,x)}{8}-\frac{9\,a\,b^2\,\mathrm{atan}\left(\frac{3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{-3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)\,\sin(c+d\,x)}{2}+\frac{3\,a\,b^2\,\mathrm{atan}\left(\frac{3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{-3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)\,\sin(3\,c+3\,d\,x)}{2}+\frac{9\,a^2\,b\,\sin(c+d\,x)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8}}{d\,\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] -((a^3*cos(c + d*x))/4 - (3*b^3*sin(c + d*x))/4 + (a^3*cos(3*c + 3*d*x))/12
 - (b^3*sin(2*c + 2*d*x))/4 + (b^3*sin(3*c + 3*d*x))/4 + (b^3*sin(4*c + 4*d
*x))/8 - (3*a*b^2*cos(3*c + 3*d*x))/4 - (3*b^3*sin(c + d*x)*log(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (3*a^2*b*sin(2*c + 2*d*x))/4 + (b^3*log(si
```

```
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/4 + (3*a*b^2*cos(c +
 d*x))/4 - (3*a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*
d*x))/8 - (9*a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2))/(2*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (
d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (3*a*b^2*atan((3*a^2
*sin(c/2 + (d*x)/2) - 2*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2))/
(2*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x
)/2)))*sin(3*c + 3*d*x))/2 + (9*a^2*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)))/8)/(d*sin(c + d*x)^3)
```

3.1075 $\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=152

$$-b^3x+\frac{a(a^2+12b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b(2a^2-b^2)\cot(c+dx)}{2d}+\frac{a(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\,c}{}$$

```
[Out] -b^3*x+1/8*a*(a^2+12*b^2)*arctanh(cos(d*x+c))/d+1/2*b*(2*a^2-b^2)*cot(d*x+c
)/d+1/8*a*(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/4*b*cot(d*x+c)*csc(d*x+c)^2
*(a+b*sin(d*x+c))^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/d
```

Rubi [A]

time = 0.32, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2968, 3127, 3126, 3110, 3100, 2814, 3855}

$$\frac{b(2a^2-b^2)\cot(c+dx)}{2d}+\frac{a(a^2+12b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{4d}-\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{4d}+b^3(-x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*x) + (a*(a^2 + 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (b*(2*a^2 - b^2
)*Cot[c + d*x])/(2*d) + (a*(a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) -
 (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(4*d) - (Cot[c + d*
x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(4*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
```

```
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^5(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{1}{4}\int$$
$$= -\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{4d} - \text{cot}$$
$$= \frac{a(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot(c+dx)\,\text{cs}}{}$$
$$= \frac{b(2a^2-b^2)\cot(c+dx)}{2d} + \frac{a(a^2-2b^2)\cot(c+dx)\,\text{csc}(}{8d}$$
$$= -b^3x + \frac{b(2a^2-b^2)\cot(c+dx)}{2d} + \frac{a(a^2-2b^2)\cot(c+}{8d}$$
$$= -b^3x + \frac{a(a^2+12b^2)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b(2a^2-b}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 690 vs. 2(152) = 304.
time = 6.17, size = 690, normalized size = 4.54

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b^3*(c + d*x)*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(d*(a + b*Sin[c + d
*x])^3)) + ((a^2*b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2
]*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + ((a
^3 - 12*a*b^2)*Csc[(c + d*x)/2]^2*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(3
2*d*(a + b*Sin[c + d*x])^3) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2*(b
 + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) - (a^3*Cs
c[(c + d*x)/2]^4*(b + a*Csc[c + d*x])^3*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c
+ d*x])^3) + ((a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Log[Cos[(c + d*x)/2]]
*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*x])^3) + ((-a^3 - 12*a*b^2)*(b + a*C
sc[c + d*x])^3*Log[Sin[(c + d*x)/2]]*Sin[c + d*x]^3)/(8*d*(a + b*Sin[c + d*
x])^3) + ((-a^3 + 12*a*b^2)*(b + a*Csc[c + d*x])^3*Sec[(c + d*x)/2]^2*Sin[c
 + d*x]^3)/(32*d*(a + b*Sin[c + d*x])^3) + (a^3*(b + a*Csc[c + d*x])^3*Sec[
```

```
(c + d*x)/2]^4*Sin[c + d*x]^3)/(64*d*(a + b*Sin[c + d*x])^3) + ((b + a*Csc[
c + d*x])^3*Sec[(c + d*x)/2]*(-(a^2*b*Sin[(c + d*x)/2]) + b^3*Sin[(c + d*x)
/2])*Sin[c + d*x]^3)/(2*d*(a + b*Sin[c + d*x])^3) + (a^2*b*(b + a*Csc[c + d
*x])^3*Sec[(c + d*x)/2]^2*Sin[c + d*x]^3*Tan[(c + d*x)/2])/(8*d*(a + b*Sin[
c + d*x])^3)
```

Maple [A]
time = 0.27, size = 166, normalized size = 1.09

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^2b\left(\cos^3(dx+c)\right)}{\sin(dx+c)^3}+3a\,b^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a^2b\left(\cos^3(dx+c)\right)}{\sin(dx+c)^3}+3a\,b^2\left(-\frac{\cos^3(dx+c)}{2\sin(dx+c)^2}-\frac{\cos(dx+c)}{2}\right.}{d}$
risch	$-b^3x-\frac{i\left(12ia\,b^2\mathrm{e}^{7i(dx+c)}+12ia\,b^2\mathrm{e}^{i(dx+c)}-12ia\,b^2\mathrm{e}^{5i(dx+c)}-ia^3\mathrm{e}^{i(dx+c)}-24b\,\mathrm{e}^{6i(dx+c)}a^2+8b^3\mathrm{e}^{6i(dx+c)}-7ia^3\mathrm{e}^{3i(dx}\right.}{}$
norman	$\frac{-\frac{a^3}{64d}+\frac{a^3\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{b^3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{b^3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-b^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3b^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3b^3x\left(\mathrm{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*
cos(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-a^2*b/sin(d*x+c)^3*cos(d*x+c)^3+3
*a*b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot
(d*x+c)))+b^3*(-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 149, normalized size = 0.98

$$-\frac{16\left(dx+c+\frac{1}{\tan(dx+c)}\right)b^3+a^3\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,ab^2\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a^2b}{\tan(dx+c)^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*(d*x + c + 1/tan(d*x + c))*b^3 + a^3*(2*(cos(d*x + c)^3 + cos(d*x
 + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + lo
g(cos(d*x + c) - 1)) - 12*a*b^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(
cos(d*x + c) + 1) - log(cos(d*x + c) - 1)) + 16*a^2*b/tan(d*x + c)^3)/d
```

Fricas [A]
time = 0.39, size = 265, normalized size = 1.74

$$-\frac{16\,b^3dx\cos\left(dx+c\right)^4-32\,b^3dx\cos\left(dx+c\right)^2+16\,b^3dx+2\left(a^3-12\,ab^2\right)\cos\left(dx+c\right)^3+2\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)-\left(\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^4+a^3+12\,ab^2-2\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^4+a^3+12\,ab^2-2\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+16\left(b^3\cos\left(dx+c\right)+\left(a^2b-b^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(16*b^3*d*x*cos(d*x + c)^4 - 32*b^3*d*x*cos(d*x + c)^2 + 16*b^3*d*x +
 2*(a^3 - 12*a*b^2)*cos(d*x + c)^3 + 2*(a^3 + 12*a*b^2)*cos(d*x + c) - ((a^
3 + 12*a*b^2)*cos(d*x + c)^4 + a^3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x
+ c)^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^3 + 12*a*b^2)*cos(d*x + c)^4 + a^
3 + 12*a*b^2 - 2*(a^3 + 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2) + 16*(b^3*cos(d*x + c) + (a^2*b - b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [**A**]
time = 0.57, size = 234, normalized size = 1.54

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-192\,(dx+c)b^3-72\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,(a^3+12\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{50\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+600\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+72\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^3*tan(1/2*d*x + 1/2*c)^4 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 72*
a*b^2*tan(1/2*d*x + 1/2*c)^2 - 192*(d*x + c)*b^3 - 72*a^2*b*tan(1/2*d*x + 1
/2*c) + 96*b^3*tan(1/2*d*x + 1/2*c) - 24*(a^3 + 12*a*b^2)*log(abs(tan(1/2*d
*x + 1/2*c))) + (50*a^3*tan(1/2*d*x + 1/2*c)^4 + 600*a*b^2*tan(1/2*d*x + 1/
2*c)^4 + 72*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 96*b^3*tan(1/2*d*x + 1/2*c)^3 -
72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/ta
n(1/2*d*x + 1/2*c)^4)/d
```

Mupad [**B**]
time = 9.87, size = 348, normalized size = 2.29

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{b^3\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+12\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2+8\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+12\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-8\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)}{d}-\frac{3\,a\,b^2\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2\,d}+\frac{3\,a^2\,b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{3\,a\,b^2\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a^2\,b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}+\frac{3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^5,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) - (a^3*cot(c/2 + (d*x)/2)^4)/(64*d) - (a^
3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*d) - (b^3*cot(c/2 + (d*x)/
2))/(2*d) + (b^3*tan(c/2 + (d*x)/2))/(2*d) - (2*b^3*atan((8*b^3*cos(c/2 + (
d*x)/2) + a^3*sin(c/2 + (d*x)/2) + 12*a*b^2*sin(c/2 + (d*x)/2))/(a^3*cos(c/
2 + (d*x)/2) - 8*b^3*sin(c/2 + (d*x)/2) + 12*a*b^2*cos(c/2 + (d*x)/2))))/d
- (3*a*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*d) + (3*a^2*b*cot
(c/2 + (d*x)/2))/(8*d) - (3*a^2*b*tan(c/2 + (d*x)/2))/(8*d) - (3*a*b^2*cot(
c/2 + (d*x)/2)^2)/(8*d) - (a^2*b*cot(c/2 + (d*x)/2)^3)/(8*d) + (3*a*b^2*tan
(c/2 + (d*x)/2)^2)/(8*d) + (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

3.1076 $\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=183

$$\frac{b(3a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a(2a^2+15b^2)\cot(c+dx)}{15d}+\frac{3b(5a^2-2b^2)\cot(c+dx)\csc(c+dx)}{40d}+\frac{a(2a$$

```
[Out] 1/8*b*(3*a^2+4*b^2)*arctanh(cos(d*x+c))/d+1/15*a*(2*a^2+15*b^2)*cot(d*x+c)/
d+3/40*b*(5*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/30*a*(2*a^2-3*b^2)*cot(d*x
+c)*csc(d*x+c)^2/d-3/20*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^2/d-1/5*
cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2968, 3127, 3126, 3110, 3100, 2827, 3852, 8, 3855}

$$\frac{a(2a^2+15b^2)\cot(c+dx)}{15d}+\frac{b(3a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{30d}+\frac{3b(5a^2-2b^2)\cot(c+dx)\csc(c+dx)}{40d}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{5d}-\frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{20d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + (a*(2*a^2 + 15*b^2)*Cot[c
 + d*x])/(15*d) + (3*b*(5*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(40*d) +
(a*(2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*d) - (3*b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(20*d) - (Cot[c + d*x]*Csc[c + d*x]^
4*(a + b*Sin[c + d*x])^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
```

```
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \csc^6(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)\\ &= -\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{5d}+\frac{1}{5}\int\\ &= -\frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{20d}-\text{co}\\ &= \frac{a(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{30d}-3b\cot(c+dx\\ &= \frac{3b(5a^2-2b^2)\cot(c+dx)\csc(c+dx)}{40d}+\frac{a(2a^2-3b^2)}{}\\ &= \frac{3b(5a^2-2b^2)\cot(c+dx)\csc(c+dx)}{40d}+\frac{a(2a^2-3b^2)}{}\\ &= \frac{b(3a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3b(5a^2-2b^2)\cot}{}\\ &= \frac{b(3a^2+4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{a(2a^2+15b^2)\cot}{15d}\end{aligned}$$

Mathematica [A]

time = 0.94, size = 344, normalized size = 1.88

$$\frac{32(2a^3+15ab^2)\cot\left(\frac{1}{2}(c+dx)\right)+30(3a^2b-4b^3)\csc^2\left(\frac{1}{2}(c+dx)\right)+360a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+480b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-360a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-480b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-90a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)+120b^3\sec^2\left(\frac{1}{2}(c+dx)\right)+45a^2b\sec^4\left(\frac{1}{2}(c+dx)\right)-16a^3\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+960ab^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-3a^3\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+a\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-45ab+(a^2-60b^2)\sin(c+dx)\right)-64a^3\tan\left(\frac{1}{2}(c+dx)\right)-480ab^2\tan\left(\frac{1}{2}(c+dx)\right)+6a^3\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (32*(2*a^3 + 15*a*b^2)*Cot[(c + d*x)/2] + 30*(3*a^2*b - 4*b^3)*Csc[(c + d*x
)/2]^2 + 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*Log[Cos[(c + d*x)/2]] -
360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c + d*x)/2]] - 90*a^2*b*
Sec[(c + d*x)/2]^2 + 120*b^3*Sec[(c + d*x)/2]^2 + 45*a^2*b*Sec[(c + d*x)/2]
^4 - 16*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 960*a*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 - 3*a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + a*Csc[(c + d*x)/
2]^4*(-45*a*b + (a^2 - 60*b^2)*Sin[c + d*x]) - 64*a^3*Tan[(c + d*x)/2] - 48
0*a*b^2*Tan[(c + d*x)/2] + 6*a^3*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*
d)
```

Maple [A]

time = 0.30, size = 187, normalized size = 1.02

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+3a^2b\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a\,b^2\left(\cos^3\right.}{\sin(dx}}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c)\right)}{15\sin(dx+c)^3}\right)+3a^2b\left(-\frac{\cos^3(dx+c)}{4\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^2}-\frac{\cos(dx+c)}{8}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)-\frac{a\,b^2\left(\cos^3\right.}{\sin(dx}}{d}$
risch	$\frac{-80ia^3e^{4i(dx+c)}-45a^2b\,e^{9i(dx+c)}+60b^3e^{9i(dx+c)}+360ia\,b^2e^{8i(dx+c)}+480ia\,b^2e^{4i(dx+c)}-270a^2b\,e^{7i(dx+c)}-120b^3e^{7i(dx+}}{}$
norman	$\frac{-\frac{a^3}{160d}+\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}-\frac{a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{80d}+\frac{a^3\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{\left(3a^2b+7b^3\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(9a^2b+24b^3\right)\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{32d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+3*
a^2*b*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos
(d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-a*b^2/sin(d*x+c)^3*cos(d*x+c)^3+b^3*
(-1/2/sin(d*x+c)^2*cos(d*x+c)^3-1/2*cos(d*x+c)-1/2*ln(csc(d*x+c)-cot(d*x+c)
)))
```

Maxima [A]

time = 0.30, size = 157, normalized size = 0.86

$$-\frac{45\,a^2b\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-60\,b^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}+\log\left(\cos\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{240\,ab^2}{\tan(dx+c)^3}+\frac{16\left(5\,\tan(dx+c)^2+3\right)a^3}{\tan(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/240*(45*a^2*b*(2*(cos(d*x + c)^3 + cos(d*x + c))/(cos(d*x + c)^4 - 2*cos
(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 60*b^3*
```

```
(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) + log(cos(d*x + c) + 1) - log(cos(d*x
+ c) - 1)) + 240*a*b^2/tan(d*x + c)^3 + 16*(5*tan(d*x + c)^2 + 3)*a^3/tan(d
*x + c)^5)/d
```

Fricas [A]
time = 0.37, size = 275, normalized size = 1.50

$$\frac{16\,(2\,a^3+15\,ab^2)\cos(dx+c)^5-80\,(a^3+3\,ab^2)\cos(dx+c)^3+15\left((3\,a^2b+4\,b^3)\cos(dx+c)^4+3\,a^2b+4\,b^3-2\,(3\,a^2b+4\,b^3)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-15\left((3\,a^2b+4\,b^3)\cos(dx+c)^4+3\,a^2b+4\,b^3-2\,(3\,a^2b+4\,b^3)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-30\left((3\,a^2b-4\,b^3)\cos(dx+c)^3+(3\,a^2b+4\,b^3)\cos(dx+c)\right)\sin(dx+c)}{240\left(d\cos(dx+c)^4-2\,d\cos(dx+c)^2+d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/240*(16*(2*a^3 + 15*a*b^2)*cos(d*x + c)^5 - 80*(a^3 + 3*a*b^2)*cos(d*x +
c)^3 + 15*((3*a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b
+ 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*((3*
a^2*b + 4*b^3)*cos(d*x + c)^4 + 3*a^2*b + 4*b^3 - 2*(3*a^2*b + 4*b^3)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((3*a^2*b - 4*b^3
)*cos(d*x + c)^3 + (3*a^2*b + 4*b^3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.52, size = 290, normalized size = 1.58

$$\frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,(3\,a^2b+4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{822\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1096\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+360\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan
(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 360*a*b^2*tan(1/2*d*x +
 1/2*c) - 120*(3*a^2*b + 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (822*a^2*b
*tan(1/2*d*x + 1/2*c)^5 + 1096*b^3*tan(1/2*d*x + 1/2*c)^5 + 60*a^3*tan(1/2*
d*x + 1/2*c)^4 + 360*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*b^3*tan(1/2*d*x + 1
```

```
/2*c)^3 - 10*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 45*a^2*b*tan(1/2*d*x + 1/2*c) - 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 9.44, size = 241, normalized size = 1.32

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{a^3}{96}+\frac{a\,b^2}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{3\,a^2\,b}{8}+\frac{b^3}{2}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^3}{3}+4\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(2\,a^3+12\,a\,b^2\right)+\frac{a^3}{5}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{32\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{a^3}{16}+\frac{3\,a\,b^2}{8}\right)}{d}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (b^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (ta
n(c/2 + (d*x)/2)^3*((a*b^2)/8 + a^3/96))/d - (log(tan(c/2 + (d*x)/2))*((3*a
^2*b)/8 + b^3/2))/d - (cot(c/2 + (d*x)/2)^5*(4*b^3*tan(c/2 + (d*x)/2)^3 + t
an(c/2 + (d*x)/2)^2*(4*a*b^2 + a^3/3) - tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 2*
a^3) + a^3/5 + (3*a^2*b*tan(c/2 + (d*x)/2))/2))/(32*d) - (tan(c/2 + (d*x)/2
)*((3*a*b^2)/8 + a^3/16))/d + (3*a^2*b*tan(c/2 + (d*x)/2)^4)/(64*d)
```

3.1077 $\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=212

$$\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{b(3a^2-b}{}$$

```
[Out] 1/16*a*(a^2+6*b^2)*arctanh(cos(d*x+c))/d+1/15*b*(6*a^2+5*b^2)*cot(d*x+c)/d+
1/16*a*(a^2+6*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/15*b*(3*a^2-b^2)*cot(d*x+c)*cs
c(d*x+c)^2/d+1/120*a*(5*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/10*b*cot(d*x
+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(
d*x+c))^3/d
```

Rubi [A]
time = 0.39, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2968, 3127, 3126, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$\frac{b(6a^2+5b^2)\cot(c+dx)}{15d}+\frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}+\frac{b(3a^2-b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{a(a^2+6b^2)\cot(c+dx)\csc(c+dx)}{16d}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{6d}-\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b*(6*a^2 + 5*b^2)*Cot[c +
 d*x])/(15*d) + (a*(a^2 + 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*(3*
a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) + (a*(5*a^2 - 6*b^2)*Cot[c +
 d*x]*Csc[c + d*x]^3)/(120*d) - (b*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c
 + d*x])^2)/(10*d) - (Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(
6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
```

```
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
```

```
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \int \csc^7(c+dx)(a+b\sin(c+dx))^3\left(1-\sin^2(c+dx)\right)$$
$$= -\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{6d}+\frac{1}{6}\int$$
$$= -\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{10d}-\frac{\cot}{}$$
$$= \frac{a(5a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{120d}-\frac{b\cot(c+dx)}{}$$
$$= \frac{b(3a^2-b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{a(5a^2-6b^2)\,\mathrm{co}}{}$$
$$= \frac{b(3a^2-b^2)\cot(c+dx)\csc^2(c+dx)}{15d}+\frac{a(5a^2-6b^2)\,\mathrm{co}}{}$$
$$= \frac{a(a^2+6b^2)\cot(c+dx)\csc(c+dx)}{16d}+\frac{b(3a^2-b^2)\cot(}{}$$
$$= \frac{a(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b(6a^2+5b^2)\cot(c}{15d}$$

Mathematica [A]

time = 1.49, size = 369, normalized size = 1.74

$$-\frac{-64(6a^2b+5b^3)\cot\left(\frac{1}{2}(c+dx)\right)-30(a^3+6ab^2)\csc^2\left(\frac{1}{2}(c+dx)\right)-120a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-720ab^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+720ab^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+30a^3\sec^2\left(\frac{1}{2}(c+dx)\right)+180ab^2\sec^2\left(\frac{1}{2}(c+dx)\right)-90ab^2\sec^4\left(\frac{1}{2}(c+dx)\right)-5a^3\sec^6\left(\frac{1}{2}(c+dx)\right)+96a^2b\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-640b^3\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+a^2\csc^6\left(\frac{1}{2}(c+dx)\right)(5a+18b\sin(c+dx))+2b\csc^4\left(\frac{1}{2}(c+dx)\right)(45ab+(-3a^2+20b^2)\sin(c+dx))+384a^2b\tan\left(\frac{1}{2}(c+dx)\right)+320b^3\tan\left(\frac{1}{2}(c+dx)\right)-36a^2b\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/1920*(-64*(6*a^2*b + 5*b^3)*Cot[(c + d*x)/2] - 30*(a^3 + 6*a*b^2)*Csc[(c
 + d*x)/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 720*a*b^2*Log[Cos[(c + d*x)/
2]] + 120*a^3*Log[Sin[(c + d*x)/2]] + 720*a*b^2*Log[Sin[(c + d*x)/2]] + 30*
a^3*Sec[(c + d*x)/2]^2 + 180*a*b^2*Sec[(c + d*x)/2]^2 - 90*a*b^2*Sec[(c + d
*x)/2]^4 - 5*a^3*Sec[(c + d*x)/2]^6 + 96*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)
/2]^4 - 640*b^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + a^2*Csc[(c + d*x)/2]^6*
(5*a + 18*b*Sin[c + d*x]) + 2*b*Csc[(c + d*x)/2]^4*(45*a*b + (-3*a^2 + 20*b
^2)*Sin[c + d*x]) + 384*a^2*b*Tan[(c + d*x)/2] + 320*b^3*Tan[(c + d*x)/2] -
 36*a^2*b*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/d
```

Maple [A]

time = 0.31, size = 224, normalized size = 1.06

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+3a^2b\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c\right.}{15\sin(dx+c}\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^3(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^3(dx+c)}{8\sin(dx+c)^4}-\frac{\cos^3(dx+c)}{16\sin(dx+c)^2}-\frac{\cos(dx+c)}{16}-\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)+3a^2b\left(-\frac{\cos^3(dx+c)}{5\sin(dx+c)^5}-\frac{2\left(\cos^3(dx+c\right.}{15\sin(dx+c}\right.}{d}$
risch	$-\frac{480ib^3e^{4i(dx+c)}+15a^3e^{11i(dx+c)}+90a\,b^2e^{11i(dx+c)}+720ib^3e^{8i(dx+c)}-240ib^3e^{10i(dx+c)}-85a^3e^{9i(dx+c)}+450a\,b^2e^{9i(dx-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^3-1/8/sin(d*x+c)^4*cos(d*x+c)^3-1/16
/sin(d*x+c)^2*cos(d*x+c)^3-1/16*cos(d*x+c)-1/16*ln(csc(d*x+c)-cot(d*x+c)))+
3*a^2*b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^3-2/15/sin(d*x+c)^3*cos(d*x+c)^3)+3*a
*b^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^3-1/8/sin(d*x+c)^2*cos(d*x+c)^3-1/8*cos(
d*x+c)-1/8*ln(csc(d*x+c)-cot(d*x+c)))-1/3*b^3/sin(d*x+c)^3*cos(d*x+c)^3)
```

Maxima [A]

time = 0.32, size = 202, normalized size = 0.95

$$-\frac{5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5-8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+90\,ab^2\left(\frac{2\left(\cos(dx+c)^3+\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{160\,b^3}{\tan(dx+c)^3}+\frac{96\left(5\,\tan(dx+c)^2+3\right)a^2b}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/480*(5*a^3*(2*(3*cos(d*x + c)^5 - 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c
) + 1) + 3*log(cos(d*x + c) - 1)) + 90*a*b^2*(2*(cos(d*x + c)^3 + cos(d*x +
 c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - log(cos(d*x + c) + 1) + log(
cos(d*x + c) - 1)) + 160*b^3/tan(d*x + c)^3 + 96*(5*tan(d*x + c)^2 + 3)*a^2
*b/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.40, size = 310, normalized size = 1.46

$$\frac{80\,a^{3}\cos\left(dx+c\right)^{3}-30\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{5}+30\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)+15\,\left(\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{6}-3\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{4}-a^{3}-6\,ab^{2}+3\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\,\left(\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{6}-3\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{4}-a^{3}-6\,ab^{2}+3\,\left(a^{3}+6\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-32\,\left(\left(6\,a^{2}b+5\,b^{3}\right)\cos\left(dx+c\right)^{5}-5\,\left(3\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{480\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(80*a^3*cos(d*x + c)^3 - 30*(a^3 + 6*a*b^2)*cos(d*x + c)^5 + 30*(a^3
+ 6*a*b^2)*cos(d*x + c) + 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6*a
*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 15*((a^3 + 6*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 6
*a*b^2)*cos(d*x + c)^4 - a^3 - 6*a*b^2 + 3*(a^3 + 6*a*b^2)*cos(d*x + c)^2)*
log(-1/2*cos(d*x + c) + 1/2) - 32*((6*a^2*b + 5*b^3)*cos(d*x + c)^5 - 5*(3*
a^2*b + b^3)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x
+ c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.53, size = 354, normalized size = 1.67

$$\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+90\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+60\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-360\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-240\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,\left(a^{3}+6\,ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{294\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1764\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+360\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+240\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-90\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 + 60*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
 1/2*c)^2 - 360*a^2*b*tan(1/2*d*x + 1/2*c) - 240*b^3*tan(1/2*d*x + 1/2*c) -
 120*(a^3 + 6*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (294*a^3*tan(1/2*d*x
+ 1/2*c)^6 + 1764*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/
2*c)^5 + 240*b^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 6
0*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan
(1/2*d*x + 1/2*c)^2 - 90*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 36*a^2*b*tan(1/2*d*
x + 1/2*c) - 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.69, size = 292, normalized size = 1.38

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^3}{2}+3\,a\,b^2\right)-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(2\,a^2\,b+\frac{8\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(12\,a^2\,b+8\,b^3\right)+\frac{a^3}{6}+\frac{6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}\right)}{64\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^3}{128}+\frac{3\,a\,b^2}{64}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{a^2\,b}{32}+\frac{b^3}{24}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{a^3}{16}+\frac{3\,a\,b^2}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2\,b}{16}+\frac{b^3}{8}\right)}{d}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(a + b*sin(c + d*x))^3)/sin(c + d*x)^7,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) - (a^3*tan(c/2 + (d*x)/2)^2)/(128*d) - (
cot(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 + a^3/2) - (a^3*tan(c/2
 + (d*x)/2)^4)/2 + tan(c/2 + (d*x)/2)^3*(2*a^2*b + (8*b^3)/3) - tan(c/2 + (
d*x)/2)^5*(12*a^2*b + 8*b^3) + a^3/6 + (6*a^2*b*tan(c/2 + (d*x)/2))/5))/(64
*d) + (tan(c/2 + (d*x)/2)^4*((3*a*b^2)/64 + a^3/128))/d + (tan(c/2 + (d*x)/
2)^3*((a^2*b)/32 + b^3/24))/d - (log(tan(c/2 + (d*x)/2))*((3*a*b^2)/8 + a^3
/16))/d - (tan(c/2 + (d*x)/2)*((3*a^2*b)/16 + b^3/8))/d + (3*a^2*b*tan(c/2
+ (d*x)/2)^5)/(160*d)
```

3.1078 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{a(4a^2-b^2)\,x}{b^5}-\frac{2a^2(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d}+\frac{(12a^2-b^2)\cos(c+dx)}{3b^4d}-\frac{2a\cos(c+dx)\sin(c+dx}{b^3d}$$

```
[Out] a*(4*a^2-b^2)*x/b^5+1/3*(12*a^2-b^2)*cos(d*x+c)/b^4/d-2*a*cos(d*x+c)*sin(d*
x+c)/b^3/d+4/3*cos(d*x+c)*sin(d*x+c)^2/b^2/d-cos(d*x+c)*sin(d*x+c)^3/b/d/(a
+b*sin(d*x+c))-2*a^2*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2
)^(1/2))/b^5/d/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2968, 3127, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2(4a^2-3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{ax(4a^2-b^2)}{b^5}+\frac{(12a^2-b^2)\cos(c+dx)}{3b^4d}-\frac{2a\sin(c+dx)\cos(c+dx)}{b^3d}-\frac{\sin^3(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{4\sin^2(c+dx)\cos(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(4*a^2 - b^2)*x)/b^5 - (2*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + ((12*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(b^3*d) + (4*Cos[c + d*x]*
Sin[c + d*x]^2)/(3*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(b*d*(a + b*Sin[c
 + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
```

```
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin^2(c+dx)\left(-3(a^2-b^2)+4(a^2-b^2)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$

$$= \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(8a\left(a^2-b^2\right.\right.}{}}{}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(c+dx)}{3b^2d} - \frac{\cos(c+dx)\sin}{bd(a+b\sin(}$$

$$= \frac{(12a^2-b^2)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4\cos(c+dx)\sin^2(}{3b^2d}$$

$$= \frac{a(4a^2-b^2)\,x}{b^5} + \frac{(12a^2-b^2)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4}{}$$

$$= \frac{a(4a^2-b^2)\,x}{b^5} + \frac{(12a^2-b^2)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4}{}$$

$$= \frac{a(4a^2-b^2)\,x}{b^5} + \frac{(12a^2-b^2)\cos(c+dx)}{3b^4d} - \frac{2a\cos(c+dx)\sin(c+dx)}{b^3d} + \frac{4}{}$$

$$= \frac{a(4a^2-b^2)\,x}{b^5} - \frac{2a^2(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{(12a^2-b^2)\cos(}{3b^4d}$$

Mathematica [A]
time = 1.93, size = 246, normalized size = 1.31

$$\frac{-\frac{48a^2(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{96a^4c-24a^2b^2c+96a^4dx-24a^2b^2dx+12ab(8a^2-b^2)\cos(c+dx)+4ab^3\cos(3(c+dx))+96a^3bc\sin(c+dx)-24ab^3c\sin(c+dx)+96a^3bdx\sin(c+dx)-24ab^3dx\sin(c+dx)+24a^2b^2\sin(2(c+dx))-2b^4\sin(2(c+dx))-b^4\sin(4(c+dx))}{a+b\sin(c+dx)}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-48*a^2*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] + (96*a^4*c - 24*a^2*b^2*c + 96*a^4*d*x - 24*a^2*b^2*d*x +
 12*a*b*(8*a^2 - b^2)*Cos[c + d*x] + 4*a*b^3*Cos[3*(c + d*x)] + 96*a^3*b*c*
Sin[c + d*x] - 24*a*b^3*c*Sin[c + d*x] + 96*a^3*b*d*x*Sin[c + d*x] - 24*a*b
^3*d*x*Sin[c + d*x] + 24*a^2*b^2*Sin[2*(c + d*x)] - 2*b^4*Sin[2*(c + d*x)]
- b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

Maple [A]
time = 0.47, size = 249, normalized size = 1.32

method	result
derivativedivides	$\dfrac{\dfrac{4\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-\frac{1}{2}b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{b^3}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+2a\left(4a^2-b^2\right)\mathrm{arct}}{b^5}}{d}$
default	$\dfrac{\dfrac{4\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-\frac{1}{2}b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{b^3}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+2a\left(4a^2-b^2\right)\mathrm{arct}}{b^5}}{d}$
risch	$\frac{4a^3x}{b^5}-\frac{ax}{b^3}+\frac{ia\,\mathrm{e}^{2i(dx+c)}}{4b^3d}+\frac{3\,\mathrm{e}^{i(dx+c)}a^2}{2b^4d}-\frac{\mathrm{e}^{i(dx+c)}}{8b^2d}+\frac{3\,\mathrm{e}^{-i(dx+c)}a^2}{2b^4d}-\frac{\mathrm{e}^{-i(dx+c)}}{8b^2d}-\frac{ia\,\mathrm{e}^{-2i(dx+c)}}{4b^3d}+\frac{}{b^5d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^5*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(3/2*a^2*b-1/2*b^3)*tan(1/2*d*x
+1/2*c)^4+3*a^2*b*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*tan(1/2*d*x+1/2*c)+3/2*a^2
*b-1/6*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(4*a^2-b^2)*arctan(tan(1/2*d*x
+1/2*c)))-4*a^2/b^5*((-1/2*b^2*tan(1/2*d*x+1/2*c)-1/2*a*b)/(a*tan(1/2*d*x+1
/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(4*a^2-3*b^2)/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.42, size = 643, normalized size = 3.42

$$\left[\frac{4\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3+6\left(4a^6-5a^4b^2+a^2b^4\right)dx+3\left(4a^5-3a^3b^2+\left(4a^4b-3a^2b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2}\right)+6\left(4a^5b-5a^3b^3+ab^5\right)\cos\left(dx+c\right)-2\left(\left(a^2b^4-b^6\right)\cos\left(dx+c\right)^3-3\left(4a^5b-5a^3b^3+ab^5\right)dx-6\left(a^4b^2-a^2b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(\left(a^2b^6-b^8\right)d\sin\left(dx+c\right)+\left(a^3b^5-ab^7\right)d\right)},\ \frac{2\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3+3\left(4a^6-5a^4b^2+a^2b^4\right)dx+3\left(4a^5-3a^3b^2+\left(4a^4b-3a^2b^3\right)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)+3\left(4a^5b-5a^3b^3+ab^5\right)\cos\left(dx+c\right)-\left(\left(a^2b^4-b^6\right)\cos\left(dx+c\right)^3-3\left(4a^5b-5a^3b^3+ab^5\right)dx-6\left(a^4b^2-a^2b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(\left(a^2b^6-b^8\right)d\sin\left(dx+c\right)+\left(a^3b^5-ab^7\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/6*(4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*
d*x + 3*(4*a^5 - 3*a^3*b^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 +
a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^
3*b^3 + a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^6 - b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d), 1/3*(2*(a^3*b^3 - a*b^5
)*cos(d*x + c)^3 + 3*(4*a^6 - 5*a^4*b^2 + a^2*b^4)*d*x + 3*(4*a^5 - 3*a^3*b
^2 + (4*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3 +
 a*b^5)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^6 -
 b^8)*d*sin(d*x + c) + (a^3*b^5 - a*b^7)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 261, normalized size = 1.39

$$\frac{\frac{3\,(4\,a^3-ab^2)(dx+c)}{b^5}-\frac{6\,(4\,a^4-3\,a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{6\,\left(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^4}+\frac{2\,\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^2-b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*(4*a^3 - a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 3*a^2*b^2)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/
2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c
)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

Mupad [B]
time = 11.85, size = 1688, normalized size = 8.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((28*a^2*tan(c/2 + (d*x)/2)^3)/b^3 - (2*(a*b^2 - 12*a^3))/(3*b^4) + (4*a^2*
tan(c/2 + (d*x)/2)^7)/b^3 + (2*tan(c/2 + (d*x)/2)^6*(a*b^2 + 4*a^3))/b^4 -
(2*tan(c/2 + (d*x)/2)^4*(a*b^2 - 12*a^3))/b^4 - (2*tan(c/2 + (d*x)/2)^2*(7*
a*b^2 - 36*a^3))/(3*b^4) + (4*tan(c/2 + (d*x)/2)*(9*a^2 - b^2))/(3*b^3) + (
4*tan(c/2 + (d*x)/2)^5*(5*a^2 - b^2))/b^3)/(d*(a + 2*b*tan(c/2 + (d*x)/2) +
 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/
2)^6 + a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d
*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) + (atan((64*a^4*tan(c/2 + (d*x)/2))/(
64*a^4 - (256*a^6)/b^2) + (256*a^6*tan(c/2 + (d*x)/2))/(256*a^6 - 64*a^4*b^
2))*(a*b^2*1i - a^3*4i)*2i)/(b^5*d) + (a^2*atan(((a^2*(-(a + b)*(a - b))^(1
/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan
(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 +
 (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10)
)/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 + (a^2*(-(a
 + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(
3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5))*1i)/(b^7
- a^2*b^5) + (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 -
8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5
*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^
2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a
^3*b^12 - 8*a^5*b^10))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)
*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7
- a^2*b^5)))/(b^7 - a^2*b^5))*1i)/(b^7 - a^2*b^5))/((64*(32*a^10 + 6*a^6*b^
4 - 32*a^8*b^2))/b^11 + (64*tan(c/2 + (d*x)/2)*(128*a^11 - 6*a^5*b^6 + 56*a
^7*b^4 - 160*a^9*b^2))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)
*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*
a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32*a^9*b^4))/b^12 + (a^2*(-(a + b)*(a
- b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b^12 - 2*a^4*b^10))/b^11 + (32*tan(c/
2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))/b^12 + (a^2*(-(a + b)*(a - b))^(1/2
)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^
14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5)))/(b^7 - a^2*b^5) + (a^2*(-(a
 + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^4*b^8 - 8*a^6*b^6 + 16*a^8*b^4
))/b^11 + (32*tan(c/2 + (d*x)/2)*(2*a^3*b^10 - 26*a^5*b^8 + 64*a^7*b^6 - 32
*a^9*b^4))/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((32*(a^2*b
^12 - 2*a^4*b^10))/b^11 + (32*tan(c/2 + (d*x)/2)*(6*a^3*b^12 - 8*a^5*b^10))
/b^12 - (a^2*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (32*tan
(c/2 + (d*x)/2)*(3*a*b^16 - 2*a^3*b^14))/b^12))/(b^7 - a^2*b^5)))/(b^7 - a^
```

```
2*b^5)))/(b^7 - a^2*b^5)))*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*2i)/(d*
(b^7 - a^2*b^5))
```

3.1079 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=153

$$-\frac{(6a^2-b^2)\,x}{2b^4}+\frac{2a(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d}-\frac{3a\cos(c+dx)}{b^3d}+\frac{3\cos(c+dx)\sin(c+dx)}{2b^2d}-\frac{\cos(c+}{bd(a-}$$

```
[Out] -1/2*(6*a^2-b^2)*x/b^4-3*a*cos(d*x+c)/b^3/d+3/2*cos(d*x+c)*sin(d*x+c)/b^2/d
-cos(d*x+c)*sin(d*x+c)^2/b/d/(a+b*sin(d*x+c))+2*a*(3*a^2-2*b^2)*arctan((b+a
*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3127, 3129, 3102, 2814, 2739, 632, 210}

$$\frac{2a(3a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\sqrt{a^2-b^2}}-\frac{x(6a^2-b^2)}{2b^4}-\frac{3a\cos(c+dx)}{b^3d}-\frac{\sin^2(c+dx)\cos(c+dx)}{bd(a+b\sin(c+dx))}+\frac{3\sin(c+dx)\cos(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((6*a^2 - b^2)*x)/b^4 + (2*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*d) - (3*a*Cos[c + d*x])/(b^3*
d) + (3*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^2
)/(b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$
$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{\sin(c+dx)\left(-2(a^2-b^2)+3(a^2-b^2)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)}$$
$$= \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))} - \frac{\int \frac{3a(a^2-b^2)-b(a^2-b^{}}{}}{}$$
$$= -\frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx)\sin^2(c+dx)}{bd(a+b\sin(c+dx))}$$
$$= -\frac{(6a^2-b^2)\,x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx}{bd(a+b}$$
$$= -\frac{(6a^2-b^2)\,x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx}{bd(a+b}$$
$$= -\frac{(6a^2-b^2)\,x}{2b^4} - \frac{3a\cos(c+dx)}{b^3d} + \frac{3\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos(c+dx}{bd(a+b}$$
$$= -\frac{(6a^2-b^2)\,x}{2b^4} + \frac{2a(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\,d} - \frac{3a\cos(c+dx)}{b^3d}$$

Mathematica [A]
time = 0.32, size = 129, normalized size = 0.84

$$\frac{2(-6a^2+b^2)(c+dx) + \frac{8a(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 8ab\cos(c+dx) - \frac{4a^2b\cos(c+dx)}{a+b\sin(c+dx)} + b^2\sin(2(c+dx))}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(-6*a^2 + b^2)*(c + d*x) + (8*a*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 8*a*b*Cos[c + d*x] - (4*a^2*b*C
os[c + d*x])/(a + b*Sin[c + d*x]) + b^2*Sin[2*(c + d*x)])/(4*b^4*d)
```

Maple [A]

time = 0.40, size = 210, normalized size = 1.37

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(6a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^4}+\frac{2a\left(\frac{-b^2\tan\left(\frac{dx}{2}\right.}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2}\right.}{}}{d}$
default	$\frac{-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(6a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^4}+\frac{2a\left(\frac{-b^2\tan\left(\frac{dx}{2}\right.}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2}\right.}{}}{d}$
risch	$-\frac{3x\,a^2}{b^4}+\frac{x}{2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8b^2d}-\frac{a\,\mathrm{e}^{i(dx+c)}}{b^3d}-\frac{a\,\mathrm{e}^{-i(dx+c)}}{b^3d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{2ia^2\left(ib+a\,\mathrm{e}^{i(dx+c)}\right)}{b^4d\left(-ib\,\mathrm{e}^{2i(dx+c)}+ib+2a\,\mathrm{e}^{i(dx+c)}\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^4*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+2*a*b*tan(1/2*d*x+1/2*c)^2-1/2*b
^2*tan(1/2*d*x+1/2*c)+2*a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(6*a^2-b^2)*arc
tan(tan(1/2*d*x+1/2*c)))+2*a/b^4*((-b^2*tan(1/2*d*x+1/2*c)-a*b)/(a*tan(1/2*
d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(3*a^2-2*b^2)/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.41, size = 568, normalized size = 3.71

$$\left[-\frac{(a^2b^3-b^5)\cos(dx+c)^3+(6a^5-7a^3b^2+ab^4)dx-(3a^4-2a^2b^2+(3a^3b-2ab^3)\sin(dx+c))\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+(6a^4b-7a^2b^3+b^5)\cos(dx+c)+((6a^4b-7a^2b^3+b^5)dx+3(a^3b^2-ab^4)\cos(dx+c))\sin(dx+c)}{2((a^2b^5-b^7)d\sin(dx+c)+(a^3b^4-ab^6)d)}, -\frac{(a^2b^3-b^5)\cos(dx+c)^3+(6a^5-7a^3b^2+ab^4)dx+2(3a^4-2a^2b^2+(3a^3b-2ab^3)\sin(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+(6a^4b-7a^2b^3+b^5)\cos(dx+c)+((6a^4b-7a^2b^3+b^5)dx+3(a^3b^2-ab^4)\cos(dx+c))\sin(dx+c)}{2((a^2b^5-b^7)d\sin(dx+c)+(a^3b^4-ab^6)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x - (
3*a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + (6*a^4*b - 7*a^2*b^3 + b^5)*cos(d*x
 + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4 - a*b^6)*d), -1/
2*((a^2*b^3 - b^5)*cos(d*x + c)^3 + (6*a^5 - 7*a^3*b^2 + a*b^4)*d*x + 2*(3*
a^4 - 2*a^2*b^2 + (3*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(
-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (6*a^4*b - 7*a^2*b^
3 + b^5)*cos(d*x + c) + ((6*a^4*b - 7*a^2*b^3 + b^5)*d*x + 3*(a^3*b^2 - a*b
^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^5 - b^7)*d*sin(d*x + c) + (a^3*b^4
- a*b^6)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 211, normalized size = 1.38

$$-\frac{\frac{(6\,a^2-b^2)(dx+c)}{b^4}-\frac{4\,(3\,a^3-2\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{4\,(ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^3}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((6*a^2 - b^2)*(d*x + c)/b^4 - 4*(3*a^3 - 2*a*b^2)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*b^4) + 4*(a*b*tan(1/2*d*x + 1/2*c) + a^2)/((a*tan(1/2*d
*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^3) + 2*(b*tan(1/2*d*x + 1/2
*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^2*b^3))/d
```

Mupad [**B**]
time = 10.40, size = 479, normalized size = 3.13

$$-\frac{\frac{6\,a^2}{b^3}+\frac{9\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(3\,a^2+b^2)}{b^3}+\frac{12\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{b^2}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(6\,a^2-b^2)}{b^3}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{\ln\left(b+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sqrt{b^2-a^2}\right)\,\left(3\,a^3\,\sqrt{b^2-a^2}-2\,a\,b^2\,\sqrt{b^2-a^2}\right)}{b^4\,d\,(a^2-b^2)}-\frac{a\,\ln\left(b+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sqrt{b^2-a^2}\right)\,\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-2\,b^2)}{d\,(b^6-a^2\,b^4)}+\frac{\mathrm{atan}\left(\frac{24\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{24\,a^3-8\,a\,b^2+\frac{144\,a^5}{b^2}}+\frac{144\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{144\,a^5+24\,a^3\,b^2-8\,a\,b^4}-\frac{8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{24\,a^3}{b^2}-8\,a+\frac{144\,a^5}{b^4}}\right)\,(a^2\,6{}\mathrm{i}-b^2\,1{}\mathrm{i})\,1{}\mathrm{i}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (atan((24*a^3*tan(c/2 + (d*x)/2))/(24*a^3 - 8*a*b^2 + (144*a^5)/b^2) + (144
*a^5*tan(c/2 + (d*x)/2))/(144*a^5 - 8*a*b^4 + 24*a^3*b^2) - (8*a*tan(c/2 +
(d*x)/2))/((24*a^3)/b^2 - 8*a + (144*a^5)/b^4))*(a^2*6i - b^2*1i)*1i)/(b^4*
d) - ((6*a^2)/b^3 + (9*a*tan(c/2 + (d*x)/2))/b^2 + (2*tan(c/2 + (d*x)/2)^4*
(3*a^2 + b^2))/b^3 + (12*a*tan(c/2 + (d*x)/2)^3)/b^2 + (3*a*tan(c/2 + (d*x)
/2)^5)/b^2 + (2*tan(c/2 + (d*x)/2)^2*(6*a^2 - b^2))/b^3)/(d*(a + 2*b*tan(c/
2 + (d*x)/2) + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*tan(
c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) -
(log(b + a*tan(c/2 + (d*x)/2) - (b^2 - a^2)^(1/2))*(3*a^3*(b^2 - a^2)^(1/2)
 - 2*a*b^2*(b^2 - a^2)^(1/2)))/(b^4*d*(a^2 - b^2)) - (a*log(b + a*tan(c/2 +
 (d*x)/2) + (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 2*b^2))/(d
*(b^6 - a^2*b^4))
```

3.1080 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=106

$$\frac{2ax}{b^3}-\frac{2(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}$$

```
[Out] 2*a*x/b^3+cos(d*x+c)*(2*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))-2*(2*a^2-b^2
)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3/d/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2942, 2814, 2739, 632, 210}

$$-\frac{2(2a^2-b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\sqrt{a^2-b^2}}+\frac{2ax}{b^3}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 - (2*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(b^3*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]*(2*a + b*Sin[c + d*x]))/(b^2*
d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}-\frac{\int \frac{-b-2a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b^2}\\
&= \frac{2ax}{b^3}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}-\frac{(2a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{b^3}\\
&= \frac{2ax}{b^3}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}-\frac{(2(2a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,d\right.}{b^3d}\\
&= \frac{2ax}{b^3}+\frac{\cos(c+dx)(2a+b\sin(c+dx))}{b^2d(a+b\sin(c+dx))}+\frac{(4(2a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x}\right.}{b^3}\\
&= \frac{2ax}{b^3}-\frac{2(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\sqrt{a^2-b^2}\,d}+\frac{\cos(c+dx)(2a+b\sin(c+dx}{b^2d(a+b\sin(c+dx))}
\end{aligned}$$

Mathematica [A]
time = 0.76, size = 130, normalized size = 1.23

$$\frac{-\frac{4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{4a^2c+4a^2dx+4ab\cos(c+dx)+4ab(c+dx)\sin(c+dx)+b^2\sin(2(c+dx))}{a+b\sin(c+dx)}}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] + (4*a^2*c + 4*a^2*d*x + 4*a*b*Cos[c + d*x] + 4*a*b*(c + d*x)*Sin
[c + d*x] + b^2*Sin[2*(c + d*x)])/(a + b*Sin[c + d*x]))/(2*b^3*d)
```

Maple [A]
time = 0.36, size = 151, normalized size = 1.42

method	result
derivativedivides	$\frac{\frac{\frac{4b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{b^3}}{d}$
default	$\frac{\frac{\frac{4b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{4\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{b^3}}{d}$
risch	$\frac{2ax}{b^3}+\frac{e^{i(dx+c)}}{2b^2d}+\frac{e^{-i(dx+c)}}{2b^2d}+\frac{2ia\left(-ia\,e^{i(dx+c)}+b\right)}{b^3d\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)}-\frac{2ia^2\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^3*(1/2*b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))-4/
b^3*((-1/2*b^2*tan(1/2*d*x+1/2*c)-1/2*a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(
1/2*d*x+1/2*c)+a)+1/2*(2*a^2-b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 479, normalized size = 4.52

$$\left[\frac{4\left(a^4-a^2b^2\right)dx+\left(2a^3-ab^2+\left(2a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2}\right)+4\left(a^3b-ab^3\right)\cos\left(dx+c\right)+2\left(2\left(a^3b-ab^3\right)dx+\left(a^2b^2-b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(\left(a^2b^4-b^6\right)d\sin\left(dx+c\right)+\left(a^3b^3-ab^5\right)d\right)}, \frac{2\left(a^4-a^2b^2\right)dx+\left(2a^3-ab^2+\left(2a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)+2\left(a^3b-ab^3\right)\cos\left(dx+c\right)+\left(2\left(a^3b-ab^3\right)dx+\left(a^2b^2-b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\left(a^2b^4-b^6\right)d\sin\left(dx+c\right)+\left(a^3b^3-ab^5\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 4*(a^3*b - a*
b^3)*cos(d*x + c) + 2*(2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos(d*x + c)
)*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5)*d), (2*
(a^4 - a^2*b^2)*d*x + (2*a^3 - a*b^2 + (2*a^2*b - b^3)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*
(a^3*b - a*b^3)*cos(d*x + c) + (2*(a^3*b - a*b^3)*d*x + (a^2*b^2 - b^4)*cos
(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*sin(d*x + c) + (a^3*b^3 - a*b^5
)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 191, normalized size = 1.80

$$\frac{2\left(\frac{(dx+c)a}{b^3}-\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(2a^2-b^2)}{\sqrt{a^2-b^2}\,b^3}+\frac{b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((d*x + c)*a/b^3 - (pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*(2*a^2 - b^2)/(sqrt(a^2 - b^2)*b
^3) + (b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*
d*x + 1/2*c) + 2*a)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^2))/d
```

Mupad [B]
time = 10.02, size = 269, normalized size = 2.54

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{b}+\frac{4\,a}{b^2}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+\frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{2\,a\,x}{b^3}-\frac{\ln\left(b+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sqrt{b^2-a^2}\right)\sqrt{-(a+b)\,(a-b)}\,(2\,a^2-b^2)}{d\,(b^5-a^2\,b^3)}-\frac{\ln\left(b+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sqrt{b^2-a^2}\right)\left(2\,a^2\sqrt{b^2-a^2}-b^2\sqrt{b^2-a^2}\right)}{b^3\,d\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3)/b + (4*a)/b^2 + (6*tan(c/2 + (d*x)/2))/b + (4*a*t
an(c/2 + (d*x)/2)^2)/b^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d
*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) + (2*a*x)/b^
3 - (log(b + a*tan(c/2 + (d*x)/2) - (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(
1/2)*(2*a^2 - b^2))/(d*(b^5 - a^2*b^3)) - (log(b + a*tan(c/2 + (d*x)/2) + (
b^2 - a^2)^(1/2))*(2*a^2*(b^2 - a^2)^(1/2) - b^2*(b^2 - a^2)^(1/2)))/(b^3*d
*(a^2 - b^2))
```

3.1081 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/a/d/(a+b*sin(d*x+c))-2*b*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2968, 3135, 12, 2826, 3855, 2739, 632, 210}

$$-\frac{2b\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\sqrt{a^2-b^2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*Sqrt[a^2 - b^2
]*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(a*d*(a + b*Sin[c + d*x
]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx \\
&= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{(a^2-b^2)\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)}{a+b\sin(c+dx)}\,dx}{a} \\
&= \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{b\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,\right.}{a^2d} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4b)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\right.}{} \\
&= -\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{ad(a+b\sin(c+dx)}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 97, normalized size = 1.05

$$
\frac{-\frac{2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a\cos(c+dx)}{a+b\sin(c+dx)}}{a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] -
Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*Cos[c + d*x])/(a + b*Sin
[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.47, size = 116, normalized size = 1.26

method	result

derivativedivides	$\frac{-\frac{4\left(\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
default	$\frac{-\frac{4\left(\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}}{d}$
risch	$\frac{2i\left(-ia\,e^{i(dx+c)}+b\right)}{abd\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)}+\frac{ib\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d\,a^2}-\frac{ib\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\right.}{b\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/a^2*((-1/2*b*tan(1/2*d*x+1/2*c)-1/2*a)/(a*tan(1/2*d*x+1/2*c)^2+2*b*
tan(1/2*d*x+1/2*c)+a)+1/2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2
*c)+2*b)/(a^2-b^2)^(1/2)))+1/a^2*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(87) = 174.
time = 0.43, size = 483, normalized size = 5.25

$$\left[-\frac{(b^2\sin(dx+c)+ab)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2(a^3-ab^2)\cos(dx+c)+(a^3-ab^2+(a^2b-b^3)\sin(dx+c))\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(a^3-ab^2+(a^2b-b^3)\sin(dx+c))\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2((a^4b-a^2b^3)d\sin(dx+c)+(a^5-a^3b^2)d)},\ \frac{2(b^2\sin(dx+c)+ab)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+2(a^3-ab^2)\cos(dx+c)-(a^3-ab^2+(a^2b-b^3)\sin(dx+c))\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a^3-ab^2+(a^2b-b^3)\sin(dx+c))\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2((a^4b-a^2b^3)d\sin(dx+c)+(a^5-a^3b^2)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*sin(d*x + c) + a*b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 2*(a^3 - a*b^2)*cos(d*x + c) + (a^3 - a*b^2 + (a^2*b - b^
3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^3 - a*b^2 + (a^2*b - b^3)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x +
 c) + (a^5 - a^3*b^2)*d), 1/2*(2*(b^2*sin(d*x + c) + a*b)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^3 - a*b^
2)*cos(d*x + c) - (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + (a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2))/((a^4*b - a^2*b^3)*d*sin(d*x + c) + (a^5 - a^3*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.41, size = 130, normalized size = 1.41

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b}{\sqrt{a^2-b^2}\,a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*b/(sqrt(a^2 - b^2)*a^2) - log(abs(tan(1/2*d*x + 1
/2*c)))/a^2 - 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 + 2
*b*tan(1/2*d*x + 1/2*c) + a)*a^2))/d
```

Mupad [B]
time = 10.16, size = 523, normalized size = 5.68

$$\frac{a^3\cos(c+dx)-ab^2-b^3\sin(c+dx)+a^3+a^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-ab^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-b^3\sin(c+dx)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)-ab^2\cos(c+dx)+a^2b\sin(c+dx)+a^2b\sin(c+dx)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+b^2\sin(c+dx)\,\mathrm{atan}\left(\frac{-a^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+ab\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3+3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}\right)\sqrt{b^2-a^2}\,2\mathrm{i}+ab\,\mathrm{atan}\left(\frac{-a^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+ab\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3+3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b-2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^2-4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^3}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{a^2d\left(a^2-b^2\right)\left(a+b\sin(c+dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (a^3*cos(c + d*x) - a*b^2 - b^3*sin(c + d*x) + a^3 + a^3*log(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2)) - a*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 - b^3*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - a*b^2*cos(
c + d*x) + a^2*b*sin(c + d*x) + b^2*sin(c + d*x)*atan((b^2*sin(c/2 + (d*x)/
2)*(b^2 - a^2)^(1/2)*4i - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*2i)/(a^3*cos(c/2 + (d*x)/2) - 4*b^3*s
in(c/2 + (d*x)/2) - 2*a*b^2*cos(c/2 + (d*x)/2) + 3*a^2*b*sin(c/2 + (d*x)/2)
))*(b^2 - a^2)^(1/2)*2i + a*b*atan((b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*4i - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + a*b*cos(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*2i)/(a^3*cos(c/2 + (d*x)/2) - 4*b^3*sin(c/2 + (d*x)/2) -
 2*a*b^2*cos(c/2 + (d*x)/2) + 3*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2
)*2i + a^2*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*
d*(a^2 - b^2)*(a + b*sin(c + d*x)))
```

3.1082 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2} dx$

Optimal. Leaf size=115

$$-\frac{2(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] 2*b*arctanh(cos(d*x+c))/a^3/d-2*cot(d*x+c)/a^2/d+cot(d*x+c)/a/d/(a+b*sin(d*
x+c))-2*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/d/
(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 210}

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}-\frac{2(a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2(a^2-b^2)-(a^2-b^2)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b(a^2-b^2)-a(a^2-b^2)\sin(c+dx)\right)}{a+b\sin(c+dx)}}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} - \frac{(a^2-2b^2)\int}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2(a^2-2b^2)}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{(4(a^2-2b^2)}{}$$

$$= -\frac{2(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx}{a^2d}$$

Mathematica [A]
time = 0.54, size = 139, normalized size = 1.21

$$-\frac{\frac{4(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\cot\left(\frac{1}{2}(c+dx)\right) - 4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Si
n[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*
x)/2])/(a^3*d)
```

Maple [A]
time = 0.49, size = 156, normalized size = 1.36

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\right.}{}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\right.}{}}{d}$
risch	$-\frac{2\left(-3a\,e^{i(dx+c)}+2ib\,e^{2i(dx+c)}-2ib+a\,e^{3i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)a^2d}+\frac{\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da}-\frac{2\ln\left(e^{i(dx+c)}+\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)-2/a^3*((b^2*tan(1/2*d*x+1/2*c)+a*b)/(a*tan(
1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2-2*b^2)/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/2/a^2/tan(1/2*d*x+1
/2*c)-2/a^3*b*ln(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(110) = 220.
time = 0.46, size = 768, normalized size = 6.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 218, normalized size = 1.90

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(a^2-2\,b^2)}{\sqrt{a^2-b^2}\,a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-14\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

Mupad [B]
time = 11.41, size = 1616, normalized size = 14.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] -(a^4*cos(c + d*x) - b^4/2 - b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a^2*b^2)/2 + (b^4*cos(2*c + 2*d*x))/2 - a^2*b^2*cos(c + d*x) - a*b^3*si
n(2*c + 2*d*x) + a^3*b*sin(2*c + 2*d*x) + a^2*b^2*log(sin(c/2 + (d*x)/2)/co
s(c/2 + (d*x)/2)) + b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
+ 2*d*x) + b^3*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d
*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*co
s(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - (a
^2*b^2*cos(2*c + 2*d*x))/2 - a*b^3*sin(c + d*x) + a^3*b*sin(c + d*x) - a^2*
b*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i - a^3*sin(c + d*x
)*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^
2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^
4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)
/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - a^2*b^2*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) - b^3*cos(2*c + 2*d*x)
*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)
*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2
*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4
*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/
2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i - 2*a*b^3*sin(c +
d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 2*a^3*b*sin(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + a*b^2*sin(c + d*x)*atan((a^3*cos(c
/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)
/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*s
in(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*4i + a^2*b*cos(2*c + 2*d*x)*atan((a^3
*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*sin(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d*x)/2) + 8*b^4*sin(c/2 +
 (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*cos(c/2 + (d*x)/2) - 8*a^2
```

```
*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/(2*a^3*d*(a^2 - b^2)*(b/4 +
 (a*sin(c + d*x))/2 - (b*cos(2*c + 2*d*x))/4))
```

3.1083 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{2b(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d}+\frac{(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}$$

```
[Out] 1/2*(a^2-6*b^2)*arctanh(cos(d*x+c))/a^4/d+3*b*cot(d*x+c)/a^3/d-3/2*cot(d*x+
c)*csc(d*x+c)/a^2/d+cot(d*x+c)*csc(d*x+c)/a/d/(a+b*sin(d*x+c))+2*b*(2*a^2-3
*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d/(a^2-b^2)^(1/2
)
```

Rubi [A]
time = 0.49, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\cot(c+dx)}{a^3d}-\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2b(2a^2-3b^2)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}+\frac{(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*Sqrt[a^2 - b^2]*d) + ((a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + (3*
b*Cot[c + d*x])/(a^3*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + (Cot[c
+ d*x]*Csc[c + d*x])/(a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
```

```
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \csc^2(c+dx)(-6b(a^2 \ldots}{\ldots}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \ldots}{\ldots}$$

$$= \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(\ldots}{\ldots}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+\ldots}{2a^2d}$$

$$= \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{3b\cot(c+dx)}{a^3d} - \frac{3\cot(c+dx)\csc(c+\ldots}{2a^2d}$$

$$= \frac{2b\left(2a^2-3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{\left(a^2-6b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \ldots$$

Mathematica [A]

time = 2.08, size = 196, normalized size = 1.25

$$\frac{-\frac{16b\left(-2a^2+3b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 8ab\cot\left(\frac{1}{2}(c+dx)\right) - a^2\csc^2\left(\frac{1}{2}(c+dx)\right) + 4\left(a^2-6b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 4\left(a^2-6b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sec^2\left(\frac{1}{2}(c+dx)\right) + \frac{8ab^2\cos(c+dx)}{a+b\sin(c+dx)} - 8ab\tan\left(\frac{1}{2}(c+dx)\right)}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]

[Out] ((-16*b*(-2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*(a^2
```

```
- 6*b^2)*Log[Cos[(c + d*x)/2]] - 4*(a^2 - 6*b^2)*Log[Sin[(c + d*x)/2]] + a^
2*Sec[(c + d*x)/2]^2 + (8*a*b^2*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*
Tan[(c + d*x)/2])/(8*a^4*d)
```

Maple [A]
time = 0.54, size = 206, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{4b\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(2a^2-3b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^4}-\frac{}{8a^2 t}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{4b\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(2a^2-3b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^4}-\frac{}{8a^2 t}}{d}$
risch	$\frac{2a^2e^{4i(dx+c)}-3iab\,e^{5i(dx+c)}+12iab\,e^{3i(dx+c)}+2a^2e^{2i(dx+c)}-9iab\,e^{i(dx+c)}+6b^2e^{4i(dx+c)}-12b^2e^{2i(dx+c)}+6b^2}{\left(e^{2i(dx+c)}-1\right)^2\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)a^3d}+\frac{\ln(e^{i(c}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^3*(1/2*a*tan(1/2*d*x+1/2*c)^2-4*b*tan(1/2*d*x+1/2*c))+4*b/a^4*((
1/2*b^2*tan(1/2*d*x+1/2*c)+1/2*a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x
+1/2*c)+a)+1/2*(2*a^2-3*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2)))-1/8/a^2/tan(1/2*d*x+1/2*c)^2+1/4*(-2*a^2+12*b^2
)/a^4*ln(tan(1/2*d*x+1/2*c))+b/a^3/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(148) = 296$.
time = 0.54, size = 1130, normalized size = 7.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c
)*sin(d*x + c) - 2*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*cos(d*x + c)^2
+ (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sq
rt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2)
)/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^5 - 7*a^3*b
^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2
+ 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^5
- 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b
 - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - a^5*b^2)*d*cos(d*x + c)^2 - (a^
7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(d*x + c)^2 - (a^6*b - a^4*b^3)*d)
*sin(d*x + c)), 1/4*(12*(a^3*b^2 - a*b^4)*cos(d*x + c)^3 - 6*(a^4*b - a^2*b
^3)*cos(d*x + c)*sin(d*x + c) + 4*(2*a^3*b - 3*a*b^3 - (2*a^3*b - 3*a*b^3)*
cos(d*x + c)^2 + (2*a^2*b^2 - 3*b^4 - (2*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) + 2*(a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c) - (a^5 - 7*a^3*
b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2
*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(1/2*cos(d*x + c) + 1/2) + (a^5 - 7*a^3*b^2 + 6*a*b^4 - (a^5 - 7*a^3*b^2 +
 6*a*b^4)*cos(d*x + c)^2 + (a^4*b - 7*a^2*b^3 + 6*b^5 - (a^4*b - 7*a^2*b^3
+ 6*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7
- a^5*b^2)*d*cos(d*x + c)^2 - (a^7 - a^5*b^2)*d + ((a^6*b - a^4*b^3)*d*cos(
d*x + c)^2 - (a^6*b - a^4*b^3)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x))**2, x)

Giac [A]
time = 0.47, size = 257, normalized size = 1.64

$$-\frac{\frac{4\,(a^2-6\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{16\,(2\,a^2b-3\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{16\,\left(b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^4}-\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*(a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 16*(2*a^2*b - 3*
b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c
)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - 16*(b^3*tan(1/2*d*x + 1/2*c) + a*b^
2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4) - (6*a^2
*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x
 + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 9.89, size = 966, normalized size = 6.15

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{2}-16\,b^2\right)+\frac{a^2}{2}-\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(a^2b+2\,b^3\right)}{a}-3\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+8\,b\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-6\,b^2\right)}{2\,a^4\,d}-\frac{b\,\mathrm{atan}\left(\frac{\frac{b\sqrt{-(a+b)(a-b)}\,(2a^2-3b^2)\left(\frac{5a^6b-12a^4b^3}{a^6}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^6-16a^4b^2+24a^2b^4)}{a^5}+\frac{b\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(2a^2-3b^2)}{a^6-a^4b^2}\right)1i}{a^6-a^4b^2}-\frac{b\sqrt{-(a+b)(a-b)}\,(2a^2-3b^2)\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^6-16a^4b^2+24a^2b^4)}{a^5}-\frac{5a^6b-12a^4b^3}{a^6}+\frac{b\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(2a^2-3b^2)}{a^6-a^4b^2}\right)1i}{a^6-a^4b^2}}{\frac{2(2a^4b-15a^2b^3+18b^5)}{a^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(18b^4-12a^2b^2)}{a^5}+\frac{b\sqrt{-(a+b)(a-b)}\,(2a^2-3b^2)\left(\frac{5a^6b-12a^4b^3}{a^6}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^6-16a^4b^2+24a^2b^4)}{a^5}+\frac{b\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(2a^2-3b^2)}{a^6-a^4b^2}\right)}{a^6-a^4b^2}+\frac{b\sqrt{-(a+b)(a-b)}\,(2a^2-3b^2)\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^6-16a^4b^2+24a^2b^4)}{a^5}-\frac{5a^6b-12a^4b^3}{a^6}+\frac{b\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(2a^2-3b^2)}{a^6-a^4b^2}\right)}{a^6-a^4b^2}}\right)\sqrt{-(a+b)(a-b)}\,(2a^2-3b^2)\,2i}{d\,(a^6-a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 16*b^2) + a
^2/2 - (4*tan(c/2 + (d*x)/2)^3*(a^2*b + 2*b^3))/a - 3*a*b*tan(c/2 + (d*x)/2
))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*ta
n(c/2 + (d*x)/2)^3)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 + (d*x
)/2))*(a^2 - 6*b^2))/(2*a^4*d) - (b*atan(((b*(-(a + b)*(a - b))^(1/2)*(2*a^
2 - 3*b^2)*((5*a^6*b - 12*a^4*b^3)/a^6 - (tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*
b^4 - 16*a^4*b^2))/a^5 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 +
(d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a
^6 - a^4*b^2) - (b*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((tan(c/2 + (d*
x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2))/a^5 - (5*a^6*b - 12*a^4*b^3)/a^6 + (
b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^
2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((2*(2*a^4*
b + 18*b^5 - 15*a^2*b^3))/a^6 + (2*tan(c/2 + (d*x)/2)*(18*b^4 - 12*a^2*b^2)
)/a^5 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((5*a^6*b - 12*a^4*b^3)
/a^6 - (tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2))/a^5 + (b*(-(a +
 b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)
*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) + (b*(-(a + b)*(a - b))
^(1/2)*(2*a^2 - 3*b^2)*((tan(c/2 + (d*x)/2)*(a^6 + 24*a^2*b^4 - 16*a^4*b^2)
```

```
)/a^5 - (5*a^6*b - 12*a^4*b^3)/a^6 + (b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(2*a^2 - 3*b^2))/(a^6 - a^4*
b^2)))/(a^6 - a^4*b^2)))*(-(a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*2i)/(d*(a
^6 - a^4*b^2))
```

3.1084 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$-\frac{2b^2(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d}-\frac{b(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(a^2-12b^2)\cot(c+dx)}{3a^4d}+\frac{2b\cot(}{}$$

```
[Out] -b*(a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(a^2-12*b^2)*cot(d*x+c)/a^4/d+
2*b*cot(d*x+c)*csc(d*x+c)/a^3/d-4/3*cot(d*x+c)*csc(d*x+c)^2/a^2/d+cot(d*x+c
)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))-2*b^2*(3*a^2-4*b^2)*arctan((b+a*tan(1/2
*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{2b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d}-\frac{2b^2(3a^2-4b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{b(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(a^2-12b^2)\cot(c+dx)}{3a^4d}+\frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*Sqrt[a^2 - b^2]*d) - (b*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) +
((a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) + (2*b*Cot[c + d*x]*Csc[c + d*x])/(
a^3*d) - (4*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) + (Cot[c + d*x]*Csc[c +
d*x]^2)/(a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
```

```
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx \\
&= \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^4(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= -\frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)(-12b(\cdots}{\cdots}}{\cdots} \\
&= \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(c+dx)}{3a^2d} + \frac{\cot(c+dx)\csc^2\cdots}{ad(a+b\sin(c\cdots} \\
&= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(\cdots}{3a^2d} \\
&= \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+dx)\csc(c+dx)}{a^3d} - \frac{4\cot(c+dx)\csc^2(\cdots}{3a^2d} \\
&= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+\cdots}{\cdots} \\
&= -\frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(a^2-12b^2\right)\cot(c+dx)}{3a^4d} + \frac{2b\cot(c+\cdots}{\cdots} \\
&= -\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b\left(a^2-4b^2\right)\tanh^{-1}(\cos(c+dx\cdots}{a^5d}
\end{aligned}
$$

Mathematica [A]

time = 6.30, size = 385, normalized size = 1.99

$$
-\frac{2b^2\left(3a^2-4b^2\right)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{\left(-a^2b+4b^3\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{\left(a^2b-4b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} - \frac{b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

[Out] (-2*b^2*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d) + ((-(a^2*b) + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((a^2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (Sec[(c + d*x)/2]*(-(a^2*Sin[(c + d*x)/2]) + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*d) - (b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)

Maple [A]

time = 0.59, size = 264, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{2b^2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(3a^2\cdots}{\cdots}\right)}{a^5}\cdots}{\cdots}$
default	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{2b^2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(3a^2\cdots}{\cdots}\right)}{a^5}\cdots}{\cdots}$
risch	$-\frac{2\left(6a^3e^{5i(dx+c)}+12ib^3e^{6i(dx+c)}-7ia^2b\,e^{2i(dx+c)}-30a\,b^2e^{5i(dx+c)}+36ib^3e^{2i(dx+c)}-36ib^3e^{4i(dx+c)}+6a\,b^2e^{7i(dx+c)}+\cdots\right.}{3\left(e^{2i(dx+c)}-1\right)^3\left(b\,e^{2i(dx+c)}-b+2ia\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8/a^4*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-2*a*b*tan(1/2*d*x+1/2*c)^2-a^2*tan(1/2*d*x+1/2*c)+12*b^2*tan(1/2*d*x+1/2*c))-2*b^2/a^5*((b^2*tan(1/2*d*x+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(3*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/24/a^2/tan(1/2*d*x+1/2*c)^3-1/8*(-a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)+1/4/a^3*b/tan(1/2*d*x+1/2*c)^2+1/a^5*b*(a^2-4*b^2)*ln(tan(1/2*d*x+1/2*c)))

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(184) = 368$.
time = 0.54, size = 1471, normalized size = 7.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(2*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 3*(3*a^2*b^3 - 4*b^
5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)
^2 + (3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 12*(a^4*b^
2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a
^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x +
c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x
 + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*b^4
 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^
2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a
^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^
3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x + c)^
4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 - a^
6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c)), -1/6*(2*(a^6 -
7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^2*b^3 - 4*b^5 + (3*a^2*b^3 -
 4*b^5)*cos(d*x + c)^4 - 2*(3*a^2*b^3 - 4*b^5)*cos(d*x + c)^2 + (3*a^3*b^2
- 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 -
b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 12*(a^4
*b^2 - a^2*b^4)*cos(d*x + c) + 3*(a^4*b^2 - 5*a^2*b^4 + 4*b^6 + (a^4*b^2 -
5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x
 + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^4*b^2 - 5*a^2*
b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - 2*(a^4*b^2 - 5
*a^2*b^4 + 4*b^6)*cos(d*x + c)^2 + (a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b -
5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
```

```
1/2) + 2*((a^5*b - 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^7*b - a^5*b^3)*d*cos(d*x +
c)^4 - 2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 + (a^7*b - a^5*b^3)*d - ((a^8 -
 a^6*b^2)*d*cos(d*x + c)^2 - (a^8 - a^6*b^2)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 329, normalized size = 1.70

$$\frac{\frac{24\,(a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{48\,(3\,a^2b^2-4\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{48\,(b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab^3)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^5}-\frac{44\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-176\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 48*(3*a^2*b^2
 - 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/2*d*x +
1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^4*tan(1/2*d*x + 1/2*c) + 36
*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(b^4*tan(1/2*d*x + 1/2*c) + a*b^3)/
((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (44*a^2*b
*tan(1/2*d*x + 1/2*c)^3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*
x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*
c) + a^3)/(a^5*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 9.99, size = 1089, normalized size = 5.64

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(8\,a\,b^2-\frac{2\,a^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(4\,a^2\,b-40\,b^3\right)+\frac{a^3}{3}-\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-a^4+12\,a^2\,b^2+16\,b^4\right)}{a}}{d\left(8\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+16\,b\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{1}{8\,a^2}-\frac{16\,a^2+32\,b^2}{64\,a^4}+\frac{2\,b^2}{a^4}\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4\,a^3\,d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-4\,b^2\right)}{a^5\,d}-[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)^2*(8*a*b^2 - (2*a^3)/
3) - tan(c/2 + (d*x)/2)^3*(4*a^2*b - 40*b^3) + a^3/3 - (4*a^2*b*tan(c/2 + (
d*x)/2))/3 + (tan(c/2 + (d*x)/2)^4*(16*b^4 - a^4 + 12*a^2*b^2))/a)/(d*(8*a^
```

5*tan(c/2 + (d*x)/2)^3 + 8*a^5*tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + (tan(c/2 + (d*x)/2)*(1/(8*a^2) - (16*a^2 + 32*b^2)/(64*a^4) + (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 + (d*x)/2))*(a^2 - 4*b^2))/(a^5*d) - (b^2*atan(((b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2) + (b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 - (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2))*1i)/(a^7 - a^5*b^2))/((4*(16*b^7 - 16*a^2*b^5 + 3*a^4*b^3))/a^8 + (4*tan(c/2 + (d*x)/2)*(16*b^6 - 12*a^2*b^4))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 + (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2) - (b^2*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((2*(8*a^5*b^4 - 4*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(a^7*b + 16*a^3*b^5 - 12*a^5*b^3))/a^7 - (b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(a^7 - a^5*b^2)))/(a^7 - a^5*b^2)))*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*2i)/(d*(a^7 - a^5*b^2))

3.1085 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=266

$$-\frac{(12a^2-b^2)\,x}{2b^5}+\frac{a(12a^4-19a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\,(a^2-b^2)^{3/2}\,d}-\frac{a(12a^2-11b^2)\cos(c+dx)}{2b^4\,(a^2-b^2)\,d}+\frac{(6a^2-5b^2)}{2}$$

```
[Out] -1/2*(12*a^2-b^2)*x/b^5+a*(12*a^4-19*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x
+1/2*c))/(a^2-b^2)^(1/2))/b^5/(a^2-b^2)^(3/2)/d-1/2*a*(12*a^2-11*b^2)*cos(d
*x+c)/b^4/(a^2-b^2)/d+1/2*(6*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)/b^3/(a^2-b^2)
/d-1/2*cos(d*x+c)*sin(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2-1/2*(4*a^2-3*b^2)*cos
(d*x+c)*sin(d*x+c)^2/b^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.56, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3127, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{(4a^2-3b^2)\sin^2(c+dx)\cos(c+dx)}{2b^2d\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{x(12a^2-b^2)}{2b^5}-\frac{a(12a^2-11b^2)\cos(c+dx)}{2b^4d\,(a^2-b^2)}+\frac{(6a^2-5b^2)\sin(c+dx)\cos(c+dx)}{2b^3d\,(a^2-b^2)}+\frac{a(12a^4-19a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\,(a^2-b^2)^{3/2}}-\frac{\sin^3(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*((12*a^2 - b^2)*x)/b^5 + (a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*(a^2 - b^2)^(3/2)*d) - (a*(12*a^
2 - 11*b^2)*Cos[c + d*x])/(2*b^4*(a^2 - b^2)*d) + ((6*a^2 - 5*b^2)*Cos[c +
d*x]*Sin[c + d*x])/(2*b^3*(a^2 - b^2)*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(2
*b*d*(a + b*Sin[c + d*x])^2) - ((4*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2
)/(2*b^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
```

```
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin^2(c+dx)\left(-3\left(a^2-b^2\right)+4\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\left(4a^2-3b^2\right)\cos(c+dx)\sin^2(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \ldots$$

$$= \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \frac{\cos(c+dx)\sin^3(c+dx)}{2bd(a+b\sin(c+dx))^2} - \ldots$$

$$= -\frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \frac{\left(6a^2-5b^2\right)\cos(c+dx)\sin(c+dx)}{2b^3\left(a^2-b^2\right)d} - \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} - \frac{a\left(12a^2-11b^2\right)\cos(c+dx)}{2b^4\left(a^2-b^2\right)d} + \ldots$$

$$= -\frac{\left(12a^2-b^2\right)x}{2b^5} + \frac{a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\left(a^2-b^2\right)^{3/2}d} - \ldots$$

Mathematica [A]

time = 4.19, size = 288, normalized size = 1.08

$$\frac{\frac{4a\left(12a^4-19a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-4ab\left(12a^4-13a^2b^2+b^4\right)(c+dx)\sin(c+dx)-2b^2\left(12a^4-13a^2b^2+b^4\right)(c+dx)\sin^2(c+dx)+\cos(c+dx)\left(-24a^5b+22a^3b^3-8ab^3\left(a^2-b^2\right)\sin^2(c+dx)+2b^4\left(a^2-b^2\right)\sin^3(c+dx)\right)-a^2\left(2\left(12a^4-13a^2b^2+b^4\right)(c+dx)+\left(18a^2b^2-17b^4\right)\sin(2(c+dx))\right)}{(a+b\sin(c+dx))^2}}{4(a-b)b^5(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((4*a*(12*a^4 - 19*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] + (-4*a*b*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*
Sin[c + d*x] - 2*b^2*(12*a^4 - 13*a^2*b^2 + b^4)*(c + d*x)*Sin[c + d*x]^2 +
 Cos[c + d*x]*(-24*a^5*b + 22*a^3*b^3 - 8*a*b^3*(a^2 - b^2)*Sin[c + d*x]^2
+ 2*b^4*(a^2 - b^2)*Sin[c + d*x]^3) - a^2*(2*(12*a^4 - 13*a^2*b^2 + b^4)*(c
 + d*x) + (18*a^2*b^2 - 17*b^4)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/
(4*(a - b)*b^5*(a + b)*d)
```

Maple [A]
time = 0.72, size = 349, normalized size = 1.31

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(12a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^5}+\frac{2a\left(-\frac{ab^2\left(5a^2-4b^2\right)(\mathrm{t}}{2\left(a^2-\right.}\right.}{}$
default	$-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(12a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^5}+\frac{2a\left(-\frac{ab^2\left(5a^2-4b^2\right)(\mathrm{t}}{2\left(a^2-\right.}\right.}{}$
risch	$-\frac{6xa^2}{b^5}+\frac{x}{2b^3}-\frac{i\mathrm{e}^{2i(dx+c)}}{8b^3d}-\frac{3a\,\mathrm{e}^{i(dx+c)}}{2b^4d}-\frac{3a\,\mathrm{e}^{-i(dx+c)}}{2b^4d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^3d}+\frac{ia^2\left(-8ia^3b\,\mathrm{e}^{3i(dx+c)}+7ia\,b^3\mathrm{e}^{3i(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^5*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+3*a*b*tan(1/2*d*x+1/2*c)^2-1/2*b
^2*tan(1/2*d*x+1/2*c)+3*a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(12*a^2-b^2)*ar
ctan(tan(1/2*d*x+1/2*c)))+2*a/b^5*((-1/2*a*b^2*(5*a^2-4*b^2)/(a^2-b^2)*tan(
1/2*d*x+1/2*c)^3-1/2*b*(6*a^4+7*a^2*b^2-10*b^4)/(a^2-b^2)*tan(1/2*d*x+1/2*c
)^2-1/2*a*b^2*(19*a^2-16*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a^2*b*(6*a^2
-5*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*
(12*a^4-19*a^2*b^2+6*b^4)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

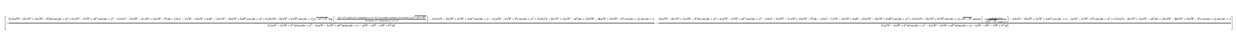
Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.45, size = 1058, normalized size = 3.98



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 +
8*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - 2*(12*a^8 - 13*a^6*b^2 - 1
1*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x + (12*a^7 - 7*a^5*b^2 - 13*a^3*b^4 + 6*a*
b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2*(12*a^6*b - 19
*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^7*b - 19*a^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d
*x + c) - 2*((a^4*b^4 - 2*a^2*b^6 + b^8)*cos(d*x + c)^3 + 2*(12*a^7*b - 25*
a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 -
 b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^7 - 2*a^2*b^9 + b^11)*d*cos(d*x +
 c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d*sin(d*x + c) - (a^6*b^5 - a^4*b^
7 - a^2*b^9 + b^11)*d), -1/2*((12*a^6*b^2 - 25*a^4*b^4 + 14*a^2*b^6 - b^8)*
d*x*cos(d*x + c)^2 + 4*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c)^3 - (12*a
^8 - 13*a^6*b^2 - 11*a^4*b^4 + 13*a^2*b^6 - b^8)*d*x - (12*a^7 - 7*a^5*b^2
- 13*a^3*b^4 + 6*a*b^6 - (12*a^5*b^2 - 19*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2
 + 2*(12*a^6*b - 19*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^7*b - 19*a
^5*b^3 + 3*a^3*b^5 + 4*a*b^7)*cos(d*x + c) - ((a^4*b^4 - 2*a^2*b^6 + b^8)*c
os(d*x + c)^3 + 2*(12*a^7*b - 25*a^5*b^3 + 14*a^3*b^5 - a*b^7)*d*x + (18*a^
6*b^2 - 36*a^4*b^4 + 19*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^4*b^
7 - 2*a^2*b^9 + b^11)*d*cos(d*x + c)^2 - 2*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*d
*sin(d*x + c) - (a^6*b^5 - a^4*b^7 - a^2*b^9 + b^11)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(251) = 502$.
time = 0.53, size = 535, normalized size = 2.01

$$\frac{\frac{2\left(12a^5-19a^3b^2+6ab^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^5-b^7\right)\sqrt{a^2-b^2}}-\frac{2\left(6a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-5a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-14ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+54a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-45a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-4b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+15a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-44ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+90a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-87a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-30ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+42a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-39a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12a^5-11a^3b^2\right)}{\left(a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}-\frac{\left(12a^2-b^2\right)(dx+c)}{b^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(12*a^5 - 19*a^3*b^2 + 6*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^5 - b^
7)*sqrt(a^2 - b^2)) - 2*(6*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 5*a^2*b^3*tan(1/2
*d*x + 1/2*c)^7 + 12*a^5*tan(1/2*d*x + 1/2*c)^6 + 5*a^3*b^2*tan(1/2*d*x + 1
/2*c)^6 - 14*a*b^4*tan(1/2*d*x + 1/2*c)^6 + 54*a^4*b*tan(1/2*d*x + 1/2*c)^5
 - 45*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^
5*tan(1/2*d*x + 1/2*c)^4 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 44*a*b^4*tan
(1/2*d*x + 1/2*c)^4 + 90*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 87*a^2*b^3*tan(1/2*
d*x + 1/2*c)^3 + 4*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a^5*tan(1/2*d*x + 1/2*c)
^2 - a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 30*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 42*
a^4*b*tan(1/2*d*x + 1/2*c) - 39*a^2*b^3*tan(1/2*d*x + 1/2*c) + 12*a^5 - 11*
a^3*b^2)/((a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2
*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (12
*a^2 - b^2)*(d*x + c)/b^5)/d
```

Mupad [B]
time = 17.72, size = 2500, normalized size = 9.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((12*a^5 - 11*a^3*b^2)/(b^4*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^7*(6*a^4 -
 5*a^2*b^2))/(b^3*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^5*(4*b^4 - 54*a^4 + 45
*a^2*b^2))/(b^3*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^3*(90*a^4 + 4*b^4 - 87*a
^2*b^2))/(b^3*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^6*(12*a^5 - 14*a*b^4 + 5*a
^3*b^2))/(b^4*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^2*(30*a*b^4 - 36*a^5 + a^3
*b^2))/(b^4*(a^2 - b^2)) + (3*tan(c/2 + (d*x)/2)*(14*a^4 - 13*a^2*b^2))/(b^
3*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^4*(11*a*b^2 - 12*a^3)*(3*a^2 + 4*b^2))
/(b^4*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/2 + (d
*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(c
/2 + (d*x)/2)^8 + a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x
```

)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan(((
(a^2*12i - b^2*1i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^
6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) - ((a^2*12i - b^2*1i)*((4
*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 +
a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^
12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - ((a^2*12i - b^2*1i)*((4
*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) +
(8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))
/(b^16 - 2*a^2*b^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)
*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288
*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12))*1i)/(2*b^5) + ((a^2*12i - b^2*1
i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4
))/(b^15 - 2*a^2*b^13 + a^4*b^11) + ((a^2*12i - b^2*1i)*((4*(4*a*b^16 - 36*
a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*
tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10
))/(b^16 - 2*a^2*b^14 + a^4*b^12) + ((a^2*12i - b^2*1i)*((4*(8*a^2*b^18 - 1
6*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*
x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b
^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*
a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^1
6 - 2*a^2*b^14 + a^4*b^12))*1i)/(2*b^5))/((8*(864*a^11 + 30*a^3*b^8 - 491*a
^5*b^6 + 1746*a^7*b^4 - 2160*a^9*b^2))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (16
*tan(c/2 + (d*x)/2)*(1728*a^12 - 6*a^2*b^10 + 169*a^4*b^8 - 1495*a^6*b^6 +
4356*a^8*b^4 - 4752*a^10*b^2))/(b^16 - 2*a^2*b^14 + a^4*b^12) + ((a^2*12i -
b^2*1i)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^
10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) - ((a^2*12i - b^2*1i)*((4*(4*a*b^16
- 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11)
+ (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^
8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - ((a^2*12i - b^2*1i)*((4*(8*a^2*b^
18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2
+ (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2
*a^2*b^14 + a^4*b^12)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14
- 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4)
)/(b^16 - 2*a^2*b^14 + a^4*b^12)))/(2*b^5) - ((a^2*12i - b^2*1i)*((4*(2*a^2
*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*
a^2*b^13 + a^4*b^11) + ((a^2*12i - b^2*1i)*((4*(4*a*b^16 - 36*a^3*b^14 + 56
*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*
x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*
a^2*b^14 + a^4*b^12) + ((a^2*12i - b^2*1i)*((4*(8*a^2*b^18 - 16*a^4*b^16 +
8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b
^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^1
2)))/(2*b^5)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 64
0*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^1
4 + a^4*b^12)))/(2*b^5)))*(a^2*12i - b^2*1i)*1i)/(b^5*d) - (a*atan(((a*(-(a
+ b)^3*(a - b)^3)^(1/2)*((4*(2*a^2*b^12 - 52*a^4*b^10 + 386*a^6*b^8 - 624*

a^8*b^6 + 288*a^10*b^4))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(2*a*b^14 - 89*a^3*b^12 + 640*a^5*b^10 - 1322*a^7*b^8 + 1056*a^9*b^6 - 288*a^11*b^4))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(4*a*b^16 - 36*a^3*b^14 + 56*a^5*b^12 - 24*a^7*b^10))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(24*a^2*b^16 - 100*a^4*b^14 + 124*a^6*b^12 - 48*a^8*b^10))/(b^16 - 2*a^2*b^14 + a^4*b^12) - (a*(-(a + b)^3*(a - b)^3)^(1/2)*((4*(8*a^2*b^18 - 16*a^4*b^16 + 8*a^6*b^14))/(b^15 - 2*a^2*b^13 + a^4*b^11) + (8*tan(c/2 + (d*x)/2)*(12*a*b^20 - 32*a^3*b^18 + 28*a^5*b^16 - 8*a^7*b^14))/(b^16 - 2*a^2*b^14 + a^4*b^12))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2))/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b^7 - a^6*b^5)))*(12*a^4 + 6*b^4 - 19*a^2*b^2)*1i)/(2*(b^11 - 3*a^2*b^9 + 3*a^4*b...

3.1086 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{3ax}{b^4}-\frac{\left(6a^4-9a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}d}+\frac{3\cos(c+dx)}{2b^3d}-\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{a(3a^2-2}{2b^3\left(a^2-b^2\right)}$$

[Out] 3*a*x/b^4-(6*a^4-9*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^4/(a^2-b^2)^(3/2)/d+3/2*cos(d*x+c)/b^3/d-1/2*cos(d*x+c)*sin(d*x+c)^2/b/d/(a+b*sin(d*x+c))^2+1/2*a*(3*a^2-2*b^2)*cos(d*x+c)/b^3/(a^2-b^2)/d/(a+b*sin(d*x+c))

Rubi [A]
time = 0.37, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3127, 3111, 3102, 2814, 2739, 632, 210}

$$\frac{a(3a^2-2b^2)\cos(c+dx)}{2b^3d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{\left(6a^4-9a^2b^2+2b^4\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d\left(a^2-b^2\right)^{3/2}}+\frac{3ax}{b^4}-\frac{\sin^2(c+dx)\cos(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{3\cos(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]

[Out] (3*a*x)/b^4 - ((6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*(a^2 - b^2)^(3/2)*d) + (3*Cos[c + d*x])/(2*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^2)/(2*b*d*(a + b*Sin[c + d*x])^2) + (a*(3*a^2 - 2*b^2)*Cos[c + d*x])/(2*b^3*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3111

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[
(-(b*c - a*d))*(A*b^2 + a^2*C)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(
b^2*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a
+ b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*(a*C*(b*c - a*d) + A*b*(a*c - b*d)
) - ((b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*Sin[e + f*x] + b*
C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} - \frac{\int \frac{\sin(c+dx)\left(-2(a^2-b^2)+3(a^2-b^2)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2b\left(a^2-b^2\right)}$$

$$= -\frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a(3a^2-2b^2)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{b(3a^4-\cdots}{\cdots}}{\cdots}$$

$$= \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a(3a^2-2b^2)\cos(c+dx)}{2b^3\left(a^2-b^2\right)d(a+b\sin(c+\cdots}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a(3a^2-2b^2)\cos(\cdots}{2b^3\left(a^2-b^2\right)d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a(3a^2-2b^2)\cos(\cdots}{2b^3\left(a^2-b^2\right)d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{3ax}{b^4} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\cos(c+dx)\sin^2(c+dx)}{2bd(a+b\sin(c+dx))^2} + \frac{a(3a^2-2b^2)\cos(\cdots}{2b^3\left(a^2-b^2\right)d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{3ax}{b^4} - \frac{(6a^4-9a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\left(a^2-b^2\right)^{3/2}d} + \frac{3\cos(c+dx)}{2b^3d} - \frac{\mathrm{co}\cdots}{2\cdots}$$

Mathematica [A]
time = 0.86, size = 159, normalized size = 0.88

$$\frac{6a(c+dx) - \frac{2(6a^4-9a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} + 2b\cos(c+dx) - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + \frac{ab(5a^2-4b^2)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))}}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (6*a*(c + d*x) - (2*(6*a^4 - 9*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 2*b*Cos[c + d*x] - (a^2*b*Cos[c
+ d*x])/(a + b*Sin[c + d*x])^2 + (a*b*(5*a^2 - 4*b^2)*Cos[c + d*x])/((a - b
)*(a + b)*(a + b*Sin[c + d*x])))/(2*b^4*d)
```

Maple [A]
time = 0.64, size = 289, normalized size = 1.61

method	result
derivativedivides	$\frac{\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+6a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2\left(\frac{-\frac{a\,b^2\left(3a^2-2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2-b^2\right)}-\frac{b\left(4a^4+5a^2b^2-6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2-b^2\right)}-\frac{a\,b^2\left(13a\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{}}{d}$
default	$\frac{\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+6a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{2\left(\frac{-\frac{a\,b^2\left(3a^2-2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2-b^2\right)}-\frac{b\left(4a^4+5a^2b^2-6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2-b^2\right)}-\frac{a\,b^2\left(13a\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{}}{d}$
risch	$\frac{3ax}{b^4}+\frac{e^{i(dx+c)}}{2b^3d}+\frac{e^{-i(dx+c)}}{2b^3d}-\frac{ia\left(-6ia^3b\,e^{3i(dx+c)}+5ia\,b^3e^{3i(dx+c)}+14ia^3b\,e^{i(dx+c)}-11ia\,b^3e^{i(dx+c)}+10a^4e^{2i(dx+\right.}{\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)^2\left(a^2-b^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^4*(b/(1+tan(1/2*d*x+1/2*c)^2)+3*a*arctan(tan(1/2*d*x+1/2*c)))-2/b^
4*((-1/2*a*b^2*(3*a^2-2*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-1/2*b*(4*a^4+5*
a^2*b^2-6*b^4)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*(13*a^2-10*b^2)/(a^
2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a^2*b*(4*a^2-3*b^2)/(a^2-b^2))/(a*tan(1/2*d*x
+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(6*a^4-9*a^2*b^2+2*b^4)/(a^2-b^2)
^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(169) = 338.
time = 0.42, size = 919, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(12*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*x*cos(d*x + c)^2 + 4*(a^4*b^3 - 2*
a^2*b^5 + b^7)*cos(d*x + c)^3 - 12*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x -
(6*a^6 - 3*a^4*b^2 - 7*a^2*b^4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*co
s(d*x + c)^2 + 2*(6*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 +
b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(6*a^6*b - 9*a^4*b^3 + a^
2*b^5 + 2*b^7)*cos(d*x + c) - 2*(12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*
a^5*b^2 - 17*a^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a
^2*b^8 + b^10)*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x
 + c) - (a^6*b^4 - a^4*b^6 - a^2*b^8 + b^10)*d), 1/2*(6*(a^5*b^2 - 2*a^3*b^
4 + a*b^6)*d*x*cos(d*x + c)^2 + 2*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^
3 - 6*(a^7 - a^5*b^2 - a^3*b^4 + a*b^6)*d*x - (6*a^6 - 3*a^4*b^2 - 7*a^2*b^
4 + 2*b^6 - (6*a^4*b^2 - 9*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(6*a^5*b - 9
*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (6*a^6*b - 9*a^4*b^3 + a^2*b^5 + 2*b^
7)*cos(d*x + c) - (12*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*x + (9*a^5*b^2 - 17*a
^3*b^4 + 8*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^6 - 2*a^2*b^8 + b^10)
*d*cos(d*x + c)^2 - 2*(a^5*b^5 - 2*a^3*b^7 + a*b^9)*d*sin(d*x + c) - (a^6*b
^4 - a^4*b^6 - a^2*b^8 + b^10)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 302, normalized size = 1.68

$$-\frac{\frac{(6a^4-9a^2b^2+2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+13a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-10ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a^4-3a^2b^2}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}-\frac{3(dx+c)a}{b^4}-\frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4 - 9*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a
```

```
^2 - b^2)) - (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)
^3 + 4*a^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^
4*tan(1/2*d*x + 1/2*c)^2 + 13*a^3*b*tan(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2
*d*x + 1/2*c) + 4*a^4 - 3*a^2*b^2)/((a^2*b^3 - b^5)*(a*tan(1/2*d*x + 1/2*c)
^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - 3*(d*x + c)*a/b^4 - 2/((tan(1/2*d*x
 + 1/2*c)^2 + 1)*b^3))/d
```

Mupad [B]
time = 15.79, size = 3031, normalized size = 16.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((6*a^4 - 5*a^2*b^2)/(b^3*(a^2 - b^2)) + (3*tan(c/2 + (d*x)/2)^4*(2*a^4 - 2
*b^4 + a^2*b^2))/(b^3*(a^2 - b^2)) + (2*tan(c/2 + (d*x)/2)^2*(6*a^4 - 7*b^4
 + 3*a^2*b^2))/(b^3*(a^2 - b^2)) - (3*tan(c/2 + (d*x)/2)*(6*a*b^2 - 7*a^3))
/(b^2*(a^2 - b^2)) - (tan(c/2 + (d*x)/2)^5*(2*a*b^2 - 3*a^3))/(b^2*(a^2 - b
^2)) - (4*tan(c/2 + (d*x)/2)^3*(5*a*b^2 - 6*a^3))/(b^2*(a^2 - b^2)))/(d*(ta
n(c/2 + (d*x)/2)^2*(3*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2) +
 a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) + (6*a*atan((192*a^2*b^7*tan(c/
2 + (d*x)/2))/((192*a^2*b^19)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (384*a^4*b^17
)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (48*a^6*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^
8) + (288*a^8*b^13)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (144*a^10*b^11)/(b^12 -
 2*a^2*b^10 + a^4*b^8)) - (144*a^6*b^3*tan(c/2 + (d*x)/2))/((192*a^2*b^19)/
(b^12 - 2*a^2*b^10 + a^4*b^8) - (384*a^4*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8
) + (48*a^6*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (288*a^8*b^13)/(b^12 - 2*
a^2*b^10 + a^4*b^8) - (144*a^10*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8))))/(b^4
*d) + (atan((((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((
8*(36*a^4*b^7 - 72*a^6*b^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (
8*tan(c/2 + (d*x)/2)*(4*a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 +
72*a^9*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*
(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^1
2 + 60*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (8*(8*a^2*b^
12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + ((-(a + b)^3
*(a - b)^3)^(1/2)*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2
*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*
b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^
2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4
*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((-(a + b)^
3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36*a^4*b^7 - 72*a^6*b
^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(4*
a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^9*b^3))/(b^13 - 2*a
```

^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(8*a^2*b^12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 60*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))*1i)/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))/((16*(54*a^8 - 12*a^2*b^6 + 72*a^4*b^4 - 117*a^6*b^2))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (16*tan(c/2 + (d*x)/2)*(216*a^9 - 72*a^3*b^6 + 396*a^5*b^4 - 540*a^7*b^2))/(b^13 - 2*a^2*b^11 + a^4*b^9) - ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36*a^4*b^7 - 72*a^6*b^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(4*a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^9*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 60*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (8*(8*a^2*b^12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) + ((-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(36*a^4*b^7 - 72*a^6*b^5 + 36*a^8*b^3))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(4*a*b^11 - 108*a^3*b^9 + 285*a^5*b^7 - 252*a^7*b^5 + 72*a^9*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*((8*(8*a^2*b^12 - 14*a^4*b^10 + 6*a^6*b^8))/(b^12 - 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(8*a*b^14 - 44*a^3*b^12 + 60*a^5*b^10 - 24*a^7*b^8))/(b^13 - 2*a^2*b^11 + a^4*b^9) + ((-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11))/(b^12 - 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 - 32*a^3*b^15 + 28*a^5*b^13 - 8*a^7*b^11))/(b^13 - 2*a^2*b^11 + a^4*b^9))*(3*a^4 + b^4 - (9*a^2*b^2)/2))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))/(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4)))*(-(a + b)^3*(a - b)^3)^(1/2)*(3*a^4 + b^4 - (9*a^2*b^2)/2)*2i)/(d*(b^10 - 3*a^2*b^8 + 3*a^4*b^6 - a^6*b^4))

3.1087 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=167

$$-\frac{x}{b^3}+\frac{a(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3\left(a^2-b^2\right)^{3/2}d}-\frac{a\cos^3(c+dx)}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2(a^2-b^2)+ab\,\mathrm{s}\right.}{2b^2\left(a^2-b^2\right)d(a+b\sin(c}$$

```
[Out] -x/b^3+a*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3
/(a^2-b^2)^(3/2)/d-1/2*a*cos(d*x+c)^3/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*co
s(d*x+c)*(2*a^2-2*b^2+a*b*sin(d*x+c))/b^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2943, 2942, 2814, 2739, 632, 210}

$$\frac{a(2a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d\left(a^2-b^2\right)^{3/2}}-\frac{a\cos^3(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)\left(2(a^2-b^2)+ab\sin(c+dx)\right)}{2b^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(x/b^3) + (a*(2*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(b^3*(a^2 - b^2)^(3/2)*d) - (a*Cos[c + d*x]^3)/(2*(a^2 - b^2)*d*(a + b
*Sin[c + d*x])^2) - (Cos[c + d*x]*(2*(a^2 - b^2) + a*b*Sin[c + d*x]))/(2*b^
2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)(2b+a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= -\frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2(a^2-b^2)+ab\sin(c+dx\right.}{2b^2(a^2-b^2)\,d(a+b\sin(c+dx))}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2(a^2-b^2)+ab\sin(\right.}{2b^2(a^2-b^2)\,d(a+b\sin(c+\right.}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2(a^2-b^2)+ab\sin(\right.}{2b^2(a^2-b^2)\,d(a+b\sin(c+\right.}$$

$$= -\frac{x}{b^3} - \frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\cos(c+dx)\left(2(a^2-b^2)+ab\sin(\right.}{2b^2(a^2-b^2)\,d(a+b\sin(c+\right.}$$

$$= -\frac{x}{b^3} + \frac{a(2a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}\,d} - \frac{a\cos^3(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+d\right.}$$

Mathematica [A]
time = 1.44, size = 289, normalized size = 1.73

$$\frac{\frac{-8(c+dx)+\frac{2a(8a^4-20a^2b^2+15b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{ab(4a^2-3b^2)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))^2}-\frac{3b(4a^4-7a^2b^2+2b^4)\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{b^3}-\frac{\frac{6ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\cos(c+dx)\left(a(2a^2+b^2)+b(a^2+2b^2)\sin(c+dx)\right)}{(a+b\sin(c+dx))^2}}{(a-b)^2(a+b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos[c +
 d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b^2 +
 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/b^3 - ((6
*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (C
os[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a + b*Sin[c
+ d*x])^2)/((a - b)^2*(a + b)^2))/(8*d)
```

Maple [A]
time = 0.53, size = 252, normalized size = 1.51

method	result

derivativedivides	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}+\frac{2\left(-\frac{a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{b(2a^4+3a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^2-b^2)}-\frac{b^2(7a^2-4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2-b^2)}-\frac{ab(2a^{}}{2(a^2}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}}{b^3}}{d}$
default	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}+\frac{2\left(-\frac{a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{b(2a^4+3a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^2-b^2)}-\frac{b^2(7a^2-4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2-b^2)}-\frac{ab(2a^{}}{2(a^2}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}}{b^3}}{d}$
risch	$-\frac{x}{b^3}+\frac{i(-4ia^3b\,e^{3i(dx+c)}+3iab^3e^{3i(dx+c)}+8ia^3b\,e^{i(dx+c)}-5iab^3e^{i(dx+c)}+6a^4e^{2i(dx+c)}-b^2a^2e^{2i(dx+c)}-2b^4e^{2i(dx+}}{(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)})^2(a^2-b^2)d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^3*arctan(tan(1/2*d*x+1/2*c))+2/b^3*((-1/2*a^2*b^2/(a^2-b^2)*tan(1
/2*d*x+1/2*c)^3-1/2*b*(2*a^4+3*a^2*b^2-2*b^4)/a/(a^2-b^2)*tan(1/2*d*x+1/2*c
)^2-1/2*b^2*(7*a^2-4*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a*b*(2*a^2-b^2)/
(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*a*(2*a^2
-3*b^2)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(157) = 314$.
time = 0.42, size = 793, normalized size = 4.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 - 4*(a^6 - a^4*b^2
- a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*c
os(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(
d*x + c) - 2*(4*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 +
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x +
 c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5
 - a^2*b^7 + b^9)*d), -1/2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x*cos(d*x + c)^
2 - 2*(a^6 - a^4*b^2 - a^2*b^4 + b^6)*d*x - (2*a^5 - a^3*b^2 - 3*a*b^4 - (2
*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(2*a^4*b - 3*a^2*b^3)*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*x + (3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
4*b^5 - 2*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)
*d*sin(d*x + c) - (a^6*b^3 - a^4*b^5 - a^2*b^7 + b^9)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 256, normalized size = 1.53

$$\frac{\frac{(2a^3-3ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^3-b^5)\sqrt{a^2-b^2}}-\frac{a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a^4-a^2b^2}{(a^3b^2-ab^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}-\frac{dx+c}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2))
- (a^3*b*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*tan(1/2*d*x
 + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 - a^2*b^2)/((a^3*b^2 - a*b
^4)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) - (d*x + c
)/b^3)/d
```

Mupad [B]
time = 13.93, size = 2709, normalized size = 16.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] (2*atan((64*a*b^5*tan(c/2 + (d*x)/2))/((176*a^3*b^12)/(b^9 - 2*a^2*b^7 + a^
4*b^5) - (160*a^5*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (48*a^7*b^8)/(b^9 - 2
*a^2*b^7 + a^4*b^5) - (64*a*b^14)/(b^9 - 2*a^2*b^7 + a^4*b^5)) - (48*a^3*b^
3*tan(c/2 + (d*x)/2))/((176*a^3*b^12)/(b^9 - 2*a^2*b^7 + a^4*b^5) - (160*a^
5*b^10)/(b^9 - 2*a^2*b^7 + a^4*b^5) + (48*a^7*b^8)/(b^9 - 2*a^2*b^7 + a^4*b
^5) - (64*a*b^14)/(b^9 - 2*a^2*b^7 + a^4*b^5))))/(b^3*d) - ((tan(c/2 + (d*x
)/2)*(7*a^2 - 4*b^2))/(b*(a^2 - b^2)) - (a*b^2 - 2*a^3)/(b^2*(a^2 - b^2)) +
 (a^2*tan(c/2 + (d*x)/2)^3)/(b*(a^2 - b^2)) + (tan(c/2 + (d*x)/2)^2*(a^2 +
2*b^2)*(2*a^2 - b^2))/(a*b^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2
+ 4*b^2) + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*
a*b*tan(c/2 + (d*x)/2))) - (a*atan(((a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^
3)^(1/2)*((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^
5) + (8*tan(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))
/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(
1/2)*((8*(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) +
(8*tan(c/2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2
*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^
2*b^12 - 8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2
+ (d*x)/2)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a
^2*b^8 + a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 -
 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*1i)/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a
^6*b^3)) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^6 -
 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2
)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*
b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^10 - 6*a^
3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12
*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(2*a
^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^12 - 8*a^4*b^10 + 4*a
^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a*b^14 - 3
2*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6)))/(2*(b
^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 -
a^6*b^3)))*1i)/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))/((16*(2*a^5 - 3
*a^3*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (16*tan(c/2 + (d*x)/2)*(8*a^6 + 12
*a^2*b^4 - 20*a^4*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(
-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 -
 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^
5*b^4 - 8*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a
```

+ b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^12 - 8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^6 - 8*a^4*b^4 + 4*a^6*b^2))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(8*a*b^8 - 29*a^3*b^6 + 28*a^5*b^4 - 8*a^7*b^2))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a*b^10 - 6*a^3*b^8 + 2*a^5*b^6))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a^2*b^10 - 20*a^4*b^8 + 8*a^6*b^6))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^12 - 8*a^4*b^10 + 4*a^6*b^8))/(b^9 - 2*a^2*b^7 + a^4*b^5) + (8*tan(c/2 + (d*x)/2)*(12*a*b^14 - 32*a^3*b^12 + 28*a^5*b^10 - 8*a^7*b^8))/(b^10 - 2*a^2*b^8 + a^4*b^6)))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))/(2*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))))*(2*a^2 - 3*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))

3.1088 $\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=154

$$-\frac{b(3a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2}+\frac{(a^2-2b^2)\,\mathrm{co}}{2a^2\left(a^2-b^2\right)d(a+}$$

```
[Out] -b*(3*a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-
b^2)^(3/2)/d-arctanh(cos(d*x+c))/a^3/d+1/2*cos(d*x+c)/a/d/(a+b*sin(d*x+c))^
2+1/2*(a^2-2*b^2)*cos(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.31, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2968, 3135, 3080, 3855, 2739, 632, 210}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(a^2-2b^2)\cos(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{b(3a^2-2b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*(a^2 - b^2)^(3/2)*d)) - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*a*
d*(a + b*Sin[c + d*x])^2) + ((a^2 - 2*b^2)*Cos[c + d*x])/(2*a^2*(a^2 - b^2)
*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(2\right.}{}}{}$$

$$= \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \csc(c+d}{a^3}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c-}{2a^2\left(a^2-b^2\right)d(a+b\sin}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(a^2-2b^2\right)\cos(c-}{2a^2\left(a^2-b^2\right)d(a+b\sin}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos}{2ad(a+}$$

Mathematica [A]
time = 0.82, size = 154, normalized size = 1.00

$$\frac{\frac{2b\left(-3a^2+2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a\cos(c+dx)\left(2a^3-3ab^2+b\left(a^2-2b^2\right)\sin(c+dx)\right)}{(a-b)(a+b)(a+b\sin(c+dx))^2}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*b*(-3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(3/2) - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + (a*Co
s[c + d*x]*(2*a^3 - 3*a*b^2 + b*(a^2 - 2*b^2)*Sin[c + d*x]))/((a - b)*(a +
b)*(a + b*Sin[c + d*x])^2))/(2*a^3*d)
```

Maple [A]
time = 0.65, size = 253, normalized size = 1.64

method	result

derivativedivides	$-\frac{2\left(\frac{-\frac{ab(3a^2-4b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{(2a^4+a^2b^2-6b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{ab(5a^2-8b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2-b^2)}-\frac{a^2(2a^2-3b^2)}{2(a^2-b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{b(3a^2-2b^2}{\quad}\right.}{a^3}}{d}$
default	$-\frac{2\left(\frac{-\frac{ab(3a^2-4b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{(2a^4+a^2b^2-6b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a^2-b^2)}-\frac{ab(5a^2-8b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^2-b^2)}-\frac{a^2(2a^2-3b^2)}{2(a^2-b^2)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{b(3a^2-2b^2}{\quad}\right.}{a^3}}{d}$
risch	$-\frac{i\left(iab^3e^{3i(dx+c)}+4ia^3be^{i(dx+c)}-7iab^3e^{i(dx+c)}+2a^4e^{2i(dx+c)}-3b^2a^2e^{2i(dx+c)}-2b^4e^{2i(dx+c)}-a^2b^2+2b^4\right)}{\left(-ibe^{2i(dx+c)}+ib+2ae^{i(dx+c)}\right)^2a^2(a^2-b^2)db}+\frac{3ib\ln\left(\right.}{\quad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3*((-1/2*a*b*(3*a^2-4*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*
a^4+a^2*b^2-6*b^4)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-1/2*a*b*(5*a^2-8*b^2)/(a^
2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a^2*(2*a^2-3*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1
/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*b*(3*a^2-2*b^2)/(a^2-b^2)^(3/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))+1/a^3*ln(tan(1/2*d*x
+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(145) = 290$.
time = 0.58, size = 996, normalized size = 6.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (3*a^4*b
 + a^2*b^3 - 2*b^5 - (3*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*
a*b^4)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) + 2*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - 2*(a^6 - a^4*b^2 -
a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^6 - a^4*b^
2 - a^2*b^4 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b -
 2*a^3*b^3 + a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 -
 2*a^5*b^4 + a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*
sin(d*x + c) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d), -1/2*((a^5*b - 3*a^3
*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) + (3*a^4*b + a^2*b^3 - 2*b^5 - (3
*a^2*b^3 - 2*b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sq
rt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
+ (2*a^6 - 5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c) - (a^6 - a^4*b^2 - a^2*b^4 +
 b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 +
a*b^5)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^6 - a^4*b^2 - a^2*b^4
 + b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3
+ a*b^5)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7*b^2 - 2*a^5*b^4
+ a^3*b^6)*d*cos(d*x + c)^2 - 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*sin(d*x + c
) - (a^9 - a^7*b^2 - a^5*b^4 + a^3*b^6)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.48, size = 277, normalized size = 1.80

$$-\frac{\frac{(3a^2b-2b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^5-a^3b^2)\sqrt{a^2-b^2}}-\frac{3a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a^4-3a^2b^2}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b - 2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^5 - a^3*b^2)*sqrt(a^2 - b^2))
```

```
- (3*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4
*tan(1/2*d*x + 1/2*c)^2 + a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*
x + 1/2*c)^2 + 5*a^3*b*tan(1/2*d*x + 1/2*c) - 8*a*b^3*tan(1/2*d*x + 1/2*c)
+ 2*a^4 - 3*a^2*b^2)/((a^5 - a^3*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1
/2*d*x + 1/2*c) + a)^2) - log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

Mupad [B]
time = 12.44, size = 1610, normalized size = 10.45

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{\frac{2\,a^2-3\,b^2}{a\,\left(a^2-b^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^4+a^2\,b^2-6\,b^4\right)}{a^3\,\left(a^2-b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,a^2\,b-8\,b^3\right)}{a^2\,\left(a^2-b^2\right)}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a^2-4\,b^2\right)}{a^2\,\left(a^2-b^2\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2+4\,b^2\right)+a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+a^2+4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(\frac{5\,a^5\,b-4\,a^3\,b^3}{a^6-a^4\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^7+14\,a^5\,b^2-20\,a^3\,b^4+8\,a\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}-\frac{b\,\left(\frac{2\,a^8\,b-2\,a^6\,b^3}{a^6-a^4\,b^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^{10}-20\,a^8\,b^2+22\,a^6\,b^4-8\,a^4\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}\right)\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}\right)\,1{}\mathrm{i}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}+\frac{b\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(\frac{5\,a^5\,b-4\,a^3\,b^3}{a^6-a^4\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^7+14\,a^5\,b^2-20\,a^3\,b^4+8\,a\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}+\frac{b\,\left(\frac{2\,a^8\,b-2\,a^6\,b^3}{a^6-a^4\,b^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^{10}-20\,a^8\,b^2+22\,a^6\,b^4-8\,a^4\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}\right)\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}\right)\,1{}\mathrm{i}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}}{\frac{2\,\left(3\,a^2\,b-2\,b^3\right)}{a^6-a^4\,b^2}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,b^4-3\,a^2\,b^2\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}-\frac{b\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(\frac{5\,a^5\,b-4\,a^3\,b^3}{a^6-a^4\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^7+14\,a^5\,b^2-20\,a^3\,b^4+8\,a\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}-\frac{b\,\left(\frac{2\,a^8\,b-2\,a^6\,b^3}{a^6-a^4\,b^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^{10}-20\,a^8\,b^2+22\,a^6\,b^4-8\,a^4\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}\right)\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}\right)}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}+\frac{b\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(\frac{5\,a^5\,b-4\,a^3\,b^3}{a^6-a^4\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^7+14\,a^5\,b^2-20\,a^3\,b^4+8\,a\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}+\frac{b\,\left(\frac{2\,a^8\,b-2\,a^6\,b^3}{a^6-a^4\,b^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^{10}-20\,a^8\,b^2+22\,a^6\,b^4-8\,a^4\,b^6\right)}{a^7-2\,a^5\,b^2+a^3\,b^4}\right)\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}\right)}{2\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}}\right)\,\left(3\,a^2-2\,b^2\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,1{}\mathrm{i}}{d\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((2*a^2 - 3*b^2)/(a*(a^2 - b^2)) + (tan(c
/2 + (d*x)/2)^2*(2*a^4 - 6*b^4 + a^2*b^2))/(a^3*(a^2 - b^2)) + (tan(c/2 + (
d*x)/2)*(5*a^2*b - 8*b^3))/(a^2*(a^2 - b^2)) + (b*tan(c/2 + (d*x)/2)^3*(3*a
^2 - 4*b^2))/(a^2*(a^2 - b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) +
a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
 + (d*x)/2))) + (b*atan(((b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6 - 2*a^7
 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((2*a^8*b - 2
*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6 + 22*a^
6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(-(a + b)
^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*1i)/(2*(a
^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) + (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*
(8*a*b^6 - 2*a^7 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) +
(b*((2*a^8*b - 2*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8
*a^4*b^6 + 22*a^6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 -
2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*
b^2)))*1i)/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))/((2*(3*a^2*b - 2*b^
3))/(a^6 - a^4*b^2) + (2*tan(c/2 + (d*x)/2)*(2*b^4 - 3*a^2*b^2))/(a^7 + a^3
*b^4 - 2*a^5*b^2) - (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((5*a^5
*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6 - 2*a^7 - 20
*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((2*a^8*b - 2*a^6*
b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6 + 22*a^6*b^4
 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(-(a + b)^3*(a
 - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))/(2*(a^9 - a^3
*b^6 + 3*a^5*b^4 - 3*a^7*b^2)) + (b*(3*a^2 - 2*b^2)*(-(a + b)^3*(a - b)^3)^
(1/2)*((5*a^5*b - 4*a^3*b^3)/(a^6 - a^4*b^2) + (tan(c/2 + (d*x)/2)*(8*a*b^6
 - 2*a^7 - 20*a^3*b^4 + 14*a^5*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*((2*a
^8*b - 2*a^6*b^3)/(a^6 - a^4*b^2) - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^4*b^6
 + 22*a^6*b^4 - 20*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(3*a^2 - 2*b^2)*(
```

```
-(a + b)^3*(a - b)^3)^(1/2))/(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))/
(2*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))))*(3*a^2 - 2*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*1i)/(d*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

3.1089 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{(2a^4-9a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d}+\frac{\cot(}{2ad(a+b}$$

```
[Out] -(2*a^4-9*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^4/(a^2-b^2)^(3/2)/d+3*b*arctanh(cos(d*x+c))/a^4/d-1/2*(5*a^2-6*b^2)*cot(d*
x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*(2*a^2-3*b^2
)*cot(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.51, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2802, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{(2a^4-9a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
```

```
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)} \\
&= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \int \frac{\csc^2(c+dx)\left(5a^4-11\right.}{} \\
&= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+}{2a^2\left(a^2-b^2\right)d(a+b\sin(} \\
&= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+}{2a^2\left(a^2-b^2\right)d(a+b\sin(} \\
&= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \\
&= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{(5a^2}{}
\end{aligned}$$

Mathematica [A]
time = 3.85, size = 195, normalized size = 0.97

$$\frac{-\frac{2\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 6b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + \frac{ab\left(-3a^2+4b^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
```

```
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [A]
time = 0.71, size = 299, normalized size = 1.48

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b(\cdots}{2a\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\cdots}{a^4}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b(\cdots}{2a\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\cdots}{a^4}}{d}$
risch	$\frac{i\left(-2ia^3b\,e^{5i(dx+c)}+3ia\,b^3e^{5i(dx+c)}+20ia^3b\,e^{3i(dx+c)}-24ia\,b^3e^{3i(dx+c)}+6a^4e^{4i(dx+c)}-3a^2b^2e^{4i(dx+c)}-6b^4e^{4i(dx+c)}-\cdots\right.}{\left(e^{2i(dx+c)}-1\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)-2/a^4*((1/2*a*b^2*(5*a^2-6*b^2)/(a^2-b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4+3*a^2*b^2-10*b^4)/(a^2-b^2)*tan(1/2*d*x+1/
2*c)^2+1/2*a*b^2*(11*a^2-14*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*
a^2-5*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1
/2*(2*a^4-9*a^2*b^2+6*b^4)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
c)+2*b)/(a^2-b^2)^(1/2)))-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1/2*d*x
+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(191) = 382$.

time = 0.64, size = 1394, normalized size = 6.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.54, size = 339, normalized size = 1.68

$$-\frac{\frac{2\left(2a^4-9a^2b^2+6b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-14ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a^4b-5a^2b^3\right)}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 10.63, size = 1762, normalized size = 8.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (a^2 - (2*tan(c/2 + (d*x)/2)*(7*a*b^3 - 6*a^
3*b))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^4 - 12*b^4 + 9*a^2*b^2))/(a^2
- b^2) + (2*tan(c/2 + (d*x)/2)^2*(a^4 - 16*b^4 + 12*a^2*b^2))/(a^2 - b^2) +
 (2*tan(c/2 + (d*x)/2)^3*(6*a^4*b - 10*b^5 + a^2*b^3))/(a*(a^2 - b^2)))/(d*
(2*a^5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*
a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (
d*x)/2)^4)) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (atan((((-(a + b)^3*(
a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6
```

*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4
*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2*a^8*b^3)/
(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20
*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^
10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4
+ 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2)
 + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(
a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (ta
n(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^
5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*
b^4 - 3*a^8*b^2))/((2*(6*a^4*b + 18*b^5 - 27*a^2*b^3))/(a^8 - a^6*b^2) + (2
*tan(c/2 + (d*x)/2)*(4*a^6 - 18*b^6 + 39*a^2*b^4 - 24*a^4*b^2))/(a^9 + a^5*
b^4 - 2*a^7*b^2) - ((-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)
/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2
)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7
*b^2) + (((2*a^10*b - 2*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a
^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(
-(a + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6
+ 3*a^6*b^4 - 3*a^8*b^2)))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a
 + b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4
 - 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7
 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*
a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8
*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1
/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)
))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*(-(a + b)^3*(a - b)^3)^(1/2)*
(a^4 + 3*b^4 - (9*a^2*b^2)/2)*2i)/(d*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^
2))

3.1090 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=269

$$\frac{b(6a^4-19a^2b^2+12b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{3/2}d}+\frac{(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{b(11a^2-12b^2)\cot(c+}{2a^4\left(a^2-b^2\right)d}$$

```
[Out] b*(6*a^4-19*a^2*b^2+12*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a^5/(a^2-b^2)^(3/2)/d+1/2*(a^2-12*b^2)*arctanh(cos(d*x+c))/a^5/d+1/2*b*(1
1*a^2-12*b^2)*cot(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cot(d*x+c)*csc(d
*x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*
(3*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.74, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{2a^2d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}+\frac{b(11a^2-12b^2)\cot(c+dx)}{2a^4d\left(a^2-b^2\right)}-\frac{(5a^2-6b^2)\cot(c+dx)\csc(c+dx)}{2a^3d\left(a^2-b^2\right)}+\frac{b(6a^4-19a^2b^2+12b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{3/2}}+\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + ((a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^5*d) + (b*(11*a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*
a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cot[c + d*
x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cot[c +
d*x]*Csc[c + d*x])/(2*a^2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx \\
&= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(4\left(a^2-b^2\right)-3\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)} \\
&= \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \int \frac{\csc^3(c+\cdots}{\cdots} \\
&= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(3a^2-\cdots}{2a^2(a\cdots} \\
&= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c\cdots}{2ad(\cdots} \\
&= \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)\csc(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c\cdots}{2ad(\cdots} \\
&= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{(5a^2-6\cdots}{\cdots} \\
&= \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^5d} + \frac{b\left(11a^2-12b^2\right)\cot(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{(5a^2-6\cdots}{\cdots} \\
&= \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{3/2}d} + \frac{\left(a^2-12b^2\right)\tanh^{-1}(\cos\cdots}{2a^5d}
\end{aligned}$$

Mathematica [A]
time = 6.23, size = 330, normalized size = 1.23

$$\frac{b\left(6a^4-19a^2b^2+12b^4\right)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{3/2}d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{\left(a^2-12b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{\left(-a^2+12b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{b^2\cos(c+dx)}{2a^3d(a+b\sin(c+dx))^2} + \frac{5a^2b^2\cos(c+dx)-6b^4\cos(c+dx)}{2a^4(a-b)(a+b)d(a+b\sin(c+dx))} - \frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/
2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(3/2)*d) + (3*
b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + ((a^2 - 12*b
^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + ((-a^2 + 12*b^2)*Log[Sin[(c + d*x)/2
]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + (b^2*Cos[c + d*x])/(2*a^3*d*
(a + b*Sin[c + d*x])^2) + (5*a^2*b^2*Cos[c + d*x] - 6*b^4*Cos[c + d*x])/(2*
a^4*(a - b)*(a + b)*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4
*d)
```

Maple [A]
time = 0.76, size = 345, normalized size = 1.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^4*(1/2*a*tan(1/2*d*x+1/2*c)^2-6*b*tan(1/2*d*x+1/2*c))+2*b/a^5*((
1/2*a*b^2*(7*a^2-8*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+1/2*b*(6*a^4+5*a^2*b
^2-14*b^4)/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(17*a^2-20*b^2)/(a^2-b^
2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(6*a^2-7*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1/2
*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(6*a^4-19*a^2*b^2+12*b^4)/(a^2-b^2)^(
3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/8/a^3/tan(
1/2*d*x+1/2*c)^2+1/4/a^5*(-2*a^2+24*b^2)*ln(tan(1/2*d*x+1/2*c))+3/2*b/a^4/t
an(1/2*d*x+1/2*c))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(254) = 508.
time = 0.86, size = 1922, normalized size = 7.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")

[Out] [-1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 - (6*a^6*b -
 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7)*cos(d*
x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)^2 + 2*(
6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 18*a^2*b^6)*cos(d*x + c) -
(a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^
4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 3
8*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 1
2*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13
*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x +
c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2
+ 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*
a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos(d*x + c)^3 - (4*a^7*b + 3
*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - 2
*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*a^7*b^4 + 2*a^5*b^6)*d*cos
(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6)*d - 2*((a^10*b - 2*a^8*b
^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*b^3 + a^6*b^5)*d)*sin(d*x
+ c)), -1/4*(2*(17*a^6*b^2 - 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c)^3 + 2*(6
*a^6*b - 13*a^4*b^3 - 7*a^2*b^5 + 12*b^7 + (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
)*cos(d*x + c)^4 - (6*a^6*b - 7*a^4*b^3 - 26*a^2*b^5 + 24*b^7)*cos(d*x + c)
^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6 - (6*a^5*b^2 - 19*a^3*b^4 + 12*a*
b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
+ b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(a^8 - 19*a^6*b^2 + 36*a^4*b^4 - 1
8*a^2*b^6)*cos(d*x + c) - (a^8 - 13*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*
b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 -
12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 1
4*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 - (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a
*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^8 - 13
*a^6*b^2 + 11*a^4*b^4 + 13*a^2*b^6 - 12*b^8 + (a^6*b^2 - 14*a^4*b^4 + 25*a^
2*b^6 - 12*b^8)*cos(d*x + c)^4 - (a^8 - 12*a^6*b^2 - 3*a^4*b^4 + 38*a^2*b^6
 - 24*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7 -
 (a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) + 2*((11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7)*cos
(d*x + c)^3 - (4*a^7*b + 3*a^5*b^3 - 19*a^3*b^5 + 12*a*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d*cos(d*x + c)^4 - (a^11 - 3*
```

```
a^7*b^4 + 2*a^5*b^6)*d*cos(d*x + c)^2 + (a^11 - a^9*b^2 - a^7*b^4 + a^5*b^6
)*d - 2*((a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c)^2 - (a^10*b - 2*a^8*
b^3 + a^6*b^5)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(c+dx\right)\csc^3\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(254) = 508.
time = 0.51, size = 526, normalized size = 1.96

$$\frac{\frac{8\left(6a^4b-19a^2b^3+12b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^7-a^5b^2\right)\sqrt{a^2-b^2}}+\frac{2a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-26a^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+24a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+20a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-60a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+32ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+53a^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-64a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-16b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+28a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+60a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-112ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+68a^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-76a^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+8a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8a^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^6+a^4b^2}{\left(a^7-a^5b^2\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2}-\frac{4\left(a^2-12b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*(6*a^4*b - 19*a^2*b^3 + 12*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - a^5*b^
2)*sqrt(a^2 - b^2)) + (2*a^6*tan(1/2*d*x + 1/2*c)^6 - 26*a^4*b^2*tan(1/2*d*
x + 1/2*c)^6 + 24*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 + 20*a^5*b*tan(1/2*d*x + 1
/2*c)^5 - 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^5*tan(1/2*d*x + 1/2*c)
^5 + 3*a^6*tan(1/2*d*x + 1/2*c)^4 + 53*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 64*
a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 16*b^6*tan(1/2*d*x + 1/2*c)^4 + 28*a^5*b*t
an(1/2*d*x + 1/2*c)^3 + 60*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 112*a*b^5*tan(1
/2*d*x + 1/2*c)^3 + 68*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 76*a^2*b^4*tan(1/2*
d*x + 1/2*c)^2 + 8*a^5*b*tan(1/2*d*x + 1/2*c) - 8*a^3*b^3*tan(1/2*d*x + 1/2
*c) - a^6 + a^4*b^2)/((a^7 - a^5*b^2)*(a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1
/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) - 4*(a^2 - 12*b^2)*log(abs(t
an(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*
d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 11.01, size = 1906, normalized size = 7.09

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (a^3/2 - (2*tan(c/2 + (d*x)/2)^5*(3*a^4*b
- 16*b^5 + 11*a^2*b^3))/(a^2 - b^2) - (2*tan(c/2 + (d*x)/2)^3*(5*a^4*b - 52
*b^5 + 41*a^2*b^3))/(a^2 - b^2) - 4*a^2*b*tan(c/2 + (d*x)/2) + (tan(c/2 + (
d*x)/2)^2*(50*a*b^4 + a^5 - 47*a^3*b^2))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^
4*(a^6 + 112*b^6 + 8*a^2*b^4 - 97*a^4*b^2))/(2*a*(a^2 - b^2)))/(d*(4*a^6*ta
n(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a
^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 + (d*x)
/2)^5)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(a^
2 - 12*b^2))/(2*a^5*d) - (b*atan(((b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 +
12*b^4 - 19*a^2*b^2)*((7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2)
- (tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a
^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 -
 a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12
*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 +
12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*1i)/(2*
(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) - (b*(-(a + b)^3*(a - b)^3)^(1/2)
*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((tan(c/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 12
4*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) - (7*a^
9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) + (b*((2*a^12*b - 2*a^10*b^
3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4
 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)
*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)
))*1i)/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))/((6*a^6*b - 144*b^7 +
240*a^2*b^5 - 91*a^4*b^3)/(a^10 - a^8*b^2) + (2*tan(c/2 + (d*x)/2)*(72*b^8
- 174*a^2*b^6 + 131*a^4*b^4 - 30*a^6*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2) + (
b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((7*a^9*b + 24
*a^5*b^5 - 32*a^7*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(a^11 + 48*a^
3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^
2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)/2)*(6*
a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a^9*b^2)
)*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^
5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^
2)) + (b*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*((tan(c
/2 + (d*x)/2)*(a^11 + 48*a^3*b^8 - 124*a^5*b^6 + 103*a^7*b^4 - 28*a^9*b^2))
/(a^11 + a^7*b^4 - 2*a^9*b^2) - (7*a^9*b + 24*a^5*b^5 - 32*a^7*b^3)/(a^10 -
 a^8*b^2) + (b*((2*a^12*b - 2*a^10*b^3)/(a^10 - a^8*b^2) - (tan(c/2 + (d*x)
/2)*(6*a^14 - 8*a^8*b^6 + 22*a^10*b^4 - 20*a^12*b^2))/(a^11 + a^7*b^4 - 2*a
^9*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^
11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3
*a^9*b^2))))*(-(a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)
/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))
```

3.1091 $\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}+\frac{4b\cos(e+fx)}{3a\left(a^2-b^2\right)f\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}-\frac{4b\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}}{}$$

```
[Out] 2/3*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(3/2)+4/3*b*cos(
f*x+e)/a/(a^2-b^2)/f/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)-4/3*b*Elli
pticE(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-
b)/(a-b))^(1/2))*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1
/2)*tan(f*x+e)/a^3/f/(a+b)^(1/2)/d^(1/2)-4/3*EllipticF(d^(1/2)*(a+b*sin(f*x
+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc
(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^2/f/(a+b)
^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2966, 2879, 3077, 2895, 3073}

$$-\frac{4b\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^3\sqrt{d}\,f\sqrt{a+b}}-\frac{4\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{3a^2\sqrt{d}\,f\sqrt{a+b}}+\frac{4b\cos(e+fx)}{3af\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}+\frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(3*a*d*f*(a + b*Sin[e + f*x])^(3/2))
+ (4*b*Cos[e + f*x])/(3*a*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin
[e + f*x]]) - (4*b*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e
+ f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt
[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3*a^3*Sq
rt[a + b]*Sqrt[d]*f) - (4*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 +
 Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]]
)/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(3
*a^2*Sqrt[a + b]*Sqrt[d]*f)
```

Rule 2879

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])^(3/2)), x_Symbol] :> Simp[2*b*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a +
b*Sin[e + f*x]]*Sqrt[d*Sin[e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(b + a
*Sin[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /;
 FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2966

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-g)*(g*
Cos[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^(m + 1)/(a
*d*f*(m + 1))), x] + Dist[g^2*((2*m + 3)/(2*a*(m + 1))), Int[(g*Cos[e + f*x
])^(p - 2)*((a + b*Sin[e + f*x])^(m + 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; F
reeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p
 + 1/2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}\,dx = \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e}}{3a}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+}{3a(a^2-b^2)f\sqrt{d\sin(e+fx}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+}{3a(a^2-b^2)f\sqrt{d\sin(e+fx}}$$

$$= \frac{2\cos(e+fx)\sqrt{d\sin(e+fx)}}{3adf(a+b\sin(e+fx))^{3/2}} + \frac{4b\cos(e+}{3a(a^2-b^2)f\sqrt{d\sin(e+fx}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3348 vs. $2(347) = 694$.

time = 29.59, size = 3348, normalized size = 9.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*Cos[e + f*x])/(3*a*(a + b*Sin[e
+ f*x])^2) - (4*b^2*Cos[e + f*x])/(3*a^2*(a^2 - b^2)*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sqrt[a + b*Sin[e + f*x]]*((2*Sqrt[a + b*Sin
[e + f*x]])/(3*a*(a^2 - b^2)*Sqrt[Sin[e + f*x]]) - (4*b*Sqrt[Sin[e + f*x]]*
Sqrt[a + b*Sin[e + f*x]])/(3*a^2*(a^2 - b^2)))*(-2*b*Sin[(e + f*x)/2]^2 - (
2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])
/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*
Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(
e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^
2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan
[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e
+ f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a^2 - b^2)*f*Sqrt[d*Sin[e + f*x]]*((2*b*C
os[e + f*x]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSin[Sqrt[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqr
t[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2]
```

)/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2
]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(
a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(3*a^2*(a
^2 - b^2)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (2*Cos[e + f*x]*Sq
rt[a + b*Sin[e + f*x]]*(-2*b*Sin[(e + f*x)/2]^2 - (2*a*(-(b*EllipticE[ArcSi
n[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[
2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*El
lipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
 b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt
[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e
 + f*x]))/(a^2 - b^2)]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]))
)/(3*a^2*(a^2 - b^2)*Sin[e + f*x]^(3/2)) + (4*Sqrt[a + b*Sin[e + f*x]]*(-2*
b*Cos[(e + f*x)/2]*Sin[(e + f*x)/2] + (a^2*Sec[(e + f*x)/2]^2*(-(b*Elliptic
E[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]
]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2])
 + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt
[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(
a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b
 + Sqrt[-a^2 + b^2]))]))/(2*Sqrt[-a^2 + b^2]*(-b + Sqrt[-a^2 + b^2])*Sqrt[(
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*((a*Tan[(e + f*x)/2
])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) + (a*((a*b*Cos[e + f*x]*Sec[(e + f*x)/2]
^2)/(a^2 - b^2) + (a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/
2])/(a^2 - b^2))*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[
(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-
a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b
^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/
(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*
Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/(Sqrt[-a^2 + b^2]*((
a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2))^(3/2)*Sqrt[(a*Tan[(
e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) - (2*a*(-1/2*(b*EllipticE[ArcSin[Sqr
t[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]],
(2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]^2) - (a^2*El
lipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 +
 b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)
/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*(b + Sqrt[-a^2
 + b^2])*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]) + (a^2*Ellip
ticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^
2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sec[(e + f*x)/2]
^2*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*(-b + Sqrt[-a^2
 + b^2])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (a*b*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2]*Sqrt[1 - (-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f
*x)/2])/(-b + Sqrt[-a^2 + b^2])])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(-b + Sq
rt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (-b + Sqrt[
-a^2 + b^2] - a*Tan[(e + f*x)/2])/(2*Sqrt[-a^2 + b^2])]) + (a^2*Sec[(e + f*

```
x)/2]^2*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e
 + f*x)/2])/(b + Sqrt[-a^2 + b^2]))])/(4*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[(b +
 Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]*Sqrt[1 - (b + Sqr
t[-a^2 + b^2] + a*Tan[(e + f*x)/2])/(2*Sqrt[-a^...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4674 vs. $2(307) = 614$.
time = 15.62, size = 4675, normalized size = 13.47

method	result	size
default	Expression too large to display	4675

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/f*(-4*cos(f*x+e)*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)
-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(
-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-(-a^2+b^2)^(1/
2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^
(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e)*a-(-a^2+
b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2
)*(-a^2+b^2)^(1/2)*b^3+4*cos(f*x+e)*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+co
s(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-
(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e)
)^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f
*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/si
n(f*x+e))^(1/2)*a^2*b^2-4*cos(f*x+e)*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(
1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+c
os(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a
-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e
))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(
f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/s
in(f*x+e))^(1/2)*b^4+2*cos(f*x+e)*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2
)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(
f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((-(cos(f*x+e)*a-(-
a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^
(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x
+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(
f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a^2*b-4*cos(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)
^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1
+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)
*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x
+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(co
```

```
s(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))
/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*a*b^2+4*cos(f*x+e)*((cos(f*x+e)*a+(-a^2
+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(
a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f
*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/si
n(f*x+e))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*
(-(cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(
1/2))/sin(f*x+e))^(1/2)*a^3*b-4*cos(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*
sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*
x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-(-a^
2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1
/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e
)*a-(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*
x+e))^(1/2)*a*b^3+2*cos(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b
*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a
^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((-(cos(f*x+e)*a-(-a^2+b^2)^(1/2)
*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e)*a-(-a^2+b^
2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*
(-a^2+b^2)^(1/2)*a^3-4*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e
)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+
(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-(-a^2+b^2)^(1
/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2
^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e)*a-(-a^2
+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/
2)*(-a^2+b^2)^(1/2)*b^3+4*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*
x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/
(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-(-a^2+b^2)
^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/
2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e)*a-(-
a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^
(1/2)*a^2*b^2-4*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin
(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b
^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((-(cos(f*x+e)*a-(-a^2+b^2)^(1/2)*sin
(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2),1/2*2^(1/2)*
((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-(cos(f*x+e)*a-(-a^2+b^2)^(
1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*b^4+
2*sin(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*si...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^2/(b^3*
d*cos(f*x + e)^4 - (3*a^2*b + 2*b^3)*d*cos(f*x + e)^2 + (3*a^2*b + b^3)*d -
 (3*a*b^2*d*cos(f*x + e)^2 - (a^3 + 3*a*b^2)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/((b*sin(f*x + e) + a)^(5/2)*sqrt(d*sin(f*x + e))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(e+f\,x)^2}{\sqrt{d\,\sin(e+f\,x)}\,\left(a+b\,\sin(e+f\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(5/2)),x)
```

```
[Out] int(cos(e + f*x)^2/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(5/2)), x)
```

3.1092 $\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{3ax}{128}-\frac{b\cos^5(c+dx)}{5d}+\frac{2b\cos^7(c+dx)}{7d}-\frac{b\cos^9(c+dx)}{9d}+\frac{3a\cos(c+dx)\sin(c+dx)}{128d}+\frac{a\cos^3(c+dx)\sin(c+a}{64d}$$

```
[Out] 3/128*a*x-1/5*b*cos(d*x+c)^5/d+2/7*b*cos(d*x+c)^7/d-1/9*b*cos(d*x+c)^9/d+3/
128*a*cos(d*x+c)*sin(d*x+c)/d+1/64*a*cos(d*x+c)^3*sin(d*x+c)/d-1/16*a*cos(d
*x+c)^5*sin(d*x+c)/d-1/8*a*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 276}

$$-\frac{a\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{a\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3a\sin(c+dx)\cos(c+dx)}{128d}+\frac{3ax}{128}-\frac{b\cos^9(c+dx)}{9d}+\frac{2b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/128 - (b*Cos[c + d*x]^5)/(5*d) + (2*b*Cos[c + d*x]^7)/(7*d) - (b*Co
s[c + d*x]^9)/(9*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (a*Cos[c +
d*x]^3*Sin[c + d*x])/(64*d) - (a*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (a*C
os[c + d*x]^5*Sin[c + d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^4(c+dx)\,dx + b\int \cos^4(c+dx)\sin^5$$

$$= -\frac{a\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3a)\int \cos^4(c+dx)\sin$$

$$= -\frac{a\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{a\cos^5(c+dx)\sin^3(c+dx}{8d}$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{a}{}$$

$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx)}{9d} + \frac{3}{}$$

$$= \frac{3ax}{128} - \frac{b\cos^5(c+dx)}{5d} + \frac{2b\cos^7(c+dx)}{7d} - \frac{b\cos^9(c+dx}{9d}$$

Mathematica [A]
time = 0.18, size = 92, normalized size = 0.64

$$\frac{7560ac+7560adx-7560b\cos(c+dx)-1680b\cos(3(c+dx))+1008b\cos(5(c+dx))+180b\cos(7(c+dx))-140b\cos(9(c+dx))-2520a\sin(4(c+dx))+315a\sin(8(c+dx))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (7560*a*c + 7560*a*d*x - 7560*b*Cos[c + d*x] - 1680*b*Cos[3*(c + d*x)] + 10
08*b*Cos[5*(c + d*x)] + 180*b*Cos[7*(c + d*x)] - 140*b*Cos[9*(c + d*x)] - 2
520*a*Sin[4*(c + d*x)] + 315*a*Sin[8*(c + d*x)])/(322560*d)
```

Maple [A]
time = 0.34, size = 124, normalized size = 0.87

method	result
risch	$\frac{3ax}{128}-\frac{3b\cos(dx+c)}{128d}-\frac{b\cos(9dx+9c)}{2304d}+\frac{a\sin(8dx+8c)}{1024d}+\frac{b\cos(7dx+7c)}{1792d}+\frac{b\cos(5dx+5c)}{320d}-\frac{a\sin(4dx+4c)}{128d}-$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+b\left(-\frac{\left(\sin^{4}(d\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^{3}(dx+c)\right)\left(\cos^{5}(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{16}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{64}+\frac{3dx}{128}+\frac{3c}{128}\right)+b\left(-\frac{\left(\sin^{4}(d\right.}{}\right.}{d}$
norman	$-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+\frac{13a\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{3ax}{128}-\frac{16b}{315d}+\frac{3a\left(\tan^{17}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{63ax\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{27ax\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b*(-1/9*sin(d*x+c
)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))
```

Maxima [A]
time = 0.28, size = 71, normalized size = 0.50

$$\frac{315\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)a-1024\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(dx+c\right)^{5}\right)b}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/322560*(315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a - 1
024*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b)/d
```

Fricas [A]
time = 0.38, size = 95, normalized size = 0.66

$$-\frac{4480\,b\cos\left(dx+c\right)^{9}-11520\,b\cos\left(dx+c\right)^{7}+8064\,b\cos\left(dx+c\right)^{5}-945\,adx-315\left(16\,a\cos\left(dx+c\right)^{7}-24\,a\cos\left(dx+c\right)^{5}+2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/40320*(4480*b*cos(d*x + c)^9 - 11520*b*cos(d*x + c)^7 + 8064*b*cos(d*x +
 c)^5 - 945*a*d*x - 315*(16*a*cos(d*x + c)^7 - 24*a*cos(d*x + c)^5 + 2*a*co
s(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(131) = 262$.
time = 1.30, size = 272, normalized size = 1.90

$$\begin{cases} \frac{3ax\sin^8(c+dx)}{128} + \frac{3ax\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{9ax\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3ax\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3ax\cos^8(c+dx)}{128} + \frac{3a\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11a\sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{11a\sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{3a\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{b\sin^4(c+dx)\cos^5(c+dx)}{5d} - \frac{4b\sin^2(c+dx)\cos^7(c+dx)}{35d} - \frac{8b\cos^9(c+dx)}{315d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin^4(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**8/128 + 3*a*x*sin(c + d*x)**6*cos(c + d*x)**
2/32 + 9*a*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*a*x*sin(c + d*x)**2*cos
(c + d*x)**6/32 + 3*a*x*cos(c + d*x)**8/128 + 3*a*sin(c + d*x)**7*cos(c + d
*x)/(128*d) + 11*a*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*a*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - 3*a*sin(c + d*x)*cos(c + d*x)**7/(128*d) -
 b*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(
c)**4*cos(c)**4, True))
```

Giac [A]
time = 0.53, size = 107, normalized size = 0.75

$$\frac{3}{128}\,ax - \frac{b\cos(9\,dx+9\,c)}{2304\,d} + \frac{b\cos(7\,dx+7\,c)}{1792\,d} + \frac{b\cos(5\,dx+5\,c)}{320\,d} - \frac{b\cos(3\,dx+3\,c)}{192\,d} - \frac{3\,b\cos(dx+c)}{128\,d} + \frac{a\sin(8\,dx+8\,c)}{1024\,d} - \frac{a\sin(4\,dx+4\,c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*a*x - 1/2304*b*cos(9*d*x + 9*c)/d + 1/1792*b*cos(7*d*x + 7*c)/d + 1/3
20*b*cos(5*d*x + 5*c)/d - 1/192*b*cos(3*d*x + 3*c)/d - 3/128*b*cos(d*x + c)
/d + 1/1024*a*sin(8*d*x + 8*c)/d - 1/128*a*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 13.32, size = 223, normalized size = 1.56

$$\frac{3\,a\,x}{128} - \frac{-\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64} - \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{32} + \frac{155\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{32} + \frac{32\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}}{3} - \frac{169\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{32} - 16\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \frac{112\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}}{5} + \frac{169\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{32} - \frac{32\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5} - \frac{155\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{32} + \frac{64\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{35} + \frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{32} + \frac{16\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35} + \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64} + \frac{16\,b}{315}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/128 - ((16*b)/315 + (3*a*tan(c/2 + (d*x)/2))/64 + (13*a*tan(c/2 + (
d*x)/2)^3)/32 - (155*a*tan(c/2 + (d*x)/2)^5)/32 + (169*a*tan(c/2 + (d*x)/2)
^7)/32 - (169*a*tan(c/2 + (d*x)/2)^11)/32 + (155*a*tan(c/2 + (d*x)/2)^13)/3
2 - (13*a*tan(c/2 + (d*x)/2)^15)/32 - (3*a*tan(c/2 + (d*x)/2)^17)/64 + (16*
b*tan(c/2 + (d*x)/2)^2)/35 + (64*b*tan(c/2 + (d*x)/2)^4)/35 - (32*b*tan(c/2
 + (d*x)/2)^6)/5 + (112*b*tan(c/2 + (d*x)/2)^8)/5 - 16*b*tan(c/2 + (d*x)/2)
^10 + (32*b*tan(c/2 + (d*x)/2)^12)/3)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.1093 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=127

$$\frac{3bx}{128}-\frac{a\cos^5(c+dx)}{5d}+\frac{a\cos^7(c+dx)}{7d}+\frac{3b\cos(c+dx)\sin(c+dx)}{128d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{64d}-\frac{b\cos^5(c+dx)}{16}$$

```
[Out] 3/128*b*x-1/5*a*cos(d*x+c)^5/d+1/7*a*cos(d*x+c)^7/d+3/128*b*cos(d*x+c)*sin(
d*x+c)/d+1/64*b*cos(d*x+c)^3*sin(d*x+c)/d-1/16*b*cos(d*x+c)^5*sin(d*x+c)/d-
1/8*b*cos(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2645, 14, 2648, 2715, 8}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b\sin(c+dx)\cos(c+dx)}{128d}+\frac{3bx}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/128 - (a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^7)/(7*d) + (3*b*Co
s[c + d*x]*Sin[c + d*x])/(128*d) + (b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) -
 (b*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) - (b*Cos[c + d*x]^5*Sin[c + d*x]^3)
/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^3(c+dx)\,dx + b\int \cos^4(c+dx)\sin^4$$

$$= -\frac{b\cos^5(c+dx)\sin^3(c+dx)}{8d} + \frac{1}{8}(3b)\int \cos^4(c+dx)\sin$$

$$= -\frac{b\cos^5(c+dx)\sin(c+dx)}{16d} - \frac{b\cos^5(c+dx)\sin^3(c+dx}{8d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{b\cos^3(c+dx)\sin(c}{64d}$$

$$= -\frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)\sin(c}{128d}$$

$$= \frac{3bx}{128} - \frac{a\cos^5(c+dx)}{5d} + \frac{a\cos^7(c+dx)}{7d} + \frac{3b\cos(c+dx)}{128}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.61

$$\frac{840bdx - 1680a\cos(c+dx) - 560a\cos(3(c+dx)) + 112a\cos(5(c+dx)) + 80a\cos(7(c+dx)) - 280b\sin(4(c+dx)) + 35b\sin(8(c+dx))}{35840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (840*b*d*x - 1680*a*Cos[c + d*x] - 560*a*Cos[3*(c + d*x)] + 112*a*Cos[5*(c + d*x)] + 80*a*Cos[7*(c + d*x)] - 280*b*Sin[4*(c + d*x)] + 35*b*Sin[8*(c + d*x)])/(35840*d)
```

Maple [A]
time = 0.26, size = 106, normalized size = 0.83

method	result
risch	$\frac{3bx}{128}-\frac{3a\cos(dx+c)}{64d}+\frac{b\sin(8dx+8c)}{1024d}+\frac{a\cos(7dx+7c)}{448d}+\frac{a\cos(5dx+5c)}{320d}-\frac{b\sin(4dx+4c)}{128d}-\frac{a\cos(3dx+3c)}{64d}$
derivativedivides	$\frac{a\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+b\left(-\frac{(\sin^3(dx+c))(\cos^5(dx+c))}{8}-\frac{\sin(dx+c)(\cos^5(dx+c))}{16}+\frac{(\cos^3(dx+c)+}{}\right.}{d}$
default	$\frac{a\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+b\left(-\frac{(\sin^3(dx+c))(\cos^5(dx+c))}{8}-\frac{\sin(dx+c)(\cos^5(dx+c))}{16}+\frac{(\cos^3(dx+c)+}{}\right.}{d}$
norman	$\frac{3bx}{128}-\frac{4a}{35d}-\frac{4a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{23b\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{3b\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{3bx\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{3bx\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128}-\frac{32c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.48

$$\frac{1024\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a+35\left(24\,dx+24\,c+\sin(8\,dx+8\,c)-8\sin(4\,dx+4\,c)\right)b}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/35840*(1024*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a + 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b)/d
```

Fricas [A]
time = 0.37, size = 84, normalized size = 0.66

$$\frac{640\,a\cos(dx+c)^7-896\,a\cos(dx+c)^5+105\,bdx+35\left(16\,b\cos(dx+c)^7-24\,b\cos(dx+c)^5+2\,b\cos(dx+c)^3+3\,b\cos(dx+c)\right)\sin(dx+c)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4480*(640*a*cos(d*x + c)^7 - 896*a*cos(d*x + c)^5 + 105*b*d*x + 35*(16*b*
cos(d*x + c)^7 - 24*b*cos(d*x + c)^5 + 2*b*cos(d*x + c)^3 + 3*b*cos(d*x + c
))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(116) = 232$.
time = 0.91, size = 248, normalized size = 1.95

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{2a\cos^7(c+dx)}{35d} + \frac{3bx\sin^8(c+dx)}{128} + \frac{3bx\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{9bx\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3bx\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3bx\cos^8(c+dx)}{128} + \frac{3b\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11b\sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{11b\sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{3b\sin(c+dx)\cos^7(c+dx)}{128d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin^3(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a*cos(c + d*x)**7/(
35*d) + 3*b*x*sin(c + d*x)**8/128 + 3*b*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 9*b*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b*x*sin(c + d*x)**2*cos(c
+ d*x)**6/32 + 3*b*x*cos(c + d*x)**8/128 + 3*b*sin(c + d*x)**7*cos(c + d*x)
/(128*d) + 11*b*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b*sin(c + d*x)
**3*cos(c + d*x)**5/(128*d) - 3*b*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(
d, 0)), (x*(a + b*sin(c))*sin(c)**3*cos(c)**4, True))
```

Giac [A]
time = 0.51, size = 92, normalized size = 0.72

$$\frac{3}{128}\,bx + \frac{a\cos(7\,dx+7\,c)}{448\,d} + \frac{a\cos(5\,dx+5\,c)}{320\,d} - \frac{a\cos(3\,dx+3\,c)}{64\,d} - \frac{3\,a\cos(dx+c)}{64\,d} + \frac{b\sin(8\,dx+8\,c)}{1024\,d} - \frac{b\sin(4\,dx+4\,c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/128*b*x + 1/448*a*cos(7*d*x + 7*c)/d + 1/320*a*cos(5*d*x + 5*c)/d - 1/64*
a*cos(3*d*x + 3*c)/d - 3/64*a*cos(d*x + c)/d + 1/1024*b*sin(8*d*x + 8*c)/d
- 1/128*b*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 13.17, size = 209, normalized size = 1.65

$$\frac{3\,b\,x}{128} - \frac{-\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64} - \frac{23\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64} + 4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + \frac{333\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{64} - \frac{671\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{64} + 4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + \frac{671\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{64} + \frac{32\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{5} - \frac{333\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{64} - \frac{4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5} + \frac{23\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{64} + \frac{32\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35} + \frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64} + \frac{4\,a}{35}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] (3*b*x)/128 - ((4*a)/35 + (3*b*tan(c/2 + (d*x)/2))/64 + (32*a*tan(c/2 + (d*
x)/2)^2)/35 - (4*a*tan(c/2 + (d*x)/2)^4)/5 + (32*a*tan(c/2 + (d*x)/2)^6)/5
+ 4*a*tan(c/2 + (d*x)/2)^8 + 4*a*tan(c/2 + (d*x)/2)^12 + (23*b*tan(c/2 + (d
*x)/2)^3)/64 - (333*b*tan(c/2 + (d*x)/2)^5)/64 + (671*b*tan(c/2 + (d*x)/2)^
7)/64 - (671*b*tan(c/2 + (d*x)/2)^9)/64 + (333*b*tan(c/2 + (d*x)/2)^11)/64
- (23*b*tan(c/2 + (d*x)/2)^13)/64 - (3*b*tan(c/2 + (d*x)/2)^15)/64)/(d*(tan
(c/2 + (d*x)/2)^2 + 1)^8)
```

3.1094 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{ax}{16}-\frac{b\cos^5(c+dx)}{5d}+\frac{b\cos^7(c+dx)}{7d}+\frac{a\cos(c+dx)\sin(c+dx)}{16d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*a*x-1/5*b*cos(d*x+c)^5/d+1/7*b*cos(d*x+c)^7/d+1/16*a*cos(d*x+c)*sin(d*
x+c)/d+1/24*a*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2648, 2715, 8, 2645, 14}

$$-\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{a\sin(c+dx)\cos(c+dx)}{16d}+\frac{ax}{16}+\frac{b\cos^7(c+dx)}{7d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/16 - (b*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^7)/(7*d) + (a*Cos[c +
 d*x]*Sin[c + d*x])/(16*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a*Co
s[c + d*x]^5*Sin[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin^2(c+dx)\,dx+b\int \cos^4(c+dx)\sin^3\\ &= -\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}a\int \cos^4(c+dx)\,dx-\\ &= \frac{a\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}\\ &= -\frac{b\cos^5(c+dx)}{5d}+\frac{b\cos^7(c+dx)}{7d}+\frac{a\cos(c+dx)\sin(c-}{16d}\\ &= \frac{ax}{16}-\frac{b\cos^5(c+dx)}{5d}+\frac{b\cos^7(c+dx)}{7d}+\frac{a\cos(c+dx)\sin}{16d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 88, normalized size = 0.85

$$\frac{420adx-315b\cos(c+dx)-105b\cos(3(c+dx))+21b\cos(5(c+dx))+15b\cos(7(c+dx))+105a\sin(2(c+dx))-105a\sin(4(c+dx))-35a\sin(6(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (420*a*d*x - 315*b*Cos[c + d*x] - 105*b*Cos[3*(c + d*x)] + 21*b*Cos[5*(c +
d*x)] + 15*b*Cos[7*(c + d*x)] + 105*a*Sin[2*(c + d*x)] - 105*a*Sin[4*(c + d
*x)] - 35*a*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A]
time = 0.19, size = 88, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx\right.}{35}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx\right.}{35}\right.}{d}$
risch	$\frac{ax}{16}-\frac{3b\cos(dx+c)}{64d}+\frac{b\cos(7dx+7c)}{448d}-\frac{a\sin(6dx+6c)}{192d}+\frac{b\cos(5dx+5c)}{320d}-\frac{a\sin(4dx+4c)}{64d}-\frac{b\cos(3dx+3c)}{64d}+$
norman	$\frac{\frac{4b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax}{16}-\frac{4b}{35d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{11a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{31a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{31a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{11a\left(\tan^{11}\left(\frac{d}{2}\right.\right.}{6d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin
(d*x+c)+1/16*d*x+1/16*c)+b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^
5))
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.63

$$\frac{35\left(4\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a+192\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)b}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6720*(35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a +
192*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b)/d
```

Fricas [A]
time = 0.36, size = 73, normalized size = 0.71

$$\frac{240\,b\cos\left(dx+c\right)^7-336\,b\cos\left(dx+c\right)^5+105\,adx-35\left(8\,a\cos\left(dx+c\right)^5-2\,a\cos\left(dx+c\right)^3-3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*b*cos(d*x + c)^7 - 336*b*cos(d*x + c)^5 + 105*a*d*x - 35*(8*a*c
os(d*x + c)^5 - 2*a*cos(d*x + c)^3 - 3*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(90) = 180$.
time = 0.62, size = 192, normalized size = 1.86

$$\begin{cases} \frac{ax\sin^6(c+dx)}{16} + \frac{3ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{ax\cos^6(c+dx)}{16} + \frac{a\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{a\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{a\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{b\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{2b\cos^7(c+dx)}{35d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin^2(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**6/16 + 3*a*x*sin(c + d*x)**4*cos(c + d*x)**2/1
6 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a*x*cos(c + d*x)**6/16 + a*s
in(c + d*x)**5*cos(c + d*x)/(16*d) + a*sin(c + d*x)**3*cos(c + d*x)**3/(6*d
) - a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*sin(c + d*x)**2*cos(c + d*x)*
*5/(5*d) - 2*b*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*
*2*cos(c)**4, True))
```

Giac [A]
time = 0.50, size = 107, normalized size = 1.04

$$\frac{1}{16}\,ax + \frac{b\cos(7\,dx + 7\,c)}{448\,d} + \frac{b\cos(5\,dx + 5\,c)}{320\,d} - \frac{b\cos(3\,dx + 3\,c)}{64\,d} - \frac{3\,b\cos(dx + c)}{64\,d} - \frac{a\sin(6\,dx + 6\,c)}{192\,d} - \frac{a\sin(4\,dx + 4\,c)}{64\,d} + \frac{a\sin(2\,dx + 2\,c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*a*x + 1/448*b*cos(7*d*x + 7*c)/d + 1/320*b*cos(5*d*x + 5*c)/d - 1/64*b
*cos(3*d*x + 3*c)/d - 3/64*b*cos(d*x + c)/d - 1/192*a*sin(6*d*x + 6*c)/d -
1/64*a*sin(4*d*x + 4*c)/d + 1/64*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 13.02, size = 181, normalized size = 1.76

$$\frac{a\,x}{16} - \frac{-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8} + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{6} + 4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} - \frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} - 4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8} + 8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6} + \frac{31\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{24} - \frac{8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{5} - \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{6} + \frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5} + \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8} + \frac{4\,b}{35}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/16 - ((4*b)/35 + (a*tan(c/2 + (d*x)/2))/8 - (11*a*tan(c/2 + (d*x)/2)^
3)/6 + (31*a*tan(c/2 + (d*x)/2)^5)/24 - (31*a*tan(c/2 + (d*x)/2)^9)/24 + (1
1*a*tan(c/2 + (d*x)/2)^11)/6 - (a*tan(c/2 + (d*x)/2)^13)/8 + (4*b*tan(c/2 +
 (d*x)/2)^2)/5 - (8*b*tan(c/2 + (d*x)/2)^4)/5 + 8*b*tan(c/2 + (d*x)/2)^6 -
4*b*tan(c/2 + (d*x)/2)^8 + 4*b*tan(c/2 + (d*x)/2)^10)/(d*(tan(c/2 + (d*x)/2
)^2 + 1)^7)
```

3.1095 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{bx}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 1/16*b*x-1/5*a*cos(d*x+c)^5/d+1/16*b*cos(d*x+c)*sin(d*x+c)/d+1/24*b*cos(d*x
+c)^3*sin(d*x+c)/d-1/6*b*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2645, 30, 2648, 2715, 8}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b\sin(c+dx)\cos(c+dx)}{16d}+\frac{bx}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*x)/16 - (a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(16*d)
+ (b*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (b*Cos[c + d*x]^5*Sin[c + d*x])/
(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos^4(c+dx)\sin(c+dx)\,dx+b\int \cos^4(c+dx)\sin^2(c\\ &= -\frac{b\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}b\int \cos^4(c+dx)\,dx-\frac{aS}{}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos^3(c+dx)\sin(c+dx)}{24d}-\frac{b\cos^5(c}{}\\ &= -\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(c+}{}\\ &= \frac{bx}{16}-\frac{a\cos^5(c+dx)}{5d}+\frac{b\cos(c+dx)\sin(c+dx)}{16d}+\frac{b\cos^3(}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 77, normalized size = 0.89

$$-\frac{-60bdx+120a\cos(c+dx)+60a\cos(3(c+dx))+12a\cos(5(c+dx))-15b\sin(2(c+dx))+15b\sin(4(c+dx))+5b\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/960*(-60*b*d*x + 120*a*Cos[c + d*x] + 60*a*Cos[3*(c + d*x)] + 12*a*Cos[5
*(c + d*x)] - 15*b*Sin[2*(c + d*x)] + 15*b*Sin[4*(c + d*x)] + 5*b*Sin[6*(c
+ d*x)])/d
```

Maple [A]
time = 0.15, size = 68, normalized size = 0.78

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+b\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5}+b\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)}{d}$
risch	$\frac{bx}{16}-\frac{a\cos(dx+c)}{8d}-\frac{b\sin(6dx+6c)}{192d}-\frac{a\cos(5dx+5c)}{80d}-\frac{b\sin(4dx+4c)}{64d}-\frac{a\cos(3dx+3c)}{16d}+\frac{b\sin(2dx+2c)}{64d}$
norman	$\frac{bx}{16}-\frac{2a}{5d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{47b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{13b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{13b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{47b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{b\left(\tan^{11}\left(\frac{dx}{2}+\right.\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a*cos(d*x+c)^5+b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.60

$$-\frac{192\,a\cos\left(dx+c\right)^5-5\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)b}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*(192*a*cos(d*x + c)^5 - 5*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*
sin(4*d*x + 4*c))*b)/d
```

Fricas [A]
time = 0.36, size = 62, normalized size = 0.71

$$-\frac{48\,a\cos\left(dx+c\right)^5-15\,bdx+5\left(8\,b\cos\left(dx+c\right)^5-2\,b\cos\left(dx+c\right)^3-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(48*a*cos(d*x + c)^5 - 15*b*d*x + 5*(8*b*cos(d*x + c)^5 - 2*b*cos(d*
x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(76) = 152$.
time = 0.41, size = 167, normalized size = 1.92

$$\begin{cases} -\frac{a\cos^5(c+dx)}{5d}+\frac{bx\sin^6(c+dx)}{16}+\frac{3bx\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{3bx\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{bx\cos^6(c+dx)}{16}+\frac{b\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{b\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))\sin(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**5/(5*d) + b*x*sin(c + d*x)**6/16 + 3*b*x*sin(c
+ d*x)**4*cos(c + d*x)**2/16 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + b
*x*cos(c + d*x)**6/16 + b*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b*sin(c + d
*x)**3*cos(c + d*x)**3/(6*d) - b*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d,
 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**4, True))
```

Giac [A]
time = 0.54, size = 92, normalized size = 1.06

$$\frac{1}{16}\,bx-\frac{a\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{a\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{a\cos\left(dx+c\right)}{8\,d}-\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{b\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*b*x - 1/80*a*cos(5*d*x + 5*c)/d - 1/16*a*cos(3*d*x + 3*c)/d - 1/8*a*co
s(d*x + c)/d - 1/192*b*sin(6*d*x + 6*c)/d - 1/64*b*sin(4*d*x + 4*c)/d + 1/6
4*b*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 12.86, size = 181, normalized size = 2.08

$$\frac{b\,x}{16}-\frac{-\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{47\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-\frac{13\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{13\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{47\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{5}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}+\frac{2\,a}{5}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*x)/16 - ((2*a)/5 + (b*tan(c/2 + (d*x)/2))/8 + (2*a*tan(c/2 + (d*x)/2)^2)
/5 + 4*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + 2*a*tan(c/2 + (d
*x)/2)^8 + 2*a*tan(c/2 + (d*x)/2)^10 - (47*b*tan(c/2 + (d*x)/2)^3)/24 + (13
*b*tan(c/2 + (d*x)/2)^5)/4 - (13*b*tan(c/2 + (d*x)/2)^7)/4 + (47*b*tan(c/2
+ (d*x)/2)^9)/24 - (b*tan(c/2 + (d*x)/2)^11)/8)/(d*(tan(c/2 + (d*x)/2)^2 +
1)^6)
```

3.1096 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{3bx}{8}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\cos^3(c+dx)}{3d}+\frac{3b\cos(c+dx)\sin(c+dx)}{8d}+\frac{b\cos^3(c+dx)\sin}{4d}$$

```
[Out] 3/8*b*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d+3/8*b*c
os(d*x+c)*sin(d*x+c)/d+1/4*b*cos(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2672, 308, 212, 2715, 8}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3b\sin(c+dx)\cos(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/8 - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d + (a*Cos[c + d
*x]^3)/(3*d) + (3*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^3*Si
n[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^3(c+dx)\cot(c+dx)\,dx + b\int \cos^4(c+dx)\,dx$$
$$= \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}(3b)\int \cos^2(c+dx)\,dx - $$
$$= \frac{3b\cos(c+dx)\sin(c+dx)}{8d} + \frac{b\cos^3(c+dx)\sin(c+dx)}{4d} + $$
$$= \frac{3bx}{8} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c+dx)}{3d} + \frac{3b\cos(c+dx)\,\mathrm{si}}{8d}$$
$$= \frac{3bx}{8} - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\cos^3(c}{3a}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 1.22

$$\frac{3b(c+dx)}{8d} + \frac{5a\cos(c+dx)}{4d} + \frac{a\cos(3(c+dx))}{12d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*(c + d*x))/(8*d) + (5*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12
*d) - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d + (b*Sin[2*
(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.14, size = 76, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3bx}{8}+\frac{5a\,e^{i(dx+c)}}{8d}+\frac{5a\,e^{-i(dx+c)}}{8d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{b\sin(4dx+4c)}{32d}+\frac{a\cos(3dx+3c)}{12d}$
norman	$\frac{\frac{3bx}{8}+\frac{8a}{3d}+\frac{5b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9bx\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{4}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b*(1/4*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]

time = 0.29, size = 81, normalized size = 0.91

$$\frac{16\left(2\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)b}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*b)/d
```

Fricas [A]

time = 0.40, size = 88, normalized size = 0.99

$$\frac{8\,a\cos\left(dx+c\right)^3+9\,bdx+24\,a\cos\left(dx+c\right)-12\,a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+12\,a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(2\,b\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x + 24*a*cos(d*x + c) - 12*a*log(1/2*cos(d
*x + c) + 1/2) + 12*a*log(-1/2*cos(d*x + c) + 1/2) + 3*(2*b*cos(d*x + c)^3
+ 3*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\cos^4\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**4*csc(c + d*x), x)
```

Giac [A]
time = 0.46, size = 145, normalized size = 1.63

$$\frac{9\,(dx+c)b+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-48\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-96\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+9\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-32\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*b + 24*a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(15*b*tan(1/2
*d*x + 1/2*c)^7 - 48*a*tan(1/2*d*x + 1/2*c)^6 - 9*b*tan(1/2*d*x + 1/2*c)^5
- 96*a*tan(1/2*d*x + 1/2*c)^4 + 9*b*tan(1/2*d*x + 1/2*c)^3 - 80*a*tan(1/2*d
*x + 1/2*c)^2 - 15*b*tan(1/2*d*x + 1/2*c) - 32*a)/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

Mupad [B]
time = 10.82, size = 242, normalized size = 2.72

$$\frac{3\,b\,\mathrm{atan}\left(\frac{9\,b^2}{16\left(\frac{3\,a\,b}{2}-\frac{9\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{3\,a\,b}{2}-\frac{9\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}+\frac{-\frac{5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{20\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{8\,a}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (3*b*atan((9*b^2)/(16*((3*a*b)/2 - (9*b^2*tan(c/2 + (d*x)/2))/16)) + (3*a*b
*tan(c/2 + (d*x)/2))/(2*((3*a*b)/2 - (9*b^2*tan(c/2 + (d*x)/2))/16))))/(4*d
) + ((8*a)/3 + (5*b*tan(c/2 + (d*x)/2))/4 + (20*a*tan(c/2 + (d*x)/2)^2)/3 +
 8*a*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 - (3*b*tan(c/2 + (d*x)
/2)^3)/4 + (3*b*tan(c/2 + (d*x)/2)^5)/4 - (5*b*tan(c/2 + (d*x)/2)^7)/4)/(d*
(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 +
 tan(c/2 + (d*x)/2)^8 + 1)) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.1097 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{3ax}{2}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}$$

```
[Out] -3/2*a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d-3/2*a*
cot(d*x+c)/d+1/2*a*cos(d*x+c)^2*cot(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {2917, 2671, 294, 327, 209, 2672, 308, 212}

$$-\frac{3a\cot(c+dx)}{2d}+\frac{a\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3ax}{2}+\frac{b\cos^3(c+dx)}{3d}+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*x)/2 - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d + (b*Cos[c +
d*x]^3)/(3*d) - (3*a*Cot[c + d*x])/(2*d) + (a*Cos[c + d*x]^2*Cot[c + d*x])/
(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^2(c+dx)\cot^2(c+dx)\,dx + b\int \cos^3(c+dx)\,\mathrm{co}$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{x}{1-}\right.}{}$$

$$= \frac{a\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{(3a)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \mathrm{c}\right.}{2d}$$

$$= \frac{b\cos(c+dx)}{d} + \frac{b\cos^3(c+dx)}{3d} - \frac{3a\cot(c+dx)}{2d} + \frac{a\,\mathrm{co}}{}$$

$$= -\frac{3ax}{2} - \frac{b\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\cos(c+dx)}{d} + \frac{b\cos}{}$$

Mathematica [A]

time = 0.25, size = 105, normalized size = 1.27

$$-\frac{3a(c+dx)}{2d} + \frac{5b\cos(c+dx)}{4d} + \frac{b\cos(3(c+dx))}{12d} - \frac{a\cot(c+dx)}{d} - \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (5*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/(1
2*d) - (a*Cot[c + d*x])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d
*x)/2]])/d - (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]

time = 0.12, size = 94, normalized size = 1.13

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(d\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(d\right.}{d}$
risch	$-\frac{3ax}{2}+\frac{ia\,\mathrm{e}^{2i(dx+c)}}{8d}+\frac{5b\,\mathrm{e}^{i(dx+c)}}{8d}+\frac{5b\,\mathrm{e}^{-i(dx+c)}}{8d}-\frac{ia\,\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2ia}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}+$
norman	$\frac{\frac{4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{2d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
-3/2*d*x-3/2*c)+b*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 91, normalized size = 1.10

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a-\left(2\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a - (2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*lo
g(cos(d*x + c) - 1))*b)/d
```

Fricas [A]
time = 0.36, size = 107, normalized size = 1.29

$$\frac{3\,a\cos\left(dx+c\right)^{3}-3\,b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\,b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a\cos\left(dx+c\right)+\left(2\,b\cos\left(dx+c\right)^{3}-9\,adx+6\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c)^3 - 3*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*
b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*a*cos(d*x + c) + (2*b*cos(d
*x + c)^3 - 9*a*d*x + 6*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)\csc^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**4*csc(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 142, normalized size = 1.71

$$-\frac{9\left(dx+c\right)a-6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{3\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + 3*(2*b*tan(1/2*d*x + 1/2*c) + a)/tan(1/2*d*x + 1/2*c) - 2*(3*a*t
an(1/2*d*x + 1/2*c)^5 + 12*b*tan(1/2*d*x + 1/2*c)^4 + 12*b*tan(1/2*d*x + 1/
2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) + 8*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 9.39, size = 241, normalized size = 2.90

$$\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{16\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}-a}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{3\,a\,\mathrm{atan}\left(\frac{9\,a^2}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\,b\,a}-\frac{6\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+6\,b\,a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] ((16*b*tan(c/2 + (d*x)/2))/3 - a - 5*a*tan(c/2 + (d*x)/2)^2 - 3*a*tan(c/2 +
 (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 + 8*b*tan(c/2 + (d*x)/2)^3 + 8*b*tan(c
/2 + (d*x)/2)^5)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(
c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (a*tan(c/2 + (d*x)/2))/(2*d)
+ (b*log(tan(c/2 + (d*x)/2)))/d + (3*a*atan((9*a^2)/(6*a*b + 9*a^2*tan(c/2
+ (d*x)/2)) - (6*a*b*tan(c/2 + (d*x)/2))/(6*a*b + 9*a^2*tan(c/2 + (d*x)/2))
))/d
```

3.1098 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{3bx}{2}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}$$

```
[Out] -3/2*b*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d-3/2*b*cot(d*x+c)/d+
1/2*b*cos(d*x+c)^2*cot(d*x+c)/d-1/2*a*cos(d*x+c)*cot(d*x+c)^2/d
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2917, 2672, 294, 327, 212, 2671, 209}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*x)/2 + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) -
 (3*b*Cot[c + d*x])/(2*d) + (b*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (a*Cos[
c + d*x]*Cot[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\cot^3(c+dx)\,dx+b\int \cos^2(c+dx)\cot^2\\ &= -\frac{a\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+dx)\right)}{d}-\frac{b\text{Subst}\left(\int \frac{x}{(1+}\right.}{}\\ &= \frac{b\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}\\ &= -\frac{3a\cos(c+dx)}{2d}-\frac{3b\cot(c+dx)}{2d}+\frac{b\cos^2(c+dx)\cot(}{2d}\\ &= -\frac{3bx}{2}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}-\frac{3b}{}\end{aligned}$$

Mathematica [A]

time = 1.00, size = 132, normalized size = 1.40

$$-\frac{3b(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{b\cot(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) - (a*Cos[c + d*x])/d - (b*Cot[c + d*x])/d - (a*Csc[(
c + d*x)/2]^2)/(8*d) + (3*a*Log[Cos[(c + d*x)/2]])/(2*d) - (3*a*Log[Sin[(c
+ d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d) - (b*Sin[2*(c + d*x)])/(4*
d)
```

Maple [A]

time = 0.16, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)s\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)s\right.}{d}$
risch	$-\frac{3bx}{2}+\frac{ib\,e^{2i(dx+c)}}{8d}-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}-\frac{ib\,e^{-2i(dx+c)}}{8d}-\frac{i\left(ia\,e^{3i(dx+c)}+ia\,e^{i(dx+c)}+2b\,e^{2i(dx+c)}-2b\right)}{d\left(e^{2i(dx+c)}-1\right)^2}$
norman	$\frac{-\frac{a}{8d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{3b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{3b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-3bx}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*
ln(csc(d*x+c)-cot(d*x+c)))+b*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*
cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]

time = 0.50, size = 101, normalized size = 1.07

$$-\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b-a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*b - a*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos
(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 139, normalized size = 1.48

$$-\frac{6\,bdx\cos\left(dx+c\right)^{2}+4\,a\cos\left(dx+c\right)^{3}-6\,bdx-6\,a\cos\left(dx+c\right)-3\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\left(b\cos\left(dx+c\right)^{3}-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*d*x*cos(d*x + c)^2 + 4*a*cos(d*x + c)^3 - 6*b*d*x - 6*a*cos(d*x +
 c) - 3*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 3*(a*cos(d*x +
 c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2) + 2*(b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 163, normalized size = 1.73

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,(dx+c)b-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-16\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x + c)*b - 12*a*log(abs(tan(1/2*d*x +
 1/2*c))) + 4*b*tan(1/2*d*x + 1/2*c) + (6*a*tan(1/2*d*x + 1/2*c)^6 + 4*b*ta
n(1/2*d*x + 1/2*c)^5 - 5*a*tan(1/2*d*x + 1/2*c)^4 - 16*b*tan(1/2*d*x + 1/2*
c)^3 - 12*a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c) - a)/(tan(1/2
*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2)/d
```

Mupad [B]
time = 9.37, size = 236, normalized size = 2.51

$$\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}-\frac{-2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\frac{17\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}}{2}+8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+9\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\right)}-\frac{3\,b\,\mathrm{atan}\left(\frac{9\,b^{2}}{9\,a\,b-9\,b^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{9\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{9\,a\,b-9\,b^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{8\,d}-\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2) + 9*a*tan(c/2
+ (d*x)/2)^2 + (17*a*tan(c/2 + (d*x)/2)^4)/2 + 8*b*tan(c/2 + (d*x)/2)^3 - 2
*b*tan(c/2 + (d*x)/2)^5)/(d*(4*tan(c/2 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^
4 + 4*tan(c/2 + (d*x)/2)^6)) - (3*b*atan((9*b^2)/(9*a*b - 9*b^2*tan(c/2 + (
d*x)/2)) + (9*a*b*tan(c/2 + (d*x)/2))/(9*a*b - 9*b^2*tan(c/2 + (d*x)/2))))/
d + (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*a*log(tan(c/2 + (d*x)/2)))/(2*d)
```

3.1099 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$ax+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

```
[Out] a*x+3/2*b*arctanh(cos(d*x+c))/d-3/2*b*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*b*cos
(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2801, 2672, 294, 327, 212, 3554, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx\\
&= a\int \cot^4(c+dx)\,dx+b\int \cos(c+dx)\cot^3(c+dx)\,dx\\
&= -\frac{a\cot^3(c+dx)}{3d}-a\int \cot^2(c+dx)\,dx-\frac{b\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos\right.}{d}\\
&= \frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+a\int 1\\
&= ax-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\\
&= ax+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\,\mathrm{c}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

time = 0.04, size = 125, normalized size = 1.52

$$-\frac{b\cos(c+dx)}{d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d}+\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A]
time = 0.14, size = 86, normalized size = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$ax-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}+\frac{12ia\,e^{4i(dx+c)}+3b\,e^{5i(dx+c)}-12ia\,e^{2i(dx+c)}+8ia-3b\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{3b\ln\left(e^{i(dx+c)}+\right.}{2d}$
norman	$\frac{ax\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+ax\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{24d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{7a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)
^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 92, normalized size = 1.12

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.
time = 0.37, size = 160, normalized size = 1.95

$$\frac{16\,a\cos\left(dx+c\right)^{3}+9\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-12\,a\cos\left(dx+c\right)+6\,\left(2\,adx\cos\left(dx+c\right)^{2}-2\,b\cos\left(dx+c\right)^{3}-2\,adx+3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.54, size = 141, normalized size = 1.72

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,\left(dx+c\right)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\frac{66\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 9.45, size = 225, normalized size = 2.74

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}-\frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{3}+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\right)}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}-\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^{2}}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^{2}+6\,b\,a}-\frac{6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^{2}+6\,b\,a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a/3 + b*tan(c/2 + (d*x)/2) - (14*a*tan(c
/2 + (d*x)/2)^2)/3 - 5*a*tan(c/2 + (d*x)/2)^4 + 17*b*tan(c/2 + (d*x)/2)^3)/
(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d*x
)/2))/(8*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*b*log(tan(c/2 + (d*x)/2))
)/(2*d) - (2*a*atan((4*a^2)/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a*b*tan
(c/2 + (d*x)/2))/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.1100 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$bx-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)}{d}-\frac{b\cot^3(c+dx)}{3d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot^3(c+dx)\csc(c$$

```
[Out] b*x-3/8*a*arctanh(cos(d*x+c))/d+b*cot(d*x+c)/d-1/3*b*cot(d*x+c)^3/d+3/8*a*c
ot(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2917, 2691, 3855, 3554, 8}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)}{3d}+\frac{b\cot(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] b*x - (3*a*ArcTanh[Cos[c + d*x]])/(8*d) + (b*Cot[c + d*x])/d - (b*Cot[c + d
*x]^3)/(3*d) + (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]^3*Cs
c[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc(c+dx)\,dx + b\int \cot^4(c+dx)\,dx$$
$$= -\frac{b\cot^3(c+dx)}{3d} - \frac{a\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3a)$$
$$= \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c+dx)}{3d} + \frac{3a\cot(c+dx)\csc(c+}{8d}$$
$$= bx - \frac{3a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{b\cot(c+dx)}{d} - \frac{b\cot^3(c}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 153, normalized size = 1.74

$$\frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{b\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{5a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
[Out] (5*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (b*Cot[c
+ d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) - (3*a*Lo
g[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (5*a*Sec[(
c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A]
time = 0.16, size = 104, normalized size = 1.18

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)\right.}{d}$

risch	$bx - \frac{-48ib\,e^{6i(dx+c)}+15a\,e^{7i(dx+c)}+96ib\,e^{4i(dx+c)}+9a\,e^{5i(dx+c)}-80ib\,e^{2i(dx+c)}+9a\,e^{3i(dx+c)}+32ib+15a\,e^{i(dx+c)}}{12d\left(e^{2i(dx+c)}-1\right)^4} -$
norman	$\frac{bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+bx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a}{64d}+\frac{7a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{7a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*co
s(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+b*(-1/3*cot(d*x+c)
^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.50, size = 107, normalized size = 1.22

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b-3\,a\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b - 3*a*(2*(
5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)
+ 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(80) = 160$.
time = 0.41, size = 180, normalized size = 2.05

$$\frac{48\,bdx\cos\left(dx+c\right)^4-96\,bdx\cos\left(dx+c\right)^2-30\,a\cos\left(dx+c\right)^3+48\,bdx+18\,a\cos\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+9\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-16\left(4\,b\cos\left(dx+c\right)^3-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*b*d*x*cos(d*x + c)^4 - 96*b*d*x*cos(d*x + c)^2 - 30*a*cos(d*x + c)
^3 + 48*b*d*x + 18*a*cos(d*x + c) - 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^
2 + a)*log(1/2*cos(d*x + c) + 1/2) + 9*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) - 16*(4*b*cos(d*x + c)^3 - 3*b*cos(d*x
 + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.48, size = 153, normalized size = 1.74

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+192\,(dx+c)b+72\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{150\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(3*a*tan(1/2*d*x + 1/2*c)^4 + 8*b*tan(1/2*d*x + 1/2*c)^3 - 24*a*tan(1
/2*d*x + 1/2*c)^2 + 192*(d*x + c)*b + 72*a*log(abs(tan(1/2*d*x + 1/2*c))) -
 120*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 - 120*b*tan(1/2
*d*x + 1/2*c)^3 - 24*a*tan(1/2*d*x + 1/2*c)^2 + 8*b*tan(1/2*d*x + 1/2*c) +
3*a)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 9.67, size = 221, normalized size = 2.51

$$\frac{3\,a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}+\frac{5\,b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}-\frac{5\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,b\,\mathrm{atan}\left(\frac{8\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3\,a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-8\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{a\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (3*a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*d) + (5*b*cot(c/2 + (d*
x)/2))/(8*d) - (5*b*tan(c/2 + (d*x)/2))/(8*d) + (2*b*atan((8*b*cos(c/2 + (d
*x)/2) + 3*a*sin(c/2 + (d*x)/2))/(3*a*cos(c/2 + (d*x)/2) - 8*b*sin(c/2 + (d
*x)/2))))/d + (a*cot(c/2 + (d*x)/2)^2)/(8*d) - (a*cot(c/2 + (d*x)/2)^4)/(64
*d) - (b*cot(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (a
*tan(c/2 + (d*x)/2)^4)/(64*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*d)
```

3.1101 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}$$

```
[Out] -3/8*b*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d+3/8*b*cot(d*x+c)*csc(d*x+
c)/d-1/4*b*cot(d*x+c)^3*csc(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.185$, Rules used = {2917, 2687, 30, 2691, 3855}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{3b\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^3(c+dx)\csc(c+dx)}{4d}+\frac{3b\cot(c+dx)\csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*b*ArcTanh[Cos[c + d*x]])/(8*d) - (a*Cot[c + d*x]^5)/(5*d) + (3*b*Cot[c
+ d*x]*Csc[c + d*x])/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^2(c+dx)\,dx + b\int \cot^4(c+dx)\,\mathrm{cs}$$
$$= -\frac{b\cot^3(c+dx)\csc(c+dx)}{4d} - \frac{1}{4}(3b)\int \cot^2(c+dx)\,\mathrm{cs}$$
$$= -\frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c+dx)\csc(c+dx)}{8d} - \frac{b\cot^3(}{}$$
$$= -\frac{3b\tanh^{-1}(\cos(c+dx))}{8d} - \frac{a\cot^5(c+dx)}{5d} + \frac{3b\cot(c-}{}$$

Mathematica [A]
time = 0.03, size = 135, normalized size = 1.82

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{5b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{5b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Cot[c + d*x]^5)/d + (5*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d
*x)/2]^4)/(64*d) - (3*b*Log[Cos[(c + d*x)/2]])/(8*d) + (3*b*Log[Sin[(c + d*
x)/2]])/(8*d) - (5*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(6
4*d)
```

Maple [A]
time = 0.18, size = 100, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+b\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$

default	$\frac{-\frac{a\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+b\left(-\frac{\cos^{5}(dx+c)}{4\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{8\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)}{d}$
risch	$-\frac{40ia\,e^{8i(dx+c)}+25b\,e^{9i(dx+c)}-10b\,e^{7i(dx+c)}+80ia\,e^{4i(dx+c)}+10b\,e^{3i(dx+c)}+8ia-25b\,e^{i(dx+c)}}{20d\left(e^{2i(dx+c)}-1\right)^{5}}+\frac{3b\ln\left(e^{i(dx+c)}-1\right)}{8d}-\cdots$
norman	$\frac{-\frac{a}{160d}+\frac{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}-\frac{a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{a\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40d}+\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d}+7b\left(\cdots\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a/sin(d*x+c)^5*cos(d*x+c)^5+b*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8
/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c
)-cot(d*x+c))))
```

Maxima [A]
time = 0.27, size = 86, normalized size = 1.16

$$-\frac{5\,b\left(\frac{2\left(5\,\cos(dx+c)^{3}-3\,\cos(dx+c)\right)}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{16\,a}{\tan(dx+c)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 16*a/
tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(66) = 132.
time = 0.37, size = 160, normalized size = 2.16

$$-\frac{16\,a\cos\left(dx+c\right)^{5}+15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+10\left(5\,b\cos\left(dx+c\right)^{3}-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*a*cos(d*x + c)^5 + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(b*cos(d*x + c)^4 - 2*b*cos(
d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 10*(5*b*cos(d*x
 + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x
+ c)^2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(66) = 132$.

time = 0.52, size = 173, normalized size = 2.34

$$\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+5\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{274\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+20\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+5\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/320*(2*a*tan(1/2*d*x + 1/2*c)^5 + 5*b*tan(1/2*d*x + 1/2*c)^4 - 10*a*tan(1
/2*d*x + 1/2*c)^3 - 40*b*tan(1/2*d*x + 1/2*c)^2 + 120*b*log(abs(tan(1/2*d*x
 + 1/2*c))) + 20*a*tan(1/2*d*x + 1/2*c) - (274*b*tan(1/2*d*x + 1/2*c)^5 + 2
0*a*tan(1/2*d*x + 1/2*c)^4 - 40*b*tan(1/2*d*x + 1/2*c)^3 - 10*a*tan(1/2*d*x
 + 1/2*c)^2 + 5*b*tan(1/2*d*x + 1/2*c) + 2*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]

time = 9.56, size = 174, normalized size = 2.35

$$\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{32\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}-\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}+\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}-\frac{{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (a*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(32*d) + (a*tan(c/
2 + (d*x)/2)^5)/(160*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*d) + (b*tan(c/2 + (d*
x)/2)^4)/(64*d) + (3*b*log(tan(c/2 + (d*x)/2)))/(8*d) - (cot(c/2 + (d*x)/2)
^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2 - a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 +
 (d*x)/2)^4 - 4*b*tan(c/2 + (d*x)/2)^3))/(32*d)
```

3.1102 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=98

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^5(c+dx)}{5d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot^3(c+\ldots$$

[Out] -1/16*a*arctanh(cos(d*x+c))/d-1/5*b*cot(d*x+c)^5/d-1/16*a*cot(d*x+c)*csc(d*x+c)/d+1/8*a*cot(d*x+c)*csc(d*x+c)^3/d-1/6*a*cot(d*x+c)^3*csc(d*x+c)^3/d

Rubi [A]
time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 30}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b\cot^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]

[Out] -1/16*(a*ArcTanh[Cos[c + d*x]])/d - (b*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(8*d) - (a*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^4(c+dx)\csc^3(c+dx)\,dx + b\int \cot^4(c+dx)\mathrm{csc}$$

$$= -\frac{a\cot^3(c+dx)\csc^3(c+dx)}{6d} - \frac{1}{2}a\int \cot^2(c+dx)\mathrm{csc}^3$$

$$= -\frac{b\cot^5(c+dx)}{5d} + \frac{a\cot(c+dx)\csc^3(c+dx)}{8d} - \frac{a\cot^3(}{}$$

$$= -\frac{b\cot^5(c+dx)}{5d} - \frac{a\cot(c+dx)\csc(c+dx)}{16d} + \frac{a\cot(c}{}$$

$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b\cot^5(c+dx)}{5d} - \frac{a\cot(c+}{}$$

Mathematica [A]
time = 0.04, size = 175, normalized size = 1.79

$$-\frac{b\cot^5(c+dx)}{5d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Cot[c + d*x]^5)/d - (a*Csc[(c + d*x)/2]^2)/(64*d) + (a*Csc[(c + d*x
)/2]^4)/(64*d) - (a*Csc[(c + d*x)/2]^6)/(384*d) - (a*Log[Cos[(c + d*x)/2]])
/(16*d) + (a*Log[Sin[(c + d*x)/2]])/(16*d) + (a*Sec[(c + d*x)/2]^2)/(64*d)
- (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.21, size = 118, normalized size = 1.20

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{b\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{b\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}}{d}$
risch	$\frac{-240ib\,e^{10i(dx+c)}+15a\,e^{11i(dx+c)}+240ib\,e^{8i(dx+c)}+235a\,e^{9i(dx+c)}-480ib\,e^{6i(dx+c)}+390a\,e^{7i(dx+c)}+480ib\,e^{4i(dx+c)}+39}{120d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{a}{384d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d}+\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{160d}+\frac{b(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(1+\tan^2(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/
sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x
+c)-cot(d*x+c)))-1/5*b/sin(d*x+c)^5*cos(d*x+c)^5)
```

Maxima [A]
time = 0.28, size = 106, normalized size = 1.08

$$\frac{5\,a\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\,b}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/480*(5*a*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d
*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) +
 1) + 3*log(cos(d*x + c) - 1)) - 96*b/tan(d*x + c)^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(88) = 176.
time = 0.39, size = 187, normalized size = 1.91

$$\frac{96\,b\cos\left(dx+c\right)^5\sin\left(dx+c\right)+30\,a\cos\left(dx+c\right)^5+80\,a\cos\left(dx+c\right)^3-30\,a\cos\left(dx+c\right)-15\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/480*(96*b*cos(d*x + c)^5*sin(d*x + c) + 30*a*cos(d*x + c)^5 + 80*a*cos(d*
x + c)^3 - 30*a*cos(d*x + c) - 15*(a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^4 +
3*a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) + 1/2) + 15*(a*cos(d*x + c)^6
- 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)
)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(88) = 176.
time = 0.49, size = 201, normalized size = 2.05

$$\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{294\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+120\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-60\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*a*tan(1/2*d*x + 1/2*c)^6 + 12*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan
(1/2*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*
c)^2 + 120*a*log(abs(tan(1/2*d*x + 1/2*c))) + 120*b*tan(1/2*d*x + 1/2*c) -
(294*a*tan(1/2*d*x + 1/2*c)^6 + 120*b*tan(1/2*d*x + 1/2*c)^5 - 15*a*tan(1/2
*d*x + 1/2*c)^4 - 60*b*tan(1/2*d*x + 1/2*c)^3 - 15*a*tan(1/2*d*x + 1/2*c)^2
 + 12*b*tan(1/2*d*x + 1/2*c) + 5*a)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.52, size = 205, normalized size = 2.09

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{128\,d}-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{128\,d}+\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{384\,d}-\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{32\,d}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,d}-\frac{{\mathrm{cot}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{2}-2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3-\frac{a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{2}+\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}+\frac{a}{6}\right)}{64\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^2)/(128*d) - (a*tan(c
/2 + (d*x)/2)^4)/(128*d) + (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (b*tan(c/2 +
(d*x)/2)^3)/(32*d) + (b*tan(c/2 + (d*x)/2)^5)/(160*d) - (cot(c/2 + (d*x)/2)
^6*(a/6 + (2*b*tan(c/2 + (d*x)/2))/5 - (a*tan(c/2 + (d*x)/2)^2)/2 - (a*tan(
c/2 + (d*x)/2)^4)/2 - 2*b*tan(c/2 + (d*x)/2)^3 + 4*b*tan(c/2 + (d*x)/2)^5))
/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/(16*d)
```

3.1103 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=114

$$-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{b\cot(c+dx)\csc(c+dx)}{16d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}$$

```
[Out] -1/16*b*arctanh(cos(d*x+c))/d-1/5*a*cot(d*x+c)^5/d-1/7*a*cot(d*x+c)^7/d-1/1
6*b*cot(d*x+c)*csc(d*x+c)/d+1/8*b*cot(d*x+c)*csc(d*x+c)^3/d-1/6*b*cot(d*x+c
)^3*csc(d*x+c)^3/d
```

Rubi [A]

time = 0.12, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2687, 14, 2691, 3853, 3855}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{a\cot^5(c+dx)}{5d}-\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}+\frac{b\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{b\cot(c+dx)\csc(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*(b*ArcTanh[Cos[c + d*x]])/d - (a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d
*x]^7)/(7*d) - (b*Cot[c + d*x]*Csc[c + d*x])/(16*d) + (b*Cot[c + d*x]*Csc[c
 + d*x]^3)/(8*d) - (b*Cot[c + d*x]^3*Csc[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\csc^4(c+dx)\,dx+b\int \cot^4(c+dx)\csc\\ &= -\frac{b\cot^3(c+dx)\csc^3(c+dx)}{6d}-\frac{1}{2}b\int \cot^2(c+dx)\csc^3\\ &= \frac{b\cot(c+dx)\csc^3(c+dx)}{8d}-\frac{b\cot^3(c+dx)\csc^3(c+dx}{6d}\\ &= -\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{b\cot(c+dx)\csc(c}{16d}\\ &= -\frac{b\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+}{7d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 239 vs. 2(114) = 228.
time = 0.07, size = 239, normalized size = 2.10

$$-\frac{2a\cot(c+dx)}{35d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{35d}+\frac{8a\cot(c+dx)\csc^4(c+dx)}{35d}-\frac{a\cot(c+dx)\csc^6(c+dx)}{7d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{16d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{16d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{b\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(35*d) - (b*Csc[(c + d*x)/2]^2)/(64*d) + (b*Csc[(c + d*
x)/2]^4)/(64*d) - (b*Csc[(c + d*x)/2]^6)/(384*d) - (a*Cot[c + d*x]*Csc[c +
d*x]^2)/(35*d) + (8*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^6)/(7*d) - (b*Log[Cos[(c + d*x)/2]])/(16*d) + (b*Log[Sin[(c +
 d*x)/2]])/(16*d) + (b*Sec[(c + d*x)/2]^2)/(64*d) - (b*Sec[(c + d*x)/2]^4)/
(64*d) + (b*Sec[(c + d*x)/2]^6)/(384*d)
```

Maple [A]
time = 0.21, size = 138, normalized size = 1.21

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+b\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+b\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+}{}\right.}{d}$
risch	$\frac{105b\,e^{13i(dx+c)}+3360ia\,e^{10i(dx+c)}+1540b\,e^{11i(dx+c)}+3360ia\,e^{8i(dx+c)}+1085b\,e^{9i(dx+c)}+6720ia\,e^{6i(dx+c)}+1344ia\,e^{4i(dx}{840d\left(e^{2i(dx+c)}-1\right)^7}$
norman	$\frac{-\frac{a}{896d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2240d}+\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{3a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{320d}-\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2240d}}{\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+b*(-
1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c
)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]
time = 0.27, size = 118, normalized size = 1.04

$$\frac{35\,b\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{96\left(7\,\tan(dx+c)^2+5\right)a}{\tan(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/3360*(35*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 96*(7*tan(d*x + c)^2 + 5)*a/tan(d*x + c)
^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(102) = 204$.

time = 0.36, size = 221, normalized size = 1.94

$$-\frac{192\,a\cos\left(dx+c\right)^7-672\,a\cos\left(dx+c\right)^5+105\left(b\cos\left(dx+c\right)^6-3\,b\cos\left(dx+c\right)^4+3\,b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\left(b\cos\left(dx+c\right)^6-3\,b\cos\left(dx+c\right)^4+3\,b\cos\left(dx+c\right)^2-b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-70\left(3\,b\cos\left(dx+c\right)^5+8\,b\cos\left(dx+c\right)^3-3\,b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3360\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3360*(192*a*cos(d*x + c)^7 - 672*a*cos(d*x + c)^5 + 105*(b*cos(d*x + c)^
6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 105*(b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 + 3*b*cos(d*x +
 c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(3*b*cos(d*x + c)
^5 + 8*b*cos(d*x + c)^3 - 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(102) = 204.
time = 0.53, size = 229, normalized size = 2.01

$$\frac{15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+840\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2178\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+315\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*(15*a*tan(1/2*d*x + 1/2*c)^7 + 35*b*tan(1/2*d*x + 1/2*c)^6 - 21*a*t
an(1/2*d*x + 1/2*c)^5 - 105*b*tan(1/2*d*x + 1/2*c)^4 - 105*a*tan(1/2*d*x +
1/2*c)^3 - 105*b*tan(1/2*d*x + 1/2*c)^2 + 840*b*log(abs(tan(1/2*d*x + 1/2*c
))) + 315*a*tan(1/2*d*x + 1/2*c) - (2178*b*tan(1/2*d*x + 1/2*c)^7 + 315*a*t
an(1/2*d*x + 1/2*c)^6 - 105*b*tan(1/2*d*x + 1/2*c)^5 - 105*a*tan(1/2*d*x +
1/2*c)^4 - 105*b*tan(1/2*d*x + 1/2*c)^3 - 21*a*tan(1/2*d*x + 1/2*c)^2 + 35*
b*tan(1/2*d*x + 1/2*c) + 15*a)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [**B**]
time = 10.99, size = 399, normalized size = 3.50

$$\frac{15\,a\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-15\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-21\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-105\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+315\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-315\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+105\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+21\,a\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-105\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-105\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+105\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+105\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+35\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}-35\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+840\,b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{13440\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^8,x)
```

```
[Out] (15*a*sin(c/2 + (d*x)/2)^14 - 15*a*cos(c/2 + (d*x)/2)^14 - 21*a*cos(c/2 + (
d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 105*a*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^10 + 315*a*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 315*a*cos(c/2 +
 (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 105*a*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d
*x)/2)^4 + 21*a*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 - 105*b*cos(c/2
+ (d*x)/2)^3*sin(c/2 + (d*x)/2)^11 - 105*b*cos(c/2 + (d*x)/2)^5*sin(c/2 + (
d*x)/2)^9 + 105*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 105*b*cos(c/2
 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 35*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*
x)/2)^13 - 35*b*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) + 840*b*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)
/(13440*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)
```

3.1104 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{b\cot^5(c+dx)}{5d}-\frac{b\cot^7(c+dx)}{7d}-\frac{3a\cot(c+dx)\csc(c+dx)}{128d}-\frac{a\cot(c+dx)\csc^3(c}{64d}$$

```
[Out] -3/128*a*arctanh(cos(d*x+c))/d-1/5*b*cot(d*x+c)^5/d-1/7*b*cot(d*x+c)^7/d-3/
128*a*cot(d*x+c)*csc(d*x+c)/d-1/64*a*cot(d*x+c)*csc(d*x+c)^3/d+1/16*a*cot(d
*x+c)*csc(d*x+c)^5/d-1/8*a*cot(d*x+c)^3*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2917, 2691, 3853, 3855, 2687, 14}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{64d}-\frac{3a\cot(c+dx)\csc(c+dx)}{128d}-\frac{b\cot^7(c+dx)}{7d}-\frac{b\cot^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(128*d) - (b*Cot[c + d*x]^5)/(5*d) - (b*Cot[c
+ d*x]^7)/(7*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(128*d) - (a*Cot[c + d*x]
*Csc[c + d*x]^3)/(64*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) - (a*Cot[c
 + d*x]^3*Csc[c + d*x]^5)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx &= a\int \cot^4(c+dx)\csc^5(c+dx)\,dx+b\int \cot^4(c+dx)\csc^4\\
&= -\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{1}{8}(3a)\int \cot^2(c+dx)\csc\\
&= \frac{a\cot(c+dx)\csc^5(c+dx)}{16d}-\frac{a\cot^3(c+dx)\csc^5(c+dx)}{8d}\\
&= -\frac{b\cot^5(c+dx)}{5d}-\frac{b\cot^7(c+dx)}{7d}-\frac{a\cot(c+dx)\csc^3(c}{64d}\\
&= -\frac{b\cot^5(c+dx)}{5d}-\frac{b\cot^7(c+dx)}{7d}-\frac{3a\cot(c+dx)\csc(c}{128d}\\
&= -\frac{3a\tanh^{-1}(\cos(c+dx))}{128d}-\frac{b\cot^5(c+dx)}{5d}-\frac{b\cot^7(c+}{7d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 279 vs. 2(136) = 272.
time = 0.06, size = 279, normalized size = 2.05

$$-\frac{2b\cot(c+dx)}{35d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{1024d}+\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{a\csc^8\left(\frac{1}{2}(c+dx)\right)}{2048d}-\frac{b\cot(c+dx)\csc^2(c+dx)}{35d}+\frac{8b\cot(c+dx)\csc^4(c+dx)}{35d}-\frac{b\cot(c+dx)\csc^6(c+dx)}{7d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{128d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{512d}-\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{1024d}-\frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{512d}+\frac{a\sec^8\left(\frac{1}{2}(c+dx)\right)}{2048d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[c + d*x])/(35*d) - (3*a*Csc[(c + d*x)/2]^2)/(512*d) + (a*Csc[(c +
 d*x)/2]^4)/(1024*d) + (a*Csc[(c + d*x)/2]^6)/(512*d) - (a*Csc[(c + d*x)/2]
^8)/(2048*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) + (8*b*Cot[c + d*x]*C
sc[c + d*x]^4)/(35*d) - (b*Cot[c + d*x]*Csc[c + d*x]^6)/(7*d) - (3*a*Log[Co
s[(c + d*x)/2]])/(128*d) + (3*a*Log[Sin[(c + d*x)/2]])/(128*d) + (3*a*Sec[(
c + d*x)/2]^2)/(512*d) - (a*Sec[(c + d*x)/2]^4)/(1024*d) - (a*Sec[(c + d*x)
/2]^6)/(512*d) + (a*Sec[(c + d*x)/2]^8)/(2048*d)
```

Maple [A]
time = 0.23, size = 156, normalized size = 1.15

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right)}{d}$
risch	$\frac{105a\,e^{15i(dx+c)}+8960ib\,e^{12i(dx+c)}-805a\,e^{13i(dx+c)}-11655a\,e^{11i(dx+c)}+8960ib\,e^{8i(dx+c)}-23485a\,e^{9i(dx+c)}-14336ib\,e}{2240d(e^{2i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/64/
sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c)^
3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x+c)))+b*(-1/7/sin(d*x+c)^7*co
s(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5))
```

Maxima [A]
time = 0.28, size = 138, normalized size = 1.01

$$\frac{35\,a\left(\frac{2\left(3\,\cos(dx+c)^7-11\,\cos(dx+c)^5-11\,\cos(dx+c)^3+3\,\cos(dx+c)\right)}{\cos(dx+c)^8-4\,\cos(dx+c)^6+6\,\cos(dx+c)^4-4\,\cos(dx+c)^2+1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{256\left(7\,\tan(dx+c)^2+5\right)b}{\tan(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8960*(35*a*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3 +
 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*
cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) -
256*(7*tan(d*x + c)^2 + 5)*b/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.36, size = 239, normalized size = 1.76

$$\frac{210\,a\cos\left(dx+c\right)^{7}-770\,a\cos\left(dx+c\right)^{5}-770\,a\cos\left(dx+c\right)^{3}+210\,a\cos\left(dx+c\right)-105\,\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+105\,\left(a\cos\left(dx+c\right)^{8}-4\,a\cos\left(dx+c\right)^{6}+6\,a\cos\left(dx+c\right)^{4}-4\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+256\,\left(2\,b\cos\left(dx+c\right)^{7}-7\,b\cos\left(dx+c\right)^{5}\right)\sin\left(dx+c\right)}{8960\,\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8960*(210*a*cos(d*x + c)^7 - 770*a*cos(d*x + c)^5 - 770*a*cos(d*x + c)^3
+ 210*a*cos(d*x + c) - 105*(a*cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos
(d*x + c)^4 - 4*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2) + 105*(a*
cos(d*x + c)^8 - 4*a*cos(d*x + c)^6 + 6*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)
^2 + a)*log(-1/2*cos(d*x + c) + 1/2) + 256*(2*b*cos(d*x + c)^7 - 7*b*cos(d*
x + c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*
x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 201, normalized size = 1.48

$$\frac{35\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-112\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-560\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+1680\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+1680\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4566\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+1680\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-560\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-112\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+35\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}}}{71680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/71680*(35*a*tan(1/2*d*x + 1/2*c)^8 + 80*b*tan(1/2*d*x + 1/2*c)^7 - 112*b*
tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 560*b*tan(1/2*d*x +
 1/2*c)^3 + 1680*a*log(abs(tan(1/2*d*x + 1/2*c))) + 1680*b*tan(1/2*d*x + 1/
2*c) - (4566*a*tan(1/2*d*x + 1/2*c)^8 + 1680*b*tan(1/2*d*x + 1/2*c)^7 - 560
*b*tan(1/2*d*x + 1/2*c)^5 - 280*a*tan(1/2*d*x + 1/2*c)^4 - 112*b*tan(1/2*d*
x + 1/2*c)^3 + 80*b*tan(1/2*d*x + 1/2*c) + 35*a)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 9.88, size = 205, normalized size = 1.51

$$\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{256\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{128\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(6\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{5}+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7}+\frac{a}{8}\right)}{256\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x)))/sin(c + d*x)^9,x)

[Out] (3*b*tan(c/2 + (d*x)/2))/(128*d) - (a*tan(c/2 + (d*x)/2)^4)/(256*d) + (a*ta
n(c/2 + (d*x)/2)^8)/(2048*d) - (b*tan(c/2 + (d*x)/2)^3)/(128*d) - (b*tan(c/
2 + (d*x)/2)^5)/(640*d) + (b*tan(c/2 + (d*x)/2)^7)/(896*d) - (cot(c/2 + (d*
x)/2)^8*(a/8 + (2*b*tan(c/2 + (d*x)/2))/7 - a*tan(c/2 + (d*x)/2)^4 - (2*b*t
an(c/2 + (d*x)/2)^3)/5 - 2*b*tan(c/2 + (d*x)/2)^5 + 6*b*tan(c/2 + (d*x)/2)^
7))/(256*d) + (3*a*log(tan(c/2 + (d*x)/2)))/(128*d)

3.1105 $\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=301

$$\frac{3abx}{64}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}+\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{3ab\cos(c+dx)\sin(c+dx)}{64d}-\frac{ab\cos(c+dx)\sin^3(c}{32d}$$

```
[Out] 3/64*a*b*x-1/105*(9*a^2+4*b^2)*cos(d*x+c)/d+1/315*(9*a^2+4*b^2)*cos(d*x+c)^
3/d-3/64*a*b*cos(d*x+c)*sin(d*x+c)/d-1/32*a*b*cos(d*x+c)*sin(d*x+c)^3/d-1/6
30*(15*a^4-44*a^2*b^2+6*b^4)*cos(d*x+c)*sin(d*x+c)^4/b^2/d-1/504*a*(10*a^2-
29*b^2)*cos(d*x+c)*sin(d*x+c)^5/b/d-5/252*(3*a^2-8*b^2)*cos(d*x+c)*sin(d*x+
c)^4*(a+b*sin(d*x+c))^2/b^2/d+1/12*a*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c
))^3/b^2/d-1/9*cos(d*x+c)*sin(d*x+c)^5*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A]
time = 0.41, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2974, 3128, 3112, 3102, 2827, 2713, 2715, 8}

$$\frac{(9a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{(9a^2+4b^2)\cos(c+dx)}{105d}-\frac{a(10a^2-29b^2)\sin^5(c+dx)\cos(c+dx)}{504bd}-\frac{5(3a^2-8b^2)\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{252b^2d}-\frac{(15a^4-44a^2b^2+6b^4)\sin^4(c+dx)\cos(c+dx)}{630b^2d}+\frac{a\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{12b^2d}-\frac{\sin^5(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{9bd}-\frac{ab\sin^3(c+dx)\cos(c+dx)}{32d}-\frac{3ab\sin(c+dx)\cos(c+dx)}{64d}+\frac{3abx}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/64 - ((9*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + ((9*a^2 + 4*b^2)*Co
s[c + d*x]^3)/(315*d) - (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(64*d) - (a*b*Cos
[c + d*x]*Sin[c + d*x]^3)/(32*d) - ((15*a^4 - 44*a^2*b^2 + 6*b^4)*Cos[c + d
*x]*Sin[c + d*x]^4)/(630*b^2*d) - (a*(10*a^2 - 29*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^5)/(504*b*d) - (5*(3*a^2 - 8*b^2)*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*
Sin[c + d*x])^2)/(252*b^2*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c
+ d*x])^3)/(12*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^3
)/(9*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{a\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^3}{12b^2d} - \frac{\cos(c-}{} \\
&= -\frac{5(3a^2-8b^2)\cos(c+dx)\sin^4(c+dx)(a+b\sin(c+dx}{252b^2d} \\
&= -\frac{a(10a^2-29b^2)\cos(c+dx)\sin^5(c+dx)}{504bd} - \frac{5(3a^2-8b}{} \\
&= -\frac{(15a^4-44a^2b^2+6b^4)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a(1}{} \\
&= -\frac{(15a^4-44a^2b^2+6b^4)\cos(c+dx)\sin^4(c+dx)}{630b^2d} - \frac{a(1}{} \\
&= -\frac{ab\cos(c+dx)\sin^3(c+dx)}{32d} - \frac{(15a^4-44a^2b^2+6b^4)\,c}{630} \\
&= -\frac{(9a^2+4b^2)\cos(c+dx)}{105d} + \frac{(9a^2+4b^2)\cos^3(c+dx)}{315d} - \\
&= \frac{3abx}{64} - \frac{(9a^2+4b^2)\cos(c+dx)}{105d} + \frac{(9a^2+4b^2)\cos^3(c+}{315d}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 144, normalized size = 0.48

$$\frac{7560abc+7560abdx-3780(2a^2+b^2)\cos(c+dx)-840(3a^2+b^2)\cos(3(c+dx))+504a^2\cos(5(c+dx))+504b^2\cos(5(c+dx))+360a^2\cos(7(c+dx))+90b^2\cos(7(c+dx))-70b^2\cos(9(c+dx))-2520ab\sin(4(c+dx))+315ab\sin(8(c+dx))}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (7560*a*b*c + 7560*a*b*d*x - 3780*(2*a^2 + b^2)*Cos[c + d*x] - 840*(3*a^2 +
 b^2)*Cos[3*(c + d*x)] + 504*a^2*Cos[5*(c + d*x)] + 504*b^2*Cos[5*(c + d*x)
```

] + 360*a^2*Cos[7*(c + d*x)] + 90*b^2*Cos[7*(c + d*x)] - 70*b^2*Cos[9*(c + d*x)] - 2520*a*b*Sin[4*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)])/(161280*d)

Maple [A]

time = 0.43, size = 161, normalized size = 0.53

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2\left(\cos^5(dx+c)\right)}{35}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^5(dx+c)\right)}{8}-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{16}+\frac{\left(\cos^3(dx\right.}{}\right.}{d}$
risch	$\frac{3abx}{64}-\frac{3a^2\cos(dx+c)}{64d}-\frac{3b^2\cos(dx+c)}{128d}-\frac{b^2\cos(9dx+9c)}{2304d}+\frac{ab\sin(8dx+8c)}{512d}+\frac{a^2\cos(7dx+7c)}{448d}+\frac{b^2\cos(7dx+}{1792d}$
norman	$\frac{13ab\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}-\frac{155ab\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{27abx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{27abx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{63abx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{189ab}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+2*a*b*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^2*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x+c)^5))

Maxima [A]

time = 0.29, size = 100, normalized size = 0.33

$$\frac{4608\left(5\cos\left(dx+c\right)^7-7\cos\left(dx+c\right)^5\right)a^2+315\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\sin\left(4\,dx+4\,c\right)\right)ab-512\left(35\cos\left(dx+c\right)^9-90\cos\left(dx+c\right)^7+63\cos\left(dx+c\right)^5\right)b^2}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/161280*(4608*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2 + 315*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b - 512*(35*cos(d*x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^2)/d

Fricas [A]

time = 0.41, size = 116, normalized size = 0.39

$$-\frac{2240\,b^2\cos\left(dx+c\right)^9-2880\left(a^2+2\,b^2\right)\cos\left(dx+c\right)^7+4032\left(a^2+b^2\right)\cos\left(dx+c\right)^5-945\,abdx-315\left(16\,ab\cos\left(dx+c\right)^7-24\,ab\cos\left(dx+c\right)^5+2\,ab\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{20160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/20160*(2240*b^2*cos(d*x + c)^9 - 2880*(a^2 + 2*b^2)*cos(d*x + c)^7 + 403
2*(a^2 + b^2)*cos(d*x + c)^5 - 945*a*b*d*x - 315*(16*a*b*cos(d*x + c)^7 - 2
4*a*b*cos(d*x + c)^5 + 2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x +
 c))/d
```

Sympy [A]
time = 1.33, size = 335, normalized size = 1.11

$$\begin{cases} - \frac{a^{2} \sin^{2}{\left(c + d x \right)} \cos^{5}{\left(c + d x \right)}}{5 d} - \frac{2 a^{2} \cos^{7}{\left(c + d x \right)}}{35 d} + \frac{3 a b x \sin^{8}{\left(c + d x \right)}}{64} + \frac{3 a b x \sin^{6}{\left(c + d x \right)} \cos^{2}{\left(c + d x \right)}}{16} + \frac{9 a b x \sin^{4}{\left(c + d x \right)} \cos^{4}{\left(c + d x \right)}}{32} + \frac{3 a b x \sin^{2}{\left(c + d x \right)} \cos^{6}{\left(c + d x \right)}}{16} + \frac{3 a b x \cos^{8}{\left(c + d x \right)}}{64} + \frac{3 a b \sin^{7}{\left(c + d x \right)} \cos{\left(c + d x \right)}}{64 d} + \frac{11 a b \sin^{5}{\left(c + d x \right)} \cos^{3}{\left(c + d x \right)}}{64 d} - \frac{11 a b \sin^{3}{\left(c + d x \right)} \cos^{5}{\left(c + d x \right)}}{64 d} - \frac{3 a b \sin{\left(c + d x \right)} \cos^{7}{\left(c + d x \right)}}{64 d} - \frac{b^{2} \sin^{4}{\left(c + d x \right)} \cos^{5}{\left(c + d x \right)}}{5 d} - \frac{4 b^{2} \sin^{2}{\left(c + d x \right)} \cos^{7}{\left(c + d x \right)}}{35 d} - \frac{8 b^{2} \cos^{9}{\left(c + d x \right)}}{315 d} & \text{for}\: d \neq 0 \\x \left(a + b \sin{\left(c \right)}\right)^{2} \sin^{3}{\left(c \right)} \cos^{4}{\left(c \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*a**2*cos(c + d*x
)**7/(35*d) + 3*a*b*x*sin(c + d*x)**8/64 + 3*a*b*x*sin(c + d*x)**6*cos(c +
d*x)**2/16 + 9*a*b*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 3*a*b*x*sin(c + d
*x)**2*cos(c + d*x)**6/16 + 3*a*b*x*cos(c + d*x)**8/64 + 3*a*b*sin(c + d*x)
**7*cos(c + d*x)/(64*d) + 11*a*b*sin(c + d*x)**5*cos(c + d*x)**3/(64*d) - 1
1*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(64*d) - 3*a*b*sin(c + d*x)*cos(c + d
*x)**7/(64*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**2*sin(c +
 d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**2*cos(c + d*x)**9/(315*d), Ne(d, 0))
, (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**4, True))
```

Giac [A]
time = 0.67, size = 142, normalized size = 0.47

$$\frac{3}{64} \, a b x - \frac{b^{2} \cos\left(9 \, d x + 9 \, c\right)}{2304 \, d} + \frac{a b \sin\left(8 \, d x + 8 \, c\right)}{512 \, d} - \frac{a b \sin\left(4 \, d x + 4 \, c\right)}{64 \, d} + \frac{{\left(4 \, a^{2} + b^{2}\right)} \cos\left(7 \, d x + 7 \, c\right)}{1792 \, d} + \frac{{\left(a^{2} + b^{2}\right)} \cos\left(5 \, d x + 5 \, c\right)}{320 \, d} - \frac{{\left(3 \, a^{2} + b^{2}\right)} \cos\left(3 \, d x + 3 \, c\right)}{192 \, d} - \frac{3 \, {\left(2 \, a^{2} + b^{2}\right)} \cos\left(d x + c\right)}{128 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/64*a*b*x - 1/2304*b^2*cos(9*d*x + 9*c)/d + 1/512*a*b*sin(8*d*x + 8*c)/d -
 1/64*a*b*sin(4*d*x + 4*c)/d + 1/1792*(4*a^2 + b^2)*cos(7*d*x + 7*c)/d + 1/
320*(a^2 + b^2)*cos(5*d*x + 5*c)/d - 1/192*(3*a^2 + b^2)*cos(3*d*x + 3*c)/d
 - 3/128*(2*a^2 + b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 12.92, size = 309, normalized size = 1.03

$$\frac{3\,a\,b\,x}{64}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(4\,a^2-16\,b^2\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(4\,a^2+\frac{32\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{28\,a^2}{5}-\frac{32\,b^2}{5}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{36\,a^2}{35}+\frac{16\,b^2}{35}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{4\,a^2}{35}+\frac{64\,b^2}{35}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{52\,a^2}{5}+\frac{112\,b^2}{5}\right)+4\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+\frac{4\,a^2}{35}+\frac{16\,b^2}{315}+\frac{13\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{16}-\frac{155\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{16}+\frac{169\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{16}-\frac{169\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{16}+\frac{155\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{16}-\frac{13\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{16}-\frac{3\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{32}+\frac{3\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a*b*x)/64 - (tan(c/2 + (d*x)/2)^10*(4*a^2 - 16*b^2) + tan(c/2 + (d*x)/2)
^12*(4*a^2 + (32*b^2)/3) + tan(c/2 + (d*x)/2)^6*((28*a^2)/5 - (32*b^2)/5) +
 tan(c/2 + (d*x)/2)^2*((36*a^2)/35 + (16*b^2)/35) + tan(c/2 + (d*x)/2)^4*((
4*a^2)/35 + (64*b^2)/35) + tan(c/2 + (d*x)/2)^8*((52*a^2)/5 + (112*b^2)/5)
+ 4*a^2*tan(c/2 + (d*x)/2)^14 + (4*a^2)/35 + (16*b^2)/315 + (13*a*b*tan(c/2
 + (d*x)/2)^3)/16 - (155*a*b*tan(c/2 + (d*x)/2)^5)/16 + (169*a*b*tan(c/2 +
(d*x)/2)^7)/16 - (169*a*b*tan(c/2 + (d*x)/2)^11)/16 + (155*a*b*tan(c/2 + (d
*x)/2)^13)/16 - (13*a*b*tan(c/2 + (d*x)/2)^15)/16 - (3*a*b*tan(c/2 + (d*x)/
2)^17)/32 + (3*a*b*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.1106 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=278

$$\frac{1}{128}\left(8a^2+3b^2\right)x-\frac{6ab\cos(c+dx)}{35d}+\frac{2ab\cos^3(c+dx)}{35d}-\frac{\left(8a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{128d}-\frac{(40a^4-140a^2b^2$$

```
[Out] 1/128*(8*a^2+3*b^2)*x-6/35*a*b*cos(d*x+c)/d+2/35*a*b*cos(d*x+c)^3/d-1/128*(
8*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/1344*(40*a^4-140*a^2*b^2+21*b^4)*cos
(d*x+c)*sin(d*x+c)^3/b^2/d-1/840*a*(20*a^2-69*b^2)*cos(d*x+c)*sin(d*x+c)^4/
b/d-1/336*(20*a^2-63*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/b^2/d+
5/56*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^3/b^2/d-1/8*cos(d*x+c)*sin(
d*x+c)^4*(a+b*sin(d*x+c))^3/b/d
```

Rubi [A]
time = 0.41, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2974, 3128, 3112, 3102, 2827, 2715, 8, 2713}

$$-\frac{a(20a^2-69b^2)\sin^4(c+dx)\cos(c+dx)}{840bd}-\frac{(20a^2-63b^2)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{336b^2d}-\frac{(8a^2+3b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{1}{128}x(8a^2+3b^2)-\frac{(40a^4-140a^2b^2+21b^4)\sin^3(c+dx)\cos(c+dx)}{1344b^2d}+\frac{5a\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{56b^2d}+\frac{2ab\cos^3(c+dx)}{35d}-\frac{6ab\cos(c+dx)}{35d}-\frac{\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((8*a^2 + 3*b^2)*x)/128 - (6*a*b*Cos[c + d*x])/(35*d) + (2*a*b*Cos[c + d*x]
^3)/(35*d) - ((8*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) - ((40*a^4
 - 140*a^2*b^2 + 21*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(1344*b^2*d) - (a*(20
*a^2 - 69*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(840*b*d) - ((20*a^2 - 63*b^2)*
Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(336*b^2*d) + (5*a*Cos[
c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(56*b^2*d) - (Cos[c + d*x]*
Sin[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(8*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
```

```
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^3}{56b^2d} - \frac{\cos(c}{}$$

$$= -\frac{(20a^2-63b^2)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+a}{336b^2d}$$

$$= -\frac{a(20a^2-69b^2)\cos(c+dx)\sin^4(c+dx)}{840bd} - \frac{(20a^2-63}{}$$

$$= -\frac{(40a^4-140a^2b^2+21b^4)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a}{}$$

$$= -\frac{(40a^4-140a^2b^2+21b^4)\cos(c+dx)\sin^3(c+dx)}{1344b^2d} - \frac{a}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sin(c+dx)}{128d} - \frac{(40a^4-140a^2b}{}$$

$$= \frac{1}{128}\left(8a^2+3b^2\right)x - \frac{6ab\cos(c+dx)}{35d} + \frac{2ab\cos^3(c+dx)}{35d}$$

Mathematica [A]
time = 0.44, size = 141, normalized size = 0.51

$$\frac{1680b^2c+3360a^2dx+1260b^2dx-5040ab\cos(c+dx)-1680ab\cos(3(c+dx))+336ab\cos(5(c+dx))+240ab\cos(7(c+dx))+840a^2\sin(2(c+dx))-840a^2\sin(4(c+dx))-420b^2\sin(4(c+dx))-280a^2\sin(6(c+dx))+\frac{105}{2}b^2\sin(8(c+dx))}{53760d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (1680*b^2*c + 3360*a^2*d*x + 1260*b^2*d*x - 5040*a*b*Cos[c + d*x] - 1680*a*
b*Cos[3*(c + d*x)] + 336*a*b*Cos[5*(c + d*x)] + 240*a*b*Cos[7*(c + d*x)] +
840*a^2*Sin[2*(c + d*x)] - 840*a^2*Sin[4*(c + d*x)] - 420*b^2*Sin[4*(c + d*
x)] - 280*a^2*Sin[6*(c + d*x)] + (105*b^2*Sin[8*(c + d*x)])/2)/(53760*d)
```

Maple [A]
time = 0.34, size = 163, normalized size = 0.59

method	result
risch	$\frac{a^2x}{16}+\frac{3b^2x}{128}-\frac{3ab\cos(dx+c)}{32d}+\frac{b^2\sin(8dx+8c)}{1024d}+\frac{ab\cos(7dx+7c)}{224d}-\frac{a^2\sin(6dx+6c)}{192d}+\frac{ab\cos(5dx+5c)}{160d}-\frac{a^2}{}$
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2(\cos^5}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}-\frac{2(\cos^5}{}\right.}{d}$
norman	$\frac{-\frac{8ab}{35d}+\frac{\left(8a^2+3b^2\right)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\left(\frac{a^2}{2}+\frac{3b^2}{16}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{7a^2}{4}+\frac{21b^2}{32}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{7a^2}{2}+\frac{21b^2}{16}\right)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d
*x+c)^5)+b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1
/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c))
```

Maxima [A]

time = 0.27, size = 101, normalized size = 0.36

$$\frac{560\,(4\,\sin(2\,dx+2\,c)^3+12\,dx+12\,c-3\,\sin(4\,dx+4\,c))a^2+6144\,(5\,\cos(dx+c)^7-7\,\cos(dx+c)^5)ab+105\,(24\,dx+24\,c+\sin(8\,dx+8\,c)-8\,\sin(4\,dx+4\,c))b^2}{107520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/107520*(560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a
^2 + 6144*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a*b + 105*(24*d*x + 24*c +
sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]

time = 0.39, size = 128, normalized size = 0.46

$$\frac{3840\,ab\cos(dx+c)^7-5376\,ab\cos(dx+c)^5+105\,(8\,a^2+3\,b^2)dx+35\,(48\,b^2\cos(dx+c)^7-8\,(8\,a^2+9\,b^2)\cos(dx+c)^5+2\,(8\,a^2+3\,b^2)\cos(dx+c)^3+3\,(8\,a^2+3\,b^2)\cos(dx+c))\sin(dx+c)}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/13440*(3840*a*b*cos(d*x + c)^7 - 5376*a*b*cos(d*x + c)^5 + 105*(8*a^2 + 3
*b^2)*d*x + 35*(48*b^2*cos(d*x + c)^7 - 8*(8*a^2 + 9*b^2)*cos(d*x + c)^5 +
```

```
2*(8*a^2 + 3*b^2)*cos(d*x + c)^3 + 3*(8*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [A]
time = 0.95, size = 420, normalized size = 1.51

$$\begin{cases} \frac{a^2 x \sin^6(c+dx)}{16} + \frac{3a^2 x \sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^2 x \sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^2 x \cos^6(c+dx)}{16} + \frac{a^2 \sin^5(c+dx)\cos(c+dx)}{16d} + \frac{a^2 \sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{a^2 \sin(c+dx)\cos^5(c+dx)}{16d} - \frac{2ab \sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{4ab\cos^7(c+dx)}{35d} + \frac{3b^2 x \sin^8(c+dx)}{128} + \frac{3b^2 x \sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{9b^2 x \sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{3b^2 x \sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{3b^2 x \cos^8(c+dx)}{128} + \frac{3b^2 \sin^7(c+dx)\cos(c+dx)}{128d} + \frac{11b^2 \sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{11b^2 \sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{3b^2 \sin(c+dx)\cos^7(c+dx)}{128d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2 \sin^2(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**6/16 + 3*a**2*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**2*x*cos(c + d*x)
**6/16 + a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**2*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 2*a*b*sin(
c + d*x)**2*cos(c + d*x)**5/(5*d) - 4*a*b*cos(c + d*x)**7/(35*d) + 3*b**2*x
*sin(c + d*x)**8/128 + 3*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**2
*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*
x)**6/32 + 3*b**2*x*cos(c + d*x)**8/128 + 3*b**2*sin(c + d*x)**7*cos(c + d*
x)/(128*d) + 11*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**2*sin(
c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**7/(
128*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**4, True))
```

Giac [A]
time = 0.66, size = 150, normalized size = 0.54

$$\frac{1}{128}(8a^2+3b^2)x + \frac{ab\cos(7dx+7c)}{224d} + \frac{ab\cos(5dx+5c)}{160d} - \frac{ab\cos(3dx+3c)}{32d} - \frac{3ab\cos(dx+c)}{32d} + \frac{b^2\sin(8dx+8c)}{1024d} - \frac{a^2\sin(6dx+6c)}{192d} + \frac{a^2\sin(2dx+2c)}{64d} - \frac{(2a^2+b^2)\sin(4dx+4c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/128*(8*a^2 + 3*b^2)*x + 1/224*a*b*cos(7*d*x + 7*c)/d + 1/160*a*b*cos(5*d*
x + 5*c)/d - 1/32*a*b*cos(3*d*x + 3*c)/d - 3/32*a*b*cos(d*x + c)/d + 1/1024
*b^2*sin(8*d*x + 8*c)/d - 1/192*a^2*sin(6*d*x + 6*c)/d + 1/64*a^2*sin(2*d*x
 + 2*c)/d - 1/128*(2*a^2 + b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 9.94, size = 139, normalized size = 0.50

$$\frac{210a^2\sin(2c+2dx) - 210a^2\sin(4c+4dx) - 70a^2\sin(6c+6dx) - 105b^2\sin(4c+4dx) + \frac{105b^2\sin(8c+8dx)}{8} - 1260ab\cos(c+dx) - 420ab\cos(3c+3dx) + 84ab\cos(5c+5dx) + 60ab\cos(7c+7dx) + 840a^2dx + 315b^2dx}{13440d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (210*a^2*sin(2*c + 2*d*x) - 210*a^2*sin(4*c + 4*d*x) - 70*a^2*sin(6*c + 6*d
*x) - 105*b^2*sin(4*c + 4*d*x) + (105*b^2*sin(8*c + 8*d*x))/8 - 1260*a*b*co
s(c + d*x) - 420*a*b*cos(3*c + 3*d*x) + 84*a*b*cos(5*c + 5*d*x) + 60*a*b*co
s(7*c + 7*d*x) + 840*a^2*d*x + 315*b^2*d*x)/(13440*d)
```

3.1107 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{abx}{8}-\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{12d}-\frac{a\cos^5(c+dx)(a+}{21d}$$

```
[Out] 1/8*a*b*x-1/105*(a^2+6*b^2)*cos(d*x+c)^5/d+1/8*a*b*cos(d*x+c)*sin(d*x+c)/d+
1/12*a*b*cos(d*x+c)^3*sin(d*x+c)/d-1/21*a*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-1
/7*cos(d*x+c)^5*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2941, 2748, 2715, 8}

$$-\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}-\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{ab\sin(c+dx)\cos(c+dx)}{8d}+\frac{abx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*b*x)/8 - ((a^2 + 6*b^2)*Cos[c + d*x]^5)/(105*d) + (a*b*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(12*d) - (a*Cos[c + d*x
]^5*(a + b*Sin[c + d*x]))/(21*d) - (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/
(7*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^2}{7d}+\frac{1}{7}\int \cos^4(c+dx)(2$$

$$= -\frac{a\cos^5(c+dx)(a+b\sin(c+dx))}{21d}-\frac{\cos^5(c+dx)(a+b}{7d}$$

$$= -\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}-\frac{a\cos^5(c+dx)(a+b\sin(c+}{21d}$$

$$= -\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}+\frac{ab\cos^3(c+dx)\sin(c+dx)}{12d}$$

$$= -\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+dx)}{8d}+$$

$$= \frac{abx}{8}-\frac{(a^2+6b^2)\cos^5(c+dx)}{105d}+\frac{ab\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A]
time = 0.32, size = 132, normalized size = 1.02

$$\frac{840abc+840abdx-105(8a^2+3b^2)\cos(c+dx)-105(4a^2+b^2)\cos(3(c+dx))-84a^2\cos(5(c+dx))+21b^2\cos(5(c+dx))+15b^2\cos(7(c+dx))+210ab\sin(2(c+dx))-210ab\sin(4(c+dx))-70ab\sin(6(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (840*a*b*c + 840*a*b*d*x - 105*(8*a^2 + 3*b^2)*Cos[c + d*x] - 105*(4*a^2 +
b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] + 21*b^2*Cos[5*(c + d*x)] +
 15*b^2*Cos[7*(c + d*x)] + 210*a*b*Sin[2*(c + d*x)] - 210*a*b*Sin[4*(c + d*
x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A]
time = 0.26, size = 105, normalized size = 0.81

method	result

derivativedivides	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5}+2ab\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)(\cdots}{7}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5}+2ab\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)(\cdots}{7}\right.}{d}$
risch	$\frac{abx}{8}-\frac{a^2\cos(dx+c)}{8d}-\frac{3b^2\cos(dx+c)}{64d}+\frac{b^2\cos(7dx+7c)}{448d}-\frac{ab\sin(6dx+6c)}{96d}-\frac{a^2\cos(5dx+5c)}{80d}+\frac{\cos(5dx+5c)b\cdots}{320d}$
norman	$\frac{-\frac{14a^2+4b^2}{35d}+\frac{abx}{8}-\frac{2a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(4a^2+4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{\left(4a^2+4b^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(6a^2-4b^2\right)\left(\tan^8\left(\frac{dx}{2}+\cdots\right.\right.}{d}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^2*cos(d*x+c)^5+2*a*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*
x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+b^2*(-1/7*sin(d*x+c)^2*c
os(d*x+c)^5-2/35*cos(d*x+c)^5))
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.63

$$-\frac{672\,a^2\cos\left(dx+c\right)^5-35\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)ab-96\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)b^2}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(672*a^2*cos(d*x + c)^5 - 35*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c
- 3*sin(4*d*x + 4*c))*a*b - 96*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^2)/d
```

Fricas [A]
time = 0.36, size = 85, normalized size = 0.66

$$\frac{120\,b^2\cos\left(dx+c\right)^7-168\left(a^2+b^2\right)\cos\left(dx+c\right)^5+105\,abdx-35\left(8\,ab\cos\left(dx+c\right)^5-2\,ab\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + b^2)*cos(d*x + c)^5 + 105*a*b*d*
x - 35*(8*a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*s
in(d*x + c))/d
```

Sympy [A]
time = 0.63, size = 223, normalized size = 1.73

$$\begin{cases} -\frac{a^2\cos^5(c+dx)}{5d}+\frac{abx\sin^6(c+dx)}{8}+\frac{3abx\sin^4(c+dx)\cos^2(c+dx)}{8}+\frac{3abx\sin^2(c+dx)\cos^4(c+dx)}{8}+\frac{abx\cos^6(c+dx)}{8}+\frac{ab\sin^5(c+dx)\cos(c+dx)}{8d}+\frac{ab\sin^3(c+dx)\cos^3(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos^5(c+dx)}{8d}-\frac{b^2\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{2b^2\cos^7(c+dx)}{35d} & \text{for } d\neq 0 \\ x(a+b\sin(c))^2\sin(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**5/(5*d) + a*b*x*sin(c + d*x)**6/8 + 3*a*b*x*
sin(c + d*x)**4*cos(c + d*x)**2/8 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**4
/8 + a*b*x*cos(c + d*x)**6/8 + a*b*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a*b
*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - a*b*sin(c + d*x)*cos(c + d*x)**5/(
8*d) - b**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 2*b**2*cos(c + d*x)**7/
(35*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**4, True))
```

Giac [A]
time = 0.51, size = 141, normalized size = 1.09

$$\frac{1}{8}\,abx + \frac{b^2\cos\left(7\,dx + 7\,c\right)}{448\,d} - \frac{ab\sin\left(6\,dx + 6\,c\right)}{96\,d} - \frac{ab\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{ab\sin\left(2\,dx + 2\,c\right)}{32\,d} - \frac{\left(4\,a^2 - b^2\right)\cos\left(5\,dx + 5\,c\right)}{320\,d} - \frac{\left(4\,a^2 + b^2\right)\cos\left(3\,dx + 3\,c\right)}{64\,d} - \frac{\left(8\,a^2 + 3\,b^2\right)\cos\left(dx + c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/
32*a*b*sin(4*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 - b^
2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^2 + b^2)*cos(3*d*x + 3*c)/d - 1/64*(8*a^2
 + 3*b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 13.01, size = 256, normalized size = 1.98

$$\frac{a\,b\,x}{8} - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(6\,a^2-4\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(4\,a^2+4\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{4\,a^2}{5}+\frac{4\,b^2}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(8\,a^2+8\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{22\,a^2}{5}-\frac{8\,b^2}{5}\right)+2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+\frac{2\,a^2}{5}+\frac{4\,b^2}{35}-\frac{11\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3}+\frac{31\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{12}-\frac{31\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{12}+\frac{11\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{3}-\frac{a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{4}+\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (a*b*x)/8 - (tan(c/2 + (d*x)/2)^8*(6*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^10*(
4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^2*((4*a^2)/5 + (4*b^2)/5) + tan(c/2 + (
d*x)/2)^6*(8*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^4*((22*a^2)/5 - (8*b^2)/5) +
 2*a^2*tan(c/2 + (d*x)/2)^12 + (2*a^2)/5 + (4*b^2)/35 - (11*a*b*tan(c/2 + (
d*x)/2)^3)/3 + (31*a*b*tan(c/2 + (d*x)/2)^5)/12 - (31*a*b*tan(c/2 + (d*x)/2
)^9)/12 + (11*a*b*tan(c/2 + (d*x)/2)^11)/3 - (a*b*tan(c/2 + (d*x)/2)^13)/4
+ (a*b*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.1108 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=116

$$\frac{3abx}{4}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{b^2\cos^5(c+dx)}{5d}+\frac{3ab\cos(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/4*a*b*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
-1/5*b^2*cos(d*x+c)^5/d+3/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/2*a*b*cos(d*x+c)^
3*sin(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 190, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2974, 3128, 3112, 3102, 2814, 3855}

$$-\frac{(a^2-12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d}-\frac{a(2a^2-27b^2)\sin(c+dx)\cos(c+dx)}{60bd}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{(a^4-14a^2b^2+3b^4)\cos(c+dx)}{15b^2d}+\frac{a\cos(c+dx)(a+b\sin(c+dx))^3}{10b^2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{5bd}+\frac{3abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/4 - (a^2*ArcTanh[Cos[c + d*x]])/d - ((a^4 - 14*a^2*b^2 + 3*b^4)*C
os[c + d*x])/(15*b^2*d) - (a*(2*a^2 - 27*b^2)*Cos[c + d*x]*Sin[c + d*x])/(6
0*b*d) - ((a^2 - 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(30*b^2*d) +
(a*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(10*b^2*d) - (Cos[c + d*x]*Sin[c +
d*x]*(a + b*Sin[c + d*x])^3)/(5*b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a\cos(c+dx)(a+b\sin(c+dx))^3}{10b^2d} - \frac{\cos(c+dx)\sin(c+\dots}{}$$

$$= -\frac{(a^2-12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{30b^2d} + \frac{a\cos(\dots}{}$$

$$= -\frac{a(2a^2-27b^2)\cos(c+dx)\sin(c+dx)}{60bd} - \frac{(a^2-12b^2)\dots}{}$$

$$= -\frac{(a^4-14a^2b^2+3b^4)\cos(c+dx)}{15b^2d} - \frac{a(2a^2-27b^2)\cos\dots}{6\dots}$$

$$= \frac{3abx}{4} - \frac{(a^4-14a^2b^2+3b^4)\cos(c+dx)}{15b^2d} - \frac{a(2a^2-27\dots}{}$$

$$= \frac{3abx}{4} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{(a^4-14a^2b^2+3b^4)\dots}{15b^2d}$$

Mathematica [A]

time = 0.35, size = 125, normalized size = 1.08

$$\frac{30(10a^2-b^2)\cos(c+dx)+5(4a^2-3b^2)\cos(3(c+dx))-3b^2\cos(5(c+dx))+15a\left(4\left(3b(c+dx)-4a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+4a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+8b\sin(2(c+dx))+b\sin(4(c+dx))\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (30*(10*a^2 - b^2)*Cos[c + d*x] + 5*(4*a^2 - 3*b^2)*Cos[3*(c + d*x)] - 3*b^
2*Cos[5*(c + d*x)] + 15*a*(4*(3*b*(c + d*x) - 4*a*Log[Cos[(c + d*x)/2]] + 4
*a*Log[Sin[(c + d*x)/2]]) + 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)]))/(24
0*d)
```

Maple [A]

time = 0.25, size = 93, normalized size = 0.80

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\left(c\dots\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\left(c\dots\right.}{d}$
risch	$\frac{3abx}{4}+\frac{5a^2e^{i(dx+c)}}{8d}-\frac{e^{i(dx+c)}b^2}{16d}+\frac{5a^2e^{-i(dx+c)}}{8d}-\frac{e^{-i(dx+c)}b^2}{16d}-\frac{a^2\ln(e^{i(dx+c)}+1)}{d}+\frac{a^2\ln(e^{i(dx+c)}-1)}{d}-\dots$
norman	$\frac{\frac{(4a^2-2b^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{40a^2-6b^2}{15d}+\frac{3abx}{4}+\frac{12a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{28a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2(22a^2-6b\dots}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/5*cos(d*x+c)^5*b^2)

Maxima [A]
time = 0.28, size = 97, normalized size = 0.84

$$-\frac{48\,b^2\cos\left(dx+c\right)^5-40\,\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^2-15\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)ab}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/240*(48*b^2*cos(d*x + c)^5 - 40*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2 - 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b)/d

Fricas [A]
time = 0.39, size = 112, normalized size = 0.97

$$-\frac{12\,b^2\cos\left(dx+c\right)^5-20\,a^2\cos\left(dx+c\right)^3-45\,abdx-60\,a^2\cos\left(dx+c\right)+30\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-30\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\,\left(2\,ab\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/60*(12*b^2*cos(d*x + c)^5 - 20*a^2*cos(d*x + c)^3 - 45*a*b*d*x - 60*a^2*cos(d*x + c) + 30*a^2*log(1/2*cos(d*x + c) + 1/2) - 30*a^2*log(-1/2*cos(d*x + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^3 + 3*a*b*cos(d*x + c))*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\cos^4\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)

[Out] Integral((a + b*sin(c + d*x))**2*cos(c + d*x)**4*csc(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(106) = 212.

time = 0.49, size = 213, normalized size = 1.84

$$\frac{45\,(dx+c)ab+60\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+60\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-360\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+120\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-280\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-75\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-80\,a^2+12\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(45*(d*x + c)*a*b + 60*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(75*a*b*
tan(1/2*d*x + 1/2*c)^9 - 120*a^2*tan(1/2*d*x + 1/2*c)^8 + 60*b^2*tan(1/2*d*
x + 1/2*c)^8 + 30*a*b*tan(1/2*d*x + 1/2*c)^7 - 360*a^2*tan(1/2*d*x + 1/2*c)
^6 - 440*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*b^2*tan(1/2*d*x + 1/2*c)^4 - 30*a
*b*tan(1/2*d*x + 1/2*c)^3 - 280*a^2*tan(1/2*d*x + 1/2*c)^2 - 75*a*b*tan(1/2
*d*x + 1/2*c) - 80*a^2 + 12*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]

time = 11.12, size = 319, normalized size = 2.75

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\,\left(4\,a^2-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,\left(\frac{44\,a^2}{3}-4\,b^2\right)+\frac{28\,a^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+12\,a^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{8\,a^2}{3}-\frac{2\,b^2}{5}+a\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-a\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{5\,a\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{2}+\frac{5\,a\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{3\,a\,b\,\mathrm{atan}\left(\frac{9\,a^2\,b^2}{4\,\left(3\,a^3\,b-\frac{9\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}\right)}+\frac{3\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\,a^3\,b-\frac{9\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^8*(4*a^2 - 2*b^2) + t
an(c/2 + (d*x)/2)^4*((44*a^2)/3 - 4*b^2) + (28*a^2*tan(c/2 + (d*x)/2)^2)/3
+ 12*a^2*tan(c/2 + (d*x)/2)^6 + (8*a^2)/3 - (2*b^2)/5 + a*b*tan(c/2 + (d*x)
/2)^3 - a*b*tan(c/2 + (d*x)/2)^7 - (5*a*b*tan(c/2 + (d*x)/2)^9)/2 + (5*a*b*
tan(c/2 + (d*x)/2))/2)/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4
 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
 + 1)) + (3*a*b*atan((9*a^2*b^2)/(4*(3*a^3*b - (9*a^2*b^2*tan(c/2 + (d*x)/2
))/4)) + (3*a^3*b*tan(c/2 + (d*x)/2))/(3*a^3*b - (9*a^2*b^2*tan(c/2 + (d*x)
/2))/4)))/(2*d)
```

3.1109 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=181

$$-\frac{3}{8}\left(4a^2-b^2\right)x-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{a(a^2+28b^2)\cos(c+dx)}{6bd}+\frac{(2a^2+39b^2)\cos(c+dx)\sin(c+dx)}{24d}+$$

```
[Out] -3/8*(4*a^2-b^2)*x-2*a*b*arctanh(cos(d*x+c))/d+1/6*a*(a^2+28*b^2)*cos(d*x+c
)/b/d+1/24*(2*a^2+39*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/12*(a^2+12*b^2)*cos(d*x
+c)*(a+b*sin(d*x+c))^2/a/b/d-1/4*cos(d*x+c)*(a+b*sin(d*x+c))^3/b/d-cot(d*x+
c)*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.34, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2973, 3128, 3112, 3102, 2814, 3855}

$$\frac{a(a^2+28b^2)\cos(c+dx)}{6bd}+\frac{(a^2+12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd}+\frac{(2a^2+39b^2)\sin(c+dx)\cos(c+dx)}{24d}-\frac{3}{8}x(4a^2-b^2)-\frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4bd}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{\cot(c+dx)(a+b\sin(c+dx))^3}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*(4*a^2 - b^2)*x)/8 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (a*(a^2 + 28*b^2
)*Cos[c + d*x])/(6*b*d) + ((2*a^2 + 39*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*
d) + ((a^2 + 12*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(12*a*b*d) - (Cos
[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^3)/(a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
```

```
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^3}{4bd} - \frac{\cot(c+dx)(a+b\text{s}}{ad}$$
$$= \frac{(a^2+12b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{12abd} - \frac{\cos(c+d}{}$$
$$= \frac{(2a^2+39b^2)\cos(c+dx)\sin(c+dx)}{24d} + \frac{(a^2+12b^2)\cos(}{}$$
$$= \frac{a(a^2+28b^2)\cos(c+dx)}{6bd} + \frac{(2a^2+39b^2)\cos(c+dx)\sin}{24d}$$
$$= -\frac{3}{8}(4a^2-b^2)\,x + \frac{a(a^2+28b^2)\cos(c+dx)}{6bd} + \frac{(2a^2+3}{}$$
$$= -\frac{3}{8}(4a^2-b^2)\,x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{a(a^2+2}{}$$

Mathematica [A]

time = 0.49, size = 167, normalized size = 0.92

$$-\frac{3a^2(c+dx)}{2d}+\frac{3b^2(c+dx)}{8d}+\frac{5ab\cos(c+dx)}{2d}+\frac{ab\cos(3(c+dx))}{6d}-\frac{a^2\cot(c+dx)}{d}-\frac{2ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\sin(2(c+dx))}{4d}+\frac{b^2\sin(2(c+dx))}{4d}+\frac{b^2\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a^2*(c + d*x))/(2*d) + (3*b^2*(c + d*x))/(8*d) + (5*a*b*Cos[c + d*x])/(
2*d) + (a*b*Cos[3*(c + d*x)])/(6*d) - (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos
[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)]
)/(4*d) + (b^2*Sin[2*(c + d*x)])/(4*d) + (b^2*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]

time = 0.21, size = 135, normalized size = 0.75

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2ab\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot\right.}{d}$
risch	$-\frac{3a^2x}{2}+\frac{3b^2x}{8}+\frac{i e^{2i(dx+c)}a^2}{8d}-\frac{i b^2 e^{2i(dx+c)}}{8d}+\frac{5ab\,e^{i(dx+c)}}{4d}+\frac{5ab\,e^{-i(dx+c)}}{4d}-\frac{i e^{-2i(dx+c)}a^2}{8d}+\frac{i b^2 e^{-2i(dx+}}{8d}$
norman	$\frac{\left(-9a^2+\frac{9b^2}{4}\right)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-6a^2+\frac{3b^2}{2}\right)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-6a^2+\frac{3b^2}{2}\right)x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3a^2}{2}+\frac{3b^2}{8}\right)x\tan\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)-3/2*d*x-3/2*c)+2*a*b*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+
c)))+b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.50, size = 127, normalized size = 0.70

$$-\frac{48\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)a^{2}-32\left(2\,\cos\left(dx+c\right)^{3}+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)ab-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)b^{2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/96*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a^2 - 32*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1)
 + 3*log(cos(d*x + c) - 1))*a*b - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*s
in(2*d*x + 2*c))*b^2)/d
```

Fricas [A]
time = 0.39, size = 155, normalized size = 0.86

$$-\frac{6\,b^{2}\cos\left(dx+c\right)^{5}-3\left(4\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{3}+24\,ab\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-24\,ab\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+9\left(4\,a^{2}-b^{2}\right)\cos\left(dx+c\right)-\left(16\,ab\cos\left(dx+c\right)^{3}-9\left(4\,a^{2}-b^{2}\right)dx+48\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/24*(6*b^2*cos(d*x + c)^5 - 3*(4*a^2 - b^2)*cos(d*x + c)^3 + 24*a*b*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 24*a*b*log(-1/2*cos(d*x + c) + 1/2)*s
in(d*x + c) + 9*(4*a^2 - b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c)^3 - 9*(4*
a^2 - b^2)*d*x + 48*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 274, normalized size = 1.51

$$\frac{48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,(4\,a^2-b^2)(dx+c)-\frac{12\,\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+192\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+160\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+64\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^2*tan(1/2*d*x + 1/2*c) -
 9*(4*a^2 - b^2)*(d*x + c) - 12*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*
d*x + 1/2*c) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 - 15*b^2*tan(1/2*d*x + 1/2*
c)^7 + 96*a*b*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 + 9*b^
2*tan(1/2*d*x + 1/2*c)^5 + 192*a*b*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*
d*x + 1/2*c)^3 - 9*b^2*tan(1/2*d*x + 1/2*c)^3 + 160*a*b*tan(1/2*d*x + 1/2*c
)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 64*a*b)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 9.54, size = 578, normalized size = 3.19

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(a^2-\frac{5b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(6a^2-\frac{5b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(8a^2+\frac{3b^2}{2}\right)-a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(2a^2-\frac{3b^2}{2}\right)+\frac{80ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+32ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+16ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{32ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{\mathrm{atan}\left(\frac{\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)\left(\frac{3b^2}{4}-3a^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)1i-\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)\left(3a^2-\frac{3b^2}{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)1i}{\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)\left(\frac{3b^2}{4}-3a^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)\left(3a^2-\frac{3b^2}{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2\,3i}{2}-\frac{b^2\,3i}{8}\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+3ab^3-12a^3b+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9a^4-\frac{9a^2b^2}{2}+\frac{9b^4}{16}\right)}\right)\left(3a^2-\frac{3b^2}{4}\right)}{d}+\frac{2ab\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^8*(a^2 - (5*b^2)/2) - tan(c/2 + (d*x)/2)^2*(6*a^2 - (5*
b^2)/2) - tan(c/2 + (d*x)/2)^4*(8*a^2 + (3*b^2)/2) - a^2 - tan(c/2 + (d*x)/
2)^6*(2*a^2 - (3*b^2)/2) + (80*a*b*tan(c/2 + (d*x)/2)^3)/3 + 32*a*b*tan(c/2
 + (d*x)/2)^5 + 16*a*b*tan(c/2 + (d*x)/2)^7 + (32*a*b*tan(c/2 + (d*x)/2))/3
)/(d*(2*tan(c/2 + (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)
^5 + 8*tan(c/2 + (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^2*tan(c/2 + (d*
x)/2))/(2*d) - (atan((((a^2*3i)/2 - (b^2*3i)/8)*((3*b^2)/4 - 3*a^2 + 6*tan(
c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) + 4*a*b*tan(c/2 + (d*x)/2))*1i - (
(a^2*3i)/2 - (b^2*3i)/8)*(3*a^2 - (3*b^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*3i
)/2 - (b^2*3i)/8) - 4*a*b*tan(c/2 + (d*x)/2))*1i)/(((a^2*3i)/2 - (b^2*3i)/8
)*((3*b^2)/4 - 3*a^2 + 6*tan(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) + 4*a
*b*tan(c/2 + (d*x)/2)) + ((a^2*3i)/2 - (b^2*3i)/8)*(3*a^2 - (3*b^2)/4 + 6*t
an(c/2 + (d*x)/2)*((a^2*3i)/2 - (b^2*3i)/8) - 4*a*b*tan(c/2 + (d*x)/2)) + 3
*a*b^3 - 12*a^3*b + 2*tan(c/2 + (d*x)/2)*(9*a^4 + (9*b^4)/16 - (9*a^2*b^2)/
2)))*(3*a^2 - (3*b^2)/4))/d + (2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1110 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=189

$$-3abx+\frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{(4a^2-23b^2)\cos(c+dx)}{6d}-\frac{b(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{3ad}-($$

```
[Out] -3*a*b*x+1/2*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/d-1/6*(4*a^2-23*b^2)*cos(d*x
+c)/d-1/3*b*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/a/d-1/6*(2*a^2-3*b^2)*cos(d*x
+c)*(a+b*sin(d*x+c))^2/a^2/d-1/2*b*cot(d*x+c)*(a+b*sin(d*x+c))^3/a^2/d-1/2*
cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2972, 3128, 3112, 3102, 2814, 3855}

$$-\frac{(4a^2-23b^2)\cos(c+dx)}{6d}-\frac{(2a^2-3b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d}-\frac{b(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{3ad}+\frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)(a+b\sin(c+dx))^3}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{2ad}-3abx$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -3*a*b*x + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((4*a^2 - 23*b^2
)*Cos[c + d*x])/(6*d) - (b*(a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(3*a*d)
 - ((2*a^2 - 3*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/(6*a^2*d) - (b*Cot
[c + d*x]*(a + b*Sin[c + d*x])^3)/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a
 + b*Sin[c + d*x])^3)/(2*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^3}{2a^2d} - \frac{\cot(c+dx)\csc(c}{}$$
$$= -\frac{(2a^2-3b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{6a^2d} - \frac{b\cot(c}{}$$
$$= -\frac{b(a^2-3b^2)\cos(c+dx)\sin(c+dx)}{3ad} - \frac{(2a^2-3b^2)\cos}{}$$
$$= -\frac{(4a^2-23b^2)\cos(c+dx)}{6d} - \frac{b(a^2-3b^2)\cos(c+dx)\sin}{3ad}$$
$$= -3abx - \frac{(4a^2-23b^2)\cos(c+dx)}{6d} - \frac{b(a^2-3b^2)\cos(c}{3a}$$
$$= -3abx + \frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{(4a^2-23}{}$$

Mathematica [A]
time = 2.26, size = 191, normalized size = 1.01

$$\frac{-6(4a^2-5b^2)\cos(c+dx)+2b^2\cos(3(c+dx))+3\left(-24abc-24abdx-8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+12a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-4ab\sin(2(c+dx))+8ab\tan\left(\frac{1}{2}(c+dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-6*(4*a^2 - 5*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] + 3*(-24*a*b*c -
24*a*b*d*x - 8*a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*a^2*Log[C
os[(c + d*x)/2]] - 8*b^2*Log[Cos[(c + d*x)/2]] - 12*a^2*Log[Sin[(c + d*x)/2
]] + 8*b^2*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Sin[2*(c
+ d*x)] + 8*a*b*Tan[(c + d*x)/2]))/(24*d)
```

Maple [A]
time = 0.26, size = 157, normalized size = 0.83

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx}{2}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx}{2}\right.\right.}{d}$
risch	$-3abx+\frac{b^2e^{3i(dx+c)}}{24d}+\frac{iab\,e^{2i(dx+c)}}{4d}-\frac{a^2e^{i(dx+c)}}{2d}+\frac{5\,e^{i(dx+c)}b^2}{8d}-\frac{a^2e^{-i(dx+c)}}{2d}+\frac{5\,e^{-i(dx+c)}b^2}{8d}-\frac{iab\,e}{}$
norman	$\frac{\frac{ab\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{8d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(23a^2-32b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(33a^2-32b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{\left(43a^2-32b^2\right)(\tan}{8d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/
2*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^
3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+b^2*(1/3*cos(d*x+c)^3+cos(d*x+c
)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.50, size = 150, normalized size = 0.79

$$-\frac{12\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab-2\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)b^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/12*(12*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a*b - 2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1)
+ 3*log(cos(d*x + c) - 1))*b^2 - 3*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1)
 - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.37, size = 210, normalized size = 1.11

$$\frac{4\,b^2\cos\left(dx+c\right)^5-36\,abdx\cos\left(dx+c\right)^2+36\,abdx-4\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)^3+6\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)+3\left(\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)^2-3\,a^2+2\,b^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(\left(3\,a^2-2\,b^2\right)\cos\left(dx+c\right)^2-3\,a^2+2\,b^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-12\left(ab\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(4*b^2*cos(d*x + c)^5 - 36*a*b*d*x*cos(d*x + c)^2 + 36*a*b*d*x - 4*(3*
a^2 - 2*b^2)*cos(d*x + c)^3 + 6*(3*a^2 - 2*b^2)*cos(d*x + c) + 3*((3*a^2 -
2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) - 3*((3*
a^2 - 2*b^2)*cos(d*x + c)^2 - 3*a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) -
 12*(a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.49, size = 252, normalized size = 1.33

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,(dx+c)ab+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,(3\,a^2-2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3\left(18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{16\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2+4\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*tan(1/2*d*x + 1/2*c)^2 - 72*(d*x + c)*a*b + 24*a*b*tan(1/2*d*x
+ 1/2*c) - 12*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3*(18*a^2*ta
n(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 16*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*a^2*tan(1/2*d*x + 1/2*c)^4 + 6*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*
d*x + 1/2*c)^2 + 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c)
- 3*a^2 + 4*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 9.44, size = 397, normalized size = 2.10

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{2}-b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{17\,a^2}{2}-16\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{35\,a^2}{2}-16\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{19\,a^2}{2}-\frac{32\,b^2}{3}\right)+\frac{a^2}{2}+20\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+12\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{6\,a\,b\,\mathrm{atan}\left(\frac{36\,a^2\,b^2}{-18\,a^3\,b+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}-\frac{12\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-18\,a^3\,b+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}+\frac{18\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-18\,a^3\,b+36\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+12\,a\,b^3}\right)}{d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*((3*a^2)/2 - b^
2))/d - (tan(c/2 + (d*x)/2)^6*((17*a^2)/2 - 16*b^2) + tan(c/2 + (d*x)/2)^4*
((35*a^2)/2 - 16*b^2) + tan(c/2 + (d*x)/2)^2*((19*a^2)/2 - (32*b^2)/3) + a^
2/2 + 20*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan
(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 +
12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)
) + (6*a*b*atan((36*a^2*b^2)/(12*a*b^3 - 18*a^3*b + 36*a^2*b^2*tan(c/2 + (d
*x)/2)) - (12*a*b^3*tan(c/2 + (d*x)/2))/(12*a*b^3 - 18*a^3*b + 36*a^2*b^2*t
an(c/2 + (d*x)/2)) + (18*a^3*b*tan(c/2 + (d*x)/2))/(12*a*b^3 - 18*a^3*b + 3
6*a^2*b^2*tan(c/2 + (d*x)/2))))/d + (a*b*tan(c/2 + (d*x)/2))/d
```

3.1111 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot}{2d}$$

```
[Out] a^2*x-3/2*b^2*x+3*a*b*arctanh(cos(d*x+c))/d-3*a*b*cos(d*x+c)/d+a^2*cot(d*x+
c)/d-3/2*b^2*cot(d*x+c)/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)/d-a*b*cos(d*x+c)*
cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d
```

Rubi [A]

time = 0.11, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 2671, 294, 327, 209, 2672, 212, 3554, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\right.$$
$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int$$
$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx\right.}{d}$$
$$= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(}{d}$$
$$= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2}{}$$
$$= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.
time = 6.19, size = 293, normalized size = 2.20

$$\frac{(2a^2-3b^2)(c+dx)}{2d}-\frac{2ab\cos(c+dx)}{d}+\frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d}-\frac{ab\csc^2\left(\frac{1}{2}(c+dx)\right)}{4d}-\frac{a^2\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d}+\frac{3ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{3ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{ab\sec^2\left(\frac{1}{2}(c+dx)\right)}{4d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}-\frac{b^2\sin(2(c+dx))}{4d}+\frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A]
time = 0.24, size = 145, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b}{d}$

risch	$a^2x-\frac{3b^2x}{2}+\frac{ib^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{ab\,\mathrm{e}^{i(dx+c)}}{d}-\frac{ab\,\mathrm{e}^{-i(dx+c)}}{d}-\frac{ib^2\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{4ia^2\mathrm{e}^{4i(dx+c)}-2ib^2\mathrm{e}^{4i(dx+c)}+2b}{}$
norman	$\frac{\left(a^2-\frac{3b^2}{2}\right)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^2-\frac{3b^2}{2}\right)x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a^2-3b^2)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{5ab\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5ab(\tan}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c))

Maxima [A]
time = 0.49, size = 138, normalized size = 1.04

$$\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\cos(dx+c)+1)-3\,\log(\cos(dx+c)-1)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d

Fricas [A]
time = 0.38, size = 218, normalized size = 1.64

$$\frac{3\,b^2\cos(dx+c)^5+4\,(2\,a^2-3\,b^2)\cos(dx+c)^3+9\,(ab\cos(dx+c)^2-ab)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,(ab\cos(dx+c)^2-ab)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3\,(2\,a^2-3\,b^2)\cos(dx+c)+3\,\left((2\,a^2-3\,b^2)dx\cos(dx+c)^2-4\,ab\cos(dx+c)^3-(2\,a^2-3\,b^2)dx+6\,ab\cos(dx+c)\right)\sin(dx+c)}{6\,(d\cos(dx+c)^2-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.48, size = 241, normalized size = 1.81

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,(2\,a^2-3\,b^2)(dx+c)+\frac{24\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}+\frac{132\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 11.75, size = 584, normalized size = 4.39

$$-\frac{\frac{5\,b^2\cos(c+d\,x)}{16}+\frac{a^2\cos(3\,c+3\,d\,x)}{3}-\frac{11\,b^2\cos(3\,c+3\,d\,x)}{32}+\frac{b^2\cos(5\,c+5\,d\,x)}{32}+\frac{a^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\sin(3\,c+3\,d\,x)}{2}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\sin(3\,c+3\,d\,x)}{4}+\frac{3\,a\,b\sin(c+d\,x)}{2}-\frac{3\,a^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\sin(c+d\,x)}{2}+\frac{9\,b^2\,\mathrm{atan}\left(\frac{-2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\sin(c+d\,x)}{4}+a\,b\sin\left(2\,c+2\,d\,x\right)-\frac{a\,b\sin(3\,c+3\,d\,x)}{2}-\frac{a\,b\sin(4\,c+4\,d\,x)}{4}+\frac{9\,a\,b\sin(c+d\,x)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{3\,a\,b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sin(3\,c+3\,d\,x)}{4}}{d\sin\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] -((5*b^2*cos(c + d*x))/16 + (a^2*cos(3*c + 3*d*x))/3 - (11*b^2*cos(3*c + 3*
d*x))/32 + (b^2*cos(5*c + 5*d*x))/32 + (a^2*atan((3*b^2*cos(c/2 + (d*x)/2)
- 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*
x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d
*x))/2 - (3*b^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) +
 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x
)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/4 + (3*a*b*sin(c + d*x)
)/2 - (3*a^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*
a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2
) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (9*b^2*atan((3*b^2*cos(c/2
 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*s
```

```
in(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*s
in(c + d*x))/4 + a*b*sin(2*c + 2*d*x) - (a*b*sin(3*c + 3*d*x))/2 - (a*b*sin
(4*c + 4*d*x))/4 + (9*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2)))/4 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d
*x))/4)/(d*sin(c + d*x)^3)
```

3.1112 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$2abx-\frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d}+\frac{17ab\cot(c+dx)}{12d}+\frac{5\cot(c+dx)\csc(c+$$

```
[Out] 2*a*b*x-3/8*(a^2-4*b^2)*arctanh(cos(d*x+c))/d-1/24*b^2*(39*a^2+2*b^2)*cos(d
*x+c)/a^2/d+17/12*a*b*cot(d*x+c)/d+5/8*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c
))^2/d+1/12*b*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3/a^2/d-1/4*cot(d*x+
c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2972, 3126, 3110, 3102, 2814, 3855}

$$-\frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d}-\frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d}+\frac{17ab\cot(c+dx)}{12d}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{4ad}+\frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{8d}+2abx$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] 2*a*b*x - (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*(39*a^2 + 2*
b^2)*Cos[c + d*x])/(24*a^2*d) + (17*a*b*Cot[c + d*x])/(12*d) + (5*Cot[c + d
*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (b*Cot[c + d*x]*Csc[c + d*
x]^2*(a + b*Sin[c + d*x])^3)/(12*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a +
 b*Sin[c + d*x])^3)/(4*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{12a^2d} - \frac{\cot(c+ \\
&= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{b\cot(c+ \\
&= \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c \\
&= -\frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \frac{5\cot \\
&= 2abx - \frac{b^2(39a^2+2b^2)\cos(c+dx)}{24a^2d} + \frac{17ab\cot(c+dx)}{12d} + \\
&= 2abx - \frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2(39a^2+2b^2}{24a}
\end{aligned}
$$

Mathematica [A]
time = 2.53, size = 270, normalized size = 1.52

$$
\frac{384abc+384abdx-192b^2\cos(c+dx)+256ab\cot\left(\frac{1}{2}(c+dx)\right)+30a^2\csc^2\left(\frac{1}{2}(c+dx)\right)-24b^2\csc^2\left(\frac{1}{2}(c+dx)\right)-3a^2\csc^4\left(\frac{1}{2}(c+dx)\right)-72a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+288b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+72a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-288b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+24b^2\sec^2\left(\frac{1}{2}(c+dx)\right)+3a^2\sec^4\left(\frac{1}{2}(c+dx)\right)+128ab\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-8ab\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-256ab\tan\left(\frac{1}{2}(c+dx)\right)}{192d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (384*a*b*c + 384*a*b*d*x - 192*b^2*Cos[c + d*x] + 256*a*b*Cot[(c + d*x)/2]
+ 30*a^2*Csc[(c + d*x)/2]^2 - 24*b^2*Csc[(c + d*x)/2]^2 - 3*a^2*Csc[(c + d*
x)/2]^4 - 72*a^2*Log[Cos[(c + d*x)/2]] + 288*b^2*Log[Cos[(c + d*x)/2]] + 72
*a^2*Log[Sin[(c + d*x)/2]] - 288*b^2*Log[Sin[(c + d*x)/2]] - 30*a^2*Sec[(c
+ d*x)/2]^2 + 24*b^2*Sec[(c + d*x)/2]^2 + 3*a^2*Sec[(c + d*x)/2]^4 + 128*a*
b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - 8*a*b*Csc[(c + d*x)/2]^4*Sin[c + d*x]
 - 256*a*b*Tan[(c + d*x)/2])/(192*d)
```

Maple [A]
time = 0.26, size = 167, normalized size = 0.94

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2ab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+2ab\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+\right.}{d}$
risch	$2abx-\frac{e^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}b^2}{2d}-\frac{-96iab\,e^{6i(dx+c)}+15a^2e^{7i(dx+c)}-12b^2e^{7i(dx+c)}+192iab\,e^{4i(dx+c)}+9a^2e^{5i(dx+}}{}$

norman	$-\frac{a^2}{64d}+\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{(3a^2-4b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{(3a^2-4b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{(15a^2-80b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{(15a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*
cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/3*cot(
d*x+c)^3+cot(d*x+c)+d*x+c)+b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+
c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.52, size = 166, normalized size = 0.93

$$\frac{32\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab-3\,a^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,b^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b - 3*a^2*
(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 +
 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*b^2*(2*cos(d*
x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*
log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.38, size = 260, normalized size = 1.46

$$\frac{96\,abdx\cos\left(dx+c\right)^4-48\,b^2\cos\left(dx+c\right)^5-192\,abdx\cos\left(dx+c\right)^2+96\,abdx-30\,(a^2-4\,b^2)\cos\left(dx+c\right)^3+18\,(a^2-4\,b^2)\cos\left(dx+c\right)-9\,\left((a^2-4\,b^2)\cos\left(dx+c\right)^4-2\,(a^2-4\,b^2)\cos\left(dx+c\right)^2+a^2-4\,b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+9\,\left((a^2-4\,b^2)\cos\left(dx+c\right)^4-2\,(a^2-4\,b^2)\cos\left(dx+c\right)^2+a^2-4\,b^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-32\,\left(4\,ab\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(96*a*b*d*x*cos(d*x + c)^4 - 48*b^2*cos(d*x + c)^5 - 192*a*b*d*x*cos(d
*x + c)^2 + 96*a*b*d*x - 30*(a^2 - 4*b^2)*cos(d*x + c)^3 + 18*(a^2 - 4*b^2)
*cos(d*x + c) - 9*((a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x +
 c)^2 + a^2 - 4*b^2)*log(1/2*cos(d*x + c) + 1/2) + 9*((a^2 - 4*b^2)*cos(d*x
 + c)^4 - 2*(a^2 - 4*b^2)*cos(d*x + c)^2 + a^2 - 4*b^2)*log(-1/2*cos(d*x +
c) + 1/2) - 32*(4*a*b*cos(d*x + c)^3 - 3*a*b*cos(d*x + c))*sin(d*x + c))/(d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.51, size = 244, normalized size = 1.37

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+384\,(dx+c)ab-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,(a^2-4\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{384\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}-\frac{150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-600\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 384*(d*x + c)*a*
b - 240*a*b*tan(1/2*d*x + 1/2*c) + 72*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1
/2*c))) - 384*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1) - (150*a^2*tan(1/2*d*x + 1/2
*c)^4 - 600*b^2*tan(1/2*d*x + 1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 2
4*a^2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1
/2*d*x + 1/2*c) + 3*a^2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 10.89, size = 825, normalized size = 4.63

$$-\frac{3\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\,a^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+21\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-24\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-21\,a^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-24\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-24\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+408\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-16\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+16\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-72\,a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,a^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+288\,b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+288\,b^2\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+224\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-224\,a\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+768\,a\,b\,\mathrm{atan}\left(\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-12\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+16\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+768\,a\,b\,\mathrm{atan}\left(\frac{3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b-12\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{-3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+16\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}\right)\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{192\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] -(3*a^2*cos(c/2 + (d*x)/2)^10 - 3*a^2*sin(c/2 + (d*x)/2)^10 + 21*a^2*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^8 + 24*a^2*cos(c/2 + (d*x)/2)^4*sin(c/2 +
 (d*x)/2)^6 - 24*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 - 21*a^2*cos
(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^2 - 24*b^2*cos(c/2 + (d*x)/2)^2*sin(c/
2 + (d*x)/2)^8 - 24*b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 + 408*b^2
*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 24*b^2*cos(c/2 + (d*x)/2)^8*si
n(c/2 + (d*x)/2)^2 - 16*a*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^9 + 16*a*
b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) - 72*a^2*log(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 - 72*a^2*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/
2)^4 + 288*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2
```

)^4*sin(c/2 + (d*x)/2)^6 + 288*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4 + 224*a*b*cos(c/2 + (d*x)/2)^3
*sin(c/2 + (d*x)/2)^7 - 224*a*b*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3 +
768*a*b*atan((3*a^2*sin(c/2 + (d*x)/2) - 12*b^2*sin(c/2 + (d*x)/2) + 16*a*
b*cos(c/2 + (d*x)/2))/(12*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2)
+ 16*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^6 +
768*a*b*atan((3*a^2*sin(c/2 + (d*x)/2) - 12*b^2*sin(c/2 + (d*x)/2) + 16*a*b
*cos(c/2 + (d*x)/2))/(12*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2)
+ 16*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/(1
92*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^4*(cos(c/2 + (d*x)/2)^2 + sin(
c/2 + (d*x)/2)^2))

3.1113 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=209

$$b^2x-\frac{3ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{(3a^4-14a^2b^2+b^4)\cot(c+dx)}{15a^2d}+\frac{b(27a^2-2b^2)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{(12a$$

```
[Out] b^2*x-3/4*a*b*arctanh(cos(d*x+c))/d-1/15*(3*a^4-14*a^2*b^2+b^4)*cot(d*x+c)/
a^2/d+1/60*b*(27*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a/d+1/30*(12*a^2-b^2)*cot
(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/a^2/d+1/10*b*cot(d*x+c)*csc(d*x+c)^
3*(a+b*sin(d*x+c))^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a
/d
```

Rubi [A]
time = 0.33, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2972, 3126, 3110, 3100, 2814, 3855}

$$\frac{b(27a^2-2b^2)\cot(c+dx)\csc(c+dx)}{60ad}+\frac{(12a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{30a^2d}+\frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{10a^2d}-\frac{(3a^4-14a^2b^2+b^4)\cot(c+dx)}{15a^2d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{5ad}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] b^2*x - (3*a*b*ArcTanh[Cos[c + d*x]])/(4*d) - ((3*a^4 - 14*a^2*b^2 + b^4)*C
ot[c + d*x])/(15*a^2*d) + (b*(27*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6
0*a*d) + ((12*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2
)/(30*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(10*a
^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(5*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c-}{}$$

$$= \frac{(12a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{30a^2d}$$

$$= \frac{b(27a^2-2b^2)\cot(c+dx)\csc(c+dx)}{60ad} + \frac{(12a^2-b^2)\cot}{}$$

$$= -\frac{(3a^4-14a^2b^2+b^4)\cot(c+dx)}{15a^2d} + \frac{b(27a^2-2b^2)\cot(}{60}$$

$$= b^2x - \frac{(3a^4-14a^2b^2+b^4)\cot(c+dx)}{15a^2d} + \frac{b(27a^2-2b^2)}{}$$

$$= b^2x - \frac{3ab\tanh^{-1}(\cos(c+dx))}{4d} - \frac{(3a^4-14a^2b^2+b^4)\,\mathrm{c}}{15a^2d}$$

Mathematica [A]

time = 1.05, size = 285, normalized size = 1.36

$$\frac{960b^2c+960b^2dx+(-96a^2+640b^2)\cot\left(\frac{1}{2}(c+dx)\right)+300ab\csc^2\left(\frac{1}{2}(c+dx)\right)-720ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+720ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-300ab\sec^2\left(\frac{1}{2}(c+dx)\right)+30ab\sec^4\left(\frac{1}{2}(c+dx)\right)-336a^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+320b^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+192a^2\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)-3a^2\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+\csc^4\left(\frac{1}{2}(c+dx)\right)\left(-30ab+(21a^2-20b^2)\sin(c+dx)\right)+96a^2\tan\left(\frac{1}{2}(c+dx)\right)-640b^2\tan\left(\frac{1}{2}(c+dx)\right)}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (960*b^2*c + 960*b^2*d*x + (-96*a^2 + 640*b^2)*Cot[(c + d*x)/2] + 300*a*b*C
sc[(c + d*x)/2]^2 - 720*a*b*Log[Cos[(c + d*x)/2]] + 720*a*b*Log[Sin[(c + d*
x)/2]] - 300*a*b*Sec[(c + d*x)/2]^2 + 30*a*b*Sec[(c + d*x)/2]^4 - 336*a^2*C
sc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^
4 + 192*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - 3*a^2*Csc[(c + d*x)/2]^6*Si
n[c + d*x] + Csc[(c + d*x)/2]^4*(-30*a*b + (21*a^2 - 20*b^2)*Sin[c + d*x])
+ 96*a^2*Tan[(c + d*x)/2] - 640*b^2*Tan[(c + d*x)/2])/(960*d)
```

Maple [A]

time = 0.25, size = 129, normalized size = 0.62

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2ab\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b^2\left(-\frac{\left(\cot^3\right.}{}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+2ab\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+b^2\left(-\frac{\left(\cot^3\right.}{}\right.}{d}$
risch	$b^2x-\frac{60ia^2e^{8i(dx+c)}-120ib^2e^{8i(dx+c)}+75ab\,e^{9i(dx+c)}+360ib^2e^{6i(dx+c)}-30ab\,e^{7i(dx+c)}+120ia^2e^{4i(dx+c)}-440ib^2e^{4i(}}{30d\left(e^{2i(dx+c)}-1\right)^5}$

norman	$\frac{b^2x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+b^2x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2}{160d}+\frac{a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+2b^2x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(3a^2-260b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{480d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^2/sin(d*x+c)^5*cos(d*x+c)^5+2*a*b*(-1/4/sin(d*x+c)^4*cos(d*x+c)
^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc
(d*x+c)-cot(d*x+c)))+b^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

Maxima [A]
time = 0.52, size = 123, normalized size = 0.59

$$\frac{40\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2-1}{\tan\left(dx+c\right)^3}\right)b^2-15\,ab\left(\frac{2\left(5\,\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{24\,a^2}{\tan\left(dx+c\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/120*(40*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^2 - 15*a*
b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) - 24*a^2/tan(d*x
 + c)^5)/d
```

Fricas [A]
time = 0.36, size = 241, normalized size = 1.15

$$-\frac{8\left(3\,a^2-20\,b^2\right)\cos\left(dx+c\right)^5+280\,b^2\cos\left(dx+c\right)^3-120\,b^2\cos\left(dx+c\right)+45\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx+c\right)^2+ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-45\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx+c\right)^2+ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-30\left(4\,b^2dx\cos\left(dx+c\right)^4-8\,b^2dx\cos\left(dx+c\right)^2-5\,ab\cos\left(dx+c\right)^3+4\,b^2dx+3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(8*(3*a^2 - 20*b^2)*cos(d*x + c)^5 + 280*b^2*cos(d*x + c)^3 - 120*b^
2*cos(d*x + c) + 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 45*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*
x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*(4*b^2*d*x*c
os(d*x + c)^4 - 8*b^2*d*x*cos(d*x + c)^2 - 5*a*b*cos(d*x + c)^3 + 4*b^2*d*x
 + 3*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^
2 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.52, size = 263, normalized size = 1.26

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+480\,(dx+c)b^2+360\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-300\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{822\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-300\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 15*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b*tan(1/2*
d*x + 1/2*c)^2 + 480*(d*x + c)*b^2 + 360*a*b*log(abs(tan(1/2*d*x + 1/2*c)))
 + 30*a^2*tan(1/2*d*x + 1/2*c) - 300*b^2*tan(1/2*d*x + 1/2*c) - (822*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 30*a^2*tan(1/2*d*x + 1/2*c)^4 - 300*b^2*tan(1/2*d*x
+ 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2
 + 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan
(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 10.10, size = 346, normalized size = 1.66

$$\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d}-\frac{a^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}+\frac{5\,b^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,d}-\frac{5\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d}+\frac{2\,b^2\,\mathrm{atan}\left(\frac{4\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+3\,a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{3\,a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-4\,b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}+\frac{a\,b\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4\,d}-\frac{a\,b\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32\,d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32\,d}+\frac{3\,a\,b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (a^2*cot(c/2 + (d*x)/2)^3)/(32*d) - (a^2*cot(c/2 + (d*x)/2)^5)/(160*d) - (b
^2*cot(c/2 + (d*x)/2)^3)/(24*d) - (a^2*tan(c/2 + (d*x)/2)^3)/(32*d) + (a^2*
tan(c/2 + (d*x)/2)^5)/(160*d) + (b^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2*co
t(c/2 + (d*x)/2))/(16*d) + (5*b^2*cot(c/2 + (d*x)/2))/(8*d) + (a^2*tan(c/2
+ (d*x)/2))/(16*d) - (5*b^2*tan(c/2 + (d*x)/2))/(8*d) + (2*b^2*atan((4*b*co
s(c/2 + (d*x)/2) + 3*a*sin(c/2 + (d*x)/2))/(3*a*cos(c/2 + (d*x)/2) - 4*b*si
n(c/2 + (d*x)/2))))/d + (a*b*cot(c/2 + (d*x)/2)^2)/(4*d) - (a*b*cot(c/2 + (
d*x)/2)^4)/(32*d) - (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) + (a*b*tan(c/2 + (d*x)
/2)^4)/(32*d) + (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d)
```

3.1114 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=236

$$-\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{2ab\cot(c+dx)}{5d}-\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d}+\frac{b(13a}{}$$

```
[Out] -1/16*(a^2+6*b^2)*arctanh(cos(d*x+c))/d-2/5*a*b*cot(d*x+c)/d-1/240*(15*a^4-
80*a^2*b^2+12*b^4)*cot(d*x+c)*csc(d*x+c)/a^2/d+1/60*b*(13*a^2-2*b^2)*cot(d*
x+c)*csc(d*x+c)^2/a/d+1/120*(35*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin
(d*x+c))^2/a^2/d+1/10*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a^2/d-1/
6*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(d*x+c))^3/a/d
```

Rubi [A]
time = 0.39, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2972, 3126, 3110, 3100, 2827, 3852, 8, 3855}

$$-\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b(13a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{60ad}+\frac{(35a^2-6b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{120a^2d}+\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{10a^2d}-\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d}-\frac{2ab\cot(c+dx)}{5d}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*((a^2 + 6*b^2)*ArcTanh[Cos[c + d*x]])/d - (2*a*b*Cot[c + d*x])/(5*d)
- ((15*a^4 - 80*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(240*a^2*d) +
(b*(13*a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a*d) + ((35*a^2 - 6*b^
2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + (b*Cot
[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(10*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
```

```
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{10a^2d} - \frac{\cot(c}{}$$
$$= \frac{(35a^2-6b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx}{120a^2d}$$
$$= \frac{b(13a^2-2b^2)\cot(c+dx)\csc^2(c+dx)}{60ad} + \frac{(35a^2-6b^2)}{}$$
$$= -\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b(}{}$$
$$= -\frac{(15a^4-80a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{240a^2d} + \frac{b(}{}$$
$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{(15a^4-80a^2b^2+}{}$$
$$= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2ab\cot(c+dx)}{5d} -$$

Mathematica [A]
time = 0.59, size = 319, normalized size = 1.35

$$\frac{-384ab\cot\left(\frac{1}{2}(c+dx)\right)-30(a^2-10b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)-120a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-720b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+720b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+30a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-300b^2\sec^2\left(\frac{1}{2}(c+dx)\right)-30a^2\sec^4\left(\frac{1}{2}(c+dx)\right)+30b^2\sec^4\left(\frac{1}{2}(c+dx)\right)+5a^2\sec^6\left(\frac{1}{2}(c+dx)\right)-1344ab\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+768ab\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)-a\csc^6\left(\frac{1}{2}(c+dx)\right)(5a+12b\sin(c+dx))+6\csc^4\left(\frac{1}{2}(c+dx)\right)(5a^2-5b^2+14ab\sin(c+dx))+384ab\tan\left(\frac{1}{2}(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-384*a*b*Cot[(c + d*x)/2] - 30*(a^2 - 10*b^2)*Csc[(c + d*x)/2]^2 - 120*a^2
*Log[Cos[(c + d*x)/2]] - 720*b^2*Log[Cos[(c + d*x)/2]] + 120*a^2*Log[Sin[(c
 + d*x)/2]] + 720*b^2*Log[Sin[(c + d*x)/2]] + 30*a^2*Sec[(c + d*x)/2]^2 - 3
00*b^2*Sec[(c + d*x)/2]^2 - 30*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[(c + d*x
)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 1344*a*b*Csc[c + d*x]^3*Sin[(c + d*x)/2
]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - a*Csc[(c + d*x)/2]^6*(5*a
```

```
 + 12*b*Sin[c + d*x]) + 6*Csc[(c + d*x)/2]^4*(5*a^2 - 5*b^2 + 14*a*b*Sin[c
+ d*x]) + 384*a*b*Tan[(c + d*x)/2])/(1920*d)
```

Maple [A]
time = 0.30, size = 198, normalized size = 0.84

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\cos^{5}(dx+c)}{6\sin(dx+c)^{6}}-\frac{\cos^{5}(dx+c)}{24\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{48\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{2ab\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+}{d}$
default	$\frac{a^{2}\left(-\frac{\cos^{5}(dx+c)}{6\sin(dx+c)^{6}}-\frac{\cos^{5}(dx+c)}{24\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{48\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{2ab\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+}{d}$
risch	$\frac{-480iab\,\mathrm{e}^{10i(dx+c)}+15a^{2}\mathrm{e}^{11i(dx+c)}-150b^{2}\mathrm{e}^{11i(dx+c)}+96iab+235a^{2}\mathrm{e}^{9i(dx+c)}+210b^{2}\mathrm{e}^{9i(dx+c)}-960iab\,\mathrm{e}^{6i(dx+c)}+390a^{2}\mathrm{e}}{}$
norman	$-\frac{a^{2}}{384d}+\frac{a^{2}\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{\left(a^{2}-6b^{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{\left(a^{2}-6b^{2}\right)\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}+\frac{\left(2a^{2}+9b^{2}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{\left(2a^{2}+9b^{2}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/4
8/sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d
*x+c)-cot(d*x+c)))-2/5*a*b/sin(d*x+c)^5*cos(d*x+c)^5+b^2*(-1/4/sin(d*x+c)^4
*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)
+3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.31, size = 180, normalized size = 0.76

$$\frac{5\,a^{2}\left(\frac{2\left(3\cos\left(dx+c\right)^{5}+8\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}-1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-30\,b^{2}\left(\frac{2\left(5\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1}+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{192\,ab}{\tan\left(dx+c\right)^{5}}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/480*(5*a^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos
(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
 + 1) + 3*log(cos(d*x + c) - 1)) - 30*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x
+ c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3
*log(cos(d*x + c) - 1)) - 192*a*b/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.36, size = 274, normalized size = 1.16

$$\frac{192\,ab\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+30\left(a^{2}-10\,b^{2}\right)\cos\left(dx+c\right)^{5}+80\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{3}-30\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)-15\left(\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{6}-3\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}-6\,b^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{6}-3\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(a^{2}+6\,b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}-6\,b^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{480\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(192*a*b*cos(d*x + c)^5*sin(d*x + c) + 30*(a^2 - 10*b^2)*cos(d*x + c)
^5 + 80*(a^2 + 6*b^2)*cos(d*x + c)^3 - 30*(a^2 + 6*b^2)*cos(d*x + c) - 15*(
(a^2 + 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*
b^2)*cos(d*x + c)^2 - a^2 - 6*b^2)*log(1/2*cos(d*x + c) + 1/2) + 15*((a^2 +
 6*b^2)*cos(d*x + c)^6 - 3*(a^2 + 6*b^2)*cos(d*x + c)^4 + 3*(a^2 + 6*b^2)*c
os(d*x + c)^2 - a^2 - 6*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^
6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 309, normalized size = 1.31

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-240\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,(a^2+6\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{294\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1764\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2
*d*x + 1/2*c)^3 - 15*a^2*tan(1/2*d*x + 1/2*c)^2 - 240*b^2*tan(1/2*d*x + 1/2
*c)^2 + 240*a*b*tan(1/2*d*x + 1/2*c) + 120*(a^2 + 6*b^2)*log(abs(tan(1/2*d*
x + 1/2*c))) - (294*a^2*tan(1/2*d*x + 1/2*c)^6 + 1764*b^2*tan(1/2*d*x + 1/2
*c)^6 + 240*a*b*tan(1/2*d*x + 1/2*c)^5 - 15*a^2*tan(1/2*d*x + 1/2*c)^4 - 24
0*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^2*tan(
1/2*d*x + 1/2*c)^2 + 30*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a*b*tan(1/2*d*x + 1
/2*c) + 5*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.67, size = 262, normalized size = 1.11

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{a^2}{16}+\frac{3\,b^2}{8}\right)}{d}+\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{2}-b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{2}+8\,b^2\right)-\frac{a^2}{6}+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}\right)}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{128}+\frac{b^2}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{128}-\frac{b^2}{64}\right)}{d}-\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{16\,d}+\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{80\,d}+\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*(a^2/16 + (3*b^2)/8))/d + (a^2*tan(c/2 + (d*x)/2)^
6)/(384*d) + (cot(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^2*(a^2/2 - b^2) + ta
n(c/2 + (d*x)/2)^4*(a^2/2 + 8*b^2) - a^2/6 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 8
*a*b*tan(c/2 + (d*x)/2)^5 - (4*a*b*tan(c/2 + (d*x)/2))/5))/(64*d) - (tan(c/
2 + (d*x)/2)^2*(a^2/128 + b^2/8))/d - (tan(c/2 + (d*x)/2)^4*(a^2/128 - b^2/
64))/d - (a*b*tan(c/2 + (d*x)/2)^3)/(16*d) + (a*b*tan(c/2 + (d*x)/2)^5)/(80
*d) + (a*b*tan(c/2 + (d*x)/2))/(8*d)
```

3.1115 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=261

$$-\frac{ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}-\frac{ab\cot(c+dx)\csc(c+dx)}{8d}-\frac{(3a^4-18a^2b^2+4b^4)\cot(c}{105a^2d}$$

```
[Out] -1/8*a*b*arctanh(cos(d*x+c))/d-1/35*(2*a^2+7*b^2)*cot(d*x+c)/d-1/8*a*b*cot(
d*x+c)*csc(d*x+c)/d-1/105*(3*a^4-18*a^2*b^2+4*b^4)*cot(d*x+c)*csc(d*x+c)^2/
a^2/d+1/420*b*(53*a^2-12*b^2)*cot(d*x+c)*csc(d*x+c)^3/a/d+2/35*(4*a^2-b^2)*
cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^2/a^2/d+2/21*b*cot(d*x+c)*csc(d*x+
c)^5*(a+b*sin(d*x+c))^3/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^
3/a/d
```

Rubi [A]
time = 0.41, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2972, 3126, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$-\frac{(2a^2+7b^2)\cot(c+dx)}{35d}+\frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad}+\frac{2(4a^2-b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{35a^2d}+\frac{2b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{21a^2d}-\frac{(3a^4-18a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{105a^2d}-\frac{ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot(c+dx)\csc(c+dx)}{8d}-\frac{\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^3}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a*b*ArcTanh[Cos[c + d*x]])/d - ((2*a^2 + 7*b^2)*Cot[c + d*x])/(35*d)
- (a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) - ((3*a^4 - 18*a^2*b^2 + 4*b^4)*Cot
[c + d*x]*Csc[c + d*x]^2)/(105*a^2*d) + (b*(53*a^2 - 12*b^2)*Cot[c + d*x]*C
sc[c + d*x]^3)/(420*a*d) + (2*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^2)/(35*a^2*d) + (2*b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*S
in[c + d*x])^3)/(21*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*
x])^3)/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
```

```
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{2b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{21a^2d} - \cot(\\
&= \frac{2(4a^2-b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx)}{35a^2d}\\
&= \frac{b(53a^2-12b^2)\cot(c+dx)\csc^3(c+dx)}{420ad} + \frac{2(4a^2-b^2)}{}\\
&= -\frac{(3a^4-18a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b(5}{}\\
&= -\frac{(3a^4-18a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{105a^2d} + \frac{b(5}{}\\
&= -\frac{ab\cot(c+dx)\csc(c+dx)}{8d} - \frac{(3a^4-18a^2b^2+4b^4)\,\mathrm{c}}{105}\\
&= -\frac{ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{(2a^2+7b^2)\cot(c+dx)}{35d} -\end{aligned}$$

Mathematica [A]
time = 0.90, size = 322, normalized size = 1.23

$$-\frac{\csc^7(c+dx)\left(840(6a^2+b^2)\cos(c+dx)+168(14a^2-b^2)\cos(3(c+dx))+336a^2\cos(5(c+dx))-504b^2\cos(5(c+dx))-48a^2\cos(7(c+dx))-168b^2\cos(7(c+dx))+3675ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)-3675ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+dx)+2170ab\sin(2(c+dx))-2205ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+2205ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(3(c+dx))+3080ab\sin(4(c+dx))+735ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))-735ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(5(c+dx))+210ab\sin(6(c+dx))-105ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))+105ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(7(c+dx))\right)}{53760d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

[Out] -1/53760*(Csc[c + d*x]^7*(840*(6*a^2 + b^2)*Cos[c + d*x] + 168*(14*a^2 - b^2)*Cos[3*(c + d*x)] + 336*a^2*Cos[5*(c + d*x)] - 504*b^2*Cos[5*(c + d*x)] - 48*a^2*Cos[7*(c + d*x)] - 168*b^2*Cos[7*(c + d*x)] + 3675*a*b*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*a*b*Log[Sin[(c + d*x)/2]]*Sin[c + d*x] + 2170*a*b*Sin[2*(c + d*x)] - 2205*a*b*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] + 2205*a*b*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3080*a*b*Sin[4*(c + d*x)] + 735*a*b*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*a*b*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 210*a*b*Sin[6*(c + d*x)] - 105*a*b*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)] + 105*a*b*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/d

Maple [A]
time = 0.30, size = 163, normalized size = 0.62

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+2ab\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+2ab\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(}{}\right.}{d}$
risch	$-2520ib^2e^{8i(dx+c)}+105ab\,e^{13i(dx+c)}-168ib^2+336ia^2e^{2i(dx+c)}+1540ab\,e^{11i(dx+c)}-48ia^2+336ib^2e^{2i(dx+c)}+1085ab\,e^{9i(}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+2*a*b*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d*x+c)))-1/5*b^2/sin(d*x+c)^5*cos(d*x+c)^5)

Maxima [A]
time = 0.29, size = 134, normalized size = 0.51

$$\frac{35\,ab\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{336\,b^2}{\tan(dx+c)^5}-\frac{48\left(7\tan(dx+c)^2+5\right)a^2}{\tan(dx+c)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/1680*(35*a*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 336*b^2/tan(d*x + c)^5 - 48*(7*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d

Fricas [A]
time = 0.39, size = 248, normalized size = 0.95

$$-\frac{48\,(2\,a^2+7\,b^2)\cos\left(dx+c\right)^7-336\,(a^2+b^2)\cos\left(dx+c\right)^5+105\,\left(ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4+3\,ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\,\left(ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4+3\,ab\cos\left(dx+c\right)^2-ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-70\,\left(3\,ab\cos\left(dx+c\right)^5+8\,ab\cos\left(dx+c\right)^3-3\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/1680*(48*(2*a^2 + 7*b^2)*cos(d*x + c)^7 - 336*(a^2 + b^2)*cos(d*x + c)^5
 + 105*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 -
a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 70*(3*a*b*cos(d*x + c)^5 + 8*a*b*cos(d*x + c)^3 - 3*a*b*
cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*c
os(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 347, normalized size = 1.33

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+84\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-420\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1680\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+840\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4356\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+840\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-105\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-420\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-210\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+84\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 21
*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 210*a*b*tan(1
/2*d*x + 1/2*c)^4 - 105*a^2*tan(1/2*d*x + 1/2*c)^3 - 420*b^2*tan(1/2*d*x +
1/2*c)^3 - 210*a*b*tan(1/2*d*x + 1/2*c)^2 + 1680*a*b*log(abs(tan(1/2*d*x +
1/2*c))) + 315*a^2*tan(1/2*d*x + 1/2*c) + 840*b^2*tan(1/2*d*x + 1/2*c) - (4
356*a*b*tan(1/2*d*x + 1/2*c)^7 + 315*a^2*tan(1/2*d*x + 1/2*c)^6 + 840*b^2*t
an(1/2*d*x + 1/2*c)^6 - 210*a*b*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*tan(1/2*d*
x + 1/2*c)^4 - 420*b^2*tan(1/2*d*x + 1/2*c)^4 - 210*a*b*tan(1/2*d*x + 1/2*c
)^3 - 21*a^2*tan(1/2*d*x + 1/2*c)^2 + 84*b^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*
b*tan(1/2*d*x + 1/2*c) + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 9.92, size = 302, normalized size = 1.16

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}+\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{a^2}{5}-\frac{4\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(3\,a^2+8\,b^2\right)-\frac{a^2}{7}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(a^2+4\,b^2\right)+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\frac{2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}\right)}{128\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^2}{128}+\frac{b^2}{16}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{a^2}{128}+\frac{b^2}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{a^2}{640}-\frac{b^2}{160}\right)}{d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}+\frac{a\,b\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^7)/(896*d) + (cot(c/2 + (d*x)/2)^7*(tan(c/2 + (d*x)
/2)^2*(a^2/5 - (4*b^2)/5) - tan(c/2 + (d*x)/2)^6*(3*a^2 + 8*b^2) - a^2/7 +
tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 2*a*b*tan(c/2 + (d*x)/2)^3 + 2*a*b*tan
(c/2 + (d*x)/2)^5 - (2*a*b*tan(c/2 + (d*x)/2))/3))/(128*d) + (tan(c/2 + (d*
x)/2)*((3*a^2)/128 + b^2/16))/d - (tan(c/2 + (d*x)/2)^3*(a^2/128 + b^2/32))
/d - (tan(c/2 + (d*x)/2)^5*(a^2/640 - b^2/160))/d - (a*b*tan(c/2 + (d*x)/2)
^2)/(64*d) - (a*b*tan(c/2 + (d*x)/2)^4)/(64*d) + (a*b*tan(c/2 + (d*x)/2)^6)
/(192*d) + (a*b*log(tan(c/2 + (d*x)/2)))/(8*d)
```

3.1116 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=354

$$\frac{1}{128}a\left(8a^2+9b^2\right)x-\frac{b(27a^2+4b^2)\cos(c+dx)}{105d}+\frac{b(27a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{a(8a^2+9b^2)\cos(c+dx)\sin(c+$$

```
[Out] 1/128*a*(8*a^2+9*b^2)*x-1/105*b*(27*a^2+4*b^2)*cos(d*x+c)/d+1/315*b*(27*a^2
+4*b^2)*cos(d*x+c)^3/d-1/128*a*(8*a^2+9*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4032
*a*(40*a^4-188*a^2*b^2+189*b^4)*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/2520*(20*a^
4-93*a^2*b^2+24*b^4)*cos(d*x+c)*sin(d*x+c)^4/b/d-1/1008*a*(20*a^2-87*b^2)*c
os(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^2/b^2/d-5/126*(a^2-4*b^2)*cos(d*x+c
)*sin(d*x+c)^3*(a+b*sin(d*x+c))^3/b^2/d+5/72*a*cos(d*x+c)*sin(d*x+c)^3*(a+b
*sin(d*x+c))^4/b^2/d-1/9*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^4/b/d
```

Rubi [A]
time = 0.59, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2974, 3128, 3112, 3102, 2827, 2715, 8, 2713}

$$\frac{b(27a^2+4b^2)\cos^3(c+dx)}{315d}-\frac{b(27a^2+4b^2)\cos(c+dx)}{105d}-\frac{5(a^2-4b^2)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^3}{126b^2d}-\frac{a(20a^2-87b^2)\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^2}{1008b^2d}-\frac{a(8a^2+9b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{1}{128}ax\left(8a^2+9b^2\right)-\frac{(20a^4-93a^2b^2+24b^4)\sin^4(c+dx)\cos(c+dx)}{2520bd}-\frac{a(40a^4-188a^2b^2+189b^4)\sin^3(c+dx)\cos(c+dx)}{4032b^2d}+\frac{5a\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^4}{72b^2d}-\frac{\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^4}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(8*a^2 + 9*b^2)*x)/128 - (b*(27*a^2 + 4*b^2)*Cos[c + d*x])/(105*d) + (b*
(27*a^2 + 4*b^2)*Cos[c + d*x]^3)/(315*d) - (a*(8*a^2 + 9*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(128*d) - (a*(40*a^4 - 188*a^2*b^2 + 189*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(4032*b^2*d) - ((20*a^4 - 93*a^2*b^2 + 24*b^4)*Cos[c + d*x]*S
in[c + d*x]^4)/(2520*b*d) - (a*(20*a^2 - 87*b^2)*Cos[c + d*x]*Sin[c + d*x]^
3*(a + b*Sin[c + d*x])^2)/(1008*b^2*d) - (5*(a^2 - 4*b^2)*Cos[c + d*x]*Sin[
c + d*x]^3*(a + b*Sin[c + d*x])^3)/(126*b^2*d) + (5*a*Cos[c + d*x]*Sin[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(72*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^4*(a
 + b*Sin[c + d*x])^4)/(9*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{5a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^4}{72b^2d} - \frac{\cos(}{}$$

$$= -\frac{5(a^2-4b^2)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx}{126b^2d}$$

$$= -\frac{a(20a^2-87b^2)\cos(c+dx)\sin^3(c+dx)(a+b\sin(c}{1008b^2d}$$

$$= -\frac{(20a^4-93a^2b^2+24b^4)\cos(c+dx)\sin^4(c+dx)}{2520bd} - \frac{a}{}$$

$$= -\frac{a(40a^4-188a^2b^2+189b^4)\cos(c+dx)\sin^3(c+dx)}{4032b^2d}$$

$$= -\frac{a(40a^4-188a^2b^2+189b^4)\cos(c+dx)\sin^3(c+dx)}{4032b^2d}$$

$$= -\frac{a(8a^2+9b^2)\cos(c+dx)\sin(c+dx)}{128d} - \frac{a(40a^4-18}{}$$

$$= \frac{1}{128}a\left(8a^2+9b^2\right)x - \frac{b(27a^2+4b^2)\cos(c+dx)}{105d} + \frac{b(2}{}$$

Mathematica [A]
time = 0.91, size = 204, normalized size = 0.58

$$\frac{15120ab^2c+10080a^3dx+11340ab^2dx-3780b(6a^2+b^2)\cos(c+dx)-840(9a^2b+b^3)\cos(3(c+dx))+1512a^2b\cos(5(c+dx))+504b^3\cos(5(c+dx))+1080a^2b\cos(7(c+dx))+90b^3\cos(7(c+dx))-70b^3\cos(9(c+dx))+2520a^3\sin(2(c+dx))-2520a^3\sin(4(c+dx))-3780ab^2\sin(4(c+dx))-840a^3\sin(6(c+dx))+\frac{945}{2}ab^2\sin(8(c+dx))}{161280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (15120*a*b^2*c + 10080*a^3*d*x + 11340*a*b^2*d*x - 3780*b*(6*a^2 + b^2)*Cos
[c + d*x] - 840*(9*a^2*b + b^3)*Cos[3*(c + d*x)] + 1512*a^2*b*Cos[5*(c + d*
```

```
x)] + 504*b^3*Cos[5*(c + d*x)] + 1080*a^2*b*Cos[7*(c + d*x)] + 90*b^3*Cos[7
*(c + d*x)] - 70*b^3*Cos[9*(c + d*x)] + 2520*a^3*Sin[2*(c + d*x)] - 2520*a^
3*Sin[4*(c + d*x)] - 3780*a*b^2*Sin[4*(c + d*x)] - 840*a^3*Sin[6*(c + d*x)]
 + (945*a*b^2*Sin[8*(c + d*x)])/2)/(161280*d)
```

Maple [A]
time = 0.49, size = 218, normalized size = 0.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos
(d*x+c)^5)+3*a*b^2*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+
c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+b^3*(
-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/315*cos(d*x
+c)^5))
```

Maxima [A]
time = 0.29, size = 140, normalized size = 0.40

$$\frac{1680\left(4\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^{3}+27648\left(5\,\cos\left(dx+c\right)^{7}-7\,\cos\left(dx+c\right)^{5}\right)a^{2}b+945\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)ab^{2}-1024\left(35\,\cos\left(dx+c\right)^{9}-90\,\cos\left(dx+c\right)^{7}+63\,\cos\left(dx+c\right)^{5}\right)b^{3}}{322560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/322560*(1680*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*
a^3 + 27648*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*a^2*b + 945*(24*d*x + 24*
c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*a*b^2 - 1024*(35*cos(d*x + c)^9
- 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^3)/d
```

Fricas [A]
time = 0.39, size = 164, normalized size = 0.46

$$-\frac{4480\,b^{3}\cos\left(dx+c\right)^{9}-5760\,(3\,a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{7}+8064\,(3\,a^{2}b+b^{3})\cos\left(dx+c\right)^{5}-315\,(8\,a^{3}+9\,ab^{2})dx-105\left(144\,ab^{2}\cos\left(dx+c\right)^{7}-8\,(8\,a^{3}+27\,ab^{2})\cos\left(dx+c\right)^{5}+2\,(8\,a^{3}+9\,ab^{2})\cos\left(dx+c\right)^{3}+3\,(8\,a^{3}+9\,ab^{2})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40320*(4480*b^3*cos(d*x + c)^9 - 5760*(3*a^2*b + 2*b^3)*cos(d*x + c)^7 +
 8064*(3*a^2*b + b^3)*cos(d*x + c)^5 - 315*(8*a^3 + 9*a*b^2)*d*x - 105*(144
*a*b^2*cos(d*x + c)^7 - 8*(8*a^3 + 27*a*b^2)*cos(d*x + c)^5 + 2*(8*a^3 + 9*
a*b^2)*cos(d*x + c)^3 + 3*(8*a^3 + 9*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 1.35, size = 505, normalized size = 1.43

$$\begin{cases} \frac{a^3x\sin^6(c+dx)}{16} + \frac{3a^3x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3a^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{a^3x\cos^6(c+dx)}{16} + \frac{a^3\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{a^3\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{a^3\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{3a^2b\sin^2(c+dx)\cos^5(c+dx)}{5d} - \frac{6a^2b\cos^7(c+dx)}{35d} + \frac{9ab^2x\sin^8(c+dx)}{128} + \frac{9ab^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{27ab^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{9ab^2x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{9ab^2x\cos^8(c+dx)}{128} + \frac{9ab^2\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{33ab^2\sin^5(c+dx)\cos^3(c+dx)}{128d} - \frac{33ab^2\sin^3(c+dx)\cos^5(c+dx)}{128d} - \frac{9ab^2\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{b^3\sin^4(c+dx)\cos^5(c+dx)}{5d} - \frac{4b^3\sin^2(c+dx)\cos^7(c+dx)}{35d} - \frac{8b^3\cos^9(c+dx)}{315d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^3\sin^2(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x*sin(c + d*x)**6/16 + 3*a**3*x*sin(c + d*x)**4*cos(c + d*x
)**2/16 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + a**3*x*cos(c + d*x)
**6/16 + a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a**3*sin(c + d*x)**3*co
s(c + d*x)**3/(6*d) - a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a**2*b*s
in(c + d*x)**2*cos(c + d*x)**5/(5*d) - 6*a**2*b*cos(c + d*x)**7/(35*d) + 9*
a*b**2*x*sin(c + d*x)**8/128 + 9*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 27*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 9*a*b**2*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 9*a*b**2*x*cos(c + d*x)**8/128 + 9*a*b**2*sin(c +
 d*x)**7*cos(c + d*x)/(128*d) + 33*a*b**2*sin(c + d*x)**5*cos(c + d*x)**3/(
128*d) - 33*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 9*a*b**2*sin(c
 + d*x)*cos(c + d*x)**7/(128*d) - b**3*sin(c + d*x)**4*cos(c + d*x)**5/(5*d
) - 4*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(35*d) - 8*b**3*cos(c + d*x)**9/
(315*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)**2*cos(c)**4, True))
```

Giac [A]
time = 0.68, size = 204, normalized size = 0.58

$$-\frac{b^3\cos(9dx+9c)}{2304d} + \frac{3ab^2\sin(8dx+8c)}{1024d} - \frac{a^3\sin(6dx+6c)}{192d} + \frac{a^3\sin(2dx+2c)}{64d} + \frac{1}{128}(8a^3+9ab^2)x + \frac{(12a^2b+b^3)\cos(7dx+7c)}{1792d} + \frac{(3a^2b+b^3)\cos(5dx+5c)}{320d} - \frac{(9a^2b+b^3)\cos(3dx+3c)}{192d} - \frac{3(6a^2b+b^3)\cos(dx+c)}{128d} - \frac{(2a^3+3ab^2)\sin(4dx+4c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2304*b^3*cos(9*d*x + 9*c)/d + 3/1024*a*b^2*sin(8*d*x + 8*c)/d - 1/192*a^
3*sin(6*d*x + 6*c)/d + 1/64*a^3*sin(2*d*x + 2*c)/d + 1/128*(8*a^3 + 9*a*b^2
)*x + 1/1792*(12*a^2*b + b^3)*cos(7*d*x + 7*c)/d + 1/320*(3*a^2*b + b^3)*co
s(5*d*x + 5*c)/d - 1/192*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/128*(6*a^2*
b + b^3)*cos(d*x + c)/d - 1/128*(2*a^3 + 3*a*b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 10.77, size = 578, normalized size = 1.63

$$\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(8a^2+9b^2)}{64\left(\frac{a^3}{8}+\frac{9ab^2}{64}\right)}\right)(8a^2+9b^2)}{64d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^3}{8}+\frac{9ab^2}{64}\right) + \frac{12a^2b}{35} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\left(\frac{a^3}{8}+\frac{9ab^2}{64}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(\frac{39ab^2}{32}-\frac{19a^3}{12}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{39ab^2}{32}-\frac{19a^3}{12}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{9a^3}{4}+\frac{465ab^2}{32}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{9a^3}{4}+\frac{465ab^2}{32}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(\frac{3a^3}{4}+\frac{507ab^2}{32}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{3a^3}{4}+\frac{507ab^2}{32}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(12a^2b-16b^3) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(12a^2b+\frac{32b^3}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{84a^2b}{5}-\frac{32b^3}{5}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{12a^2b}{35}+\frac{64b^3}{35}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{108a^2b}{35}+\frac{16b^3}{35}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(\frac{156a^2b}{5}+\frac{112b^3}{5}\right) + \frac{16b^3}{315} + 12a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+126\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+84\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+36\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)} - \frac{a(8a^2+9b^2)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (a*atan((a*tan(c/2 + (d*x)/2)*(8*a^2 + 9*b^2))/(64*((9*a*b^2)/64 + a^3/8)))
*(8*a^2 + 9*b^2))/(64*d) - (tan(c/2 + (d*x)/2)*((9*a*b^2)/64 + a^3/8) + (12
```

```
*a^2*b)/35 - tan(c/2 + (d*x)/2)^17*((9*a*b^2)/64 + a^3/8) + tan(c/2 + (d*x)
/2)^3*((39*a*b^2)/32 - (19*a^3)/12) - tan(c/2 + (d*x)/2)^15*((39*a*b^2)/32
- (19*a^3)/12) - tan(c/2 + (d*x)/2)^5*((465*a*b^2)/32 + (9*a^3)/4) + tan(c/
2 + (d*x)/2)^13*((465*a*b^2)/32 + (9*a^3)/4) + tan(c/2 + (d*x)/2)^7*((507*a
*b^2)/32 + (3*a^3)/4) - tan(c/2 + (d*x)/2)^11*((507*a*b^2)/32 + (3*a^3)/4)
+ tan(c/2 + (d*x)/2)^10*(12*a^2*b - 16*b^3) + tan(c/2 + (d*x)/2)^12*(12*a^2
*b + (32*b^3)/3) + tan(c/2 + (d*x)/2)^6*((84*a^2*b)/5 - (32*b^3)/5) + tan(c
/2 + (d*x)/2)^4*((12*a^2*b)/35 + (64*b^3)/35) + tan(c/2 + (d*x)/2)^2*((108*
a^2*b)/35 + (16*b^3)/35) + tan(c/2 + (d*x)/2)^8*((156*a^2*b)/5 + (112*b^3)/
5) + (16*b^3)/315 + 12*a^2*b*tan(c/2 + (d*x)/2)^14)/(d*(9*tan(c/2 + (d*x)/2
)^2 + 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 + 126*tan(c/2 + (d*
x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 + 84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2
 + (d*x)/2)^14 + 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 + 1)) - (a
*(8*a^2 + 9*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d)
```

3.1117 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$\frac{3}{128}b\left(8a^2+b^2\right)x-\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}+\frac{3b(8a^2+b^2)\cos(c+dx)\sin(c+dx)}{128d}+\frac{b(8a^2+b^2)\cos^3(c+d}{64d}$$

```
[Out] 3/128*b*(8*a^2+b^2)*x-1/560*a*(2*a^2+61*b^2)*cos(d*x+c)^5/d+3/128*b*(8*a^2+
b^2)*cos(d*x+c)*sin(d*x+c)/d+1/64*b*(8*a^2+b^2)*cos(d*x+c)^3*sin(d*x+c)/d-1
/112*(2*a^2+7*b^2)*cos(d*x+c)^5*(a+b*sin(d*x+c))/d-3/56*a*cos(d*x+c)^5*(a+b
*sin(d*x+c))^2/d-1/8*cos(d*x+c)^5*(a+b*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2941, 2748, 2715, 8}

$$-\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}-\frac{(2a^2+7b^2)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}+\frac{b(8a^2+b^2)\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b(8a^2+b^2)\sin(c+dx)\cos(c+dx)}{128d}+\frac{3}{128}bx\left(8a^2+b^2\right)-\frac{\cos^5(c+dx)(a+b\sin(c+dx))^3}{8d}-\frac{3a\cos^5(c+dx)(a+b\sin(c+dx))^2}{56d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(8*a^2 + b^2)*x)/128 - (a*(2*a^2 + 61*b^2)*Cos[c + d*x]^5)/(560*d) + (
3*b*(8*a^2 + b^2)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (b*(8*a^2 + b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(64*d) - ((2*a^2 + 7*b^2)*Cos[c + d*x]^5*(a + b*S
in[c + d*x]))/(112*d) - (3*a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2)/(56*d)
- (Cos[c + d*x]^5*(a + b*Sin[c + d*x])^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{\cos^5(c+dx)(a+b\sin(c+dx))^3}{8d}+\frac{1}{8}\int \cos^4(c+dx)(3\\
&= -\frac{3a\cos^5(c+dx)(a+b\sin(c+dx))^2}{56d}-\frac{\cos^5(c+dx)(a-}{8}\\
&= -\frac{(2a^2+7b^2)\cos^5(c+dx)(a+b\sin(c+dx))}{112d}-\frac{3a\cos^5(}{}\\
&= -\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}-\frac{(2a^2+7b^2)\cos^5(c+dx)}{112d}\\
&= -\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}+\frac{b(8a^2+b^2)\cos^3(c+dx)}{64d}\\
&= -\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}+\frac{3b(8a^2+b^2)\cos(c+dx)}{128d}\\
&= \frac{3}{128}b\left(8a^2+b^2\right)x-\frac{a(2a^2+61b^2)\cos^5(c+dx)}{560d}+\frac{3b(8a^2}{}\end{aligned}$$

Mathematica [A]
time = 0.61, size = 189, normalized size = 0.97

$$\frac{3360a^2bc+840b^3c+3360a^2bdx+420b^3dx-280a(8a^2+9b^2)\cos(c+dx)-280(4a^3+3ab^2)\cos(3(c+dx))-224a^3\cos(5(c+dx))+168ab^2\cos(5(c+dx))+120ab^2\cos(7(c+dx))+840a^2b\sin(2(c+dx))-840a^2b\sin(4(c+dx))-140b^3\sin(4(c+dx))-280a^2b\sin(6(c+dx))+\frac{35}{2}b^3\sin(8(c+dx))}{17920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3360*a^2*b*c + 840*b^3*c + 3360*a^2*b*d*x + 420*b^3*d*x - 280*a*(8*a^2 + 9
*b^2)*Cos[c + d*x] - 280*(4*a^3 + 3*a*b^2)*Cos[3*(c + d*x)] - 224*a^3*Cos[5
*(c + d*x)] + 168*a*b^2*Cos[5*(c + d*x)] + 120*a*b^2*Cos[7*(c + d*x)] + 840
*a^2*b*Sin[2*(c + d*x)] - 840*a^2*b*Sin[4*(c + d*x)] - 140*b^3*Sin[4*(c + d
*x)] - 280*a^2*b*Sin[6*(c + d*x)] + (35*b^3*Sin[8*(c + d*x)])/2)/(17920*d)
```

Maple [A]
time = 0.39, size = 180, normalized size = 0.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^3*cos(d*x+c)^5+3*a^2*b*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(
d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)+3*a*b^2*(-1/7*sin(d*x+
c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+b^3*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/
16*sin(d*x+c)*cos(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/
128*d*x+3/128*c))
```

Maxima [A]
time = 0.28, size = 117, normalized size = 0.60

$$-\frac{7168\,a^3\cos\left(dx+c\right)^5-560\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2b-3072\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)ab^2-35\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)b^3}{35840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35840*(7168*a^3*cos(d*x + c)^5 - 560*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12
*c - 3*sin(4*d*x + 4*c))*a^2*b - 3072*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)
*a*b^2 - 35*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A]
time = 0.38, size = 136, normalized size = 0.70

$$\frac{1920\,ab^2\cos\left(dx+c\right)^7-896\left(a^3+3\,ab^2\right)\cos\left(dx+c\right)^5+105\left(8\,a^2b+b^3\right)dx+35\left(16\,b^3\cos\left(dx+c\right)^7-8\left(8\,a^2b+3\,b^3\right)\cos\left(dx+c\right)^5+2\left(8\,a^2b+b^3\right)\cos\left(dx+c\right)^3+3\left(8\,a^2b+b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(1920*a*b^2*cos(d*x + c)^7 - 896*(a^3 + 3*a*b^2)*cos(d*x + c)^5 + 10
5*(8*a^2*b + b^3)*d*x + 35*(16*b^3*cos(d*x + c)^7 - 8*(8*a^2*b + 3*b^3)*cos
(d*x + c)^5 + 2*(8*a^2*b + b^3)*cos(d*x + c)^3 + 3*(8*a^2*b + b^3)*cos(d*x
+ c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(177) = 354.
time = 0.96, size = 456, normalized size = 2.35

$$\begin{cases} -\frac{a^3\cos^5(c+dx)}{5d}+\frac{3a^2bx\sin^6(c+dx)}{16}+\frac{9a^2bx\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{9a^2bx\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3a^2bx\cos^6(c+dx)}{16}+\frac{3a^2b\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{a^2b\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{3a^2b\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{3ab^2\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{6ab^2\cos^7(c+dx)}{35d}+\frac{3b^3x\sin^8(c+dx)}{128}+\frac{3b^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{9b^3x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{3b^3x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{3b^3x\cos^8(c+dx)}{128}+\frac{3b^3\sin^7(c+dx)\cos(c+dx)}{128d}+\frac{11b^3\sin^5(c+dx)\cos^3(c+dx)}{128d}-\frac{11b^3\sin^3(c+dx)\cos^5(c+dx)}{128d}-\frac{3b^3\sin(c+dx)\cos^7(c+dx)}{128d} & \text{for}\: d\neq 0 \\ x(a+b\sin(c))^3\sin(c)\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((-a**3*cos(c + d*x)**5/(5*d) + 3*a**2*b*x*sin(c + d*x)**6/16 + 9*
a**2*b*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a**2*b*x*sin(c + d*x)**2*co
s(c + d*x)**4/16 + 3*a**2*b*x*cos(c + d*x)**6/16 + 3*a**2*b*sin(c + d*x)**5
```

*cos(c + d*x)/(16*d) + a**2*b*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a**2*b*sin(c + d*x)*cos(c + d*x)**5/(16*d) - 3*a*b**2*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 6*a*b**2*cos(c + d*x)**7/(35*d) + 3*b**3*x*sin(c + d*x)**8/128 + 3*b**3*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**3*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*b**3*x*cos(c + d*x)**8/128 + 3*b**3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*b**3*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**3*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 3*b**3*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sin(c))**3*sin(c)*cos(c)**4, True))

Giac [A]
time = 0.64, size = 184, normalized size = 0.95

$$\frac{3\,ab^2\cos\left(7\,dx+7\,c\right)}{448\,d}+\frac{b^3\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{a^2b\sin\left(6\,dx+6\,c\right)}{64\,d}+\frac{3\,a^2b\sin\left(2\,dx+2\,c\right)}{64\,d}+\frac{3}{128}\left(8\,a^2b+b^3\right)x-\frac{\left(4\,a^3-3\,ab^2\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(4\,a^3+3\,ab^2\right)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{\left(8\,a^3+9\,ab^2\right)\cos\left(dx+c\right)}{64\,d}-\frac{\left(6\,a^2b+b^3\right)\sin\left(4\,dx+4\,c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 3/448*a*b^2*cos(7*d*x + 7*c)/d + 1/1024*b^3*sin(8*d*x + 8*c)/d - 1/64*a^2*b
*sin(6*d*x + 6*c)/d + 3/64*a^2*b*sin(2*d*x + 2*c)/d + 3/128*(8*a^2*b + b^3)
*x - 1/320*(4*a^3 - 3*a*b^2)*cos(5*d*x + 5*c)/d - 1/64*(4*a^3 + 3*a*b^2)*co
s(3*d*x + 3*c)/d - 1/64*(8*a^3 + 9*a*b^2)*cos(d*x + c)/d - 1/128*(6*a^2*b +
 b^3)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 10.92, size = 552, normalized size = 2.85

$$\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(8\,a^2+b^2\right)}{64\left(\frac{3\,a^2b}{8}+\frac{3\,b^3}{64}\right)}\right)\left(8\,a^2+b^2\right)}{64\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\,a^2b}{8}+\frac{3\,b^3}{64}\right)+10\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+\frac{12\,a\,b^2}{35}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(6\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(14\,a^3+12\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{12\,a\,b^2}{5}-\frac{26\,a^3}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{6\,a^3}{5}+\frac{96\,a\,b^2}{35}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{62\,a^3}{5}+\frac{96\,a\,b^2}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{3\,a^2b}{8}+\frac{3\,b^3}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(\frac{41\,a^2b}{8}-\frac{23\,b^3}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{41\,a^2b}{8}-\frac{23\,b^3}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{13\,a^2b}{8}+\frac{333\,b^3}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{13\,a^2b}{8}+\frac{333\,b^3}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(\frac{31\,a^2b}{8}+\frac{671\,b^3}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{31\,a^2b}{8}+\frac{671\,b^3}{64}\right)+\frac{2\,a^3}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}-\frac{3\,b\left(8\,a^2+b^2\right)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

```
[Out] (3*b*atan((3*b*tan(c/2 + (d*x)/2)*(8*a^2 + b^2))/(64*((3*a^2*b)/8 + (3*b^3)
/64)))*(8*a^2 + b^2))/(64*d) - (tan(c/2 + (d*x)/2)*((3*a^2*b)/8 + (3*b^3)/6
4) + 10*a^3*tan(c/2 + (d*x)/2)^10 + 2*a^3*tan(c/2 + (d*x)/2)^14 + (12*a*b^2
)/35 + tan(c/2 + (d*x)/2)^12*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^8*(12*
a*b^2 + 14*a^3) - tan(c/2 + (d*x)/2)^4*((12*a*b^2)/5 - (26*a^3)/5) + tan(c/
2 + (d*x)/2)^2*((96*a*b^2)/35 + (6*a^3)/5) + tan(c/2 + (d*x)/2)^6*((96*a*b^
2)/5 + (62*a^3)/5) - tan(c/2 + (d*x)/2)^15*((3*a^2*b)/8 + (3*b^3)/64) - tan
(c/2 + (d*x)/2)^3*((41*a^2*b)/8 - (23*b^3)/64) + tan(c/2 + (d*x)/2)^13*((41
*a^2*b)/8 - (23*b^3)/64) - tan(c/2 + (d*x)/2)^5*((13*a^2*b)/8 + (333*b^3)/6
4) + tan(c/2 + (d*x)/2)^11*((13*a^2*b)/8 + (333*b^3)/64) + tan(c/2 + (d*x)/
2)^7*((31*a^2*b)/8 + (671*b^3)/64) - tan(c/2 + (d*x)/2)^9*((31*a^2*b)/8 + (
671*b^3)/64) + (2*a^3)/5)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2
)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x
)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d
*x)/2)^16 + 1)) - (3*b*(8*a^2 + b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/
(64*d)
```

3.1118 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=250

$$\frac{1}{16}b\left(18a^2+b^2\right)x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(4a^4-84a^2b^2+15b^4\right)\cos}{240bd}$$

```
[Out] 1/16*b*(18*a^2+b^2)*x-a^3*arctanh(cos(d*x+c))/d-1/60*a*(2*a^4-43*a^2*b^2+36
*b^4)*cos(d*x+c)/b^2/d-1/240*(4*a^4-84*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c
)/b/d-1/120*a*(2*a^2-39*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/b^2/d-1/120*(2*a
^2-35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^3/b^2/d+1/15*a*cos(d*x+c)*(a+b*sin(d
*x+c))^4/b^2/d-1/6*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^4/b/d
```

Rubi [A]
time = 0.44, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2974, 3128, 3112, 3102, 2814, 3855}

$$-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{\left(2a^2-35b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d}-\frac{a\left(2a^2-39b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{120b^2d}+\frac{1}{16}bx\left(18a^2+b^2\right)-\frac{a\left(2a^4-43a^2b^2+36b^4\right)\cos(c+dx)}{60b^2d}-\frac{\left(4a^4-84a^2b^2+15b^4\right)\sin(c+dx)\cos(c+dx)}{240bd}+\frac{a\cos(c+dx)(a+b\sin(c+dx))^4}{15b^2d}-\frac{\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^4}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(18*a^2 + b^2)*x)/16 - (a^3*ArcTanh[Cos[c + d*x]])/d - (a*(2*a^4 - 43*a^
2*b^2 + 36*b^4)*Cos[c + d*x])/(60*b^2*d) - ((4*a^4 - 84*a^2*b^2 + 15*b^4)*C
os[c + d*x]*Sin[c + d*x])/(240*b*d) - (a*(2*a^2 - 39*b^2)*Cos[c + d*x]*(a +
 b*Sin[c + d*x])^2)/(120*b^2*d) - ((2*a^2 - 35*b^2)*Cos[c + d*x]*(a + b*Sin
[c + d*x])^3)/(120*b^2*d) + (a*Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(15*b^2
*d) - (Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^4)/(6*b*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
```

```
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a\cos(c+dx)(a+b\sin(c+dx))^4}{15b^2d} - \frac{\cos(c+dx)\sin(c+\ldots}{}$$
$$= -\frac{(2a^2-35b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{120b^2d} + \frac{a\cos(\ldots}{}$$
$$= -\frac{a(2a^2-39b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{120b^2d} - \frac{(2a^2\ldots}{}$$
$$= -\frac{(4a^4-84a^2b^2+15b^4)\cos(c+dx)\sin(c+dx)}{240bd} - \frac{a(2\ldots}{}$$
$$= -\frac{a(2a^4-43a^2b^2+36b^4)\cos(c+dx)}{60b^2d} - \frac{(4a^4-84a^2b^2\ldots}{}$$
$$= \frac{1}{16}b\left(18a^2+b^2\right)x - \frac{a(2a^4-43a^2b^2+36b^4)\cos(c+dx\ldots}{60b^2d}$$
$$= \frac{1}{16}b\left(18a^2+b^2\right)x - \frac{a^3\tanh^{-1}(\cos(c+dx))}{d} - \frac{a(2a^4-\ldots}{}$$

Mathematica [A]

time = 0.38, size = 191, normalized size = 0.76

$$\frac{1080a^2bc+60b^3c+1080a^2bdx+60b^3dx+120a(10a^2-3b^2)\cos(c+dx)+20(4a^3-9ab^2)\cos(3(c+dx))-36ab^2\cos(5(c+dx))-960a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+960a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+720a^2b\sin(2(c+dx))+15b^3\sin(2(c+dx))+90a^2b\sin(4(c+dx))-15b^3\sin(4(c+dx))-5b^3\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1080*a^2*b*c + 60*b^3*c + 1080*a^2*b*d*x + 60*b^3*d*x + 120*a*(10*a^2 - 3*
b^2)*Cos[c + d*x] + 20*(4*a^3 - 9*a*b^2)*Cos[3*(c + d*x)] - 36*a*b^2*Cos[5*
(c + d*x)] - 960*a^3*Log[Cos[(c + d*x)/2]] + 960*a^3*Log[Sin[(c + d*x)/2]]
+ 720*a^2*b*Sin[2*(c + d*x)] + 15*b^3*Sin[2*(c + d*x)] + 90*a^2*b*Sin[4*(c
+ d*x)] - 15*b^3*Sin[4*(c + d*x)] - 5*b^3*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]

time = 0.27, size = 149, normalized size = 0.60

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\ldots}{d}$
default	$\frac{a^3\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\ldots}{d}$
risch	$\frac{9a^2bx}{8}+\frac{b^3x}{16}+\frac{5a^3e^{i(dx+c)}}{8d}-\frac{3\,e^{i(dx+c)}ab^2}{16d}+\frac{5a^3e^{-i(dx+c)}}{8d}-\frac{3\,e^{-i(dx+c)}ab^2}{16d}+\frac{a^3\ln(e^{i(dx+c)}-1)}{d}-\frac{a^3\ln\ldots}{}$

norman	$\frac{\left(\frac{9}{8}a^2b+\frac{1}{16}b^3\right)x+\left(\frac{9}{8}a^2b+\frac{1}{16}b^3\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{27}{4}a^2b+\frac{3}{8}b^3\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{27}{4}a^2b+\frac{3}{8}b^3\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(1
/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-3/5*cos(d*x+c)^5
*a*b^2+b^3*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))
*sin(d*x+c)+1/16*d*x+1/16*c))
```

Maxima [A]
time = 0.28, size = 137, normalized size = 0.55

$$-\frac{576\,ab^2\cos\left(dx+c\right)^5-160\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^3-90\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2b-5\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)b^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/960*(576*a*b^2*cos(d*x + c)^5 - 160*(2*cos(d*x + c)^3 + 6*cos(d*x + c) -
 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^3 - 90*(12*d*x + 12*c
 + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^2*b - 5*(4*sin(2*d*x + 2*c)^3 +
 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*b^3)/d
```

Fricas [A]
time = 0.42, size = 150, normalized size = 0.60

$$-\frac{144\,ab^2\cos\left(dx+c\right)^5-80\,a^3\cos\left(dx+c\right)^3-240\,a^3\cos\left(dx+c\right)+120\,a^3\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-120\,a^3\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(18\,a^2b+b^3\right)dx+5\left(8\,b^3\cos\left(dx+c\right)^5-2\left(18\,a^2b+b^3\right)\cos\left(dx+c\right)^3-3\left(18\,a^2b+b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(144*a*b^2*cos(d*x + c)^5 - 80*a^3*cos(d*x + c)^3 - 240*a^3*cos(d*x
+ c) + 120*a^3*log(1/2*cos(d*x + c) + 1/2) - 120*a^3*log(-1/2*cos(d*x + c)
+ 1/2) - 15*(18*a^2*b + b^3)*d*x + 5*(8*b^3*cos(d*x + c)^5 - 2*(18*a^2*b +
b^3)*cos(d*x + c)^3 - 3*(18*a^2*b + b^3)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A]

time = 0.50, size = 427, normalized size = 1.71

$$\frac{240\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+15\,(18\,a^2b+b^3)(dx+c)-\frac{2\left(450\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-480\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+720\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+630\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+235\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1920\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+720\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+180\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-390\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3200\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1440\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-180\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+390\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-2880\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1440\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-630\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-235\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1440\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+144\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-450\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-320\,a^3+144\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/240*(240*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(18*a^2*b + b^3)*(d*x + c) - 2*(450*a^2*b*tan(1/2*d*x + 1/2*c)^11 - 15*b^3*tan(1/2*d*x + 1/2*c)^11 - 480*a^3*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^10 + 630*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 235*b^3*tan(1/2*d*x + 1/2*c)^9 - 1920*a^3*tan(1/2*d*x + 1/2*c)^8 + 720*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 180*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 390*b^3*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*tan(1/2*d*x + 1/2*c)^6 + 1440*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 180*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 390*b^3*tan(1/2*d*x + 1/2*c)^5 - 2880*a^3*tan(1/2*d*x + 1/2*c)^4 + 1440*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 630*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 235*b^3*tan(1/2*d*x + 1/2*c)^3 - 1440*a^3*tan(1/2*d*x + 1/2*c)^2 + 144*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 450*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) - 320*a^3 + 144*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d

Mupad [B]

time = 11.57, size = 690, normalized size = 2.76

$$\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{\frac{6ab^2}{5}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15a^2b}{4}-\frac{b^3}{8}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(6ab^2-4a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{6ab^2}{5}-12a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}(6ab^2-16a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}(12ab^2-24a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(12ab^2-\frac{80a^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{3a^2b}{2}-\frac{13b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(\frac{3a^2b}{2}-\frac{13b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{15a^2b}{4}-\frac{b^3}{8}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(\frac{21a^2b}{4}+\frac{47b^3}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{21a^2b}{4}+\frac{47b^3}{24}\right)-\frac{8a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b(18a^2+b^2)\left(\frac{9a^2b}{4}+\frac{b^3}{8}+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(18a^2+b^2)3i}{8}\right)}{16}+\frac{b(18a^2+b^2)\left(\frac{9a^2b}{4}+\frac{b^3}{8}+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(18a^2+b^2)3i}{8}\right)}{16}}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{81a^4b^2}{16}+\frac{9a^2b^4}{16}+\frac{b^6}{64}\right)+\frac{9a^5b}{2}+\frac{a^3b^3}{4}-\frac{b(18a^2+b^2)\left(\frac{9a^2b}{4}+\frac{b^3}{8}+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(18a^2+b^2)3i}{8}\right)1i}{16}+\frac{b(18a^2+b^2)\left(\frac{9a^2b}{4}+\frac{b^3}{8}+2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(18a^2+b^2)3i}{8}\right)1i}{16}}\right)(18a^2+b^2)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x),x)

[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - ((6*a*b^2)/5 - tan(c/2 + (d*x)/2)*((15*a^2*b)/4 - b^3/8) + tan(c/2 + (d*x)/2)^10*(6*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^2*((6*a*b^2)/5 - 12*a^3) + tan(c/2 + (d*x)/2)^8*(6*a*b^2 - 16*a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 24*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2 - (80*a^3)/3) - tan(c/2 + (d*x)/2)^5*((3*a^2*b)/2 - (13*b^3)/4) + tan(c/2 + (d*x)/2)^7*((3*a^2*b)/2 - (13*b^3)/4) + tan(c/2 + (d*x)/2)^11*((15*a^2*b)/4 - b^3/8) - tan(c/2 + (d*x)/2)^3*((21*a^2*b)/4 + (47*b^3)/24) + tan(c/2 + (d*x)/2)^9*((21*a^2*b)/4 + (47*b^3)/24) - (8*a^3)/3)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (b*atan(((b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*tan(c/2 + (d*x)/2) - (b*tan(c/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8))/16 + (b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8))/16)/(2*tan(c/2 + (d*x)/2)*(b^6/64 + (9*a^2*b^4)/16 + (81*a^4*b^2)/16) + (9*a^5*b)/2 + (a^3*b^3)/4 - (b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*

```
tan(c/2 + (d*x)/2) - (b*tan(c/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8)*1i)/16 + (
b*(18*a^2 + b^2)*((9*a^2*b)/4 + b^3/8 + 2*a^3*tan(c/2 + (d*x)/2) + (b*tan(c
/2 + (d*x)/2)*(18*a^2 + b^2)*3i)/8)*1i)/16))*(18*a^2 + b^2))/(8*d)
```

3.1119 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=229

$$-\frac{3}{8}a\left(4a^2-3b^2\right)x-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{(a^4+56a^2b^2-2b^4)\cos(c+dx)}{10bd}+\frac{a(2a^2+83b^2)\cos(c+dx)}{40d}$$

```
[Out] -3/8*a*(4*a^2-3*b^2)*x-3*a^2*b*arctanh(cos(d*x+c))/d+1/10*(a^4+56*a^2*b^2-2
*b^4)*cos(d*x+c)/b/d+1/40*a*(2*a^2+83*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/20*(a^
2+28*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/b/d+1/20*(a^2+20*b^2)*cos(d*x+c)*(a
+b*sin(d*x+c))^3/a/b/d-1/5*cos(d*x+c)*(a+b*sin(d*x+c))^4/b/d-cot(d*x+c)*(a+
b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.43, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2973, 3128, 3112, 3102, 2814, 3855}

$$\frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd}+\frac{(a^2+28b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{20bd}+\frac{a(2a^2+83b^2)\sin(c+dx)\cos(c+dx)}{40d}-\frac{3}{8}ax\left(4a^2-3b^2\right)-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{(a^4+56a^2b^2-2b^4)\cos(c+dx)}{10bd}-\frac{\cos(c+dx)(a+b\sin(c+dx))^4}{5bd}-\frac{\cot(c+dx)(a+b\sin(c+dx))^4}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(4*a^2 - 3*b^2)*x)/8 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + ((a^4 + 56
*a^2*b^2 - 2*b^4)*Cos[c + d*x])/(10*b*d) + (a*(2*a^2 + 83*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(40*d) + ((a^2 + 28*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2
)/(20*b*d) + ((a^2 + 20*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(20*a*b*d
) - (Cos[c + d*x]*(a + b*Sin[c + d*x])^4)/(5*b*d) - (Cot[c + d*x]*(a + b*Si
n[c + d*x])^4)/(a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
```

```
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{\cos(c+dx)(a+b\sin(c+dx))^4}{5bd} - \frac{\cot(c+dx)(a+b}{ad}$$

$$= \frac{(a^2+20b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{20abd} - \frac{\cos(c+}{}$$

$$= \frac{(a^2+28b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{20bd} + \frac{(a^2+2}{}$$

$$= \frac{a(2a^2+83b^2)\cos(c+dx)\sin(c+dx)}{40d} + \frac{(a^2+28b^2)\,c}{}$$

$$= \frac{(a^4+56a^2b^2-2b^4)\cos(c+dx)}{10bd} + \frac{a(2a^2+83b^2)\cos(}{40}$$

$$= -\frac{3}{8}a\left(4a^2-3b^2\right)x + \frac{(a^4+56a^2b^2-2b^4)\cos(c+dx)}{10bd}$$

$$= -\frac{3}{8}a\left(4a^2-3b^2\right)x - \frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} + \frac{(a^4}{}$$

Mathematica [A]

time = 1.95, size = 194, normalized size = 0.85

$$\frac{-240a^3c+180ab^2c-240a^3dx+180ab^2dx-20b(-30a^2+b^2)\cos(c+dx)+10(4a^2b-b^3)\cos(3(c+dx))-2b^3\cos(5(c+dx))-80a^3\cot\left(\frac{1}{2}(c+dx)\right)-480a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+480a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-40a^3\sin(2(c+dx))+120ab^2\sin(2(c+dx))+15ab^2\sin(4(c+dx))+80a^3\tan\left(\frac{1}{2}(c+dx)\right)}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-240*a^3*c + 180*a*b^2*c - 240*a^3*d*x + 180*a*b^2*d*x - 20*b*(-30*a^2 + b
^2)*Cos[c + d*x] + 10*(4*a^2*b - b^3)*Cos[3*(c + d*x)] - 2*b^3*Cos[5*(c + d
*x)] - 80*a^3*Cot[(c + d*x)/2] - 480*a^2*b*Log[Cos[(c + d*x)/2]] + 480*a^2*
b*Log[Sin[(c + d*x)/2]] - 40*a^3*Sin[2*(c + d*x)] + 120*a*b^2*Sin[2*(c + d*
x)] + 15*a*b^2*Sin[4*(c + d*x)] + 80*a^3*Tan[(c + d*x)/2])/(160*d)
```

Maple [A]

time = 0.24, size = 152, normalized size = 0.66

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^2b\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^2b\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\right.}{d}$
risch	$-\frac{3a^3x}{2}+\frac{9ab^2x}{8}+\frac{i e^{2i(dx+c)}a^3}{8d}-\frac{i e^{-2i(dx+c)}a^3}{8d}+\frac{15b\,e^{i(dx+c)}a^2}{8d}-\frac{b^3e^{i(dx+c)}}{16d}+\frac{15b\,e^{-i(dx+c)}a^2}{8d}-\frac{b^3e^{-}}{1}$

norman	$\frac{(-15a^3+\frac{45}{4}a\,b^2)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-15a^3+\frac{45}{4}a\,b^2)x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{15}{2}a^3+\frac{45}{8}a\,b^2)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{15}{2}a^3+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+
c)-3/2*d*x-3/2*c)+3*a^2*b*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*
x+c)))+3*a*b^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)
-1/5*cos(d*x+c)^5*b^3)
```

Maxima [A]
time = 0.50, size = 143, normalized size = 0.62

$$-\frac{32\,b^3\cos\left(dx+c\right)^5+80\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^2+2}{\tan\left(dx+c\right)^3+\tan\left(dx+c\right)}\right)a^3-80\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^2b-15\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)ab^2}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/160*(32*b^3*cos(d*x + c)^5 + 80*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(t
an(d*x + c)^3 + tan(d*x + c)))*a^3 - 80*(2*cos(d*x + c)^3 + 6*cos(d*x + c)
- 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))*a^2*b - 15*(12*d*x + 1
2*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a*b^2)/d
```

Fricas [A]
time = 0.38, size = 179, normalized size = 0.78

$$-\frac{30\,ab^2\cos\left(dx+c\right)^5+60\,a^2b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\,a^2b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-5\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+15\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)+\left(8\,b^3\cos\left(dx+c\right)^5-40\,a^2b\cos\left(dx+c\right)^3-120\,a^2b\cos\left(dx+c\right)+15\left(4\,a^3-3\,ab^2\right)dx\right)\sin\left(dx+c\right)}{40\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/40*(30*a*b^2*cos(d*x + c)^5 + 60*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 60*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 5*(4*a^3 - 3
*a*b^2)*cos(d*x + c)^3 + 15*(4*a^3 - 3*a*b^2)*cos(d*x + c) + (8*b^3*cos(d*x
 + c)^5 - 40*a^2*b*cos(d*x + c)^3 - 120*a^2*b*cos(d*x + c) + 15*(4*a^3 - 3*
a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.52, size = 345, normalized size = 1.51

$$\frac{120\,a^{2}b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,(4\,a^{3}-3\,ab^{2})(dx+c)-\frac{20\,(6\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3})}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-75\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+240\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-40\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-30\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+720\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+880\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-80\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-40\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+30\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+560\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-20\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+160\,a^{2}b-8\,b^{3}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/40*(120*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 20*a^3*tan(1/2*d*x + 1/2*c
) - 15*(4*a^3 - 3*a*b^2)*(d*x + c) - 20*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3
)/tan(1/2*d*x + 1/2*c) + 2*(20*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*a*b^2*tan(1/
2*d*x + 1/2*c)^9 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 40*b^3*tan(1/2*d*x +
1/2*c)^8 + 40*a^3*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^2*tan(1/2*d*x + 1/2*c)^7
+ 720*a^2*b*tan(1/2*d*x + 1/2*c)^6 + 880*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 80*
b^3*tan(1/2*d*x + 1/2*c)^4 - 40*a^3*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^2*tan(1
/2*d*x + 1/2*c)^3 + 560*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 20*a^3*tan(1/2*d*x +
 1/2*c) + 75*a*b^2*tan(1/2*d*x + 1/2*c) + 160*a^2*b - 8*b^3)/(tan(1/2*d*x +
 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 9.59, size = 674, normalized size = 2.94

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(16\,a^{2}b-\frac{4\,b^{3}}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}(a^{3}+3\,ab^{2})-10\,a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}(3\,ab^{2}-14\,a^{3})+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{15\,ab^{2}}{2}-7\,a^{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{15\,ab^{2}}{2}-a^{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}(24\,a^{2}b-4\,b^{3})+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}(88\,a^{2}b-8\,b^{3})-a^{3}+56\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+72\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{\frac{3\,a\,(4\,a^{2}-3\,b^{2})\left(\frac{9\,ab^{2}}{4}-3\,a^{3}+6\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4\,a^{2}-3\,b^{2})\,9\mathrm{i}}{4}\right)}{8}+\frac{3\,a\,(4\,a^{2}-3\,b^{2})\left(\frac{9\,ab^{2}}{4}-3\,a^{3}+6\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4\,a^{2}-3\,b^{2})\,9\mathrm{i}}{4}\right)}{8}}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9\,a^{6}-\frac{27\,a^{4}b^{2}}{2}+\frac{81\,a^{2}b^{4}}{16}\right)-18\,a^{5}b+\frac{27\,a^{3}b^{3}}{2}-\frac{a\,(4\,a^{2}-3\,b^{2})\left(\frac{9\,ab^{2}}{4}-3\,a^{3}+6\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4\,a^{2}-3\,b^{2})\,9\mathrm{i}}{4}\right)3\mathrm{i}}{8}+\frac{a\,(4\,a^{2}-3\,b^{2})\left(\frac{9\,ab^{2}}{4}-3\,a^{3}+6\,a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4\,a^{2}-3\,b^{2})\,9\mathrm{i}}{4}\right)3\mathrm{i}}{8}}\right)(4\,a^{2}-3\,b^{2})}{4\,d}+\frac{3\,a^{2}b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(16*a^2*b - (4*b^3)/5) - tan(c/2 + (d*x)/2)^8*(3*a*b^2
+ a^3) - 10*a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(3*a*b^2 - 14*a
^3) + tan(c/2 + (d*x)/2)^2*((15*a*b^2)/2 - 7*a^3) - tan(c/2 + (d*x)/2)^10*(
(15*a*b^2)/2 - a^3) + tan(c/2 + (d*x)/2)^9*(24*a^2*b - 4*b^3) + tan(c/2 + (
d*x)/2)^5*(88*a^2*b - 8*b^3) - a^3 + 56*a^2*b*tan(c/2 + (d*x)/2)^3 + 72*a^2
*b*tan(c/2 + (d*x)/2)^7)/(d*(2*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3
 + 20*tan(c/2 + (d*x)/2)^5 + 20*tan(c/2 + (d*x)/2)^7 + 10*tan(c/2 + (d*x)/2
)^9 + 2*tan(c/2 + (d*x)/2)^11)) + (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a*ata
n(((3*a*(4*a^2 - 3*b^2)*((9*a*b^2)/4 - 3*a^3 - (a*tan(c/2 + (d*x)/2)*(4*a^2
 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (d*x)/2)))/8 + (3*a*(4*a^2 - 3*b^2)*((9
*a*b^2)/4 - 3*a^3 + (a*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*t
an(c/2 + (d*x)/2)))/8)/(2*tan(c/2 + (d*x)/2)*(9*a^6 + (81*a^2*b^4)/16 - (27
*a^4*b^2)/2) - 18*a^5*b + (27*a^3*b^3)/2 - (a*(4*a^2 - 3*b^2)*((9*a*b^2)/4
```

```
- 3*a^3 - (a*tan(c/2 + (d*x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (
d*x)/2))*3i)/8 + (a*(4*a^2 - 3*b^2)*((9*a*b^2)/4 - 3*a^3 + (a*tan(c/2 + (d*
x)/2)*(4*a^2 - 3*b^2)*9i)/4 + 6*a^2*b*tan(c/2 + (d*x)/2))*3i)/8))*(4*a^2 -
3*b^2))/(4*d) + (3*a^2*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1120 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=231

$$-\frac{3}{8}b\left(12a^2-b^2\right)x+\frac{3a\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{b\left(2a^2-21b^2\right)\cos(c+dx)}{8d}$$

```
[Out] -3/8*b*(12*a^2-b^2)*x+3/2*a*(a^2-2*b^2)*arctanh(cos(d*x+c))/d-1/2*a*(a^2-17
*b^2)*cos(d*x+c)/d-1/8*b*(2*a^2-21*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(a^2-6*
b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^2/a/d-1/4*(a^2-4*b^2)*cos(d*x+c)*(a+b*sin(
d*x+c))^3/a^2/d-b*cot(d*x+c)*(a+b*sin(d*x+c))^4/a^2/d-1/2*cot(d*x+c)*csc(d*
x+c)*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.44, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2972, 3128, 3112, 3102, 2814, 3855}

$$-\frac{a\left(a^2-17b^2\right)\cos(c+dx)}{2d}-\frac{\left(a^2-4b^2\right)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d}-\frac{\left(a^2-6b^2\right)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad}-\frac{b\left(2a^2-21b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3}{8}bx\left(12a^2-b^2\right)-\frac{b\cot(c+dx)(a+b\sin(c+dx))^4}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^4}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(12*a^2 - b^2)*x)/8 + (3*a*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d)
 - (a*(a^2 - 17*b^2)*Cos[c + d*x])/(2*d) - (b*(2*a^2 - 21*b^2)*Cos[c + d*x]
*Sin[c + d*x])/(8*d) - ((a^2 - 6*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^2)/
(4*a*d) - ((a^2 - 4*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^3)/(4*a^2*d) - (
b*Cot[c + d*x]*(a + b*Sin[c + d*x])^4)/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x]
*(a + b*Sin[c + d*x])^4)/(2*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

```
&& !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^4}{a^2d} - \frac{\cot(c+dx)\csc(c$$
$$= -\frac{(a^2-4b^2)\cos(c+dx)(a+b\sin(c+dx))^3}{4a^2d} - \frac{b\cot(c+$$
$$= -\frac{(a^2-6b^2)\cos(c+dx)(a+b\sin(c+dx))^2}{4ad} - \frac{(a^2-4$$
$$= -\frac{b(2a^2-21b^2)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(a^2-6b^2)\,\mathrm{co}$$
$$= -\frac{a(a^2-17b^2)\cos(c+dx)}{2d} - \frac{b(2a^2-21b^2)\cos(c+dx}{8d}$$
$$= -\frac{3}{8}b\left(12a^2-b^2\right)x - \frac{a(a^2-17b^2)\cos(c+dx)}{2d} - \frac{b(2a^2}{}$$
$$= -\frac{3}{8}b\left(12a^2-b^2\right)x + \frac{3a(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}$$

Mathematica [A]

time = 6.13, size = 252, normalized size = 1.09

$$\frac{3b(-12a^2+b^2)(c+dx)}{8d} - \frac{a(4a^2-15b^2)\cos(c+dx)}{4d} + \frac{ab^2\cos(3(c+dx))}{4d} - \frac{3a^2b\cot\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{a^3\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{3(a^3-2ab^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{3(a^3-2ab^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^3\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b(-3a^2+b^2)\sin(2(c+dx))}{4d} + \frac{b^3\sin(4(c+dx))}{32d} + \frac{3a^2b\tan\left(\frac{1}{2}(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(-12*a^2 + b^2)*(c + d*x))/(8*d) - (a*(4*a^2 - 15*b^2)*Cos[c + d*x])/(
4*d) + (a*b^2*Cos[3*(c + d*x)])/(4*d) - (3*a^2*b*Cot[(c + d*x)/2])/(2*d) -
(a^3*Csc[(c + d*x)/2]^2)/(8*d) + (3*(a^3 - 2*a*b^2)*Log[Cos[(c + d*x)/2]])/
(2*d) - (3*(a^3 - 2*a*b^2)*Log[Sin[(c + d*x)/2]])/(2*d) + (a^3*Sec[(c + d*x
)/2]^2)/(8*d) + (b*(-3*a^2 + b^2)*Sin[2*(c + d*x)])/(4*d) + (b^3*Sin[4*(c +
 d*x)])/(32*d) + (3*a^2*b*Tan[(c + d*x)/2])/(2*d)
```

Maple [A]

time = 0.27, size = 198, normalized size = 0.86

method	result
derivativedivides	$a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx}{2}\right.\right.$
default	$a^3\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx}{2}\right.\right.$

risch	$-\frac{9a^2bx}{2}+\frac{3b^3x}{8}+\frac{a\,b^2\mathrm{e}^{3i(dx+c)}}{8d}-\frac{ib^3\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3ib\,\mathrm{e}^{2i(dx+c)}a^2}{8d}-\frac{a^3\mathrm{e}^{i(dx+c)}}{2d}+\frac{15\,\mathrm{e}^{i(dx+c)}a\,b^2}{8d}-\frac{a^3\mathrm{e}^{-i(dx}}{2d}$
norman	$\frac{(-27a^2b+\frac{9}{4}b^3)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-18a^2b+\frac{3}{2}b^3)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-18a^2b+\frac{3}{2}b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{9}{2}a^2b+\frac{3}{8}b^3)x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/
2*ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c
)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+3*a*b^2*(1/3*cos(d*x+c)^3+cos
(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.50, size = 186, normalized size = 0.81

$$-\frac{48\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2b-16\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)ab^2-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)b^3-8\,a^3\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/32*(48*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
 c)))*a^2*b - 16*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) +
1) + 3*log(cos(d*x + c) - 1))*a*b^2 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8
*sin(2*d*x + 2*c))*b^3 - 8*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos
(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 260, normalized size = 1.13

$$\frac{8\,ab^2\cos\left(dx+c\right)^5-3\,(12\,a^2b-b^3)dx\cos\left(dx+c\right)^2-8\,(a^3-2\,ab^2)\cos\left(dx+c\right)^3+3\,(12\,a^2b-b^3)dx+12\,(a^3-2\,ab^2)\cos\left(dx+c\right)-6\left(a^3-2\,ab^2-(a^3-2\,ab^2)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+6\left(a^3-2\,ab^2-(a^3-2\,ab^2)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(2\,b^3\cos\left(dx+c\right)^5-(12\,a^2b-b^3)\cos\left(dx+c\right)^3+3\,(12\,a^2b-b^3)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8*(8*a*b^2*cos(d*x + c)^5 - 3*(12*a^2*b - b^3)*d*x*cos(d*x + c)^2 - 8*(a^
3 - 2*a*b^2)*cos(d*x + c)^3 + 3*(12*a^2*b - b^3)*d*x + 12*(a^3 - 2*a*b^2)*c
os(d*x + c) - 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(1/2*co
s(d*x + c) + 1/2) + 6*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2)*log(
-1/2*cos(d*x + c) + 1/2) + (2*b^3*cos(d*x + c)^5 - (12*a^2*b - b^3)*cos(d*x
 + c)^3 + 3*(12*a^2*b - b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2
- d)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]

time = 0.53, size = 400, normalized size = 1.73

$$\frac{a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 3 \, {\left(12 \, a^{2} b - b^{3}\right)} {\left(d x + c\right)} - 12 \, {\left(a^{3} - 2 \, a b^{2}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) + \frac{18 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 36 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - a^{3}}{\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2}} + \frac{2 \, {\left(12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 5 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{7} - 8 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 48 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{6} + 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} + 3 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{5} - 24 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 96 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 3 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 24 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 80 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 12 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 5 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 8 \, a^{3} + 32 \, a b^{2}\right)}}{{\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1\right)}^{4}}}{8 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) - 3*(12*a^2
*b - b^3)*(d*x + c) - 12*(a^3 - 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (
18*a^3*tan(1/2*d*x + 1/2*c)^2 - 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*
tan(1/2*d*x + 1/2*c) - a^3)/tan(1/2*d*x + 1/2*c)^2 + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(1/2*d*x + 1/2*c)^6
+ 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3
*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4 + 96*a*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*
c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 + 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12
*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1/2*d*x + 1/2*c) - 8*a^3 + 32*a*b^2
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]

time = 9.59, size = 718, normalized size = 3.11

$$\frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(32\,a\,b^2-10\,a^3\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(48\,a\,b^2-\frac{17\,a^3}{2}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(80\,a\,b^2-27\,a^3\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(96\,a\,b^2-26\,a^3\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(6\,a^2\,b-5\,b^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(12\,a^2\,b-3\,b^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(36\,a^2\,b-5\,b^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(48\,a^2\,b+3\,b^3\right)-\frac{a^3}{2}-6\,a^2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+24\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-\frac{3\,a^3}{2}\right)}{d}+\frac{3\,a^2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{3\,b\,\mathrm{atan}\left(\frac{\frac{3\,b\,\left(12\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-3\,a^3\right)-9\,a^2\,b+\frac{3\,b^3}{4}-\frac{b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2-b^2\right)\,9{}\mathrm{i}}{4}\right)}{8}+\frac{3\,b\,\left(12\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-3\,a^3\right)-9\,a^2\,b+\frac{3\,b^3}{4}+\frac{b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2-b^2\right)\,9{}\mathrm{i}}{4}\right)}{8}}{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(81\,a^4\,b^2-\frac{27\,a^2\,b^4}{2}+\frac{9\,b^6}{16}\right)+\frac{9\,a\,b^5}{2}+27\,a^5\,b-\frac{225\,a^3\,b^3}{4}-\frac{b\,\left(12\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-3\,a^3\right)-9\,a^2\,b+\frac{3\,b^3}{4}-\frac{b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2-b^2\right)\,9{}\mathrm{i}}{4}\right)\,3{}\mathrm{i}}{8}+\frac{b\,\left(12\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-3\,a^3\right)-9\,a^2\,b+\frac{3\,b^3}{4}+\frac{b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2-b^2\right)\,9{}\mathrm{i}}{4}\right)\,3{}\mathrm{i}}{8}}\right)\,\left(12\,a^2-b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^3,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (tan(c/2 + (d*x)/2)^2*(32*a*b^2 - 10*a^3
) + tan(c/2 + (d*x)/2)^8*(48*a*b^2 - (17*a^3)/2) + tan(c/2 + (d*x)/2)^4*(80
*a*b^2 - 27*a^3) + tan(c/2 + (d*x)/2)^6*(96*a*b^2 - 26*a^3) + tan(c/2 + (d*
x)/2)^9*(6*a^2*b - 5*b^3) - tan(c/2 + (d*x)/2)^7*(12*a^2*b - 3*b^3) - tan(c
/2 + (d*x)/2)^3*(36*a^2*b - 5*b^3) - tan(c/2 + (d*x)/2)^5*(48*a^2*b + 3*b^3
) - a^3/2 - 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 16*tan
(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8 + 4*t
```

an(c/2 + (d*x)/2)^10)) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 - (3*a^3)/2))/d
+ (3*a^2*b*tan(c/2 + (d*x)/2))/(2*d) + (3*b*atan(((3*b*(12*a^2 - b^2)*(tan(
c/2 + (d*x)/2)*(6*a*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 - (b*tan(c/2 + (d*x)
/2)*(12*a^2 - b^2)*9i)/4))/8 + (3*b*(12*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a
*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 + (b*tan(c/2 + (d*x)/2)*(12*a^2 - b^2)*
9i)/4))/8)/(2*tan(c/2 + (d*x)/2)*((9*b^6)/16 - (27*a^2*b^4)/2 + 81*a^4*b^2)
 + (9*a*b^5)/2 + 27*a^5*b - (225*a^3*b^3)/4 - (b*(12*a^2 - b^2)*(tan(c/2 +
(d*x)/2)*(6*a*b^2 - 3*a^3) - 9*a^2*b + (3*b^3)/4 - (b*tan(c/2 + (d*x)/2)*(1
2*a^2 - b^2)*9i)/4)*3i)/8 + (b*(12*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(6*a*b^2
- 3*a^3) - 9*a^2*b + (3*b^3)/4 + (b*tan(c/2 + (d*x)/2)*(12*a^2 - b^2)*9i)/4
)*3i)/8))*(12*a^2 - b^2))/(4*d)

3.1121 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3}{3}$$

```
[Out] a^3*x-9/2*a*b^2*x+9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d
-9/2*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+c)^3/d+a^3*cot(d*x
+c)/d-9/2*a*b^2*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)/d-3/2*a^2*b*
cos(d*x+c)*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
```

Rubi [A]

time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 2672, 308, 212, 2671, 294, 327, 209, 3554, 8}

$$-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{9ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{9}{2}ab^2x+\frac{b^3\cos^3(c+dx)}{3d}+\frac{b^3\cos(c+dx)}{d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx) + 3ab^2\cos^2(c+dx)\cot^2(c+dx) + \right.$$

$$= a^3\int \cot^4(c+dx)\,dx + (3a^2b)\int \cos(c+dx)\cot^3(c+dx)\,dx + (3a$$

$$= -\frac{a^3\cot^3(c+dx)}{3d} - a^3\int \cot^2(c+dx)\,dx - \frac{(3a^2b)\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\right.}{d}$$

$$= \frac{a^3\cot(c+dx)}{d} + \frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{3a^2b\cos(c+dx)}{2d}$$

$$= a^3x - \frac{9a^2b\cos(c+dx)}{2d} + \frac{b^3\cos(c+dx)}{d} + \frac{b^3\cos^3(c+dx)}{3d} + \frac{a^3\,\mathrm{co}}{}$$

$$= a^3x - \frac{9}{2}ab^2x + \frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Mathematica [A]
time = 6.17, size = 355, normalized size = 1.83

$$\frac{a(2a^2-9b^2)(c+dx)}{2d} + \frac{b(-12a^2+5b^2)\cos(c+dx)}{4d} + \frac{b^3\cos(3(c+dx))}{12d} + \frac{\left(4a^3\cos\left(\frac{1}{2}(c+dx)\right) - 9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d} - \frac{3a^2b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d} + \frac{(9a^2b-2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{(-9a^2b+2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{3a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^3\sin\left(\frac{1}{2}(c+dx)\right) + 9ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d} - \frac{3ab^2\sin(2(c+dx))}{4d} + \frac{a^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A]
time = 0.26, size = 186, normalized size = 0.96

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{}$

risch	$a^3x - \frac{9a\,b^2x}{2} + \frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d} - \frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d} + \frac{5b^3\mathrm{e}^{i(dx+c)}}{8d} - \frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d} + \frac{5b^3\mathrm{e}^{-i(dx+c)}}{8d} - \frac{3ia\,b^2\mathrm{e}^{-2i}}{8d}$
norman	$\frac{(a^3-\frac{9}{2}a\,b^2)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^3-\frac{9}{2}a\,b^2)x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^3-\frac{27}{2}a\,b^2)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3a^3-\frac{27}{2}a\,b^2)x\left(\tan^7\left(\frac{d}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^2*b*(-1/2/sin(d*x+c)^2*co
s(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+3
*a*b^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
-3/2*d*x-3/2*c)+b^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))
)
```

Maxima [A]

time = 0.49, size = 187, normalized size = 0.96

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos\left(dx+c\right)^3+6\,\cos\left(dx+c\right)-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)b^3+9\,a^2b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]

time = 0.38, size = 293, normalized size = 1.51

$$\frac{18\,ab^2\cos\left(dx+c\right)^5+8\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-6\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)+2\left(2\,b^3\cos\left(dx+c\right)^5+3\left(2\,a^3-9\,ab^2\right)dx\cos\left(dx+c\right)^2-2\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^3-3\left(2\,a^3-9\,ab^2\right)dx+3\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(178) = 356.
time = 0.52, size = 421, normalized size = 2.17

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,(2\,a^3-9\,ab^2)(dx+c)-36\,(9\,a^2b-2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{198\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-44\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+135\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+156\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+132\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-324\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-351\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+156\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+126\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-540\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-315\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+148\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^3}}{72\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

Mupad [B]
time = 9.65, size = 405, normalized size = 2.09

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{9\,a^2\,b}{2}-b^3\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1{}\mathrm{i}\right)\,\left(\frac{a\,b^2\,9{}\mathrm{i}}{2}-a^3\,1{}\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{3\,a\,b^2}{2}-\frac{5\,a^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(12\,a\,b^2-4\,a^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\,(5\,a^3+12\,a\,b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(60\,a\,b^2-14\,a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(36\,a\,b^2-\frac{44\,a^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,(51\,a^2\,b-32\,b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(57\,a^2\,b-\frac{64\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,(105\,a^2\,b-32\,b^3)+\frac{a^3}{3}+3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\,(2\,a^2-9\,b^2)\,1{}\mathrm{i}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^4,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 -
 b^3))/d - (log(tan(c/2 + (d*x)/2) + 1i)*((a*b^2*9i)/2 - a^3*1i))/d + (tan(
c/2 + (d*x)/2)*((3*a*b^2)/2 - (5*a^3)/8))/d - (tan(c/2 + (d*x)/2)^2*(12*a*b
^2 - 4*a^3) - tan(c/2 + (d*x)/2)^8*(12*a*b^2 + 5*a^3) + tan(c/2 + (d*x)/2)^
4*(60*a*b^2 - 14*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - (44*a^3)/3) + tan(
```

```
c/2 + (d*x)/2)^7*(51*a^2*b - 32*b^3) + tan(c/2 + (d*x)/2)^3*(57*a^2*b - (64
*b^3)/3) + tan(c/2 + (d*x)/2)^5*(105*a^2*b - 32*b^3) + a^3/3 + 3*a^2*b*tan(
c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*t
an(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (3*a^2*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2) - 1i)*(2*a^2 - 9*b^2)*1i)/(2*d)
```

3.1122 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=187

$$\frac{3}{2}b\left(2a^2-b^2\right)x-\frac{3a(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^2(73a^2-2b^2)\cos(c+dx)}{8ad}-\frac{13b^3\cos(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/2*b*(2*a^2-b^2)*x-3/8*a*(a^2-12*b^2)*arctanh(cos(d*x+c))/d-1/8*b^2*(73*a^
2-2*b^2)*cos(d*x+c)/a/d-13/4*b^3*cos(d*x+c)*sin(d*x+c)/d+17/8*b*cot(d*x+c)*
(a+b*sin(d*x+c))^2/d+5/8*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^3/d-1/4*cot
(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]

time = 0.43, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {2972, 3126, 3112, 3102, 2814, 3855}

$$-\frac{b^2(73a^2-2b^2)\cos(c+dx)}{8ad}-\frac{3a(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{3}{2}bx\left(2a^2-b^2\right)+\frac{17b\cot(c+dx)(a+b\sin(c+dx))^2}{8d}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{4ad}+\frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{8d}-\frac{13b^3\sin(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(2*a^2 - b^2)*x)/2 - (3*a*(a^2 - 12*b^2)*ArcTanh[Cos[c + d*x]])/(8*d)
- (b^2*(73*a^2 - 2*b^2)*Cos[c + d*x])/(8*a*d) - (13*b^3*Cos[c + d*x]*Sin[c
+ d*x])/(4*d) + (17*b*Cot[c + d*x]*(a + b*Sin[c + d*x])^2)/(8*d) + (5*Cot[c
 + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^3)/(8*d) - (Cot[c + d*x]*Csc[c +
d*x]^3*(a + b*Sin[c + d*x])^4)/(4*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3\,dx &= -\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{4ad} - \frac{\int \csc^3 \\ &= \frac{5\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{8d} - \frac{\cot(c+ \\ &= \frac{17b\cot(c+dx)(a+b\sin(c+dx))^2}{8d} + \frac{5\cot(c+dx)\csc \\ &= -\frac{13b^3\cos(c+dx)\sin(c+dx)}{4d} + \frac{17b\cot(c+dx)(a+b \\ &= -\frac{b^2(73a^2-2b^2)\cos(c+dx)}{8ad} - \frac{13b^3\cos(c+dx)\sin(c- \\ &= \frac{3}{2}b(2a^2-b^2)\,x - \frac{b^2(73a^2-2b^2)\cos(c+dx)}{8ad} - \frac{13b^3\,\mathrm{co} \\ &= \frac{3}{2}b(2a^2-b^2)\,x - \frac{3a(a^2-12b^2)\tanh^{-1}(\cos(c+dx))}{8d} - \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 381 vs. 2(187) = 374.
time = 6.25, size = 381, normalized size = 2.04

$$-\frac{3b(-2a^2+b^2)(c+dx)}{2d}-\frac{3ab^2\cos(c+dx)}{d}+\frac{\left(4a^2b\cos\left(\frac{1}{2}(c+dx)\right)-b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{2d}+\frac{(5a^3-12ab^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{a^2b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a^3\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3(a^3-12ab^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{3(a^3-12ab^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{(-5a^3+12ab^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^3\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2b\sin\left(\frac{1}{2}(c+dx)\right)+b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b^3\sin(2(c+dx))}{4d}+\frac{a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(-2*a^2 + b^2)*(c + d*x))/(2*d) - (3*a*b^2*Cos[c + d*x])/d + ((4*a^2*
b*Cos[(c + d*x)/2] - b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(2*d) + ((5*a^
3 - 12*a*b^2)*Csc[(c + d*x)/2]^2)/(32*d) - (a^2*b*Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(8*d) - (a^3*Csc[(c + d*x)/2]^4)/(64*d) - (3*(a^3 - 12*a*b^2)*L
og[Cos[(c + d*x)/2]])/(8*d) + (3*(a^3 - 12*a*b^2)*Log[Sin[(c + d*x)/2]])/(8
*d) + ((-5*a^3 + 12*a*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^3*Sec[(c + d*x)/
2]^4)/(64*d) + (Sec[(c + d*x)/2]*(-4*a^2*b*Sin[(c + d*x)/2] + b^3*Sin[(c +
d*x)/2]))/(2*d) - (b^3*Sin[2*(c + d*x)])/(4*d) + (a^2*b*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2])/(8*d)
```

Maple [A]
time = 0.28, size = 226, normalized size = 1.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/4/sin(d*x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*
cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/3*co
```

```
t(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a*b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*co
s(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+b^3*(-1/sin(d*x+c)
*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.49, size = 212, normalized size = 1.13

$$\frac{16\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}-1}{\tan\left(dx+c\right)^{3}}\right)a^{2}b-8\left(3\,dx+3\,c+\frac{3\,\tan\left(dx+c\right)^{2}+2}{\tan\left(dx+c\right)^{3}+\tan\left(dx+c\right)}\right)b^{3}-a^{3}\left(\frac{2\left(5\,\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+12\,ab^{2}\left(\frac{2\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(16*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2*b - 8*(3
*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^3 -
a^3*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)
^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 12*a*b^2*(2*
cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1
) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.38, size = 331, normalized size = 1.77

$$-\frac{48\,ab^{2}\cos\left(dx+c\right)^{5}-24\,(2\,a^{2}b-b^{3})dx\cos\left(dx+c\right)^{4}+48\,(2\,a^{2}b-b^{3})dx\cos\left(dx+c\right)^{2}+10\,(a^{3}-12\,ab^{2})\cos\left(dx+c\right)^{3}-24\,(2\,a^{2}b-b^{3})dx-6\,(a^{3}-12\,ab^{2})\cos\left(dx+c\right)+3\,\left((a^{3}-12\,ab^{2})\cos\left(dx+c\right)^{4}+a^{3}-12\,ab^{2}-2\,(a^{3}-12\,ab^{2})\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,\left((a^{3}-12\,ab^{2})\cos\left(dx+c\right)^{4}+a^{3}-12\,ab^{2}-2\,(a^{3}-12\,ab^{2})\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+8\,\left(b^{3}\cos\left(dx+c\right)^{5}+4\,(2\,a^{2}b-b^{3})\cos\left(dx+c\right)^{3}-3\,(2\,a^{2}b-b^{3})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{16\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*(48*a*b^2*cos(d*x + c)^5 - 24*(2*a^2*b - b^3)*d*x*cos(d*x + c)^4 + 48
*(2*a^2*b - b^3)*d*x*cos(d*x + c)^2 + 10*(a^3 - 12*a*b^2)*cos(d*x + c)^3 -
24*(2*a^2*b - b^3)*d*x - 6*(a^3 - 12*a*b^2)*cos(d*x + c) + 3*((a^3 - 12*a*b
^2)*cos(d*x + c)^4 + a^3 - 12*a*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) - 3*((a^3 - 12*a*b^2)*cos(d*x + c)^4 + a^3 - 12*a
*b^2 - 2*(a^3 - 12*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) + 8*
(b^3*cos(d*x + c)^5 + 4*(2*a^2*b - b^3)*cos(d*x + c)^3 - 3*(2*a^2*b - b^3)*
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep

Giac [A]
time = 0.57, size = 343, normalized size = 1.83

$$\frac{a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} + 8 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 8 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 24 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 120 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 32 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 96 \, {\left(2 \, a^{2} b - b^{3}\right)} {\left(d x + c\right)} + 24 \, {\left(a^{3} - 12 \, a b^{2}\right)} \log\left({\left| \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) \right|}\right) + \frac{64 \, {\left(b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 6 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 6 \, a b^{2}\right)}}{{\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1\right)}^{2}} - \frac{50 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 600 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4} - 120 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} + 32 \, b^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{3} - 8 \, a^{3} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 24 \, a b^{2} \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 8 \, a^{2} b \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + a^{3}}{\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{4}}}{64 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/64*(a^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 120*a^2*b*tan(1/2*d*x + 1/2*c) + 32*b^3*tan(1/2*d*x + 1/2*c) + 96*(2*a^2*b - b^3)*(d*x + c) + 24*(a^3 - 12*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 64*(b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c) - 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (50*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 32*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^4)/d

Mupad [B]
time = 9.50, size = 699, normalized size = 3.74

$$\frac{a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{3\,a\,b^2}{8}-\frac{a^3}{8}\right)}{d}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(6\,a\,b^2-\frac{3\,a^3}{2}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(102\,a\,b^2-2\,a^3\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(108\,a\,b^2-\frac{15\,a^3}{4}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(26\,a^2\,b-8\,b^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(30\,a^2\,b+8\,b^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(58\,a^2\,b-32\,b^3\right)+\frac{a^3}{4}+2\,a^2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+32\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+16\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\right)}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,a^2\,b}{8}-\frac{b^3}{2}\right)}{d}+\frac{3\,a\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-12\,b^2\right)}{8\,d}+\frac{a^2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{8\,d}-\frac{3\,b\,\mathrm{atan}\left(\frac{\frac{3\,b\,\left(2\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a\,b^2-\frac{3\,a^3}{4}\right)-6\,a^2\,b+3\,b^3-b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\,9{}\mathrm{i}\right)}{2}+\frac{3\,b\,\left(2\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a\,b^2-\frac{3\,a^3}{4}\right)-6\,a^2\,b+3\,b^3+b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\,9{}\mathrm{i}\right)}{2}}{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(36\,a^4\,b^2-36\,a^2\,b^4+9\,b^6\right)+27\,a\,b^5+\frac{9\,a^5\,b}{2}-\frac{225\,a^3\,b^3}{4}-\frac{b\,\left(2\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a\,b^2-\frac{3\,a^3}{4}\right)-6\,a^2\,b+3\,b^3-b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\,9{}\mathrm{i}\right)\,3{}\mathrm{i}}{2}+\frac{b\,\left(2\,a^2-b^2\right)\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a\,b^2-\frac{3\,a^3}{4}\right)-6\,a^2\,b+3\,b^3+b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-b^2\right)\,9{}\mathrm{i}\right)\,3{}\mathrm{i}}{2}}\right)\,\left(2\,a^2-b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^5,x)

[Out] (a^3*tan(c/2 + (d*x)/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 - a^3/8))/d - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 - (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(102*a*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^4*(108*a*b^2 - (15*a^3)/4) - tan(c/2 + (d*x)/2)^3*(26*a^2*b - 8*b^3) - tan(c/2 + (d*x)/2)^7*(30*a^2*b + 8*b^3) - tan(c/2 + (d*x)/2)^5*(58*a^2*b - 32*b^3) + a^3/4 + 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (tan(c/2 + (d*x)/2)*((15*a^2*b)/8 - b^3/2))/d + (3*a*log(tan(c/2 + (d*x)/2))*(a^2 - 12*b^2))/(8*d) + (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d) - (3*b*atan(((3*b*(2*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 - b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i))/2 + (3*b*(2*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 + b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i))/2)/(2*tan(c/2 + (d*x)/2)*(9*b^6 - 36*a^2*b^4 + 36*a^4*b^2) + 27*a*b^5 + (9*a^5*b)/2 - (225*a^3*b^3)/4 - (b*(2*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 - b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i)*3i)/2 + (b*(2*a^2 - b^2)*(tan(c/2 + (d*x)/2)*(9*a*b^2 - (3*a^3)/4) - 6*a^2*b + 3*b^3 + b*tan(c/2 + (d*x)/2)*(2*a^2 - b^2)*9i)*3i)/2))*(2*a^2 - b^2))/d

3.1123 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=227

$$3ab^2x-\frac{3b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d}-\frac{a(4a^2-29b^2)\cot(c+dx)}{20d}+\frac{27b\cot(}{}$$

```
[Out] 3*a*b^2*x-3/8*b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/d-1/40*b^3*(83*a^2+2*b^2)
*cos(d*x+c)/a^2/d-1/20*a*(4*a^2-29*b^2)*cot(d*x+c)/d+27/40*b*cot(d*x+c)*csc
(d*x+c)*(a+b*sin(d*x+c))^2/d+2/5*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^3
/d+1/20*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^4/a^2/d-1/5*cot(d*x+c)*c
sc(d*x+c)^4*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.46, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2972, 3126, 3110, 3102, 2814, 3855}

$$-\frac{a(4a^2-29b^2)\cot(c+dx)}{20d}-\frac{3b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d}+\frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{20a^2d}+3ab^2x-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^4}{5ad}+\frac{2\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{5d}+\frac{27b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{40d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] 3*a*b^2*x - (3*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) - (b^3*(83*a^
2 + 2*b^2)*Cos[c + d*x])/(40*a^2*d) - (a*(4*a^2 - 29*b^2)*Cot[c + d*x])/(20
*d) + (27*b*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^2)/(40*d) + (2*C
ot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3)/(5*d) + (b*Cot[c + d*x]*
Csc[c + d*x]^3*(a + b*Sin[c + d*x])^4)/(20*a^2*d) - (Cot[c + d*x]*Csc[c + d
*x]^4*(a + b*Sin[c + d*x])^4)/(5*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
```

```
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^4}{20a^2d} - \frac{\cot(c-}{}$$
$$= \frac{2\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^3}{5d} + \frac{b\cot(c}{}$$
$$= \frac{27b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2}{40d} + \frac{2\cot(}{}$$
$$= -\frac{a(4a^2-29b^2)\cot(c+dx)}{20d} + \frac{27b\cot(c+dx)\csc(c+a}{40a}$$
$$= -\frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d} - \frac{a(4a^2-29b^2)\cot(c+d}{20d}$$
$$= 3ab^2x - \frac{b^3(83a^2+2b^2)\cos(c+dx)}{40a^2d} - \frac{a(4a^2-29b^2)\cot}{20d}$$
$$= 3ab^2x - \frac{3b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^3(83a^2-}{}$$

Mathematica [A]
time = 0.88, size = 405, normalized size = 1.78

$$\frac{960ab^2c+960ab^2dx-320b^3\cos(c+dx)-32(a^3-20ab^2)\cot\left(\frac{1}{2}(c+dx)\right)+150a^2b\csc^2\left(\frac{1}{2}(c+dx)\right)-40b^3\csc^2\left(\frac{1}{2}(c+dx)\right)-15a^2b\csc^4\left(\frac{1}{2}(c+dx)\right)-360a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+480b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+360a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-480b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-150a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)+40b^3\sec^2\left(\frac{1}{2}(c+dx)\right)+15a^2b\sec^4\left(\frac{1}{2}(c+dx)\right)-112a^3\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+320ab^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+64a^3\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right)+7a^3\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-20ab^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-a^3\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)+32a^3\tan\left(\frac{1}{2}(c+dx)\right)-640ab^2\tan\left(\frac{1}{2}(c+dx)\right)}{320d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (960*a*b^2*c + 960*a*b^2*d*x - 320*b^3*Cos[c + d*x] - 32*(a^3 - 20*a*b^2)*C
ot[(c + d*x)/2] + 150*a^2*b*Csc[(c + d*x)/2]^2 - 40*b^3*Csc[(c + d*x)/2]^2
- 15*a^2*b*Csc[(c + d*x)/2]^4 - 360*a^2*b*Log[Cos[(c + d*x)/2]] + 480*b^3*L
og[Cos[(c + d*x)/2]] + 360*a^2*b*Log[Sin[(c + d*x)/2]] - 480*b^3*Log[Sin[(c
 + d*x)/2]] - 150*a^2*b*Sec[(c + d*x)/2]^2 + 40*b^3*Sec[(c + d*x)/2]^2 + 15
*a^2*b*Sec[(c + d*x)/2]^4 - 112*a^3*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320
*a*b^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 64*a^3*Csc[c + d*x]^5*Sin[(c + d
*x)/2]^6 + 7*a^3*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a*b^2*Csc[(c + d*x)/2
]^4*Sin[c + d*x] - a^3*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 32*a^3*Tan[(c + d*
x)/2] - 640*a*b^2*Tan[(c + d*x)/2])/(320*d)
```

Maple [A]
time = 0.33, size = 192, normalized size = 0.85

method	result
derivativedivides	$\frac{-\frac{a^3\left(\cos^5(dx+c)\right)}{5\sin(dx+c)^5}+3a^2b\left(-\frac{\cos^5(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{8\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a\,b^2\left(-\right.}{d}$

default	$\frac{-\frac{a^{3}\left(\cos^{5}(dx+c)\right)}{5\sin(dx+c)^{5}}+3a^{2}b\left(-\frac{\cos^{5}(dx+c)}{4\sin(dx+c)^{4}}+\frac{\cos^{5}(dx+c)}{8\sin(dx+c)^{2}}+\frac{\left(\cos^{3}(dx+c)\right)}{8}+\frac{3\cos(dx+c)}{8}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+3a\,b^{2}\left(\right.}{d}$
risch	$3a\,b^{2}x-\frac{b^{3}\mathrm{e}^{i(dx+c)}}{2d}-\frac{b^{3}\mathrm{e}^{-i(dx+c)}}{2d}+\frac{-40ia^{3}\mathrm{e}^{8i(dx+c)}+160ia\,b^{2}-75a^{2}b\,\mathrm{e}^{9i(dx+c)}+20b^{3}\mathrm{e}^{9i(dx+c)}-560ia\,b^{2}\mathrm{e}^{2i(dx}}{}$
norman	$\frac{-\frac{a^{3}}{160d}+\frac{a^{3}\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}+\frac{\left(18a^{2}b-23b^{3}\right)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{\left(63a^{2}b-88b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{\left(117a^{2}b-172b^{3}\right)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{32d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*a^3/sin(d*x+c)^5*cos(d*x+c)^5+3*a^2*b*(-1/4/sin(d*x+c)^4*cos(d*x+
c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+3/8*ln(c
sc(d*x+c)-cot(d*x+c)))+3*a*b^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b^3*(-1
/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x
+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.50, size = 182, normalized size = 0.80

$$\frac{80\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)ab^2-15\,a^2b\left(\frac{2\left(5\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)+20\,b^3\left(\frac{2\cos(dx+c)}{\cos(dx+c)^2-1}-4\cos\left(dx+c\right)+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{16\,a^3}{\tan(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/80*(80*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a*b^2 - 15*a
^2*b*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c
)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)) + 20*b^3*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
 - 3*log(cos(d*x + c) - 1)) - 16*a^3/tan(d*x + c)^5)/d
```

Fricas [A]
time = 0.40, size = 334, normalized size = 1.47

$$-\frac{560\,ab^2\cos\left(dx+c\right)^3+16\left(a^3-20\,ab^2\right)\cos\left(dx+c\right)^5-240\,ab^2\cos\left(dx+c\right)+15\left(\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)^4+3\,a^2b-4\,b^3-2\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)^4+3\,a^2b-4\,b^3-2\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-10\left(24\,ab^2dx\cos\left(dx+c\right)^4-8\,b^3\cos\left(dx+c\right)^5-48\,ab^2dx\cos\left(dx+c\right)^2+24\,ab^2dx-5\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)^3+3\left(3\,a^2b-4\,b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{80\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/80*(560*a*b^2*cos(d*x + c)^3 + 16*(a^3 - 20*a*b^2)*cos(d*x + c)^5 - 240*
a*b^2*cos(d*x + c) + 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3
 - 2*(3*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
```

```
+ c) - 15*((3*a^2*b - 4*b^3)*cos(d*x + c)^4 + 3*a^2*b - 4*b^3 - 2*(3*a^2*b
- 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(24
*a*b^2*d*x*cos(d*x + c)^4 - 8*b^3*cos(d*x + c)^5 - 48*a*b^2*d*x*cos(d*x + c
)^2 + 24*a*b^2*d*x - 5*(3*a^2*b - 4*b^3)*cos(d*x + c)^3 + 3*(3*a^2*b - 4*b^
3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 356, normalized size = 1.57

$$\frac{2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+40\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+960\,(dx+c)ab^2+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-600\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{640\,b^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+120\,(3\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{822\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1096\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+20\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-600\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+40\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/320*(2*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*
a^3*tan(1/2*d*x + 1/2*c)^3 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a^2*b*ta
n(1/2*d*x + 1/2*c)^2 + 40*b^3*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*b^2
+ 20*a^3*tan(1/2*d*x + 1/2*c) - 600*a*b^2*tan(1/2*d*x + 1/2*c) - 640*b^3/(t
an(1/2*d*x + 1/2*c)^2 + 1) + 120*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/
2*c))) - (822*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1096*b^3*tan(1/2*d*x + 1/2*c)^
5 + 20*a^3*tan(1/2*d*x + 1/2*c)^4 - 600*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 120*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 40*b^3*tan(1/2*d*x + 1/2*c)^3 - 10*a^3*tan(1
/2*d*x + 1/2*c)^2 + 40*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2*b*tan(1/2*d*x
+ 1/2*c) + 2*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

Mupad [B]
time = 12.28, size = 1007, normalized size = 4.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^6,x)
```

```
[Out] -(2*a^3*cos(c/2 + (d*x)/2)^12 - 2*a^3*sin(c/2 + (d*x)/2)^12 + 8*a^3*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 - 10*a^3*cos(c/2 + (d*x)/2)^4*sin(c/2 +
```

```
(d*x)/2)^8 + 10*a^3*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 - 8*a^3*cos(
c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^2 - 40*b^3*cos(c/2 + (d*x)/2)^3*sin(c/
2 + (d*x)/2)^9 - 40*b^3*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 680*b^3
*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 + 40*b^3*cos(c/2 + (d*x)/2)^9*si
n(c/2 + (d*x)/2)^3 + 480*b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 480*b^3*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 - 15*a^2*b*cos(c
/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^11 + 15*a^2*b*cos(c/2 + (d*x)/2)^11*sin(c/
2 + (d*x)/2) - 40*a*b^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^10 + 560*a*
b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 - 560*a*b^2*cos(c/2 + (d*x)/2
)^8*sin(c/2 + (d*x)/2)^4 + 40*a*b^2*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2
)^2 + 105*a^2*b*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^9 + 120*a^2*b*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 - 120*a^2*b*cos(c/2 + (d*x)/2)^7*sin(c
/2 + (d*x)/2)^5 - 105*a^2*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3 + 192
0*a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 4*b^2*sin(c/2 + (d*x)/2) + 8*a*b*c
os(c/2 + (d*x)/2))/(4*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 8
*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^7 + 1920*
a*b^2*atan((3*a^2*sin(c/2 + (d*x)/2) - 4*b^2*sin(c/2 + (d*x)/2) + 8*a*b*cos
(c/2 + (d*x)/2))/(4*b^2*cos(c/2 + (d*x)/2) - 3*a^2*cos(c/2 + (d*x)/2) + 8*a
*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5 - 360*a^2
*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^5*sin(c/2
+ (d*x)/2)^7 - 360*a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2
 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/(320*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (
d*x)/2)^5*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))
```

3.1124 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=275

$$b^3x-\frac{a(a^2+18b^2)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{b(36a^4-43a^2b^2+2b^4)\cot(c+dx)}{60a^2d}-\frac{(15a^4-84a^2b^2+4b^4)\cot(c+}{240ad}$$

```
[Out] b^3*x-1/16*a*(a^2+18*b^2)*arctanh(cos(d*x+c))/d-1/60*b*(36*a^4-43*a^2*b^2+2
*b^4)*cot(d*x+c)/a^2/d-1/240*(15*a^4-84*a^2*b^2+4*b^4)*cot(d*x+c)*csc(d*x+c
)/a/d+1/120*b*(39*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^2/a^2
/d+1/120*(35*a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^3/a^2/d+1/
15*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^4/a^2/d-1/6*cot(d*x+c)*csc(d*
x+c)^5*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.49, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2972, 3126, 3110, 3100, 2814, 3855}

$$-\frac{a(a^2+18b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(35a^2-2b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3}{120a^2d}+\frac{b(39a^2-2b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2}{120a^2d}+\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^4}{15a^2d}-\frac{b(36a^4-43a^2b^2+2b^4)\cot(c+dx)}{60a^2d}-\frac{(15a^4-84a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{240ad}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^4}{6ad}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] b^3*x - (a*(a^2 + 18*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (b*(36*a^4 - 43*a
^2*b^2 + 2*b^4)*Cot[c + d*x])/(60*a^2*d) - ((15*a^4 - 84*a^2*b^2 + 4*b^4)*C
ot[c + d*x]*Csc[c + d*x])/(240*a*d) + (b*(39*a^2 - 2*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2*(a + b*Sin[c + d*x])^2)/(120*a^2*d) + ((35*a^2 - 2*b^2)*Cot[c +
d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3)/(120*a^2*d) + (b*Cot[c + d*x]*C
sc[c + d*x]^4*(a + b*Sin[c + d*x])^4)/(15*a^2*d) - (Cot[c + d*x]*Csc[c + d*
x]^5*(a + b*Sin[c + d*x])^4)/(6*a*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
```

```
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^4}{15a^2d} - \frac{\cot(c-}{}$$
$$= \frac{(35a^2-2b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx)}{120a^2d}$$
$$= \frac{b(39a^2-2b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx}{120a^2d}$$
$$= -\frac{(15a^4-84a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{240ad} + \frac{b(39}{}$$
$$= -\frac{b(36a^4-43a^2b^2+2b^4)\cot(c+dx)}{60a^2d} - \frac{(15a^4-84a^2b^2}{}$$
$$= b^3x - \frac{b(36a^4-43a^2b^2+2b^4)\cot(c+dx)}{60a^2d} - \frac{(15a^4-84}{}$$
$$= b^3x - \frac{a(a^2+18b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b(36a^4-43}{}$$

Mathematica [A]
time = 1.27, size = 408, normalized size = 1.48

$$\frac{1920b^3c+1920b^3dx-64(9a^2b-20b^3)\cot\left(\frac{1}{2}(c+dx)\right)-30(a^3-30ab^2)\csc^2\left(\frac{1}{2}(c+dx)\right)-120a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-2160ab^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2160ab^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+30a^3\sec^2\left(\frac{1}{2}(c+dx)\right)-900ab^2\sec^2\left(\frac{1}{2}(c+dx)\right)-30a^3\sec^4\left(\frac{1}{2}(c+dx)\right)+90ab^2\sec^4\left(\frac{1}{2}(c+dx)\right)+5a^3\sec^6\left(\frac{1}{2}(c+dx)\right)-2016a^2b\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+640b^3\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)-a^2\csc^6\left(\frac{1}{2}(c+dx)\right)(5a+18b\sin(c+dx))+2\csc^4\left(\frac{1}{2}(c+dx)\right)(15(a^3-3ab^2)+b(63a^2-20b^2)\sin(c+dx))+576a^2b\tan\left(\frac{1}{2}(c+dx)\right)-1280b^3\tan\left(\frac{1}{2}(c+dx)\right)+36a^2b\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (1920*b^3*c + 1920*b^3*d*x - 64*(9*a^2*b - 20*b^3)*Cot[(c + d*x)/2] - 30*(a
^3 - 30*a*b^2)*Csc[(c + d*x)/2]^2 - 120*a^3*Log[Cos[(c + d*x)/2]] - 2160*a*
b^2*Log[Cos[(c + d*x)/2]] + 120*a^3*Log[Sin[(c + d*x)/2]] + 2160*a*b^2*Log[
Sin[(c + d*x)/2]] + 30*a^3*Sec[(c + d*x)/2]^2 - 900*a*b^2*Sec[(c + d*x)/2]^
2 - 30*a^3*Sec[(c + d*x)/2]^4 + 90*a*b^2*Sec[(c + d*x)/2]^4 + 5*a^3*Sec[(c
+ d*x)/2]^6 - 2016*a^2*b*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 640*b^3*Csc[c
+ d*x]^3*Sin[(c + d*x)/2]^4 - a^2*Csc[(c + d*x)/2]^6*(5*a + 18*b*Sin[c + d*
x]) + 2*Csc[(c + d*x)/2]^4*(15*(a^3 - 3*a*b^2) + b*(63*a^2 - 20*b^2)*Sin[c
+ d*x]) + 576*a^2*b*Tan[(c + d*x)/2] - 1280*b^3*Tan[(c + d*x)/2] + 36*a^2*b
*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(1920*d)
```

Maple [A]
time = 0.33, size = 227, normalized size = 0.83

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{3a^2b\left(\cos^5(dx+c\right.}{5\sin(dx+c)^5}}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\frac{\ln(\csc(dx+c)-\cot(dx+c))}{16}\right)-\frac{3a^2b\left(\cos^5(dx+c\right.}{5\sin(dx+c)^5}}{}$
risch	$b^3x-\frac{2880ib^3e^{4i(dx+c)}+720ia^2b\,e^{10i(dx+c)}-15a^3e^{11i(dx+c)}+450a\,b^2e^{11i(dx+c)}-3200ib^3e^{6i(dx+c)}-1440ib^3e^{2i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/4
8/sin(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d
*x+c)-cot(d*x+c)))-3/5*a^2*b/sin(d*x+c)^5*cos(d*x+c)^5+3*a*b^2*(-1/4/sin(d*
x+c)^4*cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(
d*x+c)+3/8*ln(csc(d*x+c)-cot(d*x+c)))+b^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x
+c))
```

Maxima [A]

time = 0.53, size = 217, normalized size = 0.79

$$\frac{160\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)b^3+5\,a^3\left(\frac{2\left(3\,\cos(dx+c)^5+8\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-90\,ab^2\left(\frac{2\left(5\,\cos(dx+c)^3-3\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{288\,a^2b}{\tan(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/480*(160*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*b^3 + 5*a^
3*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(d*x + c)^6
 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1)) - 90*a*b^2*(2*(5*cos(d*x + c)^3 - 3*cos(d*x + c))/(co
s(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1) - 3*log(cos(
d*x + c) - 1)) - 288*a^2*b/tan(d*x + c)^5)/d
```

Fricas [A]

time = 0.40, size = 373, normalized size = 1.36

$$\frac{480\,b^3dx\cos\left(dx+c\right)^6-1440\,b^3dx\cos\left(dx+c\right)^4+1440\,b^3dx\cos\left(dx+c\right)^2+30\left(a^3-30\,ab^2\right)\cos\left(dx+c\right)^5-480\,b^3dx+80\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^3-30\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)-15\left(\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^6-3\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^4-a^3-18\,ab^2+3\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+15\left(\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^6-3\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^4-a^3-18\,ab^2+3\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+32\left(35\,b^3\cos\left(dx+c\right)^3+\left(9\,a^2b-20\,b^3\right)\cos\left(dx+c\right)^5-15\,b^3\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/480*(480*b^3*d*x*cos(d*x + c)^6 - 1440*b^3*d*x*cos(d*x + c)^4 + 1440*b^3*
d*x*cos(d*x + c)^2 + 30*(a^3 - 30*a*b^2)*cos(d*x + c)^5 - 480*b^3*d*x + 80*
(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*(a^3 + 18*a*b^2)*cos(d*x + c) - 15*((a
^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a^3 - 1
8*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
15*((a^3 + 18*a*b^2)*cos(d*x + c)^6 - 3*(a^3 + 18*a*b^2)*cos(d*x + c)^4 - a
^3 - 18*a*b^2 + 3*(a^3 + 18*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) +
1/2) + 32*(35*b^3*cos(d*x + c)^3 + (9*a^2*b - 20*b^3)*cos(d*x + c)^5 - 15*b
^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d
*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**7*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 399, normalized size = 1.45

$$\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+90\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-180\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-720\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1920\,(dx+c)b^{3}+360\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1200\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,(a^{3}+18\,ab^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{294\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+5292\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+360\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1200\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-720\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-180\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+80\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+90\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^7*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^3*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 15
*a^3*tan(1/2*d*x + 1/2*c)^4 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 180*a^2*b*t
an(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x
+ 1/2*c)^2 - 720*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 1920*(d*x + c)*b^3 + 360*a^
2*b*tan(1/2*d*x + 1/2*c) - 1200*b^3*tan(1/2*d*x + 1/2*c) + 120*(a^3 + 18*a*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - (294*a^3*tan(1/2*d*x + 1/2*c)^6 + 529
2*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 1200*b^
3*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*tan(1/2*d*x + 1/2*c)^4 - 720*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 180*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 80*b^3*tan(1/2*d*x +
1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 90*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 36*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [B]
time = 9.93, size = 446, normalized size = 1.62

$$\frac{2\,b^{3}\,\mathrm{atan}\left(\frac{4\,b^{6}}{\frac{a^{3}\,b^{3}}{4}+\frac{9\,a\,b^{5}}{2}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^{6}}+\frac{9\,a\,b^{5}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{a^{3}\,b^{3}}{4}+\frac{9\,a\,b^{5}}{2}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^{6}\right)}+\frac{a^{3}\,b^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{a^{3}\,b^{3}}{4}+\frac{9\,a\,b^{5}}{2}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^{6}\right)}\right)}{d}+\frac{a^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{384\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(\frac{a^{3}}{128}+\frac{3\,a\,b^{2}}{8}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left(\frac{3\,a\,b^{2}}{64}-\frac{a^{3}}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(\frac{3\,a^{2}\,b}{32}-\frac{b^{3}}{24}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(3\,a\,b^{2}-\frac{a^{3}}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left(\frac{a^{3}}{2}+24\,a\,b^{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(6\,a^{2}\,b-\frac{8\,b^{3}}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\left(12\,a^{2}\,b-40\,b^{3}\right)+\frac{a^{3}}{6}+\frac{6\,a^{2}\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{5}}{64\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a^{2}\,b}{16}-\frac{5\,b^{3}}{8}\right)}{d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(a^{2}+18\,b^{2}\right)}{16\,d}+\frac{3\,a^{2}\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^7,x)

[Out] (2*b^3*atan((4*b^6)/((9*a*b^5)/2 + (a^3*b^3)/4 - 4*b^6*tan(c/2 + (d*x)/2))
+ (9*a*b^5*tan(c/2 + (d*x)/2))/(2*((9*a*b^5)/2 + (a^3*b^3)/4 - 4*b^6*tan(c/
2 + (d*x)/2))) + (a^3*b^3*tan(c/2 + (d*x)/2))/(4*((9*a*b^5)/2 + (a^3*b^3)/4
 - 4*b^6*tan(c/2 + (d*x)/2)))))/d + (a^3*tan(c/2 + (d*x)/2)^6)/(384*d) - (t
an(c/2 + (d*x)/2)^2*((3*a*b^2)/8 + a^3/128))/d + (tan(c/2 + (d*x)/2)^4*((3*
a*b^2)/64 - a^3/128))/d - (tan(c/2 + (d*x)/2)^3*((3*a^2*b)/32 - b^3/24))/d
- (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - a^3/2) - tan(c/2 + (d*x)/2)^4*(24*a*b^2
+ a^3/2) - tan(c/2 + (d*x)/2)^3*(6*a^2*b - (8*b^3)/3) + tan(c/2 + (d*x)/2)^
5*(12*a^2*b - 40*b^3) + a^3/6 + (6*a^2*b*tan(c/2 + (d*x)/2))/5)/(64*d*tan(c
/2 + (d*x)/2)^6) + (tan(c/2 + (d*x)/2)*((3*a^2*b)/16 - (5*b^3)/8))/d + (a*l
og(tan(c/2 + (d*x)/2))*(a^2 + 18*b^2))/(16*d) + (3*a^2*b*tan(c/2 + (d*x)/2)
^5)/(160*d)
```

3.1125 $\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=303

$$-\frac{3b(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a(2a^2+21b^2)\cot(c+dx)}{35d}-\frac{b(105a^4-116a^2b^2+12b^4)\cot(c+dx)\csc(c}{560a^2d}$$

```
[Out] -3/16*b*(a^2+2*b^2)*arctanh(cos(d*x+c))/d-1/35*a*(2*a^2+21*b^2)*cot(d*x+c)/
d-1/560*b*(105*a^4-116*a^2*b^2+12*b^4)*cot(d*x+c)*csc(d*x+c)/a^2/d-1/140*(4
*a^4-19*a^2*b^2+2*b^4)*cot(d*x+c)*csc(d*x+c)^2/a/d+1/280*b*(53*a^2-6*b^2)*c
ot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^2/a^2/d+1/35*(8*a^2-b^2)*cot(d*x+c)
*csc(d*x+c)^4*(a+b*sin(d*x+c))^3/a^2/d+1/14*b*cot(d*x+c)*csc(d*x+c)^5*(a+b*
sin(d*x+c))^4/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.56, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2972, 3126, 3110, 3100, 2827, 3852, 8, 3855}

$$-\frac{a(2a^2+21b^2)\cot(c+dx)}{35d}-\frac{3b(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(8a^2-b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{35a^2d}+\frac{b(53a^2-6b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2}{280a^2d}+\frac{b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^4}{14a^2d}-\frac{(4a^4-19a^2b^2+2b^4)\cot(c+dx)\csc^2(c+dx)}{140ad}-\frac{b(105a^4-116a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{560a^2d}-\frac{\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^4}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(a^2 + 2*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) - (a*(2*a^2 + 21*b^2)*Cot
[c + d*x])/(35*d) - (b*(105*a^4 - 116*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c
+ d*x])/(560*a^2*d) - ((4*a^4 - 19*a^2*b^2 + 2*b^4)*Cot[c + d*x]*Csc[c + d*
x]^2)/(140*a*d) + (b*(53*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Si
n[c + d*x])^2)/(280*a^2*d) + ((8*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a
+ b*Sin[c + d*x])^3)/(35*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin
[c + d*x])^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x]
)^4)/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
```

```
0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c-}{}$$

$$= \frac{(8a^2-b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^3}{35a^2d}$$

$$= \frac{b(53a^2-6b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx}{280a^2d}$$

$$= -\frac{(4a^4-19a^2b^2+2b^4)\cot(c+dx)\csc^2(c+dx)}{140ad} + \frac{b(53}{}$$

$$= -\frac{b(105a^4-116a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{560a^2d} -$$

$$= -\frac{b(105a^4-116a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{560a^2d} -$$

$$= -\frac{3b(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{b(105a^4-116a^2}{}$$

$$= -\frac{3b(a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{16d} - \frac{a(2a^2+21b^2)\cot}{35d}$$

Mathematica [A]
time = 0.66, size = 324, normalized size = 1.07

$$-\frac{56a(14a^2-3b^2)\cos(3(c+dx))\csc^7(c+dx)+112a^3\cos(5(c+dx))\csc^7(c+dx)-504ab^2\cos(5(c+dx))\csc^7(c+dx)-16a^3\cos(7(c+dx))\csc^7(c+dx)-168ab^2\cos(7(c+dx))\csc^7(c+dx)+3360a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6720b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3360a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6720b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+70\cot(c+dx)\csc^6(c+dx)\left(12a(2a^2+b^2)+b(31a^2-18b^2)\sin(c+dx)\right)+1540a^2b\csc^7(c+dx)\sin(4(c+dx))+840b^3\csc^7(c+dx)\sin(4(c+dx))+105a^2b\csc^7(c+dx)\sin(6(c+dx))-350b^3\csc^7(c+dx)\sin(6(c+dx))}{17920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/17920*(56*a*(14*a^2 - 3*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^7 + 112*a^3*C
os[5*(c + d*x)]*Csc[c + d*x]^7 - 504*a*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^7
```

```
- 16*a^3*Cos[7*(c + d*x)]*Csc[c + d*x]^7 - 168*a*b^2*Cos[7*(c + d*x)]*Csc[c
 + d*x]^7 + 3360*a^2*b*Log[Cos[(c + d*x)/2]] + 6720*b^3*Log[Cos[(c + d*x)/2
]] - 3360*a^2*b*Log[Sin[(c + d*x)/2]] - 6720*b^3*Log[Sin[(c + d*x)/2]] + 70
*Cot[c + d*x]*Csc[c + d*x]^6*(12*a*(2*a^2 + b^2) + b*(31*a^2 - 18*b^2)*Sin[
c + d*x]) + 1540*a^2*b*Csc[c + d*x]^7*Sin[4*(c + d*x)] + 840*b^3*Csc[c + d*
x]^7*Sin[4*(c + d*x)] + 105*a^2*b*Csc[c + d*x]^7*Sin[6*(c + d*x)] - 350*b^3
*Csc[c + d*x]^7*Sin[6*(c + d*x)])/d
```

Maple [A]
time = 0.33, size = 243, normalized size = 0.80

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\ln(}{}$
default	$\frac{a^3\left(-\frac{\cos^5(dx+c)}{7\sin(dx+c)^7}-\frac{2\left(\cos^5(dx+c)\right)}{35\sin(dx+c)^5}\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{6\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{24\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{48\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{48}+\frac{\cos(dx+c)}{16}+\ln(}{}$
risch	$-\frac{630b^3e^{9i(dx+c)}+32ia^3+840b^3e^{3i(dx+c)}-350b^3e^{i(dx+c)}+1680ia\,b^2e^{12i(dx+c)}-3360ia\,b^2e^{10i(dx+c)}+5040ia\,b^2e^{8i(dx+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/7/sin(d*x+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+3*
a^2*b*(-1/6/sin(d*x+c)^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/s
in(d*x+c)^2*cos(d*x+c)^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+
c)-cot(d*x+c)))-3/5*a*b^2/sin(d*x+c)^5*cos(d*x+c)^5+b^3*(-1/4/sin(d*x+c)^4*
cos(d*x+c)^5+1/8/sin(d*x+c)^2*cos(d*x+c)^5+1/8*cos(d*x+c)^3+3/8*cos(d*x+c)+
3/8*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 208, normalized size = 0.69

$$\frac{35\,a^2b\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-70\,b^3\left(\frac{2\left(5\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^4-2\cos(dx+c)^2+1}+3\log\left(\cos\left(dx+c\right)+1\right)-3\log\left(\cos\left(dx+c\right)-1\right)\right)-\frac{672\,ab^2}{\tan(dx+c)^5}-\frac{32\left(7\tan(dx+c)^2+5\right)a^3}{\tan(dx+c)^7}}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/1120*(35*a^2*b*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/
(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x
+ c) + 1) + 3*log(cos(d*x + c) - 1)) - 70*b^3*(2*(5*cos(d*x + c)^3 - 3*cos(
d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) + 3*log(cos(d*x + c) + 1)
 - 3*log(cos(d*x + c) - 1)) - 672*a*b^2/tan(d*x + c)^5 - 32*(7*tan(d*x + c)
^2 + 5)*a^3/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.38, size = 347, normalized size = 1.15

$$-\frac{32\,(2\,a^{3}+21\,ab^{2})\cos\left(dx+c\right)^{7}-224\,(a^{3}+3\,ab^{2})\cos\left(dx+c\right)^{5}+105\,\left((a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{6}-3\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{4}-a^{2}b-2\,b^{3}+3\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-105\,\left((a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{6}-3\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{4}-a^{2}b-2\,b^{3}+3\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-70\,\left((3\,a^{2}b-10\,b^{3})\cos\left(dx+c\right)^{5}+8\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{3}-3\,(a^{2}b+2\,b^{3})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1120\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/1120*(32*(2*a^3 + 21*a*b^2)*cos(d*x + c)^7 - 224*(a^3 + 3*a*b^2)*cos(d*x
 + c)^5 + 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b + 2*b^3)*cos(d*x +
 c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 105*((a^2*b + 2*b^3)*cos(d*x + c)^6 - 3*(a^2*b +
2*b^3)*cos(d*x + c)^4 - a^2*b - 2*b^3 + 3*(a^2*b + 2*b^3)*cos(d*x + c)^2)*l
og(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*((3*a^2*b - 10*b^3)*cos(d*x +
 c)^5 + 8*(a^2*b + 2*b^3)*cos(d*x + c)^3 - 3*(a^2*b + 2*b^3)*cos(d*x + c))*
sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**8*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 456, normalized size = 1.50

$$\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+35\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+84\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+70\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-35\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-420\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-560\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+105\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+840\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+840\,(a^{2}b+2\,b^{3})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2178\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4356\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+105\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+840\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-105\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-560\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-35\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-420\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-105\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+70\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+84\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+35\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^8*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*a^3*tan(1/2*d*x + 1/2*c)^7 + 35*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 7*
a^3*tan(1/2*d*x + 1/2*c)^5 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 105*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 + 70*b^3*tan(1/2*d*x + 1/2*c)^4 - 35*a^3*tan(1/2*d*x +
 1/2*c)^3 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*
c)^2 - 560*b^3*tan(1/2*d*x + 1/2*c)^2 + 105*a^3*tan(1/2*d*x + 1/2*c) + 840*
a*b^2*tan(1/2*d*x + 1/2*c) + 840*(a^2*b + 2*b^3)*log(abs(tan(1/2*d*x + 1/2*
c))) - (2178*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4356*b^3*tan(1/2*d*x + 1/2*c)^7
 + 105*a^3*tan(1/2*d*x + 1/2*c)^6 + 840*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 105*
```

```
a^2*b*tan(1/2*d*x + 1/2*c)^5 - 560*b^3*tan(1/2*d*x + 1/2*c)^5 - 35*a^3*tan(
1/2*d*x + 1/2*c)^4 - 420*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 105*a^2*b*tan(1/2*d
*x + 1/2*c)^3 + 70*b^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c)^
2 + 84*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2*b*tan(1/2*d*x + 1/2*c) + 5*a^3
)/tan(1/2*d*x + 1/2*c)^7)/d
```

Mupad [B]
time = 9.91, size = 359, normalized size = 1.18

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{896\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^3}{128}+\frac{3\,a\,b^2}{32}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3\,a\,b^2}{160}-\frac{a^3}{640}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a^2\,b}{128}+\frac{b^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{3\,a^2\,b}{128}-\frac{b^3}{64}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3\,a^2\,b}{16}+\frac{3\,b^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{12\,a\,b^2}{5}-\frac{a^3}{5}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(3\,a^3+24\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(3\,a^2\,b-2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(3\,a^2\,b+16\,b^3\right)+\frac{a^3}{7}+a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{128\,d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\,a^3}{128}+\frac{3\,a\,b^2}{16}\right)}{d}+\frac{a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^8,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^7)/(896*d) - (tan(c/2 + (d*x)/2)^3*((3*a*b^2)/32 +
a^3/128))/d + (tan(c/2 + (d*x)/2)^5*((3*a*b^2)/160 - a^3/640))/d - (tan(c/2
 + (d*x)/2)^2*((3*a^2*b)/128 + b^3/8))/d - (tan(c/2 + (d*x)/2)^4*((3*a^2*b)
/128 - b^3/64))/d + (log(tan(c/2 + (d*x)/2))*((3*a^2*b)/16 + (3*b^3)/8))/d
- (tan(c/2 + (d*x)/2)^2*((12*a*b^2)/5 - a^3/5) - tan(c/2 + (d*x)/2)^4*(12*a
*b^2 + a^3) + tan(c/2 + (d*x)/2)^6*(24*a*b^2 + 3*a^3) - tan(c/2 + (d*x)/2)^
3*(3*a^2*b - 2*b^3) - tan(c/2 + (d*x)/2)^5*(3*a^2*b + 16*b^3) + a^3/7 + a^2
*b*tan(c/2 + (d*x)/2))/(128*d*tan(c/2 + (d*x)/2)^7) + (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/16 + (3*a^3)/128))/d + (a^2*b*tan(c/2 + (d*x)/2)^6)/(128*d)
```

3.1126 $\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=334

$$-\frac{3a(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{b(6a^2+7b^2)\cot(c+dx)}{35d}-\frac{3a(a^2+8b^2)\cot(c+dx)\csc(c+dx)}{128d}-\frac{b(24a^4$$

```
[Out] -3/128*a*(a^2+8*b^2)*arctanh(cos(d*x+c))/d-1/35*b*(6*a^2+7*b^2)*cot(d*x+c)/
d-3/128*a*(a^2+8*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/280*b*(24*a^4-25*a^2*b^2+4*
b^4)*cot(d*x+c)*csc(d*x+c)^2/a^2/d-1/2240*(35*a^4-148*a^2*b^2+24*b^4)*cot(d
*x+c)*csc(d*x+c)^3/a/d+3/560*b*(23*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)^4*(a+b*
sin(d*x+c))^2/a^2/d+1/112*(21*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)^5*(a+b*sin(d
*x+c))^3/a^2/d+1/14*b*cot(d*x+c)*csc(d*x+c)^6*(a+b*sin(d*x+c))^4/a^2/d-1/8*
cot(d*x+c)*csc(d*x+c)^7*(a+b*sin(d*x+c))^4/a/d
```

Rubi [A]
time = 0.59, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2972, 3126, 3110, 3100, 2827, 3853, 3855, 3852, 8}

$$-\frac{b(6a^2+7b^2)\cot(c+dx)}{35d}-\frac{3a(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{3a(a^2+8b^2)\cot(c+dx)\csc(c+dx)}{128d}+\frac{(21a^2-4b^2)\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3}{112a^2d}+\frac{3b(23a^2-4b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2}{560a^2d}+\frac{b\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^4}{14a^2d}-\frac{(35a^4-148a^2b^2+24b^4)\cot(c+dx)\csc^3(c+dx)}{2240ad}-\frac{b(24a^4-25a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{280a^2d}-\frac{\cot(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))^4}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (b*(6*a^2 + 7*b^2)*Cot
[c + d*x])/(35*d) - (3*a*(a^2 + 8*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) -
 (b*(24*a^4 - 25*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(280*a^2*d)
- ((35*a^4 - 148*a^2*b^2 + 24*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(2240*a*d)
+ (3*b*(23*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2)
/(560*a^2*d) + ((21*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c +
 d*x])^3)/(112*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^6*(a + b*Sin[c + d*x])
^4)/(14*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7*(a + b*Sin[c + d*x])^4)/(8*a*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
```

```
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b\cot(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^4}{14a^2d} - \frac{\cot(c-}{}$$

$$= \frac{(21a^2-4b^2)\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx)}{112a^2d}$$

$$= \frac{3b(23a^2-4b^2)\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+d}{560a^2d}$$

$$= -\frac{(35a^4-148a^2b^2+24b^4)\cot(c+dx)\csc^3(c+dx)}{2240ad} + \frac{3}{}$$

$$= -\frac{b(24a^4-25a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{(3}{}$$

$$= -\frac{b(24a^4-25a^2b^2+4b^4)\cot(c+dx)\csc^2(c+dx)}{280a^2d} - \frac{(3}{}$$

$$= -\frac{3a(a^2+8b^2)\cot(c+dx)\csc(c+dx)}{128d} - \frac{b(24a^4-25a^2}{}$$

$$= -\frac{3a(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{b(6a^2+7b^2)\cot(}{35d}$$

Mathematica [A]

time = 1.10, size = 268, normalized size = 0.80

$$-\frac{6720a(a^2+8b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-6720a(a^2+8b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\csc^8(c+dx)\left(35a(671a^2+248b^2)\cos(c+dx)+35(333a^3+104ab^2)\cos(3(c+dx))+805a^3\cos(5(c+dx))-11480ab^2\cos(5(c+dx))-105a^3\cos(7(c+dx))-840ab^2\cos(7(c+dx))+21504a^2b\sin(2(c+dx))+2688b^3\sin(2(c+dx))+16128a^2b\sin(4(c+dx))+896b^3\sin(4(c+dx))+3072a^2b\sin(6(c+dx))-896b^3\sin(6(c+dx))-384a^2b\sin(8(c+dx))-448b^3\sin(8(c+dx))\right)}{286720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/286720*(6720*a*(a^2 + 8*b^2)*Log[Cos[(c + d*x)/2]] - 6720*a*(a^2 + 8*b^2
)*Log[Sin[(c + d*x)/2]] + Csc[c + d*x]^8*(35*a*(671*a^2 + 248*b^2)*Cos[c +
d*x] + 35*(333*a^3 + 104*a*b^2)*Cos[3*(c + d*x)] + 805*a^3*Cos[5*(c + d*x)]
 - 11480*a*b^2*Cos[5*(c + d*x)] - 105*a^3*Cos[7*(c + d*x)] - 840*a*b^2*Cos[
7*(c + d*x)] + 21504*a^2*b*Sin[2*(c + d*x)] + 2688*b^3*Sin[2*(c + d*x)] + 1
6128*a^2*b*Sin[4*(c + d*x)] + 896*b^3*Sin[4*(c + d*x)] + 3072*a^2*b*Sin[6*(
c + d*x)] - 896*b^3*Sin[6*(c + d*x)] - 384*a^2*b*Sin[8*(c + d*x)] - 448*b^3
*Sin[8*(c + d*x)]))/d
```

Maple [A]
time = 0.36, size = 280, normalized size = 0.84

method	result
derivativedivides	$a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right.$
default	$a^3\left(-\frac{\cos^5(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^5(dx+c)}{16\sin(dx+c)^6}-\frac{\cos^5(dx+c)}{64\sin(dx+c)^4}+\frac{\cos^5(dx+c)}{128\sin(dx+c)^2}+\frac{\left(\cos^3(dx+c)\right)}{128}+\frac{3\cos(dx+c)}{128}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{128}\right.$
risch	$26880ia^2b\,\mathrm{e}^{12i(dx+c)}+105a^3\mathrm{e}^{15i(dx+c)}-805a^3\mathrm{e}^{13i(dx+c)}+11648ib^3\mathrm{e}^{4i(dx+c)}-3640a\,b^2\mathrm{e}^{11i(dx+c)}+31360ib^3\mathrm{e}^{8i(dx+c)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/8/sin(d*x+c)^8*cos(d*x+c)^5-1/16/sin(d*x+c)^6*cos(d*x+c)^5-1/6
4/sin(d*x+c)^4*cos(d*x+c)^5+1/128/sin(d*x+c)^2*cos(d*x+c)^5+1/128*cos(d*x+c
)^3+3/128*cos(d*x+c)+3/128*ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*(-1/7/sin(d*x
+c)^7*cos(d*x+c)^5-2/35/sin(d*x+c)^5*cos(d*x+c)^5)+3*a*b^2*(-1/6/sin(d*x+c)
^6*cos(d*x+c)^5-1/24/sin(d*x+c)^4*cos(d*x+c)^5+1/48/sin(d*x+c)^2*cos(d*x+c)
^5+1/48*cos(d*x+c)^3+1/16*cos(d*x+c)+1/16*ln(csc(d*x+c)-cot(d*x+c)))-1/5*b^
3/sin(d*x+c)^5*cos(d*x+c)^5)
```

Maxima [A]
time = 0.30, size = 248, normalized size = 0.74

$$\frac{35\,a^3\left(\frac{2\left(3\cos(dx+c)^7-11\cos(dx+c)^5-11\cos(dx+c)^3+3\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+280\,ab^2\left(\frac{2\left(3\cos(dx+c)^5+8\cos(dx+c)^3-3\cos(dx+c)\right)}{\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)-\frac{1792\,b^3}{\tan(dx+c)^5}-\frac{768\left(7\tan(dx+c)^2+5\right)a^2b}{\tan(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8960*(35*a^3*(2*(3*cos(d*x + c)^7 - 11*cos(d*x + c)^5 - 11*cos(d*x + c)^3
 + 3*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 -
4*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1))
+ 280*a*b^2*(2*(3*cos(d*x + c)^5 + 8*cos(d*x + c)^3 - 3*cos(d*x + c))/(cos(
d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) - 3*log(cos(d*x + c)
+ 1) + 3*log(cos(d*x + c) - 1)) - 1792*b^3/tan(d*x + c)^5 - 768*(7*tan(d*x
+ c)^2 + 5)*a^2*b/tan(d*x + c)^7)/d
```

Fricas [A]
time = 0.45, size = 384, normalized size = 1.15

$$\frac{210\,(a^3+8\,ab^2)\cos(dx+c)^7-70\,(11\,a^3-40\,ab^2)\cos(dx+c)^5-770\,(a^3+8\,ab^2)\cos(dx+c)^3+210\,(a^3+8\,ab^2)\cos(dx+c)-105\,\left((a^3+8\,ab^2)\cos(dx+c)^8-4\,(a^3+8\,ab^2)\cos(dx+c)^6+6\,(a^3+8\,ab^2)\cos(dx+c)^4+a^3+8\,ab^2-4\,(a^3+8\,ab^2)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+105\,\left((a^3+8\,ab^2)\cos(dx+c)^8-4\,(a^3+8\,ab^2)\cos(dx+c)^6+6\,(a^3+8\,ab^2)\cos(dx+c)^4+a^3+8\,ab^2-4\,(a^3+8\,ab^2)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+256\,\left((6\,a^2b+7\,b^3)\cos(dx+c)^7-7\,(3\,a^2b+b^3)\cos(dx+c)^5\right)\sin(dx+c)}{8960\,\left(d\cos(dx+c)^8-4\,d\cos(dx+c)^6+6\,d\cos(dx+c)^4-4\,d\cos(dx+c)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/8960*(210*(a^3 + 8*a*b^2)*cos(d*x + c)^7 - 70*(11*a^3 - 40*a*b^2)*cos(d*x
 + c)^5 - 770*(a^3 + 8*a*b^2)*cos(d*x + c)^3 + 210*(a^3 + 8*a*b^2)*cos(d*x
+ c) - 105*((a^3 + 8*a*b^2)*cos(d*x + c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)
^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^3 + 8*a*b^2)*cos(d*x +
 c)^8 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^6 + 6*(a^3 + 8*a*b^2)*cos(d*x + c)^4
 + a^3 + 8*a*b^2 - 4*(a^3 + 8*a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c)
+ 1/2) + 256*((6*a^2*b + 7*b^3)*cos(d*x + c)^7 - 7*(3*a^2*b + b^3)*cos(d*x
+ c)^5)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x
+ c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**9*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 457, normalized size = 1.37

$$\frac{35\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+240\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+560\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-336\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+448\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1680\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1680\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1680\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5040\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4480\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1680\,(a^3+8\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{4566\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+36528\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+5040\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+4480\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-1680\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1680\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-2240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-280\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1680\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-336\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+448\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+560\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+240\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+35\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8}}{71680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^9*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/71680*(35*a^3*tan(1/2*d*x + 1/2*c)^8 + 240*a^2*b*tan(1/2*d*x + 1/2*c)^7 +
 560*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 448*
b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)^4 - 1680*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2240*b^3*tan(1/2
*d*x + 1/2*c)^3 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 5040*a^2*b*tan(1/2*d*
x + 1/2*c) + 4480*b^3*tan(1/2*d*x + 1/2*c) + 1680*(a^3 + 8*a*b^2)*log(abs(t
an(1/2*d*x + 1/2*c))) - (4566*a^3*tan(1/2*d*x + 1/2*c)^8 + 36528*a*b^2*tan(
1/2*d*x + 1/2*c)^8 + 5040*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 4480*b^3*tan(1/2*d
*x + 1/2*c)^7 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 1680*a^2*b*tan(1/2*d*x
+ 1/2*c)^5 - 2240*b^3*tan(1/2*d*x + 1/2*c)^5 - 280*a^3*tan(1/2*d*x + 1/2*c)
^4 - 1680*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 336*a^2*b*tan(1/2*d*x + 1/2*c)^3 +
 448*b^3*tan(1/2*d*x + 1/2*c)^3 + 560*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^
2*b*tan(1/2*d*x + 1/2*c) + 35*a^3)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [B]

time = 9.95, size = 381, normalized size = 1.14

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2048\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^3}{256}+\frac{3\,a\,b^2}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{3\,a^2\,b}{128}+\frac{b^3}{32}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{3\,a^2\,b}{640}-\frac{b^3}{160}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^3}{128}+\frac{3\,a\,b^2}{16}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^3+6\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^2\,b+8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{6\,a^2\,b}{5}-\frac{8\,b^3}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(18\,a^2\,b+16\,b^3\right)-\frac{a^3}{8}-\frac{6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{7}-2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{256\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{9\,a^2\,b}{128}+\frac{b^3}{16}\right)}{d}-\frac{3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}+\frac{a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128\,d}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^3)/sin(c + d*x)^9,x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^8)/(2048*d) - (tan(c/2 + (d*x)/2)^4*((3*a*b^2)/128
+ a^3/256))/d - (tan(c/2 + (d*x)/2)^3*((3*a^2*b)/128 + b^3/32))/d - (tan(c/
2 + (d*x)/2)^5*((3*a^2*b)/640 - b^3/160))/d + (log(tan(c/2 + (d*x)/2))*((3*
a*b^2)/16 + (3*a^3)/128))/d + (tan(c/2 + (d*x)/2)^4*(6*a*b^2 + a^3) + tan(c
/2 + (d*x)/2)^5*(6*a^2*b + 8*b^3) + tan(c/2 + (d*x)/2)^3*((6*a^2*b)/5 - (8*
b^3)/5) - tan(c/2 + (d*x)/2)^7*(18*a^2*b + 16*b^3) - a^3/8 - (6*a^2*b*tan(c
/2 + (d*x)/2))/7 - 2*a*b^2*tan(c/2 + (d*x)/2)^2 + 6*a*b^2*tan(c/2 + (d*x)/2
)^6)/(256*d*tan(c/2 + (d*x)/2)^8) + (tan(c/2 + (d*x)/2)*((9*a^2*b)/128 + b^
3/16))/d - (3*a*b^2*tan(c/2 + (d*x)/2)^2)/(128*d) + (a*b^2*tan(c/2 + (d*x)/
2)^6)/(128*d) + (3*a^2*b*tan(c/2 + (d*x)/2)^7)/(896*d)
```

3.1127 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=307

$$-\frac{3a(8a^4-8a^2b^2+b^4)\,x}{4b^7}+\frac{6a^2(2a^4-3a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d}-\frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d}-$$

```
[Out] -3/4*a*(8*a^4-8*a^2*b^2+b^4)*x/b^7-1/5*(30*a^4-25*a^2*b^2+b^4)*cos(d*x+c)/b
^6/d+3/4*a*(4*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/b^5/d-1/5*(10*a^2-7*b^2)*cos
(d*x+c)*sin(d*x+c)^2/b^4/d+1/2*(3*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^3/
d-1/5*cos(d*x+c)*sin(d*x+c)^4/b^2/d-(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2
/d/(a+b*sin(d*x+c))+6*a^2*(2*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/2
*c))/(a^2-b^2)^(1/2))/b^7/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2971, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{3a(4a^2-3b^2)\sin(c+dx)\cos(c+dx)}{4b^5d}-\frac{(10a^2-7b^2)\sin^2(c+dx)\cos(c+dx)}{5b^4d}+\frac{(3a^2-2b^2)\sin^3(c+dx)\cos(c+dx)}{2ab^3d}+\frac{6a^2(2a^4-3a^2b^2+b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}-\frac{3ax(8a^4-8a^2b^2+b^4)}{4b^7}-\frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d}-\frac{\sin^4(c+dx)\cos(c+dx)}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(4*b^7) + (6*a^2*(2*a^4 - 3*a^2*b^2 + b^
4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d
) - ((30*a^4 - 25*a^2*b^2 + b^4)*Cos[c + d*x])/(5*b^6*d) + (3*a*(4*a^2 - 3*
b^2)*Cos[c + d*x]*Sin[c + d*x])/(4*b^5*d) - ((10*a^2 - 7*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(5*b^4*d) + ((3*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(
2*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - ((a^2 - b^2)*Cos[c +
 d*x]*Sin[c + d*x]^4)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
```

```
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= -\frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\sin^3(c+
\\
&= \frac{(3a^2-2b^2)\cos(c+dx)\sin^3(c+dx)}{2ab^3d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{(a^2-b^2
}{ab
\\
&= -\frac{(10a^2-7b^2)\cos(c+dx)\sin^2(c+dx)}{5b^4d} + \frac{(3a^2-2b^2)\cos(c+dx)\sin^3(c+}{2ab^3d}
\\
&= \frac{3a(4a^2-3b^2)\cos(c+dx)\sin(c+dx)}{4b^5d} - \frac{(10a^2-7b^2)\cos(c+dx)\sin^2(c+}{5b^4d}
\\
&= -\frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d} + \frac{3a(4a^2-3b^2)\cos(c+dx)\sin(c+dx}{4b^5d}
\\
&= -\frac{3a(8a^4-8a^2b^2+b^4)\,x}{4b^7} - \frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d} + \frac{3a(4a^2-3
\\
&= -\frac{3a(8a^4-8a^2b^2+b^4)\,x}{4b^7} - \frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d} + \frac{3a(4a^2-3
\\
&= -\frac{3a(8a^4-8a^2b^2+b^4)\,x}{4b^7} - \frac{(30a^4-25a^2b^2+b^4)\cos(c+dx)}{5b^6d} + \frac{3a(4a^2-3
\\
&= -\frac{3a(8a^4-8a^2b^2+b^4)\,x}{4b^7} + \frac{6a^2(2a^4-3a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d}
\end{aligned}
$$

Mathematica [A]

time = 3.01, size = 378, normalized size = 1.23

$$
\frac{\frac{960a^2(2a^4-3a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{960a^6c-960a^4b^2c+120a^2b^4c+960a^6dx-960a^4b^2dx+120a^2b^4dx+60ab(16a^4-14a^2b^2+b^4)\cos(c+dx)+5(8a^3b^3-5ab^5)\cos(3(c+dx))-3ab^5\cos(5(c+dx))+960a^5bc\sin(c+dx)-960a^3b^3c\sin(c+dx)+120ab^5c\sin(c+dx)+960a^5bdx\sin(c+dx)-960a^3b^3dx\sin(c+dx)+120ab^5dx\sin(c+dx)+240a^4b^2\sin(2(c+dx))-200a^2b^4\sin(2(c+dx))+5b^6\sin(2(c+dx))-10a^2b^4\sin(4(c+dx))+4b^6\sin(4(c+dx))+b^6\sin(6(c+dx))}{a+b\sin(c+dx)}}{160b^7d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]

[Out] ((960*a^2*(2*a^4 - 3*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - (960*a^6*c - 960*a^4*b^2*c + 120*a^2*b^4*c + 9
60*a^6*d*x - 960*a^4*b^2*d*x + 120*a^2*b^4*d*x + 60*a*b*(16*a^4 - 14*a^2*b^
2 + b^4)*Cos[c + d*x] + 5*(8*a^3*b^3 - 5*a*b^5)*Cos[3*(c + d*x)] - 3*a*b^5*
```

```
Cos[5*(c + d*x)] + 960*a^5*b*c*Sin[c + d*x] - 960*a^3*b^3*c*Sin[c + d*x] +
120*a*b^5*c*Sin[c + d*x] + 960*a^5*b*d*x*Sin[c + d*x] - 960*a^3*b^3*d*x*Sin
[c + d*x] + 120*a*b^5*d*x*Sin[c + d*x] + 240*a^4*b^2*Sin[2*(c + d*x)] - 200
*a^2*b^4*Sin[2*(c + d*x)] + 5*b^6*Sin[2*(c + d*x)] - 10*a^2*b^4*Sin[4*(c +
d*x)] + 4*b^6*Sin[4*(c + d*x)] + b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x])
)/(160*b^7*d)
```

Maple [A]
time = 0.63, size = 434, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/b^7*(((a^3*b^2-5/8*a*b^4)*tan(1/2*d*x+1/2*c)^9+(5/2*a^4*b-3*a^2*b^3
+1/2*b^5)*tan(1/2*d*x+1/2*c)^8+(2*a^3*b^2-1/4*a*b^4)*tan(1/2*d*x+1/2*c)^7+(
10*a^4*b-9*a^2*b^3)*tan(1/2*d*x+1/2*c)^6+(15*a^4*b-11*a^2*b^3+b^5)*tan(1/2*
d*x+1/2*c)^4+(-2*a^3*b^2+1/4*a*b^4)*tan(1/2*d*x+1/2*c)^3+(10*a^4*b-7*a^2*b^
3)*tan(1/2*d*x+1/2*c)^2+(-a^3*b^2+5/8*a*b^4)*tan(1/2*d*x+1/2*c)+5/2*a^4*b-2
*a^2*b^3+1/10*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5+3/8*a*(8*a^4-8*a^2*b^2+b^4)*a
rctan(tan(1/2*d*x+1/2*c)))+4*a^2/b^7*((-1/2*b^2*(a^2-b^2)*tan(1/2*d*x+1/2*c
)-1/2*a^3*b+1/2*a*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+3/
2*(2*a^4-3*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 649, normalized size = 2.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

[Out] [1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*(
8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 30*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^3
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6*cos(d*x + c)^5 - 10
*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x + 15*(4*a^4*
b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
, 1/20*(6*a*b^5*cos(d*x + c)^5 - 5*(4*a^3*b^3 - a*b^5)*cos(d*x + c)^3 - 15*
(8*a^6 - 8*a^4*b^2 + a^2*b^4)*d*x - 60*(2*a^5 - a^3*b^2 + (2*a^4*b - a^2*b^
3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
^2)*cos(d*x + c))) - 15*(8*a^5*b - 8*a^3*b^3 + a*b^5)*cos(d*x + c) - (4*b^6
*cos(d*x + c)^5 - 10*a^2*b^4*cos(d*x + c)^3 + 15*(8*a^5*b - 8*a^3*b^3 + a*b
^5)*d*x + 15*(4*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin
(d*x + c) + a*b^7*d)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [**A**]
time = 0.52, size = 536, normalized size = 1.75

$$-\frac{\frac{15\,(8\,a^5-8\,a^3b^2+ab^4)(dx+c)}{b^7}-\frac{120\,(2\,a^6-3\,a^4b^2+a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{40\,\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5-a^3b^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^6}+\frac{2\,\left(40\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-25\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+100\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-120\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+20\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+80\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-10\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+400\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-360\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+600\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-440\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-80\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+400\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-280\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-40\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+25\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+100\,a^4-80\,a^2b^2+4\,b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5b^6}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/20*(15*(8*a^5 - 8*a^3*b^2 + a*b^4)*(d*x + c)/b^7 - 120*(2*a^6 - 3*a^4*b^
2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d
*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 40*(a^4*b*tan(1/
2*d*x + 1/2*c) - a^2*b^3*tan(1/2*d*x + 1/2*c) + a^5 - a^3*b^2)/((a*tan(1/2*
d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^6) + 2*(40*a^3*b*tan(1/2*d
*x + 1/2*c)^9 - 25*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 100*a^4*tan(1/2*d*x + 1/2
*c)^8 - 120*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 20*b^4*tan(1/2*d*x + 1/2*c)^8
+ 80*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 10*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 400*a
^4*tan(1/2*d*x + 1/2*c)^6 - 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 600*a^4*ta
n(1/2*d*x + 1/2*c)^4 - 440*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 40*b^4*tan(1/2*
d*x + 1/2*c)^4 - 80*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^3*tan(1/2*d*x + 1
/2*c)^3 + 400*a^4*tan(1/2*d*x + 1/2*c)^2 - 280*a^2*b^2*tan(1/2*d*x + 1/2*c)

```
^2 - 40*a^3*b*tan(1/2*d*x + 1/2*c) + 25*a*b^3*tan(1/2*d*x + 1/2*c) + 100*a^
4 - 80*a^2*b^2 + 4*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

Mupad [B]
time = 12.32, size = 2390, normalized size = 7.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((2*(a*b^4 + 30*a^5 - 25*a^3*b^2))/(5*b^6) - (3*tan(c/2 + (d*x)/2)^10*(a*
b^4 - 4*a^5 + 2*a^3*b^2))/b^6 + (6*tan(c/2 + (d*x)/2)^4*(a*b^4 + 20*a^5 - 1
8*a^3*b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(a*b^4 + 30*a^5 - 25*a^3*b^2))/b^
6 + (3*tan(c/2 + (d*x)/2)^2*(9*a*b^4 + 100*a^5 - 90*a^3*b^2))/(5*b^6) + (ta
n(c/2 + (d*x)/2)*(180*a^4 + 8*b^4 - 155*a^2*b^2))/(10*b^5) + (3*tan(c/2 + (
d*x)/2)^11*(4*a^4 - 3*a^2*b^2))/(2*b^5) + (6*tan(c/2 + (d*x)/2)^8*(10*a^5 -
 7*a^3*b^2))/b^6 + (3*tan(c/2 + (d*x)/2)^7*(36*a^4 - 31*a^2*b^2))/b^5 + (ta
n(c/2 + (d*x)/2)^3*(156*a^4 - 125*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^9
*(84*a^4 + 8*b^4 - 75*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^5*(132*a^4 +
8*b^4 - 107*a^2*b^2))/b^5)/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 6*a*tan(c/2 + (
d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*ta
n(c/2 + (d*x)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12 +
10*b*tan(c/2 + (d*x)/2)^3 + 20*b*tan(c/2 + (d*x)/2)^5 + 20*b*tan(c/2 + (d*x
)/2)^7 + 10*b*tan(c/2 + (d*x)/2)^9 + 2*b*tan(c/2 + (d*x)/2)^11)) - (2*atanh
((108*a^6*(b^2 - a^2)^(1/2))/(108*a^6*b - (324*a^8)/b + (216*a^10)/b^3 - 64
8*a^7*tan(c/2 + (d*x)/2) + 216*a^5*b^2*tan(c/2 + (d*x)/2) + (432*a^9*tan(c/
2 + (d*x)/2))/b^2) - (216*a^8*(b^2 - a^2)^(1/2))/(108*a^6*b^3 - 324*a^8*b +
 (216*a^10)/b + 432*a^9*tan(c/2 + (d*x)/2) + 216*a^5*b^4*tan(c/2 + (d*x)/2)
 - 648*a^7*b^2*tan(c/2 + (d*x)/2)) + (216*a^5*tan(c/2 + (d*x)/2)*(b^2 - a^2
)^(1/2))/(108*a^6 - (324*a^8)/b^2 + (216*a^10)/b^4 + 216*a^5*b*tan(c/2 + (d
*x)/2) - (648*a^7*tan(c/2 + (d*x)/2))/b + (432*a^9*tan(c/2 + (d*x)/2))/b^3)
 - (540*a^7*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(108*a^6*b^2 - 324*a^8 +
(216*a^10)/b^2 - 648*a^7*b*tan(c/2 + (d*x)/2) + 216*a^5*b^3*tan(c/2 + (d*x)
/2) + (432*a^9*tan(c/2 + (d*x)/2))/b) + (216*a^9*tan(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2))/(216*a^10 + 108*a^6*b^4 - 324*a^8*b^2 + 432*a^9*b*tan(c/2 + (d*
x)/2) + 216*a^5*b^5*tan(c/2 + (d*x)/2) - 648*a^7*b^3*tan(c/2 + (d*x)/2)))*(
6*a^4*(b^2 - a^2)^(1/2) - 3*a^2*b^2*(b^2 - a^2)^(1/2)))/(b^7*d) - (3*a*atan
(((3*a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b
^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b
^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a
^13*b^6))/b^18 - (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (
2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 - (a
*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^
4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7)))/(
```

4*b^7) + (3*a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 + (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7)))/(4*b^7))/((4*(1728*a^16 - 81*a^6*b^10 + 999*a^8*b^8 - 3942*a^10*b^6 + 6912*a^12*b^4 - 5616*a^14*b^2))/b^17 + (4*tan(c/2 + (d*x)/2)*(6912*a^17 + 54*a^5*b^12 - 1026*a^7*b^10 + 7020*a^9*b^8 - 21600*a^11*b^6 + 32832*a^13*b^4 - 24192*a^15*b^2))/b^18 - (a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 - (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 - (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*3i)/(4*b^7) + (a*(8*a^4 + b^4 - 8*a^2*b^2)*((2*(9*a^4*b^14 - 144*a^6*b^12 + 720*a^8*b^10 - 1152*a^10*b^8 + 576*a^12*b^6))/b^17 + (2*tan(c/2 + (d*x)/2)*(18*a^3*b^16 - 441*a^5*b^14 + 2448*a^7*b^12 - 4896*a^9*b^10 + 4032*a^11*b^8 - 1152*a^13*b^6))/b^18 + (a*((2*(12*a^2*b^18 - 60*a^4*b^16 + 48*a^6*b^14))/b^17 + (2*tan(c/2 + (d*x)/2)*(96*a^3*b^18 - 288*a^5*b^16 + 192*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*(8*a^4 + b^4 - 8*a^2*b^2)*3i)/(4*b^7))*3i)/(4*b^7)))*(8*a^4 + b^4 - 8*a^2*b^2))/(2*b^7*d)

3.1128 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$\frac{(40a^4-36a^2b^2+3b^4)\,x}{8b^6}-\frac{2a(5a^4-7a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d}+\frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d}-\frac{(20a$$

```
[Out] 1/8*(40*a^4-36*a^2*b^2+3*b^4)*x/b^6+1/3*a*(15*a^2-11*b^2)*cos(d*x+c)/b^5/d-
1/8*(20*a^2-13*b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d+1/3*(5*a^2-3*b^2)*cos(d*x+c
)*sin(d*x+c)^2/a/b^3/d-1/4*cos(d*x+c)*sin(d*x+c)^3/b^2/d-(a^2-b^2)*cos(d*x+
c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))-2*a*(5*a^4-7*a^2*b^2+2*b^4)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2971, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}+\frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d}-\frac{(20a^2-13b^2)\sin(c+dx)\cos(c+dx)}{8b^4d}+\frac{(5a^2-3b^2)\sin^2(c+dx)\cos(c+dx)}{3ab^3d}-\frac{2a(5a^4-7a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}+\frac{x(40a^4-36a^2b^2+3b^4)}{8b^6}-\frac{\sin^3(c+dx)\cos(c+dx)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((40*a^4 - 36*a^2*b^2 + 3*b^4)*x)/(8*b^6) - (2*a*(5*a^4 - 7*a^2*b^2 + 2*b^4
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d)
 + (a*(15*a^2 - 11*b^2)*Cos[c + d*x])/(3*b^5*d) - ((20*a^2 - 13*b^2)*Cos[c
+ d*x]*Sin[c + d*x])/(8*b^4*d) + ((5*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]
^2)/(3*a*b^3*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - ((a^2 - b^2)*Co
s[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \sin^2(c\cdots}{\cdots}$$

$$= \frac{(5a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{3ab^3d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{(a^2-\cdots}{a\cdots}$$

$$= -\frac{(20a^2-13b^2)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{(5a^2-3b^2)\cos(c+dx)\sin^2(c\cdots}{3ab^3d}$$

$$= \frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d} - \frac{(20a^2-13b^2)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{(5\cdots}{\cdots}$$

$$= \frac{(40a^4-36a^2b^2+3b^4)\,x}{8b^6} + \frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d} - \frac{(20a^2-13b^2)\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{(40a^4-36a^2b^2+3b^4)\,x}{8b^6} + \frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d} - \frac{(20a^2-13b^2)\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{(40a^4-36a^2b^2+3b^4)\,x}{8b^6} + \frac{a(15a^2-11b^2)\cos(c+dx)}{3b^5d} - \frac{(20a^2-13b^2)\,\mathrm{c}\cdots}{\cdots}$$

$$= \frac{(40a^4-36a^2b^2+3b^4)\,x}{8b^6} - \frac{2a(5a^4-7a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d}\cdots$$

Mathematica [A]
time = 2.55, size = 325, normalized size = 1.22

$$\frac{-\frac{384a(5a^4-7a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{960a^5c-864a^3b^2c+72ab^4c+960a^5dx-864a^3b^2dx+72ab^4dx+24b(40a^4-31a^2b^2+b^4)\cos(c+dx)+(40a^2b^3-21b^5)\cos(3(c+dx))-3b^5\cos(5(c+dx))+960a^4bc\sin(c+dx)-864a^2b^3c\sin(c+dx)+72b^5c\sin(c+dx)+960a^4bdx\sin(c+dx)-864a^2b^3dx\sin(c+dx)+72b^5dx\sin(c+dx)+240a^3b^2\sin(2(c+dx))-176ab^4\sin(2(c+dx))-10ab^4\sin(4(c+dx))}{a+b\sin(c+dx)}}{192b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-384*a*(5*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/Sqrt[a^2 - b^2] + (960*a^5*c - 864*a^3*b^2*c + 72*a*b^4*c + 960
*a^5*d*x - 864*a^3*b^2*d*x + 72*a*b^4*d*x + 24*b*(40*a^4 - 31*a^2*b^2 + b^4
)*Cos[c + d*x] + (40*a^2*b^3 - 21*b^5)*Cos[3*(c + d*x)] - 3*b^5*Cos[5*(c +
d*x)] + 960*a^4*b*c*Sin[c + d*x] - 864*a^2*b^3*c*Sin[c + d*x] + 72*b^5*c*Si
n[c + d*x] + 960*a^4*b*d*x*Sin[c + d*x] - 864*a^2*b^3*d*x*Sin[c + d*x] + 72
*b^5*d*x*Sin[c + d*x] + 240*a^3*b^2*Sin[2*(c + d*x)] - 176*a*b^4*Sin[2*(c +
 d*x)] - 10*a*b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(192*b^6*d)
```

Maple [A]
time = 0.57, size = 382, normalized size = 1.43

method	result
derivativedivides	$\frac{\frac{2\left(\left(\frac{3}{2}a^2b^2-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^3b-4a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^2b^2+\frac{3}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12a^3b-8a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
default	$\frac{\frac{2\left(\left(\frac{3}{2}a^2b^2-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(4a^3b-4a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^2b^2+\frac{3}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(12a^3b-8a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{}$
risch	$\frac{5x\,a^4}{b^6}-\frac{9x\,a^2}{2b^4}+\frac{3x}{8b^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{3i\mathrm{e}^{2i(dx+c)}a^2}{8b^4d}+\frac{2a^3\mathrm{e}^{i(dx+c)}}{b^5d}-\frac{5a\,\mathrm{e}^{i(dx+c)}}{4b^3d}+\frac{2a^3\mathrm{e}^{-i(dx+c)}}{b^5d}-\frac{5a\,\mathrm{e}^{-i(d}}{4b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^6*(((3/2*a^2*b^2-5/8*b^4)*tan(1/2*d*x+1/2*c)^7+(4*a^3*b-4*a*b^3)*t
an(1/2*d*x+1/2*c)^6+(3/2*a^2*b^2+3/8*b^4)*tan(1/2*d*x+1/2*c)^5+(12*a^3*b-8*
a*b^3)*tan(1/2*d*x+1/2*c)^4+(-3/2*a^2*b^2-3/8*b^4)*tan(1/2*d*x+1/2*c)^3+(12
*a^3*b-20/3*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-3/2*a^2*b^2+5/8*b^4)*tan(1/2*d*x+
1/2*c)+4*a^3*b-8/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(40*a^4-36*a^2*b^2
+3*b^4)*arctan(tan(1/2*d*x+1/2*c)))-2*a/b^6*((-b^2*(a^2-b^2)*tan(1/2*d*x+1/
2*c)-a^3*b+a*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(5*a^4-
7*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.41, size = 604, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/24*(6*b^5*cos(d*x + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*
a^5 - 36*a^3*b^2 + 3*a*b^4)*d*x + 12*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^
3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c
))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^
3 - 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d
*x + c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d), -1/24*(6*b^5*cos(d*x
 + c)^5 - (20*a^2*b^3 - 3*b^5)*cos(d*x + c)^3 - 3*(40*a^5 - 36*a^3*b^2 + 3*
a*b^4)*d*x - 24*(5*a^4 - 2*a^2*b^2 + (5*a^3*b - 2*a*b^3)*sin(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*cos(d*x + c) + (10*a*b^4*cos(d*x + c)^3 -
 3*(40*a^4*b - 36*a^2*b^3 + 3*b^5)*d*x - 3*(20*a^3*b^2 - 13*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 449, normalized size = 1.68

$$\frac{\frac{3\,(40\,a^4-36\,a^2b^2+3\,b^4)(dx+c)}{b^6}-\frac{48\,(5\,a^5-7\,a^3b^2+2\,ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{48\,(a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4-a^2b^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^5}+\frac{2\left(36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+96\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-96\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+288\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-192\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+288\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-160\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+96\,a^3-64\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4b^5}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(40*a^4 - 36*a^2*b^2 + 3*b^4)*(d*x + c)/b^6 - 48*(5*a^5 - 7*a^3*b^2
 + 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*
x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 48*(a^3*b*tan(1/2
*d*x + 1/2*c) - a*b^3*tan(1/2*d*x + 1/2*c) + a^4 - a^2*b^2)/((a*tan(1/2*d*x
 + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^5) + 2*(36*a^2*b*tan(1/2*d*x
+ 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 96*a^3*tan(1/2*d*x + 1/2*c)^6
- 96*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b^3
*tan(1/2*d*x + 1/2*c)^5 + 288*a^3*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^2*tan(1/
```

```
2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 288*a^3*tan(1/2*d*x + 1/2*c)^2 - 160*a*b^2*tan(1/2*d*x + 1/2*c)^2
- 36*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 96*a^3 - 64
*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^5))/d
```

Mupad [B]
time = 11.79, size = 1003, normalized size = 3.76

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (60*a^4*b + (3*b^5*cos(c + d*x))/2 + 120*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)) - 44*a^2*b^3 - (21*b^5*cos(3*c + 3*d*x))/16 - (3*b^5*cos(5*c
+ 5*d*x))/16 + 9*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (93*a^
2*b^3*cos(c + d*x))/2 + 9*b^5*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2)) - 11*a*b^4*sin(2*c + 2*d*x) - (5*a*b^4*sin(4*c + 4*d*x))/8 + 60
*a^3*b^2*sin(c + d*x) - 108*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)) + (5*a^2*b^3*cos(3*c + 3*d*x))/2 - 120*a^4*atanh((2*b^2*sin(c/2 + (d*x
)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos
(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2
+ (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 -
 a^2)^(1/2) + 15*a^3*b^2*sin(2*c + 2*d*x) + 60*a^4*b*cos(c + d*x) - 44*a*b^
4*sin(c + d*x) - 108*a^2*b^3*sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)) + 48*a^2*b^2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) -
 a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c
/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2) + 120*a^4*b*
sin(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 48*a*b^3*sin(c +
 d*x)*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*
x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*co
s(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*
a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2) - 120*a^3*b*sin(c + d*x)*atanh
((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*cos(c/2 + (d*
x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + 2*a^2*b*sin(c
/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(12*b^6*d*(a + b*sin(c + d*x)))
```

3.1129 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=163

$$-\frac{a(4a^2-3b^2)\,x}{b^5}+\frac{2(4a^4-5a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d}+\frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))}-\frac{\cos(c+$$

```
[Out] -a*(4*a^2-3*b^2)*x/b^5+1/3*cos(d*x+c)^3*(4*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d
*x+c))-cos(d*x+c)*(4*a^2-b^2-2*a*b*sin(d*x+c))/b^4/d+2*(4*a^4-5*a^2*b^2+b^4
)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2942, 2944, 2814, 2739, 632, 210}

$$-\frac{ax(4a^2-3b^2)}{b^5}-\frac{\cos(c+dx)\left(4a^2-2ab\sin(c+dx)-b^2\right)}{b^4d}+\frac{2(4a^4-5a^2b^2+b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*(4*a^2 - 3*b^2)*x)/b^5) + (2*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*T
an[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^
3*(4*a + b*Sin[c + d*x]))/(3*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d*x]*(4
*a^2 - b^2 - 2*a*b*Sin[c + d*x]))/(b^4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^2(c+dx)(-b-4a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\,(4a^2-b^2-2ab\sin(c+dx}{b^4d}$$

$$= -\frac{a(4a^2-3b^2)\,x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\,(4a^2-}{}$$

$$= -\frac{a(4a^2-3b^2)\,x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\,(4a^2-}{}$$

$$= -\frac{a(4a^2-3b^2)\,x}{b^5} + \frac{\cos^3(c+dx)(4a+b\sin(c+dx))}{3b^2d(a+b\sin(c+dx))} - \frac{\cos(c+dx)\,(4a^2-}{}$$

$$= -\frac{a(4a^2-3b^2)\,x}{b^5} + \frac{2(4a^4-5a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c}{3b}$$

Mathematica [A]

time = 1.65, size = 247, normalized size = 1.52

$$\frac{\frac{48(4a^4-5a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-96a^4c+72a^2b^2c-96a^4dx+72a^2b^2dx+(-96a^3b+60ab^3)\cos(c+dx)-4ab^3\cos(3(c+dx))-96a^3bc\sin(c+dx)+72ab^3c\sin(c+dx)-96a^3bdx\sin(c+dx)+72ab^3dx\sin(c+dx)-24a^2b^2\sin(2(c+dx))+14b^4\sin(2(c+dx))+b^4\sin(4(c+dx))}{a+b\sin(c+dx)}}{24b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((48*(4*a^4 - 5*a^2*b^2 + b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/Sqrt[a^2 - b^2] + (-96*a^4*c + 72*a^2*b^2*c - 96*a^4*d*x + 72*a^2*b^2
*d*x + (-96*a^3*b + 60*a*b^3)*Cos[c + d*x] - 4*a*b^3*Cos[3*(c + d*x)] - 96*
a^3*b*c*Sin[c + d*x] + 72*a*b^3*c*Sin[c + d*x] - 96*a^3*b*d*x*Sin[c + d*x]
+ 72*a*b^3*d*x*Sin[c + d*x] - 24*a^2*b^2*Sin[2*(c + d*x)] + 14*b^4*Sin[2*(c
 + d*x)] + b^4*Sin[4*(c + d*x)])/(a + b*Sin[c + d*x]))/(24*b^5*d)
```

Maple [A]

time = 0.45, size = 276, normalized size = 1.69

method	result
derivativedivides	$-\frac{\frac{4\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2b-b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{2b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(4a^2-3b^2\right.}{}\right.}{b^5}}{d}$

default	$-\frac{\frac{4\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2b-b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{2b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(4a^2-3b^2\right)}{}\right.}{b^5}}{d}$
risch	$-\frac{4a^3x}{b^5}+\frac{3ax}{b^3}-\frac{ia\,e^{2i(dx+c)}}{4b^3d}-\frac{3\,e^{i(dx+c)}a^2}{2b^4d}+\frac{5\,e^{i(dx+c)}}{8b^2d}-\frac{3\,e^{-i(dx+c)}a^2}{2b^4d}+\frac{5\,e^{-i(dx+c)}}{8b^2d}+\frac{ia\,e^{-2i(dx+c)}}{4b^3d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/b^5*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(3/2*a^2*b-b^3)*tan(1/2*d*x+1/
2*c)^4+(3*a^2*b-b^3)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*tan(1/2*d*x+1/2*c)+3/2*
a^2*b-2/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(4*a^2-3*b^2)*arctan(tan(1/
2*d*x+1/2*c)))+4/b^5*((-1/2*b^2*(a^2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a^3*b+1/2*
a*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(4*a^4-5*a^2*b
^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.45, size = 507, normalized size = 3.11

$$\left[-\frac{4\,ab^3\cos\left(dx+c\right)^3+6\left(4\,a^4-3\,a^2b^2\right)dx+3\left(4\,a^3-ab^2+\left(4\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+6\left(4\,a^3b-3\,ab^3\right)\cos\left(dx+c\right)-2\left(b^4\cos\left(dx+c\right)^3-3\left(4\,a^3b-3\,ab^3\right)dx-3\left(2\,a^2b^2-b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(b^6d\sin\left(dx+c\right)+ab^5d\right)},-\frac{2\,ab^3\cos\left(dx+c\right)^3+3\left(4\,a^4-3\,a^2b^2\right)dx+3\left(4\,a^3-ab^2+\left(4\,a^2b-b^3\right)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+3\left(4\,a^3b-3\,ab^3\right)\cos\left(dx+c\right)-\left(b^4\cos\left(dx+c\right)^3-3\left(4\,a^3b-3\,ab^3\right)dx-3\left(2\,a^2b^2-b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3\left(b^6d\sin\left(dx+c\right)+ab^5d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*a*b^3*cos(d*x + c)^3 + 6*(4*a^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^
2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
 + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
```

```
c) - a^2 - b^2)) + 6*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - 2*(b^4*cos(d*x + c)
^3 - 3*(4*a^3*b - 3*a*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x
+ c))/(b^6*d*sin(d*x + c) + a*b^5*d), -1/3*(2*a*b^3*cos(d*x + c)^3 + 3*(4*a
^4 - 3*a^2*b^2)*d*x + 3*(4*a^3 - a*b^2 + (4*a^2*b - b^3)*sin(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
3*(4*a^3*b - 3*a*b^3)*cos(d*x + c) - (b^4*cos(d*x + c)^3 - 3*(4*a^3*b - 3*a
*b^3)*d*x - 3*(2*a^2*b^2 - b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d*sin(d*x
+ c) + a*b^5*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 300, normalized size = 1.84

$$-\frac{\frac{3\,(4\,a^3-3\,ab^2)(dx+c)}{b^5}-\frac{6\,(4\,a^4-5\,a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{6\,(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3-ab^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,a^2-4\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(4*a^3 - 3*a*b^2)*(d*x + c)/b^5 - 6*(4*a^4 - 5*a^2*b^2 + b^4)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 6*(a^2*b*tan(1/2*d*x + 1/2*c) - b^
3*tan(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/
2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*
c)^2 - 6*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 -
4*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4))/d
```

Mupad [**B**]
time = 11.53, size = 964, normalized size = 5.91

$$\frac{\frac{2\,(7\,a\,b^2-12\,a^3)}{3\,b^4}+\dots}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\dots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*(7*a*b^2 - 12*a^3))/(3*b^4) + (2*tan(c/2 + (d*x)/2)^6*(a*b^2 - 4*a^3))/
b^4 + (2*tan(c/2 + (d*x)/2)^4*(7*a*b^2 - 12*a^3))/b^4 + (2*tan(c/2 + (d*x)/
```

2)^2*(25*a*b^2 - 36*a^3))/(3*b^4) - (2*tan(c/2 + (d*x)/2)*(18*a^2 - 11*b^2)
)/(3*b^3) - (14*tan(c/2 + (d*x)/2)^3*(2*a^2 - b^2))/b^3 - (2*tan(c/2 + (d*x
)/2)^7*(2*a^2 - b^2))/b^3 - (2*tan(c/2 + (d*x)/2)^5*(10*a^2 - 7*b^2))/b^3)/
(d*(a + 2*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 6*a*tan(c/2 + (
d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8 + 6*b*tan(c/2
 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^7)) - (2*
atanh((64*a^2*(b^2 - a^2)^(1/2))/(64*a^2*b - (320*a^4)/b + (256*a^6)/b^3 -
640*a^3*tan(c/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2) + (512*a^5*tan(c/
2 + (d*x)/2))/b^2) - (256*a^4*(b^2 - a^2)^(1/2))/(64*a^2*b^3 - 320*a^4*b +
(256*a^6)/b + 512*a^5*tan(c/2 + (d*x)/2) + 128*a*b^4*tan(c/2 + (d*x)/2) - 6
40*a^3*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
))/(64*a^2 - (320*a^4)/b^2 + (256*a^6)/b^4 - (640*a^3*tan(c/2 + (d*x)/2))/b
 + (512*a^5*tan(c/2 + (d*x)/2))/b^3 + 128*a*b*tan(c/2 + (d*x)/2)) - (576*a^
3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(64*a^2*b^2 - 320*a^4 + (256*a^6)/b
^2 + 128*a*b^3*tan(c/2 + (d*x)/2) - 640*a^3*b*tan(c/2 + (d*x)/2) + (512*a^5
*tan(c/2 + (d*x)/2))/b) + (256*a^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(2
56*a^6 + 64*a^2*b^4 - 320*a^4*b^2 + 128*a*b^5*tan(c/2 + (d*x)/2) + 512*a^5*
b*tan(c/2 + (d*x)/2) - 640*a^3*b^3*tan(c/2 + (d*x)/2)))*(4*a^2*(b^2 - a^2)^
(1/2) - b^2*(b^2 - a^2)^(1/2)))/(b^5*d) - (2*a*atan((192*a^2*tan(c/2 + (d*x
)/2))/(192*a^2 - (448*a^4)/b^2 + (256*a^6)/b^4) - (448*a^4*tan(c/2 + (d*x)/
2))/(192*a^2*b^2 - 448*a^4 + (256*a^6)/b^2) + (256*a^6*tan(c/2 + (d*x)/2))/
(256*a^6 + 192*a^2*b^4 - 448*a^4*b^2))*(4*a^2 - 3*b^2))/(b^5*d)

3.1130 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{2ax}{b^3}+\frac{2\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{b^2d}-\frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}$$

```
[Out] -2*a*x/b^3-arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b^2/d-(a^2-b^2)*cos(d*x+c)/
a/b^2/d/(a+b*sin(d*x+c))+2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/b^3/d
```

Rubi [A]

time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2971, 3136, 2739, 632, 210, 3855}

$$\frac{2\sqrt{a^2-b^2}\,(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2ax}{b^3}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*x)/b^3 + (2*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) - Cos[c
+ d*x]/(b^2*d) - ((a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cos(c+dx)}{b^2d} - \frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(b^2-ab\sin(c+dx)-2a^2\sin^2(c-\right.}{a+b\sin(c+dx)}}{ab^2}$$

$$= -\frac{2ax}{b^3} - \frac{\cos(c+dx)}{b^2d} - \frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{(-2a}{}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} - \frac{(}{}$$

$$= -\frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{b^2d} - \frac{(a^2-b^2)\cos(c+dx)}{ab^2d(a+b\sin(c+dx))} + \frac{(}{}$$

$$= -\frac{2ax}{b^3} + \frac{2(2a^4-a^2b^2-b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Mathematica [A]

time = 0.51, size = 161, normalized size = 1.18

$$\frac{-\frac{2a(c+dx)}{b^3}+\frac{2(2a^4-a^2b^2-b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3\sqrt{a^2-b^2}}-\frac{\cos(c+dx)}{b^2}-\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{(-a^2+b^2)\cos(c+dx)}{ab^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(c + d*x))/b^3 + (2*(2*a^4 - a^2*b^2 - b^4)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^3*Sqrt[a^2 - b^2]) - Cos[c + d*x]/b^2 - Lo
g[Cos[(c + d*x)/2]]/a^2 + Log[Sin[(c + d*x)/2]]/a^2 + ((-a^2 + b^2)*Cos[c +
 d*x])/(a*b^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]

time = 0.55, size = 193, normalized size = 1.41

method	result
derivativedivides	$\frac{-\frac{4\left(\frac{b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\frac{4\left(-\frac{b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a^3b}{2}+\frac{ab^3}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(2a^4-a\cdots\right.}{\cdots}}{a^2b^3}}{d}$
default	$\frac{-\frac{4\left(\frac{b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\frac{4\left(-\frac{b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a^3b}{2}+\frac{ab^3}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(2a^4-a\cdots\right.}{\cdots}}{a^2b^3}}{d}$
risch	$-\frac{2ax}{b^3}-\frac{e^{i(dx+c)}}{2b^2d}-\frac{e^{-i(dx+c)}}{2b^2d}+\frac{2i\left(-a^2+b^2\right)\left(-ia\,e^{i(dx+c)}+b\right)}{b^3da\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)}+\frac{2i\sqrt{a^2-b^2}\,\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-\cdots}\right.}{b\cdots}\right.}{d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/b^3*(1/2*b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))+1
/a^2*ln(tan(1/2*d*x+1/2*c))+4/a^2/b^3*((-1/2*b^2*(a^2-b^2)*tan(1/2*d*x+1/2*
c)-1/2*a^3*b+1/2*a*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1
/2*(2*a^4-a^2*b^2-b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.50, size = 516, normalized size = 3.77

$$\left[-\frac{4\,a^4dx-\left(2\,a^3+ab^2+\left(2\,a^2b+b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\log\left(-\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)+\left(b^4\sin\left(dx+c\right)+ab^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(b^4\sin\left(dx+c\right)+ab^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(2\,a^3bdx+a^2b^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(a^2b^4d\sin\left(dx+c\right)+a^3b^3d\right)},\ -\frac{4\,a^4dx+2\left(2\,a^3+ab^2+\left(2\,a^2b+b^3\right)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)+\left(b^4\sin\left(dx+c\right)+ab^3\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(b^4\sin\left(dx+c\right)+ab^3\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+2\left(2\,a^3bdx+a^2b^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(a^2b^4d\sin\left(dx+c\right)+a^3b^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*d*x - (2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(2*a^3*b - a*b^3)*cos
(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1/2) - (b^4*s
in(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*d*x + a^2*b^
2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3*d), -1/2*(4
*a^4*d*x + 2*(2*a^3 + a*b^2 + (2*a^2*b + b^3)*sin(d*x + c))*sqrt(a^2 - b^2)
*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 2*(2*a^3*b
 - a*b^3)*cos(d*x + c) + (b^4*sin(d*x + c) + a*b^3)*log(1/2*cos(d*x + c) + 1
/2) - (b^4*sin(d*x + c) + a*b^3)*log(-1/2*cos(d*x + c) + 1/2) + 2*(2*a^3*b*
d*x + a^2*b^2*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c) + a^3*b^3
*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)\csc\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(132) = 264.
time = 0.48, size = 286, normalized size = 2.09

$$-\frac{\frac{2\,(dx+c)a}{b^3}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{2\,\left(2\,a^4-a^2b^2-b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^3}+\frac{2\,\left(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^2b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -(2*(d*x + c)*a/b^3 - log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 2*(2*a^4 - a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^2*b^3) + 2*(a^2*b*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^3*tan(1/2*d*x + 1/2*c)^2 - a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 2*a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^2*b^2))/d

Mupad [B]
time = 11.15, size = 2773, normalized size = 20.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)

[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + (atan((((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) - (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8))*1i - ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) - ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) - ((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8*b^8))/(a^3*b^8)) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) + (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) - (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8))*1i)/(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)

/(a^2*b))*(((32*(4*a^4*b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6*b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b)) + (32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^ 5) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^10 + 8*a^8 *b^8))/(a^3*b^8)) + (32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a ^8*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b^12 + 101 *a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) - (32*(28*a^8 + 2*a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(b^12 + 2*a^2 *b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3*b^8)) + ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*(((2*(b^2 - a^2)^(1/ 2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(4*b^10 + 3*a^2*b^8 - 7*a^4*b^6 + 16*a^6*b^4 - 12*a^8*b^2))/(a^2*b^5) - ((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1/2)/(a^2*b))*((32*(8*a^2*b^10 - 7*a^6*b^6))/(a^2*b^5) - ((32*(4*a^4* b^10 - 3*a^6*b^8))/(a^2*b^5) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^14 - 17*a^6 *b^12 + 2*a^8*b^10))/(a^3*b^8))*((2*(b^2 - a^2)^(1/2))/b^3 + (b^2 - a^2)^(1 /2)/(a^2*b)) + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^14 - 5*a^4*b^12 - 18*a^6*b^ 10 + 8*a^8*b^8))/(a^3*b^8)) + (32*tan(c/2 + (d*x)/2)*(6*a^4*b^10 - 11*a^2*b ^12 + 101*a^6*b^8 - 100*a^8*b^6 + 8*a^10*b^4))/(a^3*b^8)) + (32*(28*a^8 + 2 *a^2*b^6 - 28*a^4*b^4 - 6*a^6*b^2))/(a^2*b^5) - (32*tan(c/2 + (d*x)/2)*(b^1 2 + 2*a^2*b^10 + 61*a^4*b^8 - 20*a^6*b^6 - 72*a^8*b^4 + 32*a^10*b^2))/(a^3* b^8)) - (64*(28*a^6 + 2*b^6 - 12*a^2*b^4 - 18*a^4*b^2))/(a^2*b^5) - (64*tan (c/2 + (d*x)/2)*(128*a^10 + 48*a^4*b^6 - 16*a^6*b^4 - 160*a^8*b^2))/(a^3*b^ 8)))*(((b^2 - a^2)^(1/2)*4i)/b^3 + ((b^2 - a^2)^(1/2)*2i)/(a^2*b)))/d - ((2 *(2*a^2 - b^2))/(a*b^2) + (2*tan(c/2 + (d*x)/2)*(3*a^2 - b^2))/(a^2*b) + (2 *tan(c/2 + (d*x)/2)^3*(a^2 - b^2))/(a^2*b) + (2*tan(c/2 + (d*x)/2)^2*(2*a^2 - b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (4*a*atan((256*tan (c/2 + (d*x)/2))/((128*b^2)/a^2 - (384*a^2)/b^2 + (256*a*tan(c/2 + (d*x)/2))/b + (512*a^3*tan(c/2 + (d*x)/2))/b^3 - (768*a^5*tan(c/2 + (d*x)/2))/b^5 + 256) - (512*a^3)/(256*b^3 - 384*a^2*b + (128*b^5)/a^2 + 512*a^3*tan(c/2 + (d*x)/2) + 256*a*b^2*tan(c/2 + (d*x)/2) - (768*a^5*tan(c/2 + (d*x)/2))/b^2) + (768*a^5)/(256*b^5 - 384*a^2*b^3 + (128*b^7)/a^2 - 768*a^5*tan(c/2 + (d* x)/2) + 256*a*b^4*tan(c/2 + (d*x)/2) + 512*a^3*b^2*tan(c/2 + (d*x)/2)) - (2 56*a)/(256*b + 256*a*tan(c/2 + (d*x)/2) - (384*a^2)/b + (128*b^3)/a^2 + (51 2*a^3*tan(c/2 + (d*x)/2))/b^2 - (768*a^5*tan(c/2 + (d*x)/2))/b^4) - (384*a^ 2*tan(c/2 + (d*x)/2))/(256*b^2 - 384*a^2 + (128*b^4)/a^2 + (512*a^3*tan(c/2 + (d*x)/2))/b - (768*a^5*tan(c/2 + (d*x)/2))/b^3 + 256*a*b*tan(c/2 + (d*x) /2)) + (128*b*tan(c/2 + (d*x)/2))/(128*b + (256*a^2)/b - (384*a^4)/b^3 + (2 56*a^3*tan(c/2 + (d*x)/2))/b^2 + (512*a^5*tan(c/2 + (d*x)/2))/b^4 - (768*a^ 7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d)

3.1131 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=154

$$\frac{x}{b^2}-\frac{2(a^4+a^2b^2-2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}\,d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}-\frac{\text{co}}{ad(a+}$$

```
[Out] x/b^2+2*b*arctanh(cos(d*x+c))/a^3/d+(a^2-2*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin
(d*x+c))-cot(d*x+c)/a/d/(a+b*sin(d*x+c))-2*(a^4+a^2*b^2-2*b^4)*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2969, 3136, 2739, 632, 210, 3855}

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))}-\frac{2(a^4+a^2b^2-2b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] x/b^2 - (2*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d)
 + ((a^2 - 2*b^2)*Cos[c + d*x])/(a^2*b*d*(a + b*Sin[c + d*x])) - Cot[c + d*
x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)(-2b^2-ab\sin(c-}{a+b\sin(c-}}{a^2b}$$

$$= \frac{x}{b^2} + \frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int\csc(c+dx}{a^3}$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+d}{ad(a+b\sin(c}$$

$$= \frac{x}{b^2} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{(a^2-2b^2)\cos(c+dx)}{a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+d}{ad(a+b\sin(c}$$

$$= \frac{x}{b^2} - \frac{2(a^4+a^2b^2-2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx}{a^3d}$$

Mathematica [A]

time = 1.27, size = 182, normalized size = 1.18

$$\frac{\frac{2(c+dx)}{b^2} - \frac{4(a^4+a^2b^2-2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2\sqrt{a^2-b^2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{2(a^2-b^2)\cos(c+dx)}{a^2b(a+b\sin(c+dx))} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(c + d*x))/b^2 - (4*(a^4 + a^2*b^2 - 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^3*b^2*Sqrt[a^2 - b^2]) - Cot[(c + d*x)/2]/a^2 + (
4*b*Log[Cos[(c + d*x)/2]])/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*(a^2
- b^2)*Cos[c + d*x])/(a^2*b*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(
2*d)
```

Maple [A]

time = 0.53, size = 201, normalized size = 1.31

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2} + \frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2} - \frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3} - \frac{2\left(\frac{-b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ba\left(a^2-b^2\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a} + \frac{(a^4}{}\right.}{a^3b}}{d}$

default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}+\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{-b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ba\left(a^2-b^2\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^4+\right.}{}\right.}{a^3b^2}}{d}$
risch	$\frac{x}{b^2}-\frac{2i\left(3a\,b^2\mathrm{e}^{i(dx+c)}-2ib^3\mathrm{e}^{2i(dx+c)}+2ib^3+ia^2b\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}a^3-a\,b^2\mathrm{e}^{3i(dx+c)}-ia^2b-a^3\mathrm{e}^{i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)b^2\left(-ib\,\mathrm{e}^{2i(dx+c)}+ib+2a\,\mathrm{e}^{i(dx+c)}\right)a^2d}-\frac{2b\ln\left(\mathrm{e}^{\left(\right.}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)+2/b^2*arctan(tan(1/2*d*x+1/2*c))-1/2/a^2/ta
n(1/2*d*x+1/2*c)-2/a^3*b*ln(tan(1/2*d*x+1/2*c))-2/a^3/b^2*((-b^2*(a^2-b^2)*
tan(1/2*d*x+1/2*c)-b*a*(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1
/2*c)+a)+(a^4+a^2*b^2-2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.46, size = 675, normalized size = 4.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^3*b*d*x*cos(d*x + c)^2 - 2*a^3*b*d*x + 2*a^2*b^2*cos(d*x + c) - (
a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x +
c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
```

```
b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(b^4*cos(
d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - 2*(b^4
*cos(d*x + c)^2 - a*b^3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) -
2*(a^4*d*x + (a^3*b - 2*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d
*x + c)^2 - a^4*b^2*d*sin(d*x + c) - a^3*b^3*d), (a^3*b*d*x*cos(d*x + c)^2
- a^3*b*d*x + a^2*b^2*cos(d*x + c) - (a^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(d
*x + c)^2 + (a^3 + 2*a*b^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*
x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (b^4*cos(d*x + c)^2 - a*b^3*s
in(d*x + c) - b^4)*log(1/2*cos(d*x + c) + 1/2) - (b^4*cos(d*x + c)^2 - a*b^
3*sin(d*x + c) - b^4)*log(-1/2*cos(d*x + c) + 1/2) - (a^4*d*x + (a^3*b - 2*
a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^4*b^2*d*si
n(d*x + c) - a^3*b^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.47, size = 260, normalized size = 1.69

$$\frac{\frac{6\,(dx+c)}{b^2}-\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{12\,(a^4+a^2b^2-2\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^2}+\frac{4\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3b}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] 1/6*(6*(d*x + c)/b^2 - 12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 3*tan(1/2*
d*x + 1/2*c)/a^2 - 12*(a^4 + a^2*b^2 - 2*b^4)*(pi*floor(1/2*(d*x + c)/pi +
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^3*b^2) + (4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x
+ 1/2*c)^2 - 4*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*tan(1/2*d*x + 1/2*c) - 1
4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*ta
n(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b))/d
```

Mupad [B]
time = 12.09, size = 2500, normalized size = 16.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)

[Out] (2*cos(c/2 + (d*x)/2)^2)/(b*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + sin(c/2 + (d*x)/2)^3/(2*a*d*cos(c/2 + (d*x)/2)*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - cos(c/2 + (d*x)/2)^3/(2*a*d*sin(c/2 + (d*x)/2)*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (3*b*cos(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (b*sin(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 16*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 2*a^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(c/2 + (d*x)/2)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)*128i + a^5*b^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a^9*b^4*cos(c/2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*sin(c/2 + (d*x)/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 + (d*x)/2)*172i + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2)*21i))*cos(c/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*4i)/(a^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 8*a^4*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 18*a^6*b^4*sin(c/2 + (d*x)

/2)*(b^2 - a^2)^(3/2) - 26*a^6*b^6*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 1
6*a^8*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 28*a^8*b^4*sin(c/2 + (d*x)
/2)*(b^2 - a^2)^(1/2) - 64*a*b^9*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 2*a
^9*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 5*a^11*b*cos(c/2 + (d*x)/2)*(b^
2 - a^2)^(1/2))/(a^3*b^10*cos(c/2 + (d*x)/2)*80i - a*b^12*cos(c/2 + (d*x)/2
)*64i - a^12*b*sin(c/2 + (d*x)/2)*3i - b^13*sin(c/2 + (d*x)/2)*128i + a^5*b
^8*cos(c/2 + (d*x)/2)*44i - a^7*b^6*cos(c/2 + (d*x)/2)*72i + a^9*b^4*cos(c/
2 + (d*x)/2)*3i + a^11*b^2*cos(c/2 + (d*x)/2)*9i + a^2*b^11*sin(c/2 + (d*x)
/2)*192i + a^4*b^9*sin(c/2 + (d*x)/2)*56i - a^6*b^7*sin(c/2 + (d*x)/2)*172i
 + a^8*b^5*sin(c/2 + (d*x)/2)*34i + a^10*b^3*sin(c/2 + (d*x)/2)*21i))*cos(c
/2 + (d*x)/2)^2*(b^2 - a^2)^(1/2)*2i)/(b^2*d*(a*cos(c/2 + (d*x)/2)^2 + a*si
n(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))) - (2*b*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(c/2 + (d*x)/2)^2)/(a^2*d*(a*cos
(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2
 + (d*x)/2))) - (2*b^2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^3*d*(a*cos
(c/2 + (d*x)/2)^2 + a*sin(c/2 + (d*x)/2)^2 + 2*b*cos(c/2 + (d*x)/2)*sin(c/2
 + (d*x)/2))) + (atan((40*a^5*b^5*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*
a^12*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) - 128*b^10*sin(c/2 + (d*x)/2)*(b^
2 - a^2)^(3/2) - 16*a^3*b^7*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 4*a^10*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) + 4*a^5*b^7*cos(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2) + 20*a^7*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 20*a^7*b^5*co
s(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 9*a^9*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^
2)^(1/2) + 8*a^2*b^8*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(3/2) - 8*a^2*b^10*sin(
c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + 104*a^4*b^6*...

3.1132 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=158

$$\frac{6b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{\cos(c+dx)}{2a^2d\,(1-\cos^2(c+dx))}+\frac{2b\cot(c+}{a^3d}$$

```
[Out] 3/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d-1/2*cos(d*x+c)/a^2/d/(1-cos(d*x+c
)^2)+2*b*cot(d*x+c)/a^3/d-(a^2-b^2)*cos(d*x+c)/a^3/d/(a+b*sin(d*x+c))+6*b*a
rctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^4/d
```

Rubi [A]
time = 0.29, antiderivative size = 180, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2969, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))}+\frac{6b\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}-\frac{(a^2-3b^2)\cot(c+dx)}{a^3bd}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - ((a^2 - 3*b^2)*Co
t[c + d*x])/(a^3*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*b*d*(a + b*Si
n[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)(2(a^2-3b^2)-\cdots}{a+b\cdots}}{\cdots}$$

$$= -\frac{(a^2-3b^2)\cot(c+dx)}{a^3bd} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+\cdots}{2ad(a+b\sin(c+d\cdots}$$

$$= -\frac{(a^2-3b^2)\cot(c+dx)}{a^3bd} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc(c+\cdots}{2ad(a+b\sin(c+d\cdots}$$

$$= \frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{(a^2-3b^2)\cot(c+dx)}{a^3bd} + \frac{(2a^2-3b^2)\,\mathrm{co}\cdots}{2a^2bd(a+b\sin\cdots}$$

$$= \frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{(a^2-3b^2)\cot(c+dx)}{a^3bd} + \frac{(2a^2-3b^2)\,\mathrm{co}\cdots}{2a^2bd(a+b\sin\cdots}$$

$$= \frac{6b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} - \cdots$$

Mathematica [A]

time = 2.58, size = 191, normalized size = 1.21

$$\frac{48b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+8ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+12(a^2-2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12(a^2-2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)+\frac{8a(-a^2+b^2)\cos(c+dx)}{a+b\sin(c+dx)}-8ab\tan\left(\frac{1}{2}(c+dx)\right)}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 8*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 12*(a^2 - 2*b^2)*Log[Cos[(c
 + d*x)/2]] - 12*(a^2 - 2*b^2)*Log[Sin[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]
^2 + (8*a*(-a^2 + b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x]) - 8*a*b*Tan[(c +
d*x)/2])/(8*a^4*d)
```

Maple [A]

time = 0.54, size = 209, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}-\frac{1}{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+12b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{b}{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4\left(-\frac{b\left(a^2-b^2\right)\tan\left(\frac{d\cdots}{\cdots}\right.}{2}\cdots\right.}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots}\cdots}{d}$

default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}-\frac{1}{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+12b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{b}{a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4\left(-\frac{b\left(a^2-b^2\right)\tan(}{2}\right.}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$
risch	$\frac{i\left(-3ia\,b^2e^{5i(dx+c)}+12ia\,b^2e^{3i(dx+c)}+6b\,e^{2i(dx+c)}a^2-9ia\,b^2e^{i(dx+c)}+6b^3e^{4i(dx+c)}-12b^3e^{2i(dx+c)}+6b^3+2ia^3e^{5i(dx+c)}\right.}{\left(e^{2i(dx+c)}-1\right)^2b\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^3*(1/2*a*tan(1/2*d*x+1/2*c)^2-4*b*tan(1/2*d*x+1/2*c))-1/8/a^2/ta
n(1/2*d*x+1/2*c)^2+1/4/a^4*(-6*a^2+12*b^2)*ln(tan(1/2*d*x+1/2*c))+b/a^3/tan
(1/2*d*x+1/2*c)+4/a^4*((-1/2*b*(a^2-b^2)*tan(1/2*d*x+1/2*c)-1/2*a^3+1/2*a*b
^2)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+3/2*b*(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(147) = 294$.
time = 0.46, size = 804, normalized size = 5.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3
 - 6*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
```

```
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^3 - 2*a*b^
2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*
b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2
*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c)
+ 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*
sin(d*x + c)), -1/4*(6*a^2*b*cos(d*x + c)*sin(d*x + c) + 4*(a^3 - 3*a*b^2)*
cos(d*x + c)^3 + 12*(a*b*cos(d*x + c)^2 - a*b + (b^2*cos(d*x + c)^2 - b^2)*
sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 6*(a^3 - 2*a*b^2)*cos(d*x + c) + 3*(a^3 - 2*a*b^2 - (a^3
- 2*a*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^3 - 2*a*b^2 - (a^3 - 2*a
*b^2)*cos(d*x + c)^2 + (a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(a^5*d*cos(d*x + c)^2 - a^5*d + (a
^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 275, normalized size = 1.74

$$-\frac{\frac{12\,(a^2-2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{48\,(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{16\,(a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3-ab^2)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^4}-\frac{18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(12*(a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b - b^3
)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
 + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (a^2*tan(1/2*d*x + 1/2*c)^2
 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + 16*(a^2*b*tan(1/2*d*x + 1/2*c) - b^3*t
an(1/2*d*x + 1/2*c) + a^3 - a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)*a^4) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d
*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)
^2))/d
```

Mupad [B]
time = 9.73, size = 675, normalized size = 4.27

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{17\,a^2}{2}-16\,b^2\right)+\frac{a^2}{2}+\frac{4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^2\,b-2\,b^3\right)}{a}-3\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+4\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}-\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a^2-6\,b^2\right)}{2\,a^4\,d}-\frac{6\,b\,\mathrm{atanh}\left(\frac{72\,b^4\,\sqrt{b^2-a^2}}{18\,a^4\,b+72\,b^5-90\,a^2\,b^3-216\,a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+72\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{144\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}}-\frac{54\,b^2\,\sqrt{b^2-a^2}}{18\,a^2\,b-90\,b^3+\frac{72\,b^5}{a^2}+72\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{216\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{144\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}}+\frac{18\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{18\,a\,b-\frac{90\,b^3}{a}+\frac{72\,b^5}{a^3}+72\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{216\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}+\frac{144\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4}}-\frac{144\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{18\,a^3\,b-90\,a\,b^3+\frac{72\,b^5}{a}-216\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+72\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{144\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}}+\frac{144\,b^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{72\,a\,b^5+18\,a^5\,b-90\,a^3\,b^3+144\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-216\,a^2\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+72\,a^4\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{b^2-a^2}}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (tan(c/2 + (d*x)/2)^2*((17*a^2)/2 - 16*b^2
) + a^2/2 + (4*tan(c/2 + (d*x)/2)^3*(a^2*b - 2*b^3))/a - 3*a*b*tan(c/2 + (d
*x)/2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3
*b*tan(c/2 + (d*x)/2)^3)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 +
 (d*x)/2))*(3*a^2 - 6*b^2))/(2*a^4*d) - (6*b*atanh((72*b^4*(b^2 - a^2)^(1/2
))/(18*a^4*b + 72*b^5 - 90*a^2*b^3 - 216*a*b^4*tan(c/2 + (d*x)/2) + 72*a^3*
b^2*tan(c/2 + (d*x)/2) + (144*b^6*tan(c/2 + (d*x)/2))/a) - (54*b^2*(b^2 - a
^2)^(1/2))/(18*a^2*b - 90*b^3 + (72*b^5)/a^2 + 72*a*b^2*tan(c/2 + (d*x)/2)
- (216*b^4*tan(c/2 + (d*x)/2))/a + (144*b^6*tan(c/2 + (d*x)/2))/a^3) + (18*
b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(18*a*b - (90*b^3)/a + (72*b^5)/a^3
 + 72*b^2*tan(c/2 + (d*x)/2) - (216*b^4*tan(c/2 + (d*x)/2))/a^2 + (144*b^6*
tan(c/2 + (d*x)/2))/a^4) - (144*b^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(
18*a^3*b - 90*a*b^3 + (72*b^5)/a - 216*b^4*tan(c/2 + (d*x)/2) + 72*a^2*b^2*
tan(c/2 + (d*x)/2) + (144*b^6*tan(c/2 + (d*x)/2))/a^2) + (144*b^5*tan(c/2 +
 (d*x)/2)*(b^2 - a^2)^(1/2))/(72*a*b^5 + 18*a^5*b - 90*a^3*b^3 + 144*b^6*ta
n(c/2 + (d*x)/2) - 216*a^2*b^4*tan(c/2 + (d*x)/2) + 72*a^4*b^2*tan(c/2 + (d
*x)/2)))*(b^2 - a^2)^(1/2))/(a^4*d)
```

3.1133 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-$$

```
[Out] -b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(7*a^2-12*b^2)*cot(d*x+c)/a^
4/d-(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d+1/3*(3*a^2-4*b^2)*cot(d*x+c)*
csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*si
n(d*x+c))+2*(a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2803, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d}-\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd}+\frac{2(a^4-5a^2b^2+4b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \csc^3(c+dx)(6(a \cdots}{\cdots}$$

$$= -\frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\mathrm{co}\cdots}{3\cdots}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot\cdots}{3a^2bd(a+\cdots}$$

$$= \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot\cdots}{3a^2bd(a+\cdots}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot\cdots}{\cdots}$$

$$= -\frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot\cdots}{\cdots}$$

$$= \frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx)\cdots}{a^5d}$$

Mathematica [A]
time = 6.19, size = 403, normalized size = 1.69

$$\frac{2(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{(-3a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [A]
time = 0.60, size = 287, normalized size = 1.21

method	result

derivativedivides	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{1}{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a^2+12b^2}{8a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{}{4a^3\tan(}$
default	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{1}{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a^2+12b^2}{8a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{}{4a^3\tan(}$
risch	$\frac{24ib^3e^{4i(dx+c)}-\frac{14ia^2b}{3}-14a^3e^{5i(dx+c)}+14e^{3i(dx+c)}a^3+2a^3e^{7i(dx+c)}-14ia^2be^{4i(dx+c)}-4ab^2e^{7i(dx+c)}+20ab^2e^{5i(dx+}}{\left(e^{2i(dx+c)}-1\right)^3\left(b\,e^{2i(}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^4*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-2*a*b*tan(1/2*d*x+1/2*c)^2-5*a^2
*tan(1/2*d*x+1/2*c)+12*b^2*tan(1/2*d*x+1/2*c))-1/24/a^2/tan(1/2*d*x+1/2*c)^
3-1/8*(-5*a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)+1/4/a^3*b/tan(1/2*d*x+1/2*c)^2
+1/a^5*b*(3*a^2-4*b^2)*ln(tan(1/2*d*x+1/2*c))+2/a^5*((b^2*(a^2-b^2)*tan(1/2
*d*x+1/2*c)+b*a*(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a
)+(a^4-5*a^2*b^2+4*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(227) = 454$.
time = 0.46, size = 1149, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 356, normalized size = 1.50

$$\frac{\frac{24\,(3\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}+\frac{48\,(a^4-5\,a^2b^2+4\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}+\frac{48\,(a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3b-ab^3)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^5}-\frac{132\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-176\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d
```

Mupad [B]

time = 9.80, size = 973, normalized size = 4.09

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(28a^2b-40b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(8ab^2-\frac{14a^3}{3}\right)-\frac{a^3}{3}+\frac{4a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(5a^4+4a^2b^2-16b^4\right)}{a}}{d\left(8a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+16ba^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{16a^2+32b^2}{64a^4}+\frac{3}{8a^2}-\frac{2b^2}{a^4}\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4a^3d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(3a^2-4b^2\right)}{a^5d}+\frac{\mathrm{atan}\left(\text{[illegible]}\right)\sqrt{b^2-a^2}\left(a^2-4b^2\right)2i}{a^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(28*a^2*b - 40*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (14*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(5*a^4 - 16*b^4 + 4*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*
tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + tan(c/2 + (d*x)/2)
^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((16*a^2 + 32*b^2)/(64*a^4) + 3/(8*a^2)
 - (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(c/2 +
(d*x)/2))*(3*a^2 - 4*b^2))/(a^5*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2
)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b +
 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10
- 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/a^5)*1i)/a^5 + ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 - ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5)*1i)/a^5)/((4*(3*a^6*b - 16*b^7 + 32*a^2*b^5 - 19*a^4*b^3))/a^8 + (4*ta
n(c/2 + (d*x)/2)*(2*a^6 - 16*b^6 + 28*a^2*b^4 - 14*a^4*b^2))/a^7 - ((b^2 -
a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/
2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(
c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/
a^5))/a^5 + ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b
^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 -
 ((2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(
1/2)*(a^2 - 4*b^2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*2i)/(a^5*d)
```

3.1134 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=292

$$-\frac{2b(2a^4-7a^2b^2+5b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d}-\frac{(3a^4-36a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{b(11a^2-15b^2}{3a}$$

```
[Out] -1/8*(3*a^4-36*a^2*b^2+40*b^4)*arctanh(cos(d*x+c))/a^6/d-1/3*b*(11*a^2-15*b
^2)*cot(d*x+c)/a^5/d+1/8*(13*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d-1/3*(3
*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/b/d+1/4*(4*a^2-5*b^2)*cot(d*x+c)*cs
c(d*x+c)^2/a^2/b/d/(a+b*sin(d*x+c))-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*si
n(d*x+c))-2*b*(2*a^4-7*a^2*b^2+5*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-
b^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2969, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(4a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))}-\frac{b(11a^2-15b^2)\cot(c+dx)}{3a^5d}+\frac{(13a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}-\frac{(3a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{3a^3bd}-\frac{2b(2a^4-7a^2b^2+5b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{(3a^4-36a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - ((3*a^4 - 36*a^2*b^2 + 40*b^4)*ArcTanh[C
os[c + d*x]])/(8*a^6*d) - (b*(11*a^2 - 15*b^2)*Cot[c + d*x])/(3*a^5*d) + ((
13*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - ((3*a^2 - 5*b^2)*Co
t[c + d*x]*Csc[c + d*x]^2)/(3*a^3*b*d) + ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[
c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^3
)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{(4a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))} + \frac{\int \csc^4(c+\cdots}{\cdots} \\
&= -\frac{(3a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{3a^3bd} + \frac{(4a^2-5b^2)\cot(c+dx)\csc^2(c+d\cdots}{4a^2bd(a+b\sin(c+dx))} \\
&= \frac{(13a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{(3a^2-5b^2)\cot(c+dx)\csc^2(c+d\cdots}{3a^3bd} \\
&= -\frac{b(11a^2-15b^2)\cot(c+dx)}{3a^5d} + \frac{(13a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{(3\cdots}{\cdots} \\
&= -\frac{b(11a^2-15b^2)\cot(c+dx)}{3a^5d} + \frac{(13a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{(3\cdots}{\cdots} \\
&= -\frac{(3a^4-36a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b(11a^2-15b^2)\cot(c+dx)}{3a^5d} \cdots \\
&= -\frac{(3a^4-36a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{b(11a^2-15b^2)\cot(c+dx)}{3a^5d} \cdots \\
&= -\frac{2b(2a^4-7a^2b^2+5b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{(3a^4-36a^2b^2+40b^4)\,\mathrm{ta}\cdots}{8a^6d}
\end{aligned}
$$

Mathematica [A]
time = 6.28, size = 496, normalized size = 1.70

$$
-\frac{2b(2a^4-7a^2b^2+5b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{2\left(2a^2b\cos\left(\frac{1}{2}(c+dx)\right)-3b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{3a^5d} + \frac{(5a^2-12b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} + \frac{(-3a^4+36a^2b^2-40b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} + \frac{(3a^4-36a^2b^2+40b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} + \frac{(-5a^2+12b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} + \frac{2\sec\left(\frac{1}{2}(c+dx)\right)\left(2a^2b\sin\left(\frac{1}{2}(c+dx)\right)-3b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5d} + \frac{-a^2b^2\cos(c+dx)+b^4\cos(c+dx)}{a^5d(a+b\sin(c+dx))} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (2*(
2*a^2*b*Cos[(c + d*x)/2] - 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5
*d) + ((5*a^2 - 12*b^2)*Csc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2
]*Csc[(c + d*x)/2]^2)/(12*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) + ((-3*a^4
 + 36*a^2*b^2 - 40*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^6*d) + ((3*a^4 - 36*a^2
```

```
*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]])/(8*a^6*d) + ((-5*a^2 + 12*b^2)*Sec[(c
 + d*x)/2]^2)/(32*a^4*d) + Sec[(c + d*x)/2]^4/(64*a^2*d) + (2*Sec[(c + d*x)
/2]*(2*a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (-(a^2
*b^2*Cos[c + d*x]) + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

Maple [A]
time = 0.69, size = 365, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^5*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-4/3*b*tan(1/2*d*x+1/2*c)^3*a^2-
2*a^3*tan(1/2*d*x+1/2*c)^2+6*a*b^2*tan(1/2*d*x+1/2*c)^2+20*a^2*b*tan(1/2*d*
x+1/2*c)-32*b^3*tan(1/2*d*x+1/2*c))-1/64/a^2/tan(1/2*d*x+1/2*c)^4-1/32*(-4*
a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)^2+1/16/a^6*(6*a^4-72*a^2*b^2+80*b^4)*ln(
tan(1/2*d*x+1/2*c))+1/12/a^3*b/tan(1/2*d*x+1/2*c)^3-1/4*b*(5*a^2-8*b^2)/a^5
/tan(1/2*d*x+1/2*c)-4*b/a^6*((1/2*b^2*(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/2*b*a*
(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(2*a^4-7*a
^2*b^2+5*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-
b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(275) = 550$.
time = 0.63, size = 1578, normalized size = 5.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^3*b^2
 + 240*a*b^4)*cos(d*x + c)^3 + 24*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4 + 2*a
^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 - 5*b^4
)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c)
)*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
- 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3*(3*a^5 - 36*a^3*b^2 +
40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a
^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4
*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5
)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(3*a^5 - 36
*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^4 - 2*(3
*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^2*b^3 + 40*b
^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 36*a^2*b
^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2
*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4*b - 20*a^2*b^3)*cos(d*
x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*
d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x +
 c)), -1/48*(16*(11*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^5 + 2*(15*a^5 - 196*a^
3*b^2 + 240*a*b^4)*cos(d*x + c)^3 - 48*((2*a^3*b - 5*a*b^3)*cos(d*x + c)^4
+ 2*a^3*b - 5*a*b^3 - 2*(2*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + ((2*a^2*b^2 -
5*b^4)*cos(d*x + c)^4 + 2*a^2*b^2 - 5*b^4 - 2*(2*a^2*b^2 - 5*b^4)*cos(d*x +
 c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
 - b^2)*cos(d*x + c))) - 6*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c) + 3
*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x +
 c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - 36*a^
2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4 - 2*(3*a^4*
b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c)
 + 1/2) - 3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4 + (3*a^5 - 36*a^3*b^2 + 40*a*b^4
)*cos(d*x + c)^4 - 2*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*cos(d*x + c)^2 + (3*a^
4*b - 36*a^2*b^3 + 40*b^5 + (3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^4
- 2*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - 2*((49*a^4*b - 60*a^2*b^3)*cos(d*x + c)^3 - 3*(13*a^4
*b - 20*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*
d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)
^2 + a^6*b*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep

Giac [A]
time = 0.53, size = 461, normalized size = 1.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/192*(24*(3*a^4 - 36*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6
- 384*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*
a^6) - 384*(a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x + 1/2*c) + a^3*b
^2 - a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6)
 + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 24*a^6
*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a^5*b*tan
(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2*d*x + 1/2*c))/a^8 - (150*a^4*tan(1/
2*d*x + 1/2*c)^4 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d
*x + 1/2*c)^4 + 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x +
1/2*c)^3 - 24*a^4*tan(1/2*d*x + 1/2*c)^2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^
2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d

Mupad [B]
time = 9.83, size = 1158, normalized size = 3.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) + (tan(c/2 + (d*x)/2)^4*(2*a^4 + 96*b^4 - 7
8*a^2*b^2) - a^4/4 + tan(c/2 + (d*x)/2)^2*((7*a^4)/4 - (10*a^2*b^2)/3) + ta
n(c/2 + (d*x)/2)^3*(20*a*b^3 - (44*a^3*b)/3) - (4*tan(c/2 + (d*x)/2)^5*(5*a
^4*b - 8*b^5))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*
x)/2)^4 + 16*a^6*tan(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - (
tan(c/2 + (d*x)/2)^2*((32*a^2 + 64*b^2)/(512*a^4) + 1/(16*a^2) - b^2/(2*a^4
)))/d + (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^3) + (
4*b*((32*a^2 + 64*b^2)/(256*a^4) + 1/(8*a^2) - b^2/a^4))/a))/d - (b*tan(c/2
 + (d*x)/2)^3)/(12*a^3*d) + (log(tan(c/2 + (d*x)/2))*(3*a^4 + 40*b^4 - 36*a
^2*b^2))/(8*a^6*d) - (b*atan(((b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/
2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*a^9) - (
19*a^10*b + 80*a^6*b^5 - 92*a^8*b^3)/(4*a^10) + (b*(2*a^2*b - (tan(c/2 + (d
x)/2)(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))
/a^6)*1i)/a^6 - (b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((19*a^10*b + 80*a^6*b

```
^5 - 92*a^8*b^3)/(4*a^10) - (tan(c/2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224
*a^6*b^4 - 74*a^8*b^2))/(4*a^9) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^1
2 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6)
/((6*a^8*b + 200*b^9 - 460*a^2*b^7 + 347*a^4*b^5 - 93*a^6*b^3)/(2*a^10) + (
tan(c/2 + (d*x)/2)*(200*b^8 - 410*a^2*b^6 + 262*a^4*b^4 - 52*a^6*b^2))/(2*a
^9) + (b*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(3*a^10 - 1
60*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*a^9) - (19*a^10*b + 80*a^6*b^5 -
 92*a^8*b^3)/(4*a^10) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^1
0*b^2))/(4*a^9))*(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6 + (b*(b^2 - a
^2)^(1/2)*(2*a^2 - 5*b^2)*((19*a^10*b + 80*a^6*b^5 - 92*a^8*b^3)/(4*a^10) -
 (tan(c/2 + (d*x)/2)*(3*a^10 - 160*a^4*b^6 + 224*a^6*b^4 - 74*a^8*b^2))/(4*
a^9) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*
(b^2 - a^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6))*(b^2 - a^2)^(1/2)*(2*a^2 - 5
*b^2)*2i)/(a^6*d)
```

3.1135 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=331

$$\frac{3(40a^4-24a^2b^2+b^4)\,x}{8b^7}-\frac{3a(10a^4-11a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d}+\frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d}-3$$

```
[Out] 3/8*(40*a^4-24*a^2*b^2+b^4)*x/b^7+1/2*a*(30*a^2-13*b^2)*cos(d*x+c)/b^6/d-3/
8*(20*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)/b^5/d+1/2*(10*a^2-3*b^2)*cos(d*x+c)*
sin(d*x+c)^2/a/b^4/d-1/4*(15*a^2-4*b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^3/d-1
/2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^2+1/2*(7*a^2-
2*b^2)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^2/d/(a+b*sin(d*x+c))-3*a*(10*a^4-11*a^
2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d/(a^2-b^
2)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2970, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{(7a^2-2b^2)\sin^4(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d}-\frac{3(20a^2-7b^2)\sin(c+dx)\cos(c+dx)}{8b^5d}+\frac{(10a^2-3b^2)\sin^2(c+dx)\cos(c+dx)}{2ab^4d}-\frac{(15a^2-4b^2)\sin^3(c+dx)\cos(c+dx)}{4a^2b^3d}-\frac{3a(10a^4-11a^2b^2+2b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}+\frac{3x(40a^4-24a^2b^2+b^4)}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(40*a^4 - 24*a^2*b^2 + b^4)*x)/(8*b^7) - (3*a*(10*a^4 - 11*a^2*b^2 + 2*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*
d) + (a*(30*a^2 - 13*b^2)*Cos[c + d*x])/(2*b^6*d) - (3*(20*a^2 - 7*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(8*b^5*d) + ((10*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c +
d*x]^2)/(2*a*b^4*d) - ((15*a^2 - 4*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(4*a^2
*b^3*d) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a*b^2*d*(a + b*Sin[c
 + d*x])^2) + ((7*a^2 - 2*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(2*a^2*b^2*d*(a
 + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
```

```
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(7a^2-2b^2)\cos(c+dx)\sin^4(c+d}{2a^2b^2d(a+b\sin(c+dx))}$$

$$= -\frac{(15a^2-4b^2)\cos(c+dx)\sin^3(c+dx)}{4a^2b^3d} - \frac{(a^2-b^2)\cos(c+dx)\sin^4(c+}{2ab^2d(a+b\sin(c+dx))^2}$$

$$= \frac{(10a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{2ab^4d} - \frac{(15a^2-4b^2)\cos(c+dx)\sin^3(c+}{4a^2b^3d}$$

$$= -\frac{3(20a^2-7b^2)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{(10a^2-3b^2)\cos(c+dx)\sin^2(c}{2ab^4d}$$

$$= \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\cos(c+dx)\sin(c+dx)}{8b^5d} + \frac{(1}{}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)\,x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\,c}{}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)\,x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\,c}{}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)\,x}{8b^7} + \frac{a(30a^2-13b^2)\cos(c+dx)}{2b^6d} - \frac{3(20a^2-7b^2)\,c}{}$$

$$= \frac{3(40a^4-24a^2b^2+b^4)\,x}{8b^7} - \frac{3a(10a^4-11a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^7\sqrt{a^2-b^2}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1250 vs. $2(331) = 662$.
time = 7.04, size = 1250, normalized size = 3.78

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/256*((-6*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b +
a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b
^2)*Cos[c + d*x]])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 -
7*a^2*b^2 + 2*b^4)*Cos[c + d*x]])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))
))/b^3 + (6*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x])
)/(a + b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (2*(-24*(-8*a^2 + b^2)*(
c + d*x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a
*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x
] + (a*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x]])/((a - b)*(a + b)*(a +
 b*Sin[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos
[c + d*x]])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*
x)]))/b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 1
05*b^8)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
 + (-3840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x)
- 1776*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840
*a^9*b*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x]
 + 788*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*
x)*Cos[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^
6*(c + d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48
*b^10*(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b
^5*Cos[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d
*x)] - 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*C
os[5*(c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*
x)*Sin[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c +
d*x)*Sin[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(
c + d*x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] +
 1221*a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[
4*(c + d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] +
8*b^10*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c +
 d*x)] + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b
^7)/d
```

Maple [A]
time = 0.48, size = 450, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^7*((((3*a^2*b^2-5/8*b^4)*tan(1/2*d*x+1/2*c)^7+(10*a^3*b-6*a*b^3)*ta
n(1/2*d*x+1/2*c)^6+(3*a^2*b^2+3/8*b^4)*tan(1/2*d*x+1/2*c)^5+(30*a^3*b-12*a*
b^3)*tan(1/2*d*x+1/2*c)^4+(-3*a^2*b^2-3/8*b^4)*tan(1/2*d*x+1/2*c)^3+(30*a^3
*b-10*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-3*a^2*b^2+5/8*b^4)*tan(1/2*d*x+1/2*c)+1
0*a^3*b-4*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+3/8*(40*a^4-24*a^2*b^2+b^4)*arc
tan(tan(1/2*d*x+1/2*c)))-2*a/b^7*((((-9/2*a^3*b^2+2*a*b^4)*tan(1/2*d*x+1/2*c
)^3-5/2*b*(2*a^4+3*a^2*b^2-2*b^4)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*(31*a^2-16
```

```
*b^2)*tan(1/2*d*x+1/2*c)-5/2*a^2*b*(2*a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b
*tan(1/2*d*x+1/2*c)+a)^2+3/2*(10*a^4-11*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.48, size = 1110, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/8*(4*(a^3*b^5 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25
*a^2*b^6 - b^8)*d*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*
cos(d*x + c)^3 + 3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*
x - 6*(10*a^7 - a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 +
2*a*b^6)*cos(d*x + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c
))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 +
 b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^7*b
 - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(d*
x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b - 6
4*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2*b
^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2 -
2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d), -1/8*(4*(a^3*b^5
 - a*b^7)*cos(d*x + c)^5 - 3*(40*a^6*b^2 - 64*a^4*b^4 + 25*a^2*b^6 - b^8)*d
*x*cos(d*x + c)^2 - 2*(20*a^5*b^3 - 27*a^3*b^5 + 7*a*b^7)*cos(d*x + c)^3 +
3*(40*a^8 - 24*a^6*b^2 - 39*a^4*b^4 + 24*a^2*b^6 - b^8)*d*x + 12*(10*a^7 -
a^5*b^2 - 9*a^3*b^4 + 2*a*b^6 - (10*a^5*b^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x
 + c)^2 + 2*(10*a^6*b - 11*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^
2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(20*a^7
```

```
*b - 22*a^5*b^3 - a^3*b^5 + 3*a*b^7)*cos(d*x + c) - (2*(a^2*b^6 - b^8)*cos(
d*x + c)^5 - (10*a^4*b^4 - 11*a^2*b^6 + b^8)*cos(d*x + c)^3 - 6*(40*a^7*b -
 64*a^5*b^3 + 25*a^3*b^5 - a*b^7)*d*x - 3*(60*a^6*b^2 - 91*a^4*b^4 + 32*a^2
*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))/((a^2*b^9 - b^11)*d*cos(d*x + c)^2
- 2*(a^3*b^8 - a*b^10)*d*sin(d*x + c) - (a^4*b^7 - b^11)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 540, normalized size = 1.63

$$\frac{\frac{3\,(40\,a^4-24\,a^2b^2+b^4)(dx+c)}{b^7}-\frac{24\,(10\,a^5-11\,a^3b^2+2\,ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{8\left(9\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+10\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+31\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\,a^5-5\,a^3b^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2b^6}+\frac{2\left(24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-5\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+80\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-48\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-96\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-80\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+80\,a^3-32\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4b^6}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(40*a^4 - 24*a^2*b^2 + b^4)*(d*x + c)/b^7 - 24*(10*a^5 - 11*a^3*b^2
+ 2*a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
 + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^4*b*tan(1/2
*d*x + 1/2*c)^3 - 4*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/2*d*x + 1
/2*c)^2 + 15*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 10*a*b^4*tan(1/2*d*x + 1/2*c)
^2 + 31*a^4*b*tan(1/2*d*x + 1/2*c) - 16*a^2*b^3*tan(1/2*d*x + 1/2*c) + 10*a
^5 - 5*a^3*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^
2*b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 5*b^3*tan(1/2*d*x + 1/2*c)^7
+ 80*a^3*tan(1/2*d*x + 1/2*c)^6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*
b*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d
*x + 1/2*c)^4 - 96*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/
2*c)^3 - 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 80
*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 5*b^3*tan(1
/2*d*x + 1/2*c) + 80*a^3 - 32*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/
d
```

Mupad [B]
time = 16.60, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)

[Out] ((30*a^5 - 13*a^3*b^2)/b^6 + (15*tan(c/2 + (d*x)/2)^8*(10*a^5 - 6*a*b^4 + 9
*a^3*b^2))/b^6 + (3*tan(c/2 + (d*x)/2)^10*(10*a^5 - 5*a*b^4 + 9*a^3*b^2))/b
^6 + (tan(c/2 + (d*x)/2)^2*(150*a^5 - 37*a*b^4 + 15*a^3*b^2))/b^6 + (2*tan(
c/2 + (d*x)/2)^4*(150*a^5 - 59*a*b^4 + 75*a^3*b^2))/b^6 + (tan(c/2 + (d*x)/
2)*(420*a^4 - 187*a^2*b^2))/(4*b^5) + (3*tan(c/2 + (d*x)/2)^11*(20*a^4 - 7*
a^2*b^2))/(4*b^5) + (3*tan(c/2 + (d*x)/2)^7*(340*a^4 + 2*b^4 - 139*a^2*b^2)
)/(2*b^5) - (tan(c/2 + (d*x)/2)^9*(20*b^4 - 660*a^4 + 231*a^2*b^2))/(4*b^5)
 - (tan(c/2 + (d*x)/2)^5*(6*b^4 - 1380*a^4 + 623*a^2*b^2))/(2*b^5) + (tan(c
/2 + (d*x)/2)^3*(1740*a^4 + 20*b^4 - 809*a^2*b^2))/(4*b^5) - (2*tan(c/2 + (
d*x)/2)^6*(13*a*b^2 - 30*a^3)*(5*a^2 + 6*b^2))/b^6)/(d*(tan(c/2 + (d*x)/2)^
2*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^10*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)
/2)^4*(15*a^2 + 16*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 + 16*b^2) + tan(c/2
+ (d*x)/2)^6*(20*a^2 + 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 20*a*b*t
an(c/2 + (d*x)/2)^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 + 40*a*b*tan(c/2 + (d*x)/
2)^7 + 20*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*ta
n(c/2 + (d*x)/2))) + (atan((((a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14
)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (
tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*
b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*(a^4*40i + b^4*1i - a
^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 - (3*(32*a^2*b^
3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b
^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^
4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))*3i)/(8*b^7) + ((a^4*40i + b^4*
1i - a^2*b^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^
8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^1
4 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b
^18) + (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240
*a^5*b^14)/b^17 + (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^
3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (
d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))
*3i)/(8*b^7))/((108000*a^13 - 189*a^3*b^10 + (12231*a^5*b^8)/2 - 49383*a^7*
b^6 + 159840*a^9*b^4 - 221400*a^11*b^2)/b^17 - (3*(a^4*40i + b^4*1i - a^2*b
^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 72
00*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*
a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*
(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)
/b^17 - (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(
2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(3
84*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7)
+ (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 295
2*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a
*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 2
8800*a^11*b^6))/(2*b^18) + (3*(a^4*40i + b^4*1i - a^2*b^2*24i)*((12*a*b^18
```

- 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (3*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7) + (tan(c/2 + (d*x)/2)*(432000*a^14 + 54*a^2*b^12 - 2889*a^4*b^10 + 49950*a^6*b^8 - 311472*a^8*b^6 + 833760*a^10*b^4 - 993600*a^12*b^2))/b^18))*(a^4*40i + b^4*1i - a^2*b^2*24i)*3i)/(4*b^7*d) + (a*atan(((a*(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (3*a*(-(a + b)*(a - b))^(1/2)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) - (3*a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*3i)/(2*(b^9 - a^2*b^7)) + (a*(-(a + b)*(a - b))^(1/2)*(10*a^4 + 2*b^4 - 11*a^2*b^2)*(((9*a^2*b^14)/2 - 216*a^4*b^12 + 2952*a^6*b^10 - 8640*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(18*a*b^16 - 1449*a^3*b^14 + 18576*a^5*b^12 - 63648*a^7*b^10 + 77760*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (3*a*(-(a + b)*(a - b))^(1/2)*((12*a*b^18 - 204*a^3*b^16 + 240*a^5*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(384*a^2*b^18 - 2112*a^4*b^16 + 1920*a^6*b^14))/(2*b^18) + (3*a*(-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*(10*a^4 + 2*b^4 - 11*a^2*b^2))/(2*(b^9 - a^2*b^7)))*3i)/(2*(b^9 - ...

3.1136 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=284

$$\frac{a\left(9-\frac{20a^2}{b^2}\right)x}{2b^4}+\frac{(20a^4-19a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d}-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(5a^2-b^2)\cos(}{}$$

```
[Out] 1/2*a*(9-20*a^2/b^2)*x/b^4-1/6*(60*a^2-17*b^2)*cos(d*x+c)/b^5/d+(5*a^2-b^2)
*cos(d*x+c)*sin(d*x+c)/a/b^4/d-1/6*(20*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)^2/a
^2/b^3/d-1/2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^2+1
/2*(6*a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^2/d/(a+b*sin(d*x+c))+(20*a^4-1
9*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d/(a^
2-b^2)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2970, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{(6a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d}+\frac{(5a^2-b^2)\sin(c+dx)\cos(c+dx)}{ab^4d}+\frac{ax\left(9-\frac{20a^2}{b^2}\right)}{2b^4}-\frac{(20a^2-3b^2)\sin^2(c+dx)\cos(c+dx)}{6a^2b^3d}+\frac{(20a^4-19a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(9 - (20*a^2)/b^2)*x)/(2*b^4) + ((20*a^4 - 19*a^2*b^2 + 2*b^4)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*Sqrt[a^2 - b^2]*d) - ((60*a^2
 - 17*b^2)*Cos[c + d*x])/(6*b^5*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*
x])/(a*b^4*d) - ((20*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(6*a^2*b^3*d
) - ((a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a*b^2*d*(a + b*Sin[c + d*x
])^2) + ((6*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(2*a^2*b^2*d*(a + b*Sin
[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
```

```
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(6a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}$$

$$= -\frac{(20a^2-3b^2)\cos(c+dx)\sin^2(c+dx)}{6a^2b^3d} - \frac{(a^2-b^2)\cos(c+dx)\sin^3(c+}{2ab^2d(a+b\sin(c+dx))^2}$$

$$= \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{(20a^2-3b^2)\cos(c+dx)\sin^2(c+dx}{6a^2b^3d}$$

$$= -\frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\sin(c+dx)}{ab^4d} - \frac{(20a^2}{}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\,\mathrm{s}}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\,\mathrm{s}}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} - \frac{(60a^2-17b^2)\cos(c+dx)}{6b^5d} + \frac{(5a^2-b^2)\cos(c+dx)\,\mathrm{s}}{ab^4d}$$

$$= -\frac{a(20a^2-9b^2)x}{2b^6} + \frac{(20a^4-19a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}\,d} - \frac{(6}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1030 vs. $2(284) = 568$.
time = 4.28, size = 1030, normalized size = 3.63

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-12*(-48*a*(c + d*x) + (6*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 16*b*Cos[
c + d*x] + (b*(8*a^4 - 8*a^2*b^2 + b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a +
 b*Sin[c + d*x])^2) + (a*b*(-40*a^4 + 72*a^2*b^2 - 29*b^4)*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))))/b^4 + 12*((2*(2*a^2 + b^2)*ArcTa
```

```
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*Cos[c +
 d*x]*(4*a^2 - b^2 + 3*a*b*Sin[c + d*x]))/((a - b)^2*(a + b)^2*(a + b*Sin[c
 + d*x])^2)) + (6*((-6*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (Cos[c + d*x]*(-(b*(2*a^2 + b^2)) + a*(2*a^2 - 5*b^2)*S
in[c + d*x]))/(a + b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) - ((-12*(640*a
^8 - 1792*a^6*b^2 + 1680*a^4*b^4 - 560*a^2*b^6 + 35*b^8)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (3840*a^9*(c + d*x) - 6
912*a^7*b^2*(c + d*x) + 1728*a^5*b^4*(c + d*x) + 1920*a^3*b^6*(c + d*x) - 5
76*a*b^8*(c + d*x) + 3840*a^8*b*Cos[c + d*x] - 7872*a^6*b^3*Cos[c + d*x] +
4256*a^4*b^5*Cos[c + d*x] - 172*a^2*b^7*Cos[c + d*x] - 70*b^9*Cos[c + d*x]
- 1920*a^7*b^2*(c + d*x)*Cos[2*(c + d*x)] + 4416*a^5*b^4*(c + d*x)*Cos[2*(c
 + d*x)] - 3072*a^3*b^6*(c + d*x)*Cos[2*(c + d*x)] + 576*a*b^8*(c + d*x)*Co
s[2*(c + d*x)] - 320*a^6*b^3*Cos[3*(c + d*x)] + 696*a^4*b^5*Cos[3*(c + d*x)
] - 432*a^2*b^7*Cos[3*(c + d*x)] + 56*b^9*Cos[3*(c + d*x)] + 8*a^4*b^5*Cos[
5*(c + d*x)] - 16*a^2*b^7*Cos[5*(c + d*x)] + 8*b^9*Cos[5*(c + d*x)] + 7680*
a^8*b*(c + d*x)*Sin[c + d*x] - 17664*a^6*b^3*(c + d*x)*Sin[c + d*x] + 12288
*a^4*b^5*(c + d*x)*Sin[c + d*x] - 2304*a^2*b^7*(c + d*x)*Sin[c + d*x] + 288
0*a^7*b^2*Sin[2*(c + d*x)] - 6304*a^5*b^4*Sin[2*(c + d*x)] + 4022*a^3*b^6*S
in[2*(c + d*x)] - 607*a*b^8*Sin[2*(c + d*x)] + 40*a^5*b^4*Sin[4*(c + d*x)]
- 80*a^3*b^6*Sin[4*(c + d*x)] + 40*a*b^8*Sin[4*(c + d*x)])/((a^2 - b^2)^2*(
a + b*Sin[c + d*x])^2))/b^6)/(384*d)
```

Maple [A]

time = 0.75, size = 344, normalized size = 1.21

method	result
derivativedivides	$-\frac{4\left(\frac{\frac{3a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\left(3a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(6a^2b-b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+3a^2b-\frac{2b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(20a^2-9b^2 \right.}{}\right.}{b^6}$
default	$-\frac{4\left(\frac{\frac{3a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\left(3a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(6a^2b-b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{3a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+3a^2b-\frac{2b^3}{3}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a\left(20a^2-9b^2 \right.}{}\right.}{b^6}$
risch	$-\frac{10a^3x}{b^6}+\frac{9ax}{2b^4}+\frac{e^{3i(dx+c)}}{24b^3d}+\frac{3ia\,e^{-2i(dx+c)}}{8b^4d}-\frac{3\,e^{i(dx+c)}a^2}{b^5d}+\frac{5\,e^{i(dx+c)}}{8b^3d}-\frac{3\,e^{-i(dx+c)}a^2}{b^5d}+\frac{5\,e^{-i(dx+c)}}{8b^3d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/b^6*((3/4*a*b^2*tan(1/2*d*x+1/2*c)^5+(3*a^2*b-b^3)*tan(1/2*d*x+1/2*
c)^4+(6*a^2*b-b^3)*tan(1/2*d*x+1/2*c)^2-3/4*a*b^2*tan(1/2*d*x+1/2*c)+3*a^2*
b-2/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/4*a*(20*a^2-9*b^2)*arctan(tan(1/2*d
*x+1/2*c)))+4/b^6*((-1/4*a*b^2*(7*a^2-2*b^2)*tan(1/2*d*x+1/2*c)^3-1/4*b*(8*
```

```
a^4+13*a^2*b^2-6*b^4)*tan(1/2*d*x+1/2*c)^2-5/4*b^2*a*(5*a^2-2*b^2)*tan(1/2*
d*x+1/2*c)-2*a^4*b+3/4*a^2*b^3)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2
*c)+a)^2+1/4*(20*a^4-19*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(
1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 976, normalized size = 3.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(4*(a^2*b^5 - b^7)*cos(d*x + c)^5 - 6*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*
b^6)*d*x*cos(d*x + c)^2 - 8*(5*a^4*b^3 - 6*a^2*b^5 + b^7)*cos(d*x + c)^3 +
6*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 1
7*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2
0*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) + 6*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7
)*cos(d*x + c) + 2*(5*(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a
^4*b^3 + 9*a^2*b^5)*d*x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x +
c))*sin(d*x + c))/((a^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*
d*sin(d*x + c) - (a^4*b^6 - b^10)*d), 1/6*(2*(a^2*b^5 - b^7)*cos(d*x + c)^5
 - 3*(20*a^5*b^2 - 29*a^3*b^4 + 9*a*b^6)*d*x*cos(d*x + c)^2 - 4*(5*a^4*b^3
- 6*a^2*b^5 + b^7)*cos(d*x + c)^3 + 3*(20*a^7 - 9*a^5*b^2 - 20*a^3*b^4 + 9*
a*b^6)*d*x + 3*(20*a^6 + a^4*b^2 - 17*a^2*b^4 + 2*b^6 - (20*a^4*b^2 - 19*a^
2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(20*a^5*b - 19*a^3*b^3 + 2*a*b^5)*sin(d*x
 + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))) + 3*(20*a^6*b - 19*a^4*b^3 - 3*a^2*b^5 + 2*b^7)*cos(d*x + c) + (5*
```

```
(a^3*b^4 - a*b^6)*cos(d*x + c)^3 + 6*(20*a^6*b - 29*a^4*b^3 + 9*a^2*b^5)*d*
x + 3*(30*a^5*b^2 - 41*a^3*b^4 + 11*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^3*b^7 - a*b^9)*d*sin(d*x + c) - (a^4*
b^6 - b^10)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 393, normalized size = 1.38

$$-\frac{\frac{3\,(20\,a^3-9\,ab^2)(dx+c)}{b^6}-\frac{6\,(20\,a^4-19\,a^2b^2+2\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{6\left(7\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+13\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+25\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,a^4-3\,a^2b^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2b^5}+\frac{2\left(9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+72\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2-8\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^5}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*(20*a^3 - 9*a*b^2)*(d*x + c)/b^6 - 6*(20*a^4 - 19*a^2*b^2 + 2*b^4)*
(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
 b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 6*(7*a^3*b*tan(1/2*d*x + 1/2*
c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*tan(1/2*d*x + 1/2*c)^2 + 13*a
^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 25*a^3*b*tan
(1/2*d*x + 1/2*c) - 10*a*b^3*tan(1/2*d*x + 1/2*c) + 8*a^4 - 3*a^2*b^2)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^5) + 2*(9*a*b*ta
n(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 - 12*b^2*tan(1/2*d*x +
 1/2*c)^4 + 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 -
 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 - 8*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)
^3*b^5))/d
```

Mupad [B]
time = 14.34, size = 2034, normalized size = 7.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((60*a^4 - 17*a^2*b^2)/(3*b^5) - (20*tan(c/2 + (d*x)/2)^7*(a*b^2 - 5*a^3)
)/b^4 - (2*tan(c/2 + (d*x)/2)^9*(a*b^2 - 5*a^3))/b^4 - (4*tan(c/2 + (d*x)/2
)^5*(17*a*b^2 - 60*a^3))/b^4 - (4*tan(c/2 + (d*x)/2)^3*(53*a*b^2 - 165*a^3)
```

)/(3*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^4 - 6*b^4 + 21*a^2*b^2))/b^5 + (2*t
an(c/2 + (d*x)/2)^6*(40*a^4 - 17*b^4 + 42*a^2*b^2))/b^5 + (2*tan(c/2 + (d*x
)/2)^2*(120*a^4 - 25*b^4 + 46*a^2*b^2))/(3*b^5) + (2*tan(c/2 + (d*x)/2)^4*(
180*a^4 - 51*b^4 + 149*a^2*b^2))/(3*b^5) - (2*tan(c/2 + (d*x)/2)*(31*a*b^2
- 105*a^3))/(3*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2) + tan(c/2 + (
d*x)/2)^8*(5*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 + 12*b^2) + tan(c/
2 + (d*x)/2)^6*(10*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 + a^2 + 16*a*b
*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 + 16*a*b*tan(c/2 + (d*x
)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (a*atan(
(280*a^4*tan(c/2 + (d*x)/2))/(280*a^4 - 288*a^2*b^2 + (800*a^6)/b^2) - (288
*a^2*tan(c/2 + (d*x)/2))/((280*a^4)/b^2 - 288*a^2 + (800*a^6)/b^4) + (800*a
^6*tan(c/2 + (d*x)/2))/(800*a^6 - 288*a^2*b^4 + 280*a^4*b^2))*(20*a^2 - 9*b
^2))/(b^6*d) - (atan((((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2
)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 400*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x
)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))
/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*tan(c
/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a^5*b^12))/b^15 - (8*(14*a^2*b^1
4 - 20*a^4*b^12))/b^14 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2
 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2)
)/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6))*1i)/(b^8 - a^2*b^6) + ((-(a + b)*(a -
b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 4
00*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a
^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*
a^4 + b^4 - (19*a^2*b^2)/2)*((8*(14*a^2*b^14 - 20*a^4*b^12))/b^14 - (8*tan(
c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a^5*b^12))/b^15 + ((-(a + b)*(a
 - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/
b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6))*1
i)/(b^8 - a^2*b^6))/((16*(2000*a^10 + 18*a^2*b^8 - 301*a^4*b^6 + 1615*a^6*b
^4 - 3200*a^8*b^2))/b^14 + (16*tan(c/2 + (d*x)/2)*(8000*a^11 + 162*a^3*b^8
- 2259*a^5*b^6 + 9260*a^7*b^4 - 14800*a^9*b^2))/b^15 + ((-(a + b)*(a - b))^
(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(81*a^4*b^9 - 360*a^6*b^7 + 400*a
^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*a*b^13 - 238*a^3*b^11 + 1242*a^5*b
^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 + ((-(a + b)*(a - b))^(1/2)*(10*a^4
+ b^4 - (19*a^2*b^2)/2)*((8*tan(c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80
*a^5*b^12))/b^15 - (8*(14*a^2*b^14 - 20*a^4*b^12))/b^14 + ((-(a + b)*(a - b
))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 - 8*a^3*b^17))/b^15
)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6)))/(b^8
 - a^2*b^6) - ((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8
*(81*a^4*b^9 - 360*a^6*b^7 + 400*a^8*b^5))/b^14 - (8*tan(c/2 + (d*x)/2)*(4*
a*b^13 - 238*a^3*b^11 + 1242*a^5*b^9 - 1920*a^7*b^7 + 800*a^9*b^5))/b^15 +
((-(a + b)*(a - b))^(1/2)*(10*a^4 + b^4 - (19*a^2*b^2)/2)*((8*(14*a^2*b^14
- 20*a^4*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(8*a*b^16 - 76*a^3*b^14 + 80*a
^5*b^12))/b^15 + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^19 - 8*a^3*b^17))/b^15)*(10*a^4 + b^4 - (19*a^2*b^2)/2))/(b^8 -
 a^2*b^6)))/(b^8 - a^2*b^6)))/(b^8 - a^2*b^6))*(-(a + b)*(a - b))^(1/2)*(1

```
0*a^4 + b^4 - (19*a^2*b^2)/2)*2i)/(d*(b^8 - a^2*b^6))
```

3.1137 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=173

$$\frac{3(4a^2-b^2)\,x}{2b^5}-\frac{3a(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d}+\frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2}+\frac{3\cos(c+dx)\,(4a}{2b^4d(a-}$$

```
[Out] 3/2*(4*a^2-b^2)*x/b^5+1/2*cos(d*x+c)^3*(2*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*
x+c))^2+3/2*cos(d*x+c)*(4*a^2-b^2+2*a*b*sin(d*x+c))/b^4/d/(a+b*sin(d*x+c))-
3*a*(4*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d/(a
^2-b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2942, 2814, 2739, 632, 210}

$$-\frac{3a(4a^2-3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d\sqrt{a^2-b^2}}+\frac{3x(4a^2-b^2)}{2b^5}+\frac{3\cos(c+dx)\,(4a^2+2ab\sin(c+dx)-b^2)}{2b^4d(a+b\sin(c+dx))}+\frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*(4*a^2 - b^2)*x)/(2*b^5) - (3*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(b^5*Sqrt[a^2 - b^2]*d) + (Cos[c + d*x]^3*(2*a +
b*Sin[c + d*x]))/(2*b^2*d*(a + b*Sin[c + d*x])^2) + (3*Cos[c + d*x]*(4*a^2
- b^2 + 2*a*b*Sin[c + d*x]))/(2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} - \frac{3\int \frac{\cos^2(c+dx)(-2b-4a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{4b^2}$$

$$= \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2+2ab\sin(c+dx\right.}{2b^4d(a+b\sin(c+dx))}$$

$$= \frac{3(4a^2-b^2)\,x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2\right.}{2b^4d(a+b\sin}$$

$$= \frac{3(4a^2-b^2)\,x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2\right.}{2b^4d(a+b\sin}$$

$$= \frac{3(4a^2-b^2)\,x}{2b^5} + \frac{\cos^3(c+dx)(2a+b\sin(c+dx))}{2b^2d(a+b\sin(c+dx))^2} + \frac{3\cos(c+dx)\left(4a^2-b^2\right.}{2b^4d(a+b\sin}$$

$$= \frac{3(4a^2-b^2)\,x}{2b^5} - \frac{3a(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}\,d} + \frac{\cos^3(c+dx)(2a+}{2b^2d(a+b\sin}$$

Mathematica [A]
time = 2.47, size = 274, normalized size = 1.58

$$\frac{-\frac{48a(4a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{96a^4c+24a^2b^2c-12b^4c+96a^4dx+24a^2b^2dx-12b^4dx+96a^3b\cos(c+dx)+12b^2(-4a^2+b^2)(c+dx)\cos(2(c+dx))-8ab^3\cos(3(c+dx))+192a^3bc\sin(c+dx)-48ab^3c\sin(c+dx)+192a^3bdx\sin(c+dx)-48ab^3dx\sin(c+dx)+72a^2b^2\sin(2(c+dx))-10b^4\sin(2(c+dx))+b^4\sin(4(c+dx))}{(a+b\sin(c+dx))^2}}{16b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-48*a*(4*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + (96*a^4*c + 24*a^2*b^2*c - 12*b^4*c + 96*a^4*d*x + 24*a^2*
b^2*d*x - 12*b^4*d*x + 96*a^3*b*Cos[c + d*x] + 12*b^2*(-4*a^2 + b^2)*(c + d
*x)*Cos[2*(c + d*x)] - 8*a*b^3*Cos[3*(c + d*x)] + 192*a^3*b*c*Sin[c + d*x]
- 48*a*b^3*c*Sin[c + d*x] + 192*a^3*b*d*x*Sin[c + d*x] - 48*a*b^3*d*x*Sin[c
 + d*x] + 72*a^2*b^2*Sin[2*(c + d*x)] - 10*b^4*Sin[2*(c + d*x)] + b^4*Sin[4
*(c + d*x)])/(a + b*Sin[c + d*x])^2)/(16*b^5*d)
```

Maple [A]
time = 0.66, size = 288, normalized size = 1.66

method	result
derivativedivides	$\frac{\frac{4\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3ab}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\left(4a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^5}-\frac{4\left(-\frac{5a^2b^2\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{4}\right.}{}$
default	$\frac{\frac{4\left(\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}+\frac{3ab}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+3\left(4a^2-b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^5}-\frac{4\left(-\frac{5a^2b^2\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{4}\right.}{}$
risch	$\frac{6xa^2}{b^5}-\frac{3x}{2b^3}+\frac{ie^{2i(dx+c)}}{8b^3d}+\frac{3a\,e^{i(dx+c)}}{2b^4d}+\frac{3a\,e^{-i(dx+c)}}{2b^4d}-\frac{ie^{-2i(dx+c)}}{8b^3d}-\frac{i\left(-8ia^3b\,e^{3i(dx+c)}+3ia\,b^3e^{3i(dx+c)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^5*((1/4*b^2*tan(1/2*d*x+1/2*c)^3+3/2*a*b*tan(1/2*d*x+1/2*c)^2-1/4*
b^2*tan(1/2*d*x+1/2*c)+3/2*a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+3/4*(4*a^2-b^2)*
arctan(tan(1/2*d*x+1/2*c)))-4/b^5*((-5/4*a^2*b^2*tan(1/2*d*x+1/2*c)^3-1/4*b
*(6*a^4+11*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^2-1/4*b^2*(19*a^2-4*b^2)*tan
(1/2*d*x+1/2*c)-1/4*a*b*(6*a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*
x+1/2*c)+a)^2+3/4*a*(4*a^2-3*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(162) = 324$.
time = 0.42, size = 837, normalized size = 4.84

$$\left[\frac{6\left(4a^4b^2-5a^2b^4+b^6\right)dx\cos\left(dx+c\right)^2+8\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3-6\left(4a^6-a^4b^2-4a^2b^4+b^6\right)dx-3\left(4a^5+a^3b^2-3ab^4-\left(4a^3b^2-3ab^4\right)\cos\left(dx+c\right)^2+2\left(4a^4b-3a^2b^3\right)\sin\left(dx+c\right)\right)\sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2}\right)-6\left(4a^5b-3a^3b^3-ab^5\right)\cos\left(dx+c\right)-2\left(\left(a^2b^4-b^6\right)\cos\left(dx+c\right)^3+6\left(4a^5b-5a^3b^3+ab^5\right)dx+3\left(6a^4b^2-7a^2b^4+b^6\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\left(\left(a^2b^7-b^9\right)d\cos\left(dx+c\right)^2-2\left(a^3b^6-ab^8\right)d\sin\left(dx+c\right)-\left(a^4b^5-b^9\right)d\right)}, \frac{3\left(4a^4b^2-5a^2b^4+b^6\right)dx\cos\left(dx+c\right)^2+4\left(a^3b^3-ab^5\right)\cos\left(dx+c\right)^3-3\left(4a^6-a^4b^2-4a^2b^4+b^6\right)dx-3\left(4a^5+a^3b^2-3ab^4-\left(4a^3b^2-3ab^4\right)\cos\left(dx+c\right)^2+2\left(4a^4b-3a^2b^3\right)\sin\left(dx+c\right)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)-3\left(4a^5b-3a^3b^3-ab^5\right)\cos\left(dx+c\right)-\left(\left(a^2b^4-b^6\right)\cos\left(dx+c\right)^3+6\left(4a^5b-5a^3b^3+ab^5\right)dx+3\left(6a^4b^2-7a^2b^4+b^6\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2\left(\left(a^2b^7-b^9\right)d\cos\left(dx+c\right)^2-2\left(a^3b^6-ab^8\right)d\sin\left(dx+c\right)-\left(a^4b^5-b^9\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(4*a^4*b^2 - 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 8*(a^3*b^3 - a*b
^5)*cos(d*x + c)^3 - 6*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 +
 a^3*b^2 - 3*a*b^4 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 6*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - 2*((a^2*b^4 - b^6)*c
os(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*
b^4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 -
2*(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d), 1/2*(3*(4*a^4*b^2
- 5*a^2*b^4 + b^6)*d*x*cos(d*x + c)^2 + 4*(a^3*b^3 - a*b^5)*cos(d*x + c)^3
- 3*(4*a^6 - a^4*b^2 - 4*a^2*b^4 + b^6)*d*x - 3*(4*a^5 + a^3*b^2 - 3*a*b^4
- (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + 2*(4*a^4*b - 3*a^2*b^3)*sin(d*x +
c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
 c))) - 3*(4*a^5*b - 3*a^3*b^3 - a*b^5)*cos(d*x + c) - ((a^2*b^4 - b^6)*cos
(d*x + c)^3 + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)*d*x + 3*(6*a^4*b^2 - 7*a^2*b^
4 + b^6)*cos(d*x + c))*sin(d*x + c))/((a^2*b^7 - b^9)*d*cos(d*x + c)^2 - 2*
(a^3*b^6 - a*b^8)*d*sin(d*x + c) - (a^4*b^5 - b^9)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(162) = 324.
time = 0.50, size = 429, normalized size = 2.48

$$\frac{\frac{3\,(4\,a^2-b^2)(dx+c)}{b^5}-\frac{6\,(4\,a^3-3\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+15\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+54\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+45\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+90\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+29\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+42\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^4-a^2b^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2ab^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2 - b^2)*(d*x + c)/b^5 - 6*(4*a^3 - 3*a*b^2)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2
)))/(sqrt(a^2 - b^2)*b^5) + 2*(6*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 12*a^4*tan(
1/2*d*x + 1/2*c)^6 + 15*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 2*b^4*tan(1/2*d*x
+ 1/2*c)^6 + 54*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^4*tan(1/2*d*x + 1/2*c)^
4 + 45*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 4*b^4*tan(1/2*d*x + 1/2*c)^4 + 90*a
^3*b*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*tan(
1/2*d*x + 1/2*c)^2 + 29*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 42*a^3*b*tan(1/2*d*x + 1/2*c) - 4*a*b^3*tan(1/2*d*x + 1/2*c) +
 12*a^4 - a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3
+ 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*b^4))/d
```

Mupad [B]
time = 12.41, size = 1743, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((54*a^2*tan(c/2 + (d*x)/2)^5)/b^3 - (a*b^2 - 12*a^3)/b^4 + (6*a^2*tan(c/2
+ (d*x)/2)^7)/b^3 + (2*tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/b^3 + (6*tan(c/
2 + (d*x)/2)^3*(15*a^2 - 2*b^2))/b^3 + (tan(c/2 + (d*x)/2)^6*(12*a^4 - 2*b^
4 + 15*a^2*b^2))/(a*b^4) + (tan(c/2 + (d*x)/2)^2*(36*a^4 - 2*b^4 + 29*a^2*b
^2))/(a*b^4) + (tan(c/2 + (d*x)/2)^4*(3*a^2 + 4*b^2)*(12*a^2 - b^2))/(a*b^4
))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(4*a^2 +
 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*b^2) + a^2*tan(c/2 + (d*x)/2)^8 +
 a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*ta
n(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) + (atan((864*a^3*tan(c/2 +
(d*x)/2))/(216*a*b^2 - 864*a^3) - (216*a*tan(c/2 + (d*x)/2))/(216*a - (864*
a^3)/b^2))*(a^2*6i - (b^2*3i)/2)*2i)/(b^5*d) + (a*atan(((a*(-(a + b)*(a - b
))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 +
 (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4
))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*
a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 -
 (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (
```

d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5)))*3i)/(2*(b^7 - a^2*b^5)) + (a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 + (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4))/b^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5)))*3i)/(2*(b^7 - a^2*b^5)))/((16*(432*a^7 + 81*a^3*b^4 - 432*a^5*b^2))/b^11 + (16*tan(c/2 + (d*x)/2)*(1728*a^8 - 81*a^2*b^6 + 756*a^4*b^4 - 2160*a^6*b^2))/b^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 + (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5)) - (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(9*a^2*b^8 - 72*a^4*b^6 + 144*a^6*b^4))/b^11 + (8*tan(c/2 + (d*x)/2)*(18*a*b^10 - 234*a^3*b^8 + 576*a^5*b^6 - 288*a^7*b^4))/b^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*((8*(6*a*b^12 - 12*a^3*b^10))/b^11 + (8*tan(c/2 + (d*x)/2)*(36*a^2*b^12 - 48*a^4*b^10))/b^12 + (3*a*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^16 - 8*a^3*b^14))/b^12))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5))))/(2*(b^7 - a^2*b^5))))*(-(a + b)*(a - b))^(1/2)*(4*a^2 - 3*b^2)*3i)/(d*(b^7 - a^2*b^5))

3.1138 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=175

$$\frac{x}{b^3}-\frac{(2a^4-a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}+\frac{(3a^2+\ldots}{2a^2b^2d(\ldots}$$

```
[Out] x/b^3-arctanh(cos(d*x+c))/a^3/d-1/2*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d
*x+c))^2+1/2*(3*a^2+2*b^2)*cos(d*x+c)/a^2/b^2/d/(a+b*sin(d*x+c))-(2*a^4-a^2
*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^3/d/(a^2
-b^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2970, 3136, 2739, 632, 210, 3855}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(3a^2+2b^2)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}-\frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2}-\frac{(2a^4-a^2b^2+2b^4)\text{ArcTan}\left(\frac{a\tan(\frac{1}{2}(c+dx))+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d\sqrt{a^2-b^2}}+\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] x/b^3 - ((2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^3*b^3*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - ((a
^2 - b^2)*Cos[c + d*x])/(2*a*b^2*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 + 2*b^
2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(3a^2+2b^2)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\csc(c+dx)(-2b^2 ...}{}}{}$$

$$= \frac{x}{b^3} - \frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(3a^2+2b^2)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))} + \frac{\int \csc(c+d ...}{a^3}$$

$$= \frac{x}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(3a^2+2b^2)\cos ...}{2a^2b^2d(a+b\sin ...}$$

$$= \frac{x}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{(a^2-b^2)\cos(c+dx)}{2ab^2d(a+b\sin(c+dx))^2} + \frac{(3a^2+2b^2)\cos ...}{2a^2b^2d(a+b\sin ...}$$

$$= \frac{x}{b^3} - \frac{(2a^4-a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} - ...$$

Mathematica [A]
time = 1.25, size = 176, normalized size = 1.01

$$\frac{-\frac{2(2a^4-a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3\sqrt{a^2-b^2}} + 2\left(\frac{c+dx}{b^3} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}\right) + \frac{\cos(c+dx)\left(2a^3+3ab^2+b(3a^2+2b^2)\sin(c+dx)\right)}{a^2b^2(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^3*b^3*Sqrt[a^2 - b^2]) + 2*((c + d*x)/b^3 - Log[Cos[(c + d*x)/2]]/
a^3 + Log[Sin[(c + d*x)/2]]/a^3) + (Cos[c + d*x]*(2*a^3 + 3*a*b^2 + b*(3*a^
2 + 2*b^2)*Sin[c + d*x]))/(a^2*b^2*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.75, size = 240, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3} - \frac{2\left(\frac{\left(-\frac{1}{2}a^3b^2-2ab^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{b\left(2a^4+7a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2} - \frac{ab^2(7a ...}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2} ...\right)}{}}{d}$

default	$\frac{\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{\left(-\frac{1}{2}a^3b^2-2a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b\left(2a^4+7a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{a\,b^2\left(7a^2 \cdots\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2} \cdots\right.}{a^3 \cdots}}{d}$
risch	$\frac{x}{b^3}-\frac{i\left(-4ia^3b\,e^{3i(dx+c)}-ia\,b^3e^{3i(dx+c)}+8ia^3b\,e^{i(dx+c)}+7ia\,b^3e^{i(dx+c)}+6a^4e^{2i(dx+c)}+7b^2a^2e^{2i(dx+c)}+2b^4e^{2i(dx+c)}- \cdots\right.}{\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)^2a^2d\,b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^3*arctan(tan(1/2*d*x+1/2*c))+1/a^3*ln(tan(1/2*d*x+1/2*c))-2/a^3/b^
3*(((-1/2*a^3*b^2-2*a*b^4)*tan(1/2*d*x+1/2*c)^3-1/2*b*(2*a^4+7*a^2*b^2+6*b^
4)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*(7*a^2+8*b^2)*tan(1/2*d*x+1/2*c)-1/2*a^2*
b*(2*a^2+3*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2
*a^4-a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(166) = 332$.
time = 0.59, size = 1042, normalized size = 5.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^5*b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 4*(a^7 - a^3*b^4)*d*x + (2
*a^6 + a^4*b^2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x +
c)^2 + 2*(2*a^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-
```

```
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^6*b + a^4*b^3 - 3*a^2*b^5)*co
s(d*x + c) + 2*(a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4
 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 2*(a^4*b^3 - b^7 - (a
^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*
a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 -
2*(a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d), 1/2*(2*(a^5*
b^2 - a^3*b^4)*d*x*cos(d*x + c)^2 - 2*(a^7 - a^3*b^4)*d*x - (2*a^6 + a^4*b^
2 + a^2*b^4 + 2*b^6 - (2*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(2*a
^5*b - a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^6*b + a^4*b^3 - 3*a^2*b^5)
*cos(d*x + c) + (a^4*b^3 - b^7 - (a^2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^
4 - a*b^6)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^4*b^3 - b^7 - (a^
2*b^5 - b^7)*cos(d*x + c)^2 + 2*(a^3*b^4 - a*b^6)*sin(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) - (4*(a^6*b - a^4*b^3)*d*x + (3*a^5*b^2 - a^3*b^4 - 2*a*b
^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^2 - 2*(
a^6*b^4 - a^4*b^6)*d*sin(d*x + c) - (a^7*b^3 - a^3*b^7)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.49, size = 275, normalized size = 1.57

$$\frac{\frac{dx+c}{b^3}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{\left(2\,a^4-a^2b^2+2\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^3}+\frac{a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^4+3\,a^2b^2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^3b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)/b^3 + log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (2*a^4 - a^2*b^2 + 2*
b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b^3) + (a^3*b*tan(1/2*d*x +
 1/2*c)^3 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 2*a^4*tan(1/2*d*x + 1/2*c)^2 +
 7*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*b^4*tan(1/2*d*x + 1/2*c)^2 + 7*a^3*b*
tan(1/2*d*x + 1/2*c) + 8*a*b^3*tan(1/2*d*x + 1/2*c) + 2*a^4 + 3*a^2*b^2)/((
a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^3*b^2))/d
```

Mupad [B]
time = 13.50, size = 2500, normalized size = 14.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((2*a^2 + 3*b^2)/(a*b^2) + (tan(c/2 + (d*
x)/2)*(7*a^2 + 8*b^2))/(a^2*b) + (tan(c/2 + (d*x)/2)^3*(a^2 + 4*b^2))/(a^2*
b) + (tan(c/2 + (d*x)/2)^2*(2*a^4 + 6*b^4 + 7*a^2*b^2))/(a^3*b^2))/(d*(tan(
c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*t
an(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) - (2*atan((144*tan(c/2 + (
d*x)/2))/((64*b^2)/a^2 - (64*b^4)/a^4 - (64*a*tan(c/2 + (d*x)/2))/b + (64*b
*tan(c/2 + (d*x)/2))/a + (144*a^3*tan(c/2 + (d*x)/2))/b^3 - 144) + (144*a)/
(144*a*tan(c/2 + (d*x)/2) - (144*b^3)/a^2 + (64*b^5)/a^4 - (64*b^7)/a^6 - (
64*b^2*tan(c/2 + (d*x)/2))/a + (64*b^4*tan(c/2 + (d*x)/2))/a^3) + 64/(64*ta
n(c/2 + (d*x)/2) - (144*a)/b + (64*b)/a - (64*b^3)/a^3 - (64*a^2*tan(c/2 +
(d*x)/2))/b^2 + (144*a^4*tan(c/2 + (d*x)/2))/b^4) + (64*tan(c/2 + (d*x)/2))
/((64*b^2)/a^2 + (144*a^2)/b^2 - (64*a*tan(c/2 + (d*x)/2))/b + (64*a^3*tan(
c/2 + (d*x)/2))/b^3 - (144*a^5*tan(c/2 + (d*x)/2))/b^5 - 64) + 64/(64*tan(c
/2 + (d*x)/2) + (144*b)/a - (64*b^3)/a^3 + (64*b^5)/a^5 - (64*b^2*tan(c/2 +
 (d*x)/2))/a^2 - (144*a^2*tan(c/2 + (d*x)/2))/b^2) - (64*b*tan(c/2 + (d*x)/
2))/(64*b - (64*a^2)/b + (144*a^4)/b^3 - (64*a^3*tan(c/2 + (d*x)/2))/b^2 +
(64*a^5*tan(c/2 + (d*x)/2))/b^4 - (144*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*
d) + (atan((((-(a + b)*(a - b))^(1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4
 - 15*a^7*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*
a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^2))/(a^6*b^5) + ((-(a + b)*(a
 - b))^(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a
^9*b^6))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b
^8))/(a^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*
b^10))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c
/2 + (d*x)/2)*(64*a^4*b^14 - 68*a^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6
*b^8)))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^
10 + 89*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b
^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 +
73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10*b^2))/(a^6*b^8))*(a^4 + b^4
 - (a^2*b^2)/2)*1i)/(a^3*b^5 - a^5*b^3) + ((-(a + b)*(a - b))^(1/2)*((8*(14*
a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^7*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))
^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^
2))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32
*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)
*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7
*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(
a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^14 - 68*a^6*b^12 + 48*
```

a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*
x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(
a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*
x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10
*b^2))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2)*1i)/(a^3*b^5 - a^5*b^3))/((16*(
14*a^6 + 4*b^6 + 12*a^2*b^4 - 3*a^4*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(
1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^7*b^2))/(a^6*b^5) + ((-(a
+ b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8 + 14*a^6*b^6 + 16*a^8*b^4
 - 12*a^10*b^2))/(a^6*b^5) + ((-(a + b)*(a - b))^(1/2)*(a^4 + b^4 - (a^2*b^
2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6))/(a^6*b^5) + ((-(a + b)*(
a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a^6*b^5) + (8*tan(c/2 + (d*
x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/(a^6*b^8))*(a^4 + b^4 - (a
^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^14 - 68*a
^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))/(a^3*b^5 - a^5*b^3) + (8*
tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89*a^7*b^8 - 100*a^9*b^6 + 8
*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*
tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*b^8 - 68*a^6*b^6 + 48*a^8*
b^4 - 16*a^10*b^2))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^
3) - ((-(a + b)*(a - b))^(1/2)*((8*(14*a^9 + 4*a^3*b^6 + 28*a^5*b^4 - 15*a^
7*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*((8*(16*a^2*b^10 - 12*a^4*b^8
 + 14*a^6*b^6 + 16*a^8*b^4 - 12*a^10*b^2))/(a^6*b^5) - ((-(a + b)*(a - b))^
(1/2)*(a^4 + b^4 - (a^2*b^2)/2)*((8*(32*a^5*b^10 - 24*a^7*b^8 + 14*a^9*b^6)
)/(a^6*b^5) - ((-(a + b)*(a - b))^(1/2)*((8*(16*a^8*b^10 - 12*a^10*b^8))/(a
^6*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^14 - 68*a^9*b^12 + 8*a^11*b^10))/
(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d
x)/2)(64*a^4*b^14 - 68*a^6*b^12 + 48*a^8*b^10 - 16*a^10*b^8))/(a^6*b^8)))
/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^12 - 36*a^5*b^10 + 89
*a^7*b^8 - 100*a^9*b^6 + 8*a^11*b^4))/(a^6*b^8))*(a^4 + b^4 - (a^2*b^2)/2))
/(a^3*b^5 - a^5*b^3) + (8*tan(c/2 + (d*x)/2)*(4*b^12 - 4*a^2*b^10 + 73*a^4*
b^8 - 68*a^6*b^6 + 48*a^8*b^4 - 16*a^10*b^2))/(...

3.1139 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$-\frac{3(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)}{ad(a+b\sin(c+$$

```
[Out] 3*b*arctanh(cos(d*x+c))/a^4/d+1/2*(a^2-3*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d
*x+c))^2-cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2-1/2*(a^2+6*b^2)*cos(d*x+c)/a^3/b
/d/(a+b*sin(d*x+c))-3*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^4/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2969, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}-\frac{3(a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\sqrt{a^2-b^2}}-\frac{(a^2+6b^2)\cos(c+dx)}{2a^3bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*Sq
rt[a^2 - b^2]*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Cos
[c + d*x])/(2*a^2*b*d*(a + b*Sin[c + d*x])^2) - Cot[c + d*x]/(a*d*(a + b*Si
n[c + d*x])^2) - ((a^2 + 6*b^2)*Cos[c + d*x])/(2*a^3*b*d*(a + b*Sin[c + d*x
]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc(c+dx)(-6b^2-2ab\sin}{(a+b}}{}$$

$$= \frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{(a^2+6b^2)\cos(c+}{2a^3bd(a+b\sin(c+}$$

$$= \frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^2} - \frac{(a^2+6b^2)\cos(c+}{2a^3bd(a+b\sin(c+}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{(a^2-3b^2)\cos(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx}$$

$$= -\frac{3(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{a^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{(a^2-}{2a^2bd(}$$

Mathematica [A]

time = 1.90, size = 184, normalized size = 1.01

$$\frac{-\frac{6(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 6b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^2(a^2-b^2)\cos(c+dx)}{b(a+b\sin(c+dx))^2} - \frac{a(a^2+4b^2)\cos(c+dx)}{b(a+b\sin(c+dx))} + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]] - 6*b*Log[Sin[(c
 + d*x)/2]] + (a^2*(a^2 - b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])^2) - (
a*(a^2 + 4*b^2)*Cos[c + d*x])/(b*(a + b*Sin[c + d*x])) + a*Tan[(c + d*x)/2]
)/(2*a^4*d)
```

Maple [A]

time = 0.70, size = 214, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\tan(\frac{dx}{2}+\frac{c}{2})}{2a^3} - \frac{1}{2a^3\tan(\frac{dx}{2}+\frac{c}{2})} - \frac{3b\ln(\tan(\frac{dx}{2}+\frac{c}{2}))}{a^4} - \frac{4\left(\frac{-\frac{a(a^2-6b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{4} + \frac{5b(a^2+2b^2)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{4} + \frac{a(a^2+14}{}}{(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))+2b\tan(\frac{dx}{2}+\frac{c}{2})+a)^2}\right.}{a^4}}{d}$

default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}-\frac{4\left(\frac{-\frac{a\left(a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{5b\left(a^2+2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+\frac{a\left(a^2+\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{a^4}}{d}$
risch	$\frac{i\left(-2ia^3b\,e^{5i(dx+c)}-3ia\,b^3e^{5i(dx+c)}+4ia^3b\,e^{3i(dx+c)}+24ia\,b^3e^{3i(dx+c)}+2a^4e^{4i(dx+c)}+9a^2b^2e^{4i(dx+c)}+6b^4e^{4i(dx+c)}-\right.}{\left(e^{2i(dx+c)}-1\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1
/2*d*x+1/2*c))-4/a^4*((-1/4*a*(a^2-6*b^2)*tan(1/2*d*x+1/2*c)^3+5/4*b*(a^2+2
*b^2)*tan(1/2*d*x+1/2*c)^2+1/4*a*(a^2+14*b^2)*tan(1/2*d*x+1/2*c)+5/4*a^2*b)
/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/4*(a^2-2*b^2)/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(173) = 346$.
time = 0.47, size = 1064, normalized size = 5.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(2*(a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 18*(a^4*b - a^2*b^3)*
cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b^3 - 2*(a^3*b - 2*a*b^3)*cos(
```

```
d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2 - 2*b^4)*cos(d*x + c)^2)*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d
*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt
(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a
^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c) + 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2
- a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b
^2 - a*b^4)*cos(d*x + c)^2 + (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x
 + c)^2 - 2*(a^7*b - a^5*b^3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (
a^8 - a^4*b^4)*d)*sin(d*x + c)), -1/2*((a^5 + 5*a^3*b^2 - 6*a*b^4)*cos(d*x
+ c)^3 - 9*(a^4*b - a^2*b^3)*cos(d*x + c)*sin(d*x + c) + 3*(2*a^3*b - 4*a*b
^3 - 2*(a^3*b - 2*a*b^3)*cos(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4 - (a^2*b^2
 - 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(a^5 + a^3*b^2 - 2*a*b^4)*cos
(d*x + c) + 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + (
a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x
 + c) + 1/2) - 3*(2*a^3*b^2 - 2*a*b^4 - 2*(a^3*b^2 - a*b^4)*cos(d*x + c)^2
+ (a^4*b - b^5 - (a^2*b^3 - b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos
(d*x + c) + 1/2))/(2*(a^7*b - a^5*b^3)*d*cos(d*x + c)^2 - 2*(a^7*b - a^5*b^
3)*d + ((a^6*b^2 - a^4*b^4)*d*cos(d*x + c)^2 - (a^8 - a^4*b^4)*d)*sin(d*x +
 c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.50, size = 273, normalized size = 1.50

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^2b\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 + 6
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^4) - (6*b*tan(1/2*d
```

*x + 1/2*c) - a)/(a^4*tan(1/2*d*x + 1/2*c)) - 2*(a^3*tan(1/2*d*x + 1/2*c)^3
 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 10*b^3
*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) - 14*a*b^2*tan(1/2*d*x +
 1/2*c) - 5*a^2*b)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) +
a)^2*a^4))/d

Mupad [B]
time = 9.88, size = 956, normalized size = 5.25

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4a^2+32b^2)+a^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(a^2-12b^2)+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(7a^2b+10b^3)}{a}+14ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(4a^5+8a^3b^2)+2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+8a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+8a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}-\frac{3b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4d}-\frac{\mathrm{atan}\left(\frac{\frac{\sqrt{-(a+b)(a-b)}(a^2-2b^2)\left(\frac{3a^6-12a^4b^2}{a^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(12a^4b-24a^2b^3)}{a^5}-\frac{3\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(a^2-2b^2)}{2(a^6-a^4b^2)}\right)3i}{2(a^6-a^4b^2)}+\frac{\sqrt{-(a+b)(a-b)}(a^2-2b^2)\left(\frac{3a^6-12a^4b^2}{a^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(12a^4b-24a^2b^3)}{a^5}+\frac{3\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(a^2-2b^2)}{2(a^6-a^4b^2)}\right)3i}{2(a^6-a^4b^2)}}{\frac{2(9a^2b-18b^3)}{a^6}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(9a^2-18b^2)}{a^5}+\frac{3\sqrt{-(a+b)(a-b)}(a^2-2b^2)\left(\frac{3a^6-12a^4b^2}{a^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(12a^4b-24a^2b^3)}{a^5}-\frac{3\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(a^2-2b^2)}{2(a^6-a^4b^2)}\right)}{2(a^6-a^4b^2)}-\frac{3\sqrt{-(a+b)(a-b)}(a^2-2b^2)\left(\frac{3a^6-12a^4b^2}{a^6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(12a^4b-24a^2b^3)}{a^5}+\frac{3\sqrt{-(a+b)(a-b)}\left(2a^2b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a^8-8a^6b^2)}{a^5}\right)(a^2-2b^2)}{2(a^6-a^4b^2)}\right)}{2(a^6-a^4b^2)}}\right)\sqrt{-(a+b)(a-b)}(a^2-2b^2)3i}{d(a^6-a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - (tan(c/2 + (d*x)/2)^2*(4*a^2 + 32*b^2) + a^2
 - tan(c/2 + (d*x)/2)^4*(a^2 - 12*b^2) + (2*tan(c/2 + (d*x)/2)^3*(7*a^2*b +
 10*b^3))/a + 14*a*b*tan(c/2 + (d*x)/2))/(d*(2*a^5*tan(c/2 + (d*x)/2)^5 + t
an(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2) + 2*a^5*tan(c/2 + (d*x)/2) + 8*a^4*
b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (d*x)/2)^4)) - (3*b*log(tan(c/2
+ (d*x)/2)))/(a^4*d) - (atan((((-(a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a
^6 - 12*a^4*b^2)/a^6 + (tan(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 - (
3*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^
2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 - a^4*b^2)))*3i)/(2*(a^6 - a^4*b^2)) + ((-(
a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (tan(c/2 +
(d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 + (3*(-(a + b)*(a - b))^(1/2)*(2*a^2*
b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 -
a^4*b^2)))*3i)/(2*(a^6 - a^4*b^2)))/(((2*(9*a^2*b - 18*b^3))/a^6 + (2*tan(c/
2 + (d*x)/2)*(9*a^2 - 18*b^2))/a^5 + (3*(-(a + b)*(a - b))^(1/2)*(a^2 - 2*b
^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (tan(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3)
)/a^5 - (3*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 -
 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2*(a^6 - a^4*b^2))))/(2*(a^6 - a^4*b^2))
- (3*(-(a + b)*(a - b))^(1/2)*(a^2 - 2*b^2)*((3*a^6 - 12*a^4*b^2)/a^6 + (ta
n(c/2 + (d*x)/2)*(12*a^4*b - 24*a^2*b^3))/a^5 + (3*(-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^8 - 8*a^6*b^2))/a^5)*(a^2 - 2*b^2))/(2
*(a^6 - a^4*b^2))))/(2*(a^6 - a^4*b^2))))*(-(a + b)*(a - b))^(1/2)*(a^2 - 2
*b^2)*3i)/(d*(a^6 - a^4*b^2))
```

3.1140 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$\frac{3b(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d}+\frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}-\frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd}+\frac{(a^2-2}{2a^2bd(a}$$

```
[Out] 3/2*(a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d-1/2*(a^2-12*b^2)*cot(d*x+c)/a^4/b
/d+1/2*(a^2-2*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/2*cot(d*x+c)*csc
(d*x+c)/a/d/(a+b*sin(d*x+c))^2-3*b*cot(d*x+c)/a^3/d/(a+b*sin(d*x+c))+3*b*(3
*a^2-4*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d/(a^2-b^2
)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2969, 3134, 3080, 3855, 2739, 632, 210}

$$-\frac{3b\cot(c+dx)}{a^3d(a+b\sin(c+dx))}+\frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2}+\frac{3b(3a^2-4b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d}-\frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5
*Sqrt[a^2 - b^2]*d) + (3*(a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^5*d) - (
(a^2 - 12*b^2)*Cot[c + d*x])/(2*a^4*b*d) + ((a^2 - 2*b^2)*Cot[c + d*x])/(2*
a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b
*Sin[c + d*x])^2) - (3*b*Cot[c + d*x])/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)(2(a^2-6b^2)\cdots}{(a+\cdots}}{\cdots}$$

$$= \frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^2} - \frac{3b\cot(c+dx)}{a^3d(a+b\sin(c+dx\cdots}$$

$$= -\frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd} + \frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c\cdots}{2ad(a+b\sin(c+\cdots}$$

$$= -\frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd} + \frac{(a^2-2b^2)\cot(c+dx)}{2a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc(c\cdots}{2ad(a+b\sin(c+\cdots}$$

$$= \frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd} + \frac{(a^2-2b^2)\,\mathrm{c}\cdots}{2a^2bd(a+b\,\mathrm{s}\cdots}$$

$$= \frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \frac{(a^2-12b^2)\cot(c+dx)}{2a^4bd} + \frac{(a^2-2b^2)\,\mathrm{c}\cdots}{2a^2bd(a+b\,\mathrm{s}\cdots}$$

$$= \frac{3b(3a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{3(a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^5d} - \cdots$$

Mathematica [A]
time = 6.14, size = 319, normalized size = 1.46

$$\frac{3b(3a^2-4b^2)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{3(a^2-4b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} - \frac{3(a^2-4b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{-a^2\cos(c+dx)+b^2\cos(c+dx)}{2a^3d(a+b\sin(c+dx))^2} + \frac{-a^2\cos(c+dx)+6b^2\cos(c+dx)}{2a^4d(a+b\sin(c+dx))} - \frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*(3*a^2 - 4*b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x
)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) + (3*(a^2 - 4*b^2)*Log[Cos[(
c + d*x)/2]])/(2*a^5*d) - (3*(a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d)
 + Sec[(c + d*x)/2]^2/(8*a^3*d) + (-(a^2*Cos[c + d*x]) + b^2*Cos[c + d*x])/
(2*a^3*d*(a + b*Sin[c + d*x])^2) + (-(a^2*Cos[c + d*x]) + 6*b^2*Cos[c + d*x
])/(2*a^4*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [A]
time = 0.77, size = 283, normalized size = 1.30

method	result

derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^4}-\frac{1}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+24b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^5}+\frac{3b}{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4\left(\left(-\frac{3}{4}a^3b+2a\,b^3\right.\right.}{}}{}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^4}-\frac{1}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+24b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^5}+\frac{3b}{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{4\left(\left(-\frac{3}{4}a^3b+2a\,b^3\right.\right.}{}}{}$
risch	$\frac{36ib^4e^{2i(dx+c)}-2ia^4e^{6i(dx+c)}+29ib^2a^2e^{2i(dx+c)}+6b^3a\,e^{7i(dx+c)}+ia^2b^2-45ib^2e^{4i(dx+c)}a^2+15ib^2a^2e^{6i(dx+c)}-54b^3a\,e}{\left(e^{2i(dx+c)}-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^4*(1/2*a*tan(1/2*d*x+1/2*c)^2-6*b*tan(1/2*d*x+1/2*c))-1/8/a^3/ta
n(1/2*d*x+1/2*c)^2+1/4/a^5*(-6*a^2+24*b^2)*ln(tan(1/2*d*x+1/2*c))+3/2*b/a^4
/tan(1/2*d*x+1/2*c)+4/a^5*(((-3/4*a^3*b+2*a*b^3)*tan(1/2*d*x+1/2*c)^3+(-1/2
*a^4+3/4*a^2*b^2+7/2*b^4)*tan(1/2*d*x+1/2*c)^2-5/4*a*b*(a^2-4*b^2)*tan(1/2*
d*x+1/2*c)-1/4*a^2*(2*a^2-7*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1
/2*c)+a)^2+3/4*b*(3*a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(205) = 410$.
time = 0.52, size = 1560, normalized size = 7.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*(a^6 - 10*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(3*a^4*b - a^2*b^
3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b
^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2
- a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3
*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b
^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2
*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*
x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*
a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*
x + c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^
2 - 2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*
d*cos(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c)), 1/4*(4*(a^6 - 10*a^4
*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 - 6*(3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^
3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2 +
2*(3*a^3*b^2 - 4*a*b^4 - (3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) - 6*(a^6 - 7*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c) + 3*(a^6 - 4*a^4*b^2 - a
^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^
4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x + c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5
- (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - 3*(a^6 - 4*a^4*b^2 - a^2*b^4 + 4*b^6 + (a^4*b^2 - 5*a^2*b^
4 + 4*b^6)*cos(d*x + c)^4 - (a^6 - 3*a^4*b^2 - 6*a^2*b^4 + 8*b^6)*cos(d*x +
 c)^2 + 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - (a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*((a^5*b - 13*a^3
*b^3 + 12*a*b^5)*cos(d*x + c)^3 + 3*(a^5*b + 3*a^3*b^3 - 4*a*b^5)*cos(d*x +
 c))*sin(d*x + c))/((a^7*b^2 - a^5*b^4)*d*cos(d*x + c)^4 - (a^9 + a^7*b^2 -
 2*a^5*b^4)*d*cos(d*x + c)^2 + (a^9 - a^5*b^4)*d - 2*((a^8*b - a^6*b^3)*d*c
os(d*x + c)^2 - (a^8*b - a^6*b^3)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)

[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(a + b*sin(c + d*x))**3, x)

Giac [A]
time = 0.56, size = 395, normalized size = 1.81

$$-\frac{\frac{12\,(a^2-4\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{24\,(3\,a^2b-4\,b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{6\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-24\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-32\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+112\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+76\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^4}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2a^5}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/8*(12*(a^2 - 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 24*(3*a^2*b - 4*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - (6*a^4*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 32*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 + 48*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 16*b^4*tan(1/2*d*x + 1/2*c)^4 + 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 112*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1/2*d*x + 1/2*c) - a^4)/((a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2*a^5))/d

Mupad [B]
time = 9.90, size = 1100, normalized size = 5.05

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^3\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(50\,a\,b^2-9\,a^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(6\,a^2\,b-32\,b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(10\,a^2\,b-104\,b^3)-\frac{a^3}{2}+4\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(-17\,a^4+72\,a^2\,b^2+112\,b^4)}{2\,a}}{d\left(4\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8\,a^6+16\,a^4\,b^2)+16\,a^5\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+16\,a^5\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}-\frac{3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^4\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(3\,a^2-12\,b^2)}{2\,a^5\,d}-\frac{b\,\mathrm{atan}\left(\frac{\frac{b\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-4\,b^2)\left(\frac{12\,a^7\,b-24\,a^5\,b^3}{a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3\,a^7-36\,a^5\,b^2+48\,a^3\,b^4)}{a^7}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6\,a^{10}-8\,a^8\,b^2)}{a^7}\right)(3\,a^2-4\,b^2)}{2\,(a^7-a^5\,b^2)}\right)3{}\mathrm{i}}{2\,(a^7-a^5\,b^2)}-\frac{b\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-4\,b^2)\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3\,a^7-36\,a^5\,b^2+48\,a^3\,b^4)}{a^7}-\frac{12\,a^7\,b-24\,a^5\,b^3}{a^8}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6\,a^{10}-8\,a^8\,b^2)}{a^7}\right)(3\,a^2-4\,b^2)}{2\,(a^7-a^5\,b^2)}\right)3{}\mathrm{i}}{2\,(a^7-a^5\,b^2)}}{\frac{27\,a^4\,b-144\,a^2\,b^3+144\,b^5}{a^8}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(72\,b^4-54\,a^2\,b^2)}{a^7}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-4\,b^2)\left(\frac{12\,a^7\,b-24\,a^5\,b^3}{a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3\,a^7-36\,a^5\,b^2+48\,a^3\,b^4)}{a^7}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6\,a^{10}-8\,a^8\,b^2)}{a^7}\right)(3\,a^2-4\,b^2)}{2\,(a^7-a^5\,b^2)}\right)}{2\,(a^7-a^5\,b^2)}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-4\,b^2)\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(3\,a^7-36\,a^5\,b^2+48\,a^3\,b^4)}{a^7}-\frac{12\,a^7\,b-24\,a^5\,b^3}{a^8}+\frac{3\,b\sqrt{-(a+b)\,(a-b)}\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6\,a^{10}-8\,a^8\,b^2)}{a^7}\right)(3\,a^2-4\,b^2)}{2\,(a^7-a^5\,b^2)}\right)}{2\,(a^7-a^5\,b^2)}}\right)\sqrt{-(a+b)\,(a-b)}\,(3\,a^2-4\,b^2)\,3{}\mathrm{i}}{d\,(a^7-a^5\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (tan(c/2 + (d*x)/2)^2*(50*a*b^2 - 9*a^3) - tan(c/2 + (d*x)/2)^5*(6*a^2*b - 32*b^3) - tan(c/2 + (d*x)/2)^3*(10*a^2*b - 104*b^3) - a^3/2 + 4*a^2*b*tan(c/2 + (d*x)/2) + (tan(c/2 + (d*x)/2)^4*(112*b^4 - 17*a^4 + 72*a^2*b^2))/(2*a))/(d*(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 + (d*x)/2)^5)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(3*a^2 - 12*b^2))/(2*a^5*d) - (b*atan(((b*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((12*a^7*b - 24*a^5*b^3)/a^8 - (tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4 - 36*a^5*b^2))/a^7 + (3*b*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^7)*(3*a^2 - 4*b^2))/(2*(a^7 - a^5*b^2)))*3i)/(2*(a^7 - a^5*b^2)) - (b*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4 - 36*a^5*b^2))/a^7 - (12*a^7*b - 24*a^5*b^3)/a^8 + (3*b*(-(a + b)*(a

```
 - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^7)*(3*a
^2 - 4*b^2))/(2*(a^7 - a^5*b^2)))*3i)/(2*(a^7 - a^5*b^2)))/((27*a^4*b + 144
*b^5 - 144*a^2*b^3)/a^8 + (2*tan(c/2 + (d*x)/2)*(72*b^4 - 54*a^2*b^2))/a^7
+ (3*b*(-(a + b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((12*a^7*b - 24*a^5*b^3)/a^
8 - (tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4 - 36*a^5*b^2))/a^7 + (3*b*(-(a
+ b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^
7)*(3*a^2 - 4*b^2))/(2*(a^7 - a^5*b^2))))/(2*(a^7 - a^5*b^2)) + (3*b*(-(a +
 b)*(a - b))^(1/2)*(3*a^2 - 4*b^2)*((tan(c/2 + (d*x)/2)*(3*a^7 + 48*a^3*b^4
 - 36*a^5*b^2))/a^7 - (12*a^7*b - 24*a^5*b^3)/a^8 + (3*b*(-(a + b)*(a - b))
^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^10 - 8*a^8*b^2))/a^7)*(3*a^2 - 4
*b^2))/(2*(a^7 - a^5*b^2))))/(2*(a^7 - a^5*b^2))))*(-(a + b)*(a - b))^(1/2)
*(3*a^2 - 4*b^2)*3i)/(d*(a^7 - a^5*b^2))
```

3.1141 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+$$

```
[Out] -1/2*b*(9*a^2-20*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(17*a^2-60*b^2)*cot(d*x
+c)/a^5/d-(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/6*(3*a^2-5*b^2)*cot(d
*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
/(a+b*sin(d*x+c))^2+1/6*(3*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a+b*s
in(d*x+c))+(2*a^4-19*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b
^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2803, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d}-\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd}+\frac{(3a^2-20b^2)\cot(c+dx)\csc(c+dx)}{6a^3bd(a+b\sin(c+dx))}+\frac{(2a^4-19a^2b^2+20b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \csc^3(c+dx)(2 \ldots}{\ldots}$$

$$= \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{(3a^2-20b^2)\ldots}{6a^3bd(\ldots}$$

$$= -\frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \ldots$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\ldots}{6a^2bd(\ldots}$$

$$= \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{(3a^2-5b^2)\ldots}{6a^2bd(\ldots}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\ldots}{\ldots}$$

$$= -\frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} - \frac{(a^2-5b^2)\ldots}{\ldots}$$

$$= \frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b(9a^2-20b^2)\tanh^{-1}(\cos(c+\ldots}{2a^6d}$$

Mathematica [A]
time = 6.16, size = 459, normalized size = 1.59

$$\frac{(2a^4-19a^2b^2+20b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} + \frac{\left(2a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{3a^5d} + \frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} + \frac{(-9a^2b+20b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{(9a^2b-20b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{2a^4d(a+b\sin(c+dx))^2} + \frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [A]
time = 0.86, size = 360, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^5*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-3*a*b*tan(1/2*d*x+1/2*c)^2-5*a^2
*tan(1/2*d*x+1/2*c)+24*b^2*tan(1/2*d*x+1/2*c))-1/24/a^3/tan(1/2*d*x+1/2*c)^
3-1/8*(-5*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)+3/8/a^4*b/tan(1/2*d*x+1/2*c)^2
+1/2/a^6*b*(9*a^2-20*b^2)*ln(tan(1/2*d*x+1/2*c))+2/a^6*(((5/2*a^3*b^2-5*a*b
^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-a^2*b^2-18*b^4)*tan(1/2*d*x+1/2*c)^2+
1/2*a*b^2*(11*a^2-26*b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a^2-9*b^2))/(a*ta
n(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^4-19*a^2*b^2+20*b^4
)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(274) = 548$.
time = 0.65, size = 2027, normalized size = 7.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
```

```
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 +
(a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*
b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x
 + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20
*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2
 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^
6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)
^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 -
b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x
 + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 +
 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x +
 c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^
5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*c
os(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 -
58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^
4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^
4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)
^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^
5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*c
os(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)
^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*si
n(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 451, normalized size = 1.56

$$\frac{\frac{12\,(9\,a^2b-20\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6}+\frac{24\,(2\,a^4-19\,a^2b^2+20\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6}+\frac{24\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-18\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+11\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-26\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^4b-9\,a^2b^3\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^6}+\frac{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^9}-\frac{198\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-440\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+72\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 10.04, size = 1261, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8
*a^5) + 1/(4*a^3) - (9*b^2)/(2*a^5)))/d + (tan(c/2 + (d*x)/2)^6*(5*a^4 - 80
*b^4 + 16*a^2*b^2) + tan(c/2 + (d*x)/2)^4*((29*a^4)/3 - 304*b^4 + 72*a^2*b^
2) - a^4/3 + tan(c/2 + (d*x)/2)^2*((13*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 +
 (d*x)/2)^3*(156*a*b^3 - (170*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(144*b^5 -
55*a^4*b + 104*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(
c/2 + (d*x)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^
7 + 32*a^5*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/
2)^6)) + (log(tan(c/2 + (d*x)/2))*(9*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(
c/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^
4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (
```

d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2))/((18*a^6*b - 400*b^7 + 560*a^2*b^5 - 211*a^4*b^3)/a^10 + (2*tan(c/2 + (d*x)/2)*(4*a^6 - 200*b^6 + 230*a^2*b^4 - 58*a^4*b^2))/a^9 - ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^2) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)))/(a^8 - a^6*b^2)))*(-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*2i)/(d*(a^8 - a^6*b^2))

3.1142 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=340

$$-\frac{3b(2a^4-11a^2b^2+10b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7\sqrt{a^2-b^2}\,d}-\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}-\frac{b(13a^2-30\ldots}{}$$

```
[Out] -3/8*(a^4-24*a^2*b^2+40*b^4)*arctanh(cos(d*x+c))/a^7/d-1/2*b*(13*a^2-30*b^2
)*cot(d*x+c)/a^6/d+3/8*(7*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^5/d-1/2*(3*a^
2-10*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^4/b/d+1/4*(2*a^2-3*b^2)*cot(d*x+c)*csc(
d*x+c)^2/a^2/b/d/(a+b*sin(d*x+c))^2-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*si
n(d*x+c))^2+1/4*(4*a^2-15*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/b/d/(a+b*sin(d*x
+c))-3*b*(2*a^4-11*a^2*b^2+10*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2
)^(1/2))/a^7/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2969, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2}-\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d}+\frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}-\frac{(3a^2-10b^2)\cot(c+dx)\csc^2(c+dx)}{2a^4bd}+\frac{(4a^2-15b^2)\cot(c+dx)\csc^2(c+dx)}{4a^3bd(a+b\sin(c+dx))}-\frac{3b(2a^4-11a^2b^2+10b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d\sqrt{a^2-b^2}}-\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*Sqrt[a^2 - b^2]*d) - (3*(a^4 - 24*a^2*b^2 + 40*b^4)*ArcTanh
[Cos[c + d*x]])/(8*a^7*d) - (b*(13*a^2 - 30*b^2)*Cot[c + d*x])/(2*a^6*d) +
(3*(7*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - ((3*a^2 - 10*b^2
)*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^4*b*d) + ((2*a^2 - 3*b^2)*Cot[c + d*x]*
Csc[c + d*x]^2)/(4*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c +
d*x]^3)/(4*a*d*(a + b*Sin[c + d*x])^2) + ((4*a^2 - 15*b^2)*Cot[c + d*x]*Csc
[c + d*x]^2)/(4*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^4(c+}{}}{}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+dx)}{4a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2} + \frac{(4a^2-1}{4a}$$

$$= -\frac{(3a^2-10b^2)\cot(c+dx)\csc^2(c+dx)}{2a^4bd} + \frac{(2a^2-3b^2)\cot(c+dx)\csc^2(c+}{4a^2bd(a+b\sin(c+dx))^2}$$

$$= \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a^2-10b^2)\cot(c+dx)\csc^2(c+d}{2a^4bd}$$

$$= -\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a}{}$$

$$= -\frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d} + \frac{3(7a^2-20b^2)\cot(c+dx)\csc(c+dx)}{8a^5d} - \frac{(3a}{}$$

$$= -\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d}$$

$$= -\frac{3(a^4-24a^2b^2+40b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d} - \frac{b(13a^2-30b^2)\cot(c+dx)}{2a^6d}$$

$$= -\frac{3b(2a^4-11a^2b^2+10b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7\sqrt{a^2-b^2}\,d} - \frac{3(a^4-24a^2b^2+40b^4}{8a}$$

Mathematica [A]
time = 3.09, size = 347, normalized size = 1.02

$$-\frac{\frac{384b(2a^4-11a^2b^2+10b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 48(a^4-24a^2b^2+40b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 48(a^4-24a^2b^2+40b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2a\cot(c+dx)\csc^5(c+dx)\left(-4a^5+289a^3b^2-540ab^4+4(5a^5-93a^3b^2+180ab^4)\cos(2(c+dx))+(83a^3b^2-180ab^4)\cos(4(c+dx))+100a^4b\sin(c+dx)+20a^2b^3\sin(c+dx)-600b^5\sin(c+dx)-44a^4b\sin(3(c+dx))-50a^2b^3\sin(3(c+dx))+300b^5\sin(3(c+dx))+26a^2b^3\sin(5(c+dx))-60b^5\sin(5(c+dx))\right)}{(b+a\csc(c+dx))^2}}{128a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/128*((384*b*(2*a^4 - 11*a^2*b^2 + 10*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Co
```

```
s[(c + d*x)/2]] - 48*(a^4 - 24*a^2*b^2 + 40*b^4)*Log[Sin[(c + d*x)/2]] + (2
*a*Cot[c + d*x]*Csc[c + d*x]^5*(-4*a^5 + 289*a^3*b^2 - 540*a*b^4 + 4*(5*a^5
 - 93*a^3*b^2 + 180*a*b^4)*Cos[2*(c + d*x)] + (83*a^3*b^2 - 180*a*b^4)*Cos[
4*(c + d*x)] + 100*a^4*b*Sin[c + d*x] + 20*a^2*b^3*Sin[c + d*x] - 600*b^5*S
in[c + d*x] - 44*a^4*b*Sin[3*(c + d*x)] - 50*a^2*b^3*Sin[3*(c + d*x)] + 300
*b^5*Sin[3*(c + d*x)] + 26*a^2*b^3*Sin[5*(c + d*x)] - 60*b^5*Sin[5*(c + d*x
)]))/(b + a*Csc[c + d*x])^2)/(a^7*d)
```

Maple [A]
time = 0.99, size = 431, normalized size = 1.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^6*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-2*b*tan(1/2*d*x+1/2*c)^3*a^2-2*
a^3*tan(1/2*d*x+1/2*c)^2+12*a*b^2*tan(1/2*d*x+1/2*c)^2+30*a^2*b*tan(1/2*d*x
+1/2*c)-80*b^3*tan(1/2*d*x+1/2*c))-1/64/a^3/tan(1/2*d*x+1/2*c)^4-1/32*(-4*a
^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)^2+1/16/a^7*(6*a^4-144*a^2*b^2+240*b^4)*ln
(tan(1/2*d*x+1/2*c))+1/8/a^4*b/tan(1/2*d*x+1/2*c)^3-5/8*b*(3*a^2-8*b^2)/a^6
/tan(1/2*d*x+1/2*c)-2*b/a^7*(((7/2*a^3*b^2-6*a*b^4)*tan(1/2*d*x+1/2*c)^3+1/
2*b*(6*a^4+a^2*b^2-22*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(17*a^2-32*b^2)*t
an(1/2*d*x+1/2*c)+1/2*a^2*b*(6*a^2-11*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan
(1/2*d*x+1/2*c)+a)^2+3/2*(2*a^4-11*a^2*b^2+10*b^4)/(a^2-b^2)^(1/2)*arctan(1
/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(321) = 642$.
time = 0.83, size = 2592, normalized size = 7.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas ")

[Out] [1/16*(2*(83*a^6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8 - 181*a^6*b^2 + 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 + 12*(2*a^6*b - 9*a^4*b^3 - a^2*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x + c)^6 + (2*a^6*b - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*b - 16*a^4*b^3 - 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^8 - 32*a^6*b^2 + 91*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7)*cos(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*x + c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(d*x + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 - (a^11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a^8*b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c)), 1/16*(2*(83*a^6*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c)^5 + 2*(5*a^8 - 181*a^6*b^2 + 536*a^4*b^4 - 360*a^2*b^6)*cos(d*x + c)^3 - 24*(2*a^6*b - 9*a^4*b^3 - a^2*b^5 + 10*b^7 - (2*a^4*b^3 - 11*a^2*b^5 + 10*b^7)*cos(d*x + c)^6 + (2*a^6*b - 5*a^4*b^3 - 23*a^2*b^5 + 30*b^7)*cos(d*x + c)^4 - (4*a^6*b - 16*a^4*b^3 - 13*a^2*b^5 + 30*b^7)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6 + (2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^4 - 2*(2*a^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^8 - 32*a^6*b^2 + 91*a^4*b^4 - 60*a^2*b^6)*cos(d*x + c) + 3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4 + 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4 + 112*a^2*b^6 - 120*b^8)*cos(d*x +

```
c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3
 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4 - 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b
^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) -
3*(a^8 - 24*a^6*b^2 + 39*a^4*b^4 + 24*a^2*b^6 - 40*b^8 - (a^6*b^2 - 25*a^4*
b^4 + 64*a^2*b^6 - 40*b^8)*cos(d*x + c)^6 + (a^8 - 22*a^6*b^2 - 11*a^4*b^4
+ 152*a^2*b^6 - 120*b^8)*cos(d*x + c)^4 - (2*a^8 - 47*a^6*b^2 + 53*a^4*b^4
+ 112*a^2*b^6 - 120*b^8)*cos(d*x + c)^2 + 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^
5 - 40*a*b^7 + (a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^4
- 2*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*cos(d*x + c)^2)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(13*a^5*b^3 - 43*a^3*b^5 + 30*a*b^7
)*cos(d*x + c)^5 - (11*a^7*b + 21*a^5*b^3 - 152*a^3*b^5 + 120*a*b^7)*cos(d*
x + c)^3 + 3*(3*a^7*b - a^5*b^3 - 22*a^3*b^5 + 20*a*b^7)*cos(d*x + c))*sin(
d*x + c))/((a^9*b^2 - a^7*b^4)*d*cos(d*x + c)^6 - (a^11 + 2*a^9*b^2 - 3*a^7
*b^4)*d*cos(d*x + c)^4 + (2*a^11 + a^9*b^2 - 3*a^7*b^4)*d*cos(d*x + c)^2 -
(a^11 - a^7*b^4)*d - 2*((a^10*b - a^8*b^3)*d*cos(d*x + c)^4 - 2*(a^10*b - a
^8*b^3)*d*cos(d*x + c)^2 + (a^10*b - a^8*b^3)*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.63, size = 550, normalized size = 1.62

$$\frac{\frac{24\,(a^4-24\,a^2b^2+40\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^7}-\frac{192\,(2\,a^4b-11\,a^2b^3+10\,b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}-\frac{64\left(7\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-22\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+17\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-32\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^4b^2-11\,a^2b^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^7}-\frac{50\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1200\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2000\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-320\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+48\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4}{a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}+\frac{a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^8b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,a^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+48\,a^7b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+120\,a^8b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-320\,a^6b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{12}}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] 1/64*(24*(a^4 - 24*a^2*b^2 + 40*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 1
92*(2*a^4*b - 11*a^2*b^3 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a
^7) - 64*(7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
3 + 6*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 22*
b^6*tan(1/2*d*x + 1/2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) - 32*a*b^5*tan
(1/2*d*x + 1/2*c) + 6*a^4*b^2 - 11*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*
b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (50*a^4*tan(1/2*d*x + 1/2*c)^4 - 1200*
a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*
b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 8*a^4*tan(1/2
*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x +
```

1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) + (a^9*tan(1/2*d*x + 1/2*c)^4 -
8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 8*a^9*tan(1/2*d*x + 1/2*c)^2 + 48*a^7*b^2
*tan(1/2*d*x + 1/2*c)^2 + 120*a^8*b*tan(1/2*d*x + 1/2*c) - 320*a^6*b^3*tan(
1/2*d*x + 1/2*c))/a^12)/d

Mupad [B]
time = 10.21, size = 1487, normalized size = 4.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a^3*d) - (tan(c/2 + (d*x)/2)^2*((3*(a^2 + 4*b^2))/
(32*a^5) + 1/(32*a^3) - (9*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*((6*b*((3
*(a^2 + 4*b^2))/(16*a^5) + 1/(16*a^3) - (9*b^2)/(4*a^5)))/a - (192*a^2*b +
128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8*a^6)))/d - (tan(c/2 + (d*x)/2)^
3*(19*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^4*(448*a*b^4 + (15*a^5)/4 -
224*a^3*b^2) + tan(c/2 + (d*x)/2)^7*(30*a^4*b - 192*b^5 + 32*a^2*b^3) + tan
(c/2 + (d*x)/2)^5*(50*a^4*b - 832*b^5 + 280*a^2*b^3) + a^5/4 - tan(c/2 + (d
x)/2)^2((3*a^5)/2 - 5*a^3*b^2) - a^4*b*tan(c/2 + (d*x)/2) - (2*tan(c/2 +
(d*x)/2)^6*(a^6 + 176*b^6 + 152*a^2*b^4 - 114*a^4*b^2))/a)/(d*(16*a^8*tan(c
/2 + (d*x)/2)^4 + 16*a^8*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^6*(32*a^
8 + 64*a^6*b^2) + 64*a^7*b*tan(c/2 + (d*x)/2)^5 + 64*a^7*b*tan(c/2 + (d*x)/
2)^7)) - (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) + (log(tan(c/2 + (d*x)/2))*((3*
a^4)/8 + 15*b^4 - 9*a^2*b^2))/(a^7*d) + (b*atan(((b*(-(a + b)*(a - b))^(1/2
)*(((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 - (tan(c/2 + (d*x)/2)*(3*
a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) + (3*b*(-(a + b)*
(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a
^11))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 -
11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)) - (b*(-(a + b)*(a - b))^(1/2)*((tan(c/
2 + (d*x)/2)*(3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) -
((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 + (3*b*(-(a + b)*(a - b))^(
1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(2*a
^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^
2)*3i)/(2*(a^9 - a^7*b^2)))/(((9*a^8*b)/2 + 900*b^9 - 1530*a^2*b^7 + (1593*
a^4*b^5)/2 - (531*a^6*b^3)/4)/a^12 + (tan(c/2 + (d*x)/2)*(1800*b^8 - 2610*a
^2*b^6 + 1053*a^4*b^4 - 126*a^6*b^2))/(2*a^11) + (3*b*(-(a + b)*(a - b))^(1
/2)*(((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 - (tan(c/2 + (d*x)/2)*(
3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11) + (3*b*(-(a + b
)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4
a^11))(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4
- 11*a^2*b^2))/(2*(a^9 - a^7*b^2)) + (3*b*(-(a + b)*(a - b))^(1/2)*((tan(c
/2 + (d*x)/2)*(3*a^11 - 480*a^5*b^6 + 528*a^7*b^4 - 126*a^9*b^2))/(4*a^11)
- ((27*a^11*b)/4 + 60*a^7*b^5 - 51*a^9*b^3)/a^12 + (3*b*(-(a + b)*(a - b))^

```
(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(2*
a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b
^2))/(2*(a^9 - a^7*b^2))))*(-(a + b)*(a - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^
2*b^2)*3i)/(d*(a^9 - a^7*b^2))
```

3.1143 $\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=463

$$\frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d}-\frac{8(480a^4-937a^2b^2+231b^4)\cos(c+dx)(a+b}{45045b^5d}$$

```
[Out] -8/45045*(480*a^4-937*a^2*b^2+231*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^
5/d+8/3003*a*(40*a^2-81*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b
^4/d-10/1287*(16*a^2-33*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)
/b^3/d+20/143*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/13*c
os(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(3/2)/b/d+16/45045*a*(160*a^4-279*a
^2*b^2+27*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+8/45045*(320*a^6-798
*a^4*b^2+435*a^2*b^4-693*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2
))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-16/45045*a*(
160*a^6-439*a^4*b^2+306*a^2*b^4-27*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$,
Rules used = {2974, 3128, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

$$\frac{8a(40a^2-81b^2)\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{3003b^4d}-\frac{10(16a^2-33b^2)\sin^2(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{1287b^3d}-\frac{8(480a^4-937a^2b^2+231b^4)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{45045b^5d}+\frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d}+\frac{16a(160a^6-439a^4b^2+306a^2b^4-27b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{45045b^6d\sqrt{a+b\sin(c+dx)}}-\frac{8(320a^6-798a^4b^2+435a^2b^4-693b^6)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{45045b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{20a\sin^3(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{143b^2d}-\frac{2\sin^4(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{13bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (16*a*(160*a^4 - 279*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]
])/(45045*b^5*d) - (8*(480*a^4 - 937*a^2*b^2 + 231*b^4)*Cos[c + d*x]*(a + b
*Sin[c + d*x])^(3/2))/(45045*b^5*d) + (8*a*(40*a^2 - 81*b^2)*Cos[c + d*x]*S
in[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(3003*b^4*d) - (10*(16*a^2 - 33*b^2
)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(1287*b^3*d) + (2
0*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(143*b^2*d) - (
2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2))/(13*b*d) - (8*(32
0*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b^6)*EllipticE[(c - Pi/2 + d*x)/2,
(2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045*b^6*d*Sqrt[(a + b*Sin[c + d
*x])/(a + b)]) + (16*a*(160*a^6 - 439*a^4*b^2 + 306*a^2*b^4 - 27*b^6)*Ellip
ticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
```

```
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{20a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{3/2}}{143b^2d} - $$

$$= -\frac{10(16a^2-33b^2)\cos(c+dx)\sin^2(c+dx)(a+b\sin(}{1287b^3d}$$

$$= \frac{8a(40a^2-81b^2)\cos(c+dx)\sin(c+dx)(a+b\sin(c+}{3003b^4d}$$

$$= -\frac{8(480a^4-937a^2b^2+231b^4)\cos(c+dx)(a+b\sin(c}{45045b^5d}$$

$$= \frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(}}{45045b^5d}$$

$$= \frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(}}{45045b^5d}$$

$$= \frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(}}{45045b^5d}$$

$$= \frac{16a(160a^4-279a^2b^2+27b^4)\cos(c+dx)\sqrt{a+b\sin(}}{45045b^5d}$$

Mathematica [A]
time = 3.51, size = 327, normalized size = 0.71

$$\frac{\sqrt{a+b\sin(c+dx)}\left(128(320a^6-798a^4b^2+435a^2b^4-693b^6)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)-256a(160a^5-160a^4b-279a^3b^2+279a^2b^3+27ab^4-27b^5)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)-2b\cos(c+dx)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}(10240a^5-21056a^3b^2+5898ab^4-1600(2a^3b^2-3ab^4)\cos(2(c+dx))+630ab^4\cos(4(c+dx))-7680a^4b\sin(c+dx)+13592a^2b^3\sin(c+dx)-19866b^5\sin(c+dx)+1400a^2b^3\sin(3(c+dx))+5775b^5\sin(3(c+dx))+3465b^5\sin(5(c+dx)))\right)}{720720b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(128*(320*a^6 - 798*a^4*b^2 + 435*a^2*b^4 - 693*b
^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*a*(160*a^5 - 160*
a^4*b - 279*a^3*b^2 + 279*a^2*b^3 + 27*a*b^4 - 27*b^5)*EllipticF[(-2*c + Pi
 - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)]*(10240*a^5 - 21056*a^3*b^2 + 5898*a*b^4 - 1600*(2*a^3*b^2 - 3*a*b^4)
*Cos[2*(c + d*x)] + 630*a*b^4*Cos[4*(c + d*x)] - 7680*a^4*b*Sin[c + d*x] +
13592*a^2*b^3*Sin[c + d*x] - 19866*b^5*Sin[c + d*x] + 1400*a^2*b^3*Sin[3*(c
```

```
 + d*x)] + 5775*b^5*Sin[3*(c + d*x)] + 3465*b^5*Sin[5*(c + d*x)])))/(720720
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(493) = 986$.
time = 10.59, size = 1619, normalized size = 3.50

method	result	size
default	Expression too large to display	1619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/45045*(708*a^2*b^6-3512*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+2484*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+2448*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
^3*b^5-4296*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^2*b^6-216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7+4472*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-4932*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^4+4512*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^2*b^6+1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-960*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2+2772*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^8-1280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^8-2772*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8-3465*b^8*sin(d*x+c)^8+9240*b^8*sin(d
*x+c)^6-6699*b^8*sin(d*x+c)^4+924*b^8*sin(d*x+c)^2-3780*a*b^7*sin(d*x+c)^7+
```

```
35*a^2*b^6*sin(d*x+c)^6-50*a^3*b^5*sin(d*x+c)^5+10470*a*b^7*sin(d*x+c)^5+80
*a^4*b^4*sin(d*x+c)^4-232*a^2*b^6*sin(d*x+c)^4-160*a^5*b^3*sin(d*x+c)^3+454
*a^3*b^5*sin(d*x+c)^3-8322*a*b^7*sin(d*x+c)^3-640*a^6*b^2*sin(d*x+c)^2+1436
*a^4*b^4*sin(d*x+c)^2-511*a^2*b^6*sin(d*x+c)^2+160*a^5*b^3*sin(d*x+c)-404*a
^3*b^5*sin(d*x+c)+1632*a*b^7*sin(d*x+c)+640*a^6*b^2-1516*a^4*b^4)/b^7/cos(d
*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 633, normalized size = 1.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/135135*(2*sqrt(2)*(640*a^7 - 1836*a^5*b^2 + 1401*a^3*b^4 + 531*a*b^6)*sqr
t(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a
*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(
2)*(640*a^7 - 1836*a^5*b^2 + 1401*a^3*b^4 + 531*a*b^6)*sqrt(-I*b)*weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*sqrt(2)*(-320*I*a^6*
b + 798*I*a^4*b^3 - 435*I*a^2*b^5 + 693*I*b^7)*sqrt(I*b)*weierstrassZeta(-4
/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInvers
e(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*sqrt(2)*(320*I*a^6*b - 798*I*
a^4*b^3 + 435*I*a^2*b^5 - 693*I*b^7)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
 + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(315*a*b^6*cos(d*x + c)^5 - 5*(80*a^
3*b^4 - 57*a*b^6)*cos(d*x + c)^3 + 4*(160*a^5*b^2 - 279*a^3*b^4 + 27*a*b^6)
*cos(d*x + c) + (3465*b^7*cos(d*x + c)^5 + 35*(10*a^2*b^5 - 33*b^7)*cos(d*x
 + c)^3 - 6*(80*a^4*b^3 - 127*a^2*b^5 + 231*b^7)*cos(d*x + c))*sin(d*x + c)
)*sqrt(b*sin(d*x + c) + a))/(b^7*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)}\,\sin^2(c+dx)\cos^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*sin(c + d*x)**2*cos(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\sin(c+d\,x)^2\sqrt{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

3.1144 $\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$-\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}+\frac{8a(32a^4-93a^2b^2+93b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3465b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/11*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d-2/693*cos(d*x+c)^3*(8*a^2-9*b^2
-7*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/3465*cos(d*x+c)*(32*a^4-6
9*a^2*b^2+45*b^4-24*a*b*(a^2-2*b^2)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^4/
d-8/3465*a*(32*a^4-93*a^2*b^2+93*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+
b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/34
65*(32*a^6-101*a^4*b^2+114*a^2*b^4-45*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8a^2-7ab\sin(c+dx)-9b^2)}{693b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a^4-24ab(a^2-2b^2)\sin(c+dx)-69a^2b^2+45b^4)}{3465b^4d}+\frac{8a(32a^4-93a^2b^2+93b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3465b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8(32a^6-101a^4b^2+114a^2b^4-45b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3465b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(11*d) + (8*a*(32*a^4 - 93*a^2
*b^2 + 93*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[
c + d*x]])/(3465*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^6 - 1
01*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3465*b^5*d*Sqrt[a + b*Sin[c + d*x
]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^2 - 9*b^2 - 7*a*b*Sin
[c + d*x]))/(693*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4
- 69*a^2*b^2 + 45*b^4 - 24*a*b*(a^2 - 2*b^2)*Sin[c + d*x]))/(3465*b^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{2}{11}\int \frac{\cos^4(c+$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} - \frac{2\cos^3(c+dx)}{}$$

$$= -\frac{2\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{11d} + \frac{8a(32a^4-93a}{}$$

Mathematica [A]
time = 2.80, size = 326, normalized size = 0.98

$$\frac{-64a(32a^5+32a^4b-93a^3b^2-93a^2b^3+93ab^4+93b^5)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+64(32a^6-101a^4b^2+114a^2b^4-45b^6)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\left(1024a^5-2912a^3b^2+748ab^4+16(4a^3b^2-183ab^4)\cos(2(c+dx))-700ab^4\cos(4(c+dx))+256a^4b\sin(c+dx)-692a^2b^3\sin(c+dx)+990b^5\sin(c+dx)-20a^2b^3\sin(3(c+dx))-765b^5\sin(3(c+dx))-315b^5\sin(5(c+dx))\right)}{27720b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-64*a*(32*a^5 + 32*a^4*b - 93*a^3*b^2 - 93*a^2*b^3 + 93*a*b^4 + 93*b^5)*El
lipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)] + 64*(32*a^6 - 101*a^4*b^2 + 114*a^2*b^4 - 45*b^6)*EllipticF[(-2*c +
Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c
+ d*x]*(1024*a^5 - 2912*a^3*b^2 + 748*a*b^4 + 16*(4*a^3*b^2 - 183*a*b^4)*Co
s[2*(c + d*x)] - 700*a*b^4*Cos[4*(c + d*x)] + 256*a^4*b*Sin[c + d*x] - 692*
a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] - 20*a^2*b^3*Sin[3*(c + d*x)] -
```

```
765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^5*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(374) = 748$.
time = 10.25, size = 1356, normalized size = 4.08

method	result	size
default	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3465*(315*b^7*sin(d*x+c)^7-900*b^7*sin(d*x+c)^5+765*b^7*sin(d*x+c)^3-180*
b^7*sin(d*x+c)-744*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+372*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-404*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+288*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^3*b^4+456*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^2*b^5-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+128*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-96*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^5*b^2+500*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^5*b^2+896*a*b^6*sin(d*x+c)^2+16*a^4*b^3*sin(d*x+c)-4
7*a^2*b^5*sin(d*x+c)+350*a*b^6*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^5+8*a^3*b^
4*sin(d*x+c)^4-1066*a*b^6*sin(d*x+c)^4-16*a^4*b^3*sin(d*x+c)^3+52*a^2*b^5*s
in(d*x+c)^3-64*a^5*b^2*sin(d*x+c)^2+170*a^3*b^4*sin(d*x+c)^2-178*a^3*b^4-18
0*a*b^6+64*a^5*b^2-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^7-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7)/b^6/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 584, normalized size = 1.76

$$-\frac{2\left(2\sqrt{2}\left(64a^6-210a^4b^2+249a^2b^4-135b^6\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(64a^6-210a^4b^2+249a^2b^4-135b^6\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+6\sqrt{2}\left(32ia^5b-93ia^3b^3+93iab^5\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+6\sqrt{2}\left(-32ia^5b+93ia^3b^3-93iab^5\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(315b^6\cos(dx+c)^5+5(8a^2b^4-9b^6)\cos(dx+c)^3-2(32a^4b^2-69a^2b^4+45b^6)\cos(dx+c)-(35ab^5\cos(dx+c)^3-48(a^3b^3-2ab^5)\cos(dx+c))\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{10395\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/10395*(2*sqrt(2)*(64*a^6 - 210*a^4*b^2 + 249*a^2*b^4 - 135*b^6)*sqrt(I*b
)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(6
4*a^6 - 210*a^4*b^2 + 249*a^2*b^4 - 135*b^6)*sqrt(-I*b)*weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d
*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 6*sqrt(2)*(32*I*a^5*b - 93*I*a^3
*b^3 + 93*I*a*b^5)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/2
7*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2,
-8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c)
 - 2*I*a)/b)) + 6*sqrt(2)*(-32*I*a^5*b + 93*I*a^3*b^3 - 93*I*a*b^5)*sqrt(-I
*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/
b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*
b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(315*
b^6*cos(d*x + c)^5 + 5*(8*a^2*b^4 - 9*b^6)*cos(d*x + c)^3 - 2*(32*a^4*b^2 -
 69*a^2*b^4 + 45*b^6)*cos(d*x + c) - (35*a*b^5*cos(d*x + c)^3 - 48*(a^3*b^3
 - 2*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^4*sin(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(1/2), x)
```

3.1145 $\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=338

$$-\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}+\frac{8a\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35b^2d}-\frac{2\cos(c+dx)\sin(c+$$

[Out] 8/35*a*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/7*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d-2/105*(8*a^2-45*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d-2/105*a*(8*a^2-51*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/105*(8*a^4-53*a^2*b^2-60*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.59, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$,
Rules used = {2974, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d}+\frac{2a(8a^2-51b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(8a^4-53a^2b^2-60b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{105b^3d\sqrt{a+b\sin(c+dx)}}+\frac{8a\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35b^2d}-\frac{2\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{7bd}+\frac{2a\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]

[Out] (-2*(8*a^2 - 45*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(105*b^2*d) + (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(7*b*d) + (2*a*(8*a^2 - 51*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(105*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^4 - 53*a^2*b^2 - 60*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(105*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
```

```
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35b^2d} - \frac{2\cos(c+dx)\sin}{}$$

$$= -\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\,c}{}$$

$$= -\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\,c}{}$$

$$= -\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\,c}{}$$

$$= -\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\,c}{}$$

$$= -\frac{2(8a^2-45b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{105b^2d} + \frac{8a\,c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.32, size = 435, normalized size = 1.29

$$\frac{\frac{2i(-8a^2+51b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^2\sqrt{-\frac{1}{a+b}}}-\frac{8b(a^2+30b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2a(8a^2+159b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+2\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(8a^2+75b^2+15b^2\cos(2(c+dx))-6ab\sin(c+dx)\right)}{210b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 51*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) - (8*b*(a^2
 + 30*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*a*(8*a^2 + 159*b^2)*Ellip
```

```
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + 2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*
(8*a^2 + 75*b^2 + 15*b^2*Cos[2*(c + d*x)] - 6*a*b*Sin[c + d*x]))/(210*b^2*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(409) = 818$.
time = 9.08, size = 1155, normalized size = 3.42

method	result	size
default	Expression too large to display	1155

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/105*(15*b^5*sin(d*x+c)^5+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-53*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^
3+111*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a*b^4-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*b^5-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+59*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-51*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*a+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+18*a*b^4*sin(d*x+c)^
4-a^2*b^3*sin(d*x+c)^3-60*b^5*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2-63*a*b^4*
sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+45*b^5*sin(d*x+c)+4*a^3*b^2+45*a*b^4)/b^4/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\mathrm{cot}\left(c+d\,x\right)\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(1/2), x)
```

3.1146 $\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=323

$$\frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}-\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd}-\frac{\cot(c+dx)(a+b\sin(c+dx)}{ad}$$

```
[Out] -2/5*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(3/2
)/a/d+1/15*(4*a^2+15*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/b/d+1/15*(4*a
^2+57*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))
^(1/2)/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/15*a*(4*a^2+11*b^2)*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d
/(a+b*sin(d*x+c))^(1/2)-b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2973, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd}+\frac{a(4a^2+11b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+57b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd}-\frac{\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{ad}+\frac{b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((4*a^2 + 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*a*b*d) - (2*Co
s[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(5*b*d) - (Cot[c + d*x]*(a + b*Sin[c
 + d*x])^(3/2))/(a*d) - ((4*a^2 + 57*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(
a + b)]) + (a*(4*a^2 + 11*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^2*d*Sqrt[a + b*Sin[c + d*x]]) +
(b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
```

```
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{5bd} - \frac{\cot(c+dx)(a+}{}$$

$$= \frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(}{}$$

$$= \frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(}{}$$

$$= \frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(}{}$$

$$= \frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(}{}$$

$$= \frac{(4a^2+15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15abd} - \frac{2\cos(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.40, size = 422, normalized size = 1.31

$$\frac{\frac{2i(4a^2+57b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^3\sqrt{-\frac{1}{a+b}}}+\frac{184aF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(4a^2+27b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{b\sqrt{a+b\sin(c+dx)}}-\frac{4\sqrt{a+b\sin(c+dx)}(2a\cos(c+dx)+3b(5\cot(c+dx)+\sin(2(c+dx))))}{b}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 57*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^3*Sqrt[-(a + b)^(-1)]) + (184*a
*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 27*b^2)*EllipticPi[2, (-2*
```

```
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(b*Sq
rt[a + b*Sin[c + d*x]]) - (4*Sqrt[a + b*Sin[c + d*x]]*(2*a*Cos[c + d*x] + 3
*b*(5*Cot[c + d*x] + Sin[2*(c + d*x)])))/b)/(60*d)
```

Maple [A]
time = 9.57, size = 656, normalized size = 2.03

method	result
default	$-\frac{-6a\,b^{4}\sin(dx+c)(\cos^{4}(dx+c))+(2a^{3}b^{2}+21a\,b^{4})(\cos^{2}(dx+c))\sin(dx+c)+\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/15*(-6*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+21*a*b^4)*cos(d*x+c)^2*s
in(d*x+c)+(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(15*EllipticPi((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4-15*EllipticPi((b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5-4*EllipticE((b/(
a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-53*EllipticE((b/(a-
b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+57*EllipticE((b/(
a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+4*EllipticF((b/(a
-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+42*EllipticF((b/(a
-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+11*EllipticF((b/
(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-57*EllipticF((
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x+c)-8*
a^2*b^3*cos(d*x+c)^4+23*a^2*b^3*cos(d*x+c)^2)/a/b^3/sin(d*x+c)/cos(d*x+c)/(
a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin(c+dx)}\cos^2(c+dx)\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)**2*cot(c + d*x)**2, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^2\cot(c+d\,x)^2\sqrt{a+b\,\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(1/2), x)
```

3.1147 $\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=345

$$-\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}+\frac{b\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a\ldots}$$

```
[Out] 1/4*b*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+
b*sin(d*x+c))^(3/2)/a/d-1/12*(8*a^2+3*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2
)/a^2/d-1/12*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*
(a+b*sin(d*x+c))^(1/2)/a/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/12*(8*a^2+31*
b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b/d/(a+b*sin(d*x+c))^(1/2)+1/4*(12*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2972, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{(8a^2+31b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12abd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(12a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4ad\sqrt{a+b\sin(c+dx)}}+\frac{b\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/12*((8*a^2 + 3*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^2*d) + (b*
Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c +
d*x]*(a + b*Sin[c + d*x])^(3/2))/(2*a*d) + ((8*a^2 - 3*b^2)*EllipticE[(c -
Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a*b*d*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)]) - ((8*a^2 + 31*b^2)*EllipticF[(c - Pi/2 + d*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*b*d*Sqrt[a + b*Sin
[c + d*x]]) - ((12*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc(c\ \cdots}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c\ \cdots}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c\ \cdots}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c\ \cdots}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c\ \cdots}{}$$

$$= -\frac{(8a^2+3b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} + \frac{b\cot(c\ \cdots}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.30, size = 450, normalized size = 1.30

$$\frac{\frac{2i(8a^2-3b^2)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{a^2b^2\sqrt{-\frac{1}{a+b}}(-2+\csc^2(c+dx))}-\frac{4(8a\cos(c+dx)+3\cot(c+dx)(b+2a\csc(c+dx)))\sqrt{a+b\sin(c+dx)}}{a}+\frac{136bF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(64a^2+9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{a\sqrt{a+b\sin(c+dx)}}}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 - 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*(8*a*Cos[c + d*x] + 3*Cot
[c + d*x]*(b + 2*a*Csc[c + d*x]))*Sqrt[a + b*Sin[c + d*x]])/a + (136*b*Elli
pticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
```

```
b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(64*a^2 + 9*b^2)*EllipticPi[2, (-2*c + P
i - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a
+ b*Sin[c + d*x]]))/(48*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(414) = 828$.
time = 10.21, size = 1364, normalized size = 3.95

method	result	size
default	Expression too large to display	1364

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-36*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*si
n(d*x+c)^2+3*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*a*b^4*sin(d*x+c)^2-3*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*sin(d
*x+c)^2+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-42*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+31*b^3*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^2*sin(d*x+c)^2+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^
2+11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^2*sin(d*x+c)^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+8*a^2*b^3*sin(
d*x+c)^5+8*a^3*b^2*sin(d*x+c)^4+3*a*b^4*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^3-2
*a^3*b^2*sin(d*x+c)^2-3*a*b^4*sin(d*x+c)^2-9*a^2*b^3*sin(d*x+c)-6*a^3*b^2)/
a^2/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cos(d*x + c)*cot(d*x + c)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin(c + dx)}\cos(c + dx)\cot^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cos(c + d*x)*cot(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + d\,x)\,{\cot(c + d\,x)}^3\,\sqrt{a + b\,\sin(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(1/2), x)
```

3.1148 $\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=351

$$\frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}+\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d}-\frac{\cot(c+dx)\,\mathrm{cs}}{}$$

```
[Out] 1/4*b*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/3*cot(d*x+c)*csc
(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a/d+1/24*(32*a^2-3*b^2)*cot(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^2/d-1/24*(80*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+
1/24*(32*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)+1/8*b*(12*a^2-b^2)*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c
+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2804, 3126, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{(32a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{24ad\sqrt{a+b\sin(c+dx)}}+\frac{(80a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{24a^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{b(12a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{8a^2d\sqrt{a+b\sin(c+dx)}}+\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((32*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^2*d) + (b*Co
t[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(4*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(3*a*d) + ((80*a^2 + 3*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*a
^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((32*a^2 + b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a*d*Sq
rt[a + b*Sin[c + d*x]]) - (b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^2*d*Sqrt[a + b*S
in[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{4a^2d} - \frac{\cot(c+dx)\csc^2(}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} + \frac{b\cot(c+dx)\csc(c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.72, size = 473, normalized size = 1.35

$$\frac{-\frac{4\cot(c+dx)\left(-32a^2-3b^2+2ab\csc(c+dx)+8a^2\csc^2(c+dx)\right)\sqrt{a+b\sin(c+dx)}}{a^2}+\frac{\frac{2i\left(80a^2+3b^2\right)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}\left(-2+\csc^2(c+dx)\right)}-\frac{8a\left(24a^2+b^2\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2b\left(8a^2+9b^2\right)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{a^2}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 - 3*b^2 + 2*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x
]^2)*Sqrt[a + b*Sin[c + d*x]])/a^2 + (((2*I)*(80*a^2 + 3*b^2)*Cos[2*(c + d*
x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqr
t[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[
-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(
a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/
(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*
(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^
2)) - (8*a*(24*a^2 + b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
```

```
qrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*b*(8*a^2 +
 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^2)/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1494 vs. $2(420) = 840$.
time = 11.18, size = 1495, normalized size = 4.26

method	result	size
default	Expression too large to display	1495

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3-77*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a
*b^4*sin(d*x+c)^3-48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3-32*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3+7
8*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^3*sin(d*x+c)^3-b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*si
n(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2)
,(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b
^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*
sin(d*x+c)^3+32*a^3*b^2*sin(d*x+c)^5+3*a*b^4*sin(d*x+c)^5+32*a^4*b*sin(d*x+
c)^4+a^2*b^3*sin(d*x+c)^4-42*a^3*b^2*sin(d*x+c)^3-3*a*b^4*sin(d*x+c)^3-40*a
```

```
^4*b*sin(d*x+c)^2-a^2*b^3*sin(d*x+c)^2+10*a^3*b^2*sin(d*x+c)+8*a^4*b)/a^3/b
/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin{\left(c + dx \right)}} \cot^{4}{\left(c + dx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{cot}(c+d\,x)^4\,\sqrt{a+b\,\sin\left(c+d\,x\right)}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2), x)
```

3.1149 $\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=412

$$\frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d}+\frac{5(4a^2-b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d}+\frac{5b$$

```
[Out] 5/24*b*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/4*cot(d*x+c)*
csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a/d+1/192*b*(68*a^2-15*b^2)*cot(d*x+c)*
(a+b*sin(d*x+c))^(1/2)/a^3/d+5/32*(4*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^2/d-1/192*b*(68*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1
/2)-1/192*b*(196*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/64*(48*a^4+2
4*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2972, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b(196a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^2d\sqrt{a+b\sin(c+dx)}}+\frac{5(4a^2-b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d}+\frac{5b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}+\frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d}+\frac{b(68a^2-15b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(48a^4+24a^2b^2-5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (b*(68*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(192*a^3*d) + (
5*(4*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(32*a^2
*d) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*
d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(4*a*d) + (b*
(68*a^2 - 15*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*S
in[c + d*x]])/(192*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(196*a^2
+ 5*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d
*x])/(a + b)])/(192*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 + 24*a^2*b^2
 - 5*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[
c + d*x])/(a + b)])/(64*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
```

```
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
```

```
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{5b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} - \text{co}$$

$$= \frac{5(4a^2-b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{32a^2d}$$

$$= \frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5(4}{}$$

$$= \frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5(4}{}$$

$$= \frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5(4}{}$$

$$= \frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5(4}{}$$

$$= \frac{b(68a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^3d} + \frac{5(4}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.41, size = 643, normalized size = 1.56

$$\frac{\left(\frac{(68a^2b\cos(c+dx)-15b^3\cos(c+dx))\csc(c+dx)}{192a^3}+\frac{5(12a^2\cos(c+dx)+b^2\cos(c+dx))\csc^2(c+dx)}{96a^2}-\frac{b\cot(c+dx)\csc^2(c+dx)}{24a}-\frac{1}{4}\cot(c+dx)\csc^3(c+dx)\right)\sqrt{a+b\sin(c+dx)}}{d}+\frac{-\frac{2(528a^3b-20ab^3)F\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(288a^4+212a^2b^2-45b^4)\Pi\left(2;\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2i(-68a^2b^2+15b^4)\cos(c+dx)\cos(2(c+dx))\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sqrt{\frac{b-b\sin(c+dx)}{a+b}}\sqrt{-\frac{b+b\sin(c+dx)}{a-b}}}{a\sqrt{-\frac{1}{a+b}}\sqrt{1-\sin^2(c+dx)}\left(-2a^2+b^2+4a(a+b\sin(c+dx))-2(a+b\sin(c+dx))^2\right)\sqrt{-\frac{a^2-b^2-2a(a+b\sin(c+dx))+(a+b\sin(c+dx))^2}{b^2}}}}{768a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((68*a^2*b*Cos[c + d*x] - 15*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^3) +
(5*(12*a^2*Cos[c + d*x] + b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^2) - (b*C
ot[c + d*x]*Csc[c + d*x]^2)/(24*a) - (Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[
a + b*Sin[c + d*x]])/d + ((-2*(528*a^3*b - 20*a*b^3)*EllipticF[(-c + Pi/2 -
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]] - (2*(288*a^4 + 212*a^2*b^2 - 45*b^4)*EllipticPi[2, (-c + Pi/2 -
d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c
 + d*x]] - ((2*I)*(-68*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a
*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
 b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqr
t[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*
Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a +
 b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x])
 - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) +
 (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(477) = 954$.
time = 11.24, size = 1761, normalized size = 4.27

method	result	size
default	Expression too large to display	1761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-68*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-144*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b
)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+264*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-48*a^5+5*a^2*b
^3*sin(d*x+c)^5+66*a^3*b^2*sin(d*x+c)^4-15*a*b^4*sin(d*x+c)^4-5*a^2*b^3*sin
(d*x+c)^3+2*a^3*b^2*sin(d*x+c)^2-196*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-78*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^2*sin(d*x+c)^4-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
```

```
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)
^4+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi
(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*
x+c)^4+168*a^5*sin(d*x+c)^2-120*a^5*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3
*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+83*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d
*x+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-188*a^4*b*sin(d*x+c)^5+244*a^4*b*sin(d
*x+c)^3-56*a^4*b*sin(d*x+c)-68*a^3*b^2*sin(d*x+c)^6+15*a*b^4*sin(d*x+c)^6)/
a^4/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin\left(c+dx\right)}\,\cot^4\left(c+dx\right)\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x))*cot(c + d*x)**4*csc(c + d*x), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x),x)
```

```
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^5, x)
```

3.1150 $\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=484

$$-\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d}+\frac{b(108a^2-35b^2)\cot(c+dx)\csc(c+dx)\sqrt{a-}}{960a^3d}$$

[Out] 7/40*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(3/2)/a/d-1/1920*(384*a^4+332*a^2*b^2-105*b^4)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d+1/960*b*(108*a^2-35*b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+1/240*(96*a^2-35*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/1920*(384*a^4+332*a^2*b^2-105*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/1920*(384*a^4+116*a^2*b^2-35*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/128*b*(48*a^4-24*a^2*b^2+7*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 1.01, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2972, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(96a^2-35b^2)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{240a^2d}+\frac{7b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{40a^2d}-\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1920a^4d}-\frac{(384a^4+332a^2b^2-105b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1920a^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{b(48a^4-24a^2b^2+7b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{128a^4d\sqrt{a+b\sin(c+dx)}}+\frac{b(108a^2-35b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{960a^3d}+\frac{(384a^4+116a^2b^2-35b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1920a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{3/2}}{5ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]

[Out] -1/1920*((384*a^4 + 332*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^4*d) + (b*(108*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(960*a^3*d) + ((96*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(240*a^2*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2))/(5*a*d) - ((384*a^4 + 332*a^2*b^2 - 105*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1920*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((384*a^4 + 116*a^2*b^2 - 35*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1920*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(48*a^4 - 24*a^2*b^2 + 7*b^4

```
)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/(128*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
```

```
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}\,dx = \frac{7b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{40a^2d} - \cdots$$

$$= \frac{(96a^2-35b^2)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+\cdots}}{240a^2d}$$

$$= \frac{b(108a^2-35b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c\cdots}}{960a^3d}$$

$$= -\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(\cdots}}{1920a^4d}$$

$$= -\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(\cdots}}{1920a^4d}$$

$$= -\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(\cdots}}{1920a^4d}$$

$$= -\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(\cdots}}{1920a^4d}$$

$$= -\frac{(384a^4+332a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(\cdots}}{1920a^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.05, size = 545, normalized size = 1.13

$$\frac{-4\cot(c+dx)\left(384a^4+332a^2b^2-105b^4+(-216a^3b+70ab^3)\csc(c+dx)-8a^2(96a^2+7b^2)\csc^2(c+dx)+48a^3b\csc^3(c+dx)+384a^4\csc^4(c+dx)\right)\sqrt{a+b\sin(c+dx)}+b\left(-\frac{2i(384a^4+332a^2b^2-105b^4)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\cdots\right)\right)\cdots\right)\right)}{\cdots}\cdots\right)}{7680a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-4*Cot[c + d*x]*(384*a^4 + 332*a^2*b^2 - 105*b^4 + (-216*a^3*b + 70*a*b^3)
*Csc[c + d*x] - 8*a^2*(96*a^2 + 7*b^2)*Csc[c + d*x]^2 + 48*a^3*b*Csc[c + d*
x]^3 + 384*a^4*Csc[c + d*x]^4)*Sqrt[a + b*Sin[c + d*x]] + b*(((-2*I)*(384*a
^4 + 332*a^2*b^2 - 105*b^4)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*El
```

```
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*
(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*
b^2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) + (8*a*b*(108*a^2 - 35*b^2)*
EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1056*a^4 - 1052*a^2*b^2 + 315*b^4)*
EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]))/(7680*a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(545) = 1090$.

time = 14.38, size = 2075, normalized size = 4.29

method	result	size
default	Expression too large to display	2075

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/1920*(-384*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^5-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^5+
384*a^6*b+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2)
,(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-720*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*
b^3*sin(d*x+c)^5-360*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b)
)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+360*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2
))*a^2*b^5*sin(d*x+c)^5+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c)
)/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+52*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
5*b^2*sin(d*x+c)^5+437*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-105*((a+b*sin(d*x+c))/(a
```

```
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x
+c)^5+384*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^5-168*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+116*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4*b^3*sin(d*x+c)^5-402*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-35*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*
sin(d*x+c)^5+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+1152*a^6*b*sin(d*x+c)^4+124*a^4*
b^3*sin(d*x+c)^4-35*a^2*b^5*sin(d*x+c)^4-1416*a^5*b^2*sin(d*x+c)^3+14*a^3*b
^4*sin(d*x+c)^3-1152*a^6*b*sin(d*x+c)^2-8*a^4*b^3*sin(d*x+c)^2+432*a^5*b^2*
sin(d*x+c)+1368*a^5*b^2*sin(d*x+c)^5-105*a*b^6*sin(d*x+c)^5+318*a^3*b^4*sin
(d*x+c)^5-384*a^5*b^2*sin(d*x+c)^7-332*a^3*b^4*sin(d*x+c)^7+105*a*b^6*sin(d
*x+c)^7-384*a^6*b*sin(d*x+c)^6-116*a^4*b^3*sin(d*x+c)^6+35*a^2*b^5*sin(d*x+
c)^6)/a^5/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c) + a)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}{\sin\left(c+d\,x\right)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^2,x)
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(1/2))/sin(c + d*x)^6, x)
```

3.1151 $\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=528

$$\frac{8(64a^6-174a^4b^2+81a^2b^4-195b^6)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{45045b^5d}+\frac{16a(32a^4-47a^2b^2-27b^4)\cos(c+d\ldots}{45045b^5d}$$

```
[Out] 16/45045*a*(32*a^4-47*a^2*b^2-27*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^5
/d-8/45045*(160*a^4-375*a^2*b^2+117*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/
b^5/d+8/1287*a*(8*a^2-21*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2)/
b^4/d-2/2145*(80*a^2-221*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(5/2
)/b^3/d+4/39*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/b^2/d-2/15*co
s(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/b/d+8/45045*(64*a^6-174*a^4*b^
2+81*a^2*b^4-195*b^6)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+16/45045*a*(3
2*a^6-111*a^4*b^2+102*a^2*b^4-471*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a
+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/4
5045*(64*a^8-238*a^6*b^2+255*a^4*b^4-276*a^2*b^6+195*b^8)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*si
n(d*x+c))^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2974, 3128, 3102, 2832, 2831, 2742, 2740, 2734, 2732}

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (8*(64*a^6 - 174*a^4*b^2 + 81*a^2*b^4 - 195*b^6)*Cos[c + d*x]*Sqrt[a + b*Si
n[c + d*x]])/(45045*b^5*d) + (16*a*(32*a^4 - 47*a^2*b^2 - 27*b^4)*Cos[c + d
*x]*(a + b*Sin[c + d*x])^(3/2))/(45045*b^5*d) - (8*(160*a^4 - 375*a^2*b^2 +
 117*b^4)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(45045*b^5*d) + (8*a*(8*
a^2 - 21*b^2)*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(1287*b
^4*d) - (2*(80*a^2 - 221*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*(a + b*Sin[c + d*
x])^(5/2))/(2145*b^3*d) + (4*a*Cos[c + d*x]*Sin[c + d*x]^3*(a + b*Sin[c + d
*x])^(5/2))/(39*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]
)^(5/2))/(15*b*d) - (16*a*(32*a^6 - 111*a^4*b^2 + 102*a^2*b^4 - 471*b^6)*El
```

```
lipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(45045
*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^8 - 238*a^6*b^2 + 255
*a^4*b^4 - 276*a^2*b^6 + 195*b^8)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^6*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{4a\cos(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^{5/2}}{39b^2d} - \frac{2\ldots}{}$$

$$= -\frac{2(80a^2-221b^2)\cos(c+dx)\sin^2(c+dx)(a+b\sin(c\ldots}{2145b^3d}$$

$$= \frac{8a(8a^2-21b^2)\cos(c+dx)\sin(c+dx)(a+b\sin(c+\ldots}{1287b^4d}$$

$$= -\frac{8(160a^4-375a^2b^2+117b^4)\cos(c+dx)(a+b\sin(c\ldots}{45045b^5d}$$

$$= \frac{16a(32a^4-47a^2b^2-27b^4)\cos(c+dx)(a+b\sin(c+\ldots}{45045b^5d}$$

$$= \frac{8(64a^6-174a^4b^2+81a^2b^4-195b^6)\cos(c+dx)\sqrt{a+\ldots}}{45045b^5d}$$

$$= \frac{8(64a^6-174a^4b^2+81a^2b^4-195b^6)\cos(c+dx)\sqrt{a+\ldots}}{45045b^5d}$$

$$= \frac{8(64a^6-174a^4b^2+81a^2b^4-195b^6)\cos(c+dx)\sqrt{a+\ldots}}{45045b^5d}$$

$$= \frac{8(64a^6-174a^4b^2+81a^2b^4-195b^6)\cos(c+dx)\sqrt{a+\ldots}}{45045b^5d}$$

Mathematica [A]
time = 10.51, size = 382, normalized size = 0.72

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(512*(32*a^7 - 111*a^5*b^2 + 102*a^3*b^4 - 471*a*
b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 256*(64*a^7 - 64*a^6
*b - 174*a^5*b^2 + 174*a^4*b^3 + 81*a^3*b^4 - 81*a^2*b^5 - 195*a*b^6 + 195*
b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] - 2*b*Cos[c + d*x]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]*(4096*a^6 - 12416*a^4*b^2 + 8100*a^2*b^4 +
6786*b^6 + (-1280*a^4*b^2 + 3168*a^2*b^4 + 21723*b^6)*Cos[2*(c + d*x)] + 42
```

```
*(6*a^2*b^4 - 13*b^6)*Cos[4*(c + d*x)] - 3003*b^6*Cos[6*(c + d*x)] - 3072*a
^5*b*Sin[c + d*x] + 8432*a^3*b^3*Sin[c + d*x] - 41424*a*b^5*Sin[c + d*x] +
560*a^3*b^3*Sin[3*(c + d*x)] + 13776*a*b^5*Sin[3*(c + d*x)] + 7392*a*b^5*Si
n[5*(c + d*x)])))/(1441440*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1800 vs. $2(554) = 1108$.
time = 10.56, size = 1801, normalized size = 3.41

method	result	size
default	Expression too large to display	1801

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/45045*(-428*a^5*b^4+360*a^3*b^6-1704*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4-3768*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^8+25
6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^8*b-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^7*b^2-952*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^3+684*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4+102
0*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^4*b^5-3480*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6-1104*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^7+2988*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^8+
1144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^7*b^2+4584*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6+128*a^7*b^2+7644*b^9*sin(d*x+c)^7-5
109*b^9*sin(d*x+c)^5-312*b^9*sin(d*x+c)^3+780*b^9*sin(d*x+c)-3003*b^9*sin(d
```

```
*x+c)^9+780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*b^9-256*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^9+16*a^5*b^4*sin(d*x+c)^4-62*a^3*b^6*si
n(d*x+c)^4-12603*a*b^8*sin(d*x+c)^4-32*a^6*b^3*sin(d*x+c)^3+122*a^4*b^5*sin
(d*x+c)^3-8115*a^2*b^7*sin(d*x+c)^3-128*a^7*b^2*sin(d*x+c)^2+412*a^5*b^4*si
n(d*x+c)^2-305*a^3*b^6*sin(d*x+c)^2+840*a*b^8*sin(d*x+c)^2+32*a^6*b^3*sin(d
*x+c)-112*a^4*b^5*sin(d*x+c)+1512*a^2*b^7*sin(d*x+c)-6699*a*b^8*sin(d*x+c)^
8-3759*a^2*b^7*sin(d*x+c)^7+7*a^3*b^6*sin(d*x+c)^6+17682*a*b^8*sin(d*x+c)^6
-10*a^4*b^5*sin(d*x+c)^5+10362*a^2*b^7*sin(d*x+c)^5+780*a*b^8)/b^7/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 689, normalized size = 1.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/135135*(2*sqrt(2)*(128*a^8 - 492*a^6*b^2 + 561*a^4*b^4 + 114*a^2*b^6 + 58
5*b^8)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a
^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)
 + 2*sqrt(2)*(128*a^8 - 492*a^6*b^2 + 561*a^4*b^4 + 114*a^2*b^6 + 585*b^8)*
sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 12
*sqrt(2)*(-32*I*a^7*b + 111*I*a^5*b^3 - 102*I*a^3*b^5 + 471*I*a*b^7)*sqrt(I
*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b
^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 12*sqrt(2
)*(32*I*a^7*b - 111*I*a^5*b^3 + 102*I*a^3*b^5 - 471*I*a*b^7)*sqrt(-I*b)*wei
erstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, we
ierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^
```

```
3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(3003*b^8*co
s(d*x + c)^7 - 21*(3*a^2*b^6 + 208*b^8)*cos(d*x + c)^5 + 5*(16*a^4*b^4 - 27
*a^2*b^6 + 39*b^8)*cos(d*x + c)^3 - 2*(64*a^6*b^2 - 174*a^4*b^4 + 81*a^2*b^
6 - 195*b^8)*cos(d*x + c) - 2*(1848*a*b^7*cos(d*x + c)^5 + 35*(a^3*b^5 - 15
*a*b^7)*cos(d*x + c)^3 - 3*(16*a^5*b^3 - 41*a^3*b^5 + 249*a*b^7)*cos(d*x +
c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^7*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

3.1152 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=394

$$-\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}+\frac{8(32a^6-137a^4b^2+258a^2b^4+2}{15015}$$

```
[Out] -2/13*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-6/143*a*cos(d*x+c)^5*(a+b*sin(d
*x+c))^(1/2)/d-2/3003*cos(d*x+c)^3*(4*a*(2*a^2-5*b^2)-7*b*(a^2+11*b^2)*sin(
d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/15015*cos(d*x+c)*(a*(32*a^4-113*a^2*
b^2+177*b^4)-3*b*(8*a^4-27*a^2*b^2-77*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/
2)/b^4/d-8/15015*(32*a^6-137*a^4*b^2+258*a^2*b^4+231*b^6)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c
))/(a+b))^(1/2)+8/15015*a*(32*a^6-145*a^4*b^2+290*a^2*b^4-177*b^6)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/
d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(4a(2a^2-5b^2)-7b(a^2+11b^2)\sin(c+dx))}{3003b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(a(32a^4-113a^2b^2+177b^4)-3b(8a^4-27a^2b^2-77b^4)\sin(c+dx))}{15015b^4d}-\frac{8a(32a^6-145a^4b^2+290a^2b^4-177b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8(32a^6-137a^4b^2+258a^2b^4+231b^6)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}-\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-6*a*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(143*d) - (2*Cos[c + d*x]^5*
(a + b*Sin[c + d*x])^(3/2))/(13*d) + (8*(32*a^6 - 137*a^4*b^2 + 258*a^2*b^4
 + 231*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(15015*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^6 - 1
45*a^4*b^2 + 290*a^2*b^4 - 177*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^5*d*Sqrt[a + b*Sin[c + d
*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(4*a*(2*a^2 - 5*b^2) - 7
*b*(a^2 + 11*b^2)*Sin[c + d*x]))/(3003*b^2*d) + (4*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]]*(a*(32*a^4 - 113*a^2*b^2 + 177*b^4) - 3*b*(8*a^4 - 27*a^2*b^2
 - 77*b^4)*Sin[c + d*x]))/(15015*b^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
```

```
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{13d}+\frac{2}{13}\int \cos^4(c+$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

$$= -\frac{6a\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{143d}-\frac{2\cos^5(c+dx)}{}$$

Mathematica [A]

time = 8.05, size = 382, normalized size = 0.97

$$\frac{-384(32a^7+32a^6b-137a^5b^2-137a^4b^3+258a^3b^4+258a^2b^5+231ab^6+231b^7)E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+384a(32a^6-145a^4b^2+290a^2b^4-177b^6)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-3b\cos(c+dx)\left(-2048a^6+8640a^4b^2+1980a^2b^4-6622b^6+(-128a^4b^2+24512a^2b^4+8547b^6)\cos(2(c+dx))+70(86a^2b^4-11b^6)\cos(4(c+dx))-1155b^6\cos(6(c+dx))-512a^5b\sin(c+dx)+2088a^3b^3\sin(c+dx)-19492ab^5\sin(c+dx)+40a^3b^3\sin(3(c+dx))+11870ab^5\sin(3(c+dx))+5250ab^5\sin(5(c+dx))\right)}{720720b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-384*(32*a^7 + 32*a^6*b - 137*a^5*b^2 - 137*a^4*b^3 + 258*a^3*b^4 + 258*a^
2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
```

```
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(32*a^6 - 145*a^4*b^2 + 290*a^
2*b^4 - 177*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)] - 3*b*Cos[c + d*x]*(-2048*a^6 + 8640*a^4*b^2 + 198
0*a^2*b^4 - 6622*b^6 + (-128*a^4*b^2 + 24512*a^2*b^4 + 8547*b^6)*Cos[2*(c +
 d*x)] + 70*(86*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] - 1155*b^6*Cos[6*(c + d*
x)] - 512*a^5*b*Sin[c + d*x] + 2088*a^3*b^3*Sin[c + d*x] - 19492*a*b^5*Sin[
c + d*x] + 40*a^3*b^3*Sin[3*(c + d*x)] + 11870*a*b^5*Sin[3*(c + d*x)] + 525
0*a*b^5*Sin[5*(c + d*x)]))/(720720*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(432) = 864$.
time = 10.82, size = 1619, normalized size = 4.11

method	result	size
default	Expression too large to display	1619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15015*(-1016*a^2*b^6-580*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+420*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+1160*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^5+600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^2*b^6-708*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7+676*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-1580*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
4*b^4+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^2*b^6+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-96*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-924*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8-
128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
```

```
(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^8+924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*b^8+1155*b^8*sin(d*x+c)^8-3080*b^8*sin(d*x+c)^6+2
233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2+2625*a*b^7*sin(d*x+c)^7+1505*a^2*
b^6*sin(d*x+c)^6-5*a^3*b^5*sin(d*x+c)^5-7390*a*b^7*sin(d*x+c)^5+8*a^4*b^4*s
in(d*x+c)^4-4542*a^2*b^6*sin(d*x+c)^4-16*a^5*b^3*sin(d*x+c)^3+74*a^3*b^5*si
n(d*x+c)^3+6089*a*b^7*sin(d*x+c)^3-64*a^6*b^2*sin(d*x+c)^2+258*a^4*b^4*sin(
d*x+c)^2+4053*a^2*b^6*sin(d*x+c)^2+16*a^5*b^3*sin(d*x+c)-69*a^3*b^5*sin(d*x
+c)-1324*a*b^7*sin(d*x+c)+64*a^6*b^2-266*a^4*b^4)/b^6/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 632, normalized size = 1.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] -2/45045*(4*sqrt(2)*(32*a^7 - 149*a^5*b^2 + 306*a^3*b^4 - 381*a*b^6)*sqrt(I
*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 4*sqrt(2)*
(32*a^7 - 149*a^5*b^2 + 306*a^3*b^4 - 381*a*b^6)*sqrt(-I*b)*weierstrassPInv
erse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 6*sqrt(2)*(32*I*a^6*b - 137*
I*a^4*b^3 + 258*I*a^2*b^5 + 231*I*b^7)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^
2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(
4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c)
- 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 6*sqrt(2)*(-32*I*a^6*b + 137*I*a^4*b^3
- 258*I*a^2*b^5 - 231*I*b^7)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b
*sin(d*x + c) + 2*I*a)/b)) + 3*(1470*a*b^6*cos(d*x + c)^5 + 20*(2*a^3*b^4 -
```

```
5*a*b^6)*cos(d*x + c)^3 - 2*(32*a^5*b^2 - 113*a^3*b^4 + 177*a*b^6)*cos(d*x
 + c) + (1155*b^7*cos(d*x + c)^5 - 35*(a^2*b^5 + 11*b^7)*cos(d*x + c)^3 + 6
*(8*a^4*b^3 - 27*a^2*b^5 - 77*b^7)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d
*x + c) + a))/(b^6*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(3/2), x)
```

3.1153 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=390

$$-\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}-\frac{2(8a^2-77b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}+\frac{8a\cos(c$$

```
[Out] -2/315*(8*a^2-77*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d+8/63*a*cos(d*
x+c)*(a+b*sin(d*x+c))^(5/2)/b^2/d-2/9*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)
)^(5/2)/b/d-2/315*a*(8*a^2-87*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d-
2/315*(8*a^4-93*a^2*b^2+84*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/315*a*(8
*a^4-95*a^2*b^2-228*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*a^2*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4
*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a
+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2974, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{2(8a^2-77b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d}-\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d}+\frac{2a^2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{2a(8a^4-95a^2b^2-228b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^4-93a^2b^2+84b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8a\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63b^2d}-\frac{2\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a*(8*a^2 - 87*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d) -
 (2*(8*a^2 - 77*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b^2*d) +
 (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*b^2*d) - (2*Cos[c + d*x]
*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(9*b*d) + (2*(8*a^4 - 93*a^2*b^2
+ 84*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d
*x]])/(315*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^4 - 95*a^2
*b^2 - 228*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(315*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*a^2*Ellipti
cPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
```

```
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63b^2d} - \frac{2\cos(c+dx)}{}$$

$$= -\frac{2(8a^2-77b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315b^2d} + \cdots$$

$$= -\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \cdots$$

$$= -\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \cdots$$

$$= -\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \cdots$$

$$= -\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \cdots$$

$$= -\frac{2a(8a^2-87b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^2d} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.57, size = 477, normalized size = 1.22

$$\frac{-\frac{2i(8a^4-93a^2b^2+84b^4)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{ab^2\sqrt{-\frac{1}{a+b}}}-\frac{8ab(a^2+156b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(8a^4+537a^2b^2+84b^4)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\cos(c+dx)\sqrt{a+b\sin(c+dx)}\left(16a^3+556ab^2+100ab^2\cos(2(c+dx))+(-12a^2b+203b^3)\sin(c+dx)+35b^3\sin(3(c+dx))\right)}{630b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(8*a^4 - 93*a^2*b^2 + 84*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ell
ipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
- b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x])
)/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1
```

```
)]) - (8*a*b*(a^2 + 156*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)
]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^4
+ 537*a^2*b^2 + 84*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + Cos[c + d*x
]*Sqrt[a + b*Sin[c + d*x]]*(16*a^3 + 556*a*b^2 + 100*a*b^2*Cos[2*(c + d*x)]
 + (-12*a^2*b + 203*b^3)*Sin[c + d*x] + 35*b^3*Sin[3*(c + d*x)]))/(630*b^2*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(457) = 914$.

time = 10.85, size = 1405, normalized size = 3.60

method	result	size
default	Expression too large to display	1405

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-112*b^6*sin(d*x+c)^4+77*b^6*sin(d*x+c)^2+4*a^4*b
^2+164*a^2*b^4+315*a*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(
a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+8*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^5*b-6*EllipticF(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^4*b^2-95
*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*a^3*b^3+405*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*a^2*b^4-228*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a*b^5+101*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^4*b^2-177*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*a^2*b^4-315*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))
/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+85*a*b^5*sin(d*x+c)^5+53*a^2*b^4
*sin(d*x+c)^4-a^3*b^3*sin(d*x+c)^3-326*a*b^5*sin(d*x+c)^3-4*a^4*b^2*sin(d*x
+c)^2-217*a^2*b^4*sin(d*x+c)^2+a^3*b^3*sin(d*x+c)+241*a*b^5*sin(d*x+c)-84*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*b^6-8*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*(
```

```
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*a^6+84*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^6)/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)
/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^3 \cot(c+dx) \ (a+b \sin(c+dx))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(3/2), x)
```

3.1154 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=374

$$\frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}+\frac{(4a^2+35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}-\frac{2\cos(c+dx)}{}$$

```
[Out] 1/35*(4*a^2+35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a/b/d-2/7*cos(d*x+c)*
(a+b*sin(d*x+c))^(5/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a/d+1/35*(4*a^
2+65*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d+1/35*a*(4*a^2+167*b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-1/35*(4*a^4+61*a^2*b^2+40*b^4)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^2/d/(a+b*
sin(d*x+c))^(1/2)-3*a*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*
((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2973, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2+35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd}+\frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd}-\frac{a(4a^2+167b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(4a^4+61a^2b^2+40b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7bd}-\frac{\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{ad}+\frac{3ab\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 + 65*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(35*b*d) + ((4*a^2
 + 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(35*a*b*d) - (2*Cos[c +
 d*x]*(a + b*Sin[c + d*x])^(5/2))/(7*b*d) - (Cot[c + d*x]*(a + b*Sin[c + d*
x])^(5/2))/(a*d) - (a*(4*a^2 + 167*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(35*b^2*d*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)]) + ((4*a^4 + 61*a^2*b^2 + 40*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)
/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(35*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) + (3*a*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
```

```
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{7bd} - \frac{\cot(c+dx)(a}{}$$

$$= \frac{(4a^2+35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{35abd} - \frac{2\cos}{}$$

$$= \frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{(4a^2}{}$$

$$= \frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{(4a^2}{}$$

$$= \frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{(4a^2}{}$$

$$= \frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{(4a^2}{}$$

$$= \frac{(4a^2+65b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{35bd} + \frac{(4a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.77, size = 452, normalized size = 1.21

$$\frac{\frac{2i(4a^2+167b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^3\sqrt{-\frac{1}{a+b}}}+\frac{8(53a^2-20b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2a(4a^2-43b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{b\sqrt{a+b\sin(c+dx)}}-\frac{2\sqrt{a+b\sin(c+dx)}\left((4a^2-55b^2)\cos(c+dx)+b(-5b\cos(3(c+dx))+70a\cot(c+dx)+16a\sin(2(c+dx)))\right)}{b}}{140d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (8*(53*a^
```

```
2 - 20*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(4*a^2 - 43*b^2)*Ellip
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/(b*Sqrt[a + b*Sin[c + d*x]]) - (2*Sqrt[a + b*Sin[c + d*x]]*((4*a^2
- 55*b^2)*Cos[c + d*x] + b*(-5*b*Cos[3*(c + d*x)] + 70*a*Cot[c + d*x] + 16*
a*Sin[2*(c + d*x)])))/b)/(140*d)
```

Maple [A]
time = 8.98, size = 726, normalized size = 1.94

method	result
default	$-\frac{-26a\,b^4\sin(dx+c)(\cos^4(dx+c))+(2a^3b^2+31a\,b^4)(\cos^2(dx+c))\sin(dx+c)+\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\,\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/35*(-26*a*b^4*sin(d*x+c)*cos(d*x+c)^4+(2*a^3*b^2+31*a*b^4)*cos(d*x+c)^2*
sin(d*x+c)+(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b)
)^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(105*EllipticPi((b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4-105*EllipticPi((b/(a-
b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^5-4*EllipticE((
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-163*EllipticE((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+167*EllipticE
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+4*EllipticF(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b+102*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+61*Ellipti
cF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-207*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+40*Elli
pticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5)*sin(d*x+
c)+10*b^5*cos(d*x+c)^6+(-18*a^2*b^3+10*b^5)*cos(d*x+c)^4+(53*a^2*b^3-20*b^5
)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral((b*cos(d*x + c)^2*cot(d*x + c)^2*sin(d*x + c) + a*cos(d*x + c)^2*c
ot(d*x + c)^2)*sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^2 \cot(c+dx)^2 (a+b\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(3/2), x)
```

3.1155 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=383

$$-\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{(8a^2-5b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{b\cot(c+dx}{}$$

```
[Out] -1/20*(8*a^2-5*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d-1/4*b*cot(d*x+c
)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(
5/2)/a/d-1/20*(8*a^2-15*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/d-1/20*(8*
a^2-81*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c)
)^(1/2)/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/20*a*(8*a^2+37*b^2)*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b/d/(a
+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2972, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(8a^2-5b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d}-\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad}-\frac{a(8a^2+37b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{20bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-81b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{20bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\sin(c+dx)}}-\frac{b\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{4a^2d}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/20*((8*a^2 - 15*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a*d) - ((8*
a^2 - 5*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(20*a^2*d) - (b*Cot[c
 + d*x]*(a + b*Sin[c + d*x])^(5/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*
(a + b*Sin[c + d*x])^(5/2))/(2*a*d) + ((8*a^2 - 81*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(20*b*d*Sqrt[(a + b*Sin
[c + d*x])/(a + b)]) - (a*(8*a^2 + 37*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(20*b*d*Sqrt[a + b*Sin[c +
 d*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
```

```
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = -\frac{b\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{4a^2d} - \frac{\cot(c+dx)\,\mathrm{csc}}{}$$

$$= -\frac{(8a^2-5b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{20a^2d} - \frac{b\cot}{}$$

$$= -\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{(8a^2}{}$$

$$= -\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{(8a^2}{}$$

$$= -\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{(8a^2}{}$$

$$= -\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{(8a^2}{}$$

$$= -\frac{(8a^2-15b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{20ad} - \frac{(8a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.20, size = 434, normalized size = 1.13

$$\frac{\frac{2i(-8a^2+81b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^2\sqrt{-\frac{1}{a+b}}}+\frac{472abF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(112a^2+51b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+4\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}\,(-18a+8a\cos(2(c+dx))-31b\sin(c+dx)+2b\sin(3(c+dx)))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 81*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (472*
a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x
```

```
])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(112*a^2 + 51*b^2)*EllipticPi[2,
 (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] + 4*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d
*x]]*(-18*a + 8*a*Cos[2*(c + d*x)] - 31*b*Sin[c + d*x] + 2*b*Sin[3*(c + d*x
)]))/(80*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(448) = 896$.
time = 12.53, size = 1379, normalized size = 3.60

method	result	size
default	Expression too large to display	1379

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-126*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2+37*b^3*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^2*sin(d*x+c)^2+81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^
2+89*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^2*sin(d*x+c)^2-81*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+60*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b
))^(1/2))*a^3*sin(d*x+c)^2-60*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-15*Ellip
ticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*a*b^4*sin(d*x+c)^2+15*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*sin(d*x+c)^2+8*a*b^4*s
in(d*x+c)^6+24*a^2*b^3*sin(d*x+c)^5+16*a^3*b^2*sin(d*x+c)^4+17*a*b^4*sin(d*
x+c)^4+11*a^2*b^3*sin(d*x+c)^3-6*a^3*b^2*sin(d*x+c)^2-25*a*b^4*sin(d*x+c)^2
```

```
-35*a^2*b^3*sin(d*x+c)-10*a^3*b^2)/a/b^2/sin(d*x+c)^2/cos(d*x+c)/(a+b*sin(d
*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cos(d*x + c)*cot(d*x + c)^3, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)\cot(c+dx)^3(a+b\sin(c+dx))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(3/2), x)
```

3.1156 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=386

$$-\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{(32a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}+\frac{b\cot(c+dx)}{}$$

```
[Out] 1/24*(32*a^2+b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/12*b*cot(d*x+c)
*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*s
in(d*x+c))^(5/2)/a/d-1/8*b*(16*a^2+b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a
^2/d-1/8*(32*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*
sin(d*x+c))^(1/2)/a/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/8*(16*a^2+21*b^2)*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2
)/d/(a+b*sin(d*x+c))^(1/2)+1/8*b*(36*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(
1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2804, 3126, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d}+\frac{(32a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d}-\frac{(16a^2+21b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{8d\sqrt{a+b\sin(c+dx)}}+\frac{(32a^2-b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{8ad\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{b(36a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{8ad\sqrt{a+b\sin(c+dx)}}+\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{12a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/8*(b*(16*a^2 + b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^2*d) + ((3
2*a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(24*a^2*d) + (b*Cot[c
 + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(12*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(3*a*d) + ((32*a^2 - b^2)*Ellip
ticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(8*a*d*Sq
rt[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^2 + 21*b^2)*EllipticF[(c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a +
b*Sin[c + d*x]]) - (b*(36*a^2 + b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a*d*Sqrt[a + b*Sin[c + d*
x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

```
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{12a^2d} - \frac{\cot(c+dx)\csc^{}...}{}$$

$$= \frac{(32a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{24a^2d} + \frac{b\cot(c+dx)\csc...}{}$$

$$= -\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{(32a^2+b^2)\cot...}{}$$

$$= -\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{(32a^2+b^2)\cot...}{}$$

$$= -\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{(32a^2+b^2)\cot...}{}$$

$$= -\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{(32a^2+b^2)\cot...}{}$$

$$= -\frac{b(16a^2+b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{8a^2d} + \frac{(32a^2+b^2)\cot...}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.37, size = 486, normalized size = 1.26

$$\frac{\frac{2i(32a^2-b^2)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{a^2b\sqrt{-\frac{1}{a+b}}(-2+\csc^2(c+dx))}-\frac{4(16ab\cos(c+dx)+\cot(c+dx)(-32a^2+3b^2+14ab\csc(c+dx)+8a^2\csc^2(c+dx)))\sqrt{a+b\sin(c+dx)}}{3a}-\frac{8(8a^2-11b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(40a^2+3b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{a\sqrt{a+b\sin(c+dx)}}}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)*(32*a^2 - b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipti
cE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)
] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]
```

```
], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]
*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 +
 Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b*S
qrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*(16*a*b*Cos[c + d*x] + Cot[c
 + d*x]*(-32*a^2 + 3*b^2 + 14*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x]^2))*Sqr
t[a + b*Sin[c + d*x]])/(3*a) - (8*(8*a^2 - 11*b^2)*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]] + (2*b*(40*a^2 + 3*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a + b*Sin[c + d*x]])
)/(32*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(451) = 902$.
time = 12.52, size = 1511, normalized size = 3.91

method	result	size
default	Expression too large to display	1511

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3+48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-162*b^2*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3*sin(d*x+c)^3+63*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)
^3-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^5*sin(d*x+c)^3+99*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-3*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(
1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-108*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
```

```
a^2*b^3*sin(d*x+c)^3+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-
b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-3*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3+16*a^2*b^3*sin(d*x+c)^6-16*a^3*b^2*sin(d*x+c)^5+3*a*b^4*s
in(d*x+c)^5-32*a^4*b*sin(d*x+c)^4+a^2*b^3*sin(d*x+c)^4+38*a^3*b^2*sin(d*x+c
)^3-3*a*b^4*sin(d*x+c)^3+40*a^4*b*sin(d*x+c)^2-17*a^2*b^3*sin(d*x+c)^2-22*a
^3*b^2*sin(d*x+c)-8*a^4*b)/a^2/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cot(d*x + c)^4*sin(d*x + c) + a*cot(d*x + c)^4)*sqrt(b*sin(d*x
+ c) + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^4\,(a+b\,\sin\left(c+d\,x\right))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2), x)
```

3.1157 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=408

$$\frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{(20a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{32a^2d}+\frac{b}{}$$

```
[Out] 1/32*(20*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/8*b*
cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x
+c)^3*(a+b*sin(d*x+c))^(5/2)/a/d+1/64*b*(68*a^2-3*b^2)*cot(d*x+c)*(a+b*sin(
d*x+c))^(1/2)/a^2/d-1/64*b*(236*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
+1/64*b*(20*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)-3/64*(16*a^4-24*a^2*b^2
+b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/
(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.80, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2972, 3126, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}-\frac{b(20a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}+\frac{b(236a^2+3b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64a^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(20a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{32a^2d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d}+\frac{3(16a^4-24a^2b^2+b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64a^2d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (b*(68*a^2 - 3*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(64*a^2*d) + ((2
0*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(32*a^2*
d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(8*a^2*d) -
 (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(4*a*d) + (b*(236
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(64*a^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(20*a^2 + b^2)
*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
+ b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]]) + (3*(16*a^4 - 24*a^2*b^2 + b^4)*E
llipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(64*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
```

```
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \cot($$

$$= \frac{(20a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^3}{32a^2d}$$

$$= \frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{(20a^2}{}$$

$$= \frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{(20a^2}{}$$

$$= \frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{(20a^2}{}$$

$$= \frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{(20a^2}{}$$

$$= \frac{b(68a^2-3b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} + \frac{(20a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.47, size = 641, normalized size = 1.57

$$\frac{\left(\frac{3(36a^2b\cos(c+dx)+b^3\cos(c+dx))\csc(c+dx)}{64a^2}+\frac{(20a^2\cos(c+dx)-b^2\cos(c+dx))\csc^2(c+dx)}{32a}-\frac{3}{8}b\cot(c+dx)\csc^2(c+dx)-\frac{1}{4}a\cot(c+dx)\csc^3(c+dx)\right)\sqrt{a+b\sin(c+dx)}}{d}+\frac{-\frac{2(432a^3b+4ab^3)F\left(\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(96a^4+92a^2b^2+9b^4)\Pi\left(2;\frac{1}{2}\left(-c+\frac{\pi}{2}-dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2i(-236a^2b^2-3b^4)\cos(c+dx)\cos(2(c+dx))\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sqrt{\frac{b-b\sin(c+dx)}{a+b}}\sqrt{-\frac{b+b\sin(c+dx)}{a-b}}}{a\sqrt{-\frac{1}{a+b}}\sqrt{1-\sin^2(c+dx)}\left(-2a^2+b^2+4a(a+b\sin(c+dx))-2(a+b\sin(c+dx))^2\right)\sqrt{-\frac{a^2-b^2-2a(a+b\sin(c+dx))+(a+b\sin(c+dx))^2}{b^2}}}}{256a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((3*(36*a^2*b*Cos[c + d*x] + b^3*Cos[c + d*x])*Csc[c + d*x])/(64*a^2) + ((
20*a^2*Cos[c + d*x] - b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(32*a) - (3*b*Cot[c
 + d*x]*Csc[c + d*x]^2)/8 - (a*Cot[c + d*x]*Csc[c + d*x]^3)/4)*Sqrt[a + b*S
in[c + d*x]])/d + ((-2*(432*a^3*b + 4*a*b^3)*EllipticF[(-c + Pi/2 - d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
] - (2*(96*a^4 + 92*a^2*b^2 + 9*b^4)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
```

```
((2*I)*(-236*a^2*b^2 - 3*b^4)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*El
lipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a
 - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x
])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*S
qrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b
*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin
[c + d*x])^2)/b^2)]))/(256*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(473) = 946$.
time = 13.04, size = 1760, normalized size = 4.31

method	result	size
default	Expression too large to display	1760

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-
b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4+236*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+48*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*a^5*sin(d*x+c)^4-216*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+16*a^5+a^2*b^3*
sin(d*x+c)^5-134*a^3*b^2*sin(d*x+c)^4-3*a*b^4*sin(d*x+c)^4-a^2*b^3*sin(d*x+
c)^3+26*a^3*b^2*sin(d*x+c)^2-20*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4+234*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^
2*sin(d*x+c)^4-((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-48*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*s
```

```
in(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-5
6*a^5*sin(d*x+c)^2+40*a^5*sin(d*x+c)^4+72*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c
)^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+s
in(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a
,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-233*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^4-3*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4*sin(d*x+c)^4+148*a^4*b*sin(d*x+c)^5-188*a^4*b*sin(d*x+c)^3+40*
a^4*b*sin(d*x+c)+108*a^3*b^2*sin(d*x+c)^6+3*a*b^4*sin(d*x+c)^6)/a^3/sin(d*x
+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{3/2}}{\sin\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x),x)
```

```
[Out] int(((sin(c + d*x)^2 - 1)^2*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^5, x)
```

3.1158 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=484

$$-\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}+\frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{320a^2d}$$

```
[Out] 1/80*(32*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/
8*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/5*cot(d*x+c)*csc
(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/a/d-1/640*(128*a^4-116*a^2*b^2+15*b^4)*cot
(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+3/320*b*(36*a^2-5*b^2)*cot(d*x+c)*csc(
d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/640*(128*a^4-116*a^2*b^2+15*b^4)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((
a+b*sin(d*x+c))/(a+b))^(1/2)-1/640*(128*a^4+692*a^2*b^2+5*b^4)*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*P
i+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a
+b*sin(d*x+c))^(1/2)-3/128*b*(48*a^4+8*a^2*b^2-b^4)*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d
*x+c))^(1/2)
```

Rubi [A]
time = 1.03, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2972, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(32a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{80a^2d}+\frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{320a^2d}+\frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d}+\frac{(128a^4+692a^2b^2+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{640a^2d\sqrt{a+b\sin(c+dx)}}-\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^3d}-\frac{(128a^4-116a^2b^2+15b^4)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{640a^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{3b(48a^4+8a^2b^2-b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{128a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{5/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/640*((128*a^4 - 116*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*
x]])/(a^3*d) + (3*b*(36*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*S
in[c + d*x]])/(320*a^2*d) + ((32*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(
a + b*Sin[c + d*x])^(3/2))/(80*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3*(a +
 b*Sin[c + d*x])^(5/2))/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin
[c + d*x])^(5/2))/(5*a*d) - ((128*a^4 - 116*a^2*b^2 + 15*b^4)*EllipticE[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^3*d*Sqrt[(
a + b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 692*a^2*b^2 + 5*b^4)*EllipticF[
(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640
```

```
*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(48*a^4 + 8*a^2*b^2 - b^4)*Elliptic
Pi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)]
)/(128*a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
```

```
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{8a^2d} - \frac{\text{co}}{}$$

$$= \frac{(32a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+}{80a^2d}$$

$$= \frac{3b(36a^2-5b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}}{320a^2d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^3d}$$

$$= -\frac{(128a^4-116a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.04, size = 544, normalized size = 1.12

$$\frac{-4\cot(c+dx)\left(128a^4-116a^2b^2+15b^4-2ab(236a^2+5b^2)\csc(c+dx)+8a^2(-32a^2+b^2)\csc^2(c+dx)+176a^3b\csc^3(c+dx)+128a^4\csc^4(c+dx)\right)\sqrt{a+b\sin(c+dx)}+b\left(-\frac{2i(128a^4-116a^2b^2+15b^4)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^2\sqrt{-\frac{1}{a+b}}(-2+\csc^2(c+dx))}-\frac{8ab(404a^2-5b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(1312a^4+356a^2b^2-45b^4)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}\right)}{2560a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-4*Cot[c + d*x]*(128*a^4 - 116*a^2*b^2 + 15*b^4 - 2*a*b*(236*a^2 + 5*b^2)*
Csc[c + d*x] + 8*a^2*(-32*a^2 + b^2)*Csc[c + d*x]^2 + 176*a^3*b*Csc[c + d*x
]^3 + 128*a^4*Csc[c + d*x]^4)*Sqrt[a + b*Sin[c + d*x]] + b*(((-2*I)*(128*a^
4 - 116*a^2*b^2 + 15*b^4)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Elli
pticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
 b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*
x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-
1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (8*a*b*(404*a^2 - 5*b^2)*Ell
ipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
 b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(1312*a^4 + 356*a^2*b^2 - 45*b^4)*Ellip
ticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]]))/(2560*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(545) = 1090$.
time = 15.08, size = 2075, normalized size = 4.29

method	result	size
default	Expression too large to display	2075

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/640*(-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+
b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^5+12
8*a^6*b+720*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-720*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^
3*sin(d*x+c)^5+120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
```

```
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-120*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(
a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+244*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*
b^2*sin(d*x+c)^5-131*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)
^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^6*b*sin(d*x+c)^5-936*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+692*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^3*sin(d*x+c)^5+126*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(
d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5+384*a^6*b*sin(d*x+c)^4-772*a^4*b^3*si
n(d*x+c)^4+5*a^2*b^5*sin(d*x+c)^4-1032*a^5*b^2*sin(d*x+c)^3-2*a^3*b^4*sin(d
*x+c)^3-384*a^6*b*sin(d*x+c)^2+184*a^4*b^3*sin(d*x+c)^2+304*a^5*b^2*sin(d*x
+c)+856*a^5*b^2*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^5-114*a^3*b^4*sin(d*x+c)^5
-128*a^5*b^2*sin(d*x+c)^7+116*a^3*b^4*sin(d*x+c)^7-15*a*b^6*sin(d*x+c)^7-12
8*a^6*b*sin(d*x+c)^6+588*a^4*b^3*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^6)/a^4/b
/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^2,x)
```

```
[Out] \text{Hanged}
```

3.1159 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=551

$$-\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}-\frac{(240a^4-168a^2b^2+35b^4)\cot(c+dx)\csc(c}{3840a^3d}$$

```
[Out] 7/96*(4*a^2-b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2)/a^2/d+7/60*
b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/6*cot(d*x+c)*csc(d
*x+c)^5*(a+b*sin(d*x+c))^(5/2)/a/d-1/7680*b*(2064*a^4+512*a^2*b^2-105*b^4)*
cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d-1/3840*(240*a^4-168*a^2*b^2+35*b^4)
*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+1/960*b*(156*a^2-35*b^2
)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/7680*b*(2064*a^4+5
12*a^2*b^2-105*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*si
n(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/7680*b*(2544*a^4+176
*a^2*b^2-35*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(
d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/512*(64*a^6+144*a^4*b^2
-36*a^2*b^4+7*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b
*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 1.24, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2972, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{7(4a^2-b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}}{96a^2d}+\frac{b(156a^2-35b^2)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{960a^2d}+\frac{7b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{5/2}}{60a^2d}-\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{7680a^4d}-\frac{b(2064a^4+512a^2b^2-105b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{7680a^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(64a^6+144a^4b^2-36a^2b^4+7b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{512a^4d\sqrt{a+b\sin(c+dx)}}+\frac{b(2544a^4+176a^2b^2-35b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{7680a^3d\sqrt{a+b\sin(c+dx)}}-\frac{(240a^4-168a^2b^2+35b^4)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{3840a^3d}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^{5/2}}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/7680*(b*(2064*a^4 + 512*a^2*b^2 - 105*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c
 + d*x]])/(a^4*d) - ((240*a^4 - 168*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c +
d*x]*Sqrt[a + b*Sin[c + d*x]])/(3840*a^3*d) + (b*(156*a^2 - 35*b^2)*Cot[c +
 d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(960*a^2*d) + (7*(4*a^2 - b^
2)*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2))/(96*a^2*d) + (7*
b*Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2))/(60*a^2*d) - (Cot
[c + d*x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(5/2))/(6*a*d) - (b*(2064*a^4
 + 512*a^2*b^2 - 105*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt
```

```
[a + b*Sin[c + d*x]])/(7680*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*
(2544*a^4 + 176*a^2*b^2 - 35*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(7680*a^3*d*Sqrt[a + b*Sin[c + d*x]
]) + ((64*a^6 + 144*a^4*b^2 - 36*a^2*b^4 + 7*b^6)*EllipticPi[2, (c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^4*d*Sqrt
[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{3/2}\,dx = \frac{7b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{5/2}}{60a^2d} - \cdots$$

$$= \frac{7(4a^2-b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+d\cdots}{96a^2d}$$

$$= \frac{b(156a^2-35b^2)\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(\cdots}}{960a^2d}$$

$$= -\frac{(240a^4-168a^2b^2+35b^4)\cot(c+dx)\csc(c+dx)\cdots}{3840a^3d}$$

$$= -\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\,\mathrm{si}\cdots}}{7680a^4d}$$

$$= -\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\,\mathrm{si}\cdots}}{7680a^4d}$$

$$= -\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\,\mathrm{si}\cdots}}{7680a^4d}$$

$$= -\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\,\mathrm{si}\cdots}}{7680a^4d}$$

$$= -\frac{b(2064a^4+512a^2b^2-105b^4)\cot(c+dx)\sqrt{a+b\,\mathrm{si}\cdots}}{7680a^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.52, size = 771, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((((-2064*a^4*b*Cos[c + d*x] - 512*a^2*b^3*Cos[c + d*x] + 105*b^5*Cos[c + d
*x])*Csc[c + d*x])/(7680*a^4) + ((-240*a^4*Cos[c + d*x] + 168*a^2*b^2*Cos[c
```

```
 + d*x] - 35*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(3840*a^3) + ((436*a^2*b*Cos
[c + d*x] + 7*b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(960*a^2) + ((140*a^2*Cos[c
 + d*x] - 3*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/(480*a) - (13*b*Cot[c + d*x]*
Csc[c + d*x]^4)/60 - (a*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c +
d*x]])/d + ((-2*(960*a^5*b - 672*a^3*b^3 + 140*a*b^5)*EllipticF[(-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - (2*(1920*a^6 + 2256*a^4*b^2 - 1592*a^2*b^4 + 315*b^6)*Elliptic
Pi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(2064*a^4*b^2 + 512*a^2*b^4 - 105*b^6)
*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin
[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a
^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2
 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(30720*
a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(608) = 1216$.
time = 20.35, size = 2458, normalized size = 4.46

method	result	size
default	Expression too large to display	2458

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/7680*(1280*a^7-3520*a^7*sin(d*x+c)^2-480*a^7*sin(d*x+c)^6+2720*a^7*sin(d
*x+c)^4-105*a*b^6*sin(d*x+c)^6-35*a^2*b^5*sin(d*x+c)^5+14*a^3*b^4*sin(d*x+c
)^4-8*a^4*b^3*sin(d*x+c)^3+1712*a^5*b^2*sin(d*x+c)^2-960*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^
6*b*sin(d*x+c)^6+2160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-2160*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^4*b^3*sin(d*x+c)^6-540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6+540*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
```

```
+b))^(1/2))*a^2*b^5*sin(d*x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*s
in(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6+155
2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^5*b^2*sin(d*x+c)^6-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^6-2064*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^7*sin(d*x+c)^6-480*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a
-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6+960*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*El
lipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*si
n(d*x+c)^6+617*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-105*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6+25
44*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^6*b*sin(d*x+c)^6-1728*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6+176*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
4*b^3*sin(d*x+c)^6-582*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-35*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*
x+c)^6+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-8672*a^6*b*sin(d*x+c)^3+2944*a^6*b*sin
(d*x+c)-2064*a^5*b^2*sin(d*x+c)^8-512*a^3*b^4*sin(d*x+c)^8+498*a^3*b^4*sin(
d*x+c)^6+5888*a^5*b^2*sin(d*x+c)^6+184*a^4*b^3*sin(d*x+c)^5+105*a*b^6*sin(d
*x+c)^8-2544*a^6*b*sin(d*x+c)^7-176*a^4*b^3*sin(d*x+c)^7+35*a^2*b^5*sin(d*x
+c)^7+8272*a^6*b*sin(d*x+c)^5-5536*a^5*b^2*sin(d*x+c)^4)/a^5/sin(d*x+c)^6/c
os(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(3/2)*cot(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(3/2))/sin(c + d*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.1160 $\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=451

$$-\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2a\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{39d}-\frac{2\cos^5(c+dx)(a+\cdots}{15\cdots}$$

```
[Out] -2/39*a*cos(d*x+c)^5*(a+b*sin(d*x+c))^(3/2)/d-2/15*cos(d*x+c)^5*(a+b*sin(d*
x+c))^(5/2)/d-2/429*(3*a^2+13*b^2)*cos(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/d-2/
9009*cos(d*x+c)^3*(8*a^4-33*a^2*b^2-39*b^4-7*a*b*(a^2+63*b^2)*sin(d*x+c))*(
a+b*sin(d*x+c))^(1/2)/b^2/d+4/45045*cos(d*x+c)*(32*a^6-165*a^4*b^2+450*a^2*
b^4+195*b^6-24*a*b*(a^4-5*a^2*b^2-60*b^4)*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2
)/b^4/d-8/45045*a*(32*a^6-189*a^4*b^2+570*a^2*b^4+1635*b^6)*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x
+c))/(a+b))^(1/2)+8/45045*(32*a^8-197*a^6*b^2+615*a^4*b^4-255*a^2*b^6-195*b
^8)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b)
)^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.69, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2941, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2\cos^3(c+dx)\sqrt{a+b\sin(c+dx)}\,(8a^4-7ab(a^2+63b^2)\sin(c+dx)-33a^2b^2-39b^4)}{9009b^2d}+\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a^6-165a^4b^2+450a^2b^4-24ab(a^4-5a^2b^2-60b^4)\sin(c+dx)+195b^6)}{45045b^4d}+\frac{8a(32a^6-189a^4b^2+570a^2b^4+1635b^6)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{45045b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8(32a^8-197a^6b^2+615a^4b^4-255a^2b^6-195b^8)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{45045b^5d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{5/2}}{15d}-\frac{2a\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{39d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(3*a^2 + 13*b^2)*Cos[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(429*d) - (2*
a*Cos[c + d*x]^5*(a + b*Sin[c + d*x])^(3/2))/(39*d) - (2*Cos[c + d*x]^5*(a
+ b*Sin[c + d*x])^(5/2))/(15*d) + (8*a*(32*a^6 - 189*a^4*b^2 + 570*a^2*b^4
+ 1635*b^6)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c +
 d*x]])/(45045*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*(32*a^8 - 197
*a^6*b^2 + 615*a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(45045*b^5*d*Sqrt[a +
 b*Sin[c + d*x]]) - (2*Cos[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]]*(8*a^4 - 33*
a^2*b^2 - 39*b^4 - 7*a*b*(a^2 + 63*b^2)*Sin[c + d*x]))/(9009*b^2*d) + (4*Co
s[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^6 - 165*a^4*b^2 + 450*a^2*b^4 + 1
95*b^6 - 24*a*b*(a^4 - 5*a^2*b^2 - 60*b^4)*Sin[c + d*x]))/(45045*b^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
```

```
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \cos^4(c+dx)\sin(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos^5(c+dx)(a+b\sin(c+dx))^{5/2}}{15d}+\frac{2}{15}\int \cos^4(c$$

$$= -\frac{2a\cos^5(c+dx)(a+b\sin(c+dx))^{3/2}}{39d}-\frac{2\cos^5(c+$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

$$= -\frac{2(3a^2+13b^2)\cos^5(c+dx)\sqrt{a+b\sin(c+dx)}}{429d}-\frac{2}{}$$

Mathematica [A]
time = 14.99, size = 450, normalized size = 1.00

$$\frac{-256a(32a^7+32a^6b-189a^5b^2-189a^4b^3+570a^3b^4+570a^2b^5+1635ab^6+1635b^7)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+256(32a^8-197a^6b^2+615a^4b^4-255a^2b^6-195b^8)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\left(4096a^7-23936a^5b^2-36512a^3b^4+67584ab^6+8(32a^5b^2-18192a^3b^4-18741ab^6)\cos(2(c+dx))-224(161a^3b^4-54ab^6)\cos(4(c+dx))+20328ab^6\cos(6(c+dx))+1024a^6b\sin(c+dx)-5840a^4b^3\sin(c+dx)+186768a^2b^5\sin(c+dx)+8151b^7\sin(c+dx)-80a^4b^3\sin(3(c+dx))-101688a^2b^5\sin(3(c+dx))-22269b^7\sin(3(c+dx))-46536a^2b^5\sin(5(c+dx))-2457b^7\sin(5(c+dx))+3003b^7\sin(7(c+dx))\right)}{1441440b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]

[Out] (-256*a*(32*a^7 + 32*a^6*b - 189*a^5*b^2 - 189*a^4*b^3 + 570*a^3*b^4 + 570*a^2*b^5 + 1635*a*b^6 + 1635*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 256*(32*a^8 - 197*a^6*b^2 + 615*a^4*b^4 - 255*a^2*b^6 - 195*b^8)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*Cos[c + d*x]*(4096*a^7 - 23936*a^5*b^2 - 36512*a^3*b^4 + 67584*a*b^6 + 8*(32*a^5*b^2 - 18192*a^3*b^4 - 18741*a*b^6)*Cos[2*(c + d*x)] - 224*(161*a^3*b^4 - 54*a*b^6)*Cos[4*(c + d*x)] + 20328*a*b^6*Cos[6*(c + d*x)] + 1024*a^6*b*Sin[c + d*x] - 5840*a^4*b^3*Sin[c + d*x] + 186768*a^2*b^5*Sin[c + d*x] + 8151*b^7*Sin[c + d*x] - 80*a^4*b^3*Sin[3*(c + d*x)] - 101688*a^2*b^5*Sin[3*(c + d*x)] - 22269*b^7*Sin[3*(c + d*x)] - 46536*a^2*b^5*Sin[5*(c + d*x)] - 2457*b^7*Sin[5*(c + d*x)] + 3003*b^7*Sin[7*(c + d*x)]))/(1441440*b^5*d*Sqrt[a + b*Sin[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1800 vs. $2(485) = 970$.
time = 10.83, size = 1801, normalized size = 3.99

method	result	size
default	Expression too large to display	1801

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/45045*(-370*a^5*b^4-3408*a^3*b^6-3036*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4+6540*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^8+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^8*b-96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b^2-788*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^3+576*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^4+2460*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^5+5280*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6-1020*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(

```
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^7-5760*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^8+8
84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^7*b^2-4260*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^6+64*a^7*b^2-7644*b^9*sin(d*x+c)^7+5109
*b^9*sin(d*x+c)^5+312*b^9*sin(d*x+c)^3-780*b^9*sin(d*x+c)+3003*b^9*sin(d*x+
c)^9-780*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*b^9-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^9+8*a^5*b^4*sin(d*x+c)^4-13564*a^3*b^6*sin
(d*x+c)^4+19302*a*b^8*sin(d*x+c)^4-16*a^6*b^3*sin(d*x+c)^3+100*a^4*b^5*sin(
d*x+c)^3+26382*a^2*b^7*sin(d*x+c)^3-64*a^7*b^2*sin(d*x+c)^2+362*a^5*b^4*sin
(d*x+c)^2+12464*a^3*b^6*sin(d*x+c)^2-1764*a*b^8*sin(d*x+c)^2+16*a^6*b^3*sin
(d*x+c)-95*a^4*b^5*sin(d*x+c)-5484*a^2*b^7*sin(d*x+c)+10164*a*b^8*sin(d*x+c
)^8+11634*a^2*b^7*sin(d*x+c)^7+4508*a^3*b^6*sin(d*x+c)^6-26922*a*b^8*sin(d*
x+c)^6-5*a^4*b^5*sin(d*x+c)^5-32532*a^2*b^7*sin(d*x+c)^5-780*a*b^8)/b^6/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 688, normalized size = 1.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] -2/135135*(2*sqrt(2)*(64*a^8 - 402*a^6*b^2 + 1275*a^4*b^4 - 2400*a^2*b^6 -
585*b^8)*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I
*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/
```

```
b) + 2*sqrt(2)*(64*a^8 - 402*a^6*b^2 + 1275*a^4*b^4 - 2400*a^2*b^6 - 585*b^
8)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3
 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) +
 6*sqrt(2)*(32*I*a^7*b - 189*I*a^5*b^3 + 570*I*a^3*b^5 + 1635*I*a*b^7)*sqrt
(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)
/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*
b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 6*sqrt(
2)*(-32*I*a^7*b + 189*I*a^5*b^3 - 570*I*a^3*b^5 - 1635*I*a*b^7)*sqrt(-I*b)*
weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3,
 weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)
/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(3003*b^8
*cos(d*x + c)^7 - 21*(213*a^2*b^6 + 208*b^8)*cos(d*x + c)^5 - 5*(8*a^4*b^4
- 33*a^2*b^6 - 39*b^8)*cos(d*x + c)^3 + 2*(32*a^6*b^2 - 165*a^4*b^4 + 450*a
^2*b^6 + 195*b^8)*cos(d*x + c) - (7161*a*b^7*cos(d*x + c)^5 - 35*(a^3*b^5 +
 63*a*b^7)*cos(d*x + c)^3 + 48*(a^5*b^3 - 5*a^3*b^5 - 60*a*b^7)*cos(d*x + c
))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^6*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3876 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^4*sin(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))^(5/2), x)
```

3.1161 $\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=447

$$-\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}-\frac{2a(8a^2-131b^2)\cos(c+dx)(a+b\sin(c+dx))}{693b^2d}$$

```
[Out] -2/693*a*(8*a^2-131*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b^2/d-2/693*(8*a
^2-117*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/b^2/d+8/99*a*cos(d*x+c)*(a+b*
sin(d*x+c))^(7/2)/b^2/d-2/11*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(7/2)/b
/d-2/693*(8*a^4-141*a^2*b^2+36*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d
-2/693*a*(8*a^4-147*a^2*b^2+444*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b
))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+2/693
*(8*a^6-149*a^4*b^2-516*a^2*b^4-36*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*
a^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticP
i(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.90, antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2974, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2a^3\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}-\frac{2(8a^2-117b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d}-\frac{2a(8a^2-131b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d}-\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^2d}+\frac{2a(8a^4-147a^2b^2+444b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{693b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2(8a^6-149a^4b^2-516a^2b^4-36b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{693b^3d\sqrt{a+b\sin(c+dx)}}+\frac{8a\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{99b^2d}-\frac{2\sin(c+dx)\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{11bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4 - 141*a^2*b^2 + 36*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(
693*b^2*d) - (2*a*(8*a^2 - 131*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2)
)/(693*b^2*d) - (2*(8*a^2 - 117*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2
))/(693*b^2*d) + (8*a*Cos[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(99*b^2*d) -
 (2*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(11*b*d) + (2*a*(
8*a^4 - 147*a^2*b^2 + 444*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]
*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) -
 (2*(8*a^6 - 149*a^4*b^2 - 516*a^2*b^4 - 36*b^6)*EllipticF[(c - Pi/2 + d*x)
/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(693*b^3*d*Sqrt[a +
b*Sin[c + d*x]]) + (2*a^3*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
```

```
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{8a\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{99b^2d} - \frac{2\cos(c+dx)\mathrm{si}}{}$$

$$= -\frac{2(8a^2-117b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{693b^2d} + \frac{8}{}$$

$$= -\frac{2a(8a^2-131b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{693b^2d} -$$

$$= -\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+}}{693b^2d}$$

$$= -\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+}}{693b^2d}$$

$$= -\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+}}{693b^2d}$$

$$= -\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+}}{693b^2d}$$

$$= -\frac{2(8a^4-141a^2b^2+36b^4)\cos(c+dx)\sqrt{a+b\sin(c+}}{693b^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.73, size = 521, normalized size = 1.17

$$\frac{-2\left(\frac{2i(8a^4-147a^2b^2+444b^4)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^2\sqrt{-\frac{1}{a+b}}}+\frac{8b(a^4+480a^2b^2+18b^4)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2a(8a^4+1239a^2b^2+444b^4)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}\right)+\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a^4+2660a^2b^2-9b^4+4(113a^2b^2-54b^4)\cos(2(c+dx))-63b^4\cos(4(c+dx))-24a^3b\sin(c+dx)+1954ab^3\sin(c+dx)+322ab^3\sin(3(c+dx)))}{2772b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(((2*I)*(8*a^4 - 147*a^2*b^2 + 444*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
```

```
 b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^
(-1)]) + (8*b*(a^4 + 480*a^2*b^2 + 18*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]
] + (2*a*(8*a^4 + 1239*a^2*b^2 + 444*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d
*x]]) + Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^4 + 2660*a^2*b^2 - 9*b^
4 + 4*(113*a^2*b^2 - 54*b^4)*Cos[2*(c + d*x)] - 63*b^4*Cos[4*(c + d*x)] - 2
4*a^3*b*Sin[c + d*x] + 1954*a*b^3*Sin[c + d*x] + 322*a*b^3*Sin[3*(c + d*x)]
))/(2772*b^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(510) = 1020$.

time = 10.11, size = 1573, normalized size = 3.52

method	result	size
default	Expression too large to display	1573

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/693*(-693*a^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*EllipticPi(((a+b*sin(d*x+c))/(a-b))
^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+693*a^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*Ellipti
cPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))+63*b^7*sin(
d*x+c)^7-180*b^7*sin(d*x+c)^5+153*b^7*sin(d*x+c)^3-36*b^7*sin(d*x+c)-591*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^3*b^4+444*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a*b^6-149*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+1107*((a+b*sin(d*x+c))/(a-
b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E
llipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4-516*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^2*b^5-408*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a*b^6+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-6*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+155*((a+b*sin
```

```
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
5*b^2+518*a*b^6*sin(d*x+c)^2+a^4*b^3*sin(d*x+c)+754*a^2*b^5*sin(d*x+c)+224*
a*b^6*sin(d*x+c)^6+274*a^2*b^5*sin(d*x+c)^5+116*a^3*b^4*sin(d*x+c)^4-706*a*
b^6*sin(d*x+c)^4-a^4*b^3*sin(d*x+c)^3-1028*a^2*b^5*sin(d*x+c)^3-4*a^5*b^2*s
in(d*x+c)^2-505*a^3*b^4*sin(d*x+c)^2+389*a^3*b^4-36*a*b^6+4*a^5*b^2-36*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^7-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^7)/b^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^3*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral((2*a*b*cos(d*x + c)^3*cot(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c
)^5 - (a^2 + b^2)*cos(d*x + c)^3)*cot(d*x + c))*sqrt(b*sin(d*x + c) + a), x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\mathrm{cot}(c+d\,x)\,\left(a+b\,\sin\left(c+d\,x\right)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*cot(c + d*x)*(a + b*sin(c + d*x))^(5/2), x)
```

3.1162 $\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=426

$$\frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{(4a^2+6$$

```
[Out] 1/315*(20*a^2+469*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/b/d+1/63*(4*a^2+63
*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a/b/d-2/9*cos(d*x+c)*(a+b*sin(d*x+c
))^(7/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(7/2)/a/d+1/315*a*(20*a^2+759*b^2)
*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d+1/315*(20*a^4+1689*a^2*b^2-168*b^4)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^2/
d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/315*a*(20*a^4+739*a^2*b^2+816*b^4)*(sin(
1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*
c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b
^2/d/(a+b*sin(d*x+c))^(1/2)-5*a^2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a
+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2973, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd}+\frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd}+\frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd}+\frac{5a^2b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}+\frac{a(20a^4+739a^2b^2+816b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(20a^4+1689a^2b^2-168b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{9bd}-\frac{\cot(c+dx)(a+b\sin(c+dx))^{7/2}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*(20*a^2 + 759*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b*d) + ((
20*a^2 + 469*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(315*b*d) + ((4*
a^2 + 63*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(63*a*b*d) - (2*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(9*b*d) - (Cot[c + d*x]*(a + b*Sin[c +
 d*x])^(7/2))/(a*d) - ((20*a^4 + 1689*a^2*b^2 - 168*b^4)*EllipticE[(c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^2*d*Sqrt[(a + b
*Sin[c + d*x])/(a + b)]) + (a*(20*a^4 + 739*a^2*b^2 + 816*b^4)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b
^2*d*Sqrt[a + b*Sin[c + d*x]]) + (5*a^2*b*EllipticPi[2, (c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
```

```
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{2\cos(c+dx)(a+b\sin(c+dx))^{7/2}}{9bd} - \frac{\cot(c+dx)(}{}$$

$$= \frac{(4a^2+63b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{63abd} - \frac{2\,\mathrm{c}}{}$$

$$= \frac{(20a^2+469b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{315bd} + $$

$$= \frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + $$

$$= \frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + $$

$$= \frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + $$

$$= \frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + $$

$$= \frac{a(20a^2+759b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315bd} + $$

Mathematica [C] Result contains complex when optimal does not.
time = 13.35, size = 496, normalized size = 1.16

$$\frac{-\frac{2i(-20a^4-1689a^2b^2+168b^4)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{ab^2\sqrt{-\frac{1}{a+b}}}+\frac{8ab(475a^2-492b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(20a^4-1461a^2b^2-168b^4)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\sqrt{a+b\sin(c+dx)}\left((40a^3-2202ab^2)\cos(c+dx)+2b(-95ab\cos(3(c+dx))+630a^2\cot(c+dx)+(150a^2-119b^2-35b^2\cos(2(c+dx)))\sin(2(c+dx)))\right)}{1260bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)*(-20*a^4 - 1689*a^2*b^2 + 168*b^4)*(-2*a*(a - b)*EllipticE[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*
a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b
)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
```

```
b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c +
d*x]))/(a + b))]*Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b
)^(-1)]) + (8*a*b*(475*a^2 - 492*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (
2*(20*a^4 - 1461*a^2*b^2 - 168*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] -
Sqrt[a + b*Sin[c + d*x]]*((40*a^3 - 2202*a*b^2)*Cos[c + d*x] + 2*b*(-95*a*
b*Cos[3*(c + d*x)] + 630*a^2*Cot[c + d*x] + (150*a^2 - 119*b^2 - 35*b^2*Cos
[2*(c + d*x)])*Sin[2*(c + d*x)])))/(1260*b*d)
```

Maple [A]
time = 11.10, size = 864, normalized size = 2.03

method	result
default	$-\frac{70b^6\sin(dx+c)\left(\cos^6(dx+c)\right)+\left(-340a^2b^4+14b^6\right)\left(\cos^4(dx+c)\right)\sin(dx+c)+\left(10a^4b^2+57a^2b^4-84b^6\right)\left(\cos^2(dx+c)\right)\sin(dx+c)+\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/315*(70*b^6*sin(d*x+c)*cos(d*x+c)^6+(-340*a^2*b^4+14*b^6)*cos(d*x+c)^4*s
in(d*x+c)+(10*a^4*b^2+57*a^2*b^4-84*b^6)*cos(d*x+c)^2*sin(d*x+c)+(-b/(a-b)*
sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2)*(1575*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4-1575*EllipticPi((b/(a-b)*sin(d*x+c)+a/(
a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5-20*EllipticE((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6-1669*EllipticE((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+1857*EllipticE((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-168*EllipticE((b/(a-b)
*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6+20*EllipticF((b/(a-b)*s
in(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+930*EllipticF((b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+739*EllipticF((b/(a-
b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-2673*EllipticF((b
/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4+816*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+168*Elliptic
F((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6)*sin(d*x+c)+2
60*a*b^5*cos(d*x+c)^6+(-160*a^3*b^3+232*a*b^5)*cos(d*x+c)^4+(475*a^3*b^3-49
2*a*b^5)*cos(d*x+c)^2)/sin(d*x+c)/b^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)^2*cot(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] integral((2*a*b*cos(d*x + c)^2*cot(d*x + c)^2*sin(d*x + c) - (b^2*cos(d*x +
 c)^4 - (a^2 + b^2)*cos(d*x + c)^2)*cot(d*x + c)^2)*sqrt(b*sin(d*x + c) + a
), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\mathrm{cot}(c+d\,x)^2\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*cot(c + d*x)^2*(a + b*sin(c + d*x))^(5/2), x)
```

3.1163 $\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=430

$$-\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d}-\frac{(8a^2-35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{(8a^2-21b^2)}{}$$

```
[Out] -1/28*(8*a^2-35*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a/d-1/28*(8*a^2-21*b
^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-3/4*b*cot(d*x+c)*(a+b*sin(d*x+c
))^(7/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(7/2)/a/d-1/28*(8
*a^2-73*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/d-1/28*a*(8*a^2-247*b^2)*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b/d/((a
+b*sin(d*x+c))/(a+b))^(1/2)+1/28*(8*a^4+3*a^2*b^2-32*b^4)*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2
*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b/d/(a+b*sin(
d*x+c))^(1/2)+3/4*a*(4*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b
))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2972, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(8a^2-21b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d}-\frac{(8a^2-35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad}-\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d}+\frac{a(8a^2-247b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{28bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3a(4a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4d\sqrt{a+b\sin(c+dx)}}-\frac{3b\cot(c+dx)(a+b\sin(c+dx))^{7/2}}{4a^2d}-\frac{(8a^4+3a^2b^2-32b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{28bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{7/2}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/28*((8*a^2 - 73*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/d - ((8*a^2
- 35*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(28*a*d) - ((8*a^2 - 21*
b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(28*a^2*d) - (3*b*Cot[c + d*x
]*(a + b*Sin[c + d*x])^(7/2))/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]*(a + b
*Sin[c + d*x])^(7/2))/(2*a*d) + (a*(8*a^2 - 247*b^2)*EllipticE[(c - Pi/2 +
d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(28*b*d*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)]) - ((8*a^4 + 3*a^2*b^2 - 32*b^4)*EllipticF[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(28*b*d*Sqrt[a + b*
Sin[c + d*x]]) - (3*a*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*d*Sqrt[a + b*Sin[c + d*x
]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{3b\cot(c+dx)(a+b\sin(c+dx))^{7/2}}{4a^2d} - \frac{\cot(c+dx)\,\cdots}{\cdots}$$

$$= -\frac{(8a^2-21b^2)\cos(c+dx)(a+b\sin(c+dx))^{5/2}}{28a^2d} - \frac{3b\cdots}{\cdots}$$

$$= -\frac{(8a^2-35b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{28ad} - \frac{(8\cdots}{\cdots}$$

$$= -\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{(8a\cdots}{\cdots}$$

$$= -\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{(8a\cdots}{\cdots}$$

$$= -\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{(8a\cdots}{\cdots}$$

$$= -\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{(8a\cdots}{\cdots}$$

$$= -\frac{(8a^2-73b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{28d} - \frac{(8a\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.75, size = 460, normalized size = 1.07

$$\frac{\frac{2i(-8a^2+247b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^2\sqrt{-\frac{1}{a+b}}}+\frac{8b(125a^2-16b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2a(160a^2+37b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+4\sqrt{a+b\sin(c+dx)}\left((-24a^2+22b^2)\cos(c+dx)+2b^2\cos(3(c+dx))-7a\cot(c+dx)(9b+2a\csc(c+dx))-12ab\sin(2(c+dx))\right)}{112d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-8*a^2 + 247*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^
(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*Arc
Sinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*El
lipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*
```

```
Sqrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (8*b*(12
5*a^2 - 16*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*a*(160*a^2 + 37*b^2)
*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*
x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + 4*Sqrt[a + b*Sin[c + d*x]]*((-24*a
^2 + 22*b^2)*Cos[c + d*x] + 2*b^2*Cos[3*(c + d*x)] - 7*a*Cot[c + d*x]*(9*b
+ 2*a*Csc[c + d*x]) - 12*a*b*Sin[2*(c + d*x)]))/(112*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1519 vs. $2(491) = 982$.
time = 12.15, size = 1520, normalized size = 3.53

method	result	size
default	Expression too large to display	1520

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(8*b^5*sin(d*x+c)^7+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-258*b^2*((a+b*sin(d*x+
c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^
(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin
(d*x+c)^2+3*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+279*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-32*b^
5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+255*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+
c)^2-247*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2+84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^2*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-
84*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/
a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2-105*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a*b^4*sin(d*x+c
```

```
)^2+105*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/
2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*b^5*sin(d*x+c)^2+32*a*b^4*sin(d*x+c)^6+48*a^2*b^3*s
in(d*x+c)^5-32*b^5*sin(d*x+c)^5+24*a^3*b^2*sin(d*x+c)^4+7*a*b^4*sin(d*x+c)^
4+29*a^2*b^3*sin(d*x+c)^3+24*b^5*sin(d*x+c)^3-10*a^3*b^2*sin(d*x+c)^2-39*a*
b^4*sin(d*x+c)^2-77*a^2*b^3*sin(d*x+c)-14*a^3*b^2)/b^2/sin(d*x+c)^2/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cos(d*x + c)*cot(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] integral((2*a*b*cos(d*x + c)*cot(d*x + c)^3*sin(d*x + c) - (b^2*cos(d*x + c
)^3 - (a^2 + b^2)*cos(d*x + c))*cot(d*x + c)^3)*sqrt(b*sin(d*x + c) + a), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,{\mathrm{cot}\left(c+d\,x\right)}^3\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*cot(c + d*x)^3*(a + b*sin(c + d*x))^(5/2), x)
```

3.1164 $\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=429

$$-\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}-\frac{b(208a^2-25b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}+\frac{(32a^2}{}$$

```
[Out] -1/120*b*(208*a^2-25*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/24*(32*
a^2-3*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d-1/12*b*cot(d*x+c)*csc(d*
x+c)*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+
c))^(7/2)/a/d-1/40*b*(96*a^2-25*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/d-
1/40*(176*a^2-167*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b
*sin(d*x+c))^(1/2)/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/40*a*(96*a^2+179*b^2)
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(
1/2)/d/(a+b*sin(d*x+c))^(1/2)+5/8*b*(12*a^2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^
(1/2)
```

Rubi [A]

time = 0.88, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2804, 3126, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{b(208a^2-25b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d}-\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad}+\frac{(32a^2-3b^2)\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{24a^2d}-\frac{a(96a^2+179b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{40d\sqrt{a+b\sin(c+dx)}}+\frac{(176a^2-167b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{40d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{5b(12a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8d\sqrt{a+b\sin(c+dx)}}-\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{7/2}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/40*(b*(96*a^2 - 25*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a*d) - (
b*(208*a^2 - 25*b^2)*Cos[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(120*a^2*d) +
 ((32*a^2 - 3*b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(24*a^2*d) - (b
*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c
+ d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(3*a*d) + ((176*a^2 - 167
*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]]
)/(40*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (a*(96*a^2 + 179*b^2)*Ellipti
cF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(
40*d*Sqrt[a + b*Sin[c + d*x]]) - (5*b*(12*a^2 - b^2)*EllipticPi[2, (c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = -\frac{b\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \frac{\cot(c+dx)\operatorname{csc}}{}$$

$$= \frac{(32a^2-3b^2)\cot(c+dx)(a+b\sin(c+dx))^{5/2}}{24a^2d} - \frac{b\cot(c+dx)\operatorname{csc}(}{}$$

$$= -\frac{b(208a^2-25b^2)\cos(c+dx)(a+b\sin(c+dx))^{3/2}}{120a^2d} + \frac{(32a^2-3b^2}{}$$

$$= -\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b(208a^2-25b}{}$$

$$= -\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b(208a^2-25b}{}$$

$$= -\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b(208a^2-25b}{}$$

$$= -\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b(208a^2-25b}{}$$

$$= -\frac{b(96a^2-25b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{40ad} - \frac{b(208a^2-25b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.48, size = 466, normalized size = 1.09

$$\frac{\frac{2i(-176a^2+167b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}}-\frac{8a(40a^2-173b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(424a^2+117b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{4}{3}\sqrt{a+b\sin(c+dx)}\left(176ab\cos(c+dx)+5\cot(c+dx)\left(-32a^2+33b^2+26ab\csc(c+dx)+8a^2\csc^2(c+dx)\right)+24b^2\sin(2(c+dx))\right)}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(-176*a^2 + 167*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b
)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*A
rcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*
EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))
]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]) - (8*a
*(40*a^2 - 173*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(424*a^2 + 117
*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
 + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (4*Sqrt[a + b*Sin[c + d*x]]*(
176*a*b*Cos[c + d*x] + 5*Cot[c + d*x]*(-32*a^2 + 33*b^2 + 26*a*b*Csc[c + d*
x] + 8*a^2*Csc[c + d*x]^2) + 24*b^2*Sin[2*(c + d*x)]))/3)/(160*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(490) = 980$.
time = 11.21, size = 1526, normalized size = 3.56

method	result	size
default	Expression too large to display	1526

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(240*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)^3+288*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*
sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-1566*b^2*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^3*sin(d*x+c)^3+537*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+501*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin
(d*x+c)^3-528*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3+1029*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((
a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-501*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x
+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4*sin(d*x+c)^3+900*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-900*((a+
```

```
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))*a^2*b^3*sin(d*x+c)^3-75*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*si
n(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+75*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*b^5*sin(d*x+c)^3+48*a*b^4*sin(d*x+c)^7+224*a^2*b^3*sin(d*x+c
)^6+16*a^3*b^2*sin(d*x+c)^5+117*a*b^4*sin(d*x+c)^5-160*a^4*b*sin(d*x+c)^4+7
1*a^2*b^3*sin(d*x+c)^4+154*a^3*b^2*sin(d*x+c)^3-165*a*b^4*sin(d*x+c)^3+200*
a^4*b*sin(d*x+c)^2-295*a^2*b^3*sin(d*x+c)^2-170*a^3*b^2*sin(d*x+c)-40*a^4*b
)/a/b/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(c+d\,x)^4\,(a+b\,\sin(c+d\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2), x)
```

3.1165 $\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=449

$$-\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b(68a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}+\frac{(60a^2+$$

```
[Out] 5/192*b*(68*a^2+b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/96*(60*a^2+b
^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2)/a^2/d+1/24*b*cot(d*x+c)*cs
c(d*x+c)^2*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3*(a+b*si
n(d*x+c))^(7/2)/a/d-1/64*b^2*(196*a^2+5*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1
/2)/a^2/d-1/64*b*(492*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/a/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+1/64*b*(148
*a^2+169*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/d/(a+b*sin(d*x+c))^(1/2)-1/64*(48*a^4-360*a^2*b^2-5*b^4)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))
^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.96, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2972, 3126, 3128, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d}+\frac{5b(68a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}-\frac{b(148a^2+169b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64d\sqrt{a+b\sin(c+dx)}}+\frac{b(492a^2-5b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(60a^2+b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d}+\frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{7/2}}{24a^2d}+\frac{(48a^4-360a^2b^2-5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64ad\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{7/2}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/64*(b^2*(196*a^2 + 5*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^2*d)
 + (5*b*(68*a^2 + b^2)*Cot[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d)
 + ((60*a^2 + b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2))/(9
6*a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(7/2))/(24*a
^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(7/2))/(4*a*d) +
(b*(492*a^2 - 5*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a +
b*Sin[c + d*x]])/(64*a*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (b*(148*a^2
+ 169*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
 d*x])/(a + b)])/(64*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 360*a^2*b^2 -
 5*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(64*a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
```

```
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{7/2}}{24a^2d} - \text{co}$$

$$= \frac{(60a^2+b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx)}{96a^2d}$$

$$= \frac{5b(68a^2+b^2)\cot(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d} + \frac{(60}{}$$

$$= -\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} +$$

$$= -\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} +$$

$$= -\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} +$$

$$= -\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} +$$

$$= -\frac{b^2(196a^2+5b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{64a^2d} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.48, size = 655, normalized size = 1.46

$$\frac{\left(-\frac{2}{3}b^2\cos(c+dx)+\frac{5(116a^2b\cos(c+dx)-3b^3\cos(c+dx))\csc(c+dx)}{192a}+\frac{1}{96}(60a^2\cos(c+dx)-59b^2\cos(c+dx))\csc^2(c+dx)-\frac{17}{24}ab\cot(c+dx)\csc^2(c+dx)-\frac{1}{4}a^2\cot(c+dx)\csc^3(c+dx)\right)\sqrt{a+b\sin(c+dx)}}{d}+\frac{-\frac{2(688a^2b-348ab^3)F(\ldots)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(96a^4-228a^2b^2-15b^4)\Pi(\ldots)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2i(-492a^2b^2+5b^4)\cos(c+dx)\cos(2(c+dx))\left(\ldots\right)\sqrt{\frac{b-b\sin(c+dx)}{a+b}}\sqrt{-\frac{b+b\sin(c+dx)}{a-b}}}{a\sqrt{-\frac{1}{a+b}}\sqrt{1-\sin^2(c+dx)}\,(\ldots)\sqrt{-\frac{a^2-b^2-2a(a+b\sin(c+dx))+(a+b\sin(c+dx))^2}{b^2}}}}{256ad}$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((-2*b^2*Cos[c + d*x])/3 + (5*(116*a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*x]
)*Csc[c + d*x])/(192*a) + ((60*a^2*Cos[c + d*x] - 59*b^2*Cos[c + d*x])*Csc[
c + d*x]^2)/96 - (17*a*b*Cot[c + d*x]*Csc[c + d*x]^2)/24 - (a^2*Cot[c + d*x
]*Csc[c + d*x]^3)/4)*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(688*a^3*b - 348*a*
b^3)*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(96*a^4 - 228*a^2*b^2 - 15*b^4)*E
llipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(-492*a^2*b^2 + 5*b^4)*Cos[c +
d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x]
)/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2
+ 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 -
2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(256*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1776 vs. $2(510) = 1020$.
time = 13.73, size = 1777, normalized size = 3.96

method	result	size
default	Expression too large to display	1777

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-1476*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-14
4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a
-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+1032*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4-48*a^5+5*a^
2*b^3*sin(d*x+c)^5+706*a^3*b^2*sin(d*x+c)^4-15*a*b^4*sin(d*x+c)^4-133*a^2*b
^3*sin(d*x+c)^3-254*a^3*b^2*sin(d*x+c)^2+444*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(
((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-1998
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^3*b^2*sin(d*x+c)^4+507*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
```

```
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^
4*sin(d*x+c)^4+144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4+1080*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
a^3*b^2*sin(d*x+c)^4+168*a^5*sin(d*x+c)^2-120*a^5*sin(d*x+c)^4-1080*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b)
)^(1/2))*a^2*b^3*sin(d*x+c)^4+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+
c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d
*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+1491*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a^3*b^2*sin(d*x+c)^4-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-700*a^4*b*sin(d*x+c)
^5+884*a^4*b*sin(d*x+c)^3-184*a^4*b*sin(d*x+c)-452*a^3*b^2*sin(d*x+c)^6+15*
a*b^4*sin(d*x+c)^6+128*a^2*b^3*sin(d*x+c)^7)/a^2/sin(d*x+c)^4/cos(d*x+c)/(a
+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4*csc(d*x + c), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)*(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x),x)
```

```
[Out] \text{Hanged}
```

3.1166 $\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=482

$$-\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d}+\frac{b(36a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin$$

[Out] 1/64*b*(36*a^2-b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(3/2)/a^2/d+1/80*(32*a^2-b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2)/a^2/d+3/40*b*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(7/2)/a/d-1/640*(128*a^4-580*a^2*b^2+15*b^4)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/640*(128*a^4-2476*a^2*b^2-15*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/640*(128*a^4+492*a^2*b^2-5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)-3/128*b*(80*a^4-40*a^2*b^2+b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 1.04, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$,
Rules used = {2972, 3126, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(32a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}}{80a^2d}+\frac{b(36a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx))^{3/2}}{64a^2d}+\frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{7/2}}{40a^2d}-\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{640a^2d}+\frac{(128a^4+492a^2b^2-5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{640ad\sqrt{a+b\sin(c+dx)}}-\frac{(128a^4-2476a^2b^2-15b^4)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{640a^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{3b(80a^4-40a^2b^2+b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{128a^2d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{7/2}}{5ad}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]

[Out] -1/640*((128*a^4 - 580*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^2*d) + (b*(36*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]*(a + b*Sin[c + d*x])^(3/2))/(64*a^2*d) + ((32*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2))/(80*a^2*d) + (3*b*Cot[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(7/2))/(40*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(7/2))/(5*a*d) - ((128*a^4 - 2476*a^2*b^2 - 15*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(640*a^2*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((128*a^4 + 492*a^2*b^2 - 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(640*

```
a*d*Sqrt[a + b*Sin[c + d*x]]) + (3*b*(80*a^4 - 40*a^2*b^2 + b^4)*EllipticPi
[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
(128*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{3b\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{7/2}}{40a^2d} - \text{co}$$

$$= \frac{(32a^2-b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx)}{80a^2d}$$

$$= \frac{b(36a^2-b^2)\cot(c+dx)\csc(c+dx)(a+b\sin(c+dx)}{64a^2d}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^2d}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^2d}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^2d}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^2d}$$

$$= -\frac{(128a^4-580a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c}}{640a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.26, size = 545, normalized size = 1.13

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-4*Cot[c + d*x]*(128*a^4 - 1196*a^2*b^2 - 15*b^4 + (-872*a^3*b + 10*a*b^3)
*Csc[c + d*x] - 8*a^2*(32*a^2 - 31*b^2)*Csc[c + d*x]^2 + 336*a^3*b*Csc[c +
d*x]^3 + 128*a^4*Csc[c + d*x]^4)*Sqrt[a + b*Sin[c + d*x]] + b*(((-2*I)*(128
*a^4 - 2476*a^2*b^2 - 15*b^4)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*
EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/
```

(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (8*a*b*(1484*a^2 + 5*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(2272*a^4 + 1276*a^2*b^2 + 45*b^4)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]))/(2560*a^2*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. $2(543) = 1086$.
time = 13.41, size = 2075, normalized size = 4.30

method	result	size
default	Expression too large to display	2075

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/640*(128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^5-128*a^6*b-1200*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+1200*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d*x+c)^5+600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5-600*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-2604*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5+2461*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*E

```
llipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+
c)^5-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^6*b*sin(d*x+c)^5+3096*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^5-492*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^4*b^3*sin(d*x+c)^5-2466*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^5+5*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*
sin(d*x+c)^5-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1
/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^5-384*a^6*b*sin(d*x+c)^4+2652*a^4*b
^3*sin(d*x+c)^4+5*a^2*b^5*sin(d*x+c)^4+1592*a^5*b^2*sin(d*x+c)^3-258*a^3*b^
4*sin(d*x+c)^3+384*a^6*b*sin(d*x+c)^2-584*a^4*b^3*sin(d*x+c)^2-464*a^5*b^2*
sin(d*x+c)-1256*a^5*b^2*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^5+1454*a^3*b^4*sin
(d*x+c)^5+128*a^5*b^2*sin(d*x+c)^7-1196*a^3*b^4*sin(d*x+c)^7-15*a*b^6*sin(d
*x+c)^7+128*a^6*b*sin(d*x+c)^6-2068*a^4*b^3*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+
c)^6)/a^3/b/sin(d*x+c)^5/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4*csc(d*x + c)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**2*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^2*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x)^2,x)
```

```
[Out] \text{Hanged}
```

3.1167 $\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=551

$$-\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d}-\frac{(16a^4-56a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)}{256a^2d}$$

```
[Out] 1/192*b*(52*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(3/2)/a^2/d
+1/96*(28*a^2-3*b^2)*cot(d*x+c)*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2)/a^2/d+1
/12*b*cot(d*x+c)*csc(d*x+c)^4*(a+b*sin(d*x+c))^(7/2)/a^2/d-1/6*cot(d*x+c)*c
sc(d*x+c)^5*(a+b*sin(d*x+c))^(7/2)/a/d-1/1536*b*(720*a^4-176*a^2*b^2+15*b^4
)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d-1/256*(16*a^4-56*a^2*b^2+5*b^4)*c
ot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d+1/1536*b*(720*a^4-176*a^2
*b^2+15*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c
))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/1536*b*(816*a^4+1696*a^2*b^
2+5*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/
(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/512*(64*a^6+720*a^4*b^2+60*a^2*
b^4-5*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x
+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2972, 3126, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(28a^2-3b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}}{96a^2d}+\frac{b(52a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^{3/2}}{192a^2d}+\frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d}+\frac{b(816a^4+1696a^2b^2+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1536a^2d\sqrt{a+b\sin(c+dx)}}-\frac{(16a^4-56a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{256a^2d}-\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{1536a^3d}-\frac{b(720a^4-176a^2b^2+15b^4)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1536a^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(64a^6+720a^4b^2+60a^2b^4-5b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{512a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^{7/2}}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/1536*(b*(720*a^4 - 176*a^2*b^2 + 15*b^4)*Cot[c + d*x]*Sqrt[a + b*Sin[c +
 d*x]])/(a^3*d) - ((16*a^4 - 56*a^2*b^2 + 5*b^4)*Cot[c + d*x]*Csc[c + d*x]*
Sqrt[a + b*Sin[c + d*x]])/(256*a^2*d) + (b*(52*a^2 - 5*b^2)*Cot[c + d*x]*Cs
c[c + d*x]^2*(a + b*Sin[c + d*x])^(3/2))/(192*a^2*d) + ((28*a^2 - 3*b^2)*Co
t[c + d*x]*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2))/(96*a^2*d) + (b*Cot[c
 + d*x]*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^(7/2))/(12*a^2*d) - (Cot[c + d*
x]*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^(7/2))/(6*a*d) - (b*(720*a^4 - 176*a
^2*b^2 + 15*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Si
```

```
n[c + d*x]])/(1536*a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(816*a^4
+ 1696*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/(1536*a^2*d*Sqrt[a + b*Sin[c + d*x]]) + ((64*
a^6 + 720*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*EllipticPi[2, (c - Pi/2 + d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(512*a^3*d*Sqrt[a + b*Sin
[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^{5/2}\,dx = \frac{b\cot(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^{7/2}}{12a^2d} - \text{cot}\ldots$$

$$= \frac{(28a^2-3b^2)\cot(c+dx)\csc^3(c+dx)(a+b\sin(c+dx\ldots}{96a^2d}$$

$$= \frac{b(52a^2-5b^2)\cot(c+dx)\csc^2(c+dx)(a+b\sin(c+d\ldots}{192a^2d}$$

$$= -\frac{(16a^4-56a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)\sqrt{a+\ldots}}{256a^2d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c\ldots}}{1536a^3d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c\ldots}}{1536a^3d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c\ldots}}{1536a^3d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c\ldots}}{1536a^3d}$$

$$= -\frac{b(720a^4-176a^2b^2+15b^4)\cot(c+dx)\sqrt{a+b\sin(c\ldots}}{1536a^3d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.60, size = 771, normalized size = 1.40

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((((-720*a^4*b*Cos[c + d*x] + 176*a^2*b^3*Cos[c + d*x] - 15*b^5*Cos[c + d*x
])*Csc[c + d*x])/(1536*a^3) + ((-48*a^4*Cos[c + d*x] + 600*a^2*b^2*Cos[c +
```

```
d*x] + 5*b^4*Cos[c + d*x])*Csc[c + d*x]^2)/(768*a^2) + ((164*a^2*b*Cos[c +
d*x] - b^3*Cos[c + d*x])*Csc[c + d*x]^3)/(192*a) + ((28*a^2*Cos[c + d*x] -
27*b^2*Cos[c + d*x])*Csc[c + d*x]^4)/96 - (5*a*b*Cot[c + d*x]*Csc[c + d*x]^
4)/12 - (a^2*Cot[c + d*x]*Csc[c + d*x]^5)/6)*Sqrt[a + b*Sin[c + d*x]])/d +
((-2*(192*a^5*b + 3744*a^3*b^3 - 20*a*b^5)*EllipticF[(-c + Pi/2 - d*x)/2, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
- (2*(384*a^6 + 3600*a^4*b^2 + 536*a^2*b^4 - 45*b^6)*EllipticPi[2, (-c + Pi
/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*
Sin[c + d*x]] - ((2*I)*(720*a^4*b^2 - 176*a^2*b^4 + 15*b^6)*Cos[c + d*x]*Co
s[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a
+ b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a +
b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a -
 b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a -
b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(
a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a
+ b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(6144*a^3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(608) = 1216$.
time = 18.96, size = 2458, normalized size = 4.46

method	result	size
default	Expression too large to display	2458

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/1536*(-256*a^7+704*a^7*sin(d*x+c)^2+96*a^7*sin(d*x+c)^6-544*a^7*sin(d*x+c
)^4-15*a*b^6*sin(d*x+c)^6-5*a^2*b^5*sin(d*x+c)^5+2*a^3*b^4*sin(d*x+c)^4-440
*a^4*b^3*sin(d*x+c)^3-1072*a^5*b^2*sin(d*x+c)^2+192*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^6*b*s
in(d*x+c)^6-2160*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6+2160*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*
a^4*b^3*sin(d*x+c)^6-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/
(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(
a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6+180*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a^2*b^5*sin(d*x+c)^6+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
```

```
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-896*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^5*b^2*sin(d*x+c)^6-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/
(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^7*sin(d*x+c)^6+720*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*s
in(d*x+c)^6+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6-192*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi((
(a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^7*sin(d*x+c)^6
+191*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6*sin(d*x+c)^6-816*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
6*b*sin(d*x+c)^6+2592*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2*sin(d*x+c)^6-1696*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3*sin(d
*x+c)^6-186*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^3*b^4*sin(d*x+c)^6-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5*sin(d*x+c)^6+15*((
a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c
))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1
/2))*a*b^6*sin(d*x+c)^6+2656*a^6*b*sin(d*x+c)^3-896*a^6*b*sin(d*x+c)+720*a^
5*b^2*sin(d*x+c)^8-176*a^3*b^4*sin(d*x+c)^8+174*a^3*b^4*sin(d*x+c)^6-3232*a
^5*b^2*sin(d*x+c)^6+1816*a^4*b^3*sin(d*x+c)^5+15*a*b^6*sin(d*x+c)^8+816*a^6
*b*sin(d*x+c)^7-1376*a^4*b^3*sin(d*x+c)^7+5*a^2*b^5*sin(d*x+c)^7-2576*a^6*b
*sin(d*x+c)^5+3584*a^5*b^2*sin(d*x+c)^4)/a^4/sin(d*x+c)^6/cos(d*x+c)/(a+b*s
in(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^(5/2)*cot(d*x + c)^4*csc(d*x + c)^3, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)**3*(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)^3*(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x)^4*(a + b*sin(c + d*x))^(5/2))/sin(c + d*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.1168 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=471

$$\frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d}-\frac{8(480a^4-683a^2b^2+77b^4)\cos(c+dx)\sin(c+$$

```
[Out] 64/15015*a*(80*a^4-118*a^2*b^2+17*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^
6/d-8/15015*(480*a^4-683*a^2*b^2+77*b^4)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x
+c))^(1/2)/b^5/d+4/3003*a*(160*a^2-223*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*si
n(d*x+c))^(1/2)/b^4/d-10/429*(8*a^2-11*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*si
n(d*x+c))^(1/2)/b^3/d+24/143*a*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/
2)/b^2/d-2/13*cos(d*x+c)*sin(d*x+c)^5*(a+b*sin(d*x+c))^(1/2)/b/d-8/15015*(1
280*a^6-2048*a^4*b^2+453*a^2*b^4+231*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+
8/15015*a*(1280*a^6-2368*a^4*b^2+875*a^2*b^4+213*b^6)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2974, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{4a(160a^2-223b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3003b^4d}-\frac{10(8a^2-11b^2)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d}+\frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d}-\frac{8(480a^4-683a^2b^2+77b^4)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^5d}-\frac{8a(1280a^6-2368a^4b^2+875a^2b^4+213b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^7d\sqrt{a+b\sin(c+dx)}}+\frac{8(1280a^6-2048a^4b^2+453a^2b^4+231b^6)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15015b^7d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{24a\sin^4(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{143b^2d}-\frac{2\sin^5(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{13bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (64*a*(80*a^4 - 118*a^2*b^2 + 17*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]
)/(15015*b^6*d) - (8*(480*a^4 - 683*a^2*b^2 + 77*b^4)*Cos[c + d*x]*Sin[c +
d*x]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^5*d) + (4*a*(160*a^2 - 223*b^2)*Cos
[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3003*b^4*d) - (10*(8*a^
2 - 11*b^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(429*b^3*
d) + (24*a*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(143*b^2*d
) - (2*Cos[c + d*x]*Sin[c + d*x]^5*Sqrt[a + b*Sin[c + d*x]])/(13*b*d) + (8*
(1280*a^6 - 2048*a^4*b^2 + 453*a^2*b^4 + 231*b^6)*EllipticE[(c - Pi/2 + d*x
)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15015*b^7*d*Sqrt[(a + b*Sin[
c + d*x])/(a + b)]) - (8*a*(1280*a^6 - 2368*a^4*b^2 + 875*a^2*b^4 + 213*b^6
```

)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15015*b^7*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
```

```
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{24a\cos(c+dx)\sin^4(c+dx)\sqrt{a+b\sin(c+dx)}}{143b^2d} - \frac{2\cos(c+dx)\sin^5(c+\cdots}{\cdots}$$

$$= -\frac{10(8a^2-11b^2)\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{429b^3d} + \frac{24a\cos(\cdots}{\cdots}$$

$$= \frac{4a(160a^2-223b^2)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3003b^4d} - \frac{10(8a^2\cdots}{\cdots}$$

$$= -\frac{8(480a^4-683a^2b^2+77b^4)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^5d}\cdots$$

$$= \frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8(480a^4-\cdots}{\cdots}$$

$$= \frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8(480a^4-\cdots}{\cdots}$$

$$= \frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8(480a^4-\cdots}{\cdots}$$

$$= \frac{64a(80a^4-118a^2b^2+17b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15015b^6d} - \frac{8(480a^4-\cdots}{\cdots}$$

Mathematica [A]
time = 3.87, size = 382, normalized size = 0.81

$$\frac{-384(1280a^7+1280a^6b-2048a^5b^2-2048a^4b^3+453a^3b^4+453a^2b^5+231ab^6+231b^7)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+384a(1280a^6-2368a^4b^2+875a^2b^4+213b^6)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+3b\cos(c+dx)\left(81920a^6-125952a^4b^2+23760a^2b^4+6622b^6+(5120a^4b^2-5792a^2b^4-8547b^6)\cos(2(c+dx))-70(8a^2b^4-11b^6)\cos(4(c+dx))+1155b^6\cos(6(c+dx))+20480a^5b\sin(c+dx)-28608a^3b^3\sin(c+dx)+2332ab^5\sin(c+dx)-1600a^3b^3\sin(3(c+dx))+1390ab^5\sin(3(c+dx))+210ab^5\sin(5(c+dx))\right)}{720720b^7d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-384*(1280*a^7 + 1280*a^6*b - 2048*a^5*b^2 - 2048*a^4*b^3 + 453*a^3*b^4 +
453*a^2*b^5 + 231*a*b^6 + 231*b^7)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(
a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + 384*a*(1280*a^6 - 2368*a^4*b^2
 + 875*a^2*b^4 + 213*b^6)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*S
```

```
qrt[(a + b*Sin[c + d*x])/(a + b)] + 3*b*Cos[c + d*x]*(81920*a^6 - 125952*a^
4*b^2 + 23760*a^2*b^4 + 6622*b^6 + (5120*a^4*b^2 - 5792*a^2*b^4 - 8547*b^6)
*Cos[2*(c + d*x)] - 70*(8*a^2*b^4 - 11*b^6)*Cos[4*(c + d*x)] + 1155*b^6*Cos
[6*(c + d*x)] + 20480*a^5*b*Sin[c + d*x] - 28608*a^3*b^3*Sin[c + d*x] + 233
2*a*b^5*Sin[c + d*x] - 1600*a^3*b^3*Sin[3*(c + d*x)] + 1390*a*b^5*Sin[3*(c
+ d*x)] + 210*a*b^5*Sin[5*(c + d*x)]))/(720720*b^7*d*Sqrt[a + b*Sin[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(501) = 1002$.
time = 11.00, size = 1619, normalized size = 3.44

method	result	size
default	Expression too large to display	1619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/15015*(544*a^2*b^6-9472*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^3+6504*((a+b*sin(d*x+c))/(a-b))^(1/2
)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^4+3500*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^
3*b^5-1740*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^2*b^6+852*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^7+13312*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipt
icE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-10004*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^4*b^4+888*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2
)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),
((a-b)/(a+b))^(1/2))*a^2*b^6+5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7*b-3840*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b^2-924*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*b^8-5120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
```

```
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((
a-b)/(a+b))^(1/2))*a^8+924*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^8+1155*b^8*sin(d*x+c)^8-3080*b^8*sin(d*
x+c)^6+2233*b^8*sin(d*x+c)^4-308*b^8*sin(d*x+c)^2-105*a*b^7*sin(d*x+c)^7+14
0*a^2*b^6*sin(d*x+c)^6-200*a^3*b^5*sin(d*x+c)^5+410*a*b^7*sin(d*x+c)^5+320*
a^4*b^4*sin(d*x+c)^4-642*a^2*b^6*sin(d*x+c)^4-640*a^5*b^3*sin(d*x+c)^3+1244
*a^3*b^5*sin(d*x+c)^3-541*a*b^7*sin(d*x+c)^3-2560*a^6*b^2*sin(d*x+c)^2+3456
*a^4*b^4*sin(d*x+c)^2-42*a^2*b^6*sin(d*x+c)^2+640*a^5*b^3*sin(d*x+c)-1044*a
^3*b^5*sin(d*x+c)+236*a*b^7*sin(d*x+c)+2560*a^6*b^2-3776*a^4*b^4)/b^8/cos(d
*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 634, normalized size = 1.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] -2/45045*(8*sqrt(2)*(640*a^7 - 1264*a^5*b^2 + 543*a^3*b^4 + 102*a*b^6)*sqrt
(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*
b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 8*sqrt(2
)*(640*a^7 - 1264*a^5*b^2 + 543*a^3*b^4 + 102*a*b^6)*sqrt(-I*b)*weierstrass
PInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 6*sqrt(2)*(1280*I*a^6*b
- 2048*I*a^4*b^3 + 453*I*a^2*b^5 + 231*I*b^7)*sqrt(I*b)*weierstrassZeta(-4/
3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 6*sqrt(2)*(-1280*I*a^6*b + 2048*
I*a^4*b^3 - 453*I*a^2*b^5 - 231*I*b^7)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x +
c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(1260*a*b^6*cos(d*x + c)^5 - 10*(1
```

```
60*a^3*b^4 + 29*a*b^6)*cos(d*x + c)^3 + 2*(1280*a^5*b^2 - 1088*a^3*b^4 - 21
3*a*b^6)*cos(d*x + c) - (1155*b^7*cos(d*x + c)^5 - 35*(40*a^2*b^5 + 11*b^7)
*cos(d*x + c)^3 + 6*(320*a^4*b^3 - 222*a^2*b^5 - 77*b^7)*cos(d*x + c))*sin(
d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^8*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^3}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2), x)
```

3.1169 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=405

$$-\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d}+\frac{8a(120a^2-179b^2)\cos(c+dx)\sin(c+dx)\sqrt{}}{3465b^4d}$$

```
[Out] -8/3465*(160*a^4-247*a^2*b^2+45*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/
d+8/3465*a*(120*a^2-179*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b
^4/d-2/693*(80*a^2-117*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/
b^3/d+20/99*a*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b^2/d-2/11*cos
(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/2)/b/d+16/3465*a*(160*a^4-267*a^2*
b^2+69*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c)
)^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/3465*(320*a^6-614*a^4*b^2+24
9*a^2*b^4+45*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2974, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{8a(120a^2-179b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d}-\frac{2(80a^2-117b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d}-\frac{16a(160a^4-267a^2b^2+69b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3465b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d}+\frac{8(320a^6-614a^4b^2+249a^2b^4+45b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3465b^6d\sqrt{a+b\sin(c+dx)}}+\frac{20a\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{99b^2d}-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{11bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-8*(160*a^4 - 247*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(3465*b^5*d) + (8*a*(120*a^2 - 179*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(3465*b^4*d) - (2*(80*a^2 - 117*b^2)*Cos[c + d*x]*Sin[c +
 d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(693*b^3*d) + (20*a*Cos[c + d*x]*Sin[c +
d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(99*b^2*d) - (2*Cos[c + d*x]*Sin[c + d*x]^
4*Sqrt[a + b*Sin[c + d*x]])/(11*b*d) - (16*a*(160*a^4 - 267*a^2*b^2 + 69*b^
4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(
3465*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(320*a^6 - 614*a^4*b^2
+ 249*a^2*b^4 + 45*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/(3465*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{20a\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\sin(c+dx)}}{99b^2d} - \frac{2\cos(c+dx)\sin^4(c+}{11}$$

$$= -\frac{2(80a^2-117b^2)\cos(c+dx)\sin^2(c+dx)\sqrt{a+b\sin(c+dx)}}{693b^3d} + \frac{20a\cos(}{}$$

$$= \frac{8a(120a^2-179b^2)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^4d} - \frac{2(80a^2-}{}$$

$$= -\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a(120a^2-}{}$$

$$= -\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a(120a^2-}{}$$

$$= -\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a(120a^2-}{}$$

$$= -\frac{8(160a^4-247a^2b^2+45b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3465b^5d} + \frac{8a(120a^2-}{}$$

Mathematica [A]
time = 2.99, size = 326, normalized size = 0.80

$$\frac{128a(160a^5+160a^4b-267a^3b^2-267a^2b^3+69ab^4+69b^5)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-64(320a^6-614a^4b^2+249a^2b^4+45b^6)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)\left(-10240a^5+16448a^3b^2-3718ab^4-128(5a^3b^2-6ab^4)\cos(2(c+dx))+70ab^4\cos(4(c+dx))-2560a^4b\sin(c+dx)+3752a^2b^3\sin(c+dx)+990b^5\sin(c+dx)+200a^2b^3\sin(3(c+dx))-765b^5\sin(3(c+dx))-315b^5\sin(5(c+dx))\right)}{27720b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (128*a*(160*a^5 + 160*a^4*b - 267*a^3*b^2 - 267*a^2*b^3 + 69*a*b^4 + 69*b^5
)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)] - 64*(320*a^6 - 614*a^4*b^2 + 249*a^2*b^4 + 45*b^6)*EllipticF[(-2
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] + b*C
os[c + d*x]*(-10240*a^5 + 16448*a^3*b^2 - 3718*a*b^4 - 128*(5*a^3*b^2 - 6*a
*b^4)*Cos[2*(c + d*x)] + 70*a*b^4*Cos[4*(c + d*x)] - 2560*a^4*b*Sin[c + d*x
] + 3752*a^2*b^3*Sin[c + d*x] + 990*b^5*Sin[c + d*x] + 200*a^2*b^3*Sin[3*(c
```

```
 + d*x)] - 765*b^5*Sin[3*(c + d*x)] - 315*b^5*Sin[5*(c + d*x)]))/(27720*b^6
*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(439) = 878$.
time = 10.83, size = 1356, normalized size = 3.35

method	result	size
default	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/3465*(-315*b^7*sin(d*x+c)^7+900*b^7*sin(d*x+c)^5-765*b^7*sin(d*x+c)^3+18
0*b^7*sin(d*x+c)-2688*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-2456*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+1
692*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^3*b^4+996*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-732*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+1280*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-96
0*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d
*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b)
)^(1/2))*a^5*b^2+3416*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-49*a*b^6*sin(d*x+c)^2+160*a^4*b^3*sin(
d*x+c)-272*a^2*b^5*sin(d*x+c)+35*a*b^6*sin(d*x+c)^6-50*a^2*b^5*sin(d*x+c)^5
+80*a^3*b^4*sin(d*x+c)^4-166*a*b^6*sin(d*x+c)^4-160*a^4*b^3*sin(d*x+c)^3+32
2*a^2*b^5*sin(d*x+c)^3-640*a^5*b^2*sin(d*x+c)^2+908*a^3*b^4*sin(d*x+c)^2-98
8*a^3*b^4+180*a*b^6+640*a^5*b^2+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-1280*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7)/b^7/cos(d*x
+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 585, normalized size = 1.44

$$\frac{2\left(2\sqrt{2}\left(640a^6-1308a^4b^2+609a^2b^4+135b^6\right)\sqrt{i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+2\sqrt{2}\left(640a^6-1308a^4b^2+609a^2b^4+135b^6\right)\sqrt{-i b}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)-12\sqrt{2}\left(-160ia^5b+267ia^3b^3-69iab^5\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(8ia^3-9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)-12\sqrt{2}\left(160ia^5b-267ia^3b^3+69iab^5\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(4a^2-3b^2\right)}{3b^2},-\frac{8\left(-8ia^3+9iab^2\right)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)-3\left(315b^6\cos(dx+c)^5-5\left(80a^2b^4+9b^6\right)\cos(dx+c)^3+2\left(320a^4b^2-294a^2b^4-45b^6\right)\cos(dx+c)+2\left(175ab^5\cos(dx+c)^3-3\left(80a^3b^3-61ab^5\right)\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{10395\,b^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] 2/10395*(2*sqrt(2)*(640*a^6 - 1308*a^4*b^2 + 609*a^2*b^4 + 135*b^6)*sqrt(I*
b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2
)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*sqrt(2)*(
640*a^6 - 1308*a^4*b^2 + 609*a^2*b^4 + 135*b^6)*sqrt(-I*b)*weierstrassPInve
rse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*co
s(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 12*sqrt(2)*(-160*I*a^5*b + 26
7*I*a^3*b^3 - 69*I*a*b^5)*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2
)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d
*x + c) - 2*I*a)/b)) - 12*sqrt(2)*(160*I*a^5*b - 267*I*a^3*b^3 + 69*I*a*b^5
)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*
I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3
 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b))
- 3*(315*b^6*cos(d*x + c)^5 - 5*(80*a^2*b^4 + 9*b^6)*cos(d*x + c)^3 + 2*(32
0*a^4*b^2 - 294*a^2*b^4 - 45*b^6)*cos(d*x + c) + 2*(175*a*b^5*cos(d*x + c)^
3 - 3*(80*a^3*b^3 - 61*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x +
c) + a))/(b^7*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\cos^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**2*cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2), x)
```

3.1170 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=283

$$-\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d}+\frac{8(32a^4-57a^2b^2+21b^4)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{}}{315b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/63*cos(d*x+c)^3*(8*a-7*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^2/d+4/315*
cos(d*x+c)*(a*(32*a^2-33*b^2)-3*b*(8*a^2-7*b^2)*sin(d*x+c))*(a+b*sin(d*x+c)
)^(1/2)/b^4/d-8/315*(32*a^4-57*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(
1/2)+8/315*a*(32*a^4-65*a^2*b^2+33*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$,
Rules used = {2944, 2831, 2742, 2740, 2734, 2732}

$$\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(a(32a^2-33b^2)-3b(8a^2-7b^2)\sin(c+dx))}{315b^4d}-\frac{8a(32a^4-65a^2b^2+33b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{315b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8(32a^4-57a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{315b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]^3*(8*a - 7*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(63*b
^2*d) + (8*(32*a^4 - 57*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*
b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d*Sqrt[(a + b*Sin[c + d*x])/
(a + b)]) - (8*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^5*d*Sqrt[a + b
*Sin[c + d*x]]) + (4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(a*(32*a^2 - 33*
b^2) - 3*b*(8*a^2 - 7*b^2)*Sin[c + d*x]))/(315*b^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\int \frac{\cos^2(c+dx)\left(-\right.}{\sqrt{a-}}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{4\cos(c+dx)\sqrt{}}{}$$

$$= -\frac{2\cos^3(c+dx)(8a-7b\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{63b^2d} + \frac{8(32a^4-57a^2b}{}$$

Mathematica [A]
time = 2.21, size = 275, normalized size = 0.97

$$\frac{-32(32a^5+32a^4b-57a^3b^2-57a^2b^3+21ab^4+21b^5)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+32a(32a^4-65a^2b^2+33b^4)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-b\cos(c+dx)\left(-512a^4+880a^2b^2-203b^4-8(4a^2b^2-21b^4)\cos(2(c+dx))+35b^4\cos(4(c+dx))-128a^3b\sin(c+dx)+202ab^3\sin(c+dx)+10ab^3\sin(3(c+dx))\right)}{1260b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-32*(32*a^5 + 32*a^4*b - 57*a^3*b^2 - 57*a^2*b^3 + 21*a*b^4 + 21*b^5)*Elli
pticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a +
b)] + 32*a*(32*a^4 - 65*a^2*b^2 + 33*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4,
(2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-512*a^
4 + 880*a^2*b^2 - 203*b^4 - 8*(4*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)] + 35*b^
4*Cos[4*(c + d*x)] - 128*a^3*b*Sin[c + d*x] + 202*a*b^3*Sin[c + d*x] + 10*a
*b^3*Sin[3*(c + d*x)]))/(1260*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(329) = 658$.
time = 10.14, size = 1190, normalized size = 4.20

method	result	size

default	Expression too large to display	1190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-5*a*b^5*sin(d*x+c)^5+128*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^5*b-96*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
a^4*b^2-260*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*a^3*b^3+180*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^
(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^2*b^4+132*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a*b^5-84*EllipticF
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^
6-128*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/
(a-b))^(1/2)*a^6+356*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*a^4*b^2-312*EllipticE(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^2*b^4+84*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^6+8*a^2*
b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-16*a^3*b^3*sin(d*x+c)^3+34*a*b^5*sin(
d*x+c)^3-64*a^4*b^2*sin(d*x+c)^2+98*a^2*b^4*sin(d*x+c)^2+77*b^6*sin(d*x+c)^
2+16*a^3*b^3*sin(d*x+c)-29*a*b^5*sin(d*x+c)+64*a^4*b^2-106*a^2*b^4)/b^6/cos
(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 537, normalized size = 1.90

$$-\frac{2\left(4\sqrt{2}\left(32a^5-69a^3b^2+39ab^4\right)\sqrt{ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)+4\sqrt{2}\left(32a^5-69a^3b^2+39ab^4\right)\sqrt{-ib}\,\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)+6\sqrt{2}\left(32ia^4b-57ia^2b^3+21ib^5\right)\sqrt{ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(8ia^3-9iab^2)}{27b^3},\frac{3b\cos(dx+c)-3ib\sin(dx+c)-2ia}{3b}\right)\right)+6\sqrt{2}\left(-32ia^4b+57ia^2b^3-21ib^5\right)\sqrt{-ib}\,\mathrm{weierstrassZeta}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\mathrm{weierstrassPInverse}\left(-\frac{4(4a^2-3b^2)}{3b^2},-\frac{8(-8ia^3+9iab^2)}{27b^3},\frac{3b\cos(dx+c)+3ib\sin(dx+c)+2ia}{3b}\right)\right)+3\left(40ab^4\cos(dx+c)^3-2\left(32a^3b^2-33ab^4\right)\cos(dx+c)-\left(35b^5\cos(dx+c)^3-6\left(8a^2b^3-7b^5\right)\cos(dx+c)\right)\sin(dx+c)\right)\sqrt{b\sin(dx+c)+a}\right)}{945b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] -2/945*(4*sqrt(2)*(32*a^5 - 69*a^3*b^2 + 39*a*b^4)*sqrt(I*b)*weierstrassPIn
verse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 4*sqrt(2)*(32*a^5 - 69*a^3*b
^2 + 39*a*b^4)*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/
27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) +
 2*I*a)/b) + 6*sqrt(2)*(32*I*a^4*b - 57*I*a^2*b^3 + 21*I*b^5)*sqrt(I*b)*wei
erstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, wei
erstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 6*sqrt(2)*(-32*I
*a^4*b + 57*I*a^2*b^3 - 21*I*b^5)*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 -
3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a
^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) +
3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(40*a*b^4*cos(d*x + c)^3 - 2*(32*a^3*b^
2 - 33*a*b^4)*cos(d*x + c) - (35*b^5*cos(d*x + c)^3 - 6*(8*a^2*b^3 - 7*b^5)
*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^4 \sin(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(1/2), x)
```

3.1171 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=288

$$\frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d}-\frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd}+\frac{2(8a^2-21b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{15b^3d\sqrt{}}$$

```
[Out] 8/15*a*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^2/d-2/5*cos(d*x+c)*sin(d*x+c)*(a
+b*sin(d*x+c))^(1/2)/b/d-2/15*(8*a^2-21*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/
2)+2/15*a*(8*a^2-23*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((
a+b*sin(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+
1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/d/(a+b*s
in(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2974, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{2a(8a^2-23b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{15b^3d\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^2-21b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{15b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d}-\frac{2\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{5bd}+\frac{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (8*a*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(15*b^2*d) - (2*Cos[c + d*x]*Si
n[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(5*b*d) + (2*(8*a^2 - 21*b^2)*Elliptic
E[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(15*b^3*d*Sq
rt[(a + b*Sin[c + d*x])/(a + b)]) - (2*a*(8*a^2 - 23*b^2)*EllipticF[(c - Pi
/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(15*b^3*d*S
qrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
```

```
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin}}{5bd}$$

$$= \frac{8a\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{15b^2d} - \frac{2\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin}}{5bd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.49, size = 408, normalized size = 1.42

$$\frac{\frac{2i(-8a^2+21b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{ab^2\sqrt{-\frac{1}{a+b}}}+4\cos(c+dx)(4a-3b\sin(c+dx))\sqrt{a+b\sin(c+dx)}-\frac{8abF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(8a^2+9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(-8*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(a*b^2*Sqrt[-(a + b)^(-1)]) + 4*Cos[c
 + d*x]*(4*a - 3*b*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]] - (8*a*b*Elliptic
F[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])
/Sqrt[a + b*Sin[c + d*x]] - (2*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[
c + d*x]])/(30*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(363) = 726$.

time = 10.22, size = 1018, normalized size = 3.53

method	result
default	$\frac{\frac{16\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^4b}{15}-\frac{4\sqrt{\frac{a+b\sin(dx-}{a-b}}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+21*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^5+29*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*a^3*b^2-21*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+
c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-15*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elli
pticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a+15*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))+3*a*b^4*sin(d*x+c)^4-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*
x+c)^2-3*a*b^4*sin(d*x+c)^2+a^2*b^3*sin(d*x+c)+4*a^3*b^2)/a/b^4/cos(d*x+c)/
(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
[Out] integral(cos(d*x + c)^3*cot(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3442 deep
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\cot(c+d\,x)}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(1/2), x)
```

3.1172 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=285

$$-\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}-\frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad}-\frac{(4a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{\ }}{3ab^2d\sqrt{\frac{a+b\sin(c+d}{a+b}}}$$

```
[Out] -2/3*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b/d-cot(d*x+c)*(a+b*sin(d*x+c))^(1/2
)/a/d+1/3*(4*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+
b*sin(d*x+c))^(1/2)/a/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(4*a^2-7*b^2
)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^
(1/2)/b^2/d/(a+b*sin(d*x+c))^(1/2)+b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/si
n(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2973, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2-7b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3ab^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd}-\frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad}-\frac{b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (-2*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b*d) - (Cot[c + d*x]*Sqrt[a +
 b*Sin[c + d*x]])/(a*d) - ((4*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2
*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^2*d*Sqrt[(a + b*Sin[c + d*x])
/(a + b)]) + ((4*a^2 - 7*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (b
*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
```

```
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - $$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + $$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} + $$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - $$

$$= -\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3bd} - \frac{\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{ad} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 12.41, size = 416, normalized size = 1.46

$$\frac{\frac{2i(4a^2+3b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{a^2b^3\sqrt{-\frac{1}{a+b}}}-\frac{4\cot(c+dx)(3b+2a\sin(c+dx))\sqrt{a+b\sin(c+dx)}}{ab}+\frac{40F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(4a^2+9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{ab\sqrt{a+b\sin(c+dx)}}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1
)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSin
h[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ellip
ticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (
a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqr
t[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^2*b^3*Sqrt[-(a + b)^(-1)]) - (4*Co
t[c + d*x]*(3*b + 2*a*Sin[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/(a*b) + (40*E
llipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a
 + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 9*b^2)*EllipticPi[2, (-2*c +
 Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b*Sqr
t[a + b*Sin[c + d*x]]))/(12*d)
```

Maple [A]
time = 18.77, size = 704, normalized size = 2.47

method	result
default	$\frac{\sqrt{-\left(-b\sin\left(dx+c\right)-a\right)\left(\cos^2\left(dx+c\right)\right)}\left(\left(-2a^3b^2-3a\,b^4\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/3*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-2*a^3*b^2-3*a*b^4)*sin(d*x+c
)*cos(d*x+c)^2+(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(
a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(4*EllipticE((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-EllipticE((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-3*EllipticE((b/(a-b)*sin(d*x+c)
+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-4*EllipticF((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-6*EllipticF((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+7*EllipticF((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)
+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*EllipticPi((b/(a-b)*sin(d*x+c)
+a/(a-b))^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b))^(1/2))*a*b^4-3*EllipticPi((b/(a
-b)*sin(d*x+c)+a/(a-b))^(1/2),(a*b-b^2)/a/b,((a-b)/(a+b))^(1/2))*b^5)*sin(d
```

```
*x+c)+2*a^2*b^3*cos(d*x+c)^4-5*a^2*b^3*cos(d*x+c)^2)/b^3/(b*cos(d*x+c)^2*si
n(d*x+c)+a*cos(d*x+c)^2)^(1/2)/a^2/sin(d*x+c)/cos(d*x+c)/(a+b*sin(d*x+c))^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="fr
icas")
```

```
[Out] integral(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\cot(c+d\,x)^2}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(1/2), x)
```

3.1173 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=307

$$\frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}+\frac{(8a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{4a^2bd\sqrt{a+}}$$

```
[Out] 3/4*b*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)*(a+
b*sin(d*x+c))^(1/2)/a/d-1/4*(8*a^2+3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2
)+1/4*(8*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*
d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin
(d*x+c))/(a+b))^(1/2)/a/b/d/(a+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-b^2)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a
^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2972, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{(8a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{4abd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{4a^2d\sqrt{a+b\sin(c+dx)}}+\frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d}-\frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (3*b*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*d) - (Cot[c + d*x]*Csc[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(2*a*d) + ((8*a^2 + 3*b^2)*EllipticE[(c -
 Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^2*b*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) - ((8*a^2 + b^2)*EllipticF[(c - Pi/2 + d*x)/2,
 (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a*b*d*Sqrt[a + b*Sin
[c + d*x]]) - (3*(4*a^2 - b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a +
 b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
```

```
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}{2ad}$$

$$= \frac{3b\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^2d} - \frac{\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c}{2ad}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.27, size = 443, normalized size = 1.44

$$\frac{\frac{2i(8a^2+3b^2)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{a^3b^2\sqrt{-\frac{1}{a+b}}(-2+\csc^2(c+dx))}-\frac{4\cot(c+dx)(-3b+2a\csc(c+dx))\sqrt{a+b\sin(c+dx)}}{a^2}-\frac{8bF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{a\sqrt{a+b\sin(c+dx)}}+\frac{2(16a^2-9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{a^2\sqrt{a+b\sin(c+dx)}}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] (((2*I)*(8*a^2 + 3*b^2)*Cos[2*(c + d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*Ellipt
icE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]
]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)
]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1
+ Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a^3*b^
2*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*x]^2)) - (4*Cot[c + d*x]*(-3*b + 2*a*
Csc[c + d*x])*Sqrt[a + b*Sin[c + d*x]])/a^2 - (8*b*EllipticF[(-2*c + Pi - 2
*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*Sqrt[a + b*S
in[c + d*x]]) + (2*(16*a^2 - 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2
*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*Sqrt[a + b*Sin[c + d*
x]]))/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(380) = 760$.

time = 22.27, size = 913, normalized size = 2.97

method	result
default	$\frac{\sqrt{-\left(-b\sin\left(dx+c\right)-a\right)\left(\cos^{2}\left(dx+c\right)\right)}\left(\frac{2\left(\frac{a}{b}-1\right)\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{\frac{(1-\sin(dx+c))b}{a+b}}\sqrt{\frac{(-\sin(dx+c)-1)b}{a-b}}}{\sqrt{-\left(-b\sin\left(\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(2*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)
/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(d*
x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2)))+4*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(
a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*
x+c)-a)*cos(d*x+c)^2)^(1/2)*b/a*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),-
(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2))-1/2/a*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)
^(1/2)/sin(d*x+c)^2+3/4/a^2*b*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/sin(d
*x+c)+1/2/a*b*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+
c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+
c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+3
/4*b^2/a^2*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+c))
*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^
2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))-1/4*
(4*a^2+3*b^2)/a^3*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(
d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(
d*x+c)^2)^(1/2)*b*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),-(-1/b*a+1)*b/a
,((a-b)/(a+b))^(1/2)))/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
[Out] integrate(cos(d*x + c)*cot(d*x + c)^3/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,{\cot(c+d\,x)}^3}{\sqrt{a+b\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(1/2), x)
```

3.1174 $\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=353

$$\frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d}+\frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}-\frac{\cot(c+dx)\,c}{}$$

```
[Out] 1/24*(32*a^2-15*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+5/12*b*cot(d*x
+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2*(a+
b*sin(d*x+c))^(1/2)/a/d-1/24*(32*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/d/((a+b*sin(d*x+c))/(a+b))^(1/
2)-1/24*(16*a^2+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+
b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)-1/8*b*(12*a^2-5*b^2
)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2804, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(16a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{24a^2d\sqrt{a+b\sin(c+dx)}}+\frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d}+\frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d}+\frac{(32a^2-15b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{24a^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{b(12a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{8a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((32*a^2 - 15*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^3*d) + (5*b
*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(3*a*d) + ((32*a^2 - 15*b^2)*
EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(24*
a^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((16*a^2 + 5*b^2)*EllipticF[(c
- Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(24*a^2
*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(12*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(8*a^3*d*Sqrt[
a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)\sqrt{a}\cdots}{3ad}$$

$$= \frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{\cdots}}{12a^2d}$$

$$= \frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{\cdots}}{12a^2d}$$

$$= \frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{\cdots}}{12a^2d}$$

$$= \frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{\cdots}}{12a^2d}$$

$$= \frac{(32a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^3d} + \frac{5b\cot(c+dx)\csc(c+dx)\sqrt{\cdots}}{12a^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.82, size = 475, normalized size = 1.35

$$-\frac{\frac{4\cot(c+dx)(-32a^2+15b^2-10ab\csc(c+dx)+8a^2\csc^2(c+dx))\sqrt{a+b\sin(c+dx)}}{a^3}+\frac{\frac{2i(32a^2-15b^2)\cos(2(c+dx))\csc^2(c+dx)\left(2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)-b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}(-2+\csc^2(c+dx))}-\frac{8a(24a^2-5b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(-104a^2+45b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{a^3}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((-4*Cot[c + d*x]*(-32*a^2 + 15*b^2 - 10*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d
*x]^2)*Sqrt[a + b*Sin[c + d*x]])/a^3 + (((2*I)*(32*a^2 - 15*b^2)*Cos[2*(c +
 d*x)]*Csc[c + d*x]^2*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticP
i[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-(
(b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)]*(-2 + Csc[c + d*
```

```
x]^2)) - (8*a*(24*a^2 - 5*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-1
04*a^2 + 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(
a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(422) = 844$.
time = 12.52, size = 1496, normalized size = 4.24

method	result	size
default	Expression too large to display	1496

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)^3-16*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d
*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin
(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^3-42*b^2*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*sin(d*x+c)^3-5*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^
3-32*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^5*sin(d*x+c)^3+47*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-15*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
*sin(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^
2*b^3*sin(d*x+c)^3+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+
b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b
))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3-15*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))
*b^5*sin(d*x+c)^3-32*a^3*b^2*sin(d*x+c)^5+15*a*b^4*sin(d*x+c)^5-32*a^4*b*si
```

```
n(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^4+30*a^3*b^2*sin(d*x+c)^3-15*a*b^4*sin(d*x+
c)^3+40*a^4*b*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)^2+2*a^3*b^2*sin(d*x+c)-8*a^
4*b)/a^4/sin(d*x+c)^3/b/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(1/2),x)
[Out] Integral(cot(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cot(d*x + c)^4/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(1/2),x)
[Out] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(1/2), x)
```

3.1175 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=412

$$-\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}+\frac{5(12a^2-7b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+a}}{96a^3d}$$

```
[Out] -1/192*b*(188*a^2-105*b^2)*cot(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d+5/96*(12
*a^2-7*b^2)*cot(d*x+c)*csc(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/d+7/24*b*cot(d
*x+c)*csc(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3
*(a+b*sin(d*x+c))^(1/2)/a/d+1/192*b*(188*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2
*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x
),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c))/(
a+b))^(1/2)-1/192*b*(68*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/64*(
48*a^4-72*a^2*b^2+35*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2)
)*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.79, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2972, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{7b\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d}-\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d}-\frac{b(188a^2-105b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{(48a^4-72a^2b^2+35b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{64a^4d\sqrt{a+b\sin(c+dx)}}+\frac{b(68a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{192a^3d\sqrt{a+b\sin(c+dx)}}+\frac{5(12a^2-7b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{96a^3d}-\frac{\cot(c+dx)\csc^3(c+dx)\sqrt{a+b\sin(c+dx)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] -1/192*(b*(188*a^2 - 105*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(a^4*d
) + (5*(12*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])
/(96*a^3*d) + (7*b*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(2
4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(4*a*d) -
 (b*(188*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(192*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (b*(68
*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin
[c + d*x])/(a + b)])/(192*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + ((48*a^4 - 72*a
^2*b^2 + 35*b^4)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a +
 b*Sin[c + d*x])/(a + b)])/(64*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
```

```
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx = \frac{7b\cot(c+dx)\csc^2(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

$$= \frac{5(12a^2-7b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{96a^3d} + \frac{7b\cot(c+dx)}{}$$

$$= -\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5(12a^2-7b^2)\cot(c+}{}$$

$$= -\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5(12a^2-7b^2)\cot(c+}{}$$

$$= -\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5(12a^2-7b^2)\cot(c+}{}$$

$$= -\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5(12a^2-7b^2)\cot(c+}{}$$

$$= -\frac{b(188a^2-105b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{192a^4d} + \frac{5(12a^2-7b^2)\cot(c+}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.45, size = 647, normalized size = 1.57

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/Sqrt[a + b*Sin[c + d*x]],x]
```

```
[Out] ((((-188*a^2*b*Cos[c + d*x] + 105*b^3*Cos[c + d*x])*Csc[c + d*x])/(192*a^4)
 + (5*(12*a^2*Cos[c + d*x] - 7*b^2*Cos[c + d*x])*Csc[c + d*x]^2)/(96*a^3) +
 (7*b*Cot[c + d*x]*Csc[c + d*x]^2)/(24*a^2) - (Cot[c + d*x]*Csc[c + d*x]^3)
/(4*a))*Sqrt[a + b*Sin[c + d*x]])/d + ((-2*(-240*a^3*b + 140*a*b^3)*Ellipti
cF[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - (2*(288*a^4 - 620*a^2*b^2 + 315*b^4)*EllipticPi[
```

```
2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/
Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(188*a^2*b^2 - 105*b^4)*Cos[c + d*x]*Cos[
2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a +
b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a +
b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)
/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b
)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b)
)])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a
+ b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a +
b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(768*a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(477) = 954$.

time = 12.70, size = 1761, normalized size = 4.27

method	result	size
default	Expression too large to display	1761

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/192*(-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)
*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),
(a-b)/a,((a-b)/(a+b))^(1/2))*b^5*sin(d*x+c)^4-188*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+1
44*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((
a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+120*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^4+48*a^5+35*a
^2*b^3*sin(d*x+c)^5+174*a^3*b^2*sin(d*x+c)^4-105*a*b^4*sin(d*x+c)^4-35*a^2*
b^3*sin(d*x+c)^3+14*a^3*b^2*sin(d*x+c)^2+68*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-258*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^3*b^2*sin(d*x+c)^4-35*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))
^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*
sin(d*x+c)^4-144*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1
/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^4-216*((a+b*sin(d*x+c))/(
```

```
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3
*b^2*sin(d*x+c)^4-168*a^5*sin(d*x+c)^2+120*a^5*sin(d*x+c)^4+216*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1
/2))*a^2*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+
c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4+293*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*b^2*sin(d*x+c)^4-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^4-68*a^4*b*sin(d*x+c)^5+7
6*a^4*b*sin(d*x+c)^3-8*a^4*b*sin(d*x+c)-188*a^3*b^2*sin(d*x+c)^6+105*a*b^4*
sin(d*x+c)^6)/a^5/sin(d*x+c)^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cot(d*x + c)^4*csc(d*x + c)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**4*csc(c + d*x)/sqrt(a + b*sin(c + d*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c))^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\sin\left(c+d\,x\right)^2-1\right)^2}{\sin\left(c+d\,x\right)^5\,\sqrt{a+b\,\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))^(1/2)),x)
```

```
[Out] int((sin(c + d*x)^2 - 1)^2/(sin(c + d*x)^5*(a + b*sin(c + d*x))^(1/2)), x)
```

3.1176 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=466

$$-\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}-\frac{8(640a^4-592a^2b^2+15b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{1155b^6d}+\frac{8a(480a^2$$

[Out] -2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^(1/2)-8/1155*(640*a^4-592*a^2*b^2+15*b^4)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^6/d+8/1155*a*(480*a^2-419*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d-20/231*(32*a^2-27*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/b^4/d+2/33*(40*a^2-33*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/a/b^3/d-2/11*cos(d*x+c)*sin(d*x+c)^4*(a+b*sin(d*x+c))^(1/2)/b^2/d+8/1155*a*(1280*a^4-1344*a^2*b^2+123*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/1155*(1280*a^6-1664*a^4*b^2+369*a^2*b^4+15*b^6)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.77, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2971, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{8a(480a^2-419b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{1155b^5d}-\frac{20(32a^2-27b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{231b^4d}+\frac{2(40a^2-33b^2)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{33ab^3d}-\frac{8a(1280a^4-1344a^2b^2+123b^4)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1155b^7d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8(640a^4-592a^2b^2+15b^4)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{1155b^6d}+\frac{8(1280a^6-1664a^4b^2+369a^2b^4+15b^6)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{1155b^7d\sqrt{a+b\sin(c+dx)}}-\frac{2\sin^4(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{11b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]

[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (8*(640*a^4 - 592*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(1155*b^6*d) + (8*a*(480*a^2 - 419*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(1155*b^5*d) - (20*(32*a^2 - 27*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(231*b^4*d) + (2*(40*a^2 - 33*b^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(33*a*b^3*d) - (2*Cos[c + d*x]*Sin[c + d*x]^4*Sqrt[a + b*Sin[c + d*x]])/(11*b^2*d) - (8*a*(1280*a^4 - 1344*a^2*b^2 + 123*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(1155*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(1155*b^7*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^4(c+dx)\sqrt{a+b}}{11b^2d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2(40a^2-33b^2)\cos(c+dx)\sin^3(c}{33ab^3d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{20(32a^2-27b^2)\cos(c+dx)\sin^2(}{231b^4}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a(480a^2-419b^2)\cos(c+dx)\sin}{1155b}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8(640a^4-592a^2b^2+15b^4)\cos(c}{1155b^6}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8(640a^4-592a^2b^2+15b^4)\cos(c}{1155b^6}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8(640a^4-592a^2b^2+15b^4)\cos(c}{1155b^6}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{8(640a^4-592a^2b^2+15b^4)\cos(c}{1155b^6}$$

Mathematica [A]
time = 4.77, size = 326, normalized size = 0.70

$$\frac{64a(1280a^5+1280a^4b-1344a^3b^2-1344a^2b^3+123ab^4+123b^5)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-64(1280a^6-1664a^4b^2+369a^2b^4+15b^6)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+b\cos(c+dx)(-40960a^5+40448a^3b^2-2728ab^4-16(160a^3b^2-93ab^4)\cos(2(c+dx))+280ab^4\cos(4(c+dx))-10240a^4b\sin(c+dx)+8672a^2b^3\sin(c+dx)+330b^5\sin(c+dx)+800a^2b^3\sin(3(c+dx))-255b^5\sin(3(c+dx))-105b^5\sin(5(c+dx)))}{9240b^7d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (64*a*(1280*a^5 + 1280*a^4*b - 1344*a^3*b^2 - 1344*a^2*b^3 + 123*a*b^4 + 12
3*b^5)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)] - 64*(1280*a^6 - 1664*a^4*b^2 + 369*a^2*b^4 + 15*b^6)*Ellipt
```

```
icF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)
] + b*Cos[c + d*x]*(-40960*a^5 + 40448*a^3*b^2 - 2728*a*b^4 - 16*(160*a^3*b
^2 - 93*a*b^4)*Cos[2*(c + d*x)] + 280*a*b^4*Cos[4*(c + d*x)] - 10240*a^4*b*
Sin[c + d*x] + 8672*a^2*b^3*Sin[c + d*x] + 330*b^5*Sin[c + d*x] + 800*a^2*b
^3*Sin[3*(c + d*x)] - 255*b^5*Sin[3*(c + d*x)] - 105*b^5*Sin[5*(c + d*x)]))
/(9240*b^7*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. $2(498) = 996$.
time = 10.70, size = 1356, normalized size = 2.91

method	result	size
default	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/1155*(-105*b^7*sin(d*x+c)^7+300*b^7*sin(d*x+c)^5-255*b^7*sin(d*x+c)^3+60
*b^7*sin(d*x+c)-5868*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^4+492*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6-6656*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+43
92*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^4+1476*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-552*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6+5120*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-38
40*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^5*b^2+10496*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2+266*a*b^6*sin(d*x+c)^2+640*a^4*b^3*s
in(d*x+c)-692*a^2*b^5*sin(d*x+c)+140*a*b^6*sin(d*x+c)^6-200*a^2*b^5*sin(d*x
+c)^5+320*a^3*b^4*sin(d*x+c)^4-466*a*b^6*sin(d*x+c)^4-640*a^4*b^3*sin(d*x+c
)^3+892*a^2*b^5*sin(d*x+c)^3-2560*a^5*b^2*sin(d*x+c)^2+2048*a^3*b^4*sin(d*x
+c)^2-2368*a^3*b^4+60*a*b^6+2560*a^5*b^2+60*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF((
```

```
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^7-5120*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^7)/b^8
/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.31, size = 788, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] 2/3465*(2*(sqrt(2)*(2560*a^6*b - 3648*a^4*b^3 + 984*a^2*b^5 + 45*b^7)*sin(d
*x + c) + sqrt(2)*(2560*a^7 - 3648*a^5*b^2 + 984*a^3*b^4 + 45*a*b^6))*sqrt(
I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*(sqrt(2
)*(2560*a^6*b - 3648*a^4*b^3 + 984*a^2*b^5 + 45*b^7)*sin(d*x + c) + sqrt(2)
*(2560*a^7 - 3648*a^5*b^2 + 984*a^3*b^4 + 45*a*b^6))*sqrt(-I*b)*weierstrass
PInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3
*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*(sqrt(2)*(-1280*I*a^5*
b^2 + 1344*I*a^3*b^4 - 123*I*a*b^6)*sin(d*x + c) + sqrt(2)*(-1280*I*a^6*b +
 1344*I*a^4*b^3 - 123*I*a^2*b^5))*sqrt(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3
*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b)) - 6*(sqrt(2)*(1280*I*a^5*b^2 - 1344*I*a^3*b^4
+ 123*I*a*b^6)*sin(d*x + c) + sqrt(2)*(1280*I*a^6*b - 1344*I*a^4*b^3 + 123*
I*a^2*b^5))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*
I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2
*I*a)/b)) + 3*(140*a*b^6*cos(d*x + c)^5 - 2*(160*a^3*b^4 - 23*a*b^6)*cos(d*
x + c)^3 - 2*(1280*a^5*b^2 - 1344*a^3*b^4 + 123*a*b^6)*cos(d*x + c) - (105*
b^7*cos(d*x + c)^5 - 5*(40*a^2*b^5 + 3*b^7)*cos(d*x + c)^3 + 2*(320*a^4*b^3
```

```
- 246*a^2*b^5 - 15*b^7)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) +
a))/(b^9*d*sin(d*x + c) + a*b^8*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\sin(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2), x)
```

3.1177 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=401

$$-\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^5d}-\frac{16(60a^2-49b^2)}{}$$

[Out] -2*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^(1/2)+8/315*a*(160*a^2-139*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d-16/315*(60*a^2-49*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^4/d+2/63*(80*a^2-63*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a/b^3/d-2/9*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/b^2/d-8/315*(320*a^4-318*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+16/315*a*(160*a^4-199*a^2*b^2+39*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/b^6/d/(a+b*sin(d*x+c))^(1/2)

Rubi [A]
time = 0.58, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2971, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$-\frac{2(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^5d}-\frac{16(60a^2-49b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^4d}+\frac{2(80a^2-63b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{63ab^3d}-\frac{16a(160a^4-199a^2b^2+39b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^6d\sqrt{a+b\sin(c+dx)}}+\frac{8(320a^4-318a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{9b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]

[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (8*a*(160*a^2 - 139*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d) - (16*(60*a^2 - 49*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^4*d) + (2*(80*a^2 - 63*b^2)*Cos[c + d*x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(63*a*b^3*d) - (2*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(9*b^2*d) + (8*(320*a^4 - 318*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (16*a*(160*a^4 - 199*a^2*b^2 + 39*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^6*d*Sqrt[a + b*Sin[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sin^3(c+dx)\sqrt{a+b\,\mathrm{s}}}{9b^2d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{2(80a^2-63b^2)\cos(c+dx)\sin^2(c-}{63ab^3d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{16(60a^2-49b^2)\cos(c+dx)\sin(c-}{315b^4d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a}}{315b^5d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a}}{315b^5d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a}}{315b^5d}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} + \frac{8a(160a^2-139b^2)\cos(c+dx)\sqrt{a}}{315b^5d}$$

Mathematica [A]

time = 3.64, size = 275, normalized size = 0.69

$$\frac{-32(320a^5+320a^4b-318a^3b^2-318a^2b^3+21ab^4+21b^5)\,E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}+64a(160a^4-199a^2b^2+39b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}-b\cos(c+dx)\left(-5120a^4+4768a^2b^2-203b^4-8(40a^2b^2-21b^4)\cos(2(c+dx))+35b^4\cos(4(c+dx))-1280a^3b\sin(c+dx)+1012ab^3\sin(c+dx)+100ab^3\sin(3(c+dx))\right)}{1260b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-32*(320*a^5 + 320*a^4*b - 318*a^3*b^2 - 318*a^2*b^3 + 21*a*b^4 + 21*b^5)*
EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)] + 64*a*(160*a^4 - 199*a^2*b^2 + 39*b^4)*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - b*Cos[c + d*x]*(-
5120*a^4 + 4768*a^2*b^2 - 203*b^4 - 8*(40*a^2*b^2 - 21*b^4)*Cos[2*(c + d*x)
] + 35*b^4*Cos[4*(c + d*x)] - 1280*a^3*b*Sin[c + d*x] + 1012*a*b^3*Sin[c +
d*x] + 100*a*b^3*Sin[3*(c + d*x)]))/(1260*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1189 vs. $2(437) = 874$.
time = 9.47, size = 1190, normalized size = 2.97

method	result	size
default	Expression too large to display	1190

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/315*(35*b^6*sin(d*x+c)^6-50*a*b^5*sin(d*x+c)^5+1280*EllipticF(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^5*b-960*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*a^4*b^2-1592*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*a^3*b^3+1044*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2
),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^2*b^4+312*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a*b^5-84*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*b^6-1280*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*a^6+2552*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^4*b^2-1356*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*a^2*b^4+84*EllipticE
(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b
))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^
6+80*a^2*b^4*sin(d*x+c)^4-112*b^6*sin(d*x+c)^4-160*a^3*b^3*sin(d*x+c)^3+214
*a*b^5*sin(d*x+c)^3-640*a^4*b^2*sin(d*x+c)^2+476*a^2*b^4*sin(d*x+c)^2+77*b^
6*sin(d*x+c)^2+160*a^3*b^3*sin(d*x+c)-164*a*b^5*sin(d*x+c)+640*a^4*b^2-556*
a^2*b^4)/b^7/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.20, size = 723, normalized size = 1.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] -2/945*(2*(sqrt(2)*(640*a^5*b - 876*a^3*b^3 + 213*a*b^5)*sin(d*x + c) + sqr
t(2)*(640*a^6 - 876*a^4*b^2 + 213*a^2*b^4))*sqrt(I*b)*weierstrassPInverse(-
4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x
+ c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 2*(sqrt(2)*(640*a^5*b - 876*a^3*b^3
 + 213*a*b^5)*sin(d*x + c) + sqrt(2)*(640*a^6 - 876*a^4*b^2 + 213*a^2*b^4))
*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 6
*(sqrt(2)*(320*I*a^4*b^2 - 318*I*a^2*b^4 + 21*I*b^6)*sin(d*x + c) + sqrt(2)
*(320*I*a^5*b - 318*I*a^3*b^3 + 21*I*a*b^5))*sqrt(I*b)*weierstrassZeta(-4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, weierstrassPInverse(
-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x
 + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 6*(sqrt(2)*(-320*I*a^4*b^2 + 318*
I*a^2*b^4 - 21*I*b^6)*sin(d*x + c) + sqrt(2)*(-320*I*a^5*b + 318*I*a^3*b^3
- 21*I*a*b^5))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(
-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c)
+ 2*I*a)/b)) + 3*(35*b^6*cos(d*x + c)^5 - (80*a^2*b^4 - 7*b^6)*cos(d*x + c)
^3 - 2*(320*a^4*b^2 - 318*a^2*b^4 + 21*b^6)*cos(d*x + c) + 2*(25*a*b^5*cos(
d*x + c)^3 - (80*a^3*b^3 - 57*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin
(d*x + c) + a))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\sin(c+d\,x)^2}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2), x)
```

3.1178 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=261

$$-\frac{8a(32a^2-29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{8(32a^4-37a^2b^2+5b^4)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+}\right.}{35b^5d\sqrt{a+b\sin(c+dx}}$$

```
[Out] 2/7*cos(d*x+c)^3*(8*a+b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))^(1/2)-4/35*cos(d
*x+c)*(32*a^2-5*b^2-24*a*b*sin(d*x+c))*(a+b*sin(d*x+c))^(1/2)/b^4/d+8/35*a*
(32*a^2-29*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*
x+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/35*(32*a^4-37*a^2*b^2+5*
b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elliptic
F(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2942, 2944, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8a(32a^2-29b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a^2-24ab\sin(c+dx)-5b^2)}{35b^4d}+\frac{8(32a^4-37a^2b^2+5b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{35b^5d\sqrt{a+b\sin(c+dx)}}+\frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-8*a*(32*a^2 - 29*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a
 + b*Sin[c + d*x]])/(35*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(32*
a^4 - 37*a^2*b^2 + 5*b^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt
[(a + b*Sin[c + d*x])/(a + b)])/(35*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Co
s[c + d*x]^3*(8*a + b*Sin[c + d*x]))/(7*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (
4*Cos[c + d*x]*Sqrt[a + b*Sin[c + d*x]]*(32*a^2 - 5*b^2 - 24*a*b*Sin[c + d*
x]))/(35*b^4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{12\int \frac{\cos^2(c+dx)\left(-\frac{b}{2}-4a\sin(c+dx)\right)}{\sqrt{a+b\sin(c+dx)}}\,dx}{7b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a}{35b^4d}$$

$$= \frac{2\cos^3(c+dx)(8a+b\sin(c+dx))}{7b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\cos(c+dx)\sqrt{a+b\sin(c+dx)}\,(32a}{35b^4d}$$

$$= -\frac{8a(32a^2-29b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{35b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} + \frac{8(32a^4-}{}$$

Mathematica [A]
time = 2.81, size = 222, normalized size = 0.85

$$\frac{16a(32a^3+32a^2b-29ab^2-29b^3)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} - 16(32a^4-37a^2b^2+5b^4)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}} + b\cos(c+dx)\left(-256a^3+216ab^2-16ab^2\cos(2(c+dx))+(-64a^2b+45b^3)\sin(c+dx)+5b^3\sin(3(c+dx))\right)}{70b^5d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a*(32*a^3 + 32*a^2*b - 29*a*b^2 - 29*b^3)*EllipticE[(-2*c + Pi - 2*d*x)
/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)] - 16*(32*a^4 - 37*a^2
*b^2 + 5*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*S
in[c + d*x])/(a + b)] + b*Cos[c + d*x]*(-256*a^3 + 216*a*b^2 - 16*a*b^2*Cos
[2*(c + d*x)] + (-64*a^2*b + 45*b^3)*Sin[c + d*x] + 5*b^3*Sin[3*(c + d*x)])
)/(70*b^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(307) = 614$.
time = 9.13, size = 943, normalized size = 3.61

method	result
default	$\frac{\frac{2b^5\left(\sin^5(dx+c)\right)}{7}+\frac{256\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a}{35}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/35*(5*b^5*sin(d*x+c)^5+128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1
)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5-244*((a+b*sin(d*x+c))/(a-b))^(1/2)*
(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+116*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4
-128*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^4*b+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2+148*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3-96*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-20*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*b^5-8*a*b^4*sin(d*x+c)^4+16*a^2*b^3*sin(d*x+c)^3-20*b^5*sin(d*x+c)^3+
64*a^3*b^2*sin(d*x+c)^2-42*a*b^4*sin(d*x+c)^2-16*a^2*b^3*sin(d*x+c)+15*b^5*
sin(d*x+c)-64*a^3*b^2+50*a*b^4)/b^6/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 659, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] 2/105*(2*(sqrt(2)*(64*a^4*b - 82*a^2*b^3 + 15*b^5)*sin(d*x + c) + sqrt(2)*(
64*a^5 - 82*a^3*b^2 + 15*a*b^4))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2
- 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*
b*sin(d*x + c) - 2*I*a)/b) + 2*(sqrt(2)*(64*a^4*b - 82*a^2*b^3 + 15*b^5)*si
n(d*x + c) + sqrt(2)*(64*a^5 - 82*a^3*b^2 + 15*a*b^4))*sqrt(-I*b)*weierstra
ssPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*
(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*(sqrt(2)*(-32*I*a^3*
b^2 + 29*I*a*b^4)*sin(d*x + c) + sqrt(2)*(-32*I*a^4*b + 29*I*a^2*b^3))*sqrt
(I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)
/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*
b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*(sqrt
(2)*(32*I*a^3*b^2 - 29*I*a*b^4)*sin(d*x + c) + sqrt(2)*(32*I*a^4*b - 29*I*a
^2*b^3))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a
^3 + 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-
8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*
a)/b)) - 3*(8*a*b^4*cos(d*x + c)^3 + 2*(32*a^3*b^2 - 29*a*b^4)*cos(d*x + c)
 - (5*b^5*cos(d*x + c)^3 - 2*(8*a^2*b^3 - 5*b^5)*cos(d*x + c))*sin(d*x + c)
)*sqrt(b*sin(d*x + c) + a))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^4\,\sin(c+d\,x)}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(3/2), x)
```

3.1179 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}-\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d}-\frac{2(8a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b}}{3ab^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d*x+c))^(1/2)-2/3*cos(d*x+c)*(a+b*
sin(d*x+c))^(1/2)/b^2/d+2/3*(8*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b
/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a/b^3/d/((a+b*sin(d*x+c))/(a+b))^(1/2
)-2/3*(8*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/
2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*s
in(d*x+c))/(a+b))^(1/2)/b^3/d/(a+b*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/
2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d
*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/d/(a+b*sin(
d*x+c))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2971, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3b^3d\sqrt{a+b\sin(c+dx)}}-\frac{2(8a^2-3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d}+\frac{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(a*b^2*d*Sqrt[a + b*Sin[c + d*x]]) - (2*Cos[c
 + d*x]*Sqrt[a + b*Sin[c + d*x]])/(3*b^2*d) - (2*(8*a^2 - 3*b^2)*EllipticE[
(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a*b^3*d*Sqr
t[(a + b*Sin[c + d*x])/(a + b)]) + (2*(8*a^2 - 5*b^2)*EllipticF[(c - Pi/2 +
 d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*b^3*d*Sqrt[a
 + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2971

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(b^2*d*f*(m + n + 4))), x]) /; Free
```

```
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{4\int \frac{\csc(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{4\int \frac{\csc(\ldots}{\ldots}}{\ldots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} + \frac{\int \frac{\ldots}{\sqrt{a+\ldots}}}{\ldots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2(8a^2-\ldots}{\ldots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{ab^2d\sqrt{a+b\sin(c+dx)}} - \frac{2\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{3b^2d} - \frac{2(8a^2-\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.60, size = 419, normalized size = 1.42

$$-\frac{\frac{2i(-8a^2+3b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^2\sqrt{-\frac{1}{a+b}}}+\frac{8abF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^2-9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{4\cos(c+dx)(4a^2-3b^2+ab\sin(c+dx))}{\sqrt{a+b\sin(c+dx)}}}{6ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)*(-8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(
-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcS
inh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Ell
ipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
 (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*S
qrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (8*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(8*a^2 - 9*b^2)*EllipticPi[2, (-2*
c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[
a + b*Sin[c + d*x]] - (4*Cos[c + d*x]*(4*a^2 - 3*b^2 + a*b*Sin[c + d*x]))/S
qrt[a + b*Sin[c + d*x]])/(6*a*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. $2(373) = 746$.

time = 10.81, size = 1010, normalized size = 3.41

method	result
default	$-\frac{2\left(8\sqrt{\frac{a+b\sin(dx+c)}{a-b}}\,\sqrt{-\frac{(\sin(dx+c)-1)b}{a+b}}\,\sqrt{-\frac{(1+\sin(dx+c))b}{a-b}}\,\text{EllipticF}\left(\sqrt{\frac{a+b\sin(dx+c)}{a-b}},\sqrt{\frac{a-b}{a+b}}\right)a^4b-6\sqrt{\frac{a+b\sin}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b-6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+3*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2
)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4-8*((a
+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c)
)*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^5+11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^2-3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
b/(a+b))^(1/2)(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(
-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^4*Elliptic
Pi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a-3*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*b^5*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a
+b))^(1/2))-a^2*b^3*sin(d*x+c)^3-4*a^3*b^2*sin(d*x+c)^2+3*a*b^4*sin(d*x+c)^
2+a^2*b^3*sin(d*x+c)+4*a^3*b^2-3*a*b^4)/a^2/b^4/cos(d*x+c)/(a+b*sin(d*x+c))
^(1/2)/d

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")

[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^3*cot(d*x + c)/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\mathrm{cot}\left(c+d\,x\right)}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(3/2), x)
```

3.1180 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=294

$$\frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}}+\frac{(4a^2-3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-$$

```
[Out] (2*a^2-3*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-cot(d*x+c)/a/d/(a+b
*sin(d*x+c))^(1/2)-(4*a^2-3*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(
1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+(4*a^
2-b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a
+b))^(1/2)/a/b^2/d/(a+b*sin(d*x+c))^(1/2)+3*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(
1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^2/d/(a+b*sin(d*x+c))
^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {2969, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2-b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{ab^2d\sqrt{a+b\sin(c+dx)}}+\frac{(4a^2-3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{a^2b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Cos[c + d*x])/(a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - Cot[c +
 d*x]/(a*d*Sqrt[a + b*Sin[c + d*x]]) + ((4*a^2 - 3*b^2)*EllipticE[(c - Pi/2
 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(a^2*b^2*d*Sqrt[(a + b*
Sin[c + d*x])/(a + b)]) - ((4*a^2 - b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b
)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a*b^2*d*Sqrt[a + b*Sin[c +
d*x]]) - (3*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*
Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
```

```
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{3b^2}{4}-\frac{1}{2}\right.}{\sqrt{}}}{}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} + \frac{1}{2}\left(-\frac{3}{a^2}+\frac{4}{b^2}\right)\int$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{(3b)\int \frac{\csc(c+}{\sqrt{a+b\sin(}}}{2a^2}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\right.\right.}{d\sqrt{}}$$

$$= \frac{(2a^2-3b^2)\cos(c+dx)}{a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)}{ad\sqrt{a+b\sin(c+dx)}} - \frac{\left(\frac{3}{a^2}-\frac{4}{b^2}\right)E\left(\frac{1}{2}\left(c-\right.\right.}{d\sqrt{}}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 12.36, size = 433, normalized size = 1.47

$$\frac{\frac{i(-4a^2+3b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^3\sqrt{-\frac{1}{a+b}}}+\frac{4a(a^2-b^2)\cos(c+dx)}{b\sqrt{a+b\sin(c+dx)}}-2a\cot(c+dx)\sqrt{a+b\sin(c+dx)}+\frac{4a^2F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{a(4a^2-9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{b\sqrt{a+b\sin(c+dx)}}}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((I*(-4*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*
Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[S
qrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elliptic
Pi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
 b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[(
b*(1 + Sin[c + d*x]))/(-a + b)])/(b^3*Sqrt[-(a + b)^(-1)]) + (4*a*(a^2 - b^
2)*Cos[c + d*x])/(b*Sqrt[a + b*Sin[c + d*x]]) - 2*a*Cot[c + d*x]*Sqrt[a + b
*Sin[c + d*x]] + (4*a^2*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqr
t[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (a*(4*a^2 - 9*b
^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
 d*x])/(a + b)])/(b*Sqrt[a + b*Sin[c + d*x]]))/(2*a^3*d)
```

Maple [A]
time = 10.46, size = 620, normalized size = 2.11

method	result
default	$-\frac{(-2a^3b^2+3a\,b^4)\sin(dx+c)(\cos^2(dx+c))-\sqrt{-\frac{b\sin(dx+c)}{a-b}-\frac{b}{a-b}}\sqrt{-\frac{b\sin(dx+c)}{a+b}+\frac{b}{a+b}}\sqrt{\frac{b\sin(dx+c)}{a-b}+\frac{a}{a-b}}\left(3\,\mathrm{El}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -((-2*a^3*b^2+3*a*b^4)*sin(d*x+c)*cos(d*x+c)^2-(-b/(a-b)*sin(d*x+c)-b/(a-b)
)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2)*(3*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^
(1/2))*a*b^4-3*EllipticPi((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*b^5+4*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^4*b-6*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a^3*b^2-EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^2*b^3+3*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+
b))^(1/2))*a*b^4-4*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^5+7*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a^3*b^2-3*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a*b^4)*sin(d*x+c)+a^2*b^3*cos(d*x+c)^2)/b^3/sin(d*x+c)/a^3/cos(d*x+
c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="ma
xima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="fr
icas")
```

```
[Out] integral(-sqrt(b*sin(d*x + c) + a)*cos(d*x + c)^2*cot(d*x + c)^2/(b^2*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(3/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\cot(c+d\,x)^2}{(a+b\,\sin(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(3/2), x)
```

3.1181 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=366

$$\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^3bd}-\frac{(8a^2-$$

```
[Out] 1/2*(4*a^2-5*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-1/2*cot(d*x+c)*
csc(d*x+c)/a/d/(a+b*sin(d*x+c))^(1/2)-1/4*(8*a^2-15*b^2)*cot(d*x+c)*(a+b*si
n(d*x+c))^(1/2)/a^3/b/d+1/4*(8*a^2-15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^3/b/d/((a+b*sin(d*x+c))/(a+b))^(1/
2)-1/4*(8*a^2-5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a+b))^(1/2)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)+3/4*(4*a^2-5*b^2)*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos
(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^
(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2969, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-15b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{4a^3bd}-\frac{(8a^2-15b^2)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^3bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{3(4a^2-5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^3d\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((4*a^2 - 5*b^2)*Cot[c + d*x])/(2*a^2*b*d*Sqrt[a + b*Sin[c + d*x]]) - (Cot[
c + d*x]*Csc[c + d*x])/(2*a*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 15*b^2)
*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*b*d) - ((8*a^2 - 15*b^2)*Ell
ipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(4*a^3*
b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 5*b^2)*EllipticF[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^2*b*d
*Sqrt[a + b*Sin[c + d*x]]) - (3*(4*a^2 - 5*b^2)*EllipticPi[2, (c - Pi/2 + d
*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^3*d*Sqrt[a +
 b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
```

```
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}(8a^2-\cdots\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(\cdots}{\cdots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(\cdots}{\cdots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(\cdots}{\cdots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(\cdots}{\cdots}$$

$$= \frac{(4a^2-5b^2)\cot(c+dx)}{2a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\sqrt{a+b\sin(c+dx)}} - \frac{(8a^2-15b^2)\cot(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.32, size = 435, normalized size = 1.19

$$\frac{\frac{(-32a^2+60b^2)\cos(c+dx)+4a\cot(c+dx)(5b-2a\csc(c+dx))}{a^3\sqrt{a+b\sin(c+dx)}} + \frac{-\frac{2i(-8a^2+15b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)\middle|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^2\sqrt{-\frac{1}{a+b}}} - \frac{40abF\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}} + \frac{2(32a^2-45b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{a^3}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] (((-32*a^2 + 60*b^2)*Cos[c + d*x] + 4*a*Cot[c + d*x]*(5*b - 2*a*Csc[c + d*x
]))/(a^3*Sqrt[a + b*Sin[c + d*x]]) + (((-2*I)*(-8*a^2 + 15*b^2)*(-2*a*(a -
b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a +
b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Si
n[c + d*x]]], (a + b)/(a - b)] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a
 + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt
[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))
```

```
])/(a*b^2*Sqrt[-(a + b)^(-1)]) - (40*a*b*EllipticF[(-2*c + Pi - 2*d*x)/4, (
2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]]
+ (2*(32*a^2 - 45*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*
Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^3)/(16*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1348 vs. $2(435) = 870$.
time = 11.46, size = 1349, normalized size = 3.69

method	result	size
default	Expression too large to display	1349

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^4*b*sin(d*x+c)^2-18*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)^2-5*b^3*((a+
b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))
*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2
))*a^2*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*sin(d*x+c)^2+
23*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(
d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b
))^(1/2))*a^3*b^2*sin(d*x+c)^2-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^2-12*((a+b*sin(d*
x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b)
)^(1/2)*b^2*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))
^(1/2))*a^3*sin(d*x+c)^2+12*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^3*EllipticPi(((a+b*sin(d*
x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^2+15*Ellipti
cPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin(d
*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b
))^(1/2)*a*b^4*sin(d*x+c)^2-15*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a
-b)/a,((a-b)/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*b^5*sin(d*x+c)^2-8*a^3*b^2*s
in(d*x+c)^4+15*a*b^4*sin(d*x+c)^4+5*a^2*b^3*sin(d*x+c)^3+6*a^3*b^2*sin(d*x+
c)^2-15*a*b^4*sin(d*x+c)^2-5*a^2*b^3*sin(d*x+c)+2*a^3*b^2)/b^2/sin(d*x+c)^2
/a^4/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+1)),failed)
cannot
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)\cot(c+dx)^3}{(a+b\sin(c+dx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(3/2), x)
```

3.1182 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=416

$$\frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+a}}{24a^4d}$$

```
[Out] 1/3*(6*a^2-7*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(1/2)-1/3*
cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))^(1/2)+5/24*(16*a^2-21*b^2)*cot
(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a^4/d-1/12*(24*a^2-35*b^2)*cot(d*x+c)*csc(d*
x+c)*(a+b*sin(d*x+c))^(1/2)/a^3/b/d-5/24*(16*a^2-21*b^2)*(sin(1/2*c+1/4*Pi+
1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*
d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^4/d/((a+b*sin(d*x+c)
)/(a+b))^(1/2)+1/24*(32*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b)
)^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+c))^(1/2)-1/8*b*
(36*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*si
n(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.76, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2803, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}}+\frac{5(16a^2-21b^2)\cot(c+dx)\sqrt{a+b\sin(c+dx)}}{24a^4d}+\frac{5(16a^2-21b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^4d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{b(36a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(32a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{24a^3d\sqrt{a+b\sin(c+dx)}}-\frac{(24a^2-35b^2)\cot(c+dx)\csc(c+dx)\sqrt{a+b\sin(c+dx)}}{12a^3bd}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((6*a^2 - 7*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*Sqrt[a + b*Sin[c + d
*x]]) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*Sqrt[a + b*Sin[c + d*x]]) + (5
*(16*a^2 - 21*b^2)*Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(24*a^4*d) - ((24
*a^2 - 35*b^2)*Cot[c + d*x]*Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]])/(12*a^3*
b*d) + (5*(16*a^2 - 21*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sq
rt[a + b*Sin[c + d*x]])/(24*a^4*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((3
2*a^2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Si
n[c + d*x])/(a + b)])/(24*a^3*d*Sqrt[a + b*Sin[c + d*x]]) + (b*(36*a^2 - 35
*b^2)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c +
d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{3/2}}\,dx = \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{2\int \frac{\csc^3(c+dx)}{\ldots}}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} - \frac{(24a^2-35b\ldots}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5(16a^2-21\ldots}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5(16a^2-21\ldots}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5(16a^2-21\ldots}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5(16a^2-21\ldots}{\ldots}$$

$$= \frac{(6a^2-7b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd\sqrt{a+b\sin(c+dx)}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad\sqrt{a+b\sin(c+dx)}} + \frac{5(16a^2-21\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.00, size = 468, normalized size = 1.12

$$\frac{-\frac{4((-80a^2b+105b^3)\cos(c+dx)+a\cot(c+dx)(-32a^2+35b^2-14ab\csc(c+dx)+8a^2\csc^2(c+dx)))}{a^4\sqrt{a+b\sin(c+dx)}}+\frac{\frac{10i(-16a^2+21b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab\sqrt{-\frac{1}{a+b}}}-\frac{8a(24a^2-35b^2)F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2b(-296a^2+315b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{a^4}}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(3/2),x]
```

```
[Out] ((-4*((-80*a^2*b + 105*b^3)*Cos[c + d*x] + a*Cot[c + d*x]*(-32*a^2 + 35*b^2
 - 14*a*b*Csc[c + d*x] + 8*a^2*Csc[c + d*x]^2)))/(a^4*Sqrt[a + b*Sin[c + d*
x]]) + (((10*I)*(-16*a^2 + 21*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-
(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*Ellipti
```

```
cF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)
] + b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c
+ d*x]]], (a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a
 + b))]*Sqrt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b*Sqrt[-(a + b)^(-1)])
- (8*a*(24*a^2 - 35*b^2)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sq
rt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*b*(-296*a^2
 + 315*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b
*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/a^4)/(96*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(481) = 962$.
time = 11.32, size = 1496, normalized size = 3.60

method	result	size
default	Expression too large to display	1496

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(80*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^5*sin(d*x+c)^3-185*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*s
in(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3+105*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a*b^4*sin(d*x+c)^3-48*a^5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)
*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*sin(d*x+c)^3-32*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b*sin(d*x+c)^
3+150*b^2*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^3*sin(d*x+c)^3+35*b^3*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)^3-105*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a*b^4*sin(d*x+c)^3+108*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(
a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a
-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^2*sin(d*x+c)^3-108*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(
1/2))*a^2*b^3*sin(d*x+c)^3-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)
-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x
```

```
+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^4*sin(d*x+c)^3+105*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*b^5*sin(d*x+c)^3+80*a^3*b^2*sin(d*x+c)^5-105*a*b^4*sin(d*x+c)^5+
32*a^4*b*sin(d*x+c)^4-35*a^2*b^3*sin(d*x+c)^4-66*a^3*b^2*sin(d*x+c)^3+105*a
*b^4*sin(d*x+c)^3-40*a^4*b*sin(d*x+c)^2+35*a^2*b^3*sin(d*x+c)^2-14*a^3*b^2*
sin(d*x+c)+8*a^4*b)/b/a^5/sin(d*x+c)^3/cos(d*x+c)/(a+b*sin(d*x+c))^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(3/2),x)
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^4}{(a+b\,\sin\left(c+d\,x\right))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(3/2),x)
[Out] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(3/2), x)
```

3.1183 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=469

$$-\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{128a(40a^2-19b^2)\cos(c+dx}{315b^6d}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^4/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(
13*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)+128/
315*a*(40*a^2-19*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^6/d-8/315*(480*a^
2-203*b^2)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+4/63*(160*a^2
-63*b^2)*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a/b^4/d-10/9*(8*a^2
-3*b^2)*cos(d*x+c)*sin(d*x+c)^3*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d-8/315*(128
0*a^4-768*a^2*b^2+21*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(
a+b*sin(d*x+c))^(1/2)/b^7/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/315*a*(1280*a^
4-1088*a^2*b^2+123*b^4)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a
+b*sin(d*x+c))/(a+b))^(1/2)/b^7/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.75, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2970, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(13a^2-5b^2)\sin^4(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^4(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{128a(40a^2-19b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^6d}-\frac{8(480a^2-203b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{315b^5d}+\frac{4(160a^2-63b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{63ab^4d}-\frac{10(8a^2-3b^2)\sin^3(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{9a^2b^3d}-\frac{8a(1280a^4-1088a^2b^2+123b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^7d\sqrt{a+b\sin(c+dx)}}+\frac{8(1280a^4-768a^2b^2+21b^4)\sqrt{a+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{315b^7d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(13*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x]^4)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) + (128*a*(40*a^2 - 19*b^2)*Cos[c + d*x]*Sqrt[a + b*
Sin[c + d*x]])/(315*b^6*d) - (8*(480*a^2 - 203*b^2)*Cos[c + d*x]*Sin[c + d*
x]*Sqrt[a + b*Sin[c + d*x]])/(315*b^5*d) + (4*(160*a^2 - 63*b^2)*Cos[c + d*
x]*Sin[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(63*a*b^4*d) - (10*(8*a^2 - 3*b
^2)*Cos[c + d*x]*Sin[c + d*x]^3*Sqrt[a + b*Sin[c + d*x]])/(9*a^2*b^3*d) + (
8*(1280*a^4 - 768*a^2*b^2 + 21*b^4)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a
+ b)]*Sqrt[a + b*Sin[c + d*x]])/(315*b^7*d*Sqrt[(a + b*Sin[c + d*x])/(a + b
)]) - (8*a*(1280*a^4 - 1088*a^2*b^2 + 123*b^4)*EllipticF[(c - Pi/2 + d*x)/2
, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(315*b^7*d*Sqrt[a + b*
Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^4(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(13a^2-5b^2)\cos(c+dx)\sin^4(c-}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1044 vs. $2(469) = 938$.
time = 6.96, size = 1044, normalized size = 2.23

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (315*((((a^2 + 3*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(
-a + b)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*
```

```
x])/(a + b))^(3/2))/((a - b)^2*b) - (Cos[c + d*x]*(2*a*(a^2 + b^2) + b*(a^2
 + 3*b^2)*Sin[c + d*x]))/(a^2 - b^2)^2) + (315*((((32*a^4 - 57*a^2*b^2 + 21
*b^4)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(-32*a^3 + 32*a^2
*b + 33*a*b^2 - 33*b^3)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((
a + b*Sin[c + d*x])/(a + b))^(3/2))/(a - b)^2 - (b*(4*a*(8*a^4 - 13*a^2*b^2
 + 3*b^4)*Cos[c + d*x] + b*(20*a^4 - 33*a^2*b^2 + 9*b^4)*Sin[2*(c + d*x)]))
/(2*(a^2 - b^2)^2)))/b^3 - (21*((((-2048*a^6 + 4192*a^4*b^2 - 2355*a^2*b^4
+ 231*b^6)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] + a*(2048*a^5 -
2048*a^4*b - 2656*a^3*b^2 + 2656*a^2*b^3 + 603*a*b^4 - 603*b^5)*EllipticF[(
-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)])*((a + b*Sin[c + d*x])/(a + b))^(3/2))
/(a - b)^2 + (b*Cos[c + d*x]*(-64*a*b^2*(a^2 - b^2)^2*Cos[2*(c + d*x)] + b*
(1280*a^6 - 2536*a^4*b^2 + 1347*a^2*b^4 - 111*b^6)*Sin[c + d*x] + 2*(512*a^
7 - 952*a^5*b^2 + 423*a^3*b^4 + 7*a*b^6 + 6*b^3*(a^2 - b^2)^2*Sin[3*(c + d*
x)])))/(a^2 - b^2)^2))/b^5 - (5*(a + b*Sin[c + d*x])*(((-4*b*(-4096*a^7*b +
 8960*a^5*b^3 - 5884*a^3*b^5 + 1041*a*b^7)*EllipticF[(-2*c + Pi - 2*d*x)/4,
 (2*b)/(a + b)] + (65536*a^8 - 161792*a^6*b^2 + 129664*a^4*b^4 - 35109*a^2*
b^6 + 1617*b^8)*((a + b)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)] -
a*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]))*Sqrt[(a + b*Sin[c + d*x
])/(a + b)])/((a - b)^2*(a + b)^2) + b*(a + b*Sin[c + d*x])*(-128*a*(88*a^2
 - 27*b^2)*Cos[c + d*x] + 416*a*b^2*Cos[3*(c + d*x)] + (21*a*(64*a^6 - 112*
a^4*b^2 + 56*a^2*b^4 - 7*b^6)*Cos[c + d*x])/((a^2 - b^2)*(a + b*Sin[c + d*x
])^2) - (21*(1088*a^8 - 2576*a^6*b^2 + 1960*a^4*b^4 - 497*a^2*b^6 + 21*b^8)
*Cos[c + d*x])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])) - 8*b*(-276*a^2 + 35*b^
2)*Sin[2*(c + d*x)] - 56*b^3*Sin[4*(c + d*x)])))/b^7)/(10080*d*(a + b*Sin[c
 + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2032 vs. $2(499) = 998$.
time = 11.92, size = 2033, normalized size = 4.33

method	result	size
default	Expression too large to display	2033

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] 2/315*(-35*b^7*sin(d*x+c)*cos(d*x+c)^6+(120*a^2*b^5-7*b^7)*cos(d*x+c)^4*sin
(d*x+c)+(3200*a^4*b^3-1740*a^2*b^5+42*b^7)*cos(d*x+c)^2*sin(d*x+c)-4*(-b/(a
-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2)*b*(1280*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^6-2048*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^4*b^2+789*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))
^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-21*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*b^6-1280*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b
```

```
))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b+960*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2+1088*EllipticF((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3-666*EllipticF((b/(a-b)*sin(d*x+c
)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4-123*EllipticF((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5+21*EllipticF((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^6)*sin(d*x+c)+60*a*b^6*cos(d*x+c
)^6+(-320*a^3*b^4+102*a*b^6)*cos(d*x+c)^4+(2560*a^5*b^2-896*a^3*b^4-162*a*b
^6)*cos(d*x+c)^2+5120*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+
c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*b-3840*(b/(a-b)*sin(d*x+c)
+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/
(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/
2))*a^5*b^2-4352*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^3+2664*(b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-
b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^3*b^4+492*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b
))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^5-84*(b/(a-b)*sin(d*x+c)+a/(a-b))
^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1
/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6
-5120*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2
)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b)
)^(1/2),((a-b)/(a+b))^(1/2))*a^7+8192*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-
b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ellip
ticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b^2-3156*(
b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(
a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^3*b^4+84*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+
b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE(
(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^6)/(a+b*sin(d*x
+c))^(3/2)/b^8/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^3/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.35, size = 955, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] -2/945*(8*(sqrt(2)*(640*a^5*b^2 - 624*a^3*b^4 + 87*a*b^6)*cos(d*x + c)^2 -
2*sqrt(2)*(640*a^6*b - 624*a^4*b^3 + 87*a^2*b^5)*sin(d*x + c) - sqrt(2)*(64
0*a^7 + 16*a^5*b^2 - 537*a^3*b^4 + 87*a*b^6))*sqrt(I*b)*weierstrassPInverse
(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*
x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 8*(sqrt(2)*(640*a^5*b^2 - 624*a^3
*b^4 + 87*a*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(640*a^6*b - 624*a^4*b^3 + 87*a
^2*b^5)*sin(d*x + c) - sqrt(2)*(640*a^7 + 16*a^5*b^2 - 537*a^3*b^4 + 87*a*b
^6))*sqrt(-I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a
^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)
 - 6*(sqrt(2)*(-1280*I*a^4*b^3 + 768*I*a^2*b^5 - 21*I*b^7)*cos(d*x + c)^2 +
 2*sqrt(2)*(1280*I*a^5*b^2 - 768*I*a^3*b^4 + 21*I*a*b^6)*sin(d*x + c) + sqr
t(2)*(1280*I*a^6*b + 512*I*a^4*b^3 - 747*I*a^2*b^5 + 21*I*b^7))*sqrt(I*b)*w
eierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, w
eierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^
3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*(sqrt(2)*(12
80*I*a^4*b^3 - 768*I*a^2*b^5 + 21*I*b^7)*cos(d*x + c)^2 + 2*sqrt(2)*(-1280*
I*a^5*b^2 + 768*I*a^3*b^4 - 21*I*a*b^6)*sin(d*x + c) + sqrt(2)*(-1280*I*a^6
*b - 512*I*a^4*b^3 + 747*I*a^2*b^5 - 21*I*b^7))*sqrt(-I*b)*weierstrassZeta(
-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, weierstrassPInv
erse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*c
os(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) + 3*(60*a*b^6*cos(d*x + c)^5
- 2*(160*a^3*b^4 - 51*a*b^6)*cos(d*x + c)^3 + 2*(1280*a^5*b^2 - 448*a^3*b^4
 - 81*a*b^6)*cos(d*x + c) - (35*b^7*cos(d*x + c)^5 - (120*a^2*b^5 - 7*b^7)*
cos(d*x + c)^3 - 2*(1600*a^4*b^3 - 870*a^2*b^5 + 21*b^7)*cos(d*x + c))*sin(
d*x + c))*sqrt(b*sin(d*x + c) + a))/(b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(
d*x + c) - (a^2*b^8 + b^10)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^3}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2), x)
```

3.1184 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=411

$$-\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{8(32a^2-11b^2)\cos(c+dx)\sqrt{}}{21b^5d}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)*sin(d*x+c)^3/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(
11*a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)-8/21
*(32*a^2-11*b^2)*cos(d*x+c)*(a+b*sin(d*x+c))^(1/2)/b^5/d+8/21*(24*a^2-7*b^2
)*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c))^(1/2)/a/b^4/d-2/21*(80*a^2-21*b^2)
*cos(d*x+c)*sin(d*x+c)^2*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d+16/21*a*(32*a^2-1
5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/
2)/b^6/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-8/21*(64*a^4-46*a^2*b^2+3*b^4)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
b^6/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {2970, 3128, 3102, 2831, 2742, 2740, 2734, 2732}

$$\frac{2(11a^2-3b^2)\sin^3(c+dx)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\sin^3(c+dx)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{16a(32a^2-15b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{21b^6d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8(32a^2-11b^2)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{21b^5d}+\frac{8(24a^2-7b^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{21ab^4d}-\frac{2(80a^2-21b^2)\sin^2(c+dx)\cos(c+dx)\sqrt{a+b\sin(c+dx)}}{21a^2b^3d}+\frac{8(64a^4-46a^2b^2+3b^4)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{21b^6d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*b^2*d*(a + b*Sin[c + d*x]
)^(3/2)) + (2*(11*a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/(3*a^2*b^2*d*Sq
rt[a + b*Sin[c + d*x]]) - (8*(32*a^2 - 11*b^2)*Cos[c + d*x]*Sqrt[a + b*Sin[
c + d*x]])/(21*b^5*d) + (8*(24*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[
a + b*Sin[c + d*x]])/(21*a*b^4*d) - (2*(80*a^2 - 21*b^2)*Cos[c + d*x]*Sin[c
 + d*x]^2*Sqrt[a + b*Sin[c + d*x]])/(21*a^2*b^3*d) - (16*a*(32*a^2 - 15*b^2
)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(2
1*b^6*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + (8*(64*a^4 - 46*a^2*b^2 + 3*b
^4)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/
(a + b)])/(21*b^6*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)\sin^3(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(11a^2-3b^2)\cos(c+dx)\sin^3(c+}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}$$

Mathematica [A]
time = 5.65, size = 257, normalized size = 0.63

$$\frac{32a(a+b)^2\left(32a^2-15b^2\right)E\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}-16(a+b)\left(64a^4-46a^2b^2+3b^4\right)F\left(\frac{1}{4}(-2c+\pi-2dx)\middle|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}-\frac{1}{2}b\cos(c+dx)\left(1024a^4-288a^2b^2-27b^4-8\left(8a^2b^2-3b^4\right)\cos(2(c+dx))+3b^4\cos(4(c+dx))+1280a^3b\sin(c+dx)-516ab^3\sin(c+dx)+12ab^3\sin(3(c+dx))\right)}{42b^6d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (32*a*(a + b)^2*(32*a^2 - 15*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a
 + b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) - 16*(a + b)*(64*a^4 - 46*a^2*b
^2 + 3*b^4)*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c +
 d*x])/(a + b))^(3/2) - (b*Cos[c + d*x]*(1024*a^4 - 288*a^2*b^2 - 27*b^4 -
8*(8*a^2*b^2 - 3*b^4)*Cos[2*(c + d*x)] + 3*b^4*Cos[4*(c + d*x)] + 1280*a^3*
b*Sin[c + d*x] - 516*a*b^3*Sin[c + d*x] + 12*a*b^3*Sin[3*(c + d*x)]))/2)/(4
2*b^6*d*(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1641 vs. $2(445) = 890$.

time = 11.48, size = 1642, normalized size = 4.00

method	result	size
default	Expression too large to display	1642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -2/21*(6*a*b^5*sin(d*x+c)*cos(d*x+c)^4+(160*a^3*b^3-66*a*b^5)*cos(d*x+c)^2*
sin(d*x+c)+4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+
b))^(1/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*b*(64*EllipticF((b/(a-b)*sin(d
*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-48*EllipticF((b/(a-b)*sin(d
*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-46*EllipticF((b/(a-b)*sin
(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+27*EllipticF((b/(a-b)*s
in(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4+3*EllipticF((b/(a-b)*si
n(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^5-64*EllipticE((b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+94*EllipticE((b/(a-b)*sin(d*
x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^2-30*EllipticE((b/(a-b)*sin(
d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)*sin(d*x+c)+3*b^6*cos(d*x+
c)^6+(-16*a^2*b^4+3*b^6)*cos(d*x+c)^4+(128*a^4*b^2-28*a^2*b^4-6*b^6)*cos(d*
x+c)^2-256*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))
^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6+376*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2
)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*E
llipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2-12
0*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-
b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1
/2),((a-b)/(a+b))^(1/2))*a^2*b^4+256*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b
/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*Ellipt
icF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b-192*(b/(a
-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)
*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^4*b^2-184*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*
sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/
(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3+108*(b/(a-b)*s
in(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(
d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(
a+b))^(1/2))*a^2*b^4+12*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*
x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5)/(a+b*sin(d*x+c))^(3/2
)/b^7/cos(d*x+c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.27, size = 873, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] 2/63*(2*(sqrt(2)*(128*a^4*b^2 - 108*a^2*b^4 + 9*b^6)*cos(d*x + c)^2 - 2*sqr
t(2)*(128*a^5*b - 108*a^3*b^3 + 9*a*b^5)*sin(d*x + c) - sqrt(2)*(128*a^6 +
20*a^4*b^2 - 99*a^2*b^4 + 9*b^6))*sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I
*b*sin(d*x + c) - 2*I*a)/b) + 2*(sqrt(2)*(128*a^4*b^2 - 108*a^2*b^4 + 9*b^6
)*cos(d*x + c)^2 - 2*sqrt(2)*(128*a^5*b - 108*a^3*b^3 + 9*a*b^5)*sin(d*x +
c) - sqrt(2)*(128*a^6 + 20*a^4*b^2 - 99*a^2*b^4 + 9*b^6))*sqrt(-I*b)*weiers
trassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^3, 1
/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) + 12*(sqrt(2)*(32*I*a
^3*b^3 - 15*I*a*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-32*I*a^4*b^2 + 15*I*a^2*b
^4)*sin(d*x + c) + sqrt(2)*(-32*I*a^5*b - 17*I*a^3*b^3 + 15*I*a*b^5))*sqrt(
I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/
b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b
^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) + 12*(sqrt
(2)*(-32*I*a^3*b^3 + 15*I*a*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(32*I*a^4*b^2 -
 15*I*a^2*b^4)*sin(d*x + c) + sqrt(2)*(32*I*a^5*b + 17*I*a^3*b^3 - 15*I*a*b
^5))*sqrt(-I*b)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 +
 9*I*a*b^2)/b^3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*
a^3 + 9*I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b
)) + 3*(3*b^6*cos(d*x + c)^5 - (16*a^2*b^4 - 3*b^6)*cos(d*x + c)^3 + 2*(64*
a^4*b^2 - 14*a^2*b^4 - 3*b^6)*cos(d*x + c) + 2*(3*a*b^5*cos(d*x + c)^3 + (8
0*a^3*b^3 - 33*a*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c) + a))
/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2), x)
```

3.1185 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{8(32a^2-9b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{8a(32a^2-17b^2)\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{\frac{a+\cdots}{\cdots}}}{15b^5d\sqrt{a+b\sin(c+dx)}}$$

```
[Out] 2/15*cos(d*x+c)^3*(8*a+3*b*sin(d*x+c))/b^2/d/(a+b*sin(d*x+c))^(3/2)+4/15*co
s(d*x+c)*(32*a^2-9*b^2+8*a*b*sin(d*x+c))/b^4/d/(a+b*sin(d*x+c))^(1/2)-8/15*
(32*a^2-9*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x
)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x
+c))^(1/2)/b^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)+8/15*a*(32*a^2-17*b^2)*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/
b^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2942, 2831, 2742, 2740, 2734, 2732}

$$-\frac{8a(32a^2-17b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{a+b\sin(c+dx)}}+\frac{8(32a^2-9b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{4\cos(c+dx)\left(32a^2+8ab\sin(c+dx)-9b^2\right)}{15b^4d\sqrt{a+b\sin(c+dx)}}+\frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (8*(32*a^2 - 9*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b
*Sin[c + d*x]])/(15*b^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (8*a*(32*a^
2 - 17*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(15*b^5*d*Sqrt[a + b*Sin[c + d*x]]) + (2*Cos[c + d*x]^3*(
8*a + 3*b*Sin[c + d*x]))/(15*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (4*Cos[c +
 d*x]*(32*a^2 - 9*b^2 + 8*a*b*Sin[c + d*x]))/(15*b^4*d*Sqrt[a + b*Sin[c + d
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} - \frac{4\int \frac{\cos^2(c+dx)\left(-\frac{3b}{2}-4a\sin(c+dx)\right)}{(a+b\sin(c+dx))^{3/2}}\,dx}{5b^2}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{2\cos^3(c+dx)(8a+3b\sin(c+dx))}{15b^2d(a+b\sin(c+dx))^{3/2}} + \frac{4\cos(c+dx)\left(32a^2-9b^2+8ab\sin(\right.}{15b^4d\sqrt{a+b\sin(c+dx)}}$$

$$= \frac{8(32a^2-9b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{15b^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}} - \frac{8a(32a^2-17}{}$$

Mathematica [A]
time = 4.42, size = 211, normalized size = 0.83

$$\frac{-32(a+b)^2(32a^2-9b^2)\,E\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+32a(a+b)(32a^2-17b^2)\,F\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\left(\frac{a+b\sin(c+dx)}{a+b}\right)^{3/2}+2b\cos(c+dx)\left(256a^3-24ab^2-16ab^2\cos(2(c+dx))+b(320a^2-69b^2)\sin(c+dx)+3b^3\sin(3(c+dx))\right)}{60b^5d(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-32*(a + b)^2*(32*a^2 - 9*b^2)*EllipticE[(-2*c + Pi - 2*d*x)/4, (2*b)/(a +
 b)]*((a + b*Sin[c + d*x])/(a + b))^(3/2) + 32*a*(a + b)*(32*a^2 - 17*b^2)*
EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*((a + b*Sin[c + d*x])/(a +
b))^(3/2) + 2*b*Cos[c + d*x]*(256*a^3 - 24*a*b^2 - 16*a*b^2*Cos[2*(c + d*x)
] + b*(320*a^2 - 69*b^2)*Sin[c + d*x] + 3*b^3*Sin[3*(c + d*x)]))/(60*b^5*d*
(a + b*Sin[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(300) = 600$.
time = 10.70, size = 1430, normalized size = 5.63

method	result	size
default	Expression too large to display	1430

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(3*b^5*sin(d*x+c)*cos(d*x+c)^4+(80*a^2*b^3-18*b^5)*cos(d*x+c)^2*sin(d*
x+c)-4*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1
/2)*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*b*(32*EllipticE((b/(a-b)*sin(d*x+c)+
a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4-41*EllipticE((b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2+9*EllipticE((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4-32*EllipticF((b/(a-b)*sin(d*x+c)+a/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b+24*EllipticF((b/(a-b)*sin(d*x+c)+a/(
a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^2+17*EllipticF((b/(a-b)*sin(d*x+c)+a
/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^3-9*EllipticF((b/(a-b)*sin(d*x+c)+a/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*b^4)*sin(d*x+c)-8*a*b^4*cos(d*x+c)^4+(64*
a^3*b^2-2*a*b^4)*cos(d*x+c)^2+128*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a
+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF
((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b-96*(b/(a-b)*
sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin
(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^3*b^2-68*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d
*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticF((b/(a-b)
*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^3+36*(b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)
-b/(a-b))^(1/2)*EllipticF((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^
(1/2))*a*b^4-128*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/
(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x
+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5+164*(b/(a-b)*sin(d*x+c)+a/(a-b)
)^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(
1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*
b^2-36*(b/(a-b)*sin(d*x+c)+a/(a-b))^(1/2)*(-b/(a+b)*sin(d*x+c)+b/(a+b))^(1/
2)*(-b/(a-b)*sin(d*x+c)-b/(a-b))^(1/2)*EllipticE((b/(a-b)*sin(d*x+c)+a/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a*b^4)/(a+b*sin(d*x+c))^(3/2)/b^6/cos(d*x+c)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.22, size = 794, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] -2/45*(4*(sqrt(2)*(32*a^3*b^2 - 21*a*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(32*a^
4*b - 21*a^2*b^3)*sin(d*x + c) - sqrt(2)*(32*a^5 + 11*a^3*b^2 - 21*a*b^4))*
sqrt(I*b)*weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*
I*a*b^2)/b^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b) + 4*(s
qrt(2)*(32*a^3*b^2 - 21*a*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(32*a^4*b - 21*a^
2*b^3)*sin(d*x + c) - sqrt(2)*(32*a^5 + 11*a^3*b^2 - 21*a*b^4))*sqrt(-I*b)*
weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/
b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b) - 6*(sqrt(2)*(-
32*I*a^2*b^3 + 9*I*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(32*I*a^3*b^2 - 9*I*a*b^
4)*sin(d*x + c) + sqrt(2)*(32*I*a^4*b + 23*I*a^2*b^3 - 9*I*b^5))*sqrt(I*b)*
weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b^3,
weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*I*a^3 - 9*I*a*b^2)/b
^3, 1/3*(3*b*cos(d*x + c) - 3*I*b*sin(d*x + c) - 2*I*a)/b)) - 6*(sqrt(2)*(3
2*I*a^2*b^3 - 9*I*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-32*I*a^3*b^2 + 9*I*a*b^
4)*sin(d*x + c) + sqrt(2)*(-32*I*a^4*b - 23*I*a^2*b^3 + 9*I*b^5))*sqrt(-I*b
)*weierstrassZeta(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^2)/b^
3, weierstrassPInverse(-4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(-8*I*a^3 + 9*I*a*b^
2)/b^3, 1/3*(3*b*cos(d*x + c) + 3*I*b*sin(d*x + c) + 2*I*a)/b)) - 3*(8*a*b^
4*cos(d*x + c)^3 - 2*(32*a^3*b^2 - a*b^4)*cos(d*x + c) - (3*b^5*cos(d*x + c
)^3 + 2*(40*a^2*b^3 - 9*b^5)*cos(d*x + c))*sin(d*x + c))*sqrt(b*sin(d*x + c
) + a))/(b^8*d*cos(d*x + c)^2 - 2*a*b^7*d*sin(d*x + c) - (a^2*b^6 + b^8)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x))^(5/2), x)
```

3.1186 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}+\frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^2+3b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)|\frac{2b}{a+b}\right)\sqrt{a+b\sin(c+dx)}}{3a^2b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}$$

```
[Out] -2/3*(a^2-b^2)*cos(d*x+c)/a/b^2/d/(a+b*sin(d*x+c))^(3/2)+2/3*(5*a^2+3*b^2)*
cos(d*x+c)/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)-2/3*(8*a^2+3*b^2)*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi
+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^2/b^3/d/((a+b*s
in(d*x+c))/(a+b))^(1/2)+2/3*(8*a^2+b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(
a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a/b^3/d/(a+b*sin(d*x+c))^(1/2)-
2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(
cos(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b
))^(1/2)/a^2/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2970, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}}-\frac{2(8a^2+b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3ab^3d\sqrt{a+b\sin(c+dx)}}+\frac{2(8a^2+3b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{3a^2b^3d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{2\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{a^2d\sqrt{a+b\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*Cos[c + d*x])/(3*a*b^2*d*(a + b*Sin[c + d*x])^(3/2)) + (2*(
5*a^2 + 3*b^2)*Cos[c + d*x])/(3*a^2*b^2*d*Sqrt[a + b*Sin[c + d*x]]) + (2*(8
*a^2 + 3*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c
 + d*x]])/(3*a^2*b^3*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - (2*(8*a^2 + b^
2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(
a + b)])/(3*a*b^3*d*Sqrt[a + b*Sin[c + d*x]]) + (2*EllipticPi[2, (c - Pi/2
+ d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(a^2*d*Sqrt[a
+ b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
```

```
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} - \frac{4\int \frac{\csc(c+dx)(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{4\int \frac{\csc(c+dx)(\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{\int \frac{\csc(c\cdots}{\sqrt{a+b\sin\cdots}}}{a\cdots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2(8a^2+3b^2)\cdots}{\cdots}$$

$$= -\frac{2(a^2-b^2)\cos(c+dx)}{3ab^2d(a+b\sin(c+dx))^{3/2}} + \frac{2(5a^2+3b^2)\cos(c+dx)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}} + \frac{2(8a^2+3b^2)\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.50, size = 443, normalized size = 1.42

$$-\frac{\frac{i(8a^2+3b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{\frac{b(1+\sin(c+dx))}{-a+b}}}{b^2\sqrt{-\frac{1}{a+b}}}+\frac{2a^2(a^2-b^2)\cos(c+dx)}{(a+b\sin(c+dx))^{3/2}}-\frac{2a(5a^2+3b^2)\cos(c+dx)}{\sqrt{a+b\sin(c+dx)}}+\frac{4a^2bF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{a(8a^2+9b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}}{3a^3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*((I*(8*a^2 + 3*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[(b*(1 + Sin[c + d*x]))/(-a + b)])/(b^2*Sqrt[-(a + b)^(-1)]) + (2*a^2*(a^
2 - b^2)*Cos[c + d*x])/(a + b*Sin[c + d*x])^(3/2) - (2*a*(5*a^2 + 3*b^2)*Co
s[c + d*x])/Sqrt[a + b*Sin[c + d*x]] + (4*a^2*b*EllipticF[(-2*c + Pi - 2*d*
x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c +
 d*x]] + (a*(8*a^2 + 9*b^2)*EllipticPi[2, (-2*c + Pi - 2*d*x)/4, (2*b)/(a +
```

```
b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]])/(a^3*b^2
*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(388) = 776$.
time = 43.62, size = 1375, normalized size = 4.39

method	result	size
default	Expression too large to display	1375

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*(1/b^3*(2*b*(1/b*a-1)*((a+b*sin(d*x
+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)
*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c)
)/(a-b))^(1/2),((a-b)/(a+b))^(1/2)))-4*a*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(
-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2)))+1/b^3*(3*a^4-2*a^2*b^2-b^4)/a^2*(2*b*cos(d*x+c)^
2/(a^2-b^2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)+2*a/(a^2-b^2)*(1/b*a-1)
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(
-sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF((
(a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+2/(a^2-b^2)*b*(1/b*a-1)*
((a+b*sin(d*x+c))/(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-
sin(d*x+c)-1)*b)^(1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-1/b*a-1)*
EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a
+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))))+(-a^4+2*a^2*b^2-b^4)/a/b
^3*(2/3/(a^2-b^2)/b*(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)/(sin(d*x+c)+1/b
*a)^2+8/3*b*cos(d*x+c)^2/(a^2-b^2)^2*a/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1
/2)+2*(3*a^2+b^2)/(3*a^4-6*a^2*b^2+3*b^4)*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/
(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^
(1/2),((a-b)/(a+b))^(1/2))+8/3*a*b/(a^2-b^2)^2*(1/b*a-1)*((a+b*sin(d*x+c))/
(a-b))^(1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(
1/2)/(-(-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*((-1/b*a-1)*EllipticE(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))+EllipticF(((a+b*sin(d*x+c))/(a-
b))^(1/2),((a-b)/(a+b))^(1/2))))-2/a^3*(1/b*a-1)*((a+b*sin(d*x+c))/(a-b))^(
1/2)*(1/(a+b)*(1-sin(d*x+c))*b)^(1/2)*(1/(a-b)*(-sin(d*x+c)-1)*b)^(1/2)/(-(
-b*sin(d*x+c)-a)*cos(d*x+c)^2)^(1/2)*b*EllipticPi(((a+b*sin(d*x+c))/(a-b))^
(1/2),-(-1/b*a+1)*b/a,((a-b)/(a+b))^(1/2)))/cos(d*x+c)/(a+b*sin(d*x+c))^(1/
2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
[Out] integrate(cos(d*x + c)^3*cot(d*x + c)/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*cot(d*x+c)/(a+b*sin(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3433 deep
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*cot(d*x+c)/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\cot(c+d\,x)}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^3*cot(c + d*x))/(a + b*sin(c + d*x))^(5/2), x)
```

3.1187 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$\frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}-\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}}-\frac{(4a^2+15b^2)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+15b^2)\,E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{3a^3b^2d\sqrt{}}$$

```
[Out] 1/3*(2*a^2-5*b^2)*cos(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-cot(d*x+c)/a/d/
(a+b*sin(d*x+c))^(3/2)-1/3*(4*a^2+15*b^2)*cos(d*x+c)/a^3/b/d/(a+b*sin(d*x+c
))^(1/2)+1/3*(4*a^2+15*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))
*(a+b*sin(d*x+c))^(1/2)/a^3/b^2/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/3*(4*a^2
+5*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(
a+b))^(1/2)/a^2/b^2/d/(a+b*sin(d*x+c))^(1/2)+5*b*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2,
2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a^3/d/(a+b*sin(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2969, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$-\frac{5b\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{a^3d\sqrt{a+b\sin(c+dx)}}+\frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{(4a^2+5b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3a^2b^2d\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+15b^2)\cos(c+dx)}{3a^3bd\sqrt{a+b\sin(c+dx)}}-\frac{(4a^2+15b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|\frac{2b}{a+b}\right)}{3a^3b^2d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((2*a^2 - 5*b^2)*Cos[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - Cot
[c + d*x]/(a*d*(a + b*Sin[c + d*x])^(3/2)) - ((4*a^2 + 15*b^2)*Cos[c + d*x]
)/(3*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((4*a^2 + 15*b^2)*EllipticE[(c - P
i/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(3*a^3*b^2*d*Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]) + ((4*a^2 + 5*b^2)*EllipticF[(c - Pi/2 + d*x)/
2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(3*a^2*b^2*d*Sqrt[a +
 b*Sin[c + d*x]]) - (5*b*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*S
qrt[(a + b*Sin[c + d*x])/(a + b)])/(a^3*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
```

```
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc(c+dx)\left(-\frac{15b^2}{4}\right.}{}}{}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos}{3a^3bd\sqrt{a+b\sin}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos}{3a^3bd\sqrt{a+b\sin}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos}{3a^3bd\sqrt{a+b\sin}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos}{3a^3bd\sqrt{a+b\sin}}$$

$$= \frac{(2a^2-5b^2)\cos(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))^{3/2}} - \frac{(4a^2+15b^2)\cos}{3a^3bd\sqrt{a+b\sin}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.73, size = 445, normalized size = 1.29

$$\frac{\frac{2i(4a^2+15b^2)\left(-2a(a-b)E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\left(-2aF\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)+b\Pi\left(\frac{a+b}{a};i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\sin(c+dx)}\right)|\frac{a+b}{a-b}\right)\right)\right)\sec(c+dx)\sqrt{-\frac{b(-1+\sin(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sin(c+dx))}{a-b}}}{ab^2\sqrt{-\frac{1}{a+b}}}+\frac{40abF\left(\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}+\frac{2(4a^2+45b^2)\Pi\left(2;\frac{1}{4}(-2c+\pi-2dx)|\frac{2b}{a+b}\right)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}{\sqrt{a+b\sin(c+dx)}}-\frac{2(4a(a^2+10b^2)\cos(c+dx)+6a^2b\cot(c+dx)+b(4a^2+15b^2)\sin(2(c+dx)))}{(a+b\sin(c+dx))^{3/2}}}{12a^3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (((2*I)*(4*a^2 + 15*b^2)*(-2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-
1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(-2*a*EllipticF[I*ArcSi
nh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*Elli
pticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]],
(a + b)/(a - b)]))*Sec[c + d*x]*Sqrt[-((b*(-1 + Sin[c + d*x]))/(a + b))]*Sq
rt[-((b*(1 + Sin[c + d*x]))/(a - b))])/(a*b^2*Sqrt[-(a + b)^(-1)]) + (40*a*
b*EllipticF[(-2*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])
/(a + b)])/Sqrt[a + b*Sin[c + d*x]] + (2*(4*a^2 + 45*b^2)*EllipticPi[2, (-2
```

```
*c + Pi - 2*d*x)/4, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt
[a + b*Sin[c + d*x]] - (2*(4*a*(a^2 + 10*b^2)*Cos[c + d*x] + 6*a^2*b*Cot[c
+ d*x] + b*(4*a^2 + 15*b^2)*Sin[2*(c + d*x)]))/(a + b*Sin[c + d*x])^(3/2))/
(12*a^3*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(419) = 838$.
time = 11.68, size = 2112, normalized size = 6.10

method	result	size
default	Expression too large to display	2112

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/3*(-11*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-
(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-
b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+
b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^2-15*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+
b))^(1/2))*a*b^5*sin(d*x+c)^2+4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)+6*((a+b*sin(d*x+c))
/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/
2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*si
n(d*x+c)+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)-15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)-11*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^4*b^2*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a
+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b
))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)-15*((a+b*sin(d*x+c))/(a-b)
)^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ell
ipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^2*b^4
*sin(d*x+c)+15*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2
),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)+4*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)
```

```
^2+6*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^3*b^3*sin(d*x+c)^2+5*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*
x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(
d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-15*((a+b*sin
(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a
-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*
b^5*sin(d*x+c)^2-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^2+15*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x
+c)^2-4*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a^6*sin(d*x+c)+3*a^3*b^3-20*a^2*b^4*sin(d*x+c)^3+15*a*b^5*si
n(d*x+c)^2+2*a^4*b^2*sin(d*x+c)+20*a^2*b^4*sin(d*x+c)+a^3*b^3*sin(d*x+c)^2-
4*a^3*b^3*sin(d*x+c)^4-15*a*b^5*sin(d*x+c)^4-2*a^4*b^2*sin(d*x+c)^3)/a^4/si
n(d*x+c)/(a+b*sin(d*x+c))^(3/2)/b^3/cos(d*x+c)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="ma
xima")
```

```
[Out] integrate(cos(d*x + c)^2*cot(d*x + c)^2/(b*sin(d*x + c) + a)^(5/2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*cot(d*x+c)**2/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)**2*cot(c + d*x)**2/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*cot(d*x+c)^2/(a+b*sin(d*x+c))^(5/2),x, algorithm="gi
ac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\mathrm{cot}\left(c+d\,x\right)^2}{{\left(a+b\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*cot(c + d*x)^2)/(a + b*sin(c + d*x))^(5/2), x)
```

3.1188 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=407

$$\frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}}-\frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-35b^2)\cot(c+d}{12a^3bd\sqrt{a+b\sin(c+d}}$$

```
[Out] 1/6*(4*a^2-7*b^2)*cot(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-1/2*cot(d*x+c)*
csc(d*x+c)/a/d/(a+b*sin(d*x+c))^(3/2)-1/12*(8*a^2-105*b^2)*cos(d*x+c)/a^4/d
/(a+b*sin(d*x+c))^(1/2)-1/12*(8*a^2-35*b^2)*cot(d*x+c)/a^3/b/d/(a+b*sin(d*x
+c))^(1/2)+1/12*(8*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1
/2))*(a+b*sin(d*x+c))^(1/2)/a^4/b/d/((a+b*sin(d*x+c))/(a+b))^(1/2)-1/12*(8*
a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c
))/(a+b))^(1/2)/a^3/b/d/(a+b*sin(d*x+c))^(1/2)+1/4*(12*a^2-35*b^2)*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))^(1/2)/a
^4/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2969, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}}-\frac{(8a^2-105b^2)\cos(c+dx)}{12a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-105b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12a^4bd\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}-\frac{(12a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{4a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-35b^2)\cot(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}+\frac{(8a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{12a^3bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((4*a^2 - 7*b^2)*Cot[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^(3/2)) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d*(a + b*Sin[c + d*x])^(3/2)) - ((8*a^2 - 105
*b^2)*Cos[c + d*x])/(12*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 35*b^2)
*Cot[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a^2 - 105*b^2)*E
llipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[a + b*Sin[c + d*x]])/(12*a
^4*b*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) + ((8*a^2 - 35*b^2)*EllipticF[(c
 - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(12*a^
3*b*d*Sqrt[a + b*Sin[c + d*x]]) - ((12*a^2 - 35*b^2)*EllipticPi[2, (c - Pi/
2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/(4*a^4*d*Sqr
t[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2969

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b*d*(n + 1)*(m + 1)), Int[(d*Sin[e + f*x])^(n + 1)*(a + b*Sin[e +
```

```
f*x])^(m + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*
(m + 1)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n + 4))*
Sin[e + f*x]^2, x], x], x] - Simp[(a^2*(n + 1) - b^2*(m + n + 2))*Cos[e + f
*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2*b*d^2*f*(n
+ 1)*(m + 1))), x]) /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && In
tegersQ[2*m, 2*n] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} + \frac{\int \frac{\csc^2(c+dx)\left(\frac{1}{4}(8a \cdots}{\cdots}}{\cdots}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-35b^2)\,c\cdots}{12a^3bd\sqrt{a+b\cdots}}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\,c\cdots}{12a^4d\sqrt{a+b\,s\cdots}}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\,c\cdots}{12a^4d\sqrt{a+b\,s\cdots}}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\,c\cdots}{12a^4d\sqrt{a+b\,s\cdots}}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\,c\cdots}{12a^4d\sqrt{a+b\,s\cdots}}$$

$$= \frac{(4a^2-7b^2)\cot(c+dx)}{6a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc(c+dx)}{2ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-105b^2)\,c\cdots}{12a^4d\sqrt{a+b\,s\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.49, size = 622, normalized size = 1.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*((11*b*Cot[c + d*x])/(4*a^4) - (Cot[c + d*x]*Csc[
c + d*x])/(2*a^3) - (2*(a^2*Cos[c + d*x] - b^2*Cos[c + d*x]))/(3*a^3*(a + b
*Sin[c + d*x])^2) - (2*(a^2*Cos[c + d*x] - 9*b^2*Cos[c + d*x]))/(3*a^4*(a +
 b*Sin[c + d*x]))))/d + ((-280*a*b*EllipticF[(-c + Pi/2 - d*x)/2, (2*b)/(a
+ b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - (2*(-8
```

```
0*a^2 + 315*b^2)*EllipticPi[2, (-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a
+ b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin[c + d*x]] - ((2*I)*(8*a^2 - 105*
b^2)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(
a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)] + b*(2*a*EllipticF
[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]
- b*EllipticPi[(a + b)/a, I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c +
d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b
*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a + b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(
-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-(
(a^2 - b^2 - 2*a*(a + b*Sin[c + d*x]) + (a + b*Sin[c + d*x])^2)/b^2)]))/(48
*a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2616 vs. $2(472) = 944$.
time = 12.92, size = 2617, normalized size = 6.43

method	result	size
default	Expression too large to display	2617

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(-6*a^4*b^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b
))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b)
)^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-113*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*
sin(d*x+c)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(
1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/
2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2-8*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
F(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^2+36
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-
b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2+8*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(s
in(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b
*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*sin(d*x+c)^3-113*((a+b
*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*
b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2)
)*a^3*b^3*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*
b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/
(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3-8*((a+b*sin(d*x+c))/(a
-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*
EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d
*x+c)^3+78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(
```

```
-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a
-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3-36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-
(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((
a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c
)^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)
/a,((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2+35*((a+b*sin(d*x+c))/(a-b))^(1
/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellipti
cF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3
-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a*b^5*sin(d*x+c)^3+36*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(
d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3-36*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)
/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3-105*a*b^5*sin(d*x+c)^5-140*a^2*b^4*sin(
d*x+c)^4-29*a^3*b^3*sin(d*x+c)^3+105*a*b^5*sin(d*x+c)^3-10*a^4*b^2*sin(d*x+
c)^2+140*a^2*b^4*sin(d*x+c)^2+21*a^3*b^3*sin(d*x+c)+105*((a+b*sin(d*x+c))/(
a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)
*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b
^5*sin(d*x+c)^2+78*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^2+35*((a+b*sin(d*x+c))/(a-b))^
(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Ellip
ticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)
^2-105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+
sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/
(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^2+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(si
n(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b
*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^3+8*(
(a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+
c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(
1/2))*a^6*sin(d*x+c)^2+16*a^4*b^2*sin(d*x+c)^4+8*a^3*b^3*sin(d*x+c)^5)/a^5/
sin(d*x+c)^2/(a+b*sin(d*x+c))^(3/2)/b^2/cos(d*x+c)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxi
ma")
```

```
[Out] Timed out
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="fric
as")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(17),SparseUnivariatePolynomial(InnerPrimeField(17)),?^2+6*?+12)),f
ailed)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)**3/(a+b*sin(d*x+c))**(5/2),x)
```

```
[Out] Integral(cos(c + d*x)*cot(c + d*x)**3/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*cot(d*x+c)^3/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\cot(c+d\,x)^3}{(a+b\,\sin(c+d\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*cot(c + d*x)^3)/(a + b*sin(c + d*x))^(5/2), x)
```

3.1189 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=458

$$\frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}}+\frac{b(32a^2-105b^2)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{(16a^2-35b^2}{8a^4d\sqrt{a+b}$$

```
[Out] 1/3*(2*a^2-3*b^2)*cot(d*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^(3/2)-1/3*
cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b*sin(d*x+c))^(3/2)+1/8*b*(32*a^2-105*b^2)*c
os(d*x+c)/a^5/d/(a+b*sin(d*x+c))^(1/2)+1/8*(16*a^2-35*b^2)*cot(d*x+c)/a^4/d
/(a+b*sin(d*x+c))^(1/2)-1/12*(8*a^2-21*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(
a+b*sin(d*x+c))^(1/2)-1/8*(32*a^2-105*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(
b/(a+b))^(1/2))*(a+b*sin(d*x+c))^(1/2)/a^5/d/((a+b*sin(d*x+c))/(a+b))^(1/2)
+1/8*(16*a^2-35*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1
/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)*(b/(a+b))^(1/2))*((a+b*
sin(d*x+c))/(a+b))^(1/2)/a^4/d/(a+b*sin(d*x+c))^(1/2)-15/8*b*(4*a^2-7*b^2)*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(co
s(1/2*c+1/4*Pi+1/2*d*x),2,2^(1/2)*(b/(a+b))^(1/2))*((a+b*sin(d*x+c))/(a+b))
^(1/2)/a^5/d/(a+b*sin(d*x+c))^(1/2)
```

Rubi [A]
time = 0.98, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2803, 3134, 3138, 2734, 2732, 3081, 2742, 2740, 2886, 2884}

$$\frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}}+\frac{b(32a^2-105b^2)\cos(c+dx)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{(32a^2-105b^2)\sqrt{a+b\sin(c+dx)}\,E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^5d\sqrt{\frac{a+b\sin(c+dx)}{a+b}}}+\frac{15b(4a^2-7b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^5d\sqrt{a+b\sin(c+dx)}}+\frac{(16a^2-35b^2)\cot(c+dx)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(16a^2-35b^2)\sqrt{\frac{a+b\sin(c+dx)}{a+b}}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|\frac{2b}{a+b}\right)}{8a^4d\sqrt{a+b\sin(c+dx)}}-\frac{(8a^2-21b^2)\cot(c+dx)\csc(c+dx)}{12a^3bd\sqrt{a+b\sin(c+dx)}}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] ((2*a^2 - 3*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])
^(3/2)) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^(3/2))
+ (b*(32*a^2 - 105*b^2)*Cos[c + d*x])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]]) +
((16*a^2 - 35*b^2)*Cot[c + d*x])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) - ((8*a
^2 - 21*b^2)*Cot[c + d*x]*Csc[c + d*x])/(12*a^3*b*d*Sqrt[a + b*Sin[c + d*x]
]) + ((32*a^2 - 105*b^2)*EllipticE[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[
a + b*Sin[c + d*x]])/(8*a^5*d*Sqrt[(a + b*Sin[c + d*x])/(a + b)]) - ((16*a^
2 - 35*b^2)*EllipticF[(c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c
+ d*x])/(a + b)])/(8*a^4*d*Sqrt[a + b*Sin[c + d*x]]) + (15*b*(4*a^2 - 7*b^2
)*EllipticPi[2, (c - Pi/2 + d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x]
)/(a + b)])/(8*a^5*d*Sqrt[a + b*Sin[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && IntegerQ[2*m]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{5/2}}\,dx = \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{2\int \frac{\csc^3(c+dx}{}}{}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} - \frac{(8a^2-21b^2}{12a^3bd}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{(16a^2-35b}{8a^4d\sqrt{a+}}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b(32a^2-10}{8a^5d\sqrt{a+}}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b(32a^2-10}{8a^5d\sqrt{a+}}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b(32a^2-10}{8a^5d\sqrt{a+}}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b(32a^2-10}{8a^5d\sqrt{a+}}$$

$$= \frac{(2a^2-3b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))^{3/2}} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^{3/2}} + \frac{b(32a^2-10}{8a^5d\sqrt{a+}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 16.56, size = 680, normalized size = 1.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[a + b*Sin[c + d*x]]*(((32*a^2*Cos[c + d*x] - 123*b^2*Cos[c + d*x])*Cs
c[c + d*x])/(24*a^5) + (17*b*Cot[c + d*x]*Csc[c + d*x])/(12*a^4) - (Cot[c +
```

```
d*x]*Csc[c + d*x]^2)/(3*a^3) + (2*(a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x]))
/(3*a^4*(a + b*Sin[c + d*x])^2) + (8*(a^2*b*Cos[c + d*x] - 3*b^3*Cos[c + d*
x]))/(3*a^5*(a + b*Sin[c + d*x]))))/d + ((-2*(32*a^3 - 140*a*b^2)*EllipticF
[(-c + Pi/2 - d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sq
rt[a + b*Sin[c + d*x]] - (2*(152*a^2*b - 315*b^3)*EllipticPi[2, (-c + Pi/2
- d*x)/2, (2*b)/(a + b)]*Sqrt[(a + b*Sin[c + d*x])/(a + b)])/Sqrt[a + b*Sin
[c + d*x]] - ((2*I)*(-32*a^2*b + 105*b^3)*Cos[c + d*x]*Cos[2*(c + d*x)]*(2*
a*(a - b)*EllipticE[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]]
, (a + b)/(a - b)] + b*(2*a*EllipticF[I*ArcSinh[Sqrt[-(a + b)^(-1)]*Sqrt[a
+ b*Sin[c + d*x]]], (a + b)/(a - b)] - b*EllipticPi[(a + b)/a, I*ArcSinh[Sq
rt[-(a + b)^(-1)]*Sqrt[a + b*Sin[c + d*x]]], (a + b)/(a - b)]))*Sqrt[(b - b
*Sin[c + d*x])/(a + b)]*Sqrt[-((b + b*Sin[c + d*x])/(a - b))])/(a*Sqrt[-(a
+ b)^(-1)]*Sqrt[1 - Sin[c + d*x]^2]*(-2*a^2 + b^2 + 4*a*(a + b*Sin[c + d*x]
) - 2*(a + b*Sin[c + d*x])^2)*Sqrt[-((a^2 - b^2 - 2*a*(a + b*Sin[c + d*x])
+ (a + b*Sin[c + d*x])^2)/b^2)]))/(32*a^5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2869 vs. $2(519) = 1038$.
time = 15.02, size = 2870, normalized size = 6.27

method	result	size
default	Expression too large to display	2870

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(8*a^5*b+306*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))
^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(
1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4+105*((a+b*sin(d*x+c))/(a-b))
^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elli
pticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c
)^4-315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1
+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)
/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin
(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*
sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4-
180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin
(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,(
(a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^4-315*((a+b*sin(d*x+c))/(a-b))^(1/2)
*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi
(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+
c)^4-411*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(
1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b
)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(
sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a
```

+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^3
+306*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+si
n(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a
+b))^(1/2))*a^4*b^2*sin(d*x+c)^3+105*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(
d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*si
n(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3-315*((a+b*
sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b
/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))
*a^2*b^4*sin(d*x+c)^3-180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b
/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/
(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^3-315*((a+b*si
n(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(
a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(a-b)/a,((a-b)/(a+b))
^(1/2))*a^2*b^4*sin(d*x+c)^3+96*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c
)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x
+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d*x+c)^3-48*((a+b*sin(d*x+c)
)/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1
/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^6*sin(d
x+c)^3+315((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*
(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))^(1/2),(
a-b)/a,((a-b)/(a+b))^(1/2))*b^6*sin(d*x+c)^4-40*a^5*b*sin(d*x+c)^2+32*a^5*b
*sin(d*x+c)^4+96*a^3*b^3*sin(d*x+c)^6-315*a*b^5*sin(d*x+c)^6+144*a^4*b^2*si
n(d*x+c)^5-420*a^2*b^4*sin(d*x+c)^5+420*a^2*b^4*sin(d*x+c)^3-18*a^4*b^2*sin
(d*x+c)+63*a^3*b^3*sin(d*x+c)^2-159*a^3*b^3*sin(d*x+c)^4+315*a*b^5*sin(d*x+
c)^4-126*a^4*b^2*sin(d*x+c)^3+315*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x
+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d
*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^2*b^4*sin(d*x+c)^3-48*((a+b*sin(
d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-
b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5
*b*sin(d*x+c)^3+180*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b)
)^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticPi(((a+b*sin(d*x+c))/(a-b))
^(1/2),(a-b)/a,((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^3+96*((a+b*sin(d*x+c
))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(
1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^5*b*si
n(d*x+c)^4-411*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/
2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticE(((a+b*sin(d*x+c))/(a-b))^(1/2)
,((a-b)/(a+b))^(1/2))*a^3*b^3*sin(d*x+c)^4+315*((a+b*sin(d*x+c))/(a-b))^(1/
2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*Elliptic
E(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a*b^5*sin(d*x+c)^4-48
*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-1)*b/(a+b))^(1/2)*(-(1+sin(d*
x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c))/(a-b))^(1/2),((a-b)/(a+b))
^(1/2))*a^5*b*sin(d*x+c)^4-48*((a+b*sin(d*x+c))/(a-b))^(1/2)*(-(sin(d*x+c)-
1)*b/(a+b))^(1/2)*(-(1+sin(d*x+c))*b/(a-b))^(1/2)*EllipticF(((a+b*sin(d*x+c
))/(a-b))^(1/2),((a-b)/(a+b))^(1/2))*a^4*b^2*sin(d*x+c)^4)/sin(d*x+c)^3/a^6
/(a+b*sin(d*x+c))^(3/2)/b/cos(d*x+c)/d

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**(5/2),x)
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x))^(5/2),x)
[Out] \text{Hanged}
```

3.1190 $\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=510

$$\frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}}+\frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}}+\frac{8(a^2-2b^2)\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^3(a^2-b^2)\,df(a+b\sin(e+fx))^{3/2}}+$$

```
[Out] 2/7*cos(f*x+e)^3*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(7/2)+12/35*co
s(f*x+e)*(d*sin(f*x+e))^(1/2)/a^2/d/f/(a+b*sin(f*x+e))^(5/2)+8/35*(a^2-2*b^
2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^3/(a^2-b^2)/d/f/(a+b*sin(f*x+e))^(3/2)
+32/35*b*(2*a^2-b^2)*cos(f*x+e)/a^3/(a^2-b^2)^2/f/(d*sin(f*x+e))^(1/2)/(a+b
*sin(f*x+e))^(1/2)-32/35*b*(2*a^2-b^2)*EllipticE(d^(1/2)*(a+b*sin(f*x+e))^(
1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+e
))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^5/(a-b)/(a+b)^(
3/2)/f/d^(1/2)-8/35*(5*a^2-3*a*b-4*b^2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+
e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^4/(a-b)/(a+b)^
(3/2)/f/d^(1/2)
```

Rubi [A]
time = 1.20, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2966, 2968, 3135, 3072, 3077, 2895, 3073}

$$\frac{12\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^2df(a+b\sin(e+fx))^{5/2}}-\frac{32b(2a^2-b^2)\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{35a^5\sqrt{d}f(a-b)(a+b)^{3/2}}-\frac{8(5a^2-3ab-4b^2)\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{35a^4\sqrt{d}f(a-b)(a+b)^{3/2}}+\frac{32b(2a^2-b^2)\cos(e+fx)}{35a^3f(a^2-b^2)^2\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}+\frac{8(a^2-2b^2)\cos(e+fx)\sqrt{d\sin(e+fx)}}{35a^3df(a^2-b^2)(a+b\sin(e+fx))^{3/2}}+\frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (2*Cos[e + f*x]^3*Sqrt[d*Sin[e + f*x]])/(7*a*d*f*(a + b*Sin[e + f*x])^(7/2)
) + (12*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^2*d*f*(a + b*Sin[e + f*x])
^(5/2)) + (8*(a^2 - 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(35*a^3*(a^2
- b^2)*d*f*(a + b*Sin[e + f*x])^(3/2)) + (32*b*(2*a^2 - b^2)*Cos[e + f*x])/
(35*a^3*(a^2 - b^2)^2*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (3
2*b*(2*a^2 - b^2)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e +
 f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[
a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(35*a^5*(a
 - b)*(a + b)^(3/2)*Sqrt[d]*f) - (8*(5*a^2 - 3*a*b - 4*b^2)*Sqrt[(a*(1 - Cs
c[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin
[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -(
(a + b)/(a - b))]*Tan[e + f*x])/(35*a^4*(a - b)*(a + b)^(3/2)*Sqrt[d]*f)
```

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2966

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-g)*(g*
Cos[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^(m + 1)/(a
*d*f*(m + 1))), x] + Dist[g^2*((2*m + 3)/(2*a*(m + 1))), Int[(g*Cos[e + f*x
])^(p - 2)*((a + b*Sin[e + f*x])^(m + 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; F
reeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p
 + 1/2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
```

```
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}\,dx = \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{\cos^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin \cdots}}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{6\int \frac{1-\sin^2(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin \cdots}}{7a}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+ \cdots}}{35a^2df(a+b\sin(e+fx \cdots}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+ \cdots}}{35a^2df(a+b\sin(e+fx \cdots}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+ \cdots}}{35a^2df(a+b\sin(e+fx \cdots}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+ \cdots}}{35a^2df(a+b\sin(e+fx \cdots}$$

$$= \frac{2\cos^3(e+fx)\sqrt{d\sin(e+fx)}}{7adf(a+b\sin(e+fx))^{7/2}} + \frac{12\cos(e+fx)\sqrt{d\sin(e+ \cdots}}{35a^2df(a+b\sin(e+fx \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.41, size = 1670, normalized size = 3.27

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2)),
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((-2*(a^2*Cos[e + f*x] - b^2*Cos[e +
 f*x]))/(7*a*b^2*(a + b*Sin[e + f*x])^4) + (4*(5*a^2*Cos[e + f*x] + 3*b^2*C
os[e + f*x]))/(35*a^2*b^2*(a + b*Sin[e + f*x])^3) - (2*(5*a^4*Cos[e + f*x]
- 9*a^2*b^2*Cos[e + f*x] + 8*b^4*Cos[e + f*x]))/(35*a^3*b^2*(a^2 - b^2)*(a
+ b*Sin[e + f*x])^2) - (32*(2*a^2*b^2*Cos[e + f*x] - b^4*Cos[e + f*x]))/(35
*a^4*(a^2 - b^2)^2*(a + b*Sin[e + f*x]))))/(f*Sqrt[d*Sin[e + f*x]]) + (4*Sq
rt[Sin[e + f*x]]*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e +
Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]
```

```
^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e
 + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x]
)/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*S
qrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sq
rt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Cs
c[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 -
f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
 f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[(
(a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a
 + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi
/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Si
n[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^
2*b^2 - 4*b^4)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x
]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e
+ Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f
*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(
a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*
x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b
*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 -
 f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqr
t[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e
 + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)
/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[
e + f*x]]*Sqrt[a + b*Sin[e + f*x]]))))/b)))/(35*a^4*(a - b)^2*(a + b)^2*f*Sq
rt[d*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25052 vs. $2(458) = 916$.

time = 9.12, size = 25053, normalized size = 49.12

method	result	size
default	Expression too large to display	25053

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(-sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^4/(b^5
*d*cos(f*x + e)^6 - (10*a^2*b^3 + 3*b^5)*d*cos(f*x + e)^4 + (5*a^4*b + 20*a
^2*b^3 + 3*b^5)*d*cos(f*x + e)^2 - (5*a^4*b + 10*a^2*b^3 + b^5)*d - (5*a*b^
4*d*cos(f*x + e)^4 - 10*(a^3*b^2 + a*b^4)*d*cos(f*x + e)^2 + (a^5 + 10*a^3*
b^2 + 5*a*b^4)*d)*sin(f*x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/((b*sin(f*x + e) + a)^(9/2)*sqrt(d*sin(f*x + e))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(9/2)),x)
[Out] int(cos(e + f*x)^4/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(9/2)), x)
```

3.1191 $\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]
[Out] Defer[Int][(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]], x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx = \int \frac{\cos^4(c+dx)\sqrt[3]{\sin(c+dx)}}{\sqrt{a+b\sin(c+dx)}}\,dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^(1/3))/Sqrt[a + b*Sin[c + d*x]],x]
[Out] $Aborted
```

Maple [A]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^4(dx+c)\right)\left(\sin^{\frac{1}{3}}(dx+c)\right)}{\sqrt{a+b\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sin{\left(c+dx \right)}}\cos^{4}{\left(c+dx \right)}}{\sqrt{a+b\sin{\left(c+dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(1/3)/(a+b*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(sin(c + d*x)**(1/3)*cos(c + d*x)**4/sqrt(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(1/3)/(a+b*sin(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate(cos(d*x + c)^4*sin(d*x + c)^(1/3)/sqrt(b*sin(d*x + c) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^{1/3}}{\sqrt{a+b\,\sin\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^(1/3))/(a + b*sin(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^4*sin(c + d*x)^(1/3))/(a + b*sin(c + d*x))^(1/2), x)
```

3.1192 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Mathematica [A]
time = 4.88, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Maple [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos (c+d\,x)^4\sin (c+d\,x)^n\,(a+b\,\sin (c+d\,x))^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^p,x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^p, x)
```

3.1193 $\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p$$

Mathematica [A]
time = 3.98, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^{-3-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-3 - p)*(a + b*Sin[c + d*x])^p, x]
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^{-3-p}(dx+c)\right)\left(a+b\sin(dx+c)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-3-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-3-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 3)*cos(d*x + c)^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(a+b\,\sin\left(c+d\,x\right)\right)^p}{\sin\left(c+d\,x\right)^{p+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 3),x)
[Out] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 3), x)
```

3.1194 $\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=36

$$\operatorname{Int}\left(\cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p,x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Defer[Int][Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx = \int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,d$$

Mathematica [A]
time = 4.51, size = 0, normalized size = 0.00

$$\int \cos^4(c+dx)\sin^{-4-p}(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^(-4 - p)*(a + b*Sin[c + d*x])^p, x]
```

Maple [A]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^{-4-p}(dx+c)\right)(a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="m
axima")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="f
ricas")
```

```
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**(-4-p)*(a+b*sin(d*x+c))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^(-4-p)*(a+b*sin(d*x+c))^p,x, algorithm="g
iac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^(-p - 4)*cos(d*x + c)^4, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^4\,\left(a+b\,\sin\left(c+d\,x\right)\right)^p}{\sin\left(c+d\,x\right)^{p+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 4),x)
```

```
[Out] int((cos(c + d*x)^4*(a + b*sin(c + d*x))^p)/sin(c + d*x)^(p + 4), x)
```

3.1195 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=623

$$-\frac{3a(2a^4(6+5n+n^2)+3b^4(35+12n+n^2)-2a^2b^2(58+16n+n^2))\cos(c+dx)\sin^{1+n}(c+dx)}{b^2d(2+n)(4+n)(5+n)(6+n)(7+n)}+\frac{3a(3b^2(1$$

```
[Out] -3*a*(2*a^4*(n^2+5*n+6)+3*b^4*(n^2+12*n+35)-2*a^2*b^2*(n^2+16*n+58))*cos(d*
x+c)*sin(d*x+c)^(1+n)/b^2/d/(5+n)/(6+n)/(7+n)/(n^2+6*n+8)-3*(2*a^4*(n^2+5*n
+6)+b^4*(n^2+10*n+24)-2*a^2*b^2*(n^2+16*n+57))*cos(d*x+c)*sin(d*x+c)^(2+n)/
b/d/(3+n)/(4+n)/(5+n)/(6+n)/(7+n)-3*a*(a^2*(n^2+5*n+6)-b^2*(n^2+15*n+53))*c
os(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^2/b^2/d/(4+n)/(5+n)/(6+n)/(7+n)
-(a^2*(2+n)*(3+n)-b^2*(6+n)*(8+n))*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x
+c))^3/b^2/d/(5+n)/(6+n)/(7+n)+a*(3+n)*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin
(d*x+c))^4/b^2/d/(6+n)/(7+n)-cos(d*x+c)*sin(d*x+c)^(2+n)*(a+b*sin(d*x+c))^4
/b/d/(7+n)+3*a*(3*b^2*(1+n)+a^2*(6+n))*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*n
],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(6+n)/(n^3+7*n^2+14*n+8)/(co
s(d*x+c)^2)^(1/2)+3*b*(b^2*(2+n)+3*a^2*(7+n))*cos(d*x+c)*hypergeom([1/2, 1+
1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(5+n)/(7+n)/(n^2+5*n+6)/(
cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 623, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2974, 3128, 3112, 3102, 2827, 2722}

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*(2*a^4*(6 + 5*n + n^2) + 3*b^4*(35 + 12*n + n^2) - 2*a^2*b^2*(58 + 16
*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n
)*(6 + n)*(7 + n)) + (3*a*(3*b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n)
)/(d*(1 + n)*(2 + n)*(4 + n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (3*(2*a^4*(6 +
 5*n + n^2) + b^4*(24 + 10*n + n^2) - 2*a^2*b^2*(57 + 16*n + n^2))*Cos[c +
d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)) +
(3*b*(b^2*(2 + n) + 3*a^2*(7 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*
(5 + n)*(7 + n)*Sqrt[Cos[c + d*x]^2]) - (3*a*(a^2*(6 + 5*n + n^2) - b^2*(53
 + 15*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(
b^2*d*(4 + n)*(5 + n)*(6 + n)*(7 + n)) - ((a^2*(2 + n)*(3 + n) - b^2*(6 + n
```

)*(8 + n))*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b^2*d *(5 + n)*(6 + n)*(7 + n)) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^4)/(b^2*d*(6 + n)*(7 + n)) - (Cos[c + d*x]*Sin[c + d*x]^(2 + n)*(a + b*Sin[c + d*x])^4)/(b*d*(7 + n))

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
```

```
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))}{b^2d(6+n)(7+n)}$$

$$= -\frac{(a^2(2+n)(3+n)-b^2(6+n)(8+n))\cos(c+dx)\,\mathrm{s}}{b^2d(5+n)(6+n)(7+}$$

$$= -\frac{3a(a^2(6+5n+n^2)-b^2(53+15n+n^2))\cos(c+dx}{b^2d(4+n)(5+n)(6+n}$$

$$= -\frac{3(2a^4(6+5n+n^2)+b^4(24+10n+n^2)-2a^2b^2(57}{bd(3+n)(4+n)(5+n)(}$$

$$= -\frac{3a(2a^4(6+5n+n^2)+3b^4(35+12n+n^2)-2a^2b^2(}{b^2d(2+n)(4+n)(5+n}$$

$$= -\frac{3a(2a^4(6+5n+n^2)+3b^4(35+12n+n^2)-2a^2b^2(}{b^2d(2+n)(4+n)(5+n}$$

$$= -\frac{3a(2a^4(6+5n+n^2)+3b^4(35+12n+n^2)-2a^2b^2(}{b^2d(2+n)(4+n)(5+n}$$

Mathematica [A]
time = 0.62, size = 195, normalized size = 0.31

$$\frac{\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\sin^{1+n}(c+dx)\left(\frac{a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)}{1+n}+b\sin(c+dx)\left(\frac{3a^2\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)}{2+n}+b\sin(c+dx)\left(\frac{3a\,{}_2F_1\left(-\frac{3}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(c+dx)\right)}{3+n}+\frac{b\,{}_2F_1\left(-\frac{3}{2},\frac{4+n}{2};\frac{6+n}{2};\sin^2(c+dx)\right)\sin(c+dx)}{4+n}\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*((a^3*Hypergeometri
c2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2])/(1 + n) + b*Sin[c + d*x]*
((3*a^2*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2])/(2 +
 n) + b*Sin[c + d*x]*((3*a*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Si
n[c + d*x]^2])/(3 + n) + (b*Hypergeometric2F1[-3/2, (4 + n)/2, (6 + n)/2, S
in[c + d*x]^2]*Sin[c + d*x])/(4 + n)))))/d
```

Maple [F]
time = 1.32, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(dx+c\right)\right) \left(\sin^{n}\left(dx+c\right)\right) \left(a+b\sin\left(dx+c\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] integral(-(3*a*b^2*cos(d*x + c)^6 - (a^3 + 3*a*b^2)*cos(d*x + c)^4 + (b^3*c
os(d*x + c)^6 - (3*a^2*b + b^3)*cos(d*x + c)^4)*sin(d*x + c))*sin(d*x + c)^
n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5988 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^3,x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^3, x)
```

3.1196 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=487

$$-\frac{(3b^4(5+n)+2a^4(6+5n+n^2)-2a^2b^2(40+13n+n^2))\cos(c+dx)\sin^{1+n}(c+dx)}{b^2d(2+n)(4+n)(5+n)(6+n)}+\frac{3(b^2(1+n)+a^2(6+}{d(1}$$

```
[Out] -(3*b^4*(5+n)+2*a^4*(n^2+5*n+6)-2*a^2*b^2*(n^2+13*n+40))*cos(d*x+c)*sin(d*x
+c)^(1+n)/b^2/d/(5+n)/(6+n)/(n^2+6*n+8)-2*a*(a^2*(n^2+5*n+6)-b^2*(n^2+13*n+
39))*cos(d*x+c)*sin(d*x+c)^(2+n)/b/d/(3+n)/(4+n)/(5+n)/(6+n)-(a^2*(2+n)*(3+
n)-b^2*(5+n)*(7+n))*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^2/b^2/d/(4
+n)/(5+n)/(6+n)+a*(3+n)*cos(d*x+c)*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^3/b^2/
d/(5+n)/(6+n)-cos(d*x+c)*sin(d*x+c)^(2+n)*(a+b*sin(d*x+c))^3/b/d/(6+n)+3*(b
^2*(1+n)+a^2*(6+n))*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(d
*x+c)^2)*sin(d*x+c)^(1+n)/d/(4+n)/(6+n)/(n^2+3*n+2)/(cos(d*x+c)^2)^(1/2)+6*
a*b*cos(d*x+c)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^
(2+n)/d/(5+n)/(n^2+5*n+6)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.72, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2974, 3128, 3112, 3102, 2827, 2722}

$$\frac{3(a^2(n+6)+b^2(n+1))\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)(n+2)(n+4)(n+6)\sqrt{\cos^2(c+dx)}}-\frac{2a(a^2(n^2+5n+6)-b^2(n^2+13n+39))\cos(c+dx)\sin^{n+2}(c+dx)}{bd(n+3)(n+4)(n+5)(n+6)}-\frac{(a^2(n+2)(n+3)-b^2(n+5)(n+7))\cos(c+dx)\sin^{n+1}(c+dx)(a+b\sin(c+dx))^2}{b^2d(n+4)(n+5)(n+6)}-\frac{(2a^4(n^2+5n+6)-2a^2b^2(n^2+13n+40)+3b^4(n+5))\cos(c+dx)\sin^{n+1}(c+dx)}{b^2d(n+2)(n+4)(n+5)(n+6)}+\frac{a(n+3)\cos(c+dx)\sin^{n+1}(c+dx)(a+b\sin(c+dx))^3}{b^2d(n+5)(n+6)}+\frac{6ab\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)(n+3)(n+5)\sqrt{\cos^2(c+dx)}}-\frac{\cos(c+dx)\sin^{n+2}(c+dx)(a+b\sin(c+dx))^3}{bd(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((3*b^4*(5 + n) + 2*a^4*(6 + 5*n + n^2) - 2*a^2*b^2*(40 + 13*n + n^2))*Co
s[c + d*x]*Sin[c + d*x]^(1 + n))/(b^2*d*(2 + n)*(4 + n)*(5 + n)*(6 + n))) +
 (3*(b^2*(1 + n) + a^2*(6 + n))*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)
/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*(2 + n)*(4
+ n)*(6 + n)*Sqrt[Cos[c + d*x]^2]) - (2*a*(a^2*(6 + 5*n + n^2) - b^2*(39 +
13*n + n^2))*Cos[c + d*x]*Sin[c + d*x]^(2 + n))/(b*d*(3 + n)*(4 + n)*(5 + n
)*(6 + n)) + (6*a*b*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/
2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n)*(3 + n)*(5 + n)*Sqrt[Co
s[c + d*x]^2]) - ((a^2*(2 + n)*(3 + n) - b^2*(5 + n)*(7 + n))*Cos[c + d*x]*
Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^2)/(b^2*d*(4 + n)*(5 + n)*(6 + n)
) + (a*(3 + n)*Cos[c + d*x]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^3)/(b
^2*d*(5 + n)*(6 + n)) - (Cos[c + d*x]*Sin[c + d*x]^(2 + n)*(a + b*Sin[c + d
*x])^3)/(b*d*(6 + n))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a(3+n)\cos(c+dx)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^3}{b^2d(5+n)(6+n)}$$

$$= -\frac{(a^2(2+n)(3+n)-b^2(5+n)(7+n))\cos(c+dx)\sin}{b^2d(4+n)(5+n)(6+}$$

$$= -\frac{2a(a^2(6+5n+n^2)-b^2(39+13n+n^2))\cos(c+dx)}{bd(3+n)(4+n)(5+n)(6+n)}$$

$$= -\frac{(3b^4(5+n)+2a^4(6+5n+n^2)-2a^2b^2(40+13n+}{b^2d(2+n)(4+n)(5+n)(6}$$

$$= -\frac{(3b^4(5+n)+2a^4(6+5n+n^2)-2a^2b^2(40+13n+}{b^2d(2+n)(4+n)(5+n)(6}$$

$$= -\frac{(3b^4(5+n)+2a^4(6+5n+n^2)-2a^2b^2(40+13n+}{b^2d(2+n)(4+n)(5+n)(6}$$

Mathematica [A]
time = 0.24, size = 167, normalized size = 0.34

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{1+n}(c+dx)\left(a^2(6+5n+n^2)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+b(1+n)\sin(c+dx)\left(2a(3+n)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)+b(2+n)\,{}_2F_1\left(-\frac{3}{2},\frac{3+n}{2};\frac{5+n}{2};\sin^2(c+dx)\right)\sin(c+dx)\right)\right)}{d(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a^2*(6 + 5*n + n^2
)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)
*Sin[c + d*x]*(2*a*(3 + n)*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Si
n[c + d*x]^2] + b*(2 + n)*Hypergeometric2F1[-3/2, (3 + n)/2, (5 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x])))/(d*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
[Out] integral(-(b^2*cos(d*x + c)^6 - 2*a*b*cos(d*x + c)^4*sin(d*x + c) - (a^2 +
b^2)*cos(d*x + c)^4)*sin(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^2,x)
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x))^2, x)
```

3.1197 $\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=129

$$\frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)}{d(2+n)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*cos(d*x+c)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+
c)^(1+n)/d/(1+n)/(cos(d*x+c)^2)^(1/2)+b*cos(d*x+c)*hypergeom([-3/2, 1+1/2*n
],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2917, 2657}

$$\frac{a\cos(c+dx)\sin^{n+1}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)\sqrt{\cos^2(c+dx)}}+\frac{b\cos(c+dx)\sin^{n+2}(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^
2]*Sin[c + d*x]^(1 + n))/(d*(1 + n)*Sqrt[Cos[c + d*x]^2]) + (b*Cos[c + d*x]
*Hypergeometric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]
^(2 + n))/(d*(2 + n)*Sqrt[Cos[c + d*x]^2])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \cos^4(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^4(c+dx)\sin^n(c+dx)\,dx + b\int \cos^4(c+dx)\sin^1$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+}{d(1+n)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 111, normalized size = 0.86

$$\frac{\sqrt{\cos^2(c+dx)}\sec(c+dx)\sin^{1+n}(c+dx)\left(a(2+n)\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+b(1+n)\,{}_2F_1\left(-\frac{3}{2},\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)\sin(c+dx)\right)}{d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Sec[c + d*x]*Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeo
metric2F1[-3/2, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2] + b*(1 + n)*Hypergeom
etric2F1[-3/2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]))/(d*(1 +
 n)*(2 + n))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^4(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+b\sin(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4*sin(d*x + c) + a*cos(d*x + c)^4)*sin(d*x + c)^n,
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*cos(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x)),x)
```

```
[Out] int(cos(c + d*x)^4*sin(c + d*x)^n*(a + b*sin(c + d*x)), x)
```

3.1198 $\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{a\sin^{10}(c+dx)}{10d}+\frac{b\sin^{11}(c+dx)}{11d}$$

```
[Out] 1/6*a*sin(d*x+c)^6/d+1/7*b*sin(d*x+c)^7/d-1/4*a*sin(d*x+c)^8/d-2/9*b*sin(d*
x+c)^9/d+1/10*a*sin(d*x+c)^10/d+1/11*b*sin(d*x+c)^11/d
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$\frac{a\sin^{10}(c+dx)}{10d}-\frac{a\sin^8(c+dx)}{4d}+\frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^{11}(c+dx)}{11d}-\frac{2b\sin^9(c+dx)}{9d}+\frac{b\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^6)/(6*d) + (b*Sin[c + d*x]^7)/(7*d) - (a*Sin[c + d*x]^8)/(4
*d) - (2*b*Sin[c + d*x]^9)/(9*d) + (a*Sin[c + d*x]^10)/(10*d) + (b*Sin[c +
d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)(b^2-x^2)^2}{b^5}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int x^5(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^{10} d}$$
$$= \frac{\text{Subst}\left(\int \left(ab^4x^5+b^4x^6-2ab^2x^7-2b^2x^8+ax^9+x^{10}\right)\right.}{b^{10}d}$$
$$= \frac{a\sin^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{a\sin^8(c+dx)}{4d}-\frac{2b\text{s}}{}$$

Mathematica [A]
time = 0.30, size = 105, normalized size = 1.08

$$\frac{-34650a\cos(2(c+dx))+5775a\cos(6(c+dx))-693a\cos(10(c+dx))+34650b\sin(c+dx)-11550b\sin(3(c+dx))-3465b\sin(5(c+dx))+2475b\sin(7(c+dx))+385b\sin(9(c+dx))-315b\sin(11(c+dx))}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-34650*a*Cos[2*(c + d*x)] + 5775*a*Cos[6*(c + d*x)] - 693*a*Cos[10*(c + d*
x)] + 34650*b*Sin[c + d*x] - 11550*b*Sin[3*(c + d*x)] - 3465*b*Sin[5*(c + d
*x)] + 2475*b*Sin[7*(c + d*x)] + 385*b*Sin[9*(c + d*x)] - 315*b*Sin[11*(c +
 d*x)])/(3548160*d)
```

Maple [A]
time = 0.53, size = 138, normalized size = 1.42

method	result
risch	$\frac{5b\sin(dx+c)}{512d}-\frac{b\sin(11dx+11c)}{11264d}-\frac{a\cos(10dx+10c)}{5120d}+\frac{b\sin(9dx+9c)}{9216d}+\frac{5b\sin(7dx+7c)}{7168d}+\frac{5a\cos(6dx+6c)}{3072d}-\frac{b}{}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+b\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{11}-\frac{5\left(\sin^3(dx+\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^6(dx+c)\right)}{10}-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{20}-\frac{\left(\cos^6(dx+c)\right)}{60}\right)+b\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^6(dx+c)\right)}{11}-\frac{5\left(\sin^3(dx+\right.}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60
*cos(d*x+c)^6)+b*(-1/11*sin(d*x+c)^5*cos(d*x+c)^6-5/99*sin(d*x+c)^3*cos(d*x
```

```
+c)^6-5/231*sin(d*x+c)*cos(d*x+c)^6+1/231*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.74

$$\frac{1260\,b\sin\left(dx+c\right)^{11}+1386\,a\sin\left(dx+c\right)^{10}-3080\,b\sin\left(dx+c\right)^{9}-3465\,a\sin\left(dx+c\right)^{8}+1980\,b\sin\left(dx+c\right)^{7}+2310\,a\sin\left(dx+c\right)^{6}}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/13860*(1260*b*sin(d*x + c)^11 + 1386*a*sin(d*x + c)^10 - 3080*b*sin(d*x +
 c)^9 - 3465*a*sin(d*x + c)^8 + 1980*b*sin(d*x + c)^7 + 2310*a*sin(d*x + c)
^6)/d
```

Fricas [A]
time = 0.37, size = 106, normalized size = 1.09

$$-\frac{1386\,a\cos\left(dx+c\right)^{10}-3465\,a\cos\left(dx+c\right)^{8}+2310\,a\cos\left(dx+c\right)^{6}+20\,\left(63\,b\cos\left(dx+c\right)^{10}-161\,b\cos\left(dx+c\right)^{8}+113\,b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}-8\,b\right)\sin\left(dx+c\right)}{13860\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/13860*(1386*a*cos(d*x + c)^10 - 3465*a*cos(d*x + c)^8 + 2310*a*cos(d*x +
 c)^6 + 20*(63*b*cos(d*x + c)^10 - 161*b*cos(d*x + c)^8 + 113*b*cos(d*x + c
)^6 - 3*b*cos(d*x + c)^4 - 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d
```

Sympy [A]
time = 2.58, size = 136, normalized size = 1.40

$$\begin{cases} -\frac{a\sin^{4}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{6d}-\frac{a\sin^{2}\left(c+dx\right)\cos^{8}\left(c+dx\right)}{12d}-\frac{a\cos^{10}\left(c+dx\right)}{60d}+\frac{8b\sin^{11}\left(c+dx\right)}{693d}+\frac{4b\sin^{9}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{63d}+\frac{b\sin^{7}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{7d} & \text{for}\: d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)\sin^{5}\left(c\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
[Out] Piecewise((-a*sin(c + d*x)**4*cos(c + d*x)**6/(6*d) - a*sin(c + d*x)**2*cos
(c + d*x)**8/(12*d) - a*cos(c + d*x)**10/(60*d) + 8*b*sin(c + d*x)**11/(693
*d) + 4*b*sin(c + d*x)**9*cos(c + d*x)**2/(63*d) + b*sin(c + d*x)**7*cos(c
+ d*x)**4/(7*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**5*cos(c)**5, True))
```

Giac [A]
time = 0.55, size = 133, normalized size = 1.37

$$-\frac{a\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,a\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,a\cos\left(2\,dx+2\,c\right)}{512\,d}-\frac{b\sin\left(11\,dx+11\,c\right)}{11264\,d}+\frac{b\sin\left(9\,dx+9\,c\right)}{9216\,d}+\frac{5\,b\sin\left(7\,dx+7\,c\right)}{7168\,d}-\frac{b\sin\left(5\,dx+5\,c\right)}{1024\,d}-\frac{5\,b\sin\left(3\,dx+3\,c\right)}{1536\,d}+\frac{5\,b\sin\left(dx+c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*a*cos(10*d*x + 10*c)/d + 5/3072*a*cos(6*d*x + 6*c)/d - 5/512*a*cos(
2*d*x + 2*c)/d - 1/11264*b*sin(11*d*x + 11*c)/d + 1/9216*b*sin(9*d*x + 9*c)
/d + 5/7168*b*sin(7*d*x + 7*c)/d - 1/1024*b*sin(5*d*x + 5*c)/d - 5/1536*b*s
in(3*d*x + 3*c)/d + 5/512*b*sin(d*x + c)/d
```

Mupad [B]
time = 0.08, size = 71, normalized size = 0.73

$$\frac{\frac{b\sin(c+d\,x)^{11}}{11}+\frac{a\sin(c+d\,x)^{10}}{10}-\frac{2\,b\sin(c+d\,x)^9}{9}-\frac{a\sin(c+d\,x)^8}{4}+\frac{b\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^6)/6 - (a*sin(c + d*x)^8)/4 + (a*sin(c + d*x)^10)/10 + (b*
sin(c + d*x)^7)/7 - (2*b*sin(c + d*x)^9)/9 + (b*sin(c + d*x)^11)/11)/d
```

3.1199 $\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{b\sin^8(c+dx)}{4d}+\frac{a\sin^9(c+dx)}{9d}+\frac{b\sin^{10}(c+dx)}{10d}$$

```
[Out] 1/5*a*sin(d*x+c)^5/d+1/6*b*sin(d*x+c)^6/d-2/7*a*sin(d*x+c)^7/d-1/4*b*sin(d*
x+c)^8/d+1/9*a*sin(d*x+c)^9/d+1/10*b*sin(d*x+c)^10/d
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$\frac{a\sin^9(c+dx)}{9d}-\frac{2a\sin^7(c+dx)}{7d}+\frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^{10}(c+dx)}{10d}-\frac{b\sin^8(c+dx)}{4d}+\frac{b\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^5)/(5*d) + (b*Sin[c + d*x]^6)/(6*d) - (2*a*Sin[c + d*x]^7)/
(7*d) - (b*Sin[c + d*x]^8)/(4*d) + (a*Sin[c + d*x]^9)/(9*d) + (b*Sin[c + d*
x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{x^4(a+x)(b^2-x^2)^2}{b^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int x^4(a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^9 d}$$

$$= \frac{\text{Subst}\left(\int \left(ab^4x^4+b^4x^5-2ab^2x^6-2b^2x^7+ax^8+x^9\right)\,d\right.}{b^9 d}$$

$$= \frac{a\sin^5(c+dx)}{5d}+\frac{b\sin^6(c+dx)}{6d}-\frac{2a\sin^7(c+dx)}{7d}-\frac{b\,\mathrm{s}}{}$$

Mathematica [A]

time = 0.23, size = 94, normalized size = 0.97

$$\frac{-3150b\cos(2(c+dx))+525b\cos(6(c+dx))-63b\cos(10(c+dx))+7560a\sin(c+dx)-1680a\sin(3(c+dx))-1008a\sin(5(c+dx))+180a\sin(7(c+dx))+140a\sin(9(c+dx))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3150*b*Cos[2*(c + d*x)] + 525*b*Cos[6*(c + d*x)] - 63*b*Cos[10*(c + d*x)]
 + 7560*a*Sin[c + d*x] - 1680*a*Sin[3*(c + d*x)] - 1008*a*Sin[5*(c + d*x)]
+ 180*a*Sin[7*(c + d*x)] + 140*a*Sin[9*(c + d*x)])/(322560*d)
```

Maple [A]

time = 0.43, size = 120, normalized size = 1.24

method	result
risch	$\frac{3a\sin(dx+c)}{128d}-\frac{b\cos(10dx+10c)}{5120d}+\frac{a\sin(9dx+9c)}{2304d}+\frac{a\sin(7dx+7c)}{1792d}+\frac{5b\cos(6dx+6c)}{3072d}-\frac{a\sin(5dx+5c)}{320d}-\frac{a\,\mathrm{si}}{}$
derivativedivides	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{105}\right)+b\left(-\frac{\left(\sin^4(dx+c)\right)}{}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10*sin(d*x+c)^4*cos(d*
x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/60*cos(d*x+c)^6))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.74

$$\frac{126\,b\sin\left(dx+c\right)^{10}+140\,a\sin\left(dx+c\right)^{9}-315\,b\sin\left(dx+c\right)^{8}-360\,a\sin\left(dx+c\right)^{7}+210\,b\sin\left(dx+c\right)^{6}+252\,a\sin\left(dx+c\right)^{5}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1260*(126*b*sin(d*x + c)^10 + 140*a*sin(d*x + c)^9 - 315*b*sin(d*x + c)^8
 - 360*a*sin(d*x + c)^7 + 210*b*sin(d*x + c)^6 + 252*a*sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.35, size = 95, normalized size = 0.98

$$-\frac{126\,b\cos\left(dx+c\right)^{10}-315\,b\cos\left(dx+c\right)^{8}+210\,b\cos\left(dx+c\right)^{6}-4\,\left(35\,a\cos\left(dx+c\right)^{8}-50\,a\cos\left(dx+c\right)^{6}+3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/1260*(126*b*cos(d*x + c)^10 - 315*b*cos(d*x + c)^8 + 210*b*cos(d*x + c)^
6 - 4*(35*a*cos(d*x + c)^8 - 50*a*cos(d*x + c)^6 + 3*a*cos(d*x + c)^4 + 4*a
*cos(d*x + c)^2 + 8*a)*sin(d*x + c))/d
```

Sympy [A]
time = 1.78, size = 136, normalized size = 1.40

$$\begin{cases} \frac{8a\sin^{9}\left(c+dx\right)}{315d}+\frac{4a\sin^{7}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{35d}+\frac{a\sin^{5}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{5d}-\frac{b\sin^{4}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{6d}-\frac{b\sin^{2}\left(c+dx\right)\cos^{8}\left(c+dx\right)}{12d}-\frac{b\cos^{10}\left(c+dx\right)}{60d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)\sin^{4}\left(c\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**9/(315*d) + 4*a*sin(c + d*x)**7*cos(c + d*x)**
2/(35*d) + a*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) - b*sin(c + d*x)**4*cos(
c + d*x)**6/(6*d) - b*sin(c + d*x)**2*cos(c + d*x)**8/(12*d) - b*cos(c + d*
x)**10/(60*d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)**4*cos(c)**5, True))
```

Giac [A]
time = 0.57, size = 118, normalized size = 1.22

$$-\frac{b\cos\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\,b\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{a\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{a\sin\left(7\,dx+7\,c\right)}{1792\,d}-\frac{a\sin\left(5\,dx+5\,c\right)}{320\,d}-\frac{a\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{3\,a\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/5120*b*cos(10*d*x + 10*c)/d + 5/3072*b*cos(6*d*x + 6*c)/d - 5/512*b*cos(
2*d*x + 2*c)/d + 1/2304*a*sin(9*d*x + 9*c)/d + 1/1792*a*sin(7*d*x + 7*c)/d
- 1/320*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 3/128*a*sin(d*x
 + c)/d
```

Mupad [B]
time = 0.06, size = 71, normalized size = 0.73

$$\frac{\frac{b\sin(c+dx)^{10}}{10}+\frac{a\sin(c+dx)^9}{9}-\frac{b\sin(c+dx)^8}{4}-\frac{2a\sin(c+dx)^7}{7}+\frac{b\sin(c+dx)^6}{6}+\frac{a\sin(c+dx)^5}{5}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^5)/5 - (2*a*sin(c + d*x)^7)/7 + (a*sin(c + d*x)^9)/9 + (b*
sin(c + d*x)^6)/6 - (b*sin(c + d*x)^8)/4 + (b*sin(c + d*x)^10)/10)/d
```

3.1200 $\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{a\cos^8(c+dx)}{8d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^9(c+dx)}{9d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/8*a*cos(d*x+c)^8/d+1/5*b*sin(d*x+c)^5/d-2/7*b*sin(d
*x+c)^7/d+1/9*b*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2645, 14, 2644, 276}

$$\frac{a\cos^8(c+dx)}{8d}-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^9(c+dx)}{9d}-\frac{2b\sin^7(c+dx)}{7d}+\frac{b\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (a*Cos[c + d*x]^8)/(8*d) + (b*Sin[c + d*x]^5)/(
5*d) - (2*b*Sin[c + d*x]^7)/(7*d) + (b*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^3(c+dx)\,dx + b\int \cos^5(c+dx)\sin$$

$$= -\frac{a\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\Big(\int}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^5-x^7)\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int (\right.}{}$$

$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{a\cos^8(c+dx)}{8d} + \frac{b\sin^5(c+dx)}{5d} - \frac{2}{}$$

Mathematica [A]
time = 0.22, size = 105, normalized size = 1.30

$$\frac{-7560a\cos(2(c+dx)) - 1260a\cos(4(c+dx)) + 840a\cos(6(c+dx)) + 315a\cos(8(c+dx)) + 7560b\sin(c+dx) - 1680b\sin(3(c+dx)) - 1008b\sin(5(c+dx)) + 180b\sin(7(c+dx)) + 140b\sin(9(c+dx))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-7560*a*Cos[2*(c + d*x)] - 1260*a*Cos[4*(c + d*x)] + 840*a*Cos[6*(c + d*x)
] + 315*a*Cos[8*(c + d*x)] + 7560*b*Sin[c + d*x] - 1680*b*Sin[3*(c + d*x)]
- 1008*b*Sin[5*(c + d*x)] + 180*b*Sin[7*(c + d*x)] + 140*b*Sin[9*(c + d*x)]
)/(322560*d)
```

Maple [A]
time = 0.36, size = 102, normalized size = 1.26

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+b\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)\right.}{}\right.}{d}$
default	$\frac{a\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right)+b\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^6(dx+c)\right)}{9}-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)\right.}{}\right.}{d}$
risch	$\frac{3b\sin(dx+c)}{128d}+\frac{b\sin(9dx+9c)}{2304d}+\frac{a\cos(8dx+8c)}{1024d}+\frac{b\sin(7dx+7c)}{1792d}+\frac{a\cos(6dx+6c)}{384d}-\frac{b\sin(5dx+5c)}{320d}-\frac{a\cos(4dx}{256d}$
norman	$\frac{\frac{32b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{384b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{6976b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{315d}-\frac{384b\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{35d}+\frac{32b\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{4a\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b*(-1/9*sin(d*x+c
)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.89

$$\frac{280\,b\sin\left(dx+c\right)^9+315\,a\sin\left(dx+c\right)^8-720\,b\sin\left(dx+c\right)^7-840\,a\sin\left(dx+c\right)^6+504\,b\sin\left(dx+c\right)^5+630\,a\sin\left(dx+c\right)^4}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2520*(280*b*sin(d*x + c)^9 + 315*a*sin(d*x + c)^8 - 720*b*sin(d*x + c)^7
- 840*a*sin(d*x + c)^6 + 504*b*sin(d*x + c)^5 + 630*a*sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.37, size = 84, normalized size = 1.04

$$\frac{315\,a\cos\left(dx+c\right)^8-420\,a\cos\left(dx+c\right)^6+8\left(35\,b\cos\left(dx+c\right)^8-50\,b\cos\left(dx+c\right)^6+3\,b\cos\left(dx+c\right)^4+4\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*cos(d*x + c)^8 - 420*a*cos(d*x + c)^6 + 8*(35*b*cos(d*x + c)^
8 - 50*b*cos(d*x + c)^6 + 3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*si
n(d*x + c))/d
```

Sympy [A]
time = 1.27, size = 114, normalized size = 1.41

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos^6(c+dx)}{6d} - \frac{a\cos^8(c+dx)}{24d} + \frac{8b\sin^9(c+dx)}{315d} + \frac{4b\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{b\sin^5(c+dx)\cos^4(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a+b\sin(c))\sin^3(c)\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - a*cos(c + d*x)**8/(24
*d) + 8*b*sin(c + d*x)**9/(315*d) + 4*b*sin(c + d*x)**7*cos(c + d*x)**2/(35
*d) + b*sin(c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(d, 0)), (x*(a + b*sin(c))
*sin(c)**3*cos(c)**5, True))
```

Giac [A]
time = 0.50, size = 133, normalized size = 1.64

$$\frac{a\cos(8\,dx+8\,c)}{1024\,d} + \frac{a\cos(6\,dx+6\,c)}{384\,d} - \frac{a\cos(4\,dx+4\,c)}{256\,d} - \frac{3\,a\cos(2\,dx+2\,c)}{128\,d} + \frac{b\sin(9\,dx+9\,c)}{2304\,d} + \frac{b\sin(7\,dx+7\,c)}{1792\,d} - \frac{b\sin(5\,dx+5\,c)}{320\,d} - \frac{b\sin(3\,dx+3\,c)}{192\,d} + \frac{3\,b\sin(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*a*cos(8*d*x + 8*c)/d + 1/384*a*cos(6*d*x + 6*c)/d - 1/256*a*cos(4*d*
x + 4*c)/d - 3/128*a*cos(2*d*x + 2*c)/d + 1/2304*b*sin(9*d*x + 9*c)/d + 1/1
792*b*sin(7*d*x + 7*c)/d - 1/320*b*sin(5*d*x + 5*c)/d - 1/192*b*sin(3*d*x +
 3*c)/d + 3/128*b*sin(d*x + c)/d
```

Mupad [B]
time = 0.06, size = 71, normalized size = 0.88

$$\frac{\frac{b\sin(c+d\,x)^9}{9} + \frac{a\sin(c+d\,x)^8}{8} - \frac{2\,b\sin(c+d\,x)^7}{7} - \frac{a\sin(c+d\,x)^6}{3} + \frac{b\sin(c+d\,x)^5}{5} + \frac{a\sin(c+d\,x)^4}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^4)/4 - (a*sin(c + d*x)^6)/3 + (a*sin(c + d*x)^8)/8 + (b*si
n(c + d*x)^5)/5 - (2*b*sin(c + d*x)^7)/7 + (b*sin(c + d*x)^9)/9)/d
```

3.1201 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{b\cos^6(c+dx)}{6d}+\frac{b\cos^8(c+dx)}{8d}+\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^7(c+dx)}{7d}$$

```
[Out] -1/6*b*cos(d*x+c)^6/d+1/8*b*cos(d*x+c)^8/d+1/3*a*sin(d*x+c)^3/d-2/5*a*sin(d
*x+c)^5/d+1/7*a*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2644, 276, 2645, 14}

$$\frac{a\sin^7(c+dx)}{7d}-\frac{2a\sin^5(c+dx)}{5d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\cos^8(c+dx)}{8d}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cos[c + d*x]^6)/d + (b*Cos[c + d*x]^8)/(8*d) + (a*Sin[c + d*x]^3)/(
3*d) - (2*a*Sin[c + d*x]^5)/(5*d) + (a*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin^2(c+dx)\,dx + b\int \cos^5(c+dx)\sin$$

$$= \frac{a\text{Subst}\left(\int x^2(1-x^2)^2\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\text{Subst}(\int}{}$$

$$= \frac{a\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\text{Subs}}{}$$

$$= -\frac{b\cos^6(c+dx)}{6d} + \frac{b\cos^8(c+dx)}{8d} + \frac{a\sin^3(c+dx)}{3d} - \frac{2}{}$$

Mathematica [A]
time = 0.22, size = 94, normalized size = 1.16

$$-\frac{2520b\cos(2(c+dx))+420b\cos(4(c+dx))-280b\cos(6(c+dx))-105b\cos(8(c+dx))-8400a\sin(c+dx)+560a\sin(3(c+dx))+1008a\sin(5(c+dx))+240a\sin(7(c+dx))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/107520*(2520*b*Cos[2*(c + d*x)] + 420*b*Cos[4*(c + d*x)] - 280*b*Cos[6*(
c + d*x)] - 105*b*Cos[8*(c + d*x)] - 8400*a*Sin[c + d*x] + 560*a*Sin[3*(c +
 d*x)] + 1008*a*Sin[5*(c + d*x)] + 240*a*Sin[7*(c + d*x)])/d
```

Maple [A]
time = 0.30, size = 84, normalized size = 1.04

method	result

derivativedivides	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right.}{d}$
default	$\frac{a\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+c)\right)}{24}\right.}{d}$
risch	$\frac{5a\sin(dx+c)}{64d}+\frac{b\cos(8dx+8c)}{1024d}-\frac{a\sin(7dx+7c)}{448d}+\frac{b\cos(6dx+6c)}{384d}-\frac{3a\sin(5dx+5c)}{320d}-\frac{b\cos(4dx+4c)}{256d}-\frac{a\sin(3d}{192}$
norman	$\frac{\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{688a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{688a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{105d}+\frac{8a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))+b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A]

time = 0.28, size = 72, normalized size = 0.89

$$\frac{105\,b\sin\left(dx+c\right)^8+120\,a\sin\left(dx+c\right)^7-280\,b\sin\left(dx+c\right)^6-336\,a\sin\left(dx+c\right)^5+210\,b\sin\left(dx+c\right)^4+280\,a\sin\left(dx+c\right)^3}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/840*(105*b*sin(d*x + c)^8 + 120*a*sin(d*x + c)^7 - 280*b*sin(d*x + c)^6 -
 336*a*sin(d*x + c)^5 + 210*b*sin(d*x + c)^4 + 280*a*sin(d*x + c)^3)/d
```

Fricas [A]

time = 0.36, size = 73, normalized size = 0.90

$$\frac{105\,b\cos\left(dx+c\right)^8-140\,b\cos\left(dx+c\right)^6-8\,\left(15\,a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4-4\,a\cos\left(dx+c\right)^2-8\,a\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/840*(105*b*cos(d*x + c)^8 - 140*b*cos(d*x + c)^6 - 8*(15*a*cos(d*x + c)^6
 - 3*a*cos(d*x + c)^4 - 4*a*cos(d*x + c)^2 - 8*a)*sin(d*x + c))/d
```

Sympy [A]

time = 0.88, size = 114, normalized size = 1.41

$$\begin{cases}\frac{8a\sin^7\left(c+dx\right)}{105d}+\frac{4a\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{a\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d}-\frac{b\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{6d}-\frac{b\cos^8\left(c+dx\right)}{24d} & \text{for } d\neq 0\\ x\left(a+b\sin\left(c\right)\right)\sin^2\left(c\right)\cos^5\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**7/(105*d) + 4*a*sin(c + d*x)**5*cos(c + d*x)**
2/(15*d) + a*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - b*sin(c + d*x)**2*cos(
c + d*x)**6/(6*d) - b*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a + b*sin(c))*
sin(c)**2*cos(c)**5, True))
```

Giac [A]
time = 0.51, size = 118, normalized size = 1.46

$$\frac{b\cos(8\,dx+8\,c)}{1024\,d}+\frac{b\cos(6\,dx+6\,c)}{384\,d}-\frac{b\cos(4\,dx+4\,c)}{256\,d}-\frac{3\,b\cos(2\,dx+2\,c)}{128\,d}-\frac{a\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,a\sin(5\,dx+5\,c)}{320\,d}-\frac{a\sin(3\,dx+3\,c)}{192\,d}+\frac{5\,a\sin(dx+c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1024*b*cos(8*d*x + 8*c)/d + 1/384*b*cos(6*d*x + 6*c)/d - 1/256*b*cos(4*d*
x + 4*c)/d - 3/128*b*cos(2*d*x + 2*c)/d - 1/448*a*sin(7*d*x + 7*c)/d - 3/32
0*a*sin(5*d*x + 5*c)/d - 1/192*a*sin(3*d*x + 3*c)/d + 5/64*a*sin(d*x + c)/d
```

Mupad [B]
time = 0.06, size = 71, normalized size = 0.88

$$\frac{\frac{b\sin(c+d\,x)^8}{8}+\frac{a\sin(c+d\,x)^7}{7}-\frac{b\sin(c+d\,x)^6}{3}-\frac{2\,a\sin(c+d\,x)^5}{5}+\frac{b\sin(c+d\,x)^4}{4}+\frac{a\sin(c+d\,x)^3}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^3)/3 - (2*a*sin(c + d*x)^5)/5 + (a*sin(c + d*x)^7)/7 + (b*
sin(c + d*x)^4)/4 - (b*sin(c + d*x)^6)/3 + (b*sin(c + d*x)^8)/8)/d
```

3.1202 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cos(d*x+c)^6/d+1/3*b*sin(d*x+c)^3/d-2/5*b*sin(d*x+c)^5/d+1/7*b*sin(d
*x+c)^7/d
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2913, 2645, 30, 2644, 276}

$$-\frac{a\cos^6(c+dx)}{6d}+\frac{b\sin^7(c+dx)}{7d}-\frac{2b\sin^5(c+dx)}{5d}+\frac{b\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cos[c + d*x]^6)/d + (b*Sin[c + d*x]^3)/(3*d) - (2*b*Sin[c + d*x]^5)
/(5*d) + (b*Sin[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))\,dx = a\int \cos^5(c+dx)\sin(c+dx)\,dx + b\int \cos^5(c+dx)\sin^2($$
$$= -\frac{a\text{Subst}\left(\int x^5\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int x^2(1-x\right.}{}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(\right.}{d}$$
$$= -\frac{a\cos^6(c+dx)}{6d} + \frac{b\sin^3(c+dx)}{3d} - \frac{2b\sin^5(c+dx)}{5d} + \frac{b}{}$$

Mathematica [A]
time = 0.17, size = 86, normalized size = 1.32

$$-\frac{350a+525a\cos(2(c+dx))+210a\cos(4(c+dx))+35a\cos(6(c+dx))-525b\sin(c+dx)+35b\sin(3(c+dx))+63b\sin(5(c+dx))+15b\sin(7(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(350*a + 525*a*Cos[2*(c + d*x)] + 210*a*Cos[4*(c + d*x)] + 35*a*Cos
[6*(c + d*x)] - 525*b*Sin[c + d*x] + 35*b*Sin[3*(c + d*x)] + 63*b*Sin[5*(c
+ d*x)] + 15*b*Sin[7*(c + d*x)])/d
```

Maple [A]
time = 0.21, size = 64, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6}+b\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$

default	$\frac{-\frac{a\left(\cos^{6}(dx+c)\right)}{6}+b\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)}{d}$
risch	$\frac{5b\sin(dx+c)}{64d}-\frac{b\sin(7dx+7c)}{448d}-\frac{a\cos(6dx+6c)}{192d}-\frac{3b\sin(5dx+5c)}{320d}-\frac{a\cos(4dx+4c)}{32d}-\frac{b\sin(3dx+3c)}{192d}-\frac{5a\cos(2\ldots}{64\ldots}$
norman	$\frac{\frac{2a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{32b\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\ldots}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a*cos(d*x+c)^6+b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+
c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]

time = 0.28, size = 72, normalized size = 1.11

$$\frac{30\,b\sin\left(dx+c\right)^{7}+35\,a\sin\left(dx+c\right)^{6}-84\,b\sin\left(dx+c\right)^{5}-105\,a\sin\left(dx+c\right)^{4}+70\,b\sin\left(dx+c\right)^{3}+105\,a\sin\left(dx+c\right)^{2}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*b*sin(d*x + c)^7 + 35*a*sin(d*x + c)^6 - 84*b*sin(d*x + c)^5 - 10
5*a*sin(d*x + c)^4 + 70*b*sin(d*x + c)^3 + 105*a*sin(d*x + c)^2)/d
```

Fricas [A]

time = 0.36, size = 62, normalized size = 0.95

$$-\frac{35\,a\cos\left(dx+c\right)^{6}+2\,\left(15\,b\cos\left(dx+c\right)^{6}-3\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}-8\,b\right)\sin\left(dx+c\right)}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(35*a*cos(d*x + c)^6 + 2*(15*b*cos(d*x + c)^6 - 3*b*cos(d*x + c)^4 -
 4*b*cos(d*x + c)^2 - 8*b)*sin(d*x + c))/d
```

Sympy [A]

time = 0.59, size = 90, normalized size = 1.38

$$\begin{cases} -\frac{a\cos^{6}\left(c+dx\right)}{6d}+\frac{8b\sin^{7}\left(c+dx\right)}{105d}+\frac{4b\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{15d}+\frac{b\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)\sin\left(c\right)\cos^{5}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((-a*cos(c + d*x)**6/(6*d) + 8*b*sin(c + d*x)**7/(105*d) + 4*b*sin
(c + d*x)**5*cos(c + d*x)**2/(15*d) + b*sin(c + d*x)**3*cos(c + d*x)**4/(3*
d), Ne(d, 0)), (x*(a + b*sin(c))*sin(c)*cos(c)**5, True))
```

Giac [A]
time = 0.47, size = 103, normalized size = 1.58

$$-\frac{a\cos(6\,dx+6\,c)}{192\,d}-\frac{a\cos(4\,dx+4\,c)}{32\,d}-\frac{5\,a\cos(2\,dx+2\,c)}{64\,d}-\frac{b\sin(7\,dx+7\,c)}{448\,d}-\frac{3\,b\sin(5\,dx+5\,c)}{320\,d}-\frac{b\sin(3\,dx+3\,c)}{192\,d}+\frac{5\,b\sin(dx+c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*a*cos(6*d*x + 6*c)/d - 1/32*a*cos(4*d*x + 4*c)/d - 5/64*a*cos(2*d*x
+ 2*c)/d - 1/448*b*sin(7*d*x + 7*c)/d - 3/320*b*sin(5*d*x + 5*c)/d - 1/192*
b*sin(3*d*x + 3*c)/d + 5/64*b*sin(d*x + c)/d
```

Mupad [B]
time = 11.59, size = 71, normalized size = 1.09

$$\frac{\frac{b\sin(c+d\,x)^7}{7}+\frac{a\sin(c+d\,x)^6}{6}-\frac{2\,b\sin(c+d\,x)^5}{5}-\frac{a\sin(c+d\,x)^4}{2}+\frac{b\sin(c+d\,x)^3}{3}+\frac{a\sin(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x)),x)
```

```
[Out] ((a*sin(c + d*x)^2)/2 - (a*sin(c + d*x)^4)/2 + (a*sin(c + d*x)^6)/6 + (b*si
n(c + d*x)^3)/3 - (2*b*sin(c + d*x)^5)/5 + (b*sin(c + d*x)^7)/7)/d
```

3.1203 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}+\frac{b\sin^5(c+dx)}{5d}$$

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d-a*sin(d*x+c)^2/d-2/3*b*sin(d*x+c)^3/d+1/4
*a*sin(d*x+c)^4/d+1/5*b*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2916, 12, 780}

$$\frac{a\sin^4(c+dx)}{4d}-\frac{a\sin^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin^5(c+dx)}{5d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b(a+x)(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(b^4+\frac{ab^4}{x}-2ab^2x-2b^2x^2+ax^3+x^4\right)\,dx, x,\right.}{b^4d}$$
$$= \frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2}{}$$

Mathematica [A]
time = 0.03, size = 86, normalized size = 1.00

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{a\sin^2(c+dx)}{d}-\frac{2b\sin^3(c+dx)}{3d}+\frac{a\sin^4(c+dx)}{4d}+\frac{b\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d - (a*Sin[c + d*x]^2)/d - (2*b*
Sin[c + d*x]^3)/(3*d) + (a*Sin[c + d*x]^4)/(4*d) + (b*Sin[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.16, size = 65, normalized size = 0.76

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
default	$\frac{a\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+\frac{b\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}}{d}$
risch	$-iax+\frac{3a\,e^{2i(dx+c)}}{16d}+\frac{3a\,e^{-2i(dx+c)}}{16d}-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{5b\sin(dx+c)}{8d}+\frac{b\sin(5dx+5c)}{80d}+\frac{a\cos}{}$
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{116b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a(t}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/5*b*(8/3+cos(d*
x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.80

$$\frac{12\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,b\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\sin\left(dx+c\right)\right)+60\,b\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(sin(d*x + c)) + 60*b*sin(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 74, normalized size = 0.86

$$\frac{15\,a\cos\left(dx+c\right)^{4}+30\,a\cos\left(dx+c\right)^{2}+60\,a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(3\,b\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{2}+8\,b\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*cos(d*x + c)^4 + 30*a*cos(d*x + c)^2 + 60*a*log(1/2*sin(d*x + c)
) + 4*(3*b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\cos^{5}\left(c+dx\right)\csc\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cos(c + d*x)**5*csc(c + d*x), x)
```

Giac [A]
time = 0.50, size = 70, normalized size = 0.81

$$\frac{12\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-40\,b\sin\left(dx+c\right)^{3}-60\,a\sin\left(dx+c\right)^{2}+60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,b\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(12*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 40*b*sin(d*x + c)^3 - 60*
a*sin(d*x + c)^2 + 60*a*log(abs(sin(d*x + c))) + 60*b*sin(d*x + c))/d
```

Mupad [B]
time = 12.00, size = 126, normalized size = 1.47

$$\frac{a\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{a\cos\left(c+d\,x\right)^2}{2\,d}+\frac{a\cos\left(c+d\,x\right)^4}{4\,d}+\frac{8\,b\,\sin\left(c+d\,x\right)}{15\,d}+\frac{4\,b\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{b\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x),x)
```

```
[Out] (a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*log(1/cos(c/2 + (d*x)
/2)^2))/d + (a*cos(c + d*x)^2)/(2*d) + (a*cos(c + d*x)^4)/(4*d) + (8*b*sin(
c + d*x))/(15*d) + (4*b*cos(c + d*x)^2*sin(c + d*x))/(15*d) + (b*cos(c + d*
x)^4*sin(c + d*x))/(5*d)
```

3.1204 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=83

$$-\frac{a\csc(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{b\sin^2(c+dx)}{d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin^4(c+dx)}{4d}$$

```
[Out] -a*csc(d*x+c)/d+b*ln(sin(d*x+c))/d-2*a*sin(d*x+c)/d-b*sin(d*x+c)^2/d+1/3*a*
sin(d*x+c)^3/d+1/4*b*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$\frac{a\sin^3(c+dx)}{3d}-\frac{2a\sin(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{b\sin^4(c+dx)}{4d}-\frac{b\sin^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^2(a+x)(b^2-x^2)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2ab^2+\frac{ab^4}{x^2}+\frac{b^4}{x}-2b^2x+ax^2+x^3\right)dx, x,\right.}{b^3 d}$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}$$

Mathematica [A]
time = 0.03, size = 83, normalized size = 1.00

$$-\frac{a\csc(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}-\frac{2a\sin(c+dx)}{d}-\frac{b\sin^2(c+dx)}{d}+\frac{a\sin^3(c+dx)}{3d}+\frac{b\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Log[Sin[c + d*x]])/d - (2*a*Sin[c + d*x])/d - (b
*Sin[c + d*x]^2)/d + (a*Sin[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.15, size = 85, normalized size = 1.02

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+b\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+b\left(\frac{\left(\cos^4(dx+c)\right)}{4}+\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)}{d}$
risch	$-ibx+\frac{3b\,e^{2i(dx+c)}}{16d}+\frac{7ia\,e^{i(dx+c)}}{8d}-\frac{7ia\,e^{-i(dx+c)}}{8d}+\frac{3b\,e^{-2i(dx+c)}}{16d}-\frac{2ibc}{d}-\frac{2ia\,e^{i(dx+c)}}{d(e^{2i(dx+c)}-1)}+\frac{b\ln(e^{2i(d}}{d}$
norman	$\frac{-\frac{a}{2d}-\frac{13a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{43a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{43a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{13a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{4b\left(\tan^5\left(\frac{dx}{2}\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(
d*x+c))+b*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.83

$$\frac{3\,b\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,b\sin\left(dx+c\right)^{2}+12\,b\log\left(\sin\left(dx+c\right)\right)-24\,a\sin\left(dx+c\right)-\frac{12\,a}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(sin(d*x + c)) - 24*a*sin(d*x + c) - 12*a/sin(d*x + c))/d
```

Fricas [A]
time = 0.36, size = 91, normalized size = 1.10

$$\frac{32\,a\cos\left(dx+c\right)^{4}+128\,a\cos\left(dx+c\right)^{2}+96\,b\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\left(8\,b\cos\left(dx+c\right)^{4}+16\,b\cos\left(dx+c\right)^{2}-11\,b\right)\sin\left(dx+c\right)-256\,a}{96\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(32*a*cos(d*x + c)^4 + 128*a*cos(d*x + c)^2 + 96*b*log(1/2*sin(d*x + c
))*sin(d*x + c) + 3*(8*b*cos(d*x + c)^4 + 16*b*cos(d*x + c)^2 - 11*b)*sin(d
*x + c) - 256*a)/(d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 79, normalized size = 0.95

$$\frac{3\,b\sin\left(dx+c\right)^{4}+4\,a\sin\left(dx+c\right)^{3}-12\,b\sin\left(dx+c\right)^{2}+12\,b\log\left(|\sin\left(dx+c\right)|\right)-24\,a\sin\left(dx+c\right)-\frac{12\,(b\sin(dx+c)+a)}{\sin(dx+c)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*sin(d*x + c)^4 + 4*a*sin(d*x + c)^3 - 12*b*sin(d*x + c)^2 + 12*b*
log(abs(sin(d*x + c))) - 24*a*sin(d*x + c) - 12*(b*sin(d*x + c) + a)/sin(d*
x + c))/d
```

Mupad [B]
time = 11.93, size = 250, normalized size = 3.01

$$\frac{b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}-\frac{4\,b\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{d}+\frac{8\,b\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{d}-\frac{8\,b\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{d}+\frac{4\,b\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8}{d}-\frac{9\,a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{20\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{16\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{8\,a\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{3\,d\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^2,x)
```

```
[Out] (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b*log(1/cos(c/2 + (d*x)
/2)^2))/d - (4*b*cos(c/2 + (d*x)/2)^2)/d + (8*b*cos(c/2 + (d*x)/2)^4)/d - (
8*b*cos(c/2 + (d*x)/2)^6)/d + (4*b*cos(c/2 + (d*x)/2)^8)/d - (9*a*cos(c/2 +
 (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a*sin(c/2 + (d*x)/2))/(2*d*cos(c/2 +
 (d*x)/2)) + (20*a*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a*c
os(c/2 + (d*x)/2)^5)/(3*d*sin(c/2 + (d*x)/2)) + (8*a*cos(c/2 + (d*x)/2)^7)/
(3*d*sin(c/2 + (d*x)/2))
```

3.1205 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}-\frac{2b\sin(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}+\frac{b\sin^3(c+dx)}{3d}$$

```
[Out] -b*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-2*a*ln(sin(d*x+c))/d-2*b*sin(d*x+c)/d+
1/2*a*sin(d*x+c)^2/d+1/3*b*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$\frac{a\sin^2(c+dx)}{2d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{2b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (2*a*Log[Sin[c + d*x]])/
d - (2*b*Sin[c + d*x])/d + (a*Sin[c + d*x]^2)/(2*d) + (b*Sin[c + d*x]^3)/(3
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)(b^2-x^2)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2b^2+\frac{ab^4}{x^3}+\frac{b^4}{x^2}-\frac{2ab^2}{x}+ax+x^2\right)\,dx, x, b\text{s}\right.}{b^2 d}$$
$$= -\frac{b\csc(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{2a\log(\sin(c+dx))}{d}$$

Mathematica [A]
time = 0.15, size = 77, normalized size = 0.90

$$-\frac{b\csc(c+dx)}{d}-\frac{2b\sin(c+dx)}{d}+\frac{b\sin^3(c+dx)}{3d}-\frac{a(\csc^2(c+dx)+4\log(\sin(c+dx))-\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
[Out] -((b*Csc[c + d*x])/d) - (2*b*Sin[c + d*x])/d + (b*Sin[c + d*x]^3)/(3*d) - (
a*(Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.16, size = 105, normalized size = 1.22

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-\left(\cos^2(dx+c)\right)-2\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$2iax+\frac{ib\,e^{3i(dx+c)}}{24d}-\frac{a\,e^{2i(dx+c)}}{8d}+\frac{7ib\,e^{i(dx+c)}}{8d}-\frac{7ib\,e^{-i(dx+c)}}{8d}-\frac{a\,e^{-2i(dx+c)}}{8d}-\frac{ib\,e^{-3i(dx+c)}}{24d}+\frac{4iac}{d}-$
norman	$\frac{-\frac{a}{8d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{6b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{25b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{6b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(s
in(d*x+c)))+b*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.79

$$\frac{2\,b\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\sin\left(dx+c\right)\right)-12\,b\sin\left(dx+c\right)-\frac{3\,(2\,b\sin(dx+c)+a)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(sin(d*x + c)) - 12*
b*sin(d*x + c) - 3*(2*b*sin(d*x + c) + a)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 102, normalized size = 1.19

$$-\frac{6\,a\cos\left(dx+c\right)^{4}-9\,a\cos\left(dx+c\right)^{2}+24\,\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\,\left(b\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{2}-8\,b\right)\sin\left(dx+c\right)-3\,a}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a*cos(d*x + c)^4 - 9*a*cos(d*x + c)^2 + 24*(a*cos(d*x + c)^2 - a)*
log(1/2*sin(d*x + c)) + 4*(b*cos(d*x + c)^4 + 4*b*cos(d*x + c)^2 - 8*b)*sin
(d*x + c) - 3*a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.50, size = 82, normalized size = 0.95

$$\frac{2\,b\sin\left(dx+c\right)^{3}+3\,a\sin\left(dx+c\right)^{2}-12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)-12\,b\sin\left(dx+c\right)+\frac{3\left(6\,a\sin(dx+c)^{2}-2\,b\sin(dx+c)-a\right)}{\sin(dx+c)^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - 12*a*log(abs(sin(d*x + c)))
- 12*b*sin(d*x + c) + 3*(6*a*sin(d*x + c)^2 - 2*b*sin(d*x + c) - a)/sin(d*x
 + c)^2)/d
```

Mupad [B]
time = 11.89, size = 229, normalized size = 2.66

$$\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{18\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{82\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{13\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+22\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^3,x)
```

```
[Out] (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2
 + 2*b*tan(c/2 + (d*x)/2) + (3*a*tan(c/2 + (d*x)/2)^2)/2 - (13*a*tan(c/2 +
(d*x)/2)^4)/2 - (15*a*tan(c/2 + (d*x)/2)^6)/2 + 22*b*tan(c/2 + (d*x)/2)^3 +
 (82*b*tan(c/2 + (d*x)/2)^5)/3 + 18*b*tan(c/2 + (d*x)/2)^7)/(d*(4*tan(c/2 +
 (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2
 + (d*x)/2)^8)) - (a*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*a*log(tan(c/2 + (d*x)
/2)))/d
```

3.1206 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=85

$$\frac{2a\csc(c+dx)}{d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2b\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] 2*a*csc(d*x+c)/d-1/2*b*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-2*b*ln(sin(d*x+c
))/d+a*sin(d*x+c)/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2916, 12, 780}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{b\sin^2(c+dx)}{2d}-\frac{b\csc^2(c+dx)}{2d}-\frac{2b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (b*Csc[c + d*x]^2)/(2*d) - (a*Csc[c + d*x]^3)/(3*d)
- (2*b*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^4(a+x)(b^2-x^2)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(a+\frac{ab^4}{x^4}+\frac{b^4}{x^3}-\frac{2ab^2}{x^2}-\frac{2b^2}{x}+x\right)\,dx, x, b\sin(c+\right.}{bd}$$
$$= \frac{2a\csc(c+dx)}{d}-\frac{b\csc^2(c+dx)}{2d}-\frac{a\csc^3(c+dx)}{3d}-\frac{2b\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 0.89

$$\frac{2a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b(\csc^2(c+dx)+4\log(\sin(c+dx))-\sin^2(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (b*(
Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - Sin[c + d*x]^2))/(2*d)
```

Maple [A]
time = 0.17, size = 121, normalized size = 1.42

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+b\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\cos^2\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+b\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^4(dx+c)\right)}{2}-(\cos^2\right.}{d}$
risch	$2ibx-\frac{b\,e^{2i(dx+c)}}{8d}-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{b\,e^{-2i(dx+c)}}{8d}+\frac{4ibc}{d}+\frac{2i\left(6a\,e^{5i(dx+c)}-8a\,e^{3i(dx+c)}-3ib\,e\right.}{3d(e^{2i(d}}$
norman	$\frac{-\frac{a}{24d}+\frac{19a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{55a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{55a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{19a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{b\tan\left(\frac{dx}{2}+\right.}{8d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d
*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2
*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.81

$$\frac{3\,b\sin\left(dx+c\right)^{2}-12\,b\log\left(\sin\left(dx+c\right)\right)+6\,a\sin\left(dx+c\right)+\frac{12\,a\sin(dx+c)^{2}-3\,b\sin(dx+c)-2\,a}{\sin(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(sin(d*x + c)) + 6*a*sin(d*x + c) + (12*a
*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.36, size = 117, normalized size = 1.38

$$-\frac{12\,a\cos\left(dx+c\right)^{4}-48\,a\cos\left(dx+c\right)^{2}+24\,\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\left(2\,b\cos\left(dx+c\right)^{4}-3\,b\cos\left(dx+c\right)^{2}-b\right)\sin\left(dx+c\right)+32\,a}{12\,\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^4 - 48*a*cos(d*x + c)^2 + 24*(b*cos(d*x + c)^2 - b
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(2*b*cos(d*x + c)^4 - 3*b*cos(d*x
+ c)^2 - b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.45, size = 81, normalized size = 0.95

$$\frac{3\,b\sin\left(dx+c\right)^{2}-12\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)+6\,a\sin\left(dx+c\right)+\frac{22\,b\sin(dx+c)^{3}+12\,a\sin(dx+c)^{2}-3\,b\sin(dx+c)-2\,a}{\sin(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*b*sin(d*x + c)^2 - 12*b*log(abs(sin(d*x + c))) + 6*a*sin(d*x + c) +
(22*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/sin(d*
x + c)^3)/d
```

Mupad [B]

time = 11.91, size = 218, normalized size = 2.56

$$\frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{89\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}-b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{2\,b\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{2\,b\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^4,x)
```

```
[Out] ((19*a*tan(c/2 + (d*x)/2)^2)/3 - b*tan(c/2 + (d*x)/2) - a/3 + (89*a*tan(c/2
 + (d*x)/2)^4)/3 + 23*a*tan(c/2 + (d*x)/2)^6 - 2*b*tan(c/2 + (d*x)/2)^3 + 1
5*b*tan(c/2 + (d*x)/2)^5)/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*tan(c/2 + (d*x)/2
)^5 + 8*tan(c/2 + (d*x)/2)^7)) + (7*a*tan(c/2 + (d*x)/2))/(8*d) + (2*b*log(
tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (b*tan(c/2
 + (d*x)/2)^2)/(8*d) - (2*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1207 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] 2*b*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*b*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/d
+a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 780}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int \frac{(a+x)\left(b^2-x^2\right)^2}{x^5}\,dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx,x,b\sin(c+dx)\right)}{d}\\ &= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\end{aligned}$$

Mathematica [A]
time = 0.18, size = 87, normalized size = 1.07

$$\frac{2b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{a(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

Maple [A]
time = 0.18, size = 101, normalized size = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\right.}{d}$
default	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\right.}{d}$
risch	$-iax-\frac{ib\,e^{i(dx+c)}}{2d}+\frac{ib\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4i\left(3ia\,e^{6i(dx+c)}+3b\,e^{7i(dx+c)}-3ia\,e^{4i(dx+c)}-7b\,e^{5i(dx+c)}+3ia\,e^{2i(\right.}{3d\left(e^{2i(dx+c)}-1\right)^4}$
norman	$\frac{-\frac{a}{64d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{11a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24d}+\frac{5b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{15b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{4d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/3/sin(d*x+
c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.85

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,b\sin\left(dx+c\right)+\frac{24\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,b\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.38, size = 110, normalized size = 1.36

$$-\frac{12\,a\cos\left(dx+c\right)^{2}-12\,\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,\left(3\,b\cos\left(dx+c\right)^{4}-12\,b\cos\left(dx+c\right)^{2}+8\,b\right)\sin\left(dx+c\right)-9\,a}{12\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.49, size = 82, normalized size = 1.01

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,b\sin\left(dx+c\right)-\frac{25\,a\sin\left(dx+c\right)^{4}-24\,b\sin\left(dx+c\right)^{3}-12\,a\sin\left(dx+c\right)^{2}+4\,b\sin\left(dx+c\right)+3\,a}{\sin\left(dx+c\right)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

Mupad [B]
time = 12.26, size = 207, normalized size = 2.56

$$\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}+\frac{46\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{40\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{4}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}-\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d}+\frac{a\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^5,x)
```

```
[Out] (7*b*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*b*tan(
c/2 + (d*x)/2))/3 - a/4 + 3*a*tan(c/2 + (d*x)/2)^4 + (40*b*tan(c/2 + (d*x)/
2)^3)/3 + 46*b*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*tan(c
/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2 + (d
*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x)/2)^
3)/(24*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.1208 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=86

$$-\frac{a\csc(c+dx)}{d}+\frac{b\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{b\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}+\frac{b\log(\sin(c+dx))}{d}$$

```
[Out] -a*csc(d*x+c)/d+b*csc(d*x+c)^2/d+2/3*a*csc(d*x+c)^3/d-1/4*b*csc(d*x+c)^4/d-
1/5*a*csc(d*x+c)^5/d+b*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2916, 12, 780}

$$-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}-\frac{b\csc^4(c+dx)}{4d}+\frac{b\csc^2(c+dx)}{d}+\frac{b\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (b*Csc[c + d*x]^2)/d + (2*a*Csc[c + d*x]^3)/(3*d) -
 (b*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) + (b*Log[Sin[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{b\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{ab^4}{x^6}+\frac{b^4}{x^5}-\frac{2ab^2}{x^4}-\frac{2b^2}{x^3}+\frac{a}{x^2}+\frac{1}{x}\right)dx, x, b\sin(c$$
$$= -\frac{a\csc(c+dx)}{d}+\frac{b\csc^2(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{b\,\mathrm{cs}}{}$$

Mathematica [A]
time = 0.13, size = 92, normalized size = 1.07

$$-\frac{a\csc(c+dx)}{d}+\frac{2a\csc^3(c+dx)}{3d}-\frac{a\csc^5(c+dx)}{5d}+\frac{b(2\cot^2(c+dx)-\cot^4(c+dx)+4\log(\cos(c+dx))+4\log(\tan(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) + (2*a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*
d) + (b*(2*Cot[c + d*x]^2 - Cot[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Ta
n[c + d*x]]))/(4*d)
```

Maple [A]
time = 0.19, size = 121, normalized size = 1.41

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx\right.}{2}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+b\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx\right.}{2}\right.}{d}$
risch	$-ibx-\frac{2ibc}{d}-\frac{2i\left(15a\,e^{9i(dx+c)}-20a\,e^{7i(dx+c)}-30ib\,e^{8i(dx+c)}+58a\,e^{5i(dx+c)}+60ib\,e^{6i(dx+c)}-20a\,e^{3i(dx+c)}-60ib\,e^{4\right.}{15d\left(e^{2i(dx+c)}-1\right)^5}$
norman	$\frac{-\frac{a}{160d}+\frac{11a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{25a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{25a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{11a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{240d}-\frac{a\left(\tan^{12}\left(\frac{d}{}\right.\right.}{160d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/s
in(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+
b*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.84

$$\frac{60\,b\log\left(\sin\left(dx+c\right)\right)-\frac{60\,a\sin(dx+c)^4-60\,b\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,b\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*b*log(sin(d*x + c)) - (60*a*sin(d*x + c)^4 - 60*b*sin(d*x + c)^3 -
 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.36, size = 124, normalized size = 1.44

$$-\frac{60\,a\cos\left(dx+c\right)^4-80\,a\cos\left(dx+c\right)^2-60\,\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+15\,\left(4\,b\cos\left(dx+c\right)^2-3\,b\right)\sin\left(dx+c\right)+32\,a}{60\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(60*a*cos(d*x + c)^4 - 80*a*cos(d*x + c)^2 - 60*(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + b)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15*(4*b*cos(d*x
 + c)^2 - 3*b)*sin(d*x + c) + 32*a)/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2
 + d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.52, size = 84, normalized size = 0.98

$$\frac{60\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{137\,b\sin(dx+c)^5+60\,a\sin(dx+c)^4-60\,b\sin(dx+c)^3-40\,a\sin(dx+c)^2+15\,b\sin(dx+c)+12\,a}{\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(sin(d*x + c))) - (137*b*sin(d*x + c)^5 + 60*a*sin(d*x +
c)^4 - 60*b*sin(d*x + c)^3 - 40*a*sin(d*x + c)^2 + 15*b*sin(d*x + c) + 12*a
)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 11.80, size = 193, normalized size = 2.24

$$\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}+\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+\frac{a}{5}\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^6,x)
```

```
[Out] (5*a*tan(c/2 + (d*x)/2)^3)/(96*d) - (b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (
5*a*tan(c/2 + (d*x)/2))/(16*d) - (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*b*ta
n(c/2 + (d*x)/2)^2)/(16*d) - (b*tan(c/2 + (d*x)/2)^4)/(64*d) + (b*log(tan(c
/2 + (d*x)/2)))/d - (cot(c/2 + (d*x)/2)^5*(a/5 + (b*tan(c/2 + (d*x)/2))/2 -
 (5*a*tan(c/2 + (d*x)/2)^2)/3 + 10*a*tan(c/2 + (d*x)/2)^4 - 6*b*tan(c/2 + (
d*x)/2)^3))/(32*d)
```

3.1209 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=61

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc(c+dx)}{d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc^5(c+dx)}{5d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-b*csc(d*x+c)/d+2/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^
5/d
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 30, 2686, 200}

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^5(c+dx)}{5d}+\frac{2b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^2(c+dx)\,dx + b\int \cot^5(c+dx)\csc($$

$$= -\frac{a\text{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int (-1+\right.}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c\right.}{d}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc(c+dx)}{d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\,c}{}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 1.00

$$-\frac{a\cot^6(c+dx)}{6d} - \frac{b\csc(c+dx)}{d} + \frac{2b\csc^3(c+dx)}{3d} - \frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (b*Csc[c + d*x])/d + (2*b*Csc[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.22, size = 110, normalized size = 1.80

method	result

derivativedivides	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+b\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)}{d}$
default	$\frac{-\frac{a\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6}+b\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)}{d}$
risch	$-\frac{2i\left(15ia\,e^{10i(dx+c)}+15b\,e^{11i(dx+c)}-35b\,e^{9i(dx+c)}+50ia\,e^{6i(dx+c)}+78b\,e^{7i(dx+c)}-78b\,e^{5i(dx+c)}+15ia\,e^{2i(dx+c)}+35b\,e\right.}{15d\left(e^{2i(dx+c)}-1\right)^6}$
norman	$\frac{-\frac{a}{384d}+\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{3a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}-\frac{3a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{128d}+\frac{5a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{160d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a/sin(d*x+c)^6*cos(d*x+c)^6+b*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/1
5/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)
^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 1.15

$$-\frac{30\,b\sin\left(dx+c\right)^5+15\,a\sin\left(dx+c\right)^4-20\,b\sin\left(dx+c\right)^3-15\,a\sin\left(dx+c\right)^2+6\,b\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Fricas [A]
time = 0.35, size = 100, normalized size = 1.64

$$\frac{15\,a\cos\left(dx+c\right)^4-15\,a\cos\left(dx+c\right)^2+2\,\left(15\,b\cos\left(dx+c\right)^4-20\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)+5\,a}{30\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*cos(d*x + c)^4 - 15*a*cos(d*x + c)^2 + 2*(15*b*cos(d*x + c)^4 -
20*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 5*a)/(d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 70, normalized size = 1.15

$$-\frac{30\,b\sin\left(dx+c\right)^{5}+15\,a\sin\left(dx+c\right)^{4}-20\,b\sin\left(dx+c\right)^{3}-15\,a\sin\left(dx+c\right)^{2}+6\,b\sin\left(dx+c\right)+5\,a}{30\,d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(30*b*sin(d*x + c)^5 + 15*a*sin(d*x + c)^4 - 20*b*sin(d*x + c)^3 - 15
*a*sin(d*x + c)^2 + 6*b*sin(d*x + c) + 5*a)/(d*sin(d*x + c)^6)
```

Mupad [B]
time = 11.79, size = 69, normalized size = 1.13

$$-\frac{b\sin\left(c+d\,x\right)^5+\frac{a\sin\left(c+d\,x\right)^4}{2}-\frac{2\,b\sin\left(c+d\,x\right)^3}{3}-\frac{a\sin\left(c+d\,x\right)^2}{2}+\frac{b\,\sin\left(c+d\,x\right)}{5}+\frac{a}{6}}{d\sin\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^7,x)
```

```
[Out] -(a/6 + (b*sin(c + d*x))/5 - (a*sin(c + d*x)^2)/2 + (a*sin(c + d*x)^4)/2 -
(2*b*sin(c + d*x)^3)/3 + b*sin(c + d*x)^5)/(d*sin(c + d*x)^6)
```

3.1210 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$-\frac{b\cot^6(c+dx)}{6d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^7(c+dx)}{7d}$$

```
[Out] -1/6*b*cot(d*x+c)^6/d-1/3*a*csc(d*x+c)^3/d+2/5*a*csc(d*x+c)^5/d-1/7*a*csc(d
*x+c)^7/d
```

Rubi [A]
time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2686, 276, 2687, 30}

$$-\frac{a\csc^7(c+dx)}{7d}+\frac{2a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^3(c+dx)\,dx + b\int \cot^5(c+dx)\csc^2$$

$$= -\frac{a\mathrm{Subst}\left(\int x^2(-1+x^2)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\mathrm{Subst}}{}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\mathrm{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \csc(\right.}{d}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a}{}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 1.00

$$-\frac{b\cot^6(c+dx)}{6d} - \frac{a\csc^3(c+dx)}{3d} + \frac{2a\csc^5(c+dx)}{5d} - \frac{a\csc^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cot[c + d*x]^6)/d - (a*Csc[c + d*x]^3)/(3*d) + (2*a*Csc[c + d*x]^5)
/(5*d) - (a*Csc[c + d*x]^7)/(7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 0.24, size = 128, normalized size = 1.97

method	result

derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{b\left(\cos^6(dx\right.}{6\sin(dx+}}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{b\left(\cos^6(dx\right.}{6\sin(dx+}}{d}$
risch	$\frac{\frac{8ia\,e^{11i(dx+c)}}{3}+2b\,e^{12i(dx+c)}+\frac{32ia\,e^{9i(dx+c)}}{15}-2b\,e^{10i(dx+c)}+\frac{304ia\,e^{7i(dx+c)}}{35}+\frac{20b\,e^{8i(dx+c)}}{3}+\frac{32ia\,e^{5i(dx+c)}}{15}-\frac{20b\,e^{6i(dx+}}{3}}{d\left(e^{2i(dx+c)}-1\right)^7}$
norman	$-\frac{a}{896d}+\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280d}+\frac{a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}-\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{5a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}-\frac{a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{a\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{480d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105
/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c
)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.08

$$-\frac{105\,b\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,b\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,b\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Fricas [A]
time = 0.36, size = 106, normalized size = 1.63

$$\frac{70\,a\cos\left(dx+c\right)^{4}-56\,a\cos\left(dx+c\right)^{2}+35\,\left(3\,b\cos\left(dx+c\right)^{4}-3\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+16\,a}{210\,\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(70*a*cos(d*x + c)^4 - 56*a*cos(d*x + c)^2 + 35*(3*b*cos(d*x + c)^4 -
 3*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 16*a)/((d*cos(d*x + c)^6 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c)),x)
[Out] Timed out
```

Giac [A]
time = 0.53, size = 70, normalized size = 1.08

$$-\frac{105\,b\sin\left(dx+c\right)^{5}+70\,a\sin\left(dx+c\right)^{4}-105\,b\sin\left(dx+c\right)^{3}-84\,a\sin\left(dx+c\right)^{2}+35\,b\sin\left(dx+c\right)+30\,a}{210\,d\sin\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -1/210*(105*b*sin(d*x + c)^5 + 70*a*sin(d*x + c)^4 - 105*b*sin(d*x + c)^3 -
 84*a*sin(d*x + c)^2 + 35*b*sin(d*x + c) + 30*a)/(d*sin(d*x + c)^7)
```

Mupad [B]
time = 11.61, size = 70, normalized size = 1.08

$$-\frac{105\,b\sin\left(c+d\,x\right)^{5}+70\,a\sin\left(c+d\,x\right)^{4}-105\,b\sin\left(c+d\,x\right)^{3}-84\,a\sin\left(c+d\,x\right)^{2}+35\,b\,\sin\left(c+d\,x\right)+30\,a}{210\,d\sin\left(c+d\,x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^8,x)
[Out] -(30*a + 35*b*sin(c + d*x) - 84*a*sin(c + d*x)^2 + 70*a*sin(c + d*x)^4 - 10
5*b*sin(c + d*x)^3 + 105*b*sin(c + d*x)^5)/(210*d*sin(c + d*x)^7)
```

3.1211 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\cot^6(c+dx)}{6d}-\frac{a\cot^8(c+dx)}{8d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^7(c+dx)}{7d}$$

```
[Out] -1/6*a*cot(d*x+c)^6/d-1/8*a*cot(d*x+c)^8/d-1/3*b*csc(d*x+c)^3/d+2/5*b*csc(d
*x+c)^5/d-1/7*b*csc(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 14, 2686, 276}

$$-\frac{a\cot^8(c+dx)}{8d}-\frac{a\cot^6(c+dx)}{6d}-\frac{b\csc^7(c+dx)}{7d}+\frac{2b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(a*Cot[c + d*x]^6)/d - (a*Cot[c + d*x]^8)/(8*d) - (b*Csc[c + d*x]^3)/(
3*d) + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^4(c+dx)\,dx + b\int \cot^5(c+dx)\csc^3$$

$$= -\frac{a\text{Subst}\left(\int x^5(1+x^2)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\text{Subst}\big(}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^5+x^7)\,dx, x, -\cot(c+dx)\right)}{d} - \frac{b\text{Subst}\big(\int}{}$$

$$= -\frac{a\cot^6(c+dx)}{6d} - \frac{a\cot^8(c+dx)}{8d} - \frac{b\csc^3(c+dx)}{3d} + \frac{2b}{}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 1.09

$$-\frac{b\csc^3(c+dx)}{3d} + \frac{2b\csc^5(c+dx)}{5d} - \frac{b\csc^7(c+dx)}{7d} - \frac{a(6\csc^4(c+dx) - 8\csc^6(c+dx) + 3\csc^8(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(b*Csc[c + d*x]^3)/d + (2*b*Csc[c + d*x]^5)/(5*d) - (b*Csc[c + d*x]^7)
/(7*d) - (a*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(71) = 142$.
time = 0.24, size = 148, normalized size = 1.83

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(c}{}\right.}{3}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(c}{}\right.}{3}\right.}{d}$
risch	$\frac{4i(105ia\,e^{12i(dx+c)}+70b\,e^{13i(dx+c)}+140ia\,e^{10i(dx+c)}-14b\,e^{11i(dx+c)}+350ia\,e^{8i(dx+c)}+172b\,e^{9i(dx+c)}+140ia\,e^{6i(dx+c)}}{105d\left(e^{2i(dx+c)}-1\right)^8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-
1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+
c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin\left(dx+c\right)^5+210\,a\sin\left(dx+c\right)^4-336\,b\sin\left(dx+c\right)^3-280\,a\sin\left(dx+c\right)^2+120\,b\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.35, size = 109, normalized size = 1.35

$$-\frac{210\,a\cos\left(dx+c\right)^4-140\,a\cos\left(dx+c\right)^2+8\,\left(35\,b\cos\left(dx+c\right)^4-28\,b\cos\left(dx+c\right)^2+8\,b\right)\sin\left(dx+c\right)+35\,a}{840\,\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/840*(210*a*cos(d*x + c)^4 - 140*a*cos(d*x + c)^2 + 8*(35*b*cos(d*x + c)^
4 - 28*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 35*a)/(d*cos(d*x + c)^8 - 4*d
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-336\,b\sin\left(dx+c\right)^{3}-280\,a\sin\left(dx+c\right)^{2}+120\,b\sin\left(dx+c\right)+105\,a}{840\,d\sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/840*(280*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 336*b*sin(d*x + c)^3
- 280*a*sin(d*x + c)^2 + 120*b*sin(d*x + c) + 105*a)/(d*sin(d*x + c)^8)
```

Mupad [B]
time = 11.64, size = 70, normalized size = 0.86

$$-\frac{280\,b\sin\left(c+d\,x\right)^{5}+210\,a\sin\left(c+d\,x\right)^{4}-336\,b\sin\left(c+d\,x\right)^{3}-280\,a\sin\left(c+d\,x\right)^{2}+120\,b\,\sin\left(c+d\,x\right)+105\,a}{840\,d\sin\left(c+d\,x\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^9,x)
```

```
[Out] -(105*a + 120*b*sin(c + d*x) - 280*a*sin(c + d*x)^2 + 210*a*sin(c + d*x)^4
- 336*b*sin(c + d*x)^3 + 280*b*sin(c + d*x)^5)/(840*d*sin(c + d*x)^8)
```

3.1212 $\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{b\cot^6(c+dx)}{6d}-\frac{b\cot^8(c+dx)}{8d}-\frac{a\csc^5(c+dx)}{5d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^9(c+dx)}{9d}$$

```
[Out] -1/6*b*cot(d*x+c)^6/d-1/8*b*cot(d*x+c)^8/d-1/5*a*csc(d*x+c)^5/d+2/7*a*csc(d
*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2686, 276, 2687, 14}

$$-\frac{a\csc^9(c+dx)}{9d}+\frac{2a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{b\cot^8(c+dx)}{8d}-\frac{b\cot^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Cot[c + d*x]^6)/d - (b*Cot[c + d*x]^8)/(8*d) - (a*Csc[c + d*x]^5)/(
5*d) + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \cot^5(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \cot^5(c+dx)\csc^5(c+dx)\,dx + b\int \cot^5(c+dx)\csc^4$$

$$= -\frac{a\text{Subst}\left(\int x^4(-1+x^2)^2\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\text{Subst}}{}$$

$$= -\frac{a\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \csc(c+dx)\right)}{d} - \frac{b\text{Subs}}{}$$

$$= -\frac{b\cot^6(c+dx)}{6d} - \frac{b\cot^8(c+dx)}{8d} - \frac{a\csc^5(c+dx)}{5d} + \frac{2a}{}$$

Mathematica [A]
time = 0.09, size = 88, normalized size = 1.09

$$-\frac{a\csc^5(c+dx)}{5d} + \frac{2a\csc^7(c+dx)}{7d} - \frac{a\csc^9(c+dx)}{9d} - \frac{b(6\csc^4(c+dx) - 8\csc^6(c+dx) + 3\csc^8(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(a*Csc[c + d*x]^5)/d + (2*a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)
/(9*d) - (b*(6*Csc[c + d*x]^4 - 8*Csc[c + d*x]^6 + 3*Csc[c + d*x]^8))/(24*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(71) = 142$.
time = 0.27, size = 166, normalized size = 2.05

method	result
risch	$-\frac{4(504ia\,e^{13i(dx+c)}+315b\,e^{14i(dx+c)}+864ia\,e^{11i(dx+c)}+105b\,e^{12i(dx+c)}+1744ia\,e^{9i(dx+c)}+630b\,e^{10i(dx+c)}+864ia\,e^{7i}}{315d\left(e^{2i(dx+c)}-1\right)^{9}}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx}{105}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{105\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx}{105}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x+c)^7*cos(d*x+c)^6-1/105
/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d*x+c)^6-1/105/sin(d*x+c)
*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/8
/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.86

$$-\frac{630\,b\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,b\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,b\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Fricas [A]
time = 0.36, size = 115, normalized size = 1.42

$$-\frac{504\,a\cos\left(dx+c\right)^{4}-288\,a\cos\left(dx+c\right)^{2}+105\,\left(6\,b\cos\left(dx+c\right)^{4}-4\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+64\,a}{2520\,\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] -1/2520*(504*a*cos(d*x + c)^4 - 288*a*cos(d*x + c)^2 + 105*(6*b*cos(d*x + c
)^4 - 4*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 64*a)/((d*cos(d*x + c)^8 - 4*d
```

```
*cos(d*x + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c)
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**10*(a+b*sin(d*x+c)),x)
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 70, normalized size = 0.86

$$-\frac{630\,b\sin\left(dx+c\right)^{5}+504\,a\sin\left(dx+c\right)^{4}-840\,b\sin\left(dx+c\right)^{3}-720\,a\sin\left(dx+c\right)^{2}+315\,b\sin\left(dx+c\right)+280\,a}{2520\,d\sin\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^10*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2520*(630*b*sin(d*x + c)^5 + 504*a*sin(d*x + c)^4 - 840*b*sin(d*x + c)^3
 - 720*a*sin(d*x + c)^2 + 315*b*sin(d*x + c) + 280*a)/(d*sin(d*x + c)^9)
```

Mupad [**B**]
time = 11.67, size = 70, normalized size = 0.86

$$-\frac{\frac{b\sin\left(c+d\,x\right)^{5}}{4}+\frac{a\sin\left(c+d\,x\right)^{4}}{5}-\frac{b\sin\left(c+d\,x\right)^{3}}{3}-\frac{2\,a\sin\left(c+d\,x\right)^{2}}{7}+\frac{b\,\sin\left(c+d\,x\right)}{8}+\frac{a}{9}}{d\sin\left(c+d\,x\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^10,x)
[Out] -(a/9 + (b*sin(c + d*x))/8 - (2*a*sin(c + d*x)^2)/7 + (a*sin(c + d*x)^4)/5
- (b*sin(c + d*x)^3)/3 + (b*sin(c + d*x)^5)/4)/(d*sin(c + d*x)^9)
```

3.1213 $\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{b\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}+\frac{2b\csc^7(c+dx)}{7d}+\frac{a\csc^8(c+dx)}{4d}-\frac{b\csc^9(c+dx)}{9d}-\frac{a\csc^{10}(c+dx)}{10d}$$

```
[Out] -1/5*b*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d+2/7*b*csc(d*x+c)^7/d+1/4*a*csc(d
*x+c)^8/d-1/9*b*csc(d*x+c)^9/d-1/10*a*csc(d*x+c)^10/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$-\frac{a\csc^{10}(c+dx)}{10d}+\frac{a\csc^8(c+dx)}{4d}-\frac{a\csc^6(c+dx)}{6d}-\frac{b\csc^9(c+dx)}{9d}+\frac{2b\csc^7(c+dx)}{7d}-\frac{b\csc^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Csc[c + d*x]^5)/d - (a*Csc[c + d*x]^6)/(6*d) + (2*b*Csc[c + d*x]^7)
/(7*d) + (a*Csc[c + d*x]^8)/(4*d) - (b*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d
*x]^10)/(10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{11}(a+x)(b^2-x^2)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^6\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^{11}}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^6\text{Subst}\left(\int \left(\frac{ab^4}{x^{11}} + \frac{b^4}{x^{10}} - \frac{2ab^2}{x^9} - \frac{2b^2}{x^8} + \frac{a}{x^7} + \frac{1}{x^6}\right)\,dx, x, b\,\text{si}\right.}{d}$$
$$= -\frac{b\csc^5(c+dx)}{5d} - \frac{a\csc^6(c+dx)}{6d} + \frac{2b\csc^7(c+dx)}{7d} + \frac{a}{}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.91

$$-\frac{b\csc^5(c+dx)}{5d} + \frac{2b\csc^7(c+dx)}{7d} - \frac{b\csc^9(c+dx)}{9d} - \frac{a(10\csc^6(c+dx) - 15\csc^8(c+dx) + 6\csc^{10}(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^6*(a + b*Sin[c + d*x]),x]
[Out] -1/5*(b*Csc[c + d*x]^5)/d + (2*b*Csc[c + d*x]^7)/(7*d) - (b*Csc[c + d*x]^9)
/(9*d) - (a*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/(6
0*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.
time = 0.28, size = 184, normalized size = 1.90

method	result
risch	$-\frac{32i(105ia\,e^{14i(dx+c)}+63b\,e^{15i(dx+c)}+210ia\,e^{12i(dx+c)}+45b\,e^{13i(dx+c)}+378ia\,e^{10i(dx+c)}+110b\,e^{11i(dx+c)}+210ia\,e^{8i(dx}}{315d\left(e^{2i(dx+c)}-1\right)^{10}}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\text{co}}{105}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{10\sin(dx+c)^{10}}-\frac{\cos^6(dx+c)}{20\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{60\sin(dx+c)^6}\right)+b\left(-\frac{\cos^6(dx+c)}{9\sin(dx+c)^9}-\frac{\cos^6(dx+c)}{21\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{105\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{315\sin(dx+c)^3}-\frac{\text{co}}{105}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x+c)^8*cos(d*x+c)^6-1/6
0/sin(d*x+c)^6*cos(d*x+c)^6)+b*(-1/9/sin(d*x+c)^9*cos(d*x+c)^6-1/21/sin(d*x
+c)^7*cos(d*x+c)^6-1/105/sin(d*x+c)^5*cos(d*x+c)^6+1/315/sin(d*x+c)^3*cos(d
*x+c)^6-1/105/sin(d*x+c)*cos(d*x+c)^6-1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c)))
```

Maxima [**A**]
time = 0.27, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,b\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,b\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Fricas [**A**]
time = 0.36, size = 122, normalized size = 1.26

$$\frac{210\,a\cos\left(dx+c\right)^{4}-105\,a\cos\left(dx+c\right)^{2}+4\left(63\,b\cos\left(dx+c\right)^{4}-36\,b\cos\left(dx+c\right)^{2}+8\,b\right)\sin\left(dx+c\right)+21\,a}{1260\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/1260*(210*a*cos(d*x + c)^4 - 105*a*cos(d*x + c)^2 + 4*(63*b*cos(d*x + c)^
4 - 36*b*cos(d*x + c)^2 + 8*b)*sin(d*x + c) + 21*a)/(d*cos(d*x + c)^10 - 5*
d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d*x
+ c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**11*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.51, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin\left(dx+c\right)^{5}+210\,a\sin\left(dx+c\right)^{4}-360\,b\sin\left(dx+c\right)^{3}-315\,a\sin\left(dx+c\right)^{2}+140\,b\sin\left(dx+c\right)+126\,a}{1260\,d\sin\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^11*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/1260*(252*b*sin(d*x + c)^5 + 210*a*sin(d*x + c)^4 - 360*b*sin(d*x + c)^3
 - 315*a*sin(d*x + c)^2 + 140*b*sin(d*x + c) + 126*a)/(d*sin(d*x + c)^10)
```

Mupad [B]
time = 11.64, size = 70, normalized size = 0.72

$$-\frac{252\,b\sin\left(c+d\,x\right)^5+210\,a\sin\left(c+d\,x\right)^4-360\,b\sin\left(c+d\,x\right)^3-315\,a\sin\left(c+d\,x\right)^2+140\,b\,\sin\left(c+d\,x\right)+126\,a}{1260\,d\sin\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^11,x)
```

```
[Out] -(126*a + 140*b*sin(c + d*x) - 315*a*sin(c + d*x)^2 + 210*a*sin(c + d*x)^4
- 360*b*sin(c + d*x)^3 + 252*b*sin(c + d*x)^5)/(1260*d*sin(c + d*x)^10)
```

3.1214 $\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$-\frac{b\csc^6(c+dx)}{6d}-\frac{a\csc^7(c+dx)}{7d}+\frac{b\csc^8(c+dx)}{4d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{b\csc^{10}(c+dx)}{10d}-\frac{a\csc^{11}(c+dx)}{11d}$$

```
[Out] -1/6*b*csc(d*x+c)^6/d-1/7*a*csc(d*x+c)^7/d+1/4*b*csc(d*x+c)^8/d+2/9*a*csc(d
*x+c)^9/d-1/10*b*csc(d*x+c)^10/d-1/11*a*csc(d*x+c)^11/d
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 780}

$$-\frac{a\csc^{11}(c+dx)}{11d}+\frac{2a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+dx)}{7d}-\frac{b\csc^{10}(c+dx)}{10d}+\frac{b\csc^8(c+dx)}{4d}-\frac{b\csc^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6*(b*Csc[c + d*x]^6)/d - (a*Csc[c + d*x]^7)/(7*d) + (b*Csc[c + d*x]^8)/(
4*d) + (2*a*Csc[c + d*x]^9)/(9*d) - (b*Csc[c + d*x]^10)/(10*d) - (a*Csc[c +
 d*x]^11)/(11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^7(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{b^{12}(a+x)(b^2-x^2)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^7\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^{12}}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\text{Subst}\left(\int \left(\frac{ab^4}{x^{12}}+\frac{b^4}{x^{11}}-\frac{2ab^2}{x^{10}}-\frac{2b^2}{x^9}+\frac{a}{x^8}+\frac{1}{x^7}\right)\,dx, x, b\sin\right.}{d}$$

$$= -\frac{b\csc^6(c+dx)}{6d} - \frac{a\csc^7(c+dx)}{7d} + \frac{b\csc^8(c+dx)}{4d} + \frac{2a}{}$$

Mathematica [A]
time = 0.08, size = 88, normalized size = 0.91

$$-\frac{a\csc^7(c+dx)}{7d} + \frac{2a\csc^9(c+dx)}{9d} - \frac{a\csc^{11}(c+dx)}{11d} - \frac{b(10\csc^6(c+dx) - 15\csc^8(c+dx) + 6\csc^{10}(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^7*(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/7*(a*Csc[c + d*x]^7)/d + (2*a*Csc[c + d*x]^9)/(9*d) - (a*Csc[c + d*x]^11
)/(11*d) - (b*(10*Csc[c + d*x]^6 - 15*Csc[c + d*x]^8 + 6*Csc[c + d*x]^10))/
(60*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(85) = 170.
time = 0.31, size = 202, normalized size = 2.08

method	result
risch	$\frac{\frac{128ia\,e^{15i(dx+c)}}{7}+\frac{32b\,e^{16i(dx+c)}}{3}+\frac{2560ia\,e^{13i(dx+c)}}{63}+\frac{32b\,e^{14i(dx+c)}}{3}+\frac{47360ia\,e^{11i(dx+c)}}{693}+\frac{256b\,e^{12i(dx+c)}}{15}+\frac{2560ia\,e^{9i(dx+c)}}{63}}{d\left(e^{2i(dx+c)}-1\right)^{11}}$
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^9}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{231\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos}{}\right.}{23}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{11\sin(dx+c)^{11}}-\frac{5\left(\cos^6(dx+c)\right)}{99\sin(dx+c)^9}-\frac{5\left(\cos^6(dx+c)\right)}{231\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{231\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{693\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{231\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos}{}\right.}{23}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/11/sin(d*x+c)^11*cos(d*x+c)^6-5/99/sin(d*x+c)^9*cos(d*x+c)^6-5/2
31/sin(d*x+c)^7*cos(d*x+c)^6-1/231/sin(d*x+c)^5*cos(d*x+c)^6+1/693/sin(d*x+
c)^3*cos(d*x+c)^6-1/231/sin(d*x+c)*cos(d*x+c)^6-1/231*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))+b*(-1/10/sin(d*x+c)^10*cos(d*x+c)^6-1/20/sin(d*x
+c)^8*cos(d*x+c)^6-1/60/sin(d*x+c)^6*cos(d*x+c)^6))
```

Maxima [A]
time = 0.28, size = 70, normalized size = 0.72

$$-\frac{2310\,b\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,b\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,b\sin\left(dx+c\right)+1260\,a}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

Fricas [A]
time = 0.36, size = 128, normalized size = 1.32

$$\frac{1980\,a\cos\left(dx+c\right)^{4}-880\,a\cos\left(dx+c\right)^{2}+231\,\left(10\,b\cos\left(dx+c\right)^{4}-5\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+160\,a}{13860\,\left(d\cos\left(dx+c\right)^{10}-5\,d\cos\left(dx+c\right)^{8}+10\,d\cos\left(dx+c\right)^{6}-10\,d\cos\left(dx+c\right)^{4}+5\,d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/13860*(1980*a*cos(d*x + c)^4 - 880*a*cos(d*x + c)^2 + 231*(10*b*cos(d*x +
 c)^4 - 5*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 160*a)/((d*cos(d*x + c)^10 -
 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos(d
*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**12*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 70, normalized size = 0.72

$$-\frac{2310\,b\sin\left(dx+c\right)^{5}+1980\,a\sin\left(dx+c\right)^{4}-3465\,b\sin\left(dx+c\right)^{3}-3080\,a\sin\left(dx+c\right)^{2}+1386\,b\sin\left(dx+c\right)+1260\,a}{13860\,d\sin\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^12*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13860*(2310*b*sin(d*x + c)^5 + 1980*a*sin(d*x + c)^4 - 3465*b*sin(d*x +
c)^3 - 3080*a*sin(d*x + c)^2 + 1386*b*sin(d*x + c) + 1260*a)/(d*sin(d*x + c
)^11)
```

Mupad [B]
time = 11.69, size = 70, normalized size = 0.72

$$-\frac{\frac{b\sin(c+d\,x)^5}{6}+\frac{a\sin(c+d\,x)^4}{7}-\frac{b\sin(c+d\,x)^3}{4}-\frac{2\,a\sin(c+d\,x)^2}{9}+\frac{b\,\sin(c+d\,x)}{10}+\frac{a}{11}}{d\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x)))/sin(c + d*x)^12,x)
```

```
[Out] -(a/11 + (b*sin(c + d*x))/10 - (2*a*sin(c + d*x)^2)/9 + (a*sin(c + d*x)^4)/
7 - (b*sin(c + d*x)^3)/4 + (b*sin(c + d*x)^5)/6)/(d*sin(c + d*x)^11)
```

3.1215 $\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}+ab\,\mathrm{si}$$

```
[Out] 1/3*a^2*sin(d*x+c)^3/d+1/2*a*b*sin(d*x+c)^4/d-1/5*(2*a^2-b^2)*sin(d*x+c)^5/
d-2/3*a*b*sin(d*x+c)^6/d+1/7*(a^2-2*b^2)*sin(d*x+c)^7/d+1/4*a*b*sin(d*x+c)^
8/d+1/9*b^2*sin(d*x+c)^9/d
```

Rubi [A]

time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(a^2-2b^2)\sin^7(c+dx)}{7d}-\frac{(2a^2-b^2)\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^8(c+dx)}{4d}-\frac{2ab\sin^6(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}+\frac{b^2\sin^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) - ((2*a^2 - b^2)*Si
n[c + d*x]^5)/(5*d) - (2*a*b*Sin[c + d*x]^6)/(3*d) + ((a^2 - 2*b^2)*Sin[c +
 d*x]^7)/(7*d) + (a*b*Sin[c + d*x]^8)/(4*d) + (b^2*Sin[c + d*x]^9)/(9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+x)^2\left(b^2-x^2\right)^2}{b^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d} \\
&= \frac{\text{Subst}\left(\int x^2(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^7 d} \\
&= \frac{\text{Subst}\left(\int \left(a^2b^4x^2+2ab^4x^3+b^2\left(-2a^2+b^2\right)x^4-4ab^2x^5\right.\right.}{b^7 d} \\
&= \frac{a^2\sin^3(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{\left(2a^2-b^2\right)\sin^5(c+}{5d}
\end{aligned}
$$

Mathematica [A]

time = 0.55, size = 169, normalized size = 1.22

$$
\frac{-7560ab\cos(2(c+dx))-1260ab\cos(4(c+dx))+840ab\cos(6(c+dx))+315ab\cos(8(c+dx))+12600a^2\sin(c+dx)+3780b^2\sin(c+dx)-840a^2\sin(3(c+dx))-840b^2\sin(3(c+dx))-1512a^2\sin(5(c+dx))-504b^2\sin(5(c+dx))-360a^2\sin(7(c+dx))+90b^2\sin(7(c+dx))+70b^2\sin(9(c+dx))}{161280d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-7560*a*b*Cos[2*(c + d*x)] - 1260*a*b*Cos[4*(c + d*x)] + 840*a*b*Cos[6*(c
+ d*x)] + 315*a*b*Cos[8*(c + d*x)] + 12600*a^2*Sin[c + d*x] + 3780*b^2*Sin[
c + d*x] - 840*a^2*Sin[3*(c + d*x)] - 840*b^2*Sin[3*(c + d*x)] - 1512*a^2*S
in[5*(c + d*x)] - 504*b^2*Sin[5*(c + d*x)] - 360*a^2*Sin[7*(c + d*x)] + 90*
b^2*Sin[7*(c + d*x)] + 70*b^2*Sin[9*(c + d*x)])/(161280*d)
```

Maple [A]

time = 0.50, size = 155, normalized size = 1.12

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(dx+c)\right)}{8}-\frac{\left(\cos^6(dx+\right.}{24}\right.}{d}$
risch	$\frac{5a^2\sin(dx+c)}{64d}+\frac{3b^2\sin(dx+c)}{128d}+\frac{b^2\sin(9dx+9c)}{2304d}+\frac{ab\cos(8dx+8c)}{512d}-\frac{\sin(7dx+7c)a^2}{448d}+\frac{\sin(7dx+7c)b^2}{1792d}+\frac{ab\cos}{}$

norman	$\frac{\frac{8a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8a^2\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{16\left(a^2+2b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{16\left(a^2+2b^2\right)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}+\frac{8\left(31a^2-48b^2\right)\left(\tan^7\right.}{35d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))+2*a*b*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+b
^2*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105*(8/3+
cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 0.78

$$\frac{140\,b^2\sin\left(dx+c\right)^9+315\,ab\sin\left(dx+c\right)^8-840\,ab\sin\left(dx+c\right)^6+180\,\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^7+630\,ab\sin\left(dx+c\right)^4-252\,\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^5+420\,a^2\sin\left(dx+c\right)^3}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/1260*(140*b^2*sin(d*x + c)^9 + 315*a*b*sin(d*x + c)^8 - 840*a*b*sin(d*x +
 c)^6 + 180*(a^2 - 2*b^2)*sin(d*x + c)^7 + 630*a*b*sin(d*x + c)^4 - 252*(2*
a^2 - b^2)*sin(d*x + c)^5 + 420*a^2*sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.37, size = 121, normalized size = 0.88

$$\frac{315\,ab\cos\left(dx+c\right)^8-420\,ab\cos\left(dx+c\right)^6+4\,\left(35\,b^2\cos\left(dx+c\right)^8-5\,\left(9\,a^2+10\,b^2\right)\cos\left(dx+c\right)^6+3\,\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^4+4\,\left(3\,a^2+b^2\right)\cos\left(dx+c\right)^2+24\,a^2+8\,b^2\right)\sin\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1260*(315*a*b*cos(d*x + c)^8 - 420*a*b*cos(d*x + c)^6 + 4*(35*b^2*cos(d*x
 + c)^8 - 5*(9*a^2 + 10*b^2)*cos(d*x + c)^6 + 3*(3*a^2 + b^2)*cos(d*x + c)^
4 + 4*(3*a^2 + b^2)*cos(d*x + c)^2 + 24*a^2 + 8*b^2)*sin(d*x + c))/d
```

Sympy [A]
time = 1.33, size = 190, normalized size = 1.38

$$\begin{cases} \frac{8a^2\sin^7\left(c+dx\right)}{105d}+\frac{4a^2\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{a^2\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d}-\frac{ab\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{3d}-\frac{ab\cos^8\left(c+dx\right)}{12d}+\frac{8b^2\sin^9\left(c+dx\right)}{315d}+\frac{4b^2\sin^7\left(c+dx\right)\cos^2\left(c+dx\right)}{35d}+\frac{b^2\sin^5\left(c+dx\right)\cos^4\left(c+dx\right)}{5d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)^2\sin^2\left(c\right)\cos^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**7/(105*d) + 4*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(15*d) + a**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - a*b*sin(c + d
*x)**2*cos(c + d*x)**6/(3*d) - a*b*cos(c + d*x)**8/(12*d) + 8*b**2*sin(c +
d*x)**9/(315*d) + 4*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + b**2*sin(
c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**
2*cos(c)**5, True))
```

Giac [A]
time = 0.56, size = 173, normalized size = 1.25

$$\frac{ab\cos(8\,dx+8\,c)}{512\,d}+\frac{ab\cos(6\,dx+6\,c)}{192\,d}-\frac{ab\cos(4\,dx+4\,c)}{128\,d}-\frac{3\,ab\cos(2\,dx+2\,c)}{64\,d}+\frac{b^2\sin(9\,dx+9\,c)}{2304\,d}-\frac{(4\,a^2-b^2)\sin(7\,dx+7\,c)}{1792\,d}-\frac{(3\,a^2+b^2)\sin(5\,dx+5\,c)}{320\,d}-\frac{(a^2+b^2)\sin(3\,dx+3\,c)}{192\,d}+\frac{(10\,a^2+3\,b^2)\sin(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/512*a*b*cos(8*d*x + 8*c)/d + 1/192*a*b*cos(6*d*x + 6*c)/d - 1/128*a*b*cos
(4*d*x + 4*c)/d - 3/64*a*b*cos(2*d*x + 2*c)/d + 1/2304*b^2*sin(9*d*x + 9*c)
/d - 1/1792*(4*a^2 - b^2)*sin(7*d*x + 7*c)/d - 1/320*(3*a^2 + b^2)*sin(5*d*
x + 5*c)/d - 1/192*(a^2 + b^2)*sin(3*d*x + 3*c)/d + 1/128*(10*a^2 + 3*b^2)*
sin(d*x + c)/d
```

Mupad [B]
time = 11.44, size = 108, normalized size = 0.78

$$\frac{\sin(c+d\,x)^7\left(\frac{a^2}{7}-\frac{2\,b^2}{7}\right)-\sin(c+d\,x)^5\left(\frac{2\,a^2}{5}-\frac{b^2}{5}\right)+\frac{a^2\sin(c+d\,x)^3}{3}+\frac{b^2\sin(c+d\,x)^9}{9}+\frac{a\,b\sin(c+d\,x)^4}{2}-\frac{2\,a\,b\sin(c+d\,x)^6}{3}+\frac{a\,b\sin(c+d\,x)^8}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^7*(a^2/7 - (2*b^2)/7) - sin(c + d*x)^5*((2*a^2)/5 - b^2/5) +
(a^2*sin(c + d*x)^3)/3 + (b^2*sin(c + d*x)^9)/9 + (a*b*sin(c + d*x)^4)/2 -
(2*a*b*sin(c + d*x)^6)/3 + (a*b*sin(c + d*x)^8)/4)/d
```

3.1216 $\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}+2ab$$

```
[Out] 1/2*a^2*sin(d*x+c)^2/d+2/3*a*b*sin(d*x+c)^3/d-1/4*(2*a^2-b^2)*sin(d*x+c)^4/
d-4/5*a*b*sin(d*x+c)^5/d+1/6*(a^2-2*b^2)*sin(d*x+c)^6/d+2/7*a*b*sin(d*x+c)^
7/d+1/8*b^2*sin(d*x+c)^8/d
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 786}

$$\frac{(a^2-2b^2)\sin^6(c+dx)}{6d}-\frac{(2a^2-b^2)\sin^4(c+dx)}{4d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^7(c+dx)}{7d}-\frac{4ab\sin^5(c+dx)}{5d}+\frac{2ab\sin^3(c+dx)}{3d}+\frac{b^2\sin^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^2)/(2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) - ((2*a^2 - b^2)*
Sin[c + d*x]^4)/(4*d) - (4*a*b*Sin[c + d*x]^5)/(5*d) + ((a^2 - 2*b^2)*Sin[c
 + d*x]^6)/(6*d) + (2*a*b*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^8)/(8*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\mathrm{Subst}\left(\int \frac{x(a+x)^2(b^2-x^2)^2}{b}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\mathrm{Subst}\left(\int x(a+x)^2\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(a^2b^4x+2ab^4x^2+b^2\left(-2a^2+b^2\right)x^3-4ab^2x^4+\right.\right.}{b^6 d}$$
$$= \frac{a^2\sin^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{\left(2a^2-b^2\right)\sin^4(c+}{4d}$$

Mathematica [A]
time = 0.50, size = 138, normalized size = 1.00

$$-\frac{-2590b^2+840(10a^2+3b^2)\cos(2(c+dx))+420(8a^2+b^2)\cos(4(c+dx))+560a^2\cos(6(c+dx))-280b^2\cos(6(c+dx))-105b^2\cos(8(c+dx))-16800ab\sin(c+dx)+1120ab\sin(3(c+dx))+2016ab\sin(5(c+dx))+480ab\sin(7(c+dx))}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/107520*(-2590*b^2 + 840*(10*a^2 + 3*b^2)*Cos[2*(c + d*x)] + 420*(8*a^2 +
 b^2)*Cos[4*(c + d*x)] + 560*a^2*Cos[6*(c + d*x)] - 280*b^2*Cos[6*(c + d*x)
] - 105*b^2*Cos[8*(c + d*x)] - 16800*a*b*Sin[c + d*x] + 1120*a*b*Sin[3*(c +
 d*x)] + 2016*a*b*Sin[5*(c + d*x)] + 480*a*b*Sin[7*(c + d*x)])/d
```

Maple [A]
time = 0.40, size = 101, normalized size = 0.73

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6}+2ab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(\right.}{8}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6}+2ab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)+b^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^6(\right.}{8}\right.}{d}$
risch	$\frac{5ab\sin(dx+c)}{32d}+\frac{b^2\cos(8dx+8c)}{1024d}-\frac{ab\sin(7dx+7c)}{224d}-\frac{\cos(6dx+6c)a^2}{192d}+\frac{\cos(6dx+6c)b^2}{384d}-\frac{3ab\sin(5dx+5c)}{160d}-\frac{a^2}{}$
norman	$\frac{2a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(a^2+b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(a^2+b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{10\left(4a^2+4b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^2*cos(d*x+c)^6+2*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+co
s(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^
6-1/24*cos(d*x+c)^6))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.78

$$\frac{105\,b^2\sin\left(dx+c\right)^8+240\,ab\sin\left(dx+c\right)^7-672\,ab\sin\left(dx+c\right)^5+140\,\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^3-210\,\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^4+420\,a^2\sin\left(dx+c\right)^2}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/840*(105*b^2*sin(d*x + c)^8 + 240*a*b*sin(d*x + c)^7 - 672*a*b*sin(d*x +
c)^5 + 140*(a^2 - 2*b^2)*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^3 - 210*(2*a
^2 - b^2)*sin(d*x + c)^4 + 420*a^2*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.35, size = 85, normalized size = 0.62

$$\frac{105\,b^2\cos\left(dx+c\right)^8-140\,\left(a^2+b^2\right)\cos\left(dx+c\right)^6-16\,\left(15\,ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4-4\,ab\cos\left(dx+c\right)^2-8\,ab\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^2*cos(d*x + c)^8 - 140*(a^2 + b^2)*cos(d*x + c)^6 - 16*(15*a*b
*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 - 4*a*b*cos(d*x + c)^2 - 8*a*b)*sin(
d*x + c))/d
```

Sympy [A]
time = 0.95, size = 139, normalized size = 1.01

$$\begin{cases} -\frac{a^2\cos^6\left(c+dx\right)}{6d}+\frac{16ab\sin^7\left(c+dx\right)}{105d}+\frac{8ab\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{15d}+\frac{2ab\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{3d}-\frac{b^2\sin^2\left(c+dx\right)\cos^6\left(c+dx\right)}{6d}-\frac{b^2\cos^8\left(c+dx\right)}{24d} & \text{for}\ d\neq 0 \\ x\left(a+b\sin\left(c\right)\right)^2\sin\left(c\right)\cos^5\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**6/(6*d) + 16*a*b*sin(c + d*x)**7/(105*d) + 8
*a*b*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + 2*a*b*sin(c + d*x)**3*cos(c +
 d*x)**4/(3*d) - b**2*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - b**2*cos(c +
d*x)**8/(24*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)*cos(c)**5, True))
```

Giac [A]
time = 0.51, size = 152, normalized size = 1.10

$$\frac{b^2\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{ab\sin\left(7\,dx+7\,c\right)}{224\,d}-\frac{3\,ab\sin\left(5\,dx+5\,c\right)}{160\,d}-\frac{ab\sin\left(3\,dx+3\,c\right)}{96\,d}+\frac{5\,ab\sin\left(dx+c\right)}{32\,d}-\frac{\left(2\,a^2-b^2\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(8\,a^2+b^2\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{\left(10\,a^2+3\,b^2\right)\cos\left(2\,dx+2\,c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/1024*b^2*cos(8*d*x + 8*c)/d - 1/224*a*b*sin(7*d*x + 7*c)/d - 3/160*a*b*sin(5*d*x + 5*c)/d - 1/96*a*b*sin(3*d*x + 3*c)/d + 5/32*a*b*sin(d*x + c)/d - 1/384*(2*a^2 - b^2)*cos(6*d*x + 6*c)/d - 1/256*(8*a^2 + b^2)*cos(4*d*x + 4*c)/d - 1/128*(10*a^2 + 3*b^2)*cos(2*d*x + 2*c)/d

Mupad [B]
time = 11.47, size = 108, normalized size = 0.78

$$\frac{\sin\left(c+d\,x\right)^6\,\left(\frac{a^2}{6}-\frac{b^2}{3}\right)-\sin\left(c+d\,x\right)^4\,\left(\frac{a^2}{2}-\frac{b^2}{4}\right)+\frac{a^2\,\sin\left(c+d\,x\right)^2}{2}+\frac{b^2\,\sin\left(c+d\,x\right)^8}{8}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)^3}{3}-\frac{4\,a\,b\,\sin\left(c+d\,x\right)^5}{5}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)^7}{7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)

[Out] (sin(c + d*x)^6*(a^2/6 - b^2/3) - sin(c + d*x)^4*(a^2/2 - b^2/4) + (a^2*sin(c + d*x)^2)/2 + (b^2*sin(c + d*x)^8)/8 + (2*a*b*sin(c + d*x)^3)/3 - (4*a*b*sin(c + d*x)^5)/5 + (2*a*b*sin(c + d*x)^7)/7)/d

3.1217 $\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}+$$

```
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d-1/2*(2*a^2-b^2)*sin(d*x+c)^2/d-4/3*
a*b*sin(d*x+c)^3/d+1/4*(a^2-2*b^2)*sin(d*x+c)^4/d+2/5*a*b*sin(d*x+c)^5/d+1/
6*b^2*sin(d*x+c)^6/d
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 962}

$$\frac{(a^2-2b^2)\sin^4(c+dx)}{4d}-\frac{(2a^2-b^2)\sin^2(c+dx)}{2d}+\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin^5(c+dx)}{5d}-\frac{4ab\sin^3(c+dx)}{3d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d - ((2*a^2 - b^2)*Sin[c +
 d*x]^2)/(2*d) - (4*a*b*Sin[c + d*x]^3)/(3*d) + ((a^2 - 2*b^2)*Sin[c + d*x]
^4)/(4*d) + (2*a*b*Sin[c + d*x]^5)/(5*d) + (b^2*Sin[c + d*x]^6)/(6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(2ab^4 + \frac{a^2b^4}{x} - b^2(2a^2-b^2)\,x - 4ab^2x^2 + (a^2 -\right.\right.}{b^4 d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} - \frac{(2a^2-b^2)\sin^2(}{2d}$$

Mathematica [A]
time = 0.11, size = 105, normalized size = 0.81

$$\frac{60a^2\log(\sin(c+dx)) + 120ab\sin(c+dx) + 30(-2a^2+b^2)\sin^2(c+dx) - 80ab\sin^3(c+dx) + 15(a^2-2b^2)\sin^4(c+dx) + 24ab\sin^5(c+dx) + 10b^2\sin^6(c+dx)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^2*Log[Sin[c + d*x]] + 120*a*b*Sin[c + d*x] + 30*(-2*a^2 + b^2)*Sin[c
+ d*x]^2 - 80*a*b*Sin[c + d*x]^3 + 15*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 24*a*b
*Sin[c + d*x]^5 + 10*b^2*Sin[c + d*x]^6)/(60*d)
```

Maple [A]
time = 0.26, size = 81, normalized size = 0.62

method	result
derivativedivides	$\frac{a^2\left(\frac{(\cos^4(dx+c))}{4}+\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+\frac{2ab\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}-\frac{(\cos^6(dx+c))b^2}{6}}{d}$
default	$\frac{a^2\left(\frac{(\cos^4(dx+c))}{4}+\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+\frac{2ab\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)}{5}-\frac{(\cos^6(dx+c))b^2}{6}}{d}$
risch	$-ia^2x+\frac{3a^2e^{2i(dx+c)}}{16d}-\frac{5\,e^{2i(dx+c)}b^2}{128d}+\frac{3a^2e^{-2i(dx+c)}}{16d}-\frac{5\,e^{-2i(dx+c)}b^2}{128d}-\frac{2ia^2c}{d}+\frac{a^2\ln(e^{2i(dx+c)}-1)}{d}+$
norman	$\frac{-\frac{12a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{12a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(2a^2-b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(2a^2-b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4(12a^2-5b^2)\left(\tan^6\right.}{3d}}{(1+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a^2*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+2/5*a*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*cos(d*x+c)^6*b^2)

Maxima [**A**]
time = 0.28, size = 105, normalized size = 0.81

$$\frac{10\,b^2\sin\left(dx+c\right)^6+24\,ab\sin\left(dx+c\right)^5-80\,ab\sin\left(dx+c\right)^3+15\,\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^4+60\,a^2\log\left(\sin\left(dx+c\right)\right)+120\,ab\sin\left(dx+c\right)-30\,\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x + c)^3 + 15*(a^2 - 2*b^2)*sin(d*x + c)^4 + 60*a^2*log(sin(d*x + c)) + 120*a*b*sin(d*x + c) - 30*(2*a^2 - b^2)*sin(d*x + c)^2)/d

Fricas [**A**]
time = 0.38, size = 96, normalized size = 0.74

$$-\frac{10\,b^2\cos\left(dx+c\right)^6-15\,a^2\cos\left(dx+c\right)^4-30\,a^2\cos\left(dx+c\right)^2-60\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(3\,ab\cos\left(dx+c\right)^4+4\,ab\cos\left(dx+c\right)^2+8\,ab\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/60*(10*b^2*cos(d*x + c)^6 - 15*a^2*cos(d*x + c)^4 - 30*a^2*cos(d*x + c)^2 - 60*a^2*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x + c)^4 + 4*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/d

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [**A**]
time = 0.53, size = 118, normalized size = 0.91

$$\frac{10\,b^2\sin\left(dx+c\right)^6+24\,ab\sin\left(dx+c\right)^5+15\,a^2\sin\left(dx+c\right)^4-30\,b^2\sin\left(dx+c\right)^4-80\,ab\sin\left(dx+c\right)^3-60\,a^2\sin\left(dx+c\right)^2+30\,b^2\sin\left(dx+c\right)^2+60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+120\,ab\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/60*(10*b^2*sin(d*x + c)^6 + 24*a*b*sin(d*x + c)^5 + 15*a^2*sin(d*x + c)^4 - 30*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*x + c)^3 - 60*a^2*sin(d*x + c)^2 +

```
30*b^2*sin(d*x + c)^2 + 60*a^2*log(abs(sin(d*x + c))) + 120*a*b*sin(d*x + c
))/d
```

Mupad [B]
time = 11.74, size = 153, normalized size = 1.18

$$\frac{a^2 \ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)}\right)}{d}-\frac{a^2 \ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d x}{2}\right)^2}\right)}{d}+\frac{a^2\cos\left(c+d x\right)^2}{2 d}+\frac{a^2\cos\left(c+d x\right)^4}{4 d}-\frac{b^2\cos\left(c+d x\right)^6}{6 d}+\frac{16 a b\sin\left(c+d x\right)}{15 d}+\frac{8 a b\cos\left(c+d x\right)^2\sin\left(c+d x\right)}{15 d}+\frac{2 a b\cos\left(c+d x\right)^4\sin\left(c+d x\right)}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*log(1/cos(c/2 + (
d*x)/2)^2))/d + (a^2*cos(c + d*x)^2)/(2*d) + (a^2*cos(c + d*x)^4)/(4*d) - (
b^2*cos(c + d*x)^6)/(6*d) + (16*a*b*sin(c + d*x))/(15*d) + (8*a*b*cos(c + d
*x)^2*sin(c + d*x))/(15*d) + (2*a*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.1218 $\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}+$$

```
[Out] -a^2*csc(d*x+c)/d+2*a*b*ln(sin(d*x+c))/d-(2*a^2-b^2)*sin(d*x+c)/d-2*a*b*sin
(d*x+c)^2/d+1/3*(a^2-2*b^2)*sin(d*x+c)^3/d+1/2*a*b*sin(d*x+c)^4/d+1/5*b^2*s
in(d*x+c)^5/d
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(a^2-2b^2)\sin^3(c+dx)}{3d}-\frac{(2a^2-b^2)\sin(c+dx)}{d}-\frac{a^2\csc(c+dx)}{d}+\frac{ab\sin^4(c+dx)}{2d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}+\frac{b^2\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - ((2*a^2 - b^2)*Sin[
c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + ((a^2 - 2*b^2)*Sin[c + d*x]^3)/(3*
d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*Sin[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^2(a+x)^2(b^2-x^2)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2b^2\left(1-\frac{b^2}{2a^2}\right)+\frac{a^2b^4}{x^2}+\frac{2ab^4}{x}-4ab^2x+(a\right.\right.}{b^3 d}$$
$$= -\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}-\frac{(2a^2-b^2)\,\text{si}}{d}$$

Mathematica [A]
time = 0.04, size = 142, normalized size = 1.14

$$-\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{b^2\sin(c+dx)}{d}-\frac{2ab\sin^2(c+dx)}{d}+\frac{a^2\sin^3(c+dx)}{3d}-\frac{2b^2\sin^3(c+dx)}{3d}+\frac{ab\sin^4(c+dx)}{2d}+\frac{b^2\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) + (2*a*b*Log[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x]
)/d + (b^2*Sin[c + d*x])/d - (2*a*b*Sin[c + d*x]^2)/d + (a^2*Sin[c + d*x]^3
)/(3*d) - (2*b^2*Sin[c + d*x]^3)/(3*d) + (a*b*Sin[c + d*x]^4)/(2*d) + (b^2*
Sin[c + d*x]^5)/(5*d)
```

Maple [A]
time = 0.24, size = 120, normalized size = 0.96

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)\right)+2ab\left(\frac{(\cos^4(dx+c))}{4}+\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)}-\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)\right)+2ab\left(\frac{(\cos^4(dx+c))}{4}+\frac{(\cos^2(dx+c))}{2}+\ln(\sin(dx+c))\right)+}{d}$
risch	$-2iabx+\frac{3ab\,e^{2i(dx+c)}}{8d}+\frac{7ia^2e^{i(dx+c)}}{8d}-\frac{5ie^{i(dx+c)}b^2}{16d}-\frac{7ia^2e^{-i(dx+c)}}{8d}+\frac{5ie^{-i(dx+c)}b^2}{16d}+\frac{3ab\,e^{-2i(dx+c)}}{8d}$
norman	$\frac{-\frac{a^2}{2d}-\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{(7a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(7a^2-2b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(125a^2-16b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{(125a^2}{}}{(1+\tan^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*si
n(d*x+c))+2*a*b*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+1/5*b^2*
(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 105, normalized size = 0.84

$$\frac{6\,b^2\sin\left(dx+c\right)^5+15\,ab\sin\left(dx+c\right)^4-60\,ab\sin\left(dx+c\right)^2+10\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^3+60\,ab\log\left(\sin\left(dx+c\right)\right)-30\left(2\,a^2-b^2\right)\sin\left(dx+c\right)-\frac{30\,a^2}{\sin(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 - 60*a*b*sin(d*x + c)^2
+ 10*(a^2 - 2*b^2)*sin(d*x + c)^3 + 60*a*b*log(sin(d*x + c)) - 30*(2*a^2 -
b^2)*sin(d*x + c) - 30*a^2/sin(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 135, normalized size = 1.08

$$-\frac{48\,b^2\cos\left(dx+c\right)^6-16\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^4-480\,ab\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-64\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^2+640\,a^2-128\,b^2-15\left(8\,ab\cos\left(dx+c\right)^4+16\,ab\cos\left(dx+c\right)^2-11\,ab\right)\sin\left(dx+c\right)}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^2*cos(d*x + c)^6 - 16*(5*a^2 - b^2)*cos(d*x + c)^4 - 480*a*b*l
og(1/2*sin(d*x + c))*sin(d*x + c) - 64*(5*a^2 - b^2)*cos(d*x + c)^2 + 640*a
^2 - 128*b^2 - 15*(8*a*b*cos(d*x + c)^4 + 16*a*b*cos(d*x + c)^2 - 11*a*b)*s
in(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.56, size = 127, normalized size = 1.02

$$\frac{6\,b^2\sin\left(dx+c\right)^5+15\,ab\sin\left(dx+c\right)^4+10\,a^2\sin\left(dx+c\right)^3-20\,b^2\sin\left(dx+c\right)^3-60\,ab\sin\left(dx+c\right)^2+60\,ab\log\left(\left|\sin\left(dx+c\right)\right|\right)-60\,a^2\sin\left(dx+c\right)+30\,b^2\sin\left(dx+c\right)-\frac{30\,\left(2\,ab\sin(dx+c)+a^2\right)}{\sin(dx+c)}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*sin(d*x + c)^5 + 15*a*b*sin(d*x + c)^4 + 10*a^2*sin(d*x + c)^3
- 20*b^2*sin(d*x + c)^3 - 60*a*b*sin(d*x + c)^2 + 60*a*b*log(abs(sin(d*x +
c))) - 60*a^2*sin(d*x + c) + 30*b^2*sin(d*x + c) - 30*(2*a*b*sin(d*x + c) +
 a^2)/sin(d*x + c))/d
```

Mupad [B]
time = 12.02, size = 445, normalized size = 3.56

$$\frac{16\,a\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}-\frac{8\,a\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d}-\frac{16\,a\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{d}+\frac{8\,a\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d}+\frac{20\,a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{16\,a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{8\,a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{22\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{256\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{15\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{368\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{15\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{96\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{5\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{32\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{5\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{2\,a\,b\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{d}+\frac{2\,a\,b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}-\frac{9\,a^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{a^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{2\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (16*a*b*cos(c/2 + (d*x)/2)^4)/d - (8*a*b*cos(c/2 + (d*x)/2)^2)/d - (16*a*b*
cos(c/2 + (d*x)/2)^6)/d + (8*a*b*cos(c/2 + (d*x)/2)^8)/d + (20*a^2*cos(c/2
+ (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) - (16*a^2*cos(c/2 + (d*x)/2)^5)/(3*d
*sin(c/2 + (d*x)/2)) + (8*a^2*cos(c/2 + (d*x)/2)^7)/(3*d*sin(c/2 + (d*x)/2)
) - (22*b^2*cos(c/2 + (d*x)/2)^3)/(3*d*sin(c/2 + (d*x)/2)) + (256*b^2*cos(c
/2 + (d*x)/2)^5)/(15*d*sin(c/2 + (d*x)/2)) - (368*b^2*cos(c/2 + (d*x)/2)^7)
/(15*d*sin(c/2 + (d*x)/2)) + (96*b^2*cos(c/2 + (d*x)/2)^9)/(5*d*sin(c/2 + (
d*x)/2)) - (32*b^2*cos(c/2 + (d*x)/2)^11)/(5*d*sin(c/2 + (d*x)/2)) - (2*a*b
*log(1/cos(c/2 + (d*x)/2)^2))/d + (2*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/d - (9*a^2*cos(c/2 + (d*x)/2))/(2*d*sin(c/2 + (d*x)/2)) - (a^2*si
n(c/2 + (d*x)/2))/(2*d*cos(c/2 + (d*x)/2)) + (2*b^2*cos(c/2 + (d*x)/2))/(d*
sin(c/2 + (d*x)/2))
```

3.1219 $\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=127

$$-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{4ab\sin(c+dx)}{d}+\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}+$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-(2*a^2-b^2)*ln(sin(d*x+c))/d-4*a
*b*sin(d*x+c)/d+1/2*(a^2-2*b^2)*sin(d*x+c)^2/d+2/3*a*b*sin(d*x+c)^3/d+1/4*b
^2*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(a^2-2b^2)\sin^2(c+dx)}{2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}+\frac{2ab\sin^3(c+dx)}{3d}-\frac{4ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{b^2\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((2*a^2 - b^2)*Log[S
in[c + d*x]])/d - (4*a*b*Sin[c + d*x])/d + ((a^2 - 2*b^2)*Sin[c + d*x]^2)/(
2*d) + (2*a*b*Sin[c + d*x]^3)/(3*d) + (b^2*Sin[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^3(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-4ab^2 + \frac{a^2b^4}{x^3} + \frac{2ab^4}{x^2} + \frac{-2a^2b^2+b^4}{x} + (a^2-2b^2)\right.\right.}{b^2 d}$$
$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{(2a^2-b^2)\log(\text{si}}{d}$$

Mathematica [A]
time = 0.20, size = 103, normalized size = 0.81

$$\frac{-24ab\csc(c+dx) - 6a^2\csc^2(c+dx) + 12(-2a^2+b^2)\log(\sin(c+dx)) - 48ab\sin(c+dx) + 6(a^2-2b^2)\sin^2(c+dx) + 8ab\sin^3(c+dx) + 3b^2\sin^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-24*a*b*Csc[c + d*x] - 6*a^2*Csc[c + d*x]^2 + 12*(-2*a^2 + b^2)*Log[Sin[c
+ d*x]] - 48*a*b*Sin[c + d*x] + 6*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 8*a*b*Sin[
c + d*x]^3 + 3*b^2*Sin[c + d*x]^4)/(12*d)
```

Maple [A]
time = 0.27, size = 141, normalized size = 1.11

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2} - \frac{\left(\cos^4(dx+c)\right)}{2} - \left(\cos^2(dx+c)\right) - 2\ln(\sin(dx+c))\right) + 2ab\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)} - \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2} - \frac{\left(\cos^4(dx+c)\right)}{2} - \left(\cos^2(dx+c)\right) - 2\ln(\sin(dx+c))\right) + 2ab\left(-\frac{\cos^6(dx+c)}{\sin(dx+c)} - \left(\frac{8}{3} + \cos^4(dx+c) + \frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
risch	$-\frac{iab\,e^{-3i(dx+c)}}{12d} + 2ia^2x + \frac{iab\,e^{3i(dx+c)}}{12d} - \frac{a^2e^{2i(dx+c)}}{8d} + \frac{3\,e^{2i(dx+c)}b^2}{16d} - \frac{7iab\,e^{-i(dx+c)}}{4d} + \frac{7iab\,e^{i(dx+c)}}{4d} -$
norman	$\frac{-\frac{a^2}{8d} - \frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} + \frac{2\left(3a^2-2b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d} + \frac{\left(25a^2-32b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} + \frac{\left(25a^2-32b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d} - \frac{ab\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln
(sin(d*x+c)))+2*a*b*(-1/sin(d*x+c)*cos(d*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c))+b^2*(1/4*cos(d*x+c)^4+1/2*cos(d*x+c)^2+ln(sin(d*x+c)))
)
```

Maxima [A]
time = 0.28, size = 104, normalized size = 0.82

$$\frac{3\,b^2\sin\left(dx+c\right)^4+8\,ab\sin\left(dx+c\right)^3-48\,ab\sin\left(dx+c\right)+6\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^2-12\left(2\,a^2-b^2\right)\log\left(\sin\left(dx+c\right)\right)-\frac{6\left(4\,ab\sin(dx+c)+a^2\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 - 48*a*b*sin(d*x + c) + 6
*(a^2 - 2*b^2)*sin(d*x + c)^2 - 12*(2*a^2 - b^2)*log(sin(d*x + c)) - 6*(4*a
*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 160, normalized size = 1.26

$$\frac{24\,b^2\cos\left(dx+c\right)^6-24\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^4+9\left(8\,a^2-9\,b^2\right)\cos\left(dx+c\right)^2+24\,a^2+33\,b^2-96\left(\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,a^2+b^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-64\left(ab\cos\left(dx+c\right)^4+4\,ab\cos\left(dx+c\right)^2-8\,ab\right)\sin\left(dx+c\right)}{96\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*b^2*cos(d*x + c)^6 - 24*(2*a^2 - b^2)*cos(d*x + c)^4 + 9*(8*a^2 -
9*b^2)*cos(d*x + c)^2 + 24*a^2 + 33*b^2 - 96*((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a^2 + b^2)*log(1/2*sin(d*x + c)) - 64*(a*b*cos(d*x + c)^4 + 4*a*b*cos(d
*x + c)^2 - 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.51, size = 140, normalized size = 1.10

$$\frac{3\,b^2\sin\left(dx+c\right)^4+8\,ab\sin\left(dx+c\right)^3+6\,a^2\sin\left(dx+c\right)^2-12\,b^2\sin\left(dx+c\right)^2-48\,ab\sin\left(dx+c\right)-12\left(2\,a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{6\left(6\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2-4\,ab\sin(dx+c)-a^2\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^2*sin(d*x + c)^4 + 8*a*b*sin(d*x + c)^3 + 6*a^2*sin(d*x + c)^2 -
12*b^2*sin(d*x + c)^2 - 48*a*b*sin(d*x + c) - 12*(2*a^2 - b^2)*log(abs(sin(
d*x + c))) + 6*(6*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x
 + c) - a^2)/sin(d*x + c)^2)/d
```

Mupad [B]
time = 11.68, size = 331, normalized size = 2.61

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(2\,a^2-b^2\right)}{d}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(14\,a^2-16\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{15\,a^2}{2}-16\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(5\,a^2-16\,b^2\right)+\frac{a^2}{2}+48\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{296\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{272\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+36\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{d}-\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(2*a^2 - b^2))/d - (a^2*tan(c/2 + (d*x)/2)^2
)/(8*d) - (2*a^2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^6*(14*a^2 - 16*b
^2) - tan(c/2 + (d*x)/2)^8*((15*a^2)/2 - 16*b^2) - tan(c/2 + (d*x)/2)^4*(5*
a^2 - 16*b^2) + a^2/2 + 48*a*b*tan(c/2 + (d*x)/2)^3 + (296*a*b*tan(c/2 + (d
*x)/2)^5)/3 + (272*a*b*tan(c/2 + (d*x)/2)^7)/3 + 36*a*b*tan(c/2 + (d*x)/2)^
9 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 16*tan(c/2 + (d*
x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (
d*x)/2)^10)) - (log(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d - (a*b*tan(c/2 + (
d*x)/2))/d
```

3.1220 $\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}-\frac{4ab\log(\sin(c+dx))}{d}+\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{ab}{}$$

[Out] (2*a^2-b^2)*csc(d*x+c)/d-a*b*csc(d*x+c)^2/d-1/3*a^2*csc(d*x+c)^3/d-4*a*b*ln(sin(d*x+c))/d+(a^2-2*b^2)*sin(d*x+c)/d+a*b*sin(d*x+c)^2/d+1/3*b^2*sin(d*x+c)^3/d

Rubi [A]
time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 962}

$$\frac{(a^2-2b^2)\sin(c+dx)}{d}+\frac{(2a^2-b^2)\csc(c+dx)}{d}-\frac{a^2\csc^3(c+dx)}{3d}+\frac{ab\sin^2(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{4ab\log(\sin(c+dx))}{d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/d - (a*b*Csc[c + d*x]^2)/d - (a^2*Csc[c + d*x]^3)/(3*d) - (4*a*b*Log[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/d + (a*b*Sin[c + d*x]^2)/d + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^4(a+x)^2(b^2-x^2)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^4}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{a^2b^4}{x^4}+\frac{2ab^4}{x^3}+\frac{-2a^2b^2+b^4}{x^2}-\frac{4ab^2}{x}\right.\right.}{bd}$$
$$= \frac{(2a^2-b^2)\csc(c+dx)}{d} - \frac{ab\csc^2(c+dx)}{d} - \frac{a^2\csc^3(c+d}{3d}$$

Mathematica [A]

time = 0.19, size = 103, normalized size = 0.86

$$\frac{(6a^2-3b^2)\csc(c+dx)-3ab\csc^2(c+dx)-a^2\csc^3(c+dx)-12ab\log(\sin(c+dx))+3(a^2-2b^2)\sin(c+dx)+3ab\sin^2(c+dx)+b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2 - 3*b^2)*Csc[c + d*x] - 3*a*b*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^3 -
 12*a*b*Log[Sin[c + d*x]] + 3*(a^2 - 2*b^2)*Sin[c + d*x] + 3*a*b*Sin[c + d*
x]^2 + b^2*Sin[c + d*x]^3)/(3*d)
```

Maple [A]

time = 0.27, size = 176, normalized size = 1.47

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)\right)+2ab\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{(\cos^4(dx+c))}{2}-(\text{co}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(dx+c)\right)+2ab\left(-\frac{\cos^6(dx+c)}{2\sin(dx+c)^2}-\frac{(\cos^4(dx+c))}{2}-(\text{co}\right.}{d}$
norman	$\frac{-\frac{a^2}{24d}-\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(3a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(3a^2-2b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5(11a^2-10b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{(43a^2}{}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$
risch	$4iabx+\frac{ib^2e^{3i(dx+c)}}{24d}-\frac{ab\,e^{2i(dx+c)}}{4d}-\frac{ia^2e^{i(dx+c)}}{2d}+\frac{7ie^{i(dx+c)}b^2}{8d}+\frac{ia^2e^{-i(dx+c)}}{2d}-\frac{7ie^{-i(dx+c)}b^2}{8d}-\frac{ab\,e^{-2}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+2*a*b*(-1/2/sin(d*x+c)^2*cos(d*x+c)
```

```
^6-1/2*cos(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+b^2*(-1/sin(d*x+c)*cos(d
*x+c)^6-(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 103, normalized size = 0.86

$$\frac{b^2\sin(dx+c)^3+3\,ab\sin(dx+c)^2-12\,ab\log(\sin(dx+c))+3\,(a^2-2\,b^2)\sin(dx+c)-\frac{3\,ab\sin(dx+c)-3\,(2\,a^2-b^2)\sin(dx+c)^2+a^2}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(sin(d*x + c)) +
 3*(a^2 - 2*b^2)*sin(d*x + c) - (3*a*b*sin(d*x + c) - 3*(2*a^2 - b^2)*sin(d
*x + c)^2 + a^2)/sin(d*x + c)^3)/d
```

Fricas [A]
time = 0.36, size = 158, normalized size = 1.32

$$\frac{2\,b^2\cos(dx+c)^6-6\,(a^2-b^2)\cos(dx+c)^4+24\,(a^2-b^2)\cos(dx+c)^2-24\,(ab\cos(dx+c)^2-ab)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)-16\,a^2+16\,b^2-3\,(2\,ab\cos(dx+c)^4-3\,ab\cos(dx+c)^2-ab)\sin(dx+c)}{6\,(d\cos(dx+c)^2-d)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b^2*cos(d*x + c)^6 - 6*(a^2 - b^2)*cos(d*x + c)^4 + 24*(a^2 - b^2)*c
os(d*x + c)^2 - 24*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*sin(d*x + c))*sin(d*x
 + c) - 16*a^2 + 16*b^2 - 3*(2*a*b*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)^2 -
a*b)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [A]
time = 0.53, size = 127, normalized size = 1.06

$$\frac{b^2\sin(dx+c)^3+3\,ab\sin(dx+c)^2-12\,ab\log(|\sin(dx+c)|)+3\,a^2\sin(dx+c)-6\,b^2\sin(dx+c)+\frac{22\,ab\sin(dx+c)^3+6\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2-3\,ab\sin(dx+c)-a^2}{\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*sin(d*x + c)^3 + 3*a*b*sin(d*x + c)^2 - 12*a*b*log(abs(sin(d*x + c
))) + 3*a^2*sin(d*x + c) - 6*b^2*sin(d*x + c) + (22*a*b*sin(d*x + c)^3 + 6*
a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/sin(d
*x + c)^3)/d
```

Mupad [B]
time = 11.71, size = 315, normalized size = 2.62

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(6\,a^2-4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(23\,a^2-36\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(36\,a^2-44\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{158\,a^2}{3}-\frac{164\,b^2}{3}\right)-\frac{a^2}{3}-6\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+26\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+30\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+24\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}-\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7\,a^2}{8}-\frac{b^2}{2}\right)}{d}-\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{4\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{4\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(6*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^8*(23*a^2 - 36*b
^2) + tan(c/2 + (d*x)/2)^4*(36*a^2 - 44*b^2) + tan(c/2 + (d*x)/2)^6*((158*a
^2)/3 - (164*b^2)/3) - a^2/3 - 6*a*b*tan(c/2 + (d*x)/2)^3 + 26*a*b*tan(c/2
+ (d*x)/2)^5 + 30*a*b*tan(c/2 + (d*x)/2)^7 - 2*a*b*tan(c/2 + (d*x)/2))/(d*(
8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^5 + 24*tan(c/2 + (d*x)/2)^7
+ 8*tan(c/2 + (d*x)/2)^9)) - (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) + (tan(c/2 +
 (d*x)/2)*((7*a^2)/8 - b^2/2))/d - (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) - (4*a*
b*log(tan(c/2 + (d*x)/2)))/d + (4*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.1221 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}+$$

[Out] 4*a*b*csc(d*x+c)/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/4*a^2*csc(d*x+c)^4/d+(a^2-2*b^2)*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2800, 962}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]

[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2b^4}{x^5}+\frac{2ab^4}{x^4}+\frac{-2a^2b^2+b^4}{x^3}-\frac{4ab^2}{x^2}+\frac{a^2-2b^2}{x}+x\right)dx, x, b\right.}{d}$$
$$= \frac{4ab\csc(c+dx)}{d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{a^2 c}{}$$

Mathematica [A]
time = 0.49, size = 107, normalized size = 0.85

$$\frac{48ab\csc(c+dx)+6(2a^2-b^2)\csc^2(c+dx)-8ab\csc^3(c+dx)-3a^2\csc^4(c+dx)+6(2(a^2-2b^2)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

Maple [A]
time = 0.29, size = 157, normalized size = 1.25

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right.\right.}{d}$
norman	$\frac{-\frac{a^2}{64d}-\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64d}+\frac{\left(5a^2-4b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}+\frac{\left(5a^2-4b^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{\left(11a^2-72b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32d}-\frac{ab\tan(}{12}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(1+\tan\right.}$
risch	$-ia^2x+2ib^2x-\frac{e^{2i(dx+c)}b^2}{8d}-\frac{iab\,e^{i(dx+c)}}{d}+\frac{iab\,e^{-i(dx+c)}}{d}-\frac{e^{-2i(dx+c)}b^2}{8d}-\frac{2ia^2c}{d}+\frac{4ib^2c}{d}+\frac{2i(6ia^2e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos
(d*x+c)^2)*sin(d*x+c))+b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c)^4
-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

Maxima [A]
time = 0.27, size = 105, normalized size = 0.83

$$\frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\,\left(a^2-2\,b^2\right)\log\left(\sin\left(dx+c\right)\right)+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\,\left(2\,a^2-b^2\right)\sin(dx+c)^2-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

Fricas [A]
time = 0.36, size = 177, normalized size = 1.40

$$-\frac{6\,b^2\cos\left(dx+c\right)^6-15\,b^2\cos\left(dx+c\right)^4+6\,\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2-9\,a^2+3\,b^2-12\,\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^4-2\,\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2+a^2-2\,b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\,\left(3\,ab\cos\left(dx+c\right)^4-12\,ab\cos\left(dx+c\right)^2+8\,ab\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.55, size = 138, normalized size = 1.10

$$\frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\,\left(a^2-2\,b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^2\sin(dx+c)^4-50\,b^2\sin(dx+c)^4-48\,ab\sin(dx+c)^3-12\,a^2\sin(dx+c)^2+6\,b^2\sin(dx+c)^2+8\,ab\sin(dx+c)+3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

Mupad [B]
time = 11.69, size = 310, normalized size = 2.46

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5a^2}{2}-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{23a^2}{4}-4b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(3a^2+30b^2)-\frac{a^2}{4}+\frac{76ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{356ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+92ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\frac{4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^2-2b^2)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^2-2b^2)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a^2}{16}-\frac{b^2}{8}\right)}{d}-\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12d}+\frac{7ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*((5*a^2)/2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*((23*a^2)/
4 - 4*b^2) + tan(c/2 + (d*x)/2)^6*(3*a^2 + 30*b^2) - a^2/4 + (76*a*b*tan(c/
2 + (d*x)/2)^3)/3 + (356*a*b*tan(c/2 + (d*x)/2)^5)/3 + 92*a*b*tan(c/2 + (d*
x)/2)^7 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 32*ta
n(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (log(tan(c/2 + (d*x)/2)^2
+ 1)*(a^2 - 2*b^2))/d - (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (log(tan(c/2 +
(d*x)/2))*(a^2 - 2*b^2))/d + (tan(c/2 + (d*x)/2)^2*((3*a^2)/16 - b^2/8))/d
- (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (7*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

3.1222 $\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=124

$$-\frac{(a^2-2b^2)\csc(c+dx)}{d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{(2a^2-b^2)\csc^3(c+dx)}{3d}-\frac{ab\csc^4(c+dx)}{2d}-\frac{a^2\csc^5(c+dx)}{5d}+\frac{2ab}{}$$

```
[Out] -(a^2-2*b^2)*csc(d*x+c)/d+2*a*b*csc(d*x+c)^2/d+1/3*(2*a^2-b^2)*csc(d*x+c)^3
/d-1/2*a*b*csc(d*x+c)^4/d-1/5*a^2*csc(d*x+c)^5/d+2*a*b*ln(sin(d*x+c))/d+b^2
*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 962}

$$\frac{(2a^2-b^2)\csc^3(c+dx)}{3d}-\frac{(a^2-2b^2)\csc(c+dx)}{d}-\frac{a^2\csc^5(c+dx)}{5d}-\frac{ab\csc^4(c+dx)}{2d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}+\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 2*b^2)*Csc[c + d*x])/d) + (2*a*b*Csc[c + d*x]^2)/d + ((2*a^2 - b^
2)*Csc[c + d*x]^3)/(3*d) - (a*b*Csc[c + d*x]^4)/(2*d) - (a^2*Csc[c + d*x]^5
)/(5*d) + (2*a*b*Log[Sin[c + d*x]])/d + (b^2*Sin[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^6(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^6}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(1+\frac{a^2b^4}{x^6}+\frac{2ab^4}{x^5}+\frac{-2a^2b^2+b^4}{x^4}-\frac{4ab^2}{x^3}+\frac{a^2-2b^2}{x^2}+\right.\right.}{d}$$
$$= -\frac{(a^2-2b^2)\csc(c+dx)}{d}+\frac{2ab\csc^2(c+dx)}{d}+\frac{(2a^2-b^2}{}$$

Mathematica [A]

time = 0.13, size = 105, normalized size = 0.85

$$\frac{-30(a^2-2b^2)\csc(c+dx)+60ab\csc^2(c+dx)+10(2a^2-b^2)\csc^3(c+dx)-15ab\csc^4(c+dx)-6a^2\csc^5(c+dx)+30b(2a\log(\sin(c+dx))+b\sin(c+dx))}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
[Out] (-30*(a^2 - 2*b^2)*Csc[c + d*x] + 60*a*b*Csc[c + d*x]^2 + 10*(2*a^2 - b^2)*
Csc[c + d*x]^3 - 15*a*b*Csc[c + d*x]^4 - 6*a^2*Csc[c + d*x]^5 + 30*b*(2*a*L
og[Sin[c + d*x]] + b*Sin[c + d*x]))/(30*d)
```

Maple [A]

time = 0.30, size = 192, normalized size = 1.55

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+2ab\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{15\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{5\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right)+2ab\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2\right.}{}\right.}{d}$
risch	$-2iabx-\frac{ie^{i(dx+c)}b^2}{2d}+\frac{ie^{-i(dx+c)}b^2}{2d}-\frac{4iabc}{d}-\frac{2i\left(15a^2e^{9i(dx+c)}-30b^2e^{9i(dx+c)}-20a^2e^{7i(dx+c)}+100b^2e^{7i(dx+\right.}{}$
norman	$-\frac{a^2}{160d}-\frac{a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{160d}-\frac{5\left(17a^2-88b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}-\frac{5\left(17a^2-88b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d}+\frac{\left(19a^2-20b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{480d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5
/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)
)+2*a*b*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b^2*(-1/3/sin(d
*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*
x+c)^2)*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.85

$$\frac{60\,ab\log\left(\sin\left(dx+c\right)\right)+30\,b^{2}\sin\left(dx+c\right)+\frac{60\,ab\sin\left(dx+c\right)^{3}-30\,\left(a^{2}-2\,b^{2}\right)\sin\left(dx+c\right)^{4}-15\,ab\sin\left(dx+c\right)+10\,\left(2\,a^{2}-b^{2}\right)\sin\left(dx+c\right)^{2}-6\,a^{2}}{\sin\left(dx+c\right)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*(60*a*b*log(sin(d*x + c)) + 30*b^2*sin(d*x + c) + (60*a*b*sin(d*x + c)
^3 - 30*(a^2 - 2*b^2)*sin(d*x + c)^4 - 15*a*b*sin(d*x + c) + 10*(2*a^2 - b^
2)*sin(d*x + c)^2 - 6*a^2)/sin(d*x + c)^5)/d
```

Fricas [A]
time = 0.37, size = 166, normalized size = 1.34

$$-\frac{30\,b^{2}\cos\left(dx+c\right)^{6}+30\,\left(a^{2}-5\,b^{2}\right)\cos\left(dx+c\right)^{4}-40\,\left(a^{2}-5\,b^{2}\right)\cos\left(dx+c\right)^{2}-60\,\left(ab\cos\left(dx+c\right)^{4}-2\,ab\cos\left(dx+c\right)^{2}+ab\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+16\,a^{2}-80\,b^{2}+15\,\left(4\,ab\cos\left(dx+c\right)^{2}-3\,ab\right)\sin\left(dx+c\right)}{30\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/30*(30*b^2*cos(d*x + c)^6 + 30*(a^2 - 5*b^2)*cos(d*x + c)^4 - 40*(a^2 -
5*b^2)*cos(d*x + c)^2 - 60*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^2 + a*b
)*log(1/2*sin(d*x + c))*sin(d*x + c) + 16*a^2 - 80*b^2 + 15*(4*a*b*cos(d*x
+ c)^2 - 3*a*b)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 131, normalized size = 1.06

$$\frac{60\,ab\log\left(|\sin\left(dx+c\right)|\right)+30\,b^2\sin\left(dx+c\right)-\frac{137\,ab\sin(dx+c)^5+30\,a^2\sin(dx+c)^4-60\,b^2\sin(dx+c)^4-60\,ab\sin(dx+c)^3-20\,a^2\sin(dx+c)^2+10\,b^2\sin(dx+c)^2+15\,ab\sin(dx+c)+6\,a^2}{\sin(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(sin(d*x + c))) + 30*b^2*sin(d*x + c) - (137*a*b*sin(d*
x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 60*a*b*sin(d*x +
 c)^3 - 20*a^2*sin(d*x + c)^2 + 10*b^2*sin(d*x + c)^2 + 15*a*b*sin(d*x + c)
 + 6*a^2)/sin(d*x + c)^5)/d
```

Mupad [B]
time = 11.71, size = 297, normalized size = 2.40

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{5a^2}{96}-\frac{b^2}{24}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5a^2}{16}-\frac{7b^2}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(10\,a^2-92\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{22a^2}{15}-\frac{4b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{25a^2}{3}-\frac{80b^2}{3}\right)+\frac{a^2}{5}-11\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-12\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d}+\frac{3\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32\,d}+\frac{2\,ab\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{2\,ab\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((5*a^2)/96 - b^2/24))/d - (tan(c/2 + (d*x)/2)*((5*a^
2)/16 - (7*b^2)/8))/d - (tan(c/2 + (d*x)/2)^6*(10*a^2 - 92*b^2) - tan(c/2 +
 (d*x)/2)^2*((22*a^2)/15 - (4*b^2)/3) + tan(c/2 + (d*x)/2)^4*((25*a^2)/3 -
(80*b^2)/3) + a^2/5 - 11*a*b*tan(c/2 + (d*x)/2)^3 - 12*a*b*tan(c/2 + (d*x)/
2)^5 + a*b*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (
d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2)^5)/(160*d) + (3*a*b*tan(c/2 + (d*x)/2
)^2)/(8*d) - (a*b*tan(c/2 + (d*x)/2)^4)/(32*d) + (2*a*b*log(tan(c/2 + (d*x)
/2)))/d - (2*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.1223 $\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=130

$$-\frac{2ab\csc(c+dx)}{d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}+\frac{4ab\csc^3(c+dx)}{3d}+\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{2ab\csc^5(c+dx)}{5d}-$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*(a^2-2*b^2)*csc(d*x+c)^2/d+4/3*a*b*csc(d*x+c)^3/d+1
/4*(2*a^2-b^2)*csc(d*x+c)^4/d-2/5*a*b*csc(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d
+b^2*ln(sin(d*x+c))/d
```

Rubi [A]
time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(2a^2-b^2)\csc^4(c+dx)}{4d}-\frac{(a^2-2b^2)\csc^2(c+dx)}{2d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2ab\csc^5(c+dx)}{5d}+\frac{4ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{b^2\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - ((a^2 - 2*b^2)*Csc[c + d*x]^2)/(2*d) + (4*a*b*Csc
[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Csc[c + d*x]^4)/(4*d) - (2*a*b*Csc[c +
d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (b^2*Log[Sin[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^7(a+x)^2(b^2-x^2)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{b^2\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^7}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\text{Subst}\left(\int \left(\frac{a^2b^4}{x^7} + \frac{2ab^4}{x^6} + \frac{-2a^2b^2+b^4}{x^5} - \frac{4ab^2}{x^4} + \frac{a^2-2b^2}{x^3} + \frac{2a}{x^2}\right.\right.}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d} - \frac{(a^2-2b^2)\csc^2(c+dx)}{2d} + \frac{4ab\csc^3(}{3d}$$

Mathematica [A]

time = 0.12, size = 107, normalized size = 0.82

$$\frac{-120ab\csc(c+dx) - 30(a^2-2b^2)\csc^2(c+dx) + 80ab\csc^3(c+dx) + 15(2a^2-b^2)\csc^4(c+dx) - 24ab\csc^5(c+dx) - 10a^2\csc^6(c+dx) + 60b^2\log(\sin(c+dx))}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
[Out] (-120*a*b*Csc[c + d*x] - 30*(a^2 - 2*b^2)*Csc[c + d*x]^2 + 80*a*b*Csc[c + d
*x]^3 + 15*(2*a^2 - b^2)*Csc[c + d*x]^4 - 24*a*b*Csc[c + d*x]^5 - 10*a^2*Cs
c[c + d*x]^6 + 60*b^2*Log[Sin[c + d*x]])/(60*d)
```

Maple [A]

time = 0.32, size = 146, normalized size = 1.12

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6} + 2ab\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5} + \frac{\cos^6(dx+c)}{15\sin(dx+c)^3} - \frac{\cos^6(dx+c)}{5\sin(dx+c)} - \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right) + b^2\left(-\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^6(dx+c)\right)}{6\sin(dx+c)^6} + 2ab\left(-\frac{\cos^6(dx+c)}{5\sin(dx+c)^5} + \frac{\cos^6(dx+c)}{15\sin(dx+c)^3} - \frac{\cos^6(dx+c)}{5\sin(dx+c)} - \frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}\right) + b^2\left(-\right.}{d}$
risch	$-ib^2x - \frac{2ib^2c}{d} - \frac{2i\left(15ia^2e^{10i(dx+c)} - 30ib^2e^{10i(dx+c)} + 30ab\,e^{11i(dx+c)} + 90ib^2e^{8i(dx+c)} - 70ab\,e^{9i(dx+c)} + 50ia^2e^{6i(dx}\right.}{}$
norman	$\frac{-\frac{a^2}{384d} - \frac{a^2\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{384d} - \frac{\left(a^2-15b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d} - \frac{\left(a^2-15b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96d} + \frac{\left(2a^2-3b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d} + \frac{\left(2a^2-3b\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6*a^2/sin(d*x+c)^6*cos(d*x+c)^6+2*a*b*(-1/5/sin(d*x+c)^5*cos(d*x+c)
^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+b^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c
)^2+ln(sin(d*x+c))))
```

Maxima [A]
time = 0.28, size = 108, normalized size = 0.83

$$\frac{60\,b^2\log\left(\sin\left(dx+c\right)\right)-\frac{120\,ab\sin\left(dx+c\right)^5-80\,ab\sin\left(dx+c\right)^3+30\,\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^4+24\,ab\sin\left(dx+c\right)-15\,\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^2+10\,a^2}{\sin\left(dx+c\right)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*b^2*log(sin(d*x + c)) - (120*a*b*sin(d*x + c)^5 - 80*a*b*sin(d*x +
 c)^3 + 30*(a^2 - 2*b^2)*sin(d*x + c)^4 + 24*a*b*sin(d*x + c) - 15*(2*a^2 -
 b^2)*sin(d*x + c)^2 + 10*a^2)/sin(d*x + c)^6)/d
```

Fricas [A]
time = 0.36, size = 183, normalized size = 1.41

$$\frac{30\,\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^4-15\,\left(2\,a^2-7\,b^2\right)\cos\left(dx+c\right)^2+10\,a^2-45\,b^2+60\,\left(b^2\cos\left(dx+c\right)^6-3\,b^2\cos\left(dx+c\right)^4+3\,b^2\cos\left(dx+c\right)^2-b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+8\,\left(15\,ab\cos\left(dx+c\right)^4-20\,ab\cos\left(dx+c\right)^2+8\,ab\right)\sin\left(dx+c\right)}{60\,\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(30*(a^2 - 2*b^2)*cos(d*x + c)^4 - 15*(2*a^2 - 7*b^2)*cos(d*x + c)^2 +
 10*a^2 - 45*b^2 + 60*(b^2*cos(d*x + c)^6 - 3*b^2*cos(d*x + c)^4 + 3*b^2*co
s(d*x + c)^2 - b^2)*log(1/2*sin(d*x + c)) + 8*(15*a*b*cos(d*x + c)^4 - 20*a
*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x +
c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 134, normalized size = 1.03

$$\frac{60\, b^2 \log\left(\left|\sin\left(dx+c\right)\right|\right) - \frac{147\, b^2 \sin(dx+c)^6 + 120\, ab \sin(dx+c)^5 + 30\, a^2 \sin(dx+c)^4 - 60\, b^2 \sin(dx+c)^4 - 80\, ab \sin(dx+c)^3 - 30\, a^2 \sin(dx+c)^2 + 15\, b^2 \sin(dx+c)^2 + 24\, ab \sin(dx+c) + 10\, a^2}{\sin(dx+c)^6}}{60\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*b^2*log(abs(sin(d*x + c))) - (147*b^2*sin(d*x + c)^6 + 120*a*b*sin
(d*x + c)^5 + 30*a^2*sin(d*x + c)^4 - 60*b^2*sin(d*x + c)^4 - 80*a*b*sin(d*
x + c)^3 - 30*a^2*sin(d*x + c)^2 + 15*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x +
 c) + 10*a^2)/sin(d*x + c)^6)/d
```

Mupad [B]
time = 11.87, size = 274, normalized size = 2.11

$$\frac{b^2 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a^2 \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\, d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6 \left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 \left(\frac{5a^2}{2} - 12\, b^2\right) + \frac{a^2}{6} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 \left(a^2-b^2\right) - \frac{20\, a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3} + 40\, a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + \frac{4\, a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}\right)}{64\, d} - \frac{b^2 \ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 \left(\frac{a^2}{64} - \frac{b^2}{64}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 \left(\frac{5a^2}{128} - \frac{3b^2}{16}\right)}{d} + \frac{5\, a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{48\, d} - \frac{a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{80\, d} - \frac{5\, a\, b \tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (b^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*tan(c/2 + (d*x)/2)^6)/(384*d) - (cot
(c/2 + (d*x)/2)^6*(tan(c/2 + (d*x)/2)^4*((5*a^2)/2 - 12*b^2) + a^2/6 - tan(
c/2 + (d*x)/2)^2*(a^2 - b^2) - (20*a*b*tan(c/2 + (d*x)/2)^3)/3 + 40*a*b*tan
(c/2 + (d*x)/2)^5 + (4*a*b*tan(c/2 + (d*x)/2))/5))/(64*d) - (b^2*log(tan(c/
2 + (d*x)/2)^2 + 1))/d + (tan(c/2 + (d*x)/2)^4*(a^2/64 - b^2/64))/d - (tan(
c/2 + (d*x)/2)^2*((5*a^2)/128 - (3*b^2)/16))/d + (5*a*b*tan(c/2 + (d*x)/2)^
3)/(48*d) - (a*b*tan(c/2 + (d*x)/2)^5)/(80*d) - (5*a*b*tan(c/2 + (d*x)/2))/
(8*d)
```

3.1224 $\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=129

$$-\frac{b^2\csc(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}+\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{ab\cs$$

```
[Out] -b^2*csc(d*x+c)/d-a*b*csc(d*x+c)^2/d-1/3*(a^2-2*b^2)*csc(d*x+c)^3/d+a*b*csc
(d*x+c)^4/d+1/5*(2*a^2-b^2)*csc(d*x+c)^5/d-1/3*a*b*csc(d*x+c)^6/d-1/7*a^2*c
sc(d*x+c)^7/d
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(2a^2-b^2)\csc^5(c+dx)}{5d}-\frac{(a^2-2b^2)\csc^3(c+dx)}{3d}-\frac{a^2\csc^7(c+dx)}{7d}-\frac{ab\csc^6(c+dx)}{3d}+\frac{ab\csc^4(c+dx)}{d}-\frac{ab\csc^2(c+dx)}{d}-\frac{b^2\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((b^2*Csc[c + d*x])/d) - (a*b*Csc[c + d*x]^2)/d - ((a^2 - 2*b^2)*Csc[c + d
*x]^3)/(3*d) + (a*b*Csc[c + d*x]^4)/d + ((2*a^2 - b^2)*Csc[c + d*x]^5)/(5*d
) - (a*b*Csc[c + d*x]^6)/(3*d) - (a^2*Csc[c + d*x]^7)/(7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^8(a+x)^2(b^2-x^2)^2}{x^8}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^8}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\text{Subst}\left(\int \left(\frac{a^2b^4}{x^8} + \frac{2ab^4}{x^7} + \frac{-2a^2b^2+b^4}{x^6} - \frac{4ab^2}{x^5} + \frac{a^2-2b^2}{x^4} + \frac{2a}{x^3}\right.\right.}{d}$$

$$= -\frac{b^2\csc(c+dx)}{d} - \frac{ab\csc^2(c+dx)}{d} - \frac{(a^2-2b^2)\csc^3(c+}{3d}$$

Mathematica [A]
time = 0.14, size = 104, normalized size = 0.81

$$-\frac{\csc(c+dx)\left(105b^2+105ab\csc(c+dx)+35(a^2-2b^2)\csc^2(c+dx)-105ab\csc^3(c+dx)+21(-2a^2+b^2)\csc^4(c+dx)+35ab\csc^5(c+dx)+15a^2\csc^6(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/105*(Csc[c + d*x]*(105*b^2 + 105*a*b*Csc[c + d*x] + 35*(a^2 - 2*b^2)*Csc
[c + d*x]^2 - 105*a*b*Csc[c + d*x]^3 + 21*(-2*a^2 + b^2)*Csc[c + d*x]^4 + 3
5*a*b*Csc[c + d*x]^5 + 15*a^2*Csc[c + d*x]^6))/d
```

Maple [A]
time = 0.32, size = 218, normalized size = 1.69

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx\right.}{3\sin(dx+}}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{ab\left(\cos^6(dx\right.}{3\sin(dx+}}{d}$
risch	$-\frac{2i\left(105b^2e^{13i(dx+c)}-140a^2e^{11i(dx+c)}-350b^2e^{11i(dx+c)}-210iab\,e^{2i(dx+c)}-112a^2e^{9i(dx+c)}+791b^2e^{9i(dx+c)}-210iab\,e^{10}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(a^2*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/sin(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/3*a*b/sin(d*x+c)^6*cos(d*x+c)^6+b^2*(-1/5/sin(d*x+c)^5*cos(d*x+c)^6+1/15/sin(d*x+c)^3*cos(d*x+c)^6-1/5/sin(d*x+c)*cos(d*x+c)^6-1/5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))

Maxima [A]
time = 0.29, size = 106, normalized size = 0.82

$$-\frac{105\,b^2\sin\left(dx+c\right)^6+105\,ab\sin\left(dx+c\right)^5-105\,ab\sin\left(dx+c\right)^3+35\left(a^2-2\,b^2\right)\sin\left(dx+c\right)^4+35\,ab\sin\left(dx+c\right)-21\left(2\,a^2-b^2\right)\sin\left(dx+c\right)^2+15\,a^2}{105\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 - 105*a*b*sin(d*x + c)^3 + 35*(a^2 - 2*b^2)*sin(d*x + c)^4 + 35*a*b*sin(d*x + c) - 21*(2*a^2 - b^2)*sin(d*x + c)^2 + 15*a^2)/(d*sin(d*x + c)^7)

Fricas [A]
time = 0.35, size = 146, normalized size = 1.13

$$-\frac{105\,b^2\cos\left(dx+c\right)^6-35\left(a^2+7\,b^2\right)\cos\left(dx+c\right)^4+28\left(a^2+7\,b^2\right)\cos\left(dx+c\right)^2-8\,a^2-56\,b^2-35\left(3\,ab\cos\left(dx+c\right)^4-3\,ab\cos\left(dx+c\right)^2+ab\right)\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/105*(105*b^2*cos(d*x + c)^6 - 35*(a^2 + 7*b^2)*cos(d*x + c)^4 + 28*(a^2 + 7*b^2)*cos(d*x + c)^2 - 8*a^2 - 56*b^2 - 35*(3*a*b*cos(d*x + c)^4 - 3*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)*sin(d*x + c))

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A]
time = 0.53, size = 118, normalized size = 0.91

$$-\frac{105\,b^2\sin\left(dx+c\right)^6+105\,ab\sin\left(dx+c\right)^5+35\,a^2\sin\left(dx+c\right)^4-70\,b^2\sin\left(dx+c\right)^4-105\,ab\sin\left(dx+c\right)^3-42\,a^2\sin\left(dx+c\right)^2+21\,b^2\sin\left(dx+c\right)^2+35\,ab\sin\left(dx+c\right)+15\,a^2}{105\,d\sin\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/105*(105*b^2*sin(d*x + c)^6 + 105*a*b*sin(d*x + c)^5 + 35*a^2*sin(d*x +
c)^4 - 70*b^2*sin(d*x + c)^4 - 105*a*b*sin(d*x + c)^3 - 42*a^2*sin(d*x + c)
^2 + 21*b^2*sin(d*x + c)^2 + 35*a*b*sin(d*x + c) + 15*a^2)/(d*sin(d*x + c)^
7)
```

Mupad [B]
time = 11.79, size = 105, normalized size = 0.81

$$-\frac{\frac{a^2}{7}+\sin\left(c+d\,x\right)^4\,\left(\frac{a^2}{3}-\frac{2\,b^2}{3}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{2\,a^2}{5}-\frac{b^2}{5}\right)+b^2\,\sin\left(c+d\,x\right)^6+\frac{a\,b\,\sin\left(c+d\,x\right)}{3}-a\,b\,\sin\left(c+d\,x\right)^3+a\,b\,\sin\left(c+d\,x\right)^5}{d\,\sin\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] -(a^2/7 + sin(c + d*x)^4*(a^2/3 - (2*b^2)/3) - sin(c + d*x)^2*((2*a^2)/5 -
b^2/5) + b^2*sin(c + d*x)^6 + (a*b*sin(c + d*x))/3 - a*b*sin(c + d*x)^3 + a
*b*sin(c + d*x)^5)/(d*sin(c + d*x)^7)
```

3.1225 $\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=138

$$-\frac{b^2\csc^2(c+dx)}{2d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}+\frac{4ab\csc^5(c+dx)}{5d}+\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-2$$

```
[Out] -1/2*b^2*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1/4*(a^2-2*b^2)*csc(d*x+c)^4
/d+4/5*a*b*csc(d*x+c)^5/d+1/6*(2*a^2-b^2)*csc(d*x+c)^6/d-2/7*a*b*csc(d*x+c)
^7/d-1/8*a^2*csc(d*x+c)^8/d
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{(2a^2-b^2)\csc^6(c+dx)}{6d}-\frac{(a^2-2b^2)\csc^4(c+dx)}{4d}-\frac{a^2\csc^8(c+dx)}{8d}-\frac{2ab\csc^7(c+dx)}{7d}+\frac{4ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{b^2\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*(b^2*Csc[c + d*x]^2)/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - ((a^2 - 2*b^2)
*Csc[c + d*x]^4)/(4*d) + (4*a*b*Csc[c + d*x]^5)/(5*d) + ((2*a^2 - b^2)*Csc[
c + d*x]^6)/(6*d) - (2*a*b*Csc[c + d*x]^7)/(7*d) - (a^2*Csc[c + d*x]^8)/(8*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cot^5(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{b^9(a+x)^2(b^2-x^2)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)^2}{x^9}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{a^2b^4}{x^9} + \frac{2ab^4}{x^8} + \frac{-2a^2b^2+b^4}{x^7} - \frac{4ab^2}{x^6} + \frac{a^2-2b^2}{x^5} + \frac{2a}{x^4}\right.\right.}{d}$$

$$= -\frac{b^2\csc^2(c+dx)}{2d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{(a^2-2b^2)\csc^4(}{4d}$$

Mathematica [A]

time = 0.16, size = 108, normalized size = 0.78

$$-\frac{\csc^2(c+dx)\left(420b^2+560ab\csc(c+dx)+210(a^2-2b^2)\csc^2(c+dx)-672ab\csc^3(c+dx)-140(2a^2-b^2)\csc^4(c+dx)+240ab\csc^5(c+dx)+105a^2\csc^6(c+dx)\right)}{840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/840*(Csc[c + d*x]^2*(420*b^2 + 560*a*b*Csc[c + d*x] + 210*(a^2 - 2*b^2)*
Csc[c + d*x]^2 - 672*a*b*Csc[c + d*x]^3 - 140*(2*a^2 - b^2)*Csc[c + d*x]^4
+ 240*a*b*Csc[c + d*x]^5 + 105*a^2*Csc[c + d*x]^6))/d
```

Maple [A]

time = 0.39, size = 173, normalized size = 1.25

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+2ab\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(}{}\right.}{}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^6(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^6(dx+c)}{24\sin(dx+c)^6}\right)+2ab\left(-\frac{\cos^6(dx+c)}{7\sin(dx+c)^7}-\frac{\cos^6(dx+c)}{35\sin(dx+c)^5}+\frac{\cos^6(dx+c)}{105\sin(dx+c)^3}-\frac{\cos^6(dx+c)}{35\sin(dx+c)}-\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(}{}\right.}{}\right.}{d}$
risch	$\frac{2b^2e^{14i(dx+c)}-4a^2e^{12i(dx+c)}-4b^2e^{12i(dx+c)}+\frac{16iab\,e^{5i(dx+c)}}{15}-\frac{16a^2e^{10i(dx+c)}}{3}+\frac{26b^2e^{10i(dx+c)}}{3}-\frac{16iab\,e^{11i(dx+c)}}{15}-40a^2e^{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^6-1/24/sin(d*x+c)^6*cos(d*x+c)^6)+2*
a*b*(-1/7/sin(d*x+c)^7*cos(d*x+c)^6-1/35/sin(d*x+c)^5*cos(d*x+c)^6+1/105/si
n(d*x+c)^3*cos(d*x+c)^6-1/35/sin(d*x+c)*cos(d*x+c)^6-1/35*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c))-1/6*b^2/sin(d*x+c)^6*cos(d*x+c)^6)
```

Maxima [A]
time = 0.28, size = 106, normalized size = 0.77

$$-\frac{420\,b^2\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^5-672\,ab\sin\left(dx+c\right)^3+210\,(a^2-2\,b^2)\sin\left(dx+c\right)^4+240\,ab\sin\left(dx+c\right)-140\,(2\,a^2-b^2)\sin\left(dx+c\right)^2+105\,a^2}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 - 672*a*b*sin(d*x +
 c)^3 + 210*(a^2 - 2*b^2)*sin(d*x + c)^4 + 240*a*b*sin(d*x + c) - 140*(2*a^
2 - b^2)*sin(d*x + c)^2 + 105*a^2)/(d*sin(d*x + c)^8)
```

Fricas [A]
time = 0.36, size = 148, normalized size = 1.07

$$\frac{420\,b^2\cos\left(dx+c\right)^6-210\,(a^2+4\,b^2)\cos\left(dx+c\right)^4+140\,(a^2+4\,b^2)\cos\left(dx+c\right)^2-35\,a^2-140\,b^2-16\left(35\,ab\cos\left(dx+c\right)^4-28\,ab\cos\left(dx+c\right)^2+8\,ab\right)\sin\left(dx+c\right)}{840\left(d\cos\left(dx+c\right)^8-4\,d\cos\left(dx+c\right)^6+6\,d\cos\left(dx+c\right)^4-4\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/840*(420*b^2*cos(d*x + c)^6 - 210*(a^2 + 4*b^2)*cos(d*x + c)^4 + 140*(a^2
 + 4*b^2)*cos(d*x + c)^2 - 35*a^2 - 140*b^2 - 16*(35*a*b*cos(d*x + c)^4 - 2
8*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^8 - 4*d*cos(d*x
 + c)^6 + 6*d*cos(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 118, normalized size = 0.86

$$-\frac{420\,b^2\sin\left(dx+c\right)^6+560\,ab\sin\left(dx+c\right)^5+210\,a^2\sin\left(dx+c\right)^4-420\,b^2\sin\left(dx+c\right)^4-672\,ab\sin\left(dx+c\right)^3-280\,a^2\sin\left(dx+c\right)^2+140\,b^2\sin\left(dx+c\right)^2+240\,ab\sin\left(dx+c\right)+105\,a^2}{840\,d\sin\left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/840*(420*b^2*sin(d*x + c)^6 + 560*a*b*sin(d*x + c)^5 + 210*a^2*sin(d*x +
 c)^4 - 420*b^2*sin(d*x + c)^4 - 672*a*b*sin(d*x + c)^3 - 280*a^2*sin(d*x +
 c)^2 + 140*b^2*sin(d*x + c)^2 + 240*a*b*sin(d*x + c) + 105*a^2)/(d*sin(d*x
 + c)^8)
```

Mupad [B]
time = 11.78, size = 107, normalized size = 0.78

$$-\frac{\frac{a^2}{8}+\sin\left(c+d\,x\right)^4\,\left(\frac{a^2}{4}-\frac{b^2}{2}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{a^2}{3}-\frac{b^2}{6}\right)+\frac{b^2\,\sin\left(c+d\,x\right)^6}{2}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{7}-\frac{4\,a\,b\,\sin\left(c+d\,x\right)^3}{5}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)^5}{3}}{d\,\sin\left(c+d\,x\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(a + b*sin(c + d*x))^2)/sin(c + d*x)^9,x)
[Out] -(a^2/8 + sin(c + d*x)^4*(a^2/4 - b^2/2) - sin(c + d*x)^2*(a^2/3 - b^2/6) +
 (b^2*sin(c + d*x)^6)/2 + (2*a*b*sin(c + d*x))/7 - (4*a*b*sin(c + d*x)^3)/5
 + (2*a*b*sin(c + d*x)^5)/3)/(d*sin(c + d*x)^8)
```

3.1226 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=235

$$\frac{a^2(7a^4-10a^2b^2+3b^4)\log(a+b\sin(c+dx))}{b^8d}-\frac{2a(3a^4-4a^2b^2+b^4)\sin(c+dx)}{b^7d}+\frac{(5a^4-6a^2b^2+b^4)\sin^2(c$$

```
[Out] a^2*(7*a^4-10*a^2*b^2+3*b^4)*ln(a+b*sin(d*x+c))/b^8/d-2*a*(3*a^4-4*a^2*b^2+
b^4)*sin(d*x+c)/b^7/d+1/2*(5*a^4-6*a^2*b^2+b^4)*sin(d*x+c)^2/b^6/d-4/3*a*(a
^2-b^2)*sin(d*x+c)^3/b^5/d+1/4*(3*a^2-2*b^2)*sin(d*x+c)^4/b^4/d-2/5*a*sin(d
*x+c)^5/b^3/d+1/6*sin(d*x+c)^6/b^2/d+a^3*(a^2-b^2)^2/b^8/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.19, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$-\frac{4a(a^2-b^2)\sin^3(c+dx)}{3b^5d}+\frac{(3a^2-2b^2)\sin^4(c+dx)}{4b^4d}+\frac{a^2(7a^4-10a^2b^2+3b^4)\log(a+b\sin(c+dx))}{b^8d}-\frac{2a(3a^4-4a^2b^2+b^4)\sin(c+dx)}{b^7d}+\frac{(5a^4-6a^2b^2+b^4)\sin^2(c+dx)}{2b^6d}+\frac{a^3(a^2-b^2)^2}{b^8d(a+b\sin(c+dx))}-\frac{2a\sin^5(c+dx)}{5b^3d}+\frac{\sin^6(c+dx)}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(b^8*d) - (2*a*(
3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x])/(b^7*d) + ((5*a^4 - 6*a^2*b^2 + b^4)
*Sin[c + d*x]^2)/(2*b^6*d) - (4*a*(a^2 - b^2)*Sin[c + d*x]^3)/(3*b^5*d) + (
(3*a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^4*d) - (2*a*Sin[c + d*x]^5)/(5*b^3*d)
+ Sin[c + d*x]^6/(6*b^2*d) + (a^3*(a^2 - b^2)^2)/(b^8*d*(a + b*Sin[c + d*x]
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{b^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^8 d}\\ &= \frac{\text{Subst}\left(\int \left(-2a(3a^4-4a^2b^2+b^4)+(5a^4-6a^2b^2+b^4)\,x-4a(a^2-b^2)\,x^2\right.\right.}{}\\ &= \frac{a^2(7a^4-10a^2b^2+3b^4)\log(a+b\sin(c+dx))}{b^8 d}-\frac{2a(3a^4-4a^2b^2+b^4)\sin(c}{b^7 d}\end{aligned}$$

Mathematica [A]

time = 1.51, size = 264, normalized size = 1.12

$$\frac{60a^3(a^2-b^2)\left(a^2-b^2+(7a^2-3b^2)\log(a+b\sin(c+dx))\right)+60a^2b(a^2-b^2)\left(-6a^2+2b^2+(7a^2-3b^2)\log(a+b\sin(c+dx))\right)\sin(c+dx)-30ab^2(7a^4-10a^2b^2+3b^4)\sin^2(c+dx)+10b^3(7a^4-10a^2b^2+3b^4)\sin^3(c+dx)+(-35a^3b^4+50ab^6)\sin^4(c+dx)+3b^5(7a^2-10b^2)\sin^5(c+dx)-14ab^6\sin^6(c+dx)+10b^7\sin^7(c+dx)}{60b^8d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (60*a^3*(a^2 - b^2)*(a^2 - b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]]) +
 60*a^2*b*(a^2 - b^2)*(-6*a^2 + 2*b^2 + (7*a^2 - 3*b^2)*Log[a + b*Sin[c + d
*x]])*Sin[c + d*x] - 30*a*b^2*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^2 +
 10*b^3*(7*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[c + d*x]^3 + (-35*a^3*b^4 + 50*a*b
^6)*Sin[c + d*x]^4 + 3*b^5*(7*a^2 - 10*b^2)*Sin[c + d*x]^5 - 14*a*b^6*Sin[c
 + d*x]^6 + 10*b^7*Sin[c + d*x]^7)/(60*b^8*d*(a + b*Sin[c + d*x]))
```

Maple [A]

time = 0.38, size = 249, normalized size = 1.06

method	result
derivativedivides	$-\frac{-\frac{\left(\sin^6(dx+c)\right)b^5}{6}+\frac{2a\,b^4\left(\sin^5(dx+c)\right)}{5}-\frac{3a^2b^3\left(\sin^4(dx+c)\right)}{4}+\frac{b^5\left(\sin^4(dx+c)\right)}{2}+\frac{4a^3b^2\left(\sin^3(dx+c)\right)}{3}-\frac{4a\,b^4\left(\sin^3(dx+c)\right)}{3}-\frac{5a^4b(s}{}}{b^7}$
default	$-\frac{-\frac{\left(\sin^6(dx+c)\right)b^5}{6}+\frac{2a\,b^4\left(\sin^5(dx+c)\right)}{5}-\frac{3a^2b^3\left(\sin^4(dx+c)\right)}{4}+\frac{b^5\left(\sin^4(dx+c)\right)}{2}+\frac{4a^3b^2\left(\sin^3(dx+c)\right)}{3}-\frac{4a\,b^4\left(\sin^3(dx+c)\right)}{3}-\frac{5a^4b(s}{}}{b^7}$

risch	$-\frac{a\sin(5dx+5c)}{40b^3d}+\frac{3ia^5\mathrm{e}^{i(dx+c)}}{b^7d}-\frac{7ia^3\mathrm{e}^{i(dx+c)}}{2b^5d}+\frac{5ia\,\mathrm{e}^{i(dx+c)}}{8b^3d}-\frac{3ia^5\mathrm{e}^{-i(dx+c)}}{b^7d}+\frac{7ia^3\mathrm{e}^{-i(dx+c)}}{2b^5d}-\frac{5ia\,\mathrm{e}^{-i(c}}{8b^3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^7*(-1/6*sin(d*x+c)^6*b^5+2/5*a*b^4*sin(d*x+c)^5-3/4*a^2*b^3*sin(d
*x+c)^4+1/2*b^5*sin(d*x+c)^4+4/3*a^3*b^2*sin(d*x+c)^3-4/3*a*b^4*sin(d*x+c)^
3-5/2*a^4*b*sin(d*x+c)^2+3*a^2*b^3*sin(d*x+c)^2-1/2*b^5*sin(d*x+c)^2+6*a^5*
sin(d*x+c)-8*a^3*b^2*sin(d*x+c)+2*a*b^4*sin(d*x+c))+a^2/b^8*(7*a^4-10*a^2*b
^2+3*b^4)*ln(a+b*sin(d*x+c))+a^3*(a^4-2*a^2*b^2+b^4)/b^8/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 218, normalized size = 0.93

$$\frac{\frac{60\,\left(a^7-2\,a^5b^2+a^3b^4\right)}{b^9\sin(dx+c)+ab^8}+\frac{10\,b^5\sin(dx+c)^6-24\,ab^4\sin(dx+c)^5+15\,\left(3\,a^2b^3-2\,b^5\right)\sin(dx+c)^4-80\,\left(a^3b^2-ab^4\right)\sin(dx+c)^3+30\,\left(5\,a^4b-6\,a^2b^3+b^5\right)\sin(dx+c)^2-120\,\left(3\,a^5-4\,a^3b^2+ab^4\right)\sin(dx+c)}{b^7}+\frac{60\,\left(7\,a^6-10\,a^4b^2+3\,a^2b^4\right)\log(b\sin(dx+c)+a)}{b^8}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/60*(60*(a^7 - 2*a^5*b^2 + a^3*b^4)/(b^9*sin(d*x + c) + a*b^8) + (10*b^5*s
in(d*x + c)^6 - 24*a*b^4*sin(d*x + c)^5 + 15*(3*a^2*b^3 - 2*b^5)*sin(d*x +
c)^4 - 80*(a^3*b^2 - a*b^4)*sin(d*x + c)^3 + 30*(5*a^4*b - 6*a^2*b^3 + b^5)
*sin(d*x + c)^2 - 120*(3*a^5 - 4*a^3*b^2 + a*b^4)*sin(d*x + c))/b^7 + 60*(7
*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A]
time = 0.41, size = 279, normalized size = 1.19

$$\frac{112\,ab^6\cos\left(dx+c\right)^6+480\,a^7-3240\,a^5b^2+3185\,a^3b^4-487\,ab^6-8\,\left(35\,a^3b^4-8\,ab^6\right)\cos\left(dx+c\right)^4+16\,\left(105\,a^5b^2-115\,a^3b^4+16\,ab^6\right)\cos\left(dx+c\right)^2+480\,\left(7\,a^7-10\,a^5b^2+3\,a^3b^4+\left(7\,a^6b-10\,a^4b^3+3\,a^2b^5\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)-\left(80\,b^7\cos\left(dx+c\right)^6-168\,a^2b^5\cos\left(dx+c\right)^4+2880\,a^6b-3800\,a^4b^3+1007\,a^2b^5-25\,b^7+16\,\left(35\,a^4b^3-29\,a^2b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{480\,\left(b^9d\sin\left(dx+c\right)+ab^8d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/480*(112*a*b^6*cos(d*x + c)^6 + 480*a^7 - 3240*a^5*b^2 + 3185*a^3*b^4 - 4
87*a*b^6 - 8*(35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 + 16*(105*a^5*b^2 - 115*
a^3*b^4 + 16*a*b^6)*cos(d*x + c)^2 + 480*(7*a^7 - 10*a^5*b^2 + 3*a^3*b^4 +
(7*a^6*b - 10*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) -
(80*b^7*cos(d*x + c)^6 - 168*a^2*b^5*cos(d*x + c)^4 + 2880*a^6*b - 3800*a^4
*b^3 + 1007*a^2*b^5 - 25*b^7 + 16*(35*a^4*b^3 - 29*a^2*b^5)*cos(d*x + c)^2)
*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 300, normalized size = 1.28

$$\frac{\frac{60\,(7\,a^6-10\,a^4b^2+3\,a^2b^4)\log(|b\sin(dx+c)+a|)}{b^8}-\frac{60\,(7\,a^6b\sin(dx+c)-10\,a^4b^3\sin(dx+c)+3\,a^2b^5\sin(dx+c)+6\,a^7-8\,a^5b^2+2\,a^3b^4)}{(b\sin(dx+c)+a)b^8}+\frac{10\,b^{10}\sin(dx+c)^6-24\,ab^9\sin(dx+c)^5+45\,a^2b^8\sin(dx+c)^4-30\,b^{10}\sin(dx+c)^4-80\,a^3b^7\sin(dx+c)^3+80\,ab^9\sin(dx+c)^3+150\,a^4b^6\sin(dx+c)^2-180\,a^2b^8\sin(dx+c)^2+30\,b^{10}\sin(dx+c)^2-360\,a^5b^5\sin(dx+c)+480\,a^3b^7\sin(dx+c)-120\,ab^9\sin(dx+c)}{b^{12}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(60*(7*a^6 - 10*a^4*b^2 + 3*a^2*b^4)*log(abs(b*sin(d*x + c) + a))/b^8
- 60*(7*a^6*b*sin(d*x + c) - 10*a^4*b^3*sin(d*x + c) + 3*a^2*b^5*sin(d*x +
c) + 6*a^7 - 8*a^5*b^2 + 2*a^3*b^4)/((b*sin(d*x + c) + a)*b^8) + (10*b^10*s
in(d*x + c)^6 - 24*a*b^9*sin(d*x + c)^5 + 45*a^2*b^8*sin(d*x + c)^4 - 30*b^
10*sin(d*x + c)^4 - 80*a^3*b^7*sin(d*x + c)^3 + 80*a*b^9*sin(d*x + c)^3 + 1
50*a^4*b^6*sin(d*x + c)^2 - 180*a^2*b^8*sin(d*x + c)^2 + 30*b^10*sin(d*x +
c)^2 - 360*a^5*b^5*sin(d*x + c) + 480*a^3*b^7*sin(d*x + c) - 120*a*b^9*sin(
d*x + c))/b^12)/d
```

Mupad [B]
time = 0.14, size = 375, normalized size = 1.60

$$\frac{\sin(c+d\,x)^3\left(\frac{2\,a^3}{3\,b^5}+\frac{2\,a\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{3\,b}\right)}{d}-\frac{\sin(c+d\,x)^4\left(\frac{1}{2\,b^2}-\frac{3\,a^2}{4\,b^4}\right)}{d}+\frac{\sin(c+d\,x)^2\left(\frac{1}{2\,b^2}+\frac{a^2\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{2\,b^2}-\frac{a\left(\frac{2\,a^3}{b^5}+\frac{2\,a\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d}-\frac{\sin(c+d\,x)\left(\frac{a^2\left(\frac{2\,a^3}{b^5}+\frac{2\,a\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b^2}+\frac{2\,a\left(\frac{1}{b^2}+\frac{a^2\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{2\,a\left(\frac{2\,a^3}{b^5}+\frac{2\,a\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{b}\right)}{d}+\frac{\sin(c+d\,x)^6}{6\,b^2\,d}-\frac{2\,a\sin(c+d\,x)^5}{5\,b^3\,d}+\frac{\ln(a+b\sin(c+d\,x))\,(7\,a^6-10\,a^4\,b^2+3\,a^2\,b^4)}{b^8\,d}+\frac{a^7-2\,a^5\,b^2+a^3\,b^4}{b\,d\,(\sin(c+d\,x)\,b^8+a\,b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*((2*a^3)/(3*b^5) + (2*a*(2/b^2 - (3*a^2)/b^4))/(3*b)))/d -
(sin(c + d*x)^4*(1/(2*b^2) - (3*a^2)/(4*b^4)))/d + (sin(c + d*x)^2*(1/(2*b^
2) + (a^2*(2/b^2 - (3*a^2)/b^4))/(2*b^2) - (a*((2*a^3)/b^5 + (2*a*(2/b^2 -
(3*a^2)/b^4))/b))/b))/d - (sin(c + d*x)*((a^2*((2*a^3)/b^5 + (2*a*(2/b^2 -
(3*a^2)/b^4))/b))/b^2 + (2*a*(1/b^2 + (a^2*(2/b^2 - (3*a^2)/b^4))/b^2 - (2*
a*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b))/b))/b))/d + sin(c + d*x)^6
/(6*b^2*d) - (2*a*sin(c + d*x)^5)/(5*b^3*d) + (log(a + b*sin(c + d*x))*(7*a
^6 + 3*a^2*b^4 - 10*a^4*b^2))/(b^8*d) + (a^7 + a^3*b^4 - 2*a^5*b^2)/(b*d*(a
*b^7 + b^8*sin(c + d*x)))
```

3.1227 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=193

$$-\frac{2a(3a^4-4a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^7d}+\frac{(5a^4-6a^2b^2+b^4)\sin(c+dx)}{b^6d}-\frac{2a(a^2-b^2)\sin^2(c+dx)}{b^5d}-\Big($$

```
[Out] -2*a*(3*a^4-4*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^7/d+(5*a^4-6*a^2*b^2+b^4)*s
in(d*x+c)/b^6/d-2*a*(a^2-b^2)*sin(d*x+c)^2/b^5/d-1/3*(2-3*a^2/b^2)*sin(d*x+
c)^3/b^2/d-1/2*a*sin(d*x+c)^4/b^3/d+1/5*sin(d*x+c)^5/b^2/d-a^2*(a^2-b^2)^2/
b^7/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$-\frac{\left(2-\frac{3a^2}{b^2}\right)\sin^3(c+dx)}{3b^2d}-\frac{a^2(a^2-b^2)^2}{b^7d(a+b\sin(c+dx))}-\frac{2a(a^2-b^2)\sin^2(c+dx)}{b^5d}-\frac{2a(3a^4-4a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^7d}+\frac{(5a^4-6a^2b^2+b^4)\sin(c+dx)}{b^6d}-\frac{a\sin^4(c+dx)}{2b^3d}+\frac{\sin^5(c+dx)}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*(3*a^4 - 4*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^7*d) + ((5*a^4
- 6*a^2*b^2 + b^4)*Sin[c + d*x])/(b^6*d) - (2*a*(a^2 - b^2)*Sin[c + d*x]^2)
/(b^5*d) - ((2 - (3*a^2)/b^2)*Sin[c + d*x]^3)/(3*b^2*d) - (a*Sin[c + d*x]^4
)/(2*b^3*d) + Sin[c + d*x]^5/(5*b^2*d) - (a^2*(a^2 - b^2)^2)/(b^7*d*(a + b*
Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{b^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7d} \\
&= \frac{\text{Subst}\left(\int \left(5a^4\left(1+\frac{-6a^2b^2+b^4}{5a^4}\right)-4a\left(a^2-b^2\right)x+\left(3a^2-2b^2\right)x^2-2ax^3+x\right.\right.}{b^7d} \\
&= -\frac{2a\left(3a^4-4a^2b^2+b^4\right)\log(a+b\sin(c+dx))}{b^7d}+\frac{\left(5a^4-6a^2b^2+b^4\right)\sin(c+}{b^6d}
\end{aligned}
$$

Mathematica [A]
time = 1.08, size = 225, normalized size = 1.17

$$
\frac{-30a^2(a^2-b^2)\left(a^2-b^2+(6a^2-2b^2)\log(a+b\sin(c+dx))\right)-30ab(a^2-b^2)\left(-5a^2+b^2+(6a^2-2b^2)\log(a+b\sin(c+dx))\right)\sin(c+dx)+30b^2(3a^4-4a^2b^2+b^4)\sin^2(c+dx)+(-30a^3b^3+40ab^5)\sin^3(c+dx)+5b^4(3a^2-4b^2)\sin^4(c+dx)-9ab^5\sin^5(c+dx)+6b^6\sin^6(c+dx)}{30b^7d(a+b\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^2 - b^2)*(a^2 - b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]])
- 30*a*b*(a^2 - b^2)*(-5*a^2 + b^2 + (6*a^2 - 2*b^2)*Log[a + b*Sin[c + d*x]
])*Sin[c + d*x] + 30*b^2*(3*a^4 - 4*a^2*b^2 + b^4)*Sin[c + d*x]^2 + (-30*a^
3*b^3 + 40*a*b^5)*Sin[c + d*x]^3 + 5*b^4*(3*a^2 - 4*b^2)*Sin[c + d*x]^4 - 9
*a*b^5*Sin[c + d*x]^5 + 6*b^6*Sin[c + d*x]^6)/(30*b^7*d*(a + b*Sin[c + d*x]
))
```

Maple [A]
time = 0.53, size = 198, normalized size = 1.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^6*(1/5*sin(d*x+c)^5*b^4-1/2*a*b^3*sin(d*x+c)^4+a^2*b^2*sin(d*x+c)^
3-2/3*b^4*sin(d*x+c)^3-2*a^3*b*sin(d*x+c)^2+2*a*b^3*sin(d*x+c)^2+5*a^4*sin(
d*x+c)-6*a^2*b^2*sin(d*x+c)+b^4*sin(d*x+c))-2*a/b^7*(3*a^4-4*a^2*b^2+b^4)*l
n(a+b*sin(d*x+c))-a^2*(a^4-2*a^2*b^2+b^4)/b^7/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 184, normalized size = 0.95

$$-\frac{\frac{30\,(a^6-2\,a^4b^2+a^2b^4)}{b^8\sin(dx+c)+ab^7}-\frac{6\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+10\,(3\,a^2b^2-2\,b^4)\sin(dx+c)^3-60\,(a^3b-ab^3)\sin(dx+c)^2+30\,(5\,a^4-6\,a^2b^2+b^4)\sin(dx+c)}{b^6}+\frac{60\,(3\,a^5-4\,a^3b^2+ab^4)\log(b\sin(dx+c)+a)}{b^7}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(30*(a^6 - 2*a^4*b^2 + a^2*b^4)/(b^8*sin(d*x + c) + a*b^7) - (6*b^4*s
in(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 10*(3*a^2*b^2 - 2*b^4)*sin(d*x +
c)^3 - 60*(a^3*b - a*b^3)*sin(d*x + c)^2 + 30*(5*a^4 - 6*a^2*b^2 + b^4)*sin
(d*x + c))/b^6 + 60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a)/b^7
)/d
```

Fricas [A]
time = 0.39, size = 246, normalized size = 1.27

$$-\frac{48\,b^6\cos(dx+c)^6+240\,a^6-1440\,a^4b^2+1275\,a^2b^4-128\,b^6-8\,(15\,a^2b^4-2\,b^6)\cos(dx+c)^4+16\,(45\,a^4b^2-45\,a^2b^4+4\,b^6)\cos(dx+c)^2+480\,(3\,a^6-4\,a^4b^2+a^2b^4+(3\,a^5b-4\,a^3b^3+ab^5)\sin(dx+c))\log(b\sin(dx+c)+a)+\left(72\,ab^5\cos(dx+c)^4-1200\,a^5b+1440\,a^3b^3-293\,ab^5-16\,(15\,a^3b^3-11\,ab^5)\cos(dx+c)^2\right)\sin(dx+c)}{240\,(b^8d\sin(dx+c)+ab^7d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(48*b^6*cos(d*x + c)^6 + 240*a^6 - 1440*a^4*b^2 + 1275*a^2*b^4 - 128
*b^6 - 8*(15*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 + 16*(45*a^4*b^2 - 45*a^2*b^4
+ 4*b^6)*cos(d*x + c)^2 + 480*(3*a^6 - 4*a^4*b^2 + a^2*b^4 + (3*a^5*b - 4*a
^3*b^3 + a*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (72*a*b^5*cos(d*x +
 c)^4 - 1200*a^5*b + 1440*a^3*b^3 - 293*a*b^5 - 16*(15*a^3*b^3 - 11*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 249, normalized size = 1.29

$$-\frac{\frac{60\,(3\,a^5-4\,a^3b^2+ab^4)\log(|b\sin(dx+c)+a|)}{b^7}-\frac{30\,(6\,a^5b\sin(dx+c)-8\,a^3b^3\sin(dx+c)+2\,ab^5\sin(dx+c)+5\,a^6-6\,a^4b^2+a^2b^4)}{(b\sin(dx+c)+a)b^7}-\frac{6\,b^8\sin(dx+c)^5-15\,ab^7\sin(dx+c)^4+30\,a^2b^6\sin(dx+c)^3-20\,b^8\sin(dx+c)^3-60\,a^3b^5\sin(dx+c)^2+60\,ab^7\sin(dx+c)^2+150\,a^4b^4\sin(dx+c)-180\,a^2b^6\sin(dx+c)+30\,b^8\sin(dx+c)}{b^{10}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(3*a^5 - 4*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^7 - 30
*(6*a^5*b*sin(d*x + c) - 8*a^3*b^3*sin(d*x + c) + 2*a*b^5*sin(d*x + c) + 5*
a^6 - 6*a^4*b^2 + a^2*b^4)/((b*sin(d*x + c) + a)*b^7) - (6*b^8*sin(d*x + c)
^5 - 15*a*b^7*sin(d*x + c)^4 + 30*a^2*b^6*sin(d*x + c)^3 - 20*b^8*sin(d*x +
 c)^3 - 60*a^3*b^5*sin(d*x + c)^2 + 60*a*b^7*sin(d*x + c)^2 + 150*a^4*b^4*s
in(d*x + c) - 180*a^2*b^6*sin(d*x + c) + 30*b^8*sin(d*x + c))/b^10)/d
```

Mupad [B]
time = 11.46, size = 254, normalized size = 1.32

$$\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{a^3}{b^5}+\frac{a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{2}{3\,b^2}-\frac{a^2}{b^4}\right)}{d}+\frac{\sin\left(c+d\,x\right)\,\left(\frac{1}{b^2}+\frac{a^2\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{2\,a^3}{b^5}+\frac{2\,a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)}{b}\right)}{d}+\frac{\sin\left(c+d\,x\right)^5}{5\,b^2\,d}-\frac{a\sin\left(c+d\,x\right)^4}{2\,b^3\,d}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(6\,a^5-8\,a^3\,b^2+2\,a\,b^4\right)}{b^7\,d}-\frac{a^6-2\,a^4\,b^2+a^2\,b^4}{b\,d\,\left(\sin\left(c+d\,x\right)\,b^7+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*(a^3/b^5 + (a*(2/b^2 - (3*a^2)/b^4))/b))/d - (sin(c + d*x)^
3*(2/(3*b^2) - a^2/b^4))/d + (sin(c + d*x)*(1/b^2 + (a^2*(2/b^2 - (3*a^2)/b
^4))/b^2 - (2*a*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b))/b))/d + sin(
c + d*x)^5/(5*b^2*d) - (a*sin(c + d*x)^4)/(2*b^3*d) - (log(a + b*sin(c + d*
x))*(2*a*b^4 + 6*a^5 - 8*a^3*b^2))/(b^7*d) - (a^6 + a^2*b^4 - 2*a^4*b^2)/(b
*d*(a*b^6 + b^7*sin(c + d*x)))
```

3.1228 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=157

$$\frac{(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^6d}-\frac{4a(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{(3a^2-2b^2)\sin^2(c+dx)}{2b^4d}-\frac{2a\sin^3(c+d}{3b^3d}$$

```
[Out] (5*a^4-6*a^2*b^2+b^4)*ln(a+b*sin(d*x+c))/b^6/d-4*a*(a^2-b^2)*sin(d*x+c)/b^5
/d+1/2*(3*a^2-2*b^2)*sin(d*x+c)^2/b^4/d-2/3*a*sin(d*x+c)^3/b^3/d+1/4*sin(d*
x+c)^4/b^2/d+a*(a^2-b^2)^2/b^6/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 786}

$$\frac{a(a^2-b^2)^2}{b^6d(a+b\sin(c+dx))}-\frac{4a(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{(3a^2-2b^2)\sin^2(c+dx)}{2b^4d}+\frac{(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^6d}-\frac{2a\sin^3(c+dx)}{3b^3d}+\frac{\sin^4(c+dx)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 6*a^2*b^2 + b^4)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (4*a*(a^2 - b
^2)*Sin[c + d*x])/(b^5*d) + ((3*a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) - (2
*a*Sin[c + d*x]^3)/(3*b^3*d) + Sin[c + d*x]^4/(4*b^2*d) + (a*(a^2 - b^2)^2)
/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{b(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-4(a^3-ab^2)+(3a^2-2b^2)\,x-2ax^2+x^3-\frac{a(a^2-b^2)^2}{(a+x)^2}+\frac{5a^4-6a^2b^2}{a+x}\right.\right.}{b^6d}$$
$$= \frac{(5a^4-6a^2b^2+b^4)\log(a+b\sin(c+dx))}{b^6d} - \frac{4a(a^2-b^2)\sin(c+dx)}{b^5d} + \frac{(3a^2}{}$$

Mathematica [A]
time = 0.72, size = 188, normalized size = 1.20

$$\frac{12a(a^2-b^2)(a^2-b^2+(5a^2-b^2)\log(a+b\sin(c+dx)))+12b(-a^2+b^2)(4a^2+(-5a^2+b^2)\log(a+b\sin(c+dx)))\sin(c+dx)-6ab^2(5a^2-6b^2)\sin^2(c+dx)+2b^3(5a^2-6b^2)\sin^3(c+dx)-5ab^4\sin^4(c+dx)+3b^5\sin^5(c+dx)}{12b^6d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (12*a*(a^2 - b^2)*(a^2 - b^2 + (5*a^2 - b^2)*Log[a + b*Sin[c + d*x]]) + 12*
b*(-a^2 + b^2)*(4*a^2 + (-5*a^2 + b^2)*Log[a + b*Sin[c + d*x]])*Sin[c + d*x
] - 6*a*b^2*(5*a^2 - 6*b^2)*Sin[c + d*x]^2 + 2*b^3*(5*a^2 - 6*b^2)*Sin[c +
d*x]^3 - 5*a*b^4*Sin[c + d*x]^4 + 3*b^5*Sin[c + d*x]^5)/(12*b^6*d*(a + b*Si
n[c + d*x]))
```

Maple [A]
time = 0.48, size = 152, normalized size = 0.97

method	result
derivativedivides	$-\frac{-\frac{\frac{b^3(\sin^4(dx+c))}{4}+\frac{2a(\sin^3(dx+c))b^2}{3}-\frac{3a^2b(\sin^2(dx+c))}{2}+b^3(\sin^2(dx+c))+4a^3\sin(dx+c)-4a\,b^2\sin(dx+c)}{b^5}+\frac{(5a^4-6a^2b^2+b^4)}{b^6}}{d}$
default	$-\frac{-\frac{\frac{b^3(\sin^4(dx+c))}{4}+\frac{2a(\sin^3(dx+c))b^2}{3}-\frac{3a^2b(\sin^2(dx+c))}{2}+b^3(\sin^2(dx+c))+4a^3\sin(dx+c)-4a\,b^2\sin(dx+c)}{b^5}+\frac{(5a^4-6a^2b^2+b^4)}{b^6}}{d}$
risch	$\frac{12ia^2c}{b^4d}-\frac{5ia^4x}{b^6}+\frac{7ia\,\mathrm{e}^{-i(dx+c)}}{4b^3d}-\frac{3\,\mathrm{e}^{2i(dx+c)}a^2}{8b^4d}+\frac{3\,\mathrm{e}^{2i(dx+c)}}{16b^2d}-\frac{2ic}{b^2d}+\frac{6ia^2x}{b^4}-\frac{7ia\,\mathrm{e}^{i(dx+c)}}{4b^3d}-\frac{10ia^4c}{b^6d}-3$
norman	$-\frac{\frac{(150a^3-160a\,b^2)(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{3b^4d}-\frac{(150a^3-160a\,b^2)(\tan^{10}(\frac{dx}{2}+\frac{c}{2}))}{3b^4d}-\frac{(10a^3-12a\,b^2)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{b^4d}-\frac{(10a^3-12a\,b^2)(\tan^{12}}{b^4d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^5*(-1/4*b^3*sin(d*x+c)^4+2/3*a*sin(d*x+c)^3*b^2-3/2*a^2*b*sin(d*x
+c)^2+b^3*sin(d*x+c)^2+4*a^3*sin(d*x+c)-4*a*b^2*sin(d*x+c))+1/b^6*(5*a^4-6*
a^2*b^2+b^4)*ln(a+b*sin(d*x+c))+a*(a^4-2*a^2*b^2+b^4)/b^6/(a+b*sin(d*x+c)))
```

Maxima [**A**]
time = 0.28, size = 148, normalized size = 0.94

$$\frac{\frac{12\,(a^5-2\,a^3b^2+ab^4)}{b^7\sin(dx+c)+ab^6}+\frac{3\,b^3\sin(dx+c)^4-8\,ab^2\sin(dx+c)^3+6\,(3\,a^2b-2\,b^3)\sin(dx+c)^2-48\,(a^3-ab^2)\sin(dx+c)}{b^5}+\frac{12\,(5\,a^4-6\,a^2b^2+b^4)\log(b\sin(dx+c)+a)}{b^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(a^5 - 2*a^3*b^2 + a*b^4)/(b^7*sin(d*x + c) + a*b^6) + (3*b^3*sin(
d*x + c)^4 - 8*a*b^2*sin(d*x + c)^3 + 6*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 -
48*(a^3 - a*b^2)*sin(d*x + c))/b^5 + 12*(5*a^4 - 6*a^2*b^2 + b^4)*log(b*sin
(d*x + c) + a)/b^6)/d
```

Fricas [**A**]
time = 0.39, size = 203, normalized size = 1.29

$$-\frac{40\,ab^4\cos(dx+c)^4-96\,a^5+504\,a^3b^2-383\,ab^4-16\,(15\,a^3b^2-13\,ab^4)\cos(dx+c)^2-96\,(5\,a^5-6\,a^3b^2+ab^4+(5\,a^4b-6\,a^2b^3+b^5)\sin(dx+c))\log(b\sin(dx+c)+a)-\left(24\,b^5\cos(dx+c)^4-384\,a^4b+392\,a^2b^3-33\,b^5-16\,(5\,a^2b^3-3\,b^5)\cos(dx+c)^2\right)\sin(dx+c)}{96\,(b^7d\sin(dx+c)+ab^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(40*a*b^4*cos(d*x + c)^4 - 96*a^5 + 504*a^3*b^2 - 383*a*b^4 - 16*(15*
a^3*b^2 - 13*a*b^4)*cos(d*x + c)^2 - 96*(5*a^5 - 6*a^3*b^2 + a*b^4 + (5*a^4
*b - 6*a^2*b^3 + b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (24*b^5*cos(d
*x + c)^4 - 384*a^4*b + 392*a^2*b^3 - 33*b^5 - 16*(5*a^2*b^3 - 3*b^5)*cos(d
*x + c)^2)*sin(d*x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.50, size = 194, normalized size = 1.24

$$\frac{\frac{12\,(5\,a^4-6\,a^2b^2+b^4)\log(|b\sin(dx+c)+a|)}{b^6}-\frac{12\,(5\,a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+b^5\sin(dx+c)+4\,a^5-4\,a^3b^2)}{(b\sin(dx+c)+a)b^6}+\frac{3\,b^6\sin(dx+c)^4-8\,ab^5\sin(dx+c)^3+18\,a^2b^4\sin(dx+c)^2-12\,b^6\sin(dx+c)^2-48\,a^3b^3\sin(dx+c)+48\,ab^5\sin(dx+c)}{b^8}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(5*a^4 - 6*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/b^6 - 12*(5
*a^4*b*sin(d*x + c) - 6*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) + 4*a^5 - 4
*a^3*b^2)/((b*sin(d*x + c) + a)*b^6) + (3*b^6*sin(d*x + c)^4 - 8*a*b^5*sin(
d*x + c)^3 + 18*a^2*b^4*sin(d*x + c)^2 - 12*b^6*sin(d*x + c)^2 - 48*a^3*b^3
*sin(d*x + c) + 48*a*b^5*sin(d*x + c))/b^8)/d
```

Mupad [B]

time = 0.09, size = 161, normalized size = 1.03

$$\frac{\frac{\sin(c+d\,x)^4}{4\,b^2}-\sin\left(c+d\,x\right)^2\,\left(\frac{1}{b^2}-\frac{3\,a^2}{2\,b^4}\right)+\sin\left(c+d\,x\right)\,\left(\frac{2\,a^3}{b^5}+\frac{2\,a\,\left(\frac{2}{b^2}-\frac{3\,a^2}{b^4}\right)}{b}\right)-\frac{2\,a\,\sin(c+d\,x)^3}{3\,b^3}+\frac{a^5-2\,a^3\,b^2+a\,b^4}{b\,(\sin(c+d\,x)\,b^6+a\,b^5)}+\frac{\ln(a+b\,\sin(c+d\,x))\,(5\,a^4-6\,a^2\,b^2+b^4)}{b^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^4/(4*b^2) - sin(c + d*x)^2*(1/b^2 - (3*a^2)/(2*b^4)) + sin(c
+ d*x)*((2*a^3)/b^5 + (2*a*(2/b^2 - (3*a^2)/b^4))/b) - (2*a*sin(c + d*x)^3)
/(3*b^3) + (a*b^4 + a^5 - 2*a^3*b^2)/(b*(a*b^5 + b^6*sin(c + d*x))) + (log(
a + b*sin(c + d*x))*(5*a^4 + b^4 - 6*a^2*b^2))/b^6)/d
```

3.1229 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=120

$$\frac{\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sin(c+dx))}{a^2b^4d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+dx)}{2b^2d}+\frac{(a^2-b^2)^2}{ab^4d(a+b\sin(c+dx))}$$

```
[Out] ln(sin(d*x+c))/a^2/d+(a^2-b^2)*(3*a^2+b^2)*ln(a+b*sin(d*x+c))/a^2/b^4/d-2*a
*sin(d*x+c)/b^3/d+1/2*sin(d*x+c)^2/b^2/d+(a^2-b^2)^2/a/b^4/d/(a+b*sin(d*x+c
))
```

Rubi [A]

time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{(a^2-b^2)^2}{ab^4d(a+b\sin(c+dx))}+\frac{(3a^2+b^2)(a^2-b^2)\log(a+b\sin(c+dx))}{a^2b^4d}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{2a\sin(c+dx)}{b^3d}+\frac{\sin^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) + ((a^2 - b^2)*(3*a^2 + b^2)*Log[a + b*Sin[c + d*
x]])/(a^2*b^4*d) - (2*a*Sin[c + d*x])/(b^3*d) + Sin[c + d*x]^2/(2*b^2*d) +
(a^2 - b^2)^2/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{b^4}{a^2x}+x-\frac{(a^2-b^2)^2}{a(a+x)^2}+\frac{(a^2-b^2)(3a^2+b^2)}{a^2(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\log(\sin(c+dx))}{a^2 d}+\frac{(a^2-b^2)(3a^2+b^2)\log(a+b\sin(c+dx))}{a^2b^4d}-\frac{2a\sin(c+}{b^3d}$$

Mathematica [A]

time = 0.35, size = 111, normalized size = 0.92

$$\frac{\frac{2\log(\sin(c+dx))}{a^2}+\frac{2(a-b)(a+b)(3a^2+b^2)\log(a+b\sin(c+dx))}{a^2b^4}-\frac{4a\sin(c+dx)}{b^3}+\frac{\sin^2(c+dx)}{b^2}+\frac{2(a^2-b^2)^2}{ab^4(a+b\sin(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*Log[Sin[c + d*x]])/a^2 + (2*(a - b)*(a + b)*(3*a^2 + b^2)*Log[a + b*Sin
[c + d*x]])/(a^2*b^4) - (4*a*Sin[c + d*x])/b^3 + Sin[c + d*x]^2/b^2 + (2*(a
^2 - b^2)^2)/(a*b^4*(a + b*Sin[c + d*x])))/(2*d)
```

Maple [A]

time = 0.54, size = 119, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a^2}-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+2a\sin(dx+c)}{b^3}-\frac{-a^4+2a^2b^2-b^4}{a\,b^4(a+b\sin(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(a+b\sin(dx+c))}{b^4a^2}}{d}$
default	$\frac{\frac{\ln(\sin(dx+c))}{a^2}-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+2a\sin(dx+c)}{b^3}-\frac{-a^4+2a^2b^2-b^4}{a\,b^4(a+b\sin(dx+c))}+\frac{\left(3a^4-2a^2b^2-b^4\right)\ln(a+b\sin(dx+c))}{b^4a^2}}{d}$
risch	$-\frac{3ia^2x}{b^4}+\frac{2ix}{b^2}-\frac{e^{2i(dx+c)}}{8b^2d}+\frac{ia\,e^{i(dx+c)}}{b^3d}-\frac{ia\,e^{-i(dx+c)}}{b^3d}-\frac{e^{-2i(dx+c)}}{8b^2d}-\frac{6ia^2c}{b^4d}+\frac{4ic}{b^2d}+\frac{2(a^4-2a^2b^2+}{b^4da(-ib\,e^{2i(dx+c)}}$
norman	$\frac{-\frac{18a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{18a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{6a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{6a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{4\left(9a^4-8a^2b^2+3b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2b^3d}-\frac{4(6}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4\left(a(\text{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*ln(sin(d*x+c))-1/b^3*(-1/2*sin(d*x+c)^2*b+2*a*sin(d*x+c))-(-a^4+
2*a^2*b^2-b^4)/a/b^4/(a+b*sin(d*x+c))+1/b^4*(3*a^4-2*a^2*b^2-b^4)/a^2*ln(a+
b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 0.98

$$\frac{\frac{2\,(a^4-2\,a^2b^2+b^4)}{ab^5\sin(dx+c)+a^2b^4}+\frac{2\,\log(\sin(dx+c))}{a^2}+\frac{b\sin(dx+c)^2-4\,a\sin(dx+c)}{b^3}+\frac{2\,(3\,a^4-2\,a^2b^2-b^4)\log(b\sin(dx+c)+a)}{a^2b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^4 - 2*a^2*b^2 + b^4)/(a*b^5*sin(d*x + c) + a^2*b^4) + 2*log(sin(d
*x + c))/a^2 + (b*sin(d*x + c)^2 - 4*a*sin(d*x + c))/b^3 + 2*(3*a^4 - 2*a^2
*b^2 - b^4)*log(b*sin(d*x + c) + a)/(a^2*b^4))/d
```

Fricas [A]
time = 0.42, size = 189, normalized size = 1.58

$$\frac{6\,a^3b^2\cos(dx+c)^2+4\,a^5-15\,a^3b^2+4\,ab^4+4\,(3\,a^5-2\,a^3b^2-ab^4+(3\,a^4b-2\,a^2b^3-b^5)\sin(dx+c))\log(b\sin(dx+c)+a)+4\,(b^5\sin(dx+c)+ab^4)\log\left(-\frac{1}{2}\sin(dx+c)\right)-\left(2\,a^2b^3\cos(dx+c)^2+8\,a^4b-a^2b^3\right)\sin(dx+c)}{4\,(a^2b^5d\sin(dx+c)+a^3b^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^3*b^2*cos(d*x + c)^2 + 4*a^5 - 15*a^3*b^2 + 4*a*b^4 + 4*(3*a^5 - 2
*a^3*b^2 - a*b^4 + (3*a^4*b - 2*a^2*b^3 - b^5)*sin(d*x + c))*log(b*sin(d*x
+ c) + a) + 4*(b^5*sin(d*x + c) + a*b^4)*log(-1/2*sin(d*x + c)) - (2*a^2*b^
3*cos(d*x + c)^2 + 8*a^4*b - a^2*b^3)*sin(d*x + c))/(a^2*b^5*d*sin(d*x + c)
 + a^3*b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 154, normalized size = 1.28

$$\frac{\frac{2\,\log(|\sin(dx+c)|)}{a^2}+\frac{b^2\sin(dx+c)^2-4\,ab\sin(dx+c)}{b^4}+\frac{2\,(3\,a^4-2\,a^2b^2-b^4)\log(|b\sin(dx+c)+a|)}{a^2b^4}-\frac{2\,(3\,a^4b\sin(dx+c)-2\,a^2b^3\sin(dx+c)-b^5\sin(dx+c)+2\,a^5-2\,ab^4)}{(b\sin(dx+c)+a)a^2b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(sin(d*x + c)))/a^2 + (b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x + c
))/b^4 + 2*(3*a^4 - 2*a^2*b^2 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^4)
 - 2*(3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x + c) + 2*
a^5 - 2*a*b^4)/((b*sin(d*x + c) + a)*a^2*b^4))/d
```

Mupad [B]
time = 12.08, size = 338, normalized size = 2.82

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{6\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(3\,a^4-3\,a^2\,b^2+b^4)}{a^2\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(3\,a^4-2\,a^2\,b^2+b^4)}{a^2\,b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(3\,a^4-2\,a^2\,b^2+b^4)}{a^2\,b^3}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,(3\,a^2-2\,b^2)}{b^4\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,(-3\,a^4+2\,a^2\,b^2+b^4)}{a^2\,b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) - ((6*a*tan(c/2 + (d*x)/2)^2)/b^2 + (6*a*ta
n(c/2 + (d*x)/2)^4)/b^2 + (4*tan(c/2 + (d*x)/2)^3*(3*a^4 + b^4 - 3*a^2*b^2)
)/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^5*(3*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3)
+ (2*tan(c/2 + (d*x)/2)*(3*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3))/(d*(a + 2*b*t
an(c/2 + (d*x)/2) + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a
*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5
)) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a^2 - 2*b^2))/(b^4*d) - (log(a + 2*b
*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b^4 - 3*a^4 + 2*a^2*b^2))/(a
^2*b^4*d)
```

3.1230 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc(c+dx)}{a^2d}-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{2(a^4-b^4)\log(a+b\sin(c+dx))}{a^3b^3d}+\frac{\sin(c+dx)}{b^2d}-\frac{(a^2-b^2)^2}{a^2b^3d(a+b\sin(c+dx))}$$

```
[Out] -csc(d*x+c)/a^2/d-2*b*ln(sin(d*x+c))/a^3/d-2*(a^4-b^4)*ln(a+b*sin(d*x+c))/a
^3/b^3/d+sin(d*x+c)/b^2/d-(a^2-b^2)^2/a^2/b^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$-\frac{2b\log(\sin(c+dx))}{a^3d}-\frac{(a^2-b^2)^2}{a^2b^3d(a+b\sin(c+dx))}-\frac{\csc(c+dx)}{a^2d}-\frac{2(a^4-b^4)\log(a+b\sin(c+dx))}{a^3b^3d}+\frac{\sin(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]/(a^2*d)) - (2*b*Log[Sin[c + d*x]])/(a^3*d) - (2*(a^4 - b^4)*
Log[a + b*Sin[c + d*x]])/(a^3*b^3*d) + Sin[c + d*x]/(b^2*d) - (a^2 - b^2)^2
/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^2(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{a^2x^2}-\frac{2b^4}{a^3x}+\frac{(a^2-b^2)^2}{a^2(a+x)^2}-\frac{2(a^4-b^4)}{a^3(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= -\frac{\csc(c+dx)}{a^2 d}-\frac{2b\log(\sin(c+dx))}{a^3 d}-\frac{2(a^4-b^4)\log(a+b\sin(c+dx))}{a^3b^3 d}+$$

Mathematica [A]
time = 0.43, size = 95, normalized size = 0.87

$$-\frac{\frac{\csc(c+dx)}{a^2}+\frac{2b\log(\sin(c+dx))}{a^3}+2\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\log(a+b\sin(c+dx))-\frac{\sin(c+dx)}{b^2}+\frac{(a^2-b^2)^2}{a^2b^3(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
[Out] -((Csc[c + d*x]/a^2 + (2*b*Log[Sin[c + d*x]])/a^3 + 2*(a/b^3 - b/a^3)*Log[a
 + b*Sin[c + d*x]] - Sin[c + d*x]/b^2 + (a^2 - b^2)^2/(a^2*b^3*(a + b*Sin[c
 + d*x])))/d)
```

Maple [A]
time = 0.61, size = 106, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{1}{a^2\sin(dx+c)}-\frac{2b\ln(\sin(dx+c))}{a^3}+\frac{\sin(dx+c)}{b^2}+\frac{(-2a^4+2b^4)\ln(a+b\sin(dx+c))}{a^3b^3}-\frac{a^4-2a^2b^2+b^4}{b^3a^2(a+b\sin(dx+c))}}{d}$
default	$\frac{-\frac{1}{a^2\sin(dx+c)}-\frac{2b\ln(\sin(dx+c))}{a^3}+\frac{\sin(dx+c)}{b^2}+\frac{(-2a^4+2b^4)\ln(a+b\sin(dx+c))}{a^3b^3}-\frac{a^4-2a^2b^2+b^4}{b^3a^2(a+b\sin(dx+c))}}{d}$
risch	$\frac{2iax}{b^3}-\frac{i e^{i(dx+c)}}{2b^2d}+\frac{i e^{-i(dx+c)}}{2b^2d}+\frac{4iac}{b^3d}-\frac{2(a^4e^{3i(dx+c)}-2a^2b^2e^{3i(dx+c)}+2b^4e^{3i(dx+c)}-a^4e^{i(dx+c)}+2a^2b^2e^{i(dx+c)}}{a^2db^3(2a e^{3i(dx+c)}-ib e^{4i(dx+c)}-2a e^{i(dx+c)}+2ib e^{2i}}$
norman	$\frac{\frac{(12a^4-17a^2b^2+12b^4)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3b^2d}+\frac{(12a^4-17a^2b^2+12b^4)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3b^2d}-\frac{1}{2ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}+\frac{4(\tan}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a^2/sin(d*x+c)-2/a^3*b*ln(sin(d*x+c))+1/b^2*sin(d*x+c)+(-2*a^4+2*b^
4)/a^3/b^3*ln(a+b*sin(d*x+c))-1/b^3*(a^4-2*a^2*b^2+b^4)/a^2/(a+b*sin(d*x+c)
))
```

Maxima [A]
time = 0.29, size = 120, normalized size = 1.10

$$-\frac{\frac{ab^3+\left(a^4-2\,a^2b^2+2\,b^4\right)\sin(dx+c)}{a^2b^4\sin(dx+c)^2+a^3b^3\sin(dx+c)}+\frac{2\,b\log(\sin(dx+c))}{a^3}-\frac{\sin(dx+c)}{b^2}+\frac{2\,\left(a^4-b^4\right)\log(b\sin(dx+c)+a)}{a^3b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -((a*b^3 + (a^4 - 2*a^2*b^2 + 2*b^4)*sin(d*x + c))/(a^2*b^4*sin(d*x + c)^2
+ a^3*b^3*sin(d*x + c)) + 2*b*log(sin(d*x + c))/a^3 - sin(d*x + c)/b^2 + 2*
(a^4 - b^4)*log(b*sin(d*x + c) + a)/(a^3*b^3))/d
```

Fricas [A]
time = 0.41, size = 214, normalized size = 1.96

$$\frac{a^4b\cos\left(dx+c\right)^2-a^4b+a^2b^3+2\,\left(a^4b-b^5-\left(a^4b-b^5\right)\cos\left(dx+c\right)^2+\left(a^5-ab^4\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(b^5\cos\left(dx+c\right)^2-ab^4\sin\left(dx+c\right)-b^5\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+\left(a^3b^2\cos\left(dx+c\right)^2+a^5-3\,a^3b^2+2\,ab^4\right)\sin\left(dx+c\right)}{a^3b^4d\cos\left(dx+c\right)^2-a^4b^3d\sin\left(dx+c\right)-a^3b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (a^4*b*cos(d*x + c)^2 - a^4*b + a^2*b^3 + 2*(a^4*b - b^5 - (a^4*b - b^5)*co
s(d*x + c)^2 + (a^5 - a*b^4)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(b^5
*cos(d*x + c)^2 - a*b^4*sin(d*x + c) - b^5)*log(1/2*sin(d*x + c)) + (a^3*b^
2*cos(d*x + c)^2 + a^5 - 3*a^3*b^2 + 2*a*b^4)*sin(d*x + c))/(a^3*b^4*d*cos(
d*x + c)^2 - a^4*b^3*d*sin(d*x + c) - a^3*b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 131, normalized size = 1.20

$$-\frac{\frac{2\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^3}-\frac{\sin\left(dx+c\right)}{b^2}-\frac{a^3\sin\left(dx+c\right)^2+2\,a^2b\sin\left(dx+c\right)-2\,b^3\sin\left(dx+c\right)-ab^2}{\left(b\sin\left(dx+c\right)^2+a\sin\left(dx+c\right)\right)a^2b^2}+\frac{2\left(a^4-b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^3b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*b*log(abs(sin(d*x + c)))/a^3 - sin(d*x + c)/b^2 - (a^3*sin(d*x + c)^2 +
 2*a^2*b*sin(d*x + c) - 2*b^3*sin(d*x + c) - a*b^2)/((b*sin(d*x + c)^2 + a*
sin(d*x + c))*a^2*b^2) + 2*(a^4 - b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^
3))/d
```

Mupad [B]
time = 12.10, size = 313, normalized size = 2.87

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a^2-b^2\right)}{b}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^4-5\,a^2\,b^2+2\,b^4\right)}{a\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^4-9\,a^2\,b^2+4\,b^4\right)}{a\,b^2}}{d\,\left(2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,b\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}+\frac{2\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^3\,d}-\frac{2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{2\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4-b^4\right)}{a^3\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(4*a^2 - b^2))/b - 2*b*tan(c/2 + (d*x)/2) - a + (2
*tan(c/2 + (d*x)/2)^2*(4*a^4 + 2*b^4 - 5*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*
x)/2)^4*(8*a^4 + 4*b^4 - 9*a^2*b^2))/(a*b^2))/(d*(4*a^3*tan(c/2 + (d*x)/2)^
3 + 2*a^3*tan(c/2 + (d*x)/2)^5 + 2*a^3*tan(c/2 + (d*x)/2) + 4*a^2*b*tan(c/2
 + (d*x)/2)^2 + 4*a^2*b*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)/(2*a^2*
d) + (2*a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^3*d) - (2*b*log(tan(c/2 + (d*x)
/2)))/(a^3*d) - (2*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)
*(a^4 - b^4))/(a^3*b^3*d)
```

3.1231 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^4+2a^2b^2-3b^4)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{}{a^3b}$$

```
[Out] 2*b*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^2/d-(2*a^2-3*b^2)*ln(sin(d*x+c))/a^
4/d+(a^4+2*a^2*b^2-3*b^4)*ln(a+b*sin(d*x+c))/a^4/b^2/d+(a^2-b^2)^2/a^3/b^2/
d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^4+2a^2b^2-3b^4)\log(a+b\sin(c+dx))}{a^4b^2d}+\frac{(a^2-b^2)^2}{a^3b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((2*a^2 - 3*b^2)*Lo
g[Sin[c + d*x]])/(a^4*d) + ((a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x
]])/(a^4*b^2*d) + (a^2 - b^2)^2/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^3(b^2-x^2)^2}{x^3(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^3(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^3}-\frac{2b^4}{a^3x^2}+\frac{-2a^2b^2+3b^4}{a^4x}-\frac{(a^2-b^2)^2}{a^3(a+x)^2}+\frac{a^4+2a^2b^2-3b^4}{a^4(a+x)}\right)dx,x,b\sin(c$$
$$= \frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}-\frac{(2a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^4+2a^2}{}$$

Mathematica [A]
time = 0.52, size = 116, normalized size = 0.89

$$\frac{4ab\csc(c+dx)-a^2\csc^2(c+dx)-2(2a^2-3b^2)\log(\sin(c+dx))+\frac{2(a^4+2a^2b^2-3b^4)\log(a+b\sin(c+dx))}{b^2}+\frac{2a(a^2-b^2)^2}{b^2(a+b\sin(c+dx))}}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x] - a^2*Csc[c + d*x]^2 - 2*(2*a^2 - 3*b^2)*Log[Sin[c + d*
x]] + (2*(a^4 + 2*a^2*b^2 - 3*b^4)*Log[a + b*Sin[c + d*x]])/b^2 + (2*a*(a^2
 - b^2)^2)/(b^2*(a + b*Sin[c + d*x])))/(2*a^4*d)
```

Maple [A]
time = 0.54, size = 129, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{2a^2\sin(dx+c)^2}+\frac{(-2a^2+3b^2)\ln(\sin(dx+c))}{a^4}+\frac{2b}{a^3\sin(dx+c)}-\frac{-a^4+2a^2b^2-b^4}{a^3b^2(a+b\sin(dx+c))}+\frac{(a^4+2a^2b^2-3b^4)\ln(a+b\sin(dx+c))}{a^4b^2}}{d}$
default	$\frac{-\frac{1}{2a^2\sin(dx+c)^2}+\frac{(-2a^2+3b^2)\ln(\sin(dx+c))}{a^4}+\frac{2b}{a^3\sin(dx+c)}-\frac{-a^4+2a^2b^2-b^4}{a^3b^2(a+b\sin(dx+c))}+\frac{(a^4+2a^2b^2-3b^4)\ln(a+b\sin(dx+c))}{a^4b^2}}{d}$
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2d}+\frac{3b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^2d}-\frac{(4a^4-15a^2b^2+12b^4)(\tan^5(}{a^4bd}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\right.}$
risch	$-\frac{ix}{b^2}-\frac{2ic}{b^2d}+\frac{6iab^3e^{4i(dx+c)}+2a^4e^{5i(dx+c)}-4a^2b^2e^{5i(dx+c)}+6b^4e^{5i(dx+c)}-6iab^3e^{2i(dx+c)}-4a^4e^{3i(dx+c)}+12a^2b^2e^3}{(e^{2i(dx+c)}-1)^2b^2(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2/sin(d*x+c)^2+(-2*a^2+3*b^2)/a^4*ln(sin(d*x+c))+2/a^3*b/sin(d*
x+c)-(-a^4+2*a^2*b^2-b^4)/a^3/b^2/(a+b*sin(d*x+c))+(a^4+2*a^2*b^2-3*b^4)/a^
4/b^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 147, normalized size = 1.12

$$\frac{\frac{3\,ab^3\sin(dx+c)-a^2b^2+2\,\left(a^4-2\,a^2b^2+3\,b^4\right)\sin(dx+c)^2}{a^3b^3\sin(dx+c)^3+a^4b^2\sin(dx+c)^2}-\frac{2\,\left(2\,a^2-3\,b^2\right)\log(\sin(dx+c))}{a^4}+\frac{2\,\left(a^4+2\,a^2b^2-3\,b^4\right)\log(b\sin(dx+c)+a)}{a^4b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/2*((3*a*b^3*sin(d*x + c) - a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + 3*b^4)*sin(d*x
+ c)^2)/(a^3*b^3*sin(d*x + c)^3 + a^4*b^2*sin(d*x + c)^2) - 2*(2*a^2 - 3*b^
2)*log(sin(d*x + c))/a^4 + 2*(a^4 + 2*a^2*b^2 - 3*b^4)*log(b*sin(d*x + c) +
 a)/(a^4*b^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(129) = 258.
time = 0.42, size = 335, normalized size = 2.56

$$-\frac{3\,a^2b^3\sin\left(dx+c\right)+2\,a^5-5\,a^3b^2+6\,ab^4-2\,\left(a^5-2\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^2+2\,\left(a^5+2\,a^3b^2-3\,ab^4-\left(a^5+2\,a^3b^2-3\,ab^4\right)\cos\left(dx+c\right)^2+\left(a^4b+2\,a^2b^3-3\,b^5-\left(a^4b+2\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(2\,a^3b^2-3\,ab^4-\left(2\,a^3b^2-3\,ab^4\right)\cos\left(dx+c\right)^2+\left(2\,a^2b^3-3\,b^5-\left(2\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,\left(a^5b^2d\cos\left(dx+c\right)^2-a^5b^2d+\left(a^4b^3d\cos\left(dx+c\right)^2-a^4b^3d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b^3*sin(d*x + c) + 2*a^5 - 5*a^3*b^2 + 6*a*b^4 - 2*(a^5 - 2*a^3
*b^2 + 3*a*b^4)*cos(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 - 3*a*b^4 - (a^5 + 2*a^
3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (a^4*b + 2*a^2*b^3 - 3*b^5 - (a^4*b + 2*a
^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*(
2*a^3*b^2 - 3*a*b^4 - (2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^2*b^3 - 3
*b^5 - (2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x +
 c)))/(a^5*b^2*d*cos(d*x + c)^2 - a^5*b^2*d + (a^4*b^3*d*cos(d*x + c)^2 - a
^4*b^3*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 190, normalized size = 1.45

$$-\frac{\frac{2\,(2\,a^2-3\,b^2)\log(|\sin(dx+c)|)}{a^4}-\frac{2\,(a^4+2\,a^2b^2-3\,b^4)\log(|b\sin(dx+c)+a|)}{a^4b^2}+\frac{2\,(a^4\sin(dx+c)+2\,a^2b^2\sin(dx+c)-3\,b^4\sin(dx+c)+4\,a^3b-4\,ab^3)}{(b\sin(dx+c)+a)a^4b}-\frac{6\,a^2\sin(dx+c)^2-9\,b^2\sin(dx+c)^2+4\,ab\sin(dx+c)-a^2}{a^4\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^4 + 2*a^2*b^2 - 3
*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b^2) + 2*(a^4*sin(d*x + c) + 2*a^2*
b^2*sin(d*x + c) - 3*b^4*sin(d*x + c) + 4*a^3*b - 4*a*b^3)/((b*sin(d*x + c)
 + a)*a^4*b) - (6*a^2*sin(d*x + c)^2 - 9*b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x
 + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

Mupad [B]
time = 11.92, size = 280, normalized size = 2.14

$$\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{2}-8\,b^2\right)+\frac{a^2}{2}-3\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2\,a^4-5\,a^2\,b^2+2\,b^4)}{a\,b}}{d\left(4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+8\,b\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{b^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2\,a^2-3\,b^2)}{a^4\,d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^4+2\,a^2\,b^2-3\,b^4)}{a^4\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(a^3*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 8*b^2) + a^
2/2 - 3*a*b*tan(c/2 + (d*x)/2) + (4*tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 5
*a^2*b^2))/(a*b))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)
^4 + 8*a^3*b*tan(c/2 + (d*x)/2)^3)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(b^2*d)
 - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (log(tan(c/2 + (d*x)/2))*(2*a^2 - 3*b^2
))/(a^4*d) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 - 3*b^4 + 2*a^2*b^2))/(a^4*b^2*d)
```

3.1232 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=147

$$\frac{(2a^2-3b^2)\csc(c+dx)}{a^4d}+\frac{b\csc^2(c+dx)}{a^3d}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{4b(a^2-b^2)\log(\sin(c+dx))}{a^5d}-\frac{4b(a^2-b^2)\log(a+b}{a^5d}$$

[Out] (2*a^2-3*b^2)*csc(d*x+c)/a^4/d+b*csc(d*x+c)^2/a^3/d-1/3*csc(d*x+c)^3/a^2/d+4*b*(a^2-b^2)*ln(sin(d*x+c))/a^5/d-4*b*(a^2-b^2)*ln(a+b*sin(d*x+c))/a^5/d-(a^2-b^2)^2/a^4/b/d/(a+b*sin(d*x+c))

Rubi [A]
time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{b\csc^2(c+dx)}{a^3d}-\frac{\csc^3(c+dx)}{3a^2d}+\frac{4b(a^2-b^2)\log(\sin(c+dx))}{a^5d}-\frac{4b(a^2-b^2)\log(a+b\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2}{a^4bd(a+b\sin(c+dx))}+\frac{(2a^2-3b^2)\csc(c+dx)}{a^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]

[Out] ((2*a^2 - 3*b^2)*Csc[c + d*x])/(a^4*d) + (b*Csc[c + d*x]^2)/(a^3*d) - Csc[c + d*x]^3/(3*a^2*d) + (4*b*(a^2 - b^2)*Log[Sin[c + d*x]])/(a^5*d) - (4*b*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) - (a^2 - b^2)^2/(a^4*b*d*(a + b*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^4(b^2-x^2)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^4(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^4} - \frac{2b^4}{a^3x^3} + \frac{-2a^2b^2+3b^4}{a^4x^2} + \frac{4b^2(a^2-b^2)}{a^5x} + \frac{(a^2-b^2)^2}{a^4(a+x)^2} + \frac{4b^2(-a^2+b^2)}{a^5(a+x)}\right) dx}{bd}$$

$$= \frac{(2a^2-3b^2)\csc(c+dx)}{a^4 d} + \frac{b\csc^2(c+dx)}{a^3 d} - \frac{\csc^3(c+dx)}{3a^2 d} + \frac{4b(a^2-b^2)\log(}{a^5 d}$$

Mathematica [A]

time = 1.25, size = 127, normalized size = 0.86

$$\frac{3a(2a^2-3b^2)\csc(c+dx) + 3a^2b\csc^2(c+dx) - a^3\csc^3(c+dx) + 12(a-b)b(a+b)\log(\sin(c+dx)) - 12(a-b)b(a+b)\log(a+b\sin(c+dx)) - \frac{3a(a^2-b^2)^2}{b(a+b\sin(c+dx))}}{3a^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*(2*a^2 - 3*b^2)*Csc[c + d*x] + 3*a^2*b*Csc[c + d*x]^2 - a^3*Csc[c + d*
x]^3 + 12*(a - b)*b*(a + b)*Log[Sin[c + d*x]] - 12*(a - b)*b*(a + b)*Log[a
+ b*Sin[c + d*x]] - (3*a*(a^2 - b^2)^2)/(b*(a + b*Sin[c + d*x])))/(3*a^5*d)
```

Maple [A]

time = 0.55, size = 139, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{1}{3a^2\sin(dx+c)^3} - \frac{-2a^2+3b^2}{a^4\sin(dx+c)} + \frac{b}{a^3\sin(dx+c)^2} + \frac{4b(a^2-b^2)\ln(\sin(dx+c))}{a^5} - \frac{a^4-2a^2b^2+b^4}{a^4b(a+b\sin(dx+c))} - \frac{4b(a^2-b^2)\ln(a+b\sin(dx+c))}{a^5}}{d}$
default	$\frac{-\frac{1}{3a^2\sin(dx+c)^3} - \frac{-2a^2+3b^2}{a^4\sin(dx+c)} + \frac{b}{a^3\sin(dx+c)^2} + \frac{4b(a^2-b^2)\ln(\sin(dx+c))}{a^5} - \frac{a^4-2a^2b^2+b^4}{a^4b(a+b\sin(dx+c))} - \frac{4b(a^2-b^2)\ln(a+b\sin(dx+c))}{a^5}}{d}$
norman	$\frac{-\frac{1}{24ad} - \frac{\tan^{10}(\frac{dx}{2}+\frac{c}{2})}{24ad} - \frac{b(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{3a^2d} + \frac{b\tan(\frac{dx}{2}+\frac{c}{2})}{6a^2d} + \frac{b(\tan^9(\frac{dx}{2}+\frac{c}{2}))}{6a^2d} + \frac{(19a^2-24b^2)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{24a^3d} + \frac{(19a^2-24b^2)(\ta}{24a^3}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))\tan(\frac{dx}{2}+\frac{c}{2})^3(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))}$
risch	$-\frac{2i(-24iab^3e^{4i(dx+c)}+12iab^3e^{6i(dx+c)}+3a^4e^{7i(dx+c)}-12a^2b^2e^{7i(dx+c)}+12b^4e^{7i(dx+c)}+16ia^3be^{4i(dx+c)}+12iab^3e^{2i(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/a^2/sin(d*x+c)^3-(-2*a^2+3*b^2)/a^4/sin(d*x+c)+1/a^3*b/sin(d*x+c)
^2+4*b*(a^2-b^2)/a^5*ln(sin(d*x+c))-(a^4-2*a^2*b^2+b^4)/a^4/b/(a+b*sin(d*x+
c))-4*b*(a^2-b^2)/a^5*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 158, normalized size = 1.07

$$\frac{\frac{2\,a^2b^2\sin(dx+c)-a^3b-3\,(a^4-4\,a^2b^2+4\,b^4)\sin(dx+c)^3+6\,(a^3b-ab^3)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+a^5b\sin(dx+c)^3}-\frac{12\,(a^2b-b^3)\log(b\sin(dx+c)+a)}{a^5}+\frac{12\,(a^2b-b^3)\log(\sin(dx+c))}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*((2*a^2*b^2*sin(d*x + c) - a^3*b - 3*(a^4 - 4*a^2*b^2 + 4*b^4)*sin(d*x
+ c)^3 + 6*(a^3*b - a*b^3)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + a^5*b*
sin(d*x + c)^3) - 12*(a^2*b - b^3)*log(b*sin(d*x + c) + a)/a^5 + 12*(a^2*b
- b^3)*log(sin(d*x + c))/a^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(145) = 290.
time = 0.38, size = 401, normalized size = 2.73

$$\frac{5\,a^4b-6\,a^2b^3-6\,(a^4b-a^2b^3)\cos(dx+c)^2-12\,(a^2b^3-b^5+(a^2b^3-b^5)\cos(dx+c)^4-2\,(a^2b^3-b^5)\cos(dx+c)^2+(a^3b^2-ab^4-(a^3b^2-ab^4)\cos(dx+c)^2)\sin(dx+c))\log(b\sin(dx+c)+a)+12\,(a^2b^3-b^5+(a^2b^3-b^5)\cos(dx+c)^4-2\,(a^2b^3-b^5)\cos(dx+c)^2+(a^3b^2-ab^4-(a^3b^2-ab^4)\cos(dx+c)^2)\sin(dx+c))\log\left(\frac{1}{2}\sin(dx+c)\right)-(3\,a^5-14\,a^3b^2+12\,ab^4-3\,(a^5-4\,a^3b^2+4\,ab^4)\cos(dx+c)^2)\sin(dx+c)}{3\,(a^5b^2d\cos(dx+c)^4-2\,a^5b^2d\cos(dx+c)^2+a^5b^2d-(a^6bd\cos(dx+c)^2-a^6bd)\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/3*(5*a^4*b - 6*a^2*b^3 - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 12*(a^2*b^3
 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*(a^2*b^3 - b^5)*cos(d*x + c)^2
+ (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + 12*(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cos(d*x + c)^4 - 2*
(a^2*b^3 - b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4 - (a^3*b^2 - a*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x + c)) - (3*a^5 - 14*a^3*b^2 + 12*a
*b^4 - 3*(a^5 - 4*a^3*b^2 + 4*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*b^2
*d*cos(d*x + c)^4 - 2*a^5*b^2*d*cos(d*x + c)^2 + a^5*b^2*d - (a^6*b*d*cos(d
*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.47, size = 211, normalized size = 1.44

$$\frac{\frac{12\,(a^2b-b^3)\log(|\sin(dx+c)|)}{a^5}-\frac{12\,(a^2b^2-b^4)\log(|b\sin(dx+c)+a|)}{a^5b}+\frac{3\,(4\,a^2b^3\sin(dx+c)-4\,b^5\sin(dx+c)-a^5+6\,a^3b^2-5\,ab^4)}{(b\sin(dx+c)+a)a^5b}-\frac{22\,a^2b\sin(dx+c)^3-22\,b^3\sin(dx+c)^3-6\,a^3\sin(dx+c)^2+9\,ab^2\sin(dx+c)^2-3\,a^2b\sin(dx+c)+a^3}{a^5\sin(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(12*(a^2*b - b^3)*log(abs(sin(d*x + c)))/a^5 - 12*(a^2*b^2 - b^4)*log(a
bs(b*sin(d*x + c) + a))/(a^5*b) + 3*(4*a^2*b^3*sin(d*x + c) - 4*b^5*sin(d*x
 + c) - a^5 + 6*a^3*b^2 - 5*a*b^4)/((b*sin(d*x + c) + a)*a^5*b) - (22*a^2*b
*sin(d*x + c)^3 - 22*b^3*sin(d*x + c)^3 - 6*a^3*sin(d*x + c)^2 + 9*a*b^2*si
n(d*x + c)^2 - 3*a^2*b*sin(d*x + c) + a^3)/(a^5*sin(d*x + c)^3))/d
```

Mupad [B]
time = 11.72, size = 319, normalized size = 2.17

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(16\,a^2\,b-24\,b^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(8\,a\,b^2-\frac{20\,a^3}{3}\right)-\frac{a^3}{3}+\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(23\,a^4-44\,a^2\,b^2+16\,b^4)}{a}}{d\left(8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+16\,b\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{\frac{a^2}{4}+\frac{b^2}{2}}{a^4}+\frac{5}{8\,a^2}-\frac{2\,b^2}{a^4}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)(4\,a^2\,b-4\,b^3)}{a^5\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,a^3\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)(4\,a^2\,b-4\,b^3)}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(16*a^2*b - 24*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2 -
 (20*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)
^4*(23*a^4 + 16*b^4 - 44*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^
5*tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/
2)^3/(24*a^2*d) + (tan(c/2 + (d*x)/2)*((a^2/4 + b^2/2)/a^4 + 5/(8*a^2) - (2
*b^2)/a^4))/d + (log(tan(c/2 + (d*x)/2))*(4*a^2*b - 4*b^3))/(a^5*d) + (b*ta
n(c/2 + (d*x)/2)^2)/(4*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(4*a^2*b - 4*b^3))/(a^5*d)
```

3.1233 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d}+\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{(a^4-6a^2b^2+5b^4)\log}{a^6d}$$

```
[Out] -4*b*(a^2-b^2)*csc(d*x+c)/a^5/d+1/2*(2*a^2-3*b^2)*csc(d*x+c)^2/a^4/d+2/3*b*
csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4/a^2/d+(a^4-6*a^2*b^2+5*b^4)*ln(sin(d*x+
c))/a^6/d-(a^4-6*a^2*b^2+5*b^4)*ln(a+b*sin(d*x+c))/a^6/d+(a^2-b^2)^2/a^5/d/
(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}+\frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d}+\frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d}-\frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2(a^2-b^2)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{(a^2-b^2)^2}{a^5(a+x)^2} + \frac{-a^4+6}{a^6(}\right.\right.}{d}$$

$$= -\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c}{4a}$$

Mathematica [A]
time = 6.11, size = 187, normalized size = 0.99

$$-\frac{4(a-b)b(a+b)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d} - \frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d} + \frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

Maple [A]
time = 0.60, size = 172, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{1}{4a^2\sin(dx+c)^4} - \frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2} + \frac{(a^4-6a^2b^2+5b^4)\ln(\sin(dx+c))}{a^6} + \frac{2b}{3a^3\sin(dx+c)^3} - \frac{4b(a^2-b^2)}{a^5\sin(dx+c)} - \frac{(a^4-6a^2b^2+5b^4)\ln(a+b\sin}{a^6}}{d}$
default	$\frac{-\frac{1}{4a^2\sin(dx+c)^4} - \frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2} + \frac{(a^4-6a^2b^2+5b^4)\ln(\sin(dx+c))}{a^6} + \frac{2b}{3a^3\sin(dx+c)^3} - \frac{4b(a^2-b^2)}{a^5\sin(dx+c)} - \frac{(a^4-6a^2b^2+5b^4)\ln(a+b\sin}{a^6}}{d}$
norman	$\frac{-\frac{1}{64ad} - \frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad} + \frac{5b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{96a^2d} + \frac{5b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96a^2d} + \frac{(33a^2-40b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192a^3d} + \frac{(33a^2-40b^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192a^3d} - \frac{b(}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\,\mathrm{t}\right.}$
risch	$2i\left(15b^4e^{i(dx+c)}+15b^4e^{9i(dx+c)}-24a^4e^{7i(dx+c)}-60b^4e^{7i(dx+c)}+30a^4e^{5i(dx+c)}+90b^4e^{5i(dx+c)}-24a^4e^{3i(dx+c)}-60b^4e^{3i(d}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a^2/sin(d*x+c)^4-1/2*(-2*a^2+3*b^2)/a^4/sin(d*x+c)^2+(a^4-6*a^2*b
^2+5*b^4)/a^6*ln(sin(d*x+c))+2/3/a^3*b/sin(d*x+c)^3-4*b*(a^2-b^2)/a^5/sin(d
```

*x+c)-(a^4-6*a^2*b^2+5*b^4)/a^6*ln(a+b*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/a^5/(a+b*sin(d*x+c)))

Maxima [A]
time = 0.27, size = 189, normalized size = 1.01

$$\frac{\frac{5\,a^{3}b\sin\left(dx+c\right)+12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\sin\left(dx+c\right)^{4}-3\,a^{4}-6\,\left(6\,a^{3}b-5\,ab^{3}\right)\sin\left(dx+c\right)^{3}+2\,\left(6\,a^{4}-5\,a^{2}b^{2}\right)\sin\left(dx+c\right)^{2}}{a^{5}b\sin\left(dx+c\right)^{5}+a^{6}\sin\left(dx+c\right)^{4}}-\frac{12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}}+\frac{12\,\left(a^{4}-6\,a^{2}b^{2}+5\,b^{4}\right)\log\left(\sin\left(dx+c\right)\right)}{a^{6}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(182) = 364.
time = 0.40, size = 542, normalized size = 2.88

$$\frac{21\,a^{5}-82\,a^{3}b^{2}+60\,ab^{4}+12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(18\,a^{5}-77\,a^{3}b^{2}+60\,ab^{4}\right)\cos\left(dx+c\right)^{2}-12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}+\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+12\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}+\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{5}-6\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{2}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}+\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{4}-2\,\left(a^{4}b-6\,a^{2}b^{3}+5\,b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(31\,a^{4}b-30\,a^{2}b^{3}-6\,\left(6\,a^{4}b-5\,a^{2}b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{12\,\left(a^{7}d\cos\left(dx+c\right)^{4}-2\,a^{7}d\cos\left(dx+c\right)^{2}+a^{7}d+\left(a^{6}bd\cos\left(dx+c\right)^{4}-2\,a^{6}bd\cos\left(dx+c\right)^{2}+a^{6}bd\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.48, size = 278, normalized size = 1.48

$$\frac{\frac{12\,(a^4-6\,a^2b^2+5\,b^4)\log(|\sin(dx+c)|)}{a^6}-\frac{12\,(a^4b-6\,a^2b^3+5\,b^5)\log(|b\sin(dx+c)+a|)}{a^6b}+\frac{12\,(a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+5\,b^5\sin(dx+c)+2\,a^5-8\,a^3b^2+6\,ab^4)}{(b\sin(dx+c)+a)a^6}-\frac{25\,a^4\sin(dx+c)^4-150\,a^2b^2\sin(dx+c)^4+125\,b^4\sin(dx+c)^4+48\,a^3b\sin(dx+c)^3-48\,ab^3\sin(dx+c)^3-12\,a^4\sin(dx+c)^2+18\,a^2b^2\sin(dx+c)^2-8\,a^3b\sin(dx+c)+3\,a^4}{a^6\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

Mupad [B]
time = 11.81, size = 439, normalized size = 2.34

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(3\,a^4-62\,a^2\,b^2+64\,b^4)-\frac{a^4}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{11\,a^4}{4}-\frac{10\,a^2\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(20\,a\,b^3-\frac{62\,a^3\,b}{3}\right)-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(60\,a^4\,b-96\,a^2\,b^3+32\,b^5)}{a}+\frac{5\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6}}{d\,\left(16\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+32\,b\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{\frac{a^2}{16}+\frac{b^2}{8}}{a^4}+\frac{1}{8\,a^2}-\frac{b^2}{2\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b\,(32\,a^2+64\,b^2)}{64\,a^5}-\frac{b}{4\,a^3}+\frac{4\,b\,\left(\frac{\frac{a^2}{8}+\frac{b^2}{4}}{a^4}+\frac{1}{4\,a^2}-\frac{b^2}{a^4}\right)}{a}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a^4-6\,a^2\,b^2+5\,b^4)}{a^6\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{12\,a^3\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,(a^4-6\,a^2\,b^2+5\,b^4)}{a^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(3*a^4 + 64*b^4 - 62*a^2*b^2) - a^4/4 + tan(c/2 + (d*
x)/2)^2*((11*a^4)/4 - (10*a^2*b^2)/3) + tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (6
2*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 96*a^2*b^3))/a + (
5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*x)/2)^4 + 16*a^6*tan
(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^4/
(64*a^2*d) + (tan(c/2 + (d*x)/2)^2*((a^2/16 + b^2/8)/a^4 + 1/(8*a^2) - b^2/
(2*a^4)))/d - (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) - b/(4*a^
3) + (4*b*((a^2/8 + b^2/4)/a^4 + 1/(4*a^2) - b^2/a^4))/a))/d + (log(tan(c/2
 + (d*x)/2))*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d) + (b*tan(c/2 + (d*x)/2)^3)/
(12*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + 5*b^4 - 6*a^2*b^2))/(a^6*d)
```

3.1234 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=226

$$-\frac{(a^4-6a^2b^2+5b^4)\csc(c+dx)}{a^6d}-\frac{2b(a^2-b^2)\csc^2(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^3(c+dx)}{3a^4d}+\frac{b\csc^4(c+dx)}{2a^3d}-\frac{\csc^5(}{5}$$

```
[Out] -(a^4-6*a^2*b^2+5*b^4)*csc(d*x+c)/a^6/d-2*b*(a^2-b^2)*csc(d*x+c)^2/a^5/d+1/
3*(2*a^2-3*b^2)*csc(d*x+c)^3/a^4/d+1/2*b*csc(d*x+c)^4/a^3/d-1/5*csc(d*x+c)^
5/a^2/d-2*b*(a^4-4*a^2*b^2+3*b^4)*ln(sin(d*x+c))/a^7/d+2*b*(a^4-4*a^2*b^2+3
*b^4)*ln(a+b*sin(d*x+c))/a^7/d-b*(a^2-b^2)^2/a^6/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{b\csc^4(c+dx)}{2a^3d}-\frac{\csc^5(c+dx)}{5a^2d}-\frac{b(a^2-b^2)^2}{a^6d(a+b\sin(c+dx))}-\frac{2b(a^2-b^2)\csc^2(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^3(c+dx)}{3a^4d}-\frac{2b(a^4-4a^2b^2+3b^4)\log(\sin(c+dx))}{a^7d}+\frac{2b(a^4-4a^2b^2+3b^4)\log(a+b\sin(c+dx))}{a^7d}-\frac{(a^4-6a^2b^2+5b^4)\csc(c+dx)}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Csc[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Csc[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^3)/(3*a^4*d) + (b*Csc[c
 + d*x]^4)/(2*a^3*d) - Csc[c + d*x]^5/(5*a^2*d) - (2*b*(a^4 - 4*a^2*b^2 + 3
*b^4)*Log[Sin[c + d*x]])/(a^7*d) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[a + b
*Sin[c + d*x]])/(a^7*d) - (b*(a^2 - b^2)^2)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^6(b^2-x^2)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^6(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{a^2x^6} - \frac{2b^4}{a^3x^5} + \frac{-2a^2b^2+3b^4}{a^4x^4} + \frac{4b^2(a^2-b^2)}{a^5x^3} + \frac{a^4-6a^2b^2+5b^4}{a^6x^2} - \frac{2(a^4-4a^2b^2+}{a^7x}\right.\right.}{d}$$

$$= -\frac{(a^4-6a^2b^2+5b^4)\csc(c+dx)}{a^6d} - \frac{2b(a^2-b^2)\csc^2(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\,c}{3a}$$

Mathematica [A]
time = 2.81, size = 220, normalized size = 0.97

$$\frac{-30a^2(a^4-4a^2b^2+3b^4)\csc^2(c+dx)+(-40a^5b+30a^3b^3)\csc^3(c+dx)+5a^4(4a^2-3b^2)\csc^4(c+dx)+9a^5b\csc^5(c+dx)-6a^6\csc^6(c+dx)-60b^2(a^4-4a^2b^2+3b^4)\left(\log(\sin(c+dx))-\log(a+b\sin(c+dx))\right)-60ab(a^4-4a^2b^2+3b^4)\csc(c+dx)(1+\log(\sin(c+dx))-\log(a+b\sin(c+dx)))}{30a^7d(b+a\csc(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-30*a^2*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c + d*x]^2 + (-40*a^5*b + 30*a^3*b^3
)*Csc[c + d*x]^3 + 5*a^4*(4*a^2 - 3*b^2)*Csc[c + d*x]^4 + 9*a^5*b*Csc[c + d
*x]^5 - 6*a^6*Csc[c + d*x]^6 - 60*b^2*(a^4 - 4*a^2*b^2 + 3*b^4)*(Log[Sin[c
+ d*x]] - Log[a + b*Sin[c + d*x]]) - 60*a*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Csc[c
 + d*x]*(1 + Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(30*a^7*d*(b + a
*Csc[c + d*x]))
```

Maple [A]
time = 0.70, size = 207, normalized size = 0.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a^2/sin(d*x+c)^5-1/3*(-2*a^2+3*b^2)/a^4/sin(d*x+c)^3-(a^4-6*a^2*b
^2+5*b^4)/a^6/sin(d*x+c)+1/2/a^3*b/sin(d*x+c)^4-2*b*(a^2-b^2)/a^5/sin(d*x+c
)^2-2*b*(a^4-4*a^2*b^2+3*b^4)/a^7*ln(sin(d*x+c))-(a^4-2*a^2*b^2+b^4)*b/a^6/
(a+b*sin(d*x+c))+2*b*(a^4-4*a^2*b^2+3*b^4)/a^7*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 225, normalized size = 1.00

$$\frac{\frac{9 a^{4} b \sin(dx+c) - 60 \left(a^{4} b - 4 a^{2} b^{3} + 3 b^{5}\right) \sin(dx+c)^{5} - 6 a^{5} - 30 \left(a^{5} - 4 a^{3} b^{2} + 3 a b^{4}\right) \sin(dx+c)^{4} - 10 \left(4 a^{4} b - 3 a^{2} b^{3}\right) \sin(dx+c)^{3} + 5 \left(4 a^{5} - 3 a^{3} b^{2}\right) \sin(dx+c)^{2}}{a^{6} b \sin(dx+c)^{6} + a^{7} \sin(dx+c)^{5}} + \frac{60 \left(a^{4} b - 4 a^{2} b^{3} + 3 b^{5}\right) \log(b \sin(dx+c) + a)}{a^{7}} - \frac{60 \left(a^{4} b - 4 a^{2} b^{3} + 3 b^{5}\right) \log(\sin(dx+c))}{a^{7}}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/30*((9*a^4*b*sin(d*x + c) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*sin(d*x + c)^5
 - 6*a^5 - 30*(a^5 - 4*a^3*b^2 + 3*a*b^4)*sin(d*x + c)^4 - 10*(4*a^4*b - 3*
a^2*b^3)*sin(d*x + c)^3 + 5*(4*a^5 - 3*a^3*b^2)*sin(d*x + c)^2)/(a^6*b*sin(
d*x + c)^6 + a^7*sin(d*x + c)^5) + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(b*sin
(d*x + c) + a)/a^7 - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(sin(d*x + c))/a^7)/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(220) = 440$.
time = 0.42, size = 696, normalized size = 3.08

$$\frac{16 a^{6} - 105 a^{4} b^{2} + 90 a^{2} b^{4} + 30 \left(a^{6} - 4 a^{4} b^{2} + 3 a^{2} b^{4}\right) \cos(dx+c)^{4} - 5 \left(8 a^{6} - 45 a^{4} b^{2} + 36 a^{2} b^{4}\right) \cos(dx+c)^{2} + 60 \left(\left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{6} - a^{4} b^{2} + 4 a^{2} b^{4} - 3 b^{6} - 3 \left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{4} + 3 \left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{2} - \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5} + \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5}\right) \cos(dx+c)^{4} - 2 \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5}\right) \cos(dx+c)^{2}\right) \sin(dx+c)\right) \log(b \sin(dx+c) + a) - 60 \left(\left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{6} - a^{4} b^{2} + 4 a^{2} b^{4} - 3 b^{6} - 3 \left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{4} + 3 \left(a^{4} b^{2} - 4 a^{2} b^{4} + 3 b^{6}\right) \cos(dx+c)^{2} - \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5} + \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5}\right) \cos(dx+c)^{4} - 2 \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5}\right) \cos(dx+c)^{2}\right) \sin(dx+c)\right) \log\left(\frac{1}{2} \sin(dx+c)\right) + \left(91 a^{5} b - 270 a^{3} b^{3} + 180 a b^{5} + 60 \left(a^{5} b - 4 a^{3} b^{3} + 3 a b^{5}\right) \cos(dx+c)^{4} - 10 \left(16 a^{5} b - 51 a^{3} b^{3} + 36 a b^{5}\right) \cos(dx+c)^{2}\right) \sin(dx+c)}{30 \left(a^{7} b d \cos(dx+c)^{6} - 3 a^{7} b d \cos(dx+c)^{4} + 3 a^{7} b d \cos(dx+c)^{2} - a^{7} b d - \left(a^{8} d \cos(dx+c)^{4} - 2 a^{8} d \cos(dx+c)^{2} + a^{8} d\right) \sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/30*(16*a^6 - 105*a^4*b^2 + 90*a^2*b^4 + 30*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c)^4 - 5*(8*a^6 - 45*a^4*b^2 + 36*a^2*b^4)*cos(d*x + c)^2 + 60*((
a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 -
 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 +
3*b^6)*cos(d*x + c)^2 - (a^5*b - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 +
 3*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(b*sin(d*x + c) + a) - 60*((a^4*b^2 - 4*a^2*b^4 + 3*b^6)*c
os(d*x + c)^6 - a^4*b^2 + 4*a^2*b^4 - 3*b^6 - 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^
6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 4*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 - (a^5*b
 - 4*a^3*b^3 + 3*a*b^5 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^4 - 2*(
a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*sin(d*x
+ c)) + (91*a^5*b - 270*a^3*b^3 + 180*a*b^5 + 60*(a^5*b - 4*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^4 - 10*(16*a^5*b - 51*a^3*b^3 + 36*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))/(a^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*
d*cos(d*x + c)^2 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2
 + a^8*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.48, size = 332, normalized size = 1.47

$$-\frac{\frac{60\,\left(a^4b-4\,a^2b^3+3\,b^5\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^7}-\frac{60\,\left(a^4b^2-4\,a^2b^4+3\,b^6\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^7b}+\frac{30\,\left(2\,a^4b^2\sin\left(dx+c\right)-8\,a^2b^4\sin\left(dx+c\right)+6\,b^6\sin\left(dx+c\right)+3\,a^5b-10\,a^3b^3+7\,ab^5\right)}{\left(b\sin\left(dx+c\right)+a\right)a^7}-\frac{137\,a^4b\sin\left(dx+c\right)^5-548\,a^2b^3\sin\left(dx+c\right)^5+411\,b^5\sin\left(dx+c\right)^5-30\,a^5\sin\left(dx+c\right)^4+180\,a^3b^2\sin\left(dx+c\right)^4-150\,ab^4\sin\left(dx+c\right)^4-60\,a^4b\sin\left(dx+c\right)^3+60\,a^2b^3\sin\left(dx+c\right)^3+20\,a^5\sin\left(dx+c\right)^2-30\,a^3b^2\sin\left(dx+c\right)^2+15\,a^4b\sin\left(dx+c\right)-6\,a^5}{a^7\sin\left(dx+c\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*
b^2 - 4*a^2*b^4 + 3*b^6)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 30*(2*a^4*b
^2*sin(d*x + c) - 8*a^2*b^4*sin(d*x + c) + 6*b^6*sin(d*x + c) + 3*a^5*b - 1
0*a^3*b^3 + 7*a*b^5)/((b*sin(d*x + c) + a)*a^7) - (137*a^4*b*sin(d*x + c)^5
 - 548*a^2*b^3*sin(d*x + c)^5 + 411*b^5*sin(d*x + c)^5 - 30*a^5*sin(d*x + c
)^4 + 180*a^3*b^2*sin(d*x + c)^4 - 150*a*b^4*sin(d*x + c)^4 - 60*a^4*b*sin(
d*x + c)^3 + 60*a^2*b^3*sin(d*x + c)^3 + 20*a^5*sin(d*x + c)^2 - 30*a^3*b^2
*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 6*a^5)/(a^7*sin(d*x + c)^5))/d
```

Mupad [B]
time = 11.90, size = 628, normalized size = 2.78

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{64a^2+128b^2}{3072a^4}+\frac{1}{32a^2}-\frac{b^2}{6a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{b^2}{2a^4}-\frac{4b\left(\frac{b\left(64a^2+128b^2\right)}{256a^5}-\frac{b}{8a^3}+\frac{4b\left(\frac{64a^2+128b^2}{1024a^4}+\frac{3}{32a^2}-\frac{b^2}{2a^4}\right)}{a}\right)}{a}+\frac{\left(64a^2+128b^2\right)\left(\frac{64a^2+128b^2}{1024a^4}+\frac{3}{32a^2}-\frac{b^2}{2a^4}\right)}{32a^2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{23a^4b}{3}-8a^2b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{25a^5}{3}-56a^3b^2+48ab^4\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(32a^4b-184a^2b^3+160b^5\right)+\frac{a^5}{5}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{22a^5}{15}-2a^3b^2\right)-\frac{3a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(5a^6-74a^4b^2+104a^2b^4-32b^6\right)}{a}}{d\left(32a^7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32a^7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+64ba^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{b\left(64a^2+128b^2\right)}{512a^5}-\frac{b}{16a^3}+\frac{2b\left(\frac{64a^2+128b^2}{1024a^4}+\frac{3}{32a^2}-\frac{b^2}{2a^4}\right)}{a}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^4b-8a^2b^3+6b^5\right)}{a^7d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{32a^3d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(2a^4b-8a^2b^3+6b^5\right)}{a^7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 1/(32*a^2) - b^2/(6*
a^4)))/d - (tan(c/2 + (d*x)/2)*(b^2/(2*a^4) - (4*b*((b*(64*a^2 + 128*b^2))/
(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 + 128*b^2)/(1024*a^4) + 3/(32*a^2) -
b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^2)*((64*a^2 + 128*b^2)/(1024*a^4) +
3/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d - tan(c/2 + (d*x)/2)^5/(160*a^2*d)
- (tan(c/2 + (d*x)/2)^3*((23*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 + (d*x)/2)^4*(
48*a*b^4 + (25*a^5)/3 - 56*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(32*a^4*b + 160*
b^5 - 184*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((22*a^5)/15 - 2*a^3*b^2)
 - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(5*a^6 - 32*b^6
 + 104*a^2*b^4 - 74*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)^5 + 32*a^7*t
an(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) - (tan(c/2 + (d*x)/2)
^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*a^2 + 128*b^2
)/(1024*a^4) + 3/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2 + (d*x)/2))*
(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b*tan(c/2 + (d*x)/2)^4)/(32*a^3*d
) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(2*a^4*b + 6*
b^5 - 8*a^2*b^3))/(a^7*d)
```

3.1235 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=170

$$\frac{a^2\sin^{1+n}(c+dx)}{d(1+n)}+\frac{2ab\sin^{2+n}(c+dx)}{d(2+n)}-\frac{(2a^2-b^2)\sin^{3+n}(c+dx)}{d(3+n)}-\frac{4ab\sin^{4+n}(c+dx)}{d(4+n)}+\frac{(a^2-2b^2)\sin^{5+n}(c}{d(5+n)}$$

```
[Out] a^2*sin(d*x+c)^(1+n)/d/(1+n)+2*a*b*sin(d*x+c)^(2+n)/d/(2+n)-(2*a^2-b^2)*sin
(d*x+c)^(3+n)/d/(3+n)-4*a*b*sin(d*x+c)^(4+n)/d/(4+n)+(a^2-2*b^2)*sin(d*x+c)
^(5+n)/d/(5+n)+2*a*b*sin(d*x+c)^(6+n)/d/(6+n)+b^2*sin(d*x+c)^(7+n)/d/(7+n)
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 962}

$$-\frac{(2a^2-b^2)\sin^{n+3}(c+dx)}{d(n+3)}+\frac{(a^2-2b^2)\sin^{n+5}(c+dx)}{d(n+5)}+\frac{a^2\sin^{n+1}(c+dx)}{d(n+1)}+\frac{2ab\sin^{n+2}(c+dx)}{d(n+2)}-\frac{4ab\sin^{n+4}(c+dx)}{d(n+4)}+\frac{2ab\sin^{n+6}(c+dx)}{d(n+6)}+\frac{b^2\sin^{n+7}(c+dx)}{d(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (2*a*b*Sin[c + d*x]^(2 + n))/(d*(2
 + n)) - ((2*a^2 - b^2)*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (4*a*b*Sin[c +
d*x]^(4 + n))/(d*(4 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(5 + n))/(d*(5 + n)
) + (2*a*b*Sin[c + d*x]^(6 + n))/(d*(6 + n)) + (b^2*Sin[c + d*x]^(7 + n))/(
d*(7 + n))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)^2 \left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2b^4\left(\frac{x}{b}\right)^n + 2ab^5\left(\frac{x}{b}\right)^{1+n} - b^4(2a^2-b^2)\left(\frac{x}{b}\right)^{2+}\right.\right.}{}$$

$$= \frac{a^2\sin^{1+n}(c+dx)}{d(1+n)} + \frac{2ab\sin^{2+n}(c+dx)}{d(2+n)} - \frac{(2a^2-b^2)\sin}{d(3}$$

Mathematica [A]
time = 0.52, size = 139, normalized size = 0.82

$$\frac{\sin^{1+n}(c+dx)\left(\frac{a^2}{1+n}+\frac{2ab\sin(c+dx)}{2+n}-\frac{(2a^2-b^2)\sin^2(c+dx)}{3+n}-\frac{4ab\sin^3(c+dx)}{4+n}+\frac{(a^2-2b^2)\sin^4(c+dx)}{5+n}+\frac{2ab\sin^5(c+dx)}{6+n}+\frac{b^2\sin^6(c+dx)}{7+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
[Out] (Sin[c + d*x]^(1 + n)*(a^2/(1 + n) + (2*a*b*Sin[c + d*x])/(2 + n) - ((2*a^2
 - b^2)*Sin[c + d*x]^2)/(3 + n) - (4*a*b*Sin[c + d*x]^3)/(4 + n) + ((a^2 -
2*b^2)*Sin[c + d*x]^4)/(5 + n) + (2*a*b*Sin[c + d*x]^5)/(6 + n) + (b^2*Sin[
c + d*x]^6)/(7 + n)))/d
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [A]
time = 0.28, size = 178, normalized size = 1.05

$$\frac{\frac{b^2\sin(dx+c)^{n+7}}{n+7}+\frac{2ab\sin(dx+c)^{n+6}}{n+6}+\frac{a^2\sin(dx+c)^{n+5}}{n+5}-\frac{2b^2\sin(dx+c)^{n+5}}{n+5}-\frac{4ab\sin(dx+c)^{n+4}}{n+4}-\frac{2a^2\sin(dx+c)^{n+3}}{n+3}+\frac{b^2\sin(dx+c)^{n+3}}{n+3}+\frac{2ab\sin(dx+c)^{n+2}}{n+2}+\frac{a^2\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (b^2*sin(d*x + c)^(n + 7)/(n + 7) + 2*a*b*sin(d*x + c)^(n + 6)/(n + 6) + a^
2*sin(d*x + c)^(n + 5)/(n + 5) - 2*b^2*sin(d*x + c)^(n + 5)/(n + 5) - 4*a*b
*sin(d*x + c)^(n + 4)/(n + 4) - 2*a^2*sin(d*x + c)^(n + 3)/(n + 3) + b^2*si
n(d*x + c)^(n + 3)/(n + 3) + 2*a*b*sin(d*x + c)^(n + 2)/(n + 2) + a^2*sin(d
*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(170) = 340$.
time = 0.42, size = 572, normalized size = 3.36

$$-\frac{2\,(abn^6+22\,abn^5+190\,abn^4+820\,abn^3+1849\,abn^2+2038\,abn+840\,ab)\cos(dx+c)^6-16\,abn^4-256\,abn^3-2\,(abn^6+18\,abn^5+118\,abn^4+348\,abn^3+457\,abn^2+210\,abn)\cos(dx+c)^4-1376\,abn^2-2816\,abn-8\,(abn^5+16\,abn^4+86\,abn^3+176\,abn^2+105\,abn)\cos(dx+c)^2-1680\,ab+\left((b^2n^6+21\,b^2n^5+175\,b^2n^4+735\,b^2n^3+1624\,b^2n^2+1764\,b^2n+720\,b^2)\cos(dx+c)^6-8\,(a^2+b^2)n^4-\left((a^2+b^2)n^6+(23\,a^2+17\,b^2)n^5+3\,(69\,a^2+37\,b^2)n^4+5\,(185\,a^2+71\,b^2)n^3+8\,(268\,a^2+73\,b^2)n^2+1008\,a^2+144\,b^2+36\,(67\,a^2+13\,b^2)n\right)\cos(dx+c)^4-8\,(19\,a^2+13\,b^2)n^3-64\,(16\,a^2+7\,b^2)n^2-4\left((a^2+b^2)n^5+2\,(10\,a^2+7\,b^2)n^4+3\,(49\,a^2+23\,b^2)n^3+4\,(121\,a^2+37\,b^2)n^2+336\,a^2+48\,b^2+4\,(173\,a^2+35\,b^2)n\right)\cos(dx+c)^2-2688\,a^2-384\,b^2-32\,(89\,a^2+23\,b^2)n\right)\sin(dx+c)\right)\sin(dx+c)^n}{dn^7+28\,dn^6+322\,dn^5+1960\,dn^4+6769\,dn^3+13132\,dn^2+13068\,dn+5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(2*(a*b*n^6 + 22*a*b*n^5 + 190*a*b*n^4 + 820*a*b*n^3 + 1849*a*b*n^2 + 2038
*a*b*n + 840*a*b)*cos(d*x + c)^6 - 16*a*b*n^4 - 256*a*b*n^3 - 2*(a*b*n^6 +
18*a*b*n^5 + 118*a*b*n^4 + 348*a*b*n^3 + 457*a*b*n^2 + 210*a*b*n)*cos(d*x +
 c)^4 - 1376*a*b*n^2 - 2816*a*b*n - 8*(a*b*n^5 + 16*a*b*n^4 + 86*a*b*n^3 +
176*a*b*n^2 + 105*a*b*n)*cos(d*x + c)^2 - 1680*a*b + ((b^2*n^6 + 21*b^2*n^5
 + 175*b^2*n^4 + 735*b^2*n^3 + 1624*b^2*n^2 + 1764*b^2*n + 720*b^2)*cos(d*x
 + c)^6 - 8*(a^2 + b^2)*n^4 - ((a^2 + b^2)*n^6 + (23*a^2 + 17*b^2)*n^5 + 3*
(69*a^2 + 37*b^2)*n^4 + 5*(185*a^2 + 71*b^2)*n^3 + 8*(268*a^2 + 73*b^2)*n^2
 + 1008*a^2 + 144*b^2 + 36*(67*a^2 + 13*b^2)*n)*cos(d*x + c)^4 - 8*(19*a^2
+ 13*b^2)*n^3 - 64*(16*a^2 + 7*b^2)*n^2 - 4*((a^2 + b^2)*n^5 + 2*(10*a^2 +
7*b^2)*n^4 + 3*(49*a^2 + 23*b^2)*n^3 + 4*(121*a^2 + 37*b^2)*n^2 + 336*a^2 +
 48*b^2 + 4*(173*a^2 + 35*b^2)*n)*cos(d*x + c)^2 - 2688*a^2 - 384*b^2 - 32*
(89*a^2 + 23*b^2)*n)*sin(d*x + c))*sin(d*x + c)^n/(d*n^7 + 28*d*n^6 + 322*d
*n^5 + 1960*d*n^4 + 6769*d*n^3 + 13132*d*n^2 + 13068*d*n + 5040*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 18260 vs. $2(144) = 288$.
time = 21.77, size = 18260, normalized size = 107.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a + b*sin(c))**2*sin(c)**n*cos(c)**5, Eq(d, 0)), (-a**2/(6*d*
sin(c + d*x)**2) + a**2*cos(c + d*x)**2/(6*d*sin(c + d*x)**4) - a**2*cos(c
+ d*x)**4/(6*d*sin(c + d*x)**6) - 16*a*b/(15*d*sin(c + d*x)) + 8*a*b*cos(c
+ d*x)**2/(15*d*sin(c + d*x)**3) - 2*a*b*cos(c + d*x)**4/(5*d*sin(c + d*x)*
*5) + b**2*log(sin(c + d*x))/d + b**2*cos(c + d*x)**2/(2*d*sin(c + d*x)**2)
 - b**2*cos(c + d*x)**4/(4*d*sin(c + d*x)**4), Eq(n, -7)), (-8*a**2/(15*d*s
```

in(c + d*x)) + 4*a**2*cos(c + d*x)**2/(15*d*sin(c + d*x)**3) - a**2*cos(c + d*x)**4/(5*d*sin(c + d*x)**5) + 2*a*b*log(sin(c + d*x))/d + a*b*cos(c + d*x)**2/(d*sin(c + d*x)**2) - a*b*cos(c + d*x)**4/(2*d*sin(c + d*x)**4) + 8*b**2*sin(c + d*x)/(3*d) + 4*b**2*cos(c + d*x)**2/(3*d*sin(c + d*x)) - b**2*cos(c + d*x)**4/(3*d*sin(c + d*x)**3), Eq(n, -6)), (-192*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 384*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 192*a**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 192*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 384*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 192*a**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 3*a**2*tan(c/2 + d*x/2)**12/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 30*a**2*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 66*a**2*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 30*a**2*tan(c/2 + d*x/2)**2/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 3*a**2/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 16*a*b*tan(c/2 + d*x/2)**11/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 304*a*b*tan(c/2 + d*x/2)**9/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 1760*a*b*tan(c/2 + d*x/2)**7/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 1760*a*b*tan(c/2 + d*x/2)**5/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 304*a*b*tan(c/2 + d*x/2)**3/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 16*a*b*tan(c/2 + d*x/2)/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 384*b**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 768*b**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) + 384*b**2*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 384*b**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 768*b**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 384*b**2*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**4/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 24*b**2*tan(c/2 + d*x/2)**10/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x

```
/2)**4) + 432*b**2*tan(c/2 + d*x/2)**6/(192*d*tan(c/2 + d*x/2)**8 + 384*d*t
an(c/2 + d*x/2)**6 + 192*d*tan(c/2 + d*x/2)**4) - 24*b**2*tan(c/2 + d*x/2)*
*2/(192*d*tan(c/2 + d*x/2)**8 + 384*d*tan(c/2 + d*x/2)**6 + 192*d*tan(c/2 +
 d*x/2)**4), Eq(n, -5)), (-a**2*tan(c/2 + d*x/2)**12/(24*d*tan(c/2 + d*x/2)
**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 +
d*x/2)**3) + 18*a**2*tan(c/2 + d*x/2)**10/(24*d*tan(c/2 + d*x/2)**9 + 72*d*
tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3)
+ 129*a**2*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d
*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 220*a**2*
tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 +
72*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 129*a**2*tan(c/2 + d
*x/2)**4/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/
2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 18*a**2*tan(c/2 + d*x/2)**2/(24
*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**
5 + 24*d*tan(c/2 + d*x/2)**3) - a**2/(24*d*tan(c/2 + d*x/2)**9 + 72*d*tan(c
/2 + d*x/2)**7 + 72*d*tan(c/2 + d*x/2)**5 + 24*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(170) = 340.
time = 0.56, size = 575, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n
*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d
*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c
)^n*sin(d*x + c))*a^2/(n^3 + 9*n^2 + 23*n + 15) + 2*(n^2*sin(d*x + c)^n*sin
(d*x + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(
d*x + c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x
+ c)^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)
^4 + 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)
*a*b/(n^3 + 12*n^2 + 44*n + 48) + (n^2*sin(d*x + c)^n*sin(d*x + c)^7 + 8*n*
sin(d*x + c)^n*sin(d*x + c)^7 - 2*n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 15*si
n(d*x + c)^n*sin(d*x + c)^7 - 20*n*sin(d*x + c)^n*sin(d*x + c)^5 + n^2*sin(
d*x + c)^n*sin(d*x + c)^3 - 42*sin(d*x + c)^n*sin(d*x + c)^5 + 12*n*sin(d*x
 + c)^n*sin(d*x + c)^3 + 35*sin(d*x + c)^n*sin(d*x + c)^3)*b^2/(n^3 + 15*n^
2 + 71*n + 105))/d
```

Mupad [B]
time = 18.92, size = 887, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + b*sin(c + d*x))^2,x)

[Out] (sin(c + d*x)*sin(c + d*x)^n*(44*n + 12*n^2 + n^3 + 48)*(1272*a^2*n + 581*b^2*n + 4200*a^2 + 525*b^2 + 152*a^2*n^2 + 8*a^2*n^3 + 59*b^2*n^2 + 3*b^2*n^3)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (sin(c + d*x)^n*sin(5*c + 5*d*x)*(4*a^2*n - b^2*n + 28*a^2 - 21*b^2)*(324*n + 260*n^2 + 95*n^3 + 16*n^4 + n^5 + 144)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (b^2*sin(c + d*x)^n*sin(7*c + 7*d*x)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (sin(c + d*x)^n*sin(3*c + 3*d*x)*(92*n + 56*n^2 + 13*n^3 + n^4 + 48)*(184*a^2*n + 40*b^2*n + 700*a^2 - 35*b^2 + 12*a^2*n^2 + 3*b^2*n^2)*1i)/(64*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) + (a*b*sin(c + d*x)^n*(n*16958i + n^2*10137i + n^3*2788i + n^4*398i + n^5*30i + n^6*1i + 9240i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a*b*sin(c + d*x)^n*cos(6*c + 6*d*x)*(n*2038i + n^2*1849i + n^3*820i + n^4*190i + n^5*22i + n^6*1i + 840i))/(16*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a*b*sin(c + d*x)^n*cos(4*c + 4*d*x)*(n*5694i + n^2*4633i + n^3*1764i + n^4*334i + n^5*30i + n^6*1i + 2520i))/(8*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i)) - (a*b*sin(c + d*x)^n*cos(2*c + 2*d*x)*(n*20490i + n^2*9159i + n^3*1228i - n^4*62i - n^5*22i - n^6*1i + 12600i))/(16*d*(n*13068i + n^2*13132i + n^3*6769i + n^4*1960i + n^5*322i + n^6*28i + n^7*1i + 5040i))

3.1236 $\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=123

$$\frac{a\sin^{1+n}(c+dx)}{d(1+n)}+\frac{b\sin^{2+n}(c+dx)}{d(2+n)}-\frac{2a\sin^{3+n}(c+dx)}{d(3+n)}-\frac{2b\sin^{4+n}(c+dx)}{d(4+n)}+\frac{a\sin^{5+n}(c+dx)}{d(5+n)}+\frac{b\sin^{6+n}(c+dx)}{d(6+n)}$$

```
[Out] a*sin(d*x+c)^(1+n)/d/(1+n)+b*sin(d*x+c)^(2+n)/d/(2+n)-2*a*sin(d*x+c)^(3+n)/
d/(3+n)-2*b*sin(d*x+c)^(4+n)/d/(4+n)+a*sin(d*x+c)^(5+n)/d/(5+n)+b*sin(d*x+c
)^(6+n)/d/(6+n)
```

Rubi [A]
time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2916, 780}

$$\frac{a\sin^{n+1}(c+dx)}{d(n+1)}-\frac{2a\sin^{n+3}(c+dx)}{d(n+3)}+\frac{a\sin^{n+5}(c+dx)}{d(n+5)}+\frac{b\sin^{n+2}(c+dx)}{d(n+2)}-\frac{2b\sin^{n+4}(c+dx)}{d(n+4)}+\frac{b\sin^{n+6}(c+dx)}{d(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Sin[c + d*x]^(1 + n))/(d*(1 + n)) + (b*Sin[c + d*x]^(2 + n))/(d*(2 + n))
 - (2*a*Sin[c + d*x]^(3 + n))/(d*(3 + n)) - (2*b*Sin[c + d*x]^(4 + n))/(d*(
4 + n)) + (a*Sin[c + d*x]^(5 + n))/(d*(5 + n)) + (b*Sin[c + d*x]^(6 + n))/(
d*(6 + n))
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \left(\frac{x}{b}\right)^n (a+x)\left(b^2-x^2\right)^2\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(ab^4\left(\frac{x}{b}\right)^n + b^5\left(\frac{x}{b}\right)^{1+n} - 2ab^4\left(\frac{x}{b}\right)^{2+n} - 2b^5\left(\frac{x}{b}\right)^{3-}\right.\right.}{b^5 d}$$

$$= \frac{a\sin^{1+n}(c+dx)}{d(1+n)} + \frac{b\sin^{2+n}(c+dx)}{d(2+n)} - \frac{2a\sin^{3+n}(c+dx)}{d(3+n)}$$

Mathematica [A]
time = 0.15, size = 97, normalized size = 0.79

$$\frac{\sin^{1+n}(c+dx)\left(\frac{a}{1+n}+\frac{b\sin(c+dx)}{2+n}-\frac{2a\sin^2(c+dx)}{3+n}-\frac{2b\sin^3(c+dx)}{4+n}+\frac{a\sin^4(c+dx)}{5+n}+\frac{b\sin^5(c+dx)}{6+n}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^(1 + n)*(a/(1 + n) + (b*Sin[c + d*x])/(2 + n) - (2*a*Sin[c +
d*x]^2)/(3 + n) - (2*b*Sin[c + d*x]^3)/(4 + n) + (a*Sin[c + d*x]^4)/(5 + n)
 + (b*Sin[c + d*x]^5)/(6 + n)))/d
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\cos^5(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+b\sin(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [A]
time = 0.28, size = 109, normalized size = 0.89

$$\frac{\frac{b\sin(dx+c)^{n+6}}{n+6}+\frac{a\sin(dx+c)^{n+5}}{n+5}-\frac{2\,b\sin(dx+c)^{n+4}}{n+4}-\frac{2\,a\sin(dx+c)^{n+3}}{n+3}+\frac{b\sin(dx+c)^{n+2}}{n+2}+\frac{a\sin(dx+c)^{n+1}}{n+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*sin(d*x + c)^(n + 6)/(n + 6) + a*sin(d*x + c)^(n + 5)/(n + 5) - 2*b*sin(
d*x + c)^(n + 4)/(n + 4) - 2*a*sin(d*x + c)^(n + 3)/(n + 3) + b*sin(d*x + c
)^(n + 2)/(n + 2) + a*sin(d*x + c)^(n + 1)/(n + 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(123) = 246$.
time = 0.41, size = 282, normalized size = 2.29

$$-\frac{\left(\left(bn^5+15\,bn^4+85\,bn^3+225\,bn^2+274\,bn+120\,b\right)\cos\left(dx+c\right)^6-\left(bn^5+11\,bn^4+41\,bn^3+61\,bn^2+30\,bn\right)\cos\left(dx+c\right)^4-8\,bn^3-72\,bn^2-4\left(bn^4+9\,bn^3+23\,bn^2+15\,bn\right)\cos\left(dx+c\right)^2-184\,bn-\left(\left(an^5+16\,an^4+95\,an^3+260\,an^2+324\,an+144\,a\right)\cos\left(dx+c\right)^4+8\,an^3+96\,an^2+4\left(an^4+13\,an^3+56\,an^2+92\,an+48\,a\right)\cos\left(dx+c\right)^2+352\,an+384\,a\right)\sin\left(dx+c\right)-120\,b\right)\sin\left(dx+c\right)^n}{dn^6+21\,dn^5+175\,dn^4+735\,dn^3+1624\,dn^2+1764\,dn+720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((b*n^5 + 15*b*n^4 + 85*b*n^3 + 225*b*n^2 + 274*b*n + 120*b)*cos(d*x + c)^
6 - (b*n^5 + 11*b*n^4 + 41*b*n^3 + 61*b*n^2 + 30*b*n)*cos(d*x + c)^4 - 8*b*
n^3 - 72*b*n^2 - 4*(b*n^4 + 9*b*n^3 + 23*b*n^2 + 15*b*n)*cos(d*x + c)^2 - 1
84*b*n - ((a*n^5 + 16*a*n^4 + 95*a*n^3 + 260*a*n^2 + 324*a*n + 144*a)*cos(d
*x + c)^4 + 8*a*n^3 + 96*a*n^2 + 4*(a*n^4 + 13*a*n^3 + 56*a*n^2 + 92*a*n +
48*a)*cos(d*x + c)^2 + 352*a*n + 384*a)*sin(d*x + c) - 120*b)*sin(d*x + c)^
n/(d*n^6 + 21*d*n^5 + 175*d*n^4 + 735*d*n^3 + 1624*d*n^2 + 1764*d*n + 720*d
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 8675 vs. $2(104) = 208$.
time = 11.76, size = 8675, normalized size = 70.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(a + b*sin(c))*sin(c)**n*cos(c)**5, Eq(d, 0)), (-8*a/(15*d*sin
(c + d*x)) + 4*a*cos(c + d*x)**2/(15*d*sin(c + d*x)**3) - a*cos(c + d*x)**4
/(5*d*sin(c + d*x)**5) + b*log(sin(c + d*x))/d + b*cos(c + d*x)**2/(2*d*sin
(c + d*x)**2) - b*cos(c + d*x)**4/(4*d*sin(c + d*x)**4), Eq(n, -6)), (a*log
(sin(c + d*x))/d + a*cos(c + d*x)**2/(2*d*sin(c + d*x)**2) - a*cos(c + d*x)
**4/(4*d*sin(c + d*x)**4) + 8*b*sin(c + d*x)/(3*d) + 4*b*cos(c + d*x)**2/(3
*d*sin(c + d*x)) - b*cos(c + d*x)**4/(3*d*sin(c + d*x)**3), Eq(n, -5)), (-a
*tan(c/2 + d*x/2)**10/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5
+ 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/2)**8/(24*d*tan(c/2 + d*x/
2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 110*a*tan(c/
2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*t
an(c/2 + d*x/2)**3) + 110*a*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**7 +
 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 19*a*tan(c/2 + d*x/
2)**2/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 +
 d*x/2)**3) - a/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d
```

*tan(c/2 + d*x/2)**3) + 48*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)*
*7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*
x/2)**3) + 96*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**5/(24*d*tan(
c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) + 48
*b*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)*
*7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 48*b*log(tan(c/
2 + d*x/2))*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 +
d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 96*b*log(tan(c/2 + d*x/2))*tan(c/2
+ d*x/2)**5/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan
(c/2 + d*x/2)**3) - 48*b*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**3/(24*d*ta
n(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) -
3*b*tan(c/2 + d*x/2)**9/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**
5 + 24*d*tan(c/2 + d*x/2)**3) + 54*b*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*
x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(c/2 + d*x/2)**3) - 3*b*tan(c/
2 + d*x/2)/(24*d*tan(c/2 + d*x/2)**7 + 48*d*tan(c/2 + d*x/2)**5 + 24*d*tan(
c/2 + d*x/2)**3), Eq(n, -4)), (48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 +
d*x/2)**8/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c
/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c/2 + d*x/2)**2
+ 1)*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)*
*6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) + 144*a*log(tan(c
/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*ta
n(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) +
48*a*log(tan(c/2 + d*x/2)**2 + 1)*tan(c/2 + d*x/2)**2/(24*d*tan(c/2 + d*x/2
)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 +
d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**8/(24*d*tan(c/2
+ d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*ta
n(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**6/(24*d*
tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 +
24*d*tan(c/2 + d*x/2)**2) - 144*a*log(tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**
4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x
/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 48*a*log(tan(c/2 + d*x/2))*tan(c/2 + d
*x/2)**2/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/
2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3*a*tan(c/2 + d*x/2)**10/(24*d*
tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 +
24*d*tan(c/2 + d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**6/(24*d*tan(c/2 + d*x/2
)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 +
d*x/2)**2) + 63*a*tan(c/2 + d*x/2)**4/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan
(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 3
*a/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*
x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*b*tan(c/2 + d*x/2)**9/(24*d*tan(c/
2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*
tan(c/2 + d*x/2)**2) - 144*b*tan(c/2 + d*x/2)**7/(24*d*tan(c/2 + d*x/2)**8
+ 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/
2)**2) - 200*b*tan(c/2 + d*x/2)**5/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2
+ d*x/2)**6 + 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 144*b

```
*tan(c/2 + d*x/2)**3/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 +
 72*d*tan(c/2 + d*x/2)**4 + 24*d*tan(c/2 + d*x/2)**2) - 12*b*tan(c/2 + d*x/
2)/(24*d*tan(c/2 + d*x/2)**8 + 72*d*tan(c/2 + d*x/2)**6 + 72*d*tan(c/2 + d*
x/2)**4 + 24*d*tan(c/2 + d*x/2)**2), Eq(n, -3)), (-3*a*tan(c/2 + d*x/2)**10
/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2 + d*x/2)**7 + 36*d*tan(c/2 + d*x/2
)**5 + 24*d*tan(c/2 + d*x/2)**3 + 6*d*tan(c/2 + d*x/2)) - 39*a*tan(c/2 + d*
x/2)**8/(6*d*tan(c/2 + d*x/2)**9 + 24*d*tan(c/2...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(123) = 246$.
time = 0.49, size = 379, normalized size = 3.08

$$\frac{\frac{\left(n^2\sin(dx+c)^n\sin(dx+c)^5+4n\sin(dx+c)^n\sin(dx+c)^5-2n^2\sin(dx+c)^n\sin(dx+c)^3+3\sin(dx+c)^n\sin(dx+c)^5-12n\sin(dx+c)^n\sin(dx+c)^3+n^2\sin(dx+c)^n\sin(dx+c)-10\sin(dx+c)^n\sin(dx+c)^3+8n\sin(dx+c)^n\sin(dx+c)+15\sin(dx+c)^n\sin(dx+c)\right)a}{n^3+9n^2+23n+15}+\frac{\left(n^2\sin(dx+c)^n\sin(dx+c)^6+6n\sin(dx+c)^n\sin(dx+c)^6-2n^2\sin(dx+c)^n\sin(dx+c)^4+8\sin(dx+c)^n\sin(dx+c)^6-16n\sin(dx+c)^n\sin(dx+c)^4+n^2\sin(dx+c)^n\sin(dx+c)^2-24\sin(dx+c)^n\sin(dx+c)^4+10n\sin(dx+c)^n\sin(dx+c)^2+24\sin(dx+c)^n\sin(dx+c)^2\right)b}{n^3+12n^2+44n+48}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] ((n^2*sin(d*x + c)^n*sin(d*x + c)^5 + 4*n*sin(d*x + c)^n*sin(d*x + c)^5 - 2
*n^2*sin(d*x + c)^n*sin(d*x + c)^3 + 3*sin(d*x + c)^n*sin(d*x + c)^5 - 12*n
*sin(d*x + c)^n*sin(d*x + c)^3 + n^2*sin(d*x + c)^n*sin(d*x + c) - 10*sin(d
*x + c)^n*sin(d*x + c)^3 + 8*n*sin(d*x + c)^n*sin(d*x + c) + 15*sin(d*x + c
)^n*sin(d*x + c))*a/(n^3 + 9*n^2 + 23*n + 15) + (n^2*sin(d*x + c)^n*sin(d*x
 + c)^6 + 6*n*sin(d*x + c)^n*sin(d*x + c)^6 - 2*n^2*sin(d*x + c)^n*sin(d*x
+ c)^4 + 8*sin(d*x + c)^n*sin(d*x + c)^6 - 16*n*sin(d*x + c)^n*sin(d*x + c)
^4 + n^2*sin(d*x + c)^n*sin(d*x + c)^2 - 24*sin(d*x + c)^n*sin(d*x + c)^4 +
 10*n*sin(d*x + c)^n*sin(d*x + c)^2 + 24*sin(d*x + c)^n*sin(d*x + c)^2)*b/(
n^3 + 12*n^2 + 44*n + 48))/d
```

Mupad [**B**]
time = 16.34, size = 550, normalized size = 4.47

$$\frac{b\sin(c+dx)^n\left(n^5+23n^4+237n^3+1129n^2+2234n+1320\right)}{16d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{b\sin(c+dx)^n\cos(6c+6dx)\left(n^5+15n^4+85n^3+225n^2+274n+120\right)}{32d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{b\sin(c+dx)^n\cos(4c+4dx)\left(n^5+23n^4+173n^3+553n^2+762n+360\right)}{16d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{a\sin(c+dx)\sin(c+dx)^n\left(n^5\,1\mathrm{i}+n^4\,24\mathrm{i}+n^3\,263\mathrm{i}+n^2\,1476\mathrm{i}+n\,3876\mathrm{i}+3600\mathrm{i}\right)\,1\mathrm{i}}{8d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{b\sin(c+dx)^n\cos(2c+2dx)\left(-n^5-15n^4+43n^3+927n^2+2670n+1800\right)}{32d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{a\sin(c+dx)^n\sin(5c+5dx)\left(n^5\,1\mathrm{i}+n^4\,16\mathrm{i}+n^3\,95\mathrm{i}+n^2\,260\mathrm{i}+n\,324\mathrm{i}+144\mathrm{i}\right)\,1\mathrm{i}}{16d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}-\frac{a\sin(c+dx)^n\sin(3c+3dx)\left(n^5\,3\mathrm{i}+n^4\,64\mathrm{i}+n^3\,493\mathrm{i}+n^2\,1676\mathrm{i}+n\,2444\mathrm{i}+1200\mathrm{i}\right)\,1\mathrm{i}}{16d\left(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*sin(c + d*x)^n*(a + b*sin(c + d*x)),x)
```

```
[Out] (b*sin(c + d*x)^n*(2234*n + 1129*n^2 + 237*n^3 + 23*n^4 + n^5 + 1320))/(16*
d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (b*sin(c
+ d*x)^n*cos(6*c + 6*d*x)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (b*si
n(c + d*x)^n*cos(4*c + 4*d*x)*(762*n + 553*n^2 + 173*n^3 + 23*n^4 + n^5 + 3
60))/(16*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) -
(a*sin(c + d*x)*sin(c + d*x)^n*(n*3876i + n^2*1476i + n^3*263i + n^4*24i +
n^5*1i + 3600i)*1i)/(8*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 +
n^6 + 720)) - (b*sin(c + d*x)^n*cos(2*c + 2*d*x)*(2670*n + 927*n^2 + 43*n^3
 - 15*n^4 - n^5 + 1800))/(32*d*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*
```

```
n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(5*c + 5*d*x)*(n*324i + n^2*260i +
 n^3*95i + n^4*16i + n^5*1i + 144i)*1i)/(16*d*(1764*n + 1624*n^2 + 735*n^3
+ 175*n^4 + 21*n^5 + n^6 + 720)) - (a*sin(c + d*x)^n*sin(3*c + 3*d*x)*(n*24
44i + n^2*1676i + n^3*493i + n^4*64i + n^5*3i + 1200i)*1i)/(16*d*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))
```

3.1237 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=167

$$-\frac{a(a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)}+\frac{(a^2-b^2)^2\ {}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)\sin^{1+n}(c+dx)}{ab^4d(1+n)}+\frac{(a^2-2b^2)\sin^{2+n}}{b^3d(2+n}$$

```
[Out] -a*(a^2-2*b^2)*sin(d*x+c)^(1+n)/b^4/d/(1+n)+(a^2-b^2)^2*hypergeom([1, 1+n],
[2+n],-b*sin(d*x+c)/a)*sin(d*x+c)^(1+n)/a/b^4/d/(1+n)+(a^2-2*b^2)*sin(d*x+c
)^(2+n)/b^3/d/(2+n)-a*sin(d*x+c)^(3+n)/b^2/d/(3+n)+sin(d*x+c)^(4+n)/b/d/(4+
n)
```

Rubi [A]

time = 0.23, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 966, 1634, 66}

$$\frac{(a^2-b^2)^2\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ab^4d(n+1)}-\frac{a(a^2-2b^2)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{(a^2-2b^2)\sin^{n+2}(c+dx)}{b^3d(n+2)}-\frac{a\sin^{n+3}(c+dx)}{b^2d(n+3)}+\frac{\sin^{n+4}(c+dx)}{bd(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n))) + ((a^2 - b^2)^2*
Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*Sin[c + d*x]^(1 +
 n))/(a*b^4*d*(1 + n)) + ((a^2 - 2*b^2)*Sin[c + d*x]^(2 + n))/(b^3*d*(2 + n
)) - (a*Sin[c + d*x]^(3 + n))/(b^2*d*(3 + n)) + Sin[c + d*x]^(4 + n)/(b*d*(
4 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 966

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x)^(n + 1)/(g*e
^(2*p)*(m + n + 2*p + 1))), x] + Dist[1/(g*e^(2*p)*(m + n + 2*p + 1)), Int[
(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2*p)*(a + c*x^2
)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*(d + e*x)^(2*p - 1),
 x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d
^2 + a*e^2, 0] && IGtQ[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||
```

```
!IntegerQ[m])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(b^2-x^2\right)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(4+n-\frac{2(4+n)x^2}{b^2}-\frac{a(4+n)x^3}{b^4}\right)}{a+x}\,dx, x, b\sin(c+dx)\right.}{bd(4+n)}$$

$$= \frac{\sin^{4+n}(c+dx)}{bd(4+n)} + \frac{\text{Subst}\left(\int \left(-\frac{a(a^2-2b^2)(4+n)\left(\frac{x}{b}\right)^n}{b^4} - \frac{(-a^2+2b^2)(4+n)\left(\frac{x}{b}\right)^{1+n}}{b^3} - \cdots\right.\right.}{bd(4+\cdots}$$

$$= -\frac{a(a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{(a^2-2b^2)\sin^{2+n}(c+dx)}{b^3d(2+n)} - \frac{a\sin^{3+n}(c+d\cdots}{b^2d(3+n)}$$

$$= -\frac{a(a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{(a^2-b^2)^2\,{}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)\cdots}{ab^4d(1+n)}$$

Mathematica [A]
time = 0.41, size = 133, normalized size = 0.80

$$\frac{\sin^{1+n}(c+dx)\left(-\frac{a^3-2ab^2}{1+n}+\frac{(a^2-b^2)^2\,{}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)}{a(1+n)}+\frac{b(a^2-2b^2)\sin(c+dx)}{2+n}-\frac{ab^2\sin^2(c+dx)}{3+n}+\frac{b^3\sin^3(c+dx)}{4+n}\right)}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*(-((a^3 - 2*a*b^2)/(1 + n)) + ((a^2 - b^2)^2*Hypergeo
metric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a*(1 + n)) + (b*(a^2 -
2*b^2)*Sin[c + d*x])/(2 + n) - (a*b^2*Sin[c + d*x]^2)/(3 + n) + (b^3*Sin[c
+ d*x]^3)/(4 + n)))/(b^4*d)
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)}{a+b\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3881 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^n}{a+b\,\sin\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x)), x)
```

3.1238 $\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=191

$$\frac{(3a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)}+\frac{(a^2-b^2)\left(b^2n-a^2(4+n)\right)\,{}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)\sin^{1+n}(c+dx)}{a^2b^4d(1+n)}-\frac{2}{}$$

```
[Out] (3*a^2-2*b^2)*sin(d*x+c)^(1+n)/b^4/d/(1+n)+(a^2-b^2)*(b^2*n-a^2*(4+n))*hype
rgeom([1, 1+n],[2+n],-b*sin(d*x+c)/a)*sin(d*x+c)^(1+n)/a^2/b^4/d/(1+n)-2*a*
sin(d*x+c)^(2+n)/b^3/d/(2+n)+sin(d*x+c)^(3+n)/b^2/d/(3+n)+(a^2-b^2)^2*sin(d
*x+c)^(1+n)/a/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 964, 1634, 66}

$$\frac{(a^2-b^2)\left(b^2n-a^2(n+4)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{a^2b^4d(n+1)}+\frac{(3a^2-2b^2)\sin^{n+1}(c+dx)}{b^4d(n+1)}+\frac{(a^2-b^2)^2\sin^{n+1}(c+dx)}{ab^4d(a+b\sin(c+dx))}-\frac{2a\sin^{n+2}(c+dx)}{b^3d(n+2)}+\frac{\sin^{n+3}(c+dx)}{b^2d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2)*Sin[c + d*x]^(1 + n))/(b^4*d*(1 + n)) + ((a^2 - b^2)*(b^2*
n - a^2*(4 + n))*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)]*
Sin[c + d*x]^(1 + n))/(a^2*b^4*d*(1 + n)) - (2*a*Sin[c + d*x]^(2 + n))/(b^3
*d*(2 + n)) + Sin[c + d*x]^(3 + n)/(b^2*d*(3 + n)) + ((a^2 - b^2)^2*Sin[c +
 d*x]^(1 + n))/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 964

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + c*x^2)^p, d + e*x
, x], R = PolynomialRemainder[(a + c*x^2)^p, d + e*x, x]}, Simp[R*(d + e*x)
^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))), x] + Dist[1/((m + 1)*(e
*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*ExpandToSum[(m + 1)*(e*f - d*
g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^n(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(b^2-x^2)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$

$$= \frac{(a^2-b^2)^2\sin^{1+n}(c+dx)}{ab^4d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n\left(-b^3n-\frac{a^4(1+n)}{b}+2a^2b(1+n)+a\left(\frac{a^2}{b}-2b\right)x\right.}{a+x}\right.}{ab^4d}$$

$$= \frac{(a^2-b^2)^2\sin^{1+n}(c+dx)}{ab^4d(a+b\sin(c+dx))} + \frac{\text{Subst}\left(\int \left(\frac{(3a^3-2ab^2)\left(\frac{x}{b}\right)^n}{b} - 2a^2\left(\frac{x}{b}\right)^{1+n} + ab\left(\frac{x}{b}\right)\right.\right.}{}$$

$$= \frac{(3a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)} - \frac{2a\sin^{2+n}(c+dx)}{b^3d(2+n)} + \frac{\sin^{3+n}(c+dx)}{b^2d(3+n)} + \frac{(a^2-}{ab^4d}$$

$$= \frac{(3a^2-2b^2)\sin^{1+n}(c+dx)}{b^4d(1+n)} + \frac{(a^2-b^2)\left(b^2n-a^2(4+n)\right)\,{}_2F_1\left(1,1+n;2+\right.}{a^2b^4d(1+n)}$$

Mathematica [A]
time = 0.31, size = 143, normalized size = 0.75

$$\frac{\sin^{1+n}(c+dx)\left(\frac{3a^2-2b^2}{1+n}-\frac{4(a^2-b^2)\,{}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)}{1+n}+\frac{(a^2-b^2)^2\,{}_2F_1\left(2,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)}{a^2(1+n)}-\frac{2ab\sin(c+dx)}{2+n}+\frac{b^2\sin^2(c+dx)}{3+n}\right)}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sin[c + d*x]^(1 + n)*((3*a^2 - 2*b^2)/(1 + n) - (4*(a^2 - b^2)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(1 + n) + ((a^2 - b^2)^2*Hy
pergeometric2F1[2, 1 + n, 2 + n, -((b*Sin[c + d*x])/a)])/(a^2*(1 + n)) - (2
*a*b*Sin[c + d*x])/(2 + n) + (b^2*Sin[c + d*x]^2)/(3 + n)))/(b^4*d)
```

Maple [F]
time = 1.44, size = 0, normalized size = 0.00

$$\int \frac{(\cos^5(dx+c))(\sin^n(dx+c))}{(a+b\sin(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] integral(-sin(d*x + c)^n*cos(d*x + c)^5/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5990 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*cos(d*x + c)^5/(b*sin(d*x + c) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)^n}{\left(a+b\,\sin\left(c+d\,x\right)\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x))^2,x)
[Out] int((cos(c + d*x)^5*sin(c + d*x)^n)/(a + b*sin(c + d*x))^2, x)
```

3.1239 $\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=238

$$\frac{5abx}{512}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{b^2\cos^{13}(c+dx)}{13d}+\frac{5ab\cos}{}$$

```
[Out] 5/512*a*b*x-1/7*(a^2+b^2)*cos(d*x+c)^7/d+1/9*(2*a^2+3*b^2)*cos(d*x+c)^9/d-1
/11*(a^2+3*b^2)*cos(d*x+c)^11/d+1/13*b^2*cos(d*x+c)^13/d+5/512*a*b*cos(d*x+
c)*sin(d*x+c)/d+5/768*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/192*a*b*cos(d*x+c)^5*
sin(d*x+c)/d-1/32*a*b*cos(d*x+c)^7*sin(d*x+c)/d-1/12*a*b*cos(d*x+c)^7*sin(d
*x+c)^3/d-1/6*a*b*cos(d*x+c)^7*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.24, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2990, 2648, 2715, 8, 3280, 457, 78}

$$-\frac{(a^2+3b^2)\cos^{11}(c+dx)}{11d}+\frac{(2a^2+3b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^5(c+dx)\cos^7(c+dx)}{6d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{12d}-\frac{ab\sin(c+dx)\cos^7(c+dx)}{32d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{192d}+\frac{5ab\sin(c+dx)\cos^3(c+dx)}{768d}+\frac{5ab\sin(c+dx)\cos(c+dx)}{512d}+\frac{5abx}{512}+\frac{b^2\cos^{13}(c+dx)}{13d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/512 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((2*a^2 + 3*b^2)*Cos[c
 + d*x]^9)/(9*d) - ((a^2 + 3*b^2)*Cos[c + d*x]^11)/(11*d) + (b^2*Cos[c + d*
x]^13)/(13*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(512*d) + (5*a*b*Cos[c +
d*x]^3*Sin[c + d*x])/(768*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(192*d) -
(a*b*Cos[c + d*x]^7*Sin[c + d*x])/(32*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]
^3)/(12*d) - (a*b*Cos[c + d*x]^7*Sin[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3280

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^6(c+dx)\,dx + \int \cos^6(c+dx$$

$$= -\frac{ab\cos^7(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5ab)\int \cos^6(c+dx$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{12d} - \frac{ab\cos^7(c+dx)\sin^5(c$$

$$= -\frac{ab\cos^7(c+dx)\sin(c+dx)}{32d} - \frac{ab\cos^7(c+dx)\sin^3(c}{12d}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} -$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} -$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+dx)}{9d} -$$

$$= \frac{5abx}{512} - \frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(2a^2+3b^2)\cos^9(c+}{9d}$$

Mathematica [A]
time = 1.53, size = 210, normalized size = 0.88

$$\frac{360360abc + 360360abdx - 180180(2a^2+b^2)\cos(c+dx) - 15015(8a^2+3b^2)\cos(3(c+dx)) + 36036a^2\cos(5(c+dx)) + 27027b^2\cos(5(c+dx)) + 25740a^2\cos(7(c+dx)) + 7722b^2\cos(7(c+dx)) - 4004a^2\cos(9(c+dx)) - 6006b^2\cos(9(c+dx)) - 3276a^2\cos(11(c+dx)) - 819b^2\cos(11(c+dx)) + 693b^2\cos(13(c+dx)) - 135135ab\sin(4(c+dx)) + 27027ab\sin(8(c+dx)) - 3003ab\sin(12(c+dx))}{36900864d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (360360*a*b*c + 360360*a*b*d*x - 180180*(2*a^2 + b^2)*Cos[c + d*x] - 15015*
(8*a^2 + 3*b^2)*Cos[3*(c + d*x)] + 36036*a^2*Cos[5*(c + d*x)] + 27027*b^2*C
os[5*(c + d*x)] + 25740*a^2*Cos[7*(c + d*x)] + 7722*b^2*Cos[7*(c + d*x)] -
4004*a^2*Cos[9*(c + d*x)] - 6006*b^2*Cos[9*(c + d*x)] - 3276*a^2*Cos[11*(c
+ d*x)] - 819*b^2*Cos[11*(c + d*x)] + 693*b^2*Cos[13*(c + d*x)] - 135135*a*
b*Sin[4*(c + d*x)] + 27027*a*b*Sin[8*(c + d*x)] - 3003*a*b*Sin[12*(c + d*x)
])/(36900864*d)
```

Maple [A]
time = 1.15, size = 225, normalized size = 0.95

method	result

derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99}-\frac{8\left(\cos^7(dx+c)\right)}{693}\right)+2ab\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7(dx+c)\right)}{12}-\frac{\left(\sin^3(\right.}{}\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin^4(dx+c)\right)\left(\cos^7(dx+c)\right)}{11}-\frac{4\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{99}-\frac{8\left(\cos^7(dx+c)\right)}{693}\right)+2ab\left(-\frac{\left(\sin^5(dx+c)\right)\left(\cos^7(dx+c)\right)}{12}-\frac{\left(\sin^3(\right.}{}\right.}{}$
risch	$\frac{5abx}{512}-\frac{5a^2\cos(dx+c)}{512d}-\frac{5b^2\cos(dx+c)}{1024d}+\frac{b^2\cos(13dx+13c)}{53248d}-\frac{ab\sin(12dx+12c)}{12288d}-\frac{\cos(11dx+11c)a^2}{11264d}-\frac{\cos(11c}{45}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/
693*cos(d*x+c)^7)+2*a*b*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c)+b^2*(-1/13*sin(d*x+c)^
6*cos(d*x+c)^7-6/143*sin(d*x+c)^4*cos(d*x+c)^7-8/429*sin(d*x+c)^2*cos(d*x+c
)^7-16/3003*cos(d*x+c)^7))
```

Maxima [A]

time = 0.28, size = 135, normalized size = 0.57

$$-\frac{53248\left(63\cos(dx+c)^{11}-154\cos(dx+c)^9+99\cos(dx+c)^7\right)a^2-3003\left(4\sin(4dx+4c)^3+120dx+120c+9\sin(8dx+8c)-48\sin(4dx+4c)\right)ab-12288\left(231\cos(dx+c)^{13}-819\cos(dx+c)^{11}+1001\cos(dx+c)^9-429\cos(dx+c)^7\right)b^2}{36900864d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/36900864*(53248*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x +
c)^7)*a^2 - 3003*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*c + 9*sin(8*d*x + 8*
c) - 48*sin(4*d*x + 4*c))*a*b - 12288*(231*cos(d*x + c)^13 - 819*cos(d*x +
c)^11 + 1001*cos(d*x + c)^9 - 429*cos(d*x + c)^7)*b^2)/d
```

Fricas [A]

time = 0.39, size = 161, normalized size = 0.68

$$\frac{354816b^2\cos(dx+c)^{13}-419328(a^2+3b^2)\cos(dx+c)^{11}+512512(2a^2+3b^2)\cos(dx+c)^9-658944(a^2+b^2)\cos(dx+c)^7+45045abdx-3003\left(256ab\cos(dx+c)^{11}-640ab\cos(dx+c)^9+432ab\cos(dx+c)^7-8ab\cos(dx+c)^5-10ab\cos(dx+c)^3-15ab\cos(dx+c)\right)\sin(dx+c)}{4612608d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4612608*(354816*b^2*cos(d*x + c)^13 - 419328*(a^2 + 3*b^2)*cos(d*x + c)^1
1 + 512512*(2*a^2 + 3*b^2)*cos(d*x + c)^9 - 658944*(a^2 + b^2)*cos(d*x + c)
^7 + 45045*a*b*d*x - 3003*(256*a*b*cos(d*x + c)^11 - 640*a*b*cos(d*x + c)^9
 + 432*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)^3 -
15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(224) = 448$.
time = 4.70, size = 488, normalized size = 2.05

$$\begin{cases} -\frac{a^2\sin^4(c+dx)\cos^7(c+dx)}{7d} - \frac{4a^2\sin^2(c+dx)\cos^9(c+dx)}{63d} - \frac{8a^2\cos^{11}(c+dx)}{693d} + \frac{5abx\sin^{12}(c+dx)}{512} + \frac{15abx\sin^{10}(c+dx)\cos^2(c+dx)}{256} + \frac{75abx\sin^8(c+dx)\cos^4(c+dx)}{512} + \frac{25abx\sin^6(c+dx)\cos^6(c+dx)}{128} + \frac{75abx\sin^4(c+dx)\cos^8(c+dx)}{512} + \frac{15abx\sin^2(c+dx)\cos^{10}(c+dx)}{256} + \frac{5abx\cos^{12}(c+dx)}{512} + \frac{5ab\sin^{11}(c+dx)\cos(c+dx)}{512d} + \frac{85ab\sin^9(c+dx)\cos^3(c+dx)}{1536d} + \frac{33ab\sin^7(c+dx)\cos^5(c+dx)}{256d} - \frac{33ab\sin^5(c+dx)\cos^7(c+dx)}{256d} - \frac{85ab\sin^3(c+dx)\cos^9(c+dx)}{1536d} - \frac{5ab\sin(c+dx)\cos^{11}(c+dx)}{512d} - \frac{b^2\sin^6(c+dx)\cos^7(c+dx)}{7d} - \frac{2b^2\sin^4(c+dx)\cos^9(c+dx)}{21d} - \frac{8b^2\sin^2(c+dx)\cos^{11}(c+dx)}{231d} - \frac{16b^2\cos^{13}(c+dx)}{3003d} & \text{for}\ d \neq 0 \\ x(a+b\sin(c))^2\sin^5(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)**7/(7*d) - 4*a**2*sin(c + d*x
)**2*cos(c + d*x)**9/(63*d) - 8*a**2*cos(c + d*x)**11/(693*d) + 5*a*b*x*sin
(c + d*x)**12/512 + 15*a*b*x*sin(c + d*x)**10*cos(c + d*x)**2/256 + 75*a*b*
x*sin(c + d*x)**8*cos(c + d*x)**4/512 + 25*a*b*x*sin(c + d*x)**6*cos(c + d*
x)**6/128 + 75*a*b*x*sin(c + d*x)**4*cos(c + d*x)**8/512 + 15*a*b*x*sin(c +
 d*x)**2*cos(c + d*x)**10/256 + 5*a*b*x*cos(c + d*x)**12/512 + 5*a*b*sin(c
+ d*x)**11*cos(c + d*x)/(512*d) + 85*a*b*sin(c + d*x)**9*cos(c + d*x)**3/(1
536*d) + 33*a*b*sin(c + d*x)**7*cos(c + d*x)**5/(256*d) - 33*a*b*sin(c + d*
x)**5*cos(c + d*x)**7/(256*d) - 85*a*b*sin(c + d*x)**3*cos(c + d*x)**9/(153
6*d) - 5*a*b*sin(c + d*x)*cos(c + d*x)**11/(512*d) - b**2*sin(c + d*x)**6*c
os(c + d*x)**7/(7*d) - 2*b**2*sin(c + d*x)**4*cos(c + d*x)**9/(21*d) - 8*b*
*2*sin(c + d*x)**2*cos(c + d*x)**11/(231*d) - 16*b**2*cos(c + d*x)**13/(300
3*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**5*cos(c)**6, True))
```

Giac [A]
time = 0.54, size = 214, normalized size = 0.90

$$\frac{5}{512}abx + \frac{b^2\cos(13dx+13c)}{53248d} - \frac{ab\sin(12dx+12c)}{12288d} + \frac{3ab\sin(8dx+8c)}{4096d} - \frac{15ab\sin(4dx+4c)}{4096d} - \frac{(4a^2+b^2)\cos(11dx+11c)}{45056d} - \frac{(2a^2+3b^2)\cos(9dx+9c)}{18432d} + \frac{(10a^2+3b^2)\cos(7dx+7c)}{14336d} + \frac{(4a^2+3b^2)\cos(5dx+5c)}{4096d} - \frac{5(8a^2+3b^2)\cos(3dx+3c)}{12288d} - \frac{5(2a^2+b^2)\cos(dx+c)}{1024d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/512*a*b*x + 1/53248*b^2*cos(13*d*x + 13*c)/d - 1/12288*a*b*sin(12*d*x + 1
2*c)/d + 3/4096*a*b*sin(8*d*x + 8*c)/d - 15/4096*a*b*sin(4*d*x + 4*c)/d - 1
/45056*(4*a^2 + b^2)*cos(11*d*x + 11*c)/d - 1/18432*(2*a^2 + 3*b^2)*cos(9*d
*x + 9*c)/d + 1/14336*(10*a^2 + 3*b^2)*cos(7*d*x + 7*c)/d + 1/4096*(4*a^2 +
 3*b^2)*cos(5*d*x + 5*c)/d - 5/12288*(8*a^2 + 3*b^2)*cos(3*d*x + 3*c)/d - 5
/1024*(2*a^2 + b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 15.01, size = 441, normalized size = 1.85

$$\frac{5abx}{512} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}(16a^2-96b^2) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\left(\frac{16a^2}{3}-32b^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(\frac{128a^2}{3}+192b^2\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{256a^2}{63}-\frac{64b^2}{21}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{416a^2}{231}+\frac{64b^2}{77}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{96a^2}{7}+\frac{768b^2}{7}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{208a^2}{693}+\frac{32b^2}{231}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{64a^2}{21}+\frac{1216b^2}{7}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(\frac{1376a^2}{63}-\frac{512b^2}{21}\right) + \frac{32a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}}{3} + \frac{16a^2}{693} + \frac{32b^2}{3003} + \frac{95ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{384} + \frac{277ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192} - \frac{4025ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{128} + \frac{59435ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{768} - \frac{16813ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192} + \frac{16813ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{192} - \frac{59435ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}}{768} + \frac{4025ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}}{128} - \frac{277ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}}{192} - \frac{95ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{23}}{384} - \frac{5ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{25}}{256} + \frac{5ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{256}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a*b*x)/512 - (tan(c/2 + (d*x)/2)^16*(16*a^2 - 96*b^2) - tan(c/2 + (d*x)/
2)^18*((16*a^2)/3 - 32*b^2) + tan(c/2 + (d*x)/2)^14*((128*a^2)/3 + 192*b^2)
 - tan(c/2 + (d*x)/2)^6*((256*a^2)/63 - (64*b^2)/21) + tan(c/2 + (d*x)/2)^4
*((416*a^2)/231 + (64*b^2)/77) + tan(c/2 + (d*x)/2)^10*((96*a^2)/7 + (768*b
^2)/7) + tan(c/2 + (d*x)/2)^2*((208*a^2)/693 + (32*b^2)/231) - tan(c/2 + (d
*x)/2)^12*((64*a^2)/21 + (1216*b^2)/7) + tan(c/2 + (d*x)/2)^8*((1376*a^2)/6
3 - (512*b^2)/21) + (32*a^2*tan(c/2 + (d*x)/2)^20)/3 + (16*a^2)/693 + (32*b
^2)/3003 + (95*a*b*tan(c/2 + (d*x)/2)^3)/384 + (277*a*b*tan(c/2 + (d*x)/2)^
5)/192 - (4025*a*b*tan(c/2 + (d*x)/2)^7)/128 + (59435*a*b*tan(c/2 + (d*x)/2
)^9)/768 - (16813*a*b*tan(c/2 + (d*x)/2)^11)/192 + (16813*a*b*tan(c/2 + (d*
x)/2)^15)/192 - (59435*a*b*tan(c/2 + (d*x)/2)^17)/768 + (4025*a*b*tan(c/2 +
 (d*x)/2)^19)/128 - (277*a*b*tan(c/2 + (d*x)/2)^21)/192 - (95*a*b*tan(c/2 +
 (d*x)/2)^23)/384 - (5*a*b*tan(c/2 + (d*x)/2)^25)/256 + (5*a*b*tan(c/2 + (d
*x)/2))/256)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^13)
```

3.1240 $\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=250

$$\frac{(12a^2+5b^2)\,x}{1024}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{4ab\cos^9(c+dx)}{9d}-\frac{2ab\cos^{11}(c+dx)}{11d}+\frac{(12a^2+5b^2)\cos(c+dx)\sin(c+d}{1024d}$$

```
[Out] 1/1024*(12*a^2+5*b^2)*x-2/7*a*b*cos(d*x+c)^7/d+4/9*a*b*cos(d*x+c)^9/d-2/11*
a*b*cos(d*x+c)^11/d+1/1024*(12*a^2+5*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/1536*(1
2*a^2+5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/1920*(12*a^2+5*b^2)*cos(d*x+c)^5*s
in(d*x+c)/d-1/320*(44*a^2+45*b^2)*cos(d*x+c)^7*sin(d*x+c)/d+1/120*(12*a^2+2
5*b^2)*cos(d*x+c)^9*sin(d*x+c)/d-1/12*b^2*cos(d*x+c)^11*sin(d*x+c)/d
```

Rubi [A]
time = 0.23, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2990, 2645, 276, 3279, 466, 1171, 393, 205, 209}

$$\frac{(12a^2+25b^2)\sin(c+dx)\cos^9(c+dx)}{120d}-\frac{(44a^2+45b^2)\sin(c+dx)\cos^7(c+dx)}{320d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^5(c+dx)}{1920d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos^3(c+dx)}{1536d}+\frac{(12a^2+5b^2)\sin(c+dx)\cos(c+dx)}{1024d}+\frac{x(12a^2+5b^2)}{1024}-\frac{2ab\cos^{11}(c+dx)}{11d}+\frac{4ab\cos^9(c+dx)}{9d}-\frac{2ab\cos^7(c+dx)}{7d}-\frac{b^2\sin(c+dx)\cos^{11}(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a^2 + 5*b^2)*x)/1024 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (4*a*b*Cos[c + d
*x]^9)/(9*d) - (2*a*b*Cos[c + d*x]^11)/(11*d) + ((12*a^2 + 5*b^2)*Cos[c + d
*x]*Sin[c + d*x])/(1024*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])
/(1536*d) + ((12*a^2 + 5*b^2)*Cos[c + d*x]^5*Sin[c + d*x])/(1920*d) - ((44*
a^2 + 45*b^2)*Cos[c + d*x]^7*Sin[c + d*x])/(320*d) + ((12*a^2 + 25*b^2)*Cos
[c + d*x]^9*Sin[c + d*x])/(120*d) - (b^2*Cos[c + d*x]^11*Sin[c + d*x])/(12*
d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
```

```
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3279

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[x^n*((a + (a + b)*ff^2*x^2)^p/(
1 + ff^2*x^2)^((m + n)/2 + p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^5(c+dx)\,dx + \int \cos^6(c+dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^4\left(a^2+\left(a^2+b^2\right)x^2\right)}{\left(1+x^2\right)^7}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{S}}{}$$

$$= -\frac{b^2\cos^{11}(c+dx)\sin(c+dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-b^2+12b^2x^2-1}{(1+x^2}\right.}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+}{11d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d} - \frac{2ab\cos^{11}(c+}{11d}$$

$$= \frac{\left(12a^2+5b^2\right)x}{1024} - \frac{2ab\cos^7(c+dx)}{7d} + \frac{4ab\cos^9(c+dx)}{9d}$$

Mathematica [A]
time = 1.05, size = 202, normalized size = 0.81

$$\frac{166320b^2c+332640a^2dx+138600b^2dx-554400ab\cos(c+dx)-184800ab\cos(3(c+dx))+55440ab\cos(5(c+dx))+39600ab\cos(7(c+dx))-6160ab\cos(9(c+dx))-5040ab\cos(11(c+dx))+55440a^2\sin(2(c+dx))-110880a^2\sin(4(c+dx))-51975b^2\sin(4(c+dx))-27720a^2\sin(6(c+dx))+13860a^2\sin(8(c+dx))+10395b^2\sin(8(c+dx))+5544a^2\sin(10(c+dx))-1155b^2\sin(12(c+dx))}{28385280d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (166320*b^2*c + 332640*a^2*d*x + 138600*b^2*d*x - 554400*a*b*Cos[c + d*x] -
 184800*a*b*Cos[3*(c + d*x)] + 55440*a*b*Cos[5*(c + d*x)] + 39600*a*b*Cos[7
*(c + d*x)] - 6160*a*b*Cos[9*(c + d*x)] - 5040*a*b*Cos[11*(c + d*x)] + 5544
0*a^2*Sin[2*(c + d*x)] - 110880*a^2*Sin[4*(c + d*x)] - 51975*b^2*Sin[4*(c +
 d*x)] - 27720*a^2*Sin[6*(c + d*x)] + 13860*a^2*Sin[8*(c + d*x)] + 10395*b^
2*Sin[8*(c + d*x)] + 5544*a^2*Sin[10*(c + d*x)] - 1155*b^2*Sin[12*(c + d*x)
])/(28385280*d)
```

Maple [A]
time = 1.00, size = 237, normalized size = 0.95

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{160}+\frac{3dx}{256}+\frac{3}{25}\right.}{}$
risch	$\frac{3a^2x}{256}+\frac{5b^2x}{1024}-\frac{5ab\cos(dx+c)}{256d}-\frac{b^2\sin(12dx+12c)}{24576d}+\frac{a^2\sin(10dx+10c)}{5120d}-\frac{ab\cos(11dx+11c)}{5632d}-\frac{ab\cos(9dx+9c)}{4608d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/16
0*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/25
6*c)+2*a*b*(-1/11*sin(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-
8/693*cos(d*x+c)^7)+b^2*(-1/12*sin(d*x+c)^5*cos(d*x+c)^7-1/24*sin(d*x+c)^3*
cos(d*x+c)^7-1/64*cos(d*x+c)^7*sin(d*x+c)+1/384*(cos(d*x+c)^5+5/4*cos(d*x+c
)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1024*d*x+5/1024*c))
```

Maxima [A]
time = 0.29, size = 137, normalized size = 0.55

$$\frac{2772\,\left(32\,\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\,\sin\left(8\,dx+8\,c\right)-40\,\sin\left(4\,dx+4\,c\right)\right)a^{2}-81920\,\left(63\,\cos\left(dx+c\right)^{11}-154\,\cos\left(dx+c\right)^{9}+99\,\cos\left(dx+c\right)^{7}\right)ab+1155\,\left(4\,\sin\left(4\,dx+4\,c\right)^{3}+120\,dx+120\,c+9\,\sin\left(8\,dx+8\,c\right)-48\,\sin\left(4\,dx+4\,c\right)\right)b^{2}}{28385280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/28385280*(2772*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8
*c) - 40*sin(4*d*x + 4*c))*a^2 - 81920*(63*cos(d*x + c)^11 - 154*cos(d*x +
```

```
c)^9 + 99*cos(d*x + c)^7)*a*b + 1155*(4*sin(4*d*x + 4*c)^3 + 120*d*x + 120*
c + 9*sin(8*d*x + 8*c) - 48*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]
time = 0.39, size = 182, normalized size = 0.73

$$-\frac{645120\,ab\cos\left(dx+c\right)^{11}-1576960\,ab\cos\left(dx+c\right)^{9}+1013760\,ab\cos\left(dx+c\right)^{7}-3465\,(12\,a^2+5\,b^2)dx+231\,\left(1280\,b^2\cos\left(dx+c\right)^{11}-128\,(12\,a^2+25\,b^2)\cos\left(dx+c\right)^{9}+48\,(44\,a^2+45\,b^2)\cos\left(dx+c\right)^{7}-8\,(12\,a^2+5\,b^2)\cos\left(dx+c\right)^{5}-10\,(12\,a^2+5\,b^2)\cos\left(dx+c\right)^{3}-15\,(12\,a^2+5\,b^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3548160*(645120*a*b*cos(d*x + c)^11 - 1576960*a*b*cos(d*x + c)^9 + 10137
60*a*b*cos(d*x + c)^7 - 3465*(12*a^2 + 5*b^2)*d*x + 231*(1280*b^2*cos(d*x +
 c)^11 - 128*(12*a^2 + 25*b^2)*cos(d*x + c)^9 + 48*(44*a^2 + 45*b^2)*cos(d*
x + c)^7 - 8*(12*a^2 + 5*b^2)*cos(d*x + c)^5 - 10*(12*a^2 + 5*b^2)*cos(d*x
+ c)^3 - 15*(12*a^2 + 5*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(231) = 462.
time = 3.54, size = 656, normalized size = 2.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**10/256 + 15*a**2*x*sin(c + d*x)**8*cos(c
+ d*x)**2/256 + 15*a**2*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 15*a**2*x*s
in(c + d*x)**4*cos(c + d*x)**6/128 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)
**8/256 + 3*a**2*x*cos(c + d*x)**10/256 + 3*a**2*sin(c + d*x)**9*cos(c + d*
x)/(256*d) + 7*a**2*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a**2*sin(c +
d*x)**5*cos(c + d*x)**5/(10*d) - 7*a**2*sin(c + d*x)**3*cos(c + d*x)**7/(12
8*d) - 3*a**2*sin(c + d*x)*cos(c + d*x)**9/(256*d) - 2*a*b*sin(c + d*x)**4*
cos(c + d*x)**7/(7*d) - 8*a*b*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 16*a
*b*cos(c + d*x)**11/(693*d) + 5*b**2*x*sin(c + d*x)**12/1024 + 15*b**2*x*si
n(c + d*x)**10*cos(c + d*x)**2/512 + 75*b**2*x*sin(c + d*x)**8*cos(c + d*x)
**4/1024 + 25*b**2*x*sin(c + d*x)**6*cos(c + d*x)**6/256 + 75*b**2*x*sin(c
+ d*x)**4*cos(c + d*x)**8/1024 + 15*b**2*x*sin(c + d*x)**2*cos(c + d*x)**10
/512 + 5*b**2*x*cos(c + d*x)**12/1024 + 5*b**2*sin(c + d*x)**11*cos(c + d*x
)/(1024*d) + 85*b**2*sin(c + d*x)**9*cos(c + d*x)**3/(3072*d) + 33*b**2*sin
(c + d*x)**7*cos(c + d*x)**5/(512*d) - 33*b**2*sin(c + d*x)**5*cos(c + d*x)
**7/(512*d) - 85*b**2*sin(c + d*x)**3*cos(c + d*x)**9/(3072*d) - 5*b**2*sin
(c + d*x)*cos(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)
**4*cos(c)**6, True))
```

Giac [A]
time = 0.69, size = 226, normalized size = 0.90

$$\frac{1}{1024}\left(12\,a^{2}+5\,b^{2}\right)x-\frac{ab\cos\left(11\,dx+11\,c\right)}{5632\,d}-\frac{ab\cos\left(9\,dx+9\,c\right)}{4608\,d}+\frac{5\,ab\cos\left(7\,dx+7\,c\right)}{3584\,d}+\frac{ab\cos\left(5\,dx+5\,c\right)}{512\,d}-\frac{5\,ab\cos\left(3\,dx+3\,c\right)}{768\,d}-\frac{5\,ab\cos\left(dx+c\right)}{256\,d}-\frac{b^{2}\sin\left(12\,dx+12\,c\right)}{24576\,d}+\frac{a^{2}\sin\left(10\,dx+10\,c\right)}{5120\,d}-\frac{a^{2}\sin\left(6\,dx+6\,c\right)}{1024\,d}+\frac{a^{2}\sin\left(2\,dx+2\,c\right)}{512\,d}+\frac{\left(4\,a^{2}+3\,b^{2}\right)\sin\left(8\,dx+8\,c\right)}{8192\,d}-\frac{\left(32\,a^{2}+15\,b^{2}\right)\sin\left(4\,dx+4\,c\right)}{8192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1024*(12*a^2 + 5*b^2)*x - 1/5632*a*b*cos(11*d*x + 11*c)/d - 1/4608*a*b*co
s(9*d*x + 9*c)/d + 5/3584*a*b*cos(7*d*x + 7*c)/d + 1/512*a*b*cos(5*d*x + 5*
c)/d - 5/768*a*b*cos(3*d*x + 3*c)/d - 5/256*a*b*cos(d*x + c)/d - 1/24576*b^
2*sin(12*d*x + 12*c)/d + 1/5120*a^2*sin(10*d*x + 10*c)/d - 1/1024*a^2*sin(6
*d*x + 6*c)/d + 1/512*a^2*sin(2*d*x + 2*c)/d + 1/8192*(4*a^2 + 3*b^2)*sin(8
*d*x + 8*c)/d - 1/8192*(32*a^2 + 15*b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 13.55, size = 207, normalized size = 0.83

$$\frac{6930\,a^{2}\,\sin\left(2\,c+2\,d\,x\right)-13860\,a^{2}\,\sin\left(4\,c+4\,d\,x\right)-3465\,a^{2}\,\sin\left(6\,c+6\,d\,x\right)+\frac{3465\,a^{2}\,\sin\left(8\,c+8\,d\,x\right)}{2}+693\,a^{2}\,\sin\left(10\,c+10\,d\,x\right)-\frac{51975\,b^{2}\,\sin\left(4\,c+4\,d\,x\right)}{8}+\frac{10395\,b^{2}\,\sin\left(8\,c+8\,d\,x\right)}{8}-\frac{1155\,b^{2}\,\sin\left(12\,c+12\,d\,x\right)}{8}-69300\,a\,b\,\cos\left(c+d\,x\right)-23100\,a\,b\,\cos\left(3\,c+3\,d\,x\right)+6930\,a\,b\,\cos\left(5\,c+5\,d\,x\right)+4950\,a\,b\,\cos\left(7\,c+7\,d\,x\right)-770\,a\,b\,\cos\left(9\,c+9\,d\,x\right)-630\,a\,b\,\cos\left(11\,c+11\,d\,x\right)+41580\,a^{2}\,d\,x+17325\,b^{2}\,d\,x}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (6930*a^2*sin(2*c + 2*d*x) - 13860*a^2*sin(4*c + 4*d*x) - 3465*a^2*sin(6*c
+ 6*d*x) + (3465*a^2*sin(8*c + 8*d*x))/2 + 693*a^2*sin(10*c + 10*d*x) - (51
975*b^2*sin(4*c + 4*d*x))/8 + (10395*b^2*sin(8*c + 8*d*x))/8 - (1155*b^2*si
n(12*c + 12*d*x))/8 - 69300*a*b*cos(c + d*x) - 23100*a*b*cos(3*c + 3*d*x) +
 6930*a*b*cos(5*c + 5*d*x) + 4950*a*b*cos(7*c + 7*d*x) - 770*a*b*cos(9*c +
9*d*x) - 630*a*b*cos(11*c + 11*d*x) + 41580*a^2*d*x + 17325*b^2*d*x)/(35481
60*d)
```

3.1241 $\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=187

$$\frac{3abx}{128}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}+\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{b^2\cos^{11}(c+dx)}{11d}+\frac{3ab\cos(c+dx)\sin(c+dx)}{128d}+\frac{ab\,c}{}$$

```
[Out] 3/128*a*b*x-1/7*(a^2+b^2)*cos(d*x+c)^7/d+1/9*(a^2+2*b^2)*cos(d*x+c)^9/d-1/1
1*b^2*cos(d*x+c)^11/d+3/128*a*b*cos(d*x+c)*sin(d*x+c)/d+1/64*a*b*cos(d*x+c)
^3*sin(d*x+c)/d+1/80*a*b*cos(d*x+c)^5*sin(d*x+c)/d-3/40*a*b*cos(d*x+c)^7*si
n(d*x+c)/d-1/5*a*b*cos(d*x+c)^7*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2990, 2648, 2715, 8, 3280, 457, 78}

$$\frac{(a^2+2b^2)\cos^9(c+dx)}{9d}-\frac{(a^2+b^2)\cos^7(c+dx)}{7d}-\frac{ab\sin^3(c+dx)\cos^7(c+dx)}{5d}-\frac{3ab\sin(c+dx)\cos^7(c+dx)}{40d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{80d}+\frac{ab\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3ab\sin(c+dx)\cos(c+dx)}{128d}+\frac{3abx}{128}-\frac{b^2\cos^{11}(c+dx)}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*x)/128 - ((a^2 + b^2)*Cos[c + d*x]^7)/(7*d) + ((a^2 + 2*b^2)*Cos[c +
 d*x]^9)/(9*d) - (b^2*Cos[c + d*x]^11)/(11*d) + (3*a*b*Cos[c + d*x]*Sin[c +
 d*x])/(128*d) + (a*b*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (a*b*Cos[c + d*
x]^5*Sin[c + d*x])/(80*d) - (3*a*b*Cos[c + d*x]^7*Sin[c + d*x])/(40*d) - (a
*b*Cos[c + d*x]^7*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3280

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^4(c+dx)\,dx + \int \cos^6(c+dx$$

$$= -\frac{ab\cos^7(c+dx)\sin^3(c+dx)}{5d} + \frac{1}{5}(3ab)\int \cos^6(c+dx$$

$$= -\frac{3ab\cos^7(c+dx)\sin(c+dx)}{40d} - \frac{ab\cos^7(c+dx)\sin^3(}{5d}$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{80d} - \frac{3ab\cos^7(c+dx)\sin(c+}{40d}$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+dx)}{9d} -$$

$$= -\frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+dx)}{9d} -$$

$$= \frac{3abx}{128} - \frac{(a^2+b^2)\cos^7(c+dx)}{7d} + \frac{(a^2+2b^2)\cos^9(c+}{9d}$$

Mathematica [A]
time = 0.77, size = 197, normalized size = 1.05

$$\frac{83160abc+83160abdx-6930(12a^2+5b^2)\cos(c+dx)-2310(16a^2+5b^2)\cos(3(c+dx))+3465b^2\cos(5(c+dx))+5940a^2\cos(7(c+dx))+2475b^2\cos(7(c+dx))+1540a^2\cos(9(c+dx))-385b^2\cos(9(c+dx))-315b^2\cos(11(c+dx))+13860ab\sin(2(c+dx))-27720ab\sin(4(c+dx))-6930ab\sin(6(c+dx))+3465ab\sin(8(c+dx))+1386ab\sin(10(c+dx))}{3548160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (83160*a*b*c + 83160*a*b*d*x - 6930*(12*a^2 + 5*b^2)*Cos[c + d*x] - 2310*(1
6*a^2 + 5*b^2)*Cos[3*(c + d*x)] + 3465*b^2*Cos[5*(c + d*x)] + 5940*a^2*Cos[
7*(c + d*x)] + 2475*b^2*Cos[7*(c + d*x)] + 1540*a^2*Cos[9*(c + d*x)] - 385*
b^2*Cos[9*(c + d*x)] - 315*b^2*Cos[11*(c + d*x)] + 13860*a*b*Sin[2*(c + d*x
)] - 27720*a*b*Sin[4*(c + d*x)] - 6930*a*b*Sin[6*(c + d*x)] + 3465*a*b*Sin[
8*(c + d*x)] + 1386*a*b*Sin[10*(c + d*x)])/(3548160*d)
```

Maple [A]
time = 0.72, size = 171, normalized size = 0.91

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(d\right.}{}\right.}{}$

default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^7(dx+c)\right)}{9}-\frac{2\left(\cos^7(dx+c)\right)}{63}\right)+2ab\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^7(dx+c)\right)}{10}-\frac{3\left(\cos^7(dx+c)\right)\sin(dx+c)}{80}+\frac{\left(\cos^5(dx+\right.}{}\right.}{}$
risch	$\frac{3abx}{128}-\frac{3a^2\cos(dx+c)}{128d}-\frac{5b^2\cos(dx+c)}{512d}-\frac{\cos(11dx+11c)b^2}{11264d}+\frac{ab\sin(10dx+10c)}{2560d}+\frac{\cos(9dx+9c)a^2}{2304d}-\frac{b^2\cos(9dx}{921}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+2*a*b*(-1/10*si
n(d*x+c)^3*cos(d*x+c)^7-3/80*cos(d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/
4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)+b^2*(-1/11*si
n(d*x+c)^4*cos(d*x+c)^7-4/99*sin(d*x+c)^2*cos(d*x+c)^7-8/693*cos(d*x+c)^7))
```

Maxima [A]
time = 0.28, size = 115, normalized size = 0.61

$$\frac{56320\left(7\cos\left(dx+c\right)^{9}-9\cos\left(dx+c\right)^{7}\right)a^{2}+693\left(32\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\,\sin\left(8\,dx+8\,c\right)-40\,\sin\left(4\,dx+4\,c\right)\right)ab-5120\left(63\cos\left(dx+c\right)^{11}-154\cos\left(dx+c\right)^{9}+99\cos\left(dx+c\right)^{7}\right)b^{2}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/3548160*(56320*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2 + 693*(32*sin(2*
d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*
a*b - 5120*(63*cos(d*x + c)^11 - 154*cos(d*x + c)^9 + 99*cos(d*x + c)^7)*b^
2)/d
```

Fricas [A]
time = 0.40, size = 128, normalized size = 0.68

$$-\frac{40320\,b^{2}\cos\left(dx+c\right)^{11}-49280\left(a^{2}+2\,b^{2}\right)\cos\left(dx+c\right)^{9}+63360\left(a^{2}+b^{2}\right)\cos\left(dx+c\right)^{7}-10395\,abdx-693\left(128\,ab\cos\left(dx+c\right)^{9}-176\,ab\cos\left(dx+c\right)^{7}+8\,ab\cos\left(dx+c\right)^{5}+10\,ab\cos\left(dx+c\right)^{3}+15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{443520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/443520*(40320*b^2*cos(d*x + c)^11 - 49280*(a^2 + 2*b^2)*cos(d*x + c)^9 +
 63360*(a^2 + b^2)*cos(d*x + c)^7 - 10395*a*b*d*x - 693*(128*a*b*cos(d*x +
c)^9 - 176*a*b*cos(d*x + c)^7 + 8*a*b*cos(d*x + c)^5 + 10*a*b*cos(d*x + c)^
3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(178) = 356$.
time = 2.58, size = 384, normalized size = 2.05

$$\begin{cases}-\frac{a^2\sin^2(c+dx)\cos^7(c+dx)}{7d}-\frac{2a^2\cos^9(c+dx)}{63d}+\frac{3abx\sin^{10}(c+dx)}{128}+\frac{15abx\sin^8(c+dx)\cos^2(c+dx)}{128}+\frac{15abx\sin^6(c+dx)\cos^4(c+dx)}{64}+\frac{15abx\sin^4(c+dx)\cos^6(c+dx)}{64}+\frac{15abx\sin^2(c+dx)\cos^8(c+dx)}{128}+\frac{3abx\cos^{10}(c+dx)}{128}+\frac{3ab\sin^9(c+dx)\cos(c+dx)}{128d}+\frac{7ab\sin^7(c+dx)\cos^3(c+dx)}{64d}+\frac{ab\sin^5(c+dx)\cos^5(c+dx)}{5d}-\frac{7ab\sin^3(c+dx)\cos^7(c+dx)}{64d}-\frac{3ab\sin(c+dx)\cos^9(c+dx)}{128d}-\frac{b^2\sin^4(c+dx)\cos^7(c+dx)}{7d}-\frac{4b^2\sin^2(c+dx)\cos^9(c+dx)}{63d}-\frac{8b^2\cos^{11}(c+dx)}{693d} & \text{for}\: d \neq 0 \\ x(a+b\sin(c))^2\sin^3(c)\cos^6(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 2*a**2*cos(c + d*x
)**9/(63*d) + 3*a*b*x*sin(c + d*x)**10/128 + 15*a*b*x*sin(c + d*x)**8*cos(c
 + d*x)**2/128 + 15*a*b*x*sin(c + d*x)**6*cos(c + d*x)**4/64 + 15*a*b*x*sin
(c + d*x)**4*cos(c + d*x)**6/64 + 15*a*b*x*sin(c + d*x)**2*cos(c + d*x)**8/
128 + 3*a*b*x*cos(c + d*x)**10/128 + 3*a*b*sin(c + d*x)**9*cos(c + d*x)/(12
8*d) + 7*a*b*sin(c + d*x)**7*cos(c + d*x)**3/(64*d) + a*b*sin(c + d*x)**5*c
os(c + d*x)**5/(5*d) - 7*a*b*sin(c + d*x)**3*cos(c + d*x)**7/(64*d) - 3*a*b
*sin(c + d*x)*cos(c + d*x)**9/(128*d) - b**2*sin(c + d*x)**4*cos(c + d*x)**
7/(7*d) - 4*b**2*sin(c + d*x)**2*cos(c + d*x)**9/(63*d) - 8*b**2*cos(c + d*
x)**11/(693*d), Ne(d, 0)), (x*(a + b*sin(c))**2*sin(c)**3*cos(c)**6, True))
```

Giac [A]
time = 0.63, size = 217, normalized size = 1.16

$$\frac{3}{128}\,abx-\frac{b^2\cos\left(11\,dx+11\,c\right)}{11264\,d}+\frac{b^2\cos\left(5\,dx+5\,c\right)}{1024\,d}+\frac{ab\sin\left(10\,dx+10\,c\right)}{2560\,d}+\frac{ab\sin\left(8\,dx+8\,c\right)}{1024\,d}-\frac{ab\sin\left(6\,dx+6\,c\right)}{512\,d}-\frac{ab\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{ab\sin\left(2\,dx+2\,c\right)}{256\,d}+\frac{\left(4\,a^2-b^2\right)\cos\left(9\,dx+9\,c\right)}{9216\,d}+\frac{\left(12\,a^2+5\,b^2\right)\cos\left(7\,dx+7\,c\right)}{7168\,d}-\frac{\left(16\,a^2+5\,b^2\right)\cos\left(3\,dx+3\,c\right)}{1536\,d}-\frac{\left(12\,a^2+5\,b^2\right)\cos\left(dx+c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/128*a*b*x - 1/11264*b^2*cos(11*d*x + 11*c)/d + 1/1024*b^2*cos(5*d*x + 5*c
)/d + 1/2560*a*b*sin(10*d*x + 10*c)/d + 1/1024*a*b*sin(8*d*x + 8*c)/d - 1/5
12*a*b*sin(6*d*x + 6*c)/d - 1/128*a*b*sin(4*d*x + 4*c)/d + 1/256*a*b*sin(2*
d*x + 2*c)/d + 1/9216*(4*a^2 - b^2)*cos(9*d*x + 9*c)/d + 1/7168*(12*a^2 + 5
*b^2)*cos(7*d*x + 7*c)/d - 1/1536*(16*a^2 + 5*b^2)*cos(3*d*x + 3*c)/d - 1/5
12*(12*a^2 + 5*b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 14.94, size = 386, normalized size = 2.06

$$\frac{3\,a\,b\,x}{128}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}\,\left(\frac{4\,a^2}{3}+\frac{32\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(8\,a^2-48\,b^2\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{64\,a^2}{7}-\frac{48\,b^2}{7}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(\frac{32\,a^2}{3}-\frac{80\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{44\,a^2}{63}+\frac{16\,b^2}{63}\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{32\,a^2}{63}-\frac{80\,b^2}{63}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{64\,a^2}{3}+\frac{176\,b^2}{3}\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{72\,a^2}{7}+\frac{240\,b^2}{7}\right)+4\,a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}+\frac{4\,a^2}{63}+\frac{16\,b^2}{693}+\frac{a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{3323\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{320}+\frac{108\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{5}-\frac{841\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{32}+\frac{841\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{32}-\frac{108\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}}{5}+\frac{3323\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}}{320}-\frac{a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}}{2}-\frac{3\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{21}}{64}+\frac{3\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64}}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (3*a*b*x)/128 - (tan(c/2 + (d*x)/2)^16*((4*a^2)/3 + (32*b^2)/3) + tan(c/2 +
 (d*x)/2)^10*(8*a^2 - 48*b^2) + tan(c/2 + (d*x)/2)^6*((64*a^2)/7 - (48*b^2)
/7) + tan(c/2 + (d*x)/2)^14*((32*a^2)/3 - (80*b^2)/3) + tan(c/2 + (d*x)/2)^
2*((44*a^2)/63 + (16*b^2)/63) - tan(c/2 + (d*x)/2)^4*((32*a^2)/63 - (80*b^2
)/63) + tan(c/2 + (d*x)/2)^12*((64*a^2)/3 + (176*b^2)/3) + tan(c/2 + (d*x)/
2)^8*((72*a^2)/7 + (240*b^2)/7) + 4*a^2*tan(c/2 + (d*x)/2)^18 + (4*a^2)/63
+ (16*b^2)/693 + (a*b*tan(c/2 + (d*x)/2)^3)/2 - (3323*a*b*tan(c/2 + (d*x)/2
)^5)/320 + (108*a*b*tan(c/2 + (d*x)/2)^7)/5 - (841*a*b*tan(c/2 + (d*x)/2)^9
```

```
)/32 + (841*a*b*tan(c/2 + (d*x)/2)^13)/32 - (108*a*b*tan(c/2 + (d*x)/2)^15)
/5 + (3323*a*b*tan(c/2 + (d*x)/2)^17)/320 - (a*b*tan(c/2 + (d*x)/2)^19)/2 -
 (3*a*b*tan(c/2 + (d*x)/2)^21)/64 + (3*a*b*tan(c/2 + (d*x)/2))/64)/(d*(tan(
c/2 + (d*x)/2)^2 + 1)^11)
```

3.1242 $\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=201

$$\frac{1}{256}\left(10a^2+3b^2\right)x-\frac{2ab\cos^7(c+dx)}{7d}+\frac{2ab\cos^9(c+dx)}{9d}+\frac{\left(10a^2+3b^2\right)\cos(c+dx)\sin(c+dx)}{256d}+\frac{\left(10a^2+3b^2\right)}{}$$

```
[Out] 1/256*(10*a^2+3*b^2)*x-2/7*a*b*cos(d*x+c)^7/d+2/9*a*b*cos(d*x+c)^9/d+1/256*
(10*a^2+3*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/384*(10*a^2+3*b^2)*cos(d*x+c)^3*si
n(d*x+c)/d+1/480*(10*a^2+3*b^2)*cos(d*x+c)^5*sin(d*x+c)/d-1/80*(10*a^2+11*b
^2)*cos(d*x+c)^7*sin(d*x+c)/d+1/10*b^2*cos(d*x+c)^9*sin(d*x+c)/d
```

Rubi [A]

time = 0.17, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2990, 2645, 14, 3279, 466, 393, 205, 209}

$$-\frac{\left(10a^2+11b^2\right)\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{\left(10a^2+3b^2\right)\sin(c+dx)\cos^5(c+dx)}{480d}+\frac{\left(10a^2+3b^2\right)\sin(c+dx)\cos^3(c+dx)}{384d}+\frac{\left(10a^2+3b^2\right)\sin(c+dx)\cos(c+dx)}{256d}+\frac{1}{256}x\left(10a^2+3b^2\right)+\frac{2ab\cos^9(c+dx)}{9d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin(c+dx)\cos^9(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((10*a^2 + 3*b^2)*x)/256 - (2*a*b*Cos[c + d*x]^7)/(7*d) + (2*a*b*Cos[c + d*
x]^9)/(9*d) + ((10*a^2 + 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(256*d) + ((10*a
^2 + 3*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(384*d) + ((10*a^2 + 3*b^2)*Cos[c
+ d*x]^5*Sin[c + d*x])/(480*d) - ((10*a^2 + 11*b^2)*Cos[c + d*x]^7*Sin[c +
d*x])/(80*d) + (b^2*Cos[c + d*x]^9*Sin[c + d*x])/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3279

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[x^n*((a + (a + b)*ff^2*x^2)^p/(
1 + ff^2*x^2)^((m + n)/2 + p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(c+dx)\sin^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\sin^3(c+dx)\,dx + \int \cos^6(c+dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2+(a^2+b^2)x^2)}{(1+x^2)^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{S}}{}$$

$$= \frac{b^2\cos^9(c+dx)\sin(c+dx)}{10d} - \frac{\text{Subst}\left(\int \frac{b^2-10(a^2+b^2)x^2}{(1+x^2)^5}\,d\right.}{10d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} - \frac{(10a^2+11b^2}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{(10a^2+3b^2)}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{(10a^2+3b^2)}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+dx)}{9d} + \frac{(10a^2+3b^2)}{}$$

$$= \frac{1}{256}(10a^2+3b^2)\,x - \frac{2ab\cos^7(c+dx)}{7d} + \frac{2ab\cos^9(c+}{9d}$$

Mathematica [A]
time = 0.62, size = 193, normalized size = 0.96

$$\frac{6300b^2c + 12600a^2dx + 3780b^2dx - 15120ab\cos(c+dx) - 6720ab\cos(3(c+dx)) + 1080ab\cos(7(c+dx)) + 280ab\cos(9(c+dx)) + 5040a^2\sin(2(c+dx)) + 630b^2\sin(2(c+dx)) - 2520a^2\sin(4(c+dx)) - 1260b^2\sin(4(c+dx)) - 1680a^2\sin(6(c+dx)) - 315b^2\sin(6(c+dx)) - 315a^2\sin(8(c+dx)) + \frac{315}{2}b^2\sin(8(c+dx)) + 63b^2\sin(10(c+dx))}{322560d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6300*b^2*c + 12600*a^2*d*x + 3780*b^2*d*x - 15120*a*b*Cos[c + d*x] - 6720*
a*b*Cos[3*(c + d*x)] + 1080*a*b*Cos[7*(c + d*x)] + 280*a*b*Cos[9*(c + d*x)]
 + 5040*a^2*Sin[2*(c + d*x)] + 630*b^2*Sin[2*(c + d*x)] - 2520*a^2*Sin[4*(c
 + d*x)] - 1260*b^2*Sin[4*(c + d*x)] - 1680*a^2*Sin[6*(c + d*x)] - 315*b^2*
Sin[6*(c + d*x)] - 315*a^2*Sin[8*(c + d*x)] + (315*b^2*Sin[8*(c + d*x)])/2
+ 63*b^2*Sin[10*(c + d*x)])/(322560*d)
```

Maple [A]
time = 0.56, size = 183, normalized size = 0.91

method	result

derivativedivides	$\frac{a^2\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos\right.}{9}\right.}{}$
default	$\frac{a^2\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+2ab\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos\right.}{9}\right.}{}$
risch	$\frac{5a^2x}{128}+\frac{3b^2x}{256}-\frac{3ab\cos(dx+c)}{64d}+\frac{b^2\sin(10dx+10c)}{5120d}+\frac{ab\cos(9dx+9c)}{1152d}-\frac{\sin(8dx+8c)a^2}{1024d}+\frac{b^2\sin(8dx+8c)}{2048d}+3a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+
15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+2*a*b*(-1/9*sin(d*x+c)^2*cos
(d*x+c)^7-2/63*cos(d*x+c)^7)+b^2*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*cos(
d*x+c)^7*sin(d*x+c)+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*s
in(d*x+c)+3/256*d*x+3/256*c))
```

Maxima [A]
time = 0.27, size = 127, normalized size = 0.63

$$\frac{210\left(64\sin(2dx+2c)^3+120dx+120c-3\sin(8dx+8c)-24\sin(4dx+4c)\right)a^2+20480\left(7\cos(dx+c)^9-9\cos(dx+c)^7\right)ab+63\left(32\sin(2dx+2c)^5+120dx+120c+5\sin(8dx+8c)-40\sin(4dx+4c)\right)b^2}{645120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/645120*(210*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c)
 - 24*sin(4*d*x + 4*c))*a^2 + 20480*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a
*b + 63*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c + 5*sin(8*d*x + 8*c) - 40*
sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]
time = 0.38, size = 149, normalized size = 0.74

$$\frac{17920\,ab\cos(dx+c)^9-23040\,ab\cos(dx+c)^7+315\left(10a^2+3b^2\right)dx+21\left(384b^2\cos(dx+c)^9-48\left(10a^2+11b^2\right)\cos(dx+c)^7+8\left(10a^2+3b^2\right)\cos(dx+c)^5+10\left(10a^2+3b^2\right)\cos(dx+c)^3+15\left(10a^2+3b^2\right)\cos(dx+c)\right)\sin(dx+c)}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/80640*(17920*a*b*cos(d*x + c)^9 - 23040*a*b*cos(d*x + c)^7 + 315*(10*a^2
+ 3*b^2)*d*x + 21*(384*b^2*cos(d*x + c)^9 - 48*(10*a^2 + 11*b^2)*cos(d*x +
```

```
c)^7 + 8*(10*a^2 + 3*b^2)*cos(d*x + c)^5 + 10*(10*a^2 + 3*b^2)*cos(d*x + c)
^3 + 15*(10*a^2 + 3*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(184) = 368$.
time = 1.86, size = 529, normalized size = 2.63

$$\begin{cases} \frac{5a^2x\sin^8(c+dx)}{128} + \frac{5a^2x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{15a^2x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{5a^2x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{5a^2x\cos^8(c+dx)}{128} + \frac{5a^2\sin^7(c+dx)\cos(c+dx)}{128d} + \frac{55a^2\sin^5(c+dx)\cos^3(c+dx)}{384d} + \frac{73a^2\sin^3(c+dx)\cos^5(c+dx)}{384d} - \frac{5a^2\sin(c+dx)\cos^7(c+dx)}{128d} - \frac{2ab\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{4ab\cos^9(c+dx)}{63d} + \frac{3b^2x\sin^{10}(c+dx)}{256} + \frac{15b^2x\sin^8(c+dx)\cos^2(c+dx)}{256} + \frac{15b^2x\sin^6(c+dx)\cos^4(c+dx)}{128} + \frac{15b^2x\sin^4(c+dx)\cos^6(c+dx)}{128} + \frac{15b^2x\sin^2(c+dx)\cos^8(c+dx)}{256} + \frac{3b^2x\cos^{10}(c+dx)}{256} + \frac{3b^2\sin^9(c+dx)\cos(c+dx)}{256d} + \frac{7b^2\sin^7(c+dx)\cos^3(c+dx)}{128d} + \frac{b^2\sin^5(c+dx)\cos^5(c+dx)}{10d} - \frac{7b^2\sin^3(c+dx)\cos^7(c+dx)}{128d} - \frac{3b^2\sin(c+dx)\cos^9(c+dx)}{256d} & \text{for } d \neq 0 \\ x(a+b\sin(c))^2\sin^2(c)\cos^6(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**8/128 + 5*a**2*x*sin(c + d*x)**6*cos(c +
d*x)**2/32 + 15*a**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 5*a**2*x*sin(c
+ d*x)**2*cos(c + d*x)**6/32 + 5*a**2*x*cos(c + d*x)**8/128 + 5*a**2*sin(c
+ d*x)**7*cos(c + d*x)/(128*d) + 55*a**2*sin(c + d*x)**5*cos(c + d*x)**3/(3
84*d) + 73*a**2*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) - 5*a**2*sin(c + d*
x)*cos(c + d*x)**7/(128*d) - 2*a*b*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) -
4*a*b*cos(c + d*x)**9/(63*d) + 3*b**2*x*sin(c + d*x)**10/256 + 15*b**2*x*si
n(c + d*x)**8*cos(c + d*x)**2/256 + 15*b**2*x*sin(c + d*x)**6*cos(c + d*x)*
*4/128 + 15*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 15*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**8/256 + 3*b**2*x*cos(c + d*x)**10/256 + 3*b**2*sin(c
+ d*x)**9*cos(c + d*x)/(256*d) + 7*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(12
8*d) + b**2*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) - 7*b**2*sin(c + d*x)**3
*cos(c + d*x)**7/(128*d) - 3*b**2*sin(c + d*x)*cos(c + d*x)**9/(256*d), Ne(
d, 0)), (x*(a + b*sin(c))**2*sin(c)**2*cos(c)**6, True))
```

Giac [A]
time = 0.56, size = 189, normalized size = 0.94

$$\frac{1}{256}\left(10a^2+3b^2\right)x + \frac{ab\cos(9dx+9c)}{1152d} + \frac{3ab\cos(7dx+7c)}{896d} - \frac{ab\cos(3dx+3c)}{48d} - \frac{3ab\cos(dx+c)}{64d} + \frac{b^2\sin(10dx+10c)}{5120d} - \frac{(2a^2-b^2)\sin(8dx+8c)}{2048d} - \frac{(16a^2+3b^2)\sin(6dx+6c)}{3072d} - \frac{(2a^2+b^2)\sin(4dx+4c)}{256d} + \frac{(8a^2+b^2)\sin(2dx+2c)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/256*(10*a^2 + 3*b^2)*x + 1/1152*a*b*cos(9*d*x + 9*c)/d + 3/896*a*b*cos(7*
d*x + 7*c)/d - 1/48*a*b*cos(3*d*x + 3*c)/d - 3/64*a*b*cos(d*x + c)/d + 1/51
20*b^2*sin(10*d*x + 10*c)/d - 1/2048*(2*a^2 - b^2)*sin(8*d*x + 8*c)/d - 1/3
072*(16*a^2 + 3*b^2)*sin(6*d*x + 6*c)/d - 1/256*(2*a^2 + b^2)*sin(4*d*x + 4
*c)/d + 1/512*(8*a^2 + b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 11.93, size = 237, normalized size = 1.18

$$\frac{5a^2x}{128} + \frac{3b^2x}{256} + \frac{5a^2\cos(c+dx)^3\sin(c+dx)}{192d} + \frac{a^2\cos(c+dx)^5\sin(c+dx)}{48d} - \frac{a^2\cos(c+dx)^7\sin(c+dx)}{8d} + \frac{b^2\cos(c+dx)^3\sin(c+dx)}{128d} + \frac{b^2\cos(c+dx)^5\sin(c+dx)}{160d} - \frac{11b^2\cos(c+dx)^7\sin(c+dx)}{80d} + \frac{b^2\cos(c+dx)^9\sin(c+dx)}{10d} - \frac{2ab\cos(c+dx)^7}{7d} + \frac{2ab\cos(c+dx)^9}{9d} + \frac{5a^2\cos(c+dx)\sin(c+dx)}{128d} + \frac{3b^2\cos(c+dx)\sin(c+dx)}{256d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/128 + (3*b^2*x)/256 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(192*d)
 + (a^2*cos(c + d*x)^5*sin(c + d*x))/(48*d) - (a^2*cos(c + d*x)^7*sin(c + d
*x))/(8*d) + (b^2*cos(c + d*x)^3*sin(c + d*x))/(128*d) + (b^2*cos(c + d*x)^
5*sin(c + d*x))/(160*d) - (11*b^2*cos(c + d*x)^7*sin(c + d*x))/(80*d) + (b^
2*cos(c + d*x)^9*sin(c + d*x))/(10*d) - (2*a*b*cos(c + d*x)^7)/(7*d) + (2*a
*b*cos(c + d*x)^9)/(9*d) + (5*a^2*cos(c + d*x)*sin(c + d*x))/(128*d) + (3*b
^2*cos(c + d*x)*sin(c + d*x))/(256*d)
```

3.1243 $\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=152

$$\frac{5abx}{64}-\frac{(a^2+8b^2)\cos^7(c+dx)}{252d}+\frac{5ab\cos(c+dx)\sin(c+dx)}{64d}+\frac{5ab\cos^3(c+dx)\sin(c+dx)}{96d}+\frac{ab\cos^5(c+dx)}{24d}$$

```
[Out] 5/64*a*b*x-1/252*(a^2+8*b^2)*cos(d*x+c)^7/d+5/64*a*b*cos(d*x+c)*sin(d*x+c)/
d+5/96*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/24*a*b*cos(d*x+c)^5*sin(d*x+c)/d-1/3
6*a*cos(d*x+c)^7*(a+b*sin(d*x+c))/d-1/9*cos(d*x+c)^7*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2941, 2748, 2715, 8}

$$-\frac{(a^2+8b^2)\cos^7(c+dx)}{252d}-\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d}-\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{5ab\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{5ab\sin(c+dx)\cos(c+dx)}{64d}+\frac{5abx}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/64 - ((a^2 + 8*b^2)*Cos[c + d*x]^7)/(252*d) + (5*a*b*Cos[c + d*x]
*Sin[c + d*x])/(64*d) + (5*a*b*Cos[c + d*x]^3*Sin[c + d*x])/(96*d) + (a*b*C
os[c + d*x]^5*Sin[c + d*x])/(24*d) - (a*Cos[c + d*x]^7*(a + b*Sin[c + d*x])
)/(36*d) - (Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2)/(9*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
```

```
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cos^6(c+dx)\sin(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{\cos^7(c+dx)(a+b\sin(c+dx))^2}{9d} + \frac{1}{9}\int \cos^6(c+dx)(2$$

$$= -\frac{a\cos^7(c+dx)(a+b\sin(c+dx))}{36d} - \frac{\cos^7(c+dx)(a+b}{9d}$$

$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} - \frac{a\cos^7(c+dx)(a+b\sin(c+}{36d}$$

$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{ab\cos^5(c+dx)\sin(c+dx)}{24d}$$

$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos^3(c+dx)\sin(c+dx)}{96d}$$

$$= -\frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos(c+dx)\sin(c+dx)}{64d}$$

$$= \frac{5abx}{64} - \frac{(a^2+8b^2)\cos^7(c+dx)}{252d} + \frac{5ab\cos(c+dx)\sin(c-}{64d}$$

Mathematica [A]
time = 0.71, size = 161, normalized size = 1.06

$$-\frac{-1260abc-1260abdx+126(10a^2+3b^2)\cos(c+dx)+84(9a^2+2b^2)\cos(3(c+dx))+252a^2\cos(5(c+dx))+36a^2\cos(7(c+dx))-27b^2\cos(7(c+dx))-7b^2\cos(9(c+dx))-504ab\sin(2(c+dx))+252ab\sin(4(c+dx))+168ab\sin(6(c+dx))+\frac{63}{2}ab\sin(8(c+dx))}{16128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*Sin[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16128*(-1260*a*b*c - 1260*a*b*d*x + 126*(10*a^2 + 3*b^2)*Cos[c + d*x] +
84*(9*a^2 + 2*b^2)*Cos[3*(c + d*x)] + 252*a^2*Cos[5*(c + d*x)] + 36*a^2*Cos
[7*(c + d*x)] - 27*b^2*Cos[7*(c + d*x)] - 7*b^2*Cos[9*(c + d*x)] - 504*a*b*
Sin[2*(c + d*x)] + 252*a*b*Sin[4*(c + d*x)] + 168*a*b*Sin[6*(c + d*x)] + (6
3*a*b*Sin[8*(c + d*x)])/2)/d
```

Maple [A]
time = 0.47, size = 115, normalized size = 0.76

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7}+2ab\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+b}{d}$
default	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7}+2ab\left(-\frac{\left(\cos^7(dx+c)\right)\sin(dx+c)}{8}+\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{48}+\frac{5dx}{128}+\frac{5c}{128}\right)+b}{d}$
risch	$\frac{5abx}{64}-\frac{5a^2\cos(dx+c)}{64d}-\frac{3b^2\cos(dx+c)}{128d}+\frac{b^2\cos(9dx+9c)}{2304d}-\frac{ab\sin(8dx+8c)}{512d}-\frac{a^2\cos(7dx+7c)}{448d}+\frac{3b^2\cos(7dx}{1792d}$
norman	$\frac{83ab\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16d}+\frac{45abx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64}+\frac{45abx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{105abx\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16}+\frac{315abx\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{32}+\frac{315a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^2*cos(d*x+c)^7+2*a*b*(-1/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*
x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+b^2*
(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7))
```

Maxima [A]

time = 0.28, size = 92, normalized size = 0.61

$$-\frac{4608\,a^2\cos\left(dx+c\right)^7-21\,\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)ab-512\,\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)b^2}{32256\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32256*(4608*a^2*cos(d*x + c)^7 - 21*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 1
20*c - 3*sin(8*d*x + 8*c) - 24*sin(4*d*x + 4*c))*a*b - 512*(7*cos(d*x + c)^
9 - 9*cos(d*x + c)^7)*b^2)/d
```

Fricas [A]

time = 0.38, size = 97, normalized size = 0.64

$$\frac{448\,b^2\cos\left(dx+c\right)^9-576\,\left(a^2+b^2\right)\cos\left(dx+c\right)^7+315\,abdx-21\,\left(48\,ab\cos\left(dx+c\right)^7-8\,ab\cos\left(dx+c\right)^5-10\,ab\cos\left(dx+c\right)^3-15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4032\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4032*(448*b^2*cos(d*x + c)^9 - 576*(a^2 + b^2)*cos(d*x + c)^7 + 315*a*b*d
*x - 21*(48*a*b*cos(d*x + c)^7 - 8*a*b*cos(d*x + c)^5 - 10*a*b*cos(d*x + c)
^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 1.30, size = 282, normalized size = 1.86

$$\begin{cases} -\frac{a^{2}\cos^{7}(c+dx)}{7d} + \frac{5abx\sin^{8}(c+dx)}{64} + \frac{5abx\sin^{6}(c+dx)\cos^{2}(c+dx)}{16} + \frac{15abx\sin^{4}(c+dx)\cos^{4}(c+dx)}{32} + \frac{5abx\sin^{2}(c+dx)\cos^{6}(c+dx)}{16} + \frac{5abx\cos^{8}(c+dx)}{64} + \frac{5ab\sin^{7}(c+dx)\cos(c+dx)}{64d} + \frac{55ab\sin^{5}(c+dx)\cos^{3}(c+dx)}{192d} + \frac{73ab\sin^{3}(c+dx)\cos^{5}(c+dx)}{192d} - \frac{5ab\sin(c+dx)\cos^{7}(c+dx)}{64d} - \frac{b^{2}\sin^{2}(c+dx)\cos^{7}(c+dx)}{7d} - \frac{2b^{2}\cos^{9}(c+dx)}{63d} & \text{for}\ d\neq 0 \\ x(a+b\sin(c))^{2}\sin(c)\cos^{6}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((-a**2*cos(c + d*x)**7/(7*d) + 5*a*b*x*sin(c + d*x)**8/64 + 5*a*b
*x*sin(c + d*x)**6*cos(c + d*x)**2/16 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*
x)**4/32 + 5*a*b*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 5*a*b*x*cos(c + d*x
)**8/64 + 5*a*b*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a*b*sin(c + d*x)**
5*cos(c + d*x)**3/(192*d) + 73*a*b*sin(c + d*x)**3*cos(c + d*x)**5/(192*d)
- 5*a*b*sin(c + d*x)*cos(c + d*x)**7/(64*d) - b**2*sin(c + d*x)**2*cos(c +
d*x)**7/(7*d) - 2*b**2*cos(c + d*x)**9/(63*d), Ne(d, 0)), (x*(a + b*sin(c))
**2*sin(c)*cos(c)**6, True))
```

Giac [A]
time = 0.50, size = 176, normalized size = 1.16

$$\frac{5}{64}\,abx + \frac{b^{2}\cos(9\,dx+9\,c)}{2304\,d} - \frac{a^{2}\cos(5\,dx+5\,c)}{64\,d} - \frac{ab\sin(8\,dx+8\,c)}{512\,d} - \frac{ab\sin(6\,dx+6\,c)}{96\,d} - \frac{ab\sin(4\,dx+4\,c)}{64\,d} + \frac{ab\sin(2\,dx+2\,c)}{32\,d} - \frac{(4\,a^{2}-3\,b^{2})\cos(7\,dx+7\,c)}{1792\,d} - \frac{(9\,a^{2}+2\,b^{2})\cos(3\,dx+3\,c)}{192\,d} - \frac{(10\,a^{2}+3\,b^{2})\cos(dx+c)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 5/64*a*b*x + 1/2304*b^2*cos(9*d*x + 9*c)/d - 1/64*a^2*cos(5*d*x + 5*c)/d -
1/512*a*b*sin(8*d*x + 8*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d - 1/64*a*b*sin(4
*d*x + 4*c)/d + 1/32*a*b*sin(2*d*x + 2*c)/d - 1/1792*(4*a^2 - 3*b^2)*cos(7*
d*x + 7*c)/d - 1/192*(9*a^2 + 2*b^2)*cos(3*d*x + 3*c)/d - 1/128*(10*a^2 + 3
*b^2)*cos(d*x + c)/d
```

Mupad [B]
time = 15.05, size = 332, normalized size = 2.18

$$\frac{5\,a\,b\,x}{64} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,(4\,a^2+4\,b^2) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\left(\frac{4\,a^2}{7}+\frac{4\,b^2}{7}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}(12\,a^2+12\,b^2) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}(16\,a^2-12\,b^2) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\left(12\,a^2-\frac{20\,b^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}(20\,a^2+20\,b^2) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\left(\frac{44\,a^2}{7}-\frac{12\,b^2}{7}\right) + 2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16} + \frac{2\,a^2}{7} + \frac{4\,b^2}{63} - \frac{191\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{48} + \frac{83\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{16} - \frac{145\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{16} + \frac{145\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{16} - \frac{83\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{16} + \frac{191\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{48} - \frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{32} + \frac{5\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{32}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*sin(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a*b*x)/64 - (tan(c/2 + (d*x)/2)^14*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^
2*((4*a^2)/7 + (4*b^2)/7) + tan(c/2 + (d*x)/2)^6*(12*a^2 + 12*b^2) + tan(c/
2 + (d*x)/2)^8*(16*a^2 - 12*b^2) + tan(c/2 + (d*x)/2)^12*(12*a^2 - (20*b^2)
/3) + tan(c/2 + (d*x)/2)^10*(20*a^2 + 20*b^2) + tan(c/2 + (d*x)/2)^4*((44*a
^2)/7 - (12*b^2)/7) + 2*a^2*tan(c/2 + (d*x)/2)^16 + (2*a^2)/7 + (4*b^2)/63
- (191*a*b*tan(c/2 + (d*x)/2)^3)/48 + (83*a*b*tan(c/2 + (d*x)/2)^5)/16 - (1
45*a*b*tan(c/2 + (d*x)/2)^7)/16 + (145*a*b*tan(c/2 + (d*x)/2)^11)/16 - (83*
a*b*tan(c/2 + (d*x)/2)^13)/16 + (191*a*b*tan(c/2 + (d*x)/2)^15)/48 - (5*a*b
*tan(c/2 + (d*x)/2)^17)/32 + (5*a*b*tan(c/2 + (d*x)/2))/32)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^9)
```

3.1244 $\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=157

$$\frac{5abx}{8}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^5(c+dx)}{5d}-\frac{b^2\cos^7(c+dx)}{7d}+\frac{5ab\cos}{}$$

```
[Out] 5/8*a*b*x-a^2*arctanh(cos(d*x+c))/d+a^2*cos(d*x+c)/d+1/3*a^2*cos(d*x+c)^3/d
+1/5*a^2*cos(d*x+c)^5/d-1/7*b^2*cos(d*x+c)^7/d+5/8*a*b*cos(d*x+c)*sin(d*x+c
)/d+5/12*a*b*cos(d*x+c)^3*sin(d*x+c)/d+1/3*a*b*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2990, 2715, 8, 14, 213}

$$\frac{a^2\cos^5(c+dx)}{5d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos(c+dx)}{d}-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{ab\sin(c+dx)\cos^5(c+dx)}{3d}+\frac{5ab\sin(c+dx)\cos^3(c+dx)}{12d}+\frac{5ab\sin(c+dx)\cos(c+dx)}{8d}+\frac{5abx}{8}-\frac{b^2\cos^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*x)/8 - (a^2*ArcTanh[Cos[c + d*x]])/d + (a^2*Cos[c + d*x])/d + (a^2*C
os[c + d*x]^3)/(3*d) + (a^2*Cos[c + d*x]^5)/(5*d) - (b^2*Cos[c + d*x]^7)/(7
*d) + (5*a*b*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*a*b*Cos[c + d*x]^3*Sin[c
 + d*x])/(12*d) + (a*b*Cos[c + d*x]^5*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)\cot(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^6(c+dx)\,dx + \int \cos^5(c+dx)\cot(c+dx)\,(a$$

$$= \frac{ab\cos^5(c+dx)\sin(c+dx)}{3d} + \frac{1}{3}(5ab)\int \cos^4(c+dx)\,dx$$

$$= \frac{5ab\cos^3(c+dx)\sin(c+dx)}{12d} + \frac{ab\cos^5(c+dx)\sin(c+d}{3d}$$

$$= \frac{a^2\cos(c+dx)}{d} + \frac{a^2\cos^3(c+dx)}{3d} + \frac{a^2\cos^5(c+dx)}{5d} - \frac{b^2}{}$$

$$= \frac{5abx}{8} - \frac{a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{a^2\cos(c+dx)}{d} + \frac{a^2 c}{}$$

Mathematica [A]
time = 0.20, size = 166, normalized size = 1.06

$$\frac{4200abc + 4200abdx + 105(88a^2-5b^2)\cos(c+dx) + 35(28a^2-9b^2)\cos(3(c+dx)) + 84a^2\cos(5(c+dx)) - 105b^2\cos(5(c+dx)) - 15b^2\cos(7(c+dx)) - 6720a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 6720a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 3150ab\sin(2(c+dx)) + 630ab\sin(4(c+dx)) + 70ab\sin(6(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4200*a*b*c + 4200*a*b*d*x + 105*(88*a^2 - 5*b^2)*Cos[c + d*x] + 35*(28*a^2
 - 9*b^2)*Cos[3*(c + d*x)] + 84*a^2*Cos[5*(c + d*x)] - 105*b^2*Cos[5*(c + d
*x)] - 15*b^2*Cos[7*(c + d*x)] - 6720*a^2*Log[Cos[(c + d*x)/2]] + 6720*a^2*
Log[Sin[(c + d*x)/2]] + 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin[4*(c + d*x)
] + 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

Maple [A]
time = 0.34, size = 113, normalized size = 0.72

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(d}{8}\right.}{6}\right.}{d}$
risch	$\frac{5abx}{8}+\frac{11a^2e^{i(dx+c)}}{16d}-\frac{5\,e^{i(dx+c)}b^2}{128d}+\frac{11a^2e^{-i(dx+c)}}{16d}-\frac{5\,e^{-i(dx+c)}b^2}{128d}-\frac{a^2\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{a^2\ln\left(e^{i(dx+c)}-\right.}{d}$
norman	$\frac{\frac{\left(6a^2-2b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{322a^2-30b^2}{105d}+\frac{5abx}{8}+\frac{24a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{176a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{232a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\left(1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*
x+c)))+2*a*b*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+5/16*d*x+5/16*c)-1/7*cos(d*x+c)^7*b^2)
```

Maxima [A]
time = 0.28, size = 122, normalized size = 0.78

$$-\frac{480\,b^2\cos\left(dx+c\right)^7-112\,\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)a^2+35\,\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)ab}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*(480*b^2*cos(d*x + c)^7 - 112*(6*cos(d*x + c)^5 + 10*cos(d*x + c)^3
 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a
^2 + 35*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin
(2*d*x + 2*c))*a*b)/d
```

Fricas [A]
time = 0.40, size = 137, normalized size = 0.87

$$-\frac{120\,b^2\cos\left(dx+c\right)^7-168\,a^2\cos\left(dx+c\right)^5-280\,a^2\cos\left(dx+c\right)^3-525\,abdx-840\,a^2\cos\left(dx+c\right)+420\,a^2\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-420\,a^2\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-35\,\left(8\,ab\cos\left(dx+c\right)^5+10\,ab\cos\left(dx+c\right)^3+15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/840*(120*b^2*cos(d*x + c)^7 - 168*a^2*cos(d*x + c)^5 - 280*a^2*cos(d*x +
 c)^3 - 525*a*b*d*x - 840*a^2*cos(d*x + c) + 420*a^2*log(1/2*cos(d*x + c) +
```

```
 1/2) - 420*a^2*log(-1/2*cos(d*x + c) + 1/2) - 35*(8*a*b*cos(d*x + c)^5 + 1
0*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(143) = 286.
time = 0.54, size = 291, normalized size = 1.85

$$\frac{525\,(dx+c)ab+840\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\left(1155\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+840\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{12}+980\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-10080\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+2975\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-20440\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+4200\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-24640\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-2975\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-16968\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2520\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-980\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6496\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1155\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1288\,a^2+120\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(525*(d*x + c)*a*b + 840*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(1155
*a*b*tan(1/2*d*x + 1/2*c)^13 - 2520*a^2*tan(1/2*d*x + 1/2*c)^12 + 840*b^2*t
an(1/2*d*x + 1/2*c)^12 + 980*a*b*tan(1/2*d*x + 1/2*c)^11 - 10080*a^2*tan(1/
2*d*x + 1/2*c)^10 + 2975*a*b*tan(1/2*d*x + 1/2*c)^9 - 20440*a^2*tan(1/2*d*x
 + 1/2*c)^8 + 4200*b^2*tan(1/2*d*x + 1/2*c)^8 - 24640*a^2*tan(1/2*d*x + 1/2
*c)^6 - 2975*a*b*tan(1/2*d*x + 1/2*c)^5 - 16968*a^2*tan(1/2*d*x + 1/2*c)^4
+ 2520*b^2*tan(1/2*d*x + 1/2*c)^4 - 980*a*b*tan(1/2*d*x + 1/2*c)^3 - 6496*a
^2*tan(1/2*d*x + 1/2*c)^2 - 1155*a*b*tan(1/2*d*x + 1/2*c) - 1288*a^2 + 120*
b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/d
```

Mupad [B]
time = 13.71, size = 415, normalized size = 2.64

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(6\,a^2-2\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{146\,a^2}{3}-10\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{202\,a^2}{5}-6\,b^2\right)+\frac{232\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{176\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+24\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{46\,a^2}{15}-\frac{2\,b^2}{7}+\frac{7\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{85\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12}-\frac{85\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{12}-\frac{7\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{3}-\frac{11\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{4}+\frac{11\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+35\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+21\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{5\,a\,b\,\mathrm{atan}\left(\frac{25\,a^2\,b^2}{16\,\left(\frac{5\,a^3\,b}{2}-\frac{25\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}+\frac{5\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{5\,a^3\,b}{2}-\frac{25\,a^2\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)^12*(6*a^2 - 2*b^2) +
tan(c/2 + (d*x)/2)^8*((146*a^2)/3 - 10*b^2) + tan(c/2 + (d*x)/2)^4*((202*a^
2)/5 - 6*b^2) + (232*a^2*tan(c/2 + (d*x)/2)^2)/15 + (176*a^2*tan(c/2 + (d*x
)/2)^6)/3 + 24*a^2*tan(c/2 + (d*x)/2)^10 + (46*a^2)/15 - (2*b^2)/7 + (7*a*b
*tan(c/2 + (d*x)/2)^3)/3 + (85*a*b*tan(c/2 + (d*x)/2)^5)/12 - (85*a*b*tan(c
/2 + (d*x)/2)^9)/12 - (7*a*b*tan(c/2 + (d*x)/2)^11)/3 - (11*a*b*tan(c/2 + (
```

```
d*x)/2)^13)/4 + (11*a*b*tan(c/2 + (d*x)/2))/4)/(d*(7*tan(c/2 + (d*x)/2)^2 +
 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^
8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)
^14 + 1)) + (5*a*b*atan((25*a^2*b^2)/(16*((5*a^3*b)/2 - (25*a^2*b^2*tan(c/2
 + (d*x)/2))/16)) + (5*a^3*b*tan(c/2 + (d*x)/2))/(2*((5*a^3*b)/2 - (25*a^2*
b^2*tan(c/2 + (d*x)/2))/16))))/(4*d)
```

3.1245 $\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=178

$$-\frac{5}{16}\left(6a^2-b^2\right)x-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\cos^3(c+dx)}{3d}+\frac{2ab\cos^5(c+dx)}{5d}-\frac{a^2\cot(c+dx)}{d}$$

```
[Out] -5/16*(6*a^2-b^2)*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d+2/3*a*b*
cos(d*x+c)^3/d+2/5*a*b*cos(d*x+c)^5/d-a^2*cot(d*x+c)/d-1/16*(14*a^2-5*b^2)*
cos(d*x+c)*sin(d*x+c)/d-1/24*(6*a^2-5*b^2)*cos(d*x+c)^3*sin(d*x+c)/d+1/6*b^
2*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2990, 2672, 308, 212, 445, 467, 464, 209}

$$-\frac{(6a^2-5b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(14a^2-5b^2)\sin(c+dx)\cos(c+dx)}{16d}-\frac{5}{16}x(6a^2-b^2)-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\cos^5(c+dx)}{5d}+\frac{2ab\cos^3(c+dx)}{3d}+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-5*(6*a^2 - b^2)*x)/16 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c +
d*x])/d + (2*a*b*Cos[c + d*x]^3)/(3*d) + (2*a*b*Cos[c + d*x]^5)/(5*d) - (a^
2*Cot[c + d*x])/d - ((14*a^2 - 5*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (
(6*a^2 - 5*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^5*S
in[c + d*x])/(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^5(c+dx)\cot(c+dx)\,dx + \int \cos^4(c+dx)\,c$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+b^2+\frac{a^2}{x^2}}{(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{Subst}\Big(\int}{}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+(a^2+b^2)x^2}{x^2(1+x^2)^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{Subst}}{}$$

$$= \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos^3(c+dx)}{3d} + \frac{2ab\cos^5(c+dx)}{5d}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos}{}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos}{}$$

$$= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\cos}{}$$

$$= -\frac{5}{16}(6a^2-b^2)\,x - \frac{2ab\tanh^{-1}(\cos(c+dx))}{d} + \frac{2ab\cos(}{d}$$

Mathematica [A]
time = 0.28, size = 220, normalized size = 1.24

$$-\frac{15a^2(c+dx)}{8d}+\frac{5b^2(c+dx)}{16d}+\frac{11ab\cos(c+dx)}{4d}+\frac{7ab\cos(3(c+dx))}{24d}+\frac{ab\cos(5(c+dx))}{40d}-\frac{a^2\cot(c+dx)}{d}-\frac{2ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\sin(2(c+dx))}{2d}+\frac{15b^2\sin(2(c+dx))}{64d}-\frac{a^2\sin(4(c+dx))}{32d}+\frac{3b^2\sin(4(c+dx))}{64d}+\frac{b^2\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a^2*(c + d*x))/(8*d) + (5*b^2*(c + d*x))/(16*d) + (11*a*b*Cos[c + d*x]
)/(4*d) + (7*a*b*Cos[3*(c + d*x)])/(24*d) + (a*b*Cos[5*(c + d*x)])/(40*d) -
 (a^2*Cot[c + d*x])/d - (2*a*b*Log[Cos[(c + d*x)/2]])/d + (2*a*b*Log[Sin[(c
 + d*x)/2]])/d - (a^2*Sin[2*(c + d*x)])/(2*d) + (15*b^2*Sin[2*(c + d*x)])/(
64*d) - (a^2*Sin[4*(c + d*x)])/(32*d) + (3*b^2*Sin[4*(c + d*x)])/(64*d) + (
b^2*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.28, size = 165, normalized size = 0.93

method	result

derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2ab\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx\right.}{3}\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+2ab\left(\frac{\left(\cos^5(dx+c)\right)}{5}+\frac{\left(\cos^3(dx\right.}{3}\right.}{d}$
risch	$-\frac{15a^2x}{8}+\frac{5b^2x}{16}-\frac{15ib^2e^{2i(dx+c)}}{128d}+\frac{15ib^2e^{-2i(dx+c)}}{128d}+\frac{11ab\,e^{i(dx+c)}}{8d}+\frac{11ab\,e^{-i(dx+c)}}{8d}+\frac{ie^{2i(dx+c)}a^2}{4d}-\frac{}{d}$
norman	$\frac{\left(-\frac{225a^2}{8}+\frac{75b^2}{16}\right)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{225a^2}{8}+\frac{75b^2}{16}\right)x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{75a^2}{2}+\frac{25b^2}{4}\right)x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{45a^2}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*co
s(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c)+2*a*b*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c
)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*
x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]

time = 0.48, size = 172, normalized size = 0.97

$$-\frac{120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+25\,\tan(dx+c)^2+8}{\tan(dx+c)^5+2\,\tan(dx+c)^3+\tan(dx+c)}\right)a^2-64\left(6\,\cos\left(dx+c\right)^5+10\,\cos\left(dx+c\right)^3+30\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)ab+5\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)b^2}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/960*(120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(t
an(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^2 - 64*(6*cos(d*x + c)^
5 + 10*cos(d*x + c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log
(cos(d*x + c) - 1))*a*b + 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4
*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^2)/d
```

Fricas [A]

time = 0.38, size = 188, normalized size = 1.06

$$-\frac{40\,b^2\cos\left(dx+c\right)^7-10\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^5-25\left(6\,a^2-b^2\right)\cos\left(dx+c\right)^3+240\,ab\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-240\,ab\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+75\left(6\,a^2-b^2\right)\cos\left(dx+c\right)-\left(96\,ab\cos\left(dx+c\right)^5+160\,ab\cos\left(dx+c\right)^3-75\left(6\,a^2-b^2\right)dx+480\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/240*(40*b^2*cos(d*x + c)^7 - 10*(6*a^2 - b^2)*cos(d*x + c)^5 - 25*(6*a^2
 - b^2)*cos(d*x + c)^3 + 240*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 240*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 75*(6*a^2 - b^2)*cos(d
*x + c) - (96*a*b*cos(d*x + c)^5 + 160*a*b*cos(d*x + c)^3 - 75*(6*a^2 - b^2
)*d*x + 480*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(166) = 332.
time = 0.53, size = 368, normalized size = 2.07

$$\frac{480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+120\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-75\,(6\,a^2-b^2)(dx+c)-\frac{120\,\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\,\left(270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-165\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1440\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+570\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+25\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+4320\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+300\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-450\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+7360\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-300\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+450\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6720\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-570\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2976\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-270\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+165\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+736\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 120*a^2*tan(1/2*d*x + 1/2*c
) - 75*(6*a^2 - b^2)*(d*x + c) - 120*(4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan
(1/2*d*x + 1/2*c) + 2*(270*a^2*tan(1/2*d*x + 1/2*c)^11 - 165*b^2*tan(1/2*d*
x + 1/2*c)^11 + 1440*a*b*tan(1/2*d*x + 1/2*c)^10 + 570*a^2*tan(1/2*d*x + 1/
2*c)^9 + 25*b^2*tan(1/2*d*x + 1/2*c)^9 + 4320*a*b*tan(1/2*d*x + 1/2*c)^8 +
300*a^2*tan(1/2*d*x + 1/2*c)^7 - 450*b^2*tan(1/2*d*x + 1/2*c)^7 + 7360*a*b*
tan(1/2*d*x + 1/2*c)^6 - 300*a^2*tan(1/2*d*x + 1/2*c)^5 + 450*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6720*a*b*tan(1/2*d*x + 1/2*c)^4 - 570*a^2*tan(1/2*d*x + 1/2
*c)^3 - 25*b^2*tan(1/2*d*x + 1/2*c)^3 + 2976*a*b*tan(1/2*d*x + 1/2*c)^2 - 2
70*a^2*tan(1/2*d*x + 1/2*c) + 165*b^2*tan(1/2*d*x + 1/2*c) + 736*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 11.89, size = 683, normalized size = 3.84

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{7a^2}{2}+\frac{5b^2}{12}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(10\,a^2+\frac{15b^2}{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{7a^2}{2}-\frac{11b^2}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{21a^2}{2}-\frac{11b^2}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(25\,a^2-\frac{15b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{49a^2}{2}+\frac{5b^2}{12}\right)-a^2+\frac{248\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{5}+112\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{368\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+72\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+24\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\frac{184\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{15}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}-\frac{\mathrm{atan}\left([illegible]\right)\left(\frac{15a^2}{4}-\frac{5b^2}{8}\right)}{d}+\frac{2\,ab\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^10*((7*a^2)/2 + (5*b^2)/12) - tan(c/2 + (d*x)/2)^8*(10*
a^2 + (15*b^2)/2) + tan(c/2 + (d*x)/2)^12*((7*a^2)/2 - (11*b^2)/4) - tan(c/
```

```
2 + (d*x)/2)^2*((21*a^2)/2 - (11*b^2)/4) - tan(c/2 + (d*x)/2)^6*(25*a^2 - (
15*b^2)/2) - tan(c/2 + (d*x)/2)^4*((49*a^2)/2 + (5*b^2)/12) - a^2 + (248*a*
b*tan(c/2 + (d*x)/2)^3)/5 + 112*a*b*tan(c/2 + (d*x)/2)^5 + (368*a*b*tan(c/2
 + (d*x)/2)^7)/3 + 72*a*b*tan(c/2 + (d*x)/2)^9 + 24*a*b*tan(c/2 + (d*x)/2)^
11 + (184*a*b*tan(c/2 + (d*x)/2))/15)/(d*(2*tan(c/2 + (d*x)/2) + 12*tan(c/2
 + (d*x)/2)^3 + 30*tan(c/2 + (d*x)/2)^5 + 40*tan(c/2 + (d*x)/2)^7 + 30*tan(
c/2 + (d*x)/2)^9 + 12*tan(c/2 + (d*x)/2)^11 + 2*tan(c/2 + (d*x)/2)^13)) + (
a^2*tan(c/2 + (d*x)/2))/(2*d) - (atan((((a^2*15i)/8 - (b^2*5i)/16)*((5*b^2)
/8 - (15*a^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) + 4*a*b*
tan(c/2 + (d*x)/2))*1i - ((a^2*15i)/8 - (b^2*5i)/16)*((15*a^2)/4 - (5*b^2)/
8 + 6*tan(c/2 + (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) - 4*a*b*tan(c/2 + (d*x
)/2))*1i)/(((a^2*15i)/8 - (b^2*5i)/16)*((5*b^2)/8 - (15*a^2)/4 + 6*tan(c/2
+ (d*x)/2)*((a^2*15i)/8 - (b^2*5i)/16) + 4*a*b*tan(c/2 + (d*x)/2)) + ((a^2*
15i)/8 - (b^2*5i)/16)*((15*a^2)/4 - (5*b^2)/8 + 6*tan(c/2 + (d*x)/2)*((a^2*
15i)/8 - (b^2*5i)/16) - 4*a*b*tan(c/2 + (d*x)/2)) + (5*a*b^3)/2 - 15*a^3*b
+ 2*tan(c/2 + (d*x)/2)*((225*a^4)/16 + (25*b^4)/64 - (75*a^2*b^2)/16)))*((1
5*a^2)/4 - (5*b^2)/8))/d + (2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1246 $\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=180

$$-\frac{15}{4}abx+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}+\frac{b^2\cos^5(c+dx)}{5d}$$

```
[Out] -15/4*a*b*x+1/2*(5*a^2-2*b^2)*arctanh(cos(d*x+c))/d-(2*a^2-b^2)*cos(d*x+c)/
d-1/3*(a^2-b^2)*cos(d*x+c)^3/d+1/5*b^2*cos(d*x+c)^5/d-15/4*a*b*cot(d*x+c)/d
+5/4*a*b*cos(d*x+c)^2*cot(d*x+c)/d+1/2*a*b*cos(d*x+c)^4*cot(d*x+c)/d-1/2*a^
2*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2990, 2671, 294, 327, 209, 466, 1824, 212}

$$-\frac{(a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(2a^2-b^2)\cos(c+dx)}{d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{2d}-\frac{15ab\cot(c+dx)}{4d}+\frac{ab\cos^4(c+dx)\cot(c+dx)}{2d}+\frac{5ab\cos^2(c+dx)\cot(c+dx)}{4d}-\frac{15abx}{4}+\frac{b^2\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a*b*x)/4 + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*d) - ((2*a^2 - b
^2)*Cos[c + d*x])/d - ((a^2 - b^2)*Cos[c + d*x]^3)/(3*d) + (b^2*Cos[c + d*x
]^5)/(5*d) - (15*a*b*Cot[c + d*x])/(4*d) + (5*a*b*Cos[c + d*x]^2*Cot[c + d*
x])/(4*d) + (a*b*Cos[c + d*x]^4*Cot[c + d*x])/(2*d) - (a^2*Cot[c + d*x]*Csc
[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^4(c+dx)\cot^2(c+dx)\,dx + \int \cos^3(c+dx)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\text{Su}}{}$$

$$= \frac{ab\cos^4(c+dx)\cot(c+dx)}{2d} - \frac{a^2\cot(c+dx)\csc(c+dx}{2d}$$

$$= \frac{5ab\cos^2(c+dx)\cot(c+dx)}{4d} + \frac{ab\cos^4(c+dx)\cot(c+}{2d}$$

$$= -\frac{(2a^2-b^2)\cos(c+dx)}{d} - \frac{(a^2-b^2)\cos^3(c+dx)}{3d} + \frac{b^2}{}$$

$$= -\frac{15}{4}abx + \frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2d} - \frac{(2a^2-b}{}$$

Mathematica [A]

time = 6.13, size = 250, normalized size = 1.39

$$-\frac{15ab(c+dx)}{4d} - \frac{(18a^2-11b^2)\cos(c+dx)}{8d} - \frac{(4a^2-7b^2)\cos(3(c+dx))}{48d} + \frac{b^2\cos(5(c+dx))}{80d} - \frac{ab\cot\left(\frac{1}{2}(c+dx)\right)}{d} - \frac{a^2\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{(5a^2-2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{(-5a^2+2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{ab\sin(2(c+dx))}{d} - \frac{ab\sin(4(c+dx))}{16d} + \frac{ab\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-15*a*b*(c + d*x))/(4*d) - ((18*a^2 - 11*b^2)*Cos[c + d*x])/(8*d) - ((4*a^
2 - 7*b^2)*Cos[3*(c + d*x)])/(48*d) + (b^2*Cos[5*(c + d*x)])/(80*d) - (a*b*
Cot[(c + d*x)/2])/d - (a^2*Csc[(c + d*x)/2]^2)/(8*d) + ((5*a^2 - 2*b^2)*Log
[Cos[(c + d*x)/2]])/(2*d) + ((-5*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*d)
+ (a^2*Sec[(c + d*x)/2]^2)/(8*d) - (a*b*Sin[2*(c + d*x)])/d - (a*b*Sin[4*(c
 + d*x)])/(16*d) + (a*b*Tan[(c + d*x)/2])/d
```

Maple [A]

time = 0.29, size = 187, normalized size = 1.04

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(d\right.\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{2}-\frac{5\left(\cos^3(dx+c)\right)}{6}-\frac{5\cos(dx+c)}{2}-\frac{5\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(d\right.\right.}{}$
risch	$-\frac{15abx}{4}-\frac{e^{3i(dx+c)}a^2}{24d}+\frac{7\,e^{3i(dx+c)}b^2}{96d}+\frac{iab\,e^{2i(dx+c)}}{2d}-\frac{9a^2e^{i(dx+c)}}{8d}+\frac{11\,e^{i(dx+c)}b^2}{16d}-\frac{9a^2e^{-i(dx+c)}}{8d}+\frac{11\,e}{}$

norman	$\frac{ab\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2}{8d}+\frac{a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(8a^2-6b^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(205a^2-96b^2\right)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(259a^2-112b^2\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-
5/2*cos(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/sin(d*x+c)*cos(d*x+
c)^7-(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15
/8*c)+b^2*(1/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d
*x+c))))
```

Maxima [A]
time = 0.51, size = 190, normalized size = 1.06

$$-\frac{5\left(4\cos\left(dx+c\right)^3-\frac{6\cos\left(dx+c\right)}{\cos\left(dx+c\right)^2-1}+24\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)a^2+15\left(15\,dx+15\,c+\frac{15\tan\left(dx+c\right)^4+25\tan\left(dx+c\right)^2+8}{\tan\left(dx+c\right)^5+2\tan\left(dx+c\right)^3+\tan\left(dx+c\right)}\right)ab-2\left(6\cos\left(dx+c\right)^5+10\cos\left(dx+c\right)^3+30\cos\left(dx+c\right)-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)b^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/60*(5*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d
*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^2 + 15*(15
*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5 +
 2*tan(d*x + c)^3 + tan(d*x + c)))*a*b - 2*(6*cos(d*x + c)^5 + 10*cos(d*x +
 c)^3 + 30*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1))*b^2)/d
```

Fricas [A]
time = 0.39, size = 244, normalized size = 1.36

$$\frac{12\,b^2\cos\left(dx+c\right)^7-225\,abdx\cos\left(dx+c\right)^2-4\left(5\,a^2-2\,b^2\right)\cos\left(dx+c\right)^5+225\,abdx-20\left(5\,a^2-2\,b^2\right)\cos\left(dx+c\right)^3+30\left(5\,a^2-2\,b^2\right)\cos\left(dx+c\right)+15\left(\left(5\,a^2-2\,b^2\right)\cos\left(dx+c\right)^2-5\,a^2+2\,b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(\left(5\,a^2-2\,b^2\right)\cos\left(dx+c\right)^2-5\,a^2+2\,b^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(2\,ab\cos\left(dx+c\right)^5+5\,ab\cos\left(dx+c\right)^3-15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{60\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/60*(12*b^2*cos(d*x + c)^7 - 225*a*b*d*x*cos(d*x + c)^2 - 4*(5*a^2 - 2*b^2
)*cos(d*x + c)^5 + 225*a*b*d*x - 20*(5*a^2 - 2*b^2)*cos(d*x + c)^3 + 30*(5*
a^2 - 2*b^2)*cos(d*x + c) + 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a^2 + 2*
b^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^2 - 2*b^2)*cos(d*x + c)^2 - 5*a
^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 15*(2*a*b*cos(d*x + c)^5 + 5*a*b
*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(164) = 328.
time = 0.54, size = 346, normalized size = 1.92

$$\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-450\,(dx+c)ab+120\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-60\,(5\,a^2-2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{15\left(30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}+\frac{4\left(135\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-180\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+180\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+150\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-600\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+360\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-800\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+560\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-150\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+280\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-135\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-140\,a^2+92\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(15*a^2*tan(1/2*d*x + 1/2*c)^2 - 450*(d*x + c)*a*b + 120*a*b*tan(1/2*
d*x + 1/2*c) - 60*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 15*(30*a
^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d
*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^2 + 4*(135*a*b*tan(1/2*d*x + 1/2*c)
^9 - 180*a^2*tan(1/2*d*x + 1/2*c)^8 + 180*b^2*tan(1/2*d*x + 1/2*c)^8 + 150*
a*b*tan(1/2*d*x + 1/2*c)^7 - 600*a^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^2*tan(1
/2*d*x + 1/2*c)^6 - 800*a^2*tan(1/2*d*x + 1/2*c)^4 + 560*b^2*tan(1/2*d*x +
1/2*c)^4 - 150*a*b*tan(1/2*d*x + 1/2*c)^3 - 520*a^2*tan(1/2*d*x + 1/2*c)^2
+ 280*b^2*tan(1/2*d*x + 1/2*c)^2 - 135*a*b*tan(1/2*d*x + 1/2*c) - 140*a^2 +
 92*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 11.76, size = 484, normalized size = 2.69

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{5\,a^2}{2}-b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{49\,a^2}{2}-24\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{165\,a^2}{2}-48\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{127\,a^2}{6}-\frac{184\,b^2}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{223\,a^2}{3}-\frac{112\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{335\,a^2}{3}-\frac{224\,b^2}{3}\right)+\frac{a^2}{2}+38\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+60\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+40\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-14\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+40\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}+\frac{15\,a\,b\,\mathrm{atan}\left(\frac{225\,a^2\,b^2}{4\left(-\frac{75\,a^3\,b}{2}+\frac{225\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2}{4}+15\,a\,b^3\right)}-\frac{15\,a\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{-\frac{75\,a^3\,b}{2}+\frac{225\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2}{4}+15\,a\,b^3}+\frac{75\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(-\frac{75\,a^3\,b}{2}+\frac{225\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2\,b^2}{4}+15\,a\,b^3\right)}\right)}{2\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^3,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*((5*a^2)/2 - b^
2))/d - (tan(c/2 + (d*x)/2)^10*((49*a^2)/2 - 24*b^2) + tan(c/2 + (d*x)/2)^8
*((165*a^2)/2 - 48*b^2) + tan(c/2 + (d*x)/2)^2*((127*a^2)/6 - (184*b^2)/15)
 + tan(c/2 + (d*x)/2)^4*((223*a^2)/3 - (112*b^2)/3) + tan(c/2 + (d*x)/2)^6*
((335*a^2)/3 - (224*b^2)/3) + a^2/2 + 38*a*b*tan(c/2 + (d*x)/2)^3 + 60*a*b*
tan(c/2 + (d*x)/2)^5 + 40*a*b*tan(c/2 + (d*x)/2)^7 - 14*a*b*tan(c/2 + (d*x)
/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2
```

```
+ (d*x)/2)^4 + 40*tan(c/2 + (d*x)/2)^6 + 40*tan(c/2 + (d*x)/2)^8 + 20*tan(c
/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12)) + (15*a*b*atan((225*a^2*b^2)/(
4*(15*a*b^3 - (75*a^3*b)/2 + (225*a^2*b^2*tan(c/2 + (d*x)/2))/4)) - (15*a*b
^3*tan(c/2 + (d*x)/2))/(15*a*b^3 - (75*a^3*b)/2 + (225*a^2*b^2*tan(c/2 + (d
*x)/2))/4) + (75*a^3*b*tan(c/2 + (d*x)/2))/(2*(15*a*b^3 - (75*a^3*b)/2 + (2
25*a^2*b^2*tan(c/2 + (d*x)/2))/4))))/(2*d) + (a*b*tan(c/2 + (d*x)/2))/d
```

3.1247 $\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=177

$$\frac{5}{8}\left(4a^2-3b^2\right)x+\frac{5ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{5ab\cos(c+dx)}{d}-\frac{5ab\cos^3(c+dx)}{3d}+\frac{\left(2a^2-b^2\right)\cot(c+dx)}{d}-\frac{ab\,\mathrm{co}}{}$$

```
[Out] 5/8*(4*a^2-3*b^2)*x+5*a*b*arctanh(cos(d*x+c))/d-5*a*b*cos(d*x+c)/d-5/3*a*b*
cos(d*x+c)^3/d+(2*a^2-b^2)*cot(d*x+c)/d-a*b*cos(d*x+c)^3*cot(d*x+c)^2/d-1/3
*a^2*cot(d*x+c)^3/d+1/8*(4*a^2-7*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*b^2*cos(d
*x+c)^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2990, 2672, 294, 308, 212, 467, 1273, 1275, 209}

$$\frac{\left(2a^2-b^2\right)\cot(c+dx)}{d}+\frac{\left(4a^2-7b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}x\left(4a^2-3b^2\right)-\frac{a^2\cot^3(c+dx)}{3d}-\frac{5ab\cos^3(c+dx)}{3d}-\frac{5ab\cos(c+dx)}{d}-\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*x)/8 + (5*a*b*ArcTanh[Cos[c + d*x]])/d - (5*a*b*Cos[c +
d*x]])/d - (5*a*b*Cos[c + d*x]^3)/(3*d) + ((2*a^2 - b^2)*Cot[c + d*x])/d - (
a*b*Cos[c + d*x]^3*Cot[c + d*x]^2)/d - (a^2*Cot[c + d*x]^3)/(3*d) + ((4*a^2
 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^2(c+dx)\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cos^3(c+dx)\cot^3(c+dx)\,dx + \int \cos^2(c+dx)$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+(a^2+b^2)x^2}{x^4(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\text{Subst}}{}$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} - \frac{b^2\cos^3(c+dx)\sin(c+}{4d}$$

$$= -\frac{ab\cos^3(c+dx)\cot^2(c+dx)}{d} + \frac{(4a^2-7b^2)\cos(c+dx}{8d}$$

$$= -\frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(c+dx)}{3d} - \frac{ab\cos^3(c+dx)}{d}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c+dx)}{d} - \frac{5ab\cos^3(}{3d}$$

$$= \frac{5}{8}\left(4a^2-3b^2\right)x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{5ab\cos(c}{d}$$

Mathematica [A]
time = 6.21, size = 336, normalized size = 1.90

$$\frac{5(4a^2-3b^2)(c+dx)}{8d} - \frac{9ab\cos(c+dx)}{2d} - \frac{ab\cos(3(c+dx))}{6d} + \frac{\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d} - \frac{ab\csc^2\left(\frac{1}{2}(c+dx)\right)}{4d} - \frac{a^2\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d} + \frac{5ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{5ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{ab\sec^2\left(\frac{1}{2}(c+dx)\right)}{4d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d} + \frac{(a^2-2b^2)\sin(2(c+dx))}{4d} - \frac{b^2\sin(4(c+dx))}{32d} + \frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(4*a^2 - 3*b^2)*(c + d*x))/(8*d) - (9*a*b*Cos[c + d*x])/(2*d) - (a*b*Cos
[3*(c + d*x)])/(6*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*C
sc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c + d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (5*a*b*Log[Cos[(c + d*x)/2]])/d - (5*a*b*
Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/
2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*d) + ((a^2 - 2*b^
2)*Sin[2*(c + d*x)])/(4*d) - (b^2*Sin[4*(c + d*x)])/(32*d) + (a^2*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A]
time = 0.26, size = 223, normalized size = 1.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+2*a
*b*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*co
s(d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/sin(d*x+c)*cos(d*x+c)^7-(co
s(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.54, size = 189, normalized size = 1.07

$$\frac{4\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)a^2-4\left(4\,\cos\left(dx+c\right)^3-\frac{6\,\cos(dx+c)}{\cos(dx+c)^2-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)ab-3\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+25\,\tan(dx+c)^2+8}{\tan(dx+c)^5+2\,\tan(dx+c)^3+\tan(dx+c)}\right)b^2}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(4*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d
*x + c)^5 + tan(d*x + c)^3))*a^2 - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(co
s(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos
(d*x + c) - 1))*a*b - 3*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x +
c)^2 + 8)/(tan(d*x + c)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*b^2)/d
```

Fricas [A]
time = 0.39, size = 252, normalized size = 1.42

$$\frac{6\,b^2\cos\left(dx+c\right)^7-3\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^5+20\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)^3+60\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-60\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-15\left(4\,a^2-3\,b^2\right)\cos\left(dx+c\right)-\left(16\,ab\cos\left(dx+c\right)^5-15\left(4\,a^2-3\,b^2\right)dx\cos\left(dx+c\right)^2+80\,ab\cos\left(dx+c\right)^3+15\left(4\,a^2-3\,b^2\right)dx-120\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\left(d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(6*b^2*cos(d*x + c)^7 - 3*(4*a^2 - 3*b^2)*cos(d*x + c)^5 + 20*(4*a^2 -
 3*b^2)*cos(d*x + c)^3 + 60*(a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) - 60*(a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) - 15*(4*a^2 - 3*b^2)*cos(d*x + c) - (16*a*b*cos(d*x + c
)^5 - 15*(4*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 + 80*a*b*cos(d*x + c)^3 + 15*(4
*a^2 - 3*b^2)*d*x - 120*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2
- d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(167) = 334.
time = 0.53, size = 366, normalized size = 2.07

$$\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-120ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-27a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+15(4a^2-3b^2)(dx+c)+\frac{220ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+27a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{2\left(12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-27b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+144ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+336ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+304ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+27b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+112ab\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 120*a*b*l
og(abs(tan(1/2*d*x + 1/2*c))) - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/
2*d*x + 1/2*c) + 15*(4*a^2 - 3*b^2)*(d*x + c) + (220*a*b*tan(1/2*d*x + 1/2*
c)^3 + 27*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*
b*tan(1/2*d*x + 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3 - 2*(12*a^2*tan(1/2*d*
x + 1/2*c)^7 - 27*b^2*tan(1/2*d*x + 1/2*c)^7 + 144*a*b*tan(1/2*d*x + 1/2*c)
^6 + 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^2*tan(1/2*d*x + 1/2*c)^5 + 336*a*b
*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 304*a*b*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c)
+ 27*b^2*tan(1/2*d*x + 1/2*c) + 112*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 12.10, size = 665, normalized size = 3.76

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}-\frac{\frac{a^2}{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(42a^2-34b^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\frac{83a^2}{3}-14b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{182a^2}{3}-26b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{23a^2}{3}-4b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(a^2+14b^2)+\frac{248ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{644ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{3}+232ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+98ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{9a^2}{8}-\frac{b^2}{2}\right)}{d}+\frac{\mathrm{atan}\left(\frac{\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)\left(\frac{15b^2}{4}-5a^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)+10ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)1i-\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)\left(5a^2-\frac{15b^2}{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)-10ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)1i}{\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)\left(\frac{15b^2}{4}-5a^2+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)+10ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)\left(5a^2-\frac{15b^2}{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a^2 5i}{2}-\frac{b^2 15i}{8}\right)-10ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+\frac{75ab^3}{2}-50a^3b+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(25a^4-\frac{75a^2b^2}{2}+\frac{225b^4}{16}\right)}\right)\left(5a^2-\frac{15b^2}{4}\right)}{d}+\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4d}-\frac{5ab\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^4,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (a^2/3 - tan(c/2 + (d*x)/2)^4*(42*a^2 -
 34*b^2) - tan(c/2 + (d*x)/2)^8*((83*a^2)/3 - 14*b^2) - tan(c/2 + (d*x)/2)^
6*((182*a^2)/3 - 26*b^2) - tan(c/2 + (d*x)/2)^2*((23*a^2)/3 - 4*b^2) - tan(
c/2 + (d*x)/2)^10*(a^2 + 14*b^2) + (248*a*b*tan(c/2 + (d*x)/2)^3)/3 + (644*
a*b*tan(c/2 + (d*x)/2)^5)/3 + 232*a*b*tan(c/2 + (d*x)/2)^7 + 98*a*b*tan(c/2
 + (d*x)/2)^9 + 2*a*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*t
an(c/2 + (d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8
*tan(c/2 + (d*x)/2)^11)) - (tan(c/2 + (d*x)/2)*((9*a^2)/8 - b^2/2))/d + (at
an((((a^2*5i)/2 - (b^2*15i)/8)*((15*b^2)/4 - 5*a^2 + 6*tan(c/2 + (d*x)/2)*(
(a^2*5i)/2 - (b^2*15i)/8) + 10*a*b*tan(c/2 + (d*x)/2))*1i - ((a^2*5i)/2 - (
b^2*15i)/8)*(5*a^2 - (15*b^2)/4 + 6*tan(c/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*1
5i)/8) - 10*a*b*tan(c/2 + (d*x)/2))*1i)/(((a^2*5i)/2 - (b^2*15i)/8)*((15*b^
```

2)/4 - 5*a^2 + 6*tan(c/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*15i)/8) + 10*a*b*tan
(c/2 + (d*x)/2)) + ((a^2*5i)/2 - (b^2*15i)/8)*(5*a^2 - (15*b^2)/4 + 6*tan(c
/2 + (d*x)/2)*((a^2*5i)/2 - (b^2*15i)/8) - 10*a*b*tan(c/2 + (d*x)/2)) + (75
*a*b^3)/2 - 50*a^3*b + 2*tan(c/2 + (d*x)/2)*(25*a^4 + (225*b^4)/16 - (75*a^
2*b^2)/2)))*(5*a^2 - (15*b^2)/4))/d + (a*b*tan(c/2 + (d*x)/2)^2)/(4*d) - (5
*a*b*log(tan(c/2 + (d*x)/2)))/d

3.1248 $\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$5abx-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{5ab\cot(c+dx)}{d}-\frac{5ab\cot^3}{3}$$

```
[Out] 5*a*b*x-5/8*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/d+(a^2-2*b^2)*cos(d*x+c)/d-1/
3*b^2*cos(d*x+c)^3/d+5*a*b*cot(d*x+c)/d-5/3*a*b*cot(d*x+c)^3/d+a*b*cos(d*x+
c)^2*cot(d*x+c)^3/d+1/8*(9*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/4*a^2*cot(d
*x+c)*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2990, 2671, 294, 308, 209, 466, 1828, 1167, 212}

$$\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{5(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{8d}+\frac{(9a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8d}-\frac{a^2\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{5ab\cot^3(c+dx)}{3d}+\frac{5ab\cot(c+dx)}{d}+\frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d}+5abx-\frac{b^2\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] 5*a*b*x - (5*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(8*d) + ((a^2 - 2*b^2)*
Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + (5*a*b*Cot[c + d*x])/d - (5*
a*b*Cot[c + d*x]^3)/(3*d) + (a*b*Cos[c + d*x]^2*Cot[c + d*x]^3)/d + ((9*a^2
 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]
^3)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
```

```
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\cot^5(c+dx)(a+b\sin(c+dx))^2\,dx &= (2ab)\int \cos^2(c+dx)\cot^4(c+dx)\,dx + \int \cos(c+dx)\,\mathrm{co}\\
&= -\frac{\mathrm{Subst}\left(\int \frac{x^6\left(a^2+b^2-b^2x^2\right)}{\left(1-x^2\right)^3}\,dx, x, \cos(c+dx)\right)}{d} - \frac{(2ab)\mathrm{Sub}}{}\\
&= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc^3(c+d}{4d}\\
&= \frac{ab\cos^2(c+dx)\cot^3(c+dx)}{d} + \frac{\left(9a^2-4b^2\right)\cot(c+dx)\,\mathrm{c}}{8d}\\
&= \frac{5ab\cot(c+dx)}{d} - \frac{5ab\cot^3(c+dx)}{3d} + \frac{ab\cos^2(c+dx)\,\mathrm{co}}{d}\\
&= 5abx + \frac{\left(a^2-2b^2\right)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{5ab\,\mathrm{c}}{}\\
&= 5abx - \frac{5\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{8d} + \frac{\left(a^2-2b^2\right)\mathrm{c}}{d}
\end{aligned}$$

Mathematica [A]
time = 6.18, size = 337, normalized size = 1.94

$$\frac{5ab(c+dx)}{d}+\frac{(2a-3b)(2a+3b)\cos(c+dx)}{4d}-\frac{b^2\cos(3(c+dx))}{12d}+\frac{7ab\cot\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{\left(9a^2-4b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{ab\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12d}-\frac{a^2\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5\left(3a^2-4b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{5\left(3a^2-4b^2\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{\left(-9a^2+4b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{ab\sin(2(c+dx))}{2d}-\frac{7ab\tan\left(\frac{1}{2}(c+dx)\right)}{3d}+\frac{ab\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*(c + d*x))/d + ((2*a - 3*b)*(2*a + 3*b)*Cos[c + d*x])/(4*d) - (b^2*C
os[3*(c + d*x)])/(12*d) + (7*a*b*Cot[(c + d*x)/2])/(3*d) + ((9*a^2 - 4*b^2)
*Csc[(c + d*x)/2]^2)/(32*d) - (a*b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
*d) - (a^2*Csc[(c + d*x)/2]^4)/(64*d) - (5*(3*a^2 - 4*b^2)*Log[Cos[(c + d*x
)/2]])/(8*d) + (5*(3*a^2 - 4*b^2)*Log[Sin[(c + d*x)/2]])/(8*d) + ((-9*a^2 +
 4*b^2)*Sec[(c + d*x)/2]^2)/(32*d) + (a^2*Sec[(c + d*x)/2]^4)/(64*d) + (a*b
*Sin[2*(c + d*x)])/(2*d) - (7*a*b*Tan[(c + d*x)/2])/(3*d) + (a*b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*d)
```

Maple [A]
time = 0.29, size = 245, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*
cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)
))+2*a*b*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(c
os(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+b^2
*(-1/2/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(
d*x+c)-5/2*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.49, size = 205, normalized size = 1.18

$$\frac{16\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^{4}+10\,\tan\left(dx+c\right)^{2}-2}{\tan\left(dx+c\right)^{5}+\tan\left(dx+c\right)^{3}}\right)ab-4\left(4\,\cos\left(dx+c\right)^{3}-\frac{6\,\cos\left(dx+c\right)}{\cos\left(dx+c\right)^{2}-1}+24\,\cos\left(dx+c\right)-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)b^{2}-3\,a^{2}\left(\frac{2\left(9\,\cos\left(dx+c\right)^{3}-7\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/48*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(
d*x + c)^5 + tan(d*x + c)^3))*a*b - 4*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(c
os(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(co
s(d*x + c) - 1))*b^2 - 3*a^2*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*
x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) +
1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.39, size = 309, normalized size = 1.78

$$-\frac{16\,b^{2}\cos\left(dx+c\right)^{7}-240\,abdx\cos\left(dx+c\right)^{4}+480\,abdx\cos\left(dx+c\right)^{2}-16\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{5}-240\,abdx+50\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{3}-30\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)+15\left(\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{4}-2\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,b^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-15\left(\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{4}-2\left(3\,a^{2}-4\,b^{2}\right)\cos\left(dx+c\right)^{2}+3\,a^{2}-4\,b^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-16\left(3\,ab\cos\left(dx+c\right)^{5}-20\,ab\cos\left(dx+c\right)^{3}+15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/48*(16*b^2*cos(d*x + c)^7 - 240*a*b*d*x*cos(d*x + c)^4 + 480*a*b*d*x*cos
(d*x + c)^2 - 16*(3*a^2 - 4*b^2)*cos(d*x + c)^5 - 240*a*b*d*x + 50*(3*a^2 -
 4*b^2)*cos(d*x + c)^3 - 30*(3*a^2 - 4*b^2)*cos(d*x + c) + 15*((3*a^2 - 4*b
^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(
1/2*cos(d*x + c) + 1/2) - 15*((3*a^2 - 4*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4
*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*b^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*
a*b*cos(d*x + c)^5 - 20*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(164) = 328$.
time = 0.52, size = 346, normalized size = 1.99

$$\frac{3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+16ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-48a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+960(dx+c)ab-432ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+120(3a^2-4b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{128\left(3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+9b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2+7b^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3}-\frac{750a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1000b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-432ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-48a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+16ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(3*a^2*tan(1/2*d*x + 1/2*c)^4 + 16*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^
2*tan(1/2*d*x + 1/2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 960*(d*x + c)*a*
b - 432*a*b*tan(1/2*d*x + 1/2*c) + 120*(3*a^2 - 4*b^2)*log(abs(tan(1/2*d*x
+ 1/2*c))) - 128*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x + 1/2*c)
^4 + 9*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*t
an(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 + 7*b^2)/(tan(1/
2*d*x + 1/2*c)^2 + 1)^3 - (750*a^2*tan(1/2*d*x + 1/2*c)^4 - 1000*b^2*tan(1/
2*d*x + 1/2*c)^4 - 432*a*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^2*tan(1/2*d*x + 1/
2*c)^2 + 24*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a*b*tan(1/2*d*x + 1/2*c) + 3*a^
2)/tan(1/2*d*x + 1/2*c)^4)/d
```

Mupad [B]
time = 11.74, size = 479, normalized size = 2.75

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{15a^2}{8}-\frac{5b^2}{2}\right)}{d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{13a^2}{4}-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(36a^2-98b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{173a^2}{4}-\frac{242b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{303a^2}{4}-134b^2\right)-\frac{a^2}{4}+32ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+136ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{320ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-\frac{4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{4}-\frac{b^2}{8}\right)}{d}+\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12d}-\frac{10ab\,\mathrm{atan}\left(\frac{100a^2b^2}{-\frac{75a^3b}{2}+100\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^2+50ab^3}-\frac{50ab^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{-\frac{75a^3b}{2}+100\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^2+50ab^3}+\frac{75a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\left(-\frac{75a^3b}{2}+100\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^2+50ab^3\right)}\right)}{d}-\frac{9ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((15*a^2)/8 - (5*b^2)/2))/d + (a^2*tan(c/2 + (d*x)
/2)^4)/(64*d) + (tan(c/2 + (d*x)/2)^2*((13*a^2)/4 - 2*b^2) + tan(c/2 + (d*x
)/2)^8*(36*a^2 - 98*b^2) + tan(c/2 + (d*x)/2)^4*((173*a^2)/4 - (242*b^2)/3)
 + tan(c/2 + (d*x)/2)^6*((303*a^2)/4 - 134*b^2) - a^2/4 + 32*a*b*tan(c/2 +
(d*x)/2)^3 + 136*a*b*tan(c/2 + (d*x)/2)^5 + (320*a*b*tan(c/2 + (d*x)/2)^7)/
3 + 4*a*b*tan(c/2 + (d*x)/2)^9 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c
/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 + 16*ta
n(c/2 + (d*x)/2)^10)) - (tan(c/2 + (d*x)/2)^2*(a^2/4 - b^2/8))/d + (a*b*tan
(c/2 + (d*x)/2)^3)/(12*d) - (10*a*b*atan((100*a^2*b^2)/(50*a*b^3 - (75*a^3*
b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2)) - (50*a*b^3*tan(c/2 + (d*x)/2))/(50*
a*b^3 - (75*a^3*b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2)) + (75*a^3*b*tan(c/2
+ (d*x)/2))/(2*(50*a*b^3 - (75*a^3*b)/2 + 100*a^2*b^2*tan(c/2 + (d*x)/2))))
)/d - (9*a*b*tan(c/2 + (d*x)/2))/(4*d)
```

3.1249 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c+d}{}$$

```
[Out] -a^2*x+5/2*b^2*x-15/4*a*b*arctanh(cos(d*x+c))/d+15/4*a*b*cos(d*x+c)/d-a^2*c
ot(d*x+c)/d+5/2*b^2*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a^2*
cot(d*x+c)^3/d-5/6*b^2*cot(d*x+c)^3/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-1
/2*a*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 2671, 294, 308, 209, 2672, 327, 212, 3554, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}-\frac{5b^2\cot^3(c+dx)}{6d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{5b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx) + 2ab\cos(c+dx)\cot^5(c+dx) + a\right.$$

$$= a^2\int \cot^6(c+dx)\,dx + (2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx + b^2\int$$

$$= -\frac{a^2\cot^5(c+dx)}{5d} - a^2\int \cot^4(c+dx)\,dx - \frac{(2ab)\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,d\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d} + \frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d} - \frac{ab\cos(c+dx)\,\text{co}}{2d}$$

$$= -\frac{a^2\cot(c+dx)}{d} + \frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d} + \frac{a^2\cot^3(c+dx)}{3d}$$

$$= -a^2x + \frac{15ab\cos(c+dx)}{4d} - \frac{a^2\cot(c+dx)}{d} + \frac{5b^2\cot(c+dx)}{2d} + \frac{5a}{}$$

$$= -a^2x + \frac{5b^2x}{2} - \frac{15ab\tanh^{-1}(\cos(c+dx))}{4d} + \frac{15ab\cos(c+dx)}{4d} - \frac{a}{}$$

Mathematica [A]
time = 0.79, size = 351, normalized size = 1.74

$$\frac{-480a^2c + 1200b^2c - 480a^2dx + 1200b^2dx + 960ab\cos(c+dx) + (-368a^2+560b^2)\cot\left(\frac{1}{2}(c+dx)\right) + 270ab\csc^2\left(\frac{1}{2}(c+dx)\right) - 15ab\csc^4\left(\frac{1}{2}(c+dx)\right) - 1800ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 1800ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 270ab\sec^2\left(\frac{1}{2}(c+dx)\right) + 15ab\sec^4\left(\frac{1}{2}(c+dx)\right) - 328a^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) + 160b^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right) + 96a^2\csc^5(c+dx)\sin^6\left(\frac{1}{2}(c+dx)\right) + \frac{41}{2}a^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) - 10b^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) - \frac{3}{2}a^2\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx) + 120b^2\sin(2(c+dx)) + 368a^2\tan\left(\frac{1}{2}(c+dx)\right) - 560b^2\tan\left(\frac{1}{2}(c+dx)\right)}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

Maple [A]
time = 0.29, size = 216, normalized size = 1.07

method	result

derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+\right.}{8}}{}$
risch	$-a^2x+\frac{5b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{180ia^2e^{8i(dx+c)}-180ib^2e^{8i(dx+}}{}$
norman	$\frac{\left(-a^2+\frac{5b^2}{2}\right)x\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2+\frac{5b^2}{2}\right)x\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2a^2+5b^2)x\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2}{160d}+\frac{a^2\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{160d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/4/
sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/
8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/3/si
n(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5+5/4*c
os(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A]
time = 0.49, size = 183, normalized size = 0.91

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^4-5\,\tan\left(dx+c\right)^2+3}{\tan\left(dx+c\right)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^4+10\,\tan\left(dx+c\right)^2-2}{\tan\left(dx+c\right)^5+\tan\left(dx+c\right)^3}\right)b^2+15\,ab\left(\frac{2\left(9\,\cos\left(dx+c\right)^3-7\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A]
time = 0.40, size = 306, normalized size = 1.51

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx+c\right)^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-225\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(dx+c\right)^2+ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+60\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)+30\left(2\left(2\,a^2-5\,b^2\right)dx\cos\left(dx+c\right)^4-8\,ab\cos\left(dx+c\right)^5-4\left(2\,a^2-5\,b^2\right)dx\cos\left(dx+c\right)^2+25\,ab\cos\left(dx+c\right)^3+2\left(2\,a^2-5\,b^2\right)dx-15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 337, normalized size = 1.67

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1800\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-240\,(2\,a^2-5\,b^2)(dx+c)-\frac{480\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}-\frac{4110\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+330\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

Mupad [**B**]
time = 16.04, size = 888, normalized size = 4.40

$$\frac{95\,b^2\cos(c+d\,x)}{384}-\frac{5\,a^2\cos(c+d\,x)}{24}+\frac{5\,a^2\cos(3\,c+3\,d\,x)}{48}-\frac{23\,a^2\cos(5\,c+5\,d\,x)}{240}-\frac{163\,b^2\cos(3\,c+3\,d\,x)}{384}+\frac{71\,b^2\cos(5\,c+5\,d\,x)}{384}-\frac{b^2\cos(7\,c+7\,d\,x)}{128}+ \ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^6,x)
```

```
[Out] ((95*b^2*cos(c + d*x))/384 - (5*a^2*cos(c + d*x))/24 + (5*a^2*cos(3*c + 3*d
*x))/48 - (23*a^2*cos(5*c + 5*d*x))/240 - (163*b^2*cos(3*c + 3*d*x))/384 +
(71*b^2*cos(5*c + 5*d*x))/384 - (b^2*cos(7*c + 7*d*x))/128 + (5*a^2*atan((1
0*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x
)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/
2 + (d*x)/2)))*sin(3*c + 3*d*x))/8 - (a^2*atan((10*b^2*cos(c/2 + (d*x)/2) -
 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*
x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5
*d*x))/8 - (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/
2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2
+ (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/16 + (5*b^2*atan
((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (
d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos
(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/16 + (5*a*b*sin(c + d*x))/4 - (5*a^2*at
an((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 +
 (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*c
os(c/2 + (d*x)/2)))*sin(c + d*x))/4 + (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/
2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 +
 (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c +
 d*x))/8 + (5*a*b*sin(2*c + 2*d*x))/8 - (5*a*b*sin(3*c + 3*d*x))/8 - (17*a*
b*sin(4*c + 4*d*x))/32 + (a*b*sin(5*c + 5*d*x))/8 + (a*b*sin(6*c + 6*d*x))/
16 + (75*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/32 -
(75*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/64 + (
15*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(5*c + 5*d*x))/64)/(d*
sin(c + d*x)^5)
```

3.1250 $\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=175

$$-2abx+\frac{5(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{b^2\cos(c+dx)}{d}-\frac{2ab\cot(c+dx)}{d}+\frac{2ab\cot^3(c+dx)}{3d}-\frac{2ab\cot^5(c+dx)}{5d}$$

```
[Out] -2*a*b*x+5/16*(a^2-6*b^2)*arctanh(cos(d*x+c))/d+b^2*cos(d*x+c)/d-2*a*b*cot(
d*x+c)/d+2/3*a*b*cot(d*x+c)^3/d-2/5*a*b*cot(d*x+c)^5/d-1/16*(11*a^2-18*b^2)
*cot(d*x+c)*csc(d*x+c)/d+1/24*(13*a^2-6*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/6*
a^2*cot(d*x+c)*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2990, 3554, 8, 4451, 466, 1828, 1171, 396, 212}

$$\frac{5(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{16d}+\frac{(13a^2-6b^2)\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\cot(c+dx)\csc(c+dx)}{16d}-\frac{a^2\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{2ab\cot^5(c+dx)}{5d}+\frac{2ab\cot^3(c+dx)}{3d}-\frac{2ab\cot(c+dx)}{d}-2abx+\frac{b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + (5*(a^2 - 6*b^2)*ArcTanh[Cos[c + d*x]])/(16*d) + (b^2*Cos[c + d*
x])/d - (2*a*b*Cot[c + d*x])/d + (2*a*b*Cot[c + d*x]^3)/(3*d) - (2*a*b*Cot[
c + d*x]^5)/(5*d) - ((11*a^2 - 18*b^2)*Cot[c + d*x]*Csc[c + d*x])/(16*d) +
((13*a^2 - 6*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a^2*Cot[c + d*x]*C
sc[c + d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
```

```
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\,dx + \int \cot^6(c+dx)\csc(c+dx)\,($$

$$= -\frac{2ab\cot^5(c+dx)}{5d} - (2ab)\int \cot^4(c+dx)\,dx - \frac{\text{Subst}}{}$$

$$= \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+dx)}{5d} - \frac{a^2\cot(c+dx)\,c}{6d}$$

$$= -\frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot^5(c+d}{5d}$$

$$= -2abx - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(c+dx)}{3d} - \frac{2ab\cot}{}$$

$$= -2abx + \frac{b^2\cos(c+dx)}{d} - \frac{2ab\cot(c+dx)}{d} + \frac{2ab\cot^3(}{3a}$$

$$= -2abx + \frac{5(a^2-6b^2)\tanh^{-1}(\cos(c+dx))}{16d} + \frac{b^2\cos(c+}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(175) = 350.
time = 0.73, size = 384, normalized size = 2.19

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3840*a*b*c - 3840*a*b*d*x + 1920*b^2*Cos[c + d*x] - 2944*a*b*Cot[(c + d*x
)/2] - 330*a^2*Csc[(c + d*x)/2]^2 + 540*b^2*Csc[(c + d*x)/2]^2 + 60*a^2*Csc
[(c + d*x)/2]^4 - 30*b^2*Csc[(c + d*x)/2]^4 - 5*a^2*Csc[(c + d*x)/2]^6 + 60
0*a^2*Log[Cos[(c + d*x)/2]] - 3600*b^2*Log[Cos[(c + d*x)/2]] - 600*a^2*Log[
Sin[(c + d*x)/2]] + 3600*b^2*Log[Sin[(c + d*x)/2]] + 330*a^2*Sec[(c + d*x)/
2]^2 - 540*b^2*Sec[(c + d*x)/2]^2 - 60*a^2*Sec[(c + d*x)/2]^4 + 30*b^2*Sec[
(c + d*x)/2]^4 + 5*a^2*Sec[(c + d*x)/2]^6 - 2624*a*b*Csc[c + d*x]^3*Sin[(c
+ d*x)/2]^4 + 768*a*b*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + 164*a*b*Csc[(c +
d*x)/2]^4*Sin[c + d*x] - 12*a*b*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 2944*a*b*
Tan[(c + d*x)/2])/(1920*d)
```

Maple [A]
time = 0.34, size = 238, normalized size = 1.36 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/1
6/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*
x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c
)^3-cot(d*x+c)-d*x-c)+b^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*
cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(
d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.50, size = 219, normalized size = 1.25

$$-\frac{64\left(15\,dx+15\,c+\frac{15\,\tan\left(dx+c\right)^{4}-5\,\tan\left(dx+c\right)^{2}+3}{\tan\left(dx+c\right)^{5}}\right)ab-5\,a^{2}\left(\frac{2\left(33\,\cos\left(dx+c\right)^{5}-40\,\cos\left(dx+c\right)^{3}+15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\,\cos\left(dx+c\right)^{4}+3\,\cos\left(dx+c\right)^{2}-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+30\,b^{2}\left(\frac{2\left(9\,\cos\left(dx+c\right)^{3}-7\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/480*(64*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a*b - 5*a^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(
d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*l
og(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 30*b^2*(2*(9*cos(d*x + c
)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(163) = 326$.
time = 0.38, size = 360, normalized size = 2.06

$$-\frac{960\,abdx\cos\left(dx+c\right)^{6}-480\,b^{2}\cos\left(dx+c\right)^{7}-2880\,abdx\cos\left(dx+c\right)^{4}+2880\,abdx\cos\left(dx+c\right)^{2}-330\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{5}-960\,abdx+400\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{3}-150\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)-75\left(\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{6}-3\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+6\,b^{2}\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+75\left(\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{6}-3\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(a^{2}-6\,b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+6\,b^{2}\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-64\left(23\,ab\cos\left(dx+c\right)^{5}-35\,ab\cos\left(dx+c\right)^{3}+15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\left(d\cos\left(dx+c\right)^{6}-3\,d\cos\left(dx+c\right)^{4}+3\,d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/480*(960*a*b*d*x*cos(d*x + c)^6 - 480*b^2*cos(d*x + c)^7 - 2880*a*b*d*x*
cos(d*x + c)^4 + 2880*a*b*d*x*cos(d*x + c)^2 - 330*(a^2 - 6*b^2)*cos(d*x +
c)^5 - 960*a*b*d*x + 400*(a^2 - 6*b^2)*cos(d*x + c)^3 - 150*(a^2 - 6*b^2)*c
os(d*x + c) - 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x +
c)^4 + 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(1/2*cos(d*x + c) +
 1/2) + 75*((a^2 - 6*b^2)*cos(d*x + c)^6 - 3*(a^2 - 6*b^2)*cos(d*x + c)^4 +
 3*(a^2 - 6*b^2)*cos(d*x + c)^2 - a^2 + 6*b^2)*log(-1/2*cos(d*x + c) + 1/2)
```

```
 - 64*(23*a*b*cos(d*x + c)^5 - 35*a*b*cos(d*x + c)^3 + 15*a*b*cos(d*x + c))
*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(163) = 326.
time = 0.57, size = 337, normalized size = 1.93

$$\frac{5\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-280\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+225\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-480\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3840\,(dx+c)ab+2640\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-600\,(a^2-6\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{3840\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{1470\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-8820\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2640\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-225\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+480\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+280\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+45\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1920*(5*a^2*tan(1/2*d*x + 1/2*c)^6 + 24*a*b*tan(1/2*d*x + 1/2*c)^5 - 45*a
^2*tan(1/2*d*x + 1/2*c)^4 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 - 280*a*b*tan(1/2
*d*x + 1/2*c)^3 + 225*a^2*tan(1/2*d*x + 1/2*c)^2 - 480*b^2*tan(1/2*d*x + 1/
2*c)^2 - 3840*(d*x + c)*a*b + 2640*a*b*tan(1/2*d*x + 1/2*c) - 600*(a^2 - 6*
b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + 3840*b^2/(tan(1/2*d*x + 1/2*c)^2 + 1)
 + (1470*a^2*tan(1/2*d*x + 1/2*c)^6 - 8820*b^2*tan(1/2*d*x + 1/2*c)^6 - 264
0*a*b*tan(1/2*d*x + 1/2*c)^5 - 225*a^2*tan(1/2*d*x + 1/2*c)^4 + 480*b^2*tan
(1/2*d*x + 1/2*c)^4 + 280*a*b*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x +
 1/2*c)^2 - 30*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a*b*tan(1/2*d*x + 1/2*c) - 5
*a^2)/tan(1/2*d*x + 1/2*c)^6)/d
```

Mupad [**B**]
time = 15.06, size = 985, normalized size = 5.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^7,x)
```

```
[Out] (5*a^2*sin(c/2 + (d*x)/2)^14 - 5*a^2*cos(c/2 + (d*x)/2)^14 - 40*a^2*cos(c/2
 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 180*a^2*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^10 + 225*a^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 225*a^2
*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 180*a^2*cos(c/2 + (d*x)/2)^10*
```

sin(c/2 + (d*x)/2)^4 + 40*a^2*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 30*b^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 - 450*b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 480*b^2*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 4320*b^2*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 450*b^2*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x)/2)^4 - 30*b^2*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^2 + 24*a*b*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^13 - 24*a*b*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 600*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 - 600*a^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 3600*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 3600*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 - 256*a*b*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^11 + 2360*a*b*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 2360*a*b*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 + 256*a*b*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 7680*a*b*atan((5*a^2*sin(c/2 + (d*x)/2) - 30*b^2*sin(c/2 + (d*x)/2) + 32*a*b*cos(c/2 + (d*x)/2))/(30*b^2*cos(c/2 + (d*x)/2) - 5*a^2*cos(c/2 + (d*x)/2) + 32*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^8 + 7680*a*b*atan((5*a^2*sin(c/2 + (d*x)/2) - 30*b^2*sin(c/2 + (d*x)/2) + 32*a*b*cos(c/2 + (d*x)/2))/(30*b^2*cos(c/2 + (d*x)/2) - 5*a^2*cos(c/2 + (d*x)/2) + 32*a*b*sin(c/2 + (d*x)/2)))*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6)/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6*(cos(c/2 + (d*x)/2)^2 + sin(c/2 + (d*x)/2)^2))

3.1251 $\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=158

$$-b^2x+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b^2\cot(c+dx)}{d}+\frac{b^2\cot^3(c+dx)}{3d}-\frac{b^2\cot^5(c+dx)}{5d}-\frac{a^2\cot^7(c+dx)}{7d}-\frac{5ab\,\mathrm{co}}{}$$

```
[Out] -b^2*x+5/8*a*b*arctanh(cos(d*x+c))/d-b^2*cot(d*x+c)/d+1/3*b^2*cot(d*x+c)^3/
d-1/5*b^2*cot(d*x+c)^5/d-1/7*a^2*cot(d*x+c)^7/d-5/8*a*b*cot(d*x+c)*csc(d*x+
c)/d+5/12*a*b*cot(d*x+c)^3*csc(d*x+c)/d-1/3*a*b*cot(d*x+c)^5*csc(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2990, 2691, 3855, 14, 209}

$$-\frac{a^2\cot^7(c+dx)}{7d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{8d}-\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d}+\frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d}-\frac{5ab\cot(c+dx)\csc(c+dx)}{8d}-\frac{b^2\cot^5(c+dx)}{5d}+\frac{b^2\cot^3(c+dx)}{3d}-\frac{b^2\cot(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(b^2*x) + (5*a*b*ArcTanh[Cos[c + d*x]])/(8*d) - (b^2*Cot[c + d*x])/d + (b^
2*Cot[c + d*x]^3)/(3*d) - (b^2*Cot[c + d*x]^5)/(5*d) - (a^2*Cot[c + d*x]^7)
/(7*d) - (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(8*d) + (5*a*b*Cot[c + d*x]^3*Cs
c[c + d*x])/(12*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x])/(3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc(c+dx)\,dx + \int \cot^6(c+dx)\,\mathrm{c}$$

$$= -\frac{ab\cot^5(c+dx)\csc(c+dx)}{3d} - \frac{1}{3}(5ab)\int \cot^4(c+dx)$$

$$= \frac{5ab\cot^3(c+dx)\csc(c+dx)}{12d} - \frac{ab\cot^5(c+dx)\csc(c+}{3d}$$

$$= -\frac{b^2\cot(c+dx)}{d} + \frac{b^2\cot^3(c+dx)}{3d} - \frac{b^2\cot^5(c+dx)}{5d} -$$

$$= -b^2x + \frac{5ab\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b^2\cot(c+dx)}{d} + \frac{b^2}{}$$

Mathematica [A]
time = 0.99, size = 280, normalized size = 1.77

$$\frac{-14700b^2(c+dx)\csc^6(c+dx)+16800ab\left(\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)\right)-350\cot(c+dx)\csc^6(c+dx)\left(6(a^2+b^2)+17ab\sin(c+dx)\right)+\csc^7(c+dx)\left(-84(15a^2-41b^2)\cos(3(c+dx))-28(15a^2+71b^2)\cos(5(c+dx))-60a^2\cos(7(c+dx))+644b^2\cos(7(c+dx))+8820b^2c\sin(3(c+dx))+8820b^2dx\sin(3(c+dx))+980ab\sin(4(c+dx))-2940b^2c\sin(5(c+dx))-2940b^2dx\sin(5(c+dx))-1155ab\sin(6(c+dx))+420b^2c\sin(7(c+dx))+420b^2dx\sin(7(c+dx))\right)}{26880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-14700*b^2*(c + d*x)*Csc[c + d*x]^6 + 16800*a*b*(Log[Cos[(c + d*x)/2]] - L
og[Sin[(c + d*x)/2]]) - 350*Cot[c + d*x]*Csc[c + d*x]^6*(6*(a^2 + b^2) + 17
*a*b*Sin[c + d*x]) + Csc[c + d*x]^7*(-84*(15*a^2 - 41*b^2)*Cos[3*(c + d*x)]
 - 28*(15*a^2 + 71*b^2)*Cos[5*(c + d*x)] - 60*a^2*Cos[7*(c + d*x)] + 644*b^
2*Cos[7*(c + d*x)] + 8820*b^2*c*Sin[3*(c + d*x)] + 8820*b^2*d*x*Sin[3*(c +
d*x)] + 980*a*b*Sin[4*(c + d*x)] - 2940*b^2*c*Sin[5*(c + d*x)] - 2940*b^2*d
```

```
*x*Sin[5*(c + d*x)] - 1155*a*b*Sin[6*(c + d*x)] + 420*b^2*c*Sin[7*(c + d*x)
] + 420*b^2*d*x*Sin[7*(c + d*x)]))/(26880*d)
```

Maple [A]
time = 0.30, size = 172, normalized size = 1.09

method	result
derivativedivides	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2ab\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(c}{}\right.}{d}$
default	$\frac{-\frac{a^2\left(\cos^7(dx+c)\right)}{7\sin(dx+c)^7}+2ab\left(-\frac{\cos^7(dx+c)}{6\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{24\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{16\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{16}-\frac{5\left(\cos^3(dx+c)\right)}{48}-\frac{5\cos(dx+c)}{16}-\frac{5\ln(c}{}\right.}{d}$
risch	$-b^2x+\frac{24640ib^2e^{6i(dx+c)}+840ia^2e^{12i(dx+c)}+1155ab\,e^{13i(dx+c)}+4200ia^2e^{8i(dx+c)}-980ab\,e^{11i(dx+c)}-1288ib^2-169}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/7*a^2/sin(d*x+c)^7*cos(d*x+c)^7+2*a*b*(-1/6/sin(d*x+c)^6*cos(d*x+c)
^7+1/24/sin(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d
*x+c)^5-5/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c)))+b
^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.50, size = 153, normalized size = 0.97

$$-\frac{112\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)b^2-35\,ab\left(\frac{2\left(33\,\cos(dx+c)^5-40\,\cos(dx+c)^3+15\,\cos(dx+c)\right)}{\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1}+15\,\log\left(\cos\left(dx+c\right)+1\right)-15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{240\,a^2}{\tan(dx+c)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/1680*(112*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/ta
n(d*x + c)^5)*b^2 - 35*a*b*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*c
os(d*x + c))/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 1
5*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)) + 240*a^2/tan(d*x + c)^
7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(144) = 288$.
time = 0.40, size = 320, normalized size = 2.03

$$\frac{16\left(15\,a^2-161\,b^2\right)\cos\left(dx+c\right)^7+6496\,b^2\cos\left(dx+c\right)^5-5600\,b^2\cos\left(dx+c\right)^3+1680\,b^2\cos\left(dx+c\right)+525\left(ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4+3\,ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-525\left(ab\cos\left(dx+c\right)^6-3\,ab\cos\left(dx+c\right)^4+3\,ab\cos\left(dx+c\right)^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-70\left(24\,b^2dx\cos\left(dx+c\right)^6-72\,b^2dx\cos\left(dx+c\right)^4-33\,ab\cos\left(dx+c\right)^5+72\,b^2dx\cos\left(dx+c\right)^2+40\,ab\cos\left(dx+c\right)^3-24\,b^2dx-15\,ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{1680\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\left(dx+c\right)^2-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/1680*(16*(15*a^2 - 161*b^2)*cos(d*x + c)^7 + 6496*b^2*cos(d*x + c)^5 - 56
00*b^2*cos(d*x + c)^3 + 1680*b^2*cos(d*x + c) + 525*(a*b*cos(d*x + c)^6 - 3
*a*b*cos(d*x + c)^4 + 3*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 525*(a*b*cos(d*x + c)^6 - 3*a*b*cos(d*x + c)^4 + 3*a*b*co
s(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 70*(24*b^2*
d*x*cos(d*x + c)^6 - 72*b^2*d*x*cos(d*x + c)^4 - 33*a*b*cos(d*x + c)^5 + 72
*b^2*d*x*cos(d*x + c)^2 + 40*a*b*cos(d*x + c)^3 - 24*b^2*d*x - 15*a*b*cos(d
*x + c))*sin(d*x + c))/((d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*
x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(144) = 288.
time = 0.55, size = 356, normalized size = 2.25

$$\frac{15\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+84\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-630\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+315\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-980\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3150\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-13440\,(dx+c)b^{2}-8400\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-525\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9240\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{21780\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+525\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-9240\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-3150\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-315\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+980\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+630\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-84\,b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*a^2*tan(1/2*d*x + 1/2*c)^7 + 70*a*b*tan(1/2*d*x + 1/2*c)^6 - 10
5*a^2*tan(1/2*d*x + 1/2*c)^5 + 84*b^2*tan(1/2*d*x + 1/2*c)^5 - 630*a*b*tan(
1/2*d*x + 1/2*c)^4 + 315*a^2*tan(1/2*d*x + 1/2*c)^3 - 980*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3150*a*b*tan(1/2*d*x + 1/2*c)^2 - 13440*(d*x + c)*b^2 - 8400*a*
b*log(abs(tan(1/2*d*x + 1/2*c))) - 525*a^2*tan(1/2*d*x + 1/2*c) + 9240*b^2*
tan(1/2*d*x + 1/2*c) + (21780*a*b*tan(1/2*d*x + 1/2*c)^7 + 525*a^2*tan(1/2*
d*x + 1/2*c)^6 - 9240*b^2*tan(1/2*d*x + 1/2*c)^6 - 3150*a*b*tan(1/2*d*x + 1
/2*c)^5 - 315*a^2*tan(1/2*d*x + 1/2*c)^4 + 980*b^2*tan(1/2*d*x + 1/2*c)^4 +
 630*a*b*tan(1/2*d*x + 1/2*c)^3 + 105*a^2*tan(1/2*d*x + 1/2*c)^2 - 84*b^2*t
an(1/2*d*x + 1/2*c)^2 - 70*a*b*tan(1/2*d*x + 1/2*c) - 15*a^2)/tan(1/2*d*x +
 1/2*c)^7)/d
```

Mupad [**B**]
time = 11.87, size = 379, normalized size = 2.40

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{896\,d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{4\,b^4}{\frac{5\,a\,b^3}{2}-4\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{5\,a\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,\left(\frac{5\,a\,b^3}{2}-4\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^2}{128}-\frac{11\,b^2}{16}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(3\,a^2-\frac{28\,b^2}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(5\,a^2-88\,b^2\right)+\frac{a^2}{7}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-\frac{4\,b^2}{5}\right)-6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+30\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{128\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{3\,a^2}{128}-\frac{7\,b^2}{96}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{a^2}{128}-\frac{b^2}{160}\right)}{d}+\frac{15\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,d}-\frac{3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{192\,d}-\frac{5\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^7)/(896*d) - (2*b^2*atan((4*b^4)/((5*a*b^3)/2 - 4*b
^4*tan(c/2 + (d*x)/2)) + (5*a*b^3*tan(c/2 + (d*x)/2))/(2*((5*a*b^3)/2 - 4*b
^4*tan(c/2 + (d*x)/2)))))/d - (tan(c/2 + (d*x)/2)*((5*a^2)/128 - (11*b^2)/1
6))/d - (tan(c/2 + (d*x)/2)^4*(3*a^2 - (28*b^2)/3) - tan(c/2 + (d*x)/2)^6*(
5*a^2 - 88*b^2) + a^2/7 - tan(c/2 + (d*x)/2)^2*(a^2 - (4*b^2)/5) - 6*a*b*ta
n(c/2 + (d*x)/2)^3 + 30*a*b*tan(c/2 + (d*x)/2)^5 + (2*a*b*tan(c/2 + (d*x)/2
))/3)/(128*d*tan(c/2 + (d*x)/2)^7) + (tan(c/2 + (d*x)/2)^3*((3*a^2)/128 - (
7*b^2)/96))/d - (tan(c/2 + (d*x)/2)^5*(a^2/128 - b^2/160))/d + (15*a*b*tan(
c/2 + (d*x)/2)^2)/(64*d) - (3*a*b*tan(c/2 + (d*x)/2)^4)/(64*d) + (a*b*tan(c
/2 + (d*x)/2)^6)/(192*d) - (5*a*b*log(tan(c/2 + (d*x)/2)))/(8*d)
```

3.1252 $\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=159

$$\frac{5(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d}-\frac{2ab\cot^7(c+dx)}{7d}+\frac{(5a^2-88b^2)\cot(c+dx)\csc(c+dx)}{128d}-\frac{(59a^2-104b^2)\mathrm{co}}{}$$

```
[Out] 5/128*(a^2+8*b^2)*arctanh(cos(d*x+c))/d-2/7*a*b*cot(d*x+c)^7/d+1/128*(5*a^2
-88*b^2)*cot(d*x+c)*csc(d*x+c)/d-1/192*(59*a^2-104*b^2)*cot(d*x+c)*csc(d*x+
c)^3/d+1/48*(17*a^2-8*b^2)*cot(d*x+c)*csc(d*x+c)^5/d-1/8*a^2*cot(d*x+c)*csc
(d*x+c)^7/d
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2990, 2687, 30, 4451, 466, 1828, 1171, 393, 212}

$$\frac{5(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d}+\frac{(17a^2-8b^2)\cot(c+dx)\csc^5(c+dx)}{48d}-\frac{(59a^2-104b^2)\cot(c+dx)\csc^3(c+dx)}{192d}+\frac{(5a^2-88b^2)\cot(c+dx)\csc(c+dx)}{128d}-\frac{a^2\cot(c+dx)\csc^7(c+dx)}{8d}-\frac{2ab\cot^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*(a^2 + 8*b^2)*ArcTanh[Cos[c + d*x]])/(128*d) - (2*a*b*Cot[c + d*x]^7)/(7
*d) + ((5*a^2 - 88*b^2)*Cot[c + d*x]*Csc[c + d*x])/(128*d) - ((59*a^2 - 104
*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*d) + ((17*a^2 - 8*b^2)*Cot[c + d*x]
*Csc[c + d*x]^5)/(48*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^7)/(8*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^3(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^2(c+dx)\,dx + \int \cot^6(c+dx)$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^5}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\text{Su}}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{a^2\cot(c+dx)\csc^7(c+dx)}{8d} + \frac{\text{Sub}}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{(17a^2-8b^2)\cot(c+dx)\csc^5(c+}{48d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{(59a^2-104b^2)\cot(c+dx)\csc^3(c}{192d}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} + \frac{(5a^2-88b^2)\cot(c+dx)\csc(c+a}{128d}$$

$$= \frac{5(a^2+8b^2)\tanh^{-1}(\cos(c+dx))}{128d} - \frac{2ab\cot^7(c+dx)}{7d} +$$

Mathematica [A]
time = 0.57, size = 282, normalized size = 1.77

$$-\frac{7(895a^2-904b^2)\cos(3(c+dx))\csc^8(c+dx)+2779a^2\cos(5(c+dx))\csc^8(c+dx)+3416b^2\cos(5(c+dx))\csc^8(c+dx)+105a^2\cos(7(c+dx))\csc^8(c+dx)-1848b^2\cos(7(c+dx))\csc^8(c+dx)-6720a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-53760b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6720a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+53760b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+7\cot(c+dx)\csc^7(c+dx)\left(1765a^2+680b^2+1536ab\sin(c+dx)\right)+5376ab\csc^8(c+dx)\sin(4(c+dx))+2304ab\csc^8(c+dx)\sin(6(c+dx))+384ab\csc^8(c+dx)\sin(8(c+dx))}{172032d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/172032*(7*(895*a^2 - 904*b^2)*Cos[3*(c + d*x)]*Csc[c + d*x]^8 + 2779*a^2
*Cos[5*(c + d*x)]*Csc[c + d*x]^8 + 3416*b^2*Cos[5*(c + d*x)]*Csc[c + d*x]^8
 + 105*a^2*Cos[7*(c + d*x)]*Csc[c + d*x]^8 - 1848*b^2*Cos[7*(c + d*x)]*Csc[
c + d*x]^8 - 6720*a^2*Log[Cos[(c + d*x)/2]] - 53760*b^2*Log[Cos[(c + d*x)/2
]] + 6720*a^2*Log[Sin[(c + d*x)/2]] + 53760*b^2*Log[Sin[(c + d*x)/2]] + 7*C
ot[c + d*x]*Csc[c + d*x]^7*(1765*a^2 + 680*b^2 + 1536*a*b*Sin[c + d*x]) + 5
376*a*b*Csc[c + d*x]^8*Sin[4*(c + d*x)] + 2304*a*b*Csc[c + d*x]^8*Sin[6*(c
+ d*x)] + 384*a*b*Csc[c + d*x]^8*Sin[8*(c + d*x)])/d
```

Maple [A]

time = 0.38, size = 254, normalized size = 1.60

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\frac{5\left(\cos^3(dx+c)\right)}{384}-\frac{5\cos(dx+c)}{128}-\frac{5\ln(\csc(\right.}{}$
risch	$-\frac{-16128iab\,e^{6i(dx+c)}+105a^2e^{15i(dx+c)}-1848b^2e^{15i(dx+c)}-768iab\,e^{2i(dx+c)}+2779a^2e^{13i(dx+c)}+3416b^2e^{13i(dx+c)}+5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/1
92/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+
c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-2
/7*a*b/sin(d*x+c)^7*cos(d*x+c)^7+b^2*(-1/6/sin(d*x+c)^6*cos(d*x+c)^7+1/24/s
in(d*x+c)^4*cos(d*x+c)^7-1/16/sin(d*x+c)^2*cos(d*x+c)^7-1/16*cos(d*x+c)^5-5
/48*cos(d*x+c)^3-5/16*cos(d*x+c)-5/16*ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]

time = 0.29, size = 220, normalized size = 1.38

$$-\frac{7\,a^{2}\left(\frac{2\left(15\cos\left(dx+c\right)^{7}+73\cos\left(dx+c\right)^{5}-55\cos\left(dx+c\right)^{3}+15\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{8}-4\cos\left(dx+c\right)^{6}+6\cos\left(dx+c\right)^{4}-4\cos\left(dx+c\right)^{2}+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)-56\,b^{2}\left(\frac{2\left(33\cos\left(dx+c\right)^{5}-40\cos\left(dx+c\right)^{3}+15\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^{6}-3\cos\left(dx+c\right)^{4}+3\cos\left(dx+c\right)^{2}-1}+15\log\left(\cos\left(dx+c\right)+1\right)-15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{1536\,ab}{\tan\left(dx+c\right)^{7}}}{5376\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/5376*(7*a^2*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^
3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4
- 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) -
1)) - 56*b^2*(2*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))/(
cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1) + 15*log(cos(d*x
+ c) + 1) - 15*log(cos(d*x + c) - 1)) + 1536*a*b/tan(d*x + c)^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(147) = 294$.

time = 0.39, size = 338, normalized size = 2.13

$$-\frac{1536\,ab\cos\left(dx+c\right)^{7}\sin\left(dx+c\right)+42\left(5\,a^{2}-88\,b^{2}\right)\cos\left(dx+c\right)^{7}+1022\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{5}-770\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{3}+210\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)-105\left(\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{8}-4\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{6}+6\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{4}-4\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{2}+a^{2}+8\,b^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+105\left(\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{8}-4\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{6}+6\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{4}-4\left(a^{2}+8\,b^{2}\right)\cos\left(dx+c\right)^{2}+a^{2}+8\,b^{2}\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{5376\left(d\cos\left(dx+c\right)^{8}-4\,d\cos\left(dx+c\right)^{6}+6\,d\cos\left(dx+c\right)^{4}-4\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/5376*(1536*a*b*cos(d*x + c)^7*sin(d*x + c) + 42*(5*a^2 - 88*b^2)*cos(d*x
 + c)^7 + 1022*(a^2 + 8*b^2)*cos(d*x + c)^5 - 770*(a^2 + 8*b^2)*cos(d*x + c
)^3 + 210*(a^2 + 8*b^2)*cos(d*x + c) - 105*((a^2 + 8*b^2)*cos(d*x + c)^8 -
4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2)*cos(d*x + c)^4 - 4*(a^2 +
8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) + 105*((a^
2 + 8*b^2)*cos(d*x + c)^8 - 4*(a^2 + 8*b^2)*cos(d*x + c)^6 + 6*(a^2 + 8*b^2
)*cos(d*x + c)^4 - 4*(a^2 + 8*b^2)*cos(d*x + c)^2 + a^2 + 8*b^2)*log(-1/2*c
os(d*x + c) + 1/2))/(d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*cos(d*x +
c)^4 - 4*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(147) = 294.
time = 0.56, size = 402, normalized size = 2.53

$$\frac{21a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+96ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-112a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+112b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-672ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+168a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1008b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2016ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+336a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+5040b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3360ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1680\left(a^2+8b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+\frac{4566a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+36528b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+3360ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-336a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-5040b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-2016ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-168a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1008b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+672ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+112a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-112b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-96ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-21a^2}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8}}{43008d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/43008*(21*a^2*tan(1/2*d*x + 1/2*c)^8 + 96*a*b*tan(1/2*d*x + 1/2*c)^7 - 11
2*a^2*tan(1/2*d*x + 1/2*c)^6 + 112*b^2*tan(1/2*d*x + 1/2*c)^6 - 672*a*b*tan
(1/2*d*x + 1/2*c)^5 + 168*a^2*tan(1/2*d*x + 1/2*c)^4 - 1008*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 2016*a*b*tan(1/2*d*x + 1/2*c)^3 + 336*a^2*tan(1/2*d*x + 1/2*c
)^2 + 5040*b^2*tan(1/2*d*x + 1/2*c)^2 - 3360*a*b*tan(1/2*d*x + 1/2*c) - 168
0*(a^2 + 8*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (4566*a^2*tan(1/2*d*x + 1/
2*c)^8 + 36528*b^2*tan(1/2*d*x + 1/2*c)^8 + 3360*a*b*tan(1/2*d*x + 1/2*c)^7
 - 336*a^2*tan(1/2*d*x + 1/2*c)^6 - 5040*b^2*tan(1/2*d*x + 1/2*c)^6 - 2016*
a*b*tan(1/2*d*x + 1/2*c)^5 - 168*a^2*tan(1/2*d*x + 1/2*c)^4 + 1008*b^2*tan(
1/2*d*x + 1/2*c)^4 + 672*a*b*tan(1/2*d*x + 1/2*c)^3 + 112*a^2*tan(1/2*d*x +
 1/2*c)^2 - 112*b^2*tan(1/2*d*x + 1/2*c)^2 - 96*a*b*tan(1/2*d*x + 1/2*c) -
21*a^2)/tan(1/2*d*x + 1/2*c)^8)/d
```

Mupad [B]
time = 12.69, size = 343, normalized size = 2.16

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2048d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{5a^2}{128}+\frac{5b^2}{16}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(2a^2+30b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{2a^2}{3}-\frac{2b^2}{3}\right)+\frac{a^2}{8}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(a^2-6b^2\right)-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+12ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-20ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{7}\right)}{256d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a^2}{128}+\frac{15b^2}{128}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{a^2}{256}-\frac{3b^2}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{a^2}{384}-\frac{b^2}{384}\right)}{d}+\frac{3ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{64d}-\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{64d}+\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{448d}-\frac{5ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^9,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^8)/(2048*d) - (log(tan(c/2 + (d*x)/2))*((5*a^2)/128
 + (5*b^2)/16))/d - (cot(c/2 + (d*x)/2)^8*(tan(c/2 + (d*x)/2)^6*(2*a^2 + 30
*b^2) - tan(c/2 + (d*x)/2)^2*((2*a^2)/3 - (2*b^2)/3) + a^2/8 + tan(c/2 + (d
*x)/2)^4*(a^2 - 6*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x
)/2)^5 - 20*a*b*tan(c/2 + (d*x)/2)^7 + (4*a*b*tan(c/2 + (d*x)/2))/7))/(256*
d) + (tan(c/2 + (d*x)/2)^2*(a^2/128 + (15*b^2)/128))/d + (tan(c/2 + (d*x)/2
)^4*(a^2/256 - (3*b^2)/128))/d - (tan(c/2 + (d*x)/2)^6*(a^2/384 - b^2/384))
/d + (3*a*b*tan(c/2 + (d*x)/2)^3)/(64*d) - (a*b*tan(c/2 + (d*x)/2)^5)/(64*d
) + (a*b*tan(c/2 + (d*x)/2)^7)/(448*d) - (5*a*b*tan(c/2 + (d*x)/2))/(64*d)
```

3.1253 $\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=151

$$\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\cot(c+dx)\csc(c+dx)}{64d}-\frac{5ab\cot(c+$$

```
[Out] 5/64*a*b*arctanh(cos(d*x+c))/d-1/7*(a^2+b^2)*cot(d*x+c)^7/d-1/9*a^2*cot(d*x
+c)^9/d+5/64*a*b*cot(d*x+c)*csc(d*x+c)/d-5/32*a*b*cot(d*x+c)*csc(d*x+c)^3/d
+5/24*a*b*cot(d*x+c)^3*csc(d*x+c)^3/d-1/4*a*b*cot(d*x+c)^5*csc(d*x+c)^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2990, 2691, 3853, 3855, 14}

$$-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^9(c+dx)}{9d}+\frac{5ab\tanh^{-1}(\cos(c+dx))}{64d}-\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d}+\frac{5ab\cot^3(c+dx)\csc^3(c+dx)}{24d}-\frac{5ab\cot(c+dx)\csc^3(c+dx)}{32d}+\frac{5ab\cot(c+dx)\csc(c+dx)}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a*b*ArcTanh[Cos[c + d*x]])/(64*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d) -
 (a^2*Cot[c + d*x]^9)/(9*d) + (5*a*b*Cot[c + d*x]*Csc[c + d*x])/(64*d) - (5
*a*b*Cot[c + d*x]*Csc[c + d*x]^3)/(32*d) + (5*a*b*Cot[c + d*x]^3*Csc[c + d*
x]^3)/(24*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^3)/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
```

```
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^4(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^3(c+dx)\,dx + \int \cot^6(c+dx$$

$$= -\frac{ab\cot^5(c+dx)\csc^3(c+dx)}{4d} - \frac{1}{4}(5ab)\int \cot^4(c+d$$

$$= \frac{5ab\cot^3(c+dx)\csc^3(c+dx)}{24d} - \frac{ab\cot^5(c+dx)\csc^3(c}{4d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} - \frac{5ab\cot(}{}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a^2\cot^9(c+dx)}{9d} + \frac{5ab\cot(}{}$$

$$= \frac{5ab\tanh^{-1}(\cos(c+dx))}{64d} - \frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{a}{}$$

Mathematica [A]
time = 0.80, size = 204, normalized size = 1.35

$$-\frac{-40320ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+40320ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\csc^9(c+dx)\left(4032(8a^2+b^2)\cos(c+dx)+18816a^2\cos(3(c+dx))+5760a^2\cos(5(c+dx))-2304b^2\cos(5(c+dx))+576a^2\cos(7(c+dx))-1440b^2\cos(7(c+dx))-64a^2\cos(9(c+dx))-288b^2\cos(9(c+dx))+18270ab\sin(2(c+dx))+10458ab\sin(4(c+dx))+8022ab\sin(6(c+dx))+315ab\sin(8(c+dx))\right)}{516096d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/516096*(-40320*a*b*Log[Cos[(c + d*x)/2]] + 40320*a*b*Log[Sin[(c + d*x)/2
]] + Csc[c + d*x]^9*(4032*(8*a^2 + b^2)*Cos[c + d*x] + 18816*a^2*Cos[3*(c +
```

d*x)] + 5760*a^2*Cos[5*(c + d*x)] - 2304*b^2*Cos[5*(c + d*x)] + 576*a^2*Co s[7*(c + d*x)] - 1440*b^2*Cos[7*(c + d*x)] - 64*a^2*Cos[9*(c + d*x)] - 288* b^2*Cos[9*(c + d*x)] + 18270*a*b*Sin[2*(c + d*x)] + 10458*a*b*Sin[4*(c + d* x)] + 8022*a*b*Sin[6*(c + d*x)] + 315*a*b*Sin[8*(c + d*x)]))/d

Maple [A]
time = 0.37, size = 191, normalized size = 1.26

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+2ab\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{9\sin(dx+c)^9}-\frac{2\left(\cos^7(dx+c)\right)}{63\sin(dx+c)^7}\right)+2ab\left(-\frac{\cos^7(dx+c)}{8\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{48\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{192\sin(dx+c)^4}-\frac{\cos^7(dx+c)}{128\sin(dx+c)^2}-\frac{\left(\cos^5(dx+c)\right)}{128}-\right.}{d}$
risch	$-\frac{-32256ib^2e^{8i(dx+c)}+24192ia^2e^{6i(dx+c)}+24192ib^2e^{6i(dx+c)}-18270ab\,e^{7i(dx+c)}+1152ia^2e^{2i(dx+c)}-8022ab\,e^{3i(dx+c)}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7)+2* a*b*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(d*x+c)^6*cos(d*x+c)^7+1/192/si n(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*cos(d*x+c)^7-1/128*cos(d*x+c)^5- 5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(csc(d*x+c)-cot(d*x+c)))-1/7*b^ 2/sin(d*x+c)^7*cos(d*x+c)^7)

Maxima [A]
time = 0.29, size = 154, normalized size = 1.02

$$-\frac{21\,ab\left(\frac{2\left(15\,\cos\left(dx+c\right)^7+73\,\cos\left(dx+c\right)^5-55\,\cos\left(dx+c\right)^3+15\,\cos\left(dx+c\right)\right)}{\cos\left(dx+c\right)^8-4\,\cos\left(dx+c\right)^6+6\,\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right)^2+1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{1152\,b^2}{\tan\left(dx+c\right)^7}+\frac{128\left(9\,\tan\left(dx+c\right)^2+7\right)a^2}{\tan\left(dx+c\right)^9}}{8064\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="maxim a")

[Out] -1/8064*(21*a*b*(2*(15*cos(d*x + c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c) ^3 + 15*cos(d*x + c))/(cos(d*x + c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 1152*b^2/tan(d*x + c)^7 + 128*(9*tan(d*x + c)^2 + 7)*a^2/tan(d*x + c)^9)/d

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(137) = 274$.
time = 0.39, size = 291, normalized size = 1.93

$$\frac{128\left(2a^2+9b^2\right)\cos\left(dx+c\right)^9-1152\left(a^2+b^2\right)\cos\left(dx+c\right)^7+315\left(ab\cos\left(dx+c\right)^8-4ab\cos\left(dx+c\right)^6+6ab\cos\left(dx+c\right)^4-4ab\cos\left(dx+c\right)^2+ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-315\left(ab\cos\left(dx+c\right)^8-4ab\cos\left(dx+c\right)^6+6ab\cos\left(dx+c\right)^4-4ab\cos\left(dx+c\right)^2+ab\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-42\left(15ab\cos\left(dx+c\right)^7+73ab\cos\left(dx+c\right)^5-55ab\cos\left(dx+c\right)^3+15ab\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8064\left(d\cos\left(dx+c\right)^8-4d\cos\left(dx+c\right)^6+6d\cos\left(dx+c\right)^4-4d\cos\left(dx+c\right)^2+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/8064*(128*(2*a^2 + 9*b^2)*cos(d*x + c)^9 - 1152*(a^2 + b^2)*cos(d*x + c)^
7 + 315*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 -
 4*a*b*cos(d*x + c)^2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 315
*(a*b*cos(d*x + c)^8 - 4*a*b*cos(d*x + c)^6 + 6*a*b*cos(d*x + c)^4 - 4*a*b*
cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 42*(15*a*
b*cos(d*x + c)^7 + 73*a*b*cos(d*x + c)^5 - 55*a*b*cos(d*x + c)^3 + 15*a*b*c
os(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^8 - 4*d*cos(d*x + c)^6 + 6*d*co
s(d*x + c)^4 - 4*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**10*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(137) = 274.
time = 0.58, size = 408, normalized size = 2.70

$$\frac{14\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+63\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-54\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+72\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-504\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+504\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+336\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1512\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1008\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-5040\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-756\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2520\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{14258\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+756\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+2520\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-1008\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-336\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1512\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-504\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+504\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+336\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+54\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-63\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9}}{64512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^10*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/64512*(14*a^2*tan(1/2*d*x + 1/2*c)^9 + 63*a*b*tan(1/2*d*x + 1/2*c)^8 - 54
*a^2*tan(1/2*d*x + 1/2*c)^7 + 72*b^2*tan(1/2*d*x + 1/2*c)^7 - 336*a*b*tan(1
/2*d*x + 1/2*c)^6 - 504*b^2*tan(1/2*d*x + 1/2*c)^5 + 504*a*b*tan(1/2*d*x +
1/2*c)^4 + 336*a^2*tan(1/2*d*x + 1/2*c)^3 + 1512*b^2*tan(1/2*d*x + 1/2*c)^3
 + 1008*a*b*tan(1/2*d*x + 1/2*c)^2 - 5040*a*b*log(abs(tan(1/2*d*x + 1/2*c))
) - 756*a^2*tan(1/2*d*x + 1/2*c) - 2520*b^2*tan(1/2*d*x + 1/2*c) + (14258*a
*b*tan(1/2*d*x + 1/2*c)^9 + 756*a^2*tan(1/2*d*x + 1/2*c)^8 + 2520*b^2*tan(1
/2*d*x + 1/2*c)^8 - 1008*a*b*tan(1/2*d*x + 1/2*c)^7 - 336*a^2*tan(1/2*d*x +
 1/2*c)^6 - 1512*b^2*tan(1/2*d*x + 1/2*c)^6 - 504*a*b*tan(1/2*d*x + 1/2*c)^
5 + 504*b^2*tan(1/2*d*x + 1/2*c)^4 + 336*a*b*tan(1/2*d*x + 1/2*c)^3 + 54*a^
2*tan(1/2*d*x + 1/2*c)^2 - 72*b^2*tan(1/2*d*x + 1/2*c)^2 - 63*a*b*tan(1/2*d
*x + 1/2*c) - 14*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

Mupad [B]

time = 11.86, size = 373, normalized size = 2.47

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{4608\,d}-\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{128\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3a^2}{256}+\frac{5b^2}{128}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{8a^2}{3}+12\,b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a^2}{7}-\frac{4b^2}{7}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(6\,a^2+20\,b^2\right)-4\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{a^2}{9}-\frac{8\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{512\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a^2}{192}+\frac{3b^2}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{3a^2}{3584}-\frac{b^2}{896}\right)}{d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{128\,d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{192\,d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{1024\,d}-\frac{5\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^10,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^9)/(4608*d) - (b^2*tan(c/2 + (d*x)/2)^5)/(128*d) -
(tan(c/2 + (d*x)/2)*((3*a^2)/256 + (5*b^2)/128))/d - (cot(c/2 + (d*x)/2)^9*
(tan(c/2 + (d*x)/2)^6*((8*a^2)/3 + 12*b^2) - tan(c/2 + (d*x)/2)^2*((3*a^2)/
7 - (4*b^2)/7) - tan(c/2 + (d*x)/2)^8*(6*a^2 + 20*b^2) - 4*b^2*tan(c/2 + (d
*x)/2)^4 + a^2/9 - (8*a*b*tan(c/2 + (d*x)/2)^3)/3 + 4*a*b*tan(c/2 + (d*x)/2
)^5 + 8*a*b*tan(c/2 + (d*x)/2)^7 + (a*b*tan(c/2 + (d*x)/2))/2))/(512*d) + (
tan(c/2 + (d*x)/2)^3*(a^2/192 + (3*b^2)/128))/d - (tan(c/2 + (d*x)/2)^7*((3
*a^2)/3584 - b^2/896))/d + (a*b*tan(c/2 + (d*x)/2)^2)/(64*d) + (a*b*tan(c/2
 + (d*x)/2)^4)/(128*d) - (a*b*tan(c/2 + (d*x)/2)^6)/(192*d) + (a*b*tan(c/2
+ (d*x)/2)^8)/(1024*d) - (5*a*b*log(tan(c/2 + (d*x)/2)))/(64*d)
```

3.1254 $\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=210

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}-\frac{2ab\cot^7(c+dx)}{7d}-\frac{2ab\cot^9(c+dx)}{9d}+\frac{(3a^2+10b^2)\cot(c+dx)\csc(c+dx)}{256d}$$

```
[Out] 1/256*(3*a^2+10*b^2)*arctanh(cos(d*x+c))/d-2/7*a*b*cot(d*x+c)^7/d-2/9*a*b*c
ot(d*x+c)^9/d+1/256*(3*a^2+10*b^2)*cot(d*x+c)*csc(d*x+c)/d+1/384*(3*a^2-118
*b^2)*cot(d*x+c)*csc(d*x+c)^3/d-1/480*(93*a^2-170*b^2)*cot(d*x+c)*csc(d*x+c
)^5/d+1/80*(21*a^2-10*b^2)*cot(d*x+c)*csc(d*x+c)^7/d-1/10*a^2*cot(d*x+c)*cs
c(d*x+c)^9/d
```

Rubi [A]
time = 0.22, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2990, 2687, 14, 4451, 466, 1828, 1171, 393, 205, 212}

$$\frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d}+\frac{(21a^2-10b^2)\cot(c+dx)\csc^7(c+dx)}{80d}-\frac{(93a^2-170b^2)\cot(c+dx)\csc^5(c+dx)}{480d}+\frac{(3a^2-118b^2)\cot(c+dx)\csc^3(c+dx)}{384d}+\frac{(3a^2+10b^2)\cot(c+dx)\csc(c+dx)}{256d}-\frac{a^2\cot(c+dx)\csc^9(c+dx)}{10d}-\frac{2ab\cot^9(c+dx)}{9d}-\frac{2ab\cot^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + 10*b^2)*ArcTanh[Cos[c + d*x]])/(256*d) - (2*a*b*Cot[c + d*x]^7)/(
7*d) - (2*a*b*Cot[c + d*x]^9)/(9*d) + ((3*a^2 + 10*b^2)*Cot[c + d*x]*Csc[c
+ d*x])/(256*d) + ((3*a^2 - 118*b^2)*Cot[c + d*x]*Csc[c + d*x]^3)/(384*d) -
 ((93*a^2 - 170*b^2)*Cot[c + d*x]*Csc[c + d*x]^5)/(480*d) + ((21*a^2 - 10*b
^2)*Cot[c + d*x]*Csc[c + d*x]^7)/(80*d) - (a^2*Cot[c + d*x]*Csc[c + d*x]^9)
/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4451

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cot^6(c+dx)\csc^5(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^4(c+dx)\,dx + \int \cot^6(c+dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6(a^2+b^2-b^2x^2)}{(1-x^2)^6}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2ab)\text{S}}{}$$

$$= -\frac{a^2\cot(c+dx)\csc^9(c+dx)}{10d} + \frac{\text{Subst}\left(\int \frac{a^2+10a^2x^2+10a}{(1-x^2}\right.}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(21a^2-10b^2}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} - \frac{(93a^2-170}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(3a^2-118b^2}{}$$

$$= -\frac{2ab\cot^7(c+dx)}{7d} - \frac{2ab\cot^9(c+dx)}{9d} + \frac{(3a^2+10b^2)}{}$$

$$= \frac{(3a^2+10b^2)\tanh^{-1}(\cos(c+dx))}{256d} - \frac{2ab\cot^7(c+dx)}{7d}$$

Mathematica [A]
time = 1.01, size = 244, normalized size = 1.16

$$-\frac{-80640(3a^2+10b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+80640(3a^2+10b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\csc^{10}(c+dx)\left(630(1879a^2+290b^2)\cos(c+dx)+1260(519a^2-62b^2)\cos(3(c+dx))+218484a^2\cos(5(c+dx))-24360b^2\cos(5(c+dx))+9135a^2\cos(7(c+dx))-77070b^2\cos(7(c+dx))-945a^2\cos(9(c+dx))-3150b^2\cos(9(c+dx))+537600ab\sin(2(c+dx))+522240ab\sin(4(c+dx))+207360ab\sin(6(c+dx))+25600ab\sin(8(c+dx))-2560ab\sin(10(c+dx))\right)}{20643840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/20643840*(-80640*(3*a^2 + 10*b^2)*Log[Cos[(c + d*x)/2]] + 80640*(3*a^2 +
 10*b^2)*Log[Sin[(c + d*x)/2]] + Csc[c + d*x]^10*(630*(1879*a^2 + 290*b^2)*
Cos[c + d*x] + 1260*(519*a^2 - 62*b^2)*Cos[3*(c + d*x)] + 218484*a^2*Cos[5*
(c + d*x)] - 24360*b^2*Cos[5*(c + d*x)] + 9135*a^2*Cos[7*(c + d*x)] - 77070
*b^2*Cos[7*(c + d*x)] - 945*a^2*Cos[9*(c + d*x)] - 3150*b^2*Cos[9*(c + d*x)
] + 537600*a*b*Sin[2*(c + d*x)] + 522240*a*b*Sin[4*(c + d*x)] + 207360*a*b*
Sin[6*(c + d*x)] + 25600*a*b*Sin[8*(c + d*x)] - 2560*a*b*Sin[10*(c + d*x)])
)/d
```

Maple [A]
time = 0.42, size = 311, normalized size = 1.48

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\frac{3\cos}{}\right.}{}$
default	$\frac{a^2\left(-\frac{\cos^7(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^7(dx+c)\right)}{80\sin(dx+c)^8}-\frac{\cos^7(dx+c)}{160\sin(dx+c)^6}+\frac{\cos^7(dx+c)}{640\sin(dx+c)^4}-\frac{3\left(\cos^7(dx+c)\right)}{1280\sin(dx+c)^2}-\frac{3\left(\cos^5(dx+c)\right)}{1280}-\frac{\left(\cos^3(dx+c)\right)}{256}-\frac{3\cos}{}\right.}{}$
risch	$-\frac{322560iab\,e^{16i(dx+c)}-1183770a^2e^{9i(dx+c)}-182700b^2e^{9i(dx+c)}-653940a^2e^{7i(dx+c)}+78120b^2e^{7i(dx+c)}+78120b^2e^{13i(d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/80/sin(d*x+c)^8*cos(d*x+c)^7-1
/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c)^4*cos(d*x+c)^7-3/1280/sin(d
*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256*cos(d*x+c)^3-3/256*cos(d*x+c
)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*(-1/9/sin(d*x+c)^9*cos(d*x+c)^7-2/
63/sin(d*x+c)^7*cos(d*x+c)^7)+b^2*(-1/8/sin(d*x+c)^8*cos(d*x+c)^7-1/48/sin(
d*x+c)^6*cos(d*x+c)^7+1/192/sin(d*x+c)^4*cos(d*x+c)^7-1/128/sin(d*x+c)^2*co
s(d*x+c)^7-1/128*cos(d*x+c)^5-5/384*cos(d*x+c)^3-5/128*cos(d*x+c)-5/128*ln(
csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 272, normalized size = 1.30

$$-\frac{63\,a^2\left(\frac{2\left(15\cos(dx+c)^9-70\cos(dx+c)^7-128\cos(dx+c)^5+70\cos(dx+c)^3-15\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\cos(dx+c)^8+10\cos(dx+c)^6-10\cos(dx+c)^4+5\cos(dx+c)^2-1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+210\,b^2\left(\frac{2\left(15\cos(dx+c)^7+73\cos(dx+c)^5-55\cos(dx+c)^3+15\cos(dx+c)\right)}{\cos(dx+c)^8-4\cos(dx+c)^6+6\cos(dx+c)^4-4\cos(dx+c)^2+1}-15\log\left(\cos\left(dx+c\right)+1\right)+15\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{5120\left(9\tan(dx+c)^2+7\right)ab}{\tan(dx+c)^9}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/161280*(63*a^2*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x +
 c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x +
 c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15*
log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 210*b^2*(2*(15*cos(d*x
+ c)^7 + 73*cos(d*x + c)^5 - 55*cos(d*x + c)^3 + 15*cos(d*x + c))/(cos(d*x
+ c)^8 - 4*cos(d*x + c)^6 + 6*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + 1) - 15*l
og(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 5120*(9*tan(d*x + c)^2 +
 7)*a*b/tan(d*x + c)^9)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(194) = 388$.
time = 0.39, size = 455, normalized size = 2.17

$$-\frac{630\,(3\,a^2+10\,b^2)\cos(dx+c)^9-420\,(21\,a^2-58\,b^2)\cos(dx+c)^7-5376\,(3\,a^2+10\,b^2)\cos(dx+c)^5+2940\,(3\,a^2+10\,b^2)\cos(dx+c)^3-630\,(3\,a^2+10\,b^2)\cos(dx+c)-315\,\left((3\,a^2+10\,b^2)\cos(dx+c)^{10}-5\,(3\,a^2+10\,b^2)\cos(dx+c)^8+10\,(3\,a^2+10\,b^2)\cos(dx+c)^6-10\,(3\,a^2+10\,b^2)\cos(dx+c)^4+5\,(3\,a^2+10\,b^2)\cos(dx+c)^2-3\,a^2-10\,b^2\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+315\,\left((3\,a^2+10\,b^2)\cos(dx+c)^{10}-5\,(3\,a^2+10\,b^2)\cos(dx+c)^8+10\,(3\,a^2+10\,b^2)\cos(dx+c)^6-10\,(3\,a^2+10\,b^2)\cos(dx+c)^4+5\,(3\,a^2+10\,b^2)\cos(dx+c)^2-3\,a^2-10\,b^2\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+5120\,(2\,ab\cos(dx+c)^9-9\,ab\cos(dx+c)^7)\sin(dx+c)}{161280\,(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] -1/161280*(630*(3*a^2 + 10*b^2)*cos(d*x + c)^9 - 420*(21*a^2 - 58*b^2)*cos(
d*x + c)^7 - 5376*(3*a^2 + 10*b^2)*cos(d*x + c)^5 + 2940*(3*a^2 + 10*b^2)*c
os(d*x + c)^3 - 630*(3*a^2 + 10*b^2)*cos(d*x + c) - 315*((3*a^2 + 10*b^2)*c
os(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*b^2)*co
s(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^2)*cos(
d*x + c)^2 - 3*a^2 - 10*b^2)*log(1/2*cos(d*x + c) + 1/2) + 315*((3*a^2 + 10
*b^2)*cos(d*x + c)^10 - 5*(3*a^2 + 10*b^2)*cos(d*x + c)^8 + 10*(3*a^2 + 10*
b^2)*cos(d*x + c)^6 - 10*(3*a^2 + 10*b^2)*cos(d*x + c)^4 + 5*(3*a^2 + 10*b^
2)*cos(d*x + c)^2 - 3*a^2 - 10*b^2)*log(-1/2*cos(d*x + c) + 1/2) + 5120*(2*
a*b*cos(d*x + c)^9 - 9*a*b*cos(d*x + c)^7)*sin(d*x + c))/(d*cos(d*x + c)^10
 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*d*cos(d*x + c)^4 + 5*d*cos
(d*x + c)^2 - d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**11*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(194) = 388$.
time = 0.58, size = 468, normalized size = 2.23

$$\frac{126\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+560\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+630\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-2160\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3360\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5040\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+13440\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1260\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10080\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-30240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5040\,(3\,a^2+10\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+\frac{[illegible]\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+30240\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-1260\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-10080\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-13440\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-2520\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-5040\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+630\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3360\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2160\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-630\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-560\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-126\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}}}{1290240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^11*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/1290240*(126*a^2*tan(1/2*d*x + 1/2*c)^10 + 560*a*b*tan(1/2*d*x + 1/2*c)^9
 - 315*a^2*tan(1/2*d*x + 1/2*c)^8 + 630*b^2*tan(1/2*d*x + 1/2*c)^8 - 2160*a
*b*tan(1/2*d*x + 1/2*c)^7 - 630*a^2*tan(1/2*d*x + 1/2*c)^6 - 3360*b^2*tan(1
/2*d*x + 1/2*c)^6 + 2520*a^2*tan(1/2*d*x + 1/2*c)^4 + 5040*b^2*tan(1/2*d*x
+ 1/2*c)^4 + 13440*a*b*tan(1/2*d*x + 1/2*c)^3 + 1260*a^2*tan(1/2*d*x + 1/2*
c)^2 + 10080*b^2*tan(1/2*d*x + 1/2*c)^2 - 30240*a*b*tan(1/2*d*x + 1/2*c) -
5040*(3*a^2 + 10*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) + (44286*a^2*tan(1/2*d
*x + 1/2*c)^10 + 147620*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a*b*tan(1/2*d*x
 + 1/2*c)^9 - 1260*a^2*tan(1/2*d*x + 1/2*c)^8 - 10080*b^2*tan(1/2*d*x + 1/2
*c)^8 - 13440*a*b*tan(1/2*d*x + 1/2*c)^7 - 2520*a^2*tan(1/2*d*x + 1/2*c)^6
- 5040*b^2*tan(1/2*d*x + 1/2*c)^6 + 630*a^2*tan(1/2*d*x + 1/2*c)^4 + 3360*b
^2*tan(1/2*d*x + 1/2*c)^4 + 2160*a*b*tan(1/2*d*x + 1/2*c)^3 + 315*a^2*tan(1
/2*d*x + 1/2*c)^2 - 630*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*a*b*tan(1/2*d*x +
1/2*c) - 126*a^2)/tan(1/2*d*x + 1/2*c)^10)/d
```

Mupad [B]
time = 11.93, size = 394, normalized size = 1.88

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{10240\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{256}+\frac{5\,b^2}{128}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(2\,a^2+4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{4}-\frac{b^2}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{2}+\frac{8\,b^2}{3}\right)+\frac{a^2}{10}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(a^2+8\,b^2\right)-\frac{12\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{7}+\frac{32\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-24\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\frac{4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9}\right)}{1024\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{a^2}{512}+\frac{b^2}{256}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{a^2}{1024}+\frac{b^2}{128}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{a^2}{2048}+\frac{b^2}{384}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{a^2}{4096}-\frac{b^2}{2048}\right)}{d}+\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}-\frac{3\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{1792\,d}+\frac{a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{2304\,d}-\frac{3\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^11,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^10)/(10240*d) - (log(tan(c/2 + (d*x)/2))*((3*a^2)/2
56 + (5*b^2)/128))/d - (cot(c/2 + (d*x)/2)^10*(tan(c/2 + (d*x)/2)^6*(2*a^2
+ 4*b^2) - tan(c/2 + (d*x)/2)^2*(a^2/4 - b^2/2) - tan(c/2 + (d*x)/2)^4*(a^2
/2 + (8*b^2)/3) + a^2/10 + tan(c/2 + (d*x)/2)^8*(a^2 + 8*b^2) - (12*a*b*tan
(c/2 + (d*x)/2)^3)/7 + (32*a*b*tan(c/2 + (d*x)/2)^7)/3 - 24*a*b*tan(c/2 + (
d*x)/2)^9 + (4*a*b*tan(c/2 + (d*x)/2))/9))/(1024*d) + (tan(c/2 + (d*x)/2)^4
*(a^2/512 + b^2/256))/d + (tan(c/2 + (d*x)/2)^2*(a^2/1024 + b^2/128))/d - (
tan(c/2 + (d*x)/2)^6*(a^2/2048 + b^2/384))/d - (tan(c/2 + (d*x)/2)^8*(a^2/4
096 - b^2/2048))/d + (a*b*tan(c/2 + (d*x)/2)^3)/(96*d) - (3*a*b*tan(c/2 + (
d*x)/2)^7)/(1792*d) + (a*b*tan(c/2 + (d*x)/2)^9)/(2304*d) - (3*a*b*tan(c/2
+ (d*x)/2))/(128*d)
```

3.1255 $\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=198

$$\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{(2a^2+b^2)\cot^9(c+dx)}{9d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\cot(c+d}{12}$$

```
[Out] 3/128*a*b*arctanh(cos(d*x+c))/d-1/7*(a^2+b^2)*cot(d*x+c)^7/d-1/9*(2*a^2+b^2
)*cot(d*x+c)^9/d-1/11*a^2*cot(d*x+c)^11/d+3/128*a*b*cot(d*x+c)*csc(d*x+c)/d
+1/64*a*b*cot(d*x+c)*csc(d*x+c)^3/d-1/16*a*b*cot(d*x+c)*csc(d*x+c)^5/d+1/8*
a*b*cot(d*x+c)^3*csc(d*x+c)^5/d-1/5*a*b*cot(d*x+c)^5*csc(d*x+c)^5/d
```

Rubi [A]
time = 0.30, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2990, 2691, 3853, 3855, 459}

$$-\frac{(2a^2+b^2)\cot^9(c+dx)}{9d}-\frac{(a^2+b^2)\cot^7(c+dx)}{7d}-\frac{a^2\cot^{11}(c+dx)}{11d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{128d}-\frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d}+\frac{ab\cot^3(c+dx)\csc^5(c+dx)}{8d}-\frac{ab\cot(c+dx)\csc^5(c+dx)}{16d}+\frac{ab\cot(c+dx)\csc^3(c+dx)}{64d}+\frac{3ab\cot(c+dx)\csc(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a*b*ArcTanh[Cos[c + d*x]])/(128*d) - ((a^2 + b^2)*Cot[c + d*x]^7)/(7*d)
 - ((2*a^2 + b^2)*Cot[c + d*x]^9)/(9*d) - (a^2*Cot[c + d*x]^11)/(11*d) + (3*
a*b*Cot[c + d*x]*Csc[c + d*x])/(128*d) + (a*b*Cot[c + d*x]*Csc[c + d*x]^3)/
(64*d) - (a*b*Cot[c + d*x]*Csc[c + d*x]^5)/(16*d) + (a*b*Cot[c + d*x]^3*Csc
[c + d*x]^5)/(8*d) - (a*b*Cot[c + d*x]^5*Csc[c + d*x]^5)/(5*d)
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^6(c+dx)\csc^6(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \cot^6(c+dx)\csc^5(c+dx)\,dx + \int \cot^6(c+dx)$$

$$= -\frac{ab\cot^5(c+dx)\csc^5(c+dx)}{5d} - (ab)\int \cot^4(c+dx)\,\mathrm{cs}$$

$$= \frac{ab\cot^3(c+dx)\csc^5(c+dx)}{8d} - \frac{ab\cot^5(c+dx)\csc^5(c+}{5d}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2}{}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2}{}$$

$$= -\frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2a^2+b^2)\cot^9(c+dx)}{9d} - \frac{a^2}{}$$

$$= \frac{3ab\tanh^{-1}(\cos(c+dx))}{128d} - \frac{(a^2+b^2)\cot^7(c+dx)}{7d} - \frac{(2}{}$$

Mathematica [A]
time = 1.19, size = 250, normalized size = 1.26

$$-\frac{-5322240ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+5322240ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\csc^{11}(c+dx)\left(1478400(8a^2+b^2)\cos(c+dx)+42240(160a^2-b^2)\cos(3(c+dx))+1943040a^2\cos(5(c+dx))-865920b^2\cos(5(c+dx))+140800a^2\cos(7(c+dx))-499840b^2\cos(7(c+dx))-28160a^2\cos(9(c+dx))-77440b^2\cos(9(c+dx))+2560a^2\cos(11(c+dx))+7040b^2\cos(11(c+dx))+5828130ab\sin(2(c+dx))+4790016ab\sin(4(c+dx))+2302839ab\sin(6(c+dx))+110880ab\sin(8(c+dx))-10395ab\sin(10(c+dx))\right)}{227082240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*Csc[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/227082240*(-5322240*a*b*Log[Cos[(c + d*x)/2]] + 5322240*a*b*Log[Sin[(c +
 d*x)/2]] + Csc[c + d*x]^11*(1478400*(8*a^2 + b^2)*Cos[c + d*x] + 42240*(16
0*a^2 - b^2)*Cos[3*(c + d*x)] + 1943040*a^2*Cos[5*(c + d*x)] - 865920*b^2*C
os[5*(c + d*x)] + 140800*a^2*Cos[7*(c + d*x)] - 499840*b^2*Cos[7*(c + d*x)]
 - 28160*a^2*Cos[9*(c + d*x)] - 77440*b^2*Cos[9*(c + d*x)] + 2560*a^2*Cos[1
1*(c + d*x)] + 7040*b^2*Cos[11*(c + d*x)] + 5828130*a*b*Sin[2*(c + d*x)] +
4790016*a*b*Sin[4*(c + d*x)] + 2302839*a*b*Sin[6*(c + d*x)] + 110880*a*b*Si
n[8*(c + d*x)] - 10395*a*b*Sin[10*(c + d*x)]))/d
```

Maple [A]
time = 0.41, size = 247, normalized size = 1.25

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\cos^{7}(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^{7}(dx+c)\right)}{99\sin(dx+c)^{9}}-\frac{8\left(\cos^{7}(dx+c)\right)}{693\sin(dx+c)^{7}}\right)+2ab\left(-\frac{\cos^{7}(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^{7}(dx+c)\right)}{80\sin(dx+c)^{8}}-\frac{\cos^{7}(dx+c)}{160\sin(dx+c)^{6}}+\cdots\right.}{}$
default	$\frac{a^{2}\left(-\frac{\cos^{7}(dx+c)}{11\sin(dx+c)^{11}}-\frac{4\left(\cos^{7}(dx+c)\right)}{99\sin(dx+c)^{9}}-\frac{8\left(\cos^{7}(dx+c)\right)}{693\sin(dx+c)^{7}}\right)+2ab\left(-\frac{\cos^{7}(dx+c)}{10\sin(dx+c)^{10}}-\frac{3\left(\cos^{7}(dx+c)\right)}{80\sin(dx+c)^{8}}-\frac{\cos^{7}(dx+c)}{160\sin(dx+c)^{6}}+\cdots\right.}{}$
risch	$-\frac{-2280960ib^{2}e^{8i(dx+c)}+5828130ab\,e^{9i(dx+c)}-1520640ia^{2}e^{6i(dx+c)}+2027520ib^{2}e^{6i(dx+c)}+4790016ab\,e^{7i(dx+c)}+563\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/11/sin(d*x+c)^11*cos(d*x+c)^7-4/99/sin(d*x+c)^9*cos(d*x+c)^7-8
/693/sin(d*x+c)^7*cos(d*x+c)^7)+2*a*b*(-1/10/sin(d*x+c)^10*cos(d*x+c)^7-3/8
0/sin(d*x+c)^8*cos(d*x+c)^7-1/160/sin(d*x+c)^6*cos(d*x+c)^7+1/640/sin(d*x+c
)^4*cos(d*x+c)^7-3/1280/sin(d*x+c)^2*cos(d*x+c)^7-3/1280*cos(d*x+c)^5-1/256
*cos(d*x+c)^3-3/256*cos(d*x+c)-3/256*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/9/s
in(d*x+c)^9*cos(d*x+c)^7-2/63/sin(d*x+c)^7*cos(d*x+c)^7))
```

Maxima [A]
time = 0.28, size = 196, normalized size = 0.99

$$-\frac{693\,ab\left(\frac{2\left(15\,\cos(dx+c)^{9}-70\,\cos(dx+c)^{7}-128\,\cos(dx+c)^{5}+70\,\cos(dx+c)^{3}-15\,\cos(dx+c)\right)}{\cos(dx+c)^{10}-5\,\cos(dx+c)^{8}+10\,\cos(dx+c)^{6}-10\,\cos(dx+c)^{4}+5\,\cos(dx+c)^{2}-1}-15\,\log\left(\cos\left(dx+c\right)+1\right)+15\,\log\left(\cos\left(dx+c\right)-1\right)\right)+\frac{14080\left(9\,\tan(dx+c)^{2}+7\right)b^{2}}{\tan(dx+c)^{9}}+\frac{1280\left(99\,\tan(dx+c)^{4}+154\,\tan(dx+c)^{2}+63\right)a^{2}}{\tan(dx+c)^{11}}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="maxim
a")
```

```
[Out] -1/887040*(693*a*b*(2*(15*cos(d*x + c)^9 - 70*cos(d*x + c)^7 - 128*cos(d*x
+ c)^5 + 70*cos(d*x + c)^3 - 15*cos(d*x + c))/(cos(d*x + c)^10 - 5*cos(d*x
+ c)^8 + 10*cos(d*x + c)^6 - 10*cos(d*x + c)^4 + 5*cos(d*x + c)^2 - 1) - 15
```

```
*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1)) + 14080*(9*tan(d*x + c)^
2 + 7)*b^2/tan(d*x + c)^9 + 1280*(99*tan(d*x + c)^4 + 154*tan(d*x + c)^2 +
63)*a^2/tan(d*x + c)^11)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(180) = 360$.
time = 0.38, size = 363, normalized size = 1.83

$$\frac{2560\,(4\,a^2+11\,b^2)\cos(dx+c)^{11}-14080\,(4\,a^2+11\,b^2)\cos(dx+c)^9+126720\,(a^2+b^2)\cos(dx+c)^7+10395\,\left(ab\cos(dx+c)^{10}-5\,ab\cos(dx+c)^8+10\,ab\cos(dx+c)^6-10\,ab\cos(dx+c)^4+5\,ab\cos(dx+c)^2-ab\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-10395\,\left(ab\cos(dx+c)^{10}-5\,ab\cos(dx+c)^8+10\,ab\cos(dx+c)^6-10\,ab\cos(dx+c)^4+5\,ab\cos(dx+c)^2-ab\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-1386\,\left(15\,ab\cos(dx+c)^9-70\,ab\cos(dx+c)^7-128\,ab\cos(dx+c)^5+70\,ab\cos(dx+c)^3-15\,ab\cos(dx+c)\right)\sin(dx+c)}{887040\,\left(d\cos(dx+c)^{10}-5\,d\cos(dx+c)^8+10\,d\cos(dx+c)^6-10\,d\cos(dx+c)^4+5\,d\cos(dx+c)^2-d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="frica
s")
```

```
[Out] 1/887040*(2560*(4*a^2 + 11*b^2)*cos(d*x + c)^11 - 14080*(4*a^2 + 11*b^2)*co
s(d*x + c)^9 + 126720*(a^2 + b^2)*cos(d*x + c)^7 + 10395*(a*b*cos(d*x + c)^
10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10*a*b*cos(d*x + c)^4 +
 5*a*b*cos(d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 103
95*(a*b*cos(d*x + c)^10 - 5*a*b*cos(d*x + c)^8 + 10*a*b*cos(d*x + c)^6 - 10
*a*b*cos(d*x + c)^4 + 5*a*b*cos(d*x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) - 1386*(15*a*b*cos(d*x + c)^9 - 70*a*b*cos(d*x + c)^7 - 12
8*a*b*cos(d*x + c)^5 + 70*a*b*cos(d*x + c)^3 - 15*a*b*cos(d*x + c))*sin(d*x
 + c))/((d*cos(d*x + c)^10 - 5*d*cos(d*x + c)^8 + 10*d*cos(d*x + c)^6 - 10*
d*cos(d*x + c)^4 + 5*d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**12*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(180) = 360$.
time = 0.57, size = 502, normalized size = 2.54

$$\frac{315\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1386\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+1540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-3465\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-2475\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-5940\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-6930\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+3465\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+27720\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+11550\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36960\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+13860\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-166320\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-34650\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-83160\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{592266\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+34650\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+83160\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-13860\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-11550\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-36960\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-27720\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3465\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+6930\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2475\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5940\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+3465\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+385\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1540\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1386\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-315\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}}}{7096320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^12*(a+b*sin(d*x+c))^2,x, algorithm="giac"
)
```

```
[Out] 1/7096320*(315*a^2*tan(1/2*d*x + 1/2*c)^11 + 1386*a*b*tan(1/2*d*x + 1/2*c)^
10 - 385*a^2*tan(1/2*d*x + 1/2*c)^9 + 1540*b^2*tan(1/2*d*x + 1/2*c)^9 - 346
```

```
5*a*b*tan(1/2*d*x + 1/2*c)^8 - 2475*a^2*tan(1/2*d*x + 1/2*c)^7 - 5940*b^2*t
an(1/2*d*x + 1/2*c)^7 - 6930*a*b*tan(1/2*d*x + 1/2*c)^6 + 3465*a^2*tan(1/2*
d*x + 1/2*c)^5 + 27720*a*b*tan(1/2*d*x + 1/2*c)^4 + 11550*a^2*tan(1/2*d*x +
 1/2*c)^3 + 36960*b^2*tan(1/2*d*x + 1/2*c)^3 + 13860*a*b*tan(1/2*d*x + 1/2*
c)^2 - 166320*a*b*log(abs(tan(1/2*d*x + 1/2*c))) - 34650*a^2*tan(1/2*d*x +
1/2*c) - 83160*b^2*tan(1/2*d*x + 1/2*c) + (502266*a*b*tan(1/2*d*x + 1/2*c)^
11 + 34650*a^2*tan(1/2*d*x + 1/2*c)^10 + 83160*b^2*tan(1/2*d*x + 1/2*c)^10
- 13860*a*b*tan(1/2*d*x + 1/2*c)^9 - 11550*a^2*tan(1/2*d*x + 1/2*c)^8 - 369
60*b^2*tan(1/2*d*x + 1/2*c)^8 - 27720*a*b*tan(1/2*d*x + 1/2*c)^7 - 3465*a^2
*tan(1/2*d*x + 1/2*c)^6 + 6930*a*b*tan(1/2*d*x + 1/2*c)^5 + 2475*a^2*tan(1/
2*d*x + 1/2*c)^4 + 5940*b^2*tan(1/2*d*x + 1/2*c)^4 + 3465*a*b*tan(1/2*d*x +
 1/2*c)^3 + 385*a^2*tan(1/2*d*x + 1/2*c)^2 - 1540*b^2*tan(1/2*d*x + 1/2*c)^
2 - 1386*a*b*tan(1/2*d*x + 1/2*c) - 315*a^2)/tan(1/2*d*x + 1/2*c)^11)/d
```

Mupad [B]
time = 16.33, size = 448, normalized size = 2.26

$$-\frac{\frac{5\,a^2\,\cos(c+d\,x)}{96}+\frac{5\,b^2\,\cos(c+d\,x)}{768}+\frac{5\,a^2\,\cos(3\,c+3\,d\,x)}{168}+\frac{23\,a^2\,\cos(5\,c+5\,d\,x)}{2688}+\frac{5\,a^2\,\cos(7\,c+7\,d\,x)}{8064}-\frac{a^2\,\cos(9\,c+9\,d\,x)}{8064}+\frac{a^2\,\cos(11\,c+11\,d\,x)}{88704}-\frac{b^2\,\cos(3\,c+3\,d\,x)}{5376}-\frac{41\,b^2\,\cos(5\,c+5\,d\,x)}{10752}-\frac{71\,b^2\,\cos(7\,c+7\,d\,x)}{32256}-\frac{11\,b^2\,\cos(9\,c+9\,d\,x)}{32256}+\frac{b^2\,\cos(11\,c+11\,d\,x)}{32256}+\frac{841\,a\,b\,\sin(2\,c+2\,d\,x)}{32768}+\frac{27\,a\,b\,\sin(4\,c+4\,d\,x)}{1280}+\frac{3323\,a\,b\,\sin(6\,c+6\,d\,x)}{327680}+\frac{a\,b\,\sin(8\,c+8\,d\,x)}{2048}-\frac{3\,a\,b\,\sin(10\,c+10\,d\,x)}{65536}+\frac{693\,a\,b\,\sin(c+d\,x)\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{65536}-\frac{495\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(3\,c+3\,d\,x)}{65536}+\frac{495\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(5\,c+5\,d\,x)}{131072}-\frac{165\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(7\,c+7\,d\,x)}{131072}+\frac{33\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(9\,c+9\,d\,x)}{131072}-\frac{3\,a\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin(11\,c+11\,d\,x)}{131072}}{d\,\sin\left(c+d\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*(a + b*sin(c + d*x))^2)/sin(c + d*x)^12,x)
```

```
[Out] -((5*a^2*cos(c + d*x))/96 + (5*b^2*cos(c + d*x))/768 + (5*a^2*cos(3*c + 3*d
*x))/168 + (23*a^2*cos(5*c + 5*d*x))/2688 + (5*a^2*cos(7*c + 7*d*x))/8064 -
 (a^2*cos(9*c + 9*d*x))/8064 + (a^2*cos(11*c + 11*d*x))/88704 - (b^2*cos(3*
c + 3*d*x))/5376 - (41*b^2*cos(5*c + 5*d*x))/10752 - (71*b^2*cos(7*c + 7*d*
x))/32256 - (11*b^2*cos(9*c + 9*d*x))/32256 + (b^2*cos(11*c + 11*d*x))/3225
6 + (841*a*b*sin(2*c + 2*d*x))/32768 + (27*a*b*sin(4*c + 4*d*x))/1280 + (33
23*a*b*sin(6*c + 6*d*x))/327680 + (a*b*sin(8*c + 8*d*x))/2048 - (3*a*b*sin(
10*c + 10*d*x))/65536 + (693*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/65536 - (495*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
sin(3*c + 3*d*x))/65536 + (495*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
))*sin(5*c + 5*d*x))/131072 - (165*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*sin(7*c + 7*d*x))/131072 + (33*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*sin(9*c + 9*d*x))/131072 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))*sin(11*c + 11*d*x))/131072)/(d*sin(c + d*x)^11)
```

3.1256 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=525

$$\frac{a(64a^6-120a^4b^2+60a^2b^4-5b^6)\,x}{8b^9}-\frac{2a^2(8a^2-3b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^9d}+\frac{(840a^6-1435a^4b}{}$$

```
[Out] 1/8*a*(64*a^6-120*a^4*b^2+60*a^2*b^4-5*b^6)*x/b^9-2*a^2*(8*a^2-3*b^2)*(a^2-
b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^9/d+1/105*(84
0*a^6-1435*a^4*b^2+588*a^2*b^4-15*b^6)*cos(d*x+c)/b^8/d-1/8*a*(32*a^4-52*a^
2*b^2+19*b^4)*cos(d*x+c)*sin(d*x+c)/b^7/d+1/105*(280*a^4-441*a^2*b^2+150*b^
4)*cos(d*x+c)*sin(d*x+c)^2/b^6/d-1/12*(24*a^4-37*a^2*b^2+12*b^4)*cos(d*x+c)
*sin(d*x+c)^3/a/b^5/d+1/140*(224*a^4-340*a^2*b^2+105*b^4)*cos(d*x+c)*sin(d*
x+c)^4/a^2/b^4/d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+b*sin(d*x+c))-3/20*b*co
s(d*x+c)*sin(d*x+c)^5/a^2/d/(a+b*sin(d*x+c))-1/15*(20*a^4-30*a^2*b^2+9*b^4)
*cos(d*x+c)*sin(d*x+c)^5/a^2/b^3/d/(a+b*sin(d*x+c))-4/21*a*cos(d*x+c)*sin(d
*x+c)^6/b^2/d/(a+b*sin(d*x+c))+1/7*cos(d*x+c)*sin(d*x+c)^7/b/d/(a+b*sin(d*x
+c))
```

Rubi [A]

time = 1.24, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2975, 3126, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2(8a^2-3b^2)\left(a^2-b^2\right)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{3b\sin^5(c+dx)\cos(c+dx)}{20a^2d(a+b\sin(c+dx))}+\frac{(224a^4-340a^2b^2+105b^4)\sin^4(c+dx)\cos(c+dx)}{140a^2b^4d}-\frac{a(32a^4-52a^2b^2+19b^4)\sin(c+dx)\cos(c+dx)}{8b^7d}+\frac{(280a^4-441a^2b^2+150b^4)\sin^2(c+dx)\cos(c+dx)}{105b^6d}-\frac{(24a^4-37a^2b^2+12b^4)\sin^3(c+dx)\cos(c+dx)}{12ab^5d}-\frac{(20a^4-30a^2b^2+9b^4)\sin^5(c+dx)\cos(c+dx)}{15a^2b^3d(a+b\sin(c+dx))}+\frac{ax(64a^6-120a^4b^2+60a^2b^4-5b^6)}{8b^9}+\frac{(840a^6-1435a^4b^2+588a^2b^4-15b^6)\cos(c+dx)}{105b^8d}-\frac{4a\sin^6(c+dx)\cos(c+dx)}{21b^2d(a+b\sin(c+dx))}+\frac{\sin^7(c+dx)\cos(c+dx)}{7bd(a+b\sin(c+dx))}+\frac{\sin^4(c+dx)\cos(c+dx)}{4ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(64*a^6 - 120*a^4*b^2 + 60*a^2*b^4 - 5*b^6)*x)/(8*b^9) - (2*a^2*(8*a^2 -
 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) + ((840*a^6 - 1435*a^4*b^2 + 588*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(
105*b^8*d) - (a*(32*a^4 - 52*a^2*b^2 + 19*b^4)*Cos[c + d*x]*Sin[c + d*x])/(
8*b^7*d) + ((280*a^4 - 441*a^2*b^2 + 150*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/
(105*b^6*d) - ((24*a^4 - 37*a^2*b^2 + 12*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(12*a*b^5*d) + ((224*a^4 - 340*a^2*b^2 + 105*b^4)*Cos[c + d*x]*Sin[c + d*x]
^4)/(140*a^2*b^4*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d
*x])) - (3*b*Cos[c + d*x]*Sin[c + d*x]^5)/(20*a^2*d*(a + b*Sin[c + d*x])) -
 ((20*a^4 - 30*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(15*a^2*b^3*d*
(a + b*Sin[c + d*x])) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(21*b^2*d*(a + b*
Sin[c + d*x])) + (Cos[c + d*x]*Sin[c + d*x]^7)/(7*b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{4a\cos(c+dx)\,\mathrm{s}}{21b^2d(a+b\sin}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))} - \frac{3b\cos(c+dx)\sin^5(c+dx)}{20a^2d(a+b\sin(c+dx))} - \frac{(20a^4-30a^2b^2}{15a^2b}$$

$$= \frac{(224a^4-340a^2b^2+105b^4)\cos(c+dx)\sin^4(c+dx)}{140a^2b^4d} + \frac{\cos(c+dx)\sin^4(c}{4ad(a+b\sin(c+}$$

$$= -\frac{(24a^4-37a^2b^2+12b^4)\cos(c+dx)\sin^3(c+dx)}{12ab^5d} + \frac{(224a^4-340a^2b^2+}{}$$

$$= \frac{(280a^4-441a^2b^2+150b^4)\cos(c+dx)\sin^2(c+dx)}{105b^6d} - \frac{(24a^4-37a^2b^2+}{}$$

$$= -\frac{a(32a^4-52a^2b^2+19b^4)\cos(c+dx)\sin(c+dx)}{8b^7d} + \frac{(280a^4-441a^2b^2+}{}$$

$$= \frac{(840a^6-1435a^4b^2+588a^2b^4-15b^6)\cos(c+dx)}{105b^8d} - \frac{a(32a^4-52a^2b^2+1}{}$$

$$= \frac{a(64a^6-120a^4b^2+60a^2b^4-5b^6)\,x}{8b^9} + \frac{(840a^6-1435a^4b^2+588a^2b^4-15}{105b^8d}$$

$$= \frac{a(64a^6-120a^4b^2+60a^2b^4-5b^6)\,x}{8b^9} + \frac{(840a^6-1435a^4b^2+588a^2b^4-15}{105b^8d}$$

$$= \frac{a(64a^6-120a^4b^2+60a^2b^4-5b^6)\,x}{8b^9} + \frac{(840a^6-1435a^4b^2+588a^2b^4-15}{105b^8d}$$

$$= \frac{a(64a^6-120a^4b^2+60a^2b^4-5b^6)\,x}{8b^9} - \frac{2a^2(8a^2-3b^2)\,(a^2-b^2)^{3/2}\tan^{-1}\,(}{b^9d}$$

Mathematica [A]
time = 6.11, size = 531, normalized size = 1.01

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-26880*a^2*(8*a^2 - 3*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (107520*a^8*c - 201600*a^6*b^2*c + 100800*a^4*b^4*c -
```

```
 8400*a^2*b^6*c + 107520*a^8*d*x - 201600*a^6*b^2*d*x + 100800*a^4*b^4*d*x
- 8400*a^2*b^6*d*x + 840*a*b*(128*a^6 - 224*a^4*b^2 + 98*a^2*b^4 - 5*b^6)*C
os[c + d*x] + 70*(64*a^5*b^3 - 96*a^3*b^5 + 27*a*b^7)*Cos[3*(c + d*x)] - 33
6*a^3*b^5*Cos[5*(c + d*x)] + 350*a*b^7*Cos[5*(c + d*x)] + 40*a*b^7*Cos[7*(c
 + d*x)] + 107520*a^7*b*c*Sin[c + d*x] - 201600*a^5*b^3*c*Sin[c + d*x] + 10
0800*a^3*b^5*c*Sin[c + d*x] - 8400*a*b^7*c*Sin[c + d*x] + 107520*a^7*b*d*x*
Sin[c + d*x] - 201600*a^5*b^3*d*x*Sin[c + d*x] + 100800*a^3*b^5*d*x*Sin[c +
 d*x] - 8400*a*b^7*d*x*Sin[c + d*x] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 4592
0*a^4*b^4*Sin[2*(c + d*x)] + 18480*a^2*b^6*Sin[2*(c + d*x)] - 210*b^8*Sin[2
*(c + d*x)] - 1120*a^4*b^4*Sin[4*(c + d*x)] + 1428*a^2*b^6*Sin[4*(c + d*x)]
 - 210*b^8*Sin[4*(c + d*x)] + 112*a^2*b^6*Sin[6*(c + d*x)] - 90*b^8*Sin[6*(
c + d*x)] - 15*b^8*Sin[8*(c + d*x)])/(a + b*Sin[c + d*x]))/(13440*b^9*d)
```

Maple [A]
time = 0.45, size = 656, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^9*(((3/2*a^5*b^2-9/4*a^3*b^4+11/16*a*b^6)*tan(1/2*d*x+1/2*c)^13+(7
/2*a^6*b-15/2*a^4*b^3+9/2*a^2*b^5-1/2*b^7)*tan(1/2*d*x+1/2*c)^12+(6*a^5*b^2
-7*a^3*b^4+7/12*a*b^6)*tan(1/2*d*x+1/2*c)^11+(21*a^6*b-40*a^4*b^3+18*a^2*b^
5)*tan(1/2*d*x+1/2*c)^10+(15/2*a^5*b^2-29/4*a^3*b^4+85/48*a*b^6)*tan(1/2*d*
x+1/2*c)^9+(105/2*a^6*b-545/6*a^4*b^3+73/2*a^2*b^5-5/2*b^7)*tan(1/2*d*x+1/2
*c)^8+(70*a^6*b-340/3*a^4*b^3+44*a^2*b^5)*tan(1/2*d*x+1/2*c)^6+(-15/2*a^5*b
^2+29/4*a^3*b^4-85/48*a*b^6)*tan(1/2*d*x+1/2*c)^5+(105/2*a^6*b-165/2*a^4*b^
3+303/10*a^2*b^5-3/2*b^7)*tan(1/2*d*x+1/2*c)^4+(-6*a^5*b^2+7*a^3*b^4-7/12*a
*b^6)*tan(1/2*d*x+1/2*c)^3+(21*a^6*b-100/3*a^4*b^3+58/5*a^2*b^5)*tan(1/2*d*
x+1/2*c)^2+(-3/2*a^5*b^2+9/4*a^3*b^4-11/16*a*b^6)*tan(1/2*d*x+1/2*c)+7/2*a^
6*b-35/6*a^4*b^3+23/10*a^2*b^5-1/14*b^7)/(1+tan(1/2*d*x+1/2*c)^2)^7+1/16*a*
(64*a^6-120*a^4*b^2+60*a^2*b^4-5*b^6)*arctan(tan(1/2*d*x+1/2*c)))-4*(a^4-2*
a^2*b^2+b^4)*a^2/b^9*((-1/2*b^2*tan(1/2*d*x+1/2*c)-1/2*a*b)/(a*tan(1/2*d*x+
1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(8*a^2-3*b^2)/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.44, size = 871, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/840*(160*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5
 + 35*(32*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 12
0*a^6*b^2 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 420*(8*a^7 - 11*a^5*b^2 + 3*a^3*b
^4 + (8*a^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 105*(64*a^7*b - 120*a^5*b^3 + 60*a^3
*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*b^6*cos(d*
x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^7*b - 120*
a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^4 + 19*a^2
*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d), 1/840*(1
60*a*b^7*cos(d*x + c)^7 - 14*(24*a^3*b^5 - 5*a*b^7)*cos(d*x + c)^5 + 35*(32
*a^5*b^3 - 36*a^3*b^5 + 5*a*b^7)*cos(d*x + c)^3 + 105*(64*a^8 - 120*a^6*b^2
 + 60*a^4*b^4 - 5*a^2*b^6)*d*x + 840*(8*a^7 - 11*a^5*b^2 + 3*a^3*b^4 + (8*a
^6*b - 11*a^4*b^3 + 3*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 105*(64*a^7*b - 120*a^5*b^
3 + 60*a^3*b^5 - 5*a*b^7)*cos(d*x + c) - (120*b^8*cos(d*x + c)^7 - 224*a^2*
b^6*cos(d*x + c)^5 + 70*(8*a^4*b^4 - 7*a^2*b^6)*cos(d*x + c)^3 - 105*(64*a^
7*b - 120*a^5*b^3 + 60*a^3*b^5 - 5*a*b^7)*d*x - 105*(32*a^6*b^2 - 52*a^4*b^
4 + 19*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(b^10*d*sin(d*x + c) + a*b^9*d)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 965, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/840*(105*(64*a^7 - 120*a^5*b^2 + 60*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^9 - 16
80*(8*a^8 - 19*a^6*b^2 + 14*a^4*b^4 - 3*a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqr
t(a^2 - b^2)*b^9) + 1680*(a^6*b*tan(1/2*d*x + 1/2*c) - 2*a^4*b^3*tan(1/2*d*
x + 1/2*c) + a^2*b^5*tan(1/2*d*x + 1/2*c) + a^7 - 2*a^5*b^2 + a^3*b^4)/((a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^8) + 2*(2520*a^5*b
*tan(1/2*d*x + 1/2*c)^13 - 3780*a^3*b^3*tan(1/2*d*x + 1/2*c)^13 + 1155*a*b^
5*tan(1/2*d*x + 1/2*c)^13 + 5880*a^6*tan(1/2*d*x + 1/2*c)^12 - 12600*a^4*b^
2*tan(1/2*d*x + 1/2*c)^12 + 7560*a^2*b^4*tan(1/2*d*x + 1/2*c)^12 - 840*b^6*
tan(1/2*d*x + 1/2*c)^12 + 10080*a^5*b*tan(1/2*d*x + 1/2*c)^11 - 11760*a^3*b
^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x + 1/2*c)^11 + 35280*a^6*
tan(1/2*d*x + 1/2*c)^10 - 67200*a^4*b^2*tan(1/2*d*x + 1/2*c)^10 + 30240*a^2
*b^4*tan(1/2*d*x + 1/2*c)^10 + 12600*a^5*b*tan(1/2*d*x + 1/2*c)^9 - 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*x + 1/2*c)^9 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^8 - 152600*a^4*b^2*tan(1/2*d*x + 1/2*c)^8 + 61320*a
^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 4200*b^6*tan(1/2*d*x + 1/2*c)^8 + 117600*a^
6*tan(1/2*d*x + 1/2*c)^6 - 190400*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 73920*a^
2*b^4*tan(1/2*d*x + 1/2*c)^6 - 12600*a^5*b*tan(1/2*d*x + 1/2*c)^5 + 12180*a
^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 88200*a
^6*tan(1/2*d*x + 1/2*c)^4 - 138600*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 50904*a
^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 2520*b^6*tan(1/2*d*x + 1/2*c)^4 - 10080*a^5
*b*tan(1/2*d*x + 1/2*c)^3 + 11760*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 980*a*b^
5*tan(1/2*d*x + 1/2*c)^3 + 35280*a^6*tan(1/2*d*x + 1/2*c)^2 - 56000*a^4*b^2
*tan(1/2*d*x + 1/2*c)^2 + 19488*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 2520*a^5*b
*tan(1/2*d*x + 1/2*c) + 3780*a^3*b^3*tan(1/2*d*x + 1/2*c) - 1155*a*b^5*tan(
1/2*d*x + 1/2*c) + 5880*a^6 - 9800*a^4*b^2 + 3864*a^2*b^4 - 120*b^6)/((tan(
1/2*d*x + 1/2*c)^2 + 1)^7*b^8))/d
```

Mupad [B]
time = 18.11, size = 2500, normalized size = 4.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^14*(7*a*b^6 + 32*a^7 + 4*a^3*b^4 - 44*a^5*b^2))/(2*b^8
) - (2*(15*a*b^6 - 840*a^7 - 588*a^3*b^4 + 1435*a^5*b^2))/(105*b^8) + (2*ta
n(c/2 + (d*x)/2)^12*(4*a*b^6 + 168*a^7 + 72*a^3*b^4 - 255*a^5*b^2))/(3*b^8)
 - (2*tan(c/2 + (d*x)/2)^8*(15*a*b^6 - 840*a^7 - 588*a^3*b^4 + 1435*a^5*b^2
))/(3*b^8) + (tan(c/2 + (d*x)/2)^10*(25*a*b^6 + 2016*a^7 + 1212*a^3*b^4 - 3
284*a^5*b^2))/(6*b^8) - (2*tan(c/2 + (d*x)/2)^4*(80*a*b^6 - 2520*a^7 - 1992
```

*a^3*b^4 + 4465*a^5*b^2))/(15*b^8) - (tan(c/2 + (d*x)/2)^6*(605*a*b^6 - 168
00*a^7 - 12756*a^3*b^4 + 29500*a^5*b^2))/(30*b^8) - (tan(c/2 + (d*x)/2)^2*(
1215*a*b^6 - 23520*a^7 - 18396*a^3*b^4 + 41300*a^5*b^2))/(210*b^8) + (tan(c
/2 + (d*x)/2)*(10080*a^6 - 240*b^6 + 7413*a^2*b^4 - 17500*a^4*b^2))/(420*b^
7) + (tan(c/2 + (d*x)/2)^15*(32*a^6 + 19*a^2*b^4 - 52*a^4*b^2))/(4*b^7) + (
tan(c/2 + (d*x)/2)^11*(3168*a^6 + 2345*a^2*b^4 - 5532*a^4*b^2))/(12*b^7) +
(tan(c/2 + (d*x)/2)^7*(7200*a^6 + 4979*a^2*b^4 - 12212*a^4*b^2))/(12*b^7) +
(tan(c/2 + (d*x)/2)^3*(9120*a^6 + 6103*a^2*b^4 - 15460*a^4*b^2))/(60*b^7)
+ (tan(c/2 + (d*x)/2)^13*(864*a^6 - 48*b^6 + 661*a^2*b^4 - 1500*a^4*b^2))/(
12*b^7) + (tan(c/2 + (d*x)/2)^9*(6240*a^6 - 240*b^6 + 4429*a^2*b^4 - 10748*
a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^5*(24480*a^6 - 720*b^6 + 16499*a^2
*b^4 - 41220*a^4*b^2))/(60*b^7))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 8*a*tan(c
/2 + (d*x)/2)^2 + 28*a*tan(c/2 + (d*x)/2)^4 + 56*a*tan(c/2 + (d*x)/2)^6 + 7
0*a*tan(c/2 + (d*x)/2)^8 + 56*a*tan(c/2 + (d*x)/2)^10 + 28*a*tan(c/2 + (d*x
)/2)^12 + 8*a*tan(c/2 + (d*x)/2)^14 + a*tan(c/2 + (d*x)/2)^16 + 14*b*tan(c/
2 + (d*x)/2)^3 + 42*b*tan(c/2 + (d*x)/2)^5 + 70*b*tan(c/2 + (d*x)/2)^7 + 70
*b*tan(c/2 + (d*x)/2)^9 + 42*b*tan(c/2 + (d*x)/2)^11 + 14*b*tan(c/2 + (d*x)
/2)^13 + 2*b*tan(c/2 + (d*x)/2)^15)) + (a*atan(((a*(((25*a^4*b^20)/2 - 300*
a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^1
0 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20
+ 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 430
08*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) - (a*((20*a^2*b^24 - 164*a^4*b^22 +
272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 17
92*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) - (a*(32*a^2*b^3 + (
tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6 +
60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*
a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2))/(8*b^9)
+ (a*(((25*a^4*b^20)/2 - 300*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11
040*a^12*b^12 - 7680*a^14*b^10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*
(50*a^3*b^22 - 1801*a^5*b^20 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11
*b^14 - 90240*a^13*b^12 + 43008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) + (a*(
(20*a^2*b^24 - 164*a^4*b^22 + 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2
+ (d*x)/2)*(384*a^3*b^24 - 1792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/
(2*b^24) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26)
)/(2*b^24))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^
6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2
*b^4 - 120*a^4*b^2))/(8*b^9))/((16384*a^22 + 285*a^6*b^16 - 5530*a^8*b^14 +
38085*a^10*b^12 - 133328*a^12*b^10 + 269608*a^14*b^8 - 329120*a^16*b^6 + 2
39872*a^18*b^4 - 96256*a^20*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(65536*a^23 - 1
50*a^5*b^18 + 4300*a^7*b^16 - 46550*a^9*b^14 + 247840*a^11*b^12 - 745600*a^
13*b^10 + 1358720*a^15*b^8 - 1534336*a^17*b^6 + 1051648*a^19*b^4 - 401408*a
^21*b^2))/b^24 + (a*(((25*a^4*b^20)/2 - 300*a^6*b^18 + 2400*a^8*b^16 - 7520
*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^10 + 2048*a^16*b^8)/b^23 + (tan(
c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20 + 15576*a^7*b^18 - 54720*a^9*b^
16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43008*a^15*b^10 - 8192*a^17*b^8))/

(2*b^24) - (a*((20*a^2*b^24 - 164*a^4*b^22 + 272*a^6*b^20 - 128*a^8*b^18)/b
^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1792*a^5*b^22 + 2432*a^7*b^20 - 1
024*a^9*b^18))/(2*b^24) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^28
- 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/
(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 -
 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9) - (a*(((25*a^4*b^20)/2 - 300
*a^6*b^18 + 2400*a^8*b^16 - 7520*a^10*b^14 + 11040*a^12*b^12 - 7680*a^14*b^
10 + 2048*a^16*b^8)/b^23 + (tan(c/2 + (d*x)/2)*(50*a^3*b^22 - 1801*a^5*b^20
 + 15576*a^7*b^18 - 54720*a^9*b^16 + 96320*a^11*b^14 - 90240*a^13*b^12 + 43
008*a^15*b^10 - 8192*a^17*b^8))/(2*b^24) + (a*((20*a^2*b^24 - 164*a^4*b^22
+ 272*a^6*b^20 - 128*a^8*b^18)/b^23 + (tan(c/2 + (d*x)/2)*(384*a^3*b^24 - 1
792*a^5*b^22 + 2432*a^7*b^20 - 1024*a^9*b^18))/(2*b^24) + (a*(32*a^2*b^3 +
(tan(c/2 + (d*x)/2)*(192*a*b^28 - 128*a^3*b^26))/(2*b^24))*(64*a^6 - 5*b^6
+ 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120
*a^4*b^2)*1i)/(8*b^9))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^2)*1i)/(8*b
^9)))*(64*a^6 - 5*b^6 + 60*a^2*b^4 - 120*a^4*b^...

3.1257 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=471

$$-\frac{(112a^6-200a^4b^2+90a^2b^4-5b^6)\,x}{16b^8}+\frac{2a(7a^2-2b^2)\,(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{a(105a^4-170a$$

```
[Out] -1/16*(112*a^6-200*a^4*b^2+90*a^2*b^4-5*b^6)*x/b^8+2*a*(7*a^2-2*b^2)*(a^2-b
^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^8/d-1/15*a*(10
5*a^4-170*a^2*b^2+61*b^4)*cos(d*x+c)/b^7/d+1/16*(56*a^4-86*a^2*b^2+27*b^4)*
cos(d*x+c)*sin(d*x+c)/b^6/d-1/15*(35*a^4-52*a^2*b^2+15*b^4)*cos(d*x+c)*sin(
d*x+c)^2/a/b^5/d+1/24*(42*a^4-61*a^2*b^2+16*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^
2/b^4/d+1/3*cos(d*x+c)*sin(d*x+c)^3/a/d/(a+b*sin(d*x+c))-1/6*b*cos(d*x+c)*s
in(d*x+c)^4/a^2/d/(a+b*sin(d*x+c))-1/10*(14*a^4-20*a^2*b^2+5*b^4)*cos(d*x+c
)*sin(d*x+c)^4/a^2/b^3/d/(a+b*sin(d*x+c))-7/30*a*cos(d*x+c)*sin(d*x+c)^5/b^
2/d/(a+b*sin(d*x+c))+1/6*cos(d*x+c)*sin(d*x+c)^6/b/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.98, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2975, 3126, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{2a(7a^2-2b^2)\,(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{6a^2d(a+b\sin(c+dx))}+\frac{(42a^4-61a^2b^2+16b^4)\sin^3(c+dx)\cos(c+dx)}{24a^2b^4d}-\frac{a(105a^4-170a^2b^2+61b^4)\cos(c+dx)}{15b^7d}+\frac{(56a^4-86a^2b^2+27b^4)\sin(c+dx)\cos(c+dx)}{16b^6d}-\frac{(35a^4-52a^2b^2+15b^4)\sin^2(c+dx)\cos(c+dx)}{15ab^5d}-\frac{(14a^4-20a^2b^2+5b^4)\sin^4(c+dx)\cos(c+dx)}{10a^2b^3d(a+b\sin(c+dx))}-\frac{x(112a^6-200a^4b^2+90a^2b^4-5b^6)}{16b^8}-\frac{7a\sin^5(c+dx)\cos(c+dx)}{30b^2d(a+b\sin(c+dx))}+\frac{\sin^6(c+dx)\cos(c+dx)}{6bd(a+b\sin(c+dx))}+\frac{\sin^3(c+dx)\cos(c+dx)}{3ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*((112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*x)/b^8 + (2*a*(7*a^2 -
2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(b^8*d) - (a*(105*a^4 - 170*a^2*b^2 + 61*b^4)*Cos[c + d*x])/(15*b^7*d) + ((
56*a^4 - 86*a^2*b^2 + 27*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) - ((35*
a^4 - 52*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*a*b^5*d) + ((42
*a^4 - 61*a^2*b^2 + 16*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*a^2*b^4*d) + (
Cos[c + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])) - (b*Cos[c + d*x]
*Sin[c + d*x]^4)/(6*a^2*d*(a + b*Sin[c + d*x])) - ((14*a^4 - 20*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(10*a^2*b^3*d*(a + b*Sin[c + d*x])) - (7
*a*Cos[c + d*x]*Sin[c + d*x]^5)/(30*b^2*d*(a + b*Sin[c + d*x])) + (Cos[c +
d*x]*Sin[c + d*x]^6)/(6*b*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{7a\cos(c+dx)\sin}{30b^2d(a+b\sin(}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))} - \frac{b\cos(c+dx)\sin^4(c+dx)}{6a^2d(a+b\sin(c+dx))} - \frac{(14a^4-20a^2b^2+}{10a^2b^3d}$$

$$= \frac{(42a^4-61a^2b^2+16b^4)\cos(c+dx)\sin^3(c+dx)}{24a^2b^4d} + \frac{\cos(c+dx)\sin^3(c+dx}{3ad(a+b\sin(c+dx))}$$

$$= -\frac{(35a^4-52a^2b^2+15b^4)\cos(c+dx)\sin^2(c+dx)}{15ab^5d} + \frac{(42a^4-61a^2b^2+16b}{2}$$

$$= \frac{(56a^4-86a^2b^2+27b^4)\cos(c+dx)\sin(c+dx)}{16b^6d} - \frac{(35a^4-52a^2b^2+15b^4)\,c}{15a}$$

$$= -\frac{a(105a^4-170a^2b^2+61b^4)\cos(c+dx)}{15b^7d} + \frac{(56a^4-86a^2b^2+27b^4)\cos(c+}{16b^6d}$$

$$= -\frac{(112a^6-200a^4b^2+90a^2b^4-5b^6)\,x}{16b^8} - \frac{a(105a^4-170a^2b^2+61b^4)\cos(c-}{15b^7d}$$

$$= -\frac{(112a^6-200a^4b^2+90a^2b^4-5b^6)\,x}{16b^8} - \frac{a(105a^4-170a^2b^2+61b^4)\cos(c-}{15b^7d}$$

$$= -\frac{(112a^6-200a^4b^2+90a^2b^4-5b^6)\,x}{16b^8} - \frac{a(105a^4-170a^2b^2+61b^4)\cos(c-}{15b^7d}$$

$$= -\frac{(112a^6-200a^4b^2+90a^2b^4-5b^6)\,x}{16b^8} + \frac{2a(7a^2-2b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\Big(}{b^8d}$$

Mathematica [A]
time = 5.62, size = 462, normalized size = 0.98

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (3840*a*(7*a^2 - 2*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]] - (13440*a^7*c - 24000*a^5*b^2*c + 10800*a^3*b^4*c - 600*a*
b^6*c + 13440*a^7*d*x - 24000*a^5*b^2*d*x + 10800*a^3*b^4*d*x - 600*a*b^6*d
*x + 15*b*(896*a^6 - 1488*a^4*b^2 + 576*a^2*b^4 - 15*b^6)*Cos[c + d*x] + 10
*(56*a^4*b^3 - 79*a^2*b^5 + 18*b^7)*Cos[3*(c + d*x)] - 42*a^2*b^5*Cos[5*(c
```

```
+ d*x)] + 40*b^7*Cos[5*(c + d*x)] + 5*b^7*Cos[7*(c + d*x)] + 13440*a^6*b*c*
Sin[c + d*x] - 24000*a^4*b^3*c*Sin[c + d*x] + 10800*a^2*b^5*c*Sin[c + d*x]
- 600*b^7*c*Sin[c + d*x] + 13440*a^6*b*d*x*Sin[c + d*x] - 24000*a^4*b^3*d*x
*Sin[c + d*x] + 10800*a^2*b^5*d*x*Sin[c + d*x] - 600*b^7*d*x*Sin[c + d*x] +
 3360*a^5*b^2*Sin[2*(c + d*x)] - 5440*a^3*b^4*Sin[2*(c + d*x)] + 1910*a*b^6
*Sin[2*(c + d*x)] - 140*a^3*b^4*Sin[4*(c + d*x)] + 166*a*b^6*Sin[4*(c + d*x
)] + 14*a*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c + d*x]))/(1920*b^8*d)
```

Maple [**A**]
time = 0.43, size = 611, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^8*(((5/2*a^4*b^2-27/8*a^2*b^4+11/16*b^6)*tan(1/2*d*x+1/2*c)^11+(6
*a^5*b-12*a^3*b^3+6*a*b^5)*tan(1/2*d*x+1/2*c)^10+(15/2*a^4*b^2-57/8*a^2*b^4
-5/48*b^6)*tan(1/2*d*x+1/2*c)^9+(30*a^5*b-52*a^3*b^3+18*a*b^5)*tan(1/2*d*x+
1/2*c)^8+(5*a^4*b^2-15/4*a^2*b^4+15/8*b^6)*tan(1/2*d*x+1/2*c)^7+(60*a^5*b-2
80/3*a^3*b^3+92/3*a*b^5)*tan(1/2*d*x+1/2*c)^6+(-5*a^4*b^2+15/4*a^2*b^4-15/8
*b^6)*tan(1/2*d*x+1/2*c)^5+(60*a^5*b-88*a^3*b^3+28*a*b^5)*tan(1/2*d*x+1/2*c
)^4+(-15/2*a^4*b^2+57/8*a^2*b^4+5/48*b^6)*tan(1/2*d*x+1/2*c)^3+(30*a^5*b-44
*a^3*b^3+62/5*a*b^5)*tan(1/2*d*x+1/2*c)^2+(-5/2*a^4*b^2+27/8*a^2*b^4-11/16*
b^6)*tan(1/2*d*x+1/2*c)+6*a^5*b-28/3*a^3*b^3+46/15*a*b^5)/(1+tan(1/2*d*x+1/
2*c)^2)^6+1/16*(112*a^6-200*a^4*b^2+90*a^2*b^4-5*b^6)*arctan(tan(1/2*d*x+1/
2*c)))+2*a/b^8*((-b^2*(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-a^5*b+2*a^3*b^
3-a*b^5)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(7*a^6-16*a^4*b^
2+11*a^2*b^4-2*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.44, size = 814, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*cos(d*x + c)^5 + 5*
(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112*a^7 - 200*a^5*b^
2 + 90*a^3*b^4 - 5*a*b^6)*d*x - 120*(7*a^6 - 9*a^4*b^2 + 2*a^2*b^4 + (7*a^5
*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)) + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)
*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d
*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*
a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*sin(d*x
 + c) + a*b^8*d), -1/240*(40*b^7*cos(d*x + c)^7 - 2*(42*a^2*b^5 - 5*b^7)*co
s(d*x + c)^5 + 5*(56*a^4*b^3 - 58*a^2*b^5 + 5*b^7)*cos(d*x + c)^3 + 15*(112
*a^7 - 200*a^5*b^2 + 90*a^3*b^4 - 5*a*b^6)*d*x + 240*(7*a^6 - 9*a^4*b^2 + 2
*a^2*b^4 + (7*a^5*b - 9*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 15*(112*a^6*b
- 200*a^4*b^3 + 90*a^2*b^5 - 5*b^7)*cos(d*x + c) + (56*a*b^6*cos(d*x + c)^5
 - 10*(14*a^3*b^4 - 11*a*b^6)*cos(d*x + c)^3 + 15*(112*a^6*b - 200*a^4*b^3
+ 90*a^2*b^5 - 5*b^7)*d*x + 15*(56*a^5*b^2 - 86*a^3*b^4 + 27*a*b^6)*cos(d*x
 + c))*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 835, normalized size = 1.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/240*(15*(112*a^6 - 200*a^4*b^2 + 90*a^2*b^4 - 5*b^6)*(d*x + c)/b^8 - 480
*(7*a^7 - 16*a^5*b^2 + 11*a^3*b^4 - 2*a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
```

```
2 - b^2)*b^8) + 480*(a^5*b*tan(1/2*d*x + 1/2*c) - 2*a^3*b^3*tan(1/2*d*x + 1
/2*c) + a*b^5*tan(1/2*d*x + 1/2*c) + a^6 - 2*a^4*b^2 + a^2*b^4)/((a*tan(1/2
*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*b^7) + 2*(600*a^4*b*tan(1/2
*d*x + 1/2*c)^11 - 810*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*
x + 1/2*c)^11 + 1440*a^5*tan(1/2*d*x + 1/2*c)^10 - 2880*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 1440*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 1800*a^4*b*tan(1/2*d*x
+ 1/2*c)^9 - 1710*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2
*c)^9 + 7200*a^5*tan(1/2*d*x + 1/2*c)^8 - 12480*a^3*b^2*tan(1/2*d*x + 1/2*c
)^8 + 4320*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 1200*a^4*b*tan(1/2*d*x + 1/2*c)^7
 - 900*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 14
400*a^5*tan(1/2*d*x + 1/2*c)^6 - 22400*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 736
0*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 900*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*c)^5 + 14400*a^5*t
an(1/2*d*x + 1/2*c)^4 - 21120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 6720*a*b^4*t
an(1/2*d*x + 1/2*c)^4 - 1800*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1710*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 7200*a^5*tan(1/2*d*x
 + 1/2*c)^2 - 10560*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 2976*a*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 600*a^4*b*tan(1/2*d*x + 1/2*c) + 810*a^2*b^3*tan(1/2*d*x + 1/
2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 1440*a^5 - 2240*a^3*b^2 + 736*a*b^4)/
((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^7))/d
```

Mupad [B]
time = 16.38, size = 2500, normalized size = 5.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x))^2,x)
```

```
[Out] - ((2*(105*a^6 + 61*a^2*b^4 - 170*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)^
13*(27*a*b^4 + 56*a^5 - 86*a^3*b^2))/(8*b^6) + (8*tan(c/2 + (d*x)/2)^7*(61*
a*b^4 + 105*a^5 - 170*a^3*b^2))/(3*b^6) + (tan(c/2 + (d*x)/2)^11*(223*a*b^4
 + 336*a^5 - 558*a^3*b^2))/(6*b^6) + (tan(c/2 + (d*x)/2)^3*(1813*a*b^4 + 33
60*a^5 - 5370*a^3*b^2))/(30*b^6) + (tan(c/2 + (d*x)/2)^9*(2533*a*b^4 + 4200
*a^5 - 6954*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^5*(3323*a*b^4 + 5880*a
^5 - 9366*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^12*(56*a^6 + 11*b^6 + 2*
a^2*b^4 - 72*a^4*b^2))/(4*b^7) + (tan(c/2 + (d*x)/2)^10*(1008*a^6 - 5*b^6 +
 378*a^2*b^4 - 1464*a^4*b^2))/(12*b^7) + (tan(c/2 + (d*x)/2)^8*(1260*a^6 +
45*b^6 + 674*a^2*b^4 - 1984*a^4*b^2))/(6*b^7) + (tan(c/2 + (d*x)/2)^6*(1680
*a^6 - 45*b^6 + 1034*a^2*b^4 - 2776*a^4*b^2))/(6*b^7) + (tan(c/2 + (d*x)/2)
^2*(5040*a^6 - 165*b^6 + 3386*a^2*b^4 - 8440*a^4*b^2))/(60*b^7) + (tan(c/2
+ (d*x)/2)^4*(12600*a^6 + 25*b^6 + 8358*a^2*b^4 - 21240*a^4*b^2))/(60*b^7)
+ (tan(c/2 + (d*x)/2)*(1547*a*b^4 + 2520*a^5 - 4150*a^3*b^2))/(120*b^6))/(d
*(a + 2*b*tan(c/2 + (d*x)/2) + 7*a*tan(c/2 + (d*x)/2)^2 + 21*a*tan(c/2 + (d
*x)/2)^4 + 35*a*tan(c/2 + (d*x)/2)^6 + 35*a*tan(c/2 + (d*x)/2)^8 + 21*a*tan
```

(c/2 + (d*x)/2)^10 + 7*a*tan(c/2 + (d*x)/2)^12 + a*tan(c/2 + (d*x)/2)^14 + 12*b*tan(c/2 + (d*x)/2)^3 + 30*b*tan(c/2 + (d*x)/2)^5 + 40*b*tan(c/2 + (d*x)/2)^7 + 30*b*tan(c/2 + (d*x)/2)^9 + 12*b*tan(c/2 + (d*x)/2)^11 + 2*b*tan(c /2 + (d*x)/2)^13)) - (atan((((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a ^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 - (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^ 21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8* b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i)*1i)/(16*b^8) + ((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 752 0*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (((10*a*b^22 - 126*a^3* b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/ 2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b ^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^ 4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3 *b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^ 11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21))*(a^6*112i - b^6*5i + a^2* b^4*90i - a^4*b^2*200i)*1i)/(16*b^8))/((10976*a^19 + (135*a^3*b^16)/2 - (72 05*a^5*b^14)/4 + 15115*a^7*b^12 - (244853*a^9*b^10)/4 + 138577*a^11*b^8 - 1 84965*a^13*b^6 + 144788*a^15*b^4 - 61544*a^17*b^2)/b^20 + (tan(c/2 + (d*x)/ 2)*(175616*a^20 - 100*a^2*b^18 + 4150*a^4*b^16 - 61000*a^6*b^14 + 399830*a^ 8*b^12 - 1393080*a^10*b^10 + 2831960*a^12*b^8 - 3480576*a^14*b^6 + 2551808* a^16*b^4 - 1028608*a^18*b^2))/(4*b^21) + ((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 - (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7* b^16)/b^20 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (t an(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8 *b^16))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 13170 0*a^7*b^15 + 254720*a^9*b^13 - 254720*a^11*b^11 + 127232*a^13*b^9 - 25088*a ^15*b^7))/(8*b^21))*(a^6*112i - b^6*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b ^8) - ((((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2 - 4640*a^8* b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (((10*a*b^22 - 126*a^3*b^20 + 228*a^5*b^18 - 112*a^7*b^16)/b^20 + ((32*a^2*b^3 + (tan(c/ 2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(a^6*112i - b^6*5i + a^ 2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(1024*a^2*b^22 - 5632*a^4*b^20 + 8192*a^6*b^18 - 3584*a^8*b^16))/(8*b^21))*(a^6*112i - b^6*5 i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8) + (tan(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + 32364*a^5*b^17 - 131700*a^7*b^15 + 254720*a^9*b^13 - 2547

```
20*a^11*b^11 + 127232*a^13*b^9 - 25088*a^15*b^7))/(8*b^21))*(a^6*112i - b^6
*5i + a^2*b^4*90i - a^4*b^2*200i))/(16*b^8)))*(a^6*112i - b^6*5i + a^2*b^4*
90i - a^4*b^2*200i)*1i)/(8*b^8*d) - (a*atan(((a*(7*a^2 - 2*b^2)*(-(a + b)^3
*(a - b)^3)^(1/2)*(((25*a^2*b^19)/8 - (225*a^4*b^17)/2 + (2525*a^6*b^15)/2
- 4640*a^8*b^13 + 7520*a^10*b^11 - 5600*a^12*b^9 + 1568*a^14*b^7)/b^20 + (t
an(c/2 + (d*x)/2)*(50*a*b^21 - 2849*a^3*b^19 + ...
```

3.1258 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$\frac{a(24a^4-40a^2b^2+15b^4)\,x}{4b^7}-\frac{2(a^2-b^2)^{3/2}\,(6a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^5(c+dx)(6a+b\sin(c+dx}{5b^2d(a+b\sin(c+dx))}$$

```
[Out] 1/4*a*(24*a^4-40*a^2*b^2+15*b^4)*x/b^7-2*(a^2-b^2)^(3/2)*(6*a^2-b^2)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d+1/5*cos(d*x+c)^5*(6*a+b*si
n(d*x+c))/b^2/d/(a+b*sin(d*x+c))-1/6*cos(d*x+c)^3*(12*a^2-2*b^2-9*a*b*sin(d
*x+c))/b^4/d+1/4*cos(d*x+c)*(24*a^4-28*a^2*b^2+4*b^4-a*b*(12*a^2-11*b^2)*si
n(d*x+c))/b^6/d
```

Rubi [A]
time = 0.33, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2942, 2944, 2814, 2739, 632, 210}

$$-\frac{2(a^2-b^2)^{3/2}(6a^2-b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^3(c+dx)\,(2(6a^2-b^2)-9ab\sin(c+dx))}{6b^4d}+\frac{ax(24a^4-40a^2b^2+15b^4)}{4b^7}+\frac{\cos(c+dx)\,(4(6a^4-7a^2b^2+b^4)-ab(12a^2-11b^2)\sin(c+dx))}{4b^6d}+\frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*(24*a^4 - 40*a^2*b^2 + 15*b^4)*x)/(4*b^7) - (2*(a^2 - b^2)^(3/2)*(6*a^2
- b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (Cos[c +
 d*x]^5*(6*a + b*Sin[c + d*x]))/(5*b^2*d*(a + b*Sin[c + d*x])) - (Cos[c + d
*x]^3*(2*(6*a^2 - b^2) - 9*a*b*Sin[c + d*x]))/(6*b^4*d) + (Cos[c + d*x]*(4*
(6*a^4 - 7*a^2*b^2 + b^4) - a*b*(12*a^2 - 11*b^2)*Sin[c + d*x]))/(4*b^6*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\int \frac{\cos^4(c+dx)(-b-6a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2(6a^2-b^2)-9ab\sin(c+\right.}{6b^4d}$$

$$= \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+dx)\left(2(6a^2-b^2)-9ab\sin(c+\right.}{6b^4d}$$

$$= \frac{a(24a^4-40a^2b^2+15b^4)\,x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+d}{}$$

$$= \frac{a(24a^4-40a^2b^2+15b^4)\,x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+d}{}$$

$$= \frac{a(24a^4-40a^2b^2+15b^4)\,x}{4b^7} + \frac{\cos^5(c+dx)(6a+b\sin(c+dx))}{5b^2d(a+b\sin(c+dx))} - \frac{\cos^3(c+d}{}$$

$$= \frac{a(24a^4-40a^2b^2+15b^4)\,x}{4b^7} - \frac{2(a^2-b^2)^{3/2}(6a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^7d}$$

Mathematica [**A**]
time = 2.99, size = 371, normalized size = 1.61

$$\frac{-960(a^2-b^2)^{3/2}(6a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+\frac{2880a^6c-4800a^4b^2c+1800a^2b^4c+2880a^6dx-4800a^4b^2dx+1800a^2b^4dx+60ab(48a^4-74a^2b^2+23b^4)\cos(c+dx)+5(24a^3b^3-31ab^5)\cos(3(c+dx))-9ab^5\cos(5(c+dx))+2880a^5bc\sin(c+dx)-4800a^3b^3c\sin(c+dx)+1800ab^5c\sin(c+dx)+2880a^5bdx\sin(c+dx)-4800a^3b^3dx\sin(c+dx)+1800ab^5dx\sin(c+dx)+720a^4b^2\sin(2(c+dx))-1080a^2b^4\sin(2(c+dx))+295b^6\sin(2(c+dx))-30a^2b^4\sin(4(c+dx))+32b^6\sin(4(c+dx))+3b^6\sin(6(c+dx))}{a+b\sin(c+dx)}}{480b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-960*(a^2 - b^2)^(3/2)*(6*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]] + (2880*a^6*c - 4800*a^4*b^2*c + 1800*a^2*b^4*c + 2880*a^6*d*x
- 4800*a^4*b^2*d*x + 1800*a^2*b^4*d*x + 60*a*b*(48*a^4 - 74*a^2*b^2 + 23*b^
4)*Cos[c + d*x] + 5*(24*a^3*b^3 - 31*a*b^5)*Cos[3*(c + d*x)] - 9*a*b^5*Cos[
5*(c + d*x)] + 2880*a^5*b*c*Sin[c + d*x] - 4800*a^3*b^3*c*Sin[c + d*x] + 18
00*a*b^5*c*Sin[c + d*x] + 2880*a^5*b*d*x*Sin[c + d*x] - 4800*a^3*b^3*d*x*Si
n[c + d*x] + 1800*a*b^5*d*x*Sin[c + d*x] + 720*a^4*b^2*Sin[2*(c + d*x)] - 1
080*a^2*b^4*Sin[2*(c + d*x)] + 295*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*
(c + d*x)] + 32*b^6*Sin[4*(c + d*x)] + 3*b^6*Sin[6*(c + d*x)])/(a + b*Sin[c
 + d*x]))/(480*b^7*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(214) = 428.
time = 0.59, size = 468, normalized size = 2.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^7*(((a^3*b^2-9/8*a*b^4)*tan(1/2*d*x+1/2*c)^9+(5/2*a^4*b-9/2*a^2*b^
3+3/2*b^5)*tan(1/2*d*x+1/2*c)^8+(2*a^3*b^2-5/4*a*b^4)*tan(1/2*d*x+1/2*c)^7+
(10*a^4*b-15*a^2*b^3+3*b^5)*tan(1/2*d*x+1/2*c)^6+(15*a^4*b-20*a^2*b^3+14/3*
b^5)*tan(1/2*d*x+1/2*c)^4+(-2*a^3*b^2+5/4*a*b^4)*tan(1/2*d*x+1/2*c)^3+(10*a
^4*b-13*a^2*b^3+7/3*b^5)*tan(1/2*d*x+1/2*c)^2+(-a^3*b^2+9/8*a*b^4)*tan(1/2*
d*x+1/2*c)+5/2*a^4*b-7/2*a^2*b^3+23/30*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5+1/8*
a*(24*a^4-40*a^2*b^2+15*b^4)*arctan(tan(1/2*d*x+1/2*c)))-4/b^7*((-1/2*b^2*(
a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/2*a^5*b+a^3*b^3-1/2*a*b^5)/(a*tan(1
/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(6*a^6-13*a^4*b^2+8*a^2*b^4-b
^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)))
```

<u>Maxima</u> [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

<u>Fricas</u> [A]
time = 0.42, size = 697, normalized size = 3.02

$$\left[-\frac{18\,ab^5\cos(dx+c)^5-5\,(12\,a^3b^3-11\,ab^5)\cos(dx+c)^3-15\,(24\,a^6-40\,a^4b^2+15\,a^2b^4)dx-30\,(6\,a^5-7\,a^3b^2+ab^4+(6\,a^4b-7\,a^2b^3+b^5)\sin(dx+c))\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-15\,(24\,a^5b-40\,a^3b^3+15\,ab^5)\cos(dx+c)-(12\,b^6\cos(dx+c)^5-10\,(3\,a^2b^4-2\,b^6)\cos(dx+c)^3+15\,(24\,a^5b-40\,a^3b^3+15\,ab^5)dx+15\,(12\,a^4b^2-17\,a^2b^4+4\,b^6)\cos(dx+c))\sin(dx+c)}{60\,(b^8d\sin(dx+c)+ab^7d)},\ -\frac{18\,ab^5\cos(dx+c)^5-5\,(12\,a^3b^3-11\,ab^5)\cos(dx+c)^3-15\,(24\,a^6-40\,a^4b^2+15\,a^2b^4)dx-60\,(6\,a^5-7\,a^3b^2+ab^4+(6\,a^4b-7\,a^2b^3+b^5)\sin(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-15\,(24\,a^5b-40\,a^3b^3+15\,ab^5)\cos(dx+c)-(12\,b^6\cos(dx+c)^5-10\,(3\,a^2b^4-2\,b^6)\cos(dx+c)^3+15\,(24\,a^5b-40\,a^3b^3+15\,ab^5)dx+15\,(12\,a^4b^2-17\,a^2b^4+4\,b^6)\cos(dx+c))\sin(dx+c)}{60\,(b^8d\sin(dx+c)+ab^7d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(18*a*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3
- 15*(24*a^6 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 30*(6*a^5 - 7*a^3*b^2 + a*b^4
 + (6*a^4*b - 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*cos(d*x
 + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)^3 + 15
*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^4 + 4*b
^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d), -1/60*(18*a
```

```
*b^5*cos(d*x + c)^5 - 5*(12*a^3*b^3 - 11*a*b^5)*cos(d*x + c)^3 - 15*(24*a^6
 - 40*a^4*b^2 + 15*a^2*b^4)*d*x - 60*(6*a^5 - 7*a^3*b^2 + a*b^4 + (6*a^4*b
- 7*a^2*b^3 + b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*
cos(d*x + c) - (12*b^6*cos(d*x + c)^5 - 10*(3*a^2*b^4 - 2*b^6)*cos(d*x + c)
^3 + 15*(24*a^5*b - 40*a^3*b^3 + 15*a*b^5)*d*x + 15*(12*a^4*b^2 - 17*a^2*b^
4 + 4*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d*sin(d*x + c) + a*b^7*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(214) = 428.
time = 0.49, size = 593, normalized size = 2.57

$$\frac{\frac{15\,(24\,a^5-40\,a^3b^2+15\,ab^4)(dx+c)}{b^7}-\frac{120\,(6\,a^6-13\,a^4b^2+8\,a^2b^4-b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^7}+\frac{120\,\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5-2\,a^3b^2+ab^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)b^6}+\frac{2\,\left(120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-135\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+300\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-540\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+180\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+240\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-150\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1200\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1800\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+360\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+1800\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2400\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+560\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-240\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+150\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1200\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1560\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+280\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+135\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+300\,a^4-420\,a^2b^2+92\,b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/60*(15*(24*a^5 - 40*a^3*b^2 + 15*a*b^4)*(d*x + c)/b^7 - 120*(6*a^6 - 13*a
^4*b^2 + 8*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 120
*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2
*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*t
an(1/2*d*x + 1/2*c) + a)*b^6) + 2*(120*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 135*a
*b^3*tan(1/2*d*x + 1/2*c)^9 + 300*a^4*tan(1/2*d*x + 1/2*c)^8 - 540*a^2*b^2*
tan(1/2*d*x + 1/2*c)^8 + 180*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^3*b*tan(1/2
*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a^4*tan(1/2*d*x +
 1/2*c)^6 - 1800*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 360*b^4*tan(1/2*d*x + 1/2
*c)^6 + 1800*a^4*tan(1/2*d*x + 1/2*c)^4 - 2400*a^2*b^2*tan(1/2*d*x + 1/2*c)
^4 + 560*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 15
0*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1200*a^4*tan(1/2*d*x + 1/2*c)^2 - 1560*a^2
*b^2*tan(1/2*d*x + 1/2*c)^2 + 280*b^4*tan(1/2*d*x + 1/2*c)^2 - 120*a^3*b*ta
n(1/2*d*x + 1/2*c) + 135*a*b^3*tan(1/2*d*x + 1/2*c) + 300*a^4 - 420*a^2*b^2
 + 92*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^6))/d
```

Mupad [B]
time = 14.53, size = 2500, normalized size = 10.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x))^2,x)

[Out] ((2*(38*a*b^4 + 90*a^5 - 135*a^3*b^2))/(15*b^6) - (tan(c/2 + (d*x)/2)^10*(a*b^4 - 12*a^5 + 14*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^8*(9*a*b^4 + 30*a^5 - 41*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^4*(29*a*b^4 + 60*a^5 - 94*a^3*b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(38*a*b^4 + 90*a^5 - 135*a^3*b^2))/(3*b^6) + (tan(c/2 + (d*x)/2)^2*(157*a*b^4 + 300*a^5 - 470*a^3*b^2))/(5*b^6) + (tan(c/2 + (d*x)/2)*(540*a^4 + 244*b^4 - 825*a^2*b^2))/(30*b^5) + (tan(c/2 + (d*x)/2)^11*(12*a^4 + 4*b^4 - 17*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^9*(84*a^4 + 44*b^4 - 131*a^2*b^2))/(2*b^5) + (tan(c/2 + (d*x)/2)^7*(108*a^4 + 44*b^4 - 165*a^2*b^2))/b^5 + (tan(c/2 + (d*x)/2)^5*(396*a^4 + 172*b^4 - 585*a^2*b^2))/(3*b^5) + (tan(c/2 + (d*x)/2)^3*(468*a^4 + 172*b^4 - 687*a^2*b^2))/(6*b^5))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^2 + 15*a*tan(c/2 + (d*x)/2)^4 + 20*a*tan(c/2 + (d*x)/2)^6 + 15*a*tan(c/2 + (d*x)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 + a*tan(c/2 + (d*x)/2)^12 + 10*b*tan(c/2 + (d*x)/2)^3 + 20*b*tan(c/2 + (d*x)/2)^5 + 20*b*tan(c/2 + (d*x)/2)^7 + 10*b*tan(c/2 + (d*x)/2)^9 + 2*b*tan(c/2 + (d*x)/2)^11)) + (a*atan(((a*((2*(225*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 - (2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^2*b^2))/(4*b^7) + (a*((2*(225*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 - (2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^2*b^2))/(4*b^7))/((4*(1728*a^16 - 60*a^2*b^14 + 895*a^4*b^12 - 5056*a^6*b^10 + 14291*a^8*b^8 - 22310*a^10*b^6 + 19584*a^12*b^4 - 9072*a^14*b^2))/b^17 + (4*tan(c/2 + (d*x)/2)*(6912*a^17 - 450*a^3*b^14 + 6000*a^5*b^12 - 29690*a^7*b^10 + 74860*a^9*b^8 - 106592*a^11*b^6 + 86976*a^13*b^4 - 38016*a^15*b^2))/b^18 - (a*((2*(225*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 - (2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4 *b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7) + (a*((2*(225*a^4*b^14 - 1200*a^6*b^12 + 2320*a^8*b^10 - 1920*a^10*b^8 + 576*a^12*b^6))/b^17 - (2*tan(c/2 + (d*x)/2)*(16*a*b^18 - 706*a^3*b^16 + 4065*a^5*b^14 - 9360*a^7*b^12 + 10400*a^9*b^10 - 5568*a^11*b^8 + 1152*a^13*b^6))/b^18 + (a*(24*a^4 + 15*b^4 - 40*a^2*b^2)*((2*(44*a^2*b^18 - 92*a^4*b^16 + 48*a^6*b^14))/b^17 - (2*tan(c/2 + (d*x)/2)*(32*a*b^20 - 256*a^3*b^18 + 416*a^5*b^16 - 192*a^7*b^14))/b^18 + (a*(32*a^2*b^3 + (2*tan(c/2 + (d*x)/2)*(48*a*b^22 - 32*a^3*b^20))/b^18)*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7))*1i)/(4*b^7))*(24*a^4 + 15*b^4 - 40*a^2*b^2)*1i)/(4*b^7)))*(24*a^4 + 15*b^4 - 40*a^2*b^2))/(2*b^7*d) - (2*atanh((64*a^2*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(64*a^2*b^3 - 572*a^4*b + (1312*a^6)/b - (1164*a^8)/b^3 + (360*a^10)/b^5 + 2624*a^5*tan(c/2 + (d*x)/2) + 128*a*b^4*tan(c/2 + (d*x)/2) - 1144*a^3*b^2*tan(c/2 + (d*x)/2) - (2328*a^7*tan(c/2 + (d*x)/2))/b^2 + (720*a^9*tan(c/2 + (d*x)/2))/b^4) - (444*a^4*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(1312*a^6*b + 64*a^2*b^5 - 572*a^4*b^3 - (1164*a^8)/b + (360*a^10)/b^3 - 2328*a^7*tan(c/2 + (d*x)/2) + 128*a*b^6*tan(c/2 + (d*x)/2) - 1144*a^3*b^4*tan(c/2 + (d*x)/2) + 2624*a^5*b^2*tan(c/2 + (d*x)/2) + (720*a^9*tan(c/2 + (d*x)/2))/b^2) + (360*a^6*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(64*a^2*b^7 - 1164*a^8*b - 572*a^4*b^5 + 1312*a^6*b^3 + (360*a^10)/b + 720*a^9*tan(c/2 + (d*x)/2) + 128*a*b^8*tan(c/2 + (d*x)/2) - 1144*a^3*b^6*tan(c/2 + (d*x)/2) + 2624*a^5*b^4*tan(c/2 + (d*x)/2) - 2328*a^7*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(64*a^2*b^2 - 572*a^4 + (1312*a^6)/b^2 - (1164*a^8)/b^4 + (360*a^10)/b^6 + 128*a*b^3*tan(c/2 + (d*x)/2) - 1144*a^3*b*tan(c/2 + (d*x)/2) + (2624*a^5*tan(c/2 + (d*x)/2))/b - (2328*a^7*tan(c/2 + (d*x)/2))/b^3 + (720*a^9*tan(c/2 + (d*x)/2))/b^5) - (952*a^3*tan(c/2 + (d*x)/2)*(b^6 - a^6 - 3...

3.1259 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=266

$$\frac{ax}{b^3}+\frac{2a(2a^2-3b^2)\,x}{b^5}+\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2(a^2-b^2)^{3/2}\,(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{a^2b^5d}$$

```
[Out] a*x/b^3+2*a*(2*a^2-3*b^2)*x/b^5+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/b^5/d-2*(a^2-b^2)^(3/2)*(5*a^2+b^2)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/b^5/d-arctanh(cos(d*x+c))/a^2/d+cos(d
*x+c)/b^2/d+3*(a^2-b^2)*cos(d*x+c)/b^4/d-1/3*cos(d*x+c)^3/b^2/d-a*cos(d*x+c
)*sin(d*x+c)/b^3/d+(a^2-b^2)^2*cos(d*x+c)/a/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2976, 3855, 2718, 2715, 8, 2713, 2743, 12, 2739, 632, 210}

$$\frac{2(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2(a^2-b^2)^{3/2}\,(5a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}+\frac{2ax(2a^2-3b^2)}{b^5}+\frac{3(a^2-b^2)\cos(c+dx)}{b^4d}+\frac{(a^2-b^2)^2\cos(c+dx)}{ab^4d(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+dx)\cos(c+dx)}{b^3d}+\frac{ax}{b^3}-\frac{\cos^3(c+dx)}{3b^2d}+\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a*x)/b^3 + (2*a*(2*a^2 - 3*b^2)*x)/b^5 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (2*(a^2 - b^2)^(3/2)*(5*a^2
 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^5*d) - Arc
Tanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(b^2*d) + (3*(a^2 - b^2)*Cos[c +
d*x])/(b^4*d) - Cos[c + d*x]^3/(3*b^2*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(b
^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{2(-2a^3+3ab^2)}{b^5}+\frac{\csc(c+dx)}{a^2}+\frac{3(-a^2+b^2)\sin(c+dx)}{b^4}+\frac{2a\sin^2}{}\right.$$

$$= \frac{2a(2a^2-3b^2)\,x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a^2}+\frac{(2a)\int\sin^2(c+dx)\,dx}{b^3}-\frac{\int\sin^3(c+}{b^2}$$

$$= \frac{2a(2a^2-3b^2)\,x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{3(a^2-b^2)\cos(c+dx)}{b^4d}-\frac{a\cos(}{}$$

$$= \frac{ax}{b^3}+\frac{2a(2a^2-3b^2)\,x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{b^2d}+\frac{3(a^2-b^2)}{b}$$

$$= \frac{ax}{b^3}+\frac{2a(2a^2-3b^2)\,x}{b^5}-\frac{2(a^2-b^2)^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}$$

$$= \frac{ax}{b^3}+\frac{2a(2a^2-3b^2)\,x}{b^5}-\frac{2(a^2-b^2)^{3/2}(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5d}$$

$$= \frac{ax}{b^3}+\frac{2a(2a^2-3b^2)\,x}{b^5}+\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{2(a^2-b}{}$$

Mathematica [A]
time = 1.17, size = 207, normalized size = 0.78

$$\frac{\frac{12a(4a^2-5b^2)(c+dx)}{b^5}-\frac{24(a^2-b^2)^{3/2}(4a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^5}+\frac{9(4a^2-3b^2)\cos(c+dx)}{b^4}-\frac{\cos(3(c+dx))}{b^2}-\frac{12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{12(a^2-b^2)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))}-\frac{6a\sin(2(c+dx))}{b^3}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*(4*a^2 - 5*b^2)*(c + d*x))/b^5 - (24*(a^2 - b^2)^(3/2)*(4*a^2 + b^2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^5) + (9*(4*a^2 -
3*b^2)*Cos[c + d*x])/b^4 - Cos[3*(c + d*x)]/b^2 - (12*Log[Cos[(c + d*x)/2]]
)/a^2 + (12*Log[Sin[(c + d*x)/2]])/a^2 + (12*(a^2 - b^2)^2*Cos[c + d*x])/(a
*b^4*(a + b*Sin[c + d*x])) - (6*a*Sin[2*(c + d*x)])/b^3)/(12*d)
```

Maple [A]
time = 0.74, size = 314, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{4\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-\frac{3}{2}b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2b-2b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{7b^3}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+2a\left(4a^2-5\right.}{b^5}$
default	$\frac{\frac{4\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(\frac{3}{2}a^2b-\frac{3}{2}b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^2b-2b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{3a^2b}{2}-\frac{7b^3}{6}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+2a\left(4a^2-5\right.}{b^5}$
risch	$\frac{4a^3x}{b^5}-\frac{5ax}{b^3}+\frac{ia\,e^{2i(dx+c)}}{4b^3d}+\frac{3\,e^{i(dx+c)}a^2}{2b^4d}-\frac{9\,e^{i(dx+c)}}{8b^2d}+\frac{3\,e^{-i(dx+c)}a^2}{2b^4d}-\frac{9\,e^{-i(dx+c)}}{8b^2d}-\frac{ia\,e^{-2i(dx+c)}}{4b^3d}-\frac{3i}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4/b^5*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(3/2*a^2*b-3/2*b^3)*tan(1/2*d*x
+1/2*c)^4+(3*a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*tan(1/2*d*x+1/2*c)
+3/2*a^2*b-7/6*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(4*a^2-5*b^2)*arctan(t
an(1/2*d*x+1/2*c)))+1/a^2*ln(tan(1/2*d*x+1/2*c))-4/a^2/b^5*((-1/2*b^2*(a^4-
2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/2*a^5*b+a^3*b^3-1/2*a*b^5)/(a*tan(1/2*d
*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(4*a^6-7*a^4*b^2+2*a^2*b^4+b^6)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.66, size = 698, normalized size = 2.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(4*a^3*b^3*cos(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x - 3*(4*a^5 - 3*a
^3*b^2 - a*b^4 + (4*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a
*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5
)*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
 3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos
(d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
 c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d), 1/6*(4*a^3*b^3*cos
(d*x + c)^3 + 6*(4*a^6 - 5*a^4*b^2)*d*x + 6*(4*a^5 - 3*a^3*b^2 - a*b^4 + (4
*a^4*b - 3*a^2*b^3 - b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
+ c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(4*a^5*b - 5*a^3*b^3 + a*b^5)
*cos(d*x + c) - 3*(b^6*sin(d*x + c) + a*b^5)*log(1/2*cos(d*x + c) + 1/2) +
3*(b^6*sin(d*x + c) + a*b^5)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^2*b^4*cos(
d*x + c)^3 - 3*(4*a^5*b - 5*a^3*b^3)*d*x - 6*(a^4*b^2 - a^2*b^4)*cos(d*x +
c))*sin(d*x + c))/(a^2*b^6*d*sin(d*x + c) + a^3*b^5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{6}\left(c+dx\right)\csc\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 353, normalized size = 1.33

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{2}}+\frac{3\left(4a^{3}-5ab^{2}\right)\left(dx+c\right)}{b^{5}}-\frac{6\left(4a^{6}-7a^{4}b^{2}+2a^{2}b^{4}+b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}b^{5}}+\frac{2\left(3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+9a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-9b^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+18a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12b^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+9a^{2}-7b^{2}\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)^{3}b^{4}}+\frac{6\left(a^{4}b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^{2}b^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^{5}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^{5}-2a^{3}b^{2}+ab^{4}\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)a^{2}b^{4}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*(4*a^3 - 5*a*b^2)*(d*x + c)/b
^5 - 6*(4*a^6 - 7*a^4*b^2 + 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1
/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^
2 - b^2)*a^2*b^5) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*tan(1/2*d*x + 1
/2*c)^4 - 9*b^2*tan(1/2*d*x + 1/2*c)^4 + 18*a^2*tan(1/2*d*x + 1/2*c)^2 - 12
*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 9*a^2 - 7*b^2)/(
```

```
(tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^4) + 6*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2
*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*
b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^2*b^4))/d
```

Mupad [B]
time = 13.60, size = 2500, normalized size = 9.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + ((2*(12*a^4 + 3*b^4 - 13*a^2*b^2))/(3*a*b
^4) + (2*tan(c/2 + (d*x)/2)*(18*a^4 + 3*b^4 - 20*a^2*b^2))/(3*a^2*b^3) + (2
*tan(c/2 + (d*x)/2)^7*(2*a^4 + b^4 - 2*a^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (
d*x)/2)^6*(4*a^4 + b^4 - 3*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(10*
a^4 + 3*b^4 - 12*a^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^4*(12*a^4 + 3*
b^4 - 13*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(14*a^4 + 3*b^4 - 14*a
^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^2*(36*a^4 + 9*b^4 - 43*a^2*b^2))
/(3*a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 6*a
*tan(c/2 + (d*x)/2)^4 + 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2)^8 +
 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/
2)^7)) + (2*a*atan(((a*(4*a^2 - 5*b^2)*((a*(4*a^2 - 5*b^2)*((32*(4*b^16 + 7
*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^
12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 6
10*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8)
)/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^
12 + 14*a^8*b^10))/(a^2*b^11) + (a*((32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b^1
1) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^3*
b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^
4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16))*1i)/b^5)*1i
)/b^5 - (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b
^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(b^
20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b
^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16)))/b^5 - (a*
(4*a^2 - 5*b^2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 +
722*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (a*(4*a^2 - 5*b^2
)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 18
4*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18
 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b
^10 + 32*a^14*b^8))/(a^3*b^16) - (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^
4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) - (a*((32*(4*a^4*b^16 - 3*a
^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 +
2*a^8*b^18))/(a^3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(
16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*
```

b^16))*1i)/b^5)*1i)/b^5 - (32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16)))/b^5)/((64*(224*a^12 - 5*b^12 + 84*a^2*b^10 + 81*a^4*b^8 - 906*a^6*b^6 + 1482*a^8*b^4 - 960*a^10*b^2))/(a^2*b^11) - (64*tan(c/2 + (d*x)/2)*(2048*a^18 + 500*a^4*b^14 - 1200*a^6*b^12 - 4540*a^8*b^10 + 21944*a^10*b^8 - 36160*a^12*b^6 + 29696*a^14*b^4 - 12288*a^16*b^2))/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((a*(4*a^2 - 5*b^2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) + (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) + (a*((32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16))*1i)/b^5)*1i)/b^5 - (32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16))*1i)/b^5 + (a*(4*a^2 - 5*b^2)*((32*(5*a^2*b^12 - 224*a^14 - 184*a^4*b^10 + 154*a^6*b^8 + 722*a^8*b^6 - 1434*a^10*b^4 + 960*a^12*b^2))/(a^2*b^11) + (a*(4*a^2 - 5*b^2)*((32*(4*b^16 + 7*a^2*b^14 - 25*a^4*b^12 + 114*a^6*b^10 - 235*a^8*b^8 + 184*a^10*b^6 - 48*a^12*b^4))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(20*a^4*b^18 - 19*a^2*b^20 + 610*a^6*b^16 - 1596*a^8*b^14 + 1434*a^10*b^12 - 480*a^12*b^10 + 32*a^14*b^8))/(a^3*b^16) - (a*(4*a^2 - 5*b^2)*((32*(8*a^2*b^16 + 4*a^4*b^14 - 25*a^6*b^12 + 14*a^8*b^10))/(a^2*b^11) - (a*((32*(4*a^4*b^16 - 3*a^6*b^14))/(a^2*b^11) + (32*tan(c/2 + (d*x)/2)*(16*a^4*b^22 - 17*a^6*b^20 + 2*a^8*b^18))/(a^3*b^16))*(4*a^2 - 5*b^2)*1i)/b^5 + (32*tan(c/2 + (d*x)/2)*(16*a^2*b^22 + 3*a^4*b^20 - 62*a^6*b^18 + 60*a^8*b^16 - 16*a^10*b^14))/(a^3*b^16))*1i)/b^5)*1i)/b^5 - (32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 560*a^6*b^14 - 1414*a^8*b^12 + 4028*a^10*b^10 - 3792*a^12*b^8 + 1600*a^14*b^6 - 256*a^16*b^4))/(a^3*b^16))*1i)/b^5))*(4*a^2 - 5*b^2))/(b^5*d) + (atan((((4*a^2 + b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(b^20 + 4*a^2*b^18 + 390*a^4*b^16 - 56...

3.1260 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=254

$$-\frac{x}{2b^2}-\frac{3(a^2-b^2)\,x}{b^4}-\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^4d}+\frac{4(2a^6-3a^4b^2+b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d}+\frac{2b}{}$$

```
[Out] -1/2*x/b^2-3*(a^2-b^2)*x/b^4-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*
c))/(a^2-b^2)^(1/2))/a/b^4/d+2*b*arctanh(cos(d*x+c))/a^3/d-2*a*cos(d*x+c)/b
^3/d-cot(d*x+c)/a^2/d+1/2*cos(d*x+c)*sin(d*x+c)/b^2/d-(a^2-b^2)^2*cos(d*x+c
)/a^2/b^3/d/(a+b*sin(d*x+c))+4*(2*a^6-3*a^4*b^2+b^6)*arctan((b+a*tan(1/2*d*
x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^4/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2976, 3855, 3852, 8, 2718, 2715, 2743, 12, 2739, 632, 210}

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^4d}-\frac{3x(a^2-b^2)}{b^4}-\frac{(a^2-b^2)^2\cos(c+dx)}{a^2b^3d(a+b\sin(c+dx))}-\frac{\cot(c+dx)}{a^2d}+\frac{4(2a^6-3a^4b^2+b^6)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^4d\sqrt{a^2-b^2}}-\frac{2a\cos(c+dx)}{b^3d}+\frac{\sin(c+dx)\cos(c+dx)}{2b^2d}-\frac{x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*x/b^2 - (3*(a^2 - b^2)*x)/b^4 - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^4*d) + (4*(2*a^6 - 3*a^4*b^2 + b^6)*A
rcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4*Sqrt[a^2 - b^2]*d
) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*a*Cos[c + d*x])/(b^3*d) - Cot[
c + d*x]/(a^2*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - ((a^2 - b^2)^2*C
os[c + d*x])/(a^2*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{3(-a^2+b^2)}{b^4} - \frac{2b\csc(c+dx)}{a^3} + \frac{\csc^2(c+dx)}{a^2} + \frac{2a\sin(c+dx)}{b^3} - \sin^2$$

$$= -\frac{3(a^2-b^2)\,x}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a^2} + \frac{(2a)\int \sin(c+dx)\,dx}{b^3} - \frac{\int \sin^2(c+d}{b^2}$$

$$= -\frac{3(a^2-b^2)\,x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} + \frac{\cos(c+dx)\,\mathrm{s}}{2b^2}$$

$$= -\frac{x}{2b^2} - \frac{3(a^2-b^2)\,x}{b^4} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2a\cos(c+dx)}{b^3d} - \frac{\cot(c}{a^2}$$

$$= -\frac{x}{2b^2} - \frac{3(a^2-b^2)\,x}{b^4} + \frac{4(2a^6-3a^4b^2+b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + 2b\,\mathrm{t}$$

$$= -\frac{x}{2b^2} - \frac{3(a^2-b^2)\,x}{b^4} + \frac{4(2a^6-3a^4b^2+b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4\sqrt{a^2-b^2}\,d} + 2b\,\mathrm{t}$$

$$= -\frac{x}{2b^2} - \frac{3(a^2-b^2)\,x}{b^4} - \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^4d} + \frac{4(2a^6-3}{}$$

Mathematica [A]
time = 1.96, size = 215, normalized size = 0.85

$$\frac{\frac{2(-6a^2+5b^2)(c+dx)}{b^4} + \frac{8(a^2-b^2)^{3/2}(3a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^4} - \frac{8a\cos(c+dx)}{b^3} - \frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{8b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{8b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} - \frac{4(a^2-b^2)^2\cos(c+dx)}{a^2b^3(a+b\sin(c+dx))} + \frac{\sin(2(c+dx))}{b^2} + \frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(-6*a^2 + 5*b^2)*(c + d*x))/b^4 + (8*(a^2 - b^2)^(3/2)*(3*a^2 + 2*b^2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^4) - (8*a*Cos[c +
d*x])/b^3 - (2*Cot[(c + d*x)/2])/a^2 + (8*b*Log[Cos[(c + d*x)/2]])/a^3 - (8
*b*Log[Sin[(c + d*x)/2]])/a^3 - (4*(a^2 - b^2)^2*Cos[c + d*x])/(a^2*b^3*(a
+ b*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^2 + (2*Tan[(c + d*x)/2])/a^2)/(4*d)
```

Maple [A]
time = 0.85, size = 304, normalized size = 1.20

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(6a^2-5b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^4}-\frac{}{2a^2\tan\left(\right.}}{}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+2ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(6a^2-5b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^4}-\frac{}{2a^2\tan\left(\right.}}{}$
risch	$-\frac{3x\,a^2}{b^4}+\frac{5x}{2b^2}-\frac{i\,\mathrm{e}^{2i(dx+c)}}{8b^2d}-\frac{a\,\mathrm{e}^{i(dx+c)}}{b^3d}-\frac{a\,\mathrm{e}^{-i(dx+c)}}{b^3d}+\frac{i\,\mathrm{e}^{-2i(dx+c)}}{8b^2d}+\frac{2i\left(ia^4b\,\mathrm{e}^{2i(dx+c)}-2ia^2b^3\mathrm{e}^{2i(dx+c)}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)-2/b^4*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+2*a*b*
tan(1/2*d*x+1/2*c)^2-1/2*b^2*tan(1/2*d*x+1/2*c)+2*a*b)/(1+tan(1/2*d*x+1/2*c
)^2)^2+1/2*(6*a^2-5*b^2)*arctan(tan(1/2*d*x+1/2*c)))-1/2/a^2/tan(1/2*d*x+1/
2*c)-2/a^3*b*ln(tan(1/2*d*x+1/2*c))+2/a^3/b^4*((-b^2*(a^4-2*a^2*b^2+b^4)*ta
n(1/2*d*x+1/2*c)-b*a*(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1
/2*d*x+1/2*c)+a)+(3*a^6-4*a^4*b^2-a^2*b^4+2*b^6)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.63, size = 901, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(3*a^4*b^2*cos(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2
- (6*a^5*b - 5*a^3*b^3)*d*x - (3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b
^3 - 2*b^5)*cos(d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt
(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(
b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^4*b^2 + 2*a^2*
b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*log(1
/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*
log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*b^2
)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3*b^
5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d), -1/2*(3*a^4*b^2*c
os(d*x + c)^3 + (6*a^5*b - 5*a^3*b^3)*d*x*cos(d*x + c)^2 - (6*a^5*b - 5*a^3
*b^3)*d*x - 2*(3*a^4*b - a^2*b^3 - 2*b^5 - (3*a^4*b - a^2*b^3 - 2*b^5)*cos(
d*x + c)^2 + (3*a^5 - a^3*b^2 - 2*a*b^4)*sin(d*x + c))*sqrt(a^2 - b^2)*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^4*b^2 + 2*a
^2*b^4)*cos(d*x + c) - 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^6)*lo
g(1/2*cos(d*x + c) + 1/2) + 2*(b^6*cos(d*x + c)^2 - a*b^5*sin(d*x + c) - b^
6)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^3*cos(d*x + c)^3 + (6*a^6 - 5*a^4*
b^2)*d*x + (6*a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^3
*b^5*d*cos(d*x + c)^2 - a^4*b^4*d*sin(d*x + c) - a^3*b^5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 384, normalized size = 1.51

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{3\,(6\,a^2-5\,b^2)(dx+c)}{b^4}+\frac{6\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^3}-\frac{12\,(3\,a^6-4\,a^4b^2-a^2b^4+2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3b^4}-\frac{4\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+21\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b^3}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 3*(6*a^2 - 5*b^2)*(d*x + c)/b^4 + 6*(b*tan(1/2*d*x + 1/2*c)^3 + 4*a*tan(1
```

```
/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 4*a)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^2*b^3) - 12*(3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*(pi*floor(1/2*(d*x +
 c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2))
)/(sqrt(a^2 - b^2)*a^3*b^4) - (4*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 12*a^4*b*ta
n(1/2*d*x + 1/2*c)^2 + 21*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*b^5*tan(1/2*d*
x + 1/2*c)^2 - 12*a^5*tan(1/2*d*x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c
) - 14*a*b^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^3 +
 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3*b^3))/d
```

Mupad [B]
time = 13.48, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^2*d) - (a + (2*tan(c/2 + (d*x)/2)*(6*a^4 + 3*b^4 -
4*a^2*b^2))/b^3 + (2*tan(c/2 + (d*x)/2)^5*(6*a^4 + 3*b^4 - 2*a^2*b^2))/b^3
+ (4*tan(c/2 + (d*x)/2)^3*(6*a^4 + 3*b^4 - 5*a^2*b^2))/b^3 + (tan(c/2 + (d*
x)/2)^6*(6*a^4 + 4*b^4 - 7*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^2*(18*a^
4 + 4*b^4 - 5*a^2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^4*(24*a^4 + 8*b^4 - 1
3*a^2*b^2))/(a*b^2))/(d*(6*a^3*tan(c/2 + (d*x)/2)^3 + 6*a^3*tan(c/2 + (d*x)
/2)^5 + 2*a^3*tan(c/2 + (d*x)/2)^7 + 2*a^3*tan(c/2 + (d*x)/2) + 4*a^2*b*tan
(c/2 + (d*x)/2)^2 + 8*a^2*b*tan(c/2 + (d*x)/2)^4 + 4*a^2*b*tan(c/2 + (d*x)/
2)^6)) + (atan((((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^
10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8)
 - ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((((8*(16*a^8*b^13 - 12*a^10*b^11
))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^
14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*(64*a^5*b^14 - 48*a^7*b^12
 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(136*a^6*b
^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12
)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 +
315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)
/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b
^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*
x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11
 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12
))*1i)/(2*b^4) + ((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b
^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8
) + ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((8*(64*a^5*b^14 - 48*a^7*b^12 -
 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16*a^8*b^13 - 12*a^10*b^11))
/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14
))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*tan(c/2 + (d*x)/2)*(136*a^6*
b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^1
```

2)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12))*1i)/(2*b^4))/(((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) - ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12)))/(2*b^4) - ((a^2*6i - b^2*5i)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^13*b^2))/(a^6*b^8) + ((a^2*6i - b^2*5i)*(((a^2*6i - b^2*5i)*((8*(64*a^5*b^14 - 48*a^7*b^12 - 50*a^9*b^10 + 42*a^11*b^8))/(a^6*b^8) + (((8*(16*a^8*b^13 - 12*a^10*b^11))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^18 - 68*a^9*b^16 + 8*a^11*b^14))/(a^6*b^12))*(a^2*6i - b^2*5i))/(2*b^4) - (8*tan(c/2 + (d*x)/2)*(136*a^6*b^17 - 128*a^4*b^19 + 96*a^8*b^15 - 160*a^10*b^13 + 48*a^12*b^11))/(a^6*b^12)))/(2*b^4) - (8*(48*a^4*b^13 - 64*a^2*b^15 + 100*a^6*b^11 - 184*a^8*b^9 + 315*a^10*b^7 - 324*a^12*b^5 + 108*a^14*b^3))/(a^6*b^8) + (8*tan(c/2 + (d*x)/2)*(16*a^3*b^18 + 240*a^5*b^16 + 20*a^7*b^14 - 1696*a^9*b^12 + 2369*a^11*b^10 - 1020*a^13*b^8 + 72*a^15*b^6))/(a^6*b^12)))/(2*b^4) + (8*tan(c/2 + (d*x)/2)*(32*b^19 - 32*a^2*b^17 + 680*a^4*b^15 - 2560*a^6*b^13 + 2502*a^8*b^11 + 1216*a^10*b^9 - 3564*a^12*b^7 + 2160*a^14*b^5 - 432*a^16*b^3))/(a^6*b^12)))/(2*b^4) + (16*(80*b^13 - 756*a^12*b - 576*a^2*b^11 + 1056*a^4*b^9 + 214*a^6*b^7 - 2448*a^8*b^5 + 2430*a^10*b^3))/(a^6*b^8) - (16*tan(c/2 + (d*x)/2)*(1600*a^5*b^12 - 2592*a^17 - 5840*a^7*b^10 + 4504*a^9*b^8 + 8160*a^11*b^6 - 17064*a^13*b^4 + 11232*a^15*b^2))/(a^6*b^12)))*(a^2*6i - b^2*5i)*1i)/(b^4*d) - (2*b*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (atan((((3*a^2 + 2*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((8*(378*a^15 - 40*a^3*b^12 + 488*a^5*b^10 - 1158*a^7*b^8 + 541*a^9*b^6 + 1008*a^11*b^4 - 1215*a^^...

3.1261 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=251

$$\frac{2ax}{b^3}+\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{6(a^2-b^2)^{3/2}(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d}-\frac{\tanh^{-1}(\cos(c}{2a^2d}$$

```
[Out] 2*a*x/b^3+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a^2/b^3/d-6*(a^2-b^2)^(3/2)*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^
2-b^2)^(1/2))/a^4/b^3/d-1/2*arctanh(cos(d*x+c))/a^2/d+3*(a^2-b^2)*arctanh(c
os(d*x+c))/a^4/d+cos(d*x+c)/b^2/d+2*b*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d
*x+c)/a^2/d+(a^2-b^2)^2*cos(d*x+c)/a^3/b^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2976, 3855, 3852, 8, 3853, 2718, 2743, 12, 2739, 632, 210}

$$\frac{2b\cot(c+dx)}{a^3d}+\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{6(a^2+b^2)(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^3d}+\frac{3(a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(a^2-b^2)^2\cos(c+dx)}{a^3b^2d(a+b\sin(c+dx))}+\frac{2ax}{b^3}+\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*x)/b^3 + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^2*b^3*d) - (6*(a^2 - b^2)^(3/2)*(a^2 + b^2)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^3*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*
d) + (3*(a^2 - b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^2*d) +
 (2*b*Cot[c + d*x])/(a^3*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) + ((a^2
 - b^2)^2*Cos[c + d*x])/(a^3*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \left(\frac{2a}{b^3} - \frac{3(a^2-b^2)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \sin\right.\\
&= \frac{2ax}{b^3} + \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \sin(c+dx)\,dx}{b^2} - \frac{(2b)\int \csc^2(c+dx)\,dx}{a^3} - \\
&= \frac{2ax}{b^3} + \frac{3(a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^2d} - \frac{\cot(c+dx)\csc(c}{2a^2d}\\
&= \frac{2ax}{b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} + \frac{3(a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+}{b^2d}\\
&= \frac{2ax}{b^3} - \frac{6(a^2-b^2)^{3/2}(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+}{2a^2d}\\
&= \frac{2ax}{b^3} - \frac{6(a^2-b^2)^{3/2}(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} - \frac{\tanh^{-1}(\cos(c+}{2a^2d}\\
&= \frac{2ax}{b^3} + \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^3d} - \frac{6(a^2-b^2)^{3/2}(a^2+b^2)\tan}{a^4b^3}
\end{aligned}$$

Mathematica [A]

time = 6.15, size = 315, normalized size = 1.25

$$\frac{2a(c+dx)}{b^3d} - \frac{2(a^2-b^2)^{3/2}(2a^2+3b^2)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^3d} + \frac{\cos(c+dx)}{b^2d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)}{a^3d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{(5a^2-6b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(-5a^2+6b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{a^4\cos(c+dx)-2a^2b^2\cos(c+dx)+b^4\cos(c+dx)}{a^3b^2d(a+b\sin(c+dx))} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a*(c + d*x))/(b^3*d) - (2*(a^2 - b^2)^(3/2)*(2*a^2 + 3*b^2)*ArcTan[(Sec[
(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(
a^4*b^3*d) + Cos[c + d*x]/(b^2*d) + (b*Cot[(c + d*x)/2])/(a^3*d) - Csc[(c +
 d*x)/2]^2/(8*a^2*d) + ((5*a^2 - 6*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) +
((-5*a^2 + 6*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) + Sec[(c + d*x)/2]^2/(8*
```

```
a^2*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(a^
3*b^2*d*(a + b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

Maple [A]
time = 0.78, size = 290, normalized size = 1.16

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{\frac{4b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{1}{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-10a^2+12b^2\right)\ln\left(\tan\left(\frac{d}{}\right.\right.}{4a^4}}{}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{\frac{4b}{2+2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+4a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3}-\frac{1}{8a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-10a^2+12b^2\right)\ln\left(\tan\left(\frac{d}{}\right.\right.}{4a^4}}{}$
risch	$\frac{2ax}{b^3}+\frac{e^{i(dx+c)}}{2b^2d}+\frac{e^{-i(dx+c)}}{2b^2d}+\frac{i\left(6b^5e^{4i(dx+c)}-12b^5e^{2i(dx+c)}+6b^5-3ia\,b^4e^{5i(dx+c)}+4ia^5e^{3i(dx+c)}-8ia^3b^2e^{3i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^3*(1/2*a*tan(1/2*d*x+1/2*c)^2-4*b*tan(1/2*d*x+1/2*c))+4/b^3*(1/2
*b/(1+tan(1/2*d*x+1/2*c)^2)+a*arctan(tan(1/2*d*x+1/2*c)))-1/8/a^2/tan(1/2*d
*x+1/2*c)^2+1/4/a^4*(-10*a^2+12*b^2)*ln(tan(1/2*d*x+1/2*c))+b/a^3/tan(1/2*d
*x+1/2*c)-4/a^4/b^3*((-1/2*b^2*(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/2*a
^5*b+a^3*b^3-1/2*a*b^5)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1
/2*(2*a^6-a^4*b^2-4*a^2*b^4+3*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(237) = 474$.
time = 0.69, size = 1210, normalized size = 4.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^3 + 2*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 + a^3*b^2 - 3*a*
b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b + a^2*b^3 - 3*b
^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(4*a^5*b - 5*a^3*b^3 + 6*a*b^5)*cos(d*x + c) - (5*a^3
*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6
- (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) +
1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^2 + (5*a^2
*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x + c)^2 + 2*a^4*b^2*cos(d*x + c)
^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^5
*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^4*d*cos(d*x + c)^2 - a^4*b^4*d)*
sin(d*x + c)), 1/4*(8*a^6*d*x*cos(d*x + c)^2 - 8*a^6*d*x + 4*(2*a^5*b - 2*a
^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 4*(2*a^5 + a^3*b^2 - 3*a*b^4 - (2*a^5 +
a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 + (2*a^4*b + a^2*b^3 - 3*b^5 - (2*a^4*b +
 a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(4*a^5*b - 5*a^3*b^3
+ 6*a*b^5)*cos(d*x + c) - (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 - 6*a*b^5)*cos(
d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x + c)^2)*sin(d
*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^3*b^3 - 6*a*b^5 - (5*a^3*b^3 -
6*a*b^5)*cos(d*x + c)^2 + (5*a^2*b^4 - 6*b^6 - (5*a^2*b^4 - 6*b^6)*cos(d*x
+ c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*a^5*b*d*x*cos(d*x
 + c)^2 + 2*a^4*b^2*cos(d*x + c)^3 - 4*a^5*b*d*x - (2*a^4*b^2 + 3*a^2*b^4)*
cos(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^2 - a^5*b^3*d + (a^4*b^
4*d*cos(d*x + c)^2 - a^4*b^4*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**2,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep

Giac [A]
time = 0.51, size = 463, normalized size = 1.84

$$\frac{\frac{16\,(dx+c)a}{b^3}-\frac{4\,(5\,a^2-6\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{30\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}-\frac{16\,(2\,a^6-a^4b^2-4\,a^2b^4+3\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4b^3}+\frac{16\left(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^5-2\,a^3b^2+ab^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^4b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/8*(16*(d*x + c)*a/b^3 - 4*(5*a^2 - 6*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 + (30*a^2*tan(1/2*d*x + 1/2*c)^2 - 36*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^4*tan(1/2*d*x + 1/2*c)^2) - 16*(2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^3) + 16*(a^4*b*tan(1/2*d*x + 1/2*c)^3 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 - 2*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + a*b^4*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*tan(1/2*d*x + 1/2*c) + 2*a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^4*b^2))/d

Mupad [B]
time = 12.90, size = 2500, normalized size = 9.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + ((tan(c/2 + (d*x)/2)^2*(16*a^4 + 16*b^4 - 17*a^2*b^2))/b^2 - a^2/2 + (tan(c/2 + (d*x)/2)^4*(32*a^4 + 32*b^4 - 33*a^2*b^2))/(2*b^2) + 3*a*b*tan(c/2 + (d*x)/2) + (4*tan(c/2 + (d*x)/2)^5*(2*a^4 + 2*b^4 - 3*a^2*b^2))/(a*b) + (tan(c/2 + (d*x)/2)^3*(24*a^4 + 8*b^4 - 9*a^2*b^2))/(a*b))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 8*a^4*tan(c/2 + (d*x)/2)^4 + 4*a^4*tan(c/2 + (d*x)/2)^6 + 8*a^3*b*tan(c/2 + (d*x)/2)^3 + 8*a^3*b*tan(c/2 + (d*x)/2)^5)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) - (log(tan(c/2 + (d*x)/2))*(5*a^2 - 6*b^2))/(2*a^4*d) + (4*a*atan((2560*a)/((4800*a^2)/b - 2560*a*tan(c/2 + (d*x)/2) - 12160*b + (4480*b^3)/a^2 + (11520*b^5)/a^4 - (12096*b^7)/a^6 + (3456*b^9)/a^8 + (6144*b^2*tan(c/2 + (d*x)/2))/a - (5120*a^3*tan(c/2 + (d*x)/2))/b^2 - (2304*b^4*tan(c/2 + (d*x)/2))/a^3 + (3840*a^5*tan(c/2 + (d*x)/2))/b^4) - (12160*tan(c/2 + (d*x)/2))/((4480*b^2)/a^2 + (4800*a^2)/b^2 + (11520*b^4)/a^4 - (12096*b^6)/a^6 + (3456*b^8)/a^8 - (2560*a*tan(c/2 + (

d*x)/2))/b + (6144*b*tan(c/2 + (d*x)/2))/a - (2304*b^3*tan(c/2 + (d*x)/2))/
a^3 - (5120*a^3*tan(c/2 + (d*x)/2))/b^3 + (3840*a^5*tan(c/2 + (d*x)/2))/b^5
 - 12160) - 6144/(6144*tan(c/2 + (d*x)/2) - (12160*a)/b + (4480*b)/a + (115
20*b^3)/a^3 + (4800*a^3)/b^3 - (12096*b^5)/a^5 + (3456*b^7)/a^7 - (2304*b^2
*tan(c/2 + (d*x)/2))/a^2 - (2560*a^2*tan(c/2 + (d*x)/2))/b^2 - (5120*a^4*ta
n(c/2 + (d*x)/2))/b^4 + (3840*a^6*tan(c/2 + (d*x)/2))/b^6) + (5120*a^3)/(48
00*a^2*b - 12160*b^3 + (4480*b^5)/a^2 + (11520*b^7)/a^4 - (12096*b^9)/a^6 +
 (3456*b^11)/a^8 - 5120*a^3*tan(c/2 + (d*x)/2) - 2560*a*b^2*tan(c/2 + (d*x)
/2) + (6144*b^4*tan(c/2 + (d*x)/2))/a + (3840*a^5*tan(c/2 + (d*x)/2))/b^2 -
 (2304*b^6*tan(c/2 + (d*x)/2))/a^3) + (2304*b^2)/(4480*a*b + (11520*b^3)/a
 - (12160*a^3)/b - (12096*b^5)/a^3 + (4800*a^5)/b^3 + (3456*b^7)/a^5 + 6144*
a^2*tan(c/2 + (d*x)/2) - 2304*b^2*tan(c/2 + (d*x)/2) - (2560*a^4*tan(c/2 +
(d*x)/2))/b^2 - (5120*a^6*tan(c/2 + (d*x)/2))/b^4 + (3840*a^8*tan(c/2 + (d*
x)/2))/b^6) - (3840*a^5)/(4800*a^2*b^3 - 12160*b^5 + (4480*b^7)/a^2 + (1152
0*b^9)/a^4 - (12096*b^11)/a^6 + (3456*b^13)/a^8 + 3840*a^5*tan(c/2 + (d*x)/
2) - 2560*a*b^4*tan(c/2 + (d*x)/2) - 5120*a^3*b^2*tan(c/2 + (d*x)/2) + (614
4*b^6*tan(c/2 + (d*x)/2))/a - (2304*b^8*tan(c/2 + (d*x)/2))/a^3) + (4800*a^
2*tan(c/2 + (d*x)/2))/(4800*a^2 - 12160*b^2 + (4480*b^4)/a^2 + (11520*b^6)/
a^4 - (12096*b^8)/a^6 + (3456*b^10)/a^8 + (6144*b^3*tan(c/2 + (d*x)/2))/a -
 (5120*a^3*tan(c/2 + (d*x)/2))/b - (2304*b^5*tan(c/2 + (d*x)/2))/a^3 + (384
0*a^5*tan(c/2 + (d*x)/2))/b^3 - 2560*a*b*tan(c/2 + (d*x)/2)) + (11520*b^3*t
an(c/2 + (d*x)/2))/(4480*a^2*b + 11520*b^3 - (12160*a^4)/b - (12096*b^5)/a^
2 + (4800*a^6)/b^3 + (3456*b^7)/a^4 + 6144*a^3*tan(c/2 + (d*x)/2) - 2304*a*
b^2*tan(c/2 + (d*x)/2) - (2560*a^5*tan(c/2 + (d*x)/2))/b^2 - (5120*a^7*tan(
c/2 + (d*x)/2))/b^4 + (3840*a^9*tan(c/2 + (d*x)/2))/b^6) - (12096*b^5*tan(c
/2 + (d*x)/2))/(4480*a^4*b - 12096*b^5 + 11520*a^2*b^3 - (12160*a^6)/b + (3
456*b^7)/a^2 + (4800*a^8)/b^3 + 6144*a^5*tan(c/2 + (d*x)/2) - 2304*a^3*b^2*
tan(c/2 + (d*x)/2) - (2560*a^7*tan(c/2 + (d*x)/2))/b^2 - (5120*a^9*tan(c/2
+ (d*x)/2))/b^4 + (3840*a^11*tan(c/2 + (d*x)/2))/b^6) + (3456*b^7*tan(c/2 +
 (d*x)/2))/(4480*a^6*b + 3456*b^7 - 12096*a^2*b^5 + 11520*a^4*b^3 - (12160*
a^8)/b + (4800*a^10)/b^3 + 6144*a^7*tan(c/2 + (d*x)/2) - 2304*a^5*b^2*tan(c
/2 + (d*x)/2) - (2560*a^9*tan(c/2 + (d*x)/2))/b^2 - (5120*a^11*tan(c/2 + (d
*x)/2))/b^4 + (3840*a^13*tan(c/2 + (d*x)/2))/b^6) + (4480*b*tan(c/2 + (d*x)
/2))/(4480*b + 6144*a*tan(c/2 + (d*x)/2) - (12160*a^2)/b + (11520*b^3)/a^2
+ (4800*a^4)/b^3 - (12096*b^5)/a^4 + (3456*b^7)/a^6 - (2304*b^2*tan(c/2 + (
d*x)/2))/a - (2560*a^3*tan(c/2 + (d*x)/2))/b^2 - (5120*a^5*tan(c/2 + (d*x)/
2))/b^4 + (3840*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d) - (atan((((2*a^2 + 3
b^2)(-(a + b)^3*(a - b)^3)^(1/2)*((16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8
 + 232*a^8*b^6 - 224*a^10*b^4))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18
 - 189*a^2*b^16 + 180*a^4*b^14 + 70*a^6*b^12 + 194*a^8*b^10 - 653*a^10*b^8
+ 292*a^12*b^6 + 120*a^14*b^4 - 64*a^16*b^2))/(a^9*b^8) + ((2*a^2 + 3*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2)*((16*(72*a^2*b^14 - 174*a^4*b^12 + 95*a^6*b^10
 + 42*a^8*b^8 - 35*a^10*b^6 + 32*a^12*b^4 - 24*a^14*b^2))/(a^8*b^5) + (16*t
an(c/2 + (d*x)/2)*(18*a^4*b^16 + 60*a^6*b^14 - 220*a^8*b^12 + 132*a^10*b^10
 + 202*a^12*b^8 - 200*a^14*b^6 + 16*a^16*b^4))/(a^9*b^8) + ((2*a^2 + 3*b^2)

```
*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(48*a^6*b^12 - 76*a^8*b^10 + 15*a^10*b^8
 + 14*a^12*b^6))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(96*a^6*b^16 - 182*a^8*
b^14 + 73*a^10*b^12 + 30*a^12*b^10 - 16*a^14*b^8))/(a^9*b^8) + (((16*(8*a^1
0*b^10 - 6*a^12*b^8))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(32*a^10*b^14 - 34
*a^12*b^12 + 4*a^14*b^10))/(a^9*b^8))*(2*a^2 + 3*b^2)*(-(a + b)^3*(a - b)^3
)^(1/2))/(a^4*b^3)))/(a^4*b^3)))/(a^4*b^3))*1i)/(a^4*b^3) + ((2*a^2 + 3*b^2
)*(-(a + b)^3*(a - b)^3)^(1/2)*((16*(56*a^14 + 36*a^4*b^10 - 96*a^6*b^8 + 2
32*a^8*b^6 - 224*a^10*b^4))/(a^8*b^5) + (16*tan(c/2 + (d*x)/2)*(54*b^18 - 1
89*a^2*b^16 + 180*a^4*b^14 + 70*a^6*b^12 + 194*...
```

3.1262 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=287

$$-\frac{x}{b^2}-\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4(a^6-3a^2b^4+2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}\,d}+\frac{b\tanh^{-1}(\cos(c+\cdots}{a^3d}$$

```
[Out] -x/b^2-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a
^3/b^2/d+b*arctanh(cos(d*x+c))/a^3/d-2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/
a^5/d-cot(d*x+c)/a^2/d+3*(a^2-b^2)*cot(d*x+c)/a^4/d-1/3*cot(d*x+c)^3/a^2/d+
b*cot(d*x+c)*csc(d*x+c)/a^3/d-(a^2-b^2)^2*cos(d*x+c)/a^4/b/d/(a+b*sin(d*x+c
))+4*(a^6-3*a^2*b^4+2*b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/a^5/b^2/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2976, 3855, 3852, 8, 3853, 2743, 12, 2739, 632, 210}

$$\frac{b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\cot(c+dx)\csc(c+dx)}{a^3d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{2b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{3(a^2-b^2)\cot(c+dx)}{a^4d}-\frac{(a^2-b^2)^2\cos(c+dx)}{a^4bd(a+b\sin(c+dx))}-\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^2d}+\frac{4(a^6-3a^2b^4+2b^6)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5b^2d\sqrt{a^2-b^2}}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^2*d) + (4*(a^6 - 3*a^2*b^4 + 2*b^6)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^2*Sqrt[a^2 - b^2]*d) + (b*ArcTanh[Cos[c + d
*x]])/(a^3*d) - (2*b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(a^5*d) - Cot[c
 + d*x]/(a^2*d) + (3*(a^2 - b^2)*Cot[c + d*x])/(a^4*d) - Cot[c + d*x]^3/(3*
a^2*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - ((a^2 - b^2)^2*Cos[c + d*x
])/(a^4*b*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{b^2} + \frac{2(3a^2b-2b^3)\csc(c+dx)}{a^5} - \frac{3(a^2-b^2)\csc^2(c+dx)}{a^4} - \frac{2b\csc\cdots}{\cdots}\right.$$

$$= -\frac{x}{b^2} + \frac{\int \csc^4(c+dx)\,dx}{a^2} - \frac{(2b)\int \csc^3(c+dx)\,dx}{a^3} + \frac{(2b(3a^2-2b^2))\int \csc\cdots}{a^5}$$

$$= -\frac{x}{b^2} - \frac{2b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{b\cot(c+dx)\csc(c+dx)}{a^3d} - \cdots$$

$$= -\frac{x}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} - \frac{\cot\cdots}{\cdots}$$

$$= -\frac{x}{b^2} + \frac{4(a^6-3a^2b^4+2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}\,d} + \frac{b\tanh^{-1}(\cos(c+\cdots}{a^3d}$$

$$= -\frac{x}{b^2} + \frac{4(a^6-3a^2b^4+2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2\sqrt{a^2-b^2}\,d} + \frac{b\tanh^{-1}(\cos(c+\cdots}{a^3d}$$

$$= -\frac{x}{b^2} - \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^2d} + \frac{4(a^6-3a^2b^4+2b^6)\tan^{-}\cdots}{a^5b^2\sqrt{a^2-\cdots}}$$

Mathematica [A]
time = 6.25, size = 428, normalized size = 1.49

$$-\frac{c+dx}{b^2d}+\frac{2(a^2-b^2)^{3/2}(a^2+4b^2)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^2d}+\frac{\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d}+\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d}+\frac{(-5a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d}+\frac{(5a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d}-\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d}+\frac{-a^4\cos(c+dx)+2a^2b^2\cos(c+dx)-b^4\cos(c+dx)}{a^4bd(a+b\sin(c+dx))}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((c + d*x)/(b^2*d)) + (2*(a^2 - b^2)^(3/2)*(a^2 + 4*b^2)*ArcTan[(Sec[(c +
d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b
^2*d) + ((7*a^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2]
)/(6*a^4*d) + (b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c +
 d*x)/2]^2)/(24*a^2*d) + ((-5*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d)
 + ((5*a^2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^
2)/(4*a^3*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c +
```

d*x)/2]))/(6*a^4*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*C
os[c + d*x])/(a^4*b*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/(24*a^2*d)

Maple [A]
time = 0.79, size = 327, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-}{8a^4}}{}$
default	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^4}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{24a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-}{8a^4}}{}$
risch	$-\frac{x}{b^2}+\frac{2i\left(3ia^4b\,e^{6i(dx+c)}+3a^5e^{7i(dx+c)}-9a^5e^{5i(dx+c)}+9a^5e^{3i(dx+c)}-3a^5e^{i(dx+c)}+36a^3b^2e^{5i(dx+c)}-30a\,b^4e^{5i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/8/a^4*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-2*a*b*tan(1/2*d*x+1/2*c)^2-9*a^2
*tan(1/2*d*x+1/2*c)+12*b^2*tan(1/2*d*x+1/2*c))-2/b^2*arctan(tan(1/2*d*x+1/2
*c))-1/24/a^2/tan(1/2*d*x+1/2*c)^3-1/8*(-9*a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*
c)+1/4/a^3*b/tan(1/2*d*x+1/2*c)^2+1/a^5*b*(5*a^2-4*b^2)*ln(tan(1/2*d*x+1/2*
c))+2/a^5/b^2*((-b^2*(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-b*a*(a^4-2*a^2*
b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^6+2*a^4*b^2-
7*a^2*b^4+4*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a
^2-b^2)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(275) = 550$.

time = 0.65, size = 1437, normalized size = 5.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/6*(6*a^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d
*x + 2*(7*a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 3*(a^4*b + 3*a^2*b^3 - 4*b^
5 + (a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b
^5)*cos(d*x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^
4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b^4 - 4*b^
6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)
^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b^4 - 4*b^
6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*b^3 - 4*a
*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x
 + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b - 13*a^3*
b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cos(d*x +
c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x + c)^2 +
a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c)), -1/6*(6*a
^5*b*d*x*cos(d*x + c)^4 - 12*a^5*b*d*x*cos(d*x + c)^2 + 6*a^5*b*d*x + 2*(7*
a^4*b^2 - 6*a^2*b^4)*cos(d*x + c)^3 + 6*(a^4*b + 3*a^2*b^3 - 4*b^5 + (a^4*b
 + 3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 3*a^2*b^3 - 4*b^5)*cos(d*
x + c)^2 + (a^5 + 3*a^3*b^2 - 4*a*b^4 - (a^5 + 3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a
^2 - b^2)*cos(d*x + c))) - 12*(a^4*b^2 - a^2*b^4)*cos(d*x + c) + 3*(5*a^2*b
^4 - 4*b^6 + (5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos
(d*x + c)^2 + (5*a^3*b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*
sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(5*a^2*b^4 - 4*b^6 + (5*a^2*b
^4 - 4*b^6)*cos(d*x + c)^4 - 2*(5*a^2*b^4 - 4*b^6)*cos(d*x + c)^2 + (5*a^3*
b^3 - 4*a*b^5 - (5*a^3*b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(3*a^6*d*x*cos(d*x + c)^2 - 3*a^6*d*x + (3*a^5*b
- 13*a^3*b^3 + 12*a*b^5)*cos(d*x + c)^3 - 3*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*c
os(d*x + c))*sin(d*x + c))/(a^5*b^3*d*cos(d*x + c)^4 - 2*a^5*b^3*d*cos(d*x
+ c)^2 + a^5*b^3*d - (a^6*b^2*d*cos(d*x + c)^2 - a^6*b^2*d)*sin(d*x + c))]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.49, size = 399, normalized size = 1.39

$$-\frac{\frac{24\,(dx+c)}{b^2}-\frac{24\,(5\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{48\,(a^6+2\,a^4b^2-7\,a^2b^4+4\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5b^2}+\frac{48\,(a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5-2\,a^3b^2+ab^4)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^5b}+\frac{220\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-176\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(24*(d*x + c)/b^2 - 24*(5*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^5 - (a^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 27*
a^4*tan(1/2*d*x + 1/2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 - 48*(a^6 +
 2*a^4*b^2 - 7*a^2*b^4 + 4*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5*
b^2) + 48*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^
5*tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^
2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5*b) + (220*a^2*b*tan(1/2*d*x + 1/2*c)^
3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b
^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/
2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 12.78, size = 2500, normalized size = 8.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((24*a^2 + 32*b^2)/(6
4*a^4) + 3/(4*a^2) - (2*b^2)/a^4))/d - (tan(c/2 + (d*x)/2)^2*(8*a*b^2 - (26
*a^3)/3) + a^3/3 - (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/2)^4*(
7*a^4 + 16*b^4 - 20*a^2*b^2))/a + (4*tan(c/2 + (d*x)/2)^3*(4*a^4 + 10*b^4 -
 13*a^2*b^2))/b)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a^5*tan(c/2 + (d*x)/2)^
5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + (2*atan((((((((((32*(4*a^14*b^7 - 3*a
^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 +
 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 - (32*(32*a^9*b^10 - 64*a^11*b^8 + 30*a^1
3*b^6 + a^15*b^4))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*
a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4))*1i)/b^2 - (
32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7 + 5*a
^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 4
```

0*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b ^2))/(a^12*b^4))*1i)/b^2 - (32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9* b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(4*a^ 16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4))/b^2 - ((((((((32*(4* a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 + (32*(32*a^9*b^10 - 64*a^1 1*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(148*a ^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71* a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(1 6*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15* b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 + (32*(6*a^15 + 16*a^5*b^10 - 56*a^7 *b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4))/b^2)/(((((((((32*(4*a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)* (16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1i)/b^2 - (32*(32*a^ 9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10*a^14*b^7 + 4*a^16* b^5))/(a^12*b^4))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 22 0*a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + (32*tan(c/ 2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13 *b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 - (32*(6*a^15 + 16*a^ 5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^ 12*b^4))*1i)/b^2 + ((((((((32*(4*a^14*b^7 - 3*a^16*b^5))/(a^12*b^2) + (32* tan(c/2 + (d*x)/2)*(16*a^13*b^10 - 17*a^15*b^8 + 2*a^17*b^6))/(a^12*b^4))*1 i)/b^2 + (32*(32*a^9*b^10 - 64*a^11*b^8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^ 2) - (32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*b^9 + 10 *a^14*b^7 + 4*a^16*b^5))/(a^12*b^4))*1i)/b^2 - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^1 2*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 + 5*a^9*b^10 + 4 4*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b^4))*1i)/b^2 + (32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 + 97*a^9*b^6 - 84*a^11*b^4 + 20*a^13 *b^2))/(a^12*b^2) - (32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a^2*b^ 15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12*b^5 + 10*a^14*b^3))/(a^12*b^4))*1i)/b^2 - (64*(30*a^12*b - 64*b^13 + 304*a^2*b^ 11 - 604*a^4*b^9 + 613*a^6*b^7 - 280*a^8*b^5 + a^10*b^3))/(a^12*b^2) - (64* tan(c/2 + (d*x)/2)*(8*a^15 - 16*a^7*b^8 + 60*a^9*b^6 - 64*a^11*b^4 + 12*a^1 3*b^2))/(a^12*b^4))))/(b^2*d) - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log (tan(c/2 + (d*x)/2))*(5*a^2 - 4*b^2))/(a^5*d) - (atan((((a^2 + 4*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(4*a^16*b - 64*b^17 + 304*a ^2*b^15 - 540*a^4*b^13 + 405*a^6*b^11 - 124*a^8*b^9 + 78*a^10*b^7 - 72*a^12

```
*b^5 + 10*a^14*b^3))/(a^12*b^4) - (32*(6*a^15 + 16*a^5*b^10 - 56*a^7*b^8 +
97*a^9*b^6 - 84*a^11*b^4 + 20*a^13*b^2))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a +
 b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(16*a^5*b^14 - 40*a^7*b^12 +
 5*a^9*b^10 + 44*a^11*b^8 - 6*a^13*b^6 - 20*a^15*b^4 + 2*a^17*b^2))/(a^12*b
^4) - (32*(3*a^16*b - 64*a^4*b^13 + 208*a^6*b^11 - 220*a^8*b^9 + 71*a^10*b^
7 + 5*a^12*b^5 - 4*a^14*b^3))/(a^12*b^2) + ((a^2 + 4*b^2)*(-(a + b)^3*(a -
b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(148*a^10*b^11 - 64*a^8*b^13 - 97*a^12*
b^9 + 10*a^14*b^7 + 4*a^16*b^5))/(a^12*b^4) - (32*(32*a^9*b^10 - 64*a^11*b^
8 + 30*a^13*b^6 + a^15*b^4))/(a^12*b^2) + (((32*(4*a^14*b^7 - 3*a^16*b^5))/
(a^12*b^2) + (32*tan(c/2 + (d*x)/2)*(16*a^13*b^...
```

3.1263 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=303

$$-\frac{10b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}+\frac{(3a^4-20a^2b^2+15}{3a^5b}$$

```
[Out] -10*b*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/
d-5/8*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d+1/3*(3*a^4-20*a^2*
b^2+15*b^4)*cot(d*x+c)/a^5/b/d+5/8*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/
d-cot(d*x+c)/b/d/(a+b*sin(d*x+c))-1/3*(6*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a
^3/d/(a+b*sin(d*x+c))+5/12*b*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+b*sin(d*x+c))
-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.77, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{5b\cot(c+dx)\csc^2(c+dx)}{12a^2d(a+b\sin(c+dx))}-\frac{10b(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}-\frac{(6a^2-5b^2)\cot(c+dx)\csc(c+dx)}{3a^3d(a+b\sin(c+dx))}-\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}+\frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd}-\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^6*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d)
+ ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(3*a^5*b*d) + (5*(5*a^2 - 4*
b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) - Cot[c + d*x]/(b*d*(a + b*Sin[c
+ d*x])) - ((6*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^3*d*(a + b*Sin[
c + d*x])) + (5*b*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^2*d*(a + b*Sin[c + d*x
])) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} + \frac{5b\cot(c+dx)\csc^2(c+dx)}{12a^2d(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^3(c}{4ad(a+b\sin(c}$$

$$= -\frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(6a^2-5b^2)\cot(c+dx)\csc(c+dx)}{3a^3d(a+b\sin(c+dx))} + \frac{5b\cot(c}{12a^2d(a}$$

$$= \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} - \frac{\cot(c+dx)}{bd(a+b\sin(c+dx))} - \frac{(6a^2-5b}{3a^3}$$

$$= \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}$$

$$= \frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{3a^5bd} + \frac{5(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}$$

$$= -\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(3a^4-20a^2b^2+15b^4)\cot}{3a^5bd}$$

$$= -\frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(3a^4-20a^2b^2+15b^4)\cot}{3a^5bd}$$

$$= -\frac{10b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5(3a^4-12a^2b^2+8b^4)\tanh^{-1}}{8a^6d}$$

Mathematica [A]
time = 6.26, size = 487, normalized size = 1.61

$$-\frac{10b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{\left(-7a^2b\cos\left(\frac{1}{2}(c+dx)\right)+6b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{3a^5d} + \frac{3(3a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{12a^3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} - \frac{5(3a^4-12a^2b^2+8b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} + \frac{5(3a^4-12a^2b^2+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^6d} - \frac{3(3a^2-4b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^4d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(7a^2b\sin\left(\frac{1}{2}(c+dx)\right)-6b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5d} + \frac{a^4\cos(c+dx)-2a^2b^2\cos(c+dx)+b^4\cos(c+dx)}{a^5d(a+b\sin(c+dx))} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-10*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*S
in[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*d) + ((-7*a^2*b*Cos[(c + d*x)/2] +
 6*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (3*(3*a^2 - 4*b^2)*C
sc[(c + d*x)/2]^2)/(32*a^4*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12
*a^3*d) - Csc[(c + d*x)/2]^4/(64*a^2*d) - (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*L
og[Cos[(c + d*x)/2]])/(8*a^6*d) + (5*(3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(
c + d*x)/2]])/(8*a^6*d) - (3*(3*a^2 - 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^4*d)
 + Sec[(c + d*x)/2]^4/(64*a^2*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Sin[(c + d*x)
/2] - 6*b^3*Sin[(c + d*x)/2]))/(3*a^5*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Co
s[c + d*x] + b^4*Cos[c + d*x])/(a^5*d*(a + b*Sin[c + d*x])) - (b*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(12*a^3*d)
```

Maple [**A**]
time = 0.75, size = 370, normalized size = 1.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^5*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-4/3*b*tan(1/2*d*x+1/2*c)^3*a^2-
4*a^3*tan(1/2*d*x+1/2*c)^2+6*a*b^2*tan(1/2*d*x+1/2*c)^2+36*a^2*b*tan(1/2*d*
x+1/2*c)-32*b^3*tan(1/2*d*x+1/2*c))-1/64/a^2/tan(1/2*d*x+1/2*c)^4-1/32*(-8*
a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)^2+1/16/a^6*(30*a^4-120*a^2*b^2+80*b^4)*l
n(tan(1/2*d*x+1/2*c))+1/12/a^3*b/tan(1/2*d*x+1/2*c)^3-1/4*b*(9*a^2-8*b^2)/a
^5/tan(1/2*d*x+1/2*c)-4/a^6*((-1/2*b*(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)
-1/2*a^5+a^3*b^2-1/2*a*b^4)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+
a)+5/2*b*(a^4-2*a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(286) = 572.
time = 0.60, size = 1576, normalized size = 5.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/48*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*
a^3*b^2 + 48*a*b^4)*cos(d*x + c)^3 - 120*((a^3*b - a*b^3)*cos(d*x + c)^4 +
a^3*b - a*b^3 - 2*(a^3*b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x
 + c)^4 + a^2*b^2 - b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(3*a^5 - 12*
a^3*b^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5
 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*
cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(
d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 +
(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a
*b^4)*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^
3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2
)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*
cos(d*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d
*cos(d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4
- 2*a^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c)), 1/48*(16*(3*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 10*(15*a^5 - 68*a^3*b^2 + 48*a*b^4)*cos
(d*x + c)^3 + 240*((a^3*b - a*b^3)*cos(d*x + c)^4 + a^3*b - a*b^3 - 2*(a^3*
b - a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4
 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(3*a^5 - 12*a^3*b
^2 + 8*a*b^4)*cos(d*x + c) - 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^5 - 12
*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*
cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin(d*x +
 c))*log(1/2*cos(d*x + c) + 1/2) + 15*(3*a^5 - 12*a^3*b^2 + 8*a*b^4 + (3*a^
5 - 12*a^3*b^2 + 8*a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 - 12*a^3*b^2 + 8*a*b^4)
*cos(d*x + c)^2 + (3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8
*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*sin
(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 10*((17*a^4*b - 12*a^2*b^3)*cos(d
*x + c)^3 - 3*(5*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(
d*x + c)^4 - 2*a^7*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a
^6*b*d*cos(d*x + c)^2 + a^6*b*d)*sin(d*x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.52, size = 475, normalized size = 1.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/192*(120*(3*a^4 - 12*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6
- 1920*(a^4*b - 2*a^2*b^3 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6
) + 384*(a^4*b*tan(1/2*d*x + 1/2*c) - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) + b^5*
tan(1/2*d*x + 1/2*c) + a^5 - 2*a^3*b^2 + a*b^4)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^6) + (3*a^6*tan(1/2*d*x + 1/2*c)^4 - 16*a
^5*b*tan(1/2*d*x + 1/2*c)^3 - 48*a^6*tan(1/2*d*x + 1/2*c)^2 + 72*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 432*a^5*b*tan(1/2*d*x + 1/2*c) - 384*a^3*b^3*tan(1/2
*d*x + 1/2*c))/a^8 - (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 3000*a^2*b^2*tan(1/2
*d*x + 1/2*c)^4 + 2000*b^4*tan(1/2*d*x + 1/2*c)^4 + 432*a^3*b*tan(1/2*d*x +
 1/2*c)^3 - 384*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^
2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 16*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a
^4)/(a^6*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [B]
time = 12.03, size = 1117, normalized size = 3.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a^2*d) + (tan(c/2 + (d*x)/2)^4*(36*a^4 + 96*b^4 -
142*a^2*b^2) - a^4/4 + tan(c/2 + (d*x)/2)^2*((15*a^4)/4 - (10*a^2*b^2)/3) +
 tan(c/2 + (d*x)/2)^3*(20*a*b^3 - (80*a^3*b)/3) - (4*tan(c/2 + (d*x)/2)^5*(
a^4*b - 8*b^5 + 8*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*
tan(c/2 + (d*x)/2)^4 + 16*a^6*tan(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*
x)/2)^5)) - (tan(c/2 + (d*x)/2)^2*((32*a^2 + 64*b^2)/(512*a^4) + 3/(16*a^2)
 - b^2/(2*a^4)))/d + (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) -
b/(4*a^3) + (4*b*((32*a^2 + 64*b^2)/(256*a^4) + 3/(8*a^2) - b^2/a^4))/a))/d
 - (b*tan(c/2 + (d*x)/2)^3)/(12*a^3*d) + (log(tan(c/2 + (d*x)/2))*(15*a^4 +
 40*b^4 - 60*a^2*b^2))/(8*a^6*d) - (b*atan(((b*(-(a + b)^3*(a - b)^3)^(1/2)
```

```
*((tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))/
(4*a^9) - (55*a^10*b + 80*a^6*b^5 - 140*a^8*b^3)/(4*a^10) + (5*b*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^
3)^(1/2))/a^6)*5i)/a^6 - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((55*a^10*b + 80*a
^6*b^5 - 140*a^8*b^3)/(4*a^10) - (tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6
 + 320*a^6*b^4 - 170*a^8*b^2))/(4*a^9) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6)*5i)/
a^6)/((75*a^8*b + 200*b^9 - 700*a^2*b^7 + 875*a^4*b^5 - 450*a^6*b^3)/(2*a^1
0) + (tan(c/2 + (d*x)/2)*(200*b^8 - 650*a^2*b^6 + 700*a^4*b^4 - 250*a^6*b^2
))/(2*a^9) + (5*b*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(15*a^1
0 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))/(4*a^9) - (55*a^10*b + 80*a^6
*b^5 - 140*a^8*b^3)/(4*a^10) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12
 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6 + (5*b*(-
(a + b)^3*(a - b)^3)^(1/2)*((55*a^10*b + 80*a^6*b^5 - 140*a^8*b^3)/(4*a^10)
 - (tan(c/2 + (d*x)/2)*(15*a^10 - 160*a^4*b^6 + 320*a^6*b^4 - 170*a^8*b^2))
/(4*a^9) + (5*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*
a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6))*(-(a + b)^3*(a - b)^3)^(1/2
)*10i)/(a^6*d)
```

3.1264 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$-\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d}-\frac{(38a^4-1}{}$$

```
[Out] -2*(a^2-6*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^7/d+1/4*b*(15*a^4-40*a^2*b^2+24*b^4)*arctanh(cos(d*x+c))/a^7/d-1/15*
(38*a^4-135*a^2*b^2+90*b^4)*cot(d*x+c)/a^6/d+1/4*(4*a^4-17*a^2*b^2+12*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/b/d-1/30*(15*a^4-82*a^2*b^2+60*b^4)*cot(d*x+c)*cs
c(d*x+c)^2/a^4/b^2/d-1/2*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))+1/6*a*c
ot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))+1/6*(2*a^4-12*a^2*b^2+9*b^4)*
cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))+3/10*b*cot(d*x+c)*csc(d*
x+c)^3/a^2/d/(a+b*sin(d*x+c))-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin(d*x+
c))
```

Rubi [A]

time = 0.97, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}-\frac{2(a^2-6b^2)(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d}+\frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d}-\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d}+\frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd}+\frac{(2a^4-12a^2b^2+9b^4)\cot(c+dx)\csc^2(c+dx)}{6a^3b^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+}{10a^2d(a+b\sin(c+}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{(2a^4-12a^2b^2+9b^4)\,c}{6a^3b^2d(a+}$$

$$= -\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} +$$

$$= \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{(15a^4-82a^2b^2+60b^4)\cot(c+}{30a^4b^2d}$$

$$= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(}{4a^5bd}$$

$$= -\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d} + \frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(}{4a^5bd}$$

$$= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+}{15a^6d}$$

$$= \frac{b(15a^4-40a^2b^2+24b^4)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{(38a^4-135a^2b^2+90b^4)\cot(c+}{15a^6d}$$

$$= -\frac{2(a^2-6b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b(15a^4-40a^2b^2+24b^4)\,\text{ta}}{4a^7d}$$

Mathematica [A]
time = 1.09, size = 361, normalized size = 0.85

$$-\frac{1920(a^2-6b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 240b(15a^4-40a^2b^2+24b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 240b(15a^4-40a^2b^2+24b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2a\cot(c+dx)\csc^5(c+dx)\left(196a^5-735a^3b^2+540ab^4-12\left(16a^5-85a^3b^2+60ab^4\right)\cos(2(c+dx))+\left(92a^5-285a^3b^2+180ab^4\right)\cos(4(c+dx))+1162a^4b\sin(c+dx)-3060a^2b^3\sin(c+dx)+1800b^5\sin(c+dx)-562a^4b\sin(3(c+dx))+1470a^2b^3\sin(3(c+dx))-900b^5\sin(3(c+dx))+76a^4b\sin(5(c+dx))-270a^2b^3\sin(5(c+dx))+180b^5\sin(5(c+dx))\right)}{b+a\csc(c+dx)}}{960a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/960*(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)
/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Co
t[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 -
 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^
4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] +
 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c
 + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b
^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(a^7
*d)
```

Maple [A]
time = 0.85, size = 465, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^6*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-b*tan(1/2*d*x+1/2*c)^4*a^3-7/3*
a^4*tan(1/2*d*x+1/2*c)^3+4*a^2*b^2*tan(1/2*d*x+1/2*c)^3+16*a^3*b*tan(1/2*d*
x+1/2*c)^2-16*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-108*a^2*
b^2*tan(1/2*d*x+1/2*c)+80*b^4*tan(1/2*d*x+1/2*c))-1/160/a^2/tan(1/2*d*x+1/2
*c)^5-1/96*(-7*a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-108*a^2*b^
2+80*b^4)/a^6/tan(1/2*d*x+1/2*c)+1/32/a^3*b/tan(1/2*d*x+1/2*c)^4-1/2*b/a^5*
(a^2-b^2)/tan(1/2*d*x+1/2*c)^2-1/4/a^7*b*(15*a^4-40*a^2*b^2+24*b^4)*ln(tan(
1/2*d*x+1/2*c))-2/a^7*((b^2*(a^4-2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+b*a*(a^4
-2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^6-8*a
^4*b^2+13*a^2*b^4-6*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(401) = 802$.
time = 0.68, size = 2011, normalized size = 4.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
```

```
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [**A**]
time = 0.56, size = 596, normalized size = 1.41

$$-\frac{\frac{120\,(15\,a^4b-40\,a^2b^3+24\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^7}+\frac{960\,(a^6-8\,a^4b^2+13\,a^2b^4-6\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^7}+\frac{960\,(a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^5b-2\,a^3b^3+ab^5)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)a^7}-\cdots}{480\,d}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
```

```
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 12.16, size = 1424, normalized size = 3.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))^2),x)
[Out] tan(c/2 + (d*x)/2)^5/(160*a^2*d) + (tan(c/2 + (d*x)/2)*(1/(4*a^2) + b^2/(2*
a^4) - (4*b*((b*(64*a^2 + 128*b^2))/(256*a^5) - b/(8*a^3) + (4*b*((64*a^2 +
 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/a + ((64*a^2 + 128*b^
2)*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/(32*a^2)))/d
 - (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 5/(96*a^2) - b^2/
(6*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((31*a^4*b)/3 - 8*a^2*b^3) + tan(c/2 +
(d*x)/2)^4*(48*a*b^4 + (59*a^5)/3 - 72*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(124
*a^4*b + 224*b^5 - 360*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((32*a^5)/15
 - 2*a^3*b^2) - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x)/2)^6*(1
1*a^6 + 32*b^6 - 24*a^2*b^4 - 22*a^4*b^2))/a)/(d*(32*a^7*tan(c/2 + (d*x)/2)
^5 + 32*a^7*tan(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6)) + (tan(c
/2 + (d*x)/2)^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3) + (2*b*((64*
a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/d - (log(tan(c/2
 + (d*x)/2))*(15*a^4*b + 24*b^5 - 40*a^2*b^3))/(4*a^7*d) - (b*tan(c/2 + (d*
x)/2)^4)/(32*a^3*d) - (atan((((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((
4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)
/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^
2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*
(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a
- b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) +
(tan(c/2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(
2*a^11) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11)
)*(a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7)/((15*a^10*b - 1
44*b^11 + 552*a^2*b^9 - 802*a^4*b^7 + 539*a^6*b^5 - 160*a^8*b^3)/a^12 + (ta
n(c/2 + (d*x)/2)*(8*a^10 - 144*b^10 + 516*a^2*b^8 - 682*a^4*b^6 + 400*a^6*b
^4 - 98*a^8*b^2))/a^11 - ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((4*a^
13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/2 + (d*x)/2)*
(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11) + ((2*a^2*b
- (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2 - 6*b^2)*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^7))/a^7 + ((a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)
^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) + (tan(c/
2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/(2*a^11)
 - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^11))*(a^2
- 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7))/a^7))*(a^2 - 6*b^2)*(-(a + b)^
3*(a - b)^3)^(1/2)*2i)/(a^7*d)
```

3.1265 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=480

$$\frac{2b(2a^2-7b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d}$$

```
[Out] 2*b*(2*a^2-7*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/a^8/d+1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*arctanh(cos(d*x+c
))/a^8/d+1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*cot(d*x+c)/a^7/d-1/16*(27*a^4-
86*a^2*b^2+56*b^4)*cot(d*x+c)*csc(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b
^4)*cot(d*x+c)*csc(d*x+c)^2/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cot(d*x
+c)*csc(d*x+c)^3/a^4/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/b/d/(a+b*sin(d*x+c))
+1/6*a*cot(d*x+c)*csc(d*x+c)^3/b^2/d/(a+b*sin(d*x+c))+1/10*(5*a^4-20*a^2*b^
2+14*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^3/b^2/d/(a+b*sin(d*x+c))+7/30*b*cot(d*x
+c)*csc(d*x+c)^4/a^2/d/(a+b*sin(d*x+c))-1/6*cot(d*x+c)*csc(d*x+c)^5/a/d/(a+
b*sin(d*x+c))
```

Rubi [A]

time = 1.24, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{7b\cot(c+dx)\csc^4(c+dx)}{30a^2d(a+b\sin(c+dx))}+\frac{2b(2a^2-7b^2)\left(a^2-b^2\right)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(16a^4-61a^2b^2+42b^4\right)\cot(c+dx)\csc^3(c+dx)}{24a^4b^2d}+\frac{b\left(61a^4-170a^2b^2+105b^4\right)\cot(c+dx)}{15a^7d}-\frac{\left(27a^4-86a^2b^2+56b^4\right)\cot(c+dx)\csc(c+dx)}{16a^6d}+\frac{\left(15a^4-52a^2b^2+35b^4\right)\cot(c+dx)\csc^2(c+dx)}{15a^5bd}+\frac{\left(5a^4-20a^2b^2+14b^4\right)\cot(c+dx)\csc^3(c+dx)}{10a^3b^2d(a+b\sin(c+dx))}+\frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)\tanh^{-1}(\cos(c+dx))}{16a^8d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^8*d) + ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*ArcTa
nh[Cos[c + d*x]])/(16*a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Cot[c +
d*x])/(15*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cot[c + d*x]*Csc[c + d*x
])/(16*a^6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(15*a^5*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cot[c + d*x]*Csc[c + d*x]^
3)/(24*a^4*b^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*b*d*(a + b*Sin[c + d*x
])) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((5*
a^4 - 20*a^2*b^2 + 14*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^3*b^2*d*(a +
b*Sin[c + d*x])) + (7*b*Cot[c + d*x]*Csc[c + d*x]^4)/(30*a^2*d*(a + b*Sin[c
 + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{7b\cot(c+dx)\,\mathrm{cs}}{30a^2d(a+b\sin}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^3(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{(5a^4-20a^2b^2+}{10a^3b^2}$$

$$= -\frac{(16a^4-61a^2b^2+42b^4)\cot(c+dx)\csc^3(c+dx)}{24a^4b^2d} - \frac{\cot(c+dx)\csc^2(c+d}{3bd(a+b\sin(c+dx)}$$

$$= \frac{(15a^4-52a^2b^2+35b^4)\cot(c+dx)\csc^2(c+dx)}{15a^5bd} - \frac{(16a^4-61a^2b^2+42b^4)\,\mathrm{c}}{24a^4}$$

$$= -\frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx)\csc(c+dx)}{16a^6d} + \frac{(15a^4-52a^2b^2+35b^4)}{15a}$$

$$= \frac{b(61a^4-170a^2b^2+105b^4)\cot(c+dx)}{15a^7d} - \frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx}{16a^6d}$$

$$= \frac{b(61a^4-170a^2b^2+105b^4)\cot(c+dx)}{15a^7d} - \frac{(27a^4-86a^2b^2+56b^4)\cot(c+dx}{16a^6d}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b(61a^4-170a^2b^2}{1}$$

$$= \frac{(5a^6-90a^4b^2+200a^2b^4-112b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d} + \frac{b(61a^4-170a^2b^2}{1}$$

$$= \frac{2b(2a^2-7b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{(5a^6-90a^4b^2+200a^2}{}$$

Mathematica [A]
time = 1.12, size = 447, normalized size = 0.93

$$\frac{15360b(2a^2-7b^2)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+480(5a^6-90a^4b^2+200a^2b^4-112b^6)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+480(-5a^6+90a^4b^2-200a^2b^4+112b^6)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2a\cot(c+dx)\csc^6(c+dx)(590a^6-6956a^4b^2+15280a^2b^4-8400b^6-8(35a^6-1289a^4b^2+2830a^2b^4-1575b^6)\cos(2(c+dx))\cdots}{\cdots}}{7680a^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (15360*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]] + 480*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*Log[Cos
[(c + d*x)/2]] + 480*(-5*a^6 + 90*a^4*b^2 - 200*a^2*b^4 + 112*b^6)*Log[Sin[
(c + d*x)/2]] - (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(590*a^6 - 6956*a^4*b^2 +
15280*a^2*b^4 - 8400*b^6 - 8*(35*a^6 - 1289*a^4*b^2 + 2830*a^2*b^4 - 1575*b
```

```
^6)*Cos[2*(c + d*x)] + (330*a^6 - 3844*a^4*b^2 + 8720*a^2*b^4 - 5040*b^6)*C
os[4*(c + d*x)] + 488*a^4*b^2*Cos[6*(c + d*x)] - 1360*a^2*b^4*Cos[6*(c + d*
x)] + 840*b^6*Cos[6*(c + d*x)] - 3942*a^5*b*Sin[c + d*x] + 12620*a^3*b^3*Si
n[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 1967*a^5*b*Sin[3*(c + d*x)] - 6590*a
^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 571*a^5*b*Sin[5*(c
+ d*x)] + 1430*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[5*(c + d*x)]))/(b +
 a*Csc[c + d*x]))/(7680*a^8*d)
```

Maple [A]
time = 1.07, size = 572, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{a^5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{5}-\frac{3a^5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3a^3b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{28a^4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{32a^2b^3\left(\tan^3\left(\right.\right.}{3}}{}$
default	$\frac{\frac{a^5\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6}-\frac{4b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{5}-\frac{3a^5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3a^3b^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{28a^4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{32a^2b^3\left(\tan^3\left(\right.\right.}{3}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/64/a^7*(1/6*a^5*tan(1/2*d*x+1/2*c)^6-4/5*b*tan(1/2*d*x+1/2*c)^5*a^4-
3/2*a^5*tan(1/2*d*x+1/2*c)^4+3*a^3*b^2*tan(1/2*d*x+1/2*c)^4+28/3*a^4*b*tan(
1/2*d*x+1/2*c)^3-32/3*a^2*b^3*tan(1/2*d*x+1/2*c)^3+15/2*a^5*tan(1/2*d*x+1/2
*c)^2-48*a^3*b^2*tan(1/2*d*x+1/2*c)^2+40*a*b^4*tan(1/2*d*x+1/2*c)^2-88*a^4*
b*tan(1/2*d*x+1/2*c)+288*a^2*b^3*tan(1/2*d*x+1/2*c)-192*b^5*tan(1/2*d*x+1/2
*c))-1/384/a^2/tan(1/2*d*x+1/2*c)^6-1/256*(-6*a^2+12*b^2)/a^4/tan(1/2*d*x+1
/2*c)^4-1/128/a^6*(15*a^4-96*a^2*b^2+80*b^4)/tan(1/2*d*x+1/2*c)^2+1/64/a^8*
(-20*a^6+360*a^4*b^2-800*a^2*b^4+448*b^6)*ln(tan(1/2*d*x+1/2*c))+1/80/a^3*b
/tan(1/2*d*x+1/2*c)^5-1/48/a^5*b*(7*a^2-8*b^2)/tan(1/2*d*x+1/2*c)^3+1/8*b*(
11*a^4-36*a^2*b^2+24*b^4)/a^7/tan(1/2*d*x+1/2*c)+4*b/a^8*((1/2*b^2*(a^4-2*a
^2*b^2+b^4)*tan(1/2*d*x+1/2*c)+1/2*a^5*b-a^3*b^3+1/2*a*b^5)/(a*tan(1/2*d*x+
1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(2*a^6-11*a^4*b^2+16*a^2*b^4-7*b^6)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. $2(455) = 910$.
time = 0.91, size = 2588, normalized size = 5.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/480*(32*(61*a^5*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a
^7 - 3386*a^5*b^2 + 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 -
 94*a^5*b^2 + 218*a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 + 240*((2*a^5*b - 9*a
^3*b^3 + 7*a*b^5)*cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5
*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5
)*cos(d*x + c)^2 + ((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*
b^2 + 9*a^2*b^4 - 7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4
+ 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
 200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
```

```
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c)), 1/480*(32*(61*a^5
*b^2 - 170*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^7 + 2*(165*a^7 - 3386*a^5*b^2
+ 8440*a^3*b^4 - 5040*a*b^6)*cos(d*x + c)^5 - 80*(5*a^7 - 94*a^5*b^2 + 218*
a^3*b^4 - 126*a*b^6)*cos(d*x + c)^3 - 480*((2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*
cos(d*x + c)^6 - 2*a^5*b + 9*a^3*b^3 - 7*a*b^5 - 3*(2*a^5*b - 9*a^3*b^3 + 7
*a*b^5)*cos(d*x + c)^4 + 3*(2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 +
 ((2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^6 - 2*a^4*b^2 + 9*a^2*b^4 -
7*b^6 - 3*(2*a^4*b^2 - 9*a^2*b^4 + 7*b^6)*cos(d*x + c)^4 + 3*(2*a^4*b^2 - 9
*a^2*b^4 + 7*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(5*a^7 - 90*a^5*b^2
+ 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c) - 15*(5*a^7 - 90*a^5*b^2 + 200*a^3*
b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^4 - 3*
(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5*a^6*b -
90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(5*a^7 - 90*a^5*b^2 +
 200*a^3*b^4 - 112*a*b^6 - (5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*c
os(d*x + c)^6 + 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x +
c)^4 - 3*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*cos(d*x + c)^2 + (5
*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7 - (5*a^6*b - 90*a^4*b^3 + 200*a
^2*b^5 - 112*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 -
112*b^7)*cos(d*x + c)^4 - 3*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*
cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((571*a^6*b
- 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c)^5 - 40*(23*a^6*b - 68*a^4*b^3 +
42*a^2*b^5)*cos(d*x + c)^3 + 15*(27*a^6*b - 86*a^4*b^3 + 56*a^2*b^5)*cos(d*
x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^6 - 3*a^9*d*cos(d*x + c)^4 + 3*a^
9*d*cos(d*x + c)^2 - a^9*d + (a^8*b*d*cos(d*x + c)^6 - 3*a^8*b*d*cos(d*x +
c)^4 + 3*a^8*b*d*cos(d*x + c)^2 - a^8*b*d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 0.57, size = 736, normalized size = 1.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/1920*(120*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*log(abs(tan(1/2*d
*x + 1/2*c)))/a^8 - 3840*(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sq
rt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8) - 3840*(a^4*b^3*tan(1/2*d*x + 1/2*c)
- 2*a^2*b^5*tan(1/2*d*x + 1/2*c) + b^7*tan(1/2*d*x + 1/2*c) + a^5*b^2 - 2*a
^3*b^4 + a*b^6)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*
a^8) - (5*a^10*tan(1/2*d*x + 1/2*c)^6 - 24*a^9*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^10*tan(1/2*d*x + 1/2*c)^4 + 90*a^8*b^2*tan(1/2*d*x + 1/2*c)^4 + 280*a^9
*b*tan(1/2*d*x + 1/2*c)^3 - 320*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^10*t
an(1/2*d*x + 1/2*c)^2 - 1440*a^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 1200*a^6*b^4*
tan(1/2*d*x + 1/2*c)^2 - 2640*a^9*b*tan(1/2*d*x + 1/2*c) + 8640*a^7*b^3*tan
(1/2*d*x + 1/2*c) - 5760*a^5*b^5*tan(1/2*d*x + 1/2*c))/a^12 - (1470*a^6*tan
(1/2*d*x + 1/2*c)^6 - 26460*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 58800*a^2*b^4*
tan(1/2*d*x + 1/2*c)^6 - 32928*b^6*tan(1/2*d*x + 1/2*c)^6 + 2640*a^5*b*tan(
1/2*d*x + 1/2*c)^5 - 8640*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 5760*a*b^5*tan(1
/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 1440*a^4*b^2*tan(1/2*d
*x + 1/2*c)^4 - 1200*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 280*a^5*b*tan(1/2*d*x
 + 1/2*c)^3 + 320*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x + 1/2
*c)^2 - 90*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^5*b*tan(1/2*d*x + 1/2*c) -
 5*a^6)/(a^8*tan(1/2*d*x + 1/2*c)^6))/d
```

Mupad [B]
time = 12.58, size = 1810, normalized size = 3.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + b*sin(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^6/(384*a^2*d) - (tan(c/2 + (d*x)/2)^4*((128*a^2 + 256*b^
2)/(16384*a^4) + 1/(64*a^2) - b^2/(16*a^4)))/d + (tan(c/2 + (d*x)/2)^2*(3/(
128*a^2) + b^2/(8*a^4) - (2*b*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a
^3) + (4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))
/a + ((128*a^2 + 256*b^2)*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^
2/(4*a^4)))/(128*a^2)))/d + (tan(c/2 + (d*x)/2)*(b/(16*a^3) - ((128*a^2 + 2
56*b^2)*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a^3) + (4*b*((128*a^2 +
 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))/(64*a^2) + (4*b*((128
*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a - (4*b*(3/(64*a^2
```

) + b^2/(4*a^4) - (4*b*((b*(128*a^2 + 256*b^2))/(1024*a^5) - b/(16*a^3) + (4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^4)))/a))/a + ((128*a^2 + 256*b^2)*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b^2/(4*a^ 4)))/(64*a^2)))/a))/d + (tan(c/2 + (d*x)/2)^3*((b*(128*a^2 + 256*b^2))/(307 2*a^5) - b/(48*a^3) + (4*b*((128*a^2 + 256*b^2)/(4096*a^4) + 1/(16*a^2) - b ^2/(4*a^4)))/(3*a)))/d - (tan(c/2 + (d*x)/2)^3*((83*a^5*b)/15 - (14*a^3*b^3)/3) + a^6/6 + tan(c/2 + (d*x)/2)^4*(6*a^6 + (56*a^2*b^4)/3 - (79*a^4*b^2)/ 3) - tan(c/2 + (d*x)/2)^5*(112*a*b^5 + (191*a^5*b)/3 - (544*a^3*b^3)/3) - t an(c/2 + (d*x)/2)^2*((4*a^6)/3 - (7*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^6*((15 *a^6)/2 - 512*b^6 + 872*a^2*b^4 - 352*a^4*b^2) - (8*tan(c/2 + (d*x)/2)^7*(1 1*a^6*b + 16*b^7 - 8*a^2*b^5 - 20*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/15)/(d*(64*a^8*tan(c/2 + (d*x)/2)^6 + 64*a^8*tan(c/2 + (d*x)/2)^8 + 128*a ^7*b*tan(c/2 + (d*x)/2)^7)) - (b*tan(c/2 + (d*x)/2)^5)/(80*a^3*d) - (log(ta n(c/2 + (d*x)/2))*(5*a^6 - 112*b^6 + 200*a^2*b^4 - 90*a^4*b^2))/(16*a^8*d) + (b*atan(((b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(8*a^13) - (37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 266*a^12*b^3)/(8*a^14) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a ^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^8)*1i)/a^8 - (b*(2*a^2 - 7*b^2) *(-(a + b)^3*(a - b)^3)^(1/2)*((37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 26 6*a^12*b^3)/(8*a^14) - (tan(c/2 + (d*x)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8 *b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(8*a^13) + (b*(2*a^2*b - (tan(c/2 + (d *x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^8)*1i)/a^8)/((10*a^12*b + 784*b^13 - 3192*a^2*b^11 + 5062*a^ 4*b^9 - 3899*a^6*b^7 + 1470*a^8*b^5 - 235*a^10*b^3)/(4*a^14) + (tan(c/2 + (d*x)/2)*(784*b^12 - 2996*a^2*b^10 + 4362*a^4*b^8 - 2980*a^6*b^6 + 938*a^8*b ^4 - 108*a^10*b^2))/(4*a^13) + (b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1 /2)*((tan(c/2 + (d*x)/2)*(5*a^14 + 448*a^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^ 4 - 164*a^12*b^2))/(8*a^13) - (37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 266 *a^12*b^3)/(8*a^14) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14* b^2))/(8*a^13))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^8))/a^8 + (b*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((37*a^14*b - 224*a^8*b^7 + 456*a^10*b^5 - 266*a^12*b^3)/(8*a^14) - (tan(c/2 + (d*x)/2)*(5*a^14 + 448*a ^6*b^8 - 1024*a^8*b^6 + 732*a^10*b^4 - 164*a^12*b^2))/(8*a^13) + (b*(2*a^2* b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(2*a^2 - 7*b^2)* (-(a + b)^3*(a - b)^3)^(1/2))/a^8))/a^8))*(2*a^2 - 7*b^2)*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^8*d)

3.1266 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=536

$$-\frac{(448a^6-600a^4b^2+180a^2b^4-5b^6)\,x}{16b^9}+\frac{a\sqrt{a^2-b^2}\,(56a^4-47a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{a(840a^4$$

```
[Out] -1/16*(448*a^6-600*a^4*b^2+180*a^2*b^4-5*b^6)*x/b^9-1/30*a*(840*a^4-985*a^2
*b^2+213*b^4)*cos(d*x+c)/b^8/d+1/16*(224*a^4-244*a^2*b^2+43*b^4)*cos(d*x+c)
*sin(d*x+c)/b^7/d-1/30*(280*a^4-291*a^2*b^2+45*b^4)*cos(d*x+c)*sin(d*x+c)^2
/a/b^6/d+1/24*(168*a^4-169*a^2*b^2+24*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^5/
d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d/(a+b*sin(d*x+c))^2-1/10*b*cos(d*x+c)*sin(
d*x+c)^5/a^2/d/(a+b*sin(d*x+c))^2-1/60*(56*a^4-60*a^2*b^2+9*b^4)*cos(d*x+c)
*sin(d*x+c)^5/a^2/b^3/d/(a+b*sin(d*x+c))^2-4/15*a*cos(d*x+c)*sin(d*x+c)^6/b
^2/d/(a+b*sin(d*x+c))^2+1/6*cos(d*x+c)*sin(d*x+c)^7/b/d/(a+b*sin(d*x+c))^2-
1/20*(112*a^4-110*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^4/d/(a+b*si
n(d*x+c))+a*(56*a^4-47*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-
b^2)^(1/2))*(a^2-b^2)^(1/2)/b^9/d
```

Rubi [A]
time = 1.37, antiderivative size = 536, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2975, 3126, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{b\sin^5(c+dx)\cos(c+dx)}{10a^2d(a+b\sin(c+dx))^2}+\frac{a\sqrt{a^2-b^2}\,(56a^4-47a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{(112a^4-110a^2b^2+15b^4)\sin^4(c+dx)\cos(c+dx)}{20a^2b^4d(a+b\sin(c+dx))}-\frac{a(840a^4-985a^2b^2+213b^4)\cos(c+dx)}{30b^8d}+\frac{(224a^4-244a^2b^2+43b^4)\sin(c+dx)\cos(c+dx)}{16b^7d}-\frac{(280a^4-291a^2b^2+45b^4)\sin^2(c+dx)\cos(c+dx)}{30ab^6d}+\frac{(168a^4-169a^2b^2+24b^4)\sin^3(c+dx)\cos(c+dx)}{24a^2b^5d}-\frac{(56a^4-60a^2b^2+9b^4)\sin^5(c+dx)\cos(c+dx)}{60a^2b^3d(a+b\sin(c+dx))^2}-\frac{x(448a^6-600a^4b^2+180a^2b^4-5b^6)}{16b^9}-\frac{4a\sin^6(c+dx)\cos(c+dx)}{15b^2d(a+b\sin(c+dx))^2}+\frac{\sin^7(c+dx)\cos(c+dx)}{6bd(a+b\sin(c+dx))^2}+\frac{\sin^4(c+dx)\cos(c+dx)}{4ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*((448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*x)/b^9 + (a*Sqrt[a^2 -
 b^2]*(56*a^4 - 47*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(b^9*d) - (a*(840*a^4 - 985*a^2*b^2 + 213*b^4)*Cos[c + d*x])/(30
*b^8*d) + ((224*a^4 - 244*a^2*b^2 + 43*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*
b^7*d) - ((280*a^4 - 291*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30
*a*b^6*d) + ((168*a^4 - 169*a^2*b^2 + 24*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*a^2*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d*(a + b*Sin[c + d*x])^
2) - (b*Cos[c + d*x]*Sin[c + d*x]^5)/(10*a^2*d*(a + b*Sin[c + d*x])^2) - ((
56*a^4 - 60*a^2*b^2 + 9*b^4)*Cos[c + d*x]*Sin[c + d*x]^5)/(60*a^2*b^3*d*(a
+ b*Sin[c + d*x])^2) - (4*a*Cos[c + d*x]*Sin[c + d*x]^6)/(15*b^2*d*(a + b*S
in[c + d*x])^2) + (Cos[c + d*x]*Sin[c + d*x]^7)/(6*b*d*(a + b*Sin[c + d*x])
^2) - ((112*a^4 - 110*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(20*a^
2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{4a\cos(c+dx)\,\mathrm{si}}{15b^2d(a+b\sin(}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{(56a^4-60a^2b^2 -}{60a^2b^3}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^5(c+dx)}{10a^2d(a+b\sin(c+dx))^2} - \frac{(56a^4-60a^2b^2 -}{60a^2b^3}$$

$$= \frac{(168a^4-169a^2b^2+24b^4)\cos(c+dx)\sin^3(c+dx)}{24a^2b^5d} + \frac{\cos(c+dx)\sin^4(c+}{4ad(a+b\sin(c+d}$$

$$= -\frac{(280a^4-291a^2b^2+45b^4)\cos(c+dx)\sin^2(c+dx)}{30ab^6d} + \frac{(168a^4-169a^2b^2}{}$$

$$= \frac{(224a^4-244a^2b^2+43b^4)\cos(c+dx)\sin(c+dx)}{16b^7d} - \frac{(280a^4-291a^2b^2+}{}$$

$$= -\frac{a(840a^4-985a^2b^2+213b^4)\cos(c+dx)}{30b^8d} + \frac{(224a^4-244a^2b^2+43b^4)\,\mathrm{co}}{16b^7d}$$

$$= -\frac{(448a^6-600a^4b^2+180a^2b^4-5b^6)\,x}{16b^9} - \frac{a(840a^4-985a^2b^2+213b^4)\cos}{30b^8d}$$

$$= -\frac{(448a^6-600a^4b^2+180a^2b^4-5b^6)\,x}{16b^9} - \frac{a(840a^4-985a^2b^2+213b^4)\cos}{30b^8d}$$

$$= -\frac{(448a^6-600a^4b^2+180a^2b^4-5b^6)\,x}{16b^9} - \frac{a(840a^4-985a^2b^2+213b^4)\cos}{30b^8d}$$

$$= -\frac{(448a^6-600a^4b^2+180a^2b^4-5b^6)\,x}{16b^9} + \frac{a\sqrt{a^2-b^2}\,(56a^4-47a^2b^2+6b}{b^9d}$$

Mathematica [A]
time = 10.79, size = 631, normalized size = 1.18

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3840*a*(a^2 - b^2)^(5/2)*(56*a^4 - 47*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]] + ((a^2 - b^2)^2*(-107520*a^8*c + 90240*a^6*b
```

```
^2*c + 28800*a^4*b^4*c - 20400*a^2*b^6*c + 600*b^8*c - 107520*a^8*d*x + 902
40*a^6*b^2*d*x + 28800*a^4*b^4*d*x - 20400*a^2*b^6*d*x + 600*b^8*d*x - 80*a
*b*(1344*a^6 - 1464*a^4*b^2 + 202*a^2*b^4 + 33*b^6)*Cos[c + d*x] + 120*b^2*
(448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*(c + d*x)*Cos[2*(c + d*x)] +
8960*a^5*b^3*Cos[3*(c + d*x)] - 10880*a^3*b^5*Cos[3*(c + d*x)] + 2436*a*b^7
*Cos[3*(c + d*x)] - 224*a^3*b^5*Cos[5*(c + d*x)] + 188*a*b^7*Cos[5*(c + d*x
)] + 16*a*b^7*Cos[7*(c + d*x)] - 215040*a^7*b*c*Sin[c + d*x] + 288000*a^5*b
^3*c*Sin[c + d*x] - 86400*a^3*b^5*c*Sin[c + d*x] + 2400*a*b^7*c*Sin[c + d*x
] - 215040*a^7*b*d*x*Sin[c + d*x] + 288000*a^5*b^3*d*x*Sin[c + d*x] - 86400
*a^3*b^5*d*x*Sin[c + d*x] + 2400*a*b^7*d*x*Sin[c + d*x] - 80640*a^6*b^2*Sin
[2*(c + d*x)] + 99040*a^4*b^4*Sin[2*(c + d*x)] - 24600*a^2*b^6*Sin[2*(c + d
*x)] + 405*b^8*Sin[2*(c + d*x)] - 1120*a^4*b^4*Sin[4*(c + d*x)] + 1164*a^2*
b^6*Sin[4*(c + d*x)] - 140*b^8*Sin[4*(c + d*x)] + 56*a^2*b^6*Sin[6*(c + d*x
)] - 35*b^8*Sin[6*(c + d*x)] - 5*b^8*Sin[8*(c + d*x)]))/(a + b*Sin[c + d*x]
)^2)/(3840*(a - b)^2*b^9*(a + b)^2*d)
```

Maple [A]
time = 0.62, size = 696, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^9*(((15/2*a^4*b^2-27/4*a^2*b^4+11/16*b^6)*tan(1/2*d*x+1/2*c)^11+(
21*a^5*b-30*a^3*b^3+9*a*b^5)*tan(1/2*d*x+1/2*c)^10+(45/2*a^4*b^2-57/4*a^2*b
^4-5/48*b^6)*tan(1/2*d*x+1/2*c)^9+(105*a^5*b-130*a^3*b^3+27*a*b^5)*tan(1/2*
d*x+1/2*c)^8+(15*a^4*b^2-15/2*a^2*b^4+15/8*b^6)*tan(1/2*d*x+1/2*c)^7+(210*a
^5*b-700/3*a^3*b^3+46*a*b^5)*tan(1/2*d*x+1/2*c)^6+(-15*a^4*b^2+15/2*a^2*b^4
-15/8*b^6)*tan(1/2*d*x+1/2*c)^5+(210*a^5*b-220*a^3*b^3+42*a*b^5)*tan(1/2*d*
x+1/2*c)^4+(-45/2*a^4*b^2+57/4*a^2*b^4+5/48*b^6)*tan(1/2*d*x+1/2*c)^3+(105*
a^5*b-110*a^3*b^3+93/5*a*b^5)*tan(1/2*d*x+1/2*c)^2+(-15/2*a^4*b^2+27/4*a^2*
b^4-11/16*b^6)*tan(1/2*d*x+1/2*c)+21*a^5*b-70/3*a^3*b^3+23/5*a*b^5)/(1+tan(
1/2*d*x+1/2*c)^2)^6+1/16*(448*a^6-600*a^4*b^2+180*a^2*b^4-5*b^6)*arctan(tan
(1/2*d*x+1/2*c)))+2*a/b^9*((-1/2*a*b^2*(13*a^4-17*a^2*b^2+4*b^4)*tan(1/2*d*
x+1/2*c)^3-1/2*b*(14*a^6+9*a^4*b^2-33*a^2*b^4+10*b^6)*tan(1/2*d*x+1/2*c)^2-
1/2*b^2*a*(43*a^4-59*a^2*b^2+16*b^4)*tan(1/2*d*x+1/2*c)-7*a^6*b+19/2*a^4*b^
3-5/2*a^2*b^5)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(56*
a^6-103*a^4*b^2+53*a^2*b^4-6*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.50, size = 1128, normalized size = 2.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5
 + 15*(448*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2
+ 10*(224*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 -
152*a^6*b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x + 60*(56*a^7 + 9*a^5*b
^2 - 41*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c
)^2 + 2*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*
log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 30*(224*a^7*b - 188*a^5*b^3 - 3
2*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*(56*a^2*b^6
 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*cos(d*x + c
)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x + 15*(672*a^
6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^1
1*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*d), -1/240*
(64*a*b^7*cos(d*x + c)^7 - 4*(56*a^3*b^5 - 19*a*b^7)*cos(d*x + c)^5 + 15*(4
48*a^6*b^2 - 600*a^4*b^4 + 180*a^2*b^6 - 5*b^8)*d*x*cos(d*x + c)^2 + 10*(22
4*a^5*b^3 - 244*a^3*b^5 + 43*a*b^7)*cos(d*x + c)^3 - 15*(448*a^8 - 152*a^6*
b^2 - 420*a^4*b^4 + 175*a^2*b^6 - 5*b^8)*d*x - 120*(56*a^7 + 9*a^5*b^2 - 41
*a^3*b^4 + 6*a*b^6 - (56*a^5*b^2 - 47*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2 + 2
*(56*a^6*b - 47*a^4*b^3 + 6*a^2*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(224*a^7*b - 188*
a^5*b^3 - 32*a^3*b^5 + 19*a*b^7)*cos(d*x + c) - (40*b^8*cos(d*x + c)^7 - 2*
(56*a^2*b^6 - 5*b^8)*cos(d*x + c)^5 + 5*(112*a^4*b^4 - 94*a^2*b^6 + 5*b^8)*
cos(d*x + c)^3 + 30*(448*a^7*b - 600*a^5*b^3 + 180*a^3*b^5 - 5*a*b^7)*d*x +
 15*(672*a^6*b^2 - 844*a^4*b^4 + 223*a^2*b^6 - 5*b^8)*cos(d*x + c))*sin(d*x
 + c))/(b^11*d*cos(d*x + c)^2 - 2*a*b^10*d*sin(d*x + c) - (a^2*b^9 + b^11)*
d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 968, normalized size = 1.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(15*(448*a^6 - 600*a^4*b^2 + 180*a^2*b^4 - 5*b^6)*(d*x + c)/b^9 - 24
0*(56*a^7 - 103*a^5*b^2 + 53*a^3*b^4 - 6*a*b^6)*(pi*floor(1/2*(d*x + c)/pi
+ 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt
(a^2 - b^2)*b^9) + 240*(13*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 17*a^4*b^3*tan(1/
2*d*x + 1/2*c)^3 + 4*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 14*a^7*tan(1/2*d*x +
1/2*c)^2 + 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 33*a^3*b^4*tan(1/2*d*x + 1/2*
c)^2 + 10*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 43*a^6*b*tan(1/2*d*x + 1/2*c) - 59
*a^4*b^3*tan(1/2*d*x + 1/2*c) + 16*a^2*b^5*tan(1/2*d*x + 1/2*c) + 14*a^7 -
19*a^5*b^2 + 5*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*
c) + a)^2*b^8) + 2*(1800*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 1620*a^2*b^3*tan(1
/2*d*x + 1/2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 5040*a^5*tan(1/2*d*x
 + 1/2*c)^10 - 7200*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 2160*a*b^4*tan(1/2*d*
x + 1/2*c)^10 + 5400*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 3420*a^2*b^3*tan(1/2*d*
x + 1/2*c)^9 - 25*b^5*tan(1/2*d*x + 1/2*c)^9 + 25200*a^5*tan(1/2*d*x + 1/2*
c)^8 - 31200*a^3*b^2*tan(1/2*d*x + 1/2*c)^8 + 6480*a*b^4*tan(1/2*d*x + 1/2*
c)^8 + 3600*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1800*a^2*b^3*tan(1/2*d*x + 1/2*c
)^7 + 450*b^5*tan(1/2*d*x + 1/2*c)^7 + 50400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5
6000*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 + 11040*a*b^4*tan(1/2*d*x + 1/2*c)^6 -
3600*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1800*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 4
50*b^5*tan(1/2*d*x + 1/2*c)^5 + 50400*a^5*tan(1/2*d*x + 1/2*c)^4 - 52800*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 + 10080*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 5400*a^
4*b*tan(1/2*d*x + 1/2*c)^3 + 3420*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*t
an(1/2*d*x + 1/2*c)^3 + 25200*a^5*tan(1/2*d*x + 1/2*c)^2 - 26400*a^3*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 4464*a*b^4*tan(1/2*d*x + 1/2*c)^2 - 1800*a^4*b*tan(1
/2*d*x + 1/2*c) + 1620*a^2*b^3*tan(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x +
 1/2*c) + 5040*a^5 - 5600*a^3*b^2 + 1104*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 +
1)^6*b^8))/d
```

Mupad [B]
time = 48.18, size = 2500, normalized size = 4.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] - ((840*a^7 + 213*a^3*b^4 - 985*a^5*b^2)/(15*b^8) + (tan(c/2 + (d*x)/2)^14*
(31*a*b^6 + 112*a^7 - 138*a^3*b^4 + 18*a^5*b^2))/(2*b^8) + (tan(c/2 + (d*x)
/2)^12*(410*a*b^6 + 1176*a^7 - 1533*a^3*b^4 + 189*a^5*b^2))/(3*b^8) + (tan(
c/2 + (d*x)/2)^10*(2281*a*b^6 + 7056*a^7 - 8766*a^3*b^4 + 686*a^5*b^2))/(6*
b^8) + (tan(c/2 + (d*x)/2)^2*(1239*a*b^6 + 11760*a^7 - 2402*a^3*b^4 - 9310*
a^5*b^2))/(30*b^8) + (tan(c/2 + (d*x)/2)^4*(3062*a*b^6 + 17640*a^7 - 10011*
a^3*b^4 - 8365*a^5*b^2))/(15*b^8) + (tan(c/2 + (d*x)/2)^6*(14155*a*b^6 + 58
800*a^7 - 50514*a^3*b^4 - 12950*a^5*b^2))/(30*b^8) + (tan(c/2 + (d*x)/2)*(2
3520*a^6 + 6171*a^2*b^4 - 27860*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)/2)^1
5*(224*a^6 + 43*a^2*b^4 - 244*a^4*b^2))/(8*b^7) + (tan(c/2 + (d*x)/2)^13*(8
736*a^6 + 132*b^6 + 1453*a^2*b^4 - 9516*a^4*b^2))/(24*b^7) + (tan(c/2 + (d*
x)/2)^11*(38304*a^6 - 20*b^6 + 8033*a^2*b^4 - 43068*a^4*b^2))/(24*b^7) + (t
an(c/2 + (d*x)/2)^9*(84000*a^6 + 360*b^6 + 20341*a^2*b^4 - 97324*a^4*b^2))/
(24*b^7) + (tan(c/2 + (d*x)/2)^7*(104160*a^6 - 360*b^6 + 27371*a^2*b^4 - 12
3316*a^4*b^2))/(24*b^7) + (tan(c/2 + (d*x)/2)^3*(144480*a^6 - 660*b^6 + 404
47*a^2*b^4 - 173060*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)/2)^5*(372960*a^6
 + 100*b^6 + 102971*a^2*b^4 - 446580*a^4*b^2))/(120*b^7) + (tan(c/2 + (d*x)
/2)^8*(7*a^2 + 8*b^2)*(213*a*b^4 + 840*a^5 - 985*a^3*b^2))/(3*b^8))/(d*(tan
(c/2 + (d*x)/2)^2*(8*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^14*(8*a^2 + 4*b^2) +
 tan(c/2 + (d*x)/2)^4*(28*a^2 + 24*b^2) + tan(c/2 + (d*x)/2)^12*(28*a^2 + 2
4*b^2) + tan(c/2 + (d*x)/2)^6*(56*a^2 + 60*b^2) + tan(c/2 + (d*x)/2)^10*(56
*a^2 + 60*b^2) + tan(c/2 + (d*x)/2)^8*(70*a^2 + 80*b^2) + a^2*tan(c/2 + (d*
x)/2)^16 + a^2 + 28*a*b*tan(c/2 + (d*x)/2)^3 + 84*a*b*tan(c/2 + (d*x)/2)^5
+ 140*a*b*tan(c/2 + (d*x)/2)^7 + 140*a*b*tan(c/2 + (d*x)/2)^9 + 84*a*b*tan(
c/2 + (d*x)/2)^11 + 28*a*b*tan(c/2 + (d*x)/2)^13 + 4*a*b*tan(c/2 + (d*x)/2)
^15 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan((((((25*a^2*b^20)/8 - 225*a^4*b^18
 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 250
88*a^14*b^8)/b^23 - (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^
18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/
(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (ta
n(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a
^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*
b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 7
38240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 -
 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*60
0i)*1i)/(16*b^9) + ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 2756
0*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (((
10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + ((32*a^2*b^3
 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b
^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*
a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^
```

6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i)*1i)/(16*b^9))/((702464*a^19 + (645*a^3*b^16)/4 - (65155*a^5*b^14)/8 + (922065*a^7*b^12)/8 - 740668*a^9*b^10 + 2522213*a^11*b^8 - 4837780*a^13*b^6 + 5244512*a^15*b^4 - 2998016*a^17*b^2)/b^23 + ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 - (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) - ((((25*a^2*b^20)/8 - 225*a^4*b^18 + 4800*a^6*b^16 - 27560*a^8*b^14 + 65160*a^10*b^12 - 67200*a^12*b^10 + 25088*a^14*b^8)/b^23 + (((10*a*b^24 - 274*a^3*b^22 + 712*a^5*b^20 - 448*a^7*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^28 - 512*a^3*b^26))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(1536*a^2*b^24 - 13568*a^4*b^22 + 26368*a^6*b^20 - 14336*a^8*b^18))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^22 - 5929*a^3*b^20 + 119304*a^5*b^18 - 738240*a^7*b^16 + 2004800*a^9*b^14 - 2655360*a^11*b^12 + 1677312*a^13*b^10 - 401408*a^15*b^8))/(8*b^24))*(a^6*448i - b^6*5i + a^2*b^4*180i - a^4*b^2*600i))/(16*b^9) + (...

3.1267 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=485

$$\frac{a(168a^4-200a^2b^2+45b^4)\,x}{8b^8}-\frac{\sqrt{a^2-b^2}\,(42a^4-29a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^8d}+\frac{(630a^4-645a^2b^2}{30}$$

```
[Out] 1/8*a*(168*a^4-200*a^2*b^2+45*b^4)*x/b^8+1/30*(630*a^4-645*a^2*b^2+91*b^4)*
cos(d*x+c)/b^7/d-1/8*(84*a^4-79*a^2*b^2+8*b^4)*cos(d*x+c)*sin(d*x+c)/a/b^6/
d+1/30*(210*a^4-187*a^2*b^2+15*b^4)*cos(d*x+c)*sin(d*x+c)^2/a^2/b^5/d+1/3*c
os(d*x+c)*sin(d*x+c)^3/a/d/(a+b*sin(d*x+c))^2-1/12*b*cos(d*x+c)*sin(d*x+c)^
4/a^2/d/(a+b*sin(d*x+c))^2-1/60*(63*a^4-60*a^2*b^2+5*b^4)*cos(d*x+c)*sin(d*
x+c)^4/a^2/b^3/d/(a+b*sin(d*x+c))^2-7/20*a*cos(d*x+c)*sin(d*x+c)^5/b^2/d/(a
+b*sin(d*x+c))^2+1/5*cos(d*x+c)*sin(d*x+c)^6/b/d/(a+b*sin(d*x+c))^2-1/12*(6
3*a^4-54*a^2*b^2+4*b^4)*cos(d*x+c)*sin(d*x+c)^3/a^2/b^4/d/(a+b*sin(d*x+c))-
(42*a^4-29*a^2*b^2+2*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*
(a^2-b^2)^(1/2)/b^8/d
```

Rubi [A]
time = 1.08, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2975, 3126, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{b\sin^4(c+dx)\cos(c+dx)}{12a^2d(a+b\sin(c+dx))^2}-\frac{\sqrt{a^2-b^2}\,(42a^4-29a^2b^2+2b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{(63a^4-54a^2b^2+4b^4)\sin^3(c+dx)\cos(c+dx)}{12a^2b^4d(a+b\sin(c+dx))}+\frac{ax(168a^4-200a^2b^2+45b^4)}{8b^8}+\frac{(630a^4-645a^2b^2+91b^4)\cos(c+dx)}{30b^7d}-\frac{(84a^4-79a^2b^2+8b^4)\sin(c+dx)\cos(c+dx)}{8ab^6d}+\frac{(210a^4-187a^2b^2+15b^4)\sin^2(c+dx)\cos(c+dx)}{30a^2b^5d}-\frac{(63a^4-60a^2b^2+5b^4)\sin^4(c+dx)\cos(c+dx)}{60a^2b^3d(a+b\sin(c+dx))^2}-\frac{7a\sin^5(c+dx)\cos(c+dx)}{20b^2d(a+b\sin(c+dx))^2}+\frac{\sin^6(c+dx)\cos(c+dx)}{5bd(a+b\sin(c+dx))^2}+\frac{\sin^3(c+dx)\cos(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(168*a^4 - 200*a^2*b^2 + 45*b^4)*x)/(8*b^8) - (Sqrt[a^2 - b^2]*(42*a^4 -
 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8
*d) + ((630*a^4 - 645*a^2*b^2 + 91*b^4)*Cos[c + d*x])/(30*b^7*d) - ((84*a^4
 - 79*a^2*b^2 + 8*b^4)*Cos[c + d*x]*Sin[c + d*x])/(8*a*b^6*d) + ((210*a^4 -
 187*a^2*b^2 + 15*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(30*a^2*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d*(a + b*Sin[c + d*x])^2) - (b*Cos[c + d*x]*Si
n[c + d*x]^4)/(12*a^2*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 60*a^2*b^2 + 5
*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(60*a^2*b^3*d*(a + b*Sin[c + d*x])^2) -
(7*a*Cos[c + d*x]*Sin[c + d*x]^5)/(20*b^2*d*(a + b*Sin[c + d*x])^2) + (Cos[
c + d*x]*Sin[c + d*x]^6)/(5*b*d*(a + b*Sin[c + d*x])^2) - ((63*a^4 - 54*a^2
*b^2 + 4*b^4)*Cos[c + d*x]*Sin[c + d*x]^3)/(12*a^2*b^4*d*(a + b*Sin[c + d*x
]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{7a\cos(c+dx)\sin}{20b^2d(a+b\sin(c}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{(63a^4 - 60a^2b^2 +}{60a^2b^3d(}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad(a+b\sin(c+dx))^2} - \frac{b\cos(c+dx)\sin^4(c+dx)}{12a^2d(a+b\sin(c+dx))^2} - \frac{(63a^4 - 60a^2b^2 +}{60a^2b^3d(}$$

$$= \frac{(210a^4 - 187a^2b^2 + 15b^4)\cos(c+dx)\sin^2(c+dx)}{30a^2b^5d} + \frac{\cos(c+dx)\sin^3(c+}{3ad(a+b\sin(c+dx}$$

$$= -\frac{(84a^4 - 79a^2b^2 + 8b^4)\cos(c+dx)\sin(c+dx)}{8ab^6d} + \frac{(210a^4 - 187a^2b^2 + 15b}{30}$$

$$= \frac{(630a^4 - 645a^2b^2 + 91b^4)\cos(c+dx)}{30b^7d} - \frac{(84a^4 - 79a^2b^2 + 8b^4)\cos(c+dx)}{8ab^6d}$$

$$= \frac{a(168a^4 - 200a^2b^2 + 45b^4)\,x}{8b^8} + \frac{(630a^4 - 645a^2b^2 + 91b^4)\cos(c+dx)}{30b^7d} - \frac{(8}{}$$

$$= \frac{a(168a^4 - 200a^2b^2 + 45b^4)\,x}{8b^8} + \frac{(630a^4 - 645a^2b^2 + 91b^4)\cos(c+dx)}{30b^7d} - \frac{(8}{}$$

$$= \frac{a(168a^4 - 200a^2b^2 + 45b^4)\,x}{8b^8} + \frac{(630a^4 - 645a^2b^2 + 91b^4)\cos(c+dx)}{30b^7d} - \frac{(8}{}$$

$$= \frac{a(168a^4 - 200a^2b^2 + 45b^4)\,x}{8b^8} - \frac{\sqrt{a^2-b^2}\,(42a^4 - 29a^2b^2 + 2b^4)\tan^{-1}\left(\frac{b+}{}\right.}{b^8d}$$

Mathematica [A]
time = 8.61, size = 517, normalized size = 1.07

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*(42*a^4 - 29*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]] + ((a^2 - b^2)^2*(40320*a^7*c - 27840*a^5*b^2*
c - 13200*a^3*b^4*c + 5400*a*b^6*c + 40320*a^7*d*x - 27840*a^5*b^2*d*x - 13
200*a^3*b^4*d*x + 5400*a*b^6*d*x + 10*b*(4032*a^6 - 3792*a^4*b^2 + 216*a^2*
```

```
b^4 + 59*b^6)*Cos[c + d*x] - 120*a*b^2*(168*a^4 - 200*a^2*b^2 + 45*b^4)*(c
+ d*x)*Cos[2*(c + d*x)] - 3360*a^4*b^3*Cos[3*(c + d*x)] + 3580*a^2*b^5*Cos[
3*(c + d*x)] - 526*b^7*Cos[3*(c + d*x)] + 84*a^2*b^5*Cos[5*(c + d*x)] - 58*
b^7*Cos[5*(c + d*x)] - 6*b^7*Cos[7*(c + d*x)] + 80640*a^6*b*c*Sin[c + d*x]
- 96000*a^4*b^3*c*Sin[c + d*x] + 21600*a^2*b^5*c*Sin[c + d*x] + 80640*a^6*b
*d*x*Sin[c + d*x] - 96000*a^4*b^3*d*x*Sin[c + d*x] + 21600*a^2*b^5*d*x*Sin[
c + d*x] + 30240*a^5*b^2*Sin[2*(c + d*x)] - 32640*a^3*b^4*Sin[2*(c + d*x)]
+ 5675*a*b^6*Sin[2*(c + d*x)] + 420*a^3*b^4*Sin[4*(c + d*x)] - 374*a*b^6*Si
n[4*(c + d*x)] - 21*a*b^6*Sin[6*(c + d*x)]))/(a + b*Sin[c + d*x])^2)/(1920*
(a - b)^2*b^8*(a + b)^2*d)
```

Maple [A]
time = 0.60, size = 554, normalized size = 1.14 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^8*(((5*a^3*b^2-27/8*a*b^4)*tan(1/2*d*x+1/2*c)^9+(15*a^4*b-18*a^2*b
^3+3*b^5)*tan(1/2*d*x+1/2*c)^8+(10*a^3*b^2-15/4*a*b^4)*tan(1/2*d*x+1/2*c)^7
+(60*a^4*b-60*a^2*b^3+6*b^5)*tan(1/2*d*x+1/2*c)^6+(90*a^4*b-80*a^2*b^3+28/3
*b^5)*tan(1/2*d*x+1/2*c)^4+(-10*a^3*b^2+15/4*a*b^4)*tan(1/2*d*x+1/2*c)^3+(6
0*a^4*b-52*a^2*b^3+14/3*b^5)*tan(1/2*d*x+1/2*c)^2+(-5*a^3*b^2+27/8*a*b^4)*t
an(1/2*d*x+1/2*c)+15*a^4*b-14*a^2*b^3+23/15*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5
+1/8*a*(168*a^4-200*a^2*b^2+45*b^4)*arctan(tan(1/2*d*x+1/2*c)))-2/b^8*((-1/
2*a*b^2*(11*a^4-13*a^2*b^2+2*b^4)*tan(1/2*d*x+1/2*c)^3-3/2*b*(4*a^6+3*a^4*b
^2-9*a^2*b^4+2*b^6)*tan(1/2*d*x+1/2*c)^2-1/2*b^2*a*(37*a^4-47*a^2*b^2+10*b^
4)*tan(1/2*d*x+1/2*c)-6*a^6*b+15/2*a^4*b^3-3/2*a^2*b^5)/(a*tan(1/2*d*x+1/2*
c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(42*a^6-71*a^4*b^2+31*a^2*b^4-2*b^6)/(
a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.46, size = 995, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*b^7)*cos(d*x + c)^5 + 15*
(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x + c)^2 + 10*(84*a^4*b^3
- 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7 - 32*a^5*b^2 - 155*a^3*b
^4 + 45*a*b^6)*d*x - 30*(42*a^6 + 13*a^4*b^2 - 27*a^2*b^4 + 2*b^6 - (42*a^4
*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*a^5*b - 29*a^3*b^3 + 2*a*
b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - 30*(84*a^6*b - 58*a^4*b^3 - 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^
6*cos(d*x + c)^5 - 5*(42*a^3*b^4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b
 - 200*a^4*b^3 + 45*a^2*b^5)*d*x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x +
c) - (a^2*b^8 + b^10)*d), 1/120*(24*b^7*cos(d*x + c)^7 - 4*(21*a^2*b^5 - 4*
b^7)*cos(d*x + c)^5 + 15*(168*a^5*b^2 - 200*a^3*b^4 + 45*a*b^6)*d*x*cos(d*x
 + c)^2 + 10*(84*a^4*b^3 - 79*a^2*b^5 + 8*b^7)*cos(d*x + c)^3 - 15*(168*a^7
 - 32*a^5*b^2 - 155*a^3*b^4 + 45*a*b^6)*d*x - 60*(42*a^6 + 13*a^4*b^2 - 27*
a^2*b^4 + 2*b^6 - (42*a^4*b^2 - 29*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(42*
a^5*b - 29*a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 30*(84*a^6*b - 58*a^4*b^3 -
 17*a^2*b^5 + 4*b^7)*cos(d*x + c) + (42*a*b^6*cos(d*x + c)^5 - 5*(42*a^3*b^
4 - 29*a*b^6)*cos(d*x + c)^3 - 30*(168*a^6*b - 200*a^4*b^3 + 45*a^2*b^5)*d*
x - 15*(252*a^5*b^2 - 279*a^3*b^4 + 53*a*b^6)*cos(d*x + c))*sin(d*x + c))/(
b^10*d*cos(d*x + c)^2 - 2*a*b^9*d*sin(d*x + c) - (a^2*b^8 + b^10)*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 724, normalized size = 1.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(168*a^5 - 200*a^3*b^2 + 45*a*b^4)*(d*x + c)/b^8 - 120*(42*a^6 -
71*a^4*b^2 + 31*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
 arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^8
) + 120*(11*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 13*a^3*b^3*tan(1/2*d*x + 1/2*c)^
3 + 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 12*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*
b^2*tan(1/2*d*x + 1/2*c)^2 - 27*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 6*b^6*tan(
1/2*d*x + 1/2*c)^2 + 37*a^5*b*tan(1/2*d*x + 1/2*c) - 47*a^3*b^3*tan(1/2*d*x
 + 1/2*c) + 10*a*b^5*tan(1/2*d*x + 1/2*c) + 12*a^6 - 15*a^4*b^2 + 3*a^2*b^4
)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*b^7) + 2*(60
0*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 405*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a^
4*tan(1/2*d*x + 1/2*c)^8 - 2160*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*ta
n(1/2*d*x + 1/2*c)^8 + 1200*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 450*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 7200*a^4*tan(1/2*d*x + 1/2*c)^6 - 7200*a^2*b^2*tan(1/2*d
*x + 1/2*c)^6 + 720*b^4*tan(1/2*d*x + 1/2*c)^6 + 10800*a^4*tan(1/2*d*x + 1/
2*c)^4 - 9600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c
)^4 - 1200*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 450*a*b^3*tan(1/2*d*x + 1/2*c)^3
+ 7200*a^4*tan(1/2*d*x + 1/2*c)^2 - 6240*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 5
60*b^4*tan(1/2*d*x + 1/2*c)^2 - 600*a^3*b*tan(1/2*d*x + 1/2*c) + 405*a*b^3*
tan(1/2*d*x + 1/2*c) + 1800*a^4 - 1680*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1
/2*c)^2 + 1)^5*b^7))/d
```

Mupad [B]
time = 23.85, size = 2500, normalized size = 5.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x))^3,x)
```

```
[Out] ((630*a^6 + 91*a^2*b^4 - 645*a^4*b^2)/(15*b^7) + (tan(c/2 + (d*x)/2)^13*(8*
a*b^4 + 84*a^5 - 79*a^3*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^11*(17*a*b^4 +
252*a^5 - 237*a^3*b^2))/b^6 + (8*tan(c/2 + (d*x)/2)^7*(91*a*b^4 + 630*a^5 -
 645*a^3*b^2))/(3*b^6) + (9*tan(c/2 + (d*x)/2)^5*(112*a*b^4 + 700*a^5 - 733
*a^3*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^9*(448*a*b^4 + 3780*a^5 - 3723*a^3
*b^2))/(4*b^6) + (tan(c/2 + (d*x)/2)^3*(643*a*b^4 + 3780*a^5 - 3975*a^3*b^2
))/(5*b^6) + (tan(c/2 + (d*x)/2)^12*(42*a^6 + 6*b^6 - 48*a^2*b^4 + 13*a^4*b
^2))/b^7 + (3*tan(c/2 + (d*x)/2)^10*(84*a^6 + 18*b^6 - 103*a^2*b^4 + 26*a^4
*b^2))/b^7 + (2*tan(c/2 + (d*x)/2)^6*(1260*a^6 + 202*b^6 - 1187*a^2*b^4 + 5
4*a^4*b^2))/(3*b^7) + (tan(c/2 + (d*x)/2)^8*(1890*a^6 + 324*b^6 - 2149*a^2*
b^4 + 417*a^4*b^2))/(3*b^7) + (tan(c/2 + (d*x)/2)^2*(3780*a^6 + 274*b^6 - 1
223*a^2*b^4 - 2190*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)^4*(9450*a^6 + 1
010*b^6 - 6354*a^2*b^4 - 2115*a^4*b^2))/(15*b^7) + (tan(c/2 + (d*x)/2)*(133
6*a*b^4 + 8820*a^5 - 9135*a^3*b^2))/(60*b^6))/(d*(tan(c/2 + (d*x)/2)^2*(7*a
^2 + 4*b^2) + tan(c/2 + (d*x)/2)^12*(7*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*
(21*a^2 + 20*b^2) + tan(c/2 + (d*x)/2)^10*(21*a^2 + 20*b^2) + tan(c/2 + (d*
```

x)/2)^6*(35*a^2 + 40*b^2) + tan(c/2 + (d*x)/2)^8*(35*a^2 + 40*b^2) + a^2*ta
n(c/2 + (d*x)/2)^14 + a^2 + 24*a*b*tan(c/2 + (d*x)/2)^3 + 60*a*b*tan(c/2 +
(d*x)/2)^5 + 80*a*b*tan(c/2 + (d*x)/2)^7 + 60*a*b*tan(c/2 + (d*x)/2)^9 + 24
*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2)^13 + 4*a*b*tan(c/2 +
(d*x)/2))) + (a*atan(((a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^
11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19
 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219
072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 200*a^2*b
^2)*((148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2
)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) +
(a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))
*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b
^4 - 200*a^2*b^2))/(8*b^8) + (a*(((2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560
*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64
*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840*a^9*b^1
1 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 20
0*a^2*b^2)*((tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18
 - 2688*a^7*b^16))/(2*b^21) - (148*a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/
b^20 + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2
b^21))(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4
 + 45*b^4 - 200*a^2*b^2))/(8*b^8))/((296352*a^16 - 360*a^2*b^14 + 10735*a^4
*b^12 - (227213*a^6*b^10)/2 + (1089913*a^8*b^8)/2 - 1331285*a^10*b^6 + 1725
696*a^12*b^4 - 1132488*a^14*b^2)/b^20 + (tan(c/2 + (d*x)/2)*(1185408*a^17 -
 4050*a^3*b^14 + 98775*a^5*b^12 - 812015*a^7*b^10 + 3206170*a^9*b^8 - 68091
68*a^11*b^6 + 7961184*a^13*b^4 - 4826304*a^15*b^2))/b^21 + (a*(((2025*a^4*b
^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/
b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 20
1360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^2
1) + (a*(168*a^4 + 45*b^4 - 200*a^2*b^2)*((148*a^2*b^20 - 484*a^4*b^18 + 33
6*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a
^5*b^18 - 2688*a^7*b^16))/(2*b^21) + (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(
192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/
(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8) - (a*(((
2025*a^4*b^15)/2 - 9000*a^6*b^13 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*
a^12*b^7)/b^20 - (tan(c/2 + (d*x)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5
*b^15 - 201360*a^7*b^13 + 311840*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^
7))/(2*b^21) + (a*(168*a^4 + 45*b^4 - 200*a^2*b^2)*((tan(c/2 + (d*x)/2)*(12
8*a*b^22 - 1984*a^3*b^20 + 4544*a^5*b^18 - 2688*a^7*b^16))/(2*b^21) - (148*
a^2*b^20 - 484*a^4*b^18 + 336*a^6*b^16)/b^20 + (a*(32*a^2*b^3 + (tan(c/2 +
(d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(168*a^4 + 45*b^4 - 200*a^2
*b^2)*1i)/(8*b^8))*1i)/(8*b^8))*(168*a^4 + 45*b^4 - 200*a^2*b^2)*1i)/(8*b^8
)))*(168*a^4 + 45*b^4 - 200*a^2*b^2))/(4*b^8*d) + (atan((((-(a + b)*(a - b)
)^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*(((2025*a^4*b^15)/2 - 9000*a^6*b^13
 + 27560*a^8*b^11 - 33600*a^10*b^9 + 14112*a^12*b^7)/b^20 - (tan(c/2 + (d*x
)/2)*(64*a*b^19 - 6034*a^3*b^17 + 57945*a^5*b^15 - 201360*a^7*b^13 + 311840

```
*a^9*b^11 - 219072*a^11*b^9 + 56448*a^13*b^7))/(2*b^21) + ((-(a + b)*(a - b
))^(1/2)*(21*a^4 + b^4 - (29*a^2*b^2)/2)*((148*a^2*b^20 - 484*a^4*b^18 + 33
6*a^6*b^16)/b^20 - (tan(c/2 + (d*x)/2)*(128*a*b^22 - 1984*a^3*b^20 + 4544*a
^5*b^18 - 2688*a^7*b^16))/(2*b^21) + ((-(a + b)*(a - b))^(1/2)*(32*a^2*b^3
+ (tan(c/2 + (d*x)/2)*(192*a*b^25 - 128*a^3*b^23))/(2*b^21))*(21*a^4 + b^4
- (29*a^2*b^2)/2))/b^8))/b^8)*1i)/b^8 + ((-(a +...
```

3.1268 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=237

$$-\frac{15(8a^4-8a^2b^2+b^4)\,x}{8b^7}+\frac{15a(2a^4-3a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d}+\frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2}+$$

```
[Out] -15/8*(8*a^4-8*a^2*b^2+b^4)*x/b^7+1/4*cos(d*x+c)^5*(3*a+b*sin(d*x+c))/b^2/d
/(a+b*sin(d*x+c))^2+5/4*cos(d*x+c)^3*(4*a^2-b^2+a*b*sin(d*x+c))/b^4/d/(a+b*
sin(d*x+c))-15/8*cos(d*x+c)*(4*a*(2*a^2-b^2)-b*(4*a^2-b^2)*sin(d*x+c))/b^6/
d+15*a*(2*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/b^7/d/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2942, 2944, 2814, 2739, 632, 210}

$$-\frac{15\cos(c+dx)\left(4a(2a^2-b^2)-b(4a^2-b^2)\sin(c+dx)\right)}{8b^6d}+\frac{5\cos^3(c+dx)\left(4a^2+ab\sin(c+dx)-b^2\right)}{4b^4d(a+b\sin(c+dx))}+\frac{15a(2a^4-3a^2b^2+b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d\sqrt{a^2-b^2}}-\frac{15x(8a^4-8a^2b^2+b^4)}{8b^7}+\frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-15*(8*a^4 - 8*a^2*b^2 + b^4)*x)/(8*b^7) + (15*a*(2*a^4 - 3*a^2*b^2 + b^4)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*Sqrt[a^2 - b^2]*d)
+ (Cos[c + d*x]^5*(3*a + b*Sin[c + d*x]))/(4*b^2*d*(a + b*Sin[c + d*x])^2)
+ (5*Cos[c + d*x]^3*(4*a^2 - b^2 + a*b*Sin[c + d*x]))/(4*b^4*d*(a + b*Sin[c
 + d*x])) - (15*Cos[c + d*x]*(4*a*(2*a^2 - b^2) - b*(4*a^2 - b^2)*Sin[c + d
*x]))/(8*b^6*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2942

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Dist[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[
e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ
[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} - \frac{5\int \frac{\cos^4(c+dx)(-2b-6a\sin(c+dx))}{(a+b\sin(c+dx))^2}dx}{8b^2}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx\right.}{4b^4d(a+b\sin(c+dx))}$$

$$= \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+dx)\left(4a^2-b^2+ab\sin(c+dx\right.}{4b^4d(a+b\sin(c+dx))}$$

$$= -\frac{15(8a^4-8a^2b^2+b^4)\,x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+d}{4b^4}$$

$$= -\frac{15(8a^4-8a^2b^2+b^4)\,x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+d}{4b^4}$$

$$= -\frac{15(8a^4-8a^2b^2+b^4)\,x}{8b^7} + \frac{\cos^5(c+dx)(3a+b\sin(c+dx))}{4b^2d(a+b\sin(c+dx))^2} + \frac{5\cos^3(c+d}{4b^4}$$

$$= -\frac{15(8a^4-8a^2b^2+b^4)\,x}{8b^7} + \frac{15a(2a^4-3a^2b^2+b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7\sqrt{a^2-b^2}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1250 vs. $2(237) = 474$.
time = 5.70, size = 1250, normalized size = 5.27

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((18*(-8*(c + d*x) + (2*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(4*a^2 - 3*b^2)*Cos
[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x])^2) - (3*b*(4*a^4 - 7*a^2*b
^2 + 2*b^4)*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x]))))/b^3
- (10*((6*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 -
b^2] + (Cos[c + d*x]*(a*(2*a^2 + b^2) + b*(a^2 + 2*b^2)*Sin[c + d*x]))/(a +
 b*Sin[c + d*x])^2))/((a - b)^2*(a + b)^2) + (10*(-24*(-8*a^2 + b^2)*(c + d
*x) - (6*a*(64*a^6 - 168*a^4*b^2 + 140*a^2*b^4 - 35*b^6)*ArcTan[(b + a*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + 96*a*b*Cos[c + d*x] + (
a*b*(-16*a^4 + 20*a^2*b^2 - 5*b^4)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Si
n[c + d*x])^2) + (b*(112*a^6 - 220*a^4*b^2 + 115*a^2*b^4 - 10*b^6)*Cos[c +
d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])) - 8*b^2*Sin[2*(c + d*x)]))
/b^5 + ((12*a*(640*a^8 - 1920*a^6*b^2 + 2016*a^4*b^4 - 840*a^2*b^6 + 105*b^
```

```
8)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (-
3840*a^10*(c + d*x) + 7680*a^8*b^2*(c + d*x) - 2976*a^6*b^4*(c + d*x) - 177
6*a^4*b^6*(c + d*x) + 960*a^2*b^8*(c + d*x) - 48*b^10*(c + d*x) - 3840*a^9*
b*Cos[c + d*x] + 8640*a^7*b^3*Cos[c + d*x] - 5696*a^5*b^5*Cos[c + d*x] + 78
8*a^3*b^7*Cos[c + d*x] + 114*a*b^9*Cos[c + d*x] + 1920*a^8*b^2*(c + d*x)*Co
s[2*(c + d*x)] - 4800*a^6*b^4*(c + d*x)*Cos[2*(c + d*x)] + 3888*a^4*b^6*(c
+ d*x)*Cos[2*(c + d*x)] - 1056*a^2*b^8*(c + d*x)*Cos[2*(c + d*x)] + 48*b^10
*(c + d*x)*Cos[2*(c + d*x)] + 320*a^7*b^3*Cos[3*(c + d*x)] - 760*a^5*b^5*Co
s[3*(c + d*x)] + 560*a^3*b^7*Cos[3*(c + d*x)] - 120*a*b^9*Cos[3*(c + d*x)]
- 8*a^5*b^5*Cos[5*(c + d*x)] + 16*a^3*b^7*Cos[5*(c + d*x)] - 8*a*b^9*Cos[5*
(c + d*x)] - 7680*a^9*b*(c + d*x)*Sin[c + d*x] + 19200*a^7*b^3*(c + d*x)*Si
n[c + d*x] - 15552*a^5*b^5*(c + d*x)*Sin[c + d*x] + 4224*a^3*b^7*(c + d*x)*
Sin[c + d*x] - 192*a*b^9*(c + d*x)*Sin[c + d*x] - 2880*a^8*b^2*Sin[2*(c + d
*x)] + 6880*a^6*b^4*Sin[2*(c + d*x)] - 5182*a^4*b^6*Sin[2*(c + d*x)] + 1221
*a^2*b^8*Sin[2*(c + d*x)] - 36*b^10*Sin[2*(c + d*x)] - 40*a^6*b^4*Sin[4*(c
+ d*x)] + 88*a^4*b^6*Sin[4*(c + d*x)] - 56*a^2*b^8*Sin[4*(c + d*x)] + 8*b^1
0*Sin[4*(c + d*x)] + 2*a^4*b^6*Sin[6*(c + d*x)] - 4*a^2*b^8*Sin[6*(c + d*x)
] + 2*b^10*Sin[6*(c + d*x)])/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2))/b^7)/(
256*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(224) = 448$.

time = 0.53, size = 472, normalized size = 1.99 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^7*(((3*a^2*b^2-9/8*b^4)*tan(1/2*d*x+1/2*c)^7+(10*a^3*b-9*a*b^3)*t
an(1/2*d*x+1/2*c)^6+(3*a^2*b^2-1/8*b^4)*tan(1/2*d*x+1/2*c)^5+(30*a^3*b-21*a
*b^3)*tan(1/2*d*x+1/2*c)^4+(-3*a^2*b^2+1/8*b^4)*tan(1/2*d*x+1/2*c)^3+(30*a^
3*b-19*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-3*a^2*b^2+9/8*b^4)*tan(1/2*d*x+1/2*c)+
10*a^3*b-7*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+15/8*(8*a^4-8*a^2*b^2+b^4)*arc
tan(tan(1/2*d*x+1/2*c)))+2/b^7*(((-9/2*a^4*b^2+9/2*a^2*b^4)*tan(1/2*d*x+1/2
*c)^3-1/2*b*(10*a^6+9*a^4*b^2-21*a^2*b^4+2*b^6)/a*tan(1/2*d*x+1/2*c)^2-1/2*
b^2*(31*a^4-35*a^2*b^2+4*b^4)*tan(1/2*d*x+1/2*c)-1/2*a*b*(10*a^4-11*a^2*b^2
+b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+15/2*a*(2*a^4-3*
a^2*b^2+b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.44, size = 837, normalized size = 3.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos(d*x
 + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b^4 +
 b^6)*d*x + 30*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x + c)^
2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2)) + 30*(4*a^5*b - 2*a^3*b^3 - a*b^5)*cos(d*x + c) - (2
*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x + c)^3 - 30*(8*a^5*b - 8*
a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 + b^6)*cos(d*x + c))*sin
(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x + c) - (a^2*b^7 + b^9)
*d), 1/8*(4*a*b^5*cos(d*x + c)^5 - 15*(8*a^4*b^2 - 8*a^2*b^4 + b^6)*d*x*cos
(d*x + c)^2 - 10*(4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^3 + 15*(8*a^6 - 7*a^2*b
^4 + b^6)*d*x + 60*(2*a^5 + a^3*b^2 - a*b^4 - (2*a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(2*a^4*b - a^2*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(
d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^5*b - 2*a^3*b^3 - a
*b^5)*cos(d*x + c) - (2*b^6*cos(d*x + c)^5 - 5*(2*a^2*b^4 - b^6)*cos(d*x +
c)^3 - 30*(8*a^5*b - 8*a^3*b^3 + a*b^5)*d*x - 15*(12*a^4*b^2 - 11*a^2*b^4 +
 b^6)*cos(d*x + c))*sin(d*x + c))/(b^9*d*cos(d*x + c)^2 - 2*a*b^8*d*sin(d*x
 + c) - (a^2*b^7 + b^9)*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(223) = 446.
time = 0.52, size = 581, normalized size = 2.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/8*(15*(8*a^4 - 8*a^2*b^2 + b^4)*(d*x + c)/b^7 - 120*(2*a^5 - 3*a^3*b^2 + a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 8*(9*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 + 31*a^5*b*tan(1/2*d*x + 1/2*c) - 35*a^3*b^3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 10*a^6 - 11*a^4*b^2 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a*b^6) + 2*(24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 9*b^3*tan(1/2*d*x + 1/2*c)^7 + 80*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^5 - b^3*tan(1/2*d*x + 1/2*c)^5 + 240*a^3*tan(1/2*d*x + 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*tan(1/2*d*x + 1/2*c)^2 - 152*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b*tan(1/2*d*x + 1/2*c) + 9*b^3*tan(1/2*d*x + 1/2*c) + 80*a^3 - 56*a*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^6))/d

Mupad [B]
time = 16.79, size = 2529, normalized size = 10.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x))^3,x)

[Out] - ((a*b^4 + 30*a^5 - 25*a^3*b^2)/b^6 + (tan(c/2 + (d*x)/2)*(420*a^4 + 16*b^4 - 355*a^2*b^2))/(4*b^5) + (15*tan(c/2 + (d*x)/2)^11*(4*a^4 - 3*a^2*b^2))/(4*b^5) + (15*tan(c/2 + (d*x)/2)^7*(68*a^4 + 2*b^4 - 55*a^2*b^2))/(2*b^5) - (5*tan(c/2 + (d*x)/2)^9*(4*b^4 - 132*a^4 + 99*a^2*b^2))/(4*b^5) + (5*tan(c/2 + (d*x)/2)^5*(276*a^4 + 10*b^4 - 235*a^2*b^2))/(2*b^5) + (5*tan(c/2 + (d*x)/2)^3*(348*a^4 + 20*b^4 - 301*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x)/2)^10*(30*a^6 + 2*b^6 - 30*a^2*b^4 + 15*a^4*b^2))/(a*b^6) + (tan(c/2 + (d*x)/2)^2*(150*a^6 + 2*b^6 - 64*a^2*b^4 - 45*a^4*b^2))/(a*b^6) + (2*tan(c/2 + (d*x)/2)^4*(150*a^6 + 4*b^6 - 110*a^2*b^4 + 15*a^4*b^2))/(a*b^6) + (tan(c/2 + (d*x)/2)^8*(150*a^6 + 8*b^6 - 165*a^2*b^4 + 75*a^4*b^2))/(a*b^6) + (2*tan(c/2 + (d*x)/2)^6*(5*a^2 + 6*b^2)*(30*a^4 + b^4 - 25*a^2*b^2))/(a*b^6))/(d*(tan(c/2 + (d*x)/2)^2*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^10*(6*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(15*a^2 + 16*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 + 16*b^2) + tan(c/2 + (d*x)/2)^6*(20*a^2 + 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 20*a*b*tan(c/2 + (d*x)/2)^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 + 40*a*b*tan(c/2 + (d*x)/2)^7 + 20*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3375*a^3*(b^2 - a^2)^(1/2))/(2*((3375*a^3*b)/2 - (10125*a^5)/(2*b) + (3375*a^7)/b^3 - 10125*a^4*tan(c/2 +

(d*x)/2) + 3375*a^2*b^2*tan(c/2 + (d*x)/2) + (6750*a^6*tan(c/2 + (d*x)/2))
/b^2)) - (3375*a^5*(b^2 - a^2)^(1/2))/((3375*a^3*b^3)/2 - (10125*a^5*b)/2 +
(3375*a^7)/b + 6750*a^6*tan(c/2 + (d*x)/2) + 3375*a^2*b^4*tan(c/2 + (d*x)/
2) - 10125*a^4*b^2*tan(c/2 + (d*x)/2)) + (3375*a^2*tan(c/2 + (d*x)/2)*(b^2
- a^2)^(1/2))/((3375*a^3)/2 - (10125*a^5)/(2*b^2) + (3375*a^7)/b^4 + 3375*a
^2*b*tan(c/2 + (d*x)/2) - (10125*a^4*tan(c/2 + (d*x)/2))/b + (6750*a^6*tan(
c/2 + (d*x)/2))/b^3) - (16875*a^4*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(2*
((3375*a^3*b^2)/2 - (10125*a^5)/2 + (3375*a^7)/b^2 - 10125*a^4*b*tan(c/2 +
(d*x)/2) + 3375*a^2*b^3*tan(c/2 + (d*x)/2) + (6750*a^6*tan(c/2 + (d*x)/2))/
b)) + (3375*a^6*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(3375*a^7 + (3375*a^3
*b^4)/2 - (10125*a^5*b^2)/2 + 6750*a^6*b*tan(c/2 + (d*x)/2) + 3375*a^2*b^5*
tan(c/2 + (d*x)/2) - 10125*a^4*b^3*tan(c/2 + (d*x)/2)))*(30*a^3*(b^2 - a^2)
^(1/2) - 15*a*b^2*(b^2 - a^2)^(1/2)))/(b^7*d) - (atan((((a^4*8i + b^4*1i -
a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*
b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^1
4 + 61200*a^5*b^12 - 122400*a^7*b^10 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2
b^18) - (15(a^4*8i + b^4*1i - a^2*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 24
0*a^5*b^14)/b^17 - (15*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*
a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (
d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))
*15i)/(8*b^7) + ((a^4*8i + b^4*1i - a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a
^4*b^12 + 9000*a^6*b^10 - 14400*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 +
(d*x)/2)*(450*a*b^16 - 11025*a^3*b^14 + 61200*a^5*b^12 - 122400*a^7*b^10 +
100800*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) + (15*(a^4*8i + b^4*1i - a^2*b^2
8i)((60*a*b^18 - 300*a^3*b^16 + 240*a^5*b^14)/b^17 + (15*(32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i -
a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 +
1920*a^6*b^14))/(2*b^18)))/(8*b^7))*15i)/(8*b^7))/((108000*a^13 - (10125*a
^3*b^10)/2 + (124875*a^5*b^8)/2 - 246375*a^7*b^6 + 432000*a^9*b^4 - 351000*
a^11*b^2)/b^17 - (15*(a^4*8i + b^4*1i - a^2*b^2*8i)*(((225*a^2*b^14)/2 - 18
00*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*b^8 + 7200*a^10*b^6)/b^17 + (tan(c/
2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^14 + 61200*a^5*b^12 - 122400*a^7*b^1
0 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2*b^18) - (15*(a^4*8i + b^4*1i - a^2
*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 240*a^5*b^14)/b^17 - (15*(32*a^2*b^3
+ (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*a^3*b^20))/(2*b^18))*(a^4*8i + b^4*
1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^
16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7)))/(8*b^7) + (15*(a^4*8i + b^4*1i -
a^2*b^2*8i)*(((225*a^2*b^14)/2 - 1800*a^4*b^12 + 9000*a^6*b^10 - 14400*a^8*
b^8 + 7200*a^10*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(450*a*b^16 - 11025*a^3*b^1
4 + 61200*a^5*b^12 - 122400*a^7*b^10 + 100800*a^9*b^8 - 28800*a^11*b^6))/(2
b^18) + (15(a^4*8i + b^4*1i - a^2*b^2*8i)*((60*a*b^18 - 300*a^3*b^16 + 24
0*a^5*b^14)/b^17 + (15*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^22 - 128*
a^3*b^20))/(2*b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i))/(8*b^7) + (tan(c/2 + (
d*x)/2)*(960*a^2*b^18 - 2880*a^4*b^16 + 1920*a^6*b^14))/(2*b^18)))/(8*b^7))
)/(8*b^7) + (tan(c/2 + (d*x)/2)*(432000*a^14 + 3375*a^2*b^12 - 64125*a^4*b^

```
10 + 438750*a^6*b^8 - 1350000*a^8*b^6 + 2052000*a^10*b^4 - 1512000*a^12*b^2
))/b^18))*(a^4*8i + b^4*1i - a^2*b^2*8i)*15i)/(4*b^7*d)
```

3.1269 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=399

$$-\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}+\frac{\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2\sqrt{a^2-b^2}\,(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c\right.}{\sqrt{a^2-}}\right.}{ab^5d}$$

```
[Out] -1/2*x/b^3-3*(2*a^2-b^2)*x/b^5-arctanh(cos(d*x+c))/a^3/d-3*a*cos(d*x+c)/b^4
/d+1/2*cos(d*x+c)*sin(d*x+c)/b^3/d+1/2*(a^2-b^2)^2*cos(d*x+c)/a/b^4/d/(a+b*
sin(d*x+c))^2+3/2*(a^2-b^2)*cos(d*x+c)/b^4/d/(a+b*sin(d*x+c))-(a^2-b^2)*(5*
a^2+b^2)*cos(d*x+c)/a^2/b^4/d/(a+b*sin(d*x+c))+2*(10*a^6-9*a^4*b^2-b^6)*arc
tan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/b^5/d/(a^2-b^2)^(1/2)+(2*
a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a
/b^5/d-2*(5*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-
b^2)^(1/2)/a/b^5/d
```

Rubi [A]
time = 0.34, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {2976, 3855, 2718, 2715, 8, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{(2a^2+b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2(5a^2+b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{3x(2a^2-b^2)}{b^5}+\frac{(a^2-b^2)^2\cos(c+dx)}{2ab^4d(a+b\sin(c+dx))^2}-\frac{(5a^2+b^2)(a^2-b^2)\cos(c+dx)}{a^2b^4d(a+b\sin(c+dx))}+\frac{3(a^2-b^2)\cos(c+dx)}{2b^4d(a+b\sin(c+dx))}+\frac{2(10a^6-9a^4b^2-b^6)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^5d\sqrt{a^2-b^2}}-\frac{3a\cos(c+dx)}{b^4d}+\frac{\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*x/b^3 - (3*(2*a^2 - b^2)*x)/b^5 + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTa
n[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d) - (2*Sqrt[a^2 - b^2]
*(5*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d)
+ (2*(10*a^6 - 9*a^4*b^2 - b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(a^3*b^5*Sqrt[a^2 - b^2]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) - (3*a*C
os[c + d*x])/(b^4*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b^3*d) + ((a^2 - b^2)
^2*Cos[c + d*x])/(2*a*b^4*d*(a + b*Sin[c + d*x])^2) + (3*(a^2 - b^2)*Cos[c
+ d*x])/(2*b^4*d*(a + b*Sin[c + d*x])) - ((a^2 - b^2)*(5*a^2 + b^2)*Cos[c +
 d*x])/(a^2*b^4*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(\frac{3(-2a^2+b^2)}{b^5}+\frac{\csc(c+dx)}{a^3}+\frac{3a\sin(c+dx)}{b^4}-\frac{\sin^2(c+dx)}{b^3}+\frac{}{ab^5(a}\right.$$

$$= -\frac{3(2a^2-b^2)\,x}{b^5}+\frac{\int\csc(c+dx)\,dx}{a^3}+\frac{(3a)\int\sin(c+dx)\,dx}{b^4}-\frac{\int\sin^2(c+dx}{b^3}$$

$$= -\frac{3(2a^2-b^2)\,x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3a\cos(c+dx)}{b^4d}+\frac{\cos(c+dx)\sin(}{2b^3d}$$

$$= -\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{3a\cos(c+dx)}{b^4d}+\frac{\cos(c+}{}$$

$$= -\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}+\frac{2(10a^6-9a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}\,d}-\frac{\mathrm{ta}}{}$$

$$= -\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}+\frac{2(10a^6-9a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}\,d}-\frac{\mathrm{ta}}{}$$

$$= -\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}-\frac{2\sqrt{a^2-b^2}\,(5a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}+\frac{2}{}$$

$$= -\frac{x}{2b^3}-\frac{3(2a^2-b^2)\,x}{b^5}+\frac{\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{2\sqrt{}}{}$$

Mathematica [A]

time = 1.32, size = 243, normalized size = 0.61

$$\frac{\frac{2(-12a^2+5b^2)(c+dx)}{b^5}+\frac{4(12a^6-11a^4b^2+a^2b^4-2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^5\sqrt{a^2-b^2}}-\frac{12a\cos(c+dx)}{b^4}-\frac{4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{2(a^2-b^2)^2\cos(c+dx)}{ab^4(a+b\sin(c+dx))^2}+\frac{2(-7a^4+5a^2b^2+2b^4)\cos(c+dx)}{a^2b^4(a+b\sin(c+dx))}+\frac{\sin(2(c+dx))}{b^3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(-12*a^2 + 5*b^2)*(c + d*x))/b^5 + (4*(12*a^6 - 11*a^4*b^2 + a^2*b^4 -
2*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^5*Sqrt[a^2
- b^2]) - (12*a*Cos[c + d*x])/b^4 - (4*Log[Cos[(c + d*x)/2]])/a^3 + (4*Log[
Sin[(c + d*x)/2]])/a^3 + (2*(a^2 - b^2)^2*Cos[c + d*x])/(a*b^4*(a + b*Sin[c
 + d*x])^2) + (2*(-7*a^4 + 5*a^2*b^2 + 2*b^4)*Cos[c + d*x])/(a^2*b^4*(a + b
*Sin[c + d*x])) + Sin[2*(c + d*x)]/b^3)/(4*d)
```

Maple [A]

time = 1.01, size = 359, normalized size = 0.90

method	result
derivativedivides	$-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(12a^2-5b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^5}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+$
default	$-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+3ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+3ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\left(12a^2-5b^2\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^5}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}+$
risch	$-\frac{6x\,a^2}{b^5}+\frac{5x}{2b^3}-\frac{i\mathrm{e}^{2i(dx+c)}}{8b^3d}-\frac{3a\,\mathrm{e}^{i(dx+c)}}{2b^4d}-\frac{3a\,\mathrm{e}^{-i(dx+c)}}{2b^4d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8b^3d}+\frac{i(-8ia^5b\,\mathrm{e}^{3i(dx+c)}+7ia^3b^3\mathrm{e}^{3i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^5*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+3*a*b*tan(1/2*d*x+1/2*c)^2-1/2*b
^2*tan(1/2*d*x+1/2*c)+3*a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(12*a^2-5*b^2)*
arctan(tan(1/2*d*x+1/2*c)))+1/a^3*ln(tan(1/2*d*x+1/2*c))+2/a^3/b^5*(((-5/2*
a^5*b^2+1/2*a^3*b^4+2*a*b^6)*tan(1/2*d*x+1/2*c)^3-3/2*b*(2*a^6+3*a^4*b^2-3*
a^2*b^4-2*b^6)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*(19*a^4-11*a^2*b^2-8*b^4)*tan
(1/2*d*x+1/2*c)-3/2*a^2*b*(2*a^4-a^2*b^2-b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*
tan(1/2*d*x+1/2*c)+a)^2+1/2*(12*a^6-11*a^4*b^2+a^2*b^4-2*b^6)/(a^2-b^2)^(1/
2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.64, size = 1007, normalized size = 2.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(8*a^4*b^3*cos(d*x + c)^3 + 2*(12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*x +
c)^2 - 2*(12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x + (12*a^6 + 13*a^4*b^2 + 3*a^
2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5*b
 + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x +
 c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2)) - 2*(12*a^6*b + a^4*b^3 - 3*a^2*b^5)*cos(d*x + c) + 2*(
b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*log(1/2*cos(d*x
+ c) + 1/2) - 2*(b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^4*cos(d*x + c)^3 + 2*(12*a^6*b - 5
*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))*sin(d*x + c
))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*b^5 + a^3*b^
7)*d), -1/2*(4*a^4*b^3*cos(d*x + c)^3 + (12*a^5*b^2 - 5*a^3*b^4)*d*x*cos(d*
x + c)^2 - (12*a^7 + 7*a^5*b^2 - 5*a^3*b^4)*d*x - (12*a^6 + 13*a^4*b^2 + 3*
a^2*b^4 + 2*b^6 - (12*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2 + 2*(12*a^5
*b + a^3*b^3 + 2*a*b^5)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (12*a^6*b + a^4*b^3 - 3*a^2*b^5)*
cos(d*x + c) + (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) - a^2*b^5 - b^7)*
log(1/2*cos(d*x + c) + 1/2) - (b^7*cos(d*x + c)^2 - 2*a*b^6*sin(d*x + c) -
a^2*b^5 - b^7)*log(-1/2*cos(d*x + c) + 1/2) - (a^3*b^4*cos(d*x + c)^3 + 2*(
12*a^6*b - 5*a^4*b^3)*d*x + 2*(9*a^5*b^2 - 3*a^3*b^4 - a*b^6)*cos(d*x + c))
*sin(d*x + c))/(a^3*b^7*d*cos(d*x + c)^2 - 2*a^4*b^6*d*sin(d*x + c) - (a^5*
b^5 + a^3*b^7)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.51, size = 635, normalized size = 1.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (12*a^2 - 5*b^2)*(d*x + c)/b^5
+ 2*(12*a^6 - 11*a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
 - b^2)*a^3*b^5) - 2*(6*a^5*b*tan(1/2*d*x + 1/2*c)^7 - a^3*b^3*tan(1/2*d*x
+ 1/2*c)^7 - 4*a*b^5*tan(1/2*d*x + 1/2*c)^7 + 12*a^6*tan(1/2*d*x + 1/2*c)^6
 + 13*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 9*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 6
*b^6*tan(1/2*d*x + 1/2*c)^6 + 54*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*b^3*t
an(1/2*d*x + 1/2*c)^5 - 16*a*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^6*tan(1/2*d*
x + 1/2*c)^4 + 39*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x +
 1/2*c)^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 + 90*a^5*b*tan(1/2*d*x + 1/2*c)^3
 - 27*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 20*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 36
*a^6*tan(1/2*d*x + 1/2*c)^2 + 23*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^
4*tan(1/2*d*x + 1/2*c)^2 - 6*b^6*tan(1/2*d*x + 1/2*c)^2 + 42*a^5*b*tan(1/2*
d*x + 1/2*c) - 11*a^3*b^3*tan(1/2*d*x + 1/2*c) - 8*a*b^5*tan(1/2*d*x + 1/2*
c) + 12*a^6 - 3*a^4*b^2 - 3*a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1
/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) +
 a)^2*a^3*b^4))/d
```

Mupad [B]
time = 14.55, size = 2500, normalized size = 6.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) + ((3*(b^4 - 4*a^4 + a^2*b^2))/(a*b^4) + (t
an(c/2 + (d*x)/2)*(8*b^4 - 42*a^4 + 11*a^2*b^2))/(a^2*b^3) - (3*tan(c/2 + (
d*x)/2)^4*(12*a^6 - 4*b^6 - 7*a^2*b^4 + 13*a^4*b^2))/(a^3*b^4) - (tan(c/2 +
 (d*x)/2)^6*(12*a^6 - 6*b^6 - 9*a^2*b^4 + 13*a^4*b^2))/(a^3*b^4) - (tan(c/2
 + (d*x)/2)^2*(36*a^6 - 6*b^6 - 15*a^2*b^4 + 23*a^4*b^2))/(a^3*b^4) + (tan(
c/2 + (d*x)/2)^7*(4*b^4 - 6*a^4 + a^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)
^5*(16*b^4 - 54*a^4 + 9*a^2*b^2))/(a^2*b^3) + (tan(c/2 + (d*x)/2)^3*(20*b^4
 - 90*a^4 + 27*a^2*b^2))/(a^2*b^3))/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2
) + tan(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(6*a^2 + 8*
b^2) + a^2*tan(c/2 + (d*x)/2)^8 + a^2 + 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*
b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)
/2))) - (atan((((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b
^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b
^11) - ((a^2*12i - b^2*5i)*((4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 +
32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) - (
(a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11
*b^10))/(a^6*b^11) - (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan
(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2
*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20
+ 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (8*t
an(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14
 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) +
(8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 +
 5544*a^8*b^12 - 9958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^
16*b^4))/(a^6*b^16))*1i)/(2*b^5) + ((a^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6
048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 108
00*a^13*b^2))/(a^6*b^11) + ((a^2*12i - b^2*5i)*(((a^2*12i - b^2*5i)*((4*(64
*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (((4*
(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^
22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2*12i - b^2*5i))/(2*b^5) +
(8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*
b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2*b^5) + (4*(32*a^2*b^16 - 24*a^4*b^14
+ 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4
))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*
b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^
6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^
16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 15912*a^12*b^8 - 1296
0*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16))*1i)/(2*b^5))/(((a^2*12i - b^2*5i)*
((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 -
 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) - ((a^2*12i - b^2*5i)*((4*(32*
a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^10*b^8 + 1872*
a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) - ((a^2*12i - b^2*5i)*((4*(64*a^5*b^16
 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) - (((4*(32*a^8*b
^16 - 24*a^10*b^14))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a
^9*b^20 + 8*a^11*b^18))/(a^6*b^16))*(a^2*12i - b^2*5i))/(2*b^5) + (8*tan(c/
```

2 + (d*x)/2)*(64*a^4*b^22 - 68*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96 *a^12*b^14))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*a^3*b^20 - 180 *a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(4*b^20 - 4*a ^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9958*a^10*b^10 + 1 5912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16)))/(2*b^5) - ((a ^2*12i - b^2*5i)*((4*(20*a^3*b^12 - 6048*a^15 + 332*a^5*b^10 - 1947*a^7*b^8 + 3979*a^9*b^6 - 7038*a^11*b^4 + 10800*a^13*b^2))/(a^6*b^11) + ((a^2*12i - b^2*5i)*(((a^2*12i - b^2*5i)*((4*(64*a^5*b^16 - 48*a^7*b^14 + 160*a^9*b^12 - 168*a^11*b^10))/(a^6*b^11) + (((4*(32*a^8*b^16 - 24*a^10*b^14))/(a^6*b^1 1) + (8*tan(c/2 + (d*x)/2)*(64*a^7*b^22 - 68*a^9*b^20 + 8*a^11*b^18))/(a^6* b^16))*(a^2*12i - b^2*5i))/(2*b^5) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^22 - 6 8*a^6*b^20 + 192*a^8*b^18 - 280*a^10*b^16 + 96*a^12*b^14))/(a^6*b^16)))/(2* b^5) + (4*(32*a^2*b^16 - 24*a^4*b^14 + 160*a^6*b^12 + 32*a^8*b^10 - 1110*a^ 10*b^8 + 1872*a^12*b^6 - 864*a^14*b^4))/(a^6*b^11) + (8*tan(c/2 + (d*x)/2)* (4*a^3*b^20 - 180*a^5*b^18 + 725*a^7*b^16 - 3454*a^9*b^14 + 6506*a^11*b^12 - 3840*a^13*b^10 + 288*a^15*b^8))/(a^6*b^16)))/(2*b^5) + (8*tan(c/2 + (d*x) /2)*(4*b^20 - 4*a^2*b^18 + 445*a^4*b^16 - 2390*a^6*b^14 + 5544*a^8*b^12 - 9 958*a^10*b^10 + 15912*a^12*b^8 - 12960*a^14*b^6 + 3456*a^16*b^4))/(a^6*b^16)))/(2*b^5) - (8*(20*b^12 - 6048*a^12 + 132*a^2*b^10 - 837*a^4*b^8 + 2107*a ^6*b^6 - 6174*a^8*b^4 + 10800*a^10*b^2))/(a^6*b^11) + (16*tan(c/2 + (d*x)/2)*(41472*a^17 + 1100*a^5*b^12 - 7030*a^7*b^10 +...

3.1270 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=314

$$\frac{3ax}{b^4}+\frac{3\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{6(2a^6-a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d}+\frac{3b\tanh^{-1}}{}$$

```
[Out] 3*a*x/b^4+3*b*arctanh(cos(d*x+c))/a^4/d+cos(d*x+c)/b^3/d-cot(d*x+c)/a^3/d-1
/2*(a^2-b^2)^2*cos(d*x+c)/a^2/b^3/d/(a+b*sin(d*x+c))^2-3/2*(a^2-b^2)*cos(d*
x+c)/a/b^3/d/(a+b*sin(d*x+c))+2*(a^2-b^2)*(2*a^2+b^2)*cos(d*x+c)/a^3/b^3/d/
(a+b*sin(d*x+c))-6*(2*a^6-a^4*b^2-b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))/a^4/b^4/d/(a^2-b^2)^(1/2)+3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*
x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/b^4/d
```

Rubi [A]
time = 0.32, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {2976, 3855, 3852, 8, 2718, 2743, 2833, 12, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}+\frac{3(2a^2+b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}-\frac{(a^2-b^2)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2}-\frac{3(a^2-b^2)\cos(c+dx)}{2ab^3d(a+b\sin(c+dx))}+\frac{2(2a^2+b^2)(a^2-b^2)\cos(c+dx)}{a^3b^3d(a+b\sin(c+dx))}-\frac{6(2a^6-a^4b^2-b^6)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^4d\sqrt{a^2-b^2}}+\frac{3ax}{b^4}+\frac{\cos(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*x)/b^4 + (3*Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) - (6*(2*a^6 - a^4*b^2 - b^6)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*b^4*Sqrt[a^2 - b^2]*d) + (3*b*Ar
cTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b^3*d) - Cot[c + d*x]/(a^3*d)
 - ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^2*b^3*d*(a + b*Sin[c + d*x])^2) - (3*(a
^2 - b^2)*Cos[c + d*x])/(2*a*b^3*d*(a + b*Sin[c + d*x])) + (2*(a^2 - b^2)*(
2*a^2 + b^2)*Cos[c + d*x])/(a^3*b^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(\frac{3a}{b^4} - \frac{3b\csc(c+dx)}{a^4} + \frac{\csc^2(c+dx)}{a^3} - \frac{\sin(c+dx)}{b^3} - \frac{(a^2-b^2)}{a^2b^4(a+b\sin(c}\right.$$

$$= \frac{3ax}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a^3} - \frac{\int \sin(c+dx)\,dx}{b^3} - \frac{(3b)\int \csc(c+dx)\,dx}{a^4} - \frac{(a}{}$$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{(a^2-b^2)^2\cos(c+dx)}{2a^2b^3d(a+b\sin(c+dx))^2}$$

$$= \frac{3ax}{b^4} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{b^3d} - \frac{\cot(c+dx)}{a^3d} - \frac{(a^2-b^2)}{2a^2b^3d(a+}$$

$$= \frac{3ax}{b^4} - \frac{6(2a^6-a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx}{a^4d}$$

$$= \frac{3ax}{b^4} - \frac{6(2a^6-a^4b^2-b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^4\sqrt{a^2-b^2}\,d} + \frac{3b\tanh^{-1}(\cos(c+dx}{a^4d}$$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{6(2a^6-a^4b^2-b^6)\tan}{a^4b^4\sqrt{a^2}}$$

$$= \frac{3ax}{b^4} - \frac{4\left(\frac{1}{a^2}-\frac{2a^2}{b^4}+\frac{1}{b^2}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d} - \frac{\sqrt{a^2-b^2}\,(2a^2+b^2)\,t}{a^2}$$

Mathematica [A]

time = 4.98, size = 241, normalized size = 0.77

$$\frac{\frac{6a^5c}{b^4}+\frac{6a^5dx}{b^4}+\frac{6(-2a^6+a^4b^2-a^2b^4+2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-a\cot\left(\frac{1}{2}(c+dx)\right)+6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-6b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{a\cos(c+dx)(6a^5+a^3b^2-5ab^4-b(-9a^4+a^2b^2+4b^4)\sin(c+dx)+2a^3b^2\sin^2(c+dx))}{b^3(a+b\sin(c+dx))^2}+a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]

[Out] ((6*a^5*c)/b^4 + (6*a^5*d*x)/b^4 + (6*(-2*a^6 + a^4*b^2 - a^2*b^4 + 2*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]] - 6*b*Log[Sin[(c + d*x)/2]] + (a*Cos[c + d*x]*(6*a^5 + a^3*b^2 - 5*a*b^4 - b*(-9*a^4 + a^2*b^2 + 4*b^4)*Sin[c + d*x] + 2*a^3*b^2*Sin[c + d*x]^2))/(b^3*(a + b*Sin[c + d*x])^2) + a*Tan[(c + d*x)/2])/(2*a^4*d)

Maple [A]
time = 1.04, size = 326, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+6a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{1}{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}-\frac{2\left(-\frac{3a\,b^2\left(a^4+a^2b^2-2b^4\right)(}{2}\right.}{}}{}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+6a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^4}-\frac{1}{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{3b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}-\frac{2\left(-\frac{3a\,b^2\left(a^4+a^2b^2-2b^4\right)(}{2}\right.}{}}{}$
risch	$\frac{3ax}{b^4}+\frac{e^{i(dx+c)}}{2b^3d}+\frac{e^{-i(dx+c)}}{2b^3d}-\frac{i\left(21iab^5e^{i(dx+c)}-14ia^5be^{i(dx+c)}-24iab^5e^{3i(dx+c)}+ia^3b^3e^{i(dx+c)}-4ia^3b^3e^{3i(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)+2/b^4*(b/(1+tan(1/2*d*x+1/2*c)^2)+3*a*arctan(tan(1/2*d*x+1/2*c)))-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1/2*d*x+1/2*c))-2/a^4/b^4*((-3/2*a*b^2*(a^4+a^2*b^2-2*b^4)*tan(1/2*d*x+1/2*c)^3-1/2*b*(4*a^6+9*a^4*b^2-3*a^2*b^4-10*b^6)*tan(1/2*d*x+1/2*c)^2-1/2*b^2*a*(13*a^4+a^2*b^2-14*b^4)*tan(1/2*d*x+1/2*c)-2*a^6*b-1/2*a^4*b^3+5/2*a^2*b^5)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(2*a^6-a^4*b^2+a^2*b^4-2*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.62, size = 1171, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(24*a^6*b*d*x*cos(d*x + c)^2 - 24*a^6*b*d*x + 2*(9*a^5*b^2 - a^3*b^4 -
 6*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b +
a^3*b^3 + 2*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6
- (2*a^4*b^2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b
^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*
(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d
*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^5*b^2 - a^3*b^4 - 2*a
*b^6)*cos(d*x + c) + 6*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c
)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a*b^6
*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x +
c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(6*a^5*b^2*d*x*cos(d*x + c)^2 + 2*a^4*
b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3*a^2*b
^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b^5*d +
 (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), 1/2*(12*
a^6*b*d*x*cos(d*x + c)^2 - 12*a^6*b*d*x + (9*a^5*b^2 - a^3*b^4 - 6*a*b^6)*c
os(d*x + c)^3 - 3*(4*a^5*b + 2*a^3*b^3 + 4*a*b^5 - 2*(2*a^5*b + a^3*b^3 + 2
*a*b^5)*cos(d*x + c)^2 + (2*a^6 + 3*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (2*a^4*b^
2 + a^2*b^4 + 2*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(3*a^5*b^2 - a^3*b
^4 - 2*a*b^6)*cos(d*x + c) + 3*(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos
(d*x + c)^2 - a^2*b^5 - b^7)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*
(2*a*b^6*cos(d*x + c)^2 - 2*a*b^6 + (b^7*cos(d*x + c)^2 - a^2*b^5 - b^7)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + (6*a^5*b^2*d*x*cos(d*x + c)^2 +
2*a^4*b^3*cos(d*x + c)^3 - 6*(a^7 + a^5*b^2)*d*x - 3*(2*a^6*b + a^4*b^3 - 3
*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^5*b^5*d*cos(d*x + c)^2 - 2*a^5*b
^5*d + (a^4*b^6*d*cos(d*x + c)^2 - (a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 461, normalized size = 1.47

$$\frac{\frac{6\,(dx+c)a}{b^4}-\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{6\,(2\,a^6-a^4b^2+a^2b^4-2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4b^4}+\frac{2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-ab^3}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)a^4b^3}+\frac{2\left(3\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+13\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,a^6+a^4b^2-5\,a^2b^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2a^4b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(d*x + c)*a/b^4 - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + tan(1/2*d
*x + 1/2*c)/a^3 - 6*(2*a^6 - a^4*b^2 + a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^4*b^4) + (2*b^4*tan(1/2*d*x + 1/2*c)^3 - a*b^3*tan(1/
2*d*x + 1/2*c)^2 + 4*a^4*tan(1/2*d*x + 1/2*c) + 2*b^4*tan(1/2*d*x + 1/2*c)
- a*b^3)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^4*b^3) + 2*(3*a
^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^6*tan(1/2*d*x + 1/2*c)^2 + 9*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^2 - 3*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 - 10*b^6*tan(1/2*d*x + 1/2*c
)^2 + 13*a^5*b*tan(1/2*d*x + 1/2*c) + a^3*b^3*tan(1/2*d*x + 1/2*c) - 14*a*b
^5*tan(1/2*d*x + 1/2*c) + 4*a^6 + a^4*b^2 - 5*a^2*b^4)/((a*tan(1/2*d*x + 1/
2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^4*b^3))/d
```

Mupad [B]
time = 13.69, size = 2500, normalized size = 7.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) + ((tan(c/2 + (d*x)/2)^6*(6*a^4 - 12*b^4 + 5*a
^2*b^2))/b^2 - a^2 - (tan(c/2 + (d*x)/2)^2*(32*b^4 - 42*a^4 + a^2*b^2))/b^2
 + (tan(c/2 + (d*x)/2)^4*(48*a^4 - 44*b^4 + 5*a^2*b^2))/b^2 + (2*tan(c/2 +
(d*x)/2)*(6*a^5 - 7*a*b^4 + a^3*b^2))/b^3 + (4*tan(c/2 + (d*x)/2)^3*(6*a^6
- 5*b^6 - 6*a^2*b^4 + 9*a^4*b^2))/(a*b^3) + (2*tan(c/2 + (d*x)/2)^5*(6*a^6
- 10*b^6 - 5*a^2*b^4 + 9*a^4*b^2))/(a*b^3))/(d*(2*a^5*tan(c/2 + (d*x)/2)^7
+ tan(c/2 + (d*x)/2)^3*(6*a^5 + 8*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(6*a^5 +
8*a^3*b^2) + 2*a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 16*a
^4*b*tan(c/2 + (d*x)/2)^4 + 8*a^4*b*tan(c/2 + (d*x)/2)^6)) + (6*a*atan((648
0*tan(c/2 + (d*x)/2))/((6480*b^4)/a^4 - (12960*b^2)/a^2 - (5184*b^6)/a^6 +
(5184*b^8)/a^8 + (6480*a*tan(c/2 + (d*x)/2))/b - (5184*b*tan(c/2 + (d*x)/2)
```

)/a + (5184*b^3*tan(c/2 + (d*x)/2))/a^3 - (12960*a^3*tan(c/2 + (d*x)/2))/b^
3 + (6480*a^5*tan(c/2 + (d*x)/2))/b^5 + 6480) - (12960*tan(c/2 + (d*x)/2))/
((6480*b^2)/a^2 + (6480*a^2)/b^2 - (5184*b^4)/a^4 + (5184*b^6)/a^6 - (5184*
a*tan(c/2 + (d*x)/2))/b + (5184*b*tan(c/2 + (d*x)/2))/a + (6480*a^3*tan(c/2
 + (d*x)/2))/b^3 - (12960*a^5*tan(c/2 + (d*x)/2))/b^5 + (6480*a^7*tan(c/2 +
 (d*x)/2))/b^7 - 12960) - (6480*a)/(6480*b + 6480*a*tan(c/2 + (d*x)/2) - (1
2960*b^3)/a^2 + (6480*b^5)/a^4 - (5184*b^7)/a^6 + (5184*b^9)/a^8 - (5184*b^
2*tan(c/2 + (d*x)/2))/a - (12960*a^3*tan(c/2 + (d*x)/2))/b^2 + (5184*b^4*ta
n(c/2 + (d*x)/2))/a^3 + (6480*a^5*tan(c/2 + (d*x)/2))/b^4) + 5184/((6480*a)
/b - 5184*tan(c/2 + (d*x)/2) - (12960*b)/a + (6480*b^3)/a^3 - (5184*b^5)/a^
5 + (5184*b^7)/a^7 + (5184*b^2*tan(c/2 + (d*x)/2))/a^2 + (6480*a^2*tan(c/2
+ (d*x)/2))/b^2 - (12960*a^4*tan(c/2 + (d*x)/2))/b^4 + (6480*a^6*tan(c/2 +
(d*x)/2))/b^6) - 5184/(5184*tan(c/2 + (d*x)/2) - (12960*a)/b + (6480*b)/a -
 (5184*b^3)/a^3 + (6480*a^3)/b^3 + (5184*b^5)/a^5 - (5184*a^2*tan(c/2 + (d*
x)/2))/b^2 + (6480*a^4*tan(c/2 + (d*x)/2))/b^4 - (12960*a^6*tan(c/2 + (d*x)
/2))/b^6 + (6480*a^8*tan(c/2 + (d*x)/2))/b^8) + (6480*tan(c/2 + (d*x)/2))/(
(5184*b^4)/a^4 - (12960*a^2)/b^2 - (5184*b^2)/a^2 + (6480*a^4)/b^4 + (5184*
a*tan(c/2 + (d*x)/2))/b - (5184*a^3*tan(c/2 + (d*x)/2))/b^3 + (6480*a^5*tan
(c/2 + (d*x)/2))/b^5 - (12960*a^7*tan(c/2 + (d*x)/2))/b^7 + (6480*a^9*tan(c
/2 + (d*x)/2))/b^9 + 6480) + (12960*a^3)/(6480*b^3 - (12960*b^5)/a^2 + (648
0*b^7)/a^4 - (5184*b^9)/a^6 + (5184*b^11)/a^8 - 12960*a^3*tan(c/2 + (d*x)/2
) + 6480*a*b^2*tan(c/2 + (d*x)/2) - (5184*b^4*tan(c/2 + (d*x)/2))/a + (6480
*a^5*tan(c/2 + (d*x)/2))/b^2 + (5184*b^6*tan(c/2 + (d*x)/2))/a^3) - (6480*a
^5)/(6480*b^5 - (12960*b^7)/a^2 + (6480*b^9)/a^4 - (5184*b^11)/a^6 + (5184*
b^13)/a^8 + 6480*a^5*tan(c/2 + (d*x)/2) + 6480*a*b^4*tan(c/2 + (d*x)/2) - 1
2960*a^3*b^2*tan(c/2 + (d*x)/2) - (5184*b^6*tan(c/2 + (d*x)/2))/a + (5184*b
^8*tan(c/2 + (d*x)/2))/a^3) - (5184*b*tan(c/2 + (d*x)/2))/((6480*a^2)/b - 5
184*b + (5184*b^3)/a^2 - (12960*a^4)/b^3 + (6480*a^6)/b^5 + (5184*a^3*tan(c
/2 + (d*x)/2))/b^2 - (5184*a^5*tan(c/2 + (d*x)/2))/b^4 + (6480*a^7*tan(c/2
+ (d*x)/2))/b^6 - (12960*a^9*tan(c/2 + (d*x)/2))/b^8 + (6480*a^11*tan(c/2 +
 (d*x)/2))/b^10) + (5184*b^3*tan(c/2 + (d*x)/2))/(5184*b^3 - 5184*a^2*b + (
6480*a^4)/b - (12960*a^6)/b^3 + (6480*a^8)/b^5 + (5184*a^5*tan(c/2 + (d*x)/
2))/b^2 - (5184*a^7*tan(c/2 + (d*x)/2))/b^4 + (6480*a^9*tan(c/2 + (d*x)/2))
/b^6 - (12960*a^11*tan(c/2 + (d*x)/2))/b^8 + (6480*a^13*tan(c/2 + (d*x)/2))
/b^10)))/(b^4*d) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) + (atan((((-(a + b
)*(a - b))^(1/2)*(2*a^4 + 2*b^4 + a^2*b^2)*((8*tan(c/2 + (d*x)/2)*(108*b^19
 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^8*b^11 - 1836*a^10*b
^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9*b^12) - (8*(378*a^1
4 + 108*a^4*b^10 - 108*a^6*b^8 - 729*a^8*b^6 + 378*a^10*b^4 - 135*a^12*b^2)
)/(a^8*b^8) + (3*(-(a + b)*(a - b))^(1/2)*((8*(144*a^2*b^15 - 108*a^4*b^13
+ 90*a^6*b^11 - 126*a^8*b^9 + 144*a^12*b^5 - 108*a^14*b^3))/(a^8*b^8) + (8*
tan(c/2 + (d*x)/2)*(36*a^4*b^18 - 180*a^6*b^16 + 405*a^8*b^14 - 306*a^10*b^
12 + 909*a^12*b^10 - 900*a^14*b^8 + 72*a^16*b^6))/(a^9*b^12) + (3*(-(a + b)
(a - b))^(1/2)((8*(96*a^6*b^14 - 72*a^8*b^12 + 30*a^10*b^10 - 42*a^12*b^8
))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(192*a^6*b^19 - 204*a^8*b^17 + 96*a^10

```
*b^15 - 120*a^12*b^13 + 48*a^14*b^11))/(a^9*b^12) + (3*(-(a + b)*(a - b))^(
1/2)*((8*(16*a^10*b^13 - 12*a^12*b^11))/(a^8*b^8) + (8*tan(c/2 + (d*x)/2)*(
64*a^10*b^18 - 68*a^12*b^16 + 8*a^14*b^14))/(a^9*b^12))*(2*a^4 + 2*b^4 + a^
2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4 + a^2*b^2))/(2*a^4*b^4))*(2*a^4 + 2*b^4
 + a^2*b^2))/(2*a^4*b^4))*3i)/(2*a^4*b^4) - ((-(a + b)*(a - b))^(1/2)*(2*a^
4 + 2*b^4 + a^2*b^2)*((8*(378*a^14 + 108*a^4*b^10 - 108*a^6*b^8 - 729*a^8*b
^6 + 378*a^10*b^4 - 135*a^12*b^2))/(a^8*b^8) - (8*tan(c/2 + (d*x)/2)*(108*b
^19 - 108*a^2*b^17 + 135*a^4*b^15 - 270*a^6*b^13 + 1863*a^8*b^11 - 1836*a^1
0*b^9 + 864*a^12*b^7 - 1080*a^14*b^5 + 432*a^16*b^3))/(a^9*b^12) + (3*(-(a
+ b)*(a - b))^(1/2)*((8*(144*a^2*b^15 - 108*a^4*b^13 + 90*a^6*b^11 - 126*a^
8*b^9 + 144*a^12*b^5 - 108*a^14*b^3))/(a^8*b^8)...
```

3.1271 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=395

$$-\frac{x}{b^3}-\frac{6\sqrt{a^2-b^2}\,(a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{6(a^6+a^2}{}$$

```
[Out] -x/b^3-1/2*arctanh(cos(d*x+c))/a^3/d+3*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^5/
d+3*b*cot(d*x+c)/a^4/d-1/2*cot(d*x+c)*csc(d*x+c)/a^3/d+1/2*(a^2-b^2)^2*cos(
d*x+c)/a^3/b^2/d/(a+b*sin(d*x+c))^2+3/2*(a^2-b^2)*cos(d*x+c)/a^2/b^2/d/(a+b
*sin(d*x+c))-3*(a^4-b^4)*cos(d*x+c)/a^4/b^2/d/(a+b*sin(d*x+c))+6*(a^6+a^2*b
^4-2*b^6)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/b^3/d/(a^2-b
^2)^(1/2)-6*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2
-b^2)^(1/2)/a^3/b^3/d+(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))*(a^2-b^2)^(1/2)/a^3/b^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2976, 3855, 3852, 8, 3853, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{3(a^4-b^4)\cos(c+dx)}{a^4b^2d(a+b\sin(c+dx))}+\frac{3b\cot(c+dx)}{a^4d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3(a^2-b^2)\cos(c+dx)}{2a^2b^2d(a+b\sin(c+dx))}+\frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{6(a^2+b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{(2a^2+b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{(a^2-b^2)^2\cos(c+dx)}{2a^3b^2d(a+b\sin(c+dx))^2}+\frac{6(a^6+a^2b^4-2b^6)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5b^3d\sqrt{a^2-b^2}}-\frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(x/b^3) - (6*Sqrt[a^2 - b^2]*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(a^3*b^3*d) + (Sqrt[a^2 - b^2]*(2*a^2 + b^2)*ArcTan[(b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + (6*(a^6 + a^2*b^4 - 2*b^
6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*b^3*Sqrt[a^2 - b^
2]*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) + (3*(a^2 - 2*b^2)*ArcTanh[Cos[c +
d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x])/(
2*a^3*d) + ((a^2 - b^2)^2*Cos[c + d*x])/(2*a^3*b^2*d*(a + b*Sin[c + d*x])^2
) + (3*(a^2 - b^2)*Cos[c + d*x])/(2*a^2*b^2*d*(a + b*Sin[c + d*x])) - (3*(a
^4 - b^4)*Cos[c + d*x])/(a^4*b^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{b^3}-\frac{3(a^2-2b^2)\csc(c+dx)}{a^5}-\frac{3b\csc^2(c+dx)}{a^4}+\frac{\csc^3(c+dx)}{a^3}+\frac{}{a^3}\right.$$

$$= -\frac{x}{b^3}+\frac{\int\csc^3(c+dx)\,dx}{a^3}-\frac{(3b)\int\csc^2(c+dx)\,dx}{a^4}-\frac{(3(a^2-2b^2))\int\csc(c}{a^5}$$

$$= -\frac{x}{b^3}+\frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{(a}{2a^3b}$$

$$= -\frac{x}{b^3}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{3(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{3b\cot(c}{a^4}$$

$$= -\frac{x}{b^3}+\frac{6(a^6+a^2b^4-2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}$$

$$= -\frac{x}{b^3}+\frac{6(a^6+a^2b^4-2b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}$$

$$= -\frac{x}{b^3}-\frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{6(a^6+a^2b^4-2b^6)\tan^{-1}\left(\frac{b+}{}\right.}{a^5b^3\sqrt{a^2-b^2}\,d}$$

$$= -\frac{x}{b^3}-\frac{6\left(\frac{a}{b^3}-\frac{b}{a^3}\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}+\frac{\sqrt{a^2-b^2}\,(2a^2+b^2)\tan^{-1}\left(\right.}{a^3b^3d}$$

Mathematica [A]
time = 6.21, size = 384, normalized size = 0.97

$$-\frac{c+dx}{b^3 d}+\frac{(2a^6-a^4b^2+11a^2b^4-12b^6)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5b^3\sqrt{a^2-b^2}\,d}+\frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}+\frac{(5a^2-12b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d}+\frac{(-5a^2+12b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}+\frac{a^4\cos(c+dx)-2a^2b^2\cos(c+dx)+b^4\cos(c+dx)}{2a^3b^2d(a+b\sin(c+dx))^2}-\frac{3(a^4\cos(c+dx)+a^2b^2\cos(c+dx)-2b^4\cos(c+dx))}{2a^4b^2d(a+b\sin(c+dx))}-\frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((c + d*x)/(b^3*d)) + ((2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*ArcTan[(Sec
[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/
(a^5*b^3*Sqrt[a^2 - b^2]*d) + (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d
*x)/2]^2/(8*a^3*d) + ((5*a^2 - 12*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (
(-5*a^2 + 12*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*
a^3*d) + (a^4*Cos[c + d*x] - 2*a^2*b^2*Cos[c + d*x] + b^4*Cos[c + d*x])/(2*
a^3*b^2*d*(a + b*Sin[c + d*x])^2) - (3*(a^4*Cos[c + d*x] + a^2*b^2*Cos[c +
d*x] - 2*b^4*Cos[c + d*x]))/(2*a^4*b^2*d*(a + b*Sin[c + d*x])) - (3*b*Tan[(
c + d*x)/2])/(2*a^4*d)
```

Maple [A]
time = 1.03, size = 357, normalized size = 0.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^4*(1/2*a*tan(1/2*d*x+1/2*c)^2-6*b*tan(1/2*d*x+1/2*c))-2/b^3*arct
an(tan(1/2*d*x+1/2*c))-1/8/a^3/tan(1/2*d*x+1/2*c)^2+1/4/a^5*(-10*a^2+24*b^2
)*ln(tan(1/2*d*x+1/2*c))+3/2*b/a^4/tan(1/2*d*x+1/2*c)+2/a^5/b^3*(((-1/2*a^5
*b^2-7/2*a^3*b^4+4*a*b^6)*tan(1/2*d*x+1/2*c)^3-1/2*b*(2*a^6+9*a^4*b^2+3*a^2
*b^4-14*b^6)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*(7*a^4+13*a^2*b^2-20*b^4)*tan(1
/2*d*x+1/2*c)-1/2*a^2*b*(2*a^4+5*a^2*b^2-7*b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*
b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^6-a^4*b^2+11*a^2*b^4-12*b^6)/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(372) = 744$.
time = 0.66, size = 1658, normalized size = 4.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x + c)^
2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*b^2)
*d*x - (2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^4 + 12
*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(d*x + c) - (5*a^4*b^3 -
7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3 - 2*a
^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^4 - 12
*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (5*a^4*
b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x + c)^4 - (5*a^4*b^3
 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12*a*b^6 - (5*a^3*b^
4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) -
2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^5*b^2 + a^3*b^4 - 4*a*b^
6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6)*cos(d*x + c))*sin(d*x
+ c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*b^5)*d*cos(d*x + c)^2 +
(a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2 - a^6*b^4*d)*sin(d*x +
c)), -1/4*(4*a^5*b^2*d*x*cos(d*x + c)^4 - 4*(a^7 + 2*a^5*b^2)*d*x*cos(d*x +
 c)^2 - 2*(2*a^6*b + 5*a^4*b^3 - 18*a^2*b^5)*cos(d*x + c)^3 + 4*(a^7 + a^5*
b^2)*d*x + 2*(2*a^6 + 3*a^4*b^2 + 13*a^2*b^4 + 12*b^6 + (2*a^4*b^2 + a^2*b^
4 + 12*b^6)*cos(d*x + c)^4 - (2*a^6 + 5*a^4*b^2 + 14*a^2*b^4 + 24*b^6)*cos(
d*x + c)^2 + 2*(2*a^5*b + a^3*b^3 + 12*a*b^5 - (2*a^5*b + a^3*b^3 + 12*a*b^
5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 4*(a^6*b + 3*a^4*b^3 - 9*a^2*b^5)*cos(
d*x + c) - (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos(d*x +
 c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^4 - 12
*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) + (5*a^4*b^3 - 7*a^2*b^5 - 12*b^7 + (5*a^2*b^5 - 12*b^7)*cos
(d*x + c)^4 - (5*a^4*b^3 - 2*a^2*b^5 - 24*b^7)*cos(d*x + c)^2 + 2*(5*a^3*b^
4 - 12*a*b^6 - (5*a^3*b^4 - 12*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) - 2*(4*a^6*b*d*x*cos(d*x + c)^2 - 4*a^6*b*d*x + 3*(a^
5*b^2 + a^3*b^4 - 4*a*b^6)*cos(d*x + c)^3 - (3*a^5*b^2 - a^3*b^4 - 12*a*b^6
)*cos(d*x + c))*sin(d*x + c))/(a^5*b^5*d*cos(d*x + c)^4 - (a^7*b^3 + 2*a^5*
```

```
b^5)*d*cos(d*x + c)^2 + (a^7*b^3 + a^5*b^5)*d - 2*(a^6*b^4*d*cos(d*x + c)^2
 - a^6*b^4*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [A]
time = 0.53, size = 512, normalized size = 1.30

$$-\frac{\frac{8\,(dx+c)}{b^3}+\frac{4\,(5\,a^2-12\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}-\frac{8\,(2\,a^6-a^4b^2+11\,a^2b^4-12\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5b^3}-\frac{10\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-24\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-8\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-32\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-16\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-53\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+16\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-56\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-44\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+112\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-32\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+76\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^4b^2}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2a^5b^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(8*(d*x + c)/b^3 + 4*(5*a^2 - 12*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a
^5 - (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6 - 8*(
2*a^6 - a^4*b^2 + 11*a^2*b^4 - 12*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^
2)*a^5*b^3) - (10*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^4*tan(1/2*d*x +
 1/2*c)^6 - 8*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 4*a^3*b^3*tan(1/2*d*x + 1/2*c)
^5 - 32*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 16*a^6*tan(1/2*d*x + 1/2*c)^4 - 53*a
^4*b^2*tan(1/2*d*x + 1/2*c)^4 + 16*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 + 16*b^6*
tan(1/2*d*x + 1/2*c)^4 - 56*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 44*a^3*b^3*tan(1
/2*d*x + 1/2*c)^3 + 112*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 16*a^6*tan(1/2*d*x +
 1/2*c)^2 - 32*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^4*tan(1/2*d*x + 1/
2*c)^2 + 8*a^3*b^3*tan(1/2*d*x + 1/2*c) - a^4*b^2)/((a*tan(1/2*d*x + 1/2*c)
^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2*a^5*b^2))/d
```

Mupad [B]
time = 13.02, size = 2500, normalized size = 6.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (2*atan((1800*tan(c/2 + (d*x)/2))/((7120*b
^2)/a^2 - (12520*b^4)/a^4 + (24480*b^6)/a^6 - (31104*b^8)/a^8 + (13824*b^10
)/a^10 - (1120*a*tan(c/2 + (d*x)/2))/b + (2320*b*tan(c/2 + (d*x)/2))/a - (4
```

224*b^3*tan(c/2 + (d*x)/2))/a^3 + (720*a^3*tan(c/2 + (d*x)/2))/b^3 + (2304*
b^5*tan(c/2 + (d*x)/2))/a^5 - 1800) - (7120*tan(c/2 + (d*x)/2))/((24480*b^4
)/a^4 - (1800*a^2)/b^2 - (12520*b^2)/a^2 - (31104*b^6)/a^6 + (13824*b^8)/a^
8 + (2320*a*tan(c/2 + (d*x)/2))/b - (4224*b*tan(c/2 + (d*x)/2))/a + (2304*b
^3*tan(c/2 + (d*x)/2))/a^3 - (1120*a^3*tan(c/2 + (d*x)/2))/b^3 + (720*a^5*t
an(c/2 + (d*x)/2))/b^5 + 7120) + (720*a)/(720*a*tan(c/2 + (d*x)/2) - (1800*
b^3)/a^2 + (7120*b^5)/a^4 - (12520*b^7)/a^6 + (24480*b^9)/a^8 - (31104*b^11
)/a^10 + (13824*b^13)/a^12 - (1120*b^2*tan(c/2 + (d*x)/2))/a + (2320*b^4*ta
n(c/2 + (d*x)/2))/a^3 - (4224*b^6*tan(c/2 + (d*x)/2))/a^5 + (2304*b^8*tan(c
/2 + (d*x)/2))/a^7) - 4224/((7120*a)/b - 4224*tan(c/2 + (d*x)/2) - (12520*b
)/a + (24480*b^3)/a^3 - (1800*a^3)/b^3 - (31104*b^5)/a^5 + (13824*b^7)/a^7
 + (2304*b^2*tan(c/2 + (d*x)/2))/a^2 + (2320*a^2*tan(c/2 + (d*x)/2))/b^2 - (
1120*a^4*tan(c/2 + (d*x)/2))/b^4 + (720*a^6*tan(c/2 + (d*x)/2))/b^6) + 2320
/(2320*tan(c/2 + (d*x)/2) - (1800*a)/b + (7120*b)/a - (12520*b^3)/a^3 + (24
480*b^5)/a^5 - (31104*b^7)/a^7 + (13824*b^9)/a^9 - (4224*b^2*tan(c/2 + (d*x
)/2))/a^2 - (1120*a^2*tan(c/2 + (d*x)/2))/b^2 + (2304*b^4*tan(c/2 + (d*x)/2
))/a^4 + (720*a^4*tan(c/2 + (d*x)/2))/b^4) + (2304*b^2)/((24480*b^3)/a - 12
520*a*b + (7120*a^3)/b - (31104*b^5)/a^3 - (1800*a^5)/b^3 + (13824*b^7)/a^5
 - 4224*a^2*tan(c/2 + (d*x)/2) + 2304*b^2*tan(c/2 + (d*x)/2) + (2320*a^4*ta
n(c/2 + (d*x)/2))/b^2 - (1120*a^6*tan(c/2 + (d*x)/2))/b^4 + (720*a^8*tan(c/
2 + (d*x)/2))/b^6) - 1120/((7120*b^3)/a^3 - (1800*b)/a - 1120*tan(c/2 + (d*
x)/2) - (12520*b^5)/a^5 + (24480*b^7)/a^7 - (31104*b^9)/a^9 + (13824*b^11)/
a^11 + (2320*b^2*tan(c/2 + (d*x)/2))/a^2 + (720*a^2*tan(c/2 + (d*x)/2))/b^2
 - (4224*b^4*tan(c/2 + (d*x)/2))/a^4 + (2304*b^6*tan(c/2 + (d*x)/2))/a^6) -
 (24480*b^3*tan(c/2 + (d*x)/2))/(24480*b^3 - 12520*a^2*b + (7120*a^4)/b - (
31104*b^5)/a^2 - (1800*a^6)/b^3 + (13824*b^7)/a^4 - 4224*a^3*tan(c/2 + (d*x
)/2) + 2304*a*b^2*tan(c/2 + (d*x)/2) + (2320*a^5*tan(c/2 + (d*x)/2))/b^2 -
(1120*a^7*tan(c/2 + (d*x)/2))/b^4 + (720*a^9*tan(c/2 + (d*x)/2))/b^6) + (31
104*b^5*tan(c/2 + (d*x)/2))/(24480*a^2*b^3 - 31104*b^5 - 12520*a^4*b + (712
0*a^6)/b + (13824*b^7)/a^2 - (1800*a^8)/b^3 - 4224*a^5*tan(c/2 + (d*x)/2) +
 2304*a^3*b^2*tan(c/2 + (d*x)/2) + (2320*a^7*tan(c/2 + (d*x)/2))/b^2 - (112
0*a^9*tan(c/2 + (d*x)/2))/b^4 + (720*a^11*tan(c/2 + (d*x)/2))/b^6) - (13824
*b^7*tan(c/2 + (d*x)/2))/(13824*b^7 - 12520*a^6*b - 31104*a^2*b^5 + 24480*a
^4*b^3 + (7120*a^8)/b - (1800*a^10)/b^3 - 4224*a^7*tan(c/2 + (d*x)/2) + 230
4*a^5*b^2*tan(c/2 + (d*x)/2) + (2320*a^9*tan(c/2 + (d*x)/2))/b^2 - (1120*a^
11*tan(c/2 + (d*x)/2))/b^4 + (720*a^13*tan(c/2 + (d*x)/2))/b^6) + (12520*b*
tan(c/2 + (d*x)/2))/((7120*a^2)/b - 4224*a*tan(c/2 + (d*x)/2) - 12520*b + (
24480*b^3)/a^2 - (1800*a^4)/b^3 - (31104*b^5)/a^4 + (13824*b^7)/a^6 + (2304
*b^2*tan(c/2 + (d*x)/2))/a + (2320*a^3*tan(c/2 + (d*x)/2))/b^2 - (1120*a^5*
tan(c/2 + (d*x)/2))/b^4 + (720*a^7*tan(c/2 + (d*x)/2))/b^6)))/(b^3*d) - (a^
3/2 + (tan(c/2 + (d*x)/2)^2*(8*a^5 - 50*a*b^4 + 21*a^3*b^2))/b^2 - 4*a^2*b*
tan(c/2 + (d*x)/2) + (2*tan(c/2 + (d*x)/2)^5*(2*a^4 - 16*b^4 + 11*a^2*b^2))
/b + (2*tan(c/2 + (d*x)/2)^3*(14*a^4 - 52*b^4 + 21*a^2*b^2))/b + (tan(c/2 +
 (d*x)/2)^4*(16*a^6 - 112*b^6 - 24*a^2*b^4 + 73*a^4*b^2))/(2*a*b^2))/(d*(4*
a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^

4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*tan(c/2 + (d*x)/2)^5)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(5*a^2 - 12*b^2))/(2*a^5*d) - (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*tan(c/2 + (d*x)/2)*(1728*b^18 - 3888*a^2*b^16 + 3060*a^4*b^14 - 1565*a^6*b^12 + 1658*a^8*b^10 - 1361*a^10*b^8 + 484*a^12*b^6 - 120*a^14*b^4 + 32*a^16*b^2))/(a^12*b^8) - (4*(28*a^15 + 288*a^5*b^10 - 528*a^7*b^8 - 94*a^9*b^6 + 308*a^11*b^4 - 30*a^13*b^2))/(a^12*b^5) + ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(1152*a^4*b^14 - 1824*a^6*b^12 + 920*a^8*b^10 - 318*a^10*b^8 + 70*a^12*b^6 + 32*a^14*b^4 - 24*a^16*b^2))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(288*a^5*b^16 - 240*a^7*b^14 + 410*a^9*b^12 - 468*a^11*b^10 + 202*a^13*b^8 - 200*a^15*b^6 + 16*a^17*b^4))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(384*a^9*b^12 - 448*a^11*b^10 + 120*a^13*b^8 - 28*a^15*b^6))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(768*a^8*b^16 - 1136*a^10*b^14 + 484*a^12*b^12 - 120*a^14*b^10 + 32*a^16*b^8))/(a^12*b^8) + ((-(a + b)*(a - b))^(1/2)*((4*(32*a^14*b^10 - 24*a^16*b^8))/(a^12*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^14 - 136*a^15*b^12 + 16*a^17*b^10))/(a^12*b^8))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3))*(a^4 + 6*b^4 + (a^2*b^2)/2))/(a^5*b^3)))/(a^5*b^3))*1i)/(a^5*b^3) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 6*b^4 + (a^2*b^2)/2)*((4*(28*a^15 + 288*a^5*b^10 - ...

3.1272 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$\frac{5(a^2-4b^2)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}+\frac{(3a^4+35a^2b^2-60b^4)\,\mathrm{co}}{6a^5b^2d}$$

```
[Out] -5/2*b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(3*a^4+35*a^2*b^2-60*b^4
)*cot(d*x+c)/a^5/b^2/d-cos(d*x+c)/b/d/(a+b*sin(d*x+c))^2-1/2*a*cot(d*x+c)/b
^2/d/(a+b*sin(d*x+c))^2-1/3*(3*a^2-5*b^2)*cot(d*x+c)/a^3/d/(a+b*sin(d*x+c))
^2+5/6*b*cot(d*x+c)*csc(d*x+c)/a^2/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(
d*x+c)^2/a/d/(a+b*sin(d*x+c))^2-5/2*(a^2-2*b^2)*cot(d*x+c)/a^4/d/(a+b*sin(d
*x+c))+5*(a^2-4*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-
b^2)^(1/2)/a^6/d
```

Rubi [A]
time = 0.83, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{5b\cot(c+dx)\csc(c+dx)}{6a^2d(a+b\sin(c+dx))^2}+\frac{5(a^2-4b^2)\sqrt{a^2-b^2}\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d}-\frac{5(a^2-2b^2)\cot(c+dx)}{2a^4d(a+b\sin(c+dx))}-\frac{(3a^2-5b^2)\cot(c+dx)}{3a^3d(a+b\sin(c+dx))^2}+\frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d}-\frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2}-\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^2 - 4*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^6*d) - (5*b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^6*d) + (
(3*a^4 + 35*a^2*b^2 - 60*b^4)*Cot[c + d*x])/(6*a^5*b^2*d) - Cos[c + d*x]/(b
*d*(a + b*Sin[c + d*x])^2) - (a*Cot[c + d*x])/(2*b^2*d*(a + b*Sin[c + d*x])
^2) - ((3*a^2 - 5*b^2)*Cot[c + d*x])/(3*a^3*d*(a + b*Sin[c + d*x])^2) + (5*
b*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*
x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) - (5*(a^2 - 2*b^2)*Cot[c
+ d*x])/(2*a^4*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} + \frac{5b\cot(c+dx)\csc(c}{6a^2d(a+b\sin(c+}$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{(3a^2-5b^2)\cot(c+}{3a^3d(a+b\sin(c+d}$$

$$= -\frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(c+dx)}{2b^2d(a+b\sin(c+dx))^2} - \frac{(3a^2-5b^2)\cot(c+}{3a^3d(a+b\sin(c+d}$$

$$= \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(}{2b^2d(a+b\,\mathrm{s}}$$

$$= \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d} - \frac{\cos(c+dx)}{bd(a+b\sin(c+dx))^2} - \frac{a\cot(}{2b^2d(a+b\,\mathrm{s}}$$

$$= -\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d}$$

$$= -\frac{5b(3a^2-4b^2)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(3a^4+35a^2b^2-60b^4)\cot(c+dx)}{6a^5b^2d}$$

$$= \frac{5(a^2-4b^2)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{5b(3a^2-4b^2)\tanh^{-1}(\cos}{2a^6d}$$

Mathematica [A]
time = 6.19, size = 490, normalized size = 1.49

$$\frac{5(a^4-5a^2b^2+4b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} + \frac{\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-18b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^5d} + \frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d} - \frac{5(3a^2b-4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} + \frac{5(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d} - \frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2\sin\left(\frac{1}{2}(c+dx)\right)+18b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^5d} + \frac{-a^4\cos(c+dx)+2a^2b^2\cos(c+dx)-b^4\cos(c+dx)}{2a^4bd(a+b\sin(c+dx))^2} + \frac{a^4\cos(c+dx)+7a^2b^2\cos(c+dx)-8b^4\cos(c+dx)}{2a^5bd(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((7*a^2*C
os[(c + d*x)/2] - 18*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^5*d) + (3
*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(2
4*a^3*d) - (5*(3*a^2*b - 4*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + (5*(3*a^
2*b - 4*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 18*b^2*Sin[(c + d*x)
/2]))/(6*a^5*d) + (-(a^4*Cos[c + d*x]) + 2*a^2*b^2*Cos[c + d*x] - b^4*Cos[c
 + d*x])/(2*a^4*b*d*(a + b*Sin[c + d*x])^2) + (a^4*Cos[c + d*x] + 7*a^2*b^2
*Cos[c + d*x] - 8*b^4*Cos[c + d*x])/(2*a^5*b*d*(a + b*Sin[c + d*x])) + (Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [A]
time = 1.01, size = 359, normalized size = 1.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^5*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-3*a*b*tan(1/2*d*x+1/2*c)^2-9*a^2
*tan(1/2*d*x+1/2*c)+24*b^2*tan(1/2*d*x+1/2*c))-1/24/a^3/tan(1/2*d*x+1/2*c)^
3-1/8*(-9*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)+3/8/a^4*b/tan(1/2*d*x+1/2*c)^2
+5/2/a^6*b*(3*a^2-4*b^2)*ln(tan(1/2*d*x+1/2*c))+2/a^6*((-1/2*a*(a^4-11*a^2*
b^2+10*b^4)*tan(1/2*d*x+1/2*c)^3+9/2*b*(a^4+a^2*b^2-2*b^4)*tan(1/2*d*x+1/2*
c)^2+1/2*a*(a^4+25*a^2*b^2-26*b^4)*tan(1/2*d*x+1/2*c)+9/2*a^4*b-9/2*a^2*b^3
)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+5/2*(a^4-5*a^2*b^2+4*
b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2
))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(310) = 620$.

time = 0.51, size = 1553, normalized size = 4.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5 - 20*(2*a^5 + 5*a^3
*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 15*(2*(a^3*b - 4*a*b^3)*cos(d*x + c)^4 +
2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2 + ((a^2*b^2 - 4*b^4)
*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^2*b^2 - 8*b^4)*cos(d
*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2
 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2)) + 30*(a^5 + a^3*b^2 - 4*a*b^4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b
^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d
*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4
 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 10*((11*a^4*b - 1
8*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^
6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^
2)*d)*sin(d*x + c)), 1/12*(2*(3*a^5 + 35*a^3*b^2 - 60*a*b^4)*cos(d*x + c)^5
 - 20*(2*a^5 + 5*a^3*b^2 - 12*a*b^4)*cos(d*x + c)^3 - 30*(2*(a^3*b - 4*a*b^
3)*cos(d*x + c)^4 + 2*a^3*b - 8*a*b^3 - 4*(a^3*b - 4*a*b^3)*cos(d*x + c)^2
+ ((a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + a^4 - 3*a^2*b^2 - 4*b^4 - (a^4 - 2*a^
2*b^2 - 8*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(a^5 + a^3*b^2 - 4*a*b^
4)*cos(d*x + c) - 15*(6*a^3*b^2 - 8*a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x
 + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b
^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(6*a^3*b^2 - 8*
a*b^4 + 2*(3*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^4 - 4*(3*a^3*b^2 - 4*a*b^4)*co
s(d*x + c)^2 + (3*a^4*b - a^2*b^3 - 4*b^5 + (3*a^2*b^3 - 4*b^5)*cos(d*x + c
)^4 - (3*a^4*b + 2*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*
cos(d*x + c) + 1/2) - 10*((11*a^4*b - 18*a^2*b^3)*cos(d*x + c)^3 - 6*(2*a^4
*b - 3*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(2*a^7*b*d*cos(d*x + c)^4 - 4*a
^7*b*d*cos(d*x + c)^2 + 2*a^7*b*d + (a^6*b^2*d*cos(d*x + c)^4 - (a^8 + 2*a^
6*b^2)*d*cos(d*x + c)^2 + (a^8 + a^6*b^2)*d)*sin(d*x + c))]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.51, size = 478, normalized size = 1.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(60*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 120*(a^4 -
5*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - 24*(a^5*ta
n(1/2*d*x + 1/2*c)^3 - 11*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 10*a*b^4*tan(1/2
*d*x + 1/2*c)^3 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b^3*tan(1/2*d*x +
1/2*c)^2 + 18*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5*tan(1/2*d*x + 1/2*c) - 25*a^
3*b^2*tan(1/2*d*x + 1/2*c) + 26*a*b^4*tan(1/2*d*x + 1/2*c) - 9*a^4*b + 9*a^
2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^6) +
(a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2*c)^2 - 27*a^6*tan(1
/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9 - (330*a^2*b*tan(1/2
*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2
*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1/2*d*x + 1/2*c) + a^
3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

Mupad [B]
time = 12.41, size = 1082, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*a^3*d) + (tan(c/2 + (d*x)/2)^4*((77*a^4)/3 - 304*b
^4 + 200*a^2*b^2) + tan(c/2 + (d*x)/2)^6*(a^4 - 80*b^4 + 64*a^2*b^2) - a^4/
3 + tan(c/2 + (d*x)/2)^2*((25*a^4)/3 - (40*a^2*b^2)/3) - tan(c/2 + (d*x)/2)
^3*(156*a*b^3 - (338*a^3*b)/3) - (3*tan(c/2 + (d*x)/2)^5*(48*b^5 - 37*a^4*b
 + 8*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^7*tan(c/2 + (d*x
)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5*(16*a^7 + 32*a^5
```

```
*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2 + (d*x)/2)^6)) - (
tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(8*a^5) + 3/(4*a^3) - (9*b^2)/(2*a^5)
))/d + (log(tan(c/2 + (d*x)/2))*(15*a^2*b - 20*b^3))/(2*a^6*d) - (3*b*tan(c
/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^
10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4
*b^5 - 100*a^6*b^3))/a^9 - (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^
10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/(2*a^6))*5i)/(2*a^6) + ((b^2
 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan
(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4*b^5 - 100*a^6*b^3))/a^9 + (5*(2*a^2*b -
(tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*
b^2))/(2*a^6))*5i)/(2*a^6))/((75*a^6*b - 400*b^7 + 800*a^2*b^5 - 475*a^4*b^
3)/a^10 + (2*tan(c/2 + (d*x)/2)*(25*a^6 - 200*b^6 + 350*a^2*b^4 - 175*a^4*b
^2))/a^9 + (5*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((5*a^10 + 40*a^6*b^4 - 40*a^
8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*b + 80*a^4*b^5 - 100*a^6*b^3))/a^
9 - (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*(b^2 - a^
2)^(1/2)*(a^2 - 4*b^2))/(2*a^6)))/(2*a^6) - (5*(b^2 - a^2)^(1/2)*(a^2 - 4*b
^2)*((5*a^10 + 40*a^6*b^4 - 40*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(25*a^8*
b + 80*a^4*b^5 - 100*a^6*b^3))/a^9 + (5*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a
^12 - 8*a^10*b^2))/a^9)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/(2*a^6)))/(2*a^6))
)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*5i)/(a^6*d)
```

3.1273 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=355

$$-\frac{15b(a^2-2b^2)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}+\frac{(a^4-25a^2b^2}{}$$

```
[Out] -15/8*(a^4-8*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^7/d+1/2*(a^4-25*a^2*b^2+3
0*b^4)*cot(d*x+c)/a^6/b/d+15/8*(3*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^5/d-1/
2*cot(d*x+c)/b/d/(a+b*sin(d*x+c))^2-1/4*(4*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)
/a^3/d/(a+b*sin(d*x+c))^2+1/2*b*cot(d*x+c)*csc(d*x+c)^2/a^2/d/(a+b*sin(d*x+
c))^2-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d/(a+b*sin(d*x+c))^2-1/2*(7*a^2-10*b^2)
*cot(d*x+c)*csc(d*x+c)/a^4/d/(a+b*sin(d*x+c))-15*b*(a^2-2*b^2)*arctan((b+a*
tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^7/d
```

Rubi [A]
time = 1.07, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{2a^2d(a+b\sin(c+dx))^2}-\frac{15b(a^2-2b^2)\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}-\frac{(7a^2-10b^2)\cot(c+dx)\csc(c+dx)}{2a^4d(a+b\sin(c+dx))}-\frac{(4a^2-5b^2)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2}-\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}+\frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd}-\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-15*b*(a^2 - 2*b^2)*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^7*d) - (15*(a^4 - 8*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/
(8*a^7*d) + ((a^4 - 25*a^2*b^2 + 30*b^4)*Cot[c + d*x])/(2*a^6*b*d) + (15*(3
*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^5*d) - Cot[c + d*x]/(2*b*d*(a
 + b*Sin[c + d*x])^2) - ((4*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(4*a^3*
d*(a + b*Sin[c + d*x])^2) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(2*a^2*d*(a + b
*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d*(a + b*Sin[c + d*x
])^2) - ((7*a^2 - 10*b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^4*d*(a + b*Sin[c
+ d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2}+\frac{b\cot(c+dx)\csc^2(c+dx)}{2a^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^3(}{4ad(a+b\sin(c}$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{(4a^2-5b^2)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2}+\frac{b\cot(c}{2a^2d(}$$

$$= -\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{(4a^2-5b^2)\cot(c+dx)\csc(c+dx)}{4a^3d(a+b\sin(c+dx))^2}+\frac{b\cot(c}{2a^2d(}$$

$$= \frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}-\frac{\cot(c+dx)}{2bd(a+b\sin(c+dx))^2}-\frac{(4a^2-}{4}$$

$$= \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd}+\frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}$$

$$= \frac{(a^4-25a^2b^2+30b^4)\cot(c+dx)}{2a^6bd}+\frac{15(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^5d}$$

$$= -\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}+\frac{(a^4-25a^2b^2+30b^4)\cot(c}{2a^6bd}$$

$$= -\frac{15(a^4-8a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^7d}+\frac{(a^4-25a^2b^2+30b^4)\cot(c}{2a^6bd}$$

$$= -\frac{15b(a^2-2b^2)\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}-\frac{15(a^4-8a^2b^2+8b^4}{8}$$

Mathematica [A]

time = 0.91, size = 363, normalized size = 1.02

$$\frac{-\frac{1920b(a^4-3a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-240(a^4-8a^2b^2+8b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+240(a^4-8a^2b^2+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2a\cot(c+dx)\csc^5(c+dx)(44a^5-505a^3b^2+540ab^4+(-68a^5+660a^3b^2-720ab^4)\cos(2(c+dx))+(8a^5-155a^3b^2+180ab^4)\cos(4(c+dx))-176a^4b\sin(c+dx)-260a^2b^3\sin(c+dx)+600b^5\sin(c+dx)+66a^4b\sin(3(c+dx))+170a^2b^3\sin(3(c+dx))-300b^5\sin(3(c+dx))+2a^4b\sin(5(c+dx))-50a^2b^3\sin(5(c+dx))+60b^5\sin(5(c+dx)))}{(b+a\csc(c+dx))^2}}{128a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-1920*b*(a^4 - 3*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/Sqrt[a^2 - b^2] - 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Cos[(c + d*x
)/2]] + 240*(a^4 - 8*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c +
d*x]*Csc[c + d*x]^5*(44*a^5 - 505*a^3*b^2 + 540*a*b^4 + (-68*a^5 + 660*a^3*
b^2 - 720*a*b^4)*Cos[2*(c + d*x)] + (8*a^5 - 155*a^3*b^2 + 180*a*b^4)*Cos[4
*(c + d*x)] - 176*a^4*b*Sin[c + d*x] - 260*a^2*b^3*Sin[c + d*x] + 600*b^5*S
in[c + d*x] + 66*a^4*b*Sin[3*(c + d*x)] + 170*a^2*b^3*Sin[3*(c + d*x)] - 30
0*b^5*Sin[3*(c + d*x)] + 2*a^4*b*Sin[5*(c + d*x)] - 50*a^2*b^3*Sin[5*(c + d
*x)] + 60*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x])^2)/(128*a^7*d)
```

Maple [A]
time = 1.03, size = 455, normalized size = 1.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^6*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-2*b*tan(1/2*d*x+1/2*c)^3*a^2-4*
a^3*tan(1/2*d*x+1/2*c)^2+12*a*b^2*tan(1/2*d*x+1/2*c)^2+54*a^2*b*tan(1/2*d*x
+1/2*c)-80*b^3*tan(1/2*d*x+1/2*c))-1/64/a^3/tan(1/2*d*x+1/2*c)^4-1/32*(-8*a
^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)^2+1/16/a^7*(30*a^4-240*a^2*b^2+240*b^4)*l
n(tan(1/2*d*x+1/2*c))+1/8/a^4*b/tan(1/2*d*x+1/2*c)^3-1/8*b*(27*a^2-40*b^2)/
a^6/tan(1/2*d*x+1/2*c)-2/a^7*(((-3/2*a^5*b+15/2*a^3*b^3-6*a*b^5)*tan(1/2*d*
x+1/2*c)^3+(-a^6+9/2*a^4*b^2+15/2*a^2*b^4-11*b^6)*tan(1/2*d*x+1/2*c)^2-1/2*
a*b*(5*a^4-37*a^2*b^2+32*b^4)*tan(1/2*d*x+1/2*c)-1/2*a^2*(2*a^4-13*a^2*b^2+
11*b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+15/2*b*(a^4-3*
a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(334) = 668$.
time = 0.56, size = 2022, normalized size = 5.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 10*(5*a^6 -
64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 + 60*((a^2*b^3 - 2*b^5)*cos(d*x + c
)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^5)*cos(d*x + c)^4 +
(2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2 - 2*a*b^4 + (a^3*b^
2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2)*sin(d*x
 + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(a^6
 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x + c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6
)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4
+ 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*
x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*c
os(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x
+ c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*
cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 -
 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c
)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/
2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b - 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5
 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b
^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9
+ 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9
 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*
b*d)*sin(d*x + c)), -1/16*(2*(8*a^6 - 155*a^4*b^2 + 180*a^2*b^4)*cos(d*x +
c)^5 - 10*(5*a^6 - 64*a^4*b^2 + 72*a^2*b^4)*cos(d*x + c)^3 - 120*((a^2*b^3
- 2*b^5)*cos(d*x + c)^6 - a^4*b + a^2*b^3 + 2*b^5 - (a^4*b + a^2*b^3 - 6*b^
5)*cos(d*x + c)^4 + (2*a^4*b - a^2*b^3 - 6*b^5)*cos(d*x + c)^2 - 2*(a^3*b^2
 - 2*a*b^4 + (a^3*b^2 - 2*a*b^4)*cos(d*x + c)^4 - 2*(a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c))) + 30*(a^6 - 11*a^4*b^2 + 12*a^2*b^4)*cos(d*x +
 c) + 15*((a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 -
8*b^6 - (a^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 1
```

```
3*a^4*b^2 - 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a
*b^5 + (a^5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3
+ 8*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(
(a^4*b^2 - 8*a^2*b^4 + 8*b^6)*cos(d*x + c)^6 - a^6 + 7*a^4*b^2 - 8*b^6 - (a
^6 - 5*a^4*b^2 - 16*a^2*b^4 + 24*b^6)*cos(d*x + c)^4 + (2*a^6 - 13*a^4*b^2
- 8*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5 + (a^
5*b - 8*a^3*b^3 + 8*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 8*a^3*b^3 + 8*a*b^5)
*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 4*(2*(a^5*b -
 25*a^3*b^3 + 30*a*b^5)*cos(d*x + c)^5 + 5*(3*a^5*b + 16*a^3*b^3 - 24*a*b^5
)*cos(d*x + c)^3 - 15*(a^5*b + 2*a^3*b^3 - 4*a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4 + (2*a^
9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos(d*x +
c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.54, size = 603, normalized size = 1.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(120*(a^4 - 8*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 - 96
0*(a^4*b - 3*a^2*b^3 + 2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) +
 64*(3*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 1
2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 2*a^6*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^2*t
an(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 22*b^6*tan(1/2*
d*x + 1/2*c)^2 + 5*a^5*b*tan(1/2*d*x + 1/2*c) - 37*a^3*b^3*tan(1/2*d*x + 1/
2*c) + 32*a*b^5*tan(1/2*d*x + 1/2*c) + 2*a^6 - 13*a^4*b^2 + 11*a^2*b^4)/((a
*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^7) - (250*a^4*t
an(1/2*d*x + 1/2*c)^4 - 2000*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 2000*b^4*tan(
1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 320*a*b^3*tan(1/2*d
*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 48*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + a^4)/(a^7*tan(1/2*d*x + 1/2*c)^4) +
 (a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^8*b*tan(1/2*d*x + 1/2*c)^3 - 16*a^9*tan(
```

1/2*d*x + 1/2*c)^2 + 48*a^7*b^2*tan(1/2*d*x + 1/2*c)^2 + 216*a^8*b*tan(1/2*
d*x + 1/2*c) - 320*a^6*b^3*tan(1/2*d*x + 1/2*c))/a^12)/d

Mupad [B]
time = 12.55, size = 1275, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))^3),x)

[Out] tan(c/2 + (d*x)/2)^4/(64*a^3*d) - (tan(c/2 + (d*x)/2)^3*(35*a^4*b - 40*a^2*
b^3) - tan(c/2 + (d*x)/2)^4*(448*a*b^4 + (159*a^5)/4 - 424*a^3*b^2) + tan(c
/2 + (d*x)/2)^7*(6*a^4*b - 192*b^5 + 160*a^2*b^3) + tan(c/2 + (d*x)/2)^5*(1
0*a^4*b - 832*b^5 + 696*a^2*b^3) + a^5/4 - tan(c/2 + (d*x)/2)^2*((7*a^5)/2
- 5*a^3*b^2) - a^4*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 + (d*x)/2)^6*(9*a^6 +
88*b^6 + 20*a^2*b^4 - 93*a^4*b^2))/a)/(d*(16*a^8*tan(c/2 + (d*x)/2)^4 + 16*
a^8*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^6*(32*a^8 + 64*a^6*b^2) + 64*
a^7*b*tan(c/2 + (d*x)/2)^5 + 64*a^7*b*tan(c/2 + (d*x)/2)^7)) - (tan(c/2 + (
d*x)/2)^2*((3*(a^2 + 4*b^2))/(32*a^5) + 5/(32*a^3) - (9*b^2)/(8*a^5)))/d +
(tan(c/2 + (d*x)/2)*((6*b*((3*(a^2 + 4*b^2))/(16*a^5) + 5/(16*a^3) - (9*b^2
)/(4*a^5)))/a - (192*a^2*b + 128*b^3)/(256*a^6) + (9*b*(a^2 + 4*b^2))/(8*a^
6)))/d - (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) + (log(tan(c/2 + (d*x)/2))*((15
*a^4)/8 + 15*b^4 - 15*a^2*b^2))/(a^7*d) + (b*atan(((b*(b^2 - a^2)^(1/2)*(a^
2 - 2*b^2)*(((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)/a^12 - (tan(c/2 + (d*
x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^9*b^2))/(4*a^11) + (15*b
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(b^2 - a
^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7))*15i)/(2*a^7) - (b*(b^2 - a^2)^(1/2)*(a^2
- 2*b^2)*((tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^
9*b^2))/(4*a^11) - ((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)/a^12 + (15*b*(
2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))/(4*a^11))*(b^2 - a^2
)^(1/2)*(a^2 - 2*b^2))/(2*a^7))*15i)/(2*a^7))/(((225*a^8*b)/4 + 900*b^9 - 2
250*a^2*b^7 + (3825*a^4*b^5)/2 - (2475*a^6*b^3)/4)/a^12 + (tan(c/2 + (d*x)/
2)*(1800*b^8 - 4050*a^2*b^6 + 2925*a^4*b^4 - 675*a^6*b^2))/(2*a^11) + (15*b
(b^2 - a^2)^(1/2)(a^2 - 2*b^2)*(((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^9*b^3)
/a^12 - (tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 + 720*a^7*b^4 - 270*a^9*
b^2))/(4*a^11) + (15*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^
2))/(4*a^11))*(b^2 - a^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7)))/(2*a^7) + (15*b*(b
^2 - a^2)^(1/2)*(a^2 - 2*b^2)*((tan(c/2 + (d*x)/2)*(15*a^11 - 480*a^5*b^6 +
720*a^7*b^4 - 270*a^9*b^2))/(4*a^11) - ((75*a^11*b)/4 + 60*a^7*b^5 - 75*a^
9*b^3)/a^12 + (15*b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^14 - 32*a^12*b^2))
/(4*a^11))*(b^2 - a^2)^(1/2)*(a^2 - 2*b^2))/(2*a^7)))/(2*a^7)))*(b^2 - a^2)
^(1/2)*(a^2 - 2*b^2)*15i)/(a^7*d)

3.1274 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$-\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d}-$$

```
[Out] 1/8*b*(45*a^4-200*a^2*b^2+168*b^4)*arctanh(cos(d*x+c))/a^8/d-1/30*(91*a^4-6
45*a^2*b^2+630*b^4)*cot(d*x+c)/a^7/d+1/8*(8*a^4-79*a^2*b^2+84*b^4)*cot(d*x+
c)*csc(d*x+c)/a^6/b/d-1/30*(15*a^4-187*a^2*b^2+210*b^4)*cot(d*x+c)*csc(d*x+
c)^2/a^5/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/12*a*cot(
d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))^2+1/60*(5*a^4-60*a^2*b^2+63*b^4)
*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))^2+7/20*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))^2-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*sin
(d*x+c))^2+1/12*(4*a^4-54*a^2*b^2+63*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b^2/d
/(a+b*sin(d*x+c))-(2*a^4-29*a^2*b^2+42*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^8/d
```

Rubi [A]
time = 1.35, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{(4a^4-54a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{12a^4b^2d(a+b\sin(c+dx))}-\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d}-\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d}+\frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd}-\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d}+\frac{(5a^4-60a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{60a^3b^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[
n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+}{20a^2d(a+b\sin(c+d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4)}{60a^3b^2d(a+}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{(5a^4-60a^2b^2+63b^4)}{60a^3b^2d(a+}$$

$$= -\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$= \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{(15a^4-187a^2b^2+210b^4)\cot(c}{30a^5b^2d}$$

$$= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc}{8a^6bd}$$

$$= -\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d} + \frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc}{8a^6bd}$$

$$= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot}{30a^7d}$$

$$= \frac{b(45a^4-200a^2b^2+168b^4)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{(91a^4-645a^2b^2+630b^4)\cot}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\,(2a^4-29a^2b^2+42b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b(45a^4-200a^2b^2}{}$$

Mathematica [A]

time = 1.19, size = 448, normalized size = 0.91

$$\frac{-\frac{3840(2a^6-31a^4b^2+71a^2b^4-42b^6)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+480b(45a^4-200a^2b^2+168b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-480b(45a^4-200a^2b^2+168b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2a\cot(c+dx)\csc^6(c+dx)(-784a^6+3256a^4b^2+7860a^2b^4-12600b^6+2(384a^6-2131a^4b^2-6315a^2b^4+9450b^6)\cos(2(c+dx))+(-368a^6+824a^4b^2+6060a^2b^4-7560b^6)\cos(4(c+dx))+182a^4b^2\cos(6(c+dx))-1290a^2b^4\cos(6(c+dx))+1260b^6\cos(6(c+dx))-8156a^5b\sin(c+dx)+42270a^3b^3\sin(c+dx)-37800ab^5\sin(c+dx)+3956a^5b\sin(3(c+dx))-20715a^3b^3\sin(3(c+dx))+18900ab^5\sin(3(c+dx))-608a^5b\sin(5(c+dx))+3975a^3b^3\sin(5(c+dx))-3780ab^5\sin(5(c+dx)))}{(b+a\csc(c+dx))^2}}{3840a^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

Maple [A]

time = 1.11, size = 559, normalized size = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}{32a^7}}{}$
default	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}{32a^7}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^7*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-3/2*b*tan(1/2*d*x+1/2*c)^4*a^3-
7/3*a^4*tan(1/2*d*x+1/2*c)^3+8*a^2*b^2*tan(1/2*d*x+1/2*c)^3+24*a^3*b*tan(1/
2*d*x+1/2*c)^2-40*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-216*
a^2*b^2*tan(1/2*d*x+1/2*c)+240*b^4*tan(1/2*d*x+1/2*c))-1/160/a^3/tan(1/2*d*
```

```
x+1/2*c)^5-1/96*(-7*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-216*a
^2*b^2+240*b^4)/a^7/tan(1/2*d*x+1/2*c)+3/64/a^4*b/tan(1/2*d*x+1/2*c)^4-1/4/
a^6*b*(3*a^2-5*b^2)/tan(1/2*d*x+1/2*c)^2-1/8/a^8*b*(45*a^4-200*a^2*b^2+168*
b^4)*ln(tan(1/2*d*x+1/2*c))-2/a^8*(((5/2*a^5*b^2-19/2*a^3*b^4+7*a*b^6)*tan(
1/2*d*x+1/2*c)^3+1/2*b*(4*a^6-9*a^4*b^2-21*a^2*b^4+26*b^6)*tan(1/2*d*x+1/2*
c)^2+1/2*a*b^2*(11*a^4-49*a^2*b^2+38*b^4)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a
^4-17*a^2*b^2+13*b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+
1/2*(2*a^6-31*a^4*b^2+71*a^2*b^4-42*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(467) = 934$.
time = 0.74, size = 2571, normalized size = 5.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
```

+ 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(

```
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.56, size = 731, normalized size = 1.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 12.63, size = 1614, normalized size = 3.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a^3*d) - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(32
*a^5) + 1/(24*a^3) - (3*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*(1/(8*a^3) -
 (3*(a^2 + 4*b^2))/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(8
*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(1024*a^6) + (9*b*(a^2
+ 4*b^2))/(16*a^6)))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(32*a^5) + 1/(
8*a^3) - (9*b^2)/(8*a^5)))/a^2 + (3*b*(384*a^2*b + 256*b^3))/(512*a^7)))/d
- (tan(c/2 + (d*x)/2)^3*((187*a^5*b)/15 - 14*a^3*b^3) + a^6/5 + tan(c/2 + (
d*x)/2)^4*((263*a^6)/15 + 112*a^2*b^4 - (358*a^4*b^2)/3) + tan(c/2 + (d*x)/
2)^5*(1216*a*b^5 + (1519*a^5*b)/6 - 1360*a^3*b^3) - tan(c/2 + (d*x)/2)^2*((
29*a^6)/15 - (14*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^8*(22*a^6 + 448*b^6 - 368
*a^2*b^4 - 56*a^4*b^2) + tan(c/2 + (d*x)/2)^6*((125*a^6)/3 + 2176*b^6 - 211
2*a^2*b^4 + 112*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^7*(30*a^6*b + 104*b^7 + 36
*a^2*b^5 - 149*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/10)/(d*(32*a^9*ta
n(c/2 + (d*x)/2)^5 + 32*a^9*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^7*(64
*a^9 + 128*a^7*b^2) + 128*a^8*b*tan(c/2 + (d*x)/2)^6 + 128*a^8*b*tan(c/2 +
(d*x)/2)^8)) + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1/
(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(2048*a^6) + (9*b*(a^
2 + 4*b^2))/(32*a^6)))/d - (log(tan(c/2 + (d*x)/2))*(45*a^4*b + 168*b^5 - 2
00*a^2*b^3))/(8*a^8*d) - (3*b*tan(c/2 + (d*x)/2)^4)/(64*a^4*d) - (atan((((-
(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^
6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b
- 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) + ((-(a + b)*(a - b)
)^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(
a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8)*1i)/a^8 + ((-(a + b)*(a - b))^(1/2)*(a
^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a
^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*
b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c
/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^
2)/2))/a^8)*1i)/a^8)/(((45*a^10*b)/2 - 1764*b^11 + 5082*a^2*b^9 - (10649*a^
4*b^7)/2 + (9731*a^6*b^5)/4 - (1795*a^8*b^3)/4)/a^14 + (tan(c/2 + (d*x)/2)*
(16*a^10 - 3528*b^10 + 9282*a^2*b^8 - 8549*a^4*b^6 + 3185*a^6*b^4 - 406*a^8
*b^2))/(2*a^13) - ((-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)
*((2*a^14 - 84*a^8*b^6 + 121*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 +
 (d*x)/2)*(61*a^12*b - 672*a^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13)
 + ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a
^14*b^2))/(4*a^13))*(a^4 + 21*b^4 - (29*a^2*b^2)/2))/a^8))/a^8 + ((-(a + b)
*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/2)*((2*a^14 - 84*a^8*b^6 + 121
*a^10*b^4 - (169*a^12*b^2)/4)/a^14 + (tan(c/2 + (d*x)/2)*(61*a^12*b - 672*a
^6*b^7 + 1136*a^8*b^5 - 538*a^10*b^3))/(4*a^13) - ((-(a + b)*(a - b))^(1/2)
*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^16 - 32*a^14*b^2))/(4*a^13))*(a^4 + 2
1*b^4 - (29*a^2*b^2)/2))/a^8))/a^8))*(-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4
```

```
- (29*a^2*b^2)/2)*2i)/(a^8*d)
```

3.1275 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=600

$$-\frac{3b^2\sqrt{a^2-b^2}\,(4a^4-23a^2b^2+24b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^{10}d}-\frac{3b(5a^6-100a^4b^2+280a^2b^4-192b^6)\tanh^{-}}{16a^{10}d}$$

```
[Out] -3/16*b*(5*a^6-100*a^4*b^2+280*a^2*b^4-192*b^6)*arctanh(cos(d*x+c))/a^10/d+
1/70*(10*a^6-889*a^4*b^2+3255*a^2*b^4-2520*b^6)*cot(d*x+c)/a^9/d+3/16*b*(27
*a^4-116*a^2*b^2+96*b^4)*cot(d*x+c)*csc(d*x+c)/a^8/d-1/70*(205*a^4-973*a^2*
b^2+840*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^7/d+1/8*(16*a^4-81*a^2*b^2+72*b^4)*c
ot(d*x+c)*csc(d*x+c)^3/a^6/b/d-3/70*(35*a^4-185*a^2*b^2+168*b^4)*cot(d*x+c)
*csc(d*x+c)^4/a^5/b^2/d-1/5*cot(d*x+c)*csc(d*x+c)^3/b/d/(a+b*sin(d*x+c))^2+
1/10*a*cot(d*x+c)*csc(d*x+c)^4/b^2/d/(a+b*sin(d*x+c))^2+1/35*(7*a^4-35*a^2*
b^2+30*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^3/b^2/d/(a+b*sin(d*x+c))^2+3/14*b*cot
(d*x+c)*csc(d*x+c)^5/a^2/d/(a+b*sin(d*x+c))^2-1/7*cot(d*x+c)*csc(d*x+c)^6/a
/d/(a+b*sin(d*x+c))^2+1/10*(12*a^4-65*a^2*b^2+60*b^4)*cot(d*x+c)*csc(d*x+c)
^4/a^4/b^2/d/(a+b*sin(d*x+c))-3*b^2*(4*a^4-23*a^2*b^2+24*b^4)*arctan((b+a*t
an(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^10/d
```

Rubi [A]
time = 2.00, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{3b\cot(c+dx)\csc^5(c+dx)}{14a^2d(a+b\sin(c+dx))^2}+\frac{(12a^4-65a^2b^2+60b^4)\cot(c+dx)\csc^4(c+dx)}{10a^4b^2d(a+b\sin(c+dx))}-\frac{3b^2\sqrt{a^2-b^2}\,(4a^4-23a^2b^2+24b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^{10}d}+\frac{3b(27a^4-116a^2b^2+96b^4)\cot(c+dx)\csc(c+dx)}{16a^8d}-\frac{(205a^4-973a^2b^2+840b^4)\cot(c+dx)\csc^2(c+dx)}{70a^7d}+\frac{(16a^4-81a^2b^2+72b^4)\cot(c+dx)\csc^3(c+dx)}{8a^6bd}-\frac{3(35a^4-185a^2b^2+168b^4)\cot(c+dx)\csc^4(c+dx)}{70a^5b^2d}+\frac{(7a^4-35a^2b^2+30b^4)\cot(c+dx)\csc^4(c+dx)}{35a^3b^2d(a+b\sin(c+dx))^2}-\frac{3b(5a^6-100a^4b^2+280a^2b^4-192b^6)\tanh^{-1}(\cos(c+dx))}{16a^{10}d}+\frac{(10a^6-889a^4b^2+3255a^2b^4-2520b^6)\cot(c+dx)}{70a^9d}+\frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^6(c+dx)}{7ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^2*Sqrt[a^2 - b^2]*(4*a^4 - 23*a^2*b^2 + 24*b^4)*ArcTan[(b + a*Tan[(c
+ d*x)/2])/Sqrt[a^2 - b^2]])/(a^10*d) - (3*b*(5*a^6 - 100*a^4*b^2 + 280*a^2
*b^4 - 192*b^6)*ArcTanh[Cos[c + d*x]])/(16*a^10*d) + ((10*a^6 - 889*a^4*b^2
 + 3255*a^2*b^4 - 2520*b^6)*Cot[c + d*x])/(70*a^9*d) + (3*b*(27*a^4 - 116*a
^2*b^2 + 96*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^8*d) - ((205*a^4 - 973*a^
2*b^2 + 840*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(70*a^7*d) + ((16*a^4 - 81*a^
2*b^2 + 72*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(8*a^6*b*d) - (3*(35*a^4 - 185
*a^2*b^2 + 168*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(70*a^5*b^2*d) - (Cot[c +
d*x]*Csc[c + d*x]^3)/(5*b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c + d*x]*Csc[c
 + d*x]^4)/(10*b^2*d*(a + b*Sin[c + d*x])^2) + ((7*a^4 - 35*a^2*b^2 + 30*b^
4)*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a^3*b^2*d*(a + b*Sin[c + d*x])^2) + (3*
b*Cot[c + d*x]*Csc[c + d*x]^5)/(14*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c +
 d*x]*Csc[c + d*x]^6)/(7*a*d*(a + b*Sin[c + d*x])^2) + ((12*a^4 - 65*a^2*b^
```

```
2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/(10*a^4*b^2*d*(a + b*Sin[c + d*x])
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
```

```
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{3b\cot(c+dx)\,\mathrm{c}}{14a^2d(a+b\sin}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{(7a^4-35a^2b^2+}{35a^3b^2}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{5bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^4(c+dx)}{10b^2d(a+b\sin(c+dx))^2} + \frac{(7a^4-35a^2b^2+}{35a^3b^2}$$

$$= -\frac{3(35a^4-185a^2b^2+168b^4)\cot(c+dx)\csc^4(c+dx)}{70a^5b^2d} - \frac{\cot(c+dx)\csc^3(c}{5bd(a+b\sin(c+}$$

$$= \frac{(16a^4-81a^2b^2+72b^4)\cot(c+dx)\csc^3(c+dx)}{8a^6bd} - \frac{3(35a^4-185a^2b^2+168}{7}$$

$$= -\frac{(205a^4-973a^2b^2+840b^4)\cot(c+dx)\csc^2(c+dx)}{70a^7d} + \frac{(16a^4-81a^2b^2+}{}$$

$$= \frac{3b(27a^4-116a^2b^2+96b^4)\cot(c+dx)\csc(c+dx)}{16a^8d} - \frac{(205a^4-973a^2b^2+8}{}$$

$$= \frac{(10a^6-889a^4b^2+3255a^2b^4-2520b^6)\cot(c+dx)}{70a^9d} + \frac{3b(27a^4-116a^2b^2+}{}$$

$$= \frac{(10a^6-889a^4b^2+3255a^2b^4-2520b^6)\cot(c+dx)}{70a^9d} + \frac{3b(27a^4-116a^2b^2+}{}$$

$$= -\frac{3b(5a^6-100a^4b^2+280a^2b^4-192b^6)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{(10a^6-88}{}$$

$$= -\frac{3b(5a^6-100a^4b^2+280a^2b^4-192b^6)\tanh^{-1}(\cos(c+dx))}{16a^{10}d} + \frac{(10a^6-88}{}$$

$$= -\frac{3b^2\sqrt{a^2-b^2}\,(4a^4-23a^2b^2+24b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^{10}d} - \frac{3b(5a^6-}{}$$

Mathematica [A]
time = 1.91, size = 728, normalized size = 1.21

Antiderivative was successfully verified.

[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]

[Out] ((215040*b^2*(-4*a^6 + 27*a^4*b^2 - 47*a^2*b^4 + 24*b^6)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 13440*b*(-5*a^6 + 100*a^4*b^2 - 280*a^2*b^4 + 192*b^6)*Log[Cos[(c + d*x)/2]] + 13440*b*(5*a^6 - 100*a^4*b^2 + 280*a^2*b^4 - 192*b^6)*Log[Sin[(c + d*x)/2]] - (a*Csc[c + d*x]^9*(28*(200*a^8 + 795*a^6*b^2 - 1218*a^4*b^4 - 4110*a^2*b^6 + 5040*b^8)*Cos[c + d*x] + 28*(120*a^8 - 1403*a^6*b^2 + 1952*a^4*b^4 + 8700*a^2*b^6 - 10080*b^8)*Cos[3*(c + d*x)] + 1120*a^8*Cos[5*(c + d*x)] + 22948*a^6*b^2*Cos[5*(c + d*x)] - 18144*a^4*b^4*Cos[5*(c + d*x)] - 193200*a^2*b^6*Cos[5*(c + d*x)] + 201600*b^8*Cos[5*(c + d*x)] + 160*a^8*Cos[7*(c + d*x)] - 5884*a^6*b^2*Cos[7*(c + d*x)] - 5964*a^4*b^4*Cos[7*(c + d*x)] + 77700*a^2*b^6*Cos[7*(c + d*x)] - 70560*b^8*Cos[7*(c + d*x)] - 40*a^6*b^2*Cos[9*(c + d*x)] + 3556*a^4*b^4*Cos[9*(c + d*x)] - 13020*a^2*b^6*Cos[9*(c + d*x)] + 10080*b^8*Cos[9*(c + d*x)] - 9660*a^7*b*Sin[2*(c + d*x)] + 194334*a^5*b^3*Sin[2*(c + d*x)] - 592200*a^3*b^5*Sin[2*(c + d*x)] + 423360*a*b^7*Sin[2*(c + d*x)] + 6160*a^7*b*Sin[4*(c + d*x)] - 190582*a^5*b^3*Sin[4*(c + d*x)] + 585480*a^3*b^5*Sin[4*(c + d*x)] - 423360*a*b^7*Sin[4*(c + d*x)] - 3660*a^7*b*Sin[6*(c + d*x)] + 77462*a^5*b^3*Sin[6*(c + d*x)] - 246120*a^3*b^5*Sin[6*(c + d*x)] + 181440*a*b^7*Sin[6*(c + d*x)] + 160*a^7*b*Sin[8*(c + d*x)] - 11389*a^5*b^3*Sin[8*(c + d*x)] + 39900*a^3*b^5*Sin[8*(c + d*x)] - 30240*a*b^7*Sin[8*(c + d*x)]))/(b + a*Csc[c + d*x])^2)/(71680*a^10*d)

Maple [A]
time = 0.93, size = 775, normalized size = 1.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(1/128/a^9*(1/7*a^6*tan(1/2*d*x+1/2*c)^7-b*tan(1/2*d*x+1/2*c)^6*a^5-tan(1/2*d*x+1/2*c)^5*a^6+24/5*a^4*b^2*tan(1/2*d*x+1/2*c)^5+9*b*a^5*tan(1/2*d*x+1/2*c)^4-20*a^3*b^3*tan(1/2*d*x+1/2*c)^4+3*a^6*tan(1/2*d*x+1/2*c)^3-56*a^4*b^2*tan(1/2*d*x+1/2*c)^3+80*a^2*b^4*tan(1/2*d*x+1/2*c)^3-45*a^5*b*tan(1/2*d*x+1/2*c)^2+320*a^3*b^3*tan(1/2*d*x+1/2*c)^2-336*a*b^5*tan(1/2*d*x+1/2*c)^2-5*a^6*tan(1/2*d*x+1/2*c)+528*a^4*b^2*tan(1/2*d*x+1/2*c)-2160*a^2*b^4*tan(1/2*d*x+1/2*c)+1792*b^6*tan(1/2*d*x+1/2*c))-1/896/a^3/tan(1/2*d*x+1/2*c)^7-1/640*(-5*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c)^5-1/384*(9*a^4-168*a^2*b^2+240*b^4)/a^7/tan(1/2*d*x+1/2*c)^3-1/128*(-5*a^6+528*a^4*b^2-2160*a^2*b^4+1792*b^6)/a^9/tan(1/2*d*x+1/2*c)+1/128/a^4*b/tan(1/2*d*x+1/2*c)^6-1/128/a^6*b*(9*a^2-20*b^2)/tan(1/2*d*x+1/2*c)^4+1/128/a^8*b*(45*a^4-320*a^2*b^2+336*b^4)/tan(1/2*d*x+1/2*c)^2+3/16/a^10*b*(5*a^6-100*a^4*b^2+280*a^2*b^4-192*b^6)*ln(tan(1/2*d*x+1/2*c))-2*b^2/a^10*(((9/2*a^5*b^2-27/2*a^3*b^4+9*a*b^6)*tan(1/2*d*x+1/2*c)^3+1/2*b*(8*a^6-9*a^4*b^2-33*a^2*b^4+34*b^6)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(23*a^4-73*a^2*b^2+50*b^4)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(8*a^4-25*a^2*b^2+17*b^4))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(4*a^6-27*a^4*b^2+47*a^2*b^4-24*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(571) = 1142.
time = 0.90, size = 3687, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x
+ c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a*
b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640*
a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 1152
*a*b^8)*cos(d*x + c)^3 + 840*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x
 + c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 2
4*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x +
c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4 -
 23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 + 2
4*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*(4
*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2 -
 53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 420*(11*a^7*b^2 - 8*a^5*b
^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7*b^2 - 200*a^5*b^4 +
560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*
b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)
*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*co
s(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*
x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5*
```

a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 105*(10*a^7*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*a^4*b^5 + 544*a^2*b^7 - 768*b^9)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((160*a^8*b - 11389*a^6*b^3 + 39900*a^4*b^5 - 30240*a^2*b^7)*cos(d*x + c)^7 - 7*(165*a^8*b - 5207*a^6*b^3 + 17340*a^4*b^5 - 12960*a^2*b^7)*cos(d*x + c)^5 + 35*(40*a^8*b - 1097*a^6*b^3 + 3516*a^4*b^5 - 2592*a^2*b^7)*cos(d*x + c)^3 - 105*(5*a^8*b - 127*a^6*b^3 + 396*a^4*b^5 - 288*a^2*b^7)*cos(d*x + c))*sin(d*x + c))/(2*a^11*b*d*cos(d*x + c)^8 - 8*a^11*b*d*cos(d*x + c)^6 + 12*a^11*b*d*cos(d*x + c)^4 - 8*a^11*b*d*cos(d*x + c)^2 + 2*a^11*b*d + (a^10*b^2*d*cos(d*x + c)^8 - (a^12 + 4*a^10*b^2)*d*cos(d*x + c)^6 + 3*(a^12 + 2*a^10*b^2)*d*cos(d*x + c)^4 - (3*a^12 + 4*a^10*b^2)*d*cos(d*x + c)^2 + (a^12 + a^10*b^2)*d)*sin(d*x + c)), 1/1120*(16*(10*a^7*b^2 - 889*a^5*b^4 + 3255*a^3*b^6 - 2520*a*b^8)*cos(d*x + c)^9 - 4*(40*a^9 - 1381*a^7*b^2 - 9492*a^5*b^4 + 48720*a^3*b^6 - 40320*a*b^8)*cos(d*x + c)^7 - 28*(563*a^7*b^2 + 1068*a^5*b^4 - 9720*a^3*b^6 + 8640*a*b^8)*cos(d*x + c)^5 + 140*(105*a^7*b^2 + 20*a^5*b^4 - 1200*a^3*b^6 + 1152*a*b^8)*cos(d*x + c)^3 + 1680*(2*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^8 + 8*a^5*b^3 - 46*a^3*b^5 + 48*a*b^7 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^6 + 12*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^4 - 8*(4*a^5*b^3 - 23*a^3*b^5 + 24*a*b^7)*cos(d*x + c)^2 + ((4*a^4*b^4 - 23*a^2*b^6 + 24*b^8)*cos(d*x + c)^8 + 4*a^6*b^2 - 19*a^4*b^4 + a^2*b^6 + 24*b^8 - (4*a^6*b^2 - 7*a^4*b^4 - 68*a^2*b^6 + 96*b^8)*cos(d*x + c)^6 + 3*(4*a^6*b^2 - 15*a^4*b^4 - 22*a^2*b^6 + 48*b^8)*cos(d*x + c)^4 - (12*a^6*b^2 - 53*a^4*b^4 - 20*a^2*b^6 + 96*b^8)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 420*(11*a^7*b^2 - 8*a^5*b^4 - 92*a^3*b^6 + 96*a*b^8)*cos(d*x + c) - 105*(10*a^7*b^2 - 200*a^5*b^4 + 560*a^3*b^6 - 384*a*b^8 + 2*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^8 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^6 + 12*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^4 - 8*(5*a^7*b^2 - 100*a^5*b^4 + 280*a^3*b^6 - 192*a*b^8)*cos(d*x + c)^2 + (5*a^8*b - 95*a^6*b^3 + 180*a^4*b^5 + 88*a^2*b^7 - 192*b^9 + (5*a^6*b^3 - 100*a^4*b^5 + 280*a^2*b^7 - 192*b^9)*cos(d*x + c)^8 - (5*a^8*b - 80*a^6*b^3 - 120*a^4*b^5 + 928*a^2*b^7 - 768*b^9)*cos(d*x + c)^6 + 3*(5*a^8*b - 90*a^6*b^3 + 80*a^4*b^5 + 368*a^2*b^7 - 384*b^9)*

```
cos(d*x + c)^4 - (15*a^8*b - 280*a^6*b^3 + 440*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 1018, normalized size = 1.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4480*(840*(5*a^6*b - 100*a^4*b^3 + 280*a^2*b^5 - 192*b^7)*log(abs(tan(1/2
*d*x + 1/2*c)))/a^10 - 13440*(4*a^6*b^2 - 27*a^4*b^4 + 47*a^2*b^6 - 24*b^8)
*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^10) - 4480*(9*a^5*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 27*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 18*a*b^8*tan(1/2*d*x + 1/
2*c)^3 + 8*a^6*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^5*tan(1/2*d*x + 1/2*c)^
2 - 33*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 + 34*b^9*tan(1/2*d*x + 1/2*c)^2 + 23*
a^5*b^4*tan(1/2*d*x + 1/2*c) - 73*a^3*b^6*tan(1/2*d*x + 1/2*c) + 50*a*b^8*t
an(1/2*d*x + 1/2*c) + 8*a^6*b^3 - 25*a^4*b^5 + 17*a^2*b^7)/((a*tan(1/2*d*x
+ 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2*a^10) - (10890*a^6*b*tan(1/2*d
*x + 1/2*c)^7 - 217800*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 609840*a^2*b^5*tan(
1/2*d*x + 1/2*c)^7 - 418176*b^7*tan(1/2*d*x + 1/2*c)^7 - 175*a^7*tan(1/2*d*
x + 1/2*c)^6 + 18480*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 - 75600*a^3*b^4*tan(1/2
*d*x + 1/2*c)^6 + 62720*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 1575*a^6*b*tan(1/2*d
*x + 1/2*c)^5 + 11200*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 11760*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 + 105*a^7*tan(1/2*d*x + 1/2*c)^4 - 1960*a^5*b^2*tan(1/2*d*
x + 1/2*c)^4 + 2800*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*b*tan(1/2*d*x
+ 1/2*c)^3 - 700*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 35*a^7*tan(1/2*d*x + 1/2*
c)^2 + 168*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/2*d*x + 1/2*c) +
 5*a^7)/(a^10*tan(1/2*d*x + 1/2*c)^7) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 35
*a^17*b*tan(1/2*d*x + 1/2*c)^6 - 35*a^18*tan(1/2*d*x + 1/2*c)^5 + 168*a^16*
b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a^17*b*tan(1/2*d*x + 1/2*c)^4 - 700*a^15*b
^3*tan(1/2*d*x + 1/2*c)^4 + 105*a^18*tan(1/2*d*x + 1/2*c)^3 - 1960*a^16*b^2
*tan(1/2*d*x + 1/2*c)^3 + 2800*a^14*b^4*tan(1/2*d*x + 1/2*c)^3 - 1575*a^17*
b*tan(1/2*d*x + 1/2*c)^2 + 11200*a^15*b^3*tan(1/2*d*x + 1/2*c)^2 - 11760*a^
13*b^5*tan(1/2*d*x + 1/2*c)^2 - 175*a^18*tan(1/2*d*x + 1/2*c) + 18480*a^16*
```

```
b^2*tan(1/2*d*x + 1/2*c) - 75600*a^14*b^4*tan(1/2*d*x + 1/2*c) + 62720*a^12
*b^6*tan(1/2*d*x + 1/2*c))/a^21)/d
```

Mupad [B]
time = 13.29, size = 2500, normalized size = 4.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^8*(a + b*sin(c + d*x))^3),x)
[Out] (tan(c/2 + (d*x)/2)*(7/(128*a^3) + (9*b^2)/(32*a^5) - (6*b*((6*b*((3*(a^2 +
 4*b^2))/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5)
+ 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) +
 (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(12
8*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (3*b*(1536*a^2*b + 1024*b^3)
)/(8192*a^7)))/a - (3*b)/(64*a^4) + (9*b*(a^2 + 4*b^2))/(64*a^6) + ((1536*a
^2*b + 1024*b^3)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^
5)))/(128*a^3) - (3*(a^2 + 4*b^2)*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(6
4*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*
(a^2 + 4*b^2))/(64*a^6)))/a^2))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(12
8*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - ((1536*a^2*b + 1024*b^3)*((6
*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536
*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/(128*a^3) +
 (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*(
(3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2
*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4
*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 -
(3*b*(1536*a^2*b + 1024*b^3))/(8192*a^7)))/a^2))/d + tan(c/2 + (d*x)/2)^7/(
896*a^3*d) - (tan(c/2 + (d*x)/2)^5*((3*(a^2 + 4*b^2))/(640*a^5) + 1/(320*a^
3) - (9*b^2)/(160*a^5)))/d + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2))
/(128*a^5) + 3/(128*a^3) + (6*b*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*
a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a
^2 + 4*b^2))/(64*a^6)))/a - (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(128*a^5) +
 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (3*b*(1536*a^2*b + 1024*b^3))/(8192*
a^7)))/a - (3*b)/(128*a^4) + (9*b*(a^2 + 4*b^2))/(128*a^6) + ((1536*a^2*b +
 1024*b^3)*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/(
256*a^3) - (3*(a^2 + 4*b^2)*((6*b*((3*(a^2 + 4*b^2))/(128*a^5) + 1/(64*a^3)
 - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16384*a^6) + (9*b*(a^2 +
 4*b^2))/(64*a^6)))/(2*a^2)))/d + (tan(c/2 + (d*x)/2)^4*((3*b*((3*(a^2 + 4*
b^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/(2*a) - (1536*a^2*b + 102
4*b^3)/(65536*a^6) + (9*b*(a^2 + 4*b^2))/(256*a^6)))/d - (tan(c/2 + (d*x)/2
)^3*((a^2 + 4*b^2)/(128*a^5) + 1/(128*a^3) + (2*b*((6*b*((3*(a^2 + 4*b^2))/
(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a - (1536*a^2*b + 1024*b^3)/(16
384*a^6) + (9*b*(a^2 + 4*b^2))/(64*a^6)))/a - ((a^2 + 4*b^2)*((3*(a^2 + 4*b
```

```
^2))/(128*a^5) + 1/(64*a^3) - (9*b^2)/(32*a^5)))/a^2 - (b*(1536*a^2*b + 102
4*b^3))/(8192*a^7)))/d - (tan(c/2 + (d*x)/2)^3*((25*a^7*b)/7 - (24*a^5*b^3)
/5) - tan(c/2 + (d*x)/2)^5*(20*a^7*b + 96*a^3*b^5 - (556*a^5*b^3)/5) + tan(
c/2 + (d*x)/2)^6*(768*a^2*b^6 - 1024*a^4*b^4 + (1444*a^6*b^2)/5) + tan(c/2
+ (d*x)/2)^7*(8000*a*b^7 - 89*a^7*b - 10912*a^3*b^5 + 3352*a^5*b^3) + a^8/7
 + tan(c/2 + (d*x)/2)^4*((8*a^8)/7 + (96*a^4*b^4)/5 - (92*a^6*b^2)/5) - tan
(c/2 + (d*x)/2)^10*(5*a^8 - 2304*b^8 + 1664*a^2*b^6 + 1008*a^4*b^4 - 528*a^
6*b^2) + tan(c/2 + (d*x)/2)^8*(13568*b^8 - 7*a^8 - 15744*a^2*b^6 + 2096*a^4
*b^4 + 800*a^6*b^2) - tan(c/2 + (d*x)/2)^2*((5*a^8)/7 - (48*a^6*b^2)/35) -
(3*a^7*b*tan(c/2 + (d*x)/2))/7 + (tan(c/2 + (d*x)/2)^9*(4352*b^9 - 65*a^8*b
 + 2944*a^2*b^7 - 10128*a^4*b^5 + 3456*a^6*b^3))/a)/(d*(128*a^11*tan(c/2 +
(d*x)/2)^7 + 128*a^11*tan(c/2 + (d*x)/2)^11 + tan(c/2 + (d*x)/2)^9*(256*a^1
1 + 512*a^9*b^2) + 512*a^10*b*tan(c/2 + (d*x)/2)^8 + 512*a^10*b*tan(c/2 + (
d*x)/2)^10)) - (b*tan(c/2 + (d*x)/2)^6)/(128*a^4*d) + (log(tan(c/2 + (d*x)/
2))*(15*a^6*b - 576*b^7 + 840*a^2*b^5 - 300*a^4*b^3))/(16*a^10*d) - (b^2*at
an(((b^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23*a^2*b^2)*((2304*a^10
*b^8 - 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(16*a^18) + (tan(c/2 +
 (d*x)/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 + 2736*a^12*b^5 - 522*a
^14*b^3))/(8*a^17) - (3*b^2*(-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 +
(d*x)/2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 24*b^4 - 23*a^2*b^2))/
(2*a^10))*3i)/(2*a^10) + (b^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23
*a^2*b^2)*((2304*a^10*b^8 - 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(
16*a^18) + (tan(c/2 + (d*x)/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 +
2736*a^12*b^5 - 522*a^14*b^3))/(8*a^17) + (3*b^2*(-(a + b)*(a - b))^(1/2)*(
2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^20 - 64*a^18*b^2))/(8*a^17))*(4*a^4 + 2
4*b^4 - 23*a^2*b^2))/(2*a^10))*3i)/(2*a^10))/((41472*b^15 - 141696*a^2*b^13
 + 186696*a^4*b^11 - 118332*a^6*b^9 + 36495*a^8*b^7 - 4815*a^10*b^5 + 180*a
^12*b^3)/(8*a^18) + (tan(c/2 + (d*x)/2)*(20736*b^14 - 65664*a^2*b^12 + 7822
8*a^4*b^10 - 43065*a^6*b^8 + 10737*a^8*b^6 - 972*a^10*b^4))/(4*a^17) - (3*b
^2*(-(a + b)*(a - b))^(1/2)*(4*a^4 + 24*b^4 - 23*a^2*b^2)*((2304*a^10*b^8 -
 3936*a^12*b^6 + 1896*a^14*b^4 - 222*a^16*b^2)/(16*a^18) + (tan(c/2 + (d*x)
/2)*(15*a^16*b + 2304*a^8*b^9 - 4512*a^10*b^7 + 2736*a^12*b^5 - 522*a^14*b^
3))/(8*a^17) - (3*b^2*(-(a + b)*(a - b))^(1/2)*...
```

3.1276 $\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=712

$$\frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}}-\frac{20(a^2-b^2)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}}+\frac{80(3a^2+2b^2)\cos(e+fx)\sqrt{d\sin}}{693a^3b^2df(a+b\sin(e+f}$$

```
[Out] 2/11*cos(f*x+e)^5*(d*sin(f*x+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))^(11/2)-20/99*
(a^2-b^2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^2/b^2/d/f/(a+b*sin(f*x+e))^(9/2
)+80/693*(3*a^2+2*b^2)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^3/b^2/d/f/(a+b*sin
(f*x+e))^(7/2)-4/231*(5*a^4-17*a^2*b^2+16*b^4)*cos(f*x+e)*(d*sin(f*x+e))^(1
/2)/a^4/b^2/(a^2-b^2)/d/f/(a+b*sin(f*x+e))^(5/2)-8/693*(5*a^6-22*a^4*b^2+65
*a^2*b^4-32*b^6)*cos(f*x+e)*(d*sin(f*x+e))^(1/2)/a^5/b^2/(a^2-b^2)^2/d/f/(a
+b*sin(f*x+e))^(3/2)+16/693*b*(93*a^4-93*a^2*b^2+32*b^4)*cos(f*x+e)/a^5/(a^
2-b^2)^3/f/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)-16/693*b*(93*a^4-93*
a^2*b^2+32*b^4)*EllipticE(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin
(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+c
sc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^7/(a-b)^2/(a+b)^(5/2)/f/d^(1/2)-16/693
*(45*a^4-48*a^3*b-69*a^2*b^2+24*a*b^3+32*b^4)*EllipticF(d^(1/2)*(a+b*sin(f*
x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a*(1-cs
c(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/a^6/(a-b)^
2/(a+b)^(5/2)/f/d^(1/2)
```

Rubi [A]

time = 1.67, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2966, 2970, 3134, 3072, 3077, 2895, 3073}

$$-\frac{20(a^2-b^2)\cos(e+fx)\sqrt{d\sin(e+fx)}}{99a^2b^2df(a+b\sin(e+fx))^{9/2}}-\frac{4(5a^4-17a^2b^2+16b^4)\cos(e+fx)\sqrt{d\sin(e+fx)}}{231a^4b^2df(a^2-b^2)(a+b\sin(e+fx))^{5/2}}+\frac{80(3a^2+2b^2)\cos(e+fx)\sqrt{d\sin(e+fx)}}{693a^3b^2df(a+b\sin(e+fx))^{7/2}}-\frac{16b(93a^4-93a^2b^2+32b^4)\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{693a^7\sqrt{d}f(a-b)^2(a+b)^{5/2}}+\frac{16b(93a^4-93a^2b^2+32b^4)\cos(e+fx)}{693a^5f(a^2-b^2)^3\sqrt{d\sin(e+fx)}\sqrt{a+b\sin(e+fx)}}-\frac{8(5a^6-22a^4b^2+65a^2b^4-32b^6)\cos(e+fx)\sqrt{d\sin(e+fx)}}{693a^5b^2df(a^2-b^2)^2(a+b\sin(e+fx))^{3/2}}-\frac{16(45a^4-48a^3b-69a^2b^2+24ab^3+32b^4)\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{d}\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{693a^6\sqrt{d}f(a-b)^2(a+b)^{5/2}}+\frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2)),x]
```

```
[Out] (2*Cos[e + f*x]^5*Sqrt[d*Sin[e + f*x]])/(11*a*d*f*(a + b*Sin[e + f*x])^(11/
2)) - (20*(a^2 - b^2)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(99*a^2*b^2*d*f*(a
 + b*Sin[e + f*x])^(9/2)) + (80*(3*a^2 + 2*b^2)*Cos[e + f*x]*Sqrt[d*Sin[e +
 f*x]])/(693*a^3*b^2*d*f*(a + b*Sin[e + f*x])^(7/2)) - (4*(5*a^4 - 17*a^2*b
^2 + 16*b^4)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(231*a^4*b^2*(a^2 - b^2)*d*
f*(a + b*Sin[e + f*x])^(5/2)) - (8*(5*a^6 - 22*a^4*b^2 + 65*a^2*b^4 - 32*b^
6)*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]])/(693*a^5*b^2*(a^2 - b^2)^2*d*f*(a + b
*Sin[e + f*x])^(3/2)) + (16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Cos[e + f*x])/
(693*a^5*(a^2 - b^2)^3*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (
```

16*b*(93*a^4 - 93*a^2*b^2 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(693*a^7*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f) - (16*(45*a^4 - 48*a^3*b - 69*a^2*b^2 + 24*a*b^3 + 32*b^4)*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(693*a^6*(a - b)^2*(a + b)^(5/2)*Sqrt[d]*f)

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqr
t[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*Elli
pticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]
], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
&& PosQ[(a + b)/d]
```

Rule 2966

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-g)*(g*
Cos[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^(m + 1)/(a
*d*f*(m + 1))), x] + Dist[g^2*((2*m + 3)/(2*a*(m + 1))), Int[(g*Cos[e + f*x
])^(p - 2)*((a + b*Sin[e + f*x])^(m + 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; F
reeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p
 + 1/2, 0]
```

Rule 2970

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(a^2 - b^2)*Cos[e +
f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*b^2*d*f*(m +
 1))), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[(a^2*(n - m + 1) - b^2*(m + n
+ 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*((d*Sin[e + f*x])^(n + 1)/(
a^2*b^2*d*f*(m + 1)*(m + 2))), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3072

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(
x_)]]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(3/2)), x_Symbol] :> Simp[2*(A
*b - a*B)*(Cos[e + f*x]/(f*(a^2 - b^2)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[d*Sin[
```

```
e + f*x]])), x] + Dist[d/(a^2 - b^2), Int[(A*b - a*B + (a*A - b*B)*Sin[e +
f*x])/(Sqrt[a + b*Sin[e + f*x]]*(d*Sin[e + f*x])^(3/2)), x], x] /; FreeQ[{a
, b, d, e, f, A, B}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3073

```
Int[((A_) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((b_.)*sin[(e_.) + (f_.)*(x_)])
^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*A*
(c - d)*(Tan[e + f*x]/(f*b*c^2))*Rt[(c + d)/b, 2]*Sqrt[c*((1 + Csc[e + f*x]
)/(c - d))]*Sqrt[c*((1 - Csc[e + f*x])/(c + d))]*EllipticE[ArcSin[Sqrt[c +
d*Sin[e + f*x]]/Sqrt[b*Sin[e + f*x]]/Rt[(c + d)/b, 2]], -(c + d)/(c - d)],
x] /; FreeQ[{b, c, d, e, f, A, B}, x] && NeQ[c^2 - d^2, 0] && EqQ[A, B] &&
PosQ[(c + d)/b]
```

Rule 3077

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])^(3/2)*Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
 && NeQ[A, B]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{13/2}}\,dx = \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} + \frac{10\int \frac{\cos^4(e+fx)}{\sqrt{d\sin(e+fx)}\,(a+bs}}{11a}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

$$= \frac{2\cos^5(e+fx)\sqrt{d\sin(e+fx)}}{11adf(a+b\sin(e+fx))^{11/2}} - \frac{20(a^2-b^2)\cos(e+fx)\sqrt{}}{99a^2b^2df(a+b\sin(e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.68, size = 1906, normalized size = 2.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^6/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(13/2))
,x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((2*(a^4*Cos[e + f*x] - 2*a^2*b^2*Co
s[e + f*x] + b^4*Cos[e + f*x]))/(11*a*b^4*(a + b*Sin[e + f*x])^6) - (4*(18*
a^4*Cos[e + f*x] - 13*a^2*b^2*Cos[e + f*x] - 5*b^4*Cos[e + f*x]))/(99*a^2*b
^4*(a + b*Sin[e + f*x])^5) + (4*(189*a^4*Cos[e + f*x] - 3*a^2*b^2*Cos[e + f
*x] + 40*b^4*Cos[e + f*x]))/(693*a^3*b^4*(a + b*Sin[e + f*x])^4) - (4*(42*a
^6*Cos[e + f*x] - 37*a^4*b^2*Cos[e + f*x] - 17*a^2*b^4*Cos[e + f*x] + 16*b^
6*Cos[e + f*x]))/(231*a^4*b^4*(a^2 - b^2)*(a + b*Sin[e + f*x])^3) + (2*(63*
a^8*Cos[e + f*x] - 146*a^6*b^2*Cos[e + f*x] + 151*a^4*b^4*Cos[e + f*x] - 26
```

```
0*a^2*b^6*Cos[e + f*x] + 128*b^8*Cos[e + f*x]))/(693*a^5*b^4*(a^2 - b^2)^2*
(a + b*Sin[e + f*x])^2) - (16*(93*a^4*b^2*Cos[e + f*x] - 93*a^2*b^4*Cos[e +
 f*x] + 32*b^6*Cos[e + f*x]))/(693*a^6*(a^2 - b^2)^3*(a + b*Sin[e + f*x])))
)/(f*Sqrt[d*Sin[e + f*x]]) + (8*Sqrt[Sin[e + f*x]]*((4*a*(45*a^6 - 114*a^4*
b^2 + 101*a^2*b^4 - 32*b^6)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a +
 b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])
)/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqr
t[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + P
i/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[
a + b*Sin[e + f*x]]) + 4*a*(-93*a^5*b + 93*a^3*b^3 - 32*a*b^5)*((Sqrt[((a +
 b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e +
Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e
+ f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^
2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*
Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(
-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*S
ec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x
)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(93*a^4*b^2 -
 93*a^2*b^4 + 32*b^6)*((Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[
e + f*x]]) + (I*Cos[(-e + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[S
in[(-e + Pi/2 - f*x)/2]/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Si
n[e + f*x]])/(b*Sqrt[Cos[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e +
 f*x]*(a + b*Sin[e + f*x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi
/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2
*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e +
 Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/
a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqr
t[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2
 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Si
n[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e +
 f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sq
rt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])))/b)))/(693*a^6*(a - b)^3*(a + b
)^3*f*Sqrt[d*Sin[e + f*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 56845 vs. $2(648) = 1296$.

time = 2.80, size = 56846, normalized size = 79.84

method	result	size
default	Expression too large to display	56846

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*cos(f*x + e)^6/(b^7*
d*cos(f*x + e)^8 - (21*a^2*b^5 + 4*b^7)*d*cos(f*x + e)^6 + (35*a^4*b^3 + 63
*a^2*b^5 + 6*b^7)*d*cos(f*x + e)^4 - (7*a^6*b + 70*a^4*b^3 + 63*a^2*b^5 + 4
*b^7)*d*cos(f*x + e)^2 + (7*a^6*b + 35*a^4*b^3 + 21*a^2*b^5 + b^7)*d - (7*a
*b^6*d*cos(f*x + e)^6 - 7*(5*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^4 + 7*(3*a^5
*b^2 + 10*a^3*b^4 + 3*a*b^6)*d*cos(f*x + e)^2 - (a^7 + 21*a^5*b^2 + 35*a^3*
b^4 + 7*a*b^6)*d)*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e))**(13/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e))^(13/2)/(d*sin(f*x+e))^(1/2),x, algo
rithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/((b*sin(f*x + e) + a)^(13/2)*sqrt(d*sin(f*x + e)))
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^6}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(13/2)),x)
```

```
[Out] int(cos(e + f*x)^6/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^(13/2)), x)
```

3.1277 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=159

$$\frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}}+\frac{(2a^2-b^2)\,F\left(\frac{1}{4}(4e-\pi)+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{3fg^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}$$

```
[Out] 4/3*a*b*(d*sin(f*x+e))^(3/2)/d^2/f/g/(g*cos(f*x+e))^(3/2)+2/3*(a^2+b^2)*(d*
sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(3/2)-1/3*(2*a^2-b^2)*(sin(e+1/4*Pi+
f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*
f*x+2*e)^(1/2)/f/g^2/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2990, 2643, 3281, 468, 335, 243, 342, 281, 238}

$$-\frac{2(2a^2-b^2)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}+\frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}+\frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a^2 + b^2)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) + (4*
a*b*(d*Sin[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a^2
- b^2)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*S
in[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Rule 243

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4)), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3281

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[ff*c^(2*IntPart[(m - 1)/2] + 1)*((c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2])/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]
)), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
```

```
egerQ[m]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{(2ab)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{5/2}}\,dx}{d} + \int \frac{a^2+b^2\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+f}$$

$$= \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\cos^2(e+fx)^{3/4}\text{Subst}\left(\int \frac{a^2+b^2x^2}{\sqrt{dx}\,(1-x^2)^{7/}}\right.}{fg(g\cos(e+fx))^3}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{((-2a^2}{}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{(2(-2a}{}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a\right.}{}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left(2(-2a\right.}{}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} + \frac{\left((-2a^2\right.}{}$$

$$= \frac{2(a^2+b^2)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{4ab(d\sin(e+fx))^{3/2}}{3d^2fg(g\cos(e+fx))^{3/2}} - \frac{2(2a^2-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.31, size = 127, normalized size = 0.80

$$\frac{2\left(15a^2\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)+b\sin(e+fx)\left(10a+3b\cos^2(e+fx)^{3/4}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(e+fx)\right)\sin(e+fx)\right)\right)\tan(e+fx)}{15fg^2\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e +
f*x]^2] + b*Sin[e + f*x]*(10*a + 3*b*(Cos[e + f*x]^2)^(3/4)*Hypergeometric2
F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[e + f*x]))*Tan[e + f*x])/(15*f*g^2*Sq
rt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(159) = 318$.
time = 32.51, size = 372, normalized size = 2.34

method	result
default	$-\frac{\left(2\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\ \mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\mathrm{si}}{\sin(fx+e}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/3/f*(2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*
x+e)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*sin(f*x
+e)*b^2-2*cos(f*x+e)*sin(f*x+e)*2^(1/2)*a*b-cos(f*x+e)*2^(1/2)*a^2-cos(f*x+
e)*2^(1/2)*b^2+2*sin(f*x+e)*2^(1/2)*a*b+2^(1/2)*a^2+2^(1/2)*b^2)*cos(f*x+e)
*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)*2^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 155, normalized size = 0.97

$$-\frac{(2\,a^2-b^2)\sqrt{i\,dg}\,\cos(fx+e)^2\,\mathrm{ellipticF}(\cos(fx+e)+i\,\sin(fx+e),-1)+(2\,a^2-b^2)\sqrt{-i\,dg}\,\cos(fx+e)^2\,\mathrm{ellipticF}(\cos(fx+e)-i\,\sin(fx+e),-1)-2\,(2\,ab\sin(fx+e)+a^2+b^2)\sqrt{g\cos(fx+e)}\,\sqrt{d\sin(fx+e)}}{3\,dfg^3\cos(fx+e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] -1/3*((2*a^2 - b^2)*sqrt(I*d*g)*cos(f*x + e)^2*ellipticF(cos(f*x + e) + I*s
in(f*x + e), -1) + (2*a^2 - b^2)*sqrt(-I*d*g)*cos(f*x + e)^2*ellipticF(cos(
f*x + e) - I*sin(f*x + e), -1) - 2*(2*a*b*sin(f*x + e) + a^2 + b^2)*sqrt(g*
cos(f*x + e))*sqrt(d*sin(f*x + e)))/(d*f*g^3*cos(f*x + e)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\sin(e+f\,x))^2}{(g\,\cos(e+f\,x))^{5/2}\,\sqrt{d\,\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

3.1278 $\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=193

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}}-\frac{8ab\sqrt{g\cos(e+f}}{5}$$

```
[Out] 8/5*a*b*(d*sin(f*x+e))^(3/2)/d^2/f/g^3/(g*cos(f*x+e))^(1/2)+2/5*(a+b*sin(f*
x+e))^2*(d*sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(5/2)+8/5*a^2*(d*sin(f*x+
e))^(1/2)/d/f/g^3/(g*cos(f*x+e))^(1/2)+8/5*a*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/
sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)
*(d*sin(f*x+e))^(1/2)/d/f/g^4/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2967, 2917, 2643, 2651, 2652, 2719}

$$\frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}}+\frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}}-\frac{8abE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5dfg^4\sqrt{\sin(2e+2fx)}}+\frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (8*a^2*Sqrt[d*Sin[e + f*x]])/(5*d*f*g^3*Sqrt[g*Cos[e + f*x]]) + (8*a*b*(d*S
in[e + f*x])^(3/2))/(5*d^2*f*g^3*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[d*Sin[e +
f*x]]*(a + b*Sin[e + f*x])^2)/(5*d*f*g*(g*Cos[e + f*x])^(5/2)) - (8*a*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(5*d*f
*g^4*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2967

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*(g*Cos
[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^m/(d*f*g*(2*m
 + 1))), x] + Dist[2*a*(m/(g^2*(2*m + 1))), Int[(g*Cos[e + f*x])^(p + 2)*((
a + b*Sin[e + f*x])^(m - 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rubi steps

$$\int \frac{(a+b\sin(e+fx))^2}{(g\cos(e+fx))^{7/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{(4a)\int \frac{a+b\sin(e+}{(g\cos(e+fx))^{3/2}\sqrt{d}}}{5g^2}$$

$$= \frac{2\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}{5dfg(g\cos(e+fx))^{5/2}} + \frac{(4a^2)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d}}}{5g^2}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+}}{5dfg}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+}}{5dfg}$$

$$= \frac{8a^2\sqrt{d\sin(e+fx)}}{5dfg^3\sqrt{g\cos(e+fx)}} + \frac{8ab(d\sin(e+fx))^{3/2}}{5d^2fg^3\sqrt{g\cos(e+fx)}} + \frac{2\sqrt{d\sin(e+}}{5dfg}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.46, size = 105, normalized size = 0.54

$$\frac{2\left(15a^2+10ab\cos^2(e+fx)^{5/4}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\sin^2(e+fx)\right)\sin(e+fx)+3(-4a^2+b^2)\sin^2(e+fx)\right)\tan(e+fx)}{15fg^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x])^2/((g*Cos[e + f*x])^(7/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(15*a^2 + 10*a*b*(Cos[e + f*x]^2)^(5/4)*Hypergeometric2F1[3/4, 9/4, 7/4,
 Sin[e + f*x]^2]*Sin[e + f*x] + 3*(-4*a^2 + b^2)*Sin[e + f*x]^2)*Tan[e + f*
x])/(15*f*g^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(192) = 384$.
time = 0.47, size = 616, normalized size = 3.19

method	result
default	$-\frac{\left(4(\cos^3(fx+e))\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+}{\sin(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/5/f*(4*cos(f*x+e)^3*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-
8*cos(f*x+e)^3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+4*cos(f*
x+e)^2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b-8*cos(f*x+e)^2*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b+4*cos(f*x+e)^3*2^(1/2)*a
*b-4*cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*a^2+cos(f*x+e)^2*sin(f*x+e)*2^(1/2)*b^
2-2*cos(f*x+e)^2*2^(1/2)*a*b-sin(f*x+e)*2^(1/2)*a^2-sin(f*x+e)*2^(1/2)*b^2-
2*a*b*2^(1/2))*cos(f*x+e)/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2/(g*cos(f*x+e))**(7/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2/(g*cos(f*x+e))^(7/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2/((g*cos(f*x + e))^(7/2)*sqrt(d*sin(f*x + e
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \sin(e + f x))^2}{(g \cos(e + f x))^{7/2} \sqrt{d \sin(e + f x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(7/2)*(d*sin(e + f*x))^(1/2)),
x)
[Out] int((a + b*sin(e + f*x))^2/((g*cos(e + f*x))^(7/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

3.1279 $\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=76

$$-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] -a^3*ln(a+b*sin(d*x+c))/b^4/d+a^2*sin(d*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d
+1/3*sin(d*x+c)^3/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$-\frac{a^3\log(a+b\sin(c+dx))}{b^4d}+\frac{a^2\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^3*Log[a + b*Sin[c + d*x]])/(b^4*d)) + (a^2*Sin[c + d*x])/(b^3*d) - (a*
Sin[c + d*x]^2)/(2*b^2*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2 - ax + x^2 - \frac{a^3}{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= -\frac{a^3\log(a+b\sin(c+dx))}{b^4 d} + \frac{a^2\sin(c+dx)}{b^3 d} - \frac{a\sin^2(c+dx)}{2b^2 d} + \frac{\sin^3(c+d}{3bd}$$

Mathematica [A]
time = 0.15, size = 66, normalized size = 0.87

$$\frac{-6a^3\log(a+b\sin(c+dx)) + 6a^2b\sin(c+dx) - 3ab^2\sin^2(c+dx) + 2b^3\sin^3(c+dx)}{6b^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-6*a^3*Log[a + b*Sin[c + d*x]] + 6*a^2*b*Sin[c + d*x] - 3*a*b^2*Sin[c + d*
x]^2 + 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

Maple [A]
time = 0.19, size = 65, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3} - \frac{ba\left(\sin^2(dx+c)\right)}{2} + a^2\sin(dx+c)}{b^3} - \frac{a^3\ln(a+b\sin(dx+c))}{b^4}}{d}$
default	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3} - \frac{ba\left(\sin^2(dx+c)\right)}{2} + a^2\sin(dx+c)}{b^3} - \frac{a^3\ln(a+b\sin(dx+c))}{b^4}}{d}$
risch	$\frac{ia^3x}{b^4} + \frac{a\,e^{2i(dx+c)}}{8b^2d} - \frac{ie^{i(dx+c)}a^2}{2b^3d} - \frac{ie^{i(dx+c)}}{8bd} + \frac{ie^{-i(dx+c)}a^2}{2b^3d} + \frac{ie^{-i(dx+c)}}{8bd} + \frac{a\,e^{-2i(dx+c)}}{8b^2d} + \frac{2ia^3c}{b^4d} - \frac{a^3\ln}{}$
norman	$\frac{\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^3d} + \frac{2a^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d} + \frac{2\left(9a^2+4b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d} + \frac{2\left(9a^2+4b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d} - \frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d} - \frac{2a(t}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/3*sin(d*x+c)^3*b^2-1/2*b*a*sin(d*x+c)^2+a^2*sin(d*x+c))-a^3/b
^4*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.41, size = 67, normalized size = 0.88

$$-\frac{\frac{6\,a^3\log(b\sin(dx+c)+a)}{b^4}-\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*log(b*sin(d*x + c) + a)/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*b*sin
(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

Fricas [A]
time = 0.36, size = 71, normalized size = 0.93

$$\frac{3\,ab^2\cos\left(dx+c\right)^2-6\,a^3\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(b^3\cos\left(dx+c\right)^2-3\,a^2b-b^3\right)\sin\left(dx+c\right)}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*b^2*cos(d*x + c)^2 - 6*a^3*log(b*sin(d*x + c) + a) - 2*(b^3*cos(d*
x + c)^2 - 3*a^2*b - b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [A]
time = 0.50, size = 105, normalized size = 1.38

$$\begin{cases} \frac{x\sin^3\left(c\right)\cos\left(c\right)}{a} & \text{for}\: b=0\wedge d=0 \\ \frac{\sin^4\left(c+dx\right)}{4ad} & \text{for}\: b=0 \\ \frac{x\sin^3\left(c\right)\cos\left(c\right)}{a+b\sin\left(c\right)} & \text{for}\: d=0 \\ -\frac{a^3\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{b^4d}+\frac{a^2\sin\left(c+dx\right)}{b^3d}-\frac{a\sin^2\left(c+dx\right)}{2b^2d}+\frac{\sin^3\left(c+dx\right)}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**3*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)**4/(4*
a*d), Eq(b, 0)), (x*sin(c)**3*cos(c)/(a + b*sin(c)), Eq(d, 0)), (-a**3*log(
a/b + sin(c + d*x))/(b**4*d) + a**2*sin(c + d*x)/(b**3*d) - a*sin(c + d*x)*
*2/(2*b**2*d) + sin(c + d*x)**3/(3*b*d), True))
```

Giac [A]
time = 0.44, size = 68, normalized size = 0.89

$$-\frac{\frac{6\,a^3\log(|b\sin(dx+c)+a|)}{b^4}-\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)}{b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*log(abs(b*sin(d*x + c) + a))/b^4 - (2*b^2*sin(d*x + c)^3 - 3*a*
b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c))/b^3)/d
```

Mupad [B]
time = 0.07, size = 64, normalized size = 0.84

$$\frac{\frac{\sin(c+d\,x)^3}{3\,b}-\frac{a^3\,\ln(a+b\,\sin(c+d\,x))}{b^4}-\frac{a\sin(c+d\,x)^2}{2\,b^2}+\frac{a^2\,\sin(c+d\,x)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^3/(3*b) - (a^3*log(a + b*sin(c + d*x)))/b^4 - (a*sin(c + d*x)
^2)/(2*b^2) + (a^2*sin(c + d*x))/b^3)/d
```

3.1280 $\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] a^2*ln(a+b*sin(d*x+c))/b^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 45}

$$\frac{a^2\log(a+b\sin(c+dx))}{b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^2*Log[a + b*Sin[c + d*x]])/(b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c +
d*x]^2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+x+\frac{a^2}{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^3 d}$$
$$= \frac{a^2\log(a+b\sin(c+dx))}{b^3 d} - \frac{a\sin(c+dx)}{b^2 d} + \frac{\sin^2(c+dx)}{2bd}$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 0.89

$$\frac{2a^2\log(a+b\sin(c+dx)) - 2ab\sin(c+dx) + b^2\sin^2(c+dx)}{2b^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] (2*a^2*Log[a + b*Sin[c + d*x]] - 2*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(
2*b^3*d)
```

Maple [A]
time = 0.11, size = 49, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{a^2\ln(a+b\sin(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{a^2\ln(a+b\sin(dx+c))}{b^3}}{d}$
risch	$-\frac{ia^2x}{b^3}-\frac{e^{2i(dx+c)}}{8bd}+\frac{ia\,e^{i(dx+c)}}{2b^2d}-\frac{ia\,e^{-i(dx+c)}}{2b^2d}-\frac{e^{-2i(dx+c)}}{8bd}-\frac{2ia^2c}{b^3d}+\frac{a^2\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{b^3d}$
norman	$\frac{\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}+\frac{2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2d}-\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{a^2\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{b^3 d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b^2*(-1/2*sin(d*x+c)^2*b+a*sin(d*x+c))+a^2/b^3*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.38, size = 49, normalized size = 0.89

$$\frac{\frac{2\,a^2\log(b\sin(dx+c)+a)}{b^3}+\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*log(b*sin(d*x + c) + a)/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d*x +
c))/b^2)/d
```

Fricas [A]
time = 0.38, size = 47, normalized size = 0.85

$$-\frac{b^2\cos\left(dx+c\right)^2-2\,a^2\log\left(b\sin\left(dx+c\right)+a\right)+2\,ab\sin\left(dx+c\right)}{2\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(b*sin(d*x + c) + a) + 2*a*b*sin(d*x +
c))/(b^3*d)
```

Sympy [A]
time = 0.40, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x\sin^2\left(c\right)\cos\left(c\right)}{a} & \text{for}\ b=0\wedge d=0\\ \frac{\sin^3\left(c+dx\right)}{3ad} & \text{for}\ b=0\\ \frac{x\sin^2\left(c\right)\cos\left(c\right)}{a+b\sin\left(c\right)} & \text{for}\ d=0\\ \frac{a^2\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{b^3d}-\frac{a\sin\left(c+dx\right)}{b^2d}+\frac{\sin^2\left(c+dx\right)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)**2*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)**3/(3*
a*d), Eq(b, 0)), (x*sin(c)**2*cos(c)/(a + b*sin(c)), Eq(d, 0)), (a**2*log(a
/b + sin(c + d*x))/(b**3*d) - a*sin(c + d*x)/(b**2*d) + sin(c + d*x)**2/(2*
b*d), True))
```

Giac [A]
time = 0.44, size = 50, normalized size = 0.91

$$\frac{\frac{2\,a^2\log(|b\sin(dx+c)+a|)}{b^3}+\frac{b\sin(dx+c)^2-2\,a\sin(dx+c)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/b^3 + (b*sin(d*x + c)^2 - 2*a*sin(d
*x + c))/b^2)/d
```

Mupad [B]
time = 0.07, size = 47, normalized size = 0.85

$$\frac{2\,a^2\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)+b^2\,{\sin\left(c+d\,x\right)}^2-2\,a\,b\,\sin\left(c+d\,x\right)}{2\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (2*a^2*log(a + b*sin(c + d*x)) + b^2*sin(c + d*x)^2 - 2*a*b*sin(c + d*x))/(
2*b^3*d)
```

3.1281 $\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$-\frac{a\log(a+b\sin(c+dx))}{b^2d}+\frac{\sin(c+dx)}{bd}$$

```
[Out] -a*ln(a+b*sin(d*x+c))/b^2/d+sin(d*x+c)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2912, 12, 45}

$$\frac{\sin(c+dx)}{bd}-\frac{a\log(a+b\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*Log[a + b*Sin[c + d*x]])/(b^2*d)) + Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a+x}\right)\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= -\frac{a\log(a+b\sin(c+dx))}{b^2d} + \frac{\sin(c+dx)}{bd}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(a+b\sin(c+dx))}{b^2} - \frac{\sin(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] -(((a*Log[a + b*Sin[c + d*x]])/b^2 - Sin[c + d*x]/b)/d)
```

Maple [A]
time = 0.10, size = 33, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{\sin(dx+c)}{b}-\frac{a\ln(a+b\sin(dx+c))}{b^2}}{d}$	33
default	$\frac{\frac{\sin(dx+c)}{b}-\frac{a\ln(a+b\sin(dx+c))}{b^2}}{d}$	33
risch	$\frac{iax}{b^2}-\frac{i\mathrm{e}^{i(dx+c)}}{2bd}+\frac{i\mathrm{e}^{-i(dx+c)}}{2bd}+\frac{2iac}{b^2d}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}-1+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{b}\right)}{b^2d}$	94
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}+\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{a\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{b^2d}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/b*sin(d*x+c)-1/b^2*a*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 33, normalized size = 0.97

$$-\frac{\frac{a\log(b\sin(dx+c)+a)}{b^2} - \frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(b*sin(d*x + c) + a)/b^2 - sin(d*x + c)/b)/d
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.91

$$-\frac{a\log\left(b\sin\left(dx+c\right)+a\right)-b\sin\left(dx+c\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*log(b*sin(d*x + c) + a) - b*sin(d*x + c))/(b^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(27) = 54$.
time = 0.34, size = 65, normalized size = 1.91

$$\begin{cases} \frac{x\sin\left(c\right)\cos\left(c\right)}{a} & \text{for}\: b=0\wedge d=0 \\ \frac{\sin^2\left(c+dx\right)}{2ad} & \text{for}\: b=0 \\ \frac{x\sin\left(c\right)\cos\left(c\right)}{a+b\sin\left(c\right)} & \text{for}\: d=0 \\ -\frac{a\log\left(\frac{a}{b}+\sin\left(c+dx\right)\right)}{b^2d}+\frac{\sin\left(c+dx\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*sin(c)*cos(c)/a, Eq(b, 0) & Eq(d, 0)), (sin(c + d*x)**2/(2*a*d
), Eq(b, 0)), (x*sin(c)*cos(c)/(a + b*sin(c)), Eq(d, 0)), (-a*log(a/b + sin
(c + d*x))/(b**2*d) + sin(c + d*x)/(b*d), True))
```

Giac [A]
time = 0.43, size = 34, normalized size = 1.00

$$-\frac{\frac{a\log(|b\sin(dx+c)+a|)}{b^2}-\frac{\sin(dx+c)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(b*sin(d*x + c) + a))/b^2 - sin(d*x + c)/b)/d
```

Mupad [B]
time = 0.06, size = 31, normalized size = 0.91

$$-\frac{a\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)-b\,\sin\left(c+d\,x\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*sin(c + d*x))/(a + b*sin(c + d*x)),x)
[Out] -(a*log(a + b*sin(c + d*x)) - b*sin(c + d*x))/(b^2*d)
```

3.1282 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] ln(sin(d*x+c))/a/d-ln(a+b*sin(d*x+c))/a/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2800, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Maple [A]
time = 0.12, size = 33, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a}-\frac{\ln(a+b\sin(dx+c))}{a}}{d}$	33
default	$\frac{\frac{\ln(\sin(dx+c))}{a}-\frac{\ln(a+b\sin(dx+c))}{a}}{d}$	33
risch	$-\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{ad}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{ad}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/a*ln(sin(d*x+c))-1/a*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 33, normalized size = 0.97

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.91

$$-\frac{\log\left(b\sin\left(dx+c\right)+a\right)-\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)\csc\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 35, normalized size = 1.03

$$-\frac{\frac{\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a}-\frac{\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

Mupad [B]
time = 11.83, size = 48, normalized size = 1.41

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
[Out] (log(tan(c/2 + (d*x)/2)) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(a*d)
```

3.1283 $\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$-\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}$$

[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+b*ln(a+b*sin(d*x+c))/a^2/d

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2912, 12, 46}

$$-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{b\log(a+b\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c + d*x]])/(a^2*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{1}{a^2x} + \frac{1}{a^2(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{b\log(a+b\sin(c+dx))}{a^2d}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 1.00

$$-\frac{\csc(c+dx)}{ad} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{b\log(a+b\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + (b*Log[a + b*Sin[c
+ d*x]])/(a^2*d)
```

Maple [A]
time = 0.13, size = 34, normalized size = 0.68

method	result	size
derivativedivides	$-\frac{\frac{\csc(dx+c)}{a}-\frac{b\ln(a\csc(dx+c)+b)}{a^2}}{d}$	34
default	$-\frac{\frac{\csc(dx+c)}{a}-\frac{b\ln(a\csc(dx+c)+b)}{a^2}}{d}$	34
risch	$-\frac{2i e^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{a^2d}-\frac{b\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	90
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{a^2d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/a*csc(d*x+c)-1/a^2*b*ln(a*csc(d*x+c)+b))
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.94

$$\frac{\frac{b\log(b\sin(dx+c)+a)}{a^2}-\frac{b\log(\sin(dx+c))}{a^2}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(b*sin(d*x + c) + a)/a^2 - b*log(sin(d*x + c))/a^2 - 1/(a*sin(d*x + c
)))/d
```

Fricas [A]
time = 0.36, size = 56, normalized size = 1.12

$$\frac{b\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-b\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-a}{a^2d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b*log(b*sin(d*x + c) + a)*sin(d*x + c) - b*log(1/2*sin(d*x + c))*sin(d*x +
 c) - a)/(a^2*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)\csc^2\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 49, normalized size = 0.98

$$\frac{\frac{b\log(|b\sin(dx+c)+a|)}{a^2}-\frac{b\log(|\sin(dx+c)|)}{a^2}-\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (b*log(abs(b*sin(d*x + c) + a))/a^2 - b*log(abs(sin(d*x + c)))/a^2 - 1/(a*s
in(d*x + c)))/d
```

Mupad [B]
time = 11.81, size = 89, normalized size = 1.78

$$\frac{b\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^2\,d}-\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}+\frac{1}{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(a^2*d) - (b*l
og(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)/2 + 1/(2*tan(c/2 + (d
*x)/2)))/(a*d)
```

3.1284 $\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}+\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d+b^2*ln(sin(d*x+c))/a^3/d-b^2*ln(a+b
*sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2912, 12, 46}

$$\frac{b^2\log(\sin(c+dx))}{a^3d}-\frac{b^2\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{b^2\text{Subst}\left(\int \frac{1}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{1}{a^2x^2} + \frac{1}{a^3x} - \frac{1}{a^3(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} + \frac{b^2\log(\sin(c+dx))}{a^3d} - \frac{b^2\log(a+b\sin(c+d}{a^3d}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.00

$$\frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} + \frac{b^2\log(\sin(c+dx))}{a^3d} - \frac{b^2\log(a+b\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) + (b^2*Log[Sin[c + d*x]])
/(a^3*d) - (b^2*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Maple [A]
time = 0.18, size = 65, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{b^2\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}-\frac{b^2\ln(a+b\sin(dx+c))}{a^3}}{d}$
default	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{b^2\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}-\frac{b^2\ln(a+b\sin(dx+c))}{a^3}}{d}$
risch	$\frac{2i\left(-ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{b^2\ln\left(e^{2i(dx+c)}-1\right)}{a^3d}-\frac{b^2\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{a^3d}$
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d}-\frac{b^2\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\right.\right.}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sin(d*x+c)^2+b^2/a^3*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)-b^2/a^3*
ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 66, normalized size = 0.92

$$-\frac{\frac{2\,b^2\log(b\sin(dx+c)+a)}{a^3}-\frac{2\,b^2\log(\sin(dx+c))}{a^3}-\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b^2*log(b*sin(d*x + c) + a)/a^3 - 2*b^2*log(sin(d*x + c))/a^3 - (2*
b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.38, size = 100, normalized size = 1.39

$$-\frac{2\,ab\sin\left(dx+c\right)-a^2+2\left(b^2\cos\left(dx+c\right)^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\left(b^2\cos\left(dx+c\right)^2-b^2\right)\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)}{2\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 + 2*(b^2*cos(d*x + c)^2 - b^2)*log(b*sin(d*x
 + c) + a) - 2*(b^2*cos(d*x + c)^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*d*co
s(d*x + c)^2 - a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)\csc^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 71, normalized size = 0.99

$$-\frac{\frac{2\,b^2\log(|b\sin(dx+c)+a|)}{a^3}-\frac{2\,b^2\log(|\sin(dx+c)|)}{a^3}-\frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*b^2*log(abs(sin(d*x + c)))
/a^3 - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

Mupad [B]
time = 11.79, size = 132, normalized size = 1.83

$$\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^2d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8ad}-\frac{b^2\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^3d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{a}{2}-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4a^2d}+\frac{b^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (b^2*log(
a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(a^3*d) - (cot(c/2 +
(d*x)/2)^2*(a/2 - 2*b*tan(c/2 + (d*x)/2)))/(4*a^2*d) + (b^2*log(tan(c/2 + (
d*x)/2)))/(a^3*d)
```

3.1285 $\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$\frac{a(8a^4-4a^2b^2-b^4)\,x}{8b^6}-\frac{2a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d}-\frac{a(4a^2-$$

```
[Out] 1/8*a*(8*a^4-4*a^2*b^2-b^4)*x/b^6+1/15*(15*a^4-5*a^2*b^2-2*b^4)*cos(d*x+c)/
b^5/d-1/8*a*(4*a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d+1/15*(5*a^2-b^2)*cos(d*
x+c)*sin(d*x+c)^2/b^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/b^2/d+1/5*cos(d*x+c)*
sin(d*x+c)^4/b/d-2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^
2-b^2)^(1/2)/b^6/d
```

Rubi [A]

time = 0.59, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{a(4a^2-b^2)\sin(c+dx)\cos(c+dx)}{8b^4d}+\frac{(5a^2-b^2)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-\frac{2a^4\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{ax(8a^4-4a^2b^2-b^4)}{8b^6}+\frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d}-\frac{a\sin^3(c+dx)\cos(c+dx)}{4b^2d}+\frac{\sin^4(c+dx)\cos(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(8*a^4 - 4*a^2*b^2 - b^4)*x)/(8*b^6) - (2*a^4*Sqrt[a^2 - b^2]*ArcTan[(b
+ a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) + ((15*a^4 - 5*a^2*b^2 - 2*
b^4)*Cos[c + d*x])/(15*b^5*d) - (a*(4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])
/(8*b^4*d) + ((5*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3*d) - (a*Co
s[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(5*b*d
)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
```

```
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^3(c+dx)\left(-4a+b\sin(c+dx)+5a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{5b}$$

$$= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} + \frac{\int \frac{\sin^2(c+dx)(15a^2}{}}{}$$

$$= \frac{(5a^2-b^2)\cos(c+dx)\sin^2(c+dx)}{15b^3d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} + \frac{\cos(c}{}$$

$$= -\frac{a(4a^2-b^2)\cos(c+dx)\sin(c+dx)}{8b^4d} + \frac{(5a^2-b^2)\cos(c+dx)\sin^2(c+d}{15b^3d}$$

$$= \frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d} - \frac{a(4a^2-b^2)\cos(c+dx)\sin(c+dx)}{8b^4d} +$$

$$= \frac{a(8a^4-4a^2b^2-b^4)\,x}{8b^6} + \frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d} - \frac{a(4a^2-b^2)}{}$$

$$= \frac{a(8a^4-4a^2b^2-b^4)\,x}{8b^6} + \frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d} - \frac{a(4a^2-b^2)}{}$$

$$= \frac{a(8a^4-4a^2b^2-b^4)\,x}{8b^6} + \frac{(15a^4-5a^2b^2-2b^4)\cos(c+dx)}{15b^5d} - \frac{a(4a^2-b^2)}{}$$

$$= \frac{a(8a^4-4a^2b^2-b^4)\,x}{8b^6} - \frac{2a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} + \frac{(15a^4}{}$$

Mathematica [A]

time = 1.21, size = 177, normalized size = 0.75

$$\frac{-960a^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 60b(-8a^4+2a^2b^2+b^4)\cos(c+dx) - 10(4a^2b^3+b^5)\cos(3(c+dx)) + 6b^5\cos(5(c+dx)) + 15a(4(8a^4-4a^2b^2-b^4)(c+dx) - 8a^2b^2\sin(2(c+dx)) + b^4\sin(4(c+dx)))}{480b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
- 60*b*(-8*a^4 + 2*a^2*b^2 + b^4)*Cos[c + d*x] - 10*(4*a^2*b^3 + b^5)*Cos[3
```

```
*(c + d*x)] + 6*b^5*Cos[5*(c + d*x)] + 15*a*(4*(8*a^4 - 4*a^2*b^2 - b^4)*(c
 + d*x) - 8*a^2*b^2*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)]))/(480*b^6*d)
```

Maple [A]
time = 0.33, size = 355, normalized size = 1.51

method	result
risch	$\frac{a^5 x}{b^6}-\frac{a^3 x}{2b^4}-\frac{a x}{8b^2}+\frac{e^{i(dx+c)}a^4}{2b^5 d}-\frac{e^{i(dx+c)}a^2}{8b^3 d}-\frac{e^{i(dx+c)}}{16bd}+\frac{e^{-i(dx+c)}a^4}{2b^5 d}-\frac{e^{-i(dx+c)}a^2}{8b^3 d}-\frac{e^{-i(dx+c)}}{16bd}+\frac{\sqrt{-}}{}$
derivativedivides	$\frac{-\frac{2a^4\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^6}+\frac{2\left(\left(\frac{1}{2}a^3b^2-\frac{1}{8}ab^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^4b-a^2b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2+\frac{3}{4}a\right.}{}}{}$
default	$\frac{-\frac{2a^4\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^6}+\frac{2\left(\left(\frac{1}{2}a^3b^2-\frac{1}{8}ab^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^4b-a^2b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2+\frac{3}{4}a\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^4*(a^2-b^2)^(1/2)/b^6*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))+2/b^6*(((1/2*a^3*b^2-1/8*a*b^4)*tan(1/2*d*x+1/2*c)^9+(a^4*b-a
^2*b^3)*tan(1/2*d*x+1/2*c)^8+(a^3*b^2+3/4*a*b^4)*tan(1/2*d*x+1/2*c)^7+(4*a^
4*b-2*a^2*b^3-2*b^5)*tan(1/2*d*x+1/2*c)^6+(6*a^4*b+2/3*b^5-4/3*a^2*b^3)*tan
(1/2*d*x+1/2*c)^4+(-a^3*b^2-3/4*a*b^4)*tan(1/2*d*x+1/2*c)^3+(4*a^4*b-2/3*a^
2*b^3-2/3*b^5)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^3*b^2+1/8*a*b^4)*tan(1/2*d*x+1/
2*c)+a^4*b-1/3*a^2*b^3-2/15*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5+1/8*a*(8*a^4-4*
a^2*b^2-b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 427, normalized size = 1.82

$$\left[\frac{24\,b^5\cos(dx+c)^5+120\,a^4b\cos(dx+c)+60\,\sqrt{-a^2+b^2}\,a^4\log\left(\frac{(2\,a^2-b^2)\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}\right)-40\,(a^2b^3+b^5)\cos(dx+c)^3+15\,(8\,a^5-4\,a^3b^2-ab^4)dx+15\,(2\,ab^4\cos(dx+c)^3-(4\,a^3b^2+ab^4)\cos(dx+c))\sin(dx+c)}{120\,b^6d},\ \frac{24\,b^5\cos(dx+c)^5+120\,a^4b\cos(dx+c)+120\,\sqrt{a^2-b^2}\,a^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-40\,(a^2b^3+b^5)\cos(dx+c)^3+15\,(8\,a^5-4\,a^3b^2-ab^4)dx+15\,(2\,ab^4\cos(dx+c)^3-(4\,a^3b^2+ab^4)\cos(dx+c))\sin(dx+c)}{120\,b^6d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/120*(24*b^5*cos(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 60*sqrt(-a^2 + b^2
)*a^4*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 40*(a^2*b^3 + b^5)*cos(d*x
 + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3 -
 (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d), 1/120*(24*b^5*cos
(d*x + c)^5 + 120*a^4*b*cos(d*x + c) + 120*sqrt(a^2 - b^2)*a^4*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 40*(a^2*b^3 + b^5)*cos(d
*x + c)^3 + 15*(8*a^5 - 4*a^3*b^2 - a*b^4)*d*x + 15*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(218) = 436.
time = 0.45, size = 467, normalized size = 1.99

$$\frac{\frac{15\,(8\,a^5-4\,a^3b^2-ab^4)(dx+c)}{b^6}-\frac{240\,(a^6-a^4b^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-15\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-120\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+90\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+480\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-240\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-240\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+720\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-160\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+80\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-90\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-80\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-80\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,a^4-40\,a^2b^2-16\,b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5b^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 4*a^3*b^2 - a*b^4)*(d*x + c)/b^6 - 240*(a^6 - a^4*b^2)*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*
c)^9 - 15*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 - 1
20*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 90*a
*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^2*b^2*
tan(1/2*d*x + 1/2*c)^6 - 240*b^4*tan(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d
*x + 1/2*c)^4 - 160*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 80*b^4*tan(1/2*d*x + 1
/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 90*a*b^3*tan(1/2*d*x + 1/2*c)^
3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 80*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 80
*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 15*a*b^3*tan(
1/2*d*x + 1/2*c) + 120*a^4 - 40*a^2*b^2 - 16*b^4)/((tan(1/2*d*x + 1/2*c)^2
+ 1)^5*b^5))/d
```

Mupad [B]

time = 13.39, size = 376, normalized size = 1.60

$$\frac{a^4\,\cos\left(c+d\,x\right)}{b^5\,d}-\frac{\frac{\cos(c+d\,x)}{8}+\frac{\cos(3\,c+3\,d\,x)}{48}-\frac{\cos(5\,c+5\,d\,x)}{80}}{b\,d}-\frac{\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{a\,\sin(4\,c+4\,d\,x)}{32}}{b^2\,d}-\frac{\frac{a^2\,\cos(c+d\,x)}{4}+\frac{a^2\,\cos(3\,c+3\,d\,x)}{12}}{b^3\,d}-\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{a^3\,\sin(2\,c+2\,d\,x)}{4}}{b^4\,d}+\frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^6\,d}-\frac{2\,a^4\,\mathrm{atanh}\left(\frac{2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}-a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^2-2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^3}\right)\,\sqrt{b^2-a^2}}{b^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^4)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^4*cos(c + d*x))/(b^5*d) - (cos(c + d*x)/8 + cos(3*c + 3*d*x)/48 - cos(5*
c + 5*d*x)/80)/(b*d) - ((a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 -
 (a*sin(4*c + 4*d*x))/32)/(b^2*d) - ((a^2*cos(c + d*x))/4 + (a^2*cos(3*c +
3*d*x))/12)/(b^3*d) - (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (a
^3*sin(2*c + 2*d*x))/4)/(b^4*d) + (2*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/(b^6*d) - (2*a^4*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/
2) - a^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - 2*b^3*sin(c/2 + (d*x)/2) - a*b^2*c
os(c/2 + (d*x)/2) + 2*a^2*b*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^6*d)
```

3.1286 $\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=191

$$-\frac{(8a^4-4a^2b^2-b^4)\,x}{8b^5}+\frac{2a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d}+\frac{(4a^2-b^2)\cos(c+}{8b^3}$$

```
[Out] -1/8*(8*a^4-4*a^2*b^2-b^4)*x/b^5-1/3*a*(3*a^2-b^2)*cos(d*x+c)/b^4/d+1/8*(4*
a^2-b^2)*cos(d*x+c)*sin(d*x+c)/b^3/d-1/3*a*cos(d*x+c)*sin(d*x+c)^2/b^2/d+1/
4*cos(d*x+c)*sin(d*x+c)^3/b/d+2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^
2)^(1/2))*(a^2-b^2)^(1/2)/b^5/d
```

Rubi [A]

time = 0.41, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d}+\frac{(4a^2-b^2)\sin(c+dx)\cos(c+dx)}{8b^3d}-\frac{x(8a^4-4a^2b^2-b^4)}{8b^5}+\frac{2a^3\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{a\sin^2(c+dx)\cos(c+dx)}{3b^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/8*((8*a^4 - 4*a^2*b^2 - b^4)*x)/b^5 + (2*a^3*Sqrt[a^2 - b^2]*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (a*(3*a^2 - b^2)*Cos[c + d
*x])/(3*b^4*d) + ((4*a^2 - b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*b^3*d) - (a*C
os[c + d*x]*Sin[c + d*x]^2)/(3*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*
d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin^2(c+dx)\left(-3a+b\sin(c+dx)+4a\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4b}$$
$$= -\frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\int \frac{\sin(c+dx)(8a^2-$$
$$= \frac{(4a^2-b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c+dx)\sin^2(c+dx)}{3b^2d} + \frac{\cos(c+$$
$$= -\frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d} + \frac{(4a^2-b^2)\cos(c+dx)\sin(c+dx)}{8b^3d} - \frac{a\cos(c$$
$$= -\frac{(8a^4-4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d} + \frac{(4a^2-b^2)\cos(c+dx}{8b^3d}$$
$$= -\frac{(8a^4-4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d} + \frac{(4a^2-b^2)\cos(c+dx}{8b^3d}$$
$$= -\frac{(8a^4-4a^2b^2-b^4)\,x}{8b^5} - \frac{a(3a^2-b^2)\cos(c+dx)}{3b^4d} + \frac{(4a^2-b^2)\cos(c+dx}{8b^3d}$$
$$= -\frac{(8a^4-4a^2b^2-b^4)\,x}{8b^5} + \frac{2a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d} - \frac{a(3a^2$$

Mathematica [A]
time = 0.77, size = 146, normalized size = 0.76

$$\frac{-12(8a^4-4a^2b^2-b^4)(c+dx)+192a^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+24ab(-4a^2+b^2)\cos(c+dx)+8ab^3\cos(3(c+dx))+24a^2b^2\sin(2(c+dx))-3b^4\sin(4(c+dx))}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-12*(8*a^4 - 4*a^2*b^2 - b^4)*(c + d*x) + 192*a^3*Sqrt[a^2 - b^2]*ArcTan[(
b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 24*a*b*(-4*a^2 + b^2)*Cos[c + d*
x] + 8*a*b^3*Cos[3*(c + d*x)] + 24*a^2*b^2*Sin[2*(c + d*x)] - 3*b^4*Sin[4*(
c + d*x)])/(96*b^5*d)
```

Maple [A]
time = 0.27, size = 296, normalized size = 1.55

method	result
risch	$-\frac{x a^4}{b^5}+\frac{x a^2}{2b^3}+\frac{x}{8b}-\frac{a^3 e^{i(dx+c)}}{2b^4 d}+\frac{a\,e^{i(dx+c)}}{8b^2 d}-\frac{a^3 e^{-i(dx+c)}}{2b^4 d}+\frac{a\,e^{-i(dx+c)}}{8b^2 d}+\frac{i\sqrt{a^2-b^2}\,a^3\ln\left(e^{i(dx+c)}-\right.}{d\,b^5}$
derivativedivides	$\frac{\frac{2a^3\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^5}-\frac{2\left(\frac{\left(\frac{1}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2+\frac{7}{8}b^4\right)}{}\right.}{}}{}$
default	$\frac{\frac{2a^3\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^5}-\frac{2\left(\frac{\left(\frac{1}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2+\frac{7}{8}b^4\right)}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3*(a^2-b^2)^(1/2)/b^5*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))-2/b^5*(((1/2*a^2*b^2-1/8*b^4)*tan(1/2*d*x+1/2*c)^7+(a^3*b-a*b^
3)*tan(1/2*d*x+1/2*c)^6+(1/2*a^2*b^2+7/8*b^4)*tan(1/2*d*x+1/2*c)^5+(3*a^3*b
-a*b^3)*tan(1/2*d*x+1/2*c)^4+(-1/2*a^2*b^2-7/8*b^4)*tan(1/2*d*x+1/2*c)^3+(3
*a^3*b-1/3*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^2*b^2+1/8*b^4)*tan(1/2*d*x+1
/2*c)+a^3*b-1/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(8*a^4-4*a^2*b^2-b^4)
*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 380, normalized size = 1.99

$$\left[\frac{8\,ab^3\cos\left(dx+c\right)^3-24\,a^3b\cos\left(dx+c\right)+12\,\sqrt{-a^2+b^2}\,a^3\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2-2\,\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)-3\,\left(8\,a^4-4\,a^2b^2-b^4\right)dx-3\,\left(2\,b^4\cos\left(dx+c\right)^3-\left(4\,a^2b^2+b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,b^5d},\ \frac{8\,ab^3\cos\left(dx+c\right)^3-24\,a^3b\cos\left(dx+c\right)-24\,\sqrt{a^2-b^2}\,a^3\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)-3\,\left(8\,a^4-4\,a^2b^2-b^4\right)dx-3\,\left(2\,b^4\cos\left(dx+c\right)^3-\left(4\,a^2b^2+b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,b^5d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) + 12*sqrt(-a^2 + b^2)
*a^3*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 -
2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos
(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(8*a^4 - 4*a^2*b^2 - b^4
)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 + b^4)*cos(d*x + c))*sin(d*x +
 c))/(b^5*d), 1/24*(8*a*b^3*cos(d*x + c)^3 - 24*a^3*b*cos(d*x + c) - 24*sqr
t(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)
)) - 3*(8*a^4 - 4*a^2*b^2 - b^4)*d*x - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2
 + b^4)*cos(d*x + c))*sin(d*x + c))/(b^5*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(176) = 352.
time = 0.46, size = 366, normalized size = 1.92

$$-\frac{\frac{3\,(8\,a^4-4\,a^2b^2-b^4)(dx+c)}{b^5}-\frac{48\,(a^5-a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^5}+\frac{2\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+72\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,a^3-8\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4b^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(8*a^4 - 4*a^2*b^2 - b^4)*(d*x + c)/b^5 - 48*(a^5 - a^3*b^2)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/s
qrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7
 - 3*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 24*a*b^2*
tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 21*b^3*tan(1/2*d
*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 24*a*b^2*tan(1/2*d*x + 1/2*
c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 21*b^3*tan(1/2*d*x + 1/2*c)^3 + 72
*a^3*tan(1/2*d*x + 1/2*c)^2 - 8*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan
(1/2*d*x + 1/2*c) + 3*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 8*a*b^2)/((tan(1/
2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

Mupad [**B**]
time = 13.35, size = 2616, normalized size = 13.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^2*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)

[Out] (7*tan(c/2 + (d*x)/2)^3)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) - (7*tan(c/2 + (d*x)/2)^5)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) + tan(c/2 + (d*x)/2)^7/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) + (2*a)/(3*d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 + b^2)) - (2*a^3)/(d*(4*b^4*tan(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) + atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*a^3)/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 + (d*x)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^9)/b^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8 + (3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8*a^9 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/2))/(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)/b^7)))/(4*b*d) - tan(c/2 + (d*x)/2)/(4*d*(b + 4*b*tan(c/2 + (d*x)/2)^2 + 6*b*tan(c/2 + (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + b*tan(c/2 + (d*x)/2)^8)) + (2*a*tan(c/2 + (d*x)/2)^2)/(3*d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 + b^2)) + (2*a*tan(c/2 + (d*x)/2)^4)/(d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 + b^2)) + (2*a*tan(c/2 + (d*x)/2)^6)/(d*(4*b^2*tan(c/2 + (d*x)/2)^2 + 6*b^2*tan(c/2 + (d*x)/2)^4 + 4*b^2*tan(c/2 + (d*x)/2)^6 + b^2*tan(c/2 + (d*x)/2)^8 + b^2)) + (a^2*tan(c/2 + (d*x)/2))/(d*(4*b^3*tan(c/2 + (d*x)/2)^2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 + (d*x)/2)^8 + b^3)) + (a^2*atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*a^3)/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 + (d*x)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^9)/b^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8 + (3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8*a^9 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/2))/(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)/b^7))))/(b^3*d) - (2*a^4*atan((11*a^3*tan(c/2 + (d*x)/2))/(8*((a*b^2)/8 + (11*a^3)/8 + (3*a^5)/(2*b^2) - (11*a^7)/b^4 + (8*a^9)/b^6)) + (3*a^5*tan(c/2 + (d*x)/2))/(2*((a*b^4)/8 + (3*a^5)/2 + (11*a^3*b^2)/8 - (11*a^7)/b^2 + (8*a^9)/b^4)) - (11*a^7*tan(c/2 + (d*x)/2))/((a*b^6)/8 - 11*a^7 + (11*a^3*b^4)/8 + (3*a^5*b^2)/2 + (8*a^9)/b^2) + (8*a^9*tan(c/2 + (d*x)/2))/((a*b^8)/8 + 8*a^9 + (11*a^3*b^6)/8 + (3*a^5*b^4)/2 - 11*a^7*b^2) + (a*b*tan(c/2 + (d*x)/2))/(8*((a*b)/8 + (11*a^3)/(8*b) + (3*a^5)/(2*b^3) - (11*a^7)/b^5 + (8*a^9)/b^7))))/(b^5*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(d*(4*b^3*tan(c/2 + (d*x)/2)^

2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 + (d*x)/2)^8 + b^3)) - (a^2*tan(c/2 + (d*x)/2)^5)/(d*(4*b^3*tan(c/2 + (d*x)/2)^2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 + (d*x)/2)^8 + b^3)) - (a^2*tan(c/2 + (d*x)/2)^7)/(d*(4*b^3*tan(c/2 + (d*x)/2)^2 + 6*b^3*tan(c/2 + (d*x)/2)^4 + 4*b^3*tan(c/2 + (d*x)/2)^6 + b^3*tan(c/2 + (d*x)/2)^8 + b^3)) - (6*a^3*tan(c/2 + (d*x)/2)^2)/(d*(4*b^4*tan(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) - (6*a^3*tan(c/2 + (d*x)/2)^4)/(d*(4*b^4*tan(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) - (2*a^3*tan(c/2 + (d*x)/2)^6)/(d*(4*b^4*tan(c/2 + (d*x)/2)^2 + 6*b^4*tan(c/2 + (d*x)/2)^4 + 4*b^4*tan(c/2 + (d*x)/2)^6 + b^4*tan(c/2 + (d*x)/2)^8 + b^4)) - (2*a^3*atanh((a^5*(b^2 - a^2)^(1/2))/(a^5*b + (7*a^7)/b - (8*a^9)/b^3 + 14*a^6*tan(c/2 + (d*x)/2) + 2*a^4*b^2*tan(c/2 + (d*x)/2) - (16*a^8*tan(c/2 + (d*x)/2))/b^2) + (8*a^7*(b^2 - a^2)^(1/2))/(7*a^7*b + a^5*b^3 - (8*a^9)/b - 16*a^8*tan(c/2 + (d*x)/2) + 2*a^4*b^4*tan(c/2 + (d*x)/2) + 14*a^6*b^2*tan(c/2 + (d*x)/2)) + (2*a^4*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^5 + (7*a^7)/b^2 - (8*a^9)/b^4 + 2*a^4*b*tan(c/2 + (d*x)/2) + (14*a^6*tan(c/2 + (d*x)/2))/b - (16*a^8*tan(c/2 + (d*x)/2))/b^3) + (15*a^6*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(7*a^7 + a^5*b^2 - (8*a^9)/b^2 + 14*a^6*b*tan(c/2 + (d*x)/2) + 2*a^4*b^3*tan(c/2 + (d*x)/2) - (16*a^8*tan(c/2 + (d*x)/2))/b) - (8*a^8*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(a^5*b^4 - 8*a^9 + 7*a^7*b^2 - 16*a^8*b*tan(c/2 + (d*x)/2) + 2*a^4*b^5*tan(c/2 + (d*x)/2) + 14*a^6*b^3*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2))/(b^5*d)

3.1287 $\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{a(2a^2-b^2)\,x}{2b^4}-\frac{2a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{(3a^2-b^2)\cos(c+dx)}{3b^3d}-\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d}+\frac{\text{co}}{}$$

```
[Out] 1/2*a*(2*a^2-b^2)*x/b^4+1/3*(3*a^2-b^2)*cos(d*x+c)/b^3/d-1/2*a*cos(d*x+c)*s
in(d*x+c)/b^2/d+1/3*cos(d*x+c)*sin(d*x+c)^2/b/d-2*a^2*arctan((b+a*tan(1/2*d
*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/b^4/d
```

Rubi [A]
time = 0.29, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3129, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^4d}+\frac{ax(2a^2-b^2)}{2b^4}+\frac{(3a^2-b^2)\cos(c+dx)}{3b^3d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{\sin^2(c+dx)\cos(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(2*a^2 - b^2)*x)/(2*b^4) - (2*a^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*d) + ((3*a^2 - b^2)*Cos[c + d*x])/(3*b^3*d
) - (a*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^2)
/(3*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
```

```
])))
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\sin^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{\sin(c+dx)(-2a+b\sin(c+dx)+3a\sin^2(c+dx))}{a+b\sin(c+dx)}\,dx}{3b}$$
$$= -\frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab\sin(c+dx)-}{a+b\,\mathrm{si}}}{}$$
$$= \frac{(3a^2-b^2)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(c+dx)\sin^2(c+}{3bd}$$
$$= \frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-b^2)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(}{}$$
$$= \frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-b^2)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(}{}$$
$$= \frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-b^2)\cos(c+dx)}{3b^3d} - \frac{a\cos(c+dx)\sin(c+dx)}{2b^2d} + \frac{\cos(}{}$$
$$= \frac{a(2a^2-b^2)\,x}{2b^4} - \frac{2a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4d} + \frac{(3a^2-b^2)\cos(c}{3b^3d}$$

Mathematica [A]
time = 0.19, size = 130, normalized size = 0.88

$$-\frac{-12a^3c+6ab^2c-12a^3dx+6ab^2dx+24a^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+3b(-4a^2+b^2)\cos(c+dx)+b^3\cos(3(c+dx))+3ab^2\sin(2(c+dx))}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/12*(-12*a^3*c + 6*a*b^2*c - 12*a^3*d*x + 6*a*b^2*d*x + 24*a^2*Sqrt[a^2 -
 b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 3*b*(-4*a^2 + b^2)
*Cos[c + d*x] + b^3*Cos[3*(c + d*x)] + 3*a*b^2*Sin[2*(c + d*x)])/(b^4*d)
```

Maple [A]
time = 0.26, size = 184, normalized size = 1.24

method	result

derivativedivides	$\frac{-\frac{2a^2\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4}+\frac{2\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}\right.}{b^4}}{d}$
default	$\frac{-\frac{2a^2\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4}+\frac{2\left(\frac{\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}\right.}{b^4}}{d}$
risch	$\frac{a^3x}{b^4}-\frac{ax}{2b^2}+\frac{e^{i(dx+c)}a^2}{2b^3d}-\frac{e^{i(dx+c)}}{8bd}+\frac{e^{-i(dx+c)}a^2}{2b^3d}-\frac{e^{-i(dx+c)}}{8bd}-\frac{\sqrt{-a^2+b^2}\,a^2\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{-}}{}\right.}{d\,b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2*(a^2-b^2)^(1/2)/b^4*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^
2-b^2)^(1/2))+2/b^4*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(a^2*b-b^3)*tan(1/2*d*
x+1/2*c)^4+2*a^2*b*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2*tan(1/2*d*x+1/2*c)+a^2*b-
1/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(2*a^2-b^2)*arctan(tan(1/2*d*x+1/
2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 315, normalized size = 2.13

$$\left[-\frac{2b^3\cos(dx+c)^3+3ab^2\cos(dx+c)\sin(dx+c)-6a^2b\cos(dx+c)-3\sqrt{-a^2+b^2}\,a^2\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-3(2a^3-ab^2)dx}{6b^4d},\ -\frac{2b^3\cos(dx+c)^3+3ab^2\cos(dx+c)\sin(dx+c)-6a^2b\cos(dx+c)-6\sqrt{a^2-b^2}\,a^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-3(2a^3-ab^2)dx}{6b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*cos(d*x + c)*sin(d*x + c) - 6*a^2*b*c
os(d*x + c) - 3*sqrt(-a^2 + b^2)*a^2*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
```

```
 c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
)) - 3*(2*a^3 - a*b^2)*d*x)/(b^4*d), -1/6*(2*b^3*cos(d*x + c)^3 + 3*a*b^2*c
os(d*x + c)*sin(d*x + c) - 6*a^2*b*cos(d*x + c) - 6*sqrt(a^2 - b^2)*a^2*arc
tan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*a^3 - a*b^
2)*d*x)/(b^4*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 207, normalized size = 1.40

$$\frac{\frac{3\,(2\,a^3-ab^2)\,(dx+c)}{b^4}-\frac{12\,(a^4-a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2-2\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - a*b^2)*(d*x + c)/b^4 - 12*(a^4 - a^2*b^2)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^
2)))/(sqrt(a^2 - b^2)*b^4) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/
2*d*x + 1/2*c)^4 - 6*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*x + 1/2*
c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^2
 + 1)^3*b^3))/d
```

Mupad [B]
time = 12.48, size = 225, normalized size = 1.52

$$\frac{a^2\,\cos\left(c+d\,x\right)}{b^3\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{a\,\sin(2\,c+2\,d\,x)}{4}}{b^2\,d}-\frac{\frac{\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{12}}{b\,d}+\frac{2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}+\frac{2\,a^2\,\mathrm{atanh}\left(\frac{-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)\,\sqrt{b^2-a^2}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (a^2*cos(c + d*x))/(b^3*d) - (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
 + (a*sin(2*c + 2*d*x))/4)/(b^2*d) - (cos(c + d*x)/4 + cos(3*c + 3*d*x)/12)
/(b*d) + (2*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d) + (2*a
^2*atanh((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2) + a*b*cos(c/2 +
 (d*x)/2))/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) + 2*b*sin(c/2 + (d*x)/2
))))*(b^2 - a^2)^(1/2))/(b^4*d)
```

3.1288 $\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

```
[Out] -1/2*(2*a^2-b^2)*x/b^3-1/2*cos(d*x+c)*(2*a-b*sin(d*x+c))/b^2/d+2*a*arctan((
b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/b^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2944, 2814, 2739, 632, 210}

$$\frac{2a\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{x(2a^2-b^2)}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*((2*a^2 - b^2)*x)/b^3 + (2*a*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/(b^3*d) - (Cos[c + d*x]*(2*a - b*Sin[c + d*x]))/(2
*b^2*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{\int \frac{-ab-(2a^2-b^2)\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{2b^2}$$
$$= -\frac{(2a^2-b^2)\,x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{(a(a^2-b^2))\int \frac{1}{a+b\sin(c+}$$
$$= -\frac{(2a^2-b^2)\,x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}+\frac{(2a(a^2-b^2))\,\mathrm{Subst}\left(\int \right.}{}$$
$$= -\frac{(2a^2-b^2)\,x}{2b^3}-\frac{\cos(c+dx)(2a-b\sin(c+dx))}{2b^2d}-\frac{(4a(a^2-b^2))\,\mathrm{Subst}\left(\int \right.}{}$$
$$= -\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3d}-\frac{\cos(c+dx)(2a-}{2b^2d}$$

Mathematica [A]
time = 0.17, size = 104, normalized size = 1.04

$$\frac{-4a^2c+2b^2c-4a^2dx+2b^2dx+8a\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-4ab\cos(c+dx)+b^2\sin(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*Sqrt[a^2 - b^2]*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a*b*Cos[c + d*x] + b^2*Sin[2*(c
 + d*x)])/(4*b^3*d)
```

Maple [A]
time = 0.20, size = 146, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{2a\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^3}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2-b^2)\,\mathrm{ar}}{}\right.}{b^3}}{d}$
default	$\frac{\frac{2a\sqrt{a^2-b^2}\,\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^3}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2-b^2)\,\mathrm{ar}}{}\right.}{b^3}}{d}$
risch	$-\frac{x a^2}{b^3}+\frac{x}{2b}-\frac{a\,\mathrm{e}^{i(dx+c)}}{2b^2d}-\frac{a\,\mathrm{e}^{-i(dx+c)}}{2b^2d}-\frac{i\sqrt{a^2-b^2}\,a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i\left(\sqrt{a^2-b^2}-a\right)}{b}\right)}{d\,b^3}+\frac{i\sqrt{a^2-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a*(a^2-b^2)^(1/2)/b^3*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))-2/b^3*((1/2*b^2*tan(1/2*d*x+1/2*c)^3+a*b*tan(1/2*d*x+1/2*c)^2-1/
2*b^2*tan(1/2*d*x+1/2*c)+a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2-b^2)*ar
ctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 275, normalized size = 2.75

$$\left[\frac{b^2\cos(dx+c)\sin(dx+c)-(2a^2-b^2)dx-2ab\cos(dx+c)+\sqrt{-a^2+b^2}\,a\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)}{2b^3d},\ \frac{b^2\cos(dx+c)\sin(dx+c)-(2a^2-b^2)dx-2ab\cos(dx+c)-2\sqrt{a^2-b^2}\,a\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{2b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c
) + sqrt(-a^2 + b^2)*a*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^
2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(b^3*d),
1/2*(b^2*cos(d*x + c)*sin(d*x + c) - (2*a^2 - b^2)*d*x - 2*a*b*cos(d*x + c)
 - 2*sqrt(a^2 - b^2)*a*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c))))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 159, normalized size = 1.59

$$-\frac{\frac{(2\,a^2-b^2)(dx+c)}{b^3}-\frac{4\,(a^3-ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^3}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*(d*x + c)/b^3 - 4*(a^3 - a*b^2)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(
sqrt(a^2 - b^2)*b^3) + 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*
c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/
d
```

Mupad [B]
time = 12.07, size = 190, normalized size = 1.90

$$\frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{\sin(2\,c+2\,d\,x)}{4}}{b\,d}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}-\frac{a\,\cos\left(c+d\,x\right)}{b^2\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{\sqrt{b^2-a^2}\,\left(a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)\,\sqrt{b^2-a^2}}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + sin(2*c + 2*d*x)/4)/(b*d) -
(2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d) - (a*cos(c + d*
x))/(b^2*d) - (2*a*atanh((2*b^2*sin(c/2 + (d*x)/2) - a^2*sin(c/2 + (d*x)/2)
 + a*b*cos(c/2 + (d*x)/2))/((b^2 - a^2)^(1/2)*(a*cos(c/2 + (d*x)/2) + 2*b*s
in(c/2 + (d*x)/2))))*(b^2 - a^2)^(1/2))/(b^3*d)
```

3.1289 $\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -x/b-arctanh(cos(d*x+c))/a/d+2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/a/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2968, 3137, 2739, 632, 210, 3855}

$$\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(a*b*d) - ArcTanh[Cos[c + d*x]]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[C*(x
/(b*d)), x] + (Dist[(A*b^2 + a^2*C)/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f
*x]), x], x] - Dist[(c^2*C + A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f
*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}+\frac{\int \csc(c+dx)\,dx}{a}-\left(-\frac{a}{b}+\frac{b}{a}\right)\int \frac{1}{a+b\sin(c+dx)}\,dx \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\left(2\left(\frac{a}{b}-\frac{b}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c\right.\right.}{d} \\
&= -\frac{x}{b}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\left(4\left(\frac{a}{b}-\frac{b}{a}\right)\right)\operatorname{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+\right.}{d} \\
&= -\frac{x}{b}+\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{abd}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 90, normalized size = 1.20

$$-\frac{ac+adx-2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*c + a*d*x - 2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]] + b*Log[Cos[(c + d*x)/2]] - b*Log[Sin[(c + d*x)/2]])/(a*b*d))
```

Maple [A]
time = 0.28, size = 94, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\left(2a^{2}-2b^{2}\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{ab\sqrt{a^{2}-b^{2}}}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\left(2a^{2}-2b^{2}\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{ab\sqrt{a^{2}-b^{2}}}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}}{d}$
risch	$-\frac{x}{b}+\frac{i\sqrt{a^{2}-b^{2}}\,\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^{2}-b^{2}}+a\right)}{b}\right)}{dba}-\frac{i\sqrt{a^{2}-b^{2}}\,\ln\left(e^{i(dx+c)}-\frac{i\left(\sqrt{a^{2}-b^{2}}-a\right)}{b}\right)}{dba}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tan(1/2*d*x+1/2*c))+(2*a^2-2*b^2)/a/b/(a^2-b^2)^(1/2)*arctan(1/
2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/b*arctan(tan(1/2*d*x+1/2*
c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 262, normalized size = 3.49

$$\left[-\frac{2\,adx+b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\sqrt{-a^{2}+b^{2}}\log\left(-\frac{(2\,a^{2}-b^{2})\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}}\right)}{2\,abd},\ -\frac{2\,adx+b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+2\,\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^{2}-b^{2}}\cos(dx+c)}\right)}{2\,abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) - sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b*d),
 -1/2*(2*a*d*x + b*log(1/2*cos(d*x + c) + 1/2) - b*log(-1/2*cos(d*x + c) +
1/2) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(
d*x + c))))/(a*b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 94, normalized size = 1.25

$$-\frac{\frac{dx+c}{b}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a}-\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)/b - log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))*
sqrt(a^2 - b^2)/(a*b))/d
```

Mupad [B]
time = 11.97, size = 896, normalized size = 11.95

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}+\ldots$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + (2*atan((64*a^3)/(64*a^2*b - 64*b^3 + 64*a^
3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) - (64*a*b^2)/(64*a^2*b
- 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2)) + (64*b
^3*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*
```

a*b^2*tan(c/2 + (d*x)/2)) - (64*a^2*b*tan(c/2 + (d*x)/2))/(64*a^2*b - 64*b^3 + 64*a^3*tan(c/2 + (d*x)/2) - 64*a*b^2*tan(c/2 + (d*x)/2))))/(b*d) - (2*atanh((64*a^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) - (512*b^2*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3) + (512*b^4*(b^2 - a^2)^(1/2))/(256*a^4*b + 512*b^5 - 768*a^2*b^3 - 64*a^5*tan(c/2 + (d*x)/2) - 1792*a*b^4*tan(c/2 + (d*x)/2) + 832*a^3*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a) - (1280*b^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^3*b - 768*a*b^3 + (512*b^5)/a - 64*a^4*tan(c/2 + (d*x)/2) - 1792*b^4*tan(c/2 + (d*x)/2) + 832*a^2*b^2*tan(c/2 + (d*x)/2) + (1024*b^6*tan(c/2 + (d*x)/2))/a^2) + (1024*b^5*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(512*a*b^5 + 256*a^5*b - 768*a^3*b^3 - 64*a^6*tan(c/2 + (d*x)/2) + 1024*b^6*tan(c/2 + (d*x)/2) - 1792*a^2*b^4*tan(c/2 + (d*x)/2) + 832*a^4*b^2*tan(c/2 + (d*x)/2)) + (320*a*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(256*a^2*b - 768*b^3 + (512*b^5)/a^2 - 64*a^3*tan(c/2 + (d*x)/2) + 832*a*b^2*tan(c/2 + (d*x)/2) - (1792*b^4*tan(c/2 + (d*x)/2))/a + (1024*b^6*tan(c/2 + (d*x)/2))/a^3))*(b^2 - a^2)^(1/2))/(a*b*d)

3.1290 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-2*arctan((b+a*tan(1/2*d*x+1/2*c)
)/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 210}

$$-\frac{2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$
$$= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{(-a^2+b^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{(2(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\right.}{a^2d}$$
$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{(4(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2\right.}{a^2d}$$
$$= -\frac{2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A]
time = 0.17, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - a\cot\left(\frac{1}{2}(c+dx)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + a\tan\left(\frac{1}{2}(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
Tan[(c + d*x)/2])/(2*a^2*d)
```

Maple [A]
time = 0.26, size = 109, normalized size = 1.36

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2} + \frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} - \frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2} + \frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$

risch	$-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^{2}d}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^{2}d}-\frac{\sqrt{-a^{2}+b^{2}}\,\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{-a^{2}+b^{2}}}{b}\right)}{d\,a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d
*x+1/2*c))+1/2/a^2*(-4*a^2+4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 314, normalized size = 3.92

$$\left[\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\sqrt{-a^{2}+b^{2}}\log\left(\frac{\left(2a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-2ab\sin\left(dx+c\right)-a^{2}-b^{2}+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(dx+c\right)^{2}-2ab\sin\left(dx+c\right)-a^{2}-b^{2}}\right)\sin\left(dx+c\right)-2a\cos\left(dx+c\right)}{2a^{2}d\sin\left(dx+c\right)},\ \frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)}\right)\sin\left(dx+c\right)-2a\cos\left(dx+c\right)}{2a^{2}d\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^{2}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.50, size = 129, normalized size = 1.61

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 11.94, size = 204, normalized size = 2.55

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{\mathrm{atan}\left(\frac{a^3\,\sqrt{b^2-a^2}\,1\mathrm{i}-a\,b^2\,\sqrt{b^2-a^2}\,2\mathrm{i}-b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\,4\mathrm{i}+a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{b^2-a^2}\,3\mathrm{i}}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\,a^3\,b-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+2\,a\,b^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}\right)\,\sqrt{b^2-a^2}\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (atan((a^3*(b^2 - a^2)^(1/2)*1i - a*b^2*(b^2 - a^2)^(1/2)*2i - b^3*tan(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(2*a*b^3 - 2*a^3*b + a^4*tan(c/2 + (d*x)/2) + 4*b^4*tan(c/2 + (d*x)/2
) - 5*a^2*b^2*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(a^2*d) - cot(c +
d*x)/(a*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

3.1291 $\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=114

$$\frac{2b\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)
*csc(d*x+c)/a/d+2*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b
^2)^(1/2)/a^3/d
```

Rubi [A]

time = 0.29, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)}{a^2d}+\frac{2b\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3
*d) + ((a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c + d*x])/(a
^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc^3(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \frac{\csc^2(c+dx)\left(-2b-a\sin(c+dx)+b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a} \\
&= \frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \frac{\csc(c+dx)\left(-a^2+2b^2+ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{2a^2} \\
&= \frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\left(a^2-2b^2\right)\int \csc(c+dx)\,dx}{2a^3}+\frac{(b \\
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad} \\
&= \frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad} \\
&= \frac{2b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d}+\frac{\left(a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b
\end{aligned}
$$

Mathematica [A]
time = 0.60, size = 181, normalized size = 1.59

$$
\frac{16b\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+4ab\cot\left(\frac{1}{2}(c+dx)\right)-a^2\csc^2\left(\frac{1}{2}(c+dx)\right)+4a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2\sec^2\left(\frac{1}{2}(c+dx)\right)-4ab\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (16*b*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 4*
a*b*Cot[(c + d*x)/2] - a^2*Csc[(c + d*x)/2]^2 + 4*a^2*Log[Cos[(c + d*x)/2]]
 - 8*b^2*Log[Cos[(c + d*x)/2]] - 4*a^2*Log[Sin[(c + d*x)/2]] + 8*b^2*Log[Si
n[(c + d*x)/2]] + a^2*Sec[(c + d*x)/2]^2 - 4*a*b*Tan[(c + d*x)/2])/(8*a^3*d
)
```

Maple [A]
time = 0.35, size = 144, normalized size = 1.26

method	result
derivativedivides	$\frac{\frac{a\left(\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\sqrt{a^2-b^2}\ \mathrm{arc}}{}}{d}$
default	$\frac{\frac{a\left(\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-2a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\sqrt{a^2-b^2}\ \mathrm{arc}}{}}{d}$
risch	$\frac{i\left(-ia\,\mathrm{e}^{3i(dx+c)}-ia\,\mathrm{e}^{i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}-2b\right)}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{i\sqrt{a^2-b^2}\,b\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i\left(\sqrt{a^2-b^2}-a\right)}{b}\right)}{d\,a^3}+\frac{i\sqrt{a^2-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c))-1/8/a/tan(
1/2*d*x+1/2*c)^2+1/4/a^3*(-2*a^2+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2/a^2*b/ta
n(1/2*d*x+1/2*c)+2*b/a^3*(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)
+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.45, size = 472, normalized size = 4.14

$$\left[-\frac{4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)-2\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2-2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)-\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)},\ -\frac{4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,a^2\cos\left(dx+c\right)+4\left(b\cos\left(dx+c\right)^2-b\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)-\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^2-a^2+2\,b^2\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{4\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2*cos(d*x + c) - 2*(b*cos(d*x
+ c)^2 - b)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
```

```
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((
a^2 - 2*b^2)*cos(d*x + c)^2 - a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a
^3*d*cos(d*x + c)^2 - a^3*d), -1/4*(4*a*b*cos(d*x + c)*sin(d*x + c) - 2*a^2
*cos(d*x + c) + 4*(b*cos(d*x + c)^2 - b)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(1/2*cos(d*x + c) + 1/2) + ((a^2 - 2*b^2)*cos(d*x + c)^2
- a^2 + 2*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d*x + c)^2 - a^3*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 198, normalized size = 1.74

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{4\,(a^2-2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{16\,(a^2b-b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^3}+\frac{6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(a^2 - 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 16*(a^2*b - b^3)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^3) + (6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1
/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^3*tan(1/2*d*x + 1/
2*c)^2))/d
```

Mupad [B]
time = 12.45, size = 790, normalized size = 6.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)) - (a^2*(cos(c + d*x)/2 + log(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))/4 - (log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x
))/4))/((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b^2*log(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2
*d*x))/2)) + (a*b*sin(2*c + 2*d*x))/(2*((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x)
)/2)) + (b*atan((a^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d
*x)/2)*(b^2 - a^2)^(1/2)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x
)/2) + 4*a*b^4*cos(c/2 + (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*
cos(c/2 + (d*x)/2) - 12*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/
((a^3*d)/2 - (a^3*d*cos(2*c + 2*d*x))/2) - (b*cos(2*c + 2*d*x)*atan((a^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2
+ (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12
*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/((a^3*d)/2 - (a^3*d*cos
(2*c + 2*d*x))/2)
```

3.1292 $\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(a^2-3b^2)\cot(c+dx)}{3a^3d}+\frac{b\cot(c+$$

```
[Out] -1/2*b*(a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(a^2-3*b^2)*cot(d*x+c)/a^3
/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d-2*b^2*
arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^4/d
```

Rubi [A]
time = 0.42, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{2b^2\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(a^2-3b^2)\cot(c+dx)}{3a^3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^4*d) - (b*(a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((a^2 - 3*b^2)
*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^4(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^3(c+dx)\left(-3b-a\sin(c+dx)+2b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a}$$

$$= \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\int \frac{\csc^2(c+dx)\left(-2(a^2-3\ldots\right.}{\ldots}}{\ldots}$$

$$= \frac{(a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+\ldots}{3ad}$$

$$= \frac{(a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+\ldots}{3ad}$$

$$= -\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+d\ldots}{2\ldots}$$

$$= -\frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+d\ldots}{2\ldots}$$

$$= -\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b(a^2-2b^2)\tanh^{-1}(\cos(c+dx)\ldots}{2a^4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 351 vs. $2(153) = 306$.
time = 6.18, size = 351, normalized size = 2.29

$$-\frac{2b^2\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{\left(a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^3d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24ad} + \frac{(-a^2b+2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(a^2b-2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((a^2*Cos[(c + d*x)/2] - 3*b^2
*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*
a^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-(a^2*b) + 2*b^
3)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]
])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-(a^2*
```

```
Sin[(c + d*x)/2]) + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [A]

time = 0.39, size = 205, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-a^2+4b^2}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^2\tan\left(\frac{dx}{2}+\right.}}{d}$
default	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-a^2+4b^2}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^2\tan\left(\frac{dx}{2}+\right.}}{d}$
risch	$-\frac{-6ia^2e^{4i(dx+c)}+6ib^2e^{4i(dx+c)}+3be^{5i(dx+c)}a-12ib^2e^{2i(dx+c)}-2ia^2+6ib^2-3be^{i(dx+c)}a}{3da^3\left(e^{2i(dx+c)}-1\right)^3}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{2a^2d}-\frac{b^3}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^3*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-a*b*tan(1/2*d*x+1/2*c)^2-a^2*tan
(1/2*d*x+1/2*c)+4*b^2*tan(1/2*d*x+1/2*c))-1/24/a/tan(1/2*d*x+1/2*c)^3-1/8*(
-a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)+1/8/a^2*b/tan(1/2*d*x+1/2*c)^2+1/2/a^4*(
a^2-2*b^2)*b*ln(tan(1/2*d*x+1/2*c))-2*b^2/a^4*(a^2-b^2)^(1/2)*arctan(1/2*(2
*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.43, size = 591, normalized size = 3.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 6*(b^2*cos(d*x + c)^2 - b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 12*a*b^2*cos(d*x + c) - 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)

[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**4/(a + b*sin(c + d*x)), x)

Giac [A]
time = 0.47, size = 270, normalized size = 1.76

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\,(a^2b-2b^3)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{48\,(a^2b^2-b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{22a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-44b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 48*(a^2*b^2 - b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (22*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44*b^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x + 1/2*c)^3))/d

Mupad [B]
time = 13.51, size = 749, normalized size = 4.90

$$\frac{a^3\left(\frac{\cos(c+dx)}{8}+\frac{\cos(3c+3dx)}{24}\right)-a^2\left(\frac{b\sin(2c+2dx)}{8}-\frac{b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin(3c+3dx)}{16}+\frac{3b\sin(c+dx)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{16}\right)+a\left(\frac{b^2\cos(c+dx)}{8}-\frac{b^2\cos(3c+3dx)}{8}\right)+\frac{3b^3\sin(c+dx)\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{8}-\frac{b^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sin(3c+3dx)}{8}+\frac{b^2\sin(c+dx)\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}-a^2b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}+ab^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}-a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2-12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^4+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^5}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{4}-\frac{b^2\sin(3c+3dx)\,\mathrm{atan}\left(\frac{a^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,1\mathrm{i}+b^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}-a^2b^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,8\mathrm{i}+ab^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}-a^3b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5+4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b-5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^2-12\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^3+4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^4+8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^5}\right)\sqrt{b^2-a^2}\,1\mathrm{i}}{4}}{\frac{a^4d\sin(3c+3dx)}{8}-\frac{3a^4d\sin(c+dx)}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^3*(cos(c + d*x)/8 + cos(3*c + 3*d*x)/24) - a^2*((b*sin(2*c + 2*d*x))/8 -
 (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/16 + (3*b*
sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16) + a*((b^2*cos(
c + d*x))/8 - (b^2*cos(3*c + 3*d*x))/8) + (3*b^3*sin(c + d*x)*log(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*sin(3*c + 3*d*x))/8 + (b^2*sin(c + d*x)*atan((a^4*sin(c/2 + (d*x)/2
)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a^2*
b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2 + (d*x)/2)*(b^2
 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*3i)/(a^5*cos(
c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2 + (d*x)/2) + 4*
a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12*a^2*b^3*sin(c/
2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*3i)/4 - (b^2*sin(3*c + 3*d*x)*atan((a^4*si
n(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^2 - a^2)^
(1/2)*8i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i + a*b^3*cos(c/2
+ (d*x)/2)*(b^2 - a^2)^(1/2)*4i - a^3*b*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2
)*3i)/(a^5*cos(c/2 + (d*x)/2) + 8*b^5*sin(c/2 + (d*x)/2) + 4*a*b^4*cos(c/2
+ (d*x)/2) + 4*a^4*b*sin(c/2 + (d*x)/2) - 5*a^3*b^2*cos(c/2 + (d*x)/2) - 12
*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i)/4)/((a^4*d*sin(3*c + 3*
d*x))/8 - (3*a^4*d*sin(c + d*x))/8)
```

3.1293 $\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$\frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d}+\frac{(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d}-\frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d}+\frac{(a}{}$$

```
[Out] 1/8*(a^4+4*a^2*b^2-8*b^4)*arctanh(cos(d*x+c))/a^5/d-1/3*b*(a^2-3*b^2)*cot(d
*x+c)/a^4/d+1/8*(a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/d+1/3*b*cot(d*x+c)*cs
c(d*x+c)^2/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d+2*b^3*arctan((b+a*tan(1/2*
d*x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^5/d
```

Rubi [A]
time = 0.59, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d}+\frac{2b^3\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d}+\frac{(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d}+\frac{(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^5*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (b*(a
^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c + d*x]*
Csc[c + d*x]^3)/(4*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
```

```
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \frac{\csc^5(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx \\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^4(c+dx)\left(-4b-a\sin(c+dx)+3b\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4a} \\
&= \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{\int \frac{\csc^3(c+dx)(-3(a^2-}{}}{} \\
&= \frac{(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+}{} \\
&= -\frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+}{} \\
&= -\frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+}{} \\
&= \frac{(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(a^2-}{} \\
&= \frac{(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b(a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(a^2-}{} \\
&= \frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} + \frac{(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c-}{8a^5d}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 430 vs. $2(194) = 388$.
time = 6.21, size = 430, normalized size = 2.22

$$
\frac{2b^3\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} + \frac{\left(-a^2b\cos\left(\frac{1}{2}(c+dx)\right)+3b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{(a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{(a^4+4a^2b^2-8b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(-a^4-4a^2b^2+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(-a^2+4b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(a^2b\sin\left(\frac{1}{2}(c+dx)\right)-3b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*Sqrt[a^2 - b^2]*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin
[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-(a^2*b*Cos[(c + d*x)/2]) + 3
```

```
*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((a^2 - 4*b^2)*Csc[(c
+ d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^2*d
) - Csc[(c + d*x)/2]^4/(64*a*d) + ((a^4 + 4*a^2*b^2 - 8*b^4)*Log[Cos[(c + d
*x)/2]])/(8*a^5*d) + ((-a^4 - 4*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)/2]])/(8*
a^5*d) + ((-a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c + d*x)/2]^
4/(64*a*d) + (Sec[(c + d*x)/2]*(a^2*b*Sin[(c + d*x)/2] - 3*b^3*Sin[(c + d*x
)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [A]
time = 0.43, size = 252, normalized size = 1.30

method	result
derivativedivides	$\frac{\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}+2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4}-\frac{1}{64a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{(-2\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}+2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4}-\frac{1}{64a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}+\frac{(-2\cdots}{\cdots}}{\cdots}$
risch	$\frac{i\left(-12ia\,b^2e^{7i(dx+c)}+21ia^3e^{5i(dx+c)}+3ia^3e^{i(dx+c)}+12ia\,b^2e^{5i(dx+c)}-24b\,e^{6i(dx+c)}a^2+24b^3e^{6i(dx+c)}+12ia\,b^2e^{3i(dx+c}\cdots\right.}{12d\,a^4\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^4*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-2/3*b*tan(1/2*d*x+1/2*c)^3*a^2+
2*a*b^2*tan(1/2*d*x+1/2*c)^2+2*a^2*b*tan(1/2*d*x+1/2*c)-8*b^3*tan(1/2*d*x+1
/2*c))-1/64/a/tan(1/2*d*x+1/2*c)^4+1/16/a^5*(-2*a^4-8*a^2*b^2+16*b^4)*ln(ta
n(1/2*d*x+1/2*c))+1/24/a^2*b/tan(1/2*d*x+1/2*c)^3-1/8*b^2/a^3/tan(1/2*d*x+1
/2*c)^2-1/8*b*(a^2-4*b^2)/a^4/tan(1/2*d*x+1/2*c)+2*b^3/a^5*(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(179) = 358$.
time = 0.51, size = 808, normalized size = 4.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 24*(b^3*cos(d*x + c)^4 - 2*b^3
*cos(d*x + c)^2 + b^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 6*(a^4 + 4*a^2*b^2)*cos(d*x + c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*co
s(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*
x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x +
c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*
a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*
x + c)^2 + a^5*d), -1/48*(6*(a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 48*(b^3*cos(
d*x + c)^4 - 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 6*(a^4 + 4*a^2*b^2)*cos(d*x +
c) - 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4
- 2*(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
 3*((a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^4 + a^4 + 4*a^2*b^2 - 8*b^4 - 2*
(a^4 + 4*a^2*b^2 - 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16
*(3*a*b^3*cos(d*x + c) + (a^3*b - 3*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a
^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^5*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)\csc^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [**A**]
time = 0.51, size = 336, normalized size = 1.73

$$\frac{\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+24ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-96b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{24\left(a^4+4a^2b^2-8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}+\frac{384\left(a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{50a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+200a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-400b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-24a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+96ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+8a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^4}{a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 24*
a*b^2*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1
/2*d*x + 1/2*c))/a^4 - 24*(a^4 + 4*a^2*b^2 - 8*b^4)*log(abs(tan(1/2*d*x + 1
/2*c)))/a^5 + 384*(a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^
5) + (50*a^4*tan(1/2*d*x + 1/2*c)^4 + 200*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 -
400*b^4*tan(1/2*d*x + 1/2*c)^4 - 24*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 96*a*b^3
*tan(1/2*d*x + 1/2*c)^3 - 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a^3*b*tan(1
/2*d*x + 1/2*c) - 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [B]
time = 12.07, size = 873, normalized size = 4.50

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{b}{8\,a^2}-\frac{b^3}{2\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(2\,a^2\,b-8\,b^3\right)+\frac{a^3}{4}-\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}+2\,a\,b^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,a^4\,d\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(a^4+4\,a^2\,b^2-8\,b^4\right)}{8\,a^5\,d}-\frac{b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^2\,d}+\frac{b^2\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^3\,d}+\frac{b^3\,\mathrm{atan}\left(\frac{\frac{b^3\,\sqrt{b^2-a^2}\,\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a^9+32\,a^3\,b^6-32\,a^5\,b^4+2\,a^7\,b^2\right)}{4\,a^7}-\frac{a^9\,b-16\,a^5\,b^5+12\,a^7\,b^3}{4\,a^8}+\frac{b^3\,\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(24\,a^{10}-32\,a^8\,b^2\right)}{4\,a^7}\right)\,\sqrt{b^2-a^2}}{a^5}\right)\,1{}\mathrm{i}}{a^5}-\frac{b^3\,\sqrt{b^2-a^2}\,\left(\frac{a^9\,b-16\,a^5\,b^5+12\,a^7\,b^3}{4\,a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a^9+32\,a^3\,b^6-32\,a^5\,b^4+2\,a^7\,b^2\right)}{4\,a^7}+\frac{b^3\,\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(24\,a^{10}-32\,a^8\,b^2\right)}{4\,a^7}\right)\,\sqrt{b^2-a^2}}{a^5}\right)\,1{}\mathrm{i}}{a^5}}{\frac{8\,b^9-12\,a^2\,b^7+3\,a^4\,b^5+a^6\,b^3}{2\,a^8}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(8\,b^8-10\,a^2\,b^6+2\,a^4\,b^4\right)}{2\,a^7}+\frac{b^3\,\sqrt{b^2-a^2}\,\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a^9+32\,a^3\,b^6-32\,a^5\,b^4+2\,a^7\,b^2\right)}{4\,a^7}-\frac{a^9\,b-16\,a^5\,b^5+12\,a^7\,b^3}{4\,a^8}+\frac{b^3\,\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(24\,a^{10}-32\,a^8\,b^2\right)}{4\,a^7}\right)\,\sqrt{b^2-a^2}}{a^5}\right)}{a^5}+\frac{b^3\,\sqrt{b^2-a^2}\,\left(\frac{a^9\,b-16\,a^5\,b^5+12\,a^7\,b^3}{4\,a^8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(a^9+32\,a^3\,b^6-32\,a^5\,b^4+2\,a^7\,b^2\right)}{4\,a^7}+\frac{b^3\,\left(2\,a^2\,b-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(24\,a^{10}-32\,a^8\,b^2\right)}{4\,a^7}\right)\,\sqrt{b^2-a^2}}{a^5}\right)}{a^5}}\right)\,\sqrt{b^2-a^2}\,2{}\mathrm{i}}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) + (tan(c/2 + (d*x)/2)*(b/(8*a^2) - b^3/(2*a^4
)))/d - (tan(c/2 + (d*x)/2)^3*(2*a^2*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2
+ (d*x)/2))/3 + 2*a*b^2*tan(c/2 + (d*x)/2)^2)/(16*a^4*d*tan(c/2 + (d*x)/2)^
4) - (log(tan(c/2 + (d*x)/2))*(a^4 - 8*b^4 + 4*a^2*b^2))/(8*a^5*d) - (b*tan
(c/2 + (d*x)/2)^3)/(24*a^2*d) + (b^2*tan(c/2 + (d*x)/2)^2)/(8*a^3*d) + (b^3
*atan(((b^3*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a
^5*b^4 + 2*a^7*b^2))/(4*a^7) - (a^9*b - 16*a^5*b^5 + 12*a^7*b^3)/(4*a^8) +
(b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(b^2 -
 a^2)^(1/2))/a^5)*1i)/a^5 - (b^3*(b^2 - a^2)^(1/2)*((a^9*b - 16*a^5*b^5 + 1
2*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a^5*b^4 + 2
*a^7*b^2))/(4*a^7) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*
b^2))/(4*a^7))*(b^2 - a^2)^(1/2))/a^5)*1i)/a^5)/((8*b^9 - 12*a^2*b^7 + 3*a^
4*b^5 + a^6*b^3)/(2*a^8) + (tan(c/2 + (d*x)/2)*(8*b^8 - 10*a^2*b^6 + 2*a^4*
b^4))/(2*a^7) + (b^3*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b
^6 - 32*a^5*b^4 + 2*a^7*b^2))/(4*a^7) - (a^9*b - 16*a^5*b^5 + 12*a^7*b^3)/(
4*a^8) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7
))*(b^2 - a^2)^(1/2))/a^5))/a^5 + (b^3*(b^2 - a^2)^(1/2)*((a^9*b - 16*a^5*b
^5 + 12*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(a^9 + 32*a^3*b^6 - 32*a^5*b
^4 + 2*a^7*b^2))/(4*a^7) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 3
2*a^8*b^2))/(4*a^7))*(b^2 - a^2)^(1/2))/a^5))/a^5))*(b^2 - a^2)^(1/2)*2i)/(
a^5*d)
```

3.1294 $\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=238

$$-\frac{2b^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{b(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}+\frac{(2a^4+5a^2b^2-15b^4)\cot}{15a^5d}$$

```
[Out] -1/8*b*(a^4+4*a^2*b^2-8*b^4)*arctanh(cos(d*x+c))/a^6/d+1/15*(2*a^4+5*a^2*b^
2-15*b^4)*cot(d*x+c)/a^5/d-1/8*b*(a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d+1/
15*(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc(d*x+c)^3/
a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d-2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))
/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^6/d
```

Rubi [A]
time = 0.78, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2968, 3135, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2b^4\sqrt{a^2-b^2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{b(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}+\frac{(a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{15a^3d}-\frac{b(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}+\frac{(2a^4+5a^2b^2-15b^4)\cot(c+dx)}{15a^5d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^6*d) - (b*(a^4 + 4*a^2*b^2 - 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) + ((
2*a^4 + 5*a^2*b^2 - 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(a^2 - 4*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x
]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]
*Csc[c + d*x]^4)/(5*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
```

```
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^6(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^5(c+dx)(-5b-a\sin(c+dx)+4b\sin^2(c+dx))}{a+b\sin(c+dx)}\,dx}{5a}$$

$$= \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} + \frac{\int \frac{\csc^4(c+dx)(-4(a^2-$$

$$= \frac{(a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+$$

$$= -\frac{b(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{(a^2-5b^2)\cot(c+dx)\csc^2(c+dx}{15a^3d}$$

$$= \frac{(2a^4+5a^2b^2-15b^4)\cot(c+dx)}{15a^5d} - \frac{b(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} +$$

$$= \frac{(2a^4+5a^2b^2-15b^4)\cot(c+dx)}{15a^5d} - \frac{b(a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} +$$

$$= -\frac{b(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(2a^4+5a^2b^2-15b^4)\cot(c+}{15a^5d}$$

$$= -\frac{b(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} + \frac{(2a^4+5a^2b^2-15b^4)\cot(c+}{15a^5d}$$

$$= -\frac{2b^4\sqrt{a^2-b^2}\,\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{a^6d} - \frac{b(a^4+4a^2b^2-8b^4)\tanh^{-1}(\cos(}{8a^6d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 506 vs. 2(238) = 476.
time = 1.29, size = 506, normalized size = 2.13

$$\frac{-1920b^4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)+32(2a^5+5a^3b^2-15ab^4)\cot\left(\frac{1}{2}(c+dx)\right)-30a^4b\csc^2\left(\frac{1}{2}(c+dx)\right)+120a^2b^3\csc^2\left(\frac{1}{2}(c+dx)\right)+15a^4b\csc^4\left(\frac{1}{2}(c+dx)\right)-120a^4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-480a^2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+960b^5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+120a^4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+480a^2b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-960b^5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+30a^4b\sec^2\left(\frac{1}{2}(c+dx)\right)-120a^2b^3\sec^2\left(\frac{1}{2}(c+dx)\right)-15a^4b\sec^4\left(\frac{1}{2}(c+dx)\right)-16a^5\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+320a^3b^2\csc^3(c+dx)\sin^4\left(\frac{1}{2}(c+dx)\right)+a^5\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-20a^3b^2\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-3a^5\csc^6\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)-64a^5\tan\left(\frac{1}{2}(c+dx)\right)-160a^3b^2\tan\left(\frac{1}{2}(c+dx)\right)+480ab^4\tan\left(\frac{1}{2}(c+dx)\right)+6a^5\sec^4\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{960a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^2*Csc[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*b^4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + 32*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*Cot[(c + d*x)/2] - 30*a^4*b*Csc[(c + d
*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 - 1
20*a^4*b*Log[Cos[(c + d*x)/2]] - 480*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960*b^
5*Log[Cos[(c + d*x)/2]] + 120*a^4*b*Log[Sin[(c + d*x)/2]] + 480*a^2*b^3*Log
[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 30*a^4*b*Sec[(c + d*x)
/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4 - 16*a
^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[(c + d*x)/
2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] - 64*a^5*Tan[(c +
 d*x)/2] - 160*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)/2] + 6*a^
5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [A]
time = 0.50, size = 336, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}+\frac{a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{32a^5}}{}$
default	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}+\frac{a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{32a^5}}{}$
risch	$\frac{480ib^4\mathrm{e}^{2i(dx+c)}-240ia^2b^2\mathrm{e}^{6i(dx+c)}+15a^3b\,\mathrm{e}^{9i(dx+c)}-60a\,b^3\mathrm{e}^{9i(dx+c)}-80ia^2b^2\mathrm{e}^{2i(dx+c)}+120ia^2b^2\mathrm{e}^{8i(dx+c)}-120ib^4\mathrm{e}^{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^5*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-1/2*b*tan(1/2*d*x+1/2*c)^4*a^3+
1/3*a^4*tan(1/2*d*x+1/2*c)^3+4/3*a^2*b^2*tan(1/2*d*x+1/2*c)^3-4*a*b^3*tan(1
/2*d*x+1/2*c)^2-2*a^4*tan(1/2*d*x+1/2*c)-4*a^2*b^2*tan(1/2*d*x+1/2*c)+16*b^
4*tan(1/2*d*x+1/2*c))-1/160/a/tan(1/2*d*x+1/2*c)^5-1/96/a^3*(a^2+4*b^2)/tan
(1/2*d*x+1/2*c)^3-1/32*(-2*a^4-4*a^2*b^2+16*b^4)/a^5/tan(1/2*d*x+1/2*c)+1/6
4/a^2*b/tan(1/2*d*x+1/2*c)^4+1/8*b^3/a^4/tan(1/2*d*x+1/2*c)^2+1/8/a^6*b*(a^
4+4*a^2*b^2-8*b^4)*ln(tan(1/2*d*x+1/2*c))-2*b^4/a^6*(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.51, size = 959, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*x
 + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 120*(b^4*cos(d*x +
c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4
*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x +
c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^4*b + 4*a^2*b^3 - 8*
b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3 - 8
*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 30*((a^4*
b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
, -1/240*(240*a*b^4*cos(d*x + c) - 16*(2*a^5 + 5*a^3*b^2 - 15*a*b^4)*cos(d*
x + c)^5 + 80*(a^5 + a^3*b^2 - 6*a*b^4)*cos(d*x + c)^3 - 240*(b^4*cos(d*x +
 c)^4 - 2*b^4*cos(d*x + c)^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c)
 + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 15*(a^4*b + 4*a^2*b^3
- 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a^4*b + 4*a^2*b^3
 - 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 15*(a^
4*b + 4*a^2*b^3 - 8*b^5 + (a^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^4 - 2*(a
^4*b + 4*a^2*b^3 - 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(
d*x + c) - 30*((a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 + (a^4*b + 4*a^2*b^3)*cos
(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
a^6*d)*sin(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(c+dx\right)\csc^6\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2*csc(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(221) = 442.
time = 0.50, size = 444, normalized size = 1.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 10
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 120*a*b^3
*tan(1/2*d*x + 1/2*c)^2 - 60*a^4*tan(1/2*d*x + 1/2*c) - 120*a^2*b^2*tan(1/2
*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/2*c))/a^5 + 120*(a^4*b + 4*a^2*b^3
- 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 - 1920*(a^2*b^4 - b^6)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) - (274*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1
096*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a
^5*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*
tan(1/2*d*x + 1/2*c)^4 - 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 10*a^5*tan(1/
2*d*x + 1/2*c)^2 + 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*b*tan(1/2*d*x
 + 1/2*c) + 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 12.08, size = 1007, normalized size = 4.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) - (b*(1/
(32*a) + b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(1/(16*a) + b^2/(8*a^3)
+ (2*b*(b/(16*a^2) - (2*b*(1/(32*a) + b^2/(8*a^3)))/a))/a))/d + (tan(c/2 +
(d*x)/2)^3*(1/(96*a) + b^2/(24*a^3)))/d + (log(tan(c/2 + (d*x)/2))*((a^4*b)
/8 - b^5 + (a^2*b^3)/2))/(a^6*d) - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2*d) + (t
an(c/2 + (d*x)/2)^4*(2*a^4 - 16*b^4 + 4*a^2*b^2) - a^4/5 - tan(c/2 + (d*x)/
2)^2*(a^4/3 + (4*a^2*b^2)/3) + (a^3*b*tan(c/2 + (d*x)/2))/2 + 4*a*b^3*tan(c
/2 + (d*x)/2)^3)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) - (b^4*atan(((b^4*(b^2 - a
^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*b^7 - 32*a^6*b^5 + 2*a^8*b^
3))/(4*a^9) - (12*a^8*b^4 - 16*a^6*b^6 + a^10*b^2)/(4*a^10) + (b^4*(2*a^2*b
```

```
 - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2))
/a^6)*1i)/a^6 - (b^4*(b^2 - a^2)^(1/2)*((12*a^8*b^4 - 16*a^6*b^6 + a^10*b^2
)/(4*a^10) - (tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*b^7 - 32*a^6*b^5 + 2*a^8*
b^3))/(4*a^9) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2)
)/(4*a^9))*(b^2 - a^2)^(1/2))/a^6)*1i)/a^6)/((8*b^11 - 12*a^2*b^9 + 3*a^4*b
^7 + a^6*b^5)/(2*a^10) + (tan(c/2 + (d*x)/2)*(8*b^10 - 10*a^2*b^8 + 2*a^4*b
^6))/(2*a^9) + (b^4*(b^2 - a^2)^(1/2)*((tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4
*b^7 - 32*a^6*b^5 + 2*a^8*b^3))/(4*a^9) - (12*a^8*b^4 - 16*a^6*b^6 + a^10*b
^2)/(4*a^10) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))
/(4*a^9))*(b^2 - a^2)^(1/2))/a^6))/a^6 + (b^4*(b^2 - a^2)^(1/2)*((12*a^8*b^
4 - 16*a^6*b^6 + a^10*b^2)/(4*a^10) - (tan(c/2 + (d*x)/2)*(a^10*b + 32*a^4*
b^7 - 32*a^6*b^5 + 2*a^8*b^3))/(4*a^9) + (b^4*(2*a^2*b - (tan(c/2 + (d*x)/2
)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(b^2 - a^2)^(1/2))/a^6))/a^6))*(b^2 - a
^2)^(1/2)*2i)/(a^6*d)
```

3.1295 $\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{a^3(a^2-b^2)\log(a+b\sin(c+dx))}{b^6d}-\frac{a^2(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a(a^2-b^2)\sin^2(c+dx)}{2b^4d}-\frac{(a^2-b^2)\sin^3(c+dx)}{3b^3d}$$

```
[Out] a^3*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^6/d-a^2*(a^2-b^2)*sin(d*x+c)/b^5/d+1/2*a
*(a^2-b^2)*sin(d*x+c)^2/b^4/d-1/3*(a^2-b^2)*sin(d*x+c)^3/b^3/d+1/4*a*sin(d*
x+c)^4/b^2/d-1/5*sin(d*x+c)^5/b/d
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$-\frac{a^2(a^2-b^2)\sin(c+dx)}{b^5d}+\frac{a(a^2-b^2)\sin^2(c+dx)}{2b^4d}-\frac{(a^2-b^2)\sin^3(c+dx)}{3b^3d}+\frac{a^3(a^2-b^2)\log(a+b\sin(c+dx))}{b^6d}+\frac{a\sin^4(c+dx)}{4b^2d}-\frac{\sin^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a^3*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^6*d) - (a^2*(a^2 - b^2)*Sin[c
+ d*x])/(b^5*d) + (a*(a^2 - b^2)*Sin[c + d*x]^2)/(2*b^4*d) - ((a^2 - b^2)*S
in[c + d*x]^3)/(3*b^3*d) + (a*Sin[c + d*x]^4)/(4*b^2*d) - Sin[c + d*x]^5/(5
*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4\left(1-\frac{b^2}{a^2}\right)+a(a^2-b^2)x-(a^2-b^2)x^2+ax^3-x^4+\frac{a^5-a^3b^2}{a+x}\right.\right.}{b^6d}$$
$$= \frac{a^3(a^2-b^2)\log(a+b\sin(c+dx))}{b^6d} - \frac{a^2(a^2-b^2)\sin(c+dx)}{b^5d} + \frac{a(a^2-b^2)\,\mathrm{s}}{2b}$$

Mathematica [A]
time = 0.20, size = 127, normalized size = 0.85

$$\frac{\frac{60a^3(a-b)(a+b)\log(a+b\sin(c+dx))}{b^6}-\frac{60a^2(a-b)(a+b)\sin(c+dx)}{b^5}+\frac{30a(a-b)(a+b)\sin^2(c+dx)}{b^4}-\frac{20(a-b)(a+b)\sin^3(c+dx)}{b^3}+\frac{15a\sin^4(c+dx)}{b^2}-\frac{12\sin^5(c+dx)}{b}}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((60*a^3*(a - b)*(a + b)*Log[a + b*Sin[c + d*x]])/b^6 - (60*a^2*(a - b)*(a
+ b)*Sin[c + d*x])/b^5 + (30*a*(a - b)*(a + b)*Sin[c + d*x]^2)/b^4 - (20*(a
 - b)*(a + b)*Sin[c + d*x]^3)/b^3 + (15*a*Sin[c + d*x]^4)/b^2 - (12*Sin[c +
 d*x]^5)/b)/(60*d)
```

Maple [A]
time = 0.26, size = 147, normalized size = 0.99

method	result
derivativedivides	$-\frac{\frac{-\frac{(\sin^5(dx+c))b^4}{5}-\frac{a\,b^3(\sin^4(dx+c))}{4}+\frac{a^2b^2(\sin^3(dx+c))}{3}-\frac{b^4(\sin^3(dx+c))}{3}-\frac{a^3b(\sin^2(dx+c))}{2}+\frac{a\,b^3(\sin^2(dx+c))}{2}+a^4\sin(dx+c)-}{b^5}}{d}$
default	$-\frac{\frac{-\frac{(\sin^5(dx+c))b^4}{5}-\frac{a\,b^3(\sin^4(dx+c))}{4}+\frac{a^2b^2(\sin^3(dx+c))}{3}-\frac{b^4(\sin^3(dx+c))}{3}-\frac{a^3b(\sin^2(dx+c))}{2}+\frac{a\,b^3(\sin^2(dx+c))}{2}+a^4\sin(dx+c)-}{b^5}}{d}$
risch	$-\frac{3i\mathrm{e}^{i(dx+c)}a^2}{8b^3d}+\frac{3i\mathrm{e}^{-i(dx+c)}a^2}{8b^3d}-\frac{a^3\mathrm{e}^{2i(dx+c)}}{8b^4d}+\frac{a\,\mathrm{e}^{2i(dx+c)}}{16b^2d}-\frac{ia^5x}{b^6}-\frac{i\mathrm{e}^{-i(dx+c)}a^4}{2b^5d}+\frac{i\mathrm{e}^{i(dx+c)}a^4}{2b^5d}+\frac{i\mathrm{e}^{-i(dx}}{16ba}$
norman	$\frac{\frac{(8a^3-4a\,b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4}+\frac{(8a^3-4a\,b^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^4}-\frac{4(25a^4-15a^2b^2-2b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5b^5d}-\frac{4(25a^4-15a^2b^2-2b^4)(\tan^7}{5b^5d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^5*(1/5*sin(d*x+c)^5*b^4-1/4*a*b^3*sin(d*x+c)^4+1/3*a^2*b^2*sin(d*
x+c)^3-1/3*b^4*sin(d*x+c)^3-1/2*a^3*b*sin(d*x+c)^2+1/2*a*b^3*sin(d*x+c)^2+a
^4*sin(d*x+c)-a^2*b^2*sin(d*x+c))+a^3*(a^2-b^2)/b^6*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.88

$$-\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,(a^2b^2-b^4)\sin(dx+c)^3-30\,(a^3b-ab^3)\sin(dx+c)^2+60\,(a^4-a^2b^2)\sin(dx+c)}{b^5}-\frac{60\,(a^5-a^3b^2)\log(b\sin(dx+c)+a)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - b^4
)*sin(d*x + c)^3 - 30*(a^3*b - a*b^3)*sin(d*x + c)^2 + 60*(a^4 - a^2*b^2)*s
in(d*x + c))/b^5 - 60*(a^5 - a^3*b^2)*log(b*sin(d*x + c) + a)/b^6)/d
```

Fricas [A]
time = 0.37, size = 127, normalized size = 0.85

$$\frac{15\,ab^4\cos\left(dx+c\right)^4-30\,a^3b^2\cos\left(dx+c\right)^2+60\,(a^5-a^3b^2)\log\left(b\sin\left(dx+c\right)+a\right)-4\,\left(3\,b^5\cos\left(dx+c\right)^4+15\,a^4b-10\,a^2b^3-2\,b^5-(5\,a^2b^3+b^5)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b^4*cos(d*x + c)^4 - 30*a^3*b^2*cos(d*x + c)^2 + 60*(a^5 - a^3*b
^2)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4 + 15*a^4*b - 10*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 149, normalized size = 1.00

$$-\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,a^2b^2\sin(dx+c)^3-20\,b^4\sin(dx+c)^3-30\,a^3b\sin(dx+c)^2+30\,ab^3\sin(dx+c)^2+60\,a^4\sin(dx+c)-60\,a^2b^2\sin(dx+c)}{b^5}-\frac{60\,(a^5-a^3b^2)\log(|b\sin(dx+c)+a|)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] -1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*x + c)^3 - 20*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 30*a*b^3*sin(d*x + c)^2 + 60*a^4*sin(d*x + c) - 60*a^2*b^2*sin(d*x + c))/b^5 - 60*(a^5 - a^3*b^2)*log(abs(b*sin(d*x + c) + a))/b^6)/d

Mupad [B]
time = 0.08, size = 133, normalized size = 0.89

$$\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{1}{3\,b}-\frac{a^2}{3\,b^3}\right)-\frac{\sin\left(c+d\,x\right)^5}{5\,b}+\frac{a\,\sin\left(c+d\,x\right)^4}{4\,b^2}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(a^5-a^3\,b^2\right)}{b^6}-\frac{a\,\sin\left(c+d\,x\right)^2\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{2\,b}+\frac{a^2\,\sin\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)

[Out] (sin(c + d*x)^3*(1/(3*b) - a^2/(3*b^3)) - sin(c + d*x)^5/(5*b) + (a*sin(c + d*x)^4)/(4*b^2) + (log(a + b*sin(c + d*x))*(a^5 - a^3*b^2))/b^6 - (a*sin(c + d*x)^2*(1/b - a^2/b^3))/(2*b) + (a^2*sin(c + d*x)*(1/b - a^2/b^3))/b^2)/d

3.1296 $\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$-\frac{a^2(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{a(a^2-b^2)\sin(c+dx)}{b^4d}-\frac{(a^2-b^2)\sin^2(c+dx)}{2b^3d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c}{4}$$

```
[Out] -a^2*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^5/d+a*(a^2-b^2)*sin(d*x+c)/b^4/d-1/2*(a
^2-b^2)*sin(d*x+c)^2/b^3/d+1/3*a*sin(d*x+c)^3/b^2/d-1/4*sin(d*x+c)^4/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$-\frac{a^2(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d}+\frac{a(a^2-b^2)\sin(c+dx)}{b^4d}-\frac{(a^2-b^2)\sin^2(c+dx)}{2b^3d}+\frac{a\sin^3(c+dx)}{3b^2d}-\frac{\sin^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^2*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^5*d)) + (a*(a^2 - b^2)*Sin[c
 + d*x])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]^2)/(2*b^3*d) + (a*Sin[c + d*x]
^3)/(3*b^2*d) - Sin[c + d*x]^4/(4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^3\left(1-\frac{b^2}{a^2}\right)-(a^2-b^2)\,x+ax^2-x^3+\frac{a^2(-a^2+b^2)}{a+x}\right)dx, x, b\sin(c\right.}{b^5d}$$
$$= -\frac{a^2(a^2-b^2)\log(a+b\sin(c+dx))}{b^5d} + \frac{a(a^2-b^2)\sin(c+dx)}{b^4d} - \frac{(a^2-b^2)\,\text{s}}{2b}$$

Mathematica [A]
time = 0.26, size = 104, normalized size = 0.87

$$\frac{12a^2(-a^2+b^2)\log(a+b\sin(c+dx))+12ab(a^2-b^2)\sin(c+dx)+6b^2(-a^2+b^2)\sin^2(c+dx)+4ab^3\sin^3(c+dx)-3b^4\sin^4(c+dx)}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] (12*a^2*(-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + 12*a*b*(a^2 - b^2)*Sin[c + d
*x] + 6*b^2*(-a^2 + b^2)*Sin[c + d*x]^2 + 4*a*b^3*Sin[c + d*x]^3 - 3*b^4*Si
n[c + d*x]^4)/(12*b^5*d)
```

Maple [A]
time = 0.26, size = 103, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{-\frac{b^3\left(\sin^4(dx+c)\right)}{4}+\frac{a\left(\sin^3(dx+c)\right)b^2}{3}-\frac{\left(a^2-b^2\right)\left(\sin^2(dx+c)\right)b}{2}+a\left(a^2-b^2\right)\sin(dx+c)}{b^4}-\frac{a^2\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$
default	$\frac{\frac{-\frac{b^3\left(\sin^4(dx+c)\right)}{4}+\frac{a\left(\sin^3(dx+c)\right)b^2}{3}-\frac{\left(a^2-b^2\right)\left(\sin^2(dx+c)\right)b}{2}+a\left(a^2-b^2\right)\sin(dx+c)}{b^4}-\frac{a^2\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^5}}{d}$
risch	$-\frac{2ia^2c}{b^3d}+\frac{ia^4x}{b^5}+\frac{e^{2i(dx+c)}a^2}{8b^3d}-\frac{e^{2i(dx+c)}}{16bd}+\frac{ia^3e^{-i(dx+c)}}{2b^4d}+\frac{3ia\,e^{i(dx+c)}}{8b^2d}-\frac{ia^3e^{i(dx+c)}}{2b^4d}+\frac{2ia^4c}{b^5d}+\frac{e^{-2i(dx+}}{8b^3}$
norman	$\frac{-\frac{2\left(3a^2-b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{2\left(3a^2-b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{\left(2a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{\left(2a^2-2b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2a\left(a^2-b\right.}{}}{\left(1+\text{t}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(-1/4*b^3*sin(d*x+c)^4+1/3*a*sin(d*x+c)^3*b^2-1/2*(a^2-b^2)*sin(
d*x+c)^2*b+a*(a^2-b^2)*sin(d*x+c))-a^2*(a^2-b^2)/b^5*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 105, normalized size = 0.88

$$-\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\,(a^2b-b^3)\sin(dx+c)^2-12\,(a^3-ab^2)\sin(dx+c)}{b^4}+\frac{12\,(a^4-a^2b^2)\log(b\sin(dx+c)+a)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*(a^2*b - b^3)*sin
(d*x + c)^2 - 12*(a^3 - a*b^2)*sin(d*x + c))/b^4 + 12*(a^4 - a^2*b^2)*log(b
*sin(d*x + c) + a)/b^5)/d
```

Fricas [A]
time = 0.37, size = 97, normalized size = 0.82

$$-\frac{3\,b^4\cos\left(dx+c\right)^4-6\,a^2b^2\cos\left(dx+c\right)^2+12\,(a^4-a^2b^2)\log\left(b\sin\left(dx+c\right)+a\right)+4\,\left(ab^3\cos\left(dx+c\right)^2-3\,a^3b+2\,ab^3\right)\sin\left(dx+c\right)}{12\,b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*b^4*cos(d*x + c)^4 - 6*a^2*b^2*cos(d*x + c)^2 + 12*(a^4 - a^2*b^2)
*log(b*sin(d*x + c) + a) + 4*(a*b^3*cos(d*x + c)^2 - 3*a^3*b + 2*a*b^3)*sin
(d*x + c))/(b^5*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 117, normalized size = 0.98

$$-\frac{\frac{3\,b^3\sin(dx+c)^4-4\,ab^2\sin(dx+c)^3+6\,a^2b\sin(dx+c)^2-6\,b^3\sin(dx+c)^2-12\,a^3\sin(dx+c)+12\,ab^2\sin(dx+c)}{b^4}+\frac{12\,(a^4-a^2b^2)\log(|b\sin(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

[Out] -1/12*((3*b^3*sin(d*x + c)^4 - 4*a*b^2*sin(d*x + c)^3 + 6*a^2*b*sin(d*x + c
)^2 - 6*b^3*sin(d*x + c)^2 - 12*a^3*sin(d*x + c) + 12*a*b^2*sin(d*x + c))/b
^4 + 12*(a^4 - a^2*b^2)*log(abs(b*sin(d*x + c) + a))/b^5)/d

Mupad [B]
time = 11.62, size = 107, normalized size = 0.90

$$-\frac{\frac{\sin(c+d\,x)^4}{4\,b}-\sin\left(c+d\,x\right)^2\,\left(\frac{1}{2\,b}-\frac{a^2}{2\,b^3}\right)-\frac{a\,\sin(c+d\,x)^3}{3\,b^2}+\frac{\ln(a+b\,\sin(c+d\,x))\,\left(a^4-a^2\,b^2\right)}{b^5}+\frac{a\,\sin(c+d\,x)\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)

[Out] -(sin(c + d*x)^4/(4*b) - sin(c + d*x)^2*(1/(2*b) - a^2/(2*b^3)) - (a*sin(c
+ d*x)^3)/(3*b^2) + (log(a + b*sin(c + d*x))*(a^4 - a^2*b^2))/b^5 + (a*sin(
c + d*x)*(1/b - a^2/b^3))/b)/d

3.1297 $\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d}-\frac{(a^2-b^2)\sin(c+dx)}{b^3d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] a*(a^2-b^2)*ln(a+b*sin(d*x+c))/b^4/d-(a^2-b^2)*sin(d*x+c)/b^3/d+1/2*a*sin(d
*x+c)^2/b^2/d-1/3*sin(d*x+c)^3/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 786}

$$\frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d}-\frac{(a^2-b^2)\sin(c+dx)}{b^3d}+\frac{a\sin^2(c+dx)}{2b^2d}-\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((a^2 - b^2)*Sin[c + d*x]
)/(b^3*d) + (a*Sin[c + d*x]^2)/(2*b^2*d) - Sin[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^2\left(1-\frac{b^2}{a^2}\right)+ax-x^2+\frac{a^3-ab^2}{a+x}\right)dx, x, b\sin(c+dx)\right)}{b^4d}$$
$$= \frac{a(a^2-b^2)\log(a+b\sin(c+dx))}{b^4d} - \frac{(a^2-b^2)\sin(c+dx)}{b^3d} + \frac{a\sin^2(c+dx)}{2b^2d} -$$

Mathematica [A]
time = 0.15, size = 79, normalized size = 0.89

$$\frac{6a(a^2-b^2)\log(a+b\sin(c+dx))+6b(-a^2+b^2)\sin(c+dx)+3ab^2\sin^2(c+dx)-2b^3\sin^3(c+dx)}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] (6*a*(a^2 - b^2)*Log[a + b*Sin[c + d*x]] + 6*b*(-a^2 + b^2)*Sin[c + d*x] +
3*a*b^2*Sin[c + d*x]^2 - 2*b^3*Sin[c + d*x]^3)/(6*b^4*d)
```

Maple [A]
time = 0.19, size = 83, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{\frac{(\sin^3(dx+c))b^2}{3}-\frac{ba(\sin^2(dx+c))}{2}+a^2\sin(dx+c)-b^2\sin(dx+c)}{b^3}+\frac{a(a^2-b^2)\ln(a+b\sin(dx+c))}{b^4}}{d}$
default	$\frac{-\frac{\frac{(\sin^3(dx+c))b^2}{3}-\frac{ba(\sin^2(dx+c))}{2}+a^2\sin(dx+c)-b^2\sin(dx+c)}{b^3}+\frac{a(a^2-b^2)\ln(a+b\sin(dx+c))}{b^4}}{d}$
risch	$-\frac{ia^3x}{b^4}+\frac{iax}{b^2}-\frac{a\,e^{2i(dx+c)}}{8b^2d}+\frac{ie^{i(dx+c)}a^2}{2b^3d}-\frac{3ie^{i(dx+c)}}{8bd}-\frac{ie^{-i(dx+c)}a^2}{2b^3d}+\frac{3ie^{-i(dx+c)}}{8bd}-\frac{a\,e^{-2i(dx+c)}}{8b^2d}-\frac{2ia}{b^4}$
norman	$\frac{-\frac{2(a^2-b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{b^3d}-\frac{2(a^2-b^2)(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{b^3d}-\frac{2(9a^2-5b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3b^3d}-\frac{2(9a^2-5b^2)(\tan^5(\frac{dx}{2}+\frac{c}{2}))}{3b^3d}+\frac{2a(\tan^2(\frac{dx}{2}}{b^2d}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b^3*(1/3*sin(d*x+c)^3*b^2-1/2*b*a*sin(d*x+c)^2+a^2*sin(d*x+c)-b^2*s
in(d*x+c))+a*(a^2-b^2)/b^4*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.89

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,(a^2-b^2)\sin(dx+c)}{b^3}-\frac{6\,(a^3-ab^2)\log(b\sin(dx+c)+a)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*(a^2 - b^2)*sin(d*x
+ c))/b^3 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A]
time = 0.36, size = 78, normalized size = 0.88

$$-\frac{3\,ab^2\cos{(dx+c)}^2-6\,(a^3-ab^2)\log{(b\sin{(dx+c)}+a)}-2\,\left(b^3\cos{(dx+c)}^2-3\,a^2b+2\,b^3\right)\sin{(dx+c)}}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*a*b^2*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*log(b*sin(d*x + c) + a) - 2*
(b^3*cos(d*x + c)^2 - 3*a^2*b + 2*b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 85, normalized size = 0.96

$$-\frac{\frac{2\,b^2\sin(dx+c)^3-3\,ab\sin(dx+c)^2+6\,a^2\sin(dx+c)-6\,b^2\sin(dx+c)}{b^3}-\frac{6\,(a^3-ab^2)\log(|b\sin(dx+c)+a|)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*((2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 + 6*a^2*sin(d*x + c) - 6
*b^2*sin(d*x + c))/b^3 - 6*(a^3 - a*b^2)*log(abs(b*sin(d*x + c) + a))/b^4)/
d
```

Mupad [B]
time = 0.07, size = 78, normalized size = 0.88

$$\frac{\sin\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2}{b^3}\right)-\frac{\sin\left(c+d\,x\right)^3}{3\,b}+\frac{a\,\sin\left(c+d\,x\right)^2}{2\,b^2}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(a\,b^2-a^3\right)}{b^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(1/b - a^2/b^3) - sin(c + d*x)^3/(3*b) + (a*sin(c + d*x)^2)/(
2*b^2) - (log(a + b*sin(c + d*x))*(a*b^2 - a^3))/b^4)/d
```

3.1298 $\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\log(\sin(c+dx))}{ad}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] ln(sin(d*x+c))/a/d+(a^2-b^2)*ln(a+b*sin(d*x+c))/a/b^2/d-sin(d*x+c)/b/d
```

Rubi [A]
time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a*b^2*d) -
 Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\log(\sin(c+dx))}{ad} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{ab^2d} - \frac{\sin(c+dx)}{bd}$$

Mathematica [A]
time = 0.05, size = 53, normalized size = 0.90

$$\frac{b^2\log(\sin(c+dx)) + (a^2-b^2)\log(a+b\sin(c+dx)) - ab\sin(c+dx)}{ab^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Log[Sin[c + d*x]] + (a^2 - b^2)*Log[a + b*Sin[c + d*x]] - a*b*Sin[c + 
d*x])/(a*b^2*d)
```

Maple [A]
time = 0.22, size = 55, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\ln(\sin(dx+c))}{a}+\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^2a}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b}+\frac{\ln(\sin(dx+c))}{a}+\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{b^2a}}{d}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(a^2-b^2)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{a\,b^2d}-\frac{a\ln(}{}$
risch	$-\frac{iax}{b^2}+\frac{ie^{i(dx+c)}}{2bd}-\frac{ie^{-i(dx+c)}}{2bd}-\frac{2iac}{b^2d}+\frac{a\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{b^2d}-\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{ad}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*sin(d*x+c)+1/a*ln(sin(d*x+c))+1/b^2*(a^2-b^2)/a*ln(a+b*sin(d*x+c)
))
```

Maxima [A]
time = 0.29, size = 54, normalized size = 0.92

$$\frac{\frac{\log(\sin(dx+c))}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c))/a - sin(d*x + c)/b + (a^2 - b^2)*log(b*sin(d*x + c) + a)
/(a*b^2))/d
```

Fricas [A]
time = 0.42, size = 55, normalized size = 0.93

$$\frac{b^2\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right) - ab\sin\left(dx+c\right) + \left(a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (b^2*log(-1/2*sin(d*x + c)) - a*b*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x
+ c) + a))/(a*b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 56, normalized size = 0.95

$$\frac{\frac{\log(|\sin(dx+c)|)}{a} - \frac{\sin(dx+c)}{b} + \frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{ab^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(sin(d*x + c)))/a - sin(d*x + c)/b + (a^2 - b^2)*log(abs(b*sin(d*x
+ c) + a))/(a*b^2))/d
```

Mupad [B]
time = 11.86, size = 98, normalized size = 1.66

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}-\frac{\sin\left(c+d\,x\right)}{b\,d}+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(\frac{a}{b^2}-\frac{1}{a}\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) - sin(c + d*x)/(b*d) + (log(a + 2*b*tan(c/2 +
 (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a/b^2 - 1/a))/d - (a*log(tan(c/2 + (d*
x)/2)^2 + 1))/(b^2*d)
```

3.1299 $\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d-(1-b^2/a^2)*ln(a+b*sin(d*x+c))/b/d

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {2916, 12, 908}

$$-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) - ((1 - b^2/a^2)*Log[
a + b*Sin[c + d*x]])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2(b^2-x^2)}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^2}-\frac{b^2}{a^2x}+\frac{-a^2+b^2}{a^2(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\left(1-\frac{b^2}{a^2}\right)\log(a+b\sin(c+dx))}{bd}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.90

$$\frac{-ab\csc(c+dx)-b^2\log(\sin(c+dx))+(-a^2+b^2)\log(a+b\sin(c+dx))}{a^2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] (-(a*b*Csc[c + d*x]) - b^2*Log[Sin[c + d*x]] + (-a^2 + b^2)*Log[a + b*Sin[c
 + d*x]])/(a^2*b*d)
```

Maple [A]
time = 0.21, size = 59, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}+\frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{a^2b}}{d}$
default	$\frac{-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}+\frac{(-a^2+b^2)\ln(a+b\sin(dx+c))}{a^2b}}{d}$
risch	$\frac{ix}{b}+\frac{2ic}{bd}-\frac{2ie^{i(dx+c)}}{da(e^{2i(dx+c)}-1)}-\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{bd}+\frac{b\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{a^2d}-\frac{b\ln(e^{2i(dx-}}{a^2d}$
norman	$\frac{-\frac{1}{2ad}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}-\frac{(a^2-b^2)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{a^2bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))+(-a^2+b^2)/a^2/b*ln(a+b*sin(d*x
+c)))
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.95

$$-\frac{\frac{b\log(\sin(dx+c))}{a^2}+\frac{(a^2-b^2)\log(b\sin(dx+c)+a)}{a^2b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b*log(sin(d*x + c))/a^2 + (a^2 - b^2)*log(b*sin(d*x + c) + a)/(a^2*b) + 1
/(a*sin(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 69, normalized size = 1.15

$$-\frac{b^2\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+ab}{a^2bd\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b^2*log(1/2*sin(d*x + c))*sin(d*x + c) + (a^2 - b^2)*log(b*sin(d*x + c) +
 a)*sin(d*x + c) + a*b)/(a^2*b*d*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 59, normalized size = 0.98

$$-\frac{\frac{b\log(|\sin(dx+c)|)}{a^2}+\frac{(a^2-b^2)\log(|b\sin(dx+c)+a|)}{a^2b}+\frac{1}{a\sin(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(sin(d*x + c)))/a^2 + (a^2 - b^2)*log(abs(b*sin(d*x + c) + a))/(
a^2*b) + 1/(a*sin(d*x + c)))/d
```

Mupad [B]
time = 11.86, size = 118, normalized size = 1.97

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(\frac{b}{a^2}-\frac{1}{b}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{b\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(b/a^2 - 1/b))/d
- tan(c/2 + (d*x)/2)/(2*a*d) - cot(c/2 + (d*x)/2)/(2*a*d) + log(tan(c/2 + (
d*x)/2)^2 + 1)/(b*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

3.1300 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+c))/a^3/d+(a^2
-b^2)*ln(a+b*sin(d*x+c))/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{b\csc(c+dx)}{a^2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3} - \frac{b^2}{a^2x^2} + \frac{-a^2+b^2}{a^3x} + \frac{a^2-b^2}{a^3(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d} + \frac{(a^2-b^2)\log(a+b\sin}{a^3d}$$

Mathematica [A]
time = 0.11, size = 65, normalized size = 0.77

$$-\frac{-2ab\csc(c+dx)+a^2\csc^2(c+dx)+2(a^2-b^2)(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c +
 d*x]] - Log[a + b*Sin[c + d*x]]))/(a^3*d)
```

Maple [A]
time = 0.26, size = 76, normalized size = 0.90

method	result
derivativedivides	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3}}{d}$
default	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3}}{d}$
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(a^2-b^2)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{a^3d}-\frac{(a^2-b}{}$
risch	$\frac{2i\left(-ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{ad}-\frac{b^2\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)}{a^3d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sin(d*x+c)^2+1/a^3*(-a^2+b^2)*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)
+(a^2-b^2)/a^3*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.92

$$\frac{\frac{2\,(a^2-b^2)\log(b\sin(dx+c)+a)}{a^3}-\frac{2\,(a^2-b^2)\log(\sin(dx+c))}{a^3}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.38, size = 118, normalized size = 1.40

$$-\frac{2\,ab\sin\left(dx+c\right)-a^2-2\,\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\,\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 88, normalized size = 1.05

$$-\frac{\frac{2\,(a^2-b^2)\log(|\sin(dx+c)|)}{a^3}-\frac{2\,(a^2b-b^3)\log(|b\sin(dx+c)+a|)}{a^3b}-\frac{2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2
))/d
```

Mupad [B]
time = 11.76, size = 144, normalized size = 1.71

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-b^2\right)}{a^3\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}+\frac{\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-b^2\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (log(tan(
c/2 + (d*x)/2))*(a^2 - b^2))/(a^3*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2))/(4*a^
2*d*tan(c/2 + (d*x)/2)^2) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (
d*x)/2)^2)*(a^2 - b^2))/(a^3*d)
```

3.1301 $\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=282

$$\frac{(16a^6-24a^4b^2+6a^2b^4+b^6)\,x}{16b^7}-\frac{2a^3(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{a(15a^4-20a^2b^2+3b^4)\cos(c+d}{15b^6d}$$

```
[Out] 1/16*(16*a^6-24*a^4*b^2+6*a^2*b^4+b^6)*x/b^7-2*a^3*(a^2-b^2)^(3/2)*arctan((
b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d+1/15*a*(15*a^4-20*a^2*b^2+3*
b^4)*cos(d*x+c)/b^6/d-1/16*(8*a^4-10*a^2*b^2+b^4)*cos(d*x+c)*sin(d*x+c)/b^5
/d+1/15*a*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)^2/b^4/d-1/24*(6*a^2-7*b^2)*co
s(d*x+c)*sin(d*x+c)^3/b^3/d+1/5*a*cos(d*x+c)*sin(d*x+c)^4/b^2/d-1/6*cos(d*x
+c)*sin(d*x+c)^5/b/d
```

Rubi [A]
time = 0.63, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2974, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{a(5a^2-6b^2)\sin^2(c+dx)\cos(c+dx)}{15b^4d}-\frac{(6a^2-7b^2)\sin^3(c+dx)\cos(c+dx)}{24b^3d}+\frac{a(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^6d}-\frac{(8a^4-10a^2b^2+b^4)\sin(c+dx)\cos(c+dx)}{16b^5d}-\frac{2a^3(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{x(16a^6-24a^4b^2+6a^2b^4+b^6)}{16b^7}+\frac{a\sin^4(c+dx)\cos(c+dx)}{5b^2d}-\frac{\sin^5(c+dx)\cos(c+dx)}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*x)/(16*b^7) - (2*a^3*(a^2 - b^2)^(
3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) + (a*(15*a^4
 - 20*a^2*b^2 + 3*b^4)*Cos[c + d*x])/(15*b^6*d) - ((8*a^4 - 10*a^2*b^2 + b^
4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^5*d) + (a*(5*a^2 - 6*b^2)*Cos[c + d*x]*
Sin[c + d*x]^2)/(15*b^4*d) - ((6*a^2 - 7*b^2)*Cos[c + d*x]*Sin[c + d*x]^3)/
(24*b^3*d) + (a*Cos[c + d*x]*Sin[c + d*x]^4)/(5*b^2*d) - (Cos[c + d*x]*Sin[
c + d*x]^5)/(6*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
```

```
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\int \frac{\sin^3(c+dx)(6(4a^2-}{}}{}$$

$$= -\frac{(6a^2-7b^2)\cos(c+dx)\sin^3(c+dx)}{24b^3d} + \frac{a\cos(c+dx)\sin^4(c+dx)}{5b^2d} - \frac{\cos}{}$$

$$= \frac{a(5a^2-6b^2)\cos(c+dx)\sin^2(c+dx)}{15b^4d} - \frac{(6a^2-7b^2)\cos(c+dx)\sin^3(c+}{24b^3d}$$

$$= -\frac{(8a^4-10a^2b^2+b^4)\cos(c+dx)\sin(c+dx)}{16b^5d} + \frac{a(5a^2-6b^2)\cos(c+dx)}{15b^4d}$$

$$= \frac{a(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^6d} - \frac{(8a^4-10a^2b^2+b^4)\cos(c+dx)\sin}{16b^5d}$$

$$= \frac{(16a^6-24a^4b^2+6a^2b^4+b^6)\,x}{16b^7} + \frac{a(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^6d} -$$

$$= \frac{(16a^6-24a^4b^2+6a^2b^4+b^6)\,x}{16b^7} + \frac{a(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^6d} -$$

$$= \frac{(16a^6-24a^4b^2+6a^2b^4+b^6)\,x}{16b^7} + \frac{a(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^6d} -$$

$$= \frac{(16a^6-24a^4b^2+6a^2b^4+b^6)\,x}{16b^7} - \frac{2a^3(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d}$$

Mathematica [A]
time = 1.56, size = 274, normalized size = 0.97

$$\frac{960a^6c-1440a^4b^2c+360a^2b^4c+60b^6c+960a^6dx-1440a^4b^2dx+360a^2b^4dx+60b^6dx-1920a^3(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+120ab(8a^4-10a^2b^2+b^4)\cos(c+dx)+(-80a^3b^3+60ab^5)\cos(3(c+dx))+12ab^5\cos(5(c+dx))-240a^4b^2\sin(2(c+dx))+240a^2b^4\sin(2(c+dx))+15b^6\sin(2(c+dx))+30a^2b^4\sin(4(c+dx))-15b^6\sin(4(c+dx))-5b^6\sin(6(c+dx))}{960b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^6*c - 1440*a^4*b^2*c + 360*a^2*b^4*c + 60*b^6*c + 960*a^6*d*x - 1440
*a^4*b^2*d*x + 360*a^2*b^4*d*x + 60*b^6*d*x - 1920*a^3*(a^2 - b^2)^(3/2)*Ar
cTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(8*a^4 - 10*a^2*b^
2 + b^4)*Cos[c + d*x] + (-80*a^3*b^3 + 60*a*b^5)*Cos[3*(c + d*x)] + 12*a*b^
5*Cos[5*(c + d*x)] - 240*a^4*b^2*Sin[2*(c + d*x)] + 240*a^2*b^4*Sin[2*(c +
```

```
d*x)] + 15*b^6*Sin[2*(c + d*x)] + 30*a^2*b^4*Sin[4*(c + d*x)] - 15*b^6*Sin[
4*(c + d*x)] - 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

Maple [A]
time = 0.35, size = 512, normalized size = 1.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^3*(a^4-2*a^2*b^2+b^4)/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/b^7*(((1/2*a^4*b^2-5/8*a^2*b^4+1/16*b^6
)*tan(1/2*d*x+1/2*c)^11+(a^5*b-2*a^3*b^3+a*b^5)*tan(1/2*d*x+1/2*c)^10+(3/2*
a^4*b^2-7/8*a^2*b^4-47/48*b^6)*tan(1/2*d*x+1/2*c)^9+(5*a^5*b-8*a^3*b^3+a*b^
5)*tan(1/2*d*x+1/2*c)^8+(a^4*b^2-1/4*a^2*b^4+13/8*b^6)*tan(1/2*d*x+1/2*c)^7
+(10*a^5*b-40/3*a^3*b^3+2*a*b^5)*tan(1/2*d*x+1/2*c)^6+(-a^4*b^2+1/4*a^2*b^4
-13/8*b^6)*tan(1/2*d*x+1/2*c)^5+(10*a^5*b-12*a^3*b^3+2*a*b^5)*tan(1/2*d*x+1
/2*c)^4+(-3/2*a^4*b^2+7/8*a^2*b^4+47/48*b^6)*tan(1/2*d*x+1/2*c)^3+(5*a^5*b-
6*a^3*b^3+1/5*a*b^5)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^4*b^2+5/8*a^2*b^4-1/16*b^
6)*tan(1/2*d*x+1/2*c)+a^5*b-4/3*a^3*b^3+1/5*a*b^5)/(1+tan(1/2*d*x+1/2*c)^2)
^6+1/16*(16*a^6-24*a^4*b^2+6*a^2*b^4+b^6)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.41, size = 526, normalized size = 1.87

$$\left[\frac{48\,ab^5\cos\left(dx+c\right)^5-80\,a^3b^3\cos\left(dx+c\right)^3+15\left(16\,a^6-24\,a^4b^2+6\,a^2b^4+b^6\right)dx-120\left(a^5-a^3b^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2-2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}}{b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2}\right)+240\left(a^5b-a^3b^3\right)\cos\left(dx+c\right)-5\left(8\,b^6\cos\left(dx+c\right)^5-2\left(6\,a^2b^4+b^6\right)\cos\left(dx+c\right)^3+3\left(8\,a^4b^2-6\,a^2b^4-b^6\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,b^7d},\ \frac{48\,ab^5\cos\left(dx+c\right)^5-80\,a^3b^3\cos\left(dx+c\right)^3+15\left(16\,a^6-24\,a^4b^2+6\,a^2b^4+b^6\right)dx+240\left(a^5-a^3b^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^2-b^2}\cos\left(dx+c\right)}\right)+240\left(a^5b-a^3b^3\right)\cos\left(dx+c\right)-5\left(8\,b^6\cos\left(dx+c\right)^5-2\left(6\,a^2b^4+b^6\right)\cos\left(dx+c\right)^3+3\left(8\,a^4b^2-6\,a^2b^4-b^6\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,b^7d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 -
24*a^4*b^2 + 6*a^2*b^4 + b^6)*d*x - 120*(a^5 - a^3*b^2)*sqrt(-a^2 + b^2)*lo
g(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
```

```
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*b - a^3*b^3)*cos(d*x + c
) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*a^4
*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d), 1/240*(48*a*b^
5*cos(d*x + c)^5 - 80*a^3*b^3*cos(d*x + c)^3 + 15*(16*a^6 - 24*a^4*b^2 + 6*
a^2*b^4 + b^6)*d*x + 240*(a^5 - a^3*b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - a^3*b^3)*cos(d*x
+ c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 + b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 6*a^2*b^4 - b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(263) = 526.
time = 0.45, size = 726, normalized size = 2.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(16*a^6 - 24*a^4*b^2 + 6*a^2*b^4 + b^6)*(d*x + c)/b^7 - 480*(a^7
- 2*a^5*b^2 + a^3*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^7) + 2*(120
*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 150*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 15*b
^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10 - 480*a^3*b^2*
tan(1/2*d*x + 1/2*c)^10 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^10 + 360*a^4*b*tan
(1/2*d*x + 1/2*c)^9 - 210*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 235*b^5*tan(1/2*
d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 1920*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*tan(1/2*d*x + 1/
2*c)^7 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 390*b^5*tan(1/2*d*x + 1/2*c)^7
 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 3200*a^3*b^2*tan(1/2*d*x + 1/2*c)^6 +
480*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^
2*b^3*tan(1/2*d*x + 1/2*c)^5 - 390*b^5*tan(1/2*d*x + 1/2*c)^5 + 2400*a^5*ta
n(1/2*d*x + 1/2*c)^4 - 2880*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 480*a*b^4*tan(
1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 210*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 235*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a^5*tan(1/2*d*x + 1
/2*c)^2 - 1440*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 48*a*b^4*tan(1/2*d*x + 1/2*
c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 150*a^2*b^3*tan(1/2*d*x + 1/2*c) -
```

```
15*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 320*a^3*b^2 + 48*a*b^4)/((tan(1/2*d
*x + 1/2*c)^2 + 1)^6*b^6))/d
```

Mupad [B]
time = 14.64, size = 600, normalized size = 2.13

$$\frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,b\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{64\,b\,d}-\frac{\sin\left(4\,c+4\,d\,x\right)}{64\,b\,d}-\frac{\sin\left(6\,c+6\,d\,x\right)}{192\,b\,d}+\frac{a\,\cos\left(3\,c+3\,d\,x\right)}{16\,b^2\,d}+\frac{a\,\cos\left(5\,c+5\,d\,x\right)}{80\,b^2\,d}-\frac{5\,a^3\,\cos\left(c+d\,x\right)}{4\,b^4\,d}+\frac{a^5\,\cos\left(c+d\,x\right)}{b^6\,d}+\frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,b^3\,d}-\frac{3\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^5\,d}+\frac{2\,a^6\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^7\,d}-\frac{a^3\,\cos\left(3\,c+3\,d\,x\right)}{12\,b^4\,d}+\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^3\,d}+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,b^3\,d}-\frac{a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^5\,d}+\frac{a\,\cos\left(c+d\,x\right)}{8\,b^2\,d}-\frac{2\,a^3\,\mathrm{atanh}\left(\frac{2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{b^7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(8*b*d) + sin(2*c + 2*d*x)/(64*
b*d) - sin(4*c + 4*d*x)/(64*b*d) - sin(6*c + 6*d*x)/(192*b*d) + (a*cos(3*c
+ 3*d*x))/(16*b^2*d) + (a*cos(5*c + 5*d*x))/(80*b^2*d) - (5*a^3*cos(c + d*x
))/(4*b^4*d) + (a^5*cos(c + d*x))/(b^6*d) + (3*a^2*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)))/(4*b^3*d) - (3*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/(b^5*d) + (2*a^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^
7*d) - (a^3*cos(3*c + 3*d*x))/(12*b^4*d) + (a^2*sin(2*c + 2*d*x))/(4*b^3*d)
 + (a^2*sin(4*c + 4*d*x))/(32*b^3*d) - (a^4*sin(2*c + 2*d*x))/(4*b^5*d) + (
a*cos(c + d*x))/(8*b^2*d) - (2*a^3*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a
^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b
^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d
*x)/2) - 4*a^2*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)
^(1/2))/(b^7*d)
```

3.1302 $\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=235

$$-\frac{a(8a^4-12a^2b^2+3b^4)\,x}{8b^6}+\frac{2a^2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d}-\frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d}+\frac{a(\ldots$$

```
[Out] -1/8*a*(8*a^4-12*a^2*b^2+3*b^4)*x/b^6+2*a^2*(a^2-b^2)^(3/2)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^6/d-1/15*(15*a^4-20*a^2*b^2+3*b^4)*cos(
d*x+c)/b^5/d+1/8*a*(4*a^2-5*b^2)*cos(d*x+c)*sin(d*x+c)/b^4/d-1/15*(5*a^2-6*
b^2)*cos(d*x+c)*sin(d*x+c)^2/b^3/d+1/4*a*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/5*
cos(d*x+c)*sin(d*x+c)^4/b/d
```

Rubi [A]
time = 0.46, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2974, 3128, 3102, 2814, 2739, 632, 210}

$$\frac{2a^2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^6d}+\frac{a(4a^2-5b^2)\sin(c+dx)\cos(c+dx)}{8b^4d}-\frac{(5a^2-6b^2)\sin^2(c+dx)\cos(c+dx)}{15b^3d}-\frac{ax(8a^4-12a^2b^2+3b^4)}{8b^6}-\frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d}+\frac{a\sin^3(c+dx)\cos(c+dx)}{4b^2d}-\frac{\sin^4(c+dx)\cos(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/8*(a*(8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/b^6 + (2*a^2*(a^2 - b^2)^(3/2)*ArcT
an[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^6*d) - ((15*a^4 - 20*a^2*b
^2 + 3*b^4)*Cos[c + d*x])/(15*b^5*d) + (a*(4*a^2 - 5*b^2)*Cos[c + d*x]*Sin[
c + d*x])/(8*b^4*d) - ((5*a^2 - 6*b^2)*Cos[c + d*x]*Sin[c + d*x]^2)/(15*b^3
*d) + (a*Cos[c + d*x]*Sin[c + d*x]^3)/(4*b^2*d) - (Cos[c + d*x]*Sin[c + d*x
]^4)/(5*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \frac{\cos(c+dx)\sin^4(c+dx)}{5bd} - \frac{\int \frac{\sin^2(c+dx)(5(3a^2-}{}}{}$$

$$= -\frac{(5a^2-6b^2)\cos(c+dx)\sin^2(c+dx)}{15b^3d} + \frac{a\cos(c+dx)\sin^3(c+dx)}{4b^2d} - \cos$$

$$= \frac{a(4a^2-5b^2)\cos(c+dx)\sin(c+dx)}{8b^4d} - \frac{(5a^2-6b^2)\cos(c+dx)\sin^2(c+d}{15b^3d}$$

$$= -\frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d} + \frac{a(4a^2-5b^2)\cos(c+dx)\sin(c+d}{8b^4d}$$

$$= -\frac{a(8a^4-12a^2b^2+3b^4)\,x}{8b^6} - \frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d} + \frac{a(4a^2}{}$$

$$= -\frac{a(8a^4-12a^2b^2+3b^4)\,x}{8b^6} - \frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d} + \frac{a(4a^2}{}$$

$$= -\frac{a(8a^4-12a^2b^2+3b^4)\,x}{8b^6} - \frac{(15a^4-20a^2b^2+3b^4)\cos(c+dx)}{15b^5d} + \frac{a(4a^2}{}$$

$$= -\frac{a(8a^4-12a^2b^2+3b^4)\,x}{8b^6} + \frac{2a^2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^6d} - ($$

Mathematica [A]
time = 1.40, size = 186, normalized size = 0.79

$$\frac{960a^2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 60b(8a^4-10a^2b^2+b^4)\cos(c+dx) + 10(4a^2b^3-3b^5)\cos(3(c+dx)) - 6b^5\cos(5(c+dx)) - 15a(4(8a^4-12a^2b^2+3b^4)(c+dx) + (-8a^2b^2+8b^4)\sin(2(c+dx)) + b^4\sin(4(c+dx)))}{480b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (960*a^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 - 60*b*(8*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] + 10*(4*a^2*b^3 - 3*b^5)*Co
s[3*(c + d*x)] - 6*b^5*Cos[5*(c + d*x)] - 15*a*(4*(8*a^4 - 12*a^2*b^2 + 3*b
^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)] + b^4*Sin[4*(c + d*x)
]))/(480*b^6*d)
```

Maple [A]
time = 0.32, size = 363, normalized size = 1.54

method	result

derivativedivides	$\frac{\frac{2a^2(a^4-2a^2b^2+b^4)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}}-\frac{2\left(\left(\frac{1}{2}a^3b^2-\frac{5}{8}a b^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^4b-2a^2b^3+b^5\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2\right.\right.}{}}{}$
default	$\frac{\frac{2a^2(a^4-2a^2b^2+b^4)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}}-\frac{2\left(\left(\frac{1}{2}a^3b^2-\frac{5}{8}a b^4\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^4b-2a^2b^3+b^5\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b^2\right.\right.}{}}{}$
risch	$-\frac{a^5x}{b^6}+\frac{3a^3x}{2b^4}-\frac{3ax}{8b^2}-\frac{e^{i(dx+c)}a^4}{2b^5d}+\frac{5\,e^{i(dx+c)}a^2}{8b^3d}-\frac{e^{i(dx+c)}}{16bd}-\frac{e^{-i(dx+c)}a^4}{2b^5d}+\frac{5\,e^{-i(dx+c)}a^2}{8b^3d}-\frac{e^{-i(dx+c)}}{16bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*(a^4-2*a^2*b^2+b^4)/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/b^6*(((1/2*a^3*b^2-5/8*a*b^4)*tan(1/2*d*
x+1/2*c)^9+(a^4*b-2*a^2*b^3+b^5)*tan(1/2*d*x+1/2*c)^8+(a^3*b^2-1/4*a*b^4)*t
an(1/2*d*x+1/2*c)^7+(4*a^4*b-6*a^2*b^3)*tan(1/2*d*x+1/2*c)^6+(6*a^4*b-22/3*
a^2*b^3+2*b^5)*tan(1/2*d*x+1/2*c)^4+(-a^3*b^2+1/4*a*b^4)*tan(1/2*d*x+1/2*c)
^3+(4*a^4*b-14/3*a^2*b^3)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^3*b^2+5/8*a*b^4)*tan
(1/2*d*x+1/2*c)+a^4*b-4/3*a^2*b^3+1/5*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^5+1/8*a
*(8*a^4-12*a^2*b^2+3*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.39, size = 457, normalized size = 1.94

$$\left[-\frac{24\,b^5\cos(dx+c)^5-40\,a^2b^3\cos(dx+c)^3+15\,(8\,a^5-12\,a^3b^2+3\,ab^4)dx+60\,(a^4-a^2b^2)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+120\,(a^4b-a^2b^3)\cos(dx+c)+15\,(2\,ab^4\cos(dx+c)^3-(4\,a^3b^2-3\,ab^4)\cos(dx+c))\sin(dx+c)}{120\,b^6d},\ -\frac{24\,b^5\cos(dx+c)^5-40\,a^2b^3\cos(dx+c)^3+15\,(8\,a^5-12\,a^3b^2+3\,ab^4)dx+120\,(a^4-a^2b^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)+120\,(a^4b-a^2b^3)\cos(dx+c)+15\,(2\,ab^4\cos(dx+c)^3-(4\,a^3b^2-3\,ab^4)\cos(dx+c))\sin(dx+c)}{120\,b^6d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5 - 12
*a^3*b^2 + 3*a*b^4)*d*x + 60*(a^4 - a^2*b^2)*sqrt(-a^2 + b^2)*log(((2*a^2 -
 b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*s
in(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b
*sin(d*x + c) - a^2 - b^2)) + 120*(a^4*b - a^2*b^3)*cos(d*x + c) + 15*(2*a*
b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/(b^6
*d), -1/120*(24*b^5*cos(d*x + c)^5 - 40*a^2*b^3*cos(d*x + c)^3 + 15*(8*a^5
- 12*a^3*b^2 + 3*a*b^4)*d*x + 120*(a^4 - a^2*b^2)*sqrt(a^2 - b^2)*arctan(-(
a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 120*(a^4*b - a^2*b^3)
*cos(d*x + c) + 15*(2*a*b^4*cos(d*x + c)^3 - (4*a^3*b^2 - 3*a*b^4)*cos(d*x
+ c))*sin(d*x + c))/(b^6*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(218) = 436.
time = 0.47, size = 458, normalized size = 1.95

$$-\frac{\frac{15\,(8\,a^5-12\,a^3b^2+3\,ab^4)(dx+c)}{b^6}-\frac{240\,(a^6-2\,a^4b^2+a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,b^6}+\frac{2\left(60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-240\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+120\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+480\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-720\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+720\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-880\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-560\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,a^4-160\,a^2b^2+24\,b^4\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5b^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 - 12*a^3*b^2 + 3*a*b^4)*(d*x + c)/b^6 - 240*(a^6 - 2*a^4*
b^2 + a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2
*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^6) + 2*(60*a^3*b*ta
n(1/2*d*x + 1/2*c)^9 - 75*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*
x + 1/2*c)^8 - 240*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 120*b^4*tan(1/2*d*x + 1
/2*c)^8 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 30*a*b^3*tan(1/2*d*x + 1/2*c)^
7 + 480*a^4*tan(1/2*d*x + 1/2*c)^6 - 720*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 + 7
20*a^4*tan(1/2*d*x + 1/2*c)^4 - 880*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 240*b^
4*tan(1/2*d*x + 1/2*c)^4 - 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 30*a*b^3*tan(
1/2*d*x + 1/2*c)^3 + 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 560*a^2*b^2*tan(1/2*d
*x + 1/2*c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) + 75*a*b^3*tan(1/2*d*x + 1/2*
c) + 120*a^4 - 160*a^2*b^2 + 24*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^5*b^5))/
d
```

Mupad [B]
time = 13.37, size = 511, normalized size = 2.17

$$\frac{5\,a^2\,\cos\left(c+d\,x\right)}{4\,b^3\,d}-\frac{\cos\left(3\,c+3\,d\,x\right)}{16\,b\,d}-\frac{\cos\left(5\,c+5\,d\,x\right)}{80\,b\,d}-\frac{\cos\left(c+d\,x\right)}{8\,b\,d}-\frac{a^4\,\cos\left(c+d\,x\right)}{b^5\,d}-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^2\,d}-\frac{a\,\sin\left(4\,c+4\,d\,x\right)}{32\,b^2\,d}+\frac{3\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^4\,d}-\frac{2\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^6\,d}+\frac{a^2\,\cos\left(3\,c+3\,d\,x\right)}{12\,b^3\,d}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^4\,d}-\frac{3\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,b^2\,d}+\frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{b^6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (5*a^2*cos(c + d*x))/(4*b^3*d) - cos(3*c + 3*d*x)/(16*b*d) - cos(5*c + 5*d*
x)/(80*b*d) - cos(c + d*x)/(8*b*d) - (a^4*cos(c + d*x))/(b^5*d) - (a*sin(2*
c + 2*d*x))/(4*b^2*d) - (a*sin(4*c + 4*d*x))/(32*b^2*d) + (3*a^3*atan(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^4*d) - (2*a^5*atan(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/(b^6*d) + (a^2*cos(3*c + 3*d*x))/(12*b^3*d) + (a^3*si
n(2*c + 2*d*x))/(4*b^4*d) - (3*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/(4*b^2*d) + (2*a^2*atanh((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2) - a^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^
4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(
1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 +
(d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^
2*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^6*
d)
```

3.1303 $\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$\frac{(8a^4-12a^2b^2+3b^4)\,x}{8b^5}-\frac{2a(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d}-\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}+\frac{\cos(c+$$

[Out] 1/8*(8*a^4-12*a^2*b^2+3*b^4)*x/b^5-2*a*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^5/d-1/12*cos(d*x+c)^3*(4*a-3*b*sin(d*x+c))/b^2/d+1/8*cos(d*x+c)*(8*a*(a^2-b^2)-b*(4*a^2-3*b^2)*sin(d*x+c))/b^4/d

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2944, 2814, 2739, 632, 210}

$$-\frac{2a(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^5d}+\frac{\cos(c+dx)\left(8a(a^2-b^2)-b(4a^2-3b^2)\sin(c+dx)\right)}{8b^4d}+\frac{x(8a^4-12a^2b^2+3b^4)}{8b^5}-\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((8*a^4 - 12*a^2*b^2 + 3*b^4)*x)/(8*b^5) - (2*a*(a^2 - b^2)^(3/2)*ArcTan[(b
 + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^5*d) - (Cos[c + d*x]^3*(4*a - 3
*b*Sin[c + d*x]))/(12*b^2*d) + (Cos[c + d*x]*(8*a*(a^2 - b^2) - b*(4*a^2 -
3*b^2)*Sin[c + d*x]))/(8*b^4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\int \frac{\cos^2(c+dx)\left(-ab-(4a^2-3b^2)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{4b^2}$$

$$= -\frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\left(8a(a^2-b^2)-b(4a^2-3\right.}{8b^4d}$$

$$= \frac{(8a^4-12a^2b^2+3b^4)\,x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\,(}{}$$

$$= \frac{(8a^4-12a^2b^2+3b^4)\,x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\,(}{}$$

$$= \frac{(8a^4-12a^2b^2+3b^4)\,x}{8b^5} - \frac{\cos^3(c+dx)(4a-3b\sin(c+dx))}{12b^2d} + \frac{\cos(c+dx)\,(}{}$$

$$= \frac{(8a^4-12a^2b^2+3b^4)\,x}{8b^5} - \frac{2a(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^5d} - \frac{\cos^3(c+}{}$$

Mathematica [A]
time = 0.74, size = 155, normalized size = 0.97

$$\frac{-192a(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+24ab(4a^2-5b^2)\cos(c+dx)-8ab^3\cos(3(c+dx))+3(4(8a^4-12a^2b^2+3b^4)(c+dx)+(-8a^2b^2+8b^4)\sin(2(c+dx))+b^4\sin(4(c+dx)))}{96b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-192*a*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 24*a*b*(4*a^2 - 5*b^2)*Cos[c + d*x] - 8*a*b^3*Cos[3*(c + d*x)] + 3*(4*(8*
a^4 - 12*a^2*b^2 + 3*b^4)*(c + d*x) + (-8*a^2*b^2 + 8*b^4)*Sin[2*(c + d*x)]
 + b^4*Sin[4*(c + d*x)]))/(96*b^5*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(148) = 296$.
time = 0.26, size = 309, normalized size = 1.94

method	result
derivativedivides	$\frac{-\frac{2a\left(a^4-2a^2b^2+b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}}+\frac{2\left(\left(\frac{1}{2}a^2b^2-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-2a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2+\right.\right.}{}}{}$
default	$\frac{-\frac{2a\left(a^4-2a^2b^2+b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^5\sqrt{a^2-b^2}}+\frac{2\left(\left(\frac{1}{2}a^2b^2-\frac{5}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-2a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2+\right.\right.}{}}{}$
risch	$\frac{x\,a^4}{b^5}-\frac{3x\,a^2}{2b^3}+\frac{3x}{8b}+\frac{a^3\mathrm{e}^{i(dx+c)}}{2b^4d}-\frac{5a\,\mathrm{e}^{i(dx+c)}}{8b^2d}+\frac{a^3\mathrm{e}^{-i(dx+c)}}{2b^4d}-\frac{5a\,\mathrm{e}^{-i(dx+c)}}{8b^2d}-\frac{i\sqrt{a^2-b^2}\,a^3\ln\left(\mathrm{e}^{i(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a*(a^4-2*a^2*b^2+b^4)/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/b^5*(((1/2*a^2*b^2-5/8*b^4)*tan(1/2*d*x+1
/2*c)^7+(a^3*b-2*a*b^3)*tan(1/2*d*x+1/2*c)^6+(1/2*a^2*b^2+3/8*b^4)*tan(1/2*
d*x+1/2*c)^5+(3*a^3*b-4*a*b^3)*tan(1/2*d*x+1/2*c)^4+(-1/2*a^2*b^2-3/8*b^4)*
tan(1/2*d*x+1/2*c)^3+(3*a^3*b-10/3*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^2*b^
2+5/8*b^4)*tan(1/2*d*x+1/2*c)+a^3*b-4/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+1
/8*(8*a^4-12*a^2*b^2+3*b^4)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 414, normalized size = 2.60

$$\left[-\frac{8\,ab^{3}\cos\left(dx+c\right)^{3}-3\,(8\,a^{4}-12\,a^{2}b^{2}+3\,b^{4})dx+12\,(a^{3}-ab^{2})\sqrt{-a^{2}+b^{2}}\log\left(-\frac{(2\,a^{2}-b^{2})\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}}\right)-24\,(a^{3}b-ab^{3})\cos\left(dx+c\right)-3\,\left(2\,b^{4}\cos\left(dx+c\right)^{3}-(4\,a^{2}b^{2}-3\,b^{4})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,b^{5}d},-\frac{8\,ab^{3}\cos\left(dx+c\right)^{3}-3\,(8\,a^{4}-12\,a^{2}b^{2}+3\,b^{4})dx-24\,(a^{3}-ab^{2})\sqrt{a^{2}-b^{2}}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^{2}-b^{2}}\cos(dx+c)}\right)-24\,(a^{3}b-ab^{3})\cos\left(dx+c\right)-3\,\left(2\,b^{4}\cos\left(dx+c\right)^{3}-(4\,a^{2}b^{2}-3\,b^{4})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,b^{5}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x + 12*(a
^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sq
rt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 24
*(a^3*b - a*b^3)*cos(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^
4)*cos(d*x + c))*sin(d*x + c))/(b^5*d), -1/24*(8*a*b^3*cos(d*x + c)^3 - 3*(
8*a^4 - 12*a^2*b^2 + 3*b^4)*d*x - 24*(a^3 - a*b^2)*sqrt(a^2 - b^2)*arctan(-
(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 24*(a^3*b - a*b^3)*c
os(d*x + c) - 3*(2*b^4*cos(d*x + c)^3 - (4*a^2*b^2 - 3*b^4)*cos(d*x + c))*s
in(d*x + c))/(b^5*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(147) = 294$.
time = 0.46, size = 371, normalized size = 2.33

$$\frac{\frac{3\,(8\,a^{4}-12\,a^{2}b^{2}+3\,b^{4})(dx+c)}{b^{5}}-\frac{48\,(a^{5}-2\,a^{3}b^{2}+ab^{4})\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,b^{5}}+\frac{2\left(12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+24\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-48\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+9\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+72\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-96\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-9\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+72\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-80\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,a^{3}-32\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}b^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 12*a^2*b^2 + 3*b^4)*(d*x + c)/b^5 - 48*(a^5 - 2*a^3*b^2 +
a*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*b^5) + 2*(12*a^2*b*tan(1/2*d*
x + 1/2*c)^7 - 15*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^
6 - 48*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*b
```

```
^3*tan(1/2*d*x + 1/2*c)^5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 96*a*b^2*tan(1/
2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 9*b^3*tan(1/2*d*x + 1/
2*c)^3 + 72*a^3*tan(1/2*d*x + 1/2*c)^2 - 80*a*b^2*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c) + 15*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 32*a
*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*b^4))/d
```

Mupad [B]
time = 12.95, size = 453, normalized size = 2.85

$$\frac{3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,b\,d}+\frac{\sin\left(2\,c+2\,d\,x\right)}{4\,b\,d}+\frac{\sin\left(4\,c+4\,d\,x\right)}{32\,b\,d}-\frac{a\,\cos\left(3\,c+3\,d\,x\right)}{12\,b^2\,d}+\frac{a^3\,\cos\left(c+d\,x\right)}{b^4\,d}-\frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^3\,d}+\frac{2\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^5\,d}-\frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,b^3\,d}-\frac{5\,a\,\cos\left(c+d\,x\right)}{4\,b^2\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] (3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*b*d) + sin(2*c + 2*d*x)/
(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) - (a*cos(3*c + 3*d*x))/(12*b^2*d) + (a^
3*cos(c + d*x))/(b^4*d) - (3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
))/(b^3*d) + (2*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^5*d) -
(a^2*sin(2*c + 2*d*x))/(4*b^3*d) - (5*a*cos(c + d*x))/(4*b^2*d) - (2*a*atan
h((2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^2
*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)
/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2
 + (d*x)/2) - 2*a^3*b^2*cos(c/2 + (d*x)/2) - 4*a^2*b^3*sin(c/2 + (d*x)/2)))
*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^5*d)
```

3.1304 $\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=124

$$\frac{(2a^2-3b^2)\,x}{2b^3}-\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin(}{2bd}$$

[Out] 1/2*(2*a^2-3*b^2)*x/b^3-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/b^3/d-arctanh(cos(d*x+c))/a/d+a*cos(d*x+c)/b^2/d-1/2*cos(d*x+c)*sin(d*x+c)/b/d

Rubi [A]
time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2974, 3136, 2739, 632, 210, 3855}

$$-\frac{2(a^2-b^2)^{3/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^3d}+\frac{x(2a^2-3b^2)}{2b^3}+\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] ((2*a^2 - 3*b^2)*x)/(2*b^3) - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^3*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (a*Cos[c + d*x])/(b^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2974

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[a*(n + 3)*Cos[e + f*
x]*(d*Sin[e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d*f*(m + n +
 3)*(m + n + 4))), x] + (-Dist[1/(b^2*(m + n + 3)*(m + n + 4)), Int[(d*Sin[
e + f*x])^n*(a + b*Sin[e + f*x])^m*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n +
3)*(m + n + 4) + a*b*m*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3
)*(m + n + 5))*Sin[e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(d*Sin[e + f*
x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(b*d^2*f*(m + n + 4))), x]) /; Fr
eeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || Intege
rsQ[2*m, 2*n]) &&  !m < -1 &&  !LtQ[n, -1] && NeQ[m + n + 3, 0] && NeQ[m +
n + 4, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\int \frac{\csc(c+dx)\left(-2b^2-ab\sin(c+dx)-\left(2a^2-\right.\right.}{a+b\sin(c+dx)}}{2b^2}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int \csc(c+dx)\,}{a}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c}{2bd}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{a\cos(c+dx)}{b^2d} - \frac{\cos(c+dx)\sin(c}{2bd}$$

$$= \frac{(2a^2-3b^2)\,x}{2b^3} - \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^3d} - \frac{\tanh^{-1}(\cos(c+}{ad}$$

Mathematica [A]
time = 0.17, size = 143, normalized size = 1.15

$$-\frac{-4a^3c+6ab^2c-4a^3dx+6ab^2dx+8(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-4a^2b\cos(c+dx)+4b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-4b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+ab^2\sin(2(c+dx))}{4ab^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-4*a^3*c + 6*a*b^2*c - 4*a^3*d*x + 6*a*b^2*d*x + 8*(a^2 - b^2)^(3/2)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 4*a^2*b*Cos[c + d*x] + 4
*b^3*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Sin[(c + d*x)/2]] + a*b^2*Sin[2*(c +
 d*x)])/(a*b^3*d)
```

Maple [A]
time = 0.37, size = 180, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\,b^3\sqrt{a^2-b^2}}+\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.}{b^3}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{\left(-2a^4+4a^2b^2-2b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\,b^3\sqrt{a^2-b^2}}+\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\right.}{b^3}}{d}$
risch	$\frac{x\,a^2}{b^3}-\frac{3x}{2b}+\frac{a\,e^{i(dx+c)}}{2b^2d}+\frac{a\,e^{-i(dx+c)}}{2b^2d}+\frac{i\sqrt{a^2-b^2}\,a\ln\left(e^{i(dx+c)}-\frac{i\left(\sqrt{a^2-b^2}-a\right)}{b}\right)}{d\,b^3}-\frac{i\sqrt{a^2-b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tan(1/2*d*x+1/2*c))+(-2*a^4+4*a^2*b^2-2*b^4)/a/b^3/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/b^3*((1/2*b^
2*tan(1/2*d*x+1/2*c)^3+a*b*tan(1/2*d*x+1/2*c)^2-1/2*b^2*tan(1/2*d*x+1/2*c)+
a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2-3*b^2)*arctan(tan(1/2*d*x+1/2*c)
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de

Fricas [A]
time = 0.44, size = 350, normalized size = 2.82

$$\left[-\frac{ab^2\cos(dx+c)\sin(dx+c)-2a^2b\cos(dx+c)+b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(2a^3-3ab^2)dx-(-a^2+b^2)^{\frac{3}{2}}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)}{2ab^3d}, -\frac{ab^2\cos(dx+c)\sin(dx+c)-2a^2b\cos(dx+c)+b^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-b^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(2a^3-3ab^2)dx-2(a^2-b^2)^{\frac{3}{2}}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)}{2ab^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] [-1/2*(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log(1/2
*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*b^2)
*d*x - (-a^2 + b^2)^(3/2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*
x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(
-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)))/(a*b^3
d), -1/2(a*b^2*cos(d*x + c)*sin(d*x + c) - 2*a^2*b*cos(d*x + c) + b^3*log
(1/2*cos(d*x + c) + 1/2) - b^3*log(-1/2*cos(d*x + c) + 1/2) - (2*a^3 - 3*a*
b^2)*d*x - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c))))/(a*b^3*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*csc(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral(cos(c + d*x)**4*csc(c + d*x)/(a + b*sin(c + d*x)), x)

Giac [A]
time = 0.49, size = 183, normalized size = 1.48

$$\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{(2a^2-3b^2)(dx+c)}{b^3}-\frac{4(a^4-2a^2b^2+b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,ab^3}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a + (2*a^2 - 3*b^2)*(d*x + c)/b^3 - 4
*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(

```
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a*b^3) + 2*
(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/
2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^2))/d
```

Mupad [B]
time = 14.03, size = 1320, normalized size = 10.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))/(a*d) - sin(2*c + 2*d*x)/(4*b*d)
 - (3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*c
os(c/2 + (d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3
*a*b^2*sin(c/2 + (d*x)/2))))/(b*d) + (a*cos(c + d*x))/(b^2*d) + (2*a^2*atan
((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a*b^2*cos(c/2 + (
d*x)/2))/(2*b^3*cos(c/2 + (d*x)/2) - 2*a^3*sin(c/2 + (d*x)/2) + 3*a*b^2*sin
(c/2 + (d*x)/2))))/(b^3*d) + (atan((b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(3/2)*64i - a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*16i - a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*16i - a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2)*42i + a^3*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*66i - a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2)*176i + a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2)*178i - a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2)*81i - a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(3/2)*116i + a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(3/2)*72i + a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2)*148i - a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*460i + a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2)*577i - a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2)*368i + a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3
*a^2*b^4 + 3*a^4*b^2)^(1/2)*120i + a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(3/2)*32i + a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(3/2)*14i + a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2)*14i)/(64*b^15*sin(c/2 + (d*x)/2) + 32*a*b^14*cos(c/
2 + (d*x)/2) - 120*a^3*b^12*cos(c/2 + (d*x)/2) + 180*a^5*b^10*cos(c/2 + (d*
x)/2) - 137*a^7*b^8*cos(c/2 + (d*x)/2) + 54*a^9*b^6*cos(c/2 + (d*x)/2) - 9*
a^11*b^4*cos(c/2 + (d*x)/2) - 256*a^2*b^13*sin(c/2 + (d*x)/2) + 416*a^4*b^1
1*sin(c/2 + (d*x)/2) - 351*a^6*b^9*sin(c/2 + (d*x)/2) + 161*a^8*b^7*sin(c/2
 + (d*x)/2) - 37*a^10*b^5*sin(c/2 + (d*x)/2) + 3*a^12*b^3*sin(c/2 + (d*x)/2
)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i)/(a*b^3*d)
```

3.1305 $\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{ax}{b^2}+\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cos(c+dx)}{bd}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -a*x/b^2+2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/a^2/b^2/d+b*arctanh(cos(d*x+c))/a^2/d-cos(d*x+c)/b/d-cot(d*x+c)/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2973, 3136, 2739, 632, 210, 3855}

$$\frac{2\left(a^2-b^2\right)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{ax}{b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/b^2) + (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^2*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cos[c + d*x]/(
b*d) - Cot[c + d*x]/(a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2973

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((d
*Sin[e + f*x])^(n + 2)/(b*d^2*f*(m + n + 4))), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\int \frac{\csc(c+dx)\left(b^2+2ab\sin(c+dx)+a^2\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{ab}$$

$$= -\frac{ax}{b^2} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{b\int\csc(c+dx)\,dx}{a^2} + \frac{\left(a^2-b^2\right)^2\int \frac{}{a+b}}{a^2b^2}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} + \frac{\left(2\left(a^2-b^2\right)\right.}{}$$

$$= -\frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cos(c+dx)}{bd} - \frac{\cot(c+dx)}{ad} - \frac{\left(4\left(a^2-b^2\right)\right.}{}$$

$$= -\frac{ax}{b^2} + \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - $$

Mathematica [A]
time = 0.53, size = 146, normalized size = 1.40

$$-\frac{2a^3c+2a^3dx-4(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+2a^2b\cos(c+dx)+ab^2\cot\left(\frac{1}{2}(c+dx)\right)-2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-ab^2\tan\left(\frac{1}{2}(c+dx)\right)}{2a^2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(2*a^3*c + 2*a^3*d*x - 4*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]] + 2*a^2*b*Cos[c + d*x] + a*b^2*Cot[(c + d*x)/2] - 2*b
^3*Log[Cos[(c + d*x)/2]] + 2*b^3*Log[Sin[(c + d*x)/2]] - a*b^2*Tan[(c + d*x
)/2])/(a^2*b^2*d)
```

Maple [A]
time = 0.38, size = 155, normalized size = 1.49

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\right.}{b^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(4a^4-8a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2b^2\sqrt{a^2-b^2}}-\frac{2\left(\frac{b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+a\right.}{b^2}}{d}$
risch	$-\frac{ax}{b^2}-\frac{e^{i(dx+c)}}{2bd}-\frac{e^{-i(dx+c)}}{2bd}-\frac{2i}{ad\left(e^{2i(dx+c)}-1\right)}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\sqrt{-a^2+b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d
*x+1/2*c))+1/2*(4*a^4-8*a^2*b^2+4*b^4)/a^2/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-2/b^2*(b/(1+tan(1/2*d*x+1/2*c)
^2)+a*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.47, size = 396, normalized size = 3.81

$$\left[\frac{b^{3}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b^{3}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2ab^{2}\cos\left(dx+c\right)-\left(a^{2}-b^{2}\right)\sqrt{-a^{2}+b^{2}}\log\left(\frac{\left(2a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-2ab\sin\left(dx+c\right)-a^{2}-b^{2}+2\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\cos\left(dx+c\right)\right)\sqrt{-a^{2}+b^{2}}}{b^{2}\cos\left(dx+c\right)^{2}-2ab\sin\left(dx+c\right)-a^{2}-b^{2}}\right)\sin\left(dx+c\right)-2\left(a^{3}dx+a^{2}b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2a^{2}b^{2}d\sin\left(dx+c\right)}, \frac{b^{3}\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b^{3}\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2ab^{2}\cos\left(dx+c\right)-2\left(a^{2}-b^{2}\right)^{\frac{3}{2}}\arctan\left(-\frac{a\sin\left(dx+c\right)+b}{\sqrt{a^{2}-b^{2}}\cos\left(dx+c\right)}\right)\sin\left(dx+c\right)-2\left(a^{3}dx+a^{2}b\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{2a^{2}b^{2}d\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 2*a*b^2*cos(d*x + c) - (a^2 - b^2)*sqrt(-a^2 + b^
2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(
a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 2*(a^3*d*x + a^2
*b*cos(d*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c)), 1/2*(b^3*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) - b^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 2*a*b^2*cos(d*x + c) - 2*(a^2 - b^2)^(3/2)*arctan(-(a*sin(d*x + c) +
 b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 2*(a^3*d*x + a^2*b*cos(d
*x + c))*sin(d*x + c))/(a^2*b^2*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(c+dx\right)\csc^{2}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(99) = 198.
time = 0.47, size = 221, normalized size = 2.12

$$-\frac{\frac{6\left(dx+c\right)a}{b^{2}}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^{2}}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{12\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}a^{2}b^{2}}-\frac{2b^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-12a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2b^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)a^{2}b}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a/b^2 + 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/
2*d*x + 1/2*c)/a - 12*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi +
```

```
1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a
^2 - b^2)*a^2*b^2) - (2*b^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/
2*c)^2 - 12*a^2*tan(1/2*d*x + 1/2*c) + 2*b^2*tan(1/2*d*x + 1/2*c) - 3*a*b)/
((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))*a^2*b))/d
```

Mupad [B]
time = 13.78, size = 1167, normalized size = 11.22

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}-\frac{\sin(2\,c+2\,d\,x)}{2\,b\,d\,\sin(c+d\,x)}-\frac{2\,a\,\mathrm{atan}\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^3}{b^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^3\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d}+ \text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (atan((16*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)
- 4*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a
^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 12*a^3*b^
3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^5*b^7*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a^7*b^5*cos(
c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 6*a^9*b^3*cos(c/
2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 29*a^2*b^4*sin(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 18*a^4*b^2*sin(c/2
+ (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a^2*b^10*sin(c/2 + (
d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 4*a^4*b^8*sin(c/2 + (d*
x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 22*a^6*b^6*sin(c/2 + (d*x
)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 32*a^8*b^4*sin(c/2 + (d*x)
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 18*a^10*b^2*sin(c/2 + (d*x)
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a*b^5*cos(c/2 + (d*x)/2)*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 5*a^5*b*cos(c/2 + (d*x)/2)*(b^6
 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^11*b*cos(c/2 + (d*x)/2)*(b^6 -
a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*16i + a*b^14*c
os(c/2 + (d*x)/2)*8i - a^14*b*sin(c/2 + (d*x)/2)*3i - a^3*b^12*cos(c/2 + (d
*x)/2)*48i + a^5*b^10*cos(c/2 + (d*x)/2)*123i - a^7*b^8*cos(c/2 + (d*x)/2)*
167i + a^9*b^6*cos(c/2 + (d*x)/2)*126i - a^11*b^4*cos(c/2 + (d*x)/2)*51i +
a^13*b^2*cos(c/2 + (d*x)/2)*9i - a^2*b^13*sin(c/2 + (d*x)/2)*100i + a^4*b^1
1*sin(c/2 + (d*x)/2)*269i - a^6*b^9*sin(c/2 + (d*x)/2)*390i + a^8*b^7*sin(c
/2 + (d*x)/2)*323i - a^10*b^5*sin(c/2 + (d*x)/2)*151i + a^12*b^3*sin(c/2 +
(d*x)/2)*36i))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^2*b^2*d) - (b*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) - sin(2*c + 2*d*x)/(2*b*d*sin(c +
 d*x)) - (2*a*atan((a^3*cos(c/2 + (d*x)/2) + b^3*sin(c/2 + (d*x)/2))/(b^3*c
os(c/2 + (d*x)/2) - a^3*sin(c/2 + (d*x)/2))))/(b^2*d) - cot(c + d*x)/(a*d)
```

3.1306 $\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{x}{b}-\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] x/b-2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/
b/d+1/2*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d-1/2*cot(
d*x+c)*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.19, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2972, 3136, 2739, 632, 210, 3855}

$$\frac{b\cot(c+dx)}{a^2d}-\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3bd}+\frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] x/b - (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/(a^3*b*d) + ((3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) + (b*Cot[c
+ d*x])/(a^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\int \frac{\csc(c+dx)(3a^2-2b^2-ab\sin(c+dx)-2a}{a+b\sin(c+dx)}}{2a^2}$$

$$= \frac{x}{b} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(3a^2-2b^2)\int\csc(c+dx)}{2a^3}$$

$$= \frac{x}{b} + \frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c}{2ad}$$

$$= \frac{x}{b} + \frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c}{2ad}$$

$$= \frac{x}{b} - \frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3bd} + \frac{(3a^2-2b^2)\tanh^{-1}(\cos(c+a}{2a^3d}$$

Mathematica [A]

time = 1.13, size = 204, normalized size = 1.66

$$\frac{8a^3c+8a^3dx-16(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+4ab^2\cot\left(\frac{1}{2}(c+dx)\right)-a^2b\csc^2\left(\frac{1}{2}(c+dx)\right)+12a^2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-12a^2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)-4ab^2\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (8*a^3*c + 8*a^3*d*x - 16*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]] + 4*a*b^2*Cot[(c + d*x)/2] - a^2*b*Csc[(c + d*x)/2]^2 + 1
2*a^2*b*Log[Cos[(c + d*x)/2]] - 8*b^3*Log[Cos[(c + d*x)/2]] - 12*a^2*b*Log[
Sin[(c + d*x)/2]] + 8*b^3*Log[Sin[(c + d*x)/2]] + a^2*b*Sec[(c + d*x)/2]^2
- 4*a*b^2*Tan[(c + d*x)/2])/(8*a^3*b*d)
```

Maple [A]

time = 0.39, size = 180, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-8a^4+16a^2b^2-8b^4\right)a}{4a^3b\sqrt{}}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{\left(-6a^2+4b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{\left(-8a^4+16a^2b^2-8b^4\right)a}{4a^3b\sqrt{}}}{d}$
risch	$\frac{x}{b}+\frac{i\left(-ia\,e^{3i(dx+c)}-ia\,e^{i(dx+c)}+2b\,e^{2i(dx+c)}-2b\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}+\frac{3\ln\left(e^{i(dx+c)}+1\right)}{2ad}-\frac{\ln\left(e^{i(dx+c)}+1\right)b^2}{a^3d}-\frac{i\sqrt{a^2-b^2}\,\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c))-1/8/a/tan(
1/2*d*x+1/2*c)^2+1/4/a^3*(-6*a^2+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2/a^2*b/ta
n(1/2*d*x+1/2*c)+1/4*(-8*a^4+16*a^2*b^2-8*b^4)/a^3/b/(a^2-b^2)^(1/2)*arctan
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/b*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(114) = 228$.
time = 0.48, size = 572, normalized size = 4.65

$$\left[\frac{4\,a^{3} d x \cos\left(d x + c\right)^{2} - 4\,a^{3} d x - 4\,a b^{2} \cos\left(d x + c\right) \sin\left(d x + c\right) + 2\,a^{2} b \cos\left(d x + c\right) - 2\,{\left({\left(a^{2} - b^{2}\right)} \cos\left(d x + c\right)^{2} - a^{2} + b^{2}\right)} \sqrt{-a^{2} + b^{2}} \log\left(-\frac{{\left(2\,a^{2} - b^{2}\right)} \cos\left(d x + c\right)^{2} - 2\,a b \sin\left(d x + c\right) - a^{2} - b^{2} - 2\,{\left(a \cos\left(d x + c\right) \sin\left(d x + c\right) + b \cos\left(d x + c\right)\right)} \sqrt{-a^{2} + b^{2}}}{b^{2} \cos\left(d x + c\right)^{2} - 2\,a b \sin\left(d x + c\right) - a^{2} - b^{2}}\right) - {\left(3\,a^{2} b - 2\,b^{3} - {\left(3\,a^{2} b - 2\,b^{3}\right)} \cos\left(d x + c\right)^{2}\right)} \log\left(\frac{1}{2} \cos\left(d x + c\right) + \frac{1}{2}\right) + {\left(3\,a^{2} b - 2\,b^{3} - {\left(3\,a^{2} b - 2\,b^{3}\right)} \cos\left(d x + c\right)^{2}\right)} \log\left(-\frac{1}{2} \cos\left(d x + c\right) + \frac{1}{2}\right)}{4\,{\left(a^{3} b d \cos\left(d x + c\right)^{2} - a^{3} b d\right)}}, \frac{4\,a^{3} d x \cos\left(d x + c\right)^{2} - 4\,a^{3} d x - 4\,a b^{2} \cos\left(d x + c\right) \sin\left(d x + c\right) + 2\,a^{2} b \cos\left(d x + c\right) + 4\,{\left({\left(a^{2} - b^{2}\right)} \cos\left(d x + c\right)^{2} - a^{2} + b^{2}\right)} \sqrt{a^{2} - b^{2}} \arctan\left(-\frac{a \sin\left(d x + c\right) + b}{\sqrt{a^{2} - b^{2}} \cos\left(d x + c\right)}\right) - {\left(3\,a^{2} b - 2\,b^{3} - {\left(3\,a^{2} b - 2\,b^{3}\right)} \cos\left(d x + c\right)^{2}\right)} \log\left(\frac{1}{2} \cos\left(d x + c\right) + \frac{1}{2}\right) + {\left(3\,a^{2} b - 2\,b^{3} - {\left(3\,a^{2} b - 2\,b^{3}\right)} \cos\left(d x + c\right)^{2}\right)} \log\left(-\frac{1}{2} \cos\left(d x + c\right) + \frac{1}{2}\right)}{4\,{\left(a^{3} b d \cos\left(d x + c\right)^{2} - a^{3} b d\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x - 4*a*b^2*cos(d*x + c)*sin(d*x +
 c) + 2*a^2*b*cos(d*x + c) - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqr
t(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))
/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - (3*a^2*b - 2*b^3
- (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b
- 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(
a^3*b*d*cos(d*x + c)^2 - a^3*b*d), 1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*
x - 4*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*a^2*b*cos(d*x + c) + 4*((a^2 - b^
2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)
/(sqrt(a^2 - b^2)*cos(d*x + c))) - (3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos
(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^2*b - 2*b^3 - (3*a^2*b - 2*
b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*b*d*cos(d*x + c)^2
- a^3*b*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}{\left(c + d x \right)} \csc^{3}{\left(c + d x \right)}}{a + b \sin{\left(c + d x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [**A**]
time = 0.47, size = 217, normalized size = 1.76

$$\frac{\frac{8\,{\left(d x + c\right)}}{b} + \frac{a \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right)^{2} - 4\,b \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right)}{a^{2}} - \frac{4\,{\left(3\,a^{2} - 2\,b^{2}\right)} \log\left({\left| \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right) \right|}\right)}{a^{3}} - \frac{16\,{\left(a^{4} - 2\,a^{2} b^{2} + b^{4}\right)} {\left(\pi \left \lfloor \frac{d x + c}{2\,\pi} + \frac{1}{2} \right \rfloor \mathrm{sgn}\left(a\right) + \arctan\left(\frac{a \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right) + b}{\sqrt{a^{2} - b^{2}}}\right)\right)}}{\sqrt{a^{2} - b^{2}} a^{3} b} + \frac{18\,a^{2} \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right)^{2} - 12\,b^{2} \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right)^{2} + 4\,a b \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right) - a^{2}}{a^{3} \tan\left(\frac{1}{2}\,d x + \frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*(8*(d*x + c)/b + (a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(3*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(a^4 - 2*a^2*b^2 + b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^3*b) + (18*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 4*a*b*tan(1/2*d*x + 1/2*c) - a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d

Mupad [B]
time = 12.68, size = 2718, normalized size = 22.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)

[Out] (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (a^3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a^2*b*sin(c/2 + (d*x)/2))/(2*a^3*sin(c/2 + (d*x)/2) - 2*b^3*cos(c/2 + (d*x)/2) + 3*a^2*b*cos(c/2 + (d*x)/2)))*2i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) + (2*atan((32*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 14*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 14*a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 36*a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 24*a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 82*a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 63*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 11*a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 56*a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 106*a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 72*a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 16*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 19*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 13*a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*32i + a*b^14*cos(c/2 + (d*x)/2)*16i - a^14*b*sin(c/2 + (d*x)/2)*6i - a^3*b^12*cos(c/2 + (d*x)/2)*108i + a^5*b^10*cos(c/2 + (d*x)/2)*309i - a^7*b^8*cos(c/2 + (d*x)/2)*469i + a^9*b^6*cos(c/2 + (d*x)/2)*390i - a^11*b^4*cos(c/2 + (d*x)/2)*165i + a^13*b^2*cos(c/2 + (d*x)/2)*27i - a^2*b^13*sin(c/2 + (d*x)/2)*224i + a^4*b^11*sin(c/2 + (d*x)/2)*670i - a^6*b^9*sin(c/2 + (d*x)/2)*1080i + a^8*b^7*sin(c/2 + (d*x)/2)*982i - a^10*b^5*sin (c/2 + (d*x)/2)*482i + a^12*b^3*sin(c/2 + (d*x)/2)*108i))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (a^2*b*cos(c + d*x)*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) + (a^3*atan((2*a^3*cos(c/2 + (d*x)/2) + 2*b^3*sin(c/2 + (d*x)/2) - 3*a^2*b*sin(c/2 + (d*x)/2))/(2*a^3*sin(c/2 + (d*x)/2) - 2*b^3*cos(c/2 + (d*x)/2) + 3*a^2*b*cos(c/2 + (d*x)/2)))*cos(2*c + 2*d*x)*2i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*3i)/(2*(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i)) + (a*b^2*sin(2*c + 2*d*x)*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (2*cos(2*c + 2*d*x)*atan((32*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 14*a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 14*a^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 36*a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^5*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*a^7*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 24*a^9*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 82*a^2*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 63*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 2*a^2*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 11*a^4*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 56*a^6*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 106*a^8*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 72*a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 16*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 19*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + 13*a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(b^15*sin(c/2 + (d*x)/2)*32i + a*b^14*cos(c/2 + (d*x)/2)*16i - a^14*b*sin(c/2 + (d*x)/2)*6i - a^3*b^12*cos(c/2 + (d*x)/2)*108i + a^5*b^10*cos(c/2 + (d*x)/2)*309i - a^7*b^8*cos(c/2 + (d*x)/2)*469i + a^9*b^6*cos(c/2 + (d*x)/2)*390i - a^11*b^4*cos(c/2 + (d*x)/2)*165i + a^13*b^2*cos(c/2 + (d*x)/2)*27i - a^2*b^13*sin(c/2 + (d*x)/2)*224i + a^4*b^11*sin(c/2 + (d*x)/2)*670i - a^6*b^9*sin(c/2 + (d*x)/2)*1080i + a^8*b^7*sin(c/2 + (d*x)/2)*982i - a^10*b^5*sin(c/2 + (d*x)/2)*482i + a^12*b^3*sin(c/2 + (d*x)/2)*108i))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i) + (a^2*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*3i)/(2*(a^3*b*d*1i - a^3*b*d*cos(2*c + 2*d*x)*1i))

3.1307 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d}+\frac{b\cot(c+$$

```
[Out] 2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d-1/
2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(4*a^2-3*b^2)*cot(d*x+c)/a^
3/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
```

Rubi [A]
time = 0.28, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2804, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}-\frac{b(3a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin(c}{a+b\sin(c}}{6a}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} -$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} +$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(}{2a^2d}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(}{2a^2d}$$

$$= \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{(4a^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 350 vs. 2(154) = 308.
time = 6.13, size = 350, normalized size = 2.27

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^3d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24ad} + \frac{\left(-3a^2b+2b^3\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(3a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [A]
time = 0.41, size = 223, normalized size = 1.45

method	result

derivativedivides	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a^2+4b^2}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^2\tan\left(\frac{dx}{2}\right.}}{d}$
default	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{-5a^2+4b^2}{8a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^2\tan\left(\frac{dx}{2}\right.}}{d}$
risch	$-\frac{-12ia^2e^{4i(dx+c)}+6ib^2e^{4i(dx+c)}+3be^{5i(dx+c)}a+12ia^2e^{2i(dx+c)}-12ib^2e^{2i(dx+c)}-8ia^2+6ib^2-3be^{i(dx+c)}a}{3da^3\left(e^{2i(dx+c)}-1\right)^3}+\frac{3b\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^3*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-a*b*tan(1/2*d*x+1/2*c)^2-5*a^2*t
an(1/2*d*x+1/2*c)+4*b^2*tan(1/2*d*x+1/2*c))-1/24/a/tan(1/2*d*x+1/2*c)^3-1/8
*(-5*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)+1/8/a^2*b/tan(1/2*d*x+1/2*c)^2+1/2/a
^4*b*(3*a^2-2*b^2)*ln(tan(1/2*d*x+1/2*c))+1/8/a^4*(16*a^4-32*a^2*b^2+16*b^4
)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.47, size = 633, normalized size = 4.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
```

```
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)\csc^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**4*csc(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 273, normalized size = 1.77

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\left(3a^2b-2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{66a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-44b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

Mupad [B]
time = 12.42, size = 654, normalized size = 4.25

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24ad}+\frac{5\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8ad}+\frac{3b\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2a^2d}-\frac{b^3\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4d}+\frac{b\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}-\frac{b^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^2d}+\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2a^3d}+\frac{\mathrm{atan}\left(\frac{2a^5\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+8b^5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}-7a^3b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}-16a^2b^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+4ab^4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}+7a^4b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3a^4b^2-3a^2b^4+b^6}}{2\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^8-5\mathrm{i}\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^7b-16\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^6b^2+14\mathrm{i}\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5b^3+34\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^4b^4-13\mathrm{i}\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^3b^5-28\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2b^6+4\mathrm{i}\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab^7+8\mathrm{i}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^8}\right)\sqrt{-(a+b)^3(a-b)^3}\,2\mathrm{i}}{a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)

[Out] tan(c/2 + (d*x)/2)^3/(24*a*d) - cot(c/2 + (d*x)/2)^3/(24*a*d) + (5*cot(c/2 + (d*x)/2))/(8*a*d) - (5*tan(c/2 + (d*x)/2))/(8*a*d) + (3*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*a^2*d) - (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^4*d) + (b*cot(c/2 + (d*x)/2)^2)/(8*a^2*d) - (b^2*cot(c/2 + (d*x)/2))/(2*a^3*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (b^2*tan(c/2 + (d*x)/2))/(2*a^3*d) + (atan((2*a^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^5*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 7*a^3*b^2*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 16*a^2*b^3*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a*b^4*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 7*a^4*b*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^8*sin(c/2 + (d*x)/2)*2i + b^8*sin(c/2 + (d*x)/2)*8i + a*b^7*cos(c/2 + (d*x)/2)*4i - a^7*b*cos(c/2 + (d*x)/2)*5i - a^3*b^5*cos(c/2 + (d*x)/2)*13i + a^5*b^3*cos(c/2 + (d*x)/2)*14i - a^2*b^6*sin(c/2 + (d*x)/2)*28i + a^4*b^4*sin(c/2 + (d*x)/2)*34i - a^6*b^2*sin(c/2 + (d*x)/2)*16i))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^4*d)

3.1308 $\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=198

$$-\frac{2b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d}-\frac{b(4a^2-3b^2)\cot(c+dx}{3a^4d}$$

```
[Out] -2*b*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/d
-1/8*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^5/d-1/3*b*(4*a^2-3*b^2)
*cot(d*x+c)/a^4/d+1/8*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/d+1/3*b*cot(d
*x+c)*csc(d*x+c)^2/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d
```

Rubi [A]
time = 0.48, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2972, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d}-\frac{2b(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d}-\frac{b(4a^2-3b^2)\cot(c+dx)}{3a^4d}+\frac{(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d}-\frac{(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^5*d) - ((3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^5*d) - (
b*(4*a^2 - 3*b^2)*Cot[c + d*x])/(3*a^4*d) + ((5*a^2 - 4*b^2)*Cot[c + d*x]*C
sc[c + d*x])/(8*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*a^2*d) - (Cot[c
 + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\int \frac{\csc^3(c+dx)(3(5a^2-4b}{}}{}$$

$$= \frac{(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c+dx)\csc^2(c+dx)}{3a^2d} - \frac{\cot(c+}{}$$

$$= -\frac{b(4a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c}{}$$

$$= -\frac{b(4a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^3d} + \frac{b\cot(c}{}$$

$$= -\frac{(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b(4a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(}{}$$

$$= -\frac{(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d} - \frac{b(4a^2-3b^2)\cot(c+dx)}{3a^4d} + \frac{(}{}$$

$$= -\frac{2b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} - \frac{(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos}{8a^5d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 433 vs. 2(198) = 396.
time = 6.18, size = 433, normalized size = 2.19

$$-\frac{2b(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5d} + \frac{\left(-4a^2b\cos\left(\frac{1}{2}(c+dx)\right)+3b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{(5a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{(-3a^4+12a^2b^2-8b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(3a^4-12a^2b^2+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(-5a^2+4b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(4a^2b\sin\left(\frac{1}{2}(c+dx)\right)-3b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Si
n[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*d) + ((-4*a^2*b*Cos[(c + d*x)/2] +
3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + ((5*a^2 - 4*b^2)*Csc[
(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a^
2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-3*a^4 + 12*a^2*b^2 - 8*b^4)*Log[Cos
[(c + d*x)/2]])/(8*a^5*d) + ((3*a^4 - 12*a^2*b^2 + 8*b^4)*Log[Sin[(c + d*x)
/2]])/(8*a^5*d) + ((-5*a^2 + 4*b^2)*Sec[(c + d*x)/2]^2)/(32*a^3*d) + Sec[(c
 + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(4*a^2*b*Sin[(c + d*x)/2] - 3*b^3
*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24
*a^2*d)
```

Maple [A]
time = 0.46, size = 291, normalized size = 1.47

method	result
derivativedivides	$\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4}-$
default	$\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4}-$
risch	$\frac{i\left(-12ia\,b^2\mathrm{e}^{7i(dx+c)}+9ia^3\mathrm{e}^{5i(dx+c)}+15ia^3\mathrm{e}^{i(dx+c)}+12ia\,b^2\mathrm{e}^{5i(dx+c)}-48b\,\mathrm{e}^{6i(dx+c)}a^2+24b^3\mathrm{e}^{6i(dx+c)}+12ia\,b^2\mathrm{e}^{3i(dx+c}\right.}{12d\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^4*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-2/3*b*tan(1/2*d*x+1/2*c)^3*a^2-
2*a^3*tan(1/2*d*x+1/2*c)^2+2*a*b^2*tan(1/2*d*x+1/2*c)^2+10*a^2*b*tan(1/2*d*
x+1/2*c)-8*b^3*tan(1/2*d*x+1/2*c))-1/64/a/tan(1/2*d*x+1/2*c)^4-1/32*(-4*a^2
+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^2+1/16/a^5*(6*a^4-24*a^2*b^2+16*b^4)*ln(tan(
1/2*d*x+1/2*c))+1/24/a^2*b/tan(1/2*d*x+1/2*c)^3-1/8*b*(5*a^2-4*b^2)/a^4/tan
(1/2*d*x+1/2*c)-2*b*(a^4-2*a^2*b^2+b^4)/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(183) = 366$.
time = 0.56, size = 904, normalized size = 4.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 + 24*((a^2*b - b^3)*cos(d*x +
c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(3*a^4 - 4*a^2*b^2)*cos(d*x + c) +
 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 + 8*b^
4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1
/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*a^4 - 12*a^2*b^2 +
 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 - 3*(a^3*b - a*b^3)*cos(
d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x + c)^2 + a^
5*d), -1/48*(6*(5*a^4 - 4*a^2*b^2)*cos(d*x + c)^3 - 48*((a^2*b - b^3)*cos(d
*x + c)^4 + a^2*b - b^3 - 2*(a^2*b - b^3)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(3*a^4 - 4*
a^2*b^2)*cos(d*x + c) + 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4 + 3*
a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^2)*l
og(1/2*cos(d*x + c) + 1/2) - 3*((3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)^4
 + 3*a^4 - 12*a^2*b^2 + 8*b^4 - 2*(3*a^4 - 12*a^2*b^2 + 8*b^4)*cos(d*x + c)
^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((4*a^3*b - 3*a*b^3)*cos(d*x + c)^3 -
 3*(a^3*b - a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^
5*d*cos(d*x + c)^2 + a^5*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(183) = 366.
time = 0.48, size = 375, normalized size = 1.89

$$\frac{\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+120a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-96b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{24\left(3a^4-12a^2b^2+8b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}-\frac{384\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}-\frac{150a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-600a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+400b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+120a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-96ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^4}{a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*((3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 24*
a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 120*a^2*b*ta
n(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 + 24*(3*a^4 - 12*a^2*
```

b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(a^4*b - 2*a^2*b^3 + b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) - (150*a^4*tan(1/2*d*x + 1/2*c)^4 - 600*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2*d*x + 1/2*c)^4))/d

Mupad [B]

time = 12.16, size = 953, normalized size = 4.81

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{1}{8\,a}-\frac{b^2}{8\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{b}{8\,a^2}+\frac{2\,b\left(\frac{1}{4\,a}-\frac{b^2}{4\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a\,b^2-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(10\,a^2\,b-8\,b^3\right)+\frac{a^3}{4}-\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{16\,a^4\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\left(\frac{3\,a^4}{8}-\frac{3\,a^2\,b^2}{2}+b^4\right)}{a^5\,d}-\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}-\text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - (tan(c/2 + (d*x)/2)^2*(1/(8*a) - b^2/(8*a^3
)))/d + (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(1/(4*a) - b^2/(4*a^3)))/a))/
d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 2*a^3) + tan(c/2 + (d*x)/2)^3*(10*a^2*
b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (d
*x)/2)^4) + (log(tan(c/2 + (d*x)/2))*((3*a^4)/8 + b^4 - (3*a^2*b^2)/2))/(a^
5*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) - (b*atan(((b*(-(a + b)^3*(a - b
)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a^5*b^4 - 34*a^7*b
^2))/(4*a^7) - (11*a^9*b + 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) + (b*(2*a^2*b -
 (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3
)^(1/2))/a^5)*1i)/a^5 - (b*(-(a + b)^3*(a - b)^3)^(1/2)*((11*a^9*b + 16*a^5
*b^5 - 28*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a
^5*b^4 - 34*a^7*b^2))/(4*a^7) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10
- 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3)^(1/2))/a^5)*1i)/a^5)/((3*a^8
*b + 8*b^9 - 28*a^2*b^7 + 35*a^4*b^5 - 18*a^6*b^3)/(2*a^8) + (tan(c/2 + (d*
x)/2)*(8*b^8 - 26*a^2*b^6 + 28*a^4*b^4 - 10*a^6*b^2))/(2*a^7) + (b*(-(a + b
)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6 + 64*a^5*b^4
- 34*a^7*b^2))/(4*a^7) - (11*a^9*b + 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) + (b*
(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3
*(a - b)^3)^(1/2))/a^5))/a^5 + (b*(-(a + b)^3*(a - b)^3)^(1/2)*((11*a^9*b +
 16*a^5*b^5 - 28*a^7*b^3)/(4*a^8) - (tan(c/2 + (d*x)/2)*(3*a^9 - 32*a^3*b^6
 + 64*a^5*b^4 - 34*a^7*b^2))/(4*a^7) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(2
4*a^10 - 32*a^8*b^2))/(4*a^7))*(-(a + b)^3*(a - b)^3)^(1/2))/a^5))/a^5))*(-
(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^5*d)
```

3.1309 $\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{2b^2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{b(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{(3a^4-20a^2b^2+15b^4)}{15a^5d}$$

[Out] 2*b^2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+1/8*b*(3*a^4-12*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(3*a^4-20*a^2*b^2+15*b^4)*cot(d*x+c)/a^5/d-1/8*b*(5*a^2-4*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/d+1/15*(6*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d

Rubi [A]
time = 0.66, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2972, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{2b^2(a^2-b^2)^{3/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{b(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d}+\frac{(6a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{15a^3d}+\frac{b(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{15a^5d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}$$

Antiderivative was successfully verified.

[In] Int[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] (2*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^6*d) + (b*(3*a^4 - 12*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) - ((3*a^4 - 20*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (b*(5*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(8*a^4*d) + ((6*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x]^2)/(15*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad} - \frac{\int \frac{\csc^4(c+dx)(4(6a^2-5}{}}{}$$

$$= \frac{(6a^2-5b^2)\cot(c+dx)\csc^2(c+dx)}{15a^3d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c}{}$$

$$= -\frac{b(5a^2-4b^2)\cot(c+dx)\csc(c+dx)}{8a^4d} + \frac{(6a^2-5b^2)\cot(c+dx)\csc^2(c+}{15a^3d}$$

$$= -\frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{b(5a^2-4b^2)\cot(c+dx)\csc(c+dx}{8a^4d}$$

$$= -\frac{(3a^4-20a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{b(5a^2-4b^2)\cot(c+dx)\csc(c+dx}{8a^4d}$$

$$= \frac{b(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(3a^4-20a^2b^2+15b^4)\cot(c}{15a^5d}$$

$$= \frac{b(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(3a^4-20a^2b^2+15b^4)\cot(c}{15a^5d}$$

$$= \frac{2b^2(a^2-b^2)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b(3a^4-12a^2b^2+8b^4)\tanh^{-1}(\mathrm{co}}{8a^6d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 507 vs. 2(244) = 488.
time = 1.26, size = 507, normalized size = 2.08

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^4*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1920*b^2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
] - 32*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 150*a^4*b*Csc[(c
+ d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4
+ 360*a^4*b*Log[Cos[(c + d*x)/2]] - 1440*a^2*b^3*Log[Cos[(c + d*x)/2]] + 96
0*b^5*Log[Cos[(c + d*x)/2]] - 360*a^4*b*Log[Sin[(c + d*x)/2]] + 1440*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 150*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 336*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 21*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 96*a^
```

```
5*Tan[(c + d*x)/2] - 640*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*x)
/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [A]
time = 0.53, size = 379, normalized size = 1.55

method	result
derivativedivides	$\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{32a^5}$
default	$\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+4a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{32a^5}$
risch	$480ib^4e^{2i(dx+c)}-560ia^2b^2e^{2i(dx+c)}+160ia^2b^2+75a^3b\,e^{9i(dx+c)}-60a\,b^3e^{9i(dx+c)}+880ia^2b^2e^{4i(dx+c)}-720ia^2b^2e^{6i(dx+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^5*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-1/2*b*tan(1/2*d*x+1/2*c)^4*a^3-
a^4*tan(1/2*d*x+1/2*c)^3+4/3*a^2*b^2*tan(1/2*d*x+1/2*c)^3+4*a^3*b*tan(1/2*d
*x+1/2*c)^2-4*a*b^3*tan(1/2*d*x+1/2*c)^2+2*a^4*tan(1/2*d*x+1/2*c)-20*a^2*b^
2*tan(1/2*d*x+1/2*c)+16*b^4*tan(1/2*d*x+1/2*c))-1/160/a/tan(1/2*d*x+1/2*c)^
5-1/96*(-3*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^3-1/32*(2*a^4-20*a^2*b^2+16*b^
4)/a^5/tan(1/2*d*x+1/2*c)+1/64/a^2*b/tan(1/2*d*x+1/2*c)^4-1/8*b/a^4*(a^2-b^
2)/tan(1/2*d*x+1/2*c)^2-1/8/a^6*b*(3*a^4-12*a^2*b^2+8*b^4)*ln(tan(1/2*d*x+1
/2*c))+2*b^2*(a^4-2*a^2*b^2+b^4)/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(227) = 454$.

time = 0.59, size = 1051, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2
- 6*a*b^4)*cos(d*x + c)^3 + 120*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 -
 b^4 - 2*(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2
)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d
*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*
b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(3*a^4*b
 - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(
3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b - 4*a^2*b^
3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6
*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(
16*(3*a^5 - 20*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 + 80*(7*a^3*b^2 - 6*a*b^4
)*cos(d*x + c)^3 + 240*((a^2*b^2 - b^4)*cos(d*x + c)^4 + a^2*b^2 - b^4 - 2*
(a^2*b^2 - b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(3*a^4*b - 12*a^2*b^3 +
 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(3*a^4*b - 12*a^
2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 1
5*(3*a^4*b - 12*a^2*b^3 + 8*b^5 + (3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x +
c)^4 - 2*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) - 240*(a^3*b^2 - a*b^4)*cos(d*x + c) - 30*((5*a^4*b
- 4*a^2*b^3)*cos(d*x + c)^3 - (3*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x +
 c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. 2(227) = 454.
time = 0.52, size = 484, normalized size = 1.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 30
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 60*a^4*tan(1/2
*d*x + 1/2*c) - 600*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x + 1/
2*c))/a^5 - 120*(3*a^4*b - 12*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^6 + 1920*(a^4*b^2 - 2*a^2*b^4 + b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 -
 b^2)*a^6) + (822*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 3288*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*a^5*tan(1/2*d*x + 1/2*c)^4
 + 600*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x + 1/2*c)^4 -
120*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 30*
a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*t
an(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 12.18, size = 1082, normalized size = 4.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(3/
(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 1/(16*a)
+ (2*b*(b/(16*a^2) + (2*b*(3/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 +
(d*x)/2)^3*(1/(32*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2*
d) + (tan(c/2 + (d*x)/2)^2*(a^4 - (4*a^2*b^2)/3) - tan(c/2 + (d*x)/2)^4*(2*
a^4 + 16*b^4 - 20*a^2*b^2) - a^4/5 + tan(c/2 + (d*x)/2)^3*(4*a*b^3 - 4*a^3*
b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) - (log(t
an(c/2 + (d*x)/2))*((3*a^4*b)/8 + b^5 - (3*a^2*b^3)/2))/(a^6*d) + (b^2*atan
(((b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4
*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) - (16*a^6*b^6 - 28*a^8*b^4 + 11*a^
10*b^2)/(4*a^10) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6)*1i)/a^6 - (b^2*(-(a + b)^3
*(a - b)^3)^(1/2)*((16*a^6*b^6 - 28*a^8*b^4 + 11*a^10*b^2)/(4*a^10) - (tan(
c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) +
 (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a
 + b)^3*(a - b)^3)^(1/2))/a^6)*1i)/a^6)/((8*b^11 - 28*a^2*b^9 + 35*a^4*b^7
- 18*a^6*b^5 + 3*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x)/2)*(8*b^10 - 26*a^2*b
^8 + 28*a^4*b^6 - 10*a^6*b^4))/(2*a^9) + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*
((tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 + 64*a^6*b^5 - 34*a^8*b^3))/(4*
a^9) - (16*a^6*b^6 - 28*a^8*b^4 + 11*a^10*b^2)/(4*a^10) + (b^2*(2*a^2*b - (
```

```
tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)
^(1/2))/a^6))/a^6 + (b^2*(-(a + b)^3*(a - b)^3)^(1/2)*((16*a^6*b^6 - 28*a^8
*b^4 + 11*a^10*b^2)/(4*a^10) - (tan(c/2 + (d*x)/2)*(3*a^10*b - 32*a^4*b^7 +
 64*a^6*b^5 - 34*a^8*b^3))/(4*a^9) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(2
4*a^12 - 32*a^10*b^2))/(4*a^9))*(-(a + b)^3*(a - b)^3)^(1/2))/a^6))/a^6))*(
-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^6*d)
```

3.1310 $\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$-\frac{a^3(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^8d}+\frac{a^2(a^2-b^2)^2\sin(c+dx)}{b^7d}-\frac{a(a^2-b^2)^2\sin^2(c+dx)}{2b^6d}+\frac{(a^2-b^2)^2\sin^3(c}{3b^5d}$$

```
[Out] -a^3*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^8/d+a^2*(a^2-b^2)^2*sin(d*x+c)/b^7/d-
1/2*a*(a^2-b^2)^2*sin(d*x+c)^2/b^6/d+1/3*(a^2-b^2)^2*sin(d*x+c)^3/b^5/d-1/4
*a*(a^2-2*b^2)*sin(d*x+c)^4/b^4/d+1/5*(a^2-2*b^2)*sin(d*x+c)^5/b^3/d-1/6*a*
sin(d*x+c)^6/b^2/d+1/7*sin(d*x+c)^7/b/d
```

Rubi [A]

time = 0.15, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{a^2(a^2-b^2)^2\sin(c+dx)}{b^7d}-\frac{a(a^2-b^2)^2\sin^2(c+dx)}{2b^6d}+\frac{(a^2-b^2)^2\sin^3(c+dx)}{3b^5d}-\frac{a(a^2-2b^2)\sin^4(c+dx)}{4b^4d}+\frac{(a^2-2b^2)\sin^5(c+dx)}{5b^3d}-\frac{a^3(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^8d}-\frac{a\sin^6(c+dx)}{6b^2d}+\frac{\sin^7(c+dx)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^8*d)) + (a^2*(a^2 - b^2)^2
*Sin[c + d*x])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^6*d) + ((a^2
 - b^2)^2*Sin[c + d*x]^3)/(3*b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b
^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^5)/(5*b^3*d) - (a*Sin[c + d*x]^6)/(6*b^
2*d) + Sin[c + d*x]^7/(7*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{b^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{\text{Subst}\left(\int \frac{x^3(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^8 d}\\ &= \frac{\text{Subst}\left(\int \left((a^3-ab^2)^2 - a(a^2-b^2)^2 x + (a^2-b^2)^2 x^2 - a(a^2-2b^2)x^3 + (\right.\right.}{b^8 d}\\ &= -\frac{a^3(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^8 d} + \frac{a^2(a^2-b^2)^2\sin(c+dx)}{b^7 d} - \frac{a(a^2-}{}\end{aligned}$$

Mathematica [A]
time = 0.89, size = 180, normalized size = 0.85

$$\frac{-420a^3(a^2-b^2)^2\log(a+b\sin(c+dx)) + 420b(a^3-ab^2)^2\sin(c+dx) - 210ab^2(a^2-b^2)^2\sin^2(c+dx) + 140b^3(a^2-b^2)^2\sin^3(c+dx) - 105ab^4(a^2-2b^2)\sin^4(c+dx) + 84b^5(a^2-2b^2)\sin^5(c+dx) - 70ab^6\sin^6(c+dx) + 60b^7\sin^7(c+dx)}{420b^8 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-420*a^3*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 420*b*(a^3 - a*b^2)^2*Sin
[c + d*x] - 210*a*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 140*b^3*(a^2 - b^2)^2*
Sin[c + d*x]^3 - 105*a*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 + 84*b^5*(a^2 - 2*b
^2)*Sin[c + d*x]^5 - 70*a*b^6*Sin[c + d*x]^6 + 60*b^7*Sin[c + d*x]^7)/(420*
b^8*d)
```

Maple [A]
time = 0.35, size = 256, normalized size = 1.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^7*(1/7*sin(d*x+c)^7*b^6-1/6*a*sin(d*x+c)^6*b^5+1/5*a^2*b^4*sin(d*x
+c)^5-2/5*b^6*sin(d*x+c)^5-1/4*a^3*b^3*sin(d*x+c)^4+1/2*a*b^5*sin(d*x+c)^4+
1/3*a^4*b^2*sin(d*x+c)^3-2/3*a^2*b^4*sin(d*x+c)^3+1/3*b^6*sin(d*x+c)^3-1/2*
a^5*b*sin(d*x+c)^2+a^3*b^3*sin(d*x+c)^2-1/2*a*b^5*sin(d*x+c)^2+a^6*sin(d*x+
c)-2*a^4*b^2*sin(d*x+c)+a^2*b^4*sin(d*x+c))-a^3*(a^4-2*a^2*b^2+b^4)/b^8*ln(
a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 205, normalized size = 0.97

$$\frac{\frac{60\,b^6\sin(dx+c)^7-70\,ab^5\sin(dx+c)^6+84\,(a^2b^4-2\,b^6)\sin(dx+c)^5-105\,(a^3b^3-2\,ab^5)\sin(dx+c)^4+140\,(a^4b^2-2\,a^2b^4+b^6)\sin(dx+c)^3-210\,(a^5b-2\,a^3b^3+ab^5)\sin(dx+c)^2+420\,(a^6-2\,a^4b^2+a^2b^4)\sin(dx+c)}{b^7}-\frac{420\,(a^7-2\,a^5b^2+a^3b^4)\log(b\sin(dx+c)+a)}{b^8}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*(a^2*b^4 - 2*b
^6)*sin(d*x + c)^5 - 105*(a^3*b^3 - 2*a*b^5)*sin(d*x + c)^4 + 140*(a^4*b^2
- 2*a^2*b^4 + b^6)*sin(d*x + c)^3 - 210*(a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x
 + c)^2 + 420*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c))/b^7 - 420*(a^7 - 2*
a^5*b^2 + a^3*b^4)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A]
time = 0.43, size = 199, normalized size = 0.94

$$\frac{70\,ab^6\cos\left(dx+c\right)^6-105\,a^3b^4\cos\left(dx+c\right)^4+210\,(a^5b^2-a^3b^4)\cos\left(dx+c\right)^2-420\,(a^7-2\,a^5b^2+a^3b^4)\log\left(b\sin\left(dx+c\right)+a\right)-4\,\left(15\,b^7\cos\left(dx+c\right)^6-105\,a^6b+175\,a^4b^3-56\,a^2b^5-8\,b^7-3\,(7\,a^2b^5+b^7)\cos\left(dx+c\right)^4+(35\,a^4b^3-28\,a^2b^5-4\,b^7)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{420\,b^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(70*a*b^6*cos(d*x + c)^6 - 105*a^3*b^4*cos(d*x + c)^4 + 210*(a^5*b^2
- a^3*b^4)*cos(d*x + c)^2 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log(b*sin(d*x +
 c) + a) - 4*(15*b^7*cos(d*x + c)^6 - 105*a^6*b + 175*a^4*b^3 - 56*a^2*b^5
- 8*b^7 - 3*(7*a^2*b^5 + b^7)*cos(d*x + c)^4 + (35*a^4*b^3 - 28*a^2*b^5 - 4
*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^8*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 261, normalized size = 1.23

$$\frac{\frac{60\,b^6\sin(dx+c)^7-70\,ab^5\sin(dx+c)^6+84\,a^2b^4\sin(dx+c)^5-168\,b^6\sin(dx+c)^5-105\,a^3b^3\sin(dx+c)^4+210\,ab^5\sin(dx+c)^4+140\,a^4b^2\sin(dx+c)^3-280\,a^2b^4\sin(dx+c)^3+140\,b^6\sin(dx+c)^3-210\,a^5b\sin(dx+c)^2+420\,a^3b^3\sin(dx+c)^2-210\,ab^5\sin(dx+c)^2+420\,a^6\sin(dx+c)-840\,a^4b^2\sin(dx+c)+420\,a^2b^4\sin(dx+c)}{b^7}-\frac{420\,(a^7-2\,a^5b^2+a^3b^4)\log(|b\sin(dx+c)+a|)}{b^8}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*((60*b^6*sin(d*x + c)^7 - 70*a*b^5*sin(d*x + c)^6 + 84*a^2*b^4*sin(d*
x + c)^5 - 168*b^6*sin(d*x + c)^5 - 105*a^3*b^3*sin(d*x + c)^4 + 210*a*b^5*
sin(d*x + c)^4 + 140*a^4*b^2*sin(d*x + c)^3 - 280*a^2*b^4*sin(d*x + c)^3 +
140*b^6*sin(d*x + c)^3 - 210*a^5*b*sin(d*x + c)^2 + 420*a^3*b^3*sin(d*x + c
)^2 - 210*a*b^5*sin(d*x + c)^2 + 420*a^6*sin(d*x + c) - 840*a^4*b^2*sin(d*x
 + c) + 420*a^2*b^4*sin(d*x + c))/b^7 - 420*(a^7 - 2*a^5*b^2 + a^3*b^4)*log
(abs(b*sin(d*x + c) + a))/b^8)/d
```

Mupad [B]
time = 0.13, size = 236, normalized size = 1.11

$$-\frac{\sin\left(c+d\,x\right)^5\,\left(\frac{2}{5\,b}-\frac{a^2}{5\,b^3}\right)-\frac{{\sin\left(c+d\,x\right)}^7}{7\,b}-\sin\left(c+d\,x\right)^3\,\left(\frac{1}{3\,b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{3\,b^2}\right)+\frac{a\,{\sin\left(c+d\,x\right)}^6}{6\,b^2}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(a^7-2\,a^5\,b^2+a^3\,b^4\right)}{b^8}-\frac{a\,{\sin\left(c+d\,x\right)}^4\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{4\,b}+\frac{a\,{\sin\left(c+d\,x\right)}^2\,\left(\frac{1}{b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b^2}\right)}{2\,b}-\frac{a^2\,\sin\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b^2}\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^5*(2/(5*b) - a^2/(5*b^3)) - sin(c + d*x)^7/(7*b) - sin(c + d
*x)^3*(1/(3*b) - (a^2*(2/b - a^2/b^3))/(3*b^2)) + (a*sin(c + d*x)^6)/(6*b^2
) + (log(a + b*sin(c + d*x))*(a^7 + a^3*b^4 - 2*a^5*b^2))/b^8 - (a*sin(c +
d*x)^4*(2/b - a^2/b^3))/(4*b) + (a*sin(c + d*x)^2*(1/b - (a^2*(2/b - a^2/b^
3))/b^2))/(2*b) - (a^2*sin(c + d*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2))/b^2)
/d
```

3.1311 $\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{a^2(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7d}-\frac{a(a^2-b^2)^2\sin(c+dx)}{b^6d}+\frac{(a^2-b^2)^2\sin^2(c+dx)}{2b^5d}-\frac{a(a^2-2b^2)\sin^3(c+dx)}{3b^4d}$$

[Out] a^2*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^7/d-a*(a^2-b^2)^2*sin(d*x+c)/b^6/d+1/2*(a^2-b^2)^2*sin(d*x+c)^2/b^5/d-1/3*a*(a^2-2*b^2)*sin(d*x+c)^3/b^4/d+1/4*(a^2-2*b^2)*sin(d*x+c)^4/b^3/d-1/5*a*sin(d*x+c)^5/b^2/d+1/6*sin(d*x+c)^6/b/d

Rubi [A]

time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 962}

$$\frac{a^2(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7d}-\frac{a(a^2-b^2)^2\sin(c+dx)}{b^6d}+\frac{(a^2-b^2)^2\sin^2(c+dx)}{2b^5d}-\frac{a(a^2-2b^2)\sin^3(c+dx)}{3b^4d}+\frac{(a^2-2b^2)\sin^4(c+dx)}{4b^3d}-\frac{a\sin^5(c+dx)}{5b^2d}+\frac{\sin^6(c+dx)}{6bd}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]

[Out] (a^2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^7*d) - (a*(a^2 - b^2)^2*Sin[c + d*x])/(b^6*d) + ((a^2 - b^2)^2*Sin[c + d*x]^2)/(2*b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^3)/(3*b^4*d) + ((a^2 - 2*b^2)*Sin[c + d*x]^4)/(4*b^3*d) - (a*Sin[c + d*x]^5)/(5*b^2*d) + Sin[c + d*x]^6/(6*b*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{b^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(-a(a^2-b^2)^2 + (a^2-b^2)^2 x - a(a^2-2b^2)x^2 + (a^2-2b^2)x^3 - \right.\right.}{b^7 d}$$

$$= \frac{a^2(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^7 d} - \frac{a(a^2-b^2)^2\sin(c+dx)}{b^6 d} + \frac{(a^2-b^2)^2}{2}$$

Mathematica [A]
time = 0.51, size = 153, normalized size = 0.85

$$\frac{60(a^3-ab^2)^2\log(a+b\sin(c+dx)) - 60ab(a^2-b^2)^2\sin(c+dx) + 30b^2(a^2-b^2)^2\sin^2(c+dx) - 20ab^3(a^2-2b^2)\sin^3(c+dx) + 15b^4(a^2-2b^2)\sin^4(c+dx) - 12ab^5\sin^5(c+dx) + 10b^6\sin^6(c+dx)}{60b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*(a^3 - a*b^2)^2*Log[a + b*Sin[c + d*x]] - 60*a*b*(a^2 - b^2)^2*Sin[c +
d*x] + 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 - 20*a*b^3*(a^2 - 2*b^2)*Sin[c +
 d*x]^3 + 15*b^4*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 12*a*b^5*Sin[c + d*x]^5 + 1
0*b^6*Sin[c + d*x]^6)/(60*b^7*d)
```

Maple [A]
time = 0.33, size = 249, normalized size = 1.38 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^6*(-1/6*sin(d*x+c)^6*b^5+1/5*a*b^4*sin(d*x+c)^5+1/4*((-a^2*b-b*(a
^2-b^2))*b^2+2*a^2*b^3-b^3*(a^2-b^2))*sin(d*x+c)^4+1/3*(3*a*(a^2-b^2)*b^2+(
-a^2*b-b*(a^2-b^2))*a*b)*sin(d*x+c)^3+1/2*(a^2*(a^2-b^2)*b+(-a^2*b-b*(a^2-b
^2))*(a^2-b^2))*sin(d*x+c)^2+a*(a^2-b^2)^2*sin(d*x+c))+a^2*(a^4-2*a^2*b^2+b
^4)/b^7*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 172, normalized size = 0.96

$$\frac{\frac{10\,b^5\sin(dx+c)^6 - 12\,ab^4\sin(dx+c)^5 + 15\,(a^2b^3 - 2\,b^5)\sin(dx+c)^4 - 20\,(a^3b^2 - 2\,ab^4)\sin(dx+c)^3 + 30\,(a^4b - 2\,a^2b^3 + b^5)\sin(dx+c)^2 - 60\,(a^5 - 2\,a^3b^2 + ab^4)\sin(dx+c)}{b^6} + \frac{60\,(a^6 - 2\,a^4b^2 + a^2b^4)\log(b\sin(dx+c)+a)}{b^7}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*(a^2*b^3 - 2*b^
5)*sin(d*x + c)^4 - 20*(a^3*b^2 - 2*a*b^4)*sin(d*x + c)^3 + 30*(a^4*b - 2*a
^2*b^3 + b^5)*sin(d*x + c)^2 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/b
^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) + a)/b^7)/d
```

Fricas [**A**]
time = 0.38, size = 164, normalized size = 0.91

$$-\frac{10\,b^{6}\cos\left(dx+c\right)^{6}-15\,a^{2}b^{4}\cos\left(dx+c\right)^{4}+30\,\left(a^{4}b^{2}-a^{2}b^{4}\right)\cos\left(dx+c\right)^{2}-60\,\left(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)+4\,\left(3\,ab^{5}\cos\left(dx+c\right)^{4}+15\,a^{5}b-25\,a^{3}b^{3}+8\,ab^{5}-\left(5\,a^{3}b^{3}-4\,ab^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{60\,b^{7}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^6*cos(d*x + c)^6 - 15*a^2*b^4*cos(d*x + c)^4 + 30*(a^4*b^2 - a^
2*b^4)*cos(d*x + c)^2 - 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(b*sin(d*x + c) +
 a) + 4*(3*a*b^5*cos(d*x + c)^4 + 15*a^5*b - 25*a^3*b^3 + 8*a*b^5 - (5*a^3*
b^3 - 4*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^7*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 213, normalized size = 1.18

$$\frac{\frac{10\,b^{5}\sin(dx+c)^{6}-12\,ab^{4}\sin(dx+c)^{5}+15\,a^{2}b^{3}\sin(dx+c)^{4}-30\,b^{5}\sin(dx+c)^{4}-20\,a^{3}b^{2}\sin(dx+c)^{3}+40\,ab^{4}\sin(dx+c)^{3}+30\,a^{4}b\sin(dx+c)^{2}-60\,a^{2}b^{3}\sin(dx+c)^{2}+30\,b^{5}\sin(dx+c)^{2}-60\,a^{5}\sin(dx+c)+120\,a^{3}b^{2}\sin(dx+c)-60\,ab^{4}\sin(dx+c)}{b^{6}}+\frac{60\,\left(a^{6}-2\,a^{4}b^{2}+a^{2}b^{4}\right)\log(|b\sin(dx+c)+a|)}{b^{7}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*((10*b^5*sin(d*x + c)^6 - 12*a*b^4*sin(d*x + c)^5 + 15*a^2*b^3*sin(d*x
 + c)^4 - 30*b^5*sin(d*x + c)^4 - 20*a^3*b^2*sin(d*x + c)^3 + 40*a*b^4*sin(
d*x + c)^3 + 30*a^4*b*sin(d*x + c)^2 - 60*a^2*b^3*sin(d*x + c)^2 + 30*b^5*s
in(d*x + c)^2 - 60*a^5*sin(d*x + c) + 120*a^3*b^2*sin(d*x + c) - 60*a*b^4*s
in(d*x + c))/b^6 + 60*(a^6 - 2*a^4*b^2 + a^2*b^4)*log(abs(b*sin(d*x + c) +
a))/b^7)/d
```

Mupad [**B**]
time = 11.78, size = 191, normalized size = 1.06

$$\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{1}{2\,b}-\frac{a^2\,\left(\frac{1}{b}-\frac{a^2}{2\,b^3}\right)}{b^2}\right)-\sin\left(c+d\,x\right)^4\,\left(\frac{1}{2\,b}-\frac{a^2}{4\,b^3}\right)+\frac{\sin(c+d\,x)^6}{6\,b}-\frac{a\,\sin(c+d\,x)^5}{5\,b^2}+\frac{\ln(a+b\,\sin(c+d\,x))\,\left(a^6-2\,a^4\,b^2+a^2\,b^4\right)}{b^7}-\frac{a\,\sin(c+d\,x)\,\left(\frac{1}{b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b^2}\right)}{b}+\frac{a\,\sin(c+d\,x)^3\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{3\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*(1/(2*b) - (a^2*(1/b - a^2/(2*b^3)))/b^2) - sin(c + d*x)^4*
(1/(2*b) - a^2/(4*b^3)) + sin(c + d*x)^6/(6*b) - (a*sin(c + d*x)^5)/(5*b^2)
 + (log(a + b*sin(c + d*x))*(a^6 + a^2*b^4 - 2*a^4*b^2))/b^7 - (a*sin(c + d
*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2))/b + (a*sin(c + d*x)^3*(2/b - a^2/b^3
))/(3*b))/d
```

3.1312 $\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^6d}+\frac{(a^2-b^2)^2\sin(c+dx)}{b^5d}-\frac{a(a^2-2b^2)\sin^2(c+dx)}{2b^4d}+\frac{(a^2-2b^2)\sin^3(c+}{3b^3d}$$

```
[Out] -a*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/b^6/d+(a^2-b^2)^2*sin(d*x+c)/b^5/d-1/2*a*
(a^2-2*b^2)*sin(d*x+c)^2/b^4/d+1/3*(a^2-2*b^2)*sin(d*x+c)^3/b^3/d-1/4*a*sin
(d*x+c)^4/b^2/d+1/5*sin(d*x+c)^5/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 786}

$$-\frac{a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^6d}+\frac{(a^2-b^2)^2\sin(c+dx)}{b^5d}-\frac{a(a^2-2b^2)\sin^2(c+dx)}{2b^4d}+\frac{(a^2-2b^2)\sin^3(c+dx)}{3b^3d}-\frac{a\sin^4(c+dx)}{4b^2d}+\frac{\sin^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(b^6*d)) + ((a^2 - b^2)^2*Sin[c
 + d*x])/(b^5*d) - (a*(a^2 - 2*b^2)*Sin[c + d*x]^2)/(2*b^4*d) + ((a^2 - 2*b
^2)*Sin[c + d*x]^3)/(3*b^3*d) - (a*Sin[c + d*x]^4)/(4*b^2*d) + Sin[c + d*x]
^5/(5*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{b(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^6 d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2 - a(a^2-2b^2)\,x + (a^2-2b^2)\,x^2 - ax^3 + x^4 - \frac{a(a^2-b^2)^2}{a+x}\right.\right.}{b^6 d}$$
$$= -\frac{a(a^2-b^2)^2\log(a+b\sin(c+dx))}{b^6 d} + \frac{(a^2-b^2)^2\sin(c+dx)}{b^5 d} - \frac{a(a^2-2b^2)}{2b}$$

Mathematica [A]

time = 0.45, size = 128, normalized size = 0.86

$$\frac{-60a(a^2-b^2)^2\log(a+b\sin(c+dx)) + 60b(a^2-b^2)^2\sin(c+dx) - 30ab^2(a^2-2b^2)\sin^2(c+dx) + 20b^3(a^2-2b^2)\sin^3(c+dx) - 15ab^4\sin^4(c+dx) + 12b^5\sin^5(c+dx)}{60b^6 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-60*a*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]] + 60*b*(a^2 - b^2)^2*Sin[c + d
*x] - 30*a*b^2*(a^2 - 2*b^2)*Sin[c + d*x]^2 + 20*b^3*(a^2 - 2*b^2)*Sin[c +
d*x]^3 - 15*a*b^4*Sin[c + d*x]^4 + 12*b^5*Sin[c + d*x]^5)/(60*b^6*d)
```

Maple [A]

time = 0.25, size = 160, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{\frac{(\sin^5(dx+c))b^4}{5} - \frac{a\,b^3(\sin^4(dx+c))}{4} + \frac{a^2b^2(\sin^3(dx+c))}{3} - \frac{2b^4(\sin^3(dx+c))}{3} - \frac{a^3b(\sin^2(dx+c))}{2} + a\,b^3(\sin^2(dx+c)) + a^4\sin(dx+c) - 2}{b^5}}{d}$
default	$\frac{\frac{\frac{(\sin^5(dx+c))b^4}{5} - \frac{a\,b^3(\sin^4(dx+c))}{4} + \frac{a^2b^2(\sin^3(dx+c))}{3} - \frac{2b^4(\sin^3(dx+c))}{3} - \frac{a^3b(\sin^2(dx+c))}{2} + a\,b^3(\sin^2(dx+c)) + a^4\sin(dx+c) - 2}{b^5}}{d}$
risch	$\frac{7i e^{i(dx+c)}a^2}{8b^3d} - \frac{7i e^{-i(dx+c)}a^2}{8b^3d} - \frac{4ia^3c}{b^4d} + \frac{ia^5x}{b^6} + \frac{i e^{-i(dx+c)}a^4}{2b^5d} + \frac{iax}{b^2} + \frac{\sin(5dx+5c)}{80bd} - \frac{3a\,e^{-2i(dx+c)}}{16b^2d} + \frac{2a^3}{}$
norman	$\frac{\frac{2(a^4-2a^2b^2+b^4)\tan(\frac{dx}{2}+\frac{c}{2})}{b^5d} + \frac{2(a^4-2a^2b^2+b^4)(\tan^{11}(\frac{dx}{2}+\frac{c}{2}))}{b^5d} + \frac{2(15a^4-26a^2b^2+7b^4)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{3b^5d} + \frac{2(15a^4-26a^2b^2+7b}{3b^5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/b^5*(1/5*sin(d*x+c)^5*b^4-1/4*a*b^3*sin(d*x+c)^4+1/3*a^2*b^2*sin(d*x+c)^3-2/3*b^4*sin(d*x+c)^3-1/2*a^3*b*sin(d*x+c)^2+a*b^3*sin(d*x+c)^2+a^4*sin(d*x+c)-2*a^2*b^2*sin(d*x+c)+b^4*sin(d*x+c))-a*(a^4-2*a^2*b^2+b^4)/b^6*ln(a+b*sin(d*x+c)))

Maxima [A]
time = 0.30, size = 139, normalized size = 0.94

$$\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,(a^2b^2-2\,b^4)\sin(dx+c)^3-30\,(a^3b-2\,ab^3)\sin(dx+c)^2+60\,(a^4-2\,a^2b^2+b^4)\sin(dx+c)}{b^5}-\frac{60\,(a^5-2\,a^3b^2+ab^4)\log(b\sin(dx+c)+a)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*(a^2*b^2 - 2*b^4)*sin(d*x + c)^3 - 30*(a^3*b - 2*a*b^3)*sin(d*x + c)^2 + 60*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a)/b^6)/d

Fricas [A]
time = 0.38, size = 142, normalized size = 0.96

$$-\frac{15\,ab^4\cos\left(dx+c\right)^4-30\,(a^3b^2-ab^4)\cos\left(dx+c\right)^2+60\,(a^5-2\,a^3b^2+ab^4)\log\left(b\sin\left(dx+c\right)+a\right)-4\left(3\,b^5\cos\left(dx+c\right)^4+15\,a^4b-25\,a^2b^3+8\,b^5-(5\,a^2b^3-4\,b^5)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/60*(15*a*b^4*cos(d*x + c)^4 - 30*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(b*sin(d*x + c) + a) - 4*(3*b^5*cos(d*x + c)^4 + 15*a^4*b - 25*a^2*b^3 + 8*b^5 - (5*a^2*b^3 - 4*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.48, size = 165, normalized size = 1.11

$$\frac{\frac{12\,b^4\sin(dx+c)^5-15\,ab^3\sin(dx+c)^4+20\,a^2b^2\sin(dx+c)^3-40\,b^4\sin(dx+c)^3-30\,a^3b\sin(dx+c)^2+60\,ab^3\sin(dx+c)^2+60\,a^4\sin(dx+c)-120\,a^2b^2\sin(dx+c)+60\,b^4\sin(dx+c)}{b^5}-\frac{60\,(a^5-2\,a^3b^2+ab^4)\log(|b\sin(dx+c)+a|)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/60*((12*b^4*sin(d*x + c)^5 - 15*a*b^3*sin(d*x + c)^4 + 20*a^2*b^2*sin(d*x + c)^3 - 40*b^4*sin(d*x + c)^3 - 30*a^3*b*sin(d*x + c)^2 + 60*a*b^3*sin(d*x + c)^2 + 60*a^4*sin(d*x + c) - 120*a^2*b^2*sin(d*x + c) + 60*b^4*sin(d*x + c))/b^5 - 60*(a^5 - 2*a^3*b^2 + a*b^4)*log(abs(b*sin(d*x + c) + a))/b^6)/d

Mupad [B]
time = 0.07, size = 150, normalized size = 1.01

$$\frac{\sin\left(c+d\,x\right)\,\left(\frac{1}{b}-\frac{a^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{b^2}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{2}{3\,b}-\frac{a^2}{3\,b^3}\right)+\frac{{\sin\left(c+d\,x\right)}^5}{5\,b}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(a^5-2\,a^3\,b^2+a\,b^4\right)}{b^6}-\frac{a\,{\sin\left(c+d\,x\right)}^4}{4\,b^2}+\frac{a\,{\sin\left(c+d\,x\right)}^2\,\left(\frac{2}{b}-\frac{a^2}{b^3}\right)}{2\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^5*sin(c + d*x))/(a + b*sin(c + d*x)),x)

[Out] (sin(c + d*x)*(1/b - (a^2*(2/b - a^2/b^3))/b^2) - sin(c + d*x)^3*(2/(3*b) - a^2/(3*b^3)) + sin(c + d*x)^5/(5*b) - (log(a + b*sin(c + d*x))*(a*b^4 + a^5 - 2*a^3*b^2))/b^6 - (a*sin(c + d*x)^4)/(4*b^2) + (a*sin(c + d*x)^2*(2/b - a^2/b^3))/(2*b))/d

3.1313 $\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{\log(\sin(c+dx))}{ad}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\sin^3(c+dx)}{3bd}$$

```
[Out] ln(sin(d*x+c))/a/d-(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a/b^4/d+(a^2-2*b^2)*sin(d
*x+c)/b^3/d-1/2*a*sin(d*x+c)^2/b^2/d+1/3*sin(d*x+c)^3/b/d
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4d}+\frac{(a^2-2b^2)\sin(c+dx)}{b^3d}-\frac{a\sin^2(c+dx)}{2b^2d}+\frac{\log(\sin(c+dx))}{ad}+\frac{\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a*b^4*d)
 + ((a^2 - 2*b^2)*Sin[c + d*x])/(b^3*d) - (a*Sin[c + d*x]^2)/(2*b^2*d) + Si
n[c + d*x]^3/(3*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^4 d}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{ab^4 d} + \frac{(a^2-2b^2)\sin(c+dx}{b^3 d}$$

Mathematica [A]
time = 0.10, size = 101, normalized size = 0.94

$$\frac{6\left(b^4\log(\sin(c+dx))-(a^2-b^2)^2\log(a+b\sin(c+dx))\right)+6ab(a^2-2b^2)\sin(c+dx)-3a^2b^2\sin^2(c+dx)+2ab^3\sin^3(c+dx)}{6ab^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
[Out] (6*(b^4*Log[Sin[c + d*x]] - (a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]) + 6*a*b*
(a^2 - 2*b^2)*Sin[c + d*x] - 3*a^2*b^2*Sin[c + d*x]^2 + 2*a*b^3*Sin[c + d*x
]^3)/(6*a*b^4*d)
```

Maple [A]
time = 0.28, size = 105, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3}-\frac{ba\left(\sin^2(dx+c)\right)}{2}+a^2\sin(dx+c)-2b^2\sin(dx+c)}{b^3}+\frac{\ln(\sin(dx+c))}{a}+\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(a+b\sin(dx+c))}{a\,b^4}}{d}$
default	$\frac{\frac{\frac{\left(\sin^3(dx+c)\right)b^2}{3}-\frac{ba\left(\sin^2(dx+c)\right)}{2}+a^2\sin(dx+c)-2b^2\sin(dx+c)}{b^3}+\frac{\ln(\sin(dx+c))}{a}+\frac{\left(-a^4+2a^2b^2-b^4\right)\ln(a+b\sin(dx+c))}{a\,b^4}}{d}$
norman	$\frac{\frac{2\left(9a^2-14b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{2\left(9a^2-14b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}-\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}+\frac{2\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^3d}+\frac{2\left(a^2-2b^2\right)\left(\tan^7\right.}{b^3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$
risch	$-\frac{ie^{i(dx+c)}a^2}{2b^3d}-\frac{7ie^{-i(dx+c)}}{8bd}+\frac{a\,e^{2i(dx+c)}}{8b^2d}+\frac{ie^{-i(dx+c)}a^2}{2b^3d}+\frac{2ia^3c}{b^4d}-\frac{4iac}{b^2d}+\frac{ia^3x}{b^4}+\frac{a\,e^{-2i(dx+c)}}{8b^2d}-\frac{2iax}{b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(1/3*sin(d*x+c)^3*b^2-1/2*b*a*sin(d*x+c)^2+a^2*sin(d*x+c)-2*b^2*
sin(d*x+c))+1/a*ln(sin(d*x+c))+(-a^4+2*a^2*b^2-b^4)/a/b^4*ln(a+b*sin(d*x+c)
))
```

Maxima [A]
time = 0.28, size = 99, normalized size = 0.93

$$\frac{\frac{6\,\log\left(\sin\left(dx+c\right)\right)}{a}+\frac{2\,b^2\,\sin\left(dx+c\right)^3-3\,ab\sin\left(dx+c\right)^2+6\,\left(a^2-2\,b^2\right)\,\sin\left(dx+c\right)}{b^3}-\frac{6\,\left(a^4-2\,a^2b^2+b^4\right)\,\log\left(b\sin\left(dx+c\right)+a\right)}{ab^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(6*log(sin(d*x + c))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c)^2 +
 6*(a^2 - 2*b^2)*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*
x + c) + a)/(a*b^4))/d
```

Fricas [A]
time = 0.40, size = 104, normalized size = 0.97

$$\frac{3\,a^2b^2\cos\left(dx+c\right)^2+6\,b^4\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-6\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,\left(ab^3\cos\left(dx+c\right)^2-3\,a^3b+5\,ab^3\right)\sin\left(dx+c\right)}{6\,ab^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*b^2*cos(d*x + c)^2 + 6*b^4*log(-1/2*sin(d*x + c)) - 6*(a^4 - 2*a
^2*b^2 + b^4)*log(b*sin(d*x + c) + a) - 2*(a*b^3*cos(d*x + c)^2 - 3*a^3*b +
 5*a*b^3)*sin(d*x + c))/(a*b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 106, normalized size = 0.99

$$\frac{\frac{6\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}+\frac{2\,b^2\,\sin\left(dx+c\right)^3-3\,ab\sin\left(dx+c\right)^2+6\,a^2\,\sin\left(dx+c\right)-12\,b^2\,\sin\left(dx+c\right)}{b^3}-\frac{6\,\left(a^4-2\,a^2b^2+b^4\right)\,\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{ab^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*log(abs(sin(d*x + c)))/a + (2*b^2*sin(d*x + c)^3 - 3*a*b*sin(d*x + c
)^2 + 6*a^2*sin(d*x + c) - 12*b^2*sin(d*x + c))/b^3 - 6*(a^4 - 2*a^2*b^2 +
b^4)*log(abs(b*sin(d*x + c) + a))/(a*b^4))/d
```

Mupad [B]
time = 12.10, size = 254, normalized size = 2.37

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-2\,b^2\right)}{b^3}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(a^2-2\,b^2\right)}{b^3}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^2}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^2-4\,b^2\right)}{3\,b^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^2-2\,b^2\right)}{b^4\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-b^2\right)^2}{a\,b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((2*tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/b^3 +
 (2*tan(c/2 + (d*x)/2)^5*(a^2 - 2*b^2))/b^3 - (2*a*tan(c/2 + (d*x)/2)^2)/b^
2 - (2*a*tan(c/2 + (d*x)/2)^4)/b^2 + (4*tan(c/2 + (d*x)/2)^3*(3*a^2 - 4*b^2
))/(3*b^3))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 +
 (d*x)/2)^6 + 1)) + (a*log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 - 2*b^2))/(b^4*d)
 - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)
/(a*b^4*d)
```

3.1314 $\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a\sin(c+dx)}{b^2d}+\frac{\sin^2(c+dx)}{2bd}$$

```
[Out] -csc(d*x+c)/a/d-b*ln(sin(d*x+c))/a^2/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^2/b
^3/d-a*sin(d*x+c)/b^2/d+1/2*sin(d*x+c)^2/b/d
```

Rubi [A]
time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{a\sin(c+dx)}{b^2d}-\frac{\csc(c+dx)}{ad}+\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((a^2 - b^2)^2*Log[
a + b*Sin[c + d*x]])/(a^2*b^3*d) - (a*Sin[c + d*x])/(b^2*d) + Sin[c + d*x]^
2/(2*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2\left(b^2-x^2\right)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^2(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a+\frac{b^4}{ax^2}-\frac{b^4}{a^2x}+x+\frac{\left(a^2-b^2\right)^2}{a^2(a+x)}\right)\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= -\frac{\csc(c+dx)}{ad}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3d}-\frac{a}{}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.90

$$\frac{-\frac{2\csc(c+dx)}{a}-\frac{2b\log(\sin(c+dx))}{a^2}+\frac{2\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^2b^3}-\frac{2a\sin(c+dx)}{b^2}+\frac{\sin^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
[Out] ((-2*Csc[c + d*x])/a - (2*b*Log[Sin[c + d*x]])/a^2 + (2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^2*b^3) - (2*a*Sin[c + d*x])/b^2 + Sin[c + d*x]^2/b)/
(2*d)
```

Maple [A]
time = 0.28, size = 90, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\sin(dx+c))}{b^3a^2}}{d}$
default	$\frac{-\frac{1}{a\sin(dx+c)}-\frac{b\ln(\sin(dx+c))}{a^2}-\frac{-\frac{\left(\sin^2(dx+c)\right)b}{2}+a\sin(dx+c)}{b^2}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\sin(dx+c))}{b^3a^2}}{d}$
risch	$-\frac{ia^2x}{b^3}+\frac{2ix}{b}-\frac{e^{2i(dx+c)}}{8bd}+\frac{ia\,e^{i(dx+c)}}{2b^2d}-\frac{ia\,e^{-i(dx+c)}}{2b^2d}-\frac{e^{-2i(dx+c)}}{8bd}-\frac{2ia^2c}{b^3d}+\frac{4ic}{bd}-\frac{2ie^{i(dx+c)}}{da\left(e^{2i(dx+c)}-1\right)}+$
norman	$\frac{\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}-\frac{1}{2ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{2\left(a^2+b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,b^2d}-\frac{2\left(a^2+b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a\,b^2d}-\frac{\left(4a^2+3b^2\right)}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))-1/b^2*(-1/2*sin(d*x+c)^2*b+a*si
n(d*x+c))+1/b^3*(a^4-2*a^2*b^2+b^4)/a^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.95

$$-\frac{\frac{2\,b\log\left(\sin\left(dx+c\right)\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}+\frac{2}{a\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*log(sin(d*x + c))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/b^2
 + 2/(a*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(
a^2*b^3))/d
```

Fricas [A]
time = 0.40, size = 133, normalized size = 1.39

$$\frac{4\,a^3b\cos\left(dx+c\right)^2-4\,b^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-4\,a^3b-4\,ab^3+4\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-\left(2\,a^2b^2\cos\left(dx+c\right)^2-a^2b^2\right)\sin\left(dx+c\right)}{4\,a^2b^3d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*b*cos(d*x + c)^2 - 4*b^4*log(1/2*sin(d*x + c))*sin(d*x + c) - 4*
a^3*b - 4*a*b^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)*sin(d*x
 + c) - (2*a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))/(a^2*b^3*d*sin(d
*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 105, normalized size = 1.09

$$-\frac{\frac{2\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2}-\frac{b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)}{b^2}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{a^2\sin\left(dx+c\right)}-\frac{2\,\left(a^4-2\,a^2b^2+b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(sin(d*x + c)))/a^2 - (b*sin(d*x + c)^2 - 2*a*sin(d*x + c)
)/b^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)) - 2*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^2*b^3))/d
```

Mupad [B]
time = 12.03, size = 233, normalized size = 2.43

$$\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,\left(a^2-b^2\right)^2}{a^2\,b^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\,\left(a^2-2\,b^2\right)}{b^3\,d}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(2\,a^2+b^2\right)}{b^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,\left(4\,a^2+b^2\right)}{b^2}-\frac{4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{b}+1}{d\,\left(2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - b^2)^2)/(a
^2*b^3*d) - tan(c/2 + (d*x)/2)/(2*a*d) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^
2 - 2*b^2))/(b^3*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - ((2*tan(c/2 + (
d*x)/2)^2*(2*a^2 + b^2))/b^2 + (tan(c/2 + (d*x)/2)^4*(4*a^2 + b^2))/b^2 - (
4*a*tan(c/2 + (d*x)/2)^3)/b + 1)/(d*(2*a*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 +
 (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^5))
```

3.1315 $\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{a^3d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^3b^2d}+\frac{\sin(c+dx)}{bd}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(2*a^2-b^2)*ln(sin(d*x+c))/a^3/d-(a
^2-b^2)^2*ln(a+b*sin(d*x+c))/a^3/b^2/d+sin(d*x+c)/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{b\csc(c+dx)}{a^2d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^3b^2d}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{a^3d}-\frac{\csc^2(c+dx)}{2ad}+\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((2*a^2 - b^2)*Log[Sin[
c + d*x]])/(a^3*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^3*b^2*d) +
Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^3(b^2-x^2)^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^4}{ax^3}-\frac{b^4}{a^2x^2}+\frac{-2a^2b^2+b^4}{a^3x}-\frac{(a^2-b^2)^2}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{b\csc(c+dx)}{a^2 d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(2a^2-b^2)\log(\sin(c+dx))}{a^3 d}-\frac{(a^2-b^2)^2\,\mathrm{lo}}{}$$

Mathematica [A]
time = 0.27, size = 97, normalized size = 0.92

$$\frac{\frac{2b\csc(c+dx)}{a^2}-\frac{\csc^2(c+dx)}{a}+\frac{\frac{2b^2(-2a^2+b^2)\log(\sin(c+dx))-2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^3}+2b\sin(c+dx)}{b^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*b*Csc[c + d*x])/a^2 - Csc[c + d*x]^2/a + ((2*b^2*(-2*a^2 + b^2)*Log[Sin
[c + d*x]] - 2*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/a^3 + 2*b*Sin[c + d*x
])/b^2)/(2*d)
```

Maple [A]
time = 0.30, size = 99, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{(-2a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{\sin(dx+c)}{b}+\frac{(-a^4+2a^2b^2-b^4)\ln(a+b\sin(dx+c))}{a^3b^2}}{d}$
default	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{(-2a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{\sin(dx+c)}{b}+\frac{(-a^4+2a^2b^2-b^4)\ln(a+b\sin(dx+c))}{a^3b^2}}{d}$
risch	$\frac{iax}{b^2}-\frac{ie^{i(dx+c)}}{2bd}+\frac{ie^{-i(dx+c)}}{2bd}+\frac{2iac}{b^2d}+\frac{2i(-ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)})}{d\,a^2(e^{2i(dx+c)}-1)^2}-\frac{2\ln(e^{2i(dx+c)}-1)}{ad}+b^2\ln($
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^8(\frac{dx}{2}+\frac{c}{2})}{8ad}+\frac{\tan^4(\frac{dx}{2}+\frac{c}{2})}{4ad}+\frac{b\tan(\frac{dx}{2}+\frac{c}{2})}{2a^2d}+\frac{b(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{2a^2d}+\frac{(4a^2+3b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2a^2bd}+\frac{(4a^2+3b^2)(\tan^5(\frac{dx}{2}+\frac{c}{2}}{2a^2bd}}{(1+\tan^2(\frac{dx}{2}+\frac{c}{2}))^2\tan(\frac{dx}{2}+\frac{c}{2})^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2/a/sin(d*x+c)^2+1/a^3*(-2*a^2+b^2)*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)+1/b*sin(d*x+c)+(-a^4+2*a^2*b^2-b^4)/a^3/b^2*ln(a+b*sin(d*x+c)))

Maxima [A]
time = 0.31, size = 99, normalized size = 0.94

$$\frac{\frac{2\sin(dx+c)}{b}-\frac{2\,(2\,a^2-b^2)\log(\sin(dx+c))}{a^3}-\frac{2\,(a^4-2\,a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^3b^2}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(sin(d*x + c))/a^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/(a^3*b^2) + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d

Fricas [A]
time = 0.40, size = 174, normalized size = 1.66

$$\frac{a^2b^2+2\left(a^4-2\,a^2b^2+b^4-(a^4-2\,a^2b^2+b^4)\cos(dx+c)^2\right)\log(b\sin(dx+c)+a)+2\left(2\,a^2b^2-b^4-(2\,a^2b^2-b^4)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\sin(dx+c)\right)+2\left(a^3b\cos(dx+c)^2-a^3b-ab^3\right)\sin(dx+c)}{2\left(a^3b^2d\cos(dx+c)^2-a^3b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(a^2*b^2 + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 2*(2*a^2*b^2 - b^4 - (2*a^2*b^2 - b^4)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) + 2*(a^3*b*cos(d*x + c)^2 - a^3*b - a*b^3)*sin(d*x + c))/(a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)

[Out] Timed out

Giac [A]
time = 0.45, size = 130, normalized size = 1.24

$$\frac{\frac{2\sin(dx+c)}{b}-\frac{2\,(2\,a^2-b^2)\log(|\sin(dx+c)|)}{a^3}-\frac{2\,(a^4-2\,a^2b^2+b^4)\log(|b\sin(dx+c)+a|)}{a^3b^2}+\frac{6\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c)/b - 2*(2*a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^4
 - 2*a^2*b^2 + b^4)*log(abs(b*sin(d*x + c) + a))/(a^3*b^2) + (6*a^2*sin(d*x
 + c)^2 - 3*b^2*sin(d*x + c)^2 + 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c
)^2))/d
```

Mupad [B]
time = 11.98, size = 238, normalized size = 2.27

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(4\,a^2+b^2\right)}{b}}{d\,\left(4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{b^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{a^3\,d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a^3\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*tan(c/2 + (d*x)/2))/(2*a^2*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2) + (a*tan(c
/2 + (d*x)/2)^2)/2 - (2*tan(c/2 + (d*x)/2)^3*(4*a^2 + b^2))/b)/(d*(4*a^2*ta
n(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^4)) - tan(c/2 + (d*x)/2)^2/(8
*a*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) - (log(tan(c/2 + (d*x)/2)
)*(2*a^2 - b^2))/(a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d
*x)/2)^2)*(a^4 + b^4 - 2*a^2*b^2))/(a^3*b^2*d)
```

3.1316 $\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{(2a^2-b^2)\csc(c+dx)}{a^3d}+\frac{b\csc^2(c+dx)}{2a^2d}-\frac{\csc^3(c+dx)}{3ad}+\frac{b(2a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^4bd}$$

```
[Out] (2*a^2-b^2)*csc(d*x+c)/a^3/d+1/2*b*csc(d*x+c)^2/a^2/d-1/3*csc(d*x+c)^3/a/d+
b*(2*a^2-b^2)*ln(sin(d*x+c))/a^4/d+(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^4/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{b\csc^2(c+dx)}{2a^2d}+\frac{b(2a^2-b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^4bd}+\frac{(2a^2-b^2)\csc(c+dx)}{a^3d}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^2 - b^2)*Csc[c + d*x])/(a^3*d) + (b*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c
 + d*x]^3/(3*a*d) + (b*(2*a^2 - b^2)*Log[Sin[c + d*x]])/(a^4*d) + ((a^2 - b
^2)^2*Log[a + b*Sin[c + d*x]])/(a^4*b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^4(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^4(a+x)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^4} - \frac{b^4}{a^2x^3} + \frac{-2a^2b^2+b^4}{a^3x^2} + \frac{2a^2b^2-b^4}{a^4x} + \frac{(a^2-b^2)^2}{a^4(a+x)}\right)dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{(2a^2-b^2)\csc(c+dx)}{a^3d} + \frac{b\csc^2(c+dx)}{2a^2d} - \frac{\csc^3(c+dx)}{3ad} + \frac{b(2a^2-b^2)\log(\text{si}}{a^4d}$$

Mathematica [A]
time = 0.19, size = 110, normalized size = 0.92

$$\frac{6ab(2a^2-b^2)\csc(c+dx)+3a^2b^2\csc^2(c+dx)-2a^3b\csc^3(c+dx)-6b^2(-2a^2+b^2)\log(\sin(c+dx))+6(a^2-b^2)^2\log(a+b\sin(c+dx))}{6a^4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
[Out] (6*a*b*(2*a^2 - b^2)*Csc[c + d*x] + 3*a^2*b^2*Csc[c + d*x]^2 - 2*a^3*b*Csc[
c + d*x]^3 - 6*b^2*(-2*a^2 + b^2)*Log[Sin[c + d*x]] + 6*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(6*a^4*b*d)
```

Maple [A]
time = 0.31, size = 111, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{1}{3a\sin(dx+c)^3}-\frac{-2a^2+b^2}{a^3\sin(dx+c)}+\frac{(2a^2-b^2)b\ln(\sin(dx+c))}{a^4}+\frac{b}{2a^2\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{a^4b}}{d}$
default	$\frac{-\frac{1}{3a\sin(dx+c)^3}-\frac{-2a^2+b^2}{a^3\sin(dx+c)}+\frac{(2a^2-b^2)b\ln(\sin(dx+c))}{a^4}+\frac{b}{2a^2\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{a^4b}}{d}$
norman	$\frac{-\frac{1}{24ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{24ad}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^2d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8a^2d}+\frac{(5a^2-3b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6a^3d}+\frac{(5a^2-3b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6a^3d}+\frac{(7a^2-4b^2}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$
risch	$-\frac{ix}{b}-\frac{2ic}{bd}+\frac{2i\left(6a^2e^{5i(dx+c)}-3b^2e^{5i(dx+c)}-8e^{3i(dx+c)}a^2+6b^2e^{3i(dx+c)}+3iab\,e^{4i(dx+c)}+6a^2e^{i(dx+c)}-3b^2e^{i(dx+c)}-\right.}{3da^3\left(e^{2i(dx+c)}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(-1/3/a/sin(d*x+c)^3-(-2*a^2+b^2)/a^3/sin(d*x+c)+(2*a^2-b^2)/a^4*b*ln(s in(d*x+c))+1/2/a^2*b/sin(d*x+c)^2+(a^4-2*a^2*b^2+b^4)/a^4/b*ln(a+b*sin(d*x+ c)))

Maxima [A]
time = 0.28, size = 113, normalized size = 0.94

$$\frac{\frac{6\left(2a^2b-b^3\right)\log\left(\sin\left(dx+c\right)\right)}{a^4}+\frac{6\left(a^4-2a^2b^2+b^4\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^4b}+\frac{3ab\sin\left(dx+c\right)+6\left(2a^2-b^2\right)\sin\left(dx+c\right)^2-2a^2}{a^3\sin\left(dx+c\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/6*(6*(2*a^2*b - b^3)*log(sin(d*x + c))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*lo g(b*sin(d*x + c) + a)/(a^4*b) + (3*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*sin(d *x + c)^2 - 2*a^2)/(a^3*sin(d*x + c)^3))/d

Fricas [A]
time = 0.40, size = 198, normalized size = 1.65

$$-\frac{3a^2b^2\sin\left(dx+c\right)+10a^3b-6ab^3-6\left(2a^3b-ab^3\right)\cos\left(dx+c\right)^2+6\left(a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+6\left(2a^2b^2-b^4-\left(2a^2b^2-b^4\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(a^4bd\cos\left(dx+c\right)^2-a^4bd\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(3*a^2*b^2*sin(d*x + c) + 10*a^3*b - 6*a*b^3 - 6*(2*a^3*b - a*b^3)*cos (d*x + c)^2 + 6*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 6*(2*a^2*b^2 - b^4 - (2*a^2*b^ 2 - b^4)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c))/((a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep

Giac [A]
time = 0.48, size = 151, normalized size = 1.26

$$\frac{\frac{6\left(2a^2b-b^3\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^4}+\frac{6\left(a^4-2a^2b^2+b^4\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b}-\frac{22a^2b\sin\left(dx+c\right)^3-11b^3\sin\left(dx+c\right)^3-12a^3\sin\left(dx+c\right)^2+6ab^2\sin\left(dx+c\right)^2-3a^2b\sin\left(dx+c\right)+2a^3}{a^4\sin\left(dx+c\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(2*a^2*b - b^3)*log(abs(sin(d*x + c)))/a^4 + 6*(a^4 - 2*a^2*b^2 + b^
4)*log(abs(b*sin(d*x + c) + a))/(a^4*b) - (22*a^2*b*sin(d*x + c)^3 - 11*b^3
*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 6*a*b^2*sin(d*x + c)^2 - 3*a^2*b*
sin(d*x + c) + 2*a^3)/(a^4*sin(d*x + c)^3))/d
```

Mupad [B]
time = 11.93, size = 227, normalized size = 1.89

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7}{8\,a}-\frac{b^2}{2\,a^3}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{24\,a\,d}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2\,b-b^3\right)}{a^4\,d}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^2\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(7\,a^2-4\,b^2\right)-\frac{a^2}{3}+a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,a^3\,d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}+\frac{\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,{\left(a^2-b^2\right)}^2}{a^4\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(7/(8*a) - b^2/(2*a^3)))/d - tan(c/2 + (d*x)/2)^3/(24*a
*d) - log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) + (log(tan(c/2 + (d*x)/2))*(2*a^2
*b - b^3))/(a^4*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (tan(c/2 + (d*x)/
2)^2*(7*a^2 - 4*b^2) - a^2/3 + a*b*tan(c/2 + (d*x)/2))/(8*a^3*d*tan(c/2 + (
d*x)/2)^3) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2
 - b^2)^2)/(a^4*b*d)
```

3.1317 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}+\frac{b\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a^2-b^2)^2\log(\sin(c+dx$$

```
[Out] -b*(2*a^2-b^2)*csc(d*x+c)/a^4/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/a^3/d+1/3*b*cs
c(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a/d+(a^2-b^2)^2*ln(sin(d*x+c))/a^5/d-(a^2
-b^2)^2*ln(a+b*sin(d*x+c))/a^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2800, 908}

$$\frac{b\csc^3(c+dx)}{3a^2d}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5d}-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5} - \frac{b^4}{a^2x^4} + \frac{-2a^2b^2+b^4}{a^3x^3} + \frac{2a^2b^2-b^4}{a^4x^2} + \frac{(a^2-b^2)^2}{a^5x} - \frac{(a^2-b^2)^2}{a^5(a+x)}\right) dx, x, b\sin(c+d\right.}{d}$$

$$= -\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d} + \frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d} + \frac{b\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx}{4ad}$$

Mathematica [A]

time = 0.68, size = 115, normalized size = 0.78

$$\frac{12ab(-2a^2+b^2)\csc(c+dx)+6a^2(2a^2-b^2)\csc^2(c+dx)+4a^3b\csc^3(c+dx)-3a^4\csc^4(c+dx)+12(a^2-b^2)^2(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

Maple [A]

time = 0.35, size = 137, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(dx+c)^3}-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx}{a^5}}{d}$
default	$\frac{-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(dx+c)^3}-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx}{a^5}}{d}$
norman	$\frac{-\frac{1}{64ad}-\frac{\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{24a^2d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24a^2d}+\frac{(3a^2-2b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3d}+\frac{(3a^2-2b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{16a^3d}-\frac{b(7a^2-4b}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}$
risch	$\frac{2i\left(6ia^3e^{6i(dx+c)}-3iab^2e^{6i(dx+c)}-6a^2be^{7i(dx+c)}+3b^3e^{7i(dx+c)}-6ia^3e^{4i(dx+c)}+6iab^2e^{4i(dx+c)}+14a^2be^{5i(dx+c)}-9b^3e^5\right.}{3da^4\left(e^{2i(dx+c)}-1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/a/sin(d*x+c)^4-1/2*(-2*a^2+b^2)/a^3/sin(d*x+c)^2+(a^4-2*a^2*b^2+b
^4)/a^5*ln(sin(d*x+c))-(2*a^2-b^2)/a^4*b/sin(d*x+c)+1/3/a^2*b/sin(d*x+c)^3-
(a^4-2*a^2*b^2+b^4)/a^5*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.29, size = 139, normalized size = 0.94

$$-\frac{\frac{12\,(a^4-2\,a^2b^2+b^4)\log(b\sin(dx+c)+a)}{a^5}-\frac{12\,(a^4-2\,a^2b^2+b^4)\log(\sin(dx+c))}{a^5}-\frac{4\,a^2b\sin(dx+c)-12\,(2\,a^2b-b^3)\sin(dx+c)^3-3\,a^3+6\,(2\,a^3-ab^2)\sin(dx+c)^2}{a^4\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

Fricas [A]
time = 0.39, size = 271, normalized size = 1.83

$$\frac{9\,a^4-6\,a^2b^2-6\,(2\,a^4-a^2b^2)\cos(dx+c)^2-12\,\left((a^4-2\,a^2b^2+b^4)\cos(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\cos(dx+c)^2\right)\log(b\sin(dx+c)+a)+12\,\left((a^4-2\,a^2b^2+b^4)\cos(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\cos(dx+c)^2\right)\log\left(-\frac{1}{2}\sin(dx+c)\right)-4\,\left(5\,a^3b-3\,ab^3-3\,(2\,a^3b-ab^3)\cos(dx+c)^2\right)\sin(dx+c)}{12\,\left(a^5d\cos(dx+c)^4-2\,a^5d\cos(dx+c)^2+a^5d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.51, size = 201, normalized size = 1.36

$$\frac{\frac{12\,(a^4-2\,a^2b^2+b^4)\log(|\sin(dx+c)|)}{a^5}-\frac{12\,(a^4b-2\,a^2b^3+b^5)\log(|b\sin(dx+c)+a|)}{a^5b}-\frac{25\,a^4\sin(dx+c)^4-50\,a^2b^2\sin(dx+c)^4+25\,b^4\sin(dx+c)^4+24\,a^3b\sin(dx+c)^3-12\,ab^3\sin(dx+c)^3-12\,a^4\sin(dx+c)^2+6\,a^2b^2\sin(dx+c)^2-4\,a^3b\sin(dx+c)+3\,a^4}{a^5\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
 c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

Mupad [B]
time = 11.83, size = 281, normalized size = 1.90

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3}{16\,a}-\frac{b^2}{8\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,a\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b}{8\,a^2}+\frac{2\,b\,\left(\frac{3}{8\,a}-\frac{b^2}{4\,a^3}\right)}{a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a\,b^2-3\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(14\,a^2\,b-8\,b^3\right)+\frac{a^3}{4}-\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{16\,a^4\,d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}-\frac{\ln\left(a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a^5\,d}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(3/(16*a) - b^2/(8*a^3)))/d - tan(c/2 + (d*x)/2)^4/(6
4*a*d) - (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(3/(8*a) - b^2/(4*a^3)))/a))
/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^3*(14*a^2
*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (
d*x)/2)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4
 + b^4 - 2*a^2*b^2))/(a^5*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d)
```

3.1318 $\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=179

$$-\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d}-\frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d}+\frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d}+\frac{b\csc^4(c+dx)}{4a^2d}-\frac{\csc^5(c+dx)}{5ad}-$$

```
[Out] -(a^2-b^2)^2*csc(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*csc(d*x+c)^2/a^4/d+1/3*(2*a
^2-b^2)*csc(d*x+c)^3/a^3/d+1/4*b*csc(d*x+c)^4/a^2/d-1/5*csc(d*x+c)^5/a/d-b*
(a^2-b^2)^2*ln(sin(d*x+c))/a^6/d+b*(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^6/d
```

Rubi [A]

time = 0.14, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{b\csc^4(c+dx)}{4a^2d}-\frac{b(a^2-b^2)^2\log(\sin(c+dx))}{a^6d}+\frac{b(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^6d}-\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d}-\frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d}+\frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^6(b^2-x^2)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{b\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^6(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{b^4}{ax^6} - \frac{b^4}{a^2x^5} + \frac{-2a^2b^2+b^4}{a^3x^4} + \frac{2a^2b^2-b^4}{a^4x^3} + \frac{(a^2-b^2)^2}{a^5x^2} - \frac{(a^2-b^2)^2}{a^6x} + \frac{(a^2-b^2)^2}{a^6(a+x)}\right.\right.}{d}$$

$$= -\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d} - \frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d} + \frac{(2a^2-b^2)\csc^3(c+d}{3a^3d}$$

Mathematica [A]

time = 6.08, size = 179, normalized size = 1.00

$$-\frac{(a^2-b^2)^2\csc(c+dx)}{a^5d} - \frac{b(2a^2-b^2)\csc^2(c+dx)}{2a^4d} + \frac{(2a^2-b^2)\csc^3(c+dx)}{3a^3d} + \frac{b\csc^4(c+dx)}{4a^2d} - \frac{\csc^5(c+dx)}{5ad} - \frac{b(a^2-b^2)^2\log(\sin(c+dx))}{a^6d} + \frac{b(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Csc[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^3*d) + (b*Csc[c + d*x]^4)/(
4*a^2*d) - Csc[c + d*x]^5/(5*a*d) - (b*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^
6*d) + (b*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/(a^6*d)
```

Maple [A]

time = 0.38, size = 167, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{5a\sin(dx+c)^5} - \frac{-2a^2+b^2}{3a^3\sin(dx+c)^3} - \frac{a^4-2a^2b^2+b^4}{a^5\sin(dx+c)} - \frac{(2a^2-b^2)b}{2a^4\sin(dx+c)^2} + \frac{b}{4a^2\sin(dx+c)^4} - \frac{(a^4-2a^2b^2+b^4)b\ln(\sin(dx+c))}{a^6} + \frac{(a^4-2a^2b}{}}{d}$
default	$\frac{-\frac{1}{5a\sin(dx+c)^5} - \frac{-2a^2+b^2}{3a^3\sin(dx+c)^3} - \frac{a^4-2a^2b^2+b^4}{a^5\sin(dx+c)} - \frac{(2a^2-b^2)b}{2a^4\sin(dx+c)^2} + \frac{b}{4a^2\sin(dx+c)^4} - \frac{(a^4-2a^2b^2+b^4)b\ln(\sin(dx+c))}{a^6} + \frac{(a^4-2a^2b}{}}{d}$
norman	$\frac{-\frac{1}{160ad} - \frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{160ad} + \frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64a^2d} + \frac{b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{64a^2d} + \frac{(5a^2-4b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96a^3d} + \frac{(5a^2-4b^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{96a^3d} - \frac{(5a^4-}{}}{\tan\left(\frac{dx}{2}\right.}$
risch	$-\frac{2i(15a^4e^{9i(dx+c)} - 30a^2b^2e^{9i(dx+c)} + 15b^4e^{9i(dx+c)} - 20a^4e^{7i(dx+c)} + 100a^2b^2e^{7i(dx+c)} - 60b^4e^{7i(dx+c)} - 30ia^3b\,e^{2i(dx+c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/a/sin(d*x+c)^5-1/3*(-2*a^2+b^2)/a^3/sin(d*x+c)^3-(a^4-2*a^2*b^2+b
^4)/a^5/sin(d*x+c)-1/2*(2*a^2-b^2)/a^4*b/sin(d*x+c)^2+1/4/a^2*b/sin(d*x+c)^
4-(a^4-2*a^2*b^2+b^4)/a^6*b*ln(sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/a^6*b*ln(a+b
*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 170, normalized size = 0.95

$$\frac{\frac{60\,(a^4b-2\,a^2b^3+b^5)\log(b\sin(dx+c)+a)}{a^6}-\frac{60\,(a^4b-2\,a^2b^3+b^5)\log(\sin(dx+c))}{a^6}+\frac{15\,a^3b\sin(dx+c)-60\,(a^4-2\,a^2b^2+b^4)\sin(dx+c)^4-12\,a^4-30\,(2\,a^3b-ab^3)\sin(dx+c)^3+20\,(2\,a^4-a^2b^2)\sin(dx+c)^2}{a^5\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(b*sin(d*x + c) + a)/a^6 - 60*(a^4*b
- 2*a^2*b^3 + b^5)*log(sin(d*x + c))/a^6 + (15*a^3*b*sin(d*x + c) - 60*(a^4
 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 - 12*a^4 - 30*(2*a^3*b - a*b^3)*sin(d*x
+ c)^3 + 20*(2*a^4 - a^2*b^2)*sin(d*x + c)^2)/(a^5*sin(d*x + c)^5))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(171) = 342.
time = 0.40, size = 346, normalized size = 1.93

$$-\frac{32\,a^5-100\,a^3b^2+60\,ab^4+60\,(a^5-2\,a^3b^2+ab^4)\cos(dx+c)^4-20\,(4\,a^5-11\,a^3b^2+6\,ab^4)\cos(dx+c)^2-60\,\left(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)\cos(dx+c)^4-2\,(a^4b-2\,a^2b^3+b^5)\cos(dx+c)^2\right)\log(b\sin(dx+c)+a)\sin(dx+c)+60\,\left(a^4b-2\,a^2b^3+b^5+(a^4b-2\,a^2b^3+b^5)\cos(dx+c)^4-2\,(a^4b-2\,a^2b^3+b^5)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\sin(dx+c)\right)\sin(dx+c)+15\,\left(3\,a^4b-2\,a^2b^3-2\,(2\,a^4b-a^2b^3)\cos(dx+c)^2\right)\sin(dx+c)}{60\,\left(a^6d\cos(dx+c)^4-2\,a^6d\cos(dx+c)^2+a^6d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(32*a^5 - 100*a^3*b^2 + 60*a*b^4 + 60*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d
*x + c)^4 - 20*(4*a^5 - 11*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2 - 60*(a^4*b -
2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b - 2*a
^2*b^3 + b^5)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)*sin(d*x + c) + 60*(a^
4*b - 2*a^2*b^3 + b^5 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 - 2*(a^4*b
 - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))*sin(d*x + c) + 15
*(3*a^4*b - 2*a^2*b^3 - 2*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [A]
time = 0.49, size = 251, normalized size = 1.40

$$-\frac{\frac{60\,(a^4b-2\,a^2b^3+b^5)\log(|\sin(dx+c)|)}{a^6}-\frac{60\,(a^4b^2-2\,a^2b^4+b^6)\log(|b\sin(dx+c)+a|)}{a^6b}-\frac{137\,a^4b\sin(dx+c)^5-274\,a^2b^3\sin(dx+c)^5+137\,b^5\sin(dx+c)^5-60\,a^5\sin(dx+c)^4+120\,a^3b^2\sin(dx+c)^4-60\,ab^4\sin(dx+c)^4-60\,a^4b\sin(dx+c)^3+30\,a^2b^3\sin(dx+c)^3+40\,a^5\sin(dx+c)^2-20\,a^3b^2\sin(dx+c)^2+15\,a^4b\sin(dx+c)-12\,a^5}{a^6\sin(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(sin(d*x + c)))/a^6 - 60*(a^4*b^
2 - 2*a^2*b^4 + b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b) - (137*a^4*b*sin(
d*x + c)^5 - 274*a^2*b^3*sin(d*x + c)^5 + 137*b^5*sin(d*x + c)^5 - 60*a^5*s
in(d*x + c)^4 + 120*a^3*b^2*sin(d*x + c)^4 - 60*a*b^4*sin(d*x + c)^4 - 60*a
^4*b*sin(d*x + c)^3 + 30*a^2*b^3*sin(d*x + c)^3 + 40*a^5*sin(d*x + c)^2 - 2
0*a^3*b^2*sin(d*x + c)^2 + 15*a^4*b*sin(d*x + c) - 12*a^5)/(a^6*sin(d*x + c
)^5))/d
```

Mupad [B]
time = 11.92, size = 381, normalized size = 2.13

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{b^2}{8\,a^3}-\frac{5}{16\,a}+\frac{2\,b\,\left(\frac{b}{16\,a^2}+\frac{2\,b\,\left(\frac{5}{32\,a}-\frac{b^2}{8\,a^3}\right)}{a}\right)}{a}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{b}{32\,a^2}+\frac{b\,\left(\frac{5}{32\,a}-\frac{b^2}{8\,a^3}\right)}{a}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{160\,a\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{5}{96\,a}-\frac{b^2}{24\,a^3}\right)}{d}+\frac{\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^4\,b-2\,a^2\,b^3+b^5\right)}{a^6\,d}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{64\,a^2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4\,b-2\,a^2\,b^3+b^5\right)}{a^6\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{5\,a^4}{3}-\frac{4\,a^2\,b^2}{3}\right)-\frac{a^4}{5}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(10\,a^4-28\,a^2\,b^2+16\,b^4\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(4\,a\,b^3-6\,a^3\,b\right)+\frac{a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{32\,a^5\,d\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 5/(16*a) + (2*b*(b/(16*a^2) + (2*b*(5/(3
2*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(5/
(32*a) - b^2/(8*a^3)))/a))/d - tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 +
(d*x)/2)^3*(5/(96*a) - b^2/(24*a^3)))/d + (log(a + 2*b*tan(c/2 + (d*x)/2) +
 a*tan(c/2 + (d*x)/2)^2)*(a^4*b + b^5 - 2*a^2*b^3))/(a^6*d) + (b*tan(c/2 +
(d*x)/2)^4)/(64*a^2*d) - (log(tan(c/2 + (d*x)/2))*(a^4*b + b^5 - 2*a^2*b^3)
)/(a^6*d) + (tan(c/2 + (d*x)/2)^2*((5*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan
(c/2 + (d*x)/2)^4*(10*a^4 + 16*b^4 - 28*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*
a*b^3 - 6*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/
2)^5)
```

3.1319 $\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=212

$$\frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d}-\frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d}-\frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d}+\frac{(2a^2-b^2)\csc^4(c+dx)}{4a^3d}+\frac{b\csc^5(\ldots}{5\ldots}$$

```
[Out] b*(a^2-b^2)^2*csc(d*x+c)/a^6/d-1/2*(a^2-b^2)^2*csc(d*x+c)^2/a^5/d-1/3*b*(2*
a^2-b^2)*csc(d*x+c)^3/a^4/d+1/4*(2*a^2-b^2)*csc(d*x+c)^4/a^3/d+1/5*b*csc(d*
x+c)^5/a^2/d-1/6*csc(d*x+c)^6/a/d+b^2*(a^2-b^2)^2*ln(sin(d*x+c))/a^7/d-b^2*
(a^2-b^2)^2*ln(a+b*sin(d*x+c))/a^7/d
```

Rubi [A]

time = 0.16, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{b\csc^5(c+dx)}{5a^2d}+\frac{b^2(a^2-b^2)^2\log(\sin(c+dx))}{a^7d}-\frac{b^2(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^7d}+\frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d}-\frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d}-\frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d}+\frac{(2a^2-b^2)\csc^4(c+dx)}{4a^3d}-\frac{\csc^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^2*Csc[c + d*x])/(a^6*d) - ((a^2 - b^2)^2*Csc[c + d*x]^2)/(2*
a^5*d) - (b*(2*a^2 - b^2)*Csc[c + d*x]^3)/(3*a^4*d) + ((2*a^2 - b^2)*Csc[c
+ d*x]^4)/(4*a^3*d) + (b*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(6*a*d)
 + (b^2*(a^2 - b^2)^2*Log[Sin[c + d*x]])/(a^7*d) - (b^2*(a^2 - b^2)^2*Log[a
 + b*Sin[c + d*x]])/(a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{b^7(b^2-x^2)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{b^5 d}\\ &= \frac{b^2\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x^7(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b^2\text{Subst}\left(\int \left(\frac{b^4}{ax^7}-\frac{b^4}{a^2x^6}+\frac{-2a^2b^2+b^4}{a^3x^5}+\frac{2a^2b^2-b^4}{a^4x^4}+\frac{(a^2-b^2)^2}{a^5x^3}-\frac{(a^2-b^2)^2}{a^6x^2}+\frac{(a^2-b^2)^2}{a^7x}\right.\right.}{d}\\ &= \frac{b(a^2-b^2)^2\csc(c+dx)}{a^6d}-\frac{(a^2-b^2)^2\csc^2(c+dx)}{2a^5d}-\frac{b(2a^2-b^2)\csc^3(c+dx)}{3a^4d}\end{aligned}$$

Mathematica [A]
time = 2.58, size = 165, normalized size = 0.78

$$\frac{60ab(a^2-b^2)^2\csc(c+dx)-30a^2(a^2-b^2)^2\csc^2(c+dx)+20a^3b(-2a^2+b^2)\csc^3(c+dx)+15a^4(2a^2-b^2)\csc^4(c+dx)+12a^5b\csc^5(c+dx)-10a^6\csc^6(c+dx)+60(-a^2b+b^3)^2(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))}{60a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^5*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (60*a*b*(a^2 - b^2)^2*Csc[c + d*x] - 30*a^2*(a^2 - b^2)^2*Csc[c + d*x]^2 +
20*a^3*b*(-2*a^2 + b^2)*Csc[c + d*x]^3 + 15*a^4*(2*a^2 - b^2)*Csc[c + d*x]^
4 + 12*a^5*b*Csc[c + d*x]^5 - 10*a^6*Csc[c + d*x]^6 + 60*(-(a^2*b) + b^3)^2
*(Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(60*a^7*d)
```

Maple [A]
time = 0.50, size = 199, normalized size = 0.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/6/a/sin(d*x+c)^6-1/4*(-2*a^2+b^2)/a^3/sin(d*x+c)^4-1/2*(a^4-2*a^2*b
^2+b^4)/a^5/sin(d*x+c)^2-1/3*(2*a^2-b^2)/a^4*b/sin(d*x+c)^3+(a^4-2*a^2*b^2+
b^4)/a^7*b^2*ln(sin(d*x+c))+1/5/a^2*b/sin(d*x+c)^5+(a^4-2*a^2*b^2+b^4)/a^6*
b/sin(d*x+c)-(a^4-2*a^2*b^2+b^4)/a^7*b^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 206, normalized size = 0.97

$$-\frac{\frac{60\,(a^4b^2-2\,a^2b^4+b^6)\log(b\sin(dx+c)+a)}{a^7}-\frac{60\,(a^4b^2-2\,a^2b^4+b^6)\log(\sin(dx+c))}{a^7}-\frac{12\,a^4b\sin(dx+c)+60\,(a^4b-2\,a^2b^3+b^5)\sin(dx+c)^5-10\,a^5-30\,(a^5-2\,a^3b^2+ab^4)\sin(dx+c)^4-20\,(2\,a^4b-a^2b^3)\sin(dx+c)^3+15\,(2\,a^5-a^3b^2)\sin(dx+c)^2}{a^6\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(b*sin(d*x + c) + a)/a^7 - 60*(a^4
*b^2 - 2*a^2*b^4 + b^6)*log(sin(d*x + c))/a^7 - (12*a^4*b*sin(d*x + c) + 60
*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^5 - 10*a^5 - 30*(a^5 - 2*a^3*b^2 +
a*b^4)*sin(d*x + c)^4 - 20*(2*a^4*b - a^2*b^3)*sin(d*x + c)^3 + 15*(2*a^5 -
 a^3*b^2)*sin(d*x + c)^2)/(a^6*sin(d*x + c)^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(202) = 404$.
time = 0.40, size = 464, normalized size = 2.19

$$\frac{10 a^6 - 45 a^4 b^2 + 30 a^2 b^4 + 30 (a^6 - 2 a^4 b^2 + a^2 b^4) \cos(dx+c)^4 - 15 (2 a^6 - 7 a^4 b^2 + 4 a^2 b^4) \cos(dx+c)^2 - 60 \left((a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^6 - a^4 b^2 + 2 a^2 b^4 - b^6 - 3 (a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^4 + 3 (a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^2\right) \log(b \sin(dx+c) + a) + 60 \left((a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^6 - a^4 b^2 + 2 a^2 b^4 - b^6 - 3 (a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^4 + 3 (a^4 b^2 - 2 a^2 b^4 + b^6) \cos(dx+c)^2\right) \log\left(-\frac{1}{2} \sin(dx+c)\right) - 4 \left(8 a^5 b - 25 a^3 b^3 + 15 a b^5 + 15 (a^5 b - 2 a^3 b^3 + a b^5) \cos(dx+c)^4 - 5 (4 a^5 b - 11 a^3 b^3 + 6 a b^5) \cos(dx+c)^2\right) \sin(dx+c)}{60 \left(a^7 d \cos(dx+c)^6 - 3 a^7 d \cos(dx+c)^4 + 3 a^7 d \cos(dx+c)^2 - a^7 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^6 - 45*a^4*b^2 + 30*a^2*b^4 + 30*(a^6 - 2*a^4*b^2 + a^2*b^4)*cos
(d*x + c)^4 - 15*(2*a^6 - 7*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^2 - 60*((a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*
b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d
*x + c)^2)*log(b*sin(d*x + c) + a) + 60*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*
x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*log(-1/2*sin(d*x +
 c)) - 4*(8*a^5*b - 25*a^3*b^3 + 15*a*b^5 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c)^4 - 5*(4*a^5*b - 11*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c
)^2 - a^7*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.49, size = 301, normalized size = 1.42

$$\frac{\frac{60 \left(a^4 b^2 - 2 a^2 b^4 + b^6\right) \log\left(|\sin(dx+c)|\right)}{a^7} - \frac{60 \left(a^4 b^3 - 2 a^2 b^5 + b^7\right) \log\left(|b \sin(dx+c) + a|\right)}{a^7 b} - \frac{147 a^4 b^2 \sin(dx+c)^6 - 294 a^2 b^4 \sin(dx+c)^6 + 147 b^6 \sin(dx+c)^6 - 60 a^5 b \sin(dx+c)^5 + 120 a^3 b^3 \sin(dx+c)^5 - 60 a b^5 \sin(dx+c)^5 + 30 a^6 \sin(dx+c)^4 - 60 a^4 b^2 \sin(dx+c)^4 + 30 a^2 b^4 \sin(dx+c)^4 + 40 a^5 b \sin(dx+c)^3 - 20 a^3 b^3 \sin(dx+c)^3 - 30 a^6 \sin(dx+c)^2 + 15 a^4 b^2 \sin(dx+c)^2 - 12 a^5 b \sin(dx+c) + 10 a^6}{a^7 \sin(dx+c)^6}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^4*b^2 - 2*a^2*b^4 + b^6)*log(abs(sin(d*x + c)))/a^7 - 60*(a^4*b
^3 - 2*a^2*b^5 + b^7)*log(abs(b*sin(d*x + c) + a))/(a^7*b) - (147*a^4*b^2*s
in(d*x + c)^6 - 294*a^2*b^4*sin(d*x + c)^6 + 147*b^6*sin(d*x + c)^6 - 60*a^
5*b*sin(d*x + c)^5 + 120*a^3*b^3*sin(d*x + c)^5 - 60*a*b^5*sin(d*x + c)^5 +
 30*a^6*sin(d*x + c)^4 - 60*a^4*b^2*sin(d*x + c)^4 + 30*a^2*b^4*sin(d*x + c
)^4 + 40*a^5*b*sin(d*x + c)^3 - 20*a^3*b^3*sin(d*x + c)^3 - 30*a^6*sin(d*x
+ c)^2 + 15*a^4*b^2*sin(d*x + c)^2 - 12*a^5*b*sin(d*x + c) + 10*a^6)/(a^7*s
in(d*x + c)^6))/d
```

Mupad [B]
time = 12.31, size = 514, normalized size = 2.42

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{1}{64a}-\frac{b^2}{64a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{b}{96a^2}+\frac{2b\left(\frac{1}{16a}-\frac{b^2}{16a^3}\right)}{3a}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384ad}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{b}{32a^2}-\frac{2b\left(\frac{b^2}{16a^3}-\frac{5}{64a}+\frac{2b\left(\frac{b}{32a^2}+\frac{2b\left(\frac{1}{16a}-\frac{b^2}{16a^3}\right)}{a}\right)}{a}\right)}{a}+\frac{2b\left(\frac{1}{16a}-\frac{b^2}{16a^3}\right)}{a}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{b^2}{32a^3}-\frac{5}{128a}+\frac{b\left(\frac{b}{32a^2}+\frac{2b\left(\frac{1}{16a}-\frac{b^2}{16a^3}\right)}{a}\right)}{a}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^4b^2-2a^2b^4+b^6\right)}{a^7d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160a^2d}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\left(a^4b^2-2a^2b^4+b^6\right)}{a^7d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{10a^4b}{3}-\frac{8a^2b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{5a^5}{2}-12a^3b^2+8ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(20a^4b-56a^2b^3+32b^5\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(a^5-a^3b^2\right)+\frac{a^5}{6}-\frac{2a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}}{64a^6d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(sin(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(1/(64*a) - b^2/(64*a^3)))/d - (tan(c/2 + (d*x)/2)^3*
(b/(96*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/(3*a)))/d - tan(c/2 + (d*x)/2
)^6/(384*a*d) + (tan(c/2 + (d*x)/2)*(b/(32*a^2) - (2*b*(b^2/(16*a^3) - 5/(6
4*a) + (2*b*(b/(32*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/a))/a))/a + (2*b*
(1/(16*a) - b^2/(16*a^3)))/a))/d + (tan(c/2 + (d*x)/2)^2*(b^2/(32*a^3) - 5/
(128*a) + (b*(b/(32*a^2) + (2*b*(1/(16*a) - b^2/(16*a^3)))/a))/a))/d + (log
(tan(c/2 + (d*x)/2))*(b^6 - 2*a^2*b^4 + a^4*b^2))/(a^7*d) + (b*tan(c/2 + (d
*x)/2)^5)/(160*a^2*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)
/2)^2)*(b^6 - 2*a^2*b^4 + a^4*b^2))/(a^7*d) - (tan(c/2 + (d*x)/2)^3*((10*a^
4*b)/3 - (8*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*(8*a*b^4 + (5*a^5)/2 - 12*a^
3*b^2) - tan(c/2 + (d*x)/2)^5*(20*a^4*b + 32*b^5 - 56*a^2*b^3) - tan(c/2 +
(d*x)/2)^2*(a^5 - a^3*b^2) + a^5/6 - (2*a^4*b*tan(c/2 + (d*x)/2))/5)/(64*a^
6*d*tan(c/2 + (d*x)/2)^6)
```

3.1320 $\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=467

$$-\frac{(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8)\,x}{128b^9}+\frac{2a^3(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{a(105a^6-245a$$

```
[Out] -1/128*(128*a^8-320*a^6*b^2+240*a^4*b^4-40*a^2*b^6-5*b^8)*x/b^9+2*a^3*(a^2-
b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^9/d-1/105*a*(
105*a^6-245*a^4*b^2+161*a^2*b^4-15*b^6)*cos(d*x+c)/b^8/d+1/128*(64*a^6-144*
a^4*b^2+88*a^2*b^4-5*b^6)*cos(d*x+c)*sin(d*x+c)/b^7/d-1/105*a*(35*a^4-77*a^
2*b^2+45*b^4)*cos(d*x+c)*sin(d*x+c)^2/b^6/d+1/192*(48*a^4-104*a^2*b^2+59*b^
4)*cos(d*x+c)*sin(d*x+c)^3/b^5/d+1/4*cos(d*x+c)*sin(d*x+c)^4/a/d-1/140*(28*
a^4-60*a^2*b^2+35*b^4)*cos(d*x+c)*sin(d*x+c)^4/a/b^4/d-1/5*b*cos(d*x+c)*sin
(d*x+c)^5/a^2/d+1/240*(40*a^4-85*a^2*b^2+48*b^4)*cos(d*x+c)*sin(d*x+c)^5/a^
2/b^3/d-1/7*a*cos(d*x+c)*sin(d*x+c)^6/b^2/d+1/8*cos(d*x+c)*sin(d*x+c)^7/b/d
```

Rubi [A]
time = 1.17, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{b\sin^5(c+dx)\cos(c+dx)}{5a^2d}-\frac{(28a^4-60a^2b^2+35b^4)\sin^4(c+dx)\cos(c+dx)}{140ab^4d}-\frac{a(35a^4-77a^2b^2+45b^4)\sin^2(c+dx)\cos(c+dx)}{105b^6d}+\frac{(48a^4-104a^2b^2+59b^4)\sin^3(c+dx)\cos(c+dx)}{192b^5d}+\frac{(40a^4-85a^2b^2+48b^4)\sin^5(c+dx)\cos(c+dx)}{240a^2b^3d}+\frac{2a^3(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^9d}-\frac{a(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos(c+dx)}{105b^8d}+\frac{(64a^6-144a^4b^2+88a^2b^4-5b^6)\sin(c+dx)\cos(c+dx)}{128b^7d}-\frac{x(128a^8-320a^6b^2+240a^4b^4-40a^2b^6-5b^8)}{128b^9}-\frac{a\sin^6(c+dx)\cos(c+dx)}{7b^2d}+\frac{\sin^4(c+dx)\cos(c+dx)}{4ad}+\frac{\sin^7(c+dx)\cos(c+dx)}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/128*((128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*x)/b^9 +
 (2*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(b^9*d) - (a*(105*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/
(105*b^8*d) + ((64*a^6 - 144*a^4*b^2 + 88*a^2*b^4 - 5*b^6)*Cos[c + d*x]*Sin
[c + d*x])/(128*b^7*d) - (a*(35*a^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin
[c + d*x]^2)/(105*b^6*d) + ((48*a^4 - 104*a^2*b^2 + 59*b^4)*Cos[c + d*x]*Si
n[c + d*x]^3)/(192*b^5*d) + (Cos[c + d*x]*Sin[c + d*x]^4)/(4*a*d) - ((28*a^
4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a*b^4*d) - (b*Co
s[c + d*x]*Sin[c + d*x]^5)/(5*a^2*d) + ((40*a^4 - 85*a^2*b^2 + 48*b^4)*Cos[
c + d*x]*Sin[c + d*x]^5)/(240*a^2*b^3*d) - (a*Cos[c + d*x]*Sin[c + d*x]^6)/
(7*b^2*d) + (Cos[c + d*x]*Sin[c + d*x]^7)/(8*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&& !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] && !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} - \frac{a\cos(c+dx)\sin^6(}{7b^2d}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{b\cos(c+dx)\sin^5(c+dx)}{5a^2d} + \frac{(40a^4 - 85a^2b^2 +}{}$$

$$= \frac{\cos(c+dx)\sin^4(c+dx)}{4ad} - \frac{(28a^4 - 60a^2b^2 + 35b^4)\cos(c+dx)\sin^4(c+dx}{140ab^4d}$$

$$= \frac{(48a^4 - 104a^2b^2 + 59b^4)\cos(c+dx)\sin^3(c+dx)}{192b^5d} + \frac{\cos(c+dx)\sin^4(c+d}{4ad}$$

$$= -\frac{a(35a^4 - 77a^2b^2 + 45b^4)\cos(c+dx)\sin^2(c+dx)}{105b^6d} + \frac{(48a^4 - 104a^2b^2 + 5}{}$$

$$= \frac{(64a^6 - 144a^4b^2 + 88a^2b^4 - 5b^6)\cos(c+dx)\sin(c+dx)}{128b^7d} - \frac{a(35a^4 - 77a^2b}{}$$

$$= -\frac{a(105a^6 - 245a^4b^2 + 161a^2b^4 - 15b^6)\cos(c+dx)}{105b^8d} + \frac{(64a^6 - 144a^4b^2 + 8}{}$$

$$= -\frac{(128a^8 - 320a^6b^2 + 240a^4b^4 - 40a^2b^6 - 5b^8)\,x}{128b^9} - \frac{a(105a^6 - 245a^4b^2 + 1}{10}$$

$$= -\frac{(128a^8 - 320a^6b^2 + 240a^4b^4 - 40a^2b^6 - 5b^8)\,x}{128b^9} - \frac{a(105a^6 - 245a^4b^2 + 1}{10}$$

$$= -\frac{(128a^8 - 320a^6b^2 + 240a^4b^4 - 40a^2b^6 - 5b^8)\,x}{128b^9} - \frac{a(105a^6 - 245a^4b^2 + 1}{10}$$

$$= -\frac{(128a^8 - 320a^6b^2 + 240a^4b^4 - 40a^2b^6 - 5b^8)\,x}{128b^9} + \frac{2a^3(a^2-b^2)^{5/2}\tan^{-1}\Big(}{b^9d}$$

Mathematica [A]
time = 2.20, size = 403, normalized size = 0.86

$$\frac{-107520a^8c + 268800a^6b^2c - 201600a^4b^4c + 33600a^2b^6c + 4200b^8c - 107520a^8dx + 268800a^6b^2dx - 201600a^4b^4dx + 33600a^2b^6dx + 4200b^8dx + 215040a^3(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - 1680ab(64a^6 - 144a^4b^2 + 88a^2b^4 - 5b^6)\cos(c+dx) + 560(16a^5b^3 - 28a^3b^5 + 9ab^7)\cos(3(c+dx)) - 1344a^3b^5\cos(5(c+dx)) + 1680ab^7\cos(5(c+dx)) + 240ab^7\cos(7(c+dx)) + 26880a^6b^2\sin(2(c+dx)) - 53760a^4b^4\sin(2(c+dx)) + 25200a^2b^6\sin(2(c+dx)) + 1680b^8\sin(2(c+dx)) - 3360a^4b^4\sin(4(c+dx)) + 5040a^2b^6\sin(4(c+dx)) - 840b^8\sin(4(c+dx)) + 560a^2b^6\sin(6(c+dx)) - 560b^8\sin(6(c+dx)) - 105b^8\sin(8(c+dx))}{107520b^9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-107520*a^8*c + 268800*a^6*b^2*c - 201600*a^4*b^4*c + 33600*a^2*b^6*c + 42
00*b^8*c - 107520*a^8*d*x + 268800*a^6*b^2*d*x - 201600*a^4*b^4*d*x + 33600
*a^2*b^6*d*x + 4200*b^8*d*x + 215040*a^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 1680*a*b*(64*a^6 - 144*a^4*b^2 + 88*a^2*
```

```
b^4 - 5*b^6)*Cos[c + d*x] + 560*(16*a^5*b^3 - 28*a^3*b^5 + 9*a*b^7)*Cos[3*(
c + d*x)] - 1344*a^3*b^5*Cos[5*(c + d*x)] + 1680*a*b^7*Cos[5*(c + d*x)] + 2
40*a*b^7*Cos[7*(c + d*x)] + 26880*a^6*b^2*Sin[2*(c + d*x)] - 53760*a^4*b^4*
Sin[2*(c + d*x)] + 25200*a^2*b^6*Sin[2*(c + d*x)] + 1680*b^8*Sin[2*(c + d*x
)] - 3360*a^4*b^4*Sin[4*(c + d*x)] + 5040*a^2*b^6*Sin[4*(c + d*x)] - 840*b^
8*Sin[4*(c + d*x)] + 560*a^2*b^6*Sin[6*(c + d*x)] - 560*b^8*Sin[6*(c + d*x)
] - 105*b^8*Sin[8*(c + d*x)])/(107520*b^9*d)
```

Maple [A]
time = 0.29, size = 797, normalized size = 1.71 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^9*((a^7*b-7/3*a^5*b^3+23/15*a^3*b^5-1/7*a*b^7+(-1/2*a^6*b^2+9/8*a
^4*b^4-11/16*a^2*b^6+5/128*b^8)*tan(1/2*d*x+1/2*c)+(7*a^7*b-47/3*a^5*b^3+13
9/15*a^3*b^5-1/7*a*b^7)*tan(1/2*d*x+1/2*c)^2+(-5/2*a^6*b^2+37/8*a^4*b^4-61/
48*a^2*b^6-397/384*b^8)*tan(1/2*d*x+1/2*c)^3+(21*a^7*b-139/3*a^5*b^3+419/15
*a^3*b^5-3*a*b^7)*tan(1/2*d*x+1/2*c)^4+(-9/2*a^6*b^2+57/8*a^4*b^4-113/48*a^
2*b^6+895/384*b^8)*tan(1/2*d*x+1/2*c)^5+(35*a^7*b-235/3*a^5*b^3+743/15*a^3*
b^5-3*a*b^7)*tan(1/2*d*x+1/2*c)^6+(-5/2*a^6*b^2+29/8*a^4*b^4-85/48*a^2*b^6-
1765/384*b^8)*tan(1/2*d*x+1/2*c)^7+(35*a^7*b-245/3*a^5*b^3+161/3*a^3*b^5-5*
a*b^7)*tan(1/2*d*x+1/2*c)^8+(5/2*a^6*b^2-29/8*a^4*b^4+85/48*a^2*b^6+1765/38
4*b^8)*tan(1/2*d*x+1/2*c)^9+(21*a^7*b-157/3*a^5*b^3+109/3*a^3*b^5-5*a*b^7)*
tan(1/2*d*x+1/2*c)^10+(9/2*a^6*b^2-57/8*a^4*b^4+113/48*a^2*b^6-895/384*b^8)
*tan(1/2*d*x+1/2*c)^11+(7*a^7*b-19*a^5*b^3+15*a^3*b^5-a*b^7)*tan(1/2*d*x+1/
2*c)^12+(5/2*a^6*b^2-37/8*a^4*b^4+61/48*a^2*b^6+397/384*b^8)*tan(1/2*d*x+1/
2*c)^13+(a^7*b-3*a^5*b^3+3*a^3*b^5-a*b^7)*tan(1/2*d*x+1/2*c)^14+(1/2*a^6*b^
2-9/8*a^4*b^4+11/16*a^2*b^6-5/128*b^8)*tan(1/2*d*x+1/2*c)^15)/(1+tan(1/2*d*
x+1/2*c)^2)^8+1/128*(128*a^8-320*a^6*b^2+240*a^4*b^4-40*a^2*b^6-5*b^8)*arct
an(tan(1/2*d*x+1/2*c)))+2/b^9*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*a^3/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 706, normalized size = 1.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480*(a
^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4
 - 40*a^2*b^6 - 5*b^8)*d*x + 6720*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b
^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 13440*(a^7*b - 2*a^5*b^3 +
a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8*(8*a^2*b^6 + b^8)*cos
(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos(d*x + c)^3 - 3*(64*a^
6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c))*sin(d*x + c))/(b^9*
d), 1/13440*(1920*a*b^7*cos(d*x + c)^7 - 2688*a^3*b^5*cos(d*x + c)^5 + 4480
*(a^5*b^3 - a^3*b^5)*cos(d*x + c)^3 - 105*(128*a^8 - 320*a^6*b^2 + 240*a^4*
b^4 - 40*a^2*b^6 - 5*b^8)*d*x - 13440*(a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2
- b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 13440
*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c) - 35*(48*b^8*cos(d*x + c)^7 - 8
*(8*a^2*b^6 + b^8)*cos(d*x + c)^5 + 2*(48*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*cos
(d*x + c)^3 - 3*(64*a^6*b^2 - 112*a^4*b^4 + 40*a^2*b^6 + 5*b^8)*cos(d*x + c
))*sin(d*x + c))/(b^9*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. 2(440) = 880.
time = 0.49, size = 1244, normalized size = 2.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/13440*(105*(128*a^8 - 320*a^6*b^2 + 240*a^4*b^4 - 40*a^2*b^6 - 5*b^8)*(d
*x + c)/b^9 - 26880*(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*b^9) + 2*(6720*a^6*b*tan(1/2*d*x + 1/2*c)^15 - 1512
0*a^4*b^3*tan(1/2*d*x + 1/2*c)^15 + 9240*a^2*b^5*tan(1/2*d*x + 1/2*c)^15 -
525*b^7*tan(1/2*d*x + 1/2*c)^15 + 13440*a^7*tan(1/2*d*x + 1/2*c)^14 - 40320
*a^5*b^2*tan(1/2*d*x + 1/2*c)^14 + 40320*a^3*b^4*tan(1/2*d*x + 1/2*c)^14 -
13440*a*b^6*tan(1/2*d*x + 1/2*c)^14 + 33600*a^6*b*tan(1/2*d*x + 1/2*c)^13 -
 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^13 + 17080*a^2*b^5*tan(1/2*d*x + 1/2*c)
^13 + 13895*b^7*tan(1/2*d*x + 1/2*c)^13 + 94080*a^7*tan(1/2*d*x + 1/2*c)^12
 - 255360*a^5*b^2*tan(1/2*d*x + 1/2*c)^12 + 201600*a^3*b^4*tan(1/2*d*x + 1/
2*c)^12 - 13440*a*b^6*tan(1/2*d*x + 1/2*c)^12 + 60480*a^6*b*tan(1/2*d*x + 1
/2*c)^11 - 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^11 + 31640*a^2*b^5*tan(1/2*d*
x + 1/2*c)^11 - 31325*b^7*tan(1/2*d*x + 1/2*c)^11 + 282240*a^7*tan(1/2*d*x
+ 1/2*c)^10 - 703360*a^5*b^2*tan(1/2*d*x + 1/2*c)^10 + 488320*a^3*b^4*tan(1
/2*d*x + 1/2*c)^10 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^10 + 33600*a^6*b*tan(
1/2*d*x + 1/2*c)^9 - 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^9 + 23800*a^2*b^5*t
an(1/2*d*x + 1/2*c)^9 + 61775*b^7*tan(1/2*d*x + 1/2*c)^9 + 470400*a^7*tan(1
/2*d*x + 1/2*c)^8 - 1097600*a^5*b^2*tan(1/2*d*x + 1/2*c)^8 + 721280*a^3*b^4
*tan(1/2*d*x + 1/2*c)^8 - 67200*a*b^6*tan(1/2*d*x + 1/2*c)^8 - 33600*a^6*b*
tan(1/2*d*x + 1/2*c)^7 + 48720*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 23800*a^2*b
^5*tan(1/2*d*x + 1/2*c)^7 - 61775*b^7*tan(1/2*d*x + 1/2*c)^7 + 470400*a^7*t
an(1/2*d*x + 1/2*c)^6 - 1052800*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 665728*a^3
*b^4*tan(1/2*d*x + 1/2*c)^6 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^6 - 60480*a^
6*b*tan(1/2*d*x + 1/2*c)^5 + 95760*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 31640*a
^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 31325*b^7*tan(1/2*d*x + 1/2*c)^5 + 282240*a
^7*tan(1/2*d*x + 1/2*c)^4 - 622720*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 375424*
a^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 40320*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 33600
*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 62160*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 1708
0*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 13895*b^7*tan(1/2*d*x + 1/2*c)^3 + 94080
*a^7*tan(1/2*d*x + 1/2*c)^2 - 210560*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 12454
4*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 - 1920*a*b^6*tan(1/2*d*x + 1/2*c)^2 - 6720
*a^6*b*tan(1/2*d*x + 1/2*c) + 15120*a^4*b^3*tan(1/2*d*x + 1/2*c) - 9240*a^2
*b^5*tan(1/2*d*x + 1/2*c) + 525*b^7*tan(1/2*d*x + 1/2*c) + 13440*a^7 - 3136
0*a^5*b^2 + 20608*a^3*b^4 - 1920*a*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*b^8
))/d
```

Mupad [B]
time = 15.08, size = 2500, normalized size = 5.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^3)/(a + b*sin(c + d*x)),x)
```

[Out] ((2*(15*a*b^6 - 105*a^7 - 161*a^3*b^4 + 245*a^5*b^2))/(105*b^8) + (2*tan(c/2 + (d*x)/2)^14*(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2))/b^8 + (2*tan(c/2 + (d*x)/2)^12*(a*b^6 - 7*a^7 - 15*a^3*b^4 + 19*a^5*b^2))/b^8 + (2*tan(c/2 + (d*x)/2)^10*(15*a*b^6 - 63*a^7 - 109*a^3*b^4 + 157*a^5*b^2))/(3*b^8) + (2*tan(c/2 + (d*x)/2)^8*(15*a*b^6 - 105*a^7 - 161*a^3*b^4 + 245*a^5*b^2))/(3*b^8) + (2*tan(c/2 + (d*x)/2)^4*(45*a*b^6 - 315*a^7 - 419*a^3*b^4 + 695*a^5*b^2))/(15*b^8) + (2*tan(c/2 + (d*x)/2)^6*(45*a*b^6 - 525*a^7 - 743*a^3*b^4 + 1175*a^5*b^2))/(15*b^8) + (2*tan(c/2 + (d*x)/2)^2*(15*a*b^6 - 735*a^7 - 973*a^3*b^4 + 1645*a^5*b^2))/(105*b^8) + (tan(c/2 + (d*x)/2)*(64*a^6 - 5*b^6 + 88*a^2*b^4 - 144*a^4*b^2))/(64*b^7) - (tan(c/2 + (d*x)/2)^15*(64*a^6 - 5*b^6 + 88*a^2*b^4 - 144*a^4*b^2))/(64*b^7) + (tan(c/2 + (d*x)/2)^3*(960*a^6 + 397*b^6 + 488*a^2*b^4 - 1776*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^13*(960*a^6 + 397*b^6 + 488*a^2*b^4 - 1776*a^4*b^2))/(192*b^7) + (tan(c/2 + (d*x)/2)^7*(960*a^6 + 1765*b^6 + 680*a^2*b^4 - 1392*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^9*(960*a^6 + 1765*b^6 + 680*a^2*b^4 - 1392*a^4*b^2))/(192*b^7) + (tan(c/2 + (d*x)/2)^5*(1728*a^6 - 895*b^6 + 904*a^2*b^4 - 2736*a^4*b^2))/(192*b^7) - (tan(c/2 + (d*x)/2)^11*(1728*a^6 - 895*b^6 + 904*a^2*b^4 - 2736*a^4*b^2))/(192*b^7))/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) - (atan((((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 - ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i)*1i)/(128*b^9) + ((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 + ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 819200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i)*1i)/(128*b^9))/((32*a^25 - (25*a^5*b^20)/256 + (315*a^7*b^18)/256 + (3205*a^9*b^16)/256 - (39415*a^11*b^14)/256 + (10135*a^13*b^12)

/16 - (11217*a^15*b^10)/8 + (3773*a^17*b^8)/2 - (3195*a^19*b^6)/2 + 836*a^2
1*b^4 - 248*a^23*b^2)/b^23 + (tan(c/2 + (d*x)/2)*(32768*a^26 - 50*a^4*b^22
- 650*a^6*b^20 + 3850*a^8*b^18 + 24850*a^10*b^16 - 254240*a^12*b^14 + 91360
0*a^14*b^12 - 1834240*a^16*b^10 + 2293760*a^18*b^8 - 1835008*a^20*b^6 + 917
504*a^22*b^4 - 262144*a^24*b^2))/(256*b^24) + ((((25*a^2*b^24)/512 + (25*a^
4*b^22)/32 - (25*a^6*b^20)/16 - (125*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12
*b^14 + 320*a^14*b^12 - 160*a^16*b^10 + 32*a^18*b^8)/b^23 - ((((5*a*b^26)/4
 + (35*a^3*b^24)/4 - 38*a^5*b^22 + 44*a^7*b^20 - 16*a^9*b^18)/b^23 - ((32*a
^2*b^3 + (tan(c/2 + (d*x)/2)*(49152*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(
b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(128*b^9) +
 (tan(c/2 + (d*x)/2)*(32768*a^4*b^24 - 98304*a^6*b^22 + 98304*a^8*b^20 - 32
768*a^10*b^18))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i
 + a^6*b^2*320i))/(128*b^9) + (tan(c/2 + (d*x)/2)*(50*a*b^26 + 775*a^3*b^24
 - 2000*a^5*b^22 - 47584*a^7*b^20 + 278144*a^9*b^18 - 655360*a^11*b^16 + 81
9200*a^13*b^14 - 573440*a^15*b^12 + 212992*a^17*b^10 - 32768*a^19*b^8))/(51
2*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a^4*b^4*240i + a^6*b^2*320i))/(
128*b^9) - ((((25*a^2*b^24)/512 + (25*a^4*b^22)/32 - (25*a^6*b^20)/16 - (12
5*a^8*b^18)/4 + 160*a^10*b^16 - 320*a^12*b^14 + 320*a^14*b^12 - 160*a^16*b^
10 + 32*a^18*b^8)/b^23 + ((((5*a*b^26)/4 + (35*a^3*b^24)/4 - 38*a^5*b^22 +
44*a^7*b^20 - 16*a^9*b^18)/b^23 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(49152
*a*b^28 - 32768*a^3*b^26))/(512*b^24))*(b^8*5i - a^8*128i + a^2*b^6*40i - a
^4*b^4*240i + a^6*b^2*320i))/(128*b^9) + (tan(c...

3.1321 $\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=408

$$\frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^8}-\frac{2a^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^8d}+\frac{(105a^6-245a^4b^2+161a^2b^4-}{105b^7d}$$

```
[Out] 1/16*a*(16*a^6-40*a^4*b^2+30*a^2*b^4-5*b^6)*x/b^8-2*a^2*(a^2-b^2)^(5/2)*arc
tan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^8/d+1/105*(105*a^6-245*a^4*
b^2+161*a^2*b^4-15*b^6)*cos(d*x+c)/b^7/d-1/16*a*(8*a^4-18*a^2*b^2+11*b^4)*c
os(d*x+c)*sin(d*x+c)/b^6/d+1/105*(35*a^4-77*a^2*b^2+45*b^4)*cos(d*x+c)*sin(
d*x+c)^2/b^5/d+1/3*cos(d*x+c)*sin(d*x+c)^3/a/d-1/24*(6*a^4-13*a^2*b^2+8*b^4
)*cos(d*x+c)*sin(d*x+c)^3/a/b^4/d-1/4*b*cos(d*x+c)*sin(d*x+c)^4/a^2/d+1/140
*(28*a^4-60*a^2*b^2+35*b^4)*cos(d*x+c)*sin(d*x+c)^4/a^2/b^3/d-1/6*a*cos(d*x
+c)*sin(d*x+c)^5/b^2/d+1/7*cos(d*x+c)*sin(d*x+c)^6/b/d
```

Rubi [A]
time = 0.93, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3128, 3102, 2814, 2739, 632, 210}

$$-\frac{2a^2(a^2-b^2)^{5/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^8d}-\frac{b\sin^4(c+dx)\cos(c+dx)}{4a^2d}-\frac{(6a^4-13a^2b^2+8b^4)\sin^3(c+dx)\cos(c+dx)}{24ab^4d}-\frac{a(8a^4-18a^2b^2+11b^4)\sin(c+dx)\cos(c+dx)}{16b^6d}+\frac{(35a^4-77a^2b^2+45b^4)\sin^2(c+dx)\cos(c+dx)}{105b^5d}+\frac{(28a^4-60a^2b^2+35b^4)\sin^4(c+dx)\cos(c+dx)}{140a^2b^3d}+\frac{ax(16a^6-40a^4b^2+30a^2b^4-5b^6)}{16b^8}+\frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos(c+dx)}{105b^7d}-\frac{a\sin^5(c+dx)\cos(c+dx)}{6b^2d}+\frac{\sin^3(c+dx)\cos(c+dx)}{3ad}+\frac{\sin^6(c+dx)\cos(c+dx)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/(16*b^8) - (2*a^2*(a^2 - b
^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^8*d) + ((105
*a^6 - 245*a^4*b^2 + 161*a^2*b^4 - 15*b^6)*Cos[c + d*x])/(105*b^7*d) - (a*(
8*a^4 - 18*a^2*b^2 + 11*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*b^6*d) + ((35*a
^4 - 77*a^2*b^2 + 45*b^4)*Cos[c + d*x]*Sin[c + d*x]^2)/(105*b^5*d) + (Cos[c
 + d*x]*Sin[c + d*x]^3)/(3*a*d) - ((6*a^4 - 13*a^2*b^2 + 8*b^4)*Cos[c + d*x
]*Sin[c + d*x]^3)/(24*a*b^4*d) - (b*Cos[c + d*x]*Sin[c + d*x]^4)/(4*a^2*d)
+ ((28*a^4 - 60*a^2*b^2 + 35*b^4)*Cos[c + d*x]*Sin[c + d*x]^4)/(140*a^2*b^3
*d) - (a*Cos[c + d*x]*Sin[c + d*x]^5)/(6*b^2*d) + (Cos[c + d*x]*Sin[c + d*x
]^6)/(7*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} - \frac{a\cos(c+dx)\sin^5(}{6b^2d}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{b\cos(c+dx)\sin^4(c+dx)}{4a^2d} + \frac{(28a^4-60a^2b^2+}{}$$

$$= \frac{\cos(c+dx)\sin^3(c+dx)}{3ad} - \frac{(6a^4-13a^2b^2+8b^4)\cos(c+dx)\sin^3(c+dx)}{24ab^4d}$$

$$= \frac{(35a^4-77a^2b^2+45b^4)\cos(c+dx)\sin^2(c+dx)}{105b^5d} + \frac{\cos(c+dx)\sin^3(c+dx}{3ad}$$

$$= -\frac{a(8a^4-18a^2b^2+11b^4)\cos(c+dx)\sin(c+dx)}{16b^6d} + \frac{(35a^4-77a^2b^2+45b^4}{10}$$

$$= \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)\cos(c+dx)}{105b^7d} - \frac{a(8a^4-18a^2b^2+11b^4)}{16}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^8} + \frac{(105a^6-245a^4b^2+161a^2b^4-15b^6)}{105b^7d}$$

$$= \frac{a(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^8} - \frac{2a^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^8d}$$

Mathematica [A]

time = 2.07, size = 324, normalized size = 0.79

$$-\frac{-6720a^7c+16800a^5b^2c-12600a^3b^4c+2100ab^6c-6720a^7dx+16800a^5b^2dx-12600a^3b^4dx+2100ab^6dx+13440a^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)+105b(-64a^6+144a^4b^2-88a^2b^4+5b^6)\cos(c+dx)+35(16a^4b^3-28a^2b^5+9b^7)\cos(3(c+dx))-84a^2b^5\cos(5(c+dx))+105b^7\cos(5(c+dx))+15b^7\cos(7(c+dx))+1680a^5b^2\sin(2(c+dx))-3360a^3b^4\sin(2(c+dx))+1575ab^6\sin(2(c+dx))-210a^3b^4\sin(4(c+dx))+315ab^6\sin(4(c+dx))+35ab^6\sin(6(c+dx))}{6720b^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/6720*(-6720*a^7*c + 16800*a^5*b^2*c - 12600*a^3*b^4*c + 2100*a*b^6*c - 6
720*a^7*d*x + 16800*a^5*b^2*d*x - 12600*a^3*b^4*d*x + 2100*a*b^6*d*x + 1344
0*a^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] +
105*b*(-64*a^6 + 144*a^4*b^2 - 88*a^2*b^4 + 5*b^6)*Cos[c + d*x] + 35*(16*a^
4*b^3 - 28*a^2*b^5 + 9*b^7)*Cos[3*(c + d*x)] - 84*a^2*b^5*Cos[5*(c + d*x)]
+ 105*b^7*Cos[5*(c + d*x)] + 15*b^7*Cos[7*(c + d*x)] + 1680*a^5*b^2*Sin[2*(
c + d*x)] - 3360*a^3*b^4*Sin[2*(c + d*x)] + 1575*a*b^6*Sin[2*(c + d*x)] - 2
10*a^3*b^4*Sin[4*(c + d*x)] + 315*a*b^6*Sin[4*(c + d*x)] + 35*a*b^6*Sin[6*(
c + d*x)])/(b^8*d)
```

Maple [A]
time = 0.45, size = 601, normalized size = 1.47 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/b^8*(((1/2*a^5*b^2-9/8*a^3*b^4+11/16*a*b^6)*tan(1/2*d*x+1/2*c)^13+(a
^6*b-3*a^4*b^3+3*a^2*b^5-b^7)*tan(1/2*d*x+1/2*c)^12+(2*a^5*b^2-7/2*a^3*b^4+
7/12*a*b^6)*tan(1/2*d*x+1/2*c)^11+(6*a^6*b-16*a^4*b^3+12*a^2*b^5)*tan(1/2*d
*x+1/2*c)^10+(5/2*a^5*b^2-29/8*a^3*b^4+85/48*a*b^6)*tan(1/2*d*x+1/2*c)^9+(1
5*a^6*b-109/3*a^4*b^3+73/3*a^2*b^5-5*b^7)*tan(1/2*d*x+1/2*c)^8+(20*a^6*b-13
6/3*a^4*b^3+88/3*a^2*b^5)*tan(1/2*d*x+1/2*c)^6+(-5/2*a^5*b^2+29/8*a^3*b^4-8
5/48*a*b^6)*tan(1/2*d*x+1/2*c)^5+(15*a^6*b-33*a^4*b^3+101/5*a^2*b^5-3*b^7)*
tan(1/2*d*x+1/2*c)^4+(-2*a^5*b^2+7/2*a^3*b^4-7/12*a*b^6)*tan(1/2*d*x+1/2*c)
^3+(6*a^6*b-40/3*a^4*b^3+116/15*a^2*b^5)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^5*b^2
+9/8*a^3*b^4-11/16*a*b^6)*tan(1/2*d*x+1/2*c)+a^6*b-7/3*a^4*b^3+23/15*a^2*b^
5-1/7*b^7)/(1+tan(1/2*d*x+1/2*c)^2)^7+1/16*a*(16*a^6-40*a^4*b^2+30*a^2*b^4-
5*b^6)*arctan(tan(1/2*d*x+1/2*c)))-2*a^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^8/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Fricas [A]
time = 0.41, size = 619, normalized size = 1.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/1680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^
3 - a^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b
^6)*d*x - 840*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^
2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(
d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*si
n(d*x + c) - a^2 - b^2)) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
+ 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d*x + c)^3 + 3*(
8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c))/(b^8*d), -1/1
680*(240*b^7*cos(d*x + c)^7 - 336*a^2*b^5*cos(d*x + c)^5 + 560*(a^4*b^3 - a
^2*b^5)*cos(d*x + c)^3 - 105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*d
*x - 1680*(a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 1680*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c) + 35*(8*a*b^6*cos(d*x + c)^5 - 2*(6*a^3*b^4 - 5*a*b^6)*cos(d
*x + c)^3 + 3*(8*a^5*b^2 - 14*a^3*b^4 + 5*a*b^6)*cos(d*x + c))*sin(d*x + c)
)/(b^8*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 863 vs. 2(383) = 766.
time = 0.62, size = 863, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1680*(105*(16*a^7 - 40*a^5*b^2 + 30*a^3*b^4 - 5*a*b^6)*(d*x + c)/b^8 - 33
60*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*b^8) + 2*(840*a^5*b*tan(1/2*d*x + 1/2*c)^13 - 1890*a^3*b^3*tan(1/2*d
*x + 1/2*c)^13 + 1155*a*b^5*tan(1/2*d*x + 1/2*c)^13 + 1680*a^6*tan(1/2*d*x
+ 1/2*c)^12 - 5040*a^4*b^2*tan(1/2*d*x + 1/2*c)^12 + 5040*a^2*b^4*tan(1/2*d
*x + 1/2*c)^12 - 1680*b^6*tan(1/2*d*x + 1/2*c)^12 + 3360*a^5*b*tan(1/2*d*x
+ 1/2*c)^11 - 5880*a^3*b^3*tan(1/2*d*x + 1/2*c)^11 + 980*a*b^5*tan(1/2*d*x
+ 1/2*c)^11 + 10080*a^6*tan(1/2*d*x + 1/2*c)^10 - 26880*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^10 + 20160*a^2*b^4*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*b*tan(1/2*d
*x + 1/2*c)^9 - 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^9 + 2975*a*b^5*tan(1/2*d*
x + 1/2*c)^9 + 25200*a^6*tan(1/2*d*x + 1/2*c)^8 - 61040*a^4*b^2*tan(1/2*d*x
 + 1/2*c)^8 + 40880*a^2*b^4*tan(1/2*d*x + 1/2*c)^8 - 8400*b^6*tan(1/2*d*x +
 1/2*c)^8 + 33600*a^6*tan(1/2*d*x + 1/2*c)^6 - 76160*a^4*b^2*tan(1/2*d*x +
1/2*c)^6 + 49280*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4200*a^5*b*tan(1/2*d*x +
1/2*c)^5 + 6090*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 - 2975*a*b^5*tan(1/2*d*x + 1
/2*c)^5 + 25200*a^6*tan(1/2*d*x + 1/2*c)^4 - 55440*a^4*b^2*tan(1/2*d*x + 1/
2*c)^4 + 33936*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*b^6*tan(1/2*d*x + 1/2*
c)^4 - 3360*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5880*a^3*b^3*tan(1/2*d*x + 1/2*c
)^3 - 980*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 10080*a^6*tan(1/2*d*x + 1/2*c)^2 -
 22400*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12992*a^2*b^4*tan(1/2*d*x + 1/2*c)^
2 - 840*a^5*b*tan(1/2*d*x + 1/2*c) + 1890*a^3*b^3*tan(1/2*d*x + 1/2*c) - 11
55*a*b^5*tan(1/2*d*x + 1/2*c) + 1680*a^6 - 3920*a^4*b^2 + 2576*a^2*b^4 - 24
0*b^6)/((tan(1/2*d*x + 1/2*c)^2 + 1)^7*b^7))/d
```

Mupad [B]
time = 14.46, size = 2500, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x)^2)/(a + b*sin(c + d*x)),x)
```

```
[Out] ((2*(105*a^6 - 15*b^6 + 161*a^2*b^4 - 245*a^4*b^2))/(105*b^7) + (tan(c/2 +
(d*x)/2)^13*(11*a*b^4 + 8*a^5 - 18*a^3*b^2))/(8*b^6) - (tan(c/2 + (d*x)/2)^
3*(7*a*b^4 + 24*a^5 - 42*a^3*b^2))/(6*b^6) + (tan(c/2 + (d*x)/2)^11*(7*a*b^
4 + 24*a^5 - 42*a^3*b^2))/(6*b^6) - (tan(c/2 + (d*x)/2)^5*(85*a*b^4 + 120*a
^5 - 174*a^3*b^2))/(24*b^6) + (tan(c/2 + (d*x)/2)^9*(85*a*b^4 + 120*a^5 - 1
74*a^3*b^2))/(24*b^6) + (2*tan(c/2 + (d*x)/2)^12*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2))/b^7 + (4*tan(c/2 + (d*x)/2)^10*(3*a^6 + 6*a^2*b^4 - 8*a^4*b^2))/
b^7 + (8*tan(c/2 + (d*x)/2)^6*(15*a^6 + 22*a^2*b^4 - 34*a^4*b^2))/(3*b^7) +
 (4*tan(c/2 + (d*x)/2)^2*(45*a^6 + 58*a^2*b^4 - 100*a^4*b^2))/(15*b^7) + (2
*tan(c/2 + (d*x)/2)^8*(45*a^6 - 15*b^6 + 73*a^2*b^4 - 109*a^4*b^2))/(3*b^7)
 + (2*tan(c/2 + (d*x)/2)^4*(75*a^6 - 15*b^6 + 101*a^2*b^4 - 165*a^4*b^2))/(
5*b^7) - (a*tan(c/2 + (d*x)/2)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(8*b^6))/(d*(
```

7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6
+ 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/2)
^12 + tan(c/2 + (d*x)/2)^14 + 1)) + (a^2*atan(((a^2*(-(a + b)^5*(a - b)^5)^
(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^1
3 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*
(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b
^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) + (a^2*(-(a
+ b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*
b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b
^18 - 512*a^9*b^16))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b
^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8)
1i)/b^8 + (a^2(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^
17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 3
2*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a
^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9
 - 512*a^17*b^7))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^2
2 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(51
2*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a^2
(-(a + b)^5(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25
 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8)*1i)/b^8)/((32*a^22 + (55*a^6*b^16)/
4 - (585*a^8*b^14)/4 + (2445*a^10*b^12)/4 - (5511*a^12*b^10)/4 + 1874*a^14*
b^8 - 1595*a^16*b^6 + 836*a^18*b^4 - 248*a^20*b^2)/b^20 + (tan(c/2 + (d*x)/
2)*(512*a^23 - 50*a^5*b^18 + 750*a^7*b^16 - 4550*a^9*b^14 + 14770*a^11*b^12
 - 28880*a^13*b^10 + 35880*a^15*b^8 - 28672*a^17*b^6 + 14336*a^19*b^4 - 409
6*a^21*b^2))/(4*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8
 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160
*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*
b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 +
3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) + (a^2*(-(a + b)^5*(a - b)^5)^(1/2)
*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 +
 (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8
b^21) + (a^2(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2
)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)))/b^8))/b^8))/b^8 + (a^2*(-(a + b)^
5*(a - b)^5)^(1/2)*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 -
 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/
2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 +
 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21)
 - (a^2*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b
^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20
 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a^2*(-(a + b)^5*(a - b)^5)^(1
/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21)
))/b^8))/b^8))/b^8))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(b^8*d) + (a*atan(((a
*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 3
20*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a
^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 -

8960*a^13*b^11 + 3328*a^15*b^9 - 512*a^17*b^7))/(8*b^21) - (a*((10*a^2*b^22 - 38*a^4*b^20 + 44*a^6*b^18 - 16*a^8*b^16)/b^20 + (tan(c/2 + (d*x)/2)*(512*a^3*b^22 - 1536*a^5*b^20 + 1536*a^7*b^18 - 512*a^9*b^16))/(8*b^21) - (a*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^25 - 512*a^3*b^23))/(8*b^21))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2)*1i)/(16*b^8))*(16*a^6 - 5*b^6 + 30*a^2*b^4 - 40*a^4*b^2))/(16*b^8) + (a*(((25*a^4*b^19)/8 - (75*a^6*b^17)/2 + (325*a^8*b^15)/2 - 320*a^10*b^13 + 320*a^12*b^11 - 160*a^14*b^9 + 32*a^16*b^7)/b^20 + (tan(c/2 + (d*x)/2)*(50*a^3*b^21 - 881*a^5*b^19 + 4436*a^7*b^17 - 10260*a^9*b^15 + 12800*a^11*b^13 - 8960*a^13*b^11 + 3328*a^15*b...

3.1322 $\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=228

$$-\frac{(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^7}+\frac{2a(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^7d}-\frac{\cos^5(c+dx)(6a-5b\sin(c+d}{30b^2d}$$

```
[Out] -1/16*(16*a^6-40*a^4*b^2+30*a^2*b^4-5*b^6)*x/b^7+2*a*(a^2-b^2)^(5/2)*arctan
((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^7/d-1/30*cos(d*x+c)^5*(6*a-5*b
*sin(d*x+c))/b^2/d+1/24*cos(d*x+c)^3*(8*a*(a^2-b^2)-b*(6*a^2-5*b^2)*sin(d*x
+c))/b^4/d-1/16*cos(d*x+c)*(16*a*(a^2-b^2)^2-b*(8*a^4-14*a^2*b^2+5*b^4)*sin
(d*x+c))/b^6/d
```

Rubi [A]
time = 0.34, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2944, 2814, 2739, 632, 210}

$$\frac{2a(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^7d}+\frac{\cos^3(c+dx)\,(8a(a^2-b^2)-b(6a^2-5b^2)\sin(c+dx))}{24b^4d}-\frac{\cos(c+dx)\left(16a(a^2-b^2)^2-b(8a^4-14a^2b^2+5b^4)\sin(c+dx)\right)}{16b^6d}-\frac{x(16a^6-40a^4b^2+30a^2b^4-5b^6)}{16b^7}-\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*x)/b^7 + (2*a*(a^2 - b^2)
^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^7*d) - (Cos[c +
 d*x]^5*(6*a - 5*b*Sin[c + d*x]))/(30*b^2*d) + (Cos[c + d*x]^3*(8*a*(a^2 -
b^2) - b*(6*a^2 - 5*b^2)*Sin[c + d*x]))/(24*b^4*d) - (Cos[c + d*x]*(16*a*(a
^2 - b^2)^2 - b*(8*a^4 - 14*a^2*b^2 + 5*b^4)*Sin[c + d*x]))/(16*b^6*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)\sin(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\int \frac{\cos^4(c+dx)\left(-ab-\left(6a^2-5b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{6b^2}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a(a^2-b^2)-b(6a^2-\right.}{24b^4d}$$

$$= -\frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} + \frac{\cos^3(c+dx)\left(8a(a^2-b^2)-b(6a^2-\right.}{24b^4d}$$

$$= -\frac{(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} +$$

$$= -\frac{(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} +$$

$$= -\frac{(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^7} - \frac{\cos^5(c+dx)(6a-5b\sin(c+dx))}{30b^2d} +$$

$$= -\frac{(16a^6-40a^4b^2+30a^2b^4-5b^6)\,x}{16b^7} + \frac{2a(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^7d}$$

Mathematica [A]
time = 1.56, size = 275, normalized size = 1.21

$$\frac{-960a^6c+2400a^4b^2c-1800a^2b^4c+300b^6c-960a^6dx+2400a^4b^2dx-1800a^2b^4dx+300b^6dx+1920a(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-120ab(8a^4-18a^2b^2+11b^4)\cos(c+dx)+20(4a^3b^3-7ab^5)\cos(3(c+dx))-12ab^5\cos(5(c+dx))+240a^4b^2\sin(2(c+dx))-480a^2b^4\sin(2(c+dx))+225b^6\sin(2(c+dx))-30a^2b^4\sin(4(c+dx))+45b^6\sin(4(c+dx))+5b^6\sin(6(c+dx))}{960b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^6*Sin[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-960*a^6*c + 2400*a^4*b^2*c - 1800*a^2*b^4*c + 300*b^6*c - 960*a^6*d*x + 2
400*a^4*b^2*d*x - 1800*a^2*b^4*d*x + 300*b^6*d*x + 1920*a*(a^2 - b^2)^(5/2)
*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 120*a*b*(8*a^4 - 18*a^2
*b^2 + 11*b^4)*Cos[c + d*x] + 20*(4*a^3*b^3 - 7*a*b^5)*Cos[3*(c + d*x)] - 1
2*a*b^5*Cos[5*(c + d*x)] + 240*a^4*b^2*Sin[2*(c + d*x)] - 480*a^2*b^4*Sin[2
*(c + d*x)] + 225*b^6*Sin[2*(c + d*x)] - 30*a^2*b^4*Sin[4*(c + d*x)] + 45*b
^6*Sin[4*(c + d*x)] + 5*b^6*Sin[6*(c + d*x)])/(960*b^7*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(215) = 430.
time = 0.35, size = 524, normalized size = 2.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^7*(((1/2*a^4*b^2-9/8*a^2*b^4+11/16*b^6)*tan(1/2*d*x+1/2*c)^11+(a^
5*b-3*a^3*b^3+3*a*b^5)*tan(1/2*d*x+1/2*c)^10+(3/2*a^4*b^2-19/8*a^2*b^4-5/48
*b^6)*tan(1/2*d*x+1/2*c)^9+(5*a^5*b-13*a^3*b^3+9*a*b^5)*tan(1/2*d*x+1/2*c)^
8+(a^4*b^2-5/4*a^2*b^4+15/8*b^6)*tan(1/2*d*x+1/2*c)^7+(10*a^5*b-70/3*a^3*b^
3+46/3*a*b^5)*tan(1/2*d*x+1/2*c)^6+(-a^4*b^2+5/4*a^2*b^4-15/8*b^6)*tan(1/2*
d*x+1/2*c)^5+(10*a^5*b-22*a^3*b^3+14*a*b^5)*tan(1/2*d*x+1/2*c)^4+(-3/2*a^4*
b^2+19/8*a^2*b^4+5/48*b^6)*tan(1/2*d*x+1/2*c)^3+(5*a^5*b-11*a^3*b^3+31/5*a*
b^5)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^4*b^2+9/8*a^2*b^4-11/16*b^6)*tan(1/2*d*x+
1/2*c)+a^5*b-7/3*a^3*b^3+23/15*a*b^5)/(1+tan(1/2*d*x+1/2*c)^2)^6+1/16*(16*a
^6-40*a^4*b^2+30*a^2*b^4-5*b^6)*arctan(tan(1/2*d*x+1/2*c)))+2*a*(a^6-3*a^4*
b^2+3*a^2*b^4-b^6)/b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for
more de
```

Fricas [A]
time = 0.42, size = 570, normalized size = 2.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*b^5)*cos(d*x + c)^3 + 15
*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x - 120*(a^5 - 2*a^3*b^2 + a*
b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 240*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^
4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c)
)*sin(d*x + c))/(b^7*d), -1/240*(48*a*b^5*cos(d*x + c)^5 - 80*(a^3*b^3 - a*
b^5)*cos(d*x + c)^3 + 15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*d*x + 2
40*(a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) + 240*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x +
c) - 5*(8*b^6*cos(d*x + c)^5 - 2*(6*a^2*b^4 - 5*b^6)*cos(d*x + c)^3 + 3*(8*
a^4*b^2 - 14*a^2*b^4 + 5*b^6)*cos(d*x + c))*sin(d*x + c))/(b^7*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(214) = 428.
time = 0.49, size = 735, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(16*a^6 - 40*a^4*b^2 + 30*a^2*b^4 - 5*b^6)*(d*x + c)/b^7 - 480*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
```

```
a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)
*b^7) + 2*(120*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 270*a^2*b^3*tan(1/2*d*x + 1/
2*c)^11 + 165*b^5*tan(1/2*d*x + 1/2*c)^11 + 240*a^5*tan(1/2*d*x + 1/2*c)^10
 - 720*a^3*b^2*tan(1/2*d*x + 1/2*c)^10 + 720*a*b^4*tan(1/2*d*x + 1/2*c)^10
+ 360*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 570*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 - 2
5*b^5*tan(1/2*d*x + 1/2*c)^9 + 1200*a^5*tan(1/2*d*x + 1/2*c)^8 - 3120*a^3*b
^2*tan(1/2*d*x + 1/2*c)^8 + 2160*a*b^4*tan(1/2*d*x + 1/2*c)^8 + 240*a^4*b*t
an(1/2*d*x + 1/2*c)^7 - 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 450*b^5*tan(1/
2*d*x + 1/2*c)^7 + 2400*a^5*tan(1/2*d*x + 1/2*c)^6 - 5600*a^3*b^2*tan(1/2*d
*x + 1/2*c)^6 + 3680*a*b^4*tan(1/2*d*x + 1/2*c)^6 - 240*a^4*b*tan(1/2*d*x +
 1/2*c)^5 + 300*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 450*b^5*tan(1/2*d*x + 1/2*
c)^5 + 2400*a^5*tan(1/2*d*x + 1/2*c)^4 - 5280*a^3*b^2*tan(1/2*d*x + 1/2*c)^
4 + 3360*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 360*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
570*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 25*b^5*tan(1/2*d*x + 1/2*c)^3 + 1200*a
^5*tan(1/2*d*x + 1/2*c)^2 - 2640*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 1488*a*b^
4*tan(1/2*d*x + 1/2*c)^2 - 120*a^4*b*tan(1/2*d*x + 1/2*c) + 270*a^2*b^3*tan
(1/2*d*x + 1/2*c) - 165*b^5*tan(1/2*d*x + 1/2*c) + 240*a^5 - 560*a^3*b^2 +
368*a*b^4)/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*b^6))/d
```

Mupad [B]
time = 14.38, size = 2500, normalized size = 10.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^6*sin(c + d*x))/(a + b*sin(c + d*x)),x)
```

```
[Out] - ((2*(23*a*b^4 + 15*a^5 - 35*a^3*b^2))/(15*b^6) + (4*tan(c/2 + (d*x)/2)^4*
(7*a*b^4 + 5*a^5 - 11*a^3*b^2))/b^6 + (2*tan(c/2 + (d*x)/2)^8*(9*a*b^4 + 5*
a^5 - 13*a^3*b^2))/b^6 + (4*tan(c/2 + (d*x)/2)^6*(23*a*b^4 + 15*a^5 - 35*a^
3*b^2))/(3*b^6) + (2*tan(c/2 + (d*x)/2)^2*(31*a*b^4 + 25*a^5 - 55*a^3*b^2))
/(5*b^6) - (tan(c/2 + (d*x)/2)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(8*b^5) - (ta
n(c/2 + (d*x)/2)^5*(8*a^4 + 15*b^4 - 10*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x
)/2)^7*(8*a^4 + 15*b^4 - 10*a^2*b^2))/(4*b^5) + (tan(c/2 + (d*x)/2)^11*(8*a
^4 + 11*b^4 - 18*a^2*b^2))/(8*b^5) + (tan(c/2 + (d*x)/2)^3*(5*b^4 - 72*a^4
+ 114*a^2*b^2))/(24*b^5) - (tan(c/2 + (d*x)/2)^9*(5*b^4 - 72*a^4 + 114*a^2*
b^2))/(24*b^5) + (2*tan(c/2 + (d*x)/2)^10*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6)
/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/
2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/
2)^12 + 1)) - (atan((((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/
2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 - (((10
*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 - ((32*a^2*b^3 + (t
an(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i
+ a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20
- 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5
```

i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(16*b^7) + ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(16*b^7))/((32*a^19 + (55*a^3*b^16)/4 - (585*a^5*b^14)/4 + (2445*a^7*b^12)/4 - (5511*a^9*b^10)/4 + 1874*a^11*b^8 - 1595*a^13*b^6 + 836*a^15*b^4 - 248*a^17*b^2)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^20 - 50*a^2*b^18 + 750*a^4*b^16 - 4550*a^6*b^14 + 14770*a^8*b^12 - 28880*a^10*b^10 + 35880*a^12*b^8 - 28672*a^14*b^6 + 14336*a^16*b^4 - 4096*a^18*b^2))/(4*b^18) + ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 - (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 - ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) - ((((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7) + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i))/(16*b^7)))*(a^6*16i - b^6*5i + a^2*b^4*30i - a^4*b^2*40i)*1i)/(8*b^7*d) - (a*atan(((a*(-(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 + (tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^13*b^8 - 512*a^15*b^6))/(8*b^18) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*((10*a*b^20 - 38*a^3*b^18 + 44*a^5*b^16 - 16*a^7*b^14)/b^17 + (tan(c/2 + (d*x)/2)*(512*a^2*b^20 - 1536*a^4*b^18 + 1536*a^6*b^16 - 512*a^8*b^14))/(8*b^18) + (a*(-(a + b)^5*(a - b)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(768*a*b^22 - 512*a^3*b^20))/(8*b^18)))/b^7))/b^7)*1i)/b^7 + (a*(-

```
(a + b)^5*(a - b)^5)^(1/2)*(((25*a^2*b^18)/8 - (75*a^4*b^16)/2 + (325*a^6*b
^14)/2 - 320*a^8*b^12 + 320*a^10*b^10 - 160*a^12*b^8 + 32*a^14*b^6)/b^17 +
(tan(c/2 + (d*x)/2)*(50*a*b^20 - 881*a^3*b^18 + 4436*a^5*b^16 - 10260*a^7*b
^14 + 12800*a^9*b^12 - 8960*a^11*b^10 + 3328*a^...
```

3.1323 $\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{3x}{8b}-\frac{(a^2-3b^2)\,x}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\,x}{b^5}+\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{a}{}$$

[Out] -3/8*x/b-1/2*(a^2-3*b^2)*x/b^3-(a^4-3*a^2*b^2+3*b^4)*x/b^5+2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/b^5/d-arctanh(cos(d*x+c))/a/d-a*cos(d*x+c)/b^2/d-a*(a^2-3*b^2)*cos(d*x+c)/b^4/d+1/3*a*cos(d*x+c)^3/b^2/d+3/8*cos(d*x+c)*sin(d*x+c)/b/d+1/2*(a^2-3*b^2)*cos(d*x+c)*sin(d*x+c)/b^3/d+1/4*cos(d*x+c)*sin(d*x+c)^3/b/d

Rubi [A]
time = 0.19, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2976, 3855, 2718, 2715, 8, 2713, 2739, 632, 210}

$$\frac{2(a^2-b^2)^{5/2}\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ab^5d}-\frac{a(a^2-3b^2)\cos(c+dx)}{b^4d}+\frac{(a^2-3b^2)\sin(c+dx)\cos(c+dx)}{2b^3d}-\frac{x(a^2-3b^2)}{2b^3}-\frac{x(a^4-3a^2b^2+3b^4)}{b^5}+\frac{a\cos^3(c+dx)}{3b^2d}-\frac{a\cos(c+dx)}{b^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}+\frac{3\sin(c+dx)\cos(c+dx)}{8bd}-\frac{3x}{8b}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] (-3*x)/(8*b) - ((a^2 - 3*b^2)*x)/(2*b^3) - ((a^4 - 3*a^2*b^2 + 3*b^4)*x)/b^5 + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*b^5*d) - ArcTanh[Cos[c + d*x]]/(a*d) - (a*Cos[c + d*x])/(b^2*d) - (a*(a^2 - 3*b^2)*Cos[c + d*x])/(b^4*d) + (a*Cos[c + d*x]^3)/(3*b^2*d) + (3*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) + ((a^2 - 3*b^2)*Cos[c + d*x]*Sin[c + d*x])/(2*b^3*d) + (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{-a^4+3a^2b^2-3b^4}{b^5} + \frac{\csc(c+dx)}{a} + \frac{a(a^2-3b^2)\sin(c+dx)}{b^4} + \frac{(-a^2 \cdots}{\cdots} \right.$$
$$= -\frac{(a^4-3a^2b^2+3b^4)\,x}{b^5} + \frac{\int \csc(c+dx)\,dx}{a} + \frac{a\int \sin^3(c+dx)\,dx}{b^2} - \frac{\int \sin^4 \cdots}{\cdots}$$
$$= -\frac{(a^4-3a^2b^2+3b^4)\,x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{a(a^2-3b^2)\cos(c+dx)}{b^4d} \cdots$$
$$= -\frac{(a^2-3b^2)\,x}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\,x}{b^5} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{a\cos(c+ \cdots}{b^2d}$$
$$= -\frac{3x}{8b} - \frac{(a^2-3b^2)\,x}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\,x}{b^5} + \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan \cdots}{\sqrt{a^2 \cdots}}\right.}{ab^5d}$$

Mathematica [A]

time = 0.35, size = 220, normalized size = 0.87

$$-\frac{96a^5c-240a^3b^2c+180ab^4c+96a^5dx-240a^3b^2dx+180ab^4dx-192(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+24a^2b(4a^2-9b^2)\cos(c+dx)-8a^2b^3\cos(3(c+dx))+96b^5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-96b^5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-24a^3b^2\sin(2(c+dx))+48ab^4\sin(2(c+dx))+3ab^4\sin(4(c+dx))}{96ab^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*Cot[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/96*(96*a^5*c - 240*a^3*b^2*c + 180*a*b^4*c + 96*a^5*d*x - 240*a^3*b^2*d*
x + 180*a*b^4*d*x - 192*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]] + 24*a^2*b*(4*a^2 - 9*b^2)*Cos[c + d*x] - 8*a^2*b^3*Cos[3*(
c + d*x)] + 96*b^5*Log[Cos[(c + d*x)/2]] - 96*b^5*Log[Sin[(c + d*x)/2]] - 2
4*a^3*b^2*Sin[2*(c + d*x)] + 48*a*b^4*Sin[2*(c + d*x)] + 3*a*b^4*Sin[4*(c +
 d*x)])/(a*b^5*d)
```

Maple [A]

time = 0.46, size = 336, normalized size = 1.33

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{1}{2}a^2b^2-\frac{9}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-3a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^3b-7a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\cdots\right.\right.}\cdots\right.}{\cdots}$
default	$-\frac{2\left(\frac{\left(\frac{1}{2}a^2b^2-\frac{9}{8}b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^3b-3a\,b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2b^2-\frac{1}{8}b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^3b-7a\,b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right.\cdots}{\left(1+\tan^2\left(\frac{dx}{2}+\cdots\right.\right.}\cdots\right.}{\cdots}$

risch	$-\frac{x\,a^4}{b^5}+\frac{5x\,a^2}{2b^3}-\frac{15x}{8b}-\frac{a^3\mathrm{e}^{i(dx+c)}}{2b^4d}+\frac{9a\,\mathrm{e}^{i(dx+c)}}{8b^2d}-\frac{a^3\mathrm{e}^{-i(dx+c)}}{2b^4d}+\frac{9a\,\mathrm{e}^{-i(dx+c)}}{8b^2d}+\frac{i\sqrt{a^2-b^2}\,a^3\ln\left(\mathrm{e}^{i(dx}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b^5*(((1/2*a^2*b^2-9/8*b^4)*tan(1/2*d*x+1/2*c)^7+(a^3*b-3*a*b^3)*ta
n(1/2*d*x+1/2*c)^6+(1/2*a^2*b^2-1/8*b^4)*tan(1/2*d*x+1/2*c)^5+(3*a^3*b-7*a*
b^3)*tan(1/2*d*x+1/2*c)^4+(-1/2*a^2*b^2+1/8*b^4)*tan(1/2*d*x+1/2*c)^3+(3*a^
3*b-19/3*a*b^3)*tan(1/2*d*x+1/2*c)^2+(-1/2*a^2*b^2+9/8*b^4)*tan(1/2*d*x+1/2
*c)+a^3*b-7/3*a*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^4+1/8*(8*a^4-20*a^2*b^2+15*b^
4)*arctan(tan(1/2*d*x+1/2*c)))+1/a*ln(tan(1/2*d*x+1/2*c))+(2*a^6-6*a^4*b^2+
6*a^2*b^4-2*b^6)/a/b^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.63, size = 508, normalized size = 2.02

$$\left[\frac{8a^2b^3\cos(dx+c)^3-12b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3(8a^5-20a^3b^2+15ab^4)dx+12(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-24(a^4b-2a^2b^3)\cos(dx+c)-3(2ab^4\cos(dx+c)^3-(4a^3b^2-7ab^4)\cos(dx+c))\sin(dx+c)}{24ab^5d},\ \frac{8a^2b^3\cos(dx+c)^3-12b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+12b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3(8a^5-20a^3b^2+15ab^4)dx-24(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-24(a^4b-2a^2b^3)\cos(dx+c)-3(2ab^4\cos(dx+c)^3-(4a^3b^2-7ab^4)\cos(dx+c))\sin(dx+c)}{24ab^5d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(8*a^2*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b
^5*log(-1/2*cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x + 1
2*(a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*co
s(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^
2 - b^2)) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x + c)^3
 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d), 1/24*(8*a^2
```

```
*b^3*cos(d*x + c)^3 - 12*b^5*log(1/2*cos(d*x + c) + 1/2) + 12*b^5*log(-1/2*
cos(d*x + c) + 1/2) - 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*d*x - 24*(a^4 - 2*a
^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)
*cos(d*x + c))) - 24*(a^4*b - 2*a^2*b^3)*cos(d*x + c) - 3*(2*a*b^4*cos(d*x
+ c)^3 - (4*a^3*b^2 - 7*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a*b^5*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 398, normalized size = 1.58

$$\frac{\frac{24\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{3\left(8a^4-20a^2b^2+15b^4\right)(dx+c)}{b^5}+\frac{48\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,ab^5}-\frac{2\left(12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-27b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-72ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+72a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-168ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+72a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-152ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+27b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24a^3-56ab^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4b^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*log(abs(tan(1/2*d*x + 1/2*c)))/a - 3*(8*a^4 - 20*a^2*b^2 + 15*b^4)
*(d*x + c)/b^5 + 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a*b^5) - 2*(12*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 27*b^3*ta
n(1/2*d*x + 1/2*c)^7 + 24*a^3*tan(1/2*d*x + 1/2*c)^6 - 72*a*b^2*tan(1/2*d*x
 + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^
5 + 72*a^3*tan(1/2*d*x + 1/2*c)^4 - 168*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 + 72*a^3*tan(1/2
*d*x + 1/2*c)^2 - 152*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x +
 1/2*c) + 27*b^3*tan(1/2*d*x + 1/2*c) + 24*a^3 - 56*a*b^2)/((tan(1/2*d*x +
1/2*c)^2 + 1)^4*b^4))/d
```

Mupad [B]
time = 13.40, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a*d) + ((2*(7*a*b^2 - 3*a^3))/(3*b^4) + (2*tan(c/2
 + (d*x)/2)^6*(3*a*b^2 - a^3))/b^4 + (2*tan(c/2 + (d*x)/2)^4*(7*a*b^2 - 3*a
```

^3))/b^4 + (2*tan(c/2 + (d*x)/2)^2*(19*a*b^2 - 9*a^3))/(3*b^4) + (tan(c/2 + (d*x)/2)*(4*a^2 - 9*b^2))/(4*b^3) + (tan(c/2 + (d*x)/2)^3*(4*a^2 - b^2))/(4*b^3) - (tan(c/2 + (d*x)/2)^5*(4*a^2 - b^2))/(4*b^3) - (tan(c/2 + (d*x)/2)^7*(4*a^2 - 9*b^2))/(4*b^3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atan((((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 - (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5))*1i)/(8*b^5) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 + (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5))*1i)/(8*b^5))/((224*a^14 - 780*b^14 + 4445*a^2*b^12 - 10911*a^4*b^10 + 14991*a^6*b^8 - 12481*a^8*b^6 + 6312*a^10*b^4 - 1800*a^12*b^2)/b^11 + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 - (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 +

(d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5))) /(8*b^5)))/(8*b^5) - ((a^4*8i + b^4*15i - a^2*b^2*20i)*((840*a*b^14 - 112*a^15 - 3760*a^3*b^12 + (15271*a^5*b^10)/2 - (18191*a^7*b^8)/2 + (13697*a^9*b^6)/2 - 3252*a^11*b^4 + 900*a^13*b^2)/b^11 + (tan(c/2 + (d*x)/2)*(3664*b^20 - 18084*a^2*b^18 + 41125*a^4*b^16 - 56140*a^6*b^14 + 50084*a^8*b^12 - 29728*a^10*b^10 + 11392*a^12*b^8 - 2560*a^14*b^6 + 256*a^16*b^4))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((128*b^16 + 94*a^2*b^14 - (2395*a^4*b^12)/2 + 2068*a^6*b^10 - 1600*a^8*b^8 + 608*a^10*b^6 - 96*a^12*b^4)/b^11 + (tan(c/2 + (d*x)/2)*(4944*a*b^20 - 18960*a^3*b^18 + 29985*a^5*b^16 - 24664*a^7*b^14 + 10816*a^9*b^12 - 2240*a^11*b^10 + 128*a^13*b^8))/(2*b^16) + ((a^4*8i + b^4*15i - a^2*b^2*20i)*((256*a*b^16 - 452*a^3*b^14 + 340*a^5*b^12 - 112*a^7*b^10)/b^11 + (((128*a^2*b^16 - 96*a^4*b^14)/b^11 + (tan(c/2 + (d*x)/2)*(1024*a*b^22 - 1088*a^3*b^20 + 128*a^5*b^18))/(2*b^16))*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) + (tan(c/2 + (d*x)/2)*(1024*b^22 - 2368*a^2*b^20 + 2432*a^4*b^18 - 1280*a^6*b^16 + 256*a^8*b^14))/(2*b^16)))/(8*b^5)))/(8*b^5))) /(8*b^5) - (tan(c/2 + (d*x)/2)*(4500*a*b^18 - 512*a^19 - 30900*a^3*b^16 + 94700*a^5*b^14 - 170260*a^7*b^12 + 198160*a^9*b^10 - 155016*a^11*b^8 + 81600*a^13*b^6 - 27904*a^15*b^4 + 5632*a^17*b^2))/b^...

3.1324 $\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$\frac{ax}{2b^2}+\frac{a(a^2-3b^2)\,x}{b^4}-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{bd}+\frac{(a^2-3b^2)}{b}$$

```
[Out] 1/2*a*x/b^2+a*(a^2-3*b^2)*x/b^4-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/a^2/b^4/d+b*arctanh(cos(d*x+c))/a^2/d+cos(d*x+c)/b/
d+(a^2-3*b^2)*cos(d*x+c)/b^3/d-1/3*cos(d*x+c)^3/b/d-cot(d*x+c)/a/d-1/2*a*co
s(d*x+c)*sin(d*x+c)/b^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2976, 3855, 3852, 8, 2718, 2715, 2713, 2739, 632, 210}

$$-\frac{2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2b^4d}+\frac{ax(a^2-3b^2)}{b^4}+\frac{(a^2-3b^2)\cos(c+dx)}{b^3d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{a\sin(c+dx)\cos(c+dx)}{2b^2d}+\frac{ax}{2b^2}-\frac{\cot(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(2*b^2) + (a*(a^2 - 3*b^2)*x)/b^4 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b +
a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*b^4*d) + (b*ArcTanh[Cos[c + d*x]
])/(a^2*d) + Cos[c + d*x]/(b*d) + ((a^2 - 3*b^2)*Cos[c + d*x])/(b^3*d) - Co
s[c + d*x]^3/(3*b*d) - Cot[c + d*x]/(a*d) - (a*Cos[c + d*x]*Sin[c + d*x])/(
2*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{a^3-3ab^2}{b^4} - \frac{b\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a} + \frac{(-a^2+3b^2)\sin(c+dx)}{b^3}\right.$$

$$= \frac{a(a^2-3b^2)\,x}{b^4} + \frac{\int \csc^2(c+dx)\,dx}{a} + \frac{a\int \sin^2(c+dx)\,dx}{b^2} - \frac{\int \sin^3(c+dx)}{b}$$

$$= \frac{a(a^2-3b^2)\,x}{b^4} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{(a^2-3b^2)\cos(c+dx)}{b^3d} - \frac{a\cos(c}{}$$

$$= \frac{ax}{2b^2} + \frac{a(a^2-3b^2)\,x}{b^4} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{bd} + \frac{(a^2-3b^2)\,c}{b^3}$$

$$= \frac{ax}{2b^2} + \frac{a(a^2-3b^2)\,x}{b^4} - \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2b^4d} + \frac{b\tanh^{-1}(c}{a}$$

Mathematica [A]

time = 0.96, size = 208, normalized size = 1.14

$$-\frac{-12a^5c+30a^3b^2c-12a^5dx+30a^3b^2dx+24(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-3a^2b(4a^2-9b^2)\cos(c+dx)+a^2b^3\cos(3(c+dx))+6ab^4\cot\left(\frac{1}{2}(c+dx)\right)-12b^5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+12b^5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+3a^3b^2\sin(2(c+dx))-6ab^4\tan\left(\frac{1}{2}(c+dx)\right)}{12a^2b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*Cot[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/12*(-12*a^5*c + 30*a^3*b^2*c - 12*a^5*d*x + 30*a^3*b^2*d*x + 24*(a^2 - b
^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 3*a^2*b*(4*a^2
 - 9*b^2)*Cos[c + d*x] + a^2*b^3*Cos[3*(c + d*x)] + 6*a*b^4*Cot[(c + d*x)/2
] - 12*b^5*Log[Cos[(c + d*x)/2]] + 12*b^5*Log[Sin[(c + d*x)/2]] + 3*a^3*b^2
*Sin[2*(c + d*x)] - 6*a*b^4*Tan[(c + d*x)/2])/(a^2*b^4*d)
```

Maple [A]

time = 0.45, size = 264, normalized size = 1.44

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} + \frac{2\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-3b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2b-4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+a^2b-\frac{7b^3}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 b^4} +$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a} + \frac{2\left(\frac{a\,b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(a^2b-3b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(2a^2b-4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+a^2b-\frac{7b^3}{3}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 b^4} +$

risch	$\frac{a^3x}{b^4} - \frac{5ax}{2b^2} + \frac{ia\,e^{2i(dx+c)}}{8b^2d} + \frac{e^{i(dx+c)}a^2}{2b^3d} - \frac{9\,e^{i(dx+c)}}{8bd} + \frac{e^{-i(dx+c)}a^2}{2b^3d} - \frac{9\,e^{-i(dx+c)}}{8bd} - \frac{ia\,e^{-2i(dx+c)}}{8b^2d} - \frac{}{ad(e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)+2/b^4*((1/2*a*b^2*tan(1/2*d*x+1/2*c)^5+(a^2*b
-3*b^3)*tan(1/2*d*x+1/2*c)^4+(2*a^2*b-4*b^3)*tan(1/2*d*x+1/2*c)^2-1/2*a*b^2
*tan(1/2*d*x+1/2*c)+a^2*b-7/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3+1/2*a*(2*a^2-
5*b^2)*arctan(tan(1/2*d*x+1/2*c)))-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(
1/2*d*x+1/2*c))+1/2*(-4*a^6+12*a^4*b^2-12*a^2*b^4+4*b^6)/a^2/b^4/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.60, size = 549, normalized size = 3.00

$$\left[\frac{3a^3b^2\cos(dx+c)^3 + 3b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 3b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 3(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2 + 2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2 - 2ab\sin(dx+c) - a^2 - b^2}\right)\sin(dx+c) - 3(a^3b^2+2ab^4)\cos(dx+c) - \left(2a^2b^3\cos(dx+c)^3 - 3(2a^5-5a^3b^2)dx - 6(a^4b-2a^2b^3)\cos(dx+c)\right)\sin(dx+c)}{6a^2b^4d\sin(dx+c)}, \frac{3a^3b^2\cos(dx+c)^3 + 3b^5\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 3b^5\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 6(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\sin(dx+c) - 3(a^3b^2+2ab^4)\cos(dx+c) - \left(2a^2b^3\cos(dx+c)^3 - 3(2a^5-5a^3b^2)dx - 6(a^4b-2a^2b^3)\cos(dx+c)\right)\sin(dx+c)}{6a^2b^4d\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^4 - 2*a^2*b^2
 + b^4)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
+ c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a
^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x +
 c) - 3*(a^3*b^2 + 2*a*b^4)*cos(d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2
*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(
a^2*b^4*d*sin(d*x + c)), 1/6*(3*a^3*b^2*cos(d*x + c)^3 + 3*b^5*log(1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 3*b^5*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 6*(a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b
```

```
)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 3*(a^3*b^2 + 2*a*b^4)*cos(
d*x + c) - (2*a^2*b^3*cos(d*x + c)^3 - 3*(2*a^5 - 5*a^3*b^2)*d*x - 6*(a^4*b
 - 2*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^2*b^4*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**6*csc(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 302, normalized size = 1.65

$$-\frac{\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{3\,(2\,a^3-5\,ab^2)(dx+c)}{b^4}-\frac{3\,\left(2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{12\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^2b^4}-\frac{2\,\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-18\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2-14\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - 3
*(2*a^3 - 5*a*b^2)*(d*x + c)/b^4 - 3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*ta
n(1/2*d*x + 1/2*c)) + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*
(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
 b^2)))/(sqrt(a^2 - b^2)*a^2*b^4) - 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2
*tan(1/2*d*x + 1/2*c)^4 - 18*b^2*tan(1/2*d*x + 1/2*c)^4 + 12*a^2*tan(1/2*d*
x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) +
 6*a^2 - 14*b^2)/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*b^3))/d
```

Mupad [B]
time = 13.10, size = 2500, normalized size = 13.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a*d) - (3*tan(c/2 + (d*x)/2)^4 + (8*tan(c/2 + (d*x)/2
)^3*(2*a*b^2 - a^3))/b^3 + (4*tan(c/2 + (d*x)/2)^5*(3*a*b^2 - a^3))/b^3 + (
tan(c/2 + (d*x)/2)^2*(2*a^2 + 3*b^2))/b^2 - (tan(c/2 + (d*x)/2)^6*(2*a^2 -
b^2))/b^2 + (4*tan(c/2 + (d*x)/2)*(7*a*b^2 - 3*a^3))/(3*b^3) + 1)/(d*(2*a*t
an(c/2 + (d*x)/2) + 6*a*tan(c/2 + (d*x)/2)^3 + 6*a*tan(c/2 + (d*x)/2)^5 + 2
*a*tan(c/2 + (d*x)/2)^7)) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d) - (atan((((
```

-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386
*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + ((-(a +
b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9
 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)
/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*
b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) + ((-(a + b)^5*(a - b)^5)^(1/
2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) +
(((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4
*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2)
)/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b
^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*
tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1
769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/
(a^3*b^12))*1i)/(a^2*b^4) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12
 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a
^12*b^2))/(a^2*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b
^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/
(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^
14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12)
 - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^
10 - 14*a^8*b^8))/(a^2*b^8) - (((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) +
 (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12)
)*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b
^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)
))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 46
0*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 -
 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12))*1i)/(a^2*b^4))/((16*(10*b^13 - 14
*a^12*b + 36*a^2*b^11 - 231*a^4*b^9 + 391*a^6*b^7 - 297*a^8*b^5 + 105*a^10*
b^3))/(a^2*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(10*a^2*b^12 - 14*a^14
+ 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309*a^10*b^4 + 105*a^12*b^2))/
(a^2*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(16*b^15 - 52*a^2*b^13 + 50*a
^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5 - 12*a^12*b^3))/(a^2*b^8)
+ (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b^16 + 1020*a^6*b^14 - 1264*
a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14*b^6))/(a^3*b^12) + ((-(a +
 b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4*b^12 + 50*a^6*b^10 - 14*a^
8*b^8))/(a^2*b^8) + (((8*(16*a^4*b^13 - 12*a^6*b^11))/(a^2*b^8) + (8*tan(c/
2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*b^14))/(a^3*b^12))*(-(a + b
)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(64*a^2*b^19 - 148*
a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*b^11))/(a^3*b^12)))/(a^2*b^
4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 - 24*a^2*b^17 + 460*a^4*b^15
 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9 + 652*a^12*b^7 - 160*a^14*
b^5 + 16*a^16*b^3))/(a^3*b^12)))/(a^2*b^4) - ((-(a + b)^5*(a - b)^5)^(1/2)*
((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 - 309
*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(1
6*b^15 - 52*a^2*b^13 + 50*a^4*b^11 + 86*a^6*b^9 - 155*a^8*b^7 + 76*a^10*b^5

 - 12*a^12*b^3))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(84*a^2*b^18 - 360*a^4*b
^16 + 1020*a^6*b^14 - 1264*a^8*b^12 + 661*a^10*b^10 - 140*a^12*b^8 + 8*a^14
*b^6))/(a^3*b^12) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*(32*a^2*b^14 - 64*a^4
*b^12 + 50*a^6*b^10 - 14*a^8*b^8))/(a^2*b^8) - (((8*(16*a^4*b^13 - 12*a^6*b
^11))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(64*a^4*b^18 - 68*a^6*b^16 + 8*a^8*
b^14))/(a^3*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/(a^2*b^4) + (8*tan(c/2 + (
d*x)/2)*(64*a^2*b^19 - 148*a^4*b^17 + 152*a^6*b^15 - 80*a^8*b^13 + 16*a^10*
b^11))/(a^3*b^12)))/(a^2*b^4)))/(a^2*b^4) + (8*tan(c/2 + (d*x)/2)*(4*b^19 -
 24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 1769*a^8*b^11 - 1408*a^10*b^9
 + 652*a^12*b^7 - 160*a^14*b^5 + 16*a^16*b^3))/(a^3*b^12)))/(a^2*b^4) - (16
*tan(c/2 + (d*x)/2)*(32*a^18 - 500*a^4*b^14 + 2500*a^6*b^12 - 5260*a^8*b^10
 + 6036*a^10*b^8 - 4080*a^12*b^6 + 1624*a^14*b^4 - 352*a^16*b^2))/(a^3*b^12
)))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a^2*b^4*d) - (a*atan(((a*(2*a^2 - 5*b
^2)*((8*(10*a^2*b^12 - 14*a^14 + 136*a^4*b^10 - 386*a^6*b^8 + 467*a^8*b^6 -
 309*a^10*b^4 + 105*a^12*b^2))/(a^2*b^8) + (8*tan(c/2 + (d*x)/2)*(4*b^19 -
24*a^2*b^17 + 460*a^4*b^15 - 1300*a^6*b^13 + 17...

3.1325 $\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=174

$$-\frac{(2a^2-5b^2)\,x}{2b^3}+\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{a\cos(c+dx)}{b^2d}+b$$

```
[Out] -1/2*(2*a^2-5*b^2)*x/b^3+2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/
(a^2-b^2)^(1/2))/a^3/b^3/d+1/2*(5*a^2-2*b^2)*arctanh(cos(d*x+c))/a^3/d-a*co
s(d*x+c)/b^2/d+b*cot(d*x+c)/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d+1/2*cos(d*x
+c)*sin(d*x+c)/b/d
```

Rubi [A]
time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2975, 3136, 2739, 632, 210, 3855}

$$-\frac{x(2a^2-5b^2)}{2b^3}+\frac{b\cot(c+dx)}{a^2d}+\frac{2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3b^3d}+\frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{a\cos(c+dx)}{b^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*((2*a^2 - 5*b^2)*x)/b^3 + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*b^3*d) + ((5*a^2 - 2*b^2)*ArcTanh[Cos[c + d
*x]])/(2*a^3*d) - (a*Cos[c + d*x])/(b^2*d) + (b*Cot[c + d*x])/(a^2*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a*d) + (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

a^2 - b^2, 0]

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\cos(c+dx)}{2\ldots}$$

$$= -\frac{(2a^2-5b^2)\,x}{2b^3} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\cot(c+dx)}{a^2d} - \frac{\cot(c+dx)\csc(c+d\ldots}{2ad}$$

$$= -\frac{(2a^2-5b^2)\,x}{2b^3} + \frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\,c\ldots}{}$$

$$= -\frac{(2a^2-5b^2)\,x}{2b^3} + \frac{(5a^2-2b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{a\cos(c+dx)}{b^2d} + \frac{b\,c\ldots}{}$$

$$= -\frac{(2a^2-5b^2)\,x}{2b^3} + \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3b^3d} + \frac{(5a^2-2b^2)\,\mathrm{ta}\ldots}{}$$

Mathematica [A]

time = 3.50, size = 259, normalized size = 1.49

$$\frac{-8a^5c+20a^3b^2c-8a^5dx+20a^3b^2dx+16(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-8a^4b\cos(c+dx)+4ab^4\cot\left(\frac{1}{2}(c+dx)\right)-a^2b^3\csc^2\left(\frac{1}{2}(c+dx)\right)+20a^2b^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8b^5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-20a^2b^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8b^5\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a^2b^3\sec^2\left(\frac{1}{2}(c+dx)\right)+2a^3b^2\sin(2(c+dx))-4ab^4\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*Cot[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-8*a^5*c + 20*a^3*b^2*c - 8*a^5*d*x + 20*a^3*b^2*d*x + 16*(a^2 - b^2)^(5/2
)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - 8*a^4*b*Cos[c + d*x] +
 4*a*b^4*Cot[(c + d*x)/2] - a^2*b^3*Csc[(c + d*x)/2]^2 + 20*a^2*b^3*Log[Cos
[(c + d*x)/2]] - 8*b^5*Log[Cos[(c + d*x)/2]] - 20*a^2*b^3*Log[Sin[(c + d*x)
/2]] + 8*b^5*Log[Sin[(c + d*x)/2]] + a^2*b^3*Sec[(c + d*x)/2]^2 + 2*a^3*b^2
*Sin[2*(c + d*x)] - 4*a*b^4*Tan[(c + d*x)/2])/(8*a^3*b^3*d)
```

Maple [A]

time = 0.46, size = 266, normalized size = 1.53

method	result
derivativedivides	$\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2-5b^2)\arctan\left(\tan\left(\frac{d\ldots}{2}\right.\right.}{2}\ldots}{b^3}\ldots$
default	$\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2-5b^2)\arctan\left(\tan\left(\frac{d\ldots}{2}\right.\right.}{2}\ldots}{b^3}\ldots$

risch	$-\frac{x\,a^2}{b^3}+\frac{5x}{2b}+\frac{i\sqrt{a^2-b^2}\,b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}+a\right)}{b}\right)}{d\,a^3}-\frac{a\,\mathrm{e}^{i(dx+c)}}{2b^2d}-\frac{a\,\mathrm{e}^{-i(dx+c)}}{2b^2d}+\frac{i\sqrt{a^2-b^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c))-2/b^3*((1/
2*b^2*tan(1/2*d*x+1/2*c)^3+a*b*tan(1/2*d*x+1/2*c)^2-1/2*b^2*tan(1/2*d*x+1/2
*c)+a*b)/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(2*a^2-5*b^2)*arctan(tan(1/2*d*x+1/
2*c)))-1/8/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(-10*a^2+4*b^2)*ln(tan(1/2*d*x+1/
2*c))+1/2/a^2*b/tan(1/2*d*x+1/2*c)+1/4*(8*a^6-24*a^4*b^2+24*a^2*b^4-8*b^6)/
a^3/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(161) = 322$.
time = 0.67, size = 770, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 - 5*a^3*b^2)*d*x*cos(d*x + c)^2 -
2*(2*a^5 - 5*a^3*b^2)*d*x + 2*(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b
^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
+ c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^
2)) - 2*(2*a^4*b + a^2*b^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3
- 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 -
```

```
(5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*
b^2*cos(d*x + c)^3 - (a^3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b
^3*d*cos(d*x + c)^2 - a^3*b^3*d), -1/4*(4*a^4*b*cos(d*x + c)^3 + 2*(2*a^5 -
 5*a^3*b^2)*d*x*cos(d*x + c)^2 - 2*(2*a^5 - 5*a^3*b^2)*d*x - 4*(a^4 - 2*a^2
*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 2*(2*a^4*b + a^2*b
^3)*cos(d*x + c) + (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)
*log(1/2*cos(d*x + c) + 1/2) - (5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^3*b^2*cos(d*x + c)^3 - (a^
3*b^2 + 2*a*b^4)*cos(d*x + c))*sin(d*x + c))/(a^3*b^3*d*cos(d*x + c)^2 - a^
3*b^3*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(161) = 322.
time = 0.52, size = 431, normalized size = 2.48

$$\frac{\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}}-\frac{4\,\left(2\,a^{2}-5\,b^{2}\right)\left(dx+c\right)}{b^{3}}-\frac{4\,\left(5\,a^{2}-2\,b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{3}}+\frac{16\,\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{3}b^{3}}+\frac{10\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-4\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}-8\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-16\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+19\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-8\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+8\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+8\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+8\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-4\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^{2}b^{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^{2}a^{3}b^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*((a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 4*(2*a^2 -
 5*b^2)*(d*x + c)/b^3 - 4*(5*a^2 - 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^
3 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2
- b^2)*a^3*b^3) + (10*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 4*b^4*tan(1/2*d*x +
1/2*c)^6 - 8*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 4*a*b^3*tan(1/2*d*x + 1/2*c)^5
- 16*a^4*tan(1/2*d*x + 1/2*c)^4 + 19*a^2*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*b^4
*tan(1/2*d*x + 1/2*c)^4 + 8*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^3*tan(1/2*
d*x + 1/2*c)^3 - 16*a^4*tan(1/2*d*x + 1/2*c)^2 + 8*a^2*b^2*tan(1/2*d*x + 1/
2*c)^2 - 4*b^4*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^3*tan(1/2*d*x + 1/2*c) - a^2*
b^2)/((tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^2*a^3*b^2))/d
```

Mupad [**B**]
time = 13.09, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^3*(a + b*sin(c + d*x))),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 + (d*x)/2)^3*(a^2 + b^2))/b + (tan(c/2 + (d*x)/2)^2*(a*b^2 + 8*a^3))/b^2 + (tan(c/2 + (d*x)/2)^4*(a*b^2 + 16*a^3))/(2*b^2) + (2*tan(c/2 + (d*x)/2)^5*(2*a^2 - b^2))/b)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 + 8*a^2*tan(c/2 + (d*x)/2)^4 + 4*a^2*tan(c/2 + (d*x)/2)^6)) - (b*tan(c/2 + (d*x)/2))/(2*a^2*d) + (atan((((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 - (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8))*1i)/b^3 + ((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 + (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8))*1i)/b^3)/(((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) + (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 - (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 +

16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8)))/b^3 - ((a^2*1i - (b^2*5i)/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + ((a^2*1i - (b^2*5i)/2)*(((a^2*1i - (b^2*5i)/2)*((4*(64*a^5*b^12 - 208*a^7*b^10 + 160*a^9*b^8 - 28*a^11*b^6))/(a^6*b^5) - (((4*(32*a^8*b^10 - 24*a^10*b^8))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(128*a^7*b^14 - 136*a^9*b^12 + 16*a^11*b^10))/(a^6*b^8))*(a^2*1i - (b^2*5i)/2))/b^3 + (4*tan(c/2 + (d*x)/2)*(128*a^4*b^16 - 456*a^6*b^14 + 484*a^8*b^12 - 200*a^10*b^10 + 32*a^12*b^8))/(a^6*b^8)))/b^3 + (4*(184*a^4*b^12 - 32*a^2*b^14 - 360*a^6*b^10 + 78*a^8*b^8 + 240*a^10*b^6 - 152*a^12*b^4 + 24*a^14*b^2))/(a^6*b^5) - (4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^14 + 1320*a^7*b^12 - 2128*a^9*b^10 + 1242*a^11*b^8 - 280*a^13*b^6 + 16*a^15*b^4))/(a^6*b^8)))/b^3 + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8)))/b^3 + (8*(70*a^14 + 20*b^14 + 22*a^2*b^12 - 642*a^4*b^10 + 1937*a^6*b^8 - 2549*a^8*b^6 + 1695*a^10*b^4 - 553*a^12*b^2))/(a^6*b^5) - (8*tan(c/2 + (d*x)/2)*(64*a^17 + 700*a^5*b^12 - 3060*a^7*b^10 + 5412*a^9*b^8 - 4940*a^11*b^6 + 2448*a^13*b^4 - 624*a^15*b^2))/(a^6*b^8)))*(a^2*1i - (b^2*5i)/2)*2i)/(b^3*d) - (log(tan(c/2 + (d*x)/2))*((5*a^2)/2 - b^2))/(a^3*d) + (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((4*(20*a^3*b^12 - 28*a^15 + 272*a^5*b^10 - 1272*a^7*b^8 + 1709*a^9*b^6 - 998*a^11*b^4 + 270*a^13*b^2))/(a^6*b^5) + (4*tan(c/2 + (d*x)/2)*(8*b^18 - 68*a^2*b^16 + 1040*a^4*b^14 - 3900*a^6*b^12 + 5738*a^8*b^10 - 4201*a^10*b^8 + 1684*a^12*b^6 - 360*a^14*b^4 + 32*a^16*b^2))/(a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(8*a^3*b^16 - 160*a^5*b^...

3.1326 $\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{ax}{b^2}-\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{b(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{\cos(c+}{bc}$$

```
[Out] a*x/b^2-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/
a^4/b^2/d+1/2*b*arctanh(cos(d*x+c))/a^2/d-b*(3*a^2-b^2)*arctanh(cos(d*x+c))
/a^4/d+cos(d*x+c)/b/d-cot(d*x+c)/a/d+(3*a^2-b^2)*cot(d*x+c)/a^3/d-1/3*cot(d
*x+c)^3/a/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d
```

Rubi [A]

time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2976, 3855, 3852, 8, 3853, 2718, 2739, 632, 210}

$$\frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4b^2d}-\frac{b(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{(3a^2-b^2)\cot(c+dx)}{a^3d}+\frac{ax}{b^2}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/b^2 - (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*b^2*d) + (b*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b*(3*a^2 - b^2
)*ArcTanh[Cos[c + d*x]])/(a^4*d) + Cos[c + d*x]/(b*d) - Cot[c + d*x]/(a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x])/(a^3*d) - Cot[c + d*x]^3/(3*a*d) + (b*Cot[c
+ d*x]*Csc[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{a}{b^2} + \frac{(3a^2b-b^3)\csc(c+dx)}{a^4} + \frac{(-3a^2+b^2)\csc^2(c+dx)}{a^3} - \frac{b\csc^3(c+}{a^2}\right.$$

$$= \frac{ax}{b^2} + \frac{\int \csc^4(c+dx)\,dx}{a} - \frac{\int \sin(c+dx)\,dx}{b} - \frac{b\int \csc^3(c+dx)\,dx}{a^2} - \frac{(a^2-}{}$$

$$= \frac{ax}{b^2} - \frac{b(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{bd} + \frac{b\cot(c+dx)\csc(c}{2a^2d}$$

$$= \frac{ax}{b^2} + \frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+}{bd}$$

$$= \frac{ax}{b^2} - \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b(3}{}$$

Mathematica [A]
time = 6.14, size = 379, normalized size = 1.92

$$\frac{a(c+dx)}{b^2d} - \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4b^2d} + \frac{\cos(c+dx)}{bd} + \frac{\left(7a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^3d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24ad} + \frac{(-5a^2b+2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(5a^2b-2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-7a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*Cot[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(b^2*d) - (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Co
s[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*b^2*d) + Cos[c
 + d*x]/(b*d) + ((7*a^2*Cos[(c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c +
 d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a^2*d) - (Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2)/(24*a*d) + ((-5*a^2*b + 2*b^3)*Log[Cos[(c + d*x)/2]])/(
2*a^4*d) + ((5*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) - (b*Sec[(c
+ d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-7*a^2*Sin[(c + d*x)/2] + 3*b^2
*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a
*d)
```

Maple [A]
time = 0.48, size = 269, normalized size = 1.37

method	result
derivativedivides	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3} + \frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2} - \frac{}{24a\tan(}$

default	$\frac{\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-9a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{}{24a\tan}}{}$
risch	$\frac{ax}{b^2}+\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}-\frac{-18ia^2e^{4i(dx+c)}+6ib^2e^{4i(dx+c)}+3be^{5i(dx+c)}a+24ia^2e^{2i(dx+c)}-12ib^2e^{2i(dx+c)}-14}{3d\,a^3\left(e^{2i(dx+c)}-1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/8/a^3*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-a*b*tan(1/2*d*x+1/2*c)^2-9*a^2*t
an(1/2*d*x+1/2*c)+4*b^2*tan(1/2*d*x+1/2*c))+2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^
2)+a*arctan(tan(1/2*d*x+1/2*c)))-1/24/a/tan(1/2*d*x+1/2*c)^3-1/8*(-9*a^2+4*
b^2)/a^3/tan(1/2*d*x+1/2*c)+1/8/a^2*b/tan(1/2*d*x+1/2*c)^2+1/2/a^4*b*(5*a^2
-2*b^2)*ln(tan(1/2*d*x+1/2*c))+1/8*(-16*a^6+48*a^4*b^2-48*a^2*b^4+16*b^6)/a
^4/b^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.67, size = 801, normalized size = 4.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 6*(a^4 - 2*a^2*b^2 + b^4 -
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)
*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2
```

```
*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(5*a^2*b
^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) + 6*(2*a^5*d*x*cos(d*
x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*b + a^2*b^3)*cos(d*x
 + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b^2*d)*sin(d*x + c)),
 1/12*(4*(7*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3 - 12*(a^4 - 2*a^2*b^2 + b^4 -
 (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x
 + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 3*(5*a^2*b^3 - 2*
b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 3*(5*a^2*b^3 - 2*b^5 - (5*a^2*b^3 - 2*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 12*(2*a^3*b^2 - a*b^4)*cos(d*x + c) +
 6*(2*a^5*d*x*cos(d*x + c)^2 + 2*a^4*b*cos(d*x + c)^3 - 2*a^5*d*x - (2*a^4*
b + a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2*d*cos(d*x + c)^2 - a^4*b
^2*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.47, size = 317, normalized size = 1.61

$$\frac{\frac{24\,(dx+c)a}{b^2}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-27\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{48}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)b}+\frac{12\,(5\,a^2b-2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{48\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^4b^2}-\frac{110\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-44\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-27\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*a/b^2 + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3
 + 48/((tan(1/2*d*x + 1/2*c)^2 + 1)*b) + 12*(5*a^2*b - 2*b^3)*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 - 48*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/
2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2
 - b^2)))/(sqrt(a^2 - b^2)*a^4*b^2) - (110*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 4
4*b^3*tan(1/2*d*x + 1/2*c)^3 - 27*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x
+ 1/2*c)^3))/d
```

Mupad [B]
time = 15.57, size = 2500, normalized size = 12.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^6/(sin(c + d*x)^4*(a + b*sin(c + d*x))),x)

[Out] cos(c + d*x)/(32*a*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (3*sin(c + d*x))/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (7*cos(3*c + 3*d*x))/(96*a*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + sin(2*c + 2*d*x)/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - sin(3*c + 3*d*x)/(32*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - sin(4*c + 4*d*x)/(64*b*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b^2*cos(3*c + 3*d*x))/(32*a^3*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (b^2*cos(c + d*x))/(32*a^3*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b*sin(2*c + 2*d*x))/(32*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (15*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(64*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (3*a*sin(c + d*x)*atan((2*a^5*cos(c/2 + (d*x)/2) - 2*b^5*sin(c/2 + (d*x)/2) + 5*a^2*b^3*sin(c/2 + (d*x)/2))/(2*b^5*cos(c/2 + (d*x)/2) + 2*a^5*sin(c/2 + (d*x)/2) - 5*a^2*b^3*cos(c/2 + (d*x)/2))))/(16*b^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (5*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/(64*a^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) - (3*b^3*sin(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(32*a^4*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (a*sin(3*c + 3*d*x)*atan((2*a^5*cos(c/2 + (d*x)/2) - 2*b^5*sin(c/2 + (d*x)/2) + 5*a^2*b^3*sin(c/2 + (d*x)/2))/(2*b^5*cos(c/2 + (d*x)/2) + 2*a^5*sin(c/2 + (d*x)/2) - 5*a^2*b^3*cos(c/2 + (d*x)/2))))/(16*b^2*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(3*c + 3*d*x))/(32*a^4*d*((3*sin(c + d*x))/32 - sin(3*c + 3*d*x)/32)) + (sin(3*c + 3*d*x)*atan((a^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*8i + a^18*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*8i + b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*32i + a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*16i - a^7*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*12i - a^17*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^3*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*52i + a^5*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*45i - a^7*b^11*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i + a^9*b^9*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i - a^11*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*14i - a^13*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*12i + a^15*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*15i - a^2*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10 *a^6*b^4 + 5*a^8*b^2)^(3/2)*114i + a^4*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(3/2)*121i - a^6*b^2*sin (c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8* b^2)^(3/2)*50i + a^2*b^16*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10* a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*2i - a^4*b^14*sin(c/2 + (d*x)/2)*(b ^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*17i + a ^6*b^12*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b ^4 + 5*a^8*b^2)^(1/2)*60i - a^8*b^10*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^ 2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*115i + a^10*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^ (1/2)*162i - a^12*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4* b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*199i + a^14*b^4*sin(c/2 + (d*x)/2)*(b^1 0 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)*146i - a^ 16*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^ 4 + 5*a^8*b^2)^(1/2)*50i)/(32*b^23*sin(c/2 + (d*x)/2) + 16*a*b^22*cos(c/2 + (d*x)/2) - 10*a^22*b*sin(c/2 + (d*x)/2) - 172*a^3*b^20*cos(c/2 + (d*x)/2) + 825*a^5*b^18*cos(c/2 + (d*x)/2) - 2334*a^7*b^16*cos(c/2 + (d*x)/2) + 4332 *a^9*b^14*cos(c/2 + (d*x)/2) - 5528*a^11*b^12*cos(c/2 + (d*x)/2) + 4906*a^1 3*b^10*cos(c/2 + (d*x)/2) - 2975*a^15*b^8*cos(c/2 + (d*x)/2) + 1175*a^17*b^ 6*cos(c/2 + (d*x)/2) - 275*a^19*b^4*cos(c/2 + (d*x)/2) + 30*a^21*b^2*cos(c/ 2 + (d*x)/2) - 352*a^2*b^21*sin(c/2 + (d*x)/2) + 1734*a^4*b^19*sin(c/2 + (d *x)/2) - 5060*a^6*b^17*sin(c/2 + (d*x)/2) + 9738*a^8*b^15*sin(c/2 + (d*x)/2) - 12976*a^10*b^13*sin(c/2 + (d*x)/2) + 12156*a^12*b^11*sin(c/2 + (d*x)/2) - 7922*a^14*b^9*sin(c/2 + (d*x)/2) + 3470*a^16*b^7*sin(c/2 + (d*x)/2) - 96 0*a^18*b^5*sin(c/2 + (d*x)/2) + 150*a^20*b^3*sin(c/2 + (d*x)/2)))*(b^10 - a ^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a...

3.1327 $\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=195

$$-\frac{x}{b}+\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5bd}-\frac{(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^5d}+\frac{b(-2a^2+b^2)\cot(}{a^4d}$$

```
[Out] -x/b+2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5
/b/d-1/8*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^5/d+b*(-2*a^2+b^2)
*cot(d*x+c)/a^4/d+1/3*b*cot(d*x+c)^3/a^2/d+1/8*(7*a^2-4*b^2)*cot(d*x+c)*csc
(d*x+c)/a^3/d-1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

Rubi [A]
time = 0.20, antiderivative size = 275, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2976, 3855, 3852, 8, 3853, 2739, 632, 210}

$$\frac{b\cot^3(c+dx)}{3a^2d}+\frac{b\cot(c+dx)}{a^2d}+\frac{2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5bd}-\frac{b(3a^2-b^2)\cot(c+dx)}{a^4d}+\frac{(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{(3a^2-b^2)\cot(c+dx)\csc(c+dx)}{2a^3d}-\frac{(3a^4-3a^2b^2+b^4)\tanh^{-1}(\cos(c+dx))}{a^5d}-\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(x/b) + (2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^5*b*d) - (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + ((3*a^2 - b^2)*ArcTanh
[Cos[c + d*x]])/(2*a^3*d) - ((3*a^4 - 3*a^2*b^2 + b^4)*ArcTanh[Cos[c + d*x]
])/(a^5*d) + (b*Cot[c + d*x])/(a^2*d) - (b*(3*a^2 - b^2)*Cot[c + d*x])/(a^4
*d) + (b*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]*Csc[c + d*x])/(8*a*d)
+ ((3*a^2 - b^2)*Cot[c + d*x]*Csc[c + d*x])/(2*a^3*d) - (Cot[c + d*x]*Csc[c
 + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c+dx)\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{1}{b} + \frac{(3a^4-3a^2b^2+b^4)\csc(c+dx)}{a^5} + \frac{(3a^2b-b^3)\csc^2(c+dx)}{a^4} + \frac{(\cdots}{}\right.$$

$$= -\frac{x}{b} + \frac{\int \csc^5(c+dx)\,dx}{a} - \frac{b\int \csc^4(c+dx)\,dx}{a^2} + \frac{(a^2-b^2)^3 \int \frac{1}{a+b\sin(c+dx)} \cdots}{a^5 b}$$

$$= -\frac{x}{b} - \frac{(3a^4-3a^2b^2+b^4)\tanh^{-1}(\cos(c+dx))}{a^5 d} + \frac{(3a^2-b^2)\cot(c+dx)\,\mathrm{cs}\cdots}{2a^3 d}$$

$$= -\frac{x}{b} + \frac{(3a^2-b^2)\tanh^{-1}(\cos(c+dx))}{2a^3 d} - \frac{(3a^4-3a^2b^2+b^4)\tanh^{-1}(\cos(c\cdots}{a^5 d}$$

$$= -\frac{x}{b} + \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5 b d} - \frac{3\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{(\cdots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 448 vs. 2(195) = 390.
time = 6.16, size = 448, normalized size = 2.30

$$-\frac{c+dx}{bd} + \frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5bd} + \frac{\left(-7a^2b\cos\left(\frac{1}{2}(c+dx)\right)+3b^3\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{(9a^2-4b^2)\csc^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{(-15a^4+20a^2b^2-8b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(15a^4-20a^2b^2+8b^4)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8a^5d} + \frac{(-9a^2+4b^2)\sec^2\left(\frac{1}{2}(c+dx)\right)}{32a^3d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64ad} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(7a^2b\sin\left(\frac{1}{2}(c+dx)\right)-3b^3\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*Cot[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((c + d*x)/(b*d)) + (2*(a^2 - b^2)^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(
c + d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*b*d) + ((-7*a^2*b
*Cos[(c + d*x)/2] + 3*b^3*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (
(9*a^2 - 4*b^2)*Csc[(c + d*x)/2]^2)/(32*a^3*d) + (b*Cot[(c + d*x)/2]*Csc[(c
 + d*x)/2]^2)/(24*a^2*d) - Csc[(c + d*x)/2]^4/(64*a*d) + ((-15*a^4 + 20*a^2
*b^2 - 8*b^4)*Log[Cos[(c + d*x)/2]])/(8*a^5*d) + ((15*a^4 - 20*a^2*b^2 + 8*
b^4)*Log[Sin[(c + d*x)/2]])/(8*a^5*d) + ((-9*a^2 + 4*b^2)*Sec[(c + d*x)/2]^
2)/(32*a^3*d) + Sec[(c + d*x)/2]^4/(64*a*d) + (Sec[(c + d*x)/2]*(7*a^2*b*Si
n[(c + d*x)/2] - 3*b^3*Sin[(c + d*x)/2]))/(6*a^4*d) - (b*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [A]
time = 0.51, size = 320, normalized size = 1.64

method	result
derivativedivides	$\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4} - \frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3} - 4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 2ab^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 18a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4} - \cdots$

default	$\frac{\frac{\frac{a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-4a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+18a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{16a^4}-\frac{2\,\mathrm{ar}}{}}{}$
risch	$-\frac{x}{b}+\frac{i\left(-12ia\,b^2\mathrm{e}^{7i(dx+c)}-3ia^3\mathrm{e}^{5i(dx+c)}+27ia^3\mathrm{e}^{i(dx+c)}+12ia\,b^2\mathrm{e}^{5i(dx+c)}-72b\,\mathrm{e}^{6i(dx+c)}a^2+24b^3\mathrm{e}^{6i(dx+c)}+12ia\,b^2\mathrm{e}^{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/16/a^4*(1/4*a^3*tan(1/2*d*x+1/2*c)^4-2/3*b*tan(1/2*d*x+1/2*c)^3*a^2-
4*a^3*tan(1/2*d*x+1/2*c)^2+2*a*b^2*tan(1/2*d*x+1/2*c)^2+18*a^2*b*tan(1/2*d*
x+1/2*c)-8*b^3*tan(1/2*d*x+1/2*c))-2/b*arctan(tan(1/2*d*x+1/2*c))-1/64/a/ta
n(1/2*d*x+1/2*c)^4-1/32*(-8*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^2+1/16/a^5*(3
0*a^4-40*a^2*b^2+16*b^4)*ln(tan(1/2*d*x+1/2*c))+1/24/a^2*b/tan(1/2*d*x+1/2*
c)^3-1/8*b*(9*a^2-4*b^2)/a^4/tan(1/2*d*x+1/2*c)+1/16*(32*a^6-96*a^4*b^2+96*
a^2*b^4-32*b^6)/a^5/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*
b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(185) = 370$.
time = 0.68, size = 1034, normalized size = 5.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/48*(48*a^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x
+ 6*(9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - 24*((a^4 - 2*a^2*b^2 + b^4)*cos(
```

```
d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)
^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(7*a^4*b
 - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b -
 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos
(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5
 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^
3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*b^2 -
3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))
/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d), -1/48*(48*a
^5*d*x*cos(d*x + c)^4 - 96*a^5*d*x*cos(d*x + c)^2 + 48*a^5*d*x + 6*(9*a^4*b
 - 4*a^2*b^3)*cos(d*x + c)^3 + 48*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 +
 a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(7
*a^4*b - 4*a^2*b^3)*cos(d*x + c) + 3*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a
^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^
5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 3*(15*a^4*b - 20*a^2*b^3 +
 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*
a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 16*((7*a^3*
b^2 - 3*a*b^4)*cos(d*x + c)^3 - 3*(2*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x
 + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*x + c)^2 + a^5*b*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(185) = 370.
time = 0.49, size = 396, normalized size = 2.03

$$-\frac{\frac{192\,(dx+c)}{b}-\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-8\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+216\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-96\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{24\,(15\,a^4-20\,a^2b^2+8\,b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5}-\frac{384\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5b}+\frac{750\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1000\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+400\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+216\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-96\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^4}{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(192*(d*x + c)/b - (3*a^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d
*x + 1/2*c)^3 - 48*a^3*tan(1/2*d*x + 1/2*c)^2 + 24*a*b^2*tan(1/2*d*x + 1/2*
c)^2 + 216*a^2*b*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/a^4 -
24*(15*a^4 - 20*a^2*b^2 + 8*b^4)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 - 384*(
```

```
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
 + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a
^5*b) + (750*a^4*tan(1/2*d*x + 1/2*c)^4 - 1000*a^2*b^2*tan(1/2*d*x + 1/2*c)
^4 + 400*b^4*tan(1/2*d*x + 1/2*c)^4 + 216*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 96
*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 48*a^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*b^2*
tan(1/2*d*x + 1/2*c)^2 - 8*a^3*b*tan(1/2*d*x + 1/2*c) + 3*a^4)/(a^5*tan(1/2
*d*x + 1/2*c)^4))/d
```

Mupad [B]
time = 13.40, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^4/(64*a*d) - (2*atan(-(((((((4*(64*a^9*b^9 - 208*a^11*b^
7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (((4*(32*a^14*b^5 - 24*a^16*b^3))/a
^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^
12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b
^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*(24*a^16*b - 32*a^4*b^13
 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))
/a^12 + (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9
*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^
15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/
a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12
 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2
))/a^12)/b + ((((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 +
543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (((4*(64*a^9*b^9 - 208*a
^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b
^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^
2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*
a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*tan(c/2 + (d*x)/2)
*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^1
3*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^
9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*
(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8
 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)/b)/((((((((4*(64*a^9
*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (((4*(32*a^14*b^5
 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4
+ 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^1
0*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*(24*a^
16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b
^5 + 58*a^14*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*
a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)
```

1i)/b - (4(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)*1i)/b - (((((4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + (((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 - (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*1i)/b + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12)*1i)/b - (4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12)*1i)/b - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12)*1i)/b - (8*(15*a^14*b + 8*b^15 - 68*a^2*b^13 + 223*a^4*b^11 - 366*a^6*b^9 + 305*a^8*b^7 - 97*a^10*b^5 - 20*a^12*b^3))/a^12 + (8*tan(c/2 + (d*x)/2)*(8*a^7*b^8 - 4*a^15 - 20*a^9*b^6 + 12*a^11*b^4 + 4*a^13*b^2))/a^12)))/(b*d) - (tan(c/2 + (d*x)/2)^2*(1/(4*a) - b^2/(8*a^3)))/d + (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(1/(2*a) - b^2/(4*a^3)))/a))/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^3*(18*a^2*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2 + (d*x)/2)^4) + (log(tan(c/2 + (d*x)/2))*((15*a^4)/8 + b^4 - (5*a^2*b^2)/2))/(a^5*d) - (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b + 8*a^5*b^11 - 48*a^7*b^9 + 56*a^9*b^7 + 48*a^11*b^5 - 120*a^13*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(16*a^17 + 8*a^5*b^12 - 40*a^7*b^10 + 100*a^9*b^8 - 148*a^11*b^6 + 252*a^13*b^4 - 180*a^15*b^2))/a^12 - (4*(24*a^16*b - 32*a^4*b^13 + 184*a^6*b^11 - 440*a^8*b^9 + 543*a^10*b^7 - 345*a^12*b^5 + 58*a^14*b^3))/a^12 + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(64*a^9*b^9 - 208*a^11*b^7 + 240*a^13*b^5 - 93*a^15*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^8*b^10 - 456*a^10*b^8 + 604*a^12*b^6 - 335*a^14*b^4 + 62*a^16*b^2))/a^12 + (((4*(32*a^14*b^5 - 24*a^16*b^3))/a^12 + (4*tan(c/2 + (d*x)/2)*(128*a^13*b^6 - 136*a^15*b^4 + 16*a^17*b^2))/a^12)*(-(a + b)^5*(a - b)^5)^(1/2))/(a^5*b)))/(a^5*b)))/(a^5*b))*1i)/(a^5*b) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*tan(c/2 + (d*x)/2)*(62*a^16 + 8*b^16 - 68*a^2*b^14 + 255*a^4*b^12 - 550*a^6*b^10 + 873*a^8*b^8 - 1096*a^10*b^6 + 929*a^12*b^4 - 410*a^14*b^2))/a^12 - (4*(53*a^15*b + 8*a^5*b^11 - 48*...

3.1328 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=241

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{\left(23a^4-35a^2b^2+15b\right.}{15a^5d}$$

```
[Out] -2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+1
/8*b*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(23*a^4-35*a^
2*b^2+15*b^4)*cot(d*x+c)/a^5/d+1/8*b*(-9*a^2+4*b^2)*cot(d*x+c)*csc(d*x+c)/a
^4/d+1/15*(11*a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)^2/a^3/d+1/4*b*cot(d*x+c)*csc
(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

Rubi [A]
time = 0.72, antiderivative size = 307, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2805, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}-\frac{2\left(a^2-b^2\right)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}+\frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd}+\frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d}-\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d}-\frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^3b^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2805

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x]
)^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m -
 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m
*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x]
, x], x] + Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Sin[e + f
*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x]) /; FreeQ[{a, b, e, f, m}, x] &&
 NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
```

```
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx}{4a^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{(15a^4-22a^2b^2+10b^4)\,c}{30a^3}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c}{}$$

$$= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+}{}$$

$$= -\frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+}{}$$

$$= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d}$$

$$= \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d}$$

$$= -\frac{2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b(15a^4-20a^2b^2+8b^4)\tanh^{-1}(\cos(c+a}{8a^6d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 504 vs. 2(241) = 482.
time = 0.95, size = 504, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
```

```
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [A]
time = 0.57, size = 390, normalized size = 1.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/32/a^5*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-1/2*b*tan(1/2*d*x+1/2*c)^4*a^3-
7/3*a^4*tan(1/2*d*x+1/2*c)^3+4/3*a^2*b^2*tan(1/2*d*x+1/2*c)^3+8*a^3*b*tan(1
/2*d*x+1/2*c)^2-4*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-36*a
^2*b^2*tan(1/2*d*x+1/2*c)+16*b^4*tan(1/2*d*x+1/2*c))-1/160/a/tan(1/2*d*x+1/
2*c)^5-1/96*(-7*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-36*a^2*b^2
+16*b^4)/a^5/tan(1/2*d*x+1/2*c)+1/64/a^2*b/tan(1/2*d*x+1/2*c)^4-1/8/a^4*b*(
2*a^2-b^2)/tan(1/2*d*x+1/2*c)^2-1/8/a^6*b*(15*a^4-20*a^2*b^2+8*b^4)*ln(tan(
1/2*d*x+1/2*c))+1/32/a^6*(-64*a^6+192*a^4*b^2-192*a^2*b^4+64*b^6)/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(224) = 448.
time = 0.69, size = 1079, normalized size = 4.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
```

```
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(224) = 448.
time = 0.47, size = 490, normalized size = 2.03

$$\frac{\frac{6\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+240\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-120\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+660\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1080\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+480\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{5}}-\frac{120\,\left(15\,a^{4}b-20\,a^{2}b^{3}+8\,b^{5}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^{6}}-\frac{1920\,\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^{2}-b^{2}}}\right)\right)}{\sqrt{a^{2}-b^{2}}\,a^{6}}+\frac{4110\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-5480\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+2192\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-660\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+1080\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-480\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-240\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+120\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+70\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-40\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+15\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^{5}}{a^{6}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
```

```
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

Mupad [B]
time = 12.39, size = 1099, normalized size = 4.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^6*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(7/
(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 11/(16*a)
 + (2*b*(b/(16*a^2) + (2*b*(7/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/2 +
 (d*x)/2)^3*(7/(96*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64*a^2
*d) - (log(tan(c/2 + (d*x)/2))*((15*a^4*b)/8 + b^5 - (5*a^2*b^3)/2))/(a^6*d
) + (tan(c/2 + (d*x)/2)^2*((7*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan(c/2 + (
d*x)/2)^4*(22*a^4 + 16*b^4 - 36*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*a*b^3 -
8*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/2)^5) -
(atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 3
9*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b
^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a
^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6 + ((-(a + b
)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(4*a
^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))*(-
(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^4*b
^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6)/((15*a^10*b - 8*b^11 + 44*
a^2*b^9 - 99*a^4*b^7 + 113*a^6*b^5 - 65*a^8*b^3)/(2*a^10) + (tan(c/2 + (d*x
)/2)*(16*a^10 - 8*b^10 + 42*a^2*b^8 - 94*a^4*b^6 + 110*a^6*b^4 - 66*a^8*b^2
))/(2*a^9) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b
^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*
a^10*b^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)
*(31*a^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6 + ((-(a
+ b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(
4*a^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2))/(4*a^9))
*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^10*b - 32*a^
```

```
4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9)))/a^6))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^6*d)
```

3.1329 $\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=363

$$\frac{2b(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d}+\frac{b(23a^4-35a}{}$$

```
[Out] 2*b*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^7/d+
1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*arctanh(cos(d*x+c))/a^7/d+1/15*b*
(23*a^4-35*a^2*b^2+15*b^4)*cot(d*x+c)/a^6/d-1/16*(11*a^4-18*a^2*b^2+8*b^4)*
cot(d*x+c)*csc(d*x+c)/a^5/d-1/2*cot(d*x+c)*csc(d*x+c)^2/b/d+1/30*(15*a^4-22
*a^2*b^2+10*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^4/b/d+1/3*a*cot(d*x+c)*csc(d*x+c
)^3/b^2/d-1/24*(8*a^4-13*a^2*b^2+6*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^3/b^2/d+1
/5*b*cot(d*x+c)*csc(d*x+c)^4/a^2/d-1/6*cot(d*x+c)*csc(d*x+c)^5/a/d
```

Rubi [A]
time = 0.94, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^4(c+dx)}{5a^2d}+\frac{2b(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7d}+\frac{(15a^4-22a^2b^2+10b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4bd}+\frac{b(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^6d}-\frac{(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^5d}-\frac{(8a^4-13a^2b^2+6b^4)\cot(c+dx)\csc^3(c+dx)}{24a^3b^2d}+\frac{(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d}+\frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d}-\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^7*d) + ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*x]])/
(16*a^7*d) + (b*(23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^6*d) - (
(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*x])/(16*a^5*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(2*b*d) + ((15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c +
d*x]*Csc[c + d*x]^2)/(30*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^3)/(3*b^2*
d) - ((8*a^4 - 13*a^2*b^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^3*b^2
*d) + (b*Cot[c + d*x]*Csc[c + d*x]^4)/(5*a^2*d) - (Cot[c + d*x]*Csc[c + d*x
]^5)/(6*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} + \frac{b\cot(c+dx)\,\mathrm{cs}}{5a^2d}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{a\cot(c+dx)\csc^3(c+dx)}{3b^2d} - \frac{(8a^4 - 13a^2b^2}{}$$

$$= -\frac{\cot(c+dx)\csc^2(c+dx)}{2bd} + \frac{(15a^4 - 22a^2b^2 + 10b^4)\cot(c+dx)\csc^2(c+}{30a^4bd}$$

$$= -\frac{(11a^4 - 18a^2b^2 + 8b^4)\cot(c+dx)\csc(c+dx)}{16a^5d} - \frac{\cot(c+dx)\csc^2(c+dx}{2bd}$$

$$= \frac{b(23a^4 - 35a^2b^2 + 15b^4)\cot(c+dx)}{15a^6d} - \frac{(11a^4 - 18a^2b^2 + 8b^4)\cot(c+dx)}{16a^5d}$$

$$= \frac{b(23a^4 - 35a^2b^2 + 15b^4)\cot(c+dx)}{15a^6d} - \frac{(11a^4 - 18a^2b^2 + 8b^4)\cot(c+dx)}{16a^5d}$$

$$= \frac{(5a^6 - 30a^4b^2 + 40a^2b^4 - 16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b(23a^4 - 35a^2b^2 +}{15a}$$

$$= \frac{(5a^6 - 30a^4b^2 + 40a^2b^4 - 16b^6)\tanh^{-1}(\cos(c+dx))}{16a^7d} + \frac{b(23a^4 - 35a^2b^2 +}{15a}$$

$$= \frac{2b(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{(5a^6 - 30a^4b^2 + 40a^2b^4 - 16b^6)\,\mathrm{t}}{16a^7d}$$

Mathematica [A]

time = 1.02, size = 356, normalized size = 0.98

$$\frac{7680b(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + 240(5a^6-30a^4b^2+40a^2b^4-16b^6)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 240(-5a^6+30a^4b^2-40a^2b^4+16b^6)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2a\cot(c+dx)\csc^5(c+dx)\left(-295a^5+570a^3b^2-360ab^4+20(7a^5-42a^3b^2+24ab^4)\cos(2(c+dx)) - 15(11a^5-18a^3b^2+8ab^4)\cos(4(c+dx)) + 1168a^4b\sin(c+dx) - 2320a^2b^3\sin(c+dx) + 1200b^5\sin(c+dx) - 568a^4b\sin(3(c+dx)) + 1240a^2b^3\sin(3(c+dx)) - 600b^5\sin(3(c+dx)) + 184a^4b\sin(5(c+dx)) - 280a^2b^3\sin(5(c+dx)) + 120b^5\sin(5(c+dx))\right)}{3840a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (7680*b*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
+ 240*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Cos[(c + d*x)/2]] + 24
0*(-5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6)*Log[Sin[(c + d*x)/2]] + 2*a*C
ot[c + d*x]*Csc[c + d*x]^5*(-295*a^5 + 570*a^3*b^2 - 360*a*b^4 + 20*(7*a^5
- 42*a^3*b^2 + 24*a*b^4)*Cos[2*(c + d*x)] - 15*(11*a^5 - 18*a^3*b^2 + 8*a*b
^4)*Cos[4*(c + d*x)] + 1168*a^4*b*Sin[c + d*x] - 2320*a^2*b^3*Sin[c + d*x]
+ 1200*b^5*Sin[c + d*x] - 568*a^4*b*Sin[3*(c + d*x)] + 1240*a^2*b^3*Sin[3*(
c + d*x)] - 600*b^5*Sin[3*(c + d*x)] + 184*a^4*b*Sin[5*(c + d*x)] - 280*a^2
*b^3*Sin[5*(c + d*x)] + 120*b^5*Sin[5*(c + d*x)]))/(3840*a^7*d)
```

Maple [**A**]
time = 0.62, size = 486, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/64/a^6*(1/6*a^5*tan(1/2*d*x+1/2*c)^6-2/5*b*tan(1/2*d*x+1/2*c)^5*a^4-
3/2*a^5*tan(1/2*d*x+1/2*c)^4+a^3*b^2*tan(1/2*d*x+1/2*c)^4+14/3*a^4*b*tan(1/
2*d*x+1/2*c)^3-8/3*a^2*b^3*tan(1/2*d*x+1/2*c)^3+15/2*a^5*tan(1/2*d*x+1/2*c)
^2-16*a^3*b^2*tan(1/2*d*x+1/2*c)^2+8*a*b^4*tan(1/2*d*x+1/2*c)^2-44*a^4*b*ta
n(1/2*d*x+1/2*c)+72*a^2*b^3*tan(1/2*d*x+1/2*c)-32*b^5*tan(1/2*d*x+1/2*c))-1
/384/a/tan(1/2*d*x+1/2*c)^6-1/256*(-6*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^4-1
/128/a^5*(15*a^4-32*a^2*b^2+16*b^4)/tan(1/2*d*x+1/2*c)^2+1/64/a^7*(-20*a^6+
120*a^4*b^2-160*a^2*b^4+64*b^6)*ln(tan(1/2*d*x+1/2*c))+1/160/a^2*b/tan(1/2*
d*x+1/2*c)^5-1/96/a^4*b*(7*a^2-4*b^2)/tan(1/2*d*x+1/2*c)^3+1/16*b*(11*a^4-1
8*a^2*b^2+8*b^4)/a^6/tan(1/2*d*x+1/2*c)+2*b*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a
^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(340) = 680$.

time = 0.82, size = 1462, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c)^5 - 80*(5*a^6 - 1
2*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 + 240*((a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*log(-
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d
*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*
cos(d*x + c) + 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^
6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a
^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b
^6)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 +
 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16
*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c
) + 1/2) - 32*((23*a^5*b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5
*b - 13*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*
cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4
+ 3*a^7*d*cos(d*x + c)^2 - a^7*d), 1/480*(30*(11*a^6 - 18*a^4*b^2 + 8*a^2*b
^4)*cos(d*x + c)^5 - 80*(5*a^6 - 12*a^4*b^2 + 6*a^2*b^4)*cos(d*x + c)^3 - 4
80*((a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^6 - a^4*b + 2*a^2*b^3 - b^5 - 3*
(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(a^4*b - 2*a^2*b^3 + b^5)*cos(
d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c))) + 30*(5*a^6 - 14*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c) + 15*((5*a
^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^6 - 5*a^6 + 30*a^4*b^2
- 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*
x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^2)*log
(1/2*cos(d*x + c) + 1/2) - 15*((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*c
os(d*x + c)^6 - 5*a^6 + 30*a^4*b^2 - 40*a^2*b^4 + 16*b^6 - 3*(5*a^6 - 30*a^
4*b^2 + 40*a^2*b^4 - 16*b^6)*cos(d*x + c)^4 + 3*(5*a^6 - 30*a^4*b^2 + 40*a^
2*b^4 - 16*b^6)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 32*((23*a^5*
b - 35*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^5 - 5*(7*a^5*b - 13*a^3*b^3 + 6*a*b
^5)*cos(d*x + c)^3 + 15*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x +
 c))/(a^7*d*cos(d*x + c)^6 - 3*a^7*d*cos(d*x + c)^4 + 3*a^7*d*cos(d*x + c)^
2 - a^7*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 627, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/1920*((5*a^5*tan(1/2*d*x + 1/2*c)^6 - 12*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 4
5*a^5*tan(1/2*d*x + 1/2*c)^4 + 30*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 140*a^4*
b*tan(1/2*d*x + 1/2*c)^3 - 80*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 225*a^5*tan(
1/2*d*x + 1/2*c)^2 - 480*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 240*a*b^4*tan(1/2
*d*x + 1/2*c)^2 - 1320*a^4*b*tan(1/2*d*x + 1/2*c) + 2160*a^2*b^3*tan(1/2*d*
x + 1/2*c) - 960*b^5*tan(1/2*d*x + 1/2*c))/a^6 - 120*(5*a^6 - 30*a^4*b^2 +
40*a^2*b^4 - 16*b^6)*log(abs(tan(1/2*d*x + 1/2*c)))/a^7 + 3840*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^7) + (14
70*a^6*tan(1/2*d*x + 1/2*c)^6 - 8820*a^4*b^2*tan(1/2*d*x + 1/2*c)^6 + 11760
*a^2*b^4*tan(1/2*d*x + 1/2*c)^6 - 4704*b^6*tan(1/2*d*x + 1/2*c)^6 + 1320*a^
5*b*tan(1/2*d*x + 1/2*c)^5 - 2160*a^3*b^3*tan(1/2*d*x + 1/2*c)^5 + 960*a*b^
5*tan(1/2*d*x + 1/2*c)^5 - 225*a^6*tan(1/2*d*x + 1/2*c)^4 + 480*a^4*b^2*tan
(1/2*d*x + 1/2*c)^4 - 240*a^2*b^4*tan(1/2*d*x + 1/2*c)^4 - 140*a^5*b*tan(1/
2*d*x + 1/2*c)^3 + 80*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 45*a^6*tan(1/2*d*x +
 1/2*c)^2 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^5*b*tan(1/2*d*x + 1/2*
c) - 5*a^6)/(a^7*tan(1/2*d*x + 1/2*c)^6))/d
```

Mupad [B]
time = 12.34, size = 1289, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^6/(384*a*d) + (tan(c/2 + (d*x)/2)^3*(b/(96*a^2) + (2*b*(
3/(32*a) - b^2/(16*a^3)))/(3*a)))/d - (tan(c/2 + (d*x)/2)^4*(3/(128*a) - b^
2/(64*a^3)))/d - (tan(c/2 + (d*x)/2)*(b/(32*a^2) - (2*b*(b^2/(16*a^3) - 15/
(64*a) + (2*b*(b/(32*a^2) + (2*b*(3/(32*a) - b^2/(16*a^3)))/a))/a))/a + (2*
b*(3/(32*a) - b^2/(16*a^3)))/a))/d - (tan(c/2 + (d*x)/2)^2*(b^2/(32*a^3) -
15/(128*a) + (b*(b/(32*a^2) + (2*b*(3/(32*a) - b^2/(16*a^3)))/a))/a))/d - (
```

```
tan(c/2 + (d*x)/2)^3*((14*a^4*b)/3 - (8*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*
(8*a*b^4 + (15*a^5)/2 - 16*a^3*b^2) - tan(c/2 + (d*x)/2)^5*(44*a^4*b + 32*b
^5 - 72*a^2*b^3) + a^5/6 - tan(c/2 + (d*x)/2)^2*((3*a^5)/2 - a^3*b^2) - (2*
a^4*b*tan(c/2 + (d*x)/2))/5)/(64*a^6*d*tan(c/2 + (d*x)/2)^6) - (b*tan(c/2 +
 (d*x)/2)^5)/(160*a^2*d) - (log(tan(c/2 + (d*x)/2))*(5*a^6 - 16*b^6 + 40*a^
2*b^4 - 30*a^4*b^2))/(16*a^7*d) + (b*atan(((b*(-(a + b)^5*(a - b)^5)^(1/2)*
((tan(c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*
a^11*b^2))/(8*a^11) - (21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(
8*a^12) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8*a^1
1))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7)*1i)/a^7 - (b*(-(a + b)^5*(a - b)^5)^
(1/2)*((21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(8*a^12) - (tan(
c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*a^11*b
^2))/(8*a^11) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/
(8*a^11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7)*1i)/a^7)/((5*a^12*b + 16*b^13
- 88*a^2*b^11 + 198*a^4*b^9 - 231*a^6*b^7 + 145*a^8*b^5 - 45*a^10*b^3)/(4*a
^12) + (tan(c/2 + (d*x)/2)*(16*b^12 - 84*a^2*b^10 + 178*a^4*b^8 - 190*a^6*b
^6 + 102*a^8*b^4 - 22*a^10*b^2))/(4*a^11) + (b*(-(a + b)^5*(a - b)^5)^(1/2)
*((tan(c/2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72
*a^11*b^2))/(8*a^11) - (21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/
(8*a^12) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8*a^
11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7))/a^7 + (b*(-(a + b)^5*(a - b)^5)^(1
/2)*((21*a^13*b - 32*a^7*b^7 + 88*a^9*b^5 - 78*a^11*b^3)/(8*a^12) - (tan(c/
2 + (d*x)/2)*(5*a^13 + 64*a^5*b^8 - 192*a^7*b^6 + 196*a^9*b^4 - 72*a^11*b^2
))/(8*a^11) + (b*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^14 - 64*a^12*b^2))/(8
*a^11))*(-(a + b)^5*(a - b)^5)^(1/2))/a^7))/a^7))*(-(a + b)^5*(a - b)^5)^(1
/2)*2i)/(a^7*d)
```

3.1330 $\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=417

$$-\frac{2b^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{b(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d}+\frac{(15a^6-16}{}$$

```
[Out] -2*b^2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^8
/d-1/16*b*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*arctanh(cos(d*x+c))/a^8/d+1/
105*(15*a^6-161*a^4*b^2+245*a^2*b^4-105*b^6)*cot(d*x+c)/a^7/d+1/16*b*(11*a^
4-18*a^2*b^2+8*b^4)*cot(d*x+c)*csc(d*x+c)/a^6/d-1/105*(45*a^4-77*a^2*b^2+35
*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^5/d-1/3*cot(d*x+c)*csc(d*x+c)^3/b/d+1/24*(8
*a^4-13*a^2*b^2+6*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^4/b/d+1/4*a*cot(d*x+c)*csc
(d*x+c)^4/b^2/d-1/140*(35*a^4-60*a^2*b^2+28*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^
3/b^2/d+1/6*b*cot(d*x+c)*csc(d*x+c)^5/a^2/d-1/7*cot(d*x+c)*csc(d*x+c)^6/a/d
```

Rubi [A]

time = 1.17, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^5(c+dx)}{6a^2d}-\frac{2b^2(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}+\frac{(8a^4-13a^2b^2+6b^4)\cot(c+dx)\csc^3(c+dx)}{24a^4bd}+\frac{b(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^6d}-\frac{(45a^4-77a^2b^2+35b^4)\cot(c+dx)\csc^2(c+dx)}{105a^5d}-\frac{(35a^4-60a^2b^2+28b^4)\cot(c+dx)\csc^4(c+dx)}{140a^3b^2d}-\frac{b(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d}+\frac{(15a^6-161a^4b^2+245a^2b^4-105b^6)\cot(c+dx)}{105a^7d}+\frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d}-\frac{\cot(c+dx)\csc^6(c+dx)}{7ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/(a^8*d) - (b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*ArcTanh[Cos[c + d*
x]])/(16*a^8*d) + ((15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 105*b^6)*Cot[c + d
*x])/(105*a^7*d) + (b*(11*a^4 - 18*a^2*b^2 + 8*b^4)*Cot[c + d*x]*Csc[c + d*
x])/(16*a^6*d) - ((45*a^4 - 77*a^2*b^2 + 35*b^4)*Cot[c + d*x]*Csc[c + d*x]^
2)/(105*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(3*b*d) + ((8*a^4 - 13*a^2*b
^2 + 6*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(24*a^4*b*d) + (a*Cot[c + d*x]*Csc
[c + d*x]^4)/(4*b^2*d) - ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c
 + d*x]^4)/(140*a^3*b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^5)/(6*a^2*d) - (C
ot[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd}+\frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d}+\frac{b\cot(c+dx)\csc}{6a^2d}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd}+\frac{a\cot(c+dx)\csc^4(c+dx)}{4b^2d}-\frac{(35a^4-60a^2b^2}{}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{3bd}+\frac{(8a^4-13a^2b^2+6b^4)\cot(c+dx)\csc^3(c+dx)}{24a^4bd}$$

$$= -\frac{(45a^4-77a^2b^2+35b^4)\cot(c+dx)\csc^2(c+dx)}{105a^5d}-\frac{\cot(c+dx)\csc^3(c+}{3bd}$$

$$= \frac{b(11a^4-18a^2b^2+8b^4)\cot(c+dx)\csc(c+dx)}{16a^6d}-\frac{(45a^4-77a^2b^2+35b^4)}{105}$$

$$= \frac{(15a^6-161a^4b^2+245a^2b^4-105b^6)\cot(c+dx)}{105a^7d}+\frac{b(11a^4-18a^2b^2+8b^4)}{16}$$

$$= \frac{(15a^6-161a^4b^2+245a^2b^4-105b^6)\cot(c+dx)}{105a^7d}+\frac{b(11a^4-18a^2b^2+8b^4)}{16}$$

$$= -\frac{b(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d}+\frac{(15a^6-161a^4b^2}{}$$

$$= -\frac{b(5a^6-30a^4b^2+40a^2b^4-16b^6)\tanh^{-1}(\cos(c+dx))}{16a^8d}+\frac{(15a^6-161a^4b^2}{}$$

$$= -\frac{2b^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{b(5a^6-30a^4b^2+40a^2b^4-16}{16a^8d}$$

Mathematica [A]

time = 1.36, size = 442, normalized size = 1.06

$$\frac{-107520b^2(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+3360(-5a^6b+30a^4b^3-40a^2b^5+16b^7)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+3360b(5a^6-30a^4b^2+40a^2b^4-16b^6)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2a\cot(c+dx)\csc^6(c+dx)\left(1200a^6+8176a^4b^2-16240a^2b^4+8400b^6+8(225a^6-1519a^4b^2+3115a^2b^4-1575b^6)\cos(2(c+dx))+16(45a^6+329a^4b^2-665a^2b^4+315b^6)\cos(4(c+dx))+120a^6\cos(6(c+dx))-1288a^4b^2\cos(6(c+dx))+1960a^2b^4\cos(6(c+dx))-840b^6\cos(6(c+dx))-5110a^5b\sin(c+dx)+13860a^3b^3\sin(c+dx)-8400ab^5\sin(c+dx)+2135a^5b\sin(3(c+dx))-7770a^3b^3\sin(3(c+dx))+4200ab^5\sin(3(c+dx))-1155a^5b\sin(5(c+dx))+1890a^3b^3\sin(5(c+dx))-840ab^5\sin(5(c+dx))\right)}{53760a^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-107520*b^2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 3360*(-5*a^6*b + 30*a^4*b^3 - 40*a^2*b^5 + 16*b^7)*Log[Cos[(c + d*x)
/2]] + 3360*b*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*Log[Sin[(c + d*x)/
2]] - 2*a*Cot[c + d*x]*Csc[c + d*x]^6*(1200*a^6 + 8176*a^4*b^2 - 16240*a^2*
b^4 + 8400*b^6 + 8*(225*a^6 - 1519*a^4*b^2 + 3115*a^2*b^4 - 1575*b^6)*Cos[2
*(c + d*x)] + 16*(45*a^6 + 329*a^4*b^2 - 665*a^2*b^4 + 315*b^6)*Cos[4*(c +
d*x)] + 120*a^6*Cos[6*(c + d*x)] - 1288*a^4*b^2*Cos[6*(c + d*x)] + 1960*a^2
*b^4*Cos[6*(c + d*x)] - 840*b^6*Cos[6*(c + d*x)] - 5110*a^5*b*Sin[c + d*x]
+ 13860*a^3*b^3*Sin[c + d*x] - 8400*a*b^5*Sin[c + d*x] + 2135*a^5*b*Sin[3*(
c + d*x)] - 7770*a^3*b^3*Sin[3*(c + d*x)] + 4200*a*b^5*Sin[3*(c + d*x)] - 1
155*a^5*b*Sin[5*(c + d*x)] + 1890*a^3*b^3*Sin[5*(c + d*x)] - 840*a*b^5*Sin[
5*(c + d*x)]))/(53760*a^8*d)
```

Maple [A]

time = 0.70, size = 604, normalized size = 1.45

method	result
derivativedivides	$\frac{\frac{a^6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^5}{3}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^6+\frac{4a^4b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+3b\,a^5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a^3b^3\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{}$
default	$\frac{\frac{a^6\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^5}{3}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^6+\frac{4a^4b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+3b\,a^5\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2a^3b^3\left(\tan^4\left(\frac{dx}{2}+\right.\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/128/a^7*(1/7*a^6*tan(1/2*d*x+1/2*c)^7-1/3*b*tan(1/2*d*x+1/2*c)^6*a^5
-tan(1/2*d*x+1/2*c)^5*a^6+4/5*a^4*b^2*tan(1/2*d*x+1/2*c)^5+3*b*a^5*tan(1/2*
d*x+1/2*c)^4-2*a^3*b^3*tan(1/2*d*x+1/2*c)^4+3*a^6*tan(1/2*d*x+1/2*c)^3-28/3
*a^4*b^2*tan(1/2*d*x+1/2*c)^3+16/3*a^2*b^4*tan(1/2*d*x+1/2*c)^3-15*a^5*b*ta
n(1/2*d*x+1/2*c)^2+32*a^3*b^3*tan(1/2*d*x+1/2*c)^2-16*a*b^5*tan(1/2*d*x+1/2
*c)^2-5*a^6*tan(1/2*d*x+1/2*c)+88*a^4*b^2*tan(1/2*d*x+1/2*c)-144*a^2*b^4*ta
n(1/2*d*x+1/2*c)+64*b^6*tan(1/2*d*x+1/2*c))-1/896/a/tan(1/2*d*x+1/2*c)^7-1/
```

```
640*(-5*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)^5-1/384/a^5*(9*a^4-28*a^2*b^2+16*
b^4)/tan(1/2*d*x+1/2*c)^3-1/128*(-5*a^6+88*a^4*b^2-144*a^2*b^4+64*b^6)/a^7/
tan(1/2*d*x+1/2*c)+1/384/a^2*b/tan(1/2*d*x+1/2*c)^6-1/128/a^4*b*(3*a^2-2*b^
2)/tan(1/2*d*x+1/2*c)^4+1/128/a^6*b*(15*a^4-32*a^2*b^2+16*b^4)/tan(1/2*d*x+
1/2*c)^2+1/16/a^8*b*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*ln(tan(1/2*d*x+1/2
*c))-2*b^2*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.85, size = 1645, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/3360*(32*(15*a^7 - 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7
 + 224*(58*a^5*b^2 - 100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*
b^2 - 19*a^3*b^4 + 9*a*b^6)*cos(d*x + c)^3 + 1680*((a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^6 - a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b
^6)*cos(d*x + c)^4 + 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(-a^
2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) + 105*(5*a^6
*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5
- 16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*c
os(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)
^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 +
40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x
+ c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*
(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x +
c) - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b
```

```
- 12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*
b^5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x +
 c)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c)), 1/3360*(32*(15*a^7 -
 161*a^5*b^2 + 245*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^7 + 224*(58*a^5*b^2 -
100*a^3*b^4 + 45*a*b^6)*cos(d*x + c)^5 - 1120*(10*a^5*b^2 - 19*a^3*b^4 + 9*
a*b^6)*cos(d*x + c)^3 + 3360*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^6 -
a^4*b^2 + 2*a^2*b^4 - b^6 - 3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^4 +
3*(a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*si
n(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) + 105*(5*a^6*b
 - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 -
16*b^7)*cos(d*x + c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos
(d*x + c)^4 - 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2
)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 105*(5*a^6*b - 30*a^4*b^3 + 40
*a^2*b^5 - 16*b^7 - (5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x +
c)^6 + 3*(5*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^4 - 3*(5
*a^6*b - 30*a^4*b^3 + 40*a^2*b^5 - 16*b^7)*cos(d*x + c)^2)*log(-1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 3360*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)
 - 70*(3*(11*a^6*b - 18*a^4*b^3 + 8*a^2*b^5)*cos(d*x + c)^5 - 8*(5*a^6*b -
12*a^4*b^3 + 6*a^2*b^5)*cos(d*x + c)^3 + 3*(5*a^6*b - 14*a^4*b^3 + 8*a^2*b^
5)*cos(d*x + c))*sin(d*x + c))/((a^8*d*cos(d*x + c)^6 - 3*a^8*d*cos(d*x + c
)^4 + 3*a^8*d*cos(d*x + c)^2 - a^8*d)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 776, normalized size = 1.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/13440*((15*a^6*tan(1/2*d*x + 1/2*c)^7 - 35*a^5*b*tan(1/2*d*x + 1/2*c)^6 -
 105*a^6*tan(1/2*d*x + 1/2*c)^5 + 84*a^4*b^2*tan(1/2*d*x + 1/2*c)^5 + 315*a
^5*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 + 315*a^6*
tan(1/2*d*x + 1/2*c)^3 - 980*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 560*a^2*b^4*t
an(1/2*d*x + 1/2*c)^3 - 1575*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 3360*a^3*b^3*ta
n(1/2*d*x + 1/2*c)^2 - 1680*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 525*a^6*tan(1/2*
```

```
d*x + 1/2*c) + 9240*a^4*b^2*tan(1/2*d*x + 1/2*c) - 15120*a^2*b^4*tan(1/2*d*
x + 1/2*c) + 6720*b^6*tan(1/2*d*x + 1/2*c))/a^7 + 840*(5*a^6*b - 30*a^4*b^3
 + 40*a^2*b^5 - 16*b^7)*log(abs(tan(1/2*d*x + 1/2*c)))/a^8 - 26880*(a^6*b^2
 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^8)
 - (10890*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 65340*a^4*b^3*tan(1/2*d*x + 1/2*c)
^7 + 87120*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 34848*b^7*tan(1/2*d*x + 1/2*c)^
7 - 525*a^7*tan(1/2*d*x + 1/2*c)^6 + 9240*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 -
15120*a^3*b^4*tan(1/2*d*x + 1/2*c)^6 + 6720*a*b^6*tan(1/2*d*x + 1/2*c)^6 -
1575*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 3360*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 1
680*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 315*a^7*tan(1/2*d*x + 1/2*c)^4 - 980*a
^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 560*a^3*b^4*tan(1/2*d*x + 1/2*c)^4 + 315*a^
6*b*tan(1/2*d*x + 1/2*c)^3 - 210*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 105*a^7*t
an(1/2*d*x + 1/2*c)^2 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 35*a^6*b*tan(1/
2*d*x + 1/2*c) + 15*a^7)/(a^8*tan(1/2*d*x + 1/2*c)^7))/d
```

Mupad [B]
time = 12.38, size = 1513, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^8*(a + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(b^2/(32*a^3) - 5/(128*a) + (2*b*(b/(64*a^2) + (2*b*(5/
(128*a) - b^2/(32*a^3)))/a))/a + (2*b*(b/(64*a^2) - (2*b*(b^2/(32*a^3) - 9/
(128*a) + (2*b*(b/(64*a^2) + (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/a + (
2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/d + tan(c/2 + (d*x)/2)^7/(896*a*d)
+ (tan(c/2 + (d*x)/2)^4*(b/(256*a^2) + (b*(5/(128*a) - b^2/(32*a^3)))/(2*a)
))/d - (tan(c/2 + (d*x)/2)^2*(b/(128*a^2) - (b*(b^2/(32*a^3) - 9/(128*a) +
(2*b*(b/(64*a^2) + (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/a))/a + (b*(5/(128*
a) - b^2/(32*a^3)))/a))/d - (tan(c/2 + (d*x)/2)^5*(1/(128*a) - b^2/(160*a^3
)))/d - (tan(c/2 + (d*x)/2)^3*(b^2/(96*a^3) - 3/(128*a) + (2*b*(b/(64*a^2)
+ (2*b*(5/(128*a) - b^2/(32*a^3)))/a))/(3*a)))/d + (tan(c/2 + (d*x)/2)^2*(a
^6 - (4*a^4*b^2)/5) - tan(c/2 + (d*x)/2)^3*(3*a^5*b - 2*a^3*b^3) - a^6/7 -
tan(c/2 + (d*x)/2)^4*(3*a^6 + (16*a^2*b^4)/3 - (28*a^4*b^2)/3) + tan(c/2 +
(d*x)/2)^5*(16*a*b^5 + 15*a^5*b - 32*a^3*b^3) + tan(c/2 + (d*x)/2)^6*(5*a^6
 - 64*b^6 + 144*a^2*b^4 - 88*a^4*b^2) + (a^5*b*tan(c/2 + (d*x)/2))/3)/(128*
a^7*d*tan(c/2 + (d*x)/2)^7) - (b*tan(c/2 + (d*x)/2)^6)/(384*a^2*d) + (log(t
an(c/2 + (d*x)/2))*(5*a^6*b - 16*b^7 + 40*a^2*b^5 - 30*a^4*b^3))/(16*a^8*d)
 - (b^2*atan(((b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((32*a^8*b^8 - 88*a^10*b^6
+ 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 + (d*x)/2)*(5*a^14*b + 64*
a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3))/(8*a^13) + (b^2*(2*a^2
*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))*(-(a + b)^5*(a
- b)^5)^(1/2))/a^8)*1i)/a^8 + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((32*a^8*b^
```

8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 + (d*x)/2)
*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3))/(8*a^1
3) - (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/(8*a^13))
(-(a + b)^5(a - b)^5)^(1/2))/a^8)*1i)/a^8)/((16*b^15 - 88*a^2*b^13 + 198*
a^4*b^11 - 231*a^6*b^9 + 145*a^8*b^7 - 45*a^10*b^5 + 5*a^12*b^3)/(4*a^14) +
(tan(c/2 + (d*x)/2)*(16*b^14 - 84*a^2*b^12 + 178*a^4*b^10 - 190*a^6*b^8 +
102*a^8*b^6 - 22*a^10*b^4))/(4*a^13) + (b^2*(-(a + b)^5*(a - b)^5)^(1/2)*((
32*a^8*b^8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14) + (tan(c/2 +
(d*x)/2)*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5 - 72*a^12*b^3
))/(8*a^13) + (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 64*a^14*b^2))/
(8*a^13))*(-(a + b)^5*(a - b)^5)^(1/2))/a^8))/a^8 - (b^2*(-(a + b)^5*(a - b
)^5)^(1/2)*((32*a^8*b^8 - 88*a^10*b^6 + 78*a^12*b^4 - 21*a^14*b^2)/(8*a^14)
+ (tan(c/2 + (d*x)/2)*(5*a^14*b + 64*a^6*b^9 - 192*a^8*b^7 + 196*a^10*b^5
- 72*a^12*b^3))/(8*a^13) - (b^2*(2*a^2*b - (tan(c/2 + (d*x)/2)*(48*a^16 - 6
4*a^14*b^2))/(8*a^13))*(-(a + b)^5*(a - b)^5)^(1/2))/a^8))/a^8))*(-(a + b)^
5*(a - b)^5)^(1/2)*2i)/(a^8*d)

3.1331 $\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=476

$$\frac{2b^3(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^9d}+\frac{(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8)\tanh^{-1}(\cos(c+dx))}{128a^9d}-$$

```
[Out] 2*b^3*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^9/
d+1/128*(5*a^8+40*a^6*b^2-240*a^4*b^4+320*a^2*b^6-128*b^8)*arctanh(cos(d*x+
c))/a^9/d-1/105*b*(15*a^6-161*a^4*b^2+245*a^2*b^4-105*b^6)*cot(d*x+c)/a^8/d
+1/128*(5*a^6-88*a^4*b^2+144*a^2*b^4-64*b^6)*cot(d*x+c)*csc(d*x+c)/a^7/d+1/
105*b*(45*a^4-77*a^2*b^2+35*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^6/d-1/192*(59*a^
4-104*a^2*b^2+48*b^4)*cot(d*x+c)*csc(d*x+c)^3/a^5/d-1/4*cot(d*x+c)*csc(d*x+
c)^4/b/d+1/140*(35*a^4-60*a^2*b^2+28*b^4)*cot(d*x+c)*csc(d*x+c)^4/a^4/b/d+1
/5*a*cot(d*x+c)*csc(d*x+c)^5/b^2/d-1/240*(48*a^4-85*a^2*b^2+40*b^4)*cot(d*x
+c)*csc(d*x+c)^5/a^3/b^2/d+1/7*b*cot(d*x+c)*csc(d*x+c)^6/a^2/d-1/8*cot(d*x+
c)*csc(d*x+c)^7/a/d
```

Rubi [A]
time = 1.39, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2975, 3134, 3080, 3855, 2739, 632, 210}

$$\frac{b\cot(c+dx)\csc^6(c+dx)}{7a^2d}+\frac{2b^3(a^2-b^2)^{5/2}\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^9d}+\frac{(35a^4-60a^2b^2+28b^4)\cot(c+dx)\csc^4(c+dx)}{140a^4bd}+\frac{b(45a^4-77a^2b^2+35b^4)\cot(c+dx)\csc^2(c+dx)}{105a^6d}-\frac{(59a^4-104a^2b^2+48b^4)\cot(c+dx)\csc^3(c+dx)}{192a^5d}-\frac{(48a^4-85a^2b^2+40b^4)\cot(c+dx)\csc^5(c+dx)}{240a^3b^2d}-\frac{b(15a^6-161a^4b^2+245a^2b^4-105b^6)\cot(c+dx)}{105a^8d}+\frac{(5a^6-88a^4b^2+144a^2b^4-64b^6)\cot(c+dx)\csc(c+dx)}{128a^7d}+\frac{(5a^8+40a^6b^2-240a^4b^4+320a^2b^6-128b^8)\tanh^{-1}(\cos(c+dx))}{128a^9d}+\frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d}-\frac{\cot(c+dx)\csc^7(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^9*d) + ((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*ArcTa
nh[Cos[c + d*x]])/(128*a^9*d) - (b*(15*a^6 - 161*a^4*b^2 + 245*a^2*b^4 - 10
5*b^6)*Cot[c + d*x])/(105*a^8*d) + ((5*a^6 - 88*a^4*b^2 + 144*a^2*b^4 - 64*
b^6)*Cot[c + d*x]*Csc[c + d*x])/(128*a^7*d) + (b*(45*a^4 - 77*a^2*b^2 + 35*
b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(105*a^6*d) - ((59*a^4 - 104*a^2*b^2 + 48
*b^4)*Cot[c + d*x]*Csc[c + d*x]^3)/(192*a^5*d) - (Cot[c + d*x]*Csc[c + d*x]
^4)/(4*b*d) + ((35*a^4 - 60*a^2*b^2 + 28*b^4)*Cot[c + d*x]*Csc[c + d*x]^4)/
(140*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^5)/(5*b^2*d) - ((48*a^4 - 85*a
^2*b^2 + 40*b^4)*Cot[c + d*x]*Csc[c + d*x]^5)/(240*a^3*b^2*d) + (b*Cot[c +
d*x]*Csc[c + d*x]^6)/(7*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^7)/(8*a*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2975

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(d*Sin[
e + f*x])^(n + 1)*((a + b*Sin[e + f*x])^(m + 1)/(a*d*f*(n + 1))), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m + n +
2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*((a + b*Sin[e + f*x])^(m + 1)/(a^2
*d^2*f*(n + 1)*(n + 2))), x] - Simp[a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x])
^(n + 3)*((a + b*Sin[e + f*x])^(m + 1)/(b^2*d^3*f*(m + n + 5)*(m + n + 6)))
, x] + Simp[Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*((a + b*Sin[e + f*x])^(m
+ 1)/(b*d^4*f*(m + n + 6))), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
```

```
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)\csc^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} + \frac{b\cot(c+dx)\,c}{7a^2}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{a\cot(c+dx)\csc^5(c+dx)}{5b^2d} - \frac{(48a^4 - 85a^2b}{}$$

$$= -\frac{\cot(c+dx)\csc^4(c+dx)}{4bd} + \frac{(35a^4 - 60a^2b^2 + 28b^4)\cot(c+dx)\csc^4(c+}{140a^4bd}$$

$$= -\frac{(59a^4 - 104a^2b^2 + 48b^4)\cot(c+dx)\csc^3(c+dx)}{192a^5d} - \frac{\cot(c+dx)\csc^4(c}{4bd}$$

$$= \frac{b(45a^4 - 77a^2b^2 + 35b^4)\cot(c+dx)\csc^2(c+dx)}{105a^6d} - \frac{(59a^4 - 104a^2b^2 + 48}{}$$

$$= \frac{(5a^6 - 88a^4b^2 + 144a^2b^4 - 64b^6)\cot(c+dx)\csc(c+dx)}{128a^7d} + \frac{b(45a^4 - 77a}{}$$

$$= -\frac{b(15a^6 - 161a^4b^2 + 245a^2b^4 - 105b^6)\cot(c+dx)}{105a^8d} + \frac{(5a^6 - 88a^4b^2 + 14}{}$$

$$= -\frac{b(15a^6 - 161a^4b^2 + 245a^2b^4 - 105b^6)\cot(c+dx)}{105a^8d} + \frac{(5a^6 - 88a^4b^2 + 14}{}$$

$$= \frac{(5a^8 + 40a^6b^2 - 240a^4b^4 + 320a^2b^6 - 128b^8)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b(15}{}$$

$$= \frac{(5a^8 + 40a^6b^2 - 240a^4b^4 + 320a^2b^6 - 128b^8)\tanh^{-1}(\cos(c+dx))}{128a^9d} - \frac{b(15}{}$$

$$= \frac{2b^3(a^2-b^2)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^9d} + \frac{(5a^8 + 40a^6b^2 - 240a^4b^4 + 320}{1}$$

Mathematica [A]
time = 2.32, size = 593, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(Cot[c + d*x]^6*Csc[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (1720320*b^3*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]] + 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*Log[
Cos[(c + d*x)/2]] - 6720*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 -
128*b^8)*Log[Sin[(c + d*x)/2]] + a*Csc[c + d*x]^8*(-35*a*(1765*a^6 + 680*a^
```

```
4*b^2 - 1392*a^2*b^4 + 960*b^6)*Cos[c + d*x] - 35*(895*a^7 - 904*a^5*b^2 +
2736*a^3*b^4 - 1728*a*b^6)*Cos[3*(c + d*x)] - 13895*a^7*Cos[5*(c + d*x)] -
17080*a^5*b^2*Cos[5*(c + d*x)] + 62160*a^3*b^4*Cos[5*(c + d*x)] - 33600*a*b
^6*Cos[5*(c + d*x)] - 525*a^7*Cos[7*(c + d*x)] + 9240*a^5*b^2*Cos[7*(c + d*
x)] - 15120*a^3*b^4*Cos[7*(c + d*x)] + 6720*a*b^6*Cos[7*(c + d*x)] + 13440*
a^6*b*Sin[2*(c + d*x)] + 88704*a^4*b^3*Sin[2*(c + d*x)] - 174720*a^2*b^5*Si
n[2*(c + d*x)] + 94080*b^7*Sin[2*(c + d*x)] + 13440*a^6*b*Sin[4*(c + d*x)]
- 86912*a^4*b^3*Sin[4*(c + d*x)] + 183680*a^2*b^5*Sin[4*(c + d*x)] - 94080*
b^7*Sin[4*(c + d*x)] + 5760*a^6*b*Sin[6*(c + d*x)] + 42112*a^4*b^3*Sin[6*(c
 + d*x)] - 85120*a^2*b^5*Sin[6*(c + d*x)] + 40320*b^7*Sin[6*(c + d*x)] + 96
0*a^6*b*Sin[8*(c + d*x)] - 10304*a^4*b^3*Sin[8*(c + d*x)] + 15680*a^2*b^5*S
in[8*(c + d*x)] - 6720*b^7*Sin[8*(c + d*x)]))/(860160*a^9*d)
```

Maple [**A**]
time = 0.66, size = 728, normalized size = 1.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/256/a^8*(1/8*a^7*tan(1/2*d*x+1/2*c)^8-2/7*b*tan(1/2*d*x+1/2*c)^7*a^6
-2/3*a^7*tan(1/2*d*x+1/2*c)^6+2/3*a^5*b^2*tan(1/2*d*x+1/2*c)^6+2*a^6*b*tan(
1/2*d*x+1/2*c)^5-8/5*a^4*b^3*tan(1/2*d*x+1/2*c)^5+tan(1/2*d*x+1/2*c)^4*a^7-
6*a^5*b^2*tan(1/2*d*x+1/2*c)^4+4*a^3*b^4*tan(1/2*d*x+1/2*c)^4-6*a^6*b*tan(1
/2*d*x+1/2*c)^3+56/3*a^4*b^3*tan(1/2*d*x+1/2*c)^3-32/3*a^2*b^5*tan(1/2*d*x+
1/2*c)^3+2*a^7*tan(1/2*d*x+1/2*c)^2+30*a^5*b^2*tan(1/2*d*x+1/2*c)^2-64*a^3*
b^4*tan(1/2*d*x+1/2*c)^2+32*a*b^6*tan(1/2*d*x+1/2*c)^2+10*a^6*b*tan(1/2*d*x
+1/2*c)-176*a^4*b^3*tan(1/2*d*x+1/2*c)+288*a^2*b^5*tan(1/2*d*x+1/2*c)-128*b
^7*tan(1/2*d*x+1/2*c))-1/2048/a/tan(1/2*d*x+1/2*c)^8-1/1536*(-4*a^2+4*b^2)/
a^3/tan(1/2*d*x+1/2*c)^6-1/1024*(4*a^4-24*a^2*b^2+16*b^4)/a^5/tan(1/2*d*x+1
/2*c)^4-1/512*(4*a^6+60*a^4*b^2-128*a^2*b^4+64*b^6)/a^7/tan(1/2*d*x+1/2*c)^
2+1/256/a^9*(-10*a^8-80*a^6*b^2+480*a^4*b^4-640*a^2*b^6+256*b^8)*ln(tan(1/2
*d*x+1/2*c))+1/896/a^2*b/tan(1/2*d*x+1/2*c)^7-1/640/a^4*b*(5*a^2-4*b^2)/tan
(1/2*d*x+1/2*c)^5+1/384/a^6*b*(9*a^4-28*a^2*b^2+16*b^4)/tan(1/2*d*x+1/2*c)^
3-1/128*b*(5*a^6-88*a^4*b^2+144*a^2*b^4-64*b^6)/a^8/tan(1/2*d*x+1/2*c)+2*b^
3*(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/a^9/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. $2(449) = 898$.
time = 1.31, size = 2082, normalized size = 4.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(d*x + c)
^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^5
- 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x + c)^3 - 1
3440*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*b^5 + b^
7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sqrt(-
a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b
^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 210*(5*a^8 + 40*a^6*
b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 105*((5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 2
40*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 +
320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4
 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*
b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) +
105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c
)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 +
 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^
8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161*a^5*b^3 + 245*a^3*b^5 - 1
05*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^3*b^5 + 45*a*b^7)*cos(d*x
+ c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*cos(d*x + c)^3 + 105*(a^5*b
^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c)^8 -
 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4 - 4*a^9*d*cos(d*x + c)^2 +
 a^9*d), -1/26880*(210*(5*a^8 - 88*a^6*b^2 + 144*a^4*b^4 - 64*a^2*b^6)*cos(
d*x + c)^7 + 70*(73*a^8 + 584*a^6*b^2 - 1200*a^4*b^4 + 576*a^2*b^6)*cos(d*x
 + c)^5 - 70*(55*a^8 + 440*a^6*b^2 - 1104*a^4*b^4 + 576*a^2*b^6)*cos(d*x +
c)^3 + 26880*((a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^8 + a^4*b^3 - 2*a^2*
b^5 + b^7 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^6 + 6*(a^4*b^3 - 2*a
^2*b^5 + b^7)*cos(d*x + c)^4 - 4*(a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2
)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c
))) + 210*(5*a^8 + 40*a^6*b^2 - 112*a^4*b^4 + 64*a^2*b^6)*cos(d*x + c) - 10
```

```
5*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^
8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8 - 4*(5*a^8 + 4
0*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^6 + 6*(5*a^8
+ 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^4 - 4*(5*a
^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*cos(d*x + c)^2)*log(
1/2*cos(d*x + c) + 1/2) + 105*((5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8)*cos(d*x + c)^8 + 5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*
b^6 - 128*b^8 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8
)*cos(d*x + c)^6 + 6*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*
b^8)*cos(d*x + c)^4 - 4*(5*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 1
28*b^8)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2) - 256*((15*a^7*b - 161
*a^5*b^3 + 245*a^3*b^5 - 105*a*b^7)*cos(d*x + c)^7 + 7*(58*a^5*b^3 - 100*a^
3*b^5 + 45*a*b^7)*cos(d*x + c)^5 - 35*(10*a^5*b^3 - 19*a^3*b^5 + 9*a*b^7)*c
os(d*x + c)^3 + 105*(a^5*b^3 - 2*a^3*b^5 + a*b^7)*cos(d*x + c))*sin(d*x + c
))/(a^9*d*cos(d*x + c)^8 - 4*a^9*d*cos(d*x + c)^6 + 6*a^9*d*cos(d*x + c)^4
- 4*a^9*d*cos(d*x + c)^2 + a^9*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*csc(d*x+c)**9/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 948 vs. 2(449) = 898.
time = 0.51, size = 948, normalized size = 1.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*csc(d*x+c)^9/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/215040*((105*a^7*tan(1/2*d*x + 1/2*c)^8 - 240*a^6*b*tan(1/2*d*x + 1/2*c)^
7 - 560*a^7*tan(1/2*d*x + 1/2*c)^6 + 560*a^5*b^2*tan(1/2*d*x + 1/2*c)^6 + 1
680*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 1344*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 84
0*a^7*tan(1/2*d*x + 1/2*c)^4 - 5040*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 + 3360*a
^3*b^4*tan(1/2*d*x + 1/2*c)^4 - 5040*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 15680*a
^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 8960*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 1680*
a^7*tan(1/2*d*x + 1/2*c)^2 + 25200*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 - 53760*a
^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 26880*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 8400*a
^6*b*tan(1/2*d*x + 1/2*c) - 147840*a^4*b^3*tan(1/2*d*x + 1/2*c) + 241920*a^
2*b^5*tan(1/2*d*x + 1/2*c) - 107520*b^7*tan(1/2*d*x + 1/2*c))/a^8 - 1680*(5
```

```
*a^8 + 40*a^6*b^2 - 240*a^4*b^4 + 320*a^2*b^6 - 128*b^8)*log(abs(tan(1/2*d*
x + 1/2*c)))/a^9 + 430080*(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*(pi*floor
(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(
a^2 - b^2)))/(sqrt(a^2 - b^2)*a^9) + (22830*a^8*tan(1/2*d*x + 1/2*c)^8 + 18
2640*a^6*b^2*tan(1/2*d*x + 1/2*c)^8 - 1095840*a^4*b^4*tan(1/2*d*x + 1/2*c)^
8 + 1461120*a^2*b^6*tan(1/2*d*x + 1/2*c)^8 - 584448*b^8*tan(1/2*d*x + 1/2*c
)^8 - 8400*a^7*b*tan(1/2*d*x + 1/2*c)^7 + 147840*a^5*b^3*tan(1/2*d*x + 1/2*
c)^7 - 241920*a^3*b^5*tan(1/2*d*x + 1/2*c)^7 + 107520*a*b^7*tan(1/2*d*x + 1
/2*c)^7 - 1680*a^8*tan(1/2*d*x + 1/2*c)^6 - 25200*a^6*b^2*tan(1/2*d*x + 1/2
*c)^6 + 53760*a^4*b^4*tan(1/2*d*x + 1/2*c)^6 - 26880*a^2*b^6*tan(1/2*d*x +
1/2*c)^6 + 5040*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 15680*a^5*b^3*tan(1/2*d*x +
1/2*c)^5 + 8960*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 840*a^8*tan(1/2*d*x + 1/2*
c)^4 + 5040*a^6*b^2*tan(1/2*d*x + 1/2*c)^4 - 3360*a^4*b^4*tan(1/2*d*x + 1/2
*c)^4 - 1680*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 1344*a^5*b^3*tan(1/2*d*x + 1/2*
c)^3 + 560*a^8*tan(1/2*d*x + 1/2*c)^2 - 560*a^6*b^2*tan(1/2*d*x + 1/2*c)^2
+ 240*a^7*b*tan(1/2*d*x + 1/2*c) - 105*a^8)/(a^9*tan(1/2*d*x + 1/2*c)^8))/d
```

Mupad [B]
time = 12.67, size = 1861, normalized size = 3.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(sin(c + d*x)^9*(a + b*sin(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^8/(2048*a*d) + (tan(c/2 + (d*x)/2)^5*(b/(640*a^2) + (2*b
*(1/(64*a) - b^2/(64*a^3))))/(5*a)))/d - (tan(c/2 + (d*x)/2)^3*(b/(384*a^2)
- (2*b*(b^2/(64*a^3) - 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/
(64*a^3))))/a))/a))/(3*a) + (2*b*(1/(64*a) - b^2/(64*a^3))))/(3*a)))/d + (tan
(c/2 + (d*x)/2)^2*(1/(128*a) + b^2/(128*a^3) + (b*(b/(128*a^2) + (2*b*(1/(6
4*a) - b^2/(64*a^3))))/a))/a + (b*(b/(128*a^2) - (2*b*(b^2/(64*a^3) - 1/(64*
a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*a^3))))/a))/a))/a + (2*b*(
1/(64*a) - b^2/(64*a^3))))/a))/a))/d + (tan(c/2 + (d*x)/2)*(b/(128*a^2) - (2
*b*(b^2/(64*a^3) - 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*
a^3))))/a))/a))/a - (2*b*(1/(64*a) + b^2/(64*a^3) + (2*b*(b/(128*a^2) + (2*b
*(1/(64*a) - b^2/(64*a^3))))/a))/a + (2*b*(b/(128*a^2) - (2*b*(b^2/(64*a^3)
- 1/(64*a) + (2*b*(b/(128*a^2) + (2*b*(1/(64*a) - b^2/(64*a^3))))/a))/a))/a
+ (2*b*(1/(64*a) - b^2/(64*a^3))))/a))/a))/a + (2*b*(1/(64*a) - b^2/(64*a^3)
))/a))/d - (tan(c/2 + (d*x)/2)^6*(1/(384*a) - b^2/(384*a^3)))/d - (tan(c/2
+ (d*x)/2)^4*(b^2/(256*a^3) - 1/(256*a) + (b*(b/(128*a^2) + (2*b*(1/(64*a)
- b^2/(64*a^3))))/a))/(2*a)))/d - (log(tan(c/2 + (d*x)/2))*(5*a^8 - 128*b^8
+ 320*a^2*b^6 - 240*a^4*b^4 + 40*a^6*b^2))/(128*a^9*d) - (cot(c/2 + (d*x)/2
)^8*(tan(c/2 + (d*x)/2)^3*(2*a^6*b - (8*a^4*b^3)/5) - tan(c/2 + (d*x)/2)^5*
(6*a^6*b + (32*a^2*b^5)/3 - (56*a^4*b^3)/3) + tan(c/2 + (d*x)/2)^6*(32*a*b^
6 + 2*a^7 - 64*a^3*b^4 + 30*a^5*b^2) + tan(c/2 + (d*x)/2)^7*(10*a^6*b - 128
```

*b^7 + 288*a^2*b^5 - 176*a^4*b^3) + tan(c/2 + (d*x)/2)^4*(a^7 + 4*a^3*b^4 - 6*a^5*b^2) + a^7/8 - tan(c/2 + (d*x)/2)^2*((2*a^7)/3 - (2*a^5*b^2)/3) - (2*a^6*b*tan(c/2 + (d*x)/2))/7))/(256*a^8*d) - (b*tan(c/2 + (d*x)/2)^7)/(896*a^2*d) + (b^3*atan(((b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(5*a^17 + 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a^15) - (5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5 + 168*a^15*b^3)/(64*a^16) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9)*1i)/a^9 - (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5 + 168*a^15*b^3)/(64*a^16) - (tan(c/2 + (d*x)/2)*(5*a^17 + 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a^15) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9)*1i)/a^9)/((128*b^17 - 704*a^2*b^15 + 1584*a^4*b^13 - 1848*a^6*b^11 + 1155*a^8*b^9 - 345*a^10*b^7 + 25*a^12*b^5 + 5*a^14*b^3)/(32*a^16) + (tan(c/2 + (d*x)/2)*(128*b^16 - 672*a^2*b^14 + 1424*a^4*b^12 - 1530*a^6*b^10 + 846*a^8*b^8 - 206*a^10*b^6 + 10*a^12*b^4))/(32*a^15) + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(5*a^17 + 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a^15) - (5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5 + 168*a^15*b^3)/(64*a^16) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9))/a^9 + (b^3*(-(a + b)^5*(a - b)^5)^(1/2)*((5*a^17*b - 256*a^9*b^9 + 704*a^11*b^7 - 624*a^13*b^5 + 168*a^15*b^3)/(64*a^16) - (tan(c/2 + (d*x)/2)*(5*a^17 + 512*a^7*b^10 - 1536*a^9*b^8 + 1568*a^11*b^6 - 576*a^13*b^4 + 30*a^15*b^2))/(64*a^15) + (b^3*(2*a^2*b - (tan(c/2 + (d*x)/2)*(384*a^18 - 512*a^16*b^2))/(64*a^15))*(-(a + b)^5*(a - b)^5)^(1/2))/a^9))/a^9))*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(a^9*d)

3.1332 $\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a^3\log(a+b\sin(c+dx))}{b^2\left(a^2-b^2\right)d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a^3*ln(a+b*sin(d
*x+c))/b^2/(a^2-b^2)/d-sin(d*x+c)/b/d
```

Rubi [A]
time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 1643}

$$\frac{a^3\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) + (a^3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)*d) - Sin[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{b^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{2(a+b)(b-x)}+\frac{a^3}{(a-b)(a+b)(a+x)}-\frac{b^2}{2(a-b)(b+x)}\right)\,dx, x, b\sin(c+d$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)\,d}$$

Mathematica [A]
time = 0.15, size = 83, normalized size = 0.89

$$-\frac{\frac{\log(1-\sin(c+dx))}{a+b}+\frac{\log(1+\sin(c+dx))}{a-b}-\frac{2a^3\log(a+b\sin(c+dx))}{b^2(a^2-b^2)}+\frac{2\sin(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Log[1 - Sin[c + d*x]]/(a + b) + Log[1 + Sin[c + d*x]]/(a - b) - (2*a^
3*Log[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)) + (2*Sin[c + d*x])/b)/d
```

Maple [A]
time = 0.28, size = 87, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{\sin(dx+c)}{b}+\frac{a^3\ln(a+b\sin(dx+c))}{b^2(a+b)(a-b)}-\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{\sin(dx+c)}{b}+\frac{a^3\ln(a+b\sin(dx+c))}{b^2(a+b)(a-b)}-\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{a^3\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{b^2d(a^2-b^2)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)}-\frac{\ln\left(\tan\right.}{(}$
risch	$\frac{iax}{b^2}+\frac{ie^{i(dx+c)}}{2bd}-\frac{ie^{-i(dx+c)}}{2bd}+\frac{ix}{a+b}+\frac{ic}{d(a+b)}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{2ia^3x}{b^2(a^2-b^2)}-\frac{2ia^3c}{b^2d(a^2-b^2)}-\frac{\ln(e^{i(dx+c)}}{d(a+b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*sin(d*x+c)+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a-2*b)*l
n(1+sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.88

$$\frac{\frac{2\,a^3\log(b\sin(dx+c)+a)}{a^2b^2-b^4}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}-\frac{2\,\sin(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a)/(a^2*b^2 - b^4) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) - 2*sin(d*x + c)/b)/d
```

Fricas [A]
time = 0.39, size = 100, normalized size = 1.08

$$\frac{2\,a^3\log\left(b\sin\left(dx+c\right)+a\right)-\left(ab^2+b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(ab^2-b^3\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2b-b^3\right)\sin\left(dx+c\right)}{2\left(a^2b^2-b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3*log(b*sin(d*x + c) + a) - (a*b^2 + b^3)*log(sin(d*x + c) + 1) -
(a*b^2 - b^3)*log(-sin(d*x + c) + 1) - 2*(a^2*b - b^3)*sin(d*x + c))/((a^2*
b^2 - b^4)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 85, normalized size = 0.91

$$\frac{\frac{2\,a^3\log(|b\sin(dx+c)+a|)}{a^2b^2-b^4}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}-\frac{2\,\sin(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^3*log(abs(b*sin(d*x + c) + a))/(a^2*b^2 - b^4) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) - 2*sin(d*x + c)/b)/
d
```

Mupad [B]
time = 12.27, size = 134, normalized size = 1.44

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}-\frac{\sin\left(c+d\,x\right)}{b\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b^2\,d}-\frac{a^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(b^4-a^2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)) - sin(c + d*x)/(b*d) - log(tan(c/
2 + (d*x)/2) + 1)/(d*(a - b)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) -
 (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^4 - a
^2*b^2))
```

3.1333 $\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{a^2\log(a+b\sin(c+dx))}{b\,(a^2-b^2)\,d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-a^2*ln(a+b*sin(d
*x+c))/b/(a^2-b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2916, 12, 1643}

$$-\frac{a^2\log(a+b\sin(c+dx))}{bd\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) - (a^2*Log[a + b*Sin[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b}{2(a+b)(b-x)} - \frac{a^2}{(a-b)(a+b)(a+x)} + \frac{b}{2(a-b)(b+x)}\right)\,dx, x, b\sin(c+dx)\right)}{bd}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{a^2\log(a+b\sin(c+dx))}{b\,(a^2-b^2)\,d}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.90

$$\frac{-((a-b)b\log(1-\sin(c+dx))) + b(a+b)\log(1+\sin(c+dx)) - 2a^2\log(a+b\sin(c+dx))}{2(a-b)b(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a - b)*b*Log[1 - Sin[c + d*x]]) + b*(a + b)*Log[1 + Sin[c + d*x]] - 2*a
^2*Log[a + b*Sin[c + d*x]])/(2*(a - b)*b*(a + b)*d)
```

Maple [A]
time = 0.22, size = 76, normalized size = 0.95

method	result
derivativedivides	$\frac{-\frac{a^2\ln(a+b\sin(dx+c))}{(a+b)(a-b)b}+\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{a^2\ln(a+b\sin(dx+c))}{(a+b)(a-b)b}+\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)d} - \frac{a^2\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right.}{bd(a^2-b^2)}$
risch	$-\frac{ix}{b} - \frac{ix}{a-b} - \frac{ic}{d(a-b)} + \frac{ix}{a+b} + \frac{ic}{d(a+b)} + \frac{2ia^2x}{b(a^2-b^2)} + \frac{2ia^2c}{bd(a^2-b^2)} + \frac{\ln(e^{i(dx+c)}+i)}{d(a-b)} - \frac{\ln(e^{i(dx+c)}-i)}{d(a+b)} - \frac{a^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2/(a+b)/(a-b)/b*ln(a+b*sin(d*x+c))+1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(
2*a+2*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.33, size = 68, normalized size = 0.85

$$-\frac{\frac{2\,a^2\log(b\sin(dx+c)+a)}{a^2b-b^3}-\frac{\log(\sin(dx+c)+1)}{a-b}+\frac{\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a)/(a^2*b - b^3) - log(sin(d*x + c) + 1)/(
a - b) + log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 0.38, size = 74, normalized size = 0.92

$$-\frac{2\,a^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(ab-b^2\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2b-b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*log(b*sin(d*x + c) + a) - (a*b + b^2)*log(sin(d*x + c) + 1) + (
a*b - b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin^2\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.44, size = 71, normalized size = 0.89

$$-\frac{\frac{2\,a^2\log(|b\sin(dx+c)+a|)}{a^2b-b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}+\frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b))/d
```

Mupad [B]
time = 11.94, size = 117, normalized size = 1.46

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}+\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b\,d}-\frac{a^2\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{b\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a
 + b)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) - (a^2*log(a + 2*b*tan(c/2 + (
d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(b*d*(a^2 - b^2))
```

3.1334 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a*ln(a+b*sin(d*x
+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2800, 815}

$$\frac{a\log(a+b\sin(c+dx))}{d\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d
) + (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+x)(b^2-x^2)}\,dx,x,b\sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)}+\frac{a}{(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)(b+x)}\right)dx,x,b\sin(c+dx)\right)}{d}\\
&= -\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{a\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 87, normalized size = 1.18

$$\frac{(-a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+a\log(a+b\sin(c+dx))}{(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

Maple [A]
time = 0.19, size = 71, normalized size = 0.96

method	result
derivativedivides	$\frac{\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(1+\sin(dx+c))}{2a-2b}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}+\frac{ix}{a+b}+\frac{ic}{d(a+b)}-\frac{2iax}{a^2-b^2}-\frac{2iac}{d(a^2-b^2)}-\frac{\ln(e^{i(dx+c)}+i)}{d(a-b)}-\frac{\ln(e^{i(dx+c)}-i)}{d(a+b)}+\frac{a\ln\left(e^{2i(dx+c)}\ldots\right.}{d(a\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(2*a+2
*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.32, size = 65, normalized size = 0.88

$$\frac{\frac{2\,a\log(b\sin(dx+c)+a)}{a^2-b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 0.38, size = 63, normalized size = 0.85

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)-(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c + dx\right)\sec\left(c + dx\right)}{a + b\sin\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral(sin(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)

Giac [A]
time = 0.45, size = 71, normalized size = 0.96

$$\frac{\frac{2\,ab\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^2b-b^3} - \frac{\log\left(|\sin\left(dx+c\right)+1|\right)}{a-b} - \frac{\log\left(|\sin\left(dx+c\right)-1|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")

[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d

Mupad [B]
time = 12.02, size = 91, normalized size = 1.23

$$\frac{a\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^2-b^2\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,\left(a-b\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)/(cos(c + d*x)*(a + b*sin(c + d*x))),x)

[Out] (a*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2)) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b))

3.1335 $\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=93

$$-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{b^2\log(a+b\sin(c+dx))}{a\left(a^2-b^2\right)d}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+ln(sin(d*x+c))/a/d-1/2*ln(1+sin(d*x+c))/(a-b)
/d+b^2*ln(a+b*sin(d*x+c))/a/(a^2-b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2916, 12, 908}

$$\frac{b^2\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*Log[1 - Sin[c + d*x]]/((a + b)*d) + Log[Sin[c + d*x]]/(a*d) - Log[1 +
Sin[c + d*x]]/(2*(a - b)*d) + (b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{b}{x(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)} + \frac{1}{ab^2x} + \frac{1}{a(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)b^2(b+x)}\right)\,dx, x, b\sin\right.}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{b^2\log(}{}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.90

$$-\frac{\frac{\log(1-\sin(c+dx))}{a+b} - \frac{2\log(\sin(c+dx))}{a} + \frac{\log(1+\sin(c+dx))}{a-b} - \frac{2b^2\log(a+b\sin(c+dx))}{a(a^2-b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*(Log[1 - Sin[c + d*x]]/(a + b) - (2*Log[Sin[c + d*x]])/a + Log[1 + Sin
[c + d*x]]/(a - b) - (2*b^2*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)))/d
```

Maple [A]
time = 0.26, size = 87, normalized size = 0.94

method	result
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a} + \frac{b^2\ln(a+b\sin(dx+c))}{a(a+b)(a-b)} - \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{\frac{\ln(\sin(dx+c))}{a} + \frac{b^2\ln(a+b\sin(dx+c))}{a(a+b)(a-b)} - \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} + \frac{b^2\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{ad(a^2-b^2)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)d}$
risch	$\frac{ix}{a+b} + \frac{ic}{d(a+b)} + \frac{ix}{a-b} + \frac{ic}{d(a-b)} - \frac{2ib^2x}{a(a^2-b^2)} - \frac{2ib^2c}{ad(a^2-b^2)} - \frac{2ix}{a} - \frac{2ic}{ad} - \frac{\ln(e^{i(dx+c)}-i)}{d(a+b)} - \frac{\ln(e^{i(dx+c)}+}{d(a-b)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(sin(d*x+c))+b^2/a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a-2*b)*ln
(1+sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.27, size = 80, normalized size = 0.86

$$\frac{\frac{2\,b^2\log(b\sin(dx+c)+a)}{a^3-ab^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}+\frac{2\,\log(\sin(dx+c))}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a)/(a^3 - a*b^2) - log(sin(d*x + c) + 1)/(a
 - b) - log(sin(d*x + c) - 1)/(a + b) + 2*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.41, size = 93, normalized size = 1.00

$$\frac{2\,b^2\log\left(b\sin\left(dx+c\right)+a\right)+2\left(a^2-b^2\right)\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(a^2+ab\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2-ab\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*log(b*sin(d*x + c) + a) + 2*(a^2 - b^2)*log(-1/2*sin(d*x + c)) -
 (a^2 + a*b)*log(sin(d*x + c) + 1) - (a^2 - a*b)*log(-sin(d*x + c) + 1))/((
a^3 - a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 86, normalized size = 0.92

$$\frac{\frac{2\,b^3\log(|b\sin(dx+c)+a|)}{a^3b-ab^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}+\frac{2\,\log(|\sin(dx+c)|)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(b*sin(d*x + c) + a))/(a^3*b - a*b^3) - log(abs(sin(d*x +
 c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*log(abs(sin(d*x
+ c)))/a)/d
```

Mupad [B]
time = 0.17, size = 87, normalized size = 0.94

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}+\frac{b^2\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{a\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - log(sin(c + d*x) + 1)/(2*d*(a - b)) - log(sin(c +
 d*x) - 1)/(2*d*(a + b)) + (b^2*log(a + b*sin(c + d*x)))/(a*d*(a^2 - b^2))
```

3.1336 $\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{\csc(c+dx)}{ad}-\frac{\log(1-\sin(c+dx))}{2(a+b)d}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{\log(1+\sin(c+dx))}{2(a-b)d}-\frac{b^3\log(a+b\sin(c+dx))}{a^2\left(a^2-b^2\right)d}$$

```
[Out] -csc(d*x+c)/a/d-1/2*ln(1-sin(d*x+c))/(a+b)/d-b*ln(sin(d*x+c))/a^2/d+1/2*ln(
1+sin(d*x+c))/(a-b)/d-b^3*ln(a+b*sin(d*x+c))/a^2/(a^2-b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$-\frac{b^3\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\log(\sin(c+dx))}{a^2d}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}+\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - (b*Log[Sin[c
+ d*x]])/(a^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^3*Log[a + b*Sin
[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\text{Subst}\left(\int \frac{1}{x^2(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\text{Subst}\left(\int \left(\frac{1}{2b^3(a+b)(b-x)} + \frac{1}{ab^2x^2} - \frac{1}{a^2b^2x} - \frac{1}{a^2(a-b)(a+b)(a+x)} - \frac{1}{2b^3(-a+b)(b+x)}\right.\right.}{d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{\log(1+\sin(c}{2(a-b}$$

Mathematica [A]
time = 0.17, size = 97, normalized size = 0.88

$$\frac{-\frac{2\csc(c+dx)}{a} - \frac{\log(1-\sin(c+dx))}{a+b} - \frac{2b\log(\sin(c+dx))}{a^2} + \frac{\log(1+\sin(c+dx))}{a-b} + \frac{2b^3\log(a+b\sin(c+dx))}{a^2(-a^2+b^2)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*Csc[c + d*x])/a - Log[1 - Sin[c + d*x]]/(a + b) - (2*b*Log[Sin[c + d*x
]])/a^2 + Log[1 + Sin[c + d*x]]/(a - b) + (2*b^3*Log[a + b*Sin[c + d*x]])/(
a^2*(-a^2 + b^2)))/(2*d)
```

Maple [A]
time = 0.32, size = 102, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} - \frac{b^3\ln(a+b\sin(dx+c))}{a^2(a+b)(a-b)} + \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} - \frac{b^3\ln(a+b\sin(dx+c))}{a^2(a+b)(a-b)} + \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{-\frac{1}{2ad} - \frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)} - \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{(a+b)d} - \frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d} - \frac{b^3\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}{a^2d}$
risch	$-\frac{ix}{a-b} - \frac{ic}{d(a-b)} + \frac{ix}{a+b} + \frac{ic}{d(a+b)} + \frac{2ib^3x}{a^2(a^2-b^2)} + \frac{2ib^3c}{a^2d(a^2-b^2)} + \frac{2ibx}{a^2} + \frac{2ibc}{a^2d} - \frac{2ie^{i(dx+c)}}{da(e^{2i(dx+c)}-1)} + \frac{\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))-b^3/a^2/(a+b)/(a-b)*ln(a+b*sin(d*x+c))+1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1))

Maxima [A]
time = 0.29, size = 95, normalized size = 0.86

$$-\frac{\frac{2\,b^3\log(b\sin(dx+c)+a)}{a^4-a^2b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}+\frac{\log(\sin(dx+c)-1)}{a+b}+\frac{2\,b\log(\sin(dx+c))}{a^2}+\frac{2}{a\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)/(a^4 - a^2*b^2) - log(sin(d*x + c) + 1)/(a - b) + log(sin(d*x + c) - 1)/(a + b) + 2*b*log(sin(d*x + c))/a^2 + 2/(a*sin(d*x + c)))/d

Fricas [A]
time = 0.49, size = 143, normalized size = 1.30

$$-\frac{2\,b^3\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)+2\,a^3-2\,ab^2+2\,(a^2b-b^3)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-(a^3+a^2b)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+(a^3-a^2b)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)}{2\,(a^4-a^2b^2)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(2*b^3*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^3 - 2*a*b^2 + 2*(a^2*b - b^3)*log(1/2*sin(d*x + c))*sin(d*x + c) - (a^3 + a^2*b)*log(sin(d*x + c) + 1)*sin(d*x + c) + (a^3 - a^2*b)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((a^4 - a^2*b^2)*d*sin(d*x + c))

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(c+dx\right)\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*sec(d*x+c)/(a+b*sin(d*x+c)),x)

[Out] Integral(csc(c + d*x)**2*sec(c + d*x)/(a + b*sin(c + d*x)), x)

Giac [A]
time = 0.49, size = 113, normalized size = 1.03

$$-\frac{\frac{2\,b^4\log(|b\sin(dx+c)+a|)}{a^4b-a^2b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}+\frac{\log(|\sin(dx+c)-1|)}{a+b}+\frac{2\,b\log(|\sin(dx+c)|)}{a^2}-\frac{2\,(b\sin(dx+c)-a)}{a^2\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(b*sin(d*x + c) + a))/(a^4*b - a^2*b^3) - log(abs(sin(d*
x + c) + 1))/(a - b) + log(abs(sin(d*x + c) - 1))/(a + b) + 2*b*log(abs(sin
(d*x + c)))/a^2 - 2*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

Mupad [B]
time = 11.92, size = 98, normalized size = 0.89

$$-\frac{\frac{\ln(\sin(c+d\,x)-1)}{2\,(a+b)}-\frac{\ln(\sin(c+d\,x)+1)}{2\,(a-b)}+\frac{1}{a\,\sin(c+d\,x)}+\frac{b\,\ln(\sin(c+d\,x))}{a^2}+\frac{b^3\,\ln(a+b\,\sin(c+d\,x))}{a^4-a^2\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] -(log(sin(c + d*x) - 1)/(2*(a + b)) - log(sin(c + d*x) + 1)/(2*(a - b)) + 1
/(a*sin(c + d*x)) + (b*log(sin(c + d*x)))/a^2 + (b^3*log(a + b*sin(c + d*x)
))/(a^4 - a^2*b^2))/d
```

3.1337 $\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=132

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{\log(1-\sin(c+dx))}{2(a+b)d}+\frac{(a^2+b^2)\log(\sin(c+dx))}{a^3d}-\frac{\log(1+\sin(c+dx))}{2(a-b)d}+\frac{b^4\log(}{a}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/2*ln(1-sin(d*x+c))/(a+b)/d+(a^2+b
^2)*ln(sin(d*x+c))/a^3/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+b^4*ln(a+b*sin(d*x+c)
)/a^3/(a^2-b^2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$\frac{b\csc(c+dx)}{a^2d}+\frac{(a^2+b^2)\log(\sin(c+dx))}{a^3d}+\frac{b^4\log(a+b\sin(c+dx))}{a^3d\,(a^2-b^2)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - Log[1 - Sin[c + d*x]]/(
2*(a + b)*d) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) - Log[1 + Sin[c + d*
x]]/(2*(a - b)*d) + (b^4*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\text{Subst}\left(\int \frac{1}{x^3(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{2b^4(a+b)(b-x)} + \frac{1}{ab^2x^3} - \frac{1}{a^2b^2x^2} + \frac{a^2+b^2}{a^3b^4x} + \frac{1}{a^3(a-b)(a+b)(a+x)} + \frac{}{2b^4(-}\right.\right.}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{(a^2+b^2)\log(\sin(c}{a^3d}$$

Mathematica [A]
time = 0.34, size = 132, normalized size = 1.00

$$\frac{b^4\left(\frac{\csc(c+dx)}{a^2b^3} - \frac{\csc^2(c+dx)}{2ab^4} - \frac{\log(1-\sin(c+dx))}{2b^4(a+b)} + \frac{(a^2+b^2)\log(\sin(c+dx))}{a^3b^4} - \frac{\log(1+\sin(c+dx))}{2(a-b)b^4} + \frac{\log(a+b\sin(c+dx))}{a^3(a^2-b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(Csc[c + d*x]/(a^2*b^3) - Csc[c + d*x]^2/(2*a*b^4) - Log[1 - Sin[c + d
*x]]/(2*b^4*(a + b)) + ((a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*b^4) - Log[1 +
Sin[c + d*x]]/(2*(a - b)*b^4) + Log[a + b*Sin[c + d*x]]/(a^3*(a^2 - b^2))))
/d
```

Maple [A]
time = 0.37, size = 120, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{1}{2a\sin(dx+c)^2} + \frac{(a^2+b^2)\ln(\sin(dx+c))}{a^3} + \frac{b}{a^2\sin(dx+c)} + \frac{b^4\ln(a+b\sin(dx+c))}{a^3(a+b)(a-b)} - \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
default	$\frac{-\frac{1}{2a\sin(dx+c)^2} + \frac{(a^2+b^2)\ln(\sin(dx+c))}{a^3} + \frac{b}{a^2\sin(dx+c)} + \frac{b^4\ln(a+b\sin(dx+c))}{a^3(a+b)(a-b)} - \frac{\ln(1+\sin(dx+c))}{2a-2b} - \frac{\ln(\sin(dx+c)-1)}{2a+2b}}{d}$
norman	$\frac{-\frac{1}{8ad} - \frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} + \frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d} + \frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{(a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3d} + \frac{b^4\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\right.}{a^3d(a^2-b^2)}$
risch	$-\frac{2ib^2x}{a^3} - \frac{2ic}{ad} + \frac{ic}{d(a+b)} - \frac{2ix}{a} - \frac{2ib^2c}{a^3d} - \frac{2ib^4c}{a^3d(a^2-b^2)} - \frac{2ib^4x}{a^3(a^2-b^2)} + \frac{ic}{d(a-b)} + \frac{ix}{a+b} + \frac{2i\left(-ia\,e^{2i(dx+c)}\right.}{d\,a^2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sin(d*x+c)^2+(a^2+b^2)/a^3*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)+b^
4/a^3/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a-2*b)*ln(1+sin(d*x+c))-1/(2*a+2*
b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 114, normalized size = 0.86

$$\frac{\frac{2\,b^4\log(b\sin(dx+c)+a)}{a^5-a^3b^2}-\frac{\log(\sin(dx+c)+1)}{a-b}-\frac{\log(\sin(dx+c)-1)}{a+b}+\frac{2\,(a^2+b^2)\log(\sin(dx+c))}{a^3}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^4*log(b*sin(d*x + c) + a)/(a^5 - a^3*b^2) - log(sin(d*x + c) + 1)/
(a - b) - log(sin(d*x + c) - 1)/(a + b) + 2*(a^2 + b^2)*log(sin(d*x + c))/a
^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.56, size = 224, normalized size = 1.70

$$\frac{a^4-a^2b^2+2\,(b^4\cos(dx+c)^2-b^4)\log(b\sin(dx+c)+a)-2\,(a^4-b^4-(a^4-b^4)\cos(dx+c)^2)\log\left(-\frac{1}{2}\sin(dx+c)\right)+(a^4+a^3b-(a^4+a^3b)\cos(dx+c)^2)\log(\sin(dx+c)+1)+(a^4-a^3b-(a^4-a^3b)\cos(dx+c)^2)\log(-\sin(dx+c)+1)-2\,(a^3b-ab^3)\sin(dx+c)}{2\,((a^5-a^3b^2)d\cos(dx+c)^2-(a^5-a^3b^2)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^4 - a^2*b^2 + 2*(b^4*cos(d*x + c)^2 - b^4)*log(b*sin(d*x + c) + a) -
 2*(a^4 - b^4 - (a^4 - b^4)*cos(d*x + c)^2)*log(-1/2*sin(d*x + c)) + (a^4 +
 a^3*b - (a^4 + a^3*b)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + (a^4 - a^3*b
 - (a^4 - a^3*b)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^3*b - a*b^3)
*sin(d*x + c))/((a^5 - a^3*b^2)*d*cos(d*x + c)^2 - (a^5 - a^3*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)\sec(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 148, normalized size = 1.12

$$\frac{\frac{2\,b^5\log(|b\sin(dx+c)+a|)}{a^5b-a^3b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}+\frac{2\,(a^2+b^2)\log(|\sin(dx+c)|)}{a^3}-\frac{3\,a^2\sin(dx+c)^2+3\,b^2\sin(dx+c)^2-2\,ab\sin(dx+c)+a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - a^3*b^3) - log(abs(sin(d*x
 + c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b) + 2*(a^2 + b^2)*lo
g(abs(sin(d*x + c)))/a^3 - (3*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2
*a*b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d
```

Mupad [B]
time = 11.85, size = 125, normalized size = 0.95

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)}{a^3\,d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin(c+d\,x)}{a^2}}{d\sin\left(c+d\,x\right)^2}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}+\frac{b^4\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^5-a^3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x))*(a^2 + b^2))/(a^3*d) - (1/(2*a) - (b*sin(c + d*x))/a^2)/
(d*sin(c + d*x)^2) - log(sin(c + d*x) - 1)/(2*d*(a + b)) - log(sin(c + d*x)
 + 1)/(2*d*(a - b)) + (b^4*log(a + b*sin(c + d*x)))/(d*(a^5 - a^3*b^2))
```

3.1338 $\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=268

$$\frac{3bx}{2(a^2-b^2)}-\frac{a^2(2a^2+b^2)\,x}{2b^3(a^2-b^2)}+\frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}\,d}+\frac{a\cos(c+dx)}{(a^2-b^2)\,d}-\frac{a^3\cos(c+dx)}{b^2(a^2-b^2)\,d}+\frac{a\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a^2}{}$$

```
[Out] 3/2*b*x/(a^2-b^2)-1/2*a^2*(2*a^2+b^2)*x/b^3/(a^2-b^2)+2*a^5*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/d+a*cos(d*x+c)/(a^2-b
^2)/d-a^3*cos(d*x+c)/b^2/(a^2-b^2)/d+a*sec(d*x+c)/(a^2-b^2)/d+1/2*a^2*cos(d
*x+c)*sin(d*x+c)/b/(a^2-b^2)/d-3/2*b*tan(d*x+c)/(a^2-b^2)/d+1/2*b*sin(d*x+c
)^2*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$,
Rules used = {2981, 2670, 14, 2671, 294, 327, 209, 2872, 3102, 2814, 2739, 632, 210}

$$\frac{a\cos(c+dx)}{d(a^2-b^2)}-\frac{3b\tan(c+dx)}{2d(a^2-b^2)}+\frac{a\sec(c+dx)}{d(a^2-b^2)}+\frac{b\sin^2(c+dx)\tan(c+dx)}{2d(a^2-b^2)}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2bd(a^2-b^2)}+\frac{3bx}{2(a^2-b^2)}-\frac{a^2x(2a^2+b^2)}{2b^3(a^2-b^2)}+\frac{2a^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^3d(a^2-b^2)^{3/2}}-\frac{a^3\cos(c+dx)}{b^2d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*x)/(2*(a^2 - b^2)) - (a^2*(2*a^2 + b^2)*x)/(2*b^3*(a^2 - b^2)) + (2*a^
5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^3*(a^2 - b^2)^(3/2)*
d) + (a*Cos[c + d*x])/((a^2 - b^2)*d) - (a^3*Cos[c + d*x])/(b^2*(a^2 - b^2)
*d) + (a*Sec[c + d*x])/((a^2 - b^2)*d) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2
*b*(a^2 - b^2)*d) - (3*b*Tan[c + d*x])/(2*(a^2 - b^2)*d) + (b*Sin[c + d*x]^
2*Tan[c + d*x])/(2*(a^2 - b^2)*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^3(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin^2(c+dx)}{a^2 -}$$

$$= \frac{a^2\cos(c+dx)\sin(c+dx)}{2b(a^2-b^2)d} - \frac{a^2\int \frac{a+b\sin(c+dx)-2a\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{2b(a^2-b^2)} - \frac{a\text{Subst}\Big(\int}{}$$

$$= -\frac{a^3\cos(c+dx)}{b^2(a^2-b^2)d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2b(a^2-b^2)d} + \frac{b\sin^2(c+dx)\tan(c+dx}{2(a^2-b^2)d}$$

$$= -\frac{a^2(2a^2+b^2)x}{2b^3(a^2-b^2)} + \frac{a\cos(c+dx)}{(a^2-b^2)d} - \frac{a^3\cos(c+dx)}{b^2(a^2-b^2)d} + \frac{a\sec(c+dx)}{(a^2-b^2)d} + \frac{a^2}{}$$

$$= \frac{3bx}{2(a^2-b^2)} - \frac{a^2(2a^2+b^2)x}{2b^3(a^2-b^2)} + \frac{a\cos(c+dx)}{(a^2-b^2)d} - \frac{a^3\cos(c+dx)}{b^2(a^2-b^2)d} + \frac{a\sec(c}{(a^2-}$$

$$= \frac{3bx}{2(a^2-b^2)} - \frac{a^2(2a^2+b^2)x}{2b^3(a^2-b^2)} + \frac{a\cos(c+dx)}{(a^2-b^2)d} - \frac{a^3\cos(c+dx)}{b^2(a^2-b^2)d} + \frac{a\sec(c}{(a^2-}$$

$$= \frac{3bx}{2(a^2-b^2)} - \frac{a^2(2a^2+b^2)x}{2b^3(a^2-b^2)} + \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}d} + \frac{a\cos(c+}{(a^2-b^2)}$$

Mathematica [A]
time = 1.04, size = 221, normalized size = 0.82

$$\frac{\frac{-4ab^3+4a^4(c+dx)+2a^2b^2(c+dx)-6b^4(c+dx)}{-a^2b^3+b^5} + \frac{8a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^3(a^2-b^2)^{3/2}} - \frac{4a\cos(c+dx)}{b^2} + \frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{4\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\sin(2(c+dx))}{b}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*a*b^3 + 4*a^4*(c + d*x) + 2*a^2*b^2*(c + d*x) - 6*b^4*(c + d*x))/(-(a^
2*b^3) + b^5) + (8*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b
^3*(a^2 - b^2)^(3/2)) - (4*a*Cos[c + d*x])/b^2 + (4*Sin[(c + d*x)/2])/((a +
 b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (4*Sin[(c + d*x)/2])/((a - b)*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + Sin[2*(c + d*x)]/b)/(4*d)
```

Maple [A]
time = 0.39, size = 208, normalized size = 0.78

method	result

derivativedivides	$\frac{\frac{64}{(64a-64b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2+3b^2)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^3}}{d}$
default	$\frac{\frac{64}{(64a-64b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2\left(\frac{\frac{b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+ab}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(2a^2+3b^2)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^3}}{d}$
risch	$-\frac{x\,a^2}{b^3}-\frac{3x}{2b}-\frac{i e^{2i(dx+c)}}{8bd}-\frac{a\,e^{i(dx+c)}}{2b^2d}-\frac{a\,e^{-i(dx+c)}}{2b^2d}+\frac{i e^{-2i(dx+c)}}{8bd}+\frac{2i\left(ia\,e^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}-\frac{ia^5\ln\left(e^{i(}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(64/(64*a-64*b)/(tan(1/2*d*x+1/2*c)+1)-2/b^3*((1/2*b^2*tan(1/2*d*x+1/2*
c)^3+a*b*tan(1/2*d*x+1/2*c)^2-1/2*b^2*tan(1/2*d*x+1/2*c)+a*b)/(1+tan(1/2*d*
x+1/2*c)^2)^2+1/2*(2*a^2+3*b^2)*arctan(tan(1/2*d*x+1/2*c)))+2/(a-b)/(a+b)*a
^5/b^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2))-64/(64*a+64*b)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.40, size = 521, normalized size = 1.94

$$\left[\frac{\sqrt{-a^2+b^2}\,a^5\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3b^3-2ab^5-(2a^6-a^4b^2-4a^2b^4+3b^6)dx\cos(dx+c)-2(a^5b-2a^3b^3+ab^5)\cos(dx+c)^2-(2a^2b^4-2b^6-(a^4b^2-2a^2b^4+b^6)\cos(dx+c)^2)\sin(dx+c)}{2(a^4b^3-2a^2b^5+b^7)d\cos(dx+c)},\ -\frac{2\sqrt{a^2-b^2}\,a^5\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-2a^3b^3+2ab^5+(2a^6-a^4b^2-4a^2b^4+3b^6)dx\cos(dx+c)+2(a^5b-2a^3b^3+ab^5)\cos(dx+c)^2+(2a^2b^4-2b^6-(a^4b^2-2a^2b^4+b^6)\cos(dx+c)^2)\sin(dx+c)}{2(a^4b^3-2a^2b^5+b^7)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^5*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
```

```
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b^3 - 2*a*b^5 - (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*co
s(d*x + c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 - (2*a^2*b^4 - 2*
b^6 - (a^4*b^2 - 2*a^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 -
 2*a^2*b^5 + b^7)*d*cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*a^5*arctan(-(a*s
in(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^3*b^3 +
 2*a*b^5 + (2*a^6 - a^4*b^2 - 4*a^2*b^4 + 3*b^6)*d*x*cos(d*x + c) + 2*(a^5*
b - 2*a^3*b^3 + a*b^5)*cos(d*x + c)^2 + (2*a^2*b^4 - 2*b^6 - (a^4*b^2 - 2*a
^2*b^4 + b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^4*b^3 - 2*a^2*b^5 + b^7)*d*
cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 208, normalized size = 0.78

$$\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^5}{(a^2b^3-b^5)\sqrt{a^2-b^2}}+\frac{4\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}-\frac{(2a^2+3b^2)(dx+c)}{b^3}-\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^5/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) + 4*(b*t
an(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (2*a^
2 + 3*b^2)*(d*x + c)/b^3 - 2*(b*tan(1/2*d*x + 1/2*c)^3 + 2*a*tan(1/2*d*x +
1/2*c)^2 - b*tan(1/2*d*x + 1/2*c) + 2*a)/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*b^
2))/d
```

Mupad [**B**]
time = 16.81, size = 2098, normalized size = 7.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

[Out] (4*a^5*cos(c + d*x) + (5*a^5)/2 + (3*a^5*cos(2*c + 2*d*x))/2)/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^8*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^3*d*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*((11*a^4*sin(c + d*x))/8 + (3*a^4*sin(3*c + 3*d*x))/8 - 3*a^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^4*((3*a)/2 + 2*a*cos(c + d*x) + (a*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^5*((9*sin(c + d*x))/8 + sin(3*c + 3*d*x)/8 - 3*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a^7*cos(c + d*x) + a^7/2 + (a^7*cos(2*c + 2*d*x))/2)/(b^2*d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^2*(5*a^3*cos(c + d*x) + (7*a^3)/2 + (3*a^3*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + ((a^6*sin(c + d*x))/8 + (a^6*sin(3*c + 3*d*x))/8 + 3*a^6*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*((19*a^2*sin(c + d*x))/8 + (3*a^2*sin(3*c + 3*d*x))/8 - 7*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (a^5*atan((a^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*8i + a^18*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*8i - b^18*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*18i + a^3*b^15*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*42i - a^5*b^13*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*67i + a^7*b^11*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*24i + a^9*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*45i - a^11*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*46i + a^13*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*3i + a^15*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*12i - a^10*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*12i + a^2*b^16*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*93i - a^4*b^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*176i + a^6*b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*115i + a^8*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*66i - a^10*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*133i + a^12*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*36i + a^14*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*45i - a^16*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*36i - a^11*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2)*4i - a*b^17*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*9i - a^17*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*4i)/(18*b^21*sin(c/2 + (d*x)/2) + 9*a*b^20*cos(c/2 + (d*x)/2) - 60*a^3*b^18*cos(c/2 + (d*x)/2) + 160*a^5*b^16*cos(c/2 + (d*x)/2) - 200*a^7*b^14*cos(c/2 + (d*x)/2) + 70*a^9*b^12*cos(c/2 + (d*x)/2) + 116*a^11*b^10*cos(c/2 + (d*x)/2) - 160*a^13*b^8*cos(c/2 + (d*x)/2) + 80*a^15*b^6*cos(c/2 + (d*x)/2) - 15*a^17*b^4*cos(c/2 + (d*x)/2) - 120*a^2*b^19*sin(c/2 + (d*x)/2) + 320*a^4*b^17*sin(c/2 + (d*x)/2) - 400*a^6*b^15*sin(c/2 + (d*x)/2) + 140*a^8*b^13*sin(c/2 +

(d*x)/2) + 232*a^10*b^11*sin(c/2 + (d*x)/2) - 320*a^12*b^9*sin(c/2 + (d*x)/2) + 160*a^14*b^7*sin(c/2 + (d*x)/2) - 30*a^16*b^5*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i)/(b^3*d*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))

3.1339 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=183

$$-\frac{ax}{a^2-b^2}+\frac{a^3x}{b^2(a^2-b^2)}-\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a^2-b^2)^{3/2}d}+\frac{a^2\cos(c+dx)}{b(a^2-b^2)d}-\frac{b\cos(c+dx)}{(a^2-b^2)d}-\frac{b\sec(c+dx)}{(a^2-b^2)d}+\frac{a\tan(c$$

```
[Out] -a*x/(a^2-b^2)+a^3*x/b^2/(a^2-b^2)-2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a
^2-b^2)^(1/2))/b^2/(a^2-b^2)^(3/2)/d+a^2*cos(d*x+c)/b/(a^2-b^2)/d-b*cos(d*x
+c)/(a^2-b^2)/d-b*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2981, 3554, 8, 2670, 14, 2825, 12, 2814, 2739, 632, 210}

$$\frac{a^2\cos(c+dx)}{bd(a^2-b^2)}-\frac{b\cos(c+dx)}{d(a^2-b^2)}+\frac{a\tan(c+dx)}{d(a^2-b^2)}-\frac{b\sec(c+dx)}{d(a^2-b^2)}-\frac{ax}{a^2-b^2}-\frac{2a^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d(a^2-b^2)^{3/2}}+\frac{a^3x}{b^2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*x)/(a^2 - b^2)) + (a^3*x)/(b^2*(a^2 - b^2)) - (2*a^4*ArcTan[(b + a*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)*d) + (a^2*Cos[c + d
*x])/(b*(a^2 - b^2)*d) - (b*Cos[c + d*x])/((a^2 - b^2)*d) - (b*Sec[c + d*x]
)/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sin(c+dx)\tan^2(c+dx)\,d}{a^2-b^2}$$

$$= \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{a\int 1\,dx}{a^2-b^2} + \frac{a^2\int \frac{a\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + b\text{Subst}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} + \frac{a\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + b\text{Subs}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} - \frac{b\cos(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d}$$

$$= -\frac{ax}{a^2-b^2} + \frac{a^3x}{b^2\left(a^2-b^2\right)} - \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}d} + \frac{a^2\cos(c+dx)}{b\left(a^2-b^2\right)d} -$$

Mathematica [A]
time = 0.78, size = 186, normalized size = 1.02

$$\frac{\frac{b^3-a^3(c+dx)+ab^2(c+dx)}{-a^2b^2+b^4} - \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{3/2}} + \frac{\cos(c+dx)}{b} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b^3 - a^3*(c + d*x) + a*b^2*(c + d*x))/(-(a^2*b^2) + b^4) - (2*a^4*ArcTan
[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(3/2)) + Cos[c
 + d*x]/b + Sin[(c + d*x)/2]/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) + Sin[(c + d*x)/2]/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/d
```

Maple [A]
time = 0.34, size = 150, normalized size = 0.82

method	result
derivativedivides	$\frac{-\frac{32}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)b^2\sqrt{a^2-b^2}}-\frac{32}{(32a+32b)\left(\tan\right.}}{d}$
default	$\frac{-\frac{32}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\frac{2b}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2a^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)b^2\sqrt{a^2-b^2}}-\frac{32}{(32a+32b)\left(\tan\right.}}{d}$
risch	$\frac{ax}{b^2}+\frac{e^{i(dx+c)}}{2bd}+\frac{e^{-i(dx+c)}}{2bd}+\frac{-2ia+2b\,e^{i(dx+c)}}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}-\frac{a^4\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d\,b^2}+\frac{a^4\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32/(32*a-32*b)/(tan(1/2*d*x+1/2*c)+1)+2/b^2*(b/(1+tan(1/2*d*x+1/2*c)^
2)+a*arctan(tan(1/2*d*x+1/2*c)))-2/(a-b)/(a+b)*a^4/b^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-32/(32*a+32*b)/(tan(1/
2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.42, size = 431, normalized size = 2.36

$$\left[\frac{\sqrt{-a^2+b^2}\,a^4\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b^3+2b^5+2(a^5-2a^3b^2+ab^4)dx\cos(dx+c)+2(a^4b-2a^2b^3+b^5)\cos(dx+c)^2+2(a^3b^2-ab^4)\sin(dx+c)}{2(a^4b^2-2a^2b^4+b^6)d\cos(dx+c)}, \frac{\sqrt{a^2-b^2}\,a^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^2b^3+b^5+(a^5-2a^3b^2+ab^4)dx\cos(dx+c)+(a^4b-2a^2b^3+b^5)\cos(dx+c)^2+(a^3b^2-ab^4)\sin(dx+c)}{(a^4b^2-2a^2b^4+b^6)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b^3 + 2*b^5 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(a^3*b^2 - a*b^4)*sin(d*x +
c))/((a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^4*arct
an(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2
*b^3 + b^5 + (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cos(d*x + c) + (a^4*b - 2*a^2*b^
3 + b^5)*cos(d*x + c)^2 + (a^3*b^2 - a*b^4)*sin(d*x + c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 173, normalized size = 0.95

$$-\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{(a^2b^2-b^4)\sqrt{a^2-b^2}}-\frac{(dx+c)a}{b^2}+\frac{2\left(ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2-2b^2\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)(a^2b-b^3)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^4/((a^2*b^2 - b^4)*sqrt(a^2 - b^2)) - (d*x + c)
*a/b^2 + 2*(a*b*tan(1/2*d*x + 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c)^2 + a*b*t
an(1/2*d*x + 1/2*c) + a^2 - 2*b^2)/((tan(1/2*d*x + 1/2*c)^4 - 1)*(a^2*b - b
^3)))/d
```

Mupad [B]
time = 14.28, size = 1656, normalized size = 9.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*sin(c + d*x) - 6*a^5*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^7*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b^2*d*(a^2 - b^2)*(a^4 + b^4 - 2*
a^2*b^2)) + (a^6*cos(c + d*x) + a^6/2 + (a^6*cos(2*c + 2*d*x))/2)/(b*d*cos(
c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*(5*a^2*cos(c + d*x) +
(7*a^2)/2 + (3*a^2*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 +
 b^4 - 2*a^2*b^2)) - (b^2*(2*a^3*sin(c + d*x) - 6*a^3*cos(c + d*x)*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^
4 - 2*a^2*b^2)) - (b^5*(2*cos(c + d*x) + cos(2*c + 2*d*x)/2 + 3/2))/(d*cos(
c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^4*(a*sin(c + d*x) - 2*a*
cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*(4*a^4*cos(c + d*x) + (5*a^4)/2 +
 (3*a^4*cos(2*c + 2*d*x))/2))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^
2*b^2)) + (a^4*atan(((2*b^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(1/2) - 2*a^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b
^2)^(1/2) - 2*a^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3
/2) - 6*a^3*b^11*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/
2) + 15*a^5*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2
) - 20*a^7*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
 + 15*a^9*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
- 6*a^11*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) +
 3*a^6*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 1
3*a^2*b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 3
6*a^4*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 5
6*a^6*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 54
*a^8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 33*
a^10*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 12*
a^12*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^7
*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a*b^13*co
s(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^13*b*cos(c/2
 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))*1i)/((a^4 + b^4 - 2*
a^2*b^2)*(2*b^13*sin(c/2 + (d*x)/2) + a*b^12*cos(c/2 + (d*x)/2) - 6*a^3*b^1
0*cos(c/2 + (d*x)/2) + 15*a^5*b^8*cos(c/2 + (d*x)/2) - 19*a^7*b^6*cos(c/2 +
 (d*x)/2) + 12*a^9*b^4*cos(c/2 + (d*x)/2) - 3*a^11*b^2*cos(c/2 + (d*x)/2) -
 12*a^2*b^11*sin(c/2 + (d*x)/2) + 30*a^4*b^9*sin(c/2 + (d*x)/2) - 38*a^6*b^
7*sin(c/2 + (d*x)/2) + 24*a^8*b^5*sin(c/2 + (d*x)/2) - 6*a^10*b^3*sin(c/2 +
 (d*x)/2))))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b^2*d*(a^2 - b^2)*(a^4 + b^4
 - 2*a^2*b^2))
```

3.1340 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=133

$$-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}+\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d}+\frac{a\sec(c+dx)}{\left(a^2-b^2\right)d}-\frac{b\tan(c+dx)}{\left(a^2-b^2\right)d}$$

```
[Out] -a^2*x/b/(a^2-b^2)+b*x/(a^2-b^2)+2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2
-b^2)^(1/2))/b/(a^2-b^2)^(3/2)/d+a*sec(d*x+c)/(a^2-b^2)/d-b*tan(d*x+c)/(a^2
-b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2981, 2686, 8, 3554, 2814, 2739, 632, 210}

$$-\frac{b\tan(c+dx)}{d\left(a^2-b^2\right)}+\frac{a\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2x}{b\left(a^2-b^2\right)}+\frac{bx}{a^2-b^2}+\frac{2a^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a^2*x)/(b*(a^2 - b^2))) + (b*x)/(a^2 - b^2) + (2*a^3*ArcTan[(b + a*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)*d) + (a*Sec[c + d*x])/(
(a^2 - b^2)*d) - (b*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sin(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \tan^2(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{a^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\left(a^2-b^2\right)} + \frac{b\int 1\,dx}{a^2-b^2} + \frac{a\text{Subst}(\int$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{}{a}\right.}{}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\tan(c+dx)}{\left(a^2-b^2\right)d} - \frac{(4a^3)\,\text{Subst}\left(\int \right.}{}$$

$$= -\frac{a^2x}{b\left(a^2-b^2\right)} + \frac{bx}{a^2-b^2} + \frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{3/2}d} + \frac{a\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b}{}$$

Mathematica [A]

time = 0.59, size = 152, normalized size = 1.14

$$\frac{\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{-\left(\left(a^2-b^2\right)(c+dx)\cos(c+dx)\right)+b(a-b\sin(c+dx))}{(a-b)(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
 + (-((a^2 - b^2)*(c + d*x)*Cos[c + d*x]) + b*(a - b*Sin[c + d*x]))/((a - b
)*(a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])))/(b*d)
```

Maple [A]

time = 0.29, size = 130, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{16}{(16a-16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)b\sqrt{a^2-b^2}}-\frac{16}{(16a+16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
default	$\frac{-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{16}{(16a-16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)b\sqrt{a^2-b^2}}-\frac{16}{(16a+16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$

risch	$-\frac{x}{b}+\frac{2i\left(ia\,e^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}+\frac{ia^3\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)db}-\frac{ia^3\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-\right.}{b\sqrt{\ }}\right)}{\sqrt{a^2-b^2}\,(a+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/b*arctan(tan(1/2*d*x+1/2*c))+16/(16*a-16*b)/(tan(1/2*d*x+1/2*c)+1)+
2/(a-b)/(a+b)*a^3/b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))-16/(16*a+16*b)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 369, normalized size = 2.77

$$\left[\frac{\sqrt{-a^2+b^2}\,a^3\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3b-2ab^3-2(a^4-2a^2b^2+b^4)dx\cos(dx+c)-2(a^2b^2-b^4)\sin(dx+c)}{2(a^4b-2a^2b^3+b^5)d\cos(dx+c)},-\frac{\sqrt{a^2-b^2}\,a^3\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^3b+ab^3+(a^4-2a^2b^2+b^4)dx\cos(dx+c)+(a^2b^2-b^4)\sin(dx+c)}{(a^4b-2a^2b^3+b^5)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3*b - 2*a*b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x + c) - 2
*(a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x + c)),
-(sqrt(a^2 - b^2)*a^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
 + c)))*cos(d*x + c) - a^3*b + a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*d*x*cos(d*x
+ c) + (a^2*b^2 - b^4)*sin(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cos(d*x +
 c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 131, normalized size = 0.98

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3}{(a^2b-b^3)\sqrt{a^2-b^2}}-\frac{dx+c}{b}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a^3/((a^2*b - b^3)*sqrt(a^2 - b^2)) - (d*x + c)/b
+ 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1))
)/d
```

Mupad [B]
time = 14.05, size = 1538, normalized size = 11.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*cos(c + d*x) + a^5)/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2
)) - (2*a^6*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)*(
a^4 + b^4 - 2*a^2*b^2)) - (b*(a^4*sin(c + d*x) - 6*a^4*cos(c + d*x)*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b
^4 - 2*a^2*b^2)) + (b^4*(a + a*cos(c + d*x)))/(d*cos(c + d*x)*(a^2 - b^2)*(
a^4 + b^4 - 2*a^2*b^2)) - (b^2*(2*a^3*cos(c + d*x) + 2*a^3))/(d*cos(c + d*x
)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b^3*(2*a^2*sin(c + d*x) - 6*a^2*c
os(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(
a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b^5*(sin(c + d*x) - 2*cos(c + d*x)*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/(d*cos(c + d*x)*(a^2 - b^2)*(a
^4 + b^4 - 2*a^2*b^2)) - (a^3*atan(((2*b^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 2*a^14*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^
2*b^4 + 3*a^4*b^2)^(1/2) - 2*a^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(3/2) - 6*a^3*b^11*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2) + 15*a^5*b^9*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
3*a^4*b^2)^(1/2) - 20*a^7*b^7*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2) + 15*a^9*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
```

a^4*b^2)^(1/2) - 6*a^11*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 3*a^6*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) - 13*a^2*b^12*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 36*a^4*b^10*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 56*a^6*b^8*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 54*a^8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 33*a^10*b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 12*a^12*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^7*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(3/2) + a*b^13*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^13*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))*1i)/((a^4*b + b^5 - 2*a^2*b^3)*(2*b^12*sin(c/2 + (d*x)/2) + a*b^11*cos(c/2 + (d*x)/2) - 3*a^11*b*cos(c/2 + (d*x)/2) - 6*a^3*b^9*cos(c/2 + (d*x)/2) + 15*a^5*b^7*cos(c/2 + (d*x)/2) - 19*a^7*b^5*cos(c/2 + (d*x)/2) + 12*a^9*b^3*cos(c/2 + (d*x)/2) - 12*a^2*b^10*sin(c/2 + (d*x)/2) + 30*a^4*b^8*sin(c/2 + (d*x)/2) - 38*a^6*b^6*sin(c/2 + (d*x)/2) + 24*a^8*b^4*sin(c/2 + (d*x)/2) - 6*a^10*b^2*sin(c/2 + (d*x)/2))))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(b*d*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))

3.1341 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{b\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

```
[Out] -2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-b
*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.07, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$,
Rules used = {2806, 3852, 8, 2686, 2739, 632, 210}

$$-\frac{2a^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{a\tan(c+dx)}{d\,(a^2-b^2)}-\frac{b\sec(c+dx)}{d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2}-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}\\ &= -\frac{a\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{(a^2-b^2)\,d}-\frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{(a^2-b^2)\,d}\\ &= -\frac{b\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d}+\frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\tan\left(\right.\right.}{(a^2-b^2)\,d}\\ &= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}-\frac{b\sec(c+dx)}{(a^2-b^2)\,d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 152, normalized size = 1.58

$$\frac{-2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}\,(-b+b\cos(c+dx)+a\sin(c+dx))}{(a-b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [A]
time = 0.24, size = 112, normalized size = 1.17

method	result
derivativedivides	$\frac{-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
default	$\frac{-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
risch	$\frac{-2ia+2b\,e^{i(dx+c)}}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}\,a+a^2-}{b\sqrt{-a^2+b^2}}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-8/(8*a+8*b)/(tan(1/2*d*x+1/2*c)-1)
-2*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.37, size = 305, normalized size = 3.18

$$\left[\frac{\sqrt{-a^2+b^2}\,a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3+2(a^3-ab^2)\sin(dx+c)}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)},\ \frac{\sqrt{a^2-b^2}\,a^2\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^2b+b^3+(a^3-ab^2)\sin(dx+c)}{(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 107, normalized size = 1.11

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B]
time = 11.96, size = 148, normalized size = 1.54

$$\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a^2\,\mathrm{atan}\left(\frac{\frac{a^2\,\left(2\,a^2\,b-2\,b^3\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}+\frac{2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}}{2\,a^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d
*x)/2)^2 - 1)) - (2*a^2*atan(((a^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b
)^(3/2)) + (2*a^3*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3
/2)))/(2*a^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.1342 $\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}d}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

[Out] 2*a*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d+sec(d*x+c)*(a-b*sin(d*x+c))/(a^2-b^2)/d

Rubi [A]

time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2945, 12, 2739, 632, 210}

$$\frac{2ab\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2)*d) + (Sec[c + d*x]*(a - b*Sin[c + d*x]))/((a^2 - b^2)*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)\,d}-\frac{\int \frac{ab}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}\\ &= \frac{\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)\,d}+\frac{(ab)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)\,d}+\frac{(2ab)\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(c+d\right.\right.}{(a^2-b^2)\,d}\\ &= \frac{\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)\,d}-\frac{(4ab)\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx,x,2b+2a\,\text{ta}\right.}{(a^2-b^2)\,d}\\ &= \frac{2ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d}+\frac{\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 151, normalized size = 1.84

$$\frac{2ab\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}\,(a-a\cos(c+dx)-b\sin(c+dx))}{(a-b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(a - a*Cos[c + d*x] - b*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[a^
```

```
2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

Maple [A]
time = 0.22, size = 111, normalized size = 1.35

method	result
derivativedivides	$\frac{-\frac{4}{(4a+4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{(4a-4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2ab\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
default	$\frac{-\frac{4}{(4a+4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{4}{(4a-4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2ab\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}}{d}$
risch	$\frac{2i\left(ia\,e^{i(dx+c)}+b\right)}{d\left(-a^2+b^2\right)\left(e^{2i(dx+c)}+1\right)}-\frac{iba\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-a^2+b^2\right)}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}+\frac{iba\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+\right.}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4/(4*a+4*b)/(tan(1/2*d*x+1/2*c)-1)+4/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)
+2*a*b/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/
(a^2-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 308, normalized size = 3.76

$$\left[\frac{\sqrt{-a^2+b^2}\,ab\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^3-2ab^2-2(a^2b-b^3)\sin(dx+c)}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)},\ -\frac{\sqrt{a^2-b^2}\,ab\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-a^3+ab^2+(a^2b-b^3)\sin(dx+c)}{(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a*b*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^3 - 2*a*b^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
+ b^4)*d*cos(d*x + c)), -(sqrt(a^2 - b^2)*a*b*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^3 + a*b^2 + (a^2*b - b^3)*
sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)\sec^2\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 106, normalized size = 1.29

$$\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*a*b/(a^2 - b^2)^(3/2) + (b*tan(1/2*d*x + 1/2*c) -
a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B]
time = 12.02, size = 151, normalized size = 1.84

$$\frac{2\,a\,b\,\mathrm{atan}\left(\frac{\frac{a\,b\,\left(2\,a^2\,b-2\,b^3\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}+\frac{2\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}}{2\,a\,b}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (2*a*b*atan(((a*b*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a - b)^(3/2)) + (2*a^2
*b*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)^(3/2)))/(2*a*b)))
/(d*(a + b)^(3/2)*(a - b)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b*tan(c/2 + (d*x
)/2))/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.1343 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=118

$$\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{3/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{b\sec(c+dx)(b-a\sin(c+dx))}{a\left(a^2-b^2\right)d}$$

```
[Out] 2*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(3/2)/d-
arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+b*sec(d*x+c)*(b-a*sin(d*x+c))/a/(a^2
-b^2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2977, 2702, 327, 213, 2775, 12, 2739, 632, 210}

$$\frac{2b^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{3/2}}+\frac{b\sec(c+dx)(b-a\sin(c+dx))}{ad\left(a^2-b^2\right)}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(3/
2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + (b*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)\sec^2(c+dx)}{a} - \frac{b\sec^2(c+dx)}{a(a+b\sin(c+dx))}\right)dx$$

$$= \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a}$$

$$= \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a(a^2-b^2)d} + \frac{b\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a(a^2-b^2)} + \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, \ldots\right.}{ad}$$

$$= \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a(a^2-b^2)d} + \frac{b^3\int \frac{1}{a+b\sin(c+dx)}\,dx}{a(a^2-b^2)} + \frac{\text{Sub}\ldots}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a(a^2-b^2)d} + \ldots$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(c+dx)(b-a\sin(c+dx))}{a(a^2-b^2)d} - \ldots$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{3/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad} + \frac{b\sec(\ldots}{}$$

Mathematica [A]

time = 0.25, size = 191, normalized size = 1.62

$$\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx) + \sqrt{a^2-b^2}\left(-\left((a^2-b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right) + a(a-b\sin(c+dx))\right)}{a(a-b)(a+b)\sqrt{a^2-b^2}\,d\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqrt
[a^2 - b^2]*(-((a^2 - b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c
 + d*x)/2]])) + a*(a - b*Sin[c + d*x])))/(a*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]))
```

Maple [A]

time = 0.39, size = 122, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a} + \frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{2b^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)a\sqrt{a^2-b^2}} - \frac{1}{(a+b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$

default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)a\sqrt{a^2-b^2}}-\frac{1}{(a+b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
risch	$\frac{2i\left(ia\,e^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}+\frac{ib^3\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a+a^2-b^2\right)}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}-\frac{ib^3\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2-b^2}\,a-\right.}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)(a-b)da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(tan(1/2*d*x+1/2*c))+1/(a-b)/(tan(1/2*d*x+1/2*c)+1)+2*b^3/(a-b)/
(a+b)/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2))-1/(a+b)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.52, size = 457, normalized size = 3.87

$$\left[\frac{\sqrt{-a^2+b^2}\,b^3\cos(dx+c)\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4-2a^2b^2-(a^4-2a^2b^2+b^4)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(a^4-2a^2b^2+b^4)\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-2(a^3b-ab^3)\sin(dx+c)}{2(a^5-2a^3b^2+ab^4)d\cos(dx+c)},\ -\frac{2\sqrt{a^2-b^2}\,b^3\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)-2a^4+2a^2b^2+(a^4-2a^2b^2+b^4)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(a^4-2a^2b^2+b^4)\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2(a^3b-ab^3)\sin(dx+c)}{2(a^5-2a^3b^2+ab^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^3*cos(d*x + c)*log(-((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d
*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
 b^2)) + 2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(1/2*c
os(d*x + c) + 1/2) + (a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)*log(-1/2*cos(d*x
+ c) + 1/2) - 2*(a^3*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*
cos(d*x + c)), -1/2*(2*sqrt(a^2 - b^2)*b^3*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - 2*a^4 + 2*a^2*b^2 + (a^4 - 2*a^
```

```
2*b^2 + b^4)*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - (a^4 - 2*a^2*b^2 +
b^4)*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) + 2*(a^3*b - a*b^3)*sin(d*x
+ c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 135, normalized size = 1.14

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^3}{(a^3-ab^2)\sqrt{a^2-b^2}}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c
) + b)/sqrt(a^2 - b^2)))*b^3/((a^3 - a*b^2)*sqrt(a^2 - b^2)) + log(abs(tan(
1/2*d*x + 1/2*c)))/a + 2*(b*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2
*d*x + 1/2*c)^2 - 1)))/d
```

Mupad [B]
time = 12.88, size = 659, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^6*cos(c + d*x) + a^6 + a^2*b^4 - 2*a^4*b^2 + a^6*cos(c + d*x)*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)) + a^2*b^4*cos(c + d*x) - 2*a^4*b^2*cos(c + d*x) + 2*a^3
*b^3*sin(c + d*x) - a*b^5*sin(c + d*x) - a^5*b*sin(c + d*x) + b^3*cos(c + d
*x)*atan((a^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*
1i + b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*4i -
a^2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*3i + a
```

```
*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i - a^3*
b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*1i)/(a^7*cos
(c/2 + (d*x)/2) - 4*b^7*sin(c/2 + (d*x)/2) - 2*a*b^6*cos(c/2 + (d*x)/2) + 2
*a^6*b*sin(c/2 + (d*x)/2) + 4*a^3*b^4*cos(c/2 + (d*x)/2) - 3*a^5*b^2*cos(c/
2 + (d*x)/2) + 9*a^2*b^5*sin(c/2 + (d*x)/2) - 7*a^4*b^3*sin(c/2 + (d*x)/2))
)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)*2i + 3*a^2*b^4*cos(c + d*x)*log
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*a^4*b^2*cos(c + d*x)*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d*cos(c + d*x)*(a^2 - b^2)*(a^4 + b^4
- 2*a^2*b^2))
```

3.1344 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{\left(a^2-b^2\right)d}+\frac{a\tan(c+dx)}{\left(a^2-b^2\right)d}$$

```
[Out] -2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/2)
/d+b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-b*sec(d*x+c)/(a^2-b^2)/d+a*ta
n(d*x+c)/(a^2-b^2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 150, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2977, 2702, 327, 213, 2700, 14, 2775, 12, 2739, 632, 210}

$$-\frac{2b^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}-\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^
(3/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[
c + d*x])/(a^2*d) - (b^2*Sec[c + d*x]*(b - a*Sin[c + d*x]))/(a^2*(a^2 - b^2
)*d) + Tan[c + d*x]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \left(-\frac{b\csc(c+dx)\sec^2(c+dx)}{a^2}+\frac{\csc^2(c+dx)\sec^2(c+dx)}{a}+\frac{b^2\sec^2(}{a^2(a+b\sin}\right.\\
&= \frac{\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a}-\frac{b\int \csc(c+dx)\sec^2(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\mathrm{se}}{a+}}{}\\
&= -\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2\left(a^2-b^2\right)d}-\frac{b^2\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}+\frac{\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,d\right.}{}\\
&= -\frac{b\sec(c+dx)}{a^2d}-\frac{b^2\sec(c+dx)(b-a\sin(c+dx))}{a^2\left(a^2-b^2\right)d}-\frac{b^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}+\\
&= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-\frac{b^2\sec(c+dx)(b-}{a^2\left(a^2-\right.}\\
&= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b\sec(c+dx)}{a^2d}-\frac{b^2\sec(c+dx)(b-}{a^2\left(a^2-\right.}\\
&= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{b}{}
\end{aligned}
$$

Mathematica [A]
time = 0.72, size = 205, normalized size = 1.60

$$
\frac{-\frac{4b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a}+\frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}-\frac{2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-4*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)
^(3/2)) - Cot[(c + d*x)/2]/a + (2*b*Log[Cos[(c + d*x)/2]])/a^2 - (2*b*Log[S
in[(c + d*x)/2]])/a^2 + (2*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + Tan[(c + d*x)/2]/a)/(2*d)
```

Maple [A]
time = 0.46, size = 155, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{1}{(a+b)\left(\tan\left(\frac{d}{}\right.\right.}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{2b^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^2(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{1}{(a+b)\left(\tan\left(\frac{d}{}\right.\right.}}{d}$
risch	$-\frac{2\left(-ib^2e^{2i(dx+c)}+ab\,e^{3i(dx+c)}+2ia^2-ib^2-b\,e^{i(dx+c)}a\right)}{da\left(e^{2i(dx+c)}-1\right)\left(a^2-b^2\right)\left(e^{2i(dx+c)}+1\right)}-\frac{b\ln\left(e^{i(dx+c)}-1\right)}{a^2d}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}+\frac{b^4\ln\left(e^{i(dx+c}\right.}{\sqrt{-a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/(a-b)/(tan(1/2*d*x+1/2*c)+1)-1/2/a/tan(1/2*
d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c))-2/a^2/(a-b)/(a+b)*b^4/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/(a+b)/(tan(
1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(123) = 246$.
time = 0.52, size = 582, normalized size = 4.55

$$\left[\frac{\sqrt{-a^2+b^2}\,b^4\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)\sin(dx+c)+2a^5-2a^3b^2+(a^4b-2a^2b^3+b^5)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-(a^4b-2a^2b^3+b^5)\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2(2a^5-3a^3b^2+ab^4)\cos(dx+c)^2-2(a^4b-a^2b^3)\sin(dx+c)}{2(a^6-2a^4b^2+a^2b^4)d\cos(dx+c)\sin(dx+c)}, \frac{2\sqrt{a^2-b^2}\,b^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)\sin(dx+c)+2a^5-2a^3b^2+(a^4b-2a^2b^3+b^5)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-(a^4b-2a^2b^3+b^5)\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2(2a^5-3a^3b^2+ab^4)\cos(dx+c)^2-2(a^4b-a^2b^3)\sin(dx+c)}{2(a^6-2a^4b^2+a^2b^4)d\cos(dx+c)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*b^4*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2))*sin(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*c
os(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^
2 + a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)), 1/2*(2*sqrt(a^2 - b^2)*b^4*ar
ctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)*sin
(d*x + c) + 2*a^5 - 2*a^3*b^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x +
c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*(2*a^5 - 3*a^3*b^2 + a*b^4
)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4*b^2 + a^
2*b^4)*d*cos(d*x + c)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(123) = 246.
time = 0.50, size = 259, normalized size = 2.02

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^4}{(a^4-a^2b^2)\sqrt{a^2-b^2}}+\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}-\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-15a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^3-3ab^2}{(a^4-a^2b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
 1/2*c) + b)/sqrt(a^2 - b^2)))*b^4/((a^4 - a^2*b^2)*sqrt(a^2 - b^2)) + 6*b*
log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - 3*tan(1/2*d*x + 1/2*c)/a - (2*a^2*b*ta
n(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x +
1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) +
 2*b^3*tan(1/2*d*x + 1/2*c) + 3*a^3 - 3*a*b^2)/((a^4 - a^2*b^2)*(tan(1/2*d*
x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]

time = 13.68, size = 778, normalized size = 6.08

$$-\frac{\left(a\,b^6\,1{}\mathrm{i}-a^7\,\cos\left(2\,c+2\,d\,x\right)\,2{}\mathrm{i}-a^3\,b^4\,2{}\mathrm{i}+a^5\,b^2\,1{}\mathrm{i}+a\,b^6\,\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}-a^6\,b\,\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}-a^2\,b^5\,\sin\left(c+d\,x\right)\,2{}\mathrm{i}+a^4\,b^3\,\sin\left(c+d\,x\right)\,4{}\mathrm{i}-a^3\,b^4\,\cos\left(2\,c+2\,d\,x\right)\,4{}\mathrm{i}+a^5\,b^2\,\cos\left(2\,c+2\,d\,x\right)\,5{}\mathrm{i}+b^7\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}-a^2\,b^5\,\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+a^4\,b^3\,\sin\left(2\,c+2\,d\,x\right)\,2{}\mathrm{i}-a^6\,b\,\sin\left(c+d\,x\right)\,2{}\mathrm{i}-a^6\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+2\,b^4\,\sin\left(2\,c+2\,d\,x\right)\,\mathrm{atan}\left(\frac{a^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}\,1{}\mathrm{i}+b^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}\,4{}\mathrm{i}-a^2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}\,3{}\mathrm{i}+a\,b^3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}\,2{}\mathrm{i}-a^3\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^7+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6\,b-3\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5\,b^2-7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^3+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^4+9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^5-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^6-4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^7}\right)\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-a^2\,b^5\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin\left(2\,c+2\,d\,x\right)\,3{}\mathrm{i}+a^4\,b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sin\left(2\,c+2\,d\,x\right)\,3{}\mathrm{i}\right)\,1{}\mathrm{i}}{a^2\,d\,\sin\left(2\,c+2\,d\,x\right)\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] -((a*b^6*1i - a^7*cos(2*c + 2*d*x)*2i - a^3*b^4*2i + a^5*b^2*1i + a*b^6*cos
(2*c + 2*d*x)*1i - a^6*b*sin(2*c + 2*d*x)*1i - a^2*b^5*sin(c + d*x)*2i + a^
4*b^3*sin(c + d*x)*4i - a^3*b^4*cos(2*c + 2*d*x)*4i + a^5*b^2*cos(2*c + 2*d
*x)*5i + b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*1i
 - a^2*b^5*sin(2*c + 2*d*x)*1i + a^4*b^3*sin(2*c + 2*d*x)*2i - a^6*b*sin(c
+ d*x)*2i - a^6*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*
x)*1i + 2*b^4*sin(2*c + 2*d*x)*atan((a^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(1/2)*1i + b^4*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2)*4i - a^2*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^
4 + 3*a^4*b^2)^(1/2)*3i + a*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 +
 3*a^4*b^2)^(1/2)*2i - a^3*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*
a^4*b^2)^(1/2)*1i)/(a^7*cos(c/2 + (d*x)/2) - 4*b^7*sin(c/2 + (d*x)/2) - 2*a
*b^6*cos(c/2 + (d*x)/2) + 2*a^6*b*sin(c/2 + (d*x)/2) + 4*a^3*b^4*cos(c/2 +
(d*x)/2) - 3*a^5*b^2*cos(c/2 + (d*x)/2) + 9*a^2*b^5*sin(c/2 + (d*x)/2) - 7*
a^4*b^3*sin(c/2 + (d*x)/2)))*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - a^
2*b^5*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*3i + a^4*
b^3*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x)*3i)*1i)/(a^
2*d*sin(2*c + 2*d*x)*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
```

3.1345 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{3/2}d}-\frac{\left(3a^2+2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^3d}+\frac{b\cot(c+dx)}{a^2d}+\frac{\left(3a^2-b^2\right)\sec(c+dx)}{2a\left(a^2-b^2\right)d}-\frac{\csc^2(c}{}$$

```
[Out] 2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(3/2)/
d-1/2*(3*a^2+2*b^2)*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a^2/d+1/2*(3*a^2
-b^2)*sec(d*x+c)/a/(a^2-b^2)/d-1/2*csc(d*x+c)^2*sec(d*x+c)/a/d-b*tan(d*x+c)
/(a^2-b^2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 212, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2977, 2702, 327, 213, 2700, 14, 294, 2775, 12, 2739, 632, 210}

$$\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{b\tan(c+dx)}{a^2d}+\frac{b\cot(c+dx)}{a^2d}+\frac{2b^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{3/2}}+\frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3d\left(a^2-b^2\right)}+\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(
3/2)*d) - (3*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/(
a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (3*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c +
 d*x])/(a^3*d) - (Csc[c + d*x]^2*Sec[c + d*x])/(2*a*d) + (b^3*Sec[c + d*x]*
(b - a*Sin[c + d*x]))/(a^3*(a^2 - b^2)*d) - (b*Tan[c + d*x])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{b^2\csc(c+dx)\sec^2(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^2(c+dx)}{a^2} + \frac{\csc^3(c+d\right.$$

$$= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a^2} + \frac{b^2\int \mathrm{cs}}{}$$

$$= \frac{b^3\sec(c+dx)(b-a\sin(c+dx))}{a^3(a^2-b^2)\,d} + \frac{b^3\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{a^3(a^2-b^2)} + \frac{\mathrm{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\right.}{}$$

$$= \frac{b^2\sec(c+dx)}{a^3d} - \frac{\csc^2(c+dx)\sec(c+dx)}{2ad} + \frac{b^3\sec(c+dx)(b-a\sin(c+d}{a^3(a^2-b^2)\,d}$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{3\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx)}{a^3d}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{3\sec}{}$$

$$= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{3/2}\,d} - \frac{3\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+}{a^3d}$$

Mathematica [A]

time = 2.09, size = 261, normalized size = 1.44

$$\frac{\frac{16b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{3/2}}+\frac{4b\cot\left(\frac{1}{2}(c+dx)\right)}{a^2}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{a}-\frac{4(3a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{4(3a^2+2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{a}+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{8\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{4b\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((16*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)
^(3/2)) + (4*b*Cot[(c + d*x)/2])/a^2 - Csc[(c + d*x)/2]^2/a - (4*(3*a^2 + 2
*b^2)*Log[Cos[(c + d*x)/2]])/a^3 + (4*(3*a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]]
)/a^3 + Sec[(c + d*x)/2]^2/a + (8*Sin[(c + d*x)/2])/((a + b)*(Cos[(c + d*x)
/2] - Sin[(c + d*x)/2])) - (8*Sin[(c + d*x)/2])/((a - b)*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])) - (4*b*Tan[(c + d*x)/2])/a^2)/(8*d)
```

Maple [A]

time = 0.59, size = 199, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}+\frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(6a^2+4b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}+\frac{1}{(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(6a^2+4b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^3}+\frac{b}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$\frac{i\left(3ia^3e^{5i(dx+c)}-iab^2e^{5i(dx+c)}-2ia^3e^{3i(dx+c)}-2iab^2e^{3i(dx+c)}+2b^3e^{4i(dx+c)}+3ia^3e^{i(dx+c)}-ib^2ae^{i(dx+c)}-4be^{2i(dx+c)}\right.}{da^2\left(e^{2i(dx+c)}-1\right)^2(-a^2+b^2)\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c))+1/(a-b)/(t
an(1/2*d*x+1/2*c)+1)-1/8/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(6*a^2+4*b^2)*ln(ta
n(1/2*d*x+1/2*c))+1/2/a^2*b/tan(1/2*d*x+1/2*c)+2/a^3/(a-b)/(a+b)*b^5/(a^2-b
^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/(a+b)/
(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(170) = 340.
time = 0.73, size = 878, normalized size = 4.85

$$\left[-\frac{4a^6 - 4a^4b^2 - 2\left(3a^6 - 4a^4b^2 + a^2b^4\right)\cos\left(dx + c\right)^2 - 2\left(b^5\cos\left(dx + c\right)^3 - b^5\cos\left(dx + c\right)\right)\sqrt{-a^2 + b^2}\log\left(-\frac{\left(2a^2 - b^2\right)\cos\left(dx + c\right)^2 - 2ab\sin\left(dx + c\right) - a^2 - b^2 - 2\left(a\cos\left(dx + c\right)\sin\left(dx + c\right) + b\cos\left(dx + c\right)\right)\sqrt{-a^2 + b^2}}{b^2\cos\left(dx + c\right)^2 - 2ab\sin\left(dx + c\right) - a^2 - b^2}\right) + \left(\left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)^3 - \left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)\right)\log\left(\frac{1}{2}\cos\left(dx + c\right) + \frac{1}{2}\right) - \left(\left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)^3 - \left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx + c\right) + \frac{1}{2}\right) - 4\left(a^5b - a^3b^3 - \left(2a^5b - 3a^3b^3 + ab^5\right)\cos\left(dx + c\right)^2\right)\sin\left(dx + c\right)}{4\left(\left(a^7 - 2a^5b^2 + a^3b^4\right)d\cos\left(dx + c\right)^3 - \left(a^7 - 2a^5b^2 + a^3b^4\right)d\cos\left(dx + c\right)\right)}, -\frac{4a^6 - 4a^4b^2 - 2\left(3a^6 - 4a^4b^2 + a^2b^4\right)\cos\left(dx + c\right)^2 + 4\left(b^5\cos\left(dx + c\right)^3 - b^5\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\arctan\left(-\frac{a\sin\left(dx + c\right) + b}{\sqrt{a^2 - b^2}\cos\left(dx + c\right)}\right) + \left(\left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)^3 - \left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)\right)\log\left(\frac{1}{2}\cos\left(dx + c\right) + \frac{1}{2}\right) - \left(\left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)^3 - \left(3a^6 - 4a^4b^2 - a^2b^4 + 2b^6\right)\cos\left(dx + c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx + c\right) + \frac{1}{2}\right) - 4\left(a^5b - a^3b^3 - \left(2a^5b - 3a^3b^3 + ab^5\right)\cos\left(dx + c\right)^2\right)\sin\left(dx + c\right)}{4\left(\left(a^7 - 2a^5b^2 + a^3b^4\right)d\cos\left(dx + c\right)^3 - \left(a^7 - 2a^5b^2 + a^3b^4\right)d\cos\left(dx + c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 -
 2*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 -
b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*si
n(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*
sin(d*x + c) - a^2 - b^2)) + ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x
 + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d
*x + c) + 1/2) - ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3
*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1
/2) - 4*(a^5*b - a^3*b^3 - (2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)), -1/4*(4*a^6 - 4*a^4*b^2 - 2*(3*a^6 - 4*a^4*b^
2 + a^2*b^4)*cos(d*x + c)^2 + 4*(b^5*cos(d*x + c)^3 - b^5*cos(d*x + c))*sqr
t(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) +
 ((3*a^6 - 4*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2
 - a^2*b^4 + 2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - ((3*a^6 - 4
*a^4*b^2 - a^2*b^4 + 2*b^6)*cos(d*x + c)^3 - (3*a^6 - 4*a^4*b^2 - a^2*b^4 +
 2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^5*b - a^3*b^3 - (
2*a^5*b - 3*a^3*b^3 + a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^
2 + a^3*b^4)*d*cos(d*x + c)^3 - (a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(c + dx\right)\sec^2\left(c + dx\right)}{a + b\sin\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3*sec(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A]

time = 0.50, size = 245, normalized size = 1.35

$$\frac{\frac{16\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{(a^5-a^3b^2)\sqrt{a^2-b^2}}+\frac{16\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{(a^2-b^2)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2}+\frac{4(3a^2+2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{18a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(16*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - a^3*b^2)*sqrt(a^2 - b^2)) + 16*(b
*tan(1/2*d*x + 1/2*c) - a)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (a*
tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 + 4*(3*a^2 + 2*b^2)*
log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^
2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d
*x + 1/2*c)^2))/d
```

Mupad [B]

time = 15.35, size = 1570, normalized size = 8.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*((3*b^3*sin(c + d*x))/4 - (5*b^3*sin(3*c + 3*d*x))/4) - a^7*((b*sin(c
+ d*x))/2 - (b*sin(3*c + 3*d*x))/2) + a^2*(b^6/4 + (b^6*cos(2*c + 2*d*x))/4
 + (3*b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (3*b
^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) - a*((b^
7*sin(c + d*x))/4 + (b^7*sin(3*c + 3*d*x))/4) - a^6*((b^2*cos(c + d*x))/2 +
 b^2/4 - (7*b^2*cos(2*c + 2*d*x))/4 - (b^2*cos(3*c + 3*d*x))/2 + (7*b^2*cos
(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (7*b^2*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) - a^4*(b^4/4 - (b^4*c
os(c + d*x))/4 + (5*b^4*cos(2*c + 2*d*x))/4 + (b^4*cos(3*c + 3*d*x))/4 - (3
*b^4*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + (3*b^4*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8) + a^8*(cos(c
+ d*x)/4 - (3*cos(2*c + 2*d*x))/4 - cos(3*c + 3*d*x)/4 + (3*cos(c + d*x)*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (3*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/8 + 1/4) - (b^8*cos(c + d*x)*log(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (b^8*log(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2))*cos(3*c + 3*d*x))/4 + a^3*b^5*sin(3*c + 3*d*x) + (b^5*atan((3*a^6
*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^6*sin(c
/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^3*b^3*cos(c/2 +
 (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 7*a^4*b^2*sin(c/2 + (
d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 4*a*b^5*cos(c/2 + (d*x)
```

```
/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) - 3*a^5*b*cos(c/2 + (d*x)/2)*
(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^9*cos(c/2 + (d*x)/2)*3i - b^9
*sin(c/2 + (d*x)/2)*8i - a*b^8*cos(c/2 + (d*x)/2)*4i + a^8*b*sin(c/2 + (d*x
)/2)*6i + a^3*b^6*cos(c/2 + (d*x)/2)*5i + a^5*b^4*cos(c/2 + (d*x)/2)*3i - a
^7*b^2*cos(c/2 + (d*x)/2)*7i + a^2*b^7*sin(c/2 + (d*x)/2)*12i + a^4*b^5*sin
(c/2 + (d*x)/2)*4i - a^6*b^3*sin(c/2 + (d*x)/2)*14i))*cos(3*c + 3*d*x)*(-(a
 + b)^3*(a - b)^3)^(1/2)*1i)/2 - (b^5*atan((3*a^6*sin(c/2 + (d*x)/2)*(b^6 -
 a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^6*sin(c/2 + (d*x)/2)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a^3*b^3*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a
^2*b^4 + 3*a^4*b^2)^(1/2) - 7*a^4*b^2*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + 4*a*b^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
 + 3*a^4*b^2)^(1/2) - 3*a^5*b*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^9*cos(c/2 + (d*x)/2)*3i - b^9*sin(c/2 + (d*x)/2)*8i - a
*b^8*cos(c/2 + (d*x)/2)*4i + a^8*b*sin(c/2 + (d*x)/2)*6i + a^3*b^6*cos(c/2
+ (d*x)/2)*5i + a^5*b^4*cos(c/2 + (d*x)/2)*3i - a^7*b^2*cos(c/2 + (d*x)/2)*
7i + a^2*b^7*sin(c/2 + (d*x)/2)*12i + a^4*b^5*sin(c/2 + (d*x)/2)*4i - a^6*b
^3*sin(c/2 + (d*x)/2)*14i))*cos(c + d*x)*(-(a + b)^3*(a - b)^3)^(1/2)*1i)/2
)/(a^3*d*cos(c + d*x)*sin(c + d*x)^2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2))
```

3.1346 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d}-\frac{a^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}+\frac{\sec^2(c+dx)(a-b}{2\,(a^2-b}$$

```
[Out] 1/4*(2*a+b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(2*a-b)*ln(1+sin(d*x+c))/(a-b)^2
/d-a^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(
a^2-b^2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 815}

$$\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\,(a^2-b^2)}-\frac{a^3\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2(2a^2-b^2)x}{a^2-b^2}}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2}{2(a-}\right.\right.}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c}{(a^2-b^2)^2d}$$

Mathematica [A]
time = 0.35, size = 117, normalized size = 0.93

$$\frac{\frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(1+\sin(c+dx))}{(a-b)^2} - \frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(-1+\sin(c+dx))} + \frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [A]
time = 0.34, size = 121, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
default	$\frac{-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
norman	$\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d(a^2-b^2)}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2-b^2)}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{a^3\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^4-2a^2b^2+b^4)} + \frac{(2a-b)\ln\left(\tan\right.}{2(a^2-2}$

risch	$-\frac{iax}{a^2-2ab+b^2}-\frac{ibx}{2(a^2+2ab+b^2)}+\frac{ibx}{2a^2-4ab+2b^2}-\frac{iac}{(a^2+2ab+b^2)d}-\frac{iax}{a^2+2ab+b^2}+\frac{ibc}{2(a^2-2ab+b^2)d}+\frac{}{d(a^4-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/(4*a-4*b)/(1+sin(d*x+c))+1/4
*(2*a-b)/(a-b)^2*ln(1+sin(d*x+c))-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4*(2*a+b)/(a
+b)^2*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.28, size = 142, normalized size = 1.13

$$-\frac{\frac{4\,a^3\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{(2\,a-b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

Fricas [A]
time = 0.39, size = 157, normalized size = 1.25

$$-\frac{4\,a^3\cos(dx+c)^2\log(b\sin(dx+c)+a)-(2\,a^3+3\,a^2b-b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2\,a^3-3\,a^2b+b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,a^3+2\,ab^2+2\,(a^2b-b^3)\sin(dx+c)}{4\,(a^4-2\,a^2b^2+b^4)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 177, normalized size = 1.40

$$-\frac{\frac{4\,a^3 b \log(|b\sin(dx+c)+a|)}{a^4 b-2\,a^2 b^3+b^5}-\frac{(2\,a-b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(a^3\sin(dx+c)^2-a^2 b\sin(dx+c)+b^3\sin(dx+c)-ab^2\right)}{(a^4-2\,a^2 b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

Mupad [B]
time = 12.29, size = 217, normalized size = 1.72

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(2\,a-b\right)}{2\,d\,{\left(a-b\right)}^2}-\frac{\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(2\,a+b\right)}{2\,d\,{\left(a+b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(2*a - b))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*
x)/2))/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 +
(d*x)/2)^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2
+ 1)) - (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(
a^4 + b^4 - 2*a^2*b^2)) + (log(tan(c/2 + (d*x)/2) - 1)*(2*a + b))/(2*d*(a +
 b)^2)
```

3.1347 $\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a\log(1-\sin(c+dx))}{4(a+b)^2d}-\frac{a\log(1+\sin(c+dx))}{4(a-b)^2d}+\frac{a^2b\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2\,(a^2-b^2)\,d}$$

```
[Out] 1/4*a*ln(1-sin(d*x+c))/(a+b)^2/d-1/4*a*ln(1+sin(d*x+c))/(a-b)^2/d+a^2*b*ln(
a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(d*x+c)^2*(b-a*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A]

time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2916, 12, 1661, 815}

$$\frac{a^2b\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}-\frac{\sec^2(c+dx)(b-a\sin(c+dx))}{2d\,(a^2-b^2)}+\frac{a\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{a\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Sin[c + d*x]])/(4*(a
 - b)^2*d) + (a^2*b*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(b - a*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{ab^2x}{a^2-b^2}}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+\right.}{2bd}$$

$$= -\frac{\sec^2(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{2d} + \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2}\right.\right.}{}$$

$$= \frac{a\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{a^2b\log(a+b\sin(c+dx)}{(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 0.32, size = 108, normalized size = 0.93

$$-\frac{-\frac{a\log(1-\sin(c+dx))}{(a+b)^2}+\frac{a\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4a^2b\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{(a+b)(-1+\sin(c+dx))}+\frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(-((a*Log[1 - Sin[c + d*x]])/(a + b)^2) + (a*Log[1 + Sin[c + d*x]])/(a
 - b)^2 - (4*a^2*b*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a +
 b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.30, size = 112, normalized size = 0.97

method	result

derivativedivides	$\frac{\frac{a^2 b \ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{1}{(4a-4b)(1+\sin(dx+c))}-\frac{a\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{a\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
default	$\frac{\frac{a^2 b \ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{1}{(4a-4b)(1+\sin(dx+c))}-\frac{a\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{a\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)d}+\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2-b^2)d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a^2 b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^4-2a^2b^2+b^4)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d(a^2-2}$
risch	$\frac{iax}{2a^2-4ab+2b^2}+\frac{iac}{2(a^2-2ab+b^2)d}-\frac{iax}{2(a^2+2ab+b^2)}-\frac{iac}{2(a^2+2ab+b^2)d}-\frac{2ia^2bx}{a^4-2a^2b^2+b^4}-\frac{2ia^2bc}{d(a^4-2a^2b^2+b^4)}+$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2/(a+b)^2*b/(a-b)^2*ln(a+b*sin(d*x+c))-1/(4*a-4*b)/(1+sin(d*x+c))-1/4*a/(a-b)^2*ln(1+sin(d*x+c))-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4*a/(a+b)^2*ln(sin(d*x+c)-1))

Maxima [A]
time = 0.29, size = 132, normalized size = 1.14

$$\frac{\frac{4\,a^2 b\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{a\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{a\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(a\sin(dx+c)-b)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/4*(4*a^2*b*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - a*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) - 2*(a*sin(d*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d

Fricas [A]
time = 0.40, size = 154, normalized size = 1.33

$$\frac{4\,a^2b\cos(dx+c)^2\log(b\sin(dx+c)+a)-(a^3+2\,a^2b+ab^2)\cos(dx+c)^2\log(\sin(dx+c)+1)+(a^3-2\,a^2b+ab^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\,a^2b+2\,b^3+2\,(a^3-ab^2)\sin(dx+c)}{4\,(a^4-2\,a^2b^2+b^4)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/4*(4*a^2*b*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^3 + 2*a^2*b + a*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 168, normalized size = 1.45

$$\frac{\frac{4\,a^2b^2\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{a\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{a\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(a^2b\sin(dx+c)^2-a^3\sin(dx+c)+ab^2\sin(dx+c)-b^3\right)}{(a^4-2\,a^2b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(4*a^2*b^2*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - a*l
og(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + a*log(abs(sin(d*x + c) - 1)
)/(a^2 + 2*a*b + b^2) + 2*(a^2*b*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*
sin(d*x + c) - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

Mupad [B]
time = 12.22, size = 206, normalized size = 1.78

$$\frac{\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2-b^2}+\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{a^2-b^2}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{2\,d\,(a-b)^2}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{2\,d\,(a+b)^2}+\frac{a^2\,b\,\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] ((a*tan(c/2 + (d*x)/2))/(a^2 - b^2) + (a*tan(c/2 + (d*x)/2)^3)/(a^2 - b^2)
- (2*b*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(
c/2 + (d*x)/2)^2 + 1)) - (a*log(tan(c/2 + (d*x)/2) + 1))/(2*d*(a - b)^2) +
(a*log(tan(c/2 + (d*x)/2) - 1))/(2*d*(a + b)^2) + (a^2*b*log(a + 2*b*tan(c/
2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^4 - 2*a^2*b^2))
```

3.1348 $\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{b\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{b\log(1+\sin(c+dx))}{4(a-b)^2d}-\frac{ab^2\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}+\frac{\sec^2(c+dx)(a-b\sin(c+dx)}{2\,(a^2-b^2)\,d}$$

```
[Out] -1/4*b*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*b*ln(1+sin(d*x+c))/(a-b)^2/d-a*b^2*ln
(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))/(a^2-b^2)/
d
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2916, 12, 837, 815}

$$-\frac{ab^2\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\,(a^2-b^2)}-\frac{b\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{b\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(b*Log[1 - Sin[c + d*x]])/((a + b)^2*d) + (b*Log[1 + Sin[c + d*x]])/(4
*(a - b)^2*d) - (a*b^2*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) + (Sec[c
+ d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{x}{b(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{x}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} - \frac{\text{Subst}\left(\int \frac{-ab^2+b^2x}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{2(a^2-b^2)d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d} - \frac{\text{Subst}\left(\int \left(\frac{b(-a+b)}{2(a+b)(b-x)} + \frac{2ab^2}{(a-b)(a+b)(a+x)} - \right.}{2(a^2-b^2)}$$

$$= -\frac{b\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{b\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{ab^2\log(a+b\sin(c+dx}{(a^2-b^2)^2d}$$

Mathematica [A]
time = 0.26, size = 162, normalized size = 1.38

$$\frac{-\frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^2} + \frac{2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^2} - \frac{4ab^2\log(a+b\sin(c+dx))}{(a^2-b^2)^2} + \frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{1}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^2 + (2*b*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^2 - (4*a*b^2*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^2 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + 1
/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A]
time = 0.30, size = 112, normalized size = 0.96

method	result
derivativedivides	$\frac{-\frac{b^2 a\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{b\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}-\frac{b\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
default	$\frac{-\frac{b^2 a\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{b\ln(1+\sin(dx+c))}{4(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}-\frac{b\ln(\sin(dx+c)-1)}{4(a+b)^2}}{d}$
norman	$\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2-b^2\right)}-\frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}+\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a^2-b^2\right)d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d(a^2-2ab+b^2)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a^2+2ab+b^2)d}-\frac{a\,b^2\ln(\cdots}{}$
risch	$-\frac{ibx}{2(a^2-2ab+b^2)}-\frac{ibc}{2(a^2-2ab+b^2)d}+\frac{ibx}{2a^2+4ab+2b^2}+\frac{ibc}{2(a^2+2ab+b^2)d}+\frac{2ia\,b^2x}{a^4-2a^2b^2+b^4}+\frac{2ia\,b^2c}{d(a^4-2a^2b^2+b^4)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2*a/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/(4*a-4*b)/(1+sin(d*x+c))+1
/4*b/(a-b)^2*ln(1+sin(d*x+c))-1/(4*a+4*b)/(sin(d*x+c)-1)-1/4*b/(a+b)^2*ln(s
in(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 132, normalized size = 1.13

$$-\frac{\frac{4\,ab^2\log\left(b\sin\left(dx+c\right)+a\right)}{a^4-2\,a^2b^2+b^4}-\frac{b\log\left(\sin\left(dx+c\right)+1\right)}{a^2-2\,ab+b^2}+\frac{b\log\left(\sin\left(dx+c\right)-1\right)}{a^2+2\,ab+b^2}-\frac{2\,\left(b\sin\left(dx+c\right)-a\right)}{\left(a^2-b^2\right)\sin\left(dx+c\right)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a*b^2*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - b*log(sin(d
*x + c) + 1)/(a^2 - 2*a*b + b^2) + b*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b
^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A]
time = 0.40, size = 155, normalized size = 1.32

$$-\frac{4\,ab^2\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(a^2b+2\,ab^2+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(a^2b-2\,ab^2+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,a^3+2\,ab^2+2\,\left(a^2b-b^3\right)\sin\left(dx+c\right)}{4\,\left(a^4-2\,a^2b^2+b^4\right)d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*b^2*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (a^2*b + 2*a*b^2 + b
^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (a^2*b - 2*a*b^2 + b^3)*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x +
 c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 170, normalized size = 1.45

$$-\frac{\frac{4\,ab^3\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{b\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2}+\frac{b\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2}+\frac{2\left(ab^2\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)+a^3-2\,ab^2\right)}{(a^4-2\,a^2b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a*b^3*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - b*lo
g(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + b*log(abs(sin(d*x + c) - 1))
/(a^2 + 2*a*b + b^2) + 2*(a*b^2*sin(d*x + c)^2 - a^2*b*sin(d*x + c) + b^3*s
in(d*x + c) + a^3 - 2*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1))
)/d
```

Mupad [B]
time = 12.11, size = 208, normalized size = 1.78

$$\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{2\,d\,{(a-b)}^2}-\frac{\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2-b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{2\,d\,{(a+b)}^2}-\frac{a\,b^2\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2) + 1))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d*x)/2))/(
a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2 + (d*x)/2)
^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) -
(b*log(tan(c/2 + (d*x)/2) - 1))/(2*d*(a + b)^2) - (a*b^2*log(a + 2*b*tan(c/
2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^4 - 2*a^2*b^2))
```

3.1349 $\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=156

$$-\frac{(2a+3b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{\log(\sin(c+dx))}{ad}-\frac{(2a-3b)\log(1+\sin(c+dx))}{4(a-b)^2d}-\frac{b^4\log(a+b\sin(c+dx$$

```
[Out] -1/4*(2*a+3*b)*ln(1-sin(d*x+c))/(a+b)^2/d+ln(sin(d*x+c))/a/d-1/4*(2*a-3*b)*
ln(1+sin(d*x+c))/(a-b)^2/d-b^4*ln(a+b*sin(d*x+c))/a/(a^2-b^2)^2/d+1/4/(a+b)
/d/(1-sin(d*x+c))+1/4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$-\frac{b^4\log(a+b\sin(c+dx))}{ad\left(a^2-b^2\right)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(2a+3b)\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d(a-b)^2}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((2*a + 3*b)*Log[1 - Sin[c + d*x]])/((a + b)^2*d) + Log[Sin[c + d*x]]/
(a*d) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^4*Log[a +
b*Sin[c + d*x]])/(a*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) +
 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{b}{x(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2} + \frac{2a+3b}{4b^4(a+b)^2(b-x)} + \frac{1}{ab^4x} - \frac{1}{a(a-b)^2(a+b)^2(a+x)} - \frac{1}{4(a-b)b}\right.\right.}{d}$$

$$= -\frac{(2a+3b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{\log(\sin(c+dx))}{ad} - \frac{(2a-3b)\log(1+\sin}{4(a-b)^2d}$$

Mathematica [A]
time = 0.46, size = 151, normalized size = 0.97

$$\frac{b^4\left(-\frac{(2a+3b)\log(1-\sin(c+dx))}{b^4(a+b)^2} + \frac{4\log(\sin(c+dx))}{ab^4} - \frac{(2a-3b)\log(1+\sin(c+dx))}{(a-b)^2b^4} - \frac{4\log(a+b\sin(c+dx))}{a(a-b)^2(a+b)^2} - \frac{1}{b^4(a+b)(-1+\sin(c+dx))} + \frac{1}{(a-b)b^4(1+\sin(c+dx))}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^4*(-(((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(b^4*(a + b)^2)) + (4*Log[Sin[c
 + d*x]])/(a*b^4) - ((2*a - 3*b)*Log[1 + Sin[c + d*x]])/((a - b)^2*b^4) - (
4*Log[a + b*Sin[c + d*x]])/(a*(a - b)^2*(a + b)^2) - 1/(b^4*(a + b)*(-1 + S
in[c + d*x])) + 1/((a - b)*b^4*(1 + Sin[c + d*x]))))/(4*d)
```

Maple [A]
time = 0.42, size = 137, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\ln(\sin(dx+c))}{a} - \frac{b^4\ln(a+b\sin(dx+c))}{a(a+b)^2(a-b)^2} + \frac{1}{(4a-4b)(1+\sin(dx+c))} + \frac{(-2a+3b)\ln(1+\sin(dx+c))}{4(a-b)^2} - \frac{1}{(4a+4b)(\sin(dx+c)-1)} + \frac{(-2a-3b)\ln(\text{si}}{4(a+}}{d}$
default	$\frac{\frac{\ln(\sin(dx+c))}{a} - \frac{b^4\ln(a+b\sin(dx+c))}{a(a+b)^2(a-b)^2} + \frac{1}{(4a-4b)(1+\sin(dx+c))} + \frac{(-2a+3b)\ln(1+\sin(dx+c))}{4(a-b)^2} - \frac{1}{(4a+4b)(\sin(dx+c)-1)} + \frac{(-2a-3b)\ln(\text{si}}{4(a+}}{d}$
norman	$\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2-b^2\right)} - \frac{b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)} + \frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a^2-b^2\right)d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad} - \frac{(2a-3b)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d(a^2-2ab+b^2)} - \frac{(2a+3b}{2}$

risch	$\frac{iac}{(a^2+2ab+b^2)d} + \frac{i(-2ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)})}{(a^2-b^2)d(e^{2i(dx+c)}+1)^2} + \frac{2ib^4x}{a(a^4-2a^2b^2+b^4)} + \frac{iac}{(a^2-2ab+b^2)d} - \frac{3ibx}{2(a^2-2ab+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(sin(d*x+c))-1/a*b^4/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/(4*a-4
*b)/(1+sin(d*x+c))+1/4/(a-b)^2*(-2*a+3*b)*ln(1+sin(d*x+c))-1/(4*a+4*b)/(sin
(d*x+c)-1)+1/4/(a+b)^2*(-2*a-3*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 156, normalized size = 1.00

$$-\frac{\frac{4\,b^4\log(b\sin(dx+c)+a)}{a^5-2\,a^3b^2+ab^4} + \frac{(2\,a-3\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2} + \frac{(2\,a+3\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2} - \frac{2\,(b\sin(dx+c)-a)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2} - \frac{4\,\log(\sin(dx+c))}{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^4*log(b*sin(d*x + c) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + (2*a - 3*b)
*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(sin(d*x + c) -
 1)/(a^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^
2 - a^2 + b^2) - 4*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 0.69, size = 213, normalized size = 1.37

$$-\frac{4\,b^4\cos(dx+c)^2\log(b\sin(dx+c)+a) - 2\,a^4 + 2\,a^2b^2 - 4\,(a^4-2\,a^2b^2+b^4)\cos(dx+c)^2\log\left(-\frac{1}{2}\,\sin(dx+c)\right) + (2\,a^4+a^3b-4\,a^2b^2-3\,ab^3)\cos(dx+c)^2\log(\sin(dx+c)+1) + (2\,a^4-a^3b-4\,a^2b^2+3\,ab^3)\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\,(a^3b-ab^3)\sin(dx+c)}{4\,(a^5-2\,a^3b^2+ab^4)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^4*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - 2*a^4 + 2*a^2*b^2 - 4*
(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2*log(-1/2*sin(d*x + c)) + (2*a^4 + a^
3*b - 4*a^2*b^2 - 3*a*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a^4 -
a^3*b - 4*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3
*b - a*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 210, normalized size = 1.35

$$-\frac{\frac{4\,b^{5}\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{5}b-2\,a^{3}b^{3}+ab^{5}}+\frac{\left(2\,a-3\,b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{2}-2\,ab+b^{2}}+\frac{\left(2\,a+3\,b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{2}+2\,ab+b^{2}}-\frac{4\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a}-\frac{2\left(a^{3}\sin\left(dx+c\right)^{2}-2\,ab^{2}\sin\left(dx+c\right)^{2}+a^{2}b\sin\left(dx+c\right)-b^{3}\sin\left(dx+c\right)-2\,a^{3}+3\,ab^{2}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\sin\left(dx+c\right)^{2}-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^5*log(abs(b*sin(d*x + c) + a))/(a^5*b - 2*a^3*b^3 + a*b^5) + (2*a
 - 3*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (2*a + 3*b)*log(ab
s(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 4*log(abs(sin(d*x + c)))/a - 2*(
a^3*sin(d*x + c)^2 - 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(
d*x + c) - 2*a^3 + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))
/d
```

Mupad [B]
time = 12.42, size = 170, normalized size = 1.09

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b}{4\,{\left(a-b\right)}^2}-\frac{1}{2\,\left(a-b\right)}\right)}{d}-\frac{\frac{a}{2\,\left(a^2-b^2\right)}-\frac{b\,\sin\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}}{d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{1}{2\,\left(a+b\right)}\right)}{d}-\frac{b^4\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{a\,d\,{\left(a^2-b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b/(4*(a - b)^2) - 1/(2*(a - b))))/d - (a/(2*(a^2 -
b^2)) - (b*sin(c + d*x))/(2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 - 1)) + log(si
n(c + d*x))/(a*d) - (log(sin(c + d*x) - 1)*(b/(4*(a + b)^2) + 1/(2*(a + b))
))/d - (b^4*log(a + b*sin(c + d*x)))/(a*d*(a^2 - b^2)^2)
```

3.1350 $\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=171

$$-\frac{\csc(c+dx)}{ad}-\frac{(3a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(3a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d}+\frac{b^5\log(a}{a^2}$$

```
[Out] -csc(d*x+c)/a/d-1/4*(3*a+4*b)*ln(1-sin(d*x+c))/(a+b)^2/d-b*ln(sin(d*x+c))/a
^2/d+1/4*(3*a-4*b)*ln(1+sin(d*x+c))/(a-b)^2/d+b^5*ln(a+b*sin(d*x+c))/a^2/(a
^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c))-1/4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{b^5\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^2}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{1}{4d(a+b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(3a+4b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(3a-4b)\log(\sin(c+dx)+1)}{4d(a-b)^2}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - ((3*a + 4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d)
 - (b*Log[Sin[c + d*x]])/(a^2*d) + ((3*a - 4*b)*Log[1 + Sin[c + d*x]])/(4*(
a - b)^2*d) + (b^5*Log[a + b*Sin[c + d*x]])/(a^2*(a^2 - b^2)^2*d) + 1/(4*(a
 + b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^5\text{Subst}\left(\int \frac{1}{x^2(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^5\text{Subst}\left(\int \left(\frac{1}{4b^4(a+b)(b-x)^2} + \frac{3a+4b}{4b^5(a+b)^2(b-x)} + \frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2(a-b)^2(a+b)^2(a+x)}\right.}{d}$$
$$= -\frac{\csc(c+dx)}{ad} - \frac{(3a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} - \frac{b\log(\sin(c+dx))}{a^2d} + \frac{(3a}{}$$

Mathematica [A]
time = 0.53, size = 174, normalized size = 1.02

$$-\frac{\csc(c+dx)(a+b\sin(c+dx))\left(\frac{4\csc(c+dx)}{a} + \frac{(3a+4b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{4b\log(\sin(c+dx))}{a^2} - \frac{(3a-4b)\log(1+\sin(c+dx))}{(a-b)^2} - \frac{4b^5\log(a+b\sin(c+dx))}{a^2(a-b)^2(a+b)^2} + \frac{1}{(a+b)(-1+\sin(c+dx))} + \frac{1}{(a-b)(1+\sin(c+dx))}\right)}{4d(b+a\csc(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*(Csc[c + d*x]*(a + b*Sin[c + d*x])*((4*Csc[c + d*x])/a + ((3*a + 4*b)*
Log[1 - Sin[c + d*x]])/(a + b)^2 + (4*b*Log[Sin[c + d*x]])/a^2 - ((3*a - 4*
b)*Log[1 + Sin[c + d*x]])/(a - b)^2 - (4*b^5*Log[a + b*Sin[c + d*x]])/(a^2*
(a - b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Si
n[c + d*x]))))/(d*(b + a*Csc[c + d*x]))
```

Maple [A]
time = 0.49, size = 152, normalized size = 0.89

method	result
derivativedivides	$\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} + \frac{b^5\ln(a+b\sin(dx+c))}{a^2(a+b)^2(a-b)^2} - \frac{1}{(4a-4b)(1+\sin(dx+c))} + \frac{(3a-4b)\ln(1+\sin(dx+c))}{4(a-b)^2} - \frac{1}{(4a+4b)(\sin(dx+c)-1)} +}{d}$
default	$\frac{-\frac{1}{a\sin(dx+c)} - \frac{b\ln(\sin(dx+c))}{a^2} + \frac{b^5\ln(a+b\sin(dx+c))}{a^2(a+b)^2(a-b)^2} - \frac{1}{(4a-4b)(1+\sin(dx+c))} + \frac{(3a-4b)\ln(1+\sin(dx+c))}{4(a-b)^2} - \frac{1}{(4a+4b)(\sin(dx+c)-1)} +}{d}$
norman	$\frac{-\frac{1}{2ad} - \frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad} - \frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)} + \frac{\left(3a^2-b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da\left(a^2-b^2\right)} + \frac{\left(3a^2-b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da\left(a^2-b^2\right)}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{b^5\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{a^2d(a^4-2}$

risch	$-\frac{2ib^5x}{a^2(a^4-2a^2b^2+b^4)}+\frac{3iac}{2(a^2+2ab+b^2)d}+\frac{2ibc}{a^2d}+\frac{2ibx}{a^2}+\frac{2ibx}{a^2-2ab+b^2}+\frac{2ibc}{(a^2-2ab+b^2)d}-\frac{3iax}{2(a^2-2ab+b^2)}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))+b^5/a^2/(a+b)^2/(a-b)^2*ln(a+b*
sin(d*x+c))-1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(3*a-4*b)/(a-b)^2*ln(1+sin(d*x+c
))-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2*(-3*a-4*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.29, size = 200, normalized size = 1.17

$$\frac{\frac{4\,b^5\log(b\sin(dx+c)+a)}{a^6-2\,a^4b^2+a^2b^4}+\frac{(3\,a-4\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(3\,a+4\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\left(ab\sin(dx+c)-(3\,a^2-2\,b^2)\sin(dx+c)^2+2\,a^2-2\,b^2\right)}{(a^3-ab^2)\sin(dx+c)^3-(a^3-ab^2)\sin(dx+c)}-\frac{4\,b\log(\sin(dx+c))}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(b*sin(d*x + c) + a)/(a^6 - 2*a^4*b^2 + a^2*b^4) + (3*a - 4*b
)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (3*a + 4*b)*log(sin(d*x + c)
- 1)/(a^2 + 2*a*b + b^2) + 2*(a*b*sin(d*x + c) - (3*a^2 - 2*b^2)*sin(d*x +
c)^2 + 2*a^2 - 2*b^2)/((a^3 - a*b^2)*sin(d*x + c)^3 - (a^3 - a*b^2)*sin(d*x
 + c)) - 4*b*log(sin(d*x + c))/a^2)/d
```

Fricas [A]
time = 0.82, size = 287, normalized size = 1.68

$$\frac{4\,b^5\cos(dx+c)^2\log(b\sin(dx+c)+a)\sin(dx+c)+2\,a^5-2\,a^3b^2-4\,(a^4b-2\,a^2b^3+b^5)\cos(dx+c)^2\log\left(\frac{1}{2}\,\sin(dx+c)\right)\sin(dx+c)+(3\,a^5+2\,a^4b-5\,a^3b^2-4\,a^2b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)\sin(dx+c)-(3\,a^5-2\,a^4b-5\,a^3b^2+4\,a^2b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)\sin(dx+c)-2\,(3\,a^5-5\,a^3b^2+2\,ab^4)\cos(dx+c)^2-2\,(a^4b-a^2b^3)\sin(dx+c)}{4\,(a^6-2\,a^4b^2+a^2b^4)d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^5*cos(d*x + c)^2*log(b*sin(d*x + c) + a)*sin(d*x + c) + 2*a^5 - 2*
a^3*b^2 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c)^2*log(1/2*sin(d*x + c))*
sin(d*x + c) + (3*a^5 + 2*a^4*b - 5*a^3*b^2 - 4*a^2*b^3)*cos(d*x + c)^2*log
(sin(d*x + c) + 1)*sin(d*x + c) - (3*a^5 - 2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 2*(3*a^5 - 5*a^3*b^2
+ 2*a*b^4)*cos(d*x + c)^2 - 2*(a^4*b - a^2*b^3)*sin(d*x + c))/((a^6 - 2*a^4
*b^2 + a^2*b^4)*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 279, normalized size = 1.63

$$\frac{\frac{12\,b^6\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6b-2\,a^4b^3+a^2b^5}+\frac{3\,(3\,a-4\,b)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^2-2\,ab+b^2}-\frac{3\,(3\,a+4\,b)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^2+2\,ab+b^2}-\frac{12\,b\log\left(|\sin\left(dx+c\right)|\right)}{a^2}+\frac{2\left(2\,b^5\sin\left(dx+c\right)^3-9\,a^5\sin\left(dx+c\right)^2+15\,a^3b^2\sin\left(dx+c\right)^2-6\,ab^4\sin\left(dx+c\right)^2+3\,a^4b\sin\left(dx+c\right)-3\,a^2b^3\sin\left(dx+c\right)-2\,b^5\sin\left(dx+c\right)+6\,a^5-12\,a^3b^2+6\,ab^4\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\left(\sin\left(dx+c\right)^3-\sin\left(dx+c\right)\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*b^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 2*a^4*b^3 + a^2*b^5) + 3
*(3*a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - 3*(3*a + 4*b)
*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 12*b*log(abs(sin(d*x + c)
))/a^2 + 2*(2*b^5*sin(d*x + c)^3 - 9*a^5*sin(d*x + c)^2 + 15*a^3*b^2*sin(d*
x + c)^2 - 6*a*b^4*sin(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 3*a^2*b^3*sin(d*
x + c) - 2*b^5*sin(d*x + c) + 6*a^5 - 12*a^3*b^2 + 6*a*b^4)/((a^6 - 2*a^4*b
^2 + a^2*b^4)*(sin(d*x + c)^3 - sin(d*x + c))))/d
```

Mupad [B]
time = 12.40, size = 195, normalized size = 1.14

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(3\,a-4\,b\right)}{4\,d\,{\left(a-b\right)}^2}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{3}{4\,\left(a+b\right)}\right)}{d}-\frac{\frac{1}{a}+\frac{b\,\sin\left(c+d\,x\right)}{2\,\left(a^2-b^2\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(3\,a^2-2\,b^2\right)}{2\,a\,\left(a^2-b^2\right)}}{d\,\left(\sin\left(c+d\,x\right)-{\sin\left(c+d\,x\right)}^3\right)}-\frac{b\,\ln\left(\sin\left(c+d\,x\right)\right)}{a^2\,d}+\frac{b^5\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{a^2\,d\,{\left(a^2-b^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(3*a - 4*b))/(4*d*(a - b)^2) - (log(sin(c + d*x) - 1
)*(b/(4*(a + b)^2) + 3/(4*(a + b))))/d - (1/a + (b*sin(c + d*x))/(2*(a^2 -
b^2)) - (sin(c + d*x)^2*(3*a^2 - 2*b^2))/(2*a*(a^2 - b^2)))/(d*(sin(c + d*x
) - sin(c + d*x)^3)) - (b*log(sin(c + d*x)))/(a^2*d) + (b^5*log(a + b*sin(c
 + d*x)))/(a^2*d*(a^2 - b^2)^2)
```

3.1351 $\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(4a+5b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(2a^2+b^2)\log(\sin(c+dx))}{a^3d}-\frac{(4a-5b)\log(1-}{4(a-}$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/4*(4*a+5*b)*ln(1-sin(d*x+c))/(a+b
)^2/d+(2*a^2+b^2)*ln(sin(d*x+c))/a^3/d-1/4*(4*a-5*b)*ln(1+sin(d*x+c))/(a-b)
^2/d-b^6*ln(a+b*sin(d*x+c))/a^3/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c))+1/
4/(a-b)/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$\frac{b\csc(c+dx)}{a^2d}+\frac{(2a^2+b^2)\log(\sin(c+dx))}{a^3d}-\frac{b^6\log(a+b\sin(c+dx))}{a^3d\,(a^2-b^2)^2}+\frac{1}{4d(a+b)(1-\sin(c+dx))}+\frac{1}{4d(a-b)(\sin(c+dx)+1)}-\frac{(4a+5b)\log(1-\sin(c+dx))}{4d(a+b)^2}-\frac{(4a-5b)\log(\sin(c+dx)+1)}{4d(a-b)^2}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((4*a + 5*b)*Log[1 - Si
n[c + d*x]])/(4*(a + b)^2*d) + ((2*a^2 + b^2)*Log[Sin[c + d*x]])/(a^3*d) -
((4*a - 5*b)*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) - (b^6*Log[a + b*Sin[c
+ d*x]])/(a^3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])) + 1/(4*
(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)\sec^3(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b^3\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b^6\text{Subst}\left(\int \frac{1}{x^3(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b^6\text{Subst}\left(\int \left(\frac{1}{4b^5(a+b)(b-x)^2}+\frac{4a+5b}{4b^6(a+b)^2(b-x)}+\frac{1}{ab^4x^3}-\frac{1}{a^2b^4x^2}+\frac{2a^2+b^2}{a^3b^6x}-\frac{}{a^3(a-b)}\right.\right.}{d}\\ &= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(4a+5b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(2a^2+b^2}{}\end{aligned}$$

Mathematica [A]
time = 0.95, size = 168, normalized size = 0.85

$$-\frac{-\frac{4b\csc(c+dx)}{a^2}+\frac{2\csc^2(c+dx)}{a}+\frac{(4a+5b)\log(1-\sin(c+dx))}{(a+b)^2}-\frac{4(2a^2+b^2)\log(\sin(c+dx))}{a^3}+\frac{(4a-5b)\log(1+\sin(c+dx))}{(a-b)^2}+\frac{4b^6\log(a+b\sin(c+dx))}{a^3(a-b)^2(a+b)^2}+\frac{1}{(a+b)(-1+\sin(c+dx))}-\frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((-4*b*Csc[c + d*x])/a^2 + (2*Csc[c + d*x]^2)/a + ((4*a + 5*b)*Log[1 -
 Sin[c + d*x]])/(a + b)^2 - (4*(2*a^2 + b^2)*Log[Sin[c + d*x]])/a^3 + ((4*a
 - 5*b)*Log[1 + Sin[c + d*x]])/(a - b)^2 + (4*b^6*Log[a + b*Sin[c + d*x]])/
(a^3*(a - b)^2*(a + b)^2) + 1/((a + b)*(-1 + Sin[c + d*x])) - 1/((a - b)*(1
 + Sin[c + d*x])))/d
```

Maple [A]
time = 0.60, size = 172, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{(2a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}-\frac{b^6\ln(a+b\sin(dx+c))}{a^3(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(-4a+5b)\ln(1+\sin(dx+c}{4(a-b)^2}}{d}$
default	$\frac{-\frac{1}{2a\sin(dx+c)^2}+\frac{(2a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}-\frac{b^6\ln(a+b\sin(dx+c))}{a^3(a+b)^2(a-b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(-4a+5b)\ln(1+\sin(dx+c}{4(a-b)^2}}{d}$
norman	$\frac{-\frac{1}{8ad}-\frac{\tan^8(\frac{dx}{2}+\frac{c}{2})}{8ad}+\frac{b\tan(\frac{dx}{2}+\frac{c}{2})}{2a^2d}+\frac{b(\tan^7(\frac{dx}{2}+\frac{c}{2}))}{2a^2d}-\frac{(-9a^2+b^2)(\tan^4(\frac{dx}{2}+\frac{c}{2}))}{4ad(a^2-b^2)}-\frac{b(3a^2-b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2a^2d(a^2-b^2)}-\frac{b(3a^2-b^2}{2a^2}}{\tan(\frac{dx}{2}+\frac{c}{2})^2(\tan^2(\frac{dx}{2}+\frac{c}{2})-1)^2}$

risch	$\frac{5ibx}{2(a^2+2ab+b^2)}+\frac{2ib^6x}{a^3(a^4-2a^2b^2+b^4)}-\frac{5ibc}{2(a^2-2ab+b^2)d}+\frac{2ib^6c}{a^3d(a^4-2a^2b^2+b^4)}+\frac{2iac}{(a^2-2ab+b^2)d}-\frac{4ic}{ad}-\frac{2ib^2c}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sin(d*x+c)^2+(2*a^2+b^2)/a^3*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)-
b^6/a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/(4*a-4*b)/(1+sin(d*x+c))+1/4/(
a-b)^2*(-4*a+5*b)*ln(1+sin(d*x+c))-1/(4*a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2*(
-4*a-5*b)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 244, normalized size = 1.24

$$-\frac{\frac{4\,b^6\log(b\sin(dx+c)+a)}{a^7-2\,a^5b^2+a^3b^4}+\frac{(4\,a-5\,b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(4\,a+5\,b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\left((3\,a^2b-2\,b^3)\sin(dx+c)^3+a^3-ab^2-(2\,a^3-ab^2)\sin(dx+c)^2-2\,(a^2b-b^3)\sin(dx+c)\right)}{(a^4-a^2b^2)\sin(dx+c)^4-(a^4-a^2b^2)\sin(dx+c)^2}-\frac{4\,(2\,a^2+b^2)\log(\sin(dx+c))}{a^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^6*log(b*sin(d*x + c) + a)/(a^7 - 2*a^5*b^2 + a^3*b^4) + (4*a - 5*
b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(sin(d*x + c)
 - 1)/(a^2 + 2*a*b + b^2) - 2*((3*a^2*b - 2*b^3)*sin(d*x + c)^3 + a^3 - a*b
^2 - (2*a^3 - a*b^2)*sin(d*x + c)^2 - 2*(a^2*b - b^3)*sin(d*x + c))/((a^4 -
 a^2*b^2)*sin(d*x + c)^4 - (a^4 - a^2*b^2)*sin(d*x + c)^2) - 4*(2*a^2 + b^2
)*log(sin(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(185) = 370$.
time = 1.12, size = 440, normalized size = 2.23

$$-\frac{2\,a^6-2\,a^4b^2-2\,(2\,a^6-3\,a^4b^2+a^2b^4)\cos(dx+c)^2+4\,(b^6\cos(dx+c)^4-b^6\cos(dx+c)^2)\log(b\sin(dx+c)+a)-4\,((2\,a^6-3\,a^4b^2+b^6)\cos(dx+c)^4-(2\,a^6-3\,a^4b^2+b^6)\cos(dx+c)^2)\log\left(-\frac{1}{2}\sin(dx+c)\right)+((4\,a^6+3\,a^5b-6\,a^4b^2-5\,a^3b^3)\cos(dx+c)^4-(4\,a^6+3\,a^5b-6\,a^4b^2-5\,a^3b^3)\cos(dx+c)^2)\log(\sin(dx+c)+1)+((4\,a^6-3\,a^5b-6\,a^4b^2+5\,a^3b^3)\cos(dx+c)^4-(4\,a^6-3\,a^5b-6\,a^4b^2+5\,a^3b^3)\cos(dx+c)^2)\log(-\sin(dx+c)+1)-2\,(a^5b-a^3b^3-(3\,a^5b-5\,a^3b^3+2\,ab^5)\cos(dx+c)^2)\sin(dx+c)}{4\,((a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)^4-(a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^6 - 2*a^4*b^2 - 2*(2*a^6 - 3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 +
4*(b^6*cos(d*x + c)^4 - b^6*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) - 4*((2
*a^6 - 3*a^4*b^2 + b^6)*cos(d*x + c)^4 - (2*a^6 - 3*a^4*b^2 + b^6)*cos(d*x
+ c)^2)*log(-1/2*sin(d*x + c)) + ((4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)
*cos(d*x + c)^4 - (4*a^6 + 3*a^5*b - 6*a^4*b^2 - 5*a^3*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1) + ((4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x
 + c)^4 - (4*a^6 - 3*a^5*b - 6*a^4*b^2 + 5*a^3*b^3)*cos(d*x + c)^2)*log(-si
n(d*x + c) + 1) - 2*(a^5*b - a^3*b^3 - (3*a^5*b - 5*a^3*b^3 + 2*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^4 - (
a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.51, size = 275, normalized size = 1.40

$$-\frac{\frac{4\,b^{7}\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{7}b-2\,a^{5}b^{3}+a^{3}b^{5}}+\frac{\left(4\,a-5\,b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{2}-2\,ab+b^{2}}+\frac{\left(4\,a+5\,b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{2}+2\,ab+b^{2}}-\frac{2\left(2\,a^{3}\sin\left(dx+c\right)^{2}-3\,ab^{2}\sin\left(dx+c\right)^{2}+a^{2}b\sin\left(dx+c\right)-b^{3}\sin\left(dx+c\right)-3\,a^{3}+4\,ab^{2}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\left(\sin\left(dx+c\right)^{2}-1\right)}-\frac{4\left(2\,a^{2}+b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{3}}+\frac{2\left(6\,a^{2}\sin\left(dx+c\right)^{2}+3\,b^{2}\sin\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)+a^{2}\right)}{a^{3}\sin\left(dx+c\right)^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 2*a^5*b^3 + a^3*b^5) + (4
*a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + (4*a + 5*b)*log(
abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) - 2*(2*a^3*sin(d*x + c)^2 - 3*a*
b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin(d*x + c) - 3*a^3 + 4*a*b^
2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)) - 4*(2*a^2 + b^2)*log(abs
(sin(d*x + c)))/a^3 + 2*(6*a^2*sin(d*x + c)^2 + 3*b^2*sin(d*x + c)^2 - 2*a*
b*sin(d*x + c) + a^2)/(a^3*sin(d*x + c)^2))/d
```

Mupad [B]
time = 12.48, size = 240, normalized size = 1.22

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b}{4\,{\left(a-b\right)}^2}-\frac{1}{a-b}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{b}{4\,{\left(a+b\right)}^2}+\frac{1}{a+b}\right)}{d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin\left(c+d\,x\right)}{a^2}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(2\,a^2-b^2\right)}{2\,a\,\left(a^2-b^2\right)}+\frac{b\,{\sin\left(c+d\,x\right)}^3\,\left(3\,a^2-2\,b^2\right)}{2\,a^2\,\left(a^2-b^2\right)}}{d\,\left({\sin\left(c+d\,x\right)}^2-{\sin\left(c+d\,x\right)}^4\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(2\,a^2+b^2\right)}{a^3\,d}-\frac{b^6\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^7-2\,a^5\,b^2+a^3\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b/(4*(a - b)^2) - 1/(a - b)))/d - (log(sin(c + d*x)
 - 1)*(b/(4*(a + b)^2) + 1/(a + b)))/d - (1/(2*a) - (b*sin(c + d*x))/a^2 -
(sin(c + d*x)^2*(2*a^2 - b^2))/(2*a*(a^2 - b^2)) + (b*sin(c + d*x)^3*(3*a^2
 - 2*b^2))/(2*a^2*(a^2 - b^2)))/(d*(sin(c + d*x)^2 - sin(c + d*x)^4)) + (lo
g(sin(c + d*x))*(2*a^2 + b^2))/(a^3*d) - (b^6*log(a + b*sin(c + d*x)))/(d*(
a^7 + a^3*b^4 - 2*a^5*b^2))
```

3.1352 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d}+\frac{b\sec(c+dx)}{(a^2-b^2)\,d}-\frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d}-\frac{a^3\tan(c+dx)}{(a^2-b^2)^2\,d}+\frac{a\tan^3(c+d}{3\,(a^2-b^2)}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+a^
2*b*sec(d*x+c)/(a^2-b^2)^2/d+b*sec(d*x+c)/(a^2-b^2)/d-1/3*b*sec(d*x+c)^3/(a
^2-b^2)/d-a^3*tan(d*x+c)/(a^2-b^2)^2/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2806, 2687, 30, 2686, 3852, 8, 2739, 632, 210}

$$\frac{a\tan^3(c+dx)}{3d\,(a^2-b^2)}-\frac{b\sec^3(c+dx)}{3d\,(a^2-b^2)}+\frac{a^2b\sec(c+dx)}{d\,(a^2-b^2)^2}+\frac{b\sec(c+dx)}{d\,(a^2-b^2)}+\frac{2a^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{5/2}}-\frac{a^3\tan(c+dx)}{d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+}{a^2-b^2}$$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{(a^2-b^2)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{(a^2b)\int \sec(c+dx)\tan(c+dx)\,dx}{(a^2-b^2)^2}$$

$$= \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3(a^2-b^2)\,d} + \frac{a\tan^3(c+dx)}{3(a^2-b^2)\,d} + \frac{a^3\text{Subst}(\int 1\,dx, x, -\tan(c+d}{(a^2-b^2)^2\,d}$$

$$= \frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3(a^2-b^2)\,d} - \frac{a^3\tan(c+dx)}{(a^2-b^2)^2\,d} + \frac{a\tan^3(c+d}{3(a^2-b^2)}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3(a^2-b^2)\,d}$$

Mathematica [A]

time = 1.00, size = 195, normalized size = 1.10

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec^3(c+dx)\left(-16a^2b+4b^3+3b(11a^2-5b^2)\cos(c+dx)+12b(-2a^2+b^2)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-5b^3\cos(3(c+dx))+6ab^2\sin(c+dx)+8a^3\sin(3(c+dx))-2ab^2\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A]

time = 0.42, size = 214, normalized size = 1.21

method	result
derivativedivides	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\frac{16}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a+b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\frac{16}{(32a-32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{-2a+b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^4\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}}{d}$

risch	$-\frac{2\left(6ia^3e^{4i(dx+c)}-3ib^2a\,e^{4i(dx+c)}-6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+6ia^3e^{2i(dx+c)}-8a^2b\,e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4ia^3-i\right.}{3(a^4-2a^2b^2+b^4)\left(e^{2i(dx+c)}+1\right)^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-32/3/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32*b)+16/(32*a-32*b)/(tan(1/2*d*x
+1/2*c)+1)^2-1/2/(a-b)^2*(-2*a+b)/(tan(1/2*d*x+1/2*c)+1)+2*a^4/(a-b)^2/(a+b
)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)-32/3/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tan(1/2*d*x+1/2
*c)-1)^2-1/2/(a+b)^2*(-2*a-b)/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 476, normalized size = 2.69

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^4\cos(dx+c)^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)+2a^4b-4a^2b^3+2b^5-6(2a^4b-3a^2b^3+b^5)\cos(dx+c)^2-2(a^5-2a^3b^2+ab^4-(4a^5-5a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{6(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3},-\frac{3\sqrt{a^2-b^2}\,a^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)^3+a^4b-2a^2b^3+b^5-3(2a^4b-3a^2b^3+b^5)\cos(dx+c)^2-(a^5-2a^3b^2+ab^4-(4a^5-5a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{3(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.53, size = 241, normalized size = 1.36

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5a^2b+2b^3}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

Mupad [B]
time = 17.43, size = 372, normalized size = 2.10

$$\frac{\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{2(5a^2b-2b^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2ab^2-5a^3)}{3(a^4-2a^2b^2+b^4)}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2b-b^3)}{a^4-2a^2b^2+b^4}-\frac{2a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^4-2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}+\frac{2a^4\,\mathrm{atan}\left(\frac{\frac{a^4(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^4}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*a^3*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*(5*a^2*b - 2*b^3))
/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*
a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(2*a*b^2 - 5*a^3))/(3*(a^4 + b^4 - 2*a^2
*b^2)) + (4*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) -
 (2*a^2*b*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (2*a^4*ata
n(((a^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^5*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)
))/(2*a^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.1353 $\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{2a^3b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{a\sec^3(c+dx)}{3\left(a^2-b^2\right)d}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{\left(a^2-b^2\right)^2d}-\frac{b\tan^3(c+dx)}{3\left(a^2-b^2\right)d}$$

```
[Out] -2*a^3*b*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+1/3*a*sec(d*x+c)^3/(a^2-b^2)/d-a^2*sec(d*x+c)*(a-b*sin(d*x+c))/(a^2-b^2)^2
/d-1/3*b*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2981, 2686, 30, 2687, 2945, 12, 2739, 632, 210}

$$-\frac{b\tan^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a\sec^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{2a^3b\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (a*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^2*Sec[c + d*x]*(a - b*Si
n[c + d*x]))/((a^2 - b^2)^2*d) - (b*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec^2(c+d$$

$$= -\frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)^2 d} - \frac{a^2\int \frac{ab}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{a\text{Subst}\left(\int x^2\, dx\right.}{(a^2-}$$

$$= \frac{a\sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)^2 d} - \frac{b\tan^3(c+dx)}{3(a^2-b^2)d} - \frac{(a^3b}{}$$

$$= \frac{a\sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)^2 d} - \frac{b\tan^3(c+dx)}{3(a^2-b^2)d} - \frac{(2a^3}{}$$

$$= \frac{a\sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^2\sec(c+dx)(a-b\sin(c+dx))}{(a^2-b^2)^2 d} - \frac{b\tan^3(c+dx)}{3(a^2-b^2)d} + \frac{(4a^3}{}$$

$$= -\frac{2a^3b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2} d} + \frac{a\sec^3(c+dx)}{3(a^2-b^2)d} - \frac{a^2\sec(c+dx)(a-b\sin}{(a^2-b^2)^2 d}$$

Mathematica [A]
time = 1.01, size = 184, normalized size = 1.30

$$\frac{-\frac{48a^3b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\sec^3(c+dx)\left(-4a^3-8ab^2+3a(5a^2+b^2)\cos(c+dx)-12a^3\cos(2(c+dx))+5a^3\cos(3(c+dx))+ab^2\cos(3(c+dx))+6b^3\sin(c+dx)+8a^2b\sin(3(c+dx))-2b^3\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*a^3*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(-4*a^3 - 8*a*b^2 + 3*a*(5*a^2 + b^2)*Cos[c + d*x] -
 12*a^3*Cos[2*(c + d*x)] + 5*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)]
+ 6*b^3*Sin[c + d*x] + 8*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/
((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A]
time = 0.47, size = 205, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{8}{(16a-16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a-16b)}-\frac{a}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^3b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}-}{d}$

default	$\frac{-\frac{8}{(16a-16b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a-16b)}-\frac{a}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^3b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}}{d}$
risch	$-\frac{2i\left(-3ia^3e^{5i(dx+c)}-2ia^3e^{3i(dx+c)}-4iab^2e^{3i(dx+c)}-6b\,e^{4i(dx+c)}a^2+3b^3e^{4i(dx+c)}-3ia^3e^{i(dx+c)}-6b\,e^{2i(dx+c)}a^2-4a\right.}{3d(-a^2+b^2)^2\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/(16*a-16*b)/(tan(1/2*d*x+1/2*c)+1)^2+16/3/(tan(1/2*d*x+1/2*c)+1)^3/
(16*a-16*b)-1/2*a/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-2*a^3*b/(a-b)^2/(a+b)^2/(a
^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-16/3
/(tan(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tan(1/2*d*x+1/2*c)-1)^
2+1/2*a/(a+b)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 465, normalized size = 3.27

$$\left[-\frac{3\sqrt{-a^2+b^2}\,a^3b\cos(dx+c)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^5+4a^3b^2-2ab^4+6(a^5-a^3b^2)\cos(dx+c)^2+2(a^4b-2a^2b^3+b^5-(4a^4b-5a^2b^3+b^5)\cos(dx+c)^2)\sin(dx+c)}{6(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3}, \frac{3\sqrt{a^2-b^2}\,a^3b\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos(dx+c)^3+a^5-2a^3b^2+ab^4-3(a^5-a^3b^2)\cos(dx+c)^2-(a^4b-2a^2b^3+b^5-(4a^4b-5a^2b^3+b^5)\cos(dx+c)^2)\sin(dx+c)}{3(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^3*b*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^5 - a^3*b^2)*cos(d*x + c
)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
```

```
/3*(3*sqrt(a^2 - b^2)*a^3*b*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - a^3*b^2)*
cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (4*a^4*b - 5*a^2*b^3 + b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 227, normalized size = 1.60

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^3b}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^3-ab^2}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^3*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 3*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 10*
a^2*b*tan(1/2*d*x + 1/2*c)^3 + 4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*tan(1/2
*d*x + 1/2*c)^2 + 3*a^2*b*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 17.16, size = 370, normalized size = 2.61

$$\frac{\frac{2\,(2\,a^3+a\,b^2)}{3\,(a^4-2\,a^2\,b^2+b^4)}-\frac{4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(5\,a^2\,b-2\,b^3)}{3\,(a^4-2\,a^2\,b^2+b^4)}-\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{2\,a^3\,b\,\mathrm{atan}\left(\frac{\frac{a^3\,b\,(2\,a^4\,b-4\,a^2\,b^3+2\,b^5)}{(a+b)^{5/2}\,(a-b)^{5/2}}+\frac{2\,a^4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4-2\,a^2\,b^2+b^4)}{(a+b)^{5/2}\,(a-b)^{5/2}}}{2\,a^3\,b}\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(a*b^2 + 2*a^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (4*a^3*tan(c/2 + (d*x)/2
)^2)/(a^4 + b^4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(5*a^2*b - 2*b^3))/(
3*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^
2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) - (2*a^2*b*
```

```
tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 -
 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a^3*b*atan(((a^3*
b*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a^4*b*t
an(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(
2*a^3*b)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.1354 $\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=165

$$\frac{2a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}-\frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)^2\,d}+\frac{a\tan(c+dx)}{(a^2-b^2)\,d}+\frac{a\tan^3(c+dx)}{3\,(a^2-b^2)\,d}$$

```
[Out] 2*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/
d-1/3*b*sec(d*x+c)^3/(a^2-b^2)/d+a^2*sec(d*x+c)*(b-a*sin(d*x+c))/(a^2-b^2)^
2/d+a*tan(d*x+c)/(a^2-b^2)/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2981, 3852, 2686, 30, 2775, 12, 2739, 632, 210}

$$\frac{2a^2b^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\,(a^2-b^2)}+\frac{a\tan(c+dx)}{d\,(a^2-b^2)}-\frac{b\sec^3(c+dx)}{3d\,(a^2-b^2)}+\frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
5/2)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) + (a^2*Sec[c + d*x]*(b - a*S
in[c + d*x]))/((a^2 - b^2)^2*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d) + (a*Tan
[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^4(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sec^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec^3(c+dx)\tan(c+dx)\,d}{a^2-b^2}$$

$$= \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)^2 d} + \frac{a^2\int \frac{b^2}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2} - \frac{a\mathrm{Subst}\left(\int (1+\right.}{(}$$

$$= -\frac{b\sec^3(c+dx)}{3(a^2-b^2)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)^2 d} + \frac{a\tan(c+dx)}{(a^2-b^2)d} + \frac{a\,t}{3}$$

$$= -\frac{b\sec^3(c+dx)}{3(a^2-b^2)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)^2 d} + \frac{a\tan(c+dx)}{(a^2-b^2)d} + \frac{a\,t}{3}$$

$$= -\frac{b\sec^3(c+dx)}{3(a^2-b^2)d} + \frac{a^2\sec(c+dx)(b-a\sin(c+dx))}{(a^2-b^2)^2 d} + \frac{a\tan(c+dx)}{(a^2-b^2)d} + \frac{a\,t}{3}$$

$$= \frac{2a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d} - \frac{b\sec^3(c+dx)}{3(a^2-b^2)d} + \frac{a^2\sec(c+dx)(b-a\sin}{(a^2-b^2)^2 d}$$

Mathematica [A]
time = 0.90, size = 200, normalized size = 1.21

$$\frac{\frac{48a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec^3(c+dx)\left(-4a^2b-8b^3+3b(5a^2+b^2)\cos(c+dx)-12a^2b\cos(2(c+dx))+5a^2b\cos(3(c+dx))+b^3\cos(3(c+dx))-6a^3\sin(c+dx)+12ab^2\sin(c+dx)+2a^3\sin(3(c+dx))+4ab^2\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^
(5/2) - (Sec[c + d*x]^3*(-4*a^2*b - 8*b^3 + 3*b*(5*a^2 + b^2)*Cos[c + d*x]
- 12*a^2*b*Cos[2*(c + d*x)] + 5*a^2*b*Cos[3*(c + d*x)] + b^3*Cos[3*(c + d*x
)] - 6*a^3*Sin[c + d*x] + 12*a*b^2*Sin[c + d*x] + 2*a^3*Sin[3*(c + d*x)] +
4*a*b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A]
time = 0.42, size = 207, normalized size = 1.25

method	result
derivativedivides	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(8a-8b)} + \frac{4}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2} + \frac{b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} + \frac{2a^2b^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}} - \frac{}{3(}}{d}$

default	$\frac{-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}(8a-8b)}+\frac{4}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{b}{2(a-b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2a^{2}b^{2}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{a^{2}-b^{2}}}-}{d}$
risch	$\frac{-2ia^{3}e^{4i(dx+c)}+2a^{2}b\,e^{5i(dx+c)}-4ib^{2}a\,e^{2i(dx+c)}+\frac{4a^{2}b\,e^{3i(dx+c)}}{3}+\frac{8b^{3}e^{3i(dx+c)}}{3}-\frac{2ia^{3}}{3}-\frac{4ib^{2}a}{3}+2a^{2}b\,e^{i(dx+c)}}{d(-a^{2}+b^{2})^{2}\left(e^{2i(dx+c)}+1\right)^{3}}+\frac{b^{2}a^{2}\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-8/3/(tan(1/2*d*x+1/2*c)+1)^3/(8*a-8*b)+4/(8*a-8*b)/(tan(1/2*d*x+1/2*c
)+1)^2+1/2*b/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)+2*a^2*b^2/(a-b)^2/(a+b)^2/(a^2-
b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/3/(ta
n(1/2*d*x+1/2*c)-1)^3/(8*a+8*b)-4/(8*a+8*b)/(tan(1/2*d*x+1/2*c)-1)^2-1/2*b/
(a+b)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.40, size = 470, normalized size = 2.85

$$\left[-\frac{3\sqrt{-a^{2}+b^{2}}\,a^{2}b^{2}\cos(dx+c)^{3}\log\left(\frac{(2a^{2}-b^{2})\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2ab\sin(dx+c)-a^{2}-b^{2}}\right)+2a^{4}b-4a^{2}b^{3}+2b^{5}-6(a^{4}b-a^{2}b^{3})\cos(dx+c)^{2}-2(a^{5}-2a^{3}b^{2}+ab^{4}-(a^{5}+a^{3}b^{2}-2ab^{4})\cos(dx+c)^{2})\sin(dx+c)}{6(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6})d\cos(dx+c)^{3}},\ -\frac{3\sqrt{a^{2}-b^{2}}\,a^{2}b^{2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^{2}-b^{2}}\cos(dx+c)}\right)\cos(dx+c)^{3}+a^{4}b-2a^{2}b^{3}+b^{5}-3(a^{4}b-a^{2}b^{3})\cos(dx+c)^{2}-(a^{5}-2a^{3}b^{2}+ab^{4}-(a^{5}+a^{3}b^{2}-2ab^{4})\cos(dx+c)^{2})\sin(dx+c)}{3(a^{6}-3a^{4}b^{2}+3a^{2}b^{4}-b^{6})d\cos(dx+c)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^2*b^2*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(a^4*b - a^2*b^3)*cos(d*x
+ c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3),
-1/3*(3*sqrt(a^2 - b^2)*a^2*b^2*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^
```

```
2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(a^4*b - a^2
*b^3)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (a^5 + a^3*b^2 - 2*a*b^4)
*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d
*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 229, normalized size = 1.39

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b^2}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^2b-b^3}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^2*b^2/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^
2)) + (3*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^4 - 4*a^
3*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2
*d*x + 1/2*c)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b - b^3)/((a^4 - 2*a
^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 17.40, size = 375, normalized size = 2.27

$$\frac{2a^2b^2\,\mathrm{atan}\left(\frac{\frac{a^2b^2\left(2a^4b-4a^2b^3+2b^5\right)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^3b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^4-2a^2b^2+b^4\right)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^2b^2}\right)}{d\,(a+b)^{5/2}(a-b)^{5/2}}-\frac{\frac{2(2a^2b+b^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{a^4-2a^2b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^3+ab^2)}{3(a^4-2a^2b^2+b^4)}-\frac{2ab^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{4a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4-2a^2b^2+b^4}-\frac{2ab^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] (2*a^2*b^2*atan(((a^2*b^2*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a
- b)^(5/2)) + (2*a^3*b^2*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a +
b)^(5/2)*(a - b)^(5/2)))/(2*a^2*b^2)))/(d*(a + b)^(5/2)*(a - b)^(5/2)) - ((
2*(2*a^2*b + b^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*b^3*tan(c/2 + (d*x)/2)^
```

```
4)/(a^4 + b^4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(a*b^2 + 2*a^3))/(3*(a
^4 + b^4 - 2*a^2*b^2)) - (2*a*b^2*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^
2) - (4*a^2*b*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) - (2*a*b^2*tan(
c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*t
an(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.1355 $\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{2ab^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d}+\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3(a^2-b^2)d}-\frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+dx)\right)}{3(a^2-b^2)^2d}$$

```
[Out] -2*a*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+1/3*sec(d*x+c)^3*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/3*sec(d*x+c)*(3*a*b^2-b*(a
^2+2*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2945, 12, 2739, 632, 210}

$$-\frac{2ab^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}+\frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3d(a^2-b^2)}-\frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+dx)\right)}{3d(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5
/2)*d) + (Sec[c + d*x]^3*(a - b*Sin[c + d*x]))/(3*(a^2 - b^2)*d) - (Sec[c +
 d*x]*(3*a*b^2 - b*(a^2 + 2*b^2)*Sin[c + d*x]))/(3*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\int \frac{\sec^2(c+dx)\left(-ab+2b^2\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \frac{\sec(c+dx)\left(3ab^2-b(a^2+2b^2)\sin(c+\right.}{3\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2ab^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{\sec^3(c+dx)(a-b\sin(c+dx))}{3\left(a^2-b^2\right)d} - \sec(c$$

Mathematica [A]
time = 0.90, size = 203, normalized size = 1.47

$$\frac{-\frac{24ab^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\sec^3(c+dx)\left(4a^3-10ab^2-\frac{3}{2}a\left(a^2-7b^2\right)\cos(c+dx)-6ab^2\cos(2(c+dx))-\frac{1}{2}a^3\cos(3(c+dx))+\frac{7}{2}ab^2\cos(3(c+dx))-3a^2b\sin(c+dx)+6b^3\sin(c+dx)+a^2b\sin(3(c+dx))+2b^3\sin(3(c+dx))\right)}{(a-b)^2(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]

[Out] ((-24*a*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(
5/2) + (Sec[c + d*x]^3*(4*a^3 - 10*a*b^2 - (3*a*(a^2 - 7*b^2)*Cos[c + d*x])
/2 - 6*a*b^2*Cos[2*(c + d*x)] - (a^3*Cos[3*(c + d*x)])/2 + (7*a*b^2*Cos[3*(
c + d*x)])/2 - 3*a^2*b*Sin[c + d*x] + 6*b^3*Sin[c + d*x] + a^2*b*Sin[3*(c +
 d*x)] + 2*b^3*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(12*d)

Maple [A]
time = 0.40, size = 215, normalized size = 1.56

method	result
derivativedivides	$\frac{-\frac{2}{(4a-4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(4a-4b)}-\frac{-a+2b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\,b^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}-\frac{}{3(t}}{d}$
default	$\frac{-\frac{2}{(4a-4b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(4a-4b)}-\frac{-a+2b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a\,b^3\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}-\frac{}{3(}}{d}$
risch	$\frac{2i\left(3ia\,b^2\mathrm{e}^{5i(dx+c)}-4ia^3\mathrm{e}^{3i(dx+c)}+10ia\,b^2\mathrm{e}^{3i(dx+c)}+3b\,\mathrm{e}^{4i(dx+c)}a^2+3ib^2a\,\mathrm{e}^{i(dx+c)}+6b^3\mathrm{e}^{2i(dx+c)}+a^2b+2b^3\right)}{3d(-a^2+b^2)^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}-\frac{ib^3a\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/(4*a-4*b)/(tan(1/2*d*x+1/2*c)+1)^2+4/3/(tan(1/2*d*x+1/2*c)+1)^3/(4*
a-4*b)-1/2/(a-b)^2*(-a+2*b)/(tan(1/2*d*x+1/2*c)+1)-2*a*b^3/(a-b)^2/(a+b)^2/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/
3/(tan(1/2*d*x+1/2*c)-1)^3/(4*a+4*b)-2/(4*a+4*b)/(tan(1/2*d*x+1/2*c)-1)^2-1
/2*(a+2*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de

Fricas [A]
time = 0.39, size = 469, normalized size = 3.40

$$\left[-\frac{3\sqrt{-a^2+b^2}\,ab^3\cos\left(dx+c\right)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^5+4a^3b^2-2ab^4+6\,(a^3b^2-ab^4)\cos\left(dx+c\right)^2+2\left(a^4b-2a^2b^3+b^5-(a^4b+a^2b^3-2b^5)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\,(a^6-3a^4b^2+3a^2b^4-b^6)d\cos\left(dx+c\right)^3},\ \frac{3\sqrt{a^2-b^2}\,ab^3\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos\left(dx+c\right)^3+a^5-2a^3b^2+ab^4-3\,(a^3b^2-ab^4)\cos\left(dx+c\right)^2-\left(a^4b-2a^2b^3+b^5-(a^4b+a^2b^3-2b^5)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,(a^6-3a^4b^2+3a^2b^4-b^6)d\cos\left(dx+c\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a*b^3*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) - 2*a^5 + 4*a^3*b^2 - 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x +
 c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos(d*x + c)
^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1
/3*(3*sqrt(a^2 - b^2)*a*b^3*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*c
os(d*x + c)))*cos(d*x + c)^3 + a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4
)*cos(d*x + c)^2 - (a^4*b - 2*a^2*b^3 + b^5 - (a^4*b + a^2*b^3 - 2*b^5)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)\sec^4\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 240, normalized size = 1.74

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(a\right)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)ab^3}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-4a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3-4ab^2}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a*b^3/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2
)) + (3*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^2
*tan(1/2*d*x + 1/2*c)^4 - 4*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*b^3*tan(1/2*d*x + 1/2*c)
```

```
+ a^3 - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/
d
```

Mupad [B]
time = 17.06, size = 378, normalized size = 2.74

$$\frac{\frac{2\,\left(4\,a\,b^2-a^3\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,b^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(2\,a\,b^2-a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,a^2+b^2\right)}{3\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{4\,a\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^4-2\,a^2\,b^2+b^4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{2\,a\,b^3\,\mathrm{atan}\left(\frac{\frac{a\,b^3\,\left(2\,a^4\,b-4\,a^2\,b^3+2\,b^5\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}+\frac{2\,a^2\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}}{2\,a\,b^3}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^4*(a + b*sin(c + d*x))),x)
```

```
[Out] ((2*(4*a*b^2 - a^3))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (2*b^3*tan(c/2 + (d*x)/2
))/(a^4 + b^4 - 2*a^2*b^2) - (2*b^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^
2*b^2) + (2*tan(c/2 + (d*x)/2)^4*(2*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) +
 (4*b*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) - (4*
a*b^2*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/
2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (2*a*b^3*atan(
((a*b^3*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a
^2*b^3*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(
5/2)))/(2*a*b^3)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.1356 $\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{b\sec^3(c+dx)(b-a\sin(c}{3a\left(a^2-b^2\right)d}$$

```
[Out] -2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(5/2)/d
-arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d+1/3*sec(d*x+c)^3/a/d+1/3*b*sec(d*x+
c)^3*(b-a*sin(d*x+c))/a/(a^2-b^2)/d-1/3*b*sec(d*x+c)*(3*b^3+a*(2*a^2-5*b^2)
*sin(d*x+c))/a/(a^2-b^2)^2/d
```

Rubi [A]

time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {2977, 2702, 308, 213, 2775, 2945, 12, 2739, 632, 210}

$$-\frac{2b^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}+\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3ad\left(a^2-b^2\right)}-\frac{b\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3ad\left(a^2-b^2\right)^2}+\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - ArcTanh[Cos[c + d*x]]/(a*d) + Sec[c + d*x]/(a*d) + Sec[c + d*x]^3/
(3*a*d) + (b*Sec[c + d*x]^3*(b - a*Sin[c + d*x]))/(3*a*(a^2 - b^2)*d) - (b*
Sec[c + d*x]*(3*b^3 + a*(2*a^2 - 5*b^2)*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx &= \int \left(\frac{\csc(c+dx)\sec^4(c+dx)}{a}-\frac{b\sec^4(c+dx)}{a(a+b\sin(c+dx))}\right)\,dx\\
&= \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a}-\frac{b\int \frac{\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx}{a}\\
&= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d}+\frac{b\int \frac{\sec^2(c+dx)\left(-2a^2+3b^2-2ab\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{3a\left(a^2-b^2\right)}+\\
&= \frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d}-\frac{b\sec(c+dx)\left(3b^3+a(2a^2-5b^2)\sin(c\right.}{3a\left(a^2-b^2\right)^2 d}\\
&= \frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{b\sec^3(c+dx)(b-a\sin(c+dx))}{3a\left(a^2-b^2\right)d}-\frac{b\sec(c\ }{}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{b\sec^3(c+dx)(b-}{3a\left(a^2-\right.}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec^3(c+dx)}{3ad}+\frac{b\sec^3(c+dx)(b-}{3a\left(a^2-\right.}\\
&= -\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}+\frac{\sec(c+dx)}{ad}+\frac{\sec}{}
\end{aligned}$$

Mathematica [A]
time = 3.22, size = 334, normalized size = 1.72

$$\frac{-\frac{24b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}}-\frac{12\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a}+\frac{12\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a}+\frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2(7a+10b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{1}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2(-7a+10b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-24*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^
(5/2)) - (12*Log[Cos[(c + d*x)/2]])/a + (12*Log[Sin[(c + d*x)/2]])/a + 1/((
```

```
a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (2*Sin[(c + d*x)/2])/((a
+ b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + (2*(7*a + 10*b)*Sin[(c + d*
x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - (2*Sin[(c + d*x)
/2])/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) + 1/((a - b)*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*(-7*a + 10*b)*Sin[(c + d*x)/2])/((a
- b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*d)
```

Maple [A]

time = 0.63, size = 221, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-3a+4b}{2(a-b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^{5}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\right.}{2\sqrt{a^{2}-}}\right.}{(a-b)^{2}(a+b)^{2}a\sqrt{a^{2}}}}{d}$
default	$\frac{-\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{-3a+4b}{2(a-b)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2b^{5}\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\right.}{2\sqrt{a^{2}-}}\right.}{(a-b)^{2}(a+b)^{2}a\sqrt{a^{2}}}}{d}$
risch	$\frac{2i\left(-3ia^{3}e^{5i(dx+c)}+6ia\,b^{2}e^{5i(dx+c)}-10ia^{3}e^{3i(dx+c)}+16ia\,b^{2}e^{3i(dx+c)}+3b^{3}e^{4i(dx+c)}-3ia^{3}e^{i(dx+c)}+6ib^{2}a\,e^{i(dx+c)}-6b\,e^{2}\right.}{3d\left(-a^{2}+b^{2}\right)^{2}\left(e^{2i(dx+c)}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2+1/3/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3
-1/2/(a-b)^2*(-3*a+4*b)/(tan(1/2*d*x+1/2*c)+1)+1/a*ln(tan(1/2*d*x+1/2*c))-2
*b^5/(a-b)^2/(a+b)^2/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2
*b)/(a^2-b^2)^(1/2))-1/3/(a+b)/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)/(tan(1/2*
d*x+1/2*c)-1)^2-1/2*(3*a+4*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.76, size = 680, normalized size = 3.51

$$\left[-\frac{3\sqrt{-a^2+b^2}\,b^5\cos\left(dx+c\right)^3\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^6+4a^4b^2-2a^2b^4+3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\cos\left(dx+c\right)^3\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\cos\left(dx+c\right)^3\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-6\left(a^6-3a^4b^2+2a^2b^4\right)\cos\left(dx+c\right)^2+2\left(a^5b-2a^3b^3+ab^5+\left(2a^5b-7a^3b^3+5ab^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos\left(dx+c\right)^3}, \frac{6\sqrt{a^2-b^2}\,b^5\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos\left(dx+c\right)^3+2a^6-4a^4b^2+2a^2b^4-3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\cos\left(dx+c\right)^3\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+3\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\cos\left(dx+c\right)^3\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+6\left(a^6-3a^4b^2+2a^2b^4\right)\cos\left(dx+c\right)^2-2\left(a^5b-2a^3b^3+ab^5+\left(2a^5b-7a^3b^3+5ab^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{6\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos\left(dx+c\right)^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^5*cos(d*x + c)^3*log(-((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
a^2 - b^2)) - 2*a^6 + 4*a^4*b^2 - 2*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) - 6*(a^6 - 3*a
^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*
b - 7*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 +
3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3), 1/6*(6*sqrt(a^2 - b^2)*b^5*arctan(-(a
*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3 + 2*a^6 -
 4*a^4*b^2 + 2*a^2*b^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)
^3*log(1/2*cos(d*x + c) + 1/2) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(
d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2) + 6*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*c
os(d*x + c)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (2*a^5*b - 7*a^3*b^3 + 5*a*b
^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*
cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(c+dx\right)\sec^4\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 308, normalized size = 1.59

$$-\frac{\frac{6\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^5}{\left(a^5-2a^3b^2+ab^4\right)\sqrt{a^2-b^2}}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4a^3+7ab^2\right)}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(6*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^5/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(a^2 - b^2
```

```
)) - 3*log(abs(tan(1/2*d*x + 1/2*c)))/a - 2*(3*a^2*b*tan(1/2*d*x + 1/2*c)^5
 - 6*b^3*tan(1/2*d*x + 1/2*c)^5 - 6*a^3*tan(1/2*d*x + 1/2*c)^4 + 9*a*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 8*b^3*tan(1/2*d*x +
 1/2*c)^3 + 6*a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^2
+ 3*a^2*b*tan(1/2*d*x + 1/2*c) - 6*b^3*tan(1/2*d*x + 1/2*c) - 4*a^3 + 7*a*b
^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

Mupad [B]
time = 16.20, size = 2162, normalized size = 11.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] -(a^5*((15*b^5*sin(c + d*x))/4 + (7*b^5*sin(3*c + 3*d*x))/4) - a^7*((9*b^3*
sin(c + d*x))/4 + (11*b^3*sin(3*c + 3*d*x))/12) - a^3*((11*b^7*sin(c + d*x)
)/4 + (17*b^7*sin(3*c + 3*d*x))/12) + a^9*((b*sin(c + d*x))/2 + (b*sin(3*c
+ 3*d*x))/6) - a^10*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3
 + (3*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + 5/6) + a*((3*b^9*
sin(c + d*x))/4 + (5*b^9*sin(3*c + 3*d*x))/12) - a^2*((7*b^8*cos(c + d*x))/
4 + (4*b^8)/3 + b^8*cos(2*c + 2*d*x) + (7*b^8*cos(3*c + 3*d*x))/12 + (15*b^
8*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (5*b^8*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4) - a^6*((33*b^4*c
os(c + d*x))/4 + (13*b^4)/2 + (9*b^4*cos(2*c + 2*d*x))/2 + (11*b^4*cos(3*c
+ 3*d*x))/4 + (15*b^4*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/2 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))
/2) + a^8*((19*b^2*cos(c + d*x))/4 + (23*b^2)/6 + (5*b^2*cos(2*c + 2*d*x))/
2 + (19*b^2*cos(3*c + 3*d*x))/12 + (15*b^2*cos(c + d*x)*log(sin(c/2 + (d*x)
/2)/cos(c/2 + (d*x)/2)))/4 + (5*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x))/4) + a^4*((25*b^6*cos(c + d*x))/4 + (29*b^6)/6 + (7*b
^6*cos(2*c + 2*d*x))/2 + (25*b^6*cos(3*c + 3*d*x))/12 + (15*b^6*cos(c + d*x
)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (5*b^6*log(sin(c/2 + (d*x
)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2) + (3*b^10*cos(c + d*x)*log(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + (b^10*log(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (b^5*atan((4*b^6*sin(c/2 + (d*x)/2)*(
b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*
sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a
^8*b^2)^(1/2) + 2*a*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^
4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^
10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos
(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*
b^2)^(1/2) - 5*a^2*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4
*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 -
```

a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*cos(3*c + 3*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/2 + (b^5*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*cos(c + d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*3i)/2)/(a*d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))

3.1357 $\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=220

$$\frac{2b^6\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}+\frac{b(-a^2+2b^2)\sec(c+dx)}{\left(a^2-b^2\right)^2d}+\frac{b\sec^3(c+dx)(}{3a(}$$

```
[Out] 2*b^6*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(5/2)/
d+b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d+b*(-a^2+2*b^2)*sec(d*x+c)/(a^2
-b^2)^2/d+1/3*b*sec(d*x+c)^3*(-a+b*sin(d*x+c))/a/(a^2-b^2)/d+1/3*(6*a^4-10*
a^2*b^2+b^4)*tan(d*x+c)/a/(a^2-b^2)^2/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A]

time = 0.31, antiderivative size = 247, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2977, 2702, 308, 213, 2700, 276, 2775, 2945, 12, 2739, 632, 210}

$$\frac{2b^6\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2d\left(a^2-b^2\right)}+\frac{b^2\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3a^2d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3a^2d}-\frac{b\sec(c+dx)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\tan^3(c+dx)}{3ad}+\frac{2\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d) - (b*Sec[c
 + d*x])/(a^2*d) - (b*Sec[c + d*x]^3)/(3*a^2*d) - (b^2*Sec[c + d*x]^3*(b -
a*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*d) + (b^2*Sec[c + d*x]*(3*b^3 + a*(2*a^
2 - 5*b^2)*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d) + (2*Tan[c + d*x])/(a*d)
+ Tan[c + d*x]^3/(3*a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
```

```
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)\sec^4(c+dx)}{a^2} + \frac{\csc^2(c+dx)\sec^4(c+dx)}{a} + \frac{b^2\sec^4}{a^2(a+b\,\mathrm{s}}\right.$$

$$= \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2} + \frac{b^2\int}{}$$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2(a^2-b^2)\,d} - \frac{b^2\int \frac{\sec^2(c+dx)(-2a^2+3b^2-2ab\sin(c+dx))}{a+b\sin(c+dx)}}{3a^2(a^2-b^2)}$$

$$= -\frac{b^2\sec^3(c+dx)(b-a\sin(c+dx))}{3a^2(a^2-b^2)\,d} + \frac{b^2\sec(c+dx)\left(3b^3+a(2a^2-5b^2)\right.}{3a^2(a^2-b^2)^2\,d}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} - \frac{b^2\sec^3(c+dx)(b-a\,\mathrm{si}}{3a^2(a^2-b^2)}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} -$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\sec(c+dx)}{a^2d} - \frac{b\sec^3(c+dx)}{3a^2d} -$$

$$= \frac{2b^6\tan^{-1}\left(\frac{\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}}{}\right)}{a^2(a^2-b^2)^{5/2}\,d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{b\,\mathrm{s}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 450 vs. 2(220) = 440.
time = 6.32, size = 450, normalized size = 2.05

$$\frac{2b^6\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{2ad} + \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d} - \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2d} + \frac{1}{12(a+b)d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6(a+b)d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{6(a-b)d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{1}{12(a-b)d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{10a\sin\left(\frac{1}{2}(c+dx)\right)-13b\sin\left(\frac{1}{2}(c+dx)\right)}{6(a-b)^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{10a\sin\left(\frac{1}{2}(c+dx)\right)+13b\sin\left(\frac{1}{2}(c+dx)\right)}{6(a+b)^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^6*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(c + d*x)/2]))/
Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)*d) - Cot[(c + d*x)/2]/(2*a*d) + (b
*Log[Cos[(c + d*x)/2]])/(a^2*d) - (b*Log[Sin[(c + d*x)/2]])/(a^2*d) + 1/(12
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + Sin[(c + d*x)/2]/(6*(
a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + Sin[(c + d*x)/2]/(6*(a
- b)*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - 1/(12*(a - b)*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^2) + (10*a*Sin[(c + d*x)/2] - 13*b*Sin[(c + d*
x)/2])/(6*(a - b)^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (10*a*Sin[(c
 + d*x)/2] + 13*b*Sin[(c + d*x)/2])/(6*(a + b)^2*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])) + Tan[(c + d*x)/2]/(2*a*d)
```

Maple [A]
time = 0.68, size = 253, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{4a-5b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{a^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{4a-5b}{2(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{a^2}}{d}$
risch	$-\frac{2\left(9ib^4e^{2i(dx+c)}+3a^3b\,e^{7i(dx+c)}-6b^3a\,e^{7i(dx+c)}-28ia^2b^2e^{2i(dx+c)}-14ia^2b^2+7a^3e^{5i(dx+c)}b-10b^3a\,e^{5i(dx+c)}-6ia^2b^2e\right.}{3d(a^2-b^2)^2\left(e^{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2*(4*a-5*b)/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-
1/3/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2-1/2/a
/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*d*x+1/2*c))+2/a^2/(a+b)^2/(a-b)^2*b^
6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-
1/2*(4*a+5*b)/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/3/(a+b)/(tan(1/2*d*x+1/2*c)-
1)^3-1/2/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.77, size = 831, normalized size = 3.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*b^6*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2))*sin(d*x + c) - 2*a^7 + 4*a^5*b^2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*
b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(-1/2*cos(d
*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 22*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)
*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 + 7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^
6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^3*
sin(d*x + c)), -1/6*(6*sqrt(a^2 - b^2)*b^6*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3*sin(d*x + c) - 2*a^7 + 4*a^5*b^
2 - 2*a^3*b^4 - 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^3*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
^7)*cos(d*x + c)^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 2*(8*a^7 - 2
2*a^5*b^2 + 17*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 11*a^5*b^2 +
7*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 3*(a^6*b - 3*a
^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4
*b^4 - a^2*b^6)*d*cos(d*x + c)^3*sin(d*x + c))]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**A**]
time = 0.51, size = 357, normalized size = 1.62

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^6}{(a^6-2a^4b^2+a^2b^4)\sqrt{a^2-b^2}}-\frac{6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}+\frac{3\left(2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{4\left(6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+9b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+14ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-12b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4a^2b+7b^3\right)}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*b^6/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(a^2 - b
^2)) - 6*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 + 3*tan(1/2*d*x + 1/2*c)/a +
3*(2*b*tan(1/2*d*x + 1/2*c) - a)/(a^2*tan(1/2*d*x + 1/2*c)) - 4*(6*a^3*tan(
1/2*d*x + 1/2*c)^5 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*b*tan(1/2*d*x +
 1/2*c)^4 + 9*b^3*tan(1/2*d*x + 1/2*c)^4 - 8*a^3*tan(1/2*d*x + 1/2*c)^3 + 1
```

```
4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 6*a^3*tan(1/2*d*x + 1/2*c) - 9*a*b^2*tan(1/2*d*x + 1
/2*c) - 4*a^2*b + 7*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 -
 1)^3))/d
```

Mupad [B]
time = 17.38, size = 2317, normalized size = 10.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*((3*b^10)/8 + (b^10*cos(2*c + 2*d*x))/2 + (b^10*cos(4*c + 4*d*x))/8) - a
^10*((7*b*sin(c + d*x))/12 + (b*sin(2*c + 2*d*x))/3 + (b*sin(3*c + 3*d*x))/
4 + (b*sin(4*c + 4*d*x))/6 + (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
sin(2*c + 2*d*x))/4 + (b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c
 + 4*d*x))/8) - a^6*((17*b^5*sin(c + d*x))/4 + (11*b^5*sin(2*c + 2*d*x))/4
+ (9*b^5*sin(3*c + 3*d*x))/4 + (11*b^5*sin(4*c + 4*d*x))/8 + (5*b^5*log(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x))/2 + (5*b^5*log(sin(c/
2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x))/4) - a^2*((5*b^9*sin(c +
 d*x))/6 + (7*b^9*sin(2*c + 2*d*x))/12 + (b^9*sin(3*c + 3*d*x))/2 + (7*b^9*
sin(4*c + 4*d*x))/24 + (5*b^9*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*si
n(2*c + 2*d*x))/4 + (5*b^9*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4
*c + 4*d*x))/8) + a^8*((31*b^3*sin(c + d*x))/12 + (19*b^3*sin(2*c + 2*d*x))
/12 + (5*b^3*sin(3*c + 3*d*x))/4 + (19*b^3*sin(4*c + 4*d*x))/24 + (5*b^3*lo
g(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*d*x))/4 + (5*b^3*log(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x))/8) + a^4*((37*b^7*s
in(c + d*x))/12 + (25*b^7*sin(2*c + 2*d*x))/12 + (7*b^7*sin(3*c + 3*d*x))/4
 + (25*b^7*sin(4*c + 4*d*x))/24 + (5*b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*sin(2*c + 2*d*x))/2 + (5*b^7*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*sin(4*c + 4*d*x))/4) - a^7*((9*b^4)/8 + 6*b^4*cos(2*c + 2*d*x) + (23*
b^4*cos(4*c + 4*d*x))/8) + a^9*(b^2/4 + (19*b^2*cos(2*c + 2*d*x))/6 + (19*b
^2*cos(4*c + 4*d*x))/12) - a^3*((11*b^8)/8 + (8*b^8*cos(2*c + 2*d*x))/3 + (
23*b^8*cos(4*c + 4*d*x))/24) + a^5*((15*b^6)/8 + (17*b^6*cos(2*c + 2*d*x))/
3 + (59*b^6*cos(4*c + 4*d*x))/24) - a^11*((2*cos(2*c + 2*d*x))/3 + cos(4*c
+ 4*d*x)/3) + (b^11*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(2*c + 2*
d*x))/4 + (b^11*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*sin(4*c + 4*d*x)
)/8 + (b^6*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4
*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10
- 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2
+ (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^
(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10
*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5
*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 +
```

(d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*sin(2*c + 2*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/2 + (b^6*atan((4*b^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - a^6*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 2*a*b^5*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + a^5*b*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 2*a^3*b^3*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^2*b^4*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4*a^4*b^2*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^11*cos(c/2 + (d*x)/2)*1i - b^11*sin(c/2 + (d*x)/2)*4i - a*b^10*cos(c/2 + (d*x)/2)*2i + a^10*b*sin(c/2 + (d*x)/2)*2i + a^3*b^8*cos(c/2 + (d*x)/2)*7i - a^5*b^6*cos(c/2 + (d*x)/2)*11i + a^7*b^4*cos(c/2 + (d*x)/2)*10i - a^9*b^2*cos(c/2 + (d*x)/2)*5i + a^2*b^9*sin(c/2 + (d*x)/2)*15i - a^4*b^7*sin(c/2 + (d*x)/2)*24i + a^6*b^5*sin(c/2 + (d*x)/2)*21i - a^8*b^3*sin(c/2 + (d*x)/2)*10i))*sin(4*c + 4*d*x)*(-(a + b)^5*(a - b)^5)^(1/2)*1i)/4)/(a^2*d*sin(c + d*x)*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))

3.1358 $\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=332

$$-\frac{2b^7\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{b\cot(c+dx)}{a^2d}+\frac{5\sec(c+dx)}{2ad}$$

```
[Out] -2*b^7*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(5/2)
/d-5/2*arctanh(cos(d*x+c))/a/d-b^2*arctanh(cos(d*x+c))/a^3/d+b*cot(d*x+c)/a
^2/d+5/2*sec(d*x+c)/a/d+b^2*sec(d*x+c)/a^3/d+5/6*sec(d*x+c)^3/a/d+1/3*b^2*s
ec(d*x+c)^3/a^3/d-1/2*csc(d*x+c)^2*sec(d*x+c)^3/a/d+1/3*b^3*sec(d*x+c)^3*(b
-a*sin(d*x+c))/a^3/(a^2-b^2)/d-1/3*b^3*sec(d*x+c)*(3*b^3+a*(2*a^2-5*b^2)*si
n(d*x+c))/a^3/(a^2-b^2)^2/d-2*b*tan(d*x+c)/a^2/d-1/3*b*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.35, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {2977, 2702, 308, 213, 2700, 276, 294, 2775, 2945, 12, 2739, 632, 210}

$$\frac{b^2\sec^3(c+dx)}{3a^3d}+\frac{b^2\sec(c+dx)}{a^3d}-\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{b\tan^3(c+dx)}{3a^2d}-\frac{2b\tan(c+dx)}{a^2d}+\frac{b\cot(c+dx)}{a^2d}-\frac{2b^7\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}+\frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3d\left(a^2-b^2\right)}-\frac{b^3\sec(c+dx)\left(a\left(2a^2-5b^2\right)\sin(c+dx)+3b^3\right)}{3a^3d\left(a^2-b^2\right)^2}+\frac{5\sec^3(c+dx)}{6ad}+\frac{5\sec(c+dx)}{2ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b^7*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^
(5/2)*d) - (5*ArcTanh[Cos[c + d*x]])/(2*a*d) - (b^2*ArcTanh[Cos[c + d*x]])/
(a^3*d) + (b*Cot[c + d*x])/(a^2*d) + (5*Sec[c + d*x])/(2*a*d) + (b^2*Sec[c
+ d*x])/(a^3*d) + (5*Sec[c + d*x]^3)/(6*a*d) + (b^2*Sec[c + d*x]^3)/(3*a^3*
d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a*d) + (b^3*Sec[c + d*x]^3*(b - a*S
in[c + d*x]))/(3*a^3*(a^2 - b^2)*d) - (b^3*Sec[c + d*x]*(3*b^3 + a*(2*a^2 -
 5*b^2)*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d) - (2*b*Tan[c + d*x])/(a^2*d)
 - (b*Tan[c + d*x]^3)/(3*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \left(\frac{b^2\csc(c+dx)\sec^4(c+dx)}{a^3} - \frac{b\csc^2(c+dx)\sec^4(c+dx)}{a^2} + \frac{\csc^3(c+\right.$$

$$= \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a} - \frac{b\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2} + \frac{b^2\int}{}$$

$$= \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3(a^2-b^2)\,d} + \frac{b^3\int \frac{\sec^2(c+dx)(-2a^2+3b^2-2ab\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{3a^3(a^2-b^2)}$$

$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2ad} + \frac{b^3\sec^3(c+dx)(b-a\sin(c+dx))}{3a^3(a^2-b^2)\,d} - \frac{b^3\sec}{}$$

$$= \frac{b\cot(c+dx)}{a^2d} + \frac{b^2\sec(c+dx)}{a^3d} + \frac{b^2\sec^3(c+dx)}{3a^3d} - \frac{\csc^2(c+dx)\sec^3(c+}{2ad}$$

$$= -\frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\sec(c+dx)}{2ad} + \frac{b^2\sec(c+dx}{a^3d}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{b\cot(c+dx)}{a^2d} + \frac{5\mathrm{se}}{}$$

$$= -\frac{2b^7\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{5/2}\,d} - \frac{5\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{b^2\tanh^{-1}(\cos(}{a^3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 947 vs. 2(332) = 664.

time = 6.16, size = 947, normalized size = 2.85

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] 16*((a*(13*a^2 - 19*b^2)*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(96*(a^2 - b^2)
^2*d*(b + a*Csc[c + d*x])) - (b^7*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)
/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]]*Csc[c + d*x]*(a + b*Sin[c + d*x
]))/(8*a^3*(a^2 - b^2)^(5/2)*d*(b + a*Csc[c + d*x])) + (b*Cot[(c + d*x)/2]*
Csc[c + d*x]*(a + b*Sin[c + d*x]))/(32*a^2*d*(b + a*Csc[c + d*x])) - (Csc[(
c + d*x)/2]^2*Csc[c + d*x]*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c + d*
x])) + ((-5*a^2 - 2*b^2)*Csc[c + d*x]*Log[Cos[(c + d*x)/2]]*(a + b*Sin[c +
d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + ((5*a^2 + 2*b^2)*Csc[c + d*x]*Log[
Sin[(c + d*x)/2]]*(a + b*Sin[c + d*x]))/(32*a^3*d*(b + a*Csc[c + d*x])) + (
Csc[c + d*x]*Sec[(c + d*x)/2]^2*(a + b*Sin[c + d*x]))/(128*a*d*(b + a*Csc[c
```

```
+ d*x])) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a + b)*d*(b + a*Csc[c
 + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*Sin[(c +
d*x)/2]*(a + b*Sin[c + d*x]))/(96*(a + b)*d*(b + a*Csc[c + d*x])*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^3) - (Csc[c + d*x]*Sin[(c + d*x)/2]*(a + b*Sin[
c + d*x]))/(96*(a - b)*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^3) + (Csc[c + d*x]*(a + b*Sin[c + d*x]))/(192*(a - b)*d*(b + a*Csc
[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (Csc[c + d*x]*(-13*a*
Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))/(96*(a - b)
^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (Csc[c +
 d*x]*(13*a*Sin[(c + d*x)/2] + 16*b*Sin[(c + d*x)/2])*(a + b*Sin[c + d*x]))
/(96*(a + b)^2*d*(b + a*Csc[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])
) - (b*Csc[c + d*x]*(a + b*Sin[c + d*x])*Tan[(c + d*x)/2])/(32*a^2*d*(b + a
*Csc[c + d*x])))
```

Maple [A]
time = 0.82, size = 298, normalized size = 0.90

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{-10a+12b}{4(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{8a\tan\left(\cdots\right.}}{}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^2}-\frac{-10a+12b}{4(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{1}{3(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a-b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{1}{8a\tan\left(\cdots\right.}}{}$
risch	$\frac{i\left(12b^5e^{6i(dx+c)}+6b^5e^{8i(dx+c)}-12b^5e^{2i(dx+c)}-6b^5-16a^4b+28a^2b^3+32a^4b\,e^{4i(dx+c)}-16a^4b\,e^{2i(dx+c)}-15ia^5e^{i(dx+c)}-15\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^2*(1/2*a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c))-1/4/(a-b)^
2*(-10*a+12*b)/(tan(1/2*d*x+1/2*c)+1)+1/3/(a-b)/(tan(1/2*d*x+1/2*c)+1)^3-1/
2/(a-b)/(tan(1/2*d*x+1/2*c)+1)^2-1/8/a/tan(1/2*d*x+1/2*c)^2+1/4/a^3*(10*a^2
+4*b^2)*ln(tan(1/2*d*x+1/2*c))+1/2/a^2*b/tan(1/2*d*x+1/2*c)-2/a^3/(a+b)^2/(
a-b)^2*b^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2
)^(1/2))-1/4/(a+b)^2*(10*a+12*b)/(tan(1/2*d*x+1/2*c)-1)-1/3/(a+b)/(tan(1/2*
d*x+1/2*c)-1)^3-1/2/(a+b)/(tan(1/2*d*x+1/2*c)-1)^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.22, size = 1182, normalized size = 3.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 -
 a^2*b^6)*cos(d*x + c)^4 + 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^
2 + 6*(b^7*cos(d*x + c)^5 - b^7*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2)) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*
b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2
*b^8)*cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2
+ 9*a^4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4
*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a
^7*b - 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*
cos(d*x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x
 + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3
*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3), -1/12*(4*a^8 - 8*a^6*b^2
 + 4*a^4*b^4 - 6*(5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4
+ 4*(5*a^8 - 13*a^6*b^2 + 8*a^4*b^4)*cos(d*x + c)^2 - 12*(b^7*cos(d*x + c)^
5 - b^7*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(
a^2 - b^2)*cos(d*x + c))) + 3*((5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 -
2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 + a^2*b^6 - 2*b^8)*
cos(d*x + c)^3)*log(1/2*cos(d*x + c) + 1/2) - 3*((5*a^8 - 13*a^6*b^2 + 9*a^
4*b^4 + a^2*b^6 - 2*b^8)*cos(d*x + c)^5 - (5*a^8 - 13*a^6*b^2 + 9*a^4*b^4 +
 a^2*b^6 - 2*b^8)*cos(d*x + c)^3)*log(-1/2*cos(d*x + c) + 1/2) - 4*(a^7*b -
 2*a^5*b^3 + a^3*b^5 - (8*a^7*b - 22*a^5*b^3 + 17*a^3*b^5 - 3*a*b^7)*cos(d*
x + c)^4 + (4*a^7*b - 11*a^5*b^3 + 7*a^3*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^5 - (a^9 - 3*a^7*b
^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [A]
time = 0.50, size = 417, normalized size = 1.26

$$-\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)b^7}{(a^7-2a^5b^2+a^3b^4)\sqrt{a^2-b^2}}-\frac{3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a^2}-\frac{12(5a^2+2b^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{16\left(6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+12ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-8a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+14b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-18ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+6a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-7a^3+10ab^2\right)}{(a^4-2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}+\frac{3\left(30a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^2\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x
+ 1/2*c) + b)/sqrt(a^2 - b^2)))*b^7/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 -
 b^2)) - 3*(a*tan(1/2*d*x + 1/2*c)^2 - 4*b*tan(1/2*d*x + 1/2*c))/a^2 - 12*(
5*a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 16*(6*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 - 9*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^3*tan(1/2*d*x + 1/2*c)^4 +
 12*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 8*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 14*b^3*
tan(1/2*d*x + 1/2*c)^3 + 12*a^3*tan(1/2*d*x + 1/2*c)^2 - 18*a*b^2*tan(1/2*d
*x + 1/2*c)^2 + 6*a^2*b*tan(1/2*d*x + 1/2*c) - 9*b^3*tan(1/2*d*x + 1/2*c) -
 7*a^3 + 10*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3)
 + 3*(30*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 4*a*b
*tan(1/2*d*x + 1/2*c) + a^2)/(a^3*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [B]
time = 18.01, size = 2500, normalized size = 7.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] ((b^9*sin(c + d*x))/24 + (41*b^9*sin(3*c + 3*d*x))/48 + (23*b^9*sin(5*c + 5
*d*x))/48)/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4
 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a*((19*b^8)/48
- (5*b^8*cos(c + d*x))/12 + (17*b^8*cos(2*c + 2*d*x))/12 + (5*b^8*cos(3*c +
 3*d*x))/24 + (11*b^8*cos(4*c + 4*d*x))/16 + (5*b^8*cos(5*c + 5*d*x))/24 -
(5*b^8*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (5*b^8
*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (5*b^8*l
og(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32))/(d*sin(c +
 d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^8*((b*sin(3*c + 3*d*x))/6 - (b*sin
(c + d*x))/3 + (b*sin(5*c + 5*d*x))/6))/(d*sin(c + d*x)^2*((3*cos(c + d*x))
/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3
```

*a^4*b^2)) + ((3*b^10)/16 + (b^10*cos(2*c + 2*d*x))/4 + (b^10*cos(4*c + 4*d*x))/16 + (5*b^10*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 - (5*b^10*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 - (5*b^10*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32)/(a*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - ((b^11*sin(c + d*x))/8 + (3*b^11*sin(3*c + 3*d*x))/16 + (b^11*sin(5*c + 5*d*x))/16)/(a^2*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^6*((19*b^3*sin(3*c + 3*d*x))/24 - (4*b^3*sin(c + d*x))/3 + (19*b^3*sin(5*c + 5*d*x))/24))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^4*((25*b^5*sin(3*c + 3*d*x))/16 - (15*b^5*sin(c + d*x))/8 + (23*b^5*sin(5*c + 5*d*x))/16))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^2*((77*b^7*sin(3*c + 3*d*x))/48 - (23*b^7*sin(c + d*x))/24 + (59*b^7*sin(5*c + 5*d*x))/48))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^9*((5*cos(2*c + 2*d*x))/12 - (7*cos(c + d*x))/24 + (7*cos(3*c + 3*d*x))/48 + (5*cos(4*c + 4*d*x))/16 + (7*cos(5*c + 5*d*x))/48 - (5*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (5*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (5*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32 - 11/48))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^5*((7*b^4*cos(2*c + 2*d*x))/2 - b^4 - (17*b^4*cos(c + d*x))/8 + (17*b^4*cos(3*c + 3*d*x))/16 + (5*b^4*cos(4*c + 4*d*x))/2 + (17*b^4*cos(5*c + 5*d*x))/16 - (5*b^4*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/4 + (5*b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/4))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^3*((19*b^6*cos(2*c + 2*d*x))/6 - b^6/6 - (37*b^6*cos(c + d*x))/24 + (37*b^6*cos(3*c + 3*d*x))/48 + 2*b^6*cos(4*c + 4*d*x) + (37*b^6*cos(5*c + 5*d*x))/48 - (15*b^6*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + (15*b^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/16 + (15*b^6*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/16))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a^7*((23*b^2*cos(2*c + 2*d*x))/12 - (41*b^2)/48 - (31*b^2*cos(c + d*x))/24 + (31*b^2*cos(3*c + 3*d*x))/48 + (23*b^2*cos(4*c + 4*d*x))/16 + (31*b^2*cos(5*c + 5*d*x))/48 - (23*b^2*cos(c + d*x)*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/16 + (23*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/32 + (23*b^2*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/32))/(d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b^12*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/(16*a^3*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos

```
(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2
)) + (b^12*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(5*c + 5*d*x))/(16
*a^3*d*sin(c + d*x)^2*((3*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4
- 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (b^12*cos(c + d*x)*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(8*a^3*d*sin(c + d*x)^2*((3*cos(c +
 d*x))/4 + cos(3*c + 3*d*x)/4)*(a^4 + b^4 - 2*a^2*b^2)*(a^6 - b^6 + 3*a^2*b
^4 - 3*a^4*b^2)) - (b^7*atan((8*b^8*sin(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2
*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) - 5*a^8*sin(c/2 + (d*x)/2
)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + 4
*a*b^7*cos(c/2 + (d*x)/2)*(b^10 - a^10 - 5*a^2*...
```

3.1359 $\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=240

$$-\frac{(35a^2+57ab+24b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(35a^2-57ab+24b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}-\frac{a^8\log(a+b\sin(c}{b^3\,(a^2-b^2)^3}$$

```
[Out] -1/16*(35*a^2+57*a*b+24*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(35*a^2-57*a*b
+24*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-a^8*ln(a+b*sin(d*x+c))/b^3/(a^2-b^2)^3/
d+a*sin(d*x+c)/b^2/d-1/2*sin(d*x+c)^2/b/d-1/4*sec(d*x+c)^4*(b-a*sin(d*x+c))
/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(4*a^2-3*b^2)-a*(13*a^2-9*b^2)*sin(d*x+c
))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.45, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1661, 1643}

$$-\frac{(35a^2+57ab+24b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(35a^2-57ab+24b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\,(a^2-b^2)}+\frac{\sec^2(c+dx)\,(4b(4a^2-3b^2)-a(13a^2-9b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}-\frac{a^8\log(a+b\sin(c+dx))}{b^3d\,(a^2-b^2)^3}+\frac{a\sin(c+dx)}{b^2d}-\frac{\sin^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((
35*a^2 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^8*Lo
g[a + b*Sin[c + d*x]])/(b^3*(a^2 - b^2)^3*d) + (a*Sin[c + d*x])/(b^2*d) - S
in[c + d*x]^2/(2*b*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2
)*d) + (Sec[c + d*x]^2*(4*b*(4*a^2 - 3*b^2) - a*(13*a^2 - 9*b^2)*Sin[c + d*
x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^8}{b^8(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^8}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^8}{a^2-b^2}+\frac{3ab^8x}{a^2-b^2}-4b^6x^2-4b^4x^4-4b}{(a+x)(b^2-x^2)^2}\right.}{4b^5d}$$

$$= \frac{\sec^2(c+dx)\left(4b(4a^2-3b^2)-a(13a^2-9b^2)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(4b(4a^2-3b^2)-a(13a^2-9b^2)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\right.}{}$$

$$= -\frac{(35a^2+57ab+24b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{(35a^2-57ab+24b^2)\log(1}{16(a-b)^3d}$$

Mathematica [A]
time = 1.98, size = 212, normalized size = 0.88

$$\frac{-\frac{(35a^2+57ab+24b^2)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{(35a^2-57ab+24b^2)\log(1+\sin(c+dx))}{(a-b)^3}-\frac{16a^8\log(a+b\sin(c+dx))}{(a-b)^3b^3(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{13a+11b}{(a+b)^2(-1+\sin(c+dx))}+\frac{16a\sin(c+dx)}{b^2}-\frac{8\sin^2(c+dx)}{b}-\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{13a-11b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((35*a^2 + 57*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((35*a^2
 - 57*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^8*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (13*a + 11*b)/((a + b)^2*(-1 + Sin[c + d*x])) + (16*a*Sin[c + d*x])/b^2 -
 (8*Sin[c + d*x]^2)/b - 1/((a - b)*(1 + Sin[c + d*x])^2) + (13*a - 11*b)/((
a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A]
time = 0.76, size = 217, normalized size = 0.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*(-1/2*sin(d*x+c)^2*b+a*sin(d*x+c))-1/b^3*a^8/(a+b)^3/(a-b)^3*ln(
a+b*sin(d*x+c))-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(-13*a+11*b)/(a-b)^2/(1
+sin(d*x+c))+1/16*(35*a^2-57*a*b+24*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a+
8*b)/(sin(d*x+c)-1)^2-1/16*(-13*a-11*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3
*(-35*a^2-57*a*b-24*b^2)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.38, size = 316, normalized size = 1.32

$$-\frac{\frac{16\,a^{8}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}b^{3}-3\,a^{4}b^{5}+3\,a^{2}b^{7}-b^{9}}-\frac{\left(35\,a^{2}-57\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(35\,a^{2}+57\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\,\left(\left(13\,a^{3}-9\,ab^{2}\right)\sin\left(dx+c\right)^{3}+14\,a^{2}b-10\,b^{3}-4\,\left(4\,a^{2}b-3\,b^{3}\right)\sin\left(dx+c\right)^{2}-\left(11\,a^{3}-7\,ab^{2}\right)\sin\left(dx+c\right)\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right)^{4}+a^{4}-2\,a^{2}b^{2}+b^{4}-2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right)^{2}}+\frac{8\,\left(b\sin\left(dx+c\right)^{2}-2\,a\sin\left(dx+c\right)\right)}{b^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^8*log(b*sin(d*x + c) + a)/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^
9) - (35*a^2 - 57*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - 2*((13*a^3 - 9*a*b^2)*sin(d*x + c)^3 + 14*a^2*b - 10*b
^3 - 4*(4*a^2*b - 3*b^3)*sin(d*x + c)^2 - (11*a^3 - 7*a*b^2)*sin(d*x + c))/
((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 -
2*a^2*b^2 + b^4)*sin(d*x + c)^2) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))/
b^2)/d
```

Fricas [A]
time = 0.64, size = 429, normalized size = 1.79

$$-\frac{16\,a^{8}\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^{4}b^{4}-8\,a^{2}b^{6}+4\,b^{8}-8\,\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)\cos\left(dx+c\right)^{6}-\left(35\,a^{5}b^{3}+48\,a^{4}b^{4}-42\,a^{3}b^{5}-64\,a^{2}b^{6}+15\,ab^{7}+24\,b^{8}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+\left(35\,a^{5}b^{3}-48\,a^{4}b^{4}-42\,a^{3}b^{5}+64\,a^{2}b^{6}+15\,ab^{7}-24\,b^{8}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+4\,\left(a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}\right)\cos\left(dx+c\right)^{4}-8\,\left(4\,a^{4}b^{4}-7\,a^{2}b^{6}+3\,b^{8}\right)\cos\left(dx+c\right)^{2}-2\,\left(2\,a^{5}b^{3}-4\,a^{3}b^{5}+2\,ab^{7}+8\,\left(a^{7}b-3\,a^{5}b^{3}+3\,a^{3}b^{5}-ab^{7}\right)\cos\left(dx+c\right)^{4}-\left(13\,a^{5}b^{3}-22\,a^{3}b^{5}+9\,ab^{7}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,\left(a^{6}b^{3}-3\,a^{4}b^{5}+3\,a^{2}b^{7}-b^{9}\right)d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^8*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^4 - 8*a^2*b^
6 + 4*b^8 - 8*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^6 - (35*
```

```
a^5*b^3 + 48*a^4*b^4 - 42*a^3*b^5 - 64*a^2*b^6 + 15*a*b^7 + 24*b^8)*cos(d*x
 + c)^4*log(sin(d*x + c) + 1) + (35*a^5*b^3 - 48*a^4*b^4 - 42*a^3*b^5 + 64*
a^2*b^6 + 15*a*b^7 - 24*b^8)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*(a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^4 - 8*(4*a^4*b^4 - 7*a^2*b
^6 + 3*b^8)*cos(d*x + c)^2 - 2*(2*a^5*b^3 - 4*a^3*b^5 + 2*a*b^7 + 8*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^4 - (13*a^5*b^3 - 22*a^3*b^5
+ 9*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
- b^9)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**8/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 403, normalized size = 1.68

$$-\frac{\frac{16\,a^8\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^6b^3-3\,a^4b^5+3\,a^2b^7-b^9}-\frac{\left(35\,a^2-57\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(35\,a^2+57\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{8\left(b\sin\left(dx+c\right)^2-2\,a\sin\left(dx+c\right)\right)}{b^2}+\frac{2\left(36\,a^4b\sin\left(dx+c\right)^4-48\,a^2b^3\sin\left(dx+c\right)^4+18\,b^5\sin\left(dx+c\right)^4-13\,a^5\sin\left(dx+c\right)^3+22\,a^3b^2\sin\left(dx+c\right)^3-9\,ab^4\sin\left(dx+c\right)^3-56\,a^4b\sin\left(dx+c\right)^2+68\,a^2b^3\sin\left(dx+c\right)^2-24\,b^5\sin\left(dx+c\right)^2+11\,a^5\sin\left(dx+c\right)-18\,a^3b^2\sin\left(dx+c\right)+7\,ab^4\sin\left(dx+c\right)+22\,a^4b-24\,a^2b^3+8\,b^5\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^8/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^8*log(abs(b*sin(d*x + c) + a))/(a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7
 - b^9) - (35*a^2 - 57*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) + (35*a^2 + 57*a*b + 24*b^2)*log(abs(sin(d*x + c) - 1)
)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 8*(b*sin(d*x + c)^2 - 2*a*sin(d*x + c))
/b^2 + 2*(36*a^4*b*sin(d*x + c)^4 - 48*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(
d*x + c)^4 - 13*a^5*sin(d*x + c)^3 + 22*a^3*b^2*sin(d*x + c)^3 - 9*a*b^4*si
n(d*x + c)^3 - 56*a^4*b*sin(d*x + c)^2 + 68*a^2*b^3*sin(d*x + c)^2 - 24*b^5
*sin(d*x + c)^2 + 11*a^5*sin(d*x + c) - 18*a^3*b^2*sin(d*x + c) + 7*a*b^4*s
in(d*x + c) + 22*a^4*b - 24*a^2*b^3 + 8*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 14.19, size = 806, normalized size = 3.36

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{13\,b}{8\,{\left(a-b\right)}^2}+\frac{35}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(-3\,a^4+a^2\,b^2+b^4\right)}{b\,{\left(a^2-b^2\right)}^2}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^4-5\,a^2\,b^2+3\,b^4\right)}{b\,{\left(a^2-b^2\right)}^2}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(8\,a^5+7\,a^3\,b^2-11\,a\,b^4\right)}{2\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(8\,a^5+7\,a^3\,b^2-11\,a\,b^4\right)}{2\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(8\,a^5-27\,a^3\,b^2+15\,a\,b^4\right)}{4\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(24\,a^5-45\,a^3\,b^2+25\,a\,b^4\right)}{4\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(24\,a^5-45\,a^3\,b^2+25\,a\,b^4\right)}{4\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(2\,a^2-3\,b^2\right)}{b\,\left(a^2-b^2\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(2\,a^2-3\,b^2\right)}{b\,\left(a^2-b^2\right)}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(a^4-5\,a^2\,b^2+3\,b^4\right)}{b\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,a^4-27\,a^2\,b^2+15\,b^4\right)}{4\,b^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{35}{8\,\left(a+b\right)}-\frac{13\,b}{8\,{\left(a+b\right)}^2}+\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}+\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\,\left(a^2+3\,b^2\right)}{b^3\,d}+\frac{a^8\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(-a^6\,b^3+3\,a^4\,b^5-3\,a^2\,b^7+b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (13*b)/(8*(a - b)^2) + 35/(8*(a - b))))/d - ((4*tan(c/2 + (d*x)/2)^6*(b^4 - 3*a^4 + a^2*b^2))/(b*(a^2 - b^2)^2) - (2*tan(c/2 + (d*x)/2)^2*(a^4 + 3*b^4 - 5*a^2*b^2))/(b*(a^2 - b^2)^2) + (tan(c/2 + (d*x)/2)^5*(8*a^5 - 11*a*b^4 + 7*a^3*b^2))/(2*b^2*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^7*(8*a^5 - 11*a*b^4 + 7*a^3*b^2))/(2*b^2*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^11*(15*a*b^4 + 8*a^5 - 27*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(25*a*b^4 + 24*a^5 - 45*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^9*(25*a*b^4 + 24*a^5 - 45*a^3*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(c/2 + (d*x)/2)^4*(2*a^2 - 3*b^2))/(b*(a^2 - b^2)) + (4*tan(c/2 + (d*x)/2)^8*(2*a^2 - 3*b^2))/(b*(a^2 - b^2)) - (2*tan(c/2 + (d*x)/2)^10*(a^4 + 3*b^4 - 5*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (a*tan(c/2 + (d*x)/2)*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b^2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1)) - (log(tan(c/2 + (d*x)/2) - 1)*(35/(8*(a + b)) - (13*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d + (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + 3*b^2))/(b^3*d) + (a^8*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3))

3.1360 $\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=221

$$-\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{(24a^2-37ab+15b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}+\frac{a^7\log(a+b\sin(c+$$

```
[Out] -1/16*(24*a^2+37*a*b+15*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(24*a^2-37*a*b
+15*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^7*ln(a+b*sin(d*x+c))/b^2/(a^2-b^2)^3/
d-sin(d*x+c)/b/d+1/4*sec(d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+
c)^2*(4*a*(3*a^2-2*b^2)-b*(13*a^2-9*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.38, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1661, 1643}

$$-\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(24a^2-37ab+15b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d(a^2-b^2)}-\frac{\sec^2(c+dx)\left(4a(3a^2-2b^2)-b(13a^2-9b^2)\sin(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a^7\log(a+b\sin(c+dx))}{b^2d\left(a^2-b^2\right)^3}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) - ((
24*a^2 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^7*Lo
g[a + b*Sin[c + d*x]])/(b^2*(a^2 - b^2)^3*d) - Sin[c + d*x]/(b*d) + (Sec[c
+ d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(3*
a^2 - 2*b^2) - b*(13*a^2 - 9*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^7}{b^7(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^7}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{b^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^8}{a^2-b^2} - \frac{b^6(4a^2-b^2)x}{a^2-b^2} - 4b^4x^3 - 4b^2x^5}{(a+x)(b^2-x^2)^2}\, d \cdots\right.}{4b^4 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)\left(4a(3a^2-2b^2) - b(13a^2 \cdots\right.}{8(a^2-b^2)^2 d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)\left(4a(3a^2-2b^2) - b(13a^2 \cdots\right.}{8(a^2-b^2)^2 d}$$

$$= -\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{16(a+b)^3 d} - \frac{(24a^2-37ab+15b^2)\log(\cdots}{16(a-b)^3 \cdots}$$

Mathematica [A]
time = 1.50, size = 198, normalized size = 0.90

$$\frac{-\frac{(24a^2+37ab+15b^2)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{(24a^2-37ab+15b^2)\log(1+\sin(c+dx))}{(a-b)^3} + \frac{16a^7\log(a+b\sin(c+dx))}{(a-b)^3b^2(a+b)^3} + \frac{1}{(a+b)(-1+\sin(c+dx))^2} + \frac{11a+9b}{(a+b)^2(-1+\sin(c+dx))} - \frac{16\sin(c+dx)}{b} + \frac{1}{(a-b)(1+\sin(c+dx))^2} + \frac{-11a+9b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((24*a^2 + 37*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((24*a^2
 - 37*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^7*Log[a + b*Si
n[c + d*x]])/((a - b)^3*b^2*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2)
+ (11*a + 9*b)/((a + b)^2*(-1 + Sin[c + d*x])) - (16*Sin[c + d*x])/b + 1/((
a - b)*(1 + Sin[c + d*x])^2) + (-11*a + 9*b)/((a - b)^2*(1 + Sin[c + d*x]))
)/(16*d)
```

Maple [A]
time = 0.67, size = 203, normalized size = 0.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b*sin(d*x+c)+1/b^2*a^7/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a-
8*b)/(1+sin(d*x+c))^2-1/16*(11*a-9*b)/(a-b)^2/(1+sin(d*x+c))+1/16/(a-b)^3*(
-24*a^2+37*a*b-15*b^2)*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16
*(-11*a-9*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-24*a^2-37*a*b-15*b^2)*ln
(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 303, normalized size = 1.37

$$\frac{\frac{16\,a^7\log\left(b\sin\left(dx+c\right)+a\right)}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8}-\frac{\left(24\,a^2-37\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+37\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,\left(\left(13\,a^2b-9\,b^3\right)\sin\left(dx+c\right)^3+10\,a^3-6\,ab^2-4\,\left(3\,a^3-2\,ab^2\right)\sin\left(dx+c\right)^2-\left(11\,a^2b-7\,b^3\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\,\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}-\frac{16\,\sin\left(dx+c\right)}{b}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^7*log(b*sin(d*x + c) + a)/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8
) - (24*a^2 - 37*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) - 2*((13*a^2*b - 9*b^3)*sin(d*x + c)^3 + 10*a^3 - 6*a*b^2
 - 4*(3*a^3 - 2*a*b^2)*sin(d*x + c)^2 - (11*a^2*b - 7*b^3)*sin(d*x + c))/((
a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^2) - 16*sin(d*x + c)/b)/d
```

Fricas [A]
time = 0.57, size = 351, normalized size = 1.59

$$\frac{16\,a^7\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^5b^2-8\,a^3b^4+4\,ab^6-\left(24\,a^5b^2+35\,a^4b^3-24\,a^3b^4-42\,a^2b^5+8\,ab^6+15\,b^7\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(24\,a^5b^2-35\,a^4b^3-24\,a^3b^4+42\,a^2b^5+8\,ab^6-15\,b^7\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-8\,\left(3\,a^5b^2-5\,a^3b^4+2\,ab^6\right)\cos\left(dx+c\right)^2-2\,\left(2\,a^4b^3-4\,a^2b^5+2\,b^7+8\,\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)\cos\left(dx+c\right)^4-\left(13\,a^4b^3-22\,a^2b^5+9\,b^7\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^5*b^2 - 8*a^3*b^4
 + 4*a*b^6 - (24*a^5*b^2 + 35*a^4*b^3 - 24*a^3*b^4 - 42*a^2*b^5 + 8*a*b^6 +
 15*b^7)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (24*a^5*b^2 - 35*a^4*b^3 -
```

```
24*a^3*b^4 + 42*a^2*b^5 + 8*a*b^6 - 15*b^7)*cos(d*x + c)^4*log(-sin(d*x + c
) + 1) - 8*(3*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 2*(2*a^4*b^3
- 4*a^2*b^5 + 2*b^7 + 8*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^
4 - (13*a^4*b^3 - 22*a^2*b^5 + 9*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^6*b
^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [A]
time = 0.54, size = 384, normalized size = 1.74

$$\frac{\frac{16\,a^{7}\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{6}b^{2}-3\,a^{4}b^{4}+3\,a^{2}b^{6}-b^{8}}-\frac{\left(24\,a^{2}-37\,ab+15\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{\left(24\,a^{2}+37\,ab+15\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{16\,\sin\left(dx+c\right)}{b}+\frac{2\,\left(18\,a^{5}\sin\left(dx+c\right)^{4}-18\,a^{3}b^{2}\sin\left(dx+c\right)^{4}+6\,ab^{4}\sin\left(dx+c\right)^{4}-13\,a^{4}b\sin\left(dx+c\right)^{3}+22\,a^{2}b^{3}\sin\left(dx+c\right)^{3}-9\,b^{5}\sin\left(dx+c\right)^{3}-24\,a^{5}\sin\left(dx+c\right)^{2}+16\,a^{3}b^{2}\sin\left(dx+c\right)^{2}-4\,ab^{4}\sin\left(dx+c\right)^{2}+11\,a^{4}b\sin\left(dx+c\right)-18\,a^{2}b^{3}\sin\left(dx+c\right)+7\,b^{5}\sin\left(dx+c\right)+8\,a^{5}-2\,a^{3}b^{2}\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^7*log(abs(b*sin(d*x + c) + a))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6
- b^8) - (24*a^2 - 37*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) - (24*a^2 + 37*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 16*sin(d*x + c)/b + 2*(18*a^5*sin(d*x +
c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 6*a*b^4*sin(d*x + c)^4 - 13*a^4*b*sin(d*
x + c)^3 + 22*a^2*b^3*sin(d*x + c)^3 - 9*b^5*sin(d*x + c)^3 - 24*a^5*sin(d*
x + c)^2 + 16*a^3*b^2*sin(d*x + c)^2 - 4*a*b^4*sin(d*x + c)^2 + 11*a^4*b*si
n(d*x + c) - 18*a^2*b^3*sin(d*x + c) + 7*b^5*sin(d*x + c) + 8*a^5 - 2*a^3*b
^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 14.07, size = 627, normalized size = 2.84

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{11\,b}{8\,{\left(a-b\right)}^2}+\frac{3}{a-b}\right)}{d}-\frac{\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a\,b^2-4\,a^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a\,b^2-2\,a^3\right)}{{\left(a^2-b^2\right)}^2}+\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(3\,a\,b^2-4\,a^3\right)}{{\left(a^2-b^2\right)}^2}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(a\,b^2-2\,a^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(24\,a^4-29\,a^2\,b^2+9\,b^4\right)}{2\,b\,{\left(a^2-b^2\right)}^2}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(4\,a^2-5\,b^2\right)}{b\,\left(a^2-b^2\right)}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(4\,a^2-5\,b^2\right)}{b\,\left(a^2-b^2\right)}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(8\,a^4-27\,a^2\,b^2+15\,b^4\right)}{4\,b\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(8\,a^4-27\,a^2\,b^2+15\,b^4\right)}{4\,b\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{3}{a+b}-\frac{11\,b}{8\,{\left(a+b\right)}^2}+\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{a\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b^2\,d}-\frac{a^7\,\ln\left(a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(-a^6\,b^2+3\,a^4\,b^4-3\,a^2\,b^6+b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] - (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (11*b)/(8*(a - b)^2) +
3/(a - b)))/d - ((2*tan(c/2 + (d*x)/2)^4*(3*a*b^2 - 4*a^3))/(a^2 - b^2)^2 -
 (2*tan(c/2 + (d*x)/2)^2*(a*b^2 - 2*a^3))/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x
)/2)^6*(3*a*b^2 - 4*a^3))/(a^2 - b^2)^2 - (2*tan(c/2 + (d*x)/2)^8*(a*b^2 -
```

2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(24*a^4 + 9*b^4 - 2
9*a^2*b^2))/(2*b*(a^2 - b^2)^2) - (2*tan(c/2 + (d*x)/2)^3*(4*a^2 - 5*b^2))/
(b*(a^2 - b^2)) - (2*tan(c/2 + (d*x)/2)^7*(4*a^2 - 5*b^2))/(b*(a^2 - b^2))
+ (tan(c/2 + (d*x)/2)*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b*(a^4 + b^4 - 2*a^
2*b^2)) + (tan(c/2 + (d*x)/2)^9*(8*a^4 + 15*b^4 - 27*a^2*b^2))/(4*b*(a^4 +
b^4 - 2*a^2*b^2)))/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*
tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
 - (log(tan(c/2 + (d*x)/2) - 1)*(3/(a + b) - (11*b)/(8*(a + b)^2) + b^2/(4*
(a + b)^3)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/(b^2*d) - (a^7*log(a + 2
*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(b^8 - 3*a^2*b^6 + 3*a^
4*b^4 - a^6*b^2))

3.1361 $\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=208

$$-\frac{(15a^2+21ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(15a^2-21ab+8b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}-\frac{a^6\log(a+b\sin(c+}{b\,(a^2-b^2)^3\,d}$$

```
[Out] -1/16*(15*a^2+21*a*b+8*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(15*a^2-21*a*b+
8*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-a^6*ln(a+b*sin(d*x+c))/b/(a^2-b^2)^3/d-1/
4*sec(d*x+c)^4*(b-a*sin(d*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(3*a^2-2*
b^2)-a*(9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.37, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2916, 12, 1661, 1643}

$$-\frac{(15a^2+21ab+8b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(15a^2-21ab+8b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\,(a^2-b^2)}+\frac{\sec^2(c+dx)\,(4b(3a^2-2b^2)-a(9a^2-5b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}-\frac{a^6\log(a+b\sin(c+dx))}{bd\,(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((1
5*a^2 - 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^6*Log[
a + b*Sin[c + d*x]])/(b*(a^2 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d
*x]))/(4*(a^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b*(3*a^2 - 2*b^2) - a*(9*a^2 -
 5*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^6}{b^6(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^6}{a^2-b^2}+\frac{3ab^6x}{a^2-b^2}-4b^4x^2-4b^2x^4}{(a+x)(b^2-x^2)^2}\,dx,\right.}{4b^3d}$$

$$= \frac{\sec^2(c+dx)\left(4b(3a^2-2b^2)-a(9a^2-5b^2)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\frac{\cdots}{a^2\cdots}\right.}{4\cdots}$$

$$= \frac{\sec^2(c+dx)\left(4b(3a^2-2b^2)-a(9a^2-5b^2)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} - \frac{\sec^4(c+dx)\left(\frac{\cdots}{a^2\cdots}\right.}{4\cdots}$$

$$= -\frac{(15a^2+21ab+8b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{(15a^2-21ab+8b^2)\log(1+\text{s}\cdots}{16(a-b)^3d}$$

Mathematica [A]
time = 1.13, size = 187, normalized size = 0.90

$$\frac{-\frac{(15a^2+21ab+8b^2)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{(15a^2-21ab+8b^2)\log(1+\sin(c+dx))}{(a-b)^3}-\frac{16a^6\log(a+b\sin(c+dx))}{(a-b)^3b(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{9a+7b}{(a+b)^2(-1+\sin(c+dx))}-\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{9a-7b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

[Out] (-(((15*a^2 + 21*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) + ((15*a^2 - 21*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 - (16*a^6*Log[a + b*Sin[c + d*x]])/((a - b)^3*b*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (9*a + 7*b)/((a + b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (9*a - 7*b)/((a - b)^2*(1 + Sin[c + d*x]))))/(16*d)

Maple [A]
time = 0.60, size = 193, normalized size = 0.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-a^6/(a+b)^3/(a-b)^3/b*ln(a+b*sin(d*x+c))-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(-9*a+7*b)/(a-b)^2/(1+sin(d*x+c))+1/16*(15*a^2-21*a*b+8*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-9*a-7*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-15*a^2-21*a*b-8*b^2)*ln(sin(d*x+c)-1))

Maxima [A]
time = 0.32, size = 289, normalized size = 1.39

$$-\frac{\frac{16\,a^6\log(b\sin(dx+c)+a)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{(15\,a^2-21\,ab+8\,b^2)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(15\,a^2+21\,ab+8\,b^2)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left((9\,a^3-5\,ab^2)\sin(dx+c)^3+10\,a^2b-6\,b^3-4\,(3\,a^2b-2\,b^3)\sin(dx+c)^2-(7\,a^3-3\,ab^2)\sin(dx+c)\right)}{(a^4-2\,a^2b^2+b^4)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\sin(dx+c)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/16*(16*a^6*log(b*sin(d*x + c) + a)/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - (15*a^2 - 21*a*b + 8*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*((9*a^3 - 5*a*b^2)*sin(d*x + c)^3 + 10*a^2*b - 6*b^3 - 4*(3*a^2*b - 2*b^3)*sin(d*x + c)^2 - (7*a^3 - 3*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d

Fricas [A]
time = 0.60, size = 303, normalized size = 1.46

$$-\frac{16\,a^6\cos(dx+c)^4\log(b\sin(dx+c)+a)+4\,a^4b^2-8\,a^2b^4+4\,b^6-(15\,a^5b+24\,a^4b^2-10\,a^3b^3-24\,a^2b^4+3\,ab^5+8\,b^6)\cos(dx+c)^4\log(\sin(dx+c)+1)+(15\,a^5b-24\,a^4b^2-10\,a^3b^3+24\,a^2b^4+3\,ab^5-8\,b^6)\cos(dx+c)^4\log(-\sin(dx+c)+1)-8\,(3\,a^4b^2-5\,a^2b^4+2\,b^6)\cos(dx+c)^2-2\,(2\,a^5b-4\,a^3b^3+2\,ab^5-(9\,a^5b-14\,a^3b^3+5\,ab^5)\cos(dx+c)^2)\sin(dx+c)}{16\,(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/16*(16*a^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^4*b^2 - 8*a^2*b^4 + 4*b^6 - (15*a^5*b + 24*a^4*b^2 - 10*a^3*b^3 - 24*a^2*b^4 + 3*a*b^5 + 8*b^6)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (15*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 3*a*b^5 - 8*b^6)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*(3*a^4*b^2 - 5*a^2*b^4 + 2*b^6)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^3

```
 + 2*a*b^5 - (9*a^5*b - 14*a^3*b^3 + 5*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [A]
time = 0.54, size = 371, normalized size = 1.78

$$-\frac{\frac{16\,a^6\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(15\,a^2-21\,ab+8\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+21\,ab+8\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,\left(18\,a^4b\sin\left(dx+c\right)^4-18\,a^2b^3\sin\left(dx+c\right)^4+6\,b^5\sin\left(dx+c\right)^4-9\,a^5\sin\left(dx+c\right)^3+14\,a^3b^2\sin\left(dx+c\right)^3-5\,ab^4\sin\left(dx+c\right)^3-24\,a^4b\sin\left(dx+c\right)^2+16\,a^2b^3\sin\left(dx+c\right)^2-4\,b^5\sin\left(dx+c\right)^2+7\,a^5\sin\left(dx+c\right)-10\,a^3b^2\sin\left(dx+c\right)+3\,ab^4\sin\left(dx+c\right)+8\,a^4b-2\,a^2b^3\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^6*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7) - (15*a^2 - 21*a*b + 8*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) + (15*a^2 + 21*a*b + 8*b^2)*log(abs(sin(d*x + c) - 1))/(a
^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(18*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin
(d*x + c)^4 + 6*b^5*sin(d*x + c)^4 - 9*a^5*sin(d*x + c)^3 + 14*a^3*b^2*sin(
d*x + c)^3 - 5*a*b^4*sin(d*x + c)^3 - 24*a^4*b*sin(d*x + c)^2 + 16*a^2*b^3*
sin(d*x + c)^2 - 4*b^5*sin(d*x + c)^2 + 7*a^5*sin(d*x + c) - 10*a^3*b^2*sin
(d*x + c) + 3*a*b^4*sin(d*x + c) + 8*a^4*b - 2*a^2*b^3)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 13.38, size = 549, normalized size = 2.64

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{9\,b}{8\,{\left(a-b\right)}^2}+\frac{15}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(11\,a\,b^2-15\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a\,b^2-7\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(11\,a\,b^2-15\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2\,b-b^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^2\,b-2\,b^3\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(2\,a^2\,b-b^3\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,a^2-3\,b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{b\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{15}{8\,\left(a+b\right)}-\frac{9\,b}{8\,{\left(a+b\right)}^2}+\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{a^6\,\ln\left(a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6\,b-3\,a^4\,b^3+3\,a^2\,b^5-b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (9*b)/(8*(a - b)^2) + 15/
(8*(a - b))))/d - ((tan(c/2 + (d*x)/2)^3*(11*a*b^2 - 15*a^3))/(4*(a^4 + b^4
 - 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^7*(3*a*b^2 - 7*a^3))/(4*(a^4 + b^4 - 2
*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(11*a*b^2 - 15*a^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2)
 + (4*tan(c/2 + (d*x)/2)^4*(3*a^2*b - 2*b^3))/(a^4 + b^4 - 2*a^2*b^2) - (2*
tan(c/2 + (d*x)/2)^6*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) + (a*tan(c/2
```

```
+ (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d
*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x
)/2)^8 + 1)) + log(tan(c/2 + (d*x)/2)^2 + 1)/(b*d) - (log(tan(c/2 + (d*x)/2
) - 1)*(15/(8*(a + b)) - (9*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (a^6
*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6*b - b^7
+ 3*a^2*b^5 - 3*a^4*b^3))
```

3.1362 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}+\frac{a^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}+$$

```
[Out] -1/16*(8*a^2+9*a*b+3*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(8*a^2-9*a*b+3*b^
2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(
d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*(2*a^2-b^2)-b*(
9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2800, 1661, 815}

$$-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-9ab+3b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\,(a^2-b^2)}-\frac{\sec^2(c+dx)\,(4a(2a^2-b^2)-b(9a^2-5b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}+\frac{a^5\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) - ((8*a
^2 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a +
b*Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/
(4*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*
Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2}-\frac{b^4(4a^2-b^2)x}{a^2-b^2}-4b^2x^3}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\left(4a(2a^2-b^2)-b(9a^2-5b^2)\sin(c\right.}{8(a^2-b^2)^2\,d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\left(4a(2a^2-b^2)-b(9a^2-5b^2)\sin(c\right.}{8(a^2-b^2)^2\,d}$$

$$= -\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx)}{16(a-b)^3d}$$

Mathematica [A]
time = 0.93, size = 184, normalized size = 0.90

$$\frac{-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{(a+b)^3}-\frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{(a-b)^3}+\frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{7a+5b}{(a+b)^2(-1+\sin(c+dx))}+\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{-7a+5b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A]
time = 0.51, size = 189, normalized size = 0.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a-8*b)/(1+sin(d*x+c))^2-
1/16*(7*a-5*b)/(a-b)^2/(1+sin(d*x+c))+1/16/(a-b)^3*(-8*a^2+9*a*b-3*b^2)*ln(
1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-7*a-5*b)/(a+b)^2/(sin(d
*x+c)-1)+1/16/(a+b)^3*(-8*a^2-9*a*b-3*b^2)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.31, size = 288, normalized size = 1.41

$$\frac{\frac{16\,a^5\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin\left(dx+c\right)^3+6\,a^3-2\,ab^2-4\left(2\,a^3-ab^2\right)\sin\left(dx+c\right)^2-\left(7\,a^2b-3\,b^3\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.48, size = 261, normalized size = 1.28

$$\frac{16\,a^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^5-8\,a^3b^2+4\,ab^4-8\left(2\,a^5-3\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^2-2\left(2\,a^4b-4\,a^2b^3+2\,b^5-\left(9\,a^4b-14\,a^2b^3+5\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [A]
time = 0.54, size = 343, normalized size = 1.68

$$\frac{\frac{16\,a^5b\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{(8\,a^2-9\,ab+3\,b^2)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(8\,a^2+9\,ab+3\,b^2)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^5\sin(dx+c)^4-9\,a^4b\sin(dx+c)^3+14\,a^2b^3\sin(dx+c)^3-5\,b^5\sin(dx+c)^3-4\,a^5\sin(dx+c)^2-12\,a^3b^2\sin(dx+c)^2+4\,ab^4\sin(dx+c)^2+7\,a^4b\sin(dx+c)-10\,a^2b^3\sin(dx+c)+3\,b^5\sin(dx+c)+8\,a^3b^2-2\,ab^4\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 12.54, size = 498, normalized size = 2.44

$$\frac{a^5\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{1}{a+b}-\frac{7\,b}{8\,(a+b)^2}+\frac{b^2}{4\,(a+b)^3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,(a-b)^3}+\frac{7\,b}{8\,(a-b)^2}+\frac{1}{a-b}\right)}{d}-\frac{\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{a^4-2\,a^2\,b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a\,b^2-2\,a^3)}{a^4-2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(7\,a^2\,b-3\,b^3)}{4\,(a^4-2\,a^2\,b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(15\,a^2\,b-11\,b^3)}{4\,(a^4-2\,a^2\,b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(15\,a^2\,b-11\,b^3)}{4\,(a^4-2\,a^2\,b^2+b^4)}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(7\,a^2-3\,b^2)}{4\,(a^4-2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (a^5*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b) - (7*
b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (log(tan(c/2 + (d*x)/2) + 1)*(b^
2/(4*(a - b)^3) + (7*b)/(8*(a - b)^2) + 1/(a - b)))/d - ((2*a^3*tan(c/2 + (
d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (2*a^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^
4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^
2*b^2) - (tan(c/2 + (d*x)/2)^7*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^3*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 + (tan(c/2 + (d*x)/2)^5*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) -
 (b*tan(c/2 + (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2))))/(d*(6*
tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + ta
n(c/2 + (d*x)/2)^8 + 1))
```

3.1363 $\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=190

$$-\frac{a(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{a(3a-b)\log(1+\sin(c+dx))}{16(a-b)^3d}-\frac{a^4b\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-\frac{\sec^4(c+dx)(b}{4\,(a^2}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*a*(3*a-b)*ln(1+sin(d*x+c))/
(a-b)^3/d-a^4*b*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(b-a*sin(
d*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*sin(d*x
+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.30, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$,
Rules used = {2916, 12, 1661, 815}

$$-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\,(a^2-b^2)}+\frac{\sec^2(c+dx)\,(4b(2a^2-b^2)-a(5a^2-b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}-\frac{a^4b\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}-\frac{a(3a+b)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{a(3a-b)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*(a*(3*a + b)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + (a*(3*a - b)*Log[
1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (a^4*b*Log[a + b*Sin[c + d*x]])/((a^2
 - b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (S
ec[c + d*x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 -
 b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^4}{b^4(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x^4}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^4}{a^2-b^2}+\frac{3ab^4x}{a^2-b^2}-4b^2x^2}{(a+x)(b^2-x^2)^2}\,dx, x, b\right.}{4bd}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b(2a^2-b^2)-a(5a^2\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{\sec^2(c+dx)\left(4b(2a^2-b^2)-a(5a^2\right.}{8\left(a^2-b^2\right)^2 d}$$

$$= -\frac{a(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{a(3a-b)\log(1+\sin(c+dx))}{16(a-b)^3d} - \frac{a^4b\,\mathrm{l}}{}$$

Mathematica [A]
time = 1.06, size = 169, normalized size = 0.89

$$\frac{-\frac{a(3a+b)\log(1-\sin(c+dx))}{(a+b)^3}+\frac{a(3a-b)\log(1+\sin(c+dx))}{(a-b)^3}-\frac{16a^4b\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{5a+3b}{(a+b)^2(-1+\sin(c+dx))}-\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{5a-3b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x]^4)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((a*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3) + (a*(3*a - b)*Log[1 + Si
n[c + d*x]])/(a - b)^3 - (16*a^4*b*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
 b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (5*a + 3*b)/((a + b)^2*(-1 + S
in[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (5*a - 3*b)/((a - b)^2*(
1 + Sin[c + d*x])))/(16*d)
```

Maple [A]
time = 0.51, size = 175, normalized size = 0.92 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*b/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/(8*a-8*b)/(1+sin(d*x+c))
^2-1/16*(-5*a+3*b)/(a-b)^2/(1+sin(d*x+c))+1/16*a*(3*a-b)/(a-b)^3*ln(1+sin(d
*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-5*a-3*b)/(a+b)^2/(sin(d*x+c)-1
)-1/16*a*(3*a+b)/(a+b)^3*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 276, normalized size = 1.45

$$-\frac{\frac{16\,a^{4}b\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}-\frac{\left(3\,a^{2}-ab\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(3\,a^{2}+ab\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\,\left(\left(5\,a^{3}-ab^{2}\right)\sin\left(dx+c\right)^{3}+6\,a^{2}b-2\,b^{3}-4\,\left(2\,a^{2}b-b^{3}\right)\sin\left(dx+c\right)^{2}-\left(3\,a^{3}+ab^{2}\right)\sin\left(dx+c\right)\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right)^{4}+a^{4}-2\,a^{2}b^{2}+b^{4}-2\,\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)\sin\left(dx+c\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^4*b*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
 - (3*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3
*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*((5*a
^3 - a*b^2)*sin(d*x + c)^3 + 6*a^2*b - 2*b^3 - 4*(2*a^2*b - b^3)*sin(d*x +
c)^2 - (3*a^3 + a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^
4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.47, size = 261, normalized size = 1.37

$$-\frac{16\,a^{4}b\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)-\left(3\,a^{5}+8\,a^{4}b+6\,a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)+\left(3\,a^{5}-8\,a^{4}b+6\,a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^{4}b-8\,a^{2}b^{3}+4\,b^{5}-8\,\left(2\,a^{4}b-3\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}-2\,\left(2\,a^{5}-4\,a^{3}b^{2}+2\,ab^{4}-\left(5\,a^{5}-6\,a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^4*b*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^5 + 8*a^4*b +
 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^5 - 8*a^4*b
 + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b - 8*a
^2*b^3 + 4*b^5 - 8*(2*a^4*b - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - 2*(2*a^5 -
4*a^3*b^2 + 2*a*b^4 - (5*a^5 - 6*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [A]
time = 0.53, size = 333, normalized size = 1.75

$$-\frac{\frac{16\,a^4b^2\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(3\,a^2-ab\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^4b\sin\left(dx+c\right)^4-5\,a^5\sin\left(dx+c\right)^3+6\,a^3b^2\sin\left(dx+c\right)^3-ab^4\sin\left(dx+c\right)^3-4\,a^4b\sin\left(dx+c\right)^2-12\,a^2b^3\sin\left(dx+c\right)^2+4\,b^5\sin\left(dx+c\right)^2+3\,a^5\sin\left(dx+c\right)-2\,a^3b^2\sin\left(dx+c\right)-ab^4\sin\left(dx+c\right)+8\,a^2b^3-2\,b^5\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^4*b^2*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) - (3*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) + (3*a^2 + a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^4*b*sin(d*x + c)^4 - 5*a^5*sin(d*x + c)^3 + 6*a^3*b^2*si
n(d*x + c)^3 - a*b^4*sin(d*x + c)^3 - 4*a^4*b*sin(d*x + c)^2 - 12*a^2*b^3*s
in(d*x + c)^2 + 4*b^5*sin(d*x + c)^2 + 3*a^5*sin(d*x + c) - 2*a^3*b^2*sin(d
*x + c) - a*b^4*sin(d*x + c) + 8*a^2*b^3 - 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 12.45, size = 507, normalized size = 2.67

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{5\,b}{8\,{\left(a-b\right)}^2}+\frac{3}{8\,\left(a-b\right)}\right)}{d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a^3+a\,b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(7\,a\,b^2-11\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(7\,a\,b^2-11\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(2\,a^2\,b-b^3\right)}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{a^4-2\,a^2\,b^2+b^4}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{3}{8\,\left(a+b\right)}-\frac{5\,b}{8\,{\left(a+b\right)}^2}+\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{a^4\,b\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (5*b)/(8*(a - b)^2) + 3/(
8*(a - b))))/d - ((tan(c/2 + (d*x)/2)^7*(a*b^2 + 3*a^3))/(4*(a^4 + b^4 - 2*
a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(7*a*b^2 - 11*a^3))/(4*(a^4 + b^4 - 2*a^2
*b^2)) + (tan(c/2 + (d*x)/2)^5*(7*a*b^2 - 11*a^3))/(4*(a^4 + b^4 - 2*a^2*b^
2)) + (4*tan(c/2 + (d*x)/2)^4*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^2) - (2
*a^2*b*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) - (2*a^2*b*tan(c/2 + (
d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2) + (a*tan(c/2 + (d*x)/2)*(3*a^2 + b^2))/(
4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/
2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (log(tan(c/2 +
 (d*x)/2) - 1)*(3/(8*(a + b)) - (5*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d
 - (a^4*b*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.1364 $\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{b(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{(3a-b)b\log(1+\sin(c+dx))}{16(a-b)^3d}+\frac{a^3b^2\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}+\frac{\sec^4(c+dx)(a-}{4\,(a^2-}$$

```
[Out] 1/16*b*(3*a+b)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(3*a-b)*b*ln(1+sin(d*x+c))/(
a-b)^3/d+a^3*b^2*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(d*x+c)^4*(a-b*sin
(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a^3-b*(5*a^2-b^2)*sin(d*x+c))/(a^2
-b^2)^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2916, 12, 1661, 837, 815}

$$\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d\,(a^2-b^2)}+\frac{a^3b^2\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}-\frac{\sec^2(c+dx)\,(4a^3-b(5a^2-b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}+\frac{b(3a+b)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{b(3a-b)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*(3*a + b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((3*a - b)*b*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) + (a^3*b^2*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (Se
c[c + d*x]^2*(4*a^3 - b*(5*a^2 - b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^3}{b^3(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{x^3}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2(4a^2-b^2)x}{a^2-b^2}}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b(5a^2-b^2)\sin(c+dx)}{a^2-b^2}\right.}{8(a^2-b^2)\,d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{\sec^2(c+dx)\left(\frac{4a^3}{a^2-b^2}-\frac{b(5a^2-b^2)\sin(c+dx)}{a^2-b^2}\right.}{8(a^2-b^2)\,d}$$

$$= \frac{b(3a+b)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{(3a-b)b\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^3b^2\,\text{lo}}{}$$

Mathematica [A]

time = 0.84, size = 166, normalized size = 0.91

$$\frac{\frac{b(3a+b)\log(1-\sin(c+dx))}{(a+b)^3}-\frac{(3a-b)b\log(1+\sin(c+dx))}{(a-b)^3}+\frac{16a^3b^2\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{3a+b}{(a+b)^2(-1+\sin(c+dx))}+\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{-3a+b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*Tan[c + d*x]^3)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((b*(3*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - ((3*a - b)*b*Log[1 + Sin[c
 + d*x]])/(a - b)^3 + (16*a^3*b^2*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (3*a + b)/((a + b)^2*(-1 + Sin[
c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-3*a + b)/((a - b)^2*(1 +
Sin[c + d*x])))/(16*d)
```

Maple [A]

time = 0.50, size = 176, normalized size = 0.97

method	result
derivativedivides	$\frac{\frac{a^3b^2\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{3a-b}{16(a-b)^2(1+\sin(dx+c))}-\frac{(3a-b)b\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-1)^2}-\cdots}{d}$
default	$\frac{\frac{a^3b^2\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{3a-b}{16(a-b)^2(1+\sin(dx+c))}-\frac{(3a-b)b\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-1)^2}-\cdots}{d}$
norman	$\frac{-\frac{2a\,b^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^4-2a^2b^2+b^4\right)}-\frac{2a\,b^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^4-2a^2b^2+b^4\right)}+\frac{4a^3\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^4-2a^2b^2+b^4\right)}+\frac{b\left(3a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4\left(a^4-2a^2b^2+b^4\right)d}+\frac{b\left(3a^2+b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4\left(a^4-2a^2b^2+b^4\right)d}-\cdots}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{3iabx}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{3iabc}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{ib^2x}{8(a^3-3a^2b+3a\,b^2-b^3)}-\frac{2ia^3b^2x}{a^6-3a^4b^2+3a^2b^4-b^6}-\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/(a+b)^3*b^2/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a-8*b)/(1+sin(d*x+c)
)^2-1/16*(3*a-b)/(a-b)^2/(1+sin(d*x+c))-1/16*(3*a-b)/(a-b)^3*b*ln(1+sin(d*x
+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-3*a-b)/(a+b)^2/(sin(d*x+c)-1)+1/
16*(3*a+b)/(a+b)^3*b*ln(sin(d*x+c)-1))
```

Maxima [A]

time = 0.29, size = 267, normalized size = 1.47

$$\frac{\frac{16\,a^3b^2\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(3\,ab-b^2)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(3\,ab+b^2)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,a^3\sin(dx+c)^2-(5\,a^2b-b^3)\sin(dx+c)^3-2\,a^3-2\,ab^2+(3\,a^2b+b^3)\sin(dx+c)\right)}{(a^4-2\,a^2b^2+b^4)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\sin(dx+c)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*b^2*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) - (3*a*b - b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
3*a*b + b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a
^3*sin(d*x + c)^2 - (5*a^2*b - b^3)*sin(d*x + c)^3 - 2*a^3 - 2*a*b^2 + (3*a
^2*b + b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.47, size = 260, normalized size = 1.43

$$\frac{16 a^3b^2 \cos(dx+c)^4 \log(b\sin(dx+c)+a) - (3a^4b+8a^3b^2+6a^2b^3-b^5)\cos(dx+c)^4\log(\sin(dx+c)+1) + (3a^4b-8a^3b^2+6a^2b^3-b^5)\cos(dx+c)^4\log(-\sin(dx+c)+1) + 4a^5 - 8a^3b^2 + 4ab^4 - 8(a^5-a^3b^2)\cos(dx+c)^2 - 2\left(2a^4b-4a^2b^3+2b^5-(5a^4b-6a^2b^3+b^5)\cos(dx+c)^2\right)\sin(dx+c)}{16(a^6-3a^4b^2+3a^2b^4-b^6)d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*b^2*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (3*a^4*b + 8*a^3*
b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (3*a^4*b - 8*
a^3*b^2 + 6*a^2*b^3 - b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 -
8*a^3*b^2 + 4*a*b^4 - 8*(a^5 - a^3*b^2)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2
*b^3 + 2*b^5 - (5*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^2)*sin(d*x + c))/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.52, size = 326, normalized size = 1.79

$$\frac{\frac{16a^3b^3\log(|b\sin(dx+c)+a|)}{a^6b-3a^4b^3+3a^2b^5-b^7} - \frac{(3ab-b^2)\log(|\sin(dx+c)+1|)}{a^3-3a^2b+3ab^2-b^3} + \frac{(3ab+b^2)\log(|\sin(dx+c)-1|)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(6a^3b^2\sin(dx+c)^4-5a^4b\sin(dx+c)^3+6a^2b^3\sin(dx+c)^3-b^5\sin(dx+c)^3+4a^5\sin(dx+c)^2-16a^3b^2\sin(dx+c)^2+3a^4b\sin(dx+c)-2a^2b^3\sin(dx+c)-b^5\sin(dx+c)-2a^5+6a^3b^2+2ab^4\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*b^3*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^
5 - b^7) - (3*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^
2 - b^3) + (3*a*b + b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^
2 + b^3) + 2*(6*a^3*b^2*sin(d*x + c)^4 - 5*a^4*b*sin(d*x + c)^3 + 6*a^2*b^3
*sin(d*x + c)^3 - b^5*sin(d*x + c)^3 + 4*a^5*sin(d*x + c)^2 - 16*a^3*b^2*si
n(d*x + c)^2 + 3*a^4*b*sin(d*x + c) - 2*a^2*b^3*sin(d*x + c) - b^5*sin(d*x
```

```
+ c) - 2*a^5 + 6*a^3*b^2 + 2*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(s
in(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 12.45, size = 471, normalized size = 2.59

$$\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(3\,a+b\right)}{8\,d\,{\left(a+b\right)}^3}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(11\,a^2\,b-7\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{4\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{a^4-2\,a^2\,b^2+b^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(3\,a^2\,b+b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(11\,a^2\,b-7\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{2\,a\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{a^4-2\,a^2\,b^2+b^4}-\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{3\,b}{8\,{\left(a-b\right)}^2}\right)}{d}+\frac{a^3\,b^2\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2) - 1)*(3*a + b))/(8*d*(a + b)^3) - ((tan(c/2 + (d*
x)/2)^3*(11*a^2*b - 7*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) - (4*a^3*tan(c/2 +
(d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^7*(3*a^2*b + b^3)
)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(11*a^2*b - 7*b^3))/(
4*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*b^2*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*
a^2*b^2) + (2*a*b^2*tan(c/2 + (d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2) - (b*tan(
c/2 + (d*x)/2)*(3*a^2 + b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + (3*b)/(8
*(a - b)^2)))/d + (a^3*b^2*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*
x)/2)^2))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

3.1365 $\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{a(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{a(a-3b)\log(1+\sin(c+dx))}{16(a-b)^3d}-\frac{a^2b^3\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-\frac{\sec^4(c+dx)(b}{4\,(a^2}$$

```
[Out] 1/16*a*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*a*(a-3*b)*ln(1+sin(d*x+c))/(
a-b)^3/d-a^2*b^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(b-a*sin
(d*x+c))/(a^2-b^2)/d+1/8*a*sec(d*x+c)^2*(4*a*b-(a^2+3*b^2)*sin(d*x+c))/(a^2
-b^2)^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2916, 12, 1661, 837, 815}

$$-\frac{\sec^4(c+dx)(b-a\sin(c+dx))}{4d\,(a^2-b^2)}+\frac{a\sec^2(c+dx)\,(4ab-(a^2+3b^2)\sin(c+dx))}{8d\,(a^2-b^2)^2}-\frac{a^2b^3\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}+\frac{a(a+3b)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{a(a-3b)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - (a*(a - 3*b)*Log[1 +
 Sin[c + d*x]])/(16*(a - b)^3*d) - (a^2*b^3*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) - (Sec[c + d*x]^4*(b - a*Sin[c + d*x]))/(4*(a^2 - b^2)*d) + (a*
Sec[c + d*x]^2*(4*a*b - (a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^2}{b^2(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{x^2}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{b\text{Subst}\left(\int \frac{-\frac{a^2b^2}{a^2-b^2}+\frac{3ab^2x}{a^2-b^2}}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c\right.}{4d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-(a^2+3b^2)\sin(\right.}{8\left(a^2-b^2\right)^2d}$$

$$= -\frac{\sec^4(c+dx)\left(\frac{b}{a^2-b^2}-\frac{a\sin(c+dx)}{a^2-b^2}\right)}{4d} + \frac{a\sec^2(c+dx)\left(4ab-(a^2+3b^2)\sin(\right.}{8\left(a^2-b^2\right)^2d}$$

$$= \frac{a(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{a(a-3b)\log(1+\sin(c+dx))}{16(a-b)^3d} - \frac{a^2b^3\log}{}$$

Mathematica [A]

time = 0.82, size = 163, normalized size = 0.92

$$\frac{\frac{a(a+3b)\log(1-\sin(c+dx))}{(a+b)^3}-\frac{a(a-3b)\log(1+\sin(c+dx))}{(a-b)^3}-\frac{16a^2b^3\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3}+\frac{1}{(a+b)(-1+\sin(c+dx))^2}+\frac{a-b}{(a+b)^2(-1+\sin(c+dx))}-\frac{1}{(a-b)(1+\sin(c+dx))^2}+\frac{a+b}{(a-b)^2(1+\sin(c+dx))}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*Tan[c + d*x]^2)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a*(a + 3*b)*Log[1 - Sin[c + d*x]])/(a + b)^3 - (a*(a - 3*b)*Log[1 + Sin[c
 + d*x]])/(a - b)^3 - (16*a^2*b^3*Log[a + b*Sin[c + d*x]])/((a - b)^3*(a +
b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (a - b)/((a + b)^2*(-1 + Sin[c
+ d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (a + b)/((a - b)^2*(1 + Sin[c
 + d*x])))/(16*d)
```

Maple [A]

time = 0.47, size = 173, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{a^2b^3\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}-\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{-a-b}{16(a-b)^2(1+\sin(dx+c))}-\frac{a(a-3b)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-\cdots}}{d}$
default	$\frac{-\frac{a^2b^3\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}-\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{-a-b}{16(a-b)^2(1+\sin(dx+c))}-\frac{a(a-3b)\ln(1+\sin(dx+c))}{16(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-\cdots}}{d}$
norman	$\frac{\frac{2b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)d}+\frac{2b^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)d}-\frac{4a^2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^4-2a^2b^2+b^4\right)}+\frac{a\left(7a^2-3b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4\left(a^4-2a^2b^2+b^4\right)d}+\frac{a\left(7a^2-3b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4\left(a^4-2a^2b^2+b^4\right)d}+\cdots}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{3iabx}{8(a^3-3a^2b+3a\,b^2-b^3)}+\frac{ia^2x}{8a^3-24a^2b+24a\,b^2-8b^3}-\frac{ia^2c}{8(a^3+3a^2b+3a\,b^2+b^3)d}-\frac{3iabc}{8(a^3-3a^2b+3a\,b^2-b^3)d}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*b^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/(8*a-8*b)/(1+sin(d*x+c
))^2-1/16*(-a-b)/(a-b)^2/(1+sin(d*x+c))-1/16*a*(a-3*b)/(a-b)^3*ln(1+sin(d*x
+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(-a+b)/(a+b)^2/(sin(d*x+c)-1)+1/16
*a*(a+3*b)/(a+b)^3*ln(sin(d*x+c)-1))
```

Maxima [A]

time = 0.30, size = 265, normalized size = 1.49

$$-\frac{\frac{16\,a^2b^3\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{(a^2-3\,ab)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(a^2+3\,ab)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,a^2b\sin(dx+c)^2-(a^3+3\,ab^2)\sin(dx+c)^3-2\,a^2b-2\,b^3-(a^3-5\,ab^2)\sin(dx+c)\right)}{(a^4-2\,a^2b^2+b^4)\sin(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,(a^4-2\,a^2b^2+b^4)\sin(dx+c)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*a^2*b^3*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6) + (a^2 - 3*a*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) -
(a^2 + 3*a*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*
a^2*b*sin(d*x + c)^2 - (a^3 + 3*a*b^2)*sin(d*x + c)^3 - 2*a^2*b - 2*b^3 - (
a^3 - 5*a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4
- 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.46, size = 258, normalized size = 1.45

$$-\frac{16\,a^{2}b^{3}\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^{5}-6\,a^{3}b^{2}-8\,a^{2}b^{3}-3\,ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-\left(a^{5}-6\,a^{3}b^{2}+8\,a^{2}b^{3}-3\,ab^{4}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^{4}b-8\,a^{2}b^{3}+4\,b^{5}-8\left(a^{4}b-a^{2}b^{3}\right)\cos\left(dx+c\right)^{2}-2\left(2\,a^{5}-4\,a^{3}b^{2}+2\,ab^{4}-\left(a^{5}+2\,a^{3}b^{2}-3\,ab^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*a^2*b^3*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^5 - 6*a^3*b^2
 - 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^5 - 6*a^3
*b^2 + 8*a^2*b^3 - 3*a*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^4*b
 - 8*a^2*b^3 + 4*b^5 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(2*a^5 - 4*a^
3*b^2 + 2*a*b^4 - (a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))
/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)\sec^{5}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 325, normalized size = 1.83

$$-\frac{\frac{16\,a^{2}b^{4}\log(|b\sin(dx+c)+a|)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}+\frac{(a^{2}-3\,ab)\log(|\sin(dx+c)+1|)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{(a^{2}+3\,ab)\log(|\sin(dx+c)-1|)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(6\,a^{2}b^{3}\sin(dx+c)^{4}-a^{5}\sin(dx+c)^{3}-2\,a^{3}b^{2}\sin(dx+c)^{3}+3\,ab^{4}\sin(dx+c)^{3}+4\,a^{4}b\sin(dx+c)^{2}-16\,a^{2}b^{3}\sin(dx+c)^{2}-a^{5}\sin(dx+c)+6\,a^{3}b^{2}\sin(dx+c)-5\,ab^{4}\sin(dx+c)-2\,a^{4}b+6\,a^{2}b^{3}+2\,b^{5}\right)}{(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6})\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*a^2*b^4*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b
^5 - b^7) + (a^2 - 3*a*b)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3) + 2*(6*a^2*b^3*sin(d*x + c)^4 - a^5*sin(d*x + c)^3 - 2*a^3*b^2*si
n(d*x + c)^3 + 3*a*b^4*sin(d*x + c)^3 + 4*a^4*b*sin(d*x + c)^2 - 16*a^2*b^3
```

```
*sin(d*x + c)^2 - a^5*sin(d*x + c) + 6*a^3*b^2*sin(d*x + c) - 5*a*b^4*sin(d
*x + c) - 2*a^4*b + 6*a^2*b^3 + 2*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]

time = 12.51, size = 498, normalized size = 2.80

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{b^2}{4\,{\left(a-b\right)}^3}+\frac{b}{8\,{\left(a-b\right)}^2}-\frac{1}{8\,\left(a-b\right)}\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{b}{8\,{\left(a+b\right)}^2}+\frac{1}{8\,\left(a+b\right)}-\frac{b^2}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a\,b^2-7\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{a^4-2\,a^2\,b^2+b^4}-\frac{2\,b^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{a^4-2\,a^2\,b^2+b^4}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(5\,a\,b^2-a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(3\,a\,b^2-7\,a^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{4\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4}{a^4-2\,a^2\,b^2+b^4}-\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-5\,b^2\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{a^2\,b^3\,\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*(b^2/(4*(a - b)^3) + b/(8*(a - b)^2) - 1/(8*(a
 - b))))/d + (log(tan(c/2 + (d*x)/2) - 1)*(b/(8*(a + b)^2) + 1/(8*(a + b))
- b^2/(4*(a + b)^3)))/d - ((tan(c/2 + (d*x)/2)^3*(3*a*b^2 - 7*a^3))/(4*(a^4
 + b^4 - 2*a^2*b^2)) - (2*b^3*tan(c/2 + (d*x)/2)^6)/(a^4 + b^4 - 2*a^2*b^2)
 - (2*b^3*tan(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/
2)^7*(5*a*b^2 - a^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(
3*a*b^2 - 7*a^3))/(4*(a^4 + b^4 - 2*a^2*b^2)) + (4*a^2*b*tan(c/2 + (d*x)/2)
^4)/(a^4 + b^4 - 2*a^2*b^2) - (a*tan(c/2 + (d*x)/2)*(a^2 - 5*b^2))/(4*(a^4
+ b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (a^2*b^3*log(a + 2*b*
tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2))
```

3.1366 $\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{b(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3 d}+\frac{(a-3b)b\log(1+\sin(c+dx))}{16(a-b)^3 d}+\frac{ab^4\log(a+b\sin(c+dx))}{(a^2-b^2)^3 d}+\frac{\sec^4(c+dx)(a$$

```
[Out] -1/16*b*(a+3*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(a-3*b)*b*ln(1+sin(d*x+c))/
(a-b)^3/d+a*b^4*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*sec(d*x+c)^4*(a-b*sin(
d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*b^2-b*(a^2+3*b^2)*sin(d*x+c))/(a^
2-b^2)^2/d
```

Rubi [A]
time = 0.17, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2916, 12, 837, 815}

$$\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4d(a^2-b^2)}-\frac{\sec^2(c+dx)\left(4ab^2-b(a^2+3b^2)\sin(c+dx)\right)}{8d(a^2-b^2)^2}+\frac{ab^4\log(a+b\sin(c+dx))}{d(a^2-b^2)^3}-\frac{b(a+3b)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{b(a-3b)\log(\sin(c+dx)+1)}{16d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*(b*(a + 3*b)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((a - 3*b)*b*Log[
1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a*b^4*Log[a + b*Sin[c + d*x]])/((a^2
 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*d) - (S
ec[c + d*x]^2*(4*a*b^2 - b*(a^2 + 3*b^2)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
```

```
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{x}{b(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{x}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{b^2\text{Subst}\left(\int \frac{-ab^2+3b^2x}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+\right.}{4(a^2-b^2)d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)\left(4ab^2-b(a^2+3b^2)\sin(c+\right.}{8(a^2-b^2)^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4(a^2-b^2)d} - \frac{\sec^2(c+dx)\left(4ab^2-b(a^2+3b^2)\sin(c+\right.}{8(a^2-b^2)^2d}$$

$$= -\frac{b(a+3b)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{(a-3b)b\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{ab^4\,\text{lo}}{}$$

Mathematica [A]
time = 0.63, size = 244, normalized size = 1.38

$$\frac{-\frac{2b(a+3b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3}+\frac{2(a-3b)b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3}+\frac{16ab^4\log(a+b\sin(c+dx))}{(a^2-b^2)^3}+\frac{1}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{a+3b}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{1}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{a-3b}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*Tan[c + d*x])/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*b*(a + 3*b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^3 + (2*(
a - 3*b)*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*a*b^4*
Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 + 1/((a + b)*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4) + (a + 3*b)/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
```

```
2])^2) + 1/((a - b)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4) + (a - 3*b)/((
a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

Maple [A]
time = 0.48, size = 170, normalized size = 0.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*a/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a-8*b)/(1+sin(d*x+c))^
2-1/16*(-a+3*b)/(a-b)^2/(1+sin(d*x+c))+1/16*(a-3*b)/(a-b)^3*b*ln(1+sin(d*x+
c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(a+3*b)/(a+b)^2/(sin(d*x+c)-1)-1/16
*(a+3*b)/(a+b)^3*b*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 267, normalized size = 1.51

$$\frac{\frac{16\,ab^4\log\left(b\sin\left(dx+c\right)+a\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{\left(ab-3\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(ab+3\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,\left(4\,ab^2\sin\left(dx+c\right)^2-\left(a^2b+3\,b^3\right)\sin\left(dx+c\right)^3+2\,a^3-6\,ab^2-\left(a^2b-5\,b^3\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\,\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a*b^4*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
+ (a*b - 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (a*
b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b
^2*sin(d*x + c)^2 - (a^2*b + 3*b^3)*sin(d*x + c)^3 + 2*a^3 - 6*a*b^2 - (a^2
*b - 5*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.49, size = 255, normalized size = 1.44

$$\frac{16\,ab^4\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^4b-6\,a^2b^3-8\,ab^4-3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(a^4b-6\,a^2b^3+8\,ab^4-3\,b^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^5-8\,a^3b^2+4\,ab^4-8\,\left(a^3b^2-ab^4\right)\cos\left(dx+c\right)^2-2\,\left(2\,a^4b-4\,a^2b^3+2\,b^5-\left(a^4b+2\,a^2b^3-3\,b^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b^4*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (a^4*b - 6*a^2*b^3
- 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^4*b - 6*a^2*b^
3 + 8*a*b^4 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^3*
b^2 + 4*a*b^4 - 8*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b - 4*a^2*b^3
 + 2*b^5 - (a^4*b + 2*a^2*b^3 - 3*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)\sec^5\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 323, normalized size = 1.82

$$\frac{\frac{16\,ab^{5}\log(|b\sin(dx+c)+a|)}{a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}}+\frac{(ab-3\,b^{2})\log(|\sin(dx+c)+1|)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{(ab+3\,b^{2})\log(|\sin(dx+c)-1|)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\left(6\,ab^{4}\sin(dx+c)^{4}-a^{4}b\sin(dx+c)^{3}-2\,a^{2}b^{3}\sin(dx+c)^{3}+3\,b^{5}\sin(dx+c)^{3}+4\,a^{3}b^{2}\sin(dx+c)^{2}-16\,ab^{4}\sin(dx+c)^{2}-a^{4}b\sin(dx+c)+6\,a^{2}b^{3}\sin(dx+c)-5\,b^{5}\sin(dx+c)+2\,a^{5}-8\,a^{3}b^{2}+12\,ab^{4}\right)}{(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6})\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a*b^5*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) + (a*b - 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) - (a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 2*(6*a*b^4*sin(d*x + c)^4 - a^4*b*sin(d*x + c)^3 - 2*a^2*b^3*sin(d
*x + c)^3 + 3*b^5*sin(d*x + c)^3 + 4*a^3*b^2*sin(d*x + c)^2 - 16*a*b^4*sin(
d*x + c)^2 - a^4*b*sin(d*x + c) + 6*a^2*b^3*sin(d*x + c) - 5*b^5*sin(d*x +
c) + 2*a^5 - 8*a^3*b^2 + 12*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 12.40, size = 483, normalized size = 2.73

$$\frac{a\,b^{4}\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^{6}-3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}-b^{6}\right)}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}(2\,a\,b^{2}-a^{3})}{a^{4}-2\,a^{2}\,b^{2}+b^{4}}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}(2\,a\,b^{2}-a^{3})}{a^{4}-2\,a^{2}\,b^{2}+b^{4}}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}(a^{2}\,b-5\,b^{3})}{4\,(a^{4}-2\,a^{2}\,b^{2}+b^{4})}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}(7\,a^{2}\,b-3\,b^{3})}{4\,(a^{4}-2\,a^{2}\,b^{2}+b^{4})}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}(7\,a^{2}\,b-3\,b^{3})}{4\,(a^{4}-2\,a^{2}\,b^{2}+b^{4})}-\frac{4\,a\,b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}}{a^{4}-2\,a^{2}\,b^{2}+b^{4}}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a^{2}-5\,b^{2})}{4\,(a^{4}-2\,a^{2}\,b^{2}+b^{4})}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{b}{8\,(a+b)^{2}}+\frac{b^{2}}{4\,(a+b)^{3}}\right)}{d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-3\,b\right)}{8\,d\,(a-b)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] (a*b^4*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - ((2*tan(c/2 + (d*x)/2)^2*(2*a*b^2 - a^3))/(
a^4 + b^4 - 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)^6*(2*a*b^2 - a^3))/(a^4 + b^
4 - 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^7*(a^2*b - 5*b^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b
^2)) + (tan(c/2 + (d*x)/2)^5*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*a^2*b^2))
 - (4*a*b^2*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2) + (b*tan(c/2 + (d
*x)/2)*(a^2 - 5*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2)))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) - (log(tan(c/2 + (d*x)/2) - 1)*(b/(8*(a + b)^2) + b^2/(4*(a + b)^3))
)/d + (b*log(tan(c/2 + (d*x)/2) + 1)*(a - 3*b))/(8*d*(a - b)^3)
```

3.1367 $\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=233

$$-\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{\log(\sin(c+dx))}{ad}-\frac{(8a^2-21ab+15b^2)\log(1+\sin(c+dx))}{16(a-b)^3d}+\frac{b^6}{}$$

```
[Out] -1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d+ln(sin(d*x+c))/a/d-1
/16*(8*a^2-21*a*b+15*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+b^6*ln(a+b*sin(d*x+c))
/a/(a^2-b^2)^3/d+1/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*(5*a+7*b)/(a+b)^2/d/(1-
sin(d*x+c))+1/16/(a-b)/d/(1+sin(d*x+c))^2+1/16*(5*a-7*b)/(a-b)^2/d/(1+sin(d
*x+c))
```

Rubi [A]
time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 12, 908}

$$-\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(8a^2-21ab+15b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{b^6\log(a+b\sin(c+dx))}{ad\,(a^2-b^2)^3}+\frac{5a+7b}{16d(a+b)^2(1-\sin(c+dx))}+\frac{5a-7b}{16d(a-b)^2(\sin(c+dx)+1)}+\frac{1}{16d(a+b)(1-\sin(c+dx))^2}+\frac{1}{16d(a-b)(\sin(c+dx)+1)^2}+\frac{\log(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + Log
[Sin[c + d*x]]/(a*d) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c + d*x]])/(1
6*(a - b)^3*d) + (b^6*Log[a + b*Sin[c + d*x]])/(a*(a^2 - b^2)^3*d) + 1/(16*
(a + b)*d*(1 - Sin[c + d*x])^2) + (5*a + 7*b)/(16*(a + b)^2*d*(1 - Sin[c +
d*x])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/(16*(a - b)^2*
d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{b}{x(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \frac{1}{x(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3} + \frac{5a+7b}{16b^5(a+b)^2(b-x)^2} + \frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)} + \frac{1}{ab^6x} + \frac{}{a(a-b)^3}\right.\right.}{}$$

$$= -\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{\log(\sin(c+dx))}{ad} - \frac{(8a^2-2}{}$$

Mathematica [A]
time = 1.82, size = 220, normalized size = 0.94

$$\frac{b^6\left(-\frac{(8a^2+21ab+15b^2)\log(1-\sin(c+dx))}{b^6(a+b)^3} + \frac{16\log(\sin(c+dx))}{ab^6} - \frac{(8a^2-21ab+15b^2)\log(1+\sin(c+dx))}{(a-b)^3b^6} + \frac{16\log(a+b\sin(c+dx))}{a(a-b)^3(a+b)^3} + \frac{1}{b^6(a+b)(-1+\sin(c+dx))^2} + \frac{-5a-7b}{b^6(a+b)^2(-1+\sin(c+dx))} + \frac{1}{(a-b)b^6(1+\sin(c+dx))^2} + \frac{5a-7b}{(a-b)^2b^6(1+\sin(c+dx))}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^6*(-(((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/(b^6*(a + b)^3))
+ (16*Log[Sin[c + d*x]])/(a*b^6) - ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sin[c
 + d*x]])/((a - b)^3*b^6) + (16*Log[a + b*Sin[c + d*x]])/(a*(a - b)^3*(a +
b)^3) + 1/(b^6*(a + b)*(-1 + Sin[c + d*x])^2) + (-5*a - 7*b)/(b^6*(a + b)^2
*(-1 + Sin[c + d*x])) + 1/((a - b)*b^6*(1 + Sin[c + d*x])^2) + (5*a - 7*b)/
((a - b)^2*b^6*(1 + Sin[c + d*x]))))/(16*d)
```

Maple [A]
time = 0.69, size = 203, normalized size = 0.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(sin(d*x+c))+1/a*b^6/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a
-8*b)/(1+sin(d*x+c))^2-1/16*(-5*a+7*b)/(a-b)^2/(1+sin(d*x+c))+1/16/(a-b)^3*
(-8*a^2+21*a*b-15*b^2)*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16
*(5*a+7*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-8*a^2-21*a*b-15*b^2)*ln(si
n(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 299, normalized size = 1.28

$$\frac{\frac{16\,b^6\log\left(b\sin\left(dx+c\right)+a\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6}-\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(\left(3\,a^2b-7\,b^3\right)\sin\left(dx+c\right)^3+6\,a^3-10\,ab^2-4\left(a^3-2\,ab^2\right)\sin\left(dx+c\right)^2-\left(5\,a^2b-9\,b^3\right)\sin\left(dx+c\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)\sin\left(dx+c\right)^2}+\frac{16\,\log\left(\sin\left(dx+c\right)\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^6*log(b*sin(d*x + c) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
- (8*a^2 - 21*a*b + 15*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2
- b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3
*a*b^2 + b^3) + 2*((3*a^2*b - 7*b^3)*sin(d*x + c)^3 + 6*a^3 - 10*a*b^2 - 4*
(a^3 - 2*a*b^2)*sin(d*x + c)^2 - (5*a^2*b - 9*b^3)*sin(d*x + c))/((a^4 - 2*
a^2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
+ b^4)*sin(d*x + c)^2) + 16*log(sin(d*x + c))/a)/d
```

Fricas [A]
time = 1.30, size = 344, normalized size = 1.48

$$\frac{16\,b^6\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)+4\,a^6-8\,a^4b^2+4\,a^2b^4+16\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\cos\left(dx+c\right)^4\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(8\,a^6+3\,a^5b-24\,a^4b^2-10\,a^3b^3+24\,a^2b^4+15\,ab^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^6-3\,a^5b-24\,a^4b^2+10\,a^3b^3+24\,a^2b^4-15\,ab^5\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+8\left(a^6-3\,a^4b^2+2\,a^2b^4\right)\cos\left(dx+c\right)^2-2\left(2\,a^5b-4\,a^3b^3+2\,ab^5+\left(3\,a^5b-10\,a^3b^3+7\,ab^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^6*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + 4*a^6 - 8*a^4*b^2 + 4
*a^2*b^4 + 16*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c)^4*log(-1/2*s
in(d*x + c)) - (8*a^6 + 3*a^5*b - 24*a^4*b^2 - 10*a^3*b^3 + 24*a^2*b^4 + 15
*a*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^6 - 3*a^5*b - 24*a^4*b^
2 + 10*a^3*b^3 + 24*a^2*b^4 - 15*a*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 8*(a^6 - 3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)^2 - 2*(2*a^5*b - 4*a^3*b^
3 + 2*a*b^5 + (3*a^5*b - 10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2)*sin(d*x + c)
)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [A]
time = 0.48, size = 391, normalized size = 1.68

$$\frac{\frac{16\,b^7\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^7b-3\,a^5b^3+3\,a^3b^5-ab^7}-\frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,\log\left(|\sin\left(dx+c\right)|\right)}{a}+\frac{2\left(6\,a^5\sin\left(dx+c\right)^4-18\,a^3b^2\sin\left(dx+c\right)^4+18\,ab^4\sin\left(dx+c\right)^4+3\,a^4b\sin\left(dx+c\right)^3-10\,a^2b^3\sin\left(dx+c\right)^3+7\,b^5\sin\left(dx+c\right)^3-16\,a^5\sin\left(dx+c\right)^2+48\,a^3b^2\sin\left(dx+c\right)^2-44\,ab^4\sin\left(dx+c\right)^2-5\,a^4b\sin\left(dx+c\right)+14\,a^2b^3\sin\left(dx+c\right)-9\,b^5\sin\left(dx+c\right)+12\,a^5-34\,a^3b^2+28\,ab^4\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^7*log(abs(b*sin(d*x + c) + a))/(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7) - (8*a^2 - 21*a*b + 15*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - (8*a^2 + 21*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*log(abs(sin(d*x + c)))/a + 2*(6*a^5*sin
(d*x + c)^4 - 18*a^3*b^2*sin(d*x + c)^4 + 18*a*b^4*sin(d*x + c)^4 + 3*a^4*b
*sin(d*x + c)^3 - 10*a^2*b^3*sin(d*x + c)^3 + 7*b^5*sin(d*x + c)^3 - 16*a^5
*sin(d*x + c)^2 + 48*a^3*b^2*sin(d*x + c)^2 - 44*a*b^4*sin(d*x + c)^2 - 5*a
^4*b*sin(d*x + c) + 14*a^2*b^3*sin(d*x + c) - 9*b^5*sin(d*x + c) + 12*a^5 -
 34*a^3*b^2 + 28*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^
2 - 1)^2))/d
```

Mupad [B]
time = 12.43, size = 346, normalized size = 1.48

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)}{a\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{5\,b}{16\,{\left(a+b\right)}^2}+\frac{1}{2\,\left(a+b\right)}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}-\frac{5\,b}{16\,{\left(a-b\right)}^2}+\frac{1}{2\,\left(a-b\right)}\right)}{d}-\frac{\frac{5\,a\,b^2-3\,a^3}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(2\,a\,b^2-a^3\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^3\,\left(3\,a^2\,b-7\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\,\sin\left(c+d\,x\right)\,\left(5\,a^2-9\,b^2\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2+{\sin\left(c+d\,x\right)}^4-{\sin\left(c+d\,x\right)}^2\right)}-\frac{b^6\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(-a^7+3\,a^5\,b^2-3\,a^3\,b^4+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] log(sin(c + d*x))/(a*d) - (log(sin(c + d*x) - 1)*((5*b)/(16*(a + b)^2) + 1/
(2*(a + b)) + b^2/(8*(a + b)^3)))/d - (log(sin(c + d*x) + 1)*(b^2/(8*(a - b
)^3) - (5*b)/(16*(a - b)^2) + 1/(2*(a - b))))/d - ((5*a*b^2 - 3*a^3)/(4*(a^
4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^2*(2*a*b^2 - a^3))/(2*(a^4 + b^4 - 2*
a^2*b^2)) - (sin(c + d*x)^3*(3*a^2*b - 7*b^3))/(8*(a^4 + b^4 - 2*a^2*b^2))
+ (b*sin(c + d*x)*(5*a^2 - 9*b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c +
 d*x)^2 - sin(c + d*x)^2 + sin(c + d*x)^4)) - (b^6*log(a + b*sin(c + d*x)))
/(d*(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2))
```

3.1368 $\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=250

$$-\frac{\csc(c+dx)}{ad}-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{(15a^2-37ab+24b^2)\log(1+}{16(a-b)^3d}$$

```
[Out] -csc(d*x+c)/a/d-1/16*(15*a^2+37*a*b+24*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-b*ln
(sin(d*x+c))/a^2/d+1/16*(15*a^2-37*a*b+24*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d-b
^7*ln(a+b*sin(d*x+c))/a^2/(a^2-b^2)^3/d+1/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*
(7*a+9*b)/(a+b)^2/d/(1-sin(d*x+c))-1/16/(a-b)/d/(1+sin(d*x+c))^2+1/16*(-7*a
+9*b)/(a-b)^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(15a^2-37ab+24b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{b^7\log(a+b\sin(c+dx))}{a^2d\left(a^2-b^2\right)^3}-\frac{b\log(\sin(c+dx))}{a^2d}+\frac{7a+9b}{16d(a+b)^2(1-\sin(c+dx))}-\frac{7a-9b}{16d(a-b)^2(\sin(c+dx)+1)}+\frac{1}{16d(a+b)(1-\sin(c+dx))^2}-\frac{1}{16d(a-b)(\sin(c+dx)+1)^2}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]/(a*d)) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[c + d*x]])/
(16*(a + b)^3*d) - (b*Log[Sin[c + d*x]])/(a^2*d) + ((15*a^2 - 37*a*b + 24*b
^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) - (b^7*Log[a + b*Sin[c + d*x]])
/(a^2*(a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(1 - Sin[c + d*x])^2) + (7*a + 9*b
)/(16*(a + b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^
2) - (7*a - 9*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx &= \frac{b^5\text{Subst}\left(\int \frac{b^2}{x^2(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b^7\text{Subst}\left(\int \frac{1}{x^2(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}\\ &= \frac{b^7\text{Subst}\left(\int \left(\frac{1}{8b^5(a+b)(b-x)^3}+\frac{7a+9b}{16b^6(a+b)^2(b-x)^2}+\frac{15a^2+37ab+24b^2}{16b^7(a+b)^3(b-x)}+\frac{1}{ab^6x^2}-\frac{1}{a^2b^6x}\right.\right.}{}\\ &= -\frac{\csc(c+dx)}{ad}-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}-\frac{b\log(\sin(c}{a^2d}\end{aligned}$$

Mathematica [A]
time = 4.31, size = 234, normalized size = 0.94

$$\frac{b^7\left(-\frac{16\csc(c+dx)}{ab^7}-\frac{(15a^2+37ab+24b^2)\log(1-\sin(c+dx))}{b^7(a+b)^3}-\frac{16\log(\sin(c+dx))}{a^2b^6}+\frac{(15a^2-37ab+24b^2)\log(1+\sin(c+dx))}{(a-b)^3b^7}-\frac{16\log(a+b\sin(c+dx))}{a^2(a-b)^3(a+b)^3}+\frac{1}{b^7(a+b)(-1+\sin(c+dx))^2}+\frac{-7a-9b}{b^7(a+b)^2(-1+\sin(c+dx))}-\frac{1}{(a-b)b^7(1+\sin(c+dx))^2}+\frac{-7a+9b}{(a-b)^2b^7(1+\sin(c+dx))}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^7*((-16*Csc[c + d*x])/(a*b^7) - ((15*a^2 + 37*a*b + 24*b^2)*Log[1 - Sin[
c + d*x]])/(b^7*(a + b)^3) - (16*Log[Sin[c + d*x]])/(a^2*b^6) + ((15*a^2 -
37*a*b + 24*b^2)*Log[1 + Sin[c + d*x]])/((a - b)^3*b^7) - (16*Log[a + b*Sin
[c + d*x]])/(a^2*(a - b)^3*(a + b)^3) + 1/(b^7*(a + b)*(-1 + Sin[c + d*x])^
2) + (-7*a - 9*b)/(b^7*(a + b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*b^7*(1 +
 Sin[c + d*x])^2) + (-7*a + 9*b)/((a - b)^2*b^7*(1 + Sin[c + d*x]))))/(16*d
)
```

Maple [A]
time = 0.69, size = 219, normalized size = 0.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/sin(d*x+c)-1/a^2*b*ln(sin(d*x+c))-b^7/a^2/(a+b)^3/(a-b)^3*ln(a+b*
sin(d*x+c))-1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(7*a-9*b)/(a-b)^2/(1+sin(d*
x+c))+1/16*(15*a^2-37*a*b+24*b^2)/(a-b)^3*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(s
```

```
in(d*x+c)-1)^2-1/16*(7*a+9*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-15*a^2-
37*a*b-24*b^2)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.30, size = 361, normalized size = 1.44

$$-\frac{\frac{16\,b^{7}\log\left(b\sin\left(dx+c\right)+a\right)}{a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}}-\frac{\left(15\,a^{2}-37\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(15\,a^{2}+37\,ab+24\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}+\frac{2\,\left(\left(15\,a^{4}-27\,a^{2}b^{2}+8\,b^{4}\right)\sin\left(dx+c\right)^{4}+8\,a^{4}-16\,a^{2}b^{2}+8\,b^{4}-4\,\left(a^{3}b-2\,ab^{3}\right)\sin\left(dx+c\right)^{3}-\left(25\,a^{4}-45\,a^{2}b^{2}+16\,b^{4}\right)\sin\left(dx+c\right)^{2}+2\,\left(3\,a^{3}b-5\,ab^{3}\right)\sin\left(dx+c\right)\right)}{\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)^{5}-2\,\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)^{3}+\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)}+\frac{16\,b\log\left(\sin\left(dx+c\right)\right)}{a^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(16*b^7*log(b*sin(d*x + c) + a)/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^
6) - (15*a^2 - 37*a*b + 24*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) + 2*((15*a^4 - 27*a^2*b^2 + 8*b^4)*sin(d*x + c)^4 + 8*a^
4 - 16*a^2*b^2 + 8*b^4 - 4*(a^3*b - 2*a*b^3)*sin(d*x + c)^3 - (25*a^4 - 45*
a^2*b^2 + 16*b^4)*sin(d*x + c)^2 + 2*(3*a^3*b - 5*a*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^5 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x
 + c)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)) + 16*b*log(sin(d*x + c))/
a^2)/d
```

Fricas [A]
time = 1.76, size = 425, normalized size = 1.70

$$-\frac{16\,b^{7}\cos\left(dx+c\right)^{4}\log\left(b\sin\left(dx+c\right)+a\right)\sin\left(dx+c\right)-4\,a^{7}+8\,a^{5}b^{2}-4\,a^{3}b^{4}+16\,\left(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7}\right)\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-\left(15\,a^{7}+8\,a^{6}b-42\,a^{5}b^{2}-24\,a^{4}b^{3}+35\,a^{3}b^{4}+24\,a^{2}b^{5}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+\left(15\,a^{7}-8\,a^{6}b-42\,a^{5}b^{2}+24\,a^{4}b^{3}+35\,a^{3}b^{4}-24\,a^{2}b^{5}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+2\,\left(15\,a^{7}-42\,a^{5}b^{2}+35\,a^{3}b^{4}-8\,ab^{6}\right)\cos\left(dx+c\right)^{4}-2\,\left(5\,a^{7}-14\,a^{5}b^{2}+9\,a^{3}b^{4}\right)\cos\left(dx+c\right)^{2}+4\,\left(a^{6}b-2\,a^{4}b^{3}+a^{2}b^{5}+2\,\left(a^{6}b-3\,a^{4}b^{3}+2\,a^{2}b^{5}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,\left(a^{8}-3\,a^{6}b^{2}+3\,a^{4}b^{4}-a^{2}b^{6}\right)d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(16*b^7*cos(d*x + c)^4*log(b*sin(d*x + c) + a)*sin(d*x + c) - 4*a^7 +
 8*a^5*b^2 - 4*a^3*b^4 + 16*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x +
 c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) - (15*a^7 + 8*a^6*b - 42*a^5*b^2 -
 24*a^4*b^3 + 35*a^3*b^4 + 24*a^2*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
*sin(d*x + c) + (15*a^7 - 8*a^6*b - 42*a^5*b^2 + 24*a^4*b^3 + 35*a^3*b^4 -
24*a^2*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(15*a^7
- 42*a^5*b^2 + 35*a^3*b^4 - 8*a*b^6)*cos(d*x + c)^4 - 2*(5*a^7 - 14*a^5*b^2
 + 9*a^3*b^4)*cos(d*x + c)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5 + 2*(a^6*b -
3*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^4*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [A]
time = 0.48, size = 418, normalized size = 1.67

$$-\frac{\frac{16\,b^8\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^8b-3\,a^6b^3+3\,a^4b^5-a^2b^7}-\frac{\left(15\,a^2-37\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(15\,a^2+37\,ab+24\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,b\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^2}+\frac{2\left(6\,a^4b\sin\left(dx+c\right)^4-18\,a^2b^3\sin\left(dx+c\right)^4+18\,b^5\sin\left(dx+c\right)^4+7\,a^5\sin\left(dx+c\right)^3-18\,a^3b^2\sin\left(dx+c\right)^3+11\,ab^4\sin\left(dx+c\right)^3-16\,a^4b\sin\left(dx+c\right)^2+48\,a^2b^3\sin\left(dx+c\right)^2-44\,b^5\sin\left(dx+c\right)^2-9\,a^5\sin\left(dx+c\right)+22\,a^3b^2\sin\left(dx+c\right)-13\,ab^4\sin\left(dx+c\right)+12\,a^4b-34\,a^2b^3+28\,b^5\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(\sin\left(dx+c\right)^2-1\right)^2}-\frac{16\left(b\sin\left(dx+c\right)-a\right)}{a^2\sin\left(dx+c\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(16*b^8*log(abs(b*sin(d*x + c) + a))/(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 -
 a^2*b^7) - (15*a^2 - 37*a*b + 24*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*
a^2*b + 3*a*b^2 - b^3) + (15*a^2 + 37*a*b + 24*b^2)*log(abs(sin(d*x + c) -
1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*b*log(abs(sin(d*x + c)))/a^2 + 2*(
6*a^4*b*sin(d*x + c)^4 - 18*a^2*b^3*sin(d*x + c)^4 + 18*b^5*sin(d*x + c)^4
+ 7*a^5*sin(d*x + c)^3 - 18*a^3*b^2*sin(d*x + c)^3 + 11*a*b^4*sin(d*x + c)^
3 - 16*a^4*b*sin(d*x + c)^2 + 48*a^2*b^3*sin(d*x + c)^2 - 44*b^5*sin(d*x +
c)^2 - 9*a^5*sin(d*x + c) + 22*a^3*b^2*sin(d*x + c) - 13*a*b^4*sin(d*x + c)
 + 12*a^4*b - 34*a^2*b^3 + 28*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(si
n(d*x + c)^2 - 1)^2) - 16*(b*sin(d*x + c) - a)/(a^2*sin(d*x + c)))/d
```

Mupad [B]
time = 12.47, size = 373, normalized size = 1.49

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}-\frac{7\,b}{16\,{\left(a-b\right)}^2}+\frac{15}{16\,\left(a-b\right)}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{7\,b}{16\,{\left(a+b\right)}^2}+\frac{15}{16\,\left(a+b\right)}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{1}{a}-\frac{{\sin\left(c+d\,x\right)}^3\,\left(a^2\,b-2\,b^3\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\sin\left(c+d\,x\right)\,\left(3\,a^2\,b-5\,b^3\right)}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\sin\left(c+d\,x\right)}^4\,\left(15\,a^4-27\,a^2\,b^2+8\,b^4\right)}{8\,a\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(25\,a^4-45\,a^2\,b^2+16\,b^4\right)}{8\,a\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\sin\left(c+d\,x\right)}^5-2\,{\sin\left(c+d\,x\right)}^3+\sin\left(c+d\,x\right)\right)}-\frac{b\,\ln\left(\sin\left(c+d\,x\right)\right)}{a^2\,d}-\frac{b^7\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)^2*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(b^2/(8*(a - b)^3) - (7*b)/(16*(a - b)^2) + 15/(16*(
a - b))))/d - (log(sin(c + d*x) - 1)*((7*b)/(16*(a + b)^2) + 15/(16*(a + b)
) + b^2/(8*(a + b)^3)))/d - (1/a - (sin(c + d*x)^3*(a^2*b - 2*b^3))/(2*(a^4
 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)*(3*a^2*b - 5*b^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) + (sin(c + d*x)^4*(15*a^4 + 8*b^4 - 27*a^2*b^2))/(8*a*(a^4 + b^4 -
 2*a^2*b^2)) - (sin(c + d*x)^2*(25*a^4 + 16*b^4 - 45*a^2*b^2))/(8*a*(a^4 +
b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x) - 2*sin(c + d*x)^3 + sin(c + d*x)^5)) -
 (b*log(sin(c + d*x)))/(a^2*d) - (b^7*log(a + b*sin(c + d*x)))/(d*(a^8 - a^
2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))
```

3.1369 $\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=274

$$\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{a^3d}-\frac{(24a^2$$

```
[Out] b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-1/16*(24*a^2+57*a*b+35*b^2)*ln(1-si
n(d*x+c))/(a+b)^3/d+(3*a^2+b^2)*ln(sin(d*x+c))/a^3/d-1/16*(24*a^2-57*a*b+35
*b^2)*ln(1+sin(d*x+c))/(a-b)^3/d+b^8*ln(a+b*sin(d*x+c))/a^3/(a^2-b^2)^3/d+1
/16/(a+b)/d/(1-sin(d*x+c))^2+1/16*(9*a+11*b)/(a+b)^2/d/(1-sin(d*x+c))+1/16/
(a-b)/d/(1+sin(d*x+c))^2+1/16*(9*a-11*b)/(a-b)^2/d/(1+sin(d*x+c))
```

Rubi [A]
time = 0.32, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 12, 908}

$$-\frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{(24a^2-57ab+35b^2)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{b\csc(c+dx)}{a^2d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{a^3d}+\frac{b^8\log(a+b\sin(c+dx))}{a^3d\,(a^2-b^2)^3}+\frac{9a+11b}{16d(a+b)^2(1-\sin(c+dx))}+\frac{9a-11b}{16d(a-b)^2(\sin(c+dx)+1)}+\frac{1}{16d(a+b)(1-\sin(c+dx))^2}+\frac{1}{16d(a-b)(\sin(c+dx)+1)^2}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((24*a^2 + 57*a*b + 35*
b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) + ((3*a^2 + b^2)*Log[Sin[c + d
*x]])/(a^3*d) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*(a -
 b)^3*d) + (b^8*Log[a + b*Sin[c + d*x]])/(a^3*(a^2 - b^2)^3*d) + 1/(16*(a +
 b)*d*(1 - Sin[c + d*x])^2) + (9*a + 11*b)/(16*(a + b)^2*d*(1 - Sin[c + d*x
])) + 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) + (9*a - 11*b)/(16*(a - b)^2*d*
(1 + Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)\sec^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{b^3}{x^3(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\text{Subst}\left(\int \frac{1}{x^3(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\text{Subst}\left(\int \left(\frac{1}{8b^6(a+b)(b-x)^3} + \frac{9a+11b}{16b^7(a+b)^2(b-x)^2} + \frac{24a^2+57ab+35b^2}{16b^8(a+b)^3(b-x)} + \frac{1}{ab^6x^3} - \frac{1}{a^2b^6 x}\right.\right.}{}$$

$$= \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx)}{16(a+b)^3d}$$

Mathematica [A]

time = 6.09, size = 259, normalized size = 0.95

$$\frac{b^8\left(\frac{16\csc(c+dx)}{a^2b^7} - \frac{8\csc^2(c+dx)}{ab^8} - \frac{(24a^2+57ab+35b^2)\log(1-\sin(c+dx))}{b^8(a+b)^3} + \frac{16(3a^2+b^2)\log(\sin(c+dx))}{a^3b^8} - \frac{(24a^2-57ab+35b^2)\log(1+\sin(c+dx))}{(a-b)^3b^8} + \frac{16\log(a+b\sin(c+dx))}{a^3(a-b)^3(a+b)^3} + \frac{1}{b^8(a+b)(-1+\sin(c+dx))^2} + \frac{-9a-11b}{b^8(a+b)^2(-1+\sin(c+dx))} + \frac{1}{(a-b)b^8(1+\sin(c+dx))^2} + \frac{9a-11b}{(a-b)^2b^8(1+\sin(c+dx))}\right)}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^5)/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b^8*((16*Csc[c + d*x])/(a^2*b^7) - (8*Csc[c + d*x]^2)/(a*b^8) - ((24*a^2 +
 57*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/(b^8*(a + b)^3) + (16*(3*a^2 + b^2
)*Log[Sin[c + d*x]])/(a^3*b^8) - ((24*a^2 - 57*a*b + 35*b^2)*Log[1 + Sin[c
+ d*x]])/((a - b)^3*b^8) + (16*Log[a + b*Sin[c + d*x]])/(a^3*(a - b)^3*(a +
 b)^3) + 1/(b^8*(a + b)*(-1 + Sin[c + d*x])^2) + (-9*a - 11*b)/(b^8*(a + b)
^2*(-1 + Sin[c + d*x])) + 1/((a - b)*b^8*(1 + Sin[c + d*x])^2) + (9*a - 11*
b)/((a - b)^2*b^8*(1 + Sin[c + d*x]))))/(16*d)
```

Maple [A]

time = 0.94, size = 238, normalized size = 0.87

method	result
derivativedivides	$-\frac{1}{2a\sin(dx+c)^2}+\frac{(3a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{b^8\ln(a+b\sin(dx+c))}{a^3(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{-9a+11b}{16(a-b)^2(1+\sin(dx+}$

default	$-\frac{1}{2a\sin(dx+c)^2}+\frac{(3a^2+b^2)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}+\frac{b^8\ln(a+b\sin(dx+c))}{a^3(a+b)^3(a-b)^3}+\frac{1}{2(8a-8b)(1+\sin(dx+c))^2}-\frac{-9a+11b}{16(a-b)^2(1+\sin(dx+c}$
norman	$-\frac{1}{8ad}-\frac{\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2d}+\frac{b\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2d}+\frac{2(-5a^4+8a^2b^2-b^4)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad(a^4-2a^2b^2+b^4)}-\frac{(-57a^4+82a^2b^2-9b^4)(\tan^4}{8ad(a^4-2a^2b^2+b^4)}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a/sin(d*x+c)^2+(3*a^2+b^2)/a^3*ln(sin(d*x+c))+1/a^2*b/sin(d*x+c)+
b^8/a^3/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1
/16*(-9*a+11*b)/(a-b)^2/(1+sin(d*x+c))+1/16/(a-b)^3*(-24*a^2+57*a*b-35*b^2)
*ln(1+sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(9*a+11*b)/(a+b)^2/(s
in(d*x+c)-1)+1/16/(a+b)^3*(-24*a^2-57*a*b-35*b^2)*ln(sin(d*x+c)-1))
```

Maxima [A]
time = 0.32, size = 422, normalized size = 1.54

$$\frac{\frac{16\,b^8\log(b\sin(dx+c)+a)}{a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6}-\frac{(24\,a^2-57\,ab+35\,b^2)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{(24\,a^2+57\,ab+35\,b^2)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left((15\,a^4b-27\,a^2b^3+8\,b^5)\sin(dx+c)^5-4\,a^5+8\,a^3b^2-4\,ab^4-4\,(3\,a^5-5\,a^3b^2+ab^4)\sin(dx+c)^4-(25\,a^4b-45\,a^2b^3+16\,b^5)\sin(dx+c)^3+2\,(9\,a^5-15\,a^3b^2+4\,ab^4)\sin(dx+c)^2+8\,(a^4b-2\,a^2b^3+b^5)\sin(dx+c)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sin(dx+c)^6-2\,(a^6-2\,a^4b^2+a^2b^4)\sin(dx+c)^4+(a^6-2\,a^4b^2+a^2b^4)\sin(dx+c)^2}+\frac{16\,(3\,a^2+b^2)\log(\sin(dx+c))}{a^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*b^8*log(b*sin(d*x + c) + a)/(a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6
) - (24*a^2 - 57*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3) + 2*((15*a^4*b - 27*a^2*b^3 + 8*b^5)*sin(d*x + c)^5 - 4*a
^5 + 8*a^3*b^2 - 4*a*b^4 - 4*(3*a^5 - 5*a^3*b^2 + a*b^4)*sin(d*x + c)^4 - (
25*a^4*b - 45*a^2*b^3 + 16*b^5)*sin(d*x + c)^3 + 2*(9*a^5 - 15*a^3*b^2 + 4*
a*b^4)*sin(d*x + c)^2 + 8*(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6 - 2
*a^4*b^2 + a^2*b^4)*sin(d*x + c)^6 - 2*(a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x
+ c)^4 + (a^6 - 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^2) + 16*(3*a^2 + b^2)*log
(sin(d*x + c))/a^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(256) = 512$.
time = 2.45, size = 640, normalized size = 2.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^8 - 8*a^6*b^2 + 4*a^4*b^4 - 8*(3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^4 + 4*(3*a^8 - 8*a^6*b^2 + 5*a^4*b^4)*cos(d*x + c)^2 -
 16*(b^8*cos(d*x + c)^6 - b^8*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - 16*
((3*a^8 - 8*a^6*b^2 + 6*a^4*b^4 - b^8)*cos(d*x + c)^6 - (3*a^8 - 8*a^6*b^2
+ 6*a^4*b^4 - b^8)*cos(d*x + c)^4)*log(-1/2*sin(d*x + c)) + ((24*a^8 + 15*a
^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos(d*x + c)^6 -
(24*a^8 + 15*a^7*b - 64*a^6*b^2 - 42*a^5*b^3 + 48*a^4*b^4 + 35*a^3*b^5)*cos
(d*x + c)^4)*log(sin(d*x + c) + 1) + ((24*a^8 - 15*a^7*b - 64*a^6*b^2 + 42*
a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^6 - (24*a^8 - 15*a^7*b - 64
*a^6*b^2 + 42*a^5*b^3 + 48*a^4*b^4 - 35*a^3*b^5)*cos(d*x + c)^4)*log(-sin(d
*x + c) + 1) - 2*(2*a^7*b - 4*a^5*b^3 + 2*a^3*b^5 - (15*a^7*b - 42*a^5*b^3
+ 35*a^3*b^5 - 8*a*b^7)*cos(d*x + c)^4 + (5*a^7*b - 14*a^5*b^3 + 9*a^3*b^5)
*cos(d*x + c)^2)*sin(d*x + c))/((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*c
os(d*x + c)^6 - (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. 2(256) = 512.
time = 0.51, size = 589, normalized size = 2.15

$$\frac{\frac{16\,b^9\log\left(|b\sin\left(dx+c\right)+a|\right)}{a^9b-3\,a^7b^3+3\,a^5b^5-a^3b^7}-\frac{\left(24\,a^2-57\,ab+35\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(24\,a^2+57\,ab+35\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\,\left(3\,a^2+b^2\right)\log\left(|\sin\left(dx+c\right)|\right)}{a^3}+\frac{2\,\left(4\,b^8\sin\left(dx+c\right)^6+15\,a^7b\sin\left(dx+c\right)^5-42\,a^5b^3\sin\left(dx+c\right)^5+35\,a^3b^5\sin\left(dx+c\right)^5-8\,ab^7\sin\left(dx+c\right)^5-12\,a^8\sin\left(dx+c\right)^4+32\,a^6b^2\sin\left(dx+c\right)^4-24\,a^4b^4\sin\left(dx+c\right)^4+4\,a^2b^6\sin\left(dx+c\right)^4-8\,b^8\sin\left(dx+c\right)^4-25\,a^7b\sin\left(dx+c\right)^3+70\,a^5b^3\sin\left(dx+c\right)^3-61\,a^3b^5\sin\left(dx+c\right)^3+16\,ab^7\sin\left(dx+c\right)^3+18\,a^8\sin\left(dx+c\right)^2-48\,a^6b^2\sin\left(dx+c\right)^2+38\,a^4b^4\sin\left(dx+c\right)^2-8\,a^2b^6\sin\left(dx+c\right)^2+4\,b^8\sin\left(dx+c\right)^2+8\,a^7b\sin\left(dx+c\right)-24\,a^5b^3\sin\left(dx+c\right)+24\,a^3b^5\sin\left(dx+c\right)-8\,ab^7\sin\left(dx+c\right)-4\,a^8+12\,a^6b^2-12\,a^4b^4+4\,a^2b^6\right)}{\left(a^9-3\,a^7b^2+3\,a^5b^4-a^3b^6\right)\left(\sin\left(dx+c\right)^3-\sin\left(dx+c\right)\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*b^9*log(abs(b*sin(d*x + c) + a))/(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 -
a^3*b^7) - (24*a^2 - 57*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a
^2*b + 3*a*b^2 - b^3) - (24*a^2 + 57*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1
))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 16*(3*a^2 + b^2)*log(abs(sin(d*x + c))
)/a^3 + 2*(4*b^8*sin(d*x + c)^6 + 15*a^7*b*sin(d*x + c)^5 - 42*a^5*b^3*sin(
d*x + c)^5 + 35*a^3*b^5*sin(d*x + c)^5 - 8*a*b^7*sin(d*x + c)^5 - 12*a^8*si
n(d*x + c)^4 + 32*a^6*b^2*sin(d*x + c)^4 - 24*a^4*b^4*sin(d*x + c)^4 + 4*a^
2*b^6*sin(d*x + c)^4 - 8*b^8*sin(d*x + c)^4 - 25*a^7*b*sin(d*x + c)^3 + 70*
a^5*b^3*sin(d*x + c)^3 - 61*a^3*b^5*sin(d*x + c)^3 + 16*a*b^7*sin(d*x + c)^
3 + 18*a^8*sin(d*x + c)^2 - 48*a^6*b^2*sin(d*x + c)^2 + 38*a^4*b^4*sin(d*x
+ c)^2 - 8*a^2*b^6*sin(d*x + c)^2 + 4*b^8*sin(d*x + c)^2 + 8*a^7*b*sin(d*x
+ c) - 24*a^5*b^3*sin(d*x + c) + 24*a^3*b^5*sin(d*x + c) - 8*a*b^7*sin(d*x
```

```
+ c) - 4*a^8 + 12*a^6*b^2 - 12*a^4*b^4 + 4*a^2*b^6)/((a^9 - 3*a^7*b^2 + 3*a
^5*b^4 - a^3*b^6)*(sin(d*x + c)^3 - sin(d*x + c))^2))/d
```

Mupad [B]
time = 12.70, size = 412, normalized size = 1.50

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(3\,a^2+b^2\right)}{a^3\,d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{b^2}{8\,{\left(a-b\right)}^3}-\frac{9\,b}{16\,{\left(a-b\right)}^2}+\frac{3}{2\,\left(a-b\right)}\right)}{d}-\frac{\frac{1}{2\,a}-\frac{b\,\sin\left(c+d\,x\right)}{a^2}+\frac{{\sin\left(c+d\,x\right)}^4\,\left(3\,a^4-5\,a^2\,b^2+b^4\right)}{2\,a\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(9\,a^4-15\,a^2\,b^2+4\,b^4\right)}{4\,a\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^5\,\left(15\,a^4\,b-27\,a^2\,b^3+8\,b^5\right)}{8\,a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\sin\left(c+d\,x\right)}^3\,\left(25\,a^4\,b-45\,a^2\,b^3+16\,b^5\right)}{8\,a^2\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\sin\left(c+d\,x\right)}^6-2\,{\sin\left(c+d\,x\right)}^4+{\sin\left(c+d\,x\right)}^2\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{9\,b}{16\,{\left(a+b\right)}^2}+\frac{3}{2\,\left(a+b\right)}+\frac{b^2}{8\,{\left(a+b\right)}^3}\right)}{d}+\frac{b^8\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{d\,\left(a^9-3\,a^7\,b^2+3\,a^5\,b^4-a^3\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*sin(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x))*(3*a^2 + b^2))/(a^3*d) - (log(sin(c + d*x) + 1)*(b^2/(8*
(a - b)^3) - (9*b)/(16*(a - b)^2) + 3/(2*(a - b))))/d - (1/(2*a) - (b*sin(c
 + d*x))/a^2 + (sin(c + d*x)^4*(3*a^4 + b^4 - 5*a^2*b^2))/(2*a*(a^4 + b^4 -
 2*a^2*b^2)) - (sin(c + d*x)^2*(9*a^4 + 4*b^4 - 15*a^2*b^2))/(4*a*(a^4 + b^
4 - 2*a^2*b^2)) - (sin(c + d*x)^5*(15*a^4*b + 8*b^5 - 27*a^2*b^3))/(8*a^2*(
a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)^3*(25*a^4*b + 16*b^5 - 45*a^2*b^3))
/(8*a^2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + s
in(c + d*x)^6)) - (log(sin(c + d*x) - 1)*((9*b)/(16*(a + b)^2) + 3/(2*(a +
b)) + b^2/(8*(a + b)^3)))/d + (b^8*log(a + b*sin(c + d*x)))/(d*(a^9 - a^3*b
^6 + 3*a^5*b^4 - 3*a^7*b^2))
```

3.1370 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=500

$$\frac{a^4\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{a^4\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2(g}{}$$

```
[Out] -2/3*a^2*(g*cos(f*x+e))^(3/2)/b^3/f/g-2/3*(g*cos(f*x+e))^(3/2)/b/f/g+2/7*(g
*cos(f*x+e))^(7/2)/b/f/g^3+2/5*a*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b^2/f/g+a^
4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(
9/2)/(-a^2+b^2)^(1/4)/f-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))*g^(1/2)/b^(9/2)/(-a^2+b^2)^(1/4)/f+a^5*g*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^
2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e
))^(1/2)+a^5*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(s
in(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/
(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(
1/2)/b^4/f/cos(f*x+e)^(1/2)-4/5*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos
(f*x+e)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2977, 2721, 2719, 2645, 30, 2648, 14, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^4f\sqrt{\cos(e+fx)}}-\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}+\frac{a^5g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^5g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{a^4\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}-\frac{a^4\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f\sqrt[4]{b^2-a^2}}+\frac{2a\sin(e+fx)(g\cos(e+fx))^{3/2}}{5b^2fg}-\frac{4aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5b^2f\sqrt{\cos(e+fx)}}+\frac{2(g\cos(e+fx))^{7/2}}{7bfg^3}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^4*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)*f) - (a^4*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*(-a^2 + b^2)^(1/4)
*f) - (2*a^2*(g*Cos[e + f*x])^(3/2))/(3*b^3*f*g) - (2*(g*Cos[e + f*x])^(3/2
))/(3*b*f*g) + (2*(g*Cos[e + f*x])^(7/2))/(7*b*f*g^3) - (2*a^3*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b^4*f*Sqrt[Cos[e + f*x]]) - (4*a*Sqrt[
g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) + (
a^5*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)
/2, 2])/(b^5*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^5*g*Sqrt[C
os[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5
```

```
*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a*(g*Cos[e + f*x])^(3/
2)*Sin[e + f*x])/(5*b^2*f*g)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
```

```
f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^4(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a^3\sqrt{g\cos(e+fx)}}{b^4}+\frac{a^2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b^3}-\frac{a\sqrt{g\,c}}{}\right.$$

$$= -\frac{a^3\int\sqrt{g\cos(e+fx)}\,dx}{b^4}+\frac{a^4\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^4}+\frac{a^2\int\sqrt{g\cos}}{}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2fg}-\frac{(2a)\int\sqrt{g\cos(e+fx)}\,dx}{5b^2}-\frac{a^2\mathrm{Su}}{}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2a^3\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{\cos(e+fx)}}+\frac{2a(}{}$$

$$= -\frac{2a^2(g\cos(e+fx))^{3/2}}{3b^3fg}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}+\frac{2(g\cos(e+fx))^{7/2}}{7bfg^3}$$

$$= \frac{a^4\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{a^4\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(}}{\sqrt[4]{-a^2+b}}\right.}{b^{9/2}\sqrt[4]{-a^2+b^2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 36.73, size = 816, normalized size = 1.63

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^4)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/5*(a*Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*
AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I
)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1
```

```
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Lo
g[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 -
 Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - C
os[e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)
^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)
*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 +
 b^2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(b^3*f*Sqrt[Cos[e + f*x]
]) + (Sqrt[g*Cos[e + f*x]]*(-1/42*((28*a^2 + 19*b^2)*Cos[e + f*x])/b^3 + Co
s[3*(e + f*x)]/(14*b) + (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 41.27, size = 1206, normalized size = 2.41

method	result	size
default	Expression too large to display	1206

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (16/7/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/7/b*cos(
1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/b*cos(1/2*f*x+1/2*
e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/21/b*(2*cos(1/2*f*x+1/2*e)^2*g-g)
^(1/2)-4/3/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)*a^2-
4/3/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)*a^2+2/b^3*a^2*(g*(2*cos(1/2*f*x+
1/2*e)^2-1))^(1/2)+1/2/b^3*g*a^4*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3
*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8
-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/40*(g
*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*g*(-128*b^4*cos(1
/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^6+128*b^4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1
/2*e)^4+80*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/
2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*a^2*b^2+32*EllipticE(cos(1/2*f*x+1/2*e)
,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*b^4
-32*b^4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+5*a^2*sum(1/_alpha*(8*(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2
*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2
)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+
4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*ar
```

```
ctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1
/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12
*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_
alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^
(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2
*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2))/b^6/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3882 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^4/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^4*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^4*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

3.1371 $\int \frac{\sqrt{g\cos(e+fx)}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=448

$$-\frac{a^3\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}\,f}+\frac{a^3\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}\,f}+\frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg}+\frac{2a^2}{}$$

```
[Out] 2/3*a*(g*cos(f*x+e))^(3/2)/b^2/f/g-2/5*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b/f/
g-a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)
/b^(7/2)/(-a^2+b^2)^(1/4)/f+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+
b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(7/2)/(-a^2+b^2)^(1/4)/f-a^4*g*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^
2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f
*x+e))^(1/2)-a^4*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Elliptic
Pi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^
4/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*a^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e
))^(1/2)/b^3/f/cos(f*x+e)^(1/2)+4/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/f/cos
(f*x+e)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2977, 2721, 2719, 2645, 30, 2648, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{2a^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^3f\sqrt{\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^4g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt[4]{b^2-a^2}}+\frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg}-\frac{2\sin(e+fx)(g\cos(e+fx))^{3/2}}{5bfg}+\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5bf\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^3*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(b^(7/2)*(-a^2 + b^2)^(1/4)*f)) + (a^3*Sqrt[g]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(1
/4)*f) + (2*a*(g*Cos[e + f*x])^(3/2))/(3*b^2*f*g) + (2*a^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(b^3*f*Sqrt[Cos[e + f*x]]) + (4*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(5*b*f*Sqrt[Cos[e + f*x]]) - (a^4*g*Sq
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^4*g*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sq
rt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*(g*Cos[e + f*x])^(3/2)*Sin[e +
 f*x])/(5*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2\sqrt{g\cos(e+fx)}}{b^3} - \frac{a\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b^2} + \frac{\sqrt{g\cos(}{}\right.$$

$$= \frac{a^2\int\sqrt{g\cos(e+fx)}\,dx}{b^3} - \frac{a^3\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}dx}{b^3} - \frac{a\int\sqrt{g\cos(}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}\sin(e+fx)}{5bfg} + \frac{2\int\sqrt{g\cos(e+fx)}\,dx}{5b} + \frac{a\mathrm{Sub}}{}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} - \frac{2(g\,c}{}$$

$$= \frac{2a(g\cos(e+fx))^{3/2}}{3b^2fg} + \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{\cos(e+fx)}} + \frac{4\sqrt{g}}{}$$

$$= -\frac{a^3\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{a^3\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\,c}}{\sqrt[4]{-a^2}}\right.}{b^{7/2}\sqrt[4]{-a^2+b^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 36.49, size = 789, normalized size = 1.76

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*
Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[
-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
+ f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 + 2*b^2)*(a + b*Sqrt[1 - Cos[e +
f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*
```

```
(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqr
t[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos
[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(
1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^2*f*Sqrt[Cos[e + f*x]]) +
(Sqrt[g*Cos[e + f*x]]*((2*a*Cos[e + f*x])/(3*b^2) - Sin[2*(e + f*x)]/(5*b))
)/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 44.56, size = 1345, normalized size = 3.00

method	result	size
default	Expression too large to display	1345

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (4/3*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3*a/b^
2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)-1/2*g*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+
8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^
6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/12*(g*(2*cos(1/2*
f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g*(64*cos(1/2*f*x+1/2*e)^5*sin(
1/2*f*x+1/2*e)^2*b^4-64*cos(1/2*f*x+1/2*e)^5*b^4-96*cos(1/2*f*x+1/2*e)^3*si
n(1/2*f*x+1/2*e)^2*b^4-48*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*
x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^
2*b^2+16*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)
*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*b^4-48*EllipticE(co
s(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*
e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*b^4+96*cos(1/2*f*x+1/2*e)^3*b^4+32*b^4*c
os(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+48*EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2*b^2
-16*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*
cos(1/2*f*x+1/2*e)^2+1)^(1/2)*b^4+48*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*
(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*b^4-32*cos(1
/2*f*x+1/2*e)*b^4-3*sum((2*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2-sin(1/2*f*x+1/
2*e)^2*a^2-2*b^2*_alpha^2+a^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_a
lpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)
^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*
(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alph
```

```
a^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2
+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/
2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2))/b^5/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(
1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

3.1372 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=369

$$\frac{a^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}-\frac{2a\sqrt{}}{}$$

```
[Out] -2/3*(g*cos(f*x+e))^(3/2)/b/f/g+a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a
^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(5/2)/(-a^2+b^2)^(1/4)/f-a^2*arctanh(b^(1/
2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^(5/2)/(-a^2+b^2
)^(1/4)/f+a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(
sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f
/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^3*g*(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)
),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)
-2*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+
1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2977, 2721, 2719, 2645, 30, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{a^2\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt[4]{b^2-a^2}}+\frac{a^3g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^2f\sqrt{\cos(e+fx)}}-\frac{2(g\cos(e+fx))^{3/2}}{3bfg}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)*f) - (a^2*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[
g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(1/4)
*f) - (2*(g*Cos[e + f*x])^(3/2))/(3*b*f*g) - (2*a*Sqrt[g*Cos[e + f*x]]*Elli
pticE[(e + f*x)/2, 2])/(b^2*f*Sqrt[Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b - Sqrt
[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^3*g*Sqrt[Cos[e + f*x]]*EllipticP
i[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(b + Sqrt[-a^2 + b^2]
)*f*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
```

```
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a\sqrt{g\cos(e+fx)}}{b^2} + \frac{\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{b} + \frac{a^2\sqrt{g\,\mathrm{co}}}{b^2(a+b\,\mathrm{s}}\right.$$

$$= -\frac{a\int\sqrt{g\cos(e+fx)}\,dx}{b^2} + \frac{a^2\int\frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b^2} + \frac{\int\sqrt{g\cos(e+}}{}$$

$$= -\frac{\mathrm{Subst}\left(\int\sqrt{x}\,dx, x, g\cos(e+fx)\right)}{bfg} - \frac{(a^3g)\int\frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-}\right.}}{2b^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg} - \frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}} + \frac{(2a^2g)}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3bfg} - \frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}} + \frac{a^3g\sqrt{}}{}$$

$$= \frac{a^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f} - \frac{a^2\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(}}{\sqrt[4]{-a^2+b}}\right.}{b^{5/2}\sqrt[4]{-a^2+b^2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.37, size = 372, normalized size = 1.01

$$\frac{\sqrt{g\cos(e+fx)}\left(-8b^{3/2}\cos^{\frac{3}{2}}(e+fx)-\frac{a\left(8b^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(e+fx)+3\sqrt{2}\,a(a^2-b^2)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)}{(a^2-b^2)(a+b\sin(e+fx))}\right)}{12b^{5/2}f\sqrt{\cos(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*(-8*b^(3/2)*Cos[e + f*x]^(3/2) - (a*(8*b^(5/2)*Appell
F1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqr
t[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2
- b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))/((a^2 - b^2)*(a + b*Sin[e + f*x]))))/(
12*b^(5/2)*f*Sqrt[Cos[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 38.82, size = 882, normalized size = 2.39

method	result	size
default	Expression too large to display	882

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-4/3/b*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3/b*(2*co
s(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2/
b*g*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^
2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)
*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2
-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/8*(g*(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g*a*(16*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(
1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*b^2+sum(1/_
alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*Ell
ipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*
b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^
(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^
2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2
)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^
2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha
^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)
^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*
(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f
*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))/b^4/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e
)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2\,\sqrt{g\,\cos\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

3.1373 $\int \frac{\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=341

$$-\frac{a\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f}+\frac{a\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f}+\frac{2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+\right.}{bf\sqrt{\cos(e+fx)}}$$

```
[Out] -a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/b^
(3/2)/(-a^2+b^2)^(1/4)/f+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))*g^(1/2)/b^(3/2)/(-a^2+b^2)^(1/4)/f-a^2*g*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2
)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e)
)^(1/2)-a^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(si
n(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(
b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/
b/f/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{a\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}-\frac{a^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{a\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt[4]{b^2-a^2}}+\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bf\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Sqrt[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*f)) + (a*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(1/4)*
f) + (2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f
*x]]) - (a^2*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
(e + f*x)/2, 2])/(b^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^2
*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
 2])/(b^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{a+b\sin(e+fx)}\,dx = \frac{\int \sqrt{g\cos(e+fx)}\,dx}{b} - \frac{a\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{b}$$

$$= \frac{(a^2g)\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b^2} - \frac{(a^2g)\int \frac{}{\sqrt{g\,\mathrm{co}}}}{}$$

$$= \frac{2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{(2ag)\mathrm{Subst}\left(\int \frac{x^2}{(a^2-b^2)g^2+b^2x^4}\,dx,\right.}{f}$$

$$= \frac{2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{a^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2}}\right.}{b^2\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{}}$$

$$= -\frac{a\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f} + \frac{a\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(}}{\sqrt[4]{-a^2+b}}\right.}{b^{3/2}\sqrt[4]{-a^2+b^2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.28, size = 351, normalized size = 1.03

$$-\frac{\sqrt{g\cos(e+fx)}\left(8b^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(e+fx)+3\sqrt{2}\,a(a^2-b^2)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)}{12b^{3/2}(-a^2+b^2)f\sqrt{\cos(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/12*(Sqrt[g*Cos[e + f*x]]*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a
*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2
- b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^
2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x
]^2]))/(b^(3/2)*(-a^2 + b^2)*f*Sqrt[Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 42.01, size = 693, normalized size = 2.03

method	result
default	$$-\frac{ga\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^8-4b^2g\_Z^6+\left(16a^2g^2-10b^2g^2\right)\_Z^4-4b^2g^3\_Z^2+b^2g^4\right)}\frac{\left(\_R^6-\_R^4g-\_R^2g^2+g^3\right)\ln\left(\sqrt{-2\left(\sin^2\left(\frac{fx}{2}\right.\right.}\right.}{\_R^7b^2-3\_R^5b^2g+8\_R^3a^2g}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] (-1/2*g*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*
g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/
2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g
^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8*(g*(2*cos(1/2*f*x+1/2*e)^2-1
)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g*b*(1/2*(sin(1/2*f*x+1/2*e)^2-1)/b^2*(sin(1/
2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+
1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))
```

```
-1/32/b^4*sum((-2*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a^
2+2*b^2*_alpha^2-a^2)/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*
x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)
/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1
/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^
2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(
1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha
^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(1/2*f*x+1/2*
e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)\,\sqrt{g\,\cos(e+f\,x)}}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x)
```

3.1374 $\int \frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=355

$$\frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{\sqrt{b}\,\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a\sqrt[4]{-a^2+b^2}\,f}-\frac{\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{af}$$

```
[Out] arctan((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f-arctanh((g*cos(f*x+e))^(1/
2)/g^(1/2))*g^(1/2)/a/f-arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4
)/g^(1/2))*b^(1/2)*g^(1/2)/a/(-a^2+b^2)^(1/4)/f+arctanh(b^(1/2)*(g*cos(f*x+
e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)*g^(1/2)/a/(-a^2+b^2)^(1/4)/f-g*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*
e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/f/(b-(-a^2+b^2)^(1/2)
)/(g*cos(f*x+e))^(1/2)-g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1
/2)/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2977, 2645, 335, 304, 209, 212, 2780, 2886, 2884, 211, 214}

$$-\frac{\sqrt{b}\,\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}+\frac{\sqrt{b}\,\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{af\sqrt[4]{b^2-a^2}}-\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{\sqrt{g}\,\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - (Sqrt[b]*Sqrt[g]*Arc
Tan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2
+ b^2)^(1/4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
Sqrt[b]*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*
Sqrt[g])])/(a*(-a^2 + b^2)^(1/4)*f) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((b - Sqrt[-a^2 + b^2])*f*Sqrt[g
*Cos[e + f*x]]) - (g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/((b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a} - \frac{b\sqrt{g\cos(e+fx)}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int \sqrt{g\cos(e+fx)}\,\csc(e+fx)\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} + \frac{1}{2}g\int \frac{}{\sqrt{g\cos(e+fx)}\,\left(\sqrt{-\right.}$$

$$= -\frac{2\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{afg} - \frac{(2b^2g)\,\text{Subst}\left(\int \frac{x^2}{(a^2-b^2)g^2+}\right.}{}$$

$$= -\frac{g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}}; \frac{1}{2}(e+fx)\middle|\,2\right)}{\left(b-\sqrt{-a^2+b^2}\right)f\sqrt{g\cos(e+fx)}} - \frac{g\sqrt{\cos(e+fx)}}{\left(b+\sqrt{-\right.}}$$

$$= \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{\sqrt{b}\,\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right.}{a\sqrt[4]{-a^2+b^2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 23.48, size = 534, normalized size = 1.50

$$\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)\left(8abF_1\left(\tfrac{3}{4};\tfrac{1}{2},1;\tfrac{7}{4};\cos^2(e+fx),\tfrac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(e+fx)+3\left(2\sqrt{2}\sqrt{b}\,(a^2-b^2)^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\sqrt{2}\sqrt{b}\,(a^2-b^2)^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)+4a^2\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)-4b^2\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)+2a^2\log\left(1-\sqrt{\cos(e+fx)}\right)-2b^2\log\left(1-\sqrt{\cos(e+fx)}\right)-2a^2\log\left(1+\sqrt{\cos(e+fx)}\right)+2b^2\log\left(1+\sqrt{\cos(e+fx)}\right)-\sqrt{2}\sqrt{b}\,(a^2-b^2)^{3/4}\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)+\sqrt{2}\sqrt{b}\,(a^2-b^2)^{3/4}\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)}{12a(a^2-b^2)f\sqrt{\cos(e+fx)}\,(b+a\csc(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]*(8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e
+ f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*(2*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*a^2*ArcTan[Sqrt[Cos
[e + f*x]]] - 4*b^2*ArcTan[Sqrt[Cos[e + f*x]]] + 2*a^2*Log[1 - Sqrt[Cos[e +
 f*x]]] - 2*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 2*a^2*Log[1 + Sqrt[Cos[e + f*
x]]] + 2*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + b*Cos[e + f*x]] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
 + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))
```

```
*(a + b*Sqrt[Sin[e + f*x]^2]))/(12*a*(a^2 - b^2)*f*Sqrt[Cos[e + f*x]]*(b +
a*Csc[e + f*x]))
```

Maple [A]
time = 16.75, size = 186, normalized size = 0.52

method	result
default	$-\frac{\sqrt{g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sqrt{-g}+\sqrt{g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\right.\right.}}{\cos\left(\frac{f}{2}\right.}\right.}{2a\sqrt{-g}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2/a/(-g)^(1/2)*(g^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+g^(1/2)*ln(2
/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*co
s(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+2*g*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g)))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)/(a + b*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos(e+f\,x)}}{\sin(e+f\,x)\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

3.1375 $\int \frac{\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=433

$$-\frac{b\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{b^{3/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^2\sqrt[4]{-a^2+b^2}f}+\frac{b\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+f}}{\sqrt{g}}\right.}{a^2f}$$

```
[Out] -(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a/f/g-b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2
))*g^(1/2)/a^2/f+b^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/
4)/g^(1/2))*g^(1/2)/a^2/(-a^2+b^2)^(1/4)/f+b*arctanh((g*cos(f*x+e))^(1/2)/g
^(1/2))*g^(1/2)/a^2/f-b^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))*g^(1/2)/a^2/(-a^2+b^2)^(1/4)/f+b*g*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/
2)+b*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f
*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/a/f/cos(f*x
+e)^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2977, 2645, 335, 304, 209, 212, 2650, 2721, 2719, 2780, 2886, 2884, 211, 214}

$$\frac{b^{3/2}\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}-\frac{b\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{bg\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{bg\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{b^{3/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt[4]{b^2-a^2}}+\frac{b\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{\csc(e+fx)(g\cos(e+fx))^{3/2}}{afg}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((b*Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + (b^(3/2)*Sqrt
[g]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^2*(-a^2 + b^2)^(1/4)*f) + (b*Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]
)/(a^2*f) - (b^(3/2)*Sqrt[g]*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(1/4)*f) - ((g*Cos[e + f*x])^(3/2
)*Csc[e + f*x])/(a*f*g) - (Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/
(a*f*Sqrt[Cos[e + f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sq
rt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e
+ f*x]]) + (b*g*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/(a*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
```

```
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2}+\frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{a}\right.$$

$$= \frac{\int \sqrt{g\cos(e+fx)}\,\csc^2(e+fx)\,dx}{a} - \frac{b\int \sqrt{g\cos(e+fx)}\,\csc(e+fx}{a^2}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg} - \frac{\int \sqrt{g\cos(e+fx)}\,dx}{2a} + \frac{b\text{Subst}\Big(}{}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg} + \frac{(2b)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e}\right.}{a^2fg}$$

$$= -\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{afg} - \frac{\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|\,2\right)}{af\sqrt{\cos(e+fx)}}$$

$$= -\frac{b\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} + \frac{b^{3/2}\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+}}{\sqrt[4]{-a^2+b^2}\,\sqrt{}}\right.}{a^2\sqrt[4]{-a^2+b^2}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.63, size = 1550, normalized size = 3.58

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((Sqrt[g*Cos[e + f*x]]*Cot[e + f*x])/(a*f)) + (Sqrt[g*Cos[e + f*x]]*((4*a*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x
]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2))
 + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2
)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[C
os[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(
-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 +
 b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) + (5*b*(-1 +
 Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*S
```

```
qrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*
Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt
[Cos[e + f*x]]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e +
f*x]]] - 6*b^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x
]]] + 6*b^2*Log[1 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/
4)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + b*Cos[e + f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))/(12*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-1 + C
os[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*
x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a
^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)
] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Appell
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b
^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[
e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2]
 - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*Sqrt[b]
*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e +
f*x]))))/(4*a*f*Sqrt[Cos[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 78.51, size = 1266, normalized size = 2.92

method	result	size
default	Expression too large to display	1266

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/8*(4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b*(sum(1/_R/
(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^2-5*_R^2*b^2*g^2-b^2*g^3)*ln((-2*sin(1/
2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R)*(_R^6-_R^4
*g-_R^2*g^2+g^3),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z
^4-4*b^2*g^3*_Z^2+b^2*g^4))*(-g)^(1/2)*b^2*g+g^(1/2)*ln(2/(cos(1/2*f*x+1/2*
e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g
))*(-g)^(1/2)+g^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+
```

```
1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+2*g*ln(2/cos(1/2*
f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g)))+(-8*(-2*sin
(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*EllipticE(cos(1/2*f*x+1/2
*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*
(-g)^(1/2)*a*g-g^3*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2-1)^2/a*sum(
1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)
^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_
alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),
(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*
g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(
-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+
4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_al
pha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*
f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f
*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4
*_Z^2*b^2+a^2))*(-g)^(1/2))*cos(1/2*f*x+1/2*e)-16*(-2*sin(1/2*f*x+1/2*e)^4*
g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(-g)^(1/2)*a*g*sin(1/2*f*x+1/2*e)^4+16*(-2*
sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(-g)^(1/2)*a*g*sin(1/2
*f*x+1/2*e)^2-4*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(-
g)^(1/2)*a*g)/a^2/(-g)^(1/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+
sin(1/2*f*x+1/2*e)^2*g)^(3/2)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos{(e+fx)}}\csc^2{(e+fx)}}{a+b\sin{(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)**2/(a + b*sin(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos{(e+f\,x)}}}{\sin{(e+f\,x)}^2\,\left(a+b\,\sin{(e+f\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

3.1376 $\int \frac{\sqrt{g\cos(e+fx)}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=544

$$\frac{\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}+\frac{b^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^{5/2}\sqrt{g}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^3\sqrt[4]{-a^2+b^2}f}$$

```
[Out] b*(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a^2/f/g-1/2*(g*cos(f*x+e))^(3/2)*csc(f*x+
e)^2/a/f/g+1/4*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f+b^2*arctan(
(g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a^3/f-b^(5/2)*arctan(b^(1/2)*(g*cos(f
*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/a^3/(-a^2+b^2)^(1/4)/f-1/4*a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))*g^(1/2)/a/f-b^2*arctanh((g*cos(f*x+e))
^(1/2)/g^(1/2))*g^(1/2)/a^3/f+b^(5/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/
(-a^2+b^2)^(1/4)/g^(1/2))*g^(1/2)/a^3/(-a^2+b^2)^(1/4)/f-b^2*g*(cos(1/2*f*x
+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-
a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(b-(-a^2+b^2)^(1/2))/(g*cos
(f*x+e))^(1/2)-b^2*g*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/
a^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))
^(1/2)/a^2/f/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2977, 2645, 335, 304, 209, 212, 2650, 2721, 2719, 296, 2780, 2886, 2884, 211, 214}

$$\frac{b^2\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{b^2g\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{b\csc(e+fx)(g\cos(e+fx))^{3/2}}{a^2fg}+\frac{bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{a^2f\sqrt{\cos(e+fx)}}-\frac{b^{5/2}\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}+\frac{b^{5/2}\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt[4]{b^2-a^2}}+\frac{\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}-\frac{\csc^2(e+fx)(g\cos(e+fx))^{3/2}}{2afg}-\frac{\sqrt{g}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g]*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*Sqrt[g]*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (b^(5/2)*Sqrt[g]*ArcTan[(Sqrt[b]
*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(1/
4)*f) - (Sqrt[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*Sqrt
[g]*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(5/2)*Sqrt[g]*ArcTa
nh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2
 + b^2)^(1/4)*f) + (b*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f*g) - ((g*
Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f*g) + (b*Sqrt[g*Cos[e + f*x]]*Ell
ipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*
```

```
x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b - Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (b^2*g*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(b + Sqrt[-a^2 + b^2
])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^3} - \frac{b\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{a^2}\right.$$

$$= \frac{\int \sqrt{g\cos(e+fx)}\,\csc^3(e+fx)\,dx}{a} - \frac{b\int \sqrt{g\cos(e+fx)}\,\csc^2(e+\ldots}{a^2}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} + \frac{b\int \sqrt{g\cos(e+fx)}\,dx}{2a^2} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} - $$

$$= \frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2fg} - \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{2afg} + $$

$$= \frac{b^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f} - \frac{b^{5/2}\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+\ldots}}{\sqrt[4]{-a^2+b^2}\ldots}\right.}{a^3\sqrt[4]{-a^2+b^2}\,f}$$

$$= \frac{\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2\sqrt{g}\,\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order

4 in optimal.
time = 60.53, size = 1582, normalized size = 2.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*Csc[e + f*x])/(
2*a)))/f - (Sqrt[g*Cos[e + f*x]]*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(
(a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1
+ I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I
)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] -
(1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] +
 Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x
]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f
*x]^2]*(b + a*Csc[e + f*x])) - ((-a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b
*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4
)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sq
rt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b
*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 -
Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqr
t[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(
1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]
*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTa
n[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*
(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1,
 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2)
 + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqr
t[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21
*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt
[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^
2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos
[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[
Cos[e + f*x]])
```

Maple [A]
time = 17.59, size = 293, normalized size = 0.54

method	result
default	$\frac{\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{8a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}-\frac{g\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4a\sqrt{-g}}-\frac{\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g}}{16a\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (1/8/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-1/4*g/a/(-g)
^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*
x+1/2*e))-1/16/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)
-1/8*g^(1/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/16/a/(cos(1/2*f*x+1/2*e)-1)*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8*g^(1/2)/a*ln((4*g*cos(1/2*f*x+1/2*e)+
2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)-1)))
/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos(e+fx)}\,\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))*csc(e + f*x)**3/(a + b*sin(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos(e+f\,x)}}{\sin(e+f\,x)^3\,\left(a+b\,\sin(e+f\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

3.1377 $\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=621

$$\frac{a^3\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}f}+\frac{a^3\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{9/2}f}-\frac{2a^3}{}$$

```
[Out] a^3*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/b^(9/2)/f+a^3*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*c
os(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(9/2)/f+2/5*a*(g*cos(f*x+e))^(
5/2)/b^2/f/g-2/7*(g*cos(f*x+e))^(5/2)*sin(f*x+e)/b/f/g-2*a^4*g^2*(cos(1/2*f
*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))
*cos(f*x+e)^(1/2)/b^5/f/(g*cos(f*x+e))^(1/2)+2/3*a^2*g^2*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x
+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+4/21*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/
cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b
/f/(g*cos(f*x+e))^(1/2)+a^4*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/
2))*cos(f*x+e)^(1/2)/b^5/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^4*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b
^5/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*a^3*g*(g*cos(f*x+e
))^(1/2)/b^4/f+2/3*a^2*g*sin(f*x+e)*(g*cos(f*x+e))^(1/2)/b^3/f+4/21*g*sin(f
*x+e)*(g*cos(f*x+e))^(1/2)/b/f
```

Rubi [A]

time = 0.97, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {2977, 2715, 2721, 2720, 2645, 30, 2648, 2774, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{2a^4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\sqrt{g\cos(e+fx)}}-\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f}+\frac{2a^2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}}+\frac{2a^2g\sin(e+fx)\sqrt{g\cos(e+fx)}}{3b^3f}+\frac{a^4g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^4g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{a^3g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg}+\frac{4g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{21bf\sqrt{g\cos(e+fx)}}-\frac{2\sin(e+fx)(g\cos(e+fx))^{5/2}}{7bfg}+\frac{4g\sin(e+fx)\sqrt{g\cos(e+fx)}}{21bf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) + (a^3*(-a^2 + b^2)^(1/4)*g^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) - (2*a^3*g*Sqrt[g*Cos[e + f*x]])/(b^4*f) + (2*a*(g*Cos[e + f*x])^(5/2))
/(5*b^2*f*g) - (2*a^4*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^
5*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*
```

```
x)/2, 2])/(3*b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*g^2*Sqrt[Cos[e + f*x]]*Ellipt
icF[(e + f*x)/2, 2])/(21*b*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 - b^2)*g^2*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^5*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2
- b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/(b^5*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
+ (2*a^2*g*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^3*f) + (4*g*Sqrt[g*Cos[e
 + f*x]]*Sin[e + f*x])/(21*b*f) - (2*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(
7*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2(g\cos(e+fx))^{3/2}}{b^3} - \frac{a(g\cos(e+fx))^{3/2}\sin(e+fx)}{b^2} + \frac{(g\,\mathrm{co}}{}\right.$$

$$= \frac{a^2\int (g\cos(e+fx))^{3/2}\,dx}{b^3} - \frac{a^3\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int (g\cos(e+}{}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a^2g\sqrt{g\cos(e+fx)}\sin(e+fx)}{3b^3f} - \frac{2(g}{}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g\sqrt{g\cos(e+}}{3b}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} + \frac{2a^2g^2\sqrt{\cos(e+}}{3b^3f\sqrt{g}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+}}{b^5f\sqrt{g}}$$

$$= -\frac{2a^3g\sqrt{g\cos(e+fx)}}{b^4f} + \frac{2a(g\cos(e+fx))^{5/2}}{5b^2fg} - \frac{2a^4g^2\sqrt{\cos(e+}}{b^5f\sqrt{g}}$$

$$= \frac{a^3\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f} + \frac{a^3\sqrt[4]{-a^2+b^2}\,g}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.16, size = 1991, normalized size = 3.21

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/420*((g*Cos[e + f*x])^(3/2)*((-2*(70*a^3 - 19*a*b^2)*(a + b*Sqrt[1 - Cos
[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*
x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^
```

```
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a
^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] -
Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]
] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e +
 f*x]^2]*(a + b*Sin[e + f*x])) + ((210*a^3 - 21*a*b^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 +
 I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^
(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e
 + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^
2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1
[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(
2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2
))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-
a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b
^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqr
t[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-
a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e +
f*x]^2)*(a + b*Sin[e + f*x])) - (2*(-98*a^2*b - 40*b^3)*(a + b*Sqrt[1 - Cos
[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x
]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2,
1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)
*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a
^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2]
+ Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/
(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2
)*(a + b*Sin[e + f*x]))))/(b^3*f*Cos[e + f*x]^(3/2)) + ((g*Cos[e + f*x])^(3
/2)*Sec[e + f*x]*((a*Cos[2*(e + f*x)])/(5*b^2) + ((28*a^2 + 5*b^2)*Sin[e +
f*x])/(42*b^3) - Sin[3*(e + f*x)]/(14*b)))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 59.55, size = 1234, normalized size = 1.99

method	result	size
default	Expression too large to display	1234

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (8/5*g*a/b^2*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5*g*
a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5*g*a/b^2*(
2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2*g*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2)+2*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2*g^3*a^5/b^4*sum((_
R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3
)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)
-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3
*_Z^2+b^2*g^4))-2*g^3*a^3/b^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R
^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2
)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(1
6*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+32*(g*(2*cos(1/2*f*x+1/
2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^2*b*(-1/120*(sin(1/2*f*x+1/2*e)^2-1
)/b^4/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(24*b^2*cos(
1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^6-44*b^2*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1
/2*e)^4+15*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)
^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*a^2-5*EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*b^2-15*E
llipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(
1/2*f*x+1/2*e)^2)^(1/2)*a^2+9*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(
1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*b^2+16*b^2*cos(1/2*f
*x+1/2*e)*sin(1/2*f*x+1/2*e)^2)+1/64*a^2*(sin(1/2*f*x+1/2*e)^2*a^2-sin(1/2*
f*x+1/2*e)^2*b^2-a^2+b^2)/b^6*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(
4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*
b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/
2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2
*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)
^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2
/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/sin(
1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3882 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

3.1378 $\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=514

$$-\frac{a^2\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}f}-\frac{a^2\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}f}+\frac{2}{}$$

```
[Out] -a^2*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))/b^(7/2)/f-a^2*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/f-2/5*(g*cos(f*x+e))^(5
/2)/b/f/g+2*a^3*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipti
cF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(g*cos(f*x+e))^(1/2)-
2/3*a*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b
^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*
f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*
(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b^2)*g^2*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^
2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/
(g*cos(f*x+e))^(1/2)+2*a^2*g*(g*cos(f*x+e))^(1/2)/b^3/f-2/3*a*g*sin(f*x+e)*
(g*cos(f*x+e))^(1/2)/b^2/f
```

Rubi [A]
time = 0.79, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2977, 2715, 2721, 2720, 2645, 30, 2774, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2a^3g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}+\frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{a^2g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f}-\frac{a^3g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^3g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{2ag^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^2f\sqrt{g\cos(e+fx)}}-\frac{2ag\sin(e+fx)\sqrt{g\cos(e+fx)}}{3b^2f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) - (a^2*(-a^2 + b^2)^(1/4)*g^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7
/2)*f) + (2*a^2*g*Sqrt[g*Cos[e + f*x]])/(b^3*f) - (2*(g*Cos[e + f*x])^(5/2)
)/(5*b*f*g) + (2*a^3*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b^4
*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/
2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sqrt[Cos[e + f
*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(a^2 -
b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^2*Sq
```

```
rt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/
(b^4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*Sqrt
[g*Cos[e + f*x]]*Sin[e + f*x])/(3*b^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a(g\cos(e+fx))^{3/2}}{b^2}+\frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{b}+\frac{a^2(g\cos\ldots}{b^2(a+\ldots}\right.$$

$$= -\frac{a\int(g\cos(e+fx))^{3/2}\,dx}{b^2}+\frac{a^2\int\frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int(g\cos(e+fx\ldots}{\ldots}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2ag\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3b^2f}-\frac{\mathrm{Subst}\big(\int\ldots}{\ldots}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}-\frac{2ag\sqrt{g\cos(e+fx)}\,\mathrm{s}\ldots}{3b^2f}$$

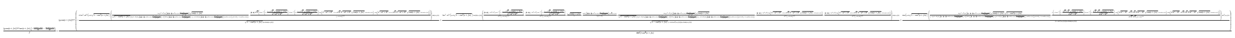
$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}-\frac{2ag^2\sqrt{\cos(e+fx)}\,F\ldots}{3b^2f\sqrt{g\cos(e\ldots}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}+\frac{2a^3g^2\sqrt{\cos(e+fx)}\,\ldots}{b^4f\sqrt{g\cos(e\ldots}}$$

$$= \frac{2a^2g\sqrt{g\cos(e+fx)}}{b^3f}-\frac{2(g\cos(e+fx))^{5/2}}{5bfg}+\frac{2a^3g^2\sqrt{\cos(e+fx)}\,\ldots}{b^4f\sqrt{g\cos(e\ldots}}$$

$$= -\frac{a^2\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2}f}-\frac{a^2\sqrt[4]{-a^2+b^2}\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 56.20, size = 1953, normalized size = 3.80



Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(-1/5*Cos[2*(e + f*x)]/b - (2*a*Sin[e
+ f*x])/(3*b^2)))/f + ((g*Cos[e + f*x])^(3/2)*((-2*(10*a^2 + 3*b^2)*(a + b*
Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1
- Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e
+ f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcT
an[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[
1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^
2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[
e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt
[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 - 3*b^2)*(a + b*Sqrt[
1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2
 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sq
rt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sq
rt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(
-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1
+ I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^
(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*
Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((
-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
```

```
]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4,
Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 +
b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f
*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b
*Sin[e + f*x]))))/(60*b^2*f*Cos[e + f*x]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 54.24, size = 1067, normalized size = 2.08

method	result	size
default	Expression too large to display	1067

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-8/5*g/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5*g/b*c
os(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+8/5*g/b*(2*cos(1/2*f
*x+1/2*e)^2*g-g)^(1/2)+2*g/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2-2*g
/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2*g^3/b^3*a^4*sum((_R^4+_R^2*g)/(_R
^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/
2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(
b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))
+2*g^3/b*a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3
*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2
*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*
g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*
f*x+1/2*e)^2)^(1/2)*g^2*a*(1/12/b^4/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+
1/2*e)^2*g)^(1/2)*(-4*b^2*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^4+3*Ellipti
cF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*
x+1/2*e)^2)^(1/2)*a^2-EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+
1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*b^2+2*b^2*cos(1/2*f*x+1/2*e)
*sin(1/2*f*x+1/2*e)^2)-1/64*a^2*(a^2-b^2)/b^6*sum(1/_alpha/(2*_alpha^2-1)*(
2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-
3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2
*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g
*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_a
lpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-
sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2
```

```
*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z
^2*b^2+a^2)))/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

3.1379 $\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=426

$$\frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}f}+\frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}f}-\frac{2(3a^2}{}$$

```
[Out] a*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(
1/4)/g^(1/2))/b^(5/2)/f+a*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*cos(f
*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/f-2/3*(3*a^2-b^2)*g^2*(cos(1
/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1
/2))*cos(f*x+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+a^2*(a^2-b^2)*g^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b-(-a^2+b^2)^(1
/2)))/(g*cos(f*x+e))^(1/2)+a^2*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(
1/2)-2/3*g*(3*a-b*sin(f*x+e))*(g*cos(f*x+e))^(1/2)/b^2/f
```

Rubi [A]
time = 0.65, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2944, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}+\frac{ag^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f}-\frac{2g^2(3a^2-b^2)\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}}+\frac{a^2g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^2g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
+ b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f) + (a*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh
[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*f)
- (2*(3*a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^3
*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Elliptic
Pi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b*(b - Sqrt[-
a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^2*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]
]*(3*a - b*Sin[e + f*x]))/(3*b^2*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{(2g^2)\int \frac{-ab-\frac{1}{2}(3a^2-b^2)}{\sqrt{g\cos(e+fx)}\,(}}{3b^2}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{(a(a^2-b^2)\,g^2)\int \frac{}{\sqrt{g\cos(}}}{b}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}\,(3a-b\sin(e+fx))}{3b^2f} + \frac{\left(a^2\sqrt{-a^2+b^2}\,g^2\right)\int \frac{}{\sqrt{g}}}{}$$

$$= -\frac{2(3a^2-b^2)\,g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}} - \frac{2g\sqrt{g\cos(e+fx)}}{3}$$

$$= -\frac{2(3a^2-b^2)\,g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^3f\sqrt{g\cos(e+fx)}} - \frac{a^2\sqrt{-a^2+b^2}\,g^2\sqrt{}}{b^3\left(b\right.}$$

$$= \frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}f} + \frac{a\sqrt[4]{-a^2+b^2}\,g^{3/2}\,\mathrm{t}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 44.78, size = 1909, normalized size = 4.48

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/6*((g*Cos[e + f*x])^(3/2)*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
```

```
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[
e + f*x])) + (3*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2
- I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2
 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*
ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/
2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2,
 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5
/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - C
os[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f
*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log
[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b
^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e +
f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqr
t[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) + (4*b*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*S
qrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2
, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*
AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/(
(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(b*f*Cos[e + f*x]^(3/2)) + (2*
(g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(3*b*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 44.18, size = 888, normalized size = 2.08

method	result	size
default	Expression too large to display	888

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] (-2*g*a/b^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2*g^3*a^3/b^2*sum((_R^4+_R
^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_
R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+
b^2*g^4))-2*g^3*a*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5
*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*co
s(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10
*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-8*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin
(1/2*f*x+1/2*e)^2)^(1/2)*g^2*b*(1/2*(sin(1/2*f*x+1/2*e)^2-1)/b^2*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2
*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*(EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-E
llipticE(cos(1/2*f*x+1/2*e),2^(1/2)))-1/16*(sin(1/2*f*x+1/2*e)^2*a^2-sin(1/
2*f*x+1/2*e)^2*b^2-a^2+b^2)/b^4*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)
*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+
2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+
1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1
/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*
e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b
^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)))/si
n(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{a+b\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x)
```

3.1380 $\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=439

$$-\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}+\frac{\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a\sqrt{b}\,f}-\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}$$

```
[Out] -g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-g^(3/2)*arctanh((g*cos(f*
x+e))^(1/2)/g^(1/2))/a/f+(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*cos(f*x
+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/f/b^(1/2)+(-a^2+b^2)^(1/4)*g^(3/2)*a
rctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/f/b^(1/2)-2
*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+
1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^2*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b-(-a^2+b^2)
^(1/2)))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(
1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)
```

Rubi [A]
time = 0.77, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2977, 2645, 327, 335, 218, 212, 209, 2774, 2946, 2721, 2720, 2781, 2886, 2884, 214, 211}

$$\frac{g^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}\,f}+\frac{g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a\sqrt{b}\,f}+\frac{g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)) + ((-a^2 + b^2)^(1/
4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
])])/(a*Sqrt[b]*f) - (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f)
+ ((-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2
 + b^2)^(1/4)*Sqrt[g])])/(a*Sqrt[b]*f) - (2*g^2*Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt[Cos[e
 + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2
- b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^2*Sqrt
[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b
*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{3/2}}{a(a+b\sin(e+fx))}\right) dx$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{af} - \frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int}{}$$

$$= -\frac{g\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}}}{b}$$

$$= -\frac{(2g)\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} + \frac{\left(\sqrt{-a^2+b^2}\,g^2\right)\int}{}$$

$$= -\frac{2g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{g^2\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g\cos}\right.}{af}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}$$

$$= -\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} + \frac{\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos}}{\sqrt[4]{-a^2}}\right.}{a\sqrt{b}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 16.45, size = 484, normalized size = 1.10

$$\frac{(g\cos(e+fx))^{3/2}\csc(e+fx)\left(8ab^{3/2}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{5}{2}}(e+fx)-5(a^2-b^2)\left(2\sqrt{2}\sqrt[4]{a^2-b^2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\sqrt{2}\sqrt[4]{a^2-b^2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)+4\sqrt{b}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)-2\sqrt{b}\log\left(1-\sqrt{\cos(e+fx)}\right)+2\sqrt{b}\log\left(1+\sqrt{\cos(e+fx)}\right)+\sqrt{2}\sqrt[4]{a^2-b^2}\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)-\sqrt{2}\sqrt[4]{a^2-b^2}\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)}{20a\sqrt{b}\,(a^2-b^2)\,f\cos^{\frac{3}{2}}(e+fx)(b+a\csc(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]*(8*a*b^(3/2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2) - 5
*(a^2 - b^2)*(2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Sqrt[2]*(a^2 - b^2)^(1/4)*ArcTan[1 +
```

```
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*Sqrt[b]*ArcTan[
Sqrt[Cos[e + f*x]]] - 2*Sqrt[b]*Log[1 - Sqrt[Cos[e + f*x]]] + 2*Sqrt[b]*Log
[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - Sq
rt[2]*(a^2 - b^2)^(1/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])*(a + b*Sqrt[Sin[e + f*x]^2]))/(
20*a*Sqrt[b]*(a^2 - b^2)*f*Cos[e + f*x]^(3/2)*(b + a*Csc[e + f*x]))
```

Maple [A]
time = 18.80, size = 216, normalized size = 0.49

method	result
default	$-g^{\frac{3}{2}} \ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)\sqrt{-g}-g^{\frac{3}{2}}\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2*(-g^(3/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)-g^(3/2)*ln(2/(cos(1/2*f*x
+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2
*e)-g))*(-g)^(1/2)+4*g*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*(-g)^(1/2)+2*g^2
*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g)
))/a/(-g)^(1/2)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}}{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

3.1381 $\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=469

$$\frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2f}+\frac{bg^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos}}{}\right.}{a^2f}$$

```
[Out] b*g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f+b*g^(3/2)*arctanh((g*c
os(f*x+e))^(1/2)/g^(1/2))/a^2/f-(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/a^2/f-(-a^2+b^2)^(1/4)*
g^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1
/2)/a^2/f+g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin
(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)-(a^2-b^2
)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b*(b-(
-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x
+e))^(1/2)-g*csc(f*x+e)*(g*cos(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.82, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {2977, 2645, 327, 335, 218, 212, 209, 2647, 2721, 2720, 2774, 2946, 2781, 2886, 2884, 214, 211}

$$-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}+\frac{bg^{3/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{\sqrt{b}\,g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f}-\frac{g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{g^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{bg^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}}-\frac{g\csc(e+fx)\sqrt{g\cos(e+fx)}}{af}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (b*g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f) - (Sqrt[b]*(-a^2 +
 b^2)^(1/4)*g^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^2*f) + (b*g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(
a^2*f) - (Sqrt[b]*(-a^2 + b^2)^(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*f) - (g*Sqrt[g*Cos[e + f*x]]*C
sc[e + f*x])/(a*f) + (g^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*
f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2
*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^2*Sqrt[Cos[e + f*x]]*Ellipti
cPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a
^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{b(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2} + \frac{(g\cos(e+fx))^{3/2}\csc^2(e+\right.$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc^2(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{3/2}\csc(e+}{a^2}$$

$$= \frac{2bg\sqrt{g\cos(e+fx)}}{a^2 f} - \frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{b\text{Subst}\left(\int\right.}{}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{(bg)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{g^2}\right)}dx, x, \right.}{a^2 f}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} - \frac{g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\right|}{af\sqrt{g\cos(e+fx)}}$$

$$= -\frac{g\sqrt{g\cos(e+fx)}\csc(e+fx)}{af} + \frac{g^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\right|}{af\sqrt{g\cos(e+fx)}}$$

$$= \frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2 f} + \frac{bg^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{a^2 f}$$

$$= \frac{bg^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2 f} - \frac{\sqrt{b}\,\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}}{\right.}}{a^2 f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.10, size = 2099, normalized size = 4.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*((g*Cos[e + f*x])^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
```

2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (6*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(a*f*Cos[e + f*x]^(3/2)) - ((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]*Sec[e + f*x])/(a*f)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 75.70, size = 1451, normalized size = 3.09

method	result	size
default	Expression too large to display	1451

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-2*g^3*b*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^
2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*
x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2
)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+2*g^3*b^3/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3
*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/
2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8
-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-g^2*b/a
^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos
(1/2*f*x+1/2*e))+1/2*g^(3/2)*b/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)-1))+1/2*g^(3/2)*b
/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1
/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-2*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*g^2*a*(1/4/a^2/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^
4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(cos(1/2*f*x+1/2*e)*(2*sin(1/2*f*x+1/2*e)
^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))-EllipticE(cos(1/2*f*x+1/2*e),2^(1/2)))-2*sin(1/2*f*x+1/2*e)^4+sin(1/2*
f*x+1/2*e)^2)-1/16*(a^2-b^2)/a^2/b^2*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(
g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2
-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2
-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1
/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f
*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*
e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^
2))-1/8/a^2/g/sin(1/2*f*x+1/2*e)^2/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f
*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*((sin(1/2*f*x+1/2*e)^2)^(1/2)*E
llipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+(sin(
1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*
x+1/2*e)^2-1)^(1/2)-2*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2-2*sin(1/2*f*x
+1/2*e)^2+cos(1/2*f*x+1/2*e)+1)-1/8/a^2/g/sin(1/2*f*x+1/2*e)^2/(2*sin(1/2*f
*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*((s
in(1/2*f*x+1/2*e)^2)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2
*f*x+1/2*e)^2-1)^(1/2)+(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1
/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-2*cos(1/2*f*x+1/2*e)*sin(1/
```

```
2*f*x+1/2*e)^2+2*sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+1/2*e)-1))/sin(1/2*f*x+1/
2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sin(e+fx)^2\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

3.1382 $\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=574

$$\frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}-\frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{b^{3/2}\sqrt[4]{-a^2+b^2}\,g^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(}{\sqrt[4]{-a^2+b}}\right.}{a^3f}$$

```
[Out] 1/4*g^(3/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-b^2*g^(3/2)*arctan((g*
cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+b^(3/2)*(-a^2+b^2)^(1/4)*g^(3/2)*arctan(b^
(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/f+1/4*g^(3/2)*arct
anh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-b^2*g^(3/2)*arctanh((g*cos(f*x+e))^(1
/2)/g^(1/2))/a^3/f+b^(3/2)*(-a^2+b^2)^(1/4)*g^(3/2)*arctanh(b^(1/2)*(g*cos(
f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/f-b*g^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(
1/2)/a^2/f/(g*cos(f*x+e))^(1/2)+b*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e)
)^(1/2)+b*(a^2-b^2)*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/
2)/a^2/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b*g*csc(f*x+e)*(
g*cos(f*x+e))^(1/2)/a^2/f-1/2*g*csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/a/f
```

Rubi [A]
time = 0.90, antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2977, 2645, 327, 335, 218, 212, 209, 2647, 2721, 2720, 294, 2774, 2946, 2781, 2886, 2884, 214, 211}

$$-\frac{b^2g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{bg^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{a^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{bg^2(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{a^2f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{bg^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)|2\right)}{a^2f\sqrt{g\cos(e+fx)}}+\frac{bg\csc(e+fx)\sqrt{g\cos(e+fx)}}{a^2f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^3f}+\frac{b^{3/2}g^{3/2}\sqrt[4]{b^2-a^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^3f}+\frac{g^{3/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}+\frac{g^{3/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}-\frac{g\csc^2(e+fx)\sqrt{g\cos(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (g^(3/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3/2)*ArcTa
n[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^(1/4)*g^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a
^3*f) + (g^(3/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2*g^(3
/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (b^(3/2)*(-a^2 + b^2)^
(1/4)*g^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/(a^3*f) + (b*g*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f) - (g*Sqr
t[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f) - (b*g^2*Sqrt[Cos[e + f*x]]*Ellip
ticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^2)*g^2*Sqr
```

```
t[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(
a^2*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (b*(a^2 - b^
2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x
)/2, 2])/(a^2*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{3/2}\csc^2(e+ \right.$$

$$= \frac{\int (g\cos(e+fx))^{3/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{3/2}\csc^2(e+}{a^2}$$

$$= -\frac{2b^2g\sqrt{g\cos(e+fx)}}{a^3f} + \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{\text{Subst}\Big(}{}$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} +$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} +$$

$$= \frac{bg\sqrt{g\cos(e+fx)}\csc(e+fx)}{a^2f} - \frac{g\sqrt{g\cos(e+fx)}\csc^2(e+fx)}{2af} -$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}$$

$$= \frac{g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} - \frac{b^2g^{3/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 61.53, size = 2129, normalized size = 3.71

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^
2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Ap
pellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
```

- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(-a^2 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Cos[e + f*x]^(3/2)) + ((g*Cos[e + f*x])^(3/2)*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a))*Sec[e +

f*x])/f

Maple [A]
time = 18.09, size = 298, normalized size = 0.52

method	result
default	$-\frac{g\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{8a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}-\frac{g^2\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4a\sqrt{-g}}-\frac{g\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}}{16a\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-1/8*g/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-1/4*g^2/a
/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1
/2*f*x+1/2*e))-1/16*g/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)+1/8*g^(3/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x
+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/8*g^(3/2)/a*ln((4*g*cos
(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2
*f*x+1/2*e)-1))+1/16*g/a/(cos(1/2*f*x+1/2*e)-1)*(-2*sin(1/2*f*x+1/2*e)^2*g+
g)^(1/2))/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}}{{\sin\left(e+f\,x\right)}^3\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

3.1383 $\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=610

$$-\frac{a^3(-a^2+b^2)^{3/4}\,g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{11/2}f}+\frac{a^3(-a^2+b^2)^{3/4}\,g^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{11/2}f}-$$

```
[Out] -a^3*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))/b^(11/2)/f+a^3*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(11/2)/f-2/3*a^3*g*(g*cos(f*
x+e))^(3/2)/b^4/f+2/7*a*(g*cos(f*x+e))^(7/2)/b^2/f/g+2/5*a^2*g*(g*cos(f*x+e
))^(3/2)*sin(f*x+e)/b^3/f+4/45*g*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/b/f-2/9*(g
*cos(f*x+e))^(7/2)*sin(f*x+e)/b/f/g+a^4*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2
)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^
(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^6/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^
(1/2)+a^4*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ell
ipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/
2)/b^6/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2*a^4*g^2*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*
cos(f*x+e))^(1/2)/b^5/f/cos(f*x+e)^(1/2)+6/5*a^2*g^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+
e))^(1/2)/b^3/f/cos(f*x+e)^(1/2)+4/15*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b
/f/cos(f*x+e)^(1/2)
```

Rubi [A]

time = 0.89, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {2977, 2715, 2721, 2719, 2645, 30, 2648, 2774, 2946, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{2a^4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^5f\sqrt{\cos(e+fx)}}-\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f}+\frac{6a^2g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5b^3f\sqrt{\cos(e+fx)}}+\frac{2a^2g\sin(e+fx)(g\cos(e+fx))^{3/2}}{5b^3f}+\frac{a^4g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^6f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^4g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^6f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{a^3g^{5/2}(b^2-a^2)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}+\frac{a^3g^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{11/2}f}+\frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg}+\frac{4g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{15bf\sqrt{\cos(e+fx)}}-\frac{2\sin(e+fx)(g\cos(e+fx))^{7/2}}{9bfg}+\frac{4g\sin(e+fx)(g\cos(e+fx))^{3/2}}{45bf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a^3*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-
a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(11/2)*f)) + (a^3*(-a^2 + b^2)^(3/4)*g^(5/2)
*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(
11/2)*f) - (2*a^3*g*(g*Cos[e + f*x])^(3/2))/(3*b^4*f) + (2*a*(g*Cos[e + f*x
])^(7/2))/(7*b^2*f*g) - (2*a^4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)
/2, 2])/(b^5*f*Sqrt[Cos[e + f*x]]) + (6*a^2*g^2*Sqrt[g*Cos[e + f*x]]*Ellipt
```

```
icE[(e + f*x)/2, 2])/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (4*g^2*Sqrt[g*Cos[e + f
*x]]*EllipticE[(e + f*x)/2, 2])/(15*b*f*Sqrt[Cos[e + f*x]]) + (a^4*(a^2 - b
^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*
x)/2, 2])/(b^6*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^4*(a^2 -
 b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b^6*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + (2*a^2*g*
(g*Cos[e + f*x])^(3/2)*Sin[e + f*x])/(5*b^3*f) + (4*g*(g*Cos[e + f*x])^(3/2
)*Sin[e + f*x])/(45*b*f) - (2*(g*Cos[e + f*x])^(7/2)*Sin[e + f*x])/(9*b*f*g
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{a^2(g\cos(e+fx))^{5/2}}{b^3} - \frac{a(g\cos(e+fx))^{5/2}\sin(e+fx)}{b^2} + \frac{(g\cos\ldots}{\ldots}\right.$$

$$= \frac{a^2\int (g\cos(e+fx))^{5/2}\,dx}{b^3} - \frac{a^3\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^3} - \frac{a\int (g\cos(e+f\ldots}{\ldots}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a^2g(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^3f} - \frac{2(g\ldots}{\ldots}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{2a^2g(g\cos(e+f\ldots}{5b\ldots}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} + \frac{6a^2g^2\sqrt{g\cos(e+\ldots}}{5b^3f\sqrt{\ldots}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+\ldots}}{b^5f\sqrt{c\ldots}}$$

$$= -\frac{2a^3g(g\cos(e+fx))^{3/2}}{3b^4f} + \frac{2a(g\cos(e+fx))^{7/2}}{7b^2fg} - \frac{2a^4g^2\sqrt{g\cos(e+\ldots}}{b^5f\sqrt{c\ldots}}$$

$$= -\frac{a^3(-a^2+b^2)^{3/4}\,g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{11/2}f} + \frac{a^3(-a^2+b^{2}\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.15, size = 790, normalized size = 1.30

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((Sin[e + f*x]*(-(((15*a^4 - 9*a^2*b^2 - 2*b^4)*Csc
[e + f*x]*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)
*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*
ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sq
```

```
rt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Co
s[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])))/(a^2 - b^2)) + ((2 + 2*I)*b^2*(-3*a^3 +
a*b^2)*((4 - 4*I)*a*Sqrt[b]*(-a^2 + b^2)^(1/4)*AppellF1[3/4, 1/2, 1, 7/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*(a
^2 - b^2)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(
1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4
)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e +
 f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])))/((-a^2 + b^2)^(5/4)*S
qrt[Sin[e + f*x]^2]))*(a + b*Sqrt[Sin[e + f*x]^2]))/(12*b^(11/2)*(a + b*Sin
[e + f*x])) + (Cos[e + f*x]^(3/2)*(90*a*b^2*Cos[2*(e + f*x)] + 21*b*(12*a^2
 + b^2)*Sin[e + f*x] - 5*(84*a^3 - 18*a*b^2 + 7*b^3*Sin[3*(e + f*x)])))/(42
*b^4)))/(15*f*Cos[e + f*x]^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 54.83, size = 2207, normalized size = 3.62

method	result	size
default	Expression too large to display	2207

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (16/7*g^2*a/b^2*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-24/
7*g^2*a/b^2*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+12/7*g^
2*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+12/7*g^2*a/
b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3*g^2*a^3/b^4*cos(1/2*f*x+1/2*e)^2
*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3*g^2*a^3/b^4*(2*cos(1/2*f*x+1/2*e)^2
*g-g)^(1/2)+2*g^2*a^3/b^4*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2*g^2*a/b^2*
(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+1/2*g^3*a^5/b^4*sum((_R^6-_R^4*g-_R^2*
g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*l
n((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R
),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z
^2+b^2*g^4))-1/2*g^3*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^
5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b
^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/420*(g*(2
*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*(-6912*cos(1/2*f*x
+1/2*e)^7*sin(1/2*f*x+1/2*e)^2*b^6+1120*cos(1/2*f*x+1/2*e)*a^2*b^4+2240*cos
(1/2*f*x+1/2*e)^5*a^2*b^4-3360*cos(1/2*f*x+1/2*e)^3*a^2*b^4+4864*cos(1/2*f*
x+1/2*e)^5*sin(1/2*f*x+1/2*e)^2*b^6-2400*cos(1/2*f*x+1/2*e)^3*sin(1/2*f*x+1
/2*e)^2*b^6+608*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2*b^6+3840*cos(1/2*f*
x+1/2*e)^9*sin(1/2*f*x+1/2*e)^2*b^6+6912*cos(1/2*f*x+1/2*e)^7*b^6-4864*cos(
```

```
1/2*f*x+1/2*e)^5*b^6+2400*cos(1/2*f*x+1/2*e)^3*b^6-608*cos(1/2*f*x+1/2*e)*b
^6-2240*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e)^2*a^2*b^4+105*sum((sin(1/2*
f*x+1/2*e)^2*(2*_alpha^2*a^2*b^2-2*_alpha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2
*b^2+2*_alpha^2*b^4+a^4-a^2*b^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*
e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e)
,-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*
_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*
e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2)
)*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_al
pha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)
^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+
1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)
/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alp
ha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f
*x+1/2*e)^2))^(1/2)-3840*cos(1/2*f*x+1/2*e)^9*b^6+400*EllipticF(cos(1/2*f*x
+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(
1/2)*sin(1/2*f*x+1/2*e)^2*b^6-1008*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/
2*e)^2*b^6-1680*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2
)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^4*b^2+2240*EllipticF(cos(1/2*f*
x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^
(1/2)*a^2*b^4-1680*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e
)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2*b^4+3360*cos(1/2*f*x+1/2*e
)^3*sin(1/2*f*x+1/2*e)^2*a^2*b^4+1008*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))
*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*b^6-1120*co
s(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2*a^2*b^4-400*EllipticF(cos(1/2*f*x+1/2
*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)
*b^6+1680*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2
)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^4*b^2-2240*Ellip
ticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f
*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^2*b^4+1680*EllipticE(cos(1/2*f*
x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^
(1/2)*sin(1/2*f*x+1/2*e)^2*a^2*b^4)/b^7/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2
*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/
2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^3*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^3*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

3.1384 $\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=501

$$\frac{a^2(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f}-\frac{a^2(-a^2+b^2)^{3/4}g^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f}+$$

```
[Out] a^2*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/b^(9/2)/f-a^2*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*c
os(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(9/2)/f+2/3*a^2*g*(g*cos(f*x+e
))^(3/2)/b^3/f-2/7*(g*cos(f*x+e))^(7/2)/b/f/g-2/5*a*g*(g*cos(f*x+e))^(3/2)*
sin(f*x+e)/b^2/f-a^3*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x
+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos
(f*x+e)^(1/2)/b^5/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^3*(a^2-b^2)
*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x
+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^5/f/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*a^3*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b
^4/f/cos(f*x+e)^(1/2)-6/5*a*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/f/cos(f
*x+e)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2977, 2715, 2721, 2719, 2645, 30, 2774, 2946, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{2a^3g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^4f\sqrt{\cos(e+fx)}}+\frac{a^2g^{5/2}(b^2-a^2)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}+\frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{a^2g^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{9/2}f}-\frac{a^3g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^3g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^5f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{6ag^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5b^2f\sqrt{\cos(e+fx)}}-\frac{2ag\sin(e+fx)(g\cos(e+fx))^{3/2}}{5b^2f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)*f) - (a^2*(-a^2 + b^2)^(3/4)*g^(5/2)*Arc
Tanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(9/2)
*f) + (2*a^2*g*(g*Cos[e + f*x])^(3/2))/(3*b^3*f) - (2*(g*Cos[e + f*x])^(7/2
))/(7*b*f*g) + (2*a^3*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(
b^4*f*Sqrt[Cos[e + f*x]]) - (6*a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*
x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (a^3*(a^2 - b^2)*g^3*Sqrt[Cos[e
```

```
+ f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^5*(b +
 Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*a*g*(g*Cos[e + f*x])^(3/2)*
Sin[e + f*x])/(5*b^2*f)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{a(g\cos(e+fx))^{5/2}}{b^2}+\frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{b}+\frac{a^2(g}{b^2(a}\right.$$

$$= -\frac{a\int (g\cos(e+fx))^{5/2}\,dx}{b^2}+\frac{a^2\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{b^2}+\frac{\int (g\cos(e+f}{}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2ag(g\cos(e+fx))^{3/2}\sin(e+fx)}{5b^2f}-\frac{\text{Subs}}{}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}-\frac{2ag(g\cos(e+fx))}{5b^2f}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}-\frac{6ag^2\sqrt{g\cos(e+f}}{5b^2f\sqrt{\cos}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}+\frac{2a^3g^2\sqrt{g\cos(e+f}}{b^4f\sqrt{\cos}}$$

$$= \frac{2a^2g(g\cos(e+fx))^{3/2}}{3b^3f}-\frac{2(g\cos(e+fx))^{7/2}}{7bfg}+\frac{2a^3g^2\sqrt{g\cos(e+f}}{b^4f\sqrt{\cos}}$$

$$= \frac{a^2(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{9/2}f}-\frac{a^2(-a^2+b^2)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 35.43, size = 824, normalized size = 1.64

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a*(g*Cos[e + f*x])^(5/2)*((-4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*App
ellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*
Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*S
qrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[S
qrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I
*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Co
s[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((5*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3
/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] -
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^
2)*(1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(5*b^3*f*Cos[e + f*x]^(5/2)
) + ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]^2*(-1/42*((-28*a^2 + 9*b^2)*Cos[e
+ f*x])/b^3 - Cos[3*(e + f*x)]/(14*b) - (a*Sin[2*(e + f*x)])/(5*b^2)))/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 44.56, size = 1382, normalized size = 2.76

method	result	size
default	Expression too large to display	1382

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-16/7*g^2/b*cos(1/2*f*x+1/2*e)^6*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+24/7*g
^2/b*cos(1/2*f*x+1/2*e)^4*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7*g^2/b*cos
(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-12/7*g^2/b*(2*cos(1/2*
f*x+1/2*e)^2*g-g)^(1/2)+4/3*g^2/b^3*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2
*e)^2*g-g)^(1/2)*a^2+4/3*g^2/b^3*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)*a^2-2*g
```

```
^2/b^3*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)*a^2+2*g^2/b*(g*(2*cos(1/2*f*x+1
/2*e)^2-1))^(1/2)-1/2*g^3/b^3*a^4*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-
3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^
8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/2*g^
3/b*a^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^
2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)
*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2
-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/40*(g*(2*cos(1/2*f*x+1/2*e)^2-
1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a*(128*cos(1/2*f*x+1/2*e)^7*b^4-256*cos(
1/2*f*x+1/2*e)^5*b^4+160*cos(1/2*f*x+1/2*e)^3*b^4+80*EllipticE(cos(1/2*f*x+
1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1
/2)*a^2*b^2-48*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)
^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*b^4-32*cos(1/2*f*x+1/2*e)*b^4+5*su
m((a^2-b^2)/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2
+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g
*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f
*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x
+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)
^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*
x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)
/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1
/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1
))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g
*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2)
)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))/b^6/(-g*(2*sin(
1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(
1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)^2*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)^2*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

3.1385 $\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=413

$$-\frac{a\left(-a^2+b^2\right)^{3/4} g^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2} f}+\frac{a\left(-a^2+b^2\right)^{3/4} g^{5/2} \tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{7/2} f}-$$

```
[Out] -a*(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))/b^(7/2)/f+a*(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*cos(
f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/f-2/15*g*(g*cos(f*x+e))^(3/
2)*(5*a-3*b*sin(f*x+e))/b^2/f+a^2*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2))
,2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+
a^2*(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticP
i(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4
/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2/5*(5*a^2-3*b^2)*g^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2
))*(g*cos(f*x+e))^(1/2)/b^3/f/cos(f*x+e)^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2944, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{a g^{5/2}\left(b^2-a^2\right)^{3/4} \text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2} f}+\frac{a g^{5/2}\left(b^2-a^2\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{7/2} f}+\frac{a^2 g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4 f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^2 g^3\left(a^2-b^2\right)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4 f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{2g^2\left(5a^2-3b^2\right)E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{5b^3 f\sqrt{\cos(e+fx)}}-\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^
2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*f)) + (a*(-a^2 + b^2)^(3/4)*g^(5/2)*ArcT
anh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*
f) - (2*(5*a^2 - 3*b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
/(5*b^3*f*Sqrt[Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*Ell
ipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b - Sqrt[-a^2
+ b^2])*f*Sqrt[g*Cos[e + f*x]]) + (a^2*(a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^4*(b + Sqrt[-a^
2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (2*g*(g*Cos[e + f*x])^(3/2)*(5*a - 3*b*
Sin[e + f*x]))/(15*b^2*f)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sin(e+fx)}{a+b\sin(e+fx)}\,dx = -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} + \frac{(2g^2)\int \frac{\sqrt{g\cos(e+fx}}{}}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{((5a^2-3b^2)\,g^2)\int \sqrt{g}}{5b^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}(5a-3b\sin(e+fx))}{15b^2f} - \frac{(a^2(a^2-b^2)\,g^3)\int \frac{}{\sqrt{g}}}{}$$

$$= -\frac{2(5a^2-3b^2)\,g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} - \frac{2g(g\cos(e+fx}{}$$

$$= -\frac{2(5a^2-3b^2)\,g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^3f\sqrt{\cos(e+fx)}} + \frac{a^2(a^2-b^2)\,g^3\sqrt{}}{b^4\left(b\right.}$$

$$= -\frac{a(-a^2+b^2)^{3/4}\,g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}f} + \frac{a(-a^2+b^2)^{3/4}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 35.87, size = 737, normalized size = 1.78

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sin[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*Cos[e + f*x]^(3/2)*(-5*a + 3*b*Sin[e + f*x]))/(
3*b^2) + ((5*a^2 - 3*b^2)*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*
x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(
a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 -
b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)
^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^
2]))/(12*b^(7/2)*(-a^2 + b^2)*(a + b*Sin[e + f*x])) + (4*a*((a*AppellF1[3/4
, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f
```

```
*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x]*(a + b*Sqrt[Sin[e + f*
x]^2]))/(b*Sqrt[Sin[e + f*x]^2]*(a + b*Sin[e + f*x]))))/(5*f*Cos[e + f*x]^(
5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 51.91, size = 1597, normalized size = 3.87

method	result	size
default	Expression too large to display	1597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] (-4/3*g^2*a/b^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-4/3
*g^2*a/b^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+2*g^2*a/b^2*(g*(2*cos(1/2*f*x
+1/2*e)^2-1))^(1/2)+1/2*g^3*a^3/b^2*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^
2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x
+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_
Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2*
g^3*a*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-
5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*c
os(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-1
0*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/12*(g*(2*cos(1/2*f*x+1/2*e)^2-1)
*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*(64*cos(1/2*f*x+1/2*e)^5*sin(1/2*f*x+1/2*e
)^2*a^2*b^4-64*cos(1/2*f*x+1/2*e)^5*a^2*b^4-96*cos(1/2*f*x+1/2*e)^3*sin(1/2
*f*x+1/2*e)^2*a^2*b^4-48*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x
+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^4
*b^2+64*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*
(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^2*b^4-48*EllipticE
(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1
/2*e)^2+1)^(1/2)*sin(1/2*f*x+1/2*e)^2*a^2*b^4+96*cos(1/2*f*x+1/2*e)^3*a^2*b
^4+32*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2*a^2*b^4+48*EllipticF(cos(1/2*
f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1
)^(1/2)*a^4*b^2-64*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e
)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*a^2*b^4+48*EllipticE(cos(1/2*f
*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)
^(1/2)*a^2*b^4-32*cos(1/2*f*x+1/2*e)*a^2*b^4-3*sum((sin(1/2*f*x+1/2*e)^2*(2
*_alpha^2*a^2*b^2-2*_alpha^2*b^4-a^4+a^2*b^2)-2*_alpha^2*a^2*b^2+2*_alpha^2
*b^4+a^4-a^2*b^2)/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2
```

```
*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_a
lpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*
b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*E
llipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^
2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2
-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2
-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1
/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*s
in(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-s
in(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^
4*b^2-4*_Z^2*b^2+a^2))*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(
1/2))/b^5/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/a^2/sin(
1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5991 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*sin(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sin(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(e + f*x)*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int((sin(e + f*x)*(g*cos(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x)
```

3.1386 $\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=425

$$\frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{ab^{3/2}f}-\frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e-}}{\sqrt{g}}\right.}{af}$$

```
[Out] g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f-(-a^2+b^2)^(3/4)*g^(5/2)*a
rctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/b^(3/2)/f-g^
(5/2)*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f+(-a^2+b^2)^(3/4)*g^(5/2)*ar
ctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/b^(3/2)/f+(a
^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(
1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b-
(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(
1/2)-2*g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/f/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2977, 2645, 327, 335, 304, 209, 212, 2774, 2946, 2721, 2719, 2780, 2886, 2884, 211, 214}

$$-\frac{g^{5/2}(b^2-a^2)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{g^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{ab^{3/2}f}+\frac{g^{5/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af}-\frac{2g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{bf\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) - ((-a^2 + b^2)^(3/4)*
g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*b^(3/2)*f) - (g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a*f) + (
(-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 +
b^2)^(1/4)*Sqrt[g])])/(a*b^(3/2)*f) - (2*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE
[(e + f*x)/2, 2])/(b*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b - S
qrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(b + Sqr
t[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
```

```
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{(g\cos(e+fx))^{5/2}\csc(e+fx)}{a} - \frac{b(g\cos(e+fx))^{5/2}}{a(a+b\sin(e+fx))}\right)dx$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc(e+fx)\,dx}{a} - \frac{b\int \frac{(g\cos(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx}{a}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3af} - \frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{afg} - \frac{g^2\int$$

$$= -\frac{g\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(e+fx)\right)}{af} - \frac{g^2\int \sqrt{g\cos(e+fx)}\,dx}{b} -$$

$$= -\frac{(2g)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{af} - \frac{((a^2-b^2)g^3)\int}{\sqrt{g}}$$

$$= -\frac{2g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{\cos(e+fx)}} - \frac{g^3\text{Subst}\left(\int \frac{1}{g-x^2}\,dx, x, \sqrt{g}\right.}{af}$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} -$$

$$= \frac{g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af} - \frac{(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g}}{\sqrt[4]{-a^2}}\right.}{ab^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 43.37, size = 484, normalized size = 1.14

$$\frac{(g\cos(e+fx))^{5/2}\csc(e+fx)\left(8ab^{5/2}F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{7}{2}}(e+fx)+7(a^2-b^2)\left(-2\sqrt{2}(a^2-b^2)^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)+2\sqrt{2}(a^2-b^2)^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)+4b^{3/2}\tan^{-1}\left(\sqrt{\cos(e+fx)}\right)+2b^{3/2}\log\left(1-\sqrt{\cos(e+fx)}\right)-2b^{3/2}\log\left(1+\sqrt{\cos(e+fx)}\right)+\sqrt{2}(a^2-b^2)^{3/4}\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)-\sqrt{2}(a^2-b^2)^{3/4}\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\left(a+b\sqrt{\sin^2(e+fx)}\right)}{28ab^{3/2}(a^2-b^2)f\cos^{\frac{5}{2}}(e+fx)(b+a\csc(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x])/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]*(8*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) +
7*(a^2 - b^2)*(-2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqr
t[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*(a^2 - b^2)^(3/4)*ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*b^(3/2)*ArcTa
```

n[Sqrt[Cos[e + f*x]]] + 2*b^(3/2)*Log[1 - Sqrt[Cos[e + f*x]]] - 2*b^(3/2)*L
og[1 + Sqrt[Cos[e + f*x]]] + Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] -
Sqrt[2]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))
/(28*a*b^(3/2)*(a^2 - b^2)*f*Cos[e + f*x]^(5/2)*(b + a*Csc[e + f*x]))

Maple [A]
time = 20.60, size = 257, normalized size = 0.60

method	result
default	$-\frac{3 g^{\frac{5}{2}} \ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right) g+g}-4 g \cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2 g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sqrt{-g}+3 g^{\frac{5}{2}} \ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)

[Out] -1/6/a/(-g)^(1/2)*(3*g^(5/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+3*g^(5/2)*
ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*
g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+8*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*(
-g)^(1/2)*g^2*sin(1/2*f*x+1/2*e)^2+6*g^3*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2
)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))-4*g^2*(-2*sin(1/2*f*x+1/2*e)^2*g+
g)^(1/2)*(-g)^(1/2))/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")

[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)*(a + b*sin(e + f*x))), x)
```

3.1387 $\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=462

$$-\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}+\frac{(-a^2+b^2)^{3/4}\,g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\sqrt{b}\,f}+\frac{bg^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos}}{}\right.}{a^2f}$$

[Out] -b*g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f+b*g^(5/2)*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f-g*(g*cos(f*x+e))^(3/2)*csc(f*x+e)/a/f+(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/f/b^(1/2)-(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/f/b^(1/2)-(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/b/f/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/b/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-g^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/a/f/cos(f*x+e)^(1/2)

Rubi [A]
time = 0.82, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 17, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {2977, 2645, 327, 335, 304, 209, 212, 2647, 2721, 2719, 2774, 2946, 2780, 2886, 2884, 211, 214}

$$\frac{g^{5/2}(b^2-a^2)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{bg^{5/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{g^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2\sqrt{b}\,f}-\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{abf\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{abf\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{bg^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f}-\frac{g^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{af\sqrt{\cos(e+fx)}}-\frac{g\csc(e+fx)(g\cos(e+fx))^{3/2}}{af}$$

Antiderivative was successfully verified.

[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]

[Out] -((b*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f)) + ((-a^2 + b^2)^(3/4)*g^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*Sqrt[b]*f) + (b*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f) - ((-a^2 + b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*Sqrt[b]*f) - (g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a*f) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*f*Sqrt[Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - ((a^2 - b^2)*g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*b*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

0] && GtQ[c + d, 0]

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^2(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(-\frac{b(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^2} + \frac{(g\cos(e+fx))^{5/2}\csc^2(e+\cdots}{a}\right.$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc^2(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{5/2}\csc(e+\cdots}{a^2}$$

$$= \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af} + \frac{b\text{Subst}\left(\cdots\right.}{}$$

$$= -\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af} + \frac{(bg)\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{g^2}}\,dx, x, g\cos(\cdots\right.}{a^2f}$$

$$= -\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af} - \frac{3g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+f\cdots\right.}{af\sqrt{\cos(e+fx)}}$$

$$= -\frac{g(g\cos(e+fx))^{3/2}\csc(e+fx)}{af} - \frac{g^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+f\cdots\right.}{af\sqrt{\cos(e+fx)}}$$

$$= -\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} + \frac{bg^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+f\cdots}}{\sqrt{g}}\right.}{a^2f}$$

$$= -\frac{bg^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f} + \frac{(-a^2+b^2)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\cdots}\right.}{a^2\sqrt{b}\,f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 58.02, size = 1465, normalized size = 3.17

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^2)/(a + b*Sin[e + f*x]),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*(-4*Cos[e + f*x]^(3/2)*Csc[e + f*x] - (5*b*Csc[e +
f*x]*(8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*(2*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*
ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
```

```
])/(a^2 - b^2)^(1/4)] + 4*a^2*ArcTan[Sqrt[Cos[e + f*x]]] - 4*b^2*ArcTan[Sqr
t[Cos[e + f*x]]] + 2*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 2*b^2*Log[1 - Sqrt[C
os[e + f*x]]] - 2*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 2*b^2*Log[1 + Sqrt[Cos[
e + f*x]]] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))*(a + b*Sqrt[Sin[e + f*x]^2]))
/(12*a*(a^2 - b^2)*(b + a*Csc[e + f*x])) + (12*a*((a*AppellF1[3/4, 1/2, 1,
7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))
/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
 f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (
1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/
(Sqrt[b]*(-a^2 + b^2)^(1/4)))*(a + b*Sqrt[Sin[e + f*x]^2]))/((b + a*Csc[e +
 f*x])*Sqrt[Sin[e + f*x]^2]) + ((-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2
)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*S
qrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e +
 f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[
e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1
/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x
]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(
-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a
^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Lo
g[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
b*Cos[e + f*x]])*(a + b*Sqrt[Sin[e + f*x]^2]))/(84*a*Sqrt[b]*(a^2 - b^2)*(a
 + b*Sin[e + f*x]))))/(4*a*f*Cos[e + f*x]^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 79.14, size = 1465, normalized size = 3.17

method	result	size
default	Expression too large to display	1465

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-1/2*g^3*b*sum((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^
2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(
1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2
*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/2*g^3*b^3/a^2*sum((_R^6-_R
^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*
```

```
b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*
2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*
b^2*g^3*_Z^2+b^2*g^4))+g^3*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/
2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))+1/2*g^(5/2)*b/a^2*ln((4*g*co
s(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/
2*f*x+1/2*e)-1))+1/2*g^(5/2)*b/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-
2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))-2*(g*(2*cos(
1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*g^3*a*(-1/4/a^2/cos(1/2*f*x
+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*(cos(1/2*f
*x+1/2*e)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(El
lipticF(cos(1/2*f*x+1/2*e),2^(1/2))-EllipticE(cos(1/2*f*x+1/2*e),2^(1/2)))-
2*sin(1/2*f*x+1/2*e)^4+sin(1/2*f*x+1/2*e)^2)+1/16/a^2/b^2*sum((-a^2+b^2)/_a
lpha*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_al
pha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)
^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/
2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alp
ha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1
/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(c
os(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^
2-4*_Z^2*b^2+a^2))-1/8/a^2/g/sin(1/2*f*x+1/2*e)^2/(2*sin(1/2*f*x+1/2*e)^2-1
)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*((sin(1/2*f*x+1/
2*e)^2)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2
-1)^(1/2)+(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2)
)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-2*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^
2-2*sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+1/2*e)+1)-1/8/a^2/g/sin(1/2*f*x+1/2*e)
^2/(2*sin(1/2*f*x+1/2*e)^2-1)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)
^2*g)^(1/2)*((sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1
/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)+(sin(1/2*f*x+1/2*e)^2)^(1/2)*Elliptic
E(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)-2*cos(1/2*f*
x+1/2*e)*sin(1/2*f*x+1/2*e)^2+2*sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+1/2*e)-1))
/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**2/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^2/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{{\sin\left(e+f\,x\right)}^2\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^2*(a + b*sin(e + f*x))), x)
```

3.1388 $\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=557

$$-\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}+\frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{\sqrt{b}\left(-a^2+b^2\right)^{3/4}g^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{}}{\sqrt[4]{-}}\right.}{a^3f}$$

```
[Out] -3/4*g^(5/2)*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f+b^2*g^(5/2)*arctan((g
*cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+3/4*g^(5/2)*arctanh((g*cos(f*x+e))^(1/2)/
g^(1/2))/a/f-b^2*g^(5/2)*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^3/f+b*g*(g
*cos(f*x+e))^(3/2)*csc(f*x+e)/a^2/f-1/2*g*(g*cos(f*x+e))^(3/2)*csc(f*x+e)^2
/a/f-(-a^2+b^2)^(3/4)*g^(5/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2
)^(1/4)/g^(1/2))*b^(1/2)/a^3/f+(-a^2+b^2)^(3/4)*g^(5/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/a^3/f+(a^2-b^2)*g^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(b-(-a^2+b^2)^(1/2
))/(g*cos(f*x+e))^(1/2)+(a^2-b^2)*g^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*
cos(f*x+e)^(1/2)/a^2/f/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b*g^2*(cos
(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^
(1/2))*(g*cos(f*x+e))^(1/2)/a^2/f/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.86, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2977, 2645, 327, 335, 304, 209, 212, 2647, 2721, 2719, 294, 2774, 2946, 2780, 2886, 2884, 211, 214}

$$\frac{b^2g^{5/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}-\frac{b^2g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f}+\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{g^3(a^2-b^2)\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{a^2f\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{bg^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{a^2f\sqrt{\cos(e+fx)}}+\frac{bg\csc(e+fx)(g\cos(e+fx))^{3/2}}{a^2f}-\frac{\sqrt{b}\,g^{5/2}(b^2-a^2)^{3/4}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^3f}+\frac{\sqrt{b}\,g^{5/2}(b^2-a^2)^{3/4}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^3f}-\frac{3g^{5/2}\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}+\frac{3g^{5/2}\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af}-\frac{g\csc^2(e+fx)(g\cos(e+fx))^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] (-3*g^(5/2)*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) + (b^2*g^(5/2)*Ar
cTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) - (Sqrt[b]*(-a^2 + b^2)^(3/4)*g
^(5/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(a^3*f) + (3*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f) - (b^2
*g^(5/2)*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f) + (Sqrt[b]*(-a^2 +
b^2)^(3/4)*g^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/
4)*Sqrt[g])])/(a^3*f) + (b*g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x])/(a^2*f) -
 (g*(g*Cos[e + f*x])^(3/2)*Csc[e + f*x]^2)/(2*a*f) + (b*g^2*Sqrt[g*Cos[e +
f*x]]*EllipticE[(e + f*x)/2, 2])/(a^2*f*Sqrt[Cos[e + f*x]]) + ((a^2 - b^2)*
```

```
g^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(a^2*(b - Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) + ((a^2 - b^2)*g^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(a^2*(b + Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2647

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[a*(a*Cos[e + f*x])^(m - 1)*((b*Sin[e + f*x])^(n + 1)/
(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\csc^3(e+fx)}{a+b\sin(e+fx)}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^{5/2}\csc(e+fx)}{a^3} - \frac{b(g\cos(e+fx))^{5/2}\csc^2(e+\right.$$

$$= \frac{\int (g\cos(e+fx))^{5/2}\csc^3(e+fx)\,dx}{a} - \frac{b\int (g\cos(e+fx))^{5/2}\csc^2(e}{a^2}$$

Sub

$$= -\frac{2b^2g(g\cos(e+fx))^{3/2}}{3a^3f} + \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} -$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx}{2af}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx}{2af}$$

$$= \frac{bg(g\cos(e+fx))^{3/2}\csc(e+fx)}{a^2f} - \frac{g(g\cos(e+fx))^{3/2}\csc^2(e+fx}{2af}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx}}{\sqrt{g}}\right.}{a^3f}$$

$$= -\frac{3g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af} + \frac{b^2g^{5/2}\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx}}{\sqrt{g}}\right.}{a^3f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 60.63, size = 1590, normalized size = 2.85

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Csc[e + f*x]^3)/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*((g*Cos[e + f*x])^(5/2)*((6*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*A
ppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
```

```
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log
[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^
2]*(b + a*Csc[e + f*x])) - ((3*a^2 - 5*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sq
rt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*A
rcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]]
)/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqr
t[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[C
os[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 -
 Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a
+ b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a
^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1
 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^
2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) +
48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[C
os[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sq
rt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]
*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 -
 b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e
+ f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(a^2*f*Cos[e + f*
x]^(5/2)) + ((g*Cos[e + f*x])^(5/2)*((b*Cot[e + f*x])/a^2 - (Cot[e + f*x]*C
sc[e + f*x])/(2*a))*Sec[e + f*x]^2)/f
```

Maple [A]
time = 20.98, size = 304, normalized size = 0.55

method	result
default	$\frac{\frac{g^2\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{8a\cos\left(\frac{fx}{2}+\frac{e}{2}\right)^2}+\frac{3g^3\ln\left(\frac{-2g+2\sqrt{-g}\,\sqrt{2\left(\cos^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g-g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)}{4a\sqrt{-g}}-\frac{g^2\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}}{16a\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (1/8*g^2/a/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+3/4*g^3/
a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(
1/2*f*x+1/2*e))-1/16*g^2/a/(cos(1/2*f*x+1/2*e)+1)*(-2*sin(1/2*f*x+1/2*e)^2*
g+g)^(1/2)+3/8*g^(5/2)/a*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+3/8*g^(5/2)/a*ln((4*g*
cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(
1/2*f*x+1/2*e)-1))+1/16*g^2/a/(cos(1/2*f*x+1/2*e)-1)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2))/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*g^2*cos(f*x + e)^2*csc(f*x + e)^3/(b*sin(f*x
+ e) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*csc(f*x+e)**3/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*csc(f*x+e)^3/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*csc(f*x + e)^3/(b*sin(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{\sin\left(e+f\,x\right)^3\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/(sin(e + f*x)^3*(a + b*sin(e + f*x))), x)
```

3.1389 $\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}-\frac{2\sqrt{g\cos(e}{bfg}$$

```
[Out] 2/5*(g*cos(f*x+e))^(5/2)/b/f/g^3-a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-
a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)-a^4*arctanh(b^(1
/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(7/2)/(-a^2+b^2)^(3/4)
/f/g^(1/2)-2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(g*cos(f*x+e))^(1/2)-4/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+a^5*(cos(1/2*f*x+
1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a
^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^4/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))
/(g*cos(f*x+e))^(1/2)+a^5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*E
llipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(
1/2)/b^4/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*a^2*(g*cos(f
*x+e))^(1/2)/b^3/f/g-2*(g*cos(f*x+e))^(1/2)/b/f/g+2/3*a*sin(f*x+e)*(g*cos(f
*x+e))^(1/2)/b^2/f/g
```

Rubi [A]
time = 0.98, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2988, 2645, 14, 2648, 2721, 2720, 30, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{2a^3\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\sqrt{g\cos(e+fx)}}-\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg}+\frac{a^5\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^5\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^4f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^4\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{7/2}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{2a\sin(e+fx)\sqrt{g\cos(e+fx)}}{3b^2fg}-\frac{4a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3b^2f\sqrt{g\cos(e+fx)}}+\frac{2(g\cos(e+fx))^{5/2}}{5bfg^3}-\frac{2\sqrt{g\cos(e+fx)}}{bfg}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(7/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*a^2*Sqrt[g*Cos[e + f*x]])/(b^3*f*g) - (2*Sqrt[g*Cos[e + f*x]])/
(b*f*g) + (2*(g*Cos[e + f*x])^(5/2))/(5*b*f*g^3) - (2*a^3*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2])/(b^4*f*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^5*S
qrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])
/(b^4*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^5*Sqrt[
```

```
Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^
4*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (2*a*Sqrt[g*Co
s[e + f*x]]*Sin[e + f*x])/(3*b^2*f*g)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
```

```
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx)}}{b^2}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3b^2fg} + \frac{a^2\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^3} -$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx))^{5/2}}{5bfg^3} + \frac{2a\sqrt{g\cos(e+f}}{3b^2}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx)}{5bfg^3}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx)}{5bfg^3}$$

$$= -\frac{2a^2\sqrt{g\cos(e+fx)}}{b^3fg} - \frac{2\sqrt{g\cos(e+fx)}}{bfg} + \frac{2(g\cos(e+fx)}{5bfg^3}$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{7/2}(-a^2+b^2)^{3/4}f\sqrt{g}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos}}{\sqrt[4]{-a^2+}}\right.}{b^{7/2}(-a^2+b^2)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.84, size = 1953, normalized size = 3.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*(Cos[2*(e + f*x)]/(5*b) + (2*a*Sin[e + f*x])/(3*b^2)))/(f*Sqr
t[g*Cos[e + f*x]]) - (Sqrt[Cos[e + f*x]]*((-2*(10*a^2 - 27*b^2)*(a + b*Sqrt
[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e +
```

```
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Co
s[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4,
3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*
x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 +
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e +
f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 -
 Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((30*a^2 + 27*b^2)*(a + b*Sqrt[1 -
 Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 -
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 +
b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[
Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
- 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f
*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^
2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I
)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/
2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos
[e + f*x]^2)*(a + b*Sin[e + f*x])) + (28*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*
(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2
*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e +
f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]
])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(
1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*S
qrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*
Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Si
n[e + f*x]))))/(60*b^2*f*Sqrt[g*Cos[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 42.81, size = 1086, normalized size = 2.13

method	result	size
default	Expression too large to display	1086

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (8/5/b*cos(1/2*f*x+1/2*e)^4/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/b*cos(
1/2*f*x+1/2*e)^2/g*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-8/5/b/g*(2*cos(1/2*f*
x+1/2*e)^2*g-g)^(1/2)-2/b^3*a^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2*a^
4/b^3*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*
g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+
1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*
_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/24*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*
x+1/2*e)^2)^(1/2)*a*(64*b^4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^4-48*Elli
pticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f
*x+1/2*e)^2-1)^(1/2)*a^2*b^2-32*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(
1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*b^4-32*b^4*cos(1/2
*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2+3*a^2*sum(1/_alpha/(2*_alpha^2-1)*(8*(g*(2
*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2
*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2
)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*si
n(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+
4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*ar
ctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1
/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12
*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_
alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^
(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2
*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2))/b^6/(-g*(2*sin(1/2
*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2
*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1390 $\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=457

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{2a\sqrt{g\cos(e+fx)}}{b^2fg}+\frac{2a^2\sqrt{\cos(e+}}{b^3f\sqrt{}}$$

```
[Out] a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/(
-a^2+b^2)^(3/4)/f/g^(1/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))/b^(5/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)+2*a^2*(cos(1/2*f*x+1/2
*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f
*x+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+4/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/b/f/
(g*cos(f*x+e))^(1/2)-a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*El
lipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1
/2)/b^3/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^4*(cos(1/2*f*
x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/f/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(g*cos(f*x+e))^(1/2)+2*a*(g*cos(f*x+e))^(1/2)/b^2/f/g-2/3*sin(f*x+e)*(g*
cos(f*x+e))^(1/2)/b/f/g
```

Rubi [A]

time = 0.76, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2988, 2648, 2721, 2720, 2645, 30, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}+\frac{2a\sqrt{g\cos(e+fx)}}{b^2fg}-\frac{2\sin(e+fx)\sqrt{g\cos(e+fx)}}{3bfg}+\frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3bf\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[
g]) + (2*a*Sqrt[g*Cos[e + f*x]])/(b^2*f*g) + (2*a^2*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(b^3*f*Sqrt[g*Cos[e + f*x]]) + (4*Sqrt[Cos[e + f*x]]
*EllipticF[(e + f*x)/2, 2])/(3*b*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e
+ f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2
 - b*(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (a^4*Sqrt[Cos[e + f*
x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 - b
*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*
Sin[e + f*x])/(3*b*f*g)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
```

```
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3bfg} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \cdots$$

$$= -\frac{2\sqrt{g\cos(e+fx)}\,\sin(e+fx)}{3bfg} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^3} - \cdots$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3bf\sqrt{g\cos(e+fx)}} - \frac{2\sqrt{g\cdots}}{\cdots}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cdots}}{\cdots}$$

$$= \frac{2a\sqrt{g\cos(e+fx)}}{b^2fg} + \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^3f\sqrt{g\cos(e+fx)}} + \frac{4\sqrt{\cdots}}{\cdots}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+\cdots}}{\sqrt[4]{-a^2+b^2}\,\cdots}\right)}{b^{5/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.22, size = 1915, normalized size = 4.19

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Cos[e + f*x]*Sin[e + f*x])/(3*b*f*Sqrt[g*Cos[e + f*x]]) + (Sqrt[Cos[e +
 f*x]]*((-2*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1
/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Co
s[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF
1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 -
 I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^
2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^
(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos
[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a
^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)
)*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + (3*a*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*
ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/
2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) +
 (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e +
f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
+ Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1
 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b
^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^
2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*
x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*
(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) - (8*b*(a + b*Sqrt[1 - Cos[e
+ f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2
])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1,
9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a
^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
- b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + S
qrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(
a + b*Sin[e + f*x]))))/(6*b*f*Sqrt[g*Cos[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 39.78, size = 934, normalized size = 2.04

method	result	size
default	Expression too large to display	934

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (2*a/b^2/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2*a^3/b^2*g*sum((_R^4+_R^2*
g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=R
ootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2
*g^4))+1/2*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/b^3*(s
in(1/2*f*x+1/2*e)^2-1)*(8*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*
f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*b^2-8*EllipticE(cos(1/2*
f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(sin(1/2*f*x+1/2*e)^2)
^(1/2)*b^2-sum(1/(2*_alpha^2-1)*(8*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha
^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*
e)^2-1)^(1/2)*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^4*b^2-8*Ellip
ticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(sin(1/2*f*x+
1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)*_alpha^2*b^2+2^(1/2)*a^2*_alpha*arctanh(1/2/(-2*sin(1/2*f*x+
1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)
^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*si
n(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-
9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin
(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(sin(1/2*f*x+1/2*e)
^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+
sin(1/2*f*x+1/2*e)^2*g)^(1/2))/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*
e)^2))^(1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
[Out] integrate(sin(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1391 $\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$-\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{2\sqrt{g\cos(e+fx)}}{bfg}-\frac{2a\sqrt{\cos(e+f}}{b^2f\sqrt{g}}$$

```
[Out] -a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(3/4)/f/g^(1/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(3/4)/f/g^(1/2)-2*a*(cos(1/2*f*x+1/2*
e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*
x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(
1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/
2))*cos(f*x+e)^(1/2)/b^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*
x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/f/(a^2-b*(b
+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-2*(g*cos(f*x+e))^(1/2)/b/f/g
```

Rubi [A]
time = 0.59, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2988, 2645, 30, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}+\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{g\cos(e+fx)}}{bfg}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sqrt[g])) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(3/4)*f*Sq
rt[g]) - (2*Sqrt[g*Cos[e + f*x]])/(b*f*g) - (2*a*Sqrt[Cos[e + f*x]]*Ellipti
cF[(e + f*x)/2, 2])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*
EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b
- Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) + (a^3*Sqrt[Cos[e + f*x]]*Elli
pticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b*(b + Sq
rt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
```

```
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= -\frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} + \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx)}}{b^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{a^3\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos\right.}}{2b^2\sqrt{-a^2+b^2}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{(2}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{bfg} - \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{g\cos(e+fx)}} + \frac{a}{b}$$

$$= -\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos}}{\sqrt[4]{-a^2+}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 38.27, size = 572, normalized size = 1.51

$$-\frac{(a^2-b^2)\left(a^2-b^2+b^2\cos^2(e+fx)\right)\sec(e+fx)\sqrt[4]{\sec^2(e+fx)}\tan^3(e+fx)\left(\frac{a^2\tan^{-1}\left(\frac{\sqrt[4]{-a^2+b^2}\sqrt[4]{1+\tan^2(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}(-a^2+b^2)^{3/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt[4]{-a^2+b^2}\sqrt[4]{1+\tan^2(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}(-a^2+b^2)^{3/4}}+\frac{a\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};-\tan^2(e+fx)\right)\tan(e+fx)}{a^2-b^2}-\frac{a^3\cot(e+fx)\Pi\left(-\frac{\sqrt{-a^2+b^2}}{b};\sin^{-1}\left(\sqrt[4]{1+\tan^2(e+fx)}\right)\middle|-1\right)\sqrt{-\tan^2(e+fx)}}{b^2(a^2-b^2)}-\frac{a^3\cot(e+fx)\Pi\left(\frac{\sqrt{-a^2+b^2}}{b};\sin^{-1}\left(\sqrt[4]{1+\tan^2(e+fx)}\right)\middle|-1\right)\sqrt{-\tan^2(e+fx)}}{b^2(a^2-b^2)}-\frac{2}{b\sqrt[4]{1+\tan^2(e+fx)}}\right)}{f\sqrt{g\cos(e+fx)}\,(b+a\csc(e+fx))\left(b(a^2-b^2)\tan^5(e+fx)+a^3\sqrt{-\tan^2(e+fx)}\sqrt{-\sec^2(e+fx)\tan^2(e+fx)}+\tan^2(e+fx)\left(-ab^2\sqrt{-\tan^2(e+fx)}\sqrt{-\sec^2(e+fx)\tan^2(e+fx)}+a^3\left(\sqrt{\sec^2(e+fx)}+\sqrt{-\tan^2(e+fx)}\sqrt{-\sec^2(e+fx)\tan^2(e+fx)}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(((a^2 - b^2)*(a^2 - b^2 + b^2*Cos[e + f*x]^2)*Sec[e + f*x]*(Sec[e + f*x]^
2)^(1/4)*Tan[e + f*x]^3*((a^2*ArcTan[((-a^2 + b^2)^(1/4)*(1 + Tan[e + f*x]^
2)^(1/4))/Sqrt[b]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - (a^2*ArcTanh[((-a^2 + b^
2)^(1/4)*(1 + Tan[e + f*x]^2)^(1/4))/Sqrt[b]])/(b^(3/2)*(-a^2 + b^2)^(3/4))
 + (a*Hypergeometric2F1[1/2, 3/4, 3/2, -Tan[e + f*x]^2]*Tan[e + f*x])/(a^2
- b^2) - (a^3*Cot[e + f*x]*EllipticPi[-(Sqrt[-a^2 + b^2]/b), ArcSin[(1 + Ta
n[e + f*x]^2)^(1/4)], -1]*Sqrt[-Tan[e + f*x]^2])/(b^2*(a^2 - b^2)) - (a^3*C
ot[e + f*x]*EllipticPi[Sqrt[-a^2 + b^2]/b, ArcSin[(1 + Tan[e + f*x]^2)^(1/4
)], -1]*Sqrt[-Tan[e + f*x]^2])/(b^2*(a^2 - b^2)) - 2/(b*(1 + Tan[e + f*x]^2
```

```
)^(1/4))))/(f*Sqrt[g*Cos[e + f*x]]*(b + a*Csc[e + f*x])*(b*(a^2 - b^2)*Tan[
e + f*x]^5 + a^3*Sqrt[-Tan[e + f*x]^2]*Sqrt[-(Sec[e + f*x]^2*Tan[e + f*x]^2
)] + Tan[e + f*x]^2*(-(a*b^2*Sqrt[-Tan[e + f*x]^2]*Sqrt[-(Sec[e + f*x]^2*Ta
n[e + f*x]^2)]) + a^3*(Sqrt[Sec[e + f*x]^2] + Sqrt[-Tan[e + f*x]^2]*Sqrt[-(
Sec[e + f*x]^2*Tan[e + f*x]^2)])))))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 39.48, size = 821, normalized size = 2.16

method	result	size
default	Expression too large to display	821

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (-2/b/g*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)+2*a^2/b*g*sum((_R^4+_R^2*g)/(_
R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf
(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4)
)+1/8*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*a*(16*(sin(
1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticF(cos(1/2
*f*x+1/2*e),2^(1/2))*b^2-sum(1/_alpha/(2*_alpha^2-1)*(8*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_a
lpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)
^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*
(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2*g*(4*_alph
a^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2
+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)
/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))*(-sin(1/2*f*x+1/
2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b
^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2),_alpha
=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x
+1/2*e)^2))^(1/2))/b^4/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(
1/2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
[Out] integrate(sin(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+fx)^2}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1392 $\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=352

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}}-\frac{a^2}{}$$

```
[Out] a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^
(3/4)/f/b^(1/2)/g^(1/2)+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(
1/4)/g^(1/2))/(-a^2+b^2)^(3/4)/f/b^(1/2)/g^(1/2)+2*(cos(1/2*f*x+1/2*e)^2)^(
1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1
/2)/b/f/(g*cos(f*x+e))^(1/2)-a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/b/f/(a^2-b^2+b*(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^2*(cos(
1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*
b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/f/(a^2-b*(b+(-a^2+b^2)^(
1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{a\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bf\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sq
rt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) + (a*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*
x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(3/4)*f*Sqrt[g])
+ (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(b*f*Sqrt[g*Cos[e + f*x]
]) - (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
f*x)/2, 2])/(b*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*Cos[e + f*x]]) - (
a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2
, 2])/(b*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{b}$$

$$= \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2b\sqrt{-a^2+b^2}} + \frac{a^2\int \frac{}{\sqrt{g\,\mathrm{co}}}}{}$$

$$= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{(2ag)\mathrm{Subst}\left(\int \frac{1}{(a^2-b^2)g^2+b^2x^4}\right.}{f}$$

$$= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bf\sqrt{g\cos(e+fx)}} - \frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{}{b-\sqrt{-}}\right.}{b\left(a^2-b^2+b\sqrt{-a^2+b^2}\right.}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{3/4}\,f\sqrt{g}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right.}{\sqrt{b}\,(-a^2+b^2)^{3/4}\,f\sqrt{g}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 9.81, size = 546, normalized size = 1.55

$$-\frac{2\sqrt{\cos(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{a\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)}{4\sqrt{2}\sqrt{b}\,(a^2-b^2)^{3/4}}+\frac{5b(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\sqrt{\cos(e+fx)}\sqrt{\sin^2(e+fx)}}{(a^2-b^2+b^2\cos^2(e+fx))\left(-5(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+2\left(2b^2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+(a^2-b^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)\cos^2(e+fx)\right)}\right)}{f\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*(a + b*Sqrt[Sin[e + f*x]^2])*((a*(-2*ArcTan[1 - (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqr
t[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[S
qrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*C
os[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*b*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Sqrt[Cos[e + f*x]]*Sqrt[Sin[e + f*x]^2])/((a^2 - b^2 + b^2*Cos[e + f*x]^2)
*(-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))))/(f
*Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 38.14, size = 687, normalized size = 1.95

method	result
default	$-2ag\left(\sum_{\_R=\mathrm{RootOf}\left(b^2\_Z^8-4b^2g\_Z^6+\left(16a^2g^2-10b^2g^2\right)\_Z^4-4b^2g^3\_Z^2+b^2g^4\right)}\frac{\left(\_R^4+\_R^2g\right)\ln\left(\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g\cdots}\right.}{\_R^7b^2-3\_R^5b^2g+8\_R^3a^2g^2-5\cdots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] (-2*a*g*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*
g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+
1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*
_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/2*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x
+1/2*e)^2)^(1/2)/b*(sin(1/2*f*x+1/2*e)^2-1)*sum(_alpha/(2*_alpha^2-1)*(8*(s
in(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos
(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b
^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*c
os(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alp
ha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arcta
```

```
nh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos
(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-
2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))
*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2
-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))/a^2/sin(1/2*f*x+1/2*e)
/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,\left(a+b\sin(e+fx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1393 $\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=369

$$-\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}}+\frac{b^{3/2}\tanh^{-1}}{a(}$$

```
[Out] -arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)+b^(3/2)*arctan(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(3/4)/f/g^(1/2)-a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)+b^(3/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(3/4)/f/g^(1/2)-b
*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2
*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/f/(a^2-b*(b-(-a^2+b^
2)^(1/2)))/(g*cos(f*x+e))^(1/2)-b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+
1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(
f*x+e)^(1/2)/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2977, 2645, 335, 218, 212, 209, 2781, 2886, 2884, 214, 211}

$$\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{af\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g])) + (b^(3/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(3/4)*f*Sqrt[g]) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*Sqrt[g]) + (b
^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b - Sqrt[-a^2 + b^2]))
*f*Sqrt[g*Cos[e + f*x]]) - (b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*
Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a\sqrt{g\cos(e+fx)}} - \frac{b}{a\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx)}\right.$$

$$= \frac{\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2+b^2}-b\cos(e+fx)\right)}\,dx}{2\sqrt{-a^2+b^2}} + \frac{b\int}{\sqrt{g\cos}}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g\cos(e+fx)}\right)}{afg} - \frac{(2b^2g)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{-a^2+b^2}};\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b\left(b-\sqrt{-a^2+b^2}\right)\right)f\sqrt{g\cos(e+fx)}} - \frac{b\sqrt{\cos(}}{(a^2-}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{af\sqrt{g}} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+f}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right.}{a\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 28.77, size = 663, normalized size = 1.80

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*(-1/8*(-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*A
rcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]]
 + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sq
rt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Co
s[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*(a^2 - b^2)^(3/4)) + (5
*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((a^2 - b^2 + b^2*Cos[e + f*x]^2)*(5*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
```

```
/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*Sqrt[Sin[e
+ f*x]^2]))*(a + b*Sqrt[Sin[e + f*x]^2]))/(f*Sqrt[g*Cos[e + f*x]]*(a + b*Si
n[e + f*x]))
```

Maple [A]
time = 15.82, size = 185, normalized size = 0.50

method	result
default	$-\frac{\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}\,-4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)+1}\right)\sqrt{-g}+\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right.}{2a\sqrt{-g}\,\sqrt{g}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERBOS
E)
```

```
[Out] -1/2/a/(-g)^(1/2)/g^(1/2)*(ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)+ln(2/(cos(1/
2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*
x+1/2*e)-g))*(-g)^(1/2)-2*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f
*x+1/2*e)^2*g+g)^(1/2)-g))*g^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)\,\sqrt{g\,\cos(e+f\,x)}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1394 $\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=448

$$\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}}-\frac{b^{5/2}\tanh^{-1}}{a^2}$$

```
[Out] b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(1/2)-b^(5/2)*arctan(b^(1/2)
*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(3/4)/f/g^(1
/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(1/2)-b^(5/2)*arctanh(b
^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(3/4)/
f/g^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2
*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(
b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/f/(a^2-b^2+b*(-a^2+b^2)^(1/
2))/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e
)^(1/2)/a/f/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-csc(f*x+e)*(g
*cos(f*x+e))^(1/2)/a/f/g
```

Rubi [A]
time = 0.64, antiderivative size = 448, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2977, 2645, 335, 218, 212, 209, 2650, 2721, 2720, 2781, 2886, 2884, 214, 211}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(b\sqrt{b^2-a^2}+a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{af\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2f\sqrt{g}\,\left(b^2-a^2\right)^{3/4}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}}-\frac{\csc(e+fx)\sqrt{g\cos(e+fx)}}{afg}+\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{af\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sqrt[g]) - (b^(5/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*Sq
rt[g]) - (b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(3/4)*f*Sqrt[g]) - (Sqrt[g*Cos[e + f*x]]*Csc
[e + f*x])/(a*f*g) + (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(a*f*Sq
rt[g*Cos[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a
^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2 + b*Sqrt[-a^2 + b^2])*f*Sqrt[g*C
os[e + f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e +
 f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
```

```
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2\sqrt{g\cos(e+fx)}} + \frac{\csc^2(e+fx)}{a\sqrt{g\cos(e+fx)}} + \frac{}{a^2\sqrt{g\cos(e}}\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^2} + \frac{b^2\int \frac{}{\sqrt{g\,c}}}{}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{2a} - \frac{b^2}{}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{(2b)\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^4}{g^2}}\,dx, x, \sqrt{g}\right.}{a^2fg}$$

$$= -\frac{\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{afg} + \frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\right.}{af\sqrt{g\cos(e+fx)}}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2f\sqrt{g}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right.}{a^2\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 60.64, size = 2093, normalized size = 4.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(Cot[e + f*x]/(a*f*Sqrt[g*Cos[e + f*x]])) - (Sqrt[Cos[e + f*x]]*((4*a*(a +
 b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt
[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos
[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*A
rcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcT
```

```
an[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[
-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*C
os[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e
 + f*x]^2]*(b + a*Csc[e + f*x])) - (b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 -
 Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)
*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqr
t[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
- (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 +
b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*
(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^
2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b
^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1
 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] -
Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*
Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] +
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a
*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]
^2)*(b + a*Csc[e + f*x])) - (6*b*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[
e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1
- Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5
/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e
+ f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1
 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3
/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*
(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 -
Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqr
t[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt
[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a
^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a*f*Sqr
t[g*Cos[e + f*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 77.55, size = 1219, normalized size = 2.72

method	result	size

default	Expression too large to display	1219

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/8*(-4*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-
g)^(1/2)*a-4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)*(-2*sin(1/2*f*x+1/2*e)^4
*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*b*(-4*
sum(1/(_R^6*b^2-3*_R^4*b^2*g+8*_R^2*a^2*g^2-5*_R^2*b^2*g^2-b^2*g^3)*_R*ln((
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R)*(
_R^2+g),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*
g^3*_Z^2+b^2*g^4))*g^(5/2)*(-g)^(1/2)*b^2+2*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(
1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(3/2)-ln(2/(cos(1/2*f*x+1/2*
e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g
))*(-g)^(1/2)*g-ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)
^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*(-g)^(1/2)*g)+(-8*(-2*sin(1/2*f*x+
1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-g)^(1/2)*EllipticF(cos(1
/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2
-1)^(1/2)*a-g^(7/2)*sin(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2-1)^2/a*sum
(1/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1
/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*
f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(
sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos
(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*
b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)
^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/
2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^
2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^
(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=Roo
tOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-g)^(1/2))*cos(1/2*f*x+1/2*e)+8*(-2*sin(1/
2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*g^(3/2)*(-g)^(1/2)*a*sin(1/2
*f*x+1/2*e)^2)/a^2/(-g)^(1/2)/g^(3/2)/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/
2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)/sin(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)^2\,\sqrt{g\,\cos(e+f\,x)}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1395 $\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=557

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}}-\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a^3\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}-\frac{3\tanh}{}$$

```
[Out] -3/4*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)-b^2*arctan((g*cos(f*x
+e))^(1/2)/g^(1/2))/a^3/f/g^(1/2)+b^(7/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/
2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^3/(-a^2+b^2)^(3/4)/f/g^(1/2)-3/4*arctanh((g*
cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(1/2)-b^2*arctanh((g*cos(f*x+e))^(1/2)/g^(
1/2))/a^3/f/g^(1/2)+b^(7/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)
^(1/4)/g^(1/2))/a^3/(-a^2+b^2)^(3/4)/f/g^(1/2)-b*(cos(1/2*f*x+1/2*e)^2)^(1/
2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2
)/a^2/f/(g*cos(f*x+e))^(1/2)-b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/a^2/f/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-b^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a^2/f/(a^2-b*(b+(-a^2+b^
2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b*csc(f*x+e)*(g*cos(f*x+e))^(1/2)/a^2/f/g-1
/2*csc(f*x+e)^2*(g*cos(f*x+e))^(1/2)/a/f/g
```

Rubi [A]
time = 0.69, antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2977, 2645, 335, 218, 212, 209, 2650, 2721, 2720, 296, 2781, 2886, 2884, 214, 211}

$$-\frac{b^2\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}-\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{a^2f\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{a^2f\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{b\csc(e+fx)\sqrt{g\cos(e+fx)}}{a^2fg}-\frac{b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)|2\right)}{a^2f\sqrt{g\cos(e+fx)}}+\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{a^3f\sqrt{g}\left(b^2-a^2\right)^{3/4}}-\frac{3\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}}-\frac{\csc^2(e+fx)\sqrt{g\cos(e+fx)}}{2afg}-\frac{3\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-3*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2*ArcTan[Sqr
t[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTan[(Sqrt[b]*Sqrt
[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 + b^2)^(3/4)*f*
Sqrt[g]) - (3*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(4*a*f*Sqrt[g]) - (b^2
*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^3*f*Sqrt[g]) + (b^(7/2)*ArcTanh[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^3*(-a^2 +
b^2)^(3/4)*f*Sqrt[g]) + (b*Sqrt[g*Cos[e + f*x]]*Csc[e + f*x])/(a^2*f*g) - (
Sqrt[g*Cos[e + f*x]]*Csc[e + f*x]^2)/(2*a*f*g) - (b*Sqrt[Cos[e + f*x]]*Elli
pticF[(e + f*x)/2, 2])/(a^2*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x
```

```
]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*
(b - Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*E
llipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a^2*(a^2 - b*(b +
 Sqrt[-a^2 + b^2]))*f*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \int \left(\frac{b^2\csc(e+fx)}{a^3\sqrt{g\cos(e+fx)}} - \frac{b\csc^2(e+fx)}{a^2\sqrt{g\cos(e+fx)}} + \frac{\csc^3(e+fx)}{a\sqrt{g\cos(e+\ldots}}\right.$$

$$= \frac{\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a} - \frac{b\int \frac{\csc^2(e+fx)}{\sqrt{g\cos(e+fx)}}\,dx}{a^2} + \frac{b^2\int \frac{\csc\ldots}{\sqrt{g\cos\ldots}}}{}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{2a^2} + \frac{b^3\int\ldots}{}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2afg}$$

$$= \frac{b\sqrt{g\cos(e+fx)}\,\csc(e+fx)}{a^2fg} - \frac{\sqrt{g\cos(e+fx)}\,\csc^2(e+fx)}{2afg}$$

$$= -\frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+\ldots}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g\ldots}}\right.}{a^3\left(-a^2+b^2\right)^{3/4}f\sqrt{g}}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{4af\sqrt{g}} - \frac{b^2\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^3f\sqrt{g}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 59.40, size = 2129, normalized size = 3.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]*((b*Csc[e + f*x])/a^2 - Csc[e + f*x]^2/(2*a)))/(f*Sqrt[g*Cos[
e + f*x]]) + (Sqrt[Cos[e + f*x]]*((-2*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
 Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]
*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2
+ b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e
+ f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x
])) - (b^2*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e
+ f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (
10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f
*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4
, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^
2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2)))
+ (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a -
 (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
+ (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2
)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))
))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) -
 (2*(3*a^2 + 3*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x
```

```
]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2
, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^
2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*S
qrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1
+ (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(
3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e +
f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*
Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(
a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/
4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(4*a^2*f*Sqrt[g*Cos[e +
 f*x]])
```

Maple [A]
time = 19.35, size = 301, normalized size = 0.54

method	result
default	$\frac{\frac{\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}}{16ag\left(\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}-\frac{3\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right)}{8a\sqrt{g}}-\frac{\sqrt{2\left(\cos^2\left(\frac{fx}{2}\right.\right.}}{8ag\cos\left(\frac{f}{}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] (1/16/a/g/(cos(1/2*f*x+1/2*e)-1)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-3/8/a/
g^(1/2)*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-2*g)/(cos(1/2*f*x+1/2*e)-1))-1/8/a/g/cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*
f*x+1/2*e)^2*g-g)^(1/2)+3/4/a/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f
*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-1/16/a/g/(cos(1/2*f*x+1/2*e)+1)
*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-3/8/a/g^(1/2)*ln((-4*g*cos(1/2*f*x+1/2
*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+
1)))/f
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^3/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)^3\,\sqrt{g\,\cos(e+f\,x)}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x)
```

3.1396 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=584

$$\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{b^{5/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}}-\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}+\frac{2a^2(g}{3b}$$

[Out] a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(5/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2/3*a^2*(g*cos(f*x+e))^(3/2)/b/(a^2-b^2)/f/g^3-2/3*b*(g*cos(f*x+e))^(3/2)/(a^2-b^2)/f/g^3-2*b/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/(a^2-b^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^3/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-4*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)+2*a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b^2/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)

Rubi [A]

time = 0.84, antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 15, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2981, 2646, 2721, 2719, 2645, 14, 2977, 30, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{2a^2(g\cos(e+fx))^{3/2}}{3bfg^3(a^2-b^2)}-\frac{2b(g\cos(e+fx))^{3/2}}{3fg^3(a^2-b^2)}-\frac{4aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2(a^2-b^2)\sqrt{\cos(e+fx)}}-\frac{2b}{fg(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2a\sin(e+fx)}{fg(a^2-b^2)\sqrt{g\cos(e+fx)}}-\frac{a^5\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3fg(a^2-b^2)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^5\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^3fg(a^2-b^2)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{a^4\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{b^{5/2}fg^{3/2}(b^2-a^2)^{5/4}}+\frac{2a^3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{b^2fg^2(a^2-b^2)\sqrt{\cos(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]

[Out] (a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(5/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a^2*(g*Cos[e + f*x])^(3/2))/(3*b*(a^2 - b^2)*f*g^3) - (2*b*(g*Cos[e + f*x])^(3/2))/(3*(a^2 - b^2)*f*g^3) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) + (2*a^3*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +

```
f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^3*(a^2 -
 b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])
/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e}}{a+b}}{(a^2}$$

$$= \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{(2a)\int \sqrt{g\cos(e+fx)}\,dx}{(a^2-b^2)\,g^2} -$$

$$= \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{a^3\int \sqrt{g\cos(e+fx)}\,dx}{b^2(a^2-b^2)\,g^2} - a$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b(g\cos(e+fx))^{3/2}}{3(a^2-b^2)\,fg^3} - \frac{4a\sqrt{}}{}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b(a^2-b^2)\,fg^3} - \frac{2b}{}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a^2(g\cos(e+fx))^{3/2}}{3b(a^2-b^2)\,fg^3} - \frac{2b}{}$$

$$= \frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{5/2}(-a^2+b^2)^{5/4}fg^{3/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(}}{\sqrt[4]{-a^2+b}}\right)}{b^{5/2}(-a^2+b^2)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 37.11, size = 820, normalized size = 1.40

$$\frac{\cos^2(e+fx)\left(\frac{2\cos(e+fx)}{3b}+\frac{2\sec(e+fx)(-b+a\sin(e+fx))}{a^2-b^2}\right)}{f(g\cos(e+fx))^{3/2}}+\frac{a\cos^{\frac{3}{2}}(e+fx)\left(\frac{4ab\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(\frac{aF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(e+fx)}{3(a^2-b^2)}+\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\left(2\tan^{-1}\left(1-\frac{(1+i)\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}}\right)-2\tan^{-1}\left(1+\frac{(1+i)\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}}\right)-\log\left(\sqrt{-a^2+b^2}-(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(e+fx)}+ib\cos(e+fx)\right)+\log\left(\sqrt{-a^2+b^2}+(1+i)\sqrt{b}\sqrt[4]{-a^2+b^2}\sqrt{\cos(e+fx)}+ib\cos(e+fx)\right)\right)}{\sqrt{b}\sqrt[4]{-a^2+b^2}}\right)\sin(e+fx)}{\sqrt{1-\cos^2(e+fx)}(a+b\sin(e+fx))}-\frac{(a^2-2b^2)\left(a+b\sqrt{1-\cos^2(e+fx)}\right)\left(8b^{5/2}F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos^{\frac{3}{2}}(e+fx)+3\sqrt{2}a(a^2-b^2)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b}\sqrt{\cos(e+fx)}}{\sqrt[4]{a^2-b^2}}\right)-\log\left(\sqrt{a^2-b^2}-\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)+\log\left(\sqrt{a^2-b^2}+\sqrt{2}\sqrt{b}\sqrt[4]{a^2-b^2}\sqrt{\cos(e+fx)}+b\cos(e+fx)\right)\right)\right)\sin^2(e+fx)}{12b^{3/2}(-a^2+b^2)(1-\cos^2(e+fx))(a+b\sin(e+fx))}\right)}{(a-b)b(a+b)f(g\cos(e+fx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^2*((2*Cos[e + f*x])/(3*b) + (2*Sec[e + f*x]*(-b + a*Sin[e + f
*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*((4*
a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b
^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^
2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 +
 b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[
b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a
^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*
x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^(5/2)*AppellF1[
3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e
 + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sq
rt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b
]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b
^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]
]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1 - Cos[e + f*x]^2)*(a + b*Si
n[e + f*x]))))/((a - b)*b*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 57.96, size = 1216, normalized size = 2.08

method	result	size
default	Expression too large to display	1216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (4/3/b/g^2*cos(1/2*f*x+1/2*e)^2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)+4/3/b/g^
2*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2)-2/b/g^2*(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2)+1/2*b/g^2/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin
(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/b/g*a^4/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*
g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*l
```

```
n((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R
),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z
^2+b^2*g^4))-1/2*b/g^2/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(
-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8*a/g*(-32*(-2*sin(1/2*f*x+1/2*e)^4*g+
sin(1/2*f*x+1/2*e)^2*g)^(1/2)*b^4*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2-1
6*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2
*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),
2^(1/2))*a^2*b^2+32*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)
^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(c
os(1/2*f*x+1/2*e),2^(1/2))*b^4+a^2*sum(1/_alpha*(8*(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1
/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alp
ha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-
1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))
*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1
/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^
2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^
2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)
^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2
))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha
^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)
^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*g*sin(1/2*f*x+1/2*e)
^2*(2*sin(1/2*f*x+1/2*e)^2-1))/(a^2-b^2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/
2*f*x+1/2*e)^2))^(1/2)/b^4/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7322 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1397 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=509

$$-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{2a}{\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}}-2a$$

```
[Out] -a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(5/4)/f/g^(3/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2*a/(a^2-b^2)/f/g/(g*
cos(f*x+e))^(1/2)-2*b*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+a^4*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g/(b-(-a
^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/
2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2)
)*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1
/2)-2*a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2
*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/b/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2
)+4*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x
+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2981, 2645, 30, 2646, 2721, 2719, 2946, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{2a^2E\left(\frac{1}{2}(e+fx)|2\right)\sqrt{g\cos(e+fx)}}{bfg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{4bE\left(\frac{1}{2}(e+fx)|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2a}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sin(e+fx)}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{b^2fg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{b^2fg\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{3/2}\left(b^2-a^2\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(5/4)*f*g^
(3/2)) + (2*a)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a^2*Sqrt[g*Cos[e
 + f*x]]*EllipticE[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]
) + (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2
*Sqrt[Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*S
qrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f
*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
```

```
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e+}}{a+b\sin}}{(a^2-}$$

$$= -\frac{2b\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int \sqrt{g\cos(e+fx)}\,dx}{b\,(a^2-b^2)\,g^2} + \cdots$$

$$= \frac{2a}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \cdots$$

$$= \frac{2a}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+\cdots\right.}{b\,(a^2-b^2)\,fg^2\sqrt{\cos(e+f\cdots}}$$

$$= \frac{2a}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2a^2\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+\cdots\right.}{b\,(a^2-b^2)\,fg^2\sqrt{\cos(e+f\cdots}}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{5/4} fg^{3/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(\cdots}}{\sqrt[4]{-a^2+b\cdots}}\right.}{b^{3/2}\left(-a^2+b^2\right)^{5/4}\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 37.16, size = 793, normalized size = 1.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) - (Cos[e + f*x]^(3/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appell
```

```
F1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos
[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt
[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]
*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*S
qrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqrt
[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*
Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e
 + f*x]^2]*(a + b*Sin[e + f*x])) - ((a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f
*x]^2])*(8*b^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(
2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*Ar
cTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt
[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[
e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Co
s[e + f*x]] + b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*b^(3/2)*(-a^2 + b^2)*(1
 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*
x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 67.40, size = 1023, normalized size = 2.01

method	result	size
default	Expression too large to display	1023

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-1/2/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)+1/2/g*a^3/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+g^3
)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*s
in(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=Ro
otOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*
g^4))+1/2/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(
1/2*f*x+1/2*e)^2*g+g)^(1/2)+32*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/
2*e)^2)^(1/2)/g*b*(1/8*(sin(1/2*f*x+1/2*e)^2-1)/b^2*(sin(1/2*f*x+1/2*e)^2)^
(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2
*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/128*a^2/(a-b)
/(a+b)/b^4*sum((-2*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+sin(1/2*f*x+1/2*e)^2*a
^2+2*b^2*_alpha^2-a^2)/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^
2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f
*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2
)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(
1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin
```

```
(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alph
a^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/8*(-sin(1/2*f*
x+1/2*e)^2+1)/(a^2-b^2)*(-(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e
)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*Ellip
ticE(cos(1/2*f*x+1/2*e),2^(1/2))+2*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1
/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2)/g/sin(1/2*f*x+1/2
*e)^2/(2*sin(1/2*f*x+1/2*e)^2-1))/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*
e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1398 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=453

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{5/4}\,fg^{3/2}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{5/4}\,fg^{3/2}}-\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}-\frac{2a\sqrt{\ }}{(}$$

```
[Out] a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2
)^(5/4)/f/g^(3/2)/b^(1/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^
2)^(1/4)/g^(1/2))/(-a^2+b^2)^(5/4)/f/g^(3/2)/b^(1/2)-2*b/(a^2-b^2)/f/g/(g*c
os(f*x+e))^(1/2)+2*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-a^3*(cos
(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2
*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g/(b-(-a^2+
b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*c
os(f*x+e)^(1/2)/b/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-2
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/
2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A]

time = 0.55, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2981, 2716, 2721, 2719, 2645, 30, 2780, 2886, 2884, 335, 304, 211, 214}

$$\frac{a^2\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2a\sin(e+fx)}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bfg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bfg\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(5/4)*f*g^(3/
2)) - (2*b)/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*
x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e + f*x]]) - (a^
3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2,
2])/(b*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) - (a^3*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*Si
n[e + f*x])/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\sqrt{g\cos(e}{a+b\sin(e}}{(a^2-b^2)}$$

$$a^3$$

$$= \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{a\int\sqrt{g\cos(e+fx)}\,dx}{(a^2-b^2)\,g^2} + -$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a\sin(e+fx)}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx}}$$

$$= -\frac{2b}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2a\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e\right.}{(a^2-b^2)\,fg^2\sqrt{\cos(e+}}$$

$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{5/4}\,fg^{3/2}} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(}}{\sqrt[4]{-a^2+b}}\right.}{\sqrt{b}\,(-a^2+b^2)^{5/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 28.97, size = 785, normalized size = 1.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2
)) - (a*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appe
llF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*C
os[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)
*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sq
rt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*
b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos
[e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b
^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
 (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sq
```

```
rt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2]
- Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] +
Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]]
+ b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f
*x]^2)*(a + b*Sin[e + f*x])))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 52.02, size = 1057, normalized size = 2.33

method	result	size
default	Expression too large to display	1057

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (1/2*b/g^2/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f
*x+1/2*e)^2*g+g)^(1/2)-1/2*b/g*a^2/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+g^
3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*
sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=R
ootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2
*g^4))-1/2*b/g^2/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin
(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/8*a/g*(32*cos(1/2*f*x+1/2*e)^3*(-g*(2*sin(1/
2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*b^2+16*(g*(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))*(si
n(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*b^2-32*cos(1/2*
f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*b^2-sum
(1/_alpha*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)
*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alph
a^2*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-
4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^
(1/2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^
2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_a
lpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/
2*e)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))
/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1
/2*f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(g*(2*c
os(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1
/2*e)^2))^(1/2)/(a^2-b^2)/b^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2
-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-sqrt(g*cos(f*x + e))*(cos(f*x + e)^2 - 1)/(b*g^2*cos(f*x + e)^2*s
in(f*x + e) + a*g^2*cos(f*x + e)^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3067 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+fx)^2}{(g\cos(e+fx))^{3/2}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1399 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=413

$$-\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+\right.}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+}}$$

```
[Out] -a*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/(-
a^2+b^2)^(5/4)/f/g^(3/2)+a*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^
(1/4)/g^(1/2))*b^(1/2)/(-a^2+b^2)^(5/4)/f/g^(3/2)+2*(a-b*sin(f*x+e))/(a^2-b
^2)/f/g/(g*cos(f*x+e))^(1/2)+a^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f
*x+e)^(1/2)/(a^2-b^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+a^2*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g/(b+(-a^2+b
^2)^(1/2))/(g*cos(f*x+e))^(1/2)+2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^
2)/f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2945, 2946, 2721, 2719, 2780, 2886, 2884, 335, 304, 211, 214}

$$-\frac{a\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2(a-b\sin(e+fx))}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt
[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2))) + (a*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[g
*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(5/4)*f*g^(3/2
)) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^
2*Sqrt[Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[
-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt
[g*Cos[e + f*x]]) + (a^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*C
os[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*
x]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2\int \frac{\sqrt{g\cos(e+fx)}\,(-ab-\frac{1}{2}b^2 \cdots}{a+b\sin(e+fx)}}{(a^2-b^2)\,g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{b\int\sqrt{g\cos(e+fx)}\,dx}{(a^2-b^2)\,g^2} + \frac{(a \cdots}{\cdots}$$

$$= \frac{2(a-b\sin(e+fx))}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}\left(\sqrt{-a^2 \cdots}\right.}}{2\,(a^2-b^2) \cdots}$$

$$= \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{(a^2-b^2)\,fg^2\sqrt{\cos(e+fx)}} + \frac{2(a-b\sin(e+f \cdots}{(a^2-b^2)\,fg\sqrt{g\cos(e \cdots}}$$

$$= \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{(a^2-b^2)\,fg^2\sqrt{\cos(e+fx)}} + \frac{a^2\sqrt{\cos(e+fx)}\,\Pi\left(\cdots \right.}{(a^2-b^2)\left(b-\sqrt{-a^2 \cdots}\right.}$$

$$= -\frac{a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{(-a^2+b^2)^{5/4}\,fg^{3/2}} + \frac{a\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b} \cdots}{\sqrt[4]{\cdots}}\right.}{(-a^2+b^2 \cdots}$$

__Mathematica__ [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 28.72, size = 783, normalized size = 1.90

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e + f*x])^(3/2)
) + (b*Cos[e + f*x]^(3/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[
e + f*x]^2]*(a + b*Sin[e + f*x])) - ((a + b*Sqrt[1 - Cos[e + f*x]^2])*(8*b^
```

```
(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Cos[e + f*x]^(3/2) + 3*Sqrt[2]*a*(a^2 - b^2)^(3/4)*(2*ArcTan[1 -
(Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] -
 Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + L
og[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] +
 b*Cos[e + f*x]]))*Sin[e + f*x]^2)/(12*Sqrt[b]*(-a^2 + b^2)*(1 - Cos[e + f*
x]^2)*(a + b*Sin[e + f*x]))))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 57.18, size = 1104, normalized size = 2.67

method	result	size
default	Expression too large to display	1104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] (-1/2/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*
f*x+1/2*e)^2*g+g)^(1/2)+1/2/g*a*b^2/(a-b)/(a+b)*sum((_R^6-_R^4*g-_R^2*g^2+g
^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2
*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=
RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^
2*g^4))+1/2/g^2*a/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/4*(sin(1/2*f*x+1/2*e)^2-1)/b/g*(32*cos(1/2*
f*x+1/2*e)^3*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*a^2*b
^2+16*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*(g*(2*
cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f*x+1
/2*e),2^(1/2))*a^2*b^2-32*cos(1/2*f*x+1/2*e)*(-g*(2*sin(1/2*f*x+1/2*e)^4-si
n(1/2*f*x+1/2*e)^2))^(1/2)*a^2*b^2-sum((2*_alpha^2*b^2-a^2)/_alpha/(2*_alph
a^2-1)*(8*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*El
lipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2
*b^2+a^2-2*b^2)/b^2)^(1/2)*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)
^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b
^2/a^2*(_alpha^2-1),2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/
2)*a^2*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a
^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alph
a^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e
)^2))^(1/2))*(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2))/(g
*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*
f*x+1/2*e)^2-1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(g*(2*cos(
1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2))/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*
```

```
e)^2))^(1/2)/(a^2-b^2)/a^2/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1)
)^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral(sqrt(g*cos(f*x + e))*sin(f*x + e)/(b*g^2*cos(f*x + e)^2*sin(f*x +
e) + a*g^2*cos(f*x + e)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{3/2}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1400 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=507

$$\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{5/4}fg^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}}+\frac{b^{5/2}\tanh^{-1}}{a(}$$

```
[Out] arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(3/2)-b^(5/2)*arctan(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(5/4)/f/g^(3/2)-ar
ctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(3/2)+b^(5/2)*arctanh(b^(1/2)*(g*
cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(5/4)/f/g^(3/2)+2/
a/f/g/(g*cos(f*x+e))^(1/2)+2*b*(b-a*sin(f*x+e))/a/(a^2-b^2)/f/g/(g*cos(f*x+
e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(si
n(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^
2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)+b^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(
1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g/(b+(-a^2+b^2)^(1/2))/(g*cos(f
*x+e))^(1/2)+2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(
sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/cos(f*x+e)
^(1/2)
```

Rubi [A]
time = 0.88, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2977, 2645, 331, 335, 304, 209, 212, 2775, 2946, 2721, 2719, 2780, 2886, 2884, 211, 214}

$$-\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{2bE\left(\frac{1}{2}(e+fx)|2\right)\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}+\frac{2b(b-a\sin(e+fx))}{afg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{fg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}+\frac{b^2\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)|2\right)}{fg\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{3/2}\left(b^2-a^2\right)^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}}+\frac{2}{afg\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) - (b^(5/2)*ArcTan[(Sqrt[
b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^(5/
4)*f*g^(3/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(3/2)) + (b^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
a*(-a^2 + b^2)^(5/4)*f*g^(3/2)) + 2/(a*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqr
t[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/((a^2 - b^2)*f*g^2*Sqrt[Cos[e
+ f*x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]),
 (e + f*x)/2, 2])/((a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*
x]]) + (b^2*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e
+ f*x)/2, 2])/((a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e + f*x]])
 + (2*b*(b - a*Sin[e + f*x]))/(a*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
```

```
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{3/2}} - \frac{b}{a(g\cos(e+fx))^{3/2}(a+b\sin(e+\ldots}\right.$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} + \frac{(2b)\int \frac{\sqrt{g\cos(e+fx)}\left(\frac{a^2}{2}+\ldots\right.}{a+b\sin(e+\ldots}}{a\left(a^2-b^2\right)\ldots}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{\text{Sub}\ldots}{\ldots}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b(b-a\sin(e+fx))}{a\left(a^2-b^2\right)fg\sqrt{g\cos(e+fx)}} - \frac{2\text{Su}\ldots}{\ldots}$$

$$= \frac{2}{afg\sqrt{g\cos(e+fx)}} + \frac{2b\sqrt{g\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\left(a^2-b^2\right)fg^2\sqrt{\cos(e+fx)}} \ldots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} + \ldots$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{3/2}} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx\ldots}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}\ldots}\right.}{a\left(-a^2+b^2\right)^{5/4}fg^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 58.85, size = 1587, normalized size = 3.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -1/2*(Cos[e + f*x]^(3/2)*((8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((a*Appel
lF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Co
s[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b
]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - (1 + I)*
```

```
Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] + Log[Sqr
t[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b
*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Cos[e + f*x]^2]*(
b + a*Csc[e + f*x])) - ((-2*a^2 + b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1
- Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan
[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - 6*Sqrt[2]*Sq
rt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^
2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] + 8*a*b*AppellF
1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[
e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^2*Log[1 - Sqrt[Cos
[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*Log[1 + Sqrt[Cos[e
+ f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] - Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + 3*Sqrt[2
]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^
2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3 - a*b^2)*(1 - Cos[
e + f*x]^2)*(b + a*Csc[e + f*x])) - (Sqrt[b]*(-1 + Cos[e + f*x]^2)*(a + b*S
qrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*(-42*Sqrt[2]*(a^2 -
b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a
^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 + (S
qrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 84*b^(3/2)*(a^2 - b
^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*AppellF1[3/4, 1/2, 1, 7/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 48*a*
b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2 - b^2)*Log[1 - Sqrt[Cos[e
+ f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[Cos[e + f*x]]] + 21*Sqrt[2]
*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2
 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] - 21*Sqrt[2]*(a^2 - b^2)
^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4
)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*(a^3 - a*b^2)*(1 - Cos[e + f*x
]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/((a - b)*(a + b)*f*(g*
Cos[e + f*x])^(3/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f
*(g*Cos[e + f*x])^(3/2))
```

Maple [A]

time = 28.87, size = 425, normalized size = 0.84

method	result
default	$-\frac{\left(4 g^{\frac{5}{2}} \ln \left(\frac{2\sqrt{-g}\,\sqrt{-2\left(\sin^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right) g+g}-2 g}{\cos\left(\frac{f x}{2}+\frac{e}{2}\right)}\right)+2\sqrt{-g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{f x}{2}+\frac{e}{2}\right)\right) g+g}+4 g \cos \cdots}{\cos\left(\frac{f x}{2}+\frac{e}{2}\right)-1}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
```

E)

[Out] 1/2*(-(4*g^(5/2)*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))+2*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^2+2*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*g^2)*sin(1/2*f*x+1/2*e)^2+2*g^(5/2)*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))-4*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)*(-g)^(1/2)*g^(3/2)+(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^2+(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*g^2)/g^(7/2)/(-g)^(1/2)/a/(2*sin(1/2*f*x+1/2*e)^2-1)/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{\frac{3}{2}}(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(csc(e + f*x)/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1401 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=627

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{5/4}fg^{3/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}-\frac{b^{7/2}\,\mathrm{ta}}{}$$

```
[Out] -b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(3/2)+b^(7/2)*arctan(b^(1/2
)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(5/4)/f/g^(
3/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(3/2)-b^(7/2)*arctanh(
b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(5/4)
/f/g^(3/2)-2*b/a^2/f/g/(g*cos(f*x+e))^(1/2)-csc(f*x+e)/a/f/g/(g*cos(f*x+e))
^(1/2)+3*sin(f*x+e)/a/f/g/(g*cos(f*x+e))^(1/2)-2*b^2*(b-a*sin(f*x+e))/a^2/(
a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*
f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*
cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g/(b-(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-
b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+
1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g/(
b+(-a^2+b^2)^(1/2))/(g*cos(f*x+e))^(1/2)-3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/
a/f/g^2/cos(f*x+e)^(1/2)-2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2
*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))*(g*cos(f*x+e))^(1/2)/a/(a^2-b^2)/
f/g^2/cos(f*x+e)^(1/2)
```

Rubi [A]
time = 0.95, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2977, 2645, 331, 335, 304, 209, 212, 2650, 2716, 2721, 2719, 2775, 2946, 2780, 2886, 2884, 211, 214}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\left(a^2-b^2\right)\sqrt{\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+fx))}{a^2fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{3/2}\left(b^2-a^2\right)^{5/4}}-\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{afg\left(a^2-b^2\right)\left(b-\sqrt{b^2-a^2}\right)\sqrt{g\cos(e+fx)}}-\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{afg\left(a^2-b^2\right)\left(\sqrt{b^2-a^2}+b\right)\sqrt{g\cos(e+fx)}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}}-\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}}-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{g\cos(e+fx)}}{afg^2\sqrt{\cos(e+fx)}}+\frac{3\sin(e+fx)}{afg\sqrt{g\cos(e+fx)}}-\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(3/2))) + (b^(7/2)*ArcT
an[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2
 + b^2)^(5/4)*f*g^(3/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f
*g^(3/2)) - (b^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(
1/4)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(5/4)*f*g^(3/2)) - (2*b)/(a^2*f*g*Sqrt[g*
Cos[e + f*x]]) - Csc[e + f*x]/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (3*Sqrt[g*Cos[
e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*f*g^2*Sqrt[Cos[e + f*x]]) - (2*b^2*
```

```
Sqrt[g*Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])/(a*(a^2 - b^2)*f*g^2*Sqrt[C
os[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b
^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b - Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos
[e + f*x]]) - (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(b + Sqrt[-a^2 + b^2])*f*g*Sqrt[g*Cos[e
 + f*x]]) + (3*Sin[e + f*x])/(a*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b^2*(b - a*S
in[e + f*x]))/(a^2*(a^2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
```

```
2*m, 2*p]
```

Rule 2780

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[a*(g/(2*b)), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[a*(g/(2*b)), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[b*(g/f), Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{3/2}} + \frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{3/2}} + \frac{}{a^2(g\cos(}\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{3/2}}\,dx}{a^2} + \frac{b^2\int \frac{}{(g\cos(e+fx))^{3/2}}}{a^2}$$

$$= -\frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{2b^2(b-a\sin(e+fx))}{a^2(a^2-b^2)fg\sqrt{g\cos(e+fx)}} + $$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} + \frac{3\sin(e}{afg\sqrt{g\,\mathrm{co}}}$$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} + \frac{3\sin(e}{afg\sqrt{g\,\mathrm{co}}}$$

$$= -\frac{2b}{a^2fg\sqrt{g\cos(e+fx)}} - \frac{\csc(e+fx)}{afg\sqrt{g\cos(e+fx)}} - \frac{3\sqrt{g\cos(e}}{af}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right.}{a^2fg^{3/2}}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{3/2}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+}}{\sqrt[4]{-a^2+b^2}\sqrt{}}\right.}{a^2(-a^2+b^2)^{5/4}fg^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 60.39, size = 1635, normalized size = 2.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -1/4*(Cos[e + f*x]^(3/2)*((-2*(6*a^3 + 2*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x
]^2])*((a*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Cos[e + f*x]^(3/2))/(3*(a^2 - b^2)) + ((1/8 + I/8)*(2*ArcTan[1
 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 +
((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
```

```
b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e +
f*x]] + Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[
e + f*x]] + I*b*Cos[e + f*x]]))/(Sqrt[b]*(-a^2 + b^2)^(1/4))))/(Sqrt[1 - Co
s[e + f*x]^2]*(b + a*Csc[e + f*x])) - ((7*a^2*b - 5*b^3)*(-1 + Cos[e + f*x]
^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*(6*Sqrt[2]*Sqrt[b]*(a^2 -
 b^2)^(3/4)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)] - 6*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[
Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 12*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]
]] + 8*a*b*AppellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Cos[e + f*x]^(3/2) + 6*a^2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*b^
2*Log[1 - Sqrt[Cos[e + f*x]]] - 6*a^2*Log[1 + Sqrt[Cos[e + f*x]]] + 6*b^2*L
og[1 + Sqrt[Cos[e + f*x]]] - 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a
^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e
+ f*x]] + 3*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)*Log[Sqrt[a^2 - b^2] + Sqrt[2]
*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(12*(a^3
- a*b^2)*(1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - ((-3*a^2*b + b^3)*(-1
 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e
+ f*x]*(-42*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqr
t[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 42*Sqrt[2]*(a^2 - b^2)^(3/4)*
(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(
1/4)] + 84*b^(3/2)*(a^2 - b^2)*ArcTan[Sqrt[Cos[e + f*x]]] - 56*a*b^(5/2)*Ap
pellF1[3/4, 1/2, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
*Cos[e + f*x]^(3/2) + 48*a*b^(5/2)*AppellF1[7/4, 1/2, 1, 11/4, Cos[e + f*x]
^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(7/2) + 42*b^(3/2)*(a^2
 - b^2)*Log[1 - Sqrt[Cos[e + f*x]]] + 42*b^(3/2)*(-a^2 + b^2)*Log[1 + Sqrt[
Cos[e + f*x]]] + 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 -
b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x
]] - 21*Sqrt[2]*(a^2 - b^2)^(3/4)*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[
2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(84*b^(
3/2)*(a^3 - a*b^2)*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[
e + f*x]))))/(a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)) + (Cos[e + f*x]^2
*(-(Cot[e + f*x]/a) + (2*Sec[e + f*x]*(-b + a*Sin[e + f*x]))/(a^2 - b^2)))/
(f*(g*Cos[e + f*x])^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 111.43, size = 1523, normalized size = 2.43

method	result	size
default	Expression too large to display	1523

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-1/g^2*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)+1/2
*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/2/g*b^5/(a-b)/(a+b)/a^2*sum
```

```
((_R^6-_R^4*g-_R^2*g^2+g^3)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^
2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*
x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2
)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/g*b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*
(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/cos(1/2*f*x+1/2*e))-1/g^(3/2)*b/(2+2^(1
/2))/(2^(1/2)-2)/a^2*ln((4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1
/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)-1))-1/g^(3/2)*b/(2+2^(1/2))/(2^
(1/2)-2)/a^2*ln((-4*g*cos(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)-2*g)/(cos(1/2*f*x+1/2*e)+1))+1/g^2*b/(2+2^(1/2))/(2^(1/2)-2)/(a
^2-b^2)*2^(1/2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)+1/8*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^3
/a/cos(1/2*f*x+1/2*e)/sin(1/2*f*x+1/2*e)^5/(2*sin(1/2*f*x+1/2*e)^2-1)^2/(a^
2-b^2)*(-4*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*a^2+4*(
-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*b^2-16*(-2*sin(1/2*
f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(3*a^2-b^2)*sin(1/2*f*x+1/2*e)
^4+16*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(3*a^2-b^2)*
sin(1/2*f*x+1/2*e)^2+cos(1/2*f*x+1/2*e)*(-24*(-2*sin(1/2*f*x+1/2*e)^4*g+sin
(1/2*f*x+1/2*e)^2*g)^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*EllipticE(cos(1/2*f
*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*a^2+8*(-2*sin(1/2*f*x+1
/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(3/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*Ellipt
icE(cos(1/2*f*x+1/2*e),2^(1/2))*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*b^2+g^2*si
n(1/2*f*x+1/2*e)^4*(2*sin(1/2*f*x+1/2*e)^2-1)^2/a^2*sum(1/_alpha*(8*(g*(2*_
alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f
*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/
a^2,2^(1/2))*_alpha^3*b^2-8*b^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(
1/2*f*x+1/2*e)^2-1)^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*
b^2)/a^2,2^(1/2))*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arct
anh(1/2/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alp
ha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2
*e)^2*_alpha^2*a^2+12*sin(1/2*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*s
in(1/2*f*x+1/2*e)^2*a^2-9*sin(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_al
pha^2-3*a^2+3*b^2))*(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1
/2))/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*s
in(1/2*f*x+1/2*e)^2+1))^(1/2),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*b^2
))/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{\frac{3}{2}}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(csc(e + f*x)**2/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)^2\,(g\,\cos(e+f\,x))^{3/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x)
```

3.1402 $\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=601

$$-\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{7/4}fg^{5/2}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}\left(-a^2+b^2\right)^{7/4}fg^{5/2}}-\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}}+$$

```
[Out] -a^4*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/b^(3/2)/
(-a^2+b^2)^(7/4)/f/g^(5/2)-a^4*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b
^2)^(1/4)/g^(1/2))/b^(3/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)-2/3*b/(a^2-b^2)/f/g/(
g*cos(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)-4/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+2*a^3*(
cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e)
,2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)-a^5*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(a^2
-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^5*(cos(1/2*f*x+1/2*e)^2)^(1
/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2
)),2^(1/2))*cos(f*x+e)^(1/2)/b^2/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)
))/(g*cos(f*x+e))^(1/2)+2*a^2*(g*cos(f*x+e))^(1/2)/b/(a^2-b^2)/f/g^3-2*b*(g
*cos(f*x+e))^(1/2)/(a^2-b^2)/f/g^3
```

Rubi [A]
time = 0.93, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 16, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$, Rules used = {2981, 2646, 2721, 2720, 2645, 14, 2988, 30, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{2a^2\sqrt{g\cos(e+fx)}}{bfg^3\left(a^2-b^2\right)}-\frac{2b\sqrt{g\cos(e+fx)}}{fg^3\left(a^2-b^2\right)}-\frac{4a\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b}{3fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}+\frac{2a\sin(e+fx)}{3fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{a^5\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^5\sqrt{\cos(e+fx)}\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{b^2fg^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^4\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{b^{3/2}fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a^3\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^4*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])
/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^4*ArcTanh[(Sqrt[b]*Sqrt[g*Cos
[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(b^(3/2)*(-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a^2*Sqrt[g*C
os[e + f*x]])/(b*(a^2 - b^2)*f*g^3) - (2*b*Sqrt[g*Cos[e + f*x]])/((a^2 - b^
2)*f*g^3) - (4*a*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^
2)*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*a^3*Sqrt[Cos[e + f*x]]*EllipticF[(e + f
*x)/2, 2])/(b^2*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*Sqrt[Cos[e +
```

```
 f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b^2*(a^2
- b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (a^5*
Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2]
)/(b^2*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*
x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\cdots}{\sqrt{g\cos(e\cdots}}}{(a\cdots}$$

$$= \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{(2a)\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3(a^2-b^2)g^2}\cdots$$

$$= \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{a^2\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, g\,\text{co}\cdots\right.}{b(a^2-b^2)fg^3\cdots}$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \cdots$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \cdots$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a^2\sqrt{g\cos(e+fx)}}{b(a^2-b^2)fg^3} - \cdots$$

$$= -\frac{a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{b^{3/2}(-a^2+b^2)^{7/4}fg^{5/2}} - \frac{a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\,\text{co}\cdots}}{\sqrt[4]{-a^2+\cdots}}\right.}{b^{3/2}(-a^2+b^2)^{7/\cdots}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 45.89, size = 1958, normalized size = 3.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^4/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) + (Cos[e + f*x]^(5/2)*((-2*(-7*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*
x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(
5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Co
```

s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e + f*x])) + ((3*a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*(((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/2 - I/2)*(-2*a^2 + b^2)*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)])/(b^(3/2)*(-a^2 + b^2)^(3/4)) + (4*Sqrt[Cos[e + f*x]])/b - (4*a*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(5*(a^2 - b^2)) + (10*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)) - ((1/4 - I/4)*(-2*a^2 + b^2)*Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])/(b^(3/2)*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(-1 + 2*Cos[e + f*x]^2)*(a + b*Sin[e + f*x])) - (4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(6*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 70.96, size = 1049, normalized size = 1.75

method	result	size
default	Expression too large to display	1049

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] (2/g^3/b*(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2)-2/g/b*a^4/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/12/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-32*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^2*a*(1/16/b^2*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))-1/256*a^4/(a-b)/(a+b)/b^4*sum(1/_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/16/(a^2-b^2)*(-1/6*cos(1/2*f*x+1/2*e)/g*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2+1/3*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))))/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^4/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^4/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1403 $\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=528

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{7/4}\,fg^{5/2}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{7/4}\,fg^{5/2}}+\frac{2a}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}}-$$

```
[Out] 2/3*a/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)-2/3*b*sin(f*x+e)/(a^2-b^2)/f/g/(g*
cos(f*x+e))^(3/2)+a^3*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/
g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)/b^(1/2)+a^3*arctanh(b^(1/2)*(g*cos(f*x+
e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)/b^(1/2)-2*a^
2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+4/3*b*
(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e
),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)+a^4*(cos(1
/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b
/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(a^2-b*(b
-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+a^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^
(1/2))*cos(f*x+e)^(1/2)/b/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*c
os(f*x+e))^(1/2)
```

Rubi [A]

time = 0.73, antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {2981, 2645, 30, 2646, 2721, 2720, 2946, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{4b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2a}{3fg(a^2-b^2)(g\cos(e+fx))^{3/2}}-\frac{2b\sin(e+fx)}{3fg(a^2-b^2)(g\cos(e+fx))^{3/2}}+\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2(a^2-b^2)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{bfg^2(a^2-b^2)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^3\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,fg^{5/2}(b^2-a^2)^{7/4}}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{\sqrt{b}\,fg^{5/2}(b^2-a^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a^3*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(
Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a^3*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e
+ f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(Sqrt[b]*(-a^2 + b^2)^(7/4)*f*g^(5/
2)) + (2*a)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) - (2*a^2*Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2])/(b*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x]
]) + (4*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^
2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqr
t[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b
)/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(b*(a^2 - b^2)*(a^2 - b*(b + Sqr
```

```
t[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (2*b*Sin[e + f*x])/(3*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{\mathrm{si}}{\sqrt{g\cos(e+}}}{(a^2}$$

$$= -\frac{2b\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{a^2\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{b(a^2-b^2)g^2}$$

$$= \frac{2a}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{2b\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx)}$$

$$= \frac{2a}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{b(a^2-b^2)fg^2\sqrt{g\cos(e+}}$$

$$= \frac{2a}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} - \frac{2a^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+\right.}{b(a^2-b^2)fg^2\sqrt{g\cos(e+}}$$

$$= \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\sqrt{b}\,(-a^2+b^2)^{7/4}fg^{5/2}} + \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e}}{\sqrt[4]{-a^2+b^2}}\right.}{\sqrt{b}\,(-a^2+b^2)^{7/4}fg}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 34.96, size = 1193, normalized size = 2.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) - (Cos[e + f*x]^(5/2)*((4*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a
```

```
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
 f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e
 + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] +
 (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]])
)/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
+ f*x])) - (2*(3*a^2 - 2*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 -
 b^2)*AppellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)]*Sqrt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*App
ellF1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
 + 2*(2*b^2*AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b
^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
f*x]^2))) + (a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b
^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^
(1/4)] - Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e
 + f*x]] + b*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b
^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 -
b^2)^(3/4)))*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/
(3*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4. time = 78.92, size = 1152, normalized size = 2.18

method	result	size
default	Expression too large to display	1152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (2/g*a^3/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*g^
2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/2)
*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g^2
-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/12/g^3*a/(a^2-b^2)/(cos(1/2*f*
x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/g^3*a*2^(1
/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)+1/12/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1/2
*f*x+1/2*e)^2*g+g)^(1/2)-1/12/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1
```

```
/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+32*(g*(2*cos(1/2*f*x+1/2*e)
^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*b/g^2*(1/64*a^2*(sin(1/2*f*x+1/2*e)^2-1)/
(a-b)/(a+b)/b^2*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+
1/2*e)^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(
g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2
*f*x+1/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)
^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/
2*f*x+1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2
-1),2^(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/8*(-sin(1/2*f*x+1
/2*e)^2+1)/(a^2-b^2)*(-1/6*cos(1/2*f*x+1/2*e)/g*(-g*(2*sin(1/2*f*x+1/2*e)^4
-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*e)^2-1/2)^2+1/3*(sin(1/2*f*x
+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e
)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2)))+1/8
*(-sin(1/2*f*x+1/2*e)^2+1)/(a^2-b^2)*(-(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(
1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+2*(-2*sin(1/2*f*x+1/2*e)^4*g+
sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)*sin(1/2*f*x+1/2*e)^2)/g/si
n(1/2*f*x+1/2*e)^2/(2*sin(1/2*f*x+1/2*e)^2-1))/sin(1/2*f*x+1/2*e)/(g*(2*cos
(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^3/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)^3/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1404 $\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))} \, dx$

Optimal. Leaf size=468

$$-\frac{a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}}-\frac{a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}}-\frac{2b}{3\left(a^2-b^2\right)fg(g\cos(e+f}$$

```
[Out] -2/3*b/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)+2/3*a*sin(f*x+e)/(a^2-b^2)/f/g/(g
*cos(f*x+e))^(3/2)-a^2*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)
/g^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)-a^2*arctanh(b^(1/2)*(g*cos(f*x
+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))*b^(1/2)/(-a^2+b^2)^(7/4)/f/g^(5/2)+2/3
*a*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/
2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x+e))^(1/2)-a^3*(co
s(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),
2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(
b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)-a^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/
cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2
^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*co
s(f*x+e))^(1/2)
```

Rubi [A]

time = 0.58, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2981, 2716, 2721, 2720, 2645, 30, 2781, 2886, 2884, 335, 218, 214, 211}

$$-\frac{a^2\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b}{3fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}+\frac{2a\sin(e+fx)}{3fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{a^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sq
rt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))) - (a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^
(5/2)) - (2*b)/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x])^(3/2)) + (2*a*Sqrt[Cos[e
 + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt[g*Cos[e + f*x
]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2 + b^2]), (e +
 f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*g^2*Sqrt[g*Cos
[e + f*x]]) - (a^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + Sqrt[-a^2 + b^2
]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b^2]))*f*g^2*Sq
rt[g*Cos[e + f*x]]) + (2*a*Sin[e + f*x])/(3*(a^2 - b^2)*f*g*(g*Cos[e + f*x]
)^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{a\int \frac{1}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{a^2\int \frac{}{\sqrt{g\cos(e}}}{(a^2}$$

$$= \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3(a^2-b^2)g^2} -$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sin(e+fx)}{3(a^2-b^2)fg(g\cos(e+}$$

$$= -\frac{2b}{3(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{2a\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e\right.}{3(a^2-b^2)fg^2\sqrt{g\cos(}}$$

$$= -\frac{a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{(-a^2+b^2)^{7/4}fg^{5/2}} - \frac{a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{(-a^2+b}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 34.32, size = 1184, normalized size = 2.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5
/2)) - (a*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)
*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]
^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
 + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos
[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
 + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]
]))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[
```

```
e + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*Appell
F1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sq
rt[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*
AppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) +
(a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
 2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log
[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b
*Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sq
rt[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))
*Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(
a + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 70.08, size = 940, normalized size = 2.01

method	result	size
default	Expression too large to display	940

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (-2/g*b*a^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2
*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1
/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*
g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))-1/12/g^3*b/(a^2-b^2)/(cos(1/2
*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/g^3*b*2
^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*
g+g)^(1/2)-1/12/g^3*b/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(
1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/g^3*b*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e
)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+8*(g*(2*cos(1/2*f*x+1/2*
e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)/g^2*a*(1/64/(a-b)/(a+b)*a^2/b^2*sum(1/_
alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)*arct
anh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)^2*a^2-3*b^2*co
s(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_alpha^2*b^2+a^2
-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)
)+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*
x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+1/2*e)^2-1))^(1
/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^(1/2))),_alpha=
RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))-1/4/(a^2-b^2)*(-1/6*cos(1/2*f*x+1/2*e)/g
*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(cos(1/2*f*x+1/2*
e)^2-1/2)^2+1/3*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1
```

```
/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)*EllipticF(cos(
1/2*f*x+1/2*e),2^(1/2))))/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))
^(1/2))/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5992 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(sin(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin \left(e+f\,x\right)^2}{{\left(g\,\cos \left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin \left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
[Out] int(sin(e + f*x)^2/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1405 $\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=432

$$\frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{(-a^2+b^2)^{7/4}fg^{5/2}}+\frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{(-a^2+b^2)^{7/4}fg^{5/2}}-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx\right.}{3(a^2-b^2)fg^2\sqrt{g\cos(e+f}}$$

```
[Out] a*b^(3/2)*arctan(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/(-a
^2+b^2)^(7/4)/f/g^(5/2)+a*b^(3/2)*arctanh(b^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^
2+b^2)^(1/4)/g^(1/2))/(-a^2+b^2)^(7/4)/f/g^(5/2)+2/3*(a-b*sin(f*x+e))/(a^2-
b^2)/f/g/(g*cos(f*x+e))^(3/2)-2/3*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*
x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f
/g^2/(g*cos(f*x+e))^(1/2)+a^2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/
2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*
x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2
)+a^2*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*
f*x+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g
^2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {2945, 2946, 2721, 2720, 2781, 2886, 2884, 335, 218, 214, 211}

$$\frac{ab^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}(b^2-a^2)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2(a^2-b^2)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{a^2b\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2(a^2-b^2)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{2(a-b\sin(e+fx))}{3fg(a^2-b^2)(g\cos(e+fx))^{3/2}}+\frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{fg^{5/2}(b^2-a^2)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g
]))/((-a^2 + b^2)^(7/4)*f*g^(5/2)) + (a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Co
s[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/((-a^2 + b^2)^(7/4)*f*g^(5/2))
- (2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt
[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (a^2*b*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)
/(b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*(a - b*Sin[e + f*x]))/(3*(a^2
 - b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{2\int \frac{-ab+\frac{1}{2}b^2\sin(e+fx)}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e \ldots}}{3\left(a^2-b^2\right)g^2}$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} - \frac{b\int \frac{1}{\sqrt{g\cos(e+fx)}}\,dx}{3\left(a^2-b^2\right)g^2} + \ldots$$

$$= \frac{2(a-b\sin(e+fx))}{3\left(a^2-b^2\right)fg(g\cos(e+fx))^{3/2}} + \frac{\left(a^2b\right)\int \frac{\ldots}{\sqrt{g\cos(e+fx)}\left(\sqrt{\ldots}\right.}}{2\left(-a^2+b \ldots\right.}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} + \frac{2(a-b\sin(e+f \ldots}{3\left(a^2-b^2\right)fg(g\cos(e+ \ldots}$$

$$= -\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3\left(a^2-b^2\right)fg^2\sqrt{g\cos(e+fx)}} - \frac{a^2b\sqrt{\cos(e+fx)}\,\Pi\left(\ldots\right.}{\left(-a^2+b^2\right)^{3/2}\left(b-\sqrt{- \ldots}\right.}$$

$$= \frac{ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{\left(-a^2+b^2\right)^{7/4}fg^{5/2}} + \frac{ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g \ldots}}{\sqrt[4]{-a^2 \ldots}}\right.}{\left(-a^2+b^2\right)^{7/4} \ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 33.97, size = 1183, normalized size = 2.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e + f*x])^(5/
2)) + (b*Cos[e + f*x]^(5/2)*((-4*a*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(
a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2,
(b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e
+ f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[
e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e +
```

```
f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2
+ b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2]
+ (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]
))/(-a^2 + b^2)^(3/4))*Sin[e + f*x])/(Sqrt[1 - Cos[e + f*x]^2]*(a + b*Sin[e
 + f*x])) + (2*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*b*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqr
t[Cos[e + f*x]]*Sqrt[1 - Cos[e + f*x]^2])/((-5*(a^2 - b^2)*AppellF1[1/4, -1
/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 2*(2*b^2*A
ppellF1[5/4, -1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)] + (a^2 - b^2)*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (
a*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] - Log[
Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*
Cos[e + f*x]] + Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqr
t[Cos[e + f*x]] + b*Cos[e + f*x]]))/(4*Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(3/4)))*
Sin[e + f*x]^2)/((1 - Cos[e + f*x]^2)*(a + b*Sin[e + f*x]))))/(3*(a - b)*(a
 + b)*f*(g*Cos[e + f*x])^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 81.33, size = 1117, normalized size = 2.59

method	result	size
default	Expression too large to display	1117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] (2/g*a*b^2/(a-b)/(a+b)*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*g+8*_R^3*a^2*
g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g^(1/
2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_Z^6+(16*a^2*g
^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/12/g^3*a/(a^2-b^2)/(cos(1/2*
f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)+1/12/g^3*a*2^
(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g
+g)^(1/2)+1/12/g^3*a/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))^2*(-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)-1/12/g^3*a*2^(1/2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)
-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-8*(g*(2*cos(1/2*f*x+1/2*e
)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)*(sin(1/2*f*x+1/2*e)^2-1)*b/g^2*(-1/16/(a
-b)/(a+b)*sum(_alpha/(2*_alpha^2-1)*(2^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/
b^2)^(1/2)*arctanh(1/2*g*(4*_alpha^2-3)/(4*a^2-3*b^2)*(4*cos(1/2*f*x+1/2*e)
^2*a^2-3*b^2*cos(1/2*f*x+1/2*e)^2+b^2*_alpha^2-3*a^2+2*b^2)*2^(1/2)/(g*(2*_
alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1
/2*e)^2))^(1/2))+8*b^2/a^2*_alpha*(_alpha^2-1)*(sin(1/2*f*x+1/2*e)^2)^(1/2)
*(-2*cos(1/2*f*x+1/2*e)^2+1)^(1/2)/(-sin(1/2*f*x+1/2*e)^2*g*(2*sin(1/2*f*x+
```

```
1/2*e)^2-1))^(1/2)*EllipticPi(cos(1/2*f*x+1/2*e),-4*b^2/a^2*(_alpha^2-1),2^
(1/2))),_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))+1/2/(a^2-b^2)*(-1/6*cos(1
/2*f*x+1/2*e)/g*(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)/(c
os(1/2*f*x+1/2*e)^2-1/2)^2+1/3*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(-2*cos(1/2*f*x
+1/2*e)^2+1)^(1/2)/(-g*(2*sin(1/2*f*x+1/2*e)^4-sin(1/2*f*x+1/2*e)^2))^(1/2)
*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2)))+1/2/(a^2-b^2)*(-(sin(1/2*f*x+1/2*e)
^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(1/2)*EllipticE(cos(1/2*f*x+1/2*e),2^(1/2))+2*(-2*sin(1/
2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)*cos(1/2*f*x+1/2*e)*sin(1/2*f
*x+1/2*e)^2)/g/sin(1/2*f*x+1/2*e)^2/(2*sin(1/2*f*x+1/2*e)^2-1))/sin(1/2*f*x
+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))/f
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3883 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(sin(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(e+f\,x)}{{\left(g\,\cos(e+f\,x)\right)}^{5/2}\,\left(a+b\,\sin(e+f\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(sin(e + f*x)/((g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1406 $\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=527

$$-\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}}+\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right)}{a\left(-a^2+b^2\right)^{7/4}fg^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}}+\frac{b^{7/2}\tanh^{-1}}{a(}$$

```
[Out] -arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(5/2)+b^(7/2)*arctan(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(7/4)/f/g^(5/2)-a
rctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a/f/g^(5/2)+b^(7/2)*arctanh(b^(1/2)*(g
*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a/(-a^2+b^2)^(7/4)/f/g^(5/2)+2
/3/a/f/g/(g*cos(f*x+e))^(3/2)+2/3*b*(b-a*sin(f*x+e))/a/(a^2-b^2)/f/g/(g*cos
(f*x+e))^(3/2)-2/3*b*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(g*cos(f*x
+e))^(1/2)+b^3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(s
in(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b
^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)+b^3*(cos(1/2*f*
x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b+(
-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/(a^2-b^2)/f/g^2/(a^2-b*(b+(-a^2+
b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 16, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$, Rules used = {2977, 2645, 331, 335, 218, 212, 209, 2775, 2946, 2721, 2720, 2781, 2886, 2884, 214, 211}

$$\frac{b^{7/2}\text{ArcTan}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3fg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2b(b-a\sin(e+fx))}{3afg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}+\frac{b^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt[4]{b^2-a^2}}\right)}{afg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}+\frac{b^3\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{fg^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}}+\frac{2}{3afg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -(ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2))) + (b^(7/2)*ArcTan[(Sq
rt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a*(-a^2 + b^2)^
(7/4)*f*g^(5/2)) - ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]]/(a*f*g^(5/2)) + (b
^(7/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])]
)/(a*(-a^2 + b^2)^(7/4)*f*g^(5/2)) + 2/(3*a*f*g*(g*Cos[e + f*x])^(3/2)) - (
2*b*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*(a^2 - b^2)*f*g^2*Sqrt
[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-a^2
 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]))*f*
g^2*Sqrt[g*Cos[e + f*x]]) + (b^3*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b + S
qrt[-a^2 + b^2]), (e + f*x)/2, 2])/((a^2 - b^2)*(a^2 - b*(b + Sqrt[-a^2 + b
```

```
^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2*b*(b - a*Sin[e + f*x]))/(3*a*(a^2 -
b^2)*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int \left(\frac{\csc(e+fx)}{a(g\cos(e+fx))^{5/2}} - \frac{b}{a(g\cos(e+fx))^{5/2}(a+b\sin(e+f$$

$$= \frac{\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{1}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{a}$$

$$= \frac{2b(b-a\sin(e+fx))}{3a(a^2-b^2)fg(g\cos(e+fx))^{3/2}} + \frac{(2b)\int \frac{-\frac{a^2}{2}+\frac{3b^2}{2}-\frac{1}{2}ab\sin(}{\sqrt{g\cos(e+fx)}\,(a+}}{3a(a^2-b^2)g^2}$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a(a^2-b^2)fg(g\cos(e+fx))^{3/2}} -$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} + \frac{2b(b-a\sin(e+fx))}{3a(a^2-b^2)fg(g\cos(e+fx))^{3/2}} -$$

$$= \frac{2}{3afg(g\cos(e+fx))^{3/2}} - \frac{2b\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3(a^2-b^2)fg^2\sqrt{g\cos(e+fx)}} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} +$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{afg^{5/2}} + \frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right.}{a(-a^2+b^2)^{7/4}fg^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 60.11, size = 2136, normalized size = 4.05

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-8*a*b*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 -
b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
+ b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*Appell
F1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2
*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
```

^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]
^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*
x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]]
)/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)
^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 +
 I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a
^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - (b^2*(-
1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e
 + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e
+ f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2
*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/
4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (1
6*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4,
1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e
+ f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/
4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2
+ b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 -
 Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(
2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[C
os[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*
(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[
Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - C
os[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(6*a^2 -
7*b^2)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*
((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2
- b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f
*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 +
 Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[C
os[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[S
qrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a
^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2
- b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f
*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1
/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - C
os[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(6*(a - b)*(a + b)*f*(g*Cos[e + f*x]
)^(5/2)) + (2*Cos[e + f*x]*(a - b*Sin[e + f*x]))/(3*(a^2 - b^2)*f*(g*Cos[e
+ f*x])^(5/2))

Maple [A]
time = 27.73, size = 627, normalized size = 1.19

method	result
default	$\frac{\left(24\ln\left(\frac{2\sqrt{-g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}-2g}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)}\right)g^{\frac{7}{2}}-12\sqrt{-g}\,\ln\left(\frac{2\sqrt{g}\,\sqrt{-2\left(\sin^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)g+g}+4g\,c}{\cos\left(\frac{fx}{2}+\frac{e}{2}\right)-1}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/6*((24*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^
(1/2)-g))*g^(7/2)-12*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*si
n(1/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3-12*(-g)^(1/2)*
ln(2/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*
g*cos(1/2*f*x+1/2*e)-g))*g^3)*sin(1/2*f*x+1/2*e)^4+(-24*ln(2/cos(1/2*f*x+1/
2*e)*((-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+12*(-g)^(1
/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2
)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3+12*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)+1)*
(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*cos(1/2*f*x+1/2*e)-g))*g^3
)*sin(1/2*f*x+1/2*e)^2+6*ln(2/cos(1/2*f*x+1/2*e)*((-g)^(1/2)*(-2*sin(1/2*f*
x+1/2*e)^2*g+g)^(1/2)-g))*g^(7/2)+4*(-g)^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)*g^(5/2)-3*(-g)^(1/2)*ln(2/(cos(1/2*f*x+1/2*e)-1)*(g^(1/2)*(-2*sin(1
/2*f*x+1/2*e)^2*g+g)^(1/2)+2*g*cos(1/2*f*x+1/2*e)-g))*g^3-3*(-g)^(1/2)*ln(2
/(cos(1/2*f*x+1/2*e)+1)*(g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g*co
s(1/2*f*x+1/2*e)-g))*g^3)/(-g)^(1/2)/g^(11/2)/a/(4*sin(1/2*f*x+1/2*e)^4-4*s
in(1/2*f*x+1/2*e)^2+1)/f
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate(csc(f*x + e)/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(e+f\,x)\,\left(g\,\cos(e+f\,x)\right)^{5/2}\,\left(a+b\,\sin(e+f\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1407 $\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=651

$$\frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}-\frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt[4]{-a^2+b^2}\,\sqrt{g}}\right)}{a^2\left(-a^2+b^2\right)^{7/4}fg^{5/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}-\frac{b^{9/2}\tanh^{-1}}{a^2}$$

```
[Out] b*arctan((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(5/2)-b^(9/2)*arctan(b^(1/2)
*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(7/4)/f/g^(5
/2)+b*arctanh((g*cos(f*x+e))^(1/2)/g^(1/2))/a^2/f/g^(5/2)-b^(9/2)*arctanh(b
^(1/2)*(g*cos(f*x+e))^(1/2)/(-a^2+b^2)^(1/4)/g^(1/2))/a^2/(-a^2+b^2)^(7/4)/
f/g^(5/2)-2/3*b/a^2/f/g/(g*cos(f*x+e))^(3/2)-csc(f*x+e)/a/f/g/(g*cos(f*x+e)
)^(3/2)+5/3*sin(f*x+e)/a/f/g/(g*cos(f*x+e))^(3/2)-2/3*b^2*(b-a*sin(f*x+e))/
a^2/(a^2-b^2)/f/g/(g*cos(f*x+e))^(3/2)+5/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos
(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/f/
g^2/(g*cos(f*x+e))^(1/2)+2/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1
/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/
g^2/(g*cos(f*x+e))^(1/2)-b^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e
)*EllipticPi(sin(1/2*f*x+1/2*e),2*b/(b-(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e
)^(1/2)/a/(a^2-b^2)/f/g^2/(a^2-b*(b-(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
-b^4*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticPi(sin(1/2*f*x
+1/2*e),2*b/(b+(-a^2+b^2)^(1/2)),2^(1/2))*cos(f*x+e)^(1/2)/a/(a^2-b^2)/f/g^
2/(a^2-b*(b+(-a^2+b^2)^(1/2)))/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 1.01, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 18, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2977, 2645, 331, 335, 218, 212, 209, 2650, 2716, 2721, 2720, 2775, 2946, 2781, 2886, 2884, 214, 211}

$$-\frac{b^{9/2}\text{ArcTan}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}+\frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b^2(b-a\sin(e+fx))}{3a^2fg\left(a^2-b^2\right)(g\cos(e+fx))^{3/2}}-\frac{b^{9/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt[4]{b^2-a^2}}\right)}{a^2fg^{5/2}\left(b^2-a^2\right)^{7/4}}-\frac{b^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b-\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{afg^2\left(a^2-b^2\right)\left(a^2-b\left(b-\sqrt{b^2-a^2}\right)\right)\sqrt{g\cos(e+fx)}}-\frac{b^4\sqrt{\cos(e+fx)}\,\Pi\left(\frac{2b}{b+\sqrt{b^2-a^2}};\frac{1}{2}(e+fx)\middle|2\right)}{afg^2\left(a^2-b^2\right)\left(a^2-b\left(\sqrt{b^2-a^2}+b\right)\right)\sqrt{g\cos(e+fx)}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}}-\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}}+\frac{5\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)}{3afg^2\sqrt{g\cos(e+fx)}}+\frac{5\sin(e+fx)}{3afg(g\cos(e+fx))^{3/2}}-\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (b*ArcTan[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^(5/2)) - (b^(9/2)*ArcTan[
(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*Sqrt[g])])/(a^2*(-a^2 +
b^2)^(7/4)*f*g^(5/2)) + (b*ArcTanh[Sqrt[g*Cos[e + f*x]]/Sqrt[g]])/(a^2*f*g^
(5/2)) - (b^(9/2)*ArcTanh[(Sqrt[b]*Sqrt[g*Cos[e + f*x]])/((-a^2 + b^2)^(1/4
)*Sqrt[g])])/(a^2*(-a^2 + b^2)^(7/4)*f*g^(5/2)) - (2*b)/(3*a^2*f*g*(g*Cos[e
 + f*x])^(3/2)) - Csc[e + f*x]/(a*f*g*(g*Cos[e + f*x])^(3/2)) + (5*Sqrt[Cos
[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*f*g^2*Sqrt[g*Cos[e + f*x]]) + (2
```

```
*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])/(3*a*(a^2 - b^2)*f*g^2*S
qrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(b - Sqrt[-
a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b - Sqrt[-a^2 + b^2]
))*f*g^2*Sqrt[g*Cos[e + f*x]]) - (b^4*Sqrt[Cos[e + f*x]]*EllipticPi[(2*b)/(
b + Sqrt[-a^2 + b^2]), (e + f*x)/2, 2])/(a*(a^2 - b^2)*(a^2 - b*(b + Sqrt[-
a^2 + b^2]))*f*g^2*Sqrt[g*Cos[e + f*x]]) + (5*Sin[e + f*x])/(3*a*f*g*(g*Cos
[e + f*x])^(3/2)) - (2*b^2*(b - a*Sin[e + f*x]))/(3*a^2*(a^2 - b^2)*f*g*(g*
Cos[e + f*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
```

```
2*m, 2*p]
```

Rule 2781

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[-a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[b*(g/f), Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2946

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2977

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^(n_))/((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[(g*cos[e +
 f*x])^p, sin[e + f*x]^n/(a + b*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/
2, 0])
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \int \left(-\frac{b\csc(e+fx)}{a^2(g\cos(e+fx))^{5/2}}+\frac{\csc^2(e+fx)}{a(g\cos(e+fx))^{5/2}}+\frac{}{a^2(g\cos(e}\right.$$

$$= \frac{\int \frac{\csc^2(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{\csc(e+fx)}{(g\cos(e+fx))^{5/2}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(g\cos(e+fx))^{5/2}(a}}{a^2}$$

$$= -\frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} - \frac{2b^2(b-a\sin(e+fx))}{3a^2(a^2-b^2)fg(g\cos(e+fx))^{3/2}} +$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\,\mathrm{si}}{3afg(g\,\mathrm{c}}$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\,\mathrm{si}}{3afg(g\,\mathrm{c}}$$

$$= -\frac{2b}{3a^2fg(g\cos(e+fx))^{3/2}} - \frac{\csc(e+fx)}{afg(g\cos(e+fx))^{3/2}} + \frac{5\sqrt{\cos(}}{3a}$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} +$$

$$= \frac{b\tan^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}}\right)}{a^2fg^{5/2}} - \frac{b^{9/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{g\cos(e+fx}}{\sqrt[4]{-a^2+b^2}\sqrt{g}}\right.}{a^2(-a^2+b^2)^{7/4}fg^{5/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 61.25, size = 2183, normalized size = 3.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/((g*Cos[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (Cos[e + f*x]^(5/2)*((-2*(10*a^3 - 18*a*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2
])*((5*a*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
+ f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a
^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
```

-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*Sqrt[b]*(2*ArcTan[1 - ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[b]*Sqrt[Cos[e + f*x]])/(-a^2 + b^2)^(1/4)] + Log[Sqrt[-a^2 + b^2] - (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]] - Log[Sqrt[-a^2 + b^2] + (1 + I)*Sqrt[b]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]] + I*b*Cos[e + f*x]]))/(-a^2 + b^2)^(3/4)))/(Sqrt[1 - Cos[e + f*x]^2]*(b + a*Csc[e + f*x])) - ((-5*a^2*b + 3*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Cos[2*(e + f*x)]*Csc[e + f*x]*((-10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (10*Sqrt[2]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) - (20*ArcTan[Sqrt[Cos[e + f*x]]])/a - (16*b*AppellF1[5/4, 1/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Cos[e + f*x]^(5/2))/(-a^2 + b^2) - (200*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) + (10*Log[1 - Sqrt[Cos[e + f*x]]])/a - (10*Log[1 + Sqrt[Cos[e + f*x]]])/a - (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4)) + (5*Sqrt[2]*(2*a^2 - b^2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(a*Sqrt[b]*(a^2 - b^2)^(3/4))))/(20*(1 - Cos[e + f*x]^2)*(-1 + 2*Cos[e + f*x]^2)*(b + a*Csc[e + f*x])) - (2*(-7*a^2*b + 9*b^3)*(-1 + Cos[e + f*x]^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Csc[e + f*x]*((5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/(Sqrt[1 - Cos[e + f*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - 2*(2*b^2*AppellF1[5/4, 1/2, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/2, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - (-2*Sqrt[2]*b^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 2*Sqrt[2]*b^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[Cos[e + f*x]])/(a^2 - b^2)^(1/4)] + 4*(a^2 - b^2)^(3/4)*ArcTan[Sqrt[Cos[e + f*x]]] - 2*(a^2 - b^2)^(3/4)*Log[1 - Sqrt[Cos[e + f*x]]] + 2*(a^2 - b^2)^(3/4)*Log[1 + Sqrt[Cos[e + f*x]]] - Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] - Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]] + Sqrt[2]*b^(3/2)*Log[Sqrt[a^2 - b^2] + Sqrt[2]*Sqrt[b]*(a^2 - b^2)^(1/4)*Sqrt[Cos[e + f*x]] + b*Cos[e + f*x]])/(8*a*(a^2 - b^2)^(3/4))))/((1 - Cos[e + f*x]^2)*(b + a*Csc[e + f*x]))))/(12*a*(a - b)*(a + b)*f*(g*Cos[e + f*x])^(5/2)) + (Cos[e + f*x]^3*(-(Csc[e +

f*x]/a) + (2*Sec[e + f*x]^2*(-b + a*Sin[e + f*x]))/(3*(a^2 - b^2))))/(f*(g*Cos[e + f*x])^(5/2))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 127.35, size = 1571, normalized size = 2.41

method	result	size
default	Expression too large to display	1571

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERB
OSE)
```

```
[Out] (1/6/g^3*b/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2
))^2*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/6/g^3*b*2^(1/2)/(2+2^(1/2))/(2^(
1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)-1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2
*g+g)^(1/2)-2/g*b^5/(a-b)/(a+b)/a^2*sum((_R^4+_R^2*g)/(_R^7*b^2-3*_R^5*b^2*
g+8*_R^3*a^2*g^2-5*_R^3*b^2*g^2-_R*b^2*g^3)*ln((-2*sin(1/2*f*x+1/2*e)^2*g+g
)^(1/2)-g^(1/2)*cos(1/2*f*x+1/2*e)*2^(1/2)-_R),_R=RootOf(b^2*_Z^8-4*b^2*g*_
Z^6+(16*a^2*g^2-10*b^2*g^2)*_Z^4-4*b^2*g^3*_Z^2+b^2*g^4))+1/6/g^3*b/(2+2^(1
/2))/(2^(1/2)-2)/(a^2-b^2)/(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))^2*(-2*sin(1/2*f
*x+1/2*e)^2*g+g)^(1/2)+1/6/g^3*b*2^(1/2)/(2+2^(1/2))/(2^(1/2)-2)/(a^2-b^2)/
(cos(1/2*f*x+1/2*e)+1/2*2^(1/2))*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-1/g^2*
b/a^2/(-g)^(1/2)*ln((-2*g+2*(-g)^(1/2)*(2*cos(1/2*f*x+1/2*e)^2*g-g)^(1/2))/
cos(1/2*f*x+1/2*e))+2/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2*ln((4*g*cos
(1/2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2
*f*x+1/2*e)-1))+2/g^(5/2)*b/(2+2^(1/2))^2/(2^(1/2)-2)^2/a^2*ln((-4*g*cos(1/
2*f*x+1/2*e)+2*g^(1/2)*(-2*sin(1/2*f*x+1/2*e)^2*g+g)^(1/2)-2*g)/(cos(1/2*f*
x+1/2*e)+1))+1/24*(g*(2*cos(1/2*f*x+1/2*e)^2-1)*sin(1/2*f*x+1/2*e)^2)^(1/2)
/g^2/a/cos(1/2*f*x+1/2*e)/(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1/2*f*x+1/2*e)^2*g
)^(3/2)/(a^2-b^2)*(40*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/
2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)*a^2*g*cos(1/2*f*x+1/2*e)*sin
(1/2*f*x+1/2*e)^2-24*EllipticF(cos(1/2*f*x+1/2*e),2^(1/2))*(sin(1/2*f*x+1/2
*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(3/2)*b^2*g*cos(1/2*f*x+1/2*e)*sin(
1/2*f*x+1/2*e)^2-80*a^2*g*sin(1/2*f*x+1/2*e)^6+48*b^2*g*sin(1/2*f*x+1/2*e)^
6+3/a^2*sum(1/_alpha/(2*_alpha^2-1)*(8*(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(
1/2)*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Elliptic
Pi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*_alpha^3*b^2-8*b
^2*_alpha*(sin(1/2*f*x+1/2*e)^2)^(1/2)*(2*sin(1/2*f*x+1/2*e)^2-1)^(1/2)*Ell
ipticPi(cos(1/2*f*x+1/2*e),(-4*_alpha^2*b^2+4*b^2)/a^2,2^(1/2))*(g*(2*_alph
a^2*b^2+a^2-2*b^2)/b^2)^(1/2)+2^(1/2)*a^2*arctanh(1/2/(-2*sin(1/2*f*x+1/2*e
)^4*g+sin(1/2*f*x+1/2*e)^2*g)^(1/2)/(g*(2*_alpha^2*b^2+a^2-2*b^2)/b^2)^(1/2
)/(4*a^2-3*b^2)*g*2^(1/2)*(-16*sin(1/2*f*x+1/2*e)^2*_alpha^2*a^2+12*sin(1/2
*f*x+1/2*e)^2*_alpha^2*b^2+4*_alpha^4*b^2+12*sin(1/2*f*x+1/2*e)^2*a^2-9*sin
(1/2*f*x+1/2*e)^2*b^2+4*_alpha^2*a^2-7*b^2*_alpha^2-3*a^2+3*b^2))*(sin(1/2*
```

```
f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2))/(g*(2*_alpha^2*b^2+a^2-2
*b^2)/b^2)^(1/2)/(sin(1/2*f*x+1/2*e)^2*g*(-2*sin(1/2*f*x+1/2*e)^2+1))^(1/2)
,_alpha=RootOf(4*_Z^4*b^2-4*_Z^2*b^2+a^2))*(-2*sin(1/2*f*x+1/2*e)^4*g+sin(1
/2*f*x+1/2*e)^2*g)^(3/2)*b^2*cos(1/2*f*x+1/2*e)+80*a^2*g*sin(1/2*f*x+1/2*e)
^4-48*b^2*g*sin(1/2*f*x+1/2*e)^4-12*a^2*g*sin(1/2*f*x+1/2*e)^2+12*b^2*g*sin
(1/2*f*x+1/2*e)^2)/sin(1/2*f*x+1/2*e)/(g*(2*cos(1/2*f*x+1/2*e)^2-1))^(1/2))
/f
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(g*cos(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(g*cos(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate(csc(f*x + e)^2/((g*cos(f*x + e))^(5/2)*(b*sin(f*x + e) + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(g\,\cos\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(g*cos(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x)
```

3.1408 $\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=926

$$\frac{a^2d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f}+\frac{d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf}-\frac{a^2d^{5/}}{}$$

```
[Out] -1/2*a^2*d^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*
sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/8*d^(5/2)*arctan(-1+2^(1/2)*d^(1
/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-
1/2*a^2*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*si
n(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/8*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)
*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-1/4
*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(
1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)-1/16*d^(5/2)*ln(g^(1/2)+co
t(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))
*g^(1/2)/b/f*2^(1/2)+1/4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*
d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^3/f*2^(1/2)+1/
16*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/
2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b/f*2^(1/2)-2*a^3*d^3*EllipticPi((g*cos(f*
x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/
2)*g^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(
1/2)+2*a^3*d^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2)
,(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(-a+b)^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-1/2*d^2*(g*cos(f*x+e))^(3/2)*(d*sin(
f*x+e))^(1/2)/b/f/g+a*d^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ell
ipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)
/b^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 1.16, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2988, 2648, 2655, 303, 1176, 631, 210, 1179, 642, 2652, 2719, 2985, 2984, 504, 1232}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (d^(5/2)*Sqrt[g]*ArcTan[1 -
```

```
(Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*
Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[
e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt
[g]*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])])/(4*Sqrt[2]*b*f) - (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Co
t[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/
(2*Sqrt[2]*b^3*f) - (d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f
) + (a^2*d^(5/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^3*f) + (d^(5/
2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*Sqrt[2]*b*f) - (2*Sqrt[2]*a^3*d^3*Sqr
t[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(S
qrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*
Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Sqrt[g]*EllipticPi
[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Si
n[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt
[d*Sin[e + f*x]]) - (d^2*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(2*b*
f*g) - (a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[
e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
```

```
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int \sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{g\cos(e+\ldots}}{a+b\ldots}}{\ldots}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{(ad^2)\int \sqrt{g\cos(e+f\ldots}}{b\ldots}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} + \frac{(a^2d^3)\int \frac{\sqrt{g\cos(e+\ldots}}{\sqrt{d\sin(e+\ldots}}}{b^3}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}\,E\ldots}{b^2f\ldots}$$

$$= -\frac{d^2(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bfg} - \frac{ad^2\sqrt{g\cos(e+fx)}\,E\ldots}{b^2f\ldots}$$

$$= -\frac{d^{5/2}\sqrt{g}\,\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+\ldots}}{\sqrt{d\sin(e+fx)}}\ldots\right.}{8\sqrt{2}\,bf}$$

$$= \frac{d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf} - \frac{d^{5/2}\sqrt{g}\,\tan\ldots}{\ldots}$$

$$= \frac{a^2d^{5/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f} + \frac{d^{5/2}\sqrt{g}\,\mathrm{ta}\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 56.82, size = 1623, normalized size = 1.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -1/2*(Sqrt[g*Cos[e + f*x]]*Cot[e + f*x]*Csc[e + f*x]*(d*Sin[e + f*x])^(5/2)
)/(b*f) + (Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((-2*b*(-(b*AppellF1
[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a
*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2)
)/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[T
an[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 -
b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*
Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
 + f*x]^2]))/(12*a*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Ta
n[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4,
 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^
(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/
2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)
^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*
a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3
/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log
[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[
2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/
2))/Sqrt[1 + Tan[e + f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f
*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))
/(4*b*f*Sqrt[Cos[e + f*x]]*Sin[e + f*x]^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4648 vs. $2(745) = 1490$.
time = 13.20, size = 4649, normalized size = 5.02

method	result	size
default	Expression too large to display	4649

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/f*(a-b)*(4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a
),1/2*2^(1/2))*cos(f*x+e)*a^3+I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2+4*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b-8*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b+4*I*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/
2)*a^2-4*I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f
*x+e)*(-a^2+b^2)^(1/2)*a^2-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*
b^2+I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)
^(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos
(f*x+e)*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
```

```
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2+4*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*
a^2+4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*cos(f*x+e)*a^2*b+4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^
2)^(1/2)+a),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2-4*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*cos(f*x+e)*a^2*b
+2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1
/2)*b^2+4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*
a*b-8*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+
4*I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*a^2-4*I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(5/2)/(b*sin(f*x + e) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(d\,\sin\left(e+f\,x\right)\right)^{5/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2))/(a + b*sin(e + f*x)), x
)
```

3.1409 $\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=578

$$-\frac{ad^{3/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2f}+\frac{ad^{3/2}\sqrt{g}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2f}+\frac{ad^{3/2}}{}$$

```
[Out] 1/2*a*d^(3/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin
(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)+1/2*a*d^(3/2)*arctan(1+2^(1/2)*d^(1/2
)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)+
1/4*a*d^(3/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^
(1/2)/(d*sin(f*x+e))^(1/2))*g^(1/2)/b^2/f*2^(1/2)-1/4*a*d^(3/2)*ln(g^(1/2)+
cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2
))*g^(1/2)/b^2/f*2^(1/2)+2*a^2*d^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/
(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e
)^(1/2)/b^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*a^2*d^2*Ellip
ticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^
(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d
*sin(f*x+e))^(1/2)-d*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Elliptic
E(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f/
sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.72, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2988, 2652, 2719, 2655, 303, 1176, 631, 210, 1179, 642, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,a^2d^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{ad^{3/2}\sqrt{g}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2f}+\frac{ad^{3/2}\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,b^2f}+\frac{ad^{3/2}\sqrt{g}\,\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,b^2f}-\frac{ad^{3/2}\sqrt{g}\,\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,b^2f}+\frac{dE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{bf\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*d^(3/2)*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqr
t[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^2*f)) + (a*d^(3/2)*Sqrt[g]*ArcTan[1
 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/
(Sqrt[2]*b^2*f) + (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (
Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2
*f) - (a*d^(3/2)*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*f) + (2*Sqr
t[2]*a^2*d^2*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b
^2*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*d^2*Sq
```

```
rt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*Sqrt[-a + b]*Sq
rt[a + b]*f*Sqrt[d*Sin[e + f*x]]) + (d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
```

0]

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{d\int\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,dx}{b} - \frac{(ad)\int\frac{\sqrt{g\cos(e+fx}}{a+b}}{b}$$

$$= -\frac{(ad^2)\int\frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} + \frac{(a^2d^2)\int\frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(}}{b^2}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{(2ad^3g)\,\mathrm{S}}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} - \frac{(ad^2g)\,\mathrm{S}}{}$$

$$= \frac{d\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{d\sin(e+fx)}}{bf\sqrt{\sin(2e+2fx)}} + \frac{(ad^{3/2}\sqrt{}}{}$$

$$= \frac{ad^{3/2}\sqrt{g}\,\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx}}{\sqrt{d\sin(e+fx)}}\right.}{2\sqrt{2}\,b^2f}$$

$$= -\frac{ad^{3/2}\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2f} + \frac{ad^{3/2}\sqrt{g}\,\mathrm{t}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 27.63, size = 176, normalized size = 0.30

$$\frac{2d\left(bF_1\left(\frac{3}{4};-\frac{3}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)-aF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)}{3\left(a^2-b^2\right)fg\sqrt[4]{\sin^2(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]
),x]
```

```
[Out] (2*d*(b*AppellF1[3/4, -3/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] - a*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqr
t[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e
+ f*x]))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3289 vs. $2(482) = 964$.
time = 0.56, size = 3290, normalized size = 5.69

method	result	size
default	Expression too large to display	3290

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(a-b)*(-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+2*cos(f*x+e)*EllipticE((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b-cos(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+
(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+cos(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b+cos(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1
/2)*a+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
```

```
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*(-a^2+b^2)^(1/2)*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-a^2+b^2)^(1/2)*b+I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-I*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2+(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2+cos(f*x+e)^2*(-a^2
+b^2)^(1/2)*2^(1/2)*b-cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b-(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^
(1/2)*a+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2
^(1/2))*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^
2+b^2)^(1/2)*a+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(e+fx))^{\frac{3}{2}}\sqrt{g\cos(e+fx)}}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*sin(e + f*x))**(3/2)*sqrt(g*cos(e + f*x))/(a + b*sin(e + f*x)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos(e+f\,x)}\,(d\,\sin(e+f\,x))^{3/2}}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x
)
```

3.1410 $\int \frac{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=509

$$\frac{\sqrt{d}\,\sqrt{g}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bf}-\frac{\sqrt{d}\,\sqrt{g}\,\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bf}-\frac{\sqrt{d}\,\sqrt{g}}{}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^
(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)-1/2*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+
e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)-1/4*ln(
g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+
e))^(1/2))*d^(1/2)*g^(1/2)/b/f*2^(1/2)+1/4*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^
(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)*g^(1/2)/b/
f*2^(1/2)-2*a*d*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2
),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/b/f/(-a+b)^
(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*a*d*EllipticPi((g*cos(f*x+e))^(1/2
)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*
sin(f*x+e)^(1/2)/b/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2988, 2655, 303, 1176, 631, 210, 1179, 642, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}\,ad\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,ad\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{\sqrt{d}\,\sqrt{g}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bf}-\frac{\sqrt{d}\,\sqrt{g}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,bf}-\frac{\sqrt{d}\,\sqrt{g}\,\log\left(-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\,\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,bf}+\frac{\sqrt{d}\,\sqrt{g}\,\log\left(\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\,\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,bf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Sqrt[d]*Sqrt[g]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]
*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*f) - (Sqrt[d]*Sqrt[g]*ArcTan[1 + (Sqrt[
2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*
b*f) - (Sqrt[d]*Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[
d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) + (Sqrt[d]*
Sqrt[g]*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
+ f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*f) - (2*Sqrt[2]*a*d*Sqrt[g]*El
lipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a +
b]*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*d*Sqrt[g]*EllipticPi[Sqrt[-a + b]
/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])]
, -1]*Sqrt[Sin[e + f*x]])/(b*Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{d\int\frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b} - \frac{(ad)\int\frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,d}{b}$$

$$= -\frac{(2d^2g)\,\text{Subst}\left(\int\frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\sin(\right.}{}$$

$$= \frac{(dg)\text{Subst}\left(\int\frac{g-dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{bf} - \frac{(dg)\text{Subst}\left(\int\frac{}{g}\right.}{}$$

$$= -\frac{\left(\sqrt{d}\sqrt{g}\right)\text{Subst}\left(\int\frac{\frac{\sqrt{2}\sqrt{g}}{\sqrt{d}}+2x}{-\frac{g}{d}-\frac{\sqrt{2}\sqrt{g}x}{\sqrt{d}}-x^2}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right.}{2\sqrt{2}\,bf}$$

$$= -\frac{\sqrt{d}\sqrt{g}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+f}}{\sqrt{d\sin(e+fx)}}\right.}{2\sqrt{2}\,bf}$$

$$= \frac{\sqrt{d}\sqrt{g}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bf} - \frac{\sqrt{d}\sqrt{g}\tan^{}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 19.68, size = 178, normalized size = 0.35

$$\frac{2\left(bF_1\left(\frac{3}{4};-\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)-aF_1\left(\frac{3}{4};\frac{1}{4},1;\frac{7}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)}{3\left(a^2-b^2\right)dfg\sin^2(e+fx)^{3/4}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),
x]
```

```
[Out] (2*(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^
2 + b^2)] - a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
```

```
2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt
[Sin[e + f*x]^2]))/(3*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x]))
```

Maple [A]
time = 0.37, size = 744, normalized size = 1.46

method	result
default	$\frac{(a-b)\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{-a^2+b^2}-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-
1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^
2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1
/2)+a),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)-a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(
-a^2+b^2)^(1/2)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*(d*sin(f*x+e))^(1/
2)*(g*cos(f*x+e))^(1/2)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))*2^(1/2)*a/b/(-a^2+b^2)^(1/2)/(-b+(-
a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin{\left(e + f x \right)}} \sqrt{g \cos{\left(e + f x \right)}}}{a + b \sin{\left(e + f x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)*(g*cos(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))/(a + b*sin(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)*(g*cos(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}\,\sqrt{d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

3.1411 $\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=208

$$\frac{2\sqrt{2}\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{\sqrt{-a+b}\sqrt{a+b}\,f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\,\Pi\left(\frac{\sqrt{-a+b}}{\sqrt{a+b}};\right.}{\sqrt{-}}$$

```
[Out] 2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2
)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/f/(-a+b)^(1/2)/(a+b)^(1/2
)/(d*sin(f*x+e))^(1/2)-2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x
+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/f/(
-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqr
t[-a + b]*Sqrt[a + b]*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[g]*Elliptic
Pi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(Sqrt[-a + b]*Sqrt[a + b]*f*Sqrt[d
*Sin[e + f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\sin(e+fx)}\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(4\sqrt{2}\,g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{((a+b)g^2+(a-b)x^4)\sqrt{1-\frac{x^4}{g^2}}}\right.}{f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{\left(2\sqrt{2}\,g\sqrt{\sin(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{a+b}\,g-\sqrt{-a+b}\,x^2\right)\sqrt{}}\right.}{\sqrt{-a+b}\,f\sqrt{d\sin(e+fx)}}$$

$$= \frac{2\sqrt{2}\,\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\right| -}{\sqrt{-a+b}\,\sqrt{a+b}\,f\sqrt{d\sin(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 32.68, size = 371, normalized size = 1.78

$$\frac{\sqrt{g\cos(e+fx)}\sqrt{\tan(e+fx)}\left(a\sqrt{\sec^2(e+fx)}+b\tan(e+fx)\right)\left(\frac{3\sqrt{2}a^{3/2}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)-\log\left(-a+\sqrt{2}\sqrt{a}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}-\sqrt{a^2-b^2}\tan(e+fx)\right)+\log\left(a+\sqrt{2}\sqrt{a}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}+\sqrt{a^2-b^2}\tan(e+fx)\right)\right)}{\sqrt[4]{a^2-b^2}}-8bF_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\tan^2(e+fx),\frac{(-a^2+b^2)\tan^2(e+fx)}{a^2}\right)\tan^{\frac{3}{2}}(e+fx)\right)}{12a^2f\sqrt{\sec^2(e+fx)}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*Sqrt[Tan[e + f*x]]*(a*Sqrt[Sec[e + f*x]^2] + b*Tan[e
+ f*x])*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^
(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2]*Tan[e + f*x]^(3/2)))/(12*a^2*f*Sqrt[Sec[e + f*x]^2]*Sqrt[d*
Sin[e + f*x]]*(a + b*Sin[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(164) = 328$.
time = 0.31, size = 590, normalized size = 2.84

method	result
default	$-\frac{\sqrt{g\cos(fx+e)}\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,(a-b)\left(2\sqrt{-a^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(g*cos(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*(a-b)*(2*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)-EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/
2*2^(1/2))*(-a^2+b^2)^(1/2)+a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))
-a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))-b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*sin(f*x+e)^2/(d*sin(f*x+e
```

```
))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))*2^(1/2)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)
^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
[Out] Integral(sqrt(g*cos(e + f*x))/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1412 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{a\sqrt{-a+b}\,\sqrt{a+b}\,df\sqrt{d\sin(e+fx)}}$$

```
[Out] -2*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(1/2)-2*b*EllipticPi((g*cos(
f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(
1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a/d/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))
^(1/2)+2*b*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a
+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a/d/f/(-a+b)^(1/2
)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*
Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*
x+e))^(1/2)/a/d^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2989, 2650, 2652, 2719, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adf\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{ad^2f\sqrt{\sin(2e+2fx)}}-\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(a*d*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*S
qrt[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*
Sqrt[a + b]*d*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[g]*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*Sqrt[-a + b]*Sqrt[a + b]*d*f*Sqrt[d*S
in[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d
*Sin[e + f*x]])/(a*d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
```

```
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+f}}{ad^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(4\sqrt{2}\,bg\sqrt{\sin(e+fx)}\right)\text{Subst}\Big(\int}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\right|2}{ad^2f\sqrt{\sin(2e+2f}}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{adfg\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\,b\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\Big(\right.}{a\sqrt{-a+b}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 52.82, size = 1619, normalized size = 5.06

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x])/(a*f*(d*Sin[e + f*x])^(
3/2)) + (Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^(3/2)*((4*a*(-(b*AppellF1[3/4, -
```

```
1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Appell
F1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Co
s[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a
^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e
+ f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^
(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
 + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*
x]^2]))/(6*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e
+ f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2
, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2
) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^
2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*A
rcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2
]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1
 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/
4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3
/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*
Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a
+ Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Ta
n[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^
2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b
^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
- b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/
Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]
*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*
f*Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2496 vs. $2(291) = 582$.

time = 0.36, size = 2497, normalized size = 7.80

method	result	size
default	Expression too large to display	2497

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)

[Out] 1/f*(a-b)*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*a-2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*(-a^2+b^2)^(1/2)*b+cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b+cos(f*x+e)*(-a^2+b^2)^(1
/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/
2))*b-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^
(1/2)+a),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/
(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b+cos(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*(-
a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
2^(1/2))(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a-2*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+(-a^2+b^2)^(1/2)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))
b+(-a^2+b^2)^(1/2)(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1
/2)-a),1/2*2^(1/2))*b-4*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f

```
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,1/2*2^(1/2))*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+
a),1/2*2^(1/2))*a*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^2*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2
)^(1/2)-a),1/2*2^(1/2))*a*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2
+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1/2)*a)*si
n(f*x+e)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/cos(f*x+e)*2^(1/2)/a/(-a
^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{\frac{3}{2}}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(g*cos(e + f*x))/((d*sin(e + f*x))**(3/2)*(a + b*sin(e + f*x))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1413 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=366

$$-\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}}+\frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{g}\,\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+}}\right.\right.}{a^2\sqrt{-a+b}\,\sqrt{a+b}\,d^2f\sqrt{d\sin(e}}$$

```
[Out] -2/3*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(3/2)+2*b*(g*cos(f*x+e))^(
3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(1/2)+2*b^2*EllipticPi((g*cos(f*x+e))^(1/2)
/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*
sin(f*x+e)^(1/2)/a^2/d^2/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*
b^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/
2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2/f/(-a+b)^(1/2)/(
a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-2*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*P
i+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x
+e))^(1/2)/a^2/d^3/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2989, 2643, 2650, 2652, 2719, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2\sqrt{g}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt
[g]*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sq
rt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*S
qrt[a + b]*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[g]*EllipticPi[
Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin
[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*Sqrt[-a + b]*Sqrt[a + b]*d^2*f*S
qrt[d*Sin[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x,
2]*Sqrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
```

```
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{b^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}}{a^2d^2}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{(2b)\int}{}$$

$$\left(4\sqrt{2}\right.$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} -$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2b\sqrt{g\,c}}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{a^2d^2fg\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}\,b^2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 52.92, size = 1645, normalized size = 4.49

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f*x
])/(3*a))*Sin[e + f*x]^3)/(f*(d*Sin[e + f*x])^(5/2)) - (b*Sqrt[g*Cos[e + f*
x]]*Sin[e + f*x]^(5/2)*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^
2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e
 + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sq
rt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x
]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2
)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] +
2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[
-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]
*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4,
1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*
x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*a^2*Cos[e + f*x
]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2))
 + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
+ f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2,
 (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*Appe
llF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan
[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan
[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4
*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan
[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(
1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[T
an[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e
 + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(
1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*
Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b
^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
```

```
 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2]))
)/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1
+ Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a^2*f*Sqrt[Cos[e + f*x]]*(d*
Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2671 vs. $2(331) = 662$.
time = 0.32, size = 2672, normalized size = 7.30

method	result	size
default	Expression too large to display	2672

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(a-b)*(-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2+3*cos(f*x+e)*sin
(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a
^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*a*b^2+3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+12*co
s(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b
^2)^(1/2)*a*b-6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/
2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
```

```
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-3*sin(f*x+e)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2
*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(
1/2)*b^2+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*a*b^2+3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3-3*sin(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*
b^2-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)
-a),1/2*2^(1/2))*a*b^2-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+12*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-6*sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+6*sin(f*x+e)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*2^(1/2)*cos
(f*x+e)^2*(-a^2+b^2)^(1/2)*a^2-6*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(
1/2)*a*b)*sin(f*x+e)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/cos(f*x+e)*2
^(1/2)/a^2/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\cos\left(e+f\,x\right)}}{{\left(d\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

3.1414 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=513

$$-\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}}+\frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}}-\frac{4(g\cos(e+fx))^{3/2}}{5ad^3fg\sqrt{d\sin(e+fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{a^3d^3fg\sqrt{d\sin(e+fx)}}-$$

```
[Out] -2/5*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(5/2)+2/3*b*(g*cos(f*x+e))
^(3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(3/2)-4/5*(g*cos(f*x+e))^(3/2)/a/d^3/f/g/
(d*sin(f*x+e))^(1/2)-2*b^2*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/g/(d*sin(f*x+e))^
(1/2)-2*b^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(
-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^3/d^3/f/(-a+b
)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+2*b^3*EllipticPi((g*cos(f*x+e))^(1
/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2
)*sin(f*x+e)^(1/2)/a^3/d^3/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)+
4/5*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*
x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a/d^4/f/sin(2*f*x+2*e
)^(1/2)+2*b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e
+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^3/d^4/f/s
in(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2989, 2650, 2652, 2719, 2643, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}b^3\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}b^3\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{a^3d^3fg\sqrt{d\sin(e+fx)}}+\frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}}-\frac{4E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5ad^4f\sqrt{\sin(2e+2fx)}}-\frac{4(g\cos(e+fx))^{3/2}}{5ad^3fg\sqrt{d\sin(e+fx)}}-\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*g*(d*Sin[e + f*x])^(5/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(3*a^2*d^2*f*g*(d*Sin[e + f*x])^(3/2)) - (4*(g*Cos[e + f
*x])^(3/2))/(5*a*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*b^2*(g*Cos[e + f*x])^(3
/2))/(a^3*d^3*f*g*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi
[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 +
 Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3
*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[g]*EllipticPi[Sqrt[-a + b]/S
qrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])],
-1]*Sqrt[Sin[e + f*x]])/(a^3*Sqrt[-a + b]*Sqrt[a + b]*d^3*f*Sqrt[d*Sin[e +
f*x]]) - (4*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e
+ f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^2*Sqrt[g*Cos[e + f*x]]*E
```

```
llipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*d^4*f*Sqrt[Sin[2*e +
 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
```

```
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{5ad^2} + \frac{b^2\int \frac{}{(d\mathrm{s}}}{}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(}{5ad^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(}{5ad^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(}{5ad^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(}{5ad^3}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{5adfg(d\sin(e+fx))^{5/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{3a^2d^2fg(d\sin(e+fx))^{3/2}} - \frac{4(}{5ad^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.05, size = 1726, normalized size = 3.36

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[g*Cos[e + f*x]]*((-2*(2*a^2*Cos[e + f*x] + 5*b^2*Cos[e + f*x])*Csc[e
+ f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^4)/(f*(d*Sin[e + f*x])^(7/2)) - (Sqrt[
g*Cos[e + f*x]]*Sin[e + f*x]^(7/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[3/
4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*Ap
pellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]
)*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(
3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b
+ 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a
^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 -
 b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
 f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2
, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] +
a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*
(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Cos[
2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2
])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b
^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4
, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x
]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x
]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]
*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (
2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f
*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]
 + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (
Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] -
 Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a +
Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[
e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^
(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^
2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[
e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Sqrt[Cos[e + f*x]]*(d*Sin[
e + f*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6207 vs. $2(483) = 966$.

time = 0.38, size = 6208, normalized size = 12.10

method	result	size
default	Expression too large to display	6208

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{7/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

3.1415 $\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=598

$$-\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}}+\frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}}-\frac{8(g\cos(e+fx))^{3/2}}{21ad^3fg(d\sin(e+fx))^{3/2}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{3a^3d^3fg(d\sin(e+fx))^{3/2}}$$

```
[Out] -2/7*(g*cos(f*x+e))^(3/2)/a/d/f/g/(d*sin(f*x+e))^(7/2)+2/5*b*(g*cos(f*x+e))
^(3/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(5/2)-8/21*(g*cos(f*x+e))^(3/2)/a/d^3/f/g
/(d*sin(f*x+e))^(3/2)-2/3*b^2*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/g/(d*sin(f*x+e
))^(3/2)+4/5*b*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/g/(d*sin(f*x+e))^(1/2)+2*b^3*
(g*cos(f*x+e))^(3/2)/a^4/d^4/f/g/(d*sin(f*x+e))^(1/2)+2*b^4*EllipticPi((g*c
os(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*
2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(
f*x+e))^(1/2)-2*b^4*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^
(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*g^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/
f/(-a+b)^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2)-4/5*b*(sin(e+1/4*Pi+f*x)^2)
^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e)
)^(1/2)*(d*sin(f*x+e))^(1/2)/a^2/d^5/f/sin(2*f*x+2*e)^(1/2)-2*b^3*(sin(e+1/
4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(
g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^4/d^5/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 1.18, antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2989, 2650, 2643, 2652, 2719, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}b^4\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}b^4\sqrt{g}\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{b-a}\sqrt{a+b}\sqrt{d\sin(e+fx)}}+\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}+\frac{2b^3(g\cos(e+fx))^{3/2}}{a^4d^4fg\sqrt{d\sin(e+fx)}}-\frac{2b^2(g\cos(e+fx))^{3/2}}{3a^3d^3fg(d\sin(e+fx))^{3/2}}+\frac{4bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{5a^2d^5f\sqrt{\sin(2e+2fx)}}+\frac{4b(g\cos(e+fx))^{3/2}}{5a^2d^4fg\sqrt{d\sin(e+fx)}}+\frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}}-\frac{8(g\cos(e+fx))^{3/2}}{21ad^3fg(d\sin(e+fx))^{3/2}}-\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*g*(d*Sin[e + f*x])^(7/2)) + (2*b*(g*Co
s[e + f*x])^(3/2))/(5*a^2*d^2*f*g*(d*Sin[e + f*x])^(5/2)) - (8*(g*Cos[e + f
*x])^(3/2))/(21*a*d^3*f*g*(d*Sin[e + f*x])^(3/2)) - (2*b^2*(g*Cos[e + f*x])
^(3/2))/(3*a^3*d^3*f*g*(d*Sin[e + f*x])^(3/2)) + (4*b*(g*Cos[e + f*x])^(3/2
))/(5*a^2*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*b^3*(g*Cos[e + f*x])^(3/2))/(a
^4*d^4*f*g*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[-(Sqrt
[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt
[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^4*Sqrt[g]*EllipticPi[Sqrt[-a + b]/Sqrt[a +
 b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqr
```

```
t[Sin[e + f*x]])/(a^4*Sqrt[-a + b]*Sqrt[a + b]*d^4*f*Sqrt[d*Sin[e + f*x]])
+ (4*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x
]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^3*Sqrt[g*Cos[e + f*x]]*Elli
pticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*
f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{b\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx}{ad}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{4\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{7ad^2} + \frac{b^2\int \cdots}{\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

$$= -\frac{2(g\cos(e+fx))^{3/2}}{7adfg(d\sin(e+fx))^{7/2}} + \frac{2b(g\cos(e+fx))^{3/2}}{5a^2d^2fg(d\sin(e+fx))^{5/2}} - \frac{\cdots}{21a\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.54, size = 1768, normalized size = 2.96

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[g*Cos[e + f*x]]/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])
),x]

[Out] (Sqrt[g*Cos[e + f*x]]*((2*(2*a^2*b*Cos[e + f*x] + 5*b^3*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^4) - (2*(4*a^2*Cos[e + f*x] + 7*b^2*Cos[e + f*x])*Csc[e + f*x
]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f*x]
*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^5)/(f*(d*Sin[e + f*x])^(9/2)) + (b*Sqr
t[g*Cos[e + f*x]]*Sin[e + f*x]^(9/2)*((-2*(4*a^3 + 10*a*b^2)*(-(b*AppellF1[
3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*
AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))
/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*
b + 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
 + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x]
+ a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]
]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b - 5*b^3)*Co
s[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]
^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 +
 b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7
/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f
*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f
*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[
2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (
Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) -
 (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/S
qrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e +
 f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x
]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
 (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a
+ Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Ta
n[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2
)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*
a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Ta
n[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*Sqrt[Cos[e + f*x]]*(d*Si
n[e + f*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6592 vs. $2(556) = 1112$.
time = 0.46, size = 6593, normalized size = 11.03

method	result	size
default	Expression too large to display	6593

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(1/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(g*cos(f*x + e))/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{g\,\cos\left(e+f\,x\right)}}{{\left(d\,\sin\left(e+f\,x\right)\right)}^{9/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(1/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

3.1416 $\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=982

$$\frac{3d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{4\sqrt{2}\,bf}+\frac{(a^2-b^2)\,d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^3f}$$

```
[Out] 3/8*d^(3/2)*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(
g*cos(f*x+e))^(1/2))/b/f*2^(1/2)+1/2*(a^2-b^2)*d^(3/2)*g^(3/2)*arctan(1-2^(
1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^3/f*2^(1/
2)-3/8*d^(3/2)*g^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2
)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)-1/2*(a^2-b^2)*d^(3/2)*g^(3/2)*arctan(1+
2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^3/f*2^
(1/2)-3/16*d^(3/2)*g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/
(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)-1/4*(a^2-b^2)*d^(3/2)*
g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2
)+d^(1/2)*tan(f*x+e))/b^3/f*2^(1/2)+3/16*d^(3/2)*g^(3/2)*ln(d^(1/2)+2^(1/2)
*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*
2^(1/2)+1/4*(a^2-b^2)*d^(3/2)*g^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2)*(d*sin(f*x
+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b^3/f*2^(1/2)+2*a*d^(3/
2)*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/b^3/f/(g*cos(
f*x+e))^(1/2)-2*a*d^(3/2)*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+co
s(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x
+e)^(1/2)/b^3/f/(g*cos(f*x+e))^(1/2)+1/2*g*(d*sin(f*x+e))^(3/2)*(g*cos(f*x+
e))^(1/2)/b/f-a*d*g*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b^2/f-1/2*a*d
^2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi
+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e
))^(1/2)
```

Rubi [A]

time = 1.11, antiderivative size = 982, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {2980, 2917, 2648, 2653, 2720, 2654, 303, 1176, 631, 210, 1179, 642, 2988, 2987, 2986, 1232}

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]
```

```
[Out] (3*d^(3/2)*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[
d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*A
```

```
rcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*
x]])])/(Sqrt[2]*b^3*f) - (3*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqr
t[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(4*Sqrt[2]*b*f) - ((a^2
 - b^2)*d^(3/2)*g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(
Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^3*f) + (2*Sqrt[2]*a*Sqrt[-a^2 +
b^2]*d^(3/2)*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
 ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f
*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*Sqrt[-a^2 + b^2]*d^(3/2)*g^2*Sqrt[Cos
[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*
x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^3*f*Sqrt[g*Cos[e + f*x]]) -
(3*d^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqr
t[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) - ((a^2 - b^2)*d
^(3/2)*g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*
Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) + (3*d^(3/2)*g^(3/
2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]
] + Sqrt[d]*Tan[e + f*x]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*d^(3/2)*g^(3/2)*L
og[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] +
Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^3*f) - (a*d*g*Sqrt[g*Cos[e + f*x]]*Sqrt
[d*Sin[e + f*x]])/(b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2))
/(2*b*f) + (a*d^2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/
(2*b^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2980

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[g^2*((a^2 - b^2)/b^2), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{((a^2-b^2)\,g^2)\int \frac{(d\mathrm{s}}{\sqrt{g\cos(e}}}{b^2}$$

$$= \frac{(ag^2)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{((a}{}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}\,(d}{2bf}$$

$$= -\frac{adg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{b^2f} + \frac{g\sqrt{g\cos(e+fx)}\,(d}{2bf}$$

$$= \frac{2\sqrt{2}\,a\sqrt{-a^2+b^2}\,d^{3/2}g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\mathrm{si}\right.}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,a\sqrt{-a^2+b^2}\,d^{3/2}g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\mathrm{si}\right.}{b^3f\sqrt{g\cos(e+fx)}}$$

$$= \frac{(a^2-b^2)\,d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^3f} - \frac{(a^2}{}$$

$$= \frac{3d^{3/2}g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{4\sqrt{2}\,bf} + \frac{(a^2-b^2)\,d}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 57.72, size = 1898, normalized size = 1.93

Warning: Unable to verify antiderivative.

[In] Integrate[((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))/(a + b*Sin[e + f*x]),x]

[Out] ((g*Cos[e + f*x])^(3/2)*Sec[e + f*x]*(d*Sin[e + f*x])^(3/2))/(2*b*f) - ((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*((10*b*(a^2 - b^2)*Sqrt[Cos[e + f*x]]*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((b*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[1 - Cos[e + f*x]^2])/(-5*(a^2 - b^2)*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (4*b^2*AppellF1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2) + (a*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2))*Sin[e + f*x]^(5/2))/((1 - Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))*(a + b*Sin[e + f*x])) + (2*a*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]^2)^(3/2)) - (a*Cos[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] + 20*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4) + 10*Sqrt[2]*a*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqrt[2]*a*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/2) + (40*b*Sqrt[Tan[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*AppellF1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sqrt[Tan[e + f*x]])/(Sqrt[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*AppellF1[5/4, 1/2, 2, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^2*AppellF1[5/4, 3/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*Tan[e + f*x]^2)*(-(b

```
^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2)))))/(10*b^2*Cos[e + f*x]^(5/2
)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Ta
n[e + f*x]^2])))/(4*b*f*Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2546 vs. 2(806) = 1612.
time = 0.60, size = 2547, normalized size = 2.59

method	result	size
default	Expression too large to display	2547

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/f*(a-b)*(4*I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*I*sin(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+I*s
in(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+
b^2)^(1/2)*b^2-4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*sin(f*x+e)*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+sin(f*x+
e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1
/2)*b^2+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(
1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
```

```
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b
+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a)
,1/2*2^(1/2))*a^3-4*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2+4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+4*si
n(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a*b-4*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3+4*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-4*sin(f*x
+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+2*
cos(f*x+e)^2*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b^2-4*cos(f*x+e)^2*(-a^2+b
^2)^(1/2)*2^(1/2)*a*b-2*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*b^2+
4*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b)*(g*cos(f*x+e))^(3/2)*(d*sin(f*x+
e))^(3/2)/sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2*2^(1/2)*a/b^3/(-a^2+b^2)^
(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

Fricas [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*(d*sin(f*x + e))^(3/2)/(b*sin(f*x + e) + a
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}\,(d\,\sin\left(e+f\,x\right))^{3/2}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x)), x
)
```

3.1417 $\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=611

$$-\frac{a\sqrt{d}\,g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f}+\frac{a\sqrt{d}\,g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f}-\frac{2\sqrt{2}}{}$$

```
[Out] -1/2*a*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos
(f*x+e))^(1/2))*d^(1/2)/b^2/f*2^(1/2)+1/2*a*g^(3/2)*arctan(1+2^(1/2)*g^(1/2
)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))*d^(1/2)/b^2/f*2^(1/2)+
1/4*a*g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e)
)^(1/2)+d^(1/2)*tan(f*x+e))*d^(1/2)/b^2/f*2^(1/2)-1/4*a*g^(3/2)*ln(d^(1/2)+
2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e
))*d^(1/2)/b^2/f*2^(1/2)-2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+c
os(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*d^(1/2
)*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e
))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-
a^2+b^2)^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/b^2/f/(g*cos(f*x+e))^(1/2)+g*(g*cos
(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f+1/2*d*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/
2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1
/2)/b/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2980, 2917, 2654, 303, 1176, 631, 210, 1179, 642, 2648, 2653, 2720, 2987, 2986, 1232}

$$-\frac{2\sqrt{2}\sqrt{d}\,g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}\,g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{g\cos(e+fx)}}-\frac{a\sqrt{d}\,g^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f}+\frac{a\sqrt{d}\,g^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}+1\right)}{\sqrt{2}\,b^2f}+\frac{a\sqrt{d}\,g^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b^2f}-\frac{a\sqrt{d}\,g^{3/2}\log\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b^2f}-\frac{dg^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2bf\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{g\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{bf}$$

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -((a*Sqrt[d]*g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqr
t[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f)) + (a*Sqrt[d]*g^(3/2)*ArcTan[1
 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/
(Sqrt[2]*b^2*f) - (2*Sqrt[2]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]
]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt
[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2
]*Sqrt[-a^2 + b^2]*Sqrt[d]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b^2*f*Sqrt[g*Cos[e + f*x]]) + (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] - (Sq
```

```
rt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e +
f*x]])/(2*Sqrt[2]*b^2*f) - (a*Sqrt[d]*g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g
]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sq
rt[2]*b^2*f) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f) - (d*g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
 f*x]]*Sqrt[d*Sin[e + f*x]])
```

<u>Rule 210</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

<u>Rule 303</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

<u>Rule 631</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 642</u>

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

<u>Rule 1176</u>

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

<u>Rule 1179</u>

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2980

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[g^2*((a^2 - b^2)/b^2), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])
```

```
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \frac{\sqrt{d\sin(e+fx)}\,(a-b\sin(e+fx))}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{((a^2-b^2)\,g^2)\int \frac{\sqrt{\cdots}}{\sqrt{g\cos(e\cdots}}}{b^2\cdots}$$

$$= \frac{(ag^2)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b^2} - \frac{g^2\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}\,dx}{bd} - \frac{\left((a^2\cdots\right.}{\cdots}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bf} - \frac{(dg^2)\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{\cdots}}}{2b}$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin\cdots\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin\cdots\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,\sqrt{-a^2+b^2}\,\sqrt{d}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin\cdots\right.}{b^2f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{a\sqrt{d}\,g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f} + \frac{a\sqrt{d}\,g^{3/2}\,\mathrm{t}\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 29.18, size = 604, normalized size = 0.99

$$-\frac{(g\cos(e+fx))^{5/2}(d\sin(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{2aF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)}{a^2-b^2}+\frac{(2a^2-b^2)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)}{-a^2b+b^3}+\frac{5\left(-5(a^2-b^2)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)(a^2-2b^2+b^2\cos^2(e+fx))+\left(-4b^2F_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+3(a^2-b^2)F_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)\sin^2(e+fx)(a^2-b^2\sin^2(e+fx))\right)}{b\left(-5(a^2-b^2)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+\left(4b^2F_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+3(-a^2+b^2)F_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)\cos^2(e+fx)\right)\sin^2(e+fx)^{3/4}(a^2-b^2\sin^2(e+fx))}\right)}{5dfg\sin^2(e+fx)^{3/4}(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] -1/5*((g*Cos[e + f*x])^(5/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x
]^2])*((2*a*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
```

```
/(-a^2 + b^2)])/(a^2 - b^2) + ((2*a^2 - b^2)*AppellF1[5/4, 3/4, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])/(-(a^2*b) + b^3) + (5*(-5*
(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]*(a^2 - 2*b^2 + b^2*Cos[e + f*x]^2) + (-4*b^2*AppellF1[5/4, 3
/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b
^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)])*Sin[e + f*x]^2*(a^2 - b^2*Sin[e + f*x]^2)))/(b*(-5*(a^2 - b^2)*Appe
llF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] +
 (4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)] + 3*(-a^2 + b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2
*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(Sin[e + f*x]^2)^(3/4)*(a^2
 - b^2*Sin[e + f*x]^2))))/(d*f*g*(Sin[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]
))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(517) = 1034$.
time = 0.38, size = 1926, normalized size = 3.15

method	result	size
default	Expression too large to display	1926

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
+1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*si
n(f*x+e)*(-a^2+b^2)^(1/2)*a-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I
,1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a+(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*sin(f*x+e)*(-a
^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+
a),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/
```

```
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*sin(f*x+e)*a^2+(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*sin(f*x+e)*b^2
-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))
*sin(f*x+e)*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-
a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*(-a^2+b^2)^(1/2)*b+(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*sin(f*x+e)*a^2
-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))
*sin(f*x+e)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
sin(f*x+e)*(-a^2+b^2)^(1/2)*a+cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*b-cos(f
*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b)*(g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)
/(-1+cos(f*x+e))/cos(f*x+e)^2*2^(1/2)*a/b^2/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)
^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin(e+fx)}\,(g\cos(e+fx))^{\frac{3}{2}}}{a+b\sin(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))*(g*cos(e + f*x))**(3/2)/(a + b*sin(e + f*x)),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}\,\sqrt{d\,\sin(e+f\,x)}}{a+b\,\sin(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

3.1418 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=577

$$\frac{g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f}-\frac{g^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f}+\frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g}{}$$

[Out] 1/2*g^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)/d^(1/2)-1/2*g^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)/d^(1/2)-1/4*g^(3/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)/d^(1/2)+1/4*g^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)/d^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a/b/f/d^(1/2)/(g*cos(f*x+e))^(1/2)-2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a/b/f/d^(1/2)/(g*cos(f*x+e))^(1/2)-g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)

Rubi [A]
time = 0.60, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2979, 2917, 2653, 2720, 2654, 303, 1176, 631, 210, 1179, 642, 2987, 2986, 1232}

$$\frac{2\sqrt{2}\,g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,g^2\sqrt{b^2-a^2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}+\frac{g^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{af\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{g^{3/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f}-\frac{g^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}+1\right)}{\sqrt{2}\,b\sqrt{d}\,f}-\frac{g^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b\sqrt{d}\,f}+\frac{g^{3/2}\log\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b\sqrt{d}\,f}$$

Antiderivative was successfully verified.

[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]

[Out] (g^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d]*f) - (g^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*Sqrt[d]*f) + (2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*b*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*b*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (g^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*S

```
qrt[d]*f) + (g^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/S
qrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*Sqrt[d]*f) + (g^2
*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqrt[g*Cos[e + f
*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2979

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[g^2*((a^2 - b^2)/(a*b*d)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e +
 f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
```

```
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{b-a\sin(e+fx)}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{ab} + \frac{((a^2-b^2)\,g^2)\int \frac{}{\sqrt{g}}}{}$$

$$= \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{g^2\int \frac{\sqrt{d\sin(e+fx}}{\sqrt{g\cos(e+fx}}}{bd}$$

$$= -\frac{(2g^3)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{\left(2\sqrt{2}\,(a^2\right.}{}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\right.\right.}{ab\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{g^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b\sqrt{d}\,f} - \frac{g^{3/2}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 21.38, size = 178, normalized size = 0.31

$$\frac{2\left(bF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)-aF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}\left(a+b\sqrt{\sin^2(e+fx)}\right)}{5\,(a^2-b^2)\,dfg\sqrt[4]{\sin^2(e+fx)}\,(a+b\sin(e+fx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x]))
),x]
```

[Out] (2*(b*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] - a*AppellF1[5/4, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*(g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sqrt[Sin[e + f*x]^2]))/(5*(a^2 - b^2)*d*f*g*(Sin[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x]))

Maple [A]
time = 0.31, size = 944, normalized size = 1.64

method	result
default	$\frac{(a-b)\left(i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)\sqrt{-a^2+b^2}\,a-i\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(f}{\sin(fx+e)}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] 1/f*(a-b)*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+a^2*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-a^2*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^2)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)/(-1+cos(f*x+e))/cos(f*x+e)^2/(d*sin(f*x+e))^(1/2)*2^(1/2)/b/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos{(e+fx)})^{\frac{3}{2}}}{\sqrt{d\sin{(e+fx)}}\,(a+b\sin{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1419 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=321

$$-\frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{-\cdots}}{}$$

```
[Out] -2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/2)/f
/(g*cos(f*x+e))^(1/2)+2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e
)^(1/2)/a^2/d^(3/2)/f/(g*cos(f*x+e))^(1/2)-2*g*(g*cos(f*x+e))^(1/2)/a/d/f/(
d*sin(f*x+e))^(1/2)+b*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ell
ipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/d/f/(g*cos(f*x+e
))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2978, 2643, 2653, 2720, 2987, 2986, 1232}

$$-\frac{2\sqrt{2}\,g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}-\frac{bg^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^2df\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqr
t[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]
])], -1])/(a^2*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a^2 + b^2
]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqr
t[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(a*d*f*Sqrt[d*Sin[e + f*x]
]) - (b*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*S
qrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
```

```
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{}{\sqrt{g\cos}}}{a}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{\left((a^2-b^2)\,g^2\sqrt{\cos(e+fx)}\right)\int \frac{}{\sqrt{\cos}}}{a^2d^2\sqrt{g\cos(e+}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{adf\sqrt{d\sin(e+fx)}} - \frac{bg^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{a^2df\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}}$$

$$-\frac{2\sqrt{2}\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\right.}{a^2d^{3/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 37.70, size = 1095, normalized size = 3.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) - ((g
*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(3/2)*((-2*b*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[
e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(
5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*
Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Co
s[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^
2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*S
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Ar
cTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos
[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 +
Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(
-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqr
```

```
t[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f
*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin
[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (2*a*Sqrt[S
in[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*
x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 -
 b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)
*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt
[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e
+ f*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(a*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*
x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2586 vs. $2(299) = 598$.
time = 0.32, size = 2587, normalized size = 8.06

method	result	size
default	Expression too large to display	2587

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(a-b)*(2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a
^2+b^2)^(1/2)*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+
b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b
-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1
/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b-cos(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2
```

```
+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1
/2*2^(1/2))*b^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))*a^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-(-a^2+b^2)^(1/2)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))
*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2
))*(-a^2+b^2)^(1/2)*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2+EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^2*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)+(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*2^(1/2)*cos(f*x+
e)*(-a^2+b^2)^(1/2)*a)*(g*cos(f*x+e))^(3/2)*sin(f*x+e)/cos(f*x+e)^2/(d*sin(
f*x+e))^(3/2)*2^(1/2)/a/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b
^2)^(1/2)-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^{\frac{3}{2}}}{(d\sin(e+fx))^{\frac{3}{2}}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/((d*sin(e + f*x))**(3/2)*(a + b*sin(e + f*
x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}}{(d\,\sin(e+f\,x))^{3/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1420 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=435

$$\frac{2\sqrt{2}\,b\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{}}{}$$

```
[Out] 2*b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-
(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/
f/(g*cos(f*x+e))^(1/2)-2*b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+c
os(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*
x+e)^(1/2)/a^3/d^(5/2)/f/(g*cos(f*x+e))^(1/2)-2/3*g*(g*cos(f*x+e))^(1/2)/a/
d/f/(d*sin(f*x+e))^(3/2)+2*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin(f*x+e)
)^(1/2)-2/3*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos
(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/d^2/f/(g*cos(f*x+e))^(1/2)/(
d*sin(f*x+e))^(1/2)+(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+
f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^3/d^2/f/(g
*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.63, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2978, 2650, 2653, 2720, 2643, 2989, 2987, 2986, 1232}

$$\frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,bg^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,bg^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}-\frac{g^2(a^2-b^2)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2g^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3ad^2f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sq
rt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x
]])], -1])/(a^3*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a^2 +
b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[
Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*d^(5/2)*f
*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*(d*Sin[e + f*x
])^(3/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) +
 (2*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqr
t[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) - ((a^2 - b^2)*g^2*EllipticF[e - Pi
/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d
*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
```

```
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{}{\sqrt{g\,c}}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{(b(a^2-}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(b(a^2-\right.}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2g^2F(e}{3ad^2f\sqrt{}}$$

$$= \frac{2\sqrt{2}\,b\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-}\right.}{a^3d^{5/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 40.04, size = 1138, normalized size = 2.62

$$\frac{(g\cos(e+fx))^{3/2}\left(\frac{2b\csc(e+fx)}{a^2}-\frac{2\csc^2(e+fx)}{3a}\right)\sin^2(e+fx)\tan(e+fx)}{f(d\sin(e+fx))^{5/2}} - \cdots$$
[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))
*Sin[e + f*x]^2*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(5/2)) - ((g*Cos[e + f*x]
)^(3/2)*Sin[e + f*x]^(5/2)*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^
2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5
*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*C
os[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2
*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sq
rt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[
e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + C
os[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-
1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt
[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*
x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[
e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[
Sin[e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2
- b^2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2
)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqr
t[1 + Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e
 + f*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Cos[e + f*x]^(3/2)*(d*Sin[e
 + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3013 vs. $2(424) = 848$.
time = 0.30, size = 3014, normalized size = 6.93

method	result	size

default	Expression too large to display	3014

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/3/f*(a-b)*(-2*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticF((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*a^2+6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-3*(-a^2+b^2)^(1/2)*cos(f
*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)
+a),1/2*2^(1/2))*a*b-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2-3*(-a^2+b
^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(
-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2
)*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a
^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b+3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^
(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3+3*cos(f*x+e
)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*a^2*b-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3-2*(-a^2+b^2)^(1/2)*sin(f*x+e
)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
```

```
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+6*sin(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-3*sin(f*x+e)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(
-a^2+b^2)^(1/2)*a*b-3*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2-3*sin(f*x+e)*(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^
2)^(1/2)*a*b-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-3*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b+3*sin(f*x
+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b
^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*b^3+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(
1/2)-a),1/2*2^(1/2))*a^2*b-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4372 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{3/2}}{(d\,\sin(e+f\,x))^{5/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

3.1421 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=525

$$-\frac{2\sqrt{2}\,b^2\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2}{}$$

```
[Out] -2*b^2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/
(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^4/d^(7/
2)/f/(g*cos(f*x+e))^(1/2)+2*b^2*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)
/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*c
os(f*x+e)^(1/2)/a^4/d^(7/2)/f/(g*cos(f*x+e))^(1/2)-2/5*g*(g*cos(f*x+e))^(1/
2)/a/d/f/(d*sin(f*x+e))^(5/2)+2/3*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin
(f*x+e))^(3/2)-8/5*g*(g*cos(f*x+e))^(1/2)/a/d^3/f/(d*sin(f*x+e))^(1/2)+2*(a
^2-b^2)*g*(g*cos(f*x+e))^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2)+2/3*b*g^2*(si
n(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1
/2))*sin(2*f*x+2*e)^(1/2)/a^2/d^3/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/
2)-b*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(
cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^4/d^3/f/(g*cos(f*x+e))^(1
/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.86, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2978, 2650, 2643, 2653, 2720, 2989, 2987, 2986, 1232}

$$-\frac{2bg^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^2d^3f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}}-\frac{2\sqrt{2}\,b^2g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}+\frac{bg^2(a^2-b^2)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^4d^3f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2g(a^2-b^2)\sqrt{g\cos(e+fx)}}{a^3d^3f\sqrt{d\sin(e+fx)}}-\frac{8g\sqrt{g\cos(e+fx)}}{5ad^3f\sqrt{d\sin(e+fx)}}-\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
 Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
f*x]])], -1])/(a^4*d^(7/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Ar
cSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^4*d^(7
/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(5*a*d*f*(d*Sin[e
+ f*x])^(5/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(3*a^2*d^2*f*(d*Sin[e + f*x])
^(3/2)) - (8*g*Sqrt[g*Cos[e + f*x]])/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*
(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*b
*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*d^3*f*Sqrt
[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b*(a^2 - b^2)*g^2*EllipticF[e - P
```

```
i/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^4*d^3*f*Sqrt[g*Cos[e + f*x]]*Sqrt[
d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{}{\sqrt{g\,\text{c}}}}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{(2bg^2)}{}$$

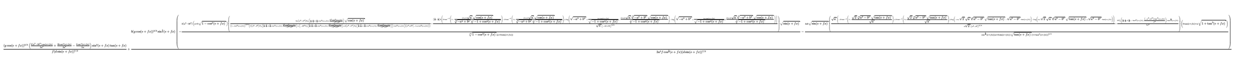
$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\,\sqrt{}}{5ad^3f}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\,\sqrt{}}{5ad^3f}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{8g\,\sqrt{}}{5ad^3f}$$

$$= -\frac{2\sqrt{2}\,b^2\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\text{si}\right.}{a^4d^{7/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 39.86, size = 1165, normalized size = 2.22



Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((2*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^3) + (2*b*Csc[
e + f*x]^2)/(3*a^2) - (2*Csc[e + f*x]^3)/(5*a))*Sin[e + f*x]^3*Tan[e + f*x]
)/(f*(d*Sin[e + f*x])^(7/2)) + (b*(g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)
*((-2*(a^2 - 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*Appe
llF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*S
qrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4,
 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*
AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2
```

```
)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e +
f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) -
 ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 +
 b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*S
qrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[
Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*
Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((
1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)
^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f
*x]^2)^(1/4)*(a + b*Sin[e + f*x])) - (4*a*b*Sqrt[Sin[e + f*x]]*((Sqrt[a]*(-
2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*Ar
cTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a +
 Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan
[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
 Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*AppellF
1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan
[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(C
os[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*x]
^2)^(3/2))))/(3*a^3*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5827 vs. $2(504) = 1008$.
time = 0.35, size = 5828, normalized size = 11.10

method	result	size
default	Expression too large to display	5828

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g \cos (e+f x))^{3/2}}{(d \sin (e+f x))^{7/2} (a+b \sin (e+f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

3.1422 $\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=688

$$\frac{2\sqrt{2}\,b^3\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3\sqrt{}}{}$$

```
[Out] 2*b^3*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(
b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)^(1/2)/a^5/d^(9/2
)/f/(g*cos(f*x+e))^(1/2)-2*b^3*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/
(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*(-a^2+b^2)^(1/2)*co
s(f*x+e)^(1/2)/a^5/d^(9/2)/f/(g*cos(f*x+e))^(1/2)-2/7*g*(g*cos(f*x+e))^(1/2
)/a/d/f/(d*sin(f*x+e))^(7/2)+2/5*b*g*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/(d*sin(
f*x+e))^(5/2)-4/7*g*(g*cos(f*x+e))^(1/2)/a/d^3/f/(d*sin(f*x+e))^(3/2)+2/3*(
a^2-b^2)*g*(g*cos(f*x+e))^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(3/2)+8/5*b*g*(g*c
os(f*x+e))^(1/2)/a^2/d^4/f/(d*sin(f*x+e))^(1/2)-2*b*(a^2-b^2)*g*(g*cos(f*x+
e))^(1/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)-4/7*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2
)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/
2)/a/d^4/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)+2/3*(a^2-b^2)*g^2*(sin
(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/
2))*sin(2*f*x+2*e)^(1/2)/a^3/d^4/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2
)+b^2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF
(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^5/d^4/f/(g*cos(f*x+e))^(
1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.12, antiderivative size = 688, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2978, 2650, 2653, 2720, 2643, 2989, 2987, 2986, 1232}

$$\frac{8bg\sqrt{g\cos(e+fx)}}{5a^2d^4f\sqrt{d\sin(e+fx)}}+\frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}}+\frac{2\sqrt{2}\,b^3g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3g^2\sqrt{b^2-a^2}\,\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}-\frac{b^2g^2(a^2-b^2)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^5d^4f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{2bg(a^2-b^2)\sqrt{g\cos(e+fx)}}{a^4d^4f\sqrt{d\sin(e+fx)}}-\frac{2g^2(a^2-b^2)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3a^3d^4f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2g(a^2-b^2)\sqrt{g\cos(e+fx)}}{3a^3d^3f(d\sin(e+fx))^{3/2}}+\frac{4g^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{7ad^4f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{4g\sqrt{g\cos(e+fx)}}{7ad^3f(d\sin(e+fx))^{3/2}}-\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b^3*Sqrt[-a^2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b -
Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f
*x]])], -1])/(a^5*d^(9/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[-a^
2 + b^2]*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), Arc
Sin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^5*d^(9/
2)*f*Sqrt[g*Cos[e + f*x]]) - (2*g*Sqrt[g*Cos[e + f*x]])/(7*a*d*f*(d*Sin[e +
 f*x])^(7/2)) + (2*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^2*f*(d*Sin[e + f*x])^
(5/2)) - (4*g*Sqrt[g*Cos[e + f*x]])/(7*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2
```

```
*(a^2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) +
 (8*b*g*Sqrt[g*Cos[e + f*x]])/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^
2 - b^2)*g*Sqrt[g*Cos[e + f*x]])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (4*g^2*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(7*a*d^4*f*Sqrt[g*Cos[
e + f*x]]*Sqrt[d*Sin[e + f*x]]) - (2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^3*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin
[e + f*x]]) - (b^2*(a^2 - b^2)*g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*
e + 2*f*x]])/(a^5*d^4*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
```

```
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{9/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{}{\sqrt{g\,\mathrm{c}}}}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{(4bg^2)}{}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g}{7ad^3f}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g}{7ad^3f}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g}{7ad^3f}$$

$$= -\frac{2g\sqrt{g\cos(e+fx)}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg\sqrt{g\cos(e+fx)}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{4g}{7ad^3f}$$

$$= \frac{2\sqrt{2}\,b^3\sqrt{-a^2+b^2}\,g^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin\right.}{a^5d^{9/2}f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 40.54, size = 1210, normalized size = 1.76

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(3/2)*((-2*b*(a^2 - 5*b^2)*Csc[e + f*x])/(5*a^4) + (2*(a^
2 - 7*b^2)*Csc[e + f*x]^2)/(21*a^3) + (2*b*Csc[e + f*x]^3)/(5*a^2) - (2*Csc
[e + f*x]^4)/(7*a))*Sin[e + f*x]^4*Tan[e + f*x])/(f*(d*Sin[e + f*x])^(9/2))
 - ((g*Cos[e + f*x])^(3/2)*Sin[e + f*x]^(9/2)*((-2*(2*a^4 + 7*a^2*b^2 - 21*
b^4)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4,
```

```
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x
]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, C
os[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3
/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b
^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b
*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1
 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2
] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 +
 b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^
2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*
(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt
[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e + f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a
 + b*Sin[e + f*x])) + (2*(2*a^3*b - 14*a*b^3)*Sqrt[Sin[e + f*x]]*((Sqrt[a]*
(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*
ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-a
 + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*T
an[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appel
lF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*T
an[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/
(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f*
x]^2)^(3/2))))/(21*a^4*f*Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(9/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6706 vs. $2(670) = 1340$.
time = 0.42, size = 6707, normalized size = 9.75

method	result	size
default	Expression too large to display	6707

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maptima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}}{(d\,\sin\left(e+f\,x\right))^{9/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

3.1423 $\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=936

$$-\frac{\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf}-\frac{(a^2-b^2)\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f}$$

```
[Out] 1/8*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f
*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/2*(a^2-b^2)*g^(5/2)*arctan(-1+2^(1/2)*d
^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(
1/2)+1/8*g^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*s
in(f*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/2*(a^2-b^2)*g^(5/2)*arctan(1+2^(1/2
)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*
2^(1/2)+1/16*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f
*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b/f*2^(1/2)+1/4*(a^2-b^2)*g^(5/2
)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin
(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(1/2)-1/16*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^
(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b/
f*2^(1/2)-1/4*(a^2-b^2)*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/
2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))*d^(1/2)/b^3/f*2^(1/2)-2*a*d*g
^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)
^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/b^3
/f/(d*sin(f*x+e))^(1/2)+2*a*d*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/
2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a
+b)^(1/2)*sin(f*x+e)^(1/2)/b^3/f/(d*sin(f*x+e))^(1/2)+1/2*g*(g*cos(f*x+e))^
(3/2)*(d*sin(f*x+e))^(1/2)/b/f-a*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*
Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*
x+e))^(1/2)/b^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.99, antiderivative size = 936, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.460$, Rules used = {2980, 2917, 2652, 2719, 2648, 2655, 303, 1176, 631, 210, 1179, 642, 2988, 2985, 2984, 504, 1232}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]),x]
```

```
[Out] -1/4*(Sqrt[d]*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sq
rt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*
```

```
ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f
*x]])])/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt
[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(4*Sqrt[2]*b*f) + ((a^2
- b^2)*Sqrt[d]*g^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(S
qrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^3*f) + (Sqrt[d]*g^(5/2)*Log[Sqrt[
g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*S
in[e + f*x]]])/(8*Sqrt[2]*b*f) + ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] +
 Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e
 + f*x]]])/(2*Sqrt[2]*b^3*f) - (Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e
 + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(8*
Sqrt[2]*b*f) - ((a^2 - b^2)*Sqrt[d]*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f
*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt
[2]*b^3*f) - (2*Sqrt[2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[-(S
qrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin
[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqr
t[2]*a*Sqrt[-a + b]*Sqrt[a + b]*d*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a +
b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt
[Sin[e + f*x]])/(b^3*f*Sqrt[d*Sin[e + f*x]]) + (g*(g*Cos[e + f*x])^(3/2)*Sq
rt[d*Sin[e + f*x]])/(2*b*f) + (a*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(b^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
```

```
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2980

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[g^2*((a^2 - b^2)/b^2), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
```

```
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}{a+b\sin(e+fx)}\,dx = \frac{g^2\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,(a-b\sin(e+fx))\,dx}{b^2} - \frac{((\cdots}{\cdots}$$

$$= \frac{(ag^2)\int \sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,dx}{b^2} - \frac{g^2\int \sqrt{g\cos(e+\cdots}{\cdots}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} - \frac{(dg^2)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\cdots}{4b}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}\,E(e\cdots}{b^2f\sqrt{\sin\cdots}}$$

$$= \frac{g(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}{2bf} + \frac{ag^2\sqrt{g\cos(e+fx)}\,E(e\cdots}{b^2f\sqrt{\sin\cdots}}$$

$$= \frac{(a^2-b^2)\sqrt{d}\,g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos\cdots}}{\sqrt{d\sin(e+\cdots}}\cdots\right.}{2\sqrt{2}\,b^3f}$$

$$= -\frac{(a^2-b^2)\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^3f} + \frac{(a^{\cdots}}{\cdots}$$

$$= -\frac{\sqrt{d}\,g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{4\sqrt{2}\,bf} - \frac{(a^2-b^2)\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 55.35, size = 1615, normalized size = 1.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]])/(a + b*Sin[e + f*x]
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sec[e + f*x]*Sqrt[d*Sin[e + f*x]])/(2*b*f) - ((g*Co
s[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]*((2*b*(-(b*AppellF1[3/4, -1/4, 1, 7/
4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/
4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]
^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(1
 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (Sqrt[Tan[e + f*x]]*((3*Sq
rt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqr
t[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*Ap
pellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a*
Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x
]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt
[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[
e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2
+ b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[
2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e +
f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]
*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a
^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]
*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[
a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a
^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*S
qrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt
[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e +
f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e
```

```
+ f*x]^2])))/(42*a*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e +
 f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(4*b*f*Cos[e + f*x
]^(5/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6310 vs. 2(757) = 1514.
time = 0.60, size = 6311, normalized size = 6.74

method	result	size
default	Expression too large to display	6311

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)*(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8571 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e))/(b*sin(f*x + e) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)),x)
```

```
[Out] int(((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x)), x
)
```

3.1424 $\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=572

$$\frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\log\left(\sqrt{g}\right.}{}$$

```
[Out] -1/2*a*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*si
n(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)-1/2*a*g^(5/2)*arctan(1+2^(1/2)*d^(1/
2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)
-1/4*a*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))
^(1/2)/(d*sin(f*x+e))^(1/2))/b^2/f*2^(1/2)/d^(1/2)+1/4*a*g^(5/2)*ln(g^(1/2)
+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/
2))/b^2/f*2^(1/2)/d^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)
/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+
b)^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(d*sin(f*x+e))^(1/2)-2*g^(5/2)*EllipticPi((
g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I
)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/b^2/f/(d*sin(f*x+e))^(1
/2)+g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*P
i+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/d/f/sin(2*f*x+2
*e)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 572, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2980, 2917, 2655, 303, 1176, 631, 210, 1179, 642, 2652, 2719, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2 f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{b^2 f\sqrt{d\sin(e+fx)}}+\frac{ag^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{ag^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,b^2\sqrt{d}\,f}+\frac{ag^{5/2}\log\left(\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,b^2\sqrt{d}\,f}-\frac{g^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{bdf\sqrt{\sin(2e+2fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b^2*Sqrt[d]*f) - (a*g^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b^
2*Sqrt[d]*f) - (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqr
t[d]*Sqrt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f)
 + (a*g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*
Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b^2*Sqrt[d]*f) + (2*Sqrt[2
]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]),
ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin
```

```
[e + f*x]])/(b^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a +
 b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b^2*f*Sqrt[d*
Sin[e + f*x]]) - (g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqr
t[d*Sin[e + f*x]])/(b*d*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2980

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/b^2, Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[g^2*((a^2 - b^2)/b^2), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
```

```
 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}\,(a-b\sin(e+fx))}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{((a^2-b^2)\,g^2)\int \frac{\sqrt{\cdots}}{\sqrt{d\sin(\cdots}}}{b\cdots}$$

$$= \frac{(ag^2)\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{b^2} - \frac{g^2\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\cdots}}{bd}$$

$$= -\frac{(2adg^3)\,\mathrm{Subst}\left(\int \frac{x^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{b^2 f} + \frac{\left(4\sqrt{2}\,(a\cdots\right.}{\cdots}$$

$$= -\frac{g^2\sqrt{g\cos(e+fx)}\,E\left(e-\frac{\pi}{4}+fx\middle|\,2\right)\sqrt{d\sin(e+fx)}}{bdf\sqrt{\sin(2e+2fx)}} + \frac{(ag^3\cdots}{\cdots}$$

$$= \frac{2\sqrt{2}\,\sqrt{-a+b}\,\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(\cdots}}{\sqrt{g}\,\sqrt{1+\mathrm{s}\cdots}}\right.\right.}{b^2 f\sqrt{d\sin(e+fx)}}$$

$$= -\frac{ag^{5/2}\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right.}{2\sqrt{2}\,b^2\sqrt{d}\,f}$$

$$= \frac{ag^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b^2\sqrt{d}\,f} - \frac{ag^{5/2}\tan^{-1}\left(1-\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.51, size = 1399, normalized size = 2.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*Sqrt[Sin[e + f*x]]*((Sqrt[Tan[e + f*x]]*((3*Sqrt[2]
*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt
[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
 - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2
 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF
1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan
[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos
[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2
)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1
+ Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e +
 f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b
^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x
]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*
Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x
]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 -
 b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^
2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2
- b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sq
rt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt
[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt
[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]
*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x
]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f
*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e +
f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Cos[e + f*x]^(
5/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5223 vs. $2(474) = 948$.
time = 0.38, size = 5224, normalized size = 9.13

method	result	size
default	Expression too large to display	5224

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1425 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$-\frac{g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bd^{3/2}f}+\frac{g^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bd^{3/2}f}+\frac{g^{5/2}\log\left(\sqrt{g}+\right.}{}$$

```
[Out] 1/2*g^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f
*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)+1/2*g^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*c
os(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)+1/4*g^(5
/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*s
in(f*x+e))^(1/2))/b/d^(3/2)/f*2^(1/2)-1/4*g^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(
1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/b/d^(3/2)/f
*2^(1/2)-2*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(1/2)-2*g^(5/2)*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a/b/d/f/(d*sin
(f*x+e))^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x
+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin
(f*x+e)^(1/2)/a/b/d/f/(d*sin(f*x+e))^(1/2)+2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2
)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/
2)*(d*sin(f*x+e))^(1/2)/a/d^2/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.72, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 16, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {2979, 2917, 2650, 2652, 2719, 2655, 303, 1176, 631, 210, 1179, 642, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{abdf\sqrt{d\sin(e+fx)}}-\frac{2g^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{ad^2f\sqrt{\sin(2e+2fx)}}-\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}}-\frac{g^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bd^{3/2}f}+\frac{g^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}+1\right)}{\sqrt{2}\,bd^{3/2}f}+\frac{g^{5/2}\log\left(-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,bd^{3/2}f}-\frac{g^{5/2}\log\left(\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}+\sqrt{g}\cot(e+fx)+\sqrt{g}\right)}{2\sqrt{2}\,bd^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((g^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[
d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(3/2)*f)) + (g^(5/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*d^(
3/2)*f) + (g^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sq
rt[g*Cos[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (g^(5/
2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x
]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*d^(3/2)*f) - (2*g*(g*Cos[e + f*x])^
(3/2))/(a*d*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g
^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]]/
(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*
```

```
Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[Sqr
t[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e
+ f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*b*d*f*Sqrt[d*Sin[e + f*x]]) - (2*g^2*
Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a*
d^2*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2979

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/(a*b), I
nt[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n*(b - a*Sin[e + f*x]), x], x]
 + Dist[g^2*((a^2 - b^2)/(a*b*d)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e +
 f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x
] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LtQ[n, -1] |
| (EqQ[p, 3/2] && EqQ[n, -2^(-1)]))
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}\,(b-a\sin(e+fx))}{(d\sin(e+fx))^{3/2}}\,dx}{ab} + \frac{((a^2-b^2)\,g^2)\int \frac{\ldots}{\sqrt{d\sin\ldots}}}{\ldots}$$

$$= \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}}\,dx}{a} - \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx}{bd} + \frac{\left((a^2-\ldots\right.}{\ldots}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{(2g^2)\int \sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\ldots}}{ad^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{g^3\text{Subst}\left(\int \frac{g-dx^2}{g^2+d^2x^4}\,dx, x, \frac{\sqrt{g\cos(e+\ldots}}{\sqrt{d\sin(e+\ldots}}\right.}{bdf}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{adf\sqrt{d\sin(e+fx)}} - \frac{2\sqrt{2}\,\sqrt{-a+b}\,\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{-\ldots}}{\sqrt{a\ldots}}\right.}{\ldots}$$

$$= \frac{g^{5/2}\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right)}{2\sqrt{2}\,bd^{3/2}f}$$

$$= -\frac{g^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,bd^{3/2}f} + \frac{g^{5/2}\tan^{-1}\left(1\ldots\right.}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.25, size = 1611, normalized size = 2.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (-2*(g*Cos[e + f*x])^(5/2)*Tan[e + f*x])/(a*f*(d*Sin[e + f*x])^(3/2)) + ((g
*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(3/2)*((2*a*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/((a^2 - b^2)*(
1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) - (b*Sqrt[Tan[e + f*x]]*((3
*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x
]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/
Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] -
Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b
*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/
a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(6*
a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e +
 f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*
Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -
Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-
a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[
e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - S
qrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqr
t[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
 Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqr
t[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]]
)/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/
4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
(2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a +
Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[
e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Ta
n[e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin
[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*f*Cos[e + f
*x]^(5/2)*(d*Sin[e + f*x])^(3/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5206 vs. $2(515) = 1030$.
time = 0.40, size = 5207, normalized size = 8.45

method	result	size
default	Expression too large to display	5207

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{(d\,\sin\left(e+f\,x\right))^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1426 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=359

$$-\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}}+\frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{-a+b}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{g}\sqrt{}}\right.\right.}{a^2d^2f\sqrt{d\sin(e+f}}$$

```
[Out] -2/3*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(3/2)+2*b*g*(g*cos(f*x+e))
^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(1/2)+2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(
1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+
b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2/f/(d*sin(f*x+e))^(1/2)-2*g^(5
/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/
2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^2/d^2
/f/(d*sin(f*x+e))^(1/2)-2*b*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*
x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))
^(1/2)/a^2/d^3/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2978, 2643, 2650, 2652, 2719, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2f\sqrt{d\sin(e+fx)}}+\frac{2bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^2d^3f\sqrt{\sin(2e+2fx)}}+\frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}}-\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(3*a*d*f*(d*Sin[e + f*x])^(3/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*Sqrt[-a
+ b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqr
t[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]]
)/(a^2*d^2*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*Sqrt[-a + b]*Sqrt[a + b]*g^
(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqr
t[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*d^2*f*Sqrt[d*Si
n[e + f*x]]) + (2*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*S
qrt[d*Sin[e + f*x]])/(a^2*d^3*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{\left(\left(a^2-b^2\right)g^2\right)\int \frac{\sqrt{g\cos(e+\ldots}}{\sqrt{d\sin(e+fx)}\,(a\ldots}}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{\left(2bg^2\right)\int \ldots}{\ldots}$$

$$\left(4\sqrt{2}\,(a\ldots\right.$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \ldots$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2bg^2\sqrt{g\,c\ldots}}{\ldots}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{3adf(d\sin(e+fx))^{3/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{a^2d^2f\sqrt{d\sin(e+fx)}} + \frac{2\sqrt{2}\,\sqrt{-\ldots}}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 54.38, size = 1656, normalized size = 4.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Csc[e + f
*x])/(3*a))*Sin[e + f*x]*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])^(5/2)) - ((g*C
os[e + f*x])^(5/2)*Sin[e + f*x]^(5/2)*((4*a*b*(-(b*AppellF1[3/4, -1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1
/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x
]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)
*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((a^2 - 2*b^2)*Sqrt[Tan
[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Ta
n[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^
2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e +
 f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x
])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Ta
n[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4,
 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^
(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1
+ b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/
2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sq
rt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^
(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)
^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^
2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*
a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3
/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log
[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2
]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2
- b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqr
t[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[
2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/
2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x
```

```
]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(
a^2*f*Cos[e + f*x]^(5/2)*(d*Sin[e + f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4667 vs. $2(324) = 648$.
time = 0.30, size = 4668, normalized size = 13.00

method	result	size
default	Expression too large to display	4668

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/3/f*(a-b)*(-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^2-6*(-a^2+b^2)^(1/
2)*cos(f*x+e)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^
2+2*2^(1/2)*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*a^2-3*sin(f*x+e)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^3+3*sin(f*x+e)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)
^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*b^3-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^
2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b+12*cos(f*x
+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti
cE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(
1/2)*a*b-6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)*a*b+3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^3+3*cos(f*x+e)*sin(f*x
```

```
+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b
^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*a*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b
+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-3*cos(f*x+e)*sin(f*x
+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/
2))*a*b^2+6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^
(1/2))*(-a^2+b^2)^(1/2)*b^2+12*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-6*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-6*(-a^2+b^2)^(1/2)*sin(f*x+e)*El
lipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2-3*sin(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b+3*sin(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^
(1/2))*a^2*b+3*(-a^2+b^2)^(1/2)*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a^2+3*(-a^2+b^2)^(1/2
)*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{5/2}}{(d\,\sin(e+f\,x))^{5/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))), x
)
```

3.1427 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=519

$$-\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}}+\frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}}-\frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}}+\frac{2(a^2-b^2)\,g(g\cos(e+fx))^{3/2}}{a^3d^3f\sqrt{d\sin(e+fx)}}$$

```
[Out] -2/5*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(5/2)+2/3*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(3/2)-4/5*g*(g*cos(f*x+e))^(3/2)/a/d^3/f/
(d*sin(f*x+e))^(1/2)+2*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin(f*
x+e))^(1/2)-2*b*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+
e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin
(f*x+e)^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2)+2*b*g^(5/2)*EllipticPi((g*cos(
f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1
/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^3/d^3/f/(d*sin(f*x+e))^(1/2
)+4/5*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4
*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a/d^4/f/sin(2*f
*x+2*e)^(1/2)-2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)
*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(
1/2)/a^3/d^4/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.77, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2978, 2650, 2652, 2719, 2643, 2989, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}\,bg^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,bg^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^3d^3f\sqrt{d\sin(e+fx)}}+\frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}}+\frac{2g^2(a^2-b^2)\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^3d^4f\sqrt{\sin(2e+2fx)}}+\frac{2g(a^2-b^2)\,(g\cos(e+fx))^{3/2}}{a^3d^3f\sqrt{d\sin(e+fx)}}-\frac{4g^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{5ad^4f\sqrt{\sin(2e+2fx)}}-\frac{4g(g\cos(e+fx))^{3/2}}{5ad^3f\sqrt{d\sin(e+fx)}}-\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(5*a*d*f*(d*Sin[e + f*x])^(5/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(3*a^2*d^2*f*(d*Sin[e + f*x])^(3/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(5*a*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*(a^2 - b^2)*g*(g*Cos[e
+ f*x])^(3/2))/(a^3*d^3*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[-a + b]
*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*
Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a
^3*d^3*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b*Sqrt[-a + b]*Sqrt[a + b]*g^(5
/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^3*d^3*f*Sqrt[d*Sin[
e + f*x]]) - (4*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(5*a*d^4*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*(a^2 - b^2)*g^2*Sq
```

```
rt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^3*
d^4*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{7/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{3/2}(a+b\sin}}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} + \frac{(b(a^2-}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\,c}{5ad^3f\,\sqrt{}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\,c}{5ad^3f\,\sqrt{}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\,c}{5ad^3f\,\sqrt{}}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{5adf(d\sin(e+fx))^{5/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{3a^2d^2f(d\sin(e+fx))^{3/2}} - \frac{4g(g\,c}{5ad^3f\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.40, size = 1734, normalized size = 3.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(7/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((2*(3*a^2*Cos[e + f*x] - 5*b^2*Cos[e + f*x])*Csc[e
 + f*x])/(5*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x])/(3*a^2) - (2*Cot[e + f*x
]*Csc[e + f*x]^2)/(5*a))*Sin[e + f*x]^2*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])
^(7/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(7/2)*((-2*(6*a^3 - 10*a*b^2
```

```
)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin
[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f
*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcT
an[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1
 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f
*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[
a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/
4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*
(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*S
qrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-3*a
^2*b + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[
1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e
 + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 +
 b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f
*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2
]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f
*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2
 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*
a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^
2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*
Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^
2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sq
rt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sq
rt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[
2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f
*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e +
 f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e
+ f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^3*f*Cos[e +
f*x]^(5/2)*(d*Sin[e + f*x])^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10137 vs. $2(489) = 978$.

time = 0.39, size = 10138, normalized size = 19.53

method	result	size
default	Expression too large to display	10138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(7/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(7/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(7/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{7/2}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(7/2)*(a + b*sin(e + f*x))), x
)
```

3.1428 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=612

$$-\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}}+\frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}}-\frac{8g(g\cos(e+fx))^{3/2}}{21ad^3f(d\sin(e+fx))^{3/2}}+\frac{2(a^2-b^2)\,g(g\cos(e+fx))^{3/2}}{3a^3d^3f(d\sin(e+fx))^{3/2}}$$

```
[Out] -2/7*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(7/2)+2/5*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(5/2)-8/21*g*(g*cos(f*x+e))^(3/2)/a/d^3/f
/(d*sin(f*x+e))^(3/2)+2/3*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin
(f*x+e))^(3/2)+4/5*b*g*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/(d*sin(f*x+e))^(1/2)-
2*b*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)+2*b^2*g
^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)
^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^4
/d^4/f/(d*sin(f*x+e))^(1/2)-2*b^2*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g
^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2
)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^4/d^4/f/(d*sin(f*x+e))^(1/2)-4/5*b*g^2*(si
n(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1
/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^2/d^5/f/sin(2*f*x+2*e)^(1/
2)+2*b*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Elliptic
E(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^4/
d^5/f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 1.03, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2978, 2650, 2643, 2652, 2719, 2989, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,b^2g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2g^{5/2}\sqrt{b-a}\,\sqrt{a+b}\,\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^4d^4f\sqrt{d\sin(e+fx)}}+\frac{4bg^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{5a^2d^5f\sqrt{\sin(2e+2fx)}}+\frac{4bg(g\cos(e+fx))^{3/2}}{5a^2d^4f\sqrt{d\sin(e+fx)}}+\frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}}-\frac{2bg^2(a^2-b^2)\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{a^4d^5f\sqrt{\sin(2e+2fx)}}-\frac{2bg(a^2-b^2)\,(g\cos(e+fx))^{3/2}}{a^4d^4f\sqrt{d\sin(e+fx)}}+\frac{2g(a^2-b^2)\,(g\cos(e+fx))^{3/2}}{3a^3d^3f(d\sin(e+fx))^{3/2}}-\frac{8g(g\cos(e+fx))^{3/2}}{21ad^3f(d\sin(e+fx))^{3/2}}-\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(7*a*d*f*(d*Sin[e + f*x])^(7/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(5*a^2*d^2*f*(d*Sin[e + f*x])^(5/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(21*a*d^3*f*(d*Sin[e + f*x])^(3/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(3*a^3*d^3*f*(d*Sin[e + f*x])^(3/2)) + (4*b*g*(g*Cos[e +
f*x])^(3/2))/(5*a^2*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*b*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[-a
 + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Sqrt[-a + b]*Sqrt[a +
b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]
```

/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^4*d^4*f*Sqrt[d*Sin[e + f*x]]) + (4*b*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(5*a^2*d^5*f*Sqrt[Sin[2*e + 2*f*x]]) - (2*b*(a^2 - b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^4*d^5*f*Sqrt[Sin[2*e + 2*f*x]])

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{9/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int\frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{9/2}}dx}{a} - \frac{((a^2-b^2)\,g^2)\int\frac{\sqrt{g\cos(e+f}}{(d\sin(e+fx))^{5/2}(a+b\sin}}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{(2bg^2)}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(}{21ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(}{21ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(}{21ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(}{21ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{7adf(d\sin(e+fx))^{7/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{5a^2d^2f(d\sin(e+fx))^{5/2}} - \frac{8g(}{21ad^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.20, size = 1776, normalized size = 2.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(9/2)*(a + b*Sin[e + f*x
]))),x]
```

```
[Out] ((g*Cos[e + f*x])^(5/2)*((-2*(3*a^2*b*Cos[e + f*x] - 5*b^3*Cos[e + f*x])*Cs
c[e + f*x])/(5*a^4) + (2*(3*a^2*Cos[e + f*x] - 7*b^2*Cos[e + f*x])*Csc[e +
f*x]^2)/(21*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^2)/(5*a^2) - (2*Cot[e + f
*x]*Csc[e + f*x]^3)/(7*a))*Sin[e + f*x]^3*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x
])^(9/2)) - (b*(g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(9/2)*((-2*(6*a^3 - 10*a
*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e +
 f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])
*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e
 + f*x])) + ((8*a^2*b - 10*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*
ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcT
an[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e
 + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + S
qrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1
, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/
2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/
2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((
-3*a^2*b + 5*b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*S
qrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -T
an[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a
^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e
 + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
 + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*
(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt
[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])
/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*S
qrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 -
Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt
[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sq
rt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])
/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (
2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
+ Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + S
qrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
 + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan
[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Si
n[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(5*a^4*f*Cos[
e + f*x]^(5/2)*(d*Sin[e + f*x])^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 10703 vs. $2(570) = 1140$.

time = 0.47, size = 10704, normalized size = 17.49

method	result	size
default	Expression too large to display	10704

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(9/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(9/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(9/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos(e+f\,x))^{5/2}}{(d\,\sin(e+f\,x))^{9/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(9/2)*(a + b*sin(e + f*x))), x
)
```

3.1429 $\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=822

$$-\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}}+\frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}}-\frac{4g(g\cos(e+fx))^{3/2}}{15ad^3f(d\sin(e+fx))^{5/2}}+\frac{2(a^2-b^2)\,g(g\cos(e+fx))}{5a^3d^3f(d\sin(e+fx))^{5/2}}$$

```
[Out] -2/9*g*(g*cos(f*x+e))^(3/2)/a/d/f/(d*sin(f*x+e))^(9/2)+2/7*b*g*(g*cos(f*x+e
))^(3/2)/a^2/d^2/f/(d*sin(f*x+e))^(7/2)-4/15*g*(g*cos(f*x+e))^(3/2)/a/d^3/f
/(d*sin(f*x+e))^(5/2)+2/5*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^3/d^3/f/(d*sin
(f*x+e))^(5/2)+8/21*b*g*(g*cos(f*x+e))^(3/2)/a^2/d^4/f/(d*sin(f*x+e))^(3/2)
-2/3*b*(a^2-b^2)*g*(g*cos(f*x+e))^(3/2)/a^4/d^4/f/(d*sin(f*x+e))^(3/2)-8/15
*g*(g*cos(f*x+e))^(3/2)/a/d^5/f/(d*sin(f*x+e))^(1/2)+4/5*(a^2-b^2)*g*(g*cos
(f*x+e))^(3/2)/a^3/d^5/f/(d*sin(f*x+e))^(1/2)+2*b^2*(a^2-b^2)*g*(g*cos(f*x+
e))^(3/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)-2*b^3*g^(5/2)*EllipticPi((g*cos(f*
x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/
2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)
+2*b^3*g^(5/2)*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2)
,(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*(-a+b)^(1/2)*(a+b)^(1/2)*sin(f*x+e)^(1
/2)/a^5/d^5/f/(d*sin(f*x+e))^(1/2)+8/15*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin
(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d
*sin(f*x+e))^(1/2)/a/d^6/f/sin(2*f*x+2*e)^(1/2)-4/5*(a^2-b^2)*g^2*(sin(e+1/
4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(
g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^3/d^6/f/sin(2*f*x+2*e)^(1/2)-2*b
^2*(a^2-b^2)*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(co
s(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/a^5/d^6/
f/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 1.37, antiderivative size = 822, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2978, 2650, 2652, 2719, 2643, 2989, 2985, 2984, 504, 1232}

$$-\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\sqrt{b-a}\sqrt{a+b}\,g^{5/2}\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}\,b^3}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2(a^2-b^2)\,g(g\cos(e+fx))^{3/2}b^2}{a^5d^5f\sqrt{d\sin(e+fx)}}+\frac{2(a^2-b^2)\,g^2\sqrt{g\cos(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,b^2}{a^5d^6f\sqrt{\sin(2e+2fx)}}-\frac{2(a^2-b^2)\,g(g\cos(e+fx))^{3/2}b}{3a^4d^4f(d\sin(e+fx))^{3/2}}+\frac{8g(g\cos(e+fx))^{3/2}b}{21a^2d^4f(d\sin(e+fx))^{3/2}}+\frac{2g(g\cos(e+fx))^{3/2}b}{7a^2d^2f(d\sin(e+fx))^{7/2}}+\frac{4(a^2-b^2)\,g(g\cos(e+fx))^{3/2}}{5a^3d^5f\sqrt{d\sin(e+fx)}}-\frac{8g(g\cos(e+fx))^{3/2}}{15ad^5f\sqrt{d\sin(e+fx)}}+\frac{4(a^2-b^2)\,g^2\sqrt{g\cos(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}}{5a^3d^6f\sqrt{\sin(2e+2fx)}}-\frac{8g^2\sqrt{g\cos(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}}{15ad^6f\sqrt{\sin(2e+2fx)}}+\frac{2(a^2-b^2)\,g(g\cos(e+fx))^{3/2}}{5a^3d^3f(d\sin(e+fx))^{5/2}}-\frac{4g(g\cos(e+fx))^{3/2}}{15ad^3f(d\sin(e+fx))^{5/2}}-\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*x])),x
]
```

```
[Out] (-2*g*(g*Cos[e + f*x])^(3/2))/(9*a*d*f*(d*Sin[e + f*x])^(9/2)) + (2*b*g*(g*
Cos[e + f*x])^(3/2))/(7*a^2*d^2*f*(d*Sin[e + f*x])^(7/2)) - (4*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^3*f*(d*Sin[e + f*x])^(5/2)) + (2*(a^2 - b^2)*g*(g*Cos
[e + f*x])^(3/2))/(5*a^3*d^3*f*(d*Sin[e + f*x])^(5/2)) + (8*b*g*(g*Cos[e +
```

```
f*x])^(3/2))/(21*a^2*d^4*f*(d*Sin[e + f*x])^(3/2)) - (2*b*(a^2 - b^2)*g*(g*
Cos[e + f*x])^(3/2))/(3*a^4*d^4*f*(d*Sin[e + f*x])^(3/2)) - (8*g*(g*Cos[e +
 f*x])^(3/2))/(15*a*d^5*f*Sqrt[d*Sin[e + f*x]]) + (4*(a^2 - b^2)*g*(g*Cos[e
 + f*x])^(3/2))/(5*a^3*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*b^2*(a^2 - b^2)*g*(
g*Cos[e + f*x])^(3/2))/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^3*Sq
rt[-a + b]*Sqrt[a + b]*g^(5/2)*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcS
in[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e +
 f*x]])/(a^5*d^5*f*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*Sqrt[-a + b]*Sqrt
[a + b]*g^(5/2)*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e +
f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^5*d^5*f
*Sqrt[d*Sin[e + f*x]]) - (8*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f
*x, 2]*Sqrt[d*Sin[e + f*x]])/(15*a*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (4*(a^2
- b^2)*g^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e +
 f*x]])/(5*a^3*d^6*f*Sqrt[Sin[2*e + 2*f*x]]) + (2*b^2*(a^2 - b^2)*g^2*Sqrt[
g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^5*d^6
*f*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2978

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] + (-Dist[b*(g^2/(a^2*d))
, Int[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] - Dist[g^2*
((a^2 - b^2)/(a^2*d^2)), Int[(g*Cos[e + f*x])^(p - 2)*((d*Sin[e + f*x])^(n
+ 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && GtQ[p, 1] && (LeQ[n, -2] || (EqQ[n,
 -3/2] && EqQ[p, 3/2]))
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^{5/2}}{(d\sin(e+fx))^{11/2}(a+b\sin(e+fx))}\,dx = \frac{g^2\int \frac{\sqrt{g\cos(e+fx)}}{(d\sin(e+fx))^{11/2}}\,dx}{a} - \frac{((a^2-b^2)\,g^2)\int \frac{\sqrt{g\cos(e+f}}{(d\sin(e+fx))^{7/2}(a+b\sin}}{a^2d^2}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{(4bg^2)}{}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

$$= -\frac{2g(g\cos(e+fx))^{3/2}}{9adf(d\sin(e+fx))^{9/2}} + \frac{2bg(g\cos(e+fx))^{3/2}}{7a^2d^2f(d\sin(e+fx))^{7/2}} - \frac{4g(}{15ad^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.72, size = 1850, normalized size = 2.25

Warning: Unable to verify antiderivative.

[In] Integrate[(g*Cos[e + f*x])^(5/2)/((d*Sin[e + f*x])^(11/2)*(a + b*Sin[e + f*x])),x]

[Out] ((g*Cos[e + f*x])^(5/2)*((2*(2*a^4*Cos[e + f*x] + 9*a^2*b^2*Cos[e + f*x] - 15*b^4*Cos[e + f*x])*Csc[e + f*x])/(15*a^5) - (2*(3*a^2*b*Cos[e + f*x] - 7*b^3*Cos[e + f*x])*Csc[e + f*x]^2)/(21*a^4) + (2*(a^2*Cos[e + f*x] - 3*b^2*Cos[e + f*x])*Csc[e + f*x]^3)/(15*a^3) + (2*b*Cot[e + f*x]*Csc[e + f*x]^3)/(7*a^2) - (2*Cot[e + f*x]*Csc[e + f*x]^4)/(9*a))*Sin[e + f*x]^4*Tan[e + f*x]^2)/(f*(d*Sin[e + f*x])^(11/2)) + ((g*Cos[e + f*x])^(5/2)*Sin[e + f*x]^(11/2)*((-2*(4*a^5 + 18*a^3*b^2 - 30*a*b^4)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^4*b + 24*a^2*b^3 - 30*b^5)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^4*b - 9*a^2*b^3 + 15*b^5)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1

```
 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(15*a^5*f*Cos[e + f*x]^(5/2)
*(d*Sin[e + f*x])^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 17101 vs. $2(779) = 1558$.
time = 0.46, size = 17102, normalized size = 20.81

method	result	size
default	Expression too large to display	17102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x,method=_R
ETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(11/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(11/2)/(a+b*sin(f*x+e)),x, al
gorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(5/2)/((b*sin(f*x + e) + a)*(d*sin(f*x + e))^(11
/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{5/2}}{(d\,\sin\left(e+f\,x\right))^{11/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(11/2)*(a + b*sin(e + f*x))),x
)
```

```
[Out] int((g*cos(e + f*x))^(5/2)/((d*sin(e + f*x))^(11/2)*(a + b*sin(e + f*x))),
x)
```

3.1430 $\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=616

$$\frac{ad^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f\sqrt{g}}-\frac{ad^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f\sqrt{g}}-\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{c}}{}$$

```
[Out] 1/2*a*d^(5/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(
f*x+e))^(1/2))/b^2/f*2^(1/2)/g^(1/2)-1/2*a*d^(5/2)*arctan(1+2^(1/2)*g^(1/2)
*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b^2/f*2^(1/2)/g^(1/2)-1
/4*a*d^(5/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))
^(1/2)+d^(1/2)*tan(f*x+e))/b^2/f*2^(1/2)/g^(1/2)+1/4*a*d^(5/2)*ln(d^(1/2)+2
^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e)
)/b^2/f*2^(1/2)/g^(1/2)-2*a^2*d^(5/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/
2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)
/b^2/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)+2*a^2*d^(5/2)*EllipticPi((d*si
n(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(
1/2)*cos(f*x+e)^(1/2)/b^2/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-d^2*(g*co
s(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/b/f/g-1/2*d^3*(sin(e+1/4*Pi+f*x)^2)^(1
/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(
1/2)/b/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2988, 2648, 2653, 2720, 2654, 303, 1176, 631, 210, 1179, 642, 2987, 2986, 1232}

$$-\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{b^2f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{ad^{5/2}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f\sqrt{g}}-\frac{ad^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}+1\right)}{\sqrt{2}\,b^2f\sqrt{g}}-\frac{ad^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b^2f\sqrt{g}}+\frac{ad^{5/2}\log\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,b^2f\sqrt{g}}+\frac{d^3\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2bf\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{d^2\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{bfg}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (a*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b^2*f*Sqrt[g]) - (a*d^(5/2)*ArcTan[1 + (Sqrt[2]
*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b^
2*f*Sqrt[g]) - (2*Sqrt[2]*a^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b
- Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e +
 f*x]])], -1])/(b^2*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*a
^2*d^(5/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSi
n[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b^2*Sqrt[-a
^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (a*d^(5/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[
```

```
g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*S
qrt[2]*b^2*f*Sqrt[g]) + (a*d^(5/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Si
n[e + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b^2*f
*Sqrt[g]) - (d^2*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(b*f*g) + (d^3*
EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(2*b*f*Sqrt[g*Cos[e +
f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}}dx}{b} - \frac{(ad)\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}}{b}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} - \frac{(ad^2)\int \frac{\sqrt{d\sin(e+fx}}{\sqrt{g\cos(e+fx}}}{b^2}$$

$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} - \frac{(2ad^3g)\,\text{Subst}\left(\int \frac{x^2}{d^2+g^2}\right.}{}$$

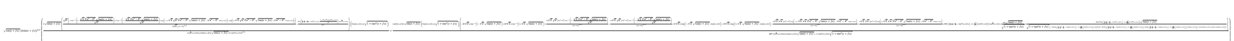
$$= -\frac{d^2\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{bfg} + \frac{d^3F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{}}{2bf\sqrt{g\cos(e+fx)}\,\sqrt{}}$$

$$= -\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\sqrt{}}\right.\right.}{b^2\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2\sqrt{2}\,a^2d^{5/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{}}{\sqrt{d}\,\sqrt{}}\right.\right.}{b^2\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{ad^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{g}\,\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,b^2f\sqrt{g}} - \frac{ad^{5/2}\tan^{-1}\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.60, size = 1318, normalized size = 2.14



Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (Sqrt[Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*((2*Sqrt[Sin[e + f*x]]*((Sqrt[a]
*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2
```

```
*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*
Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appe
llF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))
/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(1 + Tan[e + f
*x]^2)^(3/2)) + (Cos[2*(e + f*x)]*Sqrt[Sin[e + f*x]]*(b*Tan[e + f*x] + a*Sq
rt[1 + Tan[e + f*x]^2])*(-20*Sqrt[2]*a*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]
]] + 20*Sqrt[2]*a*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] + (10*Sqrt[2]*Sqrt
[a]*(2*a^2 - b^2)*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])
/Sqrt[a]])/(a^2 - b^2)^(3/4) - (10*Sqrt[2]*Sqrt[a]*(2*a^2 - b^2)*ArcTan[1 +
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(3/4)
 + 10*Sqrt[2]*a*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 10*Sqr
t[2]*a*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (5*Sqrt[2]*Sqrt
[a]*(2*a^2 - b^2)*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + (5*Sqrt[2]*Sqrt[a
]*(2*a^2 - b^2)*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(3/4) + 8*b*AppellF1[5/4, 1/
2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(5/
2) + (40*b*Sqrt[Tan[e + f*x]])/Sqrt[1 + Tan[e + f*x]^2] + (200*a^4*b*Appell
F1[1/4, 1/2, 1, 5/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sqrt[T
an[e + f*x]])/(Sqrt[1 + Tan[e + f*x]^2]*(-5*a^2*AppellF1[1/4, 1/2, 1, 5/4,
-Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + 2*(2*(a^2 - b^2)*AppellF1
[5/4, 1/2, 2, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + a^2*Ap
pellF1[5/4, 3/2, 1, 9/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2])*T
an[e + f*x]^2)*(-(b^2*Tan[e + f*x]^2) + a^2*(1 + Tan[e + f*x]^2)))))/(20*b^
2*Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]]*(-1 + Tan[e +
f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(2*f*Sqrt[g*Cos[e + f*x]]*Sin[e + f*x]^
(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2344 vs. $2(522) = 1044$.
time = 0.38, size = 2345, normalized size = 3.81

method	result	size
default	Expression too large to display	2345

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2)
```

```
)*(-a^2+b^2)^(1/2)*a*b-I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+I*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-I*sin(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*
b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a
^2+b^2)^(1/2)*a^2-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2-sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-sin(f*x+e
)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*(-a^2+b^2)^(1/2)*a^2+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b
+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b-sin(f*x+e)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*a^2-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)
^(1/2)+a),1/2*2^(1/2))*a^3+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a
/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin
```

```
(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+cos(f*x+e)^2*(-a^2+b^
2)^(1/2)*2^(1/2)*a*b-cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*b^2-cos(f*x+e)*(
-a^2+b^2)^(1/2)*2^(1/2)*a*b+cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b^2)*(d*sin
(f*x+e))^(5/2)/(-1+cos(f*x+e))/sin(f*x+e)^2/(g*cos(f*x+e))^(1/2)*2^(1/2)*a/
b^2/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin\left(e+f\,x\right))^{5/2}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1431 $\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=508

$$-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,bf\sqrt{g}}+\frac{d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,bf\sqrt{g}}+\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\mathrm{co}}}{}$$

```
[Out] -1/2*d^(3/2)*arctan(1-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/d^(1/2)/(g*cos(f
*x+e))^(1/2))/b/f*2^(1/2)/g^(1/2)+1/2*d^(3/2)*arctan(1+2^(1/2)*g^(1/2)*(d*s
in(f*x+e))^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2))/b/f*2^(1/2)/g^(1/2)+1/4*d^(3
/2)*ln(d^(1/2)-2^(1/2)*g^(1/2)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^
(1/2)*tan(f*x+e))/b/f*2^(1/2)/g^(1/2)-1/4*d^(3/2)*ln(d^(1/2)+2^(1/2)*g^(1/2
)*(d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(1/2)+d^(1/2)*tan(f*x+e))/b/f*2^(1/2)
/g^(1/2)+2*a*d^(3/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))
^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/b/f/(-a^2+b^2)^(
1/2)/(g*cos(f*x+e))^(1/2)-2*a*d^(3/2)*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/
2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)
/b/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2988, 2654, 303, 1176, 631, 210, 1179, 642, 2987, 2986, 1232}

$$\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\mathrm{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\mathrm{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{bf\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{d^{3/2}\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,bf\sqrt{g}}+\frac{d^{3/2}\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}+1\right)}{\sqrt{2}\,bf\sqrt{g}}+\frac{d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,bf\sqrt{g}}-\frac{d^{3/2}\log\left(\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}+\sqrt{d}\tan(e+fx)+\sqrt{d}\right)}{2\sqrt{2}\,bf\sqrt{g}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[
g*Cos[e + f*x]])])/(Sqrt[2]*b*f*Sqrt[g])) + (d^(3/2)*ArcTan[1 + (Sqrt[2]*Sq
rt[g]*Sqrt[d*Sin[e + f*x]])/(Sqrt[d]*Sqrt[g*Cos[e + f*x]])])/(Sqrt[2]*b*f*S
qrt[g]) + (2*Sqrt[2]*a*d^(3/2)*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[
-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])
], -1])/(b*Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*a*d^(3/2)*
Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Si
n[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(b*Sqrt[-a^2 + b^2]*f*S
qrt[g*Cos[e + f*x]]) + (d^(3/2)*Log[Sqrt[d] - (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e
 + f*x]])/Sqrt[g*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt
[g]) - (d^(3/2)*Log[Sqrt[d] + (Sqrt[2]*Sqrt[g]*Sqrt[d*Sin[e + f*x]])/Sqrt[g
*Cos[e + f*x]] + Sqrt[d]*Tan[e + f*x]])/(2*Sqrt[2]*b*f*Sqrt[g])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2654

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k*a*(b/f), Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{d\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\,dx}{b} - \frac{(ad)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\cdots}{b}$$

$$= \frac{(2d^2g)\,\mathrm{Subst}\left(\int \frac{x^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} - \frac{\left(ad\sqrt{\cos(e\cdots\right.}{\cdots}$$

$$= -\frac{d^2\mathrm{Subst}\left(\int \frac{d-gx^2}{d^2+g^2x^4}\,dx, x, \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}}\right)}{bf} + \frac{d^2\mathrm{Subst}\left(\int \frac{d\cdots}{d^2\cdots}\right.}{\cdots}$$

$$= \frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}}; \sin^{-1}\left(\frac{\sqrt{d\sin\cdots}}{\sqrt{d}\sqrt{1-\cdots}}\right.\right.}{b\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,ad^{3/2}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}}; \sin^{-1}\left(\frac{\sqrt{d\sin\cdots}}{\sqrt{d}\sqrt{1-\cdots}}\right.\right.}{b\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{g}\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{g\cos(e+fx)}}\right)}{\sqrt{2}\,bf\sqrt{g}} + \frac{d^{3/2}\tan^{-1}\left(1+\cdots\right.}{\cdots}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 21.09, size = 518, normalized size = 1.02

$$\frac{10(a^2-b^2)\cot(e+fx)(d\sin(e+fx))^{3/2}\left(a+b\sqrt{\sin^2(e+fx)}\right)\left(\frac{aF_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)}{5(a^2-b^2)F_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+\left(-4b^2F_1\left(\frac{5}{4};-\frac{1}{4},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+(-a^2+b^2)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)\cos^2(e+fx)}+\frac{bF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\sqrt{\sin^2(e+fx)}}{-5(a^2-b^2)F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+\left(4b^2F_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)+3(a^2-b^2)F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\right)\cos^2(e+fx)}\right)}{f\sqrt{g\cos(e+fx)}\,(-a+b\sin(e+fx))(a+b\sin(e+fx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (10*(a^2 - b^2)*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a + b*Sqrt[Sin[e + f*x
]^2])*((a*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2)])/(5*(a^2 - b^2)*AppellF1[1/4, -1/4, 1, 5/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-4*b^2*AppellF1[5/4, -1/4, 2, 9/4, Cos
[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-a^2 + b^2)*AppellF1[5/4
```

```
, 3/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e +
f*x]^2) + (b*AppellF1[1/4, -3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)]*Sqrt[Sin[e + f*x]^2])/(-5*(a^2 - b^2)*AppellF1[1/4, -3/4,
1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (4*b^2*AppellF
1[5/4, -3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + 3
*(a^2 - b^2)*AppellF1[5/4, 1/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2
)/(-a^2 + b^2)])*Cos[e + f*x]^2)))/(f*Sqrt[g*Cos[e + f*x]]*(-a + b*Sin[e +
f*x])*(a + b*Sin[e + f*x])^2)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 940 vs. 2(401) = 802.
time = 0.39, size = 941, normalized size = 1.85

method	result	size
default	Expression too large to display	941

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*a*(-a^2+b^2)^(1/2)-I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)-I*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*a*(-a^2+
b^2)^(1/2)+I*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2
)*a+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b
^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
*b*(-a^2+b^2)^(1/2)-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*b*(-a^2+b^2)^(1/2)+a^2*E
llipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(
1/2)+a),1/2*2^(1/2))-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b-a^2*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))+El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1
/2)-a),1/2*2^(1/2))*a*b)*(d*sin(f*x+e))^(3/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)*2^(1/2)*a/b/(-
a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin(e+fx))^{\frac{3}{2}}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral((d*sin(e + f*x))**(3/2)/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))
), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\sin(e+fx))^{3/2}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
[Out] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1432 $\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=209

$$-\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+}{}$$

```
[Out] -2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2
+b^2)^(1/2)),I)*2^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/f/(-a^2+b^2)^(1/2)/(g*cos(
f*x+e))^(1/2)+2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*d^(1/2)*cos(f*x+e)^(1/2)/f/(-a^2+b^2)^
(1/2)/(g*cos(f*x+e))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2987, 2986, 1232}

$$\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2]
)), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sq
rt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*Sqrt[d]*Sqrt[Cos[e + f*
x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sq
rt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(Sqrt[-a^2 + b^2]*f*Sqrt[g*Cos[e + f*x
]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
```

```
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\sqrt{\cos(e+fx)}\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx}{\sqrt{g\cos(e+fx)}}$$

$$= \frac{\left(2\sqrt{2}\left(1-\frac{b}{\sqrt{-a^2+b^2}}\right)d\sqrt{\cos(e+fx)}\right)\text{Subst}\left(\int \frac{}{\left(\left(b-\sqrt{\right.\right.}\right.}\right.}{f\sqrt{g\cos(e-}}$$

$$= -\frac{2\sqrt{2}\sqrt{d}\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d}}{\sqrt{d}\sqrt{}}\right.\right.}{\sqrt{-a^2+b^2}\,f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 32.42, size = 373, normalized size = 1.78

$$\frac{2\sqrt{d\sin(e+fx)}\left(a\sqrt{\sec^2(e+fx)}+b\tan(e+fx)\right)\left(\frac{\sqrt{a}\left(-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}}{\sqrt{a}}\right)+\log\left(-a+\sqrt{2}\sqrt{a}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}-\sqrt{a^2-b^2}\tan(e+fx)\right)-\log\left(a+\sqrt{2}\sqrt{a}\sqrt[4]{a^2-b^2}\sqrt{\tan(e+fx)}+\sqrt{a^2-b^2}\tan(e+fx)\right)\right)}{4\sqrt{2}(a^2-b^2)^{3/4}}-\frac{bF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\tan^2(e+fx),\frac{(-a^2+b^2)\tan^2(e+fx)}{a^2}\right)\tan^{\frac{5}{2}}(e+fx)}{5a^2}\right)}{f\sqrt{g\cos(e+fx)}\sqrt{\sec^2(e+fx)}\,(a+b\sin(e+fx))\sqrt{\tan(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/(Sqrt[g*Cos[e + f*x]]*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (2*Sqrt[d*Sin[e + f*x]]*(a*Sqrt[Sec[e + f*x]^2] + b*Tan[e + f*x])*((Sqrt[a]
*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2
*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + Log[-
a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*
```

```
Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
] + Sqrt[a^2 - b^2]*Tan[e + f*x]]])/(4*Sqrt[2]*(a^2 - b^2)^(3/4)) - (b*Appe
llF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Tan[e + f*x]^(5/2))/(5*a^2)))/(f*Sqrt[g*Cos[e + f*x]]*Sqrt[Sec[e + f*x]^2]*
(a + b*Sin[e + f*x])*Sqrt[Tan[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(173) = 346$.
time = 0.34, size = 527, normalized size = 2.52

method	result
default	$-\frac{\sqrt{d\sin(fx+e)}\left(\text{EllipticPi}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{a}{-b+\sqrt{-a^2+b^2}+a},\frac{\sqrt{2}}{2}\right)\sqrt{-a^2+b^2}+\text{Elliptic}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*(d*sin(f*x+e))^(1/2)*(EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)+EllipticP
i((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),
1/2*2^(1/2))*(-a^2+b^2)^(1/2)+a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))-b*EllipticPi((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2
))-a*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))+b*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)))*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/
2)*2^(1/2)*a/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\sin\left(e+fx\right)}}{\sqrt{g\cos\left(e+fx\right)}\,\left(a+b\sin\left(e+fx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))/(sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x))),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,\sin\left(e+f\,x\right)}}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(1/2)*(a + b*sin(e + f*x))), x
)
```

3.1433 $\int \frac{1}{\sqrt{g\cos(e+fx)}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=273

$$\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a\sqrt{-a^2+b^2}\,\sqrt{d}\,f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(\right.}{a}$$

[Out] 2*b*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a/f/(-a^2+b^2)^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2)-2*b*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a/f/(-a^2+b^2)^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2)-(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)

Rubi [A]
time = 0.36, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2989, 2653, 2720, 2987, 2986, 1232}

$$\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}+\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{af\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) + (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
```

```
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}\,dx}{a} - \frac{b\int \cdots}{\cdots}$$

$$= -\frac{\left(b\sqrt{\cos(e+fx)}\right)\int \frac{\sqrt{d\sin(e+fx)}}{\sqrt{\cos(e+fx)}\,(a+b\sin(e+\cdots}}{ad\sqrt{g\cos(e+fx)}}$$

$$= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(2e+2fx)}}{af\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}} - \frac{\left(2\sqrt{2}\,\cdots\right.}{\cdots}$$

$$= \frac{2\sqrt{2}\,b\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-}\cdots\right.}{a\sqrt{-a^2+b^2}\,\sqrt{d}\,f\sqrt{g\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 24.04, size = 652, normalized size = 2.39

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]*(((-1/8 + I/8)*b*(2*ArcTan[1 -
((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-Sin[e + f*x]^2)^
(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(
1/4)*(-Sin[e + f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/
Sqrt[-Sin[e + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*
x]])/(-Sin[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/S
qrt[-Sin[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x
]])/(-Sin[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)) + (5*a*(a^2 - b
^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 +
 b^2)]*Sqrt[Cos[e + f*x]])/((a^2 - b^2 + b^2*Cos[e + f*x]^2)*(5*(a^2 - b^2)
*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^
2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2,
 (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(Sin[e + f*x]^2)^(3/4)
```

```
))*(a + b*Sqrt[Sin[e + f*x]^2]))/(d*f*Sqrt[g*Cos[e + f*x]]*(Sin[e + f*x]^2)
^(1/4)*(a + b*Sin[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(257) = 514$.
time = 0.32, size = 631, normalized size = 2.31

method	result
default	$\frac{\left(2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right)a\sqrt{-a^2+b^2}-2\,\mathrm{EllipticF}\left(\sqrt{-\frac{-1+\cos(fx+e)-\sin(fx+e)}{\sin(fx+e)}},\frac{\sqrt{2}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*a*(-a^2+b^2)^(1/2)-2*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b-EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2
*2^(1/2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(
-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*a*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2
+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)^2/(d*sin(f*x
+e))^(1/2)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(1/2)*2^(1/2)/(-a^2+b^2)^(1/2)/(-
b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*sin(e + f*x))*sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\,\cos\left(e+f\,x\right)}\,\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
 x)
```

3.1434 $\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=320

$$-\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^2\sqrt{-a^2+b^2}\,d^{3/2}f\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+f}$$

```
[Out] -2*b^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(
-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/2)/f/(-a^2+b^2)^(1/2)
/(g*cos(f*x+e))^(1/2)+2*b^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(
f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/d^(3/
2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2*(g*cos(f*x+e))^(1/2)/a/d/f/g/(
d*sin(f*x+e))^(1/2)+b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipti
cF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/d/f/(g*cos(f*x+e))^(
1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2989, 2643, 2653, 2720, 2987, 2986, 1232}

$$-\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2d^{3/2}f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{b\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^2df\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*b^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])),
ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sq
rt[-a^2 + b^2]*d^(3/2)*f*Sqrt[g*Cos[e + f*x]]) + (2*Sqrt[2]*b^2*Sqrt[Cos[e
+ f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]
/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*d^(3/2)*f*Sq
rt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(a*d*f*g*Sqrt[d*Sin[e + f*x]
]) - (b*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^2*d*f*Sqrt[
g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
```

```
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}dx}{a} - \frac{b\int \frac{}{\sqrt{g}}}{}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{b^2\int \frac{\sqrt{d\sin}}{\sqrt{g\cos(e+f}}}{a^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} + \frac{\left(b^2\sqrt{\cos(e+fx)}\right.}{a^2}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{adfg\sqrt{d\sin(e+fx)}} - \frac{bF\left(e-\frac{\pi}{4}+fx\middle|2\right.}{a^2df\sqrt{g\cos(e+f}}$$

$$= -\frac{2\sqrt{2}\,b^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\right.}{a^2\sqrt{-a^2+b^2}\,d^{3/2}f}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 25.31, size = 715, normalized size = 2.23

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x
]])),x]
```

```
[Out] (-2*Cos[e + f*x]*Sin[e + f*x])/(a*f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(
3/2)) + (2*b*Sqrt[Cos[e + f*x]]*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2
 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a
^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*A
ppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)
] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)
/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (
b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e +
 f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x
]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 +
I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1
```

```
/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2]
 - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*
x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e +
f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos
[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sin[e + f*x]^2)/(a*f*Sq
rt[g*Cos[e + f*x]]*(1 - Cos[e + f*x]^2)^(1/4)*(d*Sin[e + f*x])^(3/2)*(a + b
*Sin[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(298) = 596$.
time = 0.34, size = 2286, normalized size = 7.14

method	result	size
default	Expression too large to display	2286

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/f*(2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2
)^(1/2)*a*b-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
F((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-
a^2+b^2)^(1/2)*b^2+cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a
^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+cos(f*x+e)*((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^2-cos(f*x
+e)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/
2))*b^3+cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/
2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+cos(f*x+e)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*(
```

```
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b-2*(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^
(1/2)*b^2+((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2
*2^(1/2))*a*b^2-((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+
a),1/2*2^(1/2))*b^3+((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1
/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2+((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*(-a^2+b^2)^(1/2)*2^(1/
2)*cos(f*x+e)*a^2-2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b)*sin(f*x+e)/(d*
sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(1/2)*2^(1/2)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b
^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\sin(e+fx))^{\frac{3}{2}}\sqrt{g\cos(e+fx)}\,(a+b\sin(e+fx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/((d*sin(e + f*x))**(3/2)*sqrt(g*cos(e + f*x))*(a + b*sin(e + f*x
))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(3/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\,\cos(e+f\,x)}\,(d\,\sin(e+f\,x))^{3/2}\,(a+b\,\sin(e+f\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
 x)
```

3.1435 $\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=424

$$\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^3\sqrt{-a^2+b^2}\,d^{5/2}f\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}$$

[Out] 2*b^3*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2*b^3*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^3/d^(5/2)/f/(-a^2+b^2)^(1/2)/(g*cos(f*x+e))^(1/2)-2/3*(g*cos(f*x+e))^(1/2)/a/d/f/g/(d*sin(f*x+e))^(3/2)+2*b*(g*cos(f*x+e))^(1/2)/a^2/d^2/f/g/(d*sin(f*x+e))^(1/2)-2/3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a/d^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)-b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^3/d^2/f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)

Rubi [A]

time = 0.77, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2989, 2650, 2653, 2720, 2643, 2987, 2986, 1232}

$$\frac{b^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{a^3d^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2b\sqrt{g\cos(e+fx)}}{a^2d^2fg\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^3d^{5/2}f\sqrt{b^2-a^2}\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3ad^2f\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),x]

[Out] (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[2]*b^3*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^3*Sqrt[-a^2 + b^2]*d^(5/2)*f*Sqrt[g*Cos[e + f*x]]) - (2*Sqrt[g*Cos[e + f*x]])/(3*a*d*f*g*(d*Sin[e + f*x])^(3/2)) + (2*b*Sqrt[g*Cos[e + f*x]])/(a^2*d^2*f*g*Sqrt[d*Sin[e + f*x]]) + (2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(3*a*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (b^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f*x]])/(a^3*d^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{1}{\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{5/2}}\,dx}{a} - \frac{b\int \cdots}{\cdots}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2\int \cdots}{\sqrt{g\cos(e+\cdots}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+\cdots}}{a^2d^2fg\sqrt{d\sin(\cdots}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+\cdots}}{a^2d^2fg\sqrt{d\sin(\cdots}}$$

$$= -\frac{2\sqrt{g\cos(e+fx)}}{3adfg(d\sin(e+fx))^{3/2}} + \frac{2b\sqrt{g\cos(e+\cdots}}{a^2d^2fg\sqrt{d\sin(\cdots}}$$

$$= \frac{2\sqrt{2}\,b^3\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\,\cdots\right.}{a^3\sqrt{-a^2+b^2}\,d^{5/2}f\,\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 40.67, size = 1140, normalized size = 2.69

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (Cos[e + f*x]*((2*b*Csc[e + f*x])/a^2 - (2*Csc[e + f*x]^2)/(3*a))*Sin[e + f
*x]^3)/(f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(5/2)) + (Sqrt[Cos[e + f*x]
]*Sin[e + f*x]^(5/2)*((-2*(2*a^2 + 3*b^2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*
((5*a*(a^2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f
*x]^2)/(-a^2 + b^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^
2 - b^2)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)] + (-4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e
 + f*x]^2)/(-a^2 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e +
 f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1
 + Cos[e + f*x]^2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[C
os[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[
1 + ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e +
f*x]^2)^(1/4))] + Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e
 + f*x]^2] - ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 +
Cos[e + f*x]^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1
+ Cos[e + f*x]^2] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])
/(-1 + Cos[e + f*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sqrt[Sin[e +
f*x]])/((1 - Cos[e + f*x]^2)^(1/4)*(a + b*Sin[e + f*x])) + (4*a*b*Sqrt[Sin[
e + f*x]]*((Sqrt[a]*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]
])/Sqrt[a]] + Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]]
 - Sqrt[a^2 - b^2]*Tan[e + f*x]] - Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(4*Sqrt[2]*(a^2 - b^
2)^(3/4)) - (b*AppellF1[5/4, 1/2, 1, 9/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Ta
n[e + f*x]^2)/a^2]*Tan[e + f*x]^(5/2))/(5*a^2))*(b*Tan[e + f*x] + a*Sqrt[1
+ Tan[e + f*x]^2]))/(Cos[e + f*x]^(5/2)*(a + b*Sin[e + f*x])*Sqrt[Tan[e + f
*x]]*(1 + Tan[e + f*x]^2)^(3/2))))/(3*a^2*f*Sqrt[g*Cos[e + f*x]]*(d*Sin[e +
 f*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2986 vs. $2(415) = 830$.

time = 0.39, size = 2987, normalized size = 7.04

method	result	size
default	Expression too large to display	2987

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/3/f*(3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b^3+3*cos(f*x+e)*sin(f*x+e
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2
```

)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*a*b^3-3*cos(f*x+e)*sin(f*x+e)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*b^4+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2
)*b^3-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a
^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^3+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4+4*cos(f*x+e
)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/
2)*a^3-4*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-a^2+b^2)^(1/2)*a^2*b+6*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^2-6*cos(f*x+e)*sin(f*x+e)*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3+3*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a)
,1/2*2^(1/2))*b^3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^
2+b^2)^(1/2)*sin(f*x+e)+3*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^3*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)-3*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^4*(-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)+3*sin(f*x+e)*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-
a^2+b^2)^(1/2)*b^3-3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(

```
-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^3+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4+4*EllipticF((-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^3*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*sin(f*x+e)-4*EllipticF((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a^2*b*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*sin(f*x+e)+6*El
lipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a*b^2*(
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*sin(f*
x+e)-6*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*b^3*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)
*sin(f*x+e)+6*(-a^2+b^2)^(1/2)*2^(1/2)*cos(f*x+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4373 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e))/(g*cos(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate(1/(sqrt(g*cos(f*x + e))*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{g\,\cos\left(e+f\,x\right)}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(1/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
 x)
```

3.1436 $\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=1064

$$-\frac{a^2d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b\,(a^2-b^2)\,fg^{3/2}}+\frac{bd^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,(a^2-b^2)\,fg^{3/2}}+\frac{a^2d^{5/2}\tan^{-1}}{}$$

```
[Out] 1/2*a^2*d^(5/2)*arctan(-1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*s
in(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/2*b*d^(5/2)*arctan(-1+2^(
1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/(a^2-b^2)/f
/g^(3/2)*2^(1/2)+1/2*a^2*d^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1
/2)/g^(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/2*b*d^(5/
2)*arctan(1+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/g^(1/2)/(d*sin(f*x+e))^(1/
2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)+1/4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/
2)-2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f
/g^(3/2)*2^(1/2)-1/4*b*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)-2^(1/2)*d^(1/2
)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)-1/
4*a^2*d^(5/2)*ln(g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^
(1/2)/(d*sin(f*x+e))^(1/2))/b/(a^2-b^2)/f/g^(3/2)*2^(1/2)+1/4*b*d^(5/2)*ln(
g^(1/2)+cot(f*x+e)*g^(1/2)+2^(1/2)*d^(1/2)*(g*cos(f*x+e))^(1/2)/(d*sin(f*x+
e))^(1/2))/(a^2-b^2)/f/g^(3/2)*2^(1/2)+2*a*d*(d*sin(f*x+e))^(3/2)/(a^2-b^2)
/f/g/(g*cos(f*x+e))^(1/2)-2*a^3*d^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)
/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)
/b/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a^3*d^3*Ellipt
icPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(
1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/b/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*si
n(f*x+e))^(1/2)-2*b*d^2*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(
1/2)+2*a*d^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+
1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/f/
g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 1.08, antiderivative size = 1064, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 17, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.460$, Rules used = {2981, 2651, 2652, 2719, 2646, 2655, 303, 1176, 631, 210, 1179, 642, 2988, 2985, 2984, 504, 1232}

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] -((a^2*d^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*S
qrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2))) + (b*d^(5/2)*ArcT
```

```
an[1 - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]]
)])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a^2*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[g*Cos[e + f*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*b*(a^2
- b^2)*f*g^(3/2)) - (b*d^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/(Sqrt[g]*Sqrt[d*Sin[e + f*x]])])/(Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) + (a
^2*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos
[e + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) - (b
*d^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] - (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e
 + f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (a^2*d
^(5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e +
 f*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*b*(a^2 - b^2)*f*g^(3/2)) + (b*d^(
5/2)*Log[Sqrt[g] + Sqrt[g]*Cot[e + f*x] + (Sqrt[2]*Sqrt[d]*Sqrt[g*Cos[e + f
*x]])/Sqrt[d*Sin[e + f*x]]])/(2*Sqrt[2]*(a^2 - b^2)*f*g^(3/2)) - (2*Sqrt[2]
*a^3*d^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e + f*x]
]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3
/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a^3*d^3*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(b*(-a + b)^(3/2)*(a + b)^(3/2
)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*d^2*Sqrt[d*Sin[e + f*x]])/((a^2 -
b^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*d*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2
)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*d^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2646

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(a*Sin[e + f*x])^(m - 1)*((b*Cos[e + f*x])^(n +
1)/(b*f*(n + 1))), x] + Dist[a^2*((m - 1)/(b^2*(n + 1))), Int[(a*Sin[e + f*
x])^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && G
tQ[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
```

```
-1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2655

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(-k)*a*(b/f), Subst[Int[x^
(k*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e
 + f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0
] && LtQ[m, 1]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2988

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] - Dist[a*(d/b), Int[(g*Cos
[e + f*x])^p*((d*Sin[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; Fre
eQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && Lt
Q[-1, p, 1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{5/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{(ad^2)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{(a^2 d}{}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= -\frac{2bd^2\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2ad(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= \frac{a^2d^{5/2}\log\left(\sqrt{g}+\sqrt{g}\,\cot(e+fx)-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{d\sin(e+fx)}}\right.}{2\sqrt{2}\,b\,(a^2-b^2)\,fg^{3/2}}$$

$$= -\frac{a^2d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\,\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{d\sin(e+fx)}}\right)}{\sqrt{2}\,b\,(a^2-b^2)\,fg^{3/2}} + \frac{bd^{5/2}\tan^{-1}}{}$$

Mathematica [F]

time = 180.01, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(d*Sin[e + f*x])^(5/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] $Aborted
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4618 vs. 2(887) = 1774.
time = 4.96, size = 4619, normalized size = 4.34

method	result	size
default	Expression too large to display	4619

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/f*((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*cos(f*x+e)*a^3-I*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^
(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2-2*sin(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2
)*b^2+2*(-a^2+b^2)^(1/2)*2^(1/2)*a*b-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b+4*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b-(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2+(-(-1+co
s(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1
/2)*b^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+
e)*(-a^2+b^2)^(1/2)*a^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
```

```
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2+(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*cos
(f*x+e)*a^2*b+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a)
,1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*cos(f*x+e)*a^2*b-2*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*b^2-2*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+4*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^3-(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^3-2*cos
(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/
(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*cos(f*x+e)*a^3-(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^2+(-(-1+cos(f*x+e)-s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipti...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(5/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(5/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(5/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,\sin\left(e+f\,x\right))^{5/2}}{(g\,\cos\left(e+f\,x\right))^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
[Out] int((d*sin(e + f*x))^(5/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1437 $\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=379

$$\frac{2\sqrt{2}\,a^2d^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,a^2d^2\Pi\left(\frac{\sqrt{-a+b}}{\sqrt{a+b}}\right.}{(-a}$$

```
[Out] -2*b*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a^2*d^2*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*s
in(f*x+e))^(1/2)-2*a^2*d^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f
*x+e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3
/2)/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a*d*(d*sin(f*x+e))^(1/2)/(
a^2-b^2)/f/g/(g*cos(f*x+e))^(1/2)-2*b*d*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1
/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin
(f*x+e))^(1/2)/(a^2-b^2)/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2981, 2643, 2651, 2652, 2719, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,a^2d^2\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,a^2d^2\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2bdE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2ad\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b(d\sin(e+fx))^{3/2}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*a^2*d^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Co
s[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
 + b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*a^2*
d^2*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[
g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b
)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*d*Sqrt[d*Sin[e + f*x]])/((a^
2 - b^2)*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*d*Sqrt[g*Cos[e + f*x]]*EllipticE[e - P
i/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x
]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{(d\sin(e+fx))^{3/2}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{(bd)\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} + \frac{(ad^2)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e}}}{a^2-b^2}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= \frac{2ad\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}}$$

$$= \frac{2\sqrt{2}\,a^2d^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\right|}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

time = 52.28, size = 1648, normalized size = 4.35

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^(3/2)/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cot[e + f*x]*(d*Sin[e + f*x])^(3/2)*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f
*(g*Cos[e + f*x])^(3/2)) - (Cos[e + f*x]^(3/2)*(d*Sin[e + f*x])^(3/2)*((4*a
*b*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Si
n[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e +
f*x])) + ((a^2 - b^2)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
 (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sq
rt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
 Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 -
b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan
[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Si
n[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + (Cos[2*(e +
f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*
b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)
*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2,
1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2)
 + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4
*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*Ar
cTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^
2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[
2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] +
Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[
e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]
*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a
^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]
*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x
]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1
/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) +
 (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*Cos[e
 + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)
```

*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*Sin[
e + f*x]^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2539 vs. $2(346) = 692$.
time = 0.46, size = 2540, normalized size = 6.70

method	result	size
default	Expression too large to display	2540

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -1/f*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a+2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^2+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-4*cos(f*x+e)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*b+2*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x

```
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a+2*(-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a+(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a^
2+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2)
)*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)*a-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))*a^2-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^
2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-4*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*(-a^2+b^2)^(1/2)*b+2*cos(f*x+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b+2*sin(f*
x+e)*2^(1/2)*(-a^2+b^2)^(1/2)*a-2*2^(1/2)*(-a^2+b^2)^(1/2)*b)*(d*sin(f*x+e)
)^(3/2)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/sin(f*x+e)^2*2^(1/2)*a/(a+b)/(-a^2+
b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\sin{(e + fx)})^{\frac{3}{2}}}{(g\cos{(e + fx)})^{\frac{3}{2}}\,(a + b\sin{(e + fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(3/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((d*sin(e + f*x))**(3/2)/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*
x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate((d*sin(f*x + e))^(3/2)/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) +
a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d\,\sin\left(e+f\,x\right)\right)}^{3/2}}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(3/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1438 $\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=374

$$-\frac{2\sqrt{2}\,abd\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,abd\Pi\left(\frac{\sqrt{-a+b}}{\sqrt{a+b}};\right.}{(-a+}$$

```
[Out] 2*a*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d/f/g/(g*cos(f*x+e))^(1/2)-2*a*b*d*Ellip
ticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)
^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*si
n(f*x+e))^(1/2)+2*a*b*d*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+
e))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)
/(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b*(d*sin(f*x+e))^(1/2)/(a^2-b
^2)/f/g/(g*cos(f*x+e))^(1/2)+2*a*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f
*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e)
)^(1/2)/(a^2-b^2)/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]

time = 0.57, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2982, 2917, 2643, 2651, 2652, 2719, 2985, 2984, 504, 1232}

$$-\frac{2aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{2a(d\sin(e+fx))^{3/2}}{dfg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,abd\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,abd\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),x]
```

```
[Out] (-2*Sqrt[2]*a*b*d*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos
[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a
+ b)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*a*b*d
*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*
Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(
3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 - b
^2)*f*g*Sqrt[g*Cos[e + f*x]]) + (2*a*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
 f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2982

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[-d/(a^2 - b^
2), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1)*(b - a*Sin[e + f*x]), x
```

```
], x] + Dist[a*b*(d/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Si
n[e + f*x])^(n - 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f,
g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n,
0]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = -\frac{d\int \frac{b-a\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} + \frac{(abd)\int \frac{\sqrt{g\,\mathrm{co}}}{\sqrt{d\sin(e+}}}{(a^2-}$$

$$= \frac{a\int \frac{\sqrt{d\sin(e+fx)}}{(g\cos(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{(bd)\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx}}}{a^2-b^2}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,dfg\sqrt{g\cos(e+f}}$$

$$= -\frac{2b\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,fg\sqrt{g\cos(e+fx)}} + \frac{2a(d\sin(e+fx))^{3/2}}{(a^2-b^2)\,dfg\sqrt{g\cos(e+f}}$$

$$= -\frac{2\sqrt{2}\,abd\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{1+\sin(e+fx)}}\right)\right|}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+f}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 41.09, size = 1274, normalized size = 3.41

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d*Sin[e + f*x]]/((g*Cos[e + f*x])^(3/2)*(a + b*Sin[e + f*x])
),x]
```

```
[Out] (2*Cos[e + f*x]*Sqrt[d*Sin[e + f*x]]*(-b + a*Sin[e + f*x]))/((a^2 - b^2)*f*
(g*Cos[e + f*x])^(3/2)) + (a*Cos[e + f*x]^(3/2)*Sqrt[d*Sin[e + f*x]]*((4*a*
(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e
 + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x
])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan
```

```
[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]
^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*A
ppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[
Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] -
 (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/S
qrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)
^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*Arc
Tan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2
)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
 f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqr
t[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Ta
n[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2
)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[
e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a
^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2
- b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(
a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2
])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((a - b)*(a + b)*f*(g*Cos[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2538 vs. $2(341) = 682$.
time = 0.43, size = 2539, normalized size = 6.79

method	result	size
default	Expression too large to display	2539

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-a^2+b^2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*a+2*cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^
2+b^2)^(1/2)*b-4*cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*a-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e)
```

)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b+cos(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b+cos(f*x+e)
*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))
*b^2-cos(f*x+e)*(-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b-cos(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^2+2*(-a^2+b^
2)^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a+2*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b-4*(-a^2+b^2)^(1/2)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*a-(-a^2+b^2)^(1/2)*(-(-1+cos(f
*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b+(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b+Ellip
ticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)
+a),1/2*2^(1/2))*b^2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)-(
-a^2+b^2)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2
)-a),1/2*2^(1/2))*a*b-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*

```
EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^
(1/2)-a),1/2*2^(1/2))*b^2+2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1/2)*a+2*sin(f*x
+e)*(-a^2+b^2)^(1/2)*2^(1/2)*b-2*(-a^2+b^2)^(1/2)*2^(1/2)*a)*(d*sin(f*x+e))
^(1/2)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/sin(f*x+e)*2^(1/2)*a/(a+b)/(-a^2+b^2
)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="maxima")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d \sin{\left(e + f x \right)}}}{\left(g \cos{\left(e + f x \right)}\right)^{\frac{3}{2}} \left(a + b \sin{\left(e + f x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**(1/2)/(g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(sqrt(d*sin(e + f*x))/((g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x))
), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^(1/2)/(g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, alg
orithm="giac")
```

```
[Out] integrate(sqrt(d*sin(f*x + e))/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,\sin\left(e+f\,x\right)}}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))),x)
```

```
[Out] int((d*sin(e + f*x))^(1/2)/((g*cos(e + f*x))^(3/2)*(a + b*sin(e + f*x))), x
)
```

3.1439 $\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=380

$$\frac{2\sqrt{2}\,b^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{1+\sin(e+fx)}}\right)\middle|-1\right)\sqrt{\sin(e+fx)}}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2\Pi\left(\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\right.}{(-a+b)^{3/}}$$

```
[Out] -2*b*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d^2/f/g/(g*cos(f*x+e))^(1/2)+2*b^2*Elli
pticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b
)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/(a+b)^(3/2)/f/g^(3/2)/(d*s
in(f*x+e))^(1/2)-2*b^2*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e
))^(1/2),(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/(-a+b)^(3/2)/
(a+b)^(3/2)/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+2*a*(d*sin(f*x+e))^(1/2)/(a^2-b^
2)/d/f/g/(g*cos(f*x+e))^(1/2)-2*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+
f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e
))^(1/2)/(a^2-b^2)/d/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2983, 2917, 2643, 2651, 2652, 2719, 2985, 2984, 504, 1232}

$$-\frac{2b(d\sin(e+fx))^{3/2}}{d^2fg\,(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{dfg^2\,(a^2-b^2)\sqrt{\sin(2e+2fx)}}+\frac{2a\sqrt{d\sin(e+fx)}}{dfg\,(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2\sqrt{2}\,b^2\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^2\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])),x]
```

```
[Out] (2*Sqrt[2]*b^2*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e
+ f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b
)^(3/2)*(a + b)^(3/2)*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*Sqrt[2]*b^2*Elli
pticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[
1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/((-a + b)^(3/2)*(a + b)^(3/2)*
f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*a*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
*f*g*Sqrt[g*Cos[e + f*x]]) - (2*b*(d*Sin[e + f*x])^(3/2))/((a^2 - b^2)*d^2*
f*g*Sqrt[g*Cos[e + f*x]]) + (2*b*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 +
f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

d, 0]

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2983

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
```

```
 Dist[b^2/(g^2*(a^2 - b^2)), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b^2\int -}{}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}\sqrt{d\sin(e+fx)}}\,dx}{a^2-b^2} - \frac{b\int}{}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin}{(a^2-b^2)\,d^2}$$

$$= \frac{2a\sqrt{d\sin(e+fx)}}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}} - \frac{2b(d\sin}{(a^2-b^2)\,d^2}$$

$$= \frac{2\sqrt{2}\,b^2\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\sin^{-1}\left(\frac{\sqrt{g\cos(e}}{\sqrt{g}\,\sqrt{1+\sin}}\right.\right.}{(-a+b)^{3/2}(a+b)^{3/2}fg^{3/2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 41.52, size = 1279, normalized size = 3.37

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
])),x]
```

```
[Out] (2*Cos[e + f*x]*Sin[e + f*x]*(a - b*Sin[e + f*x]))/((a^2 - b^2)*f*(g*Cos[e
+ f*x])^(3/2)*Sqrt[d*Sin[e + f*x]]) + (b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*
x]]*((4*a*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x
]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos
[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]
^2])*Sin[e + f*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*S
in[e + f*x])) + (Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sq
```

```
rt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Ta
n[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^
2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
+ f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqr
t[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e
+ f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(
a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[
2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/
(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sq
rt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - S
qrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[
2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqr
t[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/
(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)
*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2
*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] +
 Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sq
rt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e
+ f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[
e + f*x]^2])))/(84*a^2*b*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e
 + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/((-a + b)*(a + b
)*f*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2558 vs. $2(347) = 694$.
time = 0.33, size = 2559, normalized size = 6.73

method	result	size
default	Expression too large to display	2559

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/f*(-cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2
)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*(-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^(1/2)*a
```

b+2(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^2+b^2)^
(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*cos(f*x+e)*(-a^
2+b^2)^(1/2)*a*b+cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2
+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^2+cos(f*x+e)*((-1+cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^3-cos(f*x+e)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e
))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-cos(f*x+e
)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e)
)^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2)
)*b^3-((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(
1/2))*(-a^2+b^2)^(1/2)*b^2-((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+
b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+2*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+
e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+2*(-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^
(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(
-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^2+((-1+cos(f*x+e)+sin(f*x+e))/sin(f
x+e))^(1/2)((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^3-((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^2-((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-

```
sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*cos(f*x+e)*(-a^2+
b^2)^(1/2)*2^(1/2)*a*b+2*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^2-2*(-a^2+b^
2)^(1/2)*2^(1/2)*a*b)*cos(f*x+e)/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)*
2^(1/2)/(a+b)/(-a^2+b^2)^(1/2)/(-b+(-a^2+b^2)^(1/2)+a)/(b+(-a^2+b^2)^(1/2)-
a)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\sin{(e+fx)}}\,(g\cos{(e+fx)})^{\frac{3}{2}}\,(a+b\sin{(e+fx)})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral(1/(sqrt(d*sin(e + f*x))*(g*cos(e + f*x))**(3/2)*(a + b*sin(e + f*x
))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(g\,\cos\left(e+f\,x\right))^{3/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))),
 x)
```

3.1440 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=568

$$-\frac{2a}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}+\frac{2b^2(g\cos(e+fx))^{3/2}}{a\,(a^2-b^2)\,dfg^3\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^3\Pi\left(-\frac{\sqrt{-a+b}}{\sqrt{a+b}};\right.}{a(-a+}$$

```
[Out] 4*a*(d*sin(f*x+e))^(3/2)/(a^2-b^2)/d^3/f/g/(g*cos(f*x+e))^(1/2)+2*b^2*(g*co
s(f*x+e))^(3/2)/a/(a^2-b^2)/d/f/g^3/(d*sin(f*x+e))^(1/2)-2*a/(a^2-b^2)/d/f/
g/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)-2*b^3*EllipticPi((g*cos(f*x+e))
^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*si
n(f*x+e)^(1/2)/a/(-a+b)^(3/2)/(a+b)^(3/2)/d/f/g^(3/2)/(d*sin(f*x+e))^(1/2)+
2*b^3*EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(
1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/a/(-a+b)^(3/2)/(a+b)^(3/2)/d/f
/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/d^2/f/g/(g
*cos(f*x+e))^(1/2)+4*a*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipt
icE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a
^2-b^2)/d^2/f/g^2/sin(2*f*x+2*e)^(1/2)-2*b^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/si
n(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(
d*sin(f*x+e))^(1/2)/a/(a^2-b^2)/d^2/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2983, 2917, 2650, 2651, 2652, 2719, 2643, 2989, 2985, 2984, 504, 1232}

$$\frac{4a(d\sin(e+fx))^{3/2}}{d^3fg\,(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2b^2E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{ad^2fg^2\,(a^2-b^2)\sqrt{\sin(2e+2fx)}}-\frac{4aE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{d^2fg^2\,(a^2-b^2)\sqrt{\sin(2e+2fx)}}-\frac{2b\sqrt{d\sin(e+fx)}}{d^2fg\,(a^2-b^2)\sqrt{g\cos(e+fx)}}+\frac{2b^2(g\cos(e+fx))^{3/2}}{adfg^3\,(a^2-b^2)\sqrt{d\sin(e+fx)}}-\frac{2a}{dfg\,(a^2-b^2)\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}-\frac{2\sqrt{2}\,b^3\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adfg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}+\frac{2\sqrt{2}\,b^3\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\,\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{adfg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*a)/((a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]) + (2*b
^2*(g*Cos[e + f*x])^(3/2))/(a*(a^2 - b^2)*d*f*g^3*Sqrt[d*Sin[e + f*x]]) - (
2*Sqrt[2]*b^3*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sqrt[g*Cos[e +
 f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a +
b)^(3/2)*(a + b)^(3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (2*Sqrt[2]*b^3*E
llipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f*x]]/(Sqrt[g]*Sq
rt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a*(-a + b)^(3/2)*(a + b)^(
3/2)*d*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) - (2*b*Sqrt[d*Sin[e + f*x]])/((a^2 -
 b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]) + (4*a*(d*Sin[e + f*x])^(3/2))/((a^2 -
b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*a*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e
```

```
+ 2*f*x]]) + (2*b^2*Sqrt[g*Cos[e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
d*Sin[e + f*x]])/(a*(a^2 - b^2)*d^2*f*g^2*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2983

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{\cdots}}{(d\sin(e\cdots}}{\cdots}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{3/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{\cdots}{(g\cos\cdots}}{\cdots}$$

$$= -\frac{2a}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\cdots}}$$

$$= -\frac{2a}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\cdots}}$$

$$= -\frac{2a}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\cdots}}$$

$$= -\frac{2a}{(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.47, size = 1707, normalized size = 3.01

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2)*(a + b*Sin[e + f
*x])),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x]^2*((-2*Cot[e + f*x])/a + (2*Sec[e + f*x]*(-b +
 a*Sin[e + f*x]))/(a^2 - b^2)))/(f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^
(3/2)) - (Cos[e + f*x]^(3/2)*Sin[e + f*x]^(3/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b
*AppellF1[3/4, -1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
```

```
^2)]) + a*AppellF1[3/4, 1/4, 1, 7/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(
-a^2 + b^2)])*Cos[e + f*x]^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f
*x]^(3/2))/(3*(a^2 - b^2)*(1 - Cos[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x]))
+ ((2*a^2*b - 2*b^3)*Sqrt[Tan[e + f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 -
(Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqr
t[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] +
Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b
^2]*Tan[e + f*x]]))/(a^2 - b^2)^(1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan
[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[
e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin
[e + f*x]]*(a + b*Sin[e + f*x])*(1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b +
b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e + f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e
 + f*x]^2])*(56*b*(-3*a^2 + b^2)*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2
, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*App
ellF1[7/4, 1/2, 1, 11/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Ta
n[e + f*x]^(7/2) + 21*a^(3/2)*(4*Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Ta
n[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (
4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqr
t[a]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(
1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTa
n[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^
(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f
*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[
Tan[e + f*x]] + Tan[e + f*x]] - 2*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[
e + f*x]] + Tan[e + f*x]] - (2*Sqrt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^
(1/4) + (Sqrt[2]*b^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2
*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 -
b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a
]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^
2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])
))/(84*a^2*b^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan[e + f*x]^2])))/(a*(a - b)*(a + b)*f*(g*Co
s[e + f*x])^(3/2)*(d*Sin[e + f*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3103 vs. $2(546) = 1092$.

time = 0.35, size = 3104, normalized size = 5.46

method	result	size
default	Expression too large to display	3104

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x,method=_
RETURNVERBOSE)

[Out] 1/f*(-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*b^4+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2
*2^(1/2))*b^4-2*2^(1/2)*cos(f*x+e)*(-a^2+b^2)^(1/2)*a*b^2+4*2^(1/2)*cos(f*x
+e)*(-a^2+b^2)^(1/2)*a^3-2*2^(1/2)*(-a^2+b^2)^(1/2)*a^3+2*2^(1/2)*sin(f*x+e
)*(-a^2+b^2)^(1/2)*a^2*b-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^4+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^4-(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/si
n(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3
-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))
*a*b^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^
(1/2))*(-a^2+b^2)^(1/2)*b^3+(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2
+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^3-8*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a^3+4*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1
/2))*(-a^2+b^2)^(1/2)*a^3+2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))
*(-a^2+b^2)^(1/2)*b^3-cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+
(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3-cos(f*x+e)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(

```
f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^3-cos(
f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^
(1/2))*(-a^2+b^2)^(1/2)*b^3+cos(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/s
in(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),
a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^3-8*cos(f*x+e)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/
sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^3+4*cos(f*x+e)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^3+2*cos(f*x+e)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e
)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^3+4*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^2-2*(-(-1+cos
(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^2+4*cos(f*x+e)*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

[Out] Timed out

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(g\,\cos\left(e+f\,x\right))^{3/2}\,(d\,\sin\left(e+f\,x\right))^{3/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(3/2)*(a + b*sin(e + f*x))),
 x)
```

3.1441 $\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx$

Optimal. Leaf size=673

$$-\frac{2a}{3\left(a^2-b^2\right)dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+fx))^{3/2}}+\frac{2b^2(g\cos(e+fx))^{3/2}}{3a\left(a^2-b^2\right)dfg^3(d\sin(e+fx))^{3/2}}+\frac{}{\left(a^2-b^2\right)d^2fg\sqrt{g\,c}}$$

```
[Out] 2/3*b^2*(g*cos(f*x+e))^(3/2)/a/(a^2-b^2)/d/f/g^3/(d*sin(f*x+e))^(3/2)-2/3*a
/(a^2-b^2)/d/f/g/(d*sin(f*x+e))^(3/2)/(g*cos(f*x+e))^(1/2)-4*b*(d*sin(f*x+e
))^(3/2)/(a^2-b^2)/d^4/f/g/(g*cos(f*x+e))^(1/2)-2*b^3*(g*cos(f*x+e))^(3/2)/
a^2/(a^2-b^2)/d^2/f/g^3/(d*sin(f*x+e))^(1/2)+2*b/(a^2-b^2)/d^2/f/g/(g*cos(f
*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)+2*b^4*EllipticPi((g*cos(f*x+e))^(1/2)/g^(
1/2)/(1+sin(f*x+e))^(1/2),-(-a+b)^(1/2)/(a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(
1/2)/a^2/(-a+b)^(3/2)/(a+b)^(3/2)/d^2/f/g^(3/2)/(d*sin(f*x+e))^(1/2)-2*b^4*
EllipticPi((g*cos(f*x+e))^(1/2)/g^(1/2)/(1+sin(f*x+e))^(1/2),(-a+b)^(1/2)/(
a+b)^(1/2),I)*2^(1/2)*sin(f*x+e)^(1/2)/a^2/(-a+b)^(3/2)/(a+b)^(3/2)/d^2/f/g
^(3/2)/(d*sin(f*x+e))^(1/2)+8/3*a*(d*sin(f*x+e))^(1/2)/(a^2-b^2)/d^3/f/g/(g
*cos(f*x+e))^(1/2)-4*b*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipt
icE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(d*sin(f*x+e))^(1/2)/(a
^2-b^2)/d^3/f/g^2/sin(2*f*x+2*e)^(1/2)+2*b^3*(sin(e+1/4*Pi+f*x)^2)^(1/2)/si
n(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*(g*cos(f*x+e))^(1/2)*(
d*sin(f*x+e))^(1/2)/a^2/(a^2-b^2)/d^3/f/g^2/sin(2*f*x+2*e)^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 673, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2983, 2917, 2650, 2643, 2651, 2652, 2719, 2989, 2985, 2984, 504, 1232}

$$\frac{2\sqrt{2}\,b^4\sqrt{\sin(e+fx)}\,\Pi\left(-\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}-\frac{2\sqrt{2}\,b^4\sqrt{\sin(e+fx)}\,\Pi\left(\frac{\sqrt{b-a}}{\sqrt{a+b}};\text{ArcSin}\left(\frac{\sqrt{g\cos(e+fx)}}{\sqrt{g}\sqrt{\sin(e+fx)+1}}\right)\middle|-1\right)}{a^2d^2fg^{3/2}(b-a)^{3/2}(a+b)^{3/2}\sqrt{d\sin(e+fx)}}-\frac{4b(d\sin(e+fx))^{3/2}}{d^4fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{4bE\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{d^3fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}+\frac{8a\sqrt{d\sin(e+fx)}}{3d^3fg\left(a^2-b^2\right)\sqrt{g\cos(e+fx)}}+\frac{2b}{d^2fg\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}+\frac{2b^2(g\cos(e+fx))^{3/2}}{3adfg^3\left(a^2-b^2\right)(d\sin(e+fx))^{3/2}}-\frac{2a}{3dfg\left(a^2-b^2\right)(d\sin(e+fx))^{3/2}\sqrt{g\cos(e+fx)}}-\frac{2b^3E\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{d\sin(e+fx)}\sqrt{g\cos(e+fx)}}{a^2d^3fg^2\left(a^2-b^2\right)\sqrt{\sin(2e+2fx)}}-\frac{2b^3(g\cos(e+fx))^{3/2}}{a^2d^2fg^3\left(a^2-b^2\right)\sqrt{d\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f*x])),
x]
```

```
[Out] (-2*a)/(3*(a^2 - b^2)*d*f*g*Sqrt[g*Cos[e + f*x]]*(d*Sin[e + f*x])^(3/2)) +
(2*b^2*(g*Cos[e + f*x])^(3/2))/(3*a*(a^2 - b^2)*d*f*g^3*(d*Sin[e + f*x])^(3
/2)) + (2*b)/((a^2 - b^2)*d^2*f*g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]]
) - (2*b^3*(g*Cos[e + f*x])^(3/2))/(a^2*(a^2 - b^2)*d^2*f*g^3*Sqrt[d*Sin[e
+ f*x]]) + (2*Sqrt[2]*b^4*EllipticPi[-(Sqrt[-a + b]/Sqrt[a + b]), ArcSin[Sq
rt[g*Cos[e + f*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]
])/(a^2*(-a + b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) -
(2*Sqrt[2]*b^4*EllipticPi[Sqrt[-a + b]/Sqrt[a + b], ArcSin[Sqrt[g*Cos[e + f
*x]]/(Sqrt[g]*Sqrt[1 + Sin[e + f*x]])], -1]*Sqrt[Sin[e + f*x]])/(a^2*(-a +
```

```
b)^(3/2)*(a + b)^(3/2)*d^2*f*g^(3/2)*Sqrt[d*Sin[e + f*x]]) + (8*a*Sqrt[d*Si
n[e + f*x]])/(3*(a^2 - b^2)*d^3*f*g*Sqrt[g*Cos[e + f*x]]) - (4*b*(d*Sin[e +
 f*x])^(3/2))/((a^2 - b^2)*d^4*f*g*Sqrt[g*Cos[e + f*x]]) + (4*b*Sqrt[g*Cos[
e + f*x]]*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/((a^2 - b^2)*d
^3*f*g^2*Sqrt[Sin[2*e + 2*f*x]]) - (2*b^3*Sqrt[g*Cos[e + f*x]]*EllipticE[e
- Pi/4 + f*x, 2]*Sqrt[d*Sin[e + f*x]])/(a^2*(a^2 - b^2)*d^3*f*g^2*Sqrt[Sin[
2*e + 2*f*x]])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2643

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2650

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m + 1)/(a
*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2651

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-(b*Sin[e + f*x])^(n + 1))*((a*Cos[e + f*x])^(m + 1)
/(a*b*f*(m + 1))), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x]
)^n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m,
 -1] && IntegersQ[2*m, 2*n]
```

Rule 2652

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[Sqrt[a*Sin[e + f*x]]*(Sqrt[b*Cos[e + f*x]]/Sqrt[Sin[2*e
 + 2*f*x]]), Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2983

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/(a^2 - b^2
), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^n*(a - b*Sin[e + f*x]), x], x] -
 Dist[b^2/(g^2*(a^2 - b^2)), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e + f*x])
^n/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2
 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1]
```

Rule 2984

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[sin[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[-4*Sqrt[2]*(g/f), Su
bst[Int[x^2/(((a + b)*g^2 + (a - b)*x^4)*Sqrt[1 - x^4/g^2]), x], x, Sqrt[g*
Cos[e + f*x]]/Sqrt[1 + Sin[e + f*x]]], x] /; FreeQ[{a, b, e, f, g}, x] && N
eQ[a^2 - b^2, 0]
```

Rule 2985

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/(Sqrt[(d_)*sin[(e_.) + (f_.)*(x_)]]
*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Sin[e + f*
x]]/Sqrt[d*Sin[e + f*x]], Int[Sqrt[g*Cos[e + f*x]]/(Sqrt[Sin[e + f*x]]*(a +
 b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2,
 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
```

```
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
p, 1] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}(a+b\sin(e+fx))}\,dx = \frac{\int \frac{a-b\sin(e+fx)}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b^2\int \frac{\sqrt{}}{(d\sin(e}}{}$$

$$= \frac{a\int \frac{1}{(g\cos(e+fx))^{3/2}(d\sin(e+fx))^{5/2}}\,dx}{a^2-b^2} - \frac{b\int \frac{}{(g\cos}}{}$$

$$= -\frac{2a}{3(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+}$$

$$= -\frac{2a}{3(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+}$$

$$= -\frac{2a}{3(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+}$$

$$= -\frac{2a}{3(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+}$$

$$= -\frac{2a}{3(a^2-b^2)\,dfg\sqrt{g\cos(e+fx)}\,(d\sin(e+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 54.78, size = 1727, normalized size = 2.57

Warning: Unable to verify antiderivative.

[In] Integrate[1/((g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)*(a + b*Sin[e + f
*x])),x]

[Out] (Cos[e + f*x]^2*Sin[e + f*x]^3*((2*b*Cot[e + f*x])/a^2 - (2*Cot[e + f*x]*Cs
c[e + f*x])/(3*a) + (2*Sec[e + f*x]*(a - b*Sin[e + f*x]))/(a^2 - b^2)))/(f*
(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f*x])^(5/2)) - (b*Cos[e + f*x]^(3/2)*Sin[
e + f*x]^(5/2)*((-2*(4*a^3 - 2*a*b^2)*(-(b*AppellF1[3/4, -1/4, 1, 7/4, Cos[
e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)]) + a*AppellF1[3/4, 1/4, 1, 7
/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^(3/2)*
(a + b*Sqrt[1 - Cos[e + f*x]^2])*Sin[e + f*x]^(3/2)))/(3*(a^2 - b^2)*(1 - Co
s[e + f*x]^2)^(3/4)*(a + b*Sin[e + f*x])) + ((2*a^2*b - 2*b^3)*Sqrt[Tan[e +
f*x]]*((3*Sqrt[2]*a^(3/2)*(-2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[T
an[e + f*x]])/Sqrt[a]] + 2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]])/Sqrt[a]] - Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e
+ f*x]] - Sqrt[a^2 - b^2]*Tan[e + f*x]] + Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^
2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]]))/(a^2 - b^2)^(
1/4) - 8*b*AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e
+ f*x]^2)/a^2]*Tan[e + f*x]^(3/2))*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x
]^2]))/(12*a^2*Cos[e + f*x]^(3/2)*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(
1 + Tan[e + f*x]^2)^(3/2)) + ((-2*a^2*b + b^3)*Cos[2*(e + f*x)]*Sqrt[Tan[e
+ f*x]]*(b*Tan[e + f*x] + a*Sqrt[1 + Tan[e + f*x]^2])*(56*b*(-3*a^2 + b^2)*
AppellF1[3/4, 1/2, 1, 7/4, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*
Tan[e + f*x]^(3/2) + 24*b*(-a^2 + b^2)*AppellF1[7/4, 1/2, 1, 11/4, -Tan[e +
f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]^(7/2) + 21*a^(3/2)*(4*
Sqrt[2]*a^(3/2)*ArcTan[1 - Sqrt[2]*Sqrt[Tan[e + f*x]]] - 4*Sqrt[2]*a^(3/2)*
ArcTan[1 + Sqrt[2]*Sqrt[Tan[e + f*x]]] - (4*Sqrt[2]*a^2*ArcTan[1 - (Sqrt[2]
*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) + (2*Sqr
t[2]*b^2*ArcTan[1 - (Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]]
)/(a^2 - b^2)^(1/4) + (4*Sqrt[2]*a^2*ArcTan[1 + (Sqrt[2]*(a^2 - b^2)^(1/4)*
Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4) - (2*Sqrt[2]*b^2*ArcTan[1 +
(Sqrt[2]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]])/Sqrt[a]])/(a^2 - b^2)^(1/4)
+ 2*Sqrt[2]*a^(3/2)*Log[1 - Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - 2
*Sqrt[2]*a^(3/2)*Log[1 + Sqrt[2]*Sqrt[Tan[e + f*x]] + Tan[e + f*x]] - (2*Sq
rt[2]*a^2*Log[-a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - S
qrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (Sqrt[2]*b^2*Log[-a + Sqr
t[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e + f*x]] - Sqrt[a^2 - b^2]*Tan[e +
f*x]])/(a^2 - b^2)^(1/4) + (2*Sqrt[2]*a^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Tan[e + f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(
1/4) - (Sqrt[2]*b^2*Log[a + Sqrt[2]*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Tan[e +
f*x]] + Sqrt[a^2 - b^2]*Tan[e + f*x]])/(a^2 - b^2)^(1/4) + (8*Sqrt[a]*b*Tan
[e + f*x]^(3/2))/Sqrt[1 + Tan[e + f*x]^2])))/(84*a^2*b^2*Cos[e + f*x]^(3/2)
*Sqrt[Sin[e + f*x]]*(a + b*Sin[e + f*x])*(-1 + Tan[e + f*x]^2)*Sqrt[1 + Tan
[e + f*x]^2])))/(a^2*(-a + b)*(a + b)*f*(g*Cos[e + f*x])^(3/2)*(d*Sin[e + f

```
*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3314 vs. $2(637) = 1274$.
time = 0.45, size = 3315, normalized size = 4.93

method	result	size
default	Expression too large to display	3315

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x,method=_
RETURNVERBOSE)
```

```
[Out] 1/3/f*(-6*2^(1/2)*(-a^2+b^2)^(1/2)*a^4-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*
x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^4-6*cos(
f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2
)^(1/2)*b^4+3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(
f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/
(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^4+3*cos(f*x+e)*sin(f
*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(
1/2))*a*b^4-12*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(
-a^2+b^2)^(1/2)*a^3*b+6*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))*(-a^2+b^2)^(1/2)*a*b^3+24*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a^3*b-12*sin(f*x+e)*(-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin
(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^3+8*2^(1/2)*cos(f*x+e)^2*(
-a^2+b^2)^(1/2)*a^4+6*2^(1/2)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^3*b-2*2^(1/2)*c
os(f*x+e)^2*(-a^2+b^2)^(1/2)*a^2*b^2-3*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+
e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^5+3*cos(f*x+
```

```
e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
Pi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a)
,1/2*2^(1/2))*b^5+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-
a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^4-3*sin(f*x+e)*(-(-1+cos(
f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*b^4-6*si
n(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b
^4+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-
a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^4+3*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+
e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x
+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a*b^4-3*sin(f*x+e)*(-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^5+3*
sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/
2*2^(1/2))*b^5-12*2^(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a^3*b+6*2^
(1/2)*cos(f*x+e)*sin(f*x+e)*(-a^2+b^2)^(1/2)*a*b^3+6*cos(f*x+e)*sin(f*x+e)*
(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/
sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*
x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b^3+24*c
os(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*E
llipticE((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+
b^2)^(1/2)*a^3*b-12*cos(f*x+e)*sin(f*x+e)*(-(-1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="maxima")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))**(3/2)/(d*sin(f*x+e))**(5/2)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*cos(f*x+e))^(3/2)/(d*sin(f*x+e))^(5/2)/(a+b*sin(f*x+e)),x, a
lgorithm="giac")
```

```
[Out] integrate(1/((g*cos(f*x + e))^(3/2)*(b*sin(f*x + e) + a)*(d*sin(f*x + e))^(
5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(g\,\cos\left(e+f\,x\right)\right)}^{3/2}\,{\left(d\,\sin\left(e+f\,x\right)\right)}^{5/2}\,\left(a+b\,\sin\left(e+f\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
x)
```

```
[Out] int(1/((g*cos(e + f*x))^(3/2)*(d*sin(e + f*x))^(5/2)*(a + b*sin(e + f*x))),
 x)
```

3.1442 $\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{1+\cos(e+fx)}}\right)\middle|-1\right)}{a^2\sqrt{-a^2+b^2}\,\sqrt{d}\,f\,\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi}{}$$

```
[Out] b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f*x+e))^(1/2),-a/(b-(-
a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/f/(-a^2+b^2)^(1/2)/d^(1/2)/
(g*cos(f*x+e))^(1/2)-b*g^2*EllipticPi((d*sin(f*x+e))^(1/2)/d^(1/2)/(1+cos(f
*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)),I)*2^(1/2)*cos(f*x+e)^(1/2)/a^2/f/(-a^
2+b^2)^(1/2)/d^(1/2)/(g*cos(f*x+e))^(1/2)+g*(g*cos(f*x+e))^(1/2)*(d*sin(f*x
+e))^(1/2)/a/d/f/(a+b*sin(f*x+e))-1/2*g^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e
+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*sin(2*f*x+2*e)^(1/2)/a^2/
f/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2966, 2989, 2653, 2720, 2987, 2986, 1232}

$$\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}-\frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b+\sqrt{b^2-a^2}};\text{ArcSin}\left(\frac{\sqrt{d\sin(e+fx)}}{\sqrt{d}\,\sqrt{\cos(e+fx)+1}}\right)\middle|-1\right)}{a^2\sqrt{d}\,f\sqrt{b^2-a^2}\,\sqrt{g\cos(e+fx)}}+\frac{g^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2a^2f\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{g\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}{adf(a+b\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^2),x]
```

```
[Out] (Sqrt[2]*b*g^2*Sqrt[Cos[e + f*x]]*EllipticPi[-(a/(b - Sqrt[-a^2 + b^2])), A
rcSin[Sqrt[d*Sin[e + f*x]]/(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqr
t[-a^2 + b^2]*Sqrt[d]*f*Sqrt[g*Cos[e + f*x]]) - (Sqrt[2]*b*g^2*Sqrt[Cos[e +
 f*x]]*EllipticPi[-(a/(b + Sqrt[-a^2 + b^2])), ArcSin[Sqrt[d*Sin[e + f*x]]/
(Sqrt[d]*Sqrt[1 + Cos[e + f*x]])], -1])/(a^2*Sqrt[-a^2 + b^2]*Sqrt[d]*f*Sqr
t[g*Cos[e + f*x]]) + (g*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])/(a*d*f*(
a + b*Sin[e + f*x])) + (g^2*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[Sin[2*e + 2*f
*x]])/(2*a^2*f*Sqrt[g*Cos[e + f*x]]*Sqrt[d*Sin[e + f*x]])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 2653

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2966

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[(-g)*(g*
Cos[e + f*x])^(p - 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^(m + 1)/(a
*d*f*(m + 1))), x] + Dist[g^2*((2*m + 3)/(2*a*(m + 1))), Int[(g*Cos[e + f*x
])^(p - 2)*((a + b*Sin[e + f*x])^(m + 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; F
reeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && EqQ[m + p
 + 1/2, 0]
```

Rule 2986

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]]*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2,
2]}, Dist[2*Sqrt[2]*d*((b + q)/(f*q)), Subst[Int[1/((d*(b + q) + a*x^2)*Sqr
t[1 - x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x] -
Dist[2*Sqrt[2]*d*((b - q)/(f*q)), Subst[Int[1/((d*(b - q) + a*x^2)*Sqrt[1 -
 x^4/d^2]), x], x, Sqrt[d*Sin[e + f*x]]/Sqrt[1 + Cos[e + f*x]]], x]] /; Fre
eQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2987

```
Int[Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)
]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[Sqrt[Cos[e + f
*x]]/Sqrt[g*Cos[e + f*x]], Int[Sqrt[d*Sin[e + f*x]]/(Sqrt[Cos[e + f*x]]*(a
+ b*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2
, 0]
```

Rule 2989

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(g*
Cos[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] - Dist[b/(a*d), Int[(g*Cos[e + f
*x])^p*((d*Sin[e + f*x])^(n + 1)/(a + b*Sin[e + f*x])), x], x] /; FreeQ[{a,
 b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[-1,
```

p, 1] && LtQ[n, 0]

Rubi steps

$$\int \frac{(g\cos(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}\,dx = \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{}{\sqrt{g\cos(e+fx)}\,\sqrt{a}}}{}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2\int \frac{1}{\sqrt{g\cos(e+fx)}\,\sqrt{a}}}{2a^2}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} - \frac{\left(bg^2\sqrt{\cos(e+fx)}\right)\int -}{2a^2d\sqrt{g}}$$

$$= \frac{g\sqrt{g\cos(e+fx)}\,\sqrt{d\sin(e+fx)}}{adf(a+b\sin(e+fx))} + \frac{g^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\mathrm{si}}}{2a^2f\sqrt{g\cos(e+fx)}\,\sqrt{a}}$$

$$= \frac{\sqrt{2}\,bg^2\sqrt{\cos(e+fx)}\,\Pi\left(-\frac{a}{b-\sqrt{-a^2+b^2}};\sin^{-1}\left(\frac{\sqrt{d\sin(}}{\sqrt{d}\,\sqrt{1+\mathrm{c}}}\right.\right.}{a^2\sqrt{-a^2+b^2}\,\sqrt{d}\,f\sqrt{g\cos(e+fx)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.
time = 25.32, size = 717, normalized size = 2.17

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^(3/2)/(Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])
^2),x]
```

```
[Out] -(((g*Cos[e + f*x])^(3/2)*(a + b*Sqrt[1 - Cos[e + f*x]^2])*((5*a*(a^2 - b^2
)*AppellF1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]*Sqrt[Cos[e + f*x]])/((1 - Cos[e + f*x]^2)^(3/4)*(5*(a^2 - b^2)*AppellF
1[1/4, 3/4, 1, 5/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b^2)] + (-
4*b^2*AppellF1[5/4, 3/4, 2, 9/4, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2
 + b^2)] + 3*(a^2 - b^2)*AppellF1[5/4, 7/4, 1, 9/4, Cos[e + f*x]^2, (b^2*Co
s[e + f*x]^2)/(-a^2 + b^2)])*Cos[e + f*x]^2)*(a^2 + b^2*(-1 + Cos[e + f*x]^
2))) - ((1/8 - I/8)*b*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Cos[e + f*x]])/((
```

```
-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))] - 2*ArcTan[1 + ((1 + I)*Sqr
t[a]*Sqrt[Cos[e + f*x]])/((-a^2 + b^2)^(1/4)*(-1 + Cos[e + f*x]^2)^(1/4))]
+ Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2] - ((1
 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f*x]^2)^
(1/4)] - Log[Sqrt[-a^2 + b^2] + (I*a*Cos[e + f*x])/Sqrt[-1 + Cos[e + f*x]^2
] + ((1 + I)*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Cos[e + f*x]])/(-1 + Cos[e + f
*x]^2)^(1/4)]))/(Sqrt[a]*(-a^2 + b^2)^(3/4)))*Sin[e + f*x])/(a*f*Cos[e + f*
x]^(3/2)*(1 - Cos[e + f*x]^2)^(1/4)*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x
]))) + ((g*Cos[e + f*x])^(3/2)*Tan[e + f*x])/(a*f*Sqrt[d*Sin[e + f*x]]*(a +
 b*Sin[e + f*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3489 vs. $2(308) = 616$.

time = 0.63, size = 3490, normalized size = 10.54

method	result	size
default	Expression too large to display	3490

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x,method=_
RETURNVERBOSE)
```

```
[Out] -1/2/f*(2*2^(1/2)*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*a^2-((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-
sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2-((
-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x
+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*a*
b^2-((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/
2))*(-a^2+b^2)^(1/2)*b^2+((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^
2)^(1/2)-a),1/2*2^(1/2))*a*b^2-2*(-a^2+b^2)^(1/2)*2^(1/2)*cos(f*x+e)*a^2-si
n(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-
(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*
2^(1/2))*(-a^2+b^2)^(1/2)*a*b-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2
),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*b+((-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(-1+c
os(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x
```

```
+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*b^3-((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(
-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin
(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*b^3+2*sin
(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-
1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*a*
b-2*(-a^2+b^2)^(1/2)*sin(f*x+e)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*a^2-sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1
/2)+a),1/2*2^(1/2))*a^2*b+sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a
/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*a^2*b+2*(-(-1+cos(f*x+e)-sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(
1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*cos(f*x+e)^2*a*b+sin(f*x+e)*EllipticPi((
-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-b+(-a^2+b^2)^(1/2)+a),1/2
*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*a*b^2-sin(f*
x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((-(-1+
cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1
/2))*a*b^2-2*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF
((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/
2)*a*b-2*cos(f*x+e)^2*(-a^2+b^2)^(1/2)*2^(1/2)*a*b+2*cos(f*x+e)*(-a^2+b^2)^
(1/2)*2^(1/2)*a*b+EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
,-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*
x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/
sin(f*x+e))^(1/2)*cos(f*x+e)^2*b^3-(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f
*x+e))^(1/2)*EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),a/(-
b+(-a^2+b^2)^(1/2)+a),1/2*2^(1/2))*cos(f*x+e)^2*b^3+2*(-(-1+cos(f*x+e)-sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(
f*x+e))^(1/2),1/2*2^(1/2))*(-a^2+b^2)^(1/2)*b^2+EllipticPi((-(-1+cos(f*x+e)
-sin(f*x+e))/sin(f*x+e))^(1/2),-a/(b+(-a^2+b^2)^(1/2)-a),1/2*2^(1/2))*(-(-1
+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-a^2+b^2)^(1/2)*cos(f*x+e)
^2*b^2-EllipticPi((-(-1+cos(f*x+e)-sin(f*x+e))/...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(e+fx))^{\frac{3}{2}}}{\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(3/2)/(a+b*sin(f*x+e))**2/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Integral((g*cos(e + f*x))**(3/2)/(sqrt(d*sin(e + f*x))*(a + b*sin(e + f*x))
**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(3/2)/(a+b*sin(f*x+e))^2/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((g*cos(f*x + e))^(3/2)/((b*sin(f*x + e) + a)^2*sqrt(d*sin(f*x + e
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^{3/2}}{\sqrt{d\,\sin\left(e+f\,x\right)}\,\left(a+b\,\sin\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^2),
x)
```

```
[Out] int((g*cos(e + f*x))^(3/2)/((d*sin(e + f*x))^(1/2)*(a + b*sin(e + f*x))^2),
 x)
```

3.1443 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=82

$$-\frac{3ax}{2}+\frac{2b\cos(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}+\frac{b\sec(c+dx)}{d}+\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -3/2*a*x+2*b*cos(d*x+c)/d-1/3*b*cos(d*x+c)^3/d+b*sec(d*x+c)/d+3/2*a*tan(d*x
+c)/d-1/2*a*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2671, 294, 327, 209, 2670, 276}

$$\frac{3a\tan(c+dx)}{2d}-\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3ax}{2}-\frac{b\cos^3(c+dx)}{3d}+\frac{2b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*x)/2 + (2*b*Cos[c + d*x])/d - (b*Cos[c + d*x]^3)/(3*d) + (b*Sec[c + d
*x])/d + (3*a*Tan[c + d*x])/(2*d) - (a*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx &= a\int \sin^2(c+dx)\tan^2(c+dx)\,dx+b\int \sin^3(c+dx)\tan\\ &= \frac{a\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx,x,\tan(c+dx)\right)}{d}-\frac{b\text{Subst}\left(\int \frac{(1-x^2)}{x^2}\right.}{}\\ &= -\frac{a\sin^2(c+dx)\tan(c+dx)}{2d}+\frac{(3a)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\right.}{2d}\\ &= \frac{2b\cos(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}+\frac{b\sec(c+dx)}{d}+\frac{3a\tan}{}\\ &= -\frac{3ax}{2}+\frac{2b\cos(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 82, normalized size = 1.00

$$-\frac{3a(c+dx)}{2d}+\frac{7b\cos(c+dx)}{4d}-\frac{b\cos(3(c+dx))}{12d}+\frac{b\sec(c+dx)}{d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*a*(c + d*x))/(2*d) + (7*b*Cos[c + d*x])/(4*d) - (b*Cos[3*(c + d*x)])/(1
2*d) + (b*Sec[c + d*x])/d + (a*Sin[2*(c + d*x)])/(4*d) + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.16, size = 104, normalized size = 1.27

method	result
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\text{co}\right.}{d}$
default	$\frac{a\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\text{co}\right.}{d}$
risch	$-\frac{3ax}{2}-\frac{ia\,e^{2i(dx+c)}}{8d}+\frac{7b\,e^{i(dx+c)}}{8d}+\frac{7b\,e^{-i(dx+c)}}{8d}+\frac{ia\,e^{-2i(dx+c)}}{8d}+\frac{2ia+2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{b\cos(3dx+3c)}{12d}$
norman	$\frac{\frac{3ax}{2}-\frac{16b}{3d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3ax\left(\tan^6\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/
2*d*x-3/2*c)+b*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)
*cos(d*x+c)))
```

Maxima [A]
time = 0.50, size = 75, normalized size = 0.91

$$-\frac{3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a+2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b)/d
```

Fricas [A]
time = 0.59, size = 72, normalized size = 0.88

$$-\frac{2\,b\cos\left(dx+c\right)^4+9\,adx\cos\left(dx+c\right)-12\,b\cos\left(dx+c\right)^2-3\left(a\cos\left(dx+c\right)^2+2\,a\right)\sin\left(dx+c\right)-6\,b}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*cos(d*x + c)^4 + 9*a*d*x*cos(d*x + c) - 12*b*cos(d*x + c)^2 - 3*(
a*cos(d*x + c)^2 + 2*a)*sin(d*x + c) - 6*b)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin{(c + dx)})\sin^4{(c + dx)}\sec^2{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)**4*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.46, size = 119, normalized size = 1.45

$$-\frac{9\,(dx+c)a + \frac{12\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\,\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-10\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(9*(d*x + c)*a + 12*(a*tan(1/2*d*x + 1/2*c) + b)/(tan(1/2*d*x + 1/2*c)
^2 - 1) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 24*b
*tan(1/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - 10*b)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 18.22, size = 112, normalized size = 1.37

$$-\frac{3\,a\,x}{2} - \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \frac{32\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3} + 3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + \frac{16\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (3*a*x)/2 - ((16*b)/3 + 3*a*tan(c/2 + (d*x)/2) + 5*a*tan(c/2 + (d*x)/2)^3
 + 5*a*tan(c/2 + (d*x)/2)^5 + 3*a*tan(c/2 + (d*x)/2)^7 + (32*b*tan(c/2 + (d
*x)/2)^2)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.1444 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=65

$$-\frac{3bx}{2}+\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -3/2*b*x+a*cos(d*x+c)/d+a*sec(d*x+c)/d+3/2*b*tan(d*x+c)/d-1/2*b*sin(d*x+c)^
2*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.280$, Rules used = {2917, 2670, 14, 2671, 294, 327, 209}

$$\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{3b\tan(c+dx)}{2d}-\frac{b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*x)/2 + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (3*b*Tan[c + d*x])/(
2*d) - (b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sin(c+dx)\tan^2(c+dx)\,dx + b\int \sin^2(c+dx)\tan^2($$
$$= -\frac{a\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\right.}{}$$
$$= -\frac{b\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{a\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x\right.}{d}$$
$$= \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d} - \frac{b\sin^2(}{}$$
$$= -\frac{3bx}{2} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{3b\tan(c+dx)}{2d}$$

Mathematica [A]
time = 0.14, size = 63, normalized size = 0.97

$$-\frac{3b(c+dx)}{2d} + \frac{a\cos(c+dx)}{d} + \frac{a\sec(c+dx)}{d} + \frac{b\sin(2(c+dx))}{4d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] (-3*b*(c + d*x))/(2*d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (b*Sin[2
*(c + d*x)])/(4*d) + (b*Tan[c + d*x])/d
```

Maple [A]
time = 0.13, size = 94, normalized size = 1.45

method	result
derivativedivides	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}$
default	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}$
risch	$-\frac{3bx}{2}-\frac{ib\,e^{2i(dx+c)}}{8d}+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{ib\,e^{-2i(dx+c)}}{8d}+\frac{2i\left(-ia\,e^{i(dx+c)}+b\right)}{d\left(e^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{3bx}{2}-\frac{4a}{d}-\frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3bx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{3bx\left(\tan^6\left(\frac{dx}{2}\right.\right.}{2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b*(sin(d*x+c)^
5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.57, size = 62, normalized size = 0.95

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)b-2\,a\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*b
- 2*a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 61, normalized size = 0.94

$$-\frac{3\,bdx\cos\left(dx+c\right)-2\,a\cos\left(dx+c\right)^2-\left(b\cos\left(dx+c\right)^2+2\,b\right)\sin\left(dx+c\right)-2\,a}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(3*b*d*x*cos(d*x + c) - 2*a*cos(d*x + c)^2 - (b*cos(d*x + c)^2 + 2*b)*
sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin(c + dx))\sin^3(c + dx)\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)**3*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.49, size = 104, normalized size = 1.60

$$-\frac{3(dx+c)b + \frac{4\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1} + \frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(d*x + c)*b + 4*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^
2 - 1) + 2*(b*tan(1/2*d*x + 1/2*c)^3 - 2*a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1
/2*d*x + 1/2*c) - 2*a)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 15.97, size = 98, normalized size = 1.51

$$-\frac{3bx}{2} - \frac{3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + 4a}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - (3*b*x)/2 - (4*a + 3*b*tan(c/2 + (d*x)/2) + 4*a*tan(c/2 + (d*x)/2)^2 + 2*
b*tan(c/2 + (d*x)/2)^3 + 3*b*tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2
 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.1445 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=38

$$-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -a*x+b*cos(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]

time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2801, 3554, 8, 2670, 14}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx\\
&= a\int \tan^2(c+dx)\,dx+b\int \sin(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{a\tan(c+dx)}{d}-a\int 1\,dx-\frac{b\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}\\
&= -ax+\frac{a\tan(c+dx)}{d}-\frac{b\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d}\\
&= -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 1.24

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A]
time = 0.12, size = 59, normalized size = 1.55

method	result	size
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
default	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}+\frac{2ia+2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}$	70

norman	$\frac{ax-\frac{4b}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 1.03

$$-\frac{(dx+c-\tan(dx+c))a-b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 47, normalized size = 1.24

$$-\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\sin^2(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.49, size = 58, normalized size = 1.53

$$-\frac{(dx+c)a+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*a + 2*(a*tan(1/2*d*x + 1/2*c)^3 + a*tan(1/2*d*x + 1/2*c) + 2*b)
/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 12.48, size = 55, normalized size = 1.45

$$-a\,x-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - a*x - (4*b + 2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)/(d*(tan(c
/2 + (d*x)/2)^4 - 1))
```

3.1446 $\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=27

$$-bx+\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -b*x+a*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2917, 2686, 8, 3554}

$$\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -(b*x) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx = a\int \sec(c+dx)\tan(c+dx)\,dx + b\int \tan^2(c+dx)\,dx$$
$$= \frac{b\tan(c+dx)}{d} - b\int 1\,dx + \frac{a\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}$$
$$= -bx + \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.33

$$-\frac{b\tan^{-1}(\tan(c+dx))}{d} + \frac{a\sec(c+dx)}{d} + \frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -((b*ArcTan[Tan[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Maple [A]
time = 0.10, size = 32, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\frac{a}{\cos(dx+c)}+b(\tan(dx+c)-dx-c)}{d}$	32
default	$\frac{\frac{a}{\cos(dx+c)}+b(\tan(dx+c)-dx-c)}{d}$	32
risch	$-bx+\frac{2i(-ia\,e^{i(dx+c)}+b)}{d(e^{2i(dx+c)}+1)}$	40
norman	$\frac{bx-\frac{2a}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-bx\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a/cos(d*x+c)+b*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.52, size = 32, normalized size = 1.19

$$-\frac{(dx+c-\tan(dx+c))b - \frac{a}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*b - a/cos(d*x + c))/d
```

Fricas [A]
time = 0.40, size = 36, normalized size = 1.33

$$-\frac{bdx\cos(dx+c)-b\sin(dx+c)-a}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(b*d*x*cos(d*x + c) - b*sin(d*x + c) - a)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))\sin(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 43, normalized size = 1.59

$$-\frac{(dx+c)b+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*b + 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)^2 - 1)
)/d
```

Mupad [B]
time = 11.94, size = 41, normalized size = 1.52

$$-b\,x-\frac{2\,a+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] - b*x - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.1447 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=36

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+a*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2917, 2702, 327, 213, 3852, 8}

$$\frac{a\sec(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (a*Sec[c + d*x])/d + (b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx &= a\int \csc(c+dx)\sec^2(c+dx)\,dx+b\int \sec^2(c+dx)\,dx\\ &= \frac{a\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec(c+dx)\right)}{d}-\frac{b\text{Subst}(\int 1\,dx,x,}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}+\frac{a\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\right.}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.56

$$-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{a\sec(c+dx)}{d}+\frac{b\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
[Out] -((a*Log[Cos[(c + d*x)/2]])/d) + (a*Log[Sin[(c + d*x)/2]])/d + (a*Sec[c + d
*x])/d + (b*Tan[c + d*x])/d
```

Maple [A]
time = 0.16, size = 41, normalized size = 1.14

method	result	size
derivativedivides	$\frac{a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+b\tan(dx+c)}{d}$	41

default	$\frac{a\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+b\tan(dx+c)}{d}$	41
risch	$\frac{2i(-ia\,e^{i(dx+c)}+b)}{d(e^{2i(dx+c)}+1)}-\frac{a\ln(e^{i(dx+c)}+1)}{d}+\frac{a\ln(e^{i(dx+c)}-1)}{d}$	71
norman	$\frac{-\frac{2a}{d}-\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+b*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.33

$$\frac{a\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+2\,b\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(a*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 2
*b*tan(d*x + c))/d
```

Fricas [A]
time = 0.41, size = 65, normalized size = 1.81

$$-\frac{a\cos\left(dx+c\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,b\sin\left(dx+c\right)-2\,a}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a*cos(d*x + c)*log(-1/2*
cos(d*x + c) + 1/2) - 2*b*sin(d*x + c) - 2*a)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*csc(c + d*x)*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.46, size = 48, normalized size = 1.33

$$\frac{a \log \left(\left|\tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)\right|\right) - \frac{2 \, {\left(b \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + a\right)}}{\tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 11.94, size = 52, normalized size = 1.44

$$\frac{a\,\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a+2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)))/d - (2*a + 2*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2
+ (d*x)/2)^2 - 1))
```

3.1448 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=48

$$-\frac{b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] -b*arctanh(cos(d*x+c))/d-a*cot(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2917, 2700, 14, 2702, 327, 213}

$$\frac{a\tan(c+dx)}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*ArcTanh[Cos[c + d*x]])/d) - (a*Cot[c + d*x])/d + (b*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx &= a\int \csc^2(c+dx)\sec^2(c+dx)\,dx+b\int \csc(c+dx)\sec^2\\ &= \frac{a\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int \frac{x^2}{-1+x^2}\right.}{}\\ &= \frac{b\sec(c+dx)}{d}+\frac{a\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a\cot(c+dx)}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 1.42

$$-\frac{a\cot(c+dx)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
[Out] -((a*Cot[c + d*x])/d) - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/
2]])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.14, size = 61, normalized size = 1.27

method	result
derivativedivides	$\frac{a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)}{d}$
risch	$\frac{2b\,e^{3i(dx+c)}-4ia-2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{b\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{b\ln\left(e^{i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{a}{2d}-\frac{5a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b*(1/cos(d*x+c)+ln(csc(d*x+c)
-cot(d*x+c))))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 1.23

$$\frac{b\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 2
*a*(1/tan(d*x + c) - tan(d*x + c)))/d
```

Fricas [A]
time = 0.41, size = 96, normalized size = 2.00

$$-\frac{b\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+4\,a\cos\left(dx+c\right)^2-2\,b\sin\left(dx+c\right)-2\,a}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 4*a*cos(d*x + c)^2 - 2*b*si
n(d*x + c) - 2*a)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\csc^2\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*csc(c + d*x)**2*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(48) = 96$.
time = 0.48, size = 103, normalized size = 2.15

$$\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+10\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a*tan(1/2*d*x + 1/2*c) - (2*b*t
an(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*d*x + 1/2*c)^2 + 10*b*tan(1/2*d*x + 1/
2*c) - 3*a)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 11.91, size = 92, normalized size = 1.92

$$\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (4*b*tan(c/2 + (d*x)/2) - a + 5*a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*
x)/2) - 2*tan(c/2 + (d*x)/2)^3)) + (a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(ta
n(c/2 + (d*x)/2)))/d
```

3.1449 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=75

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot(c+dx)}{d}+\frac{3a\sec(c+dx)}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}$$

```
[Out] -3/2*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)/d+3/2*a*sec(d*x+c)/d-1/2*a*csc(d*
x+c)^2*sec(d*x+c)/d+b*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2917, 2702, 294, 327, 213, 2700, 14}

$$\frac{3a\sec(c+dx)}{2d}-\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{b\tan(c+dx)}{d}-\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x])/d + (3*a*Sec[c + d*x]
)/(2*d) - (a*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (b*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2917

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^2(c+dx)\,dx + b\int \csc^2(c+dx)\,\mathrm{sec}$$

$$= \frac{a\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{1+x}{x^2}\right.}{}$$

$$= -\frac{a\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{(3a)\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,\right.}{2d}$$

$$= -\frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+dx)}{2d} - \frac{a\csc^2(c+dx)\sec(c}{2d}$$

$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot(c+dx)}{d} + \frac{3a\sec(c+}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(75) = 150.

time = 0.26, size = 172, normalized size = 2.29

$$-\frac{2b\cot(2(c+dx))}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{a\sin\left(\frac{1}{2}(c+dx)\right)}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*b*Cot[2*(c + d*x)])/d - (a*Csc[(c + d*x)/2]^2)/(8*d) - (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]
^2)/(8*d) + (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- (a*Sin[(c + d*x)/2])/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A]
time = 0.18, size = 83, normalized size = 1.11

method	result
derivativedivides	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
default	$\frac{a\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+b\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)}{d}$
risch	$-\frac{i\left(3ia\,e^{5i(dx+c)}-2ia\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}+4b\,e^{2i(dx+c)}-4b\right)}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}-\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{2d}+\frac{3a\ln\left(e^{i(dx+c)}-1\right)}{2d}$
norman	$\frac{\frac{a}{8d}-\frac{9a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}-\frac{5b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-2a\left(\tan\right.}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d
*x+c)))+b*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))
```

Maxima [A]
time = 0.27, size = 84, normalized size = 1.12

$$\frac{a\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)-4b\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(co
s(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 4*b*(1/tan(d*x + c) - tan(d*x
+ c)))/d
```

Fricas [A]
time = 0.44, size = 128, normalized size = 1.71

$$\frac{6\,a\cos\left(dx+c\right)^{2}-3\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+3\,\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+4\,\left(2\,b\cos\left(dx+c\right)^{2}-b\right)\sin\left(dx+c\right)-4\,a}{4\,\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(6*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) + 3*(a*cos(d*x + c)^3 - a*cos(d*x + c))*log(-1/2*cos(d*x +
 c) + 1/2) + 4*(2*b*cos(d*x + c)^2 - b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)
^3 - d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.47, size = 116, normalized size = 1.55

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{16\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\frac{18\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+4\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/8*(a*tan(1/2*d*x + 1/2*c)^2 + 12*a*log(abs(tan(1/2*d*x + 1/2*c))) + 4*b*t
an(1/2*d*x + 1/2*c) - 16*(b*tan(1/2*d*x + 1/2*c) + a)/(tan(1/2*d*x + 1/2*c)
^2 - 1) - (18*a*tan(1/2*d*x + 1/2*c)^2 + 4*b*tan(1/2*d*x + 1/2*c) + a)/tan(
1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 11.93, size = 127, normalized size = 1.69

$$\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{-10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-\frac{17\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{8\,d}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^2*sin(c + d*x)^3),x)
[Out] (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2 + 2*b*tan(c/2 + (d*x)/2) - (17*a*tan(c/
2 + (d*x)/2)^2)/2 - 10*b*tan(c/2 + (d*x)/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 -
 4*tan(c/2 + (d*x)/2)^4)) + (a*tan(c/2 + (d*x)/2)^2)/(8*d) + (3*a*log(tan(c
/2 + (d*x)/2)))/(2*d)
```

3.1450 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$-3abx+\frac{(a^2+2b^2)\cos(c+dx)}{d}-\frac{b^2\cos^3(c+dx)}{3d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}$$

```
[Out] -3*a*b*x+(a^2+2*b^2)*cos(d*x+c)/d-1/3*b^2*cos(d*x+c)^3/d+(a^2+b^2)*sec(d*x+
c)/d+3*a*b*tan(d*x+c)/d-a*b*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2990, 2671, 294, 327, 209, 4442, 459}

$$\frac{(a^2+2b^2)\cos(c+dx)}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{3ab\tan(c+dx)}{d}-\frac{ab\sin^2(c+dx)\tan(c+dx)}{d}-3abx-\frac{b^2\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -3*a*b*x + ((a^2 + 2*b^2)*Cos[c + d*x])/d - (b^2*Cos[c + d*x]^3)/(3*d) + ((
a^2 + b^2)*Sec[c + d*x])/d + (3*a*b*Tan[c + d*x])/d - (a*b*Sin[c + d*x]^2*T
an[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4442

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Dist[-d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = (2ab)\int \sin^2(c+dx)\tan^2(c+dx)\,dx + \int \sin(c+dx)\,($$

$$= -\frac{\text{Subst}\left(\int \frac{(1-x^2)(a^2+b^2-b^2x^2)}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{(2a}{}$$

$$= -\frac{ab\sin^2(c+dx)\tan(c+dx)}{d} - \frac{\text{Subst}\left(\int \left(-a^2\left(1+\frac{2b}{a}\right.\right.\right.}{}$$

$$= \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(a^2+b^2)\,\text{se}}{d}$$

$$= -3abx + \frac{(a^2+2b^2)\cos(c+dx)}{d} - \frac{b^2\cos^3(c+dx)}{3d} + \frac{(}{}$$

Mathematica [A]
time = 0.32, size = 104, normalized size = 1.11

$$\frac{\sec(c+dx)\left(36a^2+45b^2-24(a^2+b^2+3ab(c+dx))\cos(c+dx)+4(3a^2+5b^2)\cos(2(c+dx))-b^2\cos(4(c+dx))+54ab\sin(c+dx)+6ab\sin(3(c+dx))\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(36*a^2 + 45*b^2 - 24*(a^2 + b^2 + 3*a*b*(c + d*x))*Cos[c + d
*x] + 4*(3*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^2*Cos[4*(c + d*x)] + 54*a*b*Si
n[c + d*x] + 6*a*b*Sin[3*(c + d*x)]))/(24*d)
```

Maple [A]
time = 0.18, size = 147, normalized size = 1.56

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2ab\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b^2\left(\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+2ab\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+b^2\left(\right.}{d}$
risch	$-3abx-\frac{iab\,e^{2i(dx+c)}}{4d}+\frac{a^2e^{i(dx+c)}}{2d}+\frac{7\,e^{i(dx+c)}b^2}{8d}+\frac{a^2e^{-i(dx+c)}}{2d}+\frac{7\,e^{-i(dx+c)}b^2}{8d}+\frac{iab\,e^{-2i(dx+c)}}{4d}+\frac{4iab}{}$
norman	$\frac{-\frac{12a^2+16b^2}{3d}+3abx-\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(12a^2+16b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{6ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{10ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10ab\left(\tan^5\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+2*a*b*(sin(d
*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+
b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)
))
```

Maxima [A]
time = 0.49, size = 97, normalized size = 1.03

$$-\frac{3\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)ab+\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b^2-3\,a^2\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(3*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*
a*b + (cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*b^2 - 3*a^2*(1/cos
(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 0.97

$$-\frac{b^2 \cos(dx+c)^4 + 9\,abdx\cos(dx+c) - 3\,(a^2+2\,b^2)\cos(dx+c)^2 - 3\,a^2 - 3\,b^2 - 3\,\left(ab\cos(dx+c)^2 + 2\,ab\right)\sin(dx+c)}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/3*(b^2*cos(d*x + c)^4 + 9*a*b*d*x*cos(d*x + c) - 3*(a^2 + 2*b^2)*cos(d*x
 + c)^2 - 3*a^2 - 3*b^2 - 3*(a*b*cos(d*x + c)^2 + 2*a*b)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin(c+dx))^2 \sin^3(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)**3*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.50, size = 172, normalized size = 1.83

$$-\frac{9\,(dx+c)ab + \frac{6\,\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\,\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2-5\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(9*(d*x + c)*a*b + 6*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + b^2)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(3*a*b*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*tan(1/2*d*x
+ 1/2*c)^4 - 3*b^2*tan(1/2*d*x + 1/2*c)^4 - 6*a^2*tan(1/2*d*x + 1/2*c)^2 -
12*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2 - 5*b^2)
/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 18.49, size = 149, normalized size = 1.59

$$-3\,a\,b\,x - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(8\,a^2+\frac{32\,b^2}{3}\right) + 4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 4\,a^2 + \frac{16\,b^2}{3} + 10\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 10\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)

[Out] - 3*a*b*x - (tan(c/2 + (d*x)/2)^2*(8*a^2 + (32*b^2)/3) + 4*a^2*tan(c/2 + (d*x)/2)^4 + 4*a^2 + (16*b^2)/3 + 10*a*b*tan(c/2 + (d*x)/2)^3 + 10*a*b*tan(c/2 + (d*x)/2)^5 + 6*a*b*tan(c/2 + (d*x)/2)^7 + 6*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)

3.1451 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$-a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -a^2*x-3/2*b^2*x+2*a*b*cos(d*x+c)/d+2*a*b*sec(d*x+c)/d+a^2*tan(d*x+c)/d+3/2
*b^2*tan(d*x+c)/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2801, 3554, 8, 2670, 14, 2671, 294, 327, 209}

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx\right.$$
$$= a^2\int \tan^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx+b^2\int$$
$$= \frac{a^2\tan(c+dx)}{d}-a^2\int 1\,dx-\frac{(2ab)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx\right.}{d}$$
$$= -a^2x+\frac{a^2\tan(c+dx)}{d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(2ab)\text{Subs}}{}$$
$$= -a^2x+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{3b^2}{}$$
$$= -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+d}{d}$$

Mathematica [A]
time = 0.32, size = 77, normalized size = 0.82

$$\frac{-4(2a^2+3b^2)(c+dx)+b\sec(c+dx)(24a+8a\cos(2(c+dx))+b\sin(3(c+dx)))+(8a^2+9b^2)\tan(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

Maple [A]
time = 0.15, size = 116, normalized size = 1.23

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos\right.}{d}$
risch	$-a^2x-\frac{3b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2+2ib^2+4b\,e^{i(dx+c)}a}{d(e^{2i(dx+c)}+1)}$
norman	$\frac{\left(a^2+\frac{3b^2}{2}\right)x+\left(-a^2-\frac{3b^2}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2-\frac{3b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(a^2+\frac{3b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8ab}{d}-\frac{2(2a}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.49, size = 83, normalized size = 0.88

$$-\frac{2\left(dx+c-\tan\left(dx+c\right)\right)a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A]
time = 0.37, size = 81, normalized size = 0.86

$$-\frac{(2\,a^2+3\,b^2)dx\cos\left(dx+c\right)-4\,ab\cos\left(dx+c\right)^2-4\,ab-\left(b^2\cos\left(dx+c\right)^2+2\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.54, size = 137, normalized size = 1.46

$$-\frac{(2\,a^2+3\,b^2)(dx+c)+\frac{4\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] -1/2*((2*a^2 + 3*b^2)*(d*x + c) + 4*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) + 2*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d

Mupad [B]
time = 17.04, size = 145, normalized size = 1.54

$$\frac{(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+(4a^2+2b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+8ab}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-x\left(a^2+\frac{3b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)

[Out] (8*a*b + tan(c/2 + (d*x)/2)^3*(4*a^2 + 2*b^2) + tan(c/2 + (d*x)/2)^5*(2*a^2 + 3*b^2) + tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2) + 8*a*b*tan(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1)) - x*(a^2 + (3*b^2)/2)

3.1452 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=42

$$-2abx+\frac{2b^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}$$

```
[Out] -2*a*b*x+2*b^2*cos(d*x+c)/d+sec(d*x+c)*(a+b*sin(d*x+c))^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2940, 12, 2718}

$$\frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d}-2abx+\frac{2b^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -2*a*b*x + (2*b^2*Cos[c + d*x])/d + (Sec[c + d*x]*(a + b*Sin[c + d*x])^2)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - \int 2b(a+b\sin(c+dx$$
$$= \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - (2b)\int (a+b\sin(c+$$
$$= -2abx + \frac{\sec(c+dx)(a+b\sin(c+dx))^2}{d} - (2b^2)\int \sin$$
$$= -2abx + \frac{2b^2\cos(c+dx)}{d} + \frac{\sec(c+dx)(a+b\sin(c+dx}{d}$$

Mathematica [A]
time = 0.23, size = 66, normalized size = 1.57

$$\frac{(2a^2+3b^2+b^2\cos(2(c+dx)))\sec(c+dx) - 2(a^2+b^2+2ab(c+dx) - 2ab\tan(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
[Out] ((2*a^2 + 3*b^2 + b^2*Cos[2*(c + d*x)])*Sec[c + d*x] - 2*(a^2 + b^2 + 2*a*b
*(c + d*x) - 2*a*b*Tan[c + d*x]))/(2*d)
```

Maple [A]
time = 0.17, size = 75, normalized size = 1.79

method	result
derivativedivides	$\frac{\frac{a^2}{\cos(dx+c)}+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
default	$\frac{\frac{a^2}{\cos(dx+c)}+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$
risch	$-2abx+\frac{e^{i(dx+c)}b^2}{2d}+\frac{e^{-i(dx+c)}b^2}{2d}+\frac{4iab+2a^2e^{i(dx+c)}+2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)}$
norman	$\frac{\frac{2a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4a^2+4b^2}{d}+2abx-\frac{(6a^2+4b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^2/cos(d*x+c)+2*a*b*(tan(d*x+c)-d*x-c)+b^2*(sin(d*x+c)^4/cos(d*x+c)+(
2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.52, size = 56, normalized size = 1.33

$$-\frac{2\,(dx+c-\tan(dx+c))ab-b^2\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*x + c - tan(d*x + c))*a*b - b^2*(1/cos(d*x + c) + cos(d*x + c)) - a^
2/cos(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.40

$$-\frac{2\,abdx\cos(dx+c)-b^2\cos(dx+c)^2-2\,ab\sin(dx+c)-a^2-b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*d*x*cos(d*x + c) - b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin(c+dx))^2\sin(c+dx)\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*sin(c + d*x)*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 82, normalized size = 1.95

$$-\frac{2\left((dx+c)ab+\frac{2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+2\,b^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((d*x + c)*a*b + (2*a*b*tan(1/2*d*x + 1/2*c)^3 + a^2*tan(1/2*d*x + 1/2*c
)^2 + 2*a*b*tan(1/2*d*x + 1/2*c) + a^2 + 2*b^2)/(tan(1/2*d*x + 1/2*c)^4 - 1
))/d
```

Mupad [B]
time = 12.25, size = 81, normalized size = 1.93

$$-\frac{2\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b^2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}-2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] - (2*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2 + 4*b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3
+ 4*a*b*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^4 - 1)) - 2*a*b*x
```

3.1453 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$-\frac{a^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{(a^2+b^2)\sec(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}$$

```
[Out] -a^2*arctanh(cos(d*x+c))/d+(a^2+b^2)*sec(d*x+c)/d+2*a*b*tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 70, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2990, 3852, 8, 3280, 457, 79, 65, 212}

$$\frac{(a^2+b^2)\sec(c+dx)}{d}-\frac{a^2\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{d}+\frac{2ab\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*Sec[c + d*x])/d - (a^2*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[
c + d*x]^2]*Sec[c + d*x])/d + (2*a*b*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3280

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx = (2ab)\int \sec^2(c+dx)\,dx + \int \csc(c+dx)\sec^2(c+dx)\,(a$$

$$= -\frac{(2ab)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d} + \frac{\left(\sqrt{\cos^2(c+d\right.}}{}$$

$$= \frac{2ab\tan(c+dx)}{d} + \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\text{Subst}}{2d}$$

$$= \frac{(a^2+b^2)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} + \frac{\left(a^2\sqrt{\cos^2(c\right.}}{}$$

$$= \frac{(a^2+b^2)\sec(c+dx)}{d} + \frac{2ab\tan(c+dx)}{d} - \frac{\left(a^2\sqrt{\cos^2(c\right.}}{}$$

$$= \frac{(a^2+b^2)\sec(c+dx)}{d} - \frac{a^2\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)\sqrt{}}{d}$$

Mathematica [A]

time = 0.15, size = 58, normalized size = 1.26

$$\frac{(a^2+b^2)\sec(c+dx) + a\left(a\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + 2b\tan(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
[Out] ((a^2 + b^2)*Sec[c + d*x] + a*(a*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x
)/2]]) + 2*b*Tan[c + d*x]))/d
```

Maple [A]

time = 0.24, size = 57, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\tan(dx+c)+\frac{b^2}{\cos(dx+c)}}{d}$
default	$\frac{a^2\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+2ab\tan(dx+c)+\frac{b^2}{\cos(dx+c)}}{d}$
risch	$\frac{4iab+2a^2e^{i(dx+c)}+2b^2e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)} - \frac{a^2\ln(e^{i(dx+c)}+1)}{d} + \frac{a^2\ln(e^{i(dx+c)}-1)}{d}$
norman	$\frac{-\frac{2a^2+2b^2}{d}-\frac{(2a^2+2b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4a^2+4b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8ab\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+2*a*b*tan(d*x+c)+b^2/cos(
d*x+c))
```

Maxima [A]
time = 0.28, size = 64, normalized size = 1.39

$$\frac{a^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)+4\,ab\tan\left(dx+c\right)+\frac{2\,b^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) +
 4*a*b*tan(d*x + c) + 2*b^2/cos(d*x + c))/d
```

Fricas [A]
time = 0.52, size = 77, normalized size = 1.67

$$-\frac{a^2\cos\left(dx+c\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^2\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-4\,ab\sin\left(dx+c\right)-2\,a^2-2\,b^2}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - a^2*cos(d*x + c)*log(-
1/2*cos(d*x + c) + 1/2) - 4*a*b*sin(d*x + c) - 2*a^2 - 2*b^2)/(d*cos(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*csc(c + d*x)*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.51, size = 57, normalized size = 1.24

$$\frac{a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{2\,\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 2*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2 +
 b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 11.85, size = 62, normalized size = 1.35

$$\frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,a^2+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+2\,b^2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^2*log(tan(c/2 + (d*x)/2)))/d - (2*a^2 + 2*b^2 + 4*a*b*tan(c/2 + (d*x)/2)
)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.1454 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=59

$$-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{(a^2+b^2)\tan(c+dx)}{d}$$

```
[Out] -2*a*b*arctanh(cos(d*x+c))/d-a^2*cot(d*x+c)/d+2*a*b*sec(d*x+c)/d+(a^2+b^2)*
tan(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2990, 2702, 327, 213, 3279, 14}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*ArcTanh[Cos[c + d*x]])/d - (a^2*Cot[c + d*x])/d + (2*a*b*Sec[c + d*
x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3279

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[x^n*((a + (a + b)*ff^2*x^2)^p/(
1 + ff^2*x^2)^((m + n)/2 + p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx &= (2ab)\int \csc(c+dx)\sec^2(c+dx)\,dx + \int \csc^2(c+dx)\,\mathrm{se}\\ &= \frac{\mathrm{Subst}\left(\int \frac{a^2+(a^2+b^2)x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\mathrm{Subst}}{}\\ &= \frac{2ab\sec(c+dx)}{d} + \frac{\mathrm{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)+\frac{a^2}{x^2}\right)\,dx, x, \mathrm{t}\right.}{d}\\ &= -\frac{2ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{2ab\sec(}{d}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 102, normalized size = 1.73

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left((2a^2+b^2)\cos(2(c+dx))-b\left(b+4a\sin(c+dx)-2a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(2(c+dx))\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^2 + b^2)*Cos[2*(
c + d*x)] - b*(b + 4*a*Sin[c + d*x] - 2*a*(Log[Cos[(c + d*x)/2]] - Log[Sin[
(c + d*x)/2]])*Sin[2*(c + d*x)])))/d
```

Maple [A]
time = 0.24, size = 75, normalized size = 1.27

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+b^2\tan(dx+c)}{d}$
default	$\frac{a^2\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+2ab\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+b^2\tan(dx+c)}{d}$
risch	$\frac{2ib^2 e^{2i(dx+c)}+4ab\, e^{3i(dx+c)}-4ia^2-2ib^2-4b\, e^{i(dx+c)}a}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{2ab\ln\left(e^{i(dx+c)}+1\right)}{d}+\frac{2ab\ln\left(e^{i(dx+c)}-1\right)}{d}$
norman	$\frac{\frac{a^2}{2d}+\frac{a^2\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2\left(a^2+b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^2+b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(5a^2+4b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\left(\tan^5\left(\frac{dx}{2}+\right.\right.}{d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+2*a*b*(1/cos(d*x+c)+ln(csc(
d*x+c)-cot(d*x+c)))+b^2*tan(d*x+c))
```

Maxima [A]
time = 0.30, size = 71, normalized size = 1.20

$$\frac{ab\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-a^2\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+b^2\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (a*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - a^2
*(1/tan(d*x + c) - tan(d*x + c)) + b^2*tan(d*x + c))/d
```

Fricas [A]
time = 0.37, size = 113, normalized size = 1.92

$$-\frac{ab\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-ab\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+\left(2a^2+b^2\right)\cos\left(dx+c\right)^2-2ab\sin\left(dx+c\right)-a^2-b^2}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -(a*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*b*cos(d*x +
 c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 + b^2)*cos(d*x + c)^
2 - 2*a*b*sin(d*x + c) - a^2 - b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(59) = 118.
time = 0.50, size = 128, normalized size = 2.17

$$\frac{12\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^2*tan(1/2*d*x + 1/2*c) - (
4*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1
/2*d*x + 1/2*c)^2 + 20*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/(tan(1/2*d*x + 1/2
*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [**B**]
time = 11.87, size = 108, normalized size = 1.83

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(5\,a^2+4\,b^2\right)-a^2+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(5*a^2 + 4*b^2) - a^2 + 8*a*b*tan(c/2 + (d*x)/2))/(d*
(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^3)) + (a^2*tan(c/2 + (d*x)/2))
/(2*d) + (2*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1455 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=100

$$-\frac{(3a^2+2b^2)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{2ab\cot(c+dx)}{d}+\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}+$$

[Out] -1/2*(3*a^2+2*b^2)*arctanh(cos(d*x+c))/d-2*a*b*cot(d*x+c)/d+1/2*(3*a^2+2*b^2)*sec(d*x+c)/d-1/2*a^2*csc(d*x+c)^2*sec(d*x+c)/d+2*a*b*tan(d*x+c)/d

Rubi [A]
time = 0.17, antiderivative size = 124, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2990, 2700, 14, 3280, 457, 79, 53, 65, 212}

$$\frac{(3a^2+2b^2)\sec(c+dx)}{2d}-\frac{(3a^2+2b^2)\sqrt{\cos^2(c+dx)}\sec(c+dx)\tanh^{-1}\left(\sqrt{\cos^2(c+dx)}\right)}{2d}-\frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{2ab\tan(c+dx)}{d}-\frac{2ab\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]

[Out] (-2*a*b*Cot[c + d*x])/d + ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*d) - ((3*a^2 + 2*b^2)*ArcTanh[Sqrt[Cos[c + d*x]^2]]*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x])/(2*d) - (a^2*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (2*a*b*Tan[c + d*x])/d

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3280

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFac
tors[Sin[e + f*x], x]}, Dist[ff*(Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x])), Su
bst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Si
n[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx &= (2ab)\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \int \csc^3(c+dx \\
&= \frac{(2ab)\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2(c+\right.}{} \\
&= \frac{(2ab)\mathrm{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(\sqrt{\cos^2\right.}{} \\
&= -\frac{2ab\cot(c+dx)}{d} - \frac{a^2\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{2ab}{} \\
&= -\frac{2ab\cot(c+dx)}{d} + \frac{(3a^2+2b^2)\sec(c+dx)}{2d} - \frac{a^2\csc^2}{} \\
&= -\frac{2ab\cot(c+dx)}{d} + \frac{(3a^2+2b^2)\sec(c+dx)}{2d} - \frac{a^2\csc^2}{} \\
&= -\frac{2ab\cot(c+dx)}{d} + \frac{(3a^2+2b^2)\sec(c+dx)}{2d} - \frac{(3a^2+}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 238 vs. 2(100) = 200.
time = 0.34, size = 238, normalized size = 2.38

$$
\frac{\csc^4(c+dx)\left(2a^2+4b^2-2(3a^2+2b^2)\cos(2(c+dx))+3a^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+2b^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-(3a^2+2b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3a^2\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2b^2\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+8ab\sin(c+dx)-8ab\sin(3(c+dx))\right)}{2d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^2 + 4*b^2 - 2*(3*a^2 + 2*b^2)*Cos[2*(c + d*x)] + 3*a^2
*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 2*b^2*Cos[3*(c + d*x)]*Log[Cos[(c
 + d*x)/2]] - (3*a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] - Log[Sin
[(c + d*x)/2]]) - 3*a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 2*b^2*Cos[
3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x] - 8*a*b*Sin[3*(c +
d*x)]))/(2*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [A]
time = 0.31, size = 116, normalized size = 1.16

method	result

derivativedivides	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^2\left(\frac{1}{\cos(dx-}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+2ab\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+b^2\left(\frac{1}{\cos(dx-}\right.}{d}$
risch	$\frac{3a^2e^{5i(dx+c)}+2b^2e^{5i(dx+c)}-2e^{3i(dx+c)}a^2-4b^2e^{3i(dx+c)}+3a^2e^{i(dx+c)}+2b^2e^{i(dx+c)}-8iab\,e^{2i(dx+c)}+8iab}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}+\frac{3a^2\ln\left(e^{i(d}\right.}{2d}$
norman	$\frac{\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ab\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2}{8d}+\frac{a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(7a^2+8b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{\left(19a^2+16b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{(3}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+2*a*b*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b^2*(1/cos(d*x+c)+ln
(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.29, size = 123, normalized size = 1.23

$$\frac{a^2\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)+2b^2\left(\frac{2}{\cos(dx+c)}-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1)\right)-8ab\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^2*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 2*b^2*(2/cos(d*x + c) - log(
cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 8*a*b*(1/tan(d*x + c) - tan(d*
x + c)))/d
```

Fricas [A]
time = 0.39, size = 186, normalized size = 1.86

$$\frac{2\left(3a^2+2b^2\right)\cos(dx+c)^2-4a^2-4b^2-\left(\left(3a^2+2b^2\right)\cos(dx+c)^3-\left(3a^2+2b^2\right)\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\left(3a^2+2b^2\right)\cos(dx+c)^3-\left(3a^2+2b^2\right)\cos(dx+c)\right)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+8\left(2ab\cos(dx+c)^2-ab\right)\sin(dx+c)}{4\left(d\cos(dx+c)^3-d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/4*(2*(3*a^2 + 2*b^2)*cos(d*x + c)^2 - 4*a^2 - 4*b^2 - ((3*a^2 + 2*b^2)*co
s(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 + 2*b^2)*cos(d*x + c)^3 - (3*a^2 + 2*b^2)*cos(d*x + c))*log(-1/2*co
s(d*x + c) + 1/2) + 8*(2*a*b*cos(d*x + c)^2 - a*b)*sin(d*x + c))/(d*cos(d*x
 + c)^3 - d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**A**]
time = 0.47, size = 157, normalized size = 1.57

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\left(3\,a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{16\left(2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2+b^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(a^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + 4*(3*a^2 + 2
*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(2*a*b*tan(1/2*d*x + 1/2*c) + a^2
 + b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (18*a^2*tan(1/2*d*x + 1/2*c)^2 + 12*
b^2*tan(1/2*d*x + 1/2*c)^2 + 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x
+ 1/2*c)^2)/d
```

Mupad [**B**]
time = 11.85, size = 148, normalized size = 1.48

$$\frac{a^2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{2}+b^2\right)}{d}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{17\,a^2}{2}+8\,b^2\right)-\frac{a^2}{2}+20\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-4\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}+\frac{a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*((3*a^2)/2 + b^
2))/d + (tan(c/2 + (d*x)/2)^2*((17*a^2)/2 + 8*b^2) - a^2/2 + 20*a*b*tan(c/2
 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 4*ta
n(c/2 + (d*x)/2)^4)) + (a*b*tan(c/2 + (d*x)/2))/d
```

3.1456 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=104

$$-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{ab\csc^2(c+dx)\sec(c+dx)}{d}$$

[Out] -3*a*b*arctanh(cos(d*x+c))/d-(2*a^2+b^2)*cot(d*x+c)/d-1/3*a^2*cot(d*x+c)^3/d+3*a*b*sec(d*x+c)/d-a*b*csc(d*x+c)^2*sec(d*x+c)/d+(a^2+b^2)*tan(d*x+c)/d

Rubi [A]
time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2990, 2702, 294, 327, 213, 3279, 459}

$$\frac{(a^2+b^2)\tan(c+dx)}{d}-\frac{(2a^2+b^2)\cot(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}+\frac{3ab\sec(c+dx)}{d}-\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{ab\csc^2(c+dx)\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*ArcTanh[Cos[c + d*x]])/d - ((2*a^2 + b^2)*Cot[c + d*x])/d - (a^2*Co
t[c + d*x]^3)/(3*d) + (3*a*b*Sec[c + d*x])/d - (a*b*Csc[c + d*x]^2*Sec[c +
d*x])/d + ((a^2 + b^2)*Tan[c + d*x])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2990

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[2*a*(b/d), I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3279

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_)*((a_) + (b_.)
*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[
e + f*x], x]}, Dist[ff^(n + 1)/f, Subst[Int[x^n*((a + (a + b)*ff^2*x^2)^p/(
1 + ff^2*x^2)^((m + n)/2 + p + 1)), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{
a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^2\,dx &= (2ab)\int \csc^3(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\,\mathrm{s}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a^2+(a^2+b^2)x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab}{}\\
&= -\frac{ab\csc^2(c+dx)\sec(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b^2}{a^2}\right)\right.\right.}{}\\
&= -\frac{(2a^2+b^2)\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} + \frac{3ab\sec(c}{d}\\
&= -\frac{3ab\tanh^{-1}(\cos(c+dx))}{d} - \frac{(2a^2+b^2)\cot(c+dx)}{d} -
\end{aligned}
$$

Mathematica [A]

time = 0.72, size = 196, normalized size = 1.88

$$
\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-4(4a^2+3b^2)\cos(2(c+dx))+(8a^2+6b^2)\cos(4(c+dx))+3b\left(2b+10a\sin(c+dx)-6a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(2(c+dx))-6a\sin(3(c+dx))+3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))\right)\right)}{192d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-4*(4*a^2 + 3*b^2)*Cos[2*(c + d*x)]
 + (8*a^2 + 6*b^2)*Cos[4*(c + d*x)] + 3*b*(2*b + 10*a*Sin[c + d*x] - 6*a*(L
og[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)] - 6*a*Sin[3*
(c + d*x)] + 3*a*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] - 3*a*Log[Sin[(c +
d*x)/2]]*Sin[4*(c + d*x)])))/(192*d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A]

time = 0.30, size = 136, normalized size = 1.31

method	result
derivativedivides	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)}{2}\right.}{d}$
default	$\frac{a^2\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+2ab\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)}{2}\right.}{d}$
risch	$\frac{6ab\,\mathrm{e}^{7i(dx+c)}-4ib^2\mathrm{e}^{4i(dx+c)}-10b\,\mathrm{e}^{5i(dx+c)}a+\frac{32ia^2\mathrm{e}^{2i(dx+c)}}{3}+8ib^2\mathrm{e}^{2i(dx+c)}+10ab\,\mathrm{e}^{3i(dx+c)}-\frac{16ia^2}{3}-4ib^2-6b\,\mathrm{e}^{i(dx+c)}a}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}$
norman	$\frac{\frac{a^2}{24d}+\frac{a^2\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{5(7a^2+6b^2)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{(11a^2+6b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{(11a^2+6b^2)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(49a^2+4}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*x+c))+2*a*b*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c)))

Maxima [**A**]
time = 0.29, size = 123, normalized size = 1.18

$$\frac{3\,ab\left(\frac{2\left(3\,\cos\left(dx+c\right)^{2}-2\right)}{\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)-6\,b^{2}\left(\frac{1}{\tan\left(dx+c\right)}-\tan\left(dx+c\right)\right)-2\,a^{2}\left(\frac{6\,\tan\left(dx+c\right)^{2}+1}{\tan\left(dx+c\right)^{3}}-3\,\tan\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(3*a*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 6*b^2*(1/tan(d*x + c) - tan(d*x + c)) - 2*a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x + c)))/d

Fricas [**A**]
time = 0.39, size = 192, normalized size = 1.85

$$-\frac{4\,{\left(4\,a^{2}+3\,b^{2}\right)}\cos\left(dx+c\right)^{4}-6\,{\left(4\,a^{2}+3\,b^{2}\right)}\cos\left(dx+c\right)^{2}+9\,{\left(ab\cos\left(dx+c\right)^{3}-ab\cos\left(dx+c\right)\right)}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,{\left(ab\cos\left(dx+c\right)^{3}-ab\cos\left(dx+c\right)\right)}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+6\,a^{2}+6\,b^{2}-6\,{\left(3\,ab\cos\left(dx+c\right)^{2}-2\,ab\right)}\sin\left(dx+c\right)}{6\,{\left(d\cos\left(dx+c\right)^{3}-d\cos\left(dx+c\right)\right)}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] -1/6*(4*(4*a^2 + 3*b^2)*cos(d*x + c)^4 - 6*(4*a^2 + 3*b^2)*cos(d*x + c)^2 + 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x + c)^3 - a*b*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*a^2 + 6*b^2 - 6*(3*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**2,x)

[Out] Timed out

Giac [A]
time = 0.47, size = 204, normalized size = 1.96

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{48\,\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{132\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 + 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) + 21*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) - 48*(a^2*tan(1/2*d*x + 1/2*c) + b^2*tan(1/2*d*x + 1/2*c) + 2
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*a
^2*tan(1/2*d*x + 1/2*c)^2 + 12*b^2*tan(1/2*d*x + 1/2*c)^2 + 6*a*b*tan(1/2*d
*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 11.86, size = 194, normalized size = 1.87

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{20\,a^2}{3}+4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(23\,a^2+20\,b^2\right)+\frac{a^2}{3}-34\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7\,a^2}{8}+\frac{b^2}{2}\right)}{d}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{3\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (tan(c/2 + (d*x)/2)^2*((20*a^2)/3 + 4*b
^2) - tan(c/2 + (d*x)/2)^4*(23*a^2 + 20*b^2) + a^2/3 - 34*a*b*tan(c/2 + (d*
x)/2)^3 + 2*a*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2
+ (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*((7*a^2)/8 + b^2/2))/d + (a*b*tan(c/2
+ (d*x)/2)^2)/(4*d) + (3*a*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1457 $\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=197

$$-\frac{9}{2}a^2bx-\frac{15b^3x}{8}+\frac{a^3\cos(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}-\frac{ab^2\cos^3(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{9}{}$$

```
[Out] -9/2*a^2*b*x-15/8*b^3*x+a^3*cos(d*x+c)/d+6*a*b^2*cos(d*x+c)/d-a*b^2*cos(d*x
+c)^3/d+a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+9/2*a^2*b*tan(d*x+c)/d+15/8*b
^3*tan(d*x+c)/d-3/2*a^2*b*sin(d*x+c)^2*tan(d*x+c)/d-5/8*b^3*sin(d*x+c)^2*ta
n(d*x+c)/d-1/4*b^3*sin(d*x+c)^4*tan(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2991, 2670, 14, 2671, 294, 327, 209, 276}

$$\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{9a^2b\tan(c+dx)}{2d}-\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}a^2bx-\frac{ab^2\cos^3(c+dx)}{d}+\frac{6ab^2\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{15b^3\tan(c+dx)}{8d}-\frac{b^3\sin^4(c+dx)\tan(c+dx)}{4d}-\frac{5b^3\sin^2(c+dx)\tan(c+dx)}{8d}-\frac{15b^3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-9*a^2*b*x)/2 - (15*b^3*x)/8 + (a^3*Cos[c + d*x])/d + (6*a*b^2*Cos[c + d*x
])/d - (a*b^2*Cos[c + d*x]^3)/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d
*x])/d + (9*a^2*b*Tan[c + d*x])/(2*d) + (15*b^3*Tan[c + d*x])/(8*d) - (3*a^
2*b*Sin[c + d*x]^2*Tan[c + d*x])/(2*d) - (5*b^3*Sin[c + d*x]^2*Tan[c + d*x]
)/(8*d) - (b^3*Sin[c + d*x]^4*Tan[c + d*x])/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\sin(c+dx)\tan^2(c+dx)+3a^2b\sin^2(c+dx)\tan\right.$$

$$= a^3\int \sin(c+dx)\tan^2(c+dx)\,dx + \left(3a^2b\right)\int \sin^2(c+$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{3a^2b\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{b^3\sin^4(c+dx)\tan(}{4d}$$

$$= \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} - \frac{ab^2\cos^3(c+dx)}{d}$$

$$= -\frac{9}{2}a^2bx + \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d} - \frac{ab^2\cos}{}$$

$$= -\frac{9}{2}a^2bx - \frac{15b^3x}{8} + \frac{a^3\cos(c+dx)}{d} + \frac{6ab^2\cos(c+dx)}{d}$$

Mathematica [A]

time = 0.54, size = 147, normalized size = 0.75

$$\frac{\sec(c+dx)\left(96a^3+360ab^2-24b(12a^2+5b^2)(c+dx)\cos(c+dx)+32(a^3+5ab^2)\cos(2(c+dx))-8ab^2\cos(4(c+dx))+216a^2b\sin(c+dx)+80b^3\sin(c+dx)+24a^2b\sin(3(c+dx))+15b^3\sin(3(c+dx))-b^3\sin(5(c+dx))\right)}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]*(96*a^3 + 360*a*b^2 - 24*b*(12*a^2 + 5*b^2)*(c + d*x)*Cos[c +
 d*x] + 32*(a^3 + 5*a*b^2)*Cos[2*(c + d*x)] - 8*a*b^2*Cos[4*(c + d*x)] + 21
6*a^2*b*Sin[c + d*x] + 80*b^3*Sin[c + d*x] + 24*a^2*b*Sin[3*(c + d*x)] + 15
*b^3*Sin[3*(c + d*x)] - b^3*Sin[5*(c + d*x)]))/(64*d)
```

Maple [A]

time = 0.19, size = 214, normalized size = 1.09

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^2b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a}{}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^2b\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a}{}$
risch	$-\frac{9a^2bx}{2}-\frac{15b^3x}{8}-\frac{3ib\,e^{2i(dx+c)}a^2}{8d}-\frac{ib^3e^{2i(dx+c)}}{4d}+\frac{a^3e^{i(dx+c)}}{2d}+\frac{21\,e^{i(dx+c)}a\,b^2}{8d}+\frac{a^3e^{-i(dx+c)}}{2d}+\frac{21\,e^{-i(}}{}$

norman	$\frac{-\frac{4a^3+16a\,b^2}{d}+\frac{3b\left(12a^2+5b^2\right)x}{8}-\frac{4a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3\left(4a^3+16a\,b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(6a^3+16a\,b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3b\left(12a\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^2*b*(sin
(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c
)+3*a*b^2*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(
d*x+c))+b^3*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*si
n(d*x+c))*cos(d*x+c)-15/8*d*x-15/8*c))
```

Maxima [A]
time = 0.55, size = 164, normalized size = 0.83

$$-\frac{12\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)a^2b+8\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)ab^2+\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}-8\,\tan\left(dx+c\right)\right)b^3-8\,a^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/8*(12*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))
*a^2*b + 8*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a*b^2 + (15*d
*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
 + c)^2 + 1) - 8*tan(d*x + c))*b^3 - 8*a^3*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A]
time = 0.41, size = 135, normalized size = 0.69

$$-\frac{8\,ab^2\cos\left(dx+c\right)^4+3\,(12\,a^2b+5\,b^3)dx\cos\left(dx+c\right)-8\,a^3-24\,ab^2-8\,(a^3+6\,ab^2)\cos\left(dx+c\right)^2+\left(2\,b^3\cos\left(dx+c\right)^4-24\,a^2b-8\,b^3-3\,(4\,a^2b+3\,b^3)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/8*(8*a*b^2*cos(d*x + c)^4 + 3*(12*a^2*b + 5*b^3)*d*x*cos(d*x + c) - 8*a^
3 - 24*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 + (2*b^3*cos(d*x + c)^4 - 2
4*a^2*b - 8*b^3 - 3*(4*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 336, normalized size = 1.71

$$-\frac{3\,(12\,a^2b+5\,b^3)(dx+c)+\frac{16\,(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\,\left(12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+7\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-24\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-136\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^3-40\,ab^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(12*a^2*b + 5*b^3)*(d*x + c) + 16*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b
^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(
12*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 7*b^3*tan(1/2*d*x + 1/2*c)^7 - 8*a^3*tan(
1/2*d*x + 1/2*c)^6 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^6 + 12*a^2*b*tan(1/2*d*x
 + 1/2*c)^5 + 15*b^3*tan(1/2*d*x + 1/2*c)^5 - 24*a^3*tan(1/2*d*x + 1/2*c)^4
 - 120*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 15*
b^3*tan(1/2*d*x + 1/2*c)^3 - 24*a^3*tan(1/2*d*x + 1/2*c)^2 - 136*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 12*a^2*b*tan(1/2*d*x + 1/2*c) - 7*b^3*tan(1/2*d*x + 1/
2*c) - 8*a^3 - 40*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 16.02, size = 323, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(9\,a^2b+\frac{15\,b^3}{4}\right)+4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\,ab^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(12\,a^3+32\,ab^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12\,a^3+48\,ab^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(24\,a^2b+10\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(9\,a^2b+\frac{15\,b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(24\,a^2b+10\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(30\,a^2b+\frac{9\,b^3}{2}\right)+4\,a^3}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(12\,a^2+5\,b^2\right)}{36\,a^2b+15\,b^3}\right)\left(12\,a^2+5\,b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a^2*b + (15*b^3)/4) + 4*a^3*tan(c/2 + (d*x)/2)^6 + 1
6*a*b^2 + tan(c/2 + (d*x)/2)^4*(32*a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^2*(
48*a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^3*(24*a^2*b + 10*b^3) + tan(c/2 + (
d*x)/2)^9*(9*a^2*b + (15*b^3)/4) + tan(c/2 + (d*x)/2)^7*(24*a^2*b + 10*b^3)
 + tan(c/2 + (d*x)/2)^5*(30*a^2*b + (9*b^3)/2) + 4*a^3)/(d*(3*tan(c/2 + (d*
x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*
x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1)) - (3*b*atan((3*b*tan(c/2 + (d*x)/2)*(
12*a^2 + 5*b^2))/(36*a^2*b + 15*b^3))*(12*a^2 + 5*b^2))/(4*d)
```

3.1458 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$-a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}+\frac{a^3\tan}{}$$

```
[Out] -a^3*x-9/2*a*b^2*x+3*a^2*b*cos(d*x+c)/d+2*b^3*cos(d*x+c)/d-1/3*b^3*cos(d*x+
c)^3/d+3*a^2*b*sec(d*x+c)/d+b^3*sec(d*x+c)/d+a^3*tan(d*x+c)/d+9/2*a*b^2*tan
(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]

time = 0.13, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2801, 3554, 8, 2670, 14, 2671, 294, 327, 209, 276}

$$\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}+\frac{2b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

IGtQ[p, 0]

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2671

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+\right.$$

$$= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3a\right.$$

$$= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{(3a^2b)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx\right.}{d}$$

$$= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{(3a^2b)\operatorname{Su}}{}$$

$$= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2}{}$$

$$= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+d}{3d}$$

Mathematica [A]

time = 0.55, size = 113, normalized size = 0.77

$$\frac{b\sec(c+dx)\left(108a^2+45b^2+4(9a^2+5b^2)\cos(2(c+dx))-b^2\cos(4(c+dx))+9ab\sin(3(c+dx))\right)+3a\left(-4(2a^2+9b^2)(c+dx)+(8a^2+27b^2)\tan(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

Maple [A]

time = 0.18, size = 169, normalized size = 1.16

method	result
derivativedivides	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3ab^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)c\right.}{d}$
default	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3ab^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)c\right.}{d}$
risch	$-a^3x-\frac{9ab^2x}{2}-\frac{3iab^2e^{2i(dx+c)}}{8d}+\frac{3be^{i(dx+c)}a^2}{2d}+\frac{7b^3e^{i(dx+c)}}{8d}+\frac{3be^{-i(dx+c)}a^2}{2d}+\frac{7b^3e^{-i(dx+c)}}{8d}+\frac{3iab^2e}{}$
norman	$\frac{a(2a^2+9b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{36a^2b+16b^3}{3d}+\frac{a(2a^2+9b^2)x}{2}-\frac{2(36a^2b+16b^3)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{3a(2a^2+5b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.48, size = 119, normalized size = 0.82

$$-\frac{6\,(dx+c-\tan\left(dx+c\right))a^{3}+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)ab^{2}+2\left(\cos\left(dx+c\right)^{3}-\frac{3}{\cos(dx+c)}-6\,\cos\left(dx+c\right)\right)b^{3}-18\,a^{2}b\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A]
time = 0.36, size = 116, normalized size = 0.79

$$-\frac{2\,b^{3}\cos\left(dx+c\right)^{4}+3\,(2\,a^{3}+9\,ab^{2})dx\cos\left(dx+c\right)-18\,a^{2}b-6\,b^{3}-6\,(3\,a^{2}b+2\,b^{3})\cos\left(dx+c\right)^{2}-3\left(3\,ab^{2}\cos\left(dx+c\right)^{2}+2\,a^{3}+6\,ab^{2}\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sin\left(c+dx\right))^{3}\sin^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sin(c + d*x)**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.50, size = 207, normalized size = 1.42

$$-\frac{3\,(2\,a^3+9\,ab^2)(dx+c)+\frac{12\,\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b+b^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\,\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,a^2b-10\,b^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 9*a*b^2)*(d*x + c) + 12*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^
2*tan(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9
*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6*b^3*tan
(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 24*b^3*tan(1/2*d*x
+ 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b - 10*b^3)/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 16.66, size = 249, normalized size = 1.71

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^3+9\,a\,b^2\right)+12\,a^2\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(2\,a^3+9\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(6\,a^3+15\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(24\,a^2\,b+\frac{32\,b^3}{3}\right)+\frac{16\,b^3}{3}+12\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+9\,b^2\right)}{2\,a^3+9\,a\,b^2}\right)\,\left(2\,a^2+9\,b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(9*a*b^2 + 2*a^3) + 12*a^2*b + tan(c/2 + (d*x)/2)^7*(9*
a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^3*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/
2)^5*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*(24*a^2*b + (32*b^3)/3) + (1
6*b^3)/3 + 12*a^2*b*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((a*tan(c/2 + (d*x
)/2)*(2*a^2 + 9*b^2))/(9*a*b^2 + 2*a^3))*(2*a^2 + 9*b^2))/d
```

3.1459 $\int \sec(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}b\left(2a^2+b^2\right)x+\frac{6ab^2\cos(c+dx)}{d}+\frac{3b^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}$$

```
[Out] -3/2*b*(2*a^2+b^2)*x+6*a*b^2*cos(d*x+c)/d+3/2*b^3*cos(d*x+c)*sin(d*x+c)/d+s
ec(d*x+c)*(a+b*sin(d*x+c))^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2940, 12, 2723}

$$-\frac{3}{2}bx\left(2a^2+b^2\right)+\frac{6ab^2\cos(c+dx)}{d}+\frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d}+\frac{3b^3\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-3*b*(2*a^2 + b^2)*x)/2 + (6*a*b^2*Cos[c + d*x])/d + (3*b^3*Cos[c + d*x]*S
in[c + d*x])/(2*d) + (Sec[c + d*x]*(a + b*Sin[c + d*x])^3)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 2940

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Dist[1/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin
[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[
m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] &
& SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d} - \int 3b(a+b\sin(c+dx)$$

$$= \frac{\sec(c+dx)(a+b\sin(c+dx))^3}{d} - (3b)\int (a+b\sin(c+d$$

$$= -\frac{3}{2}b\left(2a^2+b^2\right)x + \frac{6ab^2\cos(c+dx)}{d} + \frac{3b^3\cos(c+dx)\sin}{2d}$$

Mathematica [A]
time = 0.43, size = 91, normalized size = 1.21

$$\frac{\sec(c+dx)\left(8a^3+36ab^2+12ab^2\cos(2(c+dx))+b^3\sin(3(c+dx))\right)+3b\left(-4\left(2a^2+b^2\right)(c+dx)+\left(8a^2+3b^2\right)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (Sec[c + d*x]*(8*a^3 + 36*a*b^2 + 12*a*b^2*Cos[2*(c + d*x)] + b^3*Sin[3*(c
+ d*x)]) + 3*b*(-4*(2*a^2 + b^2)*(c + d*x) + (8*a^2 + 3*b^2)*Tan[c + d*x]))
/(8*d)
```

Maple [A]
time = 0.18, size = 132, normalized size = 1.76

method	result
derivativedivides	$\frac{\frac{a^3}{\cos(dx+c)}+3a^2b(\tan(dx+c)-dx-c)+3ab^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+b^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\,\mathrm{s}}{}\right.\right.}{d}$
default	$\frac{\frac{a^3}{\cos(dx+c)}+3a^2b(\tan(dx+c)-dx-c)+3ab^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+b^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\,\mathrm{s}}{}\right.\right.}{d}$
risch	$-3a^2bx-\frac{3b^3x}{2}-\frac{ib^3e^{2i(dx+c)}}{8d}+\frac{3\,e^{i(dx+c)}ab^2}{2d}+\frac{3\,e^{-i(dx+c)}ab^2}{2d}+\frac{ib^3e^{-2i(dx+c)}}{8d}+\frac{2i\left(-ia^3e^{i(dx+c)}-3ib^2a\right.}{d\left(e^{2i(dx+\right.}}$
norman	$\frac{-\frac{2a^3+12ab^2}{d}+\frac{3b\left(2a^2+b^2\right)x}{2}-\frac{2a^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(3a^3+12ab^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a\left(a^2+2b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3b\left(2a^2+b^2\right)}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3/cos(d*x+c)+3*a^2*b*(tan(d*x+c)-d*x-c)+3*a*b^2*(sin(d*x+c)^4/cos(d*
x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^
3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]
time = 0.50, size = 99, normalized size = 1.32

$$-\frac{6\,(dx+c-\tan\left(dx+c\right))a^{2}b+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan\left(dx+c\right)\right)b^{3}-6\,ab^{2}\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)-\frac{2\,a^{3}}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*(d*x + c - tan(d*x + c))*a^2*b + (3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*b^3 - 6*a*b^2*(1/cos(d*x + c) + cos(d*x +
c)) - 2*a^3/cos(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 90, normalized size = 1.20

$$\frac{6\,ab^{2}\cos\left(dx+c\right)^{2}-3\,(2\,a^{2}b+b^{3})dx\cos\left(dx+c\right)+2\,a^{3}+6\,ab^{2}+\left(b^{3}\cos\left(dx+c\right)^{2}+6\,a^{2}b+2\,b^{3}\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c)^2 - 3*(2*a^2*b + b^3)*d*x*cos(d*x + c) + 2*a^3 +
6*a*b^2 + (b^3*cos(d*x + c)^2 + 6*a^2*b + 2*b^3)*sin(d*x + c))/(d*cos(d*x +
 c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sin\left(c+dx\right))^{3}\sin\left(c+dx\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*sin(c + d*x)*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.
time = 0.48, size = 148, normalized size = 1.97

$$-\frac{3\,(2\,a^{2}b+b^{3})(dx+c)+\frac{4\,\left(3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}+3\,ab^{2}\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\,\left(b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-6\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,ab^{2}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(2*a^2*b + b^3)*(d*x + c) + 4*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*t
an(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(b^3*
tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x +
 1/2*c) - 6*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 16.29, size = 219, normalized size = 2.92

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(6\,a^2\,b+3\,b^3\right)+2\,a^3\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+12\,a\,b^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(4\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(6\,a^2\,b+3\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(12\,a^2\,b+2\,b^3\right)+2\,a^3}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(2\,a^2+b^2\right)}{6\,a^2\,b+3\,b^3}\right)\,\left(2\,a^2+b^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*a^2*b + 3*b^3) + 2*a^3*tan(c/2 + (d*x)/2)^4 + 12*a*b
^2 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 4*a^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*
b + 3*b^3) + tan(c/2 + (d*x)/2)^3*(12*a^2*b + 2*b^3) + 2*a^3)/(d*(tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1)) - (3*b*atan
((3*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(6*a^2*b + 3*b^3))*(2*a^2 + b^2))/d
```

3.1460 $\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$-b^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3a^2b\tan(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}$$

```
[Out] -b^3*x-a^3*arctanh(cos(d*x+c))/d+a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d+3*a^
2*b*tan(d*x+c)/d+b^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2991, 3852, 8, 2702, 327, 213, 2686, 3554}

$$\frac{a^3\sec(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^2b\tan(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\tan(c+dx)}{d}-b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*x) - (a^3*ArcTanh[Cos[c + d*x]])/d + (a^3*Sec[c + d*x])/d + (3*a*b^2*
Sec[c + d*x])/d + (3*a^2*b*Tan[c + d*x])/d + (b^3*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx &= \int \left(3a^2b\sec^2(c+dx)+a^3\csc(c+dx)\sec^2(c+dx)+3ab\right. \\
&= a^3\int \csc(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \sec^2(c+dx \\
&= \frac{b^3\tan(c+dx)}{d}-b^3\int 1\,dx+\frac{a^3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\sec\right.}{d} \\
&= -b^3x+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{3a^2b\tan(c}{d} \\
&= -b^3x-\frac{a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3\sec(c+dx)}{d}+\frac{3ab^2}{}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 83, normalized size = 1.06

$$\frac{-b^3c-b^3dx-a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+a^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(a^2+3b^2)\sec(c+dx)+b(3a^2+b^2)\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
[Out] (-(b^3*c) - b^3*d*x - a^3*Log[Cos[(c + d*x)/2]] + a^3*Log[Sin[(c + d*x)/2]]
 + a*(a^2 + 3*b^2)*Sec[c + d*x] + b*(3*a^2 + b^2)*Tan[c + d*x])/d
```

Maple [A]
time = 0.29, size = 79, normalized size = 1.01

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\tan(dx+c)+\frac{3ab^2}{\cos(dx+c)}+b^3(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^3\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a^2b\tan(dx+c)+\frac{3ab^2}{\cos(dx+c)}+b^3(\tan(dx+c)-dx-c)}{d}$
risch	$-b^3x+\frac{2i\left(-ia^3\mathrm{e}^{i(dx+c)}-3ib^2a\,\mathrm{e}^{i(dx+c)}+3a^2b+b^3\right)}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}-\frac{a^3\ln\left(\mathrm{e}^{i(dx+c)}+1\right)}{d}+\frac{a^3\ln\left(\mathrm{e}^{i(dx+c)}-1\right)}{d}$
norman	$b^3x-\frac{2a^3+6a\,b^2}{d}+2b^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b^3x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2\left(a^3+3a\,b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2(3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(a^3*(1/cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+3*a^2*b*tan(d*x+c)+3*a*b^
2/cos(d*x+c)+b^3*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 86, normalized size = 1.10

$$-\frac{2\,(dx+c-\tan\,(dx+c))b^3-a^3\left(\frac{2}{\cos(dx+c)}-\log\,(\cos\,(dx+c)+1)+\log\,(\cos\,(dx+c)-1)\right)-6\,a^2b\tan\,(dx+c)-\frac{6\,ab^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*b^3 - a^3*(2/cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)) - 6*a^2*b*tan(d*x + c) - 6*a*b^2/cos(d*x +
 c))/d
```

Fricas [A]
time = 0.38, size = 99, normalized size = 1.27

$$-\frac{2\,b^3dx\cos\,(dx+c)+a^3\cos\,(dx+c)\log\left(\frac{1}{2}\,\cos\,(dx+c)+\frac{1}{2}\right)-a^3\cos\,(dx+c)\log\left(-\frac{1}{2}\,\cos\,(dx+c)+\frac{1}{2}\right)-2\,a^3-6\,ab^2-2\,(3\,a^2b+b^3)\sin\,(dx+c)}{2\,d\cos\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d*x*cos(d*x + c) + a^3*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)
 - a^3*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2) - 2*a^3 - 6*a*b^2 - 2*(3*a
^2*b + b^3)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3434 deep
```

Giac [A]
time = 0.51, size = 86, normalized size = 1.10

$$-\frac{(dx+c)b^3 - a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + \frac{2\,\left(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3+3\,ab^2\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((d*x + c)*b^3 - a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 2*(3*a^2*b*tan(1/2*d
*x + 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*
c)^2 - 1))/d
```

Mupad [B]
time = 11.93, size = 154, normalized size = 1.97

$$\frac{a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a^2\,b+2\,b^3\right)+6\,a\,b^2+2\,a^3}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}+\frac{2\,b^3\,\mathrm{atan}\left(\frac{4\,b^6}{4\,a^3\,b^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}-\frac{4\,a^3\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^3\,b^3+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^6}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)),x)
```

```
[Out] (a^3*log(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(6*a^2*b + 2*b^3) + 6
*a*b^2 + 2*a^3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)) + (2*b^3*atan((4*b^6)/(4*a^3
*b^3 + 4*b^6*tan(c/2 + (d*x)/2)) - (4*a^3*b^3*tan(c/2 + (d*x)/2))/(4*a^3*b^
3 + 4*b^6*tan(c/2 + (d*x)/2))))/d
```

3.1461 $\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=87

$$-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}$$

```
[Out] -3*a^2*b*arctanh(cos(d*x+c))/d-a^3*cot(d*x+c)/d+3*a^2*b*sec(d*x+c)/d+b^3*se
c(d*x+c)/d+a^3*tan(d*x+c)/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2991, 3852, 8, 2702, 327, 213, 2700, 14, 2686}

$$\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*ArcTanh[Cos[c + d*x]])/d - (a^3*Cot[c + d*x])/d + (3*a^2*b*Sec[c
+ d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c +
d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3ab^2\sec^2(c+dx)+3a^2b\csc(c+dx)\sec^2(c+dx)\right.$$
$$= a^3\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \left(3a^2b\right)\int \csc(c+$$
$$= \frac{a^3\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\right.}{}$$
$$= \frac{3a^2b\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)}{d} + \frac{3ab^2\tan(c+dx)}{d}$$
$$= -\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3\cot(c+dx)}{d} + \frac{3a^2b\,\mathrm{s}}{}$$

Mathematica [A]

time = 0.29, size = 114, normalized size = 1.31

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left((2a^3+3ab^2)\cos(2(c+dx))-2b(3a^2+b^2)\sin(c+dx)-3ab\left(b+a\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(2(c+dx))\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*Sec[c + d*x]*((2*a^3 + 3*a*b^2)*Cos
[2*(c + d*x)] - 2*b*(3*a^2 + b^2)*Sin[c + d*x] - 3*a*b*(b + a*(-Log[Cos[(c
+ d*x)/2]] + Log[Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/d
```

Maple [A]

time = 0.27, size = 91, normalized size = 1.05

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a\,b^2\tan(dx+c)+\frac{b^3}{\cos(dx+c)}}{d}$
default	$\frac{a^3\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a^2b\left(\frac{1}{\cos(dx+c)}+\ln(\csc(dx+c)-\cot(dx+c))\right)+3a\,b^2\tan(dx+c)+\frac{b^3}{\cos(dx+c)}}{d}$
risch	$-\frac{2i\left(3ia^2b\,e^{3i(dx+c)}+ib^3e^{3i(dx+c)}-3ia^2b\,e^{i(dx+c)}-ib^3e^{i(dx+c)}-3a\,b^2e^{2i(dx+c)}+2a^3+3a\,b^2\right)}{d\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)}-\frac{3a^2b\ln\left(e^{i(dx+c)}+1\right)}{d}$
norman	$\frac{\frac{a^3}{2d}+\frac{a^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{2\left(3a^2b+b^3\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\left(6a^2b+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2\left(9a^2b+3b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(a^2+4b^2\right.}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*a^2*b*(1/cos(d*x+c)+ln(cs
c(d*x+c)-cot(d*x+c)))+3*a*b^2*tan(d*x+c)+b^3/cos(d*x+c))
```

Maxima [A]
time = 0.30, size = 90, normalized size = 1.03

$$\frac{3\,a^2b\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-2\,a^3\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+6\,ab^2\tan\left(dx+c\right)+\frac{2\,b^3}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(3*a^2*b*(2/cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1
)) - 2*a^3*(1/tan(d*x + c) - tan(d*x + c)) + 6*a*b^2*tan(d*x + c) + 2*b^3/c
os(d*x + c))/d
```

Fricas [A]
time = 0.56, size = 131, normalized size = 1.51

$$-\frac{3\,a^2b\cos\left(dx+c\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,a^2b\cos\left(dx+c\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\,a^3-6\,ab^2+2\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2-2\left(3\,a^2b+b^3\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/2*(3*a^2*b*cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^2
*b*cos(d*x + c)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*a^3 - 6*a*b^2
 + 2*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 2*(3*a^2*b + b^3)*sin(d*x + c))/(d*
cos(d*x + c)*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.52, size = 148, normalized size = 1.70

$$\frac{6\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{2\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^3*tan(1/2*d*x + 1/2*c) - (2
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan
(1/2*d*x + 1/2*c)^2 + 10*a^2*b*tan(1/2*d*x + 1/2*c) + 4*b^3*tan(1/2*d*x + 1
/2*c) - a^3)/(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

Mupad [B]
time = 11.91, size = 120, normalized size = 1.38

$$\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(12\,a^2\,b+4\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(5\,a^3+12\,a\,b^2\right)-a^3}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^2),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2))/(2*d) + (tan(c/2 + (d*x)/2)*(12*a^2*b + 4*b^3) + t
an(c/2 + (d*x)/2)^2*(12*a*b^2 + 5*a^3) - a^3)/(d*(2*tan(c/2 + (d*x)/2) - 2*
tan(c/2 + (d*x)/2)^3)) + (3*a^2*b*log(tan(c/2 + (d*x)/2)))/d
```

3.1462 $\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=132

$$-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3a^3\sec(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}$$

```
[Out] -3/2*a^3*arctanh(cos(d*x+c))/d-3*a*b^2*arctanh(cos(d*x+c))/d-3*a^2*b*cot(d*
x+c)/d+3/2*a^3*sec(d*x+c)/d+3*a*b^2*sec(d*x+c)/d-1/2*a^3*csc(d*x+c)^2*sec(d
*x+c)/d+3*a^2*b*tan(d*x+c)/d+b^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2991, 3852, 8, 2702, 327, 213, 2700, 14, 294}

$$\frac{3a^3\sec(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3a^2b\tan(c+dx)}{d}-\frac{3a^2b\cot(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^3\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Cos[c + d*x]])/d -
(3*a^2*b*Cot[c + d*x])/d + (3*a^3*Sec[c + d*x])/(2*d) + (3*a*b^2*Sec[c + d*
x])/d - (a^3*Csc[c + d*x]^2*Sec[c + d*x])/(2*d) + (3*a^2*b*Tan[c + d*x])/d
+ (b^3*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\sec^2(c+dx)+3ab^2\csc(c+dx)\sec^2(c+dx)+3a\right.$$
$$= a^3\int \csc^3(c+dx)\sec^2(c+dx)\,dx + \left(3a^2b\right)\int \csc^2(c+$$
$$= \frac{a^3\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(c+dx)\right)}{d} + \frac{\left(3a^2b\right)\text{Subst}\left(\right.}{}$$
$$= \frac{3ab^2\sec(c+dx)}{d} - \frac{a^3\csc^2(c+dx)\sec(c+dx)}{2d} + \frac{b^3\tan}{}$$
$$= -\frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{3a^2b\cot(c+dx)}{d} + \frac{3a^3\sec}{}$$
$$= -\frac{3a^3\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3ab^2\tanh^{-1}(\cos(c+dx))}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 267 vs. $2(132)=264$.
time = 0.41, size = 267, normalized size = 2.02

$$\frac{\csc^4(c+dx)\left(2a^3+12ab^2-6(a^3+2ab^2)\cos(2(c+dx))+3a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+6ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3a(a^2+2b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3a^3\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-6ab^2\cos(3(c+dx))\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^2b\sin(c+dx)+6b^3\sin(c+dx)-12a^2b\sin(3(c+dx))-2b^3\sin(3(c+dx))\right)}{2d\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^4*(2*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*Cos[2*(c + d*x)] + 3*
a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 6*a*b^2*Cos[3*(c + d*x)]*Log[C
os[(c + d*x)/2]] - 3*a*(a^2 + 2*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2]] -
Log[Sin[(c + d*x)/2]]) - 3*a^3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] - 6*a
*b^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 12*a^2*b*Sin[c + d*x] + 6*b^3
*Sin[c + d*x] - 12*a^2*b*Sin[3*(c + d*x)] - 2*b^3*Sin[3*(c + d*x)]))/(2*d*(
Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

Maple [A]
time = 0.31, size = 130, normalized size = 0.98

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a\,b^2\left(\frac{}{\cos}\right.}{d}$
default	$\frac{a^3\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+3a^2b\left(\frac{1}{\sin(dx+c)\cos(dx+c)}-2\cot(dx+c)\right)+3a\,b^2\left(\frac{}{\cos}\right.}{d}$
risch	$\frac{i\left(-3ia^3e^{5i(dx+c)}-6ia\,b^2e^{5i(dx+c)}+2ia^3e^{3i(dx+c)}+12ia\,b^2e^{3i(dx+c)}+2b^3e^{4i(dx+c)}-3ia^3e^{i(dx+c)}-6ib^2a\,e^{i(dx+c)}-12b\,e^{2i}\right.}{d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)}$

norman	$\frac{a^3}{8d}+\frac{a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(3a^3+12a\,b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{\left(21a^3+48a\,b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(45a^3+144a\,b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c)-cot
(d*x+c)))+3*a^2*b*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+3*a*b^2*(1/cos(d*x
+c)+ln(csc(d*x+c)-cot(d*x+c)))+b^3*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 137, normalized size = 1.04

$$\frac{a^3\left(\frac{2\left(3\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)+6\,ab^2\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-12\,a^2b\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+4\,b^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/4*(a^3*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3*log(
cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*a*b^2*(2/cos(d*x + c) - lo
g(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 12*a^2*b*(1/tan(d*x + c) - t
an(d*x + c)) + 4*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 0.43, size = 196, normalized size = 1.48

$$-\frac{4\,a^3+12\,ab^2-6\,(a^3+2\,ab^2)\cos\left(dx+c\right)^2+3\,\left((a^3+2\,ab^2)\cos\left(dx+c\right)^3-(a^3+2\,ab^2)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-3\,\left((a^3+2\,ab^2)\cos\left(dx+c\right)^3-(a^3+2\,ab^2)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+4\,\left(3\,a^2b+b^3-(6\,a^2b+b^3)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/4*(4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(d*x + c)^2 + 3*((a^3 + 2*a*b
^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) - 3*((a^3 + 2*a*b^2)*cos(d*x + c)^3 - (a^3 + 2*a*b^2)*cos(d*x + c))*log
(-1/2*cos(d*x + c) + 1/2) + 4*(3*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c)
^2)*sin(d*x + c))/(d*cos(d*x + c)^3 - d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 179, normalized size = 1.36

$$\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,(a^{3}+2\,ab^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{16\,(3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}+3\,ab^{2})}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-\frac{18\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+36\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*b*tan(1/2*d*x + 1/2*c) + 12*(a^3 +
 2*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c))) - 16*(3*a^2*b*tan(1/2*d*x + 1/2*c)
 + b^3*tan(1/2*d*x + 1/2*c) + a^3 + 3*a*b^2)/(tan(1/2*d*x + 1/2*c)^2 - 1) -
 (18*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*
b*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

Mupad [B]
time = 11.91, size = 166, normalized size = 1.26

$$\frac{a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}}{8\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{3\,a^{3}}{2}+3\,a\,b^{2}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\,\left(\frac{17\,a^{3}}{2}+24\,a\,b^{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\,(30\,a^{2}\,b+8\,b^{3})-\frac{a^{3}}{2}-6\,a^{2}\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\right)}+\frac{3\,a^{2}\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^3),x)
```

```
[Out] (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) + (log(tan(c/2 + (d*x)/2))*(3*a*b^2 + (3*a
^3)/2))/d + (tan(c/2 + (d*x)/2)^2*(24*a*b^2 + (17*a^3)/2) + tan(c/2 + (d*x)
/2)^3*(30*a^2*b + 8*b^3) - a^3/2 - 6*a^2*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/
2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^4)) + (3*a^2*b*tan(c/2 + (d*x)/2))/(2
*d)
```

3.1463 $\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=164

$$-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+$$

```
[Out] -9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/d-2*a^3*cot(d*x+c)
/d-3*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d+9/2*a^2*b*sec(d*x+c)/d+b^3*s
ec(d*x+c)/d-3/2*a^2*b*csc(d*x+c)^2*sec(d*x+c)/d+a^3*tan(d*x+c)/d+3*a*b^2*ta
n(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2991, 2702, 327, 213, 2700, 14, 294, 276}

$$\frac{a^3\tan(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{2a^3\cot(c+dx)}{d}+\frac{9a^2b\sec(c+dx)}{2d}-\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTanh[Cos[c + d*x]])/d - (2
*a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d
) + (9*a^2*b*Sec[c + d*x])/(2*d) + (b^3*Sec[c + d*x])/d - (3*a^2*b*Csc[c +
d*x]^2*Sec[c + d*x])/(2*d) + (a^3*Tan[c + d*x])/d + (3*a*b^2*Tan[c + d*x])/
d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2991

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\csc(c+dx)\sec^2(c+dx)+3ab^2\csc^2(c+dx)\sec\right.$$

$$= a^3\int \csc^4(c+dx)\sec^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^3(c+$$

$$= \frac{a^3\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx,x,\tan(c+dx)\right)}{d}+\frac{(3a^2b)\,\text{Subst}}{}$$

$$= \frac{b^3\sec(c+dx)}{d}-\frac{3a^2b\csc^2(c+dx)\sec(c+dx)}{2d}+\frac{a^3\text{Su}}{}$$

$$= -\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{2a^3\cot(c+dx)}{d}-\frac{3ab^2\,\text{co}}{}$$

$$= -\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}$$

Mathematica [A]
time = 0.99, size = 287, normalized size = 1.75

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-8(4a^3+9ab^2)\cos(2(c+dx))+4(4a^3+9ab^2)\cos(4(c+dx))+3b\left(12ab+6(5a^2+2b^2)\sin(c+dx)-2(9a^2+2b^2)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(2(c+dx))-18a^2\sin(3(c+dx))-4b^2\sin(3(c+dx))+9a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))+2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-9a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))-2b^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin(4(c+dx))\right)\right)}{384d\left(-1+\cot^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]

[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-8*(4*a^3 + 9*a*b^2)*Cos[2*(c + d*x
)] + 4*(4*a^3 + 9*a*b^2)*Cos[4*(c + d*x)] + 3*b*(12*a*b + 6*(5*a^2 + 2*b^2)
*Sin[c + d*x] - 2*(9*a^2 + 2*b^2)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x
)/2]])*Sin[2*(c + d*x)] - 18*a^2*Sin[3*(c + d*x)] - 4*b^2*Sin[3*(c + d*x)]
+ 9*a^2*Log[Cos[(c + d*x)/2]]*Sin[4*(c + d*x)] + 2*b^2*Log[Cos[(c + d*x)/2]
]*Sin[4*(c + d*x)] - 9*a^2*Log[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)] - 2*b^2*L
og[Sin[(c + d*x)/2]]*Sin[4*(c + d*x)])))/(384*d*(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A]
time = 0.32, size = 169, normalized size = 1.03

method	result
derivativedivides	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx}{}\right.}{d}$
default	$\frac{a^3\left(-\frac{1}{3\sin(dx+c)^3\cos(dx+c)}+\frac{4}{3\sin(dx+c)\cos(dx+c)}-\frac{8\cot(dx+c)}{3}\right)+3a^2b\left(-\frac{1}{2\sin(dx+c)^2\cos(dx+c)}+\frac{3}{2\cos(dx+c)}+\frac{3\ln(\csc(dx}{}\right.}{d}$
risch	$\frac{27a^2b\,e^{7i(dx+c)}+6b^3e^{7i(dx+c)}-36ib^2a\,e^{4i(dx+c)}-45a^2b\,e^{5i(dx+c)}-18b^3e^{5i(dx+c)}+32ia^3e^{2i(dx+c)}+72ib^2a\,e^{2i(dx+c)}+45}{3d\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+c)}+1\right)}$

norman	$\frac{a^3}{24d}+\frac{a^3\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{\left(9a^2b+4b^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{\left(63a^2b+16b^3\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{\left(135a^2b+48b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{9a(c}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/3/sin(d*x+c)^3/cos(d*x+c)+4/3/sin(d*x+c)/cos(d*x+c)-8/3*cot(d*
x+c))+3*a^2*b*(-1/2/sin(d*x+c)^2/cos(d*x+c)+3/2/cos(d*x+c)+3/2*ln(csc(d*x+c
)-cot(d*x+c)))+3*a*b^2*(1/sin(d*x+c)/cos(d*x+c)-2*cot(d*x+c))+b^3*(1/cos(d*
x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

Maxima [A]
time = 0.28, size = 162, normalized size = 0.99

$$\frac{9\,a^2b\left(\frac{2\left(3\,\cos(dx+c)^2-2\right)}{\cos(dx+c)^3-\cos(dx+c)}-3\,\log\left(\cos\left(dx+c\right)+1\right)+3\,\log\left(\cos\left(dx+c\right)-1\right)\right)+6\,b^3\left(\frac{2}{\cos(dx+c)}-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)-36\,ab^2\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)-4\,a^3\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan\left(dx+c\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/12*(9*a^2*b*(2*(3*cos(d*x + c)^2 - 2)/(cos(d*x + c)^3 - cos(d*x + c)) - 3
*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) + 6*b^3*(2/cos(d*x + c) -
 log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) - 36*a*b^2*(1/tan(d*x + c)
- tan(d*x + c)) - 4*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3*tan(d*x
+ c)))/d
```

Fricas [A]
time = 0.38, size = 252, normalized size = 1.54

$$-\frac{8\,\left(4\,a^3+9\,ab^2\right)\cos\left(dx+c\right)^4+12\,a^3+36\,ab^2-12\,\left(4\,a^3+9\,ab^2\right)\cos\left(dx+c\right)^2+3\,\left(\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^3-\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-3\,\left(\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^3-\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+6\,\left(6\,a^2b+2\,b^3-\left(9\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,\left(d\cos\left(dx+c\right)^3-d\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/12*(8*(4*a^3 + 9*a*b^2)*cos(d*x + c)^4 + 12*a^3 + 36*a*b^2 - 12*(4*a^3 +
 9*a*b^2)*cos(d*x + c)^2 + 3*((9*a^2*b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b +
 2*b^3)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*((9*a^2*
b + 2*b^3)*cos(d*x + c)^3 - (9*a^2*b + 2*b^3)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 6*(6*a^2*b + 2*b^3 - (9*a^2*b + 2*b^3)*cos(d*x
 + c)^2)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.70, size = 245, normalized size = 1.49

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+21\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,(9\,a^2b+2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-\frac{48\,(a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+3\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+3\,a^2b+b^3)}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1}-\frac{198\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+44\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+21\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+36\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+9\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+a^3}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 21*a^3*
tan(1/2*d*x + 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) + 12*(9*a^2*b + 2*b^3)
*log(abs(tan(1/2*d*x + 1/2*c))) - 48*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*ta
n(1/2*d*x + 1/2*c) + 3*a^2*b + b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (198*a^2
*b*tan(1/2*d*x + 1/2*c)^3 + 44*b^3*tan(1/2*d*x + 1/2*c)^3 + 21*a^3*tan(1/2*
d*x + 1/2*c)^2 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 9*a^2*b*tan(1/2*d*x + 1/
2*c) + a^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

Mupad [B]
time = 11.89, size = 218, normalized size = 1.33

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{9\,a^2\,b}{2}+b^3\right)}{d}+\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{20\,a^3}{3}+12\,a\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(23\,a^3+60\,a\,b^2)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(51\,a^2\,b+16\,b^3)+\frac{a^3}{3}+3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{7\,a^3}{8}+\frac{3\,a\,b^2}{2}\right)}{d}+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^2*sin(c + d*x)^4),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2 + b^3))/d + (a^3*tan(c/2 + (d*x)/2)^3
)/(24*d) - (tan(c/2 + (d*x)/2)^2*(12*a*b^2 + (20*a^3)/3) - tan(c/2 + (d*x)/
2)^4*(60*a*b^2 + 23*a^3) - tan(c/2 + (d*x)/2)^3*(51*a^2*b + 16*b^3) + a^3/3
 + 3*a^2*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*
x)/2)^5)) + (tan(c/2 + (d*x)/2)*((3*a*b^2)/2 + (7*a^3)/8))/d + (3*a^2*b*tan
(c/2 + (d*x)/2)^2)/(8*d)
```

3.1464 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=222

$$-\frac{x}{b^2}-\frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d}+\frac{4a^3\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(\ldots}$$

```
[Out] -x/b^2-2*a^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)
^(5/2)/d+4*a^3*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))
/b^2/(a^2-b^2)^(5/2)/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+
c)/(a-b)^2/d/(1+sin(d*x+c))-a^4*cos(d*x+c)/b/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2976, 2727, 2743, 12, 2739, 632, 210}

$$-\frac{2a^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}-\frac{a^4\cos(c+dx)}{bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{4a^3\left(a^2-2b^2\right)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}-\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(x/b^2) - (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a
^2 - b^2)^(5/2)*d) + (4*a^3*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1
 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^4*
Cos[c + d*x])/(b*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{b^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} + \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{}{b}\right.$$

$$= -\frac{x}{b^2} + \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{(2a^3(a^2-2b^2))\int \frac{1}{a+b\sin(c+dx)}}{b^2\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{}{b\,(a^2}$$

$$= -\frac{x}{b^2} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{}{b\,(a^2}$$

$$= -\frac{x}{b^2} + \frac{4a^3(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx}$$

$$= -\frac{x}{b^2} + \frac{4a^3(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx}$$

$$= -\frac{x}{b^2} - \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}d} + \frac{4a^3(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{b^2\left(a^2-b^2\right)^{5/2}d}$$

Mathematica [A]
time = 1.46, size = 236, normalized size = 1.06

$$\frac{-\frac{2ab^3+a^4(c+dx)-2a^2b^2(c+dx)+b^4(c+dx)}{(-a^2b+b^3)^2} + \frac{2a^3(a^2-4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a^2-b^2)^{5/2}} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{a^4\cos(c+dx)}{(a-b)^2b(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-((2*a*b^3 + a^4*(c + d*x) - 2*a^2*b^2*(c + d*x) + b^4*(c + d*x))/(-(a^2*b
) + b^3)^2) + (2*a^3*(a^2 - 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]/((a + b)^2*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/((a - b)^2*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) - (a^4*Cos[c + d*x])/((a - b)^2*b*(a + b)^2*(a + b*Si
n[c + d*x])))/d
```

Maple [A]
time = 0.55, size = 184, normalized size = 0.83

method	result

derivativedivides	$\frac{\frac{2a^3\left(\frac{-b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a+b)^2(a-b)^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{d}{}\right.\right.}}{d}$
default	$\frac{\frac{2a^3\left(\frac{-b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a+b)^2(a-b)^2}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{d}{}\right.\right.}}{d}$
risch	$-\frac{x}{b^2}+\frac{2i\left(3ia^2b^3e^{2i(dx+c)}-ib^5e^{2i(dx+c)}+2a\,b^4e^{3i(dx+c)}+ia^2b^3+ib^5+2a^3b^2e^{i(dx+c)}+ia^4b\,e^{2i(dx+c)}+a^5e^{3i(dx+c)}+ia\right.}{\left(e^{2i(dx+c)}+1\right)b^2\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)d\left(a^2-b^2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^3/b^2/(a+b)^2/(a-b)^2*((-b^2*tan(1/2*d*x+1/2*c)-a*b)/(a*tan(1/2*d*
x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2-4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-2/b^2*arctan(tan(1/2*d*x+1/
2*c))-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 724, normalized size = 3.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b^3 - 4*a^2*b^5 + 2*b^7 + 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*d*x*cos(d*x + c) + 2*(a^6*b - b^7)*cos(d*x + c)^2 - ((a^5*b - 4*a^3*b^3
)*cos(d*x + c)*sin(d*x + c) + (a^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(-a^2 + b
^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2
*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^5*b^2 - 2*a^3*b^4 + a*
b^6 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))
/((a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^
7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c)), -(a^4*b^3 - 2*a^2*b
^5 + b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b
- b^7)*cos(d*x + c)^2 + ((a^5*b - 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) + (a
^6 - 4*a^4*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/
(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^5*b^2 - 2*a^3*b^4 + a*b^6 - (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x*cos(d*x + c))*sin(d*x + c))/((a^6*b^3 - 3*a
^4*b^5 + 3*a^2*b^7 - b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^7*b^2 - 3*a^5*b^
4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.65, size = 264, normalized size = 1.19

$$\frac{\frac{2\left(a^5-4a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^2-2a^2b^4+b^6\right)\sqrt{a^2-b^2}}-\frac{2\left(2a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^4-2a^2b^2\right)}{\left(a^4b-2a^2b^3+b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)}-\frac{dx+c}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqr
t(a^2 - b^2)) - 2*(2*a^3*b*tan(1/2*d*x + 1/2*c)^3 + a*b^3*tan(1/2*d*x + 1/2
*c)^3 + a^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3
*tan(1/2*d*x + 1/2*c) - a^4 - 2*a^2*b^2)/((a^4*b - 2*a^2*b^3 + b^5)*(a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)) - (d*x + c)/b^2)/d
```

Mupad [B]
time = 19.77, size = 2500, normalized size = 11.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) + (2*tan(
c/2 + (d*x)/2)^2*(a^4 + 2*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) - (2*a^2*(a^2 +
 2*b^2))/(b*(a^2 - b^2)^2) - (6*a*b^2*tan(c/2 + (d*x)/2))/(a^2 - b^2)^2)/(d
*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)
/2)^3)) - (2*atan((64*a*b^23*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21
+ 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a
^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) -
(704*a^3*b^21*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19
 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440
*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (3520*a^5*b^19*
tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^
17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 45
44*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) - (9536*a^7*b^17*tan(c/2 + (d*x
)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9
*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 +
1536*a^19*b^5 - 192*a^21*b^3) + (14464*a^9*b^15*tan(c/2 + (d*x)/2))/(64*a*b
^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072
*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5
 - 192*a^21*b^3) - (11072*a^11*b^13*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^
3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 +
 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*
b^3) + (1024*a^13*b^11*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520
*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^
11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (5440*
a^15*b^9*tan(c/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 95
36*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15
*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) - (4544*a^17*b^7*tan(c
/2 + (d*x)/2))/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 +
14464*a^9*b^15 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^
17*b^7 + 1536*a^19*b^5 - 192*a^21*b^3) + (1536*a^19*b^5*tan(c/2 + (d*x)/2))
/(64*a*b^23 - 704*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15
 - 11072*a^11*b^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*
a^19*b^5 - 192*a^21*b^3) - (192*a^21*b^3*tan(c/2 + (d*x)/2))/(64*a*b^23 - 7
04*a^3*b^21 + 3520*a^5*b^19 - 9536*a^7*b^17 + 14464*a^9*b^15 - 11072*a^11*b
^13 + 1024*a^13*b^11 + 5440*a^15*b^9 - 4544*a^17*b^7 + 1536*a^19*b^5 - 192*
a^21*b^3)))/(b^2*d) + (a^3*atan(((a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a -
b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23 + 3200*a^5*b^21 -
```

9632*a^7*b^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*a^13*b^13 - 27680
*a^15*b^11 + 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5 - 64*a^23*b^3) +
32*a^2*b^24 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^18 + 6720*a^10*b^1
6 - 8064*a^12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 1440*a^18*b^8 - 320*
a^20*b^6 + 32*a^22*b^4 + (a^3*(a - 2*b)*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1
/2)*(tan(c/2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 7680*a^8*b^20 - 161
28*a^10*b^18 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a^16*b^12 - 3840*a
^18*b^10 + 768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a^3*b^25 - 1632*a^
5*b^23 + 4224*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 - 4032*a^13*b^15 +
1152*a^15*b^13 + 96*a^17*b^11 - 160*a^19*b^9 + 32*a^21*b^7 + (a^3*(a - 2*b)
*(a + 2*b)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(96*a*b^29 - 10
24*a^3*b^27 + 4960*a^5*b^25 - 14400*a^7*b^23 + 27840*a^9*b^21 - 37632*a^11*
b^19 + 36288*a^13*b^17 - 24960*a^15*b^15 + 12000*a^17*b^13 - 3840*a^19*b^11
+ 736*a^21*b^9 - 64*a^23*b^7) + 32*a^2*b^28 - 320*a^4*b^26 + 1440*a^6*b^24
- 3840*a^8*b^22 + 6720*a^10*b^20 - 8064*a^12*b^18 + 6720*a^14*b^16 - 3840*
a^16*b^14 + 1440*a^18*b^12 - 320*a^20*b^10 + 32*a^22*b^8))/(b^12 - 5*a^2*b^
10 + 10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2)))/(b^12 - 5*a^2*b^10 +
10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2))*1i)/(b^12 - 5*a^2*b^10 +
10*a^4*b^8 - 10*a^6*b^6 + 5*a^8*b^4 - a^10*b^2) + (a^3*(a - 2*b)*(a + 2*b)*
(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(64*a*b^25 - 672*a^3*b^23
+ 3200*a^5*b^21 - 9632*a^7*b^19 + 20608*a^9*b^17 - 32096*a^11*b^15 + 35776*
a^13*b^13 - 27680*a^15*b^11 + 14272*a^17*b^9 - 4608*a^19*b^7 + 832*a^21*b^5
- 64*a^23*b^3) + 32*a^2*b^24 - 320*a^4*b^22 + 1440*a^6*b^20 - 3840*a^8*b^1
8 + 6720*a^10*b^16 - 8064*a^12*b^14 + 6720*a^14*b^12 - 3840*a^16*b^10 + 144
0*a^18*b^8 - 320*a^20*b^6 + 32*a^22*b^4 - (a^3*(a - 2*b)*(a + 2*b)*(-(a + b
)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(256*a^4*b^24 - 2112*a^6*b^22 + 76
80*a^8*b^20 - 16128*a^10*b^18 + 21504*a^12*b^16 - 18816*a^14*b^14 + 10752*a
^16*b^12 - 3840*a^18*b^10 + 768*a^20*b^8 - 64*a^22*b^6) - 32*a*b^27 + 352*a
^3*b^25 - 1632*a^5*b^23 + 4224*a^7*b^21 - 6720*a^9*b^19 + 6720*a^11*b^17 -
4032*a^13*b^15 + 1152*a^15*b^13 + 96*a^17*b^11 ...

3.1465 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=212

$$\frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d}-\frac{2a^2(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}+\frac{}{2(a-b}$$

```
[Out] 2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(5/2)/d-
2*a^2*(a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-b
^2)^(5/2)/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^2/
d/(1+sin(d*x+c))+a^3*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.19, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2976, 2727, 2743, 12, 2739, 632, 210}

$$-\frac{2a^2(a^2-3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{2a^4\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{5/2}}+\frac{a^3\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(5/
2)*d) - (2*a^2*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(b*(a^2 - b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x]
)) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (a^3*Cos[c + d*x])/(
(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+dx))} + \frac{}{b\,(a^2 -}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} - \frac{(a^2(a^2-3b^2))\int \frac{1}{a+b\sin(c+dx)}\,dx}{b\,(a^2-b^2)^2} +$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\,\mathrm{c}}{(a^2-b^2)^2\,d}$$

$$= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} + \frac{a^3\,\mathrm{c}}{(a^2-b^2)^2\,d}$$

$$= -\frac{2a^2(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{5/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} +$$

$$= -\frac{2a^2(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{5/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} +$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{5/2}\,d} - \frac{2a^2(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{5/2}\,d} + \frac{}{2}$$

Mathematica [A]
time = 0.72, size = 162, normalized size = 0.76

$$\frac{\frac{6a^2b\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{a^3\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((6*a^2*b*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
- 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*Cos[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]
time = 0.47, size = 155, normalized size = 0.73

method	result

derivativedivides	$\frac{-\frac{4a^2\left(\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-\frac{3b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{-\frac{4a^2\left(\frac{-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-\frac{a}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}-\frac{3b\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
risch	$-\frac{2i\left(3ia^3b\,e^{2i(dx+c)}+a^4e^{3i(dx+c)}+a^2b^2e^{3i(dx+c)}+b^4e^{3i(dx+c)}+ia^3b+2ia\,b^3+a^4e^{i(dx+c)}+3a^2b^2e^{i(dx+c)}-b^4e^{i(dx+c)}\right)}{\left(e^{2i(dx+c)}+1\right)b\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)(a^2-b^2)^2d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-4*a^2/(a-b)^2/(a+b)^2*((-1/2*b*tan(1/2*d*x+1/2*c)-1/2*a)/(a*tan(1/2*d
*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)-3/2*b/(a^2-b^2)^(1/2)*arctan(1/2*(2*a
*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)
+1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.39, size = 534, normalized size = 2.52

$$\left[\frac{2a^5-4a^3b^2+2ab^4+2\left(a^5+a^3b^2-2ab^4\right)\cos(dx+c)^2-3\left(a^2b^2\cos(dx+c)\sin(dx+c)+a^3b\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2\left(a^4b-2a^2b^3+b^5\right)\sin(dx+c)}{2\left(\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)d\cos(dx+c)\sin(dx+c)+\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c)\right)}, \frac{a^5-2a^3b^2+ab^4+\left(a^5+a^3b^2-2ab^4\right)\cos(dx+c)^2-3\left(a^2b^2\cos(dx+c)\sin(dx+c)+a^3b\cos(dx+c)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-\left(a^4b-2a^2b^3+b^5\right)\sin(dx+c)}{\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)d\cos(dx+c)\sin(dx+c)+\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 2*(a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c
)^2 - 3*(a^2*b^2*cos(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 +
 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*co
s(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*a^2*b^3 + b^
5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(
d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)), (a^5 - 2*
a^3*b^2 + a*b^4 + (a^5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 - 3*(a^2*b^2*cos
(d*x + c)*sin(d*x + c) + a^3*b*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin
(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (a^4*b - 2*a^2*b^3 + b^5)*
sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x
 + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.58, size = 222, normalized size = 1.05

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2b}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2a^3-ab^2}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)(a^4-2a^2b^2+b^4)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a^2*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2*b*
tan(1/2*d*x + 1/2*c) - 2*b^3*tan(1/2*d*x + 1/2*c) - 2*a^3 - a*b^2)/((a*tan(
1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
- a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

Mupad [**B**]
time = 15.28, size = 276, normalized size = 1.30

$$\frac{\frac{2a\,(2a^2+b^2)}{(a^2-b^2)^2}-\frac{6a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{(a^2-b^2)^2}+\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+2b^2)}{(a^2-b^2)^2}-\frac{6ab^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4-2a^2b^2+b^4}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\frac{6a^2b\,\mathrm{atan}\left(\frac{\frac{a^2b\,(2a^4b-4a^2b^3+2b^5)}{2(a+b)^{5/2}(a-b)^{5/2}}+\frac{a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{a^2b}\right)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((2*a*(2*a^2 + b^2))/(a^2 - b^2)^2 - (6*a^2*b*tan(c/2 + (d*x)/2)^3)/(a^2 -
b^2)^2 + (2*b*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2))/(a^2 - b^2)^2 - (6*a*b^2*ta
n(c/2 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2)
 - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) + (6*a^2*b*atan(((a^
2*b*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/(2*(a + b)^(5/2)*(a - b)^(5/2)) + (a^3*b
*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)))
/(a^2*b)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.1466 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\dots}$$

```
[Out] -2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-4
*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+1
/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+
c))-a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.21, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2810, 2727, 2743, 12, 2739, 632, 210}

$$-\frac{4ab^2\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{2a^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{1}{2(a-b)^2(1+\sin(c+dx))}-\frac{}{(a^2-b^2)\,(}\right.\\
&= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2}-\frac{(2ab^2)\int \frac{1}{a+b\sin(c+dx)}\,dx}{(a^2-b^2)^2}-\frac{a^2\int \frac{1}{(a+b\sin(c+}}{a^2-b^2}\\
&= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c}{(a^2-b^2)^2\,d(a+}\\
&= \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c}{(a^2-b^2)^2\,d(a+}\\
&= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c}{2(a-b)^2d(1}\\
&= -\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c}{2(a-b)^2d(1}\\
&= -\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d}+\frac{\cos(c}{2(a+b)^2d(1}
\end{aligned}$$

Mathematica [A]
time = 0.63, size = 169, normalized size = 0.84

$$\frac{-\frac{2a(a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)-\frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]
time = 0.43, size = 162, normalized size = 0.81

method	result

derivativedivides	$-\frac{\frac{2a\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tan\left(\right.\right.}}{d}$
default	$-\frac{\frac{2a\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(a^2+2b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a-b)^2\left(\tan\left(\right.\right.}}{d}$
risch	$\frac{2i(3a^3e^{i(dx+c)}+4ia^2b\,e^{2i(dx+c)}-ib^3e^{2i(dx+c)}+2a\,b^2e^{3i(dx+c)}+2ia^2b+ib^3+e^{3i(dx+c)}a^3)}{(e^{2i(dx+c)}+1)(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)})(a^2-b^2)^2d}+\frac{a^3\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2-}}{b\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,(a+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a/(a-b)^2/(a+b)^2*((b^2*tan(1/2*d*x+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2*
c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2+2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*
tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-
1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 569, normalized size = 2.84

$$\left[-\frac{2a^4b-4a^2b^3+2b^5+2(2a^4b-a^2b^3-b^5)\cos(dx+c)^2+((a^3b+2ab^3)\cos(dx+c)\sin(dx+c)+(a^4+2a^2b^2)\cos(dx+c))\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2(a^5-2a^3b^2+ab^4)\sin(dx+c)}{2((a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c))},-\frac{a^4b-2a^2b^3+b^5+(2a^4b-a^2b^3-b^5)\cos(dx+c)^2-((a^3b+2ab^3)\cos(dx+c)\sin(dx+c)+(a^4+2a^2b^2)\cos(dx+c))\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-(a^5-2a^3b^2+ab^4)\sin(dx+c)}{(a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.56, size = 251, normalized size = 1.26

$$-\frac{2\left(\frac{(a^3+2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)(a^4-2a^2b^2+b^4)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

Mupad [B]
time = 15.96, size = 313, normalized size = 1.56

$$-\frac{\frac{6a^2b}{(a^2-b^2)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4ab^2-a^3)}{(a^2-b^2)^2}-\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+2b^2)}{a^4-2a^2b^2+b^4}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{(a^2-b^2)^2}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}-\frac{2a\,\mathrm{atan}\left(\frac{\frac{a(a^2+2b^2)(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+2b^2)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^3+4ab^2}\right)(a^2+2b^2)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)

[Out] - ((6*a^2*b)/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x)/2)*(4*a*b^2 - a^3))/(a^2 - b^2)^2 - (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2) - (2*a*tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) - (2*a*atan(((a*(a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2)) + (2*a^2*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(4*a*b^2 + 2*a^3))*(a^2 + 2*b^2))/(d*(a + b)^(5/2)*(a - b)^(5/2))

3.1467 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=133

$$\frac{2b(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d}-\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))}+\frac{\sec(c+dx)\,(2a^2+b^2-3ab\sin(c+d}{(a^2-b^2)^2\,d}$$

```
[Out] 2*b*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(5/2)/d-a*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))+sec(d*x+c)*(2*a^2+b^2-3*a
*b*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2943, 2945, 12, 2739, 632, 210}

$$\frac{2b(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{5/2}}+\frac{\sec(c+dx)\,(2a^2-3ab\sin(c+dx)+b^2)}{d\,(a^2-b^2)^2}-\frac{a\sec(c+dx)}{d\,(a^2-b^2)\,(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(5/2)*d) - (a*Sec[c + d*x])/((a^2 - b^2)*d*(a + b*Sin[c + d*x])) + (
Sec[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x]))/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\int \frac{\sec^2(c+dx)(b-2a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{-a^2+b^2}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d}$$

$$= -\frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec(c+dx)\left(2a^2+b^2-3ab\sin(c+dx)\right)}{(a^2-b^2)^2\,d}$$

$$= \frac{2b(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} - \frac{a\sec(c+dx)}{(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\text{se}}{}$$

Mathematica [A]

time = 0.65, size = 169, normalized size = 1.27

$$\frac{\frac{2b(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{ab^2\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
- b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a*b^2*C
os[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A]

time = 0.42, size = 164, normalized size = 1.23

method	result
derivativedivides	$\frac{\frac{4b\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(2a^2+b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{1}{(a-b)^2\left(\tan\right.}}{d}$

default	$\frac{\frac{4b\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{(2a^2+b^2)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}\right.\right.}}{d}$
risch	$\frac{4a^3e^{2i(dx+c)}-4ia^2b\,e^{3i(dx+c)}-2ib^3e^{3i(dx+c)}-8ia^2b\,e^{i(dx+c)}+2ib^3e^{i(dx+c)}+6a\,b^2+2a\,b^2e^{2i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)(a^2-b^2)^2d}-\frac{2ib\,a^2\ln\left(e^{i(dx+c)}\right.}{\sqrt{a^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4*b/(a-b)^2/(a+b)^2*((1/2*b^2*tan(1/2*d*x+1/2*c)+1/2*a*b)/(a*tan(1/2*d
*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(2*a^2+b^2)/(a^2-b^2)^(1/2)*arcta
n(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^2/(tan(1/2*d*x
+1/2*c)-1)+1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 0.41, size = 557, normalized size = 4.19

$$\left[\frac{2a^5-4a^3b^2+2ab^4+6(a^3b^2-ab^4)\cos(dx+c)^2-\left((2a^2b^2+b^4)\cos(dx+c)\sin(dx+c)+(2a^3b+ab^3)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2(a^4b-2a^2b^3+b^5)\sin(dx+c)}{2\left((a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c)\right)},\ \frac{a^5-2a^3b^2+ab^4+3(a^3b^2-ab^4)\cos(dx+c)^2-\left((2a^2b^2+b^4)\cos(dx+c)\sin(dx+c)+(2a^3b+ab^3)\cos(dx+c)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-(a^4b-2a^2b^3+b^5)\sin(dx+c)}{(a^6b-3a^4b^3+3a^2b^5-b^7)d\cos(dx+c)\sin(dx+c)+(a^7-3a^5b^2+3a^3b^4-ab^6)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 - 4*a^3*b^2 + 2*a*b^4 + 6*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((
2*a^2*b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c)
)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b
^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^4*b - 2*
a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d
```

```
*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)
), (a^5 - 2*a^3*b^2 + a*b^4 + 3*(a^3*b^2 - a*b^4)*cos(d*x + c)^2 - ((2*a^2*
b^2 + b^4)*cos(d*x + c)*sin(d*x + c) + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) -
(a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b
^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*c
os(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.57, size = 243, normalized size = 1.83

$$\frac{2\left(\frac{(2a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{2a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^3-2ab^2}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)(a^4-2a^2b^2+b^4)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*((2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan
(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
- b^2)) + (2*a^2*b*tan(1/2*d*x + 1/2*c)^3 + b^3*tan(1/2*d*x + 1/2*c)^3 - a^
3*tan(1/2*d*x + 1/2*c)^2 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*b^3*tan(1/2*d
*x + 1/2*c) - a^3 - 2*a*b^2)/((a*tan(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x +
 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*a^2*b^2 + b^4)))/d
```

Mupad [B]
time = 16.32, size = 310, normalized size = 2.33

$$\frac{\frac{6b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{(a^2-b^2)^2}+\frac{2a(a^2+2b^2)}{(a^2-b^2)^2}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4ab^2-a^3)}{a^4-2a^2b^2+b^4}-\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^2+b^2)}{(a^2-b^2)^2}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\frac{2b\,\mathrm{atan}\left(\frac{\frac{b(2a^2+b^2)(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2+b^2)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{4a^2b+2b^3}\right)(2a^2+b^2)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((6*b^3*tan(c/2 + (d*x)/2))/(a^2 - b^2)^2 + (2*a*(a^2 + 2*b^2))/(a^2 - b^2)
^2 - (2*tan(c/2 + (d*x)/2)^2*(4*a*b^2 - a^3))/(a^4 + b^4 - 2*a^2*b^2) - (2*
b*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 +
(d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) + (2*b*atan(
((b*(2*a^2 + b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/
2)) + (2*a*b*tan(c/2 + (d*x)/2)*(2*a^2 + b^2)*(a^4 + b^4 - 2*a^2*b^2))/((a
+ b)^(5/2)*(a - b)^(5/2)))/(4*a^2*b + 2*b^3))*(2*a^2 + b^2))/(d*(a + b)^(5/
2)*(a - b)^(5/2))
```

3.1468 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=229

$$\frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+}{2(a+b)^2d(1-}$$

[Out] 2*b^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+2*b^3*(3*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(5/2)/d-arctanh(cos(d*x+c))/a^2/d+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+b^4*cos(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sin(d*x+c))

Rubi [A]
time = 0.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2976, 3855, 2727, 2743, 12, 2739, 632, 210}

$$\frac{2b^3(3a^2-b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}+\frac{2b^3\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{b^4\cos(c+dx)}{ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]

[Out] (2*b^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)*d) + (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (b^4*Cos[c + d*x])/(a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{\csc(c+dx)}{a^2} - \frac{1}{2(a+b)^2(-1+\sin(c+dx))} - \frac{1}{2(a-b)^2(1+\sin(c+a$$

$$= \frac{\int \csc(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2} + \frac{b^3\int \frac{1}{(a+b\sin(c+dx)}}{a\,(a^2-b^2)}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+d}{2(a-b)^2d(1+\mathrm{si}}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} + \frac{\cos(c+d}{2(a-b)^2d(1+\mathrm{si}}$$

$$= \frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\mathrm{c}}{2(a+b)^2}$$

$$= \frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\mathrm{c}}{2(a+b)^2}$$

$$= \frac{2b^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} - \mathrm{ta}$$

Mathematica [A]
time = 1.62, size = 203, normalized size = 0.89

$$\frac{-\frac{2(-4a^2b^3+b^5)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{ab^4\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}}{a^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*(-4*a^2*b^3 + b^5)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2*(a^2 - b^2)^(5/2)) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))
+ (-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]] + (a*b^4*Cos[c + d*x])/((
a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/a^2)/d
```

Maple [A]
time = 0.70, size = 185, normalized size = 0.81

method	result

derivativedivides	$\frac{\frac{4b^3\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(4a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{\frac{4b^3\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(4a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2a^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
risch	$\frac{4a^4e^{2i(dx+c)}-2ia^3b\,e^{3i(dx+c)}-6ia^3b\,e^{i(dx+c)}-4ia\,b^3e^{3i(dx+c)}+4a^2b^2+2b^4e^{2i(dx+c)}+2b^4}{\left(e^{2i(dx+c)}+1\right)a\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)\left(a^2-b^2\right)^2d}+\frac{4ib^3\ln\left(e^{i(dx+c)}+\frac{i\left(\sqrt{a^2\right.}}{b}\right.}{\sqrt{a^2-b^2}\,(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(4*b^3/(a-b)^2/(a+b)^2/a^2*((1/2*b^2*tan(1/2*d*x+1/2*c)+1/2*a*b)/(a*tan
(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+1/2*(4*a^2-b^2)/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))+1/a^2*ln(tan(1/2
*d*x+1/2*c))-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^2/(tan(1/2*d*x+1/2*c)
+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(213) = 426$.
time = 0.90, size = 957, normalized size = 4.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*b^2 - a^3*b^4 - a*b^6)*cos(d
*x + c)^2 + ((4*a^2*b^4 - b^6)*cos(d*x + c)*sin(d*x + c) + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a
*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x +
c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)
) - ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + ((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^
4*b^5 - a^2*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4
 - a^3*b^6)*d*cos(d*x + c)), 1/2*(2*a^7 - 4*a^5*b^2 + 2*a^3*b^4 + 2*(2*a^5*
b^2 - a^3*b^4 - a*b^6)*cos(d*x + c)^2 - 2*((4*a^2*b^4 - b^6)*cos(d*x + c)*s
in(d*x + c) + (4*a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*
sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - ((a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + ((a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a
*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^6*b - 2*a^4*b^3 + a
^2*b^5)*sin(d*x + c))/((a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.61, size = 314, normalized size = 1.37

$$\frac{\frac{2\,(4\,a^2b^3-b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sqrt{a^2-b^2}}+\frac{2\left(2\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^5-a^3b^2-ab^4\right)}{(a^6-2\,a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqr
```

```
t(a^2 - b^2)) + 2*(2*a^4*b*tan(1/2*d*x + 1/2*c)^3 + b^5*tan(1/2*d*x + 1/2*c
)^3 - a^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + a*b^4
*tan(1/2*d*x + 1/2*c)^2 - 2*a^2*b^3*tan(1/2*d*x + 1/2*c) - b^5*tan(1/2*d*x
+ 1/2*c) - a^5 - a^3*b^2 - a*b^4)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d
*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a))
 + log(abs(tan(1/2*d*x + 1/2*c)))/a^2)/d
```

Mupad [B]
time = 14.86, size = 2076, normalized size = 9.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^2*d) + ((2*(a^4 + b^4 + a^2*b^2))/(a*(a^4 + b^4
- 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)*(b^5 + 2*a^2*b^3))/(a^2*(a^4 + b^4 -
2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^2*(b^4 - a^4 + 3*a^2*b^2))/(a*(a^4 + b^
4 - 2*a^2*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(2*a^4 + b^4))/(a^2*(a^4 + b^4
- 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*
b*tan(c/2 + (d*x)/2)^3)) - (b^3*atan(((b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)
^(1/2)*(2*a - b)*(tan(c/2 + (d*x)/2)*(2*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 1
60*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^12*b^4 - 16*a^14*b^2) - 2*a
^15*b - 4*a^3*b^13 + 28*a^5*b^11 - 74*a^7*b^9 + 96*a^9*b^7 - 64*a^11*b^5 +
20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^17
*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 - 156*a^8*b^10 +
 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) + 2*a^5*b^13 - 1
2*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^15*b^3))/(a^12 -
 a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))*1i)/(a^12 -
a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) - (b^3*(2*a +
b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^15*b - tan(c/2 + (d*x)/2)*(2
*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 160*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^
6 + 70*a^12*b^4 - 16*a^14*b^2) + 4*a^3*b^13 - 28*a^5*b^11 + 74*a^7*b^9 - 96
*a^9*b^7 + 64*a^11*b^5 - 20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5
)^(1/2)*(2*a - b)*(2*a^17*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*
a^6*b^12 - 156*a^8*b^10 + 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a
^16*b^2) + 2*a^5*b^13 - 12*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^
5 - 12*a^15*b^3))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 -
5*a^10*b^2))*1i)/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5
*a^10*b^2))/(4*a*b^13 - 32*a^3*b^11 + 88*a^5*b^9 - 112*a^7*b^7 + 68*a^9*b^5
 - 16*a^11*b^3 + 2*tan(c/2 + (d*x)/2)*(8*a^2*b^12 - 44*a^4*b^10 + 48*a^6*b^
8 + 4*a^8*b^6 - 16*a^10*b^4) + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*
(2*a - b)*(tan(c/2 + (d*x)/2)*(2*a^16 - 8*a^2*b^14 + 58*a^4*b^12 - 160*a^6*
b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^12*b^4 - 16*a^14*b^2) - 2*a^15*b -
 4*a^3*b^13 + 28*a^5*b^11 - 74*a^7*b^9 + 96*a^9*b^7 - 64*a^11*b^5 + 20*a^13
```

```
*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^17*b - ta
n(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 - 156*a^8*b^10 + 250*a^
10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) + 2*a^5*b^13 - 12*a^7*b
^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^15*b^3))/(a^12 - a^2*b^
10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))/(a^12 - a^2*b^10 +
 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2) + (b^3*(2*a + b)*(-(a +
b)^5*(a - b)^5)^(1/2)*(2*a - b)*(2*a^15*b - tan(c/2 + (d*x)/2)*(2*a^16 - 8*
a^2*b^14 + 58*a^4*b^12 - 160*a^6*b^10 + 222*a^8*b^8 - 168*a^10*b^6 + 70*a^1
2*b^4 - 16*a^14*b^2) + 4*a^3*b^13 - 28*a^5*b^11 + 74*a^7*b^9 - 96*a^9*b^7 +
 64*a^11*b^5 - 20*a^13*b^3 + (b^3*(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2
*a - b)*(2*a^17*b - tan(c/2 + (d*x)/2)*(6*a^18 - 8*a^4*b^14 + 54*a^6*b^12 -
 156*a^8*b^10 + 250*a^10*b^8 - 240*a^12*b^6 + 138*a^14*b^4 - 44*a^16*b^2) +
 2*a^5*b^13 - 12*a^7*b^11 + 30*a^9*b^9 - 40*a^11*b^7 + 30*a^13*b^5 - 12*a^1
5*b^3))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2
)))/(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)))*
(2*a + b)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a - b)*2i)/(d*(a^12 - a^2*b^10 +
5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2))
```

3.1469 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=248

$$-\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -2*b^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b^2)^(5/2)/d
-4*b^4*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a^
2-b^2)^(5/2)/d+2*b*arctanh(cos(d*x+c))/a^3/d-cot(d*x+c)/a^2/d+1/2*cos(d*x+c
)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))-b^5*cos(
d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.24, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2976, 3855, 3852, 8, 2727, 2743, 12, 2739, 632, 210}

$$\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2b^4\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{5/2}}-\frac{b^5\cos(c+dx)}{a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{\cot(c+dx)}{a^2d}-\frac{4b^4(2a^2-b^2)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*b^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(a^2 - b^2)^(5
/2)*d) - (4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^3*(a^2 - b^2)^(5/2)*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - Cot[
c + d*x]/(a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c
+ d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(a^2*(a^2 -
b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \left(-\frac{2b\csc(c+dx)}{a^3}+\frac{\csc^2(c+dx)}{a^2}-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{}{2(a}\right. \\
&= \frac{\int \csc^2(c+dx)\,dx}{a^2}+\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{(2b)\int \csc(c+dx)\,dx}{a^3}-\frac{\int \frac{1}{-1+\sin(c+}}{2(a+b} \\
&= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+d}{2(a-b)^2d(1+\mathrm{si}} \\
&= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c+dx)}{a^2d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{}{2(} \\
&= -\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c}{a} \\
&= -\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{\cot(c}{a} \\
&= -\frac{2b^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{5/2}d}-\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}d}+\frac{2}{}
\end{aligned}
$$

Mathematica [A]
time = 2.33, size = 254, normalized size = 1.02

$$
\frac{\frac{4b^4(-5a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{5/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{a^2}+\frac{4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}-\frac{4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{2b^5\cos(c+dx)}{a^2(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{a^2}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((4*b^4*(-5*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^3*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/a^2 + (4*b*Log[Cos[(c + d*x)/2]]
)/a^3 - (4*b*Log[Sin[(c + d*x)/2]])/a^3 + (2*Sin[(c + d*x)/2])/((a + b)^2*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])) - (2*b^5*Cos[c + d*x])/(a^2*(a - b)^2*(
a + b)^2*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/2]/a^2)/(2*d)
```

Maple [A]
time = 0.76, size = 215, normalized size = 0.87

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2b^4\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(5a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2a^3}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln}{}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{2b^4\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(5a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2a^3}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln}{}}{d}$
risch	$-\frac{2i\left(-2a^3b^2e^{3i(dx+c)}-2ib^5e^{4i(dx+c)}+2ib^5+2ab^4e^{3i(dx+c)}+3ia^2b^3e^{4i(dx+c)}-ia^2b^3-2a^3b^2e^{5i(dx+c)}+4a^5e^{i(dx+c)}-4i\right.}{\left(e^{2i(dx+c)}-1\right)\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)-2*b^4/(a-b)^2/(a+b)^2/a^3*((b^2*tan(1/2*d*x
+1/2*c)+a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(5*a^2-2*b^2
)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
-1/2/a^2/tan(1/2*d*x+1/2*c)-2/a^3*b*ln(tan(1/2*d*x+1/2*c))-1/(a+b)^2/(tan(1
/2*d*x+1/2*c)-1)-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(232) = 464$.
time = 0.90, size = 1355, normalized size = 5.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] [-1/2*(2*a^8 - 4*a^6*b^2 + 2*a^4*b^4 - 2*(2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a
^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x + c)^3 - (5*a^3*b^4 -
 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^7)*cos(d*x + c))*sqr
t(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
- b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/
(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*((a^6*b^2 - 3*a^
4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 -
a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*
cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*((a^6*b^2 - 3*a^4*b^4 + 3*a^2
*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*
x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))
*log(-1/2*cos(d*x + c) + 1/2) - 2*(a^7*b - 2*a^5*b^3 + a^3*b^5 + (2*a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^9*b - 3
*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c)^3 - (a^10 - 3*a^8*b^2 + 3*a^
6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x + c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b
^5 - a^3*b^7)*d*cos(d*x + c)), -(a^8 - 2*a^6*b^2 + a^4*b^4 - (2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c)^2 - ((5*a^2*b^5 - 2*b^7)*cos(d*x +
 c)^3 - (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)*sin(d*x + c) - (5*a^2*b^5 - 2*b^
7)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b
^2)*cos(d*x + c))) - ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^
3 - (a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^
6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + ((a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*cos(d*x + c)^3 - (a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)*sin(d*x + c) - (a^6*b^2 - 3*a^4*b
^4 + 3*a^2*b^6 - b^8)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (a^7*b -
 2*a^5*b^3 + a^3*b^5 + (2*a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7)*cos(d*x
+ c)^2)*sin(d*x + c))/((a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x
+ c)^3 - (a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)*sin(d*x +
c) - (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d*cos(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(232) = 464.

time = 0.59, size = 523, normalized size = 2.11

$$-\frac{\frac{20\,(5\,a^2b^4-2\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^7-2\,a^5b^2+a^3b^4)\sqrt{a^2-b^2}}-\frac{4\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-25\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-21\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-10\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-30\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+52\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+16\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+46\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+26\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^6-10\,a^4b^2+5\,a^2b^4}{(a^7-2\,a^5b^2+a^3b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{20\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}}{10\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(20*(5*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*sqrt(a^2 - b^2)) - (4*a^5*b*tan(1/2*d*x + 1/2*c)^5 - 8*a^3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 4*a*b^5*tan(1/2*d*x + 1/2*c)^5 - 25*a^6*tan(1/2*d*x + 1/
2*c)^4 - 2*a^4*b^2*tan(1/2*d*x + 1/2*c)^4 - 21*a^2*b^4*tan(1/2*d*x + 1/2*c)
^4 - 12*b^6*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*b*tan(1/2*d*x + 1/2*c)^3 - 20*a
^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 20*a^6*ta
n(1/2*d*x + 1/2*c)^2 + 52*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 + 16*a^2*b^4*tan(1
/2*d*x + 1/2*c)^2 + 12*b^6*tan(1/2*d*x + 1/2*c)^2 + 46*a^5*b*tan(1/2*d*x +
1/2*c) - 12*a^3*b^3*tan(1/2*d*x + 1/2*c) + 26*a*b^5*tan(1/2*d*x + 1/2*c) +
5*a^6 - 10*a^4*b^2 + 5*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x
 + 1/2*c)^5 + 2*b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a*t
an(1/2*d*x + 1/2*c))) + 20*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*tan(1/2
*d*x + 1/2*c)/a^2)/d
```

Mupad [B]

time = 13.33, size = 2151, normalized size = 8.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (a + (2*tan(c/2 + (d*x)/2)*(5*a^4*b + 3*b^5 - 2*a^2*b^3))/(a^4 + b^4 - 2*a^
2*b^2) + (4*tan(c/2 + (d*x)/2)^2*(b^6 - a^6 + 3*a^4*b^2))/(a*(a^2 - b^2)^2)
 - (2*b*tan(c/2 + (d*x)/2)^3*(a^4 + 3*b^4 + 2*a^2*b^2))/(a^4 + b^4 - 2*a^2*
b^2) - (tan(c/2 + (d*x)/2)^4*(5*a^6 + 4*b^6 + a^2*b^4 + 2*a^4*b^2))/(a*(a^4
 + b^4 - 2*a^2*b^2)))/(d*(2*a^3*tan(c/2 + (d*x)/2)^5 - 2*a^3*tan(c/2 + (d*x
)/2) - 4*a^2*b*tan(c/2 + (d*x)/2)^2 + 4*a^2*b*tan(c/2 + (d*x)/2)^4)) + tan(
c/2 + (d*x)/2)/(2*a^2*d) - (2*b*log(tan(c/2 + (d*x)/2)))/(a^3*d) + (b^4*ata
n(((b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(4
*a^18*b - 16*a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^
12*b^7 + 128*a^14*b^5 - 32*a^16*b^3) - 8*a^5*b^14 + 50*a^7*b^12 - 124*a^9*b
^10 + 156*a^11*b^8 - 104*a^13*b^6 + 34*a^15*b^4 - 4*a^17*b^2 + (b^4*(5*a^2
- 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^20*b - tan(c/2 + (d*x)/2)*(6*a^2
1 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^10 + 250*a^13*b^8 - 240*a^15*b^6
```

+ 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13 - 12*a^10*b^11 + 30*a^12*b^9 - 4
0*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*
a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*1i)/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a
^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) - (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b
)^5)^(1/2)*(8*a^5*b^14 - tan(c/2 + (d*x)/2)*(4*a^18*b - 16*a^4*b^15 + 104*a
^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^12*b^7 + 128*a^14*b^5 - 32*a^
16*b^3) - 50*a^7*b^12 + 124*a^9*b^10 - 156*a^11*b^8 + 104*a^13*b^6 - 34*a^1
5*b^4 + 4*a^17*b^2 + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a
^20*b - tan(c/2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^
10 + 250*a^13*b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13
 - 12*a^10*b^11 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(
a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))*1i)/(a
^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))/(16*a^2
*b^15 - 104*a^4*b^13 + 256*a^6*b^11 - 304*a^8*b^9 + 176*a^10*b^7 - 40*a^12*
b^5 - 2*tan(c/2 + (d*x)/2)*(20*a^5*b^12 - 8*a^3*b^14 + 24*a^7*b^10 - 76*a^9
*b^8 + 40*a^11*b^6) + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(ta
n(c/2 + (d*x)/2)*(4*a^18*b - 16*a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 37
2*a^10*b^9 - 288*a^12*b^7 + 128*a^14*b^5 - 32*a^16*b^3) - 8*a^5*b^14 + 50*a
^7*b^12 - 124*a^9*b^10 + 156*a^11*b^8 - 104*a^13*b^6 + 34*a^15*b^4 - 4*a^17
b^2 + (b^4(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(2*a^20*b - tan(c/
2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^12 - 156*a^11*b^10 + 250*a^13*
b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b^2) + 2*a^8*b^13 - 12*a^10*b^1
1 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 - 12*a^18*b^3))/(a^13 - a^3*b^1
0 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2)))/(a^13 - a^3*b^10 +
5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2) + (b^4*(5*a^2 - 2*b^2)*(-
(a + b)^5*(a - b)^5)^(1/2)*(8*a^5*b^14 - tan(c/2 + (d*x)/2)*(4*a^18*b - 16*
a^4*b^15 + 104*a^6*b^13 - 272*a^8*b^11 + 372*a^10*b^9 - 288*a^12*b^7 + 128*
a^14*b^5 - 32*a^16*b^3) - 50*a^7*b^12 + 124*a^9*b^10 - 156*a^11*b^8 + 104*a
^13*b^6 - 34*a^15*b^4 + 4*a^17*b^2 + (b^4*(5*a^2 - 2*b^2)*(-(a + b)^5*(a -
b)^5)^(1/2)*(2*a^20*b - tan(c/2 + (d*x)/2)*(6*a^21 - 8*a^7*b^14 + 54*a^9*b^
12 - 156*a^11*b^10 + 250*a^13*b^8 - 240*a^15*b^6 + 138*a^17*b^4 - 44*a^19*b
^2) + 2*a^8*b^13 - 12*a^10*b^11 + 30*a^12*b^9 - 40*a^14*b^7 + 30*a^16*b^5 -
 12*a^18*b^3))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a
^11*b^2)))/(a^13 - a^3*b^10 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*
b^2)))*(5*a^2 - 2*b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*2i)/(d*(a^13 - a^3*b^10
 + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^11*b^2))

3.1470 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d}+\frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{(a^2+3b^2)\tan$$

```
[Out] 2*b^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(5/2)/
d+2*b^5*(5*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/
(a^2-b^2)^(5/2)/d-1/2*arctanh(cos(d*x+c))/a^2/d-(a^2+3*b^2)*arctanh(cos(d*x
+c))/a^4/d+2*b*cot(d*x+c)/a^3/d-1/2*cot(d*x+c)*csc(d*x+c)/a^2/d+1/2*cos(d*x
+c)/(a+b)^2/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+b^6*co
s(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.26, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2976, 3855, 3852, 8, 3853, 2727, 2743, 12, 2739, 632, 210}

$$\frac{2b\cot(c+dx)}{a^3d}+\frac{2b^5\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{5/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^2d}+\frac{2b^5(5a^2-3b^2)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{5/2}}-\frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{b^6\cos(c+dx)}{a^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(
5/2)*d) + (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^2*d) - ((a^2
 + 3*b^2)*ArcTanh[Cos[c + d*x]])/(a^4*d) + (2*b*Cot[c + d*x])/(a^3*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*a^2*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c
 + d*x])) + Cos[c + d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) + (b^6*Cos[c +
d*x])/(a^3*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \int \left(\frac{(a^2+3b^2)\csc(c+dx)}{a^4} - \frac{2b\csc^2(c+dx)}{a^3} + \frac{\csc^3(c+dx)}{a^2} - \frac{}{2(a+b)^2}\right. \\
&= \frac{\int \csc^3(c+dx)\,dx}{a^2} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2} - \frac{(2b)\int \csc^2(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{-1+\sin(}}{2(a+} \\
&= -\frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^2d} + \frac{\mathrm{co}}{2(a+b)^2a} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3b^2)\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{2b\cot(c+dx}{a^3d} \\
&= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3}{} \\
&= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{(a^2+3}{} \\
&= \frac{2b^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}d} + \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}d} -
\end{aligned}$$

Mathematica [A]
time = 6.35, size = 356, normalized size = 1.21

$$\frac{6b^5(2a^2-b^2)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{5/2}d} + \frac{b\cot\left(\frac{1}{2}(c+dx)\right)}{a^3d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{3(a^2+2b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{3(a^2+2b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{b^6\cos(c+dx)}{a^3(a-b)^2(a+b)^2d(a+b\sin(c+dx))} - \frac{b\tan\left(\frac{1}{2}(c+dx)\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (6*b^5*(2*a^2 - b^2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^(5/2)*d) + (b*Cot[(c + d*x
```

```
)/2])/(a^3*d) - Csc[(c + d*x)/2]^2/(8*a^2*d) - (3*(a^2 + 2*b^2)*Log[Cos[(c
+ d*x)/2]])/(2*a^4*d) + (3*(a^2 + 2*b^2)*Log[Sin[(c + d*x)/2]])/(2*a^4*d) +
 Sec[(c + d*x)/2]^2/(8*a^2*d) + Sin[(c + d*x)/2]/((a + b)^2*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^2*d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(a^3*(a - b)^2*(a + b)^2*d*(a +
b*Sin[c + d*x])) - (b*Tan[(c + d*x)/2])/(a^3*d)
```

Maple [A]
time = 0.80, size = 261, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{4b^5\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{3\left(2a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^4(a+b)^2(a-b)^2}-\frac{}{8a^2}}{d}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^3}+\frac{4b^5\left(\frac{\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab}{2}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{3\left(2a^2-b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}\right)}{a^4(a+b)^2(a-b)^2}-\frac{}{8a^2}}{d}$
risch	$\frac{8a^4b^2-8a^2b^4+6b^6+10a^2b^4e^{2i(dx+c)}-6a^2b^4e^{6i(dx+c)}-16a^4b^2e^{2i(dx+c)}-8a^4b^2e^{4i(dx+c)}+12a^2b^4e^{4i(dx+c)}-3ia^5b\,e^{7i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^3*(1/2*a*tan(1/2*d*x+1/2*c)^2-4*b*tan(1/2*d*x+1/2*c))+4/a^4*b^5/
(a+b)^2/(a-b)^2*((1/2*b^2*tan(1/2*d*x+1/2*c)+1/2*a*b)/(a*tan(1/2*d*x+1/2*c)
^2+2*b*tan(1/2*d*x+1/2*c)+a)+3/2*(2*a^2-b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/8/a^2/tan(1/2*d*x+1/2*c)^2+1/
4/a^4*(6*a^2+12*b^2)*ln(tan(1/2*d*x+1/2*c))+b/a^3/tan(1/2*d*x+1/2*c)-1/(a+b
)^2/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(275) = 550$.
time = 1.45, size = 1844, normalized size = 6.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas
")

[Out] [-1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 - 4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6
 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2
*a*b^8)*cos(d*x + c)^2 - 6*((2*a^3*b^5 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5
 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 - b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b
^8)*cos(d*x + c))*sin(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c)
+ b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c
) - a^2 - b^2)) + 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(
d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)
 + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8
*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*l
og(1/2*cos(d*x + c) + 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*
a*b^8)*cos(d*x + c)^3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*c
os(d*x + c) + ((a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x +
c)^3 - (a^8*b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5
 - (5*a^8*b - 13*a^6*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x
+ c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*
a^6*b^5 - a^4*b^7)*d*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4
*b^7)*d*cos(d*x + c))*sin(d*x + c)), -1/4*(4*a^9 - 8*a^7*b^2 + 4*a^5*b^4 -
4*(4*a^7*b^2 - 8*a^5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^4 - 6*(a^9 - 5
*a^7*b^2 + 7*a^5*b^4 - 5*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^2 + 12*((2*a^3*b^5
 - a*b^7)*cos(d*x + c)^3 - (2*a^3*b^5 - a*b^7)*cos(d*x + c) + ((2*a^2*b^6 -
 b^8)*cos(d*x + c)^3 - (2*a^2*b^6 - b^8)*cos(d*x + c))*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 3*
((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^3 - (a^9 -
a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*b - a^6*b^3
 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*b^3 - 3*a^4
*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
```

```
+ 1/2) - 3*((a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c)^
3 - (a^9 - a^7*b^2 - 3*a^5*b^4 + 5*a^3*b^6 - 2*a*b^8)*cos(d*x + c) + ((a^8*
b - a^6*b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c)^3 - (a^8*b - a^6*
b^3 - 3*a^4*b^5 + 5*a^2*b^7 - 2*b^9)*cos(d*x + c))*sin(d*x + c))*log(-1/2*c
os(d*x + c) + 1/2) - 2*(2*a^8*b - 4*a^6*b^3 + 2*a^4*b^5 - (5*a^8*b - 13*a^6
*b^3 + 11*a^4*b^5 - 3*a^2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^11 - 3*a^9
*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^3 - (a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c) + ((a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d
*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c)
)*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 423, normalized size = 1.43

$$\frac{\frac{48\,(2\,a^2b^5-b^7)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{a^2-b^2}}+\frac{16\left(2\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^7-a^5b^2-ab^6\right)}{(a^8-2\,a^6b^2+a^4b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}+\frac{12\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}-\frac{18\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+36\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(48*(2*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 2*a^6*b^2 + a^4*b^4
)*sqrt(a^2 - b^2)) + 16*(2*a^6*b*tan(1/2*d*x + 1/2*c)^3 + b^7*tan(1/2*d*x +
 1/2*c)^3 - a^7*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 +
 a*b^6*tan(1/2*d*x + 1/2*c)^2 - 2*a^4*b^3*tan(1/2*d*x + 1/2*c) - b^7*tan(1/
2*d*x + 1/2*c) - a^7 - a^5*b^2 - a*b^6)/((a^8 - 2*a^6*b^2 + a^4*b^4)*(a*tan
(1/2*d*x + 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c)
 - a)) + 12*(a^2 + 2*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + (a^2*tan(1/2
*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c))/a^4 - (18*a^2*tan(1/2*d*x + 1
/2*c)^2 + 36*b^2*tan(1/2*d*x + 1/2*c)^2 - 8*a*b*tan(1/2*d*x + 1/2*c) + a^2)
/(a^4*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [**B**]
time = 14.09, size = 2302, normalized size = 7.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^2),x)

[Out] tan(c/2 + (d*x)/2)^2/(8*a^2*d) + ((tan(c/2 + (d*x)/2)^4*(17*a^6 - 32*b^6 + 33*a^2*b^4 - 66*a^4*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2)) - a^2/2 + (8*tan(c/2 + (d*x)/2)^2*(a^6 + 2*b^6 - 2*a^2*b^4 + 2*a^4*b^2))/(a^4 + b^4 - 2*a^2*b^2) + 3*a*b*tan(c/2 + (d*x)/2) - (4*tan(c/2 + (d*x)/2)^5*(5*a^6*b + 2*b^7 + a^2*b^5 - 2*a^4*b^3))/(a*(a^4 + b^4 - 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(a^6*b + 8*b^7 + a^2*b^5 + 14*a^4*b^3))/(a*(a^2 - b^2)^2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 - 4*a^4*tan(c/2 + (d*x)/2)^6 + 8*a^3*b*tan(c/2 + (d*x)/2)^3 - 8*a^3*b*tan(c/2 + (d*x)/2)^5)) - (b*tan(c/2 + (d*x)/2))/(a^3*d) + (log(tan(c/2 + (d*x)/2))*(3*a^2 + 6*b^2))/(2*a^4*d) - (b^5*atan(((b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144*a^20*b^2) - 24*a^21*b - 96*a^7*b^15 + 552*a^9*b^13 - 1248*a^11*b^11 + 1368*a^13*b^9 - 672*a^15*b^7 + 24*a^17*b^5 + 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))*3i)/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2) - (b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(24*a^21*b - tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144*a^20*b^2) + 96*a^7*b^15 - 552*a^9*b^13 + 1248*a^11*b^11 - 1368*a^13*b^9 + 672*a^15*b^7 - 24*a^17*b^5 - 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))*3i)/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2))/(2*tan(c/2 + (d*x)/2)*(144*a^4*b^16 - 576*a^6*b^14 + 864*a^8*b^12 - 864*a^10*b^10 + 720*a^12*b^8 - 288*a^14*b^6) + 288*a^3*b^17 - 1584*a^5*b^15 + 3168*a^7*b^13 - 2592*a^9*b^11 + 288*a^11*b^9 + 720*a^13*b^7 - 288*a^15*b^5 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^10*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144*a^20*b^2) - 24*a^21*b - 96*a^7*b^15 + 552*a^9*b^13 - 1248*a^11*b^11 + 1368*a^13*b^9 - 672*a^15*b^7 + 24*a^17*b^5 + 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 + 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^17*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2) + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*(24*a^2

```
1*b - tan(c/2 + (d*x)/2)*(24*a^22 - 192*a^6*b^16 + 1152*a^8*b^14 - 2760*a^1
0*b^12 + 3312*a^12*b^10 - 1944*a^14*b^8 + 288*a^16*b^6 + 264*a^18*b^4 - 144
*a^20*b^2) + 96*a^7*b^15 - 552*a^9*b^13 + 1248*a^11*b^11 - 1368*a^13*b^9 +
672*a^15*b^7 - 24*a^17*b^5 - 96*a^19*b^3 + (3*b^5*(2*a^2 - b^2)*(-(a + b)^5
*(a - b)^5)^(1/2)*(16*a^23*b - tan(c/2 + (d*x)/2)*(48*a^24 - 64*a^10*b^14 +
 432*a^12*b^12 - 1248*a^14*b^10 + 2000*a^16*b^8 - 1920*a^18*b^6 + 1104*a^20
*b^4 - 352*a^22*b^2) + 16*a^11*b^13 - 96*a^13*b^11 + 240*a^15*b^9 - 320*a^1
7*b^7 + 240*a^19*b^5 - 96*a^21*b^3))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*
b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6
 + 10*a^10*b^4 - 5*a^12*b^2)))*(2*a^2 - b^2)*(-(a + b)^5*(a - b)^5)^(1/2)*6
i)/(d*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)
)
```

3.1471 $\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=388

$$\frac{4a^4(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{a^4(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d}-\frac{2a^2(a^4-3a^2b^2+6b^4)\tan^{-1}}{b^2\left(a^2-b^2\right)^{7/}}$$

```
[Out] 4*a^4*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2
-b^2)^(7/2)/d-a^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/
2))/b^2/(a^2-b^2)^(7/2)/d-2*a^2*(a^4-3*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d
*x+1/2*c))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/
(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^4*cos(d*x+c)/b
/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2-3/2*a^5*cos(d*x+c)/b/(a^2-b^2)^3/d/(a+b*s
in(d*x+c))+2*a^3*(a^2-2*b^2)*cos(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.39, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2976, 2727, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{3a^5\cos(c+dx)}{2bd\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{a^4(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}+\frac{4a^4(a^2-2b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}-\frac{2a^2(a^4-3a^2b^2+6b^4)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{b^2d\left(a^2-b^2\right)^{7/2}}-\frac{a^4\cos(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{2a^3(a^2-2b^2)\cos(c+dx)}{bd\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (4*a^4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2
*(a^2 - b^2)^(7/2)*d) - (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2)*d) - (2*a^2*(a^4 - 3*a^2*b^2 + 6*b
^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^2*(a^2 - b^2)^(7/2
)*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a
 - b)^3*d*(1 + Sin[c + d*x])) - (a^4*Cos[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a
+ b*Sin[c + d*x])^2) - (3*a^5*Cos[c + d*x])/(2*b*(a^2 - b^2)^3*d*(a + b*Sin
[c + d*x])) + (2*a^3*(a^2 - 2*b^2)*Cos[c + d*x])/(b*(a^2 - b^2)^3*d*(a + b*
Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} + \frac{}{b^2\,(-}\right.$$

$$= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} + \frac{(2a^3(a^2-2b^2))\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{b^2\left(a^2-b^2\right)^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a}{2b\left(a^2-b^2\right.}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a}{2b\left(a^2-b^2\right.}$$

$$= -\frac{2a^2(a^4-3a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c}$$

$$= -\frac{2a^2(a^4-3a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c}$$

$$= \frac{4a^4(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d} - \frac{2a^2(a^4-3a^2b^2+6b^4)\tan^{-1}\left(\frac{b}{}\right.}{b^2\left(a^2-b^2\right)^{7/2}d}$$

$$= \frac{4a^4(a^2-2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{7/2}d} - \frac{a^4(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^2-b^2}}\right.}{b^2\left(a^2-b^2\right)^{7/2}d}$$

Mathematica [A]
time = 2.36, size = 195, normalized size = 0.50

$$\frac{-\frac{6a^2(a^2+4b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{a^3\cos(c+dx)(-7ab+(a^2-8b^2)\sin(c+dx))}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*a^2*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^3*
Cos[c + d*x]*(-7*a*b + (a^2 - 8*b^2)*Sin[c + d*x]))/((a - b)^3*(a + b)^3*(a
 + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.79, size = 222, normalized size = 0.57

method	result
derivativedivides	$-\frac{\frac{2a^2\left(\frac{\frac{a(a^2+6b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2}+\frac{7b(a^2+2b^2)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{2}-\frac{a(a^2-22b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{2}+\frac{7a^2b}{2}}{\left(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))+2b\tan(\frac{dx}{2}+\frac{c}{2})+a\right)^2}+\frac{3(a^2+4b^2)\arctan\left(\frac{2a\tan(}{2\sqrt{a}}\right.}{2\sqrt{a^2-b^2}}\right.}{(a-b)^3(a+b)^3}}{d}$
default	$-\frac{\frac{2a^2\left(\frac{\frac{a(a^2+6b^2)(\tan^3(\frac{dx}{2}+\frac{c}{2}))}{2}+\frac{7b(a^2+2b^2)(\tan^2(\frac{dx}{2}+\frac{c}{2}))}{2}-\frac{a(a^2-22b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{2}+\frac{7a^2b}{2}}{\left(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))+2b\tan(\frac{dx}{2}+\frac{c}{2})+a\right)^2}+\frac{3(a^2+4b^2)\arctan\left(\frac{2a\tan(}{2\sqrt{a}}\right.}{2\sqrt{a^2-b^2}}\right.}{(a-b)^3(a+b)^3}}{d}$
risch	$-\frac{i(2ib^7e^{i(dx+c)}-31ia^4b^3e^{i(dx+c)}-18ia^2b^5e^{i(dx+c)}-2ia^6b\,e^{5i(dx+c)}+2ie^{i(dx+c)}a^6b-4ib^7e^{3i(dx+c)}+9ia^4b^3e^{5i(dx+c)}+2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a^2/(a-b)^3/(a+b)^3*((1/2*a*(a^2+6*b^2)*tan(1/2*d*x+1/2*c)^3+7/2*b*
(a^2+2*b^2)*tan(1/2*d*x+1/2*c)^2-1/2*a*(a^2-22*b^2)*tan(1/2*d*x+1/2*c)+7/2*
a^2*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(a^2+4*b^2)/
(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1
/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.42, size = 906, normalized size = 2.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(11*a^6*b - 5*a^4*b^3 -
 8*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^
3 - 2*(a^5*b + 4*a^3*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*
a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*
a^3*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*
a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5
*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 +
 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), 1/2*(2*a^6*b -
6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (11*a^6*b - 5*a^4*b^3 - 8*a^2*b^5 + 2*b^7)*
cos(d*x + c)^2 + 3*((a^4*b^2 + 4*a^2*b^4)*cos(d*x + c)^3 - 2*(a^5*b + 4*a^3
*b^3)*cos(d*x + c)*sin(d*x + c) - (a^6 + 5*a^4*b^2 + 4*a^2*b^4)*cos(d*x + c
))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x +
c))) - (2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (a^7 - 11*a^5*b^2 + 4*a^3
*b^4 + 6*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4
*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^
5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*
a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.69, size = 351, normalized size = 0.90

$$-\frac{\frac{3\,(a^4+4\,a^2b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\,(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b-b^3)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+14\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+22\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,a^4b}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*(a^4 + 4*a^2*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*
tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2))))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(1/2*d*x
+ 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x
 + 1/2*c)^2 - 1)) + (a^5*tan(1/2*d*x + 1/2*c)^3 + 6*a^3*b^2*tan(1/2*d*x + 1
/2*c)^3 + 7*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^3*tan(1/2*d*x + 1/2*c)^
2 - a^5*tan(1/2*d*x + 1/2*c) + 22*a^3*b^2*tan(1/2*d*x + 1/2*c) + 7*a^4*b)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2
*d*x + 1/2*c) + a)^2))/d
```

Mupad [B]
time = 18.23, size = 585, normalized size = 1.51

$$\frac{\frac{13\,a^4\,b+2\,a^2\,b^3}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(a^5+6\,a^3\,b^2+8\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^4\,b+9\,a^2\,b^3+4\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-3\,a^4+40\,a^2\,b^2+8\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(a^2+4\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{9\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+4\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2+4\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{3\,a^2\,\mathrm{atan}\left(\frac{\frac{3\,a^2\,\left(a^2+4\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+4\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{3\,a^4+12\,a^2\,b^2}\right)\,\left(a^2+4\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] ((13*a^4*b + 2*a^2*b^3)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
(d*x)/2)^3*(8*a*b^4 + a^5 + 6*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
 + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 4*b^5 + 9*a^2*b^3))/(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(8*b^4 - 3*a^4 + 40*a^2*b^2))/(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (3*a^3*tan(c/2 + (d*x)/2)^5*(a^2 + 4*b^
2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (9*a^2*b*tan(c/2 + (d*x)/2)^4*(a^
2 + 4*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)
^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 +
 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2))) - (3*a^2*
atan(((3*a^2*(a^2 + 4*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/(2*(a
 + b)^(7/2)*(a - b)^(7/2)) + (3*a^3*tan(c/2 + (d*x)/2)*(a^2 + 4*b^2)*(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/(3*a^4 + 12*a
^2*b^2))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.1472 $\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=366

$$-\frac{2a^3(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d}+\frac{a^3(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d}+\frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{b+a\,\mathrm{ta}}{\sqrt{a}}\right.}{\left(a^2-b^2\right)^{7/2}d}$$

```
[Out] -2*a^3*(a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/b/(a^2-
b^2)^(7/2)/d+a^3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/b/(a^2-b^2)^(7/2)/d+2*a*b*(a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a
^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/
2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*a^3*cos(d*x+c)/(a^2-b^2)^2/d/(a+b
*sin(d*x+c))^2+3/2*a^4*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-a^2*(a^2-3
*b^2)*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.33, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2976, 2727, 2743, 2833, 12, 2739, 632, 210}

$$\frac{2ab(a^2+3b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{a^2(a^2-3b^2)\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{3a^4\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^3(2a^2+b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{7/2}}-\frac{2a^3(a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{bd\left(a^2-b^2\right)^{7/2}}+\frac{a^3\cos(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*a^3*(a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b*
(a^2 - b^2)^(7/2)*d) + (a^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/S
qrt[a^2 - b^2]])/(b*(a^2 - b^2)^(7/2)*d) + (2*a*b*(a^2 + 3*b^2)*ArcTan[(b +
 a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]
/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[
c + d*x])) + (a^3*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (3*a^4*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (a^2*(a^2
 - 3*b^2)*Cos[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rubi steps

$$\int \frac{\sin(c+dx)\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+dx))} + \frac{}{b\left(a^2-\right.}\right.$$

$$= -\frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{(a^2(a^2-3b^2))\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{b\left(a^2-b^2\right)^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} + \frac{a^3}{2\left(a^2-b^2\right)^2}$$

$$= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} + \frac{a^3}{2\left(a^2-b^2\right)^2}$$

$$= \frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{}{2}$$

$$= \frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{}{2}$$

$$= -\frac{2a^3(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d} + \frac{2ab(a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(\right.}{\sqrt{a^2-}}\right.}{\left(a^2-b^2\right)^{7/2}d}$$

$$= -\frac{2a^3(a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b\left(a^2-b^2\right)^{7/2}d} + \frac{a^3(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c\right.}{\sqrt{a^2-}}\right.}{b\left(a^2-b^2\right)^{7/2}d}$$

Mathematica [A]
time = 2.51, size = 204, normalized size = 0.56

$$\frac{\frac{6ab(3a^2+2b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{a^2\cos(c+dx)(2a^3+5ab^2+b(a^2+6b^2)\sin(c+dx))}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*Tan[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(3*a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (a^2
*Cos[c + d*x]*(2*a^3 + 5*a*b^2 + b*(a^2 + 6*b^2)*Sin[c + d*x]))/((a - b)^3*
(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.70, size = 243, normalized size = 0.66

method	result
derivativedivides	$-\frac{2a\left(\frac{-\frac{ab(3a^2+4b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(-a^4-5b^4-\frac{9}{2}a^2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{ba(5a^2+16b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-a^4-\frac{5a^2b^2}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}-\frac{3b(3a^2+2b\cdots}{\cdots}\right)}{(a-b)^3(a+b)^3}}{d}$
default	$-\frac{2a\left(\frac{-\frac{ab(3a^2+4b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\left(-a^4-5b^4-\frac{9}{2}a^2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{ba(5a^2+16b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}-a^4-\frac{5a^2b^2}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}-\frac{3b(3a^2+2b\cdots}{\cdots}\right)}{(a-b)^3(a+b)^3}}{d}$
risch	$-\frac{i(2iab^5e^{i(dx+c)}+39ia^3b^3e^{i(dx+c)}-6iab^5e^{5i(dx+c)}-9ia^3b^3e^{5i(dx+c)}+4iab^5e^{3i(dx+c)}+2a^6e^{4i(dx+c)}+21a^4b^2e^{4i(dx+c)}\cdots}{(e^{2i\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*a/(a-b)^3/(a+b)^3*((-1/2*a*b*(3*a^2+4*b^2)*tan(1/2*d*x+1/2*c)^3+(-a
^4-5*b^4-9/2*a^2*b^2)*tan(1/2*d*x+1/2*c)^2-1/2*b*a*(5*a^2+16*b^2)*tan(1/2*d
*x+1/2*c)-a^4-5/2*a^2*b^2)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a
)^2-3/2*b*(3*a^2+2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^3/(tan(1/2*
d*x+1/2*c)+1))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 0.42, size = 917, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(2*a^7 + 13*a^5*b^2 -
17*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^2 - 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c
)^3 - 2*(3*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^
3*b^3 + 2*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x
 + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) +
 b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b + 11*a
^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^
6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b
^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3
*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*
(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + (2*a^7 + 13*a^5*b^2 - 17*a^3*b^4
 + 2*a*b^6)*cos(d*x + c)^2 + 3*((3*a^3*b^3 + 2*a*b^5)*cos(d*x + c)^3 - 2*(3
*a^4*b^2 + 2*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (3*a^5*b + 5*a^3*b^3 + 2*
a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
 - b^2)*cos(d*x + c))) - (2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (a^6*b
+ 11*a^4*b^3 - 10*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2
- 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4
*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^
10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 377, normalized size = 1.03

$$\frac{\frac{3\,(3\,a^3b+2\,ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\,(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3-3\,ab^2)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{3\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,a^5+5\,a^3b^2}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(3*a^3*b + 2*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
```

```
- b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x
 + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*
x + 1/2*c)^2 - 1)) + (3*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 4*a^2*b^3*tan(1/2*d*
x + 1/2*c)^3 + 2*a^5*tan(1/2*d*x + 1/2*c)^2 + 9*a^3*b^2*tan(1/2*d*x + 1/2*c
)^2 + 10*a*b^4*tan(1/2*d*x + 1/2*c)^2 + 5*a^4*b*tan(1/2*d*x + 1/2*c) + 16*a
^2*b^3*tan(1/2*d*x + 1/2*c) + 2*a^5 + 5*a^3*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d
```

Mupad [B]

time = 17.85, size = 583, normalized size = 1.59

$$\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\frac{3\,a\,b\,\left(3\,a^2+2\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,a^2+2\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{9\,a^3\,b+6\,a\,b^3}\right)\,\left(3\,a^2+2\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{4\,a^5+11\,a^3\,b^2}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^3\,b^2+2\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a^4\,b+8\,a^2\,b^3+4\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^5+13\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,a^4+38\,a^2\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(3\,a^2+2\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-a^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+4\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (3*a*b*atan(((3*a*b*(3*a^2 + 2*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a^2*b*tan(c/2 + (d*x)/2)*(3*a^2 +
2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(6*a*b^3 + 9*a^3*b))*(3*a^2 + 2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((4
*a^5 + 11*a^3*b^2)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (9*tan(c/2 + (d*x)
/2)^4*(2*a*b^4 + 3*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c
/2 + (d*x)/2)^3*(3*a^4*b + 4*b^5 + 8*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2) + (2*tan(c/2 + (d*x)/2)^2*(13*a*b^4 + 2*a^5))/(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2) + (b*tan(c/2 + (d*x)/2)*(7*a^4 + 38*a^2*b^2))/(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2) - (3*a^2*b*tan(c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c
/2 + (d*x)/2)^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*
tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)))
```

3.1473 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{a^2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{7/2}d}$$

```
[Out] -4*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)
/d-a^2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^
2)^(7/2)/d-2*b^2*(3*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2
))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d*x+c)
/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c
))^2-3/2*a^3*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*a*b^3*cos(d*x+c)
/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.38, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2810, 2727, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{a^2(2a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2(3a^2+b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{a^2b\cos(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\frac{2ab^3\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{3a^3b\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{1}{2(a-b)^3(1+\sin(c+dx))} - \frac{\ldots}{(a^2-b^2)\,(\ldots}\right. \\
&= \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} - \frac{(2ab^2)\int \frac{1}{(a+b\sin(c+dx))^2}\,dx}{(a^2-b^2)^2} - \frac{a^2\int \frac{1}{(a+b\sin(\ldots}}{a^2-\ldots} \\
&= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(\ldots}{2\,(a^2-b^2)^2\,d(a\ldots} \\
&= \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} - \frac{a^2b\cos(\ldots}{2\,(a^2-b^2)^2\,d(a\ldots} \\
&= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\ldots}{2(a\ldots} \\
&= -\frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\ldots}{2(a\ldots} \\
&= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{2b^2(3a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{\ldots}{2(\ldots} \\
&= -\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{a^2(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{2b^2\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A]
time = 2.38, size = 212, normalized size = 0.61

$$
\frac{-\frac{2(2a^4+11a^2b^2+2b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) - \frac{ab\cos(c+dx)\left(4a^3+3ab^2+b(3a^2+4b^2)\sin(c+dx)\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.62, size = 258, normalized size = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(11a^2+10b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(4a^2+3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{\left(2a^4\right.}{}\right.}{(a-b)^3(a+b)^3}}{d}$
default	$-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(11a^2+10b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(4a^2+3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{\left(2a^4\right.}{}\right.}{(a-b)^3(a+b)^3}}{d}$
risch	$\frac{i\left(-2ib^5e^{i(dx+c)}-2ib^5e^{5i(dx+c)}-2ia^4b\,e^{5i(dx+c)}+24ia^4b\,e^{3i(dx+c)}+2ia^2b^3e^{3i(dx+c)}-11ia^2b^3e^{5i(dx+c)}+6a^5e^{4i(dx+c)}+33\right.}{\left(e^{2i(dx+c)}+1\right)\left(-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/(a-b)^3/(a+b)^3*(((5/2*a^3*b^2+a*b^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4
*a^4+11*a^2*b^2+6*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(11*a^2+10*b^2)*tan(1
/2*d*x+1/2*c)+1/2*a^2*b*(4*a^2+3*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*
d*x+1/2*c)+a)^2+1/2*(2*a^4+11*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1
)-1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 934, normalized size = 2.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.69, size = 384, normalized size = 1.10

$$-\frac{\frac{(2a^4+11a^2b^2+2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+6b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+11a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+10ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4a^4b+3a^2b^3}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

Mupad [B]
time = 18.21, size = 627, normalized size = 1.79

$$\frac{\frac{5\,\left(2\,a^4\,b+a^2\,b^3\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+a^3\,b^2+12\,a\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^4\,b+6\,a^2\,b^3+7\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^4\,b+11\,a^2\,b^3+2\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-2\,a^4+29\,a^2\,b^2+18\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-a^2-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+4\,b^2\right)+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-4\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{2\,a^4+11\,a^2\,b^2+2\,b^4}\right)\,\left(2\,a^4+11\,a^2\,b^2+2\,b^4\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] ((5*(2*a^4*b + a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 +
 (d*x)/2)^3*(12*a*b^4 + 2*a^5 + a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2) + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 7*b^5 + 6*a^2*b^3))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) - (3*tan(c/2 + (d*x)/2)^4*(2*a^4*b + 2*b^5 + 11*a^2*b^
3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(18*b^4 - 2
*a^4 + 29*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (a*tan(c/2 + (d*x
)/2)^5*(2*a^4 + 2*b^4 + 11*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(
d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c
/2 + (d*x)/2))) - (atan((((2*a^4 + 2*b^4 + 11*a^2*b^2)*(2*a^6*b - 2*b^7 + 6
*a^2*b^5 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (a*tan(c/2 + (d*x)
/2)*(2*a^4 + 2*b^4 + 11*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a +
 b)^(7/2)*(a - b)^(7/2)))/(2*a^4 + 2*b^4 + 11*a^2*b^2))*(2*a^4 + 2*b^4 + 11
*a^2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.1474 $\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=204

$$\frac{3ab(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}d}-\frac{a\sec(c+dx)}{2(a^2-b^2)d(a+b\sin(c+dx))^2}-\frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2d(a+b\sin(c+dx))}$$

```
[Out] 3*a*b*(2*a^2+3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b
^2)^(7/2)/d-1/2*a*sec(d*x+c)/(a^2-b^2)/d/(a+b*sin(d*x+c))^2-1/2*(3*a^2+2*b^
2)*sec(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+1/2*sec(d*x+c)*(3*a*(2*a^2+3*b
^2)-b*(11*a^2+4*b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2943, 2945, 12, 2739, 632, 210}

$$\frac{3ab(2a^2+3b^2)\,\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{\sec(c+dx)\left(3a(2a^2+3b^2)-b(11a^2+4b^2)\sin(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{(3a^2+2b^2)\sec(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{a\sec(c+dx)}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((
a^2 - b^2)^(7/2)*d) - (a*Sec[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x]
)^2) - ((3*a^2 + 2*b^2)*Sec[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x
])) + (Sec[c + d*x]*(3*a*(2*a^2 + 3*b^2) - b*(11*a^2 + 4*b^2)*Sin[c + d*x])
)/(2*(a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\sec(c+dx)\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{\int \frac{\sec^2(c+dx)(2b-3a\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx}{2(a^2-b^2)}$$

$$= -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\int}{}$$

$$= -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\mathrm{se}}{}$$

$$= -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\mathrm{se}}{}$$

$$= -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\mathrm{se}}{}$$

$$= -\frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2} - \frac{(3a^2+2b^2)\sec(c+dx)}{2(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\mathrm{se}}{}$$

$$= \frac{3ab(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} - \frac{a\sec(c+dx)}{2(a^2-b^2)\,d(a+b\sin(c+dx))^2}$$

Mathematica [A]
time = 2.24, size = 206, normalized size = 1.01

$$\frac{\frac{6ab(2a^2+3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right) + \frac{b^2\cos(c+dx)\left(a(6a^2+b^2)+b(5a^2+2b^2)\sin(c+dx)\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*Tan[c + d*x])/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*b*(2*a^2 + 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2])) - 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) + (b^2
*Cos[c + d*x]*(a*(6*a^2 + b^2) + b*(5*a^2 + 2*b^2)*Sin[c + d*x]))/((a - b)^
3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [A]
time = 0.62, size = 242, normalized size = 1.19

method	result

derivativedivides	$\frac{2b\left(\frac{\frac{7a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\left(6a^4+13a^2b^2+2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b^2\left(17a^2+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab\left(6a^2+b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{3a\left(2a^2+3b^2\right)\text{arcta}}{2\sqrt{(}}\right.}{(a-b)^3(a+b)^3}$ over d
default	$\frac{2b\left(\frac{\frac{7a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{b\left(6a^4+13a^2b^2+2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{b^2\left(17a^2+4b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{ab\left(6a^2+b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\frac{3a\left(2a^2+3b^2\right)\text{arcta}}{2\sqrt{(}}\right.}{(a-b)^3(a+b)^3}$ over d
risch	$\frac{-18ia^4b\,e^{4i(dx+c)}-27ia^2b^3e^{4i(dx+c)}-6a^3b^2e^{5i(dx+c)}-9a\,b^4e^{5i(dx+c)}-26ia^4b\,e^{2i(dx+c)}-4ib^5e^{2i(dx+c)}+8a^5e^{3i(dx+c)}+1(}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)^2d(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b/(a-b)^3/(a+b)^3*((7/2*a^2*b^2*tan(1/2*d*x+1/2*c)^3+1/2*b*(6*a^4+13
*a^2*b^2+2*b^4)/a*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(17*a^2+4*b^2)*tan(1/2*d*x+1
/2*c)+1/2*a*b*(6*a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a
)^2+3/2*a*(2*a^2+3*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+
2*b)/(a^2-b^2)^(1/2)))-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^3/(tan(1/2*
d*x+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(192) = 384$.
time = 0.42, size = 895, normalized size = 4.39

$$\left[-\frac{4a^7-12a^5b^2+12a^3b^4-4ab^6+2\left(16a^5b^2-17a^3b^4+ab^6\right)\cos(dx+c)^2-3\left(\left(2a^3b^3+3ab^5\right)\cos(dx+c)^3-2\left(2a^4b^2+3a^2b^4\right)\cos(dx+c)\sin(dx+c)-\left(2a^5b+5a^3b^3+3ab^5\right)\cos(dx+c)\right)\sqrt{-a^2+b^2}\log\left(-\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2-2\left(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c)\right)\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2\left(2a^6b-6a^4b^3+6a^2b^5-2b^7-\left(11a^4b^3-7a^2b^5-4b^7\right)\cos(dx+c)^2\right)\sin(dx+c)}{4\left(\left(a^8b^2-4a^6b^4+6a^4b^6-4a^2b^8+b^{10}\right)d\cos(dx+c)^3-2\left(a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9\right)d\cos(dx+c)\sin(dx+c)-\left(a^{10}-3a^8b^2+2a^6b^4+2a^4b^6-3a^2b^8+b^{10}\right)d\cos(dx+c)\right)},\ -\frac{2a^7-6a^5b^2+6a^3b^4-2ab^6+\left(16a^5b^2-17a^3b^4+ab^6\right)\cos(dx+c)^2+3\left(\left(2a^3b^3+3ab^5\right)\cos(dx+c)^3-2\left(2a^4b^2+3a^2b^4\right)\cos(dx+c)\sin(dx+c)-\left(2a^5b+5a^3b^3+3ab^5\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)-\left(2a^6b-6a^4b^3+6a^2b^5-2b^7-\left(11a^4b^3-7a^2b^5-4b^7\right)\cos(dx+c)^2\right)\sin(dx+c)}{2\left(\left(a^8b^2-4a^6b^4+6a^4b^6-4a^2b^8+b^{10}\right)d\cos(dx+c)^3-2\left(a^9b-4a^7b^3+6a^5b^5-4a^3b^7+ab^9\right)d\cos(dx+c)\sin(dx+c)-\left(a^{10}-3a^8b^2+2a^6b^4+2a^4b^6-3a^2b^8+b^{10}\right)d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^7 - 12*a^5*b^2 + 12*a^3*b^4 - 4*a*b^6 + 2*(16*a^5*b^2 - 17*a^3*b
^4 + a*b^6)*cos(d*x + c)^2 - 3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2
*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*
a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5
 - 4*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 -
 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*
a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^
4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)), -1/2*(2*a^7 - 6*a^5*b^2
+ 6*a^3*b^4 - 2*a*b^6 + (16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
3*((2*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3 - 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x
 + c)*sin(d*x + c) - (2*a^5*b + 5*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*sqrt(a^2
 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a
^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - (11*a^4*b^3 - 7*a^2*b^5 - 4*b^7)*cos
(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b
^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6
 - 3*a^2*b^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*sin(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.64, size = 365, normalized size = 1.79

$$\frac{\frac{3\,(2\,a^3b+3\,ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{2\,(3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^3-3\,ab^2)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}+\frac{7\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+13\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+17\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^4b^2+a^2b^4}{(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*sin(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4
```

```
- b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*x
 + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*
x + 1/2*c)^2 - 1)) + (7*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^4*b^2*tan(1/2*
d*x + 1/2*c)^2 + 13*a^2*b^4*tan(1/2*d*x + 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/
2*c)^2 + 17*a^3*b^3*tan(1/2*d*x + 1/2*c) + 4*a*b^5*tan(1/2*d*x + 1/2*c) + 6
*a^4*b^2 + a^2*b^4)/((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*(a*tan(1/2*d*x +
 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2))/d
```

Mupad [B]
time = 18.11, size = 624, normalized size = 3.06

$$\frac{3\,a\,b\,\mathrm{atan}\left(\frac{\frac{3\,a\,b\,\left(2\,a^2+3\,b^2\right)\,\left(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7\right)}{2\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}+\frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+3\,b^2\right)\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}{{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}}{6\,a^3\,b+9\,a\,b^3}\right)\,\left(2\,a^2+3\,b^2\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}-\frac{\frac{2\,a^5+12\,a^3\,b^2+a\,b^4}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,a^4\,b+7\,a^2\,b^3+6\,b^5\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-2\,a^6+24\,a^4\,b^2+21\,a^2\,b^4+2\,b^6\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4+39\,a^2\,b^2+4\,b^4\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^6-2\,a^4\,b^2+14\,a^2\,b^4+b^6\right)}{a\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}-\frac{3\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,a^2+3\,b^2\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-a^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+4\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2+4\,b^2\right)+4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(cos(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] (3*a*b*atan(((3*a*b*(2*a^2 + 3*b^2)*(2*a^6*b - 2*b^7 + 6*a^2*b^5 - 6*a^4*b^
3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (3*a^2*b*tan(c/2 + (d*x)/2)*(2*a^2 +
3*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/
(9*a*b^3 + 6*a^3*b))*(2*a^2 + 3*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2)) - ((a
*b^4 + 2*a^5 + 12*a^3*b^2)/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2
 + (d*x)/2)^3*(2*a^4*b + 6*b^5 + 7*a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2) - (tan(c/2 + (d*x)/2)^4*(2*b^6 - 2*a^6 + 21*a^2*b^4 + 24*a^4*b^2))/(a
*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)*(2*a^4 + 4*b^
4 + 39*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d*x)/2
)^2*(2*a^6 + b^6 + 14*a^2*b^4 - 2*a^4*b^2))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2)) - (3*a^2*b*tan(c/2 + (d*x)/2)^5*(2*a^2 + 3*b^2))/(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)
^2*(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*
x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)))
```

3.1475 $\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{2b^3(3a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d}+\frac{b^3(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d}+\frac{2b^3(6a^4-3a^2b^2+b^4)\tan^{-1}\left(\right.}{a^3\left(a^2-b^2\right)^{7/2}}$$

```
[Out] 2*b^3*(3*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(7/2)/d+b^3*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2)
)/a/(a^2-b^2)^(7/2)/d+2*b^3*(6*a^4-3*a^2*b^2+b^4)*arctan((b+a*tan(1/2*d*x+1
/2*c))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(7/2)/d-arctanh(cos(d*x+c))/a^3/d+1/2
*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/2*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c)
)+1/2*b^4*cos(d*x+c)/a/(a^2-b^2)^2/d/(a+b*sin(d*x+c))^2+3/2*b^4*cos(d*x+c)/
(a^2-b^2)^3/d/(a+b*sin(d*x+c))+b^4*(3*a^2-b^2)*cos(d*x+c)/a^2/(a^2-b^2)^3/d
/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2976, 3855, 2727, 2743, 2833, 12, 2739, 632, 210}

$$-\frac{\tanh^{-1}(\cos(c+dx))}{a^3d}+\frac{2b^3(3a^2-b^2)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{7/2}}+\frac{b^3(2a^2+b^2)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{ad\left(a^2-b^2\right)^{7/2}}+\frac{b^4(3a^2-b^2)\cos(c+dx)}{a^2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{3b^4\cos(c+dx)}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b^4\cos(c+dx)}{2ad\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{2b^3(6a^4-3a^2b^2+b^4)\text{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b^3*(3*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*(
a^2 - b^2)^(7/2)*d) + (b^3*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(7/2)*d) + (2*b^3*(6*a^4 - 3*a^2*b^2 + b^4)*
ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d)
 - ArcTanh[Cos[c + d*x]]/(a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^4*Cos[c + d*
x])/(2*a*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^4*Cos[c + d*x])/(2*
(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (b^4*(3*a^2 - b^2)*Cos[c + d*x])/(a
^2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \left(\frac{\csc(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+dx))} - \frac{1}{2(a-b)^3(1+\sin(c+a\right.\\
&= \frac{\int \csc(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{\int \frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3} + \frac{b^3\int \frac{1}{(a+b\sin(c+dx)}}{a\left(a^2-b^2\right)}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+d}{2(a-b)^3d(1+\mathrm{si}}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} + \frac{\cos(c+d}{2(a-b)^3d(1+\mathrm{si}}\\
&= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{}{2}\\
&= \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3d} + \frac{}{2}\\
&= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{2b^3\left(6a^4-3a^2b^2+b^4\right)\tan^{-1}\left(\frac{b+a}{\sqrt{}}\right.}{a^3\left(a^2-b^2\right)^{7/2}d}\\
&= \frac{2b^3\left(3a^2-b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a\left(a^2-b^2\right)^{7/2}d} + \frac{b^3\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{a\left(a^2-b^2\right)^{7/2}d}
\end{aligned}$$

Mathematica [A]

time = 5.39, size = 278, normalized size = 0.69

$$\frac{\frac{2b^3\left(20a^4-7a^2b^2+2b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}} - \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{b^4\cos(c+dx)}{a(a-b)^2(a+b)^2(a+b\sin(c+dx))^2} + \frac{b^4\left(9a^2-2b^2\right)\cos(c+dx)}{a^2(a-b)^3(a+b)^3(a+b\sin(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*b^3*(20*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2]])/Sqrt[a
^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)) - (2*Log[Cos[(c + d*x)/2]])/a^3 + (2*Lo
```

```
g[Sin[(c + d*x)/2]])/a^3 + (2*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])) - (2*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])) + (b^4*Cos[c + d*x])/(a*(a - b)^2*(a + b)^2*(a + b*Sin
[c + d*x])^2) + (b^4*(9*a^2 - 2*b^2)*Cos[c + d*x])/(a^2*(a - b)^3*(a + b)^3
*(a + b*Sin[c + d*x])))/(2*d)
```

Maple [A]
time = 0.97, size = 278, normalized size = 0.69

method	result
derivativedivides	$\frac{\frac{2b^3\left(\frac{\left(\frac{11}{2}a^3b^2-2ab^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(10a^4+17a^2b^2-6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ab^2\left(29a^2-8b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(10a^2-3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\cdots\right)}{(a-b)^3(a+b)^3a^3}}{d}$
default	$\frac{\frac{2b^3\left(\frac{\left(\frac{11}{2}a^3b^2-2ab^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(10a^4+17a^2b^2-6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{ab^2\left(29a^2-8b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(10a^2-3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\cdots\right)}{(a-b)^3(a+b)^3a^3}}{d}$
risch	$\frac{6ia^4b^3-36ia^4b^3e^{4i(dx+c)}+2ib^7e^{4i(dx+c)}-8ia^6be^{4i(dx+c)}-2a^5b^2e^{5i(dx+c)}-14a^3b^4e^{5i(dx+c)}+ab^6e^{5i(dx+c)}-16ia^6be^{2i(d\cdots}}{\left(e^{2i(\cdots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^3/(a-b)^3/(a+b)^3/a^3*(((11/2*a^3*b^2-2*a*b^4)*tan(1/2*d*x+1/2*c)^
3+1/2*b*(10*a^4+17*a^2*b^2-6*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(29*a^2-8*
b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(10*a^2-3*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2
*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(20*a^4-7*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arc
tan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))+1/a^3*ln(tan(1/2*d*x
+1/2*c))-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(377) = 754$.
time = 1.87, size = 1623, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^10 - 12*a^8*b^2 + 12*a^6*b^4 - 4*a^4*b^6 + 2*(10*a^8*b^2 - 2*a^6
*b^4 - 11*a^4*b^6 + 3*a^2*b^8)*cos(d*x + c)^2 - ((20*a^4*b^5 - 7*a^2*b^7 +
2*b^9)*cos(d*x + c)^3 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*s
in(d*x + c) - (20*a^6*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*s
qrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*((a^8*b^2 - 4
*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*
b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*
a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*c
os(d*x + c) + 1/2) - 2*((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*co
s(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2
*b^8 + b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*a^9*b - 6*a^
7*b^3 + 6*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b
^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^
5*b^8 + a^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4
*a^6*b^7 + a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^
9*b^4 + 2*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c)), -1/2*(2*a^10 - 6
*a^8*b^2 + 6*a^6*b^4 - 2*a^4*b^6 + (10*a^8*b^2 - 2*a^6*b^4 - 11*a^4*b^6 + 3
*a^2*b^8)*cos(d*x + c)^2 + ((20*a^4*b^5 - 7*a^2*b^7 + 2*b^9)*cos(d*x + c)^3
 - 2*(20*a^5*b^4 - 7*a^3*b^6 + 2*a*b^8)*cos(d*x + c)*sin(d*x + c) - (20*a^6
*b^3 + 13*a^4*b^5 - 5*a^2*b^7 + 2*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan
(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + ((a^8*b^2 - 4*a^6*
b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 +
 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b
^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) - ((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*cos(d
*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x +
 c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 +
b^10)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^9*b - 6*a^7*b^3 + 6
*a^5*b^5 - 2*a^3*b^7 - (6*a^7*b^3 + 5*a^5*b^5 - 13*a^3*b^7 + 2*a*b^9)*cos(d
*x + c)^2)*sin(d*x + c))/((a^11*b^2 - 4*a^9*b^4 + 6*a^7*b^6 - 4*a^5*b^8 + a
```

```
^3*b^10)*d*cos(d*x + c)^3 - 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*a^6*b^7
+ a^4*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^13 - 3*a^11*b^2 + 2*a^9*b^4 + 2
*a^7*b^6 - 3*a^5*b^8 + a^3*b^10)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.57, size = 411, normalized size = 1.02

$$\frac{\frac{(20a^4b^3-7a^2b^5+2b^7)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^9-3a^7b^2+3a^5b^4-a^3b^6)\sqrt{a^2-b^2}}+\frac{2\left(3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a^3-3ab^2\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{11a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+10a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+17a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-6b^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+29a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+10a^4b^4-3a^2b^6}{(a^9-3a^7b^2+3a^5b^4-a^3b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((20*a^4*b^3 - 7*a^2*b^5 + 2*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a)
+ arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^9 - 3*a^7*b^2 +
 3*a^5*b^4 - a^3*b^6)*sqrt(a^2 - b^2)) + 2*(3*a^2*b*tan(1/2*d*x + 1/2*c) +
b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (11*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^7*tan(1/2*d*x + 1/2*c)^3 + 10*a^4*b^4*tan(1/2*d*x + 1/2*c)^2 + 17*a^2*b
^6*tan(1/2*d*x + 1/2*c)^2 - 6*b^8*tan(1/2*d*x + 1/2*c)^2 + 29*a^3*b^5*tan(1
/2*d*x + 1/2*c) - 8*a*b^7*tan(1/2*d*x + 1/2*c) + 10*a^4*b^4 - 3*a^2*b^6)/((
a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(
1/2*d*x + 1/2*c) + a)^2) + log(abs(tan(1/2*d*x + 1/2*c)))/a^3)/d
```

Mupad [B]
time = 17.79, size = 2500, normalized size = 6.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)*(a + b*sin(c + d*x))^3),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(a^3*d) - ((2*a^6 - 3*b^6 + 10*a^2*b^4 + 6*a^4*b^2)
/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)*(2*a^6*b - 8
*b^7 + 29*a^2*b^5 + 22*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
- (2*tan(c/2 + (d*x)/2)^3*(2*a^6*b - 2*b^7 + 13*a^2*b^5 + 2*a^4*b^3))/(a^2*
```

(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^5*(6*a^6*b - 4*b
^7 + 11*a^2*b^5 + 2*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (
2*tan(c/2 + (d*x)/2)^2*(2*a^8 - 3*b^8 + 10*a^2*b^6 + 8*a^4*b^4 - 2*a^6*b^2)
)/(a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^4*(17*a^2
*b^6 - 6*b^8 - 2*a^8 + 18*a^4*b^4 + 18*a^6*b^2))/(a^3*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*
(a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/
2)^5 - 4*a*b*tan(c/2 + (d*x)/2))) + (b^3*atan(((b^3*(-(a + b)^7*(a - b)^7)^
(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*a^24 + 8*a^4*b^
20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*b^12 - 1790*a^1
4*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^22*b^2) - 2*a^2
3*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^13 + 722*a^13*b^
11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^3 + (b^3*(-(a +
 b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(
6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13*b^14 + 1512*a^
15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 288*a^23*b^4 - 62
*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*b^15 - 168*a^14
*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^22*b^5 + 18*a^24
*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b
^6 + 21*a^13*b^4 - 7*a^15*b^2))*1i)/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*
b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2) + (b^3*(-(a + b
)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*
a^24 + 8*a^4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*
b^12 - 1790*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^
22*b^2) - 2*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^1
3 + 722*a^13*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^
3 - (b^3*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c
/2 + (d*x)/2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13
*b^14 + 1512*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 28
8*a^23*b^4 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*
b^15 - 168*a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^2
2*b^5 + 18*a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*
b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2))*1i)/(a^17 - a^3*b^14 + 7*a^5
*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2))
/(2*tan(c/2 + (d*x)/2)*(2*a^3*b^18 - 41*a^5*b^16 + 225*a^7*b^14 - 715*a^9*b
^12 + 1135*a^11*b^10 - 792*a^13*b^8 + 146*a^15*b^6 + 40*a^17*b^4) + 4*a^2*b
^19 - 38*a^4*b^17 + 184*a^6*b^15 - 530*a^8*b^13 + 940*a^10*b^11 - 1034*a^12
*b^9 + 688*a^14*b^7 - 254*a^16*b^5 + 40*a^18*b^3 + (b^3*(-(a + b)^7*(a - b)
^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*a^24 + 8*a^
4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*b^12 - 1790
*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^22*b^2) - 2
*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^13 + 722*a^1
3*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^3 + (b^3*(-
(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/
2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13*b^14 + 151

2*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 288*a^23*b^4 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*b^15 - 168*a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^22*b^5 + 18*a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2)))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2) - (b^3*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(2*a^24 + 8*a^4*b^20 - 76*a^6*b^18 + 346*a^8*b^16 - 938*a^10*b^14 + 1612*a^12*b^12 - 1790*a^14*b^10 + 1276*a^16*b^8 - 566*a^18*b^6 + 148*a^20*b^4 - 22*a^22*b^2) - 2*a^23*b + 4*a^5*b^19 - 37*a^7*b^17 + 164*a^9*b^15 - 433*a^11*b^13 + 722*a^13*b^11 - 769*a^15*b^9 + 512*a^17*b^7 - 199*a^19*b^5 + 38*a^21*b^3 - (b^3*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + b^4 - (7*a^2*b^2)/2)*(tan(c/2 + (d*x)/2)*(6*a^27 + 8*a^7*b^20 - 78*a^9*b^18 + 342*a^11*b^16 - 888*a^13*b^14 + 1512*a^15*b^12 - 1764*a^17*b^10 + 1428*a^19*b^8 - 792*a^21*b^6 + 288*a^23*b^4 - 62*a^25*b^2) - 2*a^26*b + 2*a^8*b^19 - 18*a^10*b^17 + 72*a^12*b^15 - 168*a^14*b^13 + 252*a^16*b^11 - 252*a^18*b^9 + 168*a^20*b^7 - 72*a^22*b^5 + 18*a^24*b^3))/(a^17 - a^3*b^14 + 7*a^5*b^12 - 21*a^7*b^10 + 35*a^9*b^8 - 35*a^11*b^6 + 21*a^13*b^4 - 7*a^15*b^2))...

3.1476 $\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=424

$$-\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}-\frac{b^4(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{7/2}d}-\frac{2b^4(10a^4-9a^2b^2+3b^4)\tan}{a^4\left(a^2-b^2\right.}$$

```
[Out] -4*b^4*(2*a^2-b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^2/(a^
2-b^2)^(7/2)/d-b^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1
/2))/a^2/(a^2-b^2)^(7/2)/d-2*b^4*(10*a^4-9*a^2*b^2+3*b^4)*arctan((b+a*tan(1
/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/(a^2-b^2)^(7/2)/d+3*b*arctanh(cos(d*x+c
))/a^4/d-cot(d*x+c)/a^3/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d
*x+c)/(a-b)^3/d/(1+sin(d*x+c))-1/2*b^5*cos(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*si
n(d*x+c))^2-3/2*b^5*cos(d*x+c)/a/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*b^5*(2*a^
2-b^2)*cos(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2976, 3855, 3852, 8, 2727, 2743, 2833, 12, 2739, 632, 210}

$$\frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{\cot(c+dx)}{a^3d}-\frac{4b^4(2a^2-b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{b^4(2a^2+b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{7/2}}-\frac{3b^5\cos(c+dx)}{2ad\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{b^5\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}-\frac{2b^4(10a^4-9a^2b^2+3b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\left(a^2-b^2\right)^{7/2}}-\frac{2b^5(2a^2-b^2)\cos(c+dx)}{a^3d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*b^4*(2*a^2 - b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2*(a^2 - b^2)^(7/2)*d) - (b^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(7/2)*d) - (2*b^4*(10*a^4 - 9*a^2*b^2 +
 3*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4*(a^2 - b^2)^
(7/2)*d) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^3*d) + Cos
[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*
(1 + Sin[c + d*x])) - (b^5*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sin[
c + d*x])^2) - (3*b^5*Cos[c + d*x])/(2*a*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x
])) - (2*b^5*(2*a^2 - b^2)*Cos[c + d*x])/(a^3*(a^2 - b^2)^3*d*(a + b*Sin[c
+ d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
```

```
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(-\frac{3b\csc(c+dx)}{a^4} + \frac{\csc^2(c+dx)}{a^3} - \frac{1}{2(a+b)^3(-1+\sin(c+dx))} + \frac{}{2(}\right.$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a^3} + \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{(3b)\int \csc(c+dx)\,dx}{a^4} - \frac{\int \frac{1}{-1+\sin(c}}{2(a+}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+}{2(a-b)^3d(1+}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^3d} + \frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))} - \frac{}{2}$$

$$= -\frac{2b^4(10a^4-9a^2b^2+3b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{7/2}d} + \frac{3b\tanh^{-1}(\cos(c+a}{a^4d}$$

$$= -\frac{2b^4(10a^4-9a^2b^2+3b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{7/2}d} + \frac{3b\tanh^{-1}(\cos(c+a}{a^4d}$$

$$= -\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{7/2}d} - \frac{2b^4(10a^4-9a^2b^2+3b^4)\tan^{-1}}{a^4(a^2-b^2)^{7/2}}$$

$$= -\frac{4b^4(2a^2-b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{7/2}d} - \frac{b^4(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c}{\sqrt{a^2-}\right.}{a^2(a^2-b^2)^{7/2}d}$$

Mathematica [A]

time = 6.26, size = 379, normalized size = 0.89

$$4\left(-\frac{3b^4(10a^4-7a^2b^2+2b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{4a^4\left(a^2-b^2\right)^{7/2}d}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{8a^3d}+\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{4a^4d}-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4a^4d}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{4(a+b)^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{4(a-b)^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{b^5\cos(c+dx)}{8a^2(a-b)^2(a+b)^2d(a+b\sin(c+dx))^2}+\frac{-11a^2b^5\cos(c+dx)+4b^7\cos(c+dx)}{8a^3(a-b)^3(a+b)^3d(a+b\sin(c+dx))}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{8a^3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] 4*((-3*b^4*(10*a^4 - 7*a^2*b^2 + 2*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c
+ d*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(4*a^4*(a^2 - b^2)^(7/2)
*d) - Cot[(c + d*x)/2]/(8*a^3*d) + (3*b*Log[Cos[(c + d*x)/2]])/(4*a^4*d) -
(3*b*Log[Sin[(c + d*x)/2]])/(4*a^4*d) + Sin[(c + d*x)/2]/(4*(a + b)^3*d*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) + Sin[(c + d*x)/2]/(4*(a - b)^3*d*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])) - (b^5*Cos[c + d*x])/(8*a^2*(a - b)^2*(a
+ b)^2*d*(a + b*Sin[c + d*x])^2) + (-11*a^2*b^5*Cos[c + d*x] + 4*b^7*Cos[c
+ d*x])/(8*a^3*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) + Tan[(c + d*x)/
2]/(8*a^3*d))
```

Maple [A]
time = 1.10, size = 311, normalized size = 0.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)-2*b^4/(a-b)^3/(a+b)^3/a^4*(((13/2*a^3*b^2-3
*a*b^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(12*a^4+19*a^2*b^2-10*b^4)*tan(1/2*d*x+1
/2*c)^2+7/2*a*b^2*(5*a^2-2*b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(12*a^2-5*b^2)
)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+3/2*(10*a^4-7*a^2*b^2
+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(
1/2)))-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1/2*d*x+1/2*c))-1/(a+b)^3/
(tan(1/2*d*x+1/2*c)-1)-1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(399) = 798$.
time = 1.62, size = 2140, normalized size = 5.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [-1/4*(4*a^11 - 12*a^9*b^2 + 12*a^7*b^4 - 4*a^5*b^6 + 2*(4*a^9*b^2 - 4*a^7*
b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^4 - 2*(4*a^11 - 10*a
^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10)*cos(d*x + c)^2 - 3
*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 - 2*(10*a^5*b^5 - 7*a
^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b^8 + 2*b^10)*cos(d*x
 + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)*cos(d*x + c))*sin(d
*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x
 + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-
a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(2*(
a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b
^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3
*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x
+ c))*log(1/2*cos(d*x + c) + 1/2) + 6*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 -
4*a^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^
2*b^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7
- 3*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2
) - 2*(2*a^10*b - 6*a^8*b^3 + 6*a^6*b^5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^
3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(
a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2
*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + (
(a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3
 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*co
s(d*x + c))*sin(d*x + c)), -1/2*(2*a^11 - 6*a^9*b^2 + 6*a^7*b^4 - 2*a^5*b^6
 + (4*a^9*b^2 - 4*a^7*b^4 + 17*a^5*b^6 - 23*a^3*b^8 + 6*a*b^10)*cos(d*x + c
)^4 - (4*a^11 - 10*a^9*b^2 + 14*a^7*b^4 + 7*a^5*b^6 - 21*a^3*b^8 + 6*a*b^10
)*cos(d*x + c)^2 - 3*(2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c)^3 -
 2*(10*a^5*b^5 - 7*a^3*b^7 + 2*a*b^9)*cos(d*x + c) + ((10*a^4*b^6 - 7*a^2*b
^8 + 2*b^10)*cos(d*x + c)^3 - (10*a^6*b^4 + 3*a^4*b^6 - 5*a^2*b^8 + 2*b^10)
*cos(d*x + c))*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(
sqrt(a^2 - b^2)*cos(d*x + c))) - 3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*
a^3*b^8 + a*b^10)*cos(d*x + c)^3 - 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a
^3*b^8 + a*b^10)*cos(d*x + c) + ((a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b
^9 + b^11)*cos(d*x + c)^3 - (a^10*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3
```

```
*a^2*b^9 + b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
3*(2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c)^3
- 2*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*cos(d*x + c) + (
(a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*cos(d*x + c)^3 - (a^10
*b - 3*a^8*b^3 + 2*a^6*b^5 + 2*a^4*b^7 - 3*a^2*b^9 + b^11)*cos(d*x + c))*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - (2*a^10*b - 6*a^8*b^3 + 6*a^6*b^
5 - 2*a^4*b^7 + (8*a^10*b - 14*a^8*b^3 + 28*a^6*b^5 - 31*a^4*b^7 + 9*a^2*b^
9)*cos(d*x + c)^2)*sin(d*x + c))/(2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*a^
7*b^7 + a^5*b^9)*d*cos(d*x + c)^3 - 2*(a^13*b - 4*a^11*b^3 + 6*a^9*b^5 - 4*
a^7*b^7 + a^5*b^9)*d*cos(d*x + c) + ((a^12*b^2 - 4*a^10*b^4 + 6*a^8*b^6 - 4
*a^6*b^8 + a^4*b^10)*d*cos(d*x + c)^3 - (a^14 - 3*a^12*b^2 + 2*a^10*b^4 + 2
*a^8*b^6 - 3*a^6*b^8 + a^4*b^10)*d*cos(d*x + c))*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2*sec(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.71, size = 633, normalized size = 1.49

$$-\frac{\frac{6\,(10\,a^4b^4-7\,a^2b^6+2\,b^8)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\sqrt{a^2-b^2}}-\frac{2\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+10\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^7-3\,a^5b^2+3\,a^3b^4-ab^6}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{2\left(13\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+19\,a^2b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,b^9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+35\,a^3b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-14\,ab^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,a^4b^5-5\,a^2b^7\right)}{(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)^2}+\frac{6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*(10*a^4*b^4 - 7*a^2*b^6 + 2*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^10 - 3*a
^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(a^2 - b^2)) - (2*a^6*b*tan(1/2*d*x + 1/2
*c)^3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^3
 - 2*b^7*tan(1/2*d*x + 1/2*c)^3 - 5*a^7*tan(1/2*d*x + 1/2*c)^2 - 9*a^5*b^2*
tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + a*b^6*tan(1/2*d
*x + 1/2*c)^2 + 10*a^6*b*tan(1/2*d*x + 1/2*c) + 10*a^4*b^3*tan(1/2*d*x + 1/
2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 2*b^7*tan(1/2*d*x + 1/2*c) + a^7 -
3*a^5*b^2 + 3*a^3*b^4 - a*b^6)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(t
an(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))) + 2*(13*a^3*b^6*tan(1/2*d*x
+ 1/2*c)^3 - 6*a*b^8*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*b^5*tan(1/2*d*x + 1/2*
c)^2 + 19*a^2*b^7*tan(1/2*d*x + 1/2*c)^2 - 10*b^9*tan(1/2*d*x + 1/2*c)^2 +
35*a^3*b^6*tan(1/2*d*x + 1/2*c) - 14*a*b^8*tan(1/2*d*x + 1/2*c) + 12*a^4*b^
5 - 5*a^2*b^7)/((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1
```

```
/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/2*d*x + 1/2*
c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3)/d
```

Mupad [B]
time = 15.78, size = 2500, normalized size = 5.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^2*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*a^3*d) - ((tan(c/2 + (d*x)/2)^6*(5*a^8 - 12*b^8 + 25*
a^2*b^6 + 3*a^4*b^4 + 9*a^6*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - a^2
 + (tan(c/2 + (d*x)/2)^4*(9*a^8 - 20*b^8 + 55*a^2*b^6 + 23*a^4*b^4 - 7*a^6*
b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (tan(c/2 + (d*x)/2)^2*(81*a^2*b
^6 - 32*b^8 - 3*a^8 + 7*a^4*b^4 + 37*a^6*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a
^4*b^2) + (2*tan(c/2 + (d*x)/2)*(7*a*b^7 - 8*a^7*b - 18*a^3*b^5 + 4*a^5*b^3
))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^3*(2*a^8*b -
 5*b^9 + 12*a^2*b^7 + 4*a^4*b^5 + 2*a^6*b^3))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(4*a^8*b - 10*b^9 + 17*a^2*b^7 + 18*a^
4*b^5 + 16*a^6*b^3))/(a*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(2*a^5*tan
(c/2 + (d*x)/2)^7 - tan(c/2 + (d*x)/2)^3*(2*a^5 + 8*a^3*b^2) + tan(c/2 + (d
*x)/2)^5*(2*a^5 + 8*a^3*b^2) - 2*a^5*tan(c/2 + (d*x)/2) - 8*a^4*b*tan(c/2 +
 (d*x)/2)^2 + 8*a^4*b*tan(c/2 + (d*x)/2)^6)) - (3*b*log(tan(c/2 + (d*x)/2))
)/(a^4*d) - (b^4*atan(((b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 -
7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978
*a^10*b^17 - 2454*a^12*b^15 + 3936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b
^9 - 1338*a^20*b^7 + 384*a^22*b^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^
18 + 462*a^11*b^16 - 1119*a^13*b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 108
6*a^19*b^8 - 417*a^21*b^6 + 84*a^23*b^4 - 6*a^25*b^2 - (3*b^4*(-(a + b)^7*(
a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 +
8*a^10*b^20 - 78*a^12*b^18 + 342*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12
 - 1764*a^20*b^10 + 1428*a^22*b^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b
^2) - 2*a^29*b + 2*a^11*b^19 - 18*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^13
+ 252*a^19*b^11 - 252*a^21*b^9 + 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3))
/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6
 + 21*a^14*b^4 - 7*a^16*b^2)))*3i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^
8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (b^4*(-(a
 + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(
6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978*a^10*b^17 - 2454*a^12*b^15 + 39
36*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b^9 - 1338*a^20*b^7 + 384*a^22*b^
5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^18 + 462*a^11*b^16 - 1119*a^13*b
^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1086*a^19*b^8 - 417*a^21*b^6 + 84*a
^23*b^4 - 6*a^25*b^2 + (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
- 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 + 8*a^10*b^20 - 78*a^12*b^18 + 342
```

*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12 - 1764*a^20*b^10 + 1428*a^22*b^
8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b^2) - 2*a^29*b + 2*a^11*b^19 - 1
8*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^13 + 252*a^19*b^11 - 252*a^21*b^9 +
 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3))/(2*(a^18 - a^4*b^14 + 7*a^6*b^1
2 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*3
i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b
^6 + 21*a^14*b^4 - 7*a^16*b^2)))/(2*tan(c/2 + (d*x)/2)*(18*a^4*b^20 - 189*a
^6*b^18 + 765*a^8*b^16 - 1575*a^10*b^14 + 1575*a^12*b^12 - 468*a^14*b^10 -
306*a^16*b^8 + 180*a^18*b^6) + 36*a^3*b^21 - 342*a^5*b^19 + 1476*a^7*b^17 -
 3690*a^9*b^15 + 5760*a^11*b^13 - 5706*a^13*b^11 + 3492*a^15*b^9 - 1206*a^1
7*b^7 + 180*a^19*b^5 - (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
- 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 9
78*a^10*b^17 - 2454*a^12*b^15 + 3936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18
*b^9 - 1338*a^20*b^7 + 384*a^22*b^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*
b^18 + 462*a^11*b^16 - 1119*a^13*b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1
086*a^19*b^8 - 417*a^21*b^6 + 84*a^23*b^4 - 6*a^25*b^2 - (3*b^4*(-(a + b)^7
(a - b)^7)^(1/2)(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30
+ 8*a^10*b^20 - 78*a^12*b^18 + 342*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^
12 - 1764*a^20*b^10 + 1428*a^22*b^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28
*b^2) - 2*a^29*b + 2*a^11*b^19 - 18*a^13*b^17 + 72*a^15*b^15 - 168*a^17*b^1
3 + 252*a^19*b^11 - 252*a^21*b^9 + 168*a^23*b^7 - 72*a^25*b^5 + 18*a^27*b^3
))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b
^6 + 21*a^14*b^4 - 7*a^16*b^2))))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8
*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (3*b^4*(-(
a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*
(6*a^26*b + 24*a^6*b^21 - 228*a^8*b^19 + 978*a^10*b^17 - 2454*a^12*b^15 + 3
936*a^14*b^13 - 4170*a^16*b^11 + 2928*a^18*b^9 - 1338*a^20*b^7 + 384*a^22*b
^5 - 66*a^24*b^3) + 12*a^7*b^20 - 111*a^9*b^18 + 462*a^11*b^16 - 1119*a^13*
b^14 + 1716*a^15*b^12 - 1707*a^17*b^10 + 1086*a^19*b^8 - 417*a^21*b^6 + 84*
a^23*b^4 - 6*a^25*b^2 + (3*b^4*(-(a + b)^7*(a - b)^7)^(1/2)*(10*a^4 + 2*b^4
 - 7*a^2*b^2)*(tan(c/2 + (d*x)/2)*(6*a^30 + 8*a^10*b^20 - 78*a^12*b^18 + 34
2*a^14*b^16 - 888*a^16*b^14 + 1512*a^18*b^12 - 1764*a^20*b^10 + 1428*a^22*b
^8 - 792*a^24*b^6 + 288*a^26*b^4 - 62*a^28*b^2)...

3.1477 $\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=470

$$\frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d}+\frac{b^5(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d}+\frac{2b^5(15a^4-17a^2b^2+6b^4)\,\mathrm{ta}}{a^5\left(a^2-b^2\right.}$$

```
[Out] 2*b^5*(5*a^2-3*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/(a
^2-b^2)^(7/2)/d+b^5*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(
1/2))/a^3/(a^2-b^2)^(7/2)/d+2*b^5*(15*a^4-17*a^2*b^2+6*b^4)*arctan((b+a*tan
(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^5/(a^2-b^2)^(7/2)/d-1/2*arctanh(cos(d*x
+c))/a^3/d-(a^2+6*b^2)*arctanh(cos(d*x+c))/a^5/d+3*b*cot(d*x+c)/a^4/d-1/2*c
ot(d*x+c)*csc(d*x+c)/a^3/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))+1/2*cos(
d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*b^6*cos(d*x+c)/a^3/(a^2-b^2)^2/d/(a+b*s
in(d*x+c))^2+3/2*b^6*cos(d*x+c)/a^2/(a^2-b^2)^3/d/(a+b*sin(d*x+c))+b^6*(5*a
^2-3*b^2)*cos(d*x+c)/a^4/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.43, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2976, 3855, 3852, 8, 3853, 2727, 2743, 2833, 12, 2739, 632, 210}

$$\frac{3b\cot(c+dx)}{a^4d}-\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{\cot(c+dx)\csc(c+dx)}{2a^3d}+\frac{3b^6\cos(c+dx)}{2a^2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{a^5d}+\frac{b^6(5a^2-3b^2)\cos(c+dx)}{a^4d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{b^5(2a^2+b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{2b^5(5a^2-3b^2)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\left(a^2-b^2\right)^{7/2}}+\frac{b^6\cos(c+dx)}{2a^3d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{2b^5(15a^4-17a^2b^2+6b^4)\,\mathrm{ArcTan}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\left(a^2-b^2\right)^{7/2}}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*b^5*(5*a^2 - 3*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^3*(a^2 - b^2)^(7/2)*d) + (b^5*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2]
)/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(7/2)*d) + (2*b^5*(15*a^4 - 17*a^2*b^2
 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2
)^(7/2)*d) - ArcTanh[Cos[c + d*x]]/(2*a^3*d) - ((a^2 + 6*b^2)*ArcTanh[Cos[c
 + d*x]])/(a^5*d) + (3*b*Cot[c + d*x])/(a^4*d) - (Cot[c + d*x]*Csc[c + d*x]
)/(2*a^3*d) + Cos[c + d*x]/(2*(a + b)^3*d*(1 - Sin[c + d*x])) + Cos[c + d*x
]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) + (b^6*Cos[c + d*x])/(2*a^3*(a^2 - b^2
)^2*d*(a + b*Sin[c + d*x])^2) + (3*b^6*Cos[c + d*x])/(2*a^2*(a^2 - b^2)^3*d
*(a + b*Sin[c + d*x])) + (b^6*(5*a^2 - 3*b^2)*Cos[c + d*x])/(a^4*(a^2 - b^2
)^3*d*(a + b*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)\sec^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \int \left(\frac{(a^2+6b^2)\csc(c+dx)}{a^5} - \frac{3b\csc^2(c+dx)}{a^4} + \frac{\csc^3(c+dx)}{a^3} - \frac{}{2(a+b)^3(}\right.\\
&= \frac{\int \csc^3(c+dx)\,dx}{a^3} - \frac{\int \frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3} - \frac{(3b)\int \csc^2(c+dx)\,dx}{a^4} - \frac{\int \frac{1}{-1+\sin(c+}}{2(a+}\\
&= -\frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{2a^3d} + \frac{\cos}{2(a+b)^3d(}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{(a^2+6b^2)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{3b\cot(c+dx)}{a^4d}\\
&= \frac{2b^5(15a^4-17a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}\\
&= \frac{2b^5(15a^4-17a^2b^2+6b^4)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2}d} - \frac{\tanh^{-1}(\cos(c+dx))}{2a^3d}\\
&= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} + \frac{2b^5(15a^4-17a^2b^2+6b^4)\tan^{-1}\left(\right.}{a^5\left(a^2-b^2\right)^{7/2}d}\\
&= \frac{2b^5(5a^2-3b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{7/2}d} + \frac{b^5(2a^2+b^2)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right.}{a^3\left(a^2-b^2\right)^{7/2}d}\end{aligned}$$

Mathematica [A]

time = 6.92, size = 432, normalized size = 0.92

$$\frac{3b^5(14a^4-13a^2b^2+4b^4)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\left(a^2-b^2\right)^{7/2}d} + \frac{3b\cot\left(\frac{1}{2}(c+dx)\right)}{2a^4d} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} - \frac{3(a^2+4b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{3(a^2+4b^2)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^5d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d} + \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{\sin\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{b^6\cos(c+dx)}{2a^3(a-b)^2(a+b)^2d(a+b\sin(c+dx))^2} + \frac{13a^2b^6\cos(c+dx)-6b^8\cos(c+dx)}{2a^4(a-b)^3(a+b)^3d(a+b\sin(c+dx))} - \frac{3b\tan\left(\frac{1}{2}(c+dx)\right)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*Sec[c + d*x]^2)/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b^5*(14*a^4 - 13*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d
*x)/2] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*(a^2 - b^2)^(7/2)*d) +
 (3*b*Cot[(c + d*x)/2])/(2*a^4*d) - Csc[(c + d*x)/2]^2/(8*a^3*d) - (3*(a^2
+ 4*b^2)*Log[Cos[(c + d*x)/2]])/(2*a^5*d) + (3*(a^2 + 4*b^2)*Log[Sin[(c + d
*x)/2]])/(2*a^5*d) + Sec[(c + d*x)/2]^2/(8*a^3*d) + Sin[(c + d*x)/2]/((a +
b)^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - Sin[(c + d*x)/2]/((a - b)^3
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (b^6*Cos[c + d*x])/(2*a^3*(a -
b)^2*(a + b)^2*d*(a + b*Sin[c + d*x])^2) + (13*a^2*b^6*Cos[c + d*x] - 6*b^8
```

```
*Cos[c + d*x])/(2*a^4*(a - b)^3*(a + b)^3*d*(a + b*Sin[c + d*x])) - (3*b*Ta
n[(c + d*x)/2])/(2*a^4*d)
```

Maple [A]
time = 1.29, size = 355, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^4}+\frac{2b^5\left(\frac{\left(\frac{15}{2}a^3b^2-4a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7b\left(2a^4+3a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(41a^2-\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{(a-b)^3(}}{}$
default	$\frac{\frac{\frac{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-6b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a^4}+\frac{2b^5\left(\frac{\left(\frac{15}{2}a^3b^2-4a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{7b\left(2a^4+3a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(41a^2-\right.}{}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\right.}{(a-b)^3(}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4/a^4*(1/2*a*tan(1/2*d*x+1/2*c)^2-6*b*tan(1/2*d*x+1/2*c))+2*b^5/(a-b
)^3/(a+b)^3/a^5*(((15/2*a^3*b^2-4*a*b^4)*tan(1/2*d*x+1/2*c)^3+7/2*b*(2*a^4+
3*a^2*b^2-2*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(41*a^2-20*b^2)*tan(1/2*d*x
+1/2*c)+7/2*a^2*b*(2*a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*
c)+a)^2+3/2*(14*a^4-13*a^2*b^2+4*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1
/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/8/a^3/tan(1/2*d*x+1/2*c)^2+1/4/a^5*(
6*a^2+24*b^2)*ln(tan(1/2*d*x+1/2*c))+3/2*b/a^4/tan(1/2*d*x+1/2*c)-1/(a+b)^3
/(tan(1/2*d*x+1/2*c)-1)+1/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(441) = 882$.
time = 2.77, size = 2672, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] [1/4*(4*a^12 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a
^8*b^4 + 82*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12
- 31*a^10*b^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x
 + c)^2 + 3*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^
5 + 15*a^4*b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^
7 - 9*a^2*b^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^
10)*cos(d*x + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 3*(
(a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x + c)^5
- (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12)*cos(
d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 - 11*a^
2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*a
^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7 - 15*
a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x +
 c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^12
)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8 -
 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
- 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d*x + c)
 + 1/2) - 2*(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*b^3 - 2
8*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6*a^11*b
 - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos(d*x +
c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b
^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6 - 7*a^7
*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^4 + 2*a
^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*b^3 + 6
*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12*b^3 +
6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c)), 1/4*(4*a^1
2 - 12*a^10*b^2 + 12*a^8*b^4 - 4*a^6*b^6 - 2*(21*a^10*b^2 - 56*a^8*b^4 + 82
*a^6*b^6 - 65*a^4*b^8 + 18*a^2*b^10)*cos(d*x + c)^4 - 2*(3*a^12 - 31*a^10*b
^2 + 68*a^8*b^4 - 88*a^6*b^6 + 66*a^4*b^8 - 18*a^2*b^10)*cos(d*x + c)^2 - 6
*((14*a^4*b^7 - 13*a^2*b^9 + 4*b^11)*cos(d*x + c)^5 - (14*a^6*b^5 + 15*a^4*
```

```
b^7 - 22*a^2*b^9 + 8*b^11)*cos(d*x + c)^3 + (14*a^6*b^5 + a^4*b^7 - 9*a^2*b
^9 + 4*b^11)*cos(d*x + c) - 2*((14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x
 + c)^3 - (14*a^5*b^6 - 13*a^3*b^8 + 4*a*b^10)*cos(d*x + c))*sin(d*x + c))*
sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))
) - 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*cos(d*x
+ c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 + 8*b^1
2)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^4*b^8
- 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a^5*b^7
 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*a^5*b^
7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(1/2*cos(d*x + c)
 + 1/2) + 3*((a^10*b^2 - 10*a^6*b^6 + 20*a^4*b^8 - 15*a^2*b^10 + 4*b^12)*co
s(d*x + c)^5 - (a^12 + 2*a^10*b^2 - 10*a^8*b^4 + 25*a^4*b^8 - 26*a^2*b^10 +
 8*b^12)*cos(d*x + c)^3 + (a^12 + a^10*b^2 - 10*a^8*b^4 + 10*a^6*b^6 + 5*a^
4*b^8 - 11*a^2*b^10 + 4*b^12)*cos(d*x + c) - 2*((a^11*b - 10*a^7*b^5 + 20*a
^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c)^3 - (a^11*b - 10*a^7*b^5 + 20*
a^5*b^7 - 15*a^3*b^9 + 4*a*b^11)*cos(d*x + c))*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) - 2*(2*a^11*b - 6*a^9*b^3 + 6*a^7*b^5 - 2*a^5*b^7 + (12*a^9*
b^3 - 28*a^7*b^5 + 47*a^5*b^7 - 43*a^3*b^9 + 12*a*b^11)*cos(d*x + c)^4 - (6
*a^11*b - 10*a^9*b^3 + 2*a^7*b^5 + 29*a^5*b^7 - 39*a^3*b^9 + 12*a*b^11)*cos
(d*x + c)^2)*sin(d*x + c))/((a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8
+ a^5*b^10)*d*cos(d*x + c)^5 - (a^15 - 2*a^13*b^2 - 2*a^11*b^4 + 8*a^9*b^6
- 7*a^7*b^8 + 2*a^5*b^10)*d*cos(d*x + c)^3 + (a^15 - 3*a^13*b^2 + 2*a^11*b^
4 + 2*a^9*b^6 - 3*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) - 2*((a^14*b - 4*a^12*
b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^3 - (a^14*b - 4*a^12
*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c))*sin(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(441) = 882.
time = 0.66, size = 900, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(24*(14*a^4*b^5 - 13*a^2*b^7 + 4*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^11 - 3*
a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(a^2 - b^2)) + 16*(3*a^2*b*tan(1/2*d*x +
 1/2*c) + b^3*tan(1/2*d*x + 1/2*c) - a^3 - 3*a*b^2)/((a^6 - 3*a^4*b^2 + 3*a
^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)) - (6*a^10*tan(1/2*d*x + 1/2*c)^
6 + 6*a^8*b^2*tan(1/2*d*x + 1/2*c)^6 - 54*a^6*b^4*tan(1/2*d*x + 1/2*c)^6 +
66*a^4*b^6*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^8*tan(1/2*d*x + 1/2*c)^6 + 12*
a^9*b*tan(1/2*d*x + 1/2*c)^5 + 60*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 252*a^5*
b^5*tan(1/2*d*x + 1/2*c)^5 + 156*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 32*a*b^9*
tan(1/2*d*x + 1/2*c)^5 + 13*a^10*tan(1/2*d*x + 1/2*c)^4 - 15*a^8*b^2*tan(1/
2*d*x + 1/2*c)^4 + 63*a^6*b^4*tan(1/2*d*x + 1/2*c)^4 - 341*a^4*b^6*tan(1/2*
d*x + 1/2*c)^4 + 96*a^2*b^8*tan(1/2*d*x + 1/2*c)^4 + 16*b^10*tan(1/2*d*x +
1/2*c)^4 + 4*a^9*b*tan(1/2*d*x + 1/2*c)^3 + 36*a^7*b^3*tan(1/2*d*x + 1/2*c)
^3 - 132*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 188*a^3*b^7*tan(1/2*d*x + 1/2*c)^
3 + 112*a*b^9*tan(1/2*d*x + 1/2*c)^3 + 8*a^10*tan(1/2*d*x + 1/2*c)^2 - 44*a
^8*b^2*tan(1/2*d*x + 1/2*c)^2 + 84*a^6*b^4*tan(1/2*d*x + 1/2*c)^2 - 180*a^4
*b^6*tan(1/2*d*x + 1/2*c)^2 + 76*a^2*b^8*tan(1/2*d*x + 1/2*c)^2 - 8*a^9*b*t
an(1/2*d*x + 1/2*c) + 24*a^7*b^3*tan(1/2*d*x + 1/2*c) - 24*a^5*b^5*tan(1/2*
d*x + 1/2*c) + 8*a^3*b^7*tan(1/2*d*x + 1/2*c) + a^10 - 3*a^8*b^2 + 3*a^6*b^
4 - a^4*b^6)/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2
*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))^2) + 12*(a^2 +
 4*b^2)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + (a^3*tan(1/2*d*x + 1/2*c)^2 -
12*a^2*b*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 16.82, size = 2500, normalized size = 5.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*sin(c + d*x)^3*(a + b*sin(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (4*a^2*b*tan(c/2 + (d*x)/2) - a^3/2 + (5*t
an(c/2 + (d*x)/2)^2*(3*a^9 - 20*a*b^8 + 49*a^3*b^6 - 27*a^5*b^4 + 19*a^7*b^
2))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^7*(15*a
^8*b - 16*b^9 + 27*a^2*b^7 + 9*a^4*b^5 - 5*a^6*b^3))/(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2) - (4*tan(c/2 + (d*x)/2)^5*(5*a^8*b - 18*b^9 + 43*a^2*b^7 - 7*
a^4*b^5 + 7*a^6*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d
*x)/2)^3*(7*a^8*b - 52*b^9 + 115*a^2*b^7 - 27*a^4*b^5 + 47*a^6*b^3))/(a^6 -
 b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(33*a^10 - 112*b^10 +
 220*a^2*b^8 + 11*a^4*b^6 + 119*a^6*b^4 - 31*a^8*b^2))/(2*a*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)) + (tan(c/2 + (d*x)/2)^6*(17*a^10 + 112*b^10 - 120*a^2
*b^8 - 257*a^4*b^6 + 83*a^6*b^4 - 195*a^8*b^2))/(2*a*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)))/(d*(4*a^6*tan(c/2 + (d*x)/2)^2 - 4*a^6*tan(c/2 + (d*x)/2)^8
 + tan(c/2 + (d*x)/2)^4*(4*a^6 + 16*a^4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^6
```

+ 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 - 16*a^5*b*tan(c/2 + (d*x)/2)
^7)) - (3*b*tan(c/2 + (d*x)/2))/(2*a^4*d) + (log(tan(c/2 + (d*x)/2))*(3*a^2
 + 12*b^2))/(2*a^5*d) + (b^5*atan(((b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^
4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*
a^10*b^20 + 14616*a^12*b^18 - 34872*a^14*b^16 + 52800*a^16*b^14 - 52176*a^1
8*b^12 + 33168*a^20*b^10 - 12576*a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 -
168*a^28*b^2) - 24*a^29*b + 192*a^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17
- 15816*a^15*b^15 + 22800*a^17*b^13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 37
68*a^23*b^7 + 336*a^25*b^5 + 120*a^27*b^3 - (3*b^5*(-(a + b)^7*(a - b)^7)^(
1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b
^20 - 624*a^15*b^18 + 2736*a^17*b^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 1
4112*a^23*b^10 + 11424*a^25*b^8 - 6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*
b^2) - 16*a^32*b + 16*a^14*b^19 - 144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20
*b^13 + 2016*a^22*b^11 - 2016*a^24*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144
*a^30*b^3))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 -
35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*3i)/(2*(a^19 - a^5*b^14 + 7*a^7*b
^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2))
+ (b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2
+ (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*a^12*b^18 - 348
72*a^14*b^16 + 52800*a^16*b^14 - 52176*a^18*b^12 + 33168*a^20*b^10 - 12576*
a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 - 168*a^28*b^2) - 24*a^29*b + 192*a
^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17 - 15816*a^15*b^15 + 22800*a^17*b^
13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 3768*a^23*b^7 + 336*a^25*b^5 + 120*
a^27*b^3 + (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2
)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b^20 - 624*a^15*b^18 + 2736*a^17*b
^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 14112*a^23*b^10 + 11424*a^25*b^8 -
 6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*b^2) - 16*a^32*b + 16*a^14*b^19 -
 144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20*b^13 + 2016*a^22*b^11 - 2016*a^2
4*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144*a^30*b^3))/(2*(a^19 - a^5*b^14 +
 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^1
7*b^2)))*3i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 -
 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))/(2*tan(c/2 + (d*x)/2)*(576*a^5*b
^22 - 5040*a^7*b^20 + 18072*a^9*b^18 - 34128*a^11*b^16 + 35640*a^13*b^14 -
20376*a^15*b^12 + 7200*a^17*b^10 - 2952*a^19*b^8 + 1008*a^21*b^6) + 1152*a^
4*b^23 - 10368*a^6*b^21 + 41112*a^8*b^19 - 92448*a^10*b^17 + 126792*a^12*b^
15 - 105552*a^14*b^13 + 48168*a^16*b^11 - 6912*a^18*b^9 - 2952*a^20*b^7 + 1
008*a^22*b^5 - (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2
b^2)(tan(c/2 + (d*x)/2)*(24*a^30 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*
a^12*b^18 - 34872*a^14*b^16 + 52800*a^16*b^14 - 52176*a^18*b^12 + 33168*a^2
0*b^10 - 12576*a^22*b^8 + 2112*a^24*b^6 + 240*a^26*b^4 - 168*a^28*b^2) - 24
*a^29*b + 192*a^9*b^21 - 1728*a^11*b^19 + 6888*a^13*b^17 - 15816*a^15*b^15
+ 22800*a^17*b^13 - 21048*a^19*b^11 + 12048*a^21*b^9 - 3768*a^23*b^7 + 336*
a^25*b^5 + 120*a^27*b^3 - (3*b^5*(-(a + b)^7*(a - b)^7)^(1/2)*(14*a^4 + 4*b
^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(48*a^33 + 64*a^13*b^20 - 624*a^15*b^1
8 + 2736*a^17*b^16 - 7104*a^19*b^14 + 12096*a^21*b^12 - 14112*a^23*b^10 + 1

```
1424*a^25*b^8 - 6336*a^27*b^6 + 2304*a^29*b^4 - 496*a^31*b^2) - 16*a^32*b +
 16*a^14*b^19 - 144*a^16*b^17 + 576*a^18*b^15 - 1344*a^20*b^13 + 2016*a^22*
b^11 - 2016*a^24*b^9 + 1344*a^26*b^7 - 576*a^28*b^5 + 144*a^30*b^3))/(2*(a^
19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a
^15*b^4 - 7*a^17*b^2))))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 + 3
5*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (3*b^5*(-(a + b)^7*
(a - b)^7)^(1/2)*(14*a^4 + 4*b^4 - 13*a^2*b^2)*(tan(c/2 + (d*x)/2)*(24*a^30
 + 384*a^8*b^22 - 3552*a^10*b^20 + 14616*a^12*b^18 - 34872*a^14*b^16 + 5280
0*a^16*b^14 - 52176*a^18*b^12 + 33168*a^20*b^10...
```

3.1478 $\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=158

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\,\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\,\sqrt{\frac{a(1+\csc(e+fx))}{a-b}}\,F(\ldots}{\sqrt{d}\ldots}$$

[Out] sec(f*x+e)*(d*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e))^(1/2)/d/f-EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(a+b)^(1/2)*(a*(1-csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2967, 2895}

$$\frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df}-\frac{\sqrt{a+b}\,\tan(e+fx)\sqrt{\frac{a(1-\csc(e+fx))}{a+b}}\,\sqrt{\frac{a(\csc(e+fx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{d}\,\sqrt{a+b\sin(e+fx)}}{\sqrt{a+b}\,\sqrt{d\sin(e+fx)}}\right)|-\frac{a+b}{a-b}\right)}{\sqrt{d}\,f}$$

Antiderivative was successfully verified.

[In] Int[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]

[Out] (Sec[e + f*x]*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/(d*f) - (Sqrt[a + b]*Sqrt[(a*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(a*(1 + Csc[e + f*x]))/(a - b)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*Sin[e + f*x]])/(Sqrt[a + b]*Sqrt[d*Sin[e + f*x]])], -((a + b)/(a - b))]*Tan[e + f*x])/(Sqrt[d]*f)

Rule 2895

```
Int[1/(Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]]*Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]), x_Symbol] :> Simp[-2*(Tan[e + f*x]/(a*f))*Rt[(a + b)/d, 2]*Sqrt[a*((1 - Csc[e + f*x])/(a + b))]*Sqrt[a*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Sin[e + f*x]]/Sqrt[d*Sin[e + f*x]]/Rt[(a + b)/d, 2]], -(a + b)/(a - b)], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && PosQ[(a + b)/d]
```

Rule 2967

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_))/Sqrt[(d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> Simp[2*(g*Cos[e + f*x])^(p + 1)*Sqrt[d*Sin[e + f*x]]*((a + b*Sin[e + f*x])^m/(d*f*g*(2*m + 1))), x] + Dist[2*a*(m/(g^2*(2*m + 1))), Int[(g*Cos[e + f*x])^(p + 2)*((
```

```
a + b*Sin[e + f*x])^(m - 1)/Sqrt[d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, e
, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && EqQ[m + p + 3/2, 0]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)\sqrt{a+b\sin(e+fx)}}{\sqrt{d\sin(e+fx)}}\,dx = \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df} + \frac{1}{2}a\int \frac{}{\sqrt{d\sin(}}$$

$$= \frac{\sec(e+fx)\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}{df} - \frac{\sqrt{a+b}\,\sqrt{\frac{a(1}{}}}{}$$

Mathematica [A]
time = 23.75, size = 198, normalized size = 1.25

$$\frac{4a^2\sqrt{-\frac{(a+b)\cot^2\left(\frac{1}{4}(2e-\pi+2fx)\right)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{a+b\sin(e+fx)}{a(-1+\sin(e+fx))}}\right)|\frac{2a}{a-b}\right)\sec(e+fx)\sqrt{-\frac{(a+b)\sin(e+fx)(a+b\sin(e+fx))}{a^2(-1+\sin(e+fx))^2}}\,\sin^4\left(\frac{1}{4}(2e-\pi+2fx)\right)+(a+b)(a+b\sin(e+fx))\tan(e+fx)}{(a+b)f\sqrt{d\sin(e+fx)}\,\sqrt{a+b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]])/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (4*a^2*Sqrt[-(((a + b)*Cot[(2*e - Pi + 2*f*x)/4]^2)/(a - b))]*EllipticF[Arc
Sin[Sqrt[-((a + b*Sin[e + f*x])/(a*(-1 + Sin[e + f*x])))]], (2*a)/(a - b)]*
Sec[e + f*x]*Sqrt[-(((a + b)*Sin[e + f*x]*(a + b*Sin[e + f*x]))/(a^2*(-1 +
Sin[e + f*x])^2))]*Sin[(2*e - Pi + 2*f*x)/4]^4 + (a + b)*(a + b*Sin[e + f*x
])*Tan[e + f*x])/((a + b)*f*Sqrt[d*Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(142) = 284$.
time = 3.54, size = 650, normalized size = 4.11

method	result
default	$-\frac{\left(\sin(fx+e)\cos(fx+e)\sqrt{\frac{\sqrt{-a^2+b^2}\,\sin(fx+e)+b\sin(fx+e)-\cos(fx+e)a+a}{\sin(fx+e)\left(b+\sqrt{-a^2+b^2}\right)}}\,\sqrt{\frac{\cos(fx+e)a+\sqrt{-a^2+b^2}\,\sin(fx+e)}{\sqrt{-a^2+b^2}\,\sin(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(sin(f*x+e)*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-co
s(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*((cos(f*x+e)*a+(-a^2+b
^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*
(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-a^2+b^
2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(
1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a
^2+b^2)^(1/2)+sin(f*x+e)*cos(f*x+e)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x
+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*((cos(f*x+e)*a+(
-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/
2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticF((((-
a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+
b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2
))*b-sin(f*x+e)*cos(f*x+e)*2^(1/2)*b+sin(f*x+e)*2^(1/2)*b-cos(f*x+e)*2^(1/2
)*a+2^(1/2)*a)*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)/(d*sin(f*x+e))^(1/2)/(
a+b*sin(f*x+e))^(1/2)*2^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral(sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))*sec(f*x + e)^2/(d*si
n(f*x + e)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sin{(e + fx)}}\sec^2{(e + fx)}}{\sqrt{d\sin{(e + fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(1/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x))*sec(e + f*x)**2/sqrt(d*sin(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e) + a)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sin\left(e+f\,x\right)}}{{\cos\left(e+f\,x\right)}^2\,\sqrt{d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(1/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b*sin(e + f*x))^(1/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)), x)
```

3.1479 $\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=312

$$\frac{\sec(e+fx)(b+a\sin(e+fx))\sqrt{a+b\sin(e+fx)}}{f\sqrt{d\sin(e+fx)}}-\frac{(a+b)^{3/2}\sqrt{-\frac{a(-1+\csc(e+fx))}{a+b}}\sqrt{\frac{a(1+\csc(e+f}{a-b}}}{}$$

```
[Out] sec(f*x+e)*(b+a*sin(f*x+e))*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-(
a+b)^(3/2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+
e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(
f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)-b*(a+b)*EllipticE(((-b-a*csc(f*x+
e))/(a-b))^(1/2),((-a+b)/(a+b))^(1/2))*(1+sin(f*x+e))*(-a*(-1+csc(f*x+e))/(
a+b))^(1/2)*((b+a*csc(f*x+e))/(-a+b))^(1/2)*tan(f*x+e)/f/(a*(1+csc(f*x+e))/
(a-b))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [F]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]]
, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx=\int \frac{\sec^2(e+fx)(a+b\sin(e+fx))^{3/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4593 vs. 2(312) = 624.
time = 32.90, size = 4593, normalized size = 14.72

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e + f*x]], x]

[Out] (Csc[(e + f*x)/2]^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]]*((a*Sqrt[a + b*Sin[e + f*x]])/(2*Sqrt[Sin[e + f*x]]) - b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(16*f*Sqrt[d*Sin[e + f*x]]*((b*Cos[e + f*x]*Csc[(e + f*x)/2]^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]^(7/2)*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(32*Sqrt[a + b*Sin[e + f*x]]) + (7*Cos[e + f*x]*Csc[(e + f*x)/2]^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]^(5/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/32 - (Csc[(e + f*x)/2]^5*Sec[(e + f*x)/2]*Sin[e + f*x]^(7/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/8 + (Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]^3*Sin[e + f*x]^(7/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Tan[(e + f*x)/2]^

```
2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a
^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f
*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 +
b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] +
a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + S
qrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-
((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/((a + b*Sin[e + f*x])*Sqrt
[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/16 + (Csc[(e + f*x)/2]^4*
Sec[(e + f*x)/2]^2*Sin[e + f*x]^(7/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Sec[(e
 + f*x)/2]^2*Tan[(e + f*x)/2] - (a*Sqrt[-a^2 + b^2]*Sec[(e + f*x)/2]^2*Sqrt
[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[Ar
cSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sq
rt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a
*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^
2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Ta
n[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + S
qrt[-a^2 + b^2]))]]))/(2*(-b + Sqrt[-a^2 + b^2])*(a + b*Sin[e + f*x])*((a*Ta
n[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) - (2*b*Sqrt[-a^2 + b^2]*Cos
[e + f*x]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(
b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-
a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e
+ f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x
)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2
])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f
*x)/2])/(b + Sqrt[-a^2 + b^2]))]]))/((a + b*Sin[e + f*x])^2*Sqrt[(a*Tan[(e +
 f*x)/2])/(-b + Sqrt[-a^2 + b^2])]) + (Sqrt[-a^2 + b^2]*((a*b*Cos[e + f*x]*
Sec[(e + f*x)/2]^2)/(a^2 - b^2) + (a*Sec[(e + f...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2312 vs. $2(287) = 574$.

time = 0.32, size = 2313, normalized size = 7.41

method	result	size
default	Expression too large to display	2313

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/f*(2*cos(f*x+e)^2*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x
+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^
(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+
e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(
-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*s
in(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*(-a^2+b^2)
```

```
^(1/2)*b^2-2*cos(f*x+e)^2*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(
f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^
2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f
*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((
b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+
b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*a^2*b+2
*cos(f*x+e)^2*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-
a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/si
n(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*
x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)
^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)
-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*b^3-cos(f*x+e)^2*((
cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/s
in(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*
(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-
a^2+b^2)^(1/2)))^(1/2)*EllipticF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)
-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a
^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(-a^2+b^2)^(1/2)*a^2+2*cos(f*x+e)*(
(cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/
sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)
*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f
*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b
^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a
)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*(-a^2+b^2)^(1/2)*b^2-2*cos(f*x+e)*
((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)
/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2
)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(
f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+
b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+
a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*a^2*b+2*cos(f*x+e)*((cos(f*x+e)*a
+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(
1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE(((
(-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^
2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1
/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/
(b+(-a^2+b^2)^(1/2)))^(1/2)*b^3-cos(f*x+e)*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*
sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*
x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+
b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*Ellipti
cF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b
+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2
))^(1/2))*(-a^2+b^2)^(1/2)*a^2+2^(1/2)*sin(f*x+e)*cos(f*x+e)*a*b^2+2^(1/2)*
cos(f*x+e)^2*a^2*b-2^(1/2)*sin(f*x+e)*a^3-2^(1/2)*sin(f*x+e)*a*b^2+2^(1/2)*
cos(f*x+e)*a^2*b-2*2^(1/2)*a^2*b)/cos(f*x+e)/(d*sin(f*x+e))^(1/2)/(a+b*sin(
f*x+e))^(1/2)*2^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((b*sec(f*x + e)^2*sin(f*x + e) + a*sec(f*x + e)^2)*sqrt(b*sin(f*x
+ e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e))**(3/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e))^(3/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(3/2)*sec(f*x + e)^2/sqrt(d*sin(f*x + e)), x
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\sin(e+fx))^{3/2}}{\cos(e+fx)^2\sqrt{d\sin(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(3/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)),x)
[Out] int((a + b*sin(e + f*x))^(3/2)/(cos(e + f*x)^2*(d*sin(e + f*x))^(1/2)), x)
```

3.1480 $\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=366

$$\frac{5a\sec(e+fx)(b+a\sin(e+fx))\sqrt{a+b\sin(e+fx)}}{6f\sqrt{d\sin(e+fx)}}+\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{3df}$$

```
[Out] 1/3*sec(f*x+e)^3*(a+b*sin(f*x+e))^(5/2)*(d*sin(f*x+e))^(1/2)/d/f+5/6*a*sec(
f*x+e)*(b+a*sin(f*x+e))*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-5/6*a
*(a+b)^(3/2)*EllipticF(d^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*
x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+cs
c(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d^(1/2)-5/6*a*b*(a+b)*EllipticE(((-b-a*
csc(f*x+e))/(a-b))^(1/2),((-a+b)/(a+b))^(1/2))*(1+sin(f*x+e))*(-a*(-1+csc(f
*x+e))/(a+b))^(1/2)*((b+a*csc(f*x+e))/(-a+b))^(1/2)*tan(f*x+e)/f/(a*(1+csc(
f*x+e))/(a-b))^(1/2)/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)
```

Rubi [F]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]^3*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(5/2))/(3*d*f) +
(5*a*Defer[Int][(Sec[e + f*x]^2*(a + b*Sin[e + f*x])^(3/2))/Sqrt[d*Sin[e +
f*x]], x])/6
```

Rubi steps

$$\int \frac{\sec^4(e+fx)(a+b\sin(e+fx))^{5/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^3(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{5/2}}{3df}+\frac{1}{6}(5a)\int$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4665 vs. 2(366) = 732.
time = 32.70, size = 4665, normalized size = 12.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(5/2))/Sqrt[d*Sin[e + f*x]],
x]
```

```
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((Sec[e + f*x]^3*(a^2 + b^2 + 2*a*b*
Sin[e + f*x]))/3 + (Sec[e + f*x]*(5*a^2 - 2*b^2 + 5*a*b*Sin[e + f*x]))/6))/
(f*Sqrt[d*Sin[e + f*x]]) + (5*a*Csc[(e + f*x)/2]^4*Sec[(e + f*x)/2]^2*Sin[e
 + f*x]^4*Sqrt[a + b*Sin[e + f*x]]*((5*a^2*Sqrt[a + b*Sin[e + f*x]])/(12*Sq
rt[Sin[e + f*x]]) - (5*a*b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])/6)*
(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(
a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^
2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b
^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(
b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*S
qrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sq
rt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a
 + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/(9
6*f*Sqrt[d*Sin[e + f*x]]*((5*a*b*Cos[e + f*x]*Csc[(e + f*x)/2]^4*Sec[(e + f
*x)/2]^2*Sin[e + f*x]^(7/2)*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*
Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*Elliptic
E[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]
]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2])
 + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt
[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(
a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b
 + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-
b + Sqrt[-a^2 + b^2])])))/(192*Sqrt[a + b*Sin[e + f*x]]) + (35*a*Cos[e + f*
x]*Csc[(e + f*x)/2]^4*Sec[(e + f*x)/2]^2*Sin[e + f*x]^(5/2)*Sqrt[a + b*Sin[
e + f*x]]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt[(a*Sec[(e + f
*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt[(-b
+ Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqr
t[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt
[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*x)/2]
)/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2 + b^2
]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^
2])])))/192 - (5*a*Csc[(e + f*x)/2]^5*Sec[(e + f*x)/2]*Sin[e + f*x]^(7/2)*S
qrt[a + b*Sin[e + f*x]]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-a^2 + b^2]*Sqrt
[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[Ar
cSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sq
rt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a
*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^
2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Ta
n[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + S
```

```
qrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e + f*x)/2])/(-b +
Sqrt[-a^2 + b^2])])))/48 + (5*a*Csc[(e + f*x)/2]^3*Sec[(e + f*x)/2]^3*Sin[e
 + f*x]^(7/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Tan[(e + f*x)/2]^2 + (2*Sqrt[-
a^2 + b^2]*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-
(b*EllipticE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[
-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e
 + f*x)/2]) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*
x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^
2])]*Sqrt[(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e +
f*x)/2])/(b + Sqrt[-a^2 + b^2]))]))/((a + b*Sin[e + f*x])*Sqrt[(a*Tan[(e +
f*x)/2])/(-b + Sqrt[-a^2 + b^2])])))/96 + (5*a*Csc[(e + f*x)/2]^4*Sec[(e +
f*x)/2]^2*Sin[e + f*x]^(7/2)*Sqrt[a + b*Sin[e + f*x]]*(-2*b*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2] - (a*Sqrt[-a^2 + b^2]*Sec[(e + f*x)/2]^2*Sqrt[(a*Sec[(
e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*EllipticE[ArcSin[Sqrt
[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]/Sqrt[2]], (
2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]) + a*Elliptic
F[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2]]
/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[(a*Tan[(e + f*
x)/2])/(-b + Sqrt[-a^2 + b^2])]*Sqrt[-((a*Tan[(e + f*x)/2])/(b + Sqrt[-a^2
+ b^2]))]))/(2*(-b + Sqrt[-a^2 + b^2])*(a + b*Sin[e + f*x])*((a*Tan[(e + f*
x)/2])/(-b + Sqrt[-a^2 + b^2]))^(3/2)) - (2*b*Sqrt[-a^2 + b^2]*Cos[e + f*x]
*Sqrt[(a*Sec[(e + f*x)/2]^2*(a + b*Sin[e + f*x]))/(a^2 - b^2)]*(-(b*Ellipti
cE[ArcSin[Sqrt[(-b + Sqrt[-a^2 + b^2] - a*Tan[(e + f*x)/2])/Sqrt[-a^2 + b^2
]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(-b + Sqrt[-a^2 + b^2])]*Tan[(e + f*x)/2]
) + a*EllipticF[ArcSin[Sqrt[(b + Sqrt[-a^2 + b^2] + a*Tan[(e + f*x)/2])/Sqr
t[-a^2 + b^2]]/Sqrt[2]], (2*Sqrt[-a^2 + b^2])/(b + Sqrt[-a^2 + b^2])]*Sqrt[
(a*Tan[(e + f*x)/2])/(-b + Sqrt[-a^2 + b^2])]*S...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2425 vs. $2(329) = 658$.
time = 0.40, size = 2426, normalized size = 6.63

method	result	size
default	Expression too large to display	2426

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/12/f*(10*(-a^2+b^2)^(1/2)*cos(f*x+e)^4*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*s
in(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x
+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*si
n(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2
),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(
1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)
```

))^(1/2)*b^2-5*(-a^2+b^2)^(1/2)*cos(f*x+e)^4*((cos(f*x+e)*a+(-a^2+b^2)^(1/2
)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(
f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e
)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*Ellip
ticF((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/
(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1
/2))^(1/2))*a^2-10*cos(f*x+e)^4*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-
b*sin(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-
a^2+b^2)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b
*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1
/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f
*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*a
^2*b+10*cos(f*x+e)^4*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e
)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1
/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)
-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a
^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin
(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*b^3+10*(-a^2
+b^2)^(1/2)*cos(f*x+e)^3*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f
*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2
)^(1/2))/sin(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*
x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b
+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b
*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*b^2-5*(-
a^2+b^2)^(1/2)*cos(f*x+e)^3*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*si
n(f*x+e)-a)/(-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+
b^2)^(1/2))/sin(f*x+e))^(1/2)*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-co
s(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*EllipticF((((-a^2+b^2)
^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/
2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*a^2-1
0*cos(f*x+e)^3*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(
-a^2+b^2)^(1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/s
in(f*x+e))^(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f
*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2
)^(1/2))/(-a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e
)-cos(f*x+e)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*a^2*b+10*cos(f*x+e
)^3*((cos(f*x+e)*a+(-a^2+b^2)^(1/2)*sin(f*x+e)-b*sin(f*x+e)-a)/(-a^2+b^2)^(
1/2)/sin(f*x+e))^(1/2)*(a*(-1+cos(f*x+e))/(b+(-a^2+b^2)^(1/2))/sin(f*x+e))^
(1/2)*EllipticE((((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e)*a+a)/
sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2),1/2*2^(1/2)*((b+(-a^2+b^2)^(1/2))/(-
a^2+b^2)^(1/2))^(1/2))*(((-a^2+b^2)^(1/2)*sin(f*x+e)+b*sin(f*x+e)-cos(f*x+e
)*a+a)/sin(f*x+e)/(b+(-a^2+b^2)^(1/2)))^(1/2)*b^3+5*2^(1/2)*sin(f*x+e)*cos(
f*x+e)^3*a*b^2+5*2^(1/2)*cos(f*x+e)^4*a^2*b-2*2^(1/2)*cos(f*x+e)^4*b^3-5*2^
(1/2)*sin(f*x+e)*cos(f*x+e)^2*a^3+2^(1/2)*sin(f*x+e)*cos(f*x+e)^2*a*b^2+5*2
^(1/2)*cos(f*x+e)^3*a^2*b-4*2^(1/2)*cos(f*x+e)^2*a^2*b+4*2^(1/2)*cos(f*x+e)

```
^2*b^3-2*2^(1/2)*sin(f*x+e)*a^3-6*2^(1/2)*sin(f*x+e)*a*b^2-6*2^(1/2)*a^2*b-
2*2^(1/2)*b^3)/cos(f*x+e)^3/(d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(1/2)*2^(
1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
[Out] integral((2*a*b*sec(f*x + e)^4*sin(f*x + e) - (b^2*cos(f*x + e)^2 - a^2 - b
^2)*sec(f*x + e)^4)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))/(d*sin(f*
x + e)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(5/2)*sec(f*x + e)^4/sqrt(d*sin(f*x + e)), x
)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\sin\left(e+f\,x\right))^{5/2}}{{\cos\left(e+f\,x\right)}^4\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(5/2)/(cos(e + f*x)^4*(d*sin(e + f*x))^(1/2)),x)
[Out] int((a + b*sin(e + f*x))^(5/2)/(cos(e + f*x)^4*(d*sin(e + f*x))^(1/2)), x)
```

3.1481 $\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=155

$$\frac{35b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\cos^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{3a\sec^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{35b\sin(c}{8d}$$

```
[Out] 35/8*b*arctanh(sin(d*x+c))/d+1/2*a*cos(d*x+c)^2/d-3*a*ln(cos(d*x+c))/d-3/2*
a*sec(d*x+c)^2/d+1/4*a*sec(d*x+c)^4/d-35/8*b*sin(d*x+c)/d-35/24*b*sin(d*x+c
)^3/d-7/8*b*sin(d*x+c)^3*tan(d*x+c)^2/d+1/4*b*sin(d*x+c)^3*tan(d*x+c)^4/d
```

Rubi [A]

time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2913, 2670, 272, 45, 2672, 294, 308, 212}

$$\frac{a\cos^2(c+dx)}{2d}+\frac{a\sec^4(c+dx)}{4d}-\frac{3a\sec^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{35b\sin^3(c+dx)}{24d}-\frac{35b\sin(c+dx)}{8d}+\frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d}-\frac{7b\sin^3(c+dx)\tan^2(c+dx)}{8d}+\frac{35b\tanh^{-1}(\sin(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (35*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Cos[c + d*x]^2)/(2*d) - (3*a*Log[Co
s[c + d*x]])/d - (3*a*Sec[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]^4)/(4*d) - (3
5*b*Sin[c + d*x])/(8*d) - (35*b*Sin[c + d*x]^3)/(24*d) - (7*b*Sin[c + d*x]^
3*Tan[c + d*x]^2)/(8*d) + (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx &= a\int \sin^2(c+dx)\tan^5(c+dx)\,dx+b\int \sin^3(c+dx)\,\mathrm{ta} \\
&= -\frac{a\text{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\,dx,x,\cos(c+dx)\right)}{d}+\frac{b\text{Subst}\left(\int \frac{}{(1-}\right.}{} \\
&= \frac{b\sin^3(c+dx)\tan^4(c+dx)}{4d}-\frac{a\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx,x,\mathrm{co}\right.}{2d} \\
&= -\frac{7b\sin^3(c+dx)\tan^2(c+dx)}{8d}+\frac{b\sin^3(c+dx)\tan^4(c+}{4d} \\
&= \frac{a\cos^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{3a\sec^2(c+dx}{2d} \\
&= \frac{a\cos^2(c+dx)}{2d}-\frac{3a\log(\cos(c+dx))}{d}-\frac{3a\sec^2(c+dx}{2d} \\
&= \frac{35b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\cos^2(c+dx)}{2d}-\frac{3a\log(\cos}{d}
\end{aligned}
$$

Mathematica [A]

time = 0.45, size = 156, normalized size = 1.01

$$
-\frac{a(12\log(\cos(c+dx))+6\sec^2(c+dx)-\sec^4(c+dx)+2\sin^2(c+dx))}{4d}-\frac{b\sin^3(c+dx)\tan^4(c+dx)}{3d}-\frac{7b(8\sin(c+dx)\tan^4(c+dx)+5(6\sec^3(c+dx)\tan(c+dx)-8\sec(c+dx)\tan^3(c+dx)-3(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx))))}{24d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(a*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (b*Sin[c + d*x]^3*Tan[c + d*x]^4)/(3*d) - (7*b*(8*Sin[c + d
*x]*Tan[c + d*x]^4 + 5*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*Sec[c + d*x]*Tan[
c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))))/(24*d
)
```

Maple [A]

time = 0.26, size = 177, normalized size = 1.14

method	result
derivativedivides	$\frac{a\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+b\left(\frac{\sin^9(dx+c)}{4\cos(dx+c)^4}-\frac{5\left(\sin\right.}{8\cos}\right.}{d}$
default	$\frac{a\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+b\left(\frac{\sin^9(dx+c)}{4\cos(dx+c)^4}-\frac{5\left(\sin\right.}{8\cos}\right.}{d}$

risch	$3iax - \frac{ib\,e^{3i(dx+c)}}{24d} + \frac{a\,e^{2i(dx+c)}}{8d} + \frac{13ib\,e^{i(dx+c)}}{8d} - \frac{13ib\,e^{-i(dx+c)}}{8d} + \frac{a\,e^{-2i(dx+c)}}{8d} + \frac{ib\,e^{-3i(dx+c)}}{24d} + \frac{6iac}{d} +$
norman	$\frac{\frac{22a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{22a\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a}{3d}-\frac{35b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{35b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{329b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{17b\left(\tan^7\left(\frac{dx}{2}+\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin
(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+b*(1/4*sin(d*
x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*sin(d*x+c)^7-7/8*sin(
d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c)))
)
```

Maxima [A]
time = 0.31, size = 132, normalized size = 0.85

$$-\frac{16\,b\sin\left(dx+c\right)^3+24\,a\sin\left(dx+c\right)^2+3\,(24\,a-35\,b)\log\left(\sin\left(dx+c\right)+1\right)+3\,(24\,a+35\,b)\log\left(\sin\left(dx+c\right)-1\right)+144\,b\sin\left(dx+c\right)-\frac{6\left(13\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-11\,b\sin(dx+c)-10\,a\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(sin(
d*x + c) + 1) + 3*(24*a + 35*b)*log(sin(d*x + c) - 1) + 144*b*sin(d*x + c)
- 6*(13*b*sin(d*x + c)^3 + 12*a*sin(d*x + c)^2 - 11*b*sin(d*x + c) - 10*a)/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 149, normalized size = 0.96

$$\frac{24\,a\cos\left(dx+c\right)^6-3\,(24\,a-35\,b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(24\,a+35\,b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-12\,a\cos\left(dx+c\right)^4-72\,a\cos\left(dx+c\right)^2+2\left(8\,b\cos\left(dx+c\right)^6-80\,b\cos\left(dx+c\right)^4-39\,b\cos\left(dx+c\right)^2+6\,b\right)\sin\left(dx+c\right)+12\,a}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(24*a*cos(d*x + c)^6 - 3*(24*a - 35*b)*cos(d*x + c)^4*log(sin(d*x + c)
 + 1) - 3*(24*a + 35*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a*cos(d*
x + c)^4 - 72*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x + c)^6 - 80*b*cos(d*x + c)^
4 - 39*b*cos(d*x + c)^2 + 6*b)*sin(d*x + c) + 12*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**7*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 135, normalized size = 0.87

$$-\frac{16\,b\sin\left(dx+c\right)^{3}+24\,a\sin\left(dx+c\right)^{2}+3\,(24\,a-35\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,(24\,a+35\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+144\,b\sin\left(dx+c\right)-\frac{6\left(18\,a\sin(dx+c)^{4}+13\,b\sin(dx+c)^{3}-24\,a\sin(dx+c)^{2}-11\,b\sin(dx+c)+8\,a\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^7*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/48*(16*b*sin(d*x + c)^3 + 24*a*sin(d*x + c)^2 + 3*(24*a - 35*b)*log(abs(
sin(d*x + c) + 1)) + 3*(24*a + 35*b)*log(abs(sin(d*x + c) - 1)) + 144*b*sin
(d*x + c) - 6*(18*a*sin(d*x + c)^4 + 13*b*sin(d*x + c)^3 - 24*a*sin(d*x + c
)^2 - 11*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.27, size = 346, normalized size = 2.23

$$\frac{3\,a\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{-\frac{35\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}}{4}-6\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+\frac{35\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{6}+6\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{329\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}}{12}+16\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-17\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}+16\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+\frac{329\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}}{12}+6\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+\frac{35\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}}{6}-6\,a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-\frac{35\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}-1\right)}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(3\,a+\frac{35\,b}{8}\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(3\,a-\frac{35\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^7*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (6*a*tan(c/2 + (d*x)/2)^4 - 6*a*tan
(c/2 + (d*x)/2)^2 - (35*b*tan(c/2 + (d*x)/2))/4 + 16*a*tan(c/2 + (d*x)/2)^6
 + 16*a*tan(c/2 + (d*x)/2)^8 + 6*a*tan(c/2 + (d*x)/2)^10 - 6*a*tan(c/2 + (d
*x)/2)^12 + (35*b*tan(c/2 + (d*x)/2)^3)/6 + (329*b*tan(c/2 + (d*x)/2)^5)/12
 - 17*b*tan(c/2 + (d*x)/2)^7 + (329*b*tan(c/2 + (d*x)/2)^9)/12 + (35*b*tan(
c/2 + (d*x)/2)^11)/6 - (35*b*tan(c/2 + (d*x)/2)^13)/4)/(d*(tan(c/2 + (d*x)/
2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/
2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)
^14 - 1)) - (log(tan(c/2 + (d*x)/2) - 1)*(3*a + (35*b)/8))/d - (log(tan(c/2
 + (d*x)/2) + 1)*(3*a - (35*b)/8))/d
```

3.1482 $\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=135

$$\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+dx)}{2d}-\frac{3b\log(\cos(c+dx))}{d}-\frac{3b\sec^2(c+dx)}{2d}+\frac{b\sec^4(c+dx)}{4d}-\frac{15a\sin(c+}{8d}$$

```
[Out] 15/8*a*arctanh(sin(d*x+c))/d+1/2*b*cos(d*x+c)^2/d-3*b*ln(cos(d*x+c))/d-3/2*
b*sec(d*x+c)^2/d+1/4*b*sec(d*x+c)^4/d-15/8*a*sin(d*x+c)/d-5/8*a*sin(d*x+c)*
tan(d*x+c)^2/d+1/4*a*sin(d*x+c)*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2913, 2672, 294, 327, 212, 2670, 272, 45}

$$-\frac{15a\sin(c+dx)}{8d}+\frac{a\sin(c+dx)\tan^4(c+dx)}{4d}-\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\cos^2(c+dx)}{2d}+\frac{b\sec^4(c+dx)}{4d}-\frac{3b\sec^2(c+dx)}{2d}-\frac{3b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Cos[c + d*x]^2)/(2*d) - (3*b*Log[Co
s[c + d*x]])/d - (3*b*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^4)/(4*d) - (1
5*a*Sin[c + d*x])/(8*d) - (5*a*Sin[c + d*x]*Tan[c + d*x]^2)/(8*d) + (a*Sin[
c + d*x]*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \sin(c+dx)(a+b\sin(c+dx))\tan^5(c+dx)\,dx = a\int \sin(c+dx)\tan^5(c+dx)\,dx + b\int \sin^2(c+dx)\tan^5(
$$
$$= \frac{a\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\right.}{}$$
$$= \frac{a\sin(c+dx)\tan^4(c+dx)}{4d} - \frac{(5a)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \text{s}\right.}{4d}$$
$$= -\frac{5a\sin(c+dx)\tan^2(c+dx)}{8d} + \frac{a\sin(c+dx)\tan^4(c+dx}{4d}$$
$$= \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c+dx))}{d} - \frac{3b\sec^2(c+dx)}{2d} +$$
$$= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\cos^2(c+dx)}{2d} - \frac{3b\log(\cos(c}{d}$$

Mathematica [A]

time = 0.28, size = 133, normalized size = 0.99

$$-\frac{b(12\log(\cos(c+dx))+6\sec^2(c+dx)-\sec^4(c+dx)+2\sin^2(c+dx))}{4d}-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{5a(6\sec^3(c+dx)\tan(c+dx)-8\sec(c+dx)\tan^3(c+dx)-3(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/4*(b*(12*Log[Cos[c + d*x]] + 6*Sec[c + d*x]^2 - Sec[c + d*x]^4 + 2*Sin[c
 + d*x]^2))/d - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (5*a*(6*Sec[c + d*x]^3*
Tan[c + d*x] - 8*Sec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + S
ec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A]

time = 0.22, size = 167, normalized size = 1.24

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b\left(\frac{\sin^8(dx+c}{4\cos(dx+c}\right.}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b\left(\frac{\sin^8(dx+c}{4\cos(dx+c}\right.}{d}$
risch	$3ibx+\frac{b\,e^{2i(dx+c)}}{8d}+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{b\,e^{-2i(dx+c)}}{8d}+\frac{6ibc}{d}+\frac{i\left(9a\,e^{7i(dx+c)}+a\,e^{5i(dx+c)}+24ib\,e^{6i(d}\right.}{}$

norman	$\frac{\frac{12b}{d}+\frac{12b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{15a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{25a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{11a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{11a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{25a\left(\tan^{9}\left(\frac{dx}{2}\right.\right.}{4d}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin
(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+
b*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+
c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.29, size = 121, normalized size = 0.90

$$-\frac{8\,b\sin\left(dx+c\right)^{2}-3\,(5\,a-8\,b)\log\left(\sin\left(dx+c\right)+1\right)+3\,(5\,a+8\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,a\sin\left(dx+c\right)-\frac{2\left(9\,a\sin(dx+c)^{3}+12\,b\sin(dx+c)^{2}-7\,a\sin(dx+c)-10\,b\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(sin(d*x + c) + 1) + 3*(5*a +
8*b)*log(sin(d*x + c) - 1) + 16*a*sin(d*x + c) - 2*(9*a*sin(d*x + c)^3 + 12
*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) - 10*b)/(sin(d*x + c)^4 - 2*sin(d*x +
c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 138, normalized size = 1.02

$$\frac{8\,b\cos\left(dx+c\right)^{6}+3\,(5\,a-8\,b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,a+8\,b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-4\,b\cos\left(dx+c\right)^{4}-24\,b\cos\left(dx+c\right)^{2}-2\,\left(8\,a\cos\left(dx+c\right)^{4}+9\,a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)+4\,b}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cos(d*x + c)^6 + 3*(5*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) +
1) - 3*(5*a + 8*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*b*cos(d*x + c)
^4 - 24*b*cos(d*x + c)^2 - 2*(8*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^2 - 2*a
)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.58, size = 124, normalized size = 0.92

$$-\frac{8\,b\sin\left(dx+c\right)^{2}-3\,\left(5\,a-8\,b\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,\left(5\,a+8\,b\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+16\,a\sin\left(dx+c\right)-\frac{2\,\left(18\,b\sin\left(dx+c\right)^{4}+9\,a\sin\left(dx+c\right)^{3}-24\,b\sin\left(dx+c\right)^{2}-7\,a\sin\left(dx+c\right)+8\,b\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(8*b*sin(d*x + c)^2 - 3*(5*a - 8*b)*log(abs(sin(d*x + c) + 1)) + 3*(5
*a + 8*b)*log(abs(sin(d*x + c) - 1)) + 16*a*sin(d*x + c) - 2*(18*b*sin(d*x
+ c)^4 + 9*a*sin(d*x + c)^3 - 24*b*sin(d*x + c)^2 - 7*a*sin(d*x + c) + 8*b)
/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.16, size = 304, normalized size = 2.25

$$\frac{3\,b\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{15\,a}{8}+3\,b\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{15\,a}{8}-3\,b\right)}{d}-\frac{-\frac{15\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{4}-6\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{25\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4}+12\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+\frac{11\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\frac{11\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{2}+12\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{25\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{4}-6\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (3*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) - 1)*((15*a
)/8 + 3*b))/d + (log(tan(c/2 + (d*x)/2) + 1)*((15*a)/8 - 3*b))/d - ((25*a*t
an(c/2 + (d*x)/2)^3)/4 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c/2 + (d*x
)/2)^5)/2 + (11*a*tan(c/2 + (d*x)/2)^7)/2 + (25*a*tan(c/2 + (d*x)/2)^9)/4 -
 (15*a*tan(c/2 + (d*x)/2)^11)/4 - 6*b*tan(c/2 + (d*x)/2)^2 + 12*b*tan(c/2 +
 (d*x)/2)^4 + 4*b*tan(c/2 + (d*x)/2)^6 + 12*b*tan(c/2 + (d*x)/2)^8 - 6*b*ta
n(c/2 + (d*x)/2)^10)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 - 4*
tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2)^10 - tan
(c/2 + (d*x)/2)^12 - 1))
```

3.1483 $\int (a+b\sin(c+dx))\tan^5(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(1+\sin(c+dx))}{16d}-\frac{15b\sin(c+dx)}{8d}-\frac{(4a+5b\sin(c+dx))}{8d}$$

```
[Out] -1/16*(8*a+15*b)*ln(1-sin(d*x+c))/d-1/16*(8*a-15*b)*ln(1+sin(d*x+c))/d-15/8
*b*sin(d*x+c)/d-1/8*(4*a+5*b*sin(d*x+c))*tan(d*x+c)^2/d+1/4*(a+b*sin(d*x+c)
)*tan(d*x+c)^4/d
```

Rubi [A]

time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2800, 833, 788, 647, 31}

$$-\frac{(8a+15b)\log(1-\sin(c+dx))}{16d}-\frac{(8a-15b)\log(\sin(c+dx)+1)}{16d}+\frac{\tan^4(c+dx)(a+b\sin(c+dx))}{4d}-\frac{\tan^2(c+dx)(4a+5b\sin(c+dx))}{8d}-\frac{15b\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -1/16*((8*a + 15*b)*Log[1 - Sin[c + d*x]])/d - ((8*a - 15*b)*Log[1 + Sin[c
+ d*x]])/(16*d) - (15*b*Sin[c + d*x])/(8*d) - ((4*a + 5*b*Sin[c + d*x])*Tan
[c + d*x]^2)/(8*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^4)/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
```

```
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^5(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d} \\
&= \frac{(a+b\sin(c+dx))\tan^4(c+dx)}{4d} - \frac{\text{Subst}\left(\int \frac{x^3(4ab^2+5b^2x)}{(b^2-x^2)^2}\,dx, x, b\sin(c\right.}{4b^2d} \\
&= -\frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(c+dx))\tan^4(c+d}{4d} \\
&= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(}{} \\
&= -\frac{15b\sin(c+dx)}{8d} - \frac{(4a+5b\sin(c+dx))\tan^2(c+dx)}{8d} + \frac{(a+b\sin(}{} \\
&= -\frac{(8a+15b)\log(1-\sin(c+dx))}{16d} - \frac{(8a-15b)\log(1+\sin(c+dx))}{16d}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 123, normalized size = 1.06

$$-\frac{b\sin(c+dx)\tan^4(c+dx)}{d} - \frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d} - \frac{5b(6\sec^3(c+dx)\tan(c+dx)-8\sec(c+dx)\tan^3(c+dx)-3(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*b*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
```

```
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A]
time = 0.22, size = 121, normalized size = 1.04

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-15\text{s}\right.}{d}$
default	$\frac{a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-15\text{s}\right.}{d}$
risch	$iax+\frac{ib\,e^{i(dx+c)}}{2d}-\frac{ib\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}+\frac{i(16ia\,e^{6i(dx+c)}+9b\,e^{7i(dx+c)}+16ia\,e^{4i(dx+c)}+b\,e^{5i(dx+c)}+16ia\,e^{2i(d}}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{-\frac{15b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{10b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{9b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{10b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{15b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{6a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+b*(1/4*sin(d*x+c)
^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+
c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 108, normalized size = 0.93

$$-\frac{(8\,a-15\,b)\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+15\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,b\sin\left(dx+c\right)-\frac{2\left(9\,b\sin(dx+c)^3+8\,a\sin(dx+c)^2-7\,b\sin(dx+c)-6\,a\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 15*b)*log(sin(d*x + c) + 1) + (8*a + 15*b)*log(sin(d*x + c) -
 1) + 16*b*sin(d*x + c) - 2*(9*b*sin(d*x + c)^3 + 8*a*sin(d*x + c)^2 - 7*b*
sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.44, size = 114, normalized size = 0.98

$$-\frac{(8\,a-15\,b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)+(8\,a+15\,b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+16\,a\cos\left(dx+c\right)^2+2\left(8\,b\cos\left(dx+c\right)^4+9\,b\cos\left(dx+c\right)^2-2\,b\right)\sin\left(dx+c\right)-4\,a}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*((8*a - 15*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) + (8*a + 15*b)*cos
(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*cos(d*x + c)^2 + 2*(8*b*cos(d*x +
 c)^4 + 9*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.53, size = 108, normalized size = 0.93

$$-\frac{(8\,a-15\,b)\log\left(|\sin\left(dx+c\right)+1|\right)+(8\,a+15\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+16\,b\sin\left(dx+c\right)-\frac{2\left(6\,a\sin(dx+c)^4+9\,b\sin(dx+c)^3-4\,a\sin(dx+c)^2-7\,b\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 15*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 15*b)*log(abs(sin(d
*x + c) - 1)) + 16*b*sin(d*x + c) - 2*(6*a*sin(d*x + c)^4 + 9*b*sin(d*x + c
)^3 - 4*a*sin(d*x + c)^2 - 7*b*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.10, size = 261, normalized size = 2.25

$$\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-\frac{15\,b}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+\frac{15\,b}{8}\right)}{d}-\frac{\frac{15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{9\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-6\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a - (15
*b)/8))/d - (log(tan(c/2 + (d*x)/2) - 1)*(a + (15*b)/8))/d - ((15*b*tan(c/2
 + (d*x)/2))/4 + 2*a*tan(c/2 + (d*x)/2)^2 - 6*a*tan(c/2 + (d*x)/2)^4 - 6*a*
tan(c/2 + (d*x)/2)^6 + 2*a*tan(c/2 + (d*x)/2)^8 - 10*b*tan(c/2 + (d*x)/2)^3
 + (9*b*tan(c/2 + (d*x)/2)^5)/2 - 10*b*tan(c/2 + (d*x)/2)^7 + (15*b*tan(c/2
 + (d*x)/2)^9)/4)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*t
an(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1))
```

3.1484 $\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b\log(\cos(c+dx))}{d}-\frac{3a\sec(c+dx)\tan(c+dx)}{8d}-\frac{b\tan^2(c+dx)}{2d}+\frac{a\sec(c+dx)\tan}{4d}$$

[Out] 3/8*a*arctanh(sin(d*x+c))/d-b*ln(cos(d*x+c))/d-3/8*a*sec(d*x+c)*tan(d*x+c)/d-1/2*b*tan(d*x+c)^2/d+1/4*a*sec(d*x+c)*tan(d*x+c)^3/d+1/4*b*tan(d*x+c)^4/d

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2913, 2691, 3855, 3554, 3556}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}-\frac{b\tan^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]

[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Log[Cos[c + d*x]])/d - (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b*Tan[c + d*x]^2)/(2*d) + (a*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) + (b*Tan[c + d*x]^4)/(4*d)

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))\tan^4(c+dx)\,dx = a\int \sec(c+dx)\tan^4(c+dx)\,dx + b\int \tan^5(c+dx)\,dx$$

$$= \frac{a\sec(c+dx)\tan^3(c+dx)}{4d} + \frac{b\tan^4(c+dx)}{4d} - \frac{1}{4}(3a)\int \mathrm{s}$$

$$= -\frac{3a\sec(c+dx)\tan(c+dx)}{8d} - \frac{b\tan^2(c+dx)}{2d} + \frac{a\sec(c}{}$$

$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\log(\cos(c+dx))}{d} - \frac{3a\sec(c}{}$$

Mathematica [A]

time = 0.24, size = 106, normalized size = 1.03

$$\frac{a\sec(c+dx)\tan^3(c+dx)}{d} - \frac{b(4\log(\cos(c+dx)) + 2\tan^2(c+dx) - \tan^4(c+dx))}{4d} - \frac{a(6\sec^3(c+dx)\tan(c+dx) - 3(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*Sec[c + d*x]*Tan[c + d*x]^3)/d - (b*(4*Log[Cos[c + d*x]] + 2*Tan[c + d*x
]^2 - Tan[c + d*x]^4))/(4*d) - (a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTa
nh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x])))/(8*d)
```

Maple [A]

time = 0.20, size = 111, normalized size = 1.08

method	result
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4} - \frac{\sin^5(dx+c)}{8\cos(dx+c)^2} - \frac{\left(\sin^3(dx+c)\right)}{8} - \frac{3\sin(dx+c)}{8} + \frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right) + b\left(\frac{\left(\tan^4(dx+c)\right)}{4} - \frac{\left(\tan^2(dx+c)\right)}{2} -\right.}{d}$

default	$\frac{a\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}\right.}{d}$
risch	$ibx+\frac{2ibc}{d}+\frac{i\left(5a\,e^{7i(dx+c)}-3a\,e^{5i(dx+c)}+16ib\,e^{6i(dx+c)}+3a\,e^{3i(dx+c)}+16ib\,e^{4i(dx+c)}-5a\,e^{i(dx+c)}+16ib\,e^{2i(dx+c)}\right.}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{6b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6b}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin
(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+b*(1/4*tan(d*x+c)^4
-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.30, size = 100, normalized size = 0.97

$$\frac{(3a-8b)\log\left(\sin\left(dx+c\right)+1\right)-(3a+8b)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5a\sin(dx+c)^3+8b\sin(dx+c)^2-3a\sin(dx+c)-6b\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((3*a - 8*b)*log(sin(d*x + c) + 1) - (3*a + 8*b)*log(sin(d*x + c) - 1)
 + 2*(5*a*sin(d*x + c)^3 + 8*b*sin(d*x + c)^2 - 3*a*sin(d*x + c) - 6*b)/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 104, normalized size = 1.01

$$\frac{(3a-8b)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(3a+8b)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-16b\cos\left(dx+c\right)^2-2\left(5a\cos\left(dx+c\right)^2-2a\right)\sin\left(dx+c\right)+4b}{16d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*((3*a - 8*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a + 8*b)*cos(d*
x + c)^4*log(-sin(d*x + c) + 1) - 16*b*cos(d*x + c)^2 - 2*(5*a*cos(d*x + c)
^2 - 2*a)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.59, size = 100, normalized size = 0.97

$$\frac{(3\,a-8\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-(3\,a+8\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\left(6\,b\sin(dx+c)^4+5\,a\sin(dx+c)^3-4\,b\sin(dx+c)^2-3\,a\sin(dx+c)\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((3*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (3*a + 8*b)*log(abs(sin(d*x
+ c) - 1)) + 2*(6*b*sin(d*x + c)^4 + 5*a*sin(d*x + c)^3 - 4*b*sin(d*x + c)^
2 - 3*a*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.03, size = 221, normalized size = 2.15

$$\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-8\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(\frac{3\,a}{8}+b\right)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{3\,a}{8}-b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (b*log(tan(c/2 + (d*x)/2)^2 + 1))/d - ((3*a*tan(c/2 + (d*x)/2))/4 - (11*a*t
an(c/2 + (d*x)/2)^3)/4 - (11*a*tan(c/2 + (d*x)/2)^5)/4 + (3*a*tan(c/2 + (d*
x)/2)^7)/4 + 2*b*tan(c/2 + (d*x)/2)^2 - 8*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(
c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (log(tan(c/2 + (d*x)/2)
- 1)*((3*a)/8 + b))/d + (log(tan(c/2 + (d*x)/2) + 1)*((3*a)/8 - b))/d
```

3.1485 $\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=74

$$\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec(c+dx)\tan^3(c+dx)}{4d}+\frac{a\tan^4(c+dx)}{4d}$$

```
[Out] 3/8*b*arctanh(sin(d*x+c))/d-3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)*
tan(d*x+c)^3/d+1/4*a*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2913, 2687, 30, 2691, 3855}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) - (3*b*Sec[c + d*x]*Tan[c + d*x])/(8*d) +
 (b*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))\tan^3(c+dx)\,dx = a\int \sec^2(c+dx)\tan^3(c+dx)\,dx + b\int \sec(c+dx)\tan^4$$

$$= \frac{b\sec(c+dx)\tan^3(c+dx)}{4d} - \frac{1}{4}(3b)\int \sec(c+dx)\tan^2(c$$

$$= -\frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec(c+dx)\tan^3(c+dx}{4d}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3b\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A]
time = 0.20, size = 84, normalized size = 1.14

$$\frac{b\sec(c+dx)\tan^3(c+dx)}{d} + \frac{a\tan^4(c+dx)}{4d} - \frac{b\left(6\sec^3(c+dx)\tan(c+dx) - 3\left(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a*Tan[c + d*x]^4)/(4*d) - (b*(6*Sec[c
+ d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x
])))/(8*d)
```

Maple [A]
time = 0.21, size = 98, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+b\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$

default	$\frac{\frac{a\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+b\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$\frac{i\left(8ia\,e^{6i(dx+c)}+5b\,e^{7i(dx+c)}-3b\,e^{5i(dx+c)}+8ia\,e^{2i(dx+c)}+3b\,e^{3i(dx+c)}-5b\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\frac{3\ln\left(e^{i(dx+c)}+i\right)b}{8d}-\frac{3\ln(e^{i(c}}{}$
norman	$\frac{-\frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{11b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{4a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a(}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a*sin(d*x+c)^4/cos(d*x+c)^4+b*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*s
in(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 89, normalized size = 1.20

$$\frac{3\,b\log\left(\sin\left(dx+c\right)+1\right)-3\,b\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,b\sin(dx+c)^3+4\,a\sin(dx+c)^2-3\,b\sin(dx+c)-2\,a\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*b*log(sin(d*x + c) + 1) - 3*b*log(sin(d*x + c) - 1) + 2*(5*b*sin(d*
x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 93, normalized size = 1.26

$$\frac{3\,b\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,b\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-8\,a\cos\left(dx+c\right)^2-2\left(5\,b\cos\left(dx+c\right)^2-2\,b\right)\sin\left(dx+c\right)+4\,a}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(3*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) - 8*a*cos(d*x + c)^2 - 2*(5*b*cos(d*x + c)^2 - 2*b)*sin(d*x
 + c) + 4*a)/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.55, size = 81, normalized size = 1.09

$$\frac{3\,b\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\,b\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{2\,\left(5\,b\sin\left(dx+c\right)^{3}+4\,a\sin\left(dx+c\right)^{2}-3\,b\sin\left(dx+c\right)-2\,a\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(3*b*log(abs(sin(d*x + c) + 1)) - 3*b*log(abs(sin(d*x + c) - 1)) + 2*(
5*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 3*b*sin(d*x + c) - 2*a)/(sin(d*x
+ c)^2 - 1)^2)/d
```

Mupad [B]
time = 18.08, size = 144, normalized size = 1.95

$$\frac{-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\frac{11\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}-\frac{3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] (4*a*tan(c/2 + (d*x)/2)^4 - (3*b*tan(c/2 + (d*x)/2))/4 + (11*b*tan(c/2 + (d
*x)/2)^3)/4 + (11*b*tan(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)
/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)
^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (3*b*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.1486 $\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{b\tan^4(c+dx)}{4d}$$

```
[Out] -1/8*a*arctanh(sin(d*x+c))/d-1/8*a*sec(d*x+c)*tan(d*x+c)/d+1/4*a*sec(d*x+c)
^3*tan(d*x+c)/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2913, 2691, 3853, 3855, 2687, 30}

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))\tan^2(c+dx)\,dx = a\int \sec^3(c+dx)\tan^2(c+dx)\,dx + b\int \sec^2(c+dx)\tan$$

$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{1}{4}a\int \sec^3(c+dx)\,dx + \frac{bS}{}$$

$$= -\frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx}{4d}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} +$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{b\tan^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*(a*ArcTanh[Sin[c + d*x]])/d - (a*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a
*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.17, size = 88, normalized size = 1.19

method	result
derivativedivides	$\frac{a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
default	$\frac{a\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
risch	$\frac{i\left(a\,e^{7i(dx+c)}-7a\,e^{5i(dx+c)}+8ib\,e^{6i(dx+c)}+7a\,e^{3i(dx+c)}-a\,e^{i(dx+c)}+8ib\,e^{2i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{8d}+\frac{a\ln\left(e^{i(dx+\right.}}{8d}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{4b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4b\left(\tan^6\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin
(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*b*sin(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.36, size = 86, normalized size = 1.16

$$-\frac{a\log\left(\sin\left(dx+c\right)+1\right)-a\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(a\sin(dx+c)^3+4\,b\sin(dx+c)^2+a\sin(dx+c)-2\,b\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(a*log(sin(d*x + c) + 1) - a*log(sin(d*x + c) - 1) - 2*(a*sin(d*x + c
)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 91, normalized size = 1.23

$$-\frac{a\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-a\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+8\,b\cos\left(dx+c\right)^2+2\,\left(a\cos\left(dx+c\right)^2-2\,a\right)\sin\left(dx+c\right)-4\,b}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 8*b*cos(d*x + c)^2 + 2*(a*cos(d*x + c)^2 - 2*a)*sin(d*x + c)
 - 4*b)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 78, normalized size = 1.05

$$-\frac{a\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-a\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a\sin(dx+c)^3+4\,b\sin(dx+c)^2+a\sin(dx+c)-2\,b\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(a*log(abs(sin(d*x + c) + 1)) - a*log(abs(sin(d*x + c) - 1)) - 2*(a*s
in(d*x + c)^3 + 4*b*sin(d*x + c)^2 + a*sin(d*x + c) - 2*b)/(sin(d*x + c)^2
- 1)^2)/d
```

Mupad [B]
time = 18.07, size = 144, normalized size = 1.95

$$\frac{\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((a*tan(c/2 + (d*x)/2))/4 + (7*a*tan(c/2 + (d*x)/2)^3)/4 + (7*a*tan(c/2 + (
d*x)/2)^5)/4 + (a*tan(c/2 + (d*x)/2)^7)/4 + 4*b*tan(c/2 + (d*x)/2)^4)/(d*(6
*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + t
an(c/2 + (d*x)/2)^8 + 1)) - (a*atanh(tan(c/2 + (d*x)/2)))/(4*d)
```

3.1487 $\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=74

$$-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\sec^4(c+dx)}{4d}-\frac{b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] -1/8*b*arctanh(sin(d*x+c))/d+1/4*a*sec(d*x+c)^4/d-1/8*b*sec(d*x+c)*tan(d*x+
c)/d+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2913, 2686, 30, 2691, 3853, 3855}

$$\frac{a\sec^4(c+dx)}{4d}-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -1/8*(b*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+b\sin(c+dx))\tan(c+dx)\,dx &= a\int \sec^4(c+dx)\tan(c+dx)\,dx+b\int \sec^3(c+dx)\tan^2(\\ &= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d}-\frac{1}{4}b\int \sec^3(c+dx)\,dx+\frac{a\mathrm{Su}}{}\\ &= \frac{a\sec^4(c+dx)}{4d}-\frac{b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+d}{}\\ &= -\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\sec^4(c+dx)}{4d}-\frac{b\sec(c+dx)}{8}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 1.00

$$-\frac{b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\sec^4(c+dx)}{4d}-\frac{b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])*Tan[c + d*x],x]
[Out] -1/8*(b*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^4)/(4*d) - (b*Sec[c + d*
x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A]
time = 0.18, size = 80, normalized size = 1.08

method	result
derivativedivides	$\frac{\frac{a}{4\cos(dx+c)^4}+b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
default	$\frac{\frac{a}{4\cos(dx+c)^4}+b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
risch	$\frac{i\left(b\,e^{7i(dx+c)}-16ia\,e^{4i(dx+c)}-7b\,e^{5i(dx+c)}+7b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}+\frac{\ln\left(e^{i(dx+c)}-i\right)b}{8d}-\frac{\ln\left(e^{i(dx+c)}+i\right)b}{8d}$
norman	$\frac{\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{7b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{b\left(\tan^9\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a/cos(d*x+c)^4+b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/c
os(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 75, normalized size = 1.01

$$-\frac{b\log\left(\sin\left(dx+c\right)+1\right)-b\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(b\sin\left(dx+c\right)^3+b\sin\left(dx+c\right)+2\,a\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(b*log(sin(d*x + c) + 1) - b*log(sin(d*x + c) - 1) - 2*(b*sin(d*x + c
)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 80, normalized size = 1.08

$$-\frac{b\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-b\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(b\cos\left(dx+c\right)^2-2\,b\right)\sin\left(dx+c\right)-4\,a}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - b*cos(d*x + c)^4*log(-sin(d
*x + c) + 1) + 2*(b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x +
c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin\left(c+dx\right)\right)\sin\left(c+dx\right)\sec^5\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*sin(c + d*x)*sec(c + d*x)**5, x)
```

Giac [A]
time = 0.52, size = 67, normalized size = 0.91

$$-\frac{b\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-b\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(b\sin\left(dx+c\right)^{3}+b\sin\left(dx+c\right)+2\,a\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(b*log(abs(sin(d*x + c) + 1)) - b*log(abs(sin(d*x + c) - 1)) - 2*(b*s
in(d*x + c)^3 + b*sin(d*x + c) + 2*a)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 18.54, size = 158, normalized size = 2.14

$$\frac{\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}+\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((b*tan(c/2 + (d*x)/2))/4 + 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/
2)^6 + (7*b*tan(c/2 + (d*x)/2)^3)/4 + (7*b*tan(c/2 + (d*x)/2)^5)/4 + (b*tan
(c/2 + (d*x)/2)^7)/4)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (b*atanh(tan(c/2 + (
d*x)/2)))/(4*d)
```

3.1488 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=99

$$\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\sec(c+dx)\tan(c+dx)}{8d}+\frac{b\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a\tan}{}$$

[Out] 3/8*b*arctanh(sin(d*x+c))/d+a*ln(tan(d*x+c))/d+3/8*b*sec(d*x+c)*tan(d*x+c)/d+1/4*b*sec(d*x+c)^3*tan(d*x+c)/d+a*tan(d*x+c)^2/d+1/4*a*tan(d*x+c)^4/d

Rubi [A]
time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2913, 2700, 272, 45, 3853, 3855}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{a\tan^2(c+dx)}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{3b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3b\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]])/(8*d) + (a*Log[Tan[c + d*x]])/d + (3*b*Sec[c +
d*x]*Tan[c + d*x])/(8*d) + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (a*Tan[c
 + d*x]^2)/d + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
```

```
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc(c+dx)\sec^5(c+dx)\,dx + b\int \sec^5(c+dx)\,dx$$

$$= \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3b)\int \sec^3(c+dx)\,dx + \frac{a}{}$$

$$= \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} +$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3b\sec(c+dx)\tan(c+dx)}{8d} + \frac{b}{}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a\log(\tan(c+dx))}{d} + \frac{3b\sec(c+}{}$$

Mathematica [A]
time = 0.86, size = 99, normalized size = 1.00

$$-\frac{a(4\log(\cos(c+dx)) - 4\log(\sin(c+dx)) - 2\sec^2(c+dx) - \sec^4(c+dx))}{4d} + \frac{b\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{3b(\tanh^{-1}(\sin(c+dx)) + \sec(c+dx)\tan(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
[Out] -1/4*(a*(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec
[c + d*x]^4))/d + (b*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*b*(ArcTanh[Sin
[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.25, size = 82, normalized size = 0.83

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{d}$
risch	$-\frac{i\left(8ia\,e^{6i(dx+c)}+3b\,e^{7i(dx+c)}+32ia\,e^{4i(dx+c)}+11b\,e^{5i(dx+c)}+8ia\,e^{2i(dx+c)}-11b\,e^{3i(dx+c)}-3b\,e^{i(dx+c)}\right)}{4d\left(e^{2i(dx+c)}+1\right)^4}-\frac{a\ln(e^{i(dx}}{d}$
norman	$\frac{\frac{4a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{2b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{2b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5b\left(\tan\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+b*(-(-1/4*sec(d*x
+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.10

$$-\frac{(8a-3b)\log(\sin(dx+c)+1)+(8a+3b)\log(\sin(dx+c)-1)-16a\log(\sin(dx+c))+\frac{2\left(3b\sin(dx+c)^3+4a\sin(dx+c)^2-5b\sin(dx+c)-6a\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a - 3*b)*log(sin(d*x + c) + 1) + (8*a + 3*b)*log(sin(d*x + c) - 1
) - 16*a*log(sin(d*x + c)) + 2*(3*b*sin(d*x + c)^3 + 4*a*sin(d*x + c)^2 - 5
*b*sin(d*x + c) - 6*a)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 125, normalized size = 1.26

$$\frac{16a\cos(dx+c)^4\log\left(\frac{1}{2}\sin(dx+c)\right)-(8a-3b)\cos(dx+c)^4\log(\sin(dx+c)+1)-(8a+3b)\cos(dx+c)^4\log(-\sin(dx+c)+1)+8a\cos(dx+c)^2+2\left(3b\cos(dx+c)^2+2b\right)\sin(dx+c)+4a}{16d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a - 3*b)*cos(d*x + c)^
4*log(sin(d*x + c) + 1) - (8*a + 3*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)
 + 8*a*cos(d*x + c)^2 + 2*(3*b*cos(d*x + c)^2 + 2*b)*sin(d*x + c) + 4*a)/(d
*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [A]
time = 0.52, size = 113, normalized size = 1.14

$$-\frac{(8a-3b)\log(|\sin(dx+c)+1|)+(8a+3b)\log(|\sin(dx+c)-1|)-16a\log(|\sin(dx+c)|)-\frac{2\left(6a\sin(dx+c)^4-3b\sin(dx+c)^3-16a\sin(dx+c)^2+5b\sin(dx+c)+12a\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*((8*a - 3*b)*log(abs(sin(d*x + c) + 1)) + (8*a + 3*b)*log(abs(sin(d*x
 + c) - 1)) - 16*a*log(abs(sin(d*x + c))) - 2*(6*a*sin(d*x + c)^4 - 3*b*sin
(d*x + c)^3 - 16*a*sin(d*x + c)^2 + 5*b*sin(d*x + c) + 12*a)/(sin(d*x + c)^
2 - 1)^2)/d
```

Mupad [B]
time = 11.87, size = 116, normalized size = 1.17

$$\frac{-\frac{3b\sin(c+dx)^3}{8}-\frac{a\sin(c+dx)^2}{2}+\frac{5b\sin(c+dx)}{8}+\frac{3a}{4}}{d\left(\sin(c+dx)^4-2\sin(c+dx)^2+1\right)}-\frac{\ln(\sin(c+dx)+1)\left(\frac{a}{2}-\frac{3b}{16}\right)}{d}-\frac{\ln(\sin(c+dx)-1)\left(\frac{a}{2}+\frac{3b}{16}\right)}{d}+\frac{a\ln(\sin(c+dx))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] ((3*a)/4 + (5*b*sin(c + d*x))/8 - (a*sin(c + d*x)^2)/2 - (3*b*sin(c + d*x)^
3)/8)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) - (log(sin(c + d*x) + 1)*
(a/2 - (3*b)/16))/d - (log(sin(c + d*x) - 1)*(a/2 + (3*b)/16))/d + (a*log(s
in(c + d*x)))/d
```

3.1489 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{15a\csc(c+dx)}{8d}+\frac{b\log(\tan(c+dx))}{d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{a\csc(c+dx)}{4}$$

```
[Out] 15/8*a*arctanh(sin(d*x+c))/d-15/8*a*csc(d*x+c)/d+b*ln(tan(d*x+c))/d+5/8*a*c
sc(d*x+c)*sec(d*x+c)^2/d+1/4*a*csc(d*x+c)*sec(d*x+c)^4/d+b*tan(d*x+c)^2/d+1
/4*b*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2913, 2701, 294, 327, 213, 2700, 272, 45}

$$-\frac{15a\csc(c+dx)}{8d}+\frac{15a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5a\csc(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}+\frac{b\tan^2(c+dx)}{d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (15*a*ArcTanh[Sin[c + d*x]])/(8*d) - (15*a*Csc[c + d*x])/(8*d) + (b*Log[Tan
[c + d*x]])/d + (5*a*Csc[c + d*x]*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]*S
ec[c + d*x]^4)/(4*d) + (b*Tan[c + d*x]^2)/d + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^2(c+dx)\sec^5(c+dx)\,dx + b\int \csc(c+dx)\sec^5$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{(1}{}\right.}{}$$

$$= \frac{a\csc(c+dx)\sec^4(c+dx)}{4d} - \frac{(5a)\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx,\right.}{4d}$$

$$= \frac{5a\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc(c+dx)\sec^4(c+da}{4d}$$

$$= -\frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan(c+dx))}{d} + \frac{5a\csc(c+dx}{8}$$

$$= \frac{15a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{15a\csc(c+dx)}{8d} + \frac{b\log(\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.27, size = 76, normalized size = 0.66

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{b(4\log(\cos(c+dx)) - 4\log(\sin(c+dx)) - 2\sec^2(c+dx) - \sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
[Out] -((a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (b*
(4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]] - 2*Sec[c + d*x]^2 - Sec[c + d*x
]^4))/(4*d)
```

Maple [A]
time = 0.26, size = 101, normalized size = 0.88

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}\right.}{d}$
risch	$-\frac{i(15a\,e^{9i(dx+c)}+40a\,e^{7i(dx+c)}+8ib\,e^{8i(dx+c)}+18a\,e^{5i(dx+c)}+24ib\,e^{6i(dx+c)}+40a\,e^{3i(dx+c)}-24ib\,e^{4i(dx+c)}+15a\,e^{i(dx}}{4d(e^{2i(dx+c)}-1)(e^{2i(dx+c)}+1)^4}$
norman	$\frac{\frac{4b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a}{2d}+\frac{13a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5a\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5a\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(d*
x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+b*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+l
n(tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 126, normalized size = 1.10

$$\frac{(15\,a-8\,b)\log\left(\sin\left(dx+c\right)+1\right)-(15\,a+8\,b)\log\left(\sin\left(dx+c\right)-1\right)+16\,b\log\left(\sin\left(dx+c\right)\right)-\frac{2\left(15\,a\sin\left(dx+c\right)^{4}+4\,b\sin\left(dx+c\right)^{3}-25\,a\sin\left(dx+c\right)^{2}-6\,b\sin\left(dx+c\right)+8\,a\right)}{\sin\left(dx+c\right)^{5}-2\,\sin\left(dx+c\right)^{3}+\sin\left(dx+c\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*((15*a - 8*b)*log(sin(d*x + c) + 1) - (15*a + 8*b)*log(sin(d*x + c) -
1) + 16*b*log(sin(d*x + c)) - 2*(15*a*sin(d*x + c)^4 + 4*b*sin(d*x + c)^3 -
 25*a*sin(d*x + c)^2 - 6*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^5 - 2*sin(d*x
+ c)^3 + sin(d*x + c)))/d
```

Fricas [A]
time = 0.37, size = 159, normalized size = 1.38

$$\frac{16\,b\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+(15\,a-8\,b)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-(15\,a+8\,b)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-30\,a\cos\left(dx+c\right)^{4}+10\,a\cos\left(dx+c\right)^{2}+4\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sin\left(dx+c\right)+4\,a}{16\,d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a - 8*b)
*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a + 8*b)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 30*a*cos(d*x + c)^4 + 10*a*cos(
d*x + c)^2 + 4*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) + 4*a)/(d*cos(d*x + c)
^4*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.57, size = 134, normalized size = 1.17

$$\frac{(15\,a-8\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-(15\,a+8\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+16\,b\log\left(|\sin\left(dx+c\right)|\right)-\frac{16\,(b\sin(dx+c)+a)}{\sin(dx+c)}+\frac{2\left(6\,b\sin(dx+c)^4-7\,a\sin(dx+c)^3-16\,b\sin(dx+c)^2+9\,a\sin(dx+c)+12\,b\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*((15*a - 8*b)*log(abs(sin(d*x + c) + 1)) - (15*a + 8*b)*log(abs(sin(d*
x + c) - 1)) + 16*b*log(abs(sin(d*x + c))) - 16*(b*sin(d*x + c) + a)/sin(d*
x + c) + 2*(6*b*sin(d*x + c)^4 - 7*a*sin(d*x + c)^3 - 16*b*sin(d*x + c)^2 +
 9*a*sin(d*x + c) + 12*b)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 11.89, size = 130, normalized size = 1.13

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{15\,a}{16}-\frac{b}{2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{15\,a}{16}+\frac{b}{2}\right)}{d}+\frac{b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{15\,a\sin(c+d\,x)^4}{8}+\frac{b\sin(c+d\,x)^3}{2}-\frac{25\,a\sin(c+d\,x)^2}{8}-\frac{3\,b\,\sin(c+d\,x)}{4}+a}{d\,\left(\sin\left(c+d\,x\right)^5-2\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((15*a)/16 - b/2))/d - (log(sin(c + d*x) - 1)*((15*a
)/16 + b/2))/d + (b*log(sin(c + d*x)))/d - (a - (3*b*sin(c + d*x))/4 - (25*
a*sin(c + d*x)^2)/8 + (15*a*sin(c + d*x)^4)/8 + (b*sin(c + d*x)^3)/2)/(d*(s
in(c + d*x) - 2*sin(c + d*x)^3 + sin(c + d*x)^5))
```

3.1490 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}-\frac{a\cot^2(c+dx)}{2d}-\frac{15b\csc(c+dx)}{8d}+\frac{3a\log(\tan(c+dx))}{d}+\frac{5b\csc(c+dx)\sec^2(c+dx)}{8d}$$

```
[Out] 15/8*b*arctanh(sin(d*x+c))/d-1/2*a*cot(d*x+c)^2/d-15/8*b*csc(d*x+c)/d+3*a*l
n(tan(d*x+c))/d+5/8*b*csc(d*x+c)*sec(d*x+c)^2/d+1/4*b*csc(d*x+c)*sec(d*x+c)
^4/d+3/2*a*tan(d*x+c)^2/d+1/4*a*tan(d*x+c)^4/d
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2913, 2700, 272, 45, 2701, 294, 327, 213}

$$\frac{a\tan^4(c+dx)}{4d}+\frac{3a\tan^2(c+dx)}{2d}-\frac{a\cot^2(c+dx)}{2d}+\frac{3a\log(\tan(c+dx))}{d}-\frac{15b\csc(c+dx)}{8d}+\frac{15b\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\csc(c+dx)\sec^4(c+dx)}{4d}+\frac{5b\csc(c+dx)\sec^2(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (15*b*ArcTanh[Sin[c + d*x]])/(8*d) - (a*Cot[c + d*x]^2)/(2*d) - (15*b*Csc[c
 + d*x])/(8*d) + (3*a*Log[Tan[c + d*x]])/d + (5*b*Csc[c + d*x]*Sec[c + d*x]
^2)/(8*d) + (b*Csc[c + d*x]*Sec[c + d*x]^4)/(4*d) + (3*a*Tan[c + d*x]^2)/(2
*d) + (a*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^3(c+dx)\sec^5(c+dx)\,dx + b\int \csc^2(c+dx)\sec^5$$

$$= \frac{a\mathrm{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(c+dx)\right)}{d} - \frac{b\mathrm{Subst}\left(\int \frac{x^6}{(-1+x}\right.}{}$$

$$= \frac{b\csc(c+dx)\sec^4(c+dx)}{4d} + \frac{a\mathrm{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2\right.}{2d}$$

$$= \frac{5b\csc(c+dx)\sec^2(c+dx)}{8d} + \frac{b\csc(c+dx)\sec^4(c+dx)}{4d}$$

$$= -\frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+dx)}{8d} + \frac{3a\log(\tan(c+dx)}{d}$$

$$= \frac{15b\tanh^{-1}(\sin(c+dx))}{8d} - \frac{a\cot^2(c+dx)}{2d} - \frac{15b\csc(c+}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.40, size = 86, normalized size = 0.64

$$-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a(2\csc^2(c+dx)+12\log(\cos(c+dx))-12\log(\sin(c+dx))-4\sec^2(c+dx)-\sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[c + d*x]^2])/d) - (a*
(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Sec[c +
 d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.28, size = 129, normalized size = 0.96

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+b\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)}\right.}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+b\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)}\right.}{d}$
risch	$-\frac{i(24ia\,e^{10i(dx+c)}+15b\,e^{11i(dx+c)}+48ia\,e^{8i(dx+c)}+25b\,e^{9i(dx+c)}-16ia\,e^{6i(dx+c)}-22b\,e^{7i(dx+c)}+48ia\,e^{4i(dx+c)}+22b\,e^{5i}}{4d\left(e^{2i(dx+c)}-1\right)^2\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{-\frac{a}{8d}-\frac{a\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d}+\frac{13b\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5b\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{5b\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{5b\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{13}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/sin
(d*x+c)^2+3*ln(tan(d*x+c)))+b*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/c
os(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 140, normalized size = 1.04

$$-\frac{3\,(8\,a-5\,b)\log\left(\sin\left(dx+c\right)+1\right)+3\,(8\,a+5\,b)\log\left(\sin\left(dx+c\right)-1\right)-48\,a\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(15\,b\sin\left(dx+c\right)^{5}+12\,a\sin\left(dx+c\right)^{4}-25\,b\sin\left(dx+c\right)^{3}-18\,a\sin\left(dx+c\right)^{2}+8\,b\sin\left(dx+c\right)+4\,a\right)}{\sin\left(dx+c\right)^{6}-2\,\sin\left(dx+c\right)^{4}+\sin\left(dx+c\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(sin(d*x + c) + 1) + 3*(8*a + 5*b)*log(sin(d*x + c)
 - 1) - 48*a*log(sin(d*x + c)) + 2*(15*b*sin(d*x + c)^5 + 12*a*sin(d*x + c)
^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x + c) + 4*a)/(s
in(d*x + c)^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.37, size = 211, normalized size = 1.56

$$\frac{24\,a\cos\left(dx+c\right)^{4}-12\,a\cos\left(dx+c\right)^{2}+48\left(a\cos\left(dx+c\right)^{6}-a\cos\left(dx+c\right)^{4}\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-3\left((8\,a-5\,b)\cos\left(dx+c\right)^{6}-(8\,a-5\,b)\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((8\,a+5\,b)\cos\left(dx+c\right)^{6}-(8\,a+5\,b)\cos\left(dx+c\right)^{4}\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(15\,b\cos\left(dx+c\right)^{4}-5\,b\cos\left(dx+c\right)^{2}-2\,b\right)\sin\left(dx+c\right)-4\,a}{16\left(d\cos\left(dx+c\right)^{6}-d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(24*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 48*(a*cos(d*x + c)^6 - a*
cos(d*x + c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a - 5*b)*cos(d*x + c)^6 - (8*
a - 5*b)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 3*((8*a + 5*b)*cos(d*x + c
)^6 - (8*a + 5*b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(15*b*cos(d*x
+ c)^4 - 5*b*cos(d*x + c)^2 - 2*b)*sin(d*x + c) - 4*a)/(d*cos(d*x + c)^6 -
d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 133, normalized size = 0.99

$$-\frac{3\,(8\,a-5\,b)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\,(8\,a+5\,b)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-48\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{2\left(15\,b\sin(dx+c)^5+12\,a\sin(dx+c)^4-25\,b\sin(dx+c)^3-18\,a\sin(dx+c)^2+8\,b\sin(dx+c)+4\,a\right)}{\left(\sin(dx+c)^3-\sin(dx+c)\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a - 5*b)*log(abs(sin(d*x + c) + 1)) + 3*(8*a + 5*b)*log(abs(sin
(d*x + c) - 1)) - 48*a*log(abs(sin(d*x + c))) + 2*(15*b*sin(d*x + c)^5 + 12
*a*sin(d*x + c)^4 - 25*b*sin(d*x + c)^3 - 18*a*sin(d*x + c)^2 + 8*b*sin(d*x
 + c) + 4*a)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

Mupad [B]
time = 0.11, size = 146, normalized size = 1.08

$$\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a}{2}-\frac{15\,b}{16}\right)}{d}-\frac{\frac{15\,b\sin(c+d\,x)^5}{8}+\frac{3\,a\sin(c+d\,x)^4}{2}-\frac{25\,b\sin(c+d\,x)^3}{8}-\frac{9\,a\sin(c+d\,x)^2}{4}+b\,\sin\left(c+d\,x\right)+\frac{a}{2}}{d\,\left({\sin\left(c+d\,x\right)}^6-2\,{\sin\left(c+d\,x\right)}^4+{\sin\left(c+d\,x\right)}^2\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a}{2}+\frac{15\,b}{16}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x)))/d - (log(sin(c + d*x) + 1)*((3*a)/2 - (15*b)/16))/d
 - (a/2 + b*sin(c + d*x) - (9*a*sin(c + d*x)^2)/4 + (3*a*sin(c + d*x)^4)/2
- (25*b*sin(c + d*x)^3)/8 + (15*b*sin(c + d*x)^5)/8)/(d*(sin(c + d*x)^2 - 2
*sin(c + d*x)^4 + sin(c + d*x)^6)) - (log(sin(c + d*x) - 1)*((3*a)/2 + (15*
b)/16))/d
```

3.1491 $\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=155

$$\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b\cot^2(c+dx)}{2d}-\frac{35a\csc(c+dx)}{8d}-\frac{35a\csc^3(c+dx)}{24d}+\frac{3b\log(\tan(c+dx))}{d}+\frac{7a\csc^3}{}$$

```
[Out] 35/8*a*arctanh(sin(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-35/8*a*csc(d*x+c)/d-35/24
*a*csc(d*x+c)^3/d+3*b*ln(tan(d*x+c))/d+7/8*a*csc(d*x+c)^3*sec(d*x+c)^2/d+1/
4*a*csc(d*x+c)^3*sec(d*x+c)^4/d+3/2*b*tan(d*x+c)^2/d+1/4*b*tan(d*x+c)^4/d
```

Rubi [A]

time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2913, 2701, 294, 308, 213, 2700, 272, 45}

$$-\frac{35a\csc^3(c+dx)}{24d}-\frac{35a\csc(c+dx)}{8d}+\frac{35a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d}+\frac{7a\csc^3(c+dx)\sec^2(c+dx)}{8d}+\frac{b\tan^4(c+dx)}{4d}+\frac{3b\tan^2(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{3b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (35*a*ArcTanh[Sin[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/(2*d) - (35*a*Csc[c
 + d*x])/(8*d) - (35*a*Csc[c + d*x]^3)/(24*d) + (3*b*Log[Tan[c + d*x]])/d +
 (7*a*Csc[c + d*x]^3*Sec[c + d*x]^2)/(8*d) + (a*Csc[c + d*x]^3*Sec[c + d*x]
^4)/(4*d) + (3*b*Tan[c + d*x]^2)/(2*d) + (b*Tan[c + d*x]^4)/(4*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2913

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rubi steps

$$\int \csc^4(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))\,dx = a\int \csc^4(c+dx)\sec^5(c+dx)\,dx + b\int \csc^3(c+dx)\sec$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int \frac{(1}{}\right.}{}$$

$$= \frac{a\csc^3(c+dx)\sec^4(c+dx)}{4d} - \frac{(7a)\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx\right.}{4d}$$

$$= \frac{7a\csc^3(c+dx)\sec^2(c+dx)}{8d} + \frac{a\csc^3(c+dx)\sec^4(c+}{4d}$$

$$= -\frac{b\cot^2(c+dx)}{2d} + \frac{3b\log(\tan(c+dx))}{d} + \frac{7a\csc^3(c+d}{}$$

$$= -\frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c+dx)}{8d} - \frac{35a\csc^3(c+dx)}{24d}$$

$$= \frac{35a\tanh^{-1}(\sin(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{2d} - \frac{35a\csc(c}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.49, size = 90, normalized size = 0.58

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(c+dx)\right)}{3d} - \frac{b(2\csc^2(c+dx)+12\log(\cos(c+dx))-12\log(\sin(c+dx))-4\sec^2(c+dx)-\sec^4(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*Sec[c + d*x]^5*(a + b*Sin[c + d*x]),x]
[Out] -1/3*(a*Csc[c + d*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[c + d*x]^2])/d
- (b*(2*Csc[c + d*x]^2 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]] - 4*Se
c[c + d*x]^2 - Sec[c + d*x]^4))/(4*d)
```

Maple [A]
time = 0.26, size = 147, normalized size = 0.95

method	result
derivativedivides	$\frac{a\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+}{d}$
default	$\frac{a\left(\frac{1}{4\sin(dx+c)^3\cos(dx+c)^4}-\frac{7}{12\sin(dx+c)^3\cos(dx+c)^2}+\frac{35}{24\sin(dx+c)\cos(dx+c)^2}-\frac{35}{8\sin(dx+c)}+\frac{35\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+}{d}$
risch	$-\frac{i\left(105a\,e^{13i(dx+c)}+70a\,e^{11i(dx+c)}+72ib\,e^{12i(dx+c)}-329a\,e^{9i(dx+c)}+72ib\,e^{10i(dx+c)}-204a\,e^{7i(dx+c)}-192ib\,e^{8i(dx+c)}\right.}{12d\left(e^{2i(dx+c)}-1\right)^3\left(e^{2i(dx+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4/sin(d*x+c)^3/cos(d*x+c)^4-7/12/sin(d*x+c)^3/cos(d*x+c)^2+35/24/
sin(d*x+c)/cos(d*x+c)^2-35/8/sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c)))+b*(
1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/sin(d*x+c)^
2+3*ln(tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 151, normalized size = 0.97

$$\frac{3\,(35\,a-24\,b)\log\left(\sin\left(dx+c\right)+1\right)-3\,(35\,a+24\,b)\log\left(\sin\left(dx+c\right)-1\right)+144\,b\log\left(\sin\left(dx+c\right)\right)-\frac{2\,\left(105\,a\sin\left(dx+c\right)^{6}+36\,b\sin\left(dx+c\right)^{5}-175\,a\sin\left(dx+c\right)^{4}-54\,b\sin\left(dx+c\right)^{3}+56\,a\sin\left(dx+c\right)^{2}+12\,b\sin\left(dx+c\right)+8\,a\right)}{\sin\left(dx+c\right)^{7}-2\,\sin\left(dx+c\right)^{5}+\sin\left(dx+c\right)^{3}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(35*a - 24*b)*log(sin(d*x + c) + 1) - 3*(35*a + 24*b)*log(sin(d*x +
 c) - 1) + 144*b*log(sin(d*x + c)) - 2*(105*a*sin(d*x + c)^6 + 36*b*sin(d*x
 + c)^5 - 175*a*sin(d*x + c)^4 - 54*b*sin(d*x + c)^3 + 56*a*sin(d*x + c)^2
+ 12*b*sin(d*x + c) + 8*a)/(sin(d*x + c)^7 - 2*sin(d*x + c)^5 + sin(d*x + c
)^3))/d
```

Fricas [A]
time = 0.37, size = 248, normalized size = 1.60

$$-\frac{210\,a\cos\left(dx+c\right)^{6}-280\,a\cos\left(dx+c\right)^{4}+42\,a\cos\left(dx+c\right)^{2}-144\,\left(b\cos\left(dx+c\right)^{6}-b\cos\left(dx+c\right)^{4}\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)-3\,\left((35\,a-24\,b)\cos\left(dx+c\right)^{6}-(35\,a-24\,b)\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,\left((35\,a+24\,b)\cos\left(dx+c\right)^{6}-(35\,a+24\,b)\cos\left(dx+c\right)^{4}\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-12\,\left(6\,b\cos\left(dx+c\right)^{4}-3\,b\cos\left(dx+c\right)^{2}-b\right)\sin\left(dx+c\right)+12\,a}{48\,\left(d\cos\left(dx+c\right)^{6}-d\cos\left(dx+c\right)^{4}\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(210*a*cos(d*x + c)^6 - 280*a*cos(d*x + c)^4 + 42*a*cos(d*x + c)^2 -
144*(b*cos(d*x + c)^6 - b*cos(d*x + c)^4)*log(1/2*sin(d*x + c))*sin(d*x + c
) - 3*((35*a - 24*b)*cos(d*x + c)^6 - (35*a - 24*b)*cos(d*x + c)^4)*log(sin
(d*x + c) + 1)*sin(d*x + c) + 3*((35*a + 24*b)*cos(d*x + c)^6 - (35*a + 24*
b)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 12*(6*b*cos(d*x +
c)^4 - 3*b*cos(d*x + c)^2 - b)*sin(d*x + c) + 12*a)/((d*cos(d*x + c)^6 - d*
cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*sec(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]

time = 0.64, size = 160, normalized size = 1.03

$$\frac{3\,(35\,a-24\,b)\log\left(|\sin\left(dx+c\right)+1|\right)-3\,(35\,a+24\,b)\log\left(|\sin\left(dx+c\right)-1|\right)+144\,b\log\left(|\sin\left(dx+c\right)|\right)+\frac{6\left(18\,b\sin(dx+c)^4-11\,a\sin(dx+c)^3-44\,b\sin(dx+c)^2+13\,a\sin(dx+c)+28\,b\right)}{\left(\sin(dx+c)^2-1\right)^2}-\frac{8\left(33\,b\sin(dx+c)^3+18\,a\sin(dx+c)^2+3\,b\sin(dx+c)+2\,a\right)}{\sin(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*sec(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/48*(3*(35*a - 24*b)*log(abs(sin(d*x + c) + 1)) - 3*(35*a + 24*b)*log(abs(
sin(d*x + c) - 1)) + 144*b*log(abs(sin(d*x + c))) + 6*(18*b*sin(d*x + c)^4
- 11*a*sin(d*x + c)^3 - 44*b*sin(d*x + c)^2 + 13*a*sin(d*x + c) + 28*b)/(si
n(d*x + c)^2 - 1)^2 - 8*(33*b*sin(d*x + c)^3 + 18*a*sin(d*x + c)^2 + 3*b*si
n(d*x + c) + 2*a)/sin(d*x + c)^3)/d
```

Mupad [B]

time = 11.91, size = 157, normalized size = 1.01

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{35\,a}{16}-\frac{3\,b}{2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{35\,a}{16}+\frac{3\,b}{2}\right)}{d}+\frac{3\,b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\frac{35\,a\sin(c+d\,x)^6}{8}+\frac{3\,b\sin(c+d\,x)^5}{2}-\frac{175\,a\sin(c+d\,x)^4}{24}-\frac{9\,b\sin(c+d\,x)^3}{4}+\frac{7\,a\sin(c+d\,x)^2}{3}+\frac{b\,\sin(c+d\,x)}{2}+\frac{a}{3}}{d\,\left(\sin\left(c+d\,x\right)^7-2\sin\left(c+d\,x\right)^5+\sin\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))/(cos(c + d*x)^5*sin(c + d*x)^4),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((35*a)/16 - (3*b)/2))/d - (log(sin(c + d*x) - 1)*((
35*a)/16 + (3*b)/2))/d + (3*b*log(sin(c + d*x)))/d - (a/3 + (b*sin(c + d*x)
)/2 + (7*a*sin(c + d*x)^2)/3 - (175*a*sin(c + d*x)^4)/24 + (35*a*sin(c + d*
x)^6)/8 - (9*b*sin(c + d*x)^3)/4 + (3*b*sin(c + d*x)^5)/2)/(d*(sin(c + d*x)
^3 - 2*sin(c + d*x)^5 + sin(c + d*x)^7))
```

3.1492 $\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=189

$$-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(1+\sin(c+dx))}{16d}-\frac{(a^2+3b^2)\sin(c+dx)}{d}$$

```
[Out] -1/16*(15*a^2+48*a*b+35*b^2)*ln(1-sin(d*x+c))/d+1/16*(15*a^2-48*a*b+35*b^2)
*ln(1+sin(d*x+c))/d-(a^2+3*b^2)*sin(d*x+c)/d-a*b*sin(d*x+c)^2/d-1/3*b^2*sin
(d*x+c)^3/d-1/8*sec(d*x+c)^2*(11*b+9*a*sin(d*x+c))*(a+b*sin(d*x+c))/d+1/4*s
ec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2916, 12, 1659, 1824, 647, 31}

$$-\frac{(a^2+3b^2)\sin(c+dx)}{d}-\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-48ab+35b^2)\log(\sin(c+dx)+1)}{16d}-\frac{ab\sin^2(c+dx)}{d}-\frac{\sec^2(c+dx)(9a\sin(c+dx)+11b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -1/16*((15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/d + ((15*a^2 - 48*
a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - ((a^2 + 3*b^2)*Sin[c + d*x])/
d - (a*b*Sin[c + d*x]^2)/d - (b^2*Sin[c + d*x]^3)/(3*d) - (Sec[c + d*x]^2*(
11*b + 9*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a +
 b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sin(c+dx)(a+b\sin(c+dx))^2\tan^5(c+dx)\,dx &= \frac{b^5\text{Subst}\left(\int \frac{x^6(a+x)^2}{b^6(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^6(a+x)^2}{(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{bd} \\
&= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\text{Subst}\Big(\int}{} \\
&= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} \\
&= -\frac{\sec^2(c+dx)(11b+9a\sin(c+dx))(a+b\sin(c+dx))}{8d} \\
&= -\frac{(a^2+3b^2)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+}{3d} \\
&= -\frac{(a^2+3b^2)\sin(c+dx)}{d} - \frac{ab\sin^2(c+dx)}{d} - \frac{b^2\sin^3(c+}{3d} \\
&= -\frac{(15a^2+48ab+35b^2)\log(1-\sin(c+dx))}{16d} + \frac{(15a^2-}{}
\end{aligned}
$$

Mathematica [A]
time = 1.12, size = 186, normalized size = 0.98

$$
\frac{-3(15a^2+48ab+35b^2)\log(1-\sin(c+dx))+3(15a^2-48ab+35b^2)\log(1+\sin(c+dx))+\frac{3(a+b)^2}{(-1+\sin(c+dx))^2}+\frac{3(a+b)(9a+13b)}{-1+\sin(c+dx)}-48(a^2+3b^2)\sin(c+dx)-48ab\sin^2(c+dx)-16b^2\sin^3(c+dx)-\frac{3(a-b)^2}{(1+\sin(c+dx))^2}+\frac{3(9a-13b)(a-b)}{1+\sin(c+dx)}}{48d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-3*(15*a^2 + 48*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(15*a^2 - 48*a*b +
 35*b^2)*Log[1 + Sin[c + d*x]] + (3*(a + b)^2)/(-1 + Sin[c + d*x])^2 + (3*(
a + b)*(9*a + 13*b))/(-1 + Sin[c + d*x]) - 48*(a^2 + 3*b^2)*Sin[c + d*x] -
48*a*b*Sin[c + d*x]^2 - 16*b^2*Sin[c + d*x]^3 - (3*(a - b)^2)/(1 + Sin[c +
d*x])^2 + (3*(9*a - 13*b)*(a - b))/(1 + Sin[c + d*x]))/(48*d)
```

Maple [A]
time = 0.34, size = 266, normalized size = 1.41 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*s
in(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))
```

```
)+2*a*b*(1/4*sin(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*si
n(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+b^2*(1/4*sin
(d*x+c)^9/cos(d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*sin(d*x+c)^7-7/8*s
in(d*x+c)^5-35/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c
))))
```

Maxima [A]
time = 0.35, size = 180, normalized size = 0.95

$$-\frac{16\,b^{2}\sin\left(dx+c\right)^{3}+48\,ab\sin\left(dx+c\right)^{2}-3\,\left(15\,a^{2}-48\,ab+35\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(15\,a^{2}+48\,ab+35\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)+48\,\left(a^{2}+3\,b^{2}\right)\sin\left(dx+c\right)-\frac{6\,\left(24\,ab\sin\left(dx+c\right)^{2}+\left(9\,a^{2}+13\,b^{2}\right)\sin\left(dx+c\right)^{3}-20\,ab-\left(7\,a^{2}+11\,b^{2}\right)\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 - 3*(15*a^2 - 48*a*b +
 35*b^2)*log(sin(d*x + c) + 1) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(sin(d*x +
 c) - 1) + 48*(a^2 + 3*b^2)*sin(d*x + c) - 6*(24*a*b*sin(d*x + c)^2 + (9*a^
2 + 13*b^2)*sin(d*x + c)^3 - 20*a*b - (7*a^2 + 11*b^2)*sin(d*x + c))/(sin(d
*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.39, size = 198, normalized size = 1.05

$$\frac{48\,ab\cos\left(dx+c\right)^{6}-24\,ab\cos\left(dx+c\right)^{4}+3\,\left(15\,a^{2}-48\,ab+35\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(15\,a^{2}+48\,ab+35\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-144\,ab\cos\left(dx+c\right)^{2}+24\,ab+2\,\left(8\,b^{2}\cos\left(dx+c\right)^{6}-8\,\left(3\,a^{2}+10\,b^{2}\right)\cos\left(dx+c\right)^{4}-3\,\left(9\,a^{2}+13\,b^{2}\right)\cos\left(dx+c\right)^{2}+6\,a^{2}+6\,b^{2}\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/48*(48*a*b*cos(d*x + c)^6 - 24*a*b*cos(d*x + c)^4 + 3*(15*a^2 - 48*a*b +
35*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(15*a^2 + 48*a*b + 35*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 144*a*b*cos(d*x + c)^2 + 24*a*b +
2*(8*b^2*cos(d*x + c)^6 - 8*(3*a^2 + 10*b^2)*cos(d*x + c)^4 - 3*(9*a^2 + 13
*b^2)*cos(d*x + c)^2 + 6*a^2 + 6*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 198, normalized size = 1.05

$$-\frac{16\,b^{2}\sin\left(dx+c\right)^{3}+48\,ab\sin\left(dx+c\right)^{2}+48\,a^{2}\sin\left(dx+c\right)+144\,b^{2}\sin\left(dx+c\right)-3\,(15\,a^{2}-48\,ab+35\,b^{2})\log\left(|\sin\left(dx+c\right)+1|\right)+3\,(15\,a^{2}+48\,ab+35\,b^{2})\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{6\left(36\,ab\sin(dx+c)^{4}+9\,a^{2}\sin(dx+c)^{3}+13\,b^{2}\sin(dx+c)^{3}-48\,ab\sin(dx+c)^{2}-7\,a^{2}\sin(dx+c)-11\,b^{2}\sin(dx+c)+16\,ab\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^2*sin(d*x + c)^3 + 48*a*b*sin(d*x + c)^2 + 48*a^2*sin(d*x + c)
+ 144*b^2*sin(d*x + c) - 3*(15*a^2 - 48*a*b + 35*b^2)*log(abs(sin(d*x + c)
+ 1)) + 3*(15*a^2 + 48*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1)) - 6*(36*a*b
*sin(d*x + c)^4 + 9*a^2*sin(d*x + c)^3 + 13*b^2*sin(d*x + c)^3 - 48*a*b*sin
(d*x + c)^2 - 7*a^2*sin(d*x + c) - 11*b^2*sin(d*x + c) + 16*a*b)/(sin(d*x +
 c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.33, size = 433, normalized size = 2.29

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(\frac{15\,a^{2}}{8}-6\,a\,b+\frac{35\,b^{2}}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(\frac{15\,a^{2}}{8}+6\,a\,b+\frac{35\,b^{2}}{8}\right)}{d}-\frac{\left(-\frac{15\,a^{2}}{4}-\frac{35\,b^{2}}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+\left(\frac{5\,a^{2}}{2}+\frac{35\,b^{2}}{6}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\left(\frac{47\,a^{2}}{4}+\frac{329\,b^{2}}{12}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+32\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+(11\,a^{2}-17\,b^{2})\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+32\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+\left(\frac{47\,a^{2}}{4}+\frac{329\,b^{2}}{12}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+\left(\frac{5\,a^{2}}{2}+\frac{35\,b^{2}}{6}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+\left(-\frac{15\,a^{2}}{4}-\frac{35\,b^{2}}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)}+\frac{6\,a\,b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^6*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((15*a^2)/8 - 6*a*b + (35*b^2)/8))/d - (log(ta
n(c/2 + (d*x)/2) - 1)*(6*a*b + (15*a^2)/8 + (35*b^2)/8))/d - (tan(c/2 + (d*
x)/2)^7*(11*a^2 - 17*b^2) + tan(c/2 + (d*x)/2)^3*((5*a^2)/2 + (35*b^2)/6) +
 tan(c/2 + (d*x)/2)^11*((5*a^2)/2 + (35*b^2)/6) - tan(c/2 + (d*x)/2)^13*((1
5*a^2)/4 + (35*b^2)/4) + tan(c/2 + (d*x)/2)^5*((47*a^2)/4 + (329*b^2)/12) +
 tan(c/2 + (d*x)/2)^9*((47*a^2)/4 + (329*b^2)/12) - tan(c/2 + (d*x)/2)*((15
*a^2)/4 + (35*b^2)/4) - 12*a*b*tan(c/2 + (d*x)/2)^2 + 12*a*b*tan(c/2 + (d*x
)/2)^4 + 32*a*b*tan(c/2 + (d*x)/2)^6 + 32*a*b*tan(c/2 + (d*x)/2)^8 + 12*a*b
*tan(c/2 + (d*x)/2)^10 - 12*a*b*tan(c/2 + (d*x)/2)^12)/(d*(tan(c/2 + (d*x)/
2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/
2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)
^14 - 1)) + (6*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.1493 $\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=162

$$-\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d}+\frac{(15ab-4(a^2+3b^2))\log(1+\sin(c+dx))}{8d}-\frac{2ab\sin(c+dx)}{d}-b^2$$

```
[Out] -1/8*(4*a^2+15*a*b+12*b^2)*ln(1-sin(d*x+c))/d+1/8*(-4*a^2+15*a*b-12*b^2)*ln
(1+sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*b^2*sin(d*x+c)^2/d+1/4*sec(d*x+c)^4
*(a+b*sin(d*x+c))^2/d-1/4*sec(d*x+c)^2*(a+b*sin(d*x+c))*(4*a+5*b*sin(d*x+c)
)/d
```

Rubi [A]

time = 0.18, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2800, 1659, 1824, 647, 31}

$$-\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d}+\frac{(15ab-4(a^2+3b^2))\log(\sin(c+dx)+1)}{8d}-\frac{2ab\sin(c+dx)}{d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx)(a+b\sin(c+dx))(4a+5b\sin(c+dx))}{4d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -1/8*((4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]])/d + ((15*a*b - 4*(a^
2 + 3*b^2))*Log[1 + Sin[c + d*x]])/(8*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Si
n[c + d*x]^2)/(2*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[
c + d*x]^2*(a + b*Sin[c + d*x])*(4*a + 5*b*Sin[c + d*x]))/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
```

```
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^6-4ab^4x-4b^4x^2-4\right.}{(b^2-x^2)^2}\right.}{4b^2}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))(}{4d}$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx}{4d}$$

$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx}{4d}$$

$$= -\frac{(4a^2+15ab+12b^2)\log(1-\sin(c+dx))}{8d} - \frac{(4a^2-15ab+12b^2)\,\text{lo}}{8d}$$

Mathematica [A]
time = 1.44, size = 164, normalized size = 1.01

$$\frac{-2(4a^2+15ab+12b^2)\log(1-\sin(c+dx))-2(4a^2-15ab+12b^2)\log(1+\sin(c+dx))+\frac{(a+b)^2}{(-1+\sin(c+dx))^2}+\frac{(a+b)(7a+11b)}{-1+\sin(c+dx)}-32ab\sin(c+dx)-8b^2\sin^2(c+dx)+\frac{(a-b)^2}{(1+\sin(c+dx))^2}-\frac{(7a-11b)(a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-2*(4*a^2 + 15*a*b + 12*b^2)*Log[1 - Sin[c + d*x]] - 2*(4*a^2 - 15*a*b + 1
2*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(
7*a + 11*b))/(-1 + Sin[c + d*x]) - 32*a*b*Sin[c + d*x] - 8*b^2*Sin[c + d*x]
^2 + (a - b)^2/(1 + Sin[c + d*x])^2 - ((7*a - 11*b)*(a - b))/(1 + Sin[c + d
*x]))/(16*d)
```

Maple [A]
time = 0.30, size = 205, normalized size = 1.27

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\right.}{}$
default	$\frac{a^2\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\right.}{}$
norman	$\frac{\frac{\left(4a^2+12b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^2+12b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^2+3b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(a^2+3b^2\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(3a^2\right.}{}}{\left(\tan^2\left(\right.\right.}$
risch	$ia^2x-\frac{iab\,\mathrm{e}^{-i(dx+c)}}{d}+\frac{b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{iab\,\mathrm{e}^{i(dx+c)}}{d}+\frac{6ib^2c}{d}+\frac{b^2\mathrm{e}^{-2i(dx+c)}}{8d}+\frac{i\left(8ia^2\mathrm{e}^{6i(dx+c)}+12ib^2\mathrm{e}^{6i(dx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+2*a*b*(1/4*sin(
d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*si
n(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+b^2*(1/4*sin(d*x
+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d
*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.36, size = 157, normalized size = 0.97

$$-\frac{4\,b^2\sin\left(dx+c\right)^2+16\,ab\sin\left(dx+c\right)+\left(4\,a^2-15\,ab+12\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(4\,a^2+15\,ab+12\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(9\,ab\sin(dx+c)^3-7\,ab\sin(dx+c)+2\left(2\,a^2+3\,b^2\right)\sin(dx+c)^2-3\,a^2-5\,b^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(sin(d*x + c) + 1) + (4*a^2 + 15*a*b + 12*b^2)*log(sin(d*x + c) - 1) -
 2*(9*a*b*sin(d*x + c)^3 - 7*a*b*sin(d*x + c) + 2*(2*a^2 + 3*b^2)*sin(d*x +
 c)^2 - 3*a^2 - 5*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 178, normalized size = 1.10

$$\frac{4\,b^{2}\cos\left(dx+c\right)^{6}-2\,b^{2}\cos\left(dx+c\right)^{4}-\left(4\,a^{2}-15\,ab+12\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^{2}+15\,ab+12\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-4\left(2\,a^{2}+3\,b^{2}\right)\cos\left(dx+c\right)^{2}+2\,a^{2}+2\,b^{2}-2\left(8\,ab\cos\left(dx+c\right)^{4}+9\,ab\cos\left(dx+c\right)^{2}-2\,ab\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*b^2*cos(d*x + c)^6 - 2*b^2*cos(d*x + c)^4 - (4*a^2 - 15*a*b + 12*b^2
)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 15*a*b + 12*b^2)*cos(d*x
+ c)^4*log(-sin(d*x + c) + 1) - 4*(2*a^2 + 3*b^2)*cos(d*x + c)^2 + 2*a^2 +
2*b^2 - 2*(8*a*b*cos(d*x + c)^4 + 9*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c
))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.61, size = 175, normalized size = 1.08

$$-\frac{4\,b^{2}\sin\left(dx+c\right)^{2}+16\,ab\sin\left(dx+c\right)+\left(4\,a^{2}-15\,ab+12\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(4\,a^{2}+15\,ab+12\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^{2}\sin\left(dx+c\right)^{4}+9\,b^{2}\sin\left(dx+c\right)^{4}+9\,ab\sin\left(dx+c\right)^{3}-2\,a^{2}\sin\left(dx+c\right)^{2}-12\,b^{2}\sin\left(dx+c\right)^{2}-7\,ab\sin\left(dx+c\right)+4\,b^{2}\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^2*sin(d*x + c)^2 + 16*a*b*sin(d*x + c) + (4*a^2 - 15*a*b + 12*b^2
)*log(abs(sin(d*x + c) + 1)) + (4*a^2 + 15*a*b + 12*b^2)*log(abs(sin(d*x +
c) - 1)) - 2*(3*a^2*sin(d*x + c)^4 + 9*b^2*sin(d*x + c)^4 + 9*a*b*sin(d*x +
 c)^3 - 2*a^2*sin(d*x + c)^2 - 12*b^2*sin(d*x + c)^2 - 7*a*b*sin(d*x + c) +
 4*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.22, size = 377, normalized size = 2.33

$$\frac{\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\,\left(a^2+3\,b^2\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a^2-\frac{15\,a\,b}{4}+3\,b^2\right)}{d}-\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a^2+\frac{15\,a\,b}{4}+3\,b^2\right)}{d}-\frac{-\frac{15\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}}{2}+\left(-2\,a^2-6\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+\frac{25\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{2}+\left(4\,a^2+12\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+11\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7+\left(12\,a^2+4\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+11\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(4\,a^2+12\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\frac{25\,a\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\left(-2\,a^2-6\,b^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\frac{15\,a\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left(-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+2\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + 3*b^2))/d - (log(tan(c/2 + (d*x)/2) +
 1)*(a^2 - (15*a*b)/4 + 3*b^2))/d - (log(tan(c/2 + (d*x)/2) - 1)*((15*a*b)/
4 + a^2 + 3*b^2))/d - (tan(c/2 + (d*x)/2)^4*(4*a^2 + 12*b^2) - tan(c/2 + (d
*x)/2)^10*(2*a^2 + 6*b^2) - tan(c/2 + (d*x)/2)^2*(2*a^2 + 6*b^2) + tan(c/2
+ (d*x)/2)^6*(12*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^8*(4*a^2 + 12*b^2) + (25
*a*b*tan(c/2 + (d*x)/2)^3)/2 + 11*a*b*tan(c/2 + (d*x)/2)^5 + 11*a*b*tan(c/2
 + (d*x)/2)^7 + (25*a*b*tan(c/2 + (d*x)/2)^9)/2 - (15*a*b*tan(c/2 + (d*x)/2
)^11)/2 - (15*a*b*tan(c/2 + (d*x)/2))/2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c
/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2
 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1))
```

3.1494 $\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=150

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(1+\sin(c+dx))}{16d}-\frac{b^2\sin(c+dx)}{d}-\frac{\sec^2(}{}$$

```
[Out] -1/16*(3*a^2+16*a*b+15*b^2)*ln(1-sin(d*x+c))/d+1/16*(3*a^2-16*a*b+15*b^2)*l
n(1+sin(d*x+c))/d-b^2*sin(d*x+c)/d-1/8*sec(d*x+c)^2*(7*b+5*a*sin(d*x+c))*(a
+b*sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.19, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2916, 12, 1659, 1824, 647, 31}

$$-\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d}+\frac{(3a^2-16ab+15b^2)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)(5a\sin(c+dx)+7b)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{b^2\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -1/16*((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]])/d + ((3*a^2 - 16*a*
b + 15*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b^2*Sin[c + d*x])/d - (Sec[c +
 d*x]^2*(7*b + 5*a*Sin[c + d*x])*(a + b*Sin[c + d*x]))/(8*d) + (Sec[c + d*x
]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^4(a+x)^2}{b^4(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} +$$

$$= -\frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a+b\sin(c+dx))}{8d} +$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a-}{8d}$$

$$= -\frac{b^2\sin(c+dx)}{d} - \frac{\sec^2(c+dx)(7b+5a\sin(c+dx))(a-}{8d}$$

$$= -\frac{(3a^2+16ab+15b^2)\log(1-\sin(c+dx))}{16d} - \frac{(16ab-3}{}$$

Mathematica [A]

time = 0.69, size = 151, normalized size = 1.01

$$\frac{-((3a^2+16ab+15b^2)\log(1-\sin(c+dx)))+(3a^2-16ab+15b^2)\log(1+\sin(c+dx))+\frac{(a+b)^2}{(-1+\sin(c+dx))^2}+\frac{(a+b)(5a+9b)}{-1+\sin(c+dx)}-16b^2\sin(c+dx)-\frac{(a-b)^2}{(1+\sin(c+dx))^2}+\frac{(5a-9b)(a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (-((3*a^2 + 16*a*b + 15*b^2)*Log[1 - Sin[c + d*x]]) + (3*a^2 - 16*a*b + 15*
b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(5*
a + 9*b))/(-1 + Sin[c + d*x]) - 16*b^2*Sin[c + d*x] - (a - b)^2/(1 + Sin[c
+ d*x])^2 + ((5*a - 9*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]

time = 0.27, size = 200, normalized size = 1.33

method	result
derivativedivides	$a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right.}{2}\right.$

default	$\frac{a^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\frac{\left(\sin^3(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+\right.}{2}\right.}{}$
risch	$2iabx+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{4iabc}{d}+\frac{i(5a^2e^{7i(dx+c)}+9b^2e^{7i(dx+c)}-3a^2e^{5i(dx+c)}+b^2e^{5i(dx+c)}+32iab}{}$
norman	$\frac{\frac{8ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ab\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3\left(a^2+5b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{5\left(a^2+5b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5\left(a^2+5b^2\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}}{\left(\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(1/4*tan(d*
x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+b^2*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-
3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*
x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.33, size = 148, normalized size = 0.99

$$-\frac{16\,b^2\sin\left(dx+c\right)-\left(3\,a^2-16\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(3\,a^2+16\,ab+15\,b^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(16\,ab\sin(dx+c)^2+(5\,a^2+9\,b^2)\sin(dx+c)^3-12\,ab-(3\,a^2+7\,b^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(sin(d*x + c) + 1
) + (3*a^2 + 16*a*b + 15*b^2)*log(sin(d*x + c) - 1) - 2*(16*a*b*sin(d*x + c
)^2 + (5*a^2 + 9*b^2)*sin(d*x + c)^3 - 12*a*b - (3*a^2 + 7*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]

time = 0.40, size = 151, normalized size = 1.01

$$\frac{\left(3\,a^2-16\,ab+15\,b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(3\,a^2+16\,ab+15\,b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-32\,ab\cos\left(dx+c\right)^2+8\,ab-2\left(8\,b^2\cos\left(dx+c\right)^4+\left(5\,a^2+9\,b^2\right)\cos\left(dx+c\right)^2-2\,a^2-2\,b^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*((3*a^2 - 16*a*b + 15*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a
^2 + 16*a*b + 15*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 32*a*b*cos(d*
x + c)^2 + 8*a*b - 2*(8*b^2*cos(d*x + c)^4 + (5*a^2 + 9*b^2)*cos(d*x + c)^2
 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.55, size = 157, normalized size = 1.05

$$-\frac{16\,b^2\sin\left(dx+c\right)-\left(3\,a^2-16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(3\,a^2+16\,ab+15\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(12\,ab\sin\left(dx+c\right)^4+5\,a^2\sin\left(dx+c\right)^3+9\,b^2\sin\left(dx+c\right)^3-8\,ab\sin\left(dx+c\right)^2-3\,a^2\sin\left(dx+c\right)-7\,b^2\sin\left(dx+c\right)\right)}{\left(\sin\left(dx+c\right)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^2*sin(d*x + c) - (3*a^2 - 16*a*b + 15*b^2)*log(abs(sin(d*x + c)
 + 1)) + (3*a^2 + 16*a*b + 15*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(12*a*b*s
in(d*x + c)^4 + 5*a^2*sin(d*x + c)^3 + 9*b^2*sin(d*x + c)^3 - 8*a*b*sin(d*x
 + c)^2 - 3*a^2*sin(d*x + c) - 7*b^2*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/
d
```

Mupad [B]
time = 12.21, size = 332, normalized size = 2.21

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(\frac{3\,a^2}{8}-2\,a\,b+\frac{15\,b^2}{8}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\frac{3\,a^2}{8}+2\,a\,b+\frac{15\,b^2}{8}\right)}{d}+\frac{\left(-\frac{3\,a^2}{4}-\frac{15\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\left(2\,a^2+10\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(\frac{11\,a^2}{2}-\frac{9\,b^2}{2}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+12\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(2\,a^2+10\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\left(-\frac{3\,a^2}{4}-\frac{15\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{2\,a\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((3*a^2)/8 - 2*a*b + (15*b^2)/8))/d - (log(tan
(c/2 + (d*x)/2) - 1)*(2*a*b + (3*a^2)/8 + (15*b^2)/8))/d + (tan(c/2 + (d*x)
/2)^3*(2*a^2 + 10*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^2 + 10*b^2) + tan(c/2 +
(d*x)/2)^5*((11*a^2)/2 - (9*b^2)/2) - tan(c/2 + (d*x)/2)^9*((3*a^2)/4 + (15
*b^2)/4) - tan(c/2 + (d*x)/2)*((3*a^2)/4 + (15*b^2)/4) - 4*a*b*tan(c/2 + (d
*x)/2)^2 + 12*a*b*tan(c/2 + (d*x)/2)^4 + 12*a*b*tan(c/2 + (d*x)/2)^6 - 4*a*
b*tan(c/2 + (d*x)/2)^8)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2
 + 2*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10
+ 1)) + (2*a*b*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.1495 $\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{(3a-4b)b\log(1+\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(}{}$$

```
[Out] -1/8*b*(3*a+4*b)*ln(1-sin(d*x+c))/d+1/8*(3*a-4*b)*b*ln(1+sin(d*x+c))/d+1/4*
sec(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/4*sec(d*x+c)^2*(a+b*sin(d*x+c))*(2*a+3*
b*sin(d*x+c))/d
```

Rubi [A]
time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2916, 12, 1659, 647, 31}

$$-\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d}+\frac{b(3a-4b)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx)(a+b\sin(c+dx))(2a+3b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] -1/8*(b*(3*a + 4*b)*Log[1 - Sin[c + d*x]])/d + ((3*a - 4*b)*b*Log[1 + Sin[c
 + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2)/(4*d) - (Sec[c +
d*x]^2*(a + b*Sin[c + d*x])*(2*a + 3*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^3(a+x)^2}{b^3(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{x^3(a+x)^2}{(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)(-2b^4-}{(b^2-}\right.}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\,\mathrm{s}}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)(a+b\,\mathrm{s}}{}$$

$$= -\frac{b(3a+4b)\log(1-\sin(c+dx))}{8d} + \frac{(3a-4b)b\log(1+}{8d}$$

Mathematica [A]
time = 0.27, size = 129, normalized size = 1.11

$$\frac{2ab\sec(c+dx)\tan^3(c+dx)}{d}+\frac{a^2\tan^4(c+dx)}{4d}-\frac{b^2(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}-\frac{ab\left(6\sec^3(c+dx)\tan(c+dx)-3\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (2*a*b*Sec[c + d*x]*Tan[c + d*x]^3)/d + (a^2*Tan[c + d*x]^4)/(4*d) - (b^2*(
4*Log[Cos[c + d*x]] + 2*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d) - (a*b*(6*S
ec[c + d*x]^3*Tan[c + d*x] - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(4*d)
```

Maple [A]
time = 0.28, size = 136, normalized size = 1.17

method	result
derivativedivides	$\frac{\frac{a^{2}\left(\sin^{4}(dx+c)\right)}{4\cos(dx+c)^{4}}+2ab\left(\frac{\sin^{5}(dx+c)}{4\cos(dx+c)^{4}}-\frac{\sin^{5}(dx+c)}{8\cos(dx+c)^{2}}-\frac{\left(\sin^{3}(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b^{2}\left(\frac{\left(\tan^{4}(dx \cdots\right.}{4}\right.}{d}$
default	$\frac{\frac{a^{2}\left(\sin^{4}(dx+c)\right)}{4\cos(dx+c)^{4}}+2ab\left(\frac{\sin^{5}(dx+c)}{4\cos(dx+c)^{4}}-\frac{\sin^{5}(dx+c)}{8\cos(dx+c)^{2}}-\frac{\left(\sin^{3}(dx+c)\right)}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+b^{2}\left(\frac{\left(\tan^{4}(dx \cdots\right.}{4}\right.}{d}$
risch	$ib^{2}x+\frac{2ib^{2}c}{d}+\frac{i\left(4ia^{2}e^{6i(dx+c)}+8ib^{2}e^{6i(dx+c)}+5ab\,e^{7i(dx+c)}+8ib^{2}e^{4i(dx+c)}-3b\,e^{5i(dx+c)}a+4ia^{2}e^{2i(dx+c)}+8ib^{2}e^{2 \cdots\right.}{2d\left(e^{2i(dx+c)}+1\right)^{4}}$
norman	$\frac{\frac{\left(4a^{2}+4b^{2}\right)\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^{2}+4b^{2}\right)\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b^{2}\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b^{2}\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(2a^{2}+3b^{2}\right)\left(\tan^{6}\left(\frac{dx}{2}+ \cdots\right.\right.}{d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)- \cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^2*sin(d*x+c)^4/cos(d*x+c)^4+2*a*b*(1/4*sin(d*x+c)^5/cos(d*x+c)^4
-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+b^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 123, normalized size = 1.06

$$\frac{(3\,ab-4\,b^{2})\log\left(\sin\left(dx+c\right)+1\right)-(3\,ab+4\,b^{2})\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(5\,ab\sin(dx+c)^{3}-3\,ab\sin(dx+c)+2\,(a^{2}+2\,b^{2})\sin(dx+c)^{2}-a^{2}-3\,b^{2}\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)*log(sin(d*x +
c) - 1) + 2*(5*a*b*sin(d*x + c)^3 - 3*a*b*sin(d*x + c) + 2*(a^2 + 2*b^2)*si
n(d*x + c)^2 - a^2 - 3*b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 127, normalized size = 1.09

$$\frac{(3\,ab-4\,b^{2})\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-(3\,ab+4\,b^{2})\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)-4\,(a^{2}+2\,b^{2})\cos\left(dx+c\right)^{2}+2\,a^{2}+2\,b^{2}-2\,\left(5\,ab\cos\left(dx+c\right)^{2}-2\,ab\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*a*b + 4*b^2)
*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 4*(a^2 + 2*b^2)*cos(d*x + c)^2 + 2
*a^2 + 2*b^2 - 2*(5*a*b*cos(d*x + c)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.51, size = 130, normalized size = 1.12

$$\frac{\left(3\,ab-4\,b^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(3\,ab+4\,b^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{2\,\left(3\,b^2\sin(dx+c)^4+5\,ab\sin(dx+c)^3+2\,a^2\sin(dx+c)^2-2\,b^2\sin(dx+c)^2-3\,ab\sin(dx+c)-a^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*a*b - 4*b^2)*log(abs(sin(d*x + c) + 1)) - (3*a*b + 4*b^2)*log(abs(s
in(d*x + c) - 1)) + 2*(3*b^2*sin(d*x + c)^4 + 5*a*b*sin(d*x + c)^3 + 2*a^2*
sin(d*x + c)^2 - 2*b^2*sin(d*x + c)^2 - 3*a*b*sin(d*x + c) - a^2)/(sin(d*x
+ c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.09, size = 247, normalized size = 2.13

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{3\,a\,b}{4}-b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(b^2+\frac{3\,a\,b}{4}\right)}{d}+\frac{b^2\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{d}-\frac{2\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(4\,a^2+8\,b^2\right)+2\,b^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-\frac{11\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}-\frac{11\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5}{2}+\frac{3\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2}+\frac{3\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) + 1)*((3*a*b)/4 - b^2))/d - (log(tan(c/2 + (d*x)/2)
 - 1)*((3*a*b)/4 + b^2))/d + (b^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (2*b^2
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4*(4*a^2 + 8*b^2) + 2*b^2*tan(c/
2 + (d*x)/2)^6 - (11*a*b*tan(c/2 + (d*x)/2)^3)/2 - (11*a*b*tan(c/2 + (d*x)/
2)^5)/2 + (3*a*b*tan(c/2 + (d*x)/2)^7)/2 + (3*a*b*tan(c/2 + (d*x)/2))/2)/(d
*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6
+ tan(c/2 + (d*x)/2)^8 + 1))
```

3.1496 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=93

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\,(4ab+(a^2+3b^2)\sin(c+dx))}{8d}+\frac{\sec^3(c+dx)(a+b\sin(c+d$$

```
[Out] -1/8*(a^2-3*b^2)*arctanh(sin(d*x+c))/d-1/8*sec(d*x+c)^2*(4*a*b+(a^2+3*b^2)*
sin(d*x+c))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]

time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2916, 12, 1659, 792, 212}

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{\sec^2(c+dx)\,((a^2+3b^2)\sin(c+dx)+4ab)}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]])/d - (Sec[c + d*x]^2*(4*a*b + (a^
2 + 3*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Ta
n[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
```

```
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^2(a+x)^2}{b^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b^3\text{Subst}\left(\int \frac{x^2(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)}{4d} + \frac{b\text{Subst}(}{}$$
$$= -\frac{\sec^2(c+dx)\left(4ab+(a^2+3b^2)\sin(c+dx)\right)}{8d} + \frac{\sec^3(c}{}$$
$$= -\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{\sec^2(c+dx)\,(4ab+}{}$$

Mathematica [A]
time = 0.54, size = 85, normalized size = 0.91

$$-\frac{(a^2-3b^2)\tanh^{-1}(\sin(c+dx))+\frac{1}{4}\sec^4(c+dx)\left(16ab\cos(2(c+dx))+2(-3a^2+b^2+(a^2+5b^2)\cos(2(c+dx)))\sin(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -1/8*((a^2 - 3*b^2)*ArcTanh[Sin[c + d*x]] + (Sec[c + d*x]^4*(16*a*b*Cos[2*(
c + d*x)] + 2*(-3*a^2 + b^2 + (a^2 + 5*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])
)/4)/d
```

Maple [A]
time = 0.26, size = 166, normalized size = 1.78

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{ab\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{ab\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c)}{8\cos(dx+c)^2}-\right.}{d}$
risch	$\frac{i\left(a^2 e^{7i(dx+c)}+5b^2 e^{7i(dx+c)}-7a^2 e^{5i(dx+c)}-3b^2 e^{5i(dx+c)}+16iab\, e^{6i(dx+c)}+7\, e^{3i(dx+c)}a^2+3b^2 e^{3i(dx+c)}-a^2 e^{i(dx+c)}-5\right.}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{8ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ab\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(a^2-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{\left(a^2-3b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(9a^2+5b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a*b*sin(d*x+c)^4/cos(d*x+c)^4+
b^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*
x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 120, normalized size = 1.29

$$-\frac{(a^2-3\,b^2)\log(\sin(dx+c)+1)-(a^2-3\,b^2)\log(\sin(dx+c)-1)-\frac{2\left(8\,ab\sin(dx+c)^2+(a^2+5\,b^2)\sin(dx+c)^3-4\,ab+(a^2-3\,b^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*log(sin(d*x + c)
 - 1) - 2*(8*a*b*sin(d*x + c)^2 + (a^2 + 5*b^2)*sin(d*x + c)^3 - 4*a*b + (a
^2 - 3*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.36, size = 124, normalized size = 1.33

$$-\frac{(a^2-3\,b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-(a^2-3\,b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+16\,ab\cos(dx+c)^2-8\,ab+2\left((a^2+5\,b^2)\cos(dx+c)^2-2\,a^2-2\,b^2\right)\sin(dx+c)}{16\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^2 - 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^2 - 3*b^2)*c
os(d*x + c)^4*log(-sin(d*x + c) + 1) + 16*a*b*cos(d*x + c)^2 - 8*a*b + 2*((
a^2 + 5*b^2)*cos(d*x + c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^
4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.49, size = 124, normalized size = 1.33

$$-\frac{(a^2-3\,b^2)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-(a^2-3\,b^2)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(a^2\sin(dx+c)^3+5\,b^2\sin(dx+c)^3+8\,ab\sin(dx+c)^2+a^2\sin(dx+c)-3\,b^2\sin(dx+c)-4\,ab\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^2 - 3*b^2)*log(abs(sin(d*x + c) + 1)) - (a^2 - 3*b^2)*log(abs(sin
(d*x + c) - 1)) - 2*(a^2*sin(d*x + c)^3 + 5*b^2*sin(d*x + c)^3 + 8*a*b*sin(
d*x + c)^2 + a^2*sin(d*x + c) - 3*b^2*sin(d*x + c) - 4*a*b)/(sin(d*x + c)^2
 - 1)^2)/d
```

Mupad [B]
time = 17.05, size = 191, normalized size = 2.05

$$\frac{\left(\frac{a^2}{4}-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{7a^2}{4}+\frac{11b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\left(\frac{7a^2}{4}+\frac{11b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{a^2}{4}-\frac{3b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{a^2}{4}-\frac{3b^2}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^7*(a^2/4 - (3*b^2)/4) + tan(c/2 + (d*x)/2)^3*((7*a^2)/4
 + (11*b^2)/4) + tan(c/2 + (d*x)/2)^5*((7*a^2)/4 + (11*b^2)/4) + tan(c/2 +
(d*x)/2)*(a^2/4 - (3*b^2)/4) + 8*a*b*tan(c/2 + (d*x)/2)^4)/(d*(6*tan(c/2 +
(d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 + 1)) - (atanh(tan(c/2 + (d*x)/2))*(a^2/4 - (3*b^2)/4))/d
```

3.1497 $\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=72

$$-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}-\frac{\sec^2(c+dx)\left(b^2+ab\sin(c+dx)\right)}{4d}$$

```
[Out] -1/4*a*b*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^2/d-1/4*se
c(d*x+c)^2*(b^2+a*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2916, 12, 835, 653, 212}

$$-\frac{\sec^2(c+dx)\left(ab\sin(c+dx)+b^2\right)}{4d}-\frac{ab\tanh^{-1}(\sin(c+dx))}{4d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -1/4*(a*b*ArcTanh[Sin[c + d*x]])/d + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2
)/(4*d) - (Sec[c + d*x]^2*(b^2 + a*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
```

```
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x(a+x)^2}{b(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{x(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^2\text{Subst}\left(\int \frac{2b^2(a+x)}{(b^2-x^2)^2}\,d\right.}{4d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{b^4\text{Subst}\left(\int \frac{a+x}{(b^2-x^2)^2}\,d\right.}{2d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^2}{4d} - \frac{\sec^2(c+dx)\left(b^2+ab\right.}{4d}$$

$$= -\frac{ab\tanh^{-1}(\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)(a+b\sin(c+d}{4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(72) = 144.
time = 2.41, size = 215, normalized size = 2.99

$$\frac{ab(a^2-b^2)^2(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+2a^4b^2\sec^2(c+dx)+2a^4(a^2-b^2)\sec^4(c+dx)+4a^3b(a^2-b^2)\sec^3(c+dx)\tan(c+dx)+b(-6a^4b+4a^2b^3)\tan^2(c+dx)+2b^4(-a^2+b^2)\tan^4(c+dx)-2ab(a^2-b^2)\sec(c+dx)\tan(c+dx)\left(a^2+b^2+2b^2\tan^2(c+dx)\right)}{8\left(a^2-b^2\right)^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (a*b*(a^2 - b^2)^2*(Log[1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^4*
b^2*Sec[c + d*x]^2 + 2*a^4*(a^2 - b^2)*Sec[c + d*x]^4 + 4*a^3*b*(a^2 - b^2)
```

```
*Sec[c + d*x]^3*Tan[c + d*x] + b*(-6*a^4*b + 4*a^2*b^3)*Tan[c + d*x]^2 + 2*
b^4*(-a^2 + b^2)*Tan[c + d*x]^4 - 2*a*b*(a^2 - b^2)*Sec[c + d*x]*Tan[c + d*
x]*(a^2 + b^2 + 2*b^2*Tan[c + d*x]^2))/(8*(a^2 - b^2)^2*d)
```

Maple [A]

time = 0.25, size = 105, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{a^2}{4\cos(dx+c)^4}+2ab\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
default	$\frac{\frac{a^2}{4\cos(dx+c)^4}+2ab\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
risch	$\frac{i\left(4ib^2e^{6i(dx+c)}+ab\,e^{7i(dx+c)}-8ia^2e^{4i(dx+c)}-7b\,e^{5i(dx+c)}a+4ib^2e^{2i(dx+c)}+7ab\,e^{3i(dx+c)}-b\,e^{i(dx+c)}a\right)}{2d\left(e^{2i(dx+c)}+1\right)^4}-\frac{\ln(e^{i(dx+c)}}{4d}$
norman	$\frac{\frac{\left(4a^2+4b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^2+4b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^2\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(a^2+2b^2\right)\left(\tan^6\left(\frac{dx}{2}+\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^2/cos(d*x+c)^4+2*a*b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+
c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*b^2*sin
(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]

time = 0.30, size = 97, normalized size = 1.35

$$-\frac{ab\log\left(\sin\left(dx+c\right)+1\right)-ab\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(ab\sin(dx+c)^3+2\,b^2\sin(dx+c)^2+ab\sin(dx+c)+a^2-b^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a*b*log(sin(d*x + c) + 1) - a*b*log(sin(d*x + c) - 1) - 2*(a*b*sin(d*
x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]

time = 0.38, size = 104, normalized size = 1.44

$$-\frac{ab\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-ab\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,b^2\cos\left(dx+c\right)^2-2\,a^2-2\,b^2+2\left(ab\cos\left(dx+c\right)^2-2\,ab\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(a*b*cos(d*x + c)^4*log(sin(d*x + c) + 1) - a*b*cos(d*x + c)^4*log(-si
n(d*x + c) + 1) + 4*b^2*cos(d*x + c)^2 - 2*a^2 - 2*b^2 + 2*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 89, normalized size = 1.24

$$-\frac{ab\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-ab\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(ab\sin(dx+c)^3+2\,b^2\sin(dx+c)^2+ab\sin(dx+c)+a^2-b^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(a*b*log(abs(sin(d*x + c) + 1)) - a*b*log(abs(sin(d*x + c) - 1)) - 2*(
a*b*sin(d*x + c)^3 + 2*b^2*sin(d*x + c)^2 + a*b*sin(d*x + c) + a^2 - b^2)/(
sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 18.87, size = 183, normalized size = 2.54

$$\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{7\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\frac{7\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}+4\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (2*a^2*tan(c/2 + (d*x)/2)^2 + 2*a^2*tan(c/2 + (d*x)/2)^6 + 4*b^2*tan(c/2 +
(d*x)/2)^4 + (7*a*b*tan(c/2 + (d*x)/2)^3)/2 + (7*a*b*tan(c/2 + (d*x)/2)^5)/
2 + (a*b*tan(c/2 + (d*x)/2)^7)/2 + (a*b*tan(c/2 + (d*x)/2))/2)/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) - (a*b*atanh(tan(c/2 + (d*x)/2)))/(2*d)
```

3.1498 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a-3b)\log(1+\sin(c+dx))}{8d}+\frac{a\sec^2(c+dx)(2a}{4}$$

```
[Out] -1/8*a*(4*a+3*b)*ln(1-sin(d*x+c))/d+a^2*ln(sin(d*x+c))/d-1/8*a*(4*a-3*b)*ln
(1+sin(d*x+c))/d+1/4*a*sec(d*x+c)^2*(2*a+3*b*sin(d*x+c))/d+1/4*sec(d*x+c)^4
*(a^2+b^2+2*a*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.14, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2916, 12, 1819, 837, 815}

$$\frac{\sec^4(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{a^2\log(\sin(c+dx))}{d}-\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d}-\frac{a(4a-3b)\log(\sin(c+dx)+1)}{8d}+\frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/8*(a*(4*a + 3*b)*Log[1 - Sin[c + d*x]])/d + (a^2*Log[Sin[c + d*x]])/d -
(a*(4*a - 3*b)*Log[1 + Sin[c + d*x]])/(8*d) + (a*Sec[c + d*x]^2*(2*a + 3*b*
Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^4*(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\text{Subst}\left(\int \frac{b(a+x)^2}{x(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \frac{(a+x)^2}{x(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^4\text{Subst}\left(\int \frac{-}{x(}\right.}{}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+\right.}{}$$

$$= \frac{a\sec^2(c+dx)(2a+3b\sin(c+dx))}{4d} + \frac{\sec^4(c+dx)\left(a^2+\right.}{}$$

$$= -\frac{a(4a+3b)\log(1-\sin(c+dx))}{8d} + \frac{a^2\log(\sin(c+dx))}{d}$$

Mathematica [A]

time = 0.64, size = 137, normalized size = 1.09

$$\frac{-2a(4a+3b)\log(1-\sin(c+dx)) + 16a^2\log(\sin(c+dx)) - 2a(4a-3b)\log(1+\sin(c+dx)) + \frac{(a+b)^2}{(-1+\sin(c+dx))^2} - \frac{(a+b)(5a+b)}{-1+\sin(c+dx)} + \frac{(a-b)^2}{(1+\sin(c+dx))^2} + \frac{(a-b)(5a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*(4*a + 3*b)*Log[1 - Sin[c + d*x]] + 16*a^2*Log[Sin[c + d*x]] - 2*a*(4
*a - 3*b)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a + b
)*(5*a + b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((a - b
)*(5*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]
time = 0.40, size = 99, normalized size = 0.79

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+2ab\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+2ab\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{d}$
risch	$-\frac{i\left(4ia^2e^{6i(dx+c)}+3ab\,e^{7i(dx+c)}+16ia^2e^{4i(dx+c)}+8ib^2e^{4i(dx+c)}+11b\,e^{5i(dx+c)}a+4ia^2e^{2i(dx+c)}-11ab\,e^{3i(dx+c)}-3b\,e^{i}\right.}{2d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{\left(4a^2+4b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^2+4b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4b^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(2a^2+b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(2a^2+b^2\right)\left(\tan^{10}\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+2*a*b*(-(-1/4*s
ec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*b
^2/cos(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 130, normalized size = 1.03

$$\frac{8\,a^2\log\left(\sin\left(dx+c\right)\right)-\left(4\,a^2-3\,ab\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^2+3\,ab\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,ab\sin(dx+c)^3+2\,a^2\sin(dx+c)^2-5\,ab\sin(dx+c)-3\,a^2-b^2\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*a^2*log(sin(d*x + c)) - (4*a^2 - 3*a*b)*log(sin(d*x + c) + 1) - (4*a
^2 + 3*a*b)*log(sin(d*x + c) - 1) - 2*(3*a*b*sin(d*x + c)^3 + 2*a^2*sin(d*x
 + c)^2 - 5*a*b*sin(d*x + c) - 3*a^2 - b^2)/(sin(d*x + c)^4 - 2*sin(d*x + c
)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 144, normalized size = 1.14

$$\frac{8\,a^2\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(4\,a^2-3\,ab\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(4\,a^2+3\,ab\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^2\cos\left(dx+c\right)^2+2\,a^2+2\,b^2+2\left(3\,ab\cos\left(dx+c\right)^2+2\,ab\right)\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (4*a^2 - 3*a*b)*cos(d*x +
 c)^4*log(sin(d*x + c) + 1) - (4*a^2 + 3*a*b)*cos(d*x + c)^4*log(-sin(d*x +
 c) + 1) + 4*a^2*cos(d*x + c)^2 + 2*a^2 + 2*b^2 + 2*(3*a*b*cos(d*x + c)^2 +
 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.50, size = 134, normalized size = 1.06

$$\frac{8\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)-\left(4\,a^2-3\,ab\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(4\,a^2+3\,ab\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)+\frac{2\left(3\,a^2\sin(dx+c)^4-3\,ab\sin(dx+c)^3-8\,a^2\sin(dx+c)^2+5\,ab\sin(dx+c)+6\,a^2+b^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*log(abs(sin(d*x + c))) - (4*a^2 - 3*a*b)*log(abs(sin(d*x + c) +
1)) - (4*a^2 + 3*a*b)*log(abs(sin(d*x + c) - 1)) + 2*(3*a^2*sin(d*x + c)^4
- 3*a*b*sin(d*x + c)^3 - 8*a^2*sin(d*x + c)^2 + 5*a*b*sin(d*x + c) + 6*a^2
+ b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 0.13, size = 131, normalized size = 1.04

$$\frac{a^2\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}+\frac{-\frac{a^2\sin(c+d\,x)^2}{2}+\frac{3\,a^2}{4}-\frac{3\,a\,b\sin(c+d\,x)^3}{4}+\frac{5\,a\,b\,\sin(c+d\,x)}{4}+\frac{b^2}{4}}{d\,\left(\sin\left(c+d\,x\right)^4-2\sin\left(c+d\,x\right)^2+1\right)}-\frac{a\,\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(4\,a+3\,b\right)}{8\,d}-\frac{a\,\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(4\,a-3\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^2*log(sin(c + d*x)))/d + ((3*a^2)/4 + b^2/4 - (a^2*sin(c + d*x)^2)/2 + (
5*a*b*sin(c + d*x))/4 - (3*a*b*sin(c + d*x)^3)/4)/(d*(sin(c + d*x)^4 - 2*si
n(c + d*x)^2 + 1)) - (a*log(sin(c + d*x) - 1)*(4*a + 3*b))/(8*d) - (a*log(s
in(c + d*x) + 1)*(4*a - 3*b))/(8*d)
```

3.1499 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$-\frac{a^2\csc(c+dx)}{d}-\frac{(15a^2+16ab+3b^2)\log(1-\sin(c+dx))}{16d}+\frac{2ab\log(\sin(c+dx))}{d}+\frac{(15a^2-16ab+3b^2)\log(}{16d}$$

```
[Out] -a^2*csc(d*x+c)/d-1/16*(15*a^2+16*a*b+3*b^2)*ln(1-sin(d*x+c))/d+2*a*b*ln(si
n(d*x+c))/d+1/16*(15*a^2-16*a*b+3*b^2)*ln(1+sin(d*x+c))/d+1/8*b*sec(d*x+c)^
2*(8*a+(3+7*a^2/b^2)*b*sin(d*x+c))/d+1/4*b*sec(d*x+c)^4*(2*a+(a^2+b^2)*sin(
d*x+c)/b)/d
```

Rubi [A]
time = 0.23, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1819, 1816}

$$-\frac{(15a^2+16ab+3b^2)\log(1-\sin(c+dx))}{16d}+\frac{(15a^2-16ab+3b^2)\log(\sin(c+dx)+1)}{16d}+\frac{b\sec^4(c+dx)\left(\frac{(a^2+b^2)\sin(c+dx)}{b}+2a\right)}{4d}+\frac{b\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+3\right)\sin(c+dx)+8a\right)}{8d}-\frac{a^2\csc(c+dx)}{d}+\frac{2ab\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a^2*Csc[c + d*x])/d) - ((15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])
/(16*d) + (2*a*b*Log[Sin[c + d*x]])/d + ((15*a^2 - 16*a*b + 3*b^2)*Log[1 +
Sin[c + d*x]])/(16*d) + (b*Sec[c + d*x]^2*(8*a + (3 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d) + (b*Sec[c + d*x]^4*(2*a + ((a^2 + b^2)*Sin[c + d*x])/b))/(4
*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\text{Subst}\left(\int \frac{b^2(a+x)^2}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\text{Subst}\left(\int \frac{(a+x)^2}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(2a+\frac{(a^2+b^2)\sin(c+dx)}{b}\right)}{4d} - \frac{b^5\text{Subst}\left(\int \frac{-4a^2}{}\right.}{}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(}{}$$

$$= \frac{b\sec^2(c+dx)\left(8a+\left(3+\frac{7a^2}{b^2}\right)b\sin(c+dx)\right)}{8d} + \frac{b\sec^4(}{}$$

$$= -\frac{a^2\csc(c+dx)}{d} - \frac{(15a^2+16ab+3b^2)\log(1-\sin(c+}{16d}$$

Mathematica [A]
time = 1.93, size = 162, normalized size = 0.96

$$-\frac{16a^2\csc(c+dx)+(15a^2+16ab+3b^2)\log(1-\sin(c+dx))-32ab\log(\sin(c+dx))-(15a^2-16ab+3b^2)\log(1+\sin(c+dx))-\frac{(a+b)^2}{(-1+\sin(c+dx))^2}+\frac{(a+b)(7a+3b)}{-1+\sin(c+dx)}+\frac{(a-b)^2}{(1+\sin(c+dx))^2}+\frac{(7a-3b)(a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*(16*a^2*Csc[c + d*x] + (15*a^2 + 16*a*b + 3*b^2)*Log[1 - Sin[c + d*x]
] - 32*a*b*Log[Sin[c + d*x]] - (15*a^2 - 16*a*b + 3*b^2)*Log[1 + Sin[c + d*
```

```
x]] - (a + b)^2/(-1 + Sin[c + d*x])^2 + ((a + b)*(7*a + 3*b))/(-1 + Sin[c +
 d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + ((7*a - 3*b)*(a - b))/(1 + Sin[c
+ d*x]))/d
```

Maple [A]

time = 0.40, size = 153, normalized size = 0.91

method	result
derivativedivides	$\frac{a^{2}\left(\frac{1}{4 \sin(dx+c)\cos(dx+c)^{4}}+\frac{5}{8 \sin(dx+c)\cos(dx+c)^{2}}-\frac{15}{8 \sin(dx+c)}+\frac{15 \ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{1}{4 \cos(dx+c)^{4}}+\frac{1}{2 \cos(dx\cdots}{d}$
default	$\frac{a^{2}\left(\frac{1}{4 \sin(dx+c)\cos(dx+c)^{4}}+\frac{5}{8 \sin(dx+c)\cos(dx+c)^{2}}-\frac{15}{8 \sin(dx+c)}+\frac{15 \ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+2ab\left(\frac{1}{4 \cos(dx+c)^{4}}+\frac{1}{2 \cos(dx\cdots}{d}$
norman	$\frac{\frac{8ab\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8ab\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^{2}}{2d}-\frac{a^{2}\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7b^{2}\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{7b^{2}\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{\left(11a^{2}+5b^{2}\right)\cdots}{}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cdots\right.}$
risch	$-\frac{i\left(15a^{2}\mathrm{e}^{9i(dx+c)}+3b^{2}\mathrm{e}^{9i(dx+c)}+40a^{2}\mathrm{e}^{7i(dx+c)}+8b^{2}\mathrm{e}^{7i(dx+c)}+16iab\,\mathrm{e}^{8i(dx+c)}+18a^{2}\mathrm{e}^{5i(dx+c)}-22b^{2}\mathrm{e}^{5i(dx+c)}+48iab\cdots}{4d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}\left(\mathrm{e}^{2i(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(
d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(1/4/cos(d*x+c)^4+1/2/cos(d*x+
c)^2+ln(tan(d*x+c)))+b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/
8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.28, size = 163, normalized size = 0.97

$$\frac{32\,ab\log\left(\sin\left(dx+c\right)\right)+\left(15\,a^{2}-16\,ab+3\,b^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(15\,a^{2}+16\,ab+3\,b^{2}\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(8\,ab\sin(dx+c)^{3}+3\left(5\,a^{2}+b^{2}\right)\sin(dx+c)^{4}-12\,ab\sin(dx+c)-5\left(5\,a^{2}+b^{2}\right)\sin(dx+c)^{2}+8\,a^{2}\right)}{\sin(dx+c)^{5}-2\,\sin(dx+c)^{3}+\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/16*(32*a*b*log(sin(d*x + c)) + (15*a^2 - 16*a*b + 3*b^2)*log(sin(d*x + c)
 + 1) - (15*a^2 + 16*a*b + 3*b^2)*log(sin(d*x + c) - 1) - 2*(8*a*b*sin(d*x
+ c)^3 + 3*(5*a^2 + b^2)*sin(d*x + c)^4 - 12*a*b*sin(d*x + c) - 5*(5*a^2 +
b^2)*sin(d*x + c)^2 + 8*a^2)/(sin(d*x + c)^5 - 2*sin(d*x + c)^3 + sin(d*x +
 c)))/d
```

Fricas [A]

time = 0.49, size = 202, normalized size = 1.20

$$\frac{32\,ab\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+\left(15\,a^{2}-16\,ab+3\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-\left(15\,a^{2}+16\,ab+3\,b^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-6\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}+2\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+4\,a^{2}+4\,b^{2}+8\left(2\,ab\cos\left(dx+c\right)^{2}+ab\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(32*a*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + (15*a^2 -
16*a*b + 3*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - (15*a^2
 + 16*a*b + 3*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x + c) - 6*(
5*a^2 + b^2)*cos(d*x + c)^4 + 2*(5*a^2 + b^2)*cos(d*x + c)^2 + 4*a^2 + 4*b^
2 + 8*(2*a*b*cos(d*x + c)^2 + a*b)*sin(d*x + c))/(d*cos(d*x + c)^4*sin(d*x
+ c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 186, normalized size = 1.11

$$\frac{32\,ab\log\left(|\sin\left(dx+c\right)|\right)+\left(15\,a^2-16\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(15\,a^2+16\,ab+3\,b^2\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{16\,\left(2\,ab\sin(dx+c)+a^2\right)}{\sin(dx+c)}+\frac{2\,\left(12\,ab\sin(dx+c)^4-7\,a^2\sin(dx+c)^3-3\,b^2\sin(dx+c)^3-32\,ab\sin(dx+c)^2+9\,a^2\sin(dx+c)+5\,b^2\sin(dx+c)+24\,ab\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(32*a*b*log(abs(sin(d*x + c))) + (15*a^2 - 16*a*b + 3*b^2)*log(abs(sin
(d*x + c) + 1)) - (15*a^2 + 16*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1)) - 16
*(2*a*b*sin(d*x + c) + a^2)/sin(d*x + c) + 2*(12*a*b*sin(d*x + c)^4 - 7*a^2
*sin(d*x + c)^3 - 3*b^2*sin(d*x + c)^3 - 32*a*b*sin(d*x + c)^2 + 9*a^2*sin(
d*x + c) + 5*b^2*sin(d*x + c) + 24*a*b)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 11.87, size = 169, normalized size = 1.01

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{15\,a^2}{16}-a\,b+\frac{3\,b^2}{16}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{15\,a^2}{16}+a\,b+\frac{3\,b^2}{16}\right)}{d}-\frac{a^2+\sin\left(c+d\,x\right)^4\,\left(\frac{15\,a^2}{8}+\frac{3\,b^2}{8}\right)-\sin\left(c+d\,x\right)^2\,\left(\frac{25\,a^2}{8}+\frac{5\,b^2}{8}\right)-\frac{3\,a\,b\,\sin(c+d\,x)}{2}+a\,b\sin\left(c+d\,x\right)^3}{d\,\left(\sin\left(c+d\,x\right)^5-2\sin\left(c+d\,x\right)^3+\sin\left(c+d\,x\right)\right)}+\frac{2\,a\,b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (log(sin(c + d*x) + 1)*((15*a^2)/16 - a*b + (3*b^2)/16))/d - (log(sin(c + d
*x) - 1)*(a*b + (15*a^2)/16 + (3*b^2)/16))/d - (a^2 + sin(c + d*x)^4*((15*a
^2)/8 + (3*b^2)/8) - sin(c + d*x)^2*((25*a^2)/8 + (5*b^2)/8) - (3*a*b*sin(c
 + d*x))/2 + a*b*sin(c + d*x)^3)/(d*(sin(c + d*x) - 2*sin(c + d*x)^3 + sin(
c + d*x)^5)) + (2*a*b*log(sin(c + d*x)))/d
```

3.1500 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=185

$$-\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-$$

```
[Out] -2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-1/8*(12*a^2+15*a*b+4*b^2)*ln(1-s
in(d*x+c))/d+(3*a^2+b^2)*ln(sin(d*x+c))/d-1/8*(12*a^2-15*a*b+4*b^2)*ln(1+si
n(d*x+c))/d+1/4*sec(d*x+c)^4*(a^2+b^2+2*a*b*sin(d*x+c))/d+1/4*sec(d*x+c)^2*
(4*a^2+2*b^2+7*a*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.26, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1819, 1816}

$$-\frac{(12a^2+15ab+4b^2)\log(1-\sin(c+dx))}{8d}+\frac{(3a^2+b^2)\log(\sin(c+dx))}{d}-\frac{(12a^2-15ab+4b^2)\log(\sin(c+dx)+1)}{8d}+\frac{\sec^4(c+dx)(a^2+2ab\sin(c+dx)+b^2)}{4d}+\frac{\sec^2(c+dx)(2(2a^2+b^2)+7ab\sin(c+dx))}{4d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((12*a^2 + 15*a*b +
4*b^2)*Log[1 - Sin[c + d*x]])/(8*d) + ((3*a^2 + b^2)*Log[Sin[c + d*x]])/d -
 ((12*a^2 - 15*a*b + 4*b^2)*Log[1 + Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*
(a^2 + b^2 + 2*a*b*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*(2*(2*a^2 + b^2)
+ 7*a*b*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\text{Subst}\left(\int \frac{b^3(a+x)^2}{x^3(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\text{Subst}\left(\int \frac{(a+x)^2}{x^3(b^2-x^2)^3}\,dx,x,b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} - \frac{b^6\text{Subst}\left(\int \right.}{}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)}{}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d} + \frac{\sec^2(c+dx)}{}$$

$$= -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{(12a^2+15ab+}{}$$

Mathematica [A]
time = 2.42, size = 182, normalized size = 0.98

$$\frac{-32ab\csc(c+dx)-8a^2\csc^2(c+dx)-2(12a^2+15ab+4b^2)\log(1-\sin(c+dx))+16(3a^2+b^2)\log(\sin(c+dx))-2(12a^2-15ab+4b^2)\log(1+\sin(c+dx))+\frac{(a+b)^2}{(-1+\sin(c+dx))^2}-\frac{(a+b)(9a+5b)}{-1+\sin(c+dx)}+\frac{(a-b)^2}{(1+\sin(c+dx))^2}+\frac{(9a-5b)(a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-32*a*b*Csc[c + d*x] - 8*a^2*Csc[c + d*x]^2 - 2*(12*a^2 + 15*a*b + 4*b^2)*
Log[1 - Sin[c + d*x]] + 16*(3*a^2 + b^2)*Log[Sin[c + d*x]] - 2*(12*a^2 - 15
*a*b + 4*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^2/(-1 + Sin[c + d*x])^2 - ((a
```

```
 + b)*(9*a + 5*b))/(-1 + Sin[c + d*x]) + (a - b)^2/(1 + Sin[c + d*x])^2 + (
(9*a - 5*b)*(a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]
time = 0.40, size = 165, normalized size = 0.89

method	result
derivativedivides	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+2ab\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(}\right.}{d}$
default	$\frac{a^2\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+2ab\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{}{8\sin(}\right.}{d}$
risch	$-\frac{i\left(4ib^2e^{2i(dx+c)}+8ib^2e^{4i(dx+c)}+15ab\,e^{11i(dx+c)}+4ib^2e^{10i(dx+c)}+12ia^2e^{10i(dx+c)}+25ab\,e^{9i(dx+c)}+12ia^2e^{2i(dx+c)}-\right.}{2d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/s
in(d*x+c)^2+3*ln(tan(d*x+c)))+2*a*b*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*
x+c)/cos(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+b^2*(1/4/
cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 183, normalized size = 0.99

$$-\frac{(12a^2-15ab+4b^2)\log(\sin(dx+c)+1)+(12a^2+15ab+4b^2)\log(\sin(dx+c)-1)-8(3a^2+b^2)\log(\sin(dx+c))+\frac{2\left(15ab\sin(dx+c)^5-25ab\sin(dx+c)^3+2(3a^2+b^2)\sin(dx+c)^4+8ab\sin(dx+c)-3(3a^2+b^2)\sin(dx+c)^2+2a^2\right)}{\sin(dx+c)^6-2\sin(dx+c)^4+\sin(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(sin(d*x + c) + 1) + (12*a^2 + 15*a*b +
4*b^2)*log(sin(d*x + c) - 1) - 8*(3*a^2 + b^2)*log(sin(d*x + c)) + 2*(15*a*
b*sin(d*x + c)^5 - 25*a*b*sin(d*x + c)^3 + 2*(3*a^2 + b^2)*sin(d*x + c)^4 +
 8*a*b*sin(d*x + c) - 3*(3*a^2 + b^2)*sin(d*x + c)^2 + 2*a^2)/(sin(d*x + c)
^6 - 2*sin(d*x + c)^4 + sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.37, size = 285, normalized size = 1.54

$$\frac{4(3a^2+b^2)\cos(dx+c)^4-2(3a^2+b^2)\cos(dx+c)^2-2a^2-2b^2+8\left((3a^2+b^2)\cos(dx+c)^6-(3a^2+b^2)\cos(dx+c)^4\right)\log\left(\frac{1}{2}\sin(dx+c)\right)-\left((12a^2-15ab+4b^2)\cos(dx+c)^6-(12a^2-15ab+4b^2)\cos(dx+c)^4\right)\log(\sin(dx+c)+1)-\left((12a^2+15ab+4b^2)\cos(dx+c)^6-(12a^2+15ab+4b^2)\cos(dx+c)^4\right)\log(-\sin(dx+c)+1)+2\left(15ab\cos(dx+c)^4-5ab\cos(dx+c)^2-2ab\right)\sin(dx+c)}{8\left(d\cos(dx+c)^6-d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/8*(4*(3*a^2 + b^2)*cos(d*x + c)^4 - 2*(3*a^2 + b^2)*cos(d*x + c)^2 - 2*a^
2 - 2*b^2 + 8*((3*a^2 + b^2)*cos(d*x + c)^6 - (3*a^2 + b^2)*cos(d*x + c)^4)
*log(1/2*sin(d*x + c)) - ((12*a^2 - 15*a*b + 4*b^2)*cos(d*x + c)^6 - (12*a^
2 - 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((12*a^2 + 15*a
*b + 4*b^2)*cos(d*x + c)^6 - (12*a^2 + 15*a*b + 4*b^2)*cos(d*x + c)^4)*log(
-sin(d*x + c) + 1) + 2*(15*a*b*cos(d*x + c)^4 - 5*a*b*cos(d*x + c)^2 - 2*a*
b)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.56, size = 190, normalized size = 1.03

$$-\frac{(12\,a^2-15\,ab+4\,b^2)\log\left(|\sin\left(dx+c\right)+1|\right)+(12\,a^2+15\,ab+4\,b^2)\log\left(|\sin\left(dx+c\right)-1|\right)-8\,(3\,a^2+b^2)\log\left(|\sin\left(dx+c\right)|\right)+\frac{2\left(15\,ab\sin(dx+c)^5+6\,a^2\sin(dx+c)^4+2\,b^2\sin(dx+c)^4-25\,ab\sin(dx+c)^3-9\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+8\,ab\sin(dx+c)+2\,a^2\right)}{\left(\sin(dx+c)^3-\sin(dx+c)\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*((12*a^2 - 15*a*b + 4*b^2)*log(abs(sin(d*x + c) + 1)) + (12*a^2 + 15*a
*b + 4*b^2)*log(abs(sin(d*x + c) - 1)) - 8*(3*a^2 + b^2)*log(abs(sin(d*x +
c))) + 2*(15*a*b*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^4 + 2*b^2*sin(d*x + c)
^4 - 25*a*b*sin(d*x + c)^3 - 9*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
8*a*b*sin(d*x + c) + 2*a^2)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

Mupad [B]
time = 0.14, size = 194, normalized size = 1.05

$$\frac{\ln\left(\sin\left(c+d\,x\right)\right)\,\left(3\,a^2+b^2\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{3\,a^2}{2}-\frac{15\,a\,b}{8}+\frac{b^2}{2}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{3\,a^2}{2}+\frac{15\,a\,b}{8}+\frac{b^2}{2}\right)}{d}-\frac{\frac{a^2}{2}+{\sin\left(c+d\,x\right)}^4\,\left(\frac{3\,a^2}{2}+\frac{b^2}{2}\right)-{\sin\left(c+d\,x\right)}^2\,\left(\frac{9\,a^2}{4}+\frac{3\,b^2}{4}\right)+2\,a\,b\,\sin\left(c+d\,x\right)-\frac{25\,a\,b\,{\sin\left(c+d\,x\right)}^3}{4}+\frac{15\,a\,b\,{\sin\left(c+d\,x\right)}^5}{4}}{d\,\left({\sin\left(c+d\,x\right)}^6-2\,{\sin\left(c+d\,x\right)}^4+{\sin\left(c+d\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^2/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (log(sin(c + d*x))*(3*a^2 + b^2))/d - (log(sin(c + d*x) + 1)*((3*a^2)/2 - (
15*a*b)/8 + b^2/2))/d - (log(sin(c + d*x) - 1)*((15*a*b)/8 + (3*a^2)/2 + b^
2/2))/d - (a^2/2 + sin(c + d*x)^4*((3*a^2)/2 + b^2/2) - sin(c + d*x)^2*((9*
a^2)/4 + (3*b^2)/4) + 2*a*b*sin(c + d*x) - (25*a*b*sin(c + d*x)^3)/4 + (15*
a*b*sin(c + d*x)^5)/4)/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + sin(c + d*x)
^6))
```

3.1501 $\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx$

Optimal. Leaf size=202

$$-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log(1+\sin(c+dx))}{16d}-\frac{b(24a$$

```
[Out] -1/16*(a+b)*(8*a^2+37*a*b+35*b^2)*ln(1-sin(d*x+c))/d-1/16*(a-b)*(8*a^2-37*a
*b+35*b^2)*ln(1+sin(d*x+c))/d-1/8*b*(24*a^2+35*b^2)*sin(d*x+c)/d-3/2*a*b^2*
sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/d
-1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))^2*(8*a+11*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2800, 1659, 1643, 647, 31}

$$-\frac{b\left(24a^2+35b^2\right)\sin(c+dx)}{8d}-\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d}-\frac{(a-b)\left(8a^2-37ab+35b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}-\frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(8a+11b\sin(c+dx))}{8d}-\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/16*((a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]])/d - ((a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (b*(24*a^2 + 35
*b^2)*Sin[c + d*x])/(8*d) - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d
*x]^3)/(3*d) + (Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x
]^2*(a + b*Sin[c + d*x])^2*(8*a + 11*b*Sin[c + d*x]))/(8*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2(-3b^6-4ab^4x-4b^4x^2-}{(b^2-x^2)^2}\right.}{4b}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{8d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{8d}$$

$$= -\frac{b(24a^2+35b^2)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d}$$

$$= -\frac{b(24a^2+35b^2)\sin(c+dx)}{8d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d}$$

$$= -\frac{(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))}{16d} - \frac{(a-b)\left(8a^2-\right.}{}$$

Mathematica [A]
time = 0.70, size = 199, normalized size = 0.99

$$-\frac{3(a+b)\left(8a^2+37ab+35b^2\right)\log(1-\sin(c+dx))+3(a-b)\left(8a^2-37ab+35b^2\right)\log(1+\sin(c+dx))-\frac{3(a+b)^3}{(-1+\sin(c+dx))^2}-\frac{3(a+b)^2(7a+13b)}{-1+\sin(c+dx)}+144b(a^2+b^2)\sin(c+dx)+72ab^2\sin^2(c+dx)+16b^3\sin^3(c+dx)-\frac{3(a-b)^3}{(1+\sin(c+dx))^2}+\frac{3(7a-13b)(a-b)^2}{1+\sin(c+dx)}}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^5,x]
```

```
[Out] -1/48*(3*(a + b)*(8*a^2 + 37*a*b + 35*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b
)*(8*a^2 - 37*a*b + 35*b^2)*Log[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin
[c + d*x])^2 - (3*(a + b)^2*(7*a + 13*b))/(-1 + Sin[c + d*x]) + 144*b*(a^2
+ b^2)*Sin[c + d*x] + 72*a*b^2*Sin[c + d*x]^2 + 16*b^3*Sin[c + d*x]^3 - (3*
(a - b)^3)/(1 + Sin[c + d*x])^2 + (3*(7*a - 13*b)*(a - b)^2)/(1 + Sin[c + d
*x]))/d
```

Maple [A]
time = 0.36, size = 304, normalized size = 1.50 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+3*a^2*b*(1/4*si
n(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*
sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2*(1/4*s
in(d*x+c)^8/cos(d*x+c)^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4
*sin(d*x+c)^4-3/2*sin(d*x+c)^2-3*ln(cos(d*x+c)))+b^3*(1/4*sin(d*x+c)^9/cos(
d*x+c)^4-5/8*sin(d*x+c)^9/cos(d*x+c)^2-5/8*sin(d*x+c)^7-7/8*sin(d*x+c)^5-35
/24*sin(d*x+c)^3-35/8*sin(d*x+c)+35/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 217, normalized size = 1.07

$$-\frac{16\,b^3\sin\left(dx+c\right)^3+72\,ab^2\sin\left(dx+c\right)^2+3\,\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(8\,a^3+45\,a^2b+72\,ab^2+35\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right)+144\,\left(a^2b+b^3\right)\sin\left(dx+c\right)-\frac{6\,\left(\left(27\,a^2b+13\,b^3\right)\sin(dx+c)^3-6\,a^3-30\,ab^2+4\,\left(2\,a^3+9\,ab^2\right)\sin(dx+c)^2-\left(21\,a^2b+11\,b^3\right)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 3*(8*a^3 - 45*a^2*
b + 72*a*b^2 - 35*b^3)*log(sin(d*x + c) + 1) + 3*(8*a^3 + 45*a^2*b + 72*a*b
^2 + 35*b^3)*log(sin(d*x + c) - 1) + 144*(a^2*b + b^3)*sin(d*x + c) - 6*((2
7*a^2*b + 13*b^3)*sin(d*x + c)^3 - 6*a^3 - 30*a*b^2 + 4*(2*a^3 + 9*a*b^2)*s
in(d*x + c)^2 - (21*a^2*b + 11*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 238, normalized size = 1.18

$$\frac{72\,ab^2\cos\left(dx+c\right)^6-36\,ab^2\cos\left(dx+c\right)^4-3\,\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(8\,a^3+45\,a^2b+72\,ab^2+35\,b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+12\,a^3+36\,ab^2-24\,\left(2\,a^3+9\,ab^2\right)\cos\left(dx+c\right)^2+2\,\left(8\,b^3\cos\left(dx+c\right)^6-8\,\left(9\,a^2b+10\,b^3\right)\cos\left(dx+c\right)^4+18\,a^2b+6\,b^3-3\,\left(27\,a^2b+13\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/48*(72*a*b^2*cos(d*x + c)^6 - 36*a*b^2*cos(d*x + c)^4 - 3*(8*a^3 - 45*a^2
*b + 72*a*b^2 - 35*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*a^3 + 4
5*a^2*b + 72*a*b^2 + 35*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^3
 + 36*a*b^2 - 24*(2*a^3 + 9*a*b^2)*cos(d*x + c)^2 + 2*(8*b^3*cos(d*x + c)^6
 - 8*(9*a^2*b + 10*b^3)*cos(d*x + c)^4 + 18*a^2*b + 6*b^3 - 3*(27*a^2*b + 1
3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.62, size = 251, normalized size = 1.24

$$-\frac{16\,b^3\sin\left(dx+c\right)^3+72\,ab^2\sin\left(dx+c\right)^2+144\,a^2b\sin\left(dx+c\right)+144\,b^3\sin\left(dx+c\right)+3\left(8\,a^3-45\,a^2b+72\,ab^2-35\,b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(8\,a^3+45\,a^2b+72\,ab^2+35\,b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{6\left(6\,a^3\sin(dx+c)^4+54\,ab^2\sin(dx+c)^4+27\,a^2b\sin(dx+c)^3+13\,b^3\sin(dx+c)^3-4\,a^3\sin(dx+c)^2-72\,ab^2\sin(dx+c)^2-21\,a^2b\sin(dx+c)-11\,b^3\sin(dx+c)+24\,ab^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/48*(16*b^3*sin(d*x + c)^3 + 72*a*b^2*sin(d*x + c)^2 + 144*a^2*b*sin(d*x
+ c) + 144*b^3*sin(d*x + c) + 3*(8*a^3 - 45*a^2*b + 72*a*b^2 - 35*b^3)*log(
abs(sin(d*x + c) + 1)) + 3*(8*a^3 + 45*a^2*b + 72*a*b^2 + 35*b^3)*log(abs(s
in(d*x + c) - 1)) - 6*(6*a^3*sin(d*x + c)^4 + 54*a*b^2*sin(d*x + c)^4 + 27*
a^2*b*sin(d*x + c)^3 + 13*b^3*sin(d*x + c)^3 - 4*a^3*sin(d*x + c)^2 - 72*a*
b^2*sin(d*x + c)^2 - 21*a^2*b*sin(d*x + c) - 11*b^3*sin(d*x + c) + 24*a*b^2
)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.12, size = 512, normalized size = 2.53

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(a^3+9\,a\,b^2\right)}{d}-\frac{\left(-\frac{45\,a^2\,b}{4}-\frac{35\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(-2\,a^3-18\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\left(\frac{15\,a^2\,b}{2}+\frac{35\,b^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(2\,a^3+18\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{141\,a^2\,b}{4}+\frac{329\,b^3}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(16\,a^3+48\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(33\,a^2\,b-17\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(16\,a^3+48\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{141\,a^2\,b}{4}+\frac{329\,b^3}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(2\,a^3+18\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{15\,a^2\,b}{2}+\frac{35\,b^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,a^3-18\,a\,b^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\left(-\frac{45\,a^2\,b}{4}-\frac{35\,b^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-b\right)\,\left(8\,a^2-37\,a\,b+35\,b^2\right)}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+b\right)\,\left(8\,a^2+37\,a\,b+35\,b^2\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^5*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(9*a*b^2 + a^3))/d - (tan(c/2 + (d*x)/2)^4*(
18*a*b^2 + 2*a^3) - tan(c/2 + (d*x)/2)^2*(18*a*b^2 + 2*a^3) - tan(c/2 + (d*
x)/2)*((45*a^2*b)/4 + (35*b^3)/4) + tan(c/2 + (d*x)/2)^10*(18*a*b^2 + 2*a^3
) - tan(c/2 + (d*x)/2)^12*(18*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^6*(48*a*b
^2 + 16*a^3) + tan(c/2 + (d*x)/2)^8*(48*a*b^2 + 16*a^3) + tan(c/2 + (d*x)/2
```

```
)^7*(33*a^2*b - 17*b^3) + tan(c/2 + (d*x)/2)^3*((15*a^2*b)/2 + (35*b^3)/6)
+ tan(c/2 + (d*x)/2)^11*((15*a^2*b)/2 + (35*b^3)/6) - tan(c/2 + (d*x)/2)^13
*((45*a^2*b)/4 + (35*b^3)/4) + tan(c/2 + (d*x)/2)^5*((141*a^2*b)/4 + (329*b
^3)/12) + tan(c/2 + (d*x)/2)^9*((141*a^2*b)/4 + (329*b^3)/12))/(d*(tan(c/2
+ (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2
+ (d*x)/2)^8 + 3*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 - tan(c/2 +
(d*x)/2)^14 - 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(8*a^2 - 37*a*b +
35*b^2))/(8*d) - (log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(37*a*b + 8*a^2 + 35*
b^2))/(8*d)
```

3.1502 $\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=177

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(1+\sin(c+dx))}{16d}-\frac{29ab^2\sin(c}{8d}$$

```
[Out] -3/16*(a+b)*(a^2+7*a*b+8*b^2)*ln(1-sin(d*x+c))/d+3/16*(a-b)*(a^2-7*a*b+8*b^
2)*ln(1+sin(d*x+c))/d-29/8*a*b^2*sin(d*x+c)/d-1/2*b^3*sin(d*x+c)^2/d-1/8*se
c(d*x+c)^2*(8*b+5*a*sin(d*x+c))*(a+b*sin(d*x+c))^2/d+1/4*sec(d*x+c)^3*(a+b*
sin(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.24, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2916, 12, 1659, 1643, 647, 31}

$$-\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3(a-b)\left(a^2-7ab+8b^2\right)\log(\sin(c+dx)+1)}{16d}-\frac{29ab^2\sin(c+dx)}{8d}-\frac{\sec^2(c+dx)(5a\sin(c+dx)+8b)(a+b\sin(c+dx))^2}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^3}{4d}-\frac{b^3\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(a - b
)*(a^2 - 7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]])/(16*d) - (29*a*b^2*Sin[c + d
*x])/(8*d) - (b^3*Sin[c + d*x]^2)/(2*d) - (Sec[c + d*x]^2*(8*b + 5*a*Sin[c
+ d*x])*(a + b*Sin[c + d*x])^2)/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c + d*x]
)^3*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^4(a+x)^3}{b^4(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \frac{x^4(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d}$$

$$= -\frac{\sec^2(c+dx)(8b+5a\sin(c+dx))(a+b\sin(c+dx))^2}{8d}$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b}{}$$

$$= -\frac{29ab^2\sin(c+dx)}{8d} - \frac{b^3\sin^2(c+dx)}{2d} - \frac{\sec^2(c+dx)(8b}{}$$

$$= -\frac{3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))}{16d} + \frac{3(a-}{}$$

Mathematica [A]
time = 0.39, size = 174, normalized size = 0.98

$$\frac{-3(a+b)\left(a^2+7ab+8b^2\right)\log(1-\sin(c+dx))+3(a-b)\left(a^2-7ab+8b^2\right)\log(1+\sin(c+dx))+\frac{(a+b)^3}{(-1+\sin(c+dx))^2}+\frac{(a+b)^2(5a+11b)}{-1+\sin(c+dx)}-48ab^2\sin(c+dx)-8b^3\sin^2(c+dx)-\frac{(a-b)^3}{(1+\sin(c+dx))^2}+\frac{(5a-11b)(a-b)^2}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (-3*(a + b)*(a^2 + 7*a*b + 8*b^2)*Log[1 - Sin[c + d*x]] + 3*(a - b)*(a^2 -
7*a*b + 8*b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((
a + b)^2*(5*a + 11*b))/(-1 + Sin[c + d*x]) - 48*a*b^2*Sin[c + d*x] - 8*b^3*
Sin[c + d*x]^2 - (a - b)^3/(1 + Sin[c + d*x])^2 + ((5*a - 11*b)*(a - b)^2)/
(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]
time = 0.33, size = 284, normalized size = 1.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*s
in(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b*(1/4*tan(
```

```
d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c)))+3*a*b^2*(1/4*sin(d*x+c)^7/cos(d*x
+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*sin(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*
sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/4*sin(d*x+c)^8/cos(d*x+c)
^4-1/2*sin(d*x+c)^8/cos(d*x+c)^2-1/2*sin(d*x+c)^6-3/4*sin(d*x+c)^4-3/2*sin(
d*x+c)^2-3*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.34, size = 190, normalized size = 1.07

$$-\frac{8\,b^3\sin\left(dx+c\right)^2+48\,ab^2\sin\left(dx+c\right)-3\,\left(a^3-8\,a^2b+15\,ab^2-8\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\,\left(a^3+8\,a^2b+15\,ab^2+8\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\,\left(\left(5\,a^3+27\,ab^2\right)\sin(dx+c)^3-18\,a^2b-10\,b^3+12\,\left(2\,a^2b+b^3\right)\sin(dx+c)^2-3\,\left(a^3+7\,ab^2\right)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 8*a^2*b + 15*a*b^2 + 8*b^3
)*log(sin(d*x + c) - 1) - 2*((5*a^3 + 27*a*b^2)*sin(d*x + c)^3 - 18*a^2*b -
 10*b^3 + 12*(2*a^2*b + b^3)*sin(d*x + c)^2 - 3*(a^3 + 7*a*b^2)*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.40, size = 208, normalized size = 1.18

$$\frac{8\,b^3\cos\left(dx+c\right)^6-4\,b^3\cos\left(dx+c\right)^4+3\,\left(a^3-8\,a^2b+15\,ab^2-8\,b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(a^3+8\,a^2b+15\,ab^2+8\,b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+12\,a^2b+4\,b^3-24\,\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2-2\,\left(24\,ab^2\cos\left(dx+c\right)^4-2\,a^3-6\,ab^2+\left(5\,a^3+27\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(8*b^3*cos(d*x + c)^6 - 4*b^3*cos(d*x + c)^4 + 3*(a^3 - 8*a^2*b + 15*a
*b^2 - 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^3 + 8*a^2*b + 15*
a*b^2 + 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 12*a^2*b + 4*b^3 - 2
4*(2*a^2*b + b^3)*cos(d*x + c)^2 - 2*(24*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a
*b^2 + (5*a^3 + 27*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.56, size = 221, normalized size = 1.25

$$-\frac{8\,b^3\sin\left(dx+c\right)^2+48\,ab^2\sin\left(dx+c\right)-3\left(a^3-8\,a^2b+15\,ab^2-8\,b^3\right)\log\left(|\sin\left(dx+c\right)+1|\right)+3\left(a^3+8\,a^2b+15\,ab^2+8\,b^3\right)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(18\,a^2b\sin(dx+c)^4+18\,b^3\sin(dx+c)^4+5\,a^3\sin(dx+c)^3+27\,ab^2\sin(dx+c)^3-12\,a^2b\sin(dx+c)^2-24\,b^3\sin(dx+c)^2-3\,a^3\sin(dx+c)-21\,ab^2\sin(dx+c)+8\,b^3\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(8*b^3*sin(d*x + c)^2 + 48*a*b^2*sin(d*x + c) - 3*(a^3 - 8*a^2*b + 15
*a*b^2 - 8*b^3)*log(abs(sin(d*x + c) + 1)) + 3*(a^3 + 8*a^2*b + 15*a*b^2 +
8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(18*a^2*b*sin(d*x + c)^4 + 18*b^3*sin
(d*x + c)^4 + 5*a^3*sin(d*x + c)^3 + 27*a*b^2*sin(d*x + c)^3 - 12*a^2*b*sin
(d*x + c)^2 - 24*b^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - 21*a*b^2*sin(d*x
 + c) + 8*b^3)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.11, size = 449, normalized size = 2.54

$$\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+1\right)\left(3\,a^2b+3\,b^3\right)}{d}-\frac{\left(-\frac{3\,a^3}{4}-\frac{45\,ab^2}{4}\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^{11}+\left(-6\,a^2b-6\,b^3\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^{10}+\left(\frac{5\,a^3}{4}+\frac{75\,ab^2}{4}\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^9+\left(12\,a^2b+12\,b^3\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^8+\left(\frac{15\,a^3}{2}+\frac{33\,ab^2}{2}\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^7+\left(36\,a^2b+4\,b^3\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^6+\left(\frac{15\,a^3}{2}+\frac{33\,ab^2}{2}\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^5+\left(12\,a^2b+12\,b^3\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4+\left(\frac{5\,a^3}{4}+\frac{75\,ab^2}{4}\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^3+\left(-6\,a^2b-6\,b^3\right){\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+\left(-\frac{3\,a^3}{4}-\frac{45\,ab^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^{12}+2\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^{10}+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^6+{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^4+2\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2-1\right)}-\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\left(a+b\right)\left(a^2+7\,ab+8\,b^2\right)}{8\,d}+\frac{3\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\left(a-b\right)\left(a^2-7\,ab+8\,b^2\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^4*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a^2*b + 3*b^3))/d - (tan(c/2 + (d*x)/2)^5
*((33*a*b^2)/2 + (15*a^3)/2) - tan(c/2 + (d*x)/2)*((45*a*b^2)/4 + (3*a^3)/4
) + tan(c/2 + (d*x)/2)^7*((33*a*b^2)/2 + (15*a^3)/2) - tan(c/2 + (d*x)/2)^1
1*((45*a*b^2)/4 + (3*a^3)/4) + tan(c/2 + (d*x)/2)^3*((75*a*b^2)/4 + (5*a^3)
/4) + tan(c/2 + (d*x)/2)^9*((75*a*b^2)/4 + (5*a^3)/4) - tan(c/2 + (d*x)/2)^
2*(6*a^2*b + 6*b^3) - tan(c/2 + (d*x)/2)^10*(6*a^2*b + 6*b^3) + tan(c/2 + (
d*x)/2)^4*(12*a^2*b + 12*b^3) + tan(c/2 + (d*x)/2)^8*(12*a^2*b + 12*b^3) +
tan(c/2 + (d*x)/2)^6*(36*a^2*b + 4*b^3))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c
/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 2*tan(c/2
 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^12 - 1)) - (3*log(tan(c/2 + (d*x)/2) -
1)*(a + b)*(7*a*b + a^2 + 8*b^2))/(8*d) + (3*log(tan(c/2 + (d*x)/2) + 1)*(a
 - b)*(a^2 - 7*a*b + 8*b^2))/(8*d)
```

3.1503 $\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=142

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3(3a-5b)(a-b)b\log(1+\sin(c+dx))}{16d}-\frac{15b^3\sin(c+dx)}{8d}+\frac{\sec^4}{}$$

```
[Out] -3/16*b*(a+b)*(3*a+5*b)*ln(1-sin(d*x+c))/d+3/16*(3*a-5*b)*(a-b)*b*ln(1+sin(
d*x+c))/d-15/8*b^3*sin(d*x+c)/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/d-1/8*s
ec(d*x+c)^2*(a+b*sin(d*x+c))^2*(4*a+7*b*sin(d*x+c))/d
```

Rubi [A]

time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2916, 12, 1659, 788, 647, 31}

$$-\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d}+\frac{3b(3a-5b)(a-b)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}-\frac{\sec^2(c+dx)(a+b\sin(c+dx))^2(4a+7b\sin(c+dx))}{8d}-\frac{15b^3\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]])/(16*d) + (3*(3*a - 5*b)*(a
 - b)*b*Log[1 + Sin[c + d*x]])/(16*d) - (15*b^3*Sin[c + d*x])/(8*d) + (Sec[
c + d*x]^4*(a + b*Sin[c + d*x])^3)/(4*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d
*x])^2*(4*a + 7*b*Sin[c + d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^3(a+x)^3}{b^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\text{Subst}\left(\int \frac{x^3(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2(-3b^4}{(b^2}\right.}{}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{\sec^2(c+dx)(a+b\,s}{}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

$$= -\frac{15b^3\sin(c+dx)}{8d} + \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

$$= -\frac{3b(a+b)(3a+5b)\log(1-\sin(c+dx))}{16d} + \frac{3(3a-5b)}{}$$

Mathematica [**A**]
time = 0.32, size = 147, normalized size = 1.04

$$\frac{-3b(a+b)(3a+5b)\log(1-\sin(c+dx))+3(3a-5b)(a-b)b\log(1+\sin(c+dx))+\frac{(a+b)^3}{(-1+\sin(c+dx))^2}+\frac{3(a+b)^2(a+3b)}{-1+\sin(c+dx)}-16b^3\sin(c+dx)+\frac{(a-b)^3}{(1+\sin(c+dx))^2}-\frac{3(a-3b)(a-b)^2}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (-3*b*(a + b)*(3*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(3*a - 5*b)*(a - b)*b*L
og[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + (3*(a + b)^2*(a +
3*b))/(-1 + Sin[c + d*x]) - 16*b^3*Sin[c + d*x] + (a - b)^3/(1 + Sin[c + d*
x])^2 - (3*(a - 3*b)*(a - b)^2)/(1 + Sin[c + d*x]))/(16*d)
```

Maple [**A**]
time = 0.34, size = 225, normalized size = 1.58 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3*sin(d*x+c)^4/cos(d*x+c)^4+3*a^2*b*(1/4*sin(d*x+c)^5/cos(d*x+c)
^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec
(d*x+c)+tan(d*x+c)))+3*a*b^2*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+
c)))+b^3*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*s
in(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))
))
```

Maxima [**A**]
time = 0.29, size = 173, normalized size = 1.22

$$-\frac{16\,b^3\sin\left(dx+c\right)-3\left(3\,a^2b-8\,ab^2+5\,b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(3\,a^2b+8\,ab^2+5\,b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\left(5\,a^2b+3\,b^3\right)\sin(dx+c)^3-2\,a^3-18\,ab^2+4\left(a^3+6\,ab^2\right)\sin(dx+c)^2-\left(9\,a^2b+7\,b^3\right)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(sin(d*x + c)
 + 1) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(sin(d*x + c) - 1) - 2*(3*(5*a^2*b
 + 3*b^3)*sin(d*x + c)^3 - 2*a^3 - 18*a*b^2 + 4*(a^3 + 6*a*b^2)*sin(d*x + c
)^2 - (9*a^2*b + 7*b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1))/d
```

Fricas [**A**]
time = 0.37, size = 176, normalized size = 1.24

$$\frac{3\left(3\,a^2b-8\,ab^2+5\,b^3\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(3\,a^2b+8\,ab^2+5\,b^3\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,a^3+12\,ab^2-8\left(a^3+6\,ab^2\right)\cos\left(dx+c\right)^2-2\left(8\,b^3\cos\left(dx+c\right)^4-6\,a^2b-2\,b^3+3\left(5\,a^2b+3\,b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(3*(3*a^2*b - 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
3*(3*a^2*b + 8*a*b^2 + 5*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^3
 + 12*a*b^2 - 8*(a^3 + 6*a*b^2)*cos(d*x + c)^2 - 2*(8*b^3*cos(d*x + c)^4 -
6*a^2*b - 2*b^3 + 3*(5*a^2*b + 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.52, size = 188, normalized size = 1.32

$$-\frac{16\,b^{3}\sin\left(dx+c\right)-3\left(3\,a^{2}b-8\,ab^{2}+5\,b^{3}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(3\,a^{2}b+8\,ab^{2}+5\,b^{3}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(18\,ab^{2}\sin(dx+c)^{4}+15\,a^{2}b\sin(dx+c)^{3}+9\,b^{3}\sin(dx+c)^{3}+4\,a^{3}\sin(dx+c)^{2}-12\,ab^{2}\sin(dx+c)^{2}-9\,a^{2}b\sin(dx+c)-7\,b^{3}\sin(dx+c)-2\,a^{3}\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(16*b^3*sin(d*x + c) - 3*(3*a^2*b - 8*a*b^2 + 5*b^3)*log(abs(sin(d*x
+ c) + 1)) + 3*(3*a^2*b + 8*a*b^2 + 5*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(
18*a*b^2*sin(d*x + c)^4 + 15*a^2*b*sin(d*x + c)^3 + 9*b^3*sin(d*x + c)^3 +
4*a^3*sin(d*x + c)^2 - 12*a*b^2*sin(d*x + c)^2 - 9*a^2*b*sin(d*x + c) - 7*b
^3*sin(d*x + c) - 2*a^3)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.09, size = 356, normalized size = 2.51

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(4\,a^{3}+18\,a\,b^{2}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{9\,a^{2}\,b}{4}+\frac{15\,b^{3}}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(4\,a^{3}+18\,a\,b^{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(6\,a^{2}\,b+10\,b^{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(6\,a^{2}\,b+10\,b^{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{9\,a^{2}\,b}{4}+\frac{15\,b^{3}}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{33\,a^{2}\,b}{2}-\frac{9\,b^{3}}{2}\right)-6\,a\,b^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-6\,a\,b^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{3\,a\,b^{2}\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}{d}-\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(a+b\right)\,\left(3\,a+5\,b\right)}{8\,d}+\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(a-b\right)\,\left(3\,a-5\,b\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^3*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^4*(18*a*b^2 + 4*a^3) - tan(c/2 + (d*x)/2)*((9*a^2*b)/4
+ (15*b^3)/4) + tan(c/2 + (d*x)/2)^6*(18*a*b^2 + 4*a^3) + tan(c/2 + (d*x)/2
)^3*(6*a^2*b + 10*b^3) + tan(c/2 + (d*x)/2)^7*(6*a^2*b + 10*b^3) - tan(c/2
+ (d*x)/2)^9*((9*a^2*b)/4 + (15*b^3)/4) + tan(c/2 + (d*x)/2)^5*((33*a^2*b)/
2 - (9*b^3)/2) - 6*a*b^2*tan(c/2 + (d*x)/2)^2 - 6*a*b^2*tan(c/2 + (d*x)/2)^
```

```
8)/(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/
2)^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) + (3*a*b^2*log(
tan(c/2 + (d*x)/2)^2 + 1))/d - (3*b*log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(3*
a + 5*b))/(8*d) + (3*b*log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(3*a - 5*b))/(8*
d)
```

3.1504 $\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(1+\sin(c+dx))}{16d}-\frac{\sec^2(c+dx)(a+b\sin(c+d$$

```
[Out] 1/16*(a^3-9*a*b^2-8*b^3)*ln(1-sin(d*x+c))/d-1/16*(a^3-9*a*b^2+8*b^3)*ln(1+s
in(d*x+c))/d-1/8*sec(d*x+c)^2*(a+b*sin(d*x+c))*(5*a*b+(a^2+4*b^2)*sin(d*x+c
))/d+1/4*sec(d*x+c)^3*(a+b*sin(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2916, 12, 1659, 833, 647, 31}

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d}-\frac{(a^3-9ab^2+8b^3)\log(\sin(c+dx)+1)}{16d}-\frac{\sec^2(c+dx)\left((a^2+4b^2)\sin(c+dx)+5ab\right)(a+b\sin(c+dx))}{8d}+\frac{\tan(c+dx)\sec^3(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]])/(16*d) - ((a^3 - 9*a*b^2 +
8*b^3)*Log[1 + Sin[c + d*x]])/(16*d) - (Sec[c + d*x]^2*(a + b*Sin[c + d*x])
*(5*a*b + (a^2 + 4*b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^3*(a + b*Sin[c
 + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1659

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c*(p
+ 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x^2(a+x)^3}{b^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^3\text{Subst}\left(\int \frac{x^2(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^3(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)}{4d} + \frac{b\text{Subst}(}{}$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\,(5ab+(a^2+4b^2)\sin(}{8d}$$

$$= -\frac{\sec^2(c+dx)(a+b\sin(c+dx))\,(5ab+(a^2+4b^2)\sin(}{8d}$$

$$= \frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))}{16d} - \frac{(a^3-9ab^2+}{}$$

Mathematica [A]

time = 0.27, size = 140, normalized size = 0.97

$$\frac{(a^3-9ab^2-8b^3)\log(1-\sin(c+dx))-(a^3-9ab^2+8b^3)\log(1+\sin(c+dx))+\frac{(a+b)^3}{(-1+\sin(c+dx))^2}+\frac{(a+b)^2(a+7b)}{-1+\sin(c+dx)}-\frac{(a-b)^3}{(1+\sin(c+dx))^2}+\frac{(a-7b)(a-b)^2}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a^3 - 9*a*b^2 - 8*b^3)*Log[1 - Sin[c + d*x]] - (a^3 - 9*a*b^2 + 8*b^3)*Lo
g[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(a + 7*b
))/(-1 + Sin[c + d*x]) - (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - 7*b)*(a - b
)^2)/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]

time = 0.31, size = 204, normalized size = 1.42

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3ab^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c}{8\cos(dx+c}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+3ab^2\left(\frac{\sin^5(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^5(dx+c}{8\cos(dx+c}\right.}{d}$
risch	$ib^3x+\frac{2ib^3c}{d}-\frac{-ia^3e^{7i(dx+c)}-15iab^2e^{7i(dx+c)}+7ia^3e^{5i(dx+c)}+9iab^2e^{5i(dx+c)}+24be^{6i(dx+c)}a^2+16b^3e^{6i(dx+c)}-7i}{4d(e^{2i}}$

norman	$\frac{\left(12a^2b+2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(12a^2b+2b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(36a^2b+16b^3\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(36a^2b+16b^3\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*s
in(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*a^2*b*sin(d*x+c)^4/cos(d*x+c)^
4+3*a*b^2*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x+c)^2-1/8*
sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/4*tan(d*x
+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.34, size = 151, normalized size = 1.05

$$-\frac{(a^3-9\,ab^2+8\,b^3)\log\left(\sin\left(dx+c\right)+1\right)-(a^3-9\,ab^2-8\,b^3)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((a^3+15\,ab^2)\sin(dx+c)^3-6\,a^2b-6\,b^3+4\,(3\,a^2b+2\,b^3)\sin(dx+c)^2+(a^3-9\,ab^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(sin(d*x + c) + 1) - (a^3 - 9*a*b^2 - 8*b
^3)*log(sin(d*x + c) - 1) - 2*((a^3 + 15*a*b^2)*sin(d*x + c)^3 - 6*a^2*b -
6*b^3 + 4*(3*a^2*b + 2*b^3)*sin(d*x + c)^2 + (a^3 - 9*a*b^2)*sin(d*x + c))/
(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 156, normalized size = 1.08

$$-\frac{(a^3-9\,ab^2+8\,b^3)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(a^3-9\,ab^2-8\,b^3)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-12\,a^2b-4\,b^3+8\,(3\,a^2b+2\,b^3)\cos\left(dx+c\right)^2-2\,\left(2\,a^3+6\,ab^2-(a^3+15\,ab^2)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (a^3
- 9*a*b^2 - 8*b^3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 12*a^2*b - 4*b^3
 + 8*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 2*(2*a^3 + 6*a*b^2 - (a^3 + 15*a*b^
2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.52, size = 168, normalized size = 1.17

$$-\frac{\left(a^{3}-9\,ab^{2}+8\,b^{3}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-\left(a^{3}-9\,ab^{2}-8\,b^{3}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(6\,b^{3}\sin(dx+c)^{4}+a^{3}\sin(dx+c)^{3}+15\,ab^{2}\sin(dx+c)^{3}+12\,a^{2}b\sin(dx+c)^{2}-4\,b^{3}\sin(dx+c)^{2}+a^{3}\sin(dx+c)-9\,ab^{2}\sin(dx+c)-6\,a^{2}b\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*((a^3 - 9*a*b^2 + 8*b^3)*log(abs(sin(d*x + c) + 1)) - (a^3 - 9*a*b^2
- 8*b^3)*log(abs(sin(d*x + c) - 1)) - 2*(6*b^3*sin(d*x + c)^4 + a^3*sin(d*x
 + c)^3 + 15*a*b^2*sin(d*x + c)^3 + 12*a^2*b*sin(d*x + c)^2 - 4*b^3*sin(d*x
 + c)^2 + a^3*sin(d*x + c) - 9*a*b^2*sin(d*x + c) - 6*a^2*b)/(sin(d*x + c)^
2 - 1)^2)/d
```

Mupad [B]
time = 12.14, size = 299, normalized size = 2.08

$$\frac{b^{3}\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(-\frac{a^{3}}{8}+\frac{9\,a\,b^{2}}{8}+b^{3}\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(\frac{a^{3}}{8}-\frac{9\,a\,b^{2}}{8}+b^{3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{9\,a\,b^{2}}{4}-\frac{a^{3}}{4}\right)+2\,b^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+2\,b^{3}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\,\left(\frac{9\,a\,b^{2}}{4}-\frac{a^{3}}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(\frac{7\,a^{3}}{4}+\frac{33\,a\,b^{2}}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\,\left(\frac{7\,a^{3}}{4}+\frac{33\,a\,b^{2}}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\left(12\,a^{2}\,b+8\,b^{3}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^2*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (b^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) - 1)*((9*a*
b^2)/8 - a^3/8 + b^3))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a^3/8 - (9*a*b^2)/
8 + b^3))/d - (tan(c/2 + (d*x)/2)*((9*a*b^2)/4 - a^3/4) + 2*b^3*tan(c/2 + (
d*x)/2)^2 + 2*b^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/4
- a^3/4) - tan(c/2 + (d*x)/2)^3*((33*a*b^2)/4 + (7*a^3)/4) - tan(c/2 + (d*x
)/2)^5*((33*a*b^2)/4 + (7*a^3)/4) - tan(c/2 + (d*x)/2)^4*(12*a^2*b + 8*b^3)
)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2
)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.1505 $\int \sec^4(c+dx)(a+b\sin(c+dx))^3 \tan(c+dx)\,dx$

Optimal. Leaf size=90

$$-\frac{3b(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\,(b^2 -}{8d}$$

```
[Out] -3/8*b*(a^2-b^2)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(a+b*sin(d*x+c))^3/
d-3/8*sec(d*x+c)^2*(a+b*sin(d*x+c))*(b^2+a*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2916, 12, 819, 737, 212}

$$-\frac{3b(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3\sec^2(c+dx)(a+b\sin(c+dx))\,(ab\sin(c+dx)+b^2)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-3*b*(a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(a + b*Sin
[c + d*x])^3)/(4*d) - (3*Sec[c + d*x]^2*(a + b*Sin[c + d*x])*(b^2 + a*b*Sin
[c + d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{b^5\text{Subst}\left(\int \frac{x(a+x)^3}{b(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^4\text{Subst}\left(\int \frac{x(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{(3b^4)\,\text{Subst}\left(\int \frac{(a+x)^{2}}{(b^2-x^2}\right.}{4}$$

$$= \frac{\sec^4(c+dx)(a+b\sin(c+dx))^3}{4d} - \frac{3\sec^2(c+dx)(a+b}{}$$

$$= -\frac{3b(a^2-b^2)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(a+b}{4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 370 vs. 2(90) = 180.
time = 0.99, size = 370, normalized size = 4.11

$$\frac{2a(a^2-b^2)^2\sec^4(c+dx)(a+b\sin(c+dx))^4+2b(a^2-b^2)\sec^4(c+dx)(b-a\sin(c+dx))(a+b\sin(c+dx))^5+b\sec^2(c+dx)(a+b\sin(c+dx))^5\left(5a^2b+b^3-3a(a^2+b^2)\sin(c+dx)\right)+\frac{1}{2}(5a^4b+10a^2b^3+b^5)\left(3((a+b)^4\log(1-\sin(c+dx))-(a-b)^4\log(1+\sin(c+dx)))+6b^2(6a^2+b^2)\sin(c+dx)+12ab^3\sin^2(c+dx)+2b^4\sin^3(c+dx)\right)-ab(a^2+b^2)\left(6(a+b)^5\log(1-\sin(c+dx))-6(a-b)^5\log(1+\sin(c+dx))+60ab^2(2a^2+b^2)\sin(c+dx)+6b^3(10a^2+b^2)\sin^2(c+dx)+20ab^4\sin^3(c+dx)+3b^5\sin^4(c+dx)\right)}{8(a^2-b^2)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (2*a*(a^2 - b^2)^2*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^4 + 2*b*(a^2 - b^2)*
Sec[c + d*x]^4*(b - a*Sin[c + d*x])*(a + b*Sin[c + d*x])^5 + b*Sec[c + d*x]
^2*(a + b*Sin[c + d*x])^5*(5*a^2*b + b^3 - 3*a*(a^2 + b^2)*Sin[c + d*x]) +
((5*a^4*b + 10*a^2*b^3 + b^5)*(3*((a + b)^4*Log[1 - Sin[c + d*x]] - (a - b)
```

```
^4*Log[1 + Sin[c + d*x]]) + 6*b^2*(6*a^2 + b^2)*Sin[c + d*x] + 12*a*b^3*Sin
[c + d*x]^2 + 2*b^4*Sin[c + d*x]^3))/2 - a*b*(a^2 + b^2)*(6*(a + b)^5*Log[1
 - Sin[c + d*x]] - 6*(a - b)^5*Log[1 + Sin[c + d*x]] + 60*a*b^2*(2*a^2 + b^
2)*Sin[c + d*x] + 6*b^3*(10*a^2 + b^2)*Sin[c + d*x]^2 + 20*a*b^4*Sin[c + d*
x]^3 + 3*b^5*Sin[c + d*x]^4))/(8*(a^2 - b^2)^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(84) = 168$.
time = 0.31, size = 183, normalized size = 2.03

method	result
derivativedivides	$\frac{\frac{a^3}{4\cos(dx+c)^4}+3a^2b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a\,b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+b^3\left(\frac{\sin^5(dx-}{4\cos(dx+}\right.}{d}$
default	$\frac{\frac{a^3}{4\cos(dx+c)^4}+3a^2b\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a\,b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+b^3\left(\frac{\sin^5(dx-}{4\cos(dx+}\right.}{d}$
risch	$\frac{-24a\,b^2e^{6i(dx+c)}+3ia^2b\,e^{7i(dx+c)}+5ib^3e^{7i(dx+c)}+16a^3e^{4i(dx+c)}-21ia^2b\,e^{5i(dx+c)}-3ib^3e^{5i(dx+c)}-24a\,b^2e^{2i(dx+c)}+2}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{2a^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(8a^3+36a\,b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(8a^3+36a\,b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3\left(2a^3+4a\,b^2\right)\left(\tan\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3/cos(d*x+c)^4+3*a^2*b*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*
x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4*a*b^2
*sin(d*x+c)^4/cos(d*x+c)^4+b^3*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c
)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+
c))))
```

Maxima [A]
time = 0.29, size = 140, normalized size = 1.56

$$-\frac{3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^2b-b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(12\,ab^2\sin(dx+c)^2+\left(3\,a^2b+5\,b^3\right)\sin(dx+c)^3+2\,a^3-6\,ab^2+3\left(a^2b-b^3\right)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*log(sin(d*x + c) + 1) - 3*(a^2*b - b^3)*log(sin(d*x
+ c) - 1) - 2*(12*a*b^2*sin(d*x + c)^2 + (3*a^2*b + 5*b^3)*sin(d*x + c)^3 +
 2*a^3 - 6*a*b^2 + 3*(a^2*b - b^3)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*
x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 143, normalized size = 1.59

$$-\frac{3\left(a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+24\,ab^{2}\cos\left(dx+c\right)^{2}-4\,a^{3}-12\,ab^{2}-2\left(6\,a^{2}b+2\,b^{3}-\left(3\,a^{2}b+5\,b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(a^2*b - b^
3)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 24*a*b^2*cos(d*x + c)^2 - 4*a^3
- 12*a*b^2 - 2*(6*a^2*b + 2*b^3 - (3*a^2*b + 5*b^3)*cos(d*x + c)^2)*sin(d*x
 + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.53, size = 142, normalized size = 1.58

$$-\frac{3\left(a^{2}b-b^{3}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(a^{2}b-b^{3}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(3\,a^{2}b\sin\left(dx+c\right)^{3}+5\,b^{3}\sin\left(dx+c\right)^{3}+12\,ab^{2}\sin\left(dx+c\right)^{2}+3\,a^{2}b\sin\left(dx+c\right)-3\,b^{3}\sin\left(dx+c\right)+2\,a^{3}-6\,ab^{2}\right)}{\left(\sin\left(dx+c\right)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(a^2*b - b^3)*log(abs(sin(d*x + c) + 1)) - 3*(a^2*b - b^3)*log(abs
(sin(d*x + c) - 1)) - 2*(3*a^2*b*sin(d*x + c)^3 + 5*b^3*sin(d*x + c)^3 + 12
*a*b^2*sin(d*x + c)^2 + 3*a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c) + 2*a^3 -
 6*a*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 16.99, size = 228, normalized size = 2.53

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3a^2b}{4}-\frac{3b^3}{4}\right)+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{3a^2b}{4}-\frac{3b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{21a^2b}{4}+\frac{11b^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{21a^2b}{4}+\frac{11b^3}{4}\right)+12\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{3\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2-b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*a^2*b)/4 - (3*b^3)/4) + 2*a^3*tan(c/2 + (d*x)/2)^2
+ 2*a^3*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^7*((3*a^2*b)/4 - (3*b^3)/
4) + tan(c/2 + (d*x)/2)^3*((21*a^2*b)/4 + (11*b^3)/4) + tan(c/2 + (d*x)/2)^
5*((21*a^2*b)/4 + (11*b^3)/4) + 12*a*b^2*tan(c/2 + (d*x)/2)^4)/(d*(6*tan(c/
2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2
+ (d*x)/2)^8 + 1)) - (3*b*atanh(tan(c/2 + (d*x)/2))*(a^2 - b^2))/(4*d)
```

3.1506 $\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$-\frac{(8a^3+9a^2b-b^3)\log(1-\sin(c+dx))}{16d}+\frac{a^3\log(\sin(c+dx))}{d}-\frac{(8a^3-9a^2b+b^3)\log(1+\sin(c+dx))}{16d}+\frac{\sec^2}{}$$

```
[Out] -1/16*(8*a^3+9*a^2*b-b^3)*ln(1-sin(d*x+c))/d+a^3*ln(sin(d*x+c))/d-1/16*(8*a
^3-9*a^2*b+b^3)*ln(1+sin(d*x+c))/d+1/8*sec(d*x+c)^2*(4*a^3+b*(9*a^2-b^2)*si
n(d*x+c))/d+1/4*sec(d*x+c)^4*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A]
time = 0.17, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2916, 12, 1819, 837, 815}

$$\frac{a^3\log(\sin(c+dx))}{d}+\frac{\sec^4(c+dx)\left(b(3a^2+b^2)\sin(c+dx)+a(a^2+3b^2)\right)}{4d}-\frac{(8a^3+9a^2b-b^3)\log(1-\sin(c+dx))}{16d}-\frac{(8a^3-9a^2b+b^3)\log(\sin(c+dx)+1)}{16d}+\frac{\sec^2(c+dx)\left(4a^3+b(9a^2-b^2)\sin(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]])/d + (a^3*Log[Sin[c +
d*x]])/d - ((8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]])/(16*d) + (Sec[c
+ d*x]^2*(4*a^3 + b*(9*a^2 - b^2)*Sin[c + d*x]))/(8*d) + (Sec[c + d*x]^4*(a
*(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\text{Subst}\left(\int \frac{b(a+x)^3}{x(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^6\text{Subst}\left(\int \frac{(a+x)^3}{x(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\sin(c+dx)\right)}{4d} - \frac{b^{}}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b(9a^2-b^2)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+}{}$$

$$= \frac{\sec^2(c+dx)\left(4a^3+b(9a^2-b^2)\sin(c+dx)\right)}{8d} + \frac{\sec^4(c+}{}$$

$$= -\frac{(8a^3+9a^2b-b^3)\log(1-\sin(c+dx))}{16d} + \frac{a^3\log(\sin(c-}{d}$$

Mathematica [A]

time = 0.39, size = 157, normalized size = 0.95

$$\frac{-((8a^3+9a^2b-b^3)\log(1-\sin(c+dx)))+16a^3\log(\sin(c+dx))-(8a^3-9a^2b+b^3)\log(1+\sin(c+dx))+\frac{(a+b)^3}{(-1+\sin(c+dx))^2}-\frac{(5a-b)(a+b)^2}{-1+\sin(c+dx)}+\frac{(a-b)^3}{(1+\sin(c+dx))^2}+\frac{(a-b)^2(5a+b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-((8*a^3 + 9*a^2*b - b^3)*Log[1 - Sin[c + d*x]]) + 16*a^3*Log[Sin[c + d*x]
] - (8*a^3 - 9*a^2*b + b^3)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c +
 d*x])^2 - ((5*a - b)*(a + b)^2)/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c
 + d*x])^2 + ((a - b)^2*(5*a + b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]
time = 0.42, size = 167, normalized size = 1.01

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^2b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\mathrm{ta}}{8}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+c)^2}+\ln(\tan(dx+c))\right)+3a^2b\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\mathrm{ta}}{8}\right.}{d}$
risch	$\frac{i\left(-8ia^3 e^{6i(dx+c)}-9a^2b\, e^{7i(dx+c)}+b^3 e^{7i(dx+c)}-32ia^3 e^{4i(dx+c)}-48ib^2a\, e^{4i(dx+c)}-33a^2b\, e^{5i(dx+c)}-7b^3 e^{5i(dx+c)}-8ia^3\right.}{4d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{\left(4a^3+6a\,b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^3+6a\,b^2\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^3+24a\,b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(4a^3+24a\,b^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+3*a^2*b*(-(-1/4
*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3/4
*a*b^2/cos(d*x+c)^4+b^3*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos
(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 160, normalized size = 0.97

$$\frac{16\,a^3\log\left(\sin\left(dx+c\right)\right)-\left(8\,a^3-9\,a^2b+b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^3+9\,a^2b-b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(4\,a^3\sin(dx+c)^2+(9\,a^2b-b^3)\sin(dx+c)^3-6\,a^3-6\,ab^2-(15\,a^2b+b^3)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^3*log(sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)*log(sin(d*x + c) +
 1) - (8*a^3 + 9*a^2*b - b^3)*log(sin(d*x + c) - 1) - 2*(4*a^3*sin(d*x + c)
^2 + (9*a^2*b - b^3)*sin(d*x + c)^3 - 6*a^3 - 6*a*b^2 - (15*a^2*b + b^3)*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 173, normalized size = 1.05

$$\frac{16\,a^{3}\cos\left(dx+c\right)^{4}\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-\left(8\,a^{3}-9\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^{3}+9\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+8\,a^{3}\cos\left(dx+c\right)^{2}+4\,a^{3}+12\,ab^{2}+2\,\left(6\,a^{2}b+2\,b^{3}+\left(9\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3*cos(d*x + c)^4*log(1/2*sin(d*x + c)) - (8*a^3 - 9*a^2*b + b^3)
*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^3 + 9*a^2*b - b^3)*cos(d*x + c
)^4*log(-sin(d*x + c) + 1) + 8*a^3*cos(d*x + c)^2 + 4*a^3 + 12*a*b^2 + 2*(6
*a^2*b + 2*b^3 + (9*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 175, normalized size = 1.06

$$\frac{16\,a^{3}\log\left(|\sin\left(dx+c\right)|\right)-\left(8\,a^{3}-9\,a^{2}b+b^{3}\right)\log\left(|\sin\left(dx+c\right)+1|\right)-\left(8\,a^{3}+9\,a^{2}b-b^{3}\right)\log\left(|\sin\left(dx+c\right)-1|\right)+\frac{2\,\left(6\,a^{3}\sin(dx+c)^{4}-9\,a^{2}b\sin(dx+c)^{3}+b^{3}\sin(dx+c)^{3}-16\,a^{3}\sin(dx+c)^{2}+15\,a^{2}b\sin(dx+c)+b^{3}\sin(dx+c)+12\,a^{3}+6\,ab^{2}\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*a^3*log(abs(sin(d*x + c))) - (8*a^3 - 9*a^2*b + b^3)*log(abs(sin(d
*x + c) + 1)) - (8*a^3 + 9*a^2*b - b^3)*log(abs(sin(d*x + c) - 1)) + 2*(6*a
^3*sin(d*x + c)^4 - 9*a^2*b*sin(d*x + c)^3 + b^3*sin(d*x + c)^3 - 16*a^3*si
n(d*x + c)^2 + 15*a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 12*a^3 + 6*a*b^2)
/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 11.91, size = 169, normalized size = 1.02

$$\frac{a^{3}\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{a^{3}}{2}-\frac{9\,a^{2}\,b}{16}+\frac{b^{3}}{16}\right)}{d}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{a^{3}}{2}+\frac{9\,a^{2}\,b}{16}-\frac{b^{3}}{16}\right)}{d}+\frac{\frac{3\,a\,b^{2}}{4}-\sin\left(c+d\,x\right)^{3}\,\left(\frac{9\,a^{2}\,b}{8}-\frac{b^{3}}{8}\right)+\frac{3\,a^{3}}{4}+\sin\left(c+d\,x\right)\,\left(\frac{15\,a^{2}\,b}{8}+\frac{b^{3}}{8}\right)-\frac{a^{3}\,\sin(c+d\,x)^{2}}{2}}{d\,\left(\sin\left(c+d\,x\right)^{4}-2\,\sin\left(c+d\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)),x)
```

```
[Out] (a^3*log(sin(c + d*x)))/d - (log(sin(c + d*x) + 1)*(a^3/2 - (9*a^2*b)/16 +
b^3/16))/d - (log(sin(c + d*x) - 1)*((9*a^2*b)/16 + a^3/2 - b^3/16))/d + ((
3*a*b^2)/4 - sin(c + d*x)^3*((9*a^2*b)/8 - b^3/8) + (3*a^3)/4 + sin(c + d*x
)*((15*a^2*b)/8 + b^3/8) - (a^3*sin(c + d*x)^2)/2)/(d*(sin(c + d*x)^4 - 2*s
in(c + d*x)^2 + 1))
```

3.1507 $\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=171

$$-\frac{a^3\csc(c+dx)}{d}-\frac{3a(a+b)(5a+3b)\log(1-\sin(c+dx))}{16d}+\frac{3a^2b\log(\sin(c+dx))}{d}+\frac{3a(5a-3b)(a-b)\log(1+\dots}{16d}$$

```
[Out] -a^3*csc(d*x+c)/d-3/16*a*(a+b)*(5*a+3*b)*ln(1-sin(d*x+c))/d+3*a^2*b*ln(sin(
d*x+c))/d+3/16*a*(5*a-3*b)*(a-b)*ln(1+sin(d*x+c))/d+1/4*b*sec(d*x+c)^4*(3*a
^2+b^2+a*(3+a^2/b^2)*b*sin(d*x+c))/d+1/8*a*b*sec(d*x+c)^2*(12*a+(9+7*a^2/b^
2)*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.25, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1819, 1816}

$$-\frac{a^3\csc(c+dx)}{d}+\frac{b\sec^4(c+dx)\left(ab\left(\frac{a^2}{b^2}+3\right)\sin(c+dx)+3a^2+b^2\right)}{4d}+\frac{ab\sec^2(c+dx)\left(b\left(\frac{7a^2}{b^2}+9\right)\sin(c+dx)+12a\right)}{8d}+\frac{3a^2b\log(\sin(c+dx))}{d}-\frac{3a(a+b)(5a+3b)\log(1-\sin(c+dx))}{16d}+\frac{3a(5a-3b)(a-b)\log(\sin(c+dx)+1)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((a^3*Csc[c + d*x])/d) - (3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]])/(
16*d) + (3*a^2*b*Log[Sin[c + d*x]])/d + (3*a*(5*a - 3*b)*(a - b)*Log[1 + Si
n[c + d*x]])/(16*d) + (b*Sec[c + d*x]^4*(3*a^2 + b^2 + a*(3 + a^2/b^2)*b*Si
n[c + d*x]))/(4*d) + (a*b*Sec[c + d*x]^2*(12*a + (9 + (7*a^2)/b^2)*b*Sin[c
+ d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^2(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\text{Subst}\left(\int \frac{b^2(a+x)^3}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^7\text{Subst}\left(\int \frac{(a+x)^3}{x^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} -$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} +$$

$$= \frac{b\sec^4(c+dx)\left(3a^2+b^2+a\left(3+\frac{a^2}{b^2}\right)b\sin(c+dx)\right)}{4d} +$$

$$= -\frac{a^3\csc(c+dx)}{d} - \frac{3a(a+b)(5a+3b)\log(1-\sin(c+$$

$$16d$$

Mathematica [A]
time = 0.84, size = 161, normalized size = 0.94

$$-\frac{16a^3\csc(c+dx)+3a(a+b)(5a+3b)\log(1-\sin(c+dx))-48a^2b\log(\sin(c+dx))-3a(5a-3b)(a-b)\log(1+\sin(c+dx))-\frac{(a+b)^3}{(-1+\sin(c+dx))^2}+\frac{(a+b)^2(7a+b)}{-1+\sin(c+dx)}+\frac{(a-b)^3}{(1+\sin(c+dx))^2}+\frac{(a-b)^2(7a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/16*(16*a^3*Csc[c + d*x] + 3*a*(a + b)*(5*a + 3*b)*Log[1 - Sin[c + d*x]]
- 48*a^2*b*Log[Sin[c + d*x]] - 3*a*(5*a - 3*b)*(a - b)*Log[1 + Sin[c + d*x]
```

```
] - (a + b)^3/(-1 + Sin[c + d*x])^2 + ((a + b)^2*(7*a + b))/(-1 + Sin[c + d
*x]) + (a - b)^3/(1 + Sin[c + d*x])^2 + ((a - b)^2*(7*a - b))/(1 + Sin[c +
d*x]))/d
```

Maple [A]
time = 0.37, size = 170, normalized size = 0.99

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^2b\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+}\right.}{d}$
default	$\frac{a^3\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{5}{8\sin(dx+c)\cos(dx+c)^2}-\frac{15}{8\sin(dx+c)}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+3a^2b\left(\frac{1}{4\cos(dx+c)^4}+\frac{1}{2\cos(dx+}\right.}{d}$
risch	$\frac{-24iab^2e^{7i(dx+c)}-9ib^2ae^{9i(dx+c)}-18ia^3e^{5i(dx+c)}-15ia^3e^{i(dx+c)}+24a^2be^{8i(dx+c)}-24iab^2e^{3i(dx+c)}+66iab^2e^{5i(dx+c)}}{4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(d*x+c)/cos(d*x+c)^2-15/8/sin(
d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a^2*b*(1/4/cos(d*x+c)^4+1/2/cos(d*
x+c)^2+ln(tan(d*x+c)))+3*a*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x
+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*b^3/cos(d*x+c)^4)
```

Maxima [A]
time = 0.37, size = 188, normalized size = 1.10

$$\frac{48\,a^2b\log\left(\sin\left(dx+c\right)\right)+3\,\left(5\,a^3-8\,a^2b+3\,ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(5\,a^3+8\,a^2b+3\,ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(12\,a^2b\sin(dx+c)^3+3\,(5\,a^3+3\,ab^2)\sin(dx+c)^4+8\,a^3-5\,(5\,a^3+3\,ab^2)\sin(dx+c)^2-2\,(9\,a^2b+b^3)\sin(dx+c)\right)}{\sin(dx+c)^5-2\sin(dx+c)^3+\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/16*(48*a^2*b*log(sin(d*x + c)) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(sin(d*
x + c) + 1) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(sin(d*x + c) - 1) - 2*(12*a
^2*b*sin(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*sin(d*x + c)^4 + 8*a^3 - 5*(5*a^3
 + 3*a*b^2)*sin(d*x + c)^2 - 2*(9*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^
5 - 2*sin(d*x + c)^3 + sin(d*x + c)))/d
```

Fricas [A]
time = 0.39, size = 226, normalized size = 1.32

$$\frac{48\,a^2b\cos\left(dx+c\right)^4\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,\left(5\,a^3-8\,a^2b+3\,ab^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3\,\left(5\,a^3+8\,a^2b+3\,ab^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-6\,\left(5\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^4+4\,a^3+12\,ab^2+2\,\left(5\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2+4\,\left(6\,a^2b\cos\left(dx+c\right)^2+3\,a^2b+b^3\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(48*a^2*b*cos(d*x + c)^4*log(1/2*sin(d*x + c))*sin(d*x + c) + 3*(5*a^3
 - 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)*sin(d*x + c) - 3
*(5*a^3 + 8*a^2*b + 3*a*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1)*sin(d*x
+ c) - 6*(5*a^3 + 3*a*b^2)*cos(d*x + c)^4 + 4*a^3 + 12*a*b^2 + 2*(5*a^3 + 3
*a*b^2)*cos(d*x + c)^2 + 4*(6*a^2*b*cos(d*x + c)^2 + 3*a^2*b + b^3)*sin(d*x
 + c))/(d*cos(d*x + c)^4*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 210, normalized size = 1.23

$$\frac{48\,a^2b\log\left(\left|\sin\left(dx+c\right)\right|\right)+3\left(5\,a^3-8\,a^2b+3\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(5\,a^3+8\,a^2b+3\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{16\left(3\,a^2b\sin(dx+c)+a^3\right)}{\sin(dx+c)}+\frac{2\left(18\,a^2b\sin(dx+c)^4-7\,a^3\sin(dx+c)^3-9\,ab^2\sin(dx+c)^3-48\,a^2b\sin(dx+c)^2+9\,a^3\sin(dx+c)+15\,ab^2\sin(dx+c)+36\,a^2b+2\,b^3\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(48*a^2*b*log(abs(sin(d*x + c))) + 3*(5*a^3 - 8*a^2*b + 3*a*b^2)*log(a
bs(sin(d*x + c) + 1)) - 3*(5*a^3 + 8*a^2*b + 3*a*b^2)*log(abs(sin(d*x + c)
- 1)) - 16*(3*a^2*b*sin(d*x + c) + a^3)/sin(d*x + c) + 2*(18*a^2*b*sin(d*x
+ c)^4 - 7*a^3*sin(d*x + c)^3 - 9*a*b^2*sin(d*x + c)^3 - 48*a^2*b*sin(d*x +
 c)^2 + 9*a^3*sin(d*x + c) + 15*a*b^2*sin(d*x + c) + 36*a^2*b + 2*b^3)/(sin
(d*x + c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 0.13, size = 182, normalized size = 1.06

$$\frac{3\,a^2\,b\,\ln\left(\sin\left(c+d\,x\right)\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(\frac{15\,a^3}{8}+\frac{9\,a\,b^2}{8}\right)-{\sin\left(c+d\,x\right)}^2\,\left(\frac{25\,a^3}{8}+\frac{15\,a\,b^2}{8}\right)+a^3-\sin\left(c+d\,x\right)\,\left(\frac{9\,a^2\,b}{4}+\frac{b^3}{4}\right)+\frac{3\,a^2\,b\,{\sin\left(c+d\,x\right)}^3}{2}}{d\,\left({\sin\left(c+d\,x\right)}^5-2\,{\sin\left(c+d\,x\right)}^3+\sin\left(c+d\,x\right)\right)}+\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(a-b\right)\,\left(5\,a-3\,b\right)}{16\,d}-\frac{3\,a\,\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(a+b\right)\,\left(5\,a+3\,b\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^2),x)
```

```
[Out] (3*a^2*b*log(sin(c + d*x)))/d - (sin(c + d*x)^4*((9*a*b^2)/8 + (15*a^3)/8)
- sin(c + d*x)^2*((15*a*b^2)/8 + (25*a^3)/8) + a^3 - sin(c + d*x)*((9*a^2*b
)/4 + b^3/4) + (3*a^2*b*sin(c + d*x)^3)/2)/(d*(sin(c + d*x) - 2*sin(c + d*x
)^3 + sin(c + d*x)^5)) + (3*a*log(sin(c + d*x) + 1)*(a - b)*(5*a - 3*b))/(1
6*d) - (3*a*log(sin(c + d*x) - 1)*(a + b)*(5*a + 3*b))/(16*d)
```

3.1508 $\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=221

$$-\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+$$

```
[Out] -3*a^2*b*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-3/16*(a+b)*(8*a^2+7*a*b+b^2)*l
n(1-sin(d*x+c))/d+3*a*(a^2+b^2)*ln(sin(d*x+c))/d-3/16*(a-b)*(8*a^2-7*a*b+b^
2)*ln(1+sin(d*x+c))/d+1/4*b^2*sec(d*x+c)^4*(a*(3+a^2/b^2)+(1+3*a^2/b^2)*b*s
in(d*x+c))/d+1/8*b^2*sec(d*x+c)^2*(4*a*(3+2*a^2/b^2)+3*(1+7*a^2/b^2)*b*sin(
d*x+c))/d
```

Rubi [A]

time = 0.30, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 12, 1819, 1816}

$$-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))}{16d}+\frac{3a\left(a^2+b^2\right)\log(\sin(c+dx))}{d}-\frac{3(a-b)\left(8a^2-7ab+b^2\right)\log(\sin(c+dx)+1)}{16d}+\frac{b^2\sec^4(c+dx)\left(b\left(\frac{3a^2}{b^2}+1\right)\sin(c+dx)+a\left(\frac{a^2}{b^2}+3\right)\right)}{4d}+\frac{b^2\sec^2(c+dx)\left(3b\left(\frac{7a^2}{b^2}+1\right)\sin(c+dx)+4a\left(\frac{2a^2}{b^2}+3\right)\right)}{8d}-\frac{3a^2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (3*(a + b)*(8*a^2
+ 7*a*b + b^2)*Log[1 - Sin[c + d*x]])/(16*d) + (3*a*(a^2 + b^2)*Log[Sin[c +
 d*x]])/d - (3*(a - b)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]])/(16*d)
+ (b^2*Sec[c + d*x]^4*(a*(3 + a^2/b^2) + (1 + (3*a^2)/b^2)*b*Sin[c + d*x]))
/(4*d) + (b^2*Sec[c + d*x]^2*(4*a*(3 + (2*a^2)/b^2) + 3*(1 + (7*a^2)/b^2)*b
*Sin[c + d*x]))/(8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)\sec^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\text{Subst}\left(\int \frac{b^3(a+x)^3}{x^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^8\text{Subst}\left(\int \frac{(a+x)^3}{x^3(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right.}{4d}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right.}{4d}$$

$$= \frac{b^2\sec^4(c+dx)\left(a\left(3+\frac{a^2}{b^2}\right)+\left(1+\frac{3a^2}{b^2}\right)b\sin(c+dx)\right.}{4d}$$

$$= -\frac{3a^2b\csc(c+dx)}{d} - \frac{a^3\csc^2(c+dx)}{2d} - \frac{3(a+b)\left(8a^2\right.}{}$$

Mathematica [A]
time = 2.07, size = 190, normalized size = 0.86

$$\frac{-48a^2b\csc(c+dx)-8a^3\csc^2(c+dx)-3(a+b)\left(8a^2+7ab+b^2\right)\log(1-\sin(c+dx))+48a\left(a^2+b^2\right)\log(\sin(c+dx))-3(a-b)\left(8a^2-7ab+b^2\right)\log(1+\sin(c+dx))+\frac{(a+b)^3}{(-1+\sin(c+dx))^2}-\frac{3(a+b)^2(3a+b)}{-1+\sin(c+dx)}+\frac{(a-b)^3}{(1+\sin(c+dx))^2}+\frac{3(a-b)^2(3a-b)}{1+\sin(c+dx)}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*Sec[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-48*a^2*b*Csc[c + d*x] - 8*a^3*Csc[c + d*x]^2 - 3*(a + b)*(8*a^2 + 7*a*b +
 b^2)*Log[1 - Sin[c + d*x]] + 48*a*(a^2 + b^2)*Log[Sin[c + d*x]] - 3*(a - b
)*(8*a^2 - 7*a*b + b^2)*Log[1 + Sin[c + d*x]] + (a + b)^3/(-1 + Sin[c + d*x
])^2 - (3*(a + b)^2*(3*a + b))/(-1 + Sin[c + d*x]) + (a - b)^3/(1 + Sin[c +
 d*x])^2 + (3*(a - b)^2*(3*a - b))/(1 + Sin[c + d*x]))/(16*d)
```

Maple [A]
time = 0.41, size = 217, normalized size = 0.98

method	result
derivativedivides	$\frac{a^3\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^2b\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{\dots}{8\sin(d\dots}\right.\dots}{\dots}$
default	$\frac{a^3\left(\frac{1}{4\sin(dx+c)^2\cos(dx+c)^4}+\frac{3}{4\sin(dx+c)^2\cos(dx+c)^2}-\frac{3}{2\sin(dx+c)^2}+3\ln(\tan(dx+c))\right)+3a^2b\left(\frac{1}{4\sin(dx+c)\cos(dx+c)^4}+\frac{\dots}{8\sin(d\dots}\right.\dots}{\dots}$
risch	$-\frac{i\left(-16ia^3e^{6i(dx+c)}+48ia^3e^{4i(dx+c)}+45a^2b\,e^{11i(dx+c)}+3b^3e^{11i(dx+c)}+48ib^2a\,e^{4i(dx+c)}+24ia^3e^{10i(dx+c)}+75a^2b\,e^{9i(dx\dots}\right.}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/4/sin(d*x+c)^2/cos(d*x+c)^4+3/4/sin(d*x+c)^2/cos(d*x+c)^2-3/2/s
in(d*x+c)^2+3*ln(tan(d*x+c)))+3*a^2*b*(1/4/sin(d*x+c)/cos(d*x+c)^4+5/8/sin(
d*x+c)/cos(d*x+c)^2-15/8/sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c)))+3*a*b^2
*(1/4/cos(d*x+c)^4+1/2/cos(d*x+c)^2+ln(tan(d*x+c)))+b^3*(-(-1/4*sec(d*x+c)^
3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 217, normalized size = 0.98

$$-\frac{3\left(8a^3-15a^2b+8ab^2-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(8a^3+15a^2b+8ab^2+b^3\right)\log\left(\sin\left(dx+c\right)-1\right)-48\left(a^3+ab^2\right)\log\left(\sin\left(dx+c\right)\right)+\frac{2\left(3\left(15a^2b+b^3\right)\sin(dx+c)^5+12\left(a^3+ab^2\right)\sin(dx+c)^4+24a^2b\sin(dx+c)-5\left(15a^2b+b^3\right)\sin(dx+c)^3+4a^3-18\left(a^3+ab^2\right)\sin(dx+c)^2\right)}{\sin(dx+c)^6-2\sin(dx+c)^4+\sin(dx+c)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(8*a^
3 + 15*a^2*b + 8*a*b^2 + b^3)*log(sin(d*x + c) - 1) - 48*(a^3 + a*b^2)*log(
sin(d*x + c)) + 2*(3*(15*a^2*b + b^3)*sin(d*x + c)^5 + 12*(a^3 + a*b^2)*sin
(d*x + c)^4 + 24*a^2*b*sin(d*x + c) - 5*(15*a^2*b + b^3)*sin(d*x + c)^3 + 4
*a^3 - 18*(a^3 + a*b^2)*sin(d*x + c)^2)/(sin(d*x + c)^6 - 2*sin(d*x + c)^4
+ sin(d*x + c)^2))/d
```

Fricas [A]
time = 0.42, size = 337, normalized size = 1.52

$$\frac{24\left(a^3+ab^2\right)\cos\left(dx+c\right)^4-4a^3-12ab^2-12\left(a^3+ab^2\right)\cos\left(dx+c\right)^2+48\left(\left(a^3+ab^2\right)\cos\left(dx+c\right)^6-\left(a^3+ab^2\right)\cos\left(dx+c\right)^4\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-3\left(\left(8a^3-15a^2b+8ab^2-b^3\right)\cos\left(dx+c\right)^6-\left(8a^3-15a^2b+8ab^2-b^3\right)\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\left(8a^3+15a^2b+8ab^2+b^3\right)\cos\left(dx+c\right)^6-\left(8a^3+15a^2b+8ab^2+b^3\right)\cos\left(dx+c\right)^4\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\left(3\left(15a^2b+b^3\right)\cos\left(dx+c\right)^4-6a^2b-2b^3-\left(15a^2b+b^3\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\left(d\cos\left(dx+c\right)^6-d\cos\left(dx+c\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/16*(24*(a^3 + a*b^2)*cos(d*x + c)^4 - 4*a^3 - 12*a*b^2 - 12*(a^3 + a*b^2)
*cos(d*x + c)^2 + 48*((a^3 + a*b^2)*cos(d*x + c)^6 - (a^3 + a*b^2)*cos(d*x
+ c)^4)*log(1/2*sin(d*x + c)) - 3*((8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d
*x + c)^6 - (8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - 3*((8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^6 - (8*a^3 +
 15*a^2*b + 8*a*b^2 + b^3)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(3*(1
5*a^2*b + b^3)*cos(d*x + c)^4 - 6*a^2*b - 2*b^3 - (15*a^2*b + b^3)*cos(d*x
+ c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6 - d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*sec(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.57, size = 240, normalized size = 1.09

$$-\frac{3\left(8a^3-15a^2b+8ab^2-b^3\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+3\left(8a^3+15a^2b+8ab^2+b^3\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-48\left(a^3+ab^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{2\left(45a^2b\sin(dx+c)^5+3b^3\sin(dx+c)^5+12a^3\sin(dx+c)^4+12ab^2\sin(dx+c)^4-75a^2b\sin(dx+c)^3-5b^3\sin(dx+c)^3-18a^3\sin(dx+c)^2-18ab^2\sin(dx+c)^2+24a^2b\sin(dx+c)+4a^3\right)}{\left(\sin(dx+c)^3-\sin(dx+c)\right)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*sec(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/16*(3*(8*a^3 - 15*a^2*b + 8*a*b^2 - b^3)*log(abs(sin(d*x + c) + 1)) + 3*
(8*a^3 + 15*a^2*b + 8*a*b^2 + b^3)*log(abs(sin(d*x + c) - 1)) - 48*(a^3 + a
*b^2)*log(abs(sin(d*x + c))) + 2*(45*a^2*b*sin(d*x + c)^5 + 3*b^3*sin(d*x +
 c)^5 + 12*a^3*sin(d*x + c)^4 + 12*a*b^2*sin(d*x + c)^4 - 75*a^2*b*sin(d*x
+ c)^3 - 5*b^3*sin(d*x + c)^3 - 18*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x +
c)^2 + 24*a^2*b*sin(d*x + c) + 4*a^3)/(sin(d*x + c)^3 - sin(d*x + c))^2)/d
```

Mupad [**B**]
time = 11.95, size = 221, normalized size = 1.00

$$\frac{3a\,\ln\left(\sin\left(c+d\,x\right)\right)\,\left(a^2+b^2\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(\frac{3\,a^3}{2}+\frac{3\,a\,b^2}{2}\right)-{\sin\left(c+d\,x\right)}^2\,\left(\frac{9\,a^3}{4}+\frac{9\,a\,b^2}{4}\right)+{\sin\left(c+d\,x\right)}^5\,\left(\frac{45\,a^2\,b}{8}+\frac{3\,b^3}{8}\right)-{\sin\left(c+d\,x\right)}^3\,\left(\frac{75\,a^2\,b}{8}+\frac{5\,b^3}{8}\right)+\frac{a^3}{2}+3\,a^2\,b\,\sin\left(c+d\,x\right)}{d\,\left({\sin\left(c+d\,x\right)}^6-2\,{\sin\left(c+d\,x\right)}^4+{\sin\left(c+d\,x\right)}^2\right)}-\frac{3\,\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(a-b\right)\,\left(8\,a^2-7\,a\,b+b^2\right)}{16\,d}-\frac{3\,\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(a+b\right)\,\left(8\,a^2+7\,a\,b+b^2\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x))^3/(cos(c + d*x)^5*sin(c + d*x)^3),x)
```

```
[Out] (3*a*log(sin(c + d*x))*(a^2 + b^2))/d - (sin(c + d*x)^4*((3*a*b^2)/2 + (3*a
^3)/2) - sin(c + d*x)^2*((9*a*b^2)/4 + (9*a^3)/4) + sin(c + d*x)^5*((45*a^2
*b)/8 + (3*b^3)/8) - sin(c + d*x)^3*((75*a^2*b)/8 + (5*b^3)/8) + a^3/2 + 3*
a^2*b*sin(c + d*x))/(d*(sin(c + d*x)^2 - 2*sin(c + d*x)^4 + sin(c + d*x)^6)
) - (3*log(sin(c + d*x) + 1)*(a - b)*(8*a^2 - 7*a*b + b^2))/(16*d) - (3*log
(sin(c + d*x) - 1)*(a + b)*(7*a*b + 8*a^2 + b^2))/(16*d)
```

3.1509 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=295

$$-\frac{(6a^2b^2(1-n^2)-a^4(3-4n+n^2)-b^4(3+4n+n^2))\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{8d(1+n)}-\frac{abn(\cdots}{\cdots}$$

```
[Out] -1/8*(6*a^2*b^2*(-n^2+1)-a^4*(n^2-4*n+3)-b^4*(n^2+4*n+3))*hypergeom([1, 1/2
+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(1+n)-1/2*a*b*n*(a^2*(
2-n)-b^2*(2+n))*hypergeom([1, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(
2+n)/d/(2+n)+1/4*sec(d*x+c)^4*sin(d*x+c)^(1+n)*(a^4+6*a^2*b^2+b^4+4*a*b*(a^
2+b^2)*sin(d*x+c))/d+1/8*sec(d*x+c)^2*sin(d*x+c)^(1+n)*(a^4*(3-n)-6*a^2*b^2
*(1+n)-b^4*(5+n)+4*a*b*(a^2*(2-n)-b^2*(2+n))*sin(d*x+c))/d
```

Rubi [A]
time = 0.36, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 1820, 822, 371}

$$-\frac{abn(a^2(2-n)-b^2(n+2))\sin^{n+2}(c+dx)\,{}_2F_1\left(1,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{2d(n+2)}-\frac{(-(a^4(n^2-4n+3))+6a^2b^2(1-n^2)-b^4(n^2+4n+3))\sin^{n+1}(c+dx)\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{8d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^4+4ab(a^2+b^2)\sin(c+dx)+6a^2b^2+b^4\right)}{4d}+\frac{\sec^2(c+dx)\sin^{n+1}(c+dx)\left(a^4(3-n)+4ab(a^2(2-n)-b^2(n+2))\sin(c+dx)-6a^2b^2(n+1)-b^4(n+5)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/8*((6*a^2*b^2*(1 - n^2) - a^4*(3 - 4*n + n^2) - b^4*(3 + 4*n + n^2))*Hyp
ergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n
))/(d*(1 + n)) - (a*b*n*(a^2*(2 - n) - b^2*(2 + n))*Hypergeometric2F1[1, (2
 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*(2 + n)) + (
Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^4 + 6*a^2*b^2 + b^4 + 4*a*b*(a^2 + b
^2)*Sin[c + d*x]))/(4*d) + (Sec[c + d*x]^2*Sin[c + d*x]^(1 + n)*(a^4*(3 - n
) - 6*a^2*b^2*(1 + n) - b^4*(5 + n) + 4*a*b*(a^2*(2 - n) - b^2*(2 + n))*Sin
[c + d*x]))/(8*d)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
```

```
] &&  !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^4\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^4}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab(a^2+\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab(a^2+\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^4+6a^2b^2+b^4+4ab(a^2+\right.}{4d}$$

$$= -\frac{(6a^2b^2(1-n^2)-a^4(3-4n+n^2)-b^4(3+4n+n^2))}{8d(1+n}$$

Mathematica [A]
time = 0.14, size = 164, normalized size = 0.56

$$\frac{\left(6(a^2-b^2)^2\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+(a-b)^3(3a+5b)\,{}_2F_1(2,1+n;2+n;-\sin(c+dx))+(3a-5b)(a+b)^3\,{}_2F_1(2,1+n;2+n;\sin(c+dx))+2(a-b)^4\,{}_2F_1(3,1+n;2+n;-\sin(c+dx))+2(a+b)^4\,{}_2F_1(3,1+n;2+n;\sin(c+dx))\right)\sin^{1+n}(c+dx)}{16d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*(a^2 - b^2)^2*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)^3*(3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]
 + (3*a - 5*b)*(a + b)^3*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^4*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
4*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

Maple [F]
time = 1.21, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(dx+c\right)\right) \left(\sin^{n}\left(dx+c\right)\right) \left(a+b\sin\left(dx+c\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

```
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

```
[Out] integral(-(4*(a*b^3*cos(d*x + c)^2 - a^3*b - a*b^3)*sec(d*x + c)^5*sin(d*x
+ c) - (b^4*cos(d*x + c)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(d*x + c)^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^4*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^n\,{\left(a+b\,\sin\left(c+d\,x\right)\right)}^4}{{\cos\left(c+d\,x\right)}^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^4)/cos(c + d*x)^5,x)
```

```
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^4)/cos(c + d*x)^5, x)
```

3.1510 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=186

$$\frac{a(a^2(3-n)-3b^2(1+n))\ {}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{4d(1+n)}+\frac{b(3a^2(2-n)-b^2(2+n))\ {}_2F_1\left(2,\right.}{4d\left(\right.}$$

```
[Out] 1/4*a*(a^2*(3-n)-3*b^2*(1+n))*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+
c)^2)*sin(d*x+c)^(1+n)/d/(1+n)+1/4*b*(3*a^2*(2-n)-b^2*(2+n))*hypergeom([2,
1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)+1/4*sec(d*x+c)^4*
sin(d*x+c)^(1+n)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*sin(d*x+c))/d
```

Rubi [A]
time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 1820, 822, 371}

$$\frac{a(a^2(3-n)-3b^2(n+1))\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{b(3a^2(2-n)-b^2(n+2))\sin^{n+2}(c+dx)\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{4d(n+2)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(b(3a^2+b^2)\sin(c+dx)+a(a^2+3b^2)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2*(3 - n) - 3*b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (b*(3*a^2*(2 - n) -
b^2*(2 + n))*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin
[c + d*x]^(2 + n))/(4*d*(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a*
(a^2 + 3*b^2) + b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*d)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^3}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\text{s}\right.}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a(a^2+3b^2)+b(3a^2+b^2)\text{s}\right.}{4d}$$

$$= \frac{a(a^2(3-n)-3b^2(1+n))\ {}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right.}{4d(1+n)}$$

Mathematica [A]
time = 0.11, size = 158, normalized size = 0.85

$$\frac{\left(6a(a-b)(a+b)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+3(a-b)^2(a+b)\,{}_2F_1(2,1+n;2+n;-\sin(c+dx))+3(a-b)(a+b)^2\,{}_2F_1(2,1+n;2+n;\sin(c+dx))+2(a-b)^3\,{}_2F_1(3,1+n;2+n;-\sin(c+dx))+2(a+b)^3\,{}_2F_1(3,1+n;2+n;\sin(c+dx))\right)\sin^{1+n}(c+dx)}{16d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((6*a*(a - b)*(a + b)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*
x]^2] + 3*(a - b)^2*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x
]] + 3*(a - b)*(a + b)^2*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] +
 2*(a - b)^3*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^
3*Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(
16*d*(1 + n))
```

Maple [F]
time = 0.68, size = 0, normalized size = 0.00

$$\int \left(\sec^5 (dx + c)\right) \left(\sin^n (dx + c)\right) \left(a + b \sin (dx + c)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="maxima
")
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="fricas
")
[Out] integral(-((b^3*cos(d*x + c)^2 - 3*a^2*b - b^3)*sec(d*x + c)^5*sin(d*x + c)
 + (3*a*b^2*cos(d*x + c)^2 - a^3 - 3*a*b^2)*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^3*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)^3}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5,x)
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^3)/cos(c + d*x)^5, x)
```

3.1511 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=160

$$\frac{\left(a^2(3-n)-b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{4d(1+n)}+\frac{ab(2-n)\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+d\right.}{2d(2+n)}$$

```
[Out] 1/4*(a^2*(3-n)-b^2*(1+n))*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2
)*sin(d*x+c)^(1+n)/d/(1+n)+1/2*a*b*(2-n)*hypergeom([2, 1+1/2*n],[1/2*n+2],s
in(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n)+1/4*sec(d*x+c)^4*sin(d*x+c)^(1+n)*(a^
2+b^2+2*a*b*sin(d*x+c))/d
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 1820, 822, 371}

$$\frac{\left(a^2(3-n)-b^2(n+1)\right)\sin^{n+1}(c+dx)\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{4d(n+1)}+\frac{\sec^4(c+dx)\sin^{n+1}(c+dx)\left(a^2+2ab\sin(c+dx)+b^2\right)}{4d}+\frac{ab(2-n)\sin^{n+2}(c+dx)\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2*(3 - n) - b^2*(1 + n))*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]^(1 + n))/(4*d*(1 + n)) + (a*b*(2 - n)*Hypergeomet
ric2F1[2, (2 + n)/2, (4 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(2 + n))/(2*d*
(2 + n)) + (Sec[c + d*x]^4*Sin[c + d*x]^(1 + n)*(a^2 + b^2 + 2*a*b*Sin[c +
d*x]))/(4*d)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(-(c*x)^(m + 1))*(f + g*x)*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1)))
, x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*
a*(p + 1)*Q + f*(m + 2*p + 3) + g*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a,
 b, c, m}, x] && PolyQ[Pq, x] && LtQ[p, -1] &&  !GtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^2}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d}$$

$$= \frac{\sec^4(c+dx)\sin^{1+n}(c+dx)\left(a^2+b^2+2ab\sin(c+dx)\right)}{4d}$$

$$= \frac{\left(a^2(3-n)-b^2(1+n)\right)\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\text{ s}}{4d(1+n)}$$

Mathematica [A]
time = 0.15, size = 158, normalized size = 0.99

$$\frac{\left(2(3a^2-b^2)\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)+(a-b)(3a+b)\,{}_2F_1(2,1+n;2+n;-\sin(c+dx))+(3a-b)(a+b)\,{}_2F_1(2,1+n;2+n;\sin(c+dx))+2(a-b)^2\,{}_2F_1(3,1+n;2+n;-\sin(c+dx))+2(a+b)^2\,{}_2F_1(3,1+n;2+n;\sin(c+dx))\right)\sin^{1+n}(c+dx)}{16d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*(3*a^2 - b^2)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2
] + (a - b)*(3*a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]] + (
3*a - b)*(a + b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin[c + d*x]] + 2*(a -
b)^2*Hypergeometric2F1[3, 1 + n, 2 + n, -Sin[c + d*x]] + 2*(a + b)^2*Hyperg
eometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]])*Sin[c + d*x]^(1 + n))/(16*d*(1
+ n))
```

Maple [F]
time = 0.63, size = 0, normalized size = 0.00

$$\int \left(\sec^{5}\left(dx+c\right)\right)\left(\sin^{n}\left(dx+c\right)\right)\left(a+b\sin\left(dx+c\right)\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="maxima
")
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="fricas
")
[Out] integral((2*a*b*sec(d*x + c)^5*sin(d*x + c) - (b^2*cos(d*x + c)^2 - a^2 - b
^2)*sec(d*x + c)^5)*sin(d*x + c)^n, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**2,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c) + a)^2*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)^2}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5, x)
```

3.1512 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=89

$$\frac{a\,{}_2F_1\left(3,\frac{1+n}{2};\frac{3+n}{2};\sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)}+\frac{b\,{}_2F_1\left(3,\frac{2+n}{2};\frac{4+n}{2};\sin^2(c+dx)\right)\sin^{2+n}(c+dx)}{d(2+n)}$$

```
[Out] a*hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^(1+n)/d/(1+
n)+b*hypergeom([3, 1+1/2*n],[1/2*n+2],sin(d*x+c)^2)*sin(d*x+c)^(2+n)/d/(2+n
)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2916, 822, 371}

$$\frac{a\sin^{n+1}(c+dx)\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\sin^2(c+dx)\right)}{d(n+1)}+\frac{b\sin^{n+2}(c+dx)\,{}_2F_1\left(3,\frac{n+2}{2};\frac{n+4}{2};\sin^2(c+dx)\right)}{d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
(1 + n))/(d*(1 + n)) + (b*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin[c
+ d*x]^2]*Sin[c + d*x]^(2 + n))/(d*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{(ab^5)\,\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d} + \frac{b^6\text{Subst}\Big(}{}$$

$$= \frac{a\,{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(c+dx)\right)\sin^{1+n}(c+dx)}{d(1+n)} + \frac{b\,{}_2F_1\big(3}{}$$

Mathematica [A]
time = 0.08, size = 89, normalized size = 1.00

$$\frac{\sin^{1+n}(c+dx)\left(a(2+n)\,{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(c+dx)\right) + b(1+n)\,{}_2F_1\left(3, \frac{2+n}{2}; \frac{4+n}{2}; \sin^2(c+dx)\right)\sin(c+dx)\right)}{d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]^(1 + n)*(a*(2 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2,
 Sin[c + d*x]^2] + b*(1 + n)*Hypergeometric2F1[3, (2 + n)/2, (4 + n)/2, Sin
[c + d*x]^2]*Sin[c + d*x]))/(d*(1 + n)*(2 + n))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)\left(\sin^n(dx+c)\right)\left(a+b\sin(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c)^5*sin(d*x + c) + a*sec(d*x + c)^5)*sin(d*x + c)^n,
 x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5008 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x)))/cos(c + d*x)^5, x)
```

3.1513 $\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=360

$$\frac{(3a^2-9ab+8b^2)\ {}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{16(a-b)^3d(1+n)}+\frac{(3a^2+9ab+8b^2)\ {}_2F_1(1,1+n;2+n;\mathrm{s}}{16(a+b)^3d(1+}$$

```
[Out] 1/16*(3*a^2-9*a*b+8*b^2)*hypergeom([1, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(
1+n)/(a-b)^3/d/(1+n)+1/16*(3*a^2+9*a*b+8*b^2)*hypergeom([1, 1+n],[2+n],sin(
d*x+c))*sin(d*x+c)^(1+n)/(a+b)^3/d/(1+n)-b^6*hypergeom([1, 1+n],[2+n],-b*si
n(d*x+c)/a)*sin(d*x+c)^(1+n)/a/(a^2-b^2)^3/d/(1+n)+1/16*(3*a-5*b)*hypergeom
([2, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/(a-b)^2/d/(1+n)+1/16*(3*a+5*b
)*hypergeom([2, 1+n],[2+n],sin(d*x+c))*sin(d*x+c)^(1+n)/(a+b)^2/d/(1+n)+1/8
*hypergeom([3, 1+n],[2+n],-sin(d*x+c))*sin(d*x+c)^(1+n)/(a-b)/d/(1+n)+1/8*h
ypergeom([3, 1+n],[2+n],sin(d*x+c))*sin(d*x+c)^(1+n)/(a+b)/d/(1+n)
```

Rubi [A]
time = 0.38, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2916, 975, 66}

$$\frac{(3a^2-9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^3}+\frac{(3a^2+9ab+8b^2)\sin^{n+1}(c+dx)\,{}_2F_1(1,n+1;n+2;\sin(c+dx))}{16d(n+1)(a+b)^3}-\frac{b^6\sin^{n+1}(c+dx)\,{}_2F_1\left(1,n+1;n+2;-\frac{b\sin(c+dx)}{a}\right)}{ad(n+1)\left(a^2-b^2\right)^3}+\frac{(3a-5b)\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;-\sin(c+dx))}{16d(n+1)(a-b)^2}+\frac{(3a+5b)\sin^{n+1}(c+dx)\,{}_2F_1(2,n+1;n+2;\sin(c+dx))}{16d(n+1)(a+b)^2}+\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;-\sin(c+dx))}{8d(n+1)(a-b)}+\frac{\sin^{n+1}(c+dx)\,{}_2F_1(3,n+1;n+2;\sin(c+dx))}{8d(n+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]]*
Sin[c + d*x]^(1 + n))/(16*(a - b)^3*d*(1 + n)) + ((3*a^2 + 9*a*b + 8*b^2)*H
ypergeometric2F1[1, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(
a + b)^3*d*(1 + n)) - (b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*Sin[c +
d*x])/a)]*Sin[c + d*x]^(1 + n))/(a*(a^2 - b^2)^3*d*(1 + n)) + ((3*a - 5*b)*
Hypergeometric2F1[2, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(16
*(a - b)^2*d*(1 + n)) + ((3*a + 5*b)*Hypergeometric2F1[2, 1 + n, 2 + n, Sin
[c + d*x]]*Sin[c + d*x]^(1 + n))/(16*(a + b)^2*d*(1 + n)) + (Hypergeometric
2F1[3, 1 + n, 2 + n, -Sin[c + d*x]]*Sin[c + d*x]^(1 + n))/(8*(a - b)*d*(1 +
 n)) + (Hypergeometric2F1[3, 1 + n, 2 + n, Sin[c + d*x]]*Sin[c + d*x]^(1 +
n))/(8*(a + b)*d*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)\sin^n(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n}{8b^3(a+b)(b-x)^3} + \frac{(3a+5b)\left(\frac{x}{b}\right)^n}{16b^4(a+b)^2(b-x)^2} + \frac{(3a^2+9ab+8b^2)\left(\frac{x}{b}\right)^n}{16b^5(a+b)^3(b-x)} - \frac{\left(\frac{x}{b}\right)^n}{(a-b)^3(a+b}\right.\right.}{}$$

$$= \frac{((3a-5b)b)\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^2}\,dx, x, b\sin(c+dx)\right)}{16(a-b)^2d} + \frac{b^2\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n}{(b+x)^3}\,dx, x\right.}{8(a-b}$$

$$= \frac{(3a^2-9ab+8b^2)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))\sin^{1+n}(c+dx)}{16(a-b)^3d(1+n)} + \frac{(3}{}$$

Mathematica [A]
time = 0.31, size = 241, normalized size = 0.67

$$\frac{\left(\frac{(3a^2-9ab+8b^2)\,{}_2F_1(1,1+n;2+n;-\sin(c+dx))}{(a-b)^3} + \frac{(3a^2+9ab+8b^2)\,{}_2F_1(1,1+n;2+n;\sin(c+dx))}{(a+b)^3} - \frac{16b^6\,{}_2F_1\left(1,1+n;2+n;-\frac{b\sin(c+dx)}{a}\right)}{a(a-b)^3(a+b)^3} + \frac{(3a-5b)\,{}_2F_1(2,1+n;2+n;-\sin(c+dx))}{(a-b)^2} + \frac{(3a+5b)\,{}_2F_1(2,1+n;2+n;\sin(c+dx))}{(a+b)^2} + \frac{2\,{}_2F_1(3,1+n;2+n;-\sin(c+dx))}{a-b} + \frac{2\,{}_2F_1(3,1+n;2+n;\sin(c+dx))}{a+b}\right)\sin^{1+n}(c+dx)}{16d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*Sin[c + d*x]^n)/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((((3*a^2 - 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n, -Sin[c + d*x]
])/(a - b)^3 + ((3*a^2 + 9*a*b + 8*b^2)*Hypergeometric2F1[1, 1 + n, 2 + n,
Sin[c + d*x]])/(a + b)^3 - (16*b^6*Hypergeometric2F1[1, 1 + n, 2 + n, -((b*
Sin[c + d*x])/a)])/(a*(a - b)^3*(a + b)^3) + ((3*a - 5*b)*Hypergeometric2F1
```

```
[2, 1 + n, 2 + n, -Sin[c + d*x]])/(a - b)^2 + ((3*a + 5*b)*Hypergeometric2F
1[2, 1 + n, 2 + n, Sin[c + d*x]])/(a + b)^2 + (2*Hypergeometric2F1[3, 1 + n
, 2 + n, -Sin[c + d*x]])/(a - b) + (2*Hypergeometric2F1[3, 1 + n, 2 + n, Si
n[c + d*x]])/(a + b))*Sin[c + d*x]^(1 + n))/(16*d*(1 + n))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(\sec^5(dx+c))(\sin^n(dx+c))}{a+b\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="fricas")
[Out] integral(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n/(a+b*sin(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n/(a+b*sin(d*x+c)),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^n*sec(d*x + c)^5/(b*sin(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^n}{\cos\left(c+d\,x\right)^5\,\left(a+b\,\sin\left(c+d\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^n/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
[Out] int(sin(c + d*x)^n/(cos(c + d*x)^5*(a + b*sin(c + d*x))), x)
```

3.1514 $\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$

Optimal. Leaf size=487

$$\frac{3F_1\left(1+n;-p,1;2+n;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)\sin^{1+n}(c+dx)(a+b\sin(c+dx))^p\left(1+\frac{b\sin(c+dx)}{a}\right)^{-p}}{16d(1+n)}+3F$$

```
[Out] 3/16*AppellF1(1+n,-p,1,2+n,-b*sin(d*x+c)/a,-sin(d*x+c))*sin(d*x+c)^(1+n)*(a
+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+3/16*AppellF1(1+n,-p,1,2+n,
-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1
+b*sin(d*x+c)/a)^p)+3/16*AppellF1(1+n,-p,2,2+n,-b*sin(d*x+c)/a,-sin(d*x+c))
*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+3/16*Ap
pellF1(1+n,-p,2,2+n,-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d
*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)+1/8*AppellF1(1+n,-p,3,2+n,-b*sin(d*
x+c)/a,-sin(d*x+c))*sin(d*x+c)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d
*x+c)/a)^p)+1/8*AppellF1(1+n,-p,3,2+n,-b*sin(d*x+c)/a,sin(d*x+c))*sin(d*x+c
)^(1+n)*(a+b*sin(d*x+c))^p/d/(1+n)/((1+b*sin(d*x+c)/a)^p)
```

Rubi [A]

time = 0.39, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2916, 975, 140, 138, 926}

$$\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,1;n+2;-\frac{b\sin(c+dx)}{a},\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,2;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{16d(n+1)}+\frac{3\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,2;n+2;-\frac{b\sin(c+dx)}{a},\sin(c+dx)\right)}{16d(n+1)}+\frac{\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,3;n+2;-\frac{b\sin(c+dx)}{a},-\sin(c+dx)\right)}{8d(n+1)}+\frac{\sin^{n+1}(c+dx)(a+b\sin(c+dx))^p\left(\frac{b\sin(c+dx)}{a}+1\right)^{-p}F_1\left(n+1;-p,3;n+2;-\frac{b\sin(c+dx)}{a},\sin(c+dx)\right)}{8d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
```

```
[Out] (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), -Sin[c + d*x]]*Sin[
c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin[c + d*x]
)/a)^p) + (3*AppellF1[1 + n, -p, 1, 2 + n, -((b*Sin[c + d*x])/a), Sin[c + d
*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(1 + (b*Sin
[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c + d*x])/a),
-Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*(1 + n)*(
1 + (b*Sin[c + d*x])/a)^p) + (3*AppellF1[1 + n, -p, 2, 2 + n, -((b*Sin[c +
d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p)/(16*d*
(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n, -((b*S
in[c + d*x])/a), -Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d*x])^p
)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p) + (AppellF1[1 + n, -p, 3, 2 + n,
 -((b*Sin[c + d*x])/a), Sin[c + d*x]]*Sin[c + d*x]^(1 + n)*(a + b*Sin[c + d
*x])^p)/(8*d*(1 + n)*(1 + (b*Sin[c + d*x])/a)^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 926

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n, 1/(a + c*x^
2), x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[c*d^2 + a*e^2, 0] &
&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx = \frac{b^5\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b^5\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b-x)^3} + \frac{3\left(\frac{x}{b}\right)^n(a+x)^p}{16b^4(b-x)^2} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{8b^3(b+x)^3} + \frac{3\left(\frac{x}{b}\right.}{16b}\right.\right.}{d}$$

$$= \frac{(3b)\text{Subst}\left(\int \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{(b-x)^2}\,dx, x, b\sin(c+dx)\right)}{16d} + \frac{(3b)\text{Sub}}{}$$

$$= \frac{(3b)\text{Subst}\left(\int \left(\frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b-x)} + \frac{\left(\frac{x}{b}\right)^n(a+x)^p}{2b(b+x)}\right)\,dx, x, b\sin(c+\right.}{8d}$$

$$= \frac{3F_1\left(1+n; -p, 2; 2+n; -\frac{b\sin(c+dx)}{a}, -\sin(c+dx)\right)\sin}{16d(1+}$$

$$= \frac{3F_1\left(1+n; -p, 2; 2+n; -\frac{b\sin(c+dx)}{a}, -\sin(c+dx)\right)\sin}{16d(1+}$$

$$= \frac{3F_1\left(1+n; -p, 1; 2+n; -\frac{b\sin(c+dx)}{a}, -\sin(c+dx)\right)\sin}{16d(1+}$$

Mathematica [F]
time = 18.42, size = 0, normalized size = 0.00

$$\int \sec^5(c+dx)\sin^n(c+dx)(a+b\sin(c+dx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p,x]
[Out] Integrate[Sec[c + d*x]^5*Sin[c + d*x]^n*(a + b*Sin[c + d*x])^p, x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(\sec^5(dx+c)\right)\left(\sin^n(dx+c)\right)(a+b\sin(dx+c))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
[Out] int(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="maxima
")
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="fricas
")
[Out] integral((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*sin(d*x+c)**n*(a+b*sin(d*x+c))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*sin(d*x+c)^n*(a+b*sin(d*x+c))^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c) + a)^p*sin(d*x + c)^n*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^n\,\left(a+b\,\sin\left(c+d\,x\right)\right)^p}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^p)/cos(c + d*x)^5,x)
[Out] int((sin(c + d*x)^n*(a + b*sin(c + d*x))^p)/cos(c + d*x)^5, x)
```

3.1515 $\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=502

$$-\frac{3ab(-2a^2+b^2)\cos(e+fx)\sqrt{a+b\sin(e+fx)}}{5f\sqrt{d\sin(e+fx)}}+\frac{\sec^5(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}{5df}-\frac{3a\,\mathrm{s}}{}$$

```
[Out] 1/5*sec(f*x+e)^5*(a+b*sin(f*x+e))^(9/2)*(d*sin(f*x+e))^(1/2)/d/f-3/5*a*b*(-
2*a^2+b^2)*cos(f*x+e)*(a+b*sin(f*x+e))^(1/2)/f/(d*sin(f*x+e))^(1/2)-3/20*a*
sec(f*x+e)^3*(-a*(7*a^2+b^2)+2*b*(-7*a^2+b^2)*sin(f*x+e)+5*a*(a^2-b^2)*sin(
f*x+e)^2+(8*a^2*b-4*b^3)*sin(f*x+e)^3)*(d*sin(f*x+e))^(1/2)*(a+b*sin(f*x+e)
)^(1/2)/d/f-3/20*a*(a+b)^(3/2)*(5*a^2+3*a*b-4*b^2)*EllipticF(d^(1/2)*(a+b*s
in(f*x+e))^(1/2)/(a+b)^(1/2)/(d*sin(f*x+e))^(1/2),((-a-b)/(a-b))^(1/2))*(-a
*(-1+csc(f*x+e))/(a+b))^(1/2)*(a*(1+csc(f*x+e))/(a-b))^(1/2)*tan(f*x+e)/f/d
^(1/2)-3/5*b*(2*a^4-3*a^2*b^2+b^4)*EllipticE(((-b-a*csc(f*x+e))/(a-b))^(1/2
),(1-2*a/(a+b))^(1/2))*(-a*(-1+csc(f*x+e))/(a+b))^(1/2)*(d*sin(f*x+e))^(1/2
)*(-a*csc(f*x+e)^2*(1+sin(f*x+e))*(a+b*sin(f*x+e))/(a-b)^2)^(1/2)*tan(f*x+e
)/d/f/(a+b*sin(f*x+e))^(1/2)
```

Rubi [F]
time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx$$

Verification is not applicable to the result.

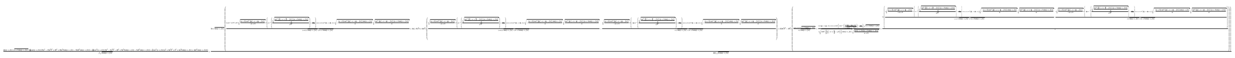
```
[In] Int[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],x]
```

```
[Out] (Sec[e + f*x]^5*Sqrt[d*Sin[e + f*x]]*(a + b*Sin[e + f*x])^(9/2))/(5*d*f) +
(9*a*Defer[Int][(Sec[e + f*x]^4*(a + b*Sin[e + f*x])^(7/2))/Sqrt[d*Sin[e +
f*x]], x])/10
```

Rubi steps

$$\int \frac{\sec^6(e+fx)(a+b\sin(e+fx))^{9/2}}{\sqrt{d\sin(e+fx)}}\,dx=\frac{\sec^5(e+fx)\sqrt{d\sin(e+fx)}\,(a+b\sin(e+fx))^{9/2}}{5df}+\frac{1}{10}(9a)\int \frac{\mathrm{s}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.45, size = 1600, normalized size = 3.19



Antiderivative was successfully verified.

```
[In] Integrate[(Sec[e + f*x]^6*(a + b*Sin[e + f*x])^(9/2))/Sqrt[d*Sin[e + f*x]],
x]
[Out] (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]]*((Sec[e + f*x]*(15*a^4 - 15*a^2*b^2
+ 4*b^4 + 24*a^3*b*Sin[e + f*x] - 12*a*b^3*Sin[e + f*x]))/20 + (Sec[e + f*x
]^3*(3*a^4 - 3*a^2*b^2 - 4*b^4 + 9*a^3*b*Sin[e + f*x] - 5*a*b^3*Sin[e + f*x
]))/10 + (Sec[e + f*x]^5*(a^4 + 6*a^2*b^2 + b^4 + 4*a^3*b*Sin[e + f*x] + 4*
a*b^3*Sin[e + f*x]))/5))/(f*Sqrt[d*Sin[e + f*x]]) + (3*a*Sqrt[Sin[e + f*x]]
*((4*a*(5*a^4 - 9*a^2*b^2 + 4*b^4)*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e
+ f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2
]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[
(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]
]*Sqrt[a + b*Sin[e + f*x]]) + 4*a*(-8*a^3*b + 4*a*b^3)*((Sqrt[((a + b)*Cot[
(-e + Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f
*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*S
in[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e
+ f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a
+ b)*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]]) - (Sqrt[((a + b)*Cot[(-e
+ Pi/2 - f*x)/2]^2)/(-a + b)]*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f
*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*S
in[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])
/(b*Sqrt[Sin[e + f*x]]*Sqrt[a + b*Sin[e + f*x]])) + 2*(8*a^2*b^2 - 4*b^4)*(
(Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Sin[e + f*x]]) + (I*Cos[(-e
 + Pi/2 - f*x)/2]*Csc[e + f*x]*EllipticE[I*ArcSinh[Sin[(-e + Pi/2 - f*x)/2]
/Sqrt[Sin[e + f*x]]], (-2*a)/(-a - b)]*Sqrt[a + b*Sin[e + f*x]])/(b*Sqrt[Co
s[(-e + Pi/2 - f*x)/2]^2*Csc[e + f*x]]*Sqrt[(Csc[e + f*x]*(a + b*Sin[e + f*
x]))/(a + b)]) + (2*a*((a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b
)]*EllipticF[ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/
a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-e + Pi/
2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/((a + b)*Sqrt[Sin[e + f*x]]*Sqrt[a
+ b*Sin[e + f*x]]) - (a*Sqrt[((a + b)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-a + b)]
*EllipticPi[-(a/b), ArcSin[Sqrt[(Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/a]/Sqrt[2]], (-2*a)/(-a + b)]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[-(((a + b)*Csc[(-e + Pi/2 - f*x)/2]^2*Sin[e + f*x])/a)]*Sqrt[(Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/a])/(b*Sqrt[Sin[e + f*x]]*Sqrt[a
 + b*Sin[e + f*x]])))/b)))/(40*f*Sqrt[d*Sin[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5433 vs. $2(461) = 922$.
time = 0.92, size = 5434, normalized size = 10.82

method	result	size
default	Expression too large to display	5434

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x,method=_RETU
RNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
[Out] integral(-(4*(a*b^3*cos(f*x + e)^2 - a^3*b - a*b^3)*sec(f*x + e)^6*sin(f*x
+ e) - (b^4*cos(f*x + e)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*co
s(f*x + e)^2)*sec(f*x + e)^6)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e))
/(d*sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e))**(9/2)/(d*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e))^(9/2)/(d*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^(9/2)*sec(f*x + e)^6/sqrt(d*sin(f*x + e)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\cos\left(e+f\,x\right)}^6\,\sqrt{d\,\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x))^(9/2)/(cos(e + f*x)^6*(d*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((a + b*sin(e + f*x))^(9/2)/(cos(e + f*x)^6*(d*sin(e + f*x))^(1/2)), x)
```

3.1516 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=458

$$-\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{2080d^3f}-\frac{9b(3bc-2ad)\cos(e+fx)\sin(e-}{208d^2j}$$

[Out] -9/2080*(64*a*b*c*d-26*a^2*d^2-b^2*(18*c^2-13*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d^3/f-9/208*b*(-2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d^2/f+3/16*cos(f*x+e)*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(7/3)/d/f-3/2080*(c+d)^2*(208*a^2*c*d^2-64*a*b*d*(3*c^2-5*d^2)+b^2*c*(54*c^2+d^2))*AppellF1(1/2,-7/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)/d^4/f/((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(1+sin(f*x+e))^(1/2)-3/2080*(c-d)*(c+d)^2*(192*a*b*c*d-208*a^2*d^2-b^2*(54*c^2+91*d^2))*AppellF1(1/2,-4/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)/d^4/f/((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(1+sin(f*x+e))^(1/2)

Rubi [A]
time = 0.81, antiderivative size = 458, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$,
Rules used = {3001, 3129, 3112, 3102, 2835, 2744, 144, 143}

$$-\frac{3(c+d)^2(208a^2cd^2-64abd(3c^2-5d^2)+b^2c(54c^2+d^2))\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{1040\sqrt{2}d^4f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{3(c-d)(c+d)^2(-208a^2d^2+192abcd-(b^2(54c^2+91d^2)))\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{1040\sqrt{2}d^4f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{9(-26a^2d^2+64abcd-(b^2(18c^2-13d^2)))\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{2080d^3f}-\frac{9b(3bc-2ad)\sin(e+fx)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{208d^2f}+\frac{3\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{7/3}}{16df}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),x]

[Out] (-9*(64*a*b*c*d - 26*a^2*d^2 - b^2*(18*c^2 - 13*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(2080*d^3*f) - (9*b*(3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(208*d^2*f) + (3*Cos[e + f*x]*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(7/3))/(16*d*f) - (3*(c + d)^2*(208*a^2*c*d^2 - 64*a*b*d*(3*c^2 - 5*d^2) + b^2*c*(54*c^2 + d^2))*AppellF1[1/2, 1/2, -7/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(192*a*b*c*d - 208*a^2*d^2 - b^2*(54*c^2 + 91*d^2))*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1/3))/(1040*Sqrt[2]*d^4*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^(1/3))

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
```

```
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^{4/3}$$

$$= \frac{3\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e}{16df}$$

$$= -\frac{9b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+}{208d^2f}$$

$$= -\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(}{2080d^3f}$$

$$= -\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(}{2080d^3f}$$

$$= -\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(}{2080d^3f}$$

$$= -\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(}{2080d^3f}$$

$$= -\frac{9(64abcd-26a^2d^2-b^2(18c^2-13d^2))\cos(}{2080d^3f}$$

Mathematica [A]
time = 6.33, size = 573, normalized size = 1.25

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] (-3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(c^2 - d^2)*(208*a^2*d^2*(4
*c^2 + 7*d^2) + 128*a*b*d*(-6*c^3 + 17*c*d^2) + b^2*(216*c^4 - 248*c^2*d^2
+ 637*d^4))*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e + f*x])/(c - d), (c +
 d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((
d*(1 + Sin[e + f*x]))/(c - d))] - 3*(208*a^2*c*d^2*(4*c^2 + 51*d^2) + 128*a
*b*d*(-6*c^4 + 21*c^2*d^2 + 40*d^4) + b^2*(216*c^5 - 392*c^3*d^2 + 3201*c*d
^4))*AppellF1[4/3, 1/2, 1/2, 7/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[
e + f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 +
```

```
Sin[e + f*x]))/(c - d))]*(c + d*Sin[e + f*x]) + 4*d^2*Cos[e + f*x]^2*(14*d^
2*(448*a*b*c*d + 208*a^2*d^2 + b^2*(4*c^2 + 91*d^2))*Cos[2*(e + f*x)] - 455
*b^2*d^4*Cos[4*(e + f*x)] + 2*(-108*b^2*c^4 + 384*a*b*c^3*d - 416*a^2*c^2*d
^2 + 152*b^2*c^2*d^2 + 2048*a*b*c*d^3 + 728*a^2*d^4 + 546*b^2*d^4 - d*(4576
*a^2*c*d^2 + 32*a*b*d*(8*c^2 + 45*d^2) + b^2*(-72*c^3 + 687*c*d^2))*Sin[e +
 f*x] + 35*b*d^3*(17*b*c + 32*a*d)*Sin[3*(e + f*x)]))))/(232960*d^5*f)
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)^2\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral(-((b^2*c + 2*a*b*d)*cos(f*x + e)^4 - (2*a*b*d + (a^2 + b^2)*c)*cos
(f*x + e)^2 + (b^2*d*cos(f*x + e)^4 - (2*a*b*c + (a^2 + b^2)*d)*cos(f*x + e
)^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2,
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^(4/3), x)
```

3.1517 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=341

$$-\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f}+\frac{3b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{7/3}}{13df}+\frac{3(c+d}{}$$

```
[Out] -3/130*(-13*a*d+6*b*c)*cos(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d^2/f+3/13*b*cos(f
*x+e)*sin(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d/f+3/130*(c+d)^2*(-13*a*c*d+6*b*c^
2-10*b*d^2)*AppellF1(1/2,-7/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*
x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^(1/3)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/3
)*2^(1/2)/(1+sin(f*x+e))^(1/2)-3/130*(c-d)*(c+d)^2*(-13*a*d+6*b*c)*AppellF1
(1/2,-4/3,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+
d*sin(f*x+e))^(1/3)/d^3/f/((c+d*sin(f*x+e))/(c+d))^(1/3)*2^(1/2)/(1+sin(f*x
+e))^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3001, 3113, 3102, 2835, 2744, 144, 143}

$$\frac{3(c+d)^2\left(-13acd+6bc^2-10bd^2\right)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{7}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}\,d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{3(c-d)(c+d)^2(6bc-13ad)\cos(e+fx)\sqrt[3]{c+d\sin(e+fx)}\,F_1\left(\frac{1}{2};\frac{1}{2},-\frac{4}{3};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{65\sqrt{2}\,d^3f\sqrt{\sin(e+fx)+1}\sqrt[3]{\frac{c+d\sin(e+fx)}{c+d}}}-\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{130d^2f}+\frac{3b\sin(e+fx)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{13df}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*(6*b*c - 13*a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(130*d^2*f) +
 (3*b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(13*d*f) + (3*(
c + d)^2*(6*b*c^2 - 13*a*c*d - 10*b*d^2)*AppellF1[1/2, 1/2, -7/3, 3/2, (1 -
 Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e
 + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*
x])/(c + d))^(1/3)) - (3*(c - d)*(c + d)^2*(6*b*c - 13*a*d)*AppellF1[1/2, 1
/2, -4/3, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e
+ f*x]*(c + d*Sin[e + f*x])^(1/3))/(65*Sqrt[2]*d^3*f*Sqrt[1 + Sin[e + f*x]]
*((c + d*Sin[e + f*x])/(c + d))^(1/3))
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
```

```
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3113

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[
(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 3)
)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*
(m + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m +
3))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^{4/3}\,(1-$$

$$= \frac{3b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{7/}}{13df}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))}{130d^2 f}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))}{130d^2 f}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))}{130d^2 f}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))}{130d^2 f}$$

$$= -\frac{3(6bc-13ad)\cos(e+fx)(c+d\sin(e+fx))}{130d^2 f}$$

Mathematica [A]

time = 3.64, size = 398, normalized size = 1.17

$$\frac{3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(12(-c^2+d^2)(-24bc^3+52ac^2d+68bcd^2+91ad^3)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sin(e+fx))}{c-d}}+3(-24bc^4+52ac^3d+84bc^2d^2+663acd^3+160bd^4)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sin(e+fx))}{c-d}}(c+d\sin(e+fx))-4d^2\cos^2(e+fx)(24bc^3-52ac^2d+128bcd^2+91ad^3+14d^2(14bc+13ad)\cos(2(e+fx))-2d(8bc^2+286acd+45bd^2)\sin(e+fx)+70bd^3\sin(3(e+fx)))\right)}{14560d^4 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(-c^2 + d^2)*(-24*b*c^3 + 52
*a*c^2*d + 68*b*c*d^2 + 91*a*d^3)*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e
```

```
 + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x
]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))] + 3*(-24*b*c^4 + 52*a
*c^3*d + 84*b*c^2*d^2 + 663*a*c*d^3 + 160*b*d^4)*AppellF1[4/3, 1/2, 1/2, 7/
3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-
1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d
*Sin[e + f*x]) - 4*d^2*Cos[e + f*x]^2*(24*b*c^3 - 52*a*c^2*d + 128*b*c*d^2
+ 91*a*d^3 + 14*d^2*(14*b*c + 13*a*d)*Cos[2*(e + f*x)] - 2*d*(8*b*c^2 + 286
*a*c*d + 45*b*d^2)*Sin[e + f*x] + 70*b*d^3*Sin[3*(e + f*x)])))/(14560*d^4*f
)
```

Maple [F]
time = 0.34, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\sin\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="maxima")
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="fricas")
[Out] integral(-(b*d*cos(f*x + e)^4 - (b*c + a*d)*cos(f*x + e)^2*sin(f*x + e) - (
a*c + b*d)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin\left(e+fx\right)\right)\left(c+d\sin\left(e+fx\right)\right)^{\frac{4}{3}}\cos^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Integral((a + b*sin(e + f*x))*(c + d*sin(e + f*x))**(4/3)*cos(e + f*x)**2,
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^(4/3),x, algorithm
="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^2\,\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^(4/3), x)
```

3.1518 $\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=125

$$\frac{3F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^{7/3}}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] 3/7*AppellF1(7/3,-1/2,-1/2,10/3,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+
d))*cos(f*x+e)*(c+d*sin(f*x+e))^(7/3)/d/f/(1+(-c-d*sin(f*x+e))/(c-d))^(1/2)
/(1+(-c-d*sin(f*x+e))/(c+d))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2783, 143}

$$\frac{3\cos(e+fx)(c+d\sin(e+fx))^{7/3}F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (3*AppellF1[7/3, -1/2, -1/2, 10/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin
[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(7/3))/(7*d*f*Sqrt[1
- (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c + d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
```

```
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^{4/3}\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int (c+dx)^{4/3}\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\sqrt{\frac{d}{c+d}-\frac{d}{c+\cdots}}\right.}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+\cdots}{c+d}}}$$

$$= \frac{3F_1\left(\frac{7}{3};-\frac{1}{2},-\frac{1}{2};\frac{10}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin\cdots}{7df\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 301 vs. 2(125) = 250.
time = 1.55, size = 301, normalized size = 2.41

$$-\frac{3\sec(e+fx)\sqrt[3]{c+d\sin(e+fx)}\left(12(4c^4+3c^2d^2-7d^4)F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sin(e+fx))}{c-d}}-3c(4c^2+51d^2)F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\sqrt{-\frac{d(-1+\sin(e+fx))}{c+d}}\sqrt{-\frac{d(1+\sin(e+fx))}{c-d}}(c+d\sin(e+fx))+4d^2\cos^2(e+fx)(-4c^2+7d^2+14d^2\cos(2(e+fx))-44cd\sin(e+fx))\right)}{1120d^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] (-3*Sec[e + f*x]*(c + d*Sin[e + f*x])^(1/3)*(12*(4*c^4 + 3*c^2*d^2 - 7*d^4)
*AppellF1[1/3, 1/2, 1/2, 4/3, (c + d*Sin[e + f*x])/(c - d), (c + d*Sin[e +
f*x])/(c + d)]*Sqrt[-((d*(-1 + Sin[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[
e + f*x]))/(c - d))] - 3*c*(4*c^2 + 51*d^2)*AppellF1[4/3, 1/2, 1/2, 7/3, (c
 + d*Sin[e + f*x])/(c - d), (c + d*Sin[e + f*x])/(c + d)]*Sqrt[-((d*(-1 + S
in[e + f*x]))/(c + d))]*Sqrt[-((d*(1 + Sin[e + f*x]))/(c - d))]*(c + d*Sin[
e + f*x]) + 4*d^2*Cos[e + f*x]^2*(-4*c^2 + 7*d^2 + 14*d^2*Cos[2*(e + f*x)]
- 44*c*d*Sin[e + f*x])))/(1120*d^3*f)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(c+d\sin(fx+e))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="maxima")
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="fricas")
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + d\sin(e + fx))^{\frac{4}{3}} \cos^2(e + fx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3),x)
[Out] Integral((c + d*sin(e + f*x))**(4/3)*cos(e + f*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3),x, algorithm="giac")
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e + f\,x)^2\,(c + d\,\sin(e + f\,x))^{4/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3),x)
[Out] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3), x)
```

3.1519 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

Rubi [A]
time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),x]
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{a+b\sin(e+fx)}\,dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$$\text{\$Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x]),
x]
[Out] $Aborted
```

Maple [A]
time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(fx+e)\right)(c+d\sin(fx+e))^{\frac{4}{3}}}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a), x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x)),x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x)), x)
```

3.1520 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

Rubi [A]
time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^2,x]
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])
^2, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^{4/3}}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^(4/3))/(a + b*Sin[e + f*x])^
2,x]
[Out] $Aborted
```

Maple [A]
time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2\left(fx+e\right)\right)\left(c+d\sin\left(fx+e\right)\right)^{\frac{4}{3}}}{\left(a+b\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**(4/3)/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^(4/3)/(a+b*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2,
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^{4/3}}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^(4/3))/(a + b*sin(e + f*x))^2, x)
```

3.1521 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=36

$$\operatorname{Int}\left(\cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx = \int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin($$

Mathematica [A]
time = 5.67, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
```

Maple [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+b\sin(fx+e))^m(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+b\,\sin\left(e+f\,x\right)\right)}^m\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)
```

3.1522 $\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$

Optimal. Leaf size=38

$$\text{Int}\left(\cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),x]
```

```
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3)
, x]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx = \int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\,\mathrm{si}$$

Mathematica [A]
time = 78.13, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^m(c+d\sin(e+fx))^{4/3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
x]
```

```
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(4/3),
 x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+b\sin(fx+e))^m(c+d\sin(fx+e))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^2*sin(f*x + e) + c*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^(1/3)*(b*sin(f*x + e) + a)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**m*(c+d*sin(f*x+e))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^m*(c+d*sin(f*x+e))^(4/3),x, algorit
hm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^(4/3)*(b*sin(f*x + e) + a)^m*cos(f*x + e)^2,
 x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos\left(e+f\,x\right)^2\,\left(a+b\,\sin\left(e+f\,x\right)\right)^m\,\left(c+d\,\sin\left(e+f\,x\right)\right)^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(4/3),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^(4/3), x)
```

3.1523 $\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=552

$$\frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3+n)))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^3f(2+n)(3+n)(4+n)}-\frac{b(3bc-2ad)\cos(e}{}$$

```
[Out] (2*a^2*d^2*(3+n)-4*a*b*c*d*(4+n)+b^2*(6*c^2-d^2*(3+n)))*cos(f*x+e)*(c+d*sin
(f*x+e))^(1+n)/d^3/f/(2+n)/(3+n)/(4+n)-b*(-2*a*d+3*b*c)*cos(f*x+e)*sin(f*x+
e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(3+n)/(4+n)+cos(f*x+e)*(a+b*sin(f*x+e))^2*(
c+d*sin(f*x+e))^(1+n)/d/f/(4+n)-(c+d)*(a^2*c*d^2*(n^2+7*n+12)-2*a*b*d*(4+n)
*(2*c^2-d^2*(2+n))+b^2*c*(6*c^2-d^2*(-n^2-n+3)))*AppellF1(1/2,-1-n,1/2,3/2,
d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^
(1/2)/d^4/f/(2+n)/(3+n)/(4+n)/(((c+d*sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(
1/2)-(c^2-d^2)*(4*a*b*c*d*(4+n)-a^2*d^2*(n^2+7*n+12)-b^2*(6*c^2+d^2*(n^2+4*
n+3)))*AppellF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*c
os(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/d^4/f/(2+n)/(3+n)/(4+n)/(((c+d*sin(f*x
+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 1.02, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3001, 3129, 3112, 3102, 2835, 2744, 144, 143}

$$-\frac{\sqrt{2}(c^2-d^2)\cos(e+fx)\left(-a^2d^2(n^2+7n+12)+4abcd(n+4)-(b^2(6c^2+d^2(n^2+4n+3)))\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}(c+d)\cos(e+fx)\left(a^2cd^2(n^2+7n+12)-2abd(n+4)(2c^2-d^2(n+2))+b^2(6c^3-cd^2(-n^2-n+3))\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^4f(n+2)(n+3)(n+4)\sqrt{\sin(e+fx)+1}}+\frac{\cos(e+fx)\left(2a^2d^2(n+3)-4abcd(n+4)+b^2(6c^2-d^2(n+3))\right)(c+d\sin(e+fx))^{n+1}}{d^3f(n+2)(n+3)(n+4)}-\frac{b(3bc-2ad)\sin(e+fx)\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{d^2f(n+3)(n+4)}+\frac{\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^{n+1}}{df(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] ((2*a^2*d^2*(3 + n) - 4*a*b*c*d*(4 + n) + b^2*(6*c^2 - d^2*(3 + n)))*Cos[e
 + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^3*f*(2 + n)*(3 + n)*(4 + n)) - (b*(
3*b*c - 2*a*d)*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2
*f*(3 + n)*(4 + n)) + (Cos[e + f*x]*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f
*x])^(1 + n))/(d*f*(4 + n)) - (Sqrt[2]*(c + d)*(a^2*c*d^2*(12 + 7*n + n^2)
 - 2*a*b*d*(4 + n)*(2*c^2 - d^2*(2 + n)) + b^2*(6*c^3 - c*d^2*(3 - n - n^2))
)*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f*x
]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 + n)*(3 + n)*(4
 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(
c^2 - d^2)*(4*a*b*c*d*(4 + n) - a^2*d^2*(12 + 7*n + n^2) - b^2*(6*c^2 + d^2
*(3 + 4*n + n^2)))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2, (d*(1
 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^4*f*(2 +
n)*(3 + n)*(4 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
```

```
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,(1$$

$$= \frac{\cos(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+}{df(4+n)}$$

$$= -\frac{b(3bc-2ad)\cos(e+fx)\sin(e+fx)(c+d\,s}{d^2f(3+n)(4+n)}$$

$$= \frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3}{d^3f(2+n)(3+}$$

$$= \frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3}{d^3f(2+n)(3+}$$

$$= \frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3}{d^3f(2+n)(3+}$$

$$= \frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3}{d^3f(2+n)(3+}$$

$$= \frac{(2a^2d^2(3+n)-4abcd(4+n)+b^2(6c^2-d^2(3}{d^3f(2+n)(3+}$$

Mathematica [F]
time = 9.16, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))^2(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+b\sin(fx+e))^2\,(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^4 - 2*a*b*cos(f*x + e)^2*sin(f*x + e) - (a^2 +
b^2)*cos(f*x + e)^2)*(d*sin(f*x + e) + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))^2*(c+d*sin(f*x+e))^n,x, algorithm="
giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(e + f x)^2 (a + b \sin(e + f x))^2 (c + d \sin(e + f x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)
```

3.1524 $\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=375

$$-\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{d^2f(2+n)(3+n)}+\frac{b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+n)}-\frac{\sqrt{2}}{}$$

```
[Out] -(2*b*c-a*d*(3+n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(2+n)/(3+n)+b*co
s(f*x+e)*sin(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+n)-(c+d)*(a*c*d*(3+n)-b*(
2*c^2-d^2*(2+n)))*AppellF1(1/2,-1-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*
sin(f*x+e))*cos(f*x+e)*(c+d*sin(f*x+e))^n*2^(1/2)/d^3/f/(2+n)/(3+n)/(((c+d*
sin(f*x+e))/(c+d))^n)/(1+sin(f*x+e))^(1/2)-(c^2-d^2)*(2*b*c-a*d*(3+n))*Appe
llF1(1/2,-n,1/2,3/2,d*(1-sin(f*x+e))/(c+d),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(
c+d*sin(f*x+e))^n*2^(1/2)/d^3/f/(2+n)/(3+n)/(((c+d*sin(f*x+e))/(c+d))^n)/(1
+sin(f*x+e))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 373, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3001, 3113, 3102, 2835, 2744, 144, 143}

$$\frac{\sqrt{2}\,(c+d)\cos(e+fx)\left(-acd(n+3)+2bc^2-bd^2(n+2)\right)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n-1;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}-\frac{\sqrt{2}\,(c^2-d^2)\cos(e+fx)(2bc-ad(n+3))(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{d^3f(n+2)(n+3)\sqrt{\sin(e+fx)+1}}-\frac{\cos(e+fx)(2bc-ad(n+3))(c+d\sin(e+fx))^{n+1}}{d^2f(n+2)(n+3)}+\frac{b\sin(e+fx)\cos(e+fx)(c+d\sin(e+fx))^{n+1}}{df(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] -(((2*b*c - a*d*(3 + n))*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d^2*f*
(2 + n)*(3 + n))) + (b*Cos[e + f*x]*Sin[e + f*x]*(c + d*Sin[e + f*x])^(1 +
n))/(d*f*(3 + n)) + (Sqrt[2]*(c + d)*(2*b*c^2 - b*d^2*(2 + n) - a*c*d*(3 +
n))*AppellF1[1/2, 1/2, -1 - n, 3/2, (1 - Sin[e + f*x])/2, (d*(1 - Sin[e + f
*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3*f*(2 + n)*(3 + n)*
Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n) - (Sqrt[2]*(c^2 -
d^2)*(2*b*c - a*d*(3 + n))*AppellF1[1/2, 1/2, -n, 3/2, (1 - Sin[e + f*x])/2
, (d*(1 - Sin[e + f*x]))/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(d^3
*f*(2 + n)*(3 + n)*Sqrt[1 + Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
```

```
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 2835

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3001

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*
((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[(a + b*Sin[e
+ f*x])^m*(c + d*Sin[e + f*x])^n*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, c
, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m,
2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&& !LtQ[m, -1]
```

Rule 3113

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[
```

```
(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 3)
)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e + f*x])^m*Simp[a*C*d + A*b*c*
(m + 3) + b*d*(C*(m + 2) + A*(m + 3))*Sin[e + f*x] - (2*a*C*d - b*c*C*(m +
3))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, m}, x] &&
NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx = \int (a+b\sin(e+fx))(c+d\sin(e+fx))^n\,(1-\mathrm{si}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)(c+d\sin(e+fx))^{1+n}}{df(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+f}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+f}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+f}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+f}{d^2f(2+n)(3+n)}$$

$$= -\frac{(2bc-ad(3+n))\cos(e+fx)(c+d\sin(e+f}{d^2f(2+n)(3+n)}$$

Mathematica [F]
time = 3.30, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(a+b\sin(e+fx))(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(a+b\sin(fx+e))(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="ma
xima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fr
icas")
```

```
[Out] integral((b*cos(f*x + e)^2*sin(f*x + e) + a*cos(f*x + e)^2)*(d*sin(f*x + e)
 + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="gi
ac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(e+fx)^2 (a+b\sin(e+fx)) (c+d\sin(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)
```

3.1525 $\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$

Optimal. Leaf size=127

$$\frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+d\sin(e+fx))^{1+n}}{df(1+n)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

```
[Out] AppellF1(1+n,-1/2,-1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*c
os(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(1+n)/(1+(-c-d*sin(f*x+e))/(c-d))^(1/2
)/(1+(-c-d*sin(f*x+e))/(c+d))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2783, 143}

$$\frac{\cos(e+fx)(c+d\sin(e+fx))^{n+1}F_1\left(n+1;-\frac{1}{2},-\frac{1}{2};n+2;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)}{df(n+1)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
```

```
[Out] (AppellF1[1 + n, -1/2, -1/2, 2 + n, (c + d*Sin[e + f*x])/(c - d), (c + d*Si
n[e + f*x])/(c + d)]*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(1 + n
)*Sqrt[1 - (c + d*Sin[e + f*x])/(c - d)]*Sqrt[1 - (c + d*Sin[e + f*x])/(c +
 d)])
```

Rule 143

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 2783

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[g*((g*Cos[e + f*x])^(p - 1)/(f*(1 - (a + b*Sin
[e + f*x])/(a - b))^((p - 1)/2)*(1 - (a + b*Sin[e + f*x])/(a + b))^((p - 1)
```

```
/2))), Subst[Int[(-b/(a - b) - b*(x/(a - b)))^((p - 1)/2)*(b/(a + b) - b*(x
/(a + b)))^((p - 1)/2)*(a + b*x)^m, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b
, e, f, g, m, p}, x] && NeQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx = \frac{\cos(e+fx)\text{Subst}\left(\int (c+dx)^n\sqrt{-\frac{d}{c-d}-\frac{dx}{c-d}}\,\sqrt{\frac{d}{c+d}-\frac{dx}{c+d}}\right.}{f\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\,\sqrt{1-\frac{c+d\sin(e+fx)}{c+d}}}$$

$$= \frac{F_1\left(1+n;-\frac{1}{2},-\frac{1}{2};2+n;\frac{c+d\sin(e+fx)}{c-d},\frac{c+d\sin(e+fx)}{c+d}\right)\cos(e+fx)(c+}{df(1+n)\sqrt{1-\frac{c+d\sin(e+fx)}{c-d}}\,\sqrt{1-\frac{c+d\sin(e+}{c+d}}}$$

Mathematica [**F**]
time = 0.23, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)(c+d\sin(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n,x]
[Out] Integrate[Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n, x]
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\cos^2(fx+e)\right)(c+d\sin(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^n,x)
```

```
[Out] int(cos(e + f*x)^2*(c + d*sin(e + f*x))^n, x)
```

3.1526 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=36

$$\operatorname{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Mathematica [A]
time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{a+b\sin(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]),x]
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x]), x]
```

Maple [A]
time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(fx+e)\right)\left(c+d\sin(fx+e)\right)^n}{a+b\sin(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="ma
xima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="fr
icas")
```

```
[Out] integral((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e)),x, algorithm="gi
ac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(e+fx)^2 (c+d\sin(e+fx))^n}{a+b\sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x)),x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x)), x)
```

3.1527 $\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=36

$$\text{Int}\left(\frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2},x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,
x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx = \int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A]
time = 41.78, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(e+fx)(c+d\sin(e+fx))^n}{(a+b\sin(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Integrate[(Cos[e + f*x]^2*(c + d*Sin[e + f*x])^n)/(a + b*Sin[e + f*x])^2, x
]
```

Maple [A]
time = 1.70, size = 0, normalized size = 0.00

$$\int \frac{(\cos^2(fx+e))\,(c+d\sin(fx+e))^n}{(a+b\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

```
[Out] int(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
maxima")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
fricas")
```

```
[Out] integral(-(d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - 2*a*b
*sin(f*x + e) - a^2 - b^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(c+d*sin(f*x+e))**n/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(c+d*sin(f*x+e))^n/(a+b*sin(f*x+e))^2,x, algorithm="
giac")
```

```
[Out] integrate((d*sin(f*x + e) + c)^n*cos(f*x + e)^2/(b*sin(f*x + e) + a)^2, x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(e+f\,x\right)^2\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^n}{{\left(a+b\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x))^2,x)
[Out] int((cos(e + f*x)^2*(c + d*sin(e + f*x))^n)/(a + b*sin(e + f*x))^2, x)
```

3.1528 $\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=188

$$\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(3aA-bB)\sin^3(c+dx)}{3d}-\frac{3(Ab+aB)\sin^4(c+dx)}{4d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(3*A*a-B*b)*sin(d*x+c)^3/
d-3/4*(A*b+B*a)*sin(d*x+c)^4/d+3/5*(A*a-B*b)*sin(d*x+c)^5/d+1/2*(A*b+B*a)*s
in(d*x+c)^6/d-1/7*(A*a-3*B*b)*sin(d*x+c)^7/d-1/8*(A*b+B*a)*sin(d*x+c)^8/d-1
/9*b*B*sin(d*x+c)^9/d
```

Rubi [A]

time = 0.17, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 786}

$$-\frac{(aB+Ab)\sin^8(c+dx)}{8d}-\frac{(aA-3bB)\sin^7(c+dx)}{7d}+\frac{(aB+Ab)\sin^6(c+dx)}{2d}+\frac{3(aA-bB)\sin^5(c+dx)}{5d}-\frac{3(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(3aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((3*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - (3*(A*b + a*B)*Sin[c + d*x]^4)/(4*d) + (3*(a*A - b*
B)*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(2*d) - ((a*A - 3*b
*B)*Sin[c + d*x]^7)/(7*d) - ((A*b + a*B)*Sin[c + d*x]^8)/(8*d) - (b*B*Sin[c
 + d*x]^9)/(9*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3\,dx, x, b\sin \cdots\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(aAb^6+b^5(Ab+aB)x+b^4(-3aA+\cdots\right.\right.}{\cdots}$$

$$= \frac{aA\sin(c+dx)}{d} + \frac{(Ab+aB)\sin^2(c+dx)}{2d} - \frac{(3\cdots}{\cdots}$$

Mathematica [A]

time = 0.87, size = 151, normalized size = 0.80

$$\frac{\sin(c+dx)\left(2520aA+1260(Ab+aB)\sin(c+dx)-840(3aA-bB)\sin^2(c+dx)-1890(Ab+aB)\sin^3(c+dx)+1512(aA-bB)\sin^4(c+dx)+1260(Ab+aB)\sin^5(c+dx)-360(aA-3bB)\sin^6(c+dx)-315(Ab+aB)\sin^7(c+dx)-280bB\sin^8(c+dx)\right)}{2520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(2520*a*A + 1260*(A*b + a*B)*Sin[c + d*x] - 840*(3*a*A - b*B)
*Sin[c + d*x]^2 - 1890*(A*b + a*B)*Sin[c + d*x]^3 + 1512*(a*A - b*B)*Sin[c
+ d*x]^4 + 1260*(A*b + a*B)*Sin[c + d*x]^5 - 360*(a*A - 3*b*B)*Sin[c + d*x]
^6 - 315*(A*b + a*B)*Sin[c + d*x]^7 - 280*b*B*Sin[c + d*x]^8))/(2520*d)
```

Maple [A]

time = 0.64, size = 128, normalized size = 0.68

method	result
derivativedivides	$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{Ab\left(\cos^8(dx+c)\right)}{8}-\frac{B\left(\cos^8(\cdots\right.}{\cdots}}{d}$
default	$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^8(dx+c)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{63}\right)-\frac{Ab\left(\cos^8(dx+c)\right)}{8}-\frac{B\left(\cos^8(\cdots\right.}{\cdots}}{d}$
risch	$\frac{35aA\sin(dx+c)}{64d}+\frac{7bB\sin(dx+c)}{128d}-\frac{Bb\sin(9dx+9c)}{2304d}-\frac{\cos(8dx+8c)Ab}{1024d}-\frac{a\cos(8dx+8c)B}{1024d}+\frac{\sin(7dx+7c)aA}{448d}-\cdots$
norman	$\frac{\frac{(2Ab+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^{16}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(3aA+Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(3aA+Bb)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(17a\cdots}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+
c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-1/8*A*b*cos(d*x+c)^8-1/8*B*cos(d*x+c)^8*
a+1/7*a*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 151, normalized size = 0.80

$$-\frac{280\,Bb\sin\left(dx+c\right)^{9}+315\,(Ba+Ab)\sin\left(dx+c\right)^{8}+360\,(Aa-3\,Bb)\sin\left(dx+c\right)^{7}-1260\,(Ba+Ab)\sin\left(dx+c\right)^{6}-1512\,(Aa-Bb)\sin\left(dx+c\right)^{5}+1890\,(Ba+Ab)\sin\left(dx+c\right)^{4}+840\,(3\,Aa-Bb)\sin\left(dx+c\right)^{3}-2520\,Aa\sin\left(dx+c\right)-1260\,(Ba+Ab)\sin\left(dx+c\right)^{2}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2520*(280*B*b*sin(d*x + c)^9 + 315*(B*a + A*b)*sin(d*x + c)^8 + 360*(A*a
 - 3*B*b)*sin(d*x + c)^7 - 1260*(B*a + A*b)*sin(d*x + c)^6 - 1512*(A*a - B*
b)*sin(d*x + c)^5 + 1890*(B*a + A*b)*sin(d*x + c)^4 + 840*(3*A*a - B*b)*sin
(d*x + c)^3 - 2520*A*a*sin(d*x + c) - 1260*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.38, size = 106, normalized size = 0.56

$$-\frac{315\,(Ba+Ab)\cos\left(dx+c\right)^{8}+8\,\left(35\,Bb\cos\left(dx+c\right)^{8}-5\,(9\,Aa+Bb)\cos\left(dx+c\right)^{6}-6\,(9\,Aa+Bb)\cos\left(dx+c\right)^{4}-8\,(9\,Aa+Bb)\cos\left(dx+c\right)^{2}-144\,Aa-16\,Bb\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/2520*(315*(B*a + A*b)*cos(d*x + c)^8 + 8*(35*B*b*cos(d*x + c)^8 - 5*(9*A
*a + B*b)*cos(d*x + c)^6 - 6*(9*A*a + B*b)*cos(d*x + c)^4 - 8*(9*A*a + B*b)
*cos(d*x + c)^2 - 144*A*a - 16*B*b)*sin(d*x + c))/d
```

Sympy [A]
time = 1.30, size = 228, normalized size = 1.21

$$\begin{cases} \frac{16Aa\sin^{7}\left(c+dx\right)}{35d}+\frac{8Aa\sin^{5}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{5d}+\frac{2Aa\sin^{3}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}+\frac{Aa\sin\left(c+dx\right)\cos^{6}\left(c+dx\right)}{d}-\frac{Ab\cos^{8}\left(c+dx\right)}{8d}-\frac{Ba\cos^{8}\left(c+dx\right)}{8d}+\frac{16Bb\sin^{9}\left(c+dx\right)}{315d}+\frac{8Bb\sin^{7}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{35d}+\frac{2Bb\sin^{5}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{5d}+\frac{Bb\sin^{3}\left(c+dx\right)\cos^{6}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x(A+B\sin\left(c\right))\left(a+b\sin\left(c\right)\right)\cos^{7}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a*sin(c + d*x)**7/(35*d) + 8*A*a*sin(c + d*x)**5*cos(c + d*
x)**2/(5*d) + 2*A*a*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a*sin(c + d*x)*co
s(c + d*x)**6/d - A*b*cos(c + d*x)**8/(8*d) - B*a*cos(c + d*x)**8/(8*d) + 1
6*B*b*sin(c + d*x)**9/(315*d) + 8*B*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d
) + 2*B*b*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + B*b*sin(c + d*x)**3*cos(c
 + d*x)**6/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**7, Tr
ue))
```

Giac [A]
time = 0.53, size = 182, normalized size = 0.97

$$-\frac{Bb\sin\left(9\,dx+9\,c\right)}{2304\,d}+\frac{7\,Aa\sin\left(3\,dx+3\,c\right)}{64\,d}-\frac{(Ba+Ab)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{(Ba+Ab)\cos\left(6\,dx+6\,c\right)}{128\,d}-\frac{7\,(Ba+Ab)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{7\,(Ba+Ab)\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{(4\,Aa-5\,Bb)\sin\left(7\,dx+7\,c\right)}{1792\,d}+\frac{(7\,Aa-2\,Bb)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{7\,(10\,Aa+Bb)\sin\left(dx+c\right)}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2304*B*b*sin(9*d*x + 9*c)/d + 7/64*A*a*sin(3*d*x + 3*c)/d - 1/1024*(B*a
+ A*b)*cos(8*d*x + 8*c)/d - 1/128*(B*a + A*b)*cos(6*d*x + 6*c)/d - 7/256*(B
*a + A*b)*cos(4*d*x + 4*c)/d - 7/128*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/179
2*(4*A*a - 5*B*b)*sin(7*d*x + 7*c)/d + 1/320*(7*A*a - 2*B*b)*sin(5*d*x + 5*
c)/d + 7/128*(10*A*a + B*b)*sin(d*x + c)/d
```

Mupad [B]
time = 0.13, size = 156, normalized size = 0.83

$$-\frac{\frac{B\,b\sin(c+d\,x)^9}{9}+\left(\frac{A\,b}{8}+\frac{B\,a}{8}\right)\,\sin\left(c+d\,x\right)^8+\left(\frac{A\,a}{7}-\frac{3\,B\,b}{7}\right)\,\sin\left(c+d\,x\right)^7+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\,\sin\left(c+d\,x\right)^6+\left(\frac{3\,B\,b}{5}-\frac{3\,A\,a}{5}\right)\,\sin\left(c+d\,x\right)^5+\left(\frac{3\,A\,b}{4}+\frac{3\,B\,a}{4}\right)\,\sin\left(c+d\,x\right)^4+\left(A\,a-\frac{B\,b}{3}\right)\,\sin\left(c+d\,x\right)^3+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\,\sin\left(c+d\,x\right)^2-A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^3*(A*a - (B*b)/3) - sin(c + d*x)^2*((A*b)/2 + (B*a)/2) - sin
(c + d*x)^6*((A*b)/2 + (B*a)/2) + sin(c + d*x)^4*((3*A*b)/4 + (3*B*a)/4) -
sin(c + d*x)^5*((3*A*a)/5 - (3*B*b)/5) + sin(c + d*x)^7*((A*a)/7 - (3*B*b)/
7) + sin(c + d*x)^8*((A*b)/8 + (B*a)/8) - A*a*sin(c + d*x) + (B*b*sin(c + d
*x)^9)/9)/d
```

3.1529 $\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}-\frac{(Ab+aB)\sin^4(c+dx)}{2d}+\frac{(aA-2bB)\,\mathrm{si}}{5d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(2*A*a-B*b)*sin(d*x+c)^3/
d-1/2*(A*b+B*a)*sin(d*x+c)^4/d+1/5*(A*a-2*B*b)*sin(d*x+c)^5/d+1/6*(A*b+B*a)
*sin(d*x+c)^6/d+1/7*b*B*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 786}

$$\frac{(aB+Ab)\sin^6(c+dx)}{6d}+\frac{(aA-2bB)\sin^5(c+dx)}{5d}-\frac{(aB+Ab)\sin^4(c+dx)}{2d}-\frac{(2aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((2*a*A - b*B)*
Sin[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(2*d) + ((a*A - 2*b*B)
*Sin[c + d*x]^5)/(5*d) + ((A*b + a*B)*Sin[c + d*x]^6)/(6*d) + (b*B*Sin[c +
d*x]^7)/(7*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2\,dx,x,b\sin\right.}{b^5d}$$
$$= \frac{\text{Subst}\left(\int \left(aAb^4+b^3(Ab+aB)x+b^2(-2aA+\right.\right.}{}$$
$$= \frac{aA\sin(c+dx)}{d} + \frac{(Ab+aB)\sin^2(c+dx)}{2d} - \frac{(2a}{}$$

Mathematica [A]

time = 0.32, size = 116, normalized size = 0.81

$$\frac{\sin(c+dx)\left(210aA+105(Ab+aB)\sin(c+dx)-70(2aA-bB)\sin^2(c+dx)-105(Ab+aB)\sin^3(c+dx)+42(aA-2bB)\sin^4(c+dx)+35(Ab+aB)\sin^5(c+dx)+30bB\sin^6(c+dx)\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sin[c + d*x]*(210*a*A + 105*(A*b + a*B)*Sin[c + d*x] - 70*(2*a*A - b*B)*Si
n[c + d*x]^2 - 105*(A*b + a*B)*Sin[c + d*x]^3 + 42*(a*A - 2*b*B)*Sin[c + d*
x]^4 + 35*(A*b + a*B)*Sin[c + d*x]^5 + 30*b*B*Sin[c + d*x]^6))/(210*d)
```

Maple [A]

time = 0.40, size = 108, normalized size = 0.76

method	result
derivativedivides	$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{Ab\left(\cos^6(dx+c)\right)}{6}-\frac{aB\left(\cos^6(dx+c)\right)}{6}+\frac{aA\left(\frac{8}{3}\right.}{}}{d}$
default	$\frac{Bb\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{Ab\left(\cos^6(dx+c)\right)}{6}-\frac{aB\left(\cos^6(dx+c)\right)}{6}+\frac{aA\left(\frac{8}{3}\right.}{}}{d}$
risch	$\frac{5aA\sin(dx+c)}{8d}+\frac{5bB\sin(dx+c)}{64d}-\frac{\sin(7dx+7c)Bb}{448d}-\frac{\cos(6dx+6c)Ab}{192d}-\frac{a\cos(6dx+6c)B}{192d}+\frac{\sin(5dx+5c)aA}{80d}-$
norman	$\frac{\frac{(2Ab+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(5aA}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(B*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-1/6*A*b*cos(d*x+c)^6-1/6*a*B*cos(d*x+c)^6+1/5*a*A*(8/3+cos
(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 116, normalized size = 0.81

$$\frac{30 Bb\sin(dx+c)^7 + 35 (Ba+Ab)\sin(dx+c)^6 + 42 (Aa-2 Bb)\sin(dx+c)^5 - 105 (Ba+Ab)\sin(dx+c)^4 - 70 (2 Aa - Bb)\sin(dx+c)^3 + 210 Aa\sin(dx+c) + 105 (Ba+Ab)\sin(dx+c)^2}{210 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/210*(30*B*b*sin(d*x + c)^7 + 35*(B*a + A*b)*sin(d*x + c)^6 + 42*(A*a - 2*
B*b)*sin(d*x + c)^5 - 105*(B*a + A*b)*sin(d*x + c)^4 - 70*(2*A*a - B*b)*sin
(d*x + c)^3 + 210*A*a*sin(d*x + c) + 105*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.44, size = 88, normalized size = 0.62

$$-\frac{35 (Ba+Ab)\cos(dx+c)^6 + 2 \left(15 Bb\cos(dx+c)^6 - 3 (7 Aa + Bb)\cos(dx+c)^4 - 4 (7 Aa + Bb)\cos(dx+c)^2 - 56 Aa - 8 Bb\right)\sin(dx+c)}{210 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/210*(35*(B*a + A*b)*cos(d*x + c)^6 + 2*(15*B*b*cos(d*x + c)^6 - 3*(7*A*a
 + B*b)*cos(d*x + c)^4 - 4*(7*A*a + B*b)*cos(d*x + c)^2 - 56*A*a - 8*B*b)*s
in(d*x + c))/d
```

Sympy [A]
time = 0.62, size = 178, normalized size = 1.24

$$\begin{cases} \frac{8Aa\sin^5(c+dx)}{15d} + \frac{4Aa\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{Aa\sin(c+dx)\cos^4(c+dx)}{d} - \frac{Ab\cos^6(c+dx)}{6d} - \frac{Ba\cos^6(c+dx)}{6d} + \frac{8Bb\sin^7(c+dx)}{105d} + \frac{4Bb\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{Bb\sin^3(c+dx)\cos^4(c+dx)}{3d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a+b\sin(c))\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a*sin(c + d*x)**5/(15*d) + 4*A*a*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**4/d - A*b*cos(c + d*x)**6/(6*d)
 - B*a*cos(c + d*x)**6/(6*d) + 8*B*b*sin(c + d*x)**7/(105*d) + 4*B*b*sin(c
+ d*x)**5*cos(c + d*x)**2/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**4/(3*d
), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))*cos(c)**5, True))
```

Giac [A]
time = 0.58, size = 145, normalized size = 1.01

$$-\frac{Bb\sin(7 dx + 7 c)}{448 d} - \frac{(Ba+Ab)\cos(6 dx + 6 c)}{192 d} - \frac{(Ba+Ab)\cos(4 dx + 4 c)}{32 d} - \frac{5 (Ba+Ab)\cos(2 dx + 2 c)}{64 d} + \frac{(4 Aa - 3 Bb)\sin(5 dx + 5 c)}{320 d} + \frac{(20 Aa - Bb)\sin(3 dx + 3 c)}{192 d} + \frac{5 (8 Aa + Bb)\sin(dx + c)}{64 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/448*B*b*sin(7*d*x + 7*c)/d - 1/192*(B*a + A*b)*cos(6*d*x + 6*c)/d - 1/32
*(B*a + A*b)*cos(4*d*x + 4*c)/d - 5/64*(B*a + A*b)*cos(2*d*x + 2*c)/d + 1/3
20*(4*A*a - 3*B*b)*sin(5*d*x + 5*c)/d + 1/192*(20*A*a - B*b)*sin(3*d*x + 3*
c)/d + 5/64*(8*A*a + B*b)*sin(d*x + c)/d
```

Mupad [B]
time = 12.06, size = 118, normalized size = 0.83

$$\frac{\frac{B\,b\,{\sin\left(c+d\,x\right)}^7}{7}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,{\sin\left(c+d\,x\right)}^6+\left(\frac{A\,a}{5}-\frac{2\,B\,b}{5}\right)\,{\sin\left(c+d\,x\right)}^5+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\,{\sin\left(c+d\,x\right)}^4+\left(\frac{B\,b}{3}-\frac{2\,A\,a}{3}\right)\,{\sin\left(c+d\,x\right)}^3+\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)\,{\sin\left(c+d\,x\right)}^2+A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*((A*b)/2 + (B*a)/2) - sin(c + d*x)^4*((A*b)/2 + (B*a)/2) -
sin(c + d*x)^3*((2*A*a)/3 - (B*b)/3) + sin(c + d*x)^5*((A*a)/5 - (2*B*b)/5)
 + sin(c + d*x)^6*((A*b)/6 + (B*a)/6) + A*a*sin(c + d*x) + (B*b*sin(c + d*x
)^7)/7)/d
```

3.1530 $\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}-\frac{(Ab+aB)\sin^4(c+dx)}{4d}-\frac{bB\sin^5(c+dx)}{5d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d-1/3*(A*a-B*b)*sin(d*x+c)^3/d-
1/4*(A*b+B*a)*sin(d*x+c)^4/d-1/5*b*B*sin(d*x+c)^5/d
```

Rubi [A]

time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 786}

$$-\frac{(aB+Ab)\sin^4(c+dx)}{4d}-\frac{(aA-bB)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}-\frac{bB\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) - ((a*A - b*B)*Si
n[c + d*x]^3)/(3*d) - ((A*b + a*B)*Sin[c + d*x]^4)/(4*d) - (b*B*Sin[c + d*x
]^5)/(5*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)\,dx, x, b\sin(\right.}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(aAb^2+b(Ab+aB)x-(aA-bB)x^2\right.\right.}{b^3d}$$
$$= \frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}-\frac{(a}{}$$

Mathematica [A]
time = 0.26, size = 80, normalized size = 0.82

$$\frac{\sin(c+dx)\left(60aA+30(Ab+aB)\sin(c+dx)-20(aA-bB)\sin^2(c+dx)-15(Ab+aB)\sin^3(c+dx)-12bB\sin^4(c+dx)\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]*(60*a*A + 30*(A*b + a*B)*Sin[c + d*x] - 20*(a*A - b*B)*Sin[c
+ d*x]^2 - 15*(A*b + a*B)*Sin[c + d*x]^3 - 12*b*B*Sin[c + d*x]^4))/(60*d)
```

Maple [A]
time = 0.30, size = 88, normalized size = 0.91

method	result
derivativedivides	$\frac{Bb\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{Ab\left(\cos^4(dx+c)\right)}{4}-\frac{aB\left(\cos^4(dx+c)\right)}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$
default	$\frac{Bb\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{Ab\left(\cos^4(dx+c)\right)}{4}-\frac{aB\left(\cos^4(dx+c)\right)}{4}+\frac{aA\left(2+\cos^2(dx+c)\right)\sin(dx}{3}}{d}$
risch	$\frac{3aA\sin(dx+c)}{4d}+\frac{bB\sin(dx+c)}{8d}-\frac{\sin(5dx+5c)Bb}{80d}-\frac{\cos(4dx+4c)Ab}{32d}-\frac{a\cos(4dx+4c)B}{32d}+\frac{aA\sin(3dx+3c)}{12d}-$
norman	$\frac{\frac{(2Ab+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8(2aA+Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8(2aA+Bb)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4(25aA-}{}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(B*b*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-1/
4*A*b*cos(d*x+c)^4-1/4*a*B*cos(d*x+c)^4+1/3*a*A*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 0.82

$$-\frac{12\,Bb\sin\left(dx+c\right)^5+15\,(Ba+Ab)\sin\left(dx+c\right)^4+20\,(Aa-Bb)\sin\left(dx+c\right)^3-60\,Aa\sin\left(dx+c\right)-30\,(Ba+Ab)\sin\left(dx+c\right)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*(B*a + A*b)*sin(d*x + c)^4 + 20*(A*a - B*
b)*sin(d*x + c)^3 - 60*A*a*sin(d*x + c) - 30*(B*a + A*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 70, normalized size = 0.72

$$-\frac{15\,(Ba+Ab)\cos{(dx+c)}^4+4\,\big(3\,Bb\cos{(dx+c)}^4-(5\,Aa+Bb)\cos{(dx+c)}^2-10\,Aa-2\,Bb\big)\sin{(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a + A*b)*cos(d*x + c)^4 + 4*(3*B*b*cos(d*x + c)^4 - (5*A*a + B
*b)*cos(d*x + c)^2 - 10*A*a - 2*B*b)*sin(d*x + c))/d
```

Sympy [A]
time = 0.27, size = 128, normalized size = 1.32

$$\begin{cases} \frac{2Aa\sin^3{(c+dx)}}{3d}+\frac{Aa\sin{(c+dx)}\cos^2{(c+dx)}}{d}-\frac{Ab\cos^4{(c+dx)}}{4d}-\frac{Ba\cos^4{(c+dx)}}{4d}+\frac{2Bb\sin^5{(c+dx)}}{15d}+\frac{Bb\sin^3{(c+dx)}\cos^2{(c+dx)}}{3d} & \text{for } d\neq 0 \\ x(A+B\sin{(c)})\,(a+b\sin{(c)})\cos^3{(c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a*sin(c + d*x)**3/(3*d) + A*a*sin(c + d*x)*cos(c + d*x)**2/d
 - A*b*cos(c + d*x)**4/(4*d) - B*a*cos(c + d*x)**4/(4*d) + 2*B*b*sin(c + d*
x)**5/(15*d) + B*b*sin(c + d*x)**3*cos(c + d*x)**2/(3*d), Ne(d, 0)), (x*(A
+ B*sin(c))*(a + b*sin(c))*cos(c)**3, True))
```

Giac [A]
time = 0.52, size = 100, normalized size = 1.03

$$-\frac{12\,Bb\sin{(dx+c)}^5+15\,Ba\sin{(dx+c)}^4+15\,Ab\sin{(dx+c)}^4+20\,Aa\sin{(dx+c)}^3-20\,Bb\sin{(dx+c)}^3-30\,Ba\sin{(dx+c)}^2-30\,Ab\sin{(dx+c)}^2-60\,Aa\sin{(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/60*(12*B*b*sin(d*x + c)^5 + 15*B*a*sin(d*x + c)^4 + 15*A*b*sin(d*x + c)^
4 + 20*A*a*sin(d*x + c)^3 - 20*B*b*sin(d*x + c)^3 - 30*B*a*sin(d*x + c)^2 -
 30*A*b*sin(d*x + c)^2 - 60*A*a*sin(d*x + c))/d
```

Mupad [B]
time = 11.99, size = 83, normalized size = 0.86

$$-\frac{\frac{B\,b\sin(c+d\,x)^5}{5}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,\sin\left(c+d\,x\right)^4+\left(\frac{A\,a}{3}-\frac{B\,b}{3}\right)\,\sin\left(c+d\,x\right)^3+\left(-\frac{A\,b}{2}-\frac{B\,a}{2}\right)\,\sin\left(c+d\,x\right)^2-A\,a\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] -(sin(c + d*x)^3*((A*a)/3 - (B*b)/3) - sin(c + d*x)^2*((A*b)/2 + (B*a)/2) +
 sin(c + d*x)^4*((A*b)/4 + (B*a)/4) - A*a*sin(c + d*x) + (B*b*sin(c + d*x)^
5)/5)/d
```

3.1531 $\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}+\frac{bB\sin^3(c+dx)}{3d}$$

```
[Out] a*A*sin(d*x+c)/d+1/2*(A*b+B*a)*sin(d*x+c)^2/d+1/3*b*B*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {2912, 45}

$$\frac{(aB+Ab)\sin^2(c+dx)}{2d}+\frac{aA\sin(c+dx)}{d}+\frac{bB\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a*A*Sin[c + d*x])/d + ((A*b + a*B)*Sin[c + d*x]^2)/(2*d) + (b*B*Sin[c + d*
x]^3)/(3*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx=\frac{\text{Subst}\left(\int (a+x)\left(A+\frac{Bx}{b}\right)\,dx,x,b\sin(c+dx)\right)}{bd}$$

$$=\frac{\text{Subst}\left(\int\left(aA+\frac{(Ab+aB)x}{b}+\frac{Bx^2}{b}\right)\,dx,x,b\sin(c+d$$

$$=\frac{aA\sin(c+dx)}{d}+\frac{(Ab+aB)\sin^2(c+dx)}{2d}+\frac{bB\,\text{si}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.87

$$\frac{\sin(c+dx)\left(6aA+3(Ab+aB)\sin(c+dx)+2bB\sin^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] (Sin[c + d*x]*(6*a*A + 3*(A*b + a*B)*Sin[c + d*x] + 2*b*B*Sin[c + d*x]^2))/
(6*d)
```

Maple [A]
time = 0.17, size = 44, normalized size = 0.85

method	result
derivativedivides	$\frac{\frac{B\left(\sin^3(dx+c)\right)b}{3}+\frac{(Ab+aB)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)a}{d}$
default	$\frac{\frac{B\left(\sin^3(dx+c)\right)b}{3}+\frac{(Ab+aB)\left(\sin^2(dx+c)\right)}{2}+A\sin(dx+c)a}{d}$
risch	$\frac{aA\sin(dx+c)}{d}+\frac{bB\sin(dx+c)}{4d}-\frac{\sin(3dx+3c)Bb}{12d}-\frac{\cos(2dx+2c)Ab}{4d}-\frac{a\cos(2dx+2c)B}{4d}$
norman	$\frac{\frac{(2Ab+2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(2Ab+2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4(3aA+2Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2aA\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2aA\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*B*sin(d*x+c)^3*b+1/2*(A*b+B*a)*sin(d*x+c)^2+A*sin(d*x+c)*a)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.87

$$\frac{2\,Bb\sin\left(dx+c\right)^3+6\,Aa\sin\left(dx+c\right)+3\left(Ba+Ab\right)\sin\left(dx+c\right)^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 6*A*a*sin(d*x + c) + 3*(B*a + A*b)*sin(d*x + c)
^2)/d
```

Fricas [A]
time = 0.35, size = 51, normalized size = 0.98

$$-\frac{3\left(Ba+Ab\right)\cos\left(dx+c\right)^2+2\left(Bb\cos\left(dx+c\right)^2-3\,Aa-Bb\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/6*(3*(B*a + A*b)*cos(d*x + c)^2 + 2*(B*b*cos(d*x + c)^2 - 3*A*a - B*b)*s
in(d*x + c))/d
```

Sympy [A]
time = 0.12, size = 75, normalized size = 1.44

$$\begin{cases} \frac{Aa\sin(c+dx)}{d} + \frac{Ab\sin^2(c+dx)}{2d} + \frac{Ba\sin^2(c+dx)}{2d} + \frac{Bb\sin^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(A + B\sin(c))(a + b\sin(c))\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a*sin(c + d*x)/d + A*b*sin(c + d*x)**2/(2*d) + B*a*sin(c + d*x
)**2/(2*d) + B*b*sin(c + d*x)**3/(3*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b
*sin(c))*cos(c), True))
```

Giac [A]
time = 0.52, size = 52, normalized size = 1.00

$$\frac{2\,Bb\sin(dx+c)^3 + 3\,Ba\sin(dx+c)^2 + 3\,Ab\sin(dx+c)^2 + 6\,Aa\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*B*b*sin(d*x + c)^3 + 3*B*a*sin(d*x + c)^2 + 3*A*b*sin(d*x + c)^2 + 6
*A*a*sin(d*x + c))/d
```

Mupad [B]
time = 12.04, size = 44, normalized size = 0.85

$$\frac{\frac{B\,b\sin(c+d\,x)^3}{3} + \left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,\sin(c+d\,x)^2 + A\,a\,\sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^2*((A*b)/2 + (B*a)/2) + A*a*sin(c + d*x) + (B*b*sin(c + d*x)^
3)/3)/d
```

3.1532 $\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(1+\sin(c+dx))}{2d}-\frac{bB\sin(c+dx)}{d}$$

```
[Out] -1/2*(a+b)*(A+B)*ln(1-sin(d*x+c))/d+1/2*(a-b)*(A-B)*ln(1+sin(d*x+c))/d-b*B*
sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2916, 788, 647, 31}

$$-\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/2*((a + b)*(A + B)*Log[1 - Sin[c + d*x]])/d + ((a - b)*(A - B)*Log[1 + S
in[c + d*x]])/(2*d) - (b*B*Sin[c + d*x])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{bB\sin(c+dx)}{d} - \frac{b\text{Subst}\left(\int \frac{-aA-bB-\left(A+\frac{aB}{b}\right)x}{b^2-x^2}\,dx$$

$$= -\frac{bB\sin(c+dx)}{d} - \frac{((a-b)(A-B))\text{Subst}\left(\int \frac{1}{-b-}\right.}{2d}$$

$$= -\frac{(a+b)(A+B)\log(1-\sin(c+dx))}{2d} + \frac{(a-b)(}{}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 1.06

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{bB\tanh^{-1}(\sin(c+dx))}{d} - \frac{Ab\log(\cos(c+dx))}{d} - \frac{aB\log(\cos(c+dx))}{d} - \frac{bB\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/d - (A*b*Log[Co
s[c + d*x]])/d - (a*B*Log[Cos[c + d*x]])/d - (b*B*Sin[c + d*x])/d
```

Maple [A]
time = 0.23, size = 71, normalized size = 1.11

method	result
derivativedivides	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))-aB\ln(\cos(dx+c))-Ab\ln(\cos(dx+c))+Bb(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{aA\ln(\sec(dx+c)+\tan(dx+c))-aB\ln(\cos(dx+c))-Ab\ln(\cos(dx+c))+Bb(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
norman	$\frac{-\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2Bb\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(Ab+aB)\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(aA-Ab-aB+Bb)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$
risch	$\frac{iBb\,e^{i(dx+c)}}{2d}+\frac{2iAbc}{d}+\frac{2iaBc}{d}+iBax+iAbx-\frac{iBb\,e^{-i(dx+c)}}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)A}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)Ab}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*ln(sec(d*x+c)+tan(d*x+c))-a*B*ln(cos(d*x+c))-A*b*ln(cos(d*x+c))+B*
b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 64, normalized size = 1.00

$$-\frac{2\,Bb\sin\left(dx+c\right)-\left(\left(A-B\right)a-\left(A-B\right)b\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(\left(A+B\right)a+\left(A+B\right)b\right)\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 0.38, size = 66, normalized size = 1.03

$$-\frac{2\,Bb\sin\left(dx+c\right)-\left(\left(A-B\right)a-\left(A-B\right)b\right)\log\left(\sin\left(dx+c\right)+1\right)+\left(\left(A+B\right)a+\left(A+B\right)b\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - ((A - B)*a - (A - B)*b)*log(sin(d*x + c) + 1) +
((A + B)*a + (A + B)*b)*log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sin\left(c+dx\right)\right)\left(a+b\sin\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A]
time = 0.52, size = 67, normalized size = 1.05

$$-\frac{2\,Bb\sin\left(dx+c\right)-\left(Aa-Ba-Ab+Bb\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\left(Aa+Ba+Ab+Bb\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*B*b*sin(d*x + c) - (A*a - B*a - A*b + B*b)*log(abs(sin(d*x + c) + 1
)) + (A*a + B*a + A*b + B*b)*log(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.12, size = 53, normalized size = 0.83

$$-\frac{B\,b\,\sin\left(c+d\,x\right)-\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(A-B\right)\,\left(a-b\right)}{2}+\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(a+b\right)\,\left(A+B\right)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x),x)
```

```
[Out] -(B*b*sin(c + d*x) - (log(sin(c + d*x) + 1)*(A - B)*(a - b))/2 + (log(sin(c
 + d*x) - 1)*(a + b)*(A + B))/2)/d
```

3.1533 $\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=59

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{2d}$$

```
[Out] 1/2*(A*a-B*b)*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)^2*(A*b+a*B+(A*a+B*b)*sin
(d*x+c))/d
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.103$, Rules used = {2916, 792, 212}

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec^2(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (Sec[c + d*x]^2*(A*b + a*B + (a
*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\sec^2(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{2d}$$
$$= \frac{(aA-bB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec^2(c+dx)(A}{}$$

Mathematica [A]
time = 0.17, size = 54, normalized size = 0.92

$$\frac{(aA-bB)\tanh^{-1}(\sin(c+dx))+\sec^2(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
[Out] ((a*A - b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]^2*(A*b + a*B + (a*A + b*B
)*Sin[c + d*x]))/(2*d)
```

Maple [A]
time = 0.30, size = 110, normalized size = 1.86

method	result
derivativedivides	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{aB}{2\cos(dx+c)^2}+\frac{Ab}{2\cos(dx+c)^2}+Bb\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c}{}\right.}{d}$
default	$\frac{aA\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{aB}{2\cos(dx+c)^2}+\frac{Ab}{2\cos(dx+c)^2}+Bb\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c}{}\right.}{d}$
risch	$-\frac{i(aA\,e^{3i(dx+c)}+Bb\,e^{3i(dx+c)}-aA\,e^{i(dx+c)}+2iAb\,e^{2i(dx+c)}-Bb\,e^{i(dx+c)}+2iBa\,e^{2i(dx+c)})}{d(e^{2i(dx+c)}+1)^2}-\frac{a\ln(e^{i(dx+c)}-i)A}{2d}+$
norman	$\frac{\frac{(aA+Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{(aA+Bb)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(Ab+aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(Ab+aB)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3(aA+Bb)\left(\tan^3\left(\frac{dx}{2}\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*a*B/
cos(d*x+c)^2+1/2*A*b/cos(d*x+c)^2+B*b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*si
n(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.29, size = 78, normalized size = 1.32

$$\frac{(Aa - Bb)\log(\sin(dx + c) + 1) - (Aa - Bb)\log(\sin(dx + c) - 1) - \frac{2(Ba + Ab + (Aa + Bb)\sin(dx + c))}{\sin(dx + c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((A*a - B*b)*log(sin(d*x + c) + 1) - (A*a - B*b)*log(sin(d*x + c) - 1)
- 2*(B*a + A*b + (A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.39, size = 92, normalized size = 1.56

$$\frac{(Aa - Bb)\cos(dx + c)^2\log(\sin(dx + c) + 1) - (Aa - Bb)\cos(dx + c)^2\log(-\sin(dx + c) + 1) + 2Ba + 2Ab + 2(Aa + Bb)\sin(dx + c)}{4d\cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*((A*a - B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a - B*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*B*a + 2*A*b + 2*(A*a + B*b)*sin(d*x + c)
)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sin(c + dx))(a + b\sin(c + dx))\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.44, size = 84, normalized size = 1.42

$$\frac{(Aa - Bb)\log(|\sin(dx + c) + 1|) - (Aa - Bb)\log(|\sin(dx + c) - 1|) - \frac{2(Aa\sin(dx + c) + Bb\sin(dx + c) + Ba + Ab)}{\sin(dx + c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/4*((A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (A*a - B*b)*log(abs(sin(d*x +
 c) - 1)) - 2*(A*a*sin(d*x + c) + B*b*sin(d*x + c) + B*a + A*b)/(sin(d*x +
c)^2 - 1))/d
```

Mupad [B]
time = 0.11, size = 63, normalized size = 1.07

$$\frac{\operatorname{atanh}(\sin\left(c+d\,x\right))\,\left(\frac{A\,a}{2}-\frac{B\,b}{2}\right)}{d}-\frac{\frac{A\,b}{2}+\frac{B\,a}{2}+\sin\left(c+d\,x\right)\,\left(\frac{A\,a}{2}+\frac{B\,b}{2}\right)}{d\,\left({\sin\left(c+d\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (atanh(sin(c + d*x))*((A*a)/2 - (B*b)/2))/d - ((A*b)/2 + (B*a)/2 + sin(c +
d*x)*((A*a)/2 + (B*b)/2))/(d*(sin(c + d*x)^2 - 1))
```

3.1534 $\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=88

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d}+\frac{(3aA-bB)\sec(c+dx)\tan(c+dx)}{8d}$$

```
[Out] 1/8*(3*A*a-B*b)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(A*b+a*B+(A*a+B*b)*s
in(d*x+c))/d+1/8*(3*A*a-B*b)*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 792, 205, 212}

$$\frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{4d}+\frac{(3aA-bB)\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4*(A*b + a*B +
(a*A + b*B)*Sin[c + d*x]))/(4*d) + ((3*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x]
)/(8*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d}$$

$$= \frac{\sec^4(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{4d}$$

$$= \frac{(3aA-bB)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\sec^4(c+dx)(}{}$$

Mathematica [A]
time = 0.42, size = 82, normalized size = 0.93

$$\frac{\sec^4(c+dx)\left(2(Ab+aB)+(3aA-bB)\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(5aA+bB)\sin(c+dx)+(-3aA+bB)\sin^3(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^4*(2*(A*b + a*B) + (3*a*A - b*B)*ArcTanh[Sin[c + d*x]]*Cos[c
+ d*x]^4 + (5*a*A + b*B)*Sin[c + d*x] + (-3*a*A + b*B)*Sin[c + d*x]^3))/(8*
d)
```

Maple [A]
time = 0.36, size = 141, normalized size = 1.60

method	result
derivativedivides	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{aB}{4\cos(dx+c)^4}+\frac{Ab}{4\cos(dx+c)^4}+Bb\left(\frac{\sin^3(dx-}{4\cos(dx+}\right.}{d}$
default	$\frac{aA\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{aB}{4\cos(dx+c)^4}+\frac{Ab}{4\cos(dx+c)^4}+Bb\left(\frac{\sin^3(dx-}{4\cos(dx+}\right.}{d}$

risch	$\frac{i(-3aA\,e^{7i(dx+c)}+Bb\,e^{7i(dx+c)}-11aA\,e^{5i(dx+c)}-7Bb\,e^{5i(dx+c)}+11aA\,e^{3i(dx+c)}-16iAb\,e^{4i(dx+c)}+7Bb\,e^{3i(dx+c)}-16i}{4d(e^{2i(dx+c)}+1)^4}$
norman	$\frac{(4Ab+4aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(4Ab+4aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(5aA+Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(5aA+Bb)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(7aA+11Bb)(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+1/4*a*B/cos(d*x+c)^4+1/4*A*b/cos(d*x+c)^4+B*b*(1/4*sin(d*x+c)^
3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+
c)+tan(d*x+c))))
```

Maxima [A]
time = 0.30, size = 112, normalized size = 1.27

$$\frac{(3\,Aa-Bb)\log\left(\sin\left(dx+c\right)+1\right)-(3\,Aa-Bb)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left((3\,Aa-Bb)\sin(dx+c)^3-2\,Ba-2\,Ab-(5\,Aa+Bb)\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*((3*A*a - B*b)*log(sin(d*x + c) + 1) - (3*A*a - B*b)*log(sin(d*x + c)
- 1) - 2*((3*A*a - B*b)*sin(d*x + c)^3 - 2*B*a - 2*A*b - (5*A*a + B*b)*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.37, size = 114, normalized size = 1.30

$$\frac{(3\,Aa-Bb)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(3\,Aa-Bb)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+4\,Ba+4\,Ab+2\left((3\,Aa-Bb)\cos\left(dx+c\right)^2+2\,Aa+2\,Bb\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*((3*A*a - B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a - B*b)*co
s(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*B*a + 4*A*b + 2*((3*A*a - B*b)*cos(
d*x + c)^2 + 2*A*a + 2*B*b)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sin\left(c+dx\right))\left(a+b\sin\left(c+dx\right)\right)\sec^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

Giac [A]
time = 0.53, size = 114, normalized size = 1.30

$$\frac{(3\,Aa-Bb)\log\left(|\sin\left(dx+c\right)+1|\right)-(3\,Aa-Bb)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(3\,Aa\sin(dx+c)^{3}-Bb\sin(dx+c)^{3}-5\,Aa\sin(dx+c)-Bb\sin(dx+c)-2\,Ba-2\,Ab\right)}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*((3*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - (3*A*a - B*b)*log(abs(sin(
d*x + c) - 1)) - 2*(3*A*a*sin(d*x + c)^3 - B*b*sin(d*x + c)^3 - 5*A*a*sin(d
*x + c) - B*b*sin(d*x + c) - 2*B*a - 2*A*b)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.14, size = 91, normalized size = 1.03

$$\frac{\left(\frac{B\,b}{8}-\frac{3\,A\,a}{8}\right)\,\sin\left(c+d\,x\right)^3+\left(\frac{5\,A\,a}{8}+\frac{B\,b}{8}\right)\,\sin\left(c+d\,x\right)+\frac{A\,b}{4}+\frac{B\,a}{4}}{d\,\left(\sin\left(c+d\,x\right)^4-2\,\sin\left(c+d\,x\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\sin\left(c+d\,x\right)\right)\,\left(\frac{3\,A\,a}{8}-\frac{B\,b}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^5,x)
```

```
[Out] ((A*b)/4 + (B*a)/4 + sin(c + d*x)*((5*A*a)/8 + (B*b)/8) - sin(c + d*x)^3*((
3*A*a)/8 - (B*b)/8))/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) + (atanh(s
in(c + d*x))*((3*A*a)/8 - (B*b)/8))/d
```

3.1535 $\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=118

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d}+\frac{(5aA-bB)\sec(c+dx}{16d}$$

```
[Out] 1/16*(5*A*a-B*b)*arctanh(sin(d*x+c))/d+1/6*sec(d*x+c)^6*(A*b+a*B+(A*a+B*b)*
sin(d*x+c))/d+1/16*(5*A*a-B*b)*sec(d*x+c)*tan(d*x+c)/d+1/24*(5*A*a-B*b)*sec
(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]

time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 792, 205, 212}

$$\frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)((aA+bB)\sin(c+dx)+aB+Ab)}{6d}+\frac{(5aA-bB)\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{(5aA-bB)\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a*A - b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^6*(A*b + a*B +
 (a*A + b*B)*Sin[c + d*x]))/(6*d) + ((5*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x
])/(16*d) + ((5*a*A - b*B)*Sec[c + d*x]^3*Tan[c + d*x])/(24*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))(A+B\sin(c+dx))\,dx = \frac{b^7\text{Subst}\left(\int \frac{(a+x)\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d}$$

$$= \frac{\sec^6(c+dx)(Ab+aB+(aA+bB)\sin(c+dx))}{6d}$$

$$= \frac{(5aA-bB)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\sec^6(c+dx)(}{}$$

Mathematica [A]
time = 0.62, size = 104, normalized size = 0.88

$$-\frac{\sec^6(c+dx)\left(-8(Ab+aB)-3(5aA-bB)\tanh^{-1}(\sin(c+dx))\cos^6(c+dx)-3(11aA+bB)\sin(c+dx)+8(5aA-bB)\sin^3(c+dx)+(-15aA+3bB)\sin^5(c+dx)\right)}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/48*(Sec[c + d*x]^6*(-8*(A*b + a*B) - 3*(5*a*A - b*B)*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^6 - 3*(11*a*A + b*B)*Sin[c + d*x] + 8*(5*a*A - b*B)*Sin[c +
 d*x]^3 + (-15*a*A + 3*b*B)*Sin[c + d*x]^5))/d
```

Maple [A]
time = 0.41, size = 169, normalized size = 1.43

method	result
derivativedivides	$\frac{aA\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{aB}{6\cos(dx+c)^6}+\frac{Ab}{6\cos(dx+c)^6}}{d}$

default	$\frac{aA\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{aB}{6\cos(dx+c)^6}+\frac{Ab}{6\cos(dx+}}{d}$
risch	$-\frac{i\left(15aA\,e^{11i(dx+c)}-3Bb\,e^{11i(dx+c)}+85aA\,e^{9i(dx+c)}-17Bb\,e^{9i(dx+c)}+198aA\,e^{7i(dx+c)}+114Bb\,e^{7i(dx+c)}-198aA\,e^{5i(}\right.}{24d(}$
norman	$\frac{(11aA+Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}+\frac{(11aA+Bb)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}+\frac{7(19aA+25Bb)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{7(19aA+25Bb)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{(71}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+1/6*a*B/cos(d*x+c)^6+1/6*A*b/cos(d*x+c)^6+
B*b*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d
*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.29, size = 143, normalized size = 1.21

$$\frac{3\,(5\,Aa-Bb)\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,Aa-Bb)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\,(5\,Aa-Bb)\sin(dx+c)^5-8\,(5\,Aa-Bb)\sin(dx+c)^3+8\,Ba+8\,Ab+3\,(11\,Aa+Bb)\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b)*log(sin(d*x +
 c) - 1) - 2*(3*(5*A*a - B*b)*sin(d*x + c)^5 - 8*(5*A*a - B*b)*sin(d*x + c)
^3 + 8*B*a + 8*A*b + 3*(11*A*a + B*b)*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A]

time = 0.39, size = 135, normalized size = 1.14

$$\frac{3\,(5\,Aa-Bb)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-3\,(5\,Aa-Bb)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+16\,Ba+16\,Ab+2\left(3\,(5\,Aa-Bb)\cos\left(dx+c\right)^4+2\,(5\,Aa-Bb)\cos\left(dx+c\right)^2+8\,Aa+8\,Bb\right)\sin\left(dx+c\right)}{96\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a - B*b
)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 16*B*a + 16*A*b + 2*(3*(5*A*a - B
*b)*cos(d*x + c)^4 + 2*(5*A*a - B*b)*cos(d*x + c)^2 + 8*A*a + 8*B*b)*sin(d*
x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.45, size = 139, normalized size = 1.18

$$\frac{3\,(5\,Aa - Bb)\log\left(|\sin\left(dx + c\right) + 1|\right) - 3\,(5\,Aa - Bb)\log\left(|\sin\left(dx + c\right) - 1|\right) - \frac{2\left(15\,Aa\sin(dx+c)^{5} - 3\,Bb\sin(dx+c)^{5} - 40\,Aa\sin(dx+c)^{3} + 8\,Bb\sin(dx+c)^{3} + 33\,Aa\sin(dx+c) + 3\,Bb\sin(dx+c) + 8\,Ba + 8\,Ab\right)}{\left(\sin(dx+c)^{2} - 1\right)^{3}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/96*(3*(5*A*a - B*b)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a - B*b)*log(abs(
sin(d*x + c) - 1)) - 2*(15*A*a*sin(d*x + c)^5 - 3*B*b*sin(d*x + c)^5 - 40*A
*a*sin(d*x + c)^3 + 8*B*b*sin(d*x + c)^3 + 33*A*a*sin(d*x + c) + 3*B*b*sin(
d*x + c) + 8*B*a + 8*A*b)/(sin(d*x + c)^2 - 1)^3)/d
```

Mupad [B]
time = 12.44, size = 120, normalized size = 1.02

$$\frac{\operatorname{atanh}(\sin\left(c + d\,x\right))\,\left(\frac{5\,A\,a}{16} - \frac{B\,b}{16}\right)}{d} - \frac{\left(\frac{5\,A\,a}{16} - \frac{B\,b}{16}\right)\,\sin\left(c + d\,x\right)^5 + \left(\frac{B\,b}{6} - \frac{5\,A\,a}{6}\right)\,\sin\left(c + d\,x\right)^3 + \left(\frac{11\,A\,a}{16} + \frac{B\,b}{16}\right)\,\sin\left(c + d\,x\right) + \frac{A\,b}{6} + \frac{B\,a}{6}}{d\,\left(\sin\left(c + d\,x\right)^6 - 3\sin\left(c + d\,x\right)^4 + 3\sin\left(c + d\,x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x)))/cos(c + d*x)^7,x)
```

```
[Out] (atanh(sin(c + d*x))*((5*A*a)/16 - (B*b)/16))/d - ((A*b)/6 + (B*a)/6 + sin(
c + d*x)*((11*A*a)/16 + (B*b)/16) - sin(c + d*x)^3*((5*A*a)/6 - (B*b)/6) +
sin(c + d*x)^5*((5*A*a)/16 - (B*b)/16))/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*
x)^4 + sin(c + d*x)^6 - 1))
```

3.1536 $\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=349

$$-\frac{(a^2-b^2)^3(Ab-aB)(a+b\sin(c+dx))^3}{3b^8d}+\frac{(a^2-b^2)^2(6aAb-7a^2B+b^2B)(a+b\sin(c+dx))^4}{4b^8d}-\frac{3(a^2-b$$

```
[Out] -1/3*(a^2-b^2)^3*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^8/d+1/4*(a^2-b^2)^2*(6*A*a*
b-7*B*a^2+B*b^2)*(a+b*sin(d*x+c))^4/b^8/d-3/5*(a^2-b^2)*(5*A*a^2*b-A*b^3-7*
B*a^3+3*B*a*b^2)*(a+b*sin(d*x+c))^5/b^8/d+1/6*(20*A*a^3*b-12*A*a*b^3-35*B*a
^4+30*B*a^2*b^2-3*B*b^4)*(a+b*sin(d*x+c))^6/b^8/d-1/7*(15*A*a^2*b-3*A*b^3-3
5*B*a^3+15*B*a*b^2)*(a+b*sin(d*x+c))^7/b^8/d+3/8*(2*A*a*b-7*B*a^2+B*b^2)*(a
+b*sin(d*x+c))^8/b^8/d-1/9*(A*b-7*B*a)*(a+b*sin(d*x+c))^9/b^8/d-1/10*B*(a+b
*sin(d*x+c))^10/b^8/d
```

Rubi [A]
time = 0.28, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$\frac{3(-7a^2B+2aAb+b^2B)(a+b\sin(c+dx))^8}{8b^8d}+\frac{(a^2-b^2)^2(-7a^2B+6aAb+b^2B)(a+b\sin(c+dx))^4}{4b^8d}-\frac{(a^2-b^2)^3(Ab-aB)(a+b\sin(c+dx))^3}{3b^8d}-\frac{(-35a^3B+15a^2Ab+15ab^2B-3Ab^3)(a+b\sin(c+dx))^7}{7b^8d}-\frac{3(a^2-b^2)(-7a^3B+5a^2Ab+3ab^2B-Ab^3)(a+b\sin(c+dx))^5}{5b^8d}+\frac{(-35a^4B+20a^3Ab+30a^2b^2B-12aAb^3-3b^4B)(a+b\sin(c+dx))^6}{6b^8d}-\frac{(Ab-7aB)(a+b\sin(c+dx))^9}{9b^8d}-\frac{B(a+b\sin(c+dx))^{10}}{10b^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/3*((a^2 - b^2)^3*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(b^8*d) + ((a^2 - b
^2)^2*(6*a*A*b - 7*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^8*d) - (3*(a
^2 - b^2)*(5*a^2*A*b - A*b^3 - 7*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)
/(5*b^8*d) + ((20*a^3*A*b - 12*a*A*b^3 - 35*a^4*B + 30*a^2*b^2*B - 3*b^4*B)
*(a + b*Sin[c + d*x])^6)/(6*b^8*d) - ((15*a^2*A*b - 3*A*b^3 - 35*a^3*B + 15
*a*b^2*B)*(a + b*Sin[c + d*x])^7)/(7*b^8*d) + (3*(2*a*A*b - 7*a^2*B + b^2*B
)*(a + b*Sin[c + d*x])^8)/(8*b^8*d) - ((A*b - 7*a*B)*(a + b*Sin[c + d*x])^9
)/(9*b^8*d) - (B*(a + b*Sin[c + d*x])^10)/(10*b^8*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3\,dx, x, b\sin\right.}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\left(-a^2+b^2\right)^3(Ab-aB)(a+x)^2}{b}+\frac{\left(-a^2+b^2\right)^2(6aAb\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)(a+b\sin(c+dx))^3}{3b^8 d}+\frac{(a}{}$$

Mathematica [A]

time = 1.00, size = 295, normalized size = 0.85

$$\frac{-3a^4\left(a^6-9a^4b^2+42a^2b^4-210b^6\right)B+2520a^2Ab^8\sin(c+dx)+1260ab^8(2Ab+aB)\sin^2(c+dx)+840b^8\left(-3a^2A+Ab^2+2abB\right)\sin^3(c+dx)+630b^8\left(-6aAb-3a^2B+b^2B\right)\sin^4(c+dx)-1512b^8\left(-a^2A+Ab^2+2abB\right)\sin^5(c+dx)+1260b^8\left(2aAb+a^2B-b^2B\right)\sin^6(c+dx)+360b^8\left(-a^2A+3Ab^2+6abB\right)\sin^7(c+dx)-315b^8\left(2aAb+a^2B-3b^2B\right)\sin^8(c+dx)-280b^9(Ab+2aB)\sin^9(c+dx)-252b^{10}B\sin^{10}(c+dx)}{2520b^8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (-3*a^4*(a^6 - 9*a^4*b^2 + 42*a^2*b^4 - 210*b^6)*B + 2520*a^2*A*b^8*Sin[c +
 d*x] + 1260*a*b^8*(2*A*b + a*B)*Sin[c + d*x]^2 + 840*b^8*(-3*a^2*A + A*b^2
 + 2*a*b*B)*Sin[c + d*x]^3 + 630*b^8*(-6*a*A*b - 3*a^2*B + b^2*B)*Sin[c + d
*x]^4 - 1512*b^8*(-(a^2*A) + A*b^2 + 2*a*b*B)*Sin[c + d*x]^5 + 1260*b^8*(2*
a*A*b + a^2*B - b^2*B)*Sin[c + d*x]^6 + 360*b^8*(-(a^2*A) + 3*A*b^2 + 6*a*b
*B)*Sin[c + d*x]^7 - 315*b^8*(2*a*A*b + a^2*B - 3*b^2*B)*Sin[c + d*x]^8 - 2
80*b^9*(A*b + 2*a*B)*Sin[c + d*x]^9 - 252*b^10*B*Sin[c + d*x]^10)/(2520*b^8
*d)
```

Maple [A]

time = 0.88, size = 229, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{a^2A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{Ba^2\left(\cos^8(dx+c)\right)}{8}-\frac{Aab\left(\cos^8(dx+c)\right)}{4}+2Bab\left(-\frac{\sin(dx+c)}{}\right.}{}$
default	$\frac{\frac{a^2A\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)}{7}-\frac{Ba^2\left(\cos^8(dx+c)\right)}{8}-\frac{Aab\left(\cos^8(dx+c)\right)}{4}+2Bab\left(-\frac{\sin(dx+c)}{}\right.}{}$

risch	$\frac{B\,b^2\cos(10dx+10c)}{5120d}-\frac{7\cos(2dx+2c)B\,a^2}{128d}-\frac{7\cos(2dx+2c)B\,b^2}{512d}-\frac{\cos(6dx+6c)B\,a^2}{128d}+\frac{\cos(6dx+6c)B\,b^2}{1024d}+\frac{7\,\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/7*a^2*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*
x+c)-1/8*B*a^2*cos(d*x+c)^8-1/4*A*a*b*cos(d*x+c)^8+2*B*a*b*(-1/9*sin(d*x+c)
*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*si
n(d*x+c))+A*b^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*c
os(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/10*sin(d*x+c)^2*cos(d*x
+c)^8-1/40*cos(d*x+c)^8))
```

Maxima [A]
time = 0.29, size = 238, normalized size = 0.68

$$-\frac{252\,Bb^2\sin\left(dx+c\right)^{10}+280\left(2\,Bab+Ab^2\right)\sin\left(dx+c\right)^9+315\left(Ba^2+2\,Aab-3\,Bb^2\right)\sin\left(dx+c\right)^8+360\left(Aa^2-6\,Bab-3\,Ab^2\right)\sin\left(dx+c\right)^7-1260\left(Ba^2+2\,Aab-Bb^2\right)\sin\left(dx+c\right)^6-1512\left(Aa^2-2\,Bab-Ab^2\right)\sin\left(dx+c\right)^5+630\left(3\,Ba^2+6\,Aab-Bb^2\right)\sin\left(dx+c\right)^4-2520\,Aa^2\sin\left(dx+c\right)+840\left(3\,Aa^2-2\,Bab-Ab^2\right)\sin\left(dx+c\right)^3-1260\left(Ba^2+2\,Aab\right)\sin\left(dx+c\right)^2}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/2520*(252*B*b^2*sin(d*x + c)^10 + 280*(2*B*a*b + A*b^2)*sin(d*x + c)^9 +
 315*(B*a^2 + 2*A*a*b - 3*B*b^2)*sin(d*x + c)^8 + 360*(A*a^2 - 6*B*a*b - 3*
A*b^2)*sin(d*x + c)^7 - 1260*(B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^6 - 151
2*(A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^5 + 630*(3*B*a^2 + 6*A*a*b - B*b^2
)*sin(d*x + c)^4 - 2520*A*a^2*sin(d*x + c) + 840*(3*A*a^2 - 2*B*a*b - A*b^2
)*sin(d*x + c)^3 - 1260*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.42, size = 174, normalized size = 0.50

$$\frac{252\,Bb^2\cos\left(dx+c\right)^{10}-315\left(Ba^2+2\,Aab+Bb^2\right)\cos\left(dx+c\right)^8-8\left(35\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^8-5\left(9\,Aa^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^6-6\left(9\,Aa^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^4-144\,Aa^2-32\,Bab-16\,Ab^2-8\left(9\,Aa^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2520*(252*B*b^2*cos(d*x + c)^10 - 315*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x +
 c)^8 - 8*(35*(2*B*a*b + A*b^2)*cos(d*x + c)^8 - 5*(9*A*a^2 + 2*B*a*b + A*b
^2)*cos(d*x + c)^6 - 6*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^4 - 144*A*a
^2 - 32*B*a*b - 16*A*b^2 - 8*(9*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*si
n(d*x + c))/d
```

Sympy [A]
time = 1.83, size = 389, normalized size = 1.11

$$\begin{cases} \frac{16Aa^2\sin^7(c+dx)}{35d} + \frac{8Aa^2\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2Aa^2\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{Aa^2\sin(c+dx)\cos^6(c+dx)}{d} - \frac{Aab\cos^8(c+dx)}{4d} + \frac{16Ab^2\sin^9(c+dx)}{315d} + \frac{8Ab^2\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{2Ab^2\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{Ab^2\sin^3(c+dx)\cos^6(c+dx)}{3d} - \frac{Ba^2\cos^8(c+dx)}{8d} + \frac{32Bab\sin^9(c+dx)}{315d} + \frac{16Bab\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{4Bab\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{2Bab\sin^3(c+dx)\cos^6(c+dx)}{3d} - \frac{Bb^2\sin^2(c+dx)\cos^8(c+dx)}{8d} - \frac{Bb^2\cos^{10}(c+dx)}{40d} & \text{for } d \neq 0 \\ x(A+B\sin(c))(a+b\sin(c))^2\cos^7(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((16*A*a**2*sin(c + d*x)**7/(35*d) + 8*A*a**2*sin(c + d*x)**5*cos(
c + d*x)**2/(5*d) + 2*A*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + A*a**2*sin
(c + d*x)*cos(c + d*x)**6/d - A*a*b*cos(c + d*x)**8/(4*d) + 16*A*b**2*sin(c
 + d*x)**9/(315*d) + 8*A*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 2*A*
b**2*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + A*b**2*sin(c + d*x)**3*cos(c +
 d*x)**6/(3*d) - B*a**2*cos(c + d*x)**8/(8*d) + 32*B*a*b*sin(c + d*x)**9/(3
15*d) + 16*B*a*b*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 4*B*a*b*sin(c + d
*x)**5*cos(c + d*x)**4/(5*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**6/(3*d
) - B*b**2*sin(c + d*x)**2*cos(c + d*x)**8/(8*d) - B*b**2*cos(c + d*x)**10/
(40*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**7, True))
```

Giac [A]
time = 0.61, size = 279, normalized size = 0.80

$$\frac{Bb^2\cos(10dx+10c)}{5120d} + \frac{7Aa^2\sin(3dx+3c)}{64d} - \frac{(Ba^2+2Aab-Bb^2)\cos(8dx+8c)}{1024d} - \frac{(8Ba^2+16Aab-Bb^2)\cos(6dx+6c)}{1024d} - \frac{(7Ba^2+14Aab+Bb^2)\cos(4dx+4c)}{256d} - \frac{7(4Ba^2+8Aab+Bb^2)\cos(2dx+2c)}{512d} - \frac{(2Bab+Ab^2)\sin(9dx+9c)}{2304d} + \frac{(4Aa^2-10Bab-5Ab^2)\sin(7dx+7c)}{1792d} + \frac{(7Aa^2-4Bab-2Ab^2)\sin(5dx+5c)}{320d} + \frac{7(10Aa^2+2Bab+Ab^2)\sin(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/5120*B*b^2*cos(10*d*x + 10*c)/d + 7/64*A*a^2*sin(3*d*x + 3*c)/d - 1/1024*
(B*a^2 + 2*A*a*b - B*b^2)*cos(8*d*x + 8*c)/d - 1/1024*(8*B*a^2 + 16*A*a*b -
 B*b^2)*cos(6*d*x + 6*c)/d - 1/256*(7*B*a^2 + 14*A*a*b + B*b^2)*cos(4*d*x +
 4*c)/d - 7/512*(4*B*a^2 + 8*A*a*b + B*b^2)*cos(2*d*x + 2*c)/d - 1/2304*(2*
B*a*b + A*b^2)*sin(9*d*x + 9*c)/d + 1/1792*(4*A*a^2 - 10*B*a*b - 5*A*b^2)*s
in(7*d*x + 7*c)/d + 1/320*(7*A*a^2 - 4*B*a*b - 2*A*b^2)*sin(5*d*x + 5*c)/d
+ 7/128*(10*A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.18, size = 236, normalized size = 0.68

$$\frac{\sin(c+dx)^2\left(\frac{Ba^2}{2}+Aba\right) - \sin(c+dx)^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right) + \sin(c+dx)^3\left(-Aa^2+\frac{2Bab}{3}+\frac{Ab^2}{3}\right) - \sin(c+dx)^5\left(-\frac{3Aa^2}{5}+\frac{6Bab}{5}+\frac{3Ab^2}{5}\right) + \sin(c+dx)^7\left(-\frac{Aa^2}{7}+\frac{6Bab}{7}+\frac{3Ab^2}{7}\right) + \sin(c+dx)^6\left(\frac{Ba^2}{2}+Aab-\frac{Bb^2}{2}\right) - \sin(c+dx)^4\left(\frac{3Ba^2}{4}+\frac{3Aab}{2}-\frac{Bb^2}{4}\right) - \sin(c+dx)^8\left(\frac{Ba^2}{8}+\frac{Aab}{4}-\frac{3Bb^2}{8}\right) - \frac{Bb^2\sin(c+dx)^{10}}{10} + Aa^2\sin(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) - sin(c + d*x)^9*((A*b^2)/9 + (2*B*a*b)
/9) + sin(c + d*x)^3*((A*b^2)/3 - A*a^2 + (2*B*a*b)/3) - sin(c + d*x)^5*((3
```

```
*A*b^2)/5 - (3*A*a^2)/5 + (6*B*a*b)/5) + sin(c + d*x)^7*((3*A*b^2)/7 - (A*a
^2)/7 + (6*B*a*b)/7) + sin(c + d*x)^6*((B*a^2)/2 - (B*b^2)/2 + A*a*b) - sin
(c + d*x)^4*((3*B*a^2)/4 - (B*b^2)/4 + (3*A*a*b)/2) - sin(c + d*x)^8*((B*a^
2)/8 - (3*B*b^2)/8 + (A*a*b)/4) - (B*b^2*sin(c + d*x)^10)/10 + A*a^2*sin(c
+ d*x))/d
```

3.1537 $\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=231

$$\frac{(a^2-b^2)^2(Ab-aB)(a+b\sin(c+dx))^3}{3b^6d}-\frac{(a^2-b^2)(4aAb-5a^2B+b^2B)(a+b\sin(c+dx))^4}{4b^6d}+\frac{2(3a^2Ab-A$$

```
[Out] 1/3*(a^2-b^2)^2*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^6/d-1/4*(a^2-b^2)*(4*A*a*b-5
*B*a^2+B*b^2)*(a+b*sin(d*x+c))^4/b^6/d+2/5*(3*A*a^2*b-A*b^3-5*B*a^3+3*B*a*b
^2)*(a+b*sin(d*x+c))^5/b^6/d-1/3*(2*A*a*b-5*B*a^2+B*b^2)*(a+b*sin(d*x+c))^6
/b^6/d+1/7*(A*b-5*B*a)*(a+b*sin(d*x+c))^7/b^6/d+1/8*B*(a+b*sin(d*x+c))^8/b^
6/d
```

Rubi [A]
time = 0.19, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$-\frac{(-5a^2B+2aAb+b^2B)(a+b\sin(c+dx))^6}{3b^6d}-\frac{(a^2-b^2)(-5a^2B+4aAb+b^2B)(a+b\sin(c+dx))^4}{4b^6d}+\frac{(a^2-b^2)^2(Ab-aB)(a+b\sin(c+dx))^3}{3b^6d}+\frac{2(-5a^3B+3a^2Ab+3ab^2B-Ab^3)(a+b\sin(c+dx))^5}{5b^6d}+\frac{(Ab-5aB)(a+b\sin(c+dx))^7}{7b^6d}+\frac{B(a+b\sin(c+dx))^8}{8b^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^6*d) - ((a^2 - b^2)
*(4*a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^6*d) + (2*(3*a^2*
A*b - A*b^3 - 5*a^3*B + 3*a*b^2*B)*(a + b*Sin[c + d*x])^5)/(5*b^6*d) - ((2*
a*A*b - 5*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^6)/(3*b^6*d) + ((A*b - 5*a*B)
*(a + b*Sin[c + d*x])^7)/(7*b^6*d) + (B*(a + b*Sin[c + d*x])^8)/(8*b^6*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2\,dx, x, b\,\text{s}\right.}{b^5d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{(-a^2+b^2)^2(Ab-aB)(a+x)^2}{b}+\frac{(-a^2+b^2)(4aA\right.\right.}{}$$

$$= \frac{(a^2-b^2)^2(Ab-aB)(a+b\sin(c+dx))^3}{3b^6d} - \frac{(a}{}$$

Mathematica [A]
time = 0.34, size = 227, normalized size = 0.98

$$\frac{a^4(3a^4-28a^2b^2+210b^4)B+840a^2Ab^6\sin(c+dx)+420ab^6(2Ab+aB)\sin^2(c+dx)+280b^6(-2a^2A+Ab^2+2abB)\sin^3(c+dx)+210b^6(-4aAb-2a^2B+b^2B)\sin^4(c+dx)+168b^6(a^2A-2Ab^2-4abB)\sin^5(c+dx)+140b^6(2aAb+a^2B-2b^2B)\sin^6(c+dx)+120b^7(Ab+2aB)\sin^7(c+dx)+105b^8B\sin^8(c+dx)}{840b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (a^4*(3*a^4 - 28*a^2*b^2 + 210*b^4)*B + 840*a^2*A*b^6*Sin[c + d*x] + 420*a*
b^6*(2*A*b + a*B)*Sin[c + d*x]^2 + 280*b^6*(-2*a^2*A + A*b^2 + 2*a*b*B)*Sin
[c + d*x]^3 + 210*b^6*(-4*a*A*b - 2*a^2*B + b^2*B)*Sin[c + d*x]^4 + 168*b^6
*(a^2*A - 2*A*b^2 - 4*a*b*B)*Sin[c + d*x]^5 + 140*b^6*(2*a*A*b + a^2*B - 2*
b^2*B)*Sin[c + d*x]^6 + 120*b^7*(A*b + 2*a*B)*Sin[c + d*x]^7 + 105*b^8*B*Si
n[c + d*x]^8)/(840*b^6*d)
```

Maple [A]
time = 0.67, size = 199, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{a^2A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{Ba^2\left(\cos^6(dx+c)\right)}{6}-\frac{Aab\left(\cos^6(dx+c)\right)}{3}+2Bab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\right.}{}$
default	$\frac{\frac{a^2A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{Ba^2\left(\cos^6(dx+c)\right)}{6}-\frac{Aab\left(\cos^6(dx+c)\right)}{3}+2Bab\left(-\frac{\sin(dx+c)\left(\cos^6(dx+c)\right)}{7}+\right.}{}$
risch	$\frac{5\sin(dx+c)a^2A}{8d}+\frac{5\sin(dx+c)Ab^2}{64d}+\frac{5\sin(dx+c)Bab}{32d}+\frac{\cos(8dx+8c)Bb^2}{1024d}-\frac{\sin(7dx+7c)Ab^2}{448d}-\frac{\sin(7dx+7c)B}{224d}$
norman	$\frac{2\left(13a^2A+4Ab^2+8Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(13a^2A+4Ab^2+8Bab\right)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(163a^2A+4Ab^2+8Bab\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/6*B*a^2*cos
(d*x+c)^6-1/3*A*a*b*cos(d*x+c)^6+2*B*a*b*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+A*b^2*(-1/7*sin(d*x+c)*cos
(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+B*b^2*(-1/8*
sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6))
```

Maxima [A]
time = 0.28, size = 184, normalized size = 0.80

$$\frac{105\,Bb^2\sin\left(dx+c\right)^8+120\,(2\,Bab+Ab^2)\sin\left(dx+c\right)^7+140\,(Ba^2+2\,Aab-2\,Bb^2)\sin\left(dx+c\right)^6+168\,(Aa^2-4\,Bab-2\,Ab^2)\sin\left(dx+c\right)^5-210\,(2\,Ba^2+4\,Aab-Bb^2)\sin\left(dx+c\right)^4+840\,Aa^2\sin\left(dx+c\right)-280\,(2\,Aa^2-2\,Bab-Ab^2)\sin\left(dx+c\right)^3+420\,(Ba^2+2\,Aab)\sin\left(dx+c\right)^2}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/840*(105*B*b^2*sin(d*x + c)^8 + 120*(2*B*a*b + A*b^2)*sin(d*x + c)^7 + 14
0*(B*a^2 + 2*A*a*b - 2*B*b^2)*sin(d*x + c)^6 + 168*(A*a^2 - 4*B*a*b - 2*A*b
^2)*sin(d*x + c)^5 - 210*(2*B*a^2 + 4*A*a*b - B*b^2)*sin(d*x + c)^4 + 840*A
*a^2*sin(d*x + c) - 280*(2*A*a^2 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 + 420*(B
*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.40, size = 147, normalized size = 0.64

$$\frac{105\,Bb^2\cos\left(dx+c\right)^8-140\,(Ba^2+2\,Aab+Bb^2)\cos\left(dx+c\right)^6-8\,\left(15\,(2\,Bab+Ab^2)\cos\left(dx+c\right)^6-3\,(7\,Aa^2+2\,Bab+Ab^2)\cos\left(dx+c\right)^4-56\,Aa^2-16\,Bab-8\,Ab^2-4\,(7\,Aa^2+2\,Bab+Ab^2)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/840*(105*B*b^2*cos(d*x + c)^8 - 140*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c
)^6 - 8*(15*(2*B*a*b + A*b^2)*cos(d*x + c)^6 - 3*(7*A*a^2 + 2*B*a*b + A*b^2
)*cos(d*x + c)^4 - 56*A*a^2 - 16*B*a*b - 8*A*b^2 - 4*(7*A*a^2 + 2*B*a*b + A
*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.94, size = 309, normalized size = 1.34

$$\begin{cases} \frac{8Aa^2\sin^5(c+dx)}{15d}+\frac{4Aa^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{Aab\cos^6(c+dx)}{3d}+\frac{8Ab^2\sin^7(c+dx)}{105d}+\frac{4Ab^2\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{Ab^2\sin^3(c+dx)\cos^4(c+dx)}{3d}-\frac{Ba^2\cos^6(c+dx)}{6d}+\frac{16Bab\sin^7(c+dx)}{105d}+\frac{8Bab\sin^5(c+dx)\cos^2(c+dx)}{15d}+\frac{2Bab\sin^3(c+dx)\cos^4(c+dx)}{3d}-\frac{Bb^2\sin^2(c+dx)\cos^6(c+dx)}{6d}-\frac{Bb^2\cos^8(c+dx)}{24d} & \text{for } d\neq 0 \\ x(A+B\sin(c))\,(a+b\sin(c))^2\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*A*a**2*sin(c + d*x)**5/(15*d) + 4*A*a**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x)**4/d - A*a*b*cos(c + d*
x)**6/(3*d) + 8*A*b**2*sin(c + d*x)**7/(105*d) + 4*A*b**2*sin(c + d*x)**5*c
os(c + d*x)**2/(15*d) + A*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - B*a*
*2*cos(c + d*x)**6/(6*d) + 16*B*a*b*sin(c + d*x)**7/(105*d) + 8*B*a*b*sin(c
 + d*x)**5*cos(c + d*x)**2/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**4
/(3*d) - B*b**2*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - B*b**2*cos(c + d*x)
**8/(24*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**5, True)
)
```

Giac [A]
time = 0.62, size = 231, normalized size = 1.00

$$\frac{Bb^2 \cos(8\,dx + 8\,c)}{1024\,d} - \frac{(2\,Ba^2 + 4\,Aab - Bb^2)\cos(6\,dx + 6\,c)}{384\,d} - \frac{(8\,Ba^2 + 16\,Aab + Bb^2)\cos(4\,dx + 4\,c)}{256\,d} - \frac{(10\,Ba^2 + 20\,Aab + 3\,Bb^2)\cos(2\,dx + 2\,c)}{128\,d} - \frac{(2\,Bab + Ab^2)\sin(7\,dx + 7\,c)}{448\,d} + \frac{(4\,Aa^2 - 6\,Bab - 3\,Ab^2)\sin(5\,dx + 5\,c)}{320\,d} + \frac{(20\,Aa^2 - 2\,Bab - Ab^2)\sin(3\,dx + 3\,c)}{192\,d} + \frac{5\,(8\,Aa^2 + 2\,Bab + Ab^2)\sin(dx + c)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1024*B*b^2*cos(8*d*x + 8*c)/d - 1/384*(2*B*a^2 + 4*A*a*b - B*b^2)*cos(6*d
*x + 6*c)/d - 1/256*(8*B*a^2 + 16*A*a*b + B*b^2)*cos(4*d*x + 4*c)/d - 1/128
*(10*B*a^2 + 20*A*a*b + 3*B*b^2)*cos(2*d*x + 2*c)/d - 1/448*(2*B*a*b + A*b^
2)*sin(7*d*x + 7*c)/d + 1/320*(4*A*a^2 - 6*B*a*b - 3*A*b^2)*sin(5*d*x + 5*c
)/d + 1/192*(20*A*a^2 - 2*B*a*b - A*b^2)*sin(3*d*x + 3*c)/d + 5/64*(8*A*a^2
 + 2*B*a*b + A*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.11, size = 180, normalized size = 0.78

$$\frac{\sin(c + d\,x)^2 \left(\frac{B\,a^2}{2} + A\,b\,a\right) + \sin(c + d\,x)^7 \left(\frac{A\,b^2}{7} + \frac{2\,B\,a\,b}{7}\right) + \sin(c + d\,x)^3 \left(-\frac{2\,A\,a^2}{3} + \frac{2\,B\,a\,b}{3} + \frac{A\,b^2}{3}\right) - \sin(c + d\,x)^5 \left(-\frac{A\,a^2}{5} + \frac{4\,B\,a\,b}{5} + \frac{2\,A\,b^2}{5}\right) - \sin(c + d\,x)^4 \left(\frac{B\,a^2}{2} + A\,a\,b - \frac{B\,b^2}{4}\right) + \sin(c + d\,x)^6 \left(\frac{B\,a^2}{6} + \frac{A\,a\,b}{3} - \frac{B\,b^2}{3}\right) + \frac{B\,b^2 \sin(c+d\,x)^8}{8} + A\,a^2 \sin(c + d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) + sin(c + d*x)^7*((A*b^2)/7 + (2*B*a*b)
/7) + sin(c + d*x)^3*((A*b^2)/3 - (2*A*a^2)/3 + (2*B*a*b)/3) - sin(c + d*x)
^5*((2*A*b^2)/5 - (A*a^2)/5 + (4*B*a*b)/5) - sin(c + d*x)^4*((B*a^2)/2 - (B
*b^2)/4 + A*a*b) + sin(c + d*x)^6*((B*a^2)/6 - (B*b^2)/3 + (A*a*b)/3) + (B*
b^2*sin(c + d*x)^8)/8 + A*a^2*sin(c + d*x))/d
```

3.1538 $\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=132

$$-\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}+\frac{(2aAb-3a^2B+b^2B)(a+b\sin(c+dx))^4}{4b^4d}-\frac{(Ab-3aB)(a+b\sin$$

```
[Out] -1/3*(a^2-b^2)*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^4/d+1/4*(2*A*a*b-3*B*a^2+B*b^
2)*(a+b*sin(d*x+c))^4/b^4/d-1/5*(A*b-3*B*a)*(a+b*sin(d*x+c))^5/b^4/d-1/6*B*
(a+b*sin(d*x+c))^6/b^4/d
```

Rubi [A]
time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$\frac{(-3a^2B+2aAb+b^2B)(a+b\sin(c+dx))^4}{4b^4d}-\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d}-\frac{(Ab-3aB)(a+b\sin(c+dx))^5}{5b^4d}-\frac{B(a+b\sin(c+dx))^6}{6b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/3*((a^2 - b^2)*(A*b - a*B)*(a + b*Sin[c + d*x])^3)/(b^4*d) + ((2*a*A*b -
 3*a^2*B + b^2*B)*(a + b*Sin[c + d*x])^4)/(4*b^4*d) - ((A*b - 3*a*B)*(a + b
*Sin[c + d*x])^5)/(5*b^4*d) - (B*(a + b*Sin[c + d*x])^6)/(6*b^4*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int(a+x)^2\left(A+\frac{Bx}{b}\right)(b^2-x^2)\,dx,x,b\,\text{si}\right.}{b^3d}$$

$$= \frac{\text{Subst}\left(\int\left(\frac{(-a^2+b^2)(Ab-aB)(a+x)^2}{b}+\frac{(2aAb-3a^2B+b}{b}\right.\right.}{}$$

$$= -\frac{(a^2-b^2)(Ab-aB)(a+b\sin(c+dx))^3}{3b^4d} + \frac{(2}{}$$

Mathematica [A]
time = 0.17, size = 111, normalized size = 0.84

$$\frac{(a+b\sin(c+dx))^3(-2a^2Ab+20Ab^3+a^3B-5ab^2B+3b(2aAb-a^2B+5b^2B)\sin(c+dx)-6b^2(2Ab-aB)\sin^2(c+dx)-10b^3B\sin^3(c+dx))}{60b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(-2*a^2*A*b + 20*A*b^3 + a^3*B - 5*a*b^2*B + 3*b*(2
*a*A*b - a^2*B + 5*b^2*B)*Sin[c + d*x] - 6*b^2*(2*A*b - a*B)*Sin[c + d*x]^2
 - 10*b^3*B*Sin[c + d*x]^3))/(60*b^4*d)
```

Maple [A]
time = 0.44, size = 169, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{Ba^2\left(\cos^4(dx+c)\right)}{4}-\frac{Aab\left(\cos^4(dx+c)\right)}{2}+2Bab\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\text{si}}{15}\right.}{}$
default	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{Ba^2\left(\cos^4(dx+c)\right)}{4}-\frac{Aab\left(\cos^4(dx+c)\right)}{2}+2Bab\left(-\frac{\left(\cos^4(dx+c)\right)\sin(dx+c)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\text{si}}{15}\right.}{}$
risch	$\frac{3\sin(dx+c)a^2A}{4d}+\frac{\sin(dx+c)Ab^2}{8d}+\frac{\sin(dx+c)Bab}{4d}+\frac{\cos(6dx+6c)Bb^2}{192d}-\frac{\sin(5dx+5c)Ab^2}{80d}-\frac{\sin(5dx+5c)Bab}{40d}$
norman	$\frac{\frac{(8Aab+4Ba^2+4Bb^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{(8Aab+4Ba^2+4Bb^2)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2(11a^2A+4Ab^2+8Bab)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2(11}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)-1/4*B*a^2*cos(d*x+c)^4-1/2*A*a*b
*cos(d*x+c)^4+2*B*a*b*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*s
in(d*x+c))+A*b^2*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*
x+c))+B*b^2*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4))
```

Maxima [A]
time = 0.27, size = 128, normalized size = 0.97

$$-\frac{10\,Bb^2\sin\left(dx+c\right)^6+12\,(2\,Bab+Ab^2)\sin\left(dx+c\right)^5+15\,(Ba^2+2\,Aab-Bb^2)\sin\left(dx+c\right)^4-60\,Aa^2\sin\left(dx+c\right)+20\,(Aa^2-2\,Bab-Ab^2)\sin\left(dx+c\right)^3-30\,(Ba^2+2\,Aab)\sin\left(dx+c\right)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 12*(2*B*a*b + A*b^2)*sin(d*x + c)^5 + 15*(
B*a^2 + 2*A*a*b - B*b^2)*sin(d*x + c)^4 - 60*A*a^2*sin(d*x + c) + 20*(A*a^2
 - 2*B*a*b - A*b^2)*sin(d*x + c)^3 - 30*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 120, normalized size = 0.91

$$\frac{10\,Bb^2\cos\left(dx+c\right)^6-15\,(Ba^2+2\,Aab+Bb^2)\cos\left(dx+c\right)^4-4\,\left(3\,(2\,Bab+Ab^2)\cos\left(dx+c\right)^4-10\,Aa^2-4\,Bab-2\,Ab^2-(5\,Aa^2+2\,Bab+Ab^2)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/60*(10*B*b^2*cos(d*x + c)^6 - 15*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^4
 - 4*(3*(2*B*a*b + A*b^2)*cos(d*x + c)^4 - 10*A*a^2 - 4*B*a*b - 2*A*b^2 - (
5*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.44, size = 228, normalized size = 1.73

$$\begin{cases} \frac{2Aa^2\sin^3(c+dx)}{3d}+\frac{Aa^2\sin(c+dx)\cos^2(c+dx)}{d}-\frac{Aab\cos^4(c+dx)}{2d}+\frac{2Ab^2\sin^5(c+dx)}{15d}+\frac{Ab^2\sin^3(c+dx)\cos^2(c+dx)}{3d}-\frac{Ba^2\cos^4(c+dx)}{4d}+\frac{4Bab\sin^5(c+dx)}{15d}+\frac{2Bab\sin^3(c+dx)\cos^2(c+dx)}{3d}-\frac{Bb^2\sin^2(c+dx)\cos^4(c+dx)}{4d}-\frac{Bb^2\cos^6(c+dx)}{12d} & \text{for}\: d\neq 0 \\ x(A+B\sin(c))\,(a+b\sin(c))^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*A*a**2*sin(c + d*x)**3/(3*d) + A*a**2*sin(c + d*x)*cos(c + d*x
)**2/d - A*a*b*cos(c + d*x)**4/(2*d) + 2*A*b**2*sin(c + d*x)**5/(15*d) + A*
b**2*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) - B*a**2*cos(c + d*x)**4/(4*d) +
 4*B*a*b*sin(c + d*x)**5/(15*d) + 2*B*a*b*sin(c + d*x)**3*cos(c + d*x)**2/(
3*d) - B*b**2*sin(c + d*x)**2*cos(c + d*x)**4/(4*d) - B*b**2*cos(c + d*x)**
6/(12*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(c))**2*cos(c)**3, True))
```

Giac [A]
time = 0.49, size = 168, normalized size = 1.27

$$-\frac{10\,Bb^2\sin\left(dx+c\right)^6+24\,Bab\sin\left(dx+c\right)^5+12\,Ab^2\sin\left(dx+c\right)^5+15\,Ba^2\sin\left(dx+c\right)^4+30\,Aab\sin\left(dx+c\right)^4-15\,Bb^2\sin\left(dx+c\right)^4+20\,Aa^2\sin\left(dx+c\right)^3-40\,Bab\sin\left(dx+c\right)^3-20\,Ab^2\sin\left(dx+c\right)^3-30\,Ba^2\sin\left(dx+c\right)^2-60\,Aab\sin\left(dx+c\right)^2-60\,Aa^2\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/60*(10*B*b^2*sin(d*x + c)^6 + 24*B*a*b*sin(d*x + c)^5 + 12*A*b^2*sin(d*x
 + c)^5 + 15*B*a^2*sin(d*x + c)^4 + 30*A*a*b*sin(d*x + c)^4 - 15*B*b^2*sin(
d*x + c)^4 + 20*A*a^2*sin(d*x + c)^3 - 40*B*a*b*sin(d*x + c)^3 - 20*A*b^2*s
in(d*x + c)^3 - 30*B*a^2*sin(d*x + c)^2 - 60*A*a*b*sin(d*x + c)^2 - 60*A*a^
2*sin(d*x + c))/d
```

Mupad [B]
time = 12.30, size = 127, normalized size = 0.96

$$\frac{\sin(c+dx)^2\left(\frac{Ba^2}{2}+Aba\right)-\sin(c+dx)^5\left(\frac{Ab^2}{5}+\frac{2Bab}{5}\right)+\sin(c+dx)^3\left(-\frac{Aa^2}{3}+\frac{2Bab}{3}+\frac{Ab^2}{3}\right)-\sin(c+dx)^4\left(\frac{Ba^2}{4}+\frac{Aab}{2}-\frac{Bb^2}{4}\right)-\frac{Bb^2\sin(c+dx)^6}{6}+Aa^2\sin(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) - sin(c + d*x)^5*((A*b^2)/5 + (2*B*a*b)
/5) + sin(c + d*x)^3*((A*b^2)/3 - (A*a^2)/3 + (2*B*a*b)/3) - sin(c + d*x)^4
*((B*a^2)/4 - (B*b^2)/4 + (A*a*b)/2) - (B*b^2*sin(c + d*x)^6)/6 + A*a^2*sin
(c + d*x))/d
```

3.1539 $\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

```
[Out] 1/3*(A*b-B*a)*(a+b*sin(d*x+c))^3/b^2/d+1/4*B*(a+b*sin(d*x+c))^4/b^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+dx))^4}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*(a + b*Sin[c + d*x])^3)/(3*b^2*d) + (B*(a + b*Sin[c + d*x])^4)
/(4*b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx &= \frac{\text{Subst}\left(\int (a+x)^2\left(A+\frac{Bx}{b}\right)\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+x)^2}{b}+\frac{B(a+x)^3}{b}\right)\,dx,x,b\sin(c\right.}{bd}\\ &= \frac{(Ab-aB)(a+b\sin(c+dx))^3}{3b^2d}+\frac{B(a+b\sin(c+}{4b^2d}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 41, normalized size = 0.76

$$\frac{(a+b\sin(c+dx))^3(4Ab-aB+3bB\sin(c+dx))}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a + b*Sin[c + d*x])^3*(4*A*b - a*B + 3*b*B*Sin[c + d*x]))/(12*b^2*d)
```

Maple [A]
time = 0.21, size = 73, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{B\,b^{2}\left(\sin^{4}(dx+c)\right)}{4}+\frac{\left(A\,b^{2}+2Bab\right)\left(\sin^{3}(dx+c)\right)}{3}+\frac{\left(2Aab+B\,a^{2}\right)\left(\sin^{2}(dx+c)\right)}{2}+\sin(dx+c)a^{2}A}{d}$
default	$\frac{\frac{B\,b^{2}\left(\sin^{4}(dx+c)\right)}{4}+\frac{\left(A\,b^{2}+2Bab\right)\left(\sin^{3}(dx+c)\right)}{3}+\frac{\left(2Aab+B\,a^{2}\right)\left(\sin^{2}(dx+c)\right)}{2}+\sin(dx+c)a^{2}A}{d}$
risch	$\frac{\sin(dx+c)a^{2}A}{d}+\frac{\sin(dx+c)A\,b^{2}}{4d}+\frac{\sin(dx+c)Bab}{2d}+\frac{\cos(4dx+4c)B\,b^{2}}{32d}-\frac{\sin(3dx+3c)A\,b^{2}}{12d}-\frac{\sin(3dx+3c)Bab}{6d}$
norman	$\frac{\frac{2\left(9a^{2}A+4A\,b^{2}+8Bab\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(9a^{2}A+4A\,b^{2}+8Bab\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(2Aab+B\,a^{2}\right)\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+2\left(2Aab+B\,a\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*B*b^2*sin(d*x+c)^4+1/3*(A*b^2+2*B*a*b)*sin(d*x+c)^3+1/2*(2*A*a*b+B
*a^2)*sin(d*x+c)^2+sin(d*x+c)*a^2*A)
```

Maxima [A]
time = 0.27, size = 74, normalized size = 1.37

$$\frac{3\,Bb^2\sin\left(dx+c\right)^4+12\,Aa^2\sin\left(dx+c\right)+4\left(2\,Bab+Ab^2\right)\sin\left(dx+c\right)^3+6\left(Ba^2+2\,Aab\right)\sin\left(dx+c\right)^2}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 12*A*a^2*sin(d*x + c) + 4*(2*B*a*b + A*b^2)*
sin(d*x + c)^3 + 6*(B*a^2 + 2*A*a*b)*sin(d*x + c)^2)/d
```

Fricas [A]
time = 0.36, size = 92, normalized size = 1.70

$$\frac{3\,Bb^2\cos\left(dx+c\right)^4-6\left(Ba^2+2\,Aab+Bb^2\right)\cos\left(dx+c\right)^2+4\left(3\,Aa^2+2\,Bab+Ab^2-\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*B*b^2*cos(d*x + c)^4 - 6*(B*a^2 + 2*A*a*b + B*b^2)*cos(d*x + c)^2 +
 4*(3*A*a^2 + 2*B*a*b + A*b^2 - (2*B*a*b + A*b^2)*cos(d*x + c)^2)*sin(d*x +
 c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.
time = 0.19, size = 117, normalized size = 2.17

$$\begin{cases} \frac{Aa^2 \sin(c+dx)}{d} + \frac{Aab \sin^2(c+dx)}{d} + \frac{Ab^2 \sin^3(c+dx)}{3d} + \frac{Ba^2 \sin^2(c+dx)}{2d} + \frac{2Bab \sin^3(c+dx)}{3d} + \frac{Bb^2 \sin^4(c+dx)}{4d} & \text{for } d \neq 0 \\ x(A + B\sin(c))\,(a + b\sin(c))^2 \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Piecewise((A*a**2*sin(c + d*x)/d + A*a*b*sin(c + d*x)**2/d + A*b**2*sin(c +
 d*x)**3/(3*d) + B*a**2*sin(c + d*x)**2/(2*d) + 2*B*a*b*sin(c + d*x)**3/(3*
d) + B*b**2*sin(c + d*x)**4/(4*d), Ne(d, 0)), (x*(A + B*sin(c))*(a + b*sin(
c))**2*cos(c), True))
```

Giac [A]
time = 0.46, size = 86, normalized size = 1.59

$$\frac{3\,Bb^2 \sin(dx+c)^4 + 8\,Bab \sin(dx+c)^3 + 4\,Ab^2 \sin(dx+c)^3 + 6\,Ba^2 \sin(dx+c)^2 + 12\,Aab \sin(dx+c)^2 + 12\,Aa^2 \sin(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/12*(3*B*b^2*sin(d*x + c)^4 + 8*B*a*b*sin(d*x + c)^3 + 4*A*b^2*sin(d*x + c
)^3 + 6*B*a^2*sin(d*x + c)^2 + 12*A*a*b*sin(d*x + c)^2 + 12*A*a^2*sin(d*x +
 c))/d
```

Mupad [B]
time = 0.07, size = 71, normalized size = 1.31

$$\frac{\sin(c+d\,x)^2 \left(\frac{B\,a^2}{2} + A\,b\,a\right) + \sin(c+d\,x)^3 \left(\frac{A\,b^2}{3} + \frac{2\,B\,a\,b}{3}\right) + \frac{B\,b^2 \sin(c+d\,x)^4}{4} + A\,a^2 \sin(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^2*((B*a^2)/2 + A*a*b) + sin(c + d*x)^3*((A*b^2)/3 + (2*B*a*b)
/3) + (B*b^2*sin(c + d*x)^4)/4 + A*a^2*sin(c + d*x))/d
```

3.1540 $\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=94

$$-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)^2(A-B)\log(1+\sin(c+dx))}{2d}-\frac{b(Ab+2aB)\sin(c+dx)}{d}-$$

```
[Out] -1/2*(a+b)^2*(A+B)*ln(1-sin(d*x+c))/d+1/2*(a-b)^2*(A-B)*ln(1+sin(d*x+c))/d-
b*(A*b+2*B*a)*sin(d*x+c)/d-1/2*b^2*B*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2916, 815, 647, 31}

$$-\frac{b(2aB+Ab)\sin(c+dx)}{d}+\frac{(a-b)^2(A-B)\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}-\frac{b^2B\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/2*((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]])/d + ((a - b)^2*(A - B)*Log[1
 + Sin[c + d*x]])/(2*d) - (b*(A*b + 2*a*B)*Sin[c + d*x])/d - (b^2*B*Sin[c +
 d*x]^2)/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(-A-\frac{2aB}{b}-\frac{Bx}{b}+\frac{b(a^2A+Ab^2+2abB)+(2a}{b(b^2-x^2}\right.\right.}{d}$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d}-\frac{b^2B\sin^2(c+dx)}{2d}+$$

$$= -\frac{b(Ab+2aB)\sin(c+dx)}{d}-\frac{b^2B\sin^2(c+dx)}{2d}-$$

$$= -\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))}{2d}+\frac{(a-b)}{}$$

Mathematica [A]
time = 0.15, size = 81, normalized size = 0.86

$$-\frac{(a+b)^2(A+B)\log(1-\sin(c+dx))-(a-b)^2(A-B)\log(1+\sin(c+dx))+2b(Ab+2aB)\sin(c+dx)+b^2B\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
[Out] -1/2*((a + b)^2*(A + B)*Log[1 - Sin[c + d*x]] - (a - b)^2*(A - B)*Log[1 + S
in[c + d*x]] + 2*b*(A*b + 2*a*B)*Sin[c + d*x] + b^2*B*Sin[c + d*x]^2)/d
```

Maple [A]
time = 0.28, size = 131, normalized size = 1.39

method	result
derivativedivides	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^2\ln(\cos(dx+c))-2Aab\ln(\cos(dx+c))+2Bab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)}{d}$
default	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))-B\,a^2\ln(\cos(dx+c))-2Aab\ln(\cos(dx+c))+2Bab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)}{d}$

norman	$\frac{-\frac{2Bb^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2Bb^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b(Ab+2aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4b(Ab+2aB)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b(Ab+2aB)\left(\tan^5\left(\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$
risch	$-\frac{ie^{-i(dx+c)}Bab}{d}+iBa^2x+2iAabx+\frac{4iAabc}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)A}{d}-\frac{a^2\ln\left(e^{i(dx+c)}-i\right)B}{d}+\frac{\cos(2dx+}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))-B*a^2*ln(cos(d*x+c))-2*A*a*b*ln(cos(d*
x+c))+2*B*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+A*b^2*(-sin(d*x+c)+ln
(sec(d*x+c)+tan(d*x+c)))+B*b^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 109, normalized size = 1.16

$$-\frac{Bb^2\sin(dx+c)^2-((A-B)a^2-2(A-B)ab+(A-B)b^2)\log(\sin(dx+c)+1)+((A+B)a^2+2(A+B)ab+(A+B)b^2)\log(\sin(dx+c)-1)+2(2Bab+Ab^2)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 - ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*lo
g(sin(d*x + c) + 1) + ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(sin(d
*x + c) - 1) + 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

Fricas [A]
time = 0.38, size = 111, normalized size = 1.18

$$\frac{Bb^2\cos(dx+c)^2+((A-B)a^2-2(A-B)ab+(A-B)b^2)\log(\sin(dx+c)+1)-((A+B)a^2+2(A+B)ab+(A+B)b^2)\log(-\sin(dx+c)+1)-2(2Bab+Ab^2)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(B*b^2*cos(d*x + c)^2 + ((A - B)*a^2 - 2*(A - B)*a*b + (A - B)*b^2)*log
(sin(d*x + c) + 1) - ((A + B)*a^2 + 2*(A + B)*a*b + (A + B)*b^2)*log(-sin(d
*x + c) + 1) - 2*(2*B*a*b + A*b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sin(c+dx))(a+b\sin(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A]
time = 0.46, size = 129, normalized size = 1.37

$$-\frac{Bb^2 \sin(dx+c)^2 + 4\,Bab\sin(dx+c) + 2\,Ab^2\sin(dx+c) - (Aa^2 - Ba^2 - 2\,Aab + 2\,Bab + Ab^2 - Bb^2)\log(|\sin(dx+c)+1|) + (Aa^2 + Ba^2 + 2\,Aab + 2\,Bab + Ab^2 + Bb^2)\log(|\sin(dx+c)-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*(B*b^2*sin(d*x + c)^2 + 4*B*a*b*sin(d*x + c) + 2*A*b^2*sin(d*x + c) -
(A*a^2 - B*a^2 - 2*A*a*b + 2*B*a*b + A*b^2 - B*b^2)*log(abs(sin(d*x + c) +
1)) + (A*a^2 + B*a^2 + 2*A*a*b + 2*B*a*b + A*b^2 + B*b^2)*log(abs(sin(d*x +
 c) - 1)))/d
```

Mupad [B]
time = 12.37, size = 80, normalized size = 0.85

$$-\frac{\sin(c+d\,x)\,\left(A\,b^2+2\,B\,a\,b\right)+\frac{\ln(\sin(c+d\,x)-1)\,(a+b)^2\,(A+B)}{2}+\frac{B\,b^2\,\sin(c+d\,x)^2}{2}-\frac{\ln(\sin(c+d\,x)+1)\,(A-B)\,(a-b)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] -(sin(c + d*x)*(A*b^2 + 2*B*a*b) + (log(sin(c + d*x) - 1)*(a + b)^2*(A + B)
)/2 + (B*b^2*sin(c + d*x)^2)/2 - (log(sin(c + d*x) + 1)*(A - B)*(a - b)^2)/
2)/d
```

3.1541 $\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=112

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(1+\sin(c+dx))}{4d}+\frac{\sec^2(c+$$

```
[Out] -1/4*(a+b)*(a*A-b*(A+2*B))*ln(1-sin(d*x+c))/d+1/4*(a-b)*(a*A+b*(A-2*B))*ln(
1+sin(d*x+c))/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))*(A*b+a*B+(A*a+B*b)*sin(d*
x+c))/d
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2916, 833, 647, 31}

$$-\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}+\frac{(a-b)(aA+b(A-2B))\log(\sin(c+dx)+1)}{4d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx))((aA+bB)\sin(c+dx)+aB+Ab)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] -1/4*((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]])/d + ((a - b)*(a*A
+ b*(A - 2*B))*Log[1 + Sin[c + d*x]])/(4*d) + (Sec[c + d*x]^2*(a + b*Sin[c
+ d*x])*(A*b + a*B + (a*A + b*B)*Sin[c + d*x]))/(2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^3\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA}{2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))(Ab+aB+(aA}{2d}$$

$$= -\frac{(a+b)(aA-b(A+2B))\log(1-\sin(c+dx))}{4d}$$

Mathematica [A]
time = 1.05, size = 174, normalized size = 1.55

$$\frac{(a^2-b^2)\left((a+b)(aA-b(A+2B))\log(1-\sin(c+dx))-(a-b)(aA+b(A-2B))\log(1+\sin(c+dx))\right)-2a^3(-Ab+aB)\sec^2(c+dx)-2(a^2-b^2)\left(a^2A+Ab^2+2abB\right)\sec(c+dx)\tan(c+dx)+\left(-6a^3Ab+4aAb^3+2b^4B\right)\tan^2(c+dx)}{4\left(-a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)*((a + b)*(a*A - b*(A + 2*B))*Log[1 - Sin[c + d*x]] - (a - b)*(
a*A + b*(A - 2*B))*Log[1 + Sin[c + d*x]]) - 2*a^3*(-(A*b) + a*B)*Sec[c + d*
x]^2 - 2*(a^2 - b^2)*(a^2*A + A*b^2 + 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x] +
(-6*a^3*A*b + 4*a*A*b^3 + 2*b^4*B)*Tan[c + d*x]^2)/(4*(-a^2 + b^2)*d)
```

Maple [A]
time = 0.37, size = 187, normalized size = 1.67

method	result

derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{B\,a^2}{2\cos(dx+c)^2}+\frac{Aab}{\cos(dx+c)^2}+2Bab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(}{}\right.}{d}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{B\,a^2}{2\cos(dx+c)^2}+\frac{Aab}{\cos(dx+c)^2}+2Bab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(}{}\right.}{d}$
risch	$-iB\,b^2x-\frac{2iB\,b^2c}{d}-\frac{i\left(A\,a^2e^{3i(dx+c)}+A\,b^2e^{3i(dx+c)}+2Bab\,e^{3i(dx+c)}-A\,a^2e^{i(dx+c)}-A\,b^2e^{i(dx+c)}+4iAab\,e^{2i(dx}\right.}{d\left(e^{2i(dx+c)}+1\right)^2}$
norman	$\frac{\frac{\left(a^2A+A\,b^2+2Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\left(a^2A+A\,b^2+2Bab\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4Aab+2B\,a^2+2B\,b^2}{d}+\frac{4\left(a^2A+A\,b^2+2Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/2*B*
a^2/cos(d*x+c)^2+A*a*b/cos(d*x+c)^2+2*B*a*b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+
1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*b^2*(1/2*sin(d*x+c)^3/cos(d
*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+B*b^2*(1/2*tan(d*x+c)
^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 122, normalized size = 1.09

$$\frac{(Aa^2-2\,Bab-(A-2\,B)b^2)\log(\sin(dx+c)+1)-(Aa^2-2\,Bab-(A+2\,B)b^2)\log(\sin(dx+c)-1)-\frac{2\,(Ba^2+2\,Aab+Bb^2+(Aa^2+2\,Bab+Ab^2)\sin(dx+c))}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*log(sin(d*x + c) + 1) - (A*a^2 - 2*B
*a*b - (A + 2*B)*b^2)*log(sin(d*x + c) - 1) - 2*(B*a^2 + 2*A*a*b + B*b^2 +
(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.43, size = 136, normalized size = 1.21

$$\frac{(Aa^2-2\,Bab-(A-2\,B)b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa^2-2\,Bab-(A+2\,B)b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,Ba^2+4\,Aab+2\,Bb^2+2\,(Aa^2+2\,Bab+Ab^2)\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - (A - 2*B)*b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1)
 - (A*a^2 - 2*B*a*b - (A + 2*B)*b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
```

```
+ 2*B*a^2 + 4*A*a*b + 2*B*b^2 + 2*(A*a^2 + 2*B*a*b + A*b^2)*sin(d*x + c))/(
d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sin(c+dx))(a+b\sin(c+dx))^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*(a + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.56, size = 146, normalized size = 1.30

$$\frac{(Aa^2-2\,Bab-Ab^2+2\,Bb^2)\log(|\sin(dx+c)+1|)-(Aa^2-2\,Bab-Ab^2-2\,Bb^2)\log(|\sin(dx+c)-1|)-\frac{2\left(Bb^2\sin(dx+c)^2+Aa^2\sin(dx+c)+2\,Bab\sin(dx+c)+Ab^2\sin(dx+c)+Ba^2+2\,Aab\right)}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/4*((A*a^2 - 2*B*a*b - A*b^2 + 2*B*b^2)*log(abs(sin(d*x + c) + 1)) - (A*a^
2 - 2*B*a*b - A*b^2 - 2*B*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(B*b^2*sin(d*
x + c)^2 + A*a^2*sin(d*x + c) + 2*B*a*b*sin(d*x + c) + A*b^2*sin(d*x + c) +
 B*a^2 + 2*A*a*b)/(sin(d*x + c)^2 - 1))/d
```

Mupad [B]
time = 12.36, size = 118, normalized size = 1.05

$$\frac{\ln(\sin(c+d\,x)+1)\,(a-b)\,(A\,a+A\,b-2\,B\,b)}{4\,d}-\frac{\sin(c+d\,x)\,\left(\frac{A\,a^2}{2}+B\,a\,b+\frac{A\,b^2}{2}\right)+\frac{B\,a^2}{2}+\frac{B\,b^2}{2}+A\,a\,b}{d\,\left(\sin(c+d\,x)^2-1\right)}+\frac{\ln(\sin(c+d\,x)-1)\,(a+b)\,(A\,b-A\,a+2\,B\,b)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (log(sin(c + d*x) + 1)*(a - b)*(A*a + A*b - 2*B*b))/(4*d) - (sin(c + d*x)*(
(A*a^2)/2 + (A*b^2)/2 + B*a*b) + (B*a^2)/2 + (B*b^2)/2 + A*a*b)/(d*(sin(c +
 d*x)^2 - 1)) + (log(sin(c + d*x) - 1)*(a + b)*(A*b - A*a + 2*B*b))/(4*d)
```

3.1542 $\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$\frac{(3a^2A-Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{4d}+\frac{\sec^2(c+$$

```
[Out] 1/8*(3*A*a^2-A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/d+1/4*sec(d*x+c)^4*(B+A*sin
(d*x+c))*(a+b*sin(d*x+c))^2/d+1/8*sec(d*x+c)^2*(2*b*(2*A*a-B*b)+(3*A*a^2+A*
b^2-2*B*a*b)*sin(d*x+c))/d
```

Rubi [A]
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2916, 835, 792, 212}

$$\frac{(3a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{\sec^2(c+dx)\left((3a^2A-2abB+Ab^2)\sin(c+dx)+2b(2aA-bB)\right)}{8d}+\frac{\sec^4(c+dx)(a+b\sin(c+dx))^2(A\sin(c+dx)+B)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((3*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (Sec[c + d*x]^4
*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(4*d) + (Sec[c + d*x]^2*(2*b*
(2*a*A - b*B) + (3*a^2*A + A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(8*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
```

```
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^5\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+d}{4d}$$

$$= \frac{\sec^4(c+dx)(B+A\sin(c+dx))(a+b\sin(c+d}{4d}$$

$$= \frac{(3a^2A-Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\text{se}}{}$$

Mathematica [A]
time = 1.28, size = 186, normalized size = 1.52

$$\frac{4(-a^2+b^2)\sec^4(c+dx)(a+b\sin(c+dx))^3(Ab-aB+(-aA+bB)\sin(c+dx))+(-3a^2A+Ab^2+2abB)\left((a^2-b^2)^2(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+2a^3b\sec^2(c+dx)-2(a^4-b^4)\sec(c+dx)\tan(c+dx)+(-6a^3b+4ab^3)\tan^2(c+dx)\right)}{16(a^2-b^2)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (4*(-a^2 + b^2)*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A)
+ b*B)*Sin[c + d*x]) + (-3*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[1 -
 Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^4 -
 b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/(16
*(a^2 - b^2)^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(116) = 232$.
time = 0.43, size = 236, normalized size = 1.93 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))+1/4*B*a^2/cos(d*x+c)^4+1/2*A*a*b/cos(d*x+c)^4+2*B*a*b*(1/4*s
in(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*l
n(sec(d*x+c)+tan(d*x+c)))+A*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+
c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+1/4*B*b^2*s
in(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 171, normalized size = 1.40

$$\frac{(3\,Aa^2-2\,Bab-Ab^2)\log\left(\sin\left(dx+c\right)+1\right)-(3\,Aa^2-2\,Bab-Ab^2)\log\left(\sin\left(dx+c\right)-1\right)+\frac{2\left(4\,Bb^2\sin(dx+c)^2-(3\,Aa^2-2\,Bab-Ab^2)\sin(dx+c)^3+2\,Ba^2+4\,Aab-2\,Bb^2+(5\,Aa^2+2\,Bab+Ab^2)\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - (3*A*a^2 - 2*B*a*
b - A*b^2)*log(sin(d*x + c) - 1) + 2*(4*B*b^2*sin(d*x + c)^2 - (3*A*a^2 - 2
*B*a*b - A*b^2)*sin(d*x + c)^3 + 2*B*a^2 + 4*A*a*b - 2*B*b^2 + (5*A*a^2 + 2
*B*a*b + A*b^2)*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 0.38, size = 173, normalized size = 1.42

$$\frac{(3\,Aa^2-2\,Bab-Ab^2)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-(3\,Aa^2-2\,Bab-Ab^2)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)-8\,Bb^2\cos\left(dx+c\right)^2+4\,Ba^2+8\,Aab+4\,Bb^2+2\left(2\,Aa^2+4\,Bab+2\,Ab^2+(3\,Aa^2-2\,Bab-Ab^2)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3
*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) - 8*B*b^2*c
os(d*x + c)^2 + 4*B*a^2 + 8*A*a*b + 4*B*b^2 + 2*(2*A*a^2 + 4*B*a*b + 2*A*b^
2 + (3*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 187, normalized size = 1.53

$$\frac{(3\,Aa^2-2\,Bab-Ab^2)\log\left(|\sin\left(dx+c\right)+1|\right)-(3\,Aa^2-2\,Bab-Ab^2)\log\left(|\sin\left(dx+c\right)-1|\right)-\frac{2\left(3\,Aa^2\sin(dx+c)^3-2\,Bab\sin(dx+c)^3-Ab^2\sin(dx+c)^3-4\,Bb^2\sin(dx+c)^2-5\,Aa^2\sin(dx+c)-2\,Bab\sin(dx+c)-Ab^2\sin(dx+c)-2\,Ba^2-4\,Aab+2\,Bb^2\right)}{\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/16*((3*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - (3*A*a^2 - 2
*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(3*A*a^2*sin(d*x + c)^3 - 2*
B*a*b*sin(d*x + c)^3 - A*b^2*sin(d*x + c)^3 - 4*B*b^2*sin(d*x + c)^2 - 5*A*
a^2*sin(d*x + c) - 2*B*a*b*sin(d*x + c) - A*b^2*sin(d*x + c) - 2*B*a^2 - 4*
A*a*b + 2*B*b^2)/(sin(d*x + c)^2 - 1)^2)/d
```

Mupad [B]
time = 12.38, size = 181, normalized size = 1.48

$$\frac{\sin\left(c+d\,x\right)\,\left(\frac{5\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}\right)+\frac{B\,a^2}{4}-\frac{B\,b^2}{4}+{\sin\left(c+d\,x\right)}^3\,\left(-\frac{3\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}\right)+\frac{B\,b^2\,{\sin(c+d\,x)}^2}{2}+\frac{A\,a\,b}{2}}{d\,\left({\sin\left(c+d\,x\right)}^4-2\,{\sin\left(c+d\,x\right)}^2+1\right)}-\frac{\mathrm{atanh}\left(\frac{4\,\sin(c+d\,x)\,\left(-\frac{3\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)}{-\frac{3\,A\,a^2}{4}+\frac{B\,a\,b}{2}+\frac{A\,b^2}{4}}\right)\,\left(-\frac{3\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^5,x)
```

```
[Out] (sin(c + d*x)*((5*A*a^2)/8 + (A*b^2)/8 + (B*a*b)/4) + (B*a^2)/4 - (B*b^2)/4
 + sin(c + d*x)^3*((A*b^2)/8 - (3*A*a^2)/8 + (B*a*b)/4) + (B*b^2*sin(c + d*
x)^2)/2 + (A*a*b)/2)/(d*(sin(c + d*x)^4 - 2*sin(c + d*x)^2 + 1)) - (atanh((
4*sin(c + d*x)*((A*b^2)/16 - (3*A*a^2)/16 + (B*a*b)/8))/((A*b^2)/4 - (3*A*a
^2)/4 + (B*a*b)/2))*((A*b^2)/8 - (3*A*a^2)/8 + (B*a*b)/4))/d
```

3.1543 $\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{(5a^2A-Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+dx))^2}{6d}+\frac{\sec^4(c+$$

```
[Out] 1/16*(5*A*a^2-A*b^2-2*B*a*b)*arctanh(sin(d*x+c))/d+1/6*sec(d*x+c)^6*(B+A*si
n(d*x+c))*(a+b*sin(d*x+c))^2/d+1/24*sec(d*x+c)^4*(2*b*(4*A*a-B*b)+(5*A*a^2+
3*A*b^2-2*B*a*b)*sin(d*x+c))/d+1/16*(5*A*a^2-A*b^2-2*B*a*b)*sec(d*x+c)*tan(
d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2916, 835, 792, 205, 212}

$$\frac{(5a^2A-2abB-Ab^2)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{\sec^4(c+dx)\left((5a^2A-2abB+3Ab^2)\sin(c+dx)+2b(4aA-bB)\right)}{24d}+\frac{(5a^2A-2abB-Ab^2)\tan(c+dx)\sec(c+dx)}{16d}+\frac{\sec^6(c+dx)(a+b\sin(c+dx))^2(A\sin(c+dx)+B)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] ((5*a^2*A - A*b^2 - 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (Sec[c + d*x]^
6*(B + A*Sin[c + d*x])*(a + b*Sin[c + d*x])^2)/(6*d) + (Sec[c + d*x]^4*(2*b
*(4*a*A - b*B) + (5*a^2*A + 3*A*b^2 - 2*a*b*B)*Sin[c + d*x]))/(24*d) + ((5*
a^2*A - A*b^2 - 2*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
```

```
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a+b\sin(c+dx))^2(A+B\sin(c+dx))\,dx = \frac{b^7\text{Subst}\left(\int \frac{(a+x)^2\left(A+\frac{Bx}{b}\right)}{(b^2-x^2)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+d}{6d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+d}{6d}$$

$$= \frac{\sec^6(c+dx)(B+A\sin(c+dx))(a+b\sin(c+d}{6d}$$

$$= \frac{(5a^2A-Ab^2-2abB)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{\text{se}}{}$$

Mathematica [A]
time = 1.03, size = 242, normalized size = 1.51

$$\frac{b\sec^6(c+dx)(a+b\sin(c+dx))^3(Ab-aB+(-aA+bB)\sin(c+dx))+\frac{1}{4}b\sec^4(c+dx)(a+b\sin(c+dx))^3(3Ab+(-5aA+2bB)\sin(c+dx))-\frac{3b(-5a^2A+Ab^2+2abB)\left((a^2-b^2)^2(\log(1-\sin(c+dx))-\log(1+\sin(c+dx)))+2a^3b\sec^2(c+dx)-2(a^4-b^4)\sec(c+dx)\tan(c+dx)+(-6a^3b+4ab^3)\tan^2(c+dx)\right)}{16(a-b)(a+b)}}{6b(-a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a + b*Sin[c + d*x])^2*(A + B*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6*(a + b*Sin[c + d*x])^3*(A*b - a*B + (-(a*A) + b*B)*Sin[c
+ d*x]) + (b*Sec[c + d*x]^4*(a + b*Sin[c + d*x])^3*(3*A*b + (-5*a*A + 2*b*B
)*Sin[c + d*x]))/4 - (3*b*(-5*a^2*A + A*b^2 + 2*a*b*B)*((a^2 - b^2)^2*(Log[
1 - Sin[c + d*x]] - Log[1 + Sin[c + d*x]]) + 2*a^3*b*Sec[c + d*x]^2 - 2*(a^
4 - b^4)*Sec[c + d*x]*Tan[c + d*x] + (-6*a^3*b + 4*a*b^3)*Tan[c + d*x]^2))/
(16*(a - b)*(a + b)))/(6*b*(-a^2 + b^2)*d)
```

Maple [A]
time = 0.39, size = 302, normalized size = 1.89

method	result
derivativedivides	$\frac{a^2 A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^2}{6\cos(dx+c)^6}+\frac{Aab}{3\cos(dx-}}{}$
default	$\frac{a^2 A\left(-\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{B\,a^2}{6\cos(dx+c)^6}+\frac{Aab}{3\cos(dx-}}{}$
risch	$-\frac{i\left(15a^2A\,e^{11i(dx+c)}-3A\,b^2e^{11i(dx+c)}-6Bab\,e^{11i(dx+c)}+85a^2A\,e^{9i(dx+c)}-17A\,b^2e^{9i(dx+c)}-34Bab\,e^{9i(dx+c)}+198A\,c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+
c)+5/16*ln(sec(d*x+c)+tan(d*x+c)))+1/6*B*a^2/cos(d*x+c)^6+1/3*A*a*b/cos(d*x
+c)^6+2*B*a*b*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+
1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c
)))+A*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16
*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+
B*b^2*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.28, size = 211, normalized size = 1.32

$$\frac{3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,Aa^2-2\,Bab-Ab^2\right)\log\left(\sin\left(dx+c\right)-1\right)-\frac{2\left(3\left(5\,Aa^2-2\,Bab-Ab^2\right)\sin(dx+c)^5+12\,Bb^2\sin(dx+c)^2-8\left(5\,Aa^2-2\,Bab-Ab^2\right)\sin(dx+c)^3+8\,Ba^2+16\,Aab-4\,Bb^2+3\left(11\,Aa^2+2\,Bab+Ab^2\right)\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(sin(d*x + c) + 1) - 3*(5*A*a^2 - 2*
B*a*b - A*b^2)*log(sin(d*x + c) - 1) - 2*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*sin
(d*x + c)^5 + 12*B*b^2*sin(d*x + c)^2 - 8*(5*A*a^2 - 2*B*a*b - A*b^2)*sin(d
```

```
*x + c)^3 + 8*B*a^2 + 16*A*a*b - 4*B*b^2 + 3*(11*A*a^2 + 2*B*a*b + A*b^2)*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 0.40, size = 203, normalized size = 1.27

$$\frac{3\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-3\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)-24\,Bb^{2}\cos\left(dx+c\right)^{2}+16\,Ba^{2}+32\,Aab+16\,Bb^{2}+2\left(3\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\cos\left(dx+c\right)^{4}+8\,Aa^{2}+16\,Bab+8\,Ab^{2}+2\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{96\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
3*(5*A*a^2 - 2*B*a*b - A*b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) - 24*B*
b^2*cos(d*x + c)^2 + 16*B*a^2 + 32*A*a*b + 16*B*b^2 + 2*(3*(5*A*a^2 - 2*B*a
*b - A*b^2)*cos(d*x + c)^4 + 8*A*a^2 + 16*B*a*b + 8*A*b^2 + 2*(5*A*a^2 - 2*
B*a*b - A*b^2)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a+b*sin(d*x+c))**2*(A+B*sin(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.53, size = 229, normalized size = 1.43

$$\frac{3\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)+1\right|\right)-3\left(5\,Aa^{2}-2\,Bab-Ab^{2}\right)\log\left(\left|\sin\left(dx+c\right)-1\right|\right)-\frac{2\left(15\,Aa^{2}\sin(dx+c)^{5}-6\,Bab\sin(dx+c)^{5}-3\,Ab^{2}\sin(dx+c)^{5}-40\,Aa^{2}\sin(dx+c)^{3}+16\,Bab\sin(dx+c)^{3}+8\,Ab^{2}\sin(dx+c)^{3}+12\,Bb^{2}\sin(dx+c)^{2}+33\,Aa^{2}\sin(dx+c)+6\,Bab\sin(dx+c)+3\,Ab^{2}\sin(dx+c)+8\,Ba^{2}+16\,Aab-4\,Bb^{2}\right)}{\left(\sin(dx+c)^{2}-1\right)^{3}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a+b*sin(d*x+c))^2*(A+B*sin(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/96*(3*(5*A*a^2 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) + 1)) - 3*(5*A*a^2
 - 2*B*a*b - A*b^2)*log(abs(sin(d*x + c) - 1)) - 2*(15*A*a^2*sin(d*x + c)^5
 - 6*B*a*b*sin(d*x + c)^5 - 3*A*b^2*sin(d*x + c)^5 - 40*A*a^2*sin(d*x + c)^
3 + 16*B*a*b*sin(d*x + c)^3 + 8*A*b^2*sin(d*x + c)^3 + 12*B*b^2*sin(d*x + c
)^2 + 33*A*a^2*sin(d*x + c) + 6*B*a*b*sin(d*x + c) + 3*A*b^2*sin(d*x + c) +
 8*B*a^2 + 16*A*a*b - 4*B*b^2)/(sin(d*x + c)^2 - 1)^3)/d
```

Mupad [B]
time = 12.45, size = 220, normalized size = 1.38

$$-\frac{\mathrm{atanh}\left(\frac{4\,\sin(c+d\,x)\,\left(-\frac{5\,A\,a^2}{32}+\frac{B\,a\,b}{16}+\frac{A\,b^2}{32}\right)}{-\frac{5\,A\,a^2}{8}+\frac{B\,a\,b}{4}+\frac{A\,b^2}{8}}\right)\,\left(-\frac{5\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(\frac{11\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)+\frac{B\,a^2}{6}-\frac{B\,b^2}{12}+{\sin\left(c+d\,x\right)}^3\,\left(-\frac{5\,A\,a^2}{6}+\frac{B\,a\,b}{3}+\frac{A\,b^2}{6}\right)-{\sin\left(c+d\,x\right)}^5\,\left(-\frac{5\,A\,a^2}{16}+\frac{B\,a\,b}{8}+\frac{A\,b^2}{16}\right)+\frac{B\,b^2\,{\sin(c+d\,x)}^2}{4}+\frac{A\,a\,b}{3}}{d\,\left({\sin\left(c+d\,x\right)}^6-3\,{\sin\left(c+d\,x\right)}^4+3\,{\sin\left(c+d\,x\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(c + d*x))*(a + b*sin(c + d*x))^2)/cos(c + d*x)^7,x)

[Out] - (atanh((4*sin(c + d*x)*((A*b^2)/32 - (5*A*a^2)/32 + (B*a*b)/16))/((A*b^2)/8 - (5*A*a^2)/8 + (B*a*b)/4))*((A*b^2)/16 - (5*A*a^2)/16 + (B*a*b)/8))/d - (sin(c + d*x)*((11*A*a^2)/16 + (A*b^2)/16 + (B*a*b)/8) + (B*a^2)/6 - (B*b^2)/12 + sin(c + d*x)^3*((A*b^2)/6 - (5*A*a^2)/6 + (B*a*b)/3) - sin(c + d*x)^5*((A*b^2)/16 - (5*A*a^2)/16 + (B*a*b)/8) + (B*b^2*sin(c + d*x)^2)/4 + (A*a*b)/3)/(d*(3*sin(c + d*x)^2 - 3*sin(c + d*x)^4 + sin(c + d*x)^6 - 1))

3.1544 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=315

$$-\frac{(a^2-b^2)^3 (Ab-aB)\log(a+b\sin(c+dx))}{b^8d}+\frac{(a^5Ab-3a^3Ab^3+3aAb^5-a^6B+3a^4b^2B-3a^2b^4B+b^6B)\,\mathrm{s}}{b^7d}$$

```
[Out] -(a^2-b^2)^3*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^8/d+(A*a^5*b-3*A*a^3*b^3+3*A*a*
b^5-B*a^6+3*B*a^4*b^2-3*B*a^2*b^4+B*b^6)*sin(d*x+c)/b^7/d-1/2*(a^4-3*a^2*b^
2+3*b^4)*(A*b-B*a)*sin(d*x+c)^2/b^6/d+1/3*(A*a^3*b-3*A*a*b^3-B*a^4+3*B*a^2*
b^2-3*B*b^4)*sin(d*x+c)^3/b^5/d-1/4*(a^2-3*b^2)*(A*b-B*a)*sin(d*x+c)^4/b^4/
d+1/5*(A*a*b-B*a^2+3*B*b^2)*sin(d*x+c)^5/b^3/d-1/6*(A*b-B*a)*sin(d*x+c)^6/b
^2/d-1/7*B*sin(d*x+c)^7/b/d
```

Rubi [A]
time = 0.26, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$-\frac{(a^2-b^2)^3 (Ab-aB)\log(a+b\sin(c+dx))}{b^8d}-\frac{(a^2-3b^2)(Ab-aB)\sin^4(c+dx)}{4b^4d}+\frac{(a^2(-B)+aAb+3b^2B)\sin^5(c+dx)}{5b^3d}-\frac{(a^4-3a^2b^2+3b^4)(Ab-aB)\sin^2(c+dx)}{2b^6d}+\frac{(a^4(-B)+a^3Ab+3a^2b^2B-3aAb^3-3b^4B)\sin^3(c+dx)}{3b^5d}+\frac{(a^6(-B)+a^5Ab+3a^4b^2B-3a^3Ab^3-3a^2b^4B+3aAb^5+b^6B)\sin(c+dx)}{b^7d}-\frac{(Ab-aB)\sin^6(c+dx)}{6b^2d}-\frac{B\sin^7(c+dx)}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)) + ((a^5*A*b
- 3*a^3*A*b^3 + 3*a*A*b^5 - a^6*B + 3*a^4*b^2*B - 3*a^2*b^4*B + b^6*B)*Sin[
c + d*x])/(b^7*d) - ((a^4 - 3*a^2*b^2 + 3*b^4)*(A*b - a*B)*Sin[c + d*x]^2)/
(2*b^6*d) + ((a^3*A*b - 3*a*A*b^3 - a^4*B + 3*a^2*b^2*B - 3*b^4*B)*Sin[c +
d*x]^3)/(3*b^5*d) - ((a^2 - 3*b^2)*(A*b - a*B)*Sin[c + d*x]^4)/(4*b^4*d) +
((a*A*b - a^2*B + 3*b^2*B)*Sin[c + d*x]^5)/(5*b^3*d) - ((A*b - a*B)*Sin[c +
 d*x]^6)/(6*b^2*d) - (B*Sin[c + d*x]^7)/(7*b*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^3}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^5Ab-3a^3Ab^3+3aAb^5-a^6B+3a^4b^2B-3a^2b^4B+b^6B}{b} - \frac{(a^4-3a^2b^2+3b^4)}{b}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)^3(Ab-aB)\log(a+b\sin(c+dx))}{b^8 d} + \frac{(a^5Ab-3a^3Ab^3}{}$$

Mathematica [A]
time = 0.60, size = 218, normalized size = 0.69

$$\frac{\frac{(Ab-aB)\left(15b^4\left(-a^2+b^2\right)\cos^4(c+dx)+10b^6\cos^6(c+dx)-60\left(a^2-b^2\right)^3\log(a+b\sin(c+dx))+60ab\left(a^4-3a^2b^2+3b^4\right)\sin(c+dx)-30b^2\left(a^2-b^2\right)^2\sin^2(c+dx)+20ab^3\left(a^2-3b^2\right)\sin^3(c+dx)+12ab^5\sin^5(c+dx)\right)}{60b}+\frac{b^6B(1225\sin(c+dx)+245\sin(3(c+dx))+49\sin(5(c+dx))+5\sin(7(c+dx)))}{2240}}{b^7 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((A*b - a*B)*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 -
 60*(a^2 - b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4
)*Sin[c + d*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^
2)*Sin[c + d*x]^3 + 12*a*b^5*Sin[c + d*x]^5))/(60*b) + (b^6*B*(1225*Sin[c +
 d*x] + 245*Sin[3*(c + d*x)] + 49*Sin[5*(c + d*x)] + 5*Sin[7*(c + d*x)]))/2
240)/(b^7*d)
```

Maple [A]
time = 0.42, size = 506, normalized size = 1.61

method	result
derivativedivides	$\frac{\frac{B a^3 b^3 \left(\sin^4(dx+c)\right)}{4} - \frac{3 B a b^5 \left(\sin^4(dx+c)\right)}{4} + \frac{A a^3 b^3 \left(\sin^3(dx+c)\right)}{3} - A a b^5 \left(\sin^3(dx+c)\right) - \frac{B a^4 b^2 \left(\sin^3(dx+c)\right)}{3} - \frac{A a^4 b^2 \left(\sin^2(dx+c\right.}{2}}{}$
default	$\frac{\frac{B a^3 b^3 \left(\sin^4(dx+c)\right)}{4} - \frac{3 B a b^5 \left(\sin^4(dx+c)\right)}{4} + \frac{A a^3 b^3 \left(\sin^3(dx+c)\right)}{3} - A a b^5 \left(\sin^3(dx+c)\right) - \frac{B a^4 b^2 \left(\sin^3(dx+c)\right)}{3} - \frac{A a^4 b^2 \left(\sin^2(dx+c\right.}{2}}{}$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^7*(3/4*A*b^6*sin(d*x+c)^4-B*b^6*sin(d*x+c)^3+3/2*B*a*b^5*sin(d*x+c
)^2-1/5*B*a^2*b^4*sin(d*x+c)^5-1/4*A*a^2*b^4*sin(d*x+c)^4+1/4*B*a^3*b^3*sin
```

```
(d*x+c)^4-3/4*B*a*b^5*sin(d*x+c)^4+1/3*A*a^3*b^3*sin(d*x+c)^3-A*a*b^5*sin(d
*x+c)^3-1/3*B*a^4*b^2*sin(d*x+c)^3-1/2*A*a^4*b^2*sin(d*x+c)^2+1/6*B*a*b^5*s
in(d*x+c)^6+1/5*A*a*b^5*sin(d*x+c)^5+A*a^5*b*sin(d*x+c)-3*A*a^3*b^3*sin(d*x
+c)+3*A*a*b^5*sin(d*x+c)+3*B*a^4*b^2*sin(d*x+c)-3*B*a^2*b^4*sin(d*x+c)+B*a^
2*b^4*sin(d*x+c)^3+3/2*A*a^2*b^4*sin(d*x+c)^2+1/2*B*a^5*b*sin(d*x+c)^2-3/2*
B*a^3*b^3*sin(d*x+c)^2-1/7*B*sin(d*x+c)^7*b^6-1/6*A*b^6*sin(d*x+c)^6+3/5*B*
b^6*sin(d*x+c)^5-B*a^6*sin(d*x+c)+B*b^6*sin(d*x+c)-3/2*A*b^6*sin(d*x+c)^2)+
(-A*a^6*b+3*A*a^4*b^3-3*A*a^2*b^5+A*b^7+B*a^7-3*B*a^5*b^2+3*B*a^3*b^4-B*a*b
^6)/b^8*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 366, normalized size = 1.16

$$-\frac{\frac{60\,Bb^6\sin(dx+c)^7-70\,(Bab^5-Ab^6)\sin(dx+c)^6+84\,(Ba^2b^4-Aab^5-3\,Bb^6)\sin(dx+c)^5-105\,(Ba^3b^3-Aa^2b^4-3\,Bab^5+3\,Ab^6)\sin(dx+c)^4+140\,(Ba^4b^2-Aa^3b^3-3\,Ba^2b^4+3\,Aab^5+3\,Bb^6)\sin(dx+c)^3-210\,(Ba^5b-Aa^4b^2-3\,Ba^3b^3+3\,Aa^2b^4+3\,Bab^5-3\,Ab^6)\sin(dx+c)^2+420\,(Ba^6-Aa^5b-3\,Ba^4b^2+3\,Aa^3b^3+3\,Ba^2b^4-3\,Aab^5-Bb^6)\sin(dx+c)}{b^7}-\frac{420\,(Ba^7-Aa^6b-3\,Ba^5b^2+3\,Aa^4b^3+3\,Ba^3b^4-3\,Aa^2b^5-Bab^6+Ab^7)\log(b\sin(dx+c)+a)}{b^8}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*(B*a*b^5 - A*b^6)*sin(d*x + c)^6 + 84
*(B*a^2*b^4 - A*a*b^5 - 3*B*b^6)*sin(d*x + c)^5 - 105*(B*a^3*b^3 - A*a^2*b^
4 - 3*B*a*b^5 + 3*A*b^6)*sin(d*x + c)^4 + 140*(B*a^4*b^2 - A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 3*B*b^6)*sin(d*x + c)^3 - 210*(B*a^5*b - A*a^4*b^2 -
3*B*a^3*b^3 + 3*A*a^2*b^4 + 3*B*a*b^5 - 3*A*b^6)*sin(d*x + c)^2 + 420*(B*a^
6 - A*a^5*b - 3*B*a^4*b^2 + 3*A*a^3*b^3 + 3*B*a^2*b^4 - 3*A*a*b^5 - B*b^6)*
sin(d*x + c))/b^7 - 420*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*
a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(b*sin(d*x + c) + a)/b^8)/d
```

Fricas [A]
time = 0.42, size = 366, normalized size = 1.16

$$-\frac{70\,(Bab^6-Ab^7)\cos(dx+c)^6-105\,(Ba^3b^4-Aa^2b^5-Bab^6+Ab^7)\cos(dx+c)^4+210\,(Ba^5b^2-Aa^4b^3-2\,Ba^3b^4+2\,Aa^2b^5+Bab^6-Ab^7)\cos(dx+c)^2-420\,(Ba^7-Aa^6b-3\,Ba^5b^2+3\,Aa^4b^3+3\,Ba^3b^4-3\,Aa^2b^5-Bab^6+Ab^7)\log(b\sin(dx+c)+a)-4\,(15\,Bb^7\cos(dx+c)^6-105\,Ba^6b+105\,Aa^5b^2+280\,Ba^4b^3-280\,Aa^3b^4-231\,Ba^2b^5+231\,Aab^6+48\,Bb^7-3\,(7\,Ba^2b^5-7\,Aab^6-6\,Bb^7)\cos(dx+c)^4+(35\,Ba^4b^3-35\,Aa^3b^4-63\,Ba^2b^5+63\,Aab^6+24\,Bb^7)\cos(dx+c)^2)\sin(dx+c)}{420\,b^8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/420*(70*(B*a*b^6 - A*b^7)*cos(d*x + c)^6 - 105*(B*a^3*b^4 - A*a^2*b^5 -
B*a*b^6 + A*b^7)*cos(d*x + c)^4 + 210*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
+ 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 - 420*(B*a^7 - A*a^6*b - 3*
B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + A*b^7)*log(
b*sin(d*x + c) + a) - 4*(15*B*b^7*cos(d*x + c)^6 - 105*B*a^6*b + 105*A*a^5*
b^2 + 280*B*a^4*b^3 - 280*A*a^3*b^4 - 231*B*a^2*b^5 + 231*A*a*b^6 + 48*B*b^
7 - 3*(7*B*a^2*b^5 - 7*A*a*b^6 - 6*B*b^7)*cos(d*x + c)^4 + (35*B*a^4*b^3 -
35*A*a^3*b^4 - 63*B*a^2*b^5 + 63*A*a*b^6 + 24*B*b^7)*cos(d*x + c)^2)*sin(d*
x + c))/(b^8*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.52, size = 511, normalized size = 1.62

$$-\frac{\frac{60 B b^6 \sin(dx+c)^7 - 70 B a b^5 \sin(dx+c)^6 + 70 A b^6 \sin(dx+c)^6 + 84 B a^2 b^4 \sin(dx+c)^5 - 84 A a b^5 \sin(dx+c)^5 - 252 B b^6 \sin(dx+c)^5 - 105 B a^3 b^3 \sin(dx+c)^4 + 105 A a^2 b^4 \sin(dx+c)^4 + 315 B a b^5 \sin(dx+c)^4 - 315 A b^6 \sin(dx+c)^4 + 140 B a^4 b^2 \sin(dx+c)^3 - 140 A a^3 b^3 \sin(dx+c)^3 - 420 B a^2 b^4 \sin(dx+c)^3 + 420 A a b^5 \sin(dx+c)^3 + 420 B b^6 \sin(dx+c)^3 - 210 B a^5 b \sin(dx+c)^2 + 210 A a^4 b^2 \sin(dx+c)^2 + 630 B a^3 b^3 \sin(dx+c)^2 - 630 A a^2 b^4 \sin(dx+c)^2 - 630 B a b^5 \sin(dx+c)^2 + 630 A b^6 \sin(dx+c)^2 + 420 B a^6 \sin(dx+c) - 420 A a^5 b \sin(dx+c) - 1260 B a^4 b^2 \sin(dx+c) + 1260 A a^3 b^3 \sin(dx+c) + 1260 B a^2 b^4 \sin(dx+c) - 1260 A a b^5 \sin(dx+c) - 420 B b^6 \sin(dx+c)}{b^7} - \frac{420 (B a^7 - A a^6 b - 3 B a^5 b^2 + 3 A a^4 b^3 + 3 B a^3 b^4 - 3 A a^2 b^5 - B a b^6 + A b^7) \log(|b \sin(dx+c) + a|)}{b^8}}{420 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/420*((60*B*b^6*sin(d*x + c)^7 - 70*B*a*b^5*sin(d*x + c)^6 + 70*A*b^6*sin
(d*x + c)^6 + 84*B*a^2*b^4*sin(d*x + c)^5 - 84*A*a*b^5*sin(d*x + c)^5 - 252
*B*b^6*sin(d*x + c)^5 - 105*B*a^3*b^3*sin(d*x + c)^4 + 105*A*a^2*b^4*sin(d*
x + c)^4 + 315*B*a*b^5*sin(d*x + c)^4 - 315*A*b^6*sin(d*x + c)^4 + 140*B*a^
4*b^2*sin(d*x + c)^3 - 140*A*a^3*b^3*sin(d*x + c)^3 - 420*B*a^2*b^4*sin(d*x
 + c)^3 + 420*A*a*b^5*sin(d*x + c)^3 + 420*B*b^6*sin(d*x + c)^3 - 210*B*a^5
*b*sin(d*x + c)^2 + 210*A*a^4*b^2*sin(d*x + c)^2 + 630*B*a^3*b^3*sin(d*x +
c)^2 - 630*A*a^2*b^4*sin(d*x + c)^2 - 630*B*a*b^5*sin(d*x + c)^2 + 630*A*b^
6*sin(d*x + c)^2 + 420*B*a^6*sin(d*x + c) - 420*A*a^5*b*sin(d*x + c) - 1260
*B*a^4*b^2*sin(d*x + c) + 1260*A*a^3*b^3*sin(d*x + c) + 1260*B*a^2*b^4*sin(
d*x + c) - 1260*A*a*b^5*sin(d*x + c) - 420*B*b^6*sin(d*x + c))/b^7 - 420*(B
*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*a^2*b^5 - B*
a*b^6 + A*b^7)*log(abs(b*sin(d*x + c) + a))/b^8)/d
```

Mupad [**B**]
time = 12.37, size = 435, normalized size = 1.38

$$\frac{\sin(c+dx)^4 \left(\frac{3A}{4b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{b}\right)}{4b}\right)}{d} - \frac{\sin(c+dx)^3 \left(\frac{B}{b} + \frac{a\left(\frac{3A}{b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{b}\right)}{b}\right)}{3b}\right)}{d} + \frac{\sin(c+dx)\left(\frac{B}{b} + \frac{a\left(\frac{3A}{b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{3A}{b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{b}\right)}{d} - \frac{\sin(c+dx)^6 \left(\frac{A}{6b} - \frac{Ba}{6b^2}\right)}{d} - \frac{\sin(c+dx)^2 \left(\frac{3A}{2b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{3A}{b} - \frac{a\left(\frac{3B}{b} + \frac{a\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)}{2b}\right)}{d} + \frac{\sin(c+dx)^5 \left(\frac{3B}{5b} + \frac{a\left(\frac{A}{b} - \frac{Ba}{b^2}\right)}{5b}\right)}{d} + \frac{\ln(a + b\sin(c+dx))\,(B a^7 - A a^6 b - 3 B a^5 b^2 + 3 A a^4 b^3 + 3 B a^3 b^4 - 3 A a^2 b^5 - B a b^6 + A b^7)}{b^8 d} - \frac{B \sin(c+dx)^7}{7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)

[Out] (sin(c + d*x)^4*((3*A)/(4*b) - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/(4*b
)))/d - (sin(c + d*x)^3*(B/b + (a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/
b^2))/b))/b))/(3*b)))/d + (sin(c + d*x)*(B/b + (a*((3*A)/b - (a*((3*B)/b +
(a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b))/b))/b))/d - (
sin(c + d*x)^6*(A/(6*b) - (B*a)/(6*b^2)))/d - (sin(c + d*x)^2*((3*A)/(2*b)
- (a*((3*B)/b + (a*((3*A)/b - (a*((3*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b
))/(2*b)))/d + (sin(c + d*x)^5*((3*B)/(5*b) + (a*(A/b - (B*a)/b^2))/(5*b)))
/d + (log(a + b*sin(c + d*x))*(A*b^7 + B*a^7 - 3*A*a^2*b^5 + 3*A*a^4*b^3 +
3*B*a^3*b^4 - 3*B*a^5*b^2 - A*a^6*b - B*a*b^6))/(b^8*d) - (B*sin(c + d*x)^7
)/(7*b*d)
```

3.1545 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=202

$$\frac{(a^2-b^2)^2(Ab-aB)\log(a+b\sin(c+dx))}{b^6d}-\frac{(a^3Ab-2aAb^3-a^4B+2a^2b^2B-b^4B)\sin(c+dx)}{b^5d}+\frac{(a^2-2b}{}$$

```
[Out] (a^2-b^2)^2*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^6/d-(A*a^3*b-2*A*a*b^3-B*a^4+2*B
*a^2*b^2-B*b^4)*sin(d*x+c)/b^5/d+1/2*(a^2-2*b^2)*(A*b-B*a)*sin(d*x+c)^2/b^4
/d-1/3*(A*a*b-B*a^2+2*B*b^2)*sin(d*x+c)^3/b^3/d+1/4*(A*b-B*a)*sin(d*x+c)^4/
b^2/d+1/5*B*sin(d*x+c)^5/b/d
```

Rubi [A]
time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$\frac{(a^2-b^2)^2(Ab-aB)\log(a+b\sin(c+dx))}{b^6d}+\frac{(a^2-2b^2)(Ab-aB)\sin^2(c+dx)}{2b^4d}-\frac{(a^2(-B)+aAb+2b^2B)\sin^3(c+dx)}{3b^3d}-\frac{(a^4(-B)+a^3Ab+2a^2b^2B-2aAb^3-b^4B)\sin(c+dx)}{b^5d}+\frac{(Ab-aB)\sin^4(c+dx)}{4b^2d}+\frac{B\sin^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 - b^2)^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^6*d) - ((a^3*A*b - 2
*a*A*b^3 - a^4*B + 2*a^2*b^2*B - b^4*B)*Sin[c + d*x])/(b^5*d) + ((a^2 - 2*b
^2)*(A*b - a*B)*Sin[c + d*x]^2)/(2*b^4*d) - ((a*A*b - a^2*B + 2*b^2*B)*Sin[
c + d*x]^3)/(3*b^3*d) + ((A*b - a*B)*Sin[c + d*x]^4)/(4*b^2*d) + (B*Sin[c +
 d*x]^5)/(5*b*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)^2}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-a^3Ab+2aAb^3+a^4B-2a^2b^2B+b^4B}{b} - \frac{(-a^2+2b^2)(Ab-aB)x}{b} - \frac{(aAb-a\right.}{}$$

$$= \frac{(a^2-b^2)^2(Ab-aB)\log(a+b\sin(c+dx))}{b^6 d} - \frac{(a^3Ab-2aAb^3-a^4}{}$$

Mathematica [A]
time = 0.29, size = 148, normalized size = 0.73

$$\frac{20(Ab-aB)\left(3b^4\cos^4(c+dx)+12(a^2-b^2)^2\log(a+b\sin(c+dx))-12ab(a^2-2b^2)\sin(c+dx)+6b^2(a^2-b^2)\sin^2(c+dx)-4ab^3\sin^3(c+dx)\right)+b^5B(150\sin(c+dx)+25\sin(3(c+dx))+3\sin(5(c+dx)))}{240b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (20*(A*b - a*B)*(3*b^4*Cos[c + d*x]^4 + 12*(a^2 - b^2)^2*Log[a + b*Sin[c +
d*x]] - 12*a*b*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b^2*(a^2 - b^2)*Sin[c + d*x]^
2 - 4*a*b^3*Sin[c + d*x]^3) + b^5*B*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)]
 + 3*Sin[5*(c + d*x)]))/(240*b^6*d)
```

Maple [A]
time = 0.31, size = 283, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^5*(-1/5*B*sin(d*x+c)^5*b^4-1/4*A*b^4*sin(d*x+c)^4+1/4*B*a*b^3*sin
(d*x+c)^4+1/3*A*a*b^3*sin(d*x+c)^3-1/3*B*a^2*b^2*sin(d*x+c)^3+2/3*B*b^4*sin
(d*x+c)^3-1/2*A*a^2*b^2*sin(d*x+c)^2+A*b^4*sin(d*x+c)^2+1/2*B*a^3*b*sin(d*x
+c)^2-B*a*b^3*sin(d*x+c)^2+A*a^3*b*sin(d*x+c)-2*A*a*b^3*sin(d*x+c)-B*a^4*si
n(d*x+c)+2*B*a^2*b^2*sin(d*x+c)-B*b^4*sin(d*x+c))+(A*a^4*b-2*A*a^2*b^3+A*b^
5-B*a^5+2*B*a^3*b^2-B*a*b^4)/b^6*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 220, normalized size = 1.09

$$\frac{\frac{12\,Bb^4\sin(dx+c)^5-15\,(Bab^3-Ab^4)\sin(dx+c)^4+20\,(Ba^2b^2-Aab^3-2\,Bb^4)\sin(dx+c)^3-30\,(Ba^3b-Aa^2b^2-2\,Bab^3+2\,Ab^4)\sin(dx+c)^2+60\,(Ba^4-Aa^3b-2\,Ba^2b^2+2\,Aab^3+Bb^4)\sin(dx+c)}{b^5}-\frac{60\,(Ba^5-Aa^4b-2\,Ba^3b^2+2\,Aa^2b^3+Bab^4-Ab^5)\log(b\sin(dx+c)+a)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*(B*a*b^3 - A*b^4)*sin(d*x + c)^4 + 20*(
B*a^2*b^2 - A*a*b^3 - 2*B*b^4)*sin(d*x + c)^3 - 30*(B*a^3*b - A*a^2*b^2 - 2
*B*a*b^3 + 2*A*b^4)*sin(d*x + c)^2 + 60*(B*a^4 - A*a^3*b - 2*B*a^2*b^2 + 2*
A*a*b^3 + B*b^4)*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*
A*a^2*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/b^6)/d
```

Fricas [A]
time = 0.41, size = 222, normalized size = 1.10

$$-\frac{15\left(Bab^4-Ab^5\right)\cos\left(dx+c\right)^4-30\left(Ba^3b^2-Aa^2b^3-Bab^4+Ab^5\right)\cos\left(dx+c\right)^2+60\left(Ba^5-Aa^4b-2\,Ba^3b^2+2\,Aa^2b^3+Bab^4-Ab^5\right)\log\left(b\sin\left(dx+c\right)+a\right)-4\left(3\,Bb^5\cos\left(dx+c\right)^4+15\,Ba^4b-15\,Aa^3b^2-25\,Ba^2b^3+25\,Aab^4+8\,Bb^5-\left(5\,Ba^2b^3-5\,Aab^4-4\,Bb^5\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{60\,b^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/60*(15*(B*a*b^4 - A*b^5)*cos(d*x + c)^4 - 30*(B*a^3*b^2 - A*a^2*b^3 - B*
a*b^4 + A*b^5)*cos(d*x + c)^2 + 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2
*b^3 + B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a) - 4*(3*B*b^5*cos(d*x + c)^4
 + 15*B*a^4*b - 15*A*a^3*b^2 - 25*B*a^2*b^3 + 25*A*a*b^4 + 8*B*b^5 - (5*B*a
^2*b^3 - 5*A*a*b^4 - 4*B*b^5)*cos(d*x + c)^2)*sin(d*x + c))/(b^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.54, size = 286, normalized size = 1.42

$$\frac{\frac{12\,Bb^4\sin(dx+c)^5-15\,Bab^3\sin(dx+c)^4+15\,Ab^4\sin(dx+c)^4+20\,Ba^2b^2\sin(dx+c)^3-20\,Aab^3\sin(dx+c)^3-40\,Bb^4\sin(dx+c)^3-30\,Ba^3b\sin(dx+c)^2+30\,Aa^2b^2\sin(dx+c)^2+60\,Bab^3\sin(dx+c)^2-60\,Ab^4\sin(dx+c)^2+60\,Ba^4\sin(dx+c)-60\,Aa^3b\sin(dx+c)-120\,Ba^2b^2\sin(dx+c)+120\,Aab^3\sin(dx+c)+60\,Bb^4\sin(dx+c)}{b^5}-\frac{60\left(Ba^5-Aa^4b-2\,Ba^3b^2+2\,Aa^2b^3+Bab^4-Ab^5\right)\log(|b\sin(dx+c)+a|)}{b^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/60*((12*B*b^4*sin(d*x + c)^5 - 15*B*a*b^3*sin(d*x + c)^4 + 15*A*b^4*sin(d
*x + c)^4 + 20*B*a^2*b^2*sin(d*x + c)^3 - 20*A*a*b^3*sin(d*x + c)^3 - 40*B*
b^4*sin(d*x + c)^3 - 30*B*a^3*b*sin(d*x + c)^2 + 30*A*a^2*b^2*sin(d*x + c)^
2 + 60*B*a*b^3*sin(d*x + c)^2 - 60*A*b^4*sin(d*x + c)^2 + 60*B*a^4*sin(d*x
+ c) - 60*A*a^3*b*sin(d*x + c) - 120*B*a^2*b^2*sin(d*x + c) + 120*A*a*b^3*s
```

```
in(d*x + c) + 60*B*b^4*sin(d*x + c))/b^5 - 60*(B*a^5 - A*a^4*b - 2*B*a^3*b^
2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)*log(abs(b*sin(d*x + c) + a))/b^6)/d
```

Mupad [B]
time = 0.10, size = 253, normalized size = 1.25

$$\frac{\sin\left(c+d\,x\right)^4\,\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^2}\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^2\,\left(\frac{A}{b}-\frac{a\,\left(\frac{2\,B}{b}+\frac{a\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{2\,b}\right)}{d}-\frac{{\sin\left(c+d\,x\right)}^3\,\left(\frac{2\,B}{3\,b}+\frac{a\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{3\,b}\right)}{d}+\frac{\sin\left(c+d\,x\right)\,\left(\frac{B}{b}+\frac{a\,\left(\frac{2\,A}{b}-\frac{a\,\left(\frac{2\,B}{b}+\frac{a\,\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)}{d}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(-B\,a^5+A\,a^4\,b+2\,B\,a^3\,b^2-2\,A\,a^2\,b^3-B\,a\,b^4+A\,b^5\right)}{b^6\,d}+\frac{B\,{\sin\left(c+d\,x\right)}^5}{5\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)^4*(A/(4*b) - (B*a)/(4*b^2)))/d - (sin(c + d*x)^2*(A/b - (a*((
2*B)/b + (a*(A/b - (B*a)/b^2))/b))/(2*b)))/d - (sin(c + d*x)^3*((2*B)/(3*b)
 + (a*(A/b - (B*a)/b^2))/(3*b)))/d + (sin(c + d*x)*(B/b + (a*((2*A)/b - (a*
((2*B)/b + (a*(A/b - (B*a)/b^2))/b))/b))/b))/d + (log(a + b*sin(c + d*x))*(
A*b^5 - B*a^5 - 2*A*a^2*b^3 + 2*B*a^3*b^2 + A*a^4*b - B*a*b^4))/(b^6*d) + (
B*sin(c + d*x)^5)/(5*b*d)
```

3.1546 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=111

$$-\frac{(a^2-b^2)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}+\frac{(aAb-a^2B+b^2B)\sin(c+dx)}{b^3d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-\frac{B}{}$$

```
[Out] -(a^2-b^2)*(A*b-B*a)*ln(a+b*sin(d*x+c))/b^4/d+(A*a*b-B*a^2+B*b^2)*sin(d*x+c
)/b^3/d-1/2*(A*b-B*a)*sin(d*x+c)^2/b^2/d-1/3*B*sin(d*x+c)^3/b/d
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$-\frac{(a^2-b^2)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}+\frac{(a^2(-B)+aAb+b^2B)\sin(c+dx)}{b^3d}-\frac{(Ab-aB)\sin^2(c+dx)}{2b^2d}-\frac{B\sin^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^4*d)) + ((a*A*b - a^
2*B + b^2*B)*Sin[c + d*x])/(b^3*d) - ((A*b - a*B)*Sin[c + d*x]^2)/(2*b^2*d)
 - (B*Sin[c + d*x]^3)/(3*b*d)
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)}{a+x}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{aAb-a^2B+b^2B}{b}+\frac{(-Ab+aB)x}{b}-\frac{Bx^2}{b}+\frac{(-a^2+b^2)(Ab-aB)}{b(a+x)}\right)dx, x\right.}{b^3d}$$
$$= -\frac{(a^2-b^2)(Ab-aB)\log(a+b\sin(c+dx))}{b^4d}+\frac{(aAb-a^2B+b^2B)}{b^3d}$$

Mathematica [A]
time = 0.27, size = 89, normalized size = 0.80

$$\frac{\left(A-\frac{aB}{b}\right)\left((-a^2+b^2)\log(a+b\sin(c+dx))+ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)\right)+\frac{1}{12}b^2B(9\sin(c+dx)+\sin(3(c+dx)))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A - (a*B)/b)*((-a^2 + b^2)*Log[a + b*Sin[c + d*x]] + a*b*Sin[c + d*x] - (
b^2*Sin[c + d*x]^2)/2) + (b^2*B*(9*Sin[c + d*x] + Sin[3*(c + d*x)]))/12)/(b
^3*d)
```

Maple [A]
time = 0.28, size = 125, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{-\frac{B\left(\sin^3(dx+c)\right)b^2}{3}-\frac{Ab^2\left(\sin^2(dx+c)\right)}{2}+\frac{Bab\left(\sin^2(dx+c)\right)}{2}+Aab\sin(dx+c)-Ba^2\sin(dx+c)+Bb^2\sin(dx+c)}{b^3}+\frac{\left(-Aa^2b+Ab^3+Ba^3\right.}{}}{d}$
default	$\frac{\frac{-\frac{B\left(\sin^3(dx+c)\right)b^2}{3}-\frac{Ab^2\left(\sin^2(dx+c)\right)}{2}+\frac{Bab\left(\sin^2(dx+c)\right)}{2}+Aab\sin(dx+c)-Ba^2\sin(dx+c)+Bb^2\sin(dx+c)}{b^3}+\frac{\left(-Aa^2b+Ab^3+Ba^3\right.}{}}{d}$
norman	$\frac{\frac{2\left(9Aab-9Ba^2+5Bb^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{2\left(9Aab-9Ba^2+5Bb^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}-\frac{2(2Ab-2aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}+\frac{2\left(Aab-Ba^2+B\right.}{b}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$-\frac{e^{-2i(dx+c)}Ba}{8b^2d}+\frac{ie^{i(dx+c)}Ba^2}{2b^3d}-\frac{2iBa^3c}{d\,b^4}+\frac{ie^{-i(dx+c)}Aa}{2b^2d}+\frac{2iBac}{d\,b^2}-\frac{3ie^{i(dx+c)}B}{8bd}-\frac{iBa^3x}{b^4}+\frac{\sin(3dx+3c}{12bd}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*(-1/3*B*sin(d*x+c)^3*b^2-1/2*A*b^2*sin(d*x+c)^2+1/2*B*a*b*sin(d*
x+c)^2+A*a*b*sin(d*x+c)-B*a^2*sin(d*x+c)+B*b^2*sin(d*x+c))+(-A*a^2*b+A*b^3+
B*a^3-B*a*b^2)/b^4*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 112, normalized size = 1.01

$$-\frac{\frac{2\,Bb^2\sin(dx+c)^3-3\,(Bab-Ab^2)\sin(dx+c)^2+6\,(Ba^2-Aab-Bb^2)\sin(dx+c)}{b^3}-\frac{6\,(Ba^3-Aa^2b-Bab^2+Ab^3)\log(b\sin(dx+c)+a)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*(B*a*b - A*b^2)*sin(d*x + c)^2 + 6*(B*a^2
 - A*a*b - B*b^2)*sin(d*x + c))/b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)
*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A]
time = 0.38, size = 112, normalized size = 1.01

$$-\frac{3\,(Bab^2-Ab^3)\cos(dx+c)^2-6\,(Ba^3-Aa^2b-Bab^2+Ab^3)\log(b\sin(dx+c)+a)-2\,(Bb^3\cos(dx+c)^2-3\,Ba^2b+3\,Aab^2+2\,Bb^3)\sin(dx+c)}{6\,b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(3*(B*a*b^2 - A*b^3)*cos(d*x + c)^2 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A
*b^3)*log(b*sin(d*x + c) + a) - 2*(B*b^3*cos(d*x + c)^2 - 3*B*a^2*b + 3*A*a
*b^2 + 2*B*b^3)*sin(d*x + c))/(b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 129, normalized size = 1.16

$$-\frac{\frac{2\,Bb^2\sin(dx+c)^3-3\,Bab\sin(dx+c)^2+3\,Ab^2\sin(dx+c)^2+6\,Ba^2\sin(dx+c)-6\,Aab\sin(dx+c)-6\,Bb^2\sin(dx+c)}{b^3}-\frac{6\,(Ba^3-Aa^2b-Bab^2+Ab^3)\log(|b\sin(dx+c)+a|)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*((2*B*b^2*sin(d*x + c)^3 - 3*B*a*b*sin(d*x + c)^2 + 3*A*b^2*sin(d*x +
c)^2 + 6*B*a^2*sin(d*x + c) - 6*A*a*b*sin(d*x + c) - 6*B*b^2*sin(d*x + c))/
b^3 - 6*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*sin(d*x + c) + a))/b^
4)/d
```

Mupad [B]
time = 0.08, size = 122, normalized size = 1.10

$$\frac{\sin(c+dx)\left(\frac{B}{b}+\frac{a\left(\frac{A}{b}-\frac{Ba}{b^2}\right)}{b}\right)}{d}-\frac{\sin(c+dx)^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)}{d}+\frac{\ln(a+b\sin(c+dx))\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)}{b^4d}-\frac{B\sin(c+dx)^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (sin(c + d*x)*(B/b + (a*(A/b - (B*a)/b^2))/b))/d - (sin(c + d*x)^2*(A/(2*b)
 - (B*a)/(2*b^2)))/d + (log(a + b*sin(c + d*x))*(A*b^3 + B*a^3 - A*a^2*b -
B*a*b^2))/(b^4*d) - (B*sin(c + d*x)^3)/(3*b*d)
```

3.1547 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

```
[Out] (A*b-B*a)*ln(a+b*sin(d*x+c))/b^2/d+B*sin(d*x+c)/b/d
```

Rubi [A]

time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {2912, 45}

$$\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/(b^2*d) + (B*Sin[c + d*x])/(b*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{a+x}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+x)}\right)\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{(Ab-aB)\log(a+b\sin(c+dx))}{b^2d}+\frac{B\sin(c+dx)}{bd}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 39, normalized size = 0.95

$$\frac{\frac{(Ab-aB)\log(a+b\sin(c+dx))}{b}+B\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
[Out] (((A*b - a*B)*Log[a + b*Sin[c + d*x]])/b + B*Sin[c + d*x])/(b*d)
```

Maple [A]
time = 0.15, size = 40, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{B\sin(dx+c)}{b}+\frac{(Ab-aB)\ln(a+b\sin(dx+c))}{b^2}}{d}$
default	$\frac{\frac{B\sin(dx+c)}{b}+\frac{(Ab-aB)\ln(a+b\sin(dx+c))}{b^2}}{d}$
norman	$\frac{\frac{2B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}+\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{(Ab-aB)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{b^2d}-\frac{(Ab-aB)\ln\left(1+\tan^2\left(\frac{d}{2}\right.\right.}{b^2d}$
risch	$-\frac{iAx}{b}+\frac{iBax}{b^2}-\frac{ie^{i(dx+c)}B}{2bd}+\frac{ie^{-i(dx+c)}B}{2bd}-\frac{2iAc}{db}+\frac{2iBac}{db^2}+\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)A}{db}-\frac{\ln\left(e^{2i(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(B/b*sin(d*x+c)+(A*b-B*a)/b^2*ln(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.98

$$\frac{\frac{B\sin(dx+c)}{b}-\frac{(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(b*sin(d*x + c) + a)/b^2)/d
```

Fricas [A]
time = 0.37, size = 38, normalized size = 0.93

$$\frac{Bb\sin(dx+c)-(Ba-Ab)\log(b\sin(dx+c)+a)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] (B*b*sin(d*x + c) - (B*a - A*b)*log(b*sin(d*x + c) + a))/(b^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(34) = 68$.
time = 0.35, size = 104, normalized size = 2.54

$$\begin{cases} \frac{x(A+B\sin(c))\cos(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\frac{A\sin(c+dx)}{d}+\frac{B\sin^2(c+dx)}{2d}}{a} & \text{for } b=0 \\ \frac{x(A+B\sin(c))\cos(c)}{a+b\sin(c)} & \text{for } d=0 \\ \frac{A\log\left(\frac{a}{b}+\sin(c+dx)\right)}{bd} - \frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{b^2d} + \frac{B\sin(c+dx)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a, Eq(b, 0) & Eq(d, 0)), ((A*sin(c + d*x
)/d + B*sin(c + d*x)**2/(2*d))/a, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)/(a +
b*sin(c)), Eq(d, 0)), (A*log(a/b + sin(c + d*x))/(b*d) - B*a*log(a/b + sin(
c + d*x))/(b**2*d) + B*sin(c + d*x)/(b*d), True))
```

Giac [A]
time = 0.47, size = 41, normalized size = 1.00

$$\frac{\frac{B\sin(dx+c)}{b} - \frac{(Ba-Ab)\log(|b\sin(dx+c)+a|)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*sin(d*x + c)/b - (B*a - A*b)*log(abs(b*sin(d*x + c) + a))/b^2)/d
```

Mupad [B]
time = 0.06, size = 41, normalized size = 1.00

$$\frac{B\,\sin\left(c+d\,x\right)}{b\,d}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(A\,b-B\,a\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x)),x)
```

```
[Out] (B*sin(c + d*x))/(b*d) + (log(a + b*sin(c + d*x))*(A*b - B*a))/(b^2*d)
```

3.1548 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)d}+\frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)d}-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] -1/2*(A+B)*ln(1-sin(d*x+c))/(a+b)/d+1/2*(A-B)*ln(1+sin(d*x+c))/(a-b)/d-(A*b
-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 815}

$$-\frac{(Ab-aB)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/2*((A + B)*Log[1 - Sin[c + d*x]])/((a + b)*d) + ((A - B)*Log[1 + Sin[c +
 d*x]])/(2*(a - b)*d) - ((A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*
d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)(b-x)} + \frac{-Ab+aB}{(a-b)b(a+b)(a+x)} + \frac{A-B}{2(a-b)b(b+x)}\right)\,dx, x, b\sin(\right.}{d}$$
$$= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)d} + \frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)d} -$$

Mathematica [**A**]
time = 0.14, size = 99, normalized size = 1.10

$$\frac{-\left((A+B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{(a+b)(A-B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+(-Ab+aB)\log(a+b\sin(c+dx))}{a-b}}{(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-((A + B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]) + ((a + b)*(A - B)*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (-(A*b) + a*B)*Log[a + b*Sin[c + d*
x]])/(a - b))/((a + b)*d)
```

Maple [**A**]
time = 0.29, size = 89, normalized size = 0.99

method	result
derivativedivides	$\frac{\frac{(-A-B)\ln(\sin(dx+c)-1)}{2a+2b}-\frac{(Ab-aB)\ln(a+b\sin(dx+c))}{(a+b)(a-b)}+\frac{(A-B)\ln(1+\sin(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{(-A-B)\ln(\sin(dx+c)-1)}{2a+2b}-\frac{(Ab-aB)\ln(a+b\sin(dx+c))}{(a+b)(a-b)}+\frac{(A-B)\ln(1+\sin(dx+c))}{2a-2b}}{d}$
norman	$\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a-b)} - \frac{(A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d(a+b)} - \frac{(Ab-aB)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}{d(a^2-b^2)}$
risch	$\frac{iBx}{a-b} + \frac{iAc}{(a+b)d} + \frac{iBc}{d(a-b)} + \frac{iBc}{(a+b)d} + \frac{2iAbc}{d(a^2-b^2)} - \frac{2iaBx}{a^2-b^2} + \frac{iAx}{a+b} - \frac{2iaBc}{d(a^2-b^2)} + \frac{2iAbx}{a^2-b^2} - \frac{iAc}{d(a-b)} + \frac{iBx}{a+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((-A-B)/(2*a+2*b)*ln(sin(d*x+c)-1)-(A*b-B*a)/(a+b)/(a-b)*ln(a+b*sin(d*x
+c))+(A-B)/(2*a-2*b)*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.88

$$\frac{\frac{2\,(Ba-Ab)\log(b\sin(dx+c)+a)}{a^2-b^2}+\frac{(A-B)\log(\sin(dx+c)+1)}{a-b}-\frac{(A+B)\log(\sin(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a)/(a^2 - b^2) + (A - B)*log(sin(d*
x + c) + 1)/(a - b) - (A + B)*log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A]
time = 0.61, size = 88, normalized size = 0.98

$$\frac{2\,(Ba-Ab)\log\left(b\sin\left(dx+c\right)+a\right)+((A-B)a+(A-B)b)\log\left(\sin\left(dx+c\right)+1\right)-((A+B)a-(A+B)b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a - A*b)*log(b*sin(d*x + c) + a) + ((A - B)*a + (A - B)*b)*log(si
n(d*x + c) + 1) - ((A + B)*a - (A + B)*b)*log(-sin(d*x + c) + 1))/((a^2 - b
^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\sin\left(c+dx\right))\sec\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 87, normalized size = 0.97

$$\frac{\frac{2\,(Bab-Ab^2)\log(|b\sin(dx+c)+a|)}{a^2b-b^3}+\frac{(A-B)\log(|\sin(dx+c)+1|)}{a-b}-\frac{(A+B)\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(B*a*b - A*b^2)*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) + (A - B)
*log(abs(sin(d*x + c) + 1))/(a - b) - (A + B)*log(abs(sin(d*x + c) - 1))/(a
 + b))/d
```

Mupad [B]
time = 0.31, size = 89, normalized size = 0.99

$$\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{A}{2}-\frac{B}{2}\right)}{d\,\left(a-b\right)}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(A\,b-B\,a\right)}{d\,\left(a^2-b^2\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{A}{2}+\frac{B}{2}\right)}{d\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(A/2 - B/2))/(d*(a - b)) - (log(a + b*sin(c + d*x))*
(A*b - B*a))/(d*(a^2 - b^2)) - (log(sin(c + d*x) - 1)*(A/2 + B/2))/(d*(a +
b))
```

3.1549 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=159

$$-\frac{(aA+b(2A+B))\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(aA-b(2A-B))\log(1+\sin(c+dx))}{4(a-b)^2d}+\frac{b^2(Ab-aB)\log(a+b\sin}{(a^2-b^2)^2\,d}$$

```
[Out] -1/4*(a*A+b*(2*A+B))*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a*A-b*(2*A-B))*ln(1+si
n(d*x+c))/(a-b)^2/d+b^2*(A*b-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d-1/2*sec(
d*x+c)^2*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

$$\frac{b^2(Ab-aB)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d\left(a^2-b^2\right)}-\frac{(aA+b(2A+B))\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(aA-b(2A-B))\log(\sin(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/4*((a*A + b*(2*A + B))*Log[1 - Sin[c + d*x]])/((a + b)^2*d) + ((a*A - b*
(2*A - B))*Log[1 + Sin[c + d*x]])/(4*(a - b)^2*d) + (b^2*(A*b - a*B)*Log[a
+ b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(A*b - a*B - (a*A -
b*B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^3\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2(a^2-b^2)d} - \frac{b\text{Subst}\left(\int \frac{-}{}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2(a^2-b^2)d} - \frac{b\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{(aA+b(2A+B))\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(aA-b(2A-B))\log}{4(a-b}$$

Mathematica [A]

time = 0.54, size = 197, normalized size = 1.24

$$\frac{-\frac{2(aA+b(2A+B))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^2}+\frac{2(aA+b(-2A+B))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^2}+\frac{4b^2(Ab-aB)\log(a+b\sin(c+dx))}{(a^2-b^2)^2}+\frac{A+B}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{-A+B}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(a*A + b*(2*A + B))*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^
2 + (2*(a*A + b*(-2*A + B))*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a -
b)^2 + (4*b^2*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (A + B)/
((a + b)*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^2))/(4*d)
```

Maple [A]

time = 0.43, size = 149, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{A+B}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-aA-2Ab-Bb)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{b^2(Ab-aB)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{A-B}{(4a-4b)(1+\sin(dx+c))}+\frac{(aA-2Ab-}{}}{d}$
default	$\frac{-\frac{A+B}{(4a+4b)(\sin(dx+c)-1)}+\frac{(-aA-2Ab-Bb)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{b^2(Ab-aB)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{A-B}{(4a-4b)(1+\sin(dx+c))}+\frac{(aA-2Ab-}{}}{d}$

norman	$\frac{\frac{(aA-Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2-b^2\right)}+\frac{(aA-Bb)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}+\frac{2(aA-Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}-\frac{(2Ab-2aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}-\frac{(2Ab-2aB)\left(\tan^4\left(\frac{}{}\right.\right.}{d\left(a^2-b^2\right)}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
risch	$\frac{iaAx}{2a^2+4ab+2b^2}+\frac{iBbx}{2a^2+4ab+2b^2}+\frac{iAbc}{(a^2+2ab+b^2)d}+\frac{iAbc}{d(a^2-2ab+b^2)}+\frac{iaAc}{2(a^2+2ab+b^2)d}+\frac{iBbc}{2(a^2+2ab+b^2)d}-\frac{}{2d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-(A+B)/(4*a+4*b)/(sin(d*x+c)-1)+1/4/(a+b)^2*(-A*a-2*A*b-B*b)*ln(sin(d*
x+c)-1)+b^2*(A*b-B*a)/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-(A-B)/(4*a-4*b)/(1
+sin(d*x+c))+1/4*(A*a-2*A*b+B*b)/(a-b)^2*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 175, normalized size = 1.10

$$-\frac{\frac{4\,(Bab^2-Ab^3)\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{(Aa-(2\,A-B)b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{(Aa+(2\,A+B)b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(Ba-Ab+(Aa-Bb)\sin(dx+c))}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/4*(4*(B*a*b^2 - A*b^3)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) -
 (A*a - (2*A - B)*b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) + (A*a + (2*
A + B)*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 2*(B*a - A*b + (A*a -
 B*b)*sin(d*x + c))/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A]
time = 0.60, size = 234, normalized size = 1.47

$$\frac{2\,Ba^3-2\,Aa^2b-2\,Bab^2+2\,Ab^3-4\,(Bab^2-Ab^3)\cos(dx+c)^2\log(b\sin(dx+c)+a)+(Aa^3+Ba^2b-(3\,A-2\,B)ab^2-(2\,A-B)b^3)\cos(dx+c)^2\log(\sin(dx+c)+1)-(Aa^3+Ba^2b-(3\,A+2\,B)ab^2+(2\,A+B)b^3)\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\,(Aa^3-Ba^2b-Aab^2+Bb^3)\sin(dx+c)}{4\,(a^4-2\,a^2b^2+b^4)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 4*(B*a*b^2 - A*b^3)*cos(d*
x + c)^2*log(b*sin(d*x + c) + a) + (A*a^3 + B*a^2*b - (3*A - 2*B)*a*b^2 - (
2*A - B)*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (A*a^3 + B*a^2*b - (3*
A + 2*B)*a*b^2 + (2*A + B)*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(
A*a^3 - B*a^2*b - A*a*b^2 + B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d
*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^3(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 260, normalized size = 1.64

$$-\frac{\frac{4\,(Bab^3-Ab^4)\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}+\frac{(Aa+2\,Ab+Bb)\log(|-\sin(dx+c)+1|)}{a^2+2\,ab+b^2}-\frac{(Aa-2\,Ab+Bb)\log(|-\sin(dx+c)-1|)}{a^2-2\,ab+b^2}+\frac{2\left(Bab^2\sin(dx+c)^2-Ab^3\sin(dx+c)^2+Aa^3\sin(dx+c)-Ba^2b\sin(dx+c)-Aab^2\sin(dx+c)+Bb^3\sin(dx+c)+Ba^3-Aa^2b-2\,Bab^2+2\,Ab^3\right)}{(a^4-2\,a^2b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/4*(4*(B*a*b^3 - A*b^4)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 +
 b^5) + (A*a + 2*A*b + B*b)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
 - (A*a - 2*A*b + B*b)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) + 2*
(B*a*b^2*sin(d*x + c)^2 - A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c) - B*a^2
*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^3 - A*a^2
*b - 2*B*a*b^2 + 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

Mupad [B]
time = 0.52, size = 197, normalized size = 1.24

$$\frac{\ln(a+b\sin(c+d\,x))\,(A\,b^3-B\,a\,b^2)}{d\,(a^4-2\,a^2\,b^2+b^4)}-\frac{\frac{A\,b-B\,a}{2\,(a^2-b^2)}-\frac{\sin(c+d\,x)\,(A\,a-B\,b)}{2\,(a^2-b^2)}}{d\cos(c+d\,x)^2}+\frac{\ln(\sin(c+d\,x)+1)\,(A\,a-b\,(2\,A-B))}{d\,(4\,a^2-8\,a\,b+4\,b^2)}-\frac{\ln(\sin(c+d\,x)-1)\,(A\,a+b\,(2\,A+B))}{d\,(4\,a^2+8\,a\,b+4\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(a + b*sin(c + d*x))*(A*b^3 - B*a*b^2))/(d*(a^4 + b^4 - 2*a^2*b^2)) - (
(A*b - B*a)/(2*(a^2 - b^2)) - (sin(c + d*x)*(A*a - B*b))/(2*(a^2 - b^2)))/(
d*cos(c + d*x)^2) + (log(sin(c + d*x) + 1)*(A*a - b*(2*A - B)))/(d*(4*a^2 -
 8*a*b + 4*b^2)) - (log(sin(c + d*x) - 1)*(A*a + b*(2*A + B)))/(d*(8*a*b +
4*a^2 + 4*b^2))
```

3.1550 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=263

$$-\frac{(3a^2A+ab(9A+B)+b^2(8A+3B))\log(1-\sin(c+dx))}{16(a+b)^3d}+\frac{(3a^2A+b^2(8A-3B)-ab(9A-B))\log(1+\mathrm{s}}{16(a-b)^3d}$$

```
[Out] -1/16*(3*a^2*A+a*b*(9*A+B)+b^2*(8*A+3*B))*ln(1-sin(d*x+c))/(a+b)^3/d+1/16*(
3*a^2*A+b^2*(8*A-3*B)-a*b*(9*A-B))*ln(1+sin(d*x+c))/(a-b)^3/d-b^4*(A*b-B*a)
*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/4*sec(d*x+c)^4*(A*b-a*B-(A*a-B*b)*sin(d
*x+c))/(a^2-b^2)/d+1/8*sec(d*x+c)^2*(4*b^2*(A*b-B*a)+(3*A*a^3-7*A*a*b^2+B*a
^2*b+3*B*b^3)*sin(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A]

time = 0.32, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

$$-\frac{(3a^2A+ab(9A+B)+b^2(8A+3B))\log(1-\sin(c+dx))}{16d(a+b)^3}+\frac{(3a^2A-ab(9A-B)+b^2(8A-3B))\log(\sin(c+dx)+1)}{16d(a-b)^3}-\frac{\sec^4(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{4d\,(a^2-b^2)}-\frac{b^4(Ab-aB)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^3}+\frac{\sec^2(c+dx)\left((3a^3A+a^2bB-7aAb^2+3b^3B)\sin(c+dx)+4b^2(Ab-aB)\right)}{8d\,(a^2-b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/16*((3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[1 - Sin[c + d*x]])/(
(a + b)^3*d) + ((3*a^2*A + b^2*(8*A - 3*B) - a*b*(9*A - B))*Log[1 + Sin[c +
 d*x]])/(16*(a - b)^3*d) - (b^4*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2
- b^2)^3*d) - (Sec[c + d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a
^2 - b^2)*d) + (Sec[c + d*x]^2*(4*b^2*(A*b - a*B) + (3*a^3*A - 7*a*A*b^2 +
a^2*b*B + 3*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
```

```
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^5\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d} - \frac{b^3\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d} + \frac{\sec^2(c+dx}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d} + \frac{\sec^2(c+dx}{}$$

$$= -\frac{(3a^2A+ab(9A+B)+b^2(8A+3B))\log(1-\sin(c+dx))}{16(a+b)^3d} + \frac{(3}{}$$

Mathematica [A]

time = 0.89, size = 321, normalized size = 1.22

$$\frac{-\frac{2(3a^2A+ab(9A+B)+b^2(8A+3B))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^3}+\frac{2(3a^2A+b^2(8A-3B)+ab(-9A+B))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^3}+\frac{16b^4(Ab-aB)\log(a+b\sin(c+dx))}{(-a^2+b^2)^3}+\frac{A+B}{(a+b)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{3aA+5Ab+aB+3bB}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{-A+B}{(a-b)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}+\frac{-3aA+5Ab+aB-3bB}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-2*(3*a^2*A + a*b*(9*A + B) + b^2*(8*A + 3*B))*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]])/(a + b)^3 + (2*(3*a^2*A + b^2*(8*A - 3*B) + a*b*(-9*A + B))
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(a - b)^3 + (16*b^4*(A*b - a*B)*
Log[a + b*Sin[c + d*x]])/(-a^2 + b^2)^3 + (A + B)/((a + b)*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^4) + (3*a*A + 5*A*b + a*B + 3*b*B)/((a + b)^2*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^2) + (-A + B)/((a - b)*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^4) + (-3*a*A + 5*A*b + a*B - 3*b*B)/((a - b)^2*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^2))/(16*d)
```

Maple [A]
time = 0.66, size = 256, normalized size = 0.97

method	result
derivativedivides	$-\frac{-A-B}{2(8a+8b)(\sin(dx+c)-1)^2}-\frac{3aA+5Ab+aB+3Bb}{16(a+b)^2(\sin(dx+c)-1)}+\frac{\left(-3a^2A-9Aab-8A\,b^2-Bab-3B\,b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^3}-\frac{(Ab-aB)b^4\ln(a+b\sin}{(a+b)^3(a-b)}$ ... over d
default	$-\frac{-A-B}{2(8a+8b)(\sin(dx+c)-1)^2}-\frac{3aA+5Ab+aB+3Bb}{16(a+b)^2(\sin(dx+c)-1)}+\frac{\left(-3a^2A-9Aab-8A\,b^2-Bab-3B\,b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^3}-\frac{(Ab-aB)b^4\ln(a+b\sin}{(a+b)^3(a-b)}$ ... over d
norman	$\frac{2(aA-Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}+\frac{2(aA-Bb)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2-b^2\right)}+\frac{\left(5a^3A-9Aa\,b^2-B\,a^2b+5B\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\left(a^4-2a^2b^2+b^4\right)}+\frac{\left(5a^3A-9Aa\,b^2-B\,a^2b+5B\right.}{4d\left(a^4-2a^2\right.}$ ...
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*(-A-B)/(8*a+8*b)/(sin(d*x+c)-1)^2-1/16*(3*A*a+5*A*b+B*a+3*B*b)/(a
+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-3*A*a^2-9*A*a*b-8*A*b^2-B*a*b-3*B*b^2)*
ln(sin(d*x+c)-1)-(A*b-B*a)*b^4/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2*(A-B)
/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(3*A*a-5*A*b-B*a+3*B*b)/(a-b)^2/(1+sin(d*x
+c))+1/16*(3*A*a^2-9*A*a*b+8*A*b^2+B*a*b-3*B*b^2)/(a-b)^3*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 367, normalized size = 1.40

$$\frac{\frac{16\left(Bab^4-Ab^5\right)\log(b\sin(dx+c)+a)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{\left(3Aa^2-(9A-B)ab+(8A-3B)b^2\right)\log(\sin(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}-\frac{\left(3Aa^2+(9A+B)ab+(8A+3B)b^2\right)\log(\sin(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}+\frac{2\left(2Ba^3-2Aa^2b-6Bab^2+6Ab^3-\left(3Aa^3+Ba^2b-7Aab^2+3Bb^3\right)\sin(dx+c)^3+4\left(Bab^2-Ab^3\right)\sin(dx+c)^2+\left(5Aa^3-Ba^2b-9Aab^2+5Bb^3\right)\sin(dx+c)\right)}{\left(a^4-2a^2b^2+b^4\right)\sin(dx+c)^4+a^4-2a^2b^2+b^4-2\left(a^4-2a^2b^2+b^4\right)\sin(dx+c)^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/16*(16*(B*a*b^4 - A*b^5)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6) + (3*A*a^2 - (9*A - B)*a*b + (8*A - 3*B)*b^2)*log(sin(d*x + c)
+ 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (3*A*a^2 + (9*A + B)*a*b + (8*A + 3*
B)*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(2*B*a^3
- 2*A*a^2*b - 6*B*a*b^2 + 6*A*b^3 - (3*A*a^3 + B*a^2*b - 7*A*a*b^2 + 3*B*b^
3)*sin(d*x + c)^3 + 4*(B*a*b^2 - A*b^3)*sin(d*x + c)^2 + (5*A*a^3 - B*a^2*b
 - 9*A*a*b^2 + 5*B*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)
^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 1.12, size = 413, normalized size = 1.57

$$\frac{4Ba^5-4Aa^4b-8Ba^3b^2+8Aa^2b^3+4Bab^4-4Ab^5+16\left(Bab^4-Ab^5\right)\cos(dx+c)^4\log(b\sin(dx+c)+a)+\left(3Aa^5+Ba^4b-10Aa^3b^2-6Ba^2b^3+(15A-8B)ab^4+(8A-3B)b^5\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-\left(3Aa^5+Ba^4b-10Aa^3b^2-6Ba^2b^3+(15A+8B)ab^4-(8A+3B)b^5\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)-8\left(Ba^3b^2-Aa^2b^3-Bab^4+Ab^5\right)\cos(dx+c)^2+2\left(2Aa^5-2Ba^4b-4Aa^3b^2+4Ba^2b^3+2Aab^4-2Bb^5+\left(3Aa^5+Ba^4b-10Aa^3b^2+2Ba^2b^3+7Aab^4-3Bb^5\right)\cos(dx+c)^2\right)\sin(dx+c)}{16\left(a^6-3a^4b^2+3a^2b^4-b^6\right)d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/16*(4*B*a^5 - 4*A*a^4*b - 8*B*a^3*b^2 + 8*A*a^2*b^3 + 4*B*a*b^4 - 4*A*b^5
 + 16*(B*a*b^4 - A*b^5)*cos(d*x + c)^4*log(b*sin(d*x + c) + a) + (3*A*a^5 +
 B*a^4*b - 10*A*a^3*b^2 - 6*B*a^2*b^3 + (15*A - 8*B)*a*b^4 + (8*A - 3*B)*b^
5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2
 - 6*B*a^2*b^3 + (15*A + 8*B)*a*b^4 - (8*A + 3*B)*b^5)*cos(d*x + c)^4*log(-
sin(d*x + c) + 1) - 8*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c
)^2 + 2*(2*A*a^5 - 2*B*a^4*b - 4*A*a^3*b^2 + 4*B*a^2*b^3 + 2*A*a*b^4 - 2*B*
b^5 + (3*A*a^5 + B*a^4*b - 10*A*a^3*b^2 + 2*B*a^2*b^3 + 7*A*a*b^4 - 3*B*b^5
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(
d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^5(c + dx)}{a + b\sin(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(254) = 508.
time = 0.53, size = 539, normalized size = 2.05

$$\frac{\frac{16\,(Bab^5-Ab^6)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{(3\,Aa^2+9\,Aab+Bab+8\,Ab^2+3\,Bb^2)\log(|-\sin(dx+c)+1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{(3\,Aa^2-9\,Aab+Bab+8\,Ab^2-3\,Bb^2)\log(|-\sin(dx+c)-1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{2\left(6\,Bab^4\sin(dx+c)^4-6\,Ab^5\sin(dx+c)^4-3\,Aa^5\sin(dx+c)^3-Ba^4b\sin(dx+c)^3+10\,Aa^3b^2\sin(dx+c)^3-2\,Ba^2b^3\sin(dx+c)^3-7\,Aab^4\sin(dx+c)^3+3\,Bb^5\sin(dx+c)^3+4\,Ba^3b^2\sin(dx+c)^2-4\,Aa^2b^3\sin(dx+c)^2-16\,Bab^4\sin(dx+c)^2+16\,Ab^5\sin(dx+c)^2+5\,Aa^5\sin(dx+c)-Ba^4b\sin(dx+c)-14\,Aa^3b^2\sin(dx+c)+6\,Ba^2b^3\sin(dx+c)+9\,Aab^4\sin(dx+c)-5\,Bb^5\sin(dx+c)+2\,Ba^5-2\,Aa^4b-8\,Ba^3b^2+8\,Aa^2b^3+12\,Bab^4-12\,Ab^5\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(\sin(dx+c)^2-1\right)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/16*(16*(B*a*b^5 - A*b^6)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
+ 3*a^2*b^5 - b^7) - (3*A*a^2 + 9*A*a*b + B*a*b + 8*A*b^2 + 3*B*b^2)*log(ab
s(-sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (3*A*a^2 - 9*A*a*b
+ B*a*b + 8*A*b^2 - 3*B*b^2)*log(abs(-sin(d*x + c) - 1))/(a^3 - 3*a^2*b + 3
*a*b^2 - b^3) + 2*(6*B*a*b^4*sin(d*x + c)^4 - 6*A*b^5*sin(d*x + c)^4 - 3*A*
a^5*sin(d*x + c)^3 - B*a^4*b*sin(d*x + c)^3 + 10*A*a^3*b^2*sin(d*x + c)^3 -
 2*B*a^2*b^3*sin(d*x + c)^3 - 7*A*a*b^4*sin(d*x + c)^3 + 3*B*b^5*sin(d*x +
c)^3 + 4*B*a^3*b^2*sin(d*x + c)^2 - 4*A*a^2*b^3*sin(d*x + c)^2 - 16*B*a*b^4
*sin(d*x + c)^2 + 16*A*b^5*sin(d*x + c)^2 + 5*A*a^5*sin(d*x + c) - B*a^4*b*
```

```
sin(d*x + c) - 14*A*a^3*b^2*sin(d*x + c) + 6*B*a^2*b^3*sin(d*x + c) + 9*A*a
*b^4*sin(d*x + c) - 5*B*b^5*sin(d*x + c) + 2*B*a^5 - 2*A*a^4*b - 8*B*a^3*b^
2 + 8*A*a^2*b^3 + 12*B*a*b^4 - 12*A*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 12.95, size = 427, normalized size = 1.62

$$\frac{\frac{B\,a^3-A\,a^2\,b-3\,B\,a\,b^2+3\,A\,b^3}{4\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{\sin(c+d\,x)\,\left(5\,A\,a^3-B\,a^2\,b-9\,A\,a\,b^2+5\,B\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{\sin(c+d\,x)^2\,\left(A\,b^3-B\,a\,b^2\right)}{2\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{\sin(c+d\,x)^3\,\left(3\,A\,a^3+B\,a^2\,b-7\,A\,a\,b^2+3\,B\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left({\cos\left(c+d\,x\right)}^2+{\sin\left(c+d\,x\right)}^4-{\sin\left(c+d\,x\right)}^2\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(3\,A\,a^2+\left(9\,A+B\right)\,a\,b+\left(8\,A+3\,B\right)\,b^2\right)}{d\,\left(16\,a^3+48\,a^2\,b+48\,a\,b^2+16\,b^3\right)}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(A\,b^5-B\,a\,b^4\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(3\,A\,a^2+\left(B-9\,A\right)\,a\,b+\left(8\,A-3\,B\right)\,b^2\right)}{d\,\left(16\,a^3-48\,a^2\,b+48\,a\,b^2-16\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + b*sin(c + d*x))),x)
```

```
[Out] ((3*A*b^3 + B*a^3 - A*a^2*b - 3*B*a*b^2)/(4*(a^4 + b^4 - 2*a^2*b^2)) + (sin
(c + d*x)*(5*A*a^3 + 5*B*b^3 - 9*A*a*b^2 - B*a^2*b))/(8*(a^4 + b^4 - 2*a^2*
b^2)) - (sin(c + d*x)^2*(A*b^3 - B*a*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2)) - (s
in(c + d*x)^3*(3*A*a^3 + 3*B*b^3 - 7*A*a*b^2 + B*a^2*b))/(8*(a^4 + b^4 - 2*
a^2*b^2)))/(d*(cos(c + d*x)^2 - sin(c + d*x)^2 + sin(c + d*x)^4)) - (log(si
n(c + d*x) - 1)*(3*A*a^2 + b^2*(8*A + 3*B) + a*b*(9*A + B)))/(d*(48*a*b^2 +
 48*a^2*b + 16*a^3 + 16*b^3)) - (log(a + b*sin(c + d*x))*(A*b^5 - B*a*b^4))
/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (log(sin(c + d*x) + 1)*(3*A*a^2
+ b^2*(8*A - 3*B) - a*b*(9*A - B)))/(d*(48*a*b^2 - 48*a^2*b + 16*a^3 - 16*b
^3))
```

3.1551 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=383

$$-\frac{(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B))\log(1-\sin(c+dx))}{32(a+b)^4d}+\frac{(5a^3A-b^3(16A-5B)$$

```
[Out] -1/32*(5*a^3*A+a^2*b*(20*A+B)+a*b^2*(29*A+4*B)+b^3*(16*A+5*B))*ln(1-sin(d*x
+c))/(a+b)^4/d+1/32*(5*a^3*A-b^3*(16*A-5*B)+a*b^2*(29*A-4*B)-a^2*b*(20*A-B)
)*ln(1+sin(d*x+c))/(a-b)^4/d+b^6*(A*b-B*a)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d
-1/6*sec(d*x+c)^6*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d+1/24*sec(d*x+c
)^4*(6*b^2*(A*b-B*a)+(5*A*a^3-11*A*a*b^2+B*a^2*b+5*B*b^3)*sin(d*x+c))/(a^2-
b^2)^2/d-1/16*sec(d*x+c)^2*(8*b^4*(A*b-B*a)-(5*A*a^5-16*A*a^3*b^2+19*A*a*b^
4+B*a^4*b-4*B*a^2*b^3-5*B*b^5)*sin(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A]
time = 0.48, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

$$-\frac{\sec^6(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{6d(a^2-b^2)}+\frac{b^6(Ab-aB)\log(a+b\sin(c+dx))}{d(a^2-b^2)^4}-\frac{(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B))\log(1-\sin(c+dx))}{32d(a+b)^4}+\frac{(5a^3A-a^2b(20A-B)+ab^2(29A-4B)-b^3(16A-5B))\log(\sin(c+dx)+1)}{32d(a-b)^4}+\frac{\sec^4(c+dx)((5a^3A+a^2bB-11aAb^2+5b^3B)\sin(c+dx)+6b^2(Ab-aB))}{24d(a^2-b^2)^2}-\frac{\sec^2(c+dx)(8b^4(Ab-aB)-(5a^5A+a^4bB-16a^3Ab^2-4a^2b^3B+19aAb^4-5b^5B)\sin(c+dx))}{16d(a^2-b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] -1/32*((5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))
*Log[1 - Sin[c + d*x]])/((a + b)^4*d) + ((5*a^3*A - b^3*(16*A - 5*B) + a*b^
2*(29*A - 4*B) - a^2*b*(20*A - B))*Log[1 + Sin[c + d*x]])/(32*(a - b)^4*d)
+ (b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (Sec[c + d*
x]^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*d) + (Sec[c + d
*x]^4*(6*b^2*(A*b - a*B) + (5*a^3*A - 11*a*A*b^2 + a^2*b*B + 5*b^3*B)*Sin[c
 + d*x]))/(24*(a^2 - b^2)^2*d) - (Sec[c + d*x]^2*(8*b^4*(A*b - a*B) - (5*a^
5*A - 16*a^3*A*b^2 + 19*a*A*b^4 + a^4*b*B - 4*a^2*b^3*B - 5*b^5*B)*Sin[c +
d*x]))/(16*(a^2 - b^2)^3*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
```

```
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{a+b\sin(c+dx)}\,dx = \frac{b^7\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)(b^2-x^2)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)d} - \frac{b^5\text{Subst}\left(\int \frac{-5}{}\right.}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)d} + \frac{\sec^4(c+dx)(}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)d} + \frac{\sec^4(c+dx)(}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)d} + \frac{\sec^4(c+dx)(}{}$$

$$= -\frac{(5a^3A+a^2b(20A+B)+ab^2(29A+4B)+b^3(16A+5B))\log(1}{32(a+b)^4d}$$

Mathematica [A]
time = 1.70, size = 565, normalized size = 1.48

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-48*(5*a^3*A + a^2*b*(20*A + B) + a*b^2*(29*A + 4*B) + b^3*(16*A + 5*B))*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(a + b)^4 + (48*(5*a^3*A + a*b^2*
(29*A - 4*B) + a^2*b*(-20*A + B) + b^3*(-16*A + 5*B))*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]])/(a - b)^4 + (768*b^6*(A*b - a*B)*Log[a + b*Sin[c + d*x
]])/(a^2 - b^2)^4 + (Sec[c + d*x]^6*(-128*a^4*A*b + 352*a^2*A*b^3 - 368*A*b
^5 + 128*a^5*B - 352*a^3*b^2*B + 368*a*b^4*B - 96*b^2*(a^2 - 3*b^2)*(-(A*b)
 + a*B)*Cos[2*(c + d*x)] - 48*b^4*(A*b - a*B)*Cos[4*(c + d*x)] + 198*a^5*A*
Sin[c + d*x] - 480*a^3*A*b^2*Sin[c + d*x] + 330*a*A*b^4*Sin[c + d*x] - 114*
a^4*b*B*Sin[c + d*x] + 264*a^2*b^3*B*Sin[c + d*x] - 198*b^5*B*Sin[c + d*x]
+ 85*a^5*A*Sin[3*(c + d*x)] - 272*a^3*A*b^2*Sin[3*(c + d*x)] + 259*a*A*b^4*
Sin[3*(c + d*x)] + 17*a^4*b*B*Sin[3*(c + d*x)] - 4*a^2*b^3*B*Sin[3*(c + d*x
)] - 85*b^5*B*Sin[3*(c + d*x)] + 15*a^5*A*Sin[5*(c + d*x)] - 48*a^3*A*b^2*S
in[5*(c + d*x)] + 57*a*A*b^4*Sin[5*(c + d*x)] + 3*a^4*b*B*Sin[5*(c + d*x)]
- 12*a^2*b^3*B*Sin[5*(c + d*x)] - 15*b^5*B*Sin[5*(c + d*x)]))/(a^2 - b^2)^3
)/(768*d)
```

Maple [A]
time = 1.18, size = 393, normalized size = 1.03

method	result
derivativedivides	$-\frac{A+B}{3(16a+16b)(\sin(dx+c)-1)^3}-\frac{-2aA-3Ab-aB-2Bb}{32(a+b)^2(\sin(dx+c)-1)^2}-\frac{5a^2A+14Aab+11Ab^2+Ba^2+4Bab+5Bb^2}{32(a+b)^3(\sin(dx+c)-1)}+\frac{(-5a^3A-20Aa^2b-29Aab^2$
default	$-\frac{A+B}{3(16a+16b)(\sin(dx+c)-1)^3}-\frac{-2aA-3Ab-aB-2Bb}{32(a+b)^2(\sin(dx+c)-1)^2}-\frac{5a^2A+14Aab+11Ab^2+Ba^2+4Bab+5Bb^2}{32(a+b)^3(\sin(dx+c)-1)}+\frac{(-5a^3A-20Aa^2b-29Aab^2$
norman	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*(A+B)/(16*a+16*b)/(sin(d*x+c)-1)^3-1/32*(-2*A*a-3*A*b-B*a-2*B*b)/
(a+b)^2/(sin(d*x+c)-1)^2-1/32*(5*A*a^2+14*A*a*b+11*A*b^2+B*a^2+4*B*a*b+5*B*
b^2)/(a+b)^3/(sin(d*x+c)-1)+1/32/(a+b)^4*(-5*A*a^3-20*A*a^2*b-29*A*a*b^2-16
*A*b^3-B*a^2*b-4*B*a*b^2-5*B*b^3)*ln(sin(d*x+c)-1)+(A*b-B*a)*b^6/(a+b)^4/(a
-b)^4*ln(a+b*sin(d*x+c))-1/3*(A-B)/(16*a-16*b)/(1+sin(d*x+c))^3-1/32*(2*A*a
-3*A*b-B*a+2*B*b)/(a-b)^2/(1+sin(d*x+c))^2-1/32*(5*A*a^2-14*A*a*b+11*A*b^2-
B*a^2+4*B*a*b-5*B*b^2)/(a-b)^3/(1+sin(d*x+c))+1/32*(5*A*a^3-20*A*a^2*b+29*A
*a*b^2-16*A*b^3+B*a^2*b-4*B*a*b^2+5*B*b^3)/(a-b)^4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 632, normalized size = 1.65

$$-\frac{\frac{96\,(Bab^6-Ab^7)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\,(5\,Aa^3-(20\,A-B)a^2b+(29\,A-4\,B)ab^2-(16\,A-5\,B)b^3)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{3\,(5\,Aa^3+(20\,A+B)a^2b+(29\,A+4\,B)ab^2+(16\,A+5\,B)b^3)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(8\,Ba^5-8\,Aa^4b-28\,Ba^3b^2+28\,Aa^2b^3+44\,Bab^4-44\,Ab^5+3\,(5\,Aa^5+Ba^4b-16\,Aa^3b^2-4\,Ba^2b^3+19\,Aab^4-5\,Bb^5)\sin(dx+c)^5+24\,(Bab^4-Ab^5)\sin(dx+c)^4-8\,(5\,Aa^5+Ba^4b-16\,Aa^3b^2-2\,Ba^2b^3+17\,Aab^4-5\,Bb^5)\sin(dx+c)^3+12\,(Ba^3b^2-Aa^2b^3-5\,Bab^4+5\,Ab^5)\sin(dx+c)^2+3\,(11\,Aa^5-Ba^4b-32\,Aa^3b^2+4\,Ba^2b^3+29\,Aab^4-11\,Bb^5)\sin(dx+c)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sin(dx+c)^6-a^6+3\,a^4b^2-3\,a^2b^4+b^6-3\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sin(dx+c)^4+3\,(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sin(dx+c)^2}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(96*(B*a*b^6 - A*b^7)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^
4*b^4 - 4*a^2*b^6 + b^8) - 3*(5*A*a^3 - (20*A - B)*a^2*b + (29*A - 4*B)*a*b
^2 - (16*A - 5*B)*b^3)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4
*a*b^3 + b^4) + 3*(5*A*a^3 + (20*A + B)*a^2*b + (29*A + 4*B)*a*b^2 + (16*A
+ 5*B)*b^3)*log(sin(d*x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4) + 2*(8*B*a^5 - 8*A*a^4*b - 28*B*a^3*b^2 + 28*A*a^2*b^3 + 44*B*a*b^4 - 44
*A*b^5 + 3*(5*A*a^5 + B*a^4*b - 16*A*a^3*b^2 - 4*B*a^2*b^3 + 19*A*a*b^4 - 5
*B*b^5)*sin(d*x + c)^5 + 24*(B*a*b^4 - A*b^5)*sin(d*x + c)^4 - 8*(5*A*a^5 +
 B*a^4*b - 16*A*a^3*b^2 - 2*B*a^2*b^3 + 17*A*a*b^4 - 5*B*b^5)*sin(d*x + c)^
3 + 12*(B*a^3*b^2 - A*a^2*b^3 - 5*B*a*b^4 + 5*A*b^5)*sin(d*x + c)^2 + 3*(11
*A*a^5 - B*a^4*b - 32*A*a^3*b^2 + 4*B*a^2*b^3 + 29*A*a*b^4 - 11*B*b^5)*sin(
d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^6 - a^6 + 3*a^4
*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sin(d*x + c)^2))/d
```

Fricas [A]
time = 2.45, size = 643, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/96*(16*B*a^7 - 16*A*a^6*b - 48*B*a^5*b^2 + 48*A*a^4*b^3 + 48*B*a^3*b^4 -
48*A*a^2*b^5 - 16*B*a*b^6 + 16*A*b^7 - 96*(B*a*b^6 - A*b^7)*cos(d*x + c)^6*
log(b*sin(d*x + c) + a) + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3
 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A - 16*B)*a*b^6 - (16*A - 5*B)*b^7)*co
s(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 -
5*B*a^4*b^3 + 35*A*a^3*b^4 + 15*B*a^2*b^5 - (35*A + 16*B)*a*b^6 + (16*A + 5
*B)*b^7)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 48*(B*a^3*b^4 - A*a^2*b^5
- B*a*b^6 + A*b^7)*cos(d*x + c)^4 - 24*(B*a^5*b^2 - A*a^4*b^3 - 2*B*a^3*b^4
 + 2*A*a^2*b^5 + B*a*b^6 - A*b^7)*cos(d*x + c)^2 + 2*(8*A*a^7 - 8*B*a^6*b -
 24*A*a^5*b^2 + 24*B*a^4*b^3 + 24*A*a^3*b^4 - 24*B*a^2*b^5 - 8*A*a*b^6 + 8*
B*b^7 + 3*(5*A*a^7 + B*a^6*b - 21*A*a^5*b^2 - 5*B*a^4*b^3 + 35*A*a^3*b^4 -
B*a^2*b^5 - 19*A*a*b^6 + 5*B*b^7)*cos(d*x + c)^4 + 2*(5*A*a^7 + B*a^6*b - 2
1*A*a^5*b^2 + 3*B*a^4*b^3 + 27*A*a^3*b^4 - 9*B*a^2*b^5 - 11*A*a*b^6 + 5*B*b
^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6
 + b^8)*d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(373) = 746$.
time = 0.54, size = 907, normalized size = 2.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/96*(96*(B*a*b^7 - A*b^8)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) + 3*(5*A*a^3 + 20*A*a^2*b + B*a^2*b + 29*A*
a*b^2 + 4*B*a*b^2 + 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 3*(5*A*a^3 - 20*A*a^2*b + B*a^2*b +
29*A*a*b^2 - 4*B*a*b^2 - 16*A*b^3 + 5*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a
^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 2*(44*B*a*b^6*sin(d*x + c)^6 -
44*A*b^7*sin(d*x + c)^6 + 15*A*a^7*sin(d*x + c)^5 + 3*B*a^6*b*sin(d*x + c)^
5 - 63*A*a^5*b^2*sin(d*x + c)^5 - 15*B*a^4*b^3*sin(d*x + c)^5 + 105*A*a^3*b
^4*sin(d*x + c)^5 - 3*B*a^2*b^5*sin(d*x + c)^5 - 57*A*a*b^6*sin(d*x + c)^5
+ 15*B*b^7*sin(d*x + c)^5 + 24*B*a^3*b^4*sin(d*x + c)^4 - 24*A*a^2*b^5*sin(
d*x + c)^4 - 156*B*a*b^6*sin(d*x + c)^4 + 156*A*b^7*sin(d*x + c)^4 - 40*A*a
^7*sin(d*x + c)^3 - 8*B*a^6*b*sin(d*x + c)^3 + 168*A*a^5*b^2*sin(d*x + c)^3
 + 24*B*a^4*b^3*sin(d*x + c)^3 - 264*A*a^3*b^4*sin(d*x + c)^3 + 24*B*a^2*b^
5*sin(d*x + c)^3 + 136*A*a*b^6*sin(d*x + c)^3 - 40*B*b^7*sin(d*x + c)^3 + 1
2*B*a^5*b^2*sin(d*x + c)^2 - 12*A*a^4*b^3*sin(d*x + c)^2 - 72*B*a^3*b^4*sin
(d*x + c)^2 + 72*A*a^2*b^5*sin(d*x + c)^2 + 192*B*a*b^6*sin(d*x + c)^2 - 19
2*A*b^7*sin(d*x + c)^2 + 33*A*a^7*sin(d*x + c) - 3*B*a^6*b*sin(d*x + c) - 1
29*A*a^5*b^2*sin(d*x + c) + 15*B*a^4*b^3*sin(d*x + c) + 183*A*a^3*b^4*sin(d
*x + c) - 45*B*a^2*b^5*sin(d*x + c) - 87*A*a*b^6*sin(d*x + c) + 33*B*b^7*si
n(d*x + c) + 8*B*a^7 - 8*A*a^6*b - 36*B*a^5*b^2 + 36*A*a^4*b^3 + 72*B*a^3*b
^4 - 72*A*a^2*b^5 - 88*B*a*b^6 + 88*A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 -
4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^3))/d
```

Mupad [**B**]
time = 13.44, size = 729, normalized size = 1.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + b*sin(c + d*x))),x)
```

```
[Out] (log(sin(c + d*x) + 1)*(5*A*a^3 - b^3*(16*A - 5*B) - a^2*b*(20*A - B) + a*b
^2*(29*A - 4*B)))/(d*(32*a^4 - 128*a^3*b - 128*a*b^3 + 32*b^4 + 192*a^2*b^2
)) - ((11*A*b^5 - 2*B*a^5 - 7*A*a^2*b^3 + 7*B*a^3*b^2 + 2*A*a^4*b - 11*B*a*
b^4)/(12*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)^4*(A*b^5 - B*
a*b^4))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)*(11*A*a^5 -
 11*B*b^5 - 32*A*a^3*b^2 + 4*B*a^2*b^3 + 29*A*a*b^4 - B*a^4*b))/(16*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^2*(5*A*b^5 - A*a^2*b^3 + B*a^
3*b^2 - 5*B*a*b^4))/(4*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)
^3*(5*A*a^5 - 5*B*b^5 - 16*A*a^3*b^2 - 2*B*a^2*b^3 + 17*A*a*b^4 + B*a^4*b))
/(6*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^5*(5*A*a^5 - 5*B*b
^5 - 16*A*a^3*b^2 - 4*B*a^2*b^3 + 19*A*a*b^4 + B*a^4*b))/(16*(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2)))/(d*(cos(c + d*x)^2 - 2*sin(c + d*x)^2 + 3*sin(c + d
*x)^4 - sin(c + d*x)^6)) - (log(sin(c + d*x) - 1)*(5*A*a^3 + b^3*(16*A + 5*
B) + a*b^2*(29*A + 4*B) + a^2*b*(20*A + B)))/(d*(128*a*b^3 + 128*a^3*b + 32
*a^4 + 32*b^4 + 192*a^2*b^2)) + (log(a + b*sin(c + d*x))*(A*b^7 - B*a*b^6))
/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.1552 $\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=324

$$\frac{(a^2-b^2)^2\,(6aAb-7a^2B+b^2B)\log(a+b\sin(c+dx))}{b^8d}-\frac{(5a^4Ab-9a^2Ab^3+3Ab^5-6a^5B+12a^3b^2B-6ab}{b^7d}$$

```
[Out] (a^2-b^2)^2*(6*A*a*b-7*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^8/d-(5*A*a^4*b-9*A
*a^2*b^3+3*A*b^5-6*B*a^5+12*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/b^7/d+1/2*(4*A*
a^3*b-6*A*a*b^3-5*B*a^4+9*B*a^2*b^2-3*B*b^4)*sin(d*x+c)^2/b^6/d-1/3*(3*A*a^
2*b-3*A*b^3-4*B*a^3+6*B*a*b^2)*sin(d*x+c)^3/b^5/d+1/4*(2*A*a*b-3*B*a^2+3*B*
b^2)*sin(d*x+c)^4/b^4/d-1/5*(A*b-2*B*a)*sin(d*x+c)^5/b^3/d-1/6*B*sin(d*x+c)
^6/b^2/d+(a^2-b^2)^3*(A*b-B*a)/b^8/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.28, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$\frac{(a^2-b^2)^3\,(Ab-aB)}{b^8d(a+b\sin(c+dx))}+\frac{(a^2-b^2)^2\,(-7a^2B+6aAb+b^2B)\log(a+b\sin(c+dx))}{b^8d}+\frac{(-3a^2B+2aAb+3b^2B)\sin^4(c+dx)}{4b^4d}-\frac{(-4a^3B+3a^2Ab+6ab^2B-3Ab^3)\sin^3(c+dx)}{3b^5d}+\frac{(-5a^4B+4a^3Ab+9a^2b^2B-6aAb^3-3b^4B)\sin^2(c+dx)}{2b^6d}-\frac{(-6a^5B+5a^4Ab+12a^3b^2B-9a^2Ab^3-6ab^4B+3Ab^5)\sin(c+dx)}{b^7d}-\frac{(Ab-2aB)\sin^5(c+dx)}{5b^3d}-\frac{B\sin^6(c+dx)}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)^2*(6*a*A*b - 7*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^8*d)
 - ((5*a^4*A*b - 9*a^2*A*b^3 + 3*A*b^5 - 6*a^5*B + 12*a^3*b^2*B - 6*a*b^4*B
)*Sin[c + d*x])/(b^7*d) + ((4*a^3*A*b - 6*a*A*b^3 - 5*a^4*B + 9*a^2*b^2*B -
 3*b^4*B)*Sin[c + d*x]^2)/(2*b^6*d) - ((3*a^2*A*b - 3*A*b^3 - 4*a^3*B + 6*a
*b^2*B)*Sin[c + d*x]^3)/(3*b^5*d) + ((2*a*A*b - 3*a^2*B + 3*b^2*B)*Sin[c +
d*x]^4)/(4*b^4*d) - ((A*b - 2*a*B)*Sin[c + d*x]^5)/(5*b^3*d) - (B*Sin[c + d
*x]^6)/(6*b^2*d) + ((a^2 - b^2)^3*(A*b - a*B))/(b^8*d*(a + b*Sin[c + d*x]))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)^3}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-5a^4Ab+9a^2Ab^3-3Ab^5+6a^5B-12a^3b^2B+6ab^4B}{b} - \frac{(-4a^3Ab+6aAb^3+5a \ldots}{\ldots}\right.\right.}{\ldots}$$

$$= \frac{(a^2-b^2)^2\,(6aAb-7a^2B+b^2B)\log(a+b\sin(c+dx))}{b^8 d} - \frac{(5a^4Ab - \ldots}{\ldots}$$

Mathematica [A]
time = 1.08, size = 396, normalized size = 1.22

$$\frac{B\left(15b^4(-a^2+b^2)\cos^4(c+dx)+10b^6\cos^6(c+dx)-60(a^2-b^2)^3\log(a+b\sin(c+dx))+60ab(a^4-3a^2b^2+3b^4)\sin(c+dx)-30b^2(a^2-b^2)^2\sin^2(c+dx)+20ab^3(a^2-3b^2)\sin^3(c+dx)+12ab^5\sin^5(c+dx)\right)+\frac{6(Ab-aB)\left(2b^6\cos^6(c+dx)+4(a^2-b^2)^2(4a^2-4b^2+15a^2\log(a+b\sin(c+dx)))+4ab\left(-11a^4+18a^2b^2-4b^4+15(a^2-b^2)^2\log(a+b\sin(c+dx))\right)\sin(c+dx)-2b^2(15a^4-29a^2b^2+8b^4)\sin^2(c+dx)+2ab^3(5a^2-7b^2)\sin^3(c+dx)-4a^2b^4\sin^4(c+dx)+b^4\cos^4(c+dx)(-a^2+4b^2+3ab\sin(c+dx))\right)}{a+b\sin(c+dx)}}{60b^8 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(15*b^4*(-a^2 + b^2)*Cos[c + d*x]^4 + 10*b^6*Cos[c + d*x]^6 - 60*(a^2 -
b^2)^3*Log[a + b*Sin[c + d*x]] + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Sin[c + d
*x] - 30*b^2*(a^2 - b^2)^2*Sin[c + d*x]^2 + 20*a*b^3*(a^2 - 3*b^2)*Sin[c +
d*x]^3 + 12*a*b^5*Sin[c + d*x]^5) + (6*(A*b - a*B)*(2*b^6*Cos[c + d*x]^6 +
4*(a^2 - b^2)^2*(4*a^2 - 4*b^2 + 15*a^2*Log[a + b*Sin[c + d*x]]) + 4*a*b*(-
11*a^4 + 18*a^2*b^2 - 4*b^4 + 15*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])*Sin
[c + d*x] - 2*b^2*(15*a^4 - 29*a^2*b^2 + 8*b^4)*Sin[c + d*x]^2 + 2*a*b^3*(5
*a^2 - 7*b^2)*Sin[c + d*x]^3 - 4*a^2*b^4*Sin[c + d*x]^4 + b^4*Cos[c + d*x]^
4*(-a^2 + 4*b^2 + 3*a*b*Sin[c + d*x])))/(a + b*Sin[c + d*x]))/(60*b^8*d)
```

Maple [A]
time = 0.51, size = 467, normalized size = 1.44

method	result
derivativedivides	$-\frac{\frac{B\left(\sin^6(dx+c)\right)b^5}{6}+\frac{A\,b^5\left(\sin^5(dx+c)\right)}{5}-\frac{2Ba\,b^4\left(\sin^5(dx+c)\right)}{5}-\frac{Aa\,b^4\left(\sin^4(dx+c)\right)}{2}+\frac{3B\,a^2b^3\left(\sin^4(dx+c)\right)}{4}-\frac{3B\,b^5\left(\sin^4(dx+c)\right)}{4}+\ldots}{\ldots}$
default	$-\frac{\frac{B\left(\sin^6(dx+c)\right)b^5}{6}+\frac{A\,b^5\left(\sin^5(dx+c)\right)}{5}-\frac{2Ba\,b^4\left(\sin^5(dx+c)\right)}{5}-\frac{Aa\,b^4\left(\sin^4(dx+c)\right)}{2}+\frac{3B\,a^2b^3\left(\sin^4(dx+c)\right)}{4}-\frac{3B\,b^5\left(\sin^4(dx+c)\right)}{4}+\ldots}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/b^7*(1/6*B*sin(d*x+c)^6*b^5+1/5*A*b^5*sin(d*x+c)^5-2/5*B*a*b^4*sin(
d*x+c)^5-1/2*A*a*b^4*sin(d*x+c)^4+3/4*B*a^2*b^3*sin(d*x+c)^4-3/4*B*b^5*sin(
d*x+c)^4+A*a^2*b^3*sin(d*x+c)^3-A*b^5*sin(d*x+c)^3-4/3*B*a^3*b^2*sin(d*x+c)
^3+2*B*a*b^4*sin(d*x+c)^3-2*A*a^3*b^2*sin(d*x+c)^2+3*A*a*b^4*sin(d*x+c)^2+5
/2*B*a^4*b*sin(d*x+c)^2-9/2*B*a^2*b^3*sin(d*x+c)^2+3/2*B*b^5*sin(d*x+c)^2+5
*A*a^4*b*sin(d*x+c)-9*A*a^2*b^3*sin(d*x+c)+3*A*b^5*sin(d*x+c)-6*B*a^5*sin(d
*x+c)+12*B*a^3*b^2*sin(d*x+c)-6*B*a*b^4*sin(d*x+c))+1/b^8*(6*A*a^5*b-12*A*a
^3*b^3+6*A*a*b^5-7*B*a^6+15*B*a^4*b^2-9*B*a^2*b^4+B*b^6)*ln(a+b*sin(d*x+c))
-1/b^8*(-A*a^6*b+3*A*a^4*b^3-3*A*a^2*b^5+A*b^7+B*a^7-3*B*a^5*b^2+3*B*a^3*b^
4-B*a*b^6)/(a+b*sin(d*x+c)))
```

<u>Maxima</u> [A]
time = 0.29, size = 377, normalized size = 1.16

$$-\frac{\frac{60\,(Ba^7-Aa^6b-3\,Ba^5b^2+3\,Aa^4b^3+3\,Ba^3b^4-3\,Aa^2b^5-Bab^6+Ab^7)}{b^9\sin(dx+c)+ab^8}+\frac{10\,Bb^5\sin(dx+c)^6-12\,(2\,Bab^4-Ab^5)\sin(dx+c)^5+15\,(3\,Ba^2b^3-2\,Aab^4-3\,Bb^5)\sin(dx+c)^4-20\,(4\,Ba^3b^2-3\,Aa^2b^3-6\,Bab^4+3\,Ab^5)\sin(dx+c)^3+30\,(5\,Ba^4b-4\,Aa^3b^2-9\,Ba^2b^3+6\,Aab^4+3\,Bb^5)\sin(dx+c)^2-60\,(6\,Ba^5-5\,Aa^4b-12\,Ba^3b^2+9\,Aa^2b^3+6\,Bab^4-3\,Ab^5)\sin(dx+c)}{b^7}+\frac{60\,(7\,Ba^6-6\,Aa^5b-15\,Ba^4b^2+12\,Aa^3b^3+9\,Ba^2b^4-6\,Aab^5-Bb^6)\log(b\sin(dx+c)+a)}{b^8}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/60*(60*(B*a^7 - A*a^6*b - 3*B*a^5*b^2 + 3*A*a^4*b^3 + 3*B*a^3*b^4 - 3*A*
a^2*b^5 - B*a*b^6 + A*b^7)/(b^9*sin(d*x + c) + a*b^8) + (10*B*b^5*sin(d*x +
 c)^6 - 12*(2*B*a*b^4 - A*b^5)*sin(d*x + c)^5 + 15*(3*B*a^2*b^3 - 2*A*a*b^4
 - 3*B*b^5)*sin(d*x + c)^4 - 20*(4*B*a^3*b^2 - 3*A*a^2*b^3 - 6*B*a*b^4 + 3*
A*b^5)*sin(d*x + c)^3 + 30*(5*B*a^4*b - 4*A*a^3*b^2 - 9*B*a^2*b^3 + 6*A*a*b
^4 + 3*B*b^5)*sin(d*x + c)^2 - 60*(6*B*a^5 - 5*A*a^4*b - 12*B*a^3*b^2 + 9*A
*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*sin(d*x + c))/b^7 + 60*(7*B*a^6 - 6*A*a^5*b
 - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4 - 6*A*a*b^5 - B*b^6)*log(b*sin
(d*x + c) + a)/b^8)/d
```

<u>Fricas</u> [A]
time = 0.44, size = 508, normalized size = 1.57

$$-\frac{480\,Ba^7-480\,Aa^6b-3720\,Ba^5b^2+3360\,Aa^4b^3+5705\,Ba^3b^4-4710\,Aa^2b^5-2402\,Bab^6+1536\,Ab^7+16\,(7\,Bab^6-6\,Ab^7)\cos(dx+c)^6-8\,(35\,Ba^3b^4-30\,Aa^2b^5-33\,Bab^6+24\,Ab^7)\cos(dx+c)^4+16\,(105\,Ba^5b^2-90\,Aa^4b^3-190\,Ba^3b^4+150\,Aa^2b^5+81\,Bab^6-48\,Ab^7)\cos(dx+c)^2+480\,(7\,Ba^7-6\,Aa^6b-15\,Ba^5b^2+12\,Aa^4b^3+9\,Ba^3b^4-6\,Aa^2b^5-Bab^6+(7\,Ba^6b-6\,Aa^5b^2-15\,Ba^4b^3+12\,Aa^3b^4+9\,Ba^2b^5-6\,Aab^6-Bb^7)\sin(dx+c))\log(b\sin(dx+c)+a)-(80\,Bb^7\cos(dx+c)^6+2880\,Ba^6b-2400\,Aa^5b^2-5720\,Ba^4b^3+4320\,Aa^3b^4+2967\,Ba^2b^5-1626\,Aab^6-190\,Bb^7-24\,(7\,Ba^2b^5-6\,Aab^6-5\,Bb^7)\cos(dx+c)^4+16\,(35\,Ba^4b^3-30\,Aa^3b^4-54\,Ba^2b^5+42\,Aab^6+15\,Bb^7)\cos(dx+c)^2)\sin(dx+c)}{480\,(b^9d\sin(dx+c)+ab^8d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/480*(480*B*a^7 - 480*A*a^6*b - 3720*B*a^5*b^2 + 3360*A*a^4*b^3 + 5705*B*
a^3*b^4 - 4710*A*a^2*b^5 - 2402*B*a*b^6 + 1536*A*b^7 + 16*(7*B*a*b^6 - 6*A*
b^7)*cos(d*x + c)^6 - 8*(35*B*a^3*b^4 - 30*A*a^2*b^5 - 33*B*a*b^6 + 24*A*b^
7)*cos(d*x + c)^4 + 16*(105*B*a^5*b^2 - 90*A*a^4*b^3 - 190*B*a^3*b^4 + 150*
```

```
A*a^2*b^5 + 81*B*a*b^6 - 48*A*b^7)*cos(d*x + c)^2 + 480*(7*B*a^7 - 6*A*a^6*
b - 15*B*a^5*b^2 + 12*A*a^4*b^3 + 9*B*a^3*b^4 - 6*A*a^2*b^5 - B*a*b^6 + (7*
B*a^6*b - 6*A*a^5*b^2 - 15*B*a^4*b^3 + 12*A*a^3*b^4 + 9*B*a^2*b^5 - 6*A*a*b
^6 - B*b^7)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (80*B*b^7*cos(d*x + c)^
6 + 2880*B*a^6*b - 2400*A*a^5*b^2 - 5720*B*a^4*b^3 + 4320*A*a^3*b^4 + 2967*
B*a^2*b^5 - 1626*A*a*b^6 - 190*B*b^7 - 24*(7*B*a^2*b^5 - 6*A*a*b^6 - 5*B*b^
7)*cos(d*x + c)^4 + 16*(35*B*a^4*b^3 - 30*A*a^3*b^4 - 54*B*a^2*b^5 + 42*A*a
*b^6 + 15*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/(b^9*d*sin(d*x + c) + a*b^8*
d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 570, normalized size = 1.76

$$-\frac{\frac{60\,(7\,Ba^6-6\,Aa^5b-15\,Ba^4b^2+12\,Aa^3b^3+9\,Ba^2b^4-6\,Aab^5-Bb^6)\log\left(|b\sin(dx+c)+a|\right)}{b^8}-\frac{60\,(7\,Ba^6b\sin(dx+c)-6\,Aa^5b^2\sin(dx+c)-15\,Ba^4b^3\sin(dx+c)+12\,Aa^3b^4\sin(dx+c)+9\,Ba^2b^5\sin(dx+c)-6\,Aab^6\sin(dx+c)-Bb^7\sin(dx+c)+6\,Ba^7-5\,Aa^6b-12\,Ba^5b^2+9\,Aa^4b^3+6\,Ba^3b^4-3\,Aa^2b^5-Ab^7)}{(b\sin(dx+c)+a)b^8}+\frac{10\,Bb^{10}\sin(dx+c)^6-24\,Bab^9\sin(dx+c)^5+12\,Ab^{10}\sin(dx+c)^5+45\,Ba^2b^8\sin(dx+c)^4-30\,Aab^9\sin(dx+c)^4-45\,Bb^{10}\sin(dx+c)^4-80\,Ba^3b^7\sin(dx+c)^3+60\,Aa^2b^8\sin(dx+c)^3+120\,Bab^9\sin(dx+c)^3-60\,Ab^{10}\sin(dx+c)^3+150\,Ba^4b^6\sin(dx+c)^2-120\,Aa^3b^7\sin(dx+c)^2-270\,Ba^2b^8\sin(dx+c)^2+180\,Aab^9\sin(dx+c)^2+90\,Bb^{10}\sin(dx+c)^2-360\,Ba^5b^5\sin(dx+c)+300\,Aa^4b^6\sin(dx+c)+720\,Ba^3b^7\sin(dx+c)-540\,Aa^2b^8\sin(dx+c)-360\,Bab^9\sin(dx+c)+180\,Ab^{10}\sin(dx+c)}{b^{12}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/60*(60*(7*B*a^6 - 6*A*a^5*b - 15*B*a^4*b^2 + 12*A*a^3*b^3 + 9*B*a^2*b^4
 - 6*A*a*b^5 - B*b^6)*log(abs(b*sin(d*x + c) + a))/b^8 - 60*(7*B*a^6*b*sin(d
*x + c) - 6*A*a^5*b^2*sin(d*x + c) - 15*B*a^4*b^3*sin(d*x + c) + 12*A*a^3*b
^4*sin(d*x + c) + 9*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) - B*b^7
*sin(d*x + c) + 6*B*a^7 - 5*A*a^6*b - 12*B*a^5*b^2 + 9*A*a^4*b^3 + 6*B*a^3*
b^4 - 3*A*a^2*b^5 - A*b^7)/((b*sin(d*x + c) + a)*b^8) + (10*B*b^10*sin(d*x
 + c)^6 - 24*B*a*b^9*sin(d*x + c)^5 + 12*A*b^10*sin(d*x + c)^5 + 45*B*a^2*b^
8*sin(d*x + c)^4 - 30*A*a*b^9*sin(d*x + c)^4 - 45*B*b^10*sin(d*x + c)^4 - 8
0*B*a^3*b^7*sin(d*x + c)^3 + 60*A*a^2*b^8*sin(d*x + c)^3 + 120*B*a*b^9*sin(
d*x + c)^3 - 60*A*b^10*sin(d*x + c)^3 + 150*B*a^4*b^6*sin(d*x + c)^2 - 120*
A*a^3*b^7*sin(d*x + c)^2 - 270*B*a^2*b^8*sin(d*x + c)^2 + 180*A*a*b^9*sin(d
*x + c)^2 + 90*B*b^10*sin(d*x + c)^2 - 360*B*a^5*b^5*sin(d*x + c) + 300*A*a
^4*b^6*sin(d*x + c) + 720*B*a^3*b^7*sin(d*x + c) - 540*A*a^2*b^8*sin(d*x +
c) - 360*B*a*b^9*sin(d*x + c) + 180*A*b^10*sin(d*x + c))/b^12)/d
```

Mupad [B]

time = 12.15, size = 682, normalized size = 2.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^7*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(A/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/(3*b^2) - (2*a*((3*B)/
b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/(3*b)))/d - (sin(c + d*
x)^5*(A/(5*b^2) - (2*B*a)/(5*b^3)))/d - (sin(c + d*x)^2*((3*B)/(2*b^2) + (a
*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((3*B)/b^2 + (2*a*(A/b
^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b))/b + (a^2*((3*B)/b^2 + (2*a*(A/b^2
- (2*B*a)/b^3))/b + (B*a^2)/b^4))/(2*b^2)))/d + (sin(c + d*x)^4*((3*B)/(4*b
^2) + (a*(A/b^2 - (2*B*a)/b^3))/(2*b) + (B*a^2)/(4*b^4)))/d - (sin(c + d*x)
*((3*A)/b^2 - (2*a*((3*B)/b^2 + (2*a*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3
))/b^2 - (2*a*((3*B)/b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b)
)/b + (a^2*((3*B)/b^2 + (2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b^2))
/b + (a^2*((3*A)/b^2 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((3*B)/b^2 +
(2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2)/b^4))/b))/b^2))/d + (log(a + b*sin(
c + d*x))*(B*b^6 - 7*B*a^6 - 12*A*a^3*b^3 - 9*B*a^2*b^4 + 15*B*a^4*b^2 + 6*
A*a*b^5 + 6*A*a^5*b))/(b^8*d) - (A*b^7 + B*a^7 - 3*A*a^2*b^5 + 3*A*a^4*b^3
+ 3*B*a^3*b^4 - 3*B*a^5*b^2 - A*a^6*b - B*a*b^6)/(b*d*(a*b^7 + b^8*sin(c +
d*x))) - (B*sin(c + d*x)^6)/(6*b^2*d)
```

3.1553 $\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=206

$$-\frac{(a^2-b^2)(4aAb-5a^2B+b^2B)\log(a+b\sin(c+dx))}{b^6d}+\frac{(3a^2Ab-2Ab^3-4a^3B+4ab^2B)\sin(c+dx)}{b^5d}-\frac{(2a\ldots}{}$$

```
[Out] -(a^2-b^2)*(4*A*a*b-5*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^6/d+(3*A*a^2*b-2*A*
b^3-4*B*a^3+4*B*a*b^2)*sin(d*x+c)/b^5/d-1/2*(2*A*a*b-3*B*a^2+2*B*b^2)*sin(d
*x+c)^2/b^4/d+1/3*(A*b-2*B*a)*sin(d*x+c)^3/b^3/d+1/4*B*sin(d*x+c)^4/b^2/d-(
a^2-b^2)^2*(A*b-B*a)/b^6/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.18, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$-\frac{(a^2-b^2)^2(Ab-aB)}{b^6d(a+b\sin(c+dx))}-\frac{(a^2-b^2)(-5a^2B+4aAb+b^2B)\log(a+b\sin(c+dx))}{b^6d}-\frac{(-3a^2B+2aAb+2b^2B)\sin^2(c+dx)}{2b^4d}+\frac{(-4a^3B+3a^2Ab+4ab^2B-2Ab^3)\sin(c+dx)}{b^5d}+\frac{(Ab-2aB)\sin^3(c+dx)}{3b^3d}+\frac{B\sin^4(c+dx)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a^2 - b^2)*(4*a*A*b - 5*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^6*d)
) + ((3*a^2*A*b - 2*A*b^3 - 4*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^5*d) - ((
2*a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x]^2)/(2*b^4*d) + ((A*b - 2*a*B)*Sin
[c + d*x]^3)/(3*b^3*d) + (B*Sin[c + d*x]^4)/(4*b^2*d) - ((a^2 - b^2)^2*(A*b
 - a*B))/(b^6*d*(a + b*Sin[c + d*x]))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)\left(b^2-x^2\right)^2}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^5 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{3a^2Ab-2Ab^3-4a^3B+4ab^2B}{b} + \frac{(-2aAb+3a^2B-2b^2B)x}{b} + \frac{(Ab-2aB)x}{b}\right.\right.}{}$$

$$= -\frac{\left(a^2-b^2\right)\left(4aAb-5a^2B+b^2B\right)\log(a+b\sin(c+dx))}{b^6 d} + \frac{(3a^2Ab}{}$$

Mathematica [A]
time = 1.43, size = 234, normalized size = 1.14

$$\frac{B\left(3b^3\cos^4(c+dx)+\frac{12(a^2-b^2)^2\log(a+b\sin(c+dx))}{b}-12a(a^2-2b^2)\sin(c+dx)+6b(a^2-b^2)\sin^2(c+dx)-4ab^2\sin^3(c+dx)\right)+4\left(A-\frac{aB}{b}\right)\left((8a^2b-4b^3)\sin(c+dx)-2ab^2\sin^2(c+dx)+\frac{b^4\cos^4(c+dx)-4(a^2-b^2)(a^2-b^2+3a^2\log(a+b\sin(c+dx))+3ab\log(a+b\sin(c+dx))\sin(c+dx))}{a+b\sin(c+dx)}\right)}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*(3*b^3*Cos[c + d*x]^4 + (12*(a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/b - 1
2*a*(a^2 - 2*b^2)*Sin[c + d*x] + 6*b*(a^2 - b^2)*Sin[c + d*x]^2 - 4*a*b^2*S
in[c + d*x]^3) + 4*(A - (a*B)/b)*((8*a^2*b - 4*b^3)*Sin[c + d*x] - 2*a*b^2*
Sin[c + d*x]^2 + (b^4*Cos[c + d*x]^4 - 4*(a^2 - b^2)*(a^2 - b^2 + 3*a^2*Log
[a + b*Sin[c + d*x]] + 3*a*b*Log[a + b*Sin[c + d*x]]*Sin[c + d*x]))/(a + b*
Sin[c + d*x])))/(12*b^5*d)
```

Maple [A]
time = 0.64, size = 258, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/b^5*(1/4*B*b^3*sin(d*x+c)^4+1/3*A*b^3*sin(d*x+c)^3-2/3*B*a*b^2*sin(d
*x+c)^3-A*a*b^2*sin(d*x+c)^2+3/2*B*a^2*b*sin(d*x+c)^2-B*b^3*sin(d*x+c)^2+3*
A*a^2*b*sin(d*x+c)-2*A*b^3*sin(d*x+c)-4*B*a^3*sin(d*x+c)+4*B*a*b^2*sin(d*x+
c))+(-4*A*a^3*b+4*A*a*b^3+5*B*a^4-6*B*a^2*b^2+B*b^4)/b^6*ln(a+b*sin(d*x+c))
-1/b^6*(A*a^4*b-2*A*a^2*b^3+A*b^5-B*a^5+2*B*a^3*b^2-B*a*b^4)/(a+b*sin(d*x+c
)))
```

Maxima [A]
time = 0.29, size = 229, normalized size = 1.11

$$\frac{\frac{12\,(Ba^5-Aa^4b-2\,Ba^3b^2+2\,Aa^2b^3+Bab^4-Ab^5)}{b^7\sin(dx+c)+ab^6}+\frac{3\,Bb^3\sin(dx+c)^4-4\,(2\,Bab^2-Ab^3)\sin(dx+c)^3+6\,(3\,Ba^2b-2\,Aab^2-2\,Bb^3)\sin(dx+c)^2-12\,(4\,Ba^3-3\,Aa^2b-4\,Bab^2+2\,Ab^3)\sin(dx+c)}{b^5}+\frac{12\,(5\,Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log(b\sin(dx+c)+a)}{b^6}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/12*(12*(B*a^5 - A*a^4*b - 2*B*a^3*b^2 + 2*A*a^2*b^3 + B*a*b^4 - A*b^5)/(b
^7*sin(d*x + c) + a*b^6) + (3*B*b^3*sin(d*x + c)^4 - 4*(2*B*a*b^2 - A*b^3)*
sin(d*x + c)^3 + 6*(3*B*a^2*b - 2*A*a*b^2 - 2*B*b^3)*sin(d*x + c)^2 - 12*(4
*B*a^3 - 3*A*a^2*b - 4*B*a*b^2 + 2*A*b^3)*sin(d*x + c))/b^5 + 12*(5*B*a^4 -
 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/b^6)/
d
```

Fricas [A]
time = 0.41, size = 322, normalized size = 1.56

$$\frac{96\,Ba^5-96\,Aa^4b-504\,Ba^3b^2+432\,Aa^2b^3+383\,Bab^4-256\,Ab^5-8\,(5\,Bab^4-4\,Ab^5)\cos(dx+c)^4+16\,(15\,Ba^3b^2-12\,Aa^2b^3-13\,Bab^4+8\,Ab^5)\cos(dx+c)^2+96\,\left(5\,Ba^5-4\,Aa^4b-6\,Ba^3b^2+4\,Aa^2b^3+Bab^4+(5\,Ba^4b-4\,Aa^3b^2-6\,Ba^2b^3+4\,Aab^4+Bb^5)\sin(dx+c)\right)\log(b\sin(dx+c)+a)+\left(24\,Bb^5\cos(dx+c)^4-384\,Ba^4b+288\,Aa^3b^2+392\,Ba^2b^3-208\,Aab^4-33\,Bb^5-16\,(5\,Ba^2b^3-4\,Aab^4-3\,Bb^5)\cos(dx+c)^2\right)\sin(dx+c)}{96\,(b^7d\sin(dx+c)+ab^6d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(96*B*a^5 - 96*A*a^4*b - 504*B*a^3*b^2 + 432*A*a^2*b^3 + 383*B*a*b^4 -
 256*A*b^5 - 8*(5*B*a*b^4 - 4*A*b^5)*cos(d*x + c)^4 + 16*(15*B*a^3*b^2 - 12
*A*a^2*b^3 - 13*B*a*b^4 + 8*A*b^5)*cos(d*x + c)^2 + 96*(5*B*a^5 - 4*A*a^4*b
 - 6*B*a^3*b^2 + 4*A*a^2*b^3 + B*a*b^4 + (5*B*a^4*b - 4*A*a^3*b^2 - 6*B*a^2
*b^3 + 4*A*a*b^4 + B*b^5)*sin(d*x + c))*log(b*sin(d*x + c) + a) + (24*B*b^5
*cos(d*x + c)^4 - 384*B*a^4*b + 288*A*a^3*b^2 + 392*B*a^2*b^3 - 208*A*a*b^4
 - 33*B*b^5 - 16*(5*B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cos(d*x + c)^2)*sin(d*
x + c))/(b^7*d*sin(d*x + c) + a*b^6*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 328, normalized size = 1.59

$$\frac{\frac{12\,(5\,Ba^4-4\,Aa^3b-6\,Ba^2b^2+4\,Aab^3+Bb^4)\log(|b\sin(dx+c)+a|)}{b^6}-\frac{12\,(5\,Ba^4b\sin(dx+c)-4\,Aa^3b^2\sin(dx+c)-6\,Ba^2b^3\sin(dx+c)+4\,Aab^4\sin(dx+c)+Bb^5\sin(dx+c)+4\,Ba^5-3\,Aa^4b-4\,Ba^3b^2+2\,Aa^2b^3+Ab^5)}{(b\sin(dx+c)+a)b^6}+\frac{3\,Bb^6\sin(dx+c)^4-8\,Bab^5\sin(dx+c)^3+4\,Ab^6\sin(dx+c)^3+18\,Ba^2b^4\sin(dx+c)^2-12\,Aab^5\sin(dx+c)^2-12\,Bb^6\sin(dx+c)^2-48\,Ba^3b^3\sin(dx+c)+36\,Aa^2b^4\sin(dx+c)+48\,Bab^5\sin(dx+c)-24\,Ab^6\sin(dx+c)}{b^8}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/12*(12*(5*B*a^4 - 4*A*a^3*b - 6*B*a^2*b^2 + 4*A*a*b^3 + B*b^4)*log(abs(b*
sin(d*x + c) + a))/b^6 - 12*(5*B*a^4*b*sin(d*x + c) - 4*A*a^3*b^2*sin(d*x +
 c) - 6*B*a^2*b^3*sin(d*x + c) + 4*A*a*b^4*sin(d*x + c) + B*b^5*sin(d*x + c
) + 4*B*a^5 - 3*A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + A*b^5)/((b*sin(d*x +
c) + a)*b^6) + (3*B*b^6*sin(d*x + c)^4 - 8*B*a*b^5*sin(d*x + c)^3 + 4*A*b^6
*sin(d*x + c)^3 + 18*B*a^2*b^4*sin(d*x + c)^2 - 12*A*a*b^5*sin(d*x + c)^2 -
 12*B*b^6*sin(d*x + c)^2 - 48*B*a^3*b^3*sin(d*x + c) + 36*A*a^2*b^4*sin(d*x
 + c) + 48*B*a*b^5*sin(d*x + c) - 24*A*b^6*sin(d*x + c))/b^8)/d
```

Mupad [B]
time = 0.12, size = 290, normalized size = 1.41

$$\frac{\sin\left(c+d\,x\right)^3\,\left(\frac{A}{3\,b^2}-\frac{2\,B\,a}{3\,b^3}\right)}{d}-\frac{\sin\left(c+d\,x\right)\,\left(\frac{2\,A}{b^2}+\frac{a^2\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b^2}-\frac{2\,a\,\left(\frac{2\,B}{b^2}+\frac{2\,a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)}{b}\right)}{d}-\frac{\sin\left(c+d\,x\right)^2\,\left(\frac{B}{b^2}+\frac{a\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{2\,b^4}\right)}{d}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(5\,B\,a^4-4\,A\,a^3\,b-6\,B\,a^2\,b^2+4\,A\,a\,b^3+B\,b^4\right)}{b^6\,d}-\frac{-B\,a^5+A\,a^4\,b+2\,B\,a^3\,b^2-2\,A\,a^2\,b^3-B\,a\,b^4+A\,b^5}{b\,d\,\left(\sin\left(c+d\,x\right)\,b^6+a\,b^5\right)}+\frac{B\,\sin\left(c+d\,x\right)^4}{4\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^5*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (sin(c + d*x)^3*(A/(3*b^2) - (2*B*a)/(3*b^3)))/d - (sin(c + d*x)*((2*A)/b^2
 + (a^2*(A/b^2 - (2*B*a)/b^3))/b^2 - (2*a*((2*B)/b^2 + (2*a*(A/b^2 - (2*B*a
)/b^3))/b + (B*a^2)/b^4))/b))/d - (sin(c + d*x)^2*(B/b^2 + (a*(A/b^2 - (2*B
*a)/b^3))/b + (B*a^2)/(2*b^4)))/d + (log(a + b*sin(c + d*x))*(5*B*a^4 + B*b
^4 - 6*B*a^2*b^2 + 4*A*a*b^3 - 4*A*a^3*b))/(b^6*d) - (A*b^5 - B*a^5 - 2*A*a
^2*b^3 + 2*B*a^3*b^2 + A*a^4*b - B*a*b^4)/(b*d*(a*b^5 + b^6*sin(c + d*x)))
+ (B*sin(c + d*x)^4)/(4*b^2*d)
```

3.1554 $\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=113

$$\frac{(2aAb-3a^2B+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}+\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}$$

```
[Out] (2*A*a*b-3*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))/b^4/d-(A*b-2*B*a)*sin(d*x+c)/b^3
/d-1/2*B*sin(d*x+c)^2/b^2/d+(a^2-b^2)*(A*b-B*a)/b^4/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2916, 786}

$$\frac{(a^2-b^2)(Ab-aB)}{b^4d(a+b\sin(c+dx))}+\frac{(-3a^2B+2aAb+b^2B)\log(a+b\sin(c+dx))}{b^4d}-\frac{(Ab-2aB)\sin(c+dx)}{b^3d}-\frac{B\sin^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*A*b - 3*a^2*B + b^2*B)*Log[a + b*Sin[c + d*x]])/(b^4*d) - ((A*b - 2*a
*B)*Sin[c + d*x])/(b^3*d) - (B*Sin[c + d*x]^2)/(2*b^2*d) + ((a^2 - b^2)*(A*
b - a*B))/(b^4*d*(a + b*Sin[c + d*x]))
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(A+\frac{Bx}{b}\right)(b^2-x^2)}{(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{b^3d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{-Ab+2aB}{b} - \frac{Bx}{b} + \frac{(-a^2+b^2)(Ab-aB)}{b(a+x)^2} + \frac{2aAb-3a^2B+b^2B}{b(a+x)}\right)\,dx,\right.}{b^3d}$$

$$= \frac{(2aAb-3a^2B+b^2B)\log(a+b\sin(c+dx))}{b^4d} - \frac{(Ab-2aB)\sin(c}{b^3d}$$

Mathematica [A]
time = 0.39, size = 111, normalized size = 0.98

$$\frac{\frac{(-a^2+b^2)B\log(a+b\sin(c+dx))}{b} + aB\sin(c+dx) - \frac{1}{2}bB\sin^2(c+dx) + \left(A-\frac{aB}{b}\right)\left(2a\log(a+b\sin(c+dx)) - b\sin(c+dx) + \frac{(a-b)(a+b)}{a+b\sin(c+dx)}\right)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((-a^2 + b^2)*B*Log[a + b*Sin[c + d*x]])/b + a*B*Sin[c + d*x] - (b*B*Sin[c
 + d*x]^2)/2 + (A - (a*B)/b)*(2*a*Log[a + b*Sin[c + d*x]] - b*Sin[c + d*x]
+ ((a - b)*(a + b))/(a + b*Sin[c + d*x])))/(b^3*d)
```

Maple [A]
time = 0.50, size = 117, normalized size = 1.04

method	result
derivativedivides	$\frac{-\frac{\frac{B\left(\sin^2(dx+c)\right)b}{2}+Ab\sin(dx+c)-2aB\sin(dx+c)}{b^3}+\frac{\left(2Aab-3B\,a^2+B\,b^2\right)\ln(a+b\sin(dx+c))}{b^4}-\frac{-A\,a^2b+A\,b^3+B\,a^3-Ba\,b^2}{b^4(a+b\sin(dx+c))}}{d}$
default	$\frac{-\frac{\frac{B\left(\sin^2(dx+c)\right)b}{2}+Ab\sin(dx+c)-2aB\sin(dx+c)}{b^3}+\frac{\left(2Aab-3B\,a^2+B\,b^2\right)\ln(a+b\sin(dx+c))}{b^4}-\frac{-A\,a^2b+A\,b^3+B\,a^3-Ba\,b^2}{b^4(a+b\sin(dx+c))}}{d}$
risch	$\frac{6iB\,a^2c}{b^4d}-\frac{2iBc}{b^2d}+\frac{ie^{i(dx+c)}A}{2b^2d}+\frac{e^{2i(dx+c)}B}{8b^2d}-\frac{ie^{i(dx+c)}Ba}{b^3d}-\frac{ie^{-i(dx+c)}A}{2b^2d}-\frac{ixB}{b^2}+\frac{ie^{-i(dx+c)}Ba}{b^3d}+\frac{e^{-2i}}{8}$
norman	$\frac{-\frac{2(6Ab-9aB)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{2(6Ab-9aB)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{(4Ab-6aB)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{(4Ab-6aB)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}-\frac{4(6A}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/b^3*(1/2*B*sin(d*x+c)^2*b+A*b*sin(d*x+c)-2*a*B*sin(d*x+c))+1/b^4*(2
*A*a*b-3*B*a^2+B*b^2)*ln(a+b*sin(d*x+c))-1/b^4*(-A*a^2*b+A*b^3+B*a^3-B*a*b^
2)/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 118, normalized size = 1.04

$$-\frac{\frac{2\,(Ba^3-Aa^2b-Bab^2+Ab^3)}{b^5\sin(dx+c)+ab^4}+\frac{Bb\sin(dx+c)^2-2\,(2\,Ba-Ab)\sin(dx+c)}{b^3}+\frac{2\,(3\,Ba^2-2\,Aab-Bb^2)\log(b\sin(dx+c)+a)}{b^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)/(b^5*sin(d*x + c) + a*b^4) + (B
*b*sin(d*x + c)^2 - 2*(2*B*a - A*b)*sin(d*x + c))/b^3 + 2*(3*B*a^2 - 2*A*a*
b - B*b^2)*log(b*sin(d*x + c) + a)/b^4)/d
```

Fricas [A]
time = 0.41, size = 178, normalized size = 1.58

$$-\frac{4\,Ba^3-4\,Aa^2b-11\,Bab^2+8\,Ab^3+2\,(3\,Bab^2-2\,Ab^3)\cos(dx+c)^2+4\,(3\,Ba^3-2\,Aa^2b-Bab^2+(3\,Ba^2b-2\,Aab^2-Bb^3)\sin(dx+c))\log(b\sin(dx+c)+a)-(2\,Bb^3\cos(dx+c)^2+8\,Ba^2b-4\,Aab^2-Bb^3)\sin(dx+c)}{4\,(b^5d\sin(dx+c)+ab^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/4*(4*B*a^3 - 4*A*a^2*b - 11*B*a*b^2 + 8*A*b^3 + 2*(3*B*a*b^2 - 2*A*b^3)*
cos(d*x + c)^2 + 4*(3*B*a^3 - 2*A*a^2*b - B*a*b^2 + (3*B*a^2*b - 2*A*a*b^2
- B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a) - (2*B*b^3*cos(d*x + c)^2 +
8*B*a^2*b - 4*A*a*b^2 - B*b^3)*sin(d*x + c))/(b^5*d*sin(d*x + c) + a*b^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 188, normalized size = 1.66

$$-\frac{\frac{(b\sin(dx+c)+a)^2\left(B-\frac{2\,(3\,Bab-Ab^2)}{(b\sin(dx+c)+a)b}\right)}{b^4}-\frac{2\,(3\,Ba^2-2\,Aab-Bb^2)\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b^4}+\frac{2\left(\frac{Ba^3b^2}{b\sin(dx+c)+a}-\frac{Aa^2b^3}{b\sin(dx+c)+a}-\frac{Bab^4}{b\sin(dx+c)+a}+\frac{Ab^5}{b\sin(dx+c)+a}\right)}{b^6}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*((b*sin(d*x + c) + a)^2*(B - 2*(3*B*a*b - A*b^2)/((b*sin(d*x + c) + a)
*b))/b^4 - 2*(3*B*a^2 - 2*A*a*b - B*b^2)*log(abs(b*sin(d*x + c) + a)/((b*si
n(d*x + c) + a)^2*abs(b)))/b^4 + 2*(B*a^3*b^2/(b*sin(d*x + c) + a) - A*a^2*
b^3/(b*sin(d*x + c) + a) - B*a*b^4/(b*sin(d*x + c) + a) + A*b^5/(b*sin(d*x
+ c) + a))/b^6)/d
```

Mupad [B]
time = 12.10, size = 128, normalized size = 1.13

$$\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(-3\,B\,a^2+2\,A\,a\,b+B\,b^2\right)}{b^4\,d}-\frac{B\,a^3-A\,a^2\,b-B\,a\,b^2+A\,b^3}{b\,d\,\left(\sin\left(c+d\,x\right)\,b^4+a\,b^3\right)}-\frac{\sin\left(c+d\,x\right)\,\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{d}-\frac{B\,{\sin\left(c+d\,x\right)}^2}{2\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (log(a + b*sin(c + d*x))*(B*b^2 - 3*B*a^2 + 2*A*a*b))/(b^4*d) - (A*b^3 + B*
a^3 - A*a^2*b - B*a*b^2)/(b*d*(a*b^3 + b^4*sin(c + d*x))) - (sin(c + d*x)*(
A/b^2 - (2*B*a)/b^3))/d - (B*sin(c + d*x)^2)/(2*b^2*d)
```

3.1555 $\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=48

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

```
[Out] B*ln(a+b*sin(d*x+c))/b^2/d+(-A*b+B*a)/b^2/d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2912, 45}

$$\frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]])/(b^2*d) - (A*b - a*B)/(b^2*d*(a + b*Sin[c + d*x
]))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2}\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+x)^2}+\frac{B}{b(a+x)}\right)\,dx,x,b\sin(c+dx)\right)}{bd}\\ &= \frac{B\log(a+b\sin(c+dx))}{b^2d}-\frac{Ab-aB}{b^2d(a+b\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 0.88

$$\frac{B\log(a+b\sin(c+dx))+\frac{-Ab+aB}{a+b\sin(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (B*Log[a + b*Sin[c + d*x]] + (-(A*b) + a*B)/(a + b*Sin[c + d*x]))/(b^2*d)
```

Maple [A]
time = 0.28, size = 47, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{B\ln(a+b\sin(dx+c))}{b^2}-\frac{Ab-aB}{b^2(a+b\sin(dx+c))}}{d}$
default	$\frac{\frac{B\ln(a+b\sin(dx+c))}{b^2}-\frac{Ab-aB}{b^2(a+b\sin(dx+c))}}{d}$
risch	$-\frac{ixB}{b^2}-\frac{2iBc}{b^2d}-\frac{2(Ab-aB)e^{i(dx+c)}}{b^2d\left(-ib\,e^{2i(dx+c)}+ib+2a\,e^{i(dx+c)}\right)}+\frac{\ln\left(e^{2i(dx+c)}-1+\frac{2ia\,e^{i(dx+c)}}{b}\right)B}{b^2d}$
norman	$\frac{\frac{2(Ab-aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{bda}+\frac{4(Ab-aB)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bda}+\frac{2(Ab-aB)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bda}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}+\frac{B\ln(a(\tan^2(\frac{dx}{2}+\frac{c}{2}))+2b\tan(\frac{dx}{2}+\frac{c}{2}}{b^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(B/b^2*ln(a+b*sin(d*x+c))-(A*b-B*a)/b^2/(a+b*sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 1.00

$$\frac{\frac{Ba-Ab}{b^3\sin(dx+c)+ab^2}+\frac{B\log(b\sin(dx+c)+a)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] ((B*a - A*b)/(b^3*sin(d*x + c) + a*b^2) + B*log(b*sin(d*x + c) + a)/b^2)/d
```

Fricas [A]
time = 0.37, size = 54, normalized size = 1.12

$$\frac{Ba-Ab+(Bb\sin(dx+c)+Ba)\log(b\sin(dx+c)+a)}{b^3d\sin(dx+c)+ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] (B*a - A*b + (B*b*sin(d*x + c) + B*a)*log(b*sin(d*x + c) + a))/(b^3*d*sin(d
*x + c) + a*b^2*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(39) = 78$.
time = 0.56, size = 178, normalized size = 3.71

$$\begin{cases} \frac{x(A+B\sin(c))\cos(c)}{a^2} & \text{for } b = 0 \wedge d = 0 \\ \frac{\frac{A\sin(c+dx)}{d} + \frac{B\sin^2(c+dx)}{2d}}{a^2} & \text{for } b = 0 \\ \frac{x(A+B\sin(c))\cos(c)}{(a+b\sin(c))^2} & \text{for } d = 0 \\ -\frac{Ab}{ab^2d+b^3d\sin(c+dx)} + \frac{Ba\log\left(\frac{a}{b}+\sin(c+dx)\right)}{ab^2d+b^3d\sin(c+dx)} + \frac{Ba}{ab^2d+b^3d\sin(c+dx)} + \frac{Bb\log\left(\frac{a}{b}+\sin(c+dx)\right)\sin(c+dx)}{ab^2d+b^3d\sin(c+dx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(A + B*sin(c))*cos(c)/a**2, Eq(b, 0) & Eq(d, 0)), ((A*sin(c +
d*x)/d + B*sin(c + d*x)**2/(2*d))/a**2, Eq(b, 0)), (x*(A + B*sin(c))*cos(c)
/(a + b*sin(c))**2, Eq(d, 0)), (-A*b/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a
*log(a/b + sin(c + d*x))/(a*b**2*d + b**3*d*sin(c + d*x)) + B*a/(a*b**2*d +
 b**3*d*sin(c + d*x)) + B*b*log(a/b + sin(c + d*x))*sin(c + d*x)/(a*b**2*d
+ b**3*d*sin(c + d*x)), True))
```

Giac [A]
time = 0.54, size = 80, normalized size = 1.67

$$-\frac{\frac{B\left(\frac{\log\left(\frac{|b\sin(dx+c)+a|}{(b\sin(dx+c)+a)^2|b|}\right)}{b} - \frac{a}{(b\sin(dx+c)+a)b}\right)}{b} + \frac{A}{(b\sin(dx+c)+a)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -(B*(log(abs(b*sin(d*x + c) + a)/((b*sin(d*x + c) + a)^2*abs(b)))/b - a/((b
*sin(d*x + c) + a)*b))/b + A/((b*sin(d*x + c) + a)*b))/d
```

Mupad [B]
time = 0.06, size = 48, normalized size = 1.00

$$\frac{B\,\ln\left(a+b\,\sin\left(c+d\,x\right)\right)}{b^2\,d} - \frac{A\,b-B\,a}{b^2\,d\,\left(a+b\,\sin\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B*sin(c + d*x)))/(a + b*sin(c + d*x))^2,x)
```

```
[Out] (B*log(a + b*sin(c + d*x)))/(b^2*d) - (A*b - B*a)/(b^2*d*(a + b*sin(c + d*x
)))
```

3.1556 $\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=135

$$-\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)^2d}+\frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)^2d}-\frac{(2aAb-a^2B-b^2B)\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d}-$$

```
[Out] -1/2*(A+B)*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*(A-B)*ln(1+sin(d*x+c))/(a-b)^2/d-
(2*A*a*b-B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+(A*b-B*a)/(a^2-b^2)/
d/(a+b*sin(d*x+c))
```

Rubi [A]

time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2916, 815}

$$\frac{Ab-aB}{d\,(a^2-b^2)\,(a+b\sin(c+dx))}-\frac{(a^2(-B)+2aAb-b^2B)\log(a+b\sin(c+dx))}{d\,(a^2-b^2)^2}-\frac{(A+B)\log(1-\sin(c+dx))}{2d(a+b)^2}+\frac{(A-B)\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/2*((A + B)*Log[1 - Sin[c + d*x]])/((a + b)^2*d) + ((A - B)*Log[1 + Sin[c
 + d*x]])/(2*(a - b)^2*d) - ((2*a*A*b - a^2*B - b^2*B)*Log[a + b*Sin[c + d*
x]])/((a^2 - b^2)^2*d) + (A*b - a*B)/((a^2 - b^2)*d*(a + b*Sin[c + d*x]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2(b^2-x^2)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{b\text{Subst}\left(\int \left(\frac{A+B}{2b(a+b)^2(b-x)} + \frac{-Ab+aB}{(a-b)b(a+b)(a+x)^2} + \frac{-2aAb+a^2B+b^2B}{(a-b)^2b(a+b)^2(a+x)} + \frac{}{2(a-}\right.\right.}{d}$$

$$= -\frac{(A+B)\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{(A-B)\log(1+\sin(c+dx))}{2(a-b)^2d} -$$

Mathematica [A]
time = 0.97, size = 178, normalized size = 1.32

$$\frac{-\frac{B((-a+b)\log(1-\sin(c+dx))+(a+b)\log(1+\sin(c+dx))-2b\log(a+b\sin(c+dx)))}{2b(-a+b)(a+b)} + b\left(A-\frac{aB}{b}\right)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2b} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-1/2*(B*((-a + b)*Log[1 - Sin[c + d*x]] + (a + b)*Log[1 + Sin[c + d*x]] -
2*b*Log[a + b*Sin[c + d*x]]))/(b*(-a + b)*(a + b)) + b*(A - (a*B)/b)*(-1/2*
Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b)
 - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a
+ b*Sin[c + d*x]))))/d
```

Maple [A]
time = 0.54, size = 128, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{(-A-B)\ln(\sin(dx+c)-1)}{2(a+b)^2}+\frac{Ab-aB}{(a+b)(a-b)(a+b\sin(dx+c))}-\frac{\left(2Aab-B\,a^2-B\,b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{(A-B)\ln(1+\sin(dx+c))}{2(a-b)^2}}{d}$
default	$\frac{\frac{(-A-B)\ln(\sin(dx+c)-1)}{2(a+b)^2}+\frac{Ab-aB}{(a+b)(a-b)(a+b\sin(dx+c))}-\frac{\left(2Aab-B\,a^2-B\,b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}+\frac{(A-B)\ln(1+\sin(dx+c))}{2(a-b)^2}}{d}$
norman	$\frac{-\frac{2b(Ab-aB)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(a^2-b^2\right)}-\frac{2b(Ab-aB)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(a^2-b^2\right)}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)}+\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d(a^2-2ab+b^2)}-\frac{(A+B)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{(a^2+2ab+b^2)d}$
risch	$-\frac{2iB\,b^2x}{a^4-2a^2b^2+b^4}+\frac{iBc}{(a^2+2ab+b^2)d}+\frac{4iAabx}{a^4-2a^2b^2+b^4}-\frac{2iB\,a^2x}{a^4-2a^2b^2+b^4}-\frac{iAc}{d(a^2-2ab+b^2)}+\frac{iBc}{d(a^2-2ab+b^2)}-\frac{}{d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(-A-B)/(a+b)^2*ln(sin(d*x+c)-1)+(A*b-B*a)/(a+b)/(a-b)/(a+b*sin(d*x
+c))-(2*A*a*b-B*a^2-B*b^2)/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))+1/2*(A-B)/(a-
b)^2*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.28, size = 147, normalized size = 1.09

$$\frac{\frac{2\,(Ba^2-2\,Aab+Bb^2)\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}+\frac{(A-B)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(A+B)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(Ba-Ab)}{a^3-ab^2+(a^2b-b^3)\sin(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(B*a^2 - 2*A*a*b + B*b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 +
 b^4) + (A - B)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (A + B)*log(sin
(d*x + c) - 1)/(a^2 + 2*a*b + b^2) - 2*(B*a - A*b)/(a^3 - a*b^2 + (a^2*b -
b^3)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(129) = 258.
time = 0.51, size = 283, normalized size = 2.10

$$-\frac{2\,Ba^3-2\,Aa^2b-2\,Bab^2+2\,Ab^3-2\,(Ba^3-2\,Aa^2b+Bab^2+(Ba^2b-2\,Aab^2+Bb^3)\sin(dx+c))\log(b\sin(dx+c)+a)-((A-B)a^3+2\,(A-B)a^2b+(A-B)ab^2+((A-B)a^2b+2\,(A-B)ab^2+(A-B)b^3)\sin(dx+c))\log(\sin(dx+c)+1)+((A+B)a^3-2\,(A+B)a^2b+(A+B)ab^2+((A+B)a^2b-2\,(A+B)ab^2+(A+B)b^3)\sin(dx+c))\log(-\sin(dx+c)+1)}{2\,((a^4b-2\,a^2b^3+b^5)d\sin(dx+c)+(a^5-2\,a^3b^2+ab^4)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] -1/2*(2*B*a^3 - 2*A*a^2*b - 2*B*a*b^2 + 2*A*b^3 - 2*(B*a^3 - 2*A*a^2*b + B*
a*b^2 + (B*a^2*b - 2*A*a*b^2 + B*b^3)*sin(d*x + c))*log(b*sin(d*x + c) + a)
 - ((A - B)*a^3 + 2*(A - B)*a^2*b + (A - B)*a*b^2 + ((A - B)*a^2*b + 2*(A -
 B)*a*b^2 + (A - B)*b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((A + B)*a^3
 - 2*(A + B)*a^2*b + (A + B)*a*b^2 + ((A + B)*a^2*b - 2*(A + B)*a*b^2 + (A
+ B)*b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*
d*sin(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sin(c+dx))\sec(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 205, normalized size = 1.52

$$\frac{\frac{2\,(Ba^2b-2\,Aab^2+Bb^3)\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5}-\frac{(A+B)\log(|-\sin(dx+c)+1|)}{a^2+2\,ab+b^2}+\frac{(A-B)\log(|-\sin(dx+c)-1|)}{a^2-2\,ab+b^2}-\frac{2\,(Ba^2b\sin(dx+c)-2\,Aab^2\sin(dx+c)+Bb^3\sin(dx+c)+2\,Ba^3-3\,Aa^2b+Ab^3)}{(a^4-2\,a^2b^2+b^4)(b\sin(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] 1/2*(2*(B*a^2*b - 2*A*a*b^2 + B*b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b -
2*a^2*b^3 + b^5) - (A + B)*log(abs(-sin(d*x + c) + 1))/(a^2 + 2*a*b + b^2)
+ (A - B)*log(abs(-sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 2*(B*a^2*b*sin(
d*x + c) - 2*A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + 2*B*a^3 - 3*A*a^2*
b + A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

Mupad [B]
time = 0.42, size = 131, normalized size = 0.97

$$\frac{A\,b-B\,a}{d\,\left(a^2-b^2\right)\,\left(a+b\,\sin\left(c+d\,x\right)\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(\frac{A}{2}+\frac{B}{2}\right)}{d\,{\left(a+b\right)}^2}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)}{d\,{\left(a^2-b^2\right)}^2}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(\frac{A}{2}-\frac{B}{2}\right)}{d\,{\left(a-b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)*(a + b*sin(c + d*x))^2),x)
```

```
[Out] (A*b - B*a)/(d*(a^2 - b^2)*(a + b*sin(c + d*x))) - (log(sin(c + d*x) - 1)*(
A/2 + B/2))/(d*(a + b)^2) + (log(a + b*sin(c + d*x))*(B*a^2 + B*b^2 - 2*A*a
*b))/(d*(a^2 - b^2)^2) + (log(sin(c + d*x) + 1)*(A/2 - B/2))/(d*(a - b)^2)
```

3.1557 $\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=228

$$-\frac{(aA+3Ab+2bB)\log(1-\sin(c+dx))}{4(a+b)^3d}+\frac{(aA-3Ab+2bB)\log(1+\sin(c+dx))}{4(a-b)^3d}+\frac{b^2(4aAb-3a^2B-b^2B)}{(a^2-}$$

```
[Out] -1/4*(A*a+3*A*b+2*B*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(A*a-3*A*b+2*B*b)*ln(
1+sin(d*x+c))/(a-b)^3/d+b^2*(4*A*a*b-3*B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2
-b^2)^3/d-1/2*b*(A*a^2+3*A*b^2-4*B*a*b)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))-1/2*
sec(d*x+c)^2*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.23, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

$$-\frac{b(a^2A-4abB+3Ab^2)}{2d(a^2-b^2)^2(a+b\sin(c+dx))}+\frac{b^2(-3a^2B+4aAb-b^2B)\log(a+b\sin(c+dx))}{d(a^2-b^2)^3}-\frac{\sec^2(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{2d(a^2-b^2)(a+b\sin(c+dx))}-\frac{(aA+3Ab+2bB)\log(1-\sin(c+dx))}{4d(a+b)^3}+\frac{(aA-3Ab+2bB)\log(\sin(c+dx)+1)}{4d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*((a*A + 3*A*b + 2*b*B)*Log[1 - Sin[c + d*x]])/((a + b)^3*d) + ((a*A -
3*A*b + 2*b*B)*Log[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^2*(4*a*A*b - 3*a
^2*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - (b*(a^2*A + 3*A*
b^2 - 4*a*b*B))/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) - (Sec[c + d*x]^2*
(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(2*(a^2 - b^2)*d*(a + b*Sin[c + d*x
]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^3\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2(b^2-x^2)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\text{Subst}\left(\int \frac{}{}\right.}{}$$

$$= -\frac{\sec^2(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{2\left(a^2-b^2\right)d(a+b\sin(c+dx))} - \frac{b\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{(aA+3Ab+2bB)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(aA-3Ab+2bB)\,\text{lo}}{4(a-}$$

Mathematica [A]

time = 1.21, size = 246, normalized size = 1.08

$$\frac{\frac{(aA-bB)((a-b)\log(1-\sin(c+dx))-(a+b)\log(1+\sin(c+dx))+2b\log(a+b\sin(c+dx)))}{(a-b)(a+b)} + \frac{\sec^2(c+dx)(Ab-aB+(-aA+bB)\sin(c+dx))}{a+b\sin(c+dx)} + b(a^2A+3Ab^2-4abB)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2b} - \frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} + \frac{1}{(a^2-b^2)(a+b\sin(c+dx))}\right)}{2\left(-a^2+b^2\right)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (((a*A - b*B)*((a - b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]
] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b)*(a + b)) + (Sec[c + d*x]^2*(A*b
- a*B + (-(a*A) + b*B)*Sin[c + d*x]))/(a + b*Sin[c + d*x]) + b*(a^2*A + 3*A
*b^2 - 4*a*b*B)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c +
 d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2
) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*(-a^2 + b^2)*d)
```

Maple [A]

time = 0.82, size = 191, normalized size = 0.84

method	result
derivativedivides	$\frac{-\frac{A+B}{4(a+b)^2(\sin(dx+c)-1)} + \frac{(-aA-3Ab-2Bb)\ln(\sin(dx+c)-1)}{4(a+b)^3} - \frac{b^2(Ab-aB)}{(a+b)^2(a-b)^2(a+b\sin(dx+c))} + \frac{b^2\left(4Aab-3B\,a^2-B\,b^2\right)\ln(a+b\sin}{(a+b)^3(a-b)^3}}{d}$

default	$\frac{-\frac{A+B}{4(a+b)^{2}(\sin(dx+c)-1)}+\frac{(-aA-3Ab-2Bb)\ln(\sin(dx+c)-1)}{4(a+b)^{3}}-\frac{b^{2}(Ab-aB)}{(a+b)^{2}(a-b)^{2}(a+b\sin(dx+c))}+\frac{b^{2}\left(4Aab-3B\,a^{2}-B\,b^{2}\right)\ln(a+b\sin(d}{(a+b)^{3}(a-b)^{3}}}{d}$
norman	$\frac{\frac{\left(3A\,a^{4}-5A\,a^{2}b^{2}-2A\,b^{4}-2B\,a^{3}b+6Ba\,b^{3}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}+\frac{\left(3A\,a^{4}-5A\,a^{2}b^{2}-2A\,b^{4}-2B\,a^{3}b+6Ba\,b^{3}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da\left(a^{4}-2a^{2}b^{2}+b^{4}\right)}+\frac{\left(A\,a^{4}+A\right.}{}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/4*(A+B)/(a+b)^2/(sin(d*x+c)-1)+1/4/(a+b)^3*(-A*a-3*A*b-2*B*b)*ln(si
n(d*x+c)-1)-b^2*(A*b-B*a)/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))+b^2*(4*A*a*b-3*B
*a^2-B*b^2)/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/4*(A-B)/(a-b)^2/(1+sin(d*x
+c))+1/4*(A*a-3*A*b+2*B*b)/(a-b)^3*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.30, size = 346, normalized size = 1.52

$$-\frac{\frac{4\left(3\,Ba^{2}b^{2}-4\,Aab^{3}+Bb^{4}\right)\log\left(b\sin\left(dx+c\right)+a\right)}{a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}}-\frac{\left(Aa-\left(3\,A-2\,B\right)b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(Aa+\left(3\,A+2\,B\right)b\right)\log\left(\sin\left(dx+c\right)-1\right)}{a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}}-\frac{2\left(Ba^{3}-2\,Aa^{2}b+3\,Bab^{2}-2\,Ab^{3}+\left(Aa^{2}b-4\,Bab^{2}+3\,Ab^{3}\right)\sin\left(dx+c\right)^{2}+\left(Aa^{3}-Ba^{2}b-Aab^{2}+Bb^{3}\right)\sin\left(dx+c\right)\right)}{a^{5}-2\,a^{3}b^{2}+ab^{4}-\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(dx+c\right)^{3}-\left(a^{5}-2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)^{2}+\left(a^{4}b-2\,a^{2}b^{3}+b^{5}\right)\sin\left(dx+c\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/4*(4*(3*B*a^2*b^2 - 4*A*a*b^3 + B*b^4)*log(b*sin(d*x + c) + a)/(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6) - (A*a - (3*A - 2*B)*b)*log(sin(d*x + c) + 1)/(a
^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + (3*A + 2*B)*b)*log(sin(d*x + c) - 1)
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(B*a^3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b
^3 + (A*a^2*b - 4*B*a*b^2 + 3*A*b^3)*sin(d*x + c)^2 + (A*a^3 - B*a^2*b - A*
a*b^2 + B*b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^4*b - 2*a^2*b^3
+ b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c)^2 + (a^4*b -
 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(219) = 438$.
time = 0.87, size = 598, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 4*A*a^2*b^3 + 2*B*a*b^4 - 2*A*b^5
- 2*(A*a^4*b - 4*B*a^3*b^2 + 2*A*a^2*b^3 + 4*B*a*b^4 - 3*A*b^5)*cos(d*x + c
)^2 - 4*((3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*cos(d*x + c)^2*sin(d*x + c) + (3
*B*a^3*b^2 - 4*A*a^2*b^3 + B*a*b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a)
 + ((A*a^4*b + 2*B*a^3*b^2 - 6*(A - B)*a^2*b^3 - 2*(4*A - 3*B)*a*b^4 - (3*A
 - 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b - 6*(A - B)*a
^3*b^2 - 2*(4*A - 3*B)*a^2*b^3 - (3*A - 2*B)*a*b^4)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((A*a^4*b + 2*B*a^3*b^2 - 6*(A + B)*a^2*b^3 + 2*(4*A + 3*B
)*a*b^4 - (3*A + 2*B)*b^5)*cos(d*x + c)^2*sin(d*x + c) + (A*a^5 + 2*B*a^4*b
 - 6*(A + B)*a^3*b^2 + 2*(4*A + 3*B)*a^2*b^3 - (3*A + 2*B)*a*b^4)*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) + 2*(A*a^5 - B*a^4*b - 2*A*a^3*b^2 + 2*B*a^2*
b^3 + A*a*b^4 - B*b^5)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^3(c + dx)}{(a + b\sin(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A]

time = 0.58, size = 335, normalized size = 1.47

$$-\frac{\frac{4\,(3\,Ba^2b^3-4\,Aab^4+Bb^5)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{(Aa-3\,Ab+2\,Bb)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(Aa+3\,Ab+2\,Bb)\log(|-\sin(dx+c)+1|)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(Aa^2b\sin(dx+c)^2-4\,Bab^2\sin(dx+c)^2+3\,Ab^3\sin(dx+c)^2+Aa^3\sin(dx+c)-Ba^2b\sin(dx+c)-Aab^2\sin(dx+c)+Bb^3\sin(dx+c)+Ba^3-2\,Aa^2b+3\,Bab^2-2\,Ab^3\right)}{(a^4-2\,a^2b^2+b^4)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/4*(4*(3*B*a^2*b^3 - 4*A*a*b^4 + B*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6
*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7) - (A*a - 3*A*b + 2*B*b)*log(abs(sin(d*x +
 c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (A*a + 3*A*b + 2*B*b)*log(abs(-
sin(d*x + c) + 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(A*a^2*b*sin(d*x + c
)^2 - 4*B*a*b^2*sin(d*x + c)^2 + 3*A*b^3*sin(d*x + c)^2 + A*a^3*sin(d*x + c
) - B*a^2*b*sin(d*x + c) - A*a*b^2*sin(d*x + c) + B*b^3*sin(d*x + c) + B*a^
3 - 2*A*a^2*b + 3*B*a*b^2 - 2*A*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x +
c)^3 + a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

Mupad [B]

time = 12.66, size = 327, normalized size = 1.43

$$\frac{\frac{\sin(c+d\,x)^2\,(A\,a^2\,b-4\,B\,a\,b^2+3\,A\,b^3)}{2\,(a^4-2\,a^2\,b^2+b^4)}-\frac{-B\,a^3+2\,A\,a^2\,b-3\,B\,a\,b^2+2\,A\,b^3}{2\,{(a^2-b^2)}^2}+\frac{\sin(c+d\,x)\,(A\,a-B\,b)}{2\,(a^2-b^2)}}{d\,\left(-b\sin\left(c+d\,x\right)^3-a\sin\left(c+d\,x\right)^2+b\,\sin\left(c+d\,x\right)+a\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(A\,a+b\,(3\,A+2\,B)\right)}{d\,\left(4\,a^3+12\,a^2\,b+12\,a\,b^2+4\,b^3\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(A\,a-b\,(3\,A-2\,B)\right)}{d\,\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}-\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(3\,B\,a^2\,b^2-4\,A\,a\,b^3+B\,b^4\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^3*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)^2*(3*A*b^3 + A*a^2*b - 4*B*a*b^2))/(2*(a^4 + b^4 - 2*a^2*b^2
)) - (2*A*b^3 - B*a^3 + 2*A*a^2*b - 3*B*a*b^2)/(2*(a^2 - b^2)^2) + (sin(c +
 d*x)*(A*a - B*b))/(2*(a^2 - b^2)))/(d*(a + b*sin(c + d*x) - a*sin(c + d*x)
^2 - b*sin(c + d*x)^3)) - (log(sin(c + d*x) - 1)*(A*a + b*(3*A + 2*B)))/(d*
(12*a*b^2 + 12*a^2*b + 4*a^3 + 4*b^3)) + (log(sin(c + d*x) + 1)*(A*a - b*(3
*A - 2*B)))/(d*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3)) - (log(a + b*sin(c +
d*x))*(B*b^4 + 3*B*a^2*b^2 - 4*A*a*b^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2))
```

3.1558 $\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=372

$$-\frac{(3a^2A+2ab(6A+B)+b^2(15A+8B))\log(1-\sin(c+dx))}{16(a+b)^4d}+\frac{(3a^2A+b^2(15A-8B)-2ab(6A-B))\log}{16(a-b)^4d}$$

```
[Out] -1/16*(3*a^2*A+2*a*b*(6*A+B)+b^2*(15*A+8*B))*ln(1-sin(d*x+c))/(a+b)^4/d+1/1
6*(3*a^2*A+b^2*(15*A-8*B)-2*a*b*(6*A-B))*ln(1+sin(d*x+c))/(a-b)^4/d-b^4*(6*
A*a*b-5*B*a^2-B*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-1/8*b*(3*A*a^4-12*A*a
^2*b^2-15*A*b^4+2*B*a^3*b+22*B*a*b^3)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-1/4*se
c(d*x+c)^4*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/8*
sec(d*x+c)^2*(b*(A*a^2+5*A*b^2-6*B*a*b)+(3*A*a^3-9*A*a*b^2+2*B*a^2*b+4*B*b^
3)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

Rubi [A]
time = 0.40, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

$$-\frac{(3a^2A+2ab(6A+B)+b^2(15A+8B))\log(1-\sin(c+dx))}{16d(a+b)^4}+\frac{(3a^2A-2ab(6A-B)+b^2(15A-8B))\log(\sin(c+dx)+1)}{16d(a-b)^4}-\frac{\sec^4(c+dx)(-(aA-bB)\sin(c+dx)-aB+Ab)}{4d(a^2-b^2)(a+b\sin(c+dx))}-\frac{b^4(-5a^2B+6aAb-b^2B)\log(a+b\sin(c+dx))}{d(a^2-b^2)^4}+\frac{\sec^2(c+dx)\left(b(a^2A-6abB+5Ab^2)+(3a^3A+2a^2bB-9aAb^2+4b^3B)\sin(c+dx)\right)}{8d(a^2-b^2)^2(a+b\sin(c+dx))}-\frac{b(3a^4A+2a^3bB-12a^2Ab^2+22ab^3B-15Ab^4)}{8d(a^2-b^2)^3(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/16*((3*a^2*A + 2*a*b*(6*A + B) + b^2*(15*A + 8*B))*Log[1 - Sin[c + d*x]]
)/((a + b)^4*d) + ((3*a^2*A + b^2*(15*A - 8*B) - 2*a*b*(6*A - B))*Log[1 + S
in[c + d*x]])/(16*(a - b)^4*d) - (b^4*(6*a*A*b - 5*a^2*B - b^2*B)*Log[a + b
*Sin[c + d*x]])/((a^2 - b^2)^4*d) - (b*(3*a^4*A - 12*a^2*A*b^2 - 15*A*b^4 +
 2*a^3*b*B + 22*a*b^3*B))/(8*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (Sec[c
 + d*x]^4*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(4*(a^2 - b^2)*d*(a + b*S
in[c + d*x])) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6*a*b*B) + (3*a^3*A -
 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(8*(a^2 - b^2)^2*d*(a + b*
Sin[c + d*x]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
```

```
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^5\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2(b^2-x^2)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{b^3\text{Subst}\left(\int \frac{-3}{}\right.}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\,(}{}$$

$$= -\frac{\sec^4(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{4(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\,(}{}$$

$$= -\frac{(3a^2A+2ab(6A+B)+b^2(15A+8B))\log(1-\sin(c+dx))}{16(a+b)^4d} + \frac{(3}{}$$

Mathematica [A]
time = 3.54, size = 370, normalized size = 0.99

$$\frac{-\frac{(3a^3A-9aAb^2+2a^2bB+4b^3B)((a-b)\log(1-\sin(c+dx))-(a+b)\log(1+\sin(c+dx))+2b\log(a+b\sin(c+dx)))}{(a-b)(a+b)}+\frac{2(-a^2+b^2)\sec^4(c+dx)(Ab-aB+(-aA+bB)\sin(c+dx))}{a+b\sin(c+dx)}+\frac{\sec^2(c+dx)(b(a^2A+5Ab^2-6abB)+(3a^3A-9aAb^2+2a^2bB+4b^3B)\sin(c+dx))}{a+b\sin(c+dx)}+b(-3a^4A+12a^2Ab^2+15Ab^4-2a^3bB-22ab^3B)\left(-\frac{\log(1-\sin(c+dx))}{2b(a+b)^2}+\frac{\log(1+\sin(c+dx))}{2(a-b)^2b}-\frac{2a\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}+\frac{1}{(a^2-b^2)(a+b\sin(c+dx))}\right)}{8(a^2-b^2)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-(((3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*((a - b)*Log[1 - Sin[c + d*
x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*b*Log[a + b*Sin[c + d*x]]))/((a - b
)*(a + b))) + (2*(-a^2 + b^2)*Sec[c + d*x]^4*(A*b - a*B + (-(a*A) + b*B)*Si
```

```
n[c + d*x]))/(a + b*Sin[c + d*x]) + (Sec[c + d*x]^2*(b*(a^2*A + 5*A*b^2 - 6
*a*b*B) + (3*a^3*A - 9*a*A*b^2 + 2*a^2*b*B + 4*b^3*B)*Sin[c + d*x]))/(a + b
*Sin[c + d*x]) + b*(-3*a^4*A + 12*a^2*A*b^2 + 15*A*b^4 - 2*a^3*b*B - 22*a*b
^3*B)*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a + b)^2) + Log[1 + Sin[c + d*x]]/(2*
(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) + 1/((a^
2 - b^2)*(a + b*Sin[c + d*x]))))/(8*(a^2 - b^2)^2*d)
```

Maple [A]
time = 1.33, size = 297, normalized size = 0.80 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/16*(-A-B)/(a+b)^2/(sin(d*x+c)-1)^2-1/16*(3*A*a+7*A*b+B*a+5*B*b)/(a+
b)^3/(sin(d*x+c)-1)+1/16/(a+b)^4*(-3*A*a^2-12*A*a*b-15*A*b^2-2*B*a*b-8*B*b^
2)*ln(sin(d*x+c)-1)-b^4*(6*A*a*b-5*B*a^2-B*b^2)/(a+b)^4/(a-b)^4*ln(a+b*sin(
d*x+c))+(A*b-B*a)*b^4/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))-1/16*(A-B)/(a-b)^2/(
1+sin(d*x+c))^2-1/16*(3*A*a-7*A*b-B*a+5*B*b)/(a-b)^3/(1+sin(d*x+c))+1/16*(3
*A*a^2-12*A*a*b+15*A*b^2+2*B*a*b-8*B*b^2)/(a-b)^4*ln(1+sin(d*x+c)))
```

Maxima [A]
time = 0.31, size = 659, normalized size = 1.77

$$\frac{\frac{16\,(5\,Ba^2b^4-6\,Aab^5+Bb^6)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{(3\,Aa^2-2\,(6\,A-B)ab+(15\,A-8\,B)b^2)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(3\,Aa^2+2\,(6\,A+B)ab+(15\,A+8\,B)b^2)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(2\,Ba^5-4\,Aa^4b-12\,Ba^3b^2+20\,Aa^2b^3-14\,Bab^4+8\,Ab^5-(3\,Aa^4b+2\,Ba^3b^2-12\,Aa^2b^3+22\,Bab^4-15\,Ab^5)\sin(dx+c)^4-(3\,Aa^5+2\,Ba^4b-12\,Aa^3b^2+2\,Ba^2b^3+9\,Aab^4-4\,Bb^5)\sin(dx+c)^3+(5\,Aa^4b+10\,Ba^3b^2-28\,Aa^2b^3+38\,Bab^4-25\,Ab^5)\sin(dx+c)^2+(5\,Aa^5-16\,Aa^3b^2+6\,Ba^2b^3+11\,Aab^4-6\,Bb^5)\sin(dx+c)\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6+(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\sin(dx+c)^5+(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\sin(dx+c)^4-2\,(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\sin(dx+c)^3-2\,(a^7-3\,a^5b^2+3\,a^3b^4-ab^6)\sin(dx+c)^2+(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)\sin(dx+c)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/16*(16*(5*B*a^2*b^4 - 6*A*a*b^5 + B*b^6)*log(b*sin(d*x + c) + a)/(a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (3*A*a^2 - 2*(6*A - B)*a*b + (15*
A - 8*B)*b^2)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 +
b^4) - (3*A*a^2 + 2*(6*A + B)*a*b + (15*A + 8*B)*b^2)*log(sin(d*x + c) - 1)
/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(2*B*a^5 - 4*A*a^4*b - 12*
B*a^3*b^2 + 20*A*a^2*b^3 - 14*B*a*b^4 + 8*A*b^5 - (3*A*a^4*b + 2*B*a^3*b^2
- 12*A*a^2*b^3 + 22*B*a*b^4 - 15*A*b^5)*sin(d*x + c)^4 - (3*A*a^5 + 2*B*a^4
*b - 12*A*a^3*b^2 + 2*B*a^2*b^3 + 9*A*a*b^4 - 4*B*b^5)*sin(d*x + c)^3 + (5*
A*a^4*b + 10*B*a^3*b^2 - 28*A*a^2*b^3 + 38*B*a*b^4 - 25*A*b^5)*sin(d*x + c)
^2 + (5*A*a^5 - 16*A*a^3*b^2 + 6*B*a^2*b^3 + 11*A*a*b^4 - 6*B*b^5)*sin(d*x
+ c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)
^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^3 - 2*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7)*sin(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 881 vs. $2(359) = 718$.
time = 1.91, size = 881, normalized size = 2.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/16*(4*B*a^7 - 4*A*a^6*b - 12*B*a^5*b^2 + 12*A*a^4*b^3 + 12*B*a^3*b^4 - 12
*A*a^2*b^5 - 4*B*a*b^6 + 4*A*b^7 - 2*(3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^
3 + 20*B*a^3*b^4 - 3*A*a^2*b^5 - 22*B*a*b^6 + 15*A*b^7)*cos(d*x + c)^4 + 2*
(A*a^6*b - 6*B*a^5*b^2 + 3*A*a^4*b^3 + 12*B*a^3*b^4 - 9*A*a^2*b^5 - 6*B*a*b
^6 + 5*A*b^7)*cos(d*x + c)^2 + 16*((5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*cos(d*
x + c)^4*sin(d*x + c) + (5*B*a^3*b^4 - 6*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^
4)*log(b*sin(d*x + c) + a) + ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*a^4*b^3 - 20*
B*a^3*b^4 + 5*(9*A - 8*B)*a^2*b^5 + 6*(8*A - 5*B)*a*b^6 + (15*A - 8*B)*b^7)
*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 - 20*B*a
^4*b^3 + 5*(9*A - 8*B)*a^3*b^4 + 6*(8*A - 5*B)*a^2*b^5 + (15*A - 8*B)*a*b^6
)*cos(d*x + c)^4)*log(sin(d*x + c) + 1) - ((3*A*a^6*b + 2*B*a^5*b^2 - 15*A*
a^4*b^3 - 20*B*a^3*b^4 + 5*(9*A + 8*B)*a^2*b^5 - 6*(8*A + 5*B)*a*b^6 + (15*
A + 8*B)*b^7)*cos(d*x + c)^4*sin(d*x + c) + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5
*b^2 - 20*B*a^4*b^3 + 5*(9*A + 8*B)*a^3*b^4 - 6*(8*A + 5*B)*a^2*b^5 + (15*A
 + 8*B)*a*b^6)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) + 2*(2*A*a^7 - 2*B*a^
6*b - 6*A*a^5*b^2 + 6*B*a^4*b^3 + 6*A*a^3*b^4 - 6*B*a^2*b^5 - 2*A*a*b^6 + 2
*B*b^7 + (3*A*a^7 + 2*B*a^6*b - 15*A*a^5*b^2 + 21*A*a^3*b^4 - 6*B*a^2*b^5 -
 9*A*a*b^6 + 4*B*b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4*sin(d*x + c) + (a^9 - 4*a^7*b^
2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sin(c + dx))\sec^5(c + dx)}{(a + b\sin(c + dx))^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sin(c + d*x))*sec(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(359) = 718$.
time = 0.56, size = 761, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/16*(16*(5*B*a^2*b^5 - 6*A*a*b^6 + B*b^7)*log(abs(b*sin(d*x + c) + a))/(a^
8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (3*A*a^2 + 12*A*a*b + 2*B*
a*b + 15*A*b^2 + 8*B*b^2)*log(abs(-sin(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4) + (3*A*a^2 - 12*A*a*b + 2*B*a*b + 15*A*b^2 - 8*B*b^2
)*log(abs(-sin(d*x + c) - 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) -
 16*(5*B*a^2*b^5*sin(d*x + c) - 6*A*a*b^6*sin(d*x + c) + B*b^7*sin(d*x + c)
 + 6*B*a^3*b^4 - 7*A*a^2*b^5 + A*b^7)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2
*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(30*B*a^2*b^4*sin(d*x + c)^4 - 36*A*a
*b^5*sin(d*x + c)^4 + 6*B*b^6*sin(d*x + c)^4 - 3*A*a^6*sin(d*x + c)^3 - 2*B
*a^5*b*sin(d*x + c)^3 + 15*A*a^4*b^2*sin(d*x + c)^3 - 12*B*a^3*b^3*sin(d*x
+ c)^3 - 5*A*a^2*b^4*sin(d*x + c)^3 + 14*B*a*b^5*sin(d*x + c)^3 - 7*A*b^6*s
in(d*x + c)^3 + 12*B*a^4*b^2*sin(d*x + c)^2 - 16*A*a^3*b^3*sin(d*x + c)^2 -
 68*B*a^2*b^4*sin(d*x + c)^2 + 88*A*a*b^5*sin(d*x + c)^2 - 16*B*b^6*sin(d*x
 + c)^2 + 5*A*a^6*sin(d*x + c) - 2*B*a^5*b*sin(d*x + c) - 17*A*a^4*b^2*sin(
d*x + c) + 20*B*a^3*b^3*sin(d*x + c) + 3*A*a^2*b^4*sin(d*x + c) - 18*B*a*b^
5*sin(d*x + c) + 9*A*b^6*sin(d*x + c) + 2*B*a^6 - 4*A*a^5*b - 14*B*a^4*b^2
+ 24*A*a^3*b^3 + 36*B*a^2*b^4 - 56*A*a*b^5 + 12*B*b^6)/((a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2))/d
```

Mupad [B]
time = 13.46, size = 615, normalized size = 1.65

$$\frac{\frac{B\,a^5-2\,A\,a^4\,b-6\,B\,a^3\,b^2+10\,A\,a^2\,b^3-7\,B\,a\,b^4+4\,A\,b^5}{4\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^4\,\left(3\,A\,a^4\,b+2\,B\,a^3\,b^2-12\,A\,a^2\,b^3+22\,B\,a\,b^4-15\,A\,b^5\right)}{8\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\sin\left(c+d\,x\right)\,\left(5\,A\,a^3-11\,A\,a\,b^2+6\,B\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}-\frac{{\sin\left(c+d\,x\right)}^3\,\left(3\,A\,a^3+2\,B\,a^2\,b-9\,A\,a\,b^2+4\,B\,b^3\right)}{8\,\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{{\sin\left(c+d\,x\right)}^2\,\left(5\,A\,a^4\,b+10\,B\,a^3\,b^2-28\,A\,a^2\,b^3+38\,B\,a\,b^4-25\,A\,b^5\right)}{8\,\left(a^2-b^2\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\,\left(b\,{\sin\left(c+d\,x\right)}^5+a\,{\sin\left(c+d\,x\right)}^4-2\,b\,{\sin\left(c+d\,x\right)}^3-2\,a\,{\sin\left(c+d\,x\right)}^2+b\,\sin\left(c+d\,x\right)+a\right)}+\frac{\ln\left(a+b\,\sin\left(c+d\,x\right)\right)\,\left(5\,B\,a^2\,b^4-6\,A\,a\,b^5+B\,b^6\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{\ln\left(\sin\left(c+d\,x\right)-1\right)\,\left(3\,A\,a^2+\left(12\,A+2\,B\right)\,a\,b+\left(15\,A+8\,B\right)\,b^2\right)}{d\,\left(16\,a^4+64\,a^3\,b+96\,a^2\,b^2+64\,a\,b^3+16\,b^4\right)}+\frac{\ln\left(\sin\left(c+d\,x\right)+1\right)\,\left(3\,A\,a^2+\left(2\,B-12\,A\right)\,a\,b+\left(15\,A-8\,B\right)\,b^2\right)}{d\,\left(16\,a^4-64\,a^3\,b+96\,a^2\,b^2-64\,a\,b^3+16\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^5*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((4*A*b^5 + B*a^5 + 10*A*a^2*b^3 - 6*B*a^3*b^2 - 2*A*a^4*b - 7*B*a*b^4)/(4*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^4*(2*B*a^3*b^2 - 12*A*
a^2*b^3 - 15*A*b^5 + 3*A*a^4*b + 22*B*a*b^4))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3
*a^4*b^2)) + (sin(c + d*x)*(5*A*a^3 + 6*B*b^3 - 11*A*a*b^2))/(8*(a^4 + b^4
- 2*a^2*b^2)) - (sin(c + d*x)^3*(3*A*a^3 + 4*B*b^3 - 9*A*a*b^2 + 2*B*a^2*b)
)/(8*(a^4 + b^4 - 2*a^2*b^2)) + (sin(c + d*x)^2*(10*B*a^3*b^2 - 28*A*a^2*b^
3 - 25*A*b^5 + 5*A*a^4*b + 38*B*a*b^4))/(8*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b
^2)))/(d*(a + b*sin(c + d*x) - 2*a*sin(c + d*x)^2 + a*sin(c + d*x)^4 - 2*b*
sin(c + d*x)^3 + b*sin(c + d*x)^5)) + (log(a + b*sin(c + d*x))*(B*b^6 + 5*B
*a^2*b^4 - 6*A*a*b^5))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
- (log(sin(c + d*x) - 1)*(3*A*a^2 + b^2*(15*A + 8*B) + a*b*(12*A + 2*B)))/(
d*(64*a*b^3 + 64*a^3*b + 16*a^4 + 16*b^4 + 96*a^2*b^2)) + (log(sin(c + d*x)
 + 1)*(3*A*a^2 + b^2*(15*A - 8*B) - a*b*(12*A - 2*B)))/(d*(16*a^4 - 64*a^3*
b - 64*a*b^3 + 16*b^4 + 96*a^2*b^2))
```

3.1559 $\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=550

$$-\frac{(5a^3A+a^2b(25A+2B)+ab^2(47A+10B)+b^3(35A+16B))\log(1-\sin(c+dx))}{32(a+b)^5d}+\frac{(5a^3A-b^3(35A-16\ldots}{}$$

```
[Out] -1/32*(5*a^3*A+a^2*b*(25*A+2*B)+a*b^2*(47*A+10*B)+b^3*(35*A+16*B))*ln(1-sin
(d*x+c))/(a+b)^5/d+1/32*(5*a^3*A-b^3*(35*A-16*B)+a*b^2*(47*A-10*B)-a^2*(25*
A*b-2*B*b))*ln(1+sin(d*x+c))/(a-b)^5/d+b^6*(8*A*a*b-7*B*a^2-B*b^2)*ln(a+b*s
in(d*x+c))/(a^2-b^2)^5/d-1/16*b*(5*A*a^6-23*A*a^4*b^2+47*A*a^2*b^4+35*A*b^6
+2*B*a^5*b-12*B*a^3*b^3-54*B*a*b^5)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))-1/6*sec(
d*x+c)^6*(A*b-a*B-(A*a-B*b)*sin(d*x+c))/(a^2-b^2)/d/(a+b*sin(d*x+c))+1/24*s
ec(d*x+c)^4*(b*(A*a^2+7*A*b^2-8*B*a*b)+(5*A*a^3-13*A*a*b^2+2*B*a^2*b+6*B*b^
3)*sin(d*x+c))/(a^2-b^2)^2/d/(a+b*sin(d*x+c))+1/48*sec(d*x+c)^2*(b*(5*A*a^4
-18*A*a^2*b^2-35*A*b^4+2*B*a^3*b+46*B*a*b^3)+3*(5*A*a^5-18*A*a^3*b^2+29*A*a
*b^4+2*B*a^4*b-10*B*a^2*b^3-8*B*b^5)*sin(d*x+c))/(a^2-b^2)^3/d/(a+b*sin(d*x
+c))
```

Rubi [A]
time = 0.65, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2916, 837, 815}

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/32*((5*a^3*A + a^2*b*(25*A + 2*B) + a*b^2*(47*A + 10*B) + b^3*(35*A + 16
*B))*Log[1 - Sin[c + d*x]])/((a + b)^5*d) + ((5*a^3*A - b^3*(35*A - 16*B) +
 a*b^2*(47*A - 10*B) - a^2*(25*A*b - 2*b*B))*Log[1 + Sin[c + d*x]])/(32*(a
- b)^5*d) + (b^6*(8*a*A*b - 7*a^2*B - b^2*B)*Log[a + b*Sin[c + d*x]])/((a^2
 - b^2)^5*d) - (b*(5*a^6*A - 23*a^4*A*b^2 + 47*a^2*A*b^4 + 35*A*b^6 + 2*a^5
*b*B - 12*a^3*b^3*B - 54*a*b^5*B))/(16*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])
) - (Sec[c + d*x]^6*(A*b - a*B - (a*A - b*B)*Sin[c + d*x]))/(6*(a^2 - b^2)*
d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^4*(b*(a^2*A + 7*A*b^2 - 8*a*b*B) +
(5*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B)*Sin[c + d*x]))/(24*(a^2 - b^2)
^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(b*(5*a^4*A - 18*a^2*A*b^2 - 3
5*A*b^4 + 2*a^3*b*B + 46*a*b^3*B) + 3*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4
+ 2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*Sin[c + d*x]))/(48*(a^2 - b^2)^3*d*(a
 + b*Sin[c + d*x]))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^7(c+dx)(A+B\sin(c+dx))}{(a+b\sin(c+dx))^2}\,dx = \frac{b^7\text{Subst}\left(\int \frac{A+\frac{Bx}{b}}{(a+x)^2(b^2-x^2)^4}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)\,d(a+b\sin(c+dx))} - \frac{b^5\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)}{}$$

$$= -\frac{\sec^6(c+dx)(Ab-aB-(aA-bB)\sin(c+dx))}{6(a^2-b^2)\,d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)}{}$$

$$= -\frac{(5a^3A+a^2b(25A+2B)+ab^2(47A+10B)+b^3(35A+16B))\,\text{l}}{32(a+b)^5d}$$

Mathematica [A]
time = 6.15, size = 766, normalized size = 1.39

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^7*(A + B*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^7*(-1/6*(Sec[c + d*x]^6*(-(A*b^2) + a*b*B - b*(-(a*A) + b*B)*Sin[c + d*x
]))/(b^8*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-1/4*(Sec[c + d*x]^4*(-6*a*b
^2*(a*A - b*B) - b^2*(-5*a^2*A + 7*A*b^2 - 2*a*b*B) - b*(-6*b^2*(a*A - b*B)
 - a*(-5*a^2*A + 7*A*b^2 - 2*a*b*B))*Sin[c + d*x]))/(b^6*(-a^2 + b^2)*(a +
b*Sin[c + d*x])) + (-1/2*(Sec[c + d*x]^2*(4*a*b^2*(5*a^3*A - 13*a*A*b^2 + 2
*a^2*b*B + 6*b^3*B) - b^2*(15*a^4*A - 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B -
 22*a*b^3*B) - b*(4*b^2*(5*a^3*A - 13*a*A*b^2 + 2*a^2*b*B + 6*b^3*B) - a*(1
5*a^4*A - 34*a^2*A*b^2 + 35*A*b^4 + 6*a^3*b*B - 22*a*b^3*B))*Sin[c + d*x]))
/(b^4*(-a^2 + b^2)*(a + b*Sin[c + d*x])) + (-6*(5*a^5*A - 18*a^3*A*b^2 + 29
*a*A*b^4 + 2*a^4*b*B - 10*a^2*b^3*B - 8*b^5*B)*(-1/2*Log[1 - Sin[c + d*x]]/
(b*(a + b)) + Log[1 + Sin[c + d*x]]/(2*(a - b)*b) - Log[a + b*Sin[c + d*x]]
/(a^2 - b^2)) + (6*a*(5*a^5*A - 18*a^3*A*b^2 + 29*a*A*b^4 + 2*a^4*b*B - 10*
a^2*b^3*B - 8*b^5*B) - 3*(5*a^6*A - 13*a^4*A*b^2 + 11*a^2*A*b^4 - 35*A*b^6
+ 2*a^5*b*B - 8*a^3*b^3*B + 38*a*b^5*B))*(-1/2*Log[1 - Sin[c + d*x]]/(b*(a
+ b)^2) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*b) - (2*a*Log[a + b*Sin[c + d*
x]])/((a - b)^2*(a + b)^2) + 1/((a^2 - b^2)*(a + b*Sin[c + d*x]))))/(2*b^2*
(-a^2 + b^2)))/(4*b^2*(-a^2 + b^2)))/(6*b^2*(-a^2 + b^2))))/d
```

Maple [A]
time = 1.89, size = 435, normalized size = 0.79

method	result
derivativedivides	$-\frac{A+B}{48(a+b)^{2}(\sin(dx+c)-1)^{3}}-\frac{-2aA-4Ab-aB-3Bb}{32(a+b)^{3}(\sin(dx+c)-1)^{2}}-\frac{5a^{2}A+18Aab+19Ab^{2}+Ba^{2}+6Bab+11Bb^{2}}{32(a+b)^{4}(\sin(dx+c)-1)}+\frac{(-5a^{3}A-25Aa^{2}b-47Aab^{2}-\cdots}{}$
default	$-\frac{A+B}{48(a+b)^{2}(\sin(dx+c)-1)^{3}}-\frac{-2aA-4Ab-aB-3Bb}{32(a+b)^{3}(\sin(dx+c)-1)^{2}}-\frac{5a^{2}A+18Aab+19Ab^{2}+Ba^{2}+6Bab+11Bb^{2}}{32(a+b)^{4}(\sin(dx+c)-1)}+\frac{(-5a^{3}A-25Aa^{2}b-47Aab^{2}-\cdots}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/48*(A+B)/(a+b)^2/(sin(d*x+c)-1)^3-1/32*(-2*A*a-4*A*b-B*a-3*B*b)/(a+
b)^3/(sin(d*x+c)-1)^2-1/32*(5*A*a^2+18*A*a*b+19*A*b^2+B*a^2+6*B*a*b+11*B*b^
2)/(a+b)^4/(sin(d*x+c)-1)+1/32/(a+b)^5*(-5*A*a^3-25*A*a^2*b-47*A*a*b^2-35*A
*b^3-2*B*a^2*b-10*B*a*b^2-16*B*b^3)*ln(sin(d*x+c)-1)+b^6*(8*A*a*b-7*B*a^2-B
```

```
*b^2)/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))-(A*b-B*a)*b^6/(a+b)^4/(a-b)^4/(a+b
*sin(d*x+c))-1/48*(A-B)/(a-b)^2/(1+sin(d*x+c))^3-1/32*(2*A*a-4*A*b-B*a+3*B*
b)/(a-b)^3/(1+sin(d*x+c))^2-1/32*(5*A*a^2-18*A*a*b+19*A*b^2-B*a^2+6*B*a*b-1
1*B*b^2)/(a-b)^4/(1+sin(d*x+c))+1/32*(5*A*a^3-25*A*a^2*b+47*A*a*b^2-35*A*b^
3+2*B*a^2*b-10*B*a*b^2+16*B*b^3)/(a-b)^5*ln(1+sin(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1083 vs. $2(537) = 1074$.
time = 0.40, size = 1083, normalized size = 1.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/96*(96*(7*B*a^2*b^6 - 8*A*a*b^7 + B*b^8)*log(b*sin(d*x + c) + a)/(a^10 -
 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(5*A*a^3 - (25
*A - 2*B)*a^2*b + (47*A - 10*B)*a*b^2 - (35*A - 16*B)*b^3)*log(sin(d*x + c)
 + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 3*(5*A*a^
3 + (25*A + 2*B)*a^2*b + (47*A + 10*B)*a*b^2 + (35*A + 16*B)*b^3)*log(sin(d
*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*
(8*B*a^7 - 16*A*a^6*b - 44*B*a^5*b^2 + 80*A*a^4*b^3 + 136*B*a^3*b^4 - 208*A
*a^2*b^5 + 92*B*a*b^6 - 48*A*b^7 + 3*(5*A*a^6*b + 2*B*a^5*b^2 - 23*A*a^4*b^
3 - 12*B*a^3*b^4 + 47*A*a^2*b^5 - 54*B*a*b^6 + 35*A*b^7)*sin(d*x + c)^6 + 3
*(5*A*a^7 + 2*B*a^6*b - 23*A*a^5*b^2 - 12*B*a^4*b^3 + 47*A*a^3*b^4 + 2*B*a^
2*b^5 - 29*A*a*b^6 + 8*B*b^7)*sin(d*x + c)^5 - 8*(5*A*a^6*b + 2*B*a^5*b^2 -
 23*A*a^4*b^3 - 19*B*a^3*b^4 + 55*A*a^2*b^5 - 55*B*a*b^6 + 35*A*b^7)*sin(d*
x + c)^4 - 4*(10*A*a^7 + 4*B*a^6*b - 46*A*a^5*b^2 - 17*B*a^4*b^3 + 86*A*a^3
*b^4 - 2*B*a^2*b^5 - 50*A*a*b^6 + 15*B*b^7)*sin(d*x + c)^3 + 3*(11*A*a^6*b
+ 10*B*a^5*b^2 - 57*A*a^4*b^3 - 76*B*a^3*b^4 + 161*A*a^2*b^5 - 126*B*a*b^6
+ 77*A*b^7)*sin(d*x + c)^2 + (33*A*a^7 + 2*B*a^6*b - 139*A*a^5*b^2 - 8*B*a^
4*b^3 + 227*A*a^3*b^4 - 38*B*a^2*b^5 - 121*A*a*b^6 + 44*B*b^7)*sin(d*x + c)
)/(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8 - (a^8*b - 4*a^6*b^3 + 6
*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)^7 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
 4*a^3*b^6 + a*b^8)*sin(d*x + c)^6 + 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a
^2*b^7 + b^9)*sin(d*x + c)^5 + 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 +
 a*b^8)*sin(d*x + c)^4 - 3*(a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9
)*sin(d*x + c)^3 - 3*(a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*sin(
d*x + c)^2 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*sin(d*x + c)
))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(537) = 1074$.
time = 4.70, size = 1244, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/96*(16*B*a^9 - 16*A*a^8*b - 64*B*a^7*b^2 + 64*A*a^6*b^3 + 96*B*a^5*b^4 -
96*A*a^4*b^5 - 64*B*a^3*b^6 + 64*A*a^2*b^7 + 16*B*a*b^8 - 16*A*b^9 - 6*(5*A
*a^8*b + 2*B*a^7*b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 - 42*B*a^
3*b^6 - 12*A*a^2*b^7 + 54*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^6 + 2*(5*A*a^8*b
 + 2*B*a^7*b^2 - 28*A*a^6*b^3 + 42*B*a^5*b^4 + 6*A*a^4*b^5 - 90*B*a^3*b^6 +
 52*A*a^2*b^7 + 46*B*a*b^8 - 35*A*b^9)*cos(d*x + c)^4 + 4*(A*a^8*b - 8*B*a^
7*b^2 + 4*A*a^6*b^3 + 24*B*a^5*b^4 - 18*A*a^4*b^5 - 24*B*a^3*b^6 + 20*A*a^2
*b^7 + 8*B*a*b^8 - 7*A*b^9)*cos(d*x + c)^2 - 96*((7*B*a^2*b^7 - 8*A*a*b^8 +
 B*b^9)*cos(d*x + c)^6*sin(d*x + c) + (7*B*a^3*b^6 - 8*A*a^2*b^7 + B*a*b^8)
*cos(d*x + c)^6)*log(b*sin(d*x + c) + a) + 3*((5*A*a^8*b + 2*B*a^7*b^2 - 28
*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A - 4*B)*a^
2*b^7 - 2*(64*A - 35*B)*a*b^8 - (35*A - 16*B)*b^9)*cos(d*x + c)^6*sin(d*x +
 c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 7
0*B*a^4*b^5 - 28*(5*A - 4*B)*a^3*b^6 - 2*(64*A - 35*B)*a^2*b^7 - (35*A - 16
*B)*a*b^8)*cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 3*((5*A*a^8*b + 2*B*a^7*
b^2 - 28*A*a^6*b^3 - 14*B*a^5*b^4 + 70*A*a^4*b^5 + 70*B*a^3*b^6 - 28*(5*A +
 4*B)*a^2*b^7 + 2*(64*A + 35*B)*a*b^8 - (35*A + 16*B)*b^9)*cos(d*x + c)^6*s
in(d*x + c) + (5*A*a^9 + 2*B*a^8*b - 28*A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5
*b^4 + 70*B*a^4*b^5 - 28*(5*A + 4*B)*a^3*b^6 + 2*(64*A + 35*B)*a^2*b^7 - (3
5*A + 16*B)*a*b^8)*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) + 2*(8*A*a^9 - 8*
B*a^8*b - 32*A*a^7*b^2 + 32*B*a^6*b^3 + 48*A*a^5*b^4 - 48*B*a^4*b^5 - 32*A*
a^3*b^6 + 32*B*a^2*b^7 + 8*A*a*b^8 - 8*B*b^9 + 3*(5*A*a^9 + 2*B*a^8*b - 28*
A*a^7*b^2 - 14*B*a^6*b^3 + 70*A*a^5*b^4 + 14*B*a^4*b^5 - 76*A*a^3*b^6 + 6*B
*a^2*b^7 + 29*A*a*b^8 - 8*B*b^9)*cos(d*x + c)^4 + 2*(5*A*a^9 + 2*B*a^8*b -
28*A*a^7*b^2 + 54*A*a^5*b^4 - 12*B*a^4*b^5 - 44*A*a^3*b^6 + 16*B*a^2*b^7 +
13*A*a*b^8 - 6*B*b^9)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b - 5*a^8*b^3 +
10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11)*d*cos(d*x + c)^6*sin(d*x + c) +
 (a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*
x + c)^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(537) = 1074.

time = 0.67, size = 1185, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(A+B*sin(d*x+c))/(a+b*sin(d*x+c))^2,x, algorithm="gi
ac")
[Out] -1/96*(96*(7*B*a^2*b^7 - 8*A*a*b^8 + B*b^9)*log(abs(b*sin(d*x + c) + a))/(a
^10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) + 3*(5*A*a^
3 + 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 + 10*B*a*b^2 + 35*A*b^3 + 16*B*b^3)
*log(abs(-sin(d*x + c) + 1))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5) - 3*(5*A*a^3 - 25*A*a^2*b + 2*B*a^2*b + 47*A*a*b^2 - 10*B*a*b^2
 - 35*A*b^3 + 16*B*b^3)*log(abs(-sin(d*x + c) - 1))/(a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) - 96*(7*B*a^2*b^7*sin(d*x + c) - 8*A*a*b
^8*sin(d*x + c) + B*b^9*sin(d*x + c) + 8*B*a^3*b^6 - 9*A*a^2*b^7 + A*b^9)/(
(a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(b*sin(d*x
+ c) + a)) + 2*(308*B*a^2*b^6*sin(d*x + c)^6 - 352*A*a*b^7*sin(d*x + c)^6 +
 44*B*b^8*sin(d*x + c)^6 + 15*A*a^8*sin(d*x + c)^5 + 6*B*a^7*b*sin(d*x + c)
^5 - 84*A*a^6*b^2*sin(d*x + c)^5 - 42*B*a^5*b^3*sin(d*x + c)^5 + 210*A*a^4*
b^4*sin(d*x + c)^5 - 78*B*a^3*b^5*sin(d*x + c)^5 - 84*A*a^2*b^6*sin(d*x + c
)^5 + 114*B*a*b^7*sin(d*x + c)^5 - 57*A*b^8*sin(d*x + c)^5 + 120*B*a^4*b^4*
sin(d*x + c)^4 - 144*A*a^3*b^5*sin(d*x + c)^4 - 1020*B*a^2*b^6*sin(d*x + c)
^4 + 1200*A*a*b^7*sin(d*x + c)^4 - 156*B*b^8*sin(d*x + c)^4 - 40*A*a^8*sin(
d*x + c)^3 - 16*B*a^7*b*sin(d*x + c)^3 + 224*A*a^6*b^2*sin(d*x + c)^3 + 48*
B*a^5*b^3*sin(d*x + c)^3 - 480*A*a^4*b^4*sin(d*x + c)^3 + 240*B*a^3*b^5*sin
(d*x + c)^3 + 160*A*a^2*b^6*sin(d*x + c)^3 - 272*B*a*b^7*sin(d*x + c)^3 + 1
36*A*b^8*sin(d*x + c)^3 + 36*B*a^6*b^2*sin(d*x + c)^2 - 48*A*a^5*b^3*sin(d*
x + c)^2 - 300*B*a^4*b^4*sin(d*x + c)^2 + 384*A*a^3*b^5*sin(d*x + c)^2 + 11
28*B*a^2*b^6*sin(d*x + c)^2 - 1392*A*a*b^7*sin(d*x + c)^2 + 192*B*b^8*sin(d
*x + c)^2 + 33*A*a^8*sin(d*x + c) - 6*B*a^7*b*sin(d*x + c) - 156*A*a^6*b^2*
sin(d*x + c) + 42*B*a^5*b^3*sin(d*x + c) + 270*A*a^4*b^4*sin(d*x + c) - 210
*B*a^3*b^5*sin(d*x + c) - 60*A*a^2*b^6*sin(d*x + c) + 174*B*a*b^7*sin(d*x +
 c) - 87*A*b^8*sin(d*x + c) + 8*B*a^8 - 16*A*a^7*b - 52*B*a^6*b^2 + 96*A*a^
5*b^3 + 180*B*a^4*b^4 - 288*A*a^3*b^5 - 400*B*a^2*b^6 + 560*A*a*b^7 - 88*B*
b^8)/((a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*(sin(
d*x + c)^2 - 1)^3))/d
```

Mupad [B]
time = 14.38, size = 1024, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(c + d*x))/(cos(c + d*x)^7*(a + b*sin(c + d*x))^2),x)
```

```
[Out] ((sin(c + d*x)*(33*A*a^5 - 44*B*b^5 - 106*A*a^3*b^2 - 6*B*a^2*b^3 + 121*A*a
*b^4 + 2*B*a^4*b))/(48*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)
^5*(5*A*a^5 - 8*B*b^5 - 18*A*a^3*b^2 - 10*B*a^2*b^3 + 29*A*a*b^4 + 2*B*a^4*
b))/(16*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (sin(c + d*x)^3*(10*A*a^5 -
15*B*b^5 - 36*A*a^3*b^2 - 13*B*a^2*b^3 + 50*A*a*b^4 + 4*B*a^4*b))/(12*(a^6
- b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (12*A*b^7 - 2*B*a^7 + 52*A*a^2*b^5 - 20*A
*a^4*b^3 - 34*B*a^3*b^4 + 11*B*a^5*b^2 + 4*A*a^6*b - 23*B*a*b^6)/(12*(a^2 -
 b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (sin(c + d*x)^6*(35*A*b^7 + 47
*A*a^2*b^5 - 23*A*a^4*b^3 - 12*B*a^3*b^4 + 2*B*a^5*b^2 + 5*A*a^6*b - 54*B*a
*b^6))/(16*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (sin(c + d*x)
^4*(35*A*b^7 + 55*A*a^2*b^5 - 23*A*a^4*b^3 - 19*B*a^3*b^4 + 2*B*a^5*b^2 + 5
*A*a^6*b - 55*B*a*b^6))/(6*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
 + (sin(c + d*x)^2*(77*A*b^7 + 161*A*a^2*b^5 - 57*A*a^4*b^3 - 76*B*a^3*b^4
+ 10*B*a^5*b^2 + 11*A*a^6*b - 126*B*a*b^6))/(16*(a^2 - b^2)*(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2)))/(d*(a + b*sin(c + d*x) - 3*a*sin(c + d*x)^2 + 3*a*si
n(c + d*x)^4 - a*sin(c + d*x)^6 - 3*b*sin(c + d*x)^3 + 3*b*sin(c + d*x)^5 -
 b*sin(c + d*x)^7)) - (log(a + b*sin(c + d*x))*(B*b^8 + 7*B*a^2*b^6 - 8*A*a
*b^7))/(d*(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
- (log(sin(c + d*x) - 1)*(5*A*a^3 + b^3*(35*A + 16*B) + a^2*b*(25*A + 2*B)
+ a*b^2*(47*A + 10*B)))/(d*(160*a*b^4 + 160*a^4*b + 32*a^5 + 32*b^5 + 320*a
^2*b^3 + 320*a^3*b^2)) + (log(sin(c + d*x) + 1)*(5*A*a^3 - b^3*(35*A - 16*B
) - a^2*b*(25*A - 2*B) + a*b^2*(47*A - 10*B)))/(d*(160*a*b^4 - 160*a^4*b +
32*a^5 - 32*b^5 - 320*a^2*b^3 + 320*a^3*b^2))
```

3.1560 $\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$

Optimal. Leaf size=40

$$\mathrm{Int}\left((g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx)),x\right)$$

```
[Out] Unintegrable((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x
]
```

```
[Out] Defer[Int][(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e +
f*x]), x]
```

Rubi steps

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx = \int (g\cos(e+fx))^{-1-m}(a+b\sin(e+$$

Mathematica [A]
time = 7.36, size = 0, normalized size = 0.00

$$\int (g\cos(e+fx))^{-1-m}(a+b\sin(e+fx))^m(A+B\sin(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]),x]
```

```
[Out] Integrate[(g*Cos[e + f*x])^(-1 - m)*(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f
*x]), x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int (g\cos(fx+e))^{-1-m}(a+b\sin(fx+e))^m(A+B\sin(fx+e))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
[Out] int((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="maxima")
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) +
a)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="fricas")
[Out] integral((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) + a
)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**(-1-m)*(a+b*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^(-1-m)*(a+b*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(g*cos(f*x + e))^(-m - 1)*(b*sin(f*x + e) +
a)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(A + B\,\sin(e + f\,x))\,\left(a + b\,\sin(e + f\,x)\right)^m}{\left(g\,\cos(e + f\,x)\right)^{m+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m)/(g*cos(e + f*x))^(m + 1),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m)/(g*cos(e + f*x))^(m + 1),
 x)
```

3.1561 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=330

$$-\frac{gF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+fx))^{-1+p}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(e+fx))}{a+b\sin(e+fx)}\right.}{(bc-ad)f(1-p)}$$

```
[Out] -g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b*s
in(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2
-1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/(1-p)+
g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(c+d*si
n(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-
1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/(1-p)
```

Rubi [A]
time = 0.29, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3003, 2782}

$$\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)}-\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e + f*
x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - Sin
[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a +
 b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)*f*(1 - p))) + (g*AppellF1[1 - p
, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d
*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*
Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^(
(1 - p)/2))/((b*c - a*d)*f*(1 - p))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 3003

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx &= \int \left(\frac{b(g\cos(e+fx))^p}{(bc-ad)(a+b\sin(e+fx))}-\frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+ \\ &= \frac{b\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{bc-ad}-\frac{d\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{bc-ad} \\ &= -\frac{gF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos}{(bc-ad)f(}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5085 vs. 2(330) = 660.
time = 33.97, size = 5085, normalized size = 15.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
[Out] Result too large to show
```

Maple [F]
time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p}{(a+b\sin(fx+e))(c+d\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*cos(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)
```

3.1562 $\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx$

Optimal. Leaf size=508

$$-\frac{bgF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos(e+fx))^{-1+p}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(1+\sin(e+f}{a+b\sin(e+f}\right.}{(bc-ad)^2f(1-p)}$$

```
[Out] -b*g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b
*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^(1
/2-1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)^2/f/(1
-p)+b*g*AppellF1(1-p,1/2-1/2*p,1/2-1/2*p,2-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(
c+d*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)))
^(1/2-1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)^2/f
/(1-p)+g*AppellF1(2-p,1/2-1/2*p,1/2-1/2*p,3-p,(c-d)/(c+d*sin(f*x+e)),(c+d)/
(c+d*sin(f*x+e)))*(g*cos(f*x+e))^(-1+p)*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e))
)^(1/2-1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2-1/2*p)/(-a*d+b*c)/f/
(2-p)/(c+d*sin(f*x+e))
```

Rubi [A]
time = 0.35, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {3003, 2782}

$$-\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{bg(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(1-p)(bc-ad)^2}+\frac{g(g\cos(e+fx))^{p-1}\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{1-p}{2}}F_1\left(2-p;\frac{1-p}{2},\frac{1-p}{2};3-p;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(2-p)(bc-ad)(c+d\sin(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] -((b*g*AppellF1[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (a + b)/(a + b*Sin[e +
f*x]), (a - b)/(a + b*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((b*(1 - S
in[e + f*x]))/(a + b*Sin[e + f*x])))^((1 - p)/2)*((b*(1 + Sin[e + f*x]))/(a
 + b*Sin[e + f*x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p))) + (b*g*AppellF1
[1 - p, (1 - p)/2, (1 - p)/2, 2 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/
(c + d*Sin[e + f*x])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(
c + d*Sin[e + f*x])))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*
x]))^((1 - p)/2))/((b*c - a*d)^2*f*(1 - p)) + (g*AppellF1[2 - p, (1 - p)/2,
 (1 - p)/2, 3 - p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c + d*Sin[e + f*x
])]*(g*Cos[e + f*x])^(-1 + p)*(-((d*(1 - Sin[e + f*x]))/(c + d*Sin[e + f*x]
)))^((1 - p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e + f*x]))^((1 - p)/2))/
((b*c - a*d)*f*(2 - p)*(c + d*Sin[e + f*x]))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
```

```
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 3003

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(g\cos(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))^2}\,dx = \int \left(\frac{b^2(g\cos(e+fx))^p}{(bc-ad)^2(a+b\sin(e+fx))} - \frac{d(g\cos(e+fx))^p}{(bc-ad)(c+d\sin(e+$$

$$= \frac{b^2\int \frac{(g\cos(e+fx))^p}{a+b\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{(bd)\int \frac{(g\cos(e+fx))^p}{c+d\sin(e+fx)}\,dx}{(bc-ad)^2} - \frac{d\int \frac{(g\cos(e+fx}{(c+d\sin(e+f}}{bc-ad}$$

$$= -\frac{bgF_1\left(1-p;\frac{1-p}{2},\frac{1-p}{2};2-p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)(g\cos}{(bc-ad)^2f}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 12568 vs. $2(508) = 1016$.
time = 54.30, size = 12568, normalized size = 24.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Cos[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])^2),
x]
```

```
[Out] Result too large to show
```

Maple [**F**]
time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{(g\cos(fx+e))^p}{(a+b\sin(fx+e))(c+d\sin(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="fricas")
```

```
[Out] integral((g*cos(f*x + e))^p/(a*c^2 + 2*b*c*d + a*d^2 - (2*b*c*d + a*d^2)*co
s(f*x + e)^2 - (b*d^2*cos(f*x + e)^2 - b*c^2 - 2*a*c*d - b*d^2)*sin(f*x + e
)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*cos(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm
="giac")
```

```
[Out] integrate((g*cos(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(g\,\cos\left(e+f\,x\right))^p}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,{\left(c+d\,\sin\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)
[Out] int((g*cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))^2), x)
```

3.1563 $\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx$

Optimal. Leaf size=308

$$-\frac{F_1\left(1+p;\frac{1+p}{2},\frac{1+p}{2};2+p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{1+p}{2}}\left(\frac{b(1+s}{a+b}\right.}{(bc-ad)f(1+p)}$$

```
[Out] -AppellF1(1+p,1/2+1/2*p,1/2+1/2*p,2+p,(a-b)/(a+b*sin(f*x+e)),(a+b)/(a+b*sin
(f*x+e)))*sec(f*x+e)*(g*sec(f*x+e))^p*(-b*(1-sin(f*x+e))/(a+b*sin(f*x+e)))^
(1/2+1/2*p)*(b*(1+sin(f*x+e))/(a+b*sin(f*x+e)))^(1/2+1/2*p)/(-a*d+b*c)/f/(1
+p)+AppellF1(1+p,1/2+1/2*p,1/2+1/2*p,2+p,(c-d)/(c+d*sin(f*x+e)),(c+d)/(c+d*
sin(f*x+e)))*sec(f*x+e)*(g*sec(f*x+e))^p*(-d*(1-sin(f*x+e))/(c+d*sin(f*x+e)
))^(1/2+1/2*p)*(d*(1+sin(f*x+e))/(c+d*sin(f*x+e)))^(1/2+1/2*p)/(-a*d+b*c)/f
/(1+p)
```

Rubi [A]
time = 0.39, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3005, 3003, 2782}

$$\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{d(1-\sin(e+fx))}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{d(\sin(e+fx)+1)}{c+d\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{c+d}{c+d\sin(e+fx)},\frac{c-d}{c+d\sin(e+fx)}\right)}{f(p+1)(bc-ad)}-\frac{\sec(e+fx)(g\sec(e+fx))^p\left(-\frac{b(1-\sin(e+fx))}{a+b\sin(e+fx)}\right)^{\frac{p+1}{2}}\left(\frac{b(\sin(e+fx)+1)}{a+b\sin(e+fx)}\right)^{\frac{p+1}{2}}F_1\left(p+1;\frac{p+1}{2},\frac{p+1}{2};p+2;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)}{f(p+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
```

```
[Out] -((AppellF1[1 + p, (1 + p)/2, (1 + p)/2, 2 + p, (a + b)/(a + b*Sin[e + f*x]
), (a - b)/(a + b*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((b*(1 -
 Sin[e + f*x]))/(a + b*Sin[e + f*x])))^((1 + p)/2)*((b*(1 + Sin[e + f*x]))/
(a + b*Sin[e + f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))) + (AppellF1[1 +
 p, (1 + p)/2, (1 + p)/2, 2 + p, (c + d)/(c + d*Sin[e + f*x]), (c - d)/(c +
 d*Sin[e + f*x])]*Sec[e + f*x]*(g*Sec[e + f*x])^p*(-((d*(1 - Sin[e + f*x]))
/(c + d*Sin[e + f*x])))^((1 + p)/2)*((d*(1 + Sin[e + f*x]))/(c + d*Sin[e +
f*x]))^((1 + p)/2))/((b*c - a*d)*f*(1 + p))
```

Rule 2782

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p)*((-b)*((1 - Sin[e + f*x])/(a + b*Sin[e + f*x])))^((
p - 1)/2)*(b*((1 + Sin[e + f*x])/(a + b*Sin[e + f*x])))^((p - 1)/2)))*Appel
lF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b*Sin[e + f*x]),
(a - b)/(a + b*Sin[e + f*x])], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 3003

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Int[Exp
andTrig[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x
], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && Integer
sQ[2*m, 2*n]
```

Rule 3005

```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dis
t[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]*(g*Sec[e + f*x])^FracPart[p
], Int[(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(g*Cos[e + f*x])^p),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(g\sec(e+fx))^p}{(a+b\sin(e+fx))(c+d\sin(e+fx))}\,dx = ((g\cos(e+fx))^p(g\sec(e+fx))^p)\int \frac{(g\cos(e+fx)}{(a+b\sin(e+fx))(c+}$$

$$= ((g\cos(e+fx))^p(g\sec(e+fx))^p)\int \Bigg(\frac{b(g\cos(e+fx))^{-}}{(bc-ad)(a+b\sin(e+}$$

$$= \frac{(b(g\cos(e+fx))^p(g\sec(e+fx))^p)\int \frac{(g\cos(e+fx))^{-p}}{a+b\sin(e+fx)}\,dx}{bc-ad} - \frac{(d(g}{}$$

$$= -\frac{F_1\left(1+p;\frac{1+p}{2},\frac{1+p}{2};2+p;\frac{a+b}{a+b\sin(e+fx)},\frac{a-b}{a+b\sin(e+fx)}\right)\sec(e+}{(bc-ad)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5113 vs. $2(308) = 616$.
time = 26.84, size = 5113, normalized size = 16.60

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Sec[e + f*x])^p/((a + b*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]
[Out] Result too large to show
```

Maple [F]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(g\sec(fx+e))^p}{(a+b\sin(fx+e))(c+d\sin(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] int((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
maxima")
```

```
[Out] integrate((g*sec(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
fricas")
```

```
[Out] integral(-(g*sec(f*x + e))^p/(b*d*cos(f*x + e)^2 - a*c - b*d - (b*c + a*d)*
sin(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(g \sec (e + fx))^p}{(a + b \sin (e + fx)) (c + d \sin (e + fx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))**p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] Integral((g*sec(e + f*x))**p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*sec(f*x+e))^p/(a+b*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="
giac")
```

```
[Out] integrate((g*sec(f*x + e))^p/((b*sin(f*x + e) + a)*(d*sin(f*x + e) + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{g}{\cos(e+f\,x)}\right)^p}{\left(a+b\,\sin\left(e+f\,x\right)\right)\,\left(c+d\,\sin\left(e+f\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g/cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))),x)
```

```
[Out] int((g/cos(e + f*x))^p/((a + b*sin(e + f*x))*(c + d*sin(e + f*x))), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```